A LABORATORY INVESTIGATION OF
PARALLEL LAMP BRANCH VOLTAGES

By

JOHN C. REIFF

BACHELOR OF SCIENCE

UNIVERSITY OF OKLAHOMA

NORMAN, OKLAHOMA

1947

Submitted to the Faculty of the Graduate School of
the Oklahoma Agricultural and Mechanical College
in Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE
1950

A LABORATORY INVESTIGATION OF

PARALLEL LAMP BRANCH VOLTAGES

JOHN C. REIFF

MASTER OF SCIENCE

1950

THESIS AND ABSTRACT APPROVED:

Herbert L. Jones
Thesis Adviser

Chas. F. Cameron
Faculty Representative

D. C. McIntosh
Dean of the Graduate School

PREFACE

The seemingly simple and generally ignored problem of incandescent lamp voltages in a bank of lamps is now of considerable importance. Year after year better lamp design is producing more and more lumen-hours of light from a given lamp. This lumen-hour rating of a lamp is dependent upon the lamp operating at rated voltage or as near to rated voltage as possible. To secure the best balance of lighting cost in terms of lighting results is of prime consideration. Therefore the circuit employed in the lamp distribution system must meet certain qualifications. In order to maintain the voltage at the lamp as closely as possible at the rated value the supplying circuit must be designed for low line voltage drops. This study covered in this thesis was instituted to determine the general principles involved in this type of design.

The main objective of the search is to find a means of predetermining lamp voltages in a bank in terms of the number of lamps in operation, rated lamp voltage, line resistance and the rated lamp current. A new method is needed to alleviate the work necessary in determining lamp voltages by present methods. The only method which produces a useable solution at the present time is the trial-and-error method. This method is a process of deriving a quantitative answer from the analysis of the special conditions of a given problem. It is highly desirable therefore, to formulate a new process or method of resolving the lamp voltages in a bank of incandescent lamps.

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Professor C. F. Cameron and Dr. H. L. Jones for their suggestions and untiring aid in checking the material compiled for this paper.

TABLE OF CONTENTS

INTRODUCTION

The problem of non-linear coupled circuits involves extensive ramifications and it is for this reason that the present discussion is limited to the familiar non-linear element, the incandescent lamp. A parallel bank of incandescent lamps will be analyzed as a non-linear coupled circuit. This analysis centers around the operating voltages of the lamps in the bank. Determination of these operating voltages has always been a difficult task and at best a long and tedious exercise.

Preceding the laboratory work a thorough investigation was made into the possibilities of a purely mathematical solution to the problem. This search indicated that known methods of solution involved equations which either had no known solution or were not expressed in a readily useable form.

Several studies have been made of this problem along strictly mathematical lines. In these studies drastic assumptions were made and elaborate tables devised to predict the various lamp branch voltages of a bank of incandescent lamps. The seemingly simple problem has been given little concentrated thought in the past, and although this paper does not completely exhaust the possibilities along these lines, it does produce a certain amount of information which leads to further development.

The cost of light per lumen-hour has become as real and significant as miles per gallon in indicating automobile operating costs. This light in lumens depends directly on the voltage impressed across the lamp. As the voltage is increased the lumens increase according to a certain relationship between operating voltage and rated voltage of the lamp. But as this voltage increase takes place the life of the lamp is decreased according to another relationship between the operating voltage and rated voltage. The maximum light and life combination of an incandescent lamp is obtained by operating it at its rated voltage. Therefore the most economical operating voltage is desirable to give the lowest overall cost per lumen-hour. The relationships discussed are shown in detail in the General Electric Lamp Bulletin, May 1946.[1]

After the apparent failure of mathematical solutions to give a readily useable answer it appeared to be desirable to conduct a laboratory investigation so as to arrive at an empirical solution. The construction of a circuit to simulate a bank of lamps equally spaced was undertaken and an investigation of the lamp voltages followed.

The value of the laboratory investigation necessarily depends upon the accuracy of the meters used and the care taken in the measurements. Therefore a precedent must be established as to the accuracy desired. In order to simulate practical engineering conditions the accuracy is considered sufficient if it compares with results obtained by universally used portable meters.

The content of this paper is divided into three chapters.

---

[1] C. E. Weitz, "The Incandescant Lamp," General Electric Lamp Bulletin, Bulletin LD-1 (May, 1946), pp. 4-19.

The first chapter attacks the problem from an analytical standpoint. The second chapter discusses the trial-and-error method of solution and the third chapter introduces the laboratory investigation and the empirical form developed which reduces the problem to an absolute practical answer.

CHAPTER I

ANALYTICAL METHOD

In order to solve any problem the first step is to analyze the circumstances surrounding the problem. This analysis should lead to the true understanding of the peculiarities of the conditions associated with the problem. During this analysis all variants and quantities of known relationship are expressed in an orderly arrangement. Since the orderly arrangement may be of an equation form, it follows that if this equation or one of similar type has been solved previously, then it is only necessary to apply this previous solution to the present equation. But if the equation has no previously known solution it is necessary either to solve the equation by mathematical means if possible or resort to some other type of procedure.

The analysis of incandescent lamp voltage involves a condition of non-linearity in voltage-current relation. This non-linearity of voltage-current relation makes it difficult to designate the current required by the lamp in terms of the lamp voltage. When the voltage-current relation is of constant value it is termed a constant resistance and handled analytically as a constant. But the resistivity of an incandescent lamp cannot be handled as a stable value due to the non-linearity of its voltage-current curve. Therefore an analytical expression must be devised in order to express the previously mentioned non-linearity in the analysis of the problem concerning parallel lamp branch voltages.

With the aid of a notational scheme the incandescent lamp current can be expressed in terms of the lamps rated voltage, rated current and actual operating voltage. This notational scheme as put forth in the General Electric Lamp Bulletin, May 1946 is as follows:

$$\frac{\text{amperes}}{\text{AMPERES}} = \left(\frac{\text{volts}}{\text{VOLTS}}\right)^t$$

In the foregoing expression lower case type indicates operating voltage and current whereas capitals denote rated voltage and current. Solving the equation shown for the operating current in terms of operating voltage, rated voltage and rated current results in

$$\text{amperes} = \text{AMPERES}\left(\frac{\text{volts}}{\text{VOLTS}}\right)^t$$

In the notational scheme the exponent t is determined empirically. If a voltage-current curve for an incandescent lamp is developed employing percentage of rated voltage as abscissas and percentage of rated current as ordinates the slope of the curve is the value of t. It is therefore evident that the value of t is continually changing and depends directly upon the value of voltage at which it is evaluated. This constantly changing value of the exponent in the notational scheme makes the possibility of a final solution by the analytical method very remote.

With the aid of the notational scheme just discussed an analysis of lamp voltages in a lamp bank will be carried out. The example that follows concerns the finding of an expression for the individual lamp voltages in terms of the number of lamp loads, the line resistance and rated lamp voltage. The development to follow arrives at only one lamp voltage and therefore will have to be repeated for determining each lamp voltage. Figure 1 represents the problem to be analyzed. It consists of a bank of five incandescent lamps separated by line resistances R_1, R_2, R_3 and R_4 which are made of equal value to represent equal spacing in the bank. The supply voltage V_1 is equal to the rated value for the lamps and is assumed to remain constant during the development of the analytical expression for the lamp voltage, V_5.

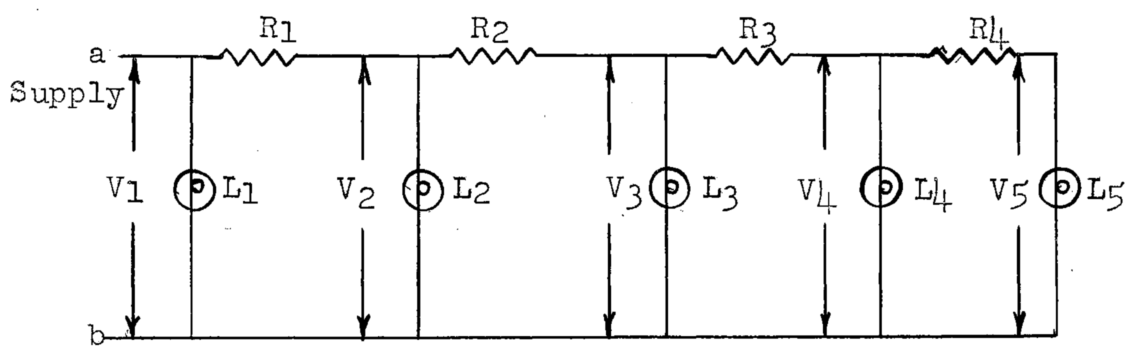


Figure 1

$R_1 = R_2 = R_3 = R_4$ line resistances between lamps simulating equal spacing in the bank.

L_1, L_2, L_3, L_4, L_5 are all identical incandescent lamps rated 100W, 120V.

V_1, V_2, V_3, V_4, V_5 are the respective lamp voltages while operating.

V_1 = 120 volts supply voltage, considered to remain constant.

I_1, I_2, I_3, I_4, I_5 are the respective lamp currents while operating.

I = rated current

V = rated voltage

Using the notational scheme mentioned the following are the lamp currents in terms of their rated current, rated voltage and operating voltage:

$$I_1 = (V_1/V)^{t_1} I$$

$$I_2 = (V_2/V)^{t_2} I$$

$$I_3 = (V_3/V)^{t_3} I$$

$$I_4 = (V_4/V)^{t_4} I$$

$$I_5 = (V_5/V)^{t_5} I$$

The supply voltage is taken as the rated voltage of the lamps and it remains constant. Therefore the expression for the current in lamp 1 is as follows:

$$I_1 = (V_1/V)^{t_1} I = (V/V)^{t_1} I = I$$

But for I_2, I_3, I_4 and I_5 the value of the exponential factor must be evaluated. The exponential factor is of the following form:

$$(V_n/V)^{t_n}$$

The value of the exponential factor cannot be given an accurate value since the value of t_n which is determined from the empirical curves previously mentioned depends on the value of V_n. Therefore the exponential factor must be carried in the given form for an analytical solution. Currents through the line resistances are as follows:

$$I_{R_1} = I_2 + I_3 + I_4 + I_5$$

$$I_{R_2} = I_3 + I_4 + I_5$$

$$I_{R_3} = I_4 + I_5$$

$$I_{R_4} = I_5$$

The currents through the line resistances give voltage drops equal to

$$V_{R_1} = R_1(I_2 + I_3 + I_4 + I_5)$$

$$V_{R_2} = R_2(I_3 + I_4 + I_5)$$

$$V_{R_3} = R_3(I_4 + I_5)$$

$$V_{R_4} = R_4(I_5)$$

Writing Kirchoffs' equation for the voltage around the outside loop of Figure 1 gives

$$V = V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4} + V_5$$

Substituting the values for the individual resistive drops in terms of current and resistance results in

$$V = R_1(I_2+I_3+I_4+I_5) + R_2(I_3+I_4+I_5) + R_3(I_4+I_5) + R_4(I_5) + V_5,$$

and multiplying and collecting terms, there results,

$$V = I_2(R_1) + I_3(R_1+R_2) + I_4(R_1+R_2+R_3) + I_5(R_1+R_2+R_3+R_4) + V_5$$

Substituting notational values for the currents gives

$$V = (V_2/V)^{t_2} I(R_1) + (V_3/V)^{t_3} I(R_1+R_2) + (V_4/V)^{t_4} I(R_1+R_2+R_3)$$
$$+ (V_5/V)^{t_5} I(R_1+R_2+R_3+R_4) + V_5$$

and solving for V_5

$$V_5 = V - \Big[(V_2/V)^{t_2} I(R_1) + (V_3/V)^{t_3} I(R_1+R_2)$$
$$+ (V_4/V)^{t_4} I(R_1+R_2+R_3) + (V_5/V)^{t_5} I(R_1+R_2+R_3+R_4)\Big]$$

Even though the preceding expression gives a representation for V_5 it contains many terms which cannot be handled analytically. No mathematical solution to the preceding equation was found during this research. The equation may be altered to the point where only one unknown voltage is involved by making further substitutions of voltage equivalents. Any additional steps of reduction are not practical due to the exponential factor which still remains. In order to arrive at a definite solution to the problem of parallel lamp branch voltages using the analytical method the last equation developed would have to be solved. To solve the complex equation for the voltage V_5 as represented previously it is necessary to resort to either an empirical method or the trial-and-error method which are more explicitly explained in other chapters of this paper.

Another analytical approach results in a repeating fraction which is a well known representation of a ladder network. The repeating fraction that follows is an analytical expression containing admittances of the incandescent lamps in terms of the notational scheme previously discussed. Since

$$\frac{\text{ohms}}{\text{OHMS}} = \left(\frac{\text{volts}}{\text{VOLTS}}\right)^m ,$$

it follows that

$$\text{ohms} = \left(\frac{\text{volts}}{\text{VOLTS}}\right)^m \text{OHMS},$$

and since

$$\text{conductivity} = y = \frac{1}{\text{ohms}} = \frac{1}{\left(\frac{\text{volts}}{\text{VOLTS}}\right)^m \text{OHMS}} = \left(\frac{\text{VOLTS}}{\text{volts}}\right)^m \text{MHOS},$$

and hence

$$y = (V/V_n)^{m_n} Y,$$

where Y represents the admittance of an incandescent lamp when operated at rated value of applied voltage and current. The lower case type indicates operating values and the capitals indicate rated values. In Figure 1 let

y_1, y_2, y_3, y_4, y_5 represent conductivity under operating conditions for the particular lamp identified by the subscript,

and if y_{ab} represents the total conductivity of the branch between points a and b the expression for the total conductivity between points a and b will be

$$y_{ab} = y_1 + \cfrac{1}{R_1 + \cfrac{1}{y_2 + \cfrac{1}{R_2 + \cfrac{1}{y_3 + \cfrac{1}{R_3 + \cfrac{1}{y_4 + \cfrac{1}{R_4 + \cfrac{1}{y_5}}}}}}}}$$

By using the notational scheme for the individual conductivities the preceding equation becomes

$$y_{ab} = (V/V_1)^{m_1}Y + \cfrac{1}{R_1 + \cfrac{1}{(V/V_2)^{m_2}Y + \cfrac{1}{R_2 + \cfrac{1}{(V/V_3)^{m_3}Y + \cfrac{1}{R_3 + \cfrac{1}{(V/V_4)^{m_4}Y + \cfrac{1}{R_4 + \cfrac{1}{(V/V_5)^{m_5}Y}}}}}}}}$$

If the preceding expression for y_{ab} could be solved for an absolute value of y_{ab} the total lamp bank current could be computed. After the total lamp bank current is known the solution for the lamp voltages presents no difficulties. The difficulty lies in the useable solution of the preceding expression for y_{ab}. It was concluded that a practical solution could only be obtained by empirical or trial-and-error methods.

Through the foregoing application of the analytical method it is readily seen that a complete solution is not possible. Since it is necessary to weigh the results of any solution against the amount of work, time, and space occupied in arriving at a useable answer, it will be found that an analytical approach is unsatisfactory.

CHAPTER II

TRIAL AND ERROR METHOD

When analytical methods fail to produce a practical solution to a problem, it becomes necessary to seek some other scheme that will lead to at least an improved understanding of the conditions involved. Very often the trial-and-error method must be resorted to in order to obtain an explanation of the phenomena under consideration.

It must be understood that the procedure of trial-and-error by itself does not in general provide complete perception of the material processes involved, and that at best it furnishes a useable solution to a problem in which the complex equations representing a solution are not known or have no solution. After circuit analysis has been performed and equations evolved which represent the functioning of the circuit, the trial-and-error method may be used to arrive at a useable solution. The actual mechanics of the method are simple but very laborious in extent and it will be found that the length of the solution depends directly on the experience of the person performing the method.

The trial-and-error method is employed extensively in circuit analysis involving non-linear circuit elements. A non-linear circuit element is an element in which a linearly applied motivating force results in a non-linearly varying movement or flow. Those who are familiar with the magnetic circuit will recognize the magnetic saturation curves for various materials as a representation of non-linearity. That is to say equal increases in magnetomotive force do not result in equal increases in flux density. The amount of increase will vary due to the difference of the slope of the saturation curve at various points of the curve. Therefore the relative permeability which is an incremental value of flux density divided by an incremental value of the motivating magnetomotive force is not of constant value. Similarly an electric circuit element whose voltage-current relation is non-linear cannot be represented as having a constant conductivity. Such a variable conductivity with applied voltage is to be found in the incandescent lamp. Consequently the solution to the problem of incandescent lamp voltages in branch circuits can be treated by the trial-and-error method.

The problem involved in determination of lamp branch voltages for the bank of lamps shown in Figure 1 will be solved by the trial-and-error method. This example will serve to indicate the amount of work, time and space required for a satisfactory solution. The incandescent lamps are the branch elements, and the line resistances are made equal to each other to simulate equal spacing of the lamps in the bank. It is evident that the lamp voltages must be determined for many values of line resistance for each combination of lamp loads. This will make it possible to find the relation between the three factors; line resistance, number of lamps in operation and the lamp voltages.

The conditions in the following solution are for only one value of line resistance and only one combination of lamp loads. The work, space and time required to obtain an accurate solution is clearly shown. The following work would have to be repeated for each combination of lamp loads as the line resistance is varied. At the present time the trial-and-error method is the only method of solution to many types of problems, and is also the only method known which will determine the lamp branch voltages in a parallel bank of lamps.

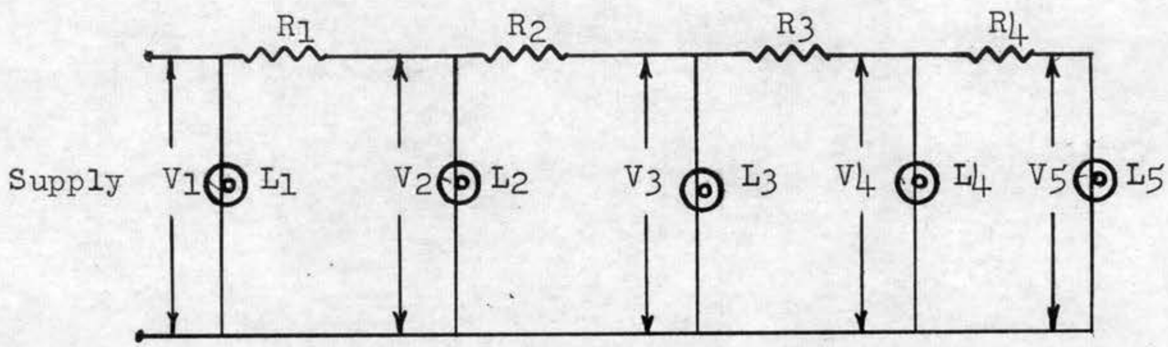


Figure 1

This example concerns a total lamp load of five lamps separated by equal resistances of 0.65 ohms in the line. The lamp bank is supplied by a source whose regulation is assumed to be zero which will give some amount of stability to the calculations.

In Figure 1 the following are the known values and conditions for the one case to be solved by the trial-and-error method.

$R_1 = R_2 = R_3 = R_4 = R_5 = 0.65$ ohms, the total resistance between lamps and is the same in order to simulate equal spacing of the lamps.

L_1, L_2, L_3, L_4, L_5 are incandescent lamps rated 100 watts, 120 volts.

V_1, V_2, V_3, V_4, V_5 are the respective lamp voltages.

V_1 = 120 volts supply voltage, considered constant.

The incandescent lamp as a circuit element is non-linear. That is to say the plot of lamp current versus lamp voltages results in a curve such as that shown in Graph 1. This is a voltage-current relation for the incandescent lamps in Figure 1. Graph 1 will be used to determine lamp current for the apparent lamp voltages necessary in the method. In order to obtain a starting point for the trial-and-error method it will be assumed that all lamps are taking rated current at their respective voltages. The only voltage known is the supply voltage of 120 volts which is the rated voltage of the lamps. The first set of apparent lamp voltages is therefore of rated value of the lamps and the lamp currents may be obtained from Graph 1 which indicates rated current corresponding to rated voltage. Rated current for the lamps of 100 watt, 120 volt rating is 0.86 amperes. Assuming that all the lamps are taking their rated current the following currents will be flowing in the indicated line resistances;

$I_{R_1} = 4 \times 0.86 = 3.44$ amperes

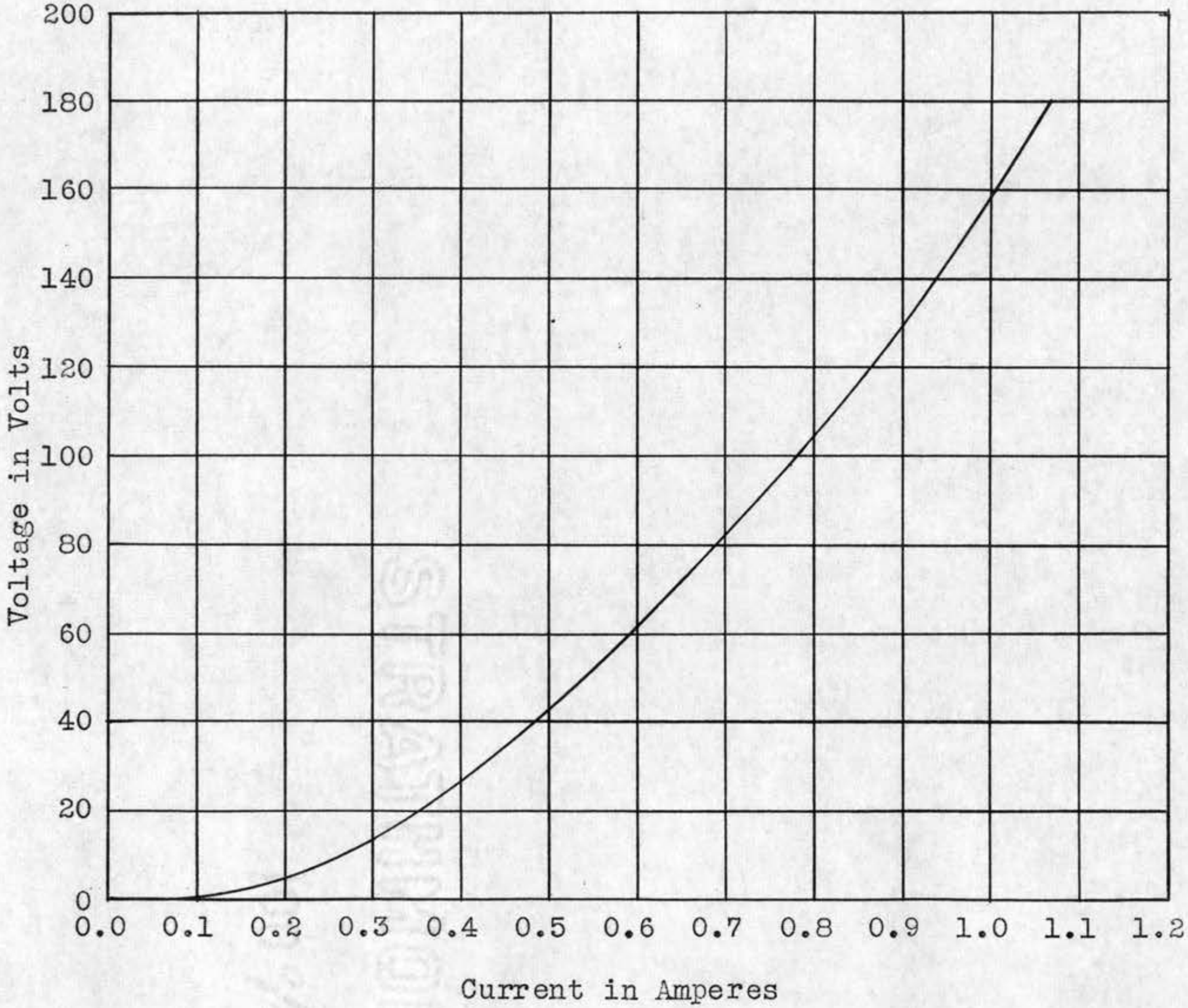
$I_{R_2} = 3 \times 0.86 = 2.58$ amperes

$I_{R_3} = 2 \times 0.86 = 1.72$ amperes

$I_{R_4} = 1 \times 0.86 = 0.86$ amperes.

With these currents flowing in the indicated line resistances there will be the line voltage drops,

$V_{R_1} = 3.44 \times 0.65 = 2.234$ volts

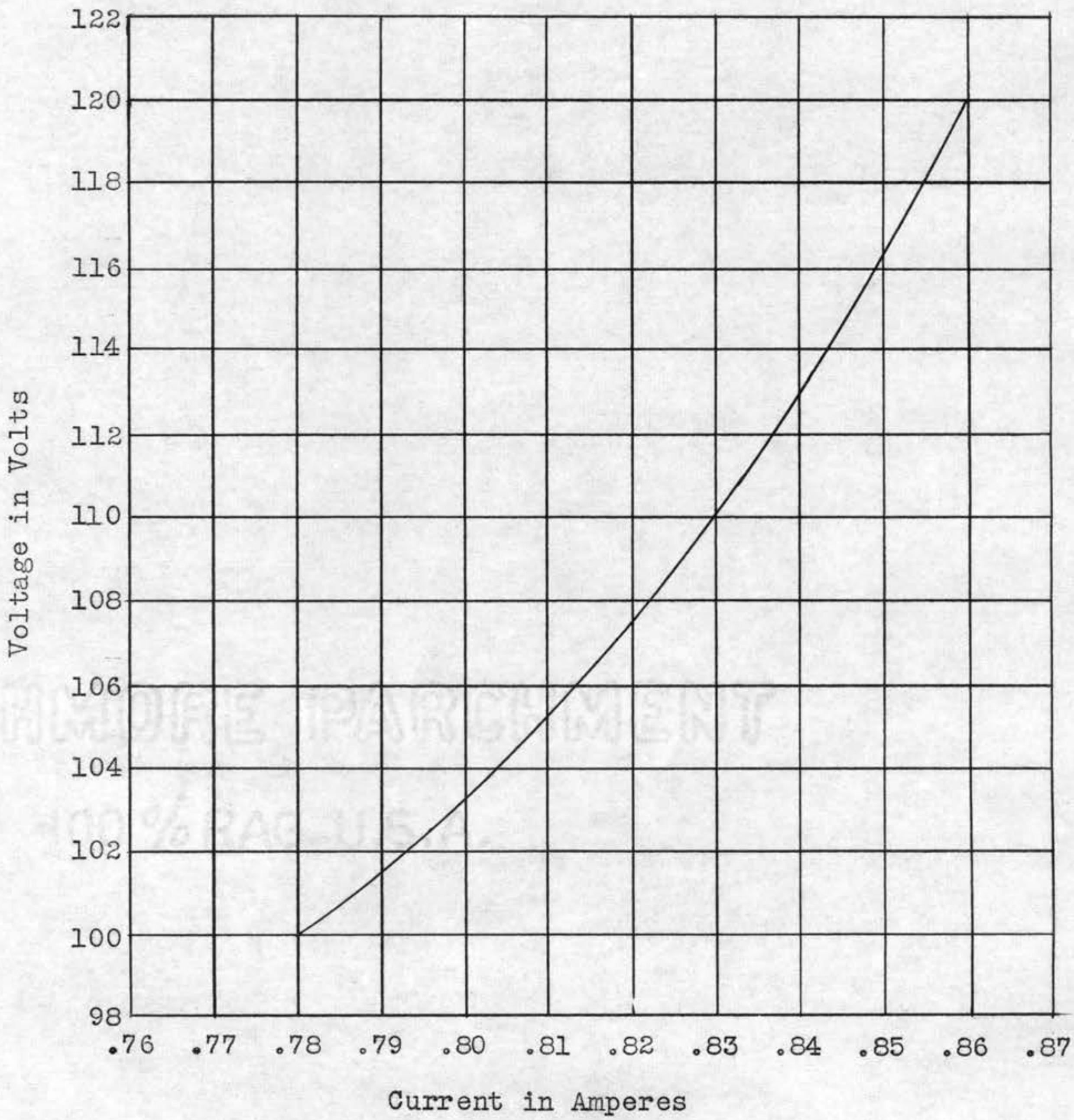


GRAPH 1

Current-Voltage Relationship in GE Incandescent Lamp 100 Watt, 120 Volt.

GRAPH 1 (Supplement)

Voltage versus Current
GE Incandescent Lamp
100 Watt, 120 Volt



$V_{R_2} = 2.58 \times 0.65 = 1.677$ volts

$V_{R_3} = 1.72 \times 0.65 = 1.117$ volts

$V_{R_4} = 0.86 \times 0.65 = 0.558$ volts.

Subtracting the line-resistance voltage drops from the supply voltage the second set of apparent lamp voltages are found to be

$V_2 = V_1 - V_{R_1} = 120.000 - 2.234 = 117.766$ volts

$V_3 = V_2 - V_{R_2} = 117.766 - 1.677 = 116.089$ volts

$V_4 = V_3 - V_{R_3} = 116.089 - 1.117 = 114.972$ volts

$V_5 = V_4 - V_{R_4} = 114.972 - -.558 = 114.414$ volts.

From these apparent lamp voltages a more accurate value for the respective currents can be read from Graph 1 mentioned previously. These currents will be,

$I_2 = 0.854$ amperes

$I_3 = 0.849$ amperes

$I_4 = 0.846$ amperes

$I_5 = 0.845$ amperes

Now the currents through the line resistances are as follows,

$I_{R_1} = 3.394$ amperes

$I_{R_2} = 2.540$ amperes

$I_{R_3} = 1.691$ amperes

$I_{R_4} = 0.845$ amperes

The preceding currents now give the voltage drops

$$V_{R_1} = 3.394 \times 0.65 = 2.205 \text{ volts}$$

$$V_{R_2} = 2.540 \times 0.65 = 1.650 \text{ volts}$$

$$V_{R_3} = 1.691 \times 0.65 = 1.100 \text{ volts}$$

$$V_{R_4} = 0.845 \times 0.65 = 0.549 \text{ volts.}$$

Subtracting these voltage drops from the supply voltage gives the third set of apparent lamp voltages,

$$V_2 = V_1 - V_{R_1} = 120.000 - 2.205 = 117.795 \text{ volts}$$

$$V_3 = V_2 - V_{R_2} = 117.795 - 1.650 = 117.144 \text{ volts}$$

$$V_4 = V_3 - V_{R_3} = 117.144 - 1.100 = 115.044 \text{ volts}$$

$$V_5 = V_4 - V_{R_4} = 115.044 - 0.549 = 114.495 \text{ volts,}$$

and using Graph 1 again the corresponding lamp currents turn out to be

$$I_2 = 0.854 \text{ amperes}$$
$$I_3 = 0.850 \text{ amperes}$$
$$I_4 = 0.846 \text{ amperes}$$
$$I_5 = 0.845 \text{ amperes.}$$

Since currents through the line resistances are

$$I_{R1} = 3.395 \text{ amperes}$$

$$I_{R2} = 2.541 \text{ amperes}$$

$$I_{R3} = 1.691 \text{ amperes}$$

$$I_{R4} = 0.845 \text{ amperes,}$$

the resistive drops become

$$V_{R_1} = 3.395 \times 0.65 = 2.205 \text{ volts}$$

$$V_{R_2} = 2.541 \times 0.65 = 1.651 \text{ volts}$$

$$V_{R_3} = 1.691 \times 0.65 = 1.100 \text{ volts}$$

$$V_{R_4} = 0.845 \times 0.65 = 0.549 \text{ volts.}$$

Subtracting the resistive drops as before results in a fourth set of apparent lamp voltages,

$$V_2 = V_1 - V_{R_1} = 120.000 - 2.205 = 117.795 \text{ volts}$$

$$V_3 = V_2 - V_{R_2} = 117.795 - 1.651 = 116.143 \text{ volts}$$

$$V_4 = V_3 - V_{R_3} = 116.143 - 1.100 = 115.043 \text{ volts}$$

$$V_5 = V_4 - V_{R_4} = 115.043 - 0.549 = 114.494 \text{ volts.}$$

A comparison of the fourth set of apparent lamp voltages with the third set indicates that further operations are not practical as far as time and space requirements are concerned. The solution is not actually complete until the sets of voltages are the same. As the resistance between the lamps is increased the number of necessary trials is also increased. Obviously the solution is also lengthened when the number of lamps is increased.

CHAPTER III

EMPIRICAL METHOD

The information obtained from observation or experience is termed empirical. Therefore it follows that the empirical method employs the results obtained from either laboratory or calculated observations. The empirical method of determining a useable solution to a problem is used extensively by all research engineers. As an example, Ohm in his study of current flow in a conductor discovered the correlation between the voltage, the current and the resistance of a conductor. This correlation is known as Ohms' Law. There are many more such empirical relations that have been produced from the research laboratories and also from practical experiences. The empirical method which employs results from laboratory observations is limited as concerns any one problem. Time and available equipment for making the previously mentioned observations control the degree of accuracy and the extent of the research. After the laboratory observations are made a mathematical correlation must be devised. This mathematical correlation must give the desired accuracy and a reasonable solution. It must be remembered that this solution must be obtained with a smaller amount of work than that required for existing solutions. One difficulty with the empirical method is the limitation caused directly by the limitations of the measuring devices employed in the laboratory.

In the problem of determining parallel lamp branch voltages the laboratory offers all the necessary equipment of lamp sockets, measuring devices and variable standard resistances. The object of the problem is to determine if possible the correlation between lamp voltages in a parallel bank of lamps, the number of lamps in the bank and the line resistance between the lamps. The circuit shown in Figure 1 was devised in order to be able to observe the three factors mentioned.

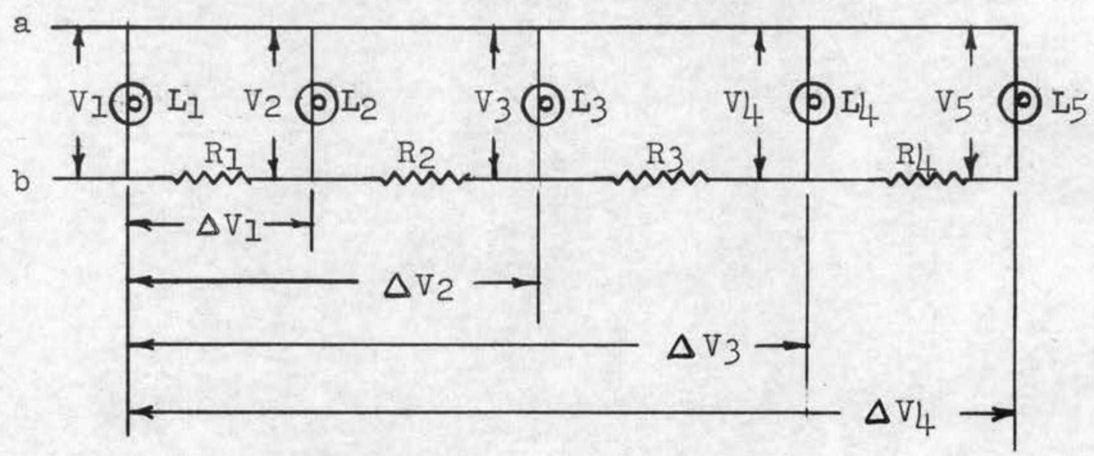


Figure 1

Voltage V_1 is the supply voltage and remains at rated voltage for the lamps throughout the observations. Resistances R_1, R_2, R_3, and R_4 are the line resistances and are made equal in order to simulate the equal spacing in the bank of lamps. The lamp voltages V_1, V_2, V_3, V_4, and V_5 are computed by subtracting the respective change in voltage, ΔV, from V_1. To keep from loading the circuit ΔV must be measured by using a high resistance voltmeter. The two stipulations observed in obtaining data from the circuit of Figure 1 are, (1) the line resistances are all equal to simulate equal spacing of the lamps in the lamp bank, (2) lamp loads are added consecutively and all lamps in one group must be operating. Table I gives results as

TABLE I

LAMPS IN OPERATION

R = 8.86	1 Lamps V_1/I_{TOTAL}	2	3	4	5
V_1	120/.86	120/1.66	120/2.38	120/2.97	120/3.44
V_2		113.0	106.6	101.5	97.0
V_3			100.0	89.0	81.5
V_4				83.0	70.0
V_5					65.0
R = 3.97					
V_1	120/.86	120/1.68	120/2.46	120/3.18	120/3.81
V_2		116.8	112.7	111.5	109.0
V_3			111.9	105.0	100.5
V_4				103.0	94.5
V_5					92.0
R = 1.51					
V_1	120/.86	120/1.68	120/2.48	120/3.26	120/4.00
V_2		118.76	117.53	116.30	115.2
V_3			117.42	114.15	111.5
V_4				112.80	109.4
V_5					108.2
R = .65					
V_1	120/.86	120/1.69	120/2.52	120/3.32	120/4.12
V_2		119.48	118.98	118.46	118.00
V_3			118.44	117.40	116.40
V_4				116.90	115.40
V_5					114.90
R = .32					
V_1	120/.86	120/1.69	120/2.52	120/3.33	120/4.15
V_2		119.76	119.48	119.22	118.97
V_3			119.22	118.70	118.17
V_4				118.44	117.64
V_5					117.38
R = .16					
V_1	120/.86	120/1.68	120/2.52	120/3.33	120/4.16
V_2		119.88	119.76	119.63	119.50
V_3			119.63	119.38	119.12
V_4				119.26	118.88
V_5					118.76
R = .08					
V_1	120/.86	120/1.68	120/2.52	120/3.35	120/4.19
V_2		119.95	119.89	119.84	119.77
V_3			119.83	119.70	119.57
V_4				119.64	119.44
V_5					119.39

obtained from the circuit of Figure 1.

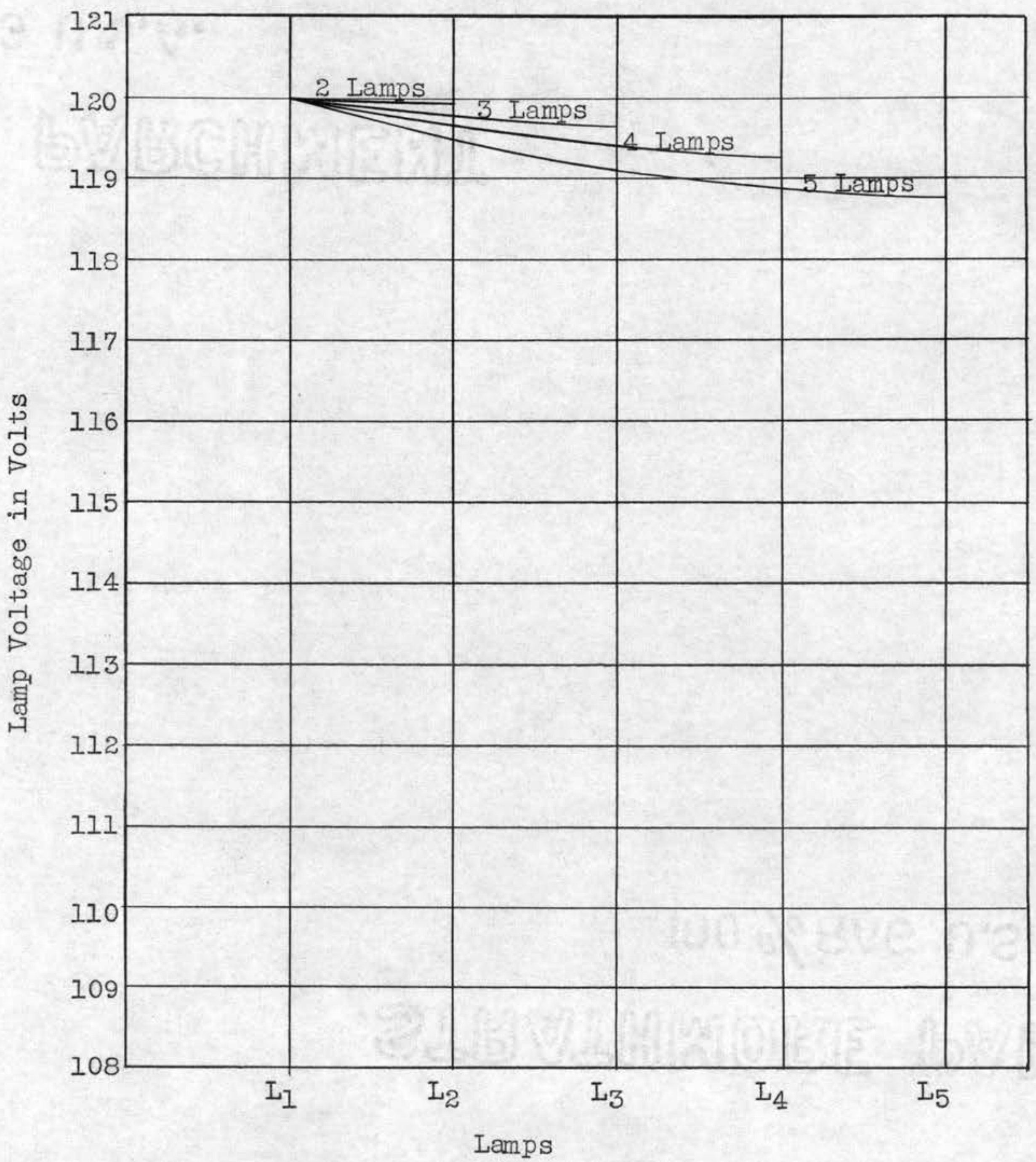
As an example of how to use Table I the voltage on the third lamp from the supply voltage will be determined when the total number of lamps in the bank is five and the line resistance is 0.65 ohms. From Table I under the group of voltages with the heading of $R = 0.65$ ohms, on the line marked V_3 and under the column headed five lamps in operation the voltage V_3 may be read. 116.40 volts is the voltage found in the preceding example.

The accuracy of Table I is well within the limits prescribed in the introduction to this paper. The line resistance R of Figure 1 is determined with a wheatstone bridge and the increments in voltage, ΔV, are made with a low voltage portable type meter. Sufficient accuracy is gained in the preceding method to compare readily with the measurements of voltage at a lamp made under actual conditions using standard portable type equipment for voltage measurement.

Table I has been established as a sufficiently accurate set of lamp voltages for all combinations of five lamp loads and line resistances ranging from 0.08 ohms to 8.86 ohms. After establishing Table I the lamp voltage variation must be analyzed. Graphs 1 through 6 indicate the variation of lamp voltages when the line resistance R is held constant and only the number of lamp loads is changed. From Graphs 1 through 6 the same information can be attained as from Table I. Graph 7 indicates the variation of voltage on lamp number two as the line resistance is varied for all four total lamp loads. It can be seen that for each combination of total lamp loads the number two lamp

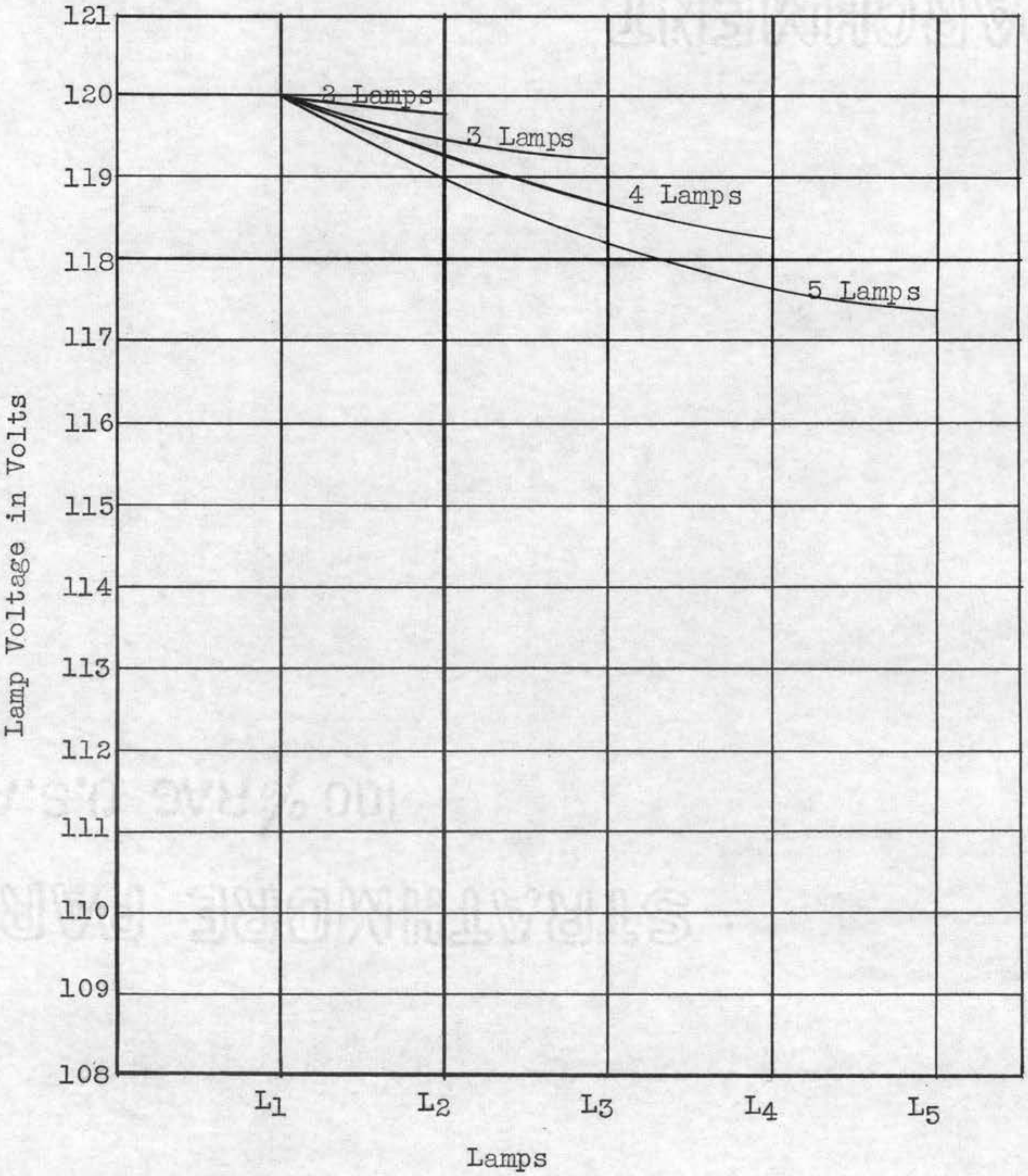
GRAPH 1

General Electric, 100 watt, 120 volt lamp voltages in the circuit of Figure 1 as the total lamp load is altered from two through five lamps. The line resistance is 0.16 ohms and the supply voltage is held constant at rated lamp value.



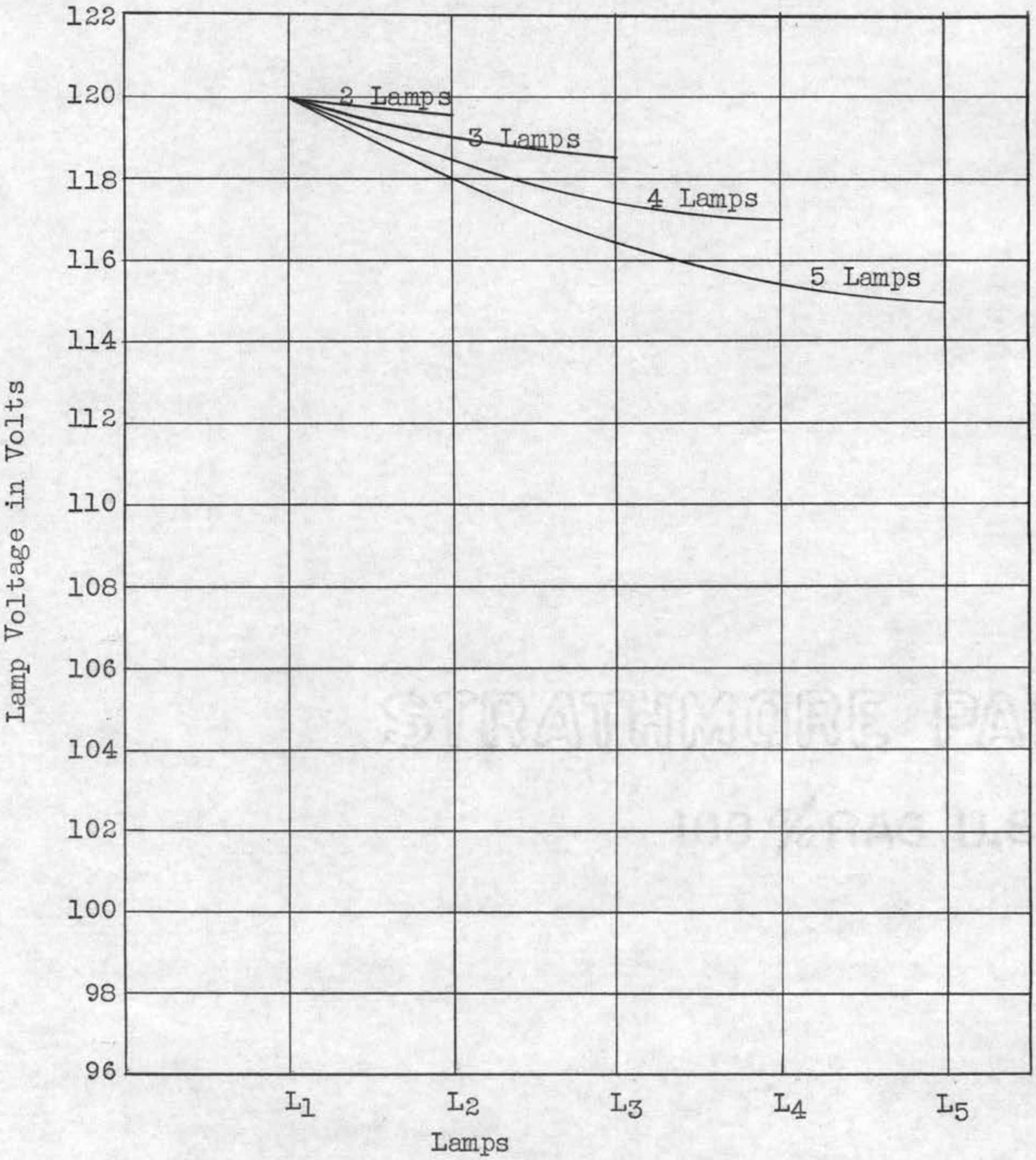
GRAPH 2

General Electric, 100 watt, 120 volt lamp voltages in the circuit of Figure 1 as the total lamp load is altered from two through five lamps. The line resistance is 0.32 ohms and the supply voltage is held constant at rated lamp value.



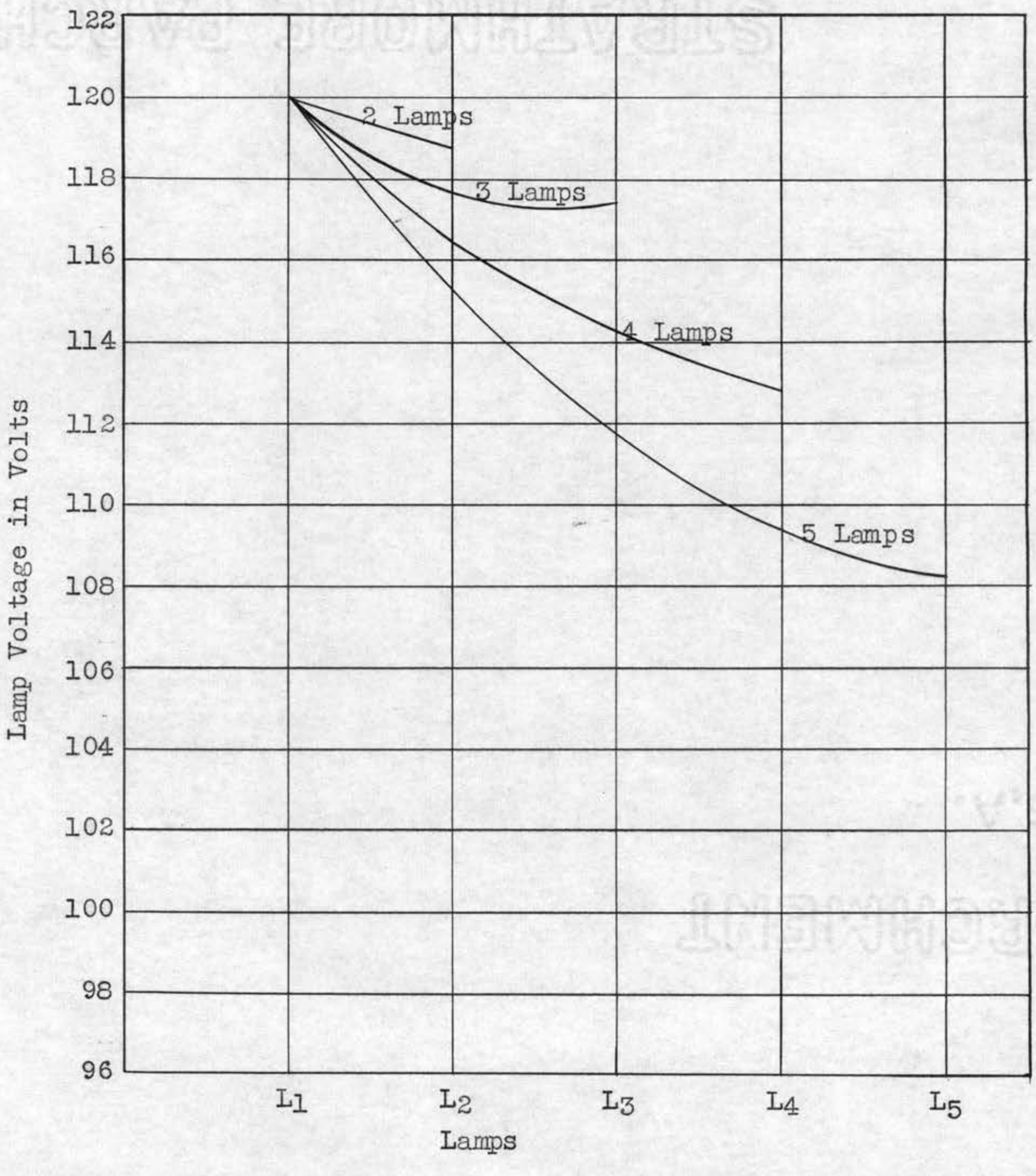
GRAPH 3

General Electric, 100 watt, 120 volt lamp voltages in the circuit of Figure 1 as the total lamp load is altered from two through five lamps. The line resistance is 0.65 ohms and the supply voltage is held constant at rated lamp value.



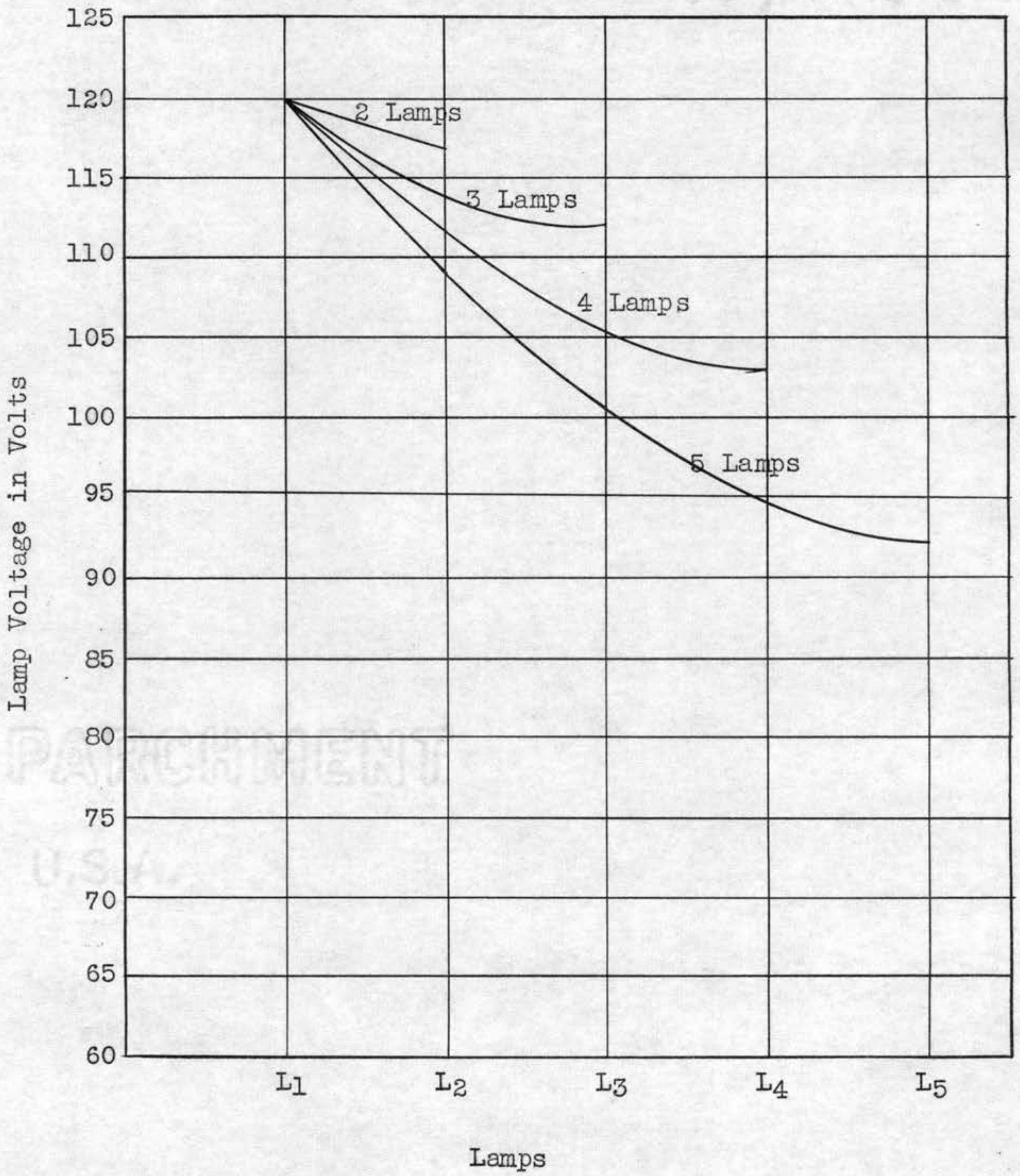
GRAPH 4

General Electric, 100 watt, 120 volt lamp voltages in the circuit of Figure 1 as the total lamp load is altered from two through five lamps. The line resistance is 1.51 ohms and the supply voltage is held constant at rated lamp value.



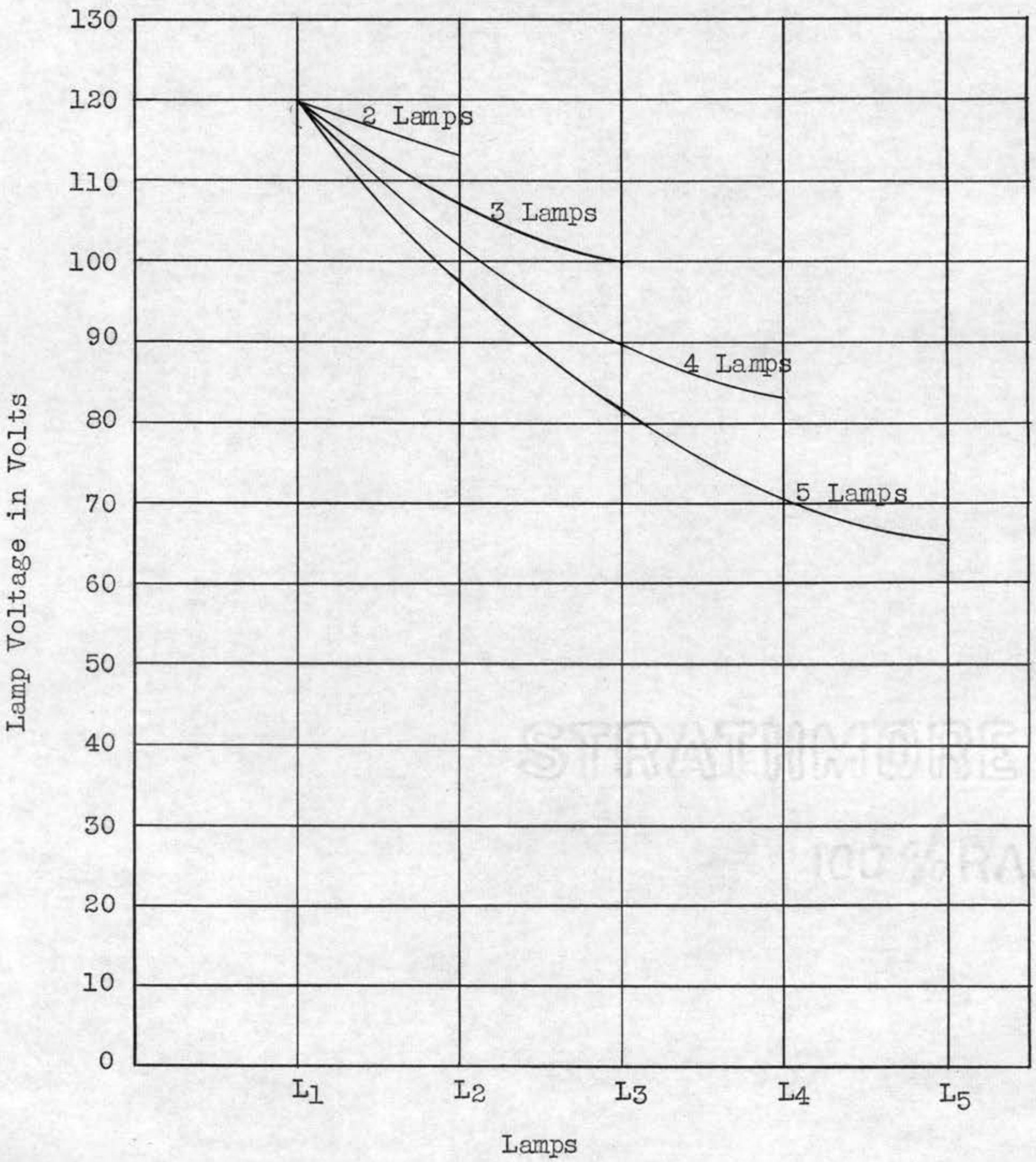
GRAPH 5

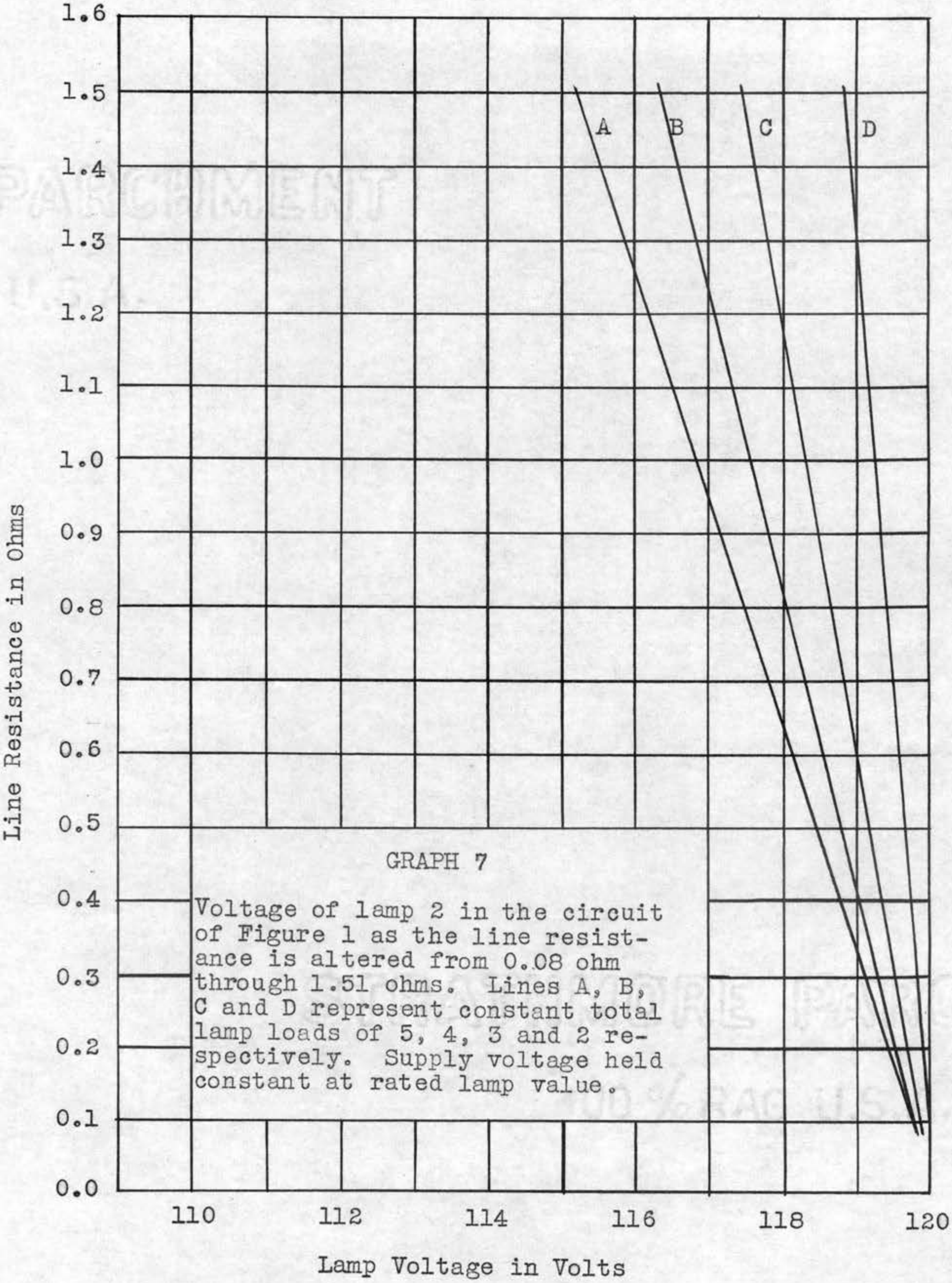
General Electric, 100 watt, 120 volt lamp voltages in the circuit of Figure 1 as the total lamp load is altered from two through five lamps. The line resistance is 3.97 ohms and the supply voltage is held constant at rated lamp value.



GRAPH 6

General Electric, 100 watt, 120 volt lamp voltages in the circuit of Figure 1 as the total lamp load is altered from two through five lamps. The line resistance is 8.86 ohms and the supply voltage is held constant at rated lamp value.





GRAPH 7

Voltage of lamp 2 in the circuit of Figure 1 as the line resistance is altered from 0.08 ohm through 1.51 ohms. Lines A, B, C and D represent constant total lamp loads of 5, 4, 3 and 2 respectively. Supply voltage held constant at rated lamp value.

voltage varies as a straight line function with respect to the line resistance. It will also be noticed that the straight lines representing the various lamp combinations are not parallel, but on the contrary diverge at a decidedly increasing rate as the line resistance is increased. This increase in separation may be characterized by the exponential function e^x. In order to find the value of x in the exponential function e^x it is only necessary to hold the line resistance and lamp number constant when the combination of lamp loads is changed. It is assumed that the solution will take the form,

$$V_L = 120e^{-nRx_L}$$

where

V_L is the lamp voltage on lamp L

120 is the rated lamp voltage

n is the number of lamps in total lamp load

R is the line resistance

x_L is a constant to be determined for lamp L.

From Graph 7 the lamp voltages for lamp number two will be indicated as a function of n, R and x_L, and following the exponential form previously devised, there results,

$$V_2 = 118.0 = 120e^{-5(0.65)x_2} \quad \text{lamp load of 5} \tag{1}$$

$$V_2 = 118.5 = 120e^{-4(0.65)x_2} \quad \text{lamp load of 4} \tag{2}$$

$$V_2 = 119.0 = 120e^{-3(0.65)x_2} \quad \text{lamp load of 3} \tag{3}$$

$$V_2 = 119.5 = 120e^{-2(0.65)x_2} \quad \text{lamp load of 2.} \tag{4}$$

Solving each of these equations for x_2 results in

$$e^{-5(0.65)x_2} = 118.0/120.0 = 0.982 \tag{1}$$

and taking the natural logarithm of both sides results in

$$5(0.65)x_2 = 0.018$$

and then solving for x_2

$$x_2 = 0.00554 \quad \text{lamp load of } 5$$

and from equation (2)

$$e^{-4(0.65)x_2} = 118.5/120.0 = 0.987 \tag{2}$$

then taking the natural logarithm of both sides gives

$$4(0.65)x_2 = 0.013$$

and solving for x_2

$$x_2 = .00500 \quad \text{lamp load of } 4$$

and from equation (3)

$$e^{-3(0.65)x_2} = 119.0/120.0 = 0.991 \tag{3}$$

taking the natural logarithm of both sides results in

$$3(0.65)x_2 = 0.009$$

which gives

$$x_2 = 0.00461 \quad \text{lamp load of } 3$$

also from equation (4)

$$e^{-2(0.65)x_2} = 119.5/120.0 = 0.995 \tag{4}$$

and taking the natural logarithm of both sides gives

$$2(0.65)x_2 = 0.00500$$

resulting in

$$x_2 = 0.00385 \quad \text{lamp load of } 2.$$

It is evident from the values obtained for x_2 under different lamp loads that a constant value of x_2 can be used. This value is taken as the average of the four values obtained. The average value of x_2 to the nearest thousandth is 0.005.

From Graph 8 the lamp voltages for lamp number 3 will be indicated as a function of n, R and x_3,

$$V_3 = 116.4 = 120e^{-5(0.65)x_3} \quad \text{lamp load of 5} \tag{5}$$

$$V_3 = 117.4 = 120e^{-4(0.65)x_3} \quad \text{lamp load of 4} \tag{6}$$

$$V_3 = 118.4 = 120e^{-3(0.65)x_3} \quad \text{lamp load of 3} \tag{7}$$

Solving these three equations for x_3 results in

$$e^{-5(0.65)x_3} = 116.4/120.0 = 0.970 \tag{5}$$

and taking the natural logarithm of both sides gives

$$5(0.65)x_3 = 0.03$$

which results in a value of

$$x_3 = 0.00923 \quad \text{lamp load of 5.}$$

Then from equation (6)

$$e^{-4(0.65)x_3} = 117.4/120.0 = 0.978 \tag{6}$$

and taking the natural logarithm of both sides results in

$$4(0.65)x_3 = 0.022$$

which produces

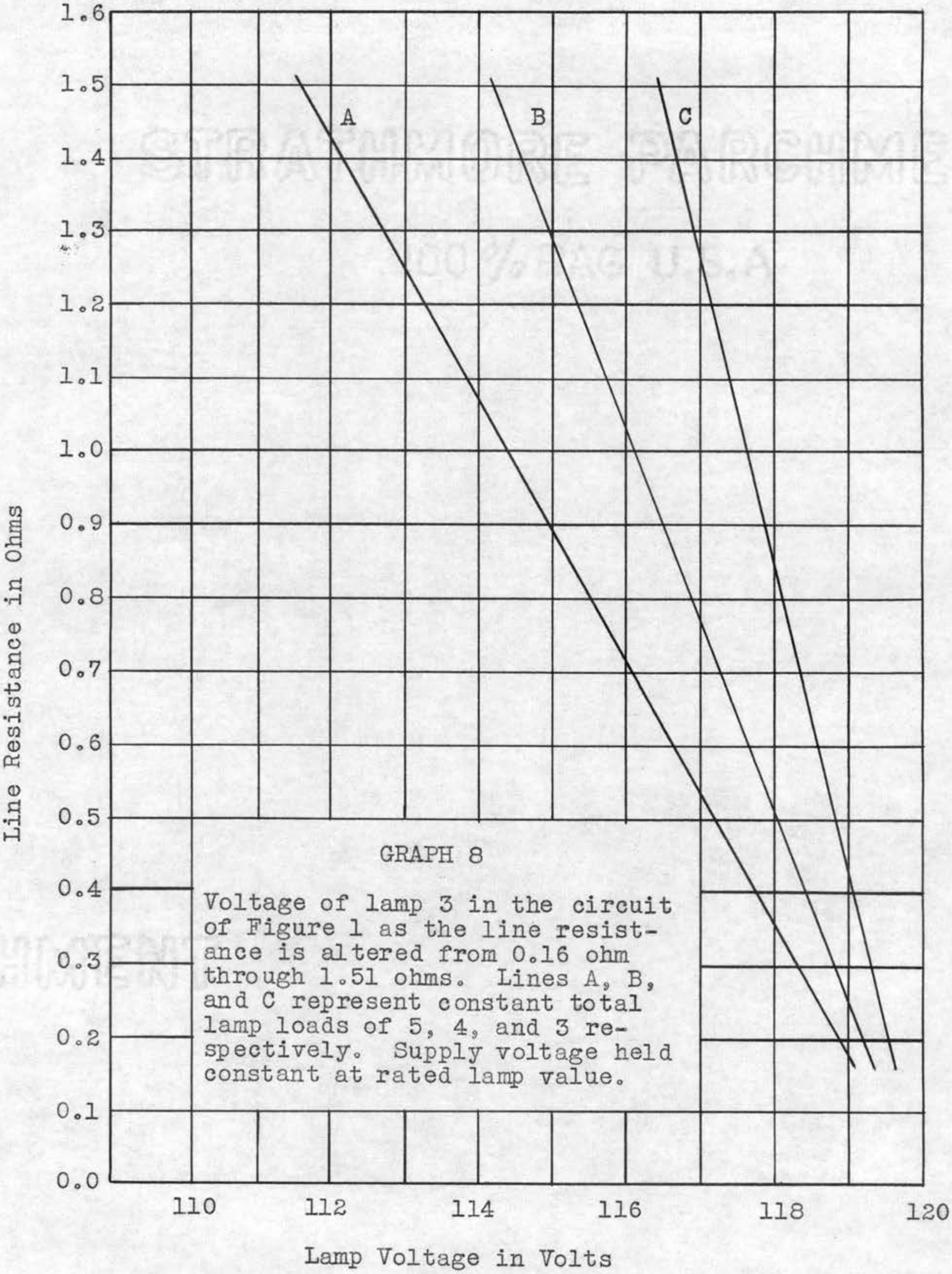
$$x_3 = 0.00845 \quad \text{lamp load of 4.}$$

Finally from equation (7)

$$e^{-3(0.65)x_3} = 118.4/120.0 = 0.987 \tag{7}$$

and taking the natural logarithm of both sides gives

$$3(0.65)x_3 = 0.013$$



GRAPH 8

Voltage of lamp 3 in the circuit of Figure 1 as the line resistance is altered from 0.16 ohm through 1.51 ohms. Lines A, B, and C represent constant total lamp loads of 5, 4, and 3 respectively. Supply voltage held constant at rated lamp value.

which resolves into

$x_3 = 0.00667$ lamp load of 3.

The average value of x_3 computed from the three values determined in the foregoing process is 0.008 carried to the nearest thousandth. Similarly the values for x_4 and x_5 can be determined. Compiling these values in a table results in the following:

L	x_L
1	0.000
2	0.005
3	0.008
4	0.011
5	0.014

where L is the number of the lamp as numbered from the source of power and x_L is the constant determined by the method just discussed. By the use of this table and the exponential form,

$$V_L = 120e^{-nRx_L}$$

the voltage on any lamp L, under any number n of lamp loads separated by equal resistance R can be computed.

To indicate the process involved in a solution using the developed empirical exponential form an example is carried out.

Example 1.

Find the voltage on the fourth lamp from the source of power in a bank of lamps containing four lamps which are rated 100 watts, 120 volts, with a line resistance of 0.65 ohms. With the given conditions,

$L = 4$

$n = 4$

$R = 0.65$ ohms

and by substituting these values in the empirical form just developed there results

$$V_4 = 120e^{-4(0.65)x_4}. \tag{8}$$

It now becomes necessary to consult the L, x_L table for the value of the exponential constant x_4. From the table corresponding to lamp 4

$x_4 = 0.011$

and substituting this value in equation (8) results in

$$V_4 = 120e^{-4(0.65)0.011}$$

which gives

$V_4 = 120 \times 0.972 = 116.8$ volts

The value determined in the example of 116.8 volts compares favorably with 116.9 volts obtained from Table I. Table II compares the empirically produced values of lamp voltages with those values from Table I.

It will be noticed that the function x_L in terms of the number of the respective lamp is increasing linearly as the lamp number is increased. Therefore the L, x_L table can be extended in the following manner.

L	x_L
1	0.000
2	0.005
3	0.008
4	0.011
5	0.014
6	0.017
7	0.020
8	0.023
9	0.026

TABLE II

	Lamp Load of 2		Lamp Load of 3		Lamp Load of 4		Lamp Load of 5	
R = .65	Computed	Table I	Computed	Table I	Computed	Table I	Computed	Table I
V_1	120.0	120.0	120.0	120.0	120.0	120.0	120.0	120.0
V_2	119.3	119.5	118.9	119.0	118.5	118.5	118.2	118.0
V_3			118.2	118.4	117.8	117.4	116.9	116.4
V_4					116.8	116.9	115.6	115.4
V_5							114.8	114.9
R = .32								
V_1	120.0	120.0	120.0	120.0	120.0	120.0	120.0	120.0
V_2	119.7	119.8	119.4	119.5	119.3	119.2	119.0	119.0
V_3			119.4	119.2	118.8	118.7	118.4	118.2
V_4					118.2	118.4	117.7	117.6
V_5							117.4	117.4
R = .16								
V_1	120.0	120.0	120.0	120.0	120.0	120.0	120.0	120.0
V_2	119.9	119.9	119.8	119.8	119.7	119.6	119.6	119.5
V_3			119.6	119.6	119.4	119.4	119.3	119.1
V_4					119.2	119.3	119.0	118.9
V_5							118.8	118.8
R = 8.86								
V_1	120.0	120.0	120.0	120.0	120.0	120.0	120.0	120.0
V_2	109.8	113.0	105.0	106.6	100.3	101.5	96.2	97.0
V_3			98.8	100.0	90.5	89.0	84.2	81.5
V_4					81.2	83.0	73.7	70.0
V_5							64.6	65.0
R = .08								
V_1	120.0	120.0	120.0	120.0	120.0	120.0	120.0	120.0
V_2	120.0	120.0	119.9	119.9	119.8	119.8	119.8	119.8
V_3			119.8	119.8	119.7	119.7	119.6	119.6
V_4					119.6	119.6	119.5	119.4
V_5							119.3	119.4

The preceding table of L, x_L has been developed empirically for an incandescent lamp rated at 100 watts, 120 volts. If the lamp under investigation has a decidedly different characteristic curve from the 100 watt, 120 volt lamp then the following method must be used to form a new L, x_L table. The example that follows will involve lamps of 100 watt, 120 volt rating in order to check the values against those already determined for x_L.

Example 2.

Determine the values for the L, x_L table previously discussed. Since the value of x_L increases linearly it is only necessary to determine two values of x_L by calculation. Therefore a three lamp circuit is used as shown in Figure 2 in order to reduce the amount of calculations. V_2 and V_3 are computed by the trial-and-error method which is not difficult when only two loops are involved.

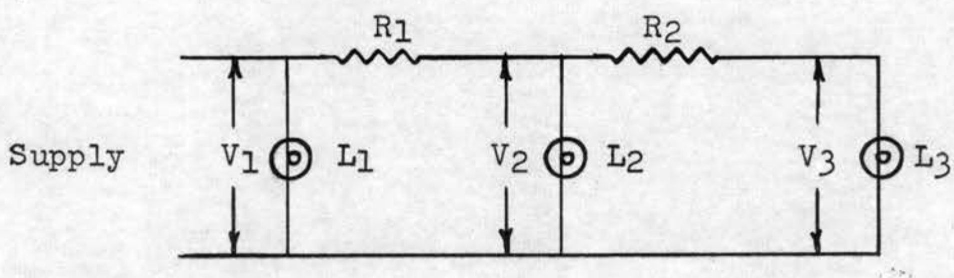


Figure 2

The respective known conditions are;

V_1 = 120 volts supply (considered constant)

L_1, L_2, L_3 are 100 watt, 120 volt incandescent lamps

$R_1 = R_2 = 0.65$ ohms line resistance

V_1, V_2, V_3 are the respective lamp voltages.

In order to determine the value of x_L in the exponential form it is necessary to know the voltage on the Lth lamp. Therefore the voltage on the Lth lamp must be determined by some known method. Since the only known process which will result in a useable solution to this type of problem is the trial-and-error method it will be applied to the example to determine V_2 and V_3.

Assuming that the lamps are all taking rated current of 0.86 amperes at the respective lamp voltages then

V_1 = 120 volts which is the supply voltage and is considered constant.

$V_2 = V_1 - V_{RI}$ where V_{RI} is the voltage drop across RI, and since the currents through the line resistances are

$$I_{R_1} = 2 \times 0.86 = 1.72 \text{ amperes}$$

$$I_{R_2} = 1 \times 0.86 = 0.86 \text{ amperes}$$

there results the line voltage drops

$$V_{R_1} = I_{R_1} \times R_1 = 1.72 \times 0.65 = 1.1 \text{ volts}$$

$$V_{R_2} = I_{R_2} \times R_2 = 0.86 \times 0.65 = 0.6 \text{ volt.}$$

Subtracting these voltage drops from the supply voltage gives

$$V_2 = V_1 - V_{R_1} = 120.0 - 1.1 = 118.9 \text{ volts}$$

$$V_3 = V_2 - V_{R_2} = 118.9 - 0.6 = 118.3 \text{ volts.}$$

Using these apparent lamp voltages and consulting the curve of lamp voltage versus lamp current more accurate values for the lamp currents are found to be

$I_2 = 0.85$ amperes

$I_3 = 0.85$ amperes

resulting in line resistance currents of

$I_{R_1} = 1.70$ amperes

$I_{R_2} = 0.85$ amperes

which will give line voltage drops of

$V_{R_1} = 1.70 \times 0.65 = 1.1$ volts

$V_{R_2} = 0.85 \times 0.65 = 0.6$ volts.

Subtracting these line voltage drops from the supply voltage as before gives

$V_2 = V_1 - V_{R_1} = 120.0 - 1.1 = 118.9$ volts

$V_3 = V_2 - V_{R_2} = 118.9 - 0.6 = 118.3$ volts.

Comparing these apparent lamp voltages with the preceding set indicates that it is not necessary to extend the process. Inserting the previously determined lamp voltages into the exponential empirical form,

$$V_L = 120e^{-nRx_L} \text{ volts}$$

which results in

$$V_2 = 118.9 = 120e^{-3(0.65)x_2},$$

or

$$e^{-3(0.65)x_2} = 118.9/120.0 = 0.990,$$

and when the natural logarithms of both sides are taken

$$3(0.65)x_2 = 0.01,$$

which gives a value for x_2

$$x_2 = 0.005.$$

Inserting the predetermined lamp voltage for lamp number three into the exponential empirical form

$$V_3 = 118.3 = 120e^{-3(0.65)x_3} \text{ volts,}$$

or

$$e^{-3(0.65)x_3} = 118.3/120.0 = 0.985$$

and when the natural logarithms of both sides are taken, there results,

$$3(0.65)x_3 = 0.015$$

which gives the value of x_3 to be

$$x_3 = 0.008.$$

With the two values of x_L developed a straight line increase in values will give the following table of L, x_L;

L	x_L
1	0.000
2	0.005
3	0.008
4	0.011
5	0.014

The foregoing table of L, x_L can be extended to include as many lamps as is necessary for the bank under consideration. This table is only useful for the specified lamp rated at 100 watts, 120 volts. This table is useful for any lamp which has similar voltage-current relationship by changing the integer, "120" in the exponential form to the rated voltage of the lamp under investigation.

CONCLUSION

The conclusion of the foregoing investigation into parallel lamp branch voltages is divided into two sections. The first section contains a discussion of the results of this thesis and the second indicates the possibilities for future study in this field.

It is the aim of this research to discover a possible means for determining the various lamp voltages in a bank of incandescent lamps. It is further desired that the method devised shall outweigh former solutions in both accuracy and ease of application. These conditions have been met in the one case investigated in this paper.

The case discussed in this research involves only a lamp bank of similar lamp loads equally spaced in the bank. Practically, this case is the most important due to the physical nature of a bank of incandescent lamps. In most installations it will be found that the lamps are spaced equally and are of the same rating.

The empirical method developed in Chapter III provides a means for obtaining the required lamp branch voltages. This method produces sufficient accuracy as can be verified by an examination of Table II, Chapter III. This table compares the computed voltage values with the values measured in the laboratory. The largest discrepancy in Table II, Chapter III is of less than one per cent variation. This variation of one per cent is negligible when the accuracy of the laboratory measuring devices is considered. The instruments used in the laboratory were two per cent portable meters.

For the engineer who desires a quick and accurate solution to the incandescent lamp bank problem this empirical method provides another process that is readily adaptable to most cases. This new process may seem much more difficult to handle than those for which experience has been a contributory factor. It will be found that this experience will also aid in the understanding of the empirical form developed in Chapter III.

It is evident from Graphs 7 and 8 that a graphical solution is possible. By extending the constant lamp load lines the lamp voltage can be determined for any value of line resistance. This is a valuable piece of information for those designing lamp banks because of the many different spacings that are encountered in a given installation. It is not feasible to compute tables for a large number of line resistances and therefore Graphs 7 and 8 provide a means for easily determining the lamp voltage for any given line resistance. These graphs could be prepared in chart or nomograph forms which could cover a wide range of lamp types. This project might be of considerable interest to lamp manufacturers.

Cases concerning other than equal spacing of the lamps in the bank will give definitely non-symmetrical results and therefore will lead to more research. Also the combining of lamp loads of different ratings or voltage-current relationships will result in varying degrees of difficulties. The solution of these cases presents many possibilities for future study.

Although the findings of this thesis are confined to the single case presented, it is obvious that they are applicable to the general problem of the non-linear circuit.

BIBLIOGRAPHY

Cameron, C. F. Lamps, Wire Size and Voltage Drop. Stillwater: Oklahoma Engineering Experiment Station, January, 1947.

Guillemin, E. A. Communication Networks, Volume II. New York: John Wiley and Sons, Inc., 1935.

Richardson, D. E. Electrical Network Calculations. New York: D. Van Nostrand Company, Inc., 1946.

Weitz, C. E. "The Incandescent Lamp." General Electric Lamp Bulletin, Bulletin LD-1 (May, 1946), 4-19.

- Patty McAffrey Howell -