THE RELATIVE REFLECTANCE OF A MONOLAYER-COVERED WATER SURFACE

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CHAPTER I

INTRODUCTION

Today's world is characterized as existing in a time of increasing population, increasing industrialization, and particularly a time of increasing water utilization. It has been estimated that between the years of 1960 and 1980 the United States will increase its average water utilization from 250 billion gallons per day to 600 billion gallons per day. In order to accomplish this, it is expected that the national water reservoir capacity will have to be raised from 275 million acre feet to around 600 million acre feet (1).[1]

The problem of evaporation from our reservoirs is very important. As an example of this, the following case is presented. During the very dry year of 1956, Lake Carl Blackwell, which supplies water for Stillwater, Oklahoma, and Oklahoma State University, lost over four times as much water by evaporation as was used for domestic purposes (2).

At least three methods have been proposed for the reduction of evaporation losses. The least theoretical method is that of placing an inflated plastic covering on the surface of a pool or lake. This method has been considered mainly in the area of swimming pool heating, where such covers reduce the evaporation and make much of the additional energy available for raising the water temperature (3).

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[1] Numbers in parentheses indicate references in the bibliography.

A second method of evaporation suppression involves reducing the surface wind over a pond by placing a network of floating wind barriers on the pond surface. This idea was considered by Professor Frank Crow at Oklahoma State University as part of his research, which was sponsored by the Bureau of Reclamation under contract No. 14-06-D-4275.

The third method and the one which has received major attention during the last fifteen years makes use of certain monomolecular surface films (monolayers), which act to increase the evaporation resistance of the water surface. The molecules of the various monolayers are long chain hydrocarbons, which have the chemical property of a hydrophobic and a hydrophilic end. Consequently, upon contact with water, they tend to form an oriented layer, one molecule thick. It has been found that monolayers of cetyl alcohol and stearyl alcohol can cause a reduction of evaporation from 25 per cent to 35 per cent, and values as high as 70 per cent have been reported (4) (5).

The purpose of the present study is to investigate analytically and experimentally the reflective properties of film-covered water surfaces. This is being done for a twofold purpose. The first is that of determining any possible change in the reflectance of a water surface when a monolayer is applied. This information will be helpful in the energy balance analysis of the evaporation suppression problem for large lakes. The second purpose is that of proposing the possibility of a new method of evaporation suppression. Such a method would be that of applying some surface film which would increase the radiant energy reflected from a lake surface so that less radiant energy would be absorbed and less energy would be made available for driving the evaporation process.

CHAPTER II

LITERATURE SURVEY

The Development of Evaporation Suppression by Monolayers

The study of the properties of monolayers on lake surfaces dates back to 1765 when Benjamin Franklin first estimated that the thickness of a monolayer must be of the order of 25 Å (6). In 1891 Agnes Pockels (7) did elementary work related to the surface tension of water contaminated by certain oils. She used a system of movable barriers in which the contaminants were confined. From her measurements she found that surface tension was inversely related to the area covered by the contaminant.

In 1917 Langmuir (8) proposed that the forces involved in formation of films of certain organic substances on water surfaces do not originate from the organic and water molecules as a whole, but rather from certain atoms in these molecules. In this work he demonstrated that certain paraffinic long chain hydrocarbons would form oriented monolayer films.

By 1925 there had developed a significant interest in the use of monolayers in the retardation of evaporation. Hestrand (9), Rideal (10), and Langmuir (11) made early attempts to evaluate evaporation suppression. These early works used various indoor laboratory devices which attempted to control specific variables, rather than to approximate the boundary conditions of a lake.

Sebba and Briscoe (12) in 1940 showed the importance of surface pressure, and they proposed a critical surface pressure below which there was little evaporation suppression. By 1943 Langmuir (13), who had won a Nobel Prize in 1932 for his surface chemistry work, was convinced that, for monolayer studies, a series of energy barriers (mathematically similar to electrical resistors) should replace the single barrier concept of Fick's diffusion law. This was further verified by Archer and LaMer (14) in 1955 when they stated that an energy of activation is required for a water molecule to be directed along a path between the monolayer molecules.

Along with the development of an analytical model for evaporation suppression, field work has been done on lakes of various sizes. Testing procedures have been developed to determine the selection of the optimum monolayer forming substance and application method. Some investigators tried to isolate the effect of single variables; such as, the surface pressure of the film and the influence of wind and wave action. In fact, in 1960 the U.S. Geological Survey (15) listed 322 references in a literature review on evaporation suppression.

Some generalizations which are widely accepted in the literature are as follows. Monolayers of cetyl alcohol and stearyl alcohol are effective in the suppression of evaporation (16). The evaporation resistance of a monolayer increases with an increase in its surface pressure and with monolayer purity (17) (18). Although the best monolayer application method has not been agreed upon, it has been found that for the maximum effect the application rate should be a function of lake size and orientation, wind velocity, air and water temperature, and other climatic conditions (4).

The Evaluation of Evaporation

In order to determine the effect of monolayers on evaporation, one must first of all be able to evaluate the evaporation of a system.

In 1915 Schmidt (19) attempted to utilize the energy balance or conservation of energy approach to estimate the yearly evaporation of an ocean. His work assumed that the difference between the net radiant energy input and the convection energy loss is used solely in creating evaporation. His work did suffer from a mistake in his calculation of the diffuse reflectance of water, so that his determined value of 0.17 was in error. His value of the emissivity, consequently, was in error also. Nonetheless, Schmidt's work did much to stimulate the analysis of water evaporation.

Angstrom (20) in 1920 applied heat radiation measurements to the study of evaporation from lakes. He used 0.94 as the value for the emissivity of water, saying that Schmidt's value was too low, probably because of wave length considerations. His values for radiation input to the earth were taken from Kimball's (21) work at the U.S. Weather Bureau. Angstrom also presented a correction factor for the effect of cloud cover on the effective back radiation from the earth:

$$R = (1.0 - k \cdot m)\, R_o,$$

where R_o is the earth radiation loss during clear sky conditions, k is the constant 0.9, and m is the cloud cover measured as a fraction of the sky covered by clouds.

Also, in 1920 Anklof (22) followed Angstrom's work by presenting data and an empirical equation for the effective back radiation from the earth. His equation considered this energy to be a function of the vapor pressure of the air, the water surface temperature, and various constants determined by curve fitting.

In 1930 Kimball and Hand (23) made tests from an airplane to determine the reflectance of various surfaces. In February, they determined the reflectance of the Chesapeake Bay under smooth conditions to be 0.097, and in May under white-cap conditions the reflectance was between 0.034 and 0.040. That is to say, they determined a decrease in reflectance with increased water waves.

By 1931 Richardson (24) had recognized that the evaporation of water was an important problem in the western United States and that this demanded investigation. He defined the term "insolation" as the exposure of the earth to the energy from the sun and the sky. He presented three different approaches to the problem of calculating evaporation, but actually his approaches required too many assumptions for terms for which there was no specific data. Consequently, his method gave only very rough estimates of evaporation.

Brunt (25) presented in 1932 a review of models for radiant heat transfer in the evaporation process. His work included comparison of the various empirical equations for back radiation from the earth to the atmosphere. And in 1939, his book included three chapters on radiation as applied to the evaporation problem (26). Specifically noted was a growing awareness of the necessity of considering the wave length distribution of solar energy. He noted the change of this distribution with a change of solar angle. Also, he emphasized the importance of the absorption spectra of water vapor, ozone, and oxygen.

Powell and Clark (27) presented measurements of the reflectance of the ocean in 1939. They determined that the reflectance on a clear day reaches a minimum value of about three to four per cent and that the percentage is increased one or two per cent when water waves exist. This was in direct contrast to the results of Kimball and Hand, whose work showed that water waves cause reflectance to be less than that from a smooth surface. Powell and Clark noted this difference, but they made no effort to explain the discrepancy.

In 1941 Holzman (28) determined that the heat balance method for determining evaporation was theoretically precise. He concluded, however, that the difficulties encountered in trying to determine accurate values for some of the parameters were sufficient to make this method useless. In his work he assumed the value of 0.906 for the emissivity of water.

Penman (29) presented in 1947 a study of evaporation which used an aerodynamic approach. He essentially used a simplified mass transport equation with the coefficient of eddy diffusion being determined from an application of the Prandtl mixing length theory. He also presented a summary of the energy balance approach and various empirical equations for evaporation as a function of wind speed and the difference between the saturation vapor pressures at the air temperature and the dew-point temperature.

In 1951 Langbein and others (30) presented work related to the evaporation of water from stock ponds in an arid area of Arizona. The problem of water inflow due to drainage of dry wash areas and the problem of seepage losses were discussed. By proper selection of pond sites which did not have continuous inflow and by making a continuous record of the change in water level, data for the inflow and the net loss due to seepage and evaporation were obtained. The method of isolating the seepage loss from the evaporation was that of plotting the change of water level, corrected for any possible inflow, as a function of an "evaporation factor." The evaporation factor, though not equal to the evaporation loss, was designed to be equal to zero when the evaporation was zero. It was determined empirically to be:

$$E = U\,(e_W - e_a)$$

where,

E = evaporation factor

U = wind velocity

e_W = partial pressure of water vapor at the surface

e_a = partial pressure of water vapor in the air above the surface.

The intersection of this line with the change of water level axis (i.e., where $E = 0.0$) was then considered to be the seepage loss. Thus, by this interpolation process, the seepage loss was isolated and the evaporation loss was determined readily.

In 1952 Ferguson (31) noted that previous energy balance work had assumed bulk approximations and that the variation of the various terms with time had not been considered. Consequently, he proposed a differential equation for the rate of evaporation from an element of pond surface. Empirical equations were assumed for various energy terms, and the differential equation was solved by numerical methods on a differential analyzer. The results of his work showed that the calculation of evaporation from merely a knowledge of the net radiation input can cause a serious error for shallow ponds. Pond depth and absolute humidity were shown to be important variables.

Sverdrup (32), in the radiation chapter of his book, gave a tabulation of the total reflectance of water surfaces for sun and sky energy inputs. He gave these reflectance values as a function of the altitude of the sun on clear days. He then postulated that the presence of water waves caused an increase in the reflectance of a lake surface, particularly at low sun angles. He further noted that on a clear day with a high sun angle, 85 per cent of the radiant energy comes from the sun and the remaining 15 per cent comes from the sky. However, on a clear day at low sun altitudes, the sky energy was noted to be as high as 40 per cent. He corrected the input energy and back radiation for cloud cover with essentially the same equation that Angstrom used in his work to correct the back radiation. Sverdrup also discussed the variation of transmitted energy through the water as a function of depth, wave length, and the lake from which the water sample originated.

The U.S. Geological Survey (33) published a report on the work done at Lake Hefner in Oklahoma City, Oklahoma, in 1954. Extensive work was done to determine the validity of various techniques for evaluating the parameters influencing evaporation. Apparatus was developed for measuring: radiant energy; wave height as a function of wind speed and velocity profile; water and air temperatures; water inflow, outflow, and total volume; and meteorological conditions, such as, cloud cover and height, humidity, and barometric pressure.

The study concluded that the energy balance approach is a valid method for evaluating the evaporation from a lake surface provided that (a) the solar radiation is determined experimentally at the location, (b) the advected energy (internal energy crossing a system boundary) terms are included, and (c) the energy storage terms are considered.

Also, the report showed that (a) reflection is a function of cloud cover and solar angle; (b) the emissivity of water is 0.970; (c) about 60 per cent of the energy irradiating a lake surface is from the atmospheric radiation, and the remainder comes directly from the sun; and (d) Bowan's ratio, which is generally considered to be a practical device for relating convected energy to the energy of evaporation, has not been sufficiently investigated to be used in precise calculations.

One chapter of this study dealt with the Cummings' radiation integrator. It was described as a thermally insulated pan, four feet in diameter, which may be used to measure the energy radiated to a lake surface. In this approach, such a pan is placed next to a lake being studied; and it is assumed that the radiant energy transferred to the pan is the same as that to the lake. This energy may be determined using the energy balance equations together with proper measurements of change of water level, net rain fall, surface and bulk temperatures, and the use of certain physical constants, such as, the heat of vaporation, water emissivity, the Bowan ratio, and others. It is assumed that the differences in the wind conditions above a pan and above a typical element of lake surface are not significant. Such differences, however, could result in different water wave magnitudes, which would possibly cause differences in the effective emissivity and reflectivity for the two surfaces. Also, the effective emissivity and reflectivity could be altered by different surface contamination on the two surfaces.

In 1958 a follow-up study to the work at Lake Hefner was done at Lake Mead (34). The purpose of this study was to evaluate the Lake Hefner techniques in an arid area. The study used approximately the same instrumentation as at Lake Hefner, and it was assumed that the values for emissivity and the values for reflectivity as a function of solar angle and cloud cover would be the same. The conclusions which are of interest were: (a) Over a year's time, the daily average radiation values of the integrator were very close to that of the flat plate radiometer and the pyrheliometer; and (b) there was excellent agreement between the water balance and the energy balance determinations of evaporation on an annual basis.

The Evaluation of Evaporation Suppression by a Monolayer

The problem of evaluating the effect of a monolayer on evaporation from lakes has received considerable attention during the last ten or so years. However, there is presently no method of evaluation which is acceptable to all researchers (35).

In 1955 Mansfield (5) reported work on the evaporation suppression effect of a cetyl alcohol monolayer in outdoor evaporation pans. These pans were 12 inches in diameter and 18 inches deep. Two pans were filled with water, and a monolayer was applied to the surface of one. By measuring the change of water level, he was able to determine an evaporation suppression of 75 per cent during the first week of a test. During the second week, the monolayer had deteriorated slightly so that the evaporation saved was down to 45 per cent; and for the third week, there was no evaporation suppression detected. The pan concept suffers from the fact that the boundary conditions of a pan are not those of a lake.

In 1958 Crow and Daniel (2) presented work using two adjacent and specially constructed 12,000 square feet ponds. The basic instrumentation was that of point gages, readable to one thousandth of an inch and operated in a stilling well outside each pond. A prefabricated plastic liner eliminated the variable of water seepage loss, and a one-month calibration run showed that the two ponds had equivalent evaporation losses. Their work with cetyl alcohol concluded that it reduced evaporation by from 33 per cent to 60 per cent, depending on the application technique and the wind conditions.

Harbeck and Koberg (36) presented in 1959 a method of evaluating the effect of a monolayer on a single large lake. Their method was essentially a combination of the energy balance equation with a simplified mass transfer equation. A calibration period was required for a lake so that the mass transfer and convection heat transfer coefficients could be determined. By studying the order of magnitudes of the various terms in the energy balance equation and by considering which terms are changed due to the presence of a monolayer, a relationship was developed involving certain energy terms realized during a time when a monolayer is on the lake surface and certain energy terms which would have been realized if the monolayer had not been present.

In 1963 Koberg and others (37) presented work done in 1959 and 1960 which used the above method for studying evaporation suppression on four one-acre farm ponds. A series of nine field tests were run, using different monolayer forming substances and different methods of application. The results showed evaporation suppression ranging from no effect up to 27 per cent decrease in evaporation. Some of the shortcomings of the Harbeck and Koberg method were indicated. For instance, the simplified energy relationship does not include the possibility of a change in the radiant energy from the sun and sky between the calibration period and the test run period. Also, it does not consider the transient nature of the change in energy storage of the water volume and the increase of evaporation over the initial conditions, which would result if the monolayer were removed or allowed to deteriorate after once being applied.

Thus it is seen that problems still exist in evaluating the actual evaporation suppression of a monolayer in a given situation. Much of the cause for this is the complexity of the measurements required. A further problem is that of maintaining a "compressed" monolayer on the entire lake surface, which is made difficult because of the wind's ability to blow the monolayer across the surface and water wave's ability to cause the monolayer to become dispersed. As the various monolayer substances have different properties, the selection of the best monolayer substance and the best application method is very complex. Certainly such a study must always include the economics of the situation so that the optimum evaporation suppression per dollar may be known.

The present study, however, is related to the problem of determining the reflectance of a monolayer-covered water surface relative to that of a water surface without a monolayer. This is being studied in order to determine if the presence of a monolayer changes the radiant energy absorption or emission. This is important because the net radiant energy input to a body of water is actually the driving potential for the evaporation process, and a change in this input would result in a direct change in the amount of water evaporated.

The Reflection of Radiant Energy

Radiant energy is generally studied by considering the optical model presented in electromagnetic theory. For this model, the energy is assumed to be a combination of individual electromagnetic waves, each having its own amplitude, angular orientation, wave length, and phase angle.

The early discoveries in electricity and magnetism were made by many persons. Among the persons who made significant contributions are Coulomb, Ampere, Faraday, Hertz, Lorentz, Fresnel, and especially Maxwell (38).

From 1888 to 1890 Drude(39) (40) (41) presented a series of papers on the determination of the optical constants (index of refraction and extinction coefficient) of metals. In particular, his papers presented equations which are deduced from the law of refraction and the reflection formulas of Fresnel. These equations related the behavior of polarized light upon reflection from various clean and contaminated surfaces. He presented experimentally determined values for the optical constants of various metals. He also considered the effect of surface layers on reflection from the metal surfaces, giving equations for the change in the polarization properties caused by such layers.

In 1900 Drude (42) published his optics book, which was significant in that it was a unified collection of the optics knowledge up to that time. Though the words "magnetic field" and "electric field" were not used, Drude did propose that a "displacement current" could exist in an "ether," and that this "displacement current" should have the same magnetic effects as the conduction currents noted in metal conductors. With these assumptions, he found a better understanding of Maxwell's equations. In his book, Drude presented Fresnel's equations for the reflection of light from a real surface. In this he proposed a transition layer, of some small thickness, across which the optical properties change from those of the first medium to those of the second medium.

Schmidt (19), in his 1915 paper which has already been mentioned, presented a table showing the variation of reflectance from a water surface with angle. His values were determined for radiation at the sodium line, using an index of refraction of 1.333. Schmidt then extended this work to determine analytically the diffuse reflectance of a water surface. It should be noted that a mistake was made, in that a ray of energy with intensity, I_0, striking a horizontal surface at an angle of incidence, θ, should have $I_0 \cos(\theta)$ as the energy per unit area, instead of the value, I_0, as Schmidt assumed. This error was pointed out and corrected by Burt (43) in 1953. He showed that Schmidt's value of 0.17 should be approximately 0.066.

In 1927 Raman and Ramdas (44) did experimental work studying the thickness of transition layers on liquid surfaces. In particular, they studied the ellipticity (which is the ratio of the amplitudes of the light vector in and at right angles to the plane of incidence) of various liquids, one being water. As one of their results, they found the ellipticity of a clean water surface to be positive; but for a water surface contaminated by a monolayer, the ellipticity was negative. Because of this, they proposed that it was unlikely that a layer absorbed from the atmosphere existed on the clean water surfaces. Although their paper expressed a desire to continue this work, it is interesting to note that in the next year Raman presented his famous paper on light scattering (the Raman Effect); and a search of the literature indicates that they presented no further work on the transition layers of surfaces.

In 1933 Strachen (45) presented analytical work on the reflectance of a surface covered by a monolayer. He considered the monolayer to act as a two-dimensional assembly of scattering centers, and his principal method of analysis was to consider the monolayer as a surface distribution of Hertzian oscillators. He considered that the total field noted during the reflection of a ray was made up of two parts. The first part was due to the incident, reflected, and refracted waves, which would be produced without a film; and the second part was produced by the oscillators. For polarized incident light, he derived first order expressions for the difference in the ratio of amplitudes, tan Ψ, and for the difference in the phase retardation, Δ, for the cases with and without a monolayer present. It is interesting to note that these equations are similar to the ones developed by Drude, though they start from different assumptions.

Also in 1933 Tronstad (46) reviewed the theoretical and experimental aspects of Drude's optical methods for thin surface films on metals. He described the method for studying the change in the state of polarized light at various angles of incidence when reflected from clean and from film-covered surfaces. He noted that the two quantities which may be determined experimentally are the phase retardation, Δ, and the ratio of the "absorption coefficients," tan Ψ. He described an optical system which included a prism to select wave length, a polarizer, a quarter-wave plate, a half-shade system, an analyzer, and a telescope. As specific examples of problems studied using this method, he mentioned normal and passive oxide films on metals, absorption of gases on metals, and monomolecular films on metals.

Tronstad (47) presented a discussion of the validity of Drude's method of studying thin films in 1935. He noted the assumption of optically sharp interfaces and that the films are assumed isotropic. He then noted the presence of a thin invisible oxide film on most real metal surfaces, and he proposed an experimental study of vacuum-evaporated metal films which could be undertaken prior to exposure to the atmosphere. The assumption was mentioned that scratches cause negligible effect so long as they are small compared to the wave length of energy being considered. The assumption of constant film thickness was indicated. Also, the possibility of double refraction caused by internal film stresses was considered to be negligible, because of the proportionality to both stress and film thickness.

In 1935 Blodgett and Langmuir (48) determined the thickness and the index of refraction of a film of barium stearate using interference methods. A procedure was presented for applying a known number of monomolecular layers of barium stearate onto a surface. Assuming uniformity of the layers, the interference method was considered to be accurate enough to measure a film thickness down to approximately 10 Å with an accuracy of about 3 Å.

Bruce (49), in his 1939 study of nonabsorbing transition layers, noted that ellipticity is found in the reflected ray at the polarization angle if the incident ray is plane polarized at the 45° azimuth, as was stated earlier by Drude (40). Bruce stated that polarization reverts back to Fresnel's model of plane polarized light outside the vicinity of the polarization angle. Drude originally assumed the index of refraction was a function of height in the transition layer. Bruce, however, following the example of Tronstad assumed that the index of refraction and the thickness of the layer were two unknown and independent constant values. After studying the equation for change of phase, he determined that Drude's form did not permit the mathematical separation of layer thickness and the index of refraction. He stated that such a separation was not possible even by use of a series of observations and the solution of simultaneous equations. He did, however, show that the minimum thickness of the layer, that will be compatible with the equations of Drude, could be found by assuming that the index of refraction of the absorbed layer is the geometric mean of the known indices of refraction for the two media.

In 1941 Centeno (50) presented a compilation from the literature of the experimental determinations for the variation of the optical properties of water with wave length. These results were determined by finding the extinction coefficient from transmission studies. Then by using this information, the reflectivity data (smoothed-mean values from the literature), and the reflection equation at normal incidence, the index of refraction was determined.

In 1945 Rothen (51) presented a method for measuring the thickness of thin films on metal surfaces. He developed apparatus for comparing the ellipticity of two film-covered surfaces. With it, he prepared a calibration curve for the change of ellipticity with film thickness, using Blodgett and Langmuir's (48) values for the film thickness of built-up barium stearate monomolecular layers. The main assumption here was that the index of refraction for the film being studied must be the same as that of barium stearate, and thus this method has severe limitations.

In 1945 Mooney (52) presented an exact theoretical treatment of transparent reflection-reducing optical coatings. He started with Maxwell's equations and applied Gauss's law and Stokes's theorem to determine the boundary conditions which must be met at the interface of the two dielectric media. These boundary conditions were found to be: (a) the continuity of the normal components of electric and magnetic induction, and (b) the continuity of the tangential components of the electric and magnetic intensities. From these, he then developed equations for the reflectivity and transmissivity of thin transparent film-covered surfaces for normal incident energy radiation.

Vasicek (53) presented in 1947 a graphical interpolation technique for determining the index of refraction and the thickness of a transparent thin film on glass. For the reflection of plane polarized light, he developed analytical equations for the phase change, Δ, and the azimuth, ψ, as a function of the angle of incidence, the index of refraction of the glass and the film, and the film thickness. A polarimeter was developed, which gave data for the orientation of the major axis of the vibration ellipse and its ellipticity, from which values of Δ and ψ could be determined. With this information and by using known values of the index of refraction of glass, he considered the inverse problem of determining the index of refraction for the film and the film thickness. This was done by assuming various values of the index of refraction of the film, and then calculating the corresponding film thickness for the known azimuth, ψ. Then, the film thickness values were used to calculate the corresponding phase changes, Δ. By plotting these Δ values versus the assumed values for the index of refraction, the actual value of the index of refraction corresponding to the measured phase change, Δ, was determined. The calculation of the corresponding film thickness was then determined.

In the above procedure, the angle of incidence was arbitrary. Vasicek continued his work to include certain simplifications for specific angles of incidence.

In 1949 Chandrasekhar (54) authored a book in which there is a very good development of the representation of polarized light. Stokes parameters were developed from a basic mathematical model of an elliptically polarized energy beam. He discussed natural light and the fact that any particular beam of light can be considered as a mixture of elliptically polarized light and an independent stream of natural light.

In 1951 Curcio and Petty (55) presented their work studying the near infrared absorption spectrum of liquid water at 20° C. They presented a graph of the variation of the absorption coefficient with wave length over the range of 0.70 microns to 2.50 microns.

Also, in 1951 Simon (56) presented a graphical method of determining the index of refraction and the index of absorption for an absorbing surface. This method used the Fresnel equations at two arbitrary angles of incidence with various values of the optical properties to construct graphs of reflectances. Such graphs were constructed for natural light and for light polarized in the s-plane and the p-plane. By taking experimental measurements of reflectance at the two angles of incidence and by going to the graph, the correct pair of values for the index of refraction and the index of absorption were determined. Simon included a procedure to correct for the polarization of the spectrometer when natural light is used.

Avery (57) proposed in 1952 the use of a method for determining the optical constants of an absorbing substance which avoided the direct measurement of reflectance. He proposed the use of the ratio of the reflectance coefficients for incident light polarized in and perpendicular to the plane of incidence. Such a ratio, when determined at two angles of incidence, was used with a ratio of Fresnel's equations and a graphical procedure similar to that of Simon (56) for the determination of the optical constants.

Heavens (58) published a book in 1955 on the optical properties of solid-thin films. In it, he used the boundary conditions mentioned by Mooney (52) to develop the equations for the reflectivity and transmissivity of film-covered surfaces. He, at first, developed equations for the reflection of a nonabsorbing surface. Then, he considered the absorbing surface by the introduction of the complex index of refraction and the complex exponential function. He then considered a nonabsorbing film on a substrate which was either an absorbing or nonabsorbing substance. Finally, he developed the reflection equation for normal incident energy which was reflected by a single-absorbing film on an absorbing substrate.

In 1959 Kislovskii (59) presented a rough graph of the variation of the reflectance, the index of refraction, and the absorption coefficient of water with wave length from 2.0 to 200 microns.

Vasicek's (60) book in 1960 on the optics of thin films presented a development of essentially the same equations as presented by Heavens (58). However, his approach was to use the methods of complex algebra to expand the complex exponential functions into terms of real valued functions.

In 1962 Shurcliff (61) authored a book on the polarization of light. In it, he described the various descriptions of polarization and the methods of analysis. Of particular interest was his practical description of the Stokes vector.

The next year Abeles (62) presented a survey of the methods for determining the optical parameters of thin films. The methods he discussed are: the photometric method, which makes use of energy reflectance measurements; the polarimetric method, which makes use of various polarization measurements; and the interference method. He considered the application of these methods to birefringent and inhomogeneous thin films as well as to ideal absorbing and nonabsorbing thin films.

In 1965 Rouard and Bousquet (63) presented a very good analysis of the state of the art for the study of thin films. They reviewed the literature on determining the optical constants and the film thicknesses for both absorbing and nonabsorbing films. Particularly, they noted that usual analytical analysis requires that ideal films be homogeneous, isotropic, and bounded by two plane and parallel surfaces. For a real film, however, they reported that this is not usually the case. It has been found that the optical properties of a thin film are dependent on the details of the method of preparation and on the thickness of the film, as well as the wave length of the energy being used in the analysis. As most of the work in the optical properties of thin films has been done for metal films which are prepared by evaporation techniques, considerable discussion centered around the structure of the film and the substrate and on the contamination caused by the formation of oxide films when the film is exposed to air.

The Optical Properties of Cetyl and Stearyl Monolayers

Little work has been found in the literature which presents the optical properties of the various monolayer forming substances in their monolayer states on water surfaces. The most commonly used evaporation reducing monolayer substances are cetyl alcohol (hexadecanol) and stearyl alcohol (octadecanol). No work has been found in the literature giving the index of refraction for these monolayers in their monolayer states.

However, a study by Hofmeister (64) in 1953 was found, which investigated the index of refraction of various fatty acid monolayers on water surfaces. Included in this study were palmitic acid and stearic acid, which are of a similar structure to that of cetyl and stearyl alcohols. Though these acid products are not found to be effective in suppressing evaporation, the trend of the results may be helpful. After making the original assumption that monolayers are transparent, Hofmeister used the methods of Drude (40) to determine the index of refraction from the measured values of ellipticity at the angle of polarization (Brewster's angle). His results showed that the index of refraction for a monolayer substance is smaller than for the same substance under bulk conditions. His work also indicated that an increase in the surface pressure increases the index of refraction. Further, he noted that monolayer substances with longer chain lengths tend to have a higher index of refraction.

A study of the absorption coefficients of cetyl alcohol and stearyl alcohol in the monolayer states has not been found in the literature. Though the work of Rouard and Bousquet (63), previously mentioned, indicated that monolayer substances will have different optical property values for the monolayer state and for bulk conditions, the following absorption spectra information for bulk conditions is reviewed.

In 1937 Fox and Martin (65) presented results of their work on the absorption spectra of cetyl and stearyl alcohols. They noted a strong absorption band at 3.4 microns and weaker bands at 2.7 and 3.0 microns.

The absorption spectra of stearic acid was noted by Freeman (66) in 1952 to include absorption bands at 7.78, 10.7, and 13.8 microns.

In 1952 Sinclair and others (67) also detected the presence of stearic acid absorption bands at 3.3, 5.8, 7.8, 10.7, and 13.8 microns.

Freeman (68) in 1953 again confirmed the stearic acid absorption bands at 5.9, 7.8, 10.3, and 13.7 microns.

In 1955 Swern and others (69) determined absorption bands for cetyl alcohol to be at 3.0 microns, and for palmitic acid the absorption bands were found to be roughly at 3.5, 5.8, 7.0, 7.8, 10.6, and 13.8 microns.

The Reflectance of Water Wave Surfaces

It has been well documented that a cetyl alcohol monolayer on a water surface has a calming effect on water waves (70) (71). The general reason postulated for this damping effect is that the magnitude of the surface stress reversals noticed as a wave passes a particular location is reduced due to the compressional elastic behavior of the surface films which have a lower surface tension than the uncontaminated water surfaces (72).

A controversy exists in the literature as to the effect of the presence of water waves on the reflectances of a water surface. As was mentioned earlier, Kimball and Hand (73) detected a decrease in the reflectance of water with an increase of wave action; and Powell and Clark (74) indicated an increase in the reflectance for a similar increase of wave action. Sverdrup (75) postulated that the presence of water waves increases the reflectance of a lake surface, particularly at low sun angles.

A partial attempt to answer this controversy will be presented in the analytical section of this thesis.

CHAPTER III

ANALYTICAL STUDIED

Radiant energy is generally studied in terms of electromagnetic theory. Using this model for analysis of the energy incident on a lake surface, it is seen that the incident energy may be broken down into components according to the angle of incidence, ϕ, the wave length, λ, and the state of polarization. If $I_{\lambda,\phi}$ denotes the total radiant intensity per differential wave length at a particular angle of incidence and if $d\omega$ represents the differential of the solid angle around the angle of incidence, the total radiant energy on a unit area of lake surface can be represented by:

$$q = \int_0^\infty \int_{\text{Hemisphere}} I_{\lambda,\phi} \cos\phi \, d\omega \, d\lambda . \qquad \text{(III-1)}$$

Of the total energy, q, a portion will be reflected; and, for a deep lake, the remainder will either be absorbed at the surface or transmitted through the water until it is absorbed. The reflected energy can be represented by:

$$Rq = \int_0^\infty \int_{\text{Hemisphere}} R_{\lambda,\phi} I_{\lambda,\phi} \cos\phi \, d\omega \, d\lambda , \qquad \text{(III-2)}$$

where $R_{\lambda,\phi}$ is called the specular-monochromatic reflectance, and where the quantity, $R_{\lambda\phi}I_{\lambda\phi}$, is dependent on the polarization as well as the wave length and angular distributions of the incoming radiant energy.

The portion of radiant energy which will be absorbed by the water and, consequently, will tend to increase the water temperature and the driving potential for evaporation is represented by:

$$q(1.0 - R).$$

The study of the reflectance of water surfaces is therefore important for a theoretical understanding of the effect of radiant energy on evaporation. Lake surfaces can be classified generally as either clean or contaminated and also as either smooth or wavy.

Since the techniques of evaporation suppression most often involve contaminating the water surfaces with a monomolecular chemical film or monolayer, theoretical considerations studying the effect of these films on the reflectance of radiant energy are presented in this chapter and in the appendices.

Since it has been shown that monolayers have a stilling effect on water surfaces, an analytical study of the influence of water waves on the reflectance of water surfaces is also presented.

The Reflectance of Thin-Film Covered Water Surfaces

As a first statement, it should be noted that the techniques of optics in electromagnetic theory will be used in the development of analytical expressions for reflectance. The energy ray therefore is represented by independent monochromatic sinusoidal waves in each of the two customary planes of polarization. These planes are the p-plane, which is parallel to the plane of incidence, and the s-plane, which is perpendicular to the plane of incidence. From basic electromagnetic theory, equations are developed in Appendix A, which may be used in the calculation of the total energy reflectance for both the conditions of unpolarized light and arbitrarily polarized light.

In terms of the physical properties of the system, it is assumed that both the water and the film may be absorbing substances; i.e., they may be opaque. It is assumed that both substances are homogeneous and isotropic, in that their optical properties are considered constant with respect to the space coordinates. The optical properties, however, are considered to be a function of wave length. It is assumed that the reflection analysis may be made at some typical surface element and that at this location the air-film interface and the film-water interface are perfectly smooth and are parallel. It is further assumed that the thin film-covered water surface acts as a specular reflector for all wave lengths under consideration.

There is found in Appendix B an analytical development of expressions for reflectance. In this appendix, it is shown that the reflectance in either plane of polarization may be considered to be a function of the following variables:

(a) the optical properties of the film and the water (index of refraction n_1, n_2, and coefficient of absorption k_1, k_2),

(b) the thickness of the film, d,

(c) the wave length of the incident energy ray, λ, and

(d) the angle of incidence of the energy ray, ϕ.

The optical properties of water are dependent on the wave length and these values are considered to be known from the literature (50). In Table I, values are presented for the reflectances of a clean water surface at 0.5 and 2.0 microns. The symbols used in Table I, R_s, R_p, and R refer respectively to the s-plane polarized, the p-plane polarized, and the unpolarized energy reflectances. The symbols ϕ, n_2, and k_2 refer, respectively, to the angle of incidence, the index of refraction, and the coefficient of absorption. The latter two are a function of wave length.

The optical properties of a particular monolayer forming substance in its oriented and compressed monolayer state are expected to be a function of wave length also, but few indications have been found in the literature as to what this functional relationship might be. From the absorption data of bulk cetyl and stearyl alcohol in the literature, rough calculations indicate that the coefficient of absorption is small, with a maximum value of approximately 0.007 at 3.4 microns (65). The thickness of the monolayer is assumed to be 24 Å, which is the average of values calculated by Hofmeister (64) for the thickness of palmetic and stearic acid monolayers.

Thus, the reflectance of an energy ray having a particular wave length and a particular angle of incidence onto a monolayer-covered surface has only two independent and unknown variables, n_1, and k_1. If more than a single monolayer is present, the thickness of the film must be considered as an additional variable. Various values of n_1, k_1, and film thickness are assumed; and a computer program was written to aid in the analysis. A copy of this program in found in Appendix C. The results for unpolarized energy at 0.5 and 2.0 microns and at various angles of incidence are also presented in Table II and Table III. In Figure 1 and Figure 2, curves are presented which show the variation of unpolarized reflectance of a monolayer as a function of the optical properties of the monolayer and the angle of incidence.

TABLE I

THE REFLECTANCE OF CLEAN WATER SURFACES

<u>0.5 Microns</u>

ϕ	R_s	R_p	R	n_2	k_2
0	0.0207	0.0207	0.0207	1.336	0.00000
20	0.0249	0.0169	0.0209	1.336	0.00000
40	0.0437	0.0060	0.0249	1.336	0.00000
60	0.1162	0.0043	0.0603	1.336	0.00000
80	0.4588	0.2390	0.3489	1.336	0.00000

<u>2.0 Microns</u>

ϕ	R_s	R_p	R	n_2	k_2
0	0.0174	0.0174	0.0174	1.304	0.00110
20	0.0210	0.0141	0.0176	1.304	0.00110
40	0.0376	0.0048	0.0212	1.304	0.00110
60	0.1037	0.0047	0.0542	1.304	0.00110
80	0.4386	0.2367	0.3377	1.304	0.00110

As it is desirable to know the change in the reflectance of water due to the application of a monolayer or a film, a second computer program was written. This program provides for the calculation of the ratio of the monochromatic reflectances at various angles of incidence for a water surface having a monolayer or film to the similar reflectance of a clean water surface. This program, together with a listing of the results, is also found in Appendix C. Table IV and Table V present the results for unpolarized incident energy. Figures 3 through 6 show graphically the variation of the unpolarized reflectance ratio with the various properties of the film at 0.5 microns.

TABLE II

THE PER CENT REFLECTANCE OF FILM-COVERED WATER SURFACES HAVING UNPOLARIZED INCIDENT ENERGY AT 0.5 MICRONS

Thickness of Film		One Molecule			Ten Molecules			One Hundred Molecules		
k_1	ϕ \ n_1	1.3	1.4	1.5	1.3	1.4	1.5	1.3	1.4	1.5
.0000	0	2.07	2.07	2.08	1.97	2.32	2.93	1.75	3.21	6.28
	20	2.09	2.09	2.10	1.99	2.33	2.91	1.86	3.06	6.05
	40	2.48	2.49	2.49	2.40	2.71	3.26	2.44	2.99	5.52
	60	6.02	6.03	6.04	5.92	6.30	6.94	5.95	6.09	7.76
	80	34.89	34.89	34.90	34.74	35.24	35.99	34.11	35.00	35.49
.0001	0	2.07	2.07	2.08	1.97	2.32	2.94	1.75	3.21	6.28
	20	2.09	2.09	2.10	1.99	2.33	2.92	1.86	3.06	6.05
	40	2.48	2.49	2.49	2.40	2.71	3.26	2.44	2.99	5.52
	60	6.02	6.03	6.04	5.92	6.29	6.94	5.95	6.08	7.72
	80	34.89	34.89	34.90	34.74	35.24	35.98	34.10	35.00	35.48
.0100	0	2.08	2.08	2.09	2.07	2.43	3.05	1.88	3.29	6.13
	20	2.10	2.10	2.11	2.09	2.43	3.03	1.97	3.16	5.93
	40	2.49	2.50	2.51	2.50	2.82	3.38	2.48	3.14	5.53
	60	6.04	6.04	6.05	6.06	6.44	7.09	5.78	6.25	7.99
	80	34.91	34.92	34.93	34.96	35.45	36.17	33.64	35.10	35.96
.1000	0	2.18	2.20	2.22	3.01	3.48	4.18	2.71	3.97	5.71
	20	2.19	2.21	2.24	3.00	3.46	4.14	2.65	3.90	5.67
	40	2.59	2.61	2.64	3.40	3.85	4.50	2.71	4.00	5.82
	60	6.17	6.19	6.22	7.31	7.78	8.47	5.60	7.40	9.42
	80	35.12	35.13	35.14	36.90	37.25	37.77	33.99	36.68	38.65
1.000	0	3.24	3.44	3.65	16.76	18.81	20.95	25.50	27.47	29.42
	20	3.25	3.45	3.67	16.67	18.72	20.87	25.56	27.52	29.46
	40	3.69	3.89	4.12	17.06	19.07	21.17	26.60	28.38	30.17
	60	7.46	7.67	7.90	21.39	22.98	24.65	32.41	33.39	34.46
	80	36.30	36.39	36.50	47.71	47.68	47.68	58.76	58.07	57.47

TABLE III

THE PER CENT REFLECTANCE OF FILM-COVERED WATER SURFACES HAVING UNPOLARIZED INCIDENT ENERGY AT 2.0 MICRONS

Thickness of Film		One Molecule			Ten Molecules			One Hundred Molecules		
k_1	ϕ \ n_1	1.3	1.4	1.5	1.3	1.4	1.5	1.3	1.4	1.5
.0000	0	1.74	1.74	1.74	1.74	1.76	1.81	1.67	3.47	6.13
	20	1.76	1.76	1.76	1.75	1.78	1.82	1.69	3.43	6.05
	40	2.12	2.12	2.12	2.11	2.14	2.18	2.05	3.76	6.34
	60	5.42	5.42	5.42	5.41	5.44	5.49	5.32	7.51	10.53
	80	33.76	33.76	33.77	33.76	33.80	33.86	33.61	36.68	39.99
.0001	0	1.74	1.74	1.74	1.74	1.76	1.81	1.67	3.47	6.14
	20	1.76	1.76	1.76	1.75	1.78	1.82	1.69	3.43	6.05
	40	2.12	2.12	2.12	2.11	2.14	2.18	2.05	3.76	6.34
	60	5.42	5.42	5.42	5.41	5.44	5.49	5.33	7.51	10.53
	80	33.76	33.76	33.76	33.76	33.80	33.86	33.61	36.68	39.99
.0100	0	1.74	1.74	1.74	1.76	1.79	1.84	1.80	3.57	6.17
	20	1.76	1.76	1.76	1.78	1.81	1.85	1.82	3.53	6.09
	40	2.12	2.12	2.12	2.14	2.17	2.21	2.20	3.87	6.40
	60	5.42	5.42	5.42	5.45	5.48	5.53	5.58	7.70	10.61
	80	33.77	33.77	33.77	33.82	33.86	33.92	34.09	36.96	40.08
.1000	0	1.77	1.77	1.77	1.99	2.06	2.15	3.30	4.91	7.15
	20	1.78	1.79	1.79	2.00	2.07	2.15	3.31	4.88	7.08
	40	2.14	2.15	2.15	2.36	2.43	2.52	3.79	5.32	7.46
	60	5.45	5.46	5.46	5.78	5.85	5.94	7.99	9.68	11.92
	80	33.83	33.83	33.83	34.38	34.41	34.48	38.11	39.70	41.48
1.000	0	2.00	2.04	2.09	4.90	5.44	6.03	28.49	30.63	32.69
	20	2.02	2.06	2.11	4.90	5.45	6.04	28.23	30.41	32.50
	40	2.39	2.43	2.48	5.36	5.91	6.52	28.38	30.50	32.54
	60	5.76	5.81	5.87	9.25	9.79	10.32	32.82	34.27	35.70
	80	34.16	34.20	34.24	37.57	37.75	37.95	57.86	57.53	57.24

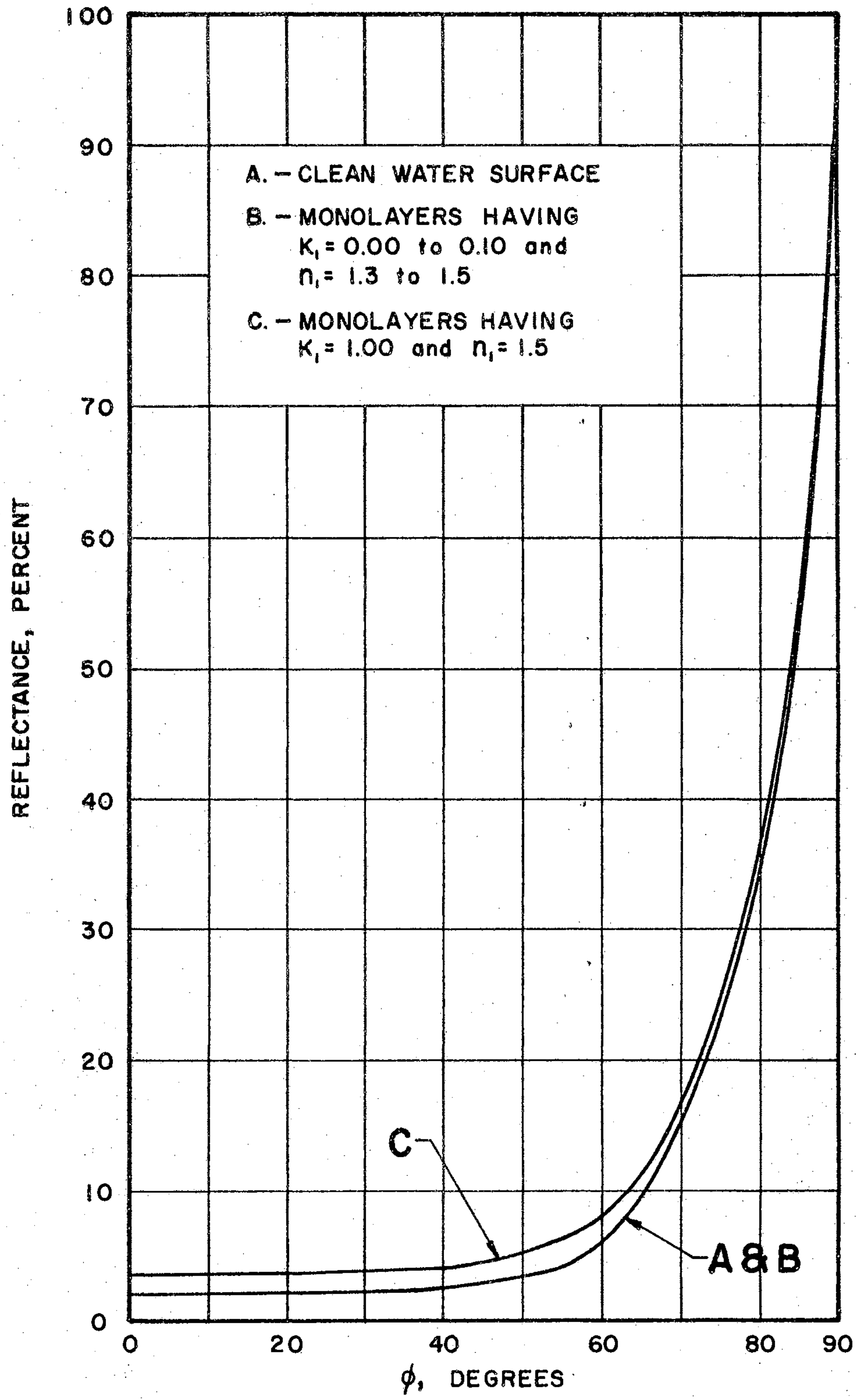

Figure 1. Unpolarized Energy Reflectance for a Monolayer-Covered Water Surface at 0.5 Microns

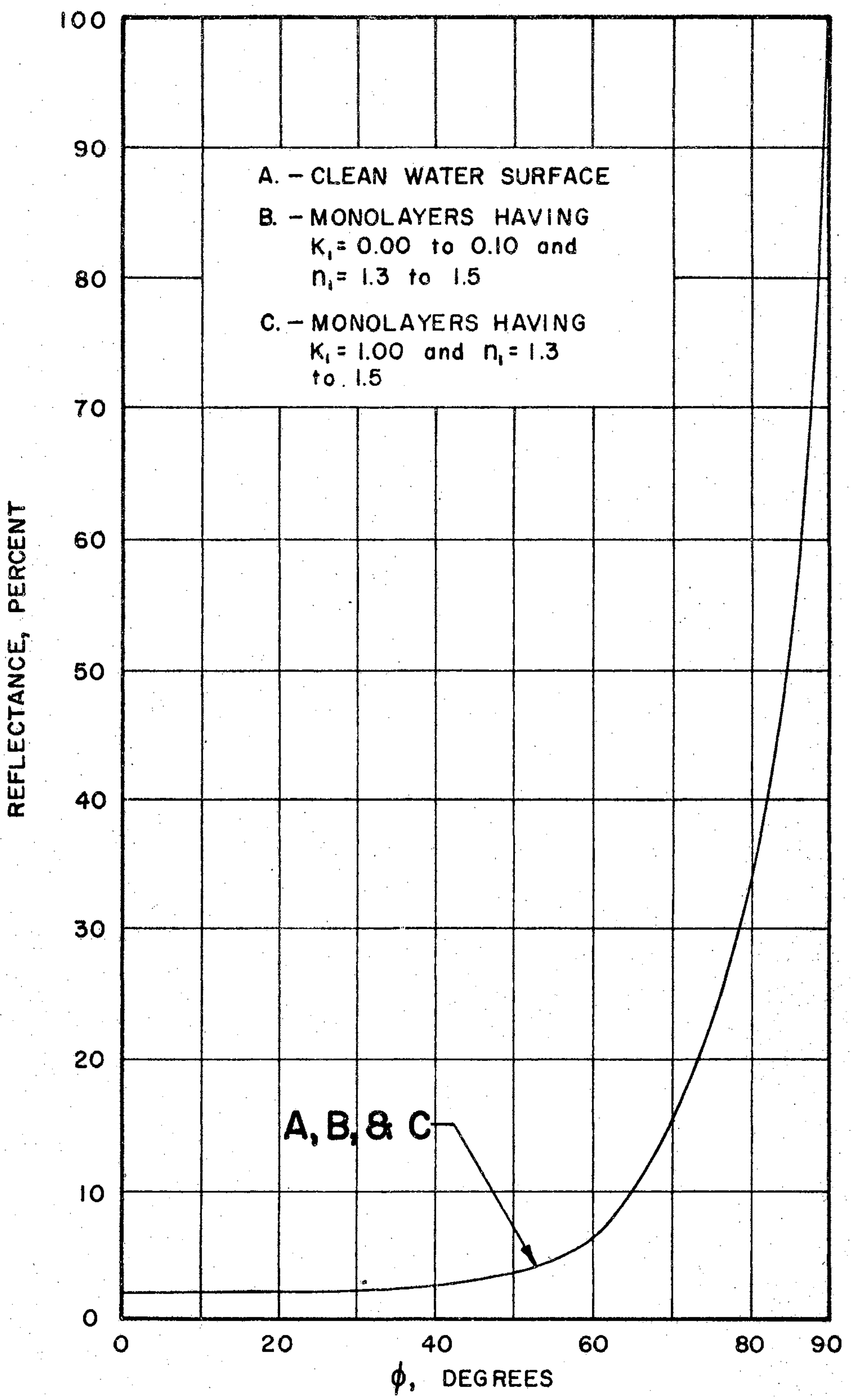

Figure 2. Unpolarized Energy Reflectance for a Monolayer-Covered Water Surface at 2.0 Microns

TABLE IV

THE RATIO OF THE REFLECTANCE OF FILM-COVERED WATER SURFACES TO THE REFLECTANCE OF WATER SURFACES WITHOUT A FILM FOR UNPOLARIZED INCIDENT ENERGY AT 0.5 MICRONS

Thickness of Film		One Molecule			Ten Molecules			One Hundred Molecules		
k_1	n_1 / ϕ	1.3	1.4	1.5	1.3	1.4	1.5	1.3	1.4	1.5
.0000	0	1.000	1.000	1.005	0.952	1.120	1.415	0.845	1.551	3.034
	20	1.000	1.000	1.005	0.952	1.115	1.392	0.890	1.464	2.895
	40	0.996	1.000	1.000	0.964	1.088	1.309	0.980	1.201	2.217
	60	0.998	1.000	1.002	0.982	1.045	1.151	0.987	1.010	1.287
	80	1.000	1.000	1.000	0.996	1.010	1.032	0.978	1.003	1.017
.0001	0	1.000	1.000	1.005	0.952	1.121	1.420	0.845	1.551	3.034
	20	1.000	1.000	1.005	0.952	1.115	1.397	0.890	1.464	2.895
	40	0.996	1.000	1.000	0.964	1.088	1.309	0.980	1.201	2.217
	60	0.998	1.000	1.002	0.982	1.043	1.151	0.987	1.008	1.287
	80	1.000	1.000	1.000	0.996	1.010	1.031	0.977	1.003	1.017
.0100	0	1.005	1.005	1.010	1.000	1.174	1.473	0.908	1.589	2.961
	20	1.005	1.005	1.010	1.000	1.163	1.450	0.943	1.512	2.837
	40	1.000	1.004	1.008	1.004	1.132	1.357	0.996	1.261	2.221
	60	1.002	1.002	1.003	1.005	1.068	1.176	0.959	1.037	1.325
	80	1.001	1.001	1.001	1.002	1.016	1.037	0.964	1.006	1.031
.1000	0	1.053	1.063	1.072	1.454	1.681	2.019	1.301	1.918	2.758
	20	1.048	1.057	1.072	1.435	1.655	1.981	1.268	1.866	2.713
	40	1.040	1.048	1.060	1.365	1.546	1.807	1.088	1.606	2.337
	60	1.023	1.027	1.032	1.212	1.290	1.405	0.929	1.227	1.562
	80	1.007	1.007	1.007	1.058	1.068	1.083	0.974	1.051	1.108
1.0000	0	1.565	1.662	1.763	8.097	9.087	10.12	12.31	13.27	14.21
	20	1.555	1.651	1.756	7.976	8.957	9.986	12.23	13.17	14.10
	40	1.482	1.562	1.655	6.851	7.659	8.502	10.68	11.40	12.12
	60	1.237	1.272	1.310	3.547	3.811	4.088	5.375	5.537	5.715
	80	1.040	1.043	1.046	1.367	1.367	1.367	1.684	1.664	1.647

TABLE V

THE RATIO OF THE REFLECTANCE OF FILM-COVERED WATER SURFACES TO THE REFLECTANCE OF WATER SURFACES WITHOUT A FILM FOR UNPOLARIZED INCIDENT ENERGY AT 2.0 MICRONS

Thickness of Film		One Molecule			Ten Molecules			One Hundred Molecules		
k_1	ϕ \ n_1	1.3	1.4	1.5	1.3	1.4	1.5	1.3	1.4	1.5
	0	1.000	1.000	1.000	1.000	1.011	1.040	0.960	1.994	3.523
	20	1.000	1.000	1.000	0.994	1.011	1.034	0.960	1.949	3.438
.0000	40	1.000	1.000	1.000	0.995	1.009	1.029	0.967	1.774	2.991
	60	1.000	1.000	1.000	0.998	1.004	1.013	0.982	1.386	1.943
	80	1.000	1.000	1.000	1.000	1.001	1.003	0.995	1.086	1.184
	0	1.000	1.000	1.000	1.000	1.011	1.040	0.960	1.994	3.529
	20	1.000	1.000	1.000	0.994	1.011	1.034	0.960	1.949	3.438
.0001	40	1.000	1.000	1.000	0.995	1.009	1.028	0.967	1.774	2.991
	60	1.000	1.000	1.000	0.998	1.004	1.013	0.983	1.386	1.943
	80	1.000	1.000	1.000	1.000	1.001	1.003	0.995	1.086	1.184
	0	1.000	1.000	1.000	1.011	1.029	1.057	1.034	2.051	3.546
	20	1.000	1.000	1.000	1.011	1.028	1.051	1.034	2.006	3.460
.0100	40	1.000	1.000	1.000	1.009	1.024	1.042	1.038	1.825	3.019
	60	1.000	1.000	1.000	1.005	1.011	1.020	1.030	1.421	1.958
	80	1.000	1.000	1.000	1.001	1.003	1.004	1.009	1.094	1.187
	0	1.017	1.017	1.017	1.144	1.184	1.236	1.897	2.822	4.109
	20	1.011	1.017	1.017	1.136	1.176	1.222	1.881	2.773	4.023
.1000	40	1.009	1.014	1.014	1.113	1.146	1.189	1.788	2.509	3.519
	60	1.006	1.007	1.007	1.066	1.079	1.096	1.474	1.786	2.199
	80	1.002	1.002	1.002	1.018	1.019	1.021	1.129	1.176	1.228
	0	1.149	1.172	1.201	2.816	3.126	3.466	16.37	17.60	18.79
	20	1.148	1.170	1.199	2.784	3.097	3.432	16.04	17.28	18.47
1.0000	40	1.127	1.146	1.170	2.528	2.788	3.075	13.39	14.39	15.35
	60	1.063	1.072	1.083	1.707	1.806	1.917	6.055	6.323	6.587
	80	1.012	1.013	1.014	1.113	1.118	1.124	1.713	1.703	1.695

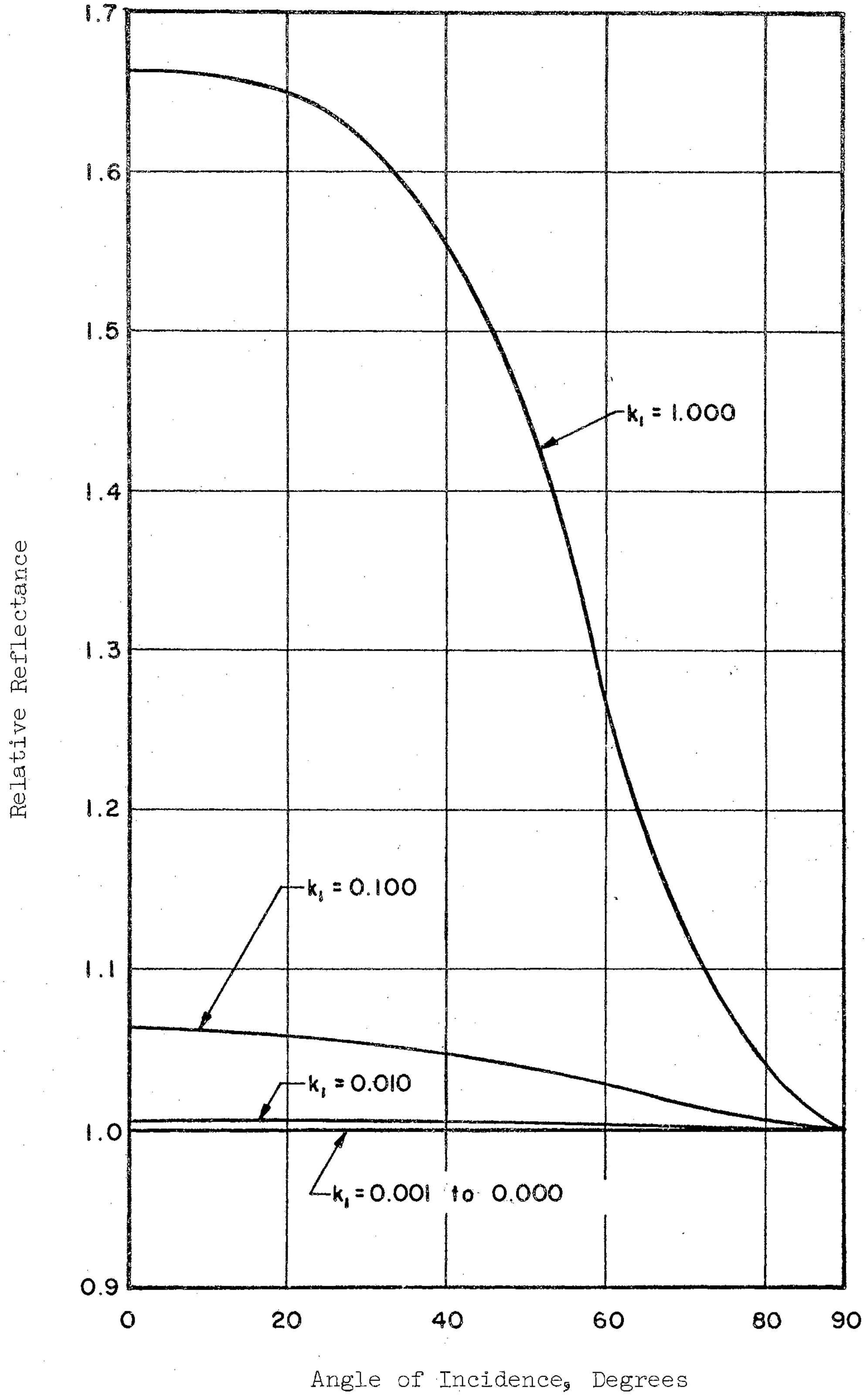

Figure 3. Relative Reflectance of Unpolarized Energy Reflected from a Monolayer-Covered Water Surface with $n_1 = 1.4$ and k_1 Variable at 0.5 Microns

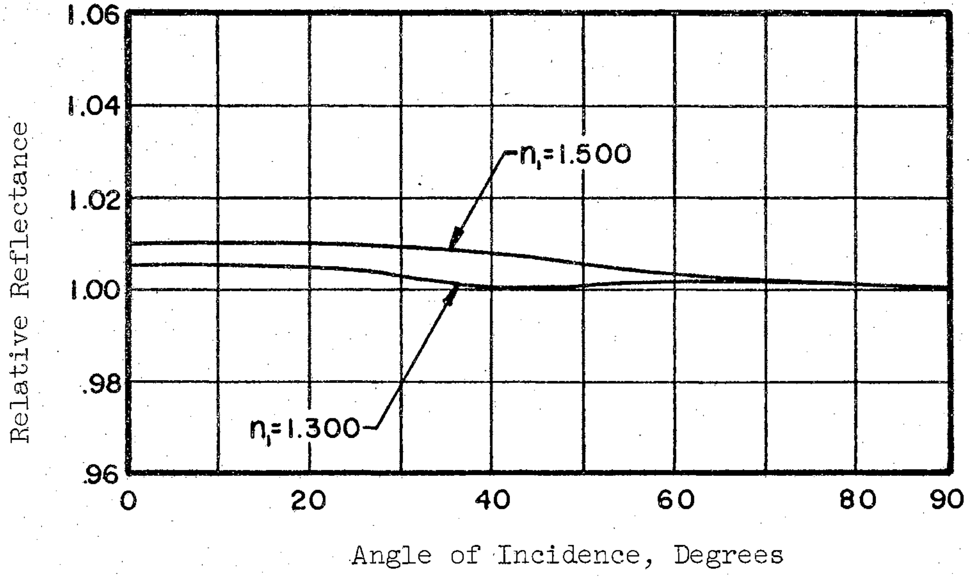

Figure 4. Relative Reflectance of Unpolarized Energy Reflected from a Monolayer-Covered Water Surface with $k_1 = 0.01$ and n_1 Variable at 0.5 Microns

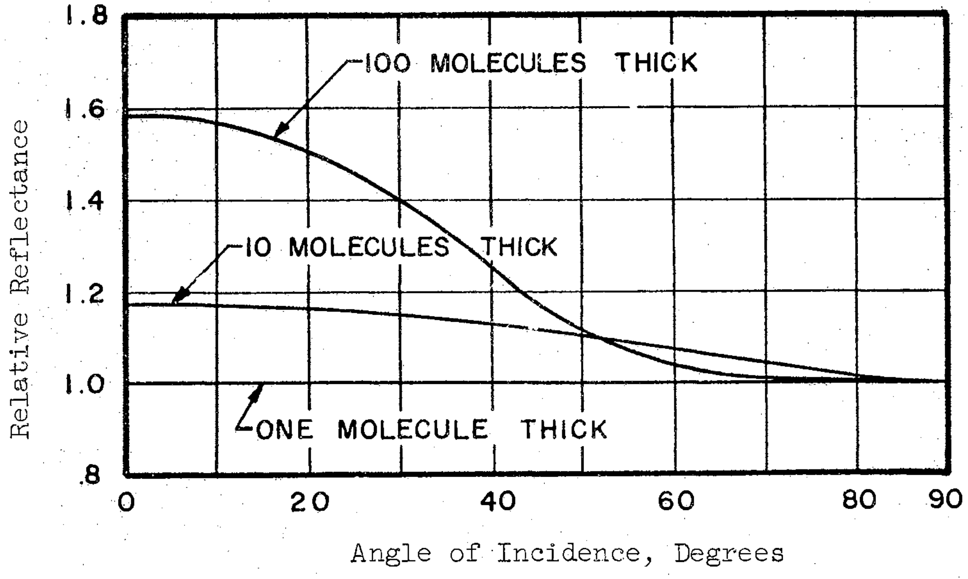

Figure 5. Relative Reflectance of Unpolarized Energy Reflected from a Film-Covered Water Surface with $k_1 = 0.00$, $n_1 = 1.4$, and with Variable Film Thickness at 0.5 Microns

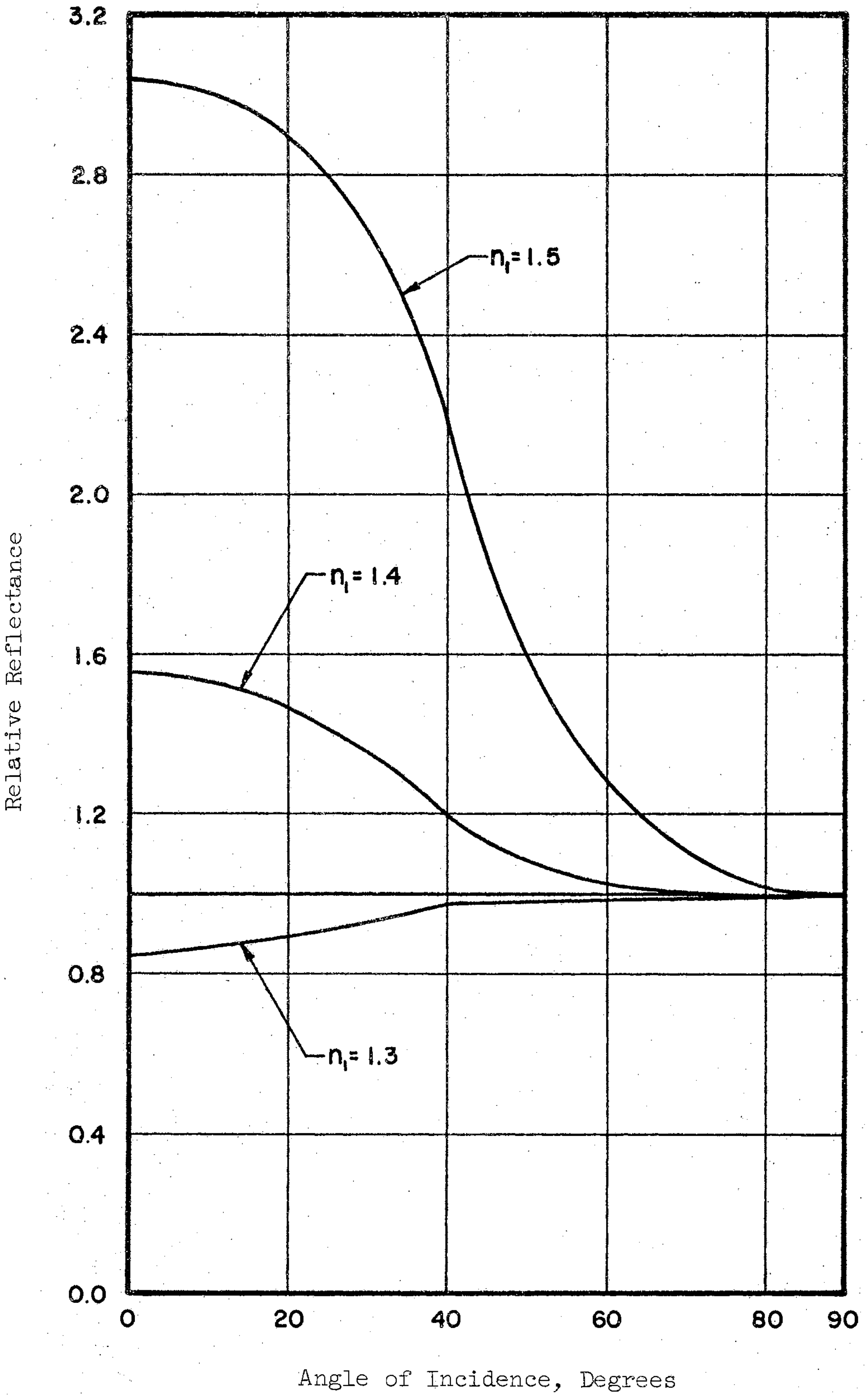

Figure 6. Relative Reflectance of Unpolarized Energy Reflected from a 100 Molecule Thick Film-Covered Water Surface with k_1 = 0.00 and n_1 Variable at 0.5 Microns

From the tables and graphs, it is seen that the reflectance of a monolayer-covered surface increases with an increase in the optical properties, n_1 and k_1, of the monolayer. It is also seen that under certain conditions, the reflectances will be less than those of a clean water surface. Further it is noted that by greatly increasing the thickness of the surface film, it is possible to increase significantly the reflectance. However, this does not always hold true, as in the case of small values of n_1 and k_1.

The Reflectance of Water Wave Surfaces

As was mentioned in the literature survey, a controversy exists as to whether water waves increase or decrease the reflectance of radiant energy. The following analytical analysis is designed to give new light to this controversy.

As assumptions for this analysis, it shall be considered that the water waves under consideration are perfectly smooth sinusoidal waves and that their wave length is very large compared to the wave length of the incident energy which is arbitrarily selected at 2.2 microns. It shall further be considered that the standard equations presented by Simon (56) for the reflectance of a single absorbing surface are known, as the equations derived in Appendix A become essentially the same as the equations of Simon, when the film thickness is set equal to zero.

The general procedure for this analysis is to assume that a water wave is irradiated by a ray of energy at some incident angle, ϕ, and that each point of the wave then is irradiated by energy at an angle, $\phi+\alpha$, where α is the angle between the horizontal and the tangent to the wave surface at that point. This is illustrated for small incident angles in Figure 7 and for incident angles large enough to cause a shadow effect in Figure 8.

Further for the analysis, the secondary reflections which could be caused by a reflected ray being directed toward another part of the water wave surface will be considered to cause only a negligible effect. It is evident that under certain circumstances this may be a poor assumption in that, by making it, the calculated reflectance of a water wave would be higher than is theoretically accurate.

If the reflectance of a surface varies from point to point, the average reflectance may be determined by performing the following integration:

$$R = \frac{1}{L} \int_0^L R(x)\, dx$$

where it is assumed that each point, x, has the same incident energy, and where L is sufficiently large so as to make the value of R representative.

In the case of an energy ray being reflected from a wave surface, it is obvious that the energy reaching each element of the wave surface is not the same. Thus, an integration technique is used that considers the ray of incident energy to be divided into n equal elements or subrays which do not necessarily interact with the same amount of surface area. This was also shown in Figure 7 and Figure 8. For each exterior point of the individual ray elements, the total angle of incidence ($\phi+\alpha$) of the

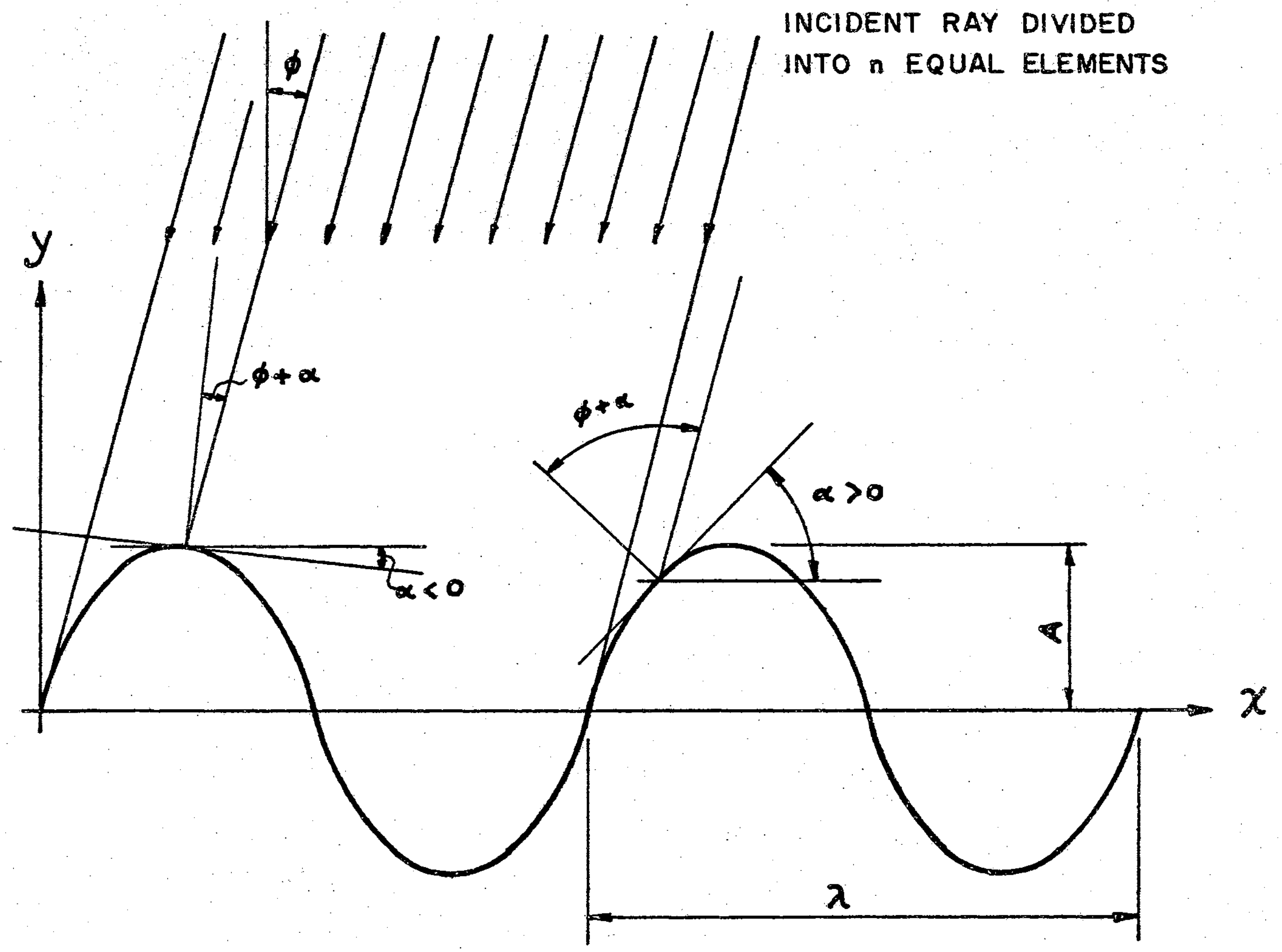

Figure 7. Energy Incident on a Water Wave Surface with No Shadow

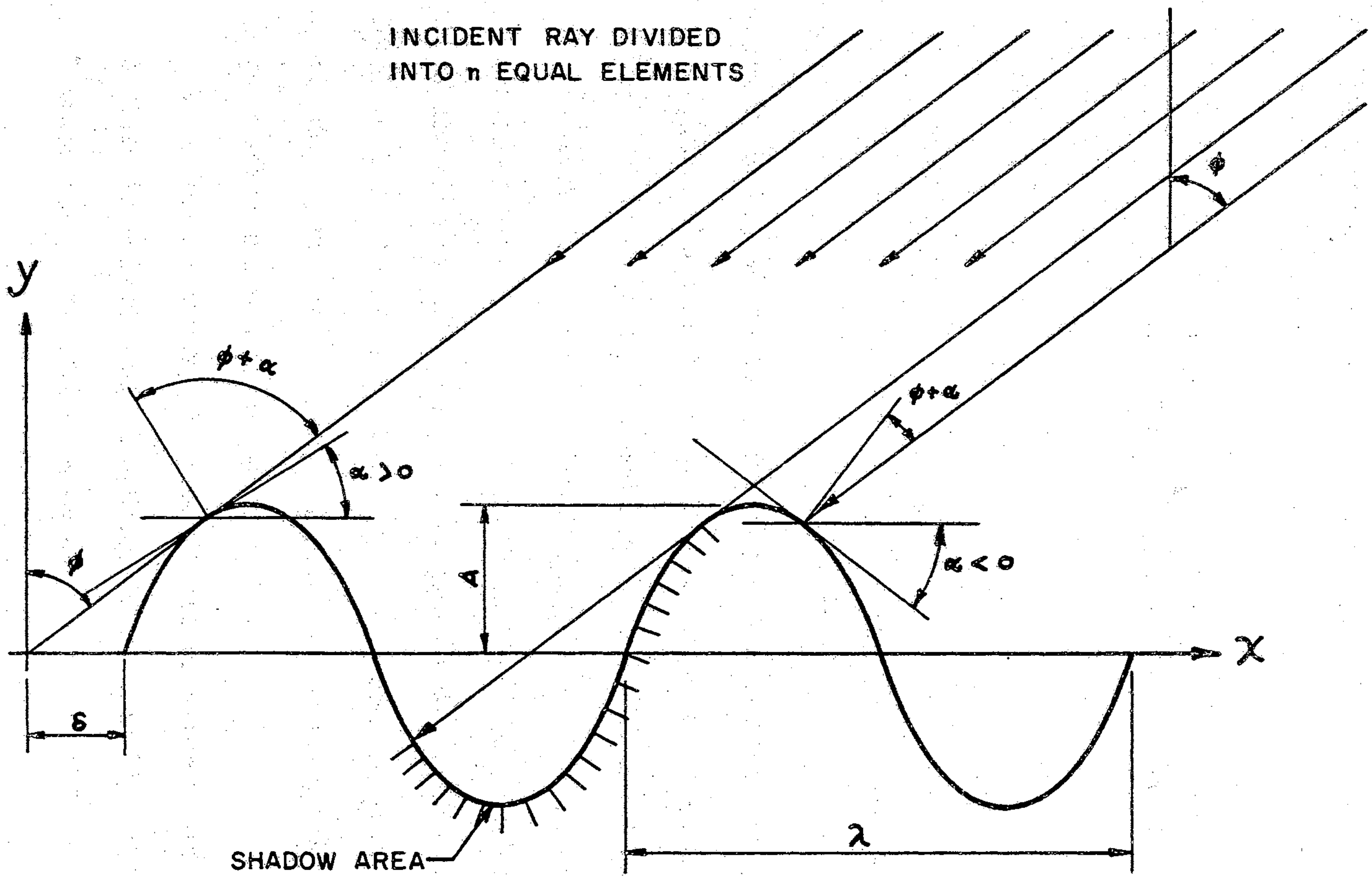

Figure 8. Energy Incident on a Water Wave Surface with a Shadow

projected exterior point is determined. With this information, the reflectance equations are used to calculate the reflectance values at these points; and the average value is assumed to be valid for the element of the ray under consideration. The integral then takes the following form:

$$R = \sum_{i=0}^{n} \left[\frac{R_{x_i} + R_{x_{i+1}}}{2} \right] \left[X_{i+1} - X_i \right] \qquad \text{(III-3)}$$

$$= \Delta X \left[\frac{1}{2} R_{x_0} + R_{x_1} + R_{x_2} + \dots + R_{x_n} + \frac{1}{2} R_{x_{n+1}} \right] \qquad \text{(III-4)}$$

where x_i and x_{i+1} denote locations for the external points of a typical ray element, where Δx is the x direction thickness of the ray element, and where the values of R_{x_i} for each external point are determined for the corresponding total angle of incidence, $\phi + \alpha$.

Appendix D contains the analytical development of equations used in the evaluation of R. A computer program, which was used in the study of the effect of the angle of incidence, ϕ, and the ratio of the water wave amplitude, A, to the water wave length, W, is also presented. The results of this study are presented in Figure 9.

As a further study, the reflectance of water waves under the condition of diffuse irradiation was investigated. Basically, this involved the evaluation of the following integral:

$$R = 2\int_0^{\pi/2} R(\phi)\ (\sin\phi\ \cos\phi)\ d\phi \qquad \text{(III-5)}$$

$$= 2\int_0^{\pi/2} R(\phi)\ \sin\phi\ (d\sin\phi)$$

Or by using numerical methods, it can be said that:

$$R = \int_0^{\pi/2} R(\phi)\, d(\sin^2\phi) \qquad \text{(III-6)}$$

$$\doteq \sum_{i=0}^{n} \left[\frac{R_{i+1} + R_i}{2}\right]\left[\sin^2\phi_{i+1} - \sin^2\phi_i\right]$$

A tabulation of the results of this study is given in Table VI.

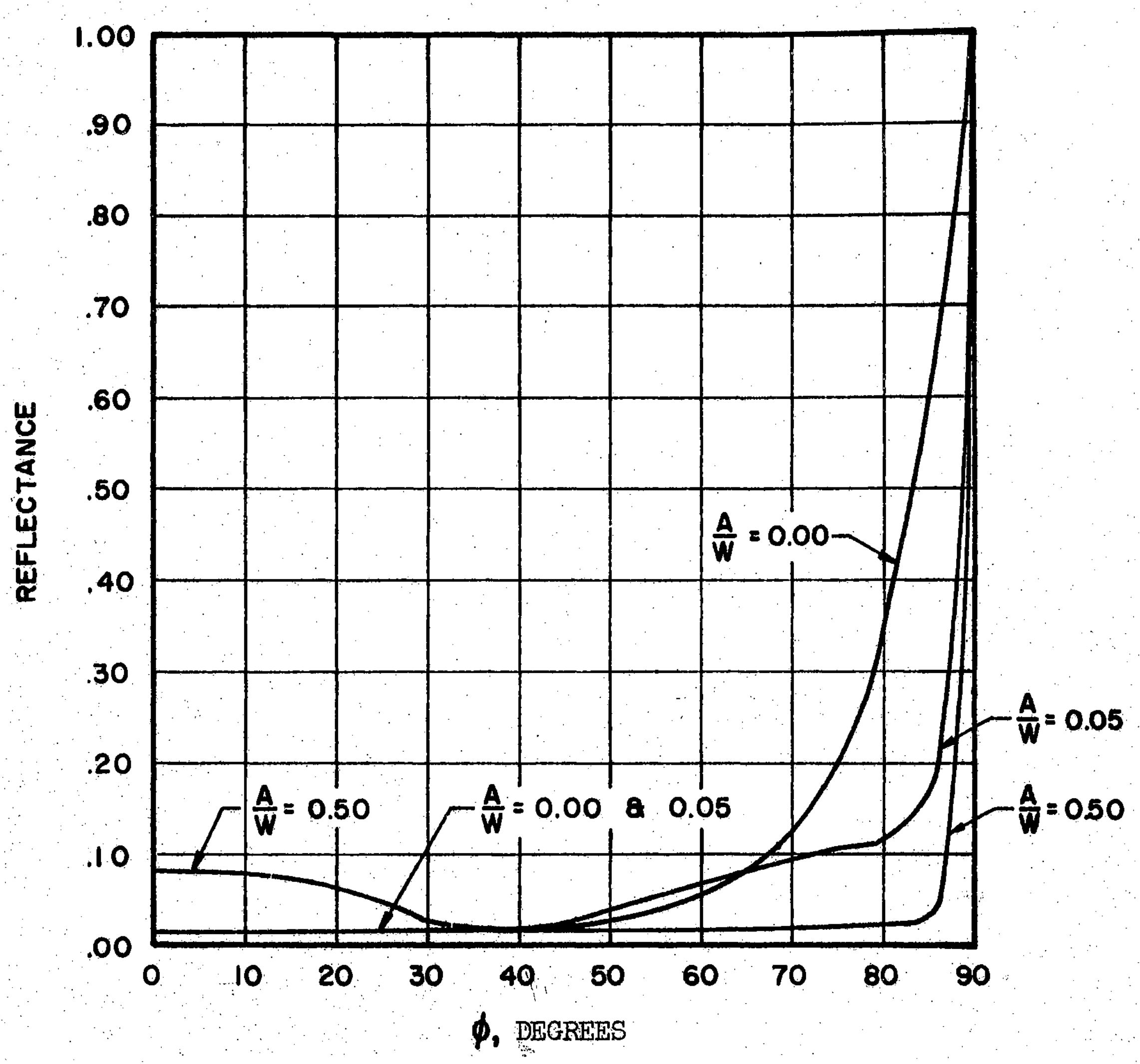

Figure 9. **Reflectance of 2.2 Micron Energy from a Water Wave** as a Function of Angle of Incidence, ϕ , and the Ratio of the Water Wave Amplitude, A, to Water Wave Length, W .

TABLE VI

THE REFLECTANCE OF A WATER WAVE SURFACE FOR DIFFUSE INCIDENT ENERGY AT 2.2 MICRONS

Ratio of the Water Wave Amplitude to the Water Wave Length	Reflectance
0.00	0.063
0.05	0.048
0.10	0.039
0.25	0.035
0.50	0.033

CHAPTER IV

EXPERIMENTAL APPARATUS

An experimental apparatus was developed in order to determine experimentally the change in the reflectance of a water surface when a monolayer film was placed on the water surface. The three basic optical setups used for experimental measurements are shown schematically in Figures 10, 11, 12. Plates I and II show pictorially the setup illustrated in Figure 11. The basic apparatus requires the following elements: (a) a monitored and chopped energy source, (b) an energy collimator, (c) a wave-length selector, (d) a polarizer, (e) a gonio-reflectometer device, which includes a water trough for the test surface, and (f) a detection system. A description of these elements follows.

Energy Sources

The two energy sources which were found to be useful in this experimental work were a carbon arc[1] and a tungsten filament lamp.[2] The carbon arc operates around 3800°K and the glass enclosed tungsten filament operates around 2300°K.

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[1] The carbon arc energy source was supplied by the Mole-Richardson Company of Hollywood, California.

[2] The tungsten lamp was a General Electric type 9 AT-8 1/4 with a vertical filament.

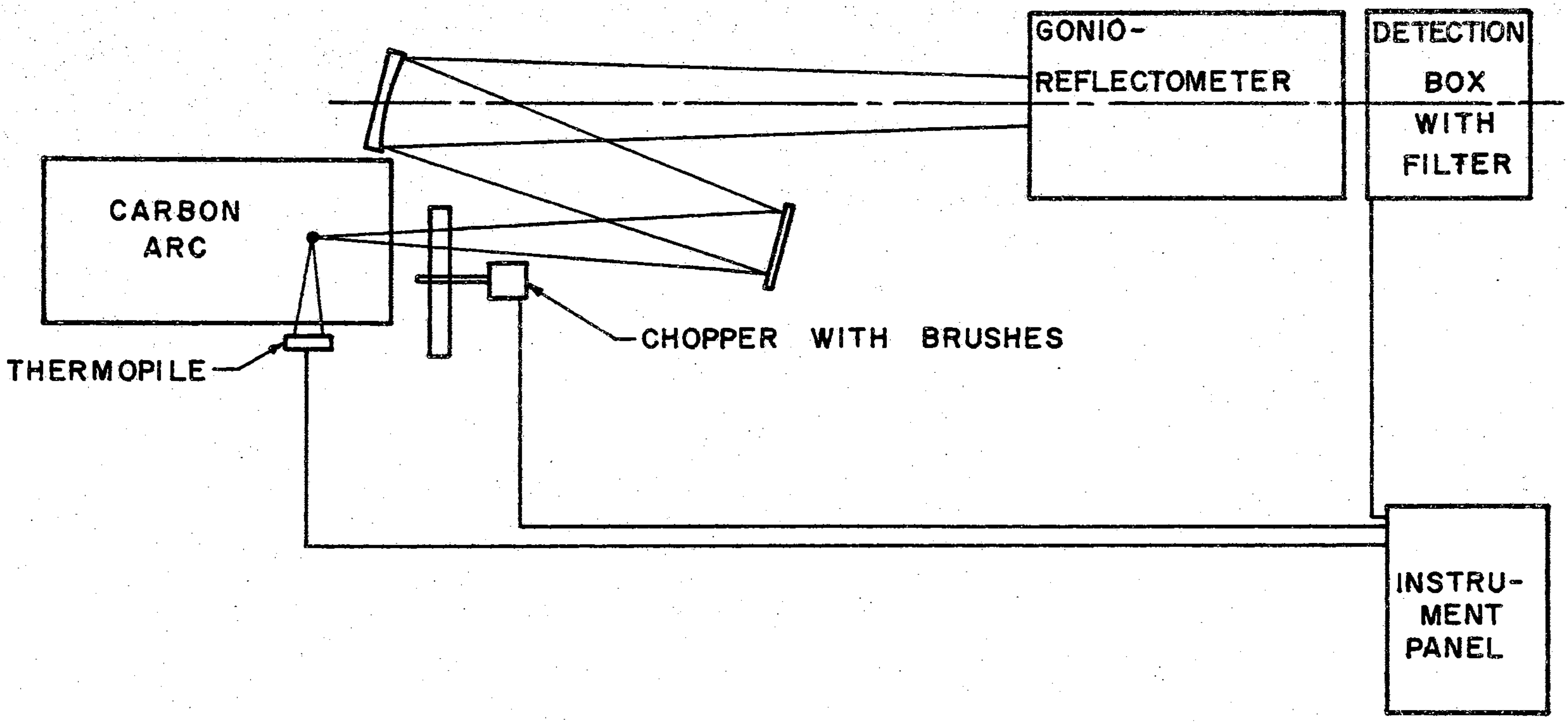


Figure 10. Optical Layout for Carbon Arc with Filters

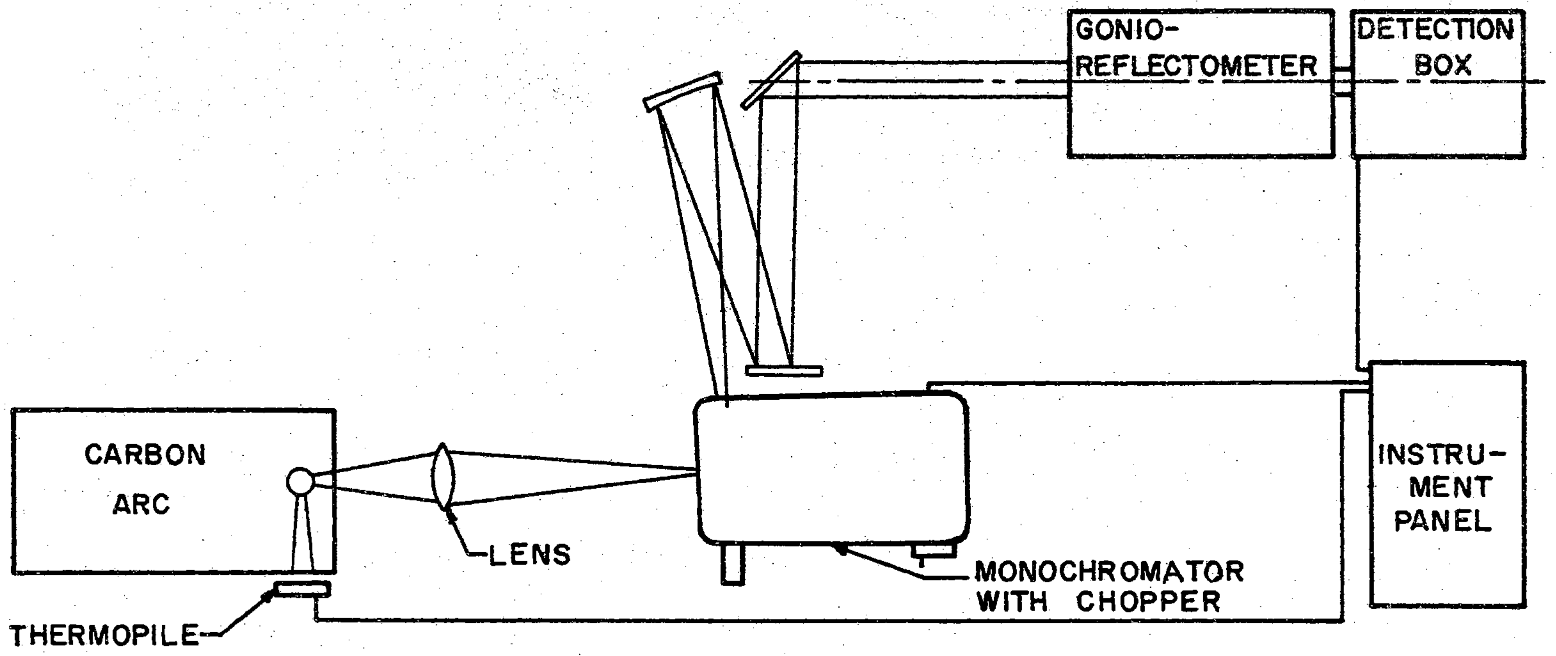


Figure 11. Optical Layout for Carbon Arc with Monochromator

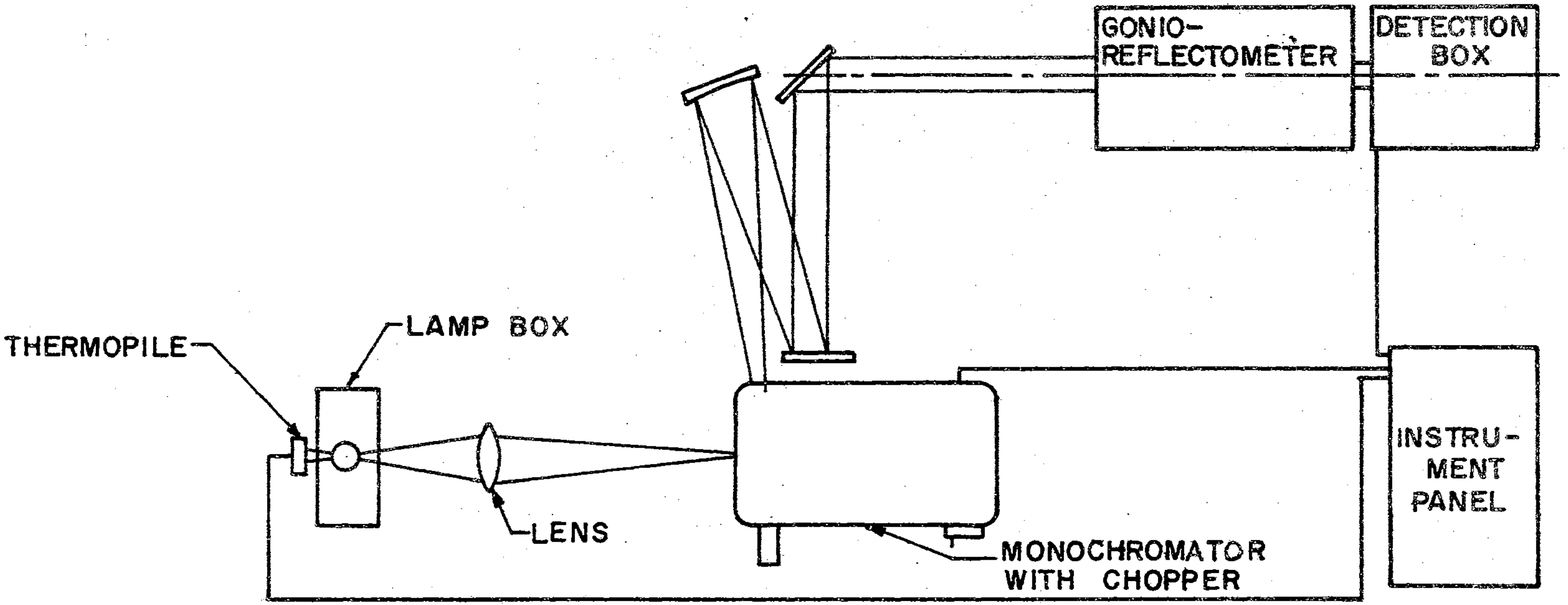


Figure 12. Optical Layout for Tungsten Source with Monochromator

PLATE I

GENERAL APPARATUS

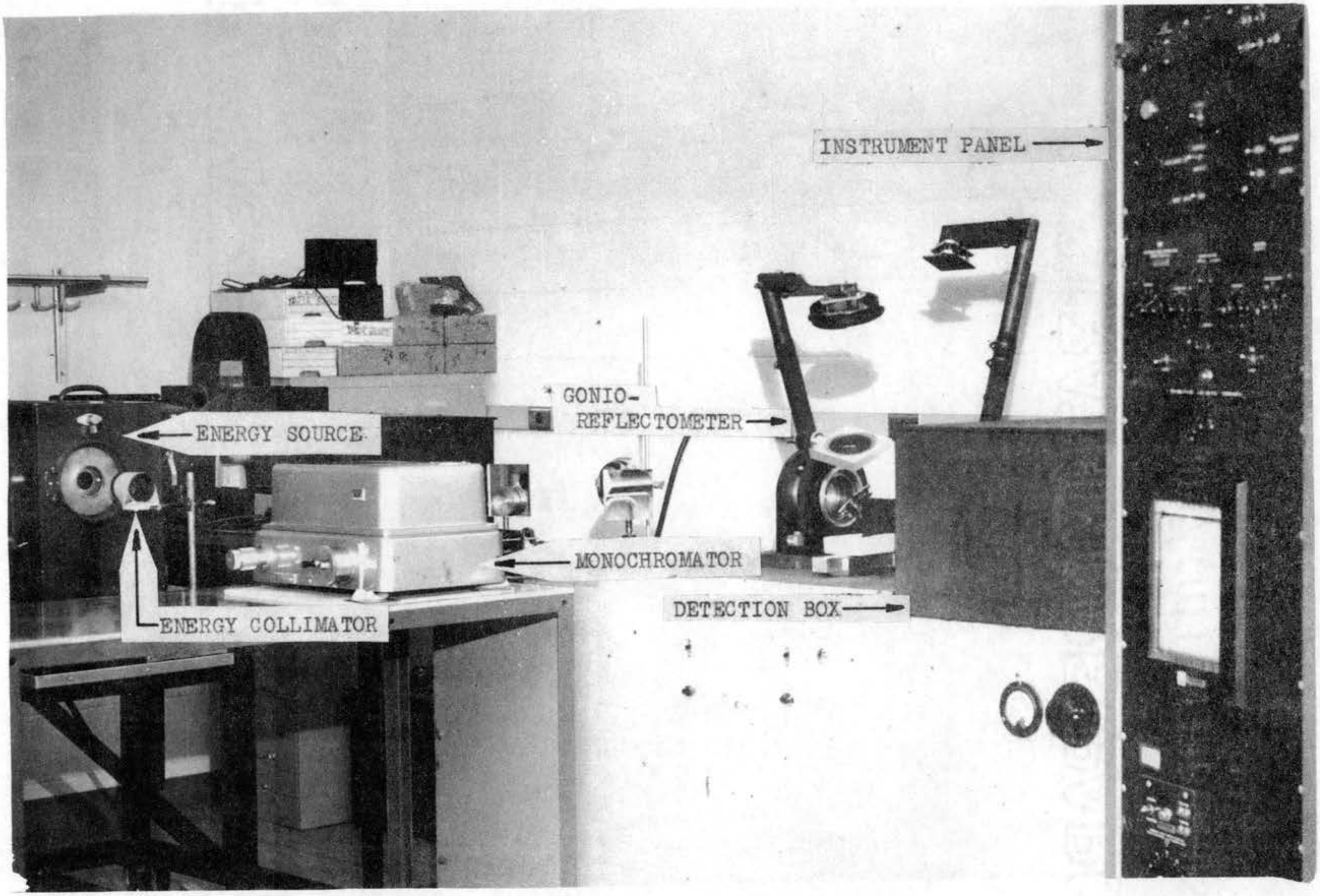
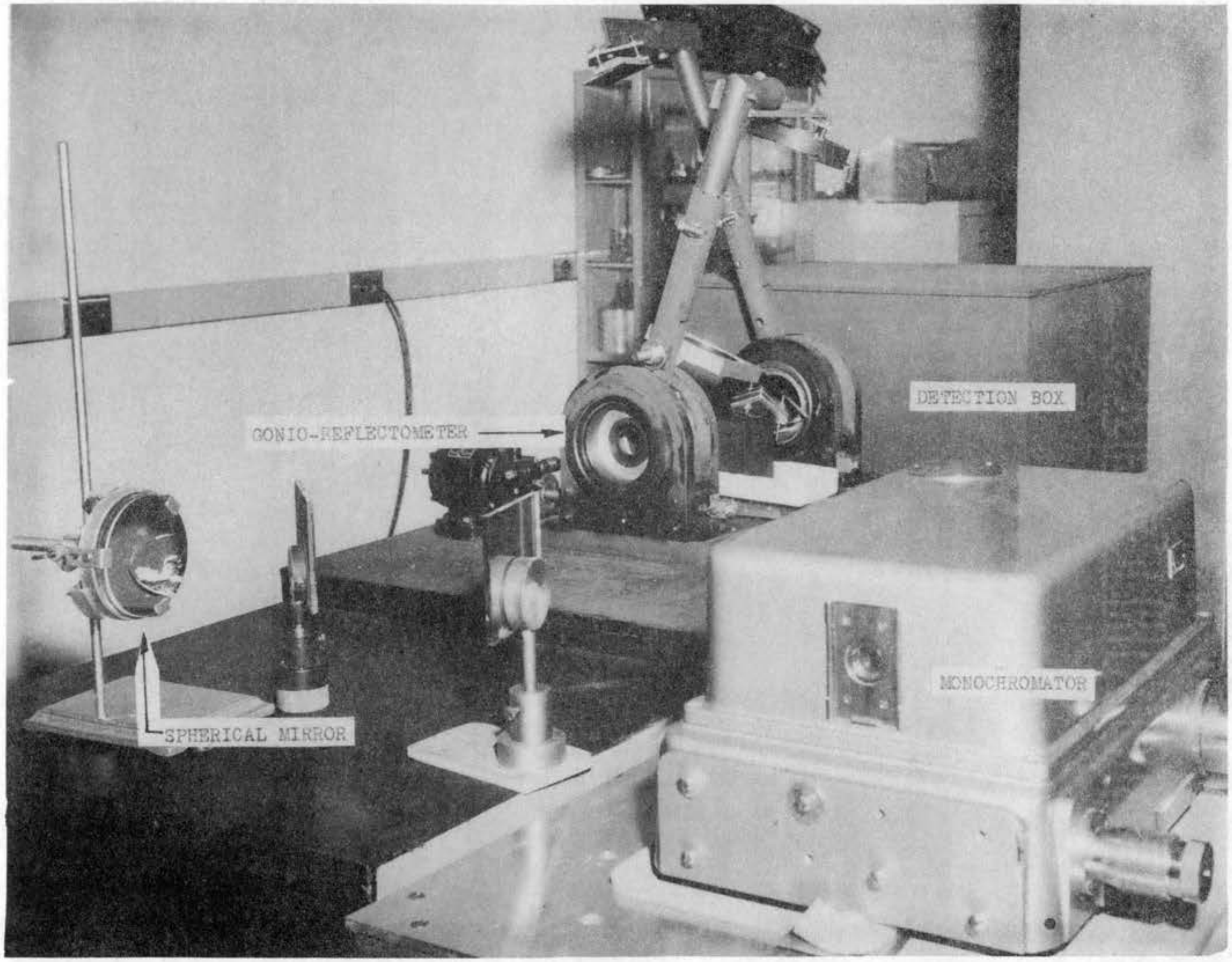


PLATE II

MONOCHROMATOR AND GONIO-REFLECTOMETER



The carbon arc was equipped with manual controls for determining the positions and the feed rates of the positive and the negative electrodes. A variable voltage power supply transformer with a built in arc-stabilizing series ballast resistance was also provided by the manufacturer. For each experimental run, care was taken to adjust the electrode positions, the electrode feed rates, and the arc voltage to achieve the optimum stability of the arc. This process was aided by the projection of a visible image of the arc onto a white paper screen. This made it possible to detect visual changes in the electrode positions and major arc fluctuations.

Since the arc, even when it was visually stable, did have a tendency to change its intensity, a thermopile[1] and a recorder[2] were used to monitor the source. By considering that the energy seen by the monitoring thermopile and the energy in the test apparatus beam originated from the same point and by assuming that the monitoring thermopile was a grey receiver and that the source was a grey emitter, it was possible to calculate the relative change in the test signal. This was done by assuming that the monitoring thermopile saw an energy fluctuation ratio of $\frac{(T + \Delta T)^4}{T^4}$ and that the energy beam for the test apparatus underwent a corresponding fluctuation as determined by Planck's Law. Calculations to correct for these changes in the energy level were made with the aid of a computer. The computed results for the carbon arc and for the tungsten energy sources are presented in Figures 13 and 14, respectively.

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[1] The thermopile had surface dimensions of 1.5 mm. by 16 mm. and was manufactured by the Eppley Laboratory of Newport, Rhode Island.

[2] The recorder used was a Speedomax H Recorder, Model S, supplied by the Leeds and Northrup Corporation.

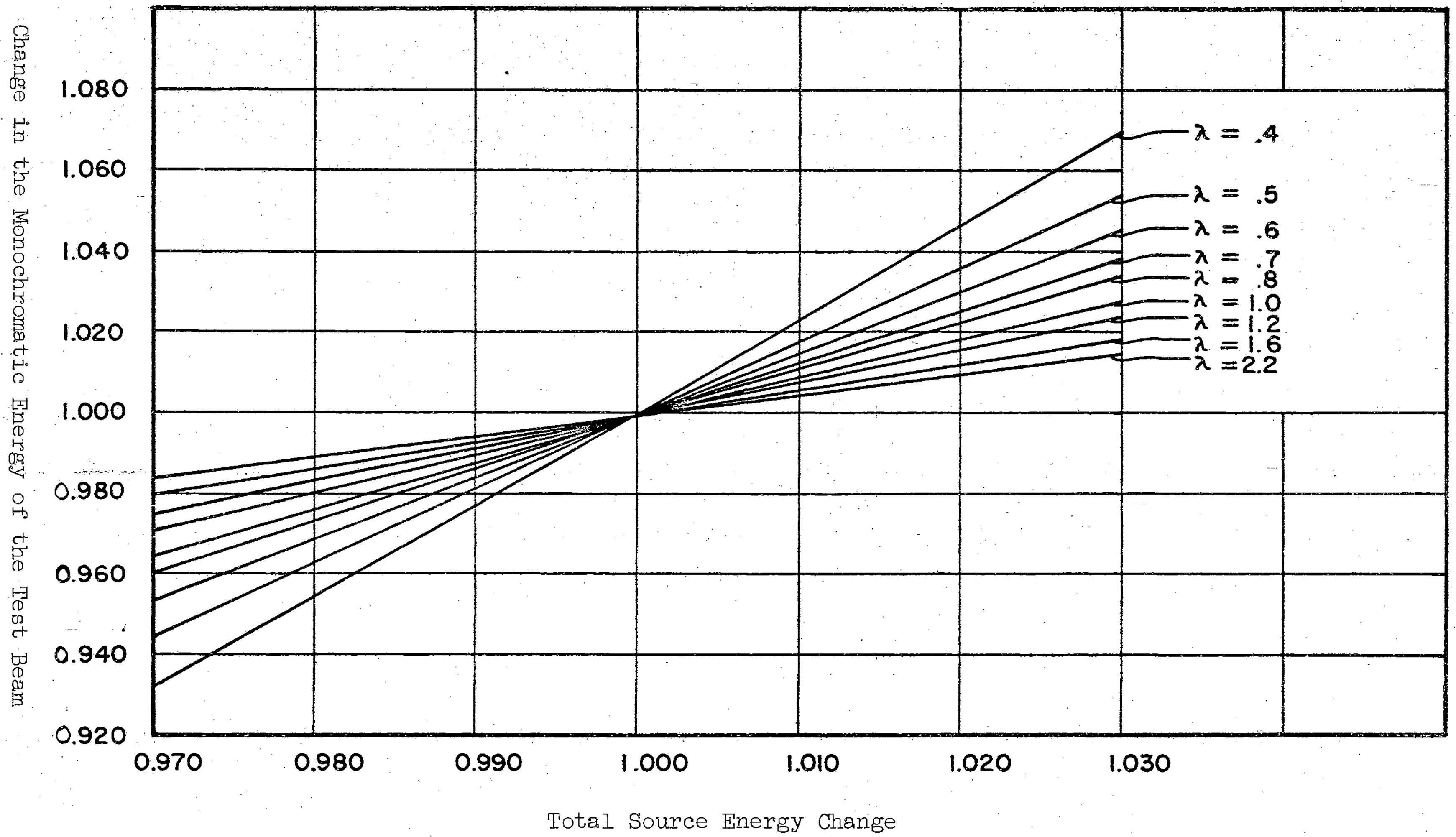

Figure 13. The Change of the Test Beam Monochromatic Energy from the Carbon Arc Source as a Function of the Wave Length and the Corresponding Change of the Total Source Energy

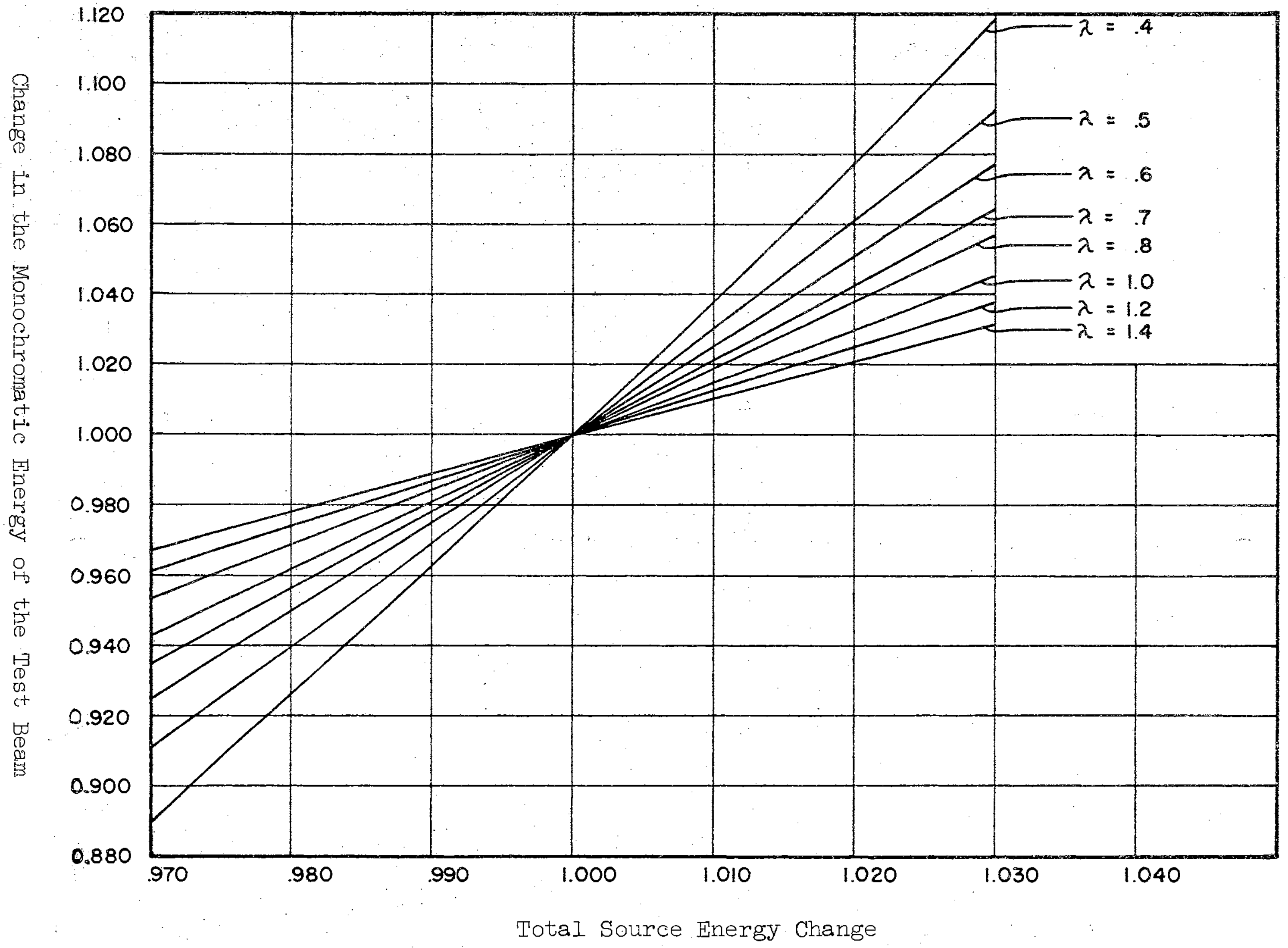

Figure 14. The Change of the Test Beam Monochromatic Energy from the Tungsten Source as a Function of the Wave Length and the Corresponding Change of the Total Source Energy

The second energy source was that of a tungsten filament lamp. A constant voltage transformer was used to smooth the line voltage, and a variable transformer was used to select the appropriate voltage across the filament. As in the case of the carbon arc, the monitoring thermopile was used to give a continuous recording of the relative source energy.

Since a pulsating energy beam, chopped at 13 cycles per second, is generally used with the available detectors and the amplification system, a signal chopper was located in the optical path. The position of the chopper used in the setup for studying wide wave bands of energy is shown in Figure 10. In the study of smaller wave bands of energy as shown in Figures 11 and 12, the chopper in the monochromator was used.

Energy Collimators

Two basic energy collimator systems were used. The first system, as shown in Figure 10, consists of a front surface spherical mirror with a 21 1/2-inch focal length. This was used to direct slightly converging energy into the gonio-reflectometer system. In the second system shown in Figures 11 and 12, a two-inch diameter glass lens with an eight-inch focal length was used to focus energy onto the entrance slit of the monochromator. The energy leaving the monochromator was then diverging, and a 21 1/2-inch spherical mirror was used to slightly reconverge it and direct it to the gonio-reflectometer system.

During use of both the above mentioned collimators, care was taken to insure that there were no losses due to the divergence of the energy beam after it reached the water surface of the gonio-reflectometer. For a constant energy source, this care in collimation insured that the total quantities of energy reaching the detection system would be decreased only by mirror and test surface absorption losses. As the mirror absorption losses are constant for reasonable time intervals, the relative values of signal would indicate the relative values of the reflectance of monolayer-covered and clean water surfaces.

Wave-Length Selectors

Two methods of wave-length selection were used. The first was that of using the optical setup shown in Figure 10 and various filters. Figures 15 and 16 show the variation of signal per wave length as a function of wave length for the various filter and source combinations. The information noted in Figure 15 was determined by using the monochromator, a fused-quartz prism, a sulfur sphere, and the Reeder thermopile. Fused-quartz crystals have been shown to have a relatively flat transmission curve over the 0.2 to 2.0 micron region, with major absorption bands at 2.2 and 2.8 microns.[1] Similarly, the information given in Figure 16 was determined using the monochromator, a NaCl prism, a sulfur sphere, and the Reeder thermopile. Crystals of NaCl have also been shown to have a flat transmission curve for the 0.4 to 14.0 micron region.[2]

The second method of wave-length selection is illustrated in Figures 11 and 12. This method made use of a monochromator[3] as is illustrated in

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[1] Optical Crystals, Bulletin 1100, Isomet Corporation, Palisades Park, New Jersey, 1963, p. 36.

[2] Ibid., p. 12.

[3] The monochromator was a Model 99 unit made by the Perkin Elmer Corporation of Norwalk, Connecticut.

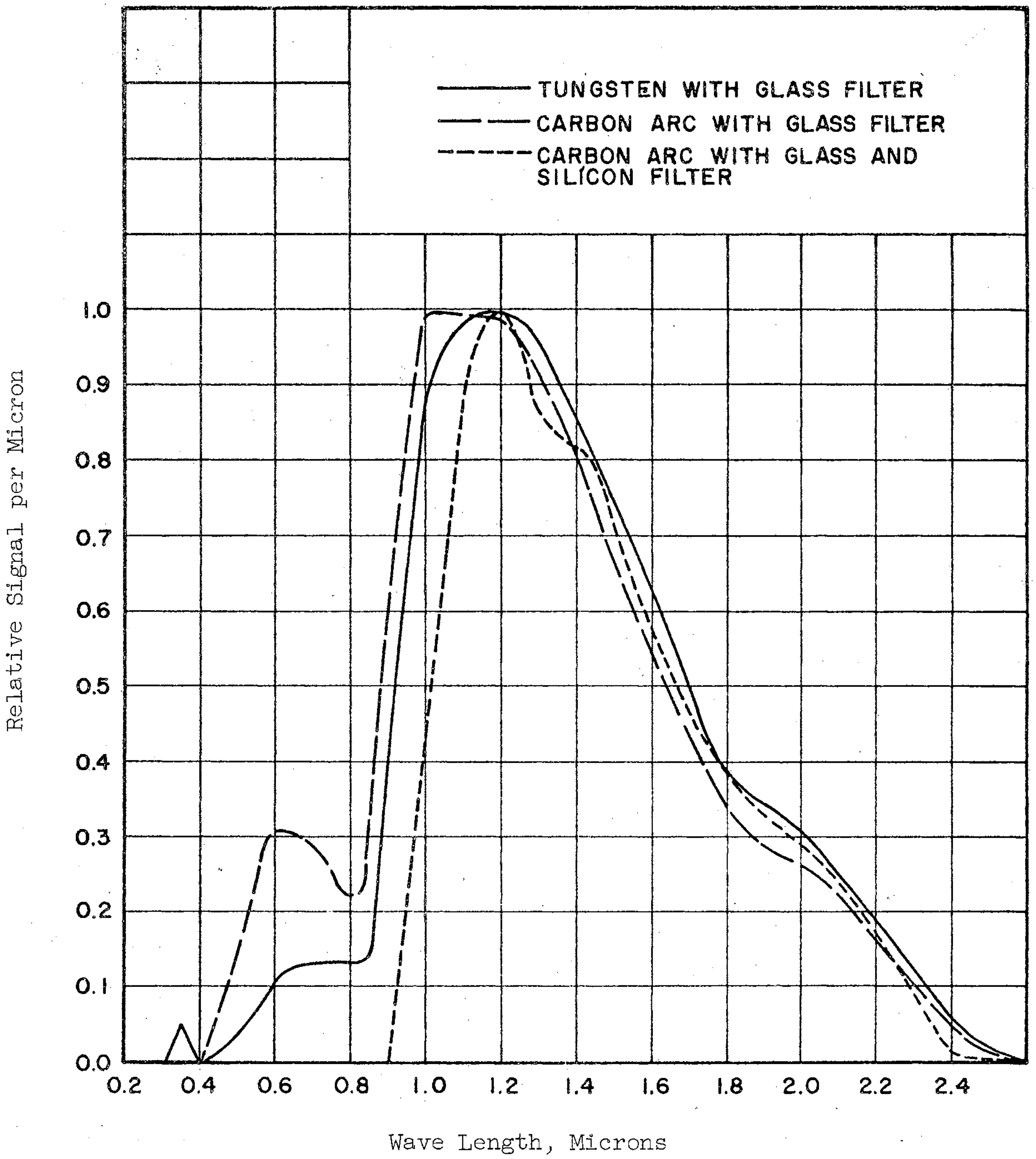

Figure 15. Energy Transmission for Short Wave Length Filters

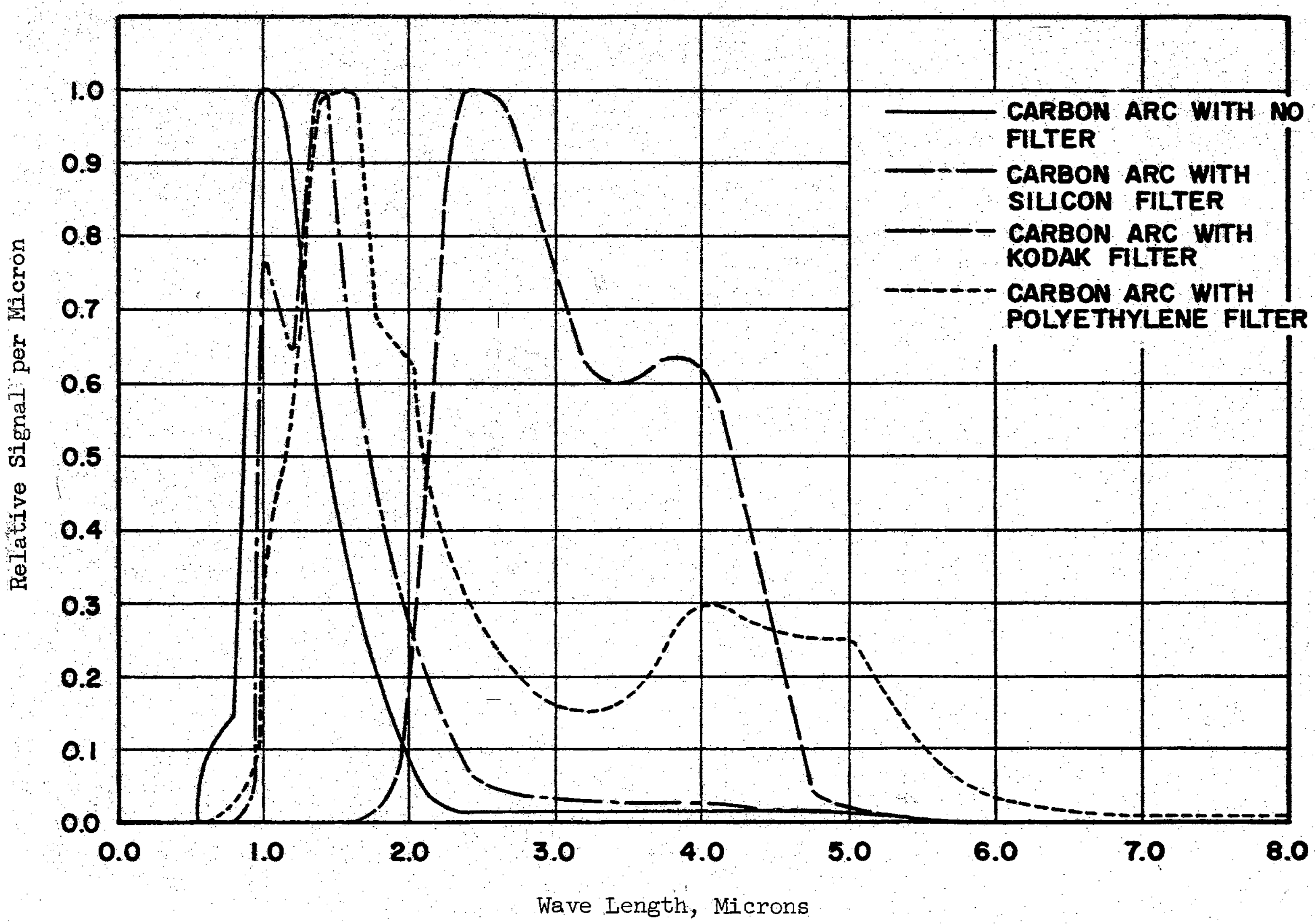

Figure 16. Energy Transmission for Long Wave Length Filters

Figure 17. Calibrated fused quartz and NaCl prisms were used in the monochromator to isolate energy in the 0.4 to 0.8 micron and 1.0 to 2.4 micron ranges, respectively.

Polarizers

Calibrated polarizers were available from a manufacturer to cover the wave-length ranges of 0.3 to 0.75 and 1.0 to 2.4 microns.[1] These polarizers were used in the optical setups which used the monochromator, as was shown in Figure 11 and Figure 12. A mounting device was positioned on the entrance arm of the gonio-reflectometer, as is shown in Figure 18. By using the two available polarizers, it was possible to have energy polarized in either the s-plane or the p-plane of the water surface over the wave-length range of 0.3 to 2.4 microns. These made possible the experimental determination of the s-plane and the p-plane reflectance ratios over this range.

Gonio-Reflectometer

The gonio-reflectometer is a device designed to measure the specular reflectance ratio of a surface as a function of the angle of incidence of the incoming energy. A side view pictorial representation of the gonio-reflectometer is shown in Figure 18 and in Plate III. As the gonio-reflectometer is a three-dimensional device, the entrance and exit portions have been separated in Figure 18.

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[1] The polarizers were Model Nos. LMA-1380 and LMA-1500, and they were purchased from Isomet Corporation of Palisades Park, New Jersey.

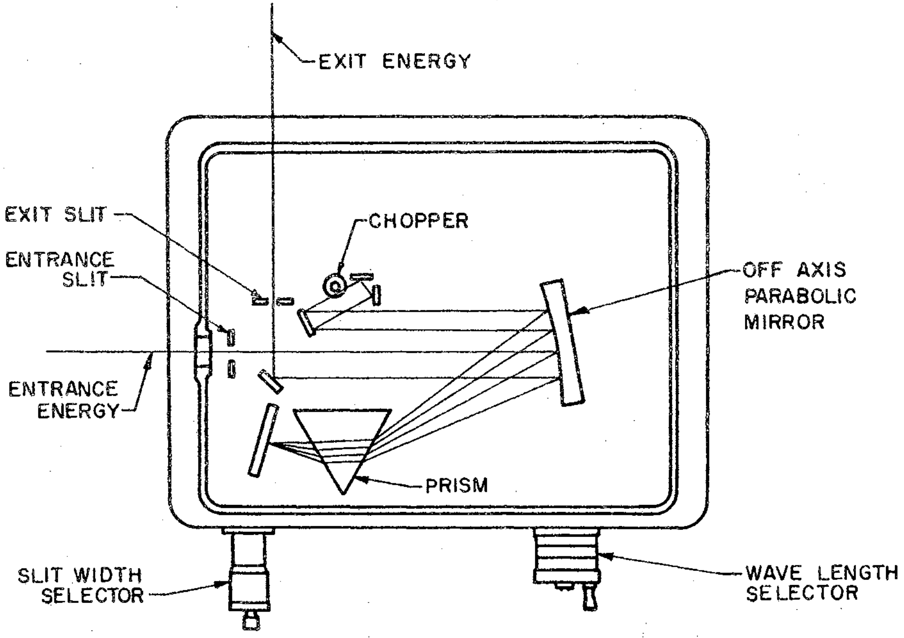


Figure 17. Monochromator Optical Schematic

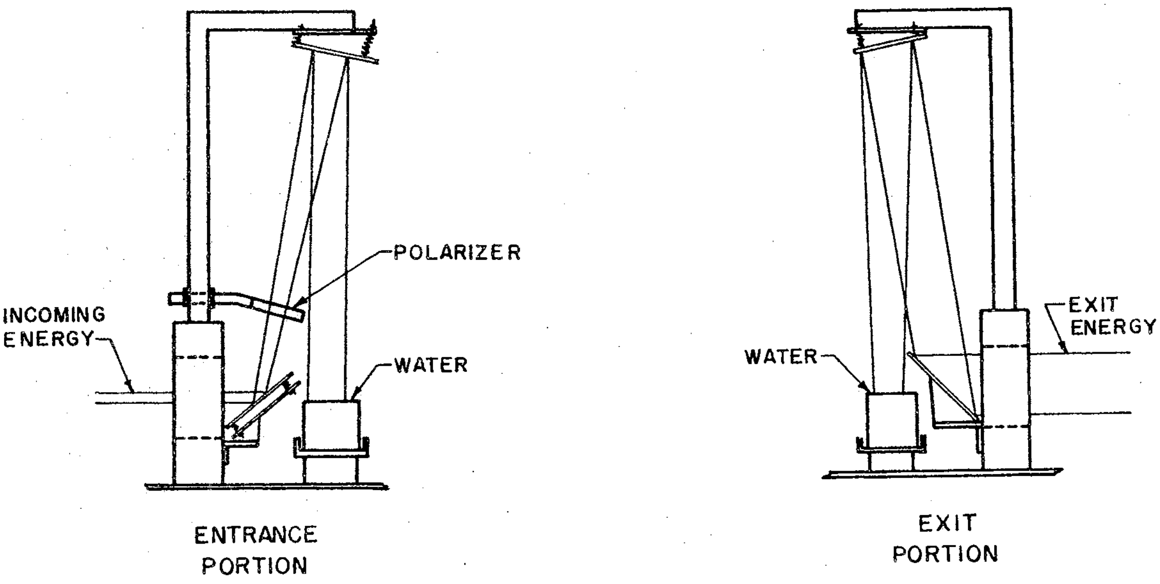
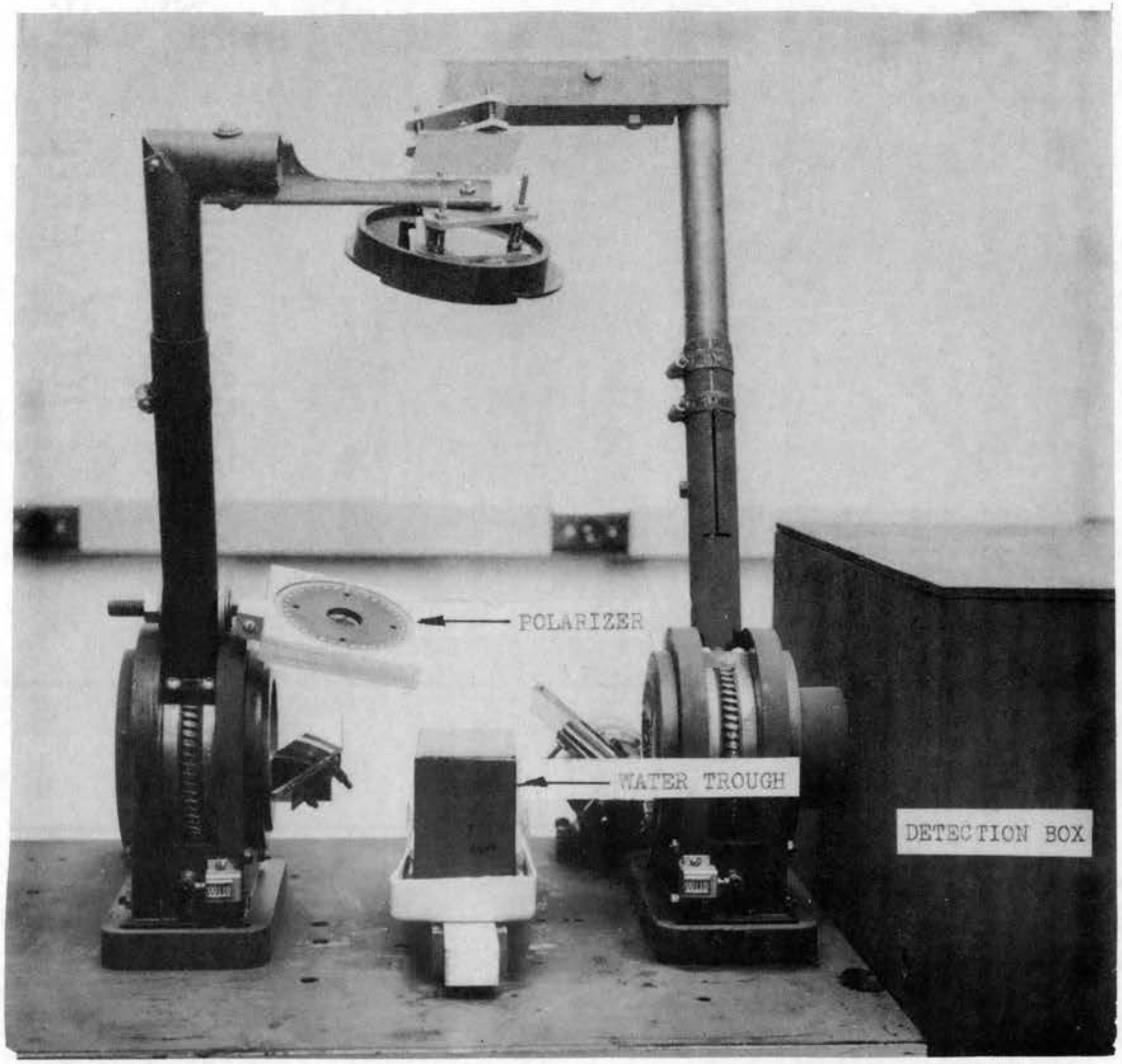


Figure 18. Gonio-Reflectometer (Side View)

PLATE III

GONIO-REFLECTOMETER WITH WATER TROUGH



The water trough and the overflow catch pan are also shown in Figure 18. The water trough was constructed from plexiglass. The sides were smooth, and the bottom was machined with a 60° v-groove mill to reduce the possibility of significant reflections from the bottom surface. To further reduce this possibility of undesirable reflected energy, the inside surfaces of the trough were painted with a waterproof flat black paint. The top edge of the trough was flat, so that a level water surface could be attained by wiping the excess water away with a teflon barrier. This procedure of wiping not only provided a reproducible location of the level water surface, but also provided a simple method of flooding and wiping to clean any contamination of the water surface.

For the experimental work undertaken, the gonio-reflectometer was arbitrarily set so as to direct the energy beam onto the water surface at an angle of 20°.

Detection System

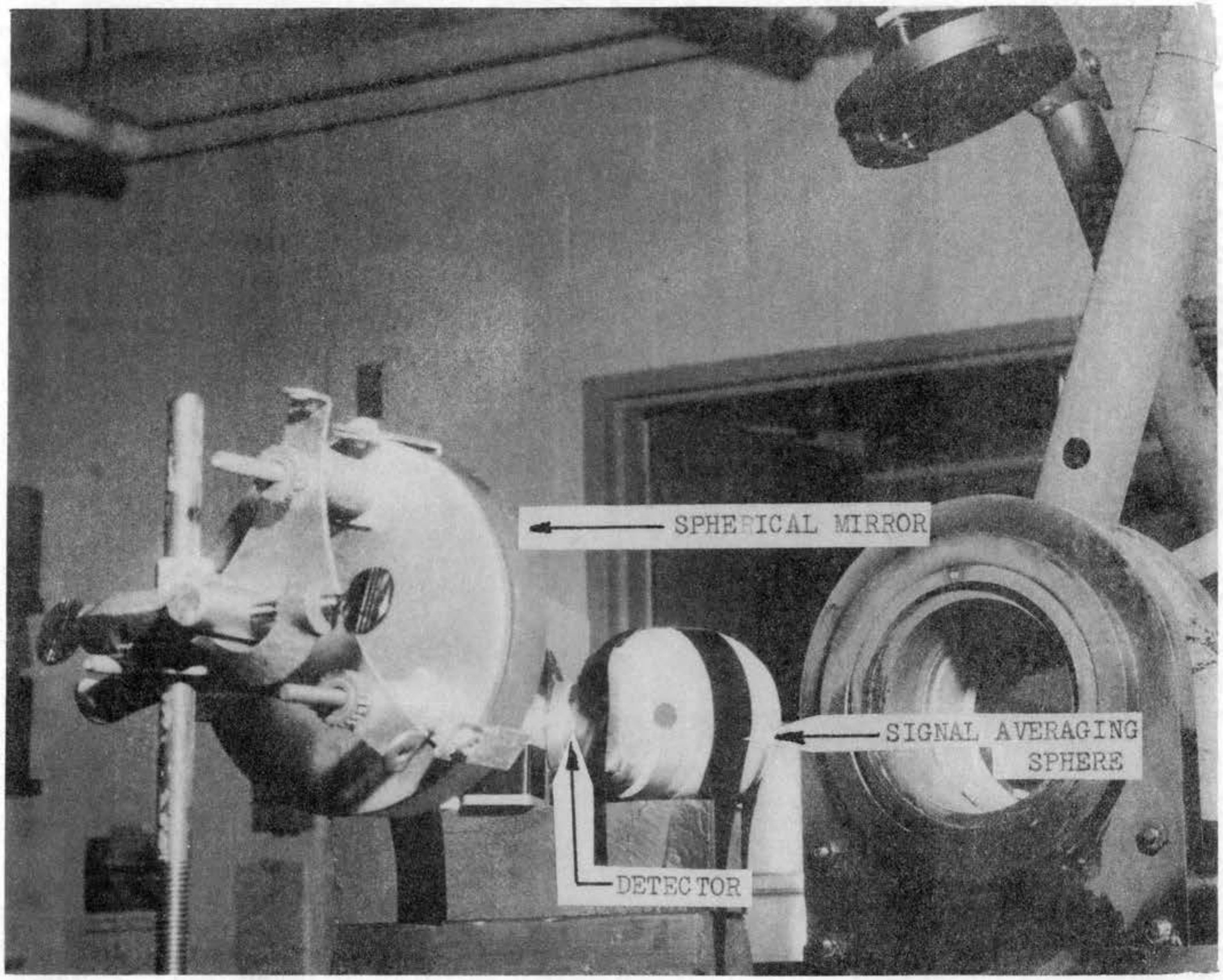
The basic layout of the detection system is shown in Figure 19. The system has four major parts. The first is that of a black detection box, which was designed to prevent stray light from being directed on the detector surface.

The spherical mirror is located in the box in order to focus the energy from the exit portion of the gonio-reflectometer into the entrance hole of the signal averaging sphere.

The signal averaging sphere shown in Plate IV is designed to reduce the variable of the focusing of the energy beam onto a detector surface. Since detectors generally have a variation in their spatial sensitivity,

PLATE IV

GONIO-REFLECTOMETER WITH DETECTOR



the presence of a signal averaging sphere did much to assure the reliability of the signals detected. The entrance hole of the sphere was sufficiently large so that the energy beam could be focused into the sphere. The inner surface of the sphere was specially selected so that the energy leaving the exit hole of the sphere would be diffuse and at the same time large enough to detect. Further, it should be noted that with a diffuse sphere the exit energy is diffuse and proportional to the energy entering the sphere and is not a function of the focusing of the incoming energy. Therefore, a detector placed in such a manner that it sees the exit energy of the sphere will produce a signal which is proportional to the energy of the energy beam which is directed into the signal averaging sphere. Work by the National Bureau of Standards indicated that a special white paint is a very good sphere coating for the 0.5 to 1.4 micron region and that a special sulfur product is very good in the infrared region.[1]

The detectors used for the various measurements were: a standard RCA 1P21 photomultiplier tube, a N-2 lead sulfide cell made by the Eastman Kodak Company, and a large area thermopile built by the Charles Reeder Company.

For the situation where the filters were used as wave-length selectors, the Reeder thermopile was used in conjunction with the sulfur-coated sphere. The energy wave bands in Figure 15 and Figure 16 were determined by using this detection setup.

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[1] Private communication from Mr. S. T. Dunn of the National Bureau of Standards indicated that the white paint manufactured by the 3M Corporation under the brand name, "Velvet Coating 101-A10," was both highly reflective and diffuse in the visible range. Also, extensive work done with a special sulfur, supplied under the brand name, "Crystex Sulfur," by the Staufer Chemical Company, 380 Madison Avenue, New York, New York, indicated that it produced a very good sphere surface out to at least 7.0 microns.

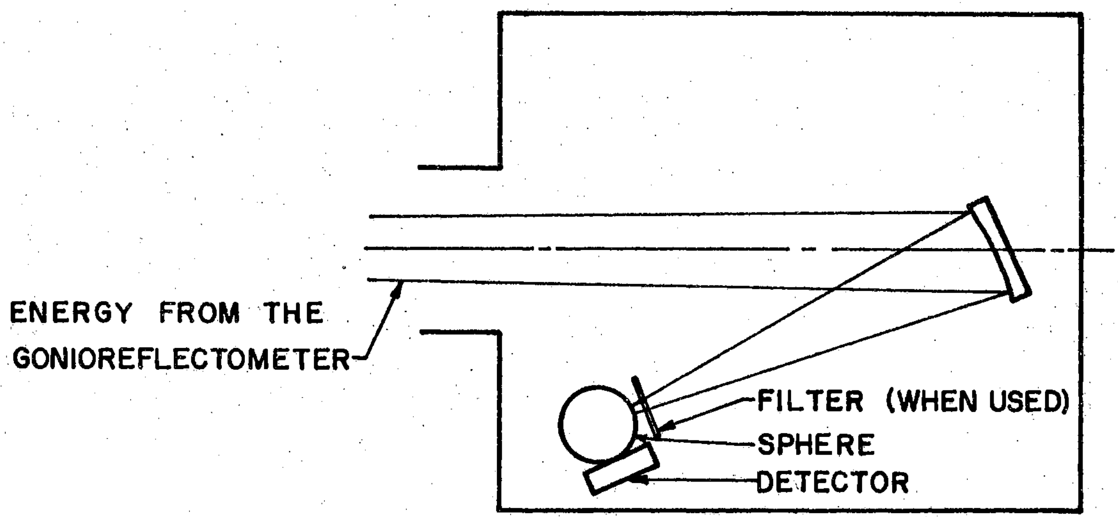

Figure 19. Detection Box (Top View)

For the case where the monochromator was used as the wave-length selector and where polarized energy reflectances were studied, the following detection combinations were used. The photomultiplier tube and the sphere coated with white paint were used in the visible region, and the lead sulfide cell and the sphere coated with sulfur were used in the near infrared region.

The amplifier[1] used in all the above detector systems had four different response levels, so that the signal noise could be significantly dampened while still maintaining the highest possible sensitivity.

In order to determine that the detection system was operating correctly, a signal attenuator was built. As shown in Plate V, it consisted

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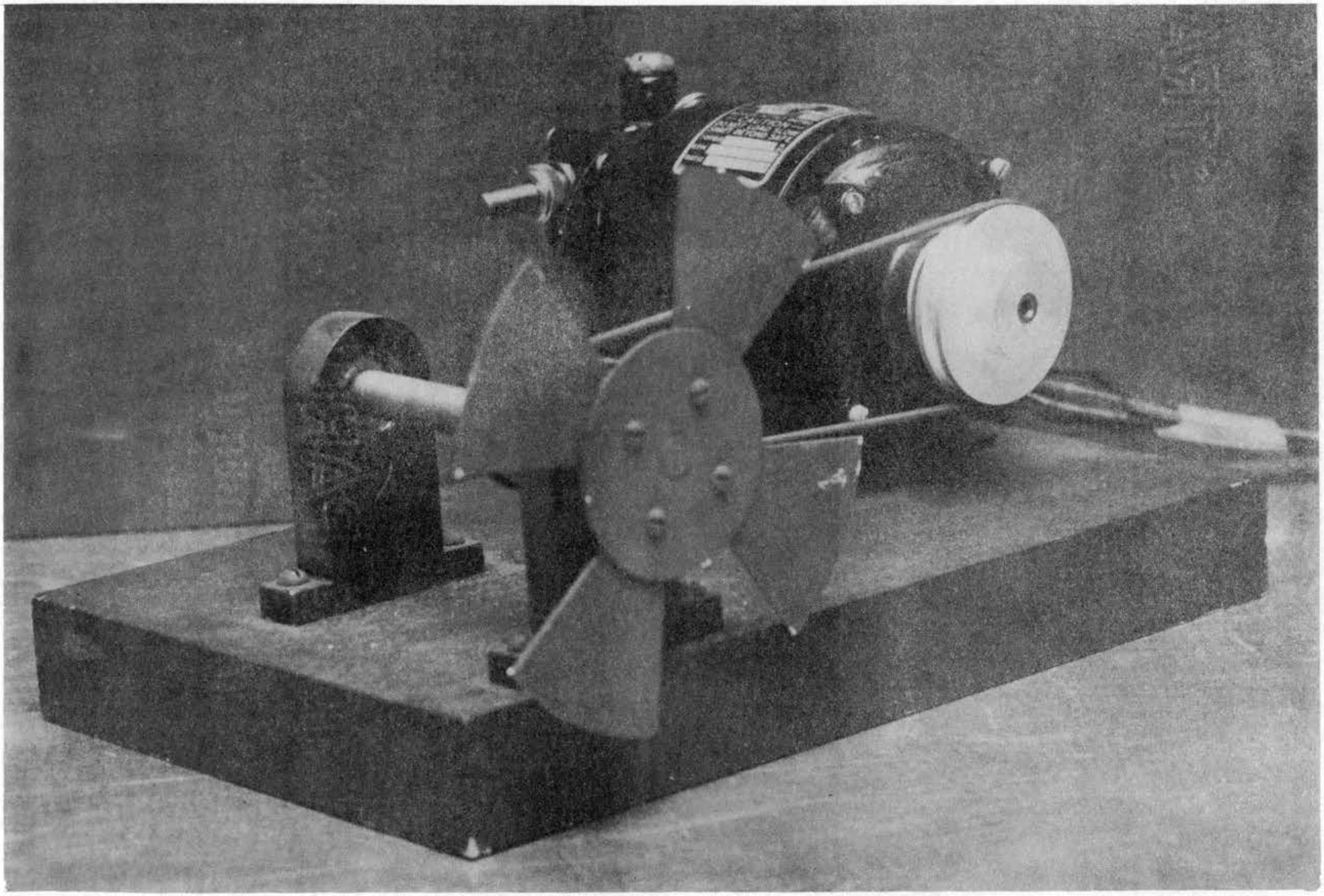
[1] The amplification system was that of a Model 107 amplifier manufactured by the Perkin Elmer Corporation of Norwalk, Connecticut.

PLATE V

SIGNAL ATTENUATOR



of a high speed chopper which when placed in the energy beam would attenuate the energy transmitted by 50 per cent.

CHAPTER V

EXPERIMENTAL TECHNIQUE

The experimental techniques for this research fell into two main areas. The first area included the work done relative to the quality of the test surfaces, which were the water surfaces and the cetyl alcohol monolayer-covered water surfaces, being investigated. The second area was that of the measurement of the relative reflectance of an energy beam incident at 20° onto the above mentioned surfaces.

Test Surface Quality

The test surfaces were designed to be perfectly smooth and clean surfaces. Distilled water was used, and a wiping and flooding process was developed to insure that the exposed surface was free from significant contamination. This process involved flooding the test surface trough with distilled water and then wiping off the excess water with five strokes of a nitric acid cleaned teflon strip. The flooding process worked to wash the old surface away and to cause a new and clean water surface to be formed. The wiping process assured the fact that such was the case, as with each wiping the old surface molecules were directed toward the overflow pan and a new water surface appeared.

The problem of the application of a cetyl alcohol monolayer onto the water surface was answered with the use of a 1.00 cc. syringe and a cetyl alcohol in methanol solution. The solution strength was 4 gm/cc.

By use of the standard indicator oils developed by the Bureau of Reclamation, it was possible to detect the presence of a compressed monolayer when one drop of solution (approximately 0.005 cc.) was applied to the 12.5 square inches of water surface (77) (78). The indicator oils had the property of forming either surface films or lenses on the surface, depending on the surface pressure of the oil and of the surface being studied. Thus, the indicator oils were, themselves, considered to be surface contaminants. Therefore, they were not used in actual test runs which determined the relative reflectance of monolayer-covered water surface. They were used, instead, to develop an assurance that the monolayer application technique produced a compressed monolayer (one with a surface pressure of approximately 40 dynes/cm.) and that the flooding and wiping process produced a clean water surface (one with a surface pressure of zero dynes/cm.).

Another interesting problem in the quality control of the test surfaces was that of the presence of very small water ripples on the water surface. These ripples were kept to a minimum by placing the energy chopping device, which had moving parts that tended to induce these ripples, on a separate mounting stand. Isomode pads[1] were used to isolate the gonio-reflectometer from the vibrations of the building floor. With large vibrations, it was possible to achieve detrimental water waves; but with care being taken to eliminate such vibrations, the ripples were not found to be detrimental.

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[1] These vibration pads were purchased from the Isomode Division of M. B. Electronics, P. O. Box 1825, New Haven, Connecticut.

The Measurement of the Relative Reflectances

The experimental apparatus and the calculations required for any change in energy source were described in the previous chapter. The technique of measuring the reflectance of energy reflected from a cetyl alcohol monolayer-covered water surface relative to that of a clean water surface is as follows.

First of all, the idea of determining the reflectance of one surface relative to that of another involves a simple analysis. The energy reaching a detector is some particular solid angle of the source energy which is transmitted through the wave-length selector, the various diaphrams, and the integrating sphere. In addition, this source energy is reduced by losses at each reflecting surface. If one reflecting surface is replaced by another surface which is positioned identically the same as the original surface, the signal indicated by a linear detection system should be the original signal times the ratio of the reflectances of the two surfaces, if the source is steady. Thus, the reflectance ratio is proportional to the signal ratio.

Consequently, in the present study, the original surface was selected as the clean water surface. After a detector signal had been determined for the water surface, a monolayer was applied; and the corresponding signal was recorded. By noting the zero of the recorder and by using detectors which were linear, it was possible to determine the reflectance ratio. Corrections were made for the variation of the level of the energy source, if such were required.

Values of the reflectance ratios for energy polarized in the s-plane and the p-plane were determined in the above mentioned manner where the incident energy was polarized respectively in the s-plane and in the p-plane. Over the range where these polarizers were considered to be good (0.4 to 2.4 microns) the monochromator and the fused quartz and NaCl prisms were used to isolate energy bands. The detectors which were used were selected because of their ability to detect low levels of energy. No values of the unpolarized reflectance ratios were taken directly with the monochromator system because of the fact that the monochromator itself acts as a significant polarizer. However, for unpolarized light, it is shown in Appendix A that the reflectance is the average of the s-plane and the p-plane reflectances.

Values of the unpolarized reflectance ratios were determined with the use of filters as wave-length selectors. The Reeder thermopile and the sulfur coated signal averaging sphere, which were used in determining the energy transmitted by the various filters, were also used in this experimental work.

CHAPTER VI

EXPERIMENTAL RESULTS

The Presentation of the Experimental Results

The average of the results for the experimental tests which used cetyl alcohol as the monolayer forming substance and which used filters as the wave-length selectors are given in Table VI. These results are consequently limited to the reflection of unpolarized energy in the given wave bands as indicated previously in Figures 15 and 16.

For energy that was polarized in the p-plane, the average of the experimental results are given in Table VIII and are shown graphically in Figure 20.

Similarly, for energy that was polarized in the s-plane, the average experimental results are presented in Table IX and in Figure 21.

In Figure 22, the average of the experimental results for the s-plane and the p-plane is presented to indicate the results which would be noticed if unpolarized monochromatic energy were incident on a cetyl alcohol monolayer-covered water surface.

The Discussion of the Experimental Results

The analysis of experimental results must answer three basic questions. First of all, the results must be shown to be reproducible. They must then be shown to be consistent with themselves. And then, they must be shown to be either consistent or inconsistent with the results predicted by the analytical model.

The question of reproducibility raises the problem of the scatter of the data. In the experimental work, a minimum of four test runs was made for each condition reported. In Tables VII, VIII, and IX, the last column represents the standard deviation, which is defined as:

$$S = \left[\frac{\sum_{i=1}^{n} (d_i)^2}{n-1}\right]^{1/2}$$

where d_i is the deviation of each point from the average, and n is the number of data points.

Thus we note a significant scatter of data, when compared with the

TABLE VII

THE AVERAGE OF EXPERIMENTAL VALUES FOR THE RELATIVE REFLECTANCE OF UNPOLARIZED ENERGY INCIDENT AT 20° USING VARIOUS FILTERS AS WAVE LENGTH SELECTORS

Source	Filter	Peak Wave Length	Wave Length Range	Number of Runs	Average Relative Reflectance	Standard Deviation
Tungsten Lamp	Glass	1.20	0.98 to 2.21	7	0.99	0.015
Carbon Arc	Glass and Silicon	1.20	0.95 to 2.16	4	0.99	0.010
Carbon Arc	Glass	1.02	0.53 to 2.13	5	1.00	0.016
Carbon Arc	Silicon	1.40	0.95 to 2.12	4	1.00	0.007
Carbon Arc	Kodak Cut On	2.40	2.00 to 4.55	5	1.00	0.013
Carbon Arc	One Sheet Polyethylene	1.55	0.95 to 5.15	5	1.00	0.014
Carbon Arc	Without Filter	1.00	0.80 to 1.78	4	1.00	0.008

TABLE VIII

THE AVERAGE OF EXPERIMENTAL VALUES FOR THE RELATIVE REFLECTANCE OF MONOCHROMATIC ENERGY INCIDENT AT 20° AND POLARIZED IN THE P-PLANE

Wave Length	Source	Detector	Number of Runs	Average Relative Reflectance	Standard Deviation
0.40	Tungsten Lamp	1P21 Tube	4	1.04	0.024
0.50	Tungsten Lamp	1P21 Tube	5	1.02	0.022
0.60	Tungsten Lamp	1P21 Tube	4	1.01	0.007
0.60	Carbon Arc	1P21 Tube	6	1.01	0.008
0.66	Carbon Arc	1P21 Tube	4	1.00	0.008
0.70	Tungsten Lamp	1P21 Tube	4	1.01	0.006
0.80	Tungsten Lamp	1P21 Tube	4	1.00	0.010
1.00	Tungsten Lamp	PbS Cell	4	1.01	0.006
1.00	Carbon Arc	PbS Cell	5	1.00	0.004
1.20	Tungsten Lamp	PbS Cell	5	1.00	0.010
1.20	Carbon Arc	PbS Cell	4	1.00	0.010
1.40	Tungsten Lamp	PbS Cell	4	1.00	0.005
1.40	Carbon Arc	PbS Cell	4	1.00	0.004
1.60	Carbon Arc	PbS Cell	4	1.00	0.003
1.80	Carbon Arc	PbS Cell	4	1.01	0.009
2.00	Carbon Arc	PbS Cell	4	1.01	0.005
2.20	Carbon Arc	PbS Cell	7	1.02	0.012

TABLE IX

THE AVERAGE OF EXPERIMENTAL VALUES FOR THE RELATIVE REFLECTANCE OF MONOCHROMATIC ENERGY INCIDENT AT 20° AND POLARIZED IN THE S-PLANE

Wave Length	Source	Detector	Number of Runs	Average Relative Reflectance	Standard Deviation
0.40	Tungsten Lamp	1P21	4	1.04	0.026
0.50	Tungsten Lamp	1P21	5	1.02	0.013
0.60	Tungsten Lamp	1P21	5	1.02	0.009
0.60	Carbon Arc	1P21	4	1.01	0.007
0.66	Carbon Arc	1P21	4	1.00	0.008
0.70	Tungsten Lamp	1P21	6	1.01	0.016
0.80	Tungsten Lamp	1P21	4	1.00	0.009
1.00	Tungsten Lamp	PbS	4	1.01	0.007
1.00	Carbon Arc	PbS	4	1.00	0.007
1.20	Tungsten Lamp	PbS	4	1.01	0.008
1.20	Carbon Arc	PbS	4	1.01	0.009
1.40	Tungsten Lamp	PbS	4	1.00	0.011
1.40	Carbon Arc	PbS	4	1.00	0.015
1.60	Carbon Arc	PbS	4	1.00	0.004
1.80	Carbon Arc	PbS	4	1.00	0.004
2.00	Carbon Arc	PbS	4	1.00	0.007
2.20	Carbon Arc	PbS	4	1.00	0.005

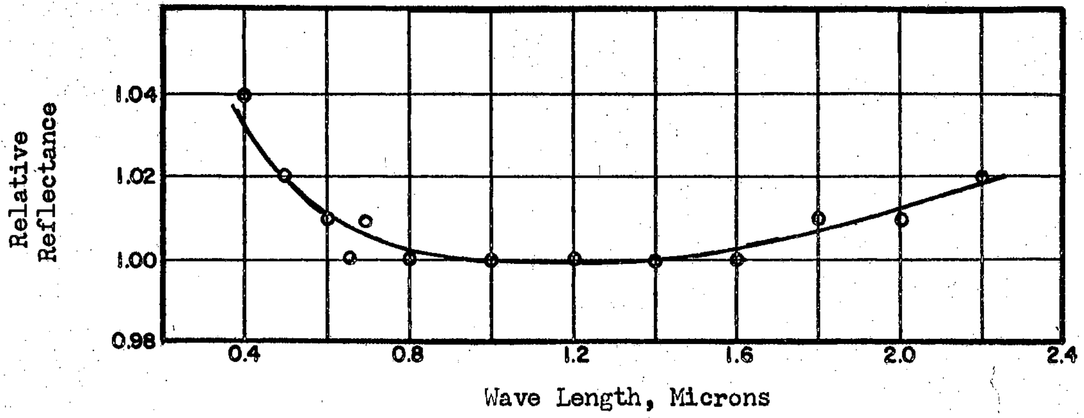

Figure 20. Average Experimental P-Plane Reflectance Ratios for a Monolayer-Covered Water Surface Relative to a Clean Water Surface when Monochromatic Energy is Incident at 20°

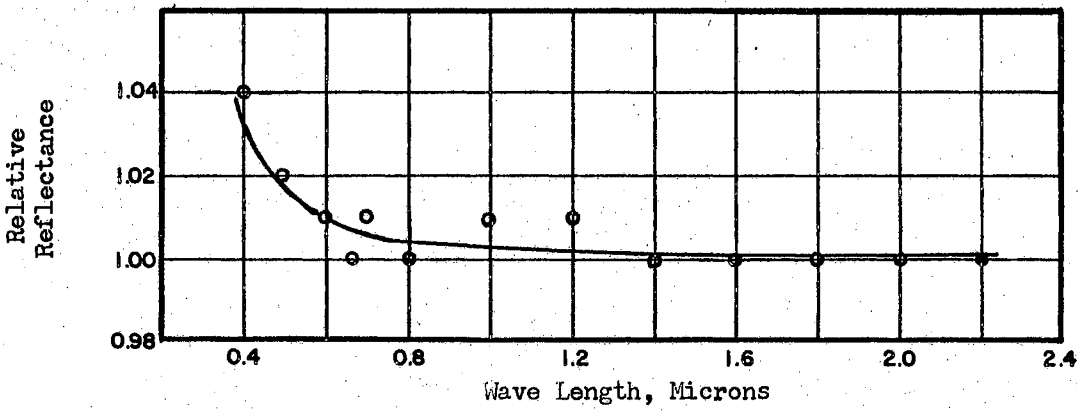

Figure 21. Average Experimental S-Plane Reflectance Ratios for a Monolayer-Covered Water Surface Relative to a Clean Water Surface when Monochromatic Energy is Incident at 20°

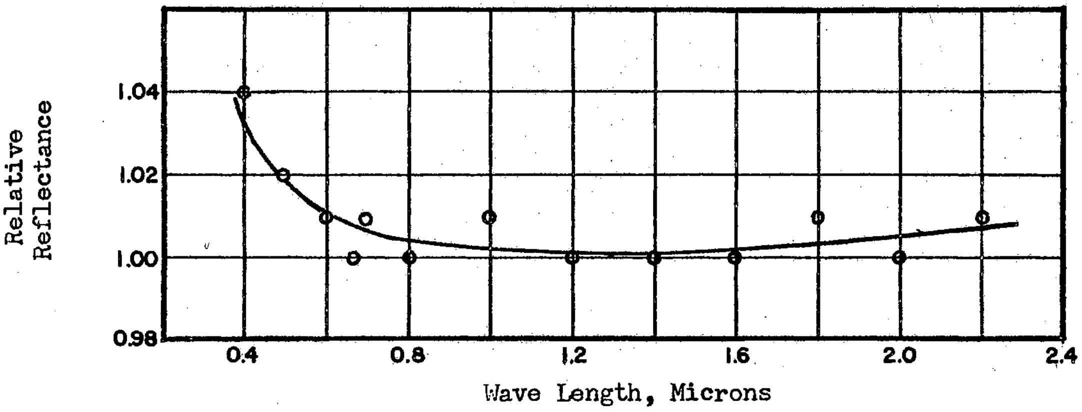

Figure 22. Average Experimental Results for the Relative Reflectance of a Monolayer-Covered Water Surface Relative to a Clean Water Surface when Unpolarized Monochromatic Energy is Incident at 20°

change in the reflectance caused by the monolayer. The following five statements present possible causes for this scatter.

1) There was a serious attempt to determine a procedure that could be used to assure the fact that the water surface being studied was clean. However, slight contamination of the water surface which was not expected may have caused this scatter.

2) Care was taken in the optical alignment to assure the fact that all the energy leaving the water surface, with the exception of that lost by mirror absorption, was directed into the signal averaging sphere of the detector system. However, it is anticipated that a larger degree of wave action and, consequently, dispersion in the reflected energy beam is expected when the water surface is clean than when a monolayer covers the surface. Thus, the possibility of irregular dispersion may contribute to the scatter of the data.

3) A careful experimental study with indicator oils was made in order to assure the fact that one drop of monolayer solution would result in a compressed monolayer on the water surface. However, it is possible that the size of the drops varied enough to cause the scatter of the data.

4) It is known that the energy source varied during the data-taking period. Though a monitoring thermopile was used and a correction procedure was developed, it is still possible that assumptions made in the correction procedure were not valid.

5) Certainly the random noise of the detection system and the fact that low levels of energy were directed into the detector system could account for much of the scatter of the data.

The second procedure for analyzing the results has to be with its self-consistency. This can be checked by noting that for a particular wave length the reflectance of unpolarized energy should be the average of the reflectances of energy polarized in the s-plane and p-plane. Then, by comparing the results tabulated in Table VII with the average of the results found for the various wave lengths in Tables VIII and IX and shown in Figure 22, the self-consistency of the results can be evaluated. By doing this, it is noted that the experimental results using the filters with unpolarized energy generally tend to result in slightly lower values for the relative reflectances, particularly in the short wave lengths. However, this lower value is well within the range of the scatter of the data; and, consequently, it is seen that the data is self-consistent.

The third procedure for analyzing the experimental results is to compare them with the results anticipated from the analytical work. It is noted that the presence of a monolayer caused a change in the experimental unpolarized reflectance of two per cent at 0.5 microns and zero per cent at 2.0 microns. This corresponds with the analytical results found in Tables IV and V for monolayers having an index of refraction between 1.3 and 1.5 and an absorption coefficient between 0.0001 and 0.1000. These optical property ranges include the values anticipated from the literature, which were (a) the index of refraction of palmatic acid being 1.396 at 0.57 microns and (b) the absorption coefficient of cetyl alcohol being 0.007 at 3.4 microns (64) (65). Further, from the analytical study, it is predicted that for constant film thickness and for constant property values, the relative reflectance will increase with a decrease in wave length. The experimental results did note this increase as the short wave lengths were approached.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The basic purpose of this investigation had been twofold. First, it was desirable to determine if the presence of a monolayer on the surface of water changed the energy reflectance. The solution to this question had been answered on the basis of two different approaches.

The primary approach was to investigate analytically and experimentally the relative increase in the reflectance of a flat water surface when a monolayer was applied. This approach assumed that the water was distilled, that the surface was clean, and that no water waves were present.

The results of the analytical study showed that the relative reflectance is increased by arbitrarily increasing the index of refraction of the film, by increasing the coefficient of absorption of the film, and by decreasing the ratio of the wave length to the film thickness. These results are tabulated in Appendix C and are illustrated in Figures 1 through 6 in Chapter III.

The results of the experimental study using cetyl alcohol as a monolayer forming substance were presented in Chapter IV. They indicated that the relative reflectance of a cetyl alcohol monolayer-covered water surface changed on the order of minus one to a positive four per cent. By considering monochromatic energy incident at 20° and in the wave-length region of 0.4 to 2.2 microns, it was shown that with a decrease of the wave length, there was a tendency to increase the relative reflectance. This is a favorable result, as it is noted that the solar energy incident on a lake surface reaches its peak value around 0.5 microns.

The secondary approach to answering the question as to the change in energy reflectance caused by a monolayer was to consider analytically the effect of the presence of water waves on the energy reflected from a water surface. This was a reasonable investigation because it is known that the presence of monolayers on lake surfaces has a tendency to reduce the magnitude of the water waves. The results of this study were presented in Chapter III. In Figure 9, the variation of the energy reflectance with the ratio of the water-wave amplitude to water-wave length and with the angle of incidence of the incoming energy was presented. This figure showed that for near normal incident energy the presence of water waves increases the reflectance of the water surface. Also, for large angles of incidence, the presence of water waves substantially decreases the energy reflectance. This study further showed that for diffuse irradiation, as would be the case for a very overcast day, the presence of water waves substantially decreases the energy reflectance. These results were presented in Table VI.

The other purpose of this investigation was to develop a new approach to the study of evaporation suppression. Since the energy which drives the evaporation process comes from absorption of the solar and sky radiation, it is felt that the reflectance of various film coverings for lake surfaces is an important, though previously neglected, study. With the variable of energy reflectance being presented for consideration in evaporation suppression studies, it is conceivable that new film-forming substances may be developed, which will have favorable reflection properties and which may be of such significance as to become the dominant factor in the evaluation of an evaporation suppressing substance.

Therefore, it is recommended that future studies be made in order to evaluate the relative reflectance properties of other film-forming substances.

Further, it is recommended that the possibility of developing new film-forming substances be considered so that substances with favorable optical properties may be found.

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APPENDIX A

INTENSITY AND REFLECTANCE RELATIONS FOR POLARIZED AND UNPOLARIZED ENERGY

The total intensity of a ray is the sum of the intensities of the component waves, whose directions may be specified along the x and y axes or some rotation of them, in a rectangular coordinate system having the z axis in the direction of propagation of the ray. This will be shown by the following development.

From electromagnetic theory considerations of the Poynting vector, the energy transferred by an electromagnetic wave is proportional to the square of the electric vector. Since this electric vector varies with time, it is obvious that the power is proportional to the time average of the square of the electric vector. It the electric vector at a point in space is denoted as $\mathcal{E}$ and if its components are denoted as $\mathcal{E}_x$ and $\mathcal{E}_y$ in the above mentioned coordinate system, at any time it may be said that:

$$\mathcal{E}^2 = \mathcal{E}_x^2 + \mathcal{E}_y^2 .$$

During some time period, τ, the power transferred is:

$$I \sim \frac{1}{\tau} \int_0^{\tau} \left(\mathcal{E}_x^2 + \mathcal{E}_y^2 \right) dt .$$

It has been said that the most general form of polarized light is that of elliptically polarized light (61). To demonstrate this, the equation of an ellipse will be derived from the basic combination of electric vectors in the two planes of polarization.

If a phase difference, ϵ , is noted between the two components of the electric vector and if a is the maximum amplitude of the vector in the x direction and b is the maximum amplitude of the vector in the y direction, it may be said that:

$$\mathcal{E}_x = a \cos \omega t$$

$$\mathcal{E}_y = b \cos (\omega t + \epsilon)$$

$$\mathcal{E}_y = b (\cos \omega t \cos \epsilon - \sin \omega t \sin \epsilon)$$

or

$$\frac{\mathcal{E}_x}{a} = \cos \omega t \tag{A-1}$$

$$\frac{\mathcal{E}_y}{b} = \cos \omega t \cos \epsilon - \sin \omega t \sin \epsilon \tag{A-2}$$

By squaring equation (A-2), one gets:

$$\left(\frac{\mathcal{E}_y}{b}\right)^2 - 2 \frac{\mathcal{E}_y}{b} \cos \omega t \cos \epsilon + \cos^2 \omega t \cos^2 \epsilon = \sin^2 \omega t \sin^2 \epsilon$$

Upon substitution of equation (A-1), it is shown that:

$$\begin{aligned}\left(\frac{\mathcal{E}_y}{b}\right)^2 - 2 \frac{\mathcal{E}_x}{a} \frac{\mathcal{E}_y}{b} \cos \epsilon + \left(\frac{\mathcal{E}_x}{a}\right)^2 \cos \epsilon &= \sin^2 \omega t \sin^2 \epsilon \\ &= \left[1 - \left(\frac{\mathcal{E}_x}{a}\right)^2\right] \sin^2 \epsilon\end{aligned}$$

By collection of terms, the general equation for an ellipse is seen:

$$\left(\frac{\mathcal{E}_y}{b}\right)^2 + \left(\frac{\mathcal{E}_x}{a}\right)^2 - 2\,\frac{\mathcal{E}_x}{a}\,\frac{\mathcal{E}_y}{b}\cos\epsilon - \sin^2\epsilon = 0\,. \qquad \text{(A-3)}$$

The task now is to show that intensity or power is a constant with respect to the transformation of the elliptic equation into new coordinates. From equation (A-3), it is seen that when $\epsilon = \frac{\pi}{2}$, the ellipse's major and minor axes line up with the coordinate axes. This could be expressed as:

$$\begin{aligned} \mathcal{E}_x &= a \sin \omega t \\ \mathcal{E}_y &= b \cos \omega t\,. \end{aligned} \qquad \text{(A-4)}$$

The symbol, $\triangleq$, denotes definition. If $\mathcal{E}^{(0)}$ is defined as:

$$\left(\mathcal{E}^{(0)}\right)^2 \triangleq a^2 + b^2,$$

and if β is defined such that:

$$a \triangleq \mathcal{E}^{(0)} \cos\beta \qquad \text{and} \qquad b \triangleq \mathcal{E}^{(0)} \sin\beta\,.$$

it is seen that:

$$\begin{aligned} \mathcal{E}_x &= \mathcal{E}^{(0)} \cos\beta \sin\omega t \\ \mathcal{E}_y &= \mathcal{E}^{(0)} \sin\beta \cos\omega t\,. \end{aligned} \qquad \text{(A-5)}$$

For transformation to another coordinate axis, for example, the r, ℓ

axes, through angle χ defined as the angle between the major axes and the axis of the ellipse, one gets:

$$\begin{aligned} \mathcal{E}_\ell &= \mathcal{E}_x \cos\chi - \mathcal{E}_y \sin\chi \\ \mathcal{E}_r &= \mathcal{E}_x \sin\chi + \mathcal{E}_y \cos\chi \ . \end{aligned} \qquad \text{(A-6)}$$

This is illustrated in Figure 23.

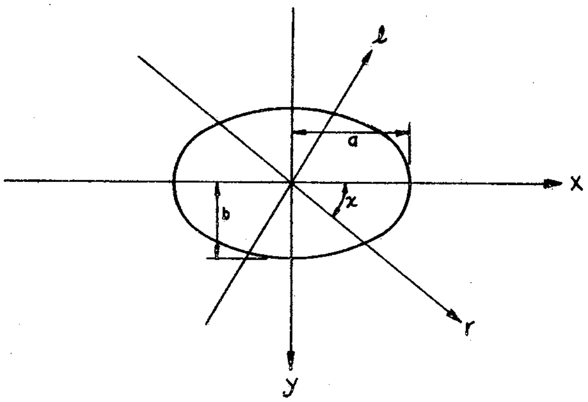

Figure 23. Elliptically Polarized Light

By making the substitution of equations (A-5) into equations (A-6), it is seen that:

$$\begin{aligned} \mathcal{E}_\ell &= \mathcal{E}^{(0)}\left[\cos\beta \cos\chi \sin\omega t - \sin\beta \sin\chi \cos\omega t\right] \\ \mathcal{E}_r &= \mathcal{E}^{(0)}\left[\cos\beta \sin\chi \sin\omega t + \sin\beta \cos\chi \cos\omega t\right] \ . \end{aligned} \qquad \text{(A-7)}$$

Now the following definitions are made:

$$\mathcal{E}_\ell^{(0)} \triangleq \mathcal{E}^{(0)}\left[\cos^2\beta\ \cos^2\chi + \sin^2\beta\ \sin^2\chi\right]^{1/2}$$

$$\mathcal{E}_r^{(0)} \triangleq \mathcal{E}^{(0)}\left[\cos^2\beta\ \sin^2\chi + \sin^2\beta\ \cos^2\chi\right]^{1/2}$$

$$\tan \epsilon_\ell \triangleq \tan\beta\ \tan\chi$$

$$\tan \epsilon_r \triangleq -\tan\beta\ \cot\chi\ .$$

These definitions may be substituted into equations (A-7) to yield:

$$\mathcal{E}_\ell = \mathcal{E}_\ell^{(0)} \frac{\left[\cos\beta\ \cos\chi\ \sin\omega t - \sin\beta\ \sin\chi\ \cos\omega t\right]}{\left[\cos^2\beta\ \cos^2\chi + \sin^2\beta\ \sin^2\chi\right]^{1/2}}$$

$$\mathcal{E}_r = \mathcal{E}_r^{(0)} \frac{\left[\cos\beta\ \sin\chi\ \sin\omega t + \sin\beta\ \cos\chi\ \cos\omega t\right]}{\left[\cos^2\beta\ \cos^2\chi + \sin^2\beta\ \sin^2\chi\right]^{1/2}}\ .$$

By considering the equation for $\mathcal{E}_\ell$, it is seen that:

$$\begin{aligned}
\mathcal{E}_\ell &= \mathcal{E}_\ell^{(0)}\left[\frac{\sin\omega t}{\sqrt{1+\tan^2\beta\tan^2\chi}} - \frac{\cos\omega t}{\sqrt{1+\cot^2\beta\ \cot^2\chi}}\right] \\
&= \mathcal{E}_\ell^{(0)}\left[\frac{\sin\omega t}{\sqrt{1+\tan^2\epsilon_\ell}} - \frac{\cos\omega t}{\sqrt{1+\cot^2\epsilon_\ell}}\right] \\
&= \mathcal{E}_\ell^{(0)}\left[\frac{\sin\omega t}{\pm\sec\epsilon_\ell} - \frac{\cos\omega t}{\pm\csc\epsilon_\ell}\right] \\
&= \mathcal{E}_\ell^{(0)}\left[\sin\omega t\ \cos\epsilon_\ell - \cos\omega t\ \sin\epsilon_\ell\right]
\end{aligned}$$

$$\mathcal{E}_\ell = \mathcal{E}_\ell^{(0)} \sin(\omega t - \epsilon_\ell)\ . \qquad \text{(A-8)}$$

By considering the equation for $\mathcal{E}_r$, it is seen that:

$$\mathcal{E}_r = \mathcal{E}_r^{(0)}\left[\frac{\sin \omega t}{\sqrt{1+\tan^2\beta \cot^2\chi}} + \frac{\cos \omega t}{\sqrt{1+\tan^2\beta \cot^2\chi}}\right]$$

$$= \mathcal{E}_r^{(0)}\left[\frac{\sin \omega t}{\sqrt{1+\tan^2 \epsilon_r}} + \frac{\cos \omega t}{\sqrt{1+\cot^2 \epsilon_r}}\right]$$

$$= \mathcal{E}_r^{(0)}\left[\frac{\sin \omega t}{\pm \sec \epsilon_r} + \frac{\cos \omega t}{\pm \csc \epsilon_r}\right]$$

$$= \mathcal{E}_r^{(0)}\left[\sin \omega t \cos \epsilon_r - \cos \omega t \sin \epsilon_r\right]$$

$$\mathcal{E}_r = \mathcal{E}_r^{(0)} \sin(\omega t - \epsilon_r). \tag{A-9}$$

It should be noted that in the expressions found two lines above equations (A-8) and (A-9), it was possible to select either a (+) or a (-). The particular combinations used were selected so that equations (A-8) and (A-9) would have the same form.

Equations (A-8) and (A-9) when squared and then integrated with respect to time yield:

$$I_r = \frac{1}{\tau}\int_0^\tau \mathcal{E}_r^{(0)^2}\, dt = \frac{1}{2}\left(\mathcal{E}_r^{(0)}\right)^2$$

$$I_\ell = \frac{1}{\tau}\int_0^\tau \mathcal{E}_\ell^{(0)^2}\, dt = \frac{1}{2}\left(\mathcal{E}_\ell^{(0)}\right)^2$$

or $$I = \frac{1}{\tau}\int_0^\tau \left(\mathcal{E}_\ell^{(0)^2} + \mathcal{E}_r^{(0)^2}\right) dt = \frac{1}{2}\left[\mathcal{E}_r^{(0)^2} + \mathcal{E}_\ell^{(0)^2}\right].$$

Now by referring back to equation (A-7), it is seen that:

$$\mathcal{E}_{\ell}^{(0)^2} = \mathcal{E}^{(0)^2}\left[\cos^2\beta\,\cos^2\chi + \sin^2\beta\,\sin^2\chi\right]$$

$$\mathcal{E}_{r}^{(0)^2} = \mathcal{E}^{(0)^2}\left[\cos^2\beta\,\sin^2\chi + \sin^2\beta\,\cos^2\chi\right]$$

$$\mathcal{E}_{r}^{(0)^2} + \mathcal{E}_{\ell}^{(0)^2} = \mathcal{E}^{(0)^2}\left(\cos^2\beta + \sin^2\beta\right)\left(\sin^2\chi + \cos^2\chi\right)$$

$$= \mathcal{E}^{(0)^2}.$$

Thus, is is seen that:

$$I = \frac{1}{2}\left(\mathcal{E}^{(0)}\right)^2 = I_{\ell} + I_r$$

and, therefore, regardless of the angle, χ, between the x axis and the r axis:

$$\boxed{I = I_{\ell} + I_r} \quad . \tag{A-10}$$

The development of expressions relating the polarized reflectances to the total reflectance for polarized incident energy now follows.

By an analysis of the meaning of the polarized reflectances, it is obvious that:

$$R_s = \frac{I_s^-}{I_s^+} \quad \text{and} \quad R_p = \frac{I_p^-}{I_p^+},$$

where the (-) superscript characterizes the reflected energy and the (+) superscript characterizes the incident energy. Equation (A-10) showed that the energy in the two planes of polarization is additive, and therefore it follows that the total incident energy is:

$$I^+ = I_s^+ + I_p^+$$

and the total reflected energy is:

$$I^- = I_s^- + I_p^- .$$

The total reflectance is defined as the ratio of the total reflected energy to the total incident energy, or:

$$\begin{aligned} R &= \frac{I_s^- + I_p^-}{I_s^+ + I_p^+} \\ &= \frac{R_s I_s^+ + R_p I_p^+}{I_s^+ + I_p^+} \\ &= R_s \left[\frac{I_s^+}{I_s^+ + I_p^+} \right] + R_p \left[\frac{I_p^+}{I_s^+ + I_p^+} \right] , \qquad \text{(A-11)} \end{aligned}$$

Thus, in order to determine the total reflectance of a surface, it is necessary to know the relationship between the magnitudes of the s-plane and the p-plane components of the incident energy.

Certainly, if the energy incident on a surface is originally unpolarized, I_s will equal I_p, and:

$$R = \frac{1}{2}(R_s + R_p). \tag{A-12}$$

If the incident energy is such that the direction of polarization giving the maximum intensity falls along the s-plane, the degree of polarization, P, is defined as:

$$P = \frac{I_s^+ - I_p^+}{I_s^+ + I_p^+}, \tag{A-13}$$

and it may be shown by algebraic manipulations that:

$$R = R_s\left(\frac{1+P}{2}\right) + R_p\left(\frac{1-P}{2}\right). \tag{A-14}$$

APPENDIX B

THE DEVELOPMENT OF REFLECTANCE EQUATIONS

The basic development of the amplitude and energy reflectance equations is well known for single absorbing surfaces at all angles of incidence and for absorbing surfaces covered by thin absorbing films at normal incidence (58) (60). This appendix presents a systematic development of such equations and extends this work to include functional equations for absorbing surfaces covered by thin absorbing films at all angles of incidence.

Amplitude Reflectances for Single Interfaces

A representation of the electric vector behavior upon reflection at an air-film interface and at a film-water interface is shown in Figure 24. The subscripts 0, 1, and 2 characterize quantities in the air, film, and water, respectively. The superscripts, +, and -, characterize vectors traveling through the media and vectors being reflected from the interfaces, respectively.

Air is considered to be a transparent media with an index of refraction, n_o. The film and water are considered to be absorbing media; and, therefore, their indices of refraction are denoted as:

$$\bar{n}_1 = n_1(1 - i k_1) \qquad \text{(B-1)}$$

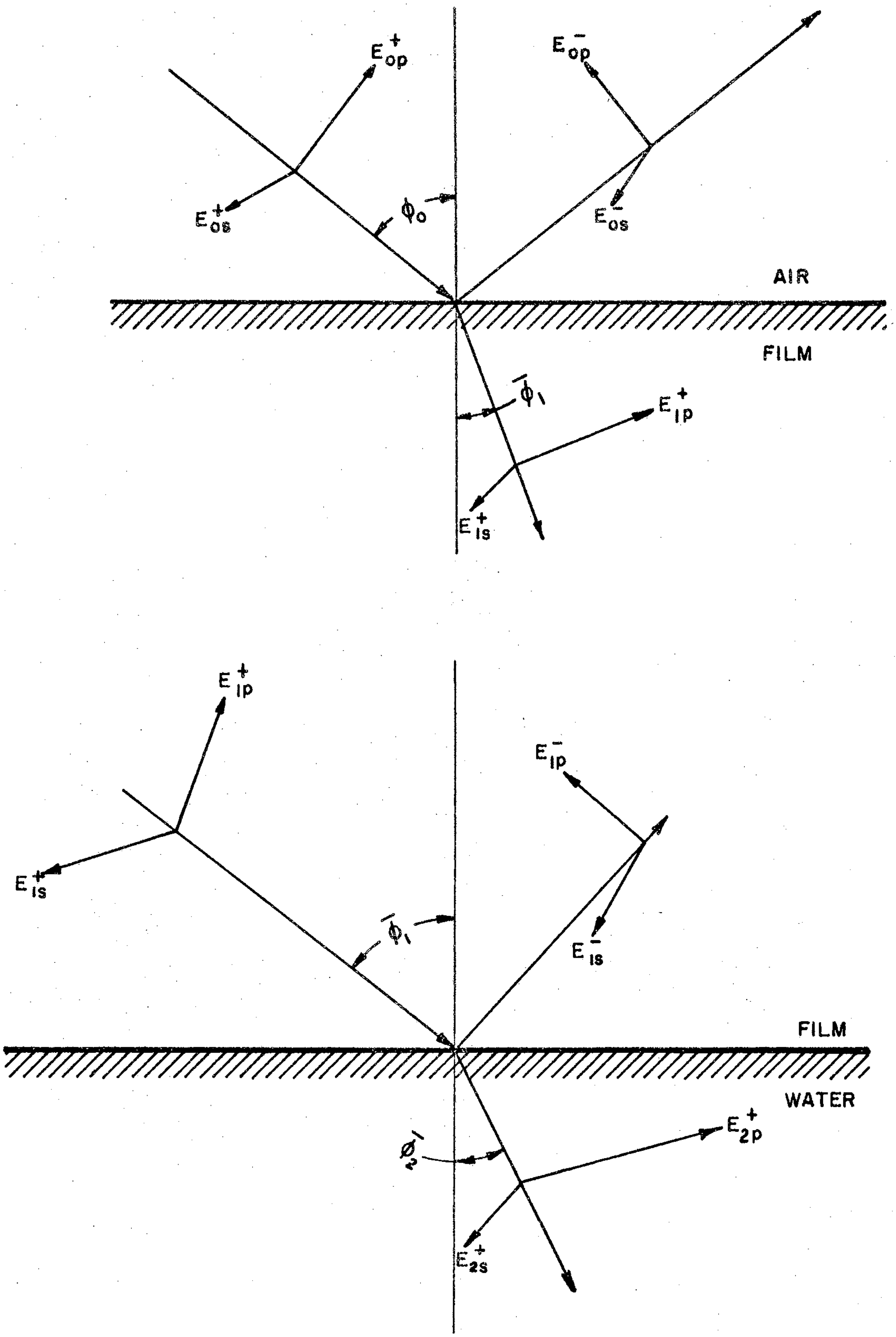

Figure 24. Reflection from Single Interfaces

$$\bar{n}_2 = n_2(1 - i k_2) \tag{B-2}$$

Snell's law at the two interfaces may be written as:

$$n_0 \sin \phi_0 = \bar{n}_1 \sin \bar{\phi}_1$$
$$\bar{n}_1 \sin \bar{\phi}_1 = \bar{n}_2 \sin \bar{\phi}_2$$

The boundary conditions for reflection are such that the tangential components of both the electric and the accompanying magnetic vectors must be continuous at the interface. By defining the x - y surface coordinates to be oriented with the x-axis in the incident plane and by assuming a sinusoidal solution to Maxwell's equations, the vectors in air at the air-film interface are:

$$E_{0x} = (E^{+}_{0p} - E^{-}_{0p}) \cos \phi_0$$
$$E_{0y} = E^{+}_{0s} + E^{-}_{0s}$$
$$H_{0x} = n_0(E^{-}_{0s} - E^{+}_{0s}) \cos \phi_0$$
$$H_{0y} = n_0(E^{+}_{0p} + E^{-}_{0p}),$$

and in the film:

$$E_{1x} = E^{+}_{1p} \cos \bar{\phi}_1$$
$$E_{1y} = E^{+}_{1s}$$
$$H_{1x} = -\bar{n}_1 E^{+}_{1s} \cos \bar{\phi}_1$$
$$H_{1y} = \bar{n}_1 E^{+}_{1p} .$$

Equating the similar vectors for the air and the film yields:

$$\left(E_{0p}^{+} - E_{0p}^{-}\right) \cos \phi_0 = E_{1p}^{+} \cos \overline{\Phi}_1$$

$$E_{0s}^{+} + E_{0s}^{-} = E_{1s}^{+}$$

$$n_0\left(E_{0s}^{+} - E_{0s}^{-}\right) \cos \phi_0 = \overline{n}_1 E_{1s}^{+} \cos \overline{\Phi}_1$$

$$n_0\left(E_{0p}^{+} + E_{0p}^{-}\right) = \overline{n}_1 E_{1p}^{+} .$$

Similarly, the equations for the film-water interface may be shown to be:

$$\left(E_{1p}^{+} - E_{1p}^{-}\right) \cos \overline{\Phi}_1 = E_{2p}^{+} \cos \overline{\Phi}_2$$

$$E_{1s}^{+} + E_{1s}^{-} = E_{2s}^{+}$$

$$\overline{n}_1\left(E_{1s}^{+} - E_{1s}^{-}\right) \cos \overline{\Phi}_1 = \overline{n}_2 E_{2s}^{+} \cos \overline{\Phi}_2$$

$$\overline{n}_1\left(E_{1p}^{+} + E_{1p}^{-}\right) = \overline{n}_2 E_{2p}^{+} .$$

By proper definition of the amplitude reflectances and by algebraic manipulations, it may be shown that:

$$\overline{r}_{1p} \triangleq \frac{E_{0p}^{-}}{E_{0p}^{+}} = \frac{\overline{n}_1 \cos \phi_0 - n_0 \cos \overline{\Phi}_1}{\overline{n}_1 \cos \phi_0 + n_0 \cos \overline{\Phi}_1} \qquad \text{(B-3)}$$

$$\overline{r}_{1s} \triangleq \frac{E_{0s}^{-}}{E_{0s}^{+}} = \frac{n_0 \cos\phi_0 - \overline{n}_1 \cos\overline{\phi}_1}{n_0 \cos\phi_0 + \overline{n}_1 \cos\overline{\phi}_1} \tag{B-4}$$

$$\overline{r}_{2p} \triangleq \frac{E_{1p}^{-}}{E_{1p}^{+}} = \frac{\overline{n}_2 \cos\overline{\phi}_1 - \overline{n}_1 \cos\overline{\phi}_2}{\overline{n}_2 \cos\overline{\phi}_1 + \overline{n}_1 \cos\overline{\phi}_2} \tag{B-5}$$

$$\overline{r}_{2s} \triangleq \frac{E_{1s}^{-}}{E_{1s}^{+}} = \frac{\overline{n}_1 \cos\overline{\phi}_1 - \overline{n}_2 \cos\overline{\phi}_2}{\overline{n}_1 \cos\overline{\phi}_1 + \overline{n}_2 \cos\overline{\phi}_2}\,. \tag{B-6}$$

These relations may be expressed as the sum of a real and an imaginary component, by considering equation (B-1):

$$\overline{n}_1 \sin\overline{\phi}_1 = n_0 \sin\phi_0 ,$$

which after squaring and making a trigonometric substitution may be shown to be:

$$\overline{n}_1^2 (1 - \cos^2\overline{\phi}_1) = n_0^2 \sin^2\phi_0$$

$$\overline{n}_1^2 \cos^2\overline{\phi}_1 = \overline{n}_1^2 - n_0^2 \sin^2\phi_0 .$$

By taking the square root and arbitrarily defining the resulting complex number, one gets:

$$\bar{n}_1 \cos \bar{\phi}_1 = \sqrt{\bar{n}_1^2 - n_0^2 \sin^2 \phi_0}$$

$$\triangleq a - bi. \qquad \text{(B-7)}$$

A similar development from equation (B-2) would yield:

$$\bar{n}_2 \cos \bar{\phi}_2 = \sqrt{\bar{n}_2^2 - n_0^2 \sin^2 \phi_0}$$

$$= a_1 - b_1 i. \qquad \text{(B-8)}$$

Explicit expressions for a, a_1, b, and b_1 will now be determined. Squaring equation (B-7) yields:

$$\bar{n}_1^2 \cos^2 \bar{\phi}_1 = \bar{n}_1^2 - n_0^2 \sin^2 \phi_0$$

$$(a - bi)^2 = n_1^2 (1 - ik_1)^2 - n_0^2 \sin^2 \phi_0$$

$$(a^2 - b^2) - 2abi = n_1^2 (1 - k_1^2 - 2ik_1) - n_0^2 \sin^2 \phi_0$$

$$= n_1^2 (1 - k_1^2) - n_0^2 \sin^2 \phi_0 - 2in_1^2 k_1.$$

Collecting the real and imaginary parts results in:

$$a^2 - b^2 = n_1^2 (1 - k_1^2) - n_0^2 \sin^2 \phi_0$$

$$ab = n_1^2 k_1, \qquad \text{(B-9)}$$

or

$$b = \frac{n_1^2 k_1}{a}.$$

Substitution for b^2 results in:

$$a^2 - \frac{n_1^4 k_1^2}{a_1^2} = n_1^2(1-k_1^2) - n_0^2 \sin^2 \phi_0,$$

or

$$a^4 - a^2[n_1^2(1-k_1^2) - n_0^2 \sin^2 \phi_0] - n_1^4 k_1^2 = 0,$$

so that

$$a^2 = \frac{1}{2}[n_1^2(1-k_1^2) - n_0^2 \sin^2 \phi_0] + \frac{1}{2}\sqrt{A} \qquad \text{(B-10)}$$

where $$A \triangleq [n_1^2(1-k_1^2) - n_0^2 \sin^2 \phi_0]^2 + 4 n_1^4 k_1^2.$$

Substitution into equation (B-9) results in:

$$\begin{aligned} b^2 &= a^2 - [n_1^2(1-k_1^2) - n_0^2 \sin^2 \phi_0] \\ &= -\frac{1}{2}[n_1^2(1-k_1^2) - n_0^2 \sin^2 \phi_0] + \frac{1}{2}\sqrt{A} \end{aligned} \qquad \text{(B-11)}$$

It is to be noted that the existence of the (+) in front of the radical is due to the requirement that the a^2 and the b^2 be greater than zero for the normal incidence case.

A similar development can be undertaken for a_1 and b_1, yielding:

$$a_1^2 = \frac{1}{2}[n_2^2(1-k_2^2) - n_0^2 \sin^2 \phi_0] + \frac{1}{2}\sqrt{B} \qquad \text{(B-12)}$$

$$b_1^2 = -\frac{1}{2}[n_2^2(1-k_2^2) - n_0^2 \sin^2 \phi_0] + \frac{1}{2}\sqrt{B} \qquad \text{(B-13)}$$

where:

$$B \triangleq \left[n_2^2(1-k_2^2) - n_0^2 \sin^2\phi_0\right]^2 + 4 n_2^4 k_2^2 .$$

It is now possible to make substitutions and algebraic manipulations of the equations for r_{1s}, r_{2s}, r_{1p}, and r_{2p}. From equation (B-4), one determines:

$$\bar{r}_{1s} = \frac{n_0 \cos\phi_0 - \bar{n}_1 \cos\bar{\phi}_1}{n_0 \cos\phi_0 + \bar{n}_1 \cos\bar{\phi}_1}$$

$$= \frac{n_0 \cos\phi_0 - a + bi}{n_0 \cos\phi_0 + a - bi} \cdot \frac{(n_0 \cos\phi_0 + a) + bi}{(n_0 \cos\phi_0 + a) + bi}$$

$$= \frac{(n_0^2 \cos^2\phi_0 - a^2 - b^2) + i\,(2 b n_0 \cos\phi_0)}{(n_0 \cos\phi_0 + a)^2 + b^2} ,$$

or

$$\boxed{\bar{r}_{1s} = g_{1s} + i h_{1s}} \qquad \text{(B-14)}$$

where:

$$g_{1s} = \frac{n_0^2 \cos^2\phi_0 - a^2 - b^2}{(n_0 \cos\phi_0 + a)^2 + b^2}$$

$$h_{1s} = \frac{2 b n_0 \cos\phi_0}{(n_0 \cos\phi_0 + a)^2 + b^2} .$$

Similarly from equation (B-6), the result is:

$$\bar{r}_{2s} = \frac{\bar{n}_1 \cos \bar{\Phi}_1 - \bar{n}_2 \cos \bar{\Phi}_2}{\bar{n}_1 \cos \bar{\Phi}_1 + \bar{n}_2 \cos \bar{\Phi}_2}$$

$$= \frac{a - bi - a_1 + b_1 i}{a - bi + a_1 - b_1 i}$$

$$= \frac{(a-a_1) - i(b-b_1)}{(a+a_1) - i(b+b_1)} \cdot \frac{(a+a_1) + i(b+b_1)}{(a+a_1) + i(b+b_1)}$$

$$= \frac{(a-a_1)(a+a_1) + (b-b_1)(b+b_1) + i[(b+b_1)(a-a_1) - (a+a_1)(b-b_1)]}{(a+a_1)^2 + (b+b_1)^2},$$

or

$$\boxed{\bar{r}_{2s} = g_{2s} + i h_{2s}} \qquad \text{(B-15)}$$

where

$$g_{2s} = \frac{(a-a_1)(a+a_1) + (b-b_1)(b+b_1)}{(a_1+a)^2 + (b_1+b)^2}$$

$$h_{2s} = \frac{(a-a_1)(b+b_1) - (a+a_1)(b-b_1)}{(a_1+a)^2 + (b_1+b)^2}$$

$$= \frac{2(ab_1 - a_1 b)}{(a_1+a)^2 + (b_1+b)^2}.$$

From equation (B-3), it may be shown that:

$$\bar{r}_{1P} = \frac{\bar{n}_1 \cos\phi_0 - n_0 \cos\bar{\phi}_1}{\bar{n}_1 \cos\phi_0 + n_0 \cos\bar{\phi}_1} \cdot \frac{\bar{n}_1}{\bar{n}_1}$$

$$= \frac{\bar{n}_1^2 \cos\phi_0 - n_0 \bar{n}_1 \cos\bar{\phi}_1}{\bar{n}_1^2 \cos\phi_0 + n_0 \bar{n}_1 \cos\bar{\phi}_1}$$

$$= \frac{n_1(1-ik_1)^2 \cos\phi_0 - n_0(a-bi)}{n_1(1-ik_1)^2 \cos\phi_0 + n_0(a-bi)}$$

$$= \frac{n_1^2(1-k_1^2)\cos\phi_0 - n_0 a - i(2n_1^2 k_1 \cos\phi_0 - n_0 b)}{n_1^2(1-k_1^2)\cos\phi_0 + n_0 a - i(2n_1^2 k_1 \cos\phi_0 + n_0 b)}$$

$$= \frac{(c-di)}{(e-fi)} \cdot \frac{(e+fi)}{(e+fi)}$$

$$= \frac{ce + fd + i(cf - ed)}{e^2 + f^2}$$

$$\boxed{\bar{r}_{1P} = g_{1P} + i h_{1P}} \quad , \qquad \text{(B-16)}$$

where:

$$c = n_1^2(1-k_1^2)\cos\phi_0 - n_0 a$$
$$d = 2n_1^2 k_1 \cos\phi_0 - n_0 b$$
$$e = n_1^2(1-k_1^2)\cos\phi_0 + n_0 a$$
$$f = 2n_1^2 k_1 \cos\phi_0 + n_0 b$$
$$g_{1p} = \frac{ce + fd}{e^2 + f^2}$$
$$h_{1p} = \frac{cf - ed}{e^2 + f^2} \quad .$$

Similar developments from equation (B-5) yield:

$$\bar{r}_{2p} = \frac{\bar{n}_2 \cos\bar{\phi}_1 - \bar{n}_1 \cos\bar{\phi}_2}{\bar{n}_2 \cos\bar{\phi}_1 + \bar{n}_1 \cos\bar{\phi}_2} \cdot \frac{\bar{n}_2}{\bar{n}_2}$$

$$= \frac{\bar{n}_2^2 \cos\bar{\phi}_1 \cdot \frac{\bar{n}_1}{\bar{n}_1} - \bar{n}_1\bar{n}_2 \cos\bar{\phi}_2}{\bar{n}_2^2 \cos\bar{\phi}_1 \cdot \frac{\bar{n}_1}{\bar{n}_1} + \bar{n}_1\bar{n}_2 \cos\bar{\phi}_2}$$

$$= \frac{\left(\frac{\bar{n}_2^2}{\bar{n}_1^2}\right)\bar{n}_1 \cos\bar{\phi}_1 - \bar{n}_2 \cos\bar{\phi}_2}{\left(\frac{\bar{n}_2^2}{\bar{n}_1^2}\right)\bar{n}_1 \cos\bar{\phi}_1 + \bar{n}_2 \cos\bar{\phi}_2}$$

$$= \frac{\left(\frac{\bar{n}_2}{\bar{n}_1}\right)^2(a-bi) - (a_1-b_1 i)}{\left(\frac{\bar{n}_2}{\bar{n}_1}\right)^2(a-bi) + (a_1-b_1 i)} \quad .$$

In order to determine an expression for $\left(\frac{\bar{n}_2}{\bar{n}_1}\right)^2$, the following analysis is undertaken.

$$\left(\frac{\bar{n}_2}{\bar{n}_1}\right)^2 = \frac{n_2^2(1-ik_2)^2}{n_1^2(1-ik_1)^2}$$

$$= \left(\frac{n_2}{n_1}\right)^2 \cdot \frac{(1-k_2^2)-i2k_2}{(1-k_1^2)-i2k_1}$$

$$= \left(\frac{n_2}{n_1}\right)^2 \cdot \frac{(1-k_2^2)-i2k_2}{(1-k_1^2)-i2k_1} \cdot \frac{(1-k_1^2)+i2k_1}{(1-k_1^2)+i2k_1}$$

$$= \left(\frac{n_2}{n_1}\right)^2 \frac{(1-k_1^2)(1-k_2^2)+4k_1k_2+2i[k_1(1-k_2^2)-k_2(1-k_1^2)]}{(1-k_1^2)^2+4k_1^2} .$$

But: $$(1+k_1^2)^2 = (1-k_1^2)^2 + 4k_1^2 ,$$

and by designating:

$$s = \left(\frac{n_2}{n_1}\right)^2 \frac{(1-k_1^2)(1-k_2^2)+4k_1k_2}{(1+k_1^2)^2} .$$

$$t = -2\left(\frac{n_2}{n_1}\right)^2 \frac{k_1(1-k_2^2)-k_2(1-k_1^2)}{(1+k_1^2)^2} ,$$

it may be shown that: $$\left(\frac{\bar{n}_2}{\bar{n}_1}\right)^2 = s - it .$$

Substitution back into the $\overline{\gamma}_{2p}$ expression shows that:

$$\overline{\gamma}_{2p} = \frac{(s-it)(a-ib)-(a_1+b_1 i)}{(s-it)(a-ib)+(a_1-b_1 i)}$$

$$= \frac{sa-tb-a_1+i(b_1-at-sb)}{sa-tb+a_1-i(b_1+at+sb)}$$

$$= \frac{(sa-tb-a_1)+i(b_1-at-sb)}{(sa-tb+a_1)-i(b_1+at+sb)} \cdot \frac{(sa-tb+a_1)+i(b_1+at+sb)}{(sa-tb+a_1)+i(b_1+at+sb)}$$

$$= \frac{(sa-tb-a_1)(sa-tb+a_1)-(b_1-at-sb)(b_1+at+sb)}{(sa-tb+a_1)^2+(b_1+at+sb)^2}$$

$$+ i\,\frac{(sa-tb-a_1)(b_1+at+sb)+(sa-tb+a_1)(b_1-at-sb)}{(sa-tb+a_1)^2+(b_1+at+sb)^2} ,$$

or

$$\boxed{\overline{\gamma}_{2p} = g_{2p} + i\,h_{2p}} \quad , \qquad \text{(B-17)}$$

where:

$$g_{2p} = \frac{(sa-tb-a_1)(sa-tb+a_1)-(b-at-sb)(b_1+at+sb)}{(sa-tb+a_1)^2+(b_1+at+sb)^2}$$

$$h_{2p} = \frac{(sa-tb-a_1)(b_1+at+sb)+(sa-tb+a_1)(b_1-at-sb)}{(sa-tb+a_1)^2+(b_1+at+sb)^2} .$$

Thus, simplified expressions have been developed for the complex amplitude reflectances for polarized energy at two different interfaces.

Amplitude Reflectances for an Air-Film-Water System

The expressions developed thus far have been for two independent interfaces. However, in the physical situation of an absorbing film covering a water surface, the reflectances are interdependent. Thus, the previously developed expressions will be considered as simply a collection of terms. From the dependent boundary conditions, the expressions for the actual reflectances will be developed.

Figure 25 shows the air-film-water system and the electric vectors in the s- and p-planes of polarization.

The boundary conditions for this air-film-water system are as follows: for the air-film interface, with the subscript, I:

$$\left(E^{+}_{op_{I}} - E^{-}_{op_{I}}\right)\cos\phi_{o} = \left(E^{+}_{1p_{I}} - E^{-}_{1p_{I}}\right)\cos\bar{\Phi}_{1}$$

$$\left(E^{+}_{op_{I}} + E^{-}_{op_{I}}\right)n_{o} = \left(E^{+}_{1p_{I}} + E^{-}_{1p_{I}}\right)\bar{n}_{1}$$

$$E^{+}_{os_{I}} + E^{-}_{os_{I}} = E^{+}_{1s_{I}} + E^{-}_{1s_{I}}$$

$$\left(E^{+}_{os_{I}} - E^{-}_{os_{I}}\right)n_{o}\cos\phi_{o} = \left(E^{+}_{1s_{I}} - E^{-}_{1s_{I}}\right)\bar{n}_{1}\cos\bar{\Phi}_{1}$$

and for the film-water interface, with the subscript II:

$$\left(E^{+}_{1s_{II}} - E^{-}_{1s_{II}}\right)\bar{n}_{1}\cos\bar{\Phi}_{1} = E^{+}_{2s_{II}}\,\bar{n}_{2}\cos\bar{\Phi}_{2}$$

$$\left(E^{+}_{1p_{II}} - E^{-}_{1p_{II}}\right)\cos\bar{\Phi}_{1} = E^{+}_{2p_{II}}\cos\bar{\Phi}_{2}$$

$$\left(E^{+}_{1p_{II}} + E^{-}_{1p_{II}}\right)\bar{n}_{1} = E^{+}_{2p_{II}}\,\bar{n}_{2}$$

$$E^{+}_{1s_{II}} + E^{-}_{1s_{II}} = E^{+}_{2s_{II}}\,.$$

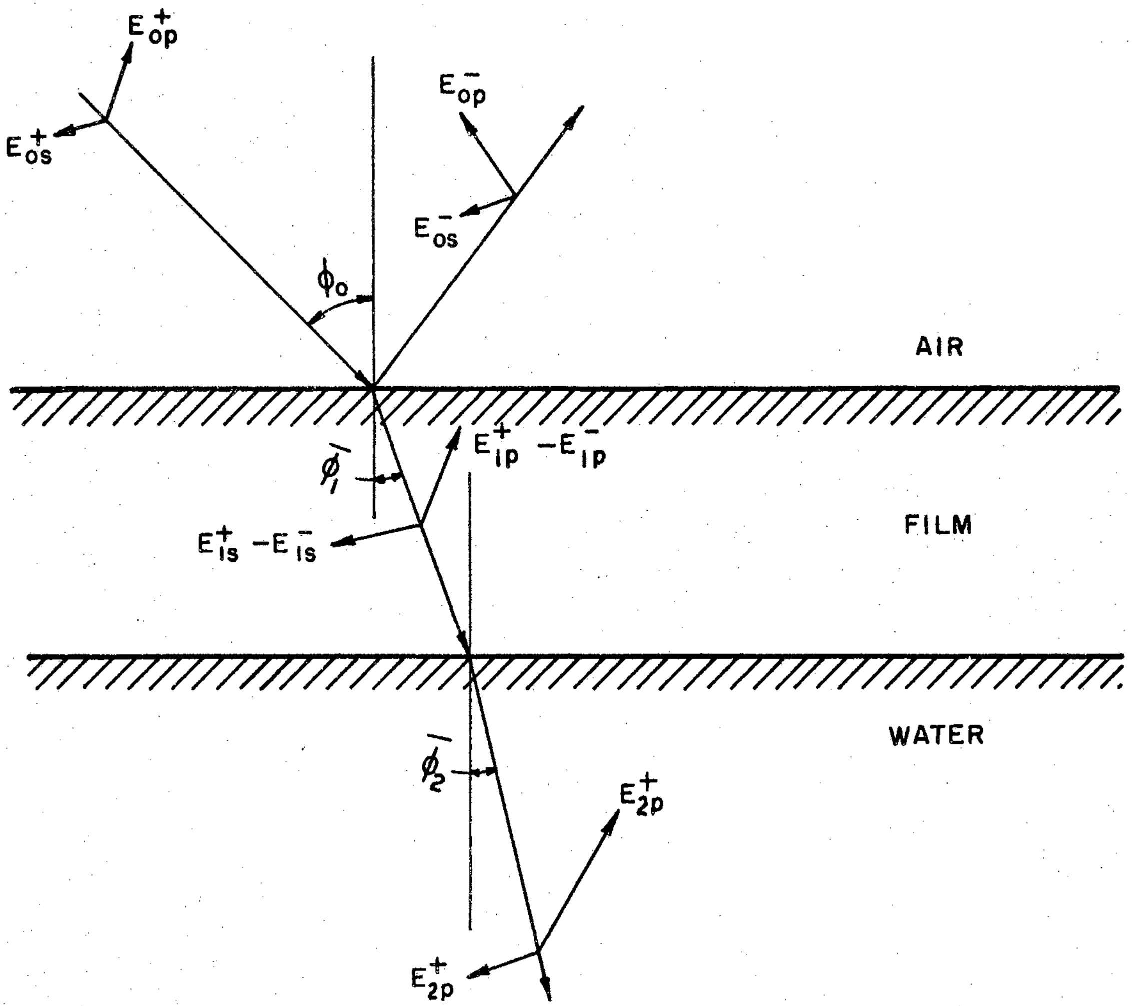

Figure 25. Reflection from an Air-Film-Water System

In order to combine the above boundary conditions, Forsterling (60) presented the following relationships between the corresponding amplitudes:

$$E^{+}_{1p_{I}} = E^{+}_{1p_{II}} \exp\left(\frac{i\,2\pi \bar{n}_1 \cos \bar{\phi}_1}{\lambda/d}\right) = E^{+}_{1p_{II}}\, e^{\frac{i\chi}{2}}$$

$$E^{-}_{1p_{I}} = E^{-}_{1p_{II}} \exp\left(\frac{-i\,2\pi \bar{n}_1 \cos \bar{\phi}_1}{\lambda/d}\right) = E^{-}_{1p_{II}}\, e^{-\frac{i\chi}{2}}$$

$$E^{+}_{1s_{I}} = E^{+}_{1s_{II}}\, e^{\frac{i\chi}{2}}$$

$$E^{-}_{1s_{I}} = E^{-}_{1s_{II}}\, e^{-\frac{i\chi}{2}}$$

where the complex phase difference, χ, is defined to be:

$$\chi \triangleq \frac{4\pi \bar{n}_1 d \cos \bar{\phi}_1}{\lambda} .$$

This allows the two sets of boundary conditions to be combined to give:

$$\left(E^{+}_{0p_{I}} - E^{-}_{0p_{I}}\right) \cos \phi_0 = \left(E^{+}_{1p_{II}}\, e^{\frac{i\chi}{2}} - E^{-}_{1p_{II}}\, e^{-\frac{i\chi}{2}}\right) \cos \bar{\phi}_1$$

$$\left(E^{+}_{0p_{I}} + E^{-}_{0p_{I}}\right) n_0 = \left(E^{+}_{1p_{II}}\, e^{\frac{i\chi}{2}} + E^{-}_{1p_{II}}\, e^{-\frac{i\chi}{2}}\right) \bar{n}_1$$

$$E^{+}_{0s_{I}} + E^{-}_{0s_{I}} = E^{+}_{1s_{II}}\, e^{\frac{i\chi}{2}} + E^{-}_{1s_{II}}\, e^{-\frac{i\chi}{2}}$$

$$\left(E^{+}_{0s_{I}} - E^{-}_{0s_{I}}\right) n_0 \cos \phi_0 = \left(E^{+}_{1s_{II}}\, e^{\frac{i\chi}{2}} - E^{-}_{1s_{II}}\, e^{-\frac{i\chi}{2}}\right) \bar{n}_1 \cos \bar{\phi}_1 .$$

Since $\bar{r}_{2p}$ and $\bar{r}_{2s}$ have already been defined by equations (A-5) and (A-6), the boundary conditions may be written as:

$$\left(E^{+}_{op_I} - E^{-}_{op_I}\right)\cos\phi_0 = \left(e^{\frac{i\chi}{2}} - \bar{r}_{2p}\, e^{-\frac{i\chi}{2}}\right)E^{+}_{1p_{II}}\cos\bar{\phi}_1 \qquad \text{(B-18)}$$

$$\left(E^{+}_{op_I} + E^{-}_{op_I}\right) n_0 = \left(e^{\frac{i\chi}{2}} + \bar{r}_{2p}\, e^{-\frac{i\chi}{2}}\right) E^{+}_{1p_{II}}\bar{n}_1 \qquad \text{(B-19)}$$

$$E^{+}_{os_I} + E^{-}_{os_I} = \left(e^{\frac{i\chi}{2}} + \bar{r}_{2s}\, e^{-\frac{i\chi}{2}}\right) E^{+}_{1s_{II}} \qquad \text{(B-20)}$$

$$\left(E^{+}_{os_I} - E^{-}_{os_I}\right) n_0\cos\phi_0 = \left(e^{\frac{i\chi}{2}} - \bar{r}_{2s}\, e^{-\frac{i\chi}{2}}\right)E^{+}_{1s_{II}}\bar{n}_1\cos\bar{\phi}_1\,. \qquad \text{(B-21)}$$

For notation purposes, the factor, β, is defined:

$$\beta = e^{\frac{i\chi}{2}}\,.$$

Division of equation (B-18) by equation (B-19) and similarly equation (B-20) by equation (B-21), the following is determined:

$$\frac{\left(E^{+}_{op_I} - E^{-}_{op_I}\right)\cos\phi_0}{\left(E^{+}_{op_I} + E^{-}_{op_I}\right) n_0} = \frac{\left(\frac{1}{\beta} - \bar{r}_{2p}\,\beta\right)\cos\bar{\phi}_1}{\left(\frac{1}{\beta} + \bar{r}_{2p}\,\beta\right)\bar{n}_1}\,,$$

and

$$\frac{E^{+}_{os_I} + E^{-}_{os_I}}{E^{+}_{os_I} - E^{-}_{os_I}} = \frac{\frac{1}{\beta} + \bar{r}_{2s}\,\beta}{\frac{1}{\beta} - \bar{r}_{2s}\,\beta}\cdot\frac{n_0\cos\phi_0}{\bar{n}_1\cos\bar{\phi}_1}\cdot\left(\frac{\beta}{\beta}\right).$$

By mathematical manipulation, it is shown that:

$$\frac{2n_0\cos\phi_0}{2\bar{n}_1\cos\bar{\phi}_1} = \frac{n_0\cos\phi_0 + \bar{n}_1\cos\bar{\phi}_1 + n_0\cos\phi_0 - \bar{n}_1\cos\bar{\phi}_1}{n_0\cos\phi_0 + \bar{n}_1\cos\bar{\phi}_1 - n_0\cos\phi_0 + \bar{n}_1\cos\bar{\phi}_1}$$

$$= \frac{1+\bar{r}_{1s}}{1-\bar{r}_{1s}} .$$

Upon substituting back, it may be seen that:

$$E^{+}_{os_I} + E^{-}_{os_I} = \frac{(1+\bar{r}_{2s}\beta^2)(1+\bar{r}_{1s})}{(1-\bar{r}_{2s}\beta^2)(1-\bar{r}_{1s})}\left(E^{+}_{os_I} - E^{-}_{os_I}\right),$$

or

$$\frac{E^{-}_{os_I}}{E^{+}_{os_I}} = \frac{(1+\beta^2\bar{r}_{2s})(1+\bar{r}_{1s}) - (1-\beta^2\bar{r}_{2s})(1-\bar{r}_{1s})}{(1+\beta^2\bar{r}_{2s})(1+\bar{r}_{1s}) + (1-\beta^2\bar{r}_{2s})(1-\bar{r}_{1s})} .$$

By defining $\bar{r}_s$, it is obvious that:

$$\bar{r}_s \triangleq \frac{E^{-}_{os_I}}{E^{+}_{os_I}}$$

$$= \frac{(1+\beta^2\bar{r}_{2s})(1+\bar{r}_{1s}) - (1-\beta^2\bar{r}_{2s})(1-\bar{r}_{1s})}{(1+\beta^2\bar{r}_{2s})(1+\bar{r}_{1s}) + (1-\beta^2\bar{r}_{2s})(1-\bar{r}_{1s})} ,$$

which after simplification yields:

$$\bar{r}_s = \frac{\bar{r}_{1s} + \bar{r}_{2s}\, e^{-i\chi}}{1 + \bar{r}_{1s}\bar{r}_{2s}\, e^{-i\chi}} \qquad \text{(B-22)}$$

A similar development in the p-plane results in:

$$\boxed{\bar{r}_p = \frac{\bar{r}_{1p} + \bar{r}_{2p}\, e^{-i\chi}}{1 + \bar{r}_{1p}\bar{r}_{2p}\, e^{-i\chi}}} \qquad \text{(B-23)}$$

The equations for $\bar{r}_s$ and $\bar{r}_p$ will now be put into a different form by noting that the complex phase difference was defined to be:

$$\chi \triangleq \frac{4\pi d \bar{n}_1 \cos \bar{\Phi}_1}{\lambda},$$

or

$$\chi = \frac{4\pi d}{\lambda}(a - bi);$$

and

$$i\chi = \frac{4\pi d}{\lambda}(ai + b)$$

$$= \frac{4\pi db}{\lambda} + \frac{4\pi ad}{\lambda} i .$$

If α and γ are defined as:

$$\alpha = \frac{2\pi bd}{\lambda}$$

$$\gamma = \frac{2\pi ad}{\lambda},$$

it follows that:

$$e^{-i\chi} = e^{-2\alpha}(\cos 2\gamma - i \sin 2\gamma).$$

It is noted that equations (B-22) and (B-23) are the same except for the subscripts, s and p; these subscripts are dropped and a typical expression for the complex reflectance in either plane is:

$$\bar{r} = \frac{\bar{r}_1 + \bar{r}_2 \, e^{-i\chi}}{1 + \bar{r}_1 \bar{r}_2 \, e^{-i\chi}} \qquad \text{(B-24)}$$

or:

$$\bar{r} = \frac{(g_1 + i h_1) + (g_2 + i h_2) e^{-2\alpha} (\cos 2\gamma - i \sin 2\gamma)}{1 + (g_1 + i h_1)(g_2 + i h_2) \, e^{-2\alpha} (\cos 2\gamma - i \sin 2\gamma)}, \qquad \text{(B-25)}$$

where the s and p subscripts for the g and h expressions have been dropped. Definition of U to be the numerator and T to be the denominator of equation (B-25), $\bar{r}$ is then defined to be:

$$\bar{r} \triangleq \frac{U}{T}.$$

This expression, then, gives the amplitude reflectance for an electromagnetic wave in either the s- or p-plane when reflected from a thin film-covered surface.

Energy Reflectances for the Air-Film-Water System

From considerations of the Poynting vector in electromagnetic theory, it is well known that the value for energy reflectance must be a real value and that it is found by multiplying the complex "amplitude reflectance" by its complex conjugate. Thus, if we define:

$$R = \bar{r}\,\bar{r}^*,$$

it follows that:

$$R = \frac{U}{T}\cdot\frac{U^*}{T^*} = \frac{U\,U^*}{T\,T^*}. \qquad \text{(B-26)}$$

The problem of evaluating U, U*, T, and T* will now be considered. By grouping terms in equation (B-25), it is seen that:

$$U = g_1 + e^{-2\alpha}(g_2\cos 2\gamma + h_2\sin 2\gamma) + i\left[h_1 + e^{-2\alpha}(h_2\cos 2\gamma - g_2\sin 2\gamma)\right]$$

$$U^* = g_1 + e^{-2\alpha}(g_2\cos 2\gamma + h_2\sin 2\gamma) - i\left[h_1 + e^{-2\alpha}(h_2\cos 2\gamma - g_2\sin 2\gamma)\right]$$

$$\begin{aligned} UU^* = {} & g_1^2 + h_1^2 + 2e^{-2\alpha}(g_1 g_2\cos 2\gamma + g_1 h_2\sin 2\gamma + h_1 h_2\cos 2\gamma \\ & - h_1 g_2\sin 2\gamma) + e^{-4\alpha}(g_2^2\cos^2 2\gamma + h_2^2\sin^2 2\gamma \\ & + h_2^2\cos^2 2\gamma + g_2^2\sin^2 2\gamma) \\ = {} & g_1^2 + h_1^2 + 2e^{-2\alpha}\left[(g_1 g_2 + h_1 h_2)\cos 2\gamma \right. \\ & \left. + (g_1 h_2 - g_2 h_1)\sin 2\gamma\right] + e^{-4\alpha}(g_2^2 + h_2^2) \\ = {} & g_1^2 + h_1^2 + e^{-2\alpha}(A\cos 2\gamma + B\sin 2\gamma) + e^{-4\alpha}(g_2^2 + h_2^2), \end{aligned}$$

where: $A \triangleq 2(g_1 g_2 + h_1 h_2)$ and $B \triangleq 2(g_1 h_2 - h_1 g_2)$.

Likewise it may be shown that:

$$T = 1 + e^{-2\alpha}\left[(g_1g_2 - h_1h_2)\cos 2\gamma + (g_1h_2 + h_1g_2)\sin 2\gamma\right]$$

$$- i e^{-2\alpha}\left[(g_1g_2 - h_1h_2)\sin 2\gamma - (g_1h_2 + g_2h_1)\cos 2\gamma\right].$$

If C and D are defined as:

$$C \triangleq 2(g_1g_2 - h_1h_2)$$

$$D \triangleq 2(g_1h_2 + h_1g_2),$$

it may be shown that:

$$T = 1 + e^{-2\alpha}\left[\frac{C}{2}\cos 2\gamma + \frac{D}{2}\sin 2\gamma\right] + i e^{-2\alpha}\left[\frac{D}{2}\cos 2\gamma - \frac{C}{2}\sin 2\gamma\right]$$

$$T^* = 1 + e^{-2\alpha}\left[\frac{C}{2}\cos 2\gamma + \frac{D}{2}\sin 2\gamma\right] - i e^{-2\alpha}\left[\frac{D}{2}\cos 2\gamma - \frac{C}{2}\sin 2\gamma\right],$$

or:

$$TT^* = 1 + e^{-2\alpha}\left[C\cos 2\gamma + D\sin 2\gamma\right] + e^{-4\alpha}\left[\frac{C^2}{4}\cos^2 2\gamma + \frac{D^2}{4}\sin^2 2\gamma\right.$$

$$\left. + \frac{D^2}{4}\cos^2 2\gamma + \frac{C^2}{4}\sin^2 2\gamma\right]$$

$$= 1 + e^{-2\alpha}(C\cos 2\gamma + D\sin 2\gamma) + \frac{1}{4}e^{-4\alpha}(D^2 + C^2).$$

However, it may be noted that:

$$\frac{C^2+D^2}{4} = \frac{4}{4}\left[g_1^2 g_2^2 - 2g_1g_2h_1h_2 + h_1^2h_2^2 + g_1^2h_2^2 + 2g_1g_2h_1h_2 + h_1^2h_2^2\right]$$

$$= (g_1^2+h_1^2)(g_2^2+h_2^2),$$

so that:

$$TT^* = 1 + e^{-2\alpha}(C\cos 2\gamma + D\sin 2\gamma) + e^{-4\alpha}(g_1^2+h_1^2)(g_2^2+h_2^2).$$

Now upon substituting back into equation (B-26), it is seen that

$$R = \frac{UU^*}{TT^*} = \frac{(g_1^2+h_1^2) + e^{-4\alpha}(g_2^2+h_2^2) + e^{-2\alpha}(A\cos 2\gamma + B\sin 2\gamma)}{1 + e^{-2\alpha}(C\cos 2\gamma + D\sin 2\gamma) + e^{-4\alpha}(g_1^2+h_1^2)(g_2^2+h_2^2)}$$

$$= \frac{(g_1^2+h_1^2)e^{2\alpha} + (g_2^2+h_2^2)e^{-2\alpha} + A\cos 2\gamma + B\sin 2\gamma}{e^{2\alpha} + e^{-2\alpha}(g_1^2+h_1^2)(g_2^2+h_2^2) + C\cos 2\gamma + D\sin 2\gamma}, \qquad \text{(B-27)}$$

where the values of the different symbols have been defined in the previous pages of this appendix. It should be mentioned, in summary, that R_s and R_p are found by giving each symbol in equation (B-27) the subscript s and p, respectively.

APPENDIX C

COMPUTER PROGRAMS FOR THIN-FILM REFLECTANCES AND RESULTS

Reflectance Program and Results

A computer program for the calculation of the s-plane, the p-plane, and the unpolarized-energy reflectances is found on the next two pages. Following it is a series of tables which present the results of the computer study. Tables X, XI, and XII give the calculated values for various reflectances at 0.5 microns; and Tables XIII, XIV, and XV give the similar reflectances at 2.0 microns.

The column headings are as follows:

PHI	The angle of the incident energy
RS	The s-plane reflectance
RP	The p-plane reflectance
R	The reflectance for unpolarized-incident energy
N1	The index of refraction of the film
K1	The coefficient of absorption of the film
N2	The index of refraction of the water
K2	The coefficient of absorption of the water
W/D	The ratio of the wave length to the film thickness.

One should further note that from the results at 0.5 microns the W/D values of 210, 21, and 2 correspond, respectively, to film thicknesses of 1, 10, and 100 molecules, where the molecular length is considered to be 24 Å.

Similarly, for the 2.0 micron results, W/D values of 835, 83, and 8 refer, respectively, to film thicknesses of 1, 10, and 100 molecules.

A Computer Program for the Calculation of the Energy Reflectances For Thin Film-Covered Water Surfaces

```
      PUNCH 105
      PUNCH 104
  100 CONTINUE
      READ 200, RN1,E1,RN2,E2,I1,IM,ID,D,W,Z
    8 DO 80 I=I1,IM,ID
      T1 = I-1
      T=(T1*3.1416)/180.0
      C1=(RN1**2)*(1.0-E1**2)-(SIN(T))**2
      D1=4.0*((RN1)**4)*(E1**2)
      C2=(RN2**2)*(1.0-E2**2)-(SIN(T))**2
      D2=4.0*((RN2)**4)*(E2**2)
      AA = ABS((C1+SQR((C1**2)+D1))/2.0)
      BB = ABS((-C1+SQR((C1**2)+D1))/2.0)
      A = SQR(AA)
      B = SQR(BB)
      AA1 = ABS((C2+SQR((C2**2)+D2))/2.0)
      BB1 = ABS((-C2+SQR((C2**2)+D2))/2.0)
      A1 = SQR(AA1)
      B1 = SQR(BB1)
      GSA=(COS(T))**2-A**2-B**2
      GSB=(COS(T)+A)**2+B**2
      GS1=GSA/GSB
      HS1=(2.0*B*COS(T))/(GSB)
      GSC=(A-A1)*(A+A1)-(B1-B)*(B1+B)
      GSD=(A+A1)**2+(B+B1)**2
      GS2=GSC/GSD
      HS2=(2.0*(A*B1-A1*B))/(GSD)
      AL=((2.0*3.1416)*B*D)/W
      GA=((2.0*3.1416)*A*D)/W
      XS=(GS1**2+HS1**2)
      YS=(GS2**2+HS2**2)
      AS2=2.0*(GS1*GS2+HS1*HS2)
      BS2=2.0*(GS1*HS2-GS2*HS1)
      CS2=2.0*(GS1*GS2-HS1*HS2)
      DS2=2.0*(GS1*HS2+GS2*HS1)
      RSA1=(XS)*(EXP(2.0*AL))+(YS)*(EXP(-2.0*AL))
      RSA2=AS2*COS(2.0*GA)+BS2*SIN(2.0*GA)
      RSA=RSA1+RSA2
      RSB1=EXP(2.0*AL)+(XS)*(YS)*(EXP(-2.0*AL))
      RSB2=CS2*COS(2.0*GA)+DS2*SIN(2.0*GA)
      RSB=RSB1+RSB2
      RS1=RSA/RSB
```

```
      GPA1=(RN1**4)*((COS(T))**2)*(1.0-(E1**2))**2-(A**2+B**2)
      GPB1=(RN1**2*COS(T)*(1.0-E1**2)+A)**2
      GPB2=(2.0*COS(T)*E1*(RN1**2)+B)**2
      GP1=Z*(GPA1+(D1)*(COS(T))**2)/(GPB1+GPB2)
      HPA1=(RN1**2)*(COS(T))*(1.0-(E1)**2)*B
      HPA2=(2.0)*(E1)*(RN1**2)*(A)*(COS(T))
      HP1=((2.0)*(Z)*(HPA1-HPA2))/(GPB1+GPB2)
      SA=(RN2**2)*((1.0-(E2**2))*(1.0-(E1**2))+4.0*E1*E2)
      SB=(RN1**2)*(1.0+(E1**2))**2
      S=SA/SB
      UA=(2.0)*(RN2**2)*(E2-E1)*(1.0+(E1)*(E2))
      U=UA/SB
      GPA3=(A*S-B*U)**2-(A1)**2
      GPA4=(A*U+B*S)**2-(B1)**2
      GPB3=(A*S-B*U+A1)**2
      GPB4=(B1+A*U+B*S)**2
      GP2=(Z*(GPA3+GPA4))/(GPB3+GPB4)
      HPA3=(A*S-B*U-A1)*(B1+A*U+B*S)
      HPA4=(A*S-B*U+A1)*(A*U+B*S-B1)
      HPB3=(A*S-B*U+A1)**2
      HPB4=(B1+A*U+B*S)**2
      HP2=Z*(HPA3-HPA4)/(HPB3+HPB4)
      XP=(GP1**2+HP1**2)
      YP=(GP2**2+HP2**2)
      AP2=2.0*(GP1*GP2+HP1*HP2)
      BP2=2.0*(GP1*HP2-GP2*HP1)
      CP2=2.0*(GP1*GP2-HP1*HP2)
      DP2=2.0*(GP1*HP2+GP2*HP1)
      RPA1=(XP)*(EXP(2.0*AL))+(YP)*(EXP(-2.0*AL))
      RPA2=AP2*COS(2.0*GA)+BP2*SIN(2.0*GA)
      RPA=RPA1+RPA2
      RPB1=EXP(2.0*AL)+(XP)*(YP)*(EXP(-2.0*AL))
      RPB2=CP2*COS(2.0*GA) + DP2*SIN(2.0*GA)
      RPB=RPB1+RPB2
      RP1=RPA/RPB
      R=(RS1+RP1)/(2.0)
      PUNCH 201, TI, RS1, RP1, R, RN1, E1, RN2, E2, W
   80 CONTINUE
      GO TO 100
  104 FORMAT(1H )
  105 FORMAT(7H    PHI,6H    RS,6X2HRP,6X1HR,6X2HN1,6X2HK1,6X2HN2,
     16X2HK2,5X3HW/D)
  200 FORMAT(4F10.5,3I3,1X,3F10.5)
  201 FORMAT(I6,3F8.4,F7.3,F9.5,F7.3,F9.5,I6)
      END
```

TABLE X

THE REFLECTANCE OF A MONOLAYER-COVERED WATER SURFACE AT 0.5 MICRONS

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0207	.0207	.0207	1.300	.00000	1.336	.00000	210
20	.0248	.0169	.0209	1.300	.00000	1.336	.00000	210
40	.0437	.0060	.0248	1.300	.00000	1.336	.00000	210
60	.1162	.0043	.0602	1.300	.00000	1.336	.00000	210
80	.4588	.2390	.3489	1.300	.00000	1.336	.00000	210
0	.0207	.0207	.0207	1.300	.00010	1.336	.00000	210
20	.0248	.0169	.0209	1.300	.00010	1.336	.00000	210
40	.0437	.0060	.0248	1.300	.00010	1.336	.00000	210
60	.1162	.0043	.0602	1.300	.00010	1.336	.00000	210
80	.4588	.2390	.3489	1.300	.00010	1.336	.00000	210
0	.0208	.0208	.0208	1.300	.01000	1.336	.00000	210
20	.0250	.0170	.0210	1.300	.01000	1.336	.00000	210
40	.0439	.0060	.0249	1.300	.01000	1.336	.00000	210
60	.1165	.0043	.0604	1.300	.01000	1.336	.00000	210
80	.4592	.2390	.3491	1.300	.01000	1.336	.00000	210
0	.0217	.0218	.0218	1.300	.10000	1.336	.00000	210
20	.0261	.0177	.0219	1.300	.10000	1.336	.00000	210
40	.0455	.0063	.0259	1.300	.10000	1.336	.00000	210
60	.1192	.0042	.0617	1.300	.10000	1.336	.00000	210
80	.4629	.2396	.3512	1.300	.10000	1.336	.00000	210
0	.0324	.0324	.0324	1.300	1.00000	1.336	.00000	210
20	.0380	.0271	.0325	1.300	1.00000	1.336	.00000	210
40	.0621	.0117	.0369	1.300	1.00000	1.336	.00000	210
60	.1465	.0026	.0746	1.300	1.00000	1.336	.00000	210
80	.4977	.2282	.3630	1.300	1.00000	1.336	.00000	210
0	.0207	.0207	.0207	1.400	.00000	1.336	.00000	210
20	.0249	.0169	.0209	1.400	.00000	1.336	.00000	210
40	.0438	.0060	.0249	1.400	.00000	1.336	.00000	210
60	.1163	.0043	.0603	1.400	.00000	1.336	.00000	210
80	.4589	.2390	.3489	1.400	.00000	1.336	.00000	210
0	.0207	.0207	.0207	1.400	.00010	1.336	.00000	210
20	.0249	.0169	.0209	1.400	.00010	1.336	.00000	210
40	.0438	.0060	.0249	1.400	.00010	1.336	.00000	210
60	.1163	.0043	.0603	1.400	.00010	1.336	.00000	210
80	.4589	.2390	.3489	1.400	.00010	1.336	.00000	210
0	.0208	.0208	.0208	1.400	.01000	1.336	.00000	210
20	.0250	.0170	.0210	1.400	.01000	1.336	.00000	210
40	.0440	.0060	.0250	1.400	.01000	1.336	.00000	210
60	.1166	.0043	.0604	1.400	.01000	1.336	.00000	210
80	.4594	.2390	.3492	1.400	.01000	1.336	.00000	210
0	.0220	.0220	.0220	1.400	.10000	1.336	.00000	210
20	.0263	.0179	.0221	1.400	.10000	1.336	.00000	210
40	.0458	.0064	.0261	1.400	.10000	1.336	.00000	210
60	.1198	.0041	.0619	1.400	.10000	1.336	.00000	210
80	.4636	.2389	.3513	1.400	.10000	1.336	.00000	210
0	.0344	.0344	.0344	1.400	1.00000	1.336	.00000	210
20	.0402	.0289	.0345	1.400	1.00000	1.336	.00000	210
40	.0650	.0129	.0389	1.400	1.00000	1.336	.00000	210
60	.1512	.0022	.0767	1.400	1.00000	1.336	.00000	210
80	.5033	.2245	.3639	1.400	1.00000	1.336	.00000	210

TABLE X (CONTINUED)

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0208	.0208	.0208	1.500	.00000	1.336	.00000	210
20	.0250	.0170	.0210	1.500	.00000	1.336	.00000	210
40	.0439	.0060	.0249	1.500	.00000	1.336	.00000	210
60	.1164	.0043	.0604	1.500	.00000	1.336	.00000	210
80	.4591	.2390	.3490	1.500	.00000	1.336	.00000	210
0	.0208	.0208	.0208	1.500	.00010	1.336	.00000	210
20	.0250	.0170	.0210	1.500	.00010	1.336	.00000	210
40	.0439	.0060	.0249	1.500	.00010	1.336	.00000	210
60	.1164	.0043	.0604	1.500	.00010	1.336	.00000	210
80	.4591	.2390	.3490	1.500	.00010	1.336	.00000	210
0	.0209	.0209	.0209	1.500	.01000	1.336	.00000	210
20	.0251	.0171	.0211	1.500	.01000	1.336	.00000	210
40	.0441	.0061	.0251	1.500	.01000	1.336	.00000	210
60	.1168	.0042	.0605	1.500	.01000	1.336	.00000	210
80	.4596	.2389	.3493	1.500	.01000	1.336	.00000	210
0	.0222	.0222	.0222	1.500	.10000	1.336	.00000	210
20	.0266	.0182	.0224	1.500	.10000	1.336	.00000	210
40	.0462	.0066	.0264	1.500	.10000	1.336	.00000	210
60	.1204	.0040	.0622	1.500	.10000	1.336	.00000	210
80	.4645	.2383	.3514	1.500	.10000	1.336	.00000	210
0	.0365	.0365	.0365	1.500	1.00000	1.336	.00000	210
20	.0425	.0308	.0367	1.500	1.00000	1.336	.00000	210
40	.0682	.0141	.0412	1.500	1.00000	1.336	.00000	210
60	.1562	.0018	.0790	1.500	1.00000	1.336	.00000	210
80	.5093	.2208	.3650	1.500	1.00000	1.336	.00000	210

TABLE XI

THE REFLECTANCE OF A TEN MOLECULE THICK FILM-COVERED WATER SURFACE AT 0.5 MICRONS

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0197	.0197	.0197	1.300	.00000	1.336	.00000	21
20	.0238	.0161	.0199	1.300	.00000	1.336	.00000	21
40	.0423	.0057	.0240	1.300	.00000	1.336	.00000	21
60	.1141	.0044	.0592	1.300	.00000	1.336	.00000	21
80	.4561	.2386	.3474	1.300	.00000	1.336	.00000	21
0	.0197	.0197	.0197	1.300	.00010	1.336	.00000	21
20	.0237	.0161	.0199	1.300	.00010	1.336	.00000	21
40	.0423	.0057	.0240	1.300	.00010	1.336	.00000	21
60	.1141	.0044	.0592	1.300	.00010	1.336	.00000	21
80	.4562	.2386	.3474	1.300	.00010	1.336	.00000	21
0	.0206	.0207	.0207	1.300	.01000	1.336	.00000	21
20	.0249	.0169	.0209	1.300	.01000	1.336	.00000	21
40	.0439	.0060	.0250	1.300	.01000	1.336	.00000	21
60	.1169	.0043	.0606	1.300	.01000	1.336	.00000	21
80	.4601	.2392	.3496	1.300	.01000	1.336	.00000	21
0	.0300	.0301	.0301	1.300	.10000	1.336	.00000	21
20	.0354	.0245	.0300	1.300	.10000	1.336	.00000	21
40	.0589	.0091	.0340	1.300	.10000	1.336	.00000	21
60	.1423	.0039	.0731	1.300	.10000	1.336	.00000	21
80	.4932	.2449	.3690	1.300	.10000	1.336	.00000	21
0	.1676	.1676	.1676	1.300	1.00000	1.336	.00000	21
20	.1830	.1503	.1667	1.300	1.00000	1.336	.00000	21
40	.2392	.1021	.1706	1.300	1.00000	1.336	.00000	21
60	.3766	.0513	.2139	1.300	1.00000	1.336	.00000	21
80	.7036	.2506	.4771	1.300	1.00000	1.336	.00000	21
0	.0232	.0232	.0232	1.400	.00000	1.336	.00000	21
20	.0276	.0189	.0233	1.400	.00000	1.336	.00000	21
40	.0474	.0068	.0271	1.400	.00000	1.336	.00000	21
60	.1218	.0041	.0630	1.400	.00000	1.336	.00000	21
80	.4656	.2392	.3524	1.400	.00000	1.336	.00000	21
0	.0232	.0233	.0232	1.400	.00010	1.336	.00000	21
20	.0276	.0189	.0233	1.400	.00010	1.336	.00000	21
40	.0474	.0068	.0271	1.400	.00010	1.336	.00000	21
60	.1218	.0041	.0629	1.400	.00010	1.336	.00000	21
80	.4656	.2392	.3524	1.400	.00010	1.336	.00000	21
0	.0243	.0243	.0243	1.400	.01000	1.336	.00000	21
20	.0288	.0198	.0243	1.400	.01000	1.336	.00000	21
40	.0491	.0072	.0282	1.400	.01000	1.336	.00000	21
60	.1249	.0039	.0644	1.400	.01000	1.336	.00000	21
80	.4698	.2392	.3545	1.400	.01000	1.336	.00000	21
0	.0348	.0348	.0348	1.400	.10000	1.336	.00000	21
20	.0407	.0286	.0346	1.400	.10000	1.336	.00000	21
40	.0658	.0111	.0385	1.400	.10000	1.336	.00000	21
60	.1527	.0028	.0778	1.400	.10000	1.336	.00000	21
80	.5052	.2397	.3725	1.400	.10000	1.336	.00000	21
0	.1881	.1881	.1881	1.400	1.00000	1.336	.00000	21
20	.2044	.1700	.1872	1.400	1.00000	1.336	.00000	21
40	.2631	.1184	.1907	1.400	1.00000	1.336	.00000	21
60	.4026	.0571	.2298	1.400	1.00000	1.336	.00000	21
80	.7211	.2324	.4768	1.400	1.00000	1.336	.00000	21

TABLE XI (CONTINUED)

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0293	.0294	.0293	1.500	.00000	1.336	.00000	21
20	.0343	.0240	.0291	1.500	.00000	1.336	.00000	21
40	.0561	.0092	.0326	1.500	.00000	1.336	.00000	21
60	.1350	.0038	.0694	1.500	.00000	1.336	.00000	21
80	.4813	.2385	.3599	1.500	.00000	1.336	.00000	21
0	.0293	.0294	.0294	1.500	.00010	1.336	.00000	21
20	.0343	.0240	.0292	1.500	.00010	1.336	.00000	21
40	.0561	.0092	.0326	1.500	.00010	1.336	.00000	21
60	.1349	.0038	.0694	1.500	.00010	1.336	.00000	21
80	.4812	.2384	.3598	1.500	.00010	1.336	.00000	21
0	.0305	.0305	.0305	1.500	.01000	1.336	.00000	21
20	.0356	.0250	.0303	1.500	.01000	1.336	.00000	21
40	.0580	.0096	.0338	1.500	.01000	1.336	.00000	21
60	.1382	.0035	.0709	1.500	.01000	1.336	.00000	21
80	.4856	.2379	.3617	1.500	.01000	1.336	.00000	21
0	.0418	.0419	.0418	1.500	.10000	1.336	.00000	21
20	.0483	.0346	.0414	1.500	.10000	1.336	.00000	21
40	.0758	.0143	.0450	1.500	.10000	1.336	.00000	21
60	.1675	.0019	.0847	1.500	.10000	1.336	.00000	21
80	.5215	.2339	.3777	1.500	.10000	1.336	.00000	21
0	.2095	.2095	.2095	1.500	1.00000	1.336	.00000	21
20	.2266	.1907	.2087	1.500	1.00000	1.336	.00000	21
40	.2875	.1359	.2117	1.500	1.00000	1.336	.00000	21
60	.4285	.0645	.2465	1.500	1.00000	1.336	.00000	21
80	.7379	.2158	.4768	1.500	1.00000	1.336	.00000	21

TABLE XII

THE REFLECTANCE OF A ONE HUNDRED MOLECULE THICK FILM-COVERED WATER SURFACE AT 0.5 MICRONS

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0175	.0175	.0175	1.300	.00000	1.336	.00000	2
20	.0222	.0150	.0186	1.300	.00000	1.336	.00000	2
40	.0430	.0058	.0244	1.300	.00000	1.336	.00000	2
60	.1147	.0043	.0595	1.300	.00000	1.336	.00000	2
80	.4448	.2373	.3411	1.300	.00000	1.336	.00000	2
0	.0175	.0175	.0175	1.300	.00010	1.336	.00000	2
20	.0221	.0150	.0186	1.300	.00010	1.336	.00000	2
40	.0430	.0058	.0244	1.300	.00010	1.336	.00000	2
60	.1147	.0043	.0595	1.300	.00010	1.336	.00000	2
80	.4448	.2373	.3410	1.300	.00010	1.336	.00000	2
0	.0187	.0188	.0188	1.300	.01000	1.336	.00000	2
20	.0235	.0159	.0197	1.300	.01000	1.336	.00000	2
40	.0436	.0059	.0248	1.300	.01000	1.336	.00000	2
60	.1112	.0044	.0578	1.300	.01000	1.336	.00000	2
80	.4365	.2362	.3364	1.300	.01000	1.336	.00000	2
0	.0270	.0271	.0271	1.300	.10000	1.336	.00000	2
20	.0314	.0215	.0265	1.300	.10000	1.336	.00000	2
40	.0476	.0066	.0271	1.300	.10000	1.336	.00000	2
60	.1065	.0055	.0560	1.300	.10000	1.336	.00000	2
80	.4358	.2440	.3399	1.300	.10000	1.336	.00000	2
0	.2550	.2550	.2550	1.300	1.00000	1.336	.00000	2
20	.2795	.2317	.2556	1.300	1.00000	1.336	.00000	2
40	.3610	.1711	.2660	1.300	1.00000	1.336	.00000	2
60	.5223	.1259	.3241	1.300	1.00000	1.336	.00000	2
80	.8010	.3742	.5876	1.300	1.00000	1.336	.00000	2
0	.0321	.0321	.0321	1.400	.00000	1.336	.00000	2
20	.0360	.0251	.0306	1.400	.00000	1.336	.00000	2
40	.0520	.0079	.0299	1.400	.00000	1.336	.00000	2
60	.1175	.0042	.0609	1.400	.00000	1.336	.00000	2
80	.4610	.2391	.3500	1.400	.00000	1.336	.00000	2
0	.0321	.0321	.0321	1.400	.00010	1.336	.00000	2
20	.0360	.0251	.0306	1.400	.00010	1.336	.00000	2
40	.0520	.0079	.0299	1.400	.00010	1.336	.00000	2
60	.1174	.0042	.0608	1.400	.00010	1.336	.00000	2
80	.4610	.2391	.3500	1.400	.00010	1.336	.00000	2
0	.0329	.0329	.0329	1.400	.01000	1.336	.00000	2
20	.0372	.0260	.0316	1.400	.01000	1.336	.00000	2
40	.0544	.0084	.0314	1.400	.01000	1.336	.00000	2
60	.1210	.0041	.0625	1.400	.01000	1.336	.00000	2
80	.4626	.2393	.3510	1.400	.01000	1.336	.00000	2
0	.0397	.0397	.0397	1.400	.10000	1.336	.00000	2
20	.0457	.0322	.0390	1.400	.10000	1.336	.00000	2
40	.0684	.0116	.0400	1.400	.10000	1.336	.00000	2
60	.1442	.0038	.0740	1.400	.10000	1.336	.00000	2
80	.4887	.2449	.3668	1.400	.10000	1.336	.00000	2
0	.2747	.2747	.2747	1.400	1.00000	1.336	.00000	2
20	.2992	.2512	.2752	1.400	1.00000	1.336	.00000	2
40	.3799	.1878	.2838	1.400	1.00000	1.336	.00000	2
60	.5383	.1296	.3339	1.400	1.00000	1.336	.00000	2
80	.8087	.3526	.5807	1.400	1.00000	1.336	.00000	2

TABLE XII (CONTINUED)

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0628	.0628	.0628	1.500	.00000	1.336	.00000	2
20	.0700	.0509	.0605	1.500	.00000	1.336	.00000	2
40	.0917	.0187	.0552	1.500	.00000	1.336	.00000	2
60	.1519	.0033	.0776	1.500	.00000	1.336	.00000	2
80	.4710	.2387	.3549	1.500	.00000	1.336	.00000	2
0	.0628	.0629	.0628	1.500	.00010	1.336	.00000	2
20	.0700	.0510	.0605	1.500	.00010	1.336	.00000	2
40	.0917	.0187	.0552	1.500	.00010	1.336	.00000	2
60	.1518	.0033	.0776	1.500	.00010	1.336	.00000	2
80	.4709	.2387	.3548	1.500	.00010	1.336	.00000	2
0	.0613	.0613	.0613	1.500	.01000	1.336	.00000	2
20	.0687	.0500	.0593	1.500	.01000	1.336	.00000	2
40	.0919	.0187	.0553	1.500	.01000	1.336	.00000	2
60	.1570	.0029	.0799	1.500	.01000	1.336	.00000	2
80	.4810	.2381	.3596	1.500	.01000	1.336	.00000	2
0	.0571	.0571	.0571	1.500	.10000	1.336	.00000	2
20	.0657	.0476	.0567	1.500	.10000	1.336	.00000	2
40	.0966	.0198	.0582	1.500	.10000	1.336	.00000	2
60	.1865	.0019	.0942	1.500	.10000	1.336	.00000	2
80	.5339	.2390	.3865	1.500	.10000	1.336	.00000	2
0	.2942	.2942	.2942	1.500	1.00000	1.336	.00000	2
20	.3186	.2705	.2946	1.500	1.00000	1.336	.00000	2
40	.3987	.2048	.3017	1.500	1.00000	1.336	.00000	2
60	.5541	.1351	.3446	1.500	1.00000	1.336	.00000	2
80	.8165	.3330	.5747	1.500	1.00000	1.336	.00000	2

TABLE XIII

THE REFLECTANCE OF A MONOLAYER-COVERED WATER SURFACE AT 2.0 MICRONS

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0174	.0174	.0174	1.300	.00000	1.304	.00110	835
20	.0210	.0141	.0176	1.300	.00000	1.304	.00110	835
40	.0376	.0048	.0212	1.300	.00000	1.304	.00110	835
60	.1037	.0047	.0542	1.300	.00000	1.304	.00110	835
80	.4386	.2367	.3376	1.300	.00000	1.304	.00110	835
0	.0174	.0174	.0174	1.300	.00010	1.304	.00110	835
20	.0210	.0141	.0176	1.300	.00010	1.304	.00110	835
40	.0376	.0048	.0212	1.300	.00010	1.304	.00110	835
60	.1037	.0047	.0542	1.300	.00010	1.304	.00110	835
80	.4386	.2367	.3376	1.300	.00010	1.304	.00110	835
0	.0174	.0174	.0174	1.300	.01000	1.304	.00110	835
20	.0210	.0141	.0176	1.300	.01000	1.304	.00110	835
40	.0376	.0048	.0212	1.300	.01000	1.304	.00110	835
60	.1037	.0047	.0542	1.300	.01000	1.304	.00110	835
80	.4387	.2367	.3377	1.300	.01000	1.304	.00110	835
0	.0177	.0177	.0177	1.300	.10000	1.304	.00110	835
20	.0213	.0143	.0178	1.300	.10000	1.304	.00110	835
40	.0380	.0048	.0214	1.300	.10000	1.304	.00110	835
60	.1044	.0046	.0545	1.300	.10000	1.304	.00110	835
80	.4397	.2368	.3383	1.300	.10000	1.304	.00110	835
0	.0200	.0200	.0200	1.300	1.00000	1.304	.00110	835
20	.0240	.0164	.0202	1.300	1.00000	1.304	.00110	835
40	.0419	.0059	.0239	1.300	1.00000	1.304	.00110	835
60	.1113	.0040	.0576	1.300	1.00000	1.304	.00110	835
80	.4496	.2336	.3416	1.300	1.00000	1.304	.00110	835
0	.0174	.0174	.0174	1.400	.00000	1.304	.00110	835
20	.0210	.0141	.0176	1.400	.00000	1.304	.00110	835
40	.0376	.0048	.0212	1.400	.00000	1.304	.00110	835
60	.1037	.0047	.0542	1.400	.00000	1.304	.00110	835
80	.4386	.2367	.3377	1.400	.00000	1.304	.00110	835
0	.0174	.0174	.0174	1.400	.00010	1.304	.00110	835
20	.0210	.0141	.0176	1.400	.00010	1.304	.00110	835
40	.0376	.0048	.0212	1.400	.00010	1.304	.00110	835
60	.1037	.0047	.0542	1.400	.00010	1.304	.00110	835
80	.4386	.2367	.3377	1.400	.00010	1.304	.00110	835
0	.0174	.0174	.0174	1.400	.01000	1.304	.00110	835
20	.0210	.0142	.0176	1.400	.01000	1.304	.00110	835
40	.0376	.0048	.0212	1.400	.01000	1.304	.00110	835
60	.1038	.0047	.0542	1.400	.01000	1.304	.00110	835
80	.4388	.2367	.3377	1.400	.01000	1.304	.00110	835
0	.0177	.0177	.0177	1.400	.10000	1.304	.00110	835
20	.0213	.0144	.0179	1.400	.10000	1.304	.00110	835
40	.0381	.0049	.0215	1.400	.10000	1.304	.00110	835
60	.1045	.0046	.0546	1.400	.10000	1.304	.00110	835
80	.4399	.2367	.3383	1.400	.10000	1.304	.00110	835
0	.0204	.0204	.0204	1.400	1.00000	1.304	.00110	835
20	.0245	.0168	.0206	1.400	1.00000	1.304	.00110	835
40	.0426	.0061	.0243	1.400	1.00000	1.304	.00110	835
60	.1125	.0038	.0581	1.400	1.00000	1.304	.00110	835
80	.4513	.2327	.3420	1.400	1.00000	1.304	.00110	835

TABLE XIII (CONTINUED)

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0174	.0174	.0174	1.500	.00000	1.304	.00110	835
20	.0210	.0141	.0176	1.500	.00000	1.304	.00110	835
40	.0376	.0048	.0212	1.500	.00000	1.304	.00110	835
60	.1037	.0047	.0542	1.500	.00000	1.304	.00110	835
80	.4386	.2367	.3377	1.500	.00000	1.304	.00110	835
0	.0174	.0174	.0174	1.500	.00010	1.304	.00110	835
20	.0210	.0141	.0176	1.500	.00010	1.304	.00110	835
40	.0376	.0048	.0212	1.500	.00010	1.304	.00110	835
60	.1037	.0047	.0542	1.500	.00010	1.304	.00110	835
80	.4386	.2367	.3377	1.500	.00010	1.304	.00110	835
0	.0174	.0174	.0174	1.500	.01000	1.304	.00110	835
20	.0211	.0142	.0176	1.500	.01000	1.304	.00110	835
40	.0377	.0048	.0212	1.500	.01000	1.304	.00110	835
60	.1038	.0047	.0542	1.500	.01000	1.304	.00110	835
80	.4388	.2367	.3377	1.500	.01000	1.304	.00110	835
0	.0177	.0177	.0177	1.500	.10000	1.304	.00110	835
20	.0214	.0144	.0179	1.500	.10000	1.304	.00110	835
40	.0382	.0049	.0215	1.500	.10000	1.304	.00110	835
60	.1047	.0046	.0546	1.500	.10000	1.304	.00110	835
80	.4401	.2365	.3383	1.500	.10000	1.304	.00110	835
0	.0209	.0209	.0209	1.500	1.00000	1.304	.00110	835
20	.0250	.0172	.0211	1.500	1.00000	1.304	.00110	835
40	.0433	.0064	.0248	1.500	1.00000	1.304	.00110	835
60	.1138	.0036	.0587	1.500	1.00000	1.304	.00110	835
80	.4531	.2317	.3424	1.500	1.00000	1.304	.00110	835

TABLE XIV

THE REFLECTANCE OF A TEN MOLECULE THICK FILM-COVERED WATER SURFACE AT 2.0 MICRONS

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0174	.0174	.0174	1.300	.00000	1.304	.00110	83
20	.0210	.0141	.0175	1.300	.00000	1.304	.00110	83
40	.0375	.0047	.0211	1.300	.00000	1.304	.00110	83
60	.1036	.0047	.0541	1.300	.00000	1.304	.00110	83
80	.4385	.2367	.3376	1.300	.00000	1.304	.00110	83
0	.0174	.0174	.0174	1.300	.00010	1.304	.00110	83
20	.0210	.0141	.0175	1.300	.00010	1.304	.00110	83
40	.0375	.0047	.0211	1.300	.00010	1.304	.00110	83
60	.1036	.0047	.0541	1.300	.00010	1.304	.00110	83
80	.4385	.2367	.3376	1.300	.00010	1.304	.00110	83
0	.0176	.0176	.0176	1.300	.01000	1.304	.00110	83
20	.0213	.0143	.0178	1.300	.01000	1.304	.00110	83
40	.0380	.0048	.0214	1.300	.01000	1.304	.00110	83
60	.1043	.0047	.0545	1.300	.01000	1.304	.00110	83
80	.4396	.2368	.3382	1.300	.01000	1.304	.00110	83
0	.0199	.0199	.0199	1.300	.10000	1.304	.00110	83
20	.0239	.0162	.0200	1.300	.10000	1.304	.00110	83
40	.0417	.0055	.0236	1.300	.10000	1.304	.00110	83
60	.1111	.0045	.0578	1.300	.10000	1.304	.00110	83
80	.4493	.2382	.3438	1.300	.10000	1.304	.00110	83
0	.0490	.0490	.0490	1.300	1.00000	1.304	.00110	83
20	.0561	.0420	.0490	1.300	1.00000	1.304	.00110	83
40	.0854	.0217	.0536	1.300	1.00000	1.304	.00110	83
60	.1805	.0044	.0925	1.300	1.00000	1.304	.00110	83
80	.5346	.2169	.3757	1.300	1.00000	1.304	.00110	83
0	.0176	.0176	.0176	1.400	.00000	1.304	.00110	83
20	.0213	.0143	.0178	1.400	.00000	1.304	.00110	83
40	.0379	.0048	.0214	1.400	.00000	1.304	.00110	83
60	.1042	.0047	.0544	1.400	.00000	1.304	.00110	83
80	.4392	.2367	.3380	1.400	.00000	1.304	.00110	83
0	.0176	.0176	.0176	1.400	.00010	1.304	.00110	83
20	.0213	.0143	.0178	1.400	.00010	1.304	.00110	83
40	.0379	.0048	.0214	1.400	.00010	1.304	.00110	83
60	.1041	.0047	.0544	1.400	.00010	1.304	.00110	83
80	.4392	.2367	.3380	1.400	.00010	1.304	.00110	83
0	.0179	.0179	.0179	1.400	.01000	1.304	.00110	83
20	.0216	.0145	.0181	1.400	.01000	1.304	.00110	83
40	.0384	.0049	.0217	1.400	.01000	1.304	.00110	83
60	.1050	.0046	.0548	1.400	.01000	1.304	.00110	83
80	.4405	.2367	.3386	1.400	.01000	1.304	.00110	83
0	.0206	.0206	.0206	1.400	.10000	1.304	.00110	83
20	.0246	.0168	.0207	1.400	.10000	1.304	.00110	83
40	.0428	.0059	.0243	1.400	.10000	1.304	.00110	83
60	.1128	.0042	.0585	1.400	.10000	1.304	.00110	83
80	.4516	.2367	.3441	1.400	.10000	1.304	.00110	83
0	.0544	.0544	.0544	1.400	1.00000	1.304	.00110	83
20	.0620	.0469	.0545	1.400	1.00000	1.304	.00110	83
40	.0931	.0252	.0591	1.400	1.00000	1.304	.00110	83
60	.1917	.0042	.0979	1.400	1.00000	1.304	.00110	83
80	.5466	.2085	.3775	1.400	1.00000	1.304	.00110	83

TABLE XIV (CONTINUED)

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0181	.0181	.0181	1.500	.00000	1.304	.00110	83
20	.0218	.0147	.0182	1.500	.00000	1.304	.00110	83
60	.1051	.0046	.0549	1.500	.00000	1.304	.00110	83
80	.4405	.2367	.3386	1.500	.00000	1.304	.00110	83
0	.0181	.0181	.0181	1.500	.00010	1.304	.00110	83
20	.0218	.0147	.0182	1.500	.00010	1.304	.00110	83
40	.0385	.0050	.0218	1.500	.00010	1.304	.00110	83
60	.1051	.0046	.0549	1.500	.00010	1.304	.00110	83
80	.4405	.2366	.3386	1.500	.00010	1.304	.00110	83
0	.0184	.0184	.0184	1.500	.01000	1.304	.00110	83
20	.0221	.0149	.0185	1.500	.01000	1.304	.00110	83
40	.0391	.0051	.0221	1.500	.01000	1.304	.00110	83
60	.1061	.0046	.0553	1.500	.01000	1.304	.00110	83
80	.4419	.2365	.3392	1.500	.01000	1.304	.00110	83
0	.0215	.0215	.0215	1.500	.10000	1.304	.00110	83
20	.0256	.0175	.0215	1.500	.10000	1.304	.00110	83
40	.0441	.0063	.0252	1.500	.10000	1.304	.00110	83
60	.1149	.0039	.0594	1.500	.10000	1.304	.00110	83
80	.4545	.2352	.3448	1.500	.10000	1.304	.00110	83
0	.0603	.0603	.0603	1.500	1.00000	1.304	.00110	83
20	.0684	.0524	.0604	1.500	1.00000	1.304	.00110	83
40	.1014	.0290	.0652	1.500	1.00000	1.304	.00110	83
60	.2036	.0042	.1039	1.500	1.00000	1.304	.00110	83
80	.5588	.2003	.3795	1.500	1.00000	1.304	.00110	83

TABLE XV

THE REFLECTANCE OF A ONE HUNDRED MOLECULE THICK FILM-COVERED WATER SURFACE AT 2.0 MICRONS

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0168	.0167	.0167	1.300	.00000	1.304	.00110	8
20	.0203	.0136	.0169	1.300	.00000	1.304	.00110	8
40	.0365	.0045	.0205	1.300	.00000	1.304	.00110	8
60	.1018	.0047	.0532	1.300	.00000	1.304	.00110	8
80	.4359	.2363	.3361	1.300	.00000	1.304	.00110	8
0	.0168	.0167	.0167	1.300	.00010	1.304	.00110	8
20	.0203	.0136	.0169	1.300	.00010	1.304	.00110	8
40	.0365	.0046	.0205	1.300	.00010	1.304	.00110	8
60	.1018	.0047	.0533	1.300	.00010	1.304	.00110	8
80	.4359	.2363	.3361	1.300	.00010	1.304	.00110	8
0	.0180	.0180	.0180	1.300	.01000	1.304	.00110	8
20	.0218	.0147	.0182	1.300	.01000	1.304	.00110	8
40	.0390	.0050	.0220	1.300	.01000	1.304	.00110	8
60	.1069	.0046	.0558	1.300	.01000	1.304	.00110	8
80	.4442	.2376	.3409	1.300	.01000	1.304	.00110	8
0	.0330	.0330	.0330	1.300	.10000	1.304	.00110	8
20	.0392	.0271	.0331	1.300	.10000	1.304	.00110	8
40	.0655	.0103	.0379	1.300	.10000	1.304	.00110	8
60	.1553	.0045	.0799	1.300	.10000	1.304	.00110	8
80	.5103	.2518	.3811	1.300	.10000	1.304	.00110	8
0	.2849	.2849	.2849	1.300	1.00000	1.304	.00110	8
20	.3069	.2578	.2823	1.300	1.00000	1.304	.00110	8
40	.3801	.1875	.2838	1.300	1.00000	1.304	.00110	8
60	.5291	.1274	.3282	1.300	1.00000	1.304	.00110	8
80	.8001	.3571	.5786	1.300	1.00000	1.304	.00110	8
0	.0347	.0347	.0347	1.400	.00000	1.304	.00110	8
20	.0404	.0283	.0343	1.400	.00000	1.304	.00110	8
40	.0644	.0107	.0376	1.400	.00000	1.304	.00110	8
60	.1469	.0034	.0751	1.400	.00000	1.304	.00110	8
80	.4935	.2400	.3668	1.400	.00000	1.304	.00110	8
0	.0347	.0347	.0347	1.400	.00010	1.304	.00110	8
20	.0404	.0283	.0343	1.400	.00010	1.304	.00110	8
40	.0644	.0108	.0376	1.400	.00010	1.304	.00110	8
60	.1468	.0033	.0751	1.400	.00010	1.304	.00110	8
80	.4936	.2400	.3668	1.400	.00010	1.304	.00110	8
0	.0357	.0357	.0357	1.400	.01000	1.304	.00110	8
20	.0415	.0291	.0353	1.400	.01000	1.304	.00110	8
40	.0663	.0112	.0387	1.400	.01000	1.304	.00110	8
60	.1508	.0031	.0770	1.400	.01000	1.304	.00110	8
80	.4992	.2400	.3696	1.400	.01000	1.304	.00110	8
0	.0491	.0491	.0491	1.400	.10000	1.304	.00110	8
20	.0571	.0406	.0488	1.400	.10000	1.304	.00110	8
40	.0895	.0169	.0532	1.400	.10000	1.304	.00110	8
60	.1913	.0024	.0968	1.400	.10000	1.304	.00110	8
80	.5495	.2446	.3970	1.400	.10000	1.304	.00110	8
0	.3063	.3063	.3063	1.400	1.00000	1.304	.00110	8
20	.3287	.2794	.3041	1.400	1.00000	1.304	.00110	8
40	.4028	.2071	.3050	1.400	1.00000	1.304	.00110	8
60	.5505	.1349	.3427	1.400	1.00000	1.304	.00110	8
80	.8117	.3390	.5753	1.400	1.00000	1.304	.00110	8

TABLE XV (CONTINUED)

PHI	RS	RP	R	N1	K1	N2	K2	W/D
0	.0614	.0613	.0613	1.500	.00000	1.304	.00110	8
20	.0700	.0509	.0605	1.500	.00000	1.304	.00110	8
40	.1046	.0222	.0634	1.500	.00000	1.304	.00110	8
60	.2087	.0019	.1053	1.500	.00000	1.304	.00110	8
80	.5631	.2368	.3999	1.500	.00000	1.304	.00110	8
0	.0614	.0613	.0614	1.500	.00010	1.304	.00110	8
20	.0700	.0509	.0605	1.500	.00010	1.304	.00110	8
40	.1046	.0223	.0634	1.500	.00010	1.304	.00110	8
60	.2087	.0019	.1053	1.500	.00010	1.304	.00110	8
80	.5631	.2366	.3999	1.500	.00010	1.304	.00110	8
0	.0617	.0617	.0617	1.500	.01000	1.304	.00110	8
20	.0705	.0513	.0609	1.500	.01000	1.304	.00110	8
40	.1055	.0225	.0640	1.500	.01000	1.304	.00110	8
60	.2107	.0016	.1061	1.500	.01000	1.304	.00110	8
80	.5657	.2359	.4008	1.500	.01000	1.304	.00110	8
0	.0715	.0715	.0715	1.500	.10000	1.304	.00110	8
20	.0817	.0599	.0708	1.500	.10000	1.304	.00110	8
40	.1219	.0273	.0746	1.500	.10000	1.304	.00110	8
60	.2378	.0006	.1192	1.500	.10000	1.304	.00110	8
80	.5954	.2342	.4148	1.500	.10000	1.304	.00110	8
0	.3269	.3269	.3269	1.500	1.00000	1.304	.00110	8
20	.3497	.3003	.3250	1.500	1.00000	1.304	.00110	8
40	.4242	.2265	.3254	1.500	1.00000	1.304	.00110	8
60	.5702	.1437	.3570	1.500	1.00000	1.304	.00110	8
80	.8220	.3227	.5724	1.500	1.00000	1.304	.00110	8

Relative Reflectance Program and Results

On the next page is found a copy of a computer program written to aid in the analysis of the effect of a film on the reflectances of a water surface. This program was used to calculate the reflectances of a film-covered water surface relative to the reflectances of a clean water surface. Following the program is found a series of tables which tabulate the results of this computer study. Tables XVI, XVII, and XVIII give the relative reflectances for energy at 0.5 microns; and Tables XIX, XX, and XXI give the similar relative reflectances for energy at 2.0 microns.

The column headings which are different from those of the previous study are as follows:

RS/RSW	The relative reflectance for s-plane energy
RP/RPW	The relative reflectance for p-plane energy
R/RW	The relative reflectance for unpolarized incident energy.

A Computer Program for Calculating the Ratio of the Reflectances of Film-Covered Water Surfaces to the Reflectances of Clean Water Surfaces

```
    READ 101,I1,B1,C1,D1,E1,F1,G1,H1,J1
    READ 101,I2,B2,C2,D2,E2,F2,G2,H2,J2
    READ 101,I3,B3,C3,D3,E3,F3,G3,H3,J3
    READ 101,I4,B4,C4,D4,E4,F4,G4,H4,J4
    READ 101,I5,B5,C5,D5,E5,F5,G5,H5,J5
 40 PUNCH 401
    PUNCH 400
    K = 1
 20 READ 101,I,B,C,D,E,F,G,H,J
    IF (I) 2,6,2
  2 IF (I-20) 3,7,3
  3 IF (I-40) 4,8,4
  4 IF (I-60) 5,9,5
  5 IF(I-80)10,10,10
  6 RAT1=B/B1
    RAT2=C/C1
    RAT3=D/D1
    GO TO 12
  7 RAT1=B/B2
    RAT2=C/C2
    RAT3=D/D2
    GO TO 12
  8 RAT1=B/B3
    RAT2=C/C3
    RAT3=D/D3
    GO TO 12
  9 RAT1=B/B4
    RAT2=C/C4
    RAT3=D/D4
    GO TO 12
 10 RAT1=B/B5
    RAT2=C/C5
    RAT3=D/D5
 12 PUNCH 300,I,RAT1,RAT2,RAT3,E,F,G,H,J
    K = K + 1
    IF(K-60)20,20,40
101 FORMAT(I6,3F8.4,F7.3,F9.5,F7.3,F9.5,I6)
300 FORMAT(I3,3F9.4,F8.3,F9.5,F7.3,F9.5,I6)
400 FORMAT(1H )
401 FORMAT(4H PHI,8H  RS/RSW,9H    RP/RPW,5X4HR/RW,5X2HN1,6X2HK1,6X2HN2
   1,6X2HK2,5X3HW/D)
    END
```

TABLE XVI

THE RELATIVE REFLECTANCE OF A MONOLAYER-COVERED WATER SURFACE AT 0.5 MICRONS

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.0000	1.0000	1.0000	1.300	.00000	1.336	.00000	210
20	.9959	1.0000	1.0000	1.300	.00000	1.336	.00000	210
40	1.0000	1.0000	.9959	1.300	.00000	1.336	.00000	210
60	1.0000	1.0000	.9983	1.300	.00000	1.336	.00000	210
80	1.0000	1.0000	1.0000	1.300	.00000	1.336	.00000	210
0	1.0000	1.0000	1.0000	1.300	.00010	1.336	.00000	210
20	.9959	1.0000	1.0000	1.300	.00010	1.336	.00000	210
40	1.0000	1.0000	.9959	1.300	.00010	1.336	.00000	210
60	1.0000	1.0000	.9983	1.300	.00010	1.336	.00000	210
80	1.0000	1.0000	1.0000	1.300	.00010	1.336	.00000	210
0	1.0048	1.0048	1.0048	1.300	.01000	1.336	.00000	210
20	1.0040	1.0059	1.0047	1.300	.01000	1.336	.00000	210
40	1.0045	1.0000	1.0000	1.300	.01000	1.336	.00000	210
60	1.0025	1.0000	1.0016	1.300	.01000	1.336	.00000	210
80	1.0008	1.0000	1.0005	1.300	.01000	1.336	.00000	210
0	1.0483	1.0531	1.0531	1.300	.10000	1.336	.00000	210
20	1.0481	1.0473	1.0478	1.300	.10000	1.336	.00000	210
40	1.0411	1.0500	1.0401	1.300	.10000	1.336	.00000	210
60	1.0258	.9767	1.0232	1.300	.10000	1.336	.00000	210
80	1.0089	1.0025	1.0065	1.300	.10000	1.336	.00000	210
0	1.5652	1.5652	1.5652	1.300	1.00000	1.336	.00000	210
20	1.5261	1.6035	1.5550	1.300	1.00000	1.336	.00000	210
40	1.4210	1.9500	1.4819	1.300	1.00000	1.336	.00000	210
60	1.2607	.6046	1.2371	1.300	1.00000	1.336	.00000	210
80	1.0847	.9548	1.0404	1.300	1.00000	1.336	.00000	210
0	1.0000	1.0000	1.0000	1.400	.00000	1.336	.00000	210
20	1.0000	1.0000	1.0000	1.400	.00000	1.336	.00000	210
40	1.0022	1.0000	1.0000	1.400	.00000	1.336	.00000	210
60	1.0008	1.0000	1.0000	1.400	.00000	1.336	.00000	210
80	1.0002	1.0000	1.0000	1.400	.00000	1.336	.00000	210
0	1.0000	1.0000	1.0000	1.400	.00010	1.336	.00000	210
20	1.0000	1.0000	1.0000	1.400	.00010	1.336	.00000	210
40	1.0022	1.0000	1.0000	1.400	.00010	1.336	.00000	210
60	1.0008	1.0000	1.0000	1.400	.00010	1.336	.00000	210
80	1.0002	1.0000	1.0000	1.400	.00010	1.336	.00000	210
0	1.0048	1.0048	1.0048	1.400	.01000	1.336	.00000	210
20	1.0040	1.0059	1.0047	1.400	.01000	1.336	.00000	210
40	1.0068	1.0000	1.0040	1.400	.01000	1.336	.00000	210
60	1.0034	1.0000	1.0016	1.400	.01000	1.336	.00000	210
80	1.0013	1.0000	1.0008	1.400	.01000	1.336	.00000	210
0	1.0628	1.0628	1.0628	1.400	.10000	1.336	.00000	210
20	1.0562	1.0591	1.0574	1.400	.10000	1.336	.00000	210
40	1.0480	1.0666	1.0481	1.400	.10000	1.336	.00000	210
60	1.0309	.9534	1.0265	1.400	.10000	1.336	.00000	210
80	1.0104	.9995	1.0068	1.400	.10000	1.336	.00000	210
0	1.6618	1.6618	1.6618	1.400	1.00000	1.336	.00000	210
20	1.6144	1.7100	1.6507	1.400	1.00000	1.336	.00000	210
40	1.4874	2.1500	1.5622	1.400	1.00000	1.336	.00000	210
60	1.3012	.5116	1.2719	1.400	1.00000	1.336	.00000	210
80	1.0969	.9393	1.0429	1.400	1.00000	1.336	.00000	210

TABLE XVI (CONTINUED)

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.0048	1.0048	1.0048	1.500	.00000	1.336	.00000	210
20	1.0040	1.0059	1.0047	1.500	.00000	1.336	.00000	210
40	1.0045	1.0000	1.0000	1.500	.00000	1.336	.00000	210
60	1.0017	1.0000	1.0016	1.500	.00000	1.336	.00000	210
80	1.0006	1.0000	1.0002	1.500	.00000	1.336	.00000	210
0	1.0048	1.0048	1.0048	1.500	.00010	1.336	.00000	210
20	1.0040	1.0059	1.0047	1.500	.00010	1.336	.00000	210
40	1.0045	1.0000	1.0000	1.500	.00010	1.336	.00000	210
60	1.0017	1.0000	1.0016	1.500	.00010	1.336	.00000	210
80	1.0006	1.0000	1.0002	1.500	.00010	1.336	.00000	210
0	1.0096	1.0096	1.0096	1.500	.01000	1.336	.00000	210
20	1.0080	1.0118	1.0095	1.500	.01000	1.336	.00000	210
40	1.0091	1.0166	1.0080	1.500	.01000	1.336	.00000	210
60	1.0051	.9767	1.0033	1.500	.01000	1.336	.00000	210
80	1.0017	.9995	1.0011	1.500	.01000	1.336	.00000	210
0	1.0724	1.0724	1.0724	1.500	.10000	1.336	.00000	210
20	1.0682	1.0769	1.0717	1.500	.10000	1.336	.00000	210
40	1.0572	1.1000	1.0602	1.500	.10000	1.336	.00000	210
60	1.0361	.9302	1.0315	1.500	.10000	1.336	.00000	210
80	1.0124	.9970	1.0071	1.500	.10000	1.336	.00000	210
0	1.7632	1.7632	1.7632	1.500	1.00000	1.336	.00000	210
20	1.7068	1.8224	1.7559	1.500	1.00000	1.336	.00000	210
40	1.5606	2.3500	1.6546	1.500	1.00000	1.336	.00000	210
60	1.3442	.4186	1.3101	1.500	1.00000	1.336	.00000	210
80	1.1100	.9238	1.0461	1.500	1.00000	1.336	.00000	210

TABLE XVII

THE RELATIVE REFLECTANCE OF A TEN MOLECULE THICK FILM-COVERED WATER SURFACE AT 0.5 MICRONS

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	.9516	.9516	.9516	1.300	.00000	1.336	.00000	21
20	.9558	.9526	.9521	1.300	.00000	1.336	.00000	21
40	.9679	.9500	.9638	1.300	.00000	1.336	.00000	21
60	.9819	1.0232	.9817	1.300	.00000	1.336	.00000	21
80	.9941	.9983	.9957	1.300	.00000	1.336	.00000	21
0	.9516	.9516	.9516	1.300	.00010	1.336	.00000	21
20	.9518	.9526	.9521	1.300	.00010	1.336	.00000	21
40	.9679	.9500	.9638	1.300	.00010	1.336	.00000	21
60	.9819	1.0232	.9817	1.300	.00010	1.336	.00000	21
80	.9943	.9983	.9957	1.300	.00010	1.336	.00000	21
0	.9951	1.0000	1.0000	1.300	.01000	1.336	.00000	21
20	1.0000	1.0000	1.0000	1.300	.01000	1.336	.00000	21
40	1.0045	1.0000	1.0040	1.300	.01000	1.336	.00000	21
60	1.0060	1.0000	1.0049	1.300	.01000	1.336	.00000	21
80	1.0028	1.0008	1.0020	1.300	.01000	1.336	.00000	21
0	1.4492	1.4541	1.4541	1.300	.10000	1.336	.00000	21
20	1.4216	1.4497	1.4354	1.300	.10000	1.336	.00000	21
40	1.3478	1.5166	1.3654	1.300	.10000	1.336	.00000	21
60	1.2246	.9069	1.2122	1.300	.10000	1.336	.00000	21
80	1.0749	1.0246	1.0576	1.300	.10000	1.336	.00000	21
0	8.0966	8.0966	8.0966	1.300	1.00000	1.336	.00000	21
20	7.3493	8.8934	7.9760	1.300	1.00000	1.336	.00000	21
40	5.4736	17.0166	6.8514	1.300	1.00000	1.336	.00000	21
60	3.2409	11.9302	3.5472	1.300	1.00000	1.336	.00000	21
80	1.5335	1.0485	1.3674	1.300	1.00000	1.336	.00000	21
0	1.1207	1.1207	1.1207	1.400	.00000	1.336	.00000	21
20	1.1084	1.1183	1.1148	1.400	.00000	1.336	.00000	21
40	1.0846	1.1333	1.0883	1.400	.00000	1.336	.00000	21
60	1.0481	.9534	1.0447	1.400	.00000	1.336	.00000	21
80	1.0148	1.0008	1.0100	1.400	.00000	1.336	.00000	21
0	1.1207	1.1256	1.1207	1.400	.00010	1.336	.00000	21
20	1.1084	1.1183	1.1148	1.400	.00010	1.336	.00000	21
40	1.0846	1.1333	1.0883	1.400	.00010	1.336	.00000	21
60	1.0481	.9534	1.0431	1.400	.00010	1.336	.00000	21
80	1.0148	1.0008	1.0100	1.400	.00010	1.336	.00000	21
0	1.1739	1.1739	1.1739	1.400	.01000	1.336	.00000	21
20	1.1566	1.1715	1.1626	1.400	.01000	1.336	.00000	21
40	1.1235	1.2000	1.1325	1.400	.01000	1.336	.00000	21
60	1.0748	.9069	1.0679	1.400	.01000	1.336	.00000	21
80	1.0239	1.0008	1.0160	1.400	.01000	1.336	.00000	21
0	1.6811	1.6811	1.6811	1.400	.10000	1.336	.00000	21
20	1.6345	1.6923	1.6555	1.400	.10000	1.336	.00000	21
40	1.5057	1.8500	1.5461	1.400	.10000	1.336	.00000	21
60	1.3141	.6511	1.2902	1.400	.10000	1.336	.00000	21
80	1.1011	1.0029	1.0676	1.400	.10000	1.336	.00000	21
0	9.0869	9.0869	9.0869	1.400	1.00000	1.336	.00000	21
20	8.2088	10.0591	8.9569	1.400	1.00000	1.336	.00000	21
40	6.0205	19.7333	7.6586	1.400	1.00000	1.336	.00000	21
60	3.4647	13.2790	3.8109	1.400	1.00000	1.336	.00000	21
80	1.5717	.9723	1.3665	1.400	1.00000	1.336	.00000	21

TABLE XVII (CONTINUED)

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.4154	1.4202	1.4154	1.500	.00000	1.336	.00000	21
20	1.3775	1.4201	1.3923	1.500	.00000	1.336	.00000	21
40	1.2837	1.5333	1.3092	1.500	.00000	1.336	.00000	21
60	1.1617	.8837	1.1509	1.500	.00000	1.336	.00000	21
80	1.0490	.9979	1.0315	1.500	.00000	1.336	.00000	21
0	1.4154	1.4202	1.4202	1.500	.00010	1.336	.00000	21
20	1.3775	1.4201	1.3971	1.500	.00010	1.336	.00000	21
40	1.2837	1.5333	1.3092	1.500	.00010	1.336	.00000	21
60	1.1609	.8837	1.1509	1.500	.00010	1.336	.00000	21
80	1.0488	.9974	1.0312	1.500	.00010	1.336	.00000	21
0	1.4734	1.4734	1.4734	1.500	.01000	1.336	.00000	21
20	1.4297	1.4792	1.4497	1.500	.01000	1.336	.00000	21
40	1.3272	1.6000	1.3574	1.500	.01000	1.336	.00000	21
60	1.1893	.8139	1.1757	1.500	.01000	1.336	.00000	21
80	1.0584	.9953	1.0366	1.500	.01000	1.336	.00000	21
0	2.0193	2.0241	2.0193	1.500	.10000	1.336	.00000	21
20	1.9397	2.0473	1.9808	1.500	.10000	1.336	.00000	21
40	1.7345	2.3833	1.8072	1.500	.10000	1.336	.00000	21
60	1.4414	.4418	1.4046	1.500	.10000	1.336	.00000	21
80	1.1366	.9786	1.0825	1.500	.10000	1.336	.00000	21
0	10.1207	10.1207	10.1207	1.500	1.00000	1.336	.00000	21
20	9.1004	11.2840	9.9856	1.500	1.00000	1.336	.00000	21
40	6.5789	22.6500	8.5020	1.500	1.00000	1.336	.00000	21
60	3.6876	15.0000	4.0878	1.500	1.00000	1.336	.00000	21
80	1.6083	.9029	1.3665	1.500	1.00000	1.336	.00000	21

TABLE XVIII

THE RELATIVE REFLECTANCE OF A ONE HUNDRED MOLECULE THICK FILM-COVERED WATER SURFACE AT 0.5 MICRONS

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	.8454	.8454	.8454	1.300	.00000	1.336	.00000	2
20	.8915	.8875	.8899	1.300	.00000	1.336	.00000	2
40	.9839	.9666	.9799	1.300	.00000	1.336	.00000	2
60	.9870	1.0000	.9867	1.300	.00000	1.336	.00000	2
80	.9694	.9928	.9776	1.300	.00000	1.336	.00000	2
0	.8454	.8454	.8454	1.300	.00010	1.336	.00000	2
20	.8875	.8875	.8899	1.300	.00010	1.336	.00000	2
40	.9839	.9666	.9799	1.300	.00010	1.336	.00000	2
60	.9870	1.0000	.9867	1.300	.00010	1.336	.00000	2
80	.9694	.9928	.9773	1.300	.00010	1.336	.00000	2
0	.9033	.9082	.9082	1.300	.01000	1.336	.00000	2
20	.9437	.9408	.9425	1.300	.01000	1.336	.00000	2
40	.9977	.9833	.9959	1.300	.01000	1.336	.00000	2
60	.9569	1.0232	.9585	1.300	.01000	1.336	.00000	2
80	.9513	.9882	.9641	1.300	.01000	1.336	.00000	2
0	1.3043	1.3091	1.3091	1.300	.10000	1.336	.00000	2
20	1.2610	1.2721	1.2679	1.300	.10000	1.336	.00000	2
40	1.0892	1.1000	1.0883	1.300	.10000	1.336	.00000	2
60	.9165	1.2790	.9286	1.300	.10000	1.336	.00000	2
80	.9498	1.0209	.9742	1.300	.10000	1.336	.00000	2
0	12.3188	12.3188	12.3188	1.300	1.00000	1.336	.00000	2
20	11.2248	13.7100	12.2296	1.300	1.00000	1.336	.00000	2
40	8.2608	28.5166	10.6827	1.300	1.00000	1.336	.00000	2
60	4.4948	29.2790	5.3747	1.300	1.00000	1.336	.00000	2
80	1.7458	1.5656	1.6841	1.300	1.00000	1.336	.00000	2
0	1.5507	1.5507	1.5507	1.400	.00000	1.336	.00000	2
20	1.4457	1.4852	1.4641	1.400	.00000	1.336	.00000	2
40	1.1899	1.3166	1.2008	1.400	.00000	1.336	.00000	2
60	1.0111	.9767	1.0099	1.400	.00000	1.336	.00000	2
80	1.0047	1.0004	1.0031	1.400	.00000	1.336	.00000	2
0	1.5507	1.5507	1.5507	1.400	.00010	1.336	.00000	2
20	1.4457	1.4852	1.4641	1.400	.00010	1.336	.00000	2
40	1.1899	1.3166	1.2008	1.400	.00010	1.336	.00000	2
60	1.0103	.9767	1.0082	1.400	.00010	1.336	.00000	2
80	1.0047	1.0004	1.0031	1.400	.00010	1.336	.00000	2
0	1.5893	1.5893	1.5893	1.400	.01000	1.336	.00000	2
20	1.4939	1.5384	1.5119	1.400	.01000	1.336	.00000	2
40	1.2448	1.4000	1.2610	1.400	.01000	1.336	.00000	2
60	1.0413	.9534	1.0364	1.400	.01000	1.336	.00000	2
80	1.0082	1.0012	1.0060	1.400	.01000	1.336	.00000	2
0	1.9178	1.9178	1.9178	1.400	.10000	1.336	.00000	2
20	1.8353	1.9053	1.8660	1.400	.10000	1.336	.00000	2
40	1.5652	1.9333	1.6064	1.400	.10000	1.336	.00000	2
60	1.2409	.8837	1.2271	1.400	.10000	1.336	.00000	2
80	1.0651	1.0246	1.0513	1.400	.10000	1.336	.00000	2
0	13.2705	13.2705	13.2705	1.400	1.00000	1.336	.00000	2
20	12.0160	14.8639	13.1674	1.400	1.00000	1.336	.00000	2
40	8.6933	31.3000	11.3975	1.400	1.00000	1.336	.00000	2
60	4.6325	30.1395	5.5373	1.400	1.00000	1.336	.00000	2
80	1.7626	1.4753	1.6643	1.400	1.00000	1.336	.00000	2

TABLE XVIII (CONTINUED)

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	3.0338	3.0338	3.0338	1.500	.00000	1.336	.00000	2
20	2.8112	3.0118	2.8947	1.500	.00000	1.336	.00000	2
40	2.0983	3.1166	2.2168	1.500	.00000	1.336	.00000	2
60	1.3072	.7674	1.2868	1.500	.00000	1.336	.00000	2
80	1.0265	.9987	1.0171	1.500	.00000	1.336	.00000	2
0	3.0338	3.0386	3.0338	1.500	.00010	1.336	.00000	2
20	2.8112	3.0177	2.8947	1.500	.00010	1.336	.00000	2
40	2.0983	3.1166	2.2168	1.500	.00010	1.336	.00000	2
60	1.3063	.7674	1.2868	1.500	.00010	1.336	.00000	2
80	1.0263	.9987	1.0169	1.500	.00010	1.336	.00000	2
0	2.9613	2.9613	2.9613	1.500	.01000	1.336	.00000	2
20	2.7590	2.9585	2.8373	1.500	.01000	1.336	.00000	2
40	2.1029	3.1166	2.2208	1.500	.01000	1.336	.00000	2
60	1.3511	.6744	1.3250	1.500	.01000	1.336	.00000	2
80	1.0483	.9962	1.0306	1.500	.01000	1.336	.00000	2
0	2.7584	2.7584	2.7584	1.500	.10000	1.336	.00000	2
20	2.6385	2.8165	2.7129	1.500	.10000	1.336	.00000	2
40	2.2105	3.3000	2.3373	1.500	.10000	1.336	.00000	2
60	1.6049	.4418	1.5621	1.500	.10000	1.336	.00000	2
80	1.1636	1.0000	1.1077	1.500	.10000	1.336	.00000	2
0	14.2125	14.2125	14.2125	1.500	1.00000	1.336	.00000	2
20	12.7951	16.0059	14.0956	1.500	1.00000	1.336	.00000	2
40	9.1235	34.1333	12.1164	1.500	1.00000	1.336	.00000	2
60	4.7685	31.4186	5.7147	1.500	1.00000	1.336	.00000	2
80	1.7796	1.3933	1.6471	1.500	1.00000	1.336	.00000	2

TABLE XIX

THE RELATIVE REFLECTANCE OF A MONOLAYER-COVERED WATER SURFACE AT 2.0 MICRONS

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.0000	1.0000	1.0000	1.300	.00000	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.300	.00000	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.300	.00000	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.300	.00000	1.304	.00110	835
80	1.0000	1.0000	.9997	1.300	.00000	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.300	.00010	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.300	.00010	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.300	.00010	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.300	.00010	1.304	.00110	835
80	1.0000	1.0000	.9997	1.300	.00010	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.300	.01000	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.300	.01000	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.300	.01000	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.300	.01000	1.304	.00110	835
80	1.0002	1.0000	1.0000	1.300	.01000	1.304	.00110	835
0	1.0172	1.0172	1.0172	1.300	.10000	1.304	.00110	835
20	1.0142	1.0141	1.0113	1.300	.10000	1.304	.00110	835
40	1.0106	1.0000	1.0094	1.300	.10000	1.304	.00110	835
60	1.0067	.9787	1.0055	1.300	.10000	1.304	.00110	835
80	1.0025	1.0004	1.0017	1.300	.10000	1.304	.00110	835
0	1.1494	1.1494	1.1494	1.300	1.00000	1.304	.00110	835
20	1.1428	1.1631	1.1477	1.300	1.00000	1.304	.00110	835
40	1.1143	1.2291	1.1273	1.300	1.00000	1.304	.00110	835
60	1.0732	.8510	1.0627	1.300	1.00000	1.304	.00110	835
80	1.0250	.9869	1.0115	1.300	1.00000	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.400	.00000	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.400	.00000	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.400	.00000	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.400	.00000	1.304	.00110	835
80	1.0000	1.0000	1.0000	1.400	.00000	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.400	.00010	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.400	.00010	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.400	.00010	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.400	.00010	1.304	.00110	835
80	1.0000	1.0000	1.0000	1.400	.00010	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.400	.01000	1.304	.00110	835
20	1.0000	1.0070	1.0000	1.400	.01000	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.400	.01000	1.304	.00110	835
60	1.0009	1.0000	1.0000	1.400	.01000	1.304	.00110	835
80	1.0004	1.0000	1.0000	1.400	.01000	1.304	.00110	835
0	1.0172	1.0172	1.0172	1.400	.10000	1.304	.00110	835
20	1.0142	1.0212	1.0170	1.400	.10000	1.304	.00110	835
40	1.0132	1.0208	1.0141	1.400	.10000	1.304	.00110	835
60	1.0077	.9787	1.0073	1.400	.10000	1.304	.00110	835
80	1.0029	1.0000	1.0017	1.400	.10000	1.304	.00110	835
0	1.1724	1.1724	1.1724	1.400	1.00000	1.304	.00110	835
20	1.1666	1.1914	1.1704	1.400	1.00000	1.304	.00110	835
40	1.1329	1.2708	1.1462	1.400	1.00000	1.304	.00110	835
60	1.0848	.8085	1.0719	1.400	1.00000	1.304	.00110	835
80	1.0289	.9831	1.0127	1.400	1.00000	1.304	.00110	835

TABLE XIX (CONTINUED)

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.0000	1.0000	1.0000	1.500	.00000	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.500	.00000	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.500	.00000	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.500	.00000	1.304	.00110	835
80	1.0000	1.0000	1.0000	1.500	.00000	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.500	.00010	1.304	.00110	835
20	1.0000	1.0000	1.0000	1.500	.00010	1.304	.00110	835
40	1.0000	1.0000	1.0000	1.500	.00010	1.304	.00110	835
60	1.0000	1.0000	1.0000	1.500	.00010	1.304	.00110	835
80	1.0000	1.0000	1.0000	1.500	.00010	1.304	.00110	835
0	1.0000	1.0000	1.0000	1.500	.01000	1.304	.00110	835
20	1.0047	1.0070	1.0000	1.500	.01000	1.304	.00110	835
40	1.0026	1.0000	1.0000	1.500	.01000	1.304	.00110	835
60	1.0009	1.0000	1.0000	1.500	.01000	1.304	.00110	835
80	1.0004	1.0000	1.0000	1.500	.01000	1.304	.00110	835
0	1.0172	1.0172	1.0172	1.500	.10000	1.304	.00110	835
20	1.0190	1.0212	1.0170	1.500	.10000	1.304	.00110	835
40	1.0159	1.0208	1.0141	1.500	.10000	1.304	.00110	835
60	1.0096	.9787	1.0073	1.500	.10000	1.304	.00110	835
80	1.0034	.9991	1.0017	1.500	.10000	1.304	.00110	835
0	1.2011	1.2011	1.2011	1.500	1.00000	1.304	.00110	835
20	1.1904	1.2198	1.1988	1.500	1.00000	1.304	.00110	835
40	1.1515	1.3333	1.1698	1.500	1.00000	1.304	.00110	835
60	1.0973	.7659	1.0830	1.500	1.00000	1.304	.00110	835
80	1.0330	.9788	1.0139	1.500	1.00000	1.304	.00110	835

TABLE XX

THE RELATIVE REFLECTANCE OF A TEN MOLECULE THICK FILM-COVERED WATER SURFACE AT 2.0 MICRONS

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.0000	1.0000	1.0000	1.300	.00000	1.304	.00110	83
20	1.0000	1.0000	.9943	1.300	.00000	1.304	.00110	83
40	.9973	.9791	.9952	1.300	.00000	1.304	.00110	83
60	.9990	1.0000	.9981	1.300	.00000	1.304	.00110	83
80	.9997	1.0000	.9997	1.300	.00000	1.304	.00110	83
0	1.0000	1.0000	1.0000	1.300	.00010	1.304	.00110	83
20	1.0000	1.0000	.9943	1.300	.00010	1.304	.00110	83
40	.9973	.9791	.9952	1.300	.00010	1.304	.00110	83
60	.9990	1.0000	.9981	1.300	.00010	1.304	.00110	83
80	.9997	1.0000	.9997	1.300	.00010	1.304	.00110	83
0	1.0114	1.0114	1.0114	1.300	.01000	1.304	.00110	83
20	1.0142	1.0141	1.0113	1.300	.01000	1.304	.00110	83
40	1.0106	1.0000	1.0094	1.300	.01000	1.304	.00110	83
60	1.0057	1.0000	1.0055	1.300	.01000	1.304	.00110	83
80	1.0022	1.0004	1.0014	1.300	.01000	1.304	.00110	83
0	1.1436	1.1436	1.1436	1.300	.10000	1.304	.00110	83
20	1.1380	1.1489	1.1363	1.300	.10000	1.304	.00110	83
40	1.1090	1.1458	1.1132	1.300	.10000	1.304	.00110	83
60	1.0713	.9574	1.0664	1.300	.10000	1.304	.00110	83
80	1.0243	1.0063	1.0180	1.300	.10000	1.304	.00110	83
0	2.8160	2.8160	2.8160	1.300	1.00000	1.304	.00110	83
20	2.6714	2.9787	2.7840	1.300	1.00000	1.304	.00110	83
40	2.2712	4.5208	2.5283	1.300	1.00000	1.304	.00110	83
60	1.7405	.9361	1.7066	1.300	1.00000	1.304	.00110	83
80	1.2188	.9163	1.1125	1.300	1.00000	1.304	.00110	83
0	1.0114	1.0114	1.0114	1.400	.00000	1.304	.00110	83
20	1.0142	1.0141	1.0113	1.400	.00000	1.304	.00110	83
40	1.0079	1.0000	1.0094	1.400	.00000	1.304	.00110	83
60	1.0048	1.0000	1.0036	1.400	.00000	1.304	.00110	83
80	1.0013	1.0000	1.0008	1.400	.00000	1.304	.00110	83
0	1.0114	1.0114	1.0114	1.400	.00010	1.304	.00110	83
20	1.0142	1.0141	1.0113	1.400	.00010	1.304	.00110	83
40	1.0079	1.0000	1.0094	1.400	.00010	1.304	.00110	83
60	1.0038	1.0000	1.0036	1.400	.00010	1.304	.00110	83
80	1.0013	1.0000	1.0008	1.400	.00010	1.304	.00110	83
0	1.0287	1.0287	1.0287	1.400	.01000	1.304	.00110	83
20	1.0285	1.0283	1.0284	1.400	.01000	1.304	.00110	83
40	1.0212	1.0208	1.0235	1.400	.01000	1.304	.00110	83
60	1.0125	.9787	1.0110	1.400	.01000	1.304	.00110	83
80	1.0043	1.0000	1.0026	1.400	.01000	1.304	.00110	83
0	1.1839	1.1839	1.1839	1.400	.10000	1.304	.00110	83
20	1.1714	1.1914	1.1761	1.400	.10000	1.304	.00110	83
40	1.1382	1.2291	1.1462	1.400	.10000	1.304	.00110	83
60	1.0877	.8936	1.0793	1.400	.10000	1.304	.00110	83
80	1.0296	1.0000	1.0189	1.400	.10000	1.304	.00110	83
0	3.1264	3.1264	3.1264	1.400	1.00000	1.304	.00110	83
20	2.9523	3.3262	3.0965	1.400	1.00000	1.304	.00110	83
40	2.4760	5.2500	2.7877	1.400	1.00000	1.304	.00110	83
60	1.8486	.8936	1.8062	1.400	1.00000	1.304	.00110	83
80	1.2462	.8808	1.1178	1.400	1.00000	1.304	.00110	83

TABLE XX (CONTINUED)

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	1.0402	1.0402	1.0402	1.500	.00000	1.304	.00110	83
20	1.0380	1.0425	1.0340	1.500	.00000	1.304	.00110	83
60	1.0135	.9787	1.0129	1.500	.00000	1.304	.00110	83
80	1.0043	1.0000	1.0026	1.500	.00000	1.304	.00110	83
0	1.0402	1.0402	1.0402	1.500	.00010	1.304	.00110	83
20	1.0380	1.0425	1.0340	1.500	.00010	1.304	.00110	83
40	1.0239	1.0416	1.0283	1.500	.00010	1.304	.00110	83
60	1.0135	.9787	1.0129	1.500	.00010	1.304	.00110	83
80	1.0043	.9995	1.0026	1.500	.00010	1.304	.00110	83
0	1.0574	1.0574	1.0574	1.500	.01000	1.304	.00110	83
20	1.0523	1.0567	1.0511	1.500	.01000	1.304	.00110	83
40	1.0398	1.0625	1.0424	1.500	.01000	1.304	.00110	83
60	1.0231	.9787	1.0202	1.500	.01000	1.304	.00110	83
80	1.0075	.9991	1.0044	1.500	.01000	1.304	.00110	83
0	1.2356	1.2356	1.2356	1.500	.10000	1.304	.00110	83
20	1.2190	1.2411	1.2215	1.500	.10000	1.304	.00110	83
40	1.1728	1.3125	1.1886	1.500	.10000	1.304	.00110	83
60	1.1080	.8297	1.0959	1.500	.10000	1.304	.00110	83
80	1.0362	.9936	1.0210	1.500	.10000	1.304	.00110	83
0	3.4655	3.4655	3.4655	1.500	1.00000	1.304	.00110	83
20	3.2571	3.7163	3.4318	1.500	1.00000	1.304	.00110	83
40	2.6968	6.0416	3.0754	1.500	1.00000	1.304	.00110	83
60	1.9633	.8936	1.9169	1.500	1.00000	1.304	.00110	83
80	1.2740	.8462	1.1237	1.500	1.00000	1.304	.00110	83

TABLE XXI

THE RELATIVE REFLECTANCE OF A ONE HUNDRED MOLECULE THICK FILM-COVERED WATER SURFACE AT 2.0 MICRONS

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	.9655	.9597	.9597	1.300	.00000	1.304	.00110	8
20	.9666	.9645	.9602	1.300	.00000	1.304	.00110	8
40	.9707	.9375	.9669	1.300	.00000	1.304	.00110	8
60	.9816	1.0000	.9815	1.300	.00000	1.304	.00110	8
80	.9938	.9983	.9952	1.300	.00000	1.304	.00110	8
0	.9655	.9597	.9597	1.300	.00010	1.304	.00110	8
20	.9666	.9645	.9602	1.300	.00010	1.304	.00110	8
40	.9707	.9583	.9669	1.300	.00010	1.304	.00110	8
60	.9816	1.0000	.9833	1.300	.00010	1.304	.00110	8
80	.9938	.9983	.9952	1.300	.00010	1.304	.00110	8
0	1.0344	1.0344	1.0344	1.300	.01000	1.304	.00110	8
20	1.0380	1.0425	1.0340	1.300	.01000	1.304	.00110	8
40	1.0372	1.0416	1.0377	1.300	.01000	1.304	.00110	8
60	1.0308	.9787	1.0295	1.300	.01000	1.304	.00110	8
80	1.0127	1.0038	1.0094	1.300	.01000	1.304	.00110	8
0	1.8965	1.8965	1.8965	1.300	.10000	1.304	.00110	8
20	1.8666	1.9219	1.8806	1.300	.10000	1.304	.00110	8
40	1.7420	2.1458	1.7877	1.300	.10000	1.304	.00110	8
60	1.4975	.9574	1.4741	1.300	.10000	1.304	.00110	8
80	1.1634	1.0637	1.1285	1.300	.10000	1.304	.00110	8
0	16.3735	16.3735	16.3735	1.300	1.00000	1.304	.00110	8
20	14.6142	18.2836	16.0397	1.300	1.00000	1.304	.00110	8
40	10.1090	39.0625	13.3867	1.300	1.00000	1.304	.00110	8
60	5.1022	27.1063	6.0553	1.300	1.00000	1.304	.00110	8
80	1.8242	1.5086	1.7133	1.300	1.00000	1.304	.00110	8
0	1.9942	1.9942	1.9942	1.400	.00000	1.304	.00110	8
20	1.9238	2.0070	1.9488	1.400	.00000	1.304	.00110	8
40	1.7127	2.2291	1.7735	1.400	.00000	1.304	.00110	8
60	1.4165	.7234	1.3856	1.400	.00000	1.304	.00110	8
80	1.1251	1.0139	1.0861	1.400	.00000	1.304	.00110	8
0	1.9942	1.9942	1.9942	1.400	.00010	1.304	.00110	8
20	1.9238	2.0070	1.9488	1.400	.00010	1.304	.00110	8
40	1.7127	2.2500	1.7735	1.400	.00010	1.304	.00110	8
60	1.4156	.7021	1.3856	1.400	.00010	1.304	.00110	8
80	1.1253	1.0139	1.0861	1.400	.00010	1.304	.00110	8
0	2.0517	2.0517	2.0517	1.400	.01000	1.304	.00110	8
20	1.9761	2.0638	2.0056	1.400	.01000	1.304	.00110	8
40	1.7632	2.3333	1.8254	1.400	.01000	1.304	.00110	8
60	1.4541	.6595	1.4206	1.400	.01000	1.304	.00110	8
80	1.1381	1.0139	1.0944	1.400	.01000	1.304	.00110	8
0	2.8218	2.8218	2.8218	1.400	.10000	1.304	.00110	8
20	2.7190	2.8794	2.7727	1.400	.10000	1.304	.00110	8
40	2.3803	3.5208	2.5094	1.400	.10000	1.304	.00110	8
60	1.8447	.5106	1.7859	1.400	.10000	1.304	.00110	8
80	1.2528	1.0333	1.1755	1.400	.10000	1.304	.00110	8
0	17.6034	17.6034	17.6034	1.400	1.00000	1.304	.00110	8
20	15.6523	19.8156	17.2784	1.400	1.00000	1.304	.00110	8
40	10.7127	43.1458	14.3867	1.400	1.00000	1.304	.00110	8
60	5.3085	28.7021	6.3228	1.400	1.00000	1.304	.00110	8
80	1.8506	1.4321	1.7035	1.400	1.00000	1.304	.00110	8

TABLE XXI (CONTINUED)

PHI	RS/RSW	RP/RPW	R/RW	N1	K1	N2	K2	W/D
0	3.5287	3.5229	3.5229	1.500	.00000	1.304	.00110	8
20	3.3333	3.6099	3.4375	1.500	.00000	1.304	.00110	8
40	2.7819	4.6250	2.9905	1.500	.00000	1.304	.00110	8
60	2.0125	.4042	1.9428	1.500	.00000	1.304	.00110	8
80	1.2838	1.0004	1.1841	1.500	.00000	1.304	.00110	8
0	3.5287	3.5229	3.5287	1.500	.00010	1.304	.00110	8
20	3.3333	3.6099	3.4375	1.500	.00010	1.304	.00110	8
40	2.7819	4.6458	2.9905	1.500	.00010	1.304	.00110	8
60	2.0125	.4042	1.9428	1.500	.00010	1.304	.00110	8
80	1.2838	.9995	1.1841	1.500	.00010	1.304	.00110	8
0	3.5459	3.5459	3.5459	1.500	.01000	1.304	.00110	8
20	3.3571	3.6382	3.4602	1.500	.01000	1.304	.00110	8
40	2.8058	4.6875	3.0188	1.500	.01000	1.304	.00110	8
60	2.0318	.3404	1.9575	1.500	.01000	1.304	.00110	8
80	1.2897	.9966	1.1868	1.500	.01000	1.304	.00110	8
0	4.1091	4.1091	4.1091	1.500	.10000	1.304	.00110	8
20	3.8904	4.2482	4.0227	1.500	.10000	1.304	.00110	8
40	3.2420	5.6875	3.5188	1.500	.10000	1.304	.00110	8
60	2.2931	.1276	2.1992	1.500	.10000	1.304	.00110	8
80	1.3575	.9894	1.2283	1.500	.10000	1.304	.00110	8
0	18.7873	18.7873	18.7873	1.500	1.00000	1.304	.00110	8
20	16.6523	21.2978	18.4659	1.500	1.00000	1.304	.00110	8
40	11.2819	47.1875	15.3490	1.500	1.00000	1.304	.00110	8
60	5.4985	30.5744	6.5867	1.500	1.00000	1.304	.00110	8
80	1.8741	1.3633	1.6949	1.500	1.00000	1.304	.00110	8

APPENDIX D

THE SOLUTION OF THE REFLECTANCE EQUATION FOR A WATER WAVE SURFACE

In Chapter III equation (III-4) was derived to be the equation for the reflectance of a water wave surface. This equation was:

$$R = \Delta x \left[\frac{1}{2} R_{x0} + R_{x_1} + R_{x2} + \dots + R_{xn} + \frac{1}{2} R_{x_{n+1}} \right] . \qquad (D-1)$$

It was mentioned that this was essentially an integration across the energy wave rather than a line integral along the wave surface since each point on the wave surface does not see the same quantity of energy. The energy wave is then considered to be divided into n equal ray elements, with exterior points defined as the point separating the ray elements.

Figure 7 showed pictorially the situation where the incident ray had a greater slope than the maximum slope of the water wave, and Figure 8 showed the situation where the ray has a smaller slope than the maximum slope of the water wave.

The general equation for the water wave surface is:

$$y = A \sin \left(\frac{2\pi x}{\lambda} - \delta \right),$$

and the equation for the projection of the energy ray through an external point is:

$$y = mx + b,$$

where the slope, m, is known to be:

$$m = \tan\left(\frac{\pi}{2} - \phi\right).$$

The basic problem is to evaluate the slope of the wave at its intersection with each projection of an exterior point. Then, the total angle of incidence, $\phi+\alpha$, may be calculated. With this information, the reflectance equations may be used to determine the values of R_{x_i} in equation (D-1).

Under the conditions shown in Figure 7 where there is no shadow, it is obvious that δ is zero. The value of the total angle of incidence for the first exterior point may be found by evaluating the slope of the wave at the point x = 0. This is accomplished by noting that the slope is:

$$\frac{dy}{dx} = \frac{2\pi A}{\lambda} \cos\left(\frac{2\pi A}{\lambda}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{2\pi A}{\lambda}.$$

The angle, α, is then:

$$\alpha = \arctan\left(\frac{2\pi A}{\lambda}\right).$$

For the second exterior point, the intersection of its projection with the wave may be determined by using Newton's second order method of tangents (76). Likewise, at this point of intersection, the slope of the wave and the total angle of incidence, $\phi+\alpha$, may be calculated. The procedure is then repeated for each exterior point.

Under the conditions shown in Figure 8, the initial exterior point is selected to be the point whose projection is tangent to the water wave. As the slopes are equal at the point of intersection, x_o , it can be said that:

$$\tan\left(\frac{\pi}{2}-\phi\right) = \frac{2\pi A}{\lambda} \cos\left(\frac{2\pi x_o}{\lambda} - \delta\right)$$

or

$$\cos\left(\frac{2\pi x_o}{\lambda} - \delta\right) = \frac{\lambda}{2\pi A} \tan\left(\frac{\pi}{2}-\phi\right) .$$

If γ is defined as follows:

$$\gamma \triangleq \left(\frac{2\pi x_o}{\lambda} - \delta\right) ,$$

it may be shown that:

$$\cos\gamma = \frac{\lambda}{2\pi A} \tan\left(\frac{\pi}{2}-\phi\right)$$

or

$$\gamma = \arccos\left[\frac{\lambda}{2\pi A} \tan\left(\frac{\pi}{2}-\phi\right)\right] .$$

At this same point of intersection, x_o, the ordinates must be equal or:

$$m x_o = A \sin(\gamma)$$

or

$$x_o = \frac{A}{m} \sin(\gamma) ,$$

which results in:

$$\delta = \frac{2\pi x_o}{\lambda} - \gamma .$$

The slope of the wave is $\frac{dy}{dx}$; and, therefore, the angle, α , is found to be:

$$\alpha = \arctan\left(\frac{dy}{dx}\right) .$$

And then the total angle of incidence for the point on the surface is $\phi + \alpha$.

Again, Newton's second order method of tangents may be used to locate the intersection of the water wave with the second exterior point; and the above procedure can be followed for the calculation of its total angle of incidence.

The reflection equations may be used for each intersection point, and the results substituted into equation (D-1). Thus, a procedure has been developed for the calculation of reflectance from a water wave.

A comparison of the reflectance of a wave with the reflectance of a plane surface can be made by evaluating the reflection equations for a water surface where the total angle of incidence is ϕ .

On the next page is found a copy of the computer program which makes use of the techniques mentioned above for the evaluation of equation (D-1).

Solution of Reflectance Equation for Water Wave Surface

```
    READ, R, E, WLENG, SLMAX, DXX
  1 READ, PHI, DIFF, AMP, DELX
    PHIR = PHI * 3.1416 / 180.0
    BETA = ( 3.1416 / 2.0 ) - PHIR
    IF (BETA - 3.1416/2.0) 2, 3, 2
  3 SLOPE = SLMAX
    GO TO 4
  2 SLOPE = (SIN(BETA))/(COS(BETA))
  4 SUM = 0.0
    Z = 0.0
    YCPT = 0.0
    C = 2.0 * 3.1416 / WLENG
    IF (SLOPE - C * AMP) 12, 13, 13
 13 X = 0.0
    DEL = 0.0
    DELTA = 0.0
    GO TO 14
 12 CC = (SLOPE)/(C * AMP)
    CCC = ( SQR(1.0 - CC**2))/CC
    GAMMA = ATN(CCC)
    X = (AMP/SLOPE)*SIN(GAMMA)
    DEL = (C*X) - GAMMA
    DELTA = DEL * 180.0 / 3.1416
    VALUE = .50
    SUM = SUM + VALUE
 10 YCPT = YCPT - (SLOPE*DELX)
    Z = Z + DELX
    X = X + DELX
    FX = AMP * SIN((C*X) - DEL) -(SLOPE * X ) - YCPT
 15 D1FX = C * AMP * COS((C*X) - DEL) - SLOPE
    D2FX = -(C**2)* AMP * SIN((C*X) - DEL)
    DNX = 2.0 * D1FX * FX
    DDX = D2FX * FX - ( 2.0 * D1FX**2)
    IF (DDX) 31, 30, 31
 30 DX = DXX
    GO TO 32
 31 DX = DNX / DDX
 32 X = X + DX
    FX = AMP * SIN((C*X) - DEL) -(SLOPE * X ) - YCPT
    ABFX = ABS (FX)
    IF (SENSE SWITCH 2) 21, 22
 21 TYPE, ABFX
 22 CONTINUE
```

```
    IF (ABFX - DIFF) 14, 14, 15
 14 TTANA = C*AMP* COS((C*X) - DEL)
    ALPHA = ATN(TTANA)
    ALPHB = ABS( ALPHA )
    CHI = 2.0 * 3.1416* X/(WLENG) - DEL
    IF (CHI - 3.1416/2.0) 25, 25, 26
 25 T = PHIR + ALPHB
    GO TO 29
 26 IF (CHI - 1.5 * 3.1416) 27, 27, 28
 27 T = ABS(PHIR - ALPHB)
    GO TO 29
 28 T = PHIR + ALPHB
 29 CONTINUE
    IF (SENSE SWITCH 3) 23, 24
 23 TYPE, X, ALPHA, CHI, PHIR, T
 24 CONTINUE
    Z7=R**2-(E**2)-(SIN(T))**2
    Z6=SQR(((Z7)**2)+4.0*(R**2)*(E**2))
    AA = ABS((Z6+Z7)/2.0)
    BB = ABS((Z6-Z7)/2.0)
    A = SQR(AA)
    B = SQR(BB)
    Z1=A**2+B**2
    Z2=2.*(A*COS(T))
    Z3=(COS(T))**2
    Z4=2.*(A*(SIN(T))**2)/(COS(T))
    Z5=((SIN(T))**4)/((COS(T))**2)
    RN=(Z1-Z2+Z3)/(Z1+Z2+Z3)
    RP=(RN)*((Z1-Z4+Z5)/(Z1+Z4+Z5))
    RT=(RN+RP)/2.0
    IF (SENSE SWITCH 1) 16, 17
 16 TYPE, Z
 17 CONTINUE
    IF (Z) 200, 8, 11
  8 VALUE = RT / 2.0
    SUM = SUM + VALUE
    GO TO 10
 11 IF ( Z - WLENG) 6, 7, 7
  6 VALUE = RT
    SUM = SUM + VALUE
    GO TO 10
  7 VALUE = RT/ 2.0
    SUM = SUM + VALUE
    RATIO = DELX / WLENG
    RUO = RATIO * SUM
    PUNCH, PHI, RUO, DELX, DELTA, AMP
    GO TO 1
200 CONTINUE
    END
```

VITA

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Candidate for the Degree of

Doctor of Philosophy

Thesis: THE RELATIVE REFLECTANCE OF A MONOLAYER-COVERED WATER SURFACE

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Biographical:

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Education: Attended grade school in Knoxville, Tennessee, and in Birmingham, Alabama; graduated from Shades Valley High School in Birmingham in 1957; received the Bachelor of Science degree from Auburn University, Auburn, Alabama, in 1961; received the Master of Science degree from Oklahoma State University, Stillwater, Oklahoma, in 1963; completed requirements for the Doctor of Philosophy degree in August of 1965.

Professional Experience: During the summer of 1960, was employed as a student engineer at the Humble Oil Refinery, Baton Rouge, Louisiana; during the summer of 1961; held the position of graduate assistant at Auburn University; was a summer research engineer at the Oak Ridge National Laboratory, Oak Ridge, Tennessee, 1963; while at Oklahoma State University, was holder of a National Defense Education Act Fellowship during the academic years, 1961 to 1964, and worked as a research assistant during the academic year, 1964-65.

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