

ULTIMATE STRENGTH DESIGN OF REINFORCED CONCRETE
COLUMN BY THE APPLICATION OF THE COMPUTER

By

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LIST OF SYMBOLS

USED IN EQUATIONS	USED IN COMPUTER PROGRAM	
A_s	ABARS	Total area of reinforcement in column in sq. in.
b	B	Side of column parallel to axis of bending
	BARS	Sizes of bars in column
CL, CL_1, CL_2	CL, CL_1, CL_2	Clear concrete cover on reinforcement in inches.
	DA	Diameter of bar or diameter of equivalent bar in inches (corner bar)
e_c	EC	Ultimate strain of concrete in in./in.
e_{sc}	ESC	Strain in compression steel (top row) in in./in.
E	ESM	Modulus of elasticity of steel in KSI
e	ECCY	Eccentricity
e_{st}	EST	Strain in tension steel (bottom row) in in./in.
e_{sy}	ESY	Strain in steel in in./in.
f_c^l	FCU	Ultimate compressive strength of concrete in KSI
f_{sc}	FSC	Stress in compression steel (top row) in KSI
f_{st}	FST	Stress in tension steel (bottom row) in KSI
f_y	FSY	Yield strength of reinforcement in KSI
M_u	MU	Moment capacity under combined axial load and bending in ft.-kips

USED IN
EQUATIONS

USED IN
COMPUTER PROGRAM

P_c	PC	Total compression in concrete in kips
p	PERC	Steel percentage in column
P_o	PO	Axial load capacity of column when centrically loaded in kips
	PSSUM	Total compression in steel (PSC PST) in kips
P_{st}	PST	Total tension in tension steel in kips
P_u	PU	Axial load capacity under combined axial load and bending in kips
t	T	Side of column perpendicular to axis of bending in in./in.
	TD	Tie diameter in inches

CHAPTER I

INTRODUCTION

The method of designing reinforced concrete columns has followed a zigzag course during the past 50 years. The original idea was based on the ultimate load carrying capacity of the members. When the elastic theory was adopted for the design of flexural members, it seemed consistent to accept the same theory for the design of compression members. As the shortcomings of the elastic theory were gradually recognized, the ultimate strength method was again introduced, although on a limited basis, for the design of members subject to axial loads only.

The proposed method of ultimate strength design by the application of the computer is applicable to various combinations of bending and axial load. Ultimate strength of the columns obtained from the proposed method of design have been compared with the ultimate strengths of the columns given in the books, Guide for Ultimate Strength Design of Reinforced Concrete by Charles S. Whitney and Edward Cohen, Ultimate Strength Design of Reinforced Concrete Columns by N. J. Everard and Edward Cohen, and been found in excellent agreement.

This study presents the method in its simplest form with the equations of the analysis, the derivations, the computer program based on the latest Fortran IV language, the tables for the practical use for the designing engineer and the interaction diagrams to aid in their

application. The interaction diagrams themselves demonstrate that their use can be quite a practical tool for a design engineer.

CHAPTER II

THEORY AND BASIC ASSUMPTIONS

The ultimate strength of eccentrically loaded columns is governed by the crushing resistance of concrete, assuming that reinforcing bars are ductile enough to deform at constant yield stress until concrete crushes in compression. ACI-ASCE Committee 327, Ultimate Strength Design, recommends a maximum strain of 0.003 at which concrete may be depended on to resist crushing. With a maximum extreme fiber strain of 0.003 in compression, and the accepted assumption that plane sections before bending remain plane after bending, it is possible for any assumed neutral axis to describe strains at all points of a cross section. The plane of the deformed section at ultimate load is established as that plane passing through the neutral axis (a line of zero strain) and the point of maximum compression strain as shown in

Figure 1.

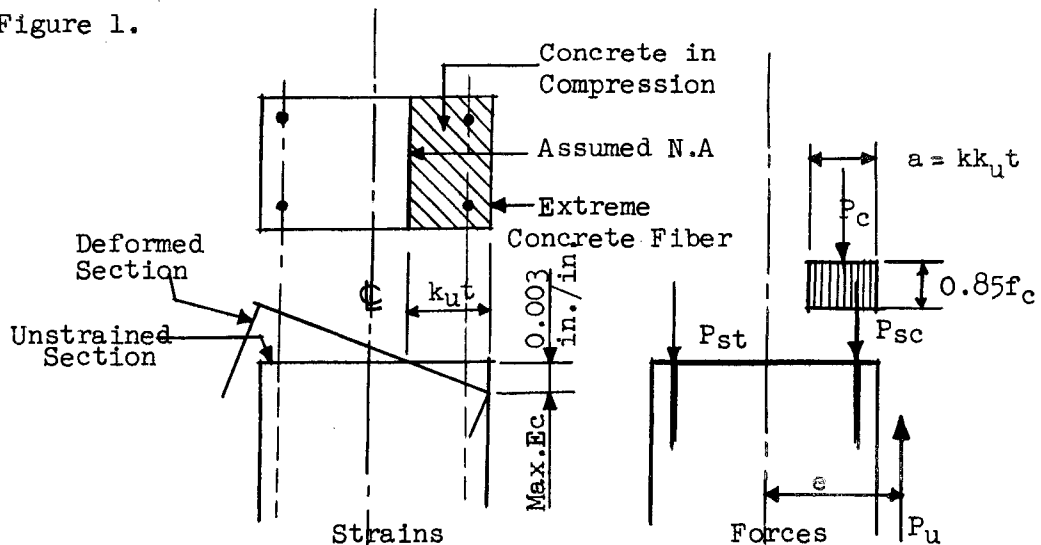


Fig. 1. Strains and resulting forces

With strains at all points established, and with knowledge of stress-strain characteristics for the reinforcement used, it is possible to evaluate the resultant force and moment that created the assumed strain distribution. For example, steel stress f_s equals the product of strain, E_s and Young's modulus E until strains exceed the yield strain of steel. In both tension and compression, for all strains beyond the yield strain, steel stress was assumed constant at the minimum yield stress specified for the type of steel considered. Concrete was assumed to have no tensile strength, and the resultant compression force on concrete was approximated using Whitney's equivalent rectangular stress distribution. Whitney's rectangular stress block is probably the most convenient aid in estimating the volume and locating the centroid for a stress "wedge" of concrete at failure conditions.

In the application of Whitney's rectangular stress distribution, restrictions of Committee 327 were observed in using an average stress of 85 percent of the cylinder strength of concrete ($0.85 f_c'$) and limiting the depth of stress block "a" in the direction of the neutral axis to 85 percent of the distance from the extreme fibre to the neutral axis, ($a=0.85 k_u t$). Forces resulting from an assumed strain distribution are shown in Figure 1. Summation of forces and moments about the center of the column yield the resultant force and moment causing the assumed strain distribution.

If a series of neutral axes are assumed perpendicular to an axis of symmetry, a series of ultimate forces and corresponding ultimate forces and corresponding ultimate moments can be obtained. The resultant moments will each act in the plane of the axis of symmetry. A

graph of the ultimate axial forces against corresponding ultimate moments is shown in Figure 2.

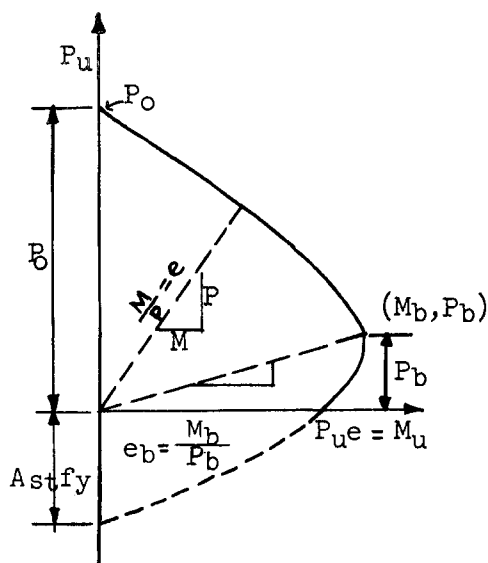


Fig. 2. Typical interaction diagram

In Figure 2, the point P_o is simply the axial load at which the concrete cross section is assumed to crush at all points simultaneously. As the load P_u is moved slightly away from the center of the cross section, the extreme fibers of concrete in the direction of eccentricity will crush before any other points of the cross section fail. Such failures are said to be compression failures. Obviously as eccentricity e increases, P_u must decrease, and M_u which equals the product of P_u and e , increases. There is a particular eccentricity for any cross section and plane of bending at which the extreme fibers of concrete in compression crush at the same instant that the extreme reinforcing bar reaches its yield strength in tension. The load and moment at such an eccentricity are called the balanced load and balanced moment at a balanced eccentricity. As eccentricity is increased beyond the balanced eccentricity, the yielding of reinforcement precedes crushing of

concrete and such failures are considered to be tension failures.

Because ultimate moments may be expressed as an axial load at some eccentricity from the center of the column, the slope with respect to the P_u axis of a line from the origin of the interaction diagram to a point of the diagram is equal to the eccentricity of load at that point of the diagram. At an infinite eccentricity, P_u is zero and the pure bending capacity of the cross section is determined. Then, as a limiting case for resultant P_u in tension, the tension capacity of reinforcing bars at their yield stress in tension establishes the strength of the cross section. The value, $A_{st}f_y$, representing ultimate capacity of reinforced concrete members in pure tension, is meaningless for columns, but it may serve as a convenient point for sketching interaction diagrams.

CHAPTER III

THE ANALYSIS AND THE DERIVATIONS

To present the application of the computer and theory stated in Chapter II, a simple rectangular section with reinforcement only in four corners has been chosen. For simplicity, the side of the section which is always in compression due to load P_u , shall be called compression side and the reinforcement on compression side shall be called compression reinforcement. Similarly the side apposite to compression side shall be called tension side and the reinforcement on tension side shall be called tension reinforcement.

The complete analysis consists of five cases.

Derivations for P_u :

Case 1.

When tension reinforcement is strained beyond yield point as shown in Figure 3, page 8 :

$$P_c = 0.85f'_c k k_u t b$$

$$e_{sc} = e_c (1 - C/k_u t)$$

$$P_{sc} = 0.5 A_s f_{sc}$$

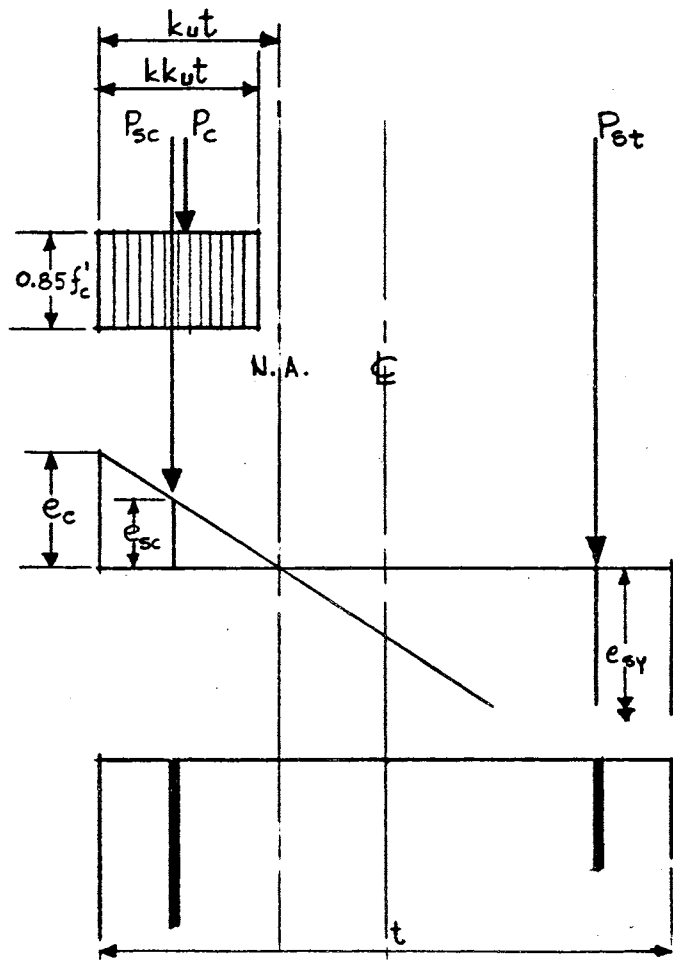
$$= 0.5 A_s e_{sc} E$$

$$= 0.5 A_s e_c E (1 - C/k_u t)$$

$$P_{st} = 0.5 A_s (-f_y)$$

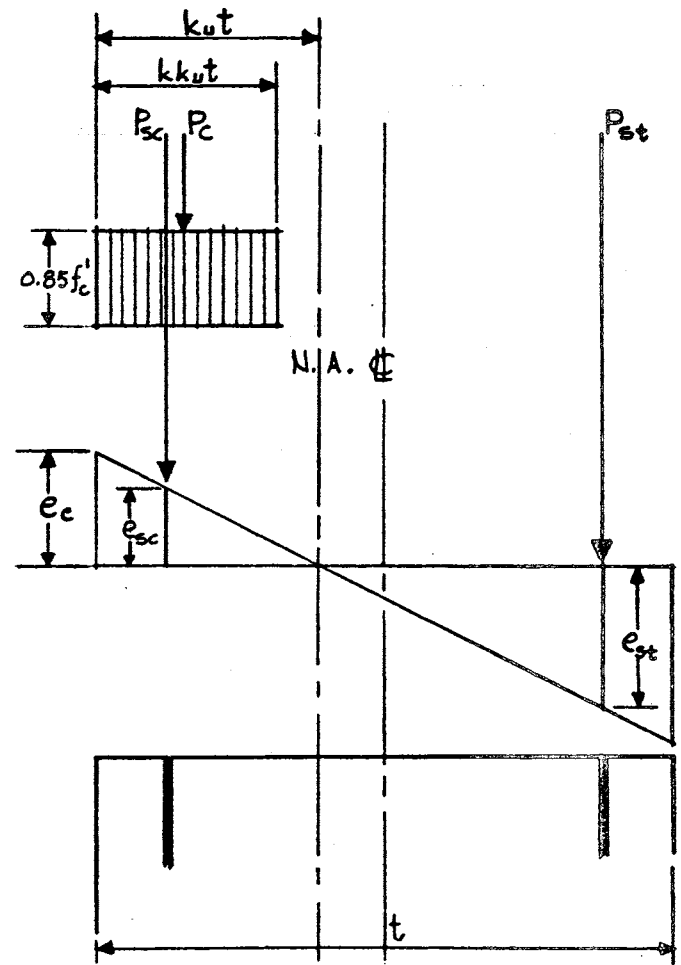
$$= -0.5 A_s f_y$$

$$P_u = P_c + P_{sc} + P_{st}$$



CASE 1

Figure 3



CASE 2

Figure 4

Case 2. (Balanced Design Condition)

When extreme fibers of concrete in compression crush (0.003 in./in.) at the same instant that the tension reinforcement reaches its yield strength, as shown in Figure 4, page 8:

$$P_c = 0.85f_c'k_k k_u t b$$

$$e_c/k_u t = e_{sy}/(t - k_u t - C)$$

$$\begin{aligned} (t-C)/k_u t &= e_{sy}/e_c + 1 \\ &= f_y/e_c E + 1 \\ &= f_y/(0.003 \times 29000) + 1 \\ &= (f_y + 87)/87 \end{aligned}$$

$$k_u t = 87(t-C)/(f_y + 87)$$

$$\begin{aligned} e_{sc} &= e_c(1 - C/k_u t) \\ &= 0.003 \left(\frac{1 - C(f_y + 87)}{87(t-C)} \right) \\ &= 0.003 \left(\frac{87(t-C) - C(f_y + 87)}{87(t-C)} \right) \\ &= 0.0003 \left(\frac{87(t-2C) - C f_y}{87(t-C)} \right) \end{aligned}$$

$$\begin{aligned} P_{sc} &= 0.5A_s f_{sc} \\ &= 0.5A_s e_{sc} E \\ &= 0.5A_s \times 29000 \left[0.003 \left(\frac{87(t-2C) - C f_y}{87(t-C)} \right) \right] \\ &= 0.5A_s \left(\frac{87(t-2C) - C f_y}{(t-C)} \right) \end{aligned}$$

$$\begin{aligned} P_{st} &= 0.5A_s (-f_y) \\ &= -0.5A_s f_y \end{aligned}$$

$$P_u = P_c + P_{sc} + P_{st}$$

$$P_b = P_u$$

Case 3.

When tension reinforcement is in tension but it has not reached the yield point as shown in Figure 5 , page 11:

$$P_c = 0.85f'_c k k_u t b$$

$$e_{sc} = e_c (1 - C/k_u t)$$

$$\begin{aligned} P_{sc} &= 0.5 A_s e_{sc} E \\ &= 0.5 A_s e_c E (1 - C/k_u t) \end{aligned}$$

$$e_{st} = -e_c (1 - (t - C)/k_u t)$$

$$\begin{aligned} P_{st} &= 0.5 A_s f_{st} \\ &= 0.5 A_s e_{st} E \\ &= -0.5 A_s e_c E (1 - (t - C)/k_u t) \end{aligned}$$

$$P_u = P_c + P_{sc} + P_{st}$$

Case 4.

When tension reinforcement is in compression as shown in Figure 6 page 11:

$$P_c = 0.85f'_c k k_u t b$$

$$e_{sc} = e_c (1 - C/k_u t)$$

$$\begin{aligned} P_{sc} &= 0.5 A_s f_{sc} \\ &= 0.5 A_s e_{sc} E \\ &= 0.5 A_s e_c E (1 - C/k_u t) \end{aligned}$$

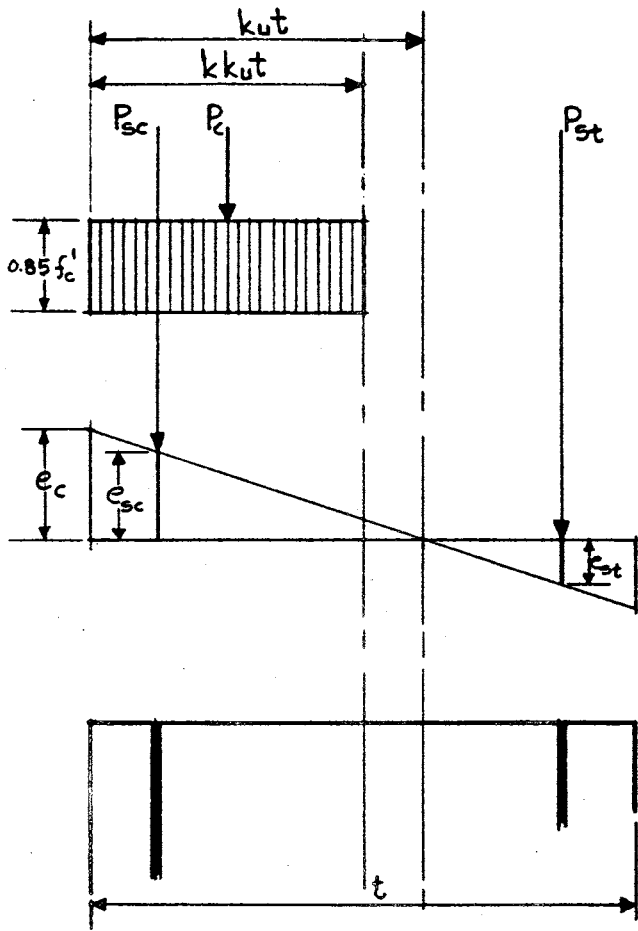
$$e_{st} = e_c (1 - (t - C)/k_u t)$$

$$\begin{aligned} P_{st} &= 0.5 A_s f_{st} \\ &= 0.5 A_s e_{st} E \\ &= 0.5 A_s e_c E (1 - (t - C)/k_u t) \end{aligned}$$

$$P_u = P_c + P_{sc} + P_{st}$$

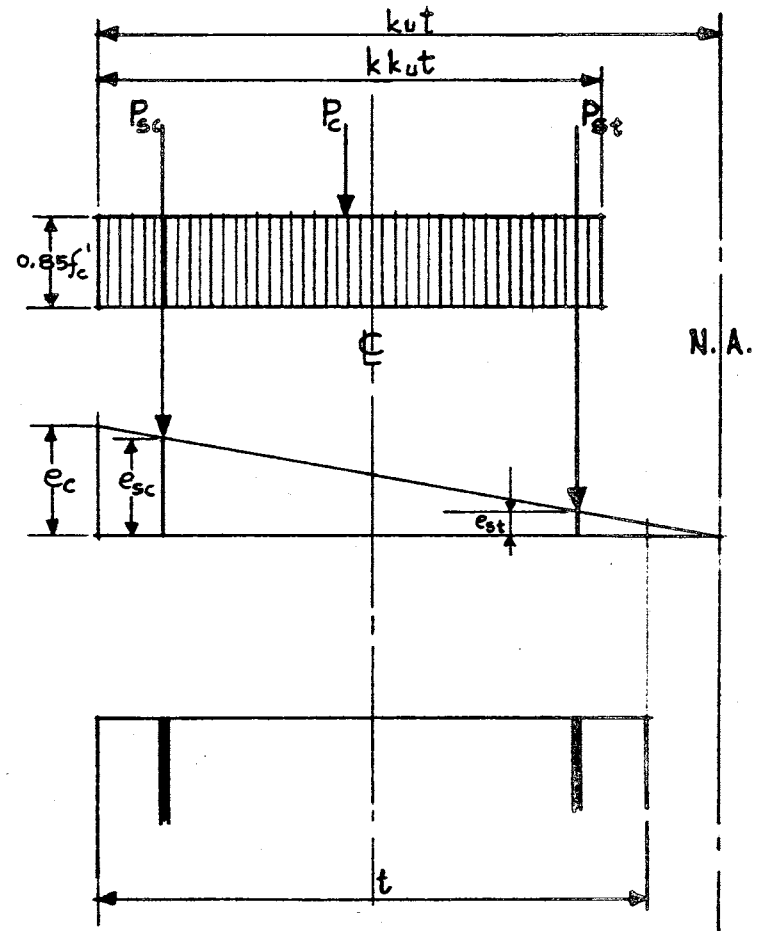
Case 5.

When the concrete section is assumed to crush at all points simultaneously due to an axial load:



CASE 3

Figure 5



CASE 4

Figure 6

$$P_c = 0.85f'_c(bt - A_s)$$

$$P_{sc} = 0.5A_s f_y$$

$$P_{st} = 0.5A_s f_y$$

$$P_u = P_c + P_{sc} + P_{st}$$

$$P_o = P_u = 0.85f'_c(bt - A_s) + A_s f_y$$

Derivation for M_u :

$$X_c = 0.0$$

$$Y_c = 0.5t - 0.5k k_u t$$

$$X_{sc} = 0.0$$

$$Y_{sc} = 0.5t - c$$

$$X_{st} = 0.0$$

$$Y_{st} = 0.5t - c$$

$$e_x = 0.0$$

$$e_y = (P_c Y_c + P_{sc} Y_{sc} - P_{st} Y_{st}) / P_u$$

$$e = \sqrt{(e_x^2 + e_y^2)}$$

$$= e_y$$

$$M_u = P_u \cdot e$$

CHAPTER IV

EXAMPLES

Example 1:

Size of column: 12" X 16"

Reinforcement: 4.00 sq. in., 2.08 percent, 4 - #9 bars

Tie diameter: 0.375 in. Bar diameter: 1.125"

Steel Strength: 40 ksi Concrete Strength: 4.00 ksi

$$\begin{aligned}CL_1 &= 1.5" + \text{tie diameter} \\ &= 1.5" + 0.375" \\ &= 1.875"\end{aligned}$$

$$\begin{aligned}CL_2 &= 1.0 \times \text{bar diameter} \\ &= 1.0 \times 1.125" \\ &= 1.125"\end{aligned}$$

$$CL_1 > CL_2$$

$$\text{Therefore, } CL = CL_1 = 1.875"$$

$$\begin{aligned}C &= CL + 0.5 (\text{bar diameter}) \\ &= 1.875" + 0.562" \\ &= 2.437"\end{aligned}$$

$$t - C = 16.0 - 2.437 = 13.563"$$

$$(t - C) / t = 13.563 / 16 = 0.85$$

$$\begin{aligned}k &= 0.85 - (f'_c - 4.0) \times 0.05 \\ &= 0.85 - (4.0 - 4.0) \times 0.05 \\ &= 0.85\end{aligned}$$

$$e_{sy} = f_y/E = 40.0/29000 = 0.00138 \text{ in./in.}$$

Case 1.

When tension reinforcement is yielded:

$$\text{Try } k_u = 0.40$$

$$\begin{aligned} e_{st} &= e_c(1 - (t-C)/k_{ut}) \\ &= 0.003(1 - 13.563/(0.4 \times 16)) \\ &= 0.003(-1.13) \\ &= -0.00339 < -e_{sy} \end{aligned}$$

Therefore $f_{st} = f_y$ (tension reinforcement has yielded)

$$\begin{aligned} P_c &= 0.85 f'_c k_u t b \\ &= 0.85 \times 4.0 \times 0.85 \times 0.40 \times 16 \times 12 \\ &= 221.95 \text{ kips} \end{aligned}$$

$$\begin{aligned} e_{sc} &= e_c(1 - C/(k_{ut})) \\ &= 0.003(1 - 2.437/(0.4 \times 16)) \\ &= 0.003(0.619) \\ &= 0.001857 \text{ in./in.} > 0.00138 \text{ in./in.} \end{aligned}$$

Therefore $f_{sc} = f_y$

$$\begin{aligned} P_{sc} &= 0.5 A_s f_y \\ &= 0.5 \times 4.0 \times 40.0 \\ &= 80.0 \text{ kips} \end{aligned}$$

$$\begin{aligned} P_{st} &= 0.5 A_s f_y \\ &= 0.5 \times 4.0 \times (-40.0) \\ &= -80.0 \text{ kips} \end{aligned}$$

$$\begin{aligned} P_u &= P_c + P_{sc} + P_{st} \\ &= 221.95 + 80.0 - 80.0 \\ &= 221.95 \text{ kips} \end{aligned}$$

factor of safety = 0.7

$$\begin{aligned} P_u &= 0.7 \times P_u \\ &= 0.7 \times 221.95 \\ &= 155.4 \text{ kips} \end{aligned}$$

$$X_c = 0.0''$$

$$\begin{aligned} Y_c &= 0.5t - 0.5 k k_u t \\ &= 0.5 \times 16 - 0.5 \times 0.85 \times 0.40 \times 16 \\ &= 5.28'' \end{aligned}$$

$$X_{sc} = 0.0''$$

$$\begin{aligned} Y_{sc} &= 0.5t - c \\ &= 0.5 \times 16 - 2.437 \\ &= 5.563'' \end{aligned}$$

$$X_{st} = 0.0''$$

$$\begin{aligned} Y_{st} &= 0.5t - c \\ &= 0.5 \times 16 - 2.437 \\ &= 5.563'' \end{aligned}$$

$$e_x = 0.0''$$

$$\begin{aligned} e_y &= (P_c Y_c + P_{sc} Y_{sc} - P_{st} Y_{st}) / P_u \\ &= (221.95 \times 5.28 + 80.0 \times 5.563 - (-80.0) \times 5.563) / 221.95 \\ &= 9.29'' \end{aligned}$$

$$e = e_y$$

$$\begin{aligned} M_u &= P_u \cdot e \\ &= (221.95 \times 9.29) / 12 \\ &= 171.9 \text{ ft. kips} \end{aligned}$$

factor of safety = 0.7

$$\begin{aligned} M_u &= 0.7 M_u \\ &= 0.7 \times 171.9 \\ &= 120.3 \text{ ft. kips} \end{aligned}$$

Case 2.

When tension reinforcement is just reached to the yield point and maximum extreme fibers strain in the compressed concrete is reached to 0.003 in./in.

$$e_c/k_u t = e_{sy}/(t-k_u t - C)$$

$$\begin{aligned} (t-C)/k_u t &= e_{sy}/e_c + 1 \\ &= f_y/e_c E + 1 \\ &= f_y (0.003 \times 29000) + 1 \\ &= (f_y + 87)/87 \end{aligned}$$

$$\begin{aligned} k_u t &= 87(t-C)/(f_y + 87) \\ &= 87(16-2.437)/(40+87) \\ &= 9.29" \end{aligned}$$

$$\begin{aligned} P_c &= 0.85 f'_c k k_u t b \\ &= 0.85 \times 4.0 \times 0.85 \times 9.29 \times 12 \\ &= 322.1 \text{ kips} \end{aligned}$$

$$\begin{aligned} e_{sc} &= e_c (1-C/k_u t) \\ &= 0.003(1-2.437/9.29) \\ &= 0.0022 > e_{sy} \end{aligned}$$

$$f_{sc} = f_y$$

$$\begin{aligned} P_{sc} &= 0.5 A_s f_y \\ &= 0.5 \times 4.0 \times 40.0 \\ &= 80.0 \text{ kips} \end{aligned}$$

$$\begin{aligned} P_{st} &= 0.5 A_s (-f_y) \\ &= 0.5 \times 4.0 \times (-40.0) \\ &= -80.0 \text{ kips} \end{aligned}$$

$$\begin{aligned} P_u &= P_c + P_{sc} + P_{st} \\ &= 322.1 + 80.0 - 80.0 \\ &= 322.1 \text{ kips} \end{aligned}$$

factor of safety = 0.7

$$P_u = 0.7 P_u$$

$$= 0.7 \times 322.1$$

$$= 225.47 \text{ kips}$$

$$X_c = 0.0''$$

$$Y_c = 0.5t - 0.5 k k_u t$$

$$= 0.5 \times 16 - 0.5 \times 0.85 \times 9.29$$

$$= 4.05''$$

$$X_{sc} = 0.0$$

$$Y_{sc} = 0.5t - c$$

$$= 0.5 \times 16 - 2.437$$

$$= 5.563''$$

$$X_{st} = 0.0''$$

$$Y_{st} = 0.5t - c$$

$$= 0.5 \times 16 - 2.437$$

$$= 5.563''$$

$$e_x = 0.0''$$

$$e_y = (P_c Y_c + P_{sc} Y_{sc} - P_{st} Y_{st}) / P_u$$

$$= (322.1 \times 4.05 + 80.0 \times 5.563 + 80.0 \times 5.563) / 322.1$$

$$= 6.88''$$

$$e = e_y$$

$$= 6.88''$$

$$M_u = P_u \cdot e$$

$$= (322.1 \times 6.88) / 12$$

$$= 184.5 \text{ ft. kips}$$

factor of safety = 0.7

$$\begin{aligned}
 M_u &= 0.7 M_u \\
 &= 0.7 \times 184.5 \\
 &= 128.1 \text{ ft kips}
 \end{aligned}$$

Case 3.

When tension reinforcement is in tension but it has not reached the yield point.

$$\text{Try } k_u = 0.80$$

$$\begin{aligned}
 e_{st} &= e_c(1-(t-C)/k_u t) \\
 &= 0.003(1-13.563/(0.8 \times 16)) \\
 &= -0.00018 > -e_{sy}
 \end{aligned}$$

$$\begin{aligned}
 P_c &= 0.85f'_c k_u t b \\
 &= 0.85 \times 4.0 \times 0.85 \times 0.80 \times 16 \times 12 \\
 &= 443.9 \text{ kips}
 \end{aligned}$$

$$\begin{aligned}
 e_{sc} &= e_c(1-C/(k_u t)) \\
 &= 0.003(1-2.437/(0.80 \times 16)) \\
 &= 0.003(0.81) \\
 &= 0.00243 > e_{sy}
 \end{aligned}$$

Therefore, $f_{sc} = f_y$

$$\begin{aligned}
 P_{sc} &= 0.5 \times 4.0 \times 40.0 \\
 &= 80.0 \text{ kips}
 \end{aligned}$$

$$\begin{aligned}
 P_{st} &= 0.5 A_s f_{st} \\
 &= 0.5 \times 4.0 \times (-0.00018)(29000) \\
 &= -10.45 \text{ kips}
 \end{aligned}$$

$$\begin{aligned}
 P_u &= P_c + P_{sc} + P_{st} \\
 &= 443.9 + 80.0 + (-10.45) \\
 &= 513.45 \text{ kips}
 \end{aligned}$$

factor of safety = 0.7

$$\begin{aligned}
 P_u &= 0.7 P_u \\
 &= 0.7 \times 513.45 \\
 &= 359.42 \text{ kips}
 \end{aligned}$$

$$X_c = 0.0''$$

$$\begin{aligned}
 Y_c &= 0.5t - 0.5 k k_u t \\
 &= 0.5 \times 16 - 0.5 \times 0.85 \times 0.8 \times 16 \\
 &= 8.0 - 5.44 \\
 &= 2.56''
 \end{aligned}$$

$$X_{sc} = 0.0''$$

$$\begin{aligned}
 Y_{sc} &= 0.5t - c \\
 &= 0.5 \times 16 - 2.437 \\
 &= 5.563''
 \end{aligned}$$

$$X_{st} = 0.0''$$

$$\begin{aligned}
 Y_{st} &= 0.5t - c \\
 &= 0.5 \times 16 - 2.437 \\
 &= 5.563''
 \end{aligned}$$

$$e_x = 0.0''$$

$$\begin{aligned}
 e_y &= (P_c Y_c + P_{sc} Y_{sc} - P_{st} Y_{st}) / P_u \\
 &= (443.9 \times 2.56 + 80.0 \times 5.563 - (-1045)(5.563)) / 513.45 \\
 &= 3.19''
 \end{aligned}$$

$$\begin{aligned}
 e &= e_y \\
 &= 3.19''
 \end{aligned}$$

$$\begin{aligned}
 M_u &= P_u \cdot e \\
 &= (513.45 \times 3.19) / 12 \\
 &= 136.5 \text{ ft. kips}
 \end{aligned}$$

$$\text{factor of safety} = 0.7$$

$$\begin{aligned}
 M_u &= 0.7 M_u \\
 &= 0.7 \times 136.5 \\
 &= 95.55 \text{ ft. kips}
 \end{aligned}$$

Case 4.

When tension reinforcement is in compression or when neutral axis lies outside the concrete column section.

$$\text{Try } k_u = 0.90$$

$$\begin{aligned}
 e_{st} &= e_c(1 - (1 - C)/k_u t) \\
 &= 0.003(1 - 13.563/(0.90 \times 16)) \\
 &= 0.003(1 - 0.94) \\
 &= 0.00018 < e_{sy}
 \end{aligned}$$

$$\begin{aligned}
 P_c &= 0.85 f_c k_u t b \\
 &= 0.85 \times 4.0 \times 0.85 \times 0.90 \times 16 \times 12 \\
 &= 499.4 \text{ kips}
 \end{aligned}$$

$$\begin{aligned}
 e_{sc} &= e_c(1 - C/(k_u t)) \\
 &= 0.003(102.437/(0.9 \times 16)) \\
 &= 0.003 \times 0.831 \\
 &= 0.002493 > e_{sy}
 \end{aligned}$$

Therefore, $f_{sc} = f_y$

$$\begin{aligned}
 P_{sc} &= 0.5 A_s f_y \\
 &= 0.5 \times 4.0 \times 40.0 \\
 &= 80.0 \text{ kips}
 \end{aligned}$$

$$\begin{aligned}
 P_{st} &= 0.5 A_s f_{st} \\
 &= 0.5 \times 4.0 \times (0.00018)(29000) \\
 &= 10.45 \text{ kips}
 \end{aligned}$$

$$\begin{aligned}
 P_u &= P_c + P_{sc} + P_{st} \\
 &= 499.4 + 80.0 + 10.45 \\
 &= 589.85 \text{ kips}
 \end{aligned}$$

factor of safety = 0.7

$$P_u = 0.7 P_u$$

$$= 0.7 \times 589.85$$

$$= 412.6 \text{ kips}$$

$$X_c = 0.0''$$

$$Y_c = 0.5t - 0.5 k_{ut}$$

$$= 0.5 \times 16 - 0.5 \times 0.85 \times 0.90 \times 16$$

$$= 1.88''$$

$$X_{sc} = 0.0''$$

$$Y_{sc} = 0.5t - C$$

$$= 0.5 \times 16 - 2.437$$

$$= 5.563''$$

$$X_{st} = 0.0''$$

$$Y_{st} = 0.5t - C$$

$$= 0.5 \times 16 - 2.437$$

$$= 5.563''$$

$$e_x = 0.0''$$

$$e_y = (P_c Y_c + P_{sc} Y_{sc} - P_{st} Y_{st}) / P_u$$

$$= (499.4 \times 1.88 + 80.0 \times 5.563 - 10.45 \times 5.563) / 589.85$$

$$= 2.245''$$

$$e = e_y$$

$$= 2.245''$$

$$M_u = P_u \cdot e$$

$$= (589.85 \times 2.245) / 12$$

$$= 110.5 \text{ ft. kips}$$

factor of safety = 0.7

$$\begin{aligned}
 M_u &= 0.7 M_u \\
 &= 0.7 \times 110.5 \\
 &= 77.3 \text{ ft. kips}
 \end{aligned}$$

Case 5.

When compression and tension both reinforcement are in compression at yield point and the whole concrete section is compressed to the 0.003 in./in. strain, i.e., when column is under concentric ultimate load.

$$P_c = 0.85f'_c(bt - A_s)$$

$$P_{sc} = 0.5A_s f_y$$

$$P_{st} = 0.5 A_s f_y$$

$$P_u = P_c + P_{sc} + P_{st}$$

$$= 0.85 \times 4.0 (12 \times 16 - 4.0) + 0.5 \times 4.0 \times 40.0 + 0.5 \times 4.0 \times 40.0$$

$$= 799.2 \text{ kips}$$

$$P_o = P_u = 799.2 \text{ kips}$$

factor of safety = 0.7

$$P_o = 0.7 P_o$$

$$= 0.7 \times 799.2$$

$$= 559.44 \text{ kips}$$

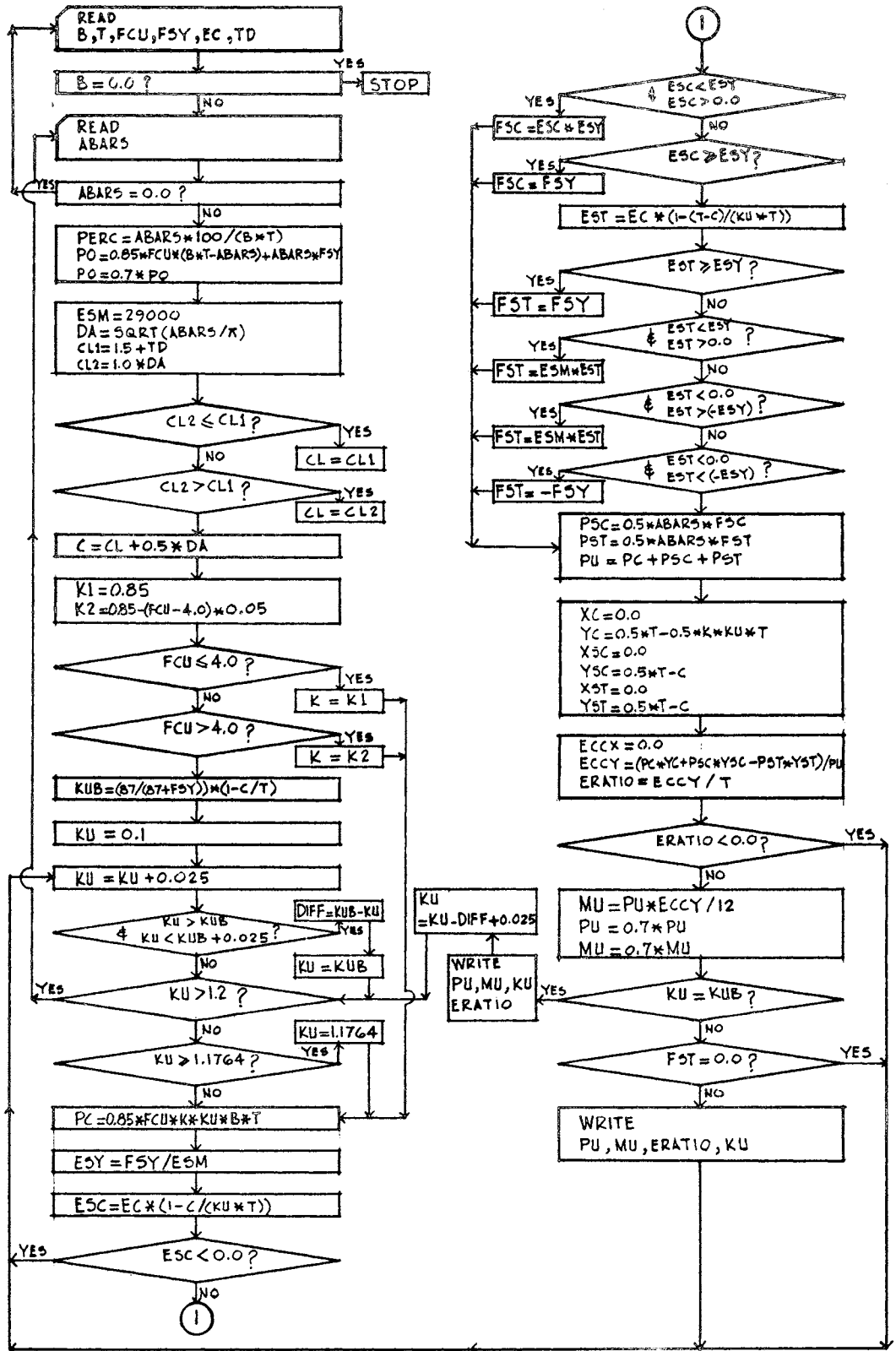
CHAPTER V

COMPUTER PROGRAMS

The computer is a very useful device in exploring the alternatives such as finding the strain in the compression and tension reinforcement of the reinforced concrete section under the set conditions and alternatives.

The procedure used in programming for the computer is the same as the procedure used in the derivation of the formulas in Chapter III. In programming for the computer all the different cases have been put together in the form of the equations with the logical and arithmetical "IF" statements in such a way that the computer itself finds the correct value of the unknowns by exploring into the alternatives set under some conditions. It finds the value of CL , coefficient k , strain in compression reinforcement Esc , strain in tension reinforcement Est , yield strains of reinforcement, the location of neutral axis for the balanced design condition, and the condition of stresses in the tension reinforcement with the use of logical and arithmetical "IF" statements.

The input that is required for the computer is the dimensions of the column section, ultimate compressive strength of concrete, yield strength of reinforcement, ultimate strain of concrete, diameter of the tie, amount of the reinforcement, and the modulus of elasticity of steel. With this data, and with a set location of neutral axis known, the computer will calculate all the required values, the value of the



FLOW DIAGRAM FOR COMPUTER PROGRAM

Figure 7

TABLE I

THE VALUES OF P_u AND M_u FOR THE VARIOUS LOCATIONS OF NEUTRAL AXIS

SIZE OF COLUMN 12. BY 16.
 STEEL STRENGTH 40.00KSI CONCRETE STRENGTH 4.00KSI
 REINFORCEMENT 2.08PERC. 4.00SQ.IN. 4-#9
 TIE DIAMETER 0.375 IN
 MAXIMUM CONCENTRIC AXIAL LOAD, E=0.0 559.4 KIPS

P _U -KIPS	M _U -FT.KIPS	E/T		KU
27.7	71.8	1.95	TENS	0.1750
50.6	82.4	1.22	TENS	0.2000
70.7	91.3	0.97	TENS	0.2250
88.6	99.0	0.84	TENS	0.2500
105.1	105.7	0.75	TENS	0.2750
116.5	109.8	0.71	TENS	0.3000
126.2	112.8	0.67	TENS	0.3250
135.9	115.6	0.64	TENS	0.3500
145.7	118.1	0.61	TENS	0.3750
155.4	120.3	0.58	TENS	0.4000
165.1	122.2	0.56	TENS	0.4250
174.8	123.9	0.53	TENS	0.4500
184.5	125.2	0.51	TENS	0.4750
194.2	126.3	0.49	TENS	0.5000
203.9	127.2	0.47	TENS	0.5250
213.6	127.7	0.45	TENS	0.5500
223.3	128.0	0.43	TENS	0.5750
225.5	128.0	0.43	YIFLD	0.5806 **
255.4	121.9	0.36	TENS	0.6250
271.5	118.4	0.33	TENS	0.6500
287.0	114.9	0.30	TENS	0.6750
302.2	111.3	0.28	TENS	0.7000
317.0	107.5	0.25	TENS	0.7250
331.5	103.7	0.23	TENS	0.7500
345.6	99.7	0.22	TENS	0.7750
359.5	95.6	0.20	TENS	0.8000
373.1	91.3	0.18	TENS	0.8250
386.5	86.9	0.17	COMP	0.8500
399.7	82.2	0.15	COMP	0.8750
412.7	77.4	0.14	COMP	0.9000
425.5	72.4	0.13	COMP	0.9250
438.1	67.2	0.12	COMP	0.9500
450.6	61.8	0.10	COMP	0.9750
463.0	56.2	0.09	COMP	1.0000
475.2	50.4	0.08	COMP	1.0250
487.3	44.3	0.07	COMP	1.0500
499.3	38.0	0.06	COMP	1.0750
511.2	31.5	0.05	COMP	1.1000
523.0	24.8	0.04	COMP	1.1250
534.7	17.8	0.02	COMP	1.1500
546.3	10.6	0.01	COMP	1.1750
547.0	10.2	0.01	COMP	1.1764

**BALANCED DESIGN CONDITION

TABLE II

THE VALUES OF P_u AND M_u FOR THE VARIOUS LOCATIONS OF NEUTRAL AXIS

SIZE OF COLUMN 12. BY 16.
 STEEL STRENGTH 40.00KSI CONCRETE STRENGTH 4.00KSI
 REINFORCEMENT 3.25PERC. 6.24SQ.IN. 4-*11
 TIE DIAMETER 0.375 IN
 MAXIMUM CONCENTRIC AXIAL LOAD, E=0.0 516.8 KIPS

PU-KIPS	MU-FT.KIPS	F/T		KU
27.2	99.1	2.74	TENS	0.2000
53.9	110.9	1.54	TENS	0.2250
77.2	120.9	1.17	TENS	0.2500
98.1	129.5	0.99	TENS	0.2750
116.5	136.8	0.88	TENS	0.3000
126.2	139.8	0.83	TENS	0.3250
135.9	142.6	0.79	TENS	0.3500
145.7	145.1	0.75	TENS	0.3750
155.4	147.3	0.71	TENS	0.4000
165.1	149.2	0.68	TENS	0.4250
174.8	150.9	0.65	TENS	0.4500
184.5	152.3	0.62	TENS	0.4750
194.2	153.4	0.59	TENS	0.5000
203.9	154.2	0.57	TENS	0.5250
213.6	154.8	0.54	TENS	0.5500
223.2	155.0	0.52	YIELD	0.5746 **
244.8	149.7	0.46	TENS	0.6000
265.1	144.7	0.41	TENS	0.6250
284.6	139.7	0.37	TENS	0.6500
303.4	134.8	0.33	TENS	0.6750
321.6	129.9	0.30	TENS	0.7000
339.1	125.0	0.28	TENS	0.7250
356.2	120.0	0.25	TENS	0.7500
372.7	115.0	0.23	TENS	0.7750
388.9	109.9	0.21	TENS	0.8000
404.6	104.7	0.19	TENS	0.8250
420.0	99.4	0.18	COMP	0.8500
435.1	94.0	0.16	COMP	0.8750
449.9	88.4	0.15	COMP	0.9000
464.4	82.7	0.13	COMP	0.9250
478.6	76.8	0.12	COMP	0.9500
492.6	70.7	0.11	COMP	0.9750
506.4	64.5	0.10	COMP	1.0000
520.0	58.0	0.08	COMP	1.0250
533.4	51.4	0.07	COMP	1.0500
546.7	44.6	0.06	COMP	1.0750
559.7	37.6	0.05	COMP	1.1000
572.7	30.4	0.04	COMP	1.1250
585.5	22.9	0.03	COMP	1.1500
598.1	15.3	0.02	COMP	1.1750
598.8	14.8	0.02	COMP	1.1764

**BALANCED DESIGN CONDITION

total compression resisted by concrete, strains and stresses in the reinforcement, total compression and tension resisted by the reinforcement and thus ultimate load capacity of the section by using the formulas given in the program. To find the ultimate moment capacity of the section for the location of the neutral axis, the computer calculates the values of the X and Y ordinates of all the forces acting on the section and then it finds the eccentricity of the ultimate load from the given formulas and thus the ultimate moment which is the product of the ultimate load P_u and the eccentricity e .

Fortran IV language has been used in the program for the computer IBM 7040. The Flow Diagram for the Program is shown on page 24, and the symbols used in the program are defined under the List of Symbols. The tables of the results obtained from the execution of the program are shown on the page 25, as samples and some of the values of the ultimate loads and ultimate moments have been compared with the values of ultimate loads and ultimate moments obtained from the examples done by hand calculations in Chapter IV.

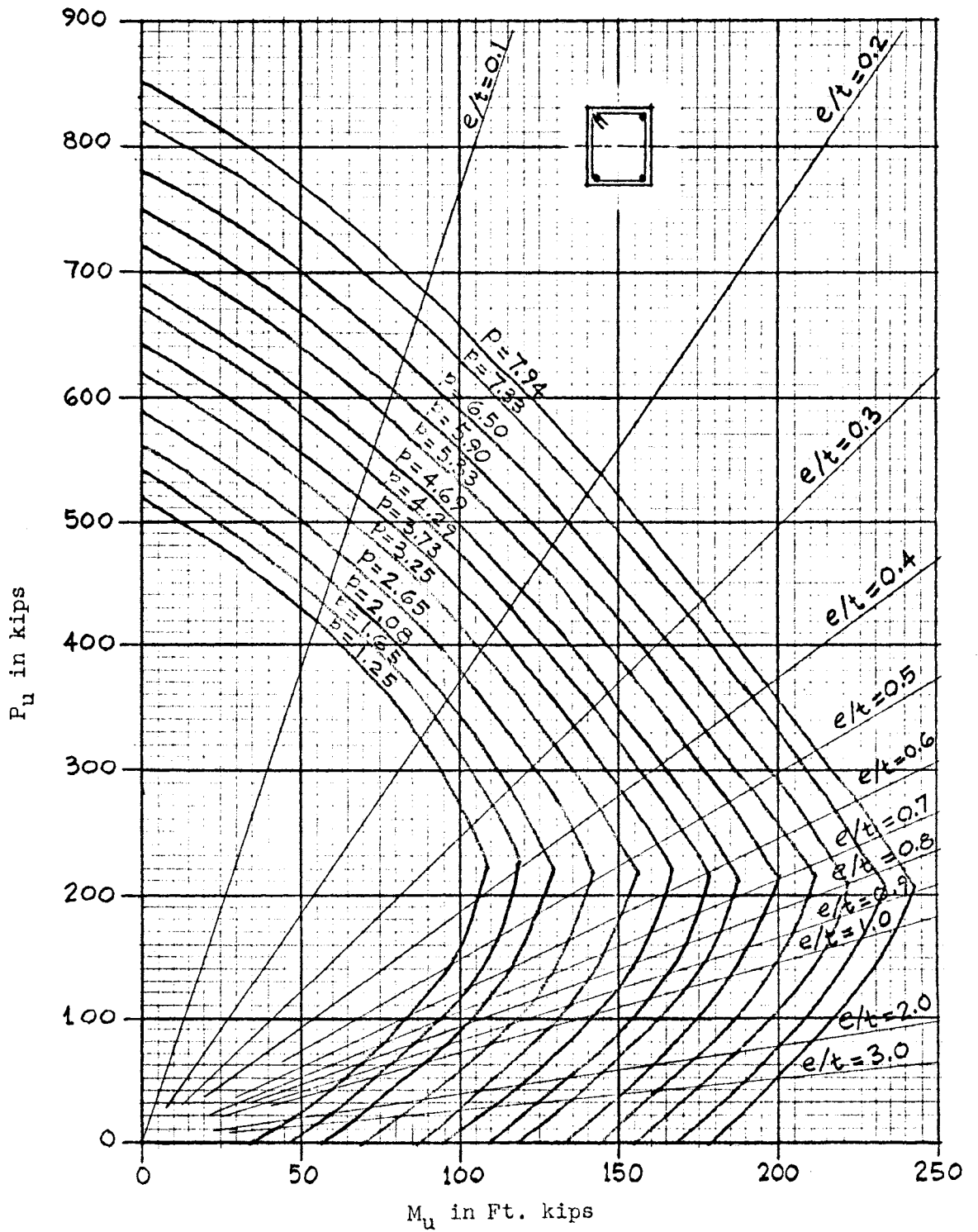
CHAPTER VI

INTERACTION DIAGRAMS

After the execution of the computer program and the values of ultimate load and ultimate moment are obtained as shown on page 25, interaction diagrams have been plotted as shown in Figure 8. These interaction diagrams describe column capacity for bending in one particular plane.

In evaluating points for an interaction diagram, a cross section of 12" X 16" has been used. Using a concrete cover of 1.5" plus the tie diameter, cross sections with bars in four corners have been studied allowing variations of f_c , f_y and steel percentage. Cylinder strengths of 3 ksi and 4 ksi and 5 ksi have been used and yield strengths of 40 ksi and 50 ksi have been used.

Data for interaction diagrams, P_u and M_u values, has been accumulated from the computer for a series of eccentricities as shown in the table on page 25. To plot the interaction diagram, the computer with the Plotter is fed with a Fortran program and the data cards with values of P_u and M_u . The interaction diagrams obtained from the Plotter are shown in Figure 8. These diagrams are very useful and a convenient tool for the ultimate strength design of columns. With the values of P_u and M_u known for a certain column, we can enter into the interaction diagram and find approximately the required percentage of reinforcement for the column. Interaction diagrams indicat-



Column Size 12" X 16"

Concrete Strength 4 ksi

Steel Strength 40 ksi

Figure 8. Interaction Diagrams

ing "balanced design condition", "compression zone" and "tension zone" can be an excellent guide to the design engineer to furnish him all the information that he needs for the design of the column.

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