A STUDY OF AUDIO FREQUENCY POWER AMPLIFIERS WITH REACTIVE LOADS

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A STUDY OF AUDIO FREQUENCY

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REACTIVE LOADS

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PREFACE

Vacuum tube power amplifiers have been used, for the past several years, in applications where the load placed on the amplifier contains a reactive component; however, due to the difficult problem of making a nonlinear analysis of such circuit operation, methods of analyses have been developed through the years practically all of which are based upon the assumption of a pure resistive load. Fortunately, many of these loads, such as the electrodynamic loudspeaker, are at least approximately resistive over some portion of the useful range of operation so that at least some indication of circuit performance can be obtained from the idealized analysis. In recent years, there has been an increasing demand for vacuum tube power amplifiers in applications where electromechanical type loads, such as electric motors used in control systems and electrodynamic shakers used for environmental testing, are used which have an appreciable reactive component over a large portion of the useful operating range. Since no general method of analysis, which can be practically applied, appears to exist at the present for circuit operation of this type, very little information concerning the design and operation of such circuits has become available.

It is the object of this work to make a study of the problem of nonlinear circuit analysis for the above latter type of operation and to study the effects of a reactive load on circuit operation using both analytical and laboratory methods. This study has been entered into by the author with the expectation of gaining a better insight of a problem which has been quite intentionally cast aside by many authors in past years and which, in more recent years, has become of considerable importance.

The author wishes to express his thanks to Professor Paul A. McCollum for his invaluable suggestions and assistance and also to the National Science Foundation for the fellowship which made it financially possible for the author to complete his graduate work at Oklahoma State University.

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CHAPTER I

INTRODUCTION

In recent years, there has been an increasing demand for vacuum tube power amplifiers in applications where electromechanical type loads, such as electric motors used in control systems and electrodynamic shakers used for environmental testing, are used which have an appreciable reactive component over a large portion of the useful range of operation. Since no general method of analysis, which can be practically applied, appears to exist, at present, for circuit operation of this type, very little information concerning the design and operation of such circuits has become available.

It is the object of this work to make a study of the problem of nonlinear circuit analysis for the above type of circuit operation and to study the effects of a reactive load on circuit operation using both analytical and laboratory methods.

In order to reach the goal set forth for this work, two chapters have been devoted to a study of the general nature of the problem of nonlinear circuit analysis; a chapter containing an analytical analysis based upon an idealized linear tube has been included in which the operation of an idealized linear tube has been compared with the operation of an actual tube determined from laboratory measurements; and a chapter in which the results of laboratory measurements made for various classes of operation of push-pull power amplifiers with reactive loads has been presented is also included.

The study begins with a general discussion of the problem of non-linear circuit analysis and of classical methods which can be applied in order to make specific studies of special types of circuits containing a nonlinear resistive circuit parameter in combination with linear re-active circuit parameters.

CHAPTER II

THE PROBLEM OF NONLINEAR CIRCUIT ANALYSIS

The purpose of this chapter is to present a brief review of the general nature of the problem of nonlinear circuit analysis and to introduce some simple methods which will be of use in the following study of circuits containing thermionic vacuum tubes which are inherently nonlinear devices.

The Nonlinear Circuit

If the three possible parameters in an electrical circuit, namely, inductance, capacitance, and resistance, are bilaterial and sensibly constant in value, a voltage applied in the circuit and the resulting current obey a linear law and the corresponding devices, or elements, which make up the circuit are called linear elements. When a circuit contains only linear elements, it can ordinarily be solved readily by the use of linear differential equations or equivalent operational techniques involving ordinary algebraic manipulations; however, often times an electrical circuit will have parameters which are functionally dependent upon some variable such as voltage, current, or time. Such nonlinear circuits are usually difficult to analyze since, in general, there is no simple set of rules or procedures to follow for solving nonlinear equations as there is in the theory of linear equations. Also, the analysis is usually further complicated by the fact that ordinarily an exact mathematical equation expressing the functional dependence of a nonlinear parameter is not known and the analysis must begin with empirical data expressed in the graphical form of a curve. The solution of a circuit of this type can usually be obtained most easily by using graphical rather than analytical techniques, although many "graphical" methods are actually a combination of the two.

Since the problem of solving a circuit containing a nonlinear resistor is similar in many ways to that for a circuit containing a vacuum tube, solutions for two simple circuits of this type have been chosen to demonstrate typical graphical and analytical methods, one of which will be modified for use at a later point on a circuit containing a vacuum tube with a reactive load.

The Terminal Characteristics of a Nonlinear Resistor

Resistance, according to Ohm's law, is the ratio of a voltage applied to a resistor to the resulting current in the resistor, $R = E/I$ = a constant. This definition applies only to linear resistors. When the resistor is of the nonlinear type, a useful concept is that of a terminal characteristic. The terminal characteristic of a two terminal resistor may be given in the form of a curve, as shown in Figure 1, where the current in the resistor has been plotted as a function of the voltage across it. Referring to Figure 1, if a point P is selected on the characteristic, the ratio e_p/i_p is called the static, or d-c, resistance at point P; and the quantity $\partial i/\partial e \rfloor_p$ (evaluated at P), which is the slope of the curve at P, is the reciprocal of which is called the dynamic, or a-c, resistance at point P. These concepts of a nonlinear resistor are also applicable to the terminal characteristic of a vacuum tube which will be discussed at a later point.

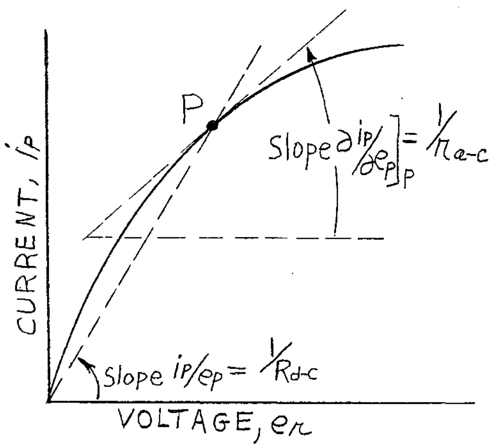

Figure 1. The terminal characteristic of a nonlinear resistor

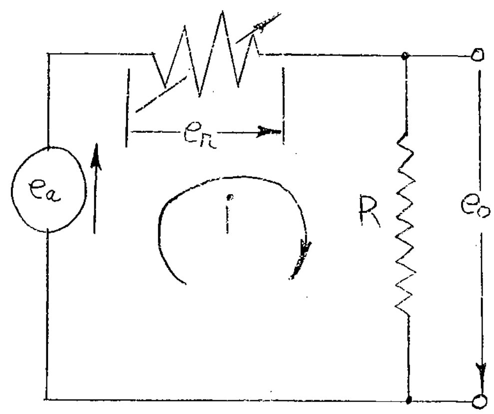

Figure 2. A simple circuit containing a nonlinear resistor

In Figure 2, a simple circuit containing a voltage source, e_a, in series with a nonlinear resistor and a linear resistor, R, is shown. The current, i, resulting from a particular applied voltage, and the output voltage, e_o, are to be determined.

Graphical Solution for Nonlinear Circuit With Pure Resistance

Since the current in the nonlinear resistor in the circuit shown in Figure 2 is assumed to be dependent only upon the voltage applied to it and the circuit contains no energy storing elements, there is a one-to-one correspondence between an applied voltage and the resulting current in the circuit. For this case, a general solution can be obtained graphically without regard to the particular nature of the voltage being applied to the circuit. In order to obtain the graphical solution, the terminal characteristic of the nonlinear resistor is constructed as shown in Figure 3. A general point p' on the curve of the current versus applied voltage, $i = f(e_a)$, is located by constructing a straight line with a slope of $(-1/R)$ through an arbitrary point A on the voltage axis, where the distance OA represents the applied voltage, and by locating point p' at the intersection of a vertical projection of point A with a horizontal projection of point P which is the point where the construction line intersects the terminal characteristic. The distance OB represents e_r, and the distance AB represents iR which is the output voltage, e_o. The construction is repeated for several points, P_k, along the length of the terminal characteristic and a smooth curve is drawn through the corresponding points p'_k as shown in Figure 4. The output voltage can be plotted versus input voltage by using horizontal distances between the two curves, $i = \phi(e_r)$ and $i = f(e_a)$, for ordinates. The completed construction is shown in Figure 4.

If the terminal characteristic is constructed for negative values of e_r, the construction can be continued into the second quadrant to include negative values of e_a.

In order to determine the output voltage waveform when the input voltage is some particular function of time, the graphical solution, $e_o = g(e_a)$, shown in Figure 4 is used to obtain a particular solution. Suppose that a constant bias voltage source, E volts, is placed in series with a sinusoidal voltage generator as shown in Figure 5a. The applied voltage is now $e_a = E + E_s \sin(wt)$. The output voltage waveform is obtained from the simple construction shown in Figure 5b. The point P is the operating point and corresponds to the value of output voltage when the dynamic portion of e_a is zero.

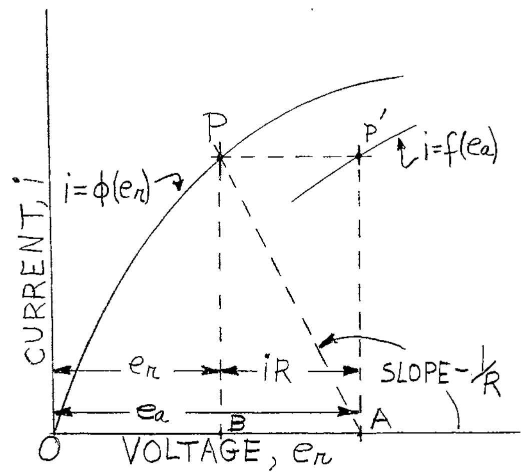

Figure 3. Graphical construction for $i = f(e_a)$

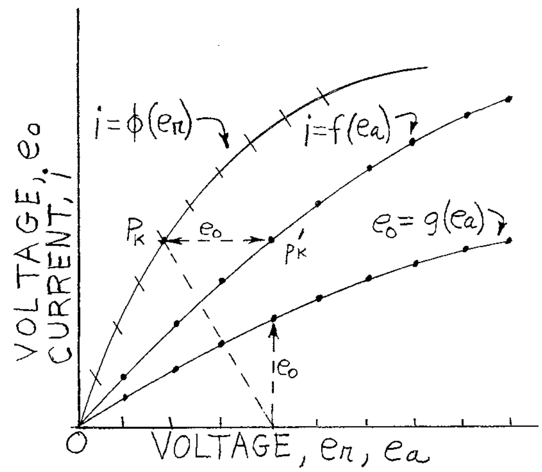

Figure 4. Completed construction for $e_o = g(e_a)$

Figure 5b shows that the output waveform is a distorted reproduction of the input waveform. From the theory of harmonic analysis, it can be shown that the output voltage waveform consists of a linear combination of sinusoidal voltages with angular frequencies of nw, $(n = 0, 1, \ldots)$, where w is the angular frequency of the driving voltage. The number and magnitudes of the harmonic components depend upon the extent of the non-linearity of the circuit. For the above pure resistive circuit, the harmonic distortion in the output voltage depends upon the location of the operating point, the size of resistor R, and the magnitude of the driving voltage. It is completely independent of the frequency of the driving voltage. The solution of this circuit is quite similar in many respects to that for a vacuum tube with a pure resistive load.

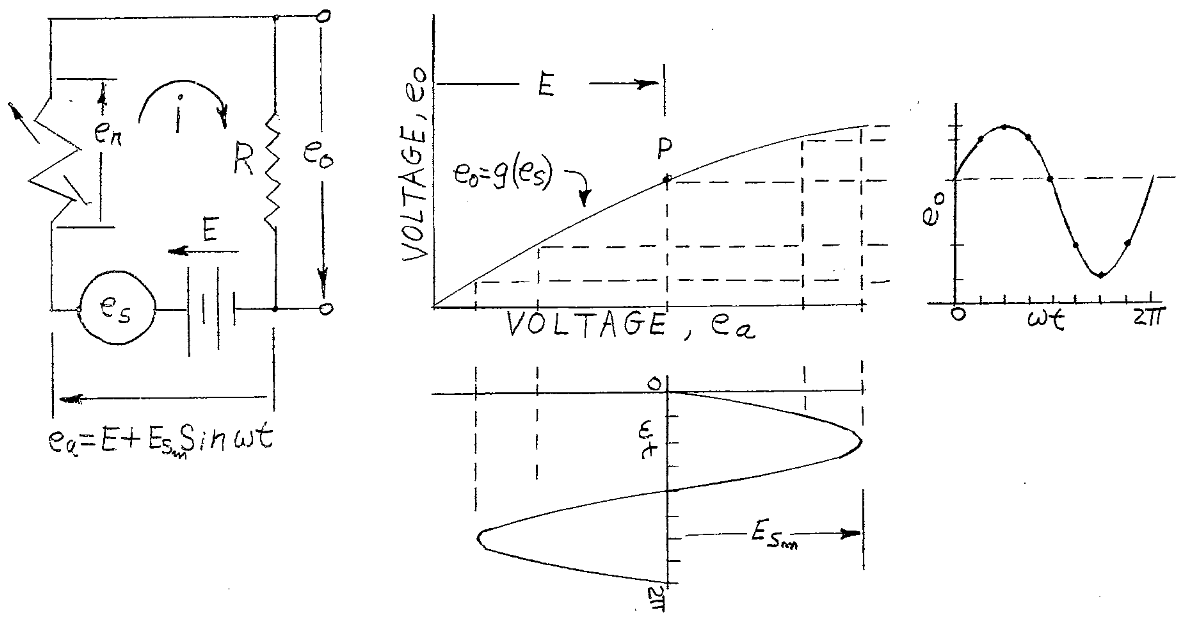

Figure 5. (a) A nonlinear resistive circuit with a sinusoidal voltage applied
(b) A particular graphical solution for obtaining the output voltage, e_o

If a capacitor or inductor is added to the circuit of Figure 2, the circuit will be capable of storing energy so that the output voltage will depend not only upon the magnitude of the load current but also upon its time history. For this case, a general solution like that for the case of pure resistance cannot be obtained; and each time that the nature of the driving voltage is altered, a completely new problem must be solved. This same complication exists in the solution of a circuit containing a vacuum tube with a reactive load. Since the solution of this type of nonlinear resistor circuit is similar in several respects to that for a vacuum tube with a reactive load, a circuit of this type will be solved in the next section. The method used to obtain the solution will be modified for use on a vacuum tube circuit in the next chapter.

Graphical Solution for Nonlinear Circuit With Reactance

The circuit shown in Figure 6 is to be solved for the current and output voltage when the driving voltage is a sinusoidal function of time. The terminal characteristic of the nonlinear resistor is shown in Figure 7a. The method of solution used here and a modification of it which will be used in the next chapter for a vacuum tube circuit are very similar to those presentd by Preisman[1] who in his work describes graphical solutions to a variety of problems of this type.

In order to begin the solution, the equations for the circuit are first written:

$$e_a = E_c + E_{sm} \sin(\omega t) = E_r + i_b R + L\frac{di_b}{dt} \qquad (1)$$

$$E_o = i_b R + L\frac{di_b}{dt} \qquad (2)$$

where i_b is the total instantaneous current in the circuit, E_r is the total instantaneous voltage across the nonlinear resistor, and E_o is the total instantaneous output voltage appearing across the reactive load. If the d-c voltage, E_c, is applied first and the circuit is given sufficient time to settle to a quiescent state before e_s is applied, the

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[1]Albert Preisman, <u>Graphical Constructions for Vacuum Tube Circuits</u> (New York, 1943), p. 105.

quiescent current is I_o, the voltage across the nonlinear resistor is E_{ro}, and the output voltage is E_{oo}. If the dynamic components of voltage and current are denoted by e_r, e_o, and i_p, Equation (1) can be written as

$$E_c + E_{s_m} \sin(\omega t) = (E_{ro} + e_r) + (I_o + i_p)R + L\frac{d(I_o + i_p)}{dt} \tag{3}$$

where $E_{ro} + e_r = E_r$ and $I_o + i_p = i_b$. Referring to the sample construction shown in Figure 7a, $E_c = E_{ro} + I_oR$ before $E_s \sin(wt)$ is applied at time equals zero. Hence,

$$E_{s_m} \sin(\omega t) = e_r + i_p R + L\frac{di_p}{dt} \tag{4}$$

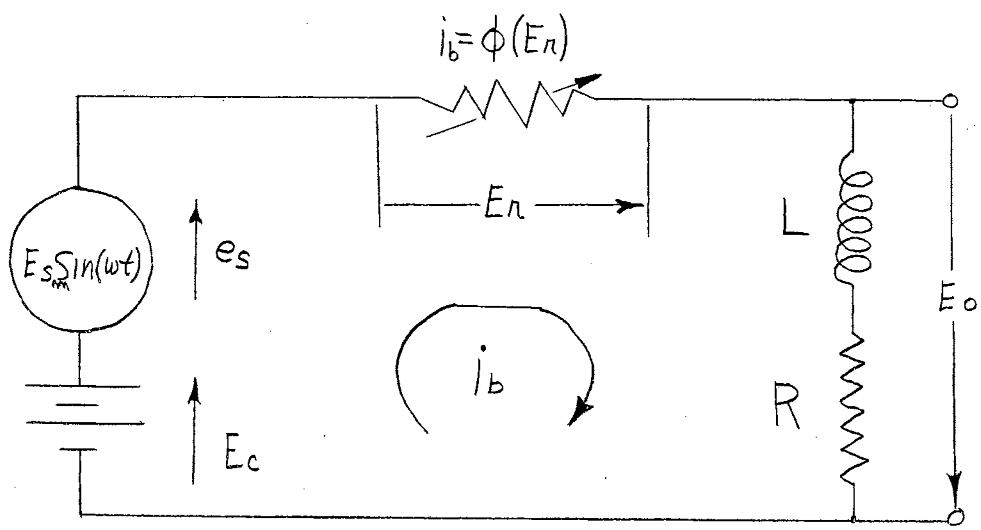

Figure 6. A simple circuit containing a nonlinear resistor and a reactive load

A plot of the current in the circuit versus the applied voltage can be made by writing Equation (4) in incremental form and solving the circuit according to the following procedure:

1. Equation (4) can be written as

$$E_{s_m} \sin \Delta_1(\omega t) \doteq e_{ro} + \Delta_1 e_r + i_{p_o} R + \Delta_1 i_p + L\frac{\Delta_1 i_p}{\Delta t} \tag{5}$$

where one cycle of the input voltage waveform has been divided into a number, N, of equal intervals of length $\Delta(wt) = 2\pi/N$. The number N must be chosen sufficiently large to give a desired accuracy.

2. Initially, the circuit is in a quiescent state so that for time $t = 0$, $E_s \sin(wt) = 0$, $e_{ro} = i_{po} = 0$, and Equation (5) can be written, for the first incremental change in the applied voltage as,

$$\Delta_1 e_s = \Delta_1 e_r + \Delta_1 i_p \left[R + L/\Delta t\right] \tag{6}$$

where $\Delta t = 2\pi/wN$. The locus of $i_p = f(e_s)$ begins at point o in Figure 7a.

3. Referring to Figure 7a, point 1 on the locus of $i_p = f(e_s)$ is located by laying off a distance corresponding to $\Delta_1 e_s$ to the right of the abscissa, E_c, (a negative $\Delta_1 e_s$ would be measured to the left of E_c) and constructing a line with a slope of $(-1/R)$ through point a as shown. This line intersects a horizontal projection of point o', where the current is I_o, at point b. Triangles abc and do'f are congruent so that the distance ca represents I_oR; and since the distance Of represents E_{ro}, the distance fc = o'b must represent $\Delta_1 e_s$ which is given by Equation (6). According to Equation (6), the distance o'b must be divided into distances o'g and gb which represent the incremental voltages across the nonlinear resistor and the reactive load, respectively.

4. Figure 7b is an enlarged view of the construction for obtaining the distances o'g and gb. Through point b, a line with a slope of $[-1/(R + L/\Delta t)]$ is drawn so that it intersects the terminial characteristic at point 1'. The vertical distance gl' represents the incremental current $\Delta_1 i_p$; the distance o'g represents $\Delta_1 e_r$; and gb represents

$\Delta_1 i_p(R + L/\Delta t)$ which is the incremental voltage across the load. By extending a horizontal line to the left through point 1', point h is located from which a vertical line can be drawn to locate point j. The distances gj and jb represent $L\Delta_1 i_p/\Delta t$ and $\Delta_1 i_p R$, respectively. The point 1' gives the value of current in the circuit after the first increment of input voltage has been applied, $i_{b1} = I_{bo} + \Delta_1 i_p$. The voltage that is being applied to the circuit is $E_c + \Delta_1 e_s$ which is represented by the distance Oa. The intersection of a vertical line through point a with a horizontal line through point 1' locates point 1 which is the second point on the locus of $i_p = f(e_s)$. The dynamic components i_p and e_s are measured along a set of coordinate axes with origin at point o. This same plot gives the total current i_b versus the total applied voltage e_g when the quantities are measured along the coordinate axes with origin at point O.

6. The third point on the locus, point 2, is located by repeating the above procedure using point 1' in the place of o', point k in place of a, point m in place of b, etc. The construction is completed when the locus forms a closed curve which is repeated for each successive cycle of the input voltage. This closed curve represents the steady-state solution of the current in the circuit as a function of the applied voltage.

7. Since the horizontal distance from the i_b axis to a point on the $i_p = f(e_s)$ locus represents a total applied voltage and a distance from the i_b axis to the terminal characteristic represents a total voltage across the nonlinear resistor, the horizontal distance from the terminal characteristic to the i_p, e_s locus represents the total output voltage

given by Equation (2). Such values of output voltage measured at each point in the construction can be plotted versus corresponding values of time (or wt) to give the output voltage waveform.

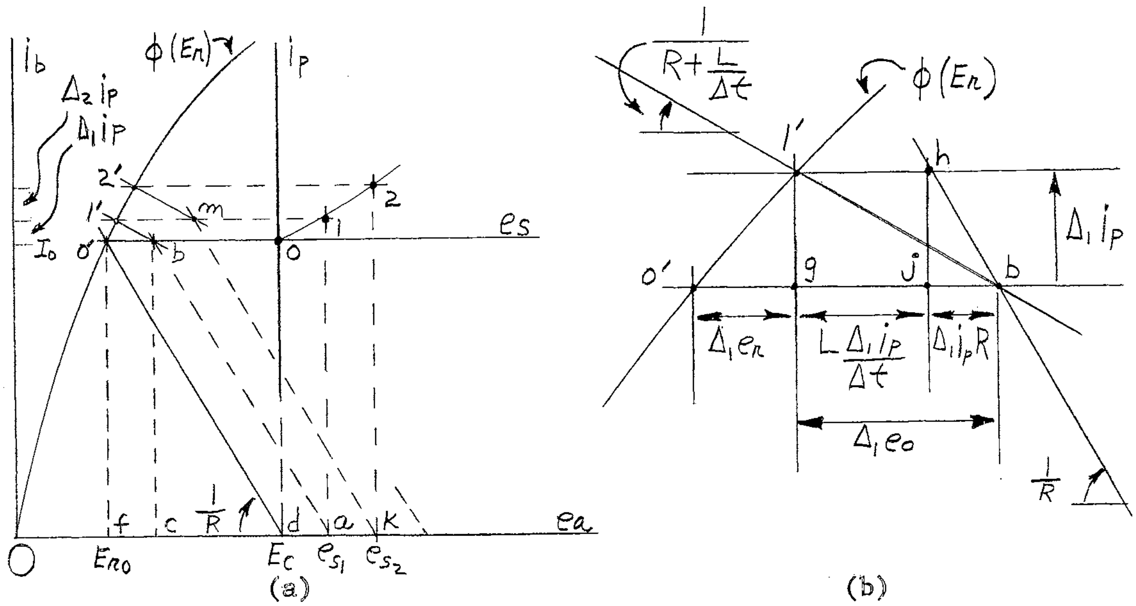

Figure 7. (a) Graphical construction for solving nonlinear resistor circuit with reactive load
(b) Enlarged view showing incremental voltages and current

The completed construction is shown in Figure 8a. In Figure 8b the value of i_b at each time point has been projected to the right and plotted versus wt to give $i_b = i_b(wt)$. The dynamic component of current is simply $i_p(wt) = i_b(wt) - Io$. A plot of $e_o(wt)$ is shown in Figure 8c. Figures 8b and 8c show that the current and output voltage waveforms possess inital transients after which they settle to the steady-state where each succeeding cycle is identical to the one proceeding it. Again, as for the case of a pure resistive circuit, the steady-state portion of the current and voltage waveforms can be decomposed into sinusoidal components with frequencies which are harmonics of the driving voltage frequency.

However, for the reactive circuit, the shape of the $i_p = f(e_s)$ locus depends upon the slope, $[-1/(R + NwL/2\pi)]$, of a construction line in Figure 7 so that the shape of the voltage and current waveforms, and hence the magnitudes of the harmonic components contained in them, depend upon the particular driving frequency being applied to the circuit.

Since the load consists of linear elements, R and L, the law of superposition holds with respect to the load voltage and current. That is if the steady-state load current is

$$i_p = \sum_{K=0}^{m} \left[a(\omega) \operatorname{Cos}(k\omega t) + b(\omega) \operatorname{Sin}(k\omega t) \right] \tag{7}$$

where w is the fundamental frequency of the applied voltage, then the steady-state output voltage must be given by

$$e_o = \sum_{K=0}^{m} Z(k\omega) \left[a(\omega) \operatorname{Cos}(k\omega t) + b(\omega) \operatorname{Sin}(k\omega t) \right] \tag{8}$$

where $Z(0) = R_{dc}$; $Z(kw) = r(kw) + jx(kw)$, the load impedance at the k^{th} harmonic; and $Z(kw)a(w)\sin(kwt)$ is interpreted to be $(r_k + jx_k) \cdot (a_k + ja_k')$ $\sin(kwt) = \sqrt{r_k^2 + x_k^2} \sqrt{a_k^2 + a_k'^2} \sin(kwt + \tan^{-1}a'/a + \tan^{-1}x_k/r_k)$. The driving voltage is a pure sinusoidal wave.

Equations (7) and (8) are the general form of the equations for the curves shown in Figures 8b and 8c. If two or more sinusoidal voltages of different frequencies are applied to the input, the output waveforms contain additional components with sum and difference frequencies. In the next chapter, it is shown that these equations also apply to a vacuum tube circuit with a reactive load when the driving voltage is a single sinusoid.

It should be noted that the function, conveniently denoted by $i_p = f(e_s)$, which is represented by the closed curve, or contour in

Figure 8a, must be of very complicated form; i_p is a double-valued function of e_s so that is is not even possible to use a power series to express i_p explicitly in terms of e_s. However, this was not the case for the nonlinear resistor circuit with pure resistance where the corresponding function, $i = f(e_a)$ was single valued. This is one of the factors which contributes greatly to the difficulty of using analytical techniques for solving a nonlinear reactive circuit of this type.

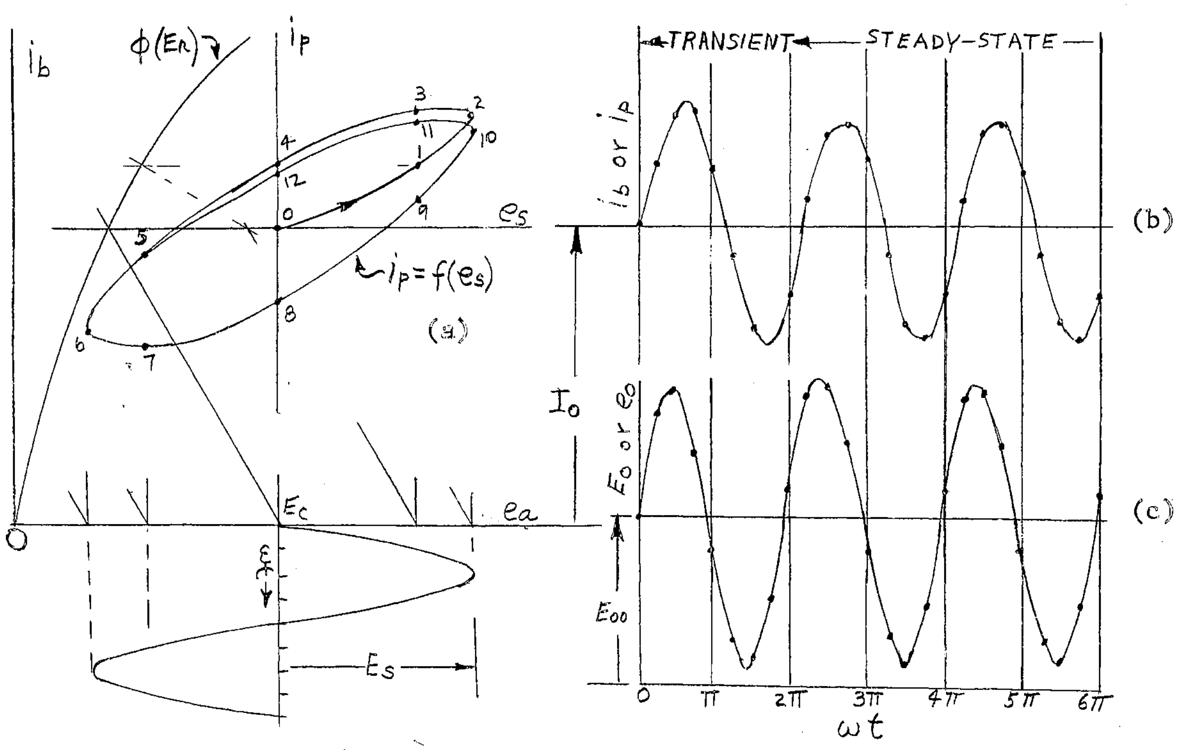

Figure 8. (a) Completed graphical construction for nonlinear resistor circuit with reactance
(b) Showing the transient and steady-state current waveform as a function of wt
(c) Showing the output voltage waveform as a distorted reproduction of the input waveform

The two circuits which have been solved graphically are very simple, but their solutions suffice to demonstrate the relative complexity of the nonlinear circuit problem as well as to demonstrate useful graphical methods which are used in the following chapter to begin a study of vacuum tube circuits with reactive loads. Although approximate solutions for these types of circuits can usually be obtained by analytical methods, such solutions are usually too cumbersome to be of much practical value. In the next section, such a method of solution is demonstrated by solving the circuit containing only pure resistance which was shown in Figure 2. A solution of this type for a circuit containing a reactance is very much more complicated than that for a pure resistance and will not be given here.

Analytical Solution for Nonlinear Circuit With Pure Resistance

In order to solve the circuit shown in Figure 2 analytically, the terminal characteristic of the nonlinear resistor must be represented by an analytical expression. Since the terminal characteristic is usually determined experimentally, it is necessary to resort to an approximate expression, obtained perhaps by determining the coefficients of a finite series, which adequately represent the terminal characteristic over a desired range of the independent variable. A finite number of terms of either a power series or a trigonometric series may be used. If the curve of the terminal characteristic has sharp bends or irregularities, a prohibitive number of terms may be required in the case of a power series. The method of least squares provides a convenient process for determining the coefficients of such series.[2]

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[2] See F. B. Hildebrand, Introduction to Numerical Analysis (New York, 1956).

Suppose that the curve shown in Figure 1 is represented by the following equation:

$$i = a_1 e_r + a_2 e_r^2 + \cdots + a_n e_r^n \tag{9}$$

where the a_k, k = 1,..., n, have been determined by a suitable method. By applying Kirchoff's law to the circuit of Figure 2,

$$e_a = e_r + iR \tag{10}$$

$$e_o = iR \tag{11}$$

and assuming that the current can be expressed as an explicit function of the input voltage in the form of a power series (which was shown by the graphical solution to be the case for the pure resistive circuit), the current can be written as

$$i = \sum_{K=1}^{n} b_K e_a^K \tag{12}$$

for n sufficiently large where the b_k are to be determined. By substituting Equation (10) into Equation (9), expanding the result, and making use of Equation (12), the following identity is obtained:

$$(b_1)e_a + (b_2)e_a^2 \equiv [a_1 - a_1 b_1 R]\, e_a + [a_2 - a_1 b_2 R - 2a_2 b_1 R + a_2 b_1^2 R^2]\, e_a^2 \tag{13}$$

where only the first and second terms of the power series are considered. By equating coefficients of like powers on the left and right sides of the identity (13), the coefficients for the first two terms of Equation (12) are found to be

$$b_1 = \frac{a_1}{1 + Ra_1} \;; \qquad b_2 = a_2 \left[\frac{(1 + a_1 R)^2 - a_1 a_2 R(2 + a_1 R)}{(1 + a_1 R)^3} \right] \tag{14}$$

Equation (12) can now be written as

$$i = b_1 e_a + b_2 e_a^2 \tag{15}$$

and the output voltage can be found by substituting Equation (15) into Equation (11). Notice that the coefficients b_k of the power series are independent of frequency and that their magnitudes depend upon the non-linearity of the terminal characteristic and the value of R, both of which were inferred from the graphical solution. If an input voltage, $E_s \text{Sin} wt$, is applied, the first two terms of Equation (15) give

$$i = \frac{E_{sm}}{2} + b_1 E_{sm} \sin(\omega t) - b_2 \frac{E_{sm}}{2} \cos(2\omega t) \qquad (16)$$

This method involves considerably more work than that for the graphical solution and it is obviously too tedious to be of much practical value for problem solving. When the circuit contains a reactance, the input voltage must be restricted to a periodic function of time (in order to use the concept of impedance) and only the steady-state solution can be considered. For this case, the current given by Equation (12) and the input voltage can be considered as parametric equations with time as the variable. The b_k will be functions of frequency so that a coefficient will be required for each frequency term arising from the expansion of $(e_a)^k$. In general, such as solution involves much more work than that for the pure resistive circuit. For this reason, vacuum tube circuits with reactive loads will be studied mainly by graphical methods supplemented by analytical techniques only when some advantage can be gained by it. An exception is the case of an idealized linear tube where the difficulty does not arise.

Summary

When a nonlinear resistor is added to an otherwise linear circuit, the problem of circuit analysis reverts from the class of well organized analyses based on linear theory to a class of analyses the discipline of which depends to a large extent upon the ingenuity of the investigator. And since the characteristic of a nonlinear resistor in such a circuit is, much of the time, available only in the graphical form of a curve determined from experimentally obtained data, graphical methods of analysis often times require the least amount of labor and provide the most direct way to obtain a solution. If the nonlinear circuit contains only pure resistance, a certain amount of generality can be retained in the solution; but when the circuit contains a reactance, the effect of changing some part of the problem can usually be determined quantitatively only by repeating the entire process of the analysis. However, the effects of varying the circuit can sometimes be predicted in a very qualitative way from an analytical solution obtained by the use of power series.

Some important distinctions between the results obtained for the circuit with pure resistance and those for the circuit with reactance are enumerated below.

For the circuit with pure resistance:

1. A general solution for output voltage across the resistive load can be obtained without regard to the particular nature of the waveform of the voltage applied to the circuit.

2. The current resulting from a sinusoidal applied voltage contains harmonic components with magnitudes which are independent of frequency.

3. The current in the circuit is a single valued function of the voltage applied to the circuit and can expressed explicitly without restriction upon the nature of the applied voltage.

4. The value of the load resistor, R, can represent the equivalent resistance of a general resistive network without altering the method of solution.

Correspondingly, for the circuit with reactance:

1. The output voltage across the reactive load depends upon the time history of the current, thus the solution depends upon the particular nature of the waveform of the applied voltage.

2. The current resulting from a sinusoidal applied voltage contains harmonic components with magnitudes which are functions of frequency.

3. The current in the circuit is a multiple-valued function of the applied voltage and the two quantities ordinarily can be related explicitly only in the form of parametric equations in terms of time.

4. The particular method of solution given here depends entirely upon the nature of the reactive load in the circuit and can be used only on relatively simple load configurations.

Since the nature of the problem of analyzing a circuit containing a vacuum tube is similar in many ways to that of a circuit containing a nonlinear resistor, some of the ideas developed in this chapter have greatly influenced the author's method of approach to the study, in the following chapters, of vacuum tube circuits containing reactive loads.

In order to gain a better insight of the operation of a vacuum tube with a reactive load, the method used here to solve the nonlinear circuit with a reactance is modified and used in the next chapter to obtain the path of operation of a vacuum tube for a particular reactive load configuration.

CHAPTER III

THE VACUUM TUBE AS A NONLINEAR CIRCUIT ELEMENT

Introduction

Although the theory of vacuum tube circuit analysis has been developed through the years to a relatively advanced state, one feature of the vacuum tube which has greatly hampered the accuracy of such analyses is the tube's inherent nonlinearity. In order to carry out an exact analysis, equations completely describing the tube's characteristic would be required. Such equations can best be approximately determined from empirically obtained data but they are much too cumbersome for practical everyday use. As a result, there are two general types of approximate analyses which are commonly applied to audio frequency amplifier circuits. One type of analysis, based on a linear equivalent circuit,[3] assumes that the vacuum tube is a linear element so that ordinary methods of linear circuit analysis are applicable. Since the vacuum tube is approximately linear only when it is operated over certain relatively small portions of its characteristics, this type of analysis is approximately valid only when small input signals are applied to the tube. The other type of analysis consists essentially

[3]See George E. Happel and Wilfred M. Hesselberth, Engineering Electronics (New York, 1953).

of graphical methods[4] which take into account the nonlinearity of the tube so that large driving signals can be applied. The first type of analysis is especially suitable for low-level voltage amplifier circuits for which the input signal is relatively small and the load of the circuit is usually designed to exhibit essentially a pure resistance over the major portion of the desired range of operating frequencies. The fact that the load ordinarily becomes reactive at the upper and lower extremes of the frequency range presents no particular difficulty in the analysis. The second type of analysis is especially applicable to audio power amplifier circuits for which the driving voltage is necessarily made large in order to obtain a large power output. In spite of the fact that power amplifiers often have electromechanical loads such as loud-speakers, electric motors, electrodynamic shakers used for enviromental testing, and many other kinds of loads which exhibit a reactance over an appreciable portion of the range of operating frequencies, the analysis nearly always assumes the load to be a pure resistance. Such a simplification can be appreciated when the peculiar problem of attempting to locate the operating path of a vacuum tube with a reactive load is considered; however, the results obtained with the simplification are strictly valid only when the load is, in fact, the pure resistance specified, since it is only for this condition that the tube actually operates over the portion of its characteristics which have been used in the analysis.

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[4]See Herbert J. Reich, Theory and Application of Electron Tubes (New York, 1944).

Since this work is primarily concerned with studying the effects of a reactive load in circuits of this latter type, audio frequency power amplifiers with reactive loads, little further discussion of methods of analysis for other cases will be made except by the way of comparing results.

In this chapter, some of the general characteristics of the operating path of a vacuum tube with a reactive load are presented by utilizing some of the ideas developed in the previous chapter to solve a simple circuit containing an inductor and a linear resistor for a load. Also, the results of analytical investigations, based on the use of power series, which have been made by others are given. A convenient geometrical picture of the tube's operation is first provided by presenting the terminal characteristic of the tube as a three-dimensional surface; also this presentation will serve to introduce much of the notation which will be used throughout latter chapters. An attempt has been made to use only those symbols approved by the I.R.E. where possible.

The Terminal Characteristic of a Vacuum Tube

The tubes used in audio power amplifiers are triodes, pentodes, and beam power tubes. Of the latter two, which have similar characteristics, only the terminal characteristic of the beam power tube will be considered. Since the plate current of a triode depends upon the net potential distribution between the cathode and plate, the plate current is a function of the two independent variables, grid voltage and plate voltage.[5] If grid current is present, it will also be a function of grid and plate voltage so that a complete geometrical representation of the tube's terminal characteristic will require two three-dimensional surfaces, one for the grid circuit terminals and the other for the plate circuit terminals. For the present, it will be assumed that the grid is biased with a negative voltage such that grid current is zero. For this case, the complete terminal characteristic of a triode can be represented by a characteristic surface like the one shown in Figure 9a. If the screen grid voltage of a beam power tube is maintained constant, the terminal characteristic for cathode, control grid, and plate can similarly be represented by a surface such as the one shown in Figure 10a. In Figure 9a, planes perpendicular to the e_c axis can be made to cut the characteristic surface at constant values of grid voltage, e_c. The intersection of all such planes, for equal increments of e_c, with the surface are curves in space which project orthogonally onto the i_b, e_b plane to give the plate family of characteristics shown in Figure 9b. A single plate family curve corresponds to the terminal characteristic of a nonlinear resistor which was shown in Figure 1. If the surface is cut by planes perpendicular to the e_b axis and the resulting curves are projected into the i_b, e_c plane, the static transfer characteristics shown in Figure 9c result. Similarly, planes prependicular to the i_b

[5]There are many factors which influence the particular nature of this function, for a complete discussion of the vacuum tube see E. Leon Chaffee, Theory of Thermionic Vacuum Tubes (New York, 1933).

axis give curves which project onto the e_b, e_c plane to give the constant current characteristics of the beam power tube. The surface shown in Figure 9a is a plot of the function $i_b = F(e_c, e_b)$. The partial derivatives of the function at a general point on the surface are $\frac{\partial i_b}{\partial e_b} = \frac{1}{r_p}$, $\frac{\partial i_b}{\partial e_c} = gm$, and $\frac{\partial e_b}{\partial e_c} = -\frac{\partial F}{\partial e_c} \Big/ \frac{\partial F}{\partial e_b} = -\mu$ or $\mu = -\frac{\partial e_b}{\partial e_c}$. r_p is the dynamic plate resistance; gm is the control-grid to plate transconductance; and μ is the amplification factor. The values of these derivatives depend upon the choice of the general point on the tube surface and can be obtained from the slopes of the curves shown in Figures 9b, 9c, and 9d, respectively.

In the audio frequency range, electron transit time and electrode lead inductance are sensibly zero. If the interelectrode capacitances of the tube, which become effective at the higher audio frequencies, are considered to be a part of the external load, then the tube is strictly a resistive type of circuit parameter. At frequencies where the effect of these capacitances is negligible, the grid voltage can be held constant at some value e_{c1} and the tube can be inserted into the circuit which was shown in Figure 2 with the plate and cathode terminals replacing those of the nonlinear resistor. The solution of this new circuit is identical to that of the original one except that the plate characteristic for the value of $e_c = e_{c1}$ is used for the terminal characteristic. If an applied voltage, corresponding e_a in Figure 2, is held constant at some value E_{bb} and the grid voltage of the tube is allowed to vary, the construction which was shown in Figure 3 can be applied by using the plate family of characteristics and by noting that the line

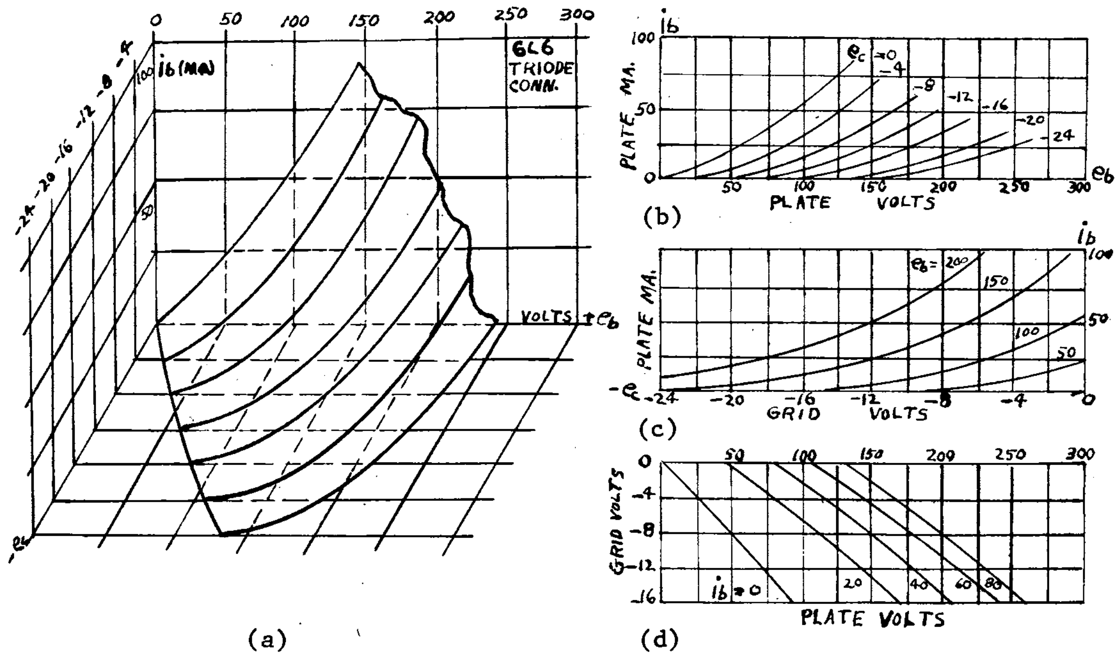


Figure 9. (a) Characteristic surface of a triode showing plate current as a function of grid and plate voltage
(b) Plate family of characteristics
(c) Static transfer characteristics
(d) Constant current characteristics

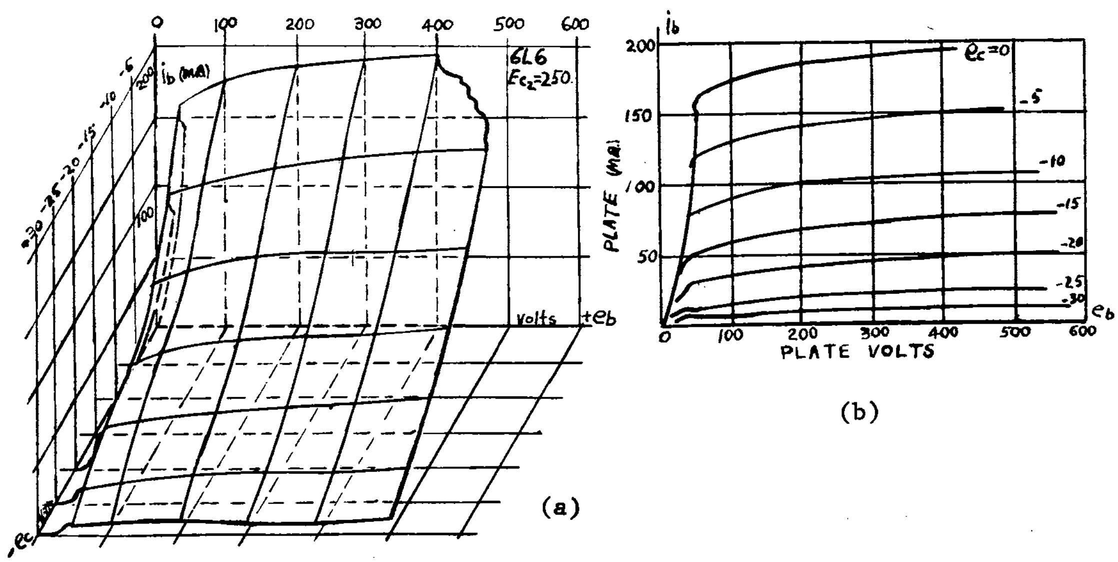


Figure 10. (a) Characteristic surface of a beam power tube
(b) Plate family of characteristics

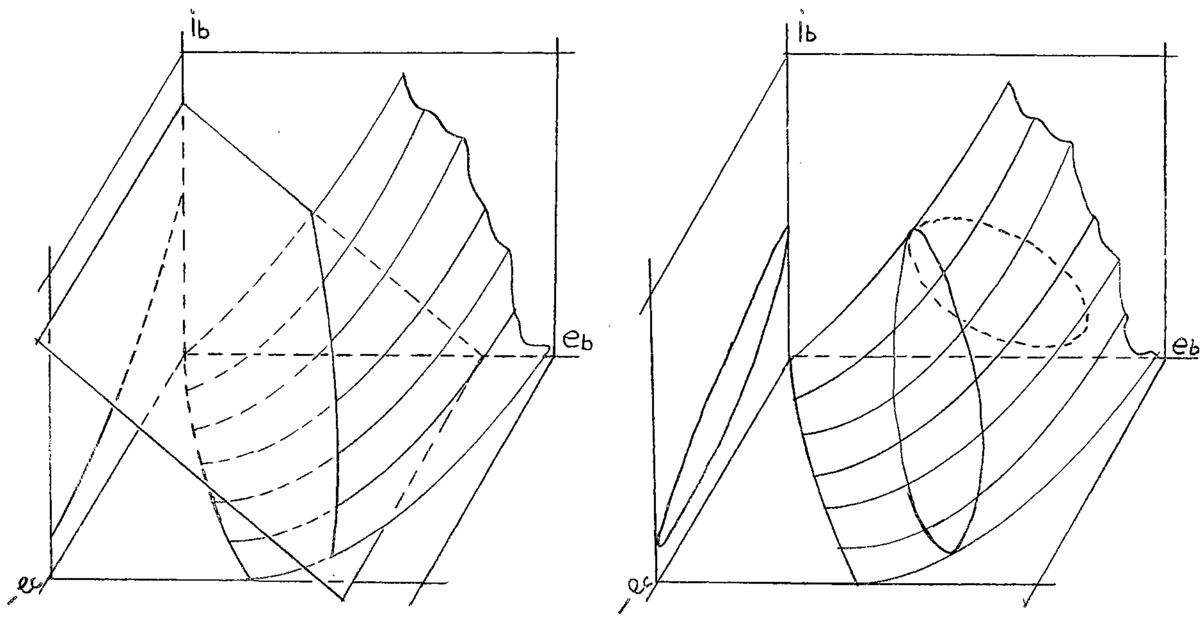
with slope $-1/R$ remains fixed through the abscissa E_{bb}. The line with slope $-1/R$ becomes the well known load-line for a vacuum tube with a pure resistive load. It is of interest to note that while the locus of points in the i_b, e_b plane which represent the tubes operating path lie along a straight line, the locus of points of operation over the characteristic surface is a single-valued curve which, illustrated in Figure 11a, is the intersection of the "load-plane" with the tube surface, and it is given by the simultaneous solution of the equations

$$i_b = F(e_c, e_b) \tag{17}$$

$$\left.\begin{aligned} i_b &= -\frac{1}{R}e_b + \frac{E_{bb}}{R} \\ e_c &= e_c \end{aligned}\right\} \tag{18}$$

When the load contains a reactance, Equation (2) must be replaced with a differential equation; and it will be shown in the following section that, for this case, the locus of points of operation over the characteristic surface is given by a multi-valued function, illustrated in Figure 11b, which is most conveniently expressed by parametric equations with time as the independent variable.

If the voltage applied to the grid of the tube with a resistive load is of the form $e_c = E_{cc} + E_g\sin(wt)$, where E_{cc} is a constant negative bias voltage, the output voltage and current waveforms are of the same nature as those which were obtained in Chapter II for the nonlinear resistor circuit with a pure resistive load, and the same discussion given there also applies in this case. The number and magnitudes of the harmonic components generated are independent of frequency.



(a) (b)

Figure 11. (a) Geometrical representation of vacuum tube operation with a resistive load
(b) Geometrical representation of vacuum tube operation with a reactive load

In the next section, a circuit containing a triode tube with a resistor and inductor for a load will be solved in order to illustrate some of the general characteristics of the operating path of a tube when the load is reactive.

Graphical Solution for Vacuum Tube Circuit With Reactive Load

If the nonlinear resistor in the circuit of Figure 6 is replaced with a triode tube and voltages are applied as shown in Figure 12, the resulting circuit can be solved by using a procedure quite similar to that for the original circuit. The bias voltage, E_{cc}, supply voltage E_{bb}, and the load resistor, R, must first be selected in order to establish a suitable quiescent point Q, as shown in Figure 13. The equations for the circuit are

$$E_{bb} = i_b R + L \frac{di_b}{dt} + e_b \tag{19}$$

$$e_c = e_g + E_{cc} = E_{gm} \sin(\omega t) + E_{cc} \tag{20}$$

In order to establish some inital conditions from which the solution may proceed, it is assumed that E_{cc} and E_{bb} have been applied for a sufficient length of time before e_g so that the circuit has reached a quiescent state given by point Q where the quiescent plate current and voltage are I_{bo} and E_{bo}, respectively. For this condition,

$$E_{bb} = I_{bo} R + E_{bo} \tag{21}$$

$$e_c = E_{cc} \tag{22}$$

and if the total plate current and voltage are written as

$$i_b = I_{bo} + i_p \tag{23}$$

$$e_b = E_{bo} + e_p \tag{24}$$

where i_p and e_p are dynamic components, Equations (23) and (24) can be substituted into Equation (19) to give

$$E_{bb} = I_{bo} R + E_{bo} + i_p R + L \frac{di_p}{dt} + e_p \tag{25}$$

Combining Equation (21) with Equation (25) gives

$$-e_p = i_p R + L \frac{di_p}{dt} \tag{26}$$

The coordinate axes in Figure 13 have been translated to the Q point from which the variables i_p and e_p in Equation (26) are measured.

Since the solution is to proceed in a step-by-step fashion, the driving voltage waveform, for one cycle, must be divided into a number, N, of intervals, of length $\Delta(wt) = 2\pi/N$, and the values of plate voltage and current at each value of grid voltage must define a point in the i_p, e_p plane which lies on a corresponding plate characteristic

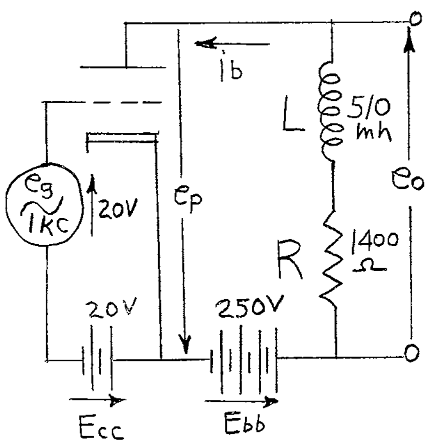


Figure 12. Triode tube with reactive load

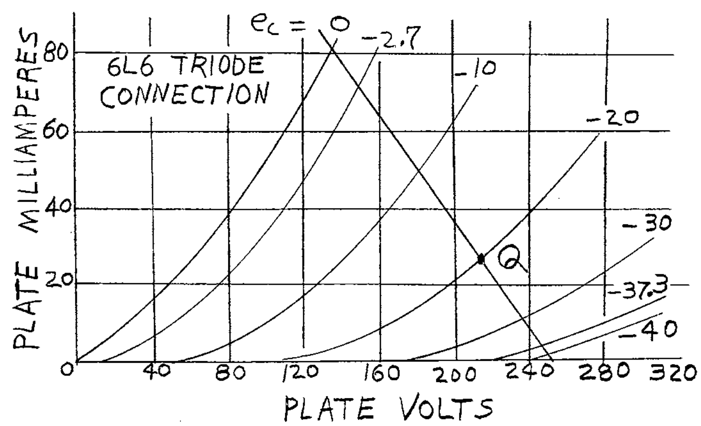


Figure 13. Special plate characteristics used in graphical solution

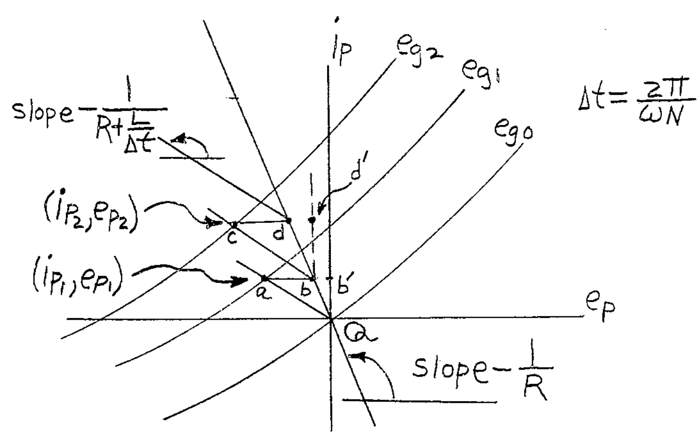


Figure 14. Sample construction for graphical solution

curve. These two requirements are most easily met by choosing a desired peak value of grid driving voltage, E_{gm}, and plotting the tube characteristics for values of grid voltage, $e_c = E_{gm}\sin(2K\pi/N) - E_{cc}$, k = 0, 1,..., N-1), as shown in Figure 13 where E_{gm} = 20 volts and N = 12. Once the preliminary setup of the problem has been made, the tube's operating path can be obtained by using the following procedure:

1. The construction begins at the Q point with the initial conditions, at time t = 0, $i_{po} = e_{po} = e_{go} = 0$.

2. Equation (8) is written in incremental form

$$-(e_{po} + \Delta_1 e_p) \doteq (i_{po} + \Delta_1 i_p)R + L\frac{\Delta_1 i_p}{\Delta t} \qquad (27)$$

which upon applying the initial conditions becomes

$$-\Delta_1 e_p = \Delta_1 i_p\left[R + L/\Delta t\right] \qquad (28)$$

Since $e_{p1} = e_{po} + \Delta_1 e_p$; $i_{p1} = i_{po} + \Delta_1 i_p$; and $e_{g1} = e_{go} + \Delta_1 e_g$, the point ($i_{p1}$, e_{p1}) is located by the intersection of a line through point Q with slope $-1/(R + L/\Delta t)$ and the plate characteristic for $e_g = e_{g1}$. A sample construction is shown in Figure 14.

3. In order to locate the point (i_{p2}, e_{p2}), the subscripts in Equation (27) are advanced to represent the next point which gives

$$-(e_{p1} + \Delta_2 e_p) = (i_{p1} + \Delta_2 i_p)R + L\frac{\Delta_2 i_p}{\Delta t} \qquad (29)$$

or

$$-e_{p2} = i_{p1}R + \Delta_2 i_p(R + L/\Delta t) \qquad (30)$$

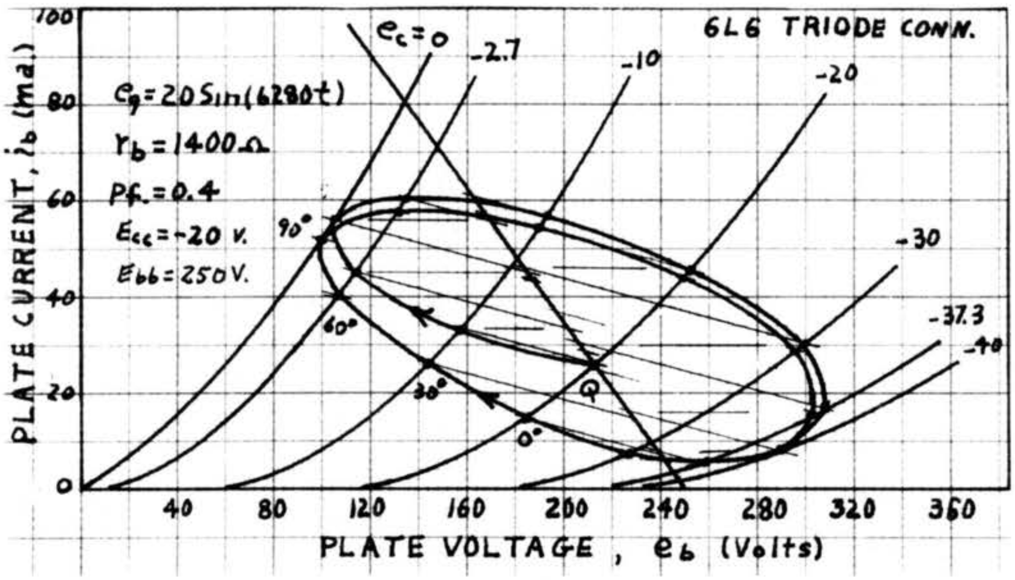
According to Equation (30), the point (i_{p2}, e_{p2}) is located by the intersection of a line through point b, Figure 14, with slope $-1/(R + L/\Delta t)$ and the plate characteristic for $e_{g2} = e_{g1} + \Delta_2 e_g$. The distance bb' represents $i_{p1}R$; ab represents $L\Delta_1 i_p/\Delta t$; dd' represents $\Delta_2 i_p R$ and cd represents $L\Delta_2 i_p/\Delta t$.

4. Successive points are located by continuing the above procedure; a line with slope $- 1/\left(R + L/\Delta t\right)$ is next drawn through point d to locate (i_{p3}, e_{p3}). By drawing a smooth curve through the points, an approximate i_p, e_p plot is obtained which spirals around the Q point for a cycle or two and then closes upon itself as shown by the completed construction in Figure 15a.

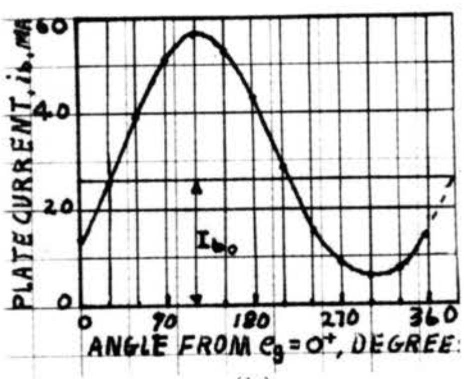
The closed contour in Figure 15a represents the steady-state path of operation from which the steady-state plate voltage and current waveforms have been plotted in Figures 15b and 15c.

Experimental verification of the graphical solution is presented in Figures 16a, 16b, and 16c which show oscillographs of the actual steady-state path of operation and the plate voltage and current waveforms of the circuit of Figure 12. By carefully scaling the voltage and current, it was possible to properly locate the oscilloscope image upon an illuminated lucite plate, placed over the face of the oscilloscope, upon which the plate characteristics for the particular tube being used were plotted.

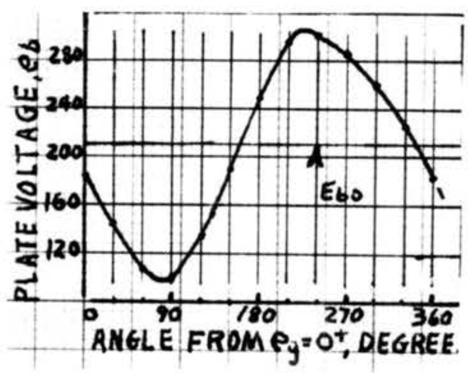
It should be noted that practical application of this method is limited to very simple types of reactive loads since higher order derivatives than the first arise when more complicated loads are considered. However, the example which has been given adequately serves the purpose of demonstrating some of the general characteristics of the operating path of a tube with a reactive load which greatly influence any attempt to formulate a practical general solution. Some inferences which may be made from the example are listed as follows:



(a)



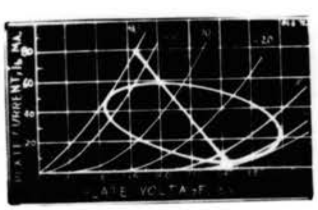
(b)



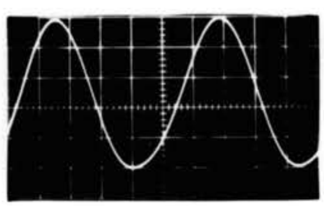
(c)

Figure 15. Graphical solution of triode tube circuit with reactive load

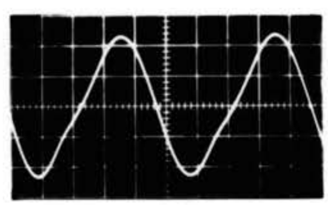
(a) Dynamic path of operation
(b) Plate current waveform
(c) Plate voltage waveform



(a)



(b)



(c)

Figure 16. Oscillograph of actual solution of triode tube circuit with reactive load

(a) Dynamic path of operation
(b) Plate current waveform
(c) Plate voltage waveform

1. In contrast with the straight-line path of operation, or load line, for a pure resistive load, the steady-state path of operation for a reactive load is a closed curve, or contour, in the i_b, e_b plane which, for the general case, must be expressed by parametric equations with time as the parameter.

2. While the load line for a pure resistive load is independent of frequency, the shape of the operating path for a reactive load is very much dependent upon the frequency of the driving voltage. The waveforms of plate voltage and current are determined by the shape of the operating path so that the magnitudes of their harmonic components are functions of frequency. For this reason, the amount of harmonic distortion in an audio amplifier with a reactive load will, in general, be different at each frequency of operation; while for the resistive load, it may be specified without regard to frequency.

3. The shape of operating path for a reactive load depends upon the nature of the family of plate characteristics over a rather appreciable region of the i_b, e_b plane. Since the operating path is traced only once in each cycle of the driving voltage instead of twice as in the case of a resistive load, all symmetry of the voltage and current waveforms is destroyed and trigonometric series employing both sine and cosine terms are required to represent them. In the case of a resistive load, the load line is traced twice in each cycle of driving voltage so that, for a suitably chosen reference, $i_p(wt) = i_p(-wt)$ and the waveforms can always be expressed in terms of a cosine series only. With respect to harmonic analysis, this means that twice as many unknowns must be solved for when the load is reactive than when it is purely resistive.

A further study of the nature of the plate current waveform for the case of a reactive load is made in the following section where the results of analytical investigations, made by others, are presented in which series expansions for plate current have been developed for resistive and reactive loads.

Series Representation of Plate Current for a Reactive Load

It has been suggested in Chapter II that a power series can be constructed to represent the terminal characteristic of a nonlinear resistor. The example which was given there illustrated the general complexity of using such a series to solve a circuit analytically. In a similar manner, a power series can be used to represent the terminal characteristic of a vacuum tube which was shown earlier in this chapter. The terminal characteristic, in this case, is represented by a three-dimensional surface, $i_b = F(e_c, e_b)$, so that a double series is required. Such a series is ordinarily constructed by utilizing the extended Taylor's theorm for two variables to expand the function $i_b = F(e_c, e_b)$ about a point Q' on the characteristic surface which projects onto the i_b, e_b plane to give the quiescent point Q. This series can be combined with other equations of a circuit to give an expression for plate current; however, the problem becomes of formidable proportions when more than the first two or three terms of the series are used. And since successive derivatives must be determined graphically, the accuracy of the method diminishes very rapidly with each step. For these reasons, this method is not very useful for problem solving; however, the coefficients of the terms of a series expansion of plate current do provide considerable information about the way in which the tube characteristics and the load affect the operation of the circuit.

When a tube with characteristics such as those shown in this chapter is connected in a circuit with a load impedance, and voltages are applied in order to establish a suitable quiescent point Q, the following equations can be written:

$$i_b = F(e_c, e_b) \tag{31}$$

$$i_b = I_{bo} + i_p \; ; \quad e_b = E_{bo} + e_p \; ; \quad e_c = E_{cc} + e_g \tag{32}$$

Expanding Equation (31) about the point Q' (I_{bo}, E_{bo}, E_{cc}) by Taylor's theorm gives

$$i_b = I_{bo} + \frac{1}{1!}\left[\frac{\partial i_b}{\partial e_b}(e_b - E_{bo}) + \frac{\partial i_b}{\partial e_c}(e_c - E_{cc})\right] + \frac{1}{2!}\left[\frac{\partial^2 i_b}{\partial e_b^2}(e_b - E_{bo})^2 + 2\frac{\partial^2 i_b}{\partial e_c \partial e_b}(e_b - E_{bo})(e_c - E_{cc}) + \frac{\partial^2 i_b}{\partial e_c^2}(e_c - E_{cc})^2\right] + \cdots \tag{33}$$

which can be expressed in terms of dynamic variables by using Equation (32) as

$$i_p = \left(\frac{\partial i_b}{\partial e_b}\right)e_p + \left(\frac{\partial i_b}{\partial e_c}\right)e_g + \frac{1}{2}\left(\frac{\partial^2 i_b}{\partial e_b^2}\right)e_p^2 + \left(\frac{\partial^2 i_b}{\partial e_c \partial e_b}\right)e_p e_g + \frac{1}{2}\left(\frac{\partial^2 i_b}{\partial e_c^2}\right)e_g^2 + \cdots \tag{34}$$

This series represents the characteristics surface about the point Q' and the partial derivatives are to be evaluated at this point.

The method of obtaining a solution can be most easily demonstrated for the simple case of a circuit with a pure resistive load; then, by using the operational notation which was introduced in conjunction with Equation (8), the method can be extended to include the more general case of a reactive load. When the load is a pure resistance, the plate current is a single-valued function of grid voltage which can be expressed in the form of a power series as

$$i_p = \sum_{n=1}^{\infty} a_n e_g^n \tag{35}$$

Upon combining the well known relation

$$e_p = -r_b i_p \tag{36}$$

with Equation (35) and substituting the result from Equation (34), and identity is obtained

$$a_1 e_g + a_2 e_g^2 + \cdots \equiv -\left(\frac{\partial i_b}{\partial e_b}\right)\sum_{n=1}^{\infty} a_n e_g^n + \left(\frac{\partial i_b}{\partial e_c}\right) e_g + \frac{1}{2}\left(\frac{\partial^2 i_b}{\partial e_b^2}\right)\left(\sum_{n=1}^{\infty} a_n e_g^n\right)^2 - \left(\frac{\partial^2 i_b}{\partial e_c \partial e_b}\right)\sum_{n=1}^{\infty} a_n e_g^{n+1} + \frac{1}{2}\left(\frac{\partial^2 i_b}{\partial e_c^2}\right) e_g^2 + \cdots \tag{37}$$

from which the coefficients a_n of Equation (35) can be determined by combining like powers of e_g on the right side of the identity and equating their coefficients with corresponding coefficients on the left side. The first two coefficients of Equation (35) are found by Llewellen[6] to be

$$a_1 = \frac{\mu}{r_p + r_b} \tag{38}$$

$$a_2 = \frac{1}{2}\left[\frac{\mu \frac{\partial \mu}{\partial e_b}\left(r_p^2 - r_b^2\right) + \frac{\partial \mu}{\partial e_c}\left(r_p + r_b\right)^2 - \mu^2 r_p \frac{\partial r_p}{\partial e_b}}{\left(r_p + r_b\right)^3}\right] \tag{39}$$

By assuming the amplification factor to be a constant, a simpler result for a_2 has been given by Reich.[7]

$$a_2 = -\frac{\mu^2 r_p}{2\left(r_p + r_b\right)^3}\frac{\partial r_p}{\partial e_b} \tag{40}$$

When the load impedance is reactive, it has been shown that the locus of operating points on the characteristic surface describe a closed curve, or contour, so that the plate current must be a multi-valued function of grid voltage. For this case, i_p cannot be expressed explicitly by a power series in the general variable, e_g. However, if e_g

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[6] F. B. Llewellyn, Bell System Technical Journal, 5 (1926), p. 433.

[7] Reich, pp. 75-77.

is restricted to a periodic function of time, such as a linear combination of sinusoidal functions, the steady-state plate current is known experimentally to be also a periodic function of time. Equation (35) can be written in a slightly different form and interpreted to be a parametric equation in the variable time. Another reason for restricting e_g to a periodic function of time is the necessity of using the concept of impedance to express the conditions imposed by the load. With the above restrictions on e_g, the steady-state plate current can be expressed by a power series with time as the independent variable as

$$i_p = \sum (A_1) e_g + \sum (A_2) e_g^2 + \cdots \tag{41}$$

where

$$e_g^m = \left(E_h \sin \omega_h t + E_k \sin \omega_k t + \cdots\right)^m \tag{42}$$

The summation symbols in Equation (41) are used to indicate that a separate coefficient (each being a function of frequency) is required for each frequency term arising in the expansion of Equation (42). The coefficients for the first two terms in the series of Equation (41) have been derived by Llewellyn and are

$$(A_1)_h = \frac{\mu}{r_p + Z_h} \tag{43}$$

$$(A_2)_{h-k} = \frac{1}{2}\left[\frac{\mu \frac{\partial \mu}{\partial e_b}\left(r_p^2 - Z_h \overline{Z}_k\right) + \frac{\partial \mu}{\partial e_c}\left(r_p + Z_h\right)\left(r_p + \overline{Z}_k\right)}{\left(r_p + Z_h\right)\left(r_p + \overline{Z}_k\right)\left(r_p + Z_{h-k}\right)}\right] \tag{44}$$

The coefficients $(A_1)_h$ and $(A_1)_k$ belong to the fundamental components of plate current, $(A_1)_h E_h \mathrm{Sin}(w_h t)$ and $(A_1)_k E_k \mathrm{Sin}(w_k t)$, which result from the first term of Equation (41). $(A_2)_{h+h}$ and $(A_2)_{k+k}$ belong to second harmonic components; $(A_2)_{h-h}$ and $(A_2)_{k-k}$ belong to direct current components; and $(A_2)_{h+k}$ and $(A_2)_{h-k}$ belong to intermodulation components all of which result from the second term of Equation (41). The coefficient $(A_2)_{h+k}$ is obtained from Equation (44) by replacing $\overline{Z}_k$ and Z_{h-k} with Z_k and Z_{h+k}, respectively. The second harmonic and direct current coefficients are obtained from the formulas for $(A_2)_{h-k}$ and $(A_2)_{h+k}$ by replacing k with h, etc. It would be noted that the above coefficients reduce to complex numbers which require the special interpretation mentioned previously. Also, it should be noted that each of the odd-power terms in Equation (41) contribute components of fundamental and odd harmonic frequencies so that the fundamental components associated with the coefficients (A_1) are good approximations to the total fundamental components only when odd-order harmonics of the third and higher order are sufficiently small. Likewise, the even-power terms contribute direct current components and even-order harmonics. The direct current components of the dynamic portion of plate current are due to plate circuit rectification which is caused by the asymmetry of the current waveform.

The grid driving voltage may be composed of any number of sinusoidal components; but in order to illustrate a relatively simple case, suppose that it consists of only two components of frequencies w_h and w_k. The first term of Equation (41) is

$$(A_1)_h E_{h_m} \sin(\omega_h t) + (A_1)_k E_{k_m} \sin(\omega_k t) \qquad (45)$$

and the second term is

$$\begin{aligned} &(A_2)_{h-h} \frac{E_{h_m}^2}{2} + (A_2)_{k-k} \frac{E_{k_m}^2}{2} - (A_2)_{h+h} \frac{E_{h_m}^2}{2} \cos(2\omega_h t) \\ &- (A_2)_{k+k} E_{k_m}^2 \cos(2\omega_k t) + (A_2)_{h-k} E_{h_m} E_{k_m} \cos(\omega_h - \omega_k)t \\ &- (A_2)_{h+k} E_{h_m} E_{k_m} \cos(\omega_h + \omega_k)t \end{aligned} \qquad (46)$$

It can be seen from Equations (43) and (44) that the coefficients depend upon the value of load impedance at various frequencies which suggests that, for the general case, the steady-state plate current of a vacuum tube with a reactive load will depend upon the value of load impedance over a wide range of frequency so that a particular solution will exist for each particular load configuration used. This makes it difficult to study the case of a reactive load in general terms.

In the following section, the general form of parametric equations of the operating path for the special case of a single sinusoidal driving voltage is presented.

Parametric Equations of the Operating Path for a Reactive Load

When the grid driving voltage is of the simple form given by

$$e_g = E_{g_m} \sin \omega t \tag{47}$$

the plate current, according to Equations (45) and (46), can be written as

$$i_p = (A_1)_h E_{g_m} \sin(\omega t) + (A_2)_{h-h} \frac{E_{g_m}^2}{2} - (A_2)_{h+h} \frac{E_{g_m}^2}{2} \cos(2\omega t) + \cdots \tag{48}$$

If all of the coefficients of like frequency terms in the series of Equation (48) are combined, the plate current can be written as

$$i_p = A_0 + A_1 \sin(\omega t) + A_2 \cos(2\omega t) + A_3 \sin(3\omega t) + \cdots \tag{49}$$

where the new coefficients, which are complex quantities (except for the zero frequency term, A_0) can be considered to operate on their sinusoidal functions to give

$$i_p = A_0 + \sqrt{a_1'^2 + a_1''^2} \sin\left(\omega t + \tan^{-1}\frac{a_1''}{a_1'}\right) + \sqrt{a_2'^2 + a_2''^2} \cos\left(2\omega t + \tan^{-1}\frac{a_2''}{a_2'}\right) + \cdots \tag{50}$$

where $A_k = a'_k + j\, a''_k$ and $a''_o = 0$. A more convenient form for Equation (50), from trigonometric identities is given by

$$i_p = a'_o + a'_1 \sin \omega t + a''_1 \cos \omega t + a'_2 \cos 2\omega t - a''_2 \sin 2\omega t + \cdots \qquad (51)$$

Equation (51) shows the necessity of using a series containing both sine and cosine terms for representing plate current when the load is reactive. For further convenience, the coefficients in Equation (51) will be denoted such that the plate current can be written as

$$i_p = a_0 + \sum_{k=1}^{m} \left[a_k \cos k\omega t + b_k \sin k\omega t \right] \qquad (52)$$

and when it is necessary to combine sine and cosine terms of like frequency, such a term will be written as

$$\sqrt{a_k^2 + b_k^2}\ \sin(k\omega t + \gamma_k) \quad ; \quad \gamma_k = \tan^{-1} \frac{a_k}{b_k} \qquad (53)$$

so that the angle γ_k represents the angular position of a phasor, representing the k^{th} harmonic, with respect to the driving sinusoid when the driving sinusoid, $E_g \sin(wt)$, passes through zero in the positive direction.

The corresponding equation for plate voltage can be obtained by simply applying the law of superposition to the current in the linear load impedance and by using the fact that the dynamic component of plate voltage must be everywhere equal in magnitude and opposite in sign to the dynamic component of load voltage.

$$-e_p = e_o = a_0 Z_0 + \sum_{k=1}^{m} \left[a_k Z_k \cos(k\omega t) + b_k Z_k \sin(k\omega t) \right] \qquad (54)$$

where Z_o is the zero frequency component of load impedance, or the resistance of the load, and Z_k is the value of load impedance at the k^{th} harmonic of the driving voltage, $Z_k = r_k + jx_k$. The parametric equations of the steady-state dynamic operating path of a vacuum tube with a reactive load and a single sinusoidal driving voltage are then

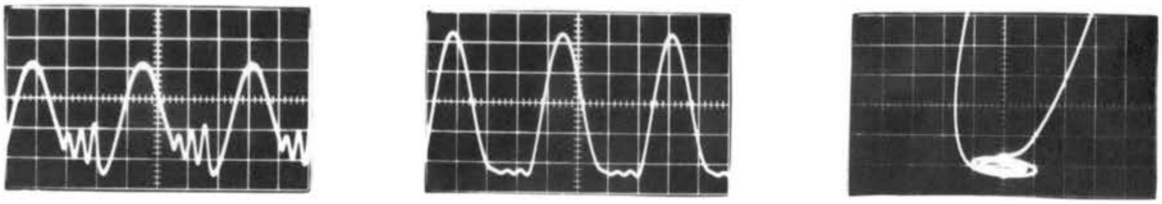
$$i_p = a_0 + \sum_{k=1}^{n}\left[a_k \cos(k\omega t) + b_k \sin(k\omega t)\right], \qquad 0 \leq \omega t < 2\pi$$

$$e_p = -a_0 Z_0 - \sum_{k=1}^{n}\left[a_k |Z_k| \cos(k\omega t + \theta_k) + b_k |Z_k| \sin(k\omega t + \theta_k)\right] \qquad (55)$$

where the highest order harmonic with not sensibly zero magnitude is of order n, and $\theta_k = \tan^{-1} x_k/r_k$. Equations (55) will be used extensively in Chapter VI.

Equations (55) apply to a very special case in which the grid driving voltage is a pure sinusoidal function of time. One condition under which such a voltage applied to the grid of a tube will not give the result indicated exists when the tube is biased such that plate current "cut-off" occurs during some part of the cycle and the load contains inductance and capacitance. When the grid voltage attempts to drive the tube into cut-off condition, the rapid change in plate current produces a damped oscillation of plate current and voltage at the damped resonant frequency of the load. Such circuit operation is shown in Figures 17a, 17b, and 17c where the plate voltage and current waveforms and the affected portion of the operating path are shown.

Under normal operation of a vacuum tube circuit with a reactive load and a single sinusoidal driving voltage, Equation (55) is valid and only components of plate current and voltage which are harmonically related to the driving frequency are present. It has been observed experimentally that the operating path is represented by a relatively smooth curve in the i_b, e_b plane when the harmonic components are rather

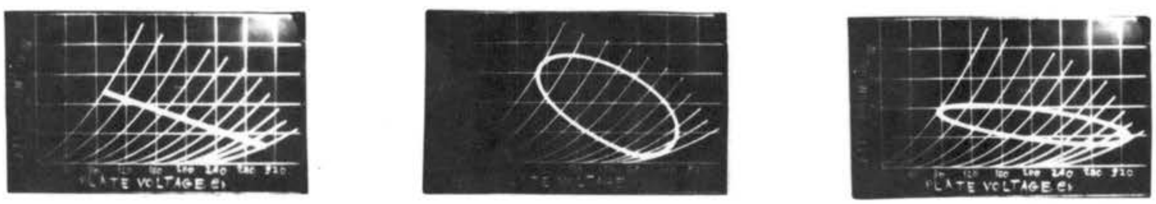


(a) (b) (c)

Figure 17. The effects of plate current "cut-off" during a portion of the operating cycle in a circuit containing a triode tube with a reactive load

(a) Plate voltage waveform
(b) Plate current waveform
(c) Portion of operating path

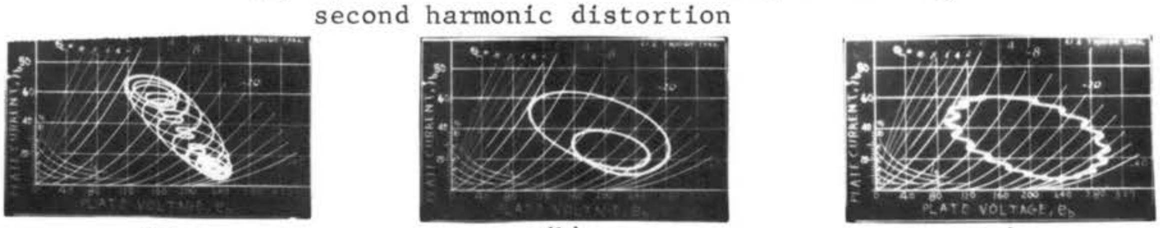
(All three images are reversed horizontally)



(a) (b) (c)

Figure 18. The path of operation of a triode tube with resistive and reactive loads with a single sine wave of driving voltage applied

(a) Normal operation with resistive load
(b) Operation with reactive load, relatively high second harmonic distortion
(c) Operation with reactive load, relatively low second harmonic distortion



(a) (b) (c)

Figure 19. The path of operation of a triode tube with a reactive load when two driving voltage sine waves of different frequency are applied simultaneously

(a) Application of 160 cps and 1000 cps
(b) Application of 500 cps and 1000 cps
(c) Application of 20,000 cps and 1000 cps

large. The operation of a triode tube under these conditions is shown in Figures 18a, 18b, and 18c. Figure 18a shows the operating path for a resistive load; Figure 19b shows the operating path for a reactive load where the second harmonic distortion of voltage and current is large; and Figure 18c shows the operating path for a reactive load where second harmonic distortion is relatively small.

In many applications of audio power amplifiers, driving voltages with sinusoidal components of two or more frequencies are applied simultaneously. For this condition of operation, the plate current of a single tube will be of the form given by Equations (45) and (46). It is interesting to note that the operating path for this type of operation involves an appreciable region of the plate family of characteristics in the i_b, e_b plane. There appears to be an infinite variety of shapes which the operating path can have, depending upon the number, magnitudes, and frequencies of the components of the driving voltage. Three examples are given in Figures 19a, 19b, and 19c. In Figure 19a, 160 cps and 1000 cps voltages have been applied to the grid of a triode tube in a ratio of 3 to 1; Figure 19b shows the operating path for the simultaneous application of 500 cps and 1000 cps voltages; and Figure 19c shows a small 20,000 cps voltage applied with a large 1000 cps voltage. No further consideration will be given in this study to the application of more than one sinusoidal driving voltage at a time.

Summary

The past two chapters have been devoted mainly to a general study of the operation of a vacuum tube circuit with a reactive load. In

Chapter II, typical methods for solving circuits containing a nonlinear resistor were presented and a comparison of the problem of nonlinear circuit analysis for the cases of pure resistive and reactive types of circuits was made. The methods and ideas presented there have been carried into the present chapter where both graphical and analytical methods have been utilized in developing some of the general characteristics of the operating path of a vacuum tube when the load is of the reactive type. Once the operating path of the tube has been established, enough information becomes available to allow most any quantity related to the operating of the circuit to be determined. The graphical method, based on a differential equation approach, which has been presented is perhaps the most easily applied method for obtaining an approximate solution; however, its practical application is limited to very simple load configurations which are not at all representative of the general electromechanical type of load which is often times the load used for audio power amplifiers. The analytical method, based on a power series approach, is a general method for determining the steady-state operating path for a periodic type of applied driving voltage. It can be used for any type of load configuration provided the load impedance as seen by the tube can be determined at the appropriate frequencies; however, this method is not at all practical for actual problem solving due to the enormous amount of work involved in order to obtain even a rough approximation to a particular solution based on the first two or three terms of a power series. It has been shown that the operating path of a vacuum tube with a reactive load depends upon the magnitudes and frequencies of the components of the driving voltage, the choice of the Q point or region of operation over the plate family of characteristics of the tube, and the value of load impedance over a wide range of frequency which will depend upon the particular load configuration used.

The following two chapters of this work are devoted to a more specific study of the actual effects of a reactive load on the power output, plate circuit efficiency, and distortion for both triode and beam power tube audio power amplifiers for various classes of operation. This study consists mainly of laboratory measurements made for a typical type of tube with a simple reactive load configuration. In particular, a comparison is made of the operation of a properly designed amplifier circuit based on the assumption of a pure resistive load with the operation of the same circuit when various amounts of reactance are added to the plate circuit load. In the following chapter, such a study is made for single-tube audio power amplifiers in class A_1 operation after a brief analysis based on an idealized linear tube is presented.

CHAPTER IV

LINEAR ANALYSIS OF SINGLE TUBE POWER AMPLIFIER FOR CLASS A_1 OPERATION

Introduction

In Chapters II and III, some indication of the general nature of the complicated processes taking place in a vacuum tube circuit with a reactive load has been given, and the typical kinds of difficulty which arise in attempts to solve this type of nonlinear circuit have been demonstrated. Also in Chapter III, it was mentioned that although a linear type of analysis can be used when the operation of the tube is confined to a limited region of its characteristic surface, a more exact method of analysis which takes into account the tube's nonlinear characteristics is required for accuracy when the operation involves an appreciable portion of the characteristic surface. Audio power amplifier circuits for which the grid driving voltage is of necessity made large in order to obtain large power output are a class of circuits which are in the latter of the two above categories; however, such a method of analysis which can be practically applied to the general case of a vacuum tube circuit with a reactive load is unknown to the author. The power series for the case of a reactive load involves graphical procedures for determining approximate values of successive derivatives, and beyond the second harmonic term, its complicated form prohibits its use; also, the graphical method, which has been presented for obtaining an approximate solution is applicable only to the very simplist of reactive loads. In view of these circumstances, the most fruitful approach to a further study of such circuits appears to be that of an analysis based upon an idealized linear tube, which has for its terminal characteristic a plane surface located tangent to the tube's actual characteristic surface at the quiescent point of operation. Such a tube will exhibit straight and equally spaced plate characteristics, and its constant values of μ, gm, and r_p will be equal to those measured at the Q point which should be chosen in a region of relative linearity. Although considerable error can be involved in the application of the results of such a linear analysis to the problem of obtaining actual numerical answers, the method is useful for determining, somewhat qualitatively, the various effects of placing a reactive element in the plate circuit load.

In this chapter, a study of the effects of a reactive load on the power output and plate circuit efficiency of a single tube triode amplifier for class A_1 operation has been made by first analyzing such a circuit for the case of an ideal linear tube and then comparing the results obtained with those obtained from actual laboratory measurements. The results of laboratory measurements made for the case of a beam power tube amplifier for class A_1 operation are also given. In order to begin an analytical study, certain basic equations must be derived and some of the properties of the dynamic path of operation of a linear tube with a reactive load must be determined. These requirements are met in the following section where the results obtained will also be of use at various points in the following chapter.

Dynamic Path of Operation of Linear Tube with Reactive Load

The dynamic path of operation of a linear tube with a reactive load will be determined by considering the circuit shown in Figure 20a. Where a suitable quiescent point Q has been chosen as shown on the linearized plate characteristics of a triode in Figure 20b.

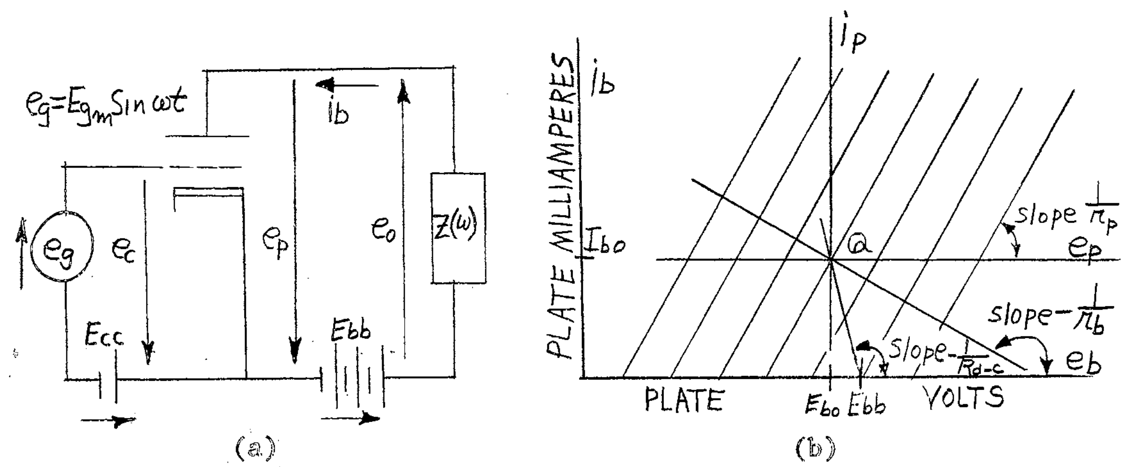


Figure 20. (a) A triode tube amplifier with general load impedance Z(w)
(b) Corresponding plate diagram for ideal tube

In Figure 20a, the load, Z(w), represents the total effective impedance as seen by the tube, and it consists of a zero frequency component, or d-c resistance, a reactive component, and a dynamic component of resistance which will, in general, vary with frequency.

In order to fix ideas, it is first assumed that the load consists of a resistor and inductor in a simple series connection. The equation of the load, in operational notation, is

$$\left[D_t + \frac{r_b}{L}\right] i_b = \frac{e_o}{L} = \frac{E_{bb} - e_b}{L} \tag{56}$$

which can be written in terms of dynamic components as

$$\left[D_t + \frac{r_b}{L}\right] i_p = -\frac{e_p}{L} \tag{57}$$

The equation of the tube's terminal characteristic is

$$i_b = F(e_c, e_b) \tag{58}$$

from which the total differential of plate current can be written as

$$\left.\begin{aligned} d(I_{bo}+i_p) &= \frac{\partial i_b}{\partial e_c}\cdot d(E_{cc}+e_g) + \frac{\partial i_b}{\partial e_b}\cdot d(E_{bo}+e_p) \\ \text{or} \quad d\,i_p &= g_m\,de_g + \frac{1}{r_p}\,de_p \end{aligned}\right\} \tag{59}$$

Since the linear tube has constant parameters, Equation (59) can be

inegrated to give

$$i_p = g_m e_g + \frac{1}{r_p} e_p = \frac{1}{r_p}\left(e_p + \mu e_g\right) \tag{60}$$

which can be combined into Equation (53) to obtain the simultaneous

differential equations

$$\left.\begin{aligned} \left[D_t + \frac{r_b+r_p}{L}\right] i_p &= \frac{\mu E_{gm}}{L}\sin\omega t \\ \left[D_t + \frac{r_b+r_p}{L}\right] e_p &= -\frac{\mu E_{gm}}{L}\sin\left(\omega t + \tan^{-1}\omega L/r_b\right) \end{aligned}\right\} \tag{61}$$

which have a solution of the form

$$\begin{aligned} i_p &= A_1 \epsilon^{-\left(\frac{r_b+r_p}{L}\right)t} + \frac{\mu E_{gm}\sin\left(\omega t - \tan^{-1}\frac{\omega L}{r_b+r_p}\right)}{\sqrt{(r_b+r_p)^2+(\omega L)^2}} \\ e_p &= A_2 \epsilon^{-\left(\frac{r_b+r_p}{L}\right)t} - \frac{\sqrt{r_b^2+(\omega L)^2}\;\mu E_{gm}\sin\left(\omega t + \tan^{-1}\frac{\omega L}{r_b} - \tan^{-1}\frac{\omega L}{r_b+r_p}\right)}{\sqrt{(r_b+r_p)^2+(\omega L)^2}} \end{aligned} \tag{62}$$

If the steady-state portions of Equation (62) are combined in such a way as to eliminate the parameter, time, the equation of an ellispe with center at the Q point, and major axis canted in the i_b, e_b plane, is obtained. Thus, the steady-state path of operation for this simple load has the shape of an ellispe.

The term "operating path" will be used hereafter to denote the steady-state locus of points (i_p, e_p) which are located relative to a set of coordinate axes with origin at the Q point in the i_b, e_b plane when the grid driving voltage is of pure sinusoidal form. The grid driving voltage, $E_{gm}\mathrm{Sin}(wt)$, will be used as a reference, and the starting point of the operating path will arbitrarily be taken to be the point (i_{po}, e_{po}) at the time that e_g passes through zero in the positive direction. With these conditions imposed, the steady-state portions of Equation (62), which can be derived by simply applying Ohm's law, become the parametric equations of the operating path and a direct extension of these equations to the more general case of the load impedance shown in Figure 20a can be made. The extended form of these equations can be conveniently written as follows:

$$\left.\begin{aligned} & e_g = E_{gm}\sin\omega t \\ & i_p = \frac{\mu E_{gm}}{|r_p + Z(\omega)|}\sin(\omega t + \gamma) = I_{pm}\sin(\omega t + \gamma) \\ & e_p = -I_{pm}|Z(\omega)|\sin(\omega t + \gamma + \theta) \;;\quad \theta = \tan^{-1}\frac{x(\omega)}{r(\omega)} \\ & Z(\omega) = r(\omega) + jx(\omega) \;;\quad \gamma = -\tan^{-1}\frac{x(\omega)}{r(\omega) + r_p} \end{aligned}\right\} \tag{63}$$

The general equation of the dynamic operating path can now be obtained by writing the general form of the equation of a canted ellipse:

$$Ae_p^2 + Be_pi_p + Ci_p^2 = 1 \tag{64}$$

The coefficients of Equation (64) can be determined by evaluating Equation (63) for three convenient values of wt, say wt = $-\gamma$, $-(\gamma + \theta)$, and $\pi/2$. The result is

$$e_p^2 + 2r(\omega)e_pi_p + |Z(\omega)|^2 i_p^2 = I_{pm}^2|Z(\omega)|^2\sin^2\theta \tag{65}$$

which, by comparison, can be shown to have the same range of values as Equation (63). Thus, Equation (65) is the equation of the dynamic operating path of a linear tube with a general load impedance, $Z(w) = r(w) + jx(w)$.

Some of the properties of the operating path, which can be derived from Equation (63), are shown in Figure 21. By equating $\frac{dip}{dep} = -\frac{\partial F/\partial ep}{\partial F/\partial ip}$ zero, where F is the left hand member of Equation (65), it is found that the maximum current is always given by the intersection of the resistive load-line and the operating path; similarly, the maximum voltage is always given by the intersection of a line with slope $-\frac{\cos\theta}{|Z(w)|}$ with the operating path. The slope of the major axis of the ellipse is for all practical purposes $-\frac{\cos\theta}{|Z(w)|}$ which can be established by using the relation, $\tan(2\alpha) = B/A-C$, where α is an acute angle, measured from the positive e_p axis, through which a set of axes i_p', e_p' must be rotated in order to transform the quadratic form of Equation (65) into a positive definite form. By using the relation, $\tan(2\alpha) = 2Z\cos(\theta)/(1-Z^2)$, or $\tan(\alpha) = \frac{(Z^2-1)}{2Z\cos\theta} + \sqrt{\left[\frac{(Z^2-1)}{2Z\cos\theta}\right]^2 + 1} \doteq |Z|/\cos\theta$, the slope of the minor axis is $|Z|/\cos\theta$. From Figure 21, the slope of the major axis must then be $-\cos\theta/|Z|$. When $x(w) = 0$, the angle $\theta = 0$, and the major axis coincides with the resistive load-line. For this case, Equation (65) becomes $e_p = -r(w)i_p$.

The form of Equation (65) reveals that the operating path of an ideal linear tube has the same shape for both inductive and capacitive reactances. However, referring back to Equation (64), it can be seen that for a positive, or inductive reactance, the operating path begins at point a, in Figure 21, and is traced in the clockwise direction; and for a negative, or capacitive reactance, the operating path begins at point b and is traced in the counter-clockwise direction. If the net load reactance is varied from some positive value, through zero, to some negative value, points a and b will approach each other along the plate characteristic curve for $e_g = 0$, meet at the Q point, and then continue on so that for some negative value of reactance each point will occupy the other point's original position. During this process, the major axis of the ellipse rotates in a clockwise direction, about point Q,[7] until the ellipse degenerates into the straight resistive load-line for zero reactance, and then rotates back in a counter-clockwise direction toward its original position. This is the type of behavior which would be expected if the load should exhibit resonance at some frequency about which the driving voltage frequency was varied.

The equations which have been developed here along with the properties of the operating path shown in Figure 21, will be of value in the next section where the effects of a reactive load on power output and plate circuit efficiency for class A_1 operation of an ideal linear triode tube are investigated.

Power Output and Plate Circuit Efficiency for an Ideal Tube

In this section, the effects of a reactive load on the power output and plate circuit efficiency of a single triode tube in class A_1 operation are predicted analytically for the case of an idealized linear tube. The results, thus obtained, are then compared with laboratory

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[7] In this discussion, it is assumed that the load is transformer coupled to the amplifier so that the Q point is determined by R_{d-c} which is a constant.

measurements made for a typical type of tube connected in an appropriate circuit of the type under investigation. In order for such a procedure to yield useful information, careful attention must be given to the establishment of suitable conditions under which the above comparison can be made. In the following study, suitable grid driving voltage and supply voltages, which are maintained at constant magnitudes, have been selected so that the effects which are observed are due only to a change in the nature of the load. To establish suitable operation voltages, the assumption first made is that the circuit has been designed for optimum operation according to methods of design based upon a pure resistive load. The results obtained will then give an indication of the effects on power output and plate circuit efficiency which should be expected to occur when the load of a properly designed amplifier becomes something other than the pure resistance for which the amplifier was designed. Ordinarily, optimum operation is interpreted to be that for which maximum power output is obtained under a given set of imposed conditions such as maximum plate dissipation, maximum permissible distortion, maximum grid current, available supply voltages, available driving power, etc. The conditions most usually imposed upon a class A_1 triode amplifier, using small type tubes, required that for an ideal tube the load resistance be approximately twice the value of plate resistance and that $E_{gm} = |E_{cc}| = 0.7\ E_{bo}/\mu$.[8] This rule has been used as general guide for selecting "typical operation conditions" for an triode tube used in this study. The tube used here is a type 6L6 beam power tube with the screen grid connected to the plate for triode operation. As in the case of most actual tubes, the optimum value of load

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[8] See Reich, pp. 228-232.

resistance for the tube varies slightly from that given above and has been found to be $2.9r_p$ or approximately three times the plate resistance measured at the Q point. This value of load resistance appears to give a reasonable compromise between power output and harmonic distortion for the operating voltages which have been chosen. Laboratory measurements for other values of load resistance have also been made in order to obtain an indication of the effects of a reactive load when both the reactive and resistive components of the load vary which is most usually the case for an electromechanical type load.

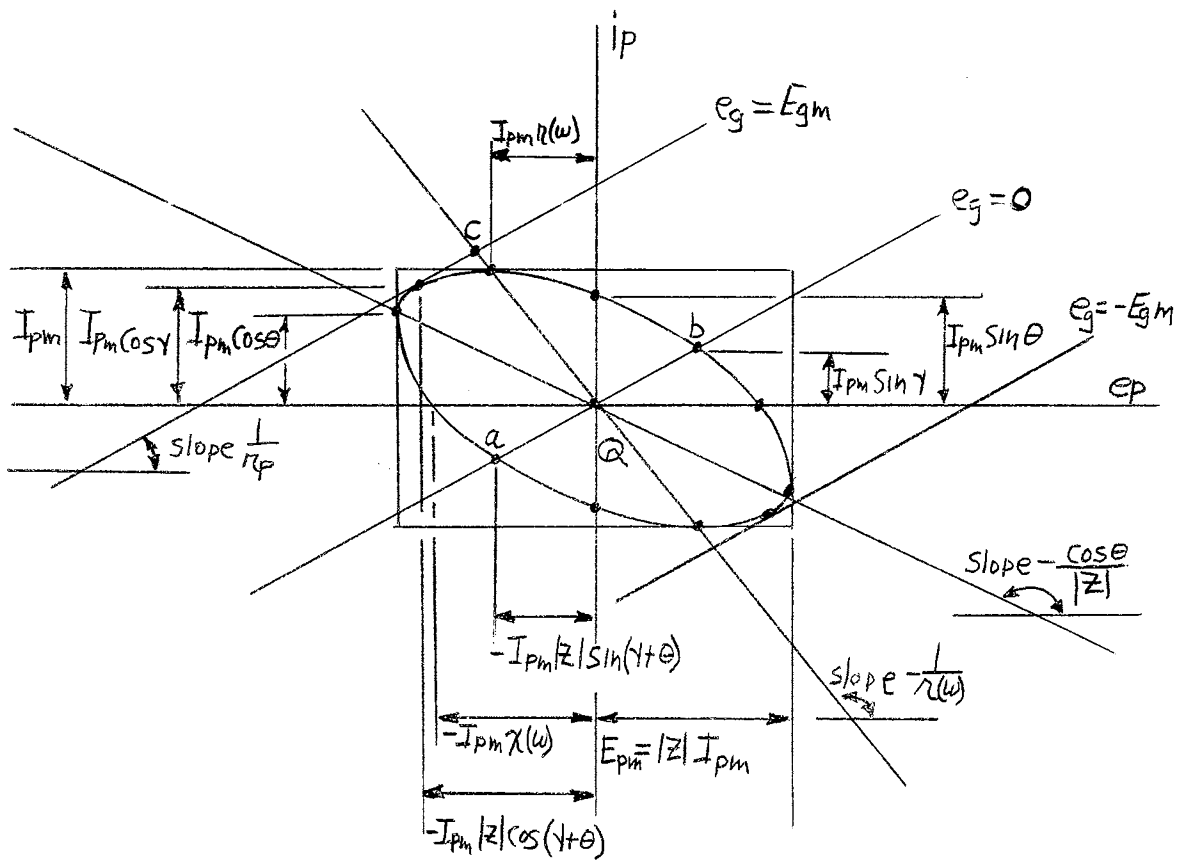


Figure 21. Properties of the dynamic operating path of an ideal linear vacuum tube with a reactive load

An analytical analysis of an ideal tube with a reactive load is begun by referring to Equation (63), which show that the presence of a reactive component in the plate circuit load causes the plate current and voltage waveforms to change in magnitude and to be shifted in phase relative to the driving sinusoid. The angular displacement between plate current and load voltage is always equal to the angle θ which is associated with the load impedance, and the angle between plate current and grid voltage, γ, depends upon both the load impedance and the value of plate resistance of the tube. For a net inductive reactance, γ is a negative angle; and for a capacitive reactance, γ is a positive angle. These relationships are illustrated in Figure 22.

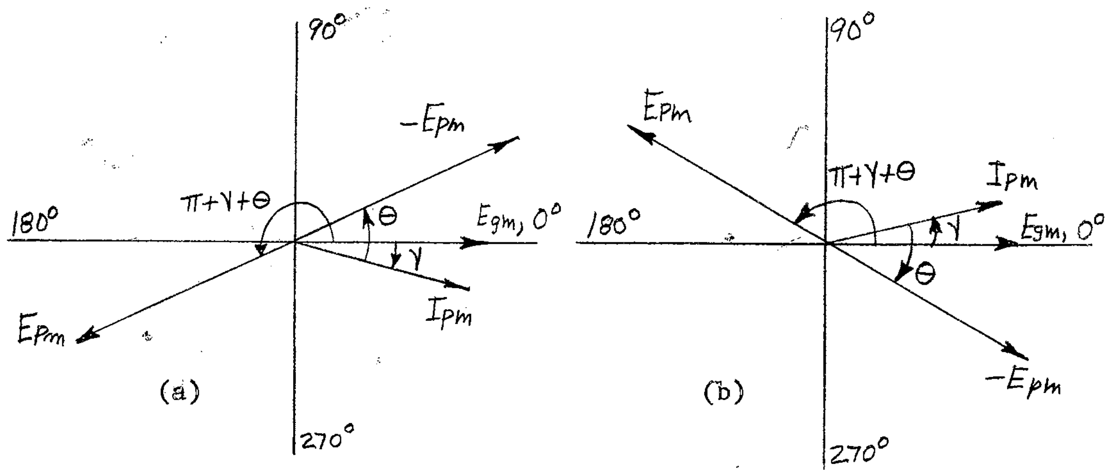


Figure 22. Phase relationships for an ideal tube with a reactive load

(a) Phasor diagram for an inductive load
(b) Phasor diagram for a capacitive load

When the load reactance is zero, the elliptical operating path degenerates into the load-line, corresponding to the value of dynamic load resistance, which intersects the plate characteristic curve for

$e_g = E_{gm}$ at point c in Figure 21. For this case, Equation (63) give

$$\left.\begin{aligned} I'_{pm} &= \frac{\mu E_{gm}}{[r(\omega)+r_p]} \\ E'_{pm} &= \frac{\mu E_{gm}\, r(\omega)}{[r(\omega)+r_p]} \end{aligned}\right\} \qquad (66)$$

When a reactance is added to the plate circuit load, the major axis of the ellipse rotates counter-clockwise from the load-line, the current diminishes to I_{pm}, and the voltage increases to E_{pm} as shown in Figure 21.

$$\left.\begin{aligned} I_{pm} &= \frac{\mu E_{gm}}{\left[(r(\omega)+r_p)^2+X(\omega)^2\right]^{1/2}} \\ E_{pm} &= \frac{\mu E_{gm}\,|Z|}{\left[(r(\omega)+r_p)^2+X(\omega)^2\right]^{1/2}} \end{aligned}\right\} \qquad (67)$$

The ratios of the magnitudes of the plate voltage and current when both reactive and resistive components of load impedance are present to the corresponding values when only the resistive component is present can be written as

$$\left.\begin{aligned} \frac{E_{pm}}{E'_{pm}} &= \frac{[(r(\omega)+r_p]\,|Z|}{\left[(r(\omega)+r_p)^2+X(\omega)^2\right]^{1/2} r(\omega)} = \frac{\cos\gamma}{\cos\theta} \\ \frac{I_{pm}}{I'_{pm}} &= \cos\gamma \end{aligned}\right\} \qquad (68)$$

The power output for the reactive load is

$$P_o = \frac{I_{pm}^2}{2}\, r(\omega) = \frac{I'^2_{pm}}{2}\, r(\omega)\cos^2\gamma = P'_o\cos^2\gamma \qquad (69)$$

which shows that the power output for a load with resistance and reactance is decreased from its value for a load with the resistive component alone by a factor of $\cos^2(\gamma)$. This power ratio has been plotted for various values of $r(w)/r_p$ in Figure 23 which shows that the relative magnitudes of the plate and load resistance have an important part in determining the loss of output power at any given value of load power factor.

The plate circuit efficiency for any amplifier is given by the ratio of the a-c power output to the corresponding d-c power input

$$\eta_p = \frac{P_o}{P_{in}} \times 100\% = \frac{I_{pm}^2 \, r(\omega)}{2 E_{bb} \cdot I_{ba}} \times 100\% \qquad (70)$$

where I_{ba} is the average d-c plate current with the grid signal applied. Since the condition of no distortion is assumed, $I_{ba} = I_{bo}$ = constant, and the ratio of the plate circuit efficiency for a load with both resistive and reactive components to that for a load with only the resistive component can be written as

$$\eta_p / \eta_p' = P_o / P_o' \qquad (71)$$

The curves in Figure 23 also represent the above ratio of plate circuit efficiencies.

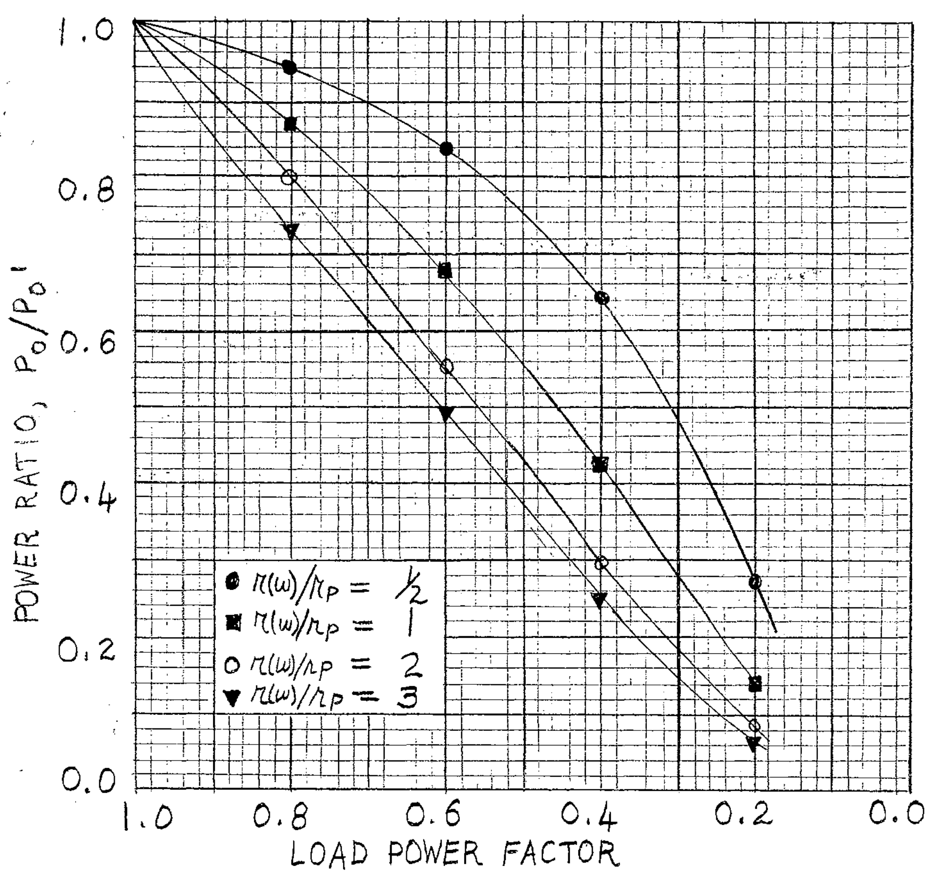


Figure 23. Ratio of power output with resistive and reactive components of load impedance to that for load with only resistive component present

The plate dissipation of a single tube triode amplifier is given by

$$P_p = P_{in} - P_o - I_{ba}^2 R_{d-c} \qquad (72)$$

For the linear tube, the d-c plate current and input power are constant so that the plate dissipation will increase by the amount that the output power decreases. However, this is of no particular importance for the case of class A_1 operation since the circuit is designed for a maximum plate dissipation which occurs when the grid driving voltage, and hence the output power, is zero.

In order to illustrate the overall effect on power output and plate circuit efficiency when both the resistive and reactive components of the load are varied relative to some reference load, the curves shown in Figure 24 have been constructed. These curves show the ratio of power output for a general reactive load to the maximum power output that the amplifier is capable of delivering for the particular operation voltages being used. The reference load is a pure resistance equal to the value of r_p measured at the Q point which is the load specified by Maxwell's maximum power transfer theorem (applicable since E_{gm} is held constant) that gives maximum power output. This power ratio has been plotted versus values of $r(w)/r_p$ for various values of load power factor according to the equation

$$P_o/P_{o\,max} = \frac{P_o'}{P_{o\,max}} \cdot \frac{P_o}{P_o'} = \frac{r(\omega)(r_b + r_p)}{r_b\,(r(\omega) + r_p)} \cos^2\theta = 4\left\{\frac{r(\omega)/r_p}{\left[1 + \frac{r(\omega)}{r_p}\right]^2 + \left[\frac{X(\omega)}{r_p}\right]^2}\right\} \qquad (73)$$

where $r_b = r_p$, the reference load. The curves can be referred to an optimum value, or design value, of load resistance by simply multiplying the ordinate by an appropriate factor

$$K = \frac{1}{4}\left[\frac{r_b'/r_p}{[1 + r_b'/r_p]^2}\right] \qquad (74)$$

where r_b' is the optimum, or design, value of load resistance. After applying Equation (74), the curves show directly the effect on the power output and plate circuit efficiency of an ideal linear tube when its load is something other than the value of pure resistance for which it was designed. In interpreting the curves, it should be remembered that all operating voltages are held constant and only the load is allowed to vary. According to Maxwell's maximum power transfer theorem, the peaks of the curves occur at $r(w)/r_p$ = pf.

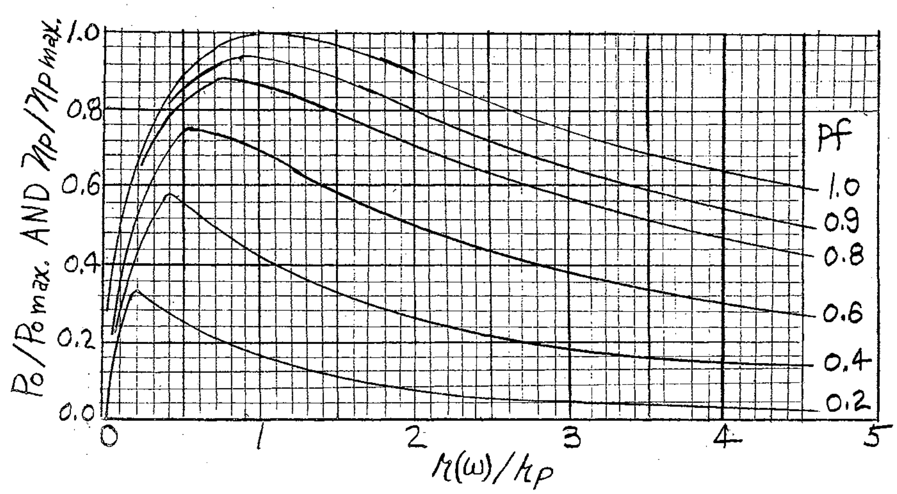


Figure 24. Ratios of power output and plate circuit efficiencies for a reactive load to maximum values for a resistive load $r_b = r_p$

In the following section, the results of laboratory measurements made for a single tube triode amplifier with a simple reactive load are presented in the form of curves of the type shown in Figure 23. Circuit operation with various power factors and ratios of load to plate resistance has been investigated. And since the differences between the results obtained from laboratory measurements and those obtained analytically for an ideal tube are due to the nonlinear characteristics of the actual tube, the results of harmonic distortion measurements are also presented in conjunction with the above study.

Operation of Triode Power Amplifier with Reactive Load

In order to study the actual operation of a single tube class A_1 triode power amplifier with a reactive load, the circuit shown in Figure 25 was constructed, and laboratory measurements were made from which power output, power input, plate circuit efficiency, plate dissipation, and harmonic distortion were determined.

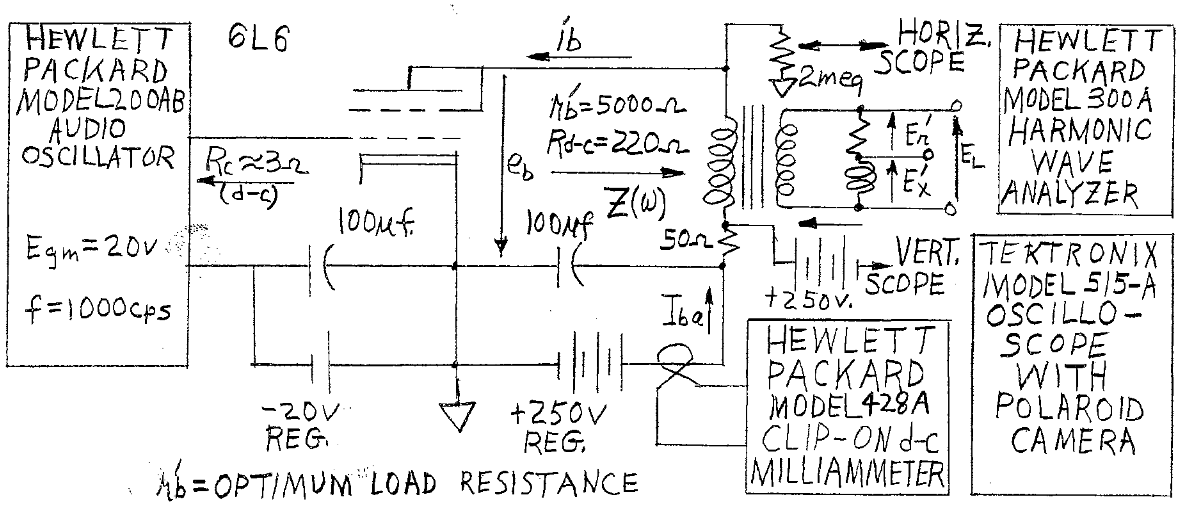


Figure 25. The circuit and equipment used to study class A_1 triode amplifier operation with a reactive load

In a previous section, it was shown that the shape of the operating path of an ideal linear tube is independent of the type of reactance exhibited by the load. Experimentally, it has been found that, for all practical purposes, this is generally true for the operating path of an actual triode tube, except at very high distortion. The fundamental components of plate voltage and current are essentially the same for equal magnitudes of either capacitive or inductive reactance; however, there is a difference in the relative magnitudes of the harmonic components. For a simple series R-C load configuration, the magnitude of the load impedance decreases with increasing harmonic frequencies; while for a simple series R-L load, the opposite is true for increasing harmonic frequencies. For these reasons, the relative magnitudes of the plate voltage harmonics will be larger for the R-L load. This behavior is predicted by the series expansions for plate current and voltage which were given in Chapter III. There are an unlimited number of load configurations which could have been used in making this study; however, since space permits the use of only one type and the inductive type of load is perhaps the type most usually encountered, the simple series R-L load configuration was chosen.

The load shown in Figure 25 consisted of a non-inductive power resistor decade box and an air-core inductor decade box. The air-core inductor was found to exhibit an appreciable a-c resistance compared to the total resistive component of the load so that the load power factor was determined under dynamic conditions from the fundamental frequency voltages E_{L1}, E'_{r1}, and E'_{x1} which are associated with the load shown in Figure 25. By constructing a vector diagram for the above voltages, Equation (75) can be derived

$$E_{r1} = \frac{E_{L1}^2 + E_{r1}'^2 - E_{x1}'^2}{2E'_{r1}} \tag{75}$$

where E_{r1} is the r.m.s. voltage across the total resistive component of the load. The load power factor is given by

$$pf = \frac{E_{h_1}}{E_{L_1}} \tag{76}$$

The error in the load power factor, as seen by the tube, and due to the d-c resistance of the plate circuit, is tolerable since the ratio of d-c resistance to the reflected load resistance is small. Also, the effects of capacitance, leakage reactance, and core losses of the output transformer were negligible at the fundamental and harmonic frequencies which were under consideration.

Other quantities of interest were calculated from the following equations:

Power input $P_{in} = E_{bb} I_{ba}$

Plate circuit efficiency $\eta_p = P_o / P_{in}$

Plate dissipation $P_p = E_{bb} I_{ba} - P_o - I_{ba}^2 (R_{d\text{-}c\ total})$ (77)

Percent current kth harmonic distortion $H_{Ik} = \frac{E'_{hk}}{E'_{h_1}} \times 100\%$

Percent voltage kth harmonic distortion $H_{Ek} = \frac{E_{Lk}}{E_{L_1}} \times 100\%$

The results which were obtained from the laboratory study are presented in graphical form in Figure 26. As an aid in interpreting the results, oscillographs of the actual operating path of the tube (over plate characteristics of the particular tube used) for various operating conditions are also presented in Figure 27 at the end of this section.

Referring to Figures 23 and 26a, a surprisingly good correspondence can be seen to exist between the power ratio curves for the actual triode tube and those predicted for an ideal linear tube. The power ratio curves for $r(w) = r_p$ differ by the least amount, while the largest difference occurs between the curves for $r(w) = \frac{1}{2}r_p$. In practically all

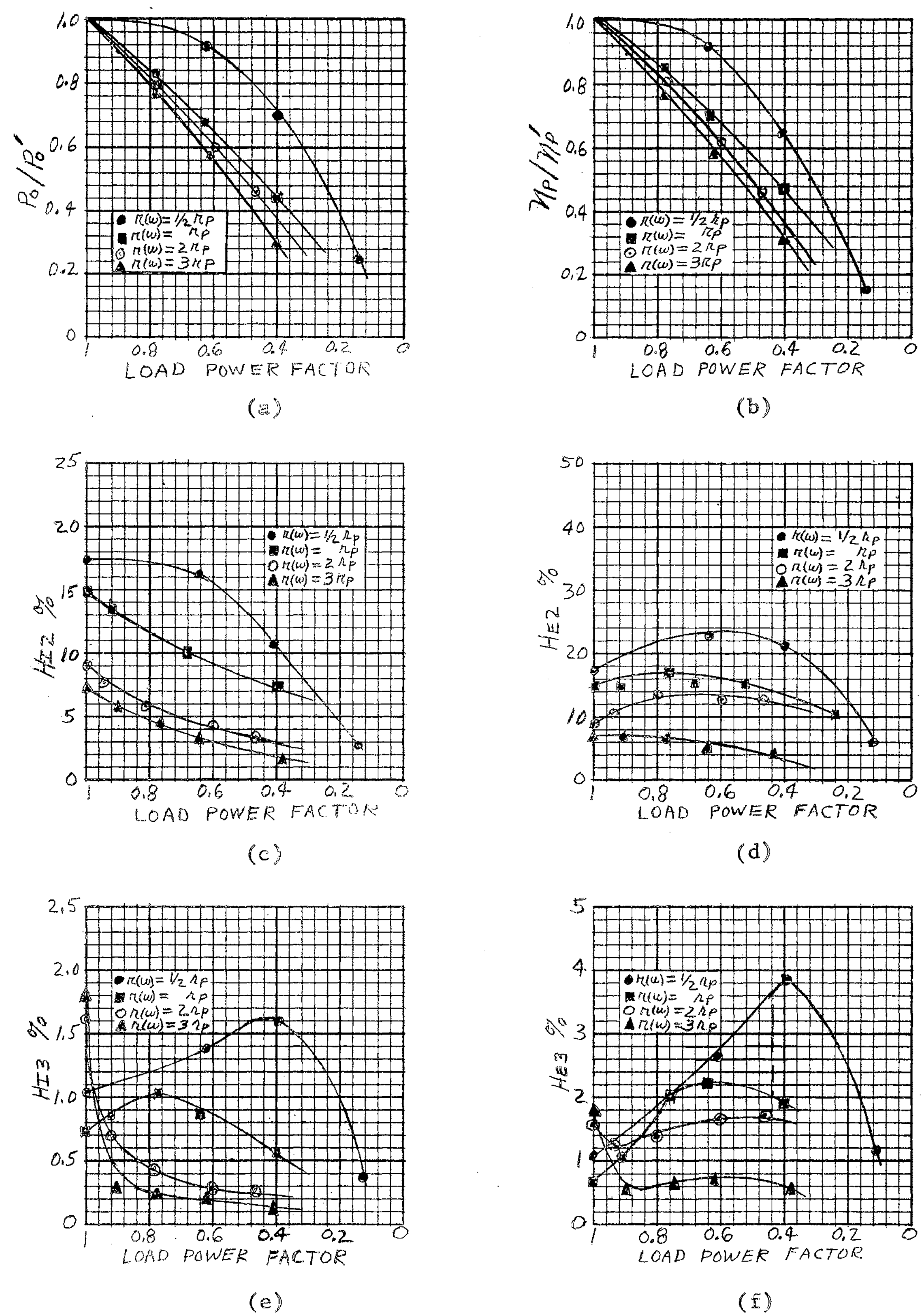


Figure 26. Results of laboratory measurements made for class A_1 operation of single tube triode power amplifier with a reactive load (Type 6L6 tube connected for triode operation)

cases, the differences between the curves show that the actual tube suffered a small loss of power output with decreasing power factor than that predicted for the linear tube. The power ratio for a linear tube given by Equation (69) can be written as

$$P_o/P_o' = \cos^2\theta = \frac{[1 + r_p/r(\omega)]^2}{[1 + r_p/r(\omega)]^2 + \left[\frac{X(\omega)}{r(\omega)}\right]^2} \tag{78}$$

which shows that for a given load power factor the power depends entirely upon the ratio of plate to load resistance. The plate resistance of the actual tube varies considerably over each cycle of the input driving voltage as shown by the operating path in Figure 27e. Furthermore, the plate resistance of the tube varies according to a periodic function of time, for a given power factor, and thus it has a constant average value. If the power ratio of the actual tube is given by a relationship similar to Equation (78), where r_p is interpreted to be the average plate resistance over a cycle of operation, then the power ratio for the actual tube would be larger for a given power factor than that for the linear tube only if the average plate resistance were greater than the value of plate resistance measured at the Q point which was used in Figure 26a to establish the ratio of plate to load resistance. By using the well known three-halves power law for a triode and assuming that the amplification factor is constant, the current of the triode can, for a first approximation, be written as

$$i_b \doteq K(e_b + \mu e_c)^{3/2} \tag{79}$$

which by taking a derivative gives

$$\frac{\partial i_b}{\partial e_b} \doteq 3/2\, K(e_b + \mu e_c)^{1/2}$$

or

$$r_p \doteq \frac{K'}{(i_b)^{1/3}} = \frac{K'}{(I_{bo} + i_p)^{1/3}} \tag{80}$$

Equation (80) shows that for equal positive and negative values of i_p, the plate resistance increases for a negative i_p by an amount greater than the amount that it decreases for a positive i_p. Thus for low distortion operation, such as that shown in Figure 27c where the plate current variation is essentially symmetrical about the Q point, the average plate resistance should be slightly greater than the plate resistance at the Q point. The load to plate resistance ratio for the case shown is $r(w)/r_p = 3.5$, and the curve for $r(w)/r_p = 3$ in Figure 26a gives slightly greater power ratios than those for a linear tube.

When the load resistance is decreased, the lower portion of the operating path, for high to medium power factors, extends into a region of the i_b, e_b plane where the plate characteristic curves become closely spaced and the plate resistance becomes relatively large. This operating condition is illustrated in Figure 27e where $r(w)/r_p = 1$ and the power factor is 0.4. Although a portion of the lower part of the operating path extends into a region of rather large plate resistance, the even harmonic distortion of the current waveform causes the positive portion of i_p to exceed the negative portion i_p by a considerable amount which tends to off-set an increase in average plate resistance.

In Figure 27f, operation for the same $r(w)/r_p = 1$ is shown where the power factor has been reduced to 0.135. Since the operating path rotates in a counter-clockwise direction with decreasing power factor, the lower portion of the operating path has rotated up out of the region of closely spaced plate characteristic curves and relatively large plate resistance. Referring to Figure 26a, the power ratio for $r(w)/r_p = 1$ does not differ greatly from that of a linear tube except at very low power factors where it begins to slightly exceed that of a linear tube. If a very low value of load resistance is used, say $r(w) = \frac{1}{2}r_p$, an appreciable portion of the lower part of the operating path, at high values of load power factor, will be located in a region of the i_b, e_b plane very near plate current cut-off where the plate resistance of the tube is extremely large. Although even-harmonic distortion of the plate current will be relatively large, it is conceivable that the average plate resistance over a cycle of operation will also be large due to an appreciable portion of the operating path being located in a region where plate resistance is approaching that of an open-circuit. The situation would not be expected to improve much with decreasing power factor until the operating path rotates sufficiently in the counter-clockwise direction. Correspondingly, the power ratio curve for $r(w)/r_p = \frac{1}{2}$ gives a somewhat high power ratio than that for a linear tube except at a very low power factor where the difference between the two becomes smaller. From the power ratio curves and the oscillographs of the operating path, it can be said that the power ratio equation for a linear tube provides a good "first approximation" for determining the reduction in power output of the actual tube relative to the power output which would be obtained if only the resistive component of the load were present. And the equation gives the best approximation when the ratio of load resistance to plate resistance is equal to or greater than unity or when the operating path does not extend appreciably into the lower region of the plate family of characteristics where the curves are closely spaced and the plate resistance is very large. The effect of varying both the reactive and resistive components of the load on the total output power of the amplifier is more difficult to predict since the variation of power output with load resistance differs considerably for various types of tubes. However, if the change in power output due to a change in load resistance from some optimum value, say $r_b = 3r_p$, to some value $r(w)$ is known, then Equation (78) provides an approximation to the reducation in power output, caused by the reactive component of the load, from the value which would be obtained with only $r(w)$ present. The total change in power output from the optimum value will be the sum of the two changes given above.

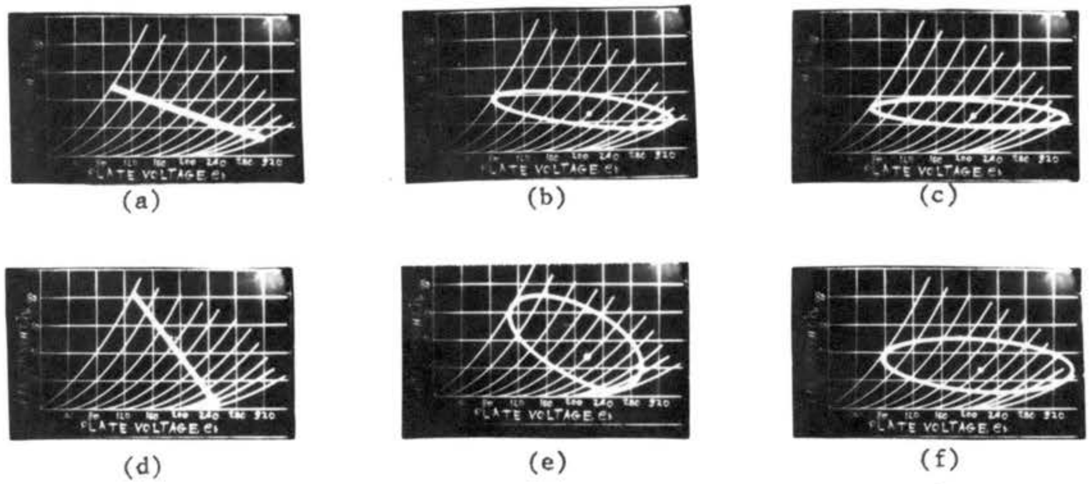
The curves in Figure 26b show the effect of a reactive component of load impedance on the ratio of the plate circuit efficiency for both reactive and resistive components present in the load to that which prevails when only the resistive component is present. The differences between these curves and those giving the power ratio are due to a slight change in the input power with power factor which is due to the effect of harmonic distortion on the average d-c plate current. For load to plate resistance ratios of 2 and 3, the harmonic distortion was relatively small and the efficiency ratio curves are practically the same as those giving the power ratio. The equation for a linear tube provides a good approximation for these resistance ratios.

Figures 26c through 26f shows the effect of the reactive load on harmonic distortion in both plate current and voltage waveforms. For relatively large values of load resistance, the operating path, or load-line, for unity power factor and a properly chosen Q point, does not extend appreciably into the region of the plate family of characteristics where the curves are closely spaced and the plate resistance is very large. This condition of operation is illustrated in Figure 27a where the load resistance $r(w) = 3.5r_p$ is slightly larger than the optimum value. When a reactive component is added to the load, the operating path rotates in a counter-clockwise direction so that its lower portion tends to rotate up away from the extremely nonlinear portion of the plate characteristics as shown in Figure 27b where $r(w) = 3.5r_p$ and pf = 0.6. The tube now operates over plate characteristics which are more uniformly spaced so that harmonic distortion should be less for this case than that for the case of unity power. This reduction in harmonic distortion is verified by the harmonic distortion curves which show that, for $r(w) = 3r_p$, the second harmonic of current, H_{I2}, was reduced from about 7 percent for unity power factor to about 2.5 percent for a power factor of 0.6, and the second harmonic of voltage, H_{E2}, was reduced from about 7 percent to about 5.2 percent. The increase in the impedance of the series R-L type load at the second harmonic frequency is demonstrated by the above figures. The third harmonic components of voltage and current were reduced by greater amounts that the second harmonic components, and higher order harmonics (measurements not shown) were negligible. In Figure 27c, a further reduction in harmonic distortion results for a power factor of 0.4; the operating path for low distortion can be seen to be nearly elliptical in shape.

For relatively small values of load resistance, the resistive load line may extend into the nonlinear lower portion of the plate characteristics as shown in Figure 27d where $r(w)/r_p = 1$. For this operating condition, the harmonic distortion for unity power factor is relatively large as shown by the harmonic distortion curves. The effect of adding a reactive component to the load is illustrated in Figure 27e where $r(w)/r_p = 1$ and pf = 0.4. For this operating condition, the second harmonic distortion of voltage and current have been reduced somewhat, but the voltage harmonic distortion is still rather large. The third harmonic of current has been reduced slightly, but the reduction is offset by the increase in load impedance at the third harmonic frequency so that the third harmonic of voltage has been increased. Figure 27f, where $r(w)/r_p = 1$ and pf = 0.135, shows by the near elliptical shape of the operating path that the harmonic distortion has been reduced appreciably due to the operating path's rotation up out of the extremely nonlinear region of the plate characteristic curves.

Measurements were made for $r(w)/r_p = \frac{1}{2}$ which indicate that this operating condition is a very undesirable one with respect to both harmonic distortion, shown by the harmonic distortion curves, and loss of total power output at unity power factor, which is illustrated qualitatively by the curves in Figure 24. The further reduction in power output due to the presence of a reactive component in the load, shown in Figure 26a, has been discussed for this case.

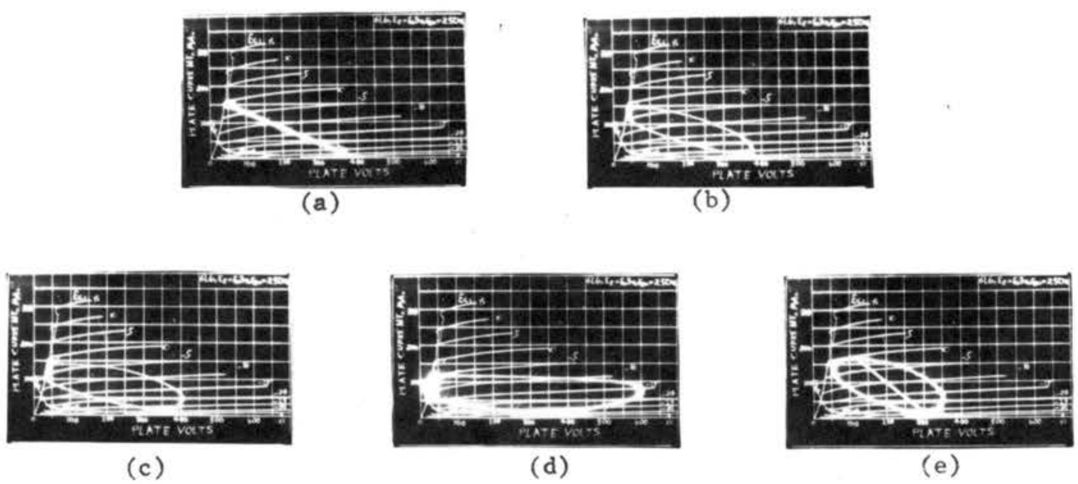
Since the relative magnitudes of the harmonic components of plate current and voltage depend considerably upon the particular load configuration used, it is difficult to make general statements for which



(a) (b) (c) (d) (e) (f)

Figure 27. The dynamic operating path of a single tube class A_1 triode power amplifier with a reactive load

(a) $r(w)/r_p$ = 3.5, pf = 1.0
(b) $r(w)/r_p$ = 3.5, pf = 0.6
(c) $r(w)/r_p$ = 3.5, pf = 0.4
(d) $r(w)/r_p$ = 1.0, pf = 1.0
(e) $r(w)/r_p$ = 1.0, pf = 0.4
(f) $r(w)/r_p$ = 1.0, pf = 0.135



(a) (b) (c) (d) (e)

Figure 28. The dynamic operating path of a single tube class A_1 beam power tube amplifier with reactive load

(a) $r(w) = r_b'$, pf = 1.0 (r_b' = ohm optimum load resistance)
(b) $r(w) = r_b'$, pf = 0.87
(c) $r(w) = r_b'$, pf = 0.69
(d) $r(w) = r_b'$, pf = 0.385
(e) $r(w) = \frac{1}{2}r_b'$, pf = 0.587

there are no exceptions; however, the results of laboratory measurements made for class A_1 operation of a triode tube with typical characteristics, and the oscillographs of its corresponding operating paths which have been presented here, indicate that harmonic distortion decreases with decreasing power factor for a triode tube provided that the resistive component of the load is sufficiently large and the quiescent point of operation is chosen such that the operating path avoids the extremely nonlinear lower portion of the plate family of characteristics.

Operation of Beam Power Tube Amplifier With Reactive Load

In order to study the operation of a single tube class A_1 beam power tube amplifier with a reactive load, a circuit similar to the one shown in Figure 25 for the triode tube was constructed. For beam power tube operation, the screen grid of the tube was connected to the 250 volt regulated supply, the grid bias voltage and peak driving voltage were set at 15 volts, and the optimum value of load resistance was 2500 ohms. Also the equations for determining quantities of interest for the beam power tube circuit are the same as those which were used for the triode tube circuit.

Operation of the beam power tube with the optimum value of pure load resistance is illustrated in Figure 28a located near the end of the previous section. Since it is desirable to have a large grid driving voltage without causing a grid current, a maximum value of total grid voltage e_{cm} = 0 volts, for a 6L6 tube, was chosen. With e_{cm} and E_{bb}

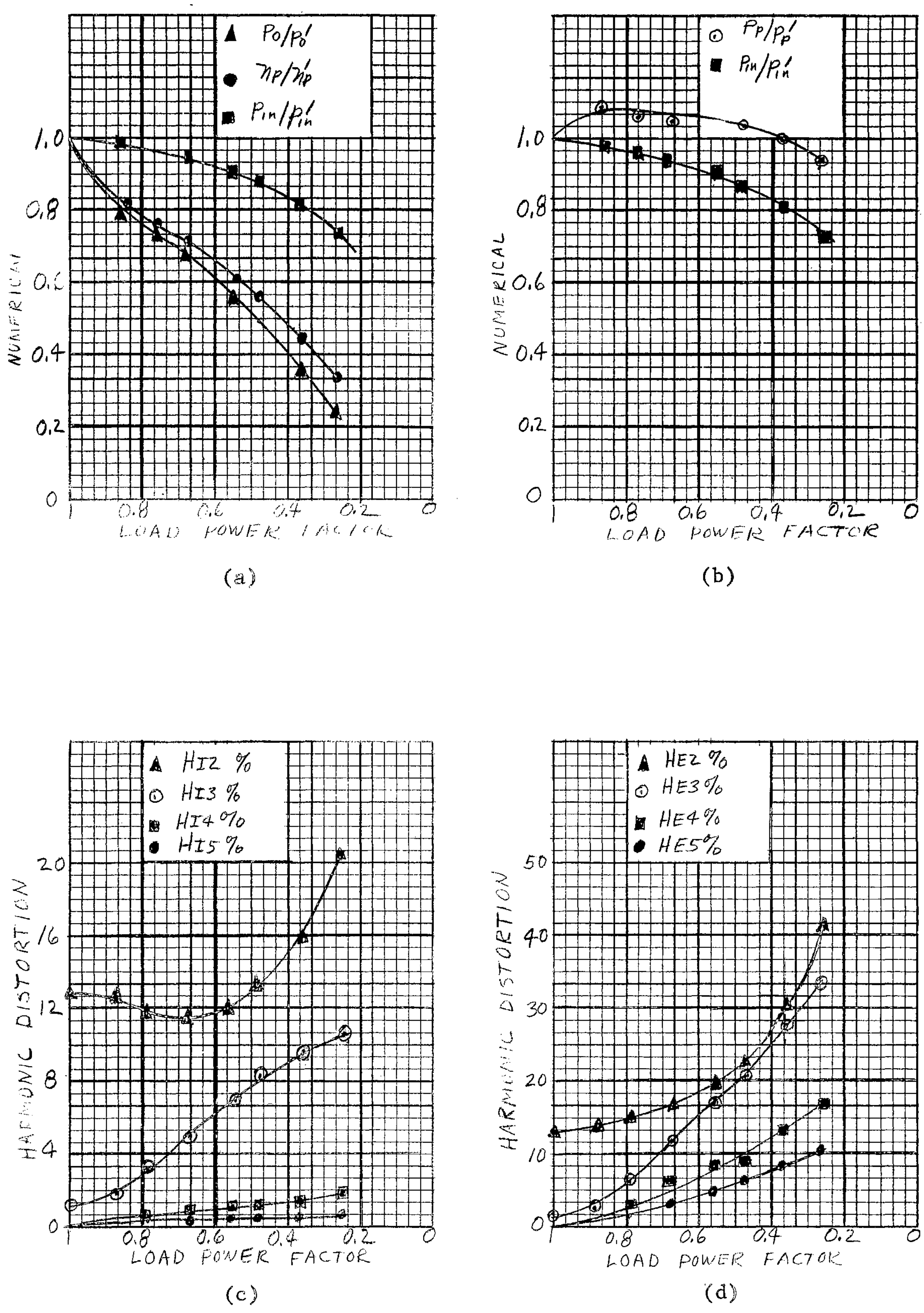


Figure 29. Results of laboratory measurements made for class A_1 operation of single beam power tube power amplifier with a reactive load (Type 6L6 tube)

fixed, it can be seen in Figure 28a that there is a limited choice in selecting a suitable value of load resistance and a quiescent point of operation. The choice is made by reaching a compromise between an attempt to obtain large power output and an attempt to prevent excessive distortion, where the plate dissipation for quiescent operation does not exceed the maximum rating of the tube. In Figure 28a, it can also be seen that, for the operating voltages used there, a decrease in the value of load resistance will cause the negative swing of the plate current to flatten so that harmonic distortion will increase. If the load resistance is increased in value, the upper end of the load line moves below the "knee" in the plate characteristic curves and the positive swing of the plate current flattens. This variation of the load resistance also has adverse effects on the power output of the amplifier. Due to these difficulties, the operation of a beam power tube with a varying load resistance is much poorer than that of a triode tube.

In Figure 29a, the ratio of power output for a load consisting of the optimum value of resistance and a reactance to the power output which was obtained for a unity power factor load consisting of only the optimum value of load resistance is shown, along with the corresponding ratio of plate circuit efficiencies, for various values of load power factor. The power ratio decreases rather rapidly with a small decrease in power factor from unity value. At lower power factors, such as the operation illustrated in Figure 28d where pf = 0.385, the power ratio decreases less rapidly. An appreciable portion of the operating path for this condition of operation is located in the lower region of the plate family of characteristics where the plate resistance of the tube is extremely large. The difference between the efficiency ratio curve and the power ratio curve is due to a reduction in input power which was caused by a change in average d-c plate current due to even-harmonic distortion.

When the power factor is decreased, the operating path rotates counter-clockwise so that its lower portion tends to rotate up out of the lower region of the plate characteristics where the curves are closely spaced; however, any resulting reduction in the harmonic distortion from this source is quickly off-set by an increase in harmonic distortion which occurs when the upper portion of the operating path rotates down below the knee in the plate characteristic curves as shown in Figure 28c. It is interesting to note that, for this case, the locus of operating points over the characteristic surface for the tube, shown in Figure 10a, must "wrap around" the side of the characteristic surface in order to reach the portion of the surface where $e_c = 0$. This causes the left side of the operating path in Figure 28c to appear to fold into i_b, e_b plane in such a way that the negative swing of the plate voltage seriously flattens. The resulting high harmonic distortion produced is shown by the harmonic distortion curves in Figures 29c and 29d.

Figure 28e shows the load line for a load resistance of one-half of the optimum value which has been superimposed upon the operating path for the same value of resistance and a power factor of 0.587. An appreciable increase in distortion resulted from this low value of load resistance. When the power factor was decreased, as shown, the distortion also decreased to a value somewhat greater than that which was measured for unity power factor with the optimum load resistance, and the corresponding decrease in power output was not appreciably large. Upon a further reduction of power factor, the upper portion of the operating path moved below the knee in the plate characteristic curves which caused a very large increase in distortion. Operation with a load resistance larger than the optimum value was found to be entirely unsatisfactory due to the large amount of harmonic distortion at unity power factor which became even larger as the power factor was decreased.

The results of laboratory measurements made for class A_1 operation of a beam power tube with typical characteristics which have been presented in this section, indicate that a single tube beam power tube amplifier which has been designed for optimum operation of the basis of of a pure resistive load, performs poorly with a reactive load due to the large distortion produced when the path of operation extends below the knee in the plate characteristic curves. Unsatisfactory operation results for all power factors when the resistive component becomes larger than the optimum value based on a resistive load (assuming that the correesponding load-line falls just above the knee); slightly poorer operation than that for unity power factor with optimum load resistance is obtained when the resistive component is slightly lower than the optimum value and the power factor varies over a limited range (power output is reduced and distortion is increased); and poor operation results when the resistive component becomes appreciably lower than the optimum value due to low power output and high distortion at unity power factor.

Summary of Results

Vacuum tubes in audio power amplifiers with reactive loads must operate over a large portion of their plate characteristic curves due to the relatively large grid driving voltages required to obtain a large power output. Since the operating path of a power tube with a reactive load may extend into extremely nonlinear regions of the plate characteristic curves, an accurate analysis of circuit operation requires a method which takes into account nonlinear operation. In the absence of such a method which can be practically applied, a linear analysis based on an ideal linear tube provides the most direct way to determine, qualitively, the general effects of a reactive load on circuit operation.

A linear analysis shows that the operating path of an ideal linear tube with a reactive load has the shape of an ellipse which has its major axis canted in the i_b, e_b plane and its origin at the quiescent point of operation. Its shape is independent of the type of load reactance. The intersection of the operating path and a load-line corresponding to the resistive component of the load gives the maximum value of plate current measured from the Q point. The intersection of the operating path and a line with a negative slope equal to the load power factor divided by the magnitude of load impedance gives a very good approximation to the maximum value of plate voltage measured from the Q point. Increasing the reactance of a reactive load causes the plate current to decrease, the plate voltage to increase, and the phase angles between the grid voltage and the plate voltage and current to change. The angle between plate voltage and current is always $\pi + \theta$ is the angle associated with the load impedance.

The ratio of power output with a reactive load to the power output which prevails with a load consisting of only the resistive component of the load is equal to the square of the cosine of the angle between plate current and grid driving voltage. This same ratio also gives the ratio of corresponding plate circuit efficiencies. When both resistive and reactive components of the load vary, the total change in power output is equal to the sum of the change in power output which results from a change in load resistance and the change which results from the addition of a reactive component given by the above ratio.

The results of laboratory measurements made for a typical type of triode tube indicate that the power ratio equation for a linear tube provides a good first approximation to the corresponding power ratio of an actual triode provided that the operating path does not extend appreciably into the extremely nonlinear lower region of the plate characteristic curves. Such a condition of operation ordinarily prevails in triode amplifiers which have been designed for optimum operation based on a pure resistive load, as long as the actual load resistance remains sufficiently large. For the type 6L6 tube connected for triode operation, the minimum load resistance for the above operating conditions was slightly larger than the value of the plate resistance measured at the Q point; the optimum load resistance was about three times the plate resistance. The actual power ratio of the triode was, for most cases, slightly higher than that predicted for a linear tube, which was apparently due to the average plate resistance over a cycle of operation being, for most cases, slightly higher than that measured at the Q point.

For the triode, the results of laboratory measurements indicated that harmonic distortion decreases with decreasing load power factor provided that the resistive component of the load does not become small enough to allow an appreciable portion of the operating path to extend into the extremely nonlinear lower portion of the plate characteristic curves. Load to plate resistance ratios below unity were found to give unsatisfactory operation largely because of high distortion and low power output at unity power factor.

The results of laboratory measurements made for a single tube class A_1 beam power tube amplifier indicated that the beam power tube performs unfavorably with a reactive load, compared to triode operation, largely because of the critical dependence of harmonic distortion on the value of the resistive component of the load. For an optimum load resistance, where the corresponding load line cuts the plate characteristic curve for e_{cmax} just above the "knee" in the curve, the amplifier gives satisfactory operation only if the power factor of the load remains near unity. With decreasing power factor, the upper portion of the operating path rotates below the "knee" in the plate curve and high harmonic distortion results. Values of load resistance slightly lower than the optimum value permit a limited variation in power factor; however, this is obtained at the expense of power output and a slightly higher harmonic distortion at unity power factor. Values of load resistance appreciably lower than the optimum value give unsatisfactory operation largely because the lower portion of the operating path extends into the lower region of the plate characteristics where the curves are very closely spaced. In general, harmonic distortion increases with decreasing power factor for the single tube class A_1 beam power tube amplifier.

Conclusions

1. The power ratio equation for a linear tube provides a good first approximation for a triode class A_1 amplifier with a reactive load provided that the operating path does not extend appreciably into the extremely nonlinear lower region of the plate characteristic curves.

2. Based on the performance of a tube with typical triode characteristics, the operation described above prevails for all power factors as long as the ratio of load to plate resistance remains above approximately unity value.

3. In general, harmonic distortion decreases with decreasing power factor for a class A_1 triode power amplifier.

4. Optimum design of a class A_1 triode power amplifier does not differ appreciably from that which is based on a pure resistive load.

5. Operation of a single tube beam power tube amplifier with a reactive load compares unfavorably with that of a triode due to the large amount of distortion produced by operation below the "knee" of the plate characteristic curves.

6. In general, harmonic distortion increases with decreasing power factor for the single tube class A_1 beam power tube amplifier.

CHAPTER V

PUSH-PULL POWER AMPLIFIERS WITH REACTIVE LOADS

Introduction

This chapter is primarily concerned with the presentation of the results of laboratory measurements which have been made for push-pull power amplifiers, in various classes of operation, which give some induction of the effects of a reactive load on circuit operation. Before presenting these results, a brief discussion of the equivalent operating path for two tubes in push-pull operation is made.

Equivalent Path of Operation Over Composite Plate Characteristics

The equivalent circuit of two ideal linear tubes in push-pull operation is shown in Figure 30 where only the dynamic components of plate voltage and current are considered. In Figure 30a, tube V_1 is represented by an equivalent generator with constant voltage μE_{gm}. The voltage generator E_{o2} represents the voltage induced in the circuit for V_1 by the unity turns ratio of the transformer between the circuit mesh for V_1 and that for V_2. A similar set of voltages for the mesh containing V_2 gives a balanced arrangement for which it can be seen that $I_{pm1} = I_{pm2}$.

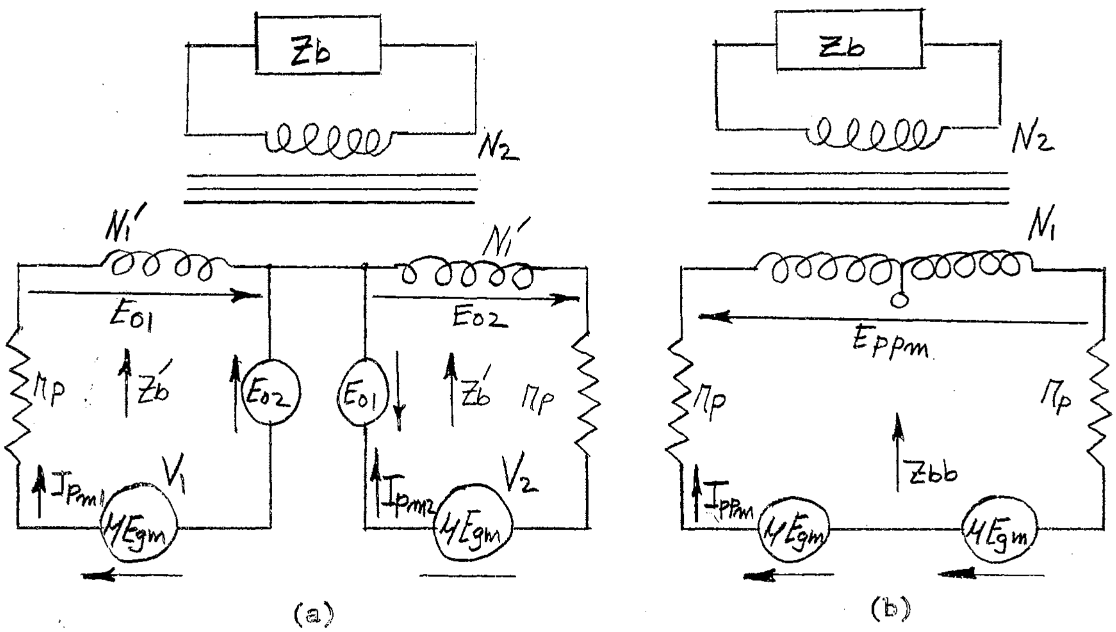


Figure 30. Equivalent circuits for ideal linear tubes in push-pull operation

For Figure 30a, the equations for one mesh of the circuit are

$$\begin{aligned} \mu E_{gm} - E_{o2} &= I_{pm1}(r_p + Z_b') \\ E_{o2} &= I_{pm2} Z_b' \\ I_{pm2} &= I_{pm1} \\ Z_b' &= \left(\frac{N_1'}{N_2}\right)^2 Z_b \end{aligned} \tag{81}$$

from which

$$\begin{aligned} I_{pm1} &= \frac{\mu E_{gm}}{|r_p + 2Z_b'|} \\ E_{pm1} &= -|Z_b'| I_{pm1} \end{aligned} \tag{82}$$

Equation (82) show that tube V_1 operates into twice the value of load impedance that it would operate into if tube V_2 were removed from its socket.

Since $|E_{o2}| = |E_{o1}|$ and $I_{pm1} = I_{pm2}$, the two meshes of the circuit in Figure 30a can be combined to give that in Figure 30b. Since $Z_{bb} = 4(N_1'/N_2)^2 Z_b$, Equation (82) can be written

$$I_{ppm} = \frac{\mu(2E_{gm})}{(2r_p + Z_{bb})} \; ; \; E_{ppm} = -|Z_{bb}| I_{ppm} \tag{83}$$

Equation (83) is normally written in the form

$$I_{ppm} = \frac{1}{2}\frac{\mu E_{gm}}{(r_p/2 + Z_b')}$$
$$E_{ppm} = -|Z_{bb}| I_{ppm} \tag{84}$$

for which the power output for both tubes is given by

$$P_o = \frac{I_{ppm}^2 r_{bb}}{2} = \frac{1}{2}\frac{\mu^2 E_{gm}^2}{|r_p/2 + Z_b'|^2}\frac{r_{bb}}{4} = \frac{\mu^2 E_g^2 r_b'}{|r_p/2 + Z_b'|^2} \tag{85}$$

which may be interpreted to show that the power output of a class A push-pull amplifier is equivalent to that of a single tube with plate resistance $r_p/2$ working into its normal load impedance $Z_b' = \frac{1}{4}Z_{bb}$. A set of composite characteristics for an equivalent single tube which is equivalent to the two actual tubes of the amplifier can be constructed from the plate characteristics of one of the tubes[9]. Since the form of the equations for the equivalent tube is the same as that for a single ideal linear tube, the equivalent operating path over the composite characteristics has the shape of an ellipse.

It should be noted that for actual tubes where harmonic distortion must be considered, the even-order harmonics cancel out in the output in the same manner as that for pure resistive loads.

$$\left.\begin{aligned} i_{b_1}(\omega t) &= I_{b_0} + \sum_{k=0}^{\infty}\left[a_k \cos(k\omega t) + b_k \sin(k\omega t)\right] \\ i_{pp} &= i_{b_1}(\omega t) - i_{b_2}(\omega t + \pi) = 2\sum_{h=1}^{\infty}\left[a_h \cos(h\omega t) + b_h \sin(h\omega t)\right] \\ h &= (2k+1), \quad k = 0, 1, 2, \cdots \end{aligned}\right\} \tag{86}$$

In the following section, the results of laboratory measurements are presented in the form of power ratio curves and harmonic distortion curves, similar to those presented in the previous chapter, for both

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[9]Samuel Seely, Electron Tube Circuits (New York, 1958), pp. 326-330.

triode and beam power tube class A_1 push-pull amplifiers. Also, oscillographs of both the operating path of an individual tube over its plate characteristics and the equivalent operating path for both tubes over the composite characteristic are shown.

Push-Pull Class A_1 Triode and Beam Power Tube Amplifiers With Reactive Loads

In order to investigate the operation of a class A_1 push-pull triode amplifier with a reactive load, a conventional circuit, using fixed bias, was constructed around a type LS-61 Linear Standard output transformer. The control grids of the two output tubes were coupled to an audio oscillator by means of a LS-10X Linear Standard input transformer. And the load connected to the secondary of the output transformer was a simple series R-L configuration consisting of resistor and inductor decade boxes.

The results of laboratory measurements made for three values of load resistance and various load power factors are presented in the form of power output and efficiency ratio curves shown in Figures 31a and 31b and harmonic distortion curves shown in Figures 31c through 31f.

As in the case of a single tube triode amplifier, the power ratio curves shown in Figure 31a show that a large ratio of load to plate resistance causes the power to decrease more rapidly with decreasing power factor. These curves are very similar to those for a single tube triode amplifier, and the discussion given for the single tube amplifier may be applied to both the power ratio and efficiency ratio curves of the present case.

It can be seen that harmonic distortion is very low for the push-pull triode amplifier which results largely from the elimination of even-order harmonics due to the balanced circuit arrangement. The second harmonic component, due to a slight circuit unbalance, was less than one percent for all operating conditions shown. As in the case of the single tube amplifier, the current harmonic distortion decreased with decreasing power factor. The slight rise in the voltage harmonic distortion for medium power factors is caused by the increase in load impedance at the higher harmonic frequencies. In general, the harmonic distortion for the class A_1 push-pull triode power amplifier can be said to decrease with decreasing power factor.

A circuit similar to the one used for the triode tube amplifier was used to investigate the operation of a class A_1 push-pull beam power tube amplifier. The results of laboratory measurements made for the optimum value of plate-to-plate load resistance and various power factors are presented in Figure 32. It can be seen in Figure 32a that the power output of the beam power tube amplifier decreases somewhat less rapidly than that for the triode amplifier due to the relatively large plate resistance of the beam power tube. However, it is very likely that the average plate resistance of the beam power tube over a cycle of operation is considerably less at low power factors than the large value of plate resistance which is measured at the Q point since a considerable portion of the operating path is located along the lower portion of the "knee" in the plate characteristics curves, for the beam power tube, where the plate resistance is very low. This would account for the lack of a great difference between the power ratio curve for the beam power tube and that for the triode.

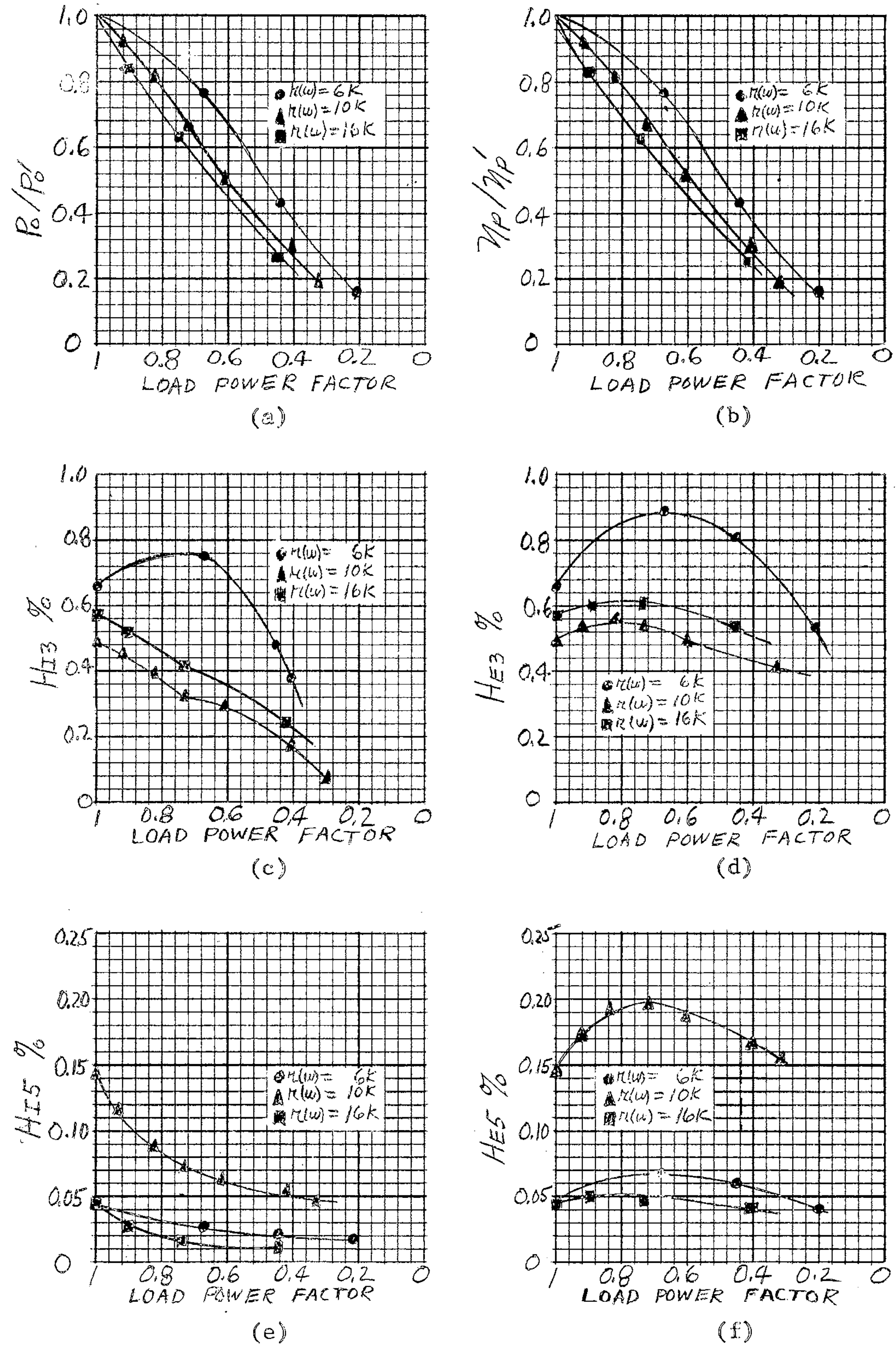


Figure 31. Operation of class A_1 push-pull triode power amplifier with reactive load (6L6 tube, triode connection, E_{bb} = 250 v, E_{cc} = -20v, E_{gm} = 20v)

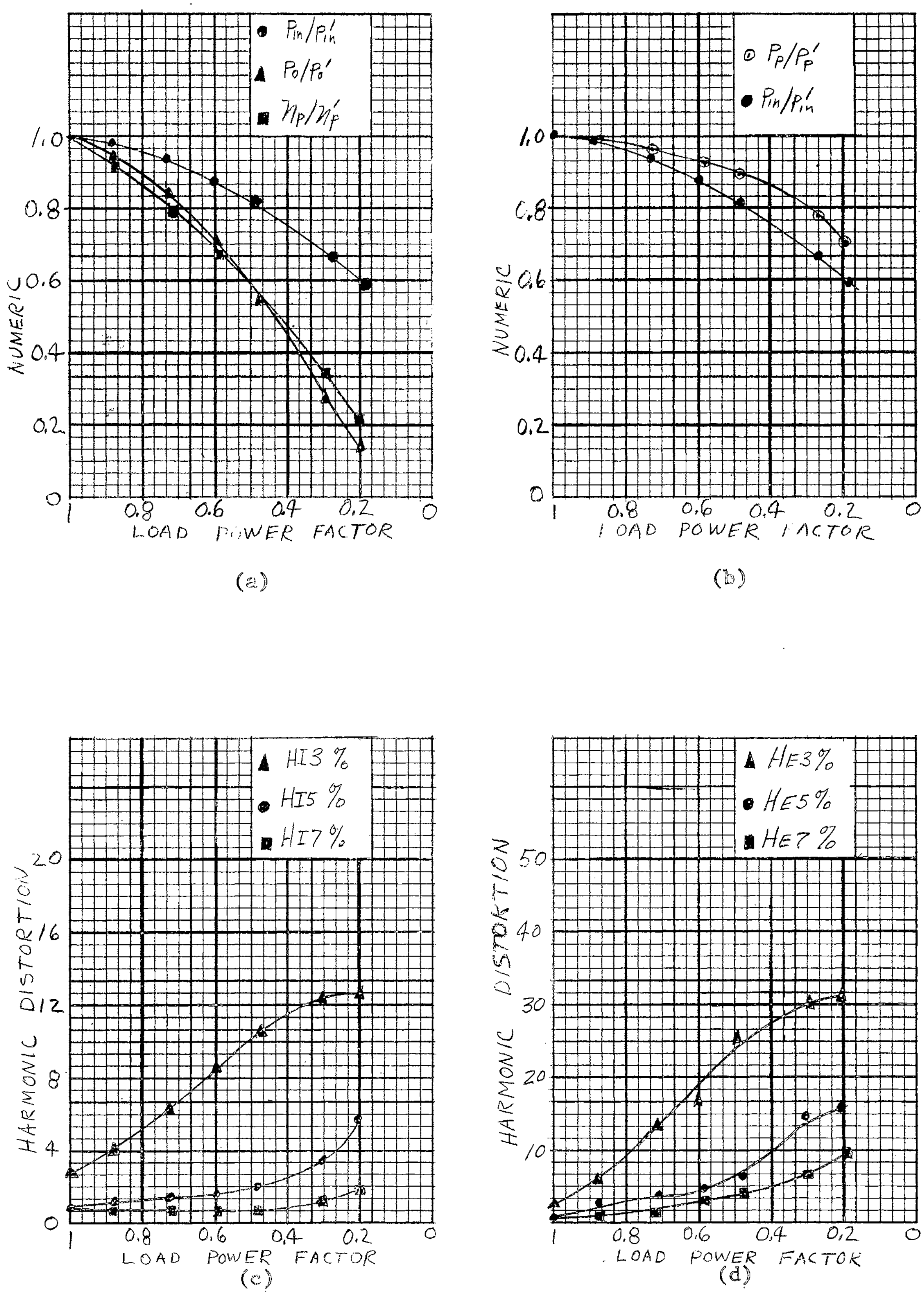
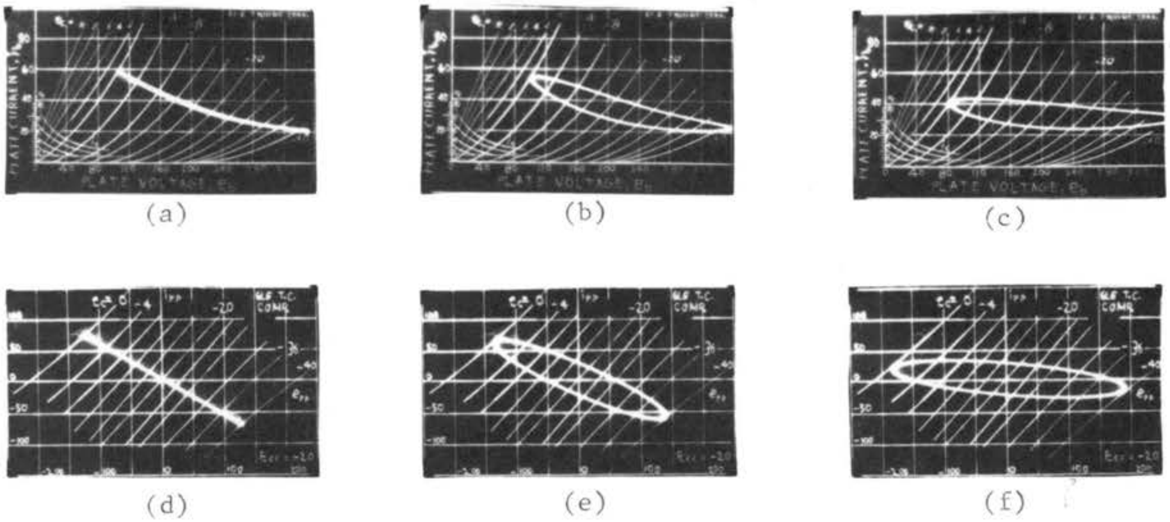


Figure 32. Operation of class A_1 push-pull beam power tube power amplifier (6L6 tube, E_{bb} = 250v, E_{c2} = 250v, E_{cc}= -15v, E_{gm} = -15 v, r_{bb}= 5K)



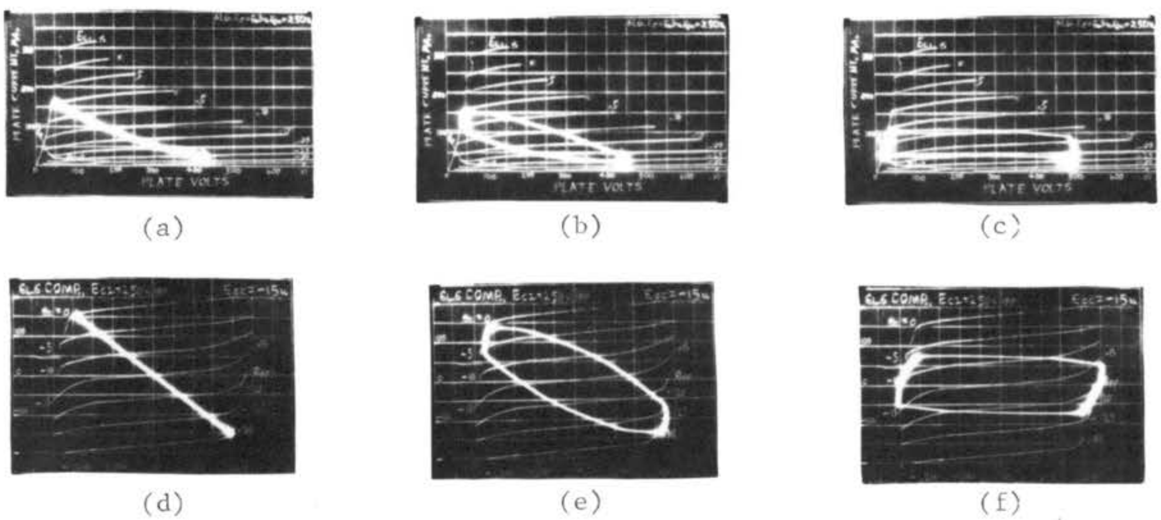
(a) (b) (c)

(d) (e) (f)

Figure 33. The dynamic operating path of a triode tube in a class A_1 push-pull power amplifier and the equivalent operating path for both tubes (6L6 tube triode connection, E_{bb} = 250 v; E_{cc} = -20 v; r_{bb} = 10k)

(a) pf = 1.0 (b) pf = 0.83 (c) pf = 0.33

(d) pf = 1.0 (e) pf = 0.83 (f) pf = 0.33



(a) (b) (c)

(d) (e) (f)

Figure 34. The dynamic operating path of a beam power tube in a class A_1 push-pull power amplifier and the equivalent operating path for both tubes (6L6 tube, E_{bb} = 250 v; E_{c2} = 250 v; E_{cc} = -15 v; r_{bb} = 5k)

(a) pf = 1.0 (b) pf = 0.74 (c) pf = 0.24

(d) pf = 1.0 (e) pf = 0.74 (f) pf = 0.24

As in the case of a single beam power tube amplifier, the harmonic distortion in a push-pull class A_1 beam power tube amplifier increases with decreasing power factor.

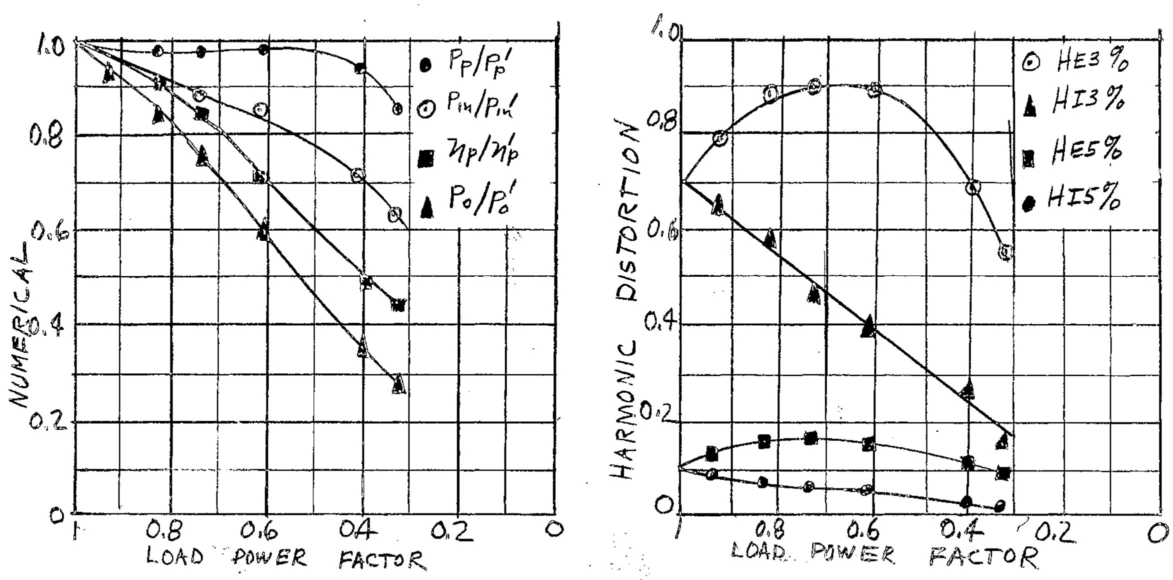
A comparison of the operating paths for triode and beam power tube push-pull class A_1 amplifiers is made in Figures 33 and 34, respectively, where optimum values of plate-to-plate load resistance were used in both cases.

Class AB_1 Operation of Beam Power Tube Amplifier With Reactive Load

The effects of a reactive load on the operation of a class AB_1 beam power tube amplifier were investigated by using the circuit which was used for class A_1 push-pull operation and changing the operating voltages as follows: $E_{bb} = 360v$, $E_{c2} = 270v$, $E_{cc} = -22v$, r_{bb} (optimum) = 6600 ohms. Since the results of laboratory measurements made for this class of operation were very similar to those obtained for class A_1 push-pull operation, the results have not been presented in order to avoid needless repetition. In general, the harmonic distortion for class AB_1 operation was very slightly larger than that for class A_1 operation, but the power ratio and efficiency ratio curves were essentially the same.

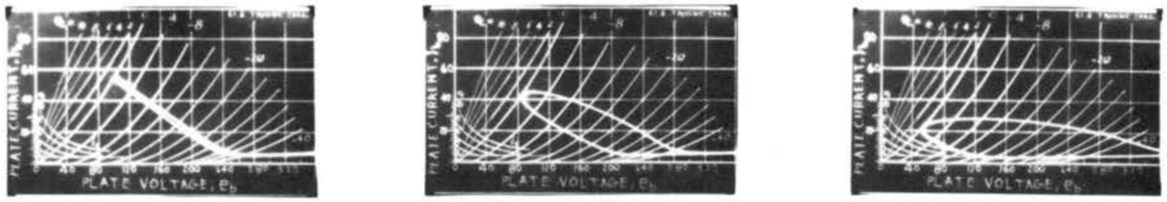
Class B Push-Pull Triode Amplifier With Reactive Load

The results of laboratory measurements made for a class B push-pull triode amplifier, where $E_{bb} = 250v$, $E_{cc} = -36v$, and r_{bb} (optimum) = 10k, are shown in Figure 35. In Figure 35a, the power ratio curve is similar to that for class A_1 operation, but the efficiency ratio curve is somewhat higher for the same power factor. The increase in efficiency of this amplifier is due to the reduction of input power with decreasing power factor. The curves show that, as the power factor is decreased, the plate dissipation is essentially constant, except at very low power factor, which indicates that the plate is called upon to dissipate some of the power returned to the circuit by the reactive component of the load. The input power decreases due to a reduction in I_{pm} since I_{ba}' for each tube is approximately I_{pm}/π and $P_{in} = E_{bb}\ I_{ba}$. The increase in efficiency of this class of operation would become especially important in high power amplifier applications.

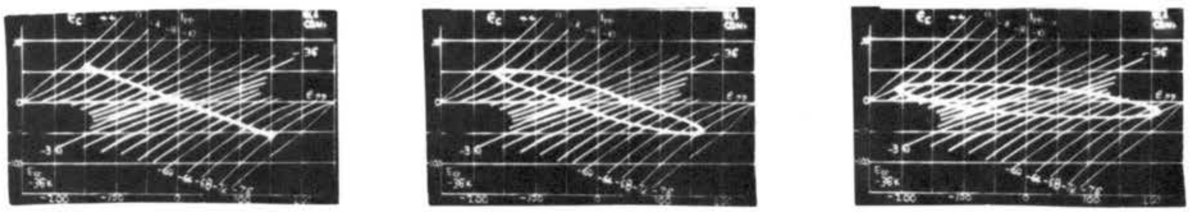


(a) (b)

Figure 35. Operation of class Ab push-pull triode power amplifier with reactive load (6L6 tube, triode connection, E_{bb} = 250v, E_{cc} = -36v, E_{gm} = 36v, r_{bb} = 10k)



(a) (b) (c)

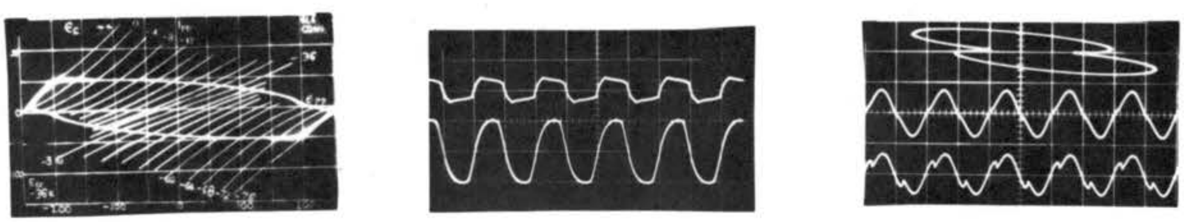


(d) (e) (f)

Figure 36. The dynamic operating path of a triode tube in a class B power amplifier and the equivalent operating path for both tubes in push-pull operation (6L6 tube, triode connection, E_{bb} = 250 v; E_{cc} = -36 v; E_{gm} = 36v; r_{bb} = 10k)

(a) pf = 1 (b) pf = 0.8 (c) pf = 0.3

(d) pf = 1 (e) pf = 0.8 (f) pf = 0.3



(a) (b) (c)

Figure 37. Illustrating the effects of improper operating conditions on the operation of a class B triode push-pull power amplifier (6L6 triode connection)

(a) Equivalent operating path with grid circuit limiting due to grid current, E_{cc} = -36 v; E_{gm} = 42 v; grid circuit resistance = 800 ohms; pf = 0.66

(b) Upper: Voltage waveform for above operation
Lower: Current waveform for above operation

(c) Upper: Equivalent operating path, E_{cc} = -45 v
Center: Current waveform
Lower: Voltage waveform

The harmonic distortion curves shown in Figure 35b show a higher harmonic distortion for class B operation than for class A operation; however, the distortion is not at all excessive. Other than the effect of the R-L load configuration on the voltage harmonics, the harmonic distortion appears to decrease with decreasing power factor.

Typical operation of the class B triode amplifier is illustrated in Figure 36 where both the operating path of a single tube and the equivalent operating path for both tubes are shown.

In Figure 37, the effects of two types of improper operation of the class B circuit are illustrated. Figure 37a shows the effect of grid circuit limiting due to grid current in a relatively high grid circuit resistance. The ends of the operating path are flattened which produces large second and third harmonic distortion. The second harmonic distortion is largely cancelled by the balanced circuit arrangement, but the effects of third and higher odd-order harmonics on the load voltage and current waveforms can be seen in Figure 37b. In Figure 37c, the effect of using too large a grid bias on the equivalent operating path is shown. This type of distortion is analogous to "notch" distortion in class B push-pull amplifiers with pure resistive loads, and it is caused by biasing the tubes too near cut-off grid voltage.

Effects of a Reactive Plate Load on Grid Circuit for Class B Operation

Ordinarily the control grid of a power tube is driven considerably positive over either a portion or all of the positive swing of the grid driving voltage when the tube is operated in class AB_2 or class B operation. The control grid is driven positive over all of the positive swing of the driving voltage when the power tube is of the special zero-bias class B type. In designing a circuit for the latter type of operation, it is important to prevent the difference between the minimum value of plate voltage and the maximum value of grid voltage from becoming too small in order to prevent excessive grid current which may cause control grid heating. Ordinarily, the minimum difference between these voltages is established in a design procedure based upon a pure resistive load. However, it has been shown that a reactive load causes the plate voltage to increase in magnitude and be shifted in phase relative to the grid driving voltage as shown in Figure 38 where a comparison is made of operation with a pure resistive load and operation with the resistive load plus a reactance.

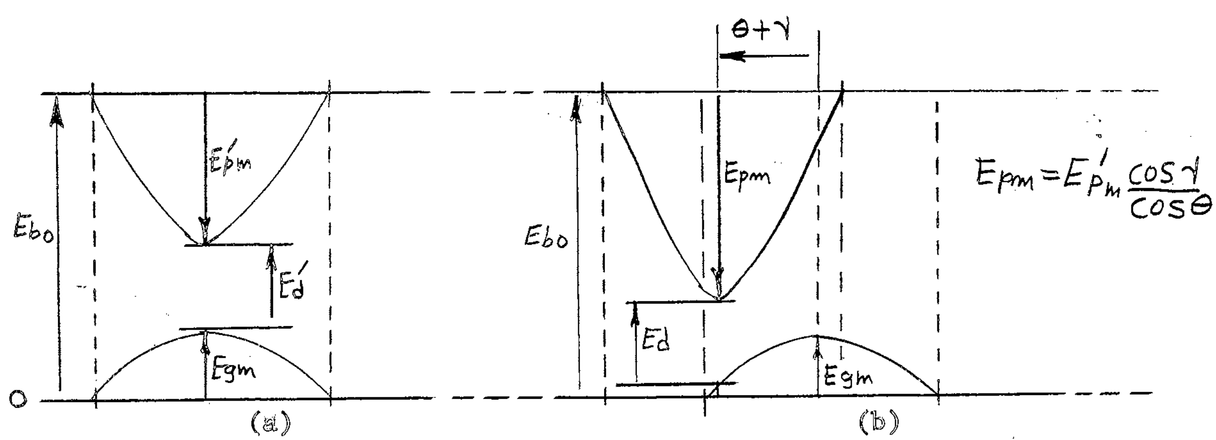


Figure 38. Comparison of class B zero-bias operation with pure resistive load and resistive load plus a reactance

(a) Pure resistive load
(b) Resistive load plus reactance

Since the average grid current, I_c, is some function of the difference between $e_{b(min)}$ and $e_{c(max)}$, the voltage E_d in Figure 38, were I_c increases as E_d decreased, it is of interest to investigate the variation of E_d with load power factor. For a linear tube with a given pure resistive load, the following conditions prevail:

$$E_d' = E_{bo} - E_{pm}' - E_{gm}$$
$$E_{pm}' = \frac{\mu E_{gm} r_b}{(r_b + r_p)} \tag{87}$$

When a reactance is added to the resistive load, the conditions become

$$E_d = E_{bo} - E_{pm} - E_{gm} \cos(\theta + \alpha)$$
$$E_{pm} = E_{pm}' \frac{\cos\alpha}{\cos\theta} \tag{88}$$

Combining Equations (87) and (88) gives the result

$$\frac{\Delta E_d}{E_{gm}} = \frac{E_d' - E_d}{E_{gm}} = \frac{\mu (r_b/r_p)}{(1 + r_b/r_p)} \left[\frac{\cos\alpha}{\cos\theta} - 1\right] + \left[\cos(\theta + \alpha) - 1\right] \tag{89}$$

which can be plotted versus power factor for a given tube, with amplification factor μ and ratio of load to plate resistance r_b/r_p, to show the variation in E_d.

Since a study was not made of zero-bias type class B tubes, this section has been presented only for the purpose of indicating the possibility of excessive grid current under the special operation conditions described above. For the 6l6 tube which has been studied, the difference between $e_{b(min)}$ and $e_{c(max)}$ is always large so that the effects of a reactive load discussed in this section do not particularly apply to this tube. It was found that the maximum variation in average grid current with power factor for a 6L6 tube in both class B triode operation and class AB_2 beam power tube operation was approximately a two percent increase at a power factor of 0.4

Summary of Results

The results of laboratory measurements made for tubes, with typical triode and beam power tube characteristics, in various classes of push-pull operation with reactive loads have been presented in this chapter. It has been found that a class A_1 push-pull triode power amplifier with a reactive load exhibits a power ratio curve, showing the effect of adding a reactance to an otherwise resistive load, which is very similar to that previously shown for a single tube class A_1 amplifier. Harmonic distortion in the class A_1 push-pull triode amplifier is considerably smaller than that for the single tube amplifier due to the balanced circuit arrangement; and in general, it decreases with decreasing power factor.

A class A_1 push-pull beam power tube amplifier was found to have a power ratio curve which decreases somewhat less rapidly than that for the triode tube amplifier; however, the harmonic distortion increases very rapidly to excessive values at low power factors. The operation of each tube in a class A_1 push-pull beam power tube amplifier is similar to that for a single tube beam power tube amplifier which has been discussed in a previous chapter.

An investigation of class AB_1 push-pull operation of beam power tubes with a reactive load revealed that this class of operation exhibits a power ratio curve which is essentially the same as that for class A_1 operation. Harmonic distortion for this class of operation was slightly higher than for class A_1 operation, but this disadvantage was off-set by a considerably higher overall power output at unity power factor. The main disadvantage of using beam power tubes with reactive loads is the large increase in harmonic distortion at low power factors.

For a class B push-pull triode power amplifier, it was found that the power ratio curve was typical of that for class A_1 operation; however, the efficiency ratio curve indicated a somewhat improved efficiency for a given power factor which was largely due to a decrease in input power with decreasing power factor. Plate dissipation was essentially constant except at very low power factor where it decreased somewhat. The plate is called upon to dissipate a portion of the power which is returned to the circuit by the reactive component of the load. The harmonic distortion for class B operation was not excessive, but it was higher than that for class A_1 operation. In general, harmonic distortion decreased with decreasing power factor for class B triode operation.

The possibility of excessive grid current in class B zero-bias type tubes occurring due to the increase of plate voltage swing with decreasing load power factor has been mentioned.

Conclusions

Comparing triode tube amplifiers with beam power tube amplifiers, the triode amplifier gives a relatively low power output and low distortion at unity power factor, while the beam power tube amplifier gives a relatively large power output with reasonable harmonic distortion at unity power factor; with decreasing power factor, both the power output and harmonic distortion of a triode amplifier decrease, and while the power output of a beam power tube amplifier decreases less rapidly than that of a triode amplifier, the harmonic distortion increases very rapidly.

With respect to power output, efficiency, and harmonic distortion for large variations in power factor, the class B push-pull triode amplifier appears to give the best operation. With respect to power output and a reasonable distortion for a very limited range in power factor about unity value, the class AB_1 beam power tube amplifier appears to give the best operation. With respect to harmonic distortion, beam power tube amplifiers give poor performance at low power factors.

CHAPTER VI

SUMMARY

In making a study of the problem of nonlinear circuit analysis for vacuum tube power amplifiers with reactive loads, it has been demonstrated that the presence of a reactive component in the plate circuit load of a vacuum tube makes a general type of solution unobtainable, and the effect of changing some part of the problem can ordinarily be determined quantitatively only by repeating the entire process of the analysis. Unlike the case of a pure resistive load, where the magnitudes of harmonic components are independent of frequency, the case of a reactive load exhibits harmonic components the magnitudes of which depend upon both the nonlinear characteristics of the tube and the values of the load impedance at each of the harmonic frequencies. For the case of a reactive load, the plate current is a multi-valued function of the grid driving voltage and the two quantities ordinarily can be related explicitly only in the form of parametric equations in terms of time. The plate voltage is also a multi-valued function of plate current and these two quantities must also be related by parametric equations in terms of time. These latter two parametric equations can be used to represent the dynamic steady-state path of operation of the tube over a set of plate characteristic curves on a plate diagram (i_b, e_b plane). Once the operating path of the tube has been established, enough information becomes available to allow most any quantity related to the operation of the circuit to be determined. Two methods of solution for determining the operating path have been presented.

The graphical method of solution, based on a differential equation approach, which has been presented is perhaps the most easily applied method for obtaining an approximate solution; however, its practical application is limited to very simple load configurations which are not at all representative of the general electromechanical type of load which is often times the load used for audio power amplifiers. The analytical method of solution, based on a power series approach, is a general method for determining the steady-state operating path for a periodic type of applied driving voltage. It can be used for any type of load configuration provided the load impedance as seen by the tube can be determined at the appropriate frequencies; however, this method is not at all practical for actual problem solving due to the enormous amount of work involved in order to obtain even a rough approximation to a particular solution based on the first two or three terms of a power series. Since an audio power amplifier is often times called upon to drive an electromechanical type of load which may exhibit a rather complicated variation of impedance with frequency, or perhaps time, a need exists for a general method for solving vacuum tube circuits with reactive loads which can be practically applied in order to obtain harmonic distortion information and an indication of other actual circuit performance.

Upon making a linear analysis, based upon an ideal linear tube, of a single power tube with a reactive load, it has been found that the ratio of power output of the amplifier with a reactive load to the power output which prevails with a load consisting of only the resistive component of the load is equal to the square of the cosine of the angle between plate current and a sinusoidal grid driving voltage. This same ratio also gives the ratio of corresponding plate circuit efficiencies. When both resistive and reactive components of the load vary, the total change in power output is equal to the sum of the change in power output which results from the change in load resistance and the change which results from the addition of the reactive component given by the above ratio. The results of laboratory measurements made for a typical type of triode tube indicate that the power ratio equation for a linear tube provides a good first approximation to the corresponding power ratio of an actual triode tube provided that the operating path does not extend appreciably into the extremely nonlinear lower region of the plate characteristic curves. Such a condition ordinarily prevails in triode amplifiers which have been designed for optimum operation based on a pure resistive load.

For the class A_1 single tube amplifier, the results of laboratory measurements indicated that, in general, harmonic distortion decreased with decreasing power factor as long as the ratio of the resistive component of the plate resistance of the tube remains above approximately unity value for a properly chosen Q point based upon a pure resistive load.

The results of laboratory measurements made for a single tube class A_1 beam power tube amplifier indicated that the beam power tube performs unfavorably with a reactive load, compared to triode tube operation, largely because of the critical dependence of harmonic distortion on the value of the resistive component of the load. With decreasing power factor, the upper portion of the operating path rotates below the "knee" in the plate characteristic curves and high harmonic distortion results. In general, harmonic distortion increases to large values with decreasing power factor for the single tube class A_1 beam power tube amplifier.

The results of laboratory measurements made for tubes with typical triode and beam power tube characteristics, in various classes of push-pull operation have been presented. It has been found that a class A_1 push-pull triode power amplifier exhibits a power ratio curve which is very similar to that for a single tube. In general, harmonic distortion decreases with decreasing power factor. A class A_1 push-pull beam power tube amplifier was found to have a power ratio curve which decreases somewhat less rapidly than that for the triode tube amplifier; however, the harmonic distortion increases very rapidly to excessive values at low power factors. An investigation of class AB_1 push-pull operation of beam power tubes with a reactive load revealed that this class of operation exhibits a power ratio curve which is essentially the same as that for class A_1 operation. Power output is higher at unity power factor, but harmonic distortion is also slightly higher than for class A_1 operation. The main disadvantage of using beam power tubes with reactive loads is the large increase in harmonic distortion at low power factors. Class B operations of triode tubes with a reactive load was found to exhibit a power ratio curve typical of that for class A_1 operation; however, the efficiency curve was somewhat higher for the same power factor. The improved efficiency results from a reduction in power input with decreasing power factor. The plate is called upon to dissipate some of the power which is returned to the circuit by the reactive component of the load so that plate dissipation remains practically constant except at very low power factors where it decreases. In general, harmonic distortion decreases with decreasing power factor for class B triode operation.

Suggestions for Future Study

1. Since audio power amplifiers are often used to drive electromechanical types of loads which may exhibit a rather complicated variation of impedance with frequency, a need exists for a general method for solving vacuum tube circuits with reactive loads which can be practically applied in order to obtain harmonic distortion information and an indication of other actual circuit performance. It is suggested that an attempt be made to overcome the inadequacies of the two methods which have been presented here by combining the two methods in such a way that quantities describing the conditions imposed by the generalized reactive load can be determined from a power series approach and can be combined with the conditions imposed by the tube's nonlinear characteristics in graphical form on a plate circuit diagram in much the same way as that for the graphical method which has been presented. The formulation of a load matrix expressing a transformation between discrete quantities of plate current and voltage over a cycle of operation, which can be determined from the theory of least squares approximation over a discrete range, appears to be a promising method of approach.

2. It is suggested that a study be made of the effects of applying feedback in amplifiers with reactive loads.

3. A useful subject for study would be the possibility of using "flexible" passive networks to compensate audio power amplifiers with reactive loads such as electrodynamic shakers used for environmental testing.

4. A subject for study which appears to have some importance, especially in high power amplifiers, is that of the possibility of excessive grid current being induced by the increase in plate voltage swing due to a reactive component of the load for the case of class AB_2 and class B operation and especially for class B zero-bias types of tubes.

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