AN INVESTMENT PROGRAMMING MODEL FOR RURAL COMMUNITY WATER SYSTEMS

NOVEMBER 1984

TECHNICAL BULLETIN T-161
AGRICULTURAL EXPERIMENT STATION
DIVISION OF AGRICULTURE
OKLAHOMA STATE UNIVERSITY

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AN INVESTMENT PROGRAMMING MODEL FOR RURAL COMMUNITY WATER SYSTEMS

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Introduction

This study investigates the economics of rural community water demand and supply in Oklahoma. It demonstrates an improved community services planning model that maximizes net social benefits when intertemporal investments, costs of operation and maintenance, and price responsiveness to water demand are considered simultaneously. The approach developed is also relevant for decision-making by planners in other public services areas.

Problem Statement

Rural water supplies are becoming increasingly scarce relative to rural demands. Continued growth of rural populations constrains the capacities of many rural water systems to provide adequate water services over a resonable planning period. This, coupled with continued industrial development of rural areas, in part dependent upon adequate supplies of water, makes critical an examination and reappraisal of current methods of rural water services planning. Furthermore, many rural communities are confronted with the problem of inadequate funds to cover both the initial investment costs and the sustaining costs of a water system.

Present methods applied to community water systems planning too frequently rely on simple rules of thumb. Average service supply cost is frequently used as a basis to set rates (prices). Future demand increases are considered, if at all, on the basis of multiplying per capita rates of consumption by projected population although economic theory and empirical results support close interrelationships among price level, consumption behavior and supply costs.

Rural community water systems financed by Federal loan programs through the Farmers Home Administration (FmHA) have been unable to plan for sufficient capacity to meet increases in water demand due to population growth since the loan programs can consider only the existing population at the time of loan initiation. As a result many rural water systems financed by the loan programs must increase capacity after relatively short periods of operation, especially in fast growing areas.

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Objectives

The purpose of this research is to provide information for the planning and management of rural water systems in Oklahoma. The primary objective is to demonstrate an improved community services planning model by incorporating factors of water demand and supply that change intertemporally. The focus of this effort is to examine growth factors that influence rural water demand and supply. Data derived from sample information on rural systems in Oklahoma are used as inputs in the planning model to determine optimum levels in system capacity, operation and consumer satisfaction.

Specific objectives are:

1. To develop deterministic community services demand and supply models for rural water services and empirically estimate those models for Oklahoma.

2. To develop programming models which address questions related to optimum timing and size of rural water system investments and optimum pricing of water resources.

3. To evaluate past public investments in rural water services using the programming models.

ECONOMICS OF WATER DEMAND AND SUPPLY IN OKLAHOMA

Optimum Capacity of Water Systems

The general concern is to build water systems that will assist in meeting a demand that grows over time especially due to growth of population. Frequently, each system is one of a sequence of sub-systems that will be built over time, and with options concerning when the sub-systems in the sequence are to be built. The larger each system, the longer it will be until another segment to the system is needed under a constant growth rate.

In determining how large to build the initial capacity or the increment (and the timing of that increment) studies have emphasized two basic factors which are nearly always in conflict:

1. It pays to build large initial capacity or increments to the system because there are usually cost savings (economies of scale) involved in increasing capacity size.

2. The commitment of resources to a capacity that will not be used for a long period of time is costly. It pays to defer investment as long as possible since future costs are more heavily discounted than present costs.

However, these two factors are sufficient only if the capacity decision is considered from a static viewpoint. In reality, growth of water demand makes the situation dynamic and the interrelationships of economies of scale, discount rate and growth must be considered simultaneously in making more realistic capacity decisions. Growth in water demand is a direct reason why a system ends up with a lack of capacity even though it started with an excess of capacity. Therefore, since the discount rate and economies of scale are taken into consideration in making capacity decisions based upon an expectation of growth, explicit inclusion of growth in the decision process is very important.

The Demand for Water in Rural Oklahoma

The value of water is defined by consumers' demand for the commodity. Consumption of water is influenced by price, consumer income, population, configuration of commercial and business uses and climatic considerations, particularly rainfall during seasons when moisture is required. Frequently water consumption is viewed as independent of price and assumes the demand per capita is fixed and that water must be found to meet "requirements". As a result water systems tend to be designed to meet such "requirements".

Price Responsiveness

Water consumption studies have shown that users are responsive to changes in price, more so than is often supposed. Where water is metered, consumers have been found to use significantly less water than those who are on a flat rate. The greater part of the difference is accounted for in the amounts used for non-household purposes such as watering lawns.

A number of published studies of the price elasticity of demand for residential water are available. Price elasticities tend to be relatively low and differ between the two major components of use, domestic use and lawn sprinkling. The elasticities also vary among the different regions of the country.

One of the first analyses was by Louis Fort (1958) based on data from a survey of water utilities. A price elasticity of demand of -0.39 was reported. Conley (1968) studied water consumption in a sample of southern California communities and reported the most likely price elasticity to be about -0.35. Howe and Linaweaver, Jr. (1967) have made the most extensive study. Data were carefully collected from a sample of water systems ranging from 34 to 2,373 dwelling units each. The overall estimated price elasticity (all uses, all regions) was found to be about -0.4. Domestic uses were found to exhibit an elasticity of -0.21, while water used for lawn sprinkling was characterized by elasticities of -0.7 in the arid west and -1.57 in the humid eastern region. Young (1971) utilized time series data to determine the price elasticity for the city of Tucson of -0.33.

These and other studies have demonstrated that consumers in fact are responsive to price changes and adjust their consumption of water accordingly. As useful as these studies are, most are based on narrow samples that limit general conclusions.

Market Demand

The market demand for rural water systems is used as the unit of analysis for this study since data are not available for individual household consuming units. Focus of the present study is the examination of factors explaining water demand behavior among rural systems in order to assist planners in the design of such systems. The market demand for water directly relates to capacity of the system and therefore is considered for purposes of planning optimum system capacity.

To predict water demand for rural areas in Oklahoma, the important variables are hypothesized to be price, number of residential taps and number of nonresidential taps. Theoretically, the marginal price of water should be used as the price variable. But, practically, it is difficult to find a representative marginal price in the aggregate for a water system. However, since most water systems issue water bills by month, it is assumed for this study that consumers respond to water consumption based upon the total monthly water bill. Average cost per thousand gallons computed for the system is assumed to be the marginal price of water for that system.

Most domestic water demand studies divide users into four or more groups such as: residential, commercial, industrial and other. In rural water systems, unlike urban water systems, there are few commercial or industrial water users. Thus, for simplifying purposes, only two groups of water users were considered in this study: residential and nonresidential. In rural areas, the majority of nonresidential users are small businesses or pasture taps for gardens and livestock. The nonresidential users, on the average, consume more water per tap than residential users. Theoretically, nonresidential users may be assumed to be more price-sensitive because their choices of whether to consume water or not is more flexible. They can also consider alternative sources of water such as ponds or wells if the costs of these alternatives are cheaper than consuming community water.

The total number of residential users in a system increases not only from an increase in the density of homes within a water district boundary but also from expansion of the water district boundary itself. However, since the objective is to plan water systems based upon a price-sensitive demand as opposed to requirement approach, it is assumed that population increases do not shift demand curves until they are willing and able to pay for community water. Aggregate demand and the number of total residential and nonresidential taps is expected to move in the same direction.

The Study Area and Data

Even though there is increasing rainfall moving from western to eastern Oklahoma, climatically the whole state of Oklahoma can be considered a semi-arid region. In 1961, the Federal government initiated the National

Rural Water Program and Congress granted authority to the Secretary of Agriculture to make loans and grants through (FmHA) for allowing organization, formation and operation of public nonprofit rural water systems.

In 1963, Nowata County Rural Water District No. 2 was organized as the first nonprofit rural water system in Oklahoma. Through mid-1979, a total of 398 rural water systems funded under this program were serving slightly over one-half million people in Oklahoma. Each water system utilizes its own pricing structure, generally decreasing block rate, while all incorporate a flat rate for the first few thousand gallons of water consumed. This information provides an opportunity to illustrate water demand relationships since each system provides water at a different price (rate). In this manner a cross-section of users, stratified by water system if needed, can be used to form the empirical counterpart of a residential water demand study without the need of resorting to strictly time series data.

In this study, data from 203 water systems were used which have the complete information needed (Rural Water Systems in Oklahoma, 1980). From these systems, the following specific data were derived:

1. AGWAD - aggregate water demand per year computed by multiplying the reported average water consumption per day by 365 for each water system. The AGWAD represents the aggregated consumer's water consumtion behavior and also implicity reflects the operating levels of a system at a particular time.

2. WAPR - water price represents the dollar value per thousand gallons of water. This variable is derived by dividing the monthly average water bill for a system by the monthly average water consumption per tap and multiplying by 1,000. For example, if the monthly average water bill per tap is $15 for a system and the monthly average water consumption per tap is 8,000 gallons, the WAPR is $1.875/1,000 gallons.

3. RESID - represents the total number of residential taps in a system at a given time.

4. NONR - total number of nonresidential taps in a system at a given time.

5. TNTAP - total number of taps (RESID plus NONR) in a system at a given time.

Empirical Estimates of Water Demand

The following water demand equations were empirically estimated:

AGWAD = f(WAPR, RESID, NONR) (1)

AGWAD = f(WAPR, TNTAP) (2)

Regression coefficients were estimated in linear and log-linear form by conventional single equation least squares methods. The estimated regression equations with standard errors of the estimates (S.E.), R^2 and sample size (n) are given below:

$$AGWAD = 25.07 - 16.04\,(WAPR) + 0.12\,(RESID) + 0.31\,(NONR) \quad (3)$$
$$S.E. \quad (6.41) \quad (2.75) \qquad (0.005) \qquad (0.05)$$

$R^2 = .78$ n = 204

$$\ell n AGWAD = -1.97 - 0.59\,\ell n\,(WAPR) + 0.95\,\ell n\,(RESID) + 0.11\,\ell n\,(NONR) \quad (4)$$
$$S.E. \quad (0.23) \quad (0.09) \qquad (0.03) \qquad (0.04)$$

$R^2 = .86$ n = 204

$$AGWAD = 26.80 - 16.91\,(WAPR) + 0.13\,(TNTAP) \quad (5)$$
$$S.E. \quad (6.61) \quad (2.83) \qquad (0.005)$$

$R^2 = .77$ n = 204

$$\ell n AGWAD = -2.38 - 0.57\,\ell n\,(WAPR) + 1.03\,\ell n\,(TNTAP) \quad (6)$$
$$S.E. = \quad (0.19) \quad (0.07) \qquad (0.03)$$

$R^2 = .87$ n = 204

All regression coefficients are statistically significant at the one percent probablilty level. In equation (3) the coefficient of WAPR shows that if the price of water increases one dollar per thousand gallons, holding other variables constant, it will reduce aggregate annual water consumption for the system about 16 million gallons. In equations (4) and (6) the coefficients of ℓn WAPR, -0.59 and -0.57, can be interpreted directly as the price elasticity of aggregate water demand for the sample of districts. This range of price elasticity for rural Oklahoma is higher than the estimated price elasticity for urban areas of about -0.4. This higher price sensitivity could be explained in that rural areas generally have alternative sources of water such as wells, streams or small ponds for nondomestic purposes whereas urban areas rely totally on public water supplies.

The coefficient of NONR in equation (3) means that if the number of nonresidential taps increases by one, holding other variables constant, aggregate water consumption will increase by 0.31 million gallons per year. The comparable amount for RESID is 0.12 million gallons.

In equation (6) the coefficient of total taps (TNTAP), 1.03, is essentially the demand elasticity of population. Statistically, 1.03 is not significantly different from one which means that if total number of taps is increased by one percent, aggregate water demand will increase approximately one percent. Thus, it can be concluded that there is a proportional one-to-one relationship between water demand and number of taps.

Water Supply Cost in Rural Oklahoma

Most rural water supply systems, in contrast to large urban water systems, are characterized by low population densities, high initial investment costs per consumer and low household per capita incomes. The basic economic problem for many rural communities is the lack of funds to finance the initial capital costs of water systems. The FmHA in the past has provided financing, and in some cases, grant funds to publicly-owned rural water systems for unincorporated communities, small towns and dispersed farm populations not exceeding 10,000.

In general, the source of water supply has a significant influence on the total water system investment cost (Sloggett 1974). The investment in treatment plants and wells represents a significant share of total water system investment cost. This cost study is limited to only those water systems purchasing treated water from neighboring systems but could be extended to systems requiring water treatment and water sources. For this study, the capital cost of water distribution is the main investment cost. The O and M (operation and maintenance) cost is hypothesized to be a direct relationship to output or amount of water delivered per unit of time.

Sample of Rural Water Systems

Sloggett (1974) surveyed 57 rural water systems in 1972 to study the economics and growth of such systems in Oklahoma. Major criteria for selection of the sample included: systems must have been in operation for at least two years to assure adequate operating records; systems must vary in size as measured in terms of number of customers (the range was from 16 to 1400 customers); systems with different sources of water supply - wells, lakes and streams, and purchase of treated water; systems with different densities of customers per mile of line and represent rural only, town only and a combination of town and rural; systems should represent all geographical areas in the state. The present study, however, was limited to the 30 out of 57 systems purchasing treated water.

The sample was resurveyed in 1981 to extend the data series and include information on changes in capacities and growth of water systems measured by the annual amount of water delivered or the number of users. One of the 30 water systems of Sloggett's sample added its own treatment facility after the original study and hence was dropped from the sample. For each system in the sample information was obtained for each year since its beginning. An example of the survey data collected by rural water district is presented in Table 1.

Investment Cost

Investment Cost Data. For the systems purchasing treated water, the main facilities are water lines, storage tanks, meters, booster pumps, office and equipment. Capacity is the outcome of certain combinations of the individual components in the distribution system. Specifically, water lines, storage tanks and booster pumps are the main facilities to determine overall capacity while office and equipment are supporting components to maintin a given capacity.

Table 1. Example of Survey Data for Rural Water District Creek #4, Oklahoma

Year	Initial Construction Cost ($)	Capital Additions ($)	Amount of Water Sold (mgy)	Number of Users	Water Purchases ($)	Salaries ($)	Utilities ($)	Office Expense ($)	Insurance and Bonds ($)	Legal and Audit ($)	Other ($)
1968	232,189		13,814	203	3729	2051	263	179	212	672	2331
1969		970	16,714	260	7012	2078	1916	214	212	655	2072
1980											

It is not easy to empirically determine the installed capacity even though information is available on each and every component of the system. Only with detailed engineering studies is it possible to determine the exact capacity of a system. Because of cost and time constraints, an alternative method was considered in determining the approximate capacity of the sample of systems. The alternative method assumes that when a system adds facilities such as parallel lines, storage tanks or booster pumps, it has reached its capacity. The volume of water flowing through the system before the addition is assumed to be the capacity of the system. The added facility has now increased system capacity which is measured again at the time of a further addition.

The initial construction cost, including various minor capital additions from year two to the year the system reached its maximum capacity, are deflated by the construction cost index to year one. The deflated cost is interpreted as the investment cost associated with capacity as measured for each water system. However, since the sample includes water systems starting operation in different years, all investment costs are deflated again to year 1965 when the year of the oldest system in the sample started operation. Because of lack of records, only 22 systems have complete data for the investment cost analysis. Data for the 22 systems are presented in (Myoung, 1983).

Investment Cost Functions. Estimated regression coefficients, standard errors of the estimate and correlation coefficients for the different capital cost models are the following:

$$\text{CAPCOST} = \underset{(46710)}{103456.4} + \underset{(2220.8)}{7973.8}\, S \qquad R^2=.59 \tag{7}$$

S.E. (46710) (2220.8)

$$\text{CAPCOST} = 189128.6 + 12231.6\, S - 17912.1\, D \qquad R^2=.66 \tag{8}$$

S.E. (60490) (2329.8) (6862.3)

$$\text{CAPCOST} = 24888.6 + 23009.5\, S - 336.9\, S^2 \qquad R^2=.51 \tag{9}$$

S.E. (7246) (7145.6) (153.5)

$$\text{CAPCOST} = 78707.0 + 25379.5\, S - 356.7\, S^2 - 16214\, D \qquad R^2=.71 \tag{10}$$

S.E. (96256) (9382.9) (247.1) (6730.8)

$$\text{ACAPCOST} = 35.63 - 0.0012\, S + 0.0000002\, S^2 - 0.757\, D \qquad R^2=.46 \tag{11}$$

S.E. (10.35) (0.0010) (0.00000003) (0.472)

where

CAPCOST = capital investment cost in 1965 dollars

ACAPCOST = average capital investment cost in 1965 dollars

S = capacity measured as millions of gallons per year

D = density in terms of number of users per mile of water line at time of capacity

The density variable D in equations (8) and (10) indicates that capital investment costs are influenced by the dispersion of users. The sign of the quadratic capacity variable, S^2, in equations (9) and (10) is negative. The average capital cost equation (11) shows the existence of economies of scale up to the capacity of 3,000 mgy. Beyond this capacity average capital costs tend to increase marginally.

Operation and Maintenance Cost

Operation and Maintenance Cost Data. The O and M costs were divided into seven categories and obtained from annual audit reports to FmHA. The information was provided by bookkeepers or managers from the individual water systems. Categories of O and M cost are as follows:

Water purchases - cost of treated water purchased for consumption within the water system.

Salaries - payments on a regularly scheduled basis to employees and managers, including employee taxes.

Utilities - cost of electricity and other utilities to operate the system.

Office expense - cost of items such as telephone, stationary and postage.

Insurance and bonds - all insurance premiums and payment of bonds for employees.

Legal and audit - all legal and auditing fees.

Other - miscellaneous and maintenance was included in this category. In some cases it was difficult to identify maintenance expenditures from investment costs. For example, costs of new meters and water line extensions were often included in the maintenance account. These items were removed and specified in capital additions if the records were sufficiently detailed to enable this adjustment. Miscellaneous items included chemicals, billing and collection fees, travel expenses, rent and equipment repair.

The seven O and M cost categories were added together to derive annual O and M cost which was paired with annual output in millions of gallons of water per year. Deflated time series on O and M cost from individual systems was combined with cross section data from the entire sample of systems to estimate an overall O and M cost function. This procedure involves two assumptions: first, that changes in relative factor prices have not resulted in any substitution between factors in the production process and, second, that changes in the system's output (amount of water supplied) have not had any influence upon factor prices. The first assumption is justified since labor has limited substitution for utilities in the pumping of water. The second assumption seems equally reasonable in open regional economies, even in the case of the very largest water system. Data for the O and M cost analysis are presented in (Myoung, 1983).

Operation and Maintenance Cost Functions. The regression coefficients, standard error of the estimates (S.E.) and the correlation coefficients for the different O and M cost models are the following:

$$OMCOST = 24278.7 + 353.7\,Q \qquad R^2=.45 \qquad (12)$$
$$S.E. \quad (2266) \quad (33.6)$$

$$OMCOST = 15118.7 + 345.2\,Q - 1130.8\,D \qquad R^2=.47 \qquad (13)$$
$$S.E. \quad (4601) \quad (33.3) \quad (496.7)$$

$$OMCOST = 7290.9 + 1086.5\,Q - 2.71\,Q^2 \qquad R^2=.64 \qquad (14)$$
$$S.E. \quad (2698) \quad (89.7) \quad (0.32)$$

$$OMCOST = 8630.1 + 1105.0\,Q - 2.77\,Q^2 - 213.7\,D \qquad R^2=.65 \qquad (15)$$
$$S.E. \quad (3868) \quad (97.7) \quad (0.34) \quad (40.9)$$

$$AOMCOST = 1611.5 - 6.3\,Q + 0.025\,Q^2 - 0.011\,QCAP - 28.3\,D$$
$$S.E. \quad (126.3) \quad (3.2) \quad (0.020) \quad (0.018) \quad (12.4)$$
$$R^2=.30 \qquad (16)$$

where

OMCOST = total operation and maintenance cost in 1965 dollars

AOMCOST = average O and M cost in 1965 dollars

Q = amount of water delivered in million gallons per year

D = density in terms of number of users per mile of water line

QCAP = Q times capacity of water system.

Equation (12) contains only a linear term in output Q. This equation yields constant marginal and average O and M costs. Equation (13) contains a term in D, density, expressed by the number of users per mile of water line. The economic interpretation of a negative coefficient on D is that the cost function is shifting downwards as density of users increases. Coefficients are all statistically significant at the two percent probably level in equations (12) and (13). However, the R^2 is only 0.45 and 0.47, respectively. Equations (14) and (15) include a quadratic term in output, Q^2, and equation (15) has a term for D. The coefficients of Q and D remain statistically significant at the three percent probability level. In both equations, the coefficients of Q^2 are statistically significant at the one percent probability level with the sign negative. This result gives continuously decreasing average variable cost and marginal cost, contrary to theoretical expectations. The R^2 increased to 0.64 and 0.65, respectively.

The quadratic function with a negative term in Q^2 may be interpreted as the first section of a third degree polynomial, the second section not being observable in practice. The reasonableness of this hypothesis can only be tested by examining the size of the larger outputs of each system relative to capacity. For this reason, average O and M cost was regressed against quantity, density and a variable measuring system capacity. These results are given in equation (16). Capacity is entered as an interaction variable with quantity since the capacity variable itself is highly correlated with quantity. The average O and M cost equation has low R^2 but the signs of the parameters are consistent with U-shaped short run O and M costs and slightly decreasing long run O and M costs. For purposes of the programming model described later, O and M costs are considered linear and proportional to quantity of water delivered.

Growth of Rural Water Systems in Oklahoma

Growth in water demand is the direct reason why excess capacity should be considered in planning of a water system. In this sample, all rural water systems have grown in number of customers to some degree. Sloggett (1974) discussed various factors contributing to growth including age of the system, per capita income in the county where the system is located, and distance of the system to the nearest growth center. In this study, only age of the system is considered paramount in describing water system growth in number of customers.

Growth of the individual water systems is computed in an index form with the initial year of the system equalling 100.

Using the overall index as a dependent variable and year (age) as an independent variable, two different models were fitted: (1) a linear model and (2) an exponential model. The results are presented in equations (17) and (18) respectively. Both equations have high R^2 and statistically significant coefficients (significant at one percent probability level):

$$Z_t = 75.6 + 14.7\, t \qquad R^2 = .99 \qquad (17)$$

$$\ell n Z_t = 4.6 + 0.0819\, \ell n t \qquad R^2 = .98 \qquad (18)$$

where

t = year (age)

Z_t = index of number of users in year (age) t

In equation (18) the coefficient of t, 0.0819, can be read directly as an annual growth rate and is equal to about eight percent.

AN INVESTMENT PROGRAMMING MODEL WITH PRICE-SENSITIVE DEMAND

In comparison with other studies, the approach developed here for planning a rural water supply system differs in several aspects. First, the optimum excess capacity for initial and additions to capacity are computed as an upper limit of the system. Economies of scale of water supply facilities are incorporated at a given discount rate to attain the optimal excess capacity design. Second, price-sensitive demands are considered in the model. They are used not only to indicate the social benefits of water demand but also to yield the socially optimal prices, reflecting the cost of investment and operation and maintenance. Third, public investment in existing rural community water services in Oklahoma under conditions of uncertain growth are evaluated by comparing those systems against the optimal prices and excess capacity designs resulting from the programming model.

Formulation of the Model

The objective of the programming model is to maximize the total discounted net benefits from investments in rural community water systems. The approach is to maximize the difference between the discounted sum of the benefits from water consumption and the sum of the discounted costs of the water system made up of investment and operation and maintenance.

Assumptions of the Model

The model presented here is based upon the fundamental assumption that the community water demand is sensitive to changes in price. Furthermore, it is assumed that aggregate demand for water varies over time and can be described by a continuous growth rate. Based on the empirical results of the previous section, it is assumed that the price elasticity of demand is constant throughout the planning period. The price-sensitive demand is then used in determining the consumers' willingness-to-pay and the total benefits of a rural water system.

In addition to the above major assumptions in the model, the following assumptions are adopted to simplify application in planning optimum water system investment:

1. Water demand in year y is a function of price in that year and no other period.

2. Capital investment costs occur as a lump sum at the time of initial construction and for any addition to capacity.

3. The O and M costs occur as a lump sum in each year of operation.

4. The capital investment costs for initial construction and any additions are a linear function of capacity and assumed to reflect economies of scale, i.e., the cost per unit of capacity is either constant or decreasing with increasing capacity.

5. The O and M costs are a linear function of output.

6. The annual social discount rate, r, is assumed to be constant over time.

7. Inflation effects on benefits and costs are not considered.

8. The planning horizon is chosen as 40 years which is the FmHA's loan repayment period for community water systems and is assumed equal to the anticipated life-time of the initial water system investment.

Benefit Function

The benefits associated with a given consumption of water in this analysis are measured by the consumers' willingness-to-pay which is denoted as the area under the demand curve up to a specific quantity demand level, say Q_y, in Figure 1. It is assumed that there is a one-to-one mapping of Q_y on $P_y(Q_y)$, the demand curve, and that when a value of Q_y is computed, the market-clearing price is also specified. For purposes of illustrating the approach, a linear demand is assumed in deriving the area under the curve although in the actual model a nonlinear demand curve is used.

Given the demand function for rural community water in year y the "willingness-to-pay" is denoted as:

$$f_y\,(Q_y) = \int_0^{Q_y} P_y\,(Q_y)\,dQ_y \tag{19}$$

where Q_y is the community water demand in year y and $P_y(Q_y)$ is the inverse demand function. For a given community the "willlingness-to-pay" is discounted to the present and summed over the entire planning period using the annual social discount factor:

$$\alpha_y = \frac{1}{(1+r)^y}$$

where r is the social discount rate. This yields the following benefit function which appears in the objective function of the programming model:

$$TB = \sum_{y=1}^{Y} \alpha_y f_y(Q_y) \tag{20}$$

where Y is the length of the plannning period in years.

Cost Function

Water system costs consist of two major components. The first is the capital cost of the proposed water system. Since it is assumed that capacity reflects economies of scale, the capital cost function is concave. The capital cost function for the water system is denoted as $S(S_\tau)$, where S_τ is the capacity added in τ^{th} time unit (initial capacity is the addition from year zero).

Additions to water systems (excluding the initial capacity) have expected lifetimes that are assumed to be longer than the planning period. Capital costs are thus annualized over the expected lifetime of the addition and then

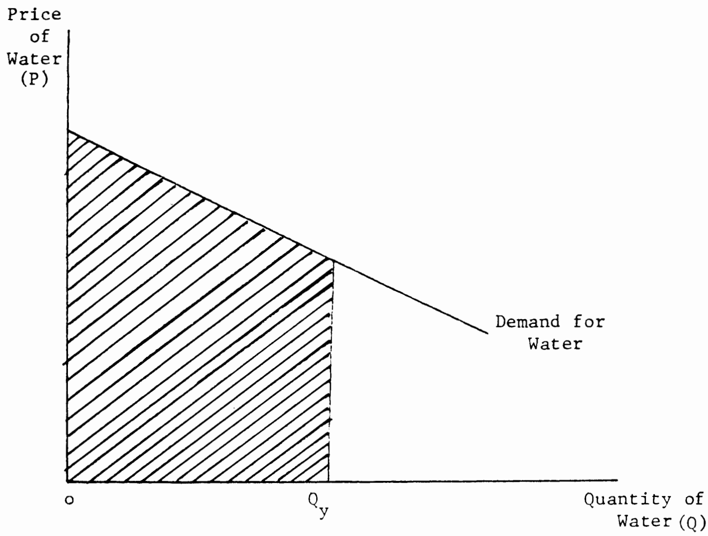

Figure 1. Willingness-to-Pay for Q_y (shaded area)
Quantity of Water

discounted to the present for the period from the time of construction to the end of the planning period. The total present worth of these annualized capital costs are the costs that appear in the objective function. For the discount rate r, capital costs are converted to annual equivalent costs by applying the capital recovery factor β:

$$\beta = \frac{r(1+r)^m}{(1+r)^m - 1} \tag{21}$$

where r is the social discount rate and m is the expected lifetime of the capital investment.

For a given or proposed water system, the total discounted capital costs are:

$$TC = \sum_{\tau=1}^{T} \sum_{y=(\tau-1)\bar{y}+1}^{Y} \alpha_y \beta\, S(S_\tau) \tag{22}$$

where:

T = number of building time units in the planning period (if planning period is 40 and $\bar{y}$ is five years then T is 8)

$\bar{y}$ = number of years in a building time unit (additions to capacity are allowed once every $\bar{y}$ years, if necessary, in order to limit the number of decision variables and constraints in the model)

τ = index of building time unit, $\tau=1,2,\ldots,T$ (begin in year $y=1$, $\bar{y}+1$, $2\bar{y}+1,\ldots,$ $(\tau-1)(\bar{y}+1)$.

The second cost component is for the expected system operation and maintenance (O and M). The O and M costs are defined as the annual costs for operation and maintenance of the system and are assumed to be a linear function of quantity of water delivered, (Q_y). It can be stated as cQ_y where c is the unit O and M costs and Q_y is the quantity of water delivered in year y.

The above O and M costs are discounted to the present and summed over the planning period. The final form of total discounted annual O and M costs is:

$$TO = \sum_{y=1}^{Y} \alpha_y\, cQ_y \tag{23}$$

Total Net Benefit

With equations (20), (22) and (23), the complete objective function for the programming model is expressed as follows:

$$\text{Max. } (TB - TC - TO) \tag{24}$$

which is to maximize equation (20) less equations (22) and (23).

Model Constraints

Having described the benefits and costs in the objective function, the necessary constraints required for a solution to the model are now expressed. The first set of constraints states that the quantity of water delivered in a specific time period cannot exceed total capacity built up to that period. This capacity constraint is stated as follows:

$$Q_y - \sum_{\tau=1}^{G} S_\tau \leq 0 \tag{25}$$

where $G = \lceil y/\bar{y} \rceil$, the ceiling of $y/\bar{y}$ which indicates the number of building time units up to year y.

The second set of constraints is the allocation constraint which requires that the actual water allocated in year y equals the water supplied in year y. This can be expressed as:

$$X_y - Q_y = 0 \tag{26}$$

where X_y is the quantity of water demanded in year y.

To assure that the capacity decision variable, S_τ, can be established at most once during any building time unit, the following constraints are needed:

$$S_\tau - \bar{s}\, Z_\tau \leq 0 \tag{27}$$

and

$$Z_\tau \leq 1 \tag{28}$$

where $\bar{s}$, a given value, is the maximum possible capacity (physical upper bound) of the water system and Z_τ is a zero-one decision variable representing the decision to add capacity in period τ (Z_τ=1) or not to add capacity in τ(Z_τ=0).

Finally, in order for solutions of this model to be meaningful, all above decisions are required to be non-negative.

Computational Considerations

The optimization model formulated above has a nonlinear objective function with several linear constraints. Since the main focus is to develop a solvable mathematical model, approximations are made to render the optimization model compatible with currently available computer techniques. Piecewise or grid linearization and fixed-charge approximation techniques are used to approximate the nonlinear objective function. The concave benefit function is linearized in the following manner. Suppose a linear demand curve is written as follows:

$$P(Q) = a + bQ \tag{29}$$

where price, P, is a function of quantity, Q. Then the area under the demand curve, B, can be expressed as follows:

$$B = \int_{o}^{Q} P(Q)dQ = Q\ (a + 0.5bQ) \tag{30}$$

Now the objective function equation (24) can be rewritten as follows using equation (30):

$$\text{Max}\ (Q(a + 0.5bQ) - S(S_{\tau}) - cQ) = NB \tag{31}$$

where NB is net social benefit. However, notice that equation (31) still contains a nonlinearity. Following Duloy and Norton (1975), this nonlinearity is removed through the use of the grid linearization technique. Grid linearization requires prior specification of a relevant range of values of the demand curve and the use of variable interpolation weights on the grid point. The interpolation weights become variables in the model and their values are jointly constrained by a set of convex combination constraints.

Implementation of the grid linearization technique is illustrated in Figure 2. Suppose that initially the demand curve defined in the price-quantity space passes through the point $(P_2, \overline{Q}_2)$ as illustrated in Figure 2. The relevant range of the demand curve is defined and truncated at points a and b. Then the relevant range of the demand curve is partitioned into segments s = 1,...,v. For each segment end point the parameters $\overline{Q}_s$ and $\overline{B}_s$ are defined to represent the cumulative known area under the aggregate demand curve for water.

The quantity of water used and the total area under the demand curve can be expressed as a weighted combination of $\overline{Q}_s$ and $\overline{B}_s$ respectively.

$$Q = \sum_{s=1}^{v} \overline{Q}_s W_s \tag{32}$$

$$B = \sum_{s=1}^{v} \overline{B}_s W_s \tag{33}$$

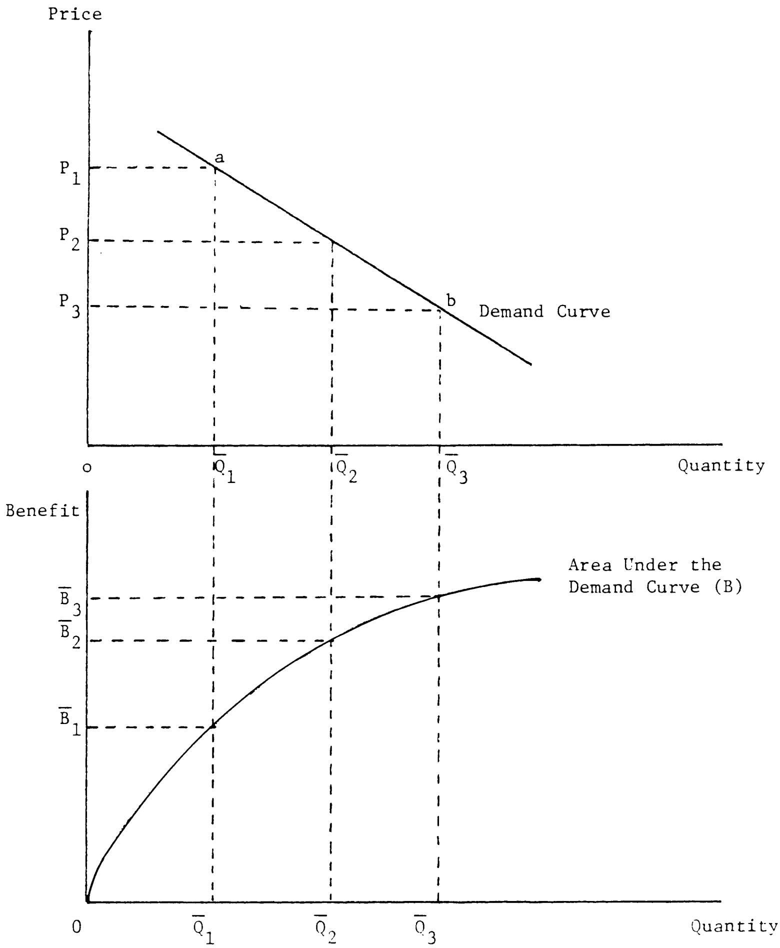

Figure 2. Grid Linearization of Demand and Benefit Functions

where W_s is a weight variable. The non-negative interpolation weight variables are defined such that $\sum_{s=1}^{v} W_s \leq 1$. Notice here that no more than two consecutive points on the quantity axis will enter the optimal basis.

For the capital investment cost function, a fixed charge (set-up cost) approximation approach is used. For example, the capital investment cost $S(S_\tau)$ becomes (see Figure 3):

$$S(S_\tau) = f Z_\tau + K S_\tau \tag{34}$$

where

f = fixed charge of the capital cost function, $S(S_\tau)$
K = slope of the capital cost function
Z_τ = binary decision variable

Solution Strategy

Substituting the linear function approximation and the fixed charge approximation into the original model reduces the model to a large-scale mixed integer linear programming problem. While a few methods exist to solve such problems, perhaps the most promising and widely used method is the branch and bound technique.

The algorithm, which is described by McMillan (1970), begins by relaxing all integer constraints thereby making the problem suitable for solution by linear programming (LP). This solution is called the optimal continuous solution. Except for trivial problems, many of the binary variables will have fractional values making the solution infeasible; i.e., non-integer values between zero and one.

The next step is to set the binary variables to either zero or one, one variable at a time, in such a way that the objective function is maximized. This is accomplished by adding a constraint to the original LP problem. Now the new LP problem restricts one of the non-integer binary variables to zero. A second new LP problem is similarly formed by restricting the same variable to one. Thus a branch is made from one binary variable and two new LP problems are created.

In the solutions of the two LP problems (called terminal nodes), the chosen binary variable will be integer (zero in one case and one in the other). Some, but probably not all, of the remaining binary variables will be non-integer; another must be selected for branching. The usual procedure is to go to the terminal node with the best objective function value and select a second variable on which to branch. The constraint restricting the first variable is retained and two new LP problems are created, one by setting the new variable to zero and the other by setting it to one. Solution of these new problems results in three terminal nodes as shown in Figure 4, one from branching on the first variable plus two from branching on the second. A search is made to find the terminal node with best functional value (in our case, the maximum). If all binary variables in this solution are integer, zero or one, then the problem is solved.

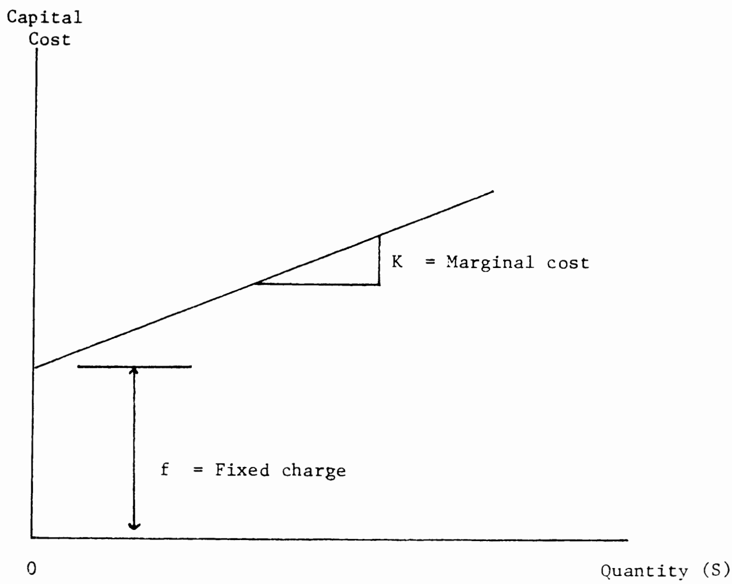

Figure 3. Fixed Charge Capital Cost Function

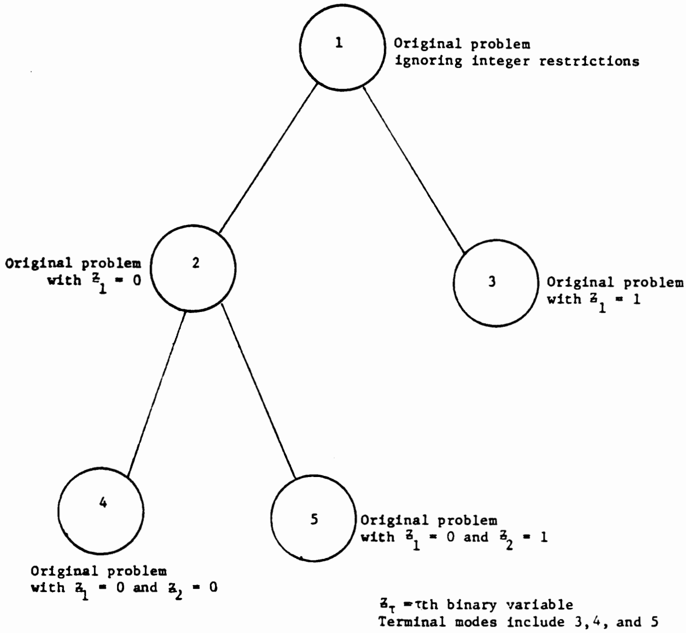

Figure 4. Mixed Programming Solution Tree

The branch and bound methodology just described can be summarized as follows:

1. Treating the binary variables as continuous solves the problem by LP.

2. If all binary variables are not integral, select one to which to branch and form two new LP problems retaining all other constraints, one with the binary variable set equal to zero and the other with it equal to one.

3. Examine the solutions (terminal nodes) and find the one with best objective function value.

4. If all binary variables for this node are integers, the problem is solved, otherwise return to step two.

At each stage of the branching process, the total number of constraints in the LP problem increases by one. It is well known that the addition of a new constraint to a LP problem will either (a) cause the objective function value to remain unchanged, or (b) cause it to deteriorate (i.e., increase for minimization problems and decrease for maximization). Thus the functional value of the optimal continuous solution is a higher bound on the feasible solution of the water system planning model. Additionally, the functional value of a terminal node is a higher bound on all other solutions that might spring from it.

Another important feature of branching and bounding has to do with infeasible solutions. As new LP problems are formulated by restricting additional binary variables, some will be infeasible and thus have no solution. For any terminal node with an infeasible solution, all problems springing from it (due to the restriction of new binary variables) will likewise be infeasible and thus can be ignored.

The Programming Tableau

To reduce the dimensions of the LP model, a five year decision time unit, τ, is used instead of an annual time unit, y. Thus, new discount rates, d_τ, and growth rates, h_τ, are computed which cover five year periods. Also, utilizing the grid linearization described, the basic linear programming model can be stated as follows:

$$\text{MAX NB} = \sum_{\tau,s} d_\tau(\overline{B}_{\tau s} W_{\tau s} - cQ_\tau) - \beta \sum_{\tau} \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y (KS_\tau + fZ_\tau) \tag{35}$$

subject to

water balance equation (WBAL)

$$-Q_{\tau} + \sum_{s} \overline{Q}_{\tau s} W_{\tau s} \leq 0 \qquad (36)$$

system capacity constraint (CAP)

$$Q_{\tau} - \sum_{\tau=1}^{G} S_{\tau} \leq 0 \qquad (37)$$

convex combination constraint (CONV)

$$\sum_{s} W_{\tau s} \leq h_{\tau} \qquad (38)$$

integer constraint (INTEGER)

$$S_{\tau} - \overline{S}Z \leq 0 \qquad (39)$$

The Kuhn-Tucker conditions for determining an optimal solution are contained in Appendix A. The variables and parameters are defined as follows:

<u>Definition of Variables</u>

$W_{\tau s}$: segment weight variable on demand and benefit function in period τ,

Q_{τ} : quantity of water supplied in period τ,

S_{τ} : capacity of water system built in period τ,

Z_{τ} : zero-one binary variable in period τ.

<u>Definition of Parameters</u>

β : capital recovery factor,

$B_{\tau s}$: area under the demand curve for segments of the initial demand function; along this segment, the willingness-to-pay is invariant under a population-induced shift in the demand curve,

d_{τ} : discount factor in period τ which is defined as

$$\left[\left(\sum_{y=(\tau-1)\overline{y}+1}^{\tau \overline{y}} (1+\alpha_{y})\right)^{1/\overline{y}} -1\right]$$

c : unit operation and maintenance costs,

K : slope of the capital cost function,

f : fixed charge of the capital cost function,

$\bar{Q}_{\tau s}$: amount of water consumed at segment s of the initial demand function,

h_τ : population growth index in period τ which can be defined as $(1+h)^{\tau y}$ where h is the annual growth rate,

$\overline{S}$: maximum possible water system capacity in an area.

A portion of the initial LP tableau (covering three periods) is presented in Table 2.

RESULTS OF THE PROGRAMMING MODEL

This section presents the results of the application of the community water pricing and investment planning model. Solutions of the mixed integer programming problem with coefficients derived from data in previous sections are presented and discussed. The effects on community water system investments from varying parameters such as the growth rate and discount rate are investigated.

Since some of the coefficients (for example, price elasticity of demand, discount rate and growth rate) used in the planning model are subject to variability, a comprehensive sensitivity analysis of the most likely combinations of input parameters is presented. The purpose is to show how sensitive water rates and investment decisions are to the discount rate and growth rate for a community's water system.

Base Results

The results presented in this section are the mathematical programming solutions obtained by using as a base the survey data given in previous sections. For the convenience of providing comparisons and sensitivity studies, these solutions will be referred hereafter as the "Base Results".

The base results consist of an optimal capacity expansion schedule for a water system growing in the number of customers at eight percent annually, the operating level of the water system over time in association with the optimal investment schedule, and the water rates at which the consumer's demands are satisfied for varying discount rates. The operating levels actually imply a set of facility policies. The optimal solutions of the base results are for the average size community of the sample survey.

Optimal Capacity Investment Schedule

The average annual growth rate of the sample of rural water districts was eight percent per year. The optimal investment decisions for the average

Table 2. Initial LP Tableau (2 Periods Only)

		RHS	O_1	S_1	Z_1	X_{11}	..	X_{1v}	Q_2	S_2	Z_2	X_{21}	..	X_{2v}
Max	E·Q	z	$-d_1C$	$-\sum_{y=1}^{Y} \alpha^{y-1}\beta K$	$-\sum_{y=1}^{Y} \alpha^{y-1} f$	d_1B_{11}	...	d_1B_{12}	$-d_2C$	$-\sum_{y=y+1}^{Y} \alpha^{y-1}\beta K$	$-\sum_{y=y+1}^{Y} \alpha^{y-1}\beta f$	d_2B_{21}	...	d_2B_{22}
WBAL	L.E.	0	-1			X_{11}	..	X_{1v}						
CAP	L.E.	0	1	-1										
CONV	L.E.	h_1				1	..	1						
INTEGER	L.E.	0		1	$-\bar{S}$									
WBAL	L.E.	0							-1			X_{21}	..	X_{2v}
CAP	L.E.	0		-1					1	-1				
CONV.	L.E.	h_2										1	..	1
INTEGER	L.E.	0								1	$-\bar{S}$			

size community at initiation of water system services with expected eight percent per year growth are shown in Table 3. The solutions indicate that the size of the initial system should be built at capacities of 136.9 mgy,[1] 108.7 mgy, and 93.8 mgy if one percent, three percent, and five percent discount rates are applied, respectively. According to the schedule of solutions these initial capacities are maintained through time unit three (15 actual years in the model) and then new facilities are added at the beginning of time unit four. The size of added capacities beginning with time unit four are 179.5 mgy, 187.2 mgy, and 162.5 mgy, respectively, for the associated discount rates. The solutions also indicate that beginning with time unit six and until the end of the planning period new additions are made for every time unit. This is because the eight percent growth in the later time units bring more capacity requirements than the early time units. In other words, capacity should be added every five years to meet eight percent annual growth for the given discount rates. Total capacities built during the entire planning period are 1320.5 mgy, 1194.7 mgy and 1003.5 mgy, respectively.

Optimal solutions associated with the higher discount rates show that water systems are not built in time unit one even though there is a demand for water. In other words, the construction of water systems should be delayed until time unit two if the discount rate is seven percent and time unit four if the discount rate is nine percent. If the discount rate goes up to 15 percent, no water system is optimum under the model conditions. That is, the expected present worth of the cost (building and operation) of the system is greater than the expected present worth of the benefit it will provide regardless of when it is built (given the discount rate is 15 percent).

The programming results correspond with the theory discussed earlier that one of the factors determining optimal capacity is the social discount rate. Suppose the discount rate is zero. Then, it would be perfectly sensible to spend a dollar now in order to save a dollar's worth of costs either in the next time period or ten years from now; or 100 years, thus, there is no limit to the size of capacity which it pays to build. With a positive discount rate, however, to save a dollar's worth of cost in a future time period we only need to spend less than a dollar now. Therefore, under a given economies of scale if the discount rate is low the size of optimal capacity is relatively large whereas if the discount rate is high the size of optimal capacity is relatively small.

In the base results the optimal size of capacity for the different discount rates shows the same trend as proposed by theory. If the discount rate is low the size of optimal capacity is relatively large and vice versa. In Table 3 the optimal initial size of water system at one percent discount rate is larger than at the three percent discount rate, which is again larger

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[1] mgy is million gallons per year.

Table 3. Optimal Capacity[a] Investment Schedule From the Basic Results at Eight Percent Growth

Discount Rate (percent)	Objective Value ($)	Building Time Unit								Total
		1	2	3	4	5	6	7	8	
1	5,534,429	136.9	-	-	179.5	-	295.2	287.6	421.3	1320.5
3	2,519,708	108.7	-	-	187.2	-	257.4	260.2	381.2	1194.7
5	1,062,444	93.8	-	-	162.5	-	208.5	218.5	320.2	1003.5
7	372,982	-	118.2	-	-	226.8	-	249.5	278.9	873.4
9	85,317	-	-	-	215.3	-	-	292.0	237.7	745.0
15	-	-	-	-	-	-	-	-	-	-

[a]Amount of system capacities in mgy.

than the optimal size at five percent discount rate. The objective function, which is the net social benefit expressed as the expected present worth of total benefits less the expected present worth of total costs during the planning period, values are also given in Table 3. Like the trend of optimal size of capacity for the different discount rates, lower discount rates give relatively higher objective function values from larger size of capacity, lower water price and higher water demand. If the discount rate goes up to 15 percent, there is no investment during the planning period and hence no net social benefits are realized.

Optimal Water Supply Schedule

There are two major factors which directly influence the short run level of water supply: size of capacity and growth in water demand. It is reasonable to say that an increse in number of customers will result in an increase in water supplied as long as excess capacity exists. However, how fast water supply should be increased depends mainly on the price elasticity of demand for water and the system's growth rate. Once water supply reaches the maximum capacity, to increase supply requires the next addition as reviewed in the previous section.

The optimal water supply schedule for the average size community in the sample with an eight percent growth rate during the planning period is presented in Table 4. As in the case of optimal investment, the various discount rates show the sensitivity on optimal water supply. For the case of a one pecent discount rate the optimal water supply increases significantly from time unit one to time unit eight. Optimal water supply increses from one time unit to the next time unit except for time unit three which is the same as that of time unit two. This is because the system reaches its maximum capacity in time unit two and additional capacity is not optimum until time unit four. It is noted that the increase of water supply in the later time units are relatively greater than those of the earlier time units. This is explained by the compounding effect of an eight percent growth rate during the whole planning period. That is, eight percent growth in earlier time units results in relatively smaller net increases in number of customers than is the case for later time units. In fact, it is probably not realistic to assume that the water system grows at a constant rate during the whole planning period, i.e., eight percent. A more realistic assumption would be for water systems with faster growth at the beginning and then slower growth during the remaining part of the planning period. Of course, the specific rate of growth depends upon the environment of individual systems.

The water supply schedule also includes solutions for various discount rates. As observed in the optimal capacity schedule, a system's water supply declines as the discount rate increases. Again there is no water supply in time unit one for the seven percent discount rate; time unit one, two and three for the nine percent discount rate; and the whole planning period for the 15 percent discount rate because no water capacity was built for these time units.

Table 4. Optimal Water Supply[a] Schedule From the Basic Results at Eight Percent Growth

Discount Rate (percent)	Water Supply Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	93.8	136.9	136.9	297.3	316.5	611.6	899.2	1320.6
3	93.8	108.7	108.7	295.9	295.9	553.3	813.5	1194.6
5	93.8	93.8	93.8	256.3	256.3	464.8	683.3	1003.5
7	--	118.2	118.2	118.2	345.1	345.1	594.6	873.1
9	--	--	--	215.3	215.3	215.3	507.3	754.0
15	--	--	--	--	--	--	--	--

[a]Amount of water supplied in mgy.

Optimal Water Rate Schedule

Optimal solutions for capacity and water supply representing different growth and social discount rates are read directly from the output of the programming model. However, the model does not provide the optimal water rate schedule directly. The optimal water rate is computed indirectly by substituting water supply for each time unit into that unit's demand equation representing a particular growth situation. To do this, it is necessary to derive the demand equation for each time unit.

Using the estimated price elasticity of demand for water and the initial average price and quantity of water demanded for the sample of rural water districts, the general demand function in rural Oklahoma was derived. The demand equation is shown at zero time unit in Table 5 and shows that if the water rate increases one dollar per mgy the quantity of water demanded will decrease about 15,000 gallons per year. The assumption is made that consumer response to price change is relatively constant during the planning period even though the water system measured in terms of number of users grows in future time units.

Growth of the water system on the price-quantity plane can be expressed by rotation of the initial demand curve as shown in Figure 5. Let D_o represent the demand curve before growth (i.e. at time unit zero), whereas D_1 represents demand after growth at time unit one. The price-quantity relationship shows that if the price level is P_1, Q_o amount of water is purchased by the given number of customers in a community (say 100 customers) at time unit zero. Assume that the number of customers increases to 200 at the end of time unit one--a 100 percent growth compared to the original number of customers. The amount of water purchased by 200 customers at time unit one would be Q_1 if the price level stays at P_1. Thus, by the assumption of constant consumer response, Q_1 should be exactly twice that of Q_o. Since this price-quantity relationship is true for each and every level of prices, the demand function for time unit one can be derived by using the information from the initial price-quantity relationship and growth in number of customers. Practically, this is derived for time unit one by dividing the slope of D_o by its growth index.

The demand equations for the different time units in Table 5 are derived in this manner--dividing the slope of the initial demand curve, 68.6, by the growth index in column two. For the Base Results, since a constant growth rate of eight percent per year is applied throughout the planning period, the demand curves become flatter and flatter as the system grows.

The optimal water rate schedule is computed by substituting the water supply into each time unit's demand equation. To analyze the optimal rate schedule, not only the relationship between optimal water supply and growth rate should be considered but also the optimal capacity schedule. This is because the water supply schedule is influenced by the optimal investment schedule. For example, in Table 6 the rate schedule for a one percent discount rate fluctuates from one time unit to another time unit depending upon timing of additional capacity. If there is pressure on capacity due to system growth it will result in addition of new capacity which allows an increase in water supply. The increased water supply brings the water

Table 5. Rotated Demand Equations for Each Time Unit at Eight Percent Annual Growth Rate

Time Unit	Growth Index (h)	Demand Equations (Inversed)
0	1.00	P = 5300 - 68.6Q
1	1.47	P = 5300 - 46.8Q
2	2.16	P = 5300 - 31.9Q
3	3.17	P = 5300 - 21.7Q
4	4.66	P = 5300 - 14.8Q
5	6.85	P = 5300 - 10.0Q
6	10.06	P = 5300 - 6.8Q
7	14.79	P = 5300 - 4.7Q
8	21.72	P = 5300 - 3.2Q

P = price per mgy dollars.
Q = quantity of water demanded in mgy.

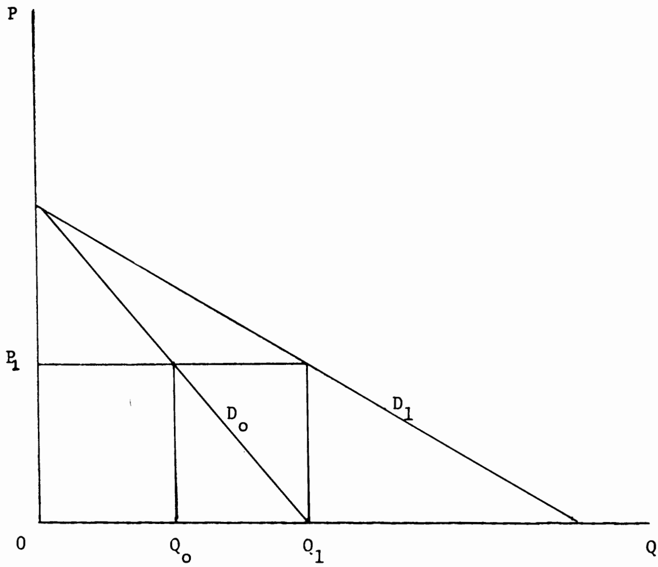

Figure 5. Rotation of Demand Curve by Growth

Table 6. Optimal Water Rate[a] Schedule From Base Results at Eight Percent Growth

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	910.2	932.9	2329.3	900.0	2135.0	1141.1	1073.8	1074.1
3	910.2	1832.5	2941.2	920.7	2341.0	1537.6	1476.6	1477.3
5	910.2	2307.8	3264.5	1506.8	2737.0	2139.4	2088.5	2088.8
7	--	1529.4	2735.1	3550.6	1489.0	2953.3	2505.4	2506.1
9	--	--	--	2113.6	3147.0	3836.0	2915.7	2916.0
15	--	--	--	--	--	--	--	--

[a]Dollars per million gallons.

rate down but not necessarily as low as if the system stayed on the same demand curve. The reason is that the slope of the new demand curve from which the optimal water rate is computed is now flatter than the previous demand curve.

In Table 3 for a one percent discount rate the initial capacity is 136.9 mgy but the actual water supply is 93.8 mgy at time unit one in Table 4. That is, 43.1 mgy excess capacity is reserved for future growth. Substituting 93.8 mgy amount of water supplied in the first time unit demand curve results in a water price of $910.20 per million gallons. In the second time unit, all of the existing capacity is utilized due to the system's growth. Therefore again substituting the optimal water suply, 136.9 mgy into the second time unit's demand equation results in $932.90 per million gallons as the water rate which is only slightly higher than that of the first time unit. In the third time unit, there is another eight percent growth in the system but additional capacity has not come into the solution set. Therefore, the amount of water supplied is restricted to the maximum capacity by raising the water rate. That is why the water supplied during the third time unit is the same as that of the second time unit but the water rate is significantly higher than that of the second time unit. Water rate is used as a means to allocate a given amount of water to more customers. In the fourth time unit there is another eight percent growth per year. Now the water system no longer relies strictly on the role of price to maintain existing capacity. Therefore a new capacity addition comes into the solution (see Table 3). With new additional capacity water supply increases and consequently the optimal water rate decreases. These interrelationships among growth rate, optimal capacity schedule, optimal water supply schedule, and optimal water rate continue until the end of the planning period for each discount rate. The above solutions are based on an eight percent growth per year. Solutions for different growth patterns are analyzed in succeeding sections.

Results and Analysis for Alternative Growth Rates

Rural community water systems have shown substantial variability in growth. In this section different environments (i.e. growth rates) are assumed to analyze the effect of growth in determining optimal solutions in terms of capacity, water supply and water rates. An important focus of this study is to determine net social benefits if decision makers would have known the system's growth at the time of initial planning.

Zero Growth Situation

Optimal Solutions. As reviewed before, economies of scale, discount rate and system growth are the main factors that determine optimum excess capacity. However, if the number of customers remains constant throughout the time period, decision makers do not need to worry about building any excess capacity or additions to capacity as long as consumer behavior is stable. Therefore the optimal capacity would be the same as the level of optimal water supply.

The solution of the model when the growth rate is zero shows this situation. The optimal capacity and the optimal water supplies are the same throughout the entire planning period as seen in Tables 7 and 8. When the discount rate is one percent, the optimal capacity is 60.8 mgy which remains constant as long as there is no growth. Like the case with growth, the optimal capacity investment decreases as the discount rate increases and there is no optimal investment if the discount rate goes beyond seven percent. Because of no excess capacity the optimal water supply is the same as the optimal capacity level (Table 8). Also, the optimal water rate for a given discount rate is the same throughout the planning period as shown in Table 9.

Equity Considerations With and Without Growth. Although the scope of this study is limited to economic efficiency it is still worthwhile to review equity aspects in terms of individual customer payments for water with and without growth.

As reviewed before, the optimal solutions of capacity, water supply, and water rate depend on system growth under given economies of scale and discount rate. Under conditions of no growth there is no excess capacity in the optimal solution and water rate is the same throughout the planning period. This means that the initial members of the system who are the only members of the system throughout the planning period pay a constant water rate during the entire planning period. For example, water rate is fixed at $2121 per million gallons during all time units when the discount rate is five percent. To review the situation of the initial members of a water system this rate can be compared to other optimal rates under conditions of growth.

As an example of comparing equity positions of initial members of water systems, Tables 10 and 11 are compared. In Table 10, with eight percent system growth, payments per user for each time unit at five percent discount rate are computed. To project the growth of users a base of 198, which is the average initial number of users of the sample system, is applied. Using optimal solutions of water supply and rate schedules, the discounted payments per user are computed and added. The total value of $1501 in Table 10 is the total amount paid by a user during the whole planning period. In Table 11 a similar procedure was applied but under conditions of constant water supply and rate schedule. The total amount paid by a user during the whole planning period and discounted to the present is compared under conditions of with and without growth. Based upon this comparison, an individual user under the growth situation is better off than under the without growth situation.

Two, Four, Six and Ten Percent Growth Rate

So far optimal solutions of the base result and zero growth situation have been reviewed. In this section optimal solutions under different rates of growth are analyzed. If decision makers correctly predict growth and plan system capacity and management accordingly, the optimal solutions would give maximum social benefits. Tabular results are presented in (Kwang, 1983) for the alternative growth rates. The following sections contain brief descriptions and analyses of the results.

Table 7. Optimal Capacity Investment Schedule From the Base Results at Zero Percent Growth[a]

Discount Rate (percent)	Objective Value ($)	Building Time Unit								
		1	2	3	4	5	6	7	8	Total
1	666,082	60.8	--	--	--	--	--	--	--	60.8
3	326,105	55.0	--	--	--	--	--	--	--	55.0
5	123,671	46.2	--	--	--	--	--	--	--	46.2
7	741	40.2	--	--	--	--	--	--	--	40.2
9	--	--	--	--	--	--	--	--	--	--
15	--	--	--	--	--	--	--	--	--	--

[a]Amount of system capacities in mgy.

Table 8. Optimal Water Supply Schedule[a] From Base Result at Zero Growth

Discount Rate (percent)	Operation Level for Each Time Unit							
	1	2	3	4	5	6	7	8
1	60.8	60.8	60.8	60.8	60.8	60.8	60.8	60.8
3	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
5	46.2	46.2	46.2	46.2	46.2	46.2	46.2	46.2
7	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2
9	--	--	--	--	--	--	--	--
15	--	--	--	--	--	--	--	--

[a] Amount of water supplied in mgy.

Table 9. Optimal Water Rate[a] Schedule From Base Result at Zero Percent Growth

Discount Rate (percent)	Optimal Water Rate for Each Time Unit							
	1	2	3	4	5	6	7	8
1	1117.0	1117.0	1117.0	1117.0	1117.0	1117.0	1117.0	1117.0
3	1516.0	1516.0	1516.0	1516.0	1516.0	1516.0	1516.0	1516.0
5	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0
7	2534.0	2534.0	2534.0	2534.0	2534.0	2534.0	2534.0	2534.0
9	--	--	--	--	--	--	--	--
15	--	--	--	--	--	--	--	--

[a]Dollarsper million gallons.

Table 10. Water Consumption Payments Per User for Each Time Unit at Eight Percent Growth and Five Percent Discount Rate

Time Unit	1	2	3	4	5	6	7	8	Total
d_c	.86494	.67780	.53115	.41622	.32616	.25558	.20027	.15693	
Water Supply (mg)	93.8	93.8	93.8	256.3	256.3	464.8	603.3	1003.5	
Water Rate ($/mg)	910.2	2307.8	3264.5	1506.8	2737.0	2139.4	2088.5	2088.5	
No. of Users	291.0	428.0	628.0	923.0	1356.0	1992.0	2928.0	4301.0	
Payment Per User (dollars discounted to present)	254.0	343.0	259.0	174.0	169.0	128.0	98.0	76.0	1501.0

Table 11. Water Consumption Payments Per User for Each Time Unit at Zero Percent Growth and Five Percent Discount Rate

Time Unit (T)	1	2	3	4	5	6	7	8	Total
d_c	.86494	.67780	.53115	.41623	.32613	.25558	.20027	.15693	
Water Supply (mg)	46.2	46.2	46.2	46.2	46.2	46.2	46.2	46.3	
Water Rate ($/mg)	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	2121.0	
No. of Users	198.0	198.0	198.0	198.0	198.0	198.0	198.0	198.0	
Payment Per User (dollars discounted to present)	428.0	335.0	263.0	206.0	161.0	126.0	99.0	78.0	1696.0

Optimal Capacity Investment Schedule. The optimal size of the initial investment increases gradually as the growth rate increases. For example, the optimal size investment with two percent growth is 66.9 mgy while it is 77.8 mgy with four percent growth rate. When the growth rate is ten percent, which is the largest growth rate studied, not only the initial investment of 93.2 mgy is larger than that of smaller growth rates, but, also, capacity additions are more frequent after time unit four. From this finding it is concluded that growth of a water system is one of the critical factors which should be considered in determining optimal investment size even though this factor is frequently ignored in much of the existing literature.

Optimal Water Supply Schedule. The five percent discount rate is used to make comparisons of solutions. Under conditions of two percent growth, the optimal water supply remains the same over the entire planning period. The initial capacity, 66.9 mgy, is fully utilized at the beginning time period and remains fully utilized with no additional capacities. Under four percent growth, capacity is increased in the sixth time unit and again is fully utilized. For the assumption of six and ten percent growth, the model results show no period with excess capacity for the five percent discount rate. It must be assumed that price is being used to allocate water under the limited capacities or until additional capacity is created.

Comparing the results for the five percent discount rate with the one percent discount rate it is noted that, under the latter condition, water systems do have excess capacities for some time units. That is, the water supplied is less than the capacity for that time unit.

Optimal Water Rate Schedule. The overwhelming result shows that price is heavily used as the allocator of water. As an example, with a two percent growth rate and a five percent discount rate, the price of water must continuously increase from time unit one to time unit eight since capacity is established in time unit one and there are no additions to capacity for the remainder of the planning period. Furthermore, water supply is at maximum capacity for each time unit. Therefore, to limit consumption of water equal to capacity requires that the price of water must increase. Further evidence of price being used as the allocator of water is seen with the four percent growth assumption. The price of water increases from $912 per million gallons in time unit one to $3,293 per million gallons in time unit five. Since capacity is added in time unit six water price is reduced to $1,505 per million gallons. Price again increases in time units seven and eight since water supplied is equal to capacity in each of these time units but growth in number of customers has occured at the four percent rate.

Another result apparent from the data is the effect of economies of scale on price. Results with the five percent discount rate show that price of water in time unit one reduces from $1,119 per million gallons for two percent growth to $912 per million gallons for four percent growth. The reason for this, in part, is due to the larger capacity installed under the four percent growth relative to the capacity installed under the two percent growth. Similarly, water price can be compared across growth rates for time unit two and at the seven percent discount rate. At two percent growth, the price is $1,922 per million gallons, at four percent growth the price is $1,322 per million gallons. The decrease in price is due, in part, to economies of scale since larger capacities were installed at each higher growth rate. Price again increases at the ten percent growth to $1,510 per million gallons but this is due in part to using price to restrict consumption at limited capacity.

Declining Growth Situation

So far the analysis has been restricted to a constant growth rate during the entire planning period. However, it is unrealistic to expect a water system to continue growing at the same rate. Rather, it is more realistic to assume that water systems grow faster during earlier time units of the planning period and then the rate of growth declines as the rural area builds up. To review optimal solutions under these assumptions of growth, three different growth patterns are studied. The first pattern is an eight percent growth rate during the first half of the planning period and then growth stops for the remainder of the planning period. The second pattern is an eight percent growth rate during the first half and then growth continues at two percent per year during the second half. The last pattern consists of an eight percent growth rate during the first half and then continues to grow at four percent per year during the second half. The tabular data again are presented in (Myoung, 1983).

Eight and Zero Percent Growth. The optimal capacity investment schedule shows no additional facility coming into the solution after the end of the fourth time unit for all discount rates. This is explained by the assumption of zero growth for the last half of the planning period. However, the solutions of initial investment for the different discount rates are the same as the solutions from the base result with eight percent growth. The optimal water supply schedule shows no change of supply level after the fourth time unit due to zero growth. Like the water supply schedule from the base result with eight percent growth, no water supply is made in the early time units if the discount rate is seven or nine percent. No water supply is realized at all if the discount rate becomes 15 percent. The optimal water rates are constant after the fourth time unit due to zero growth. The effect of price again is as an allocator of water under limited capacities. Price increases significantly in time unit three for discount rates one, three and five percent and then decreases with additions to capacity in time unit four.

Eight and Two Percent Growth. This pattern considers eight percent growth per year until the fourth time unit and then growth drops to two percent. The optimal investment schedule shows initial capacity the same as the previous case but with larger additions at time unit four due to the higher growth rate (from zero to two percent per year). An interesting difference in the solution for optimal capacity between this pattern and the previous case is the timing of initial investment when the discount rate is seven percent. When the growth pattern was eight and zero percent, the initial investment comes into the solution at the third time unit and no additional investments until the end of the planning period. When the pattern is eight and two percent a smaller investment comes into the solution in the second time unit and then two additional investments come into the solution at time units five and eight. The optimal solutions for water supply show the same levels for the first three time units for discount rates of one, three and five percent. However, water supply increases for the fourth time unit under the conditions of a slight continued growth for the latter half of the period. The effect of continued growth is to increase the optimum capacity and hence water supply for this fourth period. This also has the effect of decreasing water price for the fourth period under conditions of two percent growth in the latter half versus no growth in the latter half of the period. Water price fluctuates during the latter half of the period under conditions of two percent growth depending on when optimal capacities are added.

Eight and Four Percent Growth. The last growth pattern is the system that grows at eight percent per year during the first half of the planning period and then drops to four percent per year. In general, the solutions show the same trends as those of previous patterns except now optimal capacity investments come into the solution more often due to the higher growth rate.

Comparison of Net Social Benefits Between Actual and Optimum - The Case of Murray #1

To demonstrate the advantages of the optimal investment programming model for planning rural water systems, a comparison of results is made with an actual system, Murray #1. Using the general demand equation for water and the actual water investment and supply records of Murray #1, net social benefits are computed. Then net social benefits are computed using the optimal investment programming model and the actual rate of growth of Murray #1. Finally, the two net social benefits are compared.

Murray #1 water system started suplying water in 1967. The annual water demand, number of customers and investment record of Murray #1 are presented in Table 12. The amount of water demanded and the number of customers show dramatic increase since the system started operation. The initial number of users, 229 in 1967, increased to 934 in 1980 and results in a 12 percent annual growth rate. In addition to the initial investment, there were two expansions of capacities to meet growth of the system, 1973 and 1978.

It is assumed that the customers in Murray #1 have the same consumption behavior as explained by the general water demand equation. To reflect system growth, the general demand equation is rotated as explained previously. Specifically, the slope of the original demand equation is divided by the index of growth.

Using the rotated demand curves and the actual water demand, consumer benefits are computed. Table 13 shows the revised demand equation and the gross benefits for each year.[2] The gross benefits are discounted at five percent to compute the present worth of water consumption benefits. Also, Table 13 includes the present worth of actual O and M costs to run the water

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[2]Gross benefits are defined as total area under the demand curve before costs have been subtracted.

Table 12. Annual Water Demand, Number of Customers and Investment Record in Murray #1 Water System

Year	Water Demand (mg)	No. of Customers	Index of Growth	Investment Record ($)
1967	18.2	229	100	314,745
1968	16.8	230	100	--
1969	17.8	243	106	--
1970	17.4	252	110	--
1971	17.3	268	117	--
1972	17.4	389	170	--
1973	24.0	475	207	66,000
1974	36.0	525	229	--
1975	40.7	566	247	--
1976	39.2	599	262	--
1977	38.8	654	286	--
1978	57.1	762	333	225,000
1979	63.4	859	375	--
1980	86.9	934	408	--

Table 13. Actual Benefits and Costs in Supplying Water in Murray #1 Water System

Year	Revised Demand Equations	Water Supply (mg)	Gross Benefits ($)	Discounted Gross Benefits at 5% ($)	Discounted O&M Costs at 5% ($)	Discounted Capital Investment at 5% ($)
1967	P=4840.2-189.4X	18.2	25,355	24,148	6,311	198,799
1968	P=4840.2-189.4X	16.8	27,859	25,269	5,390	--
1969	P=4840.2-178.7X	17.8	29,536	25,514	5,439	--
1970	P=4840.2-172.2X	17.4	32,084	26,396	5,063	--
1971	P=4840.2-161.9X	17.3	35,280	27,704	4,794	--
1972	P=4840.2-111.4X	17.4	50,492	37,678	4,592	--
1973	P=4840.2-91.5X	24.0	63,461	45,101	6,033	15,741
1974	P=4840.2-82.7X	36.0	67,068	45,394	8,618	--
1975	P=4840.2-76.7X	40.7	69,943	45,086	9,280	--
1976	P=4840.2-72.3X	39.2	78,637	48,276	8,512	--
1977	P=4840.2-66.2X	38.8	88,140	51,534	8,024	--
1978	P=4840.2-56.9X	57.1	90,858	0,593	11,246	13,512
1979	P=4840.2-50.5X	63.4	103,881	55,090	11,892	--
1980	P=4840.2-46.4X	86.9	70,219	35,465	15,524	--
TOTAL				543,248	110,718	228,052

system each year and the present worth of the capital investment costs. From the information in Table 13 the net social benefits realized by the water system are computed as the total present worth of gross benefits less the total present worth of O and M and capital costs. The net social benefits equal $204,478 as computed for the actual Murray #1.

The optimum solution derived by the investment planning model is presented in Table 14. For the model solutions, the actual 12 percent growth rate is combined with the general demand equation and general O and M and capital cost functions. The optimum solution shows that 72.8 mgy capacity should have been built in the initial time unit and 55.2 mgy should have been added in the third time unit. The optimal supply schedule shows a significantly larger volume of water being supplied than for the actual system. The objective value generated by the optimal solution is $310,176 which is about 52 percent higher than that for the actual water system.

Several conclusions can be drawn from the results of these comparisons:

1. Decision makers underestimated growth of the water system and built too small an initial facility.

2. Because of an incorrect investment decision, the Murray #1 community lost considerable benefits which could have been gained if optimal decisions had been made.

3. Uncertainty relative to system growth may have been a major factor contributing to under-investments by the Murray #1 decision makers. The optimal programming model is a way to improve economic efficiency in decision making of water system investment but does not reduce the problem of uncertainty relative to system growth.

CONCLUSIONS

Summary and Policy Implications

The purpose of this research was to provide information for the planning and management of rural water systems in Oklahoma. Two major problems exist in planning of rural water services: (1) determining the optimum capacity rural water districts should build into their facilities; and (2) incorporating the effects of price-sensitive demand.

Current methods for planning rural water facilities have too frequently relied on rules of thumb such as multiplying a current rate of per capita water consumption by a projected level of population over some specified period of time. Two important economic problems are associated with this procedure. First, it assumes the demand for water is perfectly price inelastic; however, economic theory and recent empirical studies would indicate that the demand for water is price responsive. Second, this leaves no room for adjusting to different rates of population growth or different discount (interest) rates when determining the optimum timing and size of additions to capacity. Yet in determining how large to build initial or

Table 14. Optimal Investment, Operation Level and Net Social Benefit From the Programming Model

Building Time Unit	Capacity (mgy)	Operation Level (mgy)	Net Social Benefit ($)
1	72.8	41.2	--
2	--	72.8	--
3[a]	55.2	128.0	--
Total	128.0		310,176[b]

[a]Adjusted to reflect four year time unit.

[b]Program does not permit allocation of net social benefits by time unit.

increments to capacity studies have emphasized two basic factors which are nearly always in conflict: (1) it pays to build large increments to capacity because there are usually cost savings (economies of scale) involved in capacity size; and (2) the commitment of resources to a capacity that will not be used for a period of time is costly since future costs are more heavily discounted than present costs.

The primary objective of this research was to demonstrate an improved planning model by incorporating optimum timing and size of investments and price responsiveness of water demand. Specific objectives included (1) estimating functions of water demand and water cost for rural districts in Oklahoma and (2) developing and applying a mathematical programming model which maximizes social benefits from investments in rural water facilities.

Results of this research show that consumers of rural water services in Oklahoma are price sensitive--the estimated price elasticity of demand is -0.58. Thus the price of water will effect the demand for water. For economic efficiency water rates should be set equal to the marginal cost of providing additional water. Thus, the objective of determining the price of water which maximizes social benefits must take into consideration the demand for water and the cost of supplying water.

Results of the analysis of water supply costs show that there are significant economies of scale in rural water system investment and operation and maintenance. The growth analysis, which showed an overall eight percent annual growth rate measured in terms of number of customers, strongly supports the excess capacity model as a framework for planning optimum water system capacity. Failure to optimize on excess capacity may lead to under- or over-investment in community water systems and thus reduce social benefits due to inefficient allocation of resources. Under-investment for any particular community may force duplication of facilities (parallel lines) which could have been avoided if optimal capacity were planned from the beginning. Therefore, the objective of determining the optimum capacity of rural water systems which maximizes social benefits must incorporate expected growth in water demand as well as the economics of water supply.

Results of the mathematical programming model suggest the following policy decision criteria for planning rural water systems:

1. Price-sensitive consumer behavior should be considered in decisions of rural water services capacity design and water pricing.

2. The existence of economies of scale in water supply are important in determining optimum timing and size of water facility investment.

3. Predictions of growth are highly important in planning optimal water system capacity.

4. All of the above criteria should be considered simultaneously along with the discount rate in making global optimal water supply decisions for specific water districts.

Limitations and Need for Further Research

Like most, this study suffers from a number of limitations, some of which could not be avoided. Primary among these was the simplification in estimating aggregate water demand. The estimated aggregate demand functions did not consider an income effect even though income may be an important factor in explaining water consumption behavior, particularly for nonhousehold use during the summer season. An adequate measure of income for the aggregate analysis was not available.

A second shortcoming is loss of a major part of the marginal cost pricing goal in estimation of the aggregate demand function. Demand was estimated as a function of average billing price and aggregate consumption of water for the district. The general rate structure is one of declining block rates. Therefore individual consumers would theoretically equate marginal block rate price with quantity consumed. The typical block rate price for each water district was used as a surrogate of marginal price in estimation of aggregate water demand. Little difference was noted in estimated parameters when compared to the average billing price results. Evidence is scarce whether consumers adjust quantity to average billing cost or marginal cost. In any event, bias could enter in the results presented here on marginal cost pricing.

A third limitation is the linear O and M and investment cost functions adopted for water supply cost in the programming decision model. These linear cost functions may overestimate costs for small systems and underestimate costs for large systems which generally appear during the latter part of the planning period.

Finally, the purpose of this study was to provide information for the planning and management of rural water systems to achieve economic efficiency. The criteria used for this objective was marginal cost pricing. However, because of economies of scale some small water systems may operate at a level where long run marginal cost is lower than long run average cost. Under this circumstance, marginal cost pricing will not cover total water supply cost. Several alternatives are available which may allow marginal cost pricing but at the same time avoid losses due to differences between total water supply cost and total revenue collected from the marginal cost price. These kinds of pricing policies were not covered in this study and remain as further research.

In this study water supply costs cover only distribution of purchased treated water. Further, cost analysis was limited to those systems already in existence. More detailed and current costs are necessary for application of the model to actual planning conditions. Therefore, further study remains to improve the model by using engineering cost data and including other costs involved in a general water supply system.

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APPENDIX A

The Kuhn-Tucker Conditions

The Kuhn-Tucker (1950) conditions provide the necessary and sufficient conditions for determining an optimal solution. From the basic LP model the Lagrangian equation is written as follows:

$$L(W,Q,S,Z) = \sum_{\tau,s} d_\tau (\overline{B}_{\tau s} W_{\tau s} - cQ_\tau) - \beta \sum_{y=(\tau-1)y+1}^{Y} \alpha_y (KS_\tau + fZ_\tau)$$

$$- \pi_\tau (-Q_\tau + \Sigma \overline{Q}_{\tau s} W_{\tau s})$$

$$- \lambda_\tau (Q_\tau - \sum_{\tau=1}^{G} S_\tau) \qquad (10)$$

$$- \sigma_\tau (\sum_s W_{\tau s} - h_\tau)$$

$$- \mu_\tau (S_\tau - \overline{S}Z_\tau)$$

(1)

The Kuhn-Tucker conditions are met with the following results and provide an economic interpretation of each variable at the optimum.

$$\frac{\partial L}{\partial Q} = d_\tau c + \pi_\tau - \lambda_\tau \leq 0 \qquad \text{and } \frac{\partial L}{\partial Q_\tau} Q_\tau = 0 \qquad (2)$$

$$\frac{\partial L}{\partial S} = -\beta K \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y + \lambda_\tau - \mu_\tau \leq 0 \qquad \text{and } \frac{\partial L}{\partial S} S_\tau = 0 \qquad (3)$$

$$\frac{\partial L}{\partial W_s} = d_\tau \overline{B}_{\tau s} - \pi_\tau \overline{Q}_{\tau s} - \sigma_\tau \leq 0 \qquad \text{and } \frac{\partial L}{\partial W_{\tau s}} W_\tau = 0 \qquad (4)$$

$$\frac{\partial L}{\partial Z} = -\beta f \sum_{y=(\tau-1)\overline{y}+1}^{T} \alpha_y + \overline{S}\mu_\tau \leq 0 \qquad \text{and } \frac{\partial L}{\partial Z_\tau} Z_\tau = 0 \qquad (5)$$

$$\frac{\partial L}{\partial \pi_\tau} = -\,(Q_\tau + \sum_s \overline{Q}_{\tau s}\, W_{\tau s}) \quad \geq 0, \qquad \text{if} >,\ \pi_\tau = 0 \tag{6}$$

$$\frac{\partial L}{\partial \lambda_\tau} = -\,(Q_\tau - \sum_{y=1}^{\tau} S_u) \quad \geq 0, \qquad \text{if} >,\ \lambda_\tau = 0 \tag{7}$$

$$\frac{\partial L}{\partial \sigma_\tau} = -\,(\sum_s W_{\tau s} - h_\tau) \quad \geq 0, \qquad \text{if} >,\ \sigma_\tau = 0 \tag{8}$$

$$\frac{\partial L}{\partial \mu_\tau} = -\,(S_\tau - \overline{SZ}_\tau) \quad \geq 0, \qquad \text{if} >,\ \mu_\tau = 0 \tag{9}$$

The saddle point property of the function is:

$$\sum_{\tau,s} \left[d_\tau \quad (\overline{B}_{\tau s}\, W_{\tau s} - cQ_\tau) - \beta \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_y \; (KS_\tau + fZ_\tau) \right] = \sum_\tau h_\tau\, \sigma_\tau \tag{10}$$

Rewriting equation (2) gives the following:

$$\pi_\tau = \lambda_\tau - d_\tau c \tag{11}$$

where

λ_τ = shadow price of incremental capacity (i.e., marginal cost of incremental capacity),

$d_\tau c$ = discounted O and M unit cost.

Therefore, the shadow price of water, π_τ, can be interpreted as the marginal cost of supplying water which is the summation of marginal capital cost and marginal O and M cost.

Without loss of generality, assume that, of v variables $\overline{B}_{\tau s}$, only one variable is non-zero at value h_τ and others are zero. Also, at most two segment end points, $B_\tau s'$ and $B_\tau s''$, are equal to h_τ. Therefore, equation (4) becomes:

$$d_\tau B_{\tau s'}\, h_\tau - \pi_\tau \overline{Q}_{\tau s'}\ h_\tau - \sigma_\tau h_\tau = 0 \tag{12}$$

Aggregating over the planning period, equation (12) becomes:

$$\sum_\tau^T h_\tau \sigma_\tau = \sum_\tau^T \left[d_\tau \overline{B}_{\tau s'} h_\tau \;-\; \pi_\tau Q_{\tau s'} h_\tau \right] \tag{13}$$

Therefore

$$\sigma_{\tau} = d_{\tau}\overline{B}_{\tau s'} - \pi_{\tau}\overline{Q}_{\tau s'}$$

where $d_{\tau}B_{\tau s'}$ is the discounted area under a specific segment of s' of the demand curve, and $\pi_{\tau} Q_{\tau s'}$ is the total revenue from water sale. Therefore σ_{τ} can be interpreted as total consumer surplus in time τ which is the difference between the discounted area under a specific segment s' of demand curve and the total revenue from water sale.

The relationship between two shadow prices μ_{τ} and λ_{τ} can be derived by equations (3) and (5). Equation (5) can be rewritten as

$$\mu_{\tau} = \frac{\beta f \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_{y}}{\overline{S}} \qquad (14)$$

where the right hand side term is the fixed charge of investment cost embedded in the planning period per unit of maximum scale capacity. Also from equation (3),

$$\lambda_{\tau} = (\beta K) \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_{y} + \mu_{\tau} \qquad (15)$$

Substituting μ_{τ} of equation (14) into equation (15) gives:

$$\underbrace{\lambda_{\tau}}_{\text{returns from capacity built in } \tau} = \underbrace{(\beta K) \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_{y}}_{\text{discounted embedded variable cost of constructing the capacity in } \tau} + \underbrace{\left(\frac{\beta f}{\overline{S}}\right) \sum_{y=(\tau-1)\overline{y}+1}^{Y} \alpha_{y}}_{\text{discounted embedded fixed charge per unit of maximum scale of capacity}} \qquad (16)$$

In equation (16) λ_{τ} can be interpreted as returns from the capacity built in period τ. The two terms on the right hand side are the discounted variable cost of constructing capacity in τ and discounted fixed charge per unit of maximum scale capacity. The two sides should be equal at the optimal which will result in efficient allocation of resources. If we allow infinite scale of maximum capacity, i.e., $\overline{S} = \infty$ the returns will be the same as the discounted variable cost of building that capacity at optimum.

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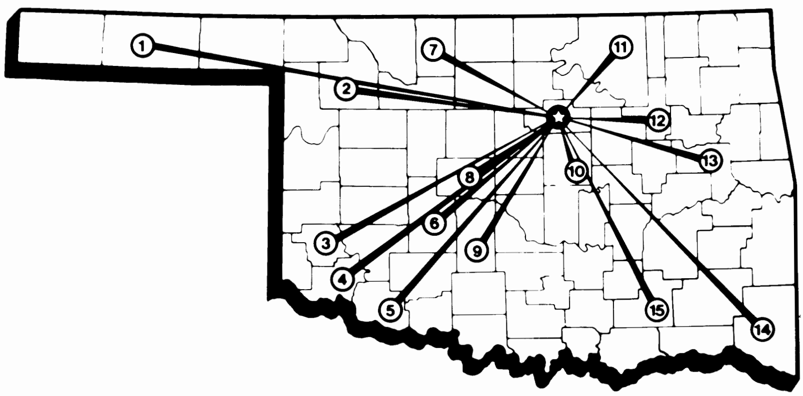

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2. Southern Great Plains Field Station — *Woodward*
3. Sandyland Research Station — *Mangum*
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