APPLICATION OF MATHEMATICAL PROGRAMMING FOR OPTIMAL INVESTMENT, WATER SUPPLY AND PRICING DECISIONS FOR RURAL WATER SYSTEMS IN OKLAHOMA

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CHAPTER I

INTRODUCTION

Overview

In general, it is argued that community services quality and availability in rural areas of the United States remain low relative to urban areas. Over the past 20 years considerable effort has been devoted by the Federal government as well as rural people to improve community services.

Jones and Gessaman (1974) identified characteristics of community services in most rural areas as follows: (1) the services are thought necessary for the public good, (2) they are available and utilized by the general public, (3) they are generally provided through relatively rigid institutional arrangements by the public sector or regulated private monopolies with high fixed investment, (4) prices of services (fees) are not set in the market and some services are provided at zero marginal cost to the consumer, (5) prices (fees) often do not allow recovery of fixed costs and may not cover variable costs, and (6) total cost to the consumer may be constant per unit of time and independent of the quantity consumed.

Rural community services have a mixture of the attributes associated with pure public and pure private goods. The market is not necessarily an effective mechanism for indicating demand or allocating resources for services possessing this mixture of attributes. In the absence of effective market mechanisms, various levels of government have carried out supply and market intervention activities designed to insure the availability of community services when and where a need has been expressed. Units of local government have been the principal providers, but are hampered by limited ability to bear the associated costs--especially where the present population density is low and delivery of services is costly or difficult.

In planning community services decision makers in local government are generally faced with the task of planning for growth. This includes estimating future growth and associated demand for community services so that optimum capacity can be built into the system. In determining the most economically efficient supply of community services, there is no simple method which can be applied in analysis of all different community services areas. In practice, limitations on available data for determining consumer market behavior and facility costs make it difficult to forecast community service demand and plan facility investment based on traditional concepts and methods used for analysis of private goods and services. Consequently, community services management requires a variety of analytical approaches depending upon the type of community service. For example, community services with relatively strong price signals such as water, electricity, and refuse collection, the individual preference and market behavior approach may be applicable while for other community services with weak price signals, such as fire protection and police protection, a political approach such as consumer voting behavior may be more expedient.

Problem Statement

Water supplies are becoming increasingly scarce relative to rural demands. Increasing rural populations with rising expectations as to adequate service, create a need for achieving increased efficiency in rural water services investment. Many rural communities are confronted with the problem of inadequate funds to cover both the initial investment costs and the sustaining costs of a water system. Continued growth of rural populations further constrain the capacity of many rural water systems to provide adequate services over a reasonable planning period. This, coupled with the continued economic development of rural areas, in part dependent upon adequate supplies of water, makes critical an examination and reappraisal of current methods of rural water services planning.

Present methods applied to community water systems planning too frequently rely on simple rules of thumb. Average service supply cost is frequently used as a basis to set rates (prices). Future demand increases are considered, if at all, on the basis of multiplying per capita rates of consumption by projected population although economic theory and empirical results support close interrelationships among price level, consumption behavior and supply costs.

Rural community water systems financed by Federal loan programs through the Farmers Home Administration have been unable to plan for sufficient capacity to meet increases in water demand due to population growth since the loan programs can consider only a fixed multiple of the existing population at the time of loan initiation. As a result many rural water systems financed by the loan programs must increase capacity after relatively short periods of operation, especially in fast growting areas.

Purpose and Objectives

The purpose of this research is to provide information for the planning and management of rural water systems in Oklahoma. The primary objective is to demonstrate an improved community services planning model by incorporating intertemporal and attitudinal correlates with decisions on rural water consumption in Oklahoma. The focus of this effort is to examine growth factors that influence rural water demand and supply. Data derived from sample information on rural systems in Oklahoma are used as inputs in the planning model to determine optimum levels in system capacity, level of operation and consumer satisfaction.

Specific objectives are:

1. To review theory on public goods and relate to conditions of demand and supply for community services in rural areas.
2. To develop deterministic community services demand and supply models for rural water services and empirically estimate those models for Oklahoma.
3. To develop programming models which address questions related to optimum timing and size of rural water system investments and optimum pricing of water resources.
4. To evaluate past public investments in rural water services using the programming models.

Plan of Presentation

The remaining text includes five chapters. Chapter II presents theoretical considerations and background material for pricing water and planning capacity of rural community water systems. Factors determining the optimum size of capacity are discussed. A selective review of previous works on consumer behavior and capacity decision making models is also included.

Chapter III contains the empirical analysis of rural water demand, cost of water supply and growth of water systems in rural Oklahoma.

Chapter IV specifies the mathematical investment programming model used in this study and the corresponding assumptions. Detailed descriptions of the analytical model - the objective function and constraints - are presented. Solution approaches and computational considerations for the dynamic mixed-integer model are also provided in this chapter.

In Chapter V, application of the model is made to typical conditions of rural water districts in Oklahoma (base results) and to three different community size water systems (small, average and large). Specifically, the optimal investment schedule, operation level and the associated water rate to maximize net social welfare for different growth situations and different discount rates are analyzed. Comparison of net social benefits from investment and pricing decisions of an actual water district and the programming optimum is made and discussed.

The major conclusions drawn from the results of the specific model application are summarized in Chapter VI. The policy implications of the findings in rural community water system management are discussed. Finally, suggestions for further work beyond the scope of this study are given.

CHAPTER II

OPTIMAL WATER RESOURCE ALLOCATION AND INVESTMENT - PRICING DECISIONS

Introduction

The economic foundation for the analytical models presented in this study are discussed in this chapter. Attention is given to the theoretical background of optimal resource allocation; pricing principles pertinent to achieving economic efficiency; and characteristics of community water price, demand and planning of community water supply. Selective reviews of previous studies in decision making for water system capacity and pricing, and the analysis of consumer behavior are presented.

Conditions for Optimal Resource Allocation

In the field of public natural resource development in general and community water resource management specifically, the objectives are not necessarily expressed in a manner as straight forward as profit maximization in the private sector of the economy. If a single social objective such as economic efficiency is postulated, the risk of ignoring other important criteria such as income distribution or regional development may occur. Nonetheless, the scope of this study is limited to the economic efficiency objective.

Principle of Equimarginal Value in Use and Marginal Cost Pricing

Allocative economic efficiency is traditionally defined as the allocation of resources in such a way that no reallocation exists which would allow gains to some without accompanying losses to others.

Let us suppose for simplicity a limited annual flow of a resource like water becomes available without cost. The problem is to allocate the supply among competitive users. Economic theory which satisfies the efficient allocation of the resource is the princple of "equimarginal value in use". The value in use of any unit of water, whether purchased by an ultimate or an intermediate consumer, is essentially measured by the maximum amount of resources (dollars) which the consumer would be willing to pay for that unit. Marginal value in use is the value of the last unit of water consumed. For any consumer, marginal value in use will ordinarily decline as the quantity of water consumed in any period increases. The principle states that resources should be so allocated that all consumers or users derive equal value in use from the marginal unit consumed or used.

From the argument developed so far it is inferred that the price should be equal to all users since otherwise a user will continue to buy additional units so long as the marginal value in use to him exceeds the price he must pay. Suppose that at a certain moment of time water price is $30 per unit. Then, if the community water system as a whole can acquire and transport another unit for, say $20, any of the individual customers to whome the unit of water is worth $30 would be happy to pay the $20 and none of the other members of the community is worse off.

On efficiency grounds, therefore, additional units should be made available so long as any member of the community is willing to pay the additional or marginal cost incurred. To meet the criterion of equimarginal value in use, the price should be made equal for all customers. The combined rule is to make the price equal to the marginal cost and equal for all customers.

One important practical consideration is that, because of differing locations, use patterns, types of service, etc., the marginal costs of serving different customers will vary. The correct solution is to arrange customers so that for each class of customers (where the classes are grouped so that all customers within any single class can be served under identical cost) the price is the same and equal to marginal cost. Between classes, however, prices should differ and equal the marginal costs involved in serving the different classes. In this study, since the majority of the rural community water system customers are residential, one class of customer is assumed for simplicity.

The Relationships Among Price, Demand and Capacity

The Role of Water Price

Howe (1971) proposed the major purposes to be accomplished through pricing include:

1. To make sure available water services are allocated to the highest value uses.
2. To adjust the quantity demanded by customers to the economically efficient quantity, i.e., the quantity for which incremental cost just equals the consumer's valuation of the last unit used.

3. To provide the proper inducement to system customers to seek the socially least cost solution for their particular circumstance.

4. To recover some portion of the costs of providing the water-related services. (p. 215)

Placing a price on water, as well as any other commodity, guarantees that only those who value additional water in excess of the price will use it, whereas those to whom it is of lower value will conserve its use. The appropriate price must be related to the appropriate measures of cost. Even though cost concepts used in economic theory are sufficiently simple, fitting the many relevant categories of water-related costs into the usual cost and pricing analysis becomes difficult. Small rural community water systems usually have three different major sources of cost: transmission, distribution and storage. (The sample of systems used in this study excludes treatment costs since most small community systems purchase treated water from nearby cities or neighboring water systems and involve only distribution and storage of water.) Further, there are some costs related just to the heavy peak demands placed on water supply systems. Some components of the system may have excess capacity at one time whereas other components may have excess capacity at another period of time. There are also economies of scale in most components of water supply systems that cause costs to depend upon the size of system capacity and the intensity of use. These are all reasons why it is difficult to be precise in specifying how water supply services should be priced.

Optimum Capacity of Water Systems

The general concern is to build water systems that will meet a demand growing over time especially due to growth of population.

Frequently, each system is one of a sequence of sub-systems that will be built over time with options concerning when (in terms of, say, the annual output capacity of the facility) the sub-systems in the sequence are to be built. The larger each system, the longer it will be until another segment to the system is needed under a constant growth rate. An example would be building additions to the initial water supply facilities for a growing rural community.

In determining how large to build the initial capacity or the increment (and the timing of that increment), studies have emphasized two basic factors which are nearly always in conflict:

1. It pays to build large initial capacity or increments to capacity because there are usually cost savings (economies of scale) involved in capacity size.

2. The commitment of resources to a capacity that will not be used for a long period of time is costly. It pays to defer investment as long as possible since future costs are more heavily discounted than present costs.

The effects of economies of scale and the discount rate on the size of capacity are portrayed in Figure 1. Under a given economies of scale factor, a, the optimal capacity decreases as the discount rate becomes higher. Similarly, for a given discount rate, r, the optimal capacity increases as the economies of scale becomes larger (toward the origin).

However, these two factors are sufficient only if the capacity decision is considered from a static viewpoint. In reality, growth of water demand makes the situation dynamic and the interrelationships of economies of scale, discount rate and growth must be considered in making capacity decisions more aqplicable. Growth in water demand is a direct reason why a system ends up with a lack of capacity even though it started with an excess of capacity. Therefore, since the discount rate

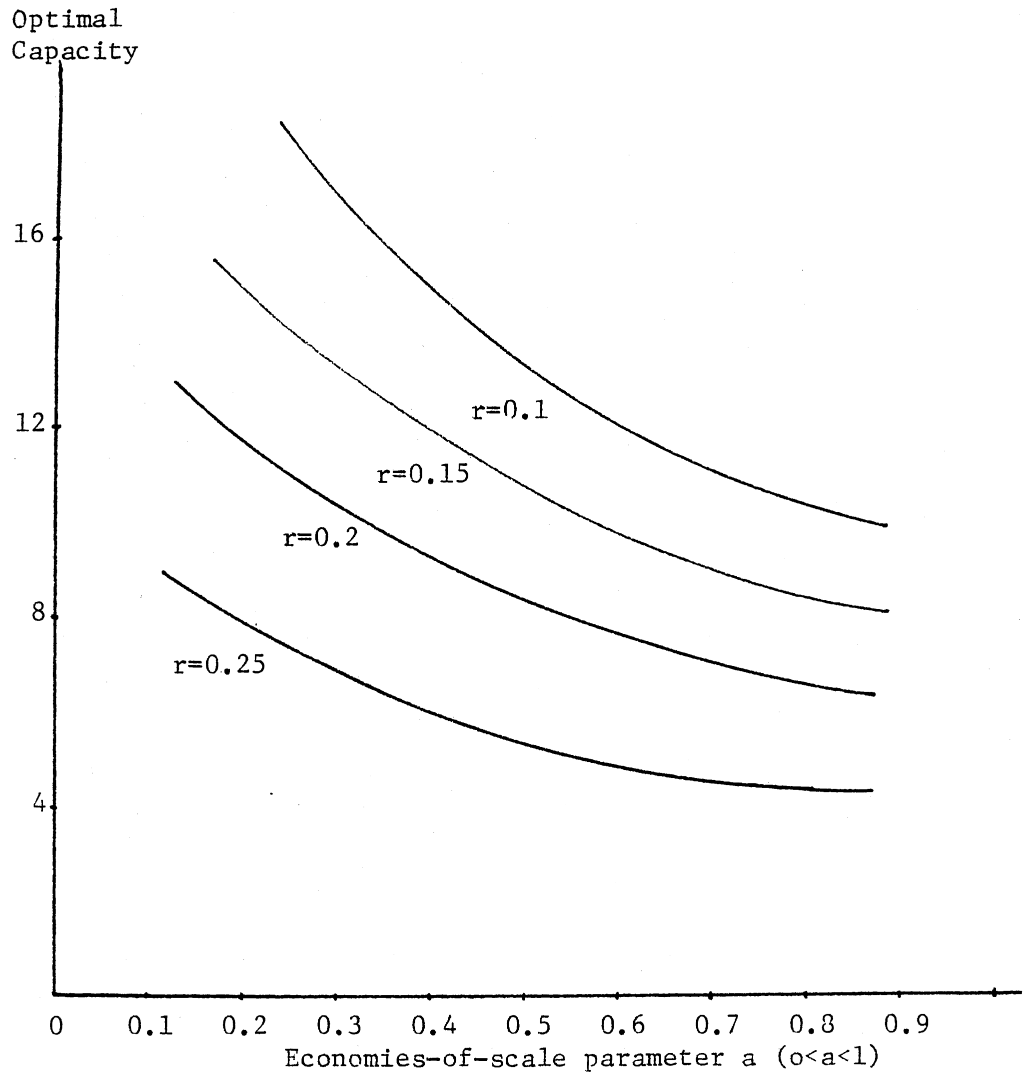

Figure 1. Relationships Among Capacity, Discount Rate and Economies of Scale Factor

and economies of scale are taken into consideration in making capacity decisions based upon an expectation of growth, explicit inclusion of growth in the decision process is very important.

The Demand for Residential Water and Water Pricing

The value of residential water is defined by consumers' demand for the commodity. Consumption of residential water is influenced by price, consumer income, population, configuration of commercial and civic uses and climatic conditions, particularly rainfall during seasons when moisture is required. The water services industry, however, frequently views water consumption as independent of price and assumes the demand per capita is fixed and that water must be found to meet "requirements". As a result water systems tend to be designed to meet such "requirements".

Water consumption studies have shown that users are responsive to changes in price, more so than is often supposed. Where water is metered, consumers have been found to use significantly less water than those who are on a flat rate. (Metering implies a conversion from a zero marginal price to a positive marginal price.) The greater part of the difference is accounted for in the amounts used for water lawns. The most striking example was the change to meters from unmetered use in Elizabeth City, North Carolina in 1931 which reduced water consumption by 83 percent (Resources 1971). Hanke (1970) also studied the impact of metering water supplies in Boulder, Colorado. In his study, lawn watering use dropped by nearly 50 percent and domestic uses declined by over 35 percent after meters were installed.

Linaweaver, Geyer, and Wolff (1976) analyzed factors influencing residential water use in a number of areas around the country. They found water sprinkling uses to be reduced 33 percent under metered conditions as compared with flat-rate pricing, although, in contrast with Hanke's results, domestic uses showed little difference.

Water pricing policies in many cities are such that it is difficult to derive inferences about consumers' willingness-to-pay. Where water is sold on a flat rate basis, the marginal price to the consumer is in effect zero. However, enough water systems, especially newly constructed systems, do charge for the marginal increment that cross section time series studies of water demand can be accomplished.

A number of published studies of the price elasticity of demand for residential water are available. Price elasticities tend to be relatively low, and differ between the two major components of use, domestic use and lawn sprinkling. The elasticities also vary among the different regions of the country.

One of the first analyses was by Louis Fort (1958) based on data from from a survey of water utilities conducted by the American Water Works Association in 1955. A price elasticity of demand of -0.39 was reported. Seidel and Baumann (1957) analyzed the same data and reported the elasticity as -1.0 in the range of 15 cents per thousand gallons to -0.12 at a price of 45 cents. Conley (1968) studied water consumption in a sample of southern California communities and reported the most likely price elasticity to be about -0.35. Howe and Linaweaver (1967) have made the most extensive study. Data were very carefully collected from a sample of water systems ranging from 34 to 2,373 dwelling units each. The overall estimated price elasticity (all uses, all regions) was found to be about -0.4. Domestic uses were found to exhibit an elasticity of -0.21, while water used for lawn sprinkling was characterized by elasticities of -0.7 in the arid west and -1.57 in the humid eastern region. Young (1971) utilized time series data to determine the price elasticity for the city of Tucson of -0.33.

These and other studies have demonstrated that consumers in fact are somewhat responsive to price changes and adjust their consumption of water accordingly. As useful as these studies are, most of them are not detailed enough and, as Howe and Linaweaver (1967) indicate, are based on such narrow samples that little use has been made of them.

Having demonstrated that demand functions can be derived for the domestic use of water and that the price elasticities of demand for household and lawn watering are significantly different and should be considered as two separate functions, it is possible to apply the willingness-to-pay concept to domestic water and to use the demand curve as representing the value of water to consumers in these uses.

Community Water Pricing

From the discussion in the previous sections, it is clear that water consumers respond to price and that economic efficiency can be achieved by setting price equal to marginal cost for all residential customers. In this section a review of different cost and pricing mechanisms related to community water services is given.

Average Cost Versus Marginal Cost Pricing

One class of customer is assumed in the rural community water service. In Figure 2, DD is the demand curve for water. Since only one

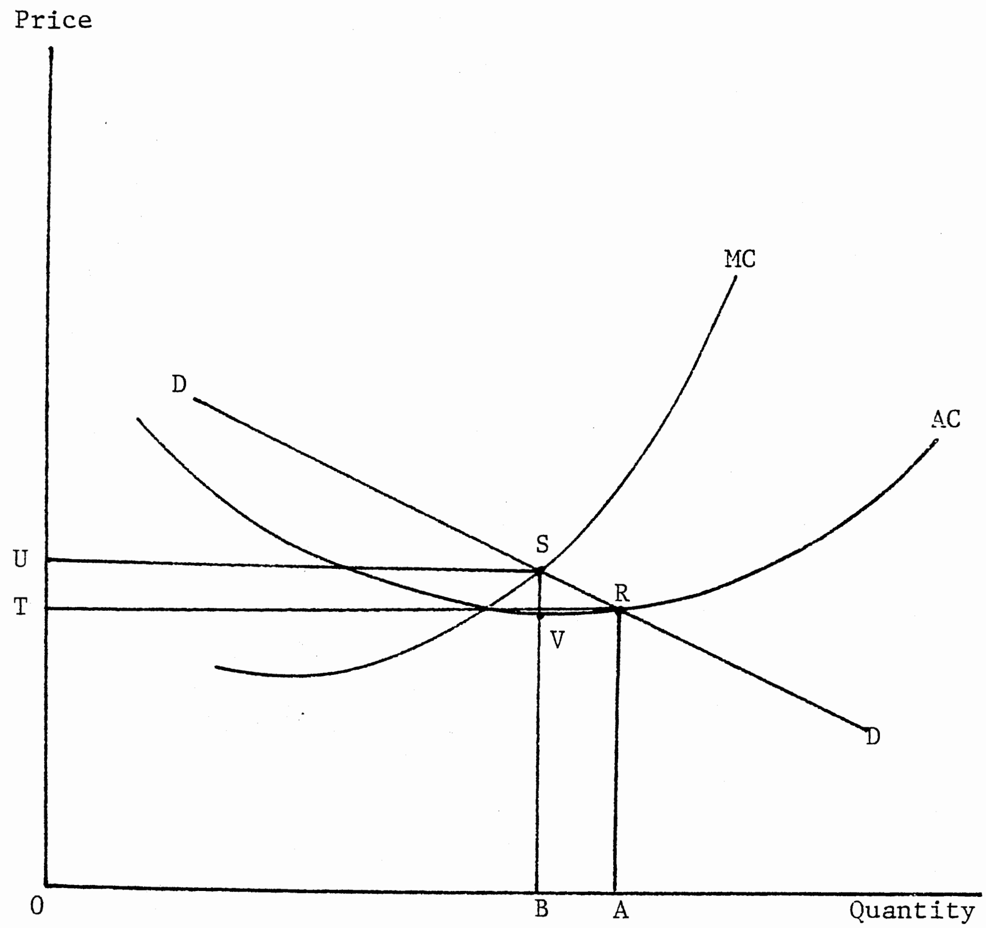

Figure 2. Solution in Range of Rising Average Cost

class of customer exists, average cost is defined as a unique function of the quantity supplied and is represented by the curve AC. The curve showing marginal cost as a function of output is labeled MC. If a single price is charged so as to cover cost while clearing the market, that price can only be equal to OT. At a price of OT the quantity OA would be demanded, the production of which involves an average cost of AR=OT.

At this solution, zero profits are earned in the economic sense; price equals unit costs including a normal interest return on capital invested. However, this is not the solution corresponding to best use of society's resources. Consider the units of output between OB and OA. For each of these units the marginal cost is greater than the amount anyone is willing to pay for the extra unit supplied, the consumer's marginal value in use. The quantity OB is demanded at the price OU=BS and, if any larger quantity is to be taken by consumers, the price will have to be reduced below BS. But the marginal cost is higher than BS throughout the range considered, hence that there are alternative uses of the resources entering into this marginal cost which consumers value more highly than they value what those resources can produce in the use considered here. The solution for the best use of resources is to produce just up to the point where the marginal cost begins to exceed the price consumers are willing to pay for the additional unit produced; that is to say, the correct output is OB at the marginal cost price BS with a profit to the community water system.

However, in small rural community water systems, a difficult management problem arises with marginal cost pricing since the demand curve frequently intersects the average cost curve in the range where the latter is still declining as in Figure 3. In this case, the average cost

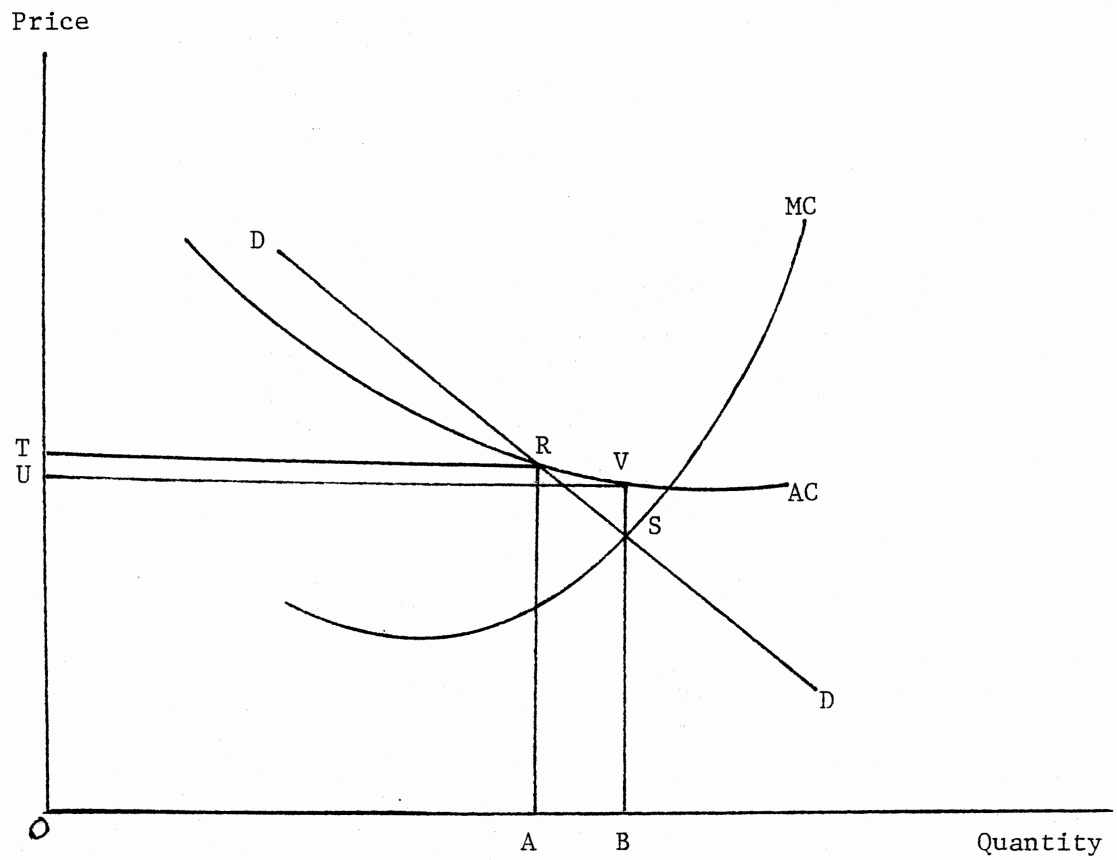

Figure 3. Solution in Range of Falling Average Cost

output and price are OA and AR, respectively, and the marginal cost output and price are OB and BS, respectively. Under these conditions, the marginal cost output is greater than the average cost output and the marginal cost price is less than the average cost price. In consequence, whereas in the previous case the community water system earned a profit at the marginal cost output and price, here it will incur a loss. The loss will be equal to the difference between average cost and price, SV, multiplied by the number of units produced (OB).

This loss from marginal cost pricing to achieve economic efficiency cannot be supported over a long period of time in the small rural community water systems. Such small water systems need a different solution to avoid losses. Hirschleifer (1969) suggests five alternative solutions. First is by means of a government contribution. Second is a voluntary contribution from members of the rural community water system. Third is setting up a descending scale of price as a function of quantity taken, but subject to the guiding rule that each customer must end up paying the same marginal price (i.e., price for the last unit consumed) and that this marginal price equal marginal cost. Fourth is price discrimination in such a way as to separate the market into two or more submarkets with prices varying from submarket to submarket. However, this is neither marginal cost nor average cost pricing, but it is a way of coping with the problem of deficits at a single price. Finally, he suggests the solution most similar to practices and procedures actually in effect in many community water systems and that is a two part tariff. Each customer is charged a single price per unit of output purchased, but in addition the customer is required to pay a lump-sum or minimum amount for the privilege of being permitted to buy at all. However, this method is different from a membership fee or benefit unit paid for capacity reservation.

Costs as a Function of Scale of Output

Small community water systems generally supply water under conditions of diminishing average cost. As discussed above, the marginal cost price will fail to generate enough revenue to cover total cost. Once major fixed facilities are in existence, there may be little extra expense required to increase output from zero up to the designed capacity. In this case, average cost will clearly be declining until capacity is reached.

The optimal solution to the investment pricing problem in these situations is shown graphically by Hirschleifer (1969). In Figure 4, the jagged average cost curve labeled AC shows a general upward trend through a series of discrete jumps, separated by regions of declining average cost. Suppose fixed capacity is such that we are operating on the notch labeled IV. The average cost curve reaches its lowest point at "designed capacity" (A_4), where a jump to notch V takes place. Corresponding to the declining average cost in this range is the short-run (i.e., relevant for this notch of fixed investment only) marginal cost curve $SRMC_4$ (dashed lines). This curve may be rising or falling, but it must be below AC throughout notch IV because AC is falling and it must equal AC where the latter reaches its local minimum at A_4. We assume it is first horizontal, then vertical which means that additional output cannot be obtained because of technical capacity limitations.

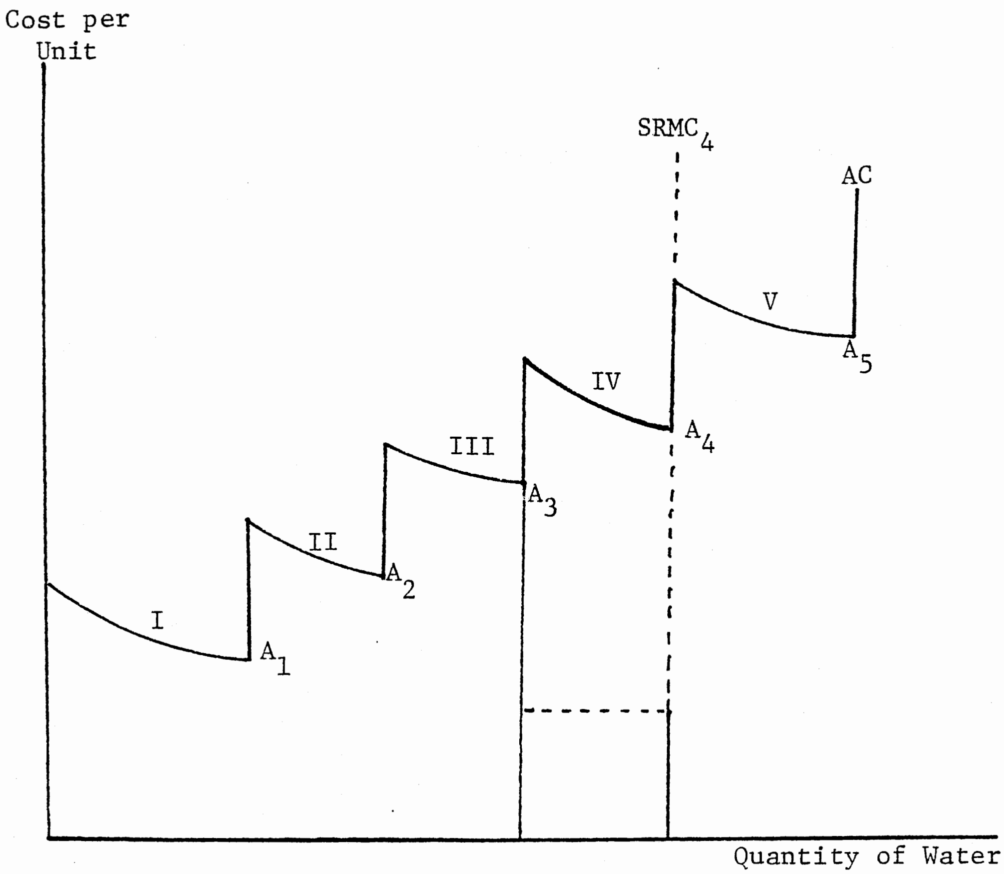

Figure 4. Short Run Average Costs of Water Supply

Turning to Figure 5, if the demand curve is $D(t_1)$ at time t_1, pricing at the intersection with SRMC will produce a loss. If the demand curve is $D(t_2)$ it will intersect the $SRMC_4$ curve in its vertical branch at A_4, so revenues will equal costs. For $D(t_3)$, marginal cost pricing will yield a profit. In this graph, the discontinuities are probably much sharper than in the real world.

The ordinary analysis of this situation in economic theory, illustrated in Figure 6, assumes complete continuity. There are assumed to be an indefinite number of short run average cost curves like those numbered in the diagram. The LRAC, or long run average cost curve, is the "envelope" of the short run average cost curves; it connects those points on the short run curves that represent the lower cost of producing any given output. There will also be long run and short run marginal cost curves. Given a demand curve like D, the intersection of LRMC and D at the point M determines the best output to produce in the long run. At optimal scale of system, the short run marginal cost $SRMC_4$ will also intersect D at the point M.

Classification of Costs: Capacity, Customer and Commodity

It is common to classify the costs of utilities into capacity (or demand), customer and commodity costs. These are usually defined as the costs that are proportional to the size of system, the number of separate services and the volume of the commodity delivered (Howe, 1979). However, even though we can theoretically classify the costs of water service in a like manner, there is no correct way to segregate total

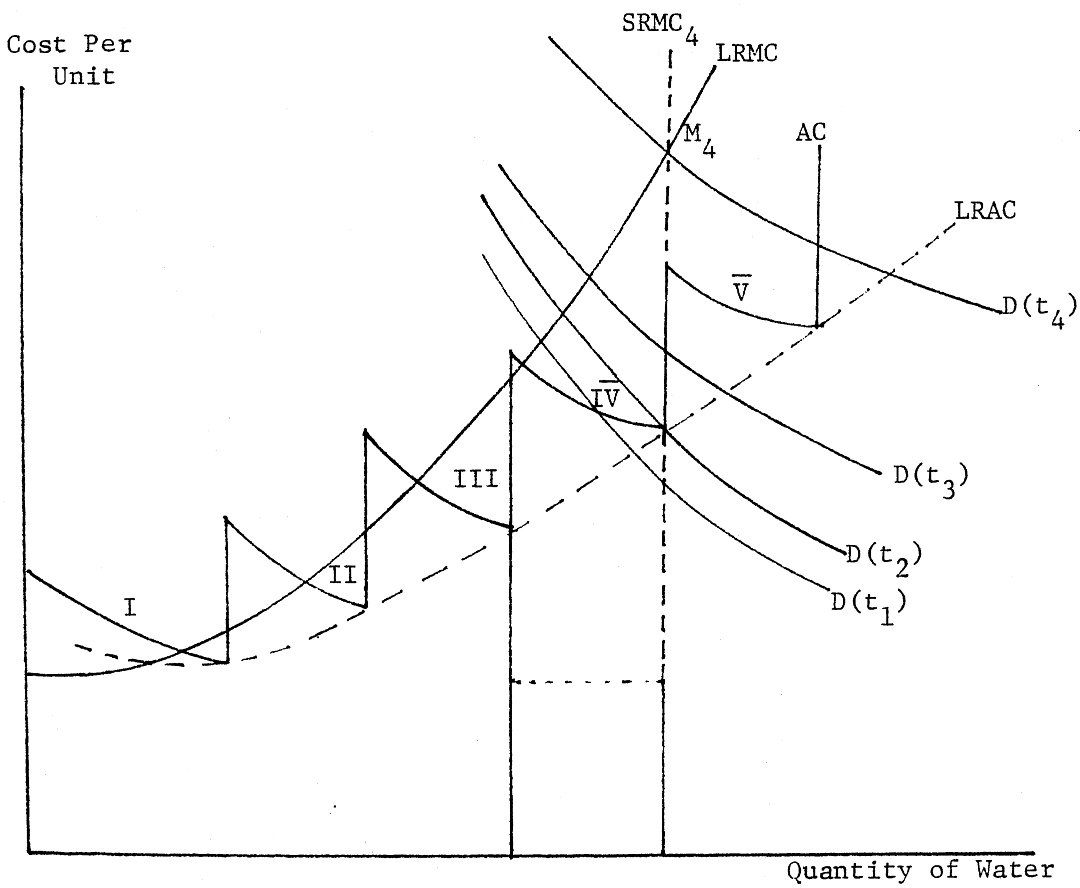

Figure 5. Long Run and Short Run Average Costs of Water Supply

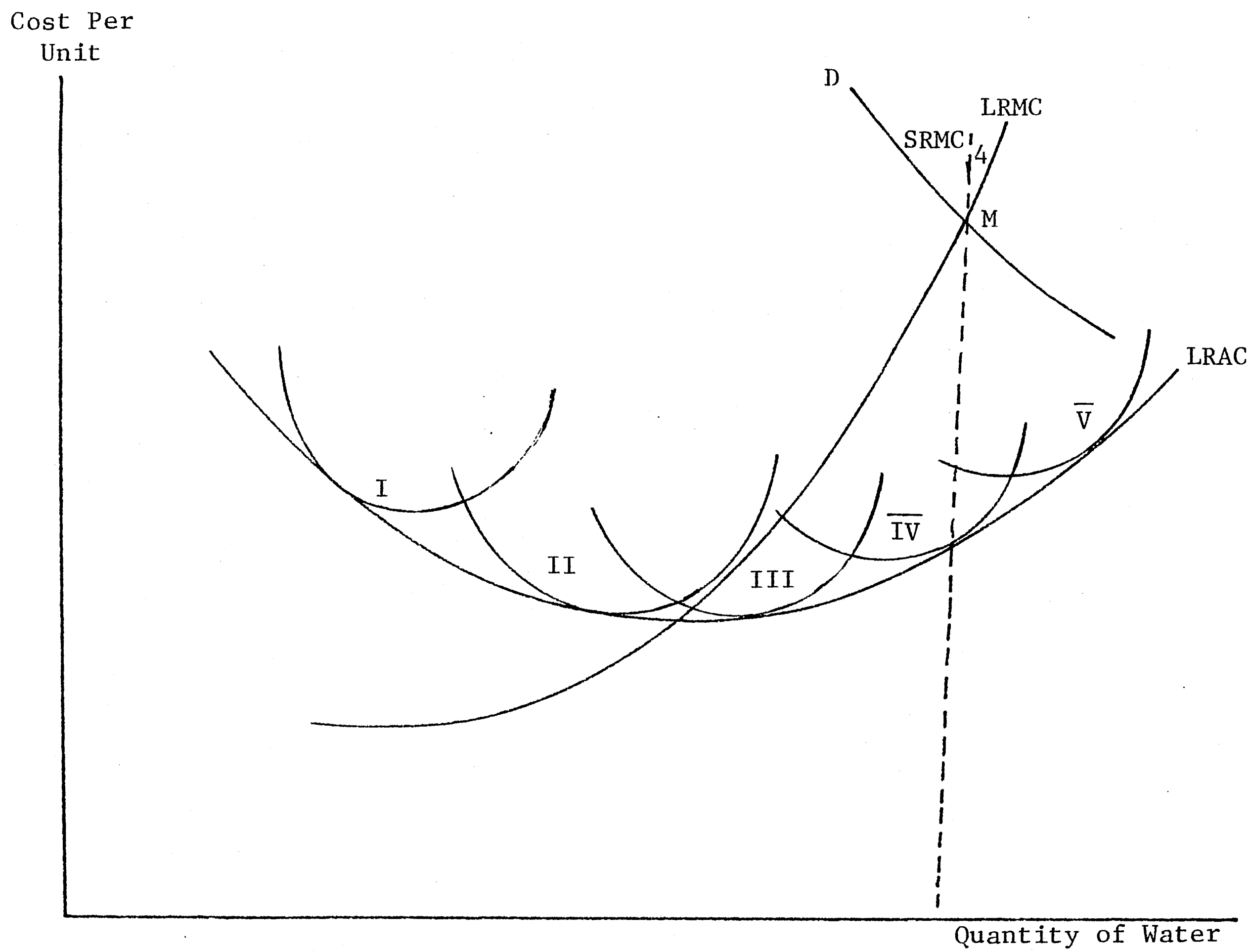

Figure 6. Theoretical Long Run and Short Run Average Costs

costs into one component due to customers, another due to capacity of water system and another due to actual deliveries of water. Rather, water costs may be regarded as varying in three dimensions: the number of customers, the total ability to serve or deliver capacity and the actual deliveries (Hirschleifer 1969). However, while total cost cannot be divided among the dimensions, the marginal cost for each is determinable: the cost of adding another customer, with capacity and deliveries constant; the cost of adding a unit of capacity, with customers and deliveries constant; and the cost of increasing delivery by a unit, with customers and capacity constant. These costs are measurable and relevant for pricing if data are available.

With regard to the capacity component of water costs, suppose a community water system accepts a new customer as a member. One of the conditions of the community water system is that the system stands ready to deliver water at any time; that is to say, it stands ready to enter instantly into a contract for delivering water at the option of the buyer. In order to meet this requirement the water system must provide some excess capacity over the actual average demand it can anticipate. From the water system's point of view this cost is the reserve capacity it holds in readiness to serve. The appropriate charge for the reservation of this capacity is the cost of providing a fractional marginal unit of capacity, the fraction being based on the system's reserve factor. In practice, all rural community water systems charge a membership fee which has the exact meaning of capacity costs.

Review of Pertinent Models of Water Resource Pricing and Investment Planning

An extensive literature on water resource investment planning and water allocation has developed over the past two decades. Most of these studies apply mathematical programming techniques to solve the regional water resource planning problems. The major approaches pertinent to this study may be divided into two groups. The first group is the dynamic, multi-period capacity models. These models generally consider a given set of possible investment projects (e.g. reservoirs, water treatment plants) and compute the minimum cost of sizing and sequencing (timing) of these investment decisions to meet a particular set of demands that vary over time. However, these studies usually attempt to meet demands that are not price-sensitive. In this sense, demands are perceived as requirements in the model.

The second group is the models that simultaneously consider the allocation and capacity expansion decisions in planning water resource systems. These studies are based on the critical assumption that water demand is sensitive to changes in price. In addition to reviewing these two major groups of studies, pertinent work that takes excess capacity (caused by economies of scale and social discount rates) into consideration while planning water system development is also discussed.

Capacity Expansion Models

Some of the early models of investment timing and sequencing were presented by Marglin (1963). The sequencing of simple independent projects with fixed scale to meet demand projections at minimum cost were first addressed by Butcher, Haimes, and Hall (1969). Erlenkotter (1973a) proposed a direct ranking approach for appropriate sequencing decisions.

Extension of the simple dynamic programming sequencing framework to incorporate capacity independence among (hydroelectric) projects was first proposed and demonstrated by Erlenkotter (1973b). Becker and Yeh (1974a, 1974b) considered the problem of project independence in developing firm water supply for a river basin. Their approach associates a "firm" yield with each reservoir configuration considered in their dynamic programming sequencing, timing and sizing model. This "firm" yield is determined by routing the most critical period flows through each candidate configuration. The complication of independent project scale decisions was addressed further by a sequence determination framework. Another approach developed by Martin (1975) utilized a dynamic programming screening technique coupled with a network-with-gains algorithm to determine the optimal capacity expansion policy for a surface water supply system. All of these dynamic programming models minimize cost of meeting a prespecified, price-insensitive, dynamic (changing over time) demand.

Another attempt at the joint treatment of scale and sequencing was made by Jakoby and Loucks (1972) in a three stage procedure. They used a static linear programming model to obtain the initial project scale decisions. These projects, with scale now fixed, are sequenced with dynamic programming. The final solution is then evaluated in a simultaneous model. Although this conjunctive use of planning models and simulation models is a useful approach, it still does not guarantee a global solution.

More recently, Steiner (1977) has formulated a mixed integer programming model to determine the capacity expansion of a regional water resources system. Although marginal water costs have been explicitly computed and used as basis for pricing water in this framework, it still treated the water demand as price-insenstive.

Water Pricing and Capacity Expansion Models

Riordan (1971a) was first to use a more general economic efficiency criterion to obtain a solution to the pricing-investment problem. In this work a price-senstitive demand for the output of the projects under consideration is introduced and a marginal cost pricing criterion is defined as required for economic efficiency. Riordan (1971b) later applied this model to an investment-pricing problem in an urban water supply facilities system using hypothetical cost and demand curves.

Cysi and Loucke (1971) also used dynamic programming and price-sensitive demand to argue that increasing block rates were welfare maximizing in the long run for water treatment facility planning. Regev and Schwartz (1973) have used discrete time control theory to formulate an interregional water investment and allocation model. Seasonal prices were explicitly considered. The results are general, but not operationally computable. Rogev and Lee (1975) also developed a planning model for a river basin development using dynamic programming methods. Their model was used to find the optimal timing and scheduling of reservoir projects in a river basin when the demand is price-sensitive. Haimes and Hainis (1974) proposed an operational framework by incorporating an input-output demand model with a dynamic programming scheduling algorithm for a regional water supply system.

More recently a price-sensitive investment model was developed by Moore (1977) as an extension of the work by Becker and Yeh (1974b) on the sequencing, timing and sizing of project investment work. Armstrong and Willis (1977) also formulated and demonstrated an investment and allocation model for water resources planning. They used the generalized Bender's decomposition approach to solve the resulting nonlinear mixed integer programming problem. Adapting the sequencing algorithm of Erlenkotter and Rogers (1977b), two general frameworks for investment planning with price-sensitive dynamic demand have been proposed and illustrated by Erlenkotter and Trippi (1976) and Erlenkotter (1977a).

Optimum Excess Capacity Model

All of the above models were demonstrated to achieve appropriate planning schedules of overall water resources allocation with relatively little attention to deriving optimum excess capacity of water supply facilities such as the size of water mains or capacity of storage tanks to meet price-sensitive, growing intertemporal water demand.

Lynn (1973) was one of the first to address the problem of optimal facility scale. His work was preceded, however, by Chenery (1952) who developed a simple model for determining the optimal excess facility expansion. Chenery's model was redefined and extended by Manne (1961) whose work has received much attention from civil engineers. However, a basic problem with Mann's model is that the mathematical expression for the optimal design period is an implicit function and in order to calculate optimal excess capacities, trial and error or numerical techniques are necessary. To overcome this limitation, Lauria, Donald and Schlenger (1977) presented an approximating equation by which optimal excess capacity design periods can be calculated directly. Whereas Manne's work is limited to capacity expansions, Thomas (1970) extended Manne's model by including the optimal scale of a system for which the level of demand exceeds the capacity of supply facilities at the beginning of the planning horizon. Thomas' model also was approximated by Lauria, Donald and Schlenger (1977). Although optimal excess capacity design periods have been explicitly computed, again in the weakness of these models is that they do not have a global solution due to assuming water demand implicitly as price-insensitive.

Distinctive Aspects of This Study

In comparison with earlier studies, the approach developed here for planning a rural water supply system differs in several aspects. First, the optimum excess capacity for initial and expansion systems are computed as an upper limit of the system. Economies of scale of water supply facilities are incorporated at a given discount rate to obtain the optimal excess capacity design. Second, price-sensitive demands are considered in the model. They are used not only to indicate the social benefits of water supply but also to yield the socially optimal prices, reflecting the cost of investments and operation and maintenance. Third, public investment in existing rural community water services in Oklahoma under various uncertain growth patterns are evaluated by comparing those systems against the optimal prices and excess capacity designs resulting from the model.

CHAPTER III

ECONOMICS OF RURAL COMMUNITY WATER DEMAND AND SUPPLY IN OKLAHOMA

Introduction

In this chapter the empirical analysis of water demand, costs of water supply and growth of water systems in rural Oklahoma are specified. Demand theory is reviewed, procedures of the demand analyses are presented and empirical results of demand estimation from cross-section data on rural water systems in Oklahoma are given. Cross-section data as well as historical data are analyzed for system cost and growth. Specifically, samples, procedure of data collection, procedure of analyses, empirical results and policy implications are discussed.

Analysis of Water Demand

Consumer and Market Demand for Water

Consumer demand is defined as the various quantities of water which a consumer is willing and able to buy as water rate (price) varies with all other factors affecting demand held constant. In community water systems, since the consuming unit is generally a household, the dwelling unit or water tap can be treated as the consumer. Consumer demand simply defines the relationship between price and the quantity purchased per unit of time while holding other factors constant. Since water is generally not considered to be an inferior good, price and quantity are expected to vary inversely and can be explained in terms of the substitution and income effects of a price change.

Market demand is a generalization of the consumer demand concept. It is defined as the quantity of a commodity which all consumers in a system are willing and able to buy as price varies while all other factors are held constant. A market demand relation can be thought of as a summation of individual demand relations. A change in price may result in changes in demand through changes in the number of consumers participating in the community water system as well as changes in the quantity purchased per customer.

Changes in Demand

It is important to distinguish between a change in quantity purchased and a change in demand. The former is a movement along the demand curve and the latter is a shift in the level of the curve. There are many factors influencing the level of demand: (1) population size and age distribution, (2) consumer income and its distribution, (3) prices and availability of other commodities, and (4) consumer tastes and preferences. These factors are called "determinantes of demand".

Since there are few replacable goods for water, it is a fair assumption that consumer's tastes and preferences for water are relatively constant or change slowly through time. Also, since the income effect on water consumption for relatively homogeneous households in rural Oklahoma is assumed trivial, we can conclude that the size of population is the most important factor explaining changes in demand for water.

A distinction between "parallel" shifts in demand and "structural changes" in demand for water can be made. We assume a simple water demand equation in which water quantity (Q) is a linear function of its price (P) and of population (N), i.e., $Q = \alpha - \beta P + \gamma N$ where α, β and γ are parameters which indicate how the variables are related. A demand curve of Q and P can be plotted for a fixed level of population. If the level of population changes, then the P-Q function shifts to a new level. This illustrates a parallel shift in demand. However, it is also possible that the parameters α, β and γ may change; that is, the coefficients relating the structure of the variables may change. In this study, no structural change in water demand is assumed but only shifts of demand due to growth of population.

The Theoretical Model of Water Demand

The market demand of rural water systems is used as the unit of analysis for this study since data are not available for individual household consuming units. Focus of the present study is the examination of factors explaining water demand behavior among rural systems in order to assist planners in the design of such systems. The market demand for water directly relates to capacity of the system. Therefore, market demand is considered for purposes of planning system capacity and not for determining simple price-quantity relationships.

As reviewed in Chapter II, previous research indicates that consumers do respond when water rates are increased. To predict water demand for rural areas in Oklahoma, the important variables are hypothesized to be price, number of residential taps and number of nonresidential taps. The aggregate water demand function can be expressed as the following:

$$Qad = f(P, N_r, N_{nr}) \qquad 3.1$$

where

Qad = average annual water quantity in millions of gallons per year (mgy)

P = average water charge per 1,000 gallons

N_r = total number of residential taps (as a surrogate for population)

N_{nr} = total number of nonresidential taps

Theoretically the marginal price of water should be used as the price variable. But practically it is difficult to find a representative marginal cost in the aggregate for a water system. However, since most water systems issue water bills by month, it is assumed for this study that consumers respond to water consumption based upon the total monthly water bill. Average cost per thousand gallons computed for the system is assumed to be the marginal price of water for that system.

Most domestic water demand studies divide users into four or more groups such as: residential, commercial, industrial and other. In rural water systems, unlike urban water systems, there are few commercial or industrial water users. Thus, for simplifying purposes, only two groups of water users will be considered in this study: residential and nonresidential. In rural areas, the majority of nonresidential users are small businesses or pasture taps for gardens and livestock. The nonresidential users, on the average, consume more water per tap than residential users. Theoretically, nonresidential users may be assumed to be more price-sensitive because their choices of whether to consume water or not is more flexible. They can also consider alternative sources of water such as ponds or wells if the costs of these alternatives are cheaper than consuming community water.

The total number of residential users in a system increases not only from an increase in the density of homes within a water district boundary but also from expansion of the water district boundary itself. However, since the objective is to plan water systems based upon a price-sensitive demand as opposed to requirement approach, it is assumed that population increases do not shift demand curves until consumers are willing and able to pay for community water. Aggregate demand and the number of total residential and nonresidential taps is expected to move in the same direction.

The Study Area and Data

Even though there is increasing rainfall moving from western to eastern Oklahoma, climatically the whole area of Oklahoma can be considered a semi-arid region. Since the Dust Bowl period, considerable legislation and assistance programs have been initiated by Federal and state governments to cope with community water problems of such regions. In 1961, the Federal government initiated the National Rural Water Program and Congress granted authority to the Secretary of Agriculture to make loans and grants through the Farmer's Home Administration (FmHA) for allowing organization, formation and operation of public nonprofit rural water systems.

In 1963, Nowata County Rural Water District No. 2 was organized as the first nonprofit rural water system in Oklahoma. Through mid-1979, a total of 398 water systems funded under this program were serving slightly over one-half million people in Oklahoma. Each water system utilizes its own pricing structure, generally decreasing block rate, while all incorporate a flat rate for the first few thousand gallons of water consumed. This information provides an excellent opportunity to illustrate water demand relationships since each system provides water at a different price (rate). In this manner a cross-section of users, stratified by water system if needed, can be used to form the empirical counterpart of a residential water demand study without the need of resorting to time series data.

In this study data from 203 water systems are used which have the complete information needed (Rural Water Systems in Oklahoma 1980). From these systems, the following specific data (see Appendix A) are derived:

1. AGWAD - The aggregate water demand (AGWAD) per year expressed as millions of gallons per year (mgy) is computed by multiplying the average water consumption per day by 365 for each water system. The AGWAD represents the aggregated consumer's water consumption behavior and also implicity reflects the operating levels of a system at a particular time.

2. WAPR - The water price (WAPR) variable represents the dollar value per thousand gallons of water. This variable is derived by dividing the monthly average water bill for a system by the monthly average water consumption per tap and multiplying by 1,000. For example, if the monthly average water bill per tap is $15 for a system and the monthly average water consumption per tap is 8,000 gallons, the WAPR is $1.875/1,000 gallons.

3. RESID - The RESID represents the total number of residential taps in a system at a given time.

4. NONR - The NONR is the total number of nonresidential taps in a system at a given time.

5. TNTAP - The TNTAP is the total number of taps (RESID plus NONR) in a system at a given time.

Empirical Estimates of Water Demand

The following water demand equations were empirically estimated:

$$AGWAD = f(WAPR, RESID, NONR) \qquad 3.2$$

$$AGWAD = f(WAPR, TNTAP) \qquad 3.3$$

Regression coefficients were estimated in linear and log-linear form by conventional single equation least squares methods. The estimated regression equations with standard errors of the estimates (S.E.), R^2 and sample size (n) are given below:

$$\begin{array}{llllll} AGWAD = & 25.07 & - 16.04\ (WAPR) & + 0.12\ (RESID) & + 0.31\ (NONR) & \\ S.E. & (6.41) & (2.75) & (0.005) & (0.05) & 3.4 \end{array}$$

$R^2 = .78 \qquad n = 204$

$$\begin{array}{llllll} \ell nAGWAD = & -1.97 & - 0.59\ell n\ (WAPR) & + 0.95\ n\ (RESID) & + 0.11\ell n\ (NONR) & \\ S.E. & (0.23) & (0.59) & (0.03) & (0.04) & 3.5 \end{array}$$

$R^2 = .86 \qquad n = 204$

$$\begin{array}{lllll} AGWAD = & 26.80 & - 16.91\ (WAPR) & + 0.13\ (TNTAP) & \\ S.E. & (6.61) & (2.83) & (0.005) & 3.6 \end{array}$$

$R^2 = .77 \qquad n = 204$

$$\begin{array}{lllll} \ell nAGWAD = & -2.38 & - 0.57\ell n\ (WAPR) & + 1.03\ell n\ (TNTAP) & \\ S.E. & (0.19) & (0.07) & (0.03) & 3.7 \end{array}$$

$R^2 = .87 \qquad n = 204$

Results show that all of the regression coefficients are statistically significant at the one percent probability level. In equation (3.4) the coefficient of WAPR shows that if the price of water increases one dollar per thousand gallons, holding other variables constant, it will reduce aggregate annual water consumption for the system about 16 million gallons. In equations (3.5) and (3.7) the coefficients of ℓn WAPR, -0.59 and -0.57, can be interpreted directly as the price elasticity of aggregate water demand in rural Oklahoma. This range of price elasticity for rural Oklahoma is higher than the estimated price elasticity for urban areas of about -0.4 (Riodan 1971b). This higher price sensitivity could be explained in that rural areas generally have alternative sources of water such as wells, streams or small ponds for domestic and nondomestic purposes whereas urban areas rely totally on public water supplies.

In Oklahoma nonresidential taps are mainly pasture taps and water consumption per tap of nonresidential users is higher than that of residential users. The coefficient of NONR in equation (3.4) means that if we increase the number of nonresidential taps by one holding other variables constant it will increase aggregate water consumption by 0.31 million gallons per year. The comparable amount of RESID is 0.12 million gallons.

In equation (3.7) the coefficient of total taps (TNTAP), 1.03, is essentially the demand elasticity of population. Statistically we can test whether this elasticity is significantly different from one.

Ho: the coefficient of ℓn(TNTAP) is equal to one

Ha: Not the null

Since the calculated t value, 0.96, is so small we fail to reject Ho. Statistically, 1.03 is not significantly different from one which means that if we increase total number of taps by one percent it will increase aggregate water demand by approximately one percent. Thus, we can conclude that there is a proportional one-to-one relationship between water demand and number of taps.

Policy Implications of Demand Analysis

In the foregoing discussion it was found that community water demand is explained by water rates and number of residential and nonresidential users. In the short run, the number of users can be assumed to be constant, thus, the above models provide demand functions permitting forecasts of price impacts on water use. The results also show an inelastic demand for water but not infinitely inelastic demand. Thus the price of water will affect the demand for water.

It was proposed in Chapter II that water rates be set equal to the marginal cost of providing additional water. There is little doubt that these marginal costs will be different for different classes of customers, for increments of water to existing customers, for extension of the service area and for peak and off-peak periods. Furthermore, the ways in which a system is designed will clearly affect the costs of supplying water to different classes of users and for different periods of time. Thus, the objective of determining the price of water which maximizes social benefits must take into consideration the demand for water and the cost of supplying water. The empirical finding on the demand for water combined with additional information on the cost of water systems will be used in the following chapter to find a practical approximation of optimum water system capacity.

Analysis of Water System Supply Cost and Growth

Background Information

Most rural water supply systems, in contrast to large urban water systems, are characterized by low population densities, high initial investment costs per consumer and low household per capita incomes. The basic economic problems for many rural communities are the lack of funds to finance the initial capital costs of water systems and the difficulty in covering costs of operation and maintenance (O and M). The Farmers Home Administration (FmHA) in the past has provided financing, and in some cases grant funds, to publicly-owned rural water systems for unincorporated communities, small towns and dispersed farm populations not exceeding 10,000.

In general, the source of water supply has a significant influence on the total water system investment cost (Sloggett 1974). The investment in treatment plants and wells represents a significant share of total water system investment cost. This cost study is limited to only those water systems purchasing treated water from neighboring systems but could be extended to systems requiring water treatment and water sources. For this study, the capital cost of water distribution is the main investment cost. The O and M cost is hypothesized to be a direct relationship to output or amount of water delivered per unit of time.

Statistically Ideal Data for Estimating Cost Functions

To understand the data deficiencies in the present study, statistically ideal data for estimating O and M and investment costs are reviewed. Theoretically, analysis of O and M costs involves the assumption that the water system's delivery is constrained by some fixed capacity limit. The ideal data for O and M costs are a series of observations on costs and output which satisfy the following conditions (Johnston 1960):

1. The basic time period for each observation should be one in which the observed output was achieved by a uniform rate of production within the period. It would not be

> desirable, for example, to have one year as the basic time period if there were substantial seasonal variations in the rate of production, for the one year figures would then be averages which might obscure the true underlying cost curve.
>
> 2. The observations on cost and output should be properly paired in the sense that the cost figure is directly associated with the output figure.
>
> 3. Output observations should be widely spread so that cost could be observed at differing rates of output.
>
> 4. The observed data should be adjusted for the influence of factors extraneous to the cost-output relationship itself. (p. 26)

To examine investment costs, or the long run relationships, essentially similar requirements apply. The basic unit of time to which individual observations relate should be short enough to avoid possible average effects and the cost-capacity observations should again be properly paired. The requirement of a wide range of output observations is more stringent than with O and M costs for now there must be observations on systems with widely different capacity limits, ideally ranging from very small to very large systems. Also, ideally, each system, of whatever scale, should be producing within that scale in the most economically efficient manner given the current state of technology and the current range of factor prices.

With data satisfying the above requirements, it would be a relatively simple matter to examine the validity and practical relevance of various hypotheses about O and M and investment costs. However, in the rural world, there are few firms whose data satisfy these requirements because few are setting their output or capacity levels to achieve a statistically desirable spread of observations. Thus, if a large cross section of firms in a given industry were examined very few would be found with any given capacity limit.

Sample of Oklahoma Rural Water System

Sloggett (1974) surveyed 57 rural water systems in 1972 to study the economics and growth of rural water systems in Oklahoma. Major criteria for selection of the sample were as follows: systems selected must have been in operation for at least two years to assure adequate operating records; different size systems measured in terms of numbers of customers were included (the range was from 16 to 1400 customers); systems included different sources of water supply - wells, lakes and streams, and purchase of treated water; and systems included different densities of customers per mile of line and represented rural only, town only and a combination of town and rural. The systems were also selected to represent geographical distribution of all rural water systems located in the state. This study, however, was limited to the 30 systems purchasing treated water.

The sample was resurveyed in 1981 to extend the data series and include information on changes in capacities and growth of water systems measured by the annual amount of water delivered or the number of users. One of the 30 water systems of Sloggett's sample added its own treatment facility after the original study and hence was dropped from the sample. For each system in the sample information was obtained for each year back to its beginning year, or to the original survey. An example of the survey data collected by rural water district is presented in Table I.

Method of Analysis and Data

There are three main problems involved in the derivation of cost functions for rural water systems. First is the determination of capacity

TABLE I

EXAMPLE OF SURVEY DATA FOR RURAL WATER DISTRICT
CREEK #4, OKLAHOMA

Year	Initial Construction Cost ($)	Capital Additions ($)	Amount of Water Sold (mgy)	Number of Users	Water Purchases ($)	Salaries ($)	Utilities ($)	Office Expense ($)	Insurance and Bonds ($)	Legal and Audit ($)	Other ($)
1968	232,189		13,814	203	3729	2051	263	179	212	672	2331
1969		970	16,714	260	7012	2078	1916	214	212	655	2072
⋮											
1980											

of existing systems. Since capacity of water distribution systems is a flow concept instead of a stock, there is no clear determination of capacity especially when the time unit of measurement is long such as a year. For this study, capacity was determined as the annual output of water the year prior to a major addition such as water storage, booster pumps to increase water pressure or parallel distribution lines. Second is adjustment of the investment and O and M cost data to remove the influence of factors other than output. The third problem is to determine statistically the best estimated fit of the data to the relationship between cost and capacity or output.

Investment Cost Data. For the systems purchasing treated water, the main facilities are water lines, storage tanks, meters, booster pumps, office and equipment. Capacity is the outcome of certain combinations of the individual components in the distribution system. Specifically, water lines, storage tanks and booster pumps are the main facilities to determine overall capacity while office and equipment are supporting components to maintain a given capacity.

As discussed previously, it is not easy to determine the installed capacity empirically even though information is available on each and every component of the system. Only with detailed engineering studies is it possible to determine the exact capacity of a system. Because of cost and time constraints, an alternative method was considered in determining the approximate capacity of the sample of systems. The alternative method assumes that when a system adds facilities such as parallel lines, storage tanks or booster pumps, it has reached its capacity. The volume of water flowing through the system before the addition(s) is assumed to be the capacity of the system. The added facility has now increased system capacity which is measured again at the time of a further addition.

From the capital improvement records, the year just before addition to facilities described above is interpreted as the year when the water system reached its maximum capacity. The amount of water delivered in that year is assumed to be the system's maximum capacity. The initial construction costs including various minor capital improvements from year two to the year the system reached its maximum capacity are deflated with an appropriate price index to year one. The construction cost index employed is presented in Table II. The deflated costs are interpreted as the investment costs equivalent to capacities measured for each water system. However, since the sample includes water systems starting operation in different years, all investment costs are deflated again to year 1965 when the year of the oldest system in the sample started operation. Because of lack of records, only 22 systems are qualified to be used for the investment cost analysis. Data for the 22 systems are presented in Table III.

Operation and Maintenance Cost Data. The O and M costs were divided into seven categories and obtained from annual audit reports to FmHA. The information was provided by bookkeepers or managers from the individual water systems. Categories of O and M cost are as follows:

Water purchases - cost of treated water purchased for consumption within the water system.

Salaries - payments on a regularly scheduled basis to employees and managers, including employee taxes.

Utilities - cost of electricity and other utilities to operate the system.

TABLE II

CONSTRUCTION COST INDEX FOR DEFLATING RURAL WATER SYSTEM INVESTMENT COSTS

Year	Index	Year	Index
1965	100	1973	182
1966	104	1974	192
1967	107	1975	208
1968	115	1976	227
1969	126	1977	246
1970	133	1978	270
1971	151	1979	290
1972	167	1980	311

Source: Based on general construction cost index compiled by Engineering News Record, McGraw Hill Publishing Co., Highstown, NJ, March 20, 1980.

TABLE III

INVESTMENT COSTS AND CAPACITIES OF SAMPLE WATER SYSTEMS, OKLAHOMA

Name	Investment Costs[a] ($)	Capacity (1,000 gal)
Kay #2	11,907	6,000
Creek #5	248,583	33,046
Nowata #5	24,984	22,000
Rogers #6	121,821	26,400
Rogers #7	176,820	11,316
Rogers #8	225,693	14,685
Washington #1	175,265	22,386
Washington #2	394,649	23,752
Mayes #2	336,370	24,813
Mayes #4	363,007	19,863
McIntosh #5	165,222	8,732
Muskogee #1	240,689	19,987
Muskogee #2	350,796	45,643
Okmulgee #1	255,052	12,134
Okmulgee #4	186,948	10,200
Murray #1	338,195	17,394
Latimer #1	586,118	30,000
Leflore #2	252,306	11,273
Leflore #3	313,627	14,067
Leflore #5	206,156	13,427
Pittsburg #7	363,363	29,823
Pittsburg #9	128,975	8,750

[a]1965 dollar value.

Office expense - cost of items such as telephone, stationary and postage.

Insurance and bonds - all insurance premiums and payment of bonds for employees.

Legal and audit - all legal and auditing fees.

Other - maintenance was included in this category. This was necessary because it was difficult to identify maintenance expenditures from available records. For example, costs of new meters and water line extensions were often included in maintenance account. These items were removed and specified in capital improvements if the records were sufficiently detailed to enable this adjustment. Miscellaneous items included in "other" were checmicals, billing and collection fees, travel expenses, rent and equipment repair.

The seven O and M cost categories were added together to derive annual O and M cost which was paired with annual output in millions of gallons of water per year. Deflated time series on O and M cost from individual systems were combined with cross section data from the entire sample of systems to estimate an overall O and M cost function. This procedure involves two assumptions: first, that changes in relative factor prices have not resulted in any substitution between factors in the production process and, second, that changes in the system's output (amount of water supplied) have not had any influence upon factor prices. The first assumption is justified since labor has limited substitution for utilities in the pumping of water. The second assumption seems equally reasonable in open regional economies, even in the case of the very largest water system. Data for the O and M cost analysis are presented in Appendix B.

Empirical Estimates of Water Supply Cost

Single-equation least-squares methods were used to estimate the parameters of O and M and investment cost functions, treating cost as the dependent variable and output, capacity and density as the independent variables. Average cost equations of O and M and investment cost are also estimated to see the existence of economies of scale for the sample of rural water districts.

Operation and Maintenance Cost. The regression coefficients, standard error of the estimate (S.E.) and the correlation coefficients for the different O and M cost models are the following:

$$\begin{array}{lllllllr}
\text{OMCOST} & = 24278.7 & + 353.7Q & & & & R^2=.45 & 3.8 \\
\text{S.E.} & (2266) & (33.6) & & & & & \\
\text{OMCOST} & = 15118.7 & + 345.2Q & - 1130.8D & & & R^2=.47 & 3.9 \\
\text{S.E.} & (4601) & (33.3) & (496.7) & & & & \\
\text{OMCOST} & = 7290.9 & + 1086.5Q & - 271Q^2 & & & R^2=.64 & 3.10 \\
\text{S.E.} & (2698) & (89.7) & (0.32) & & & & \\
\text{OMCOST} & = 8630.1 & + 1105.0Q & - 2.77Q^2 & - 213.7D & & R^2=.65 & 3.11 \\
\text{S.E.} & (3868) & (97.7) & (0.34) & (-40.9) & & & \\
\text{AOMCOST} & = 1611.5 & - 6.3Q & + 0.025Q^2 & - 0.011QCAP & - 28.3D & R^2=.30 & 3.12 \\
\text{S.E.} & (126.3) & (3.2) & (0.020) & (0.018) & (12.4) & &
\end{array}$$

where

OMCOST = total operation and maintenance cost in 1965 dollars

AOMCOST = average O and M cost in 1965 dollars

Q = amount of water delivered in million gallons per year

D = density in terms of number of users per mile of water line

QCAP = Q times capacity of water system.

Equation (3.8) contains only a linear term in output Q. This equation yields constant marginal and average O and M costs. Equation (3.9) contains a term in D, density, expressed by the number of users per mile of water line. The economic interpretation of a negative coefficient on D is that the cost function is shifting downwards as density of users increases. Coefficients are all statistically significant at the two percent probability level in equations (3.8) and (3.9). However, the R^2 is only 0.45 and 0.47, respectively. Equations (3.10) and (3.11) include a quadratic term in output, Q^2, and equation (3.11) has a term for D. The coefficients of Q and D remain statistically significant at the three percent probability level. In both equations, the coefficients of Q^2 are statistically significant at the one percent probability level with the sign negative. This result gives continuously decreasing average variable cost and marginal cost, contrary to theoretical expectations. The R^2 increased to 0.64 and 0.65, respectively.

The quadratic function with a negative term in Q^2 may be just the first section of a third degree polynomial, the second section not being observable in practice. The reasonableness of this hypothesis can only be tested by examining the size of the larger outputs of each system relative to capacity. For this reason, average O and M cost was regressed against quantity, density and a variable measuring system capacity. These results are given in equation (3.12). Capacity is entered as an interaction variable with quantity since the capacity variable itself is highly correlated with quantity. The average O and M cost equation has low R^2 but the signs of the parameters are consistent with U-shaped short run O and M costs and slightly decreasing long run O and M costs. For purposes of the programming model described later, O and M costs are considered linear and proportional to quantity of water delivered.

Investment Cost. Estimated regression coefficients, standard errors of the estimate and correlation coefficients for the different capital cost models are the following:

$$\begin{array}{ll} \text{CAPCOST} = 103456.4 + 7973.8S & R^2=.59 \quad 3.13 \\ \text{S.E.} \quad (46710) \quad (2220.8) & \end{array}$$

$$\begin{array}{ll} \text{CAPCOST} = 189128.6 + 12231.6S - 17912.1D & R^2=.66 \quad 3.14 \\ \text{S.E.} \quad (60490) \quad (2329.8) \quad (6862.3) & \end{array}$$

$$\begin{array}{ll} \text{CAPCOST} = 24888.6 + 23009.5S - 336.9S^2 & R^2=.51 \quad 3.15 \\ \text{S.E.} \quad (7246) \quad (7145.6) \quad (153.5) & \end{array}$$

$$\begin{array}{ll} \text{CAPCOST} = 78707.0 + 25379.5S - 356.7S^2 - 16214D & R^2=.71 \quad 3.16 \\ \text{S.E.} \quad (96256) \quad (9382.9) \quad (247.1) \quad (6730.8) & \end{array}$$

$$\begin{array}{ll} \text{ACAPCOST} = 35.63 - 0.0012S + 0.0000002S^2 - 0.757D & R^2=.46 \quad 3.17 \\ \text{S.E.} \quad (10.35) \quad (0.0010) \quad (0.00000003) \quad (0.472) & \end{array}$$

where

CAPCOST = capital investment cost in 1965 dollars

ACAPCOST = average capital investment cost in 1965 dollars

S = capacity measured as millions of gallons per year

D = density in terms of number of users per mile of water line at time of capacity

The density variable D in equations (3.14) and (3.16) again indicates that capital investment costs are influenced by the dispersion of users. As in the case of O and M cost, the sign of the quadratic capacity variable, S^2, in equations (3.15) and (3.16) is negative. The average capital cost equation (3.17) shows the existence of economies of scale up to the capacity of 30,000 mgy. Byond this capacity average capital costs tend to increase marginally.

Growth of Water Systems

Growth in water demand is the direct reason why excess capacity should be considered in planning of a water system. In this sample, all rural water systems have grown in number of customers to some degree. Sloggett (1974) discussed various factors contributing to growth including age of the system, per capita income in the county where the system is located, and distance of the system to the nearest growth center. In this study, only age of the system is considered paramount in describing water system growth.

Growth of the individual water systems is computed in an index form with the initial year of the system equaling 100 (Table IV). An overall index of growth for the sample of rural water districts was computed and is presented in the last column of Table IV.

Using the overall index as a dependent variable and year (age) as an independent variable, two different models were fitted: (1) a linear model and (2) an exponential model. The results are presented in equations (3.18) and (3.19) respectively. Both equations have high R^2 and statistically significant coefficients (significant at one percent probability level):

$$Z_t = 75.6 + 14.7t \qquad R^2 = .99 \qquad 3.18$$

S.E. (4.4) (.51)

$$n_t = 4.6 + 0.0819t \qquad R^2 = .98 \qquad 3.19$$

S.E.(0.02)(0.002)

where

t = year (age)

Z_t = index of number of users in year (age) t

In equation (3.19) the coefficient of t, 0.0819, can be read directly as an annual growth rate and is equal to about eight percent.

TABLE IV

INDEX OF GROWTH IN NUMBER OF CUSTOMERS FOR A SAMPLE OF RURAL WATER DISTRICTS IN OKLAHOMA

Year	Woods #1	Kay #2	Kay #3	Creek #2	Creek #5	Nowata #5	Roger #6	Roger #7	Roger #8	Wash #1	Wash #2	Mayes #2	Mayes #4	McInto #5	Muskogee #1
1	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2	106	108	102	119	108	100	105		107	102	129				
3		108		128	115	100	110		119	105					
4	118	100	127	139	122	100	139		143	107					
5	126	108		157	127	100	178		167	112			134		
6		123		177	163	100	185	140	131	115	134	250	139		
7		123		195	167	100	190	235	188	116	143	263	156		
8	132	131		215	179	100	215	257	206	138	151	333	165	159	175
9		200		230	207	97	237	275	225	149	162	336	169	163	182
10		200		246	219	97	265	295	239	160	175	353	184	179	180
11	162	138		272	235	97	294	386	252	166	195	369	206	200	201
12		138		289	244	97	332	417	260	184	230	400	223	210	201
13		138		306	264	100		498		198	238	426	244	231	232
14				323		100		504		205	248	426	275		276
15				323		100				213		445			279
16						100									

TABLE IV (Continued)

Year	Okmulgee #4	Grade Norge	Murray #1	Latimer #1	Leflore #2	Leflore #3	Leflore #5	Pitt #6	Pitt #7	Pitt #9	Push #3	Ave.
1	100	100	100	100	100	100	100	100	100	100	100	100.0
2			100		113		116				100	107.8
3		193	106						111	276	118	132.4
4			110					109	119	280	122	130.1
5			117	125			165	116	136	284	137	143.7
6	136		170	135		123		117	143	294		153.6
7	146		207	138		123			148	322	156	171.4
8	187		229	147	213	144	207		134	342		188.1
9	215		247	161	239	148	212	125	204	352		203.5
10	230		262	174	265	158	225	126	242	362		217.2
11	234		286	177	304	158	237	128	258	428	446	240.1
12	257		333	192	342	167	257	136	268	452		253.6
13	271		375	201	393	205	270	161	272	462		270.9
14	283		408	219	421	211						293.7
15					448							289.7
16					500							300.0

Policy Implications From Analysis of Supply, Cost and Growth

Results of the analysis of water supply costs show that there are significant economies of scale in rural water system investment and operation and maintenance. The growth analysis, which showed an overall eight percent annual growth rate measured in terms of number of customers, strongly supports the excess capacity model as a framework for planning optimum water system capacity. Failure to optimize on excess capacity may lead to under investment or over investment in community water systems and thus reduce social benefits due to inefficient allocation of resources. Under investment for any particular community may force duplication of facilities (parallel lines) which could have been avoided if optimal capacity were planned from the beginning. Therefore, the objective of determining the optimum capacity of rural water systems which maximize social benefits must incorporate expected growth in water demand as well as the economics of water supply.

CHAPTER IV

AN INVESTMENT PROGRAMMING MODEL FOR RURAL COMMUNITY WATER SYSTEM CAPACITIES WITH PRICE-SENSITIVE DEMAND

A mathematical programming model is developed in this chapter for planning community water system capacity when consumers' water demand is price dependent. The proposed procedure consists of selecting the optimum capacity, sequencing and timing of water system investments. The water rate decision is determined endogenously such that discounted net social benefits are maximized.

First the assumptions of the model are presented. The specific configuration of the model is then described. Computational considerations and solution strategies are discussed. The properties of mixed integer algorithms with branch and bound methods are reviewed. Finally, the basic LP model and the economic interpretation of the optimum solution with the Kuhn-Tucker conditions are presented and discussed.

Assumptions of the Model

The model presented here is based upon a fundamental assumption not ordinarily considered in most water resources capacity decision models. The assumption is that the community water demand is sensitive to changes in price. Furthermore, it is assumed that aggregate demand for water varies over time and can be described by a continuous growth rate.

Based on the empirical results of the last chapter, it is assumed that the price elasticity of demand is constant throughout the planning period. The price-sensitive demand is then used in determining the consumers' willingness-to-pay and the total benefits of a rural water system.

In addition to the above major assumptions in the model, the following assumptions are adopted to reduce needless complications in application to planning optimum water system investment:

1. Water demand in year y is a function of price in that year and no other period.

2. Capital investment costs occur as lump sums at the time of initial construction and for any additions to capacity.

3. The O and M costs occur as lump sums in each year of operations.

4. The capital investment costs for initial construction and any additions are a linear function of capacity and assumed to reflect economies of scale, i.e., the cost per unit of capacity is either constant or decreases with increasing capacity.

5. The O and M costs are a linear function of output.

6. The annual social discount rate, r, is assumed to be constant over time.

7. Inflation effects on benefits and costs are not considered.

8. The planning horizon is chosen as 40 years which is the FmHA's loan repayment period for community water systems and is assumed equal to the anticipated lifetime of the initial water system investment.

Formulation of the Model

The objective of the programming model is to maximize the total discounted net benefits from investments in rural community water systems. The approach is to maximize the difference between the discounted sum of the benefits from water consumption and the sum of the discounted costs of the water system made up of investment and operation and maintenance. This approach is described here in words, graphs and mathematical terms. In addition, a set of constraints necessary for obtaining a mathematical solution to the programming model is formulated.

Benefit Function

The benefits associated with a given consumption of water in this analysis are measured by the consumers' willingness-to-pay which is denoted as the area under the demand curve up to a specific quantity demand level, say Q_y, in Figure 7. It is assumed that there is a one-to-one mapping of Q_y on $P_y(Q_y)$, the demand curve, and that when a value of Q_y is computed, the market-clearing price is also specified. For purposes of illustrating the approach, a linear demand is assumed in deriving the area under the curve although in the actual model a nonlinear demand curve is used.

Given the demand function for rural community water in year y the "willingness-to-pay" is denoted as:

$$f_y\ (Q_y) = \int_o^{Q_y} P_y\ (Q_y)\ dQ_y \qquad 4.1$$

where Q_y is the community water demand in year y and $P_y(Q_y)$ is the inverse demand function. For a given community the "willingness-to-pay" is discounted to the present and summed over the entire planning period:

$$\alpha_y = \frac{1}{(1+r)^y}$$

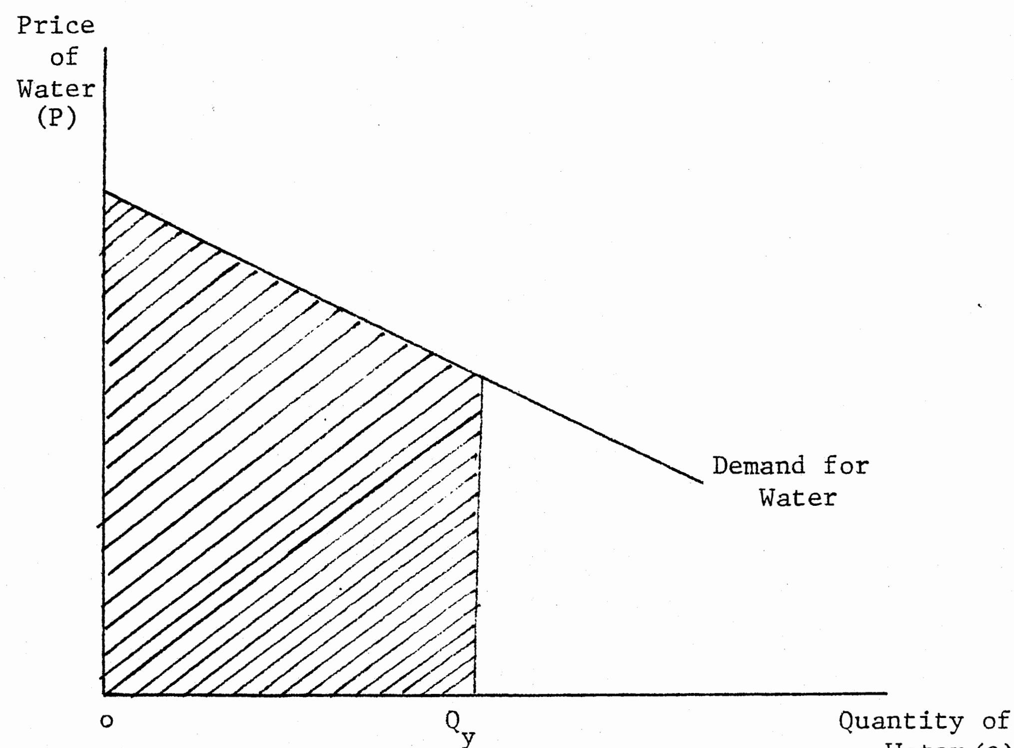

Figure 7. Willingness-to-Pay for Q_y (shaded area) Quantity of Water

where r is the social discount rate. This yields the following benefit function which appears in the objective function of the programming model:

$$TB = \sum_{y=1}^{Y} \alpha_y f_y(Q_y) \qquad 4.2$$

where Y is the length of the planning period in years.

Cost Function

Water system costs in the objective function consist of two major components. The first is the capital cost of the proposed water system. Since it is assumed that capacity reflects economies of scale, the capital cost function is concave. The capital cost function for the water system is denoted as $S(S_\tau)$, where S_τ is the capacity added in τ^{th} time unit (initial capacity is the addition from year zero).

Additions to water systems (excluding the initial capacity) have expected lifetimes that are assumed to be longer than the planning period. Capital costs are thus annualized over the expected lifetime of the addition and then discounted to the present for the period from the time of construction to the end of the planning period. The total present worth of these annualized capital costs are the costs that appear in the objective function. For the discount rate r, capital costs are converted to annual equivalent costs by applying the capital recovery factor β:

$$\beta = \frac{r(1+r)^m}{(1+r)^m-1} \qquad 4.3$$

where r is the social discount rate and m is the expected lifetime of the capital investment.

For a given or proposed water system, the total discounted capital costs are:

$$TC = \sum_{\tau=1}^{T} \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y \beta S(S_\tau) \qquad 4.4$$

where:

T = number of building time units in the planning period (if planning period is 40 and $\bar{y}$ is five years then T is 8)

$\bar{y}$ = number of years in a building time unit (additions to capacity are allowed once every $\bar{y}$ years, if necessary, in order to limit the number of decision variables and constraints in the model)

τ = index of building time unit, $\tau=1,2,\ldots,T$ (begin in year $y=1$, $\bar{y}+1$, $2\bar{y}+1,\ldots$, $(\tau-1)\bar{y}+1$).

The second cost component is for the expected system operation and maintenance (O and M). The O and M costs are defined as the annual costs for operation and maintenance of the system and are assumed to be a linear function of quantity of water delivered, (Q_y). It can be stated as cQ_y where c is the unit O and M costs and Q_y is the quantity of water delivered in year y.

The above O and M costs are discounted to the present and summed over the planning period. The final form of total discounted annual O and M costs is:

$$TO = \sum_{y=1}^{Y} \alpha_y cQ_y \qquad 4.5$$

Total Net Benefit

With equations (4.2), (4.4) and (4.5), the complete objective function for the programming model is expressed as follows:

$$\text{Max. } (TB - TC - TO) \qquad 4.6$$

which is to maximize equation (4.2) less equations (4.4) and (4.5).

Model Constraints

Having described the benefits and costs in the objective function, the necessary constraints required for a solution to the model are now expressed. The first set of constraints states that the quantity of water delivered in a specific time period cannot exceed total capacity built up to that period. This capacity constraint is stated as follows:

$$Q_y - \sum_{\tau-1}^{G} S_\tau \leq 0 \qquad 4.7$$

where $G = \lceil y/\bar{y} \rceil$, the ceiling of $y/\bar{y}$ which indicates the number of building time units up to year y.

The second set of constraints is the allocation constraint which requires that the actual water allocated in year y equals the water supplied in year y. This can be expressed as:

$$X_y - Q_y = 0 \qquad 4.8$$

where X_y is the quantity of water demanded in year y.

To assure that the capacity decision variable, S, can be established at most once during any building time unit, the following constraints are needed:

$$S_\tau - \bar{s} Z_\tau \leq 0 \qquad 4.9$$

and

$$Z_\tau \leq 1 \qquad 4.10$$

where $\bar{s}$, a given value, is the maximum possible capacity (physical upper bound) of the water system and Z_τ is a zero-one decision variable representing the decision to add capacity in period $\tau(Z_\tau=1)$ or not to add capacity in $\tau(Z_\tau=0)$.

Finally, in order for solutions of this model to be meaningful, all above decisions are required to be non-negative.

Computational Considerations

The optimization model formulated above has a nonlinear objective function with several linear constraints. Since the main focus of this chapter is to develop a solvable mathematical model, approximations are made to render the optimization model compatible with currently available computer techniques. Piecewise or grid linearization and fixed-charge approximation techniques are used to approximate the nonlinear objective function. The concave benefit function is linearized in the following manner. Suppose a linear demand curve is written as follows:

$$P(Q) = a + bQ \qquad 4.11$$

where price, P, is a function of quantity, Q. Then the area under the demand curve, B, can be expressed as follows:

$$B = \int_{0}^{Q} P(Q)dQ = Q\ (a + 1/2Q) \qquad 4.12$$

Now the objective function equation (4.6) can be rewritten as follows using equation (4.12):

$$\text{Max } (Q(a + 1/2Q) - S(S_{\tau}) - cQ) = NB \qquad 4.13$$

where NB is net social benefit. However, notice that equation (4.13) still contains a nonlinearity. Following Duloy and Norton (1975), this nonlinearity is removed through the use of the grid linearization technique. Grid linearization requires prior specification of a relevant range of values of the demand curve and the use of varible interpolation weights on the grid point. The interpolation weights become variables in the model and their values are jointly constrained by a set of convex combination constraints.

Implementation of the grid linearization technique is illustrated in Figure 8. Suppose that initially the demand curve defined in the price-quantity space passes through the point $(P_2,\bar{Q}_2)$ as illustrated in Figure 8. The relevant range of the demand curve is defined and truncated at points a and b. Then the relevant range of the demand curve is partitioned into segments s = 1,...,v. For each segment end point the parameters $\bar{Q}_s$ and $\bar{B}_s$ are defined to represent the cumulative known quantity of water sold and the cumulative known area under the aggregate demand curve for water.

The quantity of water used and the total area under the demand curve can be expressed as a weighted combination of $\bar{Q}_s$ and $\bar{B}_s$ respectively.

$$Q = \sum_{s=1}^{v} \bar{Q}_s W_s \tag{4.14}$$

$$B = \sum_{s=1}^{v} \bar{B}_s W_s \tag{4.15}$$

where W_s is a weight variable.

The non-negative interpolation weight variables are defined such that $\sum_{s=1}^{v} W_s \leq 1$. Notice here that no more than two consecutive points on the quantity axis will enter the optimal basis.

For the capital investment cost function, a fixed charge (set-up cost) approximation approach is used. For example, the capital investment cost $S(S_\tau)$ becomes (see Figure 9):

$$S(S_\tau) = f\, Z_\tau + K\, S_\tau \tag{4.16}$$

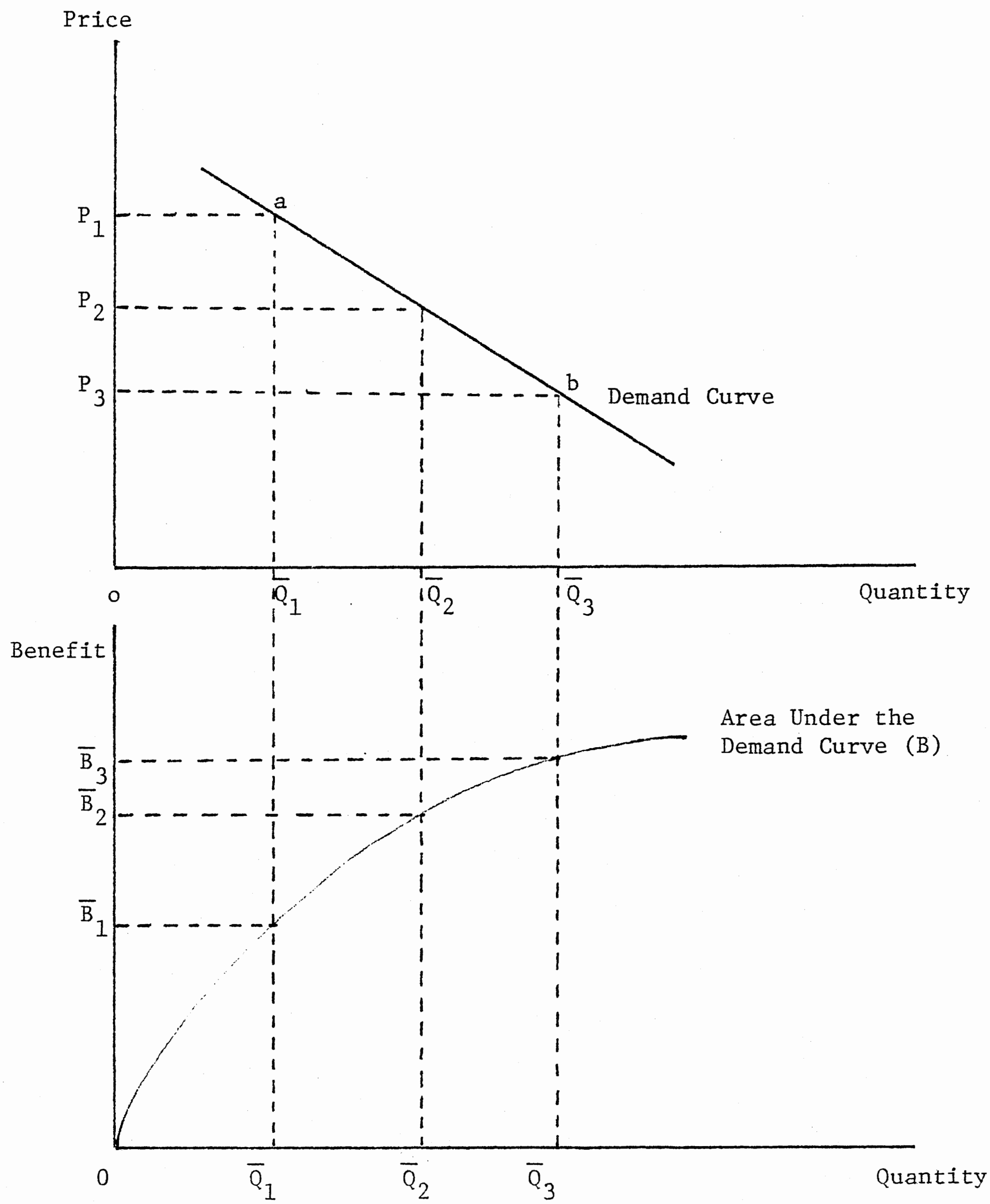

Figure 8. Grid Linearization of Demand and Benefit Functions

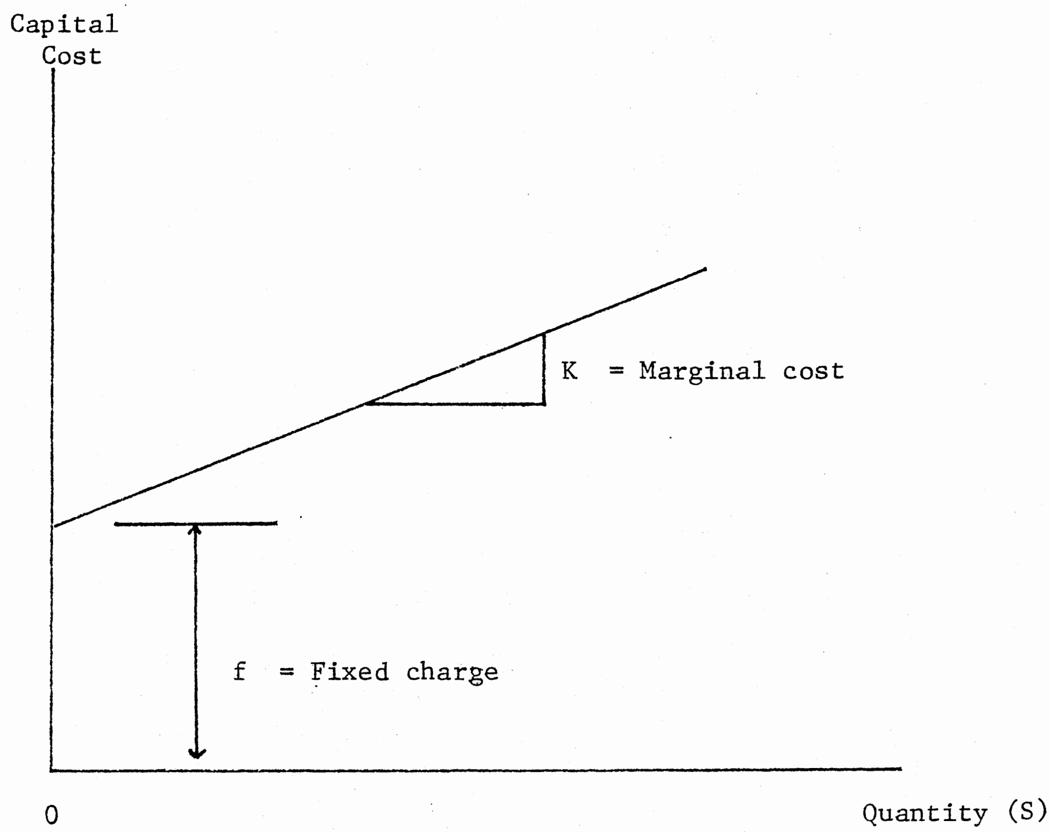

Figure 9. Fixed Charge Capital Cost Function

where

f = fixed charge of the capital cost function, $S(S_\tau)$

K = slope of the capital cost function

Z_τ = binary decision variable

Solution Strategy

Substituting the linear function approximation and the fixed charge approximation into the original model reduces the model to a large-scale mixed integer linear programming problem. While a few methods exist to solve such problems, perhaps the most promising and widely used method is the branch and bound technique.

The algorithm, which is described by McMillan (1970), begins by relaxing all integer constraints thereby making the problem suitable for solution by linear programming (LP). This solution is called the optimal continuous solution. Except for trivial problems, many of the binary variables will have fractional values making the solution infeasible; i.e., non-integer values between zero and one.

Next step is to set the binary variables to either zero or one, one variable at a time in such a way that the objective function is maximized. This is accomplished by adding a constraint to the original LP problem. Now the new LP problem restricts one of the non-integer binary variables to zero. A second new LP problem is similarly formed by restricting the same variable to one. Thus a branch is made from one binary variable and two new LP problems are created.

In the solutions of the two LP problems (called terminal nodes), the chosen binary variable will be integer (zero in one case and one in the other). However, some, but probably not all, of the remaining binary variables will be non-integer; another must be selected for branching. The usual procedure is to go to the terminal node with the best objective function value and select a second variable on which to branch. The constraint restricting the first variable is retained and two new LP problems are created, one by setting the new variable to zero and the other by setting it to one. Solution of these new problems results in three terminal nodes as shown in Figure 10, one from branching on the first variable plus two from branching on the second. A search is made to find the terminal node with best functional value (in our case, the maximum). If all binary variables in this solution are integer, zero or one, then the problem is solved.

The branch and bound methodology just described can be summarized as follows:

1. Treating the binary variables as continuous solves the problem by LP.

2. If all binary variables are not integral, select one to which to branch and form two new LP problems retaining all other constraints, one with the binary variable set equal to zero and the other with it equal to one.

3. Examine the solutions (terminal nodes) and find the one with best objective function value.

4. If all binary variables for this node are integers, the problem is solved, otherwise return to step two.

At each stage of the branching process, the total number of constraints in the LP problem increases by one. It is well known that the addition of a new constraint to a LP problem will either (a) cause the objective function value to remain unchanged, or (b) cause it to deteriorate (i.e., increase for minimization problems and decrease for maximization). Thus the functional value of the optimal continuous solution is a higher bound on the feasible solution of the water system planning

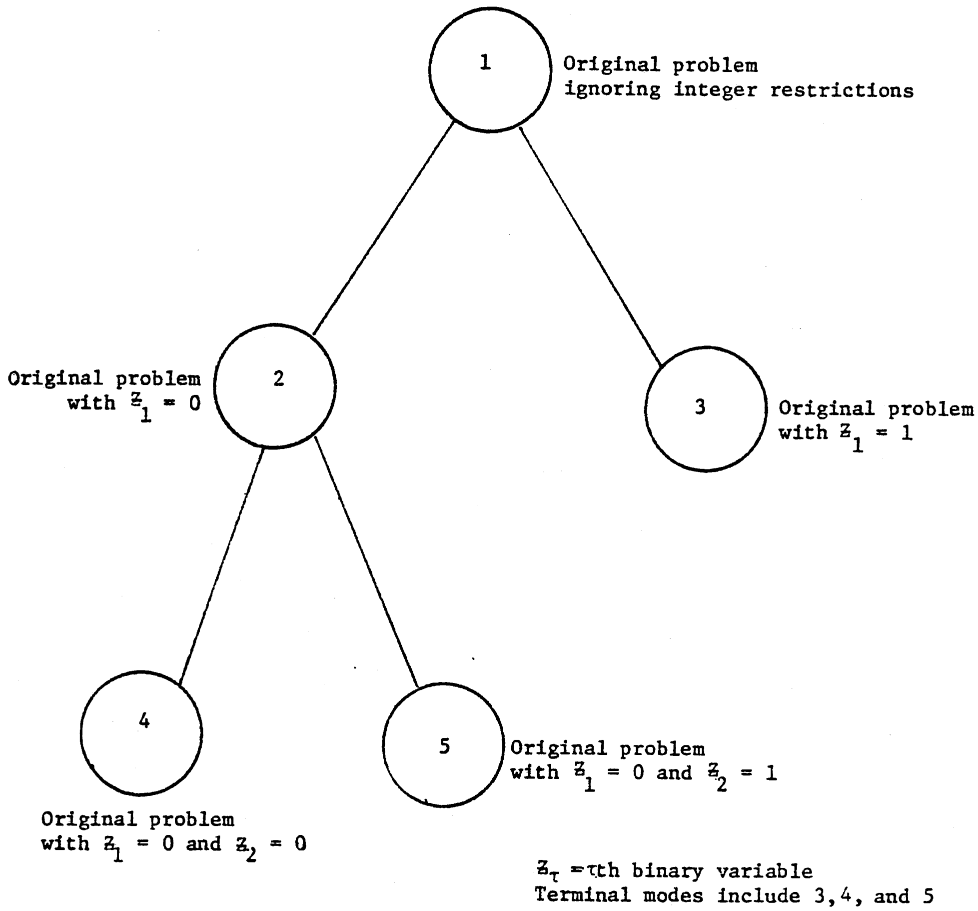

Figure 10. Mixed Programming Solution Tree

model. Additionally, the functional value of a terminal node is a higher bound on all other solutions that might spring from it.

Another important feature of branching and bounding has to do with infeasible solutions. As new LP problems are formulated by restricting additional binary variables, some will be infeasible and thus have no solution. For any terminal node with an infeasible solution, all problems springing from it (due to the restriction of new binary variables) will likewise be infeasible and thus can be ignored. A numerical example involving the use of this technique is included in Appendix C.

The MIP/370 computer program which is available at Oklahoma State University uses the branch-and-bound algorithm to find the optimal solution of the mixed integer programming problem. However, even though the well known mathematical programming software packages (i.e., IBM's MIP/370) can efficiently handle most mixed integer programming problems, solution abilities still limit the size of the problem. Hence, if the accuracy of approximation is increased (number of segments of the demand curve), the planning horizon (Y) is extended, or the time unit of the model is shortened, the mixed integer programming problem will probably exceed the size constraints of these existing computer codes.

The Basic LP Model with Economic Interpretation of the Optimal Solution

The Basic LP Model

To reduce the dimensions of the LP model, a five year decision time unit, τ, is used instead of an annual time unit, y. Thus, new discount rates, d_τ, and growth rates, h_τ, are computed which cover five year periods. Also, utilizing the grid linearization described, the basic linear programming model can be stated as follows:

$$\text{MAX NB} = \sum_{\tau,s} d_\tau \; (\overline{B}_{\tau s} W_{\tau s} - cQ_\tau) - \beta \sum_\tau \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y \; (K S_\tau + f Z_\tau) \qquad 4.17$$

subject to:

water balance equation (WBAL)

$$-Q_\tau + \sum_s \overline{Q}_{\tau s} W_{\tau s} \leq 0 \qquad [\pi] \qquad 4.18$$

system capacity constraint (CAP)

$$Q_\tau - \sum_{\tau=1}^{G} S_\tau \leq 0 \qquad [\lambda] \qquad 4.19$$

convex combination constraint (CONV)

$$\sum_s W_{\tau s} \leq h_\tau \qquad [\sigma] \qquad 4.20$$

integer constraint (INTEGER)

$$S_\tau - \overline{S} Z_\tau \leq 0 \qquad [\mu] \qquad 4.21$$

The Lagrangian multipliers are shown in brackets in the right-hand margin for each constraint. The variables and parameters are defined as follows:

<u>Definition of Variables</u>

$W_{\tau s}$: segment weight variable on demand and benefit function in period τ

Q_τ : quantity of water supplied in period τ

S_τ : capacity of water system built in period τ

Z_τ : zero-one binary variable in period τ

Definition of Parameters

β : capital recovery factor

$\bar{B}_{\tau s}$: area under the demand curve for segments of the initial demand function; along this segment, the willingness-to-pay is invariant under a population-induced shift in the demand curve

d_τ : discount factor in period τ which is defined as

$$\left[\left(\sum_{y=(\tau-1)\bar{y}+1}^{\tau\bar{y}} (1+\alpha_y)\right)^{1/\bar{y}} -1\right].$$

c : unit operation and maintenance costs,

K : slope of the capital cost function,

$\bar{Q}_{\tau s}$: amount of water consumed at segment s of the initial demand function,

h_τ : population growth index in period τ which can be defined as $(1+h)^{\tau y}$ where h is the annual growth rate,

$\bar{S}$: maximum possible water system capacity in an area.

A portion of the initial LP tableau (covering three periods) is presented in Table V.

The Kuhn-Tucker Condition

The Kuhn-Tucker (1950) conditions provide us with the necessary and sufficient conditions for determining an optimal solution[1]. From the basic LP model the Lagrangian equation is written as follows:

[1]See Appendix E for an example of the general model.

TABLE V

INITIAL LP TABLEAU (2 PERIODS ONLY)

		RHS	O_1	S_1	Z_1	X_{11}	..	X_{1v}	Q_2	S_2	Z_2	X_{21}	..	X_{2v}
Max	E·Q	z	$-d_1C$	$-\sum_{y=1}^{Y} \alpha^{y-1}\beta K$	$-\sum_{y=1}^{Y} \alpha^{y-1} f$	d_1B_{11}	...	d_1B_{12}	$-d_2C$	$-\sum_{y=y+1}^{Y} \alpha^{y-1}\beta K$	$-\sum_{y=y+1}^{Y} \alpha^{y-1}\beta f$	d_2B_{21}	...	d_2B_{22}
WBAL	L.E.	0	-1			X_{11}	..	X_{1v}						
CAP	L.E.	0	1	-1										
CONV	L.E.	h_1				1	..	1						
INTEGER	L.E.	0		1	$-\bar{S}$									
WBAL	L.E.	0							-1			X_{21}	..	X_{2v}
CAP	L.E.	0		-1					1	-1				
CONV.	L.E.	h_2										1	..	1
INTEGER	L.E.	0								1	$-\bar{S}$			

$$L(W,Q,S,Z) = \sum_{\tau,s} d_\tau \,(\overline{B}_{\tau s} W_{\tau s} - cQ_\tau) - \beta \sum_{y=(\tau-1)y+1}^{Y} \alpha_y \,(KS_\tau + fZ_\tau)$$

$$- \pi_\tau \,(-Q_\tau + \Sigma\overline{Q}_{\tau s} W_{\tau s})$$

$$- \lambda_\tau \,(Q_\tau - \sum_{\tau=1}^{G} S_\tau)$$

$$- \sigma_\tau \,(\sum_s W_{\tau s} - h_\tau)$$

$$- \mu_\tau \,(S_\tau - \overline{S}Z_\tau) \qquad 4.22$$

The Kuhn-Tucker conditions are met with the following results and provide an economic interpretation of each variable at the optimum.

$$\frac{\partial L}{\partial Q} = d_\tau c + \pi_\tau - \lambda_\tau \leq 0 \qquad \text{and } \frac{\partial L}{\partial Q_\tau} Q_\tau = 0 \qquad 4.23$$

$$\frac{\partial L}{\partial S} = -\beta K \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y + \lambda_\tau - \mu_\tau \leq 0$$

$$\text{and } \frac{\partial L}{\partial S_\tau} S_\tau = 0 \qquad 4.24$$

$$\frac{\partial L}{\partial W_{\tau s}} = d_\tau \overline{B}_{\tau s} - \pi_\tau \overline{Q}_{\tau s} - \sigma_\tau \leq 0 \qquad \text{and } \frac{\partial L}{\partial W_{\tau s}} W_\tau = 0 \qquad 4.25$$

$$\frac{\partial L}{\partial Z} = -\beta f \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y + \overline{S}\mu_\tau \leq 0$$

$$\text{and } \frac{\partial L}{\partial Z_\tau} Z_\tau = 0 \qquad 4.26$$

$$\frac{\partial L}{\partial \pi_\tau} = -\,(Q_\tau + \sum_s \overline{Q}_{\tau s} W_{\tau s}) \geq 0, \quad \text{if} >,\ \pi_\tau = 0 \qquad 4.27$$

$$\frac{\partial L}{\partial \lambda_\tau} = -\,(Q_\tau - \sum_{y=1}^{\tau} S_u) \geq 0, \quad \text{if} >,\ \lambda_\tau = 0 \qquad 4.28$$

$$\frac{\partial L}{\partial \sigma_\tau} = -\,(\sum_s W_{\tau s} - h_\tau) \geq 0, \quad \text{if} >,\ \sigma_\tau = 0 \qquad 4.29$$

$$\frac{\partial L}{\partial \mu_\tau} = -\,(S_\tau - \overline{S}Z_\tau) \geq 0, \quad \text{if} >,\ \mu_\tau = 0 \qquad 4.30$$

The saddle point property of the function is:

$$\sum_{\tau,s} \left[d_\tau \; (\overline{B}_{\tau s} \; W_{\tau s} - cQ_\tau) - \beta \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y \; (KS_\tau + fZ_\tau) \right] = \sum_\tau h_\tau \; \sigma_\tau \qquad 4.31$$

Rewriting equation (4.23) gives the following,

$$\pi_\tau = \lambda_\tau - d_\tau c$$

where

λ_τ = shadow price of incremental capacity (i.e., marginal cost of incremental capacity)

$d_\tau c$ = discounted O and M unit cost

Therefore, the shadow price of water, π_τ, can be interpreted as the marginal cost of supplying water which is the summation of marginal capital cost and marginal O and M cost.

Without loss of generality, assume that, of v variables $\overline{B}_{\tau s}$, only one variable is non-zero at value h_τ, and others are zero. Also, at most two segment end points, $\overline{B}_\tau s'$ and $\overline{B}_\tau s''$, are equal to h_τ. Therfore, the equation (4.25) becomes

$$d_\tau \overline{B}_{\tau s'} \; h_\tau - \pi_\tau \overline{Q}_{\tau s'} \; h_\tau - \sigma_\tau h_\tau = 0 \qquad 4.33$$

Aggregating over the planning period, equation (4.33) becomes

$$\sum_\tau^T h_\tau \sigma_\tau = \sum_\tau^T \left[d_\tau \overline{B}_{\tau s'} h_\tau - \pi_\tau Q_{\tau s'} h_\tau \right] \qquad 4.34$$

Therefore

$$\sigma_\tau = d_\tau \overline{B}_{\tau s'} - \pi_\tau \overline{Q}_{\tau s'}$$

where $d_\tau \overline{B}_{\tau s'}$ is the discounted area under a specific segment s′ of the demand curve, and $\pi_\tau \overline{Q}_{\tau s'}$ is the total revenue from water sale. Therefore σ_τ and be interpreted as total consumer surplus in time τ which is the difference between the discounted area under a specific segment s´ of demand curve and the total revenue from water sale.

The relationship between two shadow prices μ_τ and λ_τ can be derived by equations (4.24) and (4.26). Equation (4.26) can be rewritten as

$$\mu_\tau = \frac{\beta f \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y}{\bar{S}} \qquad 4.35$$

where the right hand side term is the fixed charge of investment cost embedded in the planning period per unit of maximum scale capacity. Also frome equation (4.24),

$$\lambda_\tau = (\beta K) \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y + \mu_\tau \qquad 4.36$$

Substituting μ of equation (4.35) into equation (4.36)

$$\underbrace{\lambda_\tau}_{\substack{\text{returns from} \\ \text{capacity} \\ \text{built in } \tau}} = \underbrace{(\beta K) \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y}_{\substack{\text{discounted embedded} \\ \text{variable cost of} \\ \text{constructing the} \\ \text{capacity in } \tau}} + \underbrace{\left(\frac{\beta f}{\bar{S}}\right) \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y}_{\substack{\text{discounted embedded} \\ \text{fixed charge per} \\ \text{unit of maximum} \\ \text{scale of capacity}}} \qquad 4.37$$

In equation (4.37) λ_τ can be interpreted as returns from the capacity built in period τ. The two terms on the right hand side are the discounted variable cost of constructing capacity in τ and discounted fixed charge per unit of maximum scale capacity. The two sides should be equal at the optimal which will result in efficient allocation of resources. If we allow infinite scale of maximum capacity, i.e., $\bar{S} = \infty$, the returns will be the same as the discounted variable cost of building that capacity at optimum.

CHAPTER V

ANALYSIS OF THE MODEL RESULTS

Introduction

This chapter presents the results of the application of the community water pricing and investment planning model. Solutions of the mixed integer programming problem with coefficients derived from the specific data in Chapter III are presented and discussed. The effects on three different community size water systems (small, average, and large) from varying parameters such as the growth rate and discount rate are investigated. The results, of course, are only as meaningful as the input data used in deriving them.

Since some of the coefficients (for example, price elasticity of demand, discount rate and growth rate) used in the planning model are subject to variability, a comprehensive sensitivity analysis of the most likely combinations of input parameters is desirable for decision making. Furthermore, such analyses will provide more insights into the usefulness of the proposed model. Therefore, a number of computer runs were made to explore the impact on benefit-maximizing investment plans and the resulting water rates from varying certain parameters in the model. The purpose is to show how sensitive water rates and investment decisions are to the discount rate and growth rate for different size community initial water systems.

Base Results

The results presented in this section are the mathematical programming solutions obtained by using as a base the survey data given in Chapter III. For the convenience of providing comparisons and sensitivity studies, these solutions will be referred hereafter as the "Base Result".

The base results consist of an optimal capacity expansion schedule of a water system, the operating level of a water system over time in association with the optimal investment schedule and the water rates at which the consumers' demands are satisfied for varying discount rates and system growth rates. The operating levels imply a set of facility policies. The optimal solutions of the base results are for the average size community of the sample survey.

Optimal Capacity Investment Schedule

In Chapter III, the average annual growth rate of the study sample showed eight percent per year. The optimal investment decisions for the average size community at the initiation of water system services with eight percent per year growth are shown in Table VI. The solutions indicate that the size of the initial system should be built at capacities of 136.9 mgy,[1] 108.7 mgy, and 93.8 mgy if one percent, three percent, and five percent discount rates are applied, respectively. According to the schedule of solutions these initial capacities are maintained through time unit three (15 actual years in the model) and then new facilities are added at the beginning of time unit four. The size of added capacities

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[1] mgy is million gallons per year.

beginning with time unit four are 179.5 mgy, 187.2 mgy, and 162.5 mgy, respectively, for the associated discount rates. The solutions also indicate that, beginning with time unit six and until the end of the planning period, new additions are made for every time unit. This is because the eight percent growth in the later time units bring more capacity requirements than the early time units. In other words, capacity should be added every five years to meet eight percent annual growth for the given discount rates. Total capacities built during the entire planning period are 1320.5 mgy, 1194.7 mgy and 1003.5 mgy, respectively.

Optimal solutions associated with the higher discount rates show that water systems are not built in time unit one even though there is a demand for water. In other words, the construction of water systems should be delayed until time unit two if the discount rate is seven percent and time unit four if the discount rate is nine percent. If the discount rate goes up to 15 percent, no water system is optimum under the model conditions. That is, the expected present worth of the cost (building and operation) of the system is greater than the expected present worth of the benefit it will provide regardless of when it is built (given discount rate of 15 percent).

The programming results correspond with the theory discussed in Chapter II that one of the factors determining the size of the optimal capacity is the social discount rate. Suppose there is no discount rate. Then, it would be perfectly sensible to spend a dollar now in order to save a dollar's worth of costs either in the next time period or ten years from now; or 100 years, thus, there is no limit to the size of capacity which it pays to build. With a positive discount rate, however, to save a dollar's worth of costs in a future time period we only need to spend less than a dollar now. Therefore, under a given economies of scale situation if the discount rate is low the size of optimal capacity is relatively large whereas if the discount rate is high the size of optimal capacity is relatively small.

In the base results the optimal size of capacities for the different discount rates shows the same trend as the proposed theory. If the discount rates are low the size of optimal capacities are relatively large and vice versa. In Table VI the optimal initial size of water system at one percent discount rate is larger than at the three percent discount rate, which is again larger than the optimal size at five percent discount rate. The objective function, which is the net social benefit expressed as the expected present worth of total benefits less the expected present worth of total costs during planning period, values are also given in Table VI. Like the trend of optimal size of capacities for the different discount rates, lower discount rates give relatively higher objective function values from larger size of capacity, lower water price and higher water demand. If the discount rates goes up to 15 percent, there is no investment during the planning period and hence no net social benefits are realized.

Optimal Water Supply Schedule

There are two major factors which directly influence the short run level of water supply: size of capacity and growth in water demand. It is reasonable to say that an increase in number of customers will result in an increase in water supplied as long as excess capacity exists. However, how fast water supply should be increased depends mainly on the system's growth rate. Once water supply reaches the maximum capacity,

TABLE VI

OPTIMAL CAPACITY[a] INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT EIGHT PERCENT GROWTH

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	5,534,429	136.9	–	–	179.5	–	295.2	287.6	421.3	1320.5
3	2,519,708	108.7	–	–	187.2	–	257.4	260.2	381.2	1194.7
5	1,062,444	93.8	–	–	162.5	–	208.5	218.5	320.2	1003.5
7	372,982	–	118.2	–	–	226.8	–	249.5	278.9	873.4
9	85,317	–	–	–	215.3	–	–	292.0	237.7	745.0
15	–	–	–	–	–	–	–	–	–	–

[a]Amount of system capacities in mgy.

to increase supply requires the next addition as reviewed in the previous section.

The optimal water supply schedule for the average sample community size with an eight percent growth rate during the planning period is presented in Table VII. As in the case of optimal investment, the various discount rates show the sensitivity on optimal water supply. For the case of a one percent discount rate the optimal water supply increases significantly from time unit one to time unit eight. Optimal water supply increases from one time unit to the next time unit except for time unit three which is the same as that of time unit two. This is because the system reaches its maximum capacity in time unit two and additional capacity is not optimum until time unit four. It is noted that the increase of water supply in the later time units are relatively greater than those of the earlier time units. This is explained by the compounding effect of an eight percent growth rate during the whole planning period. That is, eight percent growth in earlier time units results in relatively smaller net increases in number of customers than is the case for later time units. In fact, it is probably not realistic to assume that the water system grows at a constant rate during the whole planning period, i.e. eight percent. A more realistic assumption would be for water systems with fast growth at the beginning and then slower growth during the remaining part of the planning period. Of course the specific rate of growth depends upon the environment of individual systems.

The water supply schedule also includes solutions for various discount rates. As observed in the optimal capacity schedule, a system's water supply declines as the discount rate increases. Again there is

TABLE VII

OPTIMAL WATER SUPPLY[a] SCHEDULE FROM THE BASIC RESULTS
AT EIGHT PERCENT GROWTH

Discount Rate (percent)	Water Supply Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	93.8	136.9	136.9	297.3	316.5	611.6	899.2	1320.6
3	93.8	108.7	108.7	295.9	295.9	553.3	813.5	1194.6
5	93.8	93.8	93.8	256.3	256.3	464.8	683.3	1003.5
7	--	118.2	118.2	118.2	345.1	345.1	594.6	873.1
9	--	--	--	215.3	215.3	215.3	507.3	754.0
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

no water supply in time unit one for the seven percent discount rate, time unit one, two and three for the nine percent discount rate, and the whole planning period for the 15 percent discount rate because no water capacity was built for these time units.

Optimal Water Rate Schedule

Optimal solutions for capacity and water supply representing different growth and social discount rates are read directly from the output of the programming model. However, the model does not provide the optimal water rate schedule directly. The optimal water rate is computed indirectly by substituting water supply for each time unit into that unit's demand equation representing a particular growth situation. To do this, it is necessary to derive the demand equation for each time unit.

In Chapter III the general water demand function in rural Oklahoma which describes consumers' response to changes in price was derived. The demand equation is shown at zero time unit in Table VIII and shows that if the water rate increases one dollar the quantity of water demanded will decrease about 150,000 gallons per year. The assumption is made that consumer response to price change is relatively constant during the planning period even though the water system measured in terms of number of users grows in future time units.

Growth of the water system on the price-quantity plane can be expressed by rotation of the initial demand curve as shown in Figure 11. Let D_o represent the demand curve before growth (i.e. at time unit zero), whereas D_1 represents demand after growth at time unit one. The price-quantity relationship shows that if the price level is P_1, Q_o amount of

TABLE VIII

ROTATED DEMAND EQUATIONS FOR EACH TIME UNIT AT EIGHT PERCENT ANNUAL GROWTH RATE

Time Unit	Growth Index (h)	Demand Equations (Inversed)
0	1.00	P = 5300 - 68.6Q
1	1.47	P = 5300 - 46.8Q
2	2.16	P = 5300 - 31.9Q
3	3.17	P = 5300 - 21.7Q
4	4.66	P = 5300 - 14.8Q
5	6.85	P = 5300 - 10.0Q
6	10.06	P = 5300 - 6.8Q
7	14.79	P = 5300 - 4.7Q
8	21.72	P = 5300 - 3.2Q

P = price per mgy dollars.
Q = quantity of water demanded in mgy.

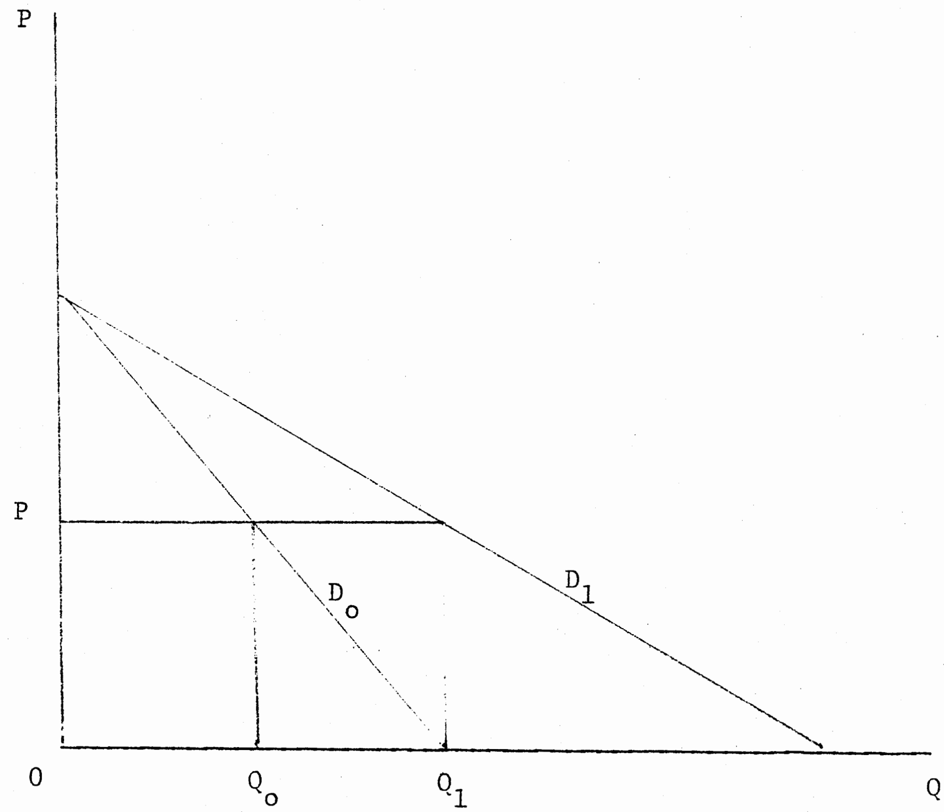

Figure 11. Rotation of Demand Curve by Growth

water is purchased by the given number of customers in a community (say 100 customers) at time unit zero. Assume that the number of customers increases to 200 at the end of time unit one--a 100 percent growth compared to the original number of customers. The amount of water purchased by 200 customers at time unit one would be Q_1 if the price level stays at P_1. Thus, by the assumption of constant consumer response, Q_1 should be exactly twice that of Q_o. Since this price-quantity relationship is true for each and every level of prices, the demand function for time unit one can be derived by using the information from the initial price-quantity relationship and growth in number of customers. Practically, this is derived for time unit one by dividing the slope of D_o by its growth index.

The demand equations for the different time units in Table VIII are derived in this manner--dividing the slope of the initial demand curve, 68.6, by the growth index in column two. For the Base Results, since a constant growth rate of eight percent per year is applied throughout the planning period, the demand curves become flatter and flatter as the system grows.

The optimal water rate schedule is computed by substituting the water supply into each time unit's demand equation. To analyze the optimal rate schedule, not only the relationship between optimal water supply and growth rate should be considered but also the optimal capacity schedule. This is because the water supply schedule is influenced by the optimal investment schedule. For example, in Table IX the rate schedule for a one percent discount rate fluctuates from one time unit to another time unit depending upon timing of additional capacity. If there is pressure on capacity due to system growth it will result in

TABLE IX

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS
AT EIGHT PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	910.2	932.9	2329.3	900.0	2135.0	1141.1	1073.8	1074.1
3	910.2	1832.5	2941.2	920.7	2341.0	1537.6	1476.6	1477.3
5	910.2	2307.8	3264.5	1506.8	2737.0	2139.4	2088.5	2088.8
7	--	1529.4	2735.1	3550.6	1489.0	2953.3	2505.4	2506.1
9	--	--	--	2113.6	3147.0	3836.0	2915.7	2916.0
15	--	--	--	--	--	--	--	--

[a]Dollar per million gallons.

addition of new capacity which allows an increase in water supply. The increased water supply brings the water rate down but not as low as if the system stayed on the same demand curve. The reason is that the slope of the new demand curve from which the optimal water rate is computed is now flatter than the previous demand curve.

In Table VI for a one percent discount rate the initial capacity is 136.9 mgy but the actual water supply is 93.8 mgy at time unit one in Table VII. That is, 43.1 mgy excess capacity is reserved for future growth. Substituting 93.8 mgy amount of water supplied in the first time unit demand curve results in a water price of $910.20 per million gallons. In the second time unit, all of the existing capacity is utilized due to the system's growth. Therefore again substituting the optimal water supply, 136.9 mgy into the second time unit's demand equation results in $932.90 per million gallons as the water rate which is higher than that of the first time unit. In the third time unit, there is another eight percent growth in the system but additional capacity has not come into the solution yet. Therefore, the amount of water supplied is restricted to the maximum capacity by raising the water rate. That is why the water supplied during the third time unit is the same as that of the second time unit but the water rate is significantly higher than that of the second time unit. Water rate is used as a means to allocate a given amount of water to more customers. In the fourth time unit there is another eight percent growth per year. Now the water system no longer relies on the role of price to maintain existing capacity. Therefore a new capacity addition comes into the solution (see Table VI). With new additional capacity water supply increases and consequently the optimal water rate decreases. These interrelationships among growth rate, optimal capacity schedule, optimal water supply schedule, and optimal water rate continue until the end of the planning period for each discount rate. Of course the above solutions are based upon eight percent growth per year. Solutions for different growth patterns are analyzed in succeeding sections.

Results and Analysis for Alternative Growth Rates

Rural community water systems have shown substantial variability in growth (see Table IV). In this section different environments (i.e. growth rates) are assumed to analyze the effect of growth in determining optimal solutions in terms of capacity, water supply and water rates. An important focus of this study is to determine net social benefits if decision makers would have known the system's growth at the time of initial planning.

Zero Growth Situation

Optimal Solutions. As reviewed before, economies of scale, discount rate and system growth are the main factors that dtermine optimum excess capacity. However, if the number of customers remains constant throughout the time period, decision makers do not need to worry about building any excess capacity or additions to capacity as long as consumer consumption behavior is stable. Therefore the optimal capacity would be the same as the level of optimal water supply.

The solution of the model when the growth rate is zero shows this situation. The optimal capacity and the optimal water supplies are the same throughout the entire planning period as seen in Tables X and XI.

TABLE X

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASE RESULTS AT ZERO PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	666,082	60.8	--	--	--	--	--	--	--	60.8
3	326,105	55.0	--	--	--	--	--	--	--	55.0
5	123,671	46.2	--	--	--	--	--	--	--	46.2
7	741	40.2	--	--	--	--	--	--	--	40.2
9	--	--	--	--	--	--	--	--	--	--
15	--	--	--	--	--	--	--	--	--	--

[a]Amount of system capacities in mgy.

TABLE XI

OPTIMAL WATER SUPPLY SCHEDULE[a] FROM BASE RESULT AT ZERO GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	60.8	60.8	60.8	60.8	60.8	60.8	60.8	60.8
3	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
5	46.2	46.2	46.2	46.2	46.2	46.2	46.2	46.2
7	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2
9	--	--	--	--	--	--	--	--
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

TABLE XII

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULT
AT ZERO PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	1117.0	1117.0	1117.0	1117.0	1117.0	1117.0	1117.0	1117.0
3	1516.0	1516.0	1516.0	1516.0	1516.0	1516.0	1516.0	1516.0
5	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0
7	2534.0	2534.0	2534.0	2534.0	2534.0	2534.0	2534.0	2534.0
9	--	--	--	--	--	--	--	--
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

When the discount rate is one percent the optimal capacity is 60.8 mgy which remains constant as long as there is no growth. Like the case with growth, the optimal capacity investment decreases as the discount rate increases and there is no optimal investment if the discount rate goes beyond seven percent. Because of no excess capacity the optimal water supply is the same as the optimal capacity level (Table XI). Also, the optimal water rates for a given discount rate are the same throughout the whole planning period as shown in Table XII.

Equity Considerations With and Without Growth. Although the scope of this study is limited to economic efficiency it is still worthwhile to review equity aspects in terms of individual customer payments for water with and without growth.

As reviewed before, the optimal solutions of capacity, operation level, and water rate depend on system growth under given economies of scale and discount rate. Under conditions of no growth there is no excess capacity in the optimal solution and water rate is the same throughout the planning period. This means that the initial members of the system who are the only members of the system throughout the planning period pay a constant water rate during the whole planning period. For example, water rate is fixed to $2121 per million gallons during all time units when the discount rate is five percent. To review the situation of the initial members of a water system this rate can be compared to other optimal rates under conditions of growth.

As an example of comparing equity positions of initial members of water systems, Tables XIII and XIV are compared. In Table XIII, with eight percent system growth, payments per user for each time unit at

TABLE XIII

WATER CONSUMPTION PAYMENTS PER USER FOR EACH TIME UNIT AT EIGHT PERCENT GROWTH AND FIVE PERCENT DISCOUNT RATE

Time Unit	1	2	3	4	5	6	7	8	Total
d_c	.86494	.67780	.53115	.41622	.32616	.25558	.20027	.15693	
Water Supply (mg)	93.8	93.8	93.8	256.3	256.3	464.8	603.3	1003.5	
Water Rate ($/mg)	910.2	2307.8	3264.5	1506.8	2737.0	2139.4	2088.5	2088.5	
No. of Users	291.0	428.0	628.0	923.0	1356.0	1992.0	2928.0	4301.0	
Payment Per User (dollars discounted to present)	254.0	343.0	259.0	174.0	169.0	128.0	98.0	76.0	1501.0

TABLE XIV

WATER CONSUMPTION PAYMENTS PER USER FOR EACH TIME UNIT AT ZERO PERCENT GROWTH AND FIVE PERCENT DISCOUNT RATE

Time Unit (T)	1	2	3	4	5	6	7	8	Total
d_c	.86494	.67780	.53115	.41623	.32613	.25558	.20027	.15693	
Water Supply (mg)	46.2	46.2	46.2	46.2	46.2	46.2	46.2	46.3	
Water Rate ($/mg)	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	
No. of Users	198.0	198.0	198.0	198.0	198.0	198.0	198.0	198.0	
Payment Per User (dollars discounted to present)	428.0	335.0	263.0	206.0	161.0	126.0	99.0	78.0	1696.0

five percent discount rate are computed. To project the growth of users, a base of 198, which is the average initial number of users of the sample system is applied. Using optimal solutions of water supply and rate schedules, the discounted payments per user are computed and added. The total value of $1501 in Table XIII is the total amount paid by a user during the whole planning period. In Table XIV a similar procedure was applied but under conditions of constant water supply and rate schedule. The total amount paid by a user during the whole planning period and discounted to the present is compared under conditions of with and without growth. Based upon this comparison, an individual user under the growth situation is better off than under the without growth situation.

Two, Four, Six and Ten Percent Growth Rate

So far optimal solutions of the base result and zero growth situation have been reviewed. In this section optimal solutions under different rates of growth are analyzed. If decision makers correctly predicted growth and planned system capacity and management accordingly, the optimal solutions would give maximum social benefits.

Optimal Capacity Investment Schedule. Tables XV, XVI, XVII and XVIII show optimal capacity investment schedules under growth conditions of two percent, four percent, six percent and ten percent respectively. To compare the effect of different growth rates on the optimal initial investment, the discount rate of five percent is chosen. The optimal size of the initial investment increases gradually as the growth rate increases. For example, the optimal size investment with two percent

TABLE XV

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT TWO PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	1,058,396	86.1	--	--	--	--	--	--	48.2	134.3
3	517,647	73.2	--	--	--	--	--	--	48.4	121.6
5	216,211	66.9	--	--	--	--	--	--		66.9
7	41,130	--	59.9	--	--	--	--	--	--	59.9
9	230	--	--	--	--	--	--	--	75.8	75.8
15	--	--	--	--	--	--	--	--	--	--

[a]Amount of system capacities in mgy.

TABLE XVI

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT FOUR PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	1,765,112	107.5	--	--	--	--	99.2	--	85.1	291.8
3	837,421	91.6	--	--	--	--	115.1	--	--	206.7
5	353,209	77.8	--	--	--	--	101.3	--	--	179.1
7	94,607	--	77.8	--	--	--	--	92.9	--	170.7
9	9,304	--	--	--	--	111.1	--	--	--	111.1
15	--	--	--	--	--	--	--	--	--	--

[a]Amount of system capacities in mgy.

TABLE XVII

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT SIX PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	3,081,815	124.8	--	--	--	148.9	--	193.9	158.1	625.7
3	1,427,597	110.1	--	--	--	150.7	--	162.1	143.0	565.9
5	601,127	82.3	--	--	114.6	--	--	216.8	--	413.7
7	192,558	--	103.6	--	--	--	160.2	--	149.9	413.7
9	33,506	--	--	--	147.1	--	--	116.6	89.2	352.9
15	--	--	--	--	--	--	--	--	--	--

[a]Amount of system capacities in mgy.

TABLE XVIII

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT TEN PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	10,076,487	165.2	--	--	243.9	249.3	402.5	647.5	1000.0	2708.4
3	4,508,708	102.7	--	164.0	--	329.0	364.1	585.8	943.8	2489.4
5	1,896,754	93.2	--	148.8	--	258.3	305.8	492.0	792.8	2090.9
7	698,512	--	142.5	--	227.7	--	331.3	428.1	689.8	1819.4
9	190,928	--	--	168.0	--	203.4	227.1	365.3	588.6	1552.4
15	--	--	--	--	--	--	--	--	--	--

[a]Amount of system capacities in mgy.

growth is 66.9 mgy while it is 77.8 mgy with four percent growth rate. When the growth rate is ten percent, which is the largest growth rate studied, not only the initial investment of 93.2 mgy is larger than that of smaller growth rates, but, also, capacity additions are more frequent after time unit four. From this finding it is concluded that growth of a water system is one of the critical factors which should be considered in determining optimal investment size even though this factor is ignored in much of the existing literature.

Optimal Water Supply Schedule. Solutions from the programming model for optimal water supply under alternative growth rates are presented in Tables XIX, XX, XXI, and XXII. Again five percent discount rate is used to make comparisons of solutions. As seen in Table XIX under conditions of two percent growth, the optimal water supply remains the same over the entire planning period. The initial capacity, 66.9 mgy, is fully utilized at the beginning time period and remains fully utilized with no additional capacities. Under four percent growth, capacity is increased in the sixth time unit and again is fully utilized. For the assumptions of six and ten percent growth, the model results show no period with excess capacity for the five percent discount rate. It must be assumed that price is being used to allocate water under the limited capacities or until additional capacity is created.

Comparing the results for the five percent discount rate with the one percent discount rate it is noted that under the latter condition water systems do have excess capacities for some time units. That is, the water supplied is less than the capacity for that time unit.

TABLE XIX

OPTIMAL WATER SUPPLY[a] SCHEDULE FROM BASE RESULTS
AT TWO PERCENT GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	70.2	77.8	86.1	86.1	86.1	86.1	86.1	134.4
3	70.2	73.2	73.2	73.2	73.2	73.2	73.2	121.6
5	66.9	66.9	66.9	66.9	66.9	66.9	66.9	66.9
7	--	59.9	59.9	59.9	59.9	59.9	59.9	59.9
9	--	--	--	--	--	--	--	75.8
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

TABLE XX

OPTIMAL WATER SUPPLY[a] SCHEDULE FROM BASE RESULTS
AT FOUR PERCENT GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	77.8	94.4	107.5	107.5	107.5	206.7	206.7	291.8
3	77.8	91.6	91.6	91.6	91.6	206.7	206.7	206.7
5	77.8	77.8	77.8	77.8	77.8	179.0	179.0	179.0
7	--	77.8	77.8	77.8	77.8	77.8	170.6	170.6
9	--	--	--	--	111.1	111.1	111.1	111.1
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

TABLE XXI

OPTIMAL WATER SUPPLY[a] SCHEDULE FROM BASE RESULTS
AT SIX PERCENT GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	85.5	114.2	124.8	124.8	273.7	273.7	467.6	625.6
3	85.5	110.1	110.1	110.1	260.8	260.8	423.0	566.0
5	82.3	82.3	82.3	196.9	196.9	196.9	413.7	413.7
7	--	103.6	103.6	103.6	103.6	263.8	263.8	413.7
9	--	--	--	147.1	147.1	147.1	263.8	352.9
15	--	--	--	--	--	--	--	--

TABLE XXII

OPTIMAL WATER SUPPLY[a] SCHEDULE FROM BASE RESULTS AT TEN PERCENT GROWTH

Objective Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	102.7	165.2	165.2	409.2	658.5	1061.0	1708.5	2708.5
3	102.7	102.7	266.7	266.7	595.7	959.8	1545.5	2489.3
5	93.2	93.2	242.0	242.0	500.3	806.2	1298.2	2091.0
7	--	142.5	142.5	370.2	370.2	701.5	1129.6	1819.5
9	--	--	168.0	160.0	371.5	598.5	963.8	1552.4
15	--	--	--	--	--	--	--	--

[a] Amount of water supplied in mgy.

Optimal Water Rate Schedule. The optimal water rates for the different growth rate assumptions are computed using the demand curves of each time unit and the associated optimal water supplies. Tables XXIII, XXIV, XXV, and XXVI present these water rates.

The overwhelming result shown in Tables XXIII through XXVI is the fact that price is heavily used as the allocator of water. As an example, for the two percent growth rate (Table XXIII) and the five percent discount rate, price of water must continuously increase from time unit one to time unit eight since capacity was established in time unit one and there are no additions to capacity for the remainder of the planning period (see Table XV). Furthermore, water supply was at the maximum capacity for each time unit (see Table XIX). Therefore to limit consumption of water equal to capacity requires that price of water must increase. Further evidence of price being used as the allocator of water is seen in Table XXIV for the four percent growth assumption. Again viewing the five percent discount results, the price of water increases from $912 per million gallons in time unit one to $3,293 per million gallons in time unit five. Since capacity is added in time unit six (see Table XVI) water price is reduced to $1,505 per million gallons. Price increases gain in time units seven and eight since water supplied is equal to capacity in each of these time units but growth in number of cusomters has occurred at the four percent rate.

Another result apparent from these tables is the effect of economies of scale on price. Again viewing the five percent discount rate results of Table XXIII with Table XXIV, price of water in time unit one reduces from $1,119 per million gallons for two percent growth to $912 per million gallons for four percent growth. The reason for

TABLE XXIII

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS AT TWO PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	913.0	912.0	909.0	1322.0	1684.0	2028.0	2338.0	1120.0
3	913.0	1172.0	1567.0	1918.0	2226.0	2518.0	2782.0	1518.0
5	1119.0	1527.0	1888.0	2209.0	2490.0	2758.0	2999.0	3219.0
7	--	1922.0	2245.0	2533.0	2784.0	3024.0	3239.0	3437.0
9	--	--	--	--	--	--	--	2943.0
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

TABLE XXIV

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS AT FOUR PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	912.0	910.0	1194.0	1925.0	2527.0	918.0	1716.0	1127.0
3	912.0	1040.0	1801.0	2424.0	2937.0	918.0	1716.0	2344.0
5	912.0	1682.0	2328.0	2857.0	3293.0	1505.0	2185.0	2740.0
7	--	1682.0	2328.0	2857.0	3293.0	3651.0	2332.0	2860.0
9	--	--	--	--	2434.0	2945.0	3367.0	3711.0
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

TABLE XV

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS AT SIX PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	914.0	915.0	1718.0	2629.0	921.0	2016.0	1138.0	1109.0
3	914.0	1072.0	2140.0	2944.0	1127.0	2170.0	1535.0	1508.0
5	1078.0	2140.0	2938.0	1086.0	2150.0	2937.0	1618.0	2528.0
7	--	1322.0	2327.0	3083.0	3642.0	2134.0	2952.0	2528.0
9	--	--	--	2152.0	2946.0	3535.0	2952.0	2936.0
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

TABLE XXVI

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS
AT TEN PER CENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	915.0	906.0	2574.0	1126.0	1086.0	1162.0	1200.0	1237.0
3	915.0	2568.0	900.0	2580.0	1488.0	1557.0	1591.0	1566.0
5	1320.0	2821.0	1307.0	2832.0	2098.0	2155.0	2184.0	2164.0
7	--	1510.0	2949.0	1524.0	2931.0	2564.0	2589.0	2571.0
9	--	--	2528.0	3668.0	2922.0	2966.0	2987.0	2971.0
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

this in part is due to the larger capacity installed under the four percent growth (Table XVI) relative to the capacity installed under the two percent growth (Table XV). Similarly, water price can be compared across growth rates for time unit two and at the seven percent discount rate. At two percent growth, the price is $1,922 per million gallons (Table XXIII), at four percent growth the price is $1,682 per million gallons (Table XXIV), and at six percent growth the price is $1,322 per million gallons (Table XXV). The decrease in price is due in part to economies of scale since larger capacities were installed at each higher growth rate. Price increases again at the ten percent growth to $1,510 per million dollars (Table XXVI) but this is due in part to using price to restrict consumption at limited capacity.

Declining Growth Situation

So far the analysis has been restricted to constant growth rate during the whole planning period. However, it is unrealistic to expect a water system to grow at a constant rate for the whole planning period. Rather, it is more realistic to assume that water systems grow faster during earlier time units of the planning period and then the rate of growth becomes moderated or stabilized. To review optimal solutions under these assumptions of growth, three different growth patterns are studied. The first pattern is an eight percent growth rate during the first half of the planning period and then growth stops for the remainder of the planning period. The second pattern is an eight percent growth rate during the first half and then growth continues at two percent per year during the second half of the planning period. The last pattern consists of an eight percent growth rate during the first half of the planning period and continues to grow at four percent per year during the second half of the planning period.

<u>Eight and Zero Percent Growth</u>. Tables XXVII, XXVIII and XXIX show the optimal solutions of investment capacity, water supply and water rate schedules, respectively. The optimal capacity investment schedule, Table XXVII, shows that there is no additional facility coming into the solution after the end of the fourth time unit for all discount rates. This is explained by the assumption of zero growth for the last half of the planning period. However, the solutions of initial investment for the different discount rates are the same as the solutions from the base result with eight percent growth (see Table VI). The optimal water supply schedule in Table XXVIII shows no change of supply level after the fourth time unit due to zero growth. Like the water supply schedule from the base result with eight percent growth, no water supply is made in the early time units if the discount rate is seven or nine percent. No water supply is realized at all if the discount rate becomes 15 percent. In Table XXIX, the optimal water rates are constant after the fourth time unit due to zero growth. The effect of price again can be seen as an allocator of water under limited capacities. Price increases significantly in time unit three for discount rates one, three and five percent and then decreases with additions to capacity in time unit four.

<u>Eight and Two Percent Growth</u>. This pattern considers eight percent growth per year until the fourth time unit and then growth drops to two percent. Table XXX which is the optimal investment schedule, shows initial capacity the same as the previous case but with larger additions

TABLE XXVII

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT EIGHT AND ZERO PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	2,540,409	136.9	--	--	146.4	--	--	--	--	283.3
3	1,314,981	108.7	--	--	147.6	--	--	--	--	256.3
5	593.953	93.8	--	--	121.5	--	--	--	--	215.3
7	208,680	--	--	159.8	--	--	--	--	--	159.8
9	47,504	--	--	--	158.9	--	--	--	--	159.8
15	--	--	--	--	--	--	--	--	--	--

[a]The first four periods (20 years) have eight percent growth and the rest of the periods have zero growth per period.

TABLE XXVIII

OPTIMAL WATER SUPPLY SCHEDULE[a] FROM BASE RESULT AT EIGHT AND ZERO PERCENT GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	93.8	136.9	136.9	283.3	283.3	283.3	283.3	283.3
3	93.8	108.7	108.7	256.3	256.3	256.3	256.3	256.3
5	93.8	93.8	93.8	215.3	215.3	215.3	215.3	215.3
7	--	--	159.8	159.8	159.8	159.8	159.8	159.8
9	--	--	--	159.8	159.8	159.8	159.8	159.8
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

TABLE XXIX

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS
AT EIGHT AND ZERO PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	910.2	932.9	2329.3	1107.2	1107.3	1107.3	1107.3	1107.3
3	910.2	1832.5	2941.2	1506.8	1506.8	1506.8	1506.8	1506.8
5	910.2	2307.8	3264.5	2113.6	2113.6	2113.6	2113.6	2113.6
7	--	--	1832.3	2935.0	2935.0	2935.0	2935.0	2935.0
9	--	--	--	2935.0	2935.0	2935.0	2935.0	2935.0
15	--	--	--	--	--	--	--	--

[a]Dollar per million gallons.

TABLE XXX

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT EIGHT AND TWO PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit 1	2	3	4	5	6	7	8	Total
1	3,531,336	136.9	-	-	160.4	-	140.4	-	113.2	550.9
3	1,692,178	108.7	-	-	188.6	-	-	163.6	-	460.9
5	743,814	93.8	-	-	162.5	-	-	135.1	-	391.4
7	255,710	-	118.2	-	-	154.8	-	-	91.2	364.2
9	57,239	-	-	-	197.6	-	-	-	113.2	310.8
15	-	-	-	-	-	-	-	-	-	-

[a]The first four periods (20 years) have eight percent growth and the rest of the periods have two percent growth per period.

at time unit four due to the higher growth rate (from zero to two percent per year). An interesting difference in the solution for optimal capacity between this pattern and the previous case is the timing of initial investment when the discount rate is seven percent. When the growth pattern was eight and zero percent, the initial investment comes into the solution at the third time unit and no additional investments until the end of the planning period. When the pattern is eight and two percent a smaller investment comes into the solution in the second time unit and then two additional investments come into the solution at time units five and eight. The optimal solutions of the water supply (Table XXXI) show the same levels for the first three time units for discount rates of one, three and five percent. However, water supply increases for the fourth time unit under the conditions of a slight continued growth for the latter half of the period. The effect of a continued growth is to increase the optimum capacity and hence water supply for this fourth period. This also has the effect of decreasing water price for the fourth period under conditions of two percent growth in the latter half (Table XXXII) versus no growth in the latter half of the period (Table XXIX). Water price fluctuates during the latter half of the period under conditions of two percent growth depending on when optimal capacities are added.

Eight and Four Percent Growth. The last growth pattern is the system that grows at eight percent per year during the first half of the planning period and then drops to four percent per year. Tables XXXIII, XXXIV and XXXV are the optimal solutions of investment capacity, water supply and water rate, respectively. In general, the solutions

TABLE XXXI

OPTIMAL WATER SUPPLY SCHEDULE[a] FROM BASE RESULT AT EIGHT AND TWO PERCENT GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	93.8	136.9	136.9	297.3	297.3	437.7	437.7	550.8
3	93.8	108.7	108.7	297.3	297.3	297.3	460.9	460.9
5	93.8	93.8	93.8	256.3	256.3	256.3	391.4	391.4
7	--	118.2	118.2	118.2	273.0	273.0	273.0	364.0
9	--	--	--	197.6	197.6	197.6	197.6	310.8
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

TABLE XXXII

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS AT EIGHT AND TWO PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	910.2	932.9	2329.3	900.0	1762.1	923.9	1535.8	1113.9
3	910.2	1832.5	2941.2	900.0	1762.1	2327.0	1336.3	1797.2
5	910.2	2307.8	3264.5	1506.8	2250.0	2737.8	1934.0	2325.4
7	--	1529.4	2735.1	3550.6	2051.3	2570.0	2952.2	2532.1
9	--	--	--	2375.5	2948.6	3324.1	3600.6	2937.9
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

TABLE XXXIII

OPTIMAL CAPACITY INVESTMENT SCHEDULE FROM THE BASIC RESULTS AT EIGHT AND FOUR PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	3,632,126	136.9	--	--	160.4	--	155.7	--	127.1	580.1
3	1,732,848	108.7	--	--	188.6	--	--	184.4	--	481.7
5	760,524	93.8	--	--	171.0	--	--	147.3	--	412.1
7	262,371	--	118.2	--	--	167.1	--	--	98.1	383.4
9	58,373	--	--	--	201.7	--	--	--	125.5	327.2
15	--	--	--	--	--	--	--	--	--	--

[a]The first four periods (20 years) have eight percent growth and the rest of the periods have four percent growth per period.

TABLE XXXIV

OPTIMAL WATER SUPPLY SCHEDULE[a] FROM BASE RESULT AT EIGHT AND FOUR PERCENT GROWTH

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	93.8	136.9	136.9	297.3	297.3	453.0	453.0	580.0
3	93.8	108.7	108,7	297.3	297.3	297.3	481.7	481.7
5	93.8	93.8	93.8	264.8	264.8	264.8	412.1	412.1
7	--	118.2	118.2	118.2	285.4	285.4	285.4	383.5
9	--	--	--	201.7	201.7	201.7	201.7	327.2
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

TABLE XXXV

OPTIMAL WATER RATE[a] SCHEDULE FROM BASE RESULTS AT EIGHT AND FOUR PERCENT GROWTH

Discount Rate (percent)	Optimal Water Rate for Each Time Used							
	1	2	3	4	5	6	7	8
1	910.2	932.9	2329.3	900.0	1821.6	905.9	1540.1	1124.0
3	910.2	1832.5	2941.2	900.0	1821.6	2416.2	1301.9	1831.8
5	910.2	2307.8	3264.5	1381.0	2201.8	2731.4	1879.6	2332.9
7	--	1529.4	2735.1	3550.6	1960.8	2531.6	2931.2	2538.8
9	--	--	--	2314.8	2940.1	3343.5	3625.9	2944.2
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

show the same trends as those of previous patterns except now optimal capacity investments come into the solution more often due to the higher growth rate.

Results and Analyses for Alternative Size Water System

So far the analyses have been based upon the average initial size of rural water districts in Oklahoma. However, the results obtained from the analyses cannot be applied directly to systems whose size deviates far from this average. To test for effects of size of rural water systems, the optimum capacity investment, operation level and water rate schedules are analyzed for large and small water systems and for the different growth environments.

Small Size Water System

The small size water system is defined as half the number of initial customers of the average size water system. Since the average initial community water system was at 49.1 mgy, the small size water system is assumed at 24.6 mgy. As with the base results, the model solutions of different discount rates and growth situations are investigated. The tabular results are presented in Appendix D.

Since the initial water system size is assumed small, investment decisions come into the solution only when the discount rate is one percent and three percent. Other than for these two low discount rate situations, construction of any size capacity is not economically feasible because the present worth of the costs of the system is greater than the present worth of the benefits even though positive water demand exists for each of the time units. For example, with a two percent growth situation, when the discount rate is three percent construction of a water system is delayed until the seventh time unit even though demand for water exists in every earlier time unit. In other words, the backlog of demand must reach a level where the present worth of the costs of the system is less than or equal to the present worth of the benefits the system provides. These results re-emphasize the effects of economies of scale on investment decisions.

The optimal water supply level and water rate schedule of the small initial water system are substantially different than for the average size system. The small water system supplies less water and the price of the water supplied is significantly higher.

Large Size Water System

The large size water system is defined as a system whose size at the beginning of the planning period is double that of the average size system. The large size initial water system is 98.2 mgy. Unlike capacity investment solutions for the small and average size water system, all discount rates except the 15 percent rate show investment in time unit one for the two percent growth situation. Even then the 15 percent discount rate shows an investment in the second time unit for the two percent growth situation. At the four percent growth situation, investment occurs from the beginning time unit at the 15 percent discount rate. For the average water system, investment did not occur for any growth rate at the 15 percent discount rate. This shows that the effect of economies of scale from the larger water system outweighs the cost of the higher discount rate.

Optimum water supply schedule of the large water system in general shows the same trends as with the average size system. Water supply is generally at the capacity level with only a few time units showing excess capacity. Water rate is hence an important allocator of water when growth in demand hits the capacity restraints. In general, water rates are lower for the large size system compared to the average size system. This again is a reflection of the economies of scale both in terms of investment cost and operation and maintenance.

Comparison of Net Social Benefits Between Actual and Optimum - The Case of Murray #1

To demonstrate the advantages of the optimal investment programming model for planning rural water systems, comparison of results with an actual system, Murray #1, is made. Using the general demand equation for water and the actual water investment and supply records of Murray #1, net social benefits are computed. Then using the optimal investment programming model and the actual rate of growth of Murray #1, the net social benefits from the optimum decisions are computed. Finally, the two net social benefits are compared.

Murray #1 water system started supplying water in 1967. The annual water demand, number of customers and investment record of Murray #1 are presented in Table XXXVI. The amount of water demanded and the number of customers show dramatic increase since the system started operation. The initial number of users, 229 in 1967, increased to 934 in 1980 and results in a 12 percent annual growth rate. In addition to the initial investment, there were two expansions of capacities to meet growth of the system, 1973 and 1978.

TABLE XXXVI

ANNUAL WATER DEMAND, NUMBER OF CUSTOMERS AND INVESTMENT RECORD IN MURRAY #1 WATER SYSTEM

Year	Water Demand (mg)	No. of Customers	Index of Growth	Investment Record ($)
1967	18.2	229	100	314,745
1968	16.8	230	100	--
1969	17.8	243	106	--
1970	17.4	252	110	--
1971	17.3	268	117	--
1972	17.4	389	170	--
1973	24.0	475	207	66,000
1974	36.0	525	229	--
1975	40.7	566	247	--
1976	39.2	599	262	--
1977	38.8	654	286	--
1978	57.1	762	333	225,000
1979	63.4	859	375	--
1980	86.9	934	408	--

It is assumed that the customers in Murray #1 have the same consumption behavior as explained by the general water demand equation. To reflect system growth, the general demand equation is rotated as explained previously. Specifically, the slope of the original demand equation is divided by the index of growth.

Using the rotated demand curves and the actual water demand, consumer benefits are computed. Table XXXVII shows the revised demand equation and the gross benefits for each year.[1] The gross benefits are discounted at five percent to compute the present worth of water consumption benefits. Also, Table XXXVII includes the present worth of actual O and M costs to run the water system each year and the present worth of the capital investment costs. From the information in Table XXXVII the net social benefits realized by the water system are computed as the total present worth of gross benefits less the total present worth of O and M and capital costs. The net social benefits equal $204,428 as computed for the actual Murray #1.

The optimum solution derived by the investment planning model is presented in Table XXXVIII. For the model soluations, the actual 12 percent growth rate is combined with the general demand equation and general O and M and capital cost functions. The optimum solution in Table XXXVIII shows that 72.8 mgy capacity should have been built in the initial time unit and 55.2 mgy should have been added in the third time unit. The optimal supply schedule shows a significantly larger volume of water being supplied than for the actual system. The objective value generated by the optimal solution is $310,176 which is about 52 percent higher than that for the actual water system.

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[1]Gross benefits are defined as total area under the demand curve before costs have been subtracted.

TABLE XXVII

ACTUAL BENEFITS AND COSTS IN SUPPLYING WATER IN
MURRAY #1 WATER SYSTEM

Year	Revised Demand Equations	Water Supply (mg)	Gross Benefits ($)	Discounted Gross Benefits at 5% ($)	Discounted O&M Costs at 5% ($)	Discounted Capital Investment at 5% ($)
1967	P=4840.2-189.4X	18.2	25,355	24,148	6,311	198,799
1968	P=4840.2-189.4X	16.8	27,859	25,269	5,390	--
1969	P=4840.2-178.7X	17.8	29,536	25,514	5,439	--
1970	P=4840.2-172.2X	17.4	32,084	26,396	5,063	--
1971	P=4840.2-161.9X	17.3	35,280	27,704	4,794	--
1972	P=4840.2-111.4X	17.4	50,492	37,678	4,592	--
1973	P=4840.2-91.5X	24.0	63,461	45,101	6,033	15,741
1974	P=4840.2-82.7X	36.0	67,068	45,394	8,618	--
1975	P=4840.2-76.7X	40.7	69,943	45,086	9,280	--
1976	P=4840.2-72.3X	39.2	78,637	48,276	8,512	--
1977	P=4840.2-66.2X	38.8	88,140	51,534	8,024	--
1978	P=4840.2-56.9X	57.1	90,858	0,593	11,246	13,512
1979	P=4840.2-50.5X	63.4	103,881	55,090	11,892	--
1980	P=4840.2-46.4X	86.9	70,219	35,465	15,524	--
TOTAL				543,248	110,718	228,052

TABLE XXXVIII

OPTIMAL INVESTMENT, OPERATION LEVEL AND NET SOCIAL BENEFIT FROM THE PROGRAMMING MODEL

Building Time Unit	Capacity (mgy)	Operation Level (mgy)	Net Social Benefit ($)
1	72.8	41.2	--
2	--	72.8	--
3[a]	55.2	128.0	--
Total	128.0		310,176[b]

[a]Adjusted to reflect four year time unit.

[b]Program does not permit allocation of net social benefits by time unit.

Several general conclusions can be drawn from the results of these comparisons:

1. Decision makers underestimated growth of the water system and built too small an initial facility.

2. Because of an incorrect investment decision, the Murray #1 community lost considerable benefits which could have been avoided or reduced if optimal decisions had been made.

3. Uncertainty relative to system growth may have been a major factor contributing to under investments by the Murray #1 decision makers. The optimal programming model is a way to improve economic efficiency in decision making of water system investment but does not reduce the problem of uncertainty relative to system growth.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

The primary objective of this study was to demonstrate an improved community services planning model by incorporating intertemporal and attitudinal correlates with decisions on rural water supply investments in Oklahoma. This was accomplished using a mathematical programming model and data collected from Oklahoma rural water systems. Specific objectives of the study included: (1) review of theory on public goods as related to rural water services, (2) estimation of aggreagate rural water demand functions and rural water supply cost functions, (3) development and application of a mathematical programming model for determining optimum timing and size of rural water system investments and optimum pricing of water, and (4) evaluation of past public investments in rural water services.

Economic Theory of Rural Water Services

In the field of public natural resource development in general and community water resource management specifically, the objective is not necessarily expressed in a manner as straight forward as profit maximization in the private sector. The scope of this study, however, is limited to the objective of economic efficiency. An important criterion of economic efficiency in allocation of goods and services is that of marginal cost pricing. By marginal cost pricing of rural water services, two general results are achieved: (1) water services are allocated to the highest value use and (2) quantity of water services demanded is adjusted so that incremental cost just equals customer valuation of the last unit used.

In small rural water systems, a difficult management problem arises with marginal cost pricing since the demand curve usually intersects the average cost curve in the range where the latter is still declining. Consequently, a small water system under the above situation will incur a loss. This study does not consider alternative pricing schemes under such situations.

Aggregate Water Demand

Aggregate water demand was found to be explained by water rates and number of users. The estimated price elasticity of aggregate water demand in rural Oklahoma is about -0.58 which supports the proposition that the price-demand relationship should be considered in planning rural water supply systems. The estimated price elasticity of -0.58 is higher than the estimated elasticity for urban areas of about -0.4 and can be explained in that rural areas generally have alternative sources of water such as wells, streams or small ponds for domestic and nondomestic purposes whereas urban areas rely almost totally on public water supplies. The demand analysis also found a proportional one-to-one relationship between water demand and number of users. That is, if the number of users doubles, water demand will also double, ceteris paribus.

Water Supply Costs

Two different sources of water supply cost - O and M and investment - are empirically analyzed. In the equation describing the relationship between total O and M cost and output the parameter for the quadratic term was estimated to be negative. It was hypothesized that the negative term in Q^2 may be just the first section of a third degree polynomial, the second section not observable from the sample data. The average O and M and investment cost curves further support the existence of economies of scale in water supply. This finding of economies of scale in water supply supports the theory for determining optimum excess capacity in water system investment.

Growth of Rural Water Systems

Average annual growth for the sample of rural water systems was computed using a growth index as the dependent variable and age of system as an independent variable. The average annual growth rate for the sample was estimated at about eight percent.

Results of the Investment Programming Model

The mathematical investment programming model developed in this study for planning rural water systems has the following distinctive aspects: first, optimal excess capacity for initial and expansions to the system are computed as an upper limit of the system. Economies of scale found empirically in water supply facilities are incorporated at given discount rates to obtain the optimal excess capacity design. Second, price-sensitive demands are considered in the model. They are used not only to indicate the social benefits of water demand but also to yield the socially optimal prices, reflecting costs of investment and operation and maintenance. Third, public investment in existing rural community water services in Oklahoma under a specific growth pattern is evaluated by comparing against the optimum system resulting from the model.

Under various input conditions, a wide range of sensitivity analyses of optimal solutions were studied. A 'base model' is defined for the average size system in Oklahoma of 49.1 million gallons of water demand in the initial year of operation and with eight percent annual growth in number of customers. The programming model is run for a planning period of 40 years with five-year increments.

The optimal capacity solutions show 136.9 mgy, 108.7 mgy and 93.8 mgy for the initial systems under conditions of one percent, three percent and five percent discount rates, respectively. According to the schedule of solutions, these initial capacities are maintained through time unit three (15 actual years in the model) and then new facilities are added at the beginning of time unit four.

The programming results correspond with the theory that one of the factors determining size of the optimal capacity is the social discount rate. In other words, under given economies of scale if the discount rate is low the size of optimal capacity is relatively large whereas if the discount rate is high the size of optimal capacity is relatively small.

Optimal water supply increases as the water system grows as long as there is excess capacity. However, once a water system reaches its capacity it cannot increase supply of water even if demand continues to grow. An increase in supply can be realized only after an addition to capacity occurs. However, the additional capacity comes into the solution only after the backlog of demand reaches a certain level. When a water system operates at capacity, water price allocates the limited amount of water. In other words, as the backlog becomes larger and larger the water rate becomes higher and higher until new additions come into the solution. In this manner, the limited supply of water is allocated to consumers on a willingness-to-pay basis.

A comparison of equity for the set of initial customers under conditions of growth and no growth of the water system is made by computing a payment per user. According to the comparison, the discounted amount of payment per user without growth is higher than the amount of payment with growth. This can be explained in that a water system with growth can take advantage of the additional economies of scale whereas a system without growth cannot.

Past Public Investments in Rural Water Services

To demonstrate the usefulness of the optimal decision model for planning rural water systems, a comparison of results with an actual system, Murray #1, is made. Using the actual growth rate of Murray #1, the optimal investment and operation schedule as well as the net social benefits are computed. The net social benefits obtained from the optimal solution are $310,176. This value is compared with the net social benefits, $204,428, which are computed using Murray #1's water supply and investment records. The social benefits generated from the optimal solution is about 52 percent higher than the benefits from the actual water supply decisions.

Conclusions

Policy Implications

In the water demand and supply cost analyses, it was found that there is price-sensitive water consumption behavior and economies of scale in water supply. These findings show close interrelationships among water price, consumption of water and water supply cost. The finding of economies of scale in water supply supports the proposition that excess capacity should be considered in water system capacity design.

From the comparison of net social benefits between actual and optimal results of water system planning for Murray #1, it is proposed that better decisions could be made in maximizing social benefits. The loss of net social benefits for Murray #1 could have been reduced if perfect information on system growth had been available and if better decisions had been made on optimal system capacity.

Based upon the results of this study, the following decision criteria for planning rural water systems are proposed:

1. Price-sensitive consumer water consumption behavior should be considered in decisions of rural water capacity design and pricing policy.

2. The existence of economies of scale in water supply costs are important in considering the above decisions.

3. Predictions of growth are highly important in planning water system capacity.

4. All of the above criteria should be considered simultaneously in making global optimal water supply decisions.

Limitations and Need for Further Research

Like most, this study suffers from a number of limitations, some of which could not be avoided. Primary among these was the simplification in estimating aggregate water demand. The estimated aggregate demand functions did not consider an income effect even though income may be an important factor in explaining water consumption behavior, particularly for nonhousehold use during the summer season. An adequate measure of income for the aggregate analysis was not available.

A second shortcoming is loss of a major part of the marginal cost pricing goal in estimation of the aggregate demand function. Demand was estimated as a function of average billing price and aggregate consumption of water for the district. The general rate structure is one of declining block rates. Therefore individual consumers would theortetically equate marginal block rate price with quantity consumed. The typical block rate price for each water district was used as a surrogate of marginal price in estimation of aggregate water demand. Little difference was noted in estimated parameters when compared to the average billing price results. Evidence is scarce whether consumers adjust quantity to average billing cost or marginal cost. In any event, bias could enter in the results presented here on marginal cost pricing.

A third limitation is that the optimal decision model, discussed in Chapter IV, adopted a linear O and M and investment cost function for water supply cost during the planning period. These linear cost functions may overestimate costs for small systems and underestimate costs for large systems which generally appear during the latter part of the planning period.

A fourth limitation is that the optimal decision model cannot be considered in its present form for other water system management issues such as peak load capacity and price decisions.

Finally, the purpose of this study was to provide information for the planning and management of rural water systems to achieve economic efficiency. The criteria used for this objective was marginal cost pricing. However, because of economies of scale some small water systems may operate at the level where long run marginal cost is lower than long run average cost. Under this circumstance, marginal cost pricing will not cover total water supply cost. As discussed in Chapter II, several alternatives are available which allow marginal cost pricing but at the same time avoid loses due to differences between total water supply cost and total revenue collected from the marginal cost price. These kinds of pricing policies were not covered in this study and remain as further research.

In this study water supply costs cover only distribution for those systems purchasing treated water. Further, cost analysis was limited to those systems in existence. More detailed and current costs are necessary for application of the model to actual planning conditions. Therefore, further study remains to improve the model by using engineering cost data and including other costs involved in a general water supply situation.

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APPENDICES

APPENDIX A

SAMPLE DATA BY OBSERVATION ON WATER

CONSUMPTION, PRICE AND

NUMBER OF TAPS

TABLE XXXIX

SAMPLE DATA BY OBSERVATION ON WATER CONSUMPTION, PRICE AND NUMBER OF TAPS

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
1	16.760	2.16	285	7	292
2	1.460	2.25	35	0	35
3	29.200	3.53	571	4	575
4	45.630	1.35	300	0	300
5	9.125	1.13	87	0	87
6	6.720	2.23	94	16	110
7	4.250	1.80	30	5	35
8	114.420	1.68	341	88	429
9	8.090	1.10	94	1	95
10	30.300	1.25	448	27	475
11	11.680	1.63	172	2	174
12	36.500	3.85	665	15	680
13	73.000	0.94	745	5	750
14	10.950	3.13	193	5	198
15	68.510	1.12	386	24	410
16	14.600	1.17	215	9	224
17	5.840	1.39	85	3	88
18	79.270	1.78	814	10	825
19	65.000	0.87	391	5	396
20	30.300	2.28	273	2	275
21	74.000	1.63	530	1	531
22	54.510	1.56	413	131	544
23	55.800	2.17	788	29	817
24	365.000	1.36	2200	0	2200
25	52.830	1.71	655	20	675
26	47.360	1.92	307	0	307
27	36.500	2.13	365	15	380
28	26.650	2.72	275	4	279
29	10.950	3.33	162	2	164
30	109.500	1.15	550	1	441

TABLE XXXIX (Continued)

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
31	3.650	2.20	60	0	60
32	2.920	1.25	67	4	71
33	78.480	2.00	380	68	448
34	13.510	2.73	78	78	156
35	121.650	1.17	460	100	560
36	73.000	1.21	634	56	690
37	161.870	0.64	1100	0	1100
38	30.420	3.53	242	14	256
39	36.500	2.00	330	31	361
40	4.140	2.10	74	1	75
41	21.290	2.14	209	2	211
42	11.130	1.29	115	1	116
43	94.900	2.17	609	2	611
44	17.520	2.36	107	58	165
45	12.050	1.63	65	35	100
46	5.340	3.15	68	34	102
47	54.750	0.86	515	50	565
48	2.920	2.92	43	7	50
49	9.230	3.75	150	54	204
50	6.210	2.75	60	31	91
51	7.300	2.10	102	18	120
52	28.470	2.15	158	60	218
53	52.300	1.78	850	100	950
54	7.300	1.40	55	1	56
55	9.125	2.90	149	12	161
56	27.680	1.38	234	9	243
57	36.500	1.99	430	20	450
58	164.250		992	101	1093
59	61.310	1.82	379	10	389
60	39.060	1.18	425	2	427
61	2.190	1.72	16	2	18

TABLE XXXIX (Continued)

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
62	29.200	0.68	189	6	195
63	10.260	3.80	77	25	102
64	13.060	1.97	60	21	81
65	60.590	1.35	884	0	884
66	6.390	1.66	114	1	115
67	182.500	0.81	975	125	1100
68	3.650	2.25	94	0	94
69	56.900	1.75	903	0	903
70	40.510	1.90	605	0	605
71	158.170	0.89	475	100	575
72	19.830	1.19	216	4	220
73	22.960	2.54	375	10	385
74	37.500	0.89	209	0	209
75	24.460	1.81	499	6	505
76	25.550	1.10	245	4	249
77	23.730	1.40	325	20	345
78	12.780	2.73	354	36	390
79	365.000	1.40	950	250	1200
80	5.457	2.42	87	0	87
81	29.200	2.57	383	1	384
82	96.730	1.50	525	2	527
83	1.460	1.90	50	0	50
84	25.550	1.75	225	5	230
85	42.560	2.75	443	8	451
86	164.250	1.33	910	18	928
87	116.440	0.79	361	39	400
88	136.880	0.72	325	10	335
89	127.750	1.12	849	11	860
90	36.500	1.58	525	25	550
91	9.130	2.00	184	3	187
92	10.980	2.25	288	1	289
93	8.500	3.15	185	4	189

TABLE XXXIX (Continued)

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
94	36.500	1.02	490	10	500
95	182.500	1.61	1299	101	1400
96	73.000	3.75	1474	12	1486
97	13.380	1.58	214	0	214
98	9.790	2.40	108	4	112
99	9.420	1.08	105	5	110
100	53.940	2.25	791	18	809
101	16.610	1.79	225	1	226
102	58.400	0.84	432	60	492
103	13.140	3.67	435	0	435
104	18.250	1.47	200	0	200
105	60.809	2.75	649	0	649
106	18.250	2.86	492	0	492
107	12.170	1.98	220	2	222
108	5.475	2.01	48	5	53
109	6.060	1.71	68	1	69
110	48.650	1.95	258	12	270
111	10.220	4.00	128	20	148
112	205.400	1.03	1387	132	1419
113	27.460	1.71	338	0	338
114	23.100	1.92	373	4	377
115	73.000	1.40	370	80	450
116	24.460	3.20	490	0	490
117	12.480	2.67	112	2	114
118	26.900	2.60	517	5	522
119	27.380	3.00	418	2	420
120	18.250	2.25	408	2	410
121	97.670	2.29	1310	11	1321
122	37.560	3.21	475	0	475
123	12.150	2.12	200	1	201
124	73.000	2.25	1342	100	1442
125	7.300	1.98	114	3	117

TABLE XXXIX (Continued)

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
126	24.330	2.83	240	0	240
127	16.430	1.72	165	0	165
128	148.190	1.07	1250	150	1400
129	12.980	1.02	203	12	215
130	5.840	1.02	38	0	38
131	34.070	2.67	520	10	530
132	9.130	2.89	104	0	104
133	28.830	2.60	390	50	440
134	42.600	1.30	667	33	700
135	10.950	1.83	162	8	170
136	10.210	3.30	160	0	160
137	21.730	1.86	204	3	207
138	109.500	1.14	982	80	1062
139	57.490	1.34	490	6	496
140	91.250	1.22	310	0	310
141	146.000	1.25	600	100	700
142	36.500	1.47	248	5	253
143	12.150	2.09	166	11	177
144	12.150	2.08	211	0	211
145	6.100	1.47	119	1	120
146	47.450	1.48	457	25	482
147	27.380	0.77	152	0	152
148	74.460	0.76	474	0	474
149	70.300	0.79	437	0	437
150	14.970	1.96	187	10	197
151	23.730	2.90	400	0	400
152	23.730	3.20	425	1	426
153	21.290	0.80	179	6	185
154	54.750	3.38	710	30	740
155	45.630	1.36	416	30	446
156	152.060	1.12	1570	130	1700
157	182.500	2.20	1350	300	1650

TABLE XXXIX (Continued)

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
158	54.750	1.29	340	25	365
159	18.250	2.87	296	4	300
160	11.320	1.66	68	8	76
161	25.550	2.50	185	0	185
162	24.090	2.67	349	5	354
163	352.590	1.52	2300	5	2305
164	132.500	1.70	1172	0	1172
165	182.500	1.74	1477	50	1527
166	21.900	1.76	250	0	250
167	38.690	1.03	600	0	600
168	73.000	2.14	497	1	498
169	10.590	1.79	140	0	140
170	51.100	1.59	405	20	425
171	10.340	1.92	141	4	145
172	10.340	1.09	140	0	140
173	18.750	2.54	263	5	268
174	84.740	1.36	690	0	690
175	127.750	1.12	675	25	700
176	7.300	1.17	101	1	102
177	48.65	1.35	187	0	187
178	365.000	1.14	2050	150	2100
179	36.500	1.50	495	12	507
180	3.830	2.22	73	2	75
181	32.850	1.25	370	0	370
182	21.900	1.10	135	0	135
183	54.750	1.28	625	125	750
184	110.800	1.08	360	40	400
185	30.400	1.71	366	4	370
186	18.250	3.29	408	22	430
187	14.600	2.94	344	6	350
188	150.940	1.89	1651	9	1660

TABLE XXXIX (Continued)

OBS	AGWAD	WAPR	RESID	NONR	TOTAL
189	310.980	1.61	975	5	980
190	25.950	1.89	337	0	337
191	5.550	3.53	87	0	87
192	2.920	3.00	50	0	50
193	44.170	3.70	721	0	721
194	18.250	2.44	337	0	337
195	36.500	1.98	569	0	569
196	2.190	1.97	31	0	31
197	18.250	2.25	225	0	225
198	32.850	1.00	198	20	218
199	29.200	2.54	196	106	302
200	2.190	2.22	115	69	184
201	1.530	2.57	15	5	20
202	14.600	0.59	90	7	97
203	11.680	2.00	58	33	91
214	19.310	1.94	198	21	219

APPENDIX B

SAMPLE DATA BY OBSERVATION ON OPERATION AND MAINTENANCE COST

TABLE XL

SAMPLE DATA BY OBSERVATION ON OPERATION AND MAINTENANCE COST

OBS	WASD mgy	COST PER mgy	COST	DENSITY PER MILE #	USERS #	WASD1
1	3.000	281.7	845	7.47	13	9.0
2	3.520	233.0	820	8.05	14	12.4
3	3.700	295.9	1095	9.20	16	13.7
4	4.200	558.6	2346	9.77	17	17.6
5	13.814	1,279.6	17677	7.59	203	190.8
6	16.714	1,351.1	22582	9.72	260	279.4
7	22.467	1,003.8	23901	9.98	267	504.8
8	21.620	1,831.9	39606	10.32	287	467.4
9	25.608	1,265.1	32397	11.91	331	605.6
10	23.987	1,172.5	28125	12.59	350	575.4
11	33.046	919.9	30399	13.53	376	1092.0
12	2.570	1,268.5	3260	5.28	29	6.6
13	2.380	1,158.8	2758	5.28	29	5.7
14	10.200	918.0	9364	7.48	168	104.0
15	10.800	925.1	9991	7.75	174	116.6
16	12.000	880.1	10561	7.97	179	144.0
17	14.400	823.9	11864	8.99	202	207.4
18	18.000	945.9	17027	7.57	223	324.0
19	19.200	816.1	15670	8.45	249	368.6
20	24.000	969.5	23267	9.37	276	576.0
21	11.316	1,420.3	16072	4.29	183	128.1
22	19.864	819.8	16284	5.96	305	394.6
23	26.190	1,144.3	29968	6.52	334	685.9
24	26.418	1,095.3	28935	6.97	357	697.9
25	33.347	1,351.7	45074	6.58	384	1112.0
26	39.907	2,265.5	90409	8.61	502	1592.6
27	43.646	1,876.9	81920	4.31	542	1905.0
28	44.376	1,801.2	79932	5.16	648	1969.2
29	50.854	1,834.4	93287	5.21	655	2586.1
30	8.211	1,082.9	8892	6.00	230	67.4
31	8.537	1,411.6	12051	6.65	255	71.2
32	10.674	2,880.3	30744	7.52	307	113.9
33	14.685	2,467.7	36238	9.33	381	215.6
34	17.897	2,327.4	41653	9.91	405	320.3
35	23.723	2,011.2	47712	10.82	442	562.8
36	30.076	1,924.6	57884	11.82	483	904.6
37	33.698	1,734.2	58440	12.56	513	1135.6
38	23.361	200.5	4683	8.74	253	545.7
39	19.098	248.9	4754	9.39	272	364.7
40	22.386	214.4	4800	9.77	283	501.1
41	42.565	120.1	5112	10.81	313	1811.8
42	17.247	315.8	5446	11.64	337	297.5

TABLE XL (Continued)

OBS	WASD mgy	COST PER mgy	COST	DENSITY PER MILE #	USERS #	WASD1
43	29.076	195.1	5674	10.90	349	845.4
44	31.582	193.0	6095	11.31	362	997.4
45	16.947	1,780.0	30165	6.68	389	287.2
46	18.924	1,619.5	30648	7.19	419	358.1
47	23.752	1,526.9	36267	8.03	468	564.2
48	28.571	1,594.9	48568	9.07	551	816.3
49	51.393	1,510.9	77651	9.11	572	2641.2
50	74.699	1,343.3	100341	9.48	595	5579.9
51	219.694	322.5	70857	6.24	866	48265.5
52	232.199	375.0	87077	6.53	905	63916.4
53	247.160	313.5	77475	7.06	979	61088.1
54	305.396	307.6	93938	7.54	1043	93266.7
55	279.398	393.1	109830	7.52	1043	78063.2
56	321.538	379.1	121888	7.70	1091	103386.6
57	12.430	5,519.0	68601	5.12	507	154.5
58	13.020	2,908.6	37870	5.74	568	169.5
59	13.682	3,625.6	49606	6.22	615	187.2
60	140.677	466.4	65605	6.81	674	1979.0
61	144.812	493.7	71497	7.44	760	20967.0
62	19.401	1,738.8	33734	5.25	268	376.4
63	22.907	1,616.5	37029	5.52	282	524.7
64	26.282	1,276.0	33552	6.07	310	690.7
65	14.708	805.1	11841	9.57	302	216.3
66	19.987	753.4	15058	9.95	314	399.5
67	12.815	1,022.9	13108	9.85	311	164.2
68	19.779	844.4	16701	8.30	347	391.2
69	18.267	1,504.6	27485	8.30	347	333.7
70	13.753	1,458.5	20430	6.53	402	189.1
71	15.944	1,568.7	25012	7.75	477	254.2
72	15.630	1,593.0	24899	7.83	482	244.3
73	23.713	1,308.7	31034	5.58	312	562.3
74	22.816	1,273.0	29044	6.17	345	520.6
75	34.727	1,795.3	62345	7.82	437	1206.0
76	45.643	1,851.9	84527	9.01	504	2083.3
77	60.394	1,286.2	77681	10.17	569	3647.4
78	54.059	1,429.4	77273	11.61	649	2922.4
79	12.134	786.2	9540	12.43	400	147.2
80	14.228	1,771.8	25209	9.76	400	202.4
81	19.108	1,523.8	29117	9.76	400	365.1
82	20.348	1,522.2	30974	9.74	399	414.0
83	26.978	1,331.5	35922	9.74	399	727.8
84	35.841	1,280.6	45898	10.35	424	1284.5
85	46.245	1,289.0	59612	11.93	448	2138.6
86	47.990	1,330.0	63829	11.81	484	2303.0
87	49.590	1,535.4	76141	12.47	511	2459.2

TABLE XL (Continued)

OBS	WASD mgy	COST PER mgy	COST	DENSITY PER MILE #	USERS #	WASD1
88	10.200	1,089.0	11108	6.37	217	104.0
89	11.000	1,659.1	18250	4.93	232	121.0
90	14.200	1,406.1	19967	6.34	298	201.6
91	14.700	1,576.1	23168	7.27	342	216.9
92	15.000	1,263.1	18947	7.78	366	225.0
93	16.300	1,603.1	27108	7.91	372	265.7
94	17.358	1,020.5	17713	3.15	252	301.3
95	17.266	1,217.0	21012	3.35	268	298.1
96	24.040	1,488.5	35784	5.19	475	577.9
97	36.010	968.7	34883	5.73	525	1296.7
98	40.699	936.1	38100	6.18	566	1656.4
99	39.198	1,127.1	44179	6.54	599	1536.5
100	38.853	1,354.2	52617	7.14	654	1509.6
101	57.139	980.5	56026	7.48	762	3264.9
102	63.372	1,215.3	77017	8.43	859	4016.0
103	86.858	1,134.5	98541	9.17	934	7544.3
104	30.000	1,239.4	37183	5.26	647	900.0
105	32.000	2,734.9	87518	5.71	708	1024.0
106	33.000	1,782.8	58834	6.18	767	1089.0
107	36.000	1,870.0	67320	6.81	845	1296.0
108	38.065	1,525.9	58083	5.73	883	1448.9
109	55.000	972.1	53463	16.95	607	3025.0
110	65.000	757.5	49240	19.10	684	4225.0
111	71.056	922.2	65530	21.92	785	5049.0
112	76.778	920.9	70706	23.51	842	5894.9
113	76.145	1,230.4	93687	18.93	895	5798.1
114	108.068	1,004.7	108571	27.90	999	11678.7
115	28.876	617.6	17835	6.01	335	833.8
116	26.725	747.4	19975	6.42	358	714.2
117	26.010	803.6	20901	6.44	359	676.5
118	23.826	1,073.4	25575	4.70	380	567.7
119	33.729	973.6	32838	5.75	465	1137.6
120	35.481	795.2	28216	9.68	449	1258.9
121	46.245	736.9	34080	10.20	473	2138.6
122	47.990	746.5	35826	11.06	513	2303.0
123	49.590	864.8	42887	11.65	540	2459.2
124	2.117	2,517.7	5330	7.78	131	4.5
125	3.954	707.1	2796	8.82	140	15.6
126	2.951	740.8	2186	8.38	141	8.7
127	3.036	1,023.1	3106	6.80	143	9.2
128	4.583	1,838.1	8424	7.23	152	21.0
129	7.376	597.2	4405	8.56	180	54.4
130	5.154	1,594.5	8218	3.82	262	26.6
131	7.380	1,436.0	10598	4.08	280	54.5
132	11,105	1,578.7	17532	4.66	320	123.3

TABLE XL (Continued)

OBS	WASD mgy	COST PER mgy	COST	DENSITY PER MILE #	USERS #	WASD1
133	30.980	1,135.3	35173	3.54	348	959.8
134	29.414	1,327.1	39035	3.19	314	865.2
135	36.078	1,273.1	45931	4.88	380	1301.6
136	49.694	1,285.5	63880	6.17	607	2469.5
137	56.255	1,275.7	71764	6.39	629	3164.6
138	58.499	1,567.3	91688	6.51	640	3422.1

APPENDIX C

ZERO-ONE MIXED INTEGER PROGRAMMING USING BRANCH AND BOUND TECHNIQUES

Consider the following 0-1 MIP problem

$$\text{Minimize } 200X_1 + 100X_2 + 75X_3 + 50X_4 + 7x_1 + 2x_2 + 5x_3 + 9x_4$$

$$\begin{array}{lrl}
\text{Subject to} & x_1 + 0.2x_2 + x_3 + 2x_4 & \geq 43 \\
& 3x_1 + 2x_3 & \geq 46 \\
& x_1 + 4x_2 & \geq 42 \\
& 0.7x_1 + x_4 & \geq 40 \\
& X_1 - 0.02x_1 & \geq 0 \\
& X_2 - 0.02x_2 & \geq 0 \\
& X_3 - 0.02x_3 & \geq 0 \\
& X_4 - 0.02x_4 & \geq 0
\end{array}$$

$0 \leq X_i \leq 1$, $x_i \geq 0$, X_i = integer, all i

Note that the X_i are integer (binary) decision variables restricted to zero or one, whereas the x_i are continuous (nonnegative) variables. Also note that the last four constraints require $X_i = 1$ if $x_i \geq 0$; hence the model is is analagous to the type of fixed-charge problem reported in this research. The last four constraints also impose an upper limit of 50 on the continuous variables since the binaries cannot exceed one.

Node 1

The first step in the solution methodology is to ignore integer restrictions on the X's. Using linear programming (LP), the following optimal continuous solution is obtained:

$Z_1 = 488.00$

Variable No.	1	2	3	4
Integer	0.31	0.13	0	0.59
Continuous	15.33	6.67	0	29.27

In this table, the optimal objective function value (Z_1 is 488.00; integer variable 1 (X_1) is 0.31; continuous variable 1 (x_1) is 15.33; and so forth.

Nodes 2 and 3

Clearly, some of the integer variables above have nonintegral value. One must be selected on which to branch (i.e., set equal to 0 or 1). Arbitrarily, X_2 is chosen. A new problem (called Node 2) is formed which is identical to the LP problem of Node 1 with addition of the new constraint $X_2 = 0$. Similarly, Node 3 is formed by adding $X_2 = 1$ to the problem of Node 1. Solution of these new problems by LP results in the following:

	$Z_2 = 568.000$				$Z_3 = 574.667$			
Variable No.	1	2	3	4	1	2	3	4
Integer	0.84	0	0	0.21	0.31	1	0	0.59
Continuous	42.00	0	0	10.60	15.33	6.67	0	29.27

As expected, objective function values (Z_2 and Z_3) increased for both problems due to the additional constraint. In Node 2 (which has best functional value), all binary variables are not integral; hence, another branch must be made.

Nodes 4 and 5

Both X_1 and X_4 in Node 2 are nonintegral; X_1 is arbitrarily chosen as the next binary variable on which to branch. Node 4 is created by adding the constraint $X_1 = 1$ to the problem of Node 2. That is, problem No. 4 is the original LP problem with two additional constraints:

$X_2 = 0$ and $X_1 = 1$. Similarly, Node 5 is formed by adding the constraint $X_1 = 0$ to the problem of Node 2. Solution of problems 4 and 5 by LP results in the following for No. 4; No. 5 however is infeasible. Because no LP solution exists, its objective function value is set equal to infinity ($Z_5 = \infty$).

$\underline{Z_4 = 600.00}$

Variable No.	1	2	3	4
Integer	1	0	0	0.1
Continuous	50.00	0	0	5.00

The terminal nodes of the solution tree now include 3, 4, and 5; branches have already been made from 1 and 2. Node 3 has best functional value; hence, its solution must be examined to determine whether the binary variables have integral value. Both X_1 and X_4 do not; one must be selected, X_1 is chosen. Nodes 6 and 7 are created by adding the constraints $X_1 = 0$ and $X_1 = 1$ to the problem of Node 3.

Additional Nodes

The process of branching on nonintegral binary variables followed by LP solution is continued until all binary varibles in the terminal nodes with best functional value are integral. At this point, the optimal solution has been found. The solution tree for this problem is shown in Figure A1. Nine interations were made (of the possible 15) to obtain optimality. Although only the final solution is "binary feasible" (i.e., with all binary variables having integral value) it is common to obtain additional feasible solutions during the branching process. Instead of continuing branching until optimality is proven, the

Node	Optimal Objective Function
1	488.00
2	568.00
3	574.67
4	600.00
5	-
6	670.50
7	700.00
8	-
9	639.40

process can be stopped if desired when an intermediate feasible solution with acceptable functional value is encountered.

A tabulation of all solutions obtained during branching for the sample problem is as follows:

	Binary Variables			
Node	1	2	3	4
1	0.31	0.13	0	0.59
2	0.84	0	0	0.21
3	0.31	1	0	0.59
4	1	0	0	0.10
5	-	-	-	-
6	0	1	0.46	0.80
7	1	1	0	0.10
8	-	-	-	-
9	1	0	0	1

	Continuous Variables			
Node	1	2	3	4
1	15.33	6.67	0	29.27
2	42.00	0	0	10.60
3	15.33	6.67	0	29.27
4	50.00	0	0	5.00
5	-	-	-	-
6	0	10.50	23.00	40.00
7	50.00	0	0	5.00
8	-	-	-	-
9	42.00	0	0	10.60

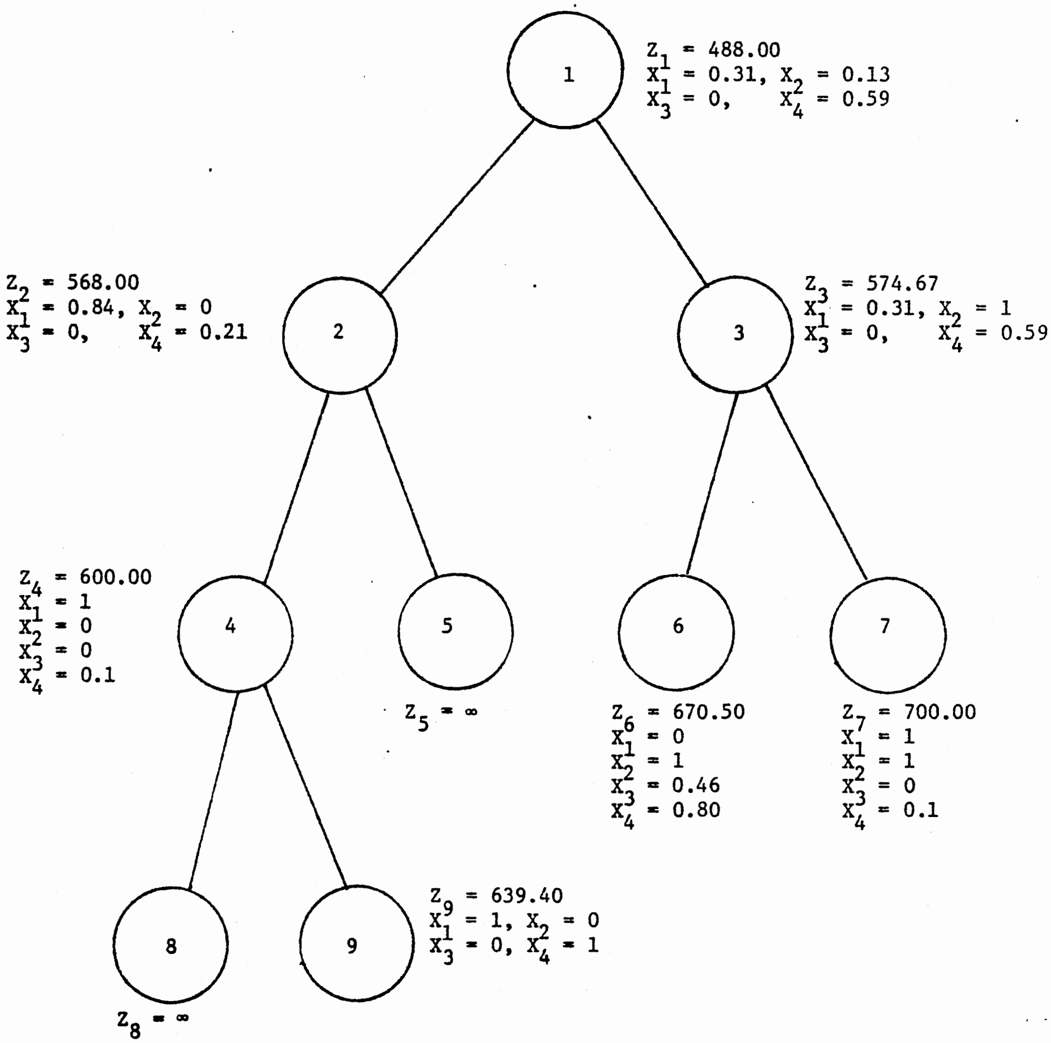

Figure 12. MIP Solution Tree for the Problem

APPENDIX D

TABLEAU RESULTS OF THE SMALL AND

LARGE SIZE WATER SYSTEMS

TABLE XLI

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF SMALL WATER SYSTEM AT TWO PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								Total
		1	2	3	4	5	6	7	8	
1	158,822	67.1	-	-	-	-	-	-	-	67.1
3	2,966	-	-	-	-	-	-	75.8	-	75.8
5	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-

[a]Amount of system capacities in mgy.

TABLE XLII

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF SMALL WATER SYSTEM AT FOUR PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	301,002	-	91.6	-	-	-	-	90.9	-	182.5
3	33,116	-	-	-	-	-	135.5	-	-	135.5
5	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-

[a]Amount of system capacities in mgy.

TABLE XLIII

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF SMALL WATER SYSTEM AT SIX PERCENT GROWTH[a]

Discount Rate (R)	Objective Value ($)	Building Time Unit								Total
		1	2	3	4	5	6	7	8	
1	600,466	-	114.2	-	-	-	167.6	-	152.7	434.5
3	102.933	-	-	-	-	196.9	-	-	156.1	353.0
5	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-

[a]Amount of system capacities in mgy.

TABLE XLIV

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF SMALL WATER SYSTEM AT EIGHT PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	1,814,717	-	136.9	-	-	208.1	-	293.9	299.4	938.3
3	249,230	-	-	-	235.0	-	-	272.3	237.7	745.0
5	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-		-	-	-	-

[a] Amount of system capacities in mgy.

TABLE XLV

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF SMALL WATER SYSTEM AT TEN PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	2,291,539	-	143.4	-	228.1	-	382.4	460.1	741.3	1955.3
3	545,131	-	-	-	290.7	-	307.8	365.3	588.6	1552.4
5	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-

[a]Amount of system capacity in mgy.

TABLE XLVI

OPTIMAL WATER SUPPLY[a] AND WATER-RATE[b] SCHEDULE OF SMALL WATER SYSTEM AT TWO PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	67.1	67.1	67.1	67.1	67.1	67.1	67.1	67.1
	(1106.3)	(1515.6)	(1877.9)	(2200.0)	(2481.8)	(2750.2)	(2991.8)	(3213.2)
3	–	–	–	–	–	–	75.8	75.8
	–	–	–	–	–	–	(2692.5)	(2942.6)
5	–	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–
9	–	–	–	–	–	–	–	–
15	–	–	–	–	–	–	–	–

[a]Amount of water supplied in mgy.

[b]Dollars per million gallons in parentheses.

TABLE XLVII

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF SMALL WATER SYSTEM AT FOUR PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	-	91.6	91.6	91.6	91.6	91.6	182.5	182.5
	-	(1040.6)	(1800,9)	(2423.8)	(2936.7)	(3358.1)	(2124.5)	(2690.3)
3	-	-	-	-	-	135.5	135.5	135.5
	-	-	-	-	-		(2942.3)	(3362.4)
5	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-

[a]Amount of water supplied in mgy.

[b]Dollarsper million gallons in parentheses.

TABLE XLVIII

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF SMALL WATER SYSTEM AT SIX PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	-	114.2	114.2	114.2	114.2	281.8	281.8	444.5
	-	(914.7)	(2022.5)	(2856.1)	(3472.8)	(1918.4)	(2792.0)	(2321.9)
3	-	-	-	-	196.9	196.9	196.9	352.9
	-	-	-	-	(2149.6)	(2937.2)	(3547.6)	(2935.6)
5	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-

[a]Amount of water supplied in mgy.

[b]Dollars per million gallons in parentheses.

TABLE XLIX

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF SMALL WATER SYSTEM AT EIGHT PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	-	136.9	136.9	136.9	345.1	345.1	638.9	938.3
	-	(932.9)	(2329.3)	(3273.9)	(1849.0)	(2953.3)	(2297.2)	(2297.4)
3	-	-	-	235.0	235.0	235.0	507.3	745.0
	-	-	-	(1822.0)	(2950.0)	(3702.0)	(2915.7)	(2916.0)
5	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-

[a]Amount of water supplied in mgy.

[b]Dollarsper million gallons in parentheses.

TABLE L

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF SMALL WATER SYSTEM AT TEN PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	-	143.4	143.4	371.5	371.5	371.5	1213.9	1955.2
	-	(1485.6)	(2933.9)	(1510.7)	(2922.4)	(2360.2)	(2386.6)	(2367.2)
3	-	-	-	290.7	290.7	598.5	963.8	1552.4
	-	-	-	(2334.9)	(3439.5)	(2965.9)	(2986.9)	(2971.4)
5	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-

[a]Amount of water supplied in mgy.

[b]Dollars per million gallons in parentheses.

TABLE LI

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF LARGE WATER SYSTEM AT TWO PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	3,037,008	95.1	-	-	-	-	-	39.3	-	134.4
3	1,809,193	87.1	-	-	-	-	-	41.5	-	128.6
5	1,102,024	77.8	-	-	-	-	-	49.8	-	127.6
7	675,585	70.2	-	-	-	-	-	51.4	-	121.6
9	405,608	68.6	-	-	-	-	-	53.0	-	121.6
15	17,028	-	54.3	-	-	-	-	-	-	54.3

[a]Amount of system capacities in mgy.

TABLE LII

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF LARGE WATER SYSTEM AT FOUR PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								Total
		1	2	3	4	5	6	7	8	
1	4,922,375	114.8	-	-	-	82.2	-	55.1	54.2	306.3
3	2,832,615	107.5	-	-	-	70.7	-	73.8	54.2	306.3
5	1,651,769	94.4	-	-	-	75.9	-	81.7	-	252.0
7	974,256	81.7	-	-	-	88.7	-	-	107.6	278.0
9	575,056	77.8	-	-	-	84.5	-	-	101.7	264.0
15	44,528	59.9	-	-	-	-	89.8	-	-	149.7

[a]Amount of system capacities in mgy.

TABLE LIII

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF LARGE WATER SYSTEM AT SIX PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	8,558,640	132.0	-	-	128.8	-	105.4	124.4	165.9	656.5
3	4,675,122	114.2	-	-	121.7	-	130.3	124.4	165.9	656.5
5	2,609,095	108.8	-	-	96.0	-	144.2	118.6	158.1	467.6
7	1,483,030	93.1	-	-	111.7	-	161.4	-	229.6	595.8
9	848,527	85.5	-	-	109.7	-	153.8	-	217.0	566.0
15	84,992	65.8	-	-	-	131.1	-	156.1	-	353.0

[a]Amount of system capacities in mgy.

TABLE LIV

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF LARGE WATER SYSTEM AT EIGHT PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								Total
		1	2	3	4	5	6	7	8	
1	15,087,504	131.3	-	152.0	-	153.7	204.8	301.8	442.1	1385.8
3	7,973,369	137.8	-	-	159.5	139.7	204.8	301.8	442.1	1385.8
5	4,288,098	99.8	-	115.5	-	201.2	195.2	287.6	421.3	1320.6
7	2,356,339	93.8	-	108.5	-	194.4	185.9	273.9	401.2	1257.7
9	1,298,954	89.4	-	103.4	-	184.0	176.6	260.2	381.2	1194.8
15	152,004	74.1	-	-	160.9	-	169.5	190.1	278.6	873.2

[a]Amount of system capacities in mgy.

TABLE LV

OPTIMAL CAPACITY INVESTMENT SCHEDULE OF LARGE WATER SYSTEM AT TEN PERCENT GROWTH[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								Total
		1	2	3	4	5	6	7	8	
1	27,045,256	157.5	-	109.2	162.7	261.6	422.4	679.5	1000.0	2792.9
3	13,936,689	142.5	-	124.2	162.7	261.6	422.4	679.5	1000.0	2792.9
5	7,285,573	119.7	-	134.5	155.0	249.3	402.5	690.8	1000.0	2751.8
7	3,868,880	102.7	-	139.3	147.6	237.6	383.3	616.6	993.6	2620.7
9	2,079,792	102.7	-	164.0	-	329.0	364.1	585.8	943.8	2489.4
15	250,092	88.8	-	-	181.7	164.8	266.1	428.1	689.8	1819.3

[a]Amount of system capacities in mgy.

TABLE LVI

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF LARGE WATER SYSTEM AT TWO PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	70.2 (912.5)	77.8 (912.1)	86.1 (908.9)	95.1 (906.4)	95.1 (1305.8)	95.1 (1686.2)	127.6 (910.6)	134.6 (1113.9)
3	70.2 (912.5)	77.8 (912.1)	86.1 (908.9)	86.1 (1322.2)	86.1 (1683.8)	86.1 (2028.2)	127.6 (910.6)	127.6 (1331.6)
5	70.2 (912.5)	77.8 (912.1)	77.8 (1332.2)	77.8 (1705.6)	77.8 (2032.4)	77.8 (2343.6)	127.6 (910.6)	127.6 (1331.6)
7	70.2 (912.5)	70.2 (1340.7)	70.2 (1719.8)	70.2 (2056.8)	70.2 (2351.6)	70.2 (2632.4)	121.6 (1117.0)	121.6 1518.2)
9	68.6 (1012.5)	68.6 (1431.0)	68.6 (1801.4)	68.6 (2130.7)	68.6 (2418.8)	68.6 (2693.2)	68.6 2940.2)	121.6 (1518.2)
15	- -	54.3 (2237.5)	54.3 (2530.7)	54.3 (2791.3)	54.3 (3019.4)	54.3 (3236.6)	54.3 (3432.1)	54.3 (3611.3)

[a]Amount of water supplied in mgy.

[b]Dollarsper million gallons in parentheses.

TABLE LVII

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF LARGE WATER SYSTEM AT FOUR PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	77.8 (912.1)	94.4 (910.4)	114.8 (914.6)	114.8 (1695.3)	170.3 (906.3)	197.0 (1123.6)	252.0 (915.2)	306.2 (921.3)
3	77.8 (912.1)	94.4 (910.4)	107.5 (1193.5)	107.5 (1924.5)	170.3 (906.3)	178.2 (1522.2)	252.0 (915.2)	306.2 (921.3)
5	77.8 (912.1)	94.4 (910.4)	94.4 (1693.9)	94.4 (2335.8)	170.3 (906.3)	170.3 (1689.6)	252.0 (915.2)	252.0 (1696.4)
7	77.8 (912.1)	81.7 (1501.0)	81.7 (2179.1)	81.7 (2734.6)	170.3 (906.3)	170.3 (1689.6)	170.3 (2336.8)	277.9 (1326.0)
9	77.8 (912.1)	77.8 (1682.3)	77.8 (2328.0)	77.8 (2857.1)	162.3 (1112.7)	162.3 (1859.2)	162.3 (2475.9)	264.0 (1524.8)
15	59.9 (1921.6)	59.9 (2514.7)	59.9 (3011.8)	59.9 (3419.1)	59.9 (3754.6)	149.7 (2126.4)	149.7 (2695.2)	149.7 (3159.3)

[a]Amount of water supplied in mgy.

[b]Dollar per million gallons in parentheses.

TABLE LVIII

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF LARGE WATER SYSTEM AT SIX PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	85.5	114.2	132.0	204.8	260.8	366.2	490.6	656.5
	(913.9)	(914.7)	(1511.6)	(917.3)	(1127.2)	(905.6)	(933.7)	(901.5)
3	85.5	114.2	114.2	204.8	236.0	366.2	490.6	656.5
	(913.9)	(914.7)	(2027.5)	(917.3)	(1524.0)	(905.6)	(933.7)	(901.5)
5	85.5	108.3	108.3	204.8	204.8	349.0	467.6	625.6
	(913.9)	(1141.3)	(2191.8)	(917.3)	(2023.2)	(1112.0)	(1138.4)	(1108.5)
7	85.5	93.1	93.1	204.8	204.8	366.2	366.2	595.8
	(913.9)	(1725.0)	(2628.0)	(917.3)	(2024.2)	(905.6)	(2040.8)	(1308.1)
9	85.5	85.5	85.5	195.2	195.2	349.0	349.0	566.0
	(913.9)	(2016.8)	(2846.2)	(1122.7)	(2176.8)	(1112.0)	(2193.9)	(1507.8)
15	65.8	65.8	65.8	65.8	196.9	196.9	352.9	352.9
	(1924.5)	(2773.3)	(3411.5)	(3891.9)	(2149.6)	(2937.2)	(2159.2)	(2935.6)

[a]Amount of water supplied in mgy.

[b]Dollars per million gallons in parentheses.

TABLE LIX

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF LARGE WATER SYSTEM AT EIGHT PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	93.8 (910.2)	131.3 (1111.5)	202.2 (912.3)	283.3 (1107.2)	437.0 (930.0)	641.8 (935.8)	943.6 (865.1)	1385.7 (865.8)
3	93.8 (910.2)	137.8 (904.2)	137.8 (2309.7)	297.3 (900.0)	437.0 (930.0)	641.8 (935.8)	943.6 (865.1)	1385.7 (865.8)
5	93.8 (910.2)	99.8 (2116.4)	202.2 (912.3)	215.3 (2113.6)	416.5 (1135.0)	611.6 (1141.1)	899.2 (1073.8)	1320.6 (1074.1)
7	93.8 (910.2)	93.8 (2307.8)	202.2 (912.3)	202.2 (2307.4)	396.6 (1334.0)	582.5 (1339.0)	856.3 (1275.4)	1257.6 (1275.7)
9	89.4 (1116.1)	89.4 (2448.1)	192.7 (118.4)	192.7 (2448.0)	376.8 (1532.0)	553.3 (1537.6)	813.5 (1476.6)	1194.6 (1477.3)
15	74.1 (1832.1)	74.1 (2936.2)	74.1 (3692.0)	235.0 (1822.0)	235.0 (2950.0)	404.4 (2550.1)	594.6 (2505.4)	873.1 (2506.1)

[a]Amount of water supplied in mgy.

[b]Dollarsper million gallons in parentheses.

TABLE LX

OPTIMAL WATER SUPPLY[a] AND WATER RATE[b] SCHEDULE OF LARGE WATER SYSTEM AT TEN PERCENT GROWTH

Discount Rate (percent)	Water Supply and Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	102.7 (914.7)	157.5 (1110.5)	266.7 (899.5)	429.4 (920.1)	691.0 (877.6)	1113.3 (958.1)	1792.8 (977.3)	2792.8 (1110.8)
3	102.7 (914.7)	142.5 (1509.5)	266.7 (899.5)	429.4 (920.1)	691.0 (877.6)	1113.3 (958.1)	1792.8 (997.3)	2792.8 (1110.8)
5	102.7 (914.7)	119.7 (21116.0)	254.1 (1107.4)	409.2 (1126.2)	658.5 (1085.6)	1061.0 (1162.1)	1751.8 (1095.7)	2751.8 (1172.3)
7	102.7 (914.7)	102.7 (2568.2)	242.0 (1307.0)	389.7 (1325.1)	627.1 (1286.6)	1010.4 (1359.4)	1627.0 (1395.2)	2620.6 (1369.1)
9	102.7 (914.7)	102.7 (2568.2)	266.7 (899.5)	266.7 (2580.0)	595.7 (1487.5)	959.8 (1556.8)	1545.5 1590.8)	2489.3 (1566.1)
15	88.8 (1508.2)	88.8 (2937.9)	88.8 (3834.8)	270.5 (2540.9)	435.4 (2513.4)	701.5 (2564.2)	1129.6 (2589.0)	1819.5 (2570.8)

[a]Amount of water supplied in mgy.

[b]Dollarsper million gallons in parentheses.

APPENDIX E

EXAMPLE OF THE REQUIREMENTS FOR OPTIMAL SOLUTION

The Requirements of Optimal Solution[1]

General Mathematical Programming Model

Let's consider the problem of finding the maximum of an objective function f(x) of n non-negative variables $x = (x_1, x_2, \ldots x_n)'$ subject to n constraints such that

$g(x) = (g_1(x), g_2(x), \ldots, g_m(s))' \geq 0,$

where $g_i(x)$ is a function of x for all i=1,2,...,m, and f(x) and g(x) satisfy the following:

Assumption 1: f(x) is differentiable and concave

Assumption 2: g(x) is differentiable and concave

More formally we have the following optimization problem. Find a vector $\bar{x} \geq 0$ that maximizes

$$f(x) \tag{1}$$

subject to the restrictions

$$g(x) \geq 0 \tag{2}$$

and

$$x \geq 0 \tag{3}$$

In order to find a way to solve this problem, we transform it into the following:

$$\text{Lagrangean form } \emptyset\ (x, e) = f(x) + e'g(x) \tag{4}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_m \end{bmatrix} \geq 0$$

and solve the following:

---

[1]T. Takayama and G.G. Judge, Spatial and Temporal Price and Allocation Models, North-Holland Publishing Company, Amsterdam, 1971.

Saddle Value Problem.

Find $(\bar{x}, \bar{e})$ that forms a saddle point of (4). The saddle point for this problem may be defined as:

Definition 1: saddle point

A pair of vectors $(\bar{x}, \bar{e})$ is called a saddle point of $\emptyset$ (x, e) in $x \geq 0$, $e \geq 0$, if the following conditions are satisfied:

$$\bar{x} \geq 0, \; \bar{e} \geq 0, \tag{5}$$

$$(x, \bar{e}) \leq \emptyset\,(\bar{x}, \bar{e}) \leq \emptyset\,(\bar{x}, e) \tag{6}$$

for all $x \geq 0$ and $e \geq 0$.

An important relationship between an optimization problem and a saddle value problem is that if there is a saddle point, then the $\bar{x}$ part of the saddle point is an optimum solution vector of the optimization problem without any qualification on f(x) and g(x).

The Kuhn-Tucker Conditions. The Kuhn-Tucker (1950) conditions provides us with the necessary and sufficient conditions for $(\bar{x}, \bar{e})$ to be a saddle point of $\emptyset$ (x,e) and these conditions are stated in the following theorems:

Theorem (Kuhn-Tucker):

For $(\bar{x}, \bar{e})$ to be a solution for the saddle value problem, the necessary conditions are

$$\bar{\emptyset}_x \leq 0 \text{ and } \bar{\emptyset}_x{}'\bar{x} = 0 \tag{7}$$

$$\bar{\emptyset}_e \geq 0 \text{ and } \bar{\emptyset}_e{}'\bar{e} = 0 \tag{8}$$

for $\bar{x} \geq 0$, $\bar{e} \geq 0$,

and the sufficient conditions are:

$$\emptyset(x, \overline{e}) \leq \emptyset\ (\overline{x}, \overline{e}) + \overline{\emptyset}'_x\ (x-\overline{x}) \tag{9}$$

$$\emptyset(\overline{x}, e) \geq \emptyset\ (\overline{x}, \overline{e}) + \overline{\emptyset}'_e\ (e-\overline{e}) \tag{10}$$

for all $x \geq 0$, $e \geq 0$,

where

$$\overline{\emptyset}_x = \left(\frac{\partial \emptyset(x,e)}{\sigma x}\right)_{(\overline{x},\overline{e})} = \left(\frac{\partial \emptyset(x,e)}{\sigma x_1} \cdots \frac{\partial x(x,e)}{\sigma x_n}\right)_{(\overline{x},\overline{e})} \tag{11}$$

and

$$\overline{\emptyset}_e = \left(\frac{\partial \emptyset(x,e)}{\partial_e}\right)_{(\overline{x},\overline{e})} = \left(\frac{\partial \emptyset(x,e)}{\partial e_1} \cdots \frac{\partial \emptyset(x,e)}{\partial e_m}\right)'_{(\overline{x},\overline{e})} \tag{12}$$

The algebracial interpretation of (7) and (8), assuming that (9) and (10) hold is as follows:

for some components of $\overline{\emptyset}_x$,

$$\text{if} \quad \frac{\partial \emptyset(\overline{x}, \overline{e})}{\partial X_i} = 0, \text{ then } \overline{x}_i \geq 0 \tag{13}$$

$$\text{if} \quad \frac{\partial \emptyset(\overline{x}, \overline{e})}{\partial X_i} < 0, \text{ then } \overline{x}_i = 0$$

for all i and j.

For some components of $\overline{\emptyset}_e$,

$$\text{if} \quad \frac{\partial \emptyset(\overline{x}, \overline{e})}{\partial e_k} = 0, \text{ then } \overline{e}_k \geq 0 \tag{14}$$

$$\text{if} \quad \frac{\partial \emptyset(\overline{x}, \overline{e})}{\partial e_\ell} > 0, \text{ then } \overline{e} = 0 \tag{15}$$

for all k and ℓ.

VITA 2

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