Research Report

GRAPH-THEORETIC CONFIRMATION OF RESTRUCTURING DURING INSIGHT

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Abstract—*The "flash of insight" sometimes observed in problem solving and in scientific discovery has been thought to be due to a sudden cognitive restructuring of the problem situation. Direct confirmation of restructuring has been difficult without an independent procedure for determining cognitive structure. Graph structures were derived from judgments of concept relatedness made by subjects who had an insight and by several groups who either did not or could not have the insight. The graphs of the solvers differed from the graphs of subjects who tried and failed, those who listened to the solvers, and those who were given the solution. When other subjects in a subsequent experiment repeatedly judged similarity of pairs of concepts, there was evidence that those connections critical to the new cognitive order were targeted long before there was the breathtaking cognitive reorganization.*

On occasion, the solution to a problem appears to arrive in a rapid and unexpected manner (Grmek, 1980; Weisburd, 1987). Gestalt psychologists believed that restructuring of the problem situation was a necessary prerequisite for successful problem solving, and that when this restructuring occurred rapidly, the subject experienced insight (Ellen, 1982; Luchins & Luchins, 1970; Scheerer, 1963; Wertheimer, 1959). To gestaltists, restructuring affected the entire problem situation; the problem was, in their words, recentered (Wertheimer, 1959). The focus of the problem structure changed because the perceived relations among the elements had changed (Ohlsson, 1984). Gestaltists argued that the restructuring was rapid because the problem solver had to overcome a fixation before the solution was available. This contention has been rightfully criticized (Dominowski, 1981; Weisberg & Alba, 1981a, 1981b) because an independent measure of fixation does not exist. In fact, a similar argument can be leveled against the explanatory power of restructuring, because an independent measure of restructuring does not exist.

It has proven difficult to disentangle empirically the cognitive structure of a problem from the actual discovery of a solution. Gestaltists used subjects' protocols as one method (Wertheimer, 1959). However, subjects' self-reports about the probability of discovering a solution have been demonstrated to be unrelated to actual insight (Metcalfe, 1986a), suggesting that self-reports following insight are equally unreliable. What is required is a measure of cognitive structure that can be used whether or not a solution to the problem has been achieved. Without such a measure, insight and restructuring are circular, and insight remains simply a term to describe the phenomenology that accompanies some successful problem solutions.

Psychological scaling provides a procedure that can address the role of restructuring as the mechanism underlying insight. The Pathfinder scaling algorithm (Schvaneveldt, Durso, & Dearholt, 1989) is one method of revealing the latent structure of the problem representation. Pathfinder uses judgments of similarity between concepts to produce a graph. Graphs have proven important in the understanding of chemical isomers, electrical circuits, Markov chains, statistical mechanics, and network flow in operational research (Harary, 1969). Pathfinder creates graphs from noisy empirical data by determining and eliminating those relations in the data that violate the assumption of triangle inequality, thus revealing the latent structure of the conceptual domain. The technique does not require subjects to report on their perception of the problem as a whole but instead produces a representation of that perception based on pair-wise similarity or dissimilarity ratings. Empirically derived graphs have been used successfully in distinguishing novices and experts (Schvaneveldt et al., 1985), in predicting the order of remembered information (Cooke, Durso, & Schvaneveldt, 1986), in predicting the usability of menus (Roske-Hofstrand & Paap, 1986), in discriminating the structure of categories (Hutchinson, 1989; Schvaneveldt et al., 1989), in accounting for the latency to categorize (Cooke, 1992), and in describing the flow of information within a city (Durso & Coggins, 1990).

Pathfinder uses the Minkowski *r* distance (Equation 1) to satisfy the requirements of a path algebra (Carre, 1979) for networks and to compute the geodesic distance between all pairs of nodes in the network:

$$W(P) = \left(\sum_{i=1}^{k} w_i^r\right)^{1/r}, \qquad (1)$$

where the weight of a path, $W(P)$, is equal to the r^{th} root of the sum of the r-power weights along the k links of the path, w_i. When $r = \infty$, the equation is appropriate for ordinal data, and the resultant network contains the minimal number of links required to maintain ordinal information (Hutchinson, 1989; Schvaneveldt, Dearholt, & Durso, 1988).

EXPERIMENT 1

Basically, we wanted to compare the graphs of people who had experienced the insight with the graphs of groups who did not. To this end, subjects made relatedness judgments of pairs of concepts at the end of the experimental session. In none of the conditions described below was the judgment task responsible for producing the insight.

To increase the occurrence of insights among the subjects in our laboratory, we used a puzzle that required the subject to

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induce a missing piece of information: "A man walks into a bar and asks for a glass of water. The bartender points a shotgun at the man. The man says, 'Thank you,' and walks out." Earlier work using this puzzle (Dayton, Durso, & Shepard, 1990) indicated that when people finally find the missing piece of information necessary to make sense of the story, the solution arrives with a suddenness and accompanying phenomenology (Metcalfe, 1986a, 1986b; Metcalfe & Wiebe, 1987) that is characteristic of insight (Wertheimer, 1959).

Subjects asked the experimenter yes-or-no questions to gain more information about the puzzle (e.g., "Was the man thirsty?" "No."). The questions and the experimenter's responses were tape-recorded.

Solvers Compared With Nonsolvers

Solvers (n = 6) were those subjects who succeeded in solving the puzzle. Nonsolvers (n = 6) were those who failed to solve the puzzle after 2 hr of questioning. Subjects made their representations of the problem available by providing relatedness judgments on a 10-point Likert scale for the 91 possible pairings of 14 terms relevant to the puzzle. Some of the terms were explicit in the puzzle (e.g., *man, bartender*), some were relevant to the solution (e.g., *surprise, remedy*), and some were merely objects found in a bar (e.g., *TV, pretzels*). The pairs were presented randomly to the subjects, and positions of members within a pair were counterbalanced.

Within each group (solvers and nonsolvers), the mean rating for each pair of terms was computed. This 14 × 14 matrix of mean ratings was then submitted to the Pathfinder algorithm. Resultant graphs appear in Figure 1. Graph distances (number of links) between all possible pairs of nodes were computed for each graph. The solvers and nonsolvers were distinctly different: The correlation between the respective graph-distance matrices was essentially zero (r = .00). Measures of the focal point of each graph (i.e., center and median) were also computed. In the solvers' graph, *remedy* and *relieved* occupy a central position. The nonsolvers' graph centers on concepts explicit in the story (*man, bartender*). People who solved the insight puzzle differed dramatically from those who actively tried to solve the puzzle but failed.

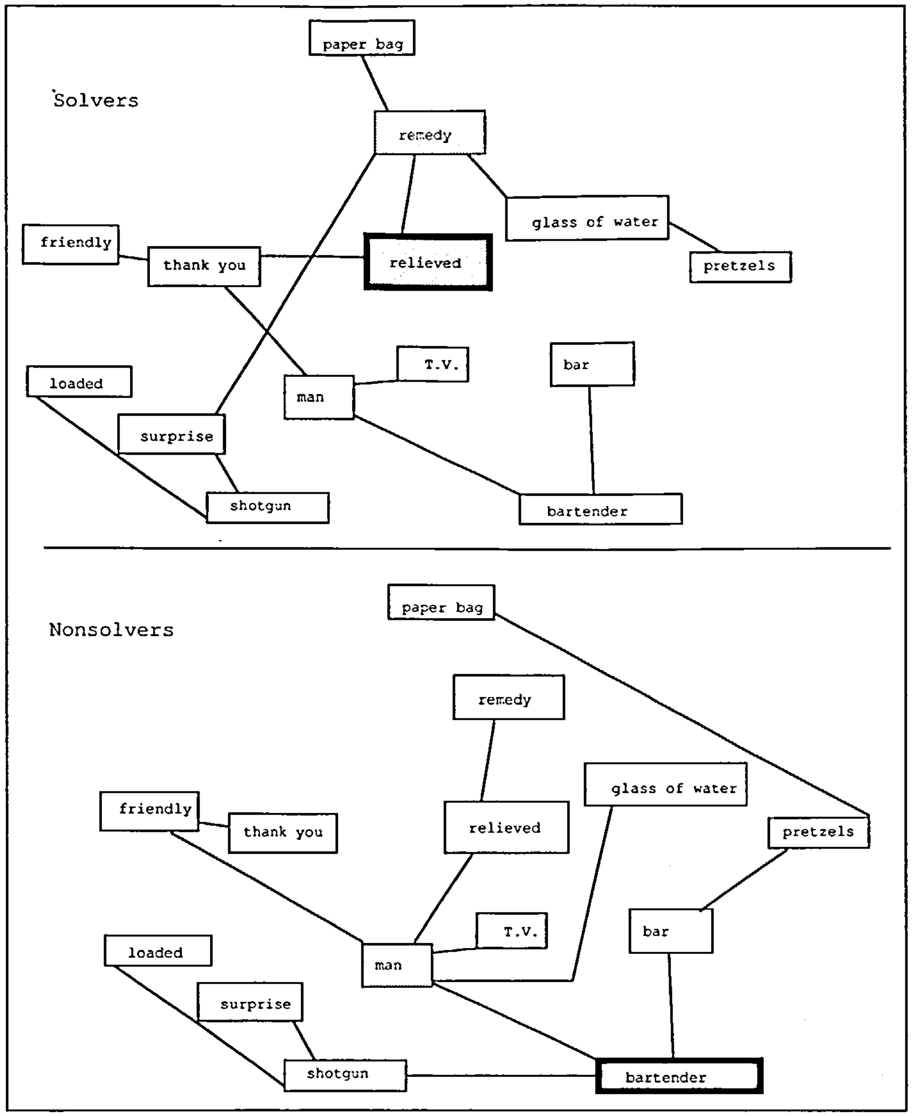

Fig. 1. Pathfinder graphs of concept associations for solvers (top) and nonsolvers (bottom). The center of each graph (indicated by a bold outline) is the node with minimum eccentricity, and the median node (indicated by shading) has the smallest average distance to all other nodes.

Comparing Other Groups With Solvers and Nonsolvers

To investigate some of the underlying reasons why solvers and nonsolvers might differ in their cognitive structure of the problem, five additional groups of 6 subjects each were given different information prior to judging relatedness. To control for the possibility that structural differences between solvers and nonsolvers might be due to differences in the questions asked, listeners attempted, but failed, to solve the puzzle after listening to the tapes of the solvers' questions up to but not including the final (i.e., solution) question. To control for the possibility that knowledge of the solution (i.e., hiccoughs) produced differences that were not due to discovering the solution, one group (hiccough narrative) was read the story except that the story began with "A man with the hiccoughs walks . . ."; thus, subjects in this group were presented with a simple narrative containing the solution, but they were not told that the story was a puzzle, nor were they asked to solve it. To determine if the process of arriving at the solution was important to the solvers' structure, another group attempted to solve the puzzle. After 10 min, but before anyone could solve the puzzle, we gave these subjects the solution. This solution-given group thus was provided with the story, an active problem-solving experience, and the solution, but unlike the solvers, they did not generate the solution on their own. It is of interest that these subjects exhibited the "aha!" phenomenology associated with insight (Metcalfe & Wiebe, 1987). Finally, two baseline control groups were included: The naive group simply gave relatedness judgments without hearing the story, and the story-only group read the original puzzle and then immediately judged relatedness without attempting to solve the puzzle first.

Table 1 presents correlations between each graph-distance matrix and the solvers' and nonsolvers' graph-distance matrices. Both the solvers and the nonsolvers had structures different from the preexperimental organization of the naive group; and the structures of both the solvers and the nonsolvers had similarities with those subjects who heard only the story. However, in every other case, the graph did not correlate with the solvers' graph (see Table 1). Neither did any of the graph-theoretic centers for these five additional groups resemble the solvers' center (see Fig. 2). Thus, the structure of subjects who listened to the solvers but did not hear the solution did not resemble the structure of the solvers. If subjects were told about hiccoughs, but did not try to solve the puzzle, their structure was nonetheless more similar to the structure of the nonsolvers than of the solvers. Apparently, having the solution available was not sufficient to yield a structure that resembled that of the solvers. Finally, even when subjects attempted to solve the puzzle and were given the solution, their graph was more similar to the nonsolvers' than the solvers'.

Only in the solvers' graph are all the remedies for hiccoughs (i.e., *paper bag, surprise, glass of water*) connected to the *remedy* node. The connection between *surprise* and *remedy* (the key to the solution) and the connection between *thank you* and *relieved* (the reason the man said "thank you") are unique to the solvers' graph. Even for subjects given the solution, *surprise* is not connected to *remedy,* suggesting these subjects did not truly appreciate the solution in the same sense as those who solved the puzzle on their own.

When the solution is handed to an active problem solver, it appears to lack the recentering power that a generated solution possesses. Other methods of passing on solutions, such as giving hints, have also been shown to be only partially effective (Dominowski & Jenrick, 1972). Apparently, one cannot easily share in the insight of a colleague, only in the results.

Table 1. *Pearson product-moment correlations between the graph-distance matrices of five groups and the solvers' and nonsolvers' graph-distance matrices*

Group	Target group: Solver	Target group: Nonsolver	More like?
Naive	.09	.15	Neither
Story only	.27*	.39*	Both
Listeners	−.02	.68*	Nonsolver°
Hiccough narrative	−.04	.23*	Nonsolver°
Solution given	.13	.38*	Nonsolver+

Note. The larger the positive correlation, the closer in a 91-d representational space are the two graphs. The more similar target group (solvers or nonsolvers) is indicated in the rightmost column.
* Correlation is significantly different from 0 at an alpha level of .05.
°,+ The two correlations are reliably ($p < .05$)° or marginally ($p < .10$)+ different using the Pearson r-to-z transform in a test for differences between correlated correlation coefficients (Meng, Rosenthal, & Rubin, 1992).

EXPERIMENT 2

In Experiment 2, we were interested in determining how critical pairs of concepts, as identified by the graphs in Experiment 1, changed within a subject as the subject heard, attempted to solve, and ultimately solved the puzzle. Thus, unlike in Experiment 1, we asked subjects to judge pairs repeatedly. Specifically, all subjects judged the pairs before they heard the puzzle (naive), after they heard the puzzle (story), after they solved the puzzle (solved), and every 10 min between hearing the story and solving the puzzle.

Rather than have subjects repeatedly judge all 91 pairs from Experiment 1, we selected pairs that, according to the graphs, should behave differently as subjects move from hearing the puzzle to solving it. These insight pairs were *surprise–remedy* and *relieved–thank you,* the only two pairs that were unique to the solvers' graph. How perception of similarity varies across time for these pairs was of critical concern. We expected that they would initially be viewed as unrelated and ultimately be viewed as related. However, how these pairs made this transition would supply information about how the restructuring of the cognitive structure proceeds. In addition to insight pairs, subjects judged two related pairs and two unrelated pairs. The related pairs (i.e., *bartender–bar; shotgun–loaded*) were connected in all the Pathfinder graphs of Experiment 1, and the unrelated pairs (i.e., *pretzel–shotgun; TV–remedy*) were never linked in the Pathfinder graphs. We expected the related pairs to be judged as similar regardless of the distance from solution, and the unrelated pairs to be judged as dissimilar at each opportunity. Finally, we added 6 filler pairs (i.e., *water–thank you, surprise–paper bag, friendly–thank you, surprise–relieved, paper bag–remedy, surprise–thank you*), bringing the total number of pairs judged at each opportunity to 12. The fillers were pairs that did not meet the criteria of the

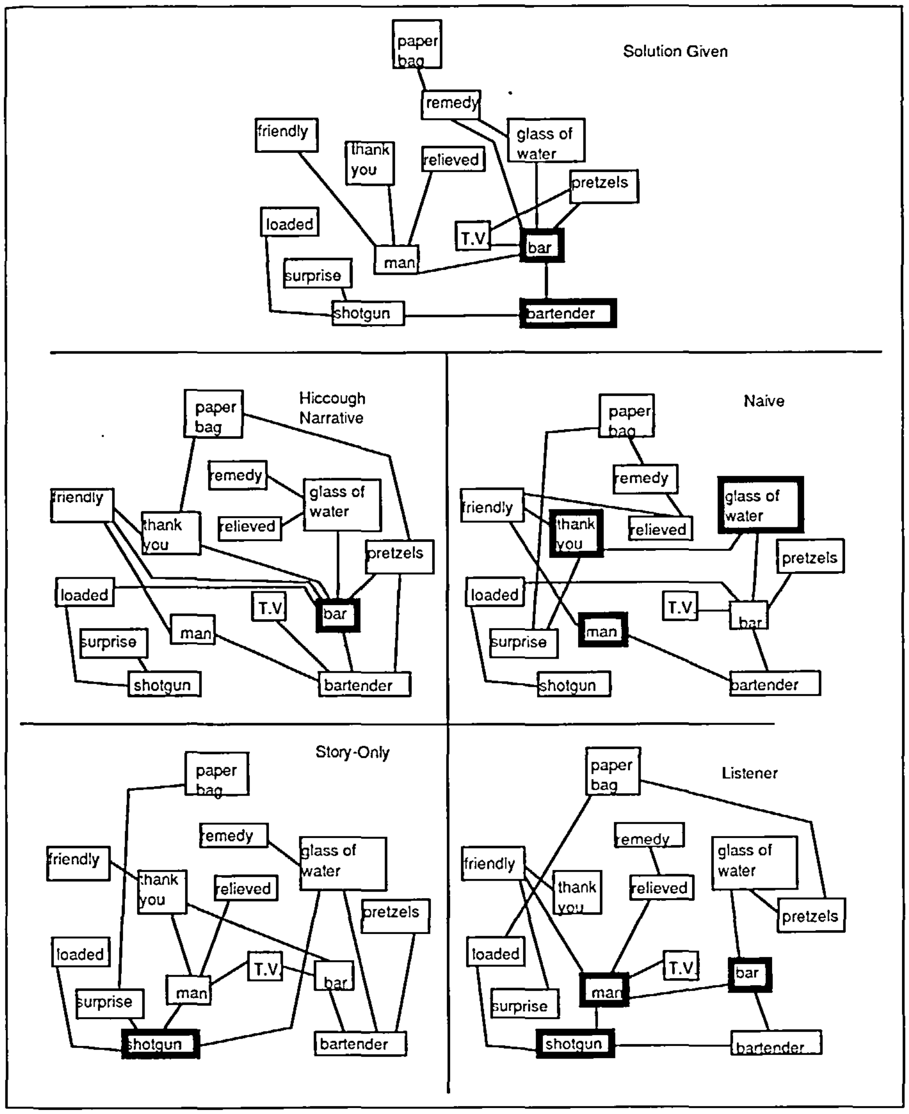

Fig. 2. Pathfinder graphs of concept associations for the control conditions: solution-given, hiccough narrative, naive, story-only, and listener groups. The bold nodes are centers; the shaded nodes are medians.

other pairs, but that we felt would help disguise the relationship of the pairs and the puzzle's solution. Pairs were presented in a random order with the constraint that the related, unrelated, and insight pairs were not in adjacent positions. Subjects judged the relatedness of pairs by placing a slash through a 5½-in. line representing a scale of relatedness divided into ⅛-in. units. Measurement was made by determining the position of the slash.

Figure 3 shows how judgments for each of the groups changed across time. Because subjects took different amounts of time to solve the puzzle, the number of similarity judgment tasks performed varied. However, all subjects made at least two sets of judgments between hearing the story and solving the puzzle: one before solving (solved-1) and one 10 min before that (solved-2). A multivariate analysis of variance confirmed that ratings depended on the interaction of phase and type of pair, $F(6, 4) = 11.71$, $p < .05$. As expected, the related pairs were viewed as similar throughout; the unrelated pairs were viewed as dissimilar throughout. Of most interest is the pattern exhibited by the insight pairs. These pairs did not leap directly from their initial dissimilar position to their ultimate similar position. Rather, the pairs that Pathfinder identified as unique to the solvers moved first to an intermediate level of similarity, $t(9) = 2.55$, $p < .05$, and then to a final, high level of similarity, $t(9) = 3.41$, $p < .01$. This move to an intermediate level could not be attributed to an averaging across subjects who moved catastrophically at different points in time. The changes apparent in the figure were confirmed by both parametric tests and sign tests. Eight of 10 subjects showed the change in similarity between story and solved-2 judgments ($p = .055$), and all but 1 showed a drop between solved-1 and solved judgments ($p = .011$), indicating that the pattern was not due to only a few individuals.

GENERAL DISCUSSION

Experiment 1 suggests a dramatic restructuring that requires more than just access to the solution and attempts to solve. Experiment 2 suggests that people gradually identify possible relationships on which to focus. Although these are not judged as clearly related until later (when the problem is solved), subjects do view them as more related than most other pairs. Other investigators (e.g.,

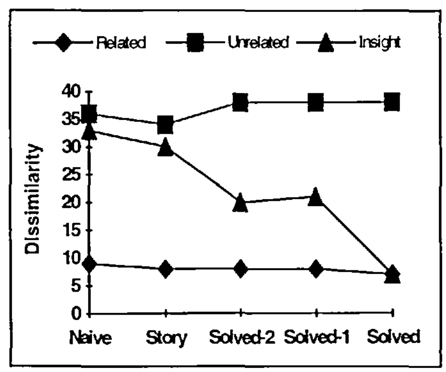

Fig. 3. Transitions in judged dissimilarity as a function of the subject's phase in the problem-solving process for concept pairs that were linked in Experiment 1 in all Pathfinder graphs (related), in no Pathfinder graphs (unrelated), or in only the solvers' Pathfinder graph (insight). Naive = before hearing the puzzle; story = after hearing the puzzle; solved-2 = 10 min before solved-1; solved-1 = before solving the puzzle; solved = after solving the puzzle. The amount of time between the story condition and the solved-2 condition depended on the amount of time the subject took to solve the puzzle.

Bowers, Regehr, & Balthazard, 1990) have also demonstrated that explicitly identifiable solutions may be preceded by an above-chance forced choice of the stimulus set relevant to the solution.

This report breaks the circularity between insight and restructuring and provides empirical evidence of the cognitive reorganization underlying insight. Those subjects who ultimately achieved the insight clearly restructured the problem to the point that elements stood in a different relation to one another. In current models of cognition, a massive reorganization and recentering can take place by the importation of some higher order abstraction or schema into the comprehension situation. Together with the goals of the situation, such importation can have dramatic effects on cognitive processing (Bransford & Johnson, 1972; Ohlsson, 1984; Smith, Adams, & Schorr, 1978). Data from our second experiment, however, provided evidence of movement toward this restructuring prior to solution. The presumed importation of a higher order abstraction does not appear to arrive without some warnings, although the warnings are apparently insufficient to produce a dramatic restructuring and may be beneath the ken of the solver (Metcalfe, 1986a, 1986b). Like dynamite, the insightful solution explodes on the solver's cognitive landscape with breathtaking suddenness, but if one looks closely, a long fuse warns of the impending reorganization.

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