DEVELOPMENT AND EVALUATION OF SPECIAL CONTROL CHARTS FOR QUALITY DATA GENERATED FROM A FIRST ORDER RESPONSE PROCESS

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CONTROL CHARTS FOR QUALITY DATA

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PREFACE

This research is concerned with the development and evaluation of special control charting techniques for quality data generated from a first order response process. The primary objectives are to present methodology for constructing the control limits of these special control charts using a conditional distribution and to use computer simulation to determine the average run length of these charts. Several SAS programs are used in the study to determine the average run length for a particular scenario. Modifications of the programs are then done to facilitate the determination of the average run length for other scenarios. Comparisons of these average run lengths with those of other control charts commonly used on continuous flow processes are then made. A FORTRAN program is also coded to calculate the control limits of these special control charts. It is found that the special control charts are capable of monitoring the mean and/or dispersion of a first order response process.

I wish to express my sincere gratitude and respect to my dissertation adviser, Dr. Kenneth E. Case, for his guidance, encouragement and assistance. For the past few years, he has shown me the real attitude of learning, and inspired me to strive for my best in all my undertakings. I also wish to express my sincere appreciation to my Ph.D. advisory committee members, Dr. Michael Branson, Dr. Joe Mize, Dr. Barry Moser and Dr. Palmer Terrell, for their interest and suggestions. I also wish to express my appreciation to all the professors who have taught me throughout my stay at O.S.U. I am glad that I have had quality education from them.

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To my parents and members of my family, goes my deepest appreciation and respect for their unending love and support. I wish to dedicate this dissertation to my fiancee, Hooi Chin Lee, who has been encouraging and supporting me since I decided to pursue graduate study. With her love, support and postponement of our marriage plans, I finally make it to the end of my formal education. Rather, I finally reach a new milestone in the learning process.

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NOTATION

ARIMA (p,d,q) = Autoregressive integrated moving average of order (p,d,q)

ARL = (i) Average run length of a control chart (a generic term); also,
(ii) Average run length of a control chart conditioned on the fact that the mean and standard deviation of the statistics used in the construction of control limits are at their expected values

ARL' = Average run length of a control chart in which the variations of the mean and standard deviation of the statistics used in the construction of control limits are taken into consideration

CL_I = Center line of the I chart

CL_{MEWMA} = Center line of the Modified EWMA chart

CL_{OPAMRY} = Center line for the t^{th} point of the OPA MRy chart

CL_{OPAY} = Center line for the t^{th} point of the OPA Y chart

E(v) = Expected value of some variable v

EWMA = Exponentially weighted moving average

FC = Computer code for filter constant of a FORP

FC1 = First filter constant of a SORP

FC2 = Second filter constant of a SORP

FORP = First order response process

I chart = Individual chart

IMA(d,q) = Integrated moving average of order (d,q)

I_n = Identity matrix of size n x n

LCL = Lower control limit of a control chart

LCL_I = LCL of the I chart

LCL_{MEWMA} = LCL of the Modified EWMA chart

$LCL_{MR(2)}$ = LCL of the MR(2) chart

LCL_{OPAMRY} = LCL for the t^{th} point of the OPA MRy chart

LCL_{OPAY} = LCL for the t^{th} point of the OPA Y chart

LCL_{Xbar} = LCL of the Xbar chart

M = Average run length of a control chart presented in the ARL table

MEAN = Computer code for μ

MR(2) = Moving range of subgroup size two of some random variate

MR(2)bar = Average of MR(2)

MRy = Moving range of subgroup size two of variate Y_t

MRybar = Average of moving ranges of subgroup size two of variate Y_t

MRz = Moving range of subgroup size two of variate Z_t

MRzbar = Average of moving ranges of subgroup size two of variate Z_t

OPA = One-period-ahead

Q = Absolute difference between the X_t and Y_{t-1} given that $Y_{t-1} = k$

R = Range of a subgroup of X variates

Rb = Average of R

$R|k$ = Range of Y_{t-1} and Y_t given that $Y_{t-1} = k$

S = Standard deviation of run length of a control chart presented in the ARL table

SEED = Seed for a random number generator

SORP = Second order response process

SDRL = Standard deviation of run length of a control chart

STD = Computer code for σ

Std(v) = Standard deviation of some variable v

UCL = Upper control limit of a control chart

UCL_I = UCL of the I chart

UCL_{MEWMA} = UCL of the Modified EWMA chart

$UCL_{MR(2)}$ = UCL of the MR(2) chart

UCL_{OPAMRY} = UCL for the t^{th} point of the OPA MRy chart

UCL_{OPAY} = UCL for the t^{th} point of the OPA Y chart

UCL_{Xbar} = UCL of the Xbar chart

V(v) = Variance of some variable v

W_t = EWMA statistic at time t

X_t = Independent but unobservable random variable at the t^{th} time period, having mean μ and standard deviation σ

Xbar = Average of a subgroup of X variates

Xbb = Average of Xbar values

Y_t = Serially correlated random variable from a first order response process at the t^{th} time period

Ybar = Average of Y_t

$Yc_{t+1}(t)$ = Forecast for the observation in period t+1 made at the end of period t

Z_t = Serially correlated random variable from a second order response process at the t^{th} time period

Zbar = Average of Z_t

$\hat{}$ = Used on some notation; indicates the estimate of that notation

d_2 = Ratio between the expected value of the average range of subgroup size n from a long series of samples from a normal universe, and the standard deviation of that universe

d_3 = Ratio between the expected value of the standard deviation of the range of subgroup size n from a long series of samples from a normal universe, and the standard deviation of that universe

$e_{t+1}(1)$ = One-step-ahead prediction errors made at time t+1; observation minus $Yc_{t+1}(t)$

$f(\cdot)$ = Density function of some random variable. It is assumed to be normal.

$g(R|k)$ = Density function of $R|k$

$h(Q)$ = Density function of Q

μ = Mean of X_t

1_n = Column vector with n elements; each element has a value of 1

$\Phi'(x)$ = The ordinate value of the standard normal distribution at x

$\Phi(x)$ = The cumulative value of the standard normal distribution at x

r = Filter constant of a FORP

σ = Standard deviation of X_t

sigma = Standard deviation of X_t; standard deviation of the process

σ_y = Asymptotic standard deviation of Y_t

σ_p = Standard deviation of the one-step-ahead prediction errors, $e_t(1)$

t = Time index

$\Theta(t)$ = EWMA of the absolute $e_t(1)$

= Indicator in the ARL table; shows that the hypothesis testing of the difference of the mean of two ARL values is significant at 5% level

CHAPTER I

BACKGROUND AND RESEARCH OBJECTIVES

Introduction

Control charts are employed for establishing and maintaining the statistical control of a process and for helping analyze process capability. Most of the existing control charting techniques are based on two major assumptions as follows:

(1) The underlying distribution from which observations are drawn is a normal distribution.

(2) The observations drawn from a process are independent of one another. Hence, the subgroups of any size n formed are also independent.

These two assumptions are not always satisfied in industry settings. Violation of the first assumption can be easily circumvented by using the Central Limit Theorem or some type of data transformation to obtain normality. Violation of the second assumption results in a serially correlated quality data stream. Serially correlated data can be easily encountered in industry processes. The measurement data generated from a continuous flow process with well mixed vessels such as is commonly encountered in chemical, refining and mining processes are often serially correlated. These continuous flow processes can be categorized in various forms. The form most commonly encountered is the first order response process. Box and Jenkins (1976) and Hunter (1986), among others, discuss the nature of first order, continuous processes as generators of serially correlated data.

The presence of serial correlation has a serious impact on the performance of traditional control charts, causing a dramatic increase in the frequency of false alarms. This proposed research is to develop and evaluate a control charting technique to deal specifically with serially correlated data generated from a first order response process.

Existing Control Charting Techniques

The concept of control charts was formally introduced in 1931 by Dr. Walter Shewhart. It is based on the principle that variation in measurement data pertaining to a process can be separated into two sources - inherent process variation due to chance causes and variation due to assignable causes. Dr. W. Edwards Deming (1982) refers to these as common cause and special cause variation, respectively. If the inherent variation can be estimated, then, using statistical procedures, it is possible to detect shifts in the mean and/or dispersion of a process. Thus, the objectives of control charts are to determine whether the process is in state of statistical control, to assist in establishing a state of statistical control, and to monitor current control of the process. The assumptions held in the application of Shewhart's control charts are data independence and normality. Development of other forms of control charts, such as the Individual chart, Moving Average chart, Cumulative Sum chart, and so on, all stem from the theory conceived by Dr. Shewhart. These charts also assume data independence and normality.

In normal practice, coupled control charts are generally used simultaneously, such as Xbar and R charts, Individual and Moving Range charts, and Moving Average and Moving Range charts, to monitor both the mean and dispersion of a process. The two underlying assumptions of data independence and normality in those control charts are not always satisfied; in these cases, the performance of these coupled control charts is seriously affected.

In situations where the normality of process data cannot be held, the Central Limit Theorem can usually be applied to justify the assumption of statistic normality. The Central Limit Theorem essentially states that, under general conditions, the distribution of subgroup means will approach normality for a large sample size. Approximate normality of data for plotting can also be achieved by some suitable forms of data transformation (Natrella, 1963) (Dudewicz, 1988). Much effort has been invested by researchers to study the effect of data non-normality on control charts.

In situations where the independence of process data does not exist, that is, the process data are serially correlated, there is no clear way to justify the use of control charts. There are some suggested ways to deal with serially correlated data, such as avoidance and compensation (Brooks and Case, 1986). Avoidance seeks to increase the sampling interval to the point that the data are sufficiently independent. Compensation seeks to remove the effect of serial correlation, back to the point of control. Both ways may be impractical in industry settings.

Currently, the approach used to deal directly with the serially correlated data is through time series analysis. A time series model is first fitted on the process data and a control chart is then applied to the residuals generated by this fitted model. There are some variations in the application of the time series approach to serially correlated data. The accuracy of a model fitting depends on both the number of observations available and the criteria used in the model fitting. This approach is quite complicated and the calculations involved are tedious. Hence, the time series approach is difficult for a non time-series expert to comprehend and use in industry.

Data from a Continuous Flow Process

Measurement data are taken on one or more characteristics of a production unit. In <u>discrete</u> processes, production units are usually independent discrete items. The measured characteristics of these items are independent of one another. Subgrouping of several measurements of the same characteristic of these items, all taken at the same time, does not affect the independence of these quality data. In a continuous process, however, there is not a well defined production unit (Wortham, 1972) (Dunn and Strenk, 1985). Almost any chemical, petroleum, bulk liquid, or other semi-homogenized product is a case of this kind. The application of traditional control charting techniques is difficult in such cases since the sampling unit is defined in terms of laboratory analysis requirements rather than in, say, shipping units of product.

This problem is even compounded by the fact that to pull n samples in a row from a continuous flow process will usually result in ranges of near zero, with the range being an almost pure measure of test variation (Walter, 1955). Thus, in continuous flow processes, the most common sampling subgroup size is one. However, even when a sample is pulled one at a time at regular intervals, the measurements of these samples are bound to be correlated to one another.

Most continuous processes have associated tanks, drums or vessels where mixing takes place. Often, as new materials are continuously added to the top of a tank, the well mixed product is drawn from the bottom, simultaneously. Due to this nature of mixing, a sample taken now has some material in it that was produced one, two, or even more sample periods earlier. This mixing prevents the samples from behaving independently, thus the data are serially correlated. The relation of the current observation of process output with the past process output can be statistically quantified.

To deal with the difficulties caused by the natural characteristics of continuous flow processes, Freund (1960) suggests the use of the acceptance control chart in batch or continuous processes. Walter (1955) suggests the use of control charts of moving averages of subgroup size four and moving ranges of subgroup size two in the continuous process control of a petroleum refinery. The use of exponentially smoothed data in control charts for continuous process control is suggested by Wortham (1972). Moving Average and Range charts, and Individual and Moving Range charts are also suggested for the monitoring of continuous flow processes (Grant and Leavenworth, 1988). Koo (1987) discusses how the Xbar chart, Individual chart and Moving Average chart can be economically used to monitor continuous flow processes. Occassione (1956) discusses how Xbar and Range Charts are applied to continuous processes. In all these cases, however, the existence of serial correlation in the data of continuous flow processes is still not addressed explicitly. Rather, all the methodologies suggested still assume the existence of normality and independence in the process data. In some cases, the dependence of process data is acknowledged but avoidance is used to deal with it.

This research specifically attempts to deal explicitly with the serially correlated data from a continuous flow process. For simplicity, only a single output of interest from a continuous flow process is considered in this research.

A typical first order continuous flow process can be depicted as in Figure 1.1. The output Y's generated by a first order process are dependent on the independent input X's and serially correlated to one another. That is, the current observation of Y is not independent of past observations of the same output. The lag i serial correlation of output Y can be calculated to determine the relationship between data that are i observations apart. The lag i serial correlation, r_i, can be estimated using the equation (Box and Jenkins, 1976).

$$r_i = \frac{\sum_{t=1}^{n-i} (Y_t - \bar{Y})*(Y_{t+i} - \bar{Y})}{\sum_{t=1}^{n} (Y_t - \bar{Y})^2} \qquad (1.1)$$

where

$$\bar{Y} = \frac{\sum_{t=1}^{n} Y_t}{n}$$

Due to the distribution of the estimated serial correlation, the estimate of the serial correlation is accurate only for a large number of observations. In an industry setting, one may or may not have the large number of observations to estimate the serial correlation of the

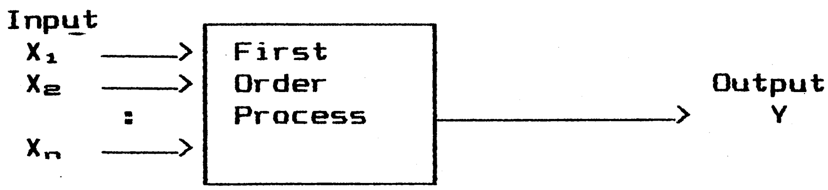

Figure 1.1. First Order Process Having Single Output Y of Interest

process output. Unfortunately, the time series analysis approach uses the estimate of the serial correlation. Hence, the time series model developed cannot correctly capture the correlation structure, especially when it is a short run process. With only few observations to begin, say 25-30, the time series analysis approach is definitely a poor one. Even with a sufficient number of observations for the estimation of serial correlation, depending on the criteria used in the model fitting, there will be several possible time series models for a given series of output. Using the model identification macro program in TIMESLAB developed by Newton (1988), this fact can be easily verified. The usual practice in the quality control discipline is to employ models which use parameters parsimoniously. However, the validity of this selection criterion in the quality control discipline has not been proved yet.

Subgroup Variation in a Continuous Flow Process

In traditional control charting, the variation of observations can be divided into two categories: within-subgroup and between-subgroup variations. This is done through the use of a concept called rational subgrouping. Within-subgroup variation is a measure of inherent or common cause variability. Common cause variation exists in all processes. It can be calculated from control chart data and is designated $\hat{\sigma}$. Between-subgroup variation is the variability from subgroup to subgroup, or the special cause variation in addition to the common cause or inherent variability. The measurement of total process variation is s. When a process is stable, or in a state of statistical control (SOSC), there is no variation from subgroup to subgroup and s is equal to $\hat{\sigma}$. However, when the process is out-of-control, the between-subgroup variation exists and it inflates the value of s well above the value of $\hat{\sigma}$.

Normally, control charts are constructed using empirical data collected from an industry process. The state of the process, either in-control or out-of-control, is usually unknown; that is why control charts are needed. Therefore, if control limits are calculated using measurement of the total variation, s, the control limits will reflect only the inherent process variability of the process if it is in control. Otherwise, the control limits will be wider since the value of s consists of both between-subgroup and within-subgroup variation. Hence, the probability of detecting an out-of-control condition would be reduced because the control limits are now wider apart. Thus, it is important to calculate the control limits using the measurement of within-subgroup variation, $\hat{\sigma}$. The usual practice is to use the subgroup range to compute the estimate of within-subgroup variation, $\hat{\sigma}$ (Patnaik, 1950) (Duncan, 1986) (Grant and Leavenworth, 1988) (Nelson, 1990).

In a continuous flow process, since observations are drawn one at a time at a regular interval, the within-subgroup variation is usually estimated by the moving ranges of size n = 2. This is to safeguard that the variation computed contains as nearly as possible only the inherent variability of the process. The moving ranges of size two are formed from a time series of output values by finding the range of the first two consecutive values, and subsequently dropping the oldest value and adding the newest value to form each successive range. The use of the moving range to estimate the process variation is also discussed by Wadsworth, et al., (1986), Gibra (1975) and A.S.T.M. Special Technical Publication (1976). The estimation of process variation using moving ranges still assumes data independence even though data are drawn from a continuous flow process. This difficulty can be dealt with by applying Hartley's Lemma (1950) which states that, if $Y_1, Y_2, \ldots, Y_k$ denote a multivariate observation from a multinormal distribution with equal variance σ^2 and equal correlation r, then the range of the Y_i is exactly distributed as the range in a subgroup of k independent normal variates with variance $\sigma^2(1-r)$ and further, is distributed independently of the mean, Ybar. This research will use Hartley's Lemma and the moving ranges of size two in the estimation of the process inherent variation.

The Nature of Continuous Well-Mixed Processes

To understand how serially correlated data may be observed, a continuous well-mixed process is considered. A typical simple chemical process is shown in Figure 1.2.

Inputs to a plug flow reactor result in output which is represented by characteristics of interest. Typical inputs to a reactor might include monomer concentration, catalyst strength and temperature; important outputs might be molecular weight of the resulting polymer, moisture content, or organic chlorine level (Brooks and Case, 1986).

That output then flows directly to an agitated (well-mixed) tank. The tank output delivers the same product, now more homogenized. That output is now designated as Y to denote that it is observed downstream of the agitated tank, as opposed to X which appears at the tank input. It is clear that the major effect of the tank will be to smooth or homogenize the variation in product coming from the reactor.

It is often not possible to measure output X. If it were, this research would not be needed. Rather, the first opportunity to measure the characteristic of interest is at downstream output Y. The mixing process results in a gradual output response relative to a change made at the input of the tank. That is, the first order output response Y at some arbitrary time s, $s > 0$, after a step disturbance in the input to the tank (output X) at time 0 follows the classical exponential response

$$Y(s) = e^{-s/\tau}Y(0) + (1-e^{-s/\tau})X(s) \qquad (1.2)$$

where τ is a constant representing the residence time of the tank and s is also the sampling interval. If the sampling interval equals to the residence time of the tank,

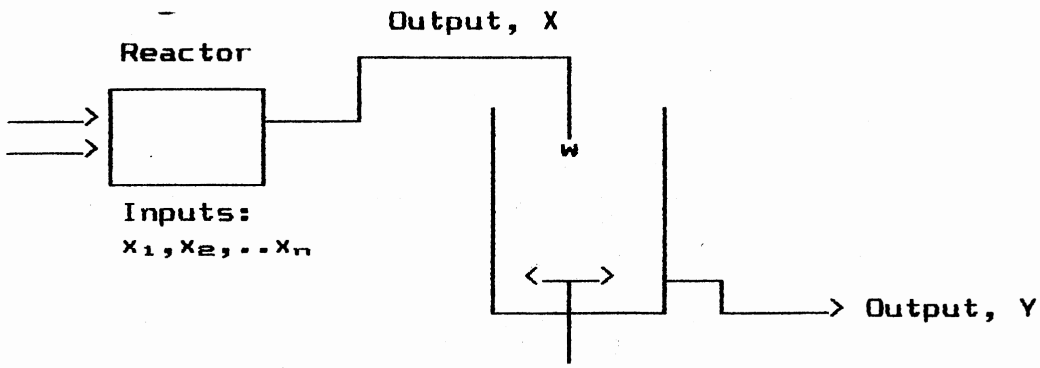

Figure 1.2: Simple Chemical Process

then only e^{-1} = 0.386 of the "old" product from time 0 would remain in the tank at time $s = \tau$. The residence time τ is equal to the ratio of occupied tank volume to the volumetric flow rate for a first order process. This equation is similar to those found in Box and Jenkins (1976), Coughanowr and Koppel (1965), and MacGregor and Tidwell (1980).

It is assumed that the length of sampling interval can be fixed. Thus, by letting $e^{-s/\tau} = r$, output Y can be observed as a single measurement at a fixed time s apart. By indexing the observed output of Y, Equation (1.2) can be rewritten as a generalized equation for any time period.

$$Y(t) = rY(t-1) + (1-r)X(t) \qquad (1.3)$$

where t = time index for output Y

r = is also known as a filter constant

Brooks and Case (1986) have shown that if the serial correlation of Y(t) is ignored, then the control charts used to monitor the process will be based on the assumption of data independence. These control charts surely generate false signals about the process and unnecessary corrective action will be implemented, thereby worsening process stability. The internal serial correlation, between contiguous members of the Y time series, is inherent in well-mixed chemical processes. Failure to meet the assumption of data independence means that all the existing common control charting techniques are not appropriate to be used. Moreover, process capability indices cannot be calculated with assurance, nor can any usual statistical inference be made concerning the data, without first considering explicitly the effect of data dependence.

In industry, most of the response processes such as refinery processes are higher order response processes. However, a higher order response process usually can be 'broken down' into smaller components of lower order response processes. As long as the physical structure permits observations to be sampled, all the lower order response processes can be monitored and controlled in isolation. Often, the lower order response processes are the first order responses processes. Each FORP can be monitored and controlled separately. By first principles, the chemical engineer is able to determine the filter constant of many FORPs and proper quality control tools can be used to monitor and control such response processes.

Research Objectives

Based on the above discussion, the scope of this research can be stated as follow:

Objective: To develop and evaluate procedures for determining a pair of new control charts capable of monitoring both the mean and dispersion of serially correlated quality data generated from a first order response process (FORP).

In order to accomplish this objective, several subobjectives must be met. The subobjectives are:

1. To derive and construct procedures for determining the appropriate control limits for the control chart that monitors the mean of serially correlated quality data generated from a FORP. To facilitate further discussion, the control chart constructed in this subobjective is denoted as an OPA Y chart.
2. To derive and construct procedures for determining the appropriate control limits for the control chart that monitors the dispersion of serially correlated quality data generated from a FORP. To facilitate further discussion, the control chart constructed in this subobjective is denoted as an OPA MRy chart.
3. To analytically show that when the proposed control chart procedures developed in subobjective (1) is applied to the serially correlated quality data generated from a FORP, it has the same Average Run Length (ARL) as an Individual chart applied to an independent normal data stream.
4. To determine the ARL of the proposed control chart procedures developed in subobjective (2) for different magnitudes of stepwise shifts in the dispersion of the input variable to a FORP using a simulation approach.
5. To determine the ARL when both the proposed control chart procedures are used simultaneously for different magnitudes of stepwise shifts in the mean of the input variable to a FORP using a simulation approach.
6. To compare the ARLs of the OPA Y chart versus those of

the I_chart when all of these charts are applied to the same FORP data stream. The shift in the mean of the input variable to a FORP data stream is either stepwise, trend or cyclical.

7. To compare the ARLs of the OPA MRy chart versus those of the MR(2) chart when these two charts are applied to the same FORP data streams. The shift in the dispersion of the input variable to a FORP data stream is a stepwise shift and the shift in the process mean is either trend or cyclical.

8. To determine and compare the ARLs of the OPA Y chart, and the combined OPA Y and OPA MRy charts for different magnitudes of stepwise shift in the mean of the input variable to a FORP data stream when the value of the supposed-to-be-known filter constant of the FORP, r, is either understated, correctly stated or overstated.

9. To determine and compare the ARLs of the OPA MRy chart for different magnitudes of stepwise shifts in the dispersion of the input variable to a FORP data stream when the value of the supposed-to-be-known filter constant of the FORP, r, is either understated, correctly stated or overstated.

10. To determine the ARLs of Montgomery's modified EWMA chart when it is applied to a FORP data stream for different magnitudes of stepwise shifts in the mean of the input variable to the FORP.

11. To compare the ARLs in subobjective (10) with those of

the OPA Y chart which is also applied to the same data stream as the modified EWMA chart.

12. To determine the ARLs of the OPA Y chart and OPA MRy chart when these charts are applied to data from a higher order response process.
13. To investigate the robustness of the proposed control chart procedure to the degree of order of a response process using the ARLs from subobjectives (3), (4) and (12).
14. To develop a simple and flexible interactive user computer program to implement the proposed control charting technique.

Research Assumptions

In order to further define and delimit this research, certain general assumptions are made. They include:

1. The serially correlated quality data are generated from a FORP.
2. The user of the proposed control charts has a prior knowledge of the FORP involved. Using first principles, it is assumed that the user knows the numerical value of the filter constant, r.
3. The input variable, X, to the FORP cannot be measured. Thus, its parameters must be determined from the observable, but serially correlated output variable, Y.
4. Only a single variable of input X is of interest to the user. The FORP does not introduce any extra variation.

Thus, by monitoring the corresponding output variable Y, the input value X can be monitored accordingly.

5. In the case of a continuous flow chemical process which exhibits a first order response behavior, the in-flow rate to the process is equal to the out-flow rate. Moreover, instantaneous and complete mixing is assumed when a change in the input is made. For a non-continuous flow FORP, similar and comparable assumptions are also made.
6. It is assumed that the input variable, X, is independent and identically distributed. The underlying distribution of variable X is normal.
7. The sampling subgroup size is one and sampling is done at a regular interval.

CHAPTER II

LITERATURE REVIEW

Introduction

Since traditional control charting was first introduced in the 1930's, various techniques have evolved to deal with different process control situations. Each control charting method is built on different sets of assumptions. The most common assumptions made among various charting techniques are the independence of data and the normality of the underlying population from which the data are drawn.

The development of various control charting techniques has been compiled by Gibra (1975) and Vance (1983). It shows the variety in the efforts to study the performance of different processes through control charting techniques. From the literature, it is clear that even though some effort has been geared to the development of control charting techniques that deal with serially correlated data, they are often developed on a generalized serially correlated data stream. Moreover, the application of these techniques is difficult.

This research concentrates on the monitoring of a correlated data stream from a FORP. Due to the inadequacy of existing control charting techniques to monitor the mean and/or dispersion of a FORP, it is imperative to develop a control charting technique that deals specifically with quality data generated from a first order response process.

Background

The concept of control charts was first introduced in the 1930's by Dr. Walter Shewhart (1931). The concept was built on the assumption that the measurable characteristic of a manufactured product is always subject to some uncontrollable variation. The variation is divided into two parts: 1) a stable system of chance which is normal to the process, and 2) variation outside of this stable pattern which can be discovered and corrected. Independence and the normality of the process data requirements are inherent in the concept and also mentioned in most statistical quality control texts (Duncan, 1986) (Grant and Leavenworth, 1988) (Wadsworth, et al., 1986).

In the 1960's, Lieberman (1965) summarized the concept of the Shewhart control charts. He points out that the Shewhart control chart has as its functions to: (1) determine the process capabilities, (2) detect and identify assignable causes of variation, and (3) provide guidance in correcting the process. The traditional control chart is a chart of data from the process, and its primary purpose is to provide a basis from which corrective action is taken. The action is taken when a plotted point falls outside a fixed limit, or a succession of points fall between some less extreme limits. Even though traditional Shewhart control charts are powerful in detecting process shifts in mean and dispersion, they fail to perform well under certain situations.

Situations such as the existence of correlated data within subgroups, correlation between subgroups, non-normality of the underlying process from which subgroups are drawn, cyclic trends in data and so on, affect the conclusion one makes about the process. The risks of Type I and/or Type II errors usually is inflated if traditional control charting techniques are used in these situations.

Correlation in Shewhart's Control Charts

Neuhardt (1987) studies the effects of correlated data within subgroups in statistical process control. Correlated data within subgroups arises because of simplicity in data collection due to multiple, but similar, measurements on a single product or multiple station machines. When each subgroup is considered as an independent sample of a multivariate random vector, the paper investigates the consequences of unrecognized or unaccounted for correlation and how the control rules should be modified if the correlation is recognized. Neuhardt realizes that the effect of correlated measurements within subgroup increases the Type I error rate for the Xbar chart. That is an out-of-control condition which occurs too frequently in the Xbar chart. However, the variance charts are not affected in the same fashion when equality of covariances of the measurements is assumed.

Alt and Deutsch (1979) show the effect of correlated observations within independent samples on the parameters of the Shewhart Xbar chart. They extend the method proposed by Page (1954) to the case where the within-subgroup observations are correlated. They find that the subgroup size n needed to detect a shift of a given magnitude increases with the extent of correlation. Both articles deal only with correlation within subgroups and not correlation between subgroups. That is, the subgroups are still assumed to be independent from one another.

Ali (1987) studies the effect of dependency between observations on the distribution of the sample mean and its rate of convergence to normality. He gives various methods to approximate the distribution of the sample mean of dependent observations generated from a stationary stochastic process and the departure from the normal distribution is numerically assessed for a range of models. It is shown that the convergence to normality is slowed by the degree of correlation, the degree of skewness in the random variables defining the stochastic process, and on the subgroup size. His results show that the central limit theorem is still valid in the sample mean of correlated data.

Throughout the development and fine-tuning of the traditional control charting techniques, several factors have been tabulated for the calculation of various control limits. These tables can be found in many statistical quality control texts (Duncan, 1986) (Grant and Leavenworth, 1988) (Wadsworth, et al., 1986). These factors are derived under the assumptions that the subgroups are drawn from a normal process and the subgroups are independent of one another. Thus, violation of these assumptions makes the factors inappropriate.

Correlation in Other Control Charts

Goodman (1982) discusses how the CUSUM can be used in a continuous flow process which generates serially correlated data. Johnson and Bagshaw (1974), and Bagshaw and Johnson (1975) study the effect of serial correlation on the performance of the CUSUM charts. They investigate the influence of serial correlation for both the fixed sample size and sequential versions of CUSUM tests. They conclude that the CUSUM test is not robust with respect to departures from data independence. Kartha and Abraham (1979) study the effect of serial correlation on the average run length of CUSUM charts. The average run length is found to be decreased by the presence of serial correlation. Nath (1976) develops the control chart for fraction defective for the case of dependent observations.

So far, not much work has been documented on the studies of the effect of data correlation in other types of control charts such as the Individual chart, Moving Range and Moving

Average charts.

Correlation in Manufacturing Data

Jacobs and Lorek (1980) show that daily manufacturing data may not be independent and normally distributed. They warn that by using the assumptions of data independence and normality in variance investigations, the results may not be valid. They conclude that care should be taken when making variance investigations, for example, setting control limits based on this type of data. They recommend that investigators either use weekly data which are more likely to be independent of one another and normal, or that they use statistical methods which explicitly take into account serial correlation or non-normality.

Hubele and Keats (1987) point out when automatic control is implemented, practically all data can be collected. In these cases, Shewhart's control charts are usually not appropriate because they require independent and normally distributed data. Consequently, time-series analysis may be more useful for process control when all the data are available in an automatic control mode.

Hahn (1977) presents an example where the data are nonindependent while the statistical analysis ignored this, thereby resulting in a wrong conclusion. He conveys the message that analyses failing to take into account nonindependent data can lead to wrong conclusions.

In industry, serially correlated data may have long been neglected. This may be attributed to the fact that it is more complicated and tedious to deal with correlated data. Thus, people often avoid them and simply ignore their existence. But, with today's computing power, correlation in quality data can be analyzed and more meaningful and accurate process control can be achieved.

Time Series Approach to Serially Correlated Quality Data

The idea of dealing with correlated data using a time series approach emerged in the 70's when Stamboulis (1971) relaxed the assumption of independence and replaced it by dependency introduced via an AR(1) model with parameter α. Vasilopoulos and Stamboulis (1978) further modified and extended the standard Shewhart control charting technique by introducing dependence via a second order autoregressive process (AR(2) Model). Curves of modified auxiliary quality control factors are presented in their paper.

The use of a time series analysis approach to deal with the correlated data is also suggested by Alwan and Roberts (1988). They propose and illustrate the use of two basic charts, the Common-Cause Chart and the Special-Cause Chart, in sorting out special causes from common causes. The autoregressive integrated moving average (ARIMA) model of Box and Jenkins is first used to model systematic nonrandom behavior of the out-of-control process. A standard control chart for residuals of the fitted model is then used to detect departures from control due to special causes.

Montgomery (1990) presents two methods for applying statistical control charts to serially correlated data. The first method is based on modeling the autocorrelative structure and applying control charts to the residuals. The second method is a simple modification of the Exponentially Weighted Moving Average (EWMA) control chart. The EWMA is found to be the optimal one-step-ahead forecast for the mean of an IMA (1,1) model. Thus, the forecast for the observation in period t+1 which is made at the end of period t is used as the center line for the control chart in period t+1. The standard deviation of these one-step-ahead prediction errors is then used to construct control limits for period t+1. The major assumption Montgomery makes in his paper is that the observations from the process can be well-modeled by an IMA (1,1) model. He claims that in practice the IMA (1,1) model is a very good approximation for forecasting the level of a time series. Alwan and Roberts (1988) also note that the IMA (1,1) model produces a forecast of the mean level of the series that is robust to the exact form of the underlying ARIMA model. Montgomery concludes that the EWMA control chart is a very useful procedure for application to serially correlated data.

Sahrmann (1979) illustrates how time series analysis can be used in coating weight control for reverse roll coating. He uses a time series analysis approach to identify sources of cyclical variation and then measures their relative contribution to overall process variation. Methods for dealing with serially correlated data are also suggested by several authors. These include, Berthouex, et al. (1976, 1978), Dooley, et al. (1986, 1990), Ermer (1979, 1980), Ermer, et al. (1979), Liao, et al. (1982), Montgomery and Friedman (1989), Notohardjono and Ermer (1983, 1986) and Yourstone and Montgomery (1989).

Although the tactical approaches to the problem of serial correlation taken by these people often differ from one another, the strategic thrust of their efforts are identical. That is, they wish to fit an appropriate time series model to the observations and then apply control charts to the stream of residuals from this model.

Need for Study

From the above discussion, it is clear that there are at least four good reasons why this research should be performed:

(1) Even though time series analysis is used to deal with correlated data, there are three major drawbacks. They are:

(i) a large number of subgroups is needed before a proper time series model can be accurately fitted,

(ii) there is usually more than one model that well fits the data, and

(iii) computation in the time series analysis is complicated and tedious.

(2) Correlated data is very common in the process industries. A simple, easy to use and easy to understand control charting technique is not now available. In the process industries, the typical method used to deal with correlated data is avoidance. Some may just ignore the fact that correlated data exist and hence, they are sure to make wrong conclusions and take unnecessary corrective action.

(3) Several researchers have investigated the effect of correlated data. However, they emphasize only on the correlation within subgroups. Often, the multivariate normal distribution is assumed to be the appropriate underlying distribution. Under these circumstances, the independence between subgroups is still assumed.

(4) The FORP is one of the most commonly encountered processes in industry. The existing control charting techniques can not monitor correlated data from a FORP effectively. Thus, it is imperative to develop a control charting technique specifically for correlated data generated from such a process.

Conclusion

The existence of correlated data in the quality control discipline can no longer be neglected. From the literature above, it is clear that the issue of correlated data has been neglected for a long time. No work has been documented on the development of a control chart specifically for serially correlated data generated from a first order response process. All recent efforts are geared to the time series analysis approach to serially correlated data. This research will satisfy one major need by providing a control charting technique which deals directly with serially correlated data generated from a first order response process.

CHAPTER III

MEAN CONTROL CHART FOR QUALITY DATA FROM A FIRST ORDER RESPONSE PROCESS

Introduction

In this chapter, a derivation of the mean and the asymptotic standard deviation of the serially correlated data stream generated from a first order response process is presented. The control charting technique specifically deals with the mean level output of a first order response process. The proposed mean control chart, denoted as an OPA Y chart, is based on conditional distribution theory. Linear model theory (Graybill, 1976) is used to derive the conditional control limits of the proposed OPA Y chart. A numerical example is then presented to illustrate how the proposed OPA Y chart can be constructed using some empirical data.

Unlike the traditional Shewhart control charts in which control limits are computed once and used for all plotted points, control limits for the OPA Y chart are computed from point to point. That is, at the current plotted point, one-period-ahead control limits are computed for the next plotted point. The process is considered out-of-control at the current time period if the current plotted point falls beyond its one-period-ahead control limits constructed one period before. The one-period-ahead control limits for the next plotted point depend on the current plotted point. In a traditional control chart, control limits are set to be three sigma away from the center line; in this proposed OPA Y chart, every pair of the conditional one-period-ahead control limits is also set at three sigma away from its center line.

Distribution of Y_t

The distribution of a serially correlated output variate from a first order response process at the t^{th} time period is derived in this section. It is assumed that the input random variable, X_t, to the first order response process, is independent and identically distributed. The underlying distribution of X_t is a normal distribution with mean, μ, and standard deviation, σ. A random vector of X can be easily formed by grouping the first (t+1) of the X's. Using linear model theory, the random vector is

$$X = (X_0, X_1, X_2, \ldots, X_t)'$$

and is distributed as a multivariate normal with mean vector, $\mu 1_{t+1}$, and covariance matrix, $\sigma^2 I_{t+1}$.

Recall that the first order response equation is

$$Y_t = rY_{t-1} + (1-r)X_t \quad (3.1)$$

Substituting

$$Y_{t-1} = rY_{t-2} + (1-r)X_{t-1} \quad (3.2)$$

into Equation (3.1) results in

$$Y_t = r\left[rY_{t-2} + (1-r)X_{t-1}\right] + (1-r)X_t$$

$$Y_t = r^2Y_{t-2} + (1-r)rX_{t-1} + (1-r)X_t$$

Continuously substituting for the first term on the right hand side of the equation results in

$$Y_t = (1-r)r^tX_0 + (1-r)r^{t-1}X_1 + (1-r)r^{t-2}X_2 + \dots + (1-r)X_t$$

$$Y_t = \sum_{k=0}^{t} (1-r)r^kX_{t-k} \tag{3.3}$$

Thus, Y_t is a linear combination of $(X_0, X_1, \dots, X_t)'$.

Letting

$$Z_1 = Y_t = (1-r)r^tX_0 + (1-r)r^{t-1}X_1 + \dots + (1-r)X_t$$

$$Z_2 = X_t = 0 + 0 + \dots + X_t$$

$$Z_3 = Y_{t-1} = (1-r)r^{t-1}X_0 + (1-r)r^{t-2}X_1 + \dots + 0X_t$$

These three equations can be written as

$$Z = AX$$

where $Z = (Z_1, Z_2, Z_3)'$

$$A = \begin{bmatrix} (1-r)r^t & (1-r)r^{t-1} & \dots & (1-r)r^1 & (1-r) \\ 0 & 0 & \dots & 0 & 1 \\ (1-r)r^{t-1} & (1-r)r^{t-2} & \dots & (1-r) & 0 \end{bmatrix}$$

The dimension of matrix A is 3 x (t+1). It can be shown that for large t, Z is distributed as a multivariate normal.

$$Z = \begin{bmatrix} Y_t \\ X_t \\ Y_{t-1} \end{bmatrix} \sim N \left[\begin{bmatrix} \mu \\ \mu \\ \mu \end{bmatrix}, \sigma^2 \begin{bmatrix} \frac{(1-r)}{(1+r)} & (1-r) & \frac{(1-r)r}{(1+r)} \\ (1-r) & 1 & 0 \\ \frac{(1-r)r}{(1+r)} & 0 & \frac{(1-r)}{(1+r)} \end{bmatrix} \right] \qquad (3.4)$$

Detailed derivation can be found in Appendix B. Using linear model theory, it is clear that the asymptotic distribution of Y_t is

$$Y_t \sim \text{Normal}\left[\mu,\ \sigma^2 \frac{(1-r)}{(1+r)}\right] \qquad (3.5)$$

It is also easily found that X_t and Y_{t-1} are independent of one another. The variance of Y_t can also be derived by viewing Equation (3.1) as a first order autoregressive process, AR(1); since it can be expressed as

$$Y_t = \alpha Y_{t-1} + e_t$$

where $\alpha = r$ and $e_t = (1-r)X_t$. The computation of the variance of Y_t follows directly from using Box and Jenkins (1976).

It may be tempting to construct a control chart for the mean of FORP data using the mean and variance as shown in (3.5). However, due to the correlated nature of Y_t, a mean chart developed using these mean and variance values is found to be inefficient in detecting process changes. It is also pointed out before that a time series approach to correlated data is quite complicated and tedious. Thus, even though a FORP can be modeled as a time series model of AR(1), in this research the time series approach is not used on the correlated data from a FORP. Rather, an one-period-ahead control chart, an OPA Y chart will be developed using the mean and variance of the conditional distribution of Y_t.

Conditional Distribution of Y_t Given Y_{t-1}

Recall that the joint distribution of $(Y_t, X_t, Y_{t-1})'$ is the expression in (3.4). If Y_{t-1} is known and it takes on the value k. Then, from linear model theory it can be shown that the joint distribution of $B = (Y_t, X_t)'$, given that Y_{t-1} equals k, is a bivariate normal.

$$B \sim \text{Normal}\left[\begin{bmatrix} rk+(1-r)\mu \\ \mu \end{bmatrix}, \sigma^2 \begin{bmatrix} (1-r)^2 & (1-r) \\ (1-r) & 1 \end{bmatrix}\right] \tag{3.6}$$

If it is further assumed that X_t takes on value c, then the conditional distribution of Y_t, given that Y_{t-1} equals k and X_t equals c, is found to be

$$P(Y_t=y_t) = \begin{cases} 1 & \text{for } y_t=rk+(1-r)c \\ 0 & \text{otherwise} \end{cases} \tag{3.7}$$

That is, when Y_{t-1} and X_t are given, the value of Y_t is deterministic.

If X_t is a random variable from a normal distribution rather than a specific value, c, then it can be shown that the distribution of Y_t, given that Y_{t-1} equals k and X_t is normally distributed with mean μ and variance σ^2, is a normal distribution with mean $rk+(1-r)\mu$ and variance $(1-r)^2\sigma^2$.

$$Y_t \mid (Y_{t-1}=k) \sim \text{Normal}\left[rk+(1-r)\mu,\ (1-r)^2\sigma^2\right] \qquad (3.8)$$

Conditional Control Limits of the OPA Y Chart

From (3.8), the one-period-ahead control limits can be constructed for the proposed OPA Y chart. The central line and control limits for the t^{th} plotted point are:

$$CL_{OPAY} = rY_{t-1} + (1-r)\mu \qquad (3.9)$$

$$UCL_{OPAY} = rY_{t-1} + (1-r)(\mu+3\sigma) \qquad (3.10)$$

$$LCL_{OPAY} = rY_{t-1} + (1-r)(\mu-3\sigma) \qquad (3.11)$$

From (3.7) it is clear that, given the observed value of a serially correlated datum Y_{t-1}, the unobservable independent variable X_t will determine the observable value of Y_t. Thus, X_t and Y_t constitute a conditional point-to-point mapping and the condition is that the value of Y_{t-1} must be known. Due to this conditional point-to-point mapping, it is found that the Type I and II errors of any plotted point on the OPA Y chart are identical to any plotted point on the Individual chart for an independent normal data stream.

To explain this fact, a phantom Individual chart is used for the X random variable. Let the upper and lower control limits of the Individual chart of X be denoted as

UCL_I and LCL_I, respectively. Since X is normally and independently distributed with mean μ and standard deviation σ, the phantom Individual chart for this X variable has the following upper and lower control limits.

$$UCL_I = \mu + 3\sigma$$

$$LCL_I = \mu - 3\sigma$$

If X at time period t, X_t, falls exactly on UCL_I, and Y_{t-1} is known and has value k, then, from Equation (3.1), Y_t is

$$Y_t = rk + (1-r)(\mu + 3\sigma) \qquad (3.12)$$

Likewise, if X_t falls exactly on LCL_I, and Y_{t-1} is known and has value k, then, from Equation (3.1), Y_t is

$$Y_t = rk + (1-r)(\mu - 3\sigma) \qquad (3.13)$$

Since (X_t, Y_t) is a point-to-point mapping, the probability of X_t falling beyond UCL_I is the same as the probability of Y_t falling beyond $rk+(1-r)(\mu+3\sigma)$. Similarly, the probability of X_t falling beyond LCL_I is the same as the probability of Y_t falling beyond $rk+(1-r)(\mu-3\sigma)$. Thus, if the one-period-ahead upper control limit of Y_t on the OPA Y chart, UCL_{OPAY}, is assigned the value $rk+(1-r)(\mu+3\sigma)$ and the lower control limit, LCL_{OPAY}, is assigned the value $rk+(1-r)(\mu-3\sigma)$, then the Type I and II errors of the proposed OPA Y chart at this plotted point Y_t are identical to the X_t point plotted on the Individual chart for an independent normal data stream. It is noted that the assignments of the UCL_{OPAY} and LCL_{OPAY} with (3.12) and (3.13), respectively, are identical to the control limits constructed in (3.10) and (3.11), respectively.

Since each plotted point on the proposed OPA Y chart has its own one-period-ahead control limits, it leads to the fact that any plotted point on the proposed OPA Y chart has identical Type I and II error risks as the Individual chart for an independent normal data stream.

To use the one-period-ahead center line and control limits as in (3.9)-(3.11) for serially correlated data observed empirically from a first order response process, the mean and variance of the unobservable variate X must be known or estimated. Using Hartley's Lemma, it is found that

$$\frac{MRybar}{d_2} = \sigma_y \sqrt{(1-r)} \qquad (3.14)$$

Equation (3.14) is also shown in the paper by Cryer, et al. (1990). From (3.5), it is found that

$$\sigma_y = \sigma \frac{\sqrt{(1-r)}}{\sqrt{(1+r)}} \qquad (3.15)$$

$$\mu_y = \mu \qquad (3.16)$$

Substituting (3.15) into (3.14) results in

$$\frac{MRybar}{d_2} = \sigma \frac{(1-r)}{\sqrt{(1+r)}}$$

$$\sigma = \frac{MRybar}{d_2} \frac{\sqrt{(1+r)}}{(1-r)} \qquad (3.17)$$

Substituting (3.16) and (3.17) into (3.9)-(3.11) and using the average of Y values, Ybar, as an estimate of μ_y, the one-period-ahead control limits of the OPA Y chart for the t^{th} time period are

$$CL_{OPAY} = rY_{t-1} + (1-r)Ybar \qquad (3.18)$$

$$UCL_{OPAY} = rY_{t-1} + (1-r)\left[Ybar + 3\,\frac{MRybar}{d_2}\,\frac{\sqrt{(1+r)}}{(1-r)}\right] \qquad (3.19)$$

$$LCL_{OPAY} = rY_{t-1} + (1-r)\left[Ybar - 3\,\frac{MRybar}{d_2}\,\frac{\sqrt{(1+r)}}{(1-r)}\right] \qquad (3.20)$$

Values of Ybar and MRybar can be calculated from the first few, say 25–30, measurements of the Y variate. Obviously, the accuracy of these estimations increases with the number of measurements used.

Numerical Illustration

A numerical example is presented in this section to illustrate how to use the proposed OPA Y chart on serially correlated data generated from a first order response process. As stated in the assumptions of this research, the user is expected to have a prior knowledge of that particular first order response process. The value of the filter constant, r, is assumed to be known using first principles.

Assume that the first order response process to be monitored has a filter constant r equal to 0.8. Suppose that 30 measurements from this process have been observed from the 1^{st} to the 30^{th} time periods. At each time period, only one measurement is collected. These 30 measurements are the Y values and they are presented in TABLE 3.1 in time order. The moving ranges of subgroup size two of Y, MRy, are calculated. The average of Y and MRy are also computed. These two quantities are essential to compute the one-period-ahead control limits. The one-period-ahead control limits are then computed for all the time periods except the 1st time period using the formulas in (3.19)-(3.20). For example, the one-period-ahead control limits for the 9th time period are computed using formulas in (3.19)-(3.20) and following quantities

Ybar = 19.761

MRybar = 0.189

d_2 = 1.128

Y_8 = 19.693

The one-period-ahead control limits for the 9th time period are found to be

Lower control limit = 19.033

Upper control limit = 20.380

All the MRy, the average of Y and MRy, and the one-period-ahead control limits are also presented in TABLE 3.1.

By plotting the Y_t values and the one-period-ahead control limits on the control chart, the required OPA Y chart is obtained. The OPA Y chart is plotted in Figure 3.1, and is easily interpreted. It should be noted that the central line of the OPA Y chart is omitted in TABLE 3.1 and Figure 3.1 to avoid confusion in the reading of table and interpretation of the control chart.

TABLE 3.1

SERIALLY CORRELATED DATA FROM A FIRST ORDER RESPONSE PROCESS WITH FILTER CONSTANT 0.8, MOVING RANGES OF TWO AND ONE-PERIOD-AHEAD CONTROL LIMITS FOR THE OPA Y CHART

Time, t	Y_t	MR_Y	LCL_{OPAY}	UCL_{OPAY}
1	20.023			
2	20.208	0.185	19.297	20.644
3	19.905	0.303	19.445	20.792
4	20.020	0.115	19.203	20.550
5	19.786	0.234	19.295	20.642
6	20.134	0.348	19.108	20.454
7	20.074	0.060	19.386	20.733
8	19.693	0.381	19.338	20.685
9	19.210	0.483	19.033	20.380
10	19.385	0.175	18.647	19.994
11	19.649	0.264	18.787	20.134
12	19.643	0.006	18.998	20.345
13	19.736	0.093	18.993	20.340
14	19.714	0.022	19.068	20.414
15	19.349	0.369	19.050	20.397
16	19.266	0.083	18.758	20.105
17	19.091	0.175	18.692	20.038
18	19.321	0.230	18.552	19.898
19	19.880	0.559	18.736	20.082
20	19.845	0.035	19.183	20.530
21	20.249	0.404	19.155	20.502
22	20.022	0.227	19.478	20.825
23	19.826	0.196	19.296	20.643
24	19.756	0.070	19.140	20.486
25	19.774	0.018	19.084	20.430
26	19.870	0.096	19.098	20.445
27	19.726	0.144	19.175	20.522
28	19.832	0.106	19.060	20.406
29	19.922	0.090	19.144	20.491
30	19.916	0.006	19.216	20.563
Average	19.761	0.189		

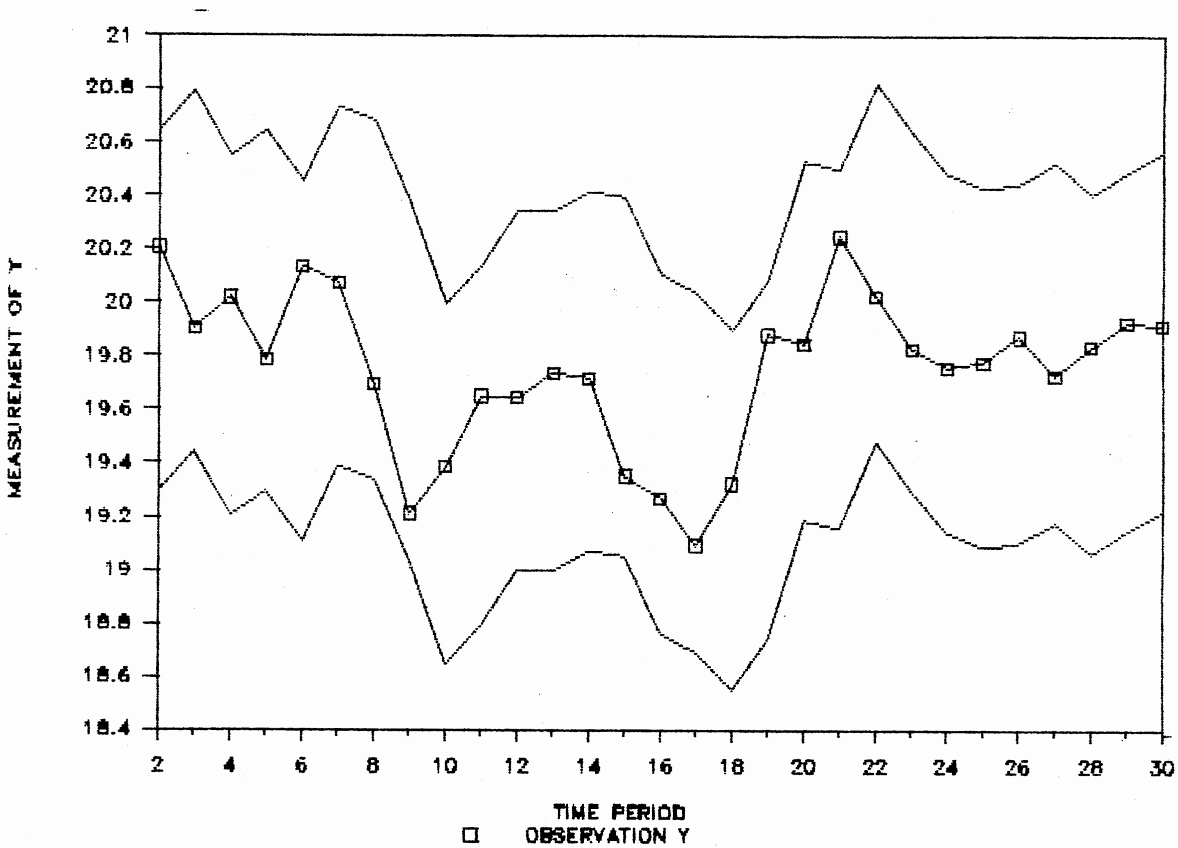

Figure 3.1. The OPA Y chart, r = 0.8

Conclusion

In this chapter, the derivation of the distribution of Y_t and the conditional distribution of Y_t given Y_{t-1} are presented. The proposed OPA Y chart is constructed using the conditional one-period-ahead control limits. The ARL of the proposed OPA Y chart is found to be identical to an Individual chart applied to an independent normal data stream. A numerical example is used to illustrate the construction of the proposed OPA Y chart to monitor the mean level of the output of a first order response process.

CHAPTER IV

DISPERSION CONTROL CHART FOR QUALITY DATA FROM A FIRST ORDER RESPONSE PROCESS

Introduction

In this chapter, a derivation of the mean and standard deviation of the conditional moving range of subgroup size two of serially correlated data generated from a first order response process is presented. The control chart that specifically deals with the dispersion of the output of a first order response process is then constructed. The proposed dispersion control chart, denoted as an OPA MRy chart, is a control chart based on the conditional distribution of the moving range of subgroup size two of serially correlated data generated from a first order response process. A numerical example is then presented to illustrate how the proposed OPA MRy chart can be constructed using empirical data.

Unlike the traditional Moving Range control charts for process dispersion in which the control limits are computed once and used for all the plotted points, the control limits for the OPA MRy chart are computed from point to point. That is, at a current plotted point, one-period-ahead control limits are computed for the next plotted point. The one-period-ahead control limits for the next plotted point depends on the current observation of the Y variate. In the traditional Moving Range chart, control limits are set to be three sigma away from the center line even though the underlying distribution of the moving range is not normal. In the proposed OPA MRy chart, this three sigma convention is still employed.

Conditional Distribution of the Moving Range of Subgroup Size Two of the Y Variate

The conditional distribution, mean and variance of the moving range of subgroup size two of serially correlated data generated from a first order response process at the t^{th} time period is derived. Recall that the first order response process that generated serially correlated output, Y, is

$$Y_t = rY_{t-1} + (1-r)X_t \tag{4.1}$$

From (3.4), it is known that X_t is independent of Y_{t-1}. If Y_{t-1} is known and takes on the value k, then

$$Y_t = rk + (1-r)X_t$$

Defining $R|k$ as the range of Y_{t-1} and Y_t given that Y_{t-1} equals k, then

$$R|k = \text{Range of } k \text{ and } Y_t$$

$$R|k = |Y_t - k| \tag{4.2}$$

Substituting (4.1) into (4.2) results in

$$R|k = |\ rk + (1-r)X_t - k\ |$$

$$R|k = |\ (1-r)X_t - (1-r)k\ |$$

$$R|k = \left| (1-r)(X_t - k) \right| \tag{4.3}$$

In a first order response process, it is found that $0 < r < 1$. Then,

$$\frac{R|k}{(1-r)} = \left| (X_t - k) \right| \tag{4.4}$$

Letting Q equal Equation (4.4) results in

$$Q = \frac{R|k}{(1-r)} = \left| (X_t - k) \right| \tag{4.5}$$

The distribution of Q is then considered. From Equation (4.5), it is found that X_t is a double valued function of Q, say Q' and Q", (Basnet and Case, 1990). That is,

$$Q' = k - Q \tag{4.6a}$$

$$Q'' = k + Q \tag{4.6b}$$

Then, the density function of Q, h(Q), is given by

$$h(Q) = \left| \frac{\delta Q'}{\delta Q} \right| f(k-Q) + \left| \frac{\delta Q''}{\delta Q} \right| f(k+Q)$$

$$h(Q) = f(k-Q) + f(k+Q) \tag{4.7}$$

where $f(\cdot)$ is defined as before.

Using Equations (4.5), (4.7), and a single variable transformation approach, the density function of $R|k$ is found to be

$$g(R|k) = \frac{1}{(1-r)} \left[f\left[k - \frac{R|k}{(1-r)}\right] + f\left[k + \frac{R|k}{(1-r)}\right] \right] \tag{4.8}$$

A detailed derivation can be found in Appendix C.

Mean and Standard Deviation of R|k

Computation of the expected value and variance of R|k from Equation (4.8) is difficult, as the results are not in a closed form. To circumvent this difficulty, the expected value and variance of the variable Q are found first. With Equation (4.7), the expected value of Q, E(Q), is found to be

$$E(Q) = \int_0^\infty Q \cdot h(Q)\, dQ \tag{4.9}$$

and the variance of Q, V(Q), is

$$V(Q) = E(Q^2) - [E(Q)]^2 \tag{4.10}$$

where

$$E(Q^2) = \int_0^\infty Q^2 \cdot h(Q)\, dQ$$

From Equation (4.5), it is clear that,

$$R|k = (1-r)Q \tag{4.11}$$

Then the expected value of R|k, E(R|k), and the variance of R|k, V(R|k) are

$$E(R|k) = (1-r)E(Q) \tag{4.12}$$

$$V(R|K) = (1-r)^2 V(Q) \tag{4.13}$$

It can be shown that the expected value of Q, E(Q), and the variance of Q, V(Q), both have a "closed" form as

$$E(Q) = 2\sigma\Phi'\left[\frac{k-\mu}{\sigma}\right] + |k-\mu|\left[1 - 2\Phi\left[\frac{-|k-\mu|}{\sigma}\right]\right] \quad (4.14)$$

$$V[Q] = \sigma^2 + (k-\mu)^2 - [E(Q)]^2 \quad (4.15)$$

where $\Phi'(x)$ and $\Phi(x)$ are as defined in the Notation. A detailed derivation can be found in Appendix C.

Substituting Equations (4.14) and (4.15) into Equations (4.12) and (4.13), the following results are obtained.

$$E(R|k) = (1-r)\left[2\sigma\Phi'\left[\frac{k-\mu}{\sigma}\right] + |k-\mu|\left[1 - 2\Phi\left[\frac{-|k-\mu|}{\sigma}\right]\right]\right] \quad (4.16)$$

$$V(R|k) = (1-r)^2\left[\sigma^2 + (k-\mu)^2 - [E(Q)]^2\right] \quad (4.17)$$

Thus, the standard deviation of $R|k$ equals

$$Std(R|k) = (1-r)\left[\sigma^2 + (k-\mu)^2 - [E(Q)]^2\right]^{1/2} \quad (4.18)$$

Conditional Control Limits of the OPA MRy Chart

From Equations (4.16) and (4.18), the one-period-ahead control limits for the t^{th} time period can be constructed for the proposed OPA MRy chart:

$$CL_{OPAMRy} = E(R|k) \quad (4.19)$$

$$UCL_{OPAMRy} = E(R|k) + 3 \cdot Std(R|k) \quad (4.20)$$

$$LCL_{OPAMRy} = E(R|k) - 3 \cdot Std(R|k) \quad (4.21)$$

In order to use the conditional one-period-ahead control limits on the proposed OPA MRy chart as shown in Equations (4.19)-(4.21) to monitor the dispersion of serially correlated data from a first order response process, the average and standard deviation of the unobservable X variate, μ and σ, must be known or estimated. Equations (4.16)-(4.18) can be used on empirical data by replacing the μ and σ with the proper estimate as in Equations (3.15)-(3.17).

The computation of the one-period-ahead control limits of the proposed OPA MRy chart may be complicated as it involves the quantities $\Phi'(x)$ and $\Phi(x)$. The calculation of $\Phi(x)$ can be aided by using a higher degree polynomial approximation (Nelson, 1983). With simple computer program, the computation of the one-period-ahead control limits of the OPA MRy chart can be greatly simplified. The FORTRAN program coded in Chapter VIII can be used to perform these computations.

Numerical Illustration

A numerical example is presented to illustrate how to use the proposed OPA MRy chart on serially correlated data generated from a first order response process. Similar to the construction of the OPA Y chart, the user is expected to have a prior knowledge of that particular first order response process. The value of the filter constant, r, is assumed to be known using first principles.

The 30 observations of Y used in the construction of the

OPA MRy chart are the same data set used in Chapter III for the construction of the OPA Y chart. From the 30 observations, the average of Y is 19.834, and of the MRy is 0.164. These two quantities are required to compute the one-period-ahead control limits. The one-period-ahead control limits are computed for all the time periods except the 1st time period using the formulas in (4.14), (4.16)-(4.18), (4.20) and (4.21). It should be noted that the FORTRAN program coded in Chapter VIII is used to perform the computation. As an example, the one-period-ahead control limits of the OPA MRy chart for 9th time period are found to be

Lower control limit = 0.0

Upper control limit = 0.586

All the observations, the MRy, the average of Y and MRy, and the one-period-ahead control limits are presented in TABLE 4.1. By plotting the MRy values and the one-period-ahead control limits on the control chart, the required OPA MRy chart is obtained. The OPA MRy chart is plotted in Figure 4.1. It should be noted that the central line of the OPA MRy chart is omitted in TABLE 4.1 and Figure 4.1 as it is not very important in interpretation of the control chart.

TABLE 4.1

SERIALLY CORRELATED DATA FROM A FIRST ORDER RESPONSE PROCESS WITH FILTER CONSTANT 0.8, MOVING RANGES OF TWO AND ONE-PERIOD-AHEAD CONTROL LIMITS FOR THE OPA MRy CHART

Time, t	Y_t	MRy	LCL_{OPAMRy}	UCL_{OPAMRy}
1	20.023			
2	20.208	0.185	0.0	0.601
3	19.905	0.303	0.0	0.629
4	20.020	0.115	0.0	0.590
5	19.786	0.234	0.0	0.600
6	20.134	0.348	0.0	0.585
7	20.074	0.060	0.0	0.616
8	19.693	0.381	0.0	0.607
9	19.210	0.483	0.0	0.586
10	19.385	0.175	0.0	0.649
11	19.649	0.264	0.0	0.616
12	19.643	0.006	0.0	0.588
13	19.736	0.093	0.0	0.588
14	19.714	0.022	0.0	0.585
15	19.349	0.365	0.0	0.586
16	19.266	0.083	0.0	0.622
17	19.091	0.175	0.0	0.638
18	19.321	0.230	0.0	0.677
19	19.880	0.559	0.0	0.627
20	19.845	0.035	0.0	0.588
21	20.249	0.404	0.0	0.587
22	20.022	0.227	0.0	0.637
23	19.826	0.196	0.0	0.601
24	19.756	0.070	0.0	0.586
25	19.774	0.018	0.0	0.585
26	19.870	0.096	0.0	0.585
27	19.726	0.144	0.0	0.588
28	19.832	0.106	0.0	0.585
29	19.922	0.090	0.0	0.586
30	19.916	0.006	0.0	0.591
Average	19.761	0.189		

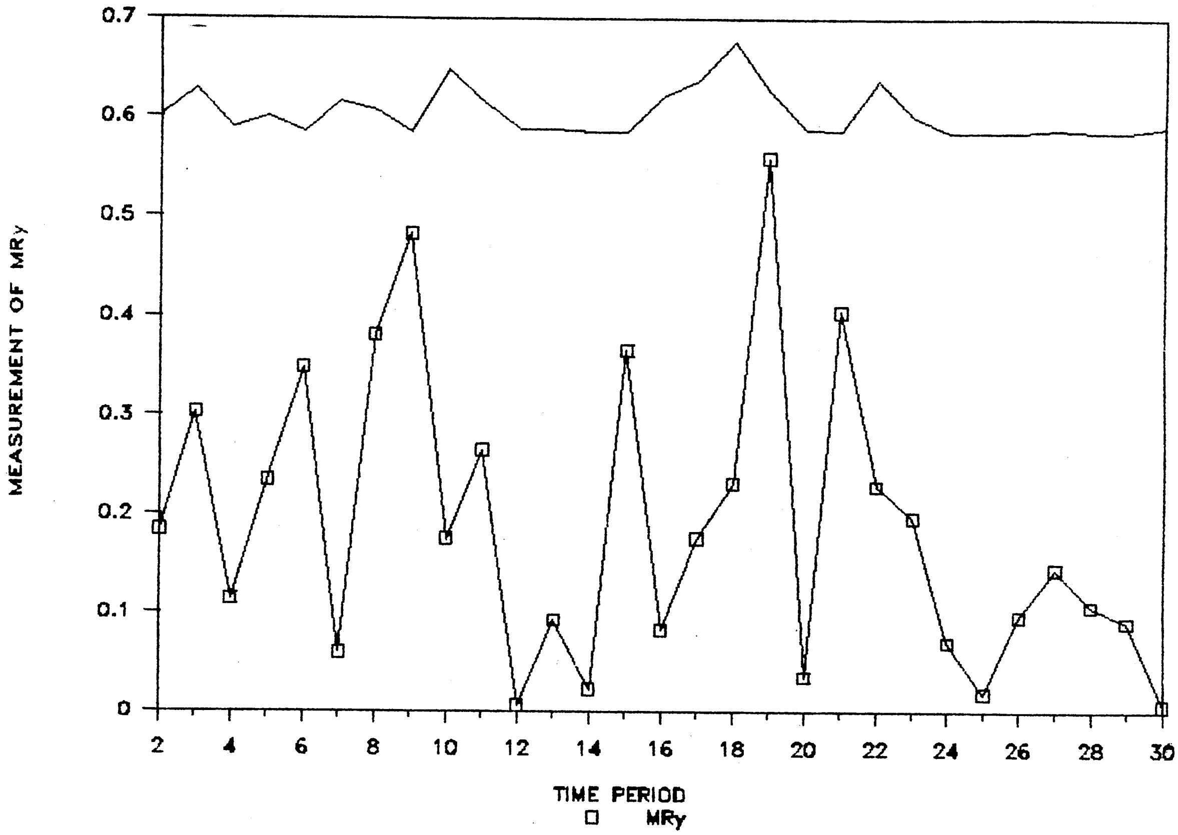

Figure 4.1. The OPA MRy chart, r = 0.8

Conclusion

In this chapter, the derivation of the conditional distribution of the moving range of (Y_{t-1}, Y_t), given that Y_{t-1} is known, is presented. The mean and standard deviation of this conditional distribution are then determined. They are used to construct the conditional one-period-ahead control limits of the proposed OPA MRy chart. A numerical example is used to illustrate the construction of the proposed OPA MRy chart to monitor the dispersion of the output of a FORP. The FORTRAN program coded in Chapter VIII is used to compute the one-period-ahead control limits of the OPA MRy chart.

CHAPTER V

DETERMINING THE AVERAGE RUN LENGTH OF A CONTROL CHART USING SIMULATION

Introduction

A desirable control chart is one that has a very low frequency of signaling a false alarm when the process is in-control and has the capabilities of swiftly signaling an out-of-control alarm when the process experiences changes. The performance measure of such capabilities is the Average Run Length, ARL, for in-control and out-of-control scenarios. In this research, the ARL of a control chart is determined by simulation.

In this chapter, the methodologies used to determine the average run length of the proposed OPA Y and OPA MRy charts, and other control charts used for comparison, are discussed. The generation of the first order response process data used in this study is presented in detail. The control limits of these control charts are constructed using the theoretical values. Several types of process shift are simulated in this research and embedded in the data stream. The process shifts are: stepwise, trend-wise, and cyclical shifts in the process mean or standard deviation. Even though there are numerous scenarios in which the ARL of control charts needs to be determined, the method to simulate all these scenarios can be easily generalized. All the computer simulations are designed using the SAS language. The procedures available in SAS greatly reduce the programming effort.

Data Generation

The purpose of this study is to develop and evaluate control charting techniques on quality data generated from a first order response process. Thus, a stream of observation data from a FORP must be generated through simulation, and control charts are then applied to this stream of data generated. In order to generalize the simulation, individual data values are generated through the SAS intrinsic normal random variate generator RANNOR. The individual data values generated are standard normal variates. To avoid computation underflow at a later stage, the individual data generated are transformed into normal variates with mean, MEAN = μ = 10.0 and standard deviation, STD = σ = 1.0. These normal variates are then input into a first order response process equation to generate the first order response process output data values, Y, which are serially correlated. The recurrence relation of a FORP data generator in computer code is,

Y2 = FC*Y1 + (1-FC)*(RANNOR(SEED)*STD + MEAN) (5.1)

where FC is the filter constant, r, of a FORP, and Y1 and Y2 are two consecutive values of Y. The values of the filter constant used in the study are 0.0, 0.3, 0.6, and 0.9. By using a filter constant equal to 0.0, an independent normal data stream is generated. Thus, Equation (5.1) can be used to generate an independent normal data stream as well.

To begin using this recurrence relation, the very first Y value used in the FORP is just a normal variate with mean equal to MEAN and standard deviation equal to STD. All subsequent Y values of the FORP are then generated using Equation (5.1). Since the very first Y value is not generated using Equation (5.1), the first few Y values generated do not truly depict a FORP. Thus, to 'warm-up' the FORP data stream, the first 50 FORP data values generated are discarded. However, the 51^{st} value generated is kept to be used later to generate the first of the observed data for control chart plotting.

To facilitate discussion, all the Y values used in the plotting of the control charts are called 'observations', this is to distinguish them from the Y values generated during the 'warm-up' period. In a real situation, the Y values obtained during the 'warm-up' period are not even observed by the user of control charts. Another assumption made at this point is that in a real situation, observation from the FORP data stream is observed at a fixed regular time interval. Correspondingly, in a simulation situation, these time periods are indexed by t, with the first observation made at time period 1. In this data generation, the 52^{nd} Y value generated is the observation made at time period 1. All the observations generated subsequently are used as individual data or, along with the 51st Y value, are used to compute moving ranges of subgroup size two. These individual values or the moving ranges are then plotted on the appropriate control chart. The appropriate control charts in this study are the OPA Y, I, OPA MRy and MR(2) charts. The first two charts are applied to the individual data, and the OPA MRy and MR(2) charts are applied to the moving ranges. Each plotted point on these charts is checked to see if it falls beyond or within the appropriate control limits.

It should be noted that only from the generation of the first observation is the effect of different type of process shift embedded into the FORP data generation process. That is, the effect of the process shift is only embedded in the generation of the 52nd Y value. Moreover, only one type of shift is embedded into a scenario under study at a time. To embed the effect of a shift in the process mean or process standard deviation into the original (10.0,1.0) normal variate, the following methods are used

(1) For a stepwise shift in the process mean, the standard deviation is multiplied by the magnitude of shift and added to the process mean. Hence, the shift in process mean is in terms of the process standard deviation. The FORP data generator in computer code becomes

```
STD   = 1.0
MEAN  = 10.0
NMEAN = MEAN + SHIFT*STD
Y2    = FC*Y1 + (1-FC)*(RANNOR(SEED)*STD + NMEAN)
```

where SHIFT is the magnitude of shift. The magnitudes of shift used in this study are: 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00. Figure 5.1 shows the stepwise shift in mean.

(2) For a shift of the process mean in trend, where trend is defined as a gradual increase in mean by a total of 3 standard deviations in 20 subgroup sampling intervals, the value of the mean used for the generation of a new X value is inflated by a rate of (3/20) standard deviation. That is, the very first observation is generated using the original mean $\mu = 10.0$. The second observation is generated using the mean $\mu = 10.0+(3/20)\sigma$. The third observation is generated using the mean $\mu = 10.0+2*(3/20)\sigma$, and so on. The mean used to generate the X values ceases to inflate further after its value has reached the maximum, $10.0+20*(3/20)\sigma$. The FORP data generator in computer code is

```
STPSZ = 3*STD/20
NMEAN = MEAN + MIN(STPSZ*PER,3*STD)
Y2    = FC*Y1 + (1-FC)*(RANNOR(SEED)*STD + NMEAN)
```

where PER is the index of the point in trend, PER = $0(1)\infty$. Figure 5.2 shows the trend-wise shift in mean.

(3) For a cyclical shift of the process mean, a cycle is defined by a sinusoidal wave with period equal to 48 subgroup sampling intervals and amplitude equal to 3 standard deviations. The value of the mean used to generate a new X value is obtained by adding to the original mean the product of a sine function and the

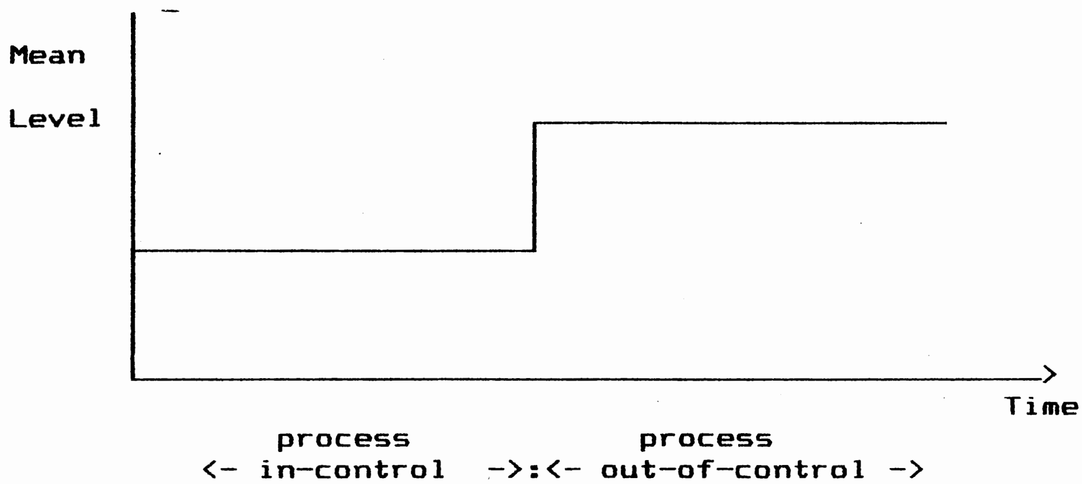

Figure 5.1. Stepwise Shift in Mean

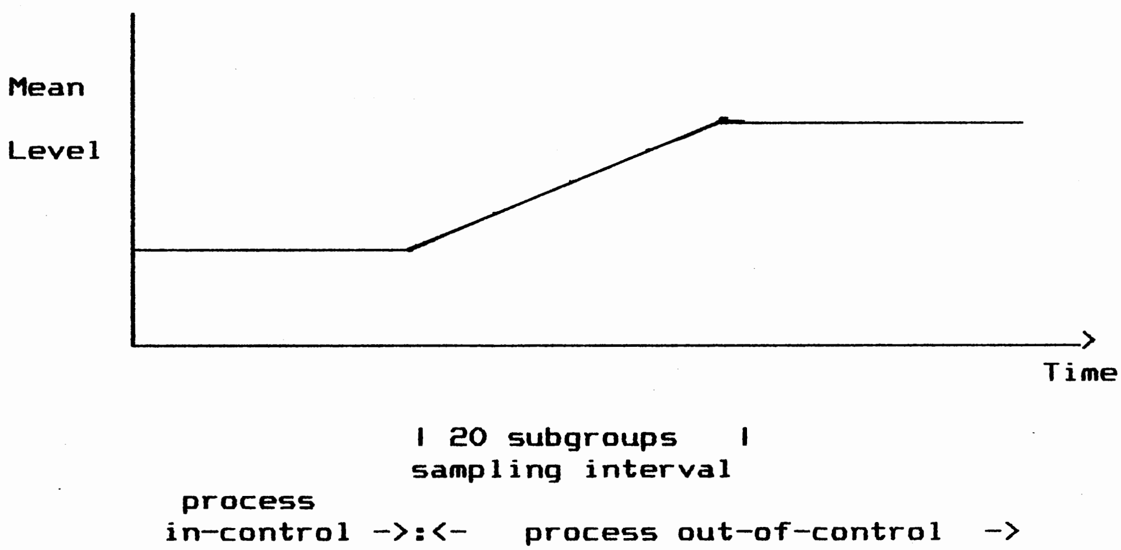

Figure 5.2. Trend-wise Shift in Mean

amplitude of the cycle. The step increment of the variable for the sine function is 2*pi/48. The initial argument for the sine function is 0.0, and it increases by a value of 2*pi/48 for each subsequent usage in the sine function. The FORP data generator in computer code becomes

```
STPSZ = 2*PI/48
AMP   = 3*STD
NMEAN = MEAN + AMP*SIN(PER)
Y2    = FC*Y1 + (1-FC)*(RANNOR(SEED)*STD + NMEAN)
PER   = PER + STPSZ
```

where PER is the argument for the sine function; it begins with a value of 0.0 and increases with a value of 2*pi/48 for subsequent steps. Figure 5.3 shows the cyclical shift in mean.

(4) For a stepwise shift in the process standard deviation, the standard deviation is multiplied by the ratio of the new standard deviation to the original standard deviation, $\sigma_{new}/\sigma_{old}$. The FORP data generator in computer code becomes

```
NSTD  = RATIO*STD
Y2    = FC*Y1 + (1-FC)*(RANNOR(SEED)*NSTD + MEAN)
```

where RATIO is the value of $\sigma_{new}/\sigma_{old}$. The ratios used in this study are: 1.00, 1.25, 1.50, 1.75, 2.00, 2.50, 3.00. Figure 5.4 shows the stepwise shift in dispersion.

If a plotted point falls within the control limits, the next plotted point is generated and checked against the control limits, and so on. If the plotted point falls beyond

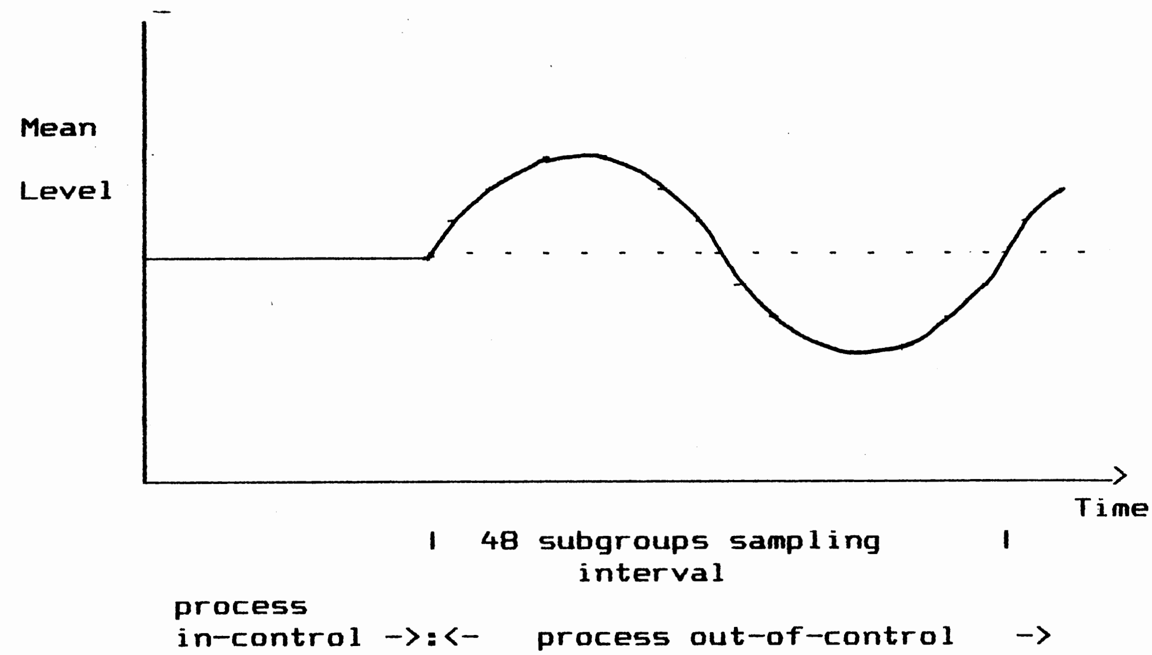

Figure 5.3. Cyclical Shift in Mean

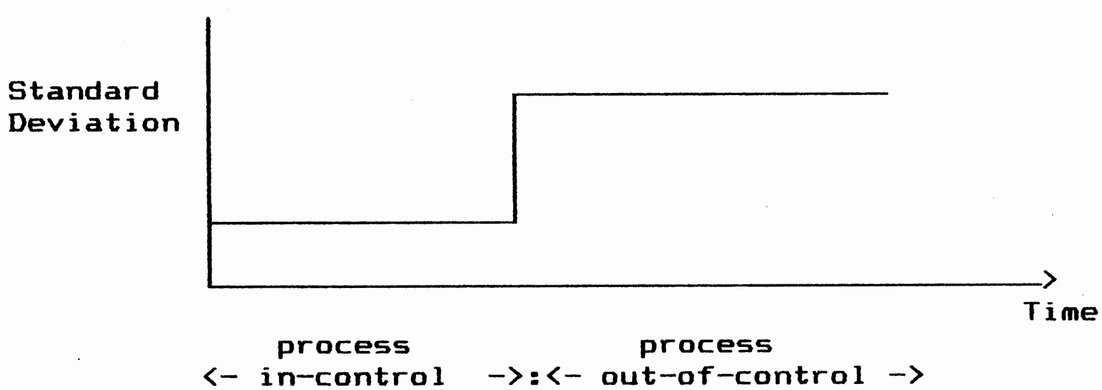

Figure 5.4. Stepwise Shift in Dispersion

the control limits, an action signal results. Then, the number of points checked since the previous action signal is recorded. This number is the 'run length' of the control chart for a particular scenario. This procedure is repeated until the $10{,}000^{th}$ run length is recorded. The average of these 10,000 run lengths is the average run length or ARL of the control chart for a particular scenario.

Control Limits

It is important to remember that the control limits of control charts discussed in this chapter are constructed based on theoretical values, not on a few initial observations collected. That is, it is assumed that the mean, μ, and standard deviation, σ, of the unobservable X variates are known. Moreover, the filter constant, r, of the FORP is also assumed to be exactly known by first principles.

OPA Y Chart

The control limits for the OPA Y chart are conditional control limits. They are conditioned on the current observation and used for the next observed value. Using Equations (3.10)-(3.11), after rearrangement of terms, the conditional control limits for the next observation of Y are

$$UCL_{OPAY} = FC*(Y2-MEAN) + MEAN + 3*STD(1-FC)$$

$$LCL_{OPAY} = FC*(Y2-MEAN) + MEAN - 3*STD(1-FC)$$

where Y2 is the current observation of Y and FC is the filter constant of the FORP.

To construct the first set of conditional control limits on the OPA Y chart, the value of Y2 is the 51^{st} Y value generated. In a real application, the average of the initial sample of observations collected is used as the first Y2. Since it is assumed that, by first principles, the filter constant can be exactly stated, the filter constant used for the construction of the conditional control limits is the same as the one used for generation of the Y data.

OPA MRy Chart

The control limits for the OPA MRy chart are conditional control limits. Similar to the control limits of the OPA Y chart, they are also conditioned on the current observation of Y which is denoted as k. Using Equations (4.20) and (4.21), the conditional control limits for the next plotted point on the OPA MRy chart are

$$UCL_{OPAMRy} = E(R|k) + 3*Std(R|k)$$

$$LCL_{OPAMRy} = E(R|k) - 3*Std(R|k)$$

where $E(R|k)$ and $Std(R|k)$ are defined by Equation (4.16) and (4.18), respectively. To construct the first set of conditional control limits on the OPA MRy chart, the value of k is the 51^{st} Y value generated.

I Chart

The purpose of applying the I chart on the FORP data stream is to investigate how the I chart performs if serial correlation in a FORP is not explicitly recognized. Whether the serial correlation in the FORP data stream is recognized or not, the expected value of the average of moving ranges of subgroup size two computed is found to be (Cryer and Ryan, 1990)

$$E\left[MR(2)bar\right] = 1.128*\sigma_y*\sqrt{(1-r)} \tag{5.2}$$

From Equation (3.15), it is known that

$$\sigma_y = \sigma \sqrt{(1-r)}/\sqrt{(1+r)} \tag{5.3}$$

Substituting (5.3) into (5.2) results in

$$E\left[MR(2)bar\right] = 1.128*\sigma*(1-r)/\sqrt{(1+r)} \tag{5.4}$$

The control limits of an I chart are

$$UCL_I = MEAN + 3*STD \tag{5.5}$$

$$LCL_I = MEAN - 3*STD \tag{5.6}$$

It is known that the STD can be estimated by

$$STD = \frac{E\left[MR(2)bar\right]}{1.128} \tag{5.7}$$

Substituting (5.7) into (5.5) and (5.6), the theoretical control limits of the I chart based on a FORP data stream are

$$UCL_I = MEAN + 2.66*E\left[MR(2)bar\right]$$

$$LCL_I = MEAN - 2.66*E\left[MR(2)bar\right]$$

These control limits are not conditional control limits. They are constructed and used for all the plotted points on the I chart under study.

<u>MR(2) Chart</u>

Whether the serial correlation in the FORP data stream is recognized or not, the expected value of the average of the moving ranges of subgroup size two computed can be determined. Using Equation (5.4) and the formula for the control limits of a traditional MR(2) chart, the theoretical control limits of the MR(2) chart based on a FORP data stream are

$$UCL_{MR(2)} = 3.267*E\left[MR(2)bar\right]$$

$$LCL_{MR(2)} = 0.0$$

These control limits are not conditional control limits. They are determined and used for all the plotted points in the MR(2) chart.

Average Run Length

The ARL is defined as the average number of subgroups that must be taken from a process until an out-of-control point is found and corrective action can be implemented. Or, equivalently, the number of plotted points on a control chart until an out-of-control point is found. The ARL is associated with the probability of a control chart detecting a process change. Normally, traditional Operating Characteristic Curves can be constructed and used to determine the ability of a control chart to detect a process change if the points plotted are independent of one another and the distribution of the points plotted is known.

However, in this research study, all the data are FORP data which are not necessarily independent. Moreover, the plotted points on certain charts are not independent of one another, such as the plotted points of the OPA MRy or the MR(2) charts. Therefore, instead of talking about the risks of failing to detect a shift of a given magnitude, a better method is to account for how many plotted points it will take to detect the shift. The simplest way to determine the ARL of a control chart on quality data from a FORP is by simulation. Through simulation, the standard deviation of the run length, SDRL, can also be determined.

In the derivation of the one-period-ahead control limits, it is found that the Type I and Type II error risks of any plotted point on the OPA Y chart are identical to the risks of the corresponding plotted point on an Individual chart for an independent normal data stream. Thus, the ARLs of a OPA Y chart for different magnitudes of shift in the mean level of the output of a first order response process are identical to the ARLs of the corresponding Individual chart.

An Individual chart is an Xbar chart with subgroup size one. The ARL of the Xbar chart has been determined by several authors such as by Graham (1986) and Champ, et al. (1987). The ARL of the Xbar chart, and hence, the ARL of the Y chart, can be found correspondingly from their work. It should be pointed out that no runs rules are used in the proposed OPA Y chart. Therefore, the corresponding ARL must be the ARL for the Xbar chart without using any runs rules. Runs rules may be incorporated into the OPA Y chart, but the procedure may be quite difficult as the one-period-ahead control limits of the OPA Y chart cause the division of control charts into proper zones, for the application of runs rules, to be more complicated.

List of Programs Coded

SAS programs have been coded to simulate the performance of the OPA Y, OPA MRy, I and MR(2) charts in various scenarios of interest. The conditions incorporated into the scenarios can be broken down in the following manner

1.a 4 filter constants exactly stated: r = 0.0, 0.3, 0.6, 0.9

1.b 1 filter constant exactly stated: r = 0.6 with

1 filter constant overstated: r = 0.7 and

1 filter constant understated: r = 0.5

2.a 7 shifts in the mean of X variates (in multiples of σ) : 0.0, 0.25, 0.50, 1.00, 1.50, 2.00, 3.00

2.b 7 shifts in the standard deviation of X variates ($\sigma_{new}/\sigma_{old}$): 1.0, 1.25, 1.50, 1.75, 2.00, 2.50, 3.00

2.c 1 shift in the mean of X variates in trend: 3σ in 20 subgroup sampling intervals

2.d 1 shift in the mean of X variates in cycle: cycle period equal to 48 subgroup sampling intervals and cycle amplitude equal to 3σ

The SAS program scenarios coded are listed in TABLE 5.1. Since the execution of the simulation of each scenario can be quite long, simulation for only one scenario is done at a time. After obtaining the ARL for one scenario, conditions are changed to form a new scenario. It is important to point out that each scenario has the same random number seed. Thus, each scenario is subject to the same stream of random numbers. An example of a SAS program and the corresponding SAS output are included in Appendix D. The SAS program is the PROGRAM 1 with filter constant 0.3 and stepwise shift in mean equal to 2σ. The ARL and SDRL are 6.1857 and 5.6497, respectively. These values are rounded and listed in the corresponding cell in TABLE 5.2.

ARL and SDRL of the OPA Y and OPA MRy Charts

Using SAS programs (1), (7) and (13), the ARL and SDRL of the OPA Y chart under a stepwise shift in process mean, the ARL and SDRL of the OPA MRy chart under a stepwise shift in process dispersion, and the ARL and SDRL of the combined OPA Y and OPA MRy charts under a stepwise shift in process mean can be determined. They are presented in TABLEs 5.2, 5.3 and 5.4, respectively.

From TABLE 5.2, it is found that the ARLs of the OPA Y chart are identical to those obtained theoretically. Without making any comparisons to other control charts, these OPA Y and OPA MRy charts appear to be capable of detecting a process shift. A special effort is made to display the ARL and SDRL of the OPA Y and OPA MRy charts under the types of process shifts shown by conditions (2.a) and (2.b). This is to illustrate the fact that, under a normal working environment in which a stepwise shift in process mean or

TABLE 5.1

LIST OF SAS PROGRAM SCENARIOS CODED

PROGRAM NUMBER	NAME OF CHART(S)	CONDITION NUMBER
1	OPA Y chart	(1.a) and (2.a)
2	OPA Y chart	(1.a) and (2.c)
3	OPA Y chart	(1.a) and (2.d)
4	I chart	(1.a) and (2.a)
5	I chart	(1.a) and (2.c)
6	I chart	(1.a) and (2.d)
7	OPA MRy chart	(1.a) and (2.b)
8	OPA MRy chart	(1.a) and (2.c)
9	OPA MRy chart	(1.a) and (2.d)
10	MR(2) chart	(1.a) and (2.b)
11	MR(2) chart	(1.a) and (2.c)
12	MR(2) chart	(1.a) and (2.d)
13	Combined OPA Y and OPA MRy charts	(1.a) and (2.a)
14	OPA Y chart	(1.b) and (2.a)
15	Combined OPA Y and OPA MRy charts	(1.b) and (2.a)
16	OPA MRy chart	(1.b) and (2.b)

TABLE 5.2

ARL OF THE OPA Y CHART WITH CONDITIONS
1.a and 2.a ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M	370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.3	M	370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.6	M	370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.9	M	370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46

PROGRAM 1 is used to generate this table

TABLE 5.3

ARL OF THE OPA MRy CHART WITH CONDITIONS
1.a and 2.b ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		$\sigma_{new}/\sigma_{old}$ (DISPERSION SHIFTS IN STEP)						
		1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.0	M	154.74	42.84	19.98	12.06	8.35	5.20	3.82
	S	155.69	43.06	19.68	11.77	7.92	4.85	3.42
0.3	M	132.81	37.60	17.46	10.61	7.51	4.70	3.48
	S	133.16	37.02	17.25	10.24	7.16	4.39	3.09
0.6	M	119.61	33.17	15.33	9.34	6.57	4.14	3.10
	S	120.70	32.51	14.81	9.00	6.26	3.77	2.71
0.9	M	110.28	28.87	13.08	7.80	5.52	3.53	2.70
	S	109.61	28.34	12.88	7.46	5.13	3.09	2.17

PROGRAM 7 is used to generate this table

TABLE 5.4

ARL OF THE COMBINED OPA Y AND OPA MRy CHARTS WITH CONDITIONS 1.a and 2.a ON FORP DATA (USING THEORETICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M	135.51	113.45	78.19	31.64	12.48	5.67	1.82
	S	136.90	113.79	79.10	31.86	12.39	5.44	1.35
0.3	M	121.26	106.82	79.95	34.50	13.06	5.69	1.80
	S	121.01	108.11	81.79	34.83	13.38	5.58	1.35
0.6	M	112.28	104.92	87.28	38.61	13.33	5.57	1.75
	S	112.96	105.06	89.45	39.37	13.74	5.58	1.30
0.9	M	108.80	115.58	107.90	39.58	12.55	5.21	1.68
	S	108.01	117.29	115.95	42.90	13.60	5.41	1.21

PROGRAM 13 is used to generate this table

dispersion is more likely than other less common process shifts, these OPA Y and OPA MRy charts do perform satisfactorily.

Conclusion

In this chapter, a simulation approach to determine the ARL of the control charts used in this study is discussed in detail. The generalized approach can be made specific to simulate the ARL of the OPA Y, OPA MRy, I, or MR(2) charts for the FORP data stream under different scenarios. All the simulation programs are coded using the SAS language. It should be noted that the control limits constructed in the simulation are based on the theoretical values of the mean and standard deviation of X variates. Moreover, the filter constant is assumed to be exactly stated. Up to this point, there are a total of 16 SAS programs coded to determine the ARL of various control charts under different scenarios.

CHAPTER VI

ANALYSIS OF SIMULATION RESULTS USING THEORETICAL FORP VALUES

Introduction

In this chapter, the performance of the proposed OPA Y and OPA MRy charts are evaluated. The ARLs of these charts are compared to the ARLs of other traditional charts which are usually employed with a continuous flow process. These traditional charts are the I and MR(2) charts. The ARLs of control charts using different scenarios (see TABLE 5.1) are determined by simulation. The ARLs determined are presented to aid comparison. Due to the cost of computing time, not all scenarios are simulated. Those scenarios which are not run are designated by the notation N/R. For some scenarios, the ARL values are too large to be computed in a CPU time of 5 minutes; those scenarios which are run but for which results are not obtained after 5 minutes CPU time are designated by the notation T/L.

Tables of Results

Generally, there are two types of ARL table. One consists of 4 rows and either 2 or 7 columns. The other consists of 3 rows and 7 columns. For example, TABLE 6.3 is a 4x2 ARL table, TABLE 6.1 a 4x7 ARL table and TABLE 6.9 a 3x7 ARL table. For a 4x2 or 4x7 ARL table, the rows show the different values of the filter constant of the FORP used, r = 0.0, 0.3, 0.6 and 0.9. The rows of the 3x7 ARL table show the values of filter constant that are either overstated, r = 0.7, exactly stated, r = 0.6, or understated, r = 0.5. The columns of a 3x7 or 4x7 ARL table show the level of shift in either the process mean or standard deviation of the X variates. And the columns of a 4x2 ARL table show the type of control chart used to monitor the process which has a shift of mean in trend or in cycle. It should be noted that each row of an ARL table consists of 2 sub-rows with the first designated as M and the second as S. The entries stand for the average run length, ARL, and the standard deviation of run length, SDRL, respectively, of a control chart for a particular combination of filter constant and process shift (scenario). So, as an example in TABLE 5.2, for a scenario in which the filter constant is 0.6 and the shift in the mean of X variates is 0.5σ, the ARL is found to be 155.53 and the SDRL is 156.60.

Hypothesis Testing

An indicator '#' is placed in front of certain ARL values in the ARL table of a control chart. This indicates a significant difference in the mean of the ARL values of the two control charts under comparison. For an in-control situation, the '#' indicates the scenario which gives the higher ARL when compared to that of the corresponding scenario in another control chart. For an out-of-control situation, the '#' indicates the scenario which gives the lower ARL. However, not all the ARLs in the evaluations are marked, as in some cases these are no corresponding ARLs to be tested with, or the test outcome is insignificant.

By the central limit theorem, the distributions of the ARLs can be approximated by normal density functions. The ARLs of the two charts under comparison are known to be independent of one another. Moreover, the true variances are unknown and not necessarily equal. The objective of hypothesis testing is to be able to claim that when the process is in-control, the OPA Y or OPA MRy chart has a larger ARL value or when the process is out-of-control, the OPA Y or OPA MRy chart has a lower ARL value. Thus, the hypothesis testing of the difference in the mean of the two ARLs for an in-control situation is

$H_o: U1 - U2 = 0$

$H_a: U1 - U2 > 0$

and for an out-of-control situation is

$H_o: U1 - U2 = 0$

$H_a: U1 - U2 < 0$

where

U1 = Mean of the ARL of the OPA Y or OPA MRy chart

U2 = Mean of the ARL of the other chart under comparison

The test statistic is

$$t' = \frac{(M_1 - M_2)}{\sqrt{\dfrac{(S_1)^2}{n_1} + \dfrac{(S_1)^2}{n_2}}}$$

where

M_1 = Observed ARL of the OPA Y or OPA MRy chart

M_2 = Observed ARL of the other chart under comparison

S_1 = Observed SDRL of the OPA Y or OPA MRy chart

S_2 = Observed SDRL of the other chart under comparison

n_1 = Number of run lengths generated for the OPA or OPA MRy chart

n_2 = Number of run lengths generated for the other chart under comparison

This test is known as Aspin-Welch test (Duncan, 1986). Since the number of run lengths generated in this research is 10000, 5000 or 1000, the test statistic t' can be approximated as a standard normal random variable.

For an in-control process, the null hypothesis will be rejected if the test statistic is greater than Z_α. In this study, α is taken at the 5% level. When the null hypothesis is rejected, it can be concluded that the ARL of the OPA Y chart or OPA MRy chart is favorable compared to that of the other chart under comparison.

For an out-of-control process, the null hypothesis will be rejected if the test statistic is less than Z_α. When the null hypothesis is rejected, it can be concluded that the ARL of the OPA Y chart or OPA MRy chart is favorable compared to that of the other chart under comparison.

Comparison of the ARL

To analyze and compare various ARLs under different scenarios, the ARLs of comparable charts are grouped under various 'evaluation' headings. A brief description is included for each evaluation. The description includes the objective of the ARL comparison, the features of the data streams used, the magnitude of process shift either in mean or dispersion, how the control limits are constructed, the type of process shift, and whether the filter constant, r, is explicitly recognized or not. The filter constant, r, is not explicit recognized if the user mistakenly considers a FORP data stream as an independent data stream. The details of how the computer simulation is carried out are documented in the previous chapter. The tabulated ARL results are presented and followed by a brief analysis of the comparison in terms of the ARL of the control charts involved.

To determine the preference of a control chart over others on a FORP data stream with a particular filter constant, the ARL of a control chart on an in-control situation is first considered. If this ARL is considered too low, the control chart will not be used on that particular FORP data stream. If the ARL is deemed acceptable, say 80 and above, the ARL of the out-of-control situation is examined. The preferred control chart should be the one with a large ARL when the process is in-control and a low ARL when the process is out-of-control. It should be noted that the preference of a control chart on FORP data also depends on the filter constant of the response process.

Evaluation 6.1

The characteristics of this evaluation are:

Objective:	To compare the ARL of the OPA Y chart and the I chart.
Data Streams:	FORP data for which r equals 0.0, 0.3, 0.6 and 0.9.
Process shift:	Mean level is shifted by 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00 in terms of process standard deviation.
Type of Shift:	Stepwise.
Control limits:	The control limits of these charts are based on theoretical values.
Filter constant:	Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA Y and I charts are presented in TABLEs 6.1 and 6.2, respectively. The SAS programs used to generate these ARL values are PROGRAMs 1 and 4; the program listings are found in Appendix E.

The ARLs in TABLE 6.1 clearly show that the performance of the OPA Y chart is the same regardless of the values of the filter constant of a FORP. From Chapter III, it is known that the ARLs of the OPA Y chart are similar to the ARLs of the I chart on an independent data stream. This fact is clearly illustrated by the similar values of ARL in the first row of both the TABLEs 6.1 and 6.2. Minor discrepancies are due to round-off errors.

TABLE 6.1

ARL OF THE OPA Y CHART WITH CONDITIONS
1.a and 2.a ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M	370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.3	M	#370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.6	M	#370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.9	M	#370.36	283.08	155.53	43.82	14.57	6.19	2.02
	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46

PROGRAM 1 is used to generate this table

TABLE 6.2

ARL OF THE I CHART WITH CONDITIONS
1.a and 2.a ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M	370.93		155.81		14.59		
	S	372.88	N/R	156.72	N/R	14.04	N/R	N/R
0.3	M	84.70	#60.62		#9.99		#2.63	
	S	84.23	60.45	N/R	9.04	N/R	1.72	N/R
0.6	M	21.92	#16.66		#4.36		#1.94	
	S	21.62	16.29	N/R	3.19	N/R	0.97	N/R
0.9	M	6.22		#4.28		#2.09		
	S	7.25	N/R	4.19	N/R	1.30	N/R	N/R

PROGRAM 4 is used to generate this table

Comparing the ARL for an out-of-control process in both TABLEs 6.1 and 6.2, the ARLs of the OPA Y chart are found to be favorably significant at the 5% level when the process is in-control for filter constant greater than 0.0. Even though the I chart has favorable ARLs when the process is out-of-control for filter constant greater than 0.0, the low ARL for an in-control process makes this chart impractical. That is, the I chart signals alarm more frequently as the filter constant of the FORP gets larger in value. For a FORP with filter constant of 0.6, it is found that the I chart on the average signals a false alarm every 21.92 plotted points when the process is indeed in-control. Thus, with this performance of the I chart when the process is in-control, it is clear that the I chart is not a useful control chart for a FORP correlated data stream.

Evaluation 6.2

The characteristics of this evaluation are:

Objective:	To compare the ARL of the OPA Y chart and the I chart.
Data Streams:	FORP data for which r equals 0.0, 0.3, 0.6 and 0.9.
Process shift:	The gradual shift in the mean is an increase with a total of 3 process standard deviations in 20 subgroup sampling intervals. Once the maximum magnitude of the shift is reached, the process mean remains at this level.
Type of Shift:	Trend-wise.
Control limits:	The control limits of these charts are based on theoretical values.

Filter constant: Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA Y and I charts are presented in TABLE 6.3. The SAS programs used to generate these ARL values are PROGRAMs 2 and 5; the program listings are found in Appendix E.

From TABLE 6.3, it is found that the I chart is more powerful in detecting a shift of the process mean in trend for different values of the filter constant of a FORP as the low ARLs of the I chart are significant at 5% level. Due to the development of the OPA Y chart in which (X_t, Y_t) is a point-to-point mapping, the ability of this chart to detect a process shift is not affected by the different values of the filter constant.

Evaluation 6.3

The characteristics of this evaluation are:

Objective: To compare the ARL of the OPA Y chart and the I chart.

Data Streams: FORP data for which r equals 0.0, 0.3, 0.6 and 0.9.

Process shift: The mean is shifted in a cyclical manner. The cycle has a period which is equal to the sampling interval of 48 subgroups. The amplitude of the cycle has a magnitude of 3 process standard deviations.

Type of Shift: Cycle.

Control limits: The control limits of these charts are based on theoretical values.

TABLE 6.3

ARL OF THE OPA Y AND I CHARTS WITH CONDITIONS 1.a and 2.c ON FORP DATA (USING THEORETICAL VALUES)

FILTER CONSTANT		MEAN SHIFTS IN TREND	
		OPA Y chart	I CHART
0.0	M	14.55	14.57
	S	4.13	4.14
0.3	M	14.55	#10.11
	S	4.13	3.45
0.6	M	14.55	#7.06
	S	4.13	3.14
0.9	M	14.55	#4.04
	S	4.13	3.02

PROGRAMs 2 and 5 are used to generate this table

Filter constant: Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA Y and I charts are presented in TABLE 6.4. The SAS programs used to generate these ARL values are PROGRAMs 3 and 6; the program listings are found in Appendix E.

From TABLE 6.4, it is found that the I chart is more powerful in detecting a shift of process mean in cycle as the low ARLs of the I chart are significant at 5% level for filter constant greater that 0.0. Even though the I chart is more sensitive to changes of process mean in step, trend, or cycle regardless of the magnitude of the filter constant of a FORP, its ARL for an in-control FORP data stream is generally considered to be too low for a filter constant greater than 0.3. Hence, as a whole, the I chart is not practical in monitoring a correlated FORP.

Generally speaking, the ARL of the proposed OPA Y chart for different magnitudes and types of shift in process mean are acceptable. The robustness of the OPA Y chart to different magnitudes of the filter constant makes it even more favorable over the I chart. At this stage, it is clear that the OPA Y chart is a more favorable control chart for a FORP data stream than the I chart.

TABLE 6.4

ARL OF THE OPA Y AND I CHARTS WITH CONDITIONS 1.a and 2.d ON FORP DATA (USING THEORETICAL VALUES)

FILTER CONSTANT		MEAN SHIFTS IN CYCLE	
		OPA Y chart	I CHART
0.0	M	8.31	8.31
	S	2.66	2.66
0.3	M	8.31	#6.06
	S	2.66	1.81
0.6	M	8.31	#4.73
	S	2.66	1.70
0.9	M	8.31	#3.11
	S	2.66	1.94

PROGRAMs 3 and 6 are used to generate this table

Evaluation 6.4

The characteristics of this evaluation are:

Objective:	To compare the ARL of the OPA MRy chart and the MR(2) chart.
Data Streams:	FORP data for which r equals 0.0, 0.3, 0.6 and 0.9
Process shift:	Ratios of new standard deviation to old standard deviation are 1.00, 1.25, 1.50, 1.75, 2.00, 2.50 and 3.00.
Type of Shift:	Stepwise.
Control limits:	The control limits of these charts are based on theoretical values.
Filter constant:	Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA MRy and MR(2) charts are presented in TABLEs 6.5 and 6.6, respectively. The SAS programs used to generate these ARL values are PROGRAMs 7 and 10; the program listings are found in Appendix E.

From TABLEs 6.5 and 6.6, it is clear that the ARLs of the OPA MRy chart are significantly higher than those of the MR(2) chart for all the scenarios as the ARLs of the OPA MRy chart are significant at 5% level for an in-control process and insignificant for out-of-control process. The abilities of these two charts to detect any magnitude of stepwise shift in dispersion for any values of filter constant are quite close.

It is interesting to realize that the ARLs of the OPA MRy chart with filter constant equal to 0.0 under different

TABLE 6.5

ARL OF THE OPA MRy CHART WITH CONDITIONS
1.a and 2.b ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		$\sigma_{new} / \sigma_{old}$ (DISPERSION SHIFTS IN STEP)						
		1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.0	M	#154.74	42.84	19.98	12.06	8.35	5.20	3.82
	S	155.69	43.06	19.68	11.77	7.92	4.85	3.42
0.3	M	#132.81	37.60	17.46	10.61	7.51	4.70	3.48
	S	133.16	37.02	17.25	10.24	7.16	4.39	3.09
0.6	M	#119.61	33.17	15.33	9.34	6.57	4.14	3.10
	S	120.70	32.51	14.81	9.00	6.26	3.77	2.71
0.9	M	110.28	28.87	13.08	7.80	5.52	3.53	2.70
	S	109.61	28.34	12.88	7.46	5.13	3.09	2.17

PROGRAM 7 is used to generate this table

TABLE 6.6

ARL OF THE MR(2) CHART WITH CONDITIONS
1.a and 2.b ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		σ_{new} / σ_{old} (DISPERSION SHIFTS IN STEP)						
		1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.0	M	120.52	#31.58	#14.88	#9.23	#6.66	#4.39	#3.36
	S	120.39	30.55	13.84	8.22	5.69	3.51	2.58
0.3	M	112.81	#29.62	#13.77	#8.38	#6.13	#3.99	#3.11
	S	111.06	29.01	12.79	7.38	5.14	3.30	2.42
0.6	M	109.89	#28.49	#13.06	#7.92	#5.67	#3.73	#2.85
	S	107.66	28.32	12.31	7.24	4.89	3.15	2.25
0.9	M	110.09	#27.55	#12.40	#7.43	#5.29	#3.44	2.66
	S	109.03	26.80	11.88	6.81	4.74	2.87	2.11

PROGRAM 10 is used to generate this table

shifts in process dispersion are not identical to the ARLs of the traditional MR(2) chart; since the filter constant equals 0.0 and the Y values are independent of one another. The moving ranges of subgroup size two of the Y values formed must have the same probability density function as the traditional moving ranges of subgroup size two from an independent data stream. However, the conditional distribution of the moving range given that the first Y value in a moving subgroup of size two is known, is different from the unconditional distribution of the moving range of subgroup size two from a stream of Y values. Thus the ARL of the OPA MRy chart, which is constructed based on conditional control limits, is sure to be different from the ARL of the traditional MR(2) chart.

At this stage, the benefit of using the OPA MRy chart on a correlated FORP data stream is not obvious yet as it is masked by the tedious construction required for the OPA MRy chart. That is, even though the OPA MRy chart has appealing ARL values for all scenarios, the computational tedium may detract from its use.

Evaluation 6.5

The characteristics of this evaluation are:

Objective: To compare the ARL of the OPA MRy chart and the MR(2) chart.

Data Streams: FORP data for which r equals 0.0, 0.3, 0.6 and 0.9.

Process shift: The gradual shift in the mean is an

	increase with a total of 3 process standard deviations in 20 subgroup sampling intervals. Once the maximum magnitude of the shift is reached, the process mean remains at this level.
Type of Shift:	Trend-wise.
Control limits:	The control limits of these charts are based on theoretical values.
Filter constant:	Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA MRy and MR(2) charts are presented in TABLE 6.7. The SAS programs used to generate these ARL values are PROGRAMs 8 and 11; the program listings are found in Appendix E.

From TABLE 6.7, it is found that the MR(2) chart is not sensitive to shifts of process mean in trend for filter constants equal to 0.0, 0.3 or 0.6. For these values of the filter constant, the OPA MRy chart has a substantially lower ARL when it is applied to the similar FORP data stream. The ARL of the OPA MRy chart on a FORP data stream with shift of process mean in trend increases as the value of filter constant increases. It is interesting to know that for filter constant equal to 0.9, the MR(2) chart has a lower ARL, which is significant at 5% level, than that of the OPA MRy chart. Recall that the shift in the mean of input variate X is at a rate of $3\sigma/20$ per subgroup sampling interval for 20 subgroup sampling intervals. When this magnitude of shift rate is 'translated' into the Y variate, it is found that the magnitude of shift in the mean of output

TABLE 6.7

ARL OF THE OPA MRy AND MR(2) CHARTS WITH CONDITIONS 1.a and 2.c ON FORP DATA (USING THEORETICAL VALUES)

FILTER CONSTANT		MEAN SHIFTS IN TREND	
		OPA MRy	MR(2)
0.0	M	#20.01	119.05
	S	8.30	120.39
0.3	M	#23.35	110.77
	S	10.22	110.92
0.6	M	#28.62	102.43
	S	14.03	105.95
0.9	M	49.62	#40.79
	S	38.74	71.88

PROGRAMs 8 and 11 are used to generate this table

variate Y depends on the filter constant of the FORP.

Shift rate in X variate = $3\sigma/20$

$$\text{Shift rate in Y variate} = \frac{3\sigma_y}{20} \quad \frac{\sqrt{(1+r)}}{\sqrt{(1-r)}}$$

$r = 0.0$ Shift rate in Y variate = $0.150\sigma_y$

$r = 0.3$ Shift rate in Y variate = $0.204\sigma_y$

$r = 0.6$ Shift rate in Y variate = $0.300\sigma_y$

$r = 0.9$ Shift rate in Y variate = $0.654\sigma_y$

Thus, it can be seen that the shift rate in the Y variate increases drastically for r=0.9. This explains the sudden decrease in the ARL of the MR(2) chart for r=0.9. Overall, the OPA MRy chart is still a more favorable control chart for a FORP data stream with a shift of process mean in trend as the ARLs are comparatively small for all values of the filter constant.

Evaluation 6.6

The characteristics of this evaluation are:

Objective: To compare the ARL of the OPA MRy chart and the MR(2) chart.

Data Streams: FORP data for which r equals 0.0, 0.3, 0.6 and 0.9.

Process shift: The mean is shifted in a cyclical manner. The cycle has a period which is equal to the sampling interval of 48 subgroups. The amplitude of the cycle has a magnitude of 3 process standard deviations.

Type of Shift: Cycle.

Control limits: The control limits of these charts are based on theoretical values.

Filter constant: Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA MRy and MR(2) charts are presented in TABLE 6.8. The SAS programs used to generate these ARL values are PROGRAMs 9 and 12; the program listings are found in Appendix E.

In TABLE 6.8, it is clear that the OPA MRy chart is more powerful than the MR(2) chart in detecting a cyclical shift of process mean for filter constant equals to 0.0, 0.3 or 0.6 as these ARLs of the OPA MRy chart are significant at the 5% level. The ARL of the OPA MRy chart increases as the value of filter constant increases, but decreases as the filter constant equals 0.9. Again, it is interesting to find that the ARL of the OPA MRy chart is higher than that of the MR(2) chart when the filter constant is 0.9. This is due to the fact that the 'translation' of the shift in X variate into Y variate is not linear; rather, is an exponential relation which depends on the filter constant.

From TABLEs 6.5 to 6.8, it is clear that the OPA MRy chart is a more favorable control chart for a FORP data stream than the MR(2) chart. Even though the OPA MRy chart may be a bit tedious to construct, this extra effort can be easily justified by the gains in abilities to detect shifts in process mean in trend or cycle. Moreover, with a computer program, the construction of the OPA MRy chart can be made very easy.

TABLE 6.8

ARL OF THE OPA MRy AND MR(2) CHARTS WITH CONDITIONS 1.a and 2.d ON FORP DATA (USING THEORETICAL VALUES)

FILTER CONSTANT		MEAN SHIFTS IN CYCLE OPA MRy	MR(2)
0.0	M	#17.39	103.02
	S	14.55	101.88
0.3	M	#23.84	79.71
	S	22.23	78.50
0.6	M	#36.56	42.48
	S	35.95	39.93
0.9	M	17.44	#10.38
	S	12.54	6.90

PROGRAMs 9 and 12 are used to generate this table

Evaluation 6.7

The characteristics of this evaluation are:

Objective:	To compare the ARL of the OPA Y charts when the value of the supposed-to-be-known filter constant of a FORP is either understated, correctly stated or overstated.
Data Streams:	FORP data for which r equals 0.6.
Process shift:	Mean level is shifted by 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00 in terms of the process standard deviation.
Type of Shift:	Stepwise.
Control limits:	The control limits of these charts are based on theoretical values.
Filter constant:	Using first principles, the r is explicitly recognized and used in the construction of control limits. However, the r used is either overstated as 0.7, exactly stated as 0.6 or understated as 0.5.

The ARL values for the OPA Y charts are presented in TABLE 6.9. The SAS program used to generate these ARL values is PROGRAM 14; the program listing is found in Appendix E.

From TABLE 6.9, it is obvious that the filter constant should be exactly stated in order for the OPA Y chart to perform as intended for a FORP data stream with stepwise shift in the process mean. If the filter constant is overstated, the ARLs of the OPA Y chart decrease. If the filter constant is understated, the ARLs of the OPA Y chart inflate. The change of the ARL is not proportional to the deviation of the supposed-to-be-known filter constant from its true value. If the detection of any magnitude of shift

TABLE 6.9

ARL OF THE OPA Y CHART WITH CONDITIONS
1.b and 2.a ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.7	M	39.07	35.28		12.17		2.93	
OS	S	38.36	34.95	N/R	12.67	N/R	2.73	N/R
0.6	M	370.36	283.08	155.53	43.82	14.57	6.19	2.02
ES	S	372.62	284.72	156.60	43.29	14.02	5.65	1.46
0.5	M				156.82		11.39	
US	S	N/R	T/L	N/R	157.03	N/R	9.76	N/R

OS = Overstated ES = Exactly stated US = Understated
PROGRAM 14 is used to generate this table

in the process mean is deemed more important than signaling a false alarm when the process is in-control, overstating the filter constant is more favorable than understating the filter constant.

Evaluation 6.8

The characteristics of this evaluation are:

Objective:	To compare the ARL of the combined OPA Y and OPA MRy charts when the value of the supposed-to-be-known filter constant of the FORP is either understated, correctly stated or overstated.
Data Streams:	FORP data for which r equals 0.6.
Process shift:	Mean level is shifted by 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00 in terms of the process standard deviation.
Type of Shift:	Stepwise.
Control limits:	The control limits of these charts are based on theoretical values.
Filter constant:	Using first principles, the r is explicitly recognized and used in the construction of control limits. However, the r used is either overstated as 0.7, exactly stated as 0.6 or understated as 0.5.

The ARL values for the combined OPA Y and OPA MRy charts are presented in TABLE 6.10. The SAS program used to generate these ARL values is PROGRAM 15; the program listing can is found in Appendix E.

From TABLE 6.10, it is obvious that the filter constant should be exactly stated in order for the combined OPA Y and OPA MRy charts to perform as intended for a FORP data stream

TABLE 6.10

ARL OF THE COMBINED OPA Y AND OPA MRy CHARTS WITH CONDITIONS 1.b and 2.a ON FORP DATA (USING THEORETICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.7	M	19.00	18.69		10.25		2.63	
OS	S	18.35	18.50	N/R	11.42	N/R	2.60	N/R
0.6	M	112.28	104.92	87.28	38.61	13.33	5.57	1.75
ES	S	112.96	105.06	89.45	39.37	13.74	5.58	1.30
0.5	M				142.37		10.78	
US	S	N/R	T/L	N/R	143.30	N/R	9.82	N/R

OS = Overstated ES = Exactly stated US = Understated
PROGRAM 15 is used to generate this table

with stepwise shift in the process mean. If the filter constant is overstated, the ARLs of the combined OPA Y and OPA MRy charts decrease. If the filter constant is understated, the ARLs of the combined OPA Y and OPA MRy charts inflate. The change of the ARL is not proportional to the deviation of the supposed-to-be-known filter constant from its true value. The ARL of the combined charts with true filter constant equal to 0.6, but overstated as 0.7, for an in-control process is only 19. This is considered too small as an ARL for an in-control process.

Evaluation 6.9

The characteristics of this evaluation are:

Objective:	To compare the ARL of the OPA MRy charts when the value of the supposed-to-be-known filter constant of the FORP is either understated, correctly stated or overstated.
Data Streams:	FORP data for which r equals 0.6.
Process shift:	Ratios of new standard deviation to old standard deviation are 1.00, 1.25, 1.50, 1.75, 2.00, 2.50 and 3.00.
Type of Shift:	Stepwise.
Control limits:	The control limits of these charts are based on theoretical values.
Filter constant:	Using first principles, the r is explicitly recognized and used in the construction of control limits. However, the r used is either overstated as 0.7, exactly stated as 0.6 or understated as 0.5.

The ARL values for the OPA MRy chart are presented in TABLE 6.11. The SAS program used to generate these ARL values is

TABLE 6.11

ARL OF THE OPA MRy CHART WITH CONDITIONS
1.b and 2.b ON FORP DATA
(USING THEORETICAL VALUES)

FILTER CONSTANT		$\sigma_{new}/\sigma_{old}$ (DISPERSION SHIFT IN STEP)						
		1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.7	M	20.01	9.24	5.74	N/R	3.44		2.15
OS	S	19.42	8.87	5.26		2.95	N/R	1.63
0.6	M	119.61	33.17	15.33	9.34	6.57	4.14	3.10
ES	S	120.70	32.51	14.81	9.00	6.26	3.77	2.71
0.5	M		153.23	49.39		14.43		4.93
US	S	T/L	153.64	48.58	N/R	14.39	N/R	4.76

OS = Overstated ES = Exactly stated US = Understated
PROGRAM 16 is used to generate this table

PROGRAM 16; the program listing is found in Appendix E.

From TABLE 6.11, it is obvious that the filter constant should be exactly stated in order for the OPA MRy chart to perform as intended for a FORP data stream with a stepwise shift in process dispersion. If the filter constant is overstated, the ARLs of the OPA MRy chart decrease. If the filter constant is understated, the ARLs of the MRy chart inflate. The change of the ARL is not proportional to the deviation of the supposed-to-be-known filter constant from its true value. The ARL of the OPA MRy chart with true filter constant equal to 0.6, but overstated as 0.7, for an in-control process is only 20.01. This is considered too small as an ARL for an in-control process. The ARL of the OPA MRy chart with true filter constant equal to 0.6, but understated as 0.5, for an in-control process is too large to be determined in a CPU time of 5 minutes.

Whether or not the ARLs inflate or decrease in a similar pattern when the true filter constant has a value other than 0.6 but is overstated or understated is not investigated in this research. From TABLEs 6.9-6.11, it is important to realize that, if the true filter constant equals 0.6 but is overstated as 0.7, the overstating of the filter constant causes the ARL of an in-control process to decrease unfavorably. If the true filter constant equals 0.6 but is understated as 0.5, the understating of the filter constant causes the ARL of an in-control process to inflate considerably. Therefore, the best practice is to exactly state the filter constant using first principles.

Conclusion

In these ARL evaluations, substantial computer simulation has been carried out. The ARL and SDRL for the OPA Y, OPA MRy, I, and MR(2) charts on a FORP data stream with different types and different magnitudes of process shifts have been determined and presented. From the above comparison, there is clear indication that the OPA Y and OPA MRy charts perform favorably on a FORP data stream. It is also important to note that the filter constant be exactly stated in order to have the OPA Y or OPA MRy chart perform as desired.

CHAPTER VII

ANALYSIS OF SIMULATION RESULTS USING EMPIRICAL FORP AND SORP VALUES

Introduction

In the previous two chapters, the ARL of the OPA Y and OPA MRy charts are evaluated and compared with those of the I and MR(2) charts. One distinguishing feature of the evaluations is that the control limits of the control charts are constructed using theoretical mean and standard deviation values of the X variates. One has the advantage of using these theoretical values because the relationship between the X (input) and Y (output) variates of a FORP are well defined. Those control charts used in the evaluations are constructed based on theoretical values of the mean and the average of the moving ranges of subgroup size two of the Y variates.

However, if the response process is not a FORP but rather a second order response process, SORP, or if the construction of a particular control chart cannot be based on the theoretical mean and standard deviation values of the X variates, then a different approach to computing the control limits of the control chart has to be used. In this case, the control limits of the control chart are constructed based on the statistics from an initial small sample, say 30 subgroups. In other words, the control limits are constructed based on empirical data.

The modified EWMA chart is a control chart suggested by Montgomery (1990) as a useful tool to monitor a correlated data stream. The construction of control limits for the Modified EWMA chart is based on statistics derived from an initial small sample of the correlated data stream. Therefore, to compare the ARLs of the Modified EWMA and OPA Y charts on a FORP data stream, the control limits of the OPA Y chart are constructed based on empirical data from a FORP. Otherwise, the comparison of ARLs for the Modified EWMA and OPA Y chart is not compatible.

To investigate how the OPA Y or OPA MRy chart performs on a SORP data stream, the control limits of the OPA Y or OPA MRy chart must be constructed based on the empirical data from the output of a SORP. This scenario corresponds to a situation in which the user of the OPA Y or OPA MRy chart mistakenly considers a SORP data stream as if it is a FORP stream. Nevertheless, it is assumed that the user still exactly states the filter constant. Using the SORP values, the OPA Y chart or OPA MRy chart is set up as if the chart will be applied on a FORP data stream. In fact, the chart constructed is applied on a SORP data stream. The ARL of the OPA Y and OPA MRy charts on a SORP data stream under different magnitudes of process shift in the X variates (input of the SORP) are determined using simulation.

To compare the ARLs of the OPA Y or OPA MRy chart on a SORP, compatible to the ARLs of the OPA Y or OPA MRy chart on a FORP, the ARL of the OPA Y or OPA MRy chart on a FORP has to be determined again. This time, the control limits of the OPA Y or OPA MRy chart on a FORP are constructed based on empirical data from the output of a FORP and the ARLs of these charts on FORP data are determined using simulation as in Chapters V and VI.

Data Generation

FORP Data

The FORP data stream used to determine the ARL of the OPA Y, OPA MRy and Modified EWMA chart are generated using the method described in Chapter V. In addition, an initial small sample needs to be generated in order to estimate the mean and average of the moving ranges of the Y variates. This is done by generating 30 observations after the first 50 Y values are discarded for 'warming-up' purpose. The first 30 observations are used to compute 30 moving ranges of subgroup size two. The average of the moving ranges, MRybar is then calculated from these 30 moving ranges. An estimate of the mean of the Y variates, Ybar, is also computed from these 30 observations. These MRybar and Ybar are statistics derived from the empirical data and are used to compute the control limits of control charts.

In this study, the first plotted point on the control chart is obtained from the 31st observation. That is, the first 30 observations are used only to estimate parameters for the construction of control limits, and are not plotted on the control charts.

SORP Data

The generation of a SORP data stream is just an extension of the generation of a FORP data stream. A SORP can be depicted as two FORPs in series (Box and Jenkins, 1976). The input to the first FORP is the X variates, and the output of the first FORP becomes the input to the second FORP. The output of the second FORP is the output of the SORP. To facilitate further discussion, the output of a SORP is denoted as a Z variate. In a real process, the filter constant for the two FORPs can be different. However, in this research, for simplicity, it is assumed that the filter constants of the first and second FORP in a SORP are the same. In computer codes, a SORP generator can be described by a pair of recurrence relations:

```
Y2 = FC*Y1 + (1-FC)*(RANNOR(SEED)*STD + MEAN)      (7.1)
Z2 = FC*Z1 + (1-FC)*Y2                             (7.2)
```

where FC is the filter constant, r, of a first and second FORP, Y1 and Y2 are two consecutive values of Y, output of the first FORP, and Z1 and Z2 are the two consecutive values of Z, output of the SORP. The values of the filter constant used in the study are 0.0, 0.3, 0.6, and 0.9. By using a filter constant equal to 0.0, an independent normal data stream is generated. Thus, Equations (7.1) and (7.2) can be used to generate an independent normal data stream as well as an SORP data stream.

To use these recurrence relations, the first Y and Z values used in the first and second FORP are two identical and independent normal variates with mean equal to μ = MEAN and standard deviation equal to σ = STD. For this study, MEAN is set to equal 10.0, and STD equals 1.0. All subsequent Y values of the first FORP are then generated using Equation (7.1). Every subsequent Y value generated is substituted into Equation (7.2) to generate a corresponding Z value.

Since the very first Y and Z values are not generated using Equations (7.1) and (7.2), respectively, the first few Z values generated do not truly depict a SORP. Thus, to 'warm-up' the SORP data generator, the first 50 SORP data values generated are discarded. However, the 51^{st} value generated is used to generate the first 'observation' of the initial sample. In this study, 30 observations are generated in order to estimate the mean and average of the moving ranges of the Z variates. This is done by generating 30 observations of Z values after the first 50 Z values are discarded for 'warming-up' purposes. These first 30 observations of Z, along with the 51^{st} Z value are used to compute 30 moving ranges of subgroup size two. The average of the moving ranges, MRzbar, is then calculated from these 30 moving ranges. An estimate of the mean of the Z variates, Zbar, is also computed from these 30 observations. These MRzbar and Zbar statistics are derived from the empirical data and are used to compute the control limits of control charts.

To embed the effect of a shift in process mean or process standard deviation into the (10.0,1.0) normal variate, for the generation of SORP data, the following methods are used:

(1) For a stepwise shift in the process mean, the standard deviation is multiplied by the magnitude of shift and added to the process mean. Hence, the shift of process mean is in terms of the process standard deviation. The SORP data generator in computer codes becomes

```
NMEAN = MEAN + SHIFT*STD
Y2    = FC*Y1 + (1-FC)*(RANNOR(SEED)*STD + NMEAN)
Z2    = FC*Z1 + (1-FC)*Y2
```

where SHIFT is the magnitude of shift. The magnitudes of shift used in this study are: 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00. Figure 5.1 shows the stepwise shift in mean.

(2) For a stepwise shift in the process standard deviation, the standard deviation is multiplied by the ratio of the new standard deviation to the original standard deviation, $\sigma_{new}/\sigma_{old}$. The SORP data generator becomes:

```
NSTD  = RATIO*STD
Y2    = FC*Y1 + (1-FC)*(RANNOR(SEED)*NSTD + MEAN)
Z2    = FC*Z1 + (1-FC)*Y2
```

where RATIO is the value of $\sigma_{new}/\sigma_{old}$. The ratios used in this study are: 1.00, 1.25, 1.50, 1.75, 2.00, 2.50, 3.00. Figure 5.4 shows the stepwise shift in dispersion.

It should be noted that only from the generation of the first plotted point onward is the effect of the process shift embedded into the SORP data generation process. That is, the effect of process shift is only embedded beginning with the generation of the 31st observation of Z. Moreover, only one type of shift is embedded in a scenario under study.

Control Limits

In this section, the construction of the control limits for the control charts used in the study are discussed. It is important to remember that the control limits are constructed based on empirical data. That is, it is assumed that the mean, μ, and standard deviation, σ, of the unobservable X variates are unknown, and they need to be estimated from empirical data. However, the filter constant, r, of the FORP is still assumed to be exactly known by first principles. In the case of a SORP data stream, the two filter constants, FC1 and FC2, used for data generation are assumed to be equal. It is assumed that the user has mistakenly considered a SORP as a FORP and applies the OPA Y or OPA MRy chart on this SORP data stream. It is further assumed that the user has exactly stated one of the filter constants, FC1, but used it as the filter constant for a FORP. This stated filter constant, FC1, is used for the construction of control limits based on empirical data which have been mistakenly considered as FORP data.

Modified EWMA Chart

The control limits of the Modified EWMA chart are one-step-ahead control limits based on the prediction error made. The EWMA statistic is

$$W_t = \alpha \, Y_t + (1-\alpha) \, W_{t-1} \tag{7.3}$$

where $0 \leq \alpha < 1$.

If the observations from the process can be modeled by an ARIMA (0,1,1) = IMA(1,1) model, then the EWMA is the optimal one-step-ahead forecast for the mean of this process (Montgomery, 1990). Using the procedure presented by Montgomery, the one-step-ahead control limits can be constructed.

If $Yc_{t+1}(t)$ is the forecast for the observation in period t+1 made at the end of period t, then

$$Yc_{t+1}(t) = W_t \tag{7.4}$$

is used as the center line for the control chart in period t+1. The sequence of one-step-ahead prediction errors

$$e_{t+1}(1) = Y_{t+1} - Yc_{t+1}(t)$$

or

$$e_{t+1}(1) = Y_{t+1} - W_t \tag{7.5}$$

are independently and identically distributed if the underlying process is really IMA(1,1). There are two procedures that could be used to estimate the standard deviation of the one-step-ahead prediction error, σ_p. However, in this study only one method is adopted to estimate σ_p. The method used is to apply an EWMA to the absolute value of the prediction error as follows,

$$\Theta(t) = \delta|e_t(1)| + (1-\delta)\ \Theta(t-1), \quad 0 \le \delta < 1 \qquad (7.6)$$

Since the mean absolute deviation of a normal distribution is related to the standard deviation by $\sigma \approx 1.25\ \Theta(t)$ (Montgomery and Johnson, 1976), the standard deviation of the prediction error at time t can be estimated by

$$\hat{\sigma}_p(t) = 1.25\ \Theta(t) \qquad (7.7)$$

The control limits of the Modified EWMA chart for period t+1, calculated at the end of period t, are

$$UCL_{mewma} = W_t + 3\ \hat{\sigma}_p(t)$$

$$CL_{mewma} = W_t$$

$$LCL_{mewma} = W_t - 3\ \hat{\sigma}_p(t) \qquad (7.8)$$

The starting values of the control chart are obtained by treating the first 30 observations of Y after the 'warm-up' process. The EWMA is used for the first 30 periods with W_o equal to Ybar from the first 30 observations. Solving Equation (7.3) successively, W_{30} is obtained. Using Equation (7.5), a series of one-step prediction errors $e_1(1)$, $e_2(1)$, ..., $e_{30}(1)$ are also obtained. As specified by Montgomery (1990), the sample standard deviation of these prediction errors is used to provide starting values for Equation (7.6) at time origin t = 0. That is, the sample standard deviation of $e_i(1)$, i = 1(1)30, is used as $\Theta(0)$ in Equation (7.6). Solving Equation (7.6) successively, $\Theta(30)$ is obtained. Consequently, $\hat{\sigma}_p(30)$ is also obtained through Equation (7.7). Substituting W_{30} and $\hat{\sigma}_p(30)$ into Equation

(7.8), the control limits for the first plotted point on the Modified EWMA chart can be calculated. The first plotted point is the 31st observation recorded.

The control limits for the second plotted point can be obtained after the 31st observation is recorded. Using Equations (7.3), (7.5), (7.6), (7.7) and (7.8), the control limits for the second plotted point are computed. This procedure is repeated to construct subsequent control limits on the control chart.

As suggested by Montgomery (1990), the value α used for the EWMA Equation (7.3) is 0.20, and the value of δ used for smoothing the error estimates in Equation (7.6) is 0.25 for the first 30 observations and 0.10 for all subsequent observations. The large initial value for δ is used to induce a more rapid rate of smoothing during the time periods following control start-up. This ensures that starting values for the EWMA and the error estimates are quickly updated.

<u>OPA Y Chart Using Y Data</u>

The control limits for the OPA Y chart are conditional control limits. They are conditioned on the current observation. Using Equations (3.10) and (3.11), after rearrangement of terms, the conditional control limits for the next plotted point on the control chart are

$$UCL_{OPAY} = FC*(W-MEAN) + MEAN + 3*STD(1-FC) \qquad (7.9)$$

$$LCL_{OPAY} = FC*(W-MEAN) + MEAN - 3*STD(1-FC) \qquad (7.10)$$

where W is the current value on the control chart and FC is the filter constant of the response process.

For a FORP data stream, the W is replaced by the current observation of Y, Y2, MEAN is replaced by Ybar, and STD is replaced by the following equation (from Equation (3.17))

$$\sigma = \frac{MRybar * \sqrt{(1+r)}}{1.128 * (1-r)} \qquad (7.11)$$

Equation (7.11) is derived from the fact that,

$$\sigma_y^2 = \sigma^2 * (1-r)/(1+r) \qquad (7.12)$$

and

$$MRybar = 1.128 * \sigma_y * \sqrt{(1-r)} \qquad (7.13)$$

Taking the square root of Equation (7.12) and substituting into Equation (7.13) results in

$$MRybar = 1.128 * \sigma * (1-r)/\sqrt{(1+r)} \qquad (7.14)$$

Then rearranging Equation (7.14) results in Equation (7.11).

<u>OPA MRy Chart Using Y Data</u>

The control limits for the OPA MRy chart are conditional control limits. Similar to the control limits of the OPA Y chart, they are also conditioned on the current plotted point on the OPA Y chart which is denoted as k. Using Equations (4.20) and (4.21), the conditional control limits for the next plotted point on the OPA MRy chart are

$$UCL_{OPAMRY} = E(R|k) + 3 \cdot Std(R|k) \qquad (7.15)$$

$$LCL_{OPAMRY} = E(R|k) - 3 \cdot Std(R|k) \qquad (7.16)$$

where $E(R|k)$ and $Std(R|k)$ are defined as in (4.16) and (4.17), and both involve the μ and σ of the X variates, plus the current observation.

For a FORP data stream, the k is replaced by the current observation of Y, Y2, μ is replaced by Ybar, and σ is replaced by Equation (7.11).

<u>OPA Y Chart Using Z Data</u>

For a SORP data stream, the conditional control limits for the next plotted point on the OPA Y chart are the Equations (7.9) and (7.10); except that the W is replaced by the current observation of Z, Z2, MEAN is replaced by Zbar, and STD is replaced by the following equation (from Equation (3.17))

$$\sigma = \frac{\text{MRzbar} * \sqrt{(1+\text{FC1})}}{1.128 * (1-\text{FC1})} \qquad (7.17)$$

It is seen that the estimate of the STD expressed in Equation (7.17) is not correct as Z is an output from a SORP.

<u>OPA MRy Chart Using Z Data</u>

For a SORP data stream, the conditional control limits for the next plotted point on the OPA MRy chart are the Equations (7.15) and (7.16); except that the k is replaced by the current observation of Z, Z2, μ is replaced by Zbar, and σ is replaced by Equation (7.17).

List of Programs Coded

SAS programs have been coded to simulate the performance of the OPA Y, OPA MRy and Modified EWMA charts in various scenarios of interest. The conditions incorporated into the scenarios can be broken down in the following manner

1.a 4 filter constants exactly stated: r = 0.0, 0.3, 0.6, 0.9

2.a 7 shifts in the mean of X variates (in multiple of σ) : 0.0, 0.25, 0.50, 1.00, 1.50, 2.00, 3.00

2.b 7 shifts in standard deviation of X variates ($\sigma_{new}/\sigma_{old}$): 1.0, 1.25, 1.50, 1.75, 2.00, 2.50, 3.00

For consistency, these conditions are labeled in such a way to match with those in Chapter V.

The SAS program scenarios coded are listed in TABLE 7.1. Since the execution of the simulation of each scenario can be quite long, only simulation for one scenario is done at a time. Due to the length of the simulation time, the ARLs for some scenarios are the averages of 10000 or, in some cases, 5000 or 1000 run lengths simulated. After obtaining the ARL for one scenario, parameters are changed to form a new scenario. It is important to point out that each scenario has the same random number seed. Thus, each scenario is subject to the same stream of random numbers.

Comparison of the ARL

To analyze and compare various ARLs under different scenarios, the ARL of comparable charts are grouped under various evaluation headings. A brief description is included for each evaluation. The description includes the objective of the ARL comparison, the features of the data streams used, the magnitude of process shift either in mean or dispersion,

TABLE 7.1

LIST OF SAS PROGRAM SCENARIOS CODED

PROGRAM NUMBER	NAME OF CHART(S)	CONDITION NUMBER
17	Modified EWMA chart on FORP data stream	(1.a) and (2.a)
18	OPA Y chart on FORP data stream	(1.a) and (2.a)
19	OPA MRy chart on FORP data stream	(1.a) and (2.b)
20	OPA Y chart on SORP data stream	(1.a) and (2.a)
21	OPA MRy chart on SORP data stream	(1.a) and (2.b)

how the control limits are constructed, type of process shift , and whether the filter constant is explicitly recognized or not. The tabulated ARL results are presented and followed by an analysis of the comparison in terms of the ARL of the control charts involved. Statistically significant ARLs are marked with '#'.

Due to the effect of variation in control limits introduced by using empirical values (see Appendix F), the ARL of the OPA Y and OPA MRy charts on a FORP data stream presented in this chapter are found to be different from those of similar charts in Chapter VI.

<u>Evaluation 7.1</u>

The characteristics of this evaluation are:

Objective:	To compare the ARL of the Modified EWMA chart and the OPA Y chart.
Data Streams:	FORP data for which r equals 0.0, 0.3, 0.6 and 0.9.
Process shift:	Mean level is shifted by 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00 in terms of process standard deviation.
Type of Shift:	Stepwise.
Control limits:	Control limits of these charts are based on the initial 30 empirical values of the Y variate.
Filter constant:	Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits in the Y chart.
EWMA parameter:	The α used in the EWMA computation is 0.2. The δ used for smoothing the error estimates is 0.25 for the initial

30 observations and 0.10 for all subsequent observations.

The ARL values for the OPA Y and the Modified EWMA charts are presented in TABLEs 7.2 and 7.3, respectively. The SAS programs used to generate these ARL values are PROGRAMs 17 and 18; the program listings are found in Appendix E.

It is seen from TABLE 7.2 that the Modified EWMA chart is insensitive to a magnitude of shift less than 3σ. It should be remembered that the variation due to using empirical data is contained in the ARLs. Examining TABLE 7.3, it is readily seen that the ARLs are large, significant at 5% level, for small magnitudes of shift in the process mean, but the ARLs are acceptable for larger magnitudes of process shift. When comparing TABLEs 7.2 and 7.3, it is clear that for a magnitude of mean shift greater or equal to 1.5 sigma, the OPA Y chart performs better than the Modified EWMA chart as the ARLs of the OPA Y chart are significant at the 5% level. It is interesting to find that the ARL of the Modified EWMA chart does not change considerably for small magnitudes of mean shift. This is not the case for the OPA Y chart. Clearly, a data stream from a first order response process is not suitable to be modeled as an IMA(1,1) time-series model.

An IMA(1,1) time series model can be written in terms of the observations, W's, and the random errors, e's, in the form (Box and Jenkins, 1976)

$$W_t = W_{t-1} + e_t - \Theta e_{t-1} \qquad (7.18)$$

TABLE 7.2

ARL OF THE MODIFIED EWMA CHART WITH CONDITIONS
1.a AND 2.a ON FORP DATA
(USING EMPIRICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M	130.18	129.30	129.37	#127.10	121.18	110.59	73.71
	S	134.79	134.50	134.93	135.25	135.00	133.96	120.58
0.3	M	131.32	130.69	#130.32	127.39	118.75	104.58	64.55
	S	132.29	135.96	136.37	136.54	136.18	133.93	116.51
0.6	M	150.34	#150.64	148.38	143.10	132.09	111.12	61.57
	S	157.43	158.40	158.06	158.62	158.13	152.75	125.53
0.9	M	217.20	215.39	#214.72	208.75	183.34	146.27	81.90
	S	226.91	227.50	228.27	229.73	226.09	215.64	175.69

PROGRAM 17 is used to generate this table

TABLE 7.3

ARL OF THE OPA Y CHART WITH CONDITIONS
1.a AND 2.a ON FORP DATA
(USING EMPIRICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M				391.76			#2.46
	S	T/L	T/L	N/R	17795.1	N/R	N/R	3.40
0.3	M			739.85		#32.59	#10.53	
	S	T/L	N/R	10275.4	N/R	104.87	44.31	N/R
0.6	M		1059.55		#113.84		#11.09	
	S	T/L	7468.27	N/R	566.84	N/R	96.22	N/R
0.9	M			490.24		#25.99		#2.31
	S	T/L	N/R	3297.0	N/R	67.08	N/R	2.65

PROGRAM 18 is used to generate this table

where θ is some constant. Recall that, from Chapter III, a FORP can be viewed as an AR(1) time series model. An AR(1) time series model can be written in terms of the observations, V's, and the random errors, e's, in the form

$$V_t = \alpha V_{t-1} + e_t \tag{7.19}$$

Obviously, an AR(1) time series model is different from the IMA(1,1) time series model. This explains why the Modified EWMA chart, which assumes that the observations from a process can be well-modeled by an IMA(1,1) time series model, does not perform satisfactorily on a FORP data stream.

Evaluation 7.2

The characteristics of this evaluation are:

Objective:	To compare the ARL of the OPA Y chart on FORP and SORP data streams.
Data Streams:	FORP data with r equal to 0.0, 0.3, 0.6 and 0.9. SORP data with FC1 equal to FC2 equal to 0.0, 0.3, 0.6 and 0.9.
Process shift:	Mean level is shifted by 0.0, 0.25, 0.50, 1.00, 1.50, 2.00 and 3.00 in terms of process standard deviation.
Type of Shift:	Stepwise.
Control limits:	Control limits of these charts are based on the initial 30 empirical values.
Filter constant:	Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits in the OPA Y chart on FORP and SORP data streams.

The ARL values for the OPA Y charts on FORP are already presented in TABLE 7.3, but to aid comparison they are listed here again as TABLE 7.4. The ARL values for the OPA Y chart on SORP data streams are presented in TABLE 7.5. The SAS programs used to generate these ARL values are PROGRAMs 18 and 20; the program listings can be found in Appendix E.

The first row of TABLEs 7.4 and 7.5 are supposed to be similar, as when the filter constant is equal to zero, the SORP and FORP are identical data streams. The discrepancies and normal statistical variation are due to the number of simulation runs performed. From TABLE 7.5, it is noted that the ARLs for the OPA Y chart are large, significant at the 5% level, for a magnitude of mean shift up to about 0.5 sigma when the filter constant of the SORP is 0.9. However, for filter constants equal to 0.3 or 0.6, the OPA Y chart is quite sensitive to the process shift in a SORP data stream as the ARLs of the OPA Y chart are significant at the 5% level.

In TABLE 7.5, the ARLs for an in-control process with filter constant equal to 0.9 seems to be very 'strange' as compared to other ARLs in the same row. This may be due to the number of simulation runs at this scenario. Another 'strange' pattern observed in TABLE 7.5 is that the SDRLs are not strictly decreasing with the increase in the magnitude of the process mean for filter constants equal to 0.0 and 0.9. The reason for such irregularity is not known. The OPA Y chart is found to be somewhat robust to a SORP data stream in the sense that the Y chart still performs well under different magnitudes of shift for a filter constant that is neither too large nor too small.

TABLE 7.4

ARL OF THE OPA Y CHART WITH CONDITIONS
1.a AND 2.a ON FORP DATA
(USING EMPIRICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M				391.76			2.46
	S	T/L	T/L	N/R	17795.1	N/R	N/R	3.40
0.3	M			739.85		32.59	10.53	
	S	T/L	N/R	10275.4	N/R	104.87	44.31	N/R
0.6	M		1059.55		113.84		11.09	
	S	T/L	7468.27	N/R	566.84	N/R	96.22	N/R
0.9	M			490.24		25.99		2.31
	S	T/L	N/R	3297.0	N/R	67.08	N/R	2.65

PROGRAM 18 is used to generate this table

TABLE 7.5

ARL OF THE OPA Y CHART WITH CONDITIONS
1.a AND 2.a ON SORP DATA[a]
(FC1=FC2; USING EMPIRICAL VALUES)

FILTER CONSTANT		STEP SHIFT IN MEAN (IN MULTIPLE OF SIGMA)						
		0.00	0.25	0.50	1.00	1.50	2.00	3.00
0.0	M		[b]2873.4	815.29	416.33	37.98	12.67	2.69
	S	T/L	25164.1	7801.57	16181.2	262.87	62.26	4.93
0.3	M	266.39	157.45	#87.75	18.64	#6.43	#3.24	1.55
	S	1503.82	455.72	267.6	37.25	12.53	3.07	0.84
0.6	M	103.78	#69.52	31.79	#8.46	3.90	#2.53	1.59
	S	579.70	192.76	76.26	22.53	3.12	1.47	0.69
0.9	M	[b]406.27	498.91	483.56	7.77	#4.66	3.42	2.33
	S	4996.63	15459.4	32540.	6.70	2.91	1.90	1.09

[a] all scenarios are simulated for 5000 times, except as marked with b
[b] simulated 1000 times
PROGRAM 20 is used to generate this table

Evaluation 7.3

The characteristics of this evaluation are:

Objective: To compare the ARL of the OPA MRy chart on FORP and SORP data streams.

Data Streams: FORP data with r equal to 0.0, 0.3, 0.6 and 0.9. SORP data with FC1 equal to FC2 equal to 0.0, 0.3, 0.6 and 0.9.

Process shift: Ratios of the new standard deviation to the old standard deviation are 1.00, 1.25, 1.50, 1.75, 2.00, 2.50 and 3.00.

Type of Shift: Stepwise.

Control limits: Control limits of these charts are based on the initial 30 empirical values.

Filter constant: Filter constant is explicitly and correctly recognized using first principles. It is used in the construction of control limits.

The ARL values for the OPA MRy charts on FORP and SORP data streams are presented in TABLEs 7.6 and 7.7, respectively. The SAS programs used to generate these ARL values are PROGRAMs 19 and 21; the program listings are found in Appendix E.

The first rows of TABLEs 7.6 and 7.7 are supposed to be similar, as when the filter constant is equal to zero, the SORP and FORP are identical data streams. The discrepancies and normal statistical variation are due to the number of simulation runs performed. From TABLE 7.7, it is noted that the ARLs for the OPA MRy chart are large when the filter constant of the SORP is 0.9 as the ARLs are significant the at the 5% level. However, for filter constants equal to 0.3

TABLE 7.6

ARL OF THE OPA MRy CHART WITH CONDITIONS
1.a AND 2.b ON FORP DATA
(USING EMPIRICAL VALUES)

FILTER CONSTANT		$\sigma_{new}/\sigma_{old}$ (DISPERSION SHIFTS IN STEP)						
		1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.0	M	390.81		24.28		9.16		
	S	2942.20	N/R	35.57	N/R	10.89	N/R	N/R
0.3	M		52.13		11.99		4.98	
	S	N/R	107.01	N/R	15.38	N/R	5.01	N/R
0.6	M	239.16		#17.95		#7.05		
	S	675.40	N/R	25.54	N/R	7.65	N/R	N/R
0.9	M		#39.63		#8.59		#3.70	
	S	N/R	67.72	N/R	10.41	N/R	3.54	N/R

PROGRAM 19 is used to generate this table

TABLE 7.7

ARL OF THE OPA MRy CHART WITH CONDITIONS
1.a AND 2.b ON SORP DATA[a]
(FC1=FC2; USING EMPIRICAL VALUES)

FILTER CONSTANT		$\sigma_{new}/\sigma_{old}$ (DISPERSION SHIFTS IN STEP)						
		1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.0	M	439.41	63.16	23.65	13.64	9.12	5.49	3.91
	S	3354.06	141.56	36.74	18.11	10.55	5.74	3.82
0.3	M	196.52	#44.47	19.21	#11.04	7.75	#4.70	3.53
	S	813.13	73.38	26.02	13.45	8.72	4.65	3.35
0.6	M	#345.17	56.57	23.92	13.21	8.77	5.20	3.67
	S	1803.70	109.51	36.18	16.90	9.49	5.27	3.57
0.9	M		[b]478.28	106.00	48.56	25.59	13.60	9.09
	S	T/L	5331.19	608.42	235.37	65.33	18.60	13.05

[a] all scenarios are simulated 5000 times, except as marked with b

[b] simulated 1000 times

PROGRAM 21 is used to generate this table

the OPA MRy chart is quite sensitive to the process shift as the ARLs are significant at the 5% level. Overall, the OPA MRy chart is found to be quite robust to the SORP data stream in the sense that the MRy chart performs quite well under different magnitudes of shift.

Conclusion

The ARL of the OPA Y, OPA MRy and Modified EWMA charts on a FORP data stream under different magnitudes of process shift are determined through simulation. The control limits of control charts in this chapter are based on empirical values. It is found that the OPA Y and OPA MRy charts are quite robust to a SORP data stream inasmuch their ARLs are acceptable for various magnitude of process shift. It is also found that the Modified EWMA chart is not suitable for a FORP data stream. The effects of variation in empirical determined control limits on the ARL of a control chart is also discussed in Appendix F. This explains the discrepancies between the ARLs of similar charts determined in previous chapters versus those in this chapter.

CHAPTER VIII

COMPUTER PROGRAM TO IMPLEMENT THE OPA Y AND OPA MRy CHARTS

Introduction

A FORTRAN program is coded to implement the proposed special control charting techniques for quality data obtained from a FORP. It is assumed that the user has prior knowledge of the filter constant of the FORP. The main purpose of the program is to illustrate the fact that the conditional control limits of the OPA Y and OPA MRy charts can be easily constructed, and manual construction of these control charts should not be regarded as an obstacle for the implementation of these useful tools on FORP data.

Program Algorithm

The user has the option of entering the observations via the keyboard or letting the program read the observed values from a disk file. The disk file should be named 'INPUT'. If the observations are to be read from an ASCII disk file, the values should be arranged in a column with the first value being the filter constant, r, and subsequent values being the observations in time order. If the observations have to be entered via keyboard, the user has to first input the known value of the filter constant, r, and the number of observations, N, required to initiate the construction of the OPA Y and OPA MRy charts. The user is then prompted to enter the observations one by one. The program then computes the average of observations and average of moving ranges of subgroup size two of these observations. These quantities are denoted as Ybar and MRybar, respectively. An estimate of the standard deviation of the X variates of a FORP, sigma, is then computed using the following equation

$$\text{sigma} = \frac{\text{MRybar} * \text{SQRT}(1.0 + r)}{1.128 * (1.0 - r)} \tag{8.1}$$

The program constructs conditional control limits for each observation entered except the first observation. Since the control limits are conditioned on the previous observation, the first set of control limits is the control limits for second observation, and is constructed conditioned on the first observation. The equations used to construct the control limits of the OPA Y and OPA MRy charts are Equations (3.19)-(3.20) and (4.20)-(4.21), respectively.

The construction of conditional control limits, and display of the OPA Y and OPA MRy values and the corresponding control limits, are performed in a subroutine called CHART. The user has the option of storing the results in a disk file. The output disk file is named 'RESULT'. Whether the user makes this selection or not, the results are always displayed on the computer monitor screen as well.

After all the control limits and observations are displayed, the user is given an opportunity to decide whether to terminate the program or to continue to enter observations. If the user decides to enter more observations, and the array used to store the observation is not yet full, an observation can be accepted and the program will compute the moving range and its control limits based upon the previously computed Ybar and MRbar. That is, the estimate of mean and standard deviation are based only upon the initial N observations. The value of the observation entered, its moving range, and corresponding control limits for the OPA Y and OPA MRy charts are displayed or stored in the disk file as well, before the user is prompted to enter the next observation. Otherwise, the program terminates. A brief flow-chart of this main program is found in Figure 8.1.

Subroutine CHART is coded to construct the conditional control limits of the OPA Y and OPA MRy charts. The argument parameters needing to be passed from the main program are the index of the plotted point, the current and previous observations, the current moving range, the Ybar, MRybar, filter constant r, and a constant term B which has been computed in the main program.

The subroutine begins by constructing the conditional control limits of the OPA Y chart using Equations (3.19) and (3.20). From Equation (4.16), it is found that the construction of the conditional control limits of the OPA MRy

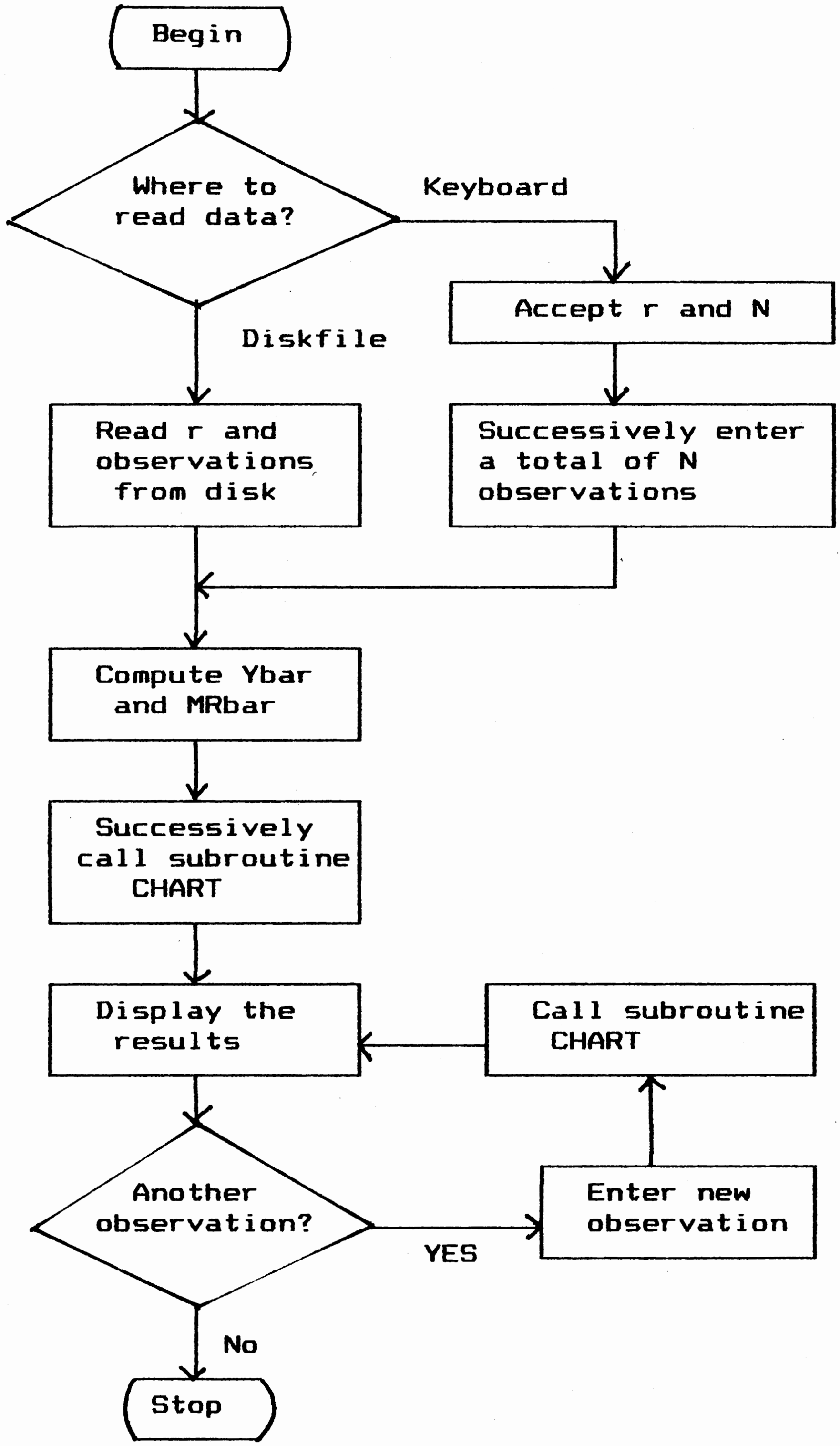

Figure 8.1. Flow-chart of Main Program for Constructing the OPA Y and OPA MRy Charts

chart involves a lot of components. They are broken down into several FORTRAN statements. The ordinate of standard normal distribution is coded as a function PHIS and the cumulative of the standard normal is coded as a function PHI. The computations in both functions are in double precision to reduce round-off computation errors. The computation of PHI is approximated using polynomial equation 29.2.19 in the text by Abramowitz and Stegun (1965). The computer codes can also be found in a program coded by Nelson (1983). This eliminates the need for numerical integration for the determination of the cumulative standard normal.

If the control limits of the OPA MRy charts are less than zero, they are set at the value zero. The current observation and its moving range are tested to see whether they fall within or beyond their respective control limits. If one falls beyond the control limits, a mark '*' will be printed beside the value during output to indicate an out-of-control situation is signaled. The flow-chart of this subroutine can be found on Figure 8.2. The FORTRAN program can be found in Appendix G.

Example

After the program is compiled and linked, the executive file can be used. After loading the .EXE file into RAM, the user is prompted to select the option on how the observations are to be input.

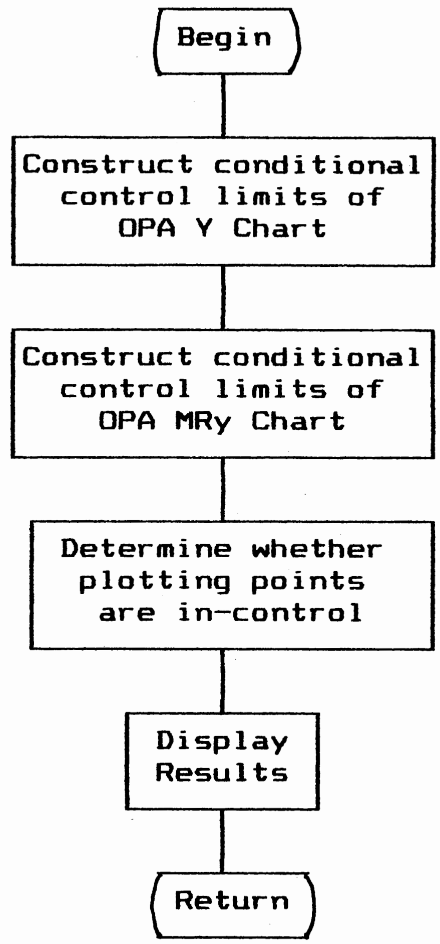

Figure 8.2. Flow-chart for Subroutine CHART

PROGRAM TO CONSTRUCT THE Y AND MRy CHARTS

ENTER D IF THE DATA ARE STORED IN DISK FILE

_

If the user has stored the observations in a disk file, and the disk file is in the current active disk drive, the user will be prompted to confirm that the disk file is in proper format and with the proper file name. Then the user is prompted to select the option whether to store the results in a disk file or not. If the user opts to enter observations via the keyboard, the user is prompted to enter the value of the filter constant.

ENTER THE VALUE OF FILTER CONSTANT

_

Assuming that the user enters 0.8 as the filter constant, the user is then prompted to enter the number of data to be entered.

ENTER THE NUMBER OF DATA POINTS

_

Assume also that the user enters 30 as the number of data values to be entered, the program then asks for the data to be input one by one.

ENTER THE DATA ONE BY ONE

_

Assume that the FORP data to be entered are 20.133, 20.241, 20.107, ..., 21.932 and 22.189. Assume also the input X variate to the FORP is known to follow a normal distribution with mean 20 and standard deviation 1. However, from time period 26 onward, the X variate has a new normal distribution with mean 23 and standard deviation 1. That is, the X variate is out-of-control from time period 26 onward. After entering these values, the user is prompted to select the option of storing the results in a disk file as well.

ENTER D IF WANT TO STORE RESULTS IN DISK

_

After making the selection, the program continues all other calculations. The results are displayed and the user is prompted to enter more data or terminate from the program.

```
NUMBER OF INITIAL OBSERVATIONS =            30
Ybar    =          20.1719700
MRybar  =    1.648965E-001
Y(1)    =          20.1330000
```

No.	Y	LCL	UCL	MRy	LCL	UCL
2	20.241	19.552	20.729	.108	.000	.512
3	20.107	19.639	20.816	.134	.000	.512
4	20.025	19.532	20.708	.082	.000	.512
5	20.093	19.466	20.643	.068	.000	.517
6	20.148	19.520	20.697	.055	.000	.513
7	19.974	19.564	20.741	.174	.000	.511
8	19.791	19.425	20.602	.183	.000	.521
9	19.793	19.279	20.456	.002	.000	.548
10	19.823	19.280	20.457	.030	.000	.547
11	19.850	19.304	20.481	.027	.000	.542
12	19.968	19.326	20.503	.118	.000	.538
13	19.844	19.420	20.597	.124	.000	.522
14	19.629	19.321	20.498	.215	.000	.539
15	19.772	19.149	20.326	.143	.000	.581
16	19.526	19.264	20.440	.246	.000	.551
17	19.716	19.067	20.244	.190	.000	.606
18	19.705	19.219	20.396	.011	.000	.562
19	19.697	19.210	20.387	.008	.000	.564
20	19.842	19.204	20.380	.145	.000	.566
21	19.895	19.320	20.496	.053	.000	.539

22	19.791	19.362	20.539	.104	.000	.531
23	19.844	19.279	20.456	.053	.000	.548
24	20.057	19.321	20.498	.213	.000	.539
25	19.975	19.492	20.668	.082	.000	.515
26	20.835*	19.426	20.603	.860*	.000	.521
27	21.425*	20.114	21.291	.590	.000	.611
28	21.539	20.586	21.763	.114	.000	.778
29	21.932*	20.677	21.854	.393	.000	.810
30	22.189*	20.992	22.168	.257	.000	.914

MORE DATA TO ENTER ? Y - TO CONTINUE

_

From the display above, it is clear that the OPA MRy chart indicates an out-of-control signal at time period 26. The OPA Y chart also signals out-of-control condition beginning from time period 26. If the user decides to continue, a letter Y needs to be entered, and the user is prompted to enter the new observation. If there are already 500 observations, a message will be displayed and the user has to restart the program.

SORRY ! THERE ARE ALREADY 500 DATA POINTS
START AGAIN

This concludes the discussion of the FORTRAN program developed in this chapter.

Conclusion

There is opportunity to enhance the FORTRAN program discussed in this chapter. For example, the estimates of Ybar and MRybar may need to be updated as more observations are entered at a later time. Or the program may need to provide flexibility to the user who wishes to try several filter constants due to not being very sure about the exact filter constant of the FORP. Or, one may wish to increase the visual capability of the program by graphically plotting the OPA Y and OPA MRy charts. The intention of this chapter is to illustrate that the methodologies to construct the OPA Y and OPA MRy charts can be programmed. Hence, the OPA Y and OPA MRy charts should be implemented as useful control charts to monitor a FORP.

CHAPTER IX

CONTRIBUTIONS AND FUTURE RESEARCH

Summary of Study

The purpose of this research is to develop procedures for constructing special control charts to monitor the mean and dispersion of an unobservable input variate, X, to a FORP by plotting the corresponding observable output variate, Y. The procedures for constructing the two control charts, the OPA Y and OPA MRy charts, are successfully developed and evaluated. The OPA Y and OPA MRy charts can be used simultaneously to monitor the mean and/or dispersion of a FORP.

The performance measure used for evaluation is the ARL of a control chart. These control charts are evaluated based on their abilities to detect various types and magnitudes of process shifts. This research concentrates on six types of process shifts. They are process mean shifts in step, trend or cycle and process dispersion shifts in step only. Other scenarios are also studied, such as when the filter constant is overstated or understated, and when SORP data are mistakenly assumed to be FORP data.

The abilities of these control charts are compared to that of the I and MR(2), and Modified EWMA charts. The I and MR(2) charts are usually used in continuous flow processes without explicitly recognizing the existence of serial correlation within data. The Modified EWMA chart, which assumes that the observations from a process can be well-modeled by an IMA(1,1) time series model, is a recent development as a robust control chart for various time-series models, not including a FORP.

Comparison shows that the overall abilities of the OPA Y and OPA MRy charts in detecting process changes are desirable. The analyses show that the OPA Y and OPA MRy charts are useful tools to monitor the process mean and dispersion of a FORP. The OPA Y and OPA MRy charts are also found to be robust to a SORP data stream. The filter constant of a FORP should be exactly stated in order for the OPA Y and OPA MRy charts to perform as intended, however. Compared to the Modified EWMA chart, the OPA Y chart is found to be more favorable. It is also shown that the OPA Y chart on a FORP has the same ARL as an I chart applied to an independent normal data stream, if it were observable. This fact is also verified in the simulation output.

Contributions

The major contribution of this research to the statistical quality control discipline is the provision of a control charting technique which deals directly with serially correlated data generated from a FORP. The OPA Y and OPA MRy control charting techniques proposed in this study provide a useful tool to monitor and control a FORP without using any unfeasible methods, such as avoidance and compensation as discussed in Chapter I, to circumvent the existence of correlation in the output data of a FORP.

In the existing control charting techniques, a prior knowledge of the process to be monitored is usually not required. The control limits of control chart can be computed by the first few observations obtained, and further observations collected can be easily plotted on the control chart established. Surely, when an out-of-control condition is signaled in such a control chart, the user needs to have some knowledge of the process being monitored in order to identify the assignable cause and take necessary corrective action. Sometimes, the task of searching for an assignable cause can be done by other technical personnel. As a whole, there is usually lack of communication between the technical personnel who design the process and the control chart user who constructs the chart and identifies any out-of-control signal. On the contrary, in using the OPA Y and MRy charts, the control chart user needs to know the filter constant of a FORP and hence the observation sampling interval. This forces the control chart user to communicate with the technical personnel who design the FORP to be monitored. Thus, the proposed OPA Y and MRy charts indirectly help to bring the technical and quality control chart personnel together and foster better communication.

The OPA Y and MRy charts are process specific. That is, these control charts should only be applied on a FORP data stream. This motivates the control chart user to acquire some knowledge of the process to be monitored before begins the construction of control charts. Existing control charting techniques (including time-series approach) are generally data specific. That is, some control charts are suitable for variable data, some are suitable for attribute data, some are suitable for serially correlated data and so on. The process specific feature in the OPA Y and MRy control charting techniques is unique among all the existing control charting techniques for serially correlated quality data. A time-series approach to serially correlated data does not require the control chart user to know from where the serially correlated data are generated; however, this is not the case for the construction of the OPA Y and OPA MRy charts. Having more knowledge of the processes, the control chart personnel can prioritize the processes to be monitored.

Another contribution of this research is the realization of variation in the empirically determined control limits on the ARL of a control chart. This will affect the way the control chart user determines a control chart parameters such as sample size, sampling interval and control limit multiplier.

Future Research

This research has developed an initial phase for development of control charts using conditional control limits to monitor correlated data streams; however, there are tremendous possibilities for expansion. Future research areas include:

1. Expand the model of control chart construction to data from a SORP or a series of first order response processes.
2. Allow more types of process shift to be present in the data, either consecutively or concurrently, and evaluate the performance of the OPA Y and OPA MRy charts.
3. Develop a user friendly software package with graphical capability that will enhance the applicability of these proposed control charts.
4. Expand the control chart construction so that the filter constant is also estimated from the empirical data and not from first principles.

Another 'by-product' of this research is the realization of the effects of variation in control limits on ARL determination. This opens an area for future research to investigate how the number of subgroups, m, and subgroup size, n, affect the ARL of a control chart.

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APPENDICES

APPENDIX A

SOME USEFUL LINEAR MODEL THEOREMS

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The following linear model theorems and definition are taken from the text by Graybill (1976). They are useful in the derivation of the conditional distribution of variable Y.

<u>Theorem</u> <u>A.1</u>

Let the px1 random vector X be distributed $N(x:\mu,\Sigma)$, and partition X, μ and Σ as

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \qquad (A.1)$$

where X_1 and μ_1 are qx1 vectors, Σ_{11} is a qxq matrix with $(0 < q < p)$, and the size of the remaining vectors and matrices are thus determined. The random vector X_1 is normally distributed with mean μ_1 and covariance matrix Σ_{11}, that is X is distributed $N(X_1:\mu_1,\Sigma_{11})$.

<u>Theorem</u> <u>A.2</u>

Let the px1 random vector X be distributed $N(x:\mu,\Sigma)$, where Σ has rank p, let B be any q x p matrix of constants, and let b be any q x 1 vector of constants. Then the q x 1 vector Y defined by

$Y = BX + b$ is distributed $N(y:B\mu + b, B\Sigma B')$

Theorem A.3

Let the px1 vector X be distributed $N(x:\mu,\Sigma)$, where Σ has rank p. Let X, μ, Σ be partitioned as in (A.1), where X_1 has size qx1, where $0 < q < p$. The conditional distribution of X_1 given $X_2 = C_2$, where C_2 is a vector of constants, is normal with mean $\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(C_2-\mu_2)$ and covariance matrix $\Sigma_{11\cdot 2}$, where $\Sigma_{11\cdot 2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$.

Definition A.4

Let the (p+1)x1 random vector X' have a multivariate normal distribution with mean μ' and covariance matrix Σ'. The simple correlation coefficient of any two random variables X_i and X_j in X' is denoted by β_{ij} and defined by

$$\beta_{ij} = \frac{\mathrm{Cov}(X_i, X_j)}{\sqrt{\mathrm{Var}(X_i)\mathrm{Var}(X_j)}} = \frac{\sigma_{ij}}{\sqrt{(\sigma_{ii}\sigma_{jj})}}$$

$$i = 0,\ldots, p \qquad j = 0,\ldots, p \qquad i \neq j$$

if $\sigma_{ii} > 0$ and $\sigma_{jj} > 0$.

If $\sigma_{ii} = 0$ or $\sigma_{jj} = 0$, β_{ij} is not defined.

The single variable transformation theorem is taken from a text by Mood, et al. (1974).

Theorem A.5

Suppose X is a continuous random variable with probability density function $f_x(\cdot)$. Set $\mathfrak{X} = \{x : f_X(x) > 0\}$. Assume that:

(i) $y=g(x)$ defines a one-to-one transformation of $\mathfrak{X}$ onto $\mathfrak{Y}$.

(ii) The derivative of $x = g^{-1}(y)$ with respect to y is continuous and nonzero for $y \in \mathfrak{Y}$, where $g^{-1}(y)$ is the inverse function of $g(x)$; that is $g^{-1}(y)$ is that x for which $g(x) = y$. Then $Y = g(X)$ is a continuous random function variable with density

$$f_Y(y) = \left| \frac{d}{dy} g^{-1}(y) \right| f_X\left[g^{-1}(y)\right]$$

APPENDIX B

SOME MATHEMATICAL DERIVATIONS RELATED TO THE OPA Y CHART

SOME MATHEMATICAL DERIVATIONS
RELATED TO THE OPA Y CHART

Mean and Asymptotic Standard Deviation of Y_t

It is assumed that the input random variable, X_t, is independent and identically distributed. The underlying distribution is a normal distribution with mean, μ, and standard deviation, σ. The $(n+1) \times 1$ random vector of X can be easily formed by grouping the first (n+1) of the X's. Using linear model theory, the random vector of X is distributed as a multivariate normal with mean vector, $\mu 1_{n+1}$, and covariance matrix, $\sigma^2 I_{n+1}$. That is,

$$X = \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \sim N_{n+1}\left[\mu 1_{n+1},\ \sigma^2 I_{n+1}\right]$$

Recall that the first order response equation is

$$Y_t = rY_{t-1} + (1-r)X_t \tag{B.1}$$

Substituting

$$Y_{t-1} = rY_{t-2} + (1-r)X_{t-1} \tag{B.2}$$

into Equation (B.1) results in

$$Y_t = r[rY_{t-2} + (1-r)X_{t-1}] + (1-r)X_t$$

$$Y_t = r^2Y_{t-2} + (1-r)rX_{t-1} + (1-r)X_t$$

Continuously substituting the Y term on the right hand side of the equation, results in

$$Y_t = (1-r)r^tX_0 + (1-r)r^{t-1}X_1 + (1-r)r^{t-2}X_2 + \dots + (1-r)X_t$$

$$Y_t = \sum_{k=0}^{t} (1-r)r^k X_{t-k} \tag{B.3}$$

Thus, Y_t is a linear combination of $(X_1, X_2, \ldots, X_n)'$.

Letting

$$Z_1 = Y_t = (1-r)r^t X_0 + (1-r)r^{t-1}X_1 + \ldots + (1-r)X_t$$

$$Z_2 = X_t = 0 + 0 + \ldots + X_t$$

$$Z_3 = Y_{t-1} = (1-r)r^{t-1}X_0 + (1-r)r^{t-2}X_1 + \ldots + 0X_t$$

These three equations can be written as

$$Z = AX$$

where

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}$$

$$A = \begin{bmatrix} (1-r)r^t & (1-r)r^{t-1} & \ldots & (1-r)r & (1-r) \\ 0 & 0 & \ldots & 0 & 1 \\ (1-r)r^{t-1} & (1-r)r^{t-2} & \ldots & (1-r) & 0 \end{bmatrix}$$

The dimension of matrix A is 3 x (t+1). Using Theorem A.2, the distribution of Z is

$$Z = AX \sim N_3(\mu A 1_{t+1}, \sigma^2 A I_{t+1} A')$$

$$\text{Mean of } Z = \mu \begin{bmatrix} (1-r)r^t & (1-r)r^{t-1} & \ldots & (1-r)r & (1-r) \\ 0 & 0 & \ldots & 0 & 1 \\ (1-r)r^{t-1} & (1-r)r^{t-2} & \ldots & (1-r) & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$= \mu \begin{bmatrix} \sum_{k=0}^{t} (1-r)r^k \\ 1 \\ \sum_{k=0}^{t-1} (1-r)r^k \end{bmatrix}$$

For large t,

$$\sum_{k=0}^{t} (1-r)r^k = (1-r)\frac{1}{(1-r)} = 1$$

and

$$\sum_{k=0}^{t-1} (1-r)r^k = (1-r)\frac{1}{(1-r)} = 1$$

Therefore, for large t,

$$\text{Mean of } Z = \mu \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The covariance matrix of $Z = \sigma^2 A I_{t+1} A' = \sigma^2 AA'$

$$= \sigma^2 \begin{bmatrix} (1-r)r^t & (1-r)r^{t-1} & \cdots & (1-r)r & (1-r) \\ 0 & 0 & \cdots & 0 & 1 \\ (1-r)r^{t-1} & (1-r)r^{t-2} & \cdots & (1-r) & 0 \end{bmatrix} \times$$

$$\begin{bmatrix} (1-r)r^t & 0 & (1-r)r^{t-1} \\ (1-r)r^{t-1} & 0 & (1-r)r^{t-2} \\ \vdots & \vdots & \vdots \\ (1-r)r & 0 & (1-r) \\ (1-r) & 1 & 0 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \sum_{k=0}^{t} (1-r)^2 r^{2k} & (1-r) & \sum_{k=1}^{t} (1-r)^2 r^{2k-1} \\ (1-r) & 1 & 0 \\ \sum_{k=1}^{t} (1-r)^2 r^{2k-1} & 0 & \sum_{k=0}^{t-1} (1-r)^2 r^{2k} \end{bmatrix}$$

For large t

$$\sum_{k=0}^{t} (1-r)^2 r^{2k} = (1-r)^2 \frac{1}{1 - r^2} = (1-r)/(1+r)$$

$$\sum_{k=0}^{t-1} (1-r)^2 r^{2k} = (1-r)^2 \frac{1}{1 - r^2} = (1-r)/(1+r)$$

$$\sum_{k=1}^{t} (1-r)^2 r^{2k-1} = (1-r)^2 r \sum_{k=1}^{t} r^{2k-2} = \frac{(1-r)^2 r}{1 - r^2} = \frac{(1-r)r}{(1+r)}$$

For large t, the covariance matrix of Z

$$= \quad \sigma^2 \begin{bmatrix} (1-r)/(1+r) & (1-r) & (1-r)r/(1+r) \\ (1-r) & 1 & 0 \\ (1-r)r/(1+r) & 0 & (1-r)/(1+r) \end{bmatrix}$$

Thus, Z is asymptotically distributed as a multivariate normal with mean vector equal to $(\mu\ \mu\ \mu)'$ and covariance matrix equal to

$$\sigma^2 \begin{bmatrix} (1-r)/(1+r) & (1-r) & (1-r)r/(1+r) \\ (1-r) & 1 & 0 \\ (1-r)r/(1+r) & 0 & (1-r)/(1+r) \end{bmatrix}$$

Using Theorem A.1, it is clear that the distribution of Y_t is

$$Y_t \sim \text{Normal}\left[\mu,\ \sigma^2(1-r)/(1+r)\right] \tag{B.4}$$

Assume that Y_{t-1} is known and it takes on the value k. Then, from Theorem A.3, the joint distribution of Y_t and X_t given Y_{t-1} equals k, is a bivariate normal with mean equal to

$$\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(C_2-\mu_2)$$

$$= \begin{bmatrix} \mu \\ \mu \end{bmatrix} + \sigma^2 \begin{bmatrix} (1-r)r/(1+r) \\ 0 \end{bmatrix} \frac{(1+r)}{(1-r)\sigma^2} (k-\mu)$$

$$= \begin{bmatrix} \mu + r(k-\mu) \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} rk + (1-r)\mu \\ \mu \end{bmatrix}$$

and covariance matrix equal to

$$\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

$$= \sigma^2 \begin{bmatrix} \dfrac{(1-r)}{(1+r)} & (1-r) \\ (1-r) & 1 \end{bmatrix} - \sigma^2 \begin{bmatrix} \dfrac{(1-r)r}{(1+r)} \\ 0 \end{bmatrix} \frac{(1+r)}{(1-r)} \begin{bmatrix} \dfrac{(1-r)r}{(1+r)} & 0 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \dfrac{(1-r)}{(1+r)} & (1-r) \\ (1-r) & 1 \end{bmatrix} - \sigma^2 \begin{bmatrix} \dfrac{(1-r)r^2}{(1+r)} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} (1-r)^2 & (1-r) \\ (1-r) & 1 \end{bmatrix}$$

Using Theorem A.1, the conditional distribution of Y_t is

$$Y_t \mid (Y_{t-1}=k) \sim \text{Normal}\left[rk+(1-r)\mu,\ (1-r)^2\sigma^2\right] \qquad \text{(B.5)}$$

APPENDIX C

SOME MATHEMATICAL DERIVATIONS RELATED TO THE OPA MRy CHART

SOME MATHEMATICAL DERIVATIONS RELATED TO THE OPA MRy CHART

Conditional Distribution of the Moving Range of Subgroup Size Two of the Y Variate

Recall that the first order response process that generates serially correlated output, Y, is

$$Y_t = rY_{t-1} + (1-r)X_t \tag{C.1}$$

In Appendix B, it is known that X_t is independent of Y_{t-1}. If Y_{t-1} is known and takes on the value k, then

$$Y_t = rk + (1-r)X_t \tag{C.2}$$

Defining $R|k$ as the range of Y_{t-1} and Y_t given that Y_{t-1} equals k, then

$$R|k = \text{Range of } k \text{ and } Y_t$$

$$R|k = |Y_t - k| \tag{C.3}$$

Substituting (C.1) into (C.3) results

$$R|k = |\ rk + (1-r)X_t - k\ |$$

$$R|k = |\ (1-r)X_t - (1-r)k\ |$$

$$R|k = |\ (1-r)(X_t - k)\ | \tag{C.4}$$

In a first order response process, it is found that $0 < r < 1$. Then,

$$\frac{R|k}{(1-r)} = |\ (X_t - k)\ | \tag{C.5}$$

Letting Q equal Equation (C.5) results in

$$Q = \frac{R|k}{(1-r)} = |\ (X_t - k)\ | \tag{C.6}$$

The distribution of Q is then considered. From Equation (C.6), it is found that X_t is a double valued function of Q, say Q' and Q", (Basnet and Case, 1990). That is,

$$Q' = k - Q \tag{C.7a}$$
$$Q'' = k + Q \tag{C.7b}$$

Then, the density function of Q, h(Q), is given by

$$h(Q) = \left| \frac{\delta Q'}{\delta Q} \right| f(k-Q) + \left| \frac{\delta Q''}{\delta Q} \right| f(k+Q)$$

$$h(Q) = f(k-Q) + f(k+Q) \tag{C.8}$$

where $f(\cdot)$ is the probability density of X_t which appears before in Equation (C.1).

Considering Equation (C.6),

$$Q = \frac{R|k}{(1-r)}$$

Taking the derivative with respect to $R|k$, and considering the absolute value, results in

$$|J| = \left| \frac{dQ}{d(R|k)} \right| = 1/(1-r) \tag{C.9}$$

Using Theorem A.5, the density function of $R|k$ is

$$g(R|k) = h\left[\frac{R|k}{(1-r)}\right] |J|$$

Thus,

$$g(R|k) = \frac{1}{(1-r)} \left[f\left[k - \frac{R|k}{(1-r)}\right] + f\left[k + \frac{R|k}{(1-r)}\right] \right] \tag{C.10}$$

Mean and Standard Deviation of R|k

With Equation (C.8), the expected value of Q, E(Q), is found to be

$$E(Q) = \int_0^\infty Q \cdot h(Q)\, dQ \tag{C.11}$$

Substituting Equation (C.8) into (C.11) results,

$$E(Q) = \int_0^\infty Q\, f(k-Q)\, dQ + \int_0^\infty Q\, f(k+Q)\, dQ \tag{C.12}$$

Considering the first component of Equation (C.12) and substituting f(k−Q) with the normal random variable equation results in

$$\int_0^\infty Q \frac{1}{\sqrt{(2\pi)}\ \sigma} \operatorname{Exp}\left[\frac{-1}{2\sigma^2}(k-Q-\mu)^2\right] dQ \tag{C.13}$$

Substituting $y = k - \mu$, and $Q = Q - y + y$ into (C.13) results in

$$\int_0^\infty (Q - y + y) \frac{1}{\sqrt{(2\pi)}\ \sigma} \operatorname{Exp}\left[\frac{-1}{2\sigma^2}(y-Q)^2\right] dQ$$

$$= \int_0^\infty (Q - y) \frac{1}{\sqrt{(2\pi)}\ \sigma} \operatorname{Exp}\left[\frac{-1}{2\sigma^2}(Q-y)^2\right] dQ$$

$$+ \int_0^\infty y \frac{1}{\sqrt{(2\pi)}\ \sigma} \operatorname{Exp}\left[\frac{-1}{2\sigma^2}(Q-y)^2\right] dQ \tag{C.14}$$

The second part is equal to y multiplied by the area from $[0,\infty)$ under a normal curve which has mean y and variance σ^2. Letting this quantity be denoted as $y*P[X > 0|\mu=y, \sigma=\sigma]$. In addition to that, by integrating the first part of (C.14) results in

$$\int_0^\infty (Q - y + y)\frac{1}{\sqrt{(2\pi)}\ \sigma}\ \mathrm{Exp}\left[\frac{-1}{2\sigma^2}(y-Q)^2\right] dQ$$

$$= \frac{\sigma}{\sqrt{(2\pi)}}\ \mathrm{Exp}\left[\frac{-1}{2\sigma^2}\ y^2\right] + y\ P[X > 0|\mu=y,\sigma=\sigma] \qquad (C.15)$$

The second component of Equation (C.12) can also be simplified by substituting $z = \mu - k$ and $Q = Q - z + z$. After performing similar steps, the second component of Equation (C.12) becomes

$$\frac{\sigma}{\sqrt{(2\pi)}}\ \mathrm{Exp}\left[\frac{-1}{2\sigma^2}\ z^2\right] + z\ P[X > 0|\mu=z,\sigma=\sigma] \qquad (C.16)$$

Substituting (C.15) and (C.16) into (C.12) after standardizing $P[X > 0|\mu=y,\sigma=\sigma]$ and $P[X > 0|\mu=z,\sigma=\sigma]$ results in

$$E(Q) = \frac{\sigma}{\sqrt{(2\pi)}}\left[\mathrm{Exp}\left[\frac{-y^2}{2\sigma^2}\right] + \mathrm{Exp}\left[\frac{-z^2}{2\sigma^2}\right]\right] + y\ P[Z > (-y/\sigma)]$$

$$+ z\ P[Z > (-z/\sigma)] \qquad (C.17)$$

where Z is a standard normal variate.

Replacing $y = k - \mu$ and $z = \mu - k$ back into (C.17) results in

$$E(Q) = \frac{\sigma}{\sqrt{(2\pi)}} \, 2 \, \text{Exp}\left[\frac{-(k-\mu)^2}{2\sigma^2}\right] + (k-\mu) \, P[Z > -(k-\mu)/\sigma]$$

$$- (k-\mu) \, P[Z > (k-\mu)/\sigma]$$

It is known that

$$P[Z > -(k-\mu)/\sigma] = \Phi[(k-\mu)/\sigma]$$

$$P[Z > (k-\mu)/\sigma] = \Phi[(\mu-k)/\sigma]$$

Therefore,

$$E(Q) = \frac{2\sigma}{\sqrt{(2\pi)}} \, \text{Exp}\left[\frac{-(k-\mu)^2}{2\sigma^2}\right] + (k-\mu)\left[\Phi[(k-\mu)/\sigma] - \Phi[(\mu-k)/\sigma]\right]$$

Due to the symmetric properties of normal random variate, it can be shown that

$$(k-\mu)\left[\Phi[(k-\mu)/\sigma] - \Phi[(\mu-k)/\sigma]\right]$$

$$= |k-\mu| * \left[1 - 2\,\Phi(-|k-\mu|/\sigma)\right]$$

where $|a|$ = absolute value of a. Hence,

$$E(Q) = 2\sigma \, \Phi'\left[\frac{k-\mu}{\sigma}\right] + |k-\mu| \left[1 - 2\,\Phi(-|k-\mu|/\sigma)\right] \qquad \text{(C.18)}$$

where

$\Phi'(a)$ is the ordinate value of standard normal at point a.

The variance of Q, V(Q), is

$$V(Q) = E(Q^2) - [E(Q)]^2 \qquad \text{(C.19)}$$

where

$$E(Q^2) = \int_0^\infty Q^2 \cdot h(Q)\, dQ \qquad (C.20)$$

Substituting Equation (C.8) into (C.20) results in

$$E(Q^2) = \int_0^\infty Q^2 f(k-Q)\, dQ + \int_0^\infty Q^2 f(k+Q)\, dQ \qquad (C.21)$$

Considering the first component of Equation (C.21) and substituting f(k−Q) with the normal random variable equation results in

$$\int_0^\infty Q^2 \frac{1}{\sqrt{(2\pi)}\ \sigma} \text{Exp} \left[\frac{-1}{2\sigma^2} (k-Q-\mu)^2 \right] dQ \qquad (C.22)$$

Substituting $y = k - \mu$, and $Q^2 = (Q - y + y)Q$ into (C.22) results in

$$\int_0^\infty (Q - y + y)Q \frac{1}{\sqrt{(2\pi)}\ \sigma} \text{Exp} \left[\frac{-1}{2\sigma^2} (y-Q)^2 \right] dQ$$

$$= \int_0^\infty (Q - y)Q \frac{1}{\sqrt{(2\pi)}\ \sigma} \text{Exp} \left[\frac{-1}{2\sigma^2} (Q-y)^2 \right] dQ$$

$$+ \int_0^\infty y\ Q \frac{1}{\sqrt{(2\pi)}\ \sigma} \text{Exp} \left[\frac{-1}{2\sigma^2} (Q-y)^2 \right] dQ \qquad (C.23)$$

The first part can be evaluated using integration by parts. It is found that

$$\int_0^{\infty} (Q - y)Q \frac{1}{\sqrt{(2\pi)}\ \sigma} \mathrm{Exp}\left[\frac{-1}{2\sigma^2} (Q-y)^2\right] dQ$$

$$= \sigma^2 \int_0^{\infty} \frac{1}{\sqrt{(2\pi)}\ \sigma} \mathrm{Exp}\left[\frac{-1}{2\sigma^2} (Q-y)^2\right] dQ$$

$$= \sigma^2 P[X > 0 | \mu=y, \sigma=\sigma]$$

$$= \sigma^2 P[Z > -y/\sigma]$$

Substituting $Q = Q - y + y$ into the second part of (C.23) results in

$$y \int_0^{\infty} (Q - y + y) \frac{1}{\sqrt{(2\pi)}\ \sigma} \mathrm{Exp}\left[\frac{-1}{2\sigma^2} (Q-y)^2\right] dQ$$

$$= y \int_0^{\infty} (Q - y) \frac{1}{\sqrt{(2\pi)}\ \sigma} \mathrm{Exp}\left[\frac{-1}{2\sigma^2} (Q-y)^2\right] dQ$$

$$+ \int_0^{\infty} y^2 \frac{1}{\sqrt{(2\pi)}\ \sigma} \mathrm{Exp}\left[\frac{-1}{2\sigma^2} (Q-y)^2\right] dQ$$

$$= \frac{y}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \mathrm{Exp}\left[\frac{-y^2}{2\sigma^2}\right] + y^2 P[X > 0 | \mu=y, \sigma=\sigma]$$

$$= \frac{y}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \mathrm{Exp}\left[\frac{-y^2}{2\sigma^2}\right] + y^2 P[Z > -y/\sigma]$$

Thus, (C.23) becomes

$$\int_0^{\infty} (Q - y + y)Q \frac{1}{\sqrt{(2\pi)}\ \sigma} \text{Exp}\left[\frac{-1}{2\sigma^2}(y-Q)^2\right] dQ$$

$$= \sigma^2 P[Z > -y/\sigma] + \frac{y}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \text{Exp}\left[\frac{-y^2}{2\sigma^2}\right] + y^2 P[Z > -y/\sigma]$$

$$= (\sigma^2+y^2)P[Z > -y/\sigma] + \frac{y}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \text{Exp}\left[\frac{-y^2}{2\sigma^2}\right] \qquad \text{(C.24)}$$

The second component of Equation (C.21) can also be simplified by substituting $z = \mu - k$ and $Q^2 = (Q - z + z)Q$ and $Q = Q - z + z$. After performing similar steps, the second component of Equation (C.21) becomes

$$(\sigma^2+z^2)P[Z > -z/\sigma] + \frac{z}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \text{Exp}\left[\frac{-z^2}{2\sigma^2}\right] \qquad \text{(C.25)}$$

Substituting (C.24) and (C.25) into (C.21) results in

$$E(Q^2) = (\sigma^2+y^2)P[Z > -y/\sigma] + \frac{y}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \text{Exp}\left[\frac{-y^2}{2\sigma^2}\right]$$

$$+ (\sigma^2+z^2)P[Z > -z/\sigma] + \frac{z}{\sqrt{(2\pi)}\ \sigma} \sigma^2 \text{Exp}\left[\frac{-z^2}{2\sigma^2}\right] \qquad \text{(C.26)}$$

Replacing $y = k - \mu$ and $z = \mu - k$ back into (C.26), and realizing that $y = - z$ results,

$$E(Q^2) = \left[\sigma^2 + (k-\mu)^2\right]\left[P[Z > (-y/\sigma)] + P[Z > (y/\sigma)]\right]$$

Due to symmetry of the normal curve,

$P[Z > (-y/\sigma)] + P[Z > (y/\sigma)] = 1$

Therefore,

$$E(Q^2) = \sigma^2 + (k-\mu)^2 \qquad (C.27)$$

Hence,

$$V(Q) = \left[\sigma^2 + (k-\mu)^2\right] - \{E(Q)\}^2 \qquad (C.28)$$

From Equation (C.6), it is clear that

$$R|k = (1-r)Q \qquad (C.29)$$

Then, the expected value of $R|k$, $E(R|k)$, and the variance of $R|k$, $V(R|k)$ are

$$E(R|k) = (1-r)E(Q) \qquad (C.30)$$

$$V(R|K) = (1-r)^2V(Q) \qquad (C.31)$$

Thus,

$$E(R|k) = (1-r)\left[2\sigma\Phi'\left[\frac{k-\mu}{\sigma}\right] + |k-\mu|\left[1- 2\Phi\left[\frac{-|k-\mu|}{\sigma}\right]\right]\right] \qquad (C.32)$$

$$V(R|k) = (1-r)^2\left[\sigma^2 + (k-\mu)^2 - [E(Q)]^2\right] \qquad (C.33)$$

$$Std(R|k) = (1-r)\left[\sigma^2 + (k-\mu)^2 - [E(Q)]^2\right]^{1/2} \qquad (C.34)$$

APPENDIX D

EXAMPLE OF A SAS PROGRAM

AND SAS OUTPUT

```
**** TSO FOREGROUND HARDCOPY ****
DSNAME=U11563A.EXAMPLE.DATA

//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                        PROGRAM 1
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED  = 12345;
   FC    = 0.3;                          /* FILTER CONSTANT */
   FC1   = 1 - FC;
   STD   = 1;                            /* INITIAL STD. DEV. OF X */
   MEAN  = 10;                           /* INITIAL MEAN OF X */
   SHIFT = 2.0;                          /* SHIFT IN TERM OF SIGMA X */
   NMEAN = MEAN + SHIFT*STD;             /* NEW MEAN OF X */
   NSIM  = 10000;                        /* NUMBER OF SIMULATION */
   QQUY  = MEAN + 3*FC1*STD;             /* CONSTANT TERM OF UCLOPAY */
   QQLY  = MEAN - 3*FC1*STD;             /* CONSTANT TERM OF LCLOPAY */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;      /* GENERATE INITIAL VALUE */
      DO K = 1 TO 50;                    /* WARM UP FORP GENERATOR */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM   = 0;
LAB:  AO    = Y1 - MEAN;
      UCLY  = QQUY + FC*AO;              /* UCL OF OPA Y CHART */
      LCLY  = QQLY + FC*AO;              /* LCL OF OPA Y CHART */
      Y2    = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLY < Y2 < UCLY THEN DO;       /* POINT IS WITHIN CL'S */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                     /* AVERAGE RUN LENGTH */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                              /* STATISTICS OF ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3. ';
   TITLE3 'SHIFT IN MEAN = 2.0 SIGMA';
//
```

ARL OF THE OPA Y CHART ON FORP DATA
WITH FILTER CONSTANT, FC = 0.3.
SHIFT IN MEAN = 2.0 SIGMA

Analysis Variable : ARL

N	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis
10000	6.1857000	5.6497440	1.0000000	52.0000000	2.0119625	5.9831888

APPENDIX E

LISTINGS OF SAS PROGRAM USED TO DETERMINE THE ARL OF CONTROL CHARTS

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 1
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter Constant */
   FC1    = 1 - FC;
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   SHIFT  = 2.0;                          /* Shift in term of sigma x */
   NMEAN  = MEAN + SHIFT*STD;             /* New mean of x */
   NSIM   = 10000;                        /* Number of Simulation */
   QQUY   = MEAN + 3*FC1*STD;             /* Constant term of UCLopay */
   QQLY   = MEAN - 3*FC1*STD;             /* Constant term of LCLopay */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
LAB:  A0     = Y1 - MEAN;
      UCLY   = QQUY + FC*A0;              /* UCL of OPA Y chart */
      LCLY   = QQLY + FC*A0;              /* LCL of OPA Y chart */
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLY < Y2 < UCLY THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3. ';
   TITLE3 'SHIFT IN MEAN = 2.0 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 2
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED  = 12345;
   FC    = 0.3;                           /* Filter Constant */
   FC1   = 1 - FC;
   STD   = 1;                             /* Initial Std. Dev. of X */
   MEAN  = 10;                            /* Initial Mean of X */
   AMP   = 3*STD;                         /* Amplitude of trend */
   STPSZ = AMP/20;                        /* Step = Amp/20 subgps */
   QQUY  = MEAN + 3*FC1*STD;              /* Constant term of UCLopay */
   QQLY  = MEAN - 3*FC1*STD;              /* Constant term of LCLopay */
   NSIM  = 10000;                         /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM   = 0;
      PER   = 0;                          /* Period = 0 */
LAB:  A0    = Y1 - MEAN;
      UCLY  = QQUY + FC*A0;               /* UCL of OPA Y chart */
      LCLY  = QQLY + FC*A0;               /* LCL of OPA Y chart */
      NMEAN = MEAN+MIN(STPSZ*PER,AMP); /* New Mean of X, trend */
      Y2    = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLY < Y2 < UCLY THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + 1;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN TREND: TREND PERIOD = 20 SUBGROUPS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 3
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter Constant */
   FC1    = 1 - FC;
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   STPSZ  = (22/7)/24;                    /* Step = 2*PI/48 subgps */
   AMP    = 3*STD;                        /* Amplitude of cycle */
   QQUY   = MEAN + 3*FC1*STD;             /* Constant term of UCLopay */
   QQLY   = MEAN - 3*FC1*STD;             /* Constant term of LCLopay */
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
      PER    = 0;                         /* Period = 0 */
LAB:  A0     = Y1 - MEAN;
      UCLY   = QQUY + FC*A0;              /* UCL of OPA Y chart */
      LCLY   = QQLY + FC*A0;              /* LCL of OPA Y chart */
      NMEAN  = MEAN + AMP*SIN(PER);       /* New Mean of X, cyclical */
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLY < Y2 < UCLY THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + STPSZ;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN CYCLE; CYCLE PERIOD = 48 SUBGROUPS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 4
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter Constant */
   FC1    = 1 - FC;
   FC2    = 1 + FC;
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   SHIFT  = 2.0;                          /* Shift in term of sigma X */
   NMEAN  = MEAN + SHIFT*STD;             /* New Mean of x */
   MRBAR  = 1.128*FC1*STD/SQRT(FC2);      /* Theo. MRBAR */
   LCLI   = MEAN - 2.66*MRBAR;            /* LCL of I chart */
   UCLI   = MEAN + 2.66*MRBAR;            /* UCL of I chart */
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD+MEAN;         /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM = 0;
LAB:  Y2  = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLI < Y2 < UCLI THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE TRADITIONAL I CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3';
   TITLE3 'SHIFT IN MEAN = 2.0 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 5
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                        /* Filter Constant */
   FC1    = 1 - FC;
   FC2    = 1 + FC;
   STD    = 1;                          /* Initial Std. Dev. of X */
   MEAN   = 10;                         /* Initial Mean of X */
   AMP    = 3*STD;                      /* Amplitude of trend */
   STPSZ  = AMP/20;                     /* Step = Amp/20 subgps */
   MRBAR  = 1.128*FC1*STD/SQRT(FC2);    /* Theo. Std. Dev. of Y */
   LCLI   = MEAN - 2.66*MRBAR;          /* LCL of I chart */
   UCLI   = MEAN + 2.66*MRBAR;          /* UCL of I chart */
   NSIM   = 10000;                      /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;     /* Generate initial value */
      DO K = 1 TO 50;                   /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
      PER    = 0;                       /* Period = 0 */
LAB:  NMEAN = MEAN+MIN(STPSZ*PER,AMP); /* New Mean of X, trend */
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLI < Y2 < UCLI THEN DO;      /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + 1;
         GO TO LAB;
      END;
      ARL = NUM + 1;                    /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                             /* Statistics of ARL */
   TITLE1 'ARL OF THE TRADITIONAL I CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN TREND: TREND PERIOD = 20 SUBGROUPS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                              PROGRAM 6
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter Constant */
   FC1    = 1 - FC;
   FC2    = 1 + FC;
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   STPSZ  = (22/7)/24;                    /* Step = 2*PI/48 subgps */
   AMP    = 3*STD;                        /* Amplitude of cycle */
   MRBAR  = 1.128*FC1*STD/SQRT(FC2);      /* Theo. MRBAR */
   LCLI   = MEAN - 2.66*MRBAR;            /* LCL of I chart */
   UCLI   = MEAN + 2.66*MRBAR;            /* UCL of I chart */
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
      PER    = 0;                         /* Period = 0 */
LAB:  NMEAN = MEAN + AMP*SIN(PER);        /* New Mean of X, cyclical */
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLI < Y2 < UCLI THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + STPSZ;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE TRADITIONAL I CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN CYCLE: CYCLE PERIOD = 48 SUBGROUPS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                        PROGRAM 7
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter Constant */
   FC1    = 1 - FC;
   B1     = 1/SQRT(2*22/7);               /* 1/SQRT(2*pi) */
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   SHIFT  = 3.0;                          /* Sigmanew over Sigmaold */
   NSTD   = SHIFT*STD;
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate 1st value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
LAB:  A1     = ABS(Y1 - MEAN);
      A2     = A1 / STD;
      B2     = B1 * EXP(-0.5*A2*A2);      /* Ordinate of Std. Normal */
      B3     = 1 - 2*PROBNORM(-A2);       /* PROBNORM = CDF of Normal */
      ERK    = 2*STD*B2 + A1* B3;         /* Expected of R given K */
      ER     = FC1 * ERK;                 /* Exp. of Range given K */
      B4     = SQRT(STD*STD + A1*A1 - ERK*ERK);
      SR     = FC1 * B4;                  /* Std. of Range given K */
      LCL    = ER - 3*SR;                 /* LCL of OPA MRy chart */
      UCL    = ER + 3*SR;                 /* UCL of OPA MRy chart */
      IF LCL < 0 THEN LCL = 0;
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*NSTD + MEAN);
      RY     = ABS(Y2-Y1);
      IF LCL < RY < UCL THEN DO;          /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA MRy CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3';
   TITLE3 'SHIFT RATIO IN DISPERSION = 3.0';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 8
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED  = 12345;
   FC    = 0.3;                        /* Filter Constant */
   FC1   = 1 - FC;
   B1    = 1/SQRT(2*22/7);             /* 1/SQRT(2*pi) */
   STD   = 1;                          /* Initial Std. Dev. of X */
   MEAN  = 10;                         /* Initial Mean of X */
   AMP   = 3*STD;                      /* Amplitude of trend */
   STPSZ = AMP/20;                     /* Step = Amp/20 subgps */
   NSIM  = 10000;                      /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;    /* Generate initial value */
      DO K = 1 TO 50;                  /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM = 0;
      PER = 0;                         /* Period = 0 */
LAB:  A1  = ABS(Y1 - MEAN);
      A2  = A1/ STD;
      B2  = B1 * EXP(-0.5*A2*A2);      /* Ordinate of Std. Normal */
      B3  = 1 - 2*PROBNORM(-A2);       /* PROBNORM = CDF of Normal */
      ERK = 2*STD*B2 + A1*B3;          /* Expected of R given K */
      ER  = FC1 * ERK;                 /* Exp. of Range given K */
      B4  = SQRT(STD*STD + A1*A1 - ERK*ERK);
      SR  = FC1 * B4;                  /* Std. of Range given K */
      LCL = ER - 3*SR;                 /* LCL of OPA MRy chart */
      UCL = ER + 3*SR;                 /* UCL of OPA MRy chart */
      IF LCL < 0 THEN LCL = 0;
      NMEAN = MEAN+MIN(STPSZ*PER,AMP); /* New Mean of X, trend */
      Y2    = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      MRY   = ABS(Y2-Y1);              /* Moving Range */
      IF LCL < MRY < UCL THEN DO;      /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + 1;
         GO TO LAB;
      END;
      ARL = NUM + 1;                   /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                            /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA MRy CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3. ';
   TITLE3 'MEAN VARIES IN TREND: TREND PERIOD = 20 SUBGROUPS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                           PROGRAM 9
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                         /* Filter Constant */
   FC1    = 1 - FC;
   B1     = 1/SQRT(2*22/7);              /* 1/SQRT(2*pi) */
   STD    = 1;                           /* Initial Std. Dev. of X */
   MEAN   = 10;                          /* Initial Mean of X */
   STPSZ  = (22/7)/24;                   /* Step = 2*PI/48 subgps */
   AMP    = 3*STD;                       /* Amplitude of Simulation */
   NSIM   = 10000;                       /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;      /* Generate initial value */
      DO K = 1 TO 50;                    /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM = 0;
      PER = 0;                           /* Period = 0 */
LAB:  A1   = ABS(Y1 - MEAN);
      A2   = A1/ STD;
      B2   = B1 * EXP(-0.5*A2*A2);       /* Ordinate of Std. Normal */
      B3   = 1 - 2*PROBNORM(-A2);        /* PROBNORM = CDF of Normal */
      ERK  = 2*STD*B2 + A1*B3;           /* Expected of R given K */
      ER   = FC1 * ERK;                  /* Exp. of Range given K */
      B4   = SQRT(STD*STD + A1*A1 - ERK*ERK);
      SR   = FC1 * B4;                   /* Std. of Range given K */
      LCL  = ER - 3*SR;                  /* LCL of OPA MRy chart */
      UCL  = ER + 3*SR;                  /* UCL of OPA MRy chart */
      IF LCL < 0 THEN LCL = 0;
      NMEAN = MEAN + AMP*SIN(PER);       /* New Mean of X, cyclical */
      Y2   = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      RY   = ABS(Y2-Y1);                 /* Moving Range */
      IF LCL < RY < UCL THEN DO;         /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + STPSZ;
         GO TO LAB;
      END;
      ARL = NUM + 1;                     /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                              /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA MRy CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN CYCLE: CYCLE PERIOD = 48 SUBGOUPRS';
   TITLE4 'AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 10
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter Constant */
   FC1    = 1 - FC;
   FC2    = 1 + FC;
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   SHIFT  = 1.0;                          /* Sigmanew over Sigmaold */
   NSTD   = SHIFT*STD;
   MRBAR  = STD*1.128*FC1/SQRT(FC2);      /* Theoretical MRbar */
   LCLMR  = 0.0;
   UCLMR  = 3.267*MRBAR;                  /* D4*MRbar = MR chart UCL */
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD+MEAN;         /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM = 0;
LAB:  Y2  = FC*Y1 + FC1*(RANNOR(SEED)*NSTD + MEAN);
      MR  = ABS(Y2-Y1);                   /* Moving Range */
      IF LCLMR < MR < UCLMR THEN DO;      /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE TRADITIONAL MR(2) CHART ON FORP DATA';
   TITLE2 'FILTER CONSTANT, FC = 0.3';
   TITLE3 'SHIFT RATIO IN DISPERSION = 1.0';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 11
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED  = 12345;
   FC    = 0.3;                          /* Filter Constant */
   FC1   = 1 - FC;
   FC2   = 1 + FC;
   STD   = 1;                            /* Initial Std. Dev. of X */
   MEAN  = 10;                           /* Initial Mean of X */
   AMP   = 3*STD;                        /* Amplitude of trend */
   STPSZ = AMP/20;                       /* Step = Amp/20 subgps */
   MRBAR = STD*1.128*FC1/SQRT(FC2);      /* Theoretical MRbar */
   LCLMR = 0.0;
   UCLMR = 3.267*MRBAR;                  /* D4*MRbar = MR chart UCL */
   NSIM  = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;      /* Generate initial value */
      DO K = 1 TO 50;                    /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM   = 0;
      PER   = 0;                         /* Period = 0 */
LAB:  NMEAN = MEAN+MIN(STPSZ*PER,AMP); /* New Mean of X, trend */
      Y2    = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      MR    = ABS(Y2-Y1);                /* Moving Range */
      IF LCLMR < MR < UCLMR THEN DO;     /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + 1;
         GO TO LAB;
      END;
      ARL = NUM + 1;                     /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                              /* Statistics of ARL */
   TITLE1 'ARL OF THE TRADITIONAL MR(2) CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN TREND: TREND PERIOD = 20 SUBGOUPRS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 12
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL ;
   SEED  = 12345;
   FC    = 0.3;                          /* Filter Constant */
   FC1   = 1 - FC;
   FC2   = 1 + FC;
   STD   = 1;                            /* Initial Std. Dev. of X */
   MEAN  = 10;                           /* Initial Mean of X */
   STPSZ = (22/7)/24;                    /* Step = 2*PI/48 subgps */
   AMP   = 3*STD;                        /* Amplitude of Simulation */
   MRBAR = STD*1.128*FC1/SQRT(FC2);      /* Theoretical MRbar */
   LCLMR = 0.0;
   UCLMR = 3.267*MRBAR;                  /* D4*MRbar = MR chart UCL */
   NSIM  = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;      /* Generate initial value */
      DO K = 1 TO 50;                    /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM   = 0;
      PER   = 0;                         /* Period = 0 */
LAB:  NMEAN = MEAN + AMP*SIN(PER);       /* New Mean of X, cyclical */
      Y2    = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      MR    = ABS(Y2-Y1);                /* Moving Range */
      IF LCLMR < MR < UCLMR THEN DO;     /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         PER = PER + STPSZ;
         GO TO LAB;
      END;
      ARL = NUM + 1;                     /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                              /* Statistics of ARL */
   TITLE1 'ARL OF THE TRADITIONAL MR(2) CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'MEAN VARIES IN CYCLE: CYCLE PERIOD = 48 SUBGOUPRS';
   TITLE4 'AND AMPLITUDE = 3*SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                           PROGRAM 13
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter constant */
   FC1    = 1 - FC;
   B1     = 1/SQRT(2*22/7);               /* 1/SQRT(2*PI) */
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   SFSTD  = 1.0;                          /* Sigmanew over Sigmaold */
   SFMU   = 2;                            /* Mean shift in sigma X */
   NMEAN  = MEAN + SFMU*STD;
   NSTD   = SFSTD*STD;
   QQUY   = MEAN + 3*FC1*STD;
   QQLY   = MEAN - 3*FC1*STD;
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
LAB:  A1     = Y1 - MEAN;
      A2     = ABS(A1) / STD;
      B2     = B1 * EXP(-0.5*A2*A2);      /* Ordinate of Std. Normal */
      B3     = 1 - 2*PROBNORM(-A2);       /* PROBNORM = CDF of Normal */
      ERK    = 2*STD*B2 + ABS(A1)*B3;     /* Expected of R given K */
      ER     = FC1 * ERK;                 /* Exp. of Range given K */
      B4     = SQRT(STD*STD + A1*A1 - ERK*ERK);
      SR     = FC1 * B4;                  /* Std. of Range given K */
      LCLMR  = ER - 3*SR;                 /* LCL of OPA MRy Chart */
      UCLMR  = ER + 3*SR;                 /* UCL of OPA MRy Chart */
      IF LCLMR < 0 THEN LCLMR = 0;
      UCLY   = QQUY + FC*A1;              /* ULC of OPA Y Chart */
      LCLY   = QQLY + FC*A1;              /* LCL of OPA Y Chart */
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*NSTD + NMEAN);
      MRY    = ABS(Y2-Y1);
      IF (LCLMR < MRY < UCLMR) AND (LCLY < Y2 <UCLY) THEN DO;
         NUM = NUM + 1;                   /* Point is in-control */
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;
      OUTPUT;                             /* Average Run Length */
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE JOINT OPA Y AND OPA MRy CHARTS ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3.';
   TITLE3 'SHIFT IN MEAN = 2 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 14
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC1    = 0.6;                        /* Actual Filter Constant */
   FC1A   = 1 - FC1;
   FC2    = 0.5;                        /* Assumed Filter Constant */
   FC2A   = 1 - FC2;
   STD    = 1;                          /* Initial Std. Dev. of X */
   MEAN   = 10;                         /* Initial Mean of X */
   SHIFT  = 2.0;                        /* Shift in term of sigma x */
   NMEAN  = MEAN + SHIFT*STD;           /* New mean of x */
   QQUY   = MEAN + 3*FC2A*STD;          /* Constant term of UCLopay */
   QQLY   = MEAN - 3*FC2A*STD;          /* Constant term of LCLopay */
   NSIM   = 10000;                      /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;     /* Generate initial value */
      DO K = 1 TO 50;                   /* Warm up FORP generator */
         Y2 = FC1*Y1 + FC1A*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
LAB:  A0     = Y1 - MEAN;
      UCLY   = QQUY + FC2*A0;           /* UCL of OPA Y chart */
      LCLY   = QQLY + FC2*A0;           /* LCL of OPA Y chart */
      Y2     = FC1*Y1 + FC1A*(RANNOR(SEED)*STD + NMEAN);
      IF LCLY < Y2 < UCLY THEN DO;      /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                    /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                             /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON FORP DATA WITH';
   TITLE2 'ACTUAL FILTER CONSTANT, FC1 = 0.6.  ASSUMED FILTER,';
   TITLE3 'CONSTANT, FC2 = 0.5.  SHIFT IN MEAN = 2.0 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 15
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED  = 12345;
   FC1   = 0.6;                           /* Actual filter constant */
   FC1A  = 1 - FC1;
   FC2   = 0.5;                           /* Assumed filter constant */
   FC2A  = 1 - FC2;
   B1    = 1/SQRT(2*22/7);                /* 1/SQRT(2*PI) */
   STD   = 1;                             /* Initial Std. Dev. of X */
   MEAN  = 10;                            /* Initial Mean of X */
   SFSTD = 1.0;                           /* Sigmanew over Sigmaold */
   SFMU  = 2;                             /* Mean shift in Sigma X */
   NMEAN = MEAN + SFMU*STD;
   NSTD  = SFSTD*STD;
   QQUY  = MEAN + 3*FC2A*STD;
   QQLY  = MEAN - 3*FC2A*STD;
   NSIM  = 10000;                         /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC1*Y1 + FC1A*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM   = 0;
LAB:  A1    = Y1 - MEAN;
      A2    = ABS(A1) / STD;
      B2    = B1 * EXP(-0.5*A2*A2);       /* Ordinate of Std. Normal */
      B3    = 1 - 2*PROBNORM(-A2);        /* PROBNORM = CDF of Normal */
      ERK   = 2*STD*B2 + ABS(A1)*B3;      /* Expected of R given K */
      ER    = FC2A * ERK;                 /* Exp. of Range given K */
      B4    = SQRT(STD*STD + A1*A1 - ERK*ERK);
      SR    = FC2A * B4;                  /* Std. of Range given K */
      LCLMR = ER - 3*SR;                  /* LCL of OPA MRy Chart */
      UCLMR = ER + 3*SR;                  /* UCL of OPA MRy Chart */
      IF LCLMR < 0 THEN LCLMR = 0;
      UCLY  = QQUY + FC2*A1;              /* ULC of OPA Y Chart */
      LCLY  = QQLY + FC2*A1;              /* LCL of OPA Y Chart */
      Y2    = FC1*Y1 + FC1A*(RANNOR(SEED)*NSTD + NMEAN);
      MRY   = ABS(Y2-Y1);
      IF (LCLMR < MRY < UCLMR) AND (LCLY < Y2 <UCLY) THEN DO;
         NUM = NUM + 1;                   /* Point is in-control */
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;
      OUTPUT;                             /* Average Run Length */
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE JOINT OPA Y AND OPA MRy CHARTS ON FORP DATA';
   TITLE2 'WITH ACTUAL FILTER CONSTANT, FC1 = 0.6.';
   TITLE3 'ASSUMED FILTER CONSTANT, FC2 = 0.5.';
   TITLE4 'SHIFT IN MEAN = 2 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 16
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC1    = 0.6;                        /* Actual Filter Constant */
   FC1A   = 1 - FC1;
   FC2    = 0.5;                        /* Assumed Filter Constant */
   FC2A   = 1-FC2;
   B1     = 1/SQRT(2*22/7);             /* 1/SQRT(2*pi) */
   STD    = 1;                          /* Initial Std. Dev. of X */
   MEAN   = 10;                         /* Initial Mean of X */
   SHIFT  = 3.0;                        /* Sigmanew over Sigmaold */
   NSTD   = SHIFT*STD;
   NSIM   = 10000;                      /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;     /* Generate 1st value */
      DO K = 1 TO 50;                   /* Warm up FORP generator */
         Y2 = FC1*Y1 + FC1A*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      NUM    = 0;
LAB:  A1     = ABS(Y1 - MEAN);
      A2     = A1 / STD;
      B2     = B1 * EXP(-0.5*A2*A2);    /* Ordinate of Std. Normal */
      B3     = 1 - 2*PROBNORM(-A2);     /* PROBNORM = CDF of Normal */
      ERK    = 2*STD*B2 + A1*B3;        /* Expected of R given K */
      ER     = FC2A * ERK;              /* Exp. of Range given K */
      B4     = SQRT(STD*STD + A1*A1 - ERK*ERK);
      SR     = FC2A * B4;               /* Std. of Range given K */
      LCL    = ER - 3*SR;               /* LCL of OPA MRy chart */
      UCL    = ER + 3*SR;               /* UCL of OPA MRy chart */
      IF LCL < 0 THEN LCL = 0;
      Y2 = FC1*Y1 + FC1A*(RANNOR(SEED)*NSTD + MEAN);
      RY = ABS(Y2-Y1);
      IF LCL < RY < UCL THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                    /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                             /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA MRy CHART ON FORP DATA WITH';
   TITLE2 'ACTUAL FILTER CONSTANT, FC1 = 0.6.';
   TITLE3 'ASSUMED FILTER CONSTANT, FC2 = 0.5.';
   TITLE4 'SHIFT RATIO IN DISPERSION = 3.0';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                         PROGRAM 17
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   ARRAY X(30) ERR(30);                   /* X(t) = Observed value */
   KEEP ARL ;
   SEED   = 12345;
   FC     = 0.3;                          /* Filter constant */
   FC1    = 1 - FC;
   STD    = 1;                            /* Initial Std. Dev. of X */
   MEAN   = 10;                           /* Initial Mean of X */
   NSIM   = 10000;                        /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                     /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      XSUM  = 0.0;
      DO L = 1 TO 30;                     /* Observe 1st 30 values */
         Y2   = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         X(L) = Y2;
         XSUM = XSUM + X(L);
         Y1   = Y2;
      END;
      ZO      = XSUM/30;                  /* ZO = Xbar */
      ERR(1) = X(1) - ZO;                 /* Error(t) = X(t) - Z(t-1) */
      SUMERR = ERR(1);
      SSQERR = ERR(1)*ERR(1);
      Z1      = 0.8*ZO + 0.2*X(1);        /* EWMA, r=0.2, Z(t) =    */
      DO M = 2 TO 30;                     /* (1-r)*Z(t-1) + r*X(t) */
         ERR(M) = X(M) - Z1;              /* Error(t) = X(t) - Z(t-1) */
         Z2      = 0.8*Z1 + 0.2*X(M);
         SUMERR = SUMERR + ERR(M);
         SSQERR = SSQERR + ERR(M)*ERR(M);
         Z1      = Z2;
      END;                                /* Delta(0) = Std. of ERR */
      DO = (SSQERR - (SUMERR*SUMERR/30))/29;
      D1 = 0.25*ABS(ERR(1)) + 0.75*DO; /* Delta, q=0.25 */
      DO N = 2 TO 30;               /* D(t)=q*|ERR(t)|+(1-q)*D(t-1) */
         D2 = 0.25*ABS(ERR(N)) + 0.75*D1;
         D1 = D2;
      END;
LAB:  UCL = Z1 + 3*1.25*D1;               /* UCL, Modified EWMA Chart */
      LCL = Z1 - 3*1.25*D1;               /* LCL, Modified EWMA CHArt */
      Y2  = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCL < Y2 < UCL THEN DO;          /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         Z2  = 0.8*Z1 + 0.2*Y2;           /* Next forecast point */
         ER  = Y2 - Z1;
         D2  = 0.1*ABS(ER) + 0.9*D1;      /* Next delta value,q=0.10 */
         Z1  = Z2;
         D1  = D2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                      /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                               /* Statistics of ARL */
   TITLE1 'ARL OF THE MODIFIED EWMA CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3. ';
   TITLE3 'SHIFT IN MEAN = 2.0 SIGMA';
   TITLE4 'LAMBDA OF EWMA CHART = 0.20. ALPHA FOR SMOOTHING THE';
   TITLE5 'ERROR ESTIMATES=0.25 (1st 30 OBS) =0.10 (THE REST)';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                           PROGRAM 18
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.3;                        /* Filter Constant */
   FC1    = 1 - FC;
   FC2    = SQRT(1+FC);
   STD    = 1;                          /* Initial Std. Dev. of X */
   MEAN   = 10;                         /* Initial Mean of X */
   SHIFT  = 2.0;                        /* Shift in term of sigma x */
   NMEAN  = MEAN + SHIFT*STD;           /* New mean of X */
   NSIM   = 10000;                      /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD+MEAN;       /* Generate initial value */
      DO K = 1 TO 50;                   /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      MRSUM = 0.0;
      YSUM  = 0.0;
      DO L = 1 TO 30;
         Y2     = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         YSUM   = YSUM + Y2;
         MRSUM  = MRSUM + ABS(Y2-Y1);
         Y1     = Y2;
      END;
      AVG    = YSUM/30;                 /* Empirically compute */
      MRBAR  = MRSUM/30;                /* AVG and MRBAR */
      SSTD   = MRBAR*FC2/(1.128*FC1);
      QQUY   = AVG + 3*FC1*SSTD;        /* Constant term of UCLopay */
      QQLY   = AVG - 3*FC1*SSTD;        /* Constant term of LCLopay */
      NUM    = 0;
LAB:  AO     = Y1 - AVG;
      UCLY   = QQUY + FC*AO;            /* UCL of OPA Y chart */
      LCLY   = QQLY + FC*AO;            /* LCL of OPA Y chart */
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*STD + NMEAN);
      IF LCLY < Y2 < UCLY THEN DO;      /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                    /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD;
   VAR ARL;                             /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.3. ';
   TITLE3 'SHIFT IN MEAN = 2.0 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 17
//*
/*ROUTE PRINT LOCAL
//   EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC     = 0.0;                           /* Filter Constant */
   FC1    = 1 - FC;
   FC2    = SQRT(1+FC);
   B1     = 1/SQRT(2*22/7);                /* 1/SQRT(2*pi) */
   STD    = 1;                             /* Initial Std. Dev. of X */
   MEAN   = 10;                            /* Initial Mean of X */
   SHIFT  = 2.0;                           /* Sigmanew over Sigmaold */
   NSTD   = SHIFT*STD;
   NSIM   = 10000;                         /* Number of Simulation */
   DO I = 1 TO NSIM;
      Y1 = RANNOR(SEED)*STD + MEAN;        /* Generate 1st value */
      DO K = 1 TO 50;                      /* Warm up FORP generator */
         Y2 = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         Y1 = Y2;
      END;
      MRSUM = 0.0;
      YSUM  = 0.0;
      DO L = 1 TO 30;
         Y2    = FC*Y1 + FC1*(RANNOR(SEED)*STD + MEAN);
         YSUM  = YSUM + Y2;
         MRSUM = MRSUM + ABS(Y2-Y1);
         Y1 = Y2;
      END;
      AVG    = YSUM/30;                    /* Empirically compute */
      MRBAR = MRSUM/30;                    /* AVG and MRBAR */
      SSTD   = MRBAR*FC2/(1.128*FC1);
      NUM    = 0;
LAB:  A1     = ABS(Y1 - AVG);
      A2     = A1 / SSTD;
      B2     = B1 * EXP(-0.5*A2*A2);       /* Ordinate of Std. Normal */
      B3     = 1 - 2*PROBNORM(-A2);        /* PROBNORM = CDF of Normal */
      ERK    = 2*SSTD*B2 + A1* B3;         /* Expected of R given K */
      ER     = FC1 * ERK;                  /* Exp.of Range given K */
   B4     = SQRT(SSTD*SSTD + A1*A1 - ERK*ERK);
      SR     = FC1 * B4;                   /* Std. of Range given K */
      LCL    = ER - 3*SR;                  /* LCL of OPA MRy chart */
      UCL    = ER + 3*SR;                  /* UCL of OPA MRy chart */
      IF LCL < 0 THEN LCL = 0;
      Y2     = FC*Y1 + FC1*(RANNOR(SEED)*NSTD + MEAN);
      RY     = ABS(Y2-Y1);
      IF LCL < RY < UCL THEN DO;           /* Point is within CL's */
         NUM = NUM + 1;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                       /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                                /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA MRy CHART ON FORP DATA';
   TITLE2 'WITH FILTER CONSTANT, FC = 0.0';
   TITLE3 'SHIFT RATIO IN DISPERSION = 2.0';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                       PROGRAM 20
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED  = 12345;
   FC1   = 0.3;                        /* 1st Filter Constant */
   FC1A  = 1 - FC1;
   FC2   = 0.3;                        /* 2nd Filter Constant */
   FC2A  = 1 - FC2;
   STD   = 1;                          /* Initial Std. Dev. of X */
   MEAN  = 10;                         /* Initial Mean of X */
   SHIFT = 2.0;                        /* Shift in term of sigma x */
   NMEAN = MEAN + SHIFT*STD;           /* New mean of X */
   NSIM  = 5000;                       /* Number of Simulation */
   DO I = 1 TO NSIM;
      Z1 = RANNOR(SEED)*STD + MEAN;    /* Generate initial value */
      Y1 = RANNOR(SEED)*STD + MEAN;    /* Generate initial value */
      DO K = 1 TO 50;                  /* Warm up SORP generator */
         Y2 = FC2*Y1 + FC2A*(RANNOR(SEED)*STD + MEAN);
         Z2 = FC1*Z1 + FC1A*Y2;
         Z1 = Z2;
         Y1 = Y2;
      END;
      MRSUM = 0.0;                     /* Compute Average and Std. */
      YSUM  = 0.0;                     /* Dev. of the process as */
      DO L = 1 TO 30;                  /* if the process is FORP */
         Y2    = FC2*Y1 + FC2A*(RANNOR(SEED)*STD + MEAN);
         Z2    = FC1*Z1 + FC1A*Y2;
         YSUM  = YSUM + Z2;
         MRSUM = MRSUM + ABS(Z2-Z1);
         Z1    = Z2;
         Y1    = Y2;
      END;
      AVG   = YSUM/30;
      MRBAR = MRSUM/30;
      SIG   = MRBAR*SQRT(1+FC1)/(1.128*FC1A);
      QQUY  = AVG + 3*FC1A*SIG;        /* Constant term of UCLopay */
      QQLY  = AVG - 3*FC1A*SIG;        /* Constant term of LCLopay */
      NUM   = 0;
LAB:  A0    = Z1 - AVG;
      UCLY  = QQUY + FC1*A0;           /* UCL of OPA Y chart */
      LCLY  = QQLY + FC1*A0;           /* LCL of OPA Y chart */
      Y2    = FC2*Y1 + FC2A*(RANNOR(SEED)*STD + NMEAN);
      Z2    = FC1*Z1 + FC1A*Y2;
      IF LCLY < Z2 < UCLY THEN DO;     /* Point is within CL's */
         NUM = NUM + 1;
         Z1  = Z2;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                   /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                            /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA Y CHART ON SORP DATA WITH';
   TITLE2 '1st FILTER CONSTANT, FC1 = 0.3, 2nd FILTER CONSTANT,';
   TITLE3 'FC2 = 0.3.  SHIFT IN MEAN = 2.0 SIGMA';
//
```

```
//U11563A JOB (11563,440-88-2421),CLASS=4,TIME=(5,0),MSGCLASS=X
//*
//*                          PROGRAM 21
//*
/*ROUTE PRINT LOCAL
//  EXEC SAS
//SYSIN DD *
DATA PHD;
   KEEP ARL;
   SEED   = 12345;
   FC1    = 0.3;                        /* Filter Constant */
   FC1A   = 1 - FC1;
   FC2    = 0.3;
   FC2A   = 1-FC2;
   B1     = 1/SQRT(2*22/7);             /* 1/SQRT(2*pi) */
   STD    = 1;                          /* Initial Std. Dev. of X */
   MEAN   = 10;                         /* Initial Mean of X */
   SHIFT  = 3.0;                        /* Sigmanew over Sigmaold */
   NSTD   = SHIFT*STD;
   NSIM   = 5000;                       /* Number of Simulation */
   DO I = 1 TO NSIM;
      Z1 = RANNOR(SEED)*STD + MEAN;     /* Generate 1st value */
      Y1 = RANNOR(SEED)*STD + MEAN;     /* Generate 1st value */
      DO K = 1 TO 50;                   /* Warm up SORP generator */
         Y2 = FC2*Y1 + FC2A*(RANNOR(SEED)*STD + MEAN);
         Z2 = FC1*Z1 + FC1A*Y2;
         Z1 = Z2;
         Y1 = Y2;
      END;
      MRSUM = 0.0;                      /* Compute Average and Std. */
      YSUM  = 0.0;                      /* Dev. of the process as */
      DO L = 1 TO 30;                   /* if the process is SORP */
         Y2    = FC2*Y1 + FC2A*(RANNOR(SEED)*STD + MEAN);
         Z2    = FC1*Z1 + FC1A*Y2;
         YSUM  = YSUM + Z2;
         MRSUM = MRSUM + ABS(Z2-Z1);
         Z1 = Z2;
         Y1 = Y2;
      END;
      AVG   = YSUM/30;
      MRBAR = MRSUM/30;
      SIG   = MRBAR*SQRT(1+FC1)/(1.128*FC1A);
      NUM   = 0;
LAB:  A1    = ABS(Z1 - AVG);
      A2    = A1 / SIG;
      B2    = B1 * EXP(-0.5*A2*A2);     /* Ordinate of Std. Normal */
      B3    = 1 - 2*PROBNORM(-A2);      /* PROBNORM = CDF of Normal */
      ERK   = 2*SIG*B2 + A1*B3;         /* Expected of R given K */
      ER    = FC1A * ERK;               /* Exp. of Range given K */
      B4    = SQRT(SIG*SIG + A1*A1 - ERK*ERK);
      SR    = FC1A * B4;                /* Std. of Range given K */
      LCL   = ER - 3*SR;                /* LCL of OPA MRy chart */
      UCL   = ER + 3*SR;                /* UCL of OPA MRy chart */
      IF LCL < 0 THEN LCL = 0;
      Y2 = FC2*Y1 + FC2A*(RANNOR(SEED)*NSTD + MEAN);
      Z2 = FC1*Z1 + FC1A*Y2;
      RZ = ABS(Z2-Z1);
      IF LCL < RZ < UCL THEN DO;        /* Point is within CL's */
         NUM = NUM + 1;
         Z1  = Z2;
         Y1  = Y2;
         GO TO LAB;
      END;
      ARL = NUM + 1;                    /* Average Run Length */
      OUTPUT;
   END;
PROC MEANS N MEAN STD SKEWNESS KURTOSIS MAX MIN;
   VAR ARL;                             /* Statistics of ARL */
   TITLE1 'ARL OF THE OPA MRy CHART ON SORP DATA WITH';
   TITLE2 '1st FILTER CONSTANT, FC1 = 0.3, 2nd FILTER CONSTANT,';
   TITLE3 'FC2 = 0.3.  SHIFT RATIO IN DISPERSION = 3.0';
//
```

APPENDIX F

THE EFFECTS OF VARIATION IN EMPIRICALLY DETERMINED CONTROL LIMITS ON THE AVERAGE RUN LENGTH

The Effects of Variation in Empirically Determined Control Limits on the ARL

To facilitate discussion in this section, an independent normal data stream with mean μ and variance σ^2 is considered. To construct an Xbar chart on these independent data, the normal practice is to group data into m subgroups of size n, say m=30 and n=4. Then, the average and range of each subgroup, Xbar and R, are computed respectively. The averages of these subgroup averages and ranges are also calculated, and are denoted as Xbb and Rb respectively. The control limits of the Xbar chart are constructed as follows,

$$UCL_{xbar} = Xbb + 0.73*Rb \quad (F.1)$$

$$LCL_{xbar} = Xbb - 0.73*Rb \quad (F.2)$$

Since each plotted point on the Xbar chart is normally distributed with mean μ and standard deviation 0.5σ (since n=4), the probability for a point (Xbar) to fall within the control limits can be easily computed, and is designated as Pa.

$$Pa = \int_{LCL_{xbar}}^{UCL_{xbar}} f(Xbar)\, d(Xbar) \quad (F.3)$$

where $f(\cdot)$ is a p.d.f. of a normal distribution. Thus, the number of points plotted until an out-of-control signal is found, even though the process is in-control, without using any runs rules, follows a geometric distribution with parameter p = (1-Pa). That is,

$$P(X) = (1 - Pa)Pa^{x-1} \qquad X = 1, 2, \ldots \qquad (F.4)$$

where X is the number of points plotted until an out-of-control signal is found.

It is well known that the expected value of a geometric random variable is p^{-1}. Thus, the ARL of the Xbar chart is equal to

$$ARL = (1 - Pa)^{-1} \qquad (F.5)$$

For an in-control process, (1 - Pa) is the Type I error and, when the process is out-of-control, Pa is the Type II error. It is known that Xbb is normally distributed with mean equal to μ and variance equal to $\sigma^2/(mn)$.

$$Xbb \sim \text{Normal}\left[\mu,\ \sigma^2/(mn)\right] \qquad (F.6)$$

Using the Central Limit Theorem, Rb is seen to be normally distributed with mean $d_2*\sigma$ and variance $(d_3*\sigma)^2/m$.

$$Rb \sim \text{Normal}\left[d_2*\sigma,\ (d_3*\sigma)^2/m\right] \qquad (F.7)$$

It is also a well known fact that Xbar and R are independent of one another; this leads to the fact that Xbb and Rb are also independent of one another.

If the Xbb and Rb are at their mean values, that is $Xbb = \mu$ and $Rb = d_2*\sigma$ where d_2 equals 2.059 for n=4, it is readily known that Pa is 0.0027 for an in-control process. The corresponding ARL is about 370. This ARL of 370 is to be interpreted as the average number of plotted points on the Xbar chart until an out-of-control signal is found when the process is, in fact, in-control. It should be noted that this ARL value of 370 is the average run length, conditioned on the fact that Xbb and Rb are at their mean values. As mentioned earlier, the distribution of run length for such a control chart with Xbb and Rb at their mean values is indeed a geometric distribution. When runs rules are applied to the control chart, the run length distribution is no longer a geometric distribution, but still has a long tail skewed to the right. Some researchers have approximated the distribution of run length when runs rules are used (Champ and Woodall, 1990). The approximate distribution of run length still assumes that both Xbb and Rb are at their mean values.

In reality, if one uses the Xbar chart with control limits empirically constructed using statistics Xbb and Rb derived from the initial m subgroups of size n, with Xbb and Rb varying according to their own distribution, the average run length one expects is different from the value 370. Certainly, ARL' has its own distribution as well. This ARL' can be determined by considering the distribution of Xbb and Rb.

For given values of Xbb and Rb, say c and d, respectively, the upper and lower control limits of the Xbar chart, UCL_{xbar} and LCL_{xbar}, can be defined in terms of the values of c and d. The probability of a plotted point falls within these control limits, Pa, is then determined. Pa is also a function of c and d. And such, the ARL of the control chart with these given Xbb and Rb values can be determined as follows,

$$ARL'|(c,d) = [1 - Pa(c,d)]^{-1} \qquad (F.8)$$

where

$$Pa(c,d) = \int_{LCL_{xbar}(c,d)}^{UCL_{xbar}(c,d)} f(Xbar)\ d(Xbar) \qquad (F.9)$$

and

$$UCL_{xbar}(c,d) = c + 0.73*d$$

$$LCL_{xbar}(c,d) = c - 0.73*d \qquad (F.10)$$

Therefore, the unconditional ARL' of the control chart can be obtained by considering all possible values of Xbb and Rb. This can be done by integrating over the entire ranges of Xbb and Rb.

$$ARL' = \int_{0}^{\infty} \int_{-\infty}^{\infty} [1 - Pa(c,d)]^{-1}\ g(c)\ h(d)\ dc\ dd \qquad (F.11)$$

where g(·) and h(·) are the p.d.f. of Xbb and Rb, respectively.

Using some type of numerical integration, it is able to verify that the ARL' obtained using Equation (F.11) is different from the expected 370 for an in-control process. This illustrates the fact that, all the while, the well known 370 ARL of the Xbar chart when the process is in-control is correct only based on the mean values of Xbb and Rb. That is, the possible variations of Xbb and Rb are not taken into consideration at all. If the variation of the Xbb and Rb are taken into consideration, the ARL obtained is certainly different. Since in a real practical application of the Xbar chart the Xbb and Rb values are computed from an initial m subgroups of size n, the ARL computed from Equation (F.11) depicts a more truthful situation.

Equation (F.11) is only used to compute the ARL for a Xbar chart on independent normal data when the process is in-control. However, the approach can be extended to other control charts, such as the Range chart, the MR(2) chart, or a control chart for a correlated data stream. In the Xbar chart, since the plotted points are independent of one another and normally distributed, the f(·) in Equation (F.9) can sufficiently describe the characteristic of these plotted points. If the plotted points are not independent of one another, such as moving ranges of an independent data stream or individual data from a FORP data stream, the p.d.f. f(·) in Equation (F.9) does not capture the correlation between plotted points. Hence, the average run length of such a control chart, with the consideration of variation of control limits, must be determined by other methods. At present, simulation seems to be the only way to determine how the number of subgroups, m, and the subgroup size, n, affect the average run length of a control chart for which control limits are constructed on the statistics derived from the initial m subgroups of size n.

APPENDIX G

LISTING OF FORTRAN PROGRAM TO CALCULATE

THE CONTROL LIMITS OF THE OPA Y

AND OPA MRy CHARTS

```
C*******************************************************************
C Objective:  This program constructs the OPA and OPA MRy charts    *
C                                                                  *
C Data input: Initial data can be read from a disk file named INPUT or *
C             entered via the keyboard.                            *
C             If data are in disk file, they have to be arranged in *
C             column with the value of filter constant in the first *
C             row, followed by the observations.                   *
C             If data is entered via keyboard, the user has to supply *
C             the value of filter constant, the number of observations *
C             to be entered and the observations.                  *
C                                                                  *
C Results output: All the results will be displayed on the computer *
C                 monitor screen.  But, the user has the option to *
C                 store the results in disk file as well.          *
C                 Only the upper and lower control limits of the   *
C                 control charts and the plotting points are listed. *
C                 Out-of-control points are indicated with a '*'.  *
C                                                                  *
C Additional data: After the initial observations have been used to *
C                  construct the control charts, the user can further *
C                  enter additional observation.                   *
C                                                                  *
C Assumption: It is assumed that the observations are from a FORP and *
C             the filter constant is known and is between 0 and 1. *
C             It is assumed that there are not more than 500       *
C             observations.                                        *
C                                                                  *
C*******************************************************************
C
      CHARACTER DAT*1, ANS*1
      COMMON YBAR,SIGMA,RHO,B
      REAL MRBAR, MRSUM, LL, LCL, MR(500), Y(500)
      D2 = 1.128
C
C    INPUT OBSERVATIONS AND COMPUTE THE YBAR AND MRBAR
C
      WRITE (*,*) 'PROGRAM TO CONSTRUCT THE OPA Y AND OPA MRy CHARTS'
      WRITE (*,*)
      WRITE (*,*) 'ENTER D IF THE DATA ARE STORED IN DISK FILE'
      READ (*,170) DAT
      IF (.NOT. ((DAT .EQ. 'D') .OR. (DAT .EQ. 'd'))) GOTO 50
      WRITE (*,*) 'THE DATA IN THE FILE SHOULD BE IN COLUMN'
      WRITE (*,*) 'THE FIRST VALUE SHOULD BE THE FILTER CONSTANT.'
      WRITE (*,*) 'ALSO, THE FILE NAME SHOULD BE "INPUT" '
      WRITE (*,*) 'IS THE FILE IN THE CORRECT FORM ?  Y - YES'
      WRITE (*,*) 'IF NO, THE PROGRAM WILL TERMINATE.'
      READ (*,170) ANS
      IF (.NOT. ((ANS .EQ. 'Y') .OR. (ANS .EQ. 'y'))) GOTO 200
      OPEN(5,FILE='INPUT')
      READ (5,*) RHO
      WRITE (*,*) 'FILTER CONSTANT', RHO
      N = 1
      READ (5,*) Y(N)
      WRITE (*,*) N,Y(N)
      YSUM   = Y(N)
      MRSUM  = 0.0
      N      = N + 1
20    READ (5,*,END=30) Y(N)
      WRITE (*,*) N, Y(N)
      MR(N) = ABS(Y(N) - Y(N-1))
      YSUM   = YSUM + Y(N)
      MRSUM  = MRSUM + MR(N)
      N      = N + 1
      GOTO 20
30    CLOSE (5)
      N = N - 1
      GOTO 80
50    WRITE (*,*) 'ENTER THE VALUE OF FILTER CONSTANT'
      READ (*,*) RHO
      WRITE (*,*) 'ENTER NUMBER OF DATA POINTS'
      READ (*,*) N
      WRITE (*,*) 'ENTER THE DATA ONE BY ONE'
      READ (*,*) Y(1)
      YSUM   = Y(1)
      MRSUM  = 0.0
      DO 70 I = 2, N
         READ (*,*) Y(I)
         MR(I) = ABS(Y(I) - Y(I-1))
         YSUM   = YSUM + Y(I)
         MRSUM  = MRSUM + MR(I)
70    CONTINUE
80    YBAR   = YSUM/FLOAT(N)
      MRBAR  = MRSUM/FLOAT(N-1)
C
C    OPTION TO STORE RESULTS IN DISK FILE
C
```

```
      WRITE (*,*)
      WRITE (*,*) 'ENTER D IF WANT TO STORE THE RESULTS TO DISK'
      READ (*,170) DAT
      WRITE (*,*) 'NUMBER OF INITIAL OBSERVATIONS = ',N
      WRITE (*,*) 'Ybar    = ', YBAR
      WRITE (*,*) 'MRybar = ', MRBAR
      WRITE (*,*) 'r       = ', RHO
      WRITE (*,*) 'Y(1)    = ', Y(1)
C
C   DISPLAY THE HEADINGS AND THE VALUES OF YBAR AND MRBAR
C
      WRITE (*,90)
90    FORMAT (/1X,'No.',7X,'Y',7X,'LCL',5X,'UCL',8X,
     * 'MRy',6X,'LCL',5X,'UCL'/)
      IF ((DAT .EQ. 'D') .OR. (DAT .EQ. 'd'))THEN
         OPEN (5,FILE='RESULT',STATUS='NEW')
         WRITE (5,*) 'NUMBER OF INITIAL OBSERVATIONS = ',N
         WRITE (5,*) 'Ybar    = ', YBAR
         WRITE (5,*) 'MRybar = ', MRBAR
         WRITE (5,*) 'Y(1)    = ', Y(1)
         WRITE (5,90)
      ENDIF
      A      = 1.0+RHO
      B      = 1.0-RHO
      SIGMA = MRBAR*SQRT(A)/(D2*B)
C
C   CONSTRUCT THE CONDITIONAL CONTROL LIMITS FOR THE Y AND MRy CHARTS
C   AND DISPLAY THEM WITH THE CORRESPONDING Y AND MRy VALUES
C
      DO 100 I = 2, N
         CALL CHART (DAT,I,Y(I),Y(I-1),MR(I))
100   CONTINUE
C
C    OPTION TO ENTER MORE OBSERVATION
C
150   WRITE (*,160)
160   FORMAT (/' MORE DATA TO ENTER ?  Y - TO CONTINUE')
      READ (*,170) ANS
170   FORMAT (A1)
      IF (ANS .NE. 'Y') GOTO 200
      N = N + 1
      IF (N .GT. 500) THEN
        WRITE (*,*) 'SORRY ! THERE ARE ALREADY 500 DATA POINTS'
        WRITE (*,*) 'START AGAIN'
        GOTO 200
      ENDIF
      WRITE (*,*)
      WRITE (*,*) 'ENTER THE NEW DATA'
      READ (*,*) Y(N)
      MR(N) = ABS(Y(N) - Y(N-1))
      WRITE (*,190)
190   FORMAT (/1X,'No.',7X,'Y',7X,'LCL',5X,'UCL',8X,
     * 'MRy',6X,'LCL',5X,'UCL'/)
C
      CALL CHART (DAT,N,Y(N),Y(N-1),MR(N))
      GOTO 150
200   IF ((DAT .EQ. 'D') .OR. (DAT .EQ. 'd')) CLOSE(5)
      STOP
      END
C**********************************************************************
C                                                                     *
C   SUBROUTINE TO COMPUTE THE CONDITIONAL CONTROL LIMITS OF THE OPA   *
C   Y AND OPA MRy CHARTS AND DISPLAY THEM ALONG WITH THE CORRESPONDING *
C   Y AND MR(2) OF Y VALUES                                           *
C                                                                     *
C**********************************************************************
C
      SUBROUTINE CHART(DAT,I,Y,Y1,MR)
      COMMON YBAR,SIGMA,RHO,B
      CHARACTER*1 DAT
      REAL MR, LCL, LL
      YCL = RHO*Y1 + B*YBAR
      LCL = YCL - 3*B*SIGMA
      UCL = YCL + 3*B*SIGMA
C
      C  = ABS(Y1 - YBAR)
      D  = PHI(-C/SIGMA)
      E  = PHIS(C/SIGMA)
      F  = 2.0*SIGMA*E + C*(1.0 - 2.0*D)
      EK = B*F
      SK = B*SQRT(SIGMA*SIGMA + C*C - F*F)
      UL = EK + 3*SK
      LL = EK - 3*SK
      IF (UL .LE. 0) UL = 0.0
      IF (LL .LE. 0) LL = 0.0
      IF ((Y .GE. LCL) .AND. (Y .LE. UCL)) THEN
         CT1 = ' '
```

```
      ELSE
         CT1 = '*'
      ENDIF
      IF ((MR .GE. LL) .AND. (MR .LE. UL)) THEN
         CT2 = ' '
      ELSE
         CT2 = '*'
      ENDIF
      IF ((DAT .EQ. 'D') .OR. (DAT .EQ. 'd')) THEN
         WRITE (5,80) I,Y,CT1,LCL,UCL,MR,CT2,LL,UL
      ENDIF
      WRITE (*,80) I,Y,CT1,LCL,UCL,MR,CT2,LL,UL
80    FORMAT(1X,I3,3X,F7.3,A1,2(1X,F7.3),4X,F7.3,A1,2(1X,F7.3))
      RETURN
      END
C***********************************************************************
C                                                                      *
C   SUBPROGRAM TO COMPUTE THE CUMULATIVE OF A STANDARD NORMAL          *
C   (USED EQUATION 26.2.19 OF ABRAMOWITZ AND STEGUN'S TEXT (1965)      *
C                                                                      *
C***********************************************************************
C
      FUNCTION PHI(X)
      DOUBLE PRECISION C(6),PH
      DATA C/4.9867347D-2, 2.11410061D-2, 3.2776263D-3,
     *       3.80036D-5, 4.88906D-5, 5.383D-6/
      IF (X .LT. 0) THEN
         Y   = DBLE(-X)
         PH  = (((((C(6)*Y + C(5))*Y + C(4))*Y + C(3))*Y
     *         + C(2))*Y + C(1))*Y
         PHI = 1.0 - SNGL(1D0 - 0.5D0*(1D0 + PH)**(-16D0))
      ELSE
         Y   = DBLE(X)
         PH  = (((((C(6)*Y + C(5))*Y + C(4))*Y + C(3))*Y
     *         + C(2))*Y + C(1))*Y
         PHI = SNGL(1D0 - 0.5D0*(1D0 + PH)**(-16D0))
      ENDIF
      RETURN
      END
C***********************************************************************
C                                                                      *
C   SUBPROGRAM TO COMPUTE THE ORDINATE VALUE OF STANDARD NORMAL        *
C                                                                      *
C***********************************************************************
C
      FUNCTION PHIS(X)
      DOUBLE PRECISION Y
      Y    = DBLE(X)
      PHIS = SNGL(0.39894228D0*DEXP(-Y*Y/2D0))
      RETURN
      END
```

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VITA

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Doctor of Philosophy

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