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A MULTIPLE DISCRIMINANT ANALYSIS OF

TECHNICAL INDICATORS ON THE NYSE

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A MULTIPLE DISCRIMINANT ANALYSIS

OF TECHNICAL INDICATORS

ON THE NYSE

The random walk hypothesis states that no statistically significant correlation exists between past public information and future price changes. The efficient market theory states that past public information cannot be used to obtain extraordiary returns on a consistent basis. This dissertation employs Multiple Discriminant Analysis (MDA) to study the relationship between technical indicators and future market movements on the NYSE. Thy null hypothesis is that no statistically significant relationship exists between future market actions and past technical indicators. If the model shows statistically significant dependence such that the null hypothesis is rejected, then the predictive power of MDA is examined in an attempt to use the dependence to formulate profitable trading models.

The data employed include daily technical indicators from 1962 through 1973 as derived from NYSE transactions. The technical indicators include volume; the proportion of stocks advancing, declining, and remaining unchanged; odd-lot purchase sales, and short-sales; new highs and lows; average volume; money flow; and the advance/decline line. The data are adjusted for growth in the NYSE over time. The indicators listed above are the input data for the MDA model. Specifically, two sets of data are employed to test the null hypotheses, a "Regular" variable set consisting of the technical indicators outlined above, and

a "Percentage" variable set consisting of percentage changes of the technical indicators.

The theory of MDA is examined extensively. The topics described and later employed in this study include statistical tests of significance, assumptions of the model, classification procedures, stepwise analysis, bias, relative importance of individual variables, and effects of multicollinearity.

The application of MDA to technical indicators involves examining the equality of the variance-covariance matrices by Box's test. If the matrices are significantly different, then a non-linear discriminant function is applicable. In addition, a complete stepwise procedure is theoretically superior to a forward or backward stepwise technique. Other important considerations include testing the discreteness of the criterion variable and correctly adjusting for the effects of bias. The MULDIS computer program developed at the University of Wisconsin is used to implement the MDA technique.

Initially, the data are segregated according to bull- and bear-market periods. The results show that a statistical and classificatory dependence does exist between past technical indicators and future market movements for the original sample. Also, the test for dispersion equality shows that a quadratic MDA function is appropriate. However, when secondary samples are classified the discriminant function is found to be non-stationary over time. To examine the non-stationarity condition the Lachenbruch holdout method is employed; the data are found to be stationary for one-year periods. The one-year data samples show statistical and classificatory dependence for both the original and secondary samples.

The discreteness of the price relatives is examined by comparing classification results from MDA to the results from a multiple regression model. The MDA model is superior. Finally, various trading rules are developed and tested on the yearly samples. The trading rules provide extraordinary returns before commissions but inferior returns after commissions.

Dedicated to:

Wendy,

my parents,

and my special friends.

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A MULTIPLE DISCRIMINANT ANALYSIS OF TECHNICAL INDICATORS ON THE NYSE

CHAPTER I

INTRODUCTION

The use of technical indicators to forecast market movements is a controversial topic in the academic and investment communities. Most academicians argue that price movements form a random walk and/or efficient market with respect to technical information, while practitioners claim that proper use of technical indicators results in the recognition of price patterns that enable superior returns to be achieved relative to a buy and hold criterion. Previous studies of technical indicators have concentrated on statistical dependence tests and univariate analysis, while almost completely ignoring trading models and multivariate analyses. This study corrects these and other deficiencies of past research by investigating the relationship between daily technical indicators and future market movements in the context of a multivariate-trading model.

Previous Research

Academicians use economic theory to argue that supply and demand forces should be in equilibrium. When an exogenous event creates a disequilibrium, the market then adjusts rapidly to restore an equilibrium position. Because of the speed of adjustment of the market mechanism, little or no opportunity exists for investors to profit from disequilibrium situations, even if they are recognized. In contrast, practitioners argue that supply and demand considerations, via technical indicators, present information concerning investors' psychological moods, and can be employed to predict short-term trends.

The academic position follows from research claiming that prices follow a random walk and/or efficient market model.[1] This model has been vigorously examined for the "narrow" view, i.e., past price information only, but only recently substantial interest has arisen concerning the "broad" view, i.e., the use of all past public information. In fact, studies of the "broad" view have mainly concentrated on fundamental information such as dividends, earnings and stock splits. The limited number of "broad" view studies that have dealt with technical indicators have mainly been concerned with statistical dependence and investigations of statistical dependence; moreover, as mentioned above, these studies are univariate in nature.

The technical studies to date concerning volume, odd-lot data, and the number of advances/declines/unchanged shares generally show that some statistically significant dependence exists between the individual technical indicators and price movements. Theil and Leenders [A] and Fama [A 1965a] investigated advance/decline/unchanged data on the Amsterdam and New York Stock Exchanges, respectively, and found some

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[1]Simply stated, a "random walk" occurs when future price changes are independent from past information; an "efficient market" is when extraordinary profits cannot consistently be obtained from past public information after all business costs are considered.

dependence. Similarly, Wu [A 1972], Kewley and Stevenson [A 1969] and Klein [A] found some relationships between odd-lot data and price changes, while Crouch [A 1970a and A 1970b] and Ying [A 1966] supported the relationship of volume to prices. For the most part, broad view tests of market efficiency, whether based on fundamental or technical data, have ignored the trading model approach.

Emery [A] provided the only attempt to analyze short-term technical indicators in a multivariate context. Emery employed a factor analysis/regression model to forecast future price changes and to make investment decisions, determining that the returns after commissions are inferior to a buy and hold policy.

Unfortunately, Emery's methodology was weak in several areas, inviting criticisms that cast doubt on the validity of the approach and hence on the results. Furthermore, Emery's data base contained a substantial number of major errors. Evaluation of Emery's work is developed in Chapter II.

Purpose and Methodology of the Study

The foregoing comments suggests that a disagreement exists between academicians and practitioners concerning the ability of technical indicators to forecast market movements. This study attempts to determine the relationship, if any, between these indicators and future market movements. To achieve this goal, the technical data are examined by various statistical tests and trading models. The statistical tool employed is multiple discriminant analysis. Since discriminant analysis is multivariate in nature, it should provide superior results to previous studies that have been mostly univariate in nature.

Multiple Discriminant Analysis (MDA) is a statistical technique that maximizes the separation between two (or more) a priori groups. This separation is accomplished by maximizing the ratio of the between group variation to the within group variation. The resulting parameters of the discriminant equation reflect this maximizing procedure, providing the best statistical separation of the groups as well as the optimal classification of observations into their respective groups.

MDA has been employed by several authors to investigate financial variables, with Altman's articles [B 1968 and B 1973] being the most widely known examples. The present paper is the initial application of MDA to technical data. Furthermore, the methodology in this study has several unique aspects. Specifically, it examines the equality of the variance-covariance matrix, classifies the observations by a non-linear process if the foregoing matrices are unequal, and provides a complete stepwise option. These procedures help to alleviate both academic and practitioner criticisms; academic criticisms are met by investigations of the underlying assumptions of the model, and practitioner criticisms are met by providing a more realistic model of the environment, namely, a multivariate non-linear model capable of investigating extreme price movements.[2]

The technical data employed in this study are volume; proportions of stocks advancing/declining/unchanged; odd-lot purchases, sales, and short-sales; the number of new highs and new lows; and several other variables normally used by practitioners, such as money flow. The data

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[2]Extreme price movements are defined as the highest and lowest price relatives for a given period of analysis. According to technicians, these extreme price relatives are more likely to be associated with unusual movements in technical indicators than are "average" price relatives.

are analyzed in a two-group context on a daily basis, with the initial phase of the investigation concentrating on bull and bear markets; a second phase of the analysis employs yearly clusterings of the daily data.

The two groups consist of the 50-best and 50-worst price relative observations from the time period in question. Each data set is analyzed in the following manner:

1) The statistical relationship between technical indicators and future market movements is investigated
2) The ability of the model to predict (classify) future market movements based on past technical information is examined
3) The ability of the model to classify successfully observations from secondary samples is tested
4) If classification dependence is evident, then a trading rule based on a reduced space MDA formulation is employed to determine if the model obtains above average returns before commissions
5) If (4) is true, then the same trading rule is examined for above average returns after commissions

Organization of the Study

Chapter II briefly explains the various aspects of the random walk and efficient market models as well as reviewing the literature involving technical indicators. Chapter III presents an integrated review of the theory of multiple discriminant analysis. Chapter IV discusses the methodology of the current investigation. Chapter V analyzes the results concerning the relationship of technical indicators to future market movements. Chapter VI presents a short summary of the methodology and results, gives the conclusions based on the results, and provides implications and suggestions for future research efforts.

CHAPTER II

THE EFFICIENT MARKET HYPOTHESIS AND TECHNICAL INDICATOR STUDIES

Introduction

This chapter briefly reviews the Random Walk and Efficient Market Hypotheses and then evaluates the methodology and results of various studies dealing with technical indicators. An in-depth theoretical approach to the random walk/efficient market theories is provided by Fama [A 1970], while Fielitz [A 1974] provided an extensive review of the literature of the efficient market hypothesis. Pinches [A] summarized the literature involving technical analysis.

The Random Walk and Efficient Market Theories

The random walk hypothesis indicates that no statistical dependence exists between past information and future price changes, while the efficient market theory says that trading models cannot provide extraordinary profits on a consistent basis. (Extraordinary profits occur when a trading model outperforms a buy and hold strategy.)

Traditionally, the above hypotheses are divided into three areas of investigation, namely the narrow, broad, and monopolistic information forms of the two theories. The narrow-form tests examine the relationship between past price information and future price changes; the broad-form tests determine the relationship between all past information and future price changes; while the monopolistic information tests investigate the possibility that certain groups possess information or special abilities that will provide extraordinary returns on a consistent basis.

Narrow view

Initial investigations of the random walk hypothesis employed serial correlation (see Kendall [A], Cowles [A], Moore [A] and Fama [A 1965a]), runs tests (Alexander [A 1961], Moore [A], and Fama [A 1965a]), and spectral analysis (Granger and Morgenstern [A 1963] and Godfrey, Granger, and Morgenstern [A]). These and other investigations (clustering, Markovian) revealed some minor statistical dependence, indicating that a strict mathematicical interpretation of the random walk hypothesis is not valid. However, the uncovered dependencies are at best weak.

Criticisms of the above studies included the linear nature of the tests and the investigation of dependence throughout the data while ignoring subsets of the data where dependence is more likely to occur. Alexander [A 1961 and A 1964] directed himself to such criticisms by developing a trading model involving filter rules for a market index, while Fama and Blume [A] applied the filter approach to individual stock price relatives. Levy [A 1967a and A 1967b] and Jensen and Bennington [A] investigated the relative strength concept and Levy [A 1971] also examined the profitability of chart patterns. In general, the studies showed that profits were realized before commissions, but not after commissions; i.e., the efficient markets hypothesis holds.

Broad view

The broad-form tests investigated the statistical and economic relationship between many types of past information and future price changes. The methodology employed by Fama, Fisher, Jensen, and Roll [A] has dominated the literature in this area. FFJR used regression analysis to remove the effects of market movements, enabling them to concentrate on the residual effects of stock splits. A similar methodology is employed by Millar and Fielitz [A] to examine stock splits, Ball and Brown [A] to investigate earnings, and Pettit [A] to determine the effects of dividends on performance. These studies concluded that a relationship does exist between split, dividend, etc. information and cummulative price changes. However, most of the price increase occurs before the information is available to the public; also, once the information is generally known, prices adjust extremely rapidly to new information.

Two other major areas of investigation examine quarterly earnings reports and the new issue market. Jones and Litzenberger [A], Latane, Tuttle and Jones [A], Latane, Joy, and Jones [A], and Latane, Jones, and Rieke [A], claimed that information regarding quarterly earnings can lead to abnormal returns. Similarly, Reilly and Hatfield [A], and Reilly [A], among others, indicated that the price performance of a stock after public listing was such that extraordinary returns to the investor could be achieved. However, further investigantion of both topics is necessary to substantiate the validity of the methodology and results, and to show that the extraordinary returns are consistent over time.

Monopolistic Information

Certain groups may possess monopolistic information that can be turned into extraordinary profits. Investigations of the existence of such groups have included financial services, mutual funds, and insiders. Diefenbach [A] showed that market positions established on the recommendations of financial services do not outperform the market. Similarly, Friend and Vickers [A], Sharpe [A], and Jensen [A] showed that the risk-return performances of mutual funds are inferior to the rish-return characteristics of the market. On the other hand, Wu [A 1963 and A 1965], Rogoff [A], Neiderhoffer and Osborne [A], and Lorie and Neiderhoffer [A] reported that some "insiders" possess sufficient monopolistic information to obtain extraordinary returns on a consistent basis. However, since the information associated with the "insiders" is not available to the general investing public, the efficient markets model seems to hold for the majority of investors.

Technical Indicator Studies

The previous sections summarize the Random Walk and Efficient Market Theories. The remaining parts of this chapter investigate the methodology and results involving technical indicators. The studies discussed here provide valuable information concerning the relationship between technical indicators and stock price changes. Moreover, they serve as a focal point for isolating problems in methodology and gaps in the literature.

A logical division of investigations involving technical market indicators is to distinguish between the univariate and multivariate approaches. Univariate studies have considered volume, odd-lot statistics, and advance/decline/unchanged data. Perhaps the major criticism of these studies is the univariate methodology employed; i.e., since the stock market is obviously multivariate in nature and since technicians generally employ a wide variety of technical indicators in their work, a univariate approach is a somewhat simplistic view of the problem at hand.

There are only two multivariate studies involving technical indicators. Gup [A] presented a very limited investigation of three monthly technical indicators, with low R^2 values. Emery [A] provided a more sophisticated regression/factor analysis approach to examine the effects of ten widely used technical indicators. However, major criticisms of Emery's work cast serious doubt on the reliability of his results. These criticisms are fully discussed later in this chapter.

Univariate Studies

Volume

Granger and Morgenstern [A 1963] and Godfrey, Granger and Morgenstern [A] used the coherence diagram from spectral analysis to examine the relationship between price and volume for weekly, daily, and transaction data. Generally, they concluded that little or no relationship exists between the price and volume series. However, in a later study Granger and Morgenstern [A 1970, Ch. 8] suggested that an unlagged relationship exists between the daily volume and the extent of the price change (as opposed to direction) as measured by the square of the daily price change or the high minus low price difference.

Crough [A 1970b] provided an empirical examination of the correlation between the absolute price change and volume and concluded that a price-volume relationship exists. He [A 1970a] also conducted a non-linear test of the price-volume relationship by analyzing the lagged effect of abnormal volume on price changes. While a statistically significant relationship appeared to exist between abnormal volume and price changes, no patterns in terms of the length of the lag or the direction of the relationship developed. Moreover, the dependencies seem to be too small to develop a profitable trading model.

Ying [A 1966] analyzed price-volume relationships for the daily Standard and Poor's Index for six years, and determined that a coincident price change/volume relationship does indeed exist. Proceeding to examine the lag structure of the data, the author discovered a significant one-day lag relationship between volume and price change; he also found a lag between certain volume characteristics and the resultant four-day cummulative price change.

Finally, Osborne [A] showed that certain relationships exist between volume and price movements. Specifically, Osborne showed that "important upward price moves end on large volume" and that the volume sequence leads the price sequence.

The number of investigations of the price-volume relationship is limited and the conclusions are contradictory. Crouch [A 1970b] argued for a coincident relationship, while Granger and Morgenstern [A 1963] and Godfrey, Granger and Morgenstern [A] disputed the relationship. The lagged effect of volume on prices was supported by the serial correlation studies of Crouch [A 1970a], Ying [A 1966], and Osborne [A], and to a lesser extent by Granger and Morgenstern [A 1970]. None of these studies attempted to develop a trading model to determine whether or not the statistical dependence can be used profitably, before or after commissions.

Odd-lot data

The "Odd-Lot Theory" attempts to describe the relationship between the small investor and the market. Kewley and Stevenson [A 1967] presented one of the early studies of odd-lotters by examining absolute values (of ten day moving averages) of the "Balance Ratio" (odd-lot sales/odd lot-purchases) in relation to price changes. They found that little or no correlation existed between the balance ratio and market prices. Furthermore, a four-week moving average or a change in the Balance Ratio from less than one to greater than one, or vice-versa, does not contain any predictive information.

Drew [A] criticized the Kewley and Stevenson results for their failure to use relative percentage changes in the balance ratio. Considering Drew's criticisms, Kewley and Stevenson [A 1969] re-examined their data by employing relative changes in a four-week moving average of the Balance Ratio. The results showed that acting on buy signals provides performance resulted substantially above the market before commissions for subsequent periods of 4, 12, and 26 weeks; however, acting on sell signals produces returns which are inferior to the return from the market.

Klein [A] examined odd-lot activity for the 20 most active NYSE stocks. Using monthly data, Klein determined that the purchases of odd-lotters are profitable but that sales are transacted at the wrong time.

Wu [A 1972] employed regression analysis to determine that (a) a significant relationship between net weekly odd-lot purchases and price changes exists, as well as between changes in odd-lot short sales and price changes; and, (b) net odd-lot purchases are related to market turning points.

None of the above studies examines the valuation or predictive effect of odd-lot data on prices for daily data. However, Kewley and Stevenson [A 1969], Klein [A], and Wu [A 1972] all reported that weekly and monthly odd-lot information can provide superior purchasing decisions (before commissions). Since all of the above studies examined long bull-market periods, one may hypothesize that odd-lot sales data could provide superior information in bear markets, but this interpretation has not been substantiated.

Advances/declines/unchanged data

The information theory approach used by Theil and leenders [A] and Fama [A 1965b] to analyze advances/declines/unchanged data on the Amsterdam and New York Stock Exchanges, respectively, led to similar analysis of the London Exchange by Dryden [A 1968], and the NYSE and ASE by Phillippatos and Nawrocki [A 1973a and A 1973b]. The information theory technique employs equation II-1 to determine the best estimate at time t of the A/D/U proportions:

II-1 $$p_{it} = a\, q_{i,t-1} + (1 - a)\, \overline{q_i}$$

where $\overline{q_i}$ is the long run mean of proportion i.

The fraction 'a' is chosen to minimize the information inaccuracy; 'a' also indicates the degree of dependence between day t and day t-1, so that the higher the value of 'a' the greater the dependence.

Thiel and Leenders determined that a = .47 for the Amsterdam Exchange, Fama obtained a = .3 for the NYSE (data from the 1950's), Dryden had a = .58 for the London Exchange, Phillippatos and Nawrocki obtained a = .42 for the NYSE (data from 1960's) and a = .55 for the ASE. Obviously, the day-to-day dependence was non-trival. Furthermore, each study also determined that a one-day lag period was optimal to minimize the information inaccuracy.

The existence of a one-day lag in the A/D/U data suggested that a trading model should be investigated. Furthermore, the trading model could be improved by considering the multivariate nature of A/D/U data instead of each variable separately as the above studies have done. Unfortunately, no one attempted to devise such a model.[1]

Zakon and Pennypacker [A] analyzed the advance/decline line by employing correlation and regression analysis. Their results showed that the A/D line is a coincident indicator of price movements. However, no strong leading relationships appear to exist. The generality of these conclusions is severely limited by the restrictive assumption of linearity imposed by the authors; thus, other non-linear relationships may exist.

Conclusions for univariate technical studies

Each of the three sets of data examined above provides evidence concerning the statistical dependence in technical and price-series data. The relationship between volume and price is the weakest; however, sufficient evidence still exists to show that the statistical aspects of the random walk hypothesis are invalid for price-volume data. Evidence from odd-lot and A/D/U data also supports the case of dependence between technical and price data. There are two main drawbacks of the univariate. In general they, one, fail to develop trading models to test the real world validity of their statistical dependencies, and two, ignore the multivariate aspects of the variables.

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[1]Dryden [A 1968 and A 1969] attempted to consider these interrelationships by means of a Markov process with transition matrices. He concluded that the probability of a share being in the same state tomorrow as it is today is high, i.e., the diagonal elements in the transition matrix are large. Furthermore, the results for subgroups showed only moderate instability. However, Fielitz [A 1971] showed that dependence on the NYSE, as measured by a Markov process, is much weaker than on the London market. Furthermore, the data from the NYSE formed nonstationary transition matrices. Finally, Fielitz and Bhargava [A] showed that the aggregate formulation employed by Dryden was not appropriate since the Markov process for each security is not likely to be homogeneous.

Multivariate Studies

Gup: regression analysis

Gup [A] employed multiple regression analysis to investigate the effect of monthly short interest, mutual fund cash ratios, and the odd-lot balance ratio on prices. After examining the coincident relationship between these variables and price changes, Gup obtained an R^2 of .30. Regressing the independent variables against the next period's price changes resulted in an R^2 of .13.

Although the dependence in Gup's article was statistically significant, it has little or no practical importance. Also, the limited number of variables, and hence information, reduced the benefit of the multivariate approach. Furthermore, the use of monthly data reduced the possibility that a significant relationship exists between the independent and dependent variables, since dynamic economic events could easily overwhelm the monthly profit potential for the three variables employed. Finally, non-stationarity was ignored as well as the possibility that dependence exists for subsets of the data base instead of throughout the data.

Emery: factor analysis/regression methodology

Emery [A] provided the only in-depth multivariate analysis of technical indicators that appears in the literature. The author attempted to examine whether (a) the information set of technical indicators contains adequate information to forecast prices, and (b) whether forecasts obtained by a factor analysis/regression methodology provided superior trading results when compared to buy and hold returns.

To obtain the inputs for the regression model, both principal components (orthogonal) and cluster analysis (non-orthogonal) techniques were applied to data for a set of technical indicators. The cluster analysis results provided three oblique clusters to explain 98 percent of the variability in the daily technical indicator data set. The output from the three clusters and, alternatively, from the first three principal components, became the input data for the linear regression model.

The forecasts resulting from the regression model were robust in terms of the forecast lag (one day is optimal), the number of trading days employed in the original data set, and the length of the information decay period. The results of the trading rule, on an after commission basis, showed that the rule did not provide significantly greater returns than the buy and hold criterion.[2]

Criticisms of Emery's methodology and data

Emery's article contained two major areas of criticism; namely, the methodology employed, and the data base. Criticisms of Emery's methodology can be broken down into several areas. First, Emery employed the restrictive assumption that the data only forms a linear relationship with prices. Second, Emery utilized the entire sample of technical data in his factor analysis/regression model, ignoring the possibility that a more restrictive set of trading days may provide more important information than all days taken together. Third, Emery failed to consider certain variables normally used by technicians. Fourth, both the factor

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[2]The author implied that a one-day lag using principal components provides superior returns to the cluster analysis procedure. However, the author failed to present the returns for the principal components technique.

and regression methods employed a best-fit approach for the data in question; the best-fit approach biases the conclusions when the data are sample sensitive or are non-stationary over time. Fifth, Hanstian [B] and Glass and Taylor [B] pointed out that the choice of the factor analytic procedure employed can have substantial effects on the computations of the factor loadings, which could cause some concern for the reliability of the procedure employed by Emery. Finally, the factor analysis/regression approach assumed that the data came from one underlying population. If the data possess characteristics of two or more populations, then the factor analysis/regression methodology provides biased results.[3]

The other major criticism of Emery's study related to the number and magnitude of the errors in the data base. Specifically, a verification of the data uncovered the errors outlined in Table II-1. Major errors were arbitrarily defined as errors of 10 percent or more from the true observations, with errors of less than 10 percent being considered as minor in nature. The data were divided into nearly equal sets, showing how the errors appear throughout the data as well as in their summation. In addition to the errors listed in Table II-1, data for five days are completely missing, or are duplicates, in Emery's data set.

The extent and absolute size of the errors poses grave doubts concerning Emery's conclusions. The errors in the dependent variable, the price index, are especially disturbing.

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[3]Joreskog [B] and Rasch [B] pointed out the necessity for one underlying population for factor analysis. When more than one population exists, Joreskog suggested that each population should be examined by obtaining separate factor solutions for each group. Furthermore, Rasch [B] showed that the stability of the factor loadings under changes in the underlying population is in question.

TABLE II-1

ERRORS IN EMERY DATA BASE[a]

	Price Index		Other Variables	
	Major	Minor	Major	Minor
Set A	3	2	49	40
Set B	19	4	101	82
Set C	33	7	111	92
Set D	31	6	35	63
TOTALS	86	19	296	277

[a]Original data obtained from John Emery. Errors detected by crosschecking data sources, as explained in text.

Conclusions

Univariate studies ignored the multivariate nature of the market. Furthermore, concentration of the statistical dependence between technical indicators and prices, to the exclusion of examining trading models, resulted in an important omission in the literature.

Although Emery's methodology and results are interesting, some of the assumptions of the model and the errors in the data are cause for concern for his results and conclusions. A more appropriate multivariate procedure and cleaner data are needed to investigate the relationship between technical indicators and price-index changes.

CHAPTER III

THEORY OF MULTIPLE DISCRIMINANT ANALYSIS

The statistical technique used in this study is Multiple Discriminant Analysis (MDA). This chapter describes the theoretical aspects of MDA. The topics discussed include: statistical tests of significance, assumptions of the model, classification procedures, step-wise analysis, bias, relative importance of individual variables, and effects of multicollineality.

MDA was introduced by Fisher [B 1936] in his famous study on Iris plants. Although there was some theoretical interest in the technique, especially in terms of the D^2 concept (i.e. the squared distance between the groups), practical applications of MDA were generally ignored until the 1960's. Since then, MDA has become popular in the areas of psychology, marketing, and finance.

General Theoretical Aspects of MDA

Discriminant analysis maximizes the ratio of between group variation to within group variation. For a two-group, reduced-space linear discriminant analysis,[1] the between group variation can be expressed as:

III-1 $$(\bar{Z}_1 - \bar{Z}_2)^2$$

where $\bar{Z}_g$ represents the mean of the g^{th} group.

[1] This initial discussion employs the term "reduced space," which is the discriminant procedure normally found in the literature. Reduced-space refers to the mapping from p dimensions (variables) to r dimensions (G - 1 groups). The reduced-space distinction is necessary due to the concept of "test-space" as described later in this chapter.

Similarly, the within group variation is given by:

III-2 $$\sum_{n=1}^{N_1} (Z_{1n} - \bar{Z}_1)^2 + \sum_{n=1}^{N_2} (Z_{2n} - \bar{Z}_2)^2$$

where N_1 and N_2 are the number of observations in groups 1 and 2, respectively.

The basic discriminant analysis statement is given by:

III-3 $$\text{Max } H = \frac{(\bar{Z}_1 - \bar{Z}_2)^2}{\sum_{g=1}^{2} \sum_{n=1}^{N_g} (Z_{gn} - \bar{Z}_g)^2}$$

To obtain the Z values and means of each group, p independent variables $X_1, X_2, \ldots X_p$ are employed. Upon obtaining $\bar{X}_{1i}$ and $\bar{X}_{2i}$, the means for variable i of groups 1 and 2, respectively, the distance between the two groups for variable i is determined by:

III-4 $$d_i = \bar{X}_{1i} - \bar{X}_{2i} \qquad i = 1, 2, \ldots p.$$

Using the concept of maximizing the distance between groups relative to the variation within groups, the linear function that discriminates best between groups is given by:[2]

III-5 $$Z = k_1 d_1 + k_2 d_2 + \ldots + k_p d_p.$$

When the X_{in} values are substituted for the d_i values, this function can be used to classify the observations.

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[2]There are various methods of calculating the coefficients, depending on the matrices employed. The results differ only by a constant term, which guarantees the proportional character of the coefficients. Eisenbeis and Avery [B, p. 4-9] discuss these various methods at some length. For various approaches to the derivation of the discriminant model for G groups, see Cooley and Lohnes [B 1962, Ch. 6] and [B 1971, Ch. 9], Smith [C 1965], and Anderson [B 1951].

Discriminant analysis, as a multivariate technique, considers the interrelationships of the variables, as well as their individual discriminatory power. Thus, a variable may not discriminate on a univariate basis, but may contribute added discriminatory power when correlated with a variable that does discriminate.[3] This simultaneous investigation of group differences sets multivariate methods apart from univariate methods.

Reduced-space operations

One benefit widely proclaimed for the two-group linear discriminant model is that it can reduce the analysis from the p-variable space (test space) to one variable space (reduced space). This dimension reduction transforms the multivariate problem into a univariate one by employing the discriminant function:

III-6 $$Z = k_1X_1 + k_2X_2 + . . . + k_pX_p.$$

Classification of the observations into groups is then accomplished by comparing the Z-values with an appropriate cutoff point. Linear reduced-space and test-space discriminant operations give equivalent results, when the group dispersion matrices are equal, since the linear transformation preserves relative distances and positions as well as the overall variance structure. The reduced-space model is often found in the literature, probably because of its simplicity and the ease in interpreting the results. However, linear reduced-space operations are optimal only when

[3] See Cooley and Lohnes [B 1965, p. 121] for a brief but convincing example. Also, see Altman [C 1968, p. 594] for an example of how intercorrelations can affect actual data. This topic is related to the multicollineality question, which is discussed in some detail later in this chapter.

the variance-covariance matrices are equal and when the data come from a multivariate normal distribution. Discussion of violations of these assumptions will be dealt with at length later in this chapter.

The G-group, linear, reduced-space operation provides advantages similar to the two-group analysis. Specifically, a linear transformation from p dimensions (variables) to r dimensions (G-1 groups) exists. As with the two-group case, the variance structures, distances, and relative positions among observations and groups are preserved by the reduced-space transformation when the variance-covariance matrices are equal. Thus, when the dispersion matrices are equal one may use the reduced-space means and dispersions to help explain group overlaps. Moreover, the number of dimensions may be reduced below r if some of the discriminant functions provide insignificant information. Eisenbeis and Avery [B, p. 9-11, 57, 63-67] provide further discussion concerning the G-group case.

Samples versus the population

The above discussion specifies the discriminant function for the entire population, as a consequence, the coefficients are the population parameters. Since most studies employ sample data, the population parameters must be estimated. Using the test-space information an equivalent expression for sample data for the two-group linear case is given by:[4]

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[4]For the linear, G-group case the relevant expression is:

$$\max\ (Q/W)$$

where Q and W are matrices and an element of Q is:

$$(Q)_{ij} = \sum_{g=1}^{G} N_g\ (\overline{X}_{gi} - \overline{X}_i)\ (\overline{X}_{gj} - \overline{X}_j)$$

N_g is the number of observations in group g = 1, . . . G, i,j = 1, 2, . . . p variables and an element of W is:

$$(W)_{ij} = \sum_{g=1}^{G} \sum_{n=1}^{N_g} (X_{gin} - \overline{X}_{gi})\ (X_{gjn} - \overline{X}_{gj}).$$

For ease of exposition the remainder of the discussion in this chapter

III-7 $$\text{Max. } H = \frac{\frac{N_1N_2}{N_1+N_2} K'dd'K}{K'S_wK}$$

where K, K', d, d', and S_w are matrices,

and $d = (\overline{X}_1 - \overline{X}_2)$

K is the coefficient matrix

S_w is the pooled within-groups dispension matrix, with an element of S_w being

$$(S_w)_{ij} = \frac{1}{N_1+N_2-2} \sum_{g=1}^{2} \sum_{n=1}^{N_g} (X_{gin} - \overline{X}_{gi})(X_{gjn} - \overline{X}_{gj})$$

with N_g being the number of observations in group g,

and i,j = 1, 2, . . . p variables.

If one had access to the populations, then determining whether or not the derived function could actually discriminate between these populations would be a simple matter. However, in most studies the populations are only sampled, and therefor various biases are encountered. Accordingly, an estimation is required as to whether the apparent difference between the populations (found by the derived function) arises merely by chance.

The D^2 statistic

Several statistics may be used to test the null hypothesis of no significant statistical difference between the means of two (or more) groups. Of these, the D^2 value is intuitively appealing, since it deals with the estimated distance between populations. Additionally, this measure of distance can be used to form several different tests of

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assumes two groups. However, the statements made here can be generalized to the G-group case.

significance and to develop confidence limits. When the variables are independent D^2 can be measured by:

$$D^2 = \frac{1}{P} \sum_{i=1}^{P} \left(\frac{\overline{X}_{1i} - \overline{X}_{2i}}{\sigma_i} \right)^2 \qquad \text{III-8}$$

When the variables are dependent Rao [B 1948a, p. 63-64] presents the following statement of D^2:

$$D^2 = \sum_{i=1}^{P} \sum_{j=1}^{P} W_p^{\ ij} (\overline{X}_{i1} - \overline{X}_{i2}) (\overline{X}_{j1} - \overline{X}_{j2}) \qquad \text{III-9}$$

where $W_p^{\ ij}$ is the matrix reciprocal of W_{ij}; i, j = 1, 2, . . . p

and

$$W_{ij} = \frac{\sum_{t=1}^{N_1} (X_{i1t} - \overline{X}_{i1})(X_{j1t} - \overline{X}_{j1}) + \sum_{t=1}^{N_2} (X_{i2t} - \overline{X}_{i2}) (X_{j2t} - \overline{X}_{j2})}{(N_1 + N_2 - 2)} \qquad \text{III-10}$$

An alternative formulation for the dependent case, can be found in Hodges [B, p. 22]:

$$D^2 = \frac{1}{|R|} \sum_{i=1}^{P} \sum_{j=1}^{P} R_{ij} \left(\frac{\overline{X}_{1i} - \overline{X}_{2i}}{\sigma_1} \right) \left(\frac{\overline{X}_{1j} - \overline{X}_{2j}}{\sigma_j} \right) \qquad \text{III-11}$$

where R_{ij} is the correlation between variables i and j.

The above expressions for D^2 are consistent point estimates of the population quantity Δ^2.

Another statistic that is frequently employed to test the null hypothesis of no significant difference between the means of two-(or more) groups is Hotelling's T^2. Note that for the two-group case, the D^2 and T^2 statistics are equivalent.[5]

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[5]See Hodges [B, Ch. 3, 4] for a good discussion of both the D^2 and T^2 statistics, and for a limited discussion of the optimum properties of T^2. In addition, Porebski [B] has an excellent mathematical summary of the various statistics that can be used to test the null hypothesis of no significant difference between group mean vectors. These include W_2^2 (within the group sum of squares), R^2, D^2, T^2, Wilk's Λ, Hsu's V, T_o^2,

Tests of significance

The D^2 value can be used to form an F-statistic to test the significance of the difference between the group mean vectors:

III-12 $$F = \frac{N_1 + N_2 - p - 1}{p} \quad \frac{N_1 N_2}{N_1 + N_2} \quad \frac{D^2}{(N_1 + N_2 - 2)}$$

where F has p and $N_1 + N_2 - p - 1$ degrees of freedom.

If the F value is significant at some α level of significance (i.e. F-calculated greater than F-table, α) then the null hypothesis of no significant difference between the population means is rejected.

A statistically significant F-value does not necessarily mean that the function has practical significance. This interpretation follows because a large sample size will virtually insure a significant F-value, even though an extensive amount of overlap between the population may be present. The problem of statistical versus practical significance in MDA can be resolved by classifying the original observations and then testing the proportion of correct classifications against the proportion expected by chance alone. In addition, the effects of bias (to be discussed in more detail later in this chapter) can be investigated by classifying a new data set. The proportion correctly classified for this secondary sample is tested for significance in the same manner as the original classification results are tested.[6]

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and ϕ. The last four statistics can be used for three or more groups, with T^2 and Wilk's Λ being the most popular. Porebski concluded that each statistic is associated with a discriminant function involving a different constant. Therefore, these constants account entirely for the differences between the statistics.

[6]If a secondary sample is not available, then there are several ways to test the original sample for significance; see Frank, Massey, and Morrision [B] and Lachenbruch [B 1967]. However, their validation procedures may remove only part of the total bias that is inherent in the original sample, as discussed in more detail later in this chapter.

Test space procedure

The reduced-space format employed in the literature and discussed above has certain shortcomings. The basis of these shortcomings is the widespread assumption that the reduced-space formulation is equivalent to the test-space or original-space procedure. This assumption is not correct and could cause potential problems in the discriminant-analysis process. Specifically, various tests of significance should be performed in test-space. These include tests for the significance of the discriminant function mean differences (which is related to D^2), tests for multivariate normality, and tests for the equality of the variance-covariance matrices. Unfortunately, little research has been completed to indicate the effects of using reduced-space instead of test-space data to perform the tests of significance cited above. Also, the bias created on the classification results could be substantial if these tests of significance are ignored. Thus, indiscriminately employing reduced-space procedures may bias the tests of significance and/or the classification results.

The discussion in this paper assumes the use of test-space unless stated otherwise.

Regression analysis

As introduced by Fisher [B 1936], two-group linear MDA is essentially equivalent to point-biserial regression analysis, i.e. the dependent variable is given values of 0 or 1 in order to designate group membership of the observations. Thus, for the two-group linear case, the results from a point-biserial regression approach differ from a multiple discriminant function only by a constant.

The relationship between two-group linear MDA and the linear regression model (MRA) is often mistated in the literature. In particular, results from MDA do not give grouping results equivalent to those from a multiple-regression model.[7] Also, theorems that are valid for regression analysis are not necessarily applicable for the discriminant model (e.g., the coefficients in MDA do not possess the same properties as their counterparts in regression).

One primary assumption for the usual regression model is that the dependent variable, Y, is normally distributed. For point-biserial regression and MDA, Y is equal to 0 or 1 and consequently is not normally distributed. Consequently when observations are drawn from one continuous population then MRA should be employed, while observations from two (or more) different and discrete populations require the MDA technique.

Lubin [B, p. 91] reported that MRA is appropriate when both the independent and dependent variables are quantitative, while MDA requires quantitative independent variables but a qualitative dependent variable. A more descriptive comparison is made by Altman [B, p. 4] who pointed out that 'regression analysis can be used to assign specific persons _within_ a group to _certain tasks_', but that one should first use MDA to assign people to the specific groups in question.

Finally, the distinction between MRA and MDA can be sharpened if it is realized that, "regression analysis attempts to estimate the coefficients which best describe the interrelationships among variables for

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[7] Grouping results are equivalent when the point-biserial designation of the dependent variable is used in the regression procedure. But when the dependent variable is treated as a continuous variable in the regression procedure, the results may differ from those of MDA.

all members of the population, (while) discriminant analysis attempts to estimate coefficients which best classify observations into one of the a priori underlying populations or groupings. "(Altman [B, p. 4-5]).[8]

MRA and MDA have different purposes and, consequently, the technique used should be chosen to implement the intent of the study.

Assumptions of the Linear Discriminant Model

Like other statistical models, MDA is based on several assumptions. Such assumptions are either definitional in character, i.e., they describe the model; or they are theoretical assumptions to permit tractible probability distributions to be substituted for more general functions.

Two definitional assumptions

As listed by Eisenbeis and Avery [B, p. 1], the two definitional assumptions of MDA are that "(1) the groups being investigated are discrete and identifiable, (2) each observation in each group can be described by a set of measurements on characteristics or variables. . ."

The first definitional assumption indicates that groups need to be identifiable, such that each observation can be assigned to a group in a logical and non-random fashion. Eisenbeis and Avery [B, p. 36] disapprove of non-discrete groupings (criterion variables). However, the implicit assumption for the model is that the criterion variable is robust for non-discrete groupings. Reinhart and Latane [C] presented a

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[8]Another description of this relationship indicates that MDA maximizes the difference between means, while MRA maximizes the multiple correlation coefficient between the independent and dependent variables. See Garrett [B].

method for comparing the classification ability of MRA to that of MDA.[9] Superiority of the regression model would indicate that discriminant analysis is inappropriate. However, if MDA gives superior results then this approach suggests that the discriminant model is robust for non-discrete data.

The second difinitional asusmption implies the need for a set of measurements to describe each observation. Thus, a direct relationship between the set of measurements (independent variables) and the observation (criterion variables) is hypothesized; if no relationship exists, then this assumption is violated (i.e., the measurements do not describe the observation). The consequence of using non-related variables is that the discriminatory ability of the function is impaired.

Another violation of the second assumption occurs if inaccurate or missing measurements for some of the independent variables are present. An analogous situation in regression analysis results in biased and inconsistent estimates. (See Johnston [B, p. 281-291]). A similar consequence would be expected for the discriminant model.

Assumption of a multivariate normal distribution

A third assumption for MDA is that the independent variables form a multivariate normal distribution.[10] Thus, each independent variable

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[9] As discussed earlier, MRA is not really intended to classify observations and therefore such a comparison is biased in favor of the MDA model. However, such a comparison does give an indication of whether or not the observations come from one underlying population with a continuous dependent variable (MRA) or several populations with discrete groupings (MDA).

[10] Recall that the dependent variable is normally distributed in the regression model, but is fixed for discriminant analysis. Furthermore, for regression analysis the distribution of the independent variables is irrelevant, but for MDA a multivariate normal distribution is needed. Fisher [B 1938, p. 377].

is assumed to be normally distributed., as well as each marginal distribution of these variables.[11] Because of the complexity of the multivariate normal distribution, it is difficult to test whether a sample (or population) actually conforms to this distribution. Indeed, the costs of such efforts normally will outweigh the benefits, since Gilbert [B] showed for a two-group linear rule that violations of the assumption of multivariate normality seem to be quite robust for classification results. Although individual tests could be performed on the normality of the individual variables, and perhaps even on the bivariate distributions (as a partial test of the assumption), once again the cost would seem to outweigh the benefits. The main consequence of non-multivariate normal data in MDA seems to be its influence on certain tests of significance. For example, Ito [B] found that tests for the quality of the variance-covariance matrices (discussed below) are highly sensitive to non-multivariate normal data; specifically, if non-multivariate normal data are present the null hypothesis of no difference between the dispersion matrices will be rejected when in reality there is no significant difference.

Eisenbeis and Avery [B, p. 37] showed that if Z is normally distributed (the Z values should form a normal distribution in each group), then classification can be performed in linear reduced space (provided the variance-covariance matrices are equal, as will be discussed shortly). Numerous tests for the normality of Z are available; however, the necessity of utilizing such procedures in practice is questionable on at

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[11]See Mood and Graybill [B, p. 207-209], especially definition 9.5 and theorem 9.7. Many authors have assumed that only the Z-values need to be normally distributed. This topic is discussed in this section.

least two counts. First, the generalized Central Limit Theorem (See Cramer [B]) suggests that as the number of independent random variables with finite variances increases (under the generalized Central Limit Theorem the variables do not have to identically distributed), the Z scores will approach a normal distribution. Second, a large sample size will often cause the rejection of the null hypothesis of no significant difference between the normal distributions and the sample in question when the true difference between the distributions is so small that the effects of classification results are negligible. Thus, the usefulness of applying formal tests to an investigation of the normality of Z seems limited, and the assumption is made in this study that classification results are robust in regard to departures from normality.[12]

Equal dispersion matrices

A fourth assumption involved in MDA concerns the equality of the dispersion matrices. The most commonly used test to examine the equality of the variance-covariance matrices is an F-test, due to Box [B].[13] (See Eisenbeis and Avery [B, p. 29]). Rejection of the null hypothesis of

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[12] Concern about violations of the normality assumption is also reduced when one realizes the actual test used for the significance of correct proportions assumes a non-parametric, but symmetric distribution (Lubin [B, p. 102]). Also, the distribution of the proportions of secondary samples are independent of the distribution from the original sample.

[13] Several other tests to examine dispersion equality appeared prior to Box's article. Plaket [B] gives an exact test for dispersion matrices using Wilk's Λ statistic; however, since Λ can only be approximately tabulated, the resultant test is not optimal. (Also see Rao [B 1948a, p. 67-71] for an example of using Λ to test dispersion matrices.) Wilks [B] developed an exact test for k=2,3 (k=number of variables) using the Incomplete Beta Function to fit the probability interval. For $k \geq 3$ Wilks developed a statistic that is approximately distributed as χ^2. (See Bock [B, p. 823-825] for an application of the χ^2 test for dispersion equality). Finally, Box [B] developed χ^2 and F-tests, determining that the F-test performed better than the χ^2 test.

equality implies that the linear discriminant function is no longer applicable.

Smith [B], Lubin [B], Anderson [B 1958], Massey [B] all discussed the non-linear discriminant fuction; however, it was not until Eisenbeis and Avery [B] promoted the concept and programmed the model that the technique became available for practical application. Other than the examples given by Eisenbeis and Avery, no non-linear studies have appeared in the literature; in fact, most authors do not even test for the equality of variances, much less for the equality of the dispersion matrices, <u>or</u> use non-linear classification procedures.

The test for equality of the variance-covariance matrices requires assuming a multivariate normal distribution and is quite sensitive to non-normality. (Ito [B]). Further, a large sample size also seems to cause unwarranted rejection of the hypothesis, as seen by examples from Eisenbeis and Avery [B, p. 53, 56]. Finally, results from the data used in this study indicate that the number of variables used influences the test of significance for dispersion matrices.

Unequal dispersions may bias the test of significance for mean vectors in the linear model. For two groups, the bias usually results in accepting the null hypothesis of no significant difference between group means more often than would be expected if the theoretically correct test of significance is employed. (See Holloway and Dunn [B, p. 136] and Eisenbeis and Avery [B, p. 8, 37].) The theoretically correct test for examining the significance of mean vectors when there are unequal dispersion matrices is an F-test with (p, N-p-1) degrees of freedom (where N is the number of observations in each group and p is the number of variables). The test if presented by Eisenbeis and Avery [B, p. 25-27] and consists of using Hotelling's T^2 value to generate the needed F-statistic. The test for examining the statistical difference between mean vectors when the dispersion matrices are equal is given by equation III-12. The extent of the bias caused by using equation III-12 when the dispersion matrices are unequal has been determined by Holloway and Dunn [B]; they showed that the bias usually involves accepting the null hypothesis more frequently than is necessary and consequently researchers often disregard the effect of the bias.

Classification may also be affected by differences in the dispersion matrices. If the dispersion matrices are not equal, a quadratic discriminant function is theoretically superior to a linear rule and will give improved classification results (both in terms of the proportion of observations correctly classified, and the accuracy of correct group classification). Conditions under which improved classification results occur for the quadratic rule versus the linear rule have not been investigated, but most likely hinge on true differences in the dispersion matrices (rather than statistical differences created by sample size and/or non-multivariate normal data), the extent of group overlap, and the number of groups.

Since linear discriminant analysis is inappropriate when there are unequal dispersions, the linear reduced-space Z-function is biased. The question of the applicability of a quadratic reduced-space operation is then raised. Little is known about the G-group, non-linear reduced-space operations.[14] For the two-group case, the non-linear reduced-space

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[14]Eisenbeis and Avery [B] treated the non-linear discriminant procedure mainly as a test-space technique where both tests of significance and classification rules rely on the test-space formulation.

operation does consider unequal dispersions; however, even in this special case the variance properties of the quadratic Z-scores have not been completely explored. Despite this lack of inforamtion, Eisenbeis and Avery [B, p. 57-62] claim that quadratic reduced-space procedures are preferable to the linear reduced-space method when the dispersion matrices are unequal.[15]

Classification and Prediction

Since a large sample size will virtually insure the statistical significance of the discriminant function, the practicial significance of this same function needs to be examined. To accomplish this, the observations are classified into their respective groups, so that the absolute number and/or the percentage of correct classifications and the extent of group overlap may be examined. The hypothetical example in Table III-1 shows how these results can be summarized in a classification matrix; the highlighted diagonal represents those observations which are correctly classified.

TABLE III-1: HYPOTHETICAL CLASSIFICATION MATRIX

		Predicted Groups		
		Group A	Group B	Total
Actual Groups	Group A	25	5	30
	Group B	2	28	30
	Total	27	33	60

[15]For the two or G-group case, if the dispersions are proportional then the quadratic test- and reduced-space operations will give equivalent classification results given the standard linear transformation (the linear transformation is the reduction from the original p dimensions to r dimensions, where p is the number of variables and r=G-1 with G being the number of groups). Eisenbeis and Avery [B, p. 57, 61-63] give more insight to the two group case.

Classification procedures

Classification in test-space is usually performed by using likelihood ratios. For a two-group classification rule, one may designate $f_1(x)$ and $f_2(x)$ as being likelihood functions for groups 1 and 2, respectively; the assumption is made that $f_1(x)$ and $f_2(x)$ possess characteristics identifiable with a specific type of known density function. Thus, for the likelihood functions $f_1(x)$ and $f_2(x)$, an observation is assigned to group 1 if:

III-13 $$\frac{f_1(x)}{f_2(x)} \geq \frac{\pi_2}{\pi_1}$$

where π_g is the a priori probability that an observation comes from group g. Otherwise the observation is assigned to group 2.[16]

Since most samples are too small to obtain adequate estimates of the likelihood function, f(x), it is customary to provide a theoretical distribution for the function. If one assumes that the two populations are multivariate normal and that they have equal dispersion matrices, then one can substitute the density functions $N_1(\mu_1,\Sigma)$ and $N_2(\mu_2,\Sigma)$ into eqation III-13, take the natural logs of both sides, and rearrange terms to obtain the following linear classification rule when the population parameters are used (see Eisenbeis and Avery [B, p. 12-16])[17]:

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[16]This classification scheme is general and thus is applicable to linear or quadratic, test or reduced, space. This (and subsequent) classification functions can be amended to include costs of misclassification.

[17]This rule gives equivalent results to the linear reduced-space cut-off procedure when the proper conditions hold, i.e. when the groups are characterized by multivariate normal distributions with equal dispersion matrices.

Assign to population 1 if:

III-14 $$X'\beta - \tfrac{1}{2}(\mu_1 + \mu_2)'\beta \geq \ln p$$

where $\beta = \Sigma^{-1}(\mu_1 - \mu_2)$.

When the population parameters are unknown, the sample estimates are substituted for the population values μ_1, μ_2, and Σ. Thus, the linear classification rule for the sample is:

Assign to population 1 if:

III-15 $$X'B_o - \tfrac{1}{2}(\overline{X}_1 - \overline{X}_2)'B_o \geq \frac{\ln p}{N_1 + N_2 - 2}$$

where $B_o = W^{-1}(\overline{X}_1 - \overline{X}_2)$

$\overline{X}_1$, $\overline{X}_2$ are the sample means

and where W is the pooled within group dispersion.

For linear reduced-space, classification into groups can also be accomplished by utilizing equation III-6 and comparing the Z-value for the observation in question with an appropriate cutoff value. Welsh [B] showed that this procedure provides optimal classification results when the parameters are the true parameters of the population, the distributions are multivariate normal, and the dispersion matrices are equal. The sampling bias that occurs for the original sample, i.e. when the sample parameters differ from the population parameters, can be determined by the Lachenbruch holdout method;[18] the total bias can be determined by classifying a secondary sample. If the concern is only with the sample at hand, then consideration of bias is irrelevant, since the same sample that formed the equation is classified. (See the section on 'Bias' for a complete discussion of these topics).

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[18]The Lachenbruch holdout method is discussed later in this chapter.

As was previously discussed, the classification results from MDA are robust to non-multivariate normal data. However, unequal dispersion matrices may create a substantial bias on the classification results if linear formulations are employed. To eliminate this bias the quadratic classification procedure should be used when the dispersion matrices are substantially different.

The two-group quadradic classification rule is obtained by substituting the multivariate normal density functions $N_1 (\mu_1,\Sigma_1)$ and $N_2 (\mu_2,\Sigma_2)$, with $\Sigma_1 \neq \Sigma_2$, into equation III-13, taking the natural logs of both sides, and rearranging terms. (See Eisenbeis and Avery [B, p. 16].) The resulting quadratic rule becomes:[19]

Assign to population 1 if:

III-16

$$X'(\Sigma_1^{-1} - \Sigma_2^{-1})X - 2(\mu_1'\Sigma_1^{-1} - \mu_2'\Sigma_2^{-1})X + \mu_1'\Sigma_1^{-1}\mu_1 - \mu_2'\Sigma_2^{-1}\mu_2 \leq \ln |\Sigma_2 \cdot \Sigma_1^{-1}| - 2 \ln p.$$

When the population parameters are estimated from sample data, the sample dispersions S_1 and S_2 are the best estimates of Σ_1 and Σ_2, and $\bar{X}_1$ and $\bar{X}_2$ are the best estimates of μ_1 and μ_2. (The appropriate substitution for sample data is straight forward). The quadratic reduced-space procedure parallels the linear method; however, as mentioned earlier less is known about the theoretical applicability of the procedure. Subsequent discussion will elaborate on the quadratic reduced-space method.

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[19]There are three possible classification rules for G groups. These rules use 1) Chi-square scores, 2) probabilities of group membership, and 3) p-space linear discriminant functions. For development of these rules see Eisenbeis and Avery [B, p. 17-20]. For a verbal explanation and an application see Smith [C 1965].

Linear versus quadratic classification results

Use of the quadratic rule in place of the linear rule depends on the equality of the group dispersion matrices. Eisenbeis and Avery [B, p. 37-52] provided the only examples of linear vs. quadratic results that appear in the literature. Specifically, Eisenbeis and Avery [B, p. 38] contended that:

> For given differences between group variable means, the predictive power of the linear rules relative to quadratic rules decreases as the difference between the group dispersions increases. The reason for this is that the formation of the linear rules requires that the group dispersion matrices be pooled to estimate the "hypothesized common" within-groups dispersion matrix. The effect is to bias the classification results away from the group with the smaller dispersion matrix. . .

Other factors that may have a substantial influence on the relative power of linear versus quadratic rules are the extent of group mean separation and/or the extent of group overlap, the number of groups used, and the extent of true differences in the dispersion matrices (rather than measured differences from Box's test of equality, which is sensitive to non-multivariate normal data and to sample size, as discussed earlier).

A priori probabilities

The *a priori* probabilities mentioned above in connection with equations III-13 through III-16 refer to the probabilities that given observations belong to the population in question. Empirical studies often use equal *a priori* probabilities, which can bias the results if the true probabilities in the underlying population are not equal. Another commonly employed method is to use the probabilities found in the sample being investigated. This creates a bias if the probabilities of the population or secondary samples differ from the probabilities of the original sample. However, if the relative probabilities for the populations are unknown, the best method to obtain the *a priori* probabilities is from the sample. One must then assume, of course, that the sample constitutes a representative selection from the populations in question.

Eisenbeis and Avery [B, p. 52-53] contended that incorrect *a priori* probabilities can have significant effects on the classification results of secondary samples. This result occurs because the *a priori* probabilities affect the constants in the classification equations, which in turn affect the cutoff values.

Testing the classification matrix for significance

The usual test of significance for the proportion of correct classifications is a t-test:

III-17
$$t = \frac{P_c - .5}{\sqrt{\frac{.5(1 - .5)}{N}}}$$

where P_c = the proportion correct

N = sample size

and .5 is the proportion of correct observations expected by chance.

The t-test is appropriate when the binomial distribution created by the classificatory procedure is symmetric. This occurs when N is large. (See Lubin [B, p. 102]). Other tests of significance include a Chi-squared test (Press [B, p. 382]) and a binomial test (Lubin [B, p. 101]).

As will be discussed in more detail shortly, if the original sample is used to test the significance of the proportions, then the test results are biased and will reject the null hypothesis of no significant difference in classification results more often than warranted by the data. Therefore, the t-test should also be performed on a secondary sample, with any conclusions concerning the predictive ability depending on the results of this latter sample rather than on these from the original sample. If a secondary sample is unavailable then part of the sample bias can be determined by using the methods discussed by Lachenbruch [B 1967] or Frank, Massey, and Morrison [B].

Stepwise Procedures

In the past several years an increase in the use of stepwise options for both regression and discriminant analysis has occurred. Examples of the stepwise procedure for MDA can be found in Altman [C 1968], Edmister [C], Simkowitz and Monroe [C], Van Matre [C], and Wood [C]. With proper use and foresight, the stepwise option can provide benefits to the user. Improper use can reduce the discriminatory power of the function and introduce a significant search bias.

Benefits of a stepwise procedure

Depending on the research design and purpose of the project, the investigator may wish to limit the number of variables that will appear in the final function. The stepwise option is appealing for several reasons. First, computer routines for inverting matrices become increasingly inefficient as the size of the matrix increases. (MDA requires matrix inversions). Thus, the stepwise procedure can aid in reduction of the size of the matrix and therefore help to reduce computer time. Second, variables that add an insignificant amount of information to the function can be eliminated. In fact Rao [B 1966] shows that adding insignificant variables can actually reduce the discriminatory power of the function below a given level of statistical significance.[20] Finally, stepwise analysis simplifies the function.

Several methods have been suggested in the literature for eliminating unnecessary variables from a function. Weiner and Dunn [B] compare four such methods: a) t-test of individual variable means, b) comparing standardized coefficients, c) stepwise selection, and d) random selection. They find the stepwise procedure to be the best.

Three methods are used to implement the stepwise concept: the forward stepwise, the backward stepwise, and the complete stepwise procedures.[21] Of these, the forward stepwise is dominant in the Finance literature, since the BMD07M computer package has been used by most researchers.[22] The forward and complete stepwise procedures add one or q variables at a time to the existing set of p variables, while the backward procedure removes one or q variables at a time from the function. The forward stepwise method selects the variables that maximize the F-ratio for p + q variables. This optimal set of p + q variables becomes the basic set that may be enlarged by adding another set of variables, via the stepwise criteria of maximizing the F-ratio. The procedure is terminated when the addition of new variables does not satisfy the

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[20]The problem of multicollinearity also is of concern. The existence of multicollinearity creates instability for the (relative) parameters of the function. To the extent that variables do not provide additional information for the analysis, these variables can be eliminated by a stepwise procedure and multicollinearity reduced, making the parameters more stable. For further discussion on the advantages and disadvantages of multicollinearity see a later section of this chapter.

[21]All the stepwise procedures used in MDA are based on a linear discriminant function; quadratic stepwise procedures have not yet been developed.

[22]The BMD07M program actually uses a combination of the forward and backward procedures.

appropriate significance test. The backward stepwise method is similar to the forward procedure except that it removes variables from the function instead of adding them. The complete stepwise method selects the variable combination from the total variable set that maximizes the F-ratio. Unlike the forward or backward procedures the complete stepwise variable set for p + q variables does not necessarily contain the same variables as found in the set of p variables. Rather, the complete stepwise method chooses those p + q variables that maximize the F-ratio, without constraints as to which variables must be included or excluded. Thus, the complete stepwise method provides optimal results for any given set of p + q variables. To obtain the F-ratio, which show the best combination of variables for a given variable set size, the D^2 statistic to measure the distance between the mean vectors of p and p + q variables is employed. (See Rao [B 1948a, p. 64] for the F-test based on the D^2 statistic; equation III-12 indicates the F-test for the entire variable set.)

To analyze the effect of the added or deleted variable on the function, comparing the resultant F-values between p and p + q variables is inappropriate, since the degrees of freedom are different in each instance. If comparison is desired, the percentage significance levels should be used. Such a procedure eliminates the problem of degrees of freedom and allows as many insignificant variables as possible to be eliminated, until some predetermined significance level is reached.

Bias

The practical significance, as well as statistical significance, of MDA as a prediction technique is affected by the creation of a 'Best Fit' bias. Specifically, sample data are employed to determine the parameters of the function which often results in parameters that are sample sensitive. A second type of bias, search bias, occurs when a stepwise procedure selects the "best" set of variables to generate the parameters of the function.

The recommended procedure to account for the effect of these biases is to classify a secondary (new) sample from another time period; the extent of the total bias will then be revealed. Other procedures currently in use to estimate bias may actually under-estimate the extent of the total bias. Such procedures include the split sample and simulated sample approaches of Frank, Massey, and Morrison [B] and the holdout method of Lachenbruch [B 1967]. Even the secondary sample approach will underestimate the total bias if the new sample is taken from the same time period and the discriminant function is not stable over time.

Sampling bias

Classification of the observations employed to generate the discriminant function will minimize the misclassification rate for the sample in question but not necessarily for the population. Recently Lachenbruch [B 1967] developed a method to adjust the misclassification rate from the original sample in MDA for sample sensitivity. The Lachenbruch holdout method eliminates one observation at a time from the original sample; the parameters of the function are determined from the remaining observations and the holdout value is then classified. This procedure is repeated in turn for each observation. The misclassification rate for the holdout sample can then be used as the adjusted misclassification rate for the population. Although search bias plus any bias related to nonstationarity over time are not considered by the Lachenbruch method, this technique is preferred to the split sample and simulated sample approaches for determining the effects of sample bias. This preference results from the sequential removal of one observation at a time by the Lachenbruch method, which creates considerably less sample sensitivity effects than the other methods when the discriminant function parameters are being estimated.

Search bias

A procedure exists for limiting the adverse effect of search bias on the classification results from utilizing the stepwise method. Specifically, a significance level for variable selection is chosen such that the misclassification effect due to the search bias is minimized. This method differs from the usual method where a 5 percent or 10 percent significance level is employed to determine whether or not additional variables should be added to the function based on the concept that a statistical difference between p and p + 1 variables must exist at a 5 percent or 10 percent level of significance to indicate a significant gain in information as portrayed by the discriminant function. However, this approach is inadequate, for two reasons. First, a combination of several variables may add a significant amount of information, even though individual variables fail the significance test. Second, the use of a 5 percent or 10 percent significance level in the statistical estimation of the discriminant function is inappropriate when classification results have priority over the statistical difference between group means.

The procedure recommended above eliminates the first problem in that the meaning of the F-statistic is redefined in order to examine the decrease in discriminatory power from the total variable set, not just one variable. This new definition allows the researcher to measure the difference between the chosen set of variables and the total set; if a significant statistical difference exists between the two sets of variables then additional variables need to be added to the stepwise set.

The second problem relates to the sensitivity of the classification results as the significance level changes. While no simulation studies have been published regarding the effect of variable elimination on classification results, Eisenbeis and Avery [B, p. 77] suggest that a 99 percent significance level be used in order to reduce the amount of information that is lost from the stepwise process. Results obtained in this study show that the 99 percent rule is superior to the use of a smaller significance level.

The Relative Importance of the Independent Variables

Few theoretical studies on MDA have discussed methods for determining the relative importance of the independent variables, since the main purpose of MDA is to discriminate between groups. Furthermore, the statistical technique most appropriate for determining the importance of the independent variables is multiple regression analysis.

Two problems relate to interpreting the relative importance of the MDA variables. First, the coefficients in the estimated discriminant function are not directly comparable when the variables are measured in different unit sizes.[23] Also, because only the ratios of the discriminant function coefficients are unique, no universally acceptable method exists for evaluating a variable's contribution to a particular function or its

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[23]A similar situation occurs for multiple regression analysis. For MRA "beta" (adjusted) coefficients are obtained by adjusting the original equation: $Y/\sigma_y = a/\sigma_y + b_1\ (\sigma_1/\sigma_y)\ (X_1/\sigma_1) + b_2\ (\sigma_2/\sigma_y)\ (X_2/\sigma_2)$. However, for MRA the dependent variable Y does not exist in the same form as in MRA, thus σ_y can not be used to obtain the relative contribution of the variables.

relative contribution vis-a-vis the other variables. Second, the parameters and relative rankings of these parameters are unstable when substantial multicollinearity exists. Because of these problems, no theoretically correct method is available for determining either the relative ranking or the percentage contribution of the independent variables to the linear discriminant function. Additionally, Massey [B, p. 29-36] reports that non-linear coefficents are difficult, if not impossible, to interpret.

When authors do attempt to consider the importance of the independent variables, one of two techniques is usually employed. One method is to scale the coefficients by multiplying each coefficient by its own standard deviation. This gives the percentage contribution of each variable. Although widely employed, this technique is highly sensitive to sample differences and to multicollinearity.[24] Moreover, this method tends to favor a variable for possessing a high standard deviation.

The second technique used is to rank the variables according to their order of inclusion (or exclusion) from the stepwise function. Since variable inclusion is based on the highest F-ratio, the order of inclusion indicates which variable will contribute the greatest amount of additional information in discriminating among the groups. In addition, if the stepwise analysis is used to remove insignificant variables from the function, the ranking and relative percentage contributions of the included variables should be more stable, for two major reasons. First, the effect of sampling bias and search bias on ranking and parameter instability should be reduced when insignificant variables are removed. Second, the

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[24] High multicollinearity has significant effects on the stability of individual coefficients.

removal of multicollinear variables should substantially reduce individual parameter instability, as has often been demonstrated in regression studies. Unfortunately, the stability of the discriminant parameters, even after insignificant variables are removed via the stepwise method, often leaves much to be desired.[25] Thus, the problems involved in determining the relative importance of the variables in discriminant analysis remain.

Multicollinearity

Interpretation of the effects of multicollinearity on MDA depend in large part on the use intended for the analysis.[26] The theoretical development of MDA has evolved around a desire to develop methods to correctly classify observations. In this case a certain amount of multicollinearity may aid this process by increasing the ability of the function to predict group membership. An example should help clarify such a situation; suppose that a sociologist wishes to classify individuals as either Japanese or "white American." Variables measuring height and skin color[27] are used to classify a sample. Separately, each variable may correctly classify (say) 75 percent of the sample. Combined, the discriminant function may classify 95 percent of the observations correctly. This increase in discriminatory power occurs because of the collinearity of the independent variables, e.g. only some Japanese are both tall and

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[25]Although individual parameter values may still be unstable, the classification results from the discriminant function could be stable, especially if relatively few observations are near the discriminant border. Furthermore, the remaining parameter instability results from the residual sample and search bias effects.

[26]Farrar and Gluber [B] discuss multicollinearity in relation to regression analysis. They review the historical approaches to the problem and the present procedures to detect the evidence, severity, and location of any multicollinearity.

[27]Assuming that skin color can be quantified.

have a light skin tone. Thus, it is evident that multicollinearity can be beneficial in MDA.

While most researchers are interested in the classificatory ability of the discriminant function, they may also be interested in the relative importance of the variables. Multicollinearity among the explanatory variables creates unstable individual parameters and thus reduces the possibility of adequately describing the contribution of individual variables to the discriminatory ability of the function. (As previously noted, this contribution can not be adequately explained with certainty anyway). Since multicollinearity may aid discrimination, but creates instability for ranking the variables, a conflict develops when both are attempted at the same time.[28]

Although multicollinearity is often thought of as 'existing or not existing', there are, in reality, degrees of interdependence. At one extreme complete independence exists and the discriminatory power of the function depends on the sum of the discriminating ability of the individual variables; consequently, the problem of instability of the coefficients is reduced. At the other extreme, complete dependence is present (at least between two variables or for some linear combination of the variables). In this case, a singular matrix is present which can not be inverted, and the coefficients of the discriminant function cannot be determined. This situation necessitates the removal of the offending variables to permit the computations.

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[28]For regression analysis, multicollinearity does not affect the estimated $\hat{y}$ values, but often seriously affects the stability of the parameters of the independent variables. Furthermore, the standard error of the estimate and the predictability of new observations may not be affected. See Neter and Wasserman [B, p. 341] and Spurr and Bonini [B, p. 610-612].

The situation of near perfect multicollinearity is more troublesome. A decision must be made to determine when the interdependence is 'harmful' to the model. Determination of the 'harmfulness' of multicollinearity, of course, once again involves consideration of the purpose or intent of the model development.

Because of the apparent similarity of MDA to MRA, many authors have assumed that <u>any</u> multicollinearity in MDA is inherently bad. However, if the intent of the analysis is to provide classification results, multicollinearity may be helpful. Ideally, one should determine if a highly collinear variable is aiding the discriminatory ability of the function or if it is only measuring 'the same thing' as another variable. In the latter case the increase in information (or D^2) obtained by adding this variable to the function will be small; in the former case D^2 should increase significantly.

Cochran [B] studied the performance of MDA under conditons of independence and interdependence. Although the results are not exhaustive, they do provide some support for the use of correlated variables in MDA. In particular, Cochran [B, p. 185-86] compares D^2 values of correlated variables to those of independent ones and concludes that:

1. The D^2 values may be larger when the correlations between variables are high and positive (if they measure different things)
2. A variate or set of variates which has negative correlations with most of the other variates will probably have a larger D^2 value (although negative correlations are unlikely in practice)
3. For a covariate with no discriminating ability, any correlation is helpful, although small correlations do not help much.

The preceeding, including Cochran's [B] study, indicates that collinearity will aid classification results for the original sample when the addition of collinear variables adds information. The effect on the classification results of secondary samples is less straightforward. If we continue to assume that the additional collinear variables provide increased information (and if the secondary sample is not sample sensitive or unstable over time) then this new information obviously will aid the classification of secondary samples. When the assumption of little or no sample sensitivity and/or unstableness over time is relaxed then, of course, the classification results from a discriminant function using relatively stable samples should outperform the results from unstable samples. But this comparison is invalid for the purpose at hand. What is needed is a comparison of classification results from the secondary sample when the collinear variables are included versus the situation when such variables are excluded. Generally, inclusion of collinear variables will provide superior classification results, since additional relevant information is avaiable. Of course, the greater the sample sensitivity or instability over time then the larger the bias between the original and secondary samples.

Although the inclusion of moderate to highly collinear variables may increase the classificatory ability of the discriminant function, this collinearity conflicts with the goal of providing relatively stable individual parameters. Unfortunately, research concerning the effects of collinearity on the MDA technique is not definitive. Hence, no acceptable method to treat all cases of collinearity in MDA exists. Essentially, the researcher employing MDA is faced with a choice of eliminating as much of the collinearity as possible to provide stability of the parameters, versus allowing certain collinear variables to remain in the function to aid the classification results.

The appropriate procedure seems to be a compromise. The first step is eliminating insignificant variables, which are often highly collinear with other variables. The second step is eliminating variables that possess very high correlations, say above .95. Such a procedure will eliminate the collinearity that is most destructive to the stability of the parameters, while causing the least damage to the classificatory ability of the function.

Summary of MDA Application Problems

To summarize the MDA technique, a list of universal criticisms and a list of frequent misapplications of MDA are presented. In this way the theoretical aspects of MDA can be summarized in a practical format. Moreover, care is taken in this paper to insure that the omissions and misapplications outlined below are avoided. A list of the authors who incorrectly applied the MDA technique follows each item.

Criticisms and/or ommissions that are universal to Finance Studies employing MDA are 1) the test for the equality of the variance-covariance matrix is not performed; 2) non-linear classification (when appropriate) is not utilized; 3) reduced-space operations may be incorrectly substituted for a test-space procedure when the variance-covariance matrices are unequal; and 4) the complete stepwise option is ignored in favor of the forward stepwise procedure.

In addition to the above theoretical omissions, several misapplications of MDA to practical situations may be cited. These are listed below:

1. secondary samples are not tested to eliminate the 'best fit' bias and/or the split-sample approach to detect such a bias is incorrectly applied. Altman [C 1968], Van Matre [C], Wood [C], Edmister [C], Stevens [C]
2. Averages instead of actual data are employed in the analysis. Altman [C 1973], Stevens [C]
3. Changing parameters over time are ignored, or a non-constant time period is employed. Altman [C 1968 and C 1973], Van Matre [C], Simkowitz and Monroe [C], Edmister [C], Stevens [C]
4. Equal a priori probabilities are employed when they are inappropriate. This procedure decreases the possibility of a Type I error, but substantially increases the Type II error. Altman [C 1968 and C 1973], Simkowitz and Monroe [C]
5. An incorrect procedure is used to select the variables used in the function or the stepwise method is incorrectly interpreted to obtain the best combination of variables for classification purposes. Altman [C 1968], Van Matre [C], Wood [C], Simkowitz and Monroe, [C], Pinches and Mingo [C]
6. An inappropriate approach to the multicollinearity problem is used. Pinches and Mingo [C], Edmister [C], Stevens [C]

CHAPTER IV

RESEARCH METHODOLOGY AND DATA

This study investigates the ability of supply and demand factors, as measured by various daily technical indicators, to predict the direction of the Standard and Poor's 500 Index. In this chapter a description of the variables, data, methodology, statistical model, computer program, and trading rules employed in the research are given.

Variables

A basic tenet of economics concerns the relationship of supply and demand to prices. Most authorities agree that supply and demand forces can be readily used to explain past movements of the stock market. However, using past supply and demand information to predict future prices is an entirely different problem.

One means of measuring supply and demand is to investigate the information provided by "technical" variables such as volume, number of stocks advancing/declining, odd lot data, new highs and lows, etc. The specific variables used in the analysis are listed and described below. Brief explanations regarding certain variables follow the listing.

Two sets of technical variables are employed in the analysis, actual values of the supply and demand relationships, and percentage change values of these relationships. Since the two sets of variables measure similar characteristics in the underlying market mechanism, combining all the variables into one set is inappropriate. Accordingly, these two groups are analyzed separately. Henceforth, the actual value variables will be designated as the "Regular" set, while the percentage change values will be the "Percentage" variables.

Regular variables:

(1) VOL = daily volume on the NYSE/number of shares outstanding on NYSE

(2) ADV = the proportion of the total number of traded stocks that advanced

(3) DEC = the proportion of stocks that declined

(4) UNCH = the proportion of stocks that remained the same

(5) ODDP = odd-lot purchases/NYSE volume for the day

(6) ODDS = odd-lot sales/NYSE volume for the day

(7) ODSS = odd-lot short sales/NYSE volume for the day

(8) HIGH = number of new highs/number of issues on NYSE

(9) LOWS = number of new lows/number of issues on NYSE

(10) AVOL = volume NYSE/5 day moving total of NYSE volume

(11) MMON = money flow, or VOL (times) the price change in the Standard and Poor's 500

(12) AVDC = number of stocks advancing/number of stocks declining

(13) SLPU = odd-lot sales/odd-lot purchases

Percentage variables:

The percentage variables consist of the percentage changes for variables (1) to (7); for example:

(14) $\%VOL = \frac{VOL_j - VOL_{j-1}}{VOL_{j-1}}$ where j refers to the time period.

Thus, variables (15) to (20) are %ADV, %DEC, %UNCH, %ODDS, and %ODSS. In addition, for completeness, variables (10) to (13) are repeated in the set of percentage variables.

Explanation of the Variables

Variables (1) to (9) are standardized to remove the effects of an expanding market over time. Without this adjustment, the data are not comparable from one time period to the next. The first nine variables are commonly associated with technical analysis. Various articles have discussed the relationship of these variables to price movements. In particular, Osborne [A], Ying [A 1966], and Crouch [A 1970a and A 1970b] give ample evidence that a relationship between volume and price changes exists.[1] Specifically, Osborne claims that the volume sequence leads the price sequence, and both Ying and Crouch show that abnormal volume can be related to price changes.

Support for the use of advance/decline/no-change data is presented by Osborne [A], Krow [A], Thiel and Leenders [A], Zakon and Pennypacker [A], Dryden [A 1968 and A 1969], Harlow [A], and Philippatos and Nawrocki [A 1973a and A 1973b]. Fama [A 1965b] reports that little or no dependence exists between prices and advance/decline data.

Odd lot statistics provide information concerning small investors' intentions. Krow [A], Kewley and Stevenson [A 1967], and Stevenson [A 1973] generally criticize the ability of odd-lot data to provide any meaningful information in regard to price forecasts, while Lambert [A] and Wu [A 1972 and A 1973] claim such data provides substantial information to the investor.

Finally, while the author is not aware of studies of the relationship of new highs and new lows to the price series, (except for their inclusion in Emery's [A] multivariate analysis), these statistics are

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[1]Hagin [A] claims a much smaller relationship than do the other authors, while Godfrey, Granger, and Morgenstern [A] and Granger and Morgenstern [A 1970, ch. 8] essentially contend that <u>no</u> relationship is present.

included here because technicians often include new highs and new lows in their decision models.

Variables (10) to (13) (i.e., AVOL, MMON, AVDC, and SLPU) are included in the model because they are widely employed by technicians. A version of AVOL can be found in the Sibbet-Hadady Publications [A] and The Financial Weekly [A]. The money flow variable, MMON, is found in Financial Weekly and Chartcraft [A], while a similar variable can be found in Sibbet-Hadady Publications and The Mansfield Trend of Speculative Activity [A]. AVDC can be found in The Financial Weekly (as a moving average).[2] Finally, SLPU is described in the ISL Daily Stock Price Index [A], Long Term Technical Trends (published by Stone and Mead) [A], and The Financial Weekly [A].

Excluded Variables

Since the advance/decline/no-change variables are measured in proportions, any one of these variables is a linear combination of the other two. Theoretically, a matrix containing all three variables can not be inverted, since it is singular. In addition, correlation analysis of the sample data shows that the simple correlation coefficient between the advance and decline proportions usually exceeds .99. Thus, since these two variables provide almost identical information to the user, only the ADV or the DEC variable is included in the model for any given data set. ADV is used during bull-market periods and DEC is employed for bear-markets periods.

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[2]Although some may argue that ADV provides the same information as does AVDC, on an empirical basis the inclusion of both variables: 1) increases the F value of the function such that the difference between group means in increased, and 2) both variables appear early in the complete stepwise procedure, showing that they provide information which aids in discrimination and classification.

For the percentage set of variables, no indicators correspond to the HIGH and LOWS that appear in the regular set. They do not so correspond because many times each of these variables is either close to, or equal to, zero. Low values for these indicators create extremely large percentage changes given small increases in the variable itself (percentage changes approach infinity when either of the variables increase from a value approaching zero). Such large values are counter-productive to the purpose of the study, since accurate classification on new data are hampered by the erratic contribution of these variables to the overall discriminatory ability of the function.

Variables (1) to (9), their percentage equivalents, and variables (10) to (13) provide a good representation of possible variables that a technician might employ in analysis of market trends. However, several other possible variables appear regularly in published technical services and newsletters. Three of the most common variables not included here are:

MONY = VOL (times) Standard and Poor's Price Index

% MONY = percentage change in MONY

SSSL = odd-lot short-sales/odd-lot sales.

Casual inspection suggests that MONY is very similar to MMON. However, MONY uses the Standard and Poor's Price Index, while MMON employs the change in this index. Therefore, the two variables measure different things, as evidenced by their low correlations of .3 to .4 over the different time periods used in this study. Since MONY is a combination of VOL and the Standard and Poor's Index, and since the level of the price index will not change drastically from day to day, MONY and VOL measure the same information. This interpretation is supported by high correlations greater than .975 between these variables for different time periods. Consequently, MONY is not used in the analysis.

Similarly, %MONY and %VOL measure the same information. The correlations between these variables over the different time periods studied here are mostly greater than .99, with a few exceptions where the correlation is greater than .97. Thus, %MONY is also not utilized.

Finally, SSSL can be compared with ODSS. Since SSSL = odd-lot short-sales/odd-lot sales, and ODSS = odd-lot short-sales/NYSE volume for the day, the numerators are equivalent. The critical comparison then lies in the denominator. Logically, odd-lot sales would be expected to be highly correlated with the NYSE volume. In fact, the correlation coefficient is greater than .9. This correlation, in turn, creates a correlation coefficient greater than .98 for ODSS and SSSL. Thus, the variable SSSL is not included in the analysis.

For each of the three variables discussed above, a second (empirical) reason to eliminate them from the analysis is that each variable (if they are included in the total set) enters the discriminant function near the end of the stepwise sequence, indicating that little or no information is added by their presence.

Data

The data employed to generate the variables discussed in the last section are obtained from the ISL Daily Stock Price Index [A] and from John Emery, The University of Arizona. Daily observations on volume (NYSE), number of stocks advancing, declining, and remaining unchanged; odd-lot purchases, sales, and short-sales; the number of new highs and new lows; and the Standard and Poor's 500 stock-price index are obtained from January 1961 to October 1965, and from April 1972 to March 1974, from the ISL Index. Emery provided data on these variables from October 1965 to March 1972. The actual variables used in the analysis are obtained by taking the raw data and making the transformation indicated above (i.e. VOL = volume on NYSE/number of shares outstanding).[3]

The Hypothesis

The basic null hypothesis of this study is simply stated: Past technical data cannot be used to predict future market actions. The basic hypothesis is restated in the following ways: First, the statistical aspect of the discriminant function is tested for the null hypothesis is: No statistically significant differences in future market actions are explained on the basis of technical indicators. Second, if the model shows statistically significant dependence, then the predictive power of the discriminant function is examined and attempts are then made to use this dependence to formulate a trading model.

A statistical test of the relationship between technical data and future market movements is accomplished by analyzing the statistical difference between the group means, computed via a discriminant function, of predicted 'up days' versus predicted 'down days'.[4] This test involves obtaining and checking the statistical significance of an F-value based on the mean difference between the overall group means. A significant F-value implies that the technical variables are useful in segregating

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[3]The data are checked extensively for errors and corrected when such errors are detected. Crosscheck references include ISL data, Standard and Poor's Statistics [A], and Barron's [A].

[4]To compute the desired discriminant functions the technical data are lagged one period; thus, the discriminant function relates the technical data in period t to the price index in period t + 1. This one-period lag is maintained in all subsequent analysis.

subsequent movements of the market, to show that dependence does exist in the data (i.e., past technical data are related to future price changes).

The statistical analysis is extended by classifying the observations into groups. Since the testing of group-mean differences can show a statistical significance because of a large sample size when no practical significance exists, classification of the data into predicted groups is a further test of dependence. As discussed in Chapter III, the t-test is utilized to determine whether group designation can be predicted on a better than chance basis.

The classification analysis is extended by testing data from another time period in order to determine whether or not the function is stationary over time. (Nonstationarity can be a major problem since the parameters in the discriminant function may change over time and/or the variables in the function may change). If the function is stationary, then a trading model can be developed. The trading model employes the discriminant function and data from future time periods to predict buy and sell days. If trading decisions made on the basis of the buy-sell predictions of the model are profitable before commisions, then profitability of the model on an after-commission basis is investigated. If the model is profitable on a consistent basis after commission charges, then the market may be classified as inefficient with regard to technical data.

Multivariate approach

The multivariate model outlined above is an obvious extension of previous univariate procedures. Theoretically, a combination of technical variables should improve the ability of the function to discriminate between the up-down price change groups since more information will be provided concerning the state of the market. In addition, the interaction effect between variables can help improve the classification ability of the model.

In applying a multivariate model, a complete stepwise procedure is used to reduce the size of the variable set and to eliminate insignificant variables.[5] The complete stepwise method is superior to a forward or backward procedure since the former method determines the best variable set (highest F-ratio) at each level by maximizing the difference between the means to the within group variance.

The selection of the cutoff point for the number of variables to be included in the stepwise function follows the suggestion of Eisenbeis and Avery [B, p. 77], who report that a variable subset possessing a 99 percent significance level when compared to the entire variable set is the appropriate size for the discriminant function. Thus, the complete stepwise set utilized contains about 99 percent of the information of the entire set. Such a high significance level is employed to minimize the loss of information and, consequently, to keep the misclassification rate as low as possible.

<u>The assumptions for MDA</u>

The two definitional assumptions for MDA, as outlined in Chapter III, are:

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[5]The inclusion of insignificant variables can reduce the statistical significance between group means because of a reduction in the degrees of freedom, resulting from the presence of the variables.

(1) the groups being investigated are discrete and identifiable;

(2) each observation in each group can be described by a set of measurements on m characteristics or variables.

In addition, the linear statistical model is based on the following theoretical assumptions:

(3) the data of each group form a multivariate normal distribution;

(4) the dispersion matrices are equal.

The first assumption, that groups are discrete and identifiable, needs to be examined, since the price relative variable (the discriminatory criterion variable) is essentially continuous. Although Eisenbeis and Avery [B, p. 36] are critical of the use of non-discrete groups, they provide little evidence for the complete avoidance of such studies (see Chapter III). Thus, the data needs to be analyzed to determine if the MDA methodology is appropriate.

An initial statistical test that investigates the applicability of MDA is to examine the difference between group means for statistical significance. If the means are significantly different, then one can hypothesize that the data are from two separate populations. In such a case MDA is appropriate. However, this simple test is affected by sample size; thus, one needs to compare classification results.

As previously discussed, Reinhart and Latane [C] introduce a method for evaluating the applicability of MDA by employing a multi-group analysis to compare the classification results of MDA and MRA. The technique presenting superior classification results is designated the appropriate method of analysis. If MRA is chosen, then the data are assumed to be continuous, while if MDA is superior, one can argue that the discriminant model is robust in terms of non-discrete groupings.

In this study, the Reinhart and Latane method is applied to three-group schemes defined on the basis of large, small, and middle groups of price relatives for given time periods. The percentages of misclassification for these groups for MDA versus MRA are then compared. The superior technique will have the smallest classification error.

The second assumption for MDA listed above indicates that a set of measurements is needed to describe each observation. Two sets of measurements (variables) are suggested to explain the behavior of the price relatives, and these variables are discussed earlier in this chapter. The empirical evidence regarding these variables will be investigated in the next chapter.

The third assumption for MDA is that the data are multivariate normal in all groups. Chapter III discusses the problem of testing for a multivariate normal distribution and concludes that an investigation of multivariate normality is extremely difficult. However, Gilbert [B] has shown that non-multivariate normal data do not seriously affect classification results for the two-group linear case. Thus, the assumption is made here that the results are robust to any deviations from multivariate normality that may exist.

The final assumption, that of equal variance-covariance matrices, applies to the linear discriminant function. If the matrices are significantly different, then the non-linear discriminant function should be employed. The test to examine the equality of the variance-covariance matrices is due to Box [B]. Chapter III discusses some possible shortcomings in the Box method. However, no investigations of these problems have appeared to date in the literature. Accordingly, projecting the true effects of the situation discussed in Chapter III on the selection of a linear versus a non-linear function is difficult. In the absense of contrary information, the Box test for the equality of variance-covariance matrices will be used; if the null hypothesis of equal variance-covariance matrices is rejected, then the non-linear function will be utilized.

Computer program

The computer program utilized to analyze the technical data is the MULDIS program, developed at The University of Wisconsin and the Federal Deposit Insurance Corporation by Eisenbeis and Avery [B]. MULDIS is unique in several ways (as compared to discriminant analysis programs in the BIOMED and SPSS packages). The program contains a test for the equality of the variance-covariance matrices, a non-linear classification procedure, complete and backward stepwise options (as well as the usual forward procedure), test-space as well as reduced-space operations, various significance tests, a Lachenbruch holdout option, selection of '*a priori*' probabilities, and several other options. MULDIS is currently the most complete and sophisticated discriminant package available.

Empirical Tests

Previous sections have suggested that a multivariate discriminant model would provide additional insights to the random walk/efficient market hypothesis. Investigations of the explanatory and the predictive power of such a model must consider potential changes in the character of the data. One convenient starting point for the analysis builds on the assumption that bull markets reflect attitudes which are fundamentally different from prevailing sentiments in bear market periods. Thus, the technical indicators and price relatives used in this study are analyzed initially in terms of bull and bear markets. For each bull-bear market period studied, the price relatives are ranked from highest to lowest. The top 50 prices relatives are designated as the "UP" group and the bottom 50 as the "DOWN" group. The remaining observations are considered insignificant, since their price relatives are near 1.0. The objective is to identify the best and worst price relative, since these are the most important movements of each period.

The complete stepwise discriminant procedure is applied to the "UP", "DOWN" groups in an attempt to explain and forecast market movements based on technical indicators. The stepwise procedure gives the best combination of technical variables in terms of their relationship to the future direction of the market. Statistical dependence in the data is examined by testing the difference between group means. This procedure is completed for both sets of variables and for each bull and bear market period. Table IV-1 indicates the time periods corresponding to each bull and bear market studied, and the related price movement of the Standard and Poor's Index.

If statistical dependence is found in the data (i.e. a significant difference between group means exists), then the next step is to determine the classification results for the best and worst price relatives of the same period. If the classification results are statistically significant then two types of secondary tests can be attempted. One test is to classify the best and worst price relatives for the next bull and bear market periods. The other test, which is somewhat less meaningful, is to classify the entire data set from the same time period to show whether or not the equation possesses discriminatory ability for the entire data set or only for the largest and smallest price relatives.

TABLE IV-1

DATES AND INDEX VALUES FOR
BULL AND BEAR MARKETS

Market	Dates	S&P Index
Bull	Jan 1961/Dec 1961	57.57 - 72.64
Bear	Dec 1961/June 1962	72.64 - 52.32
Bull	June 1962/Feb 1966	52.32 - 94.06
Bear	Feb 1966/Oct 1966	94.06 - 73.20
Bull	Oct 1966/Nov 1968	73.20 - 108.37
Bear	Mar 1968/May 1970	108.37 - 69.29
Bull	June 1970/Jan 1973	69.29 - 120.24

Since the time intervals used to describe the various bull and bear market periods (see Table IV-1) generally cover many months, the relationships among the variables are likely to be non-stationary over time. Thus, attempts are made to determine the optimal time period of analysis, i.e., the length of the time period such that the relationship among the variables remains stationary, if indeed any such period exists. Investigation of the stationarity of the discriminant function is accomplished by comparing the classification results over various time periods. If the discriminant function is used to classify data from a new data set, and if the data have a misclassification rate that is not significantly different from the original data, then one may conclude that the discriminant function has a stationary time period. Moreover, for meaningful stationarity to exist, the time period over which the function is stationary should be consistent for all data periods under review.

Comparison of the classification rates of the original and the future time periods is complicated by the downward bias created by the MDA procedure for the misclassification rate of the original data set. This "best fit bias" can be eliminated by employing the Lachenbruch hold-out procedure, which removes the observation being classified from the matrix that determines the parameters of the function. With the "best-fit bias" eliminated, the misclassification rates may be compared on a consistent basis.

For purposes of illustration the analysis above shows that the discriminant function is stationary for a period of one year. The data under review are then analyzed by the same test procedures that are applied to the bull/bear market time periods. More specifically, statistical significance between group means is analyzed, classification results for the original data are investigated, and the ability of the discriminant function to classify the next year's data is determined. If the results of this analysis are positive, then one may attempt to develop a trading model to exploit any dependencies in the data.

The above progression of events leading to the development of trading rules makes two very critical assumptions. One, it assumes that equality of the variance-covariance matrices has been tested and, if the matrices are unequal, a non-linear discriminant function has been employed in the above investigations. Two, the groups are assumed to be discrete. As previously discussed, Eisenbeis and Avery believes that an essentially continous variable, such as price relatives, should not be used to designate groups. However, MDA may be robust for non-discrete data. To test the applicability of MDA one can compare the MDA results to those of MRA. If the former is superior, then MDA is robust; if the latter, then MRA are made on the basis of three-group classification results (following the method of Reinhart and Latane [C]), with the method presenting the smallest misclassification rate being chosen as the superior technique.

The Trading Rule

Assuming that some form of the MDA model can successfully classify observations from future time periods, trading rules are developed to determine if the discovered dependencies can be put to profitable use. A successful MDA model (in terms of classification results) could possibly be unsuccessful as a trading model. This situation would occur if there is a predominant 'whipsaw' effect,[6] if observations from the middle

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[6] A 'whipsaw' effect occurs when the technical data indicate the market should be heading in one direction when, in actuality, it consistently goes in the opposite direction.

group showed false buy and sell signals, and/or if any dependencies in the data as signaled by the MDA model are insufficient to overcome commissions charges.[7]

To establish a trading rule based on the MDA output, use of the reduced-space[8] equation in place of the test-space formulation is necessary, for several reasons:

1. Formulating a trading rule based on Chi-square (or similar) test-space data is difficult.
2. The Chi-square (and hence the percentage significance values for the probability of group membership) are based on a N-dimensional model that determines the relative distance from the group centroid for each observation; such a model causes problems when an observation is unimportant in terms of its future price relative but has an unusual value for one of the variables. Such an occurance causes a large Chi-square value,[9] and thus, one variable could trigger a buy/sell transaction when not warranted.
3. A practical aspect of using a reduced-space formulation instead of a test-space rule must also be considered; the reduced-space Z-value may be readily calculated while test-space information would normally require another computer run for each day's data.

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[7]A plausible estimate of commission costs would be a 1 percent charge on both a buy and a sell order.

[8]If appropriate, the quadratic reduced-space equation may be employed.

[9]One of the main problems is that the test-space formulation is unweighted, allowing one variable (which may be relatively insignificant) to dominate the Chi-square value. Another problem is that an observation lying outside the normal clustering composition of <u>both</u> groups will receive extremely high Chi-square values for each group and, consequently, a 99 percent probability of belonging to the nearest group.

The Z scores obtained from the two-group discriminant model are analyzed to determine a suitable filter rule. After obtaining the variance of the Z's for each group, one obtains an acceptable percentage cutoff for the Z distributions such that there is as little overlap as possible for the tails of the distributions. Figure IV-1 represents a diagram of the potential situation.

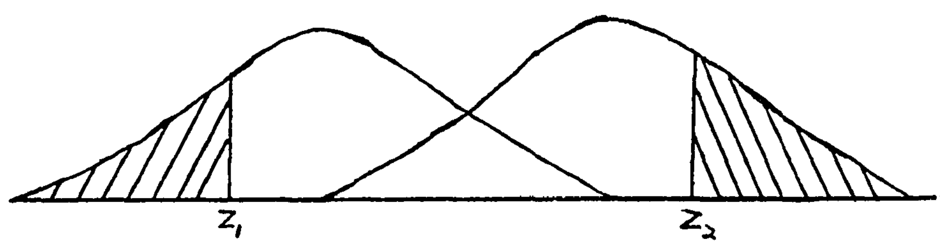

Figure IV-1

The Z_1-and Z_2-values indicate the cutoff values, with the shaded portions showing the filter values that would initiate a trade.[10] Applying the function and appropriate Z-values to subsequent data indicates whether the trading model is profitable.

The above procedure suggests several possible filter rules. One possibility is to test the method for the one-day, no commission situation. Such a rule would be appropriate for analyzing the ability of floor traders to profit from the trading model. A second situation is developed in terms of the following rule. Buy on an 'UP' signal (indicated by an appropriate Z-value) and hold the investment until a 'DOWN' signal is given, at which time sell and go short. Separate long and short loss should be maintained to indicate whether the rule is superior in one or

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[10]The distributions would have equal standard deviations if the linear discriminant function is used and equal standard deviations for a non-linear formulation.

both of these categories.[11] The preceeding rule would be profitable (at least before commissions) if there are few 'whip-saw' events, if the model is able to predict future large market moves, and/or if trends in the market are identified by the buy and sell signals.[12] To concentrate on longer term trends and to eliminate some of the 'whip-saw' effects a third, and stricter trading model requiring two verifications of the market movement is utilized. Here, a buy transaction is initiated after two consecutive[13] 'UP' signals are given, and the investment is terminated (and a short position taken) when two consecutive sell signals are produced.

If the above rules are profitable before commissions, then the after commission results should be examined for profitability and consistency to determine if the market is efficient.

The next chapter presents the results of the investigation outlined above. A summary of the sequence of the procedure follows:

A) Bull-Bear Market Analysis:

1. statistical significance tests for group mean differences;
2. classification results, original data;
3. classification results, subsequent market periods;

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[11]Logically, a trading rule developed in a bull-market situation should provide superior buy signals (at least in a continuing bull market); the ability of the rule to provide accurate sell signals is more doubtful.

[12]Z-values falling between the extremes are neutral. Note that trends in the data that are indicated by the model are not pointed out in the classification process. Thus, investigating the possibility that the MDA model is able to point out psychological trends (moods) is one more reason to study the trading model as well as the classification results.

[13]Here consecutive means two signals of one type occuring without an intervening opposite signal. A 'no transaction' signal (a Z-value between the buy and sell Z-values) may occur between two buy or sell indicators.

B) Examination of stationarity of data

C) Stationary time period market analysis:

  1. Statistical significance tests for group mean differences;

  2. classification results, original data;

  3. classification results, subsequent periods;

D) Examination of the discreteness of the price relatives

E) Development of Trading Rules

  1. Before and after commissions tests

  2. Experimentation with various cutoff Z-values

CHAPTER V

EMPIRICAL RESULTS

The previous five chapters form the theoretical foundation for the utilization of a multiple-discriminant analysis approach to investigate empirically technical indicators. Chapter II presents a review of efficient market theory and previous investigations of technical indicators; Chapter III details the multiple discriminant analysis model; and Chapter IV describes the methodology of the present study. In this chapter the multiple-discriminant analysis model is applied to various data sets of daily technical indicators, and the results are described and analyzed.

Bull- and Bear-Market Results

Employing the technical variables discussed in Chapter IV, the multivariate-discriminant model, and the 50-best and 50-worst daily price relatives for each bull- and bear-market period (the time period of each market is identified in Table IV-1), the two group statistical and classification discriminant analysis results are obtained for each market period. To determine whether or not a statistical difference exists between the 50-best and 50-worst daily price relatives for each period, the group means are compared. A statistically significant difference between group means indicates that the characteristics of the technical variables subsequent to 'UP' days are significantly different from the variable characteristics related to subsequent 'DOWN' days.[1]

Original Data

Table V-1 presents the significance levels for the statistical differences between group means for both the Regular and Percentage variable sets for each market period. The table also shows the significance levels before and after the complete stepwise procedure. The results show that the significance level is substantially reduced by using the complete stepwise set in place of the full set.[2] The table also shows that all variable sets except one are statistically significant at the 5 percent level after the complete stepwise procedure is employed. Moreover, even when the full set of variables is employed, only one of the Regular sets is not significant at the 5 percent level. Finally, the Regular set of variables has a smaller significance level than the Percentage set for most all the cases examined.

Since the sample size affects the significance results (larger sample sizes may provide statistically significant differences between groups when no real difference exists), the practical significance of the discriminant function is determined by classifying the data into groups.

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[1]Recall that the technical data are lagged one day. Thus, the technical data at time t are related to the price index at t + 1.

[2]Recall that the criterion to determine the optimum number of variables for a complete stepwise analysis is to determine when the decrease in discriminating power from the total variable set is still significant at the 99 percent level. This criterion eliminates insignificant variables and has minor effects on the classification results. The reduction in the significance levels depicted in Table V-1 shows the dual effect of eliminating insignificant variables, via the complete stepwise technique, and increasing the degrees of freedom.

TABLE V-1

STATISTICAL SIGNIFICANCE LEVELS FOR THE DIFFERENCES BETWEEN GROUP MEANS FOR BULL AND BEAR MARKET PERIODS

Market Period	Variable Set	Full Set Significance Level Percentage	Complete Stepwise Set Number Variables	Complete Stepwise Set Significance Level Percentage
1961	Regular	3.52	6	.128
Bull	Percentage	6.84	7	1.037
1962	Regular	2.85	8	.333
Bear	Percentage	9.33	7	2.610
1962-66	Regular	12.39	6	.665
Bull	Percentage	14.80	5	.705
1966	Regular	.0439	8	.00282
Bear	Percentage	.370	7	.0276
1966-68	Regular	.182	9	.0335
Bull	Percentage	40.60	5	5.073
1968-70	Regular	.0122	8	.000745
Bear	Percentage	.00427	7	.000226
1970-73	Regular	.000136	8	.00000350
Bull	Percentage	.0901	6	1.155

Tables V-2 and V-3 present the best and worst classification results among all the market periods for the Regular and Percentage variable sets. Both linear and quadratic results are presented for comparative purposes. Also, the results of Box's text of variance-covariance matrix equality are given.

For the Regular variable set the best classification results for the 1966 bear market occurs with the quadratic rule, which misclassifies 16 percent of the observations versus 27 percent for the linear rule. For the percentage variable set the quadratic rule also provides the best results by misclassifying 20 percent of the price relatives versus 29 percent for the linear rule. The worst classification results for the Regular variable set occur in the 1962-66 bull market period, where the quadratic rule misclassifies 29 percent and the linear rule 35 percent. The worst classification results for the percentage variables set occurs in the 1962 bear market where the quadratic rule misclassifies 29 percent of the observations versus 33 percent for the linear rule. The superiority of the quadratic method over the linear rule is confirmed in each case by the Box test results that show the variance-covariance matrices are not equal.

Table V-4 presents the complete results of the misclassification rates and dispersion tests for the linear and quadratic schemes, for each time period, for the two variable sets, and for the full and complete stepwise variable sets. Table V-4, with Table V-2 and V-3, show that, in general, the quadratic rule is preferable to the linear function, since the dispersion test is significant at the 5 percent level for all

TABLE V-2

BEST CLASSIFICATION RESULTS FOR FULL VARIABLE SET:
REGULAR AND PERCENTAGE VARIABLES
BULL AND BEAR MARKETS

Regular Set For 1966 Bear Market

		Quadratic Predicted Groupings				Linear Predicted Groupings		
		Up	Down	Total		Up	Down	Total
Actual Groupings	up	36	14	50	up	35	15	50
	down	2	48	50	down	12	38	50
	Total	38	62	100	Total	47	53	100

16 percent misclassified

27 percent misclassified

F-test of Dispersion matrix equality rejected at the 0.0 percent level.

Percentage Set For 1966 Bear Market

		Quadratic Predicted Groupings				Linear Predicted Groupings		
		Up	Down	Total		Up	Down	Total
Actual Groupings	up	35	15	50	up	32	18	50
	down	5	45	50	down	11	39	50
	Total	40	60	100	Total	43	57	100

20 percent misclassified

29 percent misclassified

F-test of Dispersion matrix equality rejected at the 0.0 percent level.

TABLE V-3

WORST CLASSIFICATION RESULTS FOR FULL VARIABLE SET:
REGULAR AND PERCENTAGE VARIABLES
BULL AND BEAR MARKETS

Regular Set For 1962-66 Bull Market

		Quadratic Predicted Groupings				Linear Predicted Groupings		
		Up	Down	Total		Up	Down	Total
Actual Groupings	up	44	6	50	up	31	19	50
	down	23	27	50	down	16	34	50
	Total	67	33	100	Total	47	53	100

29 percent misclassified — 35 percent misclassified

F-test of Dispersion matrix equality rejected at the 0.0 percent level.

Percentage Set For 1962 Bear Market

		Quadratic Predicted Groupings				Linear Predicted Groupings		
		Up	Down	Total		Up	Down	Total
Actual Groupings	up	28	22	50	up	36	14	50
	down	7	43	50	down	19	31	50
	Total	35	65	100	Total	55	45	100

29 percent misclassified — 33 percent misclassified

F-test of Dispersion matrix equality rejected at the 0.0 percent level.

TABLE V-4

MISCLASSIFICATION RATES AND DISPERSION TEST RESULTS
FOR BULL AND BEAR MARKETS: 50 BEST, 50 WORST
ORIGINAL OBSERVATIONS

Market Period	Variable Set	FULL SET: Quadratic Misclass. Rate Percentage	FULL SET: Linear Misclass. Rate Percentage	FULL SET: Dispersion Test Significance Level	COMPLETE STEPWISE SET: Quadratic Misclass. Rate Percentage	COMPLETE STEPWISE SET: Linear Misclass. Rate Percentage	COMPLETE STEPWISE SET: Dispersion Test Significance Level
1961	Regular	18	29	12.00	30	33	62.10
Bull	Percentage	22	31	2.49	26	31	1.08
1962	Regular	23	30	0.0	32	34	0.0
Bear	Percentage	29	33	0.0	36	30	0.0
1962-66	Regular	29	35	0.0	31	34	.209E-10
Bull	Percentage	27	30	0.0	49*	30	0.0
1966	Regular	16	27	0.0	21	21	2.63
Bear	Percentage	20	29	0.0	23	30	.00000187
1966-68	Regular	23	25	0.0	22	26	.0000528
Bull	Percentage	28	35	0.0	33	36	.230
1968-70	Regular	22	25	.000794	21	27	4.68
Bear	Percentage	13	26	.000727	24	24	.0155
1970-73	Regular	22	27	0.0	25	24	.219E-10
Bull	Percentage	30	27	0.0	43*	32	0.0

Note: all unstarred (*) misclassification rates are statistically significant at the .005 level.

but one case,[3] and since the misclassification rates for the quadratic rule are smaller than those provided by the linear rule.[4] The results of Table V-4 also show that the Regular variable sets are consistently superior to the Percentage sets.

Additional analysis of Table V-4 provides further interesting insights. For example, comparisons of the linear misclassification rates before and after the complete stepwise procedure show that in some cases an increase in the misclassification rates exists for the stepwise procedure, while in other cases a decrease is found in the rate of misclassification. The cases having a decrease in the misclassification rate from the full to the stepwise variable sets are probably attributable to the elimination of insignificant variable sets from the function. In the full variable set such insignificant variables may actually reduce the amount of useable information in the data, allowing the elimination of such variables in the stepwise process to provide superior classification results.

On the other hand, Table V-4 shows that there are several situations where the linear misclassification rate of the full variable set is less than the rate of the stepwise variable set. These cases suggest that a loss of information has occurred when the variable set is reduced.[5]

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[3]Note that the dispersion-test significance level rises from the full set to the complete stepwise set, indicating that the test is sensitive to the number of variables.

[4]In only one case for the full variable sets is the linear case superior.

[5]Recall that the selection of the variable set ignores the effects on the misclassification rate and concentrates on the F-value. The F-value measures the ratio of the between group variation to the within group variation. Certain types of data mentioned below may reduce the F-value only slightly, but greatly affect classification. Moreover, the

Two possible explanations may account for this loss in information. First multicollinearity that aided discrimination in the full set is reduced or eliminated when a variable is dropped from the function. Second, the linear formulation does not necessarily minimize the linear misclassification rate when the quadratic formulation is appropriate. In this case, the linear results in a complete stepwise procedure are biased upward from the true minimum misclassification rate.[6]

Another interesting insight is available in Table V-4. Comparisons of the quadratic results before and after the complete stepwise process reveal several cases of moderate to substantial increases in the misclassification rate. However, the increases in the misclassification rates for the quadratic rules after the stepwise process are not too surprising in that the stepwise procedure is linear in nature. In other words, the stepwise process maximizes a linear F-ratio instead of a quadratic ratio, which may increase the quadratic misclassification rate.[7] In addition, the use of the linear F-ratio combined with the 99 percent significance level used to chose the variable set and the number of variables in the set may have undesirable effects on the quadratic

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F-values are not comparable from one set size to another, since the degrees of freedom change. Therefore, the percentage significance levels should be compared. However, the significance levels are not strictly comparable when the differences are relatively small, since rounding errors and changes in degrees of freedom also contribute to the change in significance values. The effects on classification of changes in degrees of freedom and significance levels is unknown.

[6]The conditions when this bias could be significant are unknown.

[7]Unfortunately, non-linear F-tests and non-linear stepwise options are not available.

misclassification rate.[8] Finally, the linear stepwise procedure may actually reduce multicollinearity that is beneficial to quadratic classification.

Thus, several reasons may explain why the linear stepwise procedure theoretically may create inferior classification results when the final set of variables selected by the linear stepwise procedure is applied to a quadratic classification rule.[9] However, Table V-4 shows that in most cases, even after the stepwise procedure, the quadratic classifications are still superior to the linear results. Moreover, only a few situations exist wherein the quadratic misclassifications are substantially increased after the stepwise procedure.

In summary, Table V-4 shows that the quadratic function consistently outperforms the linear formulation both before and after the complete stepwise method. In addition, the complete stepwise method is theoreteically justifiable if one wishes to reduce the variable size of the function. Empirically, the linear stepwise procedure has little effect on the linear misclassification rate. The linear stepwise effect on the quadratic rate is moderately greater than on the linear rate but the quadratic procedure still is generally superior to the linear technique.

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[8]In this case, the 99 percent significance level is determined by comparing the <u>linear</u> F-ratios of the full variable set with each subsequent smaller variable set in the complete stepwise process. Thus, the size of the 'optimal' variable set, as well as the variables selected, are influenced by the <u>linear</u> F-values.

[9]The relative effect of the linear stepwise technique on the quadratic results can be noted by reflecting that only one variable set and market period gives superior linear results before the stepwise process, while three are superior afterwards.

Classifying other observations

Since Table V-4 shows that the 50-best and 50-worst observations from the bull and bear markets can be successfully classified by the MDA model (as compared to a chance model), it appears that classification dependencies do exist. However, these results provide little information concerning the overall predictive ability of the MDA model.[10] Table V-5 and V-6 present the classification results for the entire data set (i.e. all Standard and Poor's Index values, not just the best and worst) for each market period based on discriminant function rules formed from the 50-best/50-worst observations. Table V-5 shows that quadratic misclassification rates range from 26 percent for the 1966 Bear market Regular set to 50 percent for the 1962-66 Bull market Regular set, with eight of the fourteen misclassification rates statistically significant at the .0005 level. Similarly, Table V-6 shows quadratic misclassification rates of 28 percent for the 1966 Bear market Percentage set to 57 percent for the 1962-66 Bull market Regular set, with six of the fourteen rates statistically significant at the .0005 level.[11] The shorter-time periods

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[10]Recall that there is a bias when the original observations are classified, thus new data must be employed to test the predictive ability of the function.

[11]Several cases exist in Tables V-5 and V-6 where the linear misclassification results are superior to the quadratic results. These situations arise for the longer market periods. This effect can be explained by comparing the sample employed to generate the parameters of the discriminant function, with the sample which is subsequently classified. The original sample is obtained from the 50-best 50-worst observations of the period in question. When the total sample is classified, the size of the residual (middle) group between the 50-best 50-worst observations is extremely important. Since the residual group often possesses different characteristics than the original sample, observations from this group are more likely to be misclassified. The quadratic function is more sensitive to differences between the two groups since the quadratic procedure employes the individual dispersion matrices in generating the function, while the linear procedure uses the

TABLE V-5

MISCLASSIFICATION RATES FOR ALL OBSERVATIONS FOR BULL AND BEAR MARKETS: LINEAR AND QUADRATIC FUNCTIONS, FULL VARIABLE SET

Market Period	Variable Set	Misclassification Rates Percentage: Linear All Observ	Misclassification Rates Percentage: Quadratic All Observ
1961 Bull	Regular	40**	36*
	Percentage	44	45
1962 Bear	Regular	38**	32*
	Percentage	39**	29*
1962-66 Bull	Regular	54	50
	Percentage	46	41*
1966 Bear	Regular	37**	26*
	Percentage	32*	28*
1966-68 Bull	Regular	44	47
	Percentage	42**	48
1968-70 Bear	Regular	40*	41*
	Percentage	39*	37*
1970-73 Bull	Regular	39*	49
	Percentage	39*	43**

* Statistically significant at the .0005 level.
** Statistically significant at the .005 level.

TABLE V-6

CLASSIFICATION RESULTS OF ALL OBSERVATIONS FOR BULL AND BEAR MARKETS: LINEAR AND QUADRATIC FUNCTIONS, COMPLETE STEPWISE SET

Market Period	Variable Set	Misclassification Rate Percentage	
		Linear	Quadratic
1961 Bull	Regular	42	40**
	Percentage	44	44
1962 Bear	Regular	44	36*
	Percentage	36*	32*
1962-66 Bull	Regular	54	57
	Percentage	46	38*
1966 Bear	Regular	33*	31*
	Percentage	33*	28*
1966-68 Bull	Regular	43**	45
	Percentage	42**	48
1968-70 Bear	Regular	42**	42**
	Percentage	38*	38*
1970-73 Bull	Regular	39*	47
	Percentage	43**	52

* Statistically significant at the .0005 level.
** Statistically significant at the .005 level.

posses respectible quadratic rates for the Full variable sets (26 percent for the 1966 Bear market Regular set to 36 percent for the 1961 Bull market Regular variable set) while the misclassification rates become large for the longer periods (41 percent for the 1968-70 Bear market Regular variable set to 50 percent for the 1962-66 Bear market Regular set). The deterioration in results for longer periods may be a function of the non-stationarity in the discriminant equation, bias in regard to a greater number of observations in the middle group as the time period increases, or both. These effects will be investigated shortly.

Table V-7 shows the results of classifying the 50-best and 50-worst observations of future market periods, based on classification rules from previous market periods. Only three of the twenty-two misclassification rates are statistically significant at the .01 level. Thus, Table V-7 shows that the classification results are poor, indicating considerable non-stationarity in the data over time. The Regular variable set is generally able to limit the misclassification rate to approximately 40 percent when the first three market period discriminant functions are needed to forecast the next period's best-worst observations. However, the other results are inferior and/or inconsistent. Thus, apparently the data are subject to non-stationarity between different Bull-Bear market periods, and over long periods of time. However, such non-stationarity exhibited over longer periods is not unreasonable. The next section further investigates the non-stationarity condition.

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pooled matirx. Therefore, the very characteristics that provide the superiority of the quadratic function also will likely reduce the effectiveness of that rule for new data from a different population. This situation occurs when a variable has a high variability in one group and a low variability in the other group. Classification of new observations that lie between the two groups are biased toward the high variability group because of the Chi-square rule for group membership.

TABLE V-7

CLASSIFICATION RESULTS FOR SUBSEQUENT BULL AND BEAR MARKET PERIODS BASED ON PREVIOUS MARKET PERIODS: BEST AND WORST OBSERVATIONS, QUADRATIC FUNCTION, COMPLETE STEPWISE SETS

Original Data Market Period	Variable Set	Classified Period	Misclassification Rate Percentage
1961 Bull	Regular	1962 Bear	44
		1962-66 Bull	51
	Percentage	1962 Bear	49
		1962-66 Bull	46
1962	Regular	1962-66 Bull	41
		1966 Bear	41
	Percentage	1962-66 Bull	36*
		1966 Bear	41
1962-66 Bull	Regular	1966 Bear	40
		1966-68 Bull	47
	Percentage	1966 Bear	52
		1966-68 Bull	48
1966	Regular	1966-68 Bull	55
		1968-70 Bear	43
	Percentage	1966-68 Bull	37*
		1968-70 Bear	42
1966-68 Bull	Regular	1968-70 Bear	50
		1970-73 Bull	48
	Percentage	1968-70 Bear	41
		1970-73 Bull	38
1968-70 Bear	Regular	1970-73 Bull	50
	Percentage	1970-73 Bull	29*

* Statistically significant at the .01 level.

Stationarity Tests

Comparative classification results for Bull and Bear markets show that the MDA model is inadequate as a predictive devise for price relatives over different kinds of market periods. In addition, even between like-market periods the predictive results are poor, indicating the data are nonstationary for the long-period comparisons. To investigate the possibility that the discriminant functions may be stationary over shorter time periods, comparisons of the misclassification rates may be made for several lengths of time to determine if a stationary length of time is to be found wherein the misclassification rates compare favorably with the original rates. For such comparisons to be useful, the misclassification rates for the original data needs to be adjusted to eliminate the 'best fit' bias inherent in MDA. To accomplish this adjustment the Lachenbruch holdout method is employed. The Lachenbruch holdout method eliminates one observation at a time, obtains the parameters of the discriminant function from the remaining observations, and then classifies the holdout observation. This sequence is repeated for all the observations, which eliminates the 'best-fit' bias.

The 50-best and 50-worst[12] observations over a one-year period[13] are used to obtain the parameters of the discriminant function, and subsequent data are classified according to these parameters.[14]

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[12]The 50-50 grouping follows the concept that the most important observations for prediction purposes are those with large price changes, while small price changes are relatively unimportant to the trader.

[13]Data for one year is a compromise between too little data, which would make an adequate function difficult to form, and too much data, which would cause problems of changing conditions over time.

[14]A three-group stationary analysis of 50-150-50 observations also is investigated to determine if the entire data set is stationary. The results indicate that the data are stationary; however, the Lachenbruch

Table V-8 presents the two-group misclassification rates for the original sample, the Lachenbruch holdout, and the subsequent best-worst samples six and twelve months into the future.[15] These results show that the data are stationary for a period of one year, since the increases in misclassification rates from the Lachenbruch holdout method on the original data compared to the future 12-month predicted results are negligible.[16] The 12-month stationarity period is long enough to verify adequately the results, renewing hope that the discriminant model may be used to successfully classsify future data and perhaps to develop a profitable trading model;[17] stationarity of less than six months causes difficulty in substantiating the validity of the classification results, because of an inadequate number of observations.

Examination of One Year Data Samples

Given the above results concerning the stationarity of the two-group, best-worst data for a period of one year, an obvious extention of

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holdout and secondary sample misclassification rates are approximately 40 percent, which is no better than a random classification scheme. This occurs because of an extensive overlap between the middle group and the adjacent large and small price relative groups. Although the overlap is a logical consequence of the type of data employed, it is not a critical problem, since in the analysis pursued in the rest of the study only the two-extreme groups are used to determine the prediction equations.

[15]The observations for the two-group, six and twelve month analyses are put in the same relative proportions. Thus, the 50-best 50-worst observations are classified for the twelve month sample, while the six month sample uses the 25-best and 25-worst observations.

[16]The large increase in misclassification results for the 1972 regular variable set is due to the changing market conditions from 1972 to 1973.

[17]The characteristics of the data seem to change slowly enough that the misclassification rate is steady up to one year in the future. Thus, the one-year period could be more important than the 'psychological' designations of bull and bear markets, which are found to generate inadequate prediction functions.

TABLE V-8

STATIONARITY RESULTS FOR THE TWO
GROUP DISCRIMINANT FUNCTION

Year	Period	Misclassification Results Percentage			
		Original Data	Lachenbruck Holdout	Next 6 Months	Next 12 Months
1967	Regular	28	33	34	37
	Percentage	26	32	34	32
1969	Regular	18	28	30	30
	Percentage	19	27	26	25
1971	Regular	30	36	34	35
	Percentage	27	33	34	34
1972	Regular	19	31	50***	48***
	Percentage	31	44**	36*	40*

Note: All unstarred (*) misclassification rates are statistically significant at the .005 level.

* Statistically significant at the .05 level.
** Statistically significant at the .10 level.
*** Not statistically significant at the .10 level.

the model is to analyze the original bull-bear market period observations and subsequent secondary samples after they are regrouped to a yearly basis. Table V-9 presents the significance level for the difference between group means, the dispersion test significance level, the linear and quadratic misclassification rates for the previous year's data, and thy misclassification rate for the subsequent year's data based on the initial year's discriminant function. This information is presented for fairly recent time periods, and for the Regular and Percentage variable sets. The results show that both statistical and classificatory ability is present for the original sample. In addition, Table V-9 shows that a quadratic formulation is generally superior to a linear formulation, and that misclassification rates for the subsequent best-worst data are generally in the range of 25 percent to 40 percent, with all misclassification rates statistically significant at the .05 level. Additionally, Percentage-variable set results are comparable to the Regular variable set results for the initial data year, and the Percentage set is superior when the subsequent yearly data are classified.

Since the misclassification rates on the future data are superior to those available from chance prediction, this finding suggests that trading rules might be devised that capitalize on the classification dependencies. However, before such rules are developed, one final underlying assumption of the multiple discriminant model must be investigated.

Examining the Discreteness of the Price Relatives

Since the criterion variable is assumed to be discrete for MDA, investigation is required of the appropriateness of such a model when the variable determining the grouping is semi-continuous. One method often

TABLE V-9

SIGNIFICANCE LEVELS AND MISCLASSIFICATION RATES FOR SELECTED YEARLY DATA

Year	Variable Set	Mean Significance Level Percentage	Dispersion Significance Level Percentage	Misclassification Rate Percentage: Original Sample Quad	Misclassification Rate Percentage: Original Sample Linear	Misclassification Rate Percentage: Next 12 Month 50 Best-50 Worst Observations
1967	Regular	.032	0.0	19	28	37**
	Percentage	2.19	.14 E-08	24	26	32*
1968	Regular	.37	.82 E-05	25	26	32*
	Percentage	.23	.22 E-05	25	21	31*
1969	Regular	.0049	.00011	18	18	30*
	Percentage	.0031	.00013	19	19	25*
1970	Regular	.0065	0.0	28	25	40***
	Percentage	.012	0.0	27	24	38***
1971	Regular	2.91	0.0	25	30	35**
	Percentage	1.45	0.0	36	27	34**
1972	Regular	.016	.000016	16	19	48
	Percentage	.79	0.0	24	31	40***

Note: All original sample misclassification rates are statistically significant at the .0005 level.

* Statistically significant at the .0005 level.
** Statistically significant at the .005 level.
*** Statistically significant at the .05 level.

used to support a discrete model is to investigate the statistical significance of the group mean differences. If the results indicate that significant differences exist between group means, the MDA approach is deemed applicable. Table V-9 shows that the significance levels support the two-group discrete model used here.

A second and more complete method of analyzing whether or not price relatives form a discrete or continuous variable is to compare the multiple discriminant results against those from a multiple regression model. This procedure is outlined in Chapters III and IV and is due to Reinhart and Latane' [C]. To investigate the discrete versus continuous nature of the price relatives all of the data must be employed. Since the above investigations of the technical data have dealt with high and low price relatives, a natural extension that permits the analysis of all the data to determine its discreteness is to employ high, medium, and small group segregations, i.e. a three group analysis.

The misclassification results for the three-group linear and quadratic discriminant model and the comparative three-group regression model are presented in Table V-10. This table shows the superiority in terms of classification (although sometimes small) of the discriminant model. Specifically, the MDA misclassification results are statistically significant at a smaller significance level for four of the nine cases presented. The statistical significance level is equivalent for MDA and MRA for the remaining cases, although the numerical misclassification rate is smaller for MDA for all but one case. Any hesitancy to label MDA as the superior method for the data studied here can be resolved by analyzing two other indicators, as presented in Table V-11, namely the R^2 values for the regression results, and the inability of MRA to

TABLE V-10

DISCRIMINANT AND REGRESSION CLASSIFICATION RESULTS
FOR THREE GROUP YEARLY DATA

Year	Variable Set	MDA Quadratic Misclass. Rate Percentage	MDA Linear Misclass. Rate Percentage	MRA Linear Misclass. Rate Percentage
1967	Regular	28*	36*	34*
1968	Regular	35*	42***	43
1969	Regular	32*	35*	40**
1969	Percentage	37*	39*	39*
1970	Regular	40**	35*	40**
1971	Regular	37*	39*	40**
1972	Regular	41**	40**	41**
1972	Percentage	41**	41**	40**
1973	Regular	33*	37*	39*

* Statistically significant at the .0005 level.
** Statistically significant at the .005 level.
*** Statistically significant at the .01 level.

TABLE V-11

R^2 VALUES AND ERROR TERMS FOR YEARLY REGRESSION RESULTS

Year	Variable Set	R^2 Value	Number of Positive Error Terms for Highest 50 Price Relatives	Number of Negative Error Terms for Lowest 50 Price Relatives
1967	Regular	.20	47	50
1968	Regular	.13	49	49
1969	Regular	.19	50	50
1969	Percentage	.15	49	50
1970	Regular	.20	48	50
1971	Regular	.10	50	50
1972	Regular	.13	50	50
1972	Percentage	.11	50	50
1973	Regular	.14	50	50

successfully classify the observations in the high- and low-price relative groups. The low R^2 values show that the regression model explains only a small fraction of the total variation of the price relatives. The second indication that the MRA model is inappropriate is the consistently positive error terms for the large price relative group and the negative error terms for the small price relative group, as shown in Table V-11. This problem is a natural consequence of the regression model when it is applied to data that actually form several groups. MRA generates a function based on the assumption that the observations all form one population. This obscures important information that is included in the high and low price relatives, and violates a basic assumption of the regression model. Regression analysis requires the assumption that the expected value for the error terms is zero and that these error terms are randomly distributed about the regression line for <u>each</u> X_i value. The pattern of error terms shown in Table V-11 for the regression model shows that this assumption is not met.

Trading Rules

The preceding analysis has determined that a discriminant model can successfully classify future data and that a discrete two-group model is the appropriate tool to be used in such an investigation. The obvious extention of this research is to develop a trading model that determines if one can utilize the dependencies in thy data to obtain trading profits greater than those available from a buy-and-hold strategy.

As outlined in Chapter IV, the reduced-space function is employed in order to develop filter trading rules with operational buy and sell signals. Data for each year from 1967 to 1973, consisting of daily price values and Regular and Percentage variable technical data lagged one day to the price relative values, are used to generate the parameters of a discriminant function for each year. Observations for the subsequent year are then employed sequentially to examine the profitability of the rule. When technical data for each day are substituted into the reduced-space function given in equation V-1, a Z-value is computed as:

V-1 $$Z = k_1x_1 + k_2x_2 + \ldots + k_nx_n$$

Group membership is determined by comparing the computed Z-value to a specified cutoff Z-value. The cutoff values depend on the separation of group means and the size of the variance of each group, Specifically, cutoff Z-values are developed for the 'UP' and 'DOWN' groups in order to reduce both overlap and frequent whipsaw effects. To eliminate most of the group overlap, Z-values corresponding to the 25th and 75th percentiles of the two-group Z values are arbitrarily chosen *a priori* as the cutoff values. Ex-post examination of the distribution of the Z-values for the original data confirms the hypothesis that most group overlap is eliminated by the selection of these cutoff values.

Three trading rules are developed. Rule 1 initiates a buy signal when a Z-value falls in the top 25 percent of all Z observations and a short selling signal is initiated when the Z-value falls in the bottom 25 percent of the distribution. Under Rule 1, the trade is terminated when an opposite signal is generated *or* when a 'no trade' Z-value is obtained. This rule will most likely generate an extensive number of trades and therefore is assumed to be appropriate only for floor traders who are exempt from commissions.

Rule 2 generates buy and short-sale signals in the same manner as Rule 1. However, to perpetuate the trade, the transaction is kept active until an opposite signal is generated. Therefore, this rule will benefit from trends in the data as well as from the extreme Z-values which attempt to predict the large price changes. If the daily technical data can help discern short-term trends, then this information, combined with the possibility of predicting large price changes, may enable an investor to outperform a buy and hold strategy. Possible real world situations that limit the profitability of thy model are 'whipsaw' effects, non-stationarity in the data (even if limited), observations from the middle group that generate false buy and sell signals, and commission charges.

Rule 3 is more restrictive than Rule 2, and attempts to reduce possible whipsaw effects, and thus lengthen trends. If the number of trades is reduced then the total commission charge will also decline. The reduction in the whipsaw effect is attempted by requiring a verification of the buy and short-sale signals. Specifically, a purchase is made when two buy signals are generated without an intervening short-sale signal. Likewise the buy transaction is terminated only when there are two sale indicators without an intervening buy signal. A corresponding rule applied for short sales.

Tables V-12 through V-15 present the results of these trading rules before and after commissions.[18] Any trends of at least one week which occur at the end of the year are terminated (with a commission

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[18]Commissions are calculated by taking 1 percent of the value of the market index for both buy and sell transactions.

charge) at the end of that period in order to avoid overlapping periods and to provide valid comparisons between periods.

Table V-12 presents the results of the 25 percent cutoff value, before commissions, for the Regular and Percentage variable sets from 1969 to 1973. Clearly, all three rules provide superior results when compared to the buy and hold strategy. Moreover, the rules are profitable in either a bull or bear market. The Percentage variable sets often outperform the Regular sets.

A study of the specific rules shows that Rule 1 (assumed to be applicable to floor traders only, because of the substantial number of trades) is extremely profitable. The profitability of Rule 1 demonstrates the ability of the discriminant model to turn the classificatory dependencies into a trading model that provides superior profits. Likewise, Rule 2 is profitable both on an aggregate basis and for individual periods and provides returns similar to Rule 1.[19] The individual buy and sell signals for both Rules 1 and 2 provide fluctuating returns, based on the type of market. Bull markets (when the buy-and-hold return rises) are consistently associated with very profitable buy signals, which outperform the sell-short signals. Periods when the short-sale indicators are superior are related to bear markets. Rule 3, which is more restrictive in terms of initiating trades, is consistently profitable but has inferior returns when compared to Rules 1 and 2. Thus, profitable intermediate fluctuations are identified by Rule 2 but ignored by Rule 3.

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[19]Since the before commission returns for Rule 2 are similar to the returns of Rule 1 the results tend to support the original assumption that observations falling in the middle cancel each other and thus are unimportant in the final analysis.

TABLE V-12

TRADING RULE RESULTS BEFORE COMMISSIONS
FOR A 25-PERCENT CUTOFF Z VALUE

Time Period Original Sample Secondary Sample	Original Data Type Market	Variable Set	Buy & Hold Return	Rule 1 Total Return	Rule 1 Buy	Rule 1 Sell Short	Rule 2 Total Return	Rule 2 Buy	Rule 2 Sell Short	Rule 3 Total Return	Rule 3 Buy	Rule 3 Sell Short
1967/68	Bull	Regular	7.75	23.77	15.18	8.59	9.95	8.85	1.10	8.06	8.28	- .22
1967/68	Bull	Percentage	7.75	20.17	10.55	9.62	20.69	15.03	5.66	14.10	11.11	2.99
1968/69	Bear	Regular	-11.87	21.43	8.11	13.32	21.59	4.86	16.73	10.73	- .56	11.29
1968/69	Bear	Percentage	-11.87	32.14	10.93	21.21	30.26	8.76	21.50	10.86	.32	10.54
1969/70	Bear	Regular	- .85	39.82	25.23	14.59	34.55	16.94	17.61	23.45	11.46	11.99
1969/70	Bear	Percentage	- .85	57.31	30.93	26.38	60.77	29.96	30.81	48.75	24.60	24.15
1970/71	Bull	Regular	10.94	18.95	7.56	11.39	17.50	14.15	3.35	13.16	11.45	1.71
1970/71	Bull	Percentage	10.94	31.87	24.83	7.04	27.98	19.46	8.52	7.22	8.14	- .92
1971/72	Bull	Regular	14.77	21.95	9.86	12.09	14.64	13.41	1.23	- .65	6.33	- 6.98
1971/72	Bull	Percentage	14.77	18.04	11.07	6.97	22.69	18.66	4.03	.08	7.23	- 7.15
1972/73	Bull	Regular	-11.55	5.69	- 5.43	11.12	4.47	- 8.54	13.01	- 5.58	-13.94	8.36
1972/73	Bull	Percentage	-11.55	34.57	11.08	23.49	34.53	6.49	28.04	2.25	- 9.93	12.18
TOTALS		Regular	9.19	131.61	60.51	71.10	102.70	49.67	53.03	49.17	23.02	26.15
		Percentage	9.19	194.10	99.39	94.71	196.92	98.36	98.56	83.26	41.47	41.79

Table V-13 presents the after-commission results for Rules 2 and 3 for 25-percent cutoff Z-values. (Rule 1 is considered applicable only to floor traders, and thus after-commission returns are not computed). The effects of commissions are dramatic. Total returns for Rules 2 and 3 are consistently negative, despite the type of market. The losses for the buy signals are less than those for the sell signals during bull markets (and vice versa for bear markets), but these buy-and-sell signals also are consistently negative. Notably, after commissions the losses from Rule 3 are less than those from Rule 2; this observation provides a measure of the impact of commissions resulting from whipsaw effects. Similarly, the Regular variable data sets are superior to the Percentage variable data sets after commissions (the reverse is true before commissions). This finding indicates that the Percentage variable set is better able to discern short-term variabilities in investor attitudes (i.e. it is more 'sensitive' to change in the market) but that this sensitivity increases the number of trades and hence commissions. Thus, the model can indicate profitable situations before commisisons, but the commissions more than comsume the extraordinary profits achieved.

The results of Table V-13 suggest that a more restrictive filter will produce less trades, which may in turn result in profitable returns after commissions. In an attempt to reduce excessive trades, a 10-percent cutoff value is arbitrarily selected to replace the 25-percent value. Also, since the Regular variable set generally outperforms the Percentage variable set after commissions in Table V-13, the Regular variable set is used to test the applicability of the 10-percent cutoff value.

Table V-14 shows that the returns before commissions are still superior to a buy-and-hold strategy, but, as expected, are inferior to

TABLE V-13

TRADING RULE RESULTS AFTER COMMISSIONS
FOR A 25-PERCENT CUTOFF Z VALUE

Time Period Original Sample Secondary Sample	Type Market	Variable Set	Buy & Hold Return	Rule 2			Rule 3		
				Total Return	Buy	Sell Short	Total Return	Buy	Sell Short
1967/68	Bull	Regular	7.75	-29.84	-10.19	-19.65	- 5.25	.63	- 5.88
1967/68	Bull	Percentage	7.75	-38.33	-14.63	-23.70	- 3.94	1.01	- 4.95
1968/69	Bear	Regular	-11.87	-29.02	-20.49	- 8.53	- 8.45	-10.18	1.73
1968/69	Bear	Percentage	-11.87	-61.32	-36.20	-25.12	-34.56	-21.49	-13.07
1969/70	Bear	Regular	- .85	- 8.77	- 4.89	- 3.88	7.07	3.16	3.91
1969/70	Bear	Percentage	- .85	-35.49	-18.47	-17.02	19.32	9.64	9.68
1970/71	Bull	Regular	10.94	-58.16	-22.80	-35.36	-16.07	- 4.21	-11.86
1970/71	Bull	Percentage	10.94	-49.88	-19.65	-30.23	- 6.04	.51	- 6.55
1971/72	Bull	Regular	14.77	-72.50	-30.17	-42.33	-41.77	-13.13	-28.64
1971/72	Bull	Percentage	14.77	-97.34	-40.38	-56.96	-52.27	-19.01	-33.26
1972/73	Bull	Regular	-11.55	-49.78	-36.77	-13.01	-18.41	-20.21	1.80
1972/73	Bull	Percentage	-11.55	-88.72	-55.41	-33.31	-46.94	-33.45	-13.49
TOTALS		Regular	9.19	-248.07	-125.31	-122.76	-82.88	-43.94	-38.94
		Percentage	9.19	-371.08	-184.74	-186.34	-124.43	-62.79	-61.64

TABLE V-14

TRADING RULE RESULTS BEFORE COMMISSIONS
FOR A 10-PERCENT CUTOFF Z VALUE
REGULAR SET

Time Period Original Sample Secondary Sample	Market Period	Buy & Hold Return	Rule 1			Rule 2			Rule 3		
			Total Return	Buy	Sell Short	Total Return	Buy	Sell Short	Total Return	Buy	Sell Short
1967/68	Bull	7.75	10.98	7.40	3.58	9.54	10.05	- .51	3.24	8.10	- 4.86
1968/69	Bear	-11.87	25.81	6.97	18.84	25.35	6.51	18.84	12.77	.42	12.35
1969/70	Bear	- .85	20.38	17.58	2.80	4.78	5.62	- .84	3.88	3.14	.74
1970/71	Bull	10.94	- 0.28	- 0.38	.10	- 2.42	2.39	- 4.81	.19	2.27	- 2.08
1971/72	Bull	14.77	.87	1.26	- .39	- 5.99	.77	- 6.76	3.32	6.55	- 3.23
1972/73	Bull	-11.55	4.91	.02	4.89	6.22	- 7.67	13.89	4.62	-8.63	13.25
TOTAL		9.19	62.67	32.85	29.82	37.48	17.67	19.81	28.02	11.85	16.17

the returns generated from the 25-percent cutoff. Table V-15 shows that after commissions the 10-percent cutoff rule still does not provide profitable returns, nor does it outperform a buy and hold strategy. When the 10-percent cutoff results are compared to the 25-percent cutoff returns, the 10-percent cutoff returns are superior on an overall basis (i.e. the total amount of losses is reduced). However, since the overall returns are negative, the conclusion that the model cannot consistently produce extraordinary profits after commissions follows, i.e., the market is efficient on an after-commission basis.

TABLE V-15

TRADING RULE RESULTS AFTER COMMISSIONS
FOR A 10-PERCENT CUTOFF Z VALUE
REGULAR SET

Time Period Original Sample Secondary Sample	Market Period	Buy & Hold Return	Rule 2			Rule 3		
			Total Return	Buy	Sell Short	Total Return	Buy	Sell Short
1967/68	Bull	7.75	- 9.36	.56	- 9.92	- 8.13	2.35	-10.48
1968/69	Bear	-11.87	-29.66	-20.14	- 9.52	-11.54	-10.82	- .72
1969/70	Bear	- .85	- 4.94	.71	- 5.65	- .94	- .20	- .74
1970/71	Bull	10.94	-17.92	- 5.38	-12.54	- 3.83	.23	- 4.06
1971/72	Bull	14.77	-19.19	- 5.76	-13.43	- 1.23	4.21	- 5.44
1972/73	Bull	-11.55	-11.58	-16.78	5.20	- 8.18	-14.94	6.76
TOTAL		9.19	-92.65	-46.79	-45.86	-33.85	-19.17	-14.68

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

This study examines the ability of technical indicators to predict the future movement of the stock market. Since practitioners consistently employ technical indicators in an attempt to forecast the market, while academicians insist that the market is efficient with regard to the ability of technical data to provide abnormal profits, the results of this study are meaningful to both.

Chapter II reviews the random walk and efficient-market models for describing stock-price movements. Various subgroups of these models are defined depending on the type of information utilized. Specifically, the narrow view considers past price movements only, the broad view considers all past information, and the monopolistic-information view considers whether or not certain professional groups can obtain extraordinary profits.

The narrow view of the random-walk hypothesis is often examined by various statistical methods, including employing serial correlation, runs tests, and spectral analytic techniques. These linear methods show that prices do not follow a strict mathematical random walk. In addition, other statistical and probabilistic tests examining reversals and continuations, non-homogeneous dispersions, and clustering of stock prices show the existence of certain short-term irregularities and long-term trends. Moreover, various tests of technical trading models concerning only prices show definite dependence in stock market data. However, after new samples, commission charges, and risk differences are considered, extraordinary profits are not available on a consistent basis from using these techniques, i.e., the efficient markets hypothesis holds in regards to past price data.

Examination of the broad view of the random walk hypothesis necessitates the analysis of price movements in conjunction with all other past information. Various statistical evaluations have been made employing such techniques as regression analysis, factor and cluster analysis, discriminant analysis, information theory, and non-parametric rank-sort techniques, among others. The results show that definite dependence does exist between past information and future prices. Additionally, the results achieved from the few trading rules tested show that a mathematical random walk model is not valid. However, the market is still efficient after commissions and consistency of results over time are considered, i.e., trading schemes developed from ad-hoc decision rules, or the few trading models that have evolved from statistical dependencies are not able to generate abnormal profits.

A brief look at the monopolistic information viewpoint indicates a limited number of groups (namely that insiders and specialists) do have sufficient information to obtain extraordinary profits. However, the professional analysts and mutual funds do not seem to possess monopolistic information or superior forecasting ability.

Many of these studies that employ technical variables concentrate mainly on the statistical aspects of the data, e.g., Fama's [A 1965b] and Theil and Leenders' [A] studies on the intertemporal dependence of the number of stocks advancing/declining/remaining unchanged. Often, such studies generally do not attempt to convert any uncovered statistical dependencies into trading models. Moreover, many of the statistical, technical-variable oriented efforts often investigate only one technical variable at a time, or at most a few related technical variables. But the multivariate aspect of the market system dictates a multivariate analysis.

The only extensive multivariate study to date is by Emery [A]. Emery determined that a factor analysis/regression model of technical indicators can not be utilized to obtain extraordinary profits. However, two major criticisms are directed at Emery's model and results. First, the factor analysis/regression approach requires assuming the data come from one underlying population instead of two or more populations. Second, Emery's data has extensive errors. The non-linear MDA model used here considers the multi-group aspects of the data and is more successful in explaining the relationship between technical indicators and the price mechanism.

The Discriminant Model and Methodology

To investigate the relationship between technical indicators and the market price mechanism daily data are examined by means of a two-group discriminant model. The groups are designated by isolating for given time periods the 50-best and 50-worst price relatives for the period. From these data, and various technical market indicators, the discriminant function is derived. Given the above, the strict random walk model is examined by an F-test of the difference between the group means, and by a determination of the ability of the function to classify samples from original and secondary data. In addition, a model that is successfully able to classify new data is examined further by developing and evaluating trading rules that investigate the profit potential before and after commissions for technical data in a multivariate context.

To perform the above analyses two sets of variables are utilized, namely a 'Regular' variable set that includes the absolute values of various technical indicators, and a 'Percentage' variable set that examines the percentage changes for the variables included in the 'Regular' set. Also, attention is given to the non-linear discriminant tests and classification procedures, complete stepwise methods versus the usual forward stepwise process, test-space versus reduced-space procedures, and the assumption of discrete groupings.

Results

The initial analysis considers bull- and bear-market segregations of the data in an attempt to investigate the relationship between investor attitudes in the original and adjacent market periods. When the discriminant function is applied to the original data, a statistically significant difference is found between group means; moreover, the function is able to classify successfully the original sample. The relatively low misclassification rates for the original sample show that a definite relationship between daily technical indicators and subsequent high and low price relatives exists.

However, when the discriminant function for each bull and bear market period is applied to two types of secondary-data observations, the relationship between the technical indicators and price relatives no longer exists. The explanation for these results is two fold, since two different types of secondary-data sample observations are classified. The first type of sample includes all the price relatives in the period, while the second sample includes the best and worst observations from the subsequent market periods. Thus, poor classification occurs in the first sample because of the inability of the model to discriminate between observations from the middle group (which normally possess different characteristics than the observations from the original sample) and the extreme groups. Additionally, the price relatives from the middle group cluster near 1.0, to make the group selection almost arbitrary. For the second sample, the poor classifications are due to nonstationarity in the data. Thus, the characteristics of the technical indicators between market periods change to such an extent that a multivariate discriminant model cannot adequately employ the best-worst observations in the original market period to classify successfully observations in subsequent market periods.

Since the poor classification results are at least partly due to the instability of the discriminant function over different market periods, one must determine if the data are stable over shorter periods, and if so the length of that stability. Using one year's worth of data to form an initial discriminant function, the best and worst observations for the subsequent 6 and 12 month periods are classified. This process is repeated sequentially from 1967 to 1972 with an initial discriminant function estimated for data for one year to predict best and worst group membership for data over succeeding 6 and 12 month periods. The results show that little difference exists between the Lachenbruch holdout classification results for the initial observations and the classification results for the subsequent 12 month best and worst data, i.e., the relationship between technical indicators and future market movements appears to be stable for periods up to one year.

The discriminant model is reexamined by employing secondary samples of one year's duration, where the secondary samples include the best and worst price relatives from the classification results for the secondary samples of the yearly data are superior to a random selection model, showing that nonstationarity of the data is a major factor in the disappointing results for the bull-bear market-type data. Thus, a relationship between technical indicators and subsequent market action does appear to exist, and the parameters of this relationship are stable for periods up to one year.

The ultimate test of the model is to examine its ability to generate profits that are superior to market buy and hold returns. All of the trading rules employed in this study provide returns that are vastly superior to the buy and hold profits <u>before</u> commissions. However, the model's returns are inferior <u>after</u> commissions. This is true for all three trading rules, for both variable sets, and for both a 25- and 10-percent cutoff Z-value. More specifically, Rule 2, where only one confirmation is needed, generates the best results before commissions for non-floor traders and, in fact, provides returns comparable to Rule 1 (which is only feasible for the floor trader). The Percentage-variable sets generate higher returns than the Regular sets before commissions, but provide lower returns after commissions. Thus, signals given by the Percentage-variable sets are more sensitive to minor changes in the market, but the extra profits do not compensate for the additional commissions created from a greater number of trades. Finally, the substitution of a 10 percent cutoff Z-value for the 25 percent level does achieve its purpose of reducing the number of trades and the amount of commission basis.

The trading rule results before commissions show the rule is superior to the buy and hold criterion both in an absolute profit sense and in its consistency. A strict mathematical random walk model then is invalid. However, the inability of the model to produce consistently positive returns after commissions substantiates the efficient market model.

This study casts serious doubts as to the utility of technical indicators as a means of generating superior profits after commissions. This conclusion is strengthened by the research designed used here. It offers several advantages over previous efforts in this area. First, the discriminant model considers large market movements instead of the entire set of price chances of specific securities and thus is better able to consider the major effects of technical indicators on the market. Second, the model is multivariate in nature, conforming to the real world interrelationships of the market. And third, the discriminant procedure utilized here examines both the linear and nonlinear aspects of the data.

The success of the MDA model for classification purposes and for obtaining extraordinary profits before commissions, shows that floor traders may profit from the information provided by technical indicators in combination with a discriminant model. Also, the information provided by the model may be useful as a timing device in order to determine when to buy or sell, once the basic buy/sell decision has been made. Thus, on average, profits may be increased (or losses reduced) by using the model in conjunction with more fundamental type analyses, however, the results show that technical indicators in and of themselves do not provide sufficient information to 'beat the market'.

Future Research

One possible extension of the research presented in this paper is to develop a discriminant model for individual securities. Such an investigation would determine whether a model based on technical data of individual securities could outperform the market (after adjusting for risk). Certain characteristics of individual stocks may provide information that the market as a whole obscures. In addition, use of individual technical data may possibly determine superior performers on a relative basis. Moreover, additional information concerning the timing of buy/sell orders (even if a trading model is unprofitable after commissions) may be uncovered by examining an individual security model.

However, investigating such a model is beset with severe problems in developing a research methodology. Many of the variables employed in the present study could not be used when individual securities are investigated, e.g., the number of securities advancing, declining, and remaining unchanged. Additionally, both cross-sectional and longitudinal studies would present serious difficulties for obtaining reasonably consistent values for the variables; i.e., the variability of the data could be so large that the parameters would be highly unstable over time and/or over various securities.

Extending the concepts presented in this paper is another area for research. However, the results presented here show that improvements in the model by including other variables, using other cutoff values, and developing alternative trading rules would have little chance of success. Specifically, the results from the model are relatively insensitive to the full versus complete stepwise variable set formulations, the Regular versus Percentage sets, changes in the cutoff value, and the various trading rules. Moreover, the results obtained in this paper, plus those presented by Emery, show that applying other standard statistical techniques to the same data would be useless. However, one major area of improvement may lie with the development of a fully dynamic model, i.e., a model that continually updates its parameters as new information is provided. Thus, the one-year update rule employed here may be inefficient.

Implications for the Statistical Theory of Discriminant Analysis

This paper has indicated several areas where more research into the statistical aspects of MDA is needed. First, simulation studies employing the non-linear discriminant function are necessary to determine the magnitude of the bias involved in using the linear function when the variance-covariance matrices are unequal. Second, the use of the complete stepwise function as compared to the forward stepwise procedure should be analyzed, especially in terms of the effects on the classification results. Third, the test-space versus reduced-space results should be compared for various options of the model, e.g., the quadratic versus linear formulations of the model. Fourth, sensitivity of the parameters to secondary samples needs investigation for various circumstances of the model. And finally, the effect of various degrees of multicollineality on the results need to be determined.

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