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IMMUNIZATION, STOCHASTIC PROCESS RISK, AND OPTIMAL OBJECTIVE FUNCTIONS: REEXAMINATION OF THE DURATION VECTOR MODEL WITH MONTE CARLO SAMPLING

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A Dissertation APPROVED FOR THE
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CHAPTER I

Introduction

The subject of bond portfolio immunization has been a fruitful area of research since the Fisher and Weil (1971) study. Immunization is appealing to practitioners because it is viewed as an easy to apply, but superior alternative to maturity matching for the purpose of controlling interest rate risk in bond portfolios. The models most commonly employed are called single factor duration models (SFDMs) because they compute a single index which is set equal to the length of the planning horizon to effect the interest rate risk immunization. The most familiar SFDM is the Macaulay (1938) duration model. Such models can not completely eliminate risk because some of their assumptions do not hold in practice. Much of the research in the area has been directed toward methods or strategies for minimizing immunization error due to violation of SFDM assumptions. It includes Bierwag, Kaufman, and Toevs (1983), Fong and Vasicek (1984), and Fooladi and Roberts (1992).

The failure of SFDMs is most commonly attributed to the problem of stochastic process risk. That is, extant duration indexes are computed using either yield to maturity or a single stochastic variable, based on a priori assumptions about the nature of term structure innovations, in their discount functions. A number of multiple factor deterministic and stochastic models have been derived in response to the realization that the term structure of interest rates is too complex a function to be summarized by a single index. However, evaluation has shown these models either to be intractable or as failing to improve upon the empirical results of simple duration matching. Models in this category include Brennan and Schwartz (1983) and Nelson and Schaefer (1983).

An important exception is the Chambers, Carleton, and McEnally (CCM) (1988) duration vector model. This model is significant because it signaled, for many, the passing of the torch from simple single factor immunization models to a marginally more complicated, but significantly more effective, multiple factor one. The importance attributed to the CCM model derives from their empirical results, which can be summarized in three conclusions:

(1) That their duration vector with two elements (constraints) resulted in smaller immunization error than did the single factor Fisher and Weil duration model.

(2) That the inclusion of additional vector elements sequentially reduced immunization error.

(3) That optimal immunization error reduction is achieved with a 5-7 element duration vector.

The CCM (1988) conclusions were based on empirical observations of average shortfall of holding period returns from target returns in simulated portfolios generated in turn by the single factor model and by duration vectors with sequentially two through seven terms. While the CCM theory is not challenged here, their empirical methodology leaves their conclusions open to question. These unanswered questions, which are discussed in the next section, provide the motivation for this study.

The Problem

The CCM model restricts the portfolio to adherence to J constraints of the form H^j (j=1...J), where H is the time remaining to the end of the holding period, and J is the degree of the duration vector. The first constraint is $D(1)=H^1$, the second is $D(2)=H^2$, and so on through $D(J)=H^J$, where the D(j) are duration type measures derived from their model. Functionally, D(1) is equivalent to simple Fisher and Weil duration and, as is the case in all SFDMs, is initially set equal to the length of the planning horizon. This fact, along with the objective function employed by CCM to select unique portfolios from the universe of bonds introduces a bias that confounds the empirical results.

CCM employs the following objective function for both the single factor model and the duration vectors.

Minimize Σy_i^2 (i=1...I), where

y_i = proportion of security i in the portfolio,

I = the number of securities selected, and

$\Sigma y_i = 1$.

The effect of this objective function is to maximize the number of bonds included in the portfolio. With only one duration constraint, as is the case for the single factor alternative to which their model is compared, the procedure selects the entire sample of bonds. The two element duration vector, because it adds a second constraint, reduces the number of securities selected. Because the right hand side value of the first constraint in the duration vector remains invariant [i.e., D(1)=H] as J is increased, selected bond maturities, and durations, must be increasingly compressed around the horizon length, H.

Interestingly, both Bierwag, Kaufman, and Toevs (BKT) (1983) and Fong and Vasicek (FV) (1984) have shown that immunization error due to the inability of SFDMs to completely summarize the term structure of interest rates (stochastic process risk) can be reduced by compressing the maturities of constituent bonds around the horizon toward a bullet portfolio. BKT (1983) examined the issue by simulating the performances of portfolios that were immunized by SFDMs derived from incorrectly specified stochastic processes. Their simulations showed the immunization error associated with these incorrect models to be smaller the closer the concentration of maturities about the horizon length.

Fong and Vasicek (1984) developed a formal procedure for achieving maximum compression of duration matched portfolios, and thereby for minimizing immunization error due to stochastic process risk. They showed that the change in end of horizon value for a portfolio immunized via Macaulay duration matching resulting from a change in the term structure is approximated by the equation

$$DV_H / V_H = -M^2 \, ds \tag{1}$$

where

V_H = promised end of horizon portfolio value,

DV_H = the difference between realized and promised end of horizon value,

ds = the change in slope of the yield curve,

and,

$$M^2 = \Sigma(t_i - H)^2 \, C_i P_0(t_i) / V_0, \tag{2}$$

where

t_i = times when cash flows, C_i, are received,

H = holding period length, and is equal to the portfolio duration,

$P_0(t_i)$ = the present value of one dollar to be received at time t_i, and

V_0 = the weighted average present value of the portfolio.

It is evident from equation 1, that immunization error due to incorrect specification of the stochastic process can be minimized by minimizing the value of M^2. Although the efficacy of M^2 minimization has not been empirically verified, it has been recognized as a potentially optimal objective function for selecting duration matched portfolios. [See, for example, Fabozzi and Fabozzi (1989)].

To the extent that the BKT (1983) and FV (1984) recommendations are valid procedures for reducing immunization error due to stochastic process risk, the CCM (1988) results, given their empirical methodology, are entirely predictable. It is, therefore, impossible to determine the extent to which the observed immunization error reduction associated with increasing the degree of the duration vector is attributable to their model rather than to the portfolio concentration effect. Succinctly stated, use of the quadratic minimization objective introduces a potential bias in favor of the duration vector model. For this reason, replication of the CCM study while controlling for this factor would represent a significant contribution to the body of knowledge on both the efficacies of SFDMs in general and on the CCM duration vector model in particular.

While reexamination of the CCM (1988) results is the primary motivation for this research, evaluation of this issue raises another unresolved question concerning immunization by single factor duration matching. That question concerns the efficacy of

M^2 minimization as an optimal strategy for selecting immunized portfolios. As indicated earlier, the procedure has not been subjected to rigorous empirical scrutiny. In light of the fact that alternative objective functions such as the one employed by CCM (1988) and bond price convexity maximization have appeared in the literature, the efficacy of the M^2 objective is a question that begs analysis.

Purpose of the Study

The first objective of this study is to reexamine the efficacy of the CCM duration vector model while controlling for the portfolio concentration effect. The specific hypotheses to be evaluated are that:

(1) A two constraint duration vector outperforms single factor Macaulay duration matching as an immunization strategy, (2) The sequential inclusion of additional duration vector constraints materially improves immunization performance, and

(3) A seven constraint duration vector is optimal for achieving immunization error reduction.

The second purpose of the study is to empirically evaluate the efficacy of the M^2 minimization objective function for selecting portfolios immunized by Macaulay duration matching. This is accomplished by comparing the M^2 objective with a set of theoretically supportable alternatives. The hypothesis to be evaluated is that

M^2 minimization outperforms alternative objective functions for selecting immunized portfolios.

Measuring Performance

The important issue in empirical studies of immunization efficacy, given the inability of extant models to completely eliminate risk, reduces to a comparison of the performances of alternative models or strategies for their implementation. If one or more extant models could guarantee complete immunization, the only relevant performance measure would be expected holding period return. Given the current state of the art, the choice of performance measures is a nontrivial problem. As a practical matter, evaluation of alternative immunization strategies should analyze both expected return and risk measures. Selection of the appropriate risk measure is a complicated exercise. It requires limiting assumptions about the class of investors that employs the procedure.

Khang (1983) has justified immunization as a minimax (or maximin) strategy. The objective that follows from such a strategy is to minimize the maximum shortfall of realizable holding period return from a predetermined target. While this objective may be appropriate for a small percentage of immunizers, it is not likely to account for the broad appeal of the concept. The existing research recognizes this fact and employs a variety of risk-return measures in empirical analysis. These measures include mean residual returns, mean absolute deviation of residual returns, and frequency of holding period returns below target returns. This study will evaluate each of these performance measures as well as mean and dispersion of holding period returns along with a version of Sharpe's return to variability measure, (Residual Returns / Absolute Deviations of Residual Return).

The Empirical Method

Empirical evaluation of the hypotheses of interest in this study requires that comparative analyses of the selected performance measures be performed on the alternative models or strategies. The ideal procedure would be to observe a large number of sample realizations of the performance measures and to employ classical statistics to draw inferences concerning the relative efficacies of the alternatives under consideration. The nature of the immunization problem, however, severely restricts our ability to accomplish this.

Immunization is traditionally evaluated as a long term hedging strategy. Fisher and Weil (1971) and Bierwag, Kaufman, Schweitzer, and Toevs (BKST) (1981) evaluated immunization strategies for holding periods of five, ten, and fifteen years. Five year horizons appear to be the minimum standard employed in most other studies. An important exception is the CCM (1988) study that employed quarterly holding periods over sixteen quarters.

Because only risk-free and option-free bonds are appropriate for empirical evaluation, research is limited to time series analysis. This, in turn necessitates sampling from historical databases of term structure information. The Fisher and Weil (1971) and BKST (1981) studies employ the Durand data. Most of the empirical studies that followed have employed data from the Center for Research on Security Prices (CRSP) U.S. Government Bond File. This file is perceived as containing superior data relative to the Durand file, but it is also limited in that data are available for only about sixty five years.

The limitation on the availability of historical data poses three problems. The first is a problem associated with any statistical inference based on sample observations. It implicitly assumes stationarity of the stochastic processes driving the variables of interest. The second problem relates to the need to observe nonoverlapping (independent) holding periods. With only sixty five years of time series data, a maximum of thirteen 5-year independent holding periods can be observed. Maximum sample sizes are proportionately smaller for holding periods of 10 and 15 years. The third problem is that only a single sample of observations on the variables of interest can be generated for each strategy being studied. This restricts our ability to perform important replications or repeat tests.

Given the sampling limitations imposed by the available data, our ability to draw strong inferences from the results of classical methods is compromised. As an example, early immunization studies were forced to observe samples of overlapping holding periods. As a result, they were precluded from performing tests of statistical significance. They relied on a "preponderance of the evidence" logic to conclude that immunization strategies outperformed the benchmark, maturity matching, to which they were compared. While such logic is appropriate under the assumptions of negligible differences in implementation costs, tractability, and investor appeal, it is not sufficient when there exist material differences on one or more of these considerations across competing strategies. It is important, therefore, to employ a methodology that allows robust conclusions to be drawn about the materiality of observed differences among the performance measures generated from the strategies under investigation.

An empirical methodology that overcomes all of the restrictions imposed by the limited data availability is Monte Carlo experimentation, or simulation. In comparing the performances of alternative portfolio strategies, it is desirable to measure the outcomes of implementations of those strategies under a variety of term structure realizations. Implementation of immunization strategies requires both the reinvestment of intermediate cash receipts and complete portfolio rebalancing at regular discrete intervals. For portfolios of risk free and option free bonds, the only stochastic variables affecting horizon value are the prices of the relevant securities at the reinvestment, rebalancing, and horizon dates. These prices, in turn, may be specified as functions of unanticipated changes in the term structure of interest rates. While the stochastic process governing term structure innovations is not known, a variety of conditions can be simulated under alternative distributional assumptions. Large numbers of samples of investigator determined size can be generated by this methodology. Summary statistics derived from these samples are evaluated in this research to draw inferences regarding the relative immunization performances of the competing strategies.

Significance of the Study

In spite of the failure of single factor immunization models to completely eliminate interest rate risk, the technique is used by insurance companies and institutional fund managers with billions of dollars under their control. Any advances that materially improve performance are of obvious interest. However, the potential benefits of such advances must be weighed against increased costs of implementation. It is, therefore, imperative that advantages claimed for more complicated strategies, such as the duration vector model, be carefully scrutinized. It is also important to explicitly recognize that the performance of the Macaulay duration matching strategy might be affected by the objective function employed in portfolio selection. In general, objective functions that result in fewer security holdings or less trading volume are preferred, ceteris paribus.

This study contributes to the literature by addressing both issues above. Even more importantly, perhaps, it designs an experimental procedure that overcomes the sampling problems of all previous immunization studies.

Limitations of the Tests

Though the experimental methods employed in this study are designed to allow stronger inferences than those of previous immunization studies, there are important limitations. The most important of which are due to the assumptions imposed on the Monte Carlo sampling procedure and to the exclusion of explicit consideration of the effects of taxes and transaction costs on the response variables.

To generate the simulated price and yield data analyzed in this study, it is necessary to make specific assumptions about the nature of the stochastic component of the portfolio return generating process. Inferences derived from the statistical results may not be generalizable to conditions not assumed. While the experiment is designed to subject the alternative strategies to a variety of possible term structure innovations, the true stochastic process will not necessarily be encompassed.

The holding period returns evaluated in this study are before taxes and transaction costs. The differential effects of these omissions on the observed response variables are unknown. Therefore, the conclusions reached as a result of the empirical analysis may not hold when these factors are considered. In general, strategies that require the inclusion of larger numbers of securities in the immunized portfolios, or that demand a higher volume of trading at rebalancing dates, can be expected to incur higher transaction costs. Since both of these are characteristics of the duration vector model, failure to consider transaction costs potentially biases the results in its favor.

The differential effect of taxes does not appear to be a significant factor. Assuming equal investment under all competing strategies, there does not appear to be any systematic tax bias favoring either strategy regarding coupon income. Because the duration vector strategies will normally require a higher volume of trading at rebalancing dates than the single factor strategy, there is likely to be differential capital gains (or losses) tax effects. To the extent that gains and losses are equally likely, however, failure to explicitly consider taxes in the study will not bias the results.

Organization of the Study

Chapter II of this study provides a review of selected literature. Since the empirical analysis pursues two distinct lines of inquiry, the remainder of the study is separated into two parts. Part One addresses the examination of the three hypotheses concerning the CCM duration vector model. It includes chapters III through V. The CCM (1988) tests are replicated in Chapter III with the min M^2 objective function for the duration strategy. Chapter IV describes the Monte Carlo sampling design and sample generation procedure. Chapter V presents the design and implementation of the Monte Carlo sampling tests of the CCM duration vector.

Part Two addresses the hypothesis regarding the optimality of M^2 minimization as an objective function for selecting portfolios immunized by Macaulay duration matching. It is embodied in Chapter VI, which presents the Monte Carlo tests of the alternative single factor models. In addition to "minimize M^2" and the CCM quadratic minimization function, the alternatives include Convexity maximization, M^2 maximization, and two maturity constrained duration strategies suggested by Fooladi and Roberts (1992). Chapter VII summarizes the study, and presents conclusions and suggestions for further research.

CHAPTER II

REVIEW OF SELECTED LITERATURE

Introduction

The concept of immunization as a vehicle for managing interest rate risk in bond portfolios has developed heuristically over time. A large volume of studies has appeared in the literature. Included in those studies are a number of alternative duration type models that may be employed to minimize interest rate risk in portfolios of coupon bonds. Most of these models can be categorized into one of three groups: (1) Single factor models derived from duration measures that assume that random shifts and/or twists in the term structure can be fully captured in one parameter; (2) equilibrium type multiple factor stochastic models of the term structure ; and (3) deterministic multiple factor models that require two or more parameters to explain term structure movements .

Models in the first category include, of course, Fisher and Weil (1971) and Macaulay (1938) duration matching. The Fisher and Weil model explicitly assumes that the term structure of interest rates is limited to parallel (additive) shifts, while the Macaulay model derives its duration measure using yield to maturity. Others include duration measures by Bierwag (1977)(1978) and Khang (1979). The Bierwag model employs a duration measure derived under the assumption of multiplicative term structure changes. The implication of such an assumption is that longer term rates are more volatile than are short term ones. The assumptions of both models are contradicted by the empirical observation that short term rates tend to display greater volatility than long term. The Khang model is designed to address this contradiction. It derives duration measures for both additive and multiplicative processes under the assumption of term dependent interest rate changes. A common characteristic of all the SFDMs is that they implicitly assume perfect correlation of term structure changes throughout the range of maturities. The greater the deviation of actual term structure innovations from those assumed by a given SFDM, the larger the resulting immunization error. This is the essence of stochastic process risk.

Another restriction of the SFDMs above is that only a single instantaneous term structure shift of the assumed nature can occur. An important study on the implementation of single factor models, which addresses multiple term structure changes over a given horizon, is Bierwag (1979). Bierwag showed that the appropriate adjustment for multiple changes is to periodically rebalance the portfolio to maintain the duration-horizon match. Bierwag stressed that this is only a locally optimal strategy since it ensures immunization only when term structure changes are small.

Examples in the second category above include models by Cox, Ingersoll, and Ross (1979), Brennan and Schwartz (1983), and Nelson and Schaefer (1983).

The CCM duration vector model is the most significant model in the third category. Since it is the subject of this study, it is reviewed in considerable detail.

The CCM Duration Vector Model

The duration vector model is an extension of an approach attributable to Cooper (1977) who assumed that the term structure adheres to one of four a priori functional forms. CCM (1988) relax the Cooper (1977) assumptions by taking advantage of the well known mathematical theorem that any smooth function, f(x), can be approximated by a polynomial of the following form:

$$f(x)\ f_n(x)=a_0+a_1x+a_2x^2+a_3x^3+...+a_nx^n \quad (1)$$

They employ an exponential polynomial representation of the term structure under the assumption of continuous compounding. From this representation, they derive, via calculus, a set of sensitivity measures to be used as constraints in selecting immunized bond portfolios. They call the set of sensitivity measures a duration vector.

Chambers, Carleton, and McEnally (1988) derive their model from a polynomial representation that has certain desirable properties. The most important consideration in its selection is that the polynomial is of a form that produces simple derivatives. They assume that the term structure of interest rates can be expressed by the following function:

$$B(H) = \exp[- X_j H^j] \quad (2)$$

where;

H = time to maturity of a zero-coupon bond (in years),

$B(H)$ = the price of a zero-coupon bond with maturity H,

J = the length of the polynomial, and

X_j = the jth polynomial coefficient, (j=1...J).

It is important to note that the authors do not suggest this functional form as a behavioral representation of the true term structure. It is rather an approximation method that depends upon approximation theorems for its validity. Bond price sensitivity to changes in the term structure can be expressed in terms of the J polynomial coefficients. The partial derivative of bond price with respect to a given polynomial coefficient is:

$$dB(H)/dX_j = -H^j\, B(H), \text{ where } j = 1,...,J. \quad (3)$$

The percent change in bond price resulting from a change in a factor driving term structure shifts (as represented by a polynomial coefficient) can be expressed as:

$$dB(H)/dX_j[1/B(H)] = -H^j, \qquad j = 1,\ldots,J. \qquad (4)$$

The negative sign in equation (3) represents the inverse relationship between bond price and interest rate changes. It can be ignored, and the CCM duration vector follows:

$$D_H(j) = H^j \qquad j = 1,\ldots,J. \qquad (5)$$

where;

$D_H(j)$ = the jth duration measure of a discount bond with maturity H.

For coupon bonds, each coupon payment is treated by this model as a discount bond. The duration vector thus becomes a weighted average of the individual coupons' duration vectors. It is expressed as:

$$D(j) = \Sigma w_t t^j, \qquad j=1,\ldots,J. \qquad (6)$$

where;

$D(j)$ = the jth duration measure for a coupon bond with payments occurring at times t.

$w_t = C_t \exp[-R(t)t]/P_B$

C_t = the cash flow promised (coupon or maturity value) at t.

P_B = current price of the coupon bond.

$R(t)$ = the instantaneous interest rate at time t.

Note the similarity between the duration vector weights and the weights in conventional single factor duration measures. It is instructive to also note that each element of the duration vector for a certain coupon payment is expressed as a power of time and represents the sensitivity of bond price to changes in a polynomial coefficient (a term structure factor). The value weighted sum of these vector elements over all coupons plus maturity payments represents the sensitivity of the price of a coupon bond to changes in the term structure. As such, the duration vector, D(j) can be thought of as an explicit measure of interest rate risk.

The CCM coupon bond immunization procedure involves selecting and weighting the portfolio so as to constrain each weighted average duration vector element for the constituent bonds to be equal to the corresponding duration vector element of a pure discount bond with maturity equal to the planned holding period. An additional constraint in the model is that the sum of the weights must equal one (the portfolio is fully invested). The system of equations representing these constraints are summarized below.

$$\Sigma\Sigma Y_i D_i(j) = d(j) \qquad (i=1, \ ,I) \qquad (j=1, \ ,J) \qquad (7)$$

$$y_i = 1.$$

Where,

y_i = the percentage position (long or short) in bond i.

$D_i(j)$ = the jth duration measure for bond i.

I = the total number of bonds in which either a long or short position is taken in the portfolio.

J = the number of terms in the polynomial employed.

d(j) = the jth duration measure for a pure discount bond with maturity equal to the planned holding period.

Expressed in matrix notation, the constraints are

$$\underline{A}'\underline{Y} = \underline{b} \tag{8}$$

where;

$\underline{Y}$ = a vector of portfolio weights,

$[y_1 \quad y_i]$,

$\underline{A}$ = a matrix of duration vector elements for J polynomial coefficients and I constituent coupon bonds with a column of ones representing the coefficients in the second equation in 7 above.

$$\begin{vmatrix} 1 & D_1(1) & D_1(2) \ldots D_1(J) \\ 1 & D_2(1) & D_2(2) \ldots D_2(J) \\ . & . & . \;\ldots\; . \\ . & . & . \;\ldots\; . \\ . & . & . \;\ldots\; . \\ 1 & D_i(1) & D_i(2) \ldots D_j(J) \end{vmatrix}$$

and, $\underline{b}$ = a vector of J duration measures for a pure discount bond with maturity equal to the holding period.

$$\begin{vmatrix} 1 \\ d(1) \\ d(2) \\ . \\ . \\ . \\ d(J) \end{vmatrix}$$

Note that the weights are not constrained to be nonnegative as in the case of extant duration models.

The validity of the CCM duration vector model as an alternative to Macaulay duration matching turns on its ability to approximate the true term structure with a finite and small number of terms. The strategy should also be capable of implementation without the inclusion of short selling or any other strategy which, itself, could be independently employed as a risk hedging procedure. Exclusion of short selling, given selection from a finite universe of securities, may well limit the degree of the attainable duration vector. This, in turn, might alter the CCM empirical results and conclusions.

The Empirical Methods

Although the literature on duration and immunization is expansive, there is little attention given to empirical design and analysis. Of the numerous studies on immunization efficacy, few are empirical in nature. Fisher and Weil (1971) tested their duration model, which assumes additive term structure shifts, against maturity matching. Bierwag, Kaufman, Schweitzer, and Toevs [BKST] (1981) compared five alternative duration models. They included Macaulay duration, Fisher and Weil duration, Bierwag (1977) duration (multiplicative stochastic process), and two Khang (1979) duration measures (additive term dependent stochastic processes).[1]

Both Fisher and Weil and BKST generated sample observations on return relatives from overlapping holding periods of 5, 10, or 15 years. Since the samples were

[1]These models are summarized in Appendix I.

not constituted of independent observations, no rigorous statistical tests were performed in either study. Neither study included an objective function as part of the immunization strategies. Both employed one-year rebalancing intervals. Conclusions regarding the immunization efficacies of the strategies under consideration were reached by visual inspection of sample mean and dispersion measures.

Brennan and Schwartz (1983) and Nelson and Schaefer (1983) both tested their models against Fisher and Weil duration over five year holding periods. Like the previous studies, they employed overlapping holding periods and were precluded from rigorous statistical testing. Based on visual inspection of summary statistics, they both concluded that simple single factor immunization worked about as well as their more complicated models. Other empirical tests of immunization strategies were performed by Ingersoll (1983) Gultekin and Rogalski (1984), and Bierwag, Kaufman, and Latta (1987). These studies also observed overlapping holding periods and performed no rigorous statistical tests.

CCM (1988) employed an experimental design that represented a significant departure from previous studies. Rather than simulating returns on portfolios of coupon bonds over holding periods of five years or more, they investigated the return performances of portfolios composed of short term notes over quarterly holding periods. They defined an immunization strategy as an attempt to replicate the performance of a zero-coupon security with maturity equal to the relevant holding period length. The response variables were observed for both overlapping and nonoverlapping periods. Difference in means tests, using the t-statistic, were performed on shortfall from target returns for the nonoverlapping samples. However, these tests can be considered weak at best because the samples were constituted of only five observations. Evaluation of such small samples brings both the power and the robustness of the tests into question.

The most recent and most comprehensive tests of single factor immunization strategies are by Fooladi and Roberts (1992). These tests focused on the differential performances of Macaulay duration matched portfolios selected according to different criteria. They did not, however, consider multiple factor models such as the CCM (1988) duration vector. It does consider, peripherally, the minimize M^2 objective function and the issue of short selling. These are two issues of particular importance in this dissertation.

The Fooladi and Roberts (1992) results with regard to both issues were far from conclusive. Their conclusions generally support the efficacy of the M^2 objective function as a means of reducing immunization error, but they do not support its overall optimality. They made no conclusive statements about the short selling issue. Like the other studies above, Fooladi and Roberts employed overlapping observation periods and did not perform tests of statistical significance.

PART ONE

A Reexamination of the Duration Vector Model

Part One presents two separate tests for reexamining the duration vector models. The first test replicates the CCM (1988) multi period test with a larger sample and the min M^2 objective function for implementing the Macaulay duration strategy. The second test evaluates samples of holding period returns over five years using the Monte Carlo sampling procedure. Details of these tests are presented in Chapter III and Chapter V. Chapter IV includes a description of the procedures for generating Monte Carlo samples.

CHAPTER III

REPLICATION OF THE CCM DURATION VECTOR TESTS

Introduction

There are several aspects of the CCM (1988) experimental design and procedures that may be called into question. They include (1) using the quadratic minimization objective function to select portfolios, (2) permitting negative portfolio weights indicating short selling, and (3) evaluating quarterly holding periods rather than the traditional five years or more. Issues one and two above are addressed in this chapter.

The CCM Methodology

The CCM duration vector model, like extant SFDMs. is a constrained optimization strategy. It differs from single factor models in that an objective function must necessarily be specified to select unique portfolios. CCM justifies the use of the objective function, minimize Σy_i^2, as a procedure for minimizing idiosyncratic risk. This objective function would be of greatest value, of course, if term structure changes were characterized by independent random variations across the maturity range. It would be of least value if term structure innovations were perfectly correlated, as assumed by the SFDMs. Of course, the actual relationship of yield structures is probably somewhere between these two extremes. Exactly where remains an empirical question.

The procedure employed by CCM to test the efficacy of their model compares the abilities of Macaulay duration and duration vectors of two through seven degrees to generate target holding period returns. By minimizing the sum-of-squared portfolio weights, their selection procedure attempts to maximize the number of securities included. With no constraints, Σy_i^2 is minimized when equal proportions, 1/I, of each security is included in the portfolio. With only one constraint, as is the case of the single factor model, the procedure includes duration value weighted proportions of every note in each sample. When a second constraint is added, a two degree vector, fewer notes can be included. As vector constraints are incrementally added, the number of includable notes declines, resulting in a larger Σy_i^2 and an increasing concentration of note maturities around the holding period length. The consequence is that for each sample evaluated, the single factor portfolio has maximum dispersion around the horizon, and the seven-degree vector has minimum dispersion.

While CCM does not report the number of notes actually included in each portfolio, they do report the sums of squared portfolio weights. These data clearly illustrate the point. These sums averaged about 0.095 for Macaulay duration and 0.691 for the seven degree vector. A larger sum means that the portfolio is concentrated in fewer securities.

Short Selling

The differential portfolio compression would have been even more dramatic if CCM had not allowed short selling. The effect of short selling is to include negative weighted securities, thereby increasing the total numbers represented in some of the portfolios. While a necessity, perhaps, for avoiding infeasibilities in solving the multiple constraint models, failure to restrict the portfolio weights to nonnegative values might pose both statistical and practical problems.

Allowance of short selling in an immunization experiment is itself a source of potential controversy. Most immunization studies exclude it for two good reasons. First, short selling may be employed as a hedging strategy quite independent of any immunization model. It should be excluded from empirical tests of particular immunization strategies to avoid confounding the results. Secondly, short selling introduces complications, such as margin requirements, that induce a different set of transaction costs and risks. For certain institutional investors, such as pension funds, short selling may be precluded by regulatory authority. It does not follow, therefore, that immunizers would view a strategy that requires short selling with indifference.

For the reasons cited above, this study will restrict short selling in portfolio selection, and will employ the min M^2 objective function to implement the duration strategy. This objective function should minimize the effect of differential portfolio concentration and allow stronger inferences about the relative performances of the alternative strategies. It should also be noted that strict adherence to the no short selling restriction might not be possible because of infeasibilities.

The CCM Optimization Models

In effect, CCM evaluated seven immunization strategies. For ease of exposition, these alternatives are defined as Strategy 1 through Strategy 7. Complete definitions of the alternative strategies, as employed in the CCM study, follow below. All variables are as previously defined.

Strategy 1 Minimize Σy_i^2 ,

Subject to $\Sigma y_i Dur_i = h$,

$\Sigma y_i = 1$

Strategy 2	Minimize	Σy_i^2 ,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i=1$
Strategy 3	Minimize	Σy_i^2 ,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma\, y_i D_i(2)=(h-1)^2$,
		$\Sigma\, y_i D_i(3)=(h-1)^3$,
		$\Sigma\, y_i=1$
Strategy 4	Minimize	Σy_i^2 ,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma\, y_i D_i(2)=(h-1)^2$,
		$\Sigma\, y_i D_i(3)=(h-1)^3$,
		$\Sigma\, y_i D_i(4)=(h-1)^4$,
		$\Sigma\, y_i=1$
Strategy 5	Minimize	$\Sigma\, y_i^2$,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma\, y_i D_i(2)=(h-1)^2$,
		$\Sigma\, y_i D_i(3)=(h-1)^3$,
		$\Sigma\, y_i D_i(4)=(h-1)^4$,
		$\Sigma\, y_i D_i(5)=(h-1)^5$,
		$\Sigma\, y_i=1$

Strategy 6	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i D_i(5)=(h-1)^5$,
		$\Sigma y_i D_i(6)=(h-1)^6$,
		$\Sigma y_i=1$
Strategy 7	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i D_i(5)=(h-1)^5$,
		$\Sigma y_i D_i(6)=(h-1)^6$,
		$\Sigma y_i D_i(7)=(h-1)^7$,
		$\Sigma y_i=1$

For an initial planning horizon of H periods, each of these models is solved for the optimal weights, y_i, at each time s (s=0,...,H-1). The initial portfolios are selected from the universe of available bonds using the time s=0 weights. The selection process is repeated at each time s=1 through s=H-1 to maintain the constraint equalities.

Note that the first constraint in each duration vector strategy is $\Sigma y_i D_i(1)=h-1$.

$D_i(1)$ is functionally equivalent to Dur_i in Strategy 1. Even if higher terms of the duration vector had no immunization effect, their inclusion causes the portfolio weights of the D(1) constraint to be increasingly compressed around the value, h-1. Considering the results of both BKT(1983) and Fong and Vasicek (1984), as well as those of Fooladi and Roberts (1992), this could explain some portion of the immunization error reduction that CCM attribute to incrementally higher degrees of the duration vector. How much, if any, of the results observed by CCM is due to the effect of portfolio compression is the empirical question addressed in this study.

The Test Design

CCM (1988) performed both single period and multi period tests of the models. The alternatives included nine strategies; naive unconstrained, maturity matching, Macaulay duration matching, and six duration vectors of two through seven terms. The sample consisted of the prices of Treasury notes appearing in the Wall Street Journal from November 15, 1976, to August 15, 1980. Current prices were measured as the average of published bid and asked prices. Form these data, CCM was able to derive fifteen independent quarterly single period returns and fifteen overlapping multi period returns.

The procedure for performing the single period tests is straightforward and consists of four steps: (1) The duration vector for each security is derived using its yield to maturity, (2) the portfolios are selected and weighted using the quadratic minimization objective function, (3) the returns on each portfolio are observed for one-quarter periods, and (4) the observed returns are compared to a predetermined target return. The target return on each portfolio is the estimated spot rate on a 3-month, zero coupon security derived from the prevailing term structure.

The multiperiod test follows the same procedure as the single period one with the exception that each portfolio is rebalanced at the end of each quarter to maintain the duration vector constraints. In addition to the fifteen overlapping multiperiod observations, five nonoverlapping holding periods of nine months each were also observed. Summary statistics were evaluated for these independent observations.

This study uses the same data source and repeats the identical multiperiod immunization tests, on nonoverlapping observation periods, as CCM with the following exceptions:

(1) The min M^2 objective function is employed to select the Macaulay duration portfolios, (2) negative portfolio weights representing short selling are strictly limited, and (3) target returns are defined as the yield on instruments with maturity equal to the relevant nine-month holding period. The single period tests and the tests employing overlapping periods are not repeated in this dissertation. Neither the naive unconstrained nor the maturity matching tests are included.

The summary statistics used to measure relative performances of the alternative strategies are difference in average residual holding period returns and difference in mean absolute deviation of holding period returns. Residual return is defined as

Q= (target holding period return) - (realized holding period return),

and mean absolute deviation is defined as

$MAD = \Sigma(|Q_i - Q|/n)$,

where i is a sample observation on Q, Q is the sample mean from an alternative strategy, and n is the total number of observations.

For ease of exposition, the alternative strategies are subsequently referred to by number. Each strategy is represented by a specific optimization model. Complete model descriptions are provided in the next section of this chapter. Summary descriptions of the strategies are included in the following table. Pairwise tests are performed to assess the relative efficacies of the alternative strategies.

Table 1

Alternative Strategies Examined

Strategy	Description	Constraints
1	Macaulay Duration	D=h (h=holding period length)
2	two-degree duration vector	$D_1=h$; $D_2=h^2$
3	three-degree duration vector	$D_1=h$; $D_2=h^2$; $D_3=h^3$
4	four-degree duration vector	$D_1=h$; $D_2=h^2$; $D_3=h^3$; $D_4=h^4$
5	five-degree duration vector	$D_1=h$; ; $D_5=h^5$
6	six-degree duration vector	$D_1=h$; ; $D_6=h^6$
7	seven-degree duration vector	$D_1=h$; ; $D_7=h^7$

The differences in means are evaluated using one tailed paired sample t-tests. Under the null hypothesis of no difference in means, the test statistic is $t=d\sqrt{n} / S_d$ with n-1 degrees of freedom. Where,

$d= \Sigma d_i/n$ is the mean of the sample differences on the variable of interest,

d_i is the difference in the variable of interest for the matched pair on observation i (i=1,…,n), n is the number of sample observations on the matched pairs,

$S_d = \sqrt{[(d_i^2) - n\bar{d}^2 / n-1]}$ is the standard deviation of the matched pairs differences.

The Alternative Strategies

The constrained optimization models related to each strategy, as employed in this study, are specified below. Portfolios are selected by these models at the beginning of the planning horizon and at each rebalancing date.

Strategy 1 Minimize ΣM_i^2,

Subject to $\Sigma y_i Dur_i = h$,

$\Sigma y_i = 1, y_i\ 0$

Strategy 2 Minimize Σy_i^2,

Subject to $\Sigma y_i D_i(1) = h-1$,

$\Sigma y_i D_i(2) = (h-1)^2$,

$\Sigma y_i = 1, y_i\ 0$

Strategy 3 Minimize Σy_i^2,

Subject to $\Sigma y_i D_i(1) = h-1$,

$\Sigma y_i D_i(2) = (h-1)^2$,

$\Sigma y_i D_i(3) = (h-1)^3$,

$\Sigma y_i = 1, y_i\ 0$

Strategy 4 Minimize Σy_i^2,

Subject to $\Sigma y_i D_i(1) = h-1$,

$\Sigma y_i D_i(2) = (h-1)^2$,

$\Sigma y_i D_i(3) = (h-1)^3$,

		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i=1, y_i\ 0$
Strategy 5	**Minimize**	Σy_i^2 ,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i D_i(5)=(h-1)^5$,
		$\Sigma y_i=1, y_i\ 0$
Strategy 6	**Minimize**	Σy_i^2 ,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i D_i(5)=(h-1)^5$,
		$\Sigma y_i D_i(6)=(h-1)^6$,
		$\Sigma y_i=1, y_i\ 0$
Strategy 7	**Minimize**	Σy_i^2 ,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,

$$\Sigma y_i D_i(5)=(h-1)^5,$$

$$\Sigma y_i D_i(6)=(h-1)^6,$$

$$\Sigma y_i D_i(7)=(h-1)^7,$$

$$\Sigma y_i=1, y_i \; 0$$

Note the differences in these optimization models and those of CCM.

Hypotheses Tested

Given the size of the sample and considering the probability of type II error, the following hypotheses are evaluated at a significance level of 0.05. The subscripts i and j are used to denote strategies being compared. In all matched pairs, i subscripts represent the strategy with fewer constraints.

H_0: $\mu(Q_i) - \mu(Q_j) = 0$ (i=1, ,6); (j=2, ,7); (i j)

H_1: $\mu(Q_i) - \mu(Q_j) > 0$

H_0: $\mu(Q_1) - \mu(Q_j) = 0$ (j=3, ,7)

H_1: $\mu(Q_1) - \mu(Q_j) > 0$

H_0: $MAD(Q_i) - MAD(Q_j) = 0$ (i=1, ,6); (j=2, ,7)

H_1: $MAD(Q_i) - MAD(Q_j) > 0$

H_0: $MAD(Q_1) - MAD(Q_j) = 0$ (j=3, ,7)

H_1: $MAD(Q_1) - MAD(Q_j) > 0$

This format results in twenty two hypotheses to be tested (eleven for each summary statistic). Since superior performance would be indicated by smaller values of both summary statistics, rejection of the null hypothesis in either test would support the conclusion that the more complex strategy outperforms the simpler one to which it is compared. Conversely, failure to reject either null would indicate no apparent advantage to the more complex strategy.

The Sample

The sample consists of ninety three quarters of Treasury notes prices and yields from the Wall Street Journal. This allows 31 independent observations on 9-month holding period returns.

The observation period in this study is from February 15, 1970 through August 15, 1992. Quarterly prices of Treasury notes are taken from the Wall Street Journal on the first publication date following the fifteenth day of February, May, August, and November of each year in the observation period. These observation dates were chosen based on the CCM observation that large numbers of notes mature in these months. Table 2 summarizes the sample sizes for each of the ninety three quarters. Sample prices are recorded as the average between bid and asked prices appearing on the relevant dates.

The Test Procedure

The procedure for performing the CCM replication involves four steps. First, the price and yield data are divided into groups of three-quarter periods. Each one of these groups constitutes an independent observation period. There are thirty one such periods comprising the sample evaluated in this test. The target yield is taken as the yield on a note maturing nine months from the beginning of the relevant observation period. Portfolios of notes are selected by the alternative strategies at the beginning of each period and are rebalanced at the ends of the first and second quarters.

The next step in the process is to simulate the portfolio performances over the three-quarter observation periods. This is accomplished by the following procedure. The

Table 2

Sample Size Descriptions

Obs Number	Beginning Date	Number of Notes	Target Yield	Target Value
1	2-15-70	20	7.51%	$105.74
2	11-15-70	25	5.68%	$104.31
3	8-15-71	25	5.13%	$104.16
4	5-15-72	26	4.45%	$103.37
5	2-15-73	26	5.80%	$104.41
6	11-15-73	25	7.79%	$105.96
7	8-15-74	26	9.39%	$106.98
8	5-15-75	31	6.08%	$104.63
9	2-15-76	38	5.58%	$104.25
10	11-15-76	40	5.17%	$103.93
11	8-15-77	40	6.35%	$104.84
12	5-15-78	45	7.61%	$105.81
13	2-15-79	46	10.01%	$107.70
14	11-15-79	45	11.83%	$109.13
15	8-15-80	48	9.34%	$107.24
16	5-15-81	50	16.04%	$112.52
17	2-15-82	55	14.60%	$111.35
18	11-15-82	57	9.05%	$106.86
19	8-15-83	61	10.30%	$107.93
20	5=15=84	61	11.39%	$108.79
21	2-15-85	62	9.10%	$106.98
22	11-15-85	60	7.78%	$105.95
23	8-15-86	63	5.87%	$104.46
24	5-15-87	58	6.92%	$105.28
25	2-16-88	58	6.64%	$105.06
26	11-15-88	52	8.11%	$106.21
27	8-15-89	61	8.46%	$106.48
28	5-15-90	72	8.29%	$106.34
29	2-15=91	74	6.29%	$104.79
30	11-15-91	75	4.88%	$103.71
31	8-15-92	69	3.43%	$102.59

duration, M^2, and D(1) through D(7) are computed from the price and yield data for each note in the sample. The values of these measures are used in the optimization models above to select portfolios at the beginning of each three-quarter observation period (H=3), and at the ends of quarter one (h=2) and quarter 2 (h=1). The value of each portfolio is then computed at the end of each quarter by solving the following equation.

$V_s = V_{s-1}\, y_i P_{i,s}$,

where V_s = the value of a portfolio at the end of period s,

V_{s-1} = portfolio value at the beginning of period s,

y_i = the portfolio proportion (or weight) of bond i in the portfolio selected at time s-1.

$P_{i,s}$ = the price of bond i at the end of period s, and

s=1,2,3.

The values at the end of period 3, V_3, are used to derive the variables of interest. Finally, the differences in means and the differences in MADs are computed and evaluated using the paired sample t-test.

All computations of duration, M^2, and D(1)-D(7) are performed by Lotus 123 software.[2] The optimization models are solved using linear and nonlinear solvers of the GAMS (General Algebraic Modeling System) software.[3]

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[2]Lotus 123 is a trademark of the Lotus Development Corporation.

[3]GAMS (General Algebraic Modeling System) is a programming language copyrighted by The International Bank for Reconstruction and Development/The World Bank, and published by The Scientific Press, Redwood City, CA.

The Data and Results

The sample of notes, described earlier in this chapter, were used to simulate bond portfolio performances for nonoverlapping nine month holding periods. Resulting terminal values from the alternative strategies are summarized in Table 3 below.[4] Paired sample differences and test statistics were derived from these values. Summary statistics on these measurement variables are presented in Table 4. Underlined t-values denote significance at the 0.05 level.

Panel 1 of Table 4 summarizes statistics on the difference in mean terminal values for paired strategies as indicated. Although relative immunization efficacy does not necessarily depend on the magnitude of terminal portfolio values, clearly higher value is preferred, ceteris paribus. Therefore, superior performance of a strategy over the one to which it is compared is indicated by a greater terminal value. The critical value of the t-statistic for rejection of the null hypothesis is -1.697 at the 0.05 significance level. We are able to reject the null for only the pair of strategies 4 and 5 - indicating that strategy 5 generated terminal values that are significantly higher than those generated by strategy 4. Neither of the null hypotheses concerning the pairing of strategy 1 with strategies 2 through 7 can be rejected. We conclude that the duration vector strategies do not outperform simple Macaulay duration with regard to expected portfolio return.

Panel 2 of Table 4 summarizes the differences in mean residual returns for the paired samples. Superior performance of an immunization strategy is indicated by smaller

[4]Values of residual returns, absolute deviations of returns, and return to volatility are in Appendix II.

TABLE 3
Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	107.31	106.54	105.71	105.70	105.70	105.67	105.70
2	104.28	103.79	103.39	103.54	103.54	103.64	103.65
3	103.97	102.38	102.19	102.61	102.61	102.60	102.61
4	103.41	103.54	103.88	103.94	103.91	103.94	103.92
5	103.50	103.61	103.61	103.60	103.59	103.60	103.60
6	105.96	105.57	105.74	105.60	105.60	105.62	105.62
7	106.97	106.67	106.66	106.65	106.63	106.63	106.63
8	104.47	105.09	105.20	105.16	105.21	105.18	105.18
9	104.27	104.51	104.61	104.54	104.54	104.54	104.54
10	104.07	103.84	103.89	103.84	103.84	103.85	103.84
11	104.84	104.87	104.69	104.77	104.77	104.77	104.77
12	105.77	105.86	105.81	105.81	105.80	105.81	105.81
13	108.40	107.39	106.08	107.15	106.08	108.21	107.14
14	109.31	109.87	109.70	109.70	109.70	109.68	109.68
15	107.28	107.68	107.87	107.87	107.87	107.87	107.87
16	112.41	112.17	112.67	112.70	112.71	112.71	112.70
17	108.67	111.12	110.88	110.91	110.91	110.91	110.91
18	107.13	106.27	105.56	105.66	105.55	105.54	105.53
19	107.93	107.33	107.21	107.21	107.05	107.24	107.22
20	108.51	109.04	109.30	109.33	109.34	109.31	109.30
21	106.95	107.75	107.31	107.32	107.31	107.28	107.28
22	105.96	107.22	107.15	107.03	106.97	106.93	106.94
23	105.46	105.56	107.96	107.93	107.92	107.90	107.89
24	104.87	106.08	106.50	106.51	106.50	106.55	106.50
25	107.46	105.61	106.22	106.28	106.17	106.32	106.17
26	106.37	108.18	110.87	110.81	110.84	110.81	110.84
27	106.13	108.74	110.24	110.27	110.26	110.27	110.27
28	107.20	105.72	104.89	104.88	104.89	104.90	104.91
29	104.79	105.27	104.96	104.91	104.92	104.96	104.96
30	103.84	103.71	103.78	103.83	103.81	103.75	103.77
31	102.62	102.62	102.68	102.68	102.70	102.68	102.68
MEAN	106.13	106.25	106.36	106.41	106.36	106.44	106.40
STD	2.15	2.40	2.64	2.62	2.61	2.63	2.61

TABLE 4

Summary Statistics for Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	2.253	1.545	-0.10	-1.48	0.052	0.011	3.799	3.698	2.215	2.267	2.279
STD	15.55	7.988	1.552	6.575	1.050	1.791	16.73	15.75	16.44	16.39	16.21
t	1.122	1.498	-0.50	-1.74	0.386	0.049	1.758	1.818	1.043	1.071	1.088

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	-0.11	-0.11	-0.05	0.048	-0.07	0.039	-0.22	-0.27	-0.23	-0.30	-0.26
STD	1.039	0.818	0.211	0.194	0.385	0.194	1.580	1.505	1.565	1.481	1.509
t	-0.60	-0.78	-1.33	1.400	-1.12	1.117	-0.80	-1.03	-0.81	-1.16	-0.99

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	-0.16	-0.19	-0.00	0.020	-0.05	0.036	-0.35	-0.36	-0.34	-0.39	-0.35
STD	0.916	0.639	0.130	0.095	0.279	0.190	1.283	1.228	1.268	1.179	1.226
t	-0.98	-1.72	-0.05	1.215	-1.05	1.065	-1.55	-1.63	-1.49	-1.85	-1.61

PANEL 4: DIFFEERENCE IN RETURN TO VOLATILITY

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	-0.13	-0.00	-0.07	0.062	-0.10	0.045	-0.13	-0.20	-0.14	-0.24	-0.20
STD	2.094	0.677	0.212	0.202	0.411	0.213	2.252	2.223	2.262	2.205	2.241
t	-0.35	-0.04	-1.84	1.724	-1.38	1.195	-0.34	-0.52	-0.35	-0.62	-0.50

values. Since the more complex strategies are expected to generate smaller residuals, the critical value for rejection of the null hypothesis for any pair is 1.697 at the 0.05 level. Based on the results in Panel 2, we are unable to reject either null. We conclude, therefore, that the more complex duration vector strategies do not outperform simple duration with regard to size of expected residual returns.

Panel 3 of Table 4 summarizes the differences in mean absolute deviation (MAD) of returns for the paired samples. Superior performance of an immunization strategy is indicated by smaller values. The more complex strategies are expected to generate smaller MADS. Therefore, the critical value for rejection of the null hypothesis for any pair is again 1.697 at the 0.05 level. Based on the results in Panel 3, we are unable to reject either null. We conclude, therefore, that the more complex duration vector strategies do not outperform simple duration with regard to the variation of portfolio returns.

Finally, Panel 4 of Table 4 summarizes the differences in return to volatility of the paired strategies. Superior performance is indicated by higher values of this measure. Since more complex strategies are expected to generate higher risk adjusted returns, the critical t-value for rejection of the null hypotheses of no difference in means is -1.697. We are able to reject the null only for the pairing of strategies 3 and 4. We can not reject the null for any pairings of strategy 1 with either of the duration vector strategies. Therefore, the superiority of the more complex duration vector strategies is not supported by the return to variation tests.

Chapter Summary

The composite results of all the tests indicate that the duration vector model does not significantly improve immunization efficacy over simple Macaulay duration matching. Neither of the three CCM conclusions, enumerated in chapter 1, are supported by the results here. It should be noted that the results in this study do not necessarily refute CCM. However, they do suggest that at least a part of the empirical success of their model can be attributed to either their failure to prohibit short selling, or to their failure to employ an optimal objective function in the Macaulay model.

It is important to recall that the immunization error of the simple duration model is primarily due to the fact that unanticipated term structure changes are not limited to parallel shifts. The greatest differences between simple duration and the more complex models should be observed in empirical tests where nonparallel term structure changes occur. With observation periods of only nine months and portfolio rebalancing every three months, this is not likely to be the case. Over most very short segments, the term structure is approximately linear. Therefore, changes will also be approximately linear. Better insight into the incremental value of the more complex duration vector strategies can be gained through empirical tests with longer observation intervals. The Monte Carlo Sampling procedure is employed in the next two chapters to accomplish this.

CHAPTER IV

MONTE CARLO SAMPLING METHODOLOGY

Introduction

The Monte Carlo sampling procedure affords us experimental design advantages that are not available with traditional methods. A major advantage is that we have complete control over sample size. Perhaps equally as important, we can perform as many replications or repeat tests as desired.

To generate the desired samples, it is necessary to specify the return generating process, its stochastic components, the nature of the probability distribution, values of its defining parameters, and initial conditions. Each of these requirements is addressed in this chapter.

Sample Size

Paired sample t-tests are employed in evaluating the statistical hypotheses in this study. This test assumes normality of the underlying distributions of differences in means. Because the distributions of holding period returns from the alternative strategies are not known, we rely on the Central Limit Theorem. It is, therefore, necessary that the sample sizes be large. While the t-test is robust for samples of as little as twenty-five to thirty observations, larger samples are desirable so as to improve the power of the tests.

The procedure in this study tests more complicated and computationally costly strategies against simpler ones. Because of this, it is desirable that the likelihood of rejecting the null hypotheses (i.e., no difference in means) when it is true be minimized. Therefore, all tests are evaluated at the 0.01 significance level. A tradeoff to selecting such a small significance level for the tests is that, ceteris paribus, the probabilities of type II errors are increased. Absent prior knowledge on either population or sample variance, optimal sample size can not be specified. However, as a tradeoff between the time to complete a test and the power of the tests, a sample size of 60 observations is chosen. Samples of size sixty observations on bond prices and yields are generated by the Monte Carlo method. These data are generated at one-year intervals for five year observation periods.

The Return Generating Process

The empirical procedure in this study requires that the return functions from alternative immunization strategies be simulated under assumed distributions of unanticipated term structure changes.

Assume that at time zero, the spot rate function representing the term structure of interest rates is

$$[R_0(0,1), R_0(0,2), R_0(0,3), \ldots, R_0(0,t), \ldots], \text{ where} \quad (1)$$

$R_0(0,t)$ is the spot yield on a single payment security at time s=0, with maturity of t periods. And in general, the term structure at any time s is

$$[R_s(s,1), R_s(s,2), R_s(s,3), \ldots, R_s(s,t), \ldots], \text{ where} \quad (2)$$

$R_s(s,t)$ is the spot yield on a t-period single payment security at time s.

If an investor acquires a portfolio of bonds at time zero with promised income stream

$$[C_0(1), C_0(2), \ldots, C_0(T)], \quad (3)$$

where T is the number periods until the last scheduled payment, the value of this portfolio can be represented as

$$V_0 = \Sigma C_0(t)[1+R_0(0,t)]^{-t} = \Sigma C_0(t)[1+r]^{-t}, \qquad (4)$$

where r is the portfolio yield to maturity.

The spot rate function can be expressed in terms of forward rates as

$$[1+R_0(0,t)]^t=[1+R_0(0,1)][1+r_0(1,t)]^{t-1} \qquad (5)$$

where $r_0(1,t)$ is the forward rate spanning the interval (1,t) at time s=0. Under the pure expectations theory of the term structure, these forward rates represent the time zero unbiased expectations of spot rates at time s=1. If t=2, then $r_0(1,2)$ is the forward rate for the interval (1,2), and is the yield expected to prevail on a 1-year security one period hence. That is,

$$[1+R_0(0,2)]^2=[1+R_0(0,1)][1+r_0(1,2)]$$

The two-year forward rate at time s=0 is

$$[1+R_0(0,3)]^3=[1+R_0(0,1)][1+r_0(1,3)]^2,$$

and $r_0(1,3)$ is the periodic rate expected to prevail one year from now on a 2-year security. The forward rate for any interval (1,t) can be employed to represent the expected t-1 period return at time s=0. And the forward rate function can be expressed in general as

$$[1+R_s(s,t)]^t=[1+R_s(s,s+1)][1+r_s(s+1,t)]^{t-1} \qquad (6)$$

where, $r_s(s+1,t)$ is the t-1 period forward rate for the interval (s+1,t). This forward rate function is employed in this study to represent the t-1 period return at any time s.

Now assume that an immunizer has a planned holding period, H, and invests the amount V_0 at time s=0 to acquire a portfolio with the scheduled cash flow stream. We want to model the terminal value at the end of horizon H. At s=1, the investor must revise the initial portfolio to, at a minimum, reinvest the cash income $C_0(1)$. Assume that the investor completely rebalances the portfolio to maintain the duration constraints by reinvesting all accumulated wealth, V_1. The new scheduled income stream becomes

$$[C_1(2), C_1(3), C_1(4), \ldots, C_1(t), \ldots] \quad (7)$$

The value of this stream, at time s=1 is

$$V_1 = \Sigma C_1(t)[1+R_1(1,t)]^{-(t-1)}. \quad (8)$$

This grows to $C_1(2) + \Sigma C_1(t)[1+R_2(2,t)]^{-(t-2)}$ at time s=2.

After the portfolio revision at time s=2, value becomes

$$V_2 = \Sigma C_2(t)[1+R_2(2,t)]^{-(t-2)}. \quad (9)$$

In general the values at any time s before and after rebalancing are respectively,

$$V_s = C_{s-1}(s) + \Sigma C_{s-1}[1+R_s(s,t)]^{-(t-s)}, \text{ and} \quad (10)$$

$$V_s = \Sigma C_s(t)[1+R_s(s,t)]^{-(t-s)}. \quad (11)$$

Terminal value at time s=H is

$$V_H = C_{H-1}(H) + \Sigma C_{H-1}(t)[1+R_H(H,t)]^{-(t-H)}. \quad (12)$$

The Simulation Model

To specify the process to be simulated, it is necessary to express the spot rate function, $R_s(s,t)$, in terms of its implied forward rates, $r_s(s+1,t)$, and an unanticipated rate change, $\epsilon_{s+1}(s+1,t)$. Under the expectations theory, the unanticipated rate changes can be defined as the set of values,

$$[R_{s+1}(s+1,t) - r_s(s+1,t)] = \epsilon_{s+1}(s+1,t). \quad (13)$$

The stochastic component of the terminal value function to be simulated is

$$R_{s+1}(s+1,t) = r_s(s+1,t) + \epsilon_{s+1}(s+1,t), \qquad (14)$$

where, $r_s(s+1,t)$ is derived from the spot rate function at time s.

The random values, $\epsilon_{s+1}(s+1,t)$, are generated in this research using pseudo random numbers.

The computer package, @Risk, is used to generate random values of unanticipated term structure changes. The package requires that the type of distribution of the stochastic variable be specified. Since there is no direct information available on the distribution of the variable, there appears to be two reasonable alternative approaches. We can generate samples from a variety of known distributions and hope that at least one of them approximates the distribution of the stochastic variable, or we can select a specific distribution that is a reasonable representation of term structure innovations. Knowledge of the term structure and the generally accepted role of expectations in its determination leads us, initially, to select the latter alternative.

In the return generating model above, we have attached informational content to the forward rate structure. By construction, we implicitly assume that the expected value of unanticipated rate changes is zero. We also assume that realizations of positive and negative values of unanticipated changes, $\epsilon_{s+1}(s+1,t)$, are equally likely, and that values closer to zero occur with greater probability than do more extreme ones. It follows that the normal probability distribution with mean zero is a good representation of the process. A normal distribution with zero mean is initially selected for the generation of samples.

In addition to the normal distribution and zero mean specifications, the @Risk package requires that the variance be specified also. A reasonable estimate of the variance of the distribution is difficult. Fortunately, exact specification of variances is not a requisite for the validity of the tests in this study. The primary requirement is that the alternative strategies be investigated under conditions that are at least as extreme as are likely to be encountered in practice. Studies frequently assume annual term structure changes of 300 to 500 basis points over the entire holding period. Sufficiently small standard deviations are chosen here, given the initial yield structures, to avoid the generation of negative rates. Ultimately, several different data sets with different distribution assumptions are generated and evaluated in this research.

The initial Monte Carlo sample of size 60 is generated under the assumption that the unanticipated yield structure change for each maturity, denominated in years, is independent and distributed N(0,0.01). As is shown later, this specification results in sample yields of extreme volatility across maturities. Repeat tests are performed with distribution assumptions that result in yield structures that are less volatile and more representative of commonly observed reality. This is accomplished by including parameters to reflect correlations of term structure changes across maturities in the @Risk functions.

It is important to note that the initial assumption of independence above applies only to unanticipated term structure changes. Because term structures are observed in practice to take on a limited number of well defined shapes, there is clearly some correlations among changes across instruments of different maturities. However, the initial assumption is that these interdependencies are accounted for in the forward rate structures. As it turns out, however, samples yield structures generated under this assumption bear little resemblance to any normally observed in practice. Subsequent samples, generated with correlation constraints imposed across maturities, are more realistic.

Sample Generation Procedure

To initialize the simulation process, actual price and yield data on U.S. Treasury bonds with annual maturities of one to thirty years are taken from the Wall Street Journal. Hypothetical data are substituted where there are missing maturities. The following steps are executed to generate the sampling distributions of the measurement variables:

1. Starting at time s=0, select initial portfolio for each alternative model. Each portfolio will have its own promised cash flow stream, $[C_0(1), C_0(2), C_0(3), \ldots, C_0(t), \ldots]$.

2. Compute the forward rate structure, $r_0(1,t)$ from the initial yield structure. $t=2,3,4,\ldots$,(each t represents a one-year period).

3. Generate random values of $\epsilon_1(1,t)$. $t=2,3,4,\ldots$

4. Compute $R_1(1,t)=r_0(1,t)+\epsilon_1(1,t)$ (the new spot rate function).

5. Compute $V_1=C_0(1)+\Sigma C_0(t)[1+R_1(1,t)]^{-(t-1)}$.

6. Rebalance each portfolio according to its immunization rule to get

   $V_1= C_0(t)[1+R_1(1,t)]^{-(t-1)}$.

7. Compute forward rates, $r_1(2,t)$, $t=3,4,5,\ldots$, from $R_1(1,t)$ generated in step 4 above.

8. Generate random values, $\epsilon_2(2,t)$, $t=3,4,5,\ldots$.

9. Compute $R_2(2,t)=r_1(2,t)+\epsilon_2(2,t)$ (new spot rate function).

10. Compute $V_2=C_1(2)+\Sigma C_1(t)[1+R_2(2,t)]^{-(t-2)}$.

11. Rebalance each portfolio to get $V_2 = \Sigma C_2(t)[1+R_2(2,t)]^{-(t-2)}$.

12. Repeat steps 7-11 recursively for each rebalancing date, s=3 through s=H-1.

13. Terminate at s=H by repeating steps 7 through 10. At time s=H, terminal value is $V_H=C_{H-1}(H)+ \Sigma C_{H-1}(t)[1+R_H(H,t)]^{-(t-H)}$. These steps constitute a simulation run from which a single observation is derived.

14. For each alternative strategy, record V_H and compute

$Q=V_H-V_{target}$.

15. Perform 60 simulation runs from the initial conditions to get samples for each strategy.

16. Change distribution assumptions and repeat simulation to get a second sample of 60 observations.

17. Repeat step 16 under different distribution assumptions or initial conditions.

18. Derive summary statistics representing the measurement variables for both V_H and Q from each sample.

19. Use these values to perform statistical tests of the alternative strategies.

For computational convenience, we assume an initial par yield curve such that all coupons, $C_0(t)$, occur at discrete six-month intervals, and that all maturities occur at one-year intervals. This assumption conveniently limits the rebalancing requirements to uniform one-year periods.

Actual sample data, analysis, and results are included in the next chapter.

CHAPTER V

MONTE CARLO TESTS OF THE CCM DURATION VECTOR

Introduction

The purpose of this chapter is to evaluate the hypotheses regarding the CCM duration vector models versus the Macaulay duration matching model selected with the min M^2 objective function. We reiterate the limitations of extant tests that utilize actual data. All previous empirical tests of immunization strategies suffer from one or more of the following problems: (1) Overlapping observation periods of portfolio returns which violate independence requirements of statistical tests, (2) very small samples of nonoverlapping observation periods that, when coupled with the unknown nature of the underlying population distribution, severely weakens statistical tests, and (3) short observation periods that are inconsistent with the original purpose of immunization models. The procedures employed in this chapter should provide much more useful information on the alternative strategies evaluated and on the general subject of bond portfolio immunization efficacy.

The Alternative Strategies

The strategies to be evaluated were described in Chapter III. The optimization models representing those strategies are repeated here for readers' convenience.

Strategy 1	Minimize	ΣM_i^2,
	Subject to	$\Sigma y_i Dur_i = h$,
		$\Sigma y_i = 1,\ y_i \geq 0$
Strategy 2	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1) = h-1$,
		$\Sigma y_i D_i(2) = (h-1)^2$,
		$\Sigma y_i = 1,\ y_i \geq 0$

Strategy 3	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i=1,\ y_i \geq 0$
Strategy 4	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i=1,\ y_i \geq 0$
Strategy 5	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i D_i(5)=(h-1)^5$,
		$\Sigma y_i=1,\ y_i \geq 0$
Strategy 6	Minimize	Σy_i^2,
	Subject to	$\Sigma y_i D_i(1)=h-1$,
		$\Sigma y_i D_i(2)=(h-1)^2$,
		$\Sigma y_i D_i(3)=(h-1)^3$,
		$\Sigma y_i D_i(4)=(h-1)^4$,
		$\Sigma y_i D_i(5)=(h-1)^5$,
		$\Sigma y_i D_i(6)=(h-1)^6$,
		$\Sigma y_i=1,\ y_i \geq 0$

Strategy 7 Minimize Σy_i^2,

Subject to $\Sigma y_i D_i(1)=h-1$,

$\Sigma y_i D_i(2)=(h-1)^2$,

$\Sigma y_i D_i(3)=(h-1)^3$,

$\Sigma y_i D_i(4)=(h-1)^4$,

$\Sigma y_i D_i(5)=(h-1)^5$,

$\Sigma y_i D_i(6)=(h-1)^6$,

$\Sigma y_i D_i(7)=(h-1)^7$,

$\Sigma y_i=1, \; y_i\geq 0$

Portfolios are selected from the universe of available bonds by solving these models for the optimal weights.

Difference in Means Tests

The Monte Carlo procedures described in Chapter IV generates paired samples of size 60, for the five-year planning horizon, from which sample estimates of the following means are computed on each of the seven alternative strategies:

Variable	Description
μ(HPR)	Mean holding period Return
μ(Q)	Mean residual holding period return
MAD(HPR)	Mean absolute deviation of HPR
MAD(Q)	Mean absolute deviation of residual HPR
μ(R/V)	Mean HPR divided by the standard deviation of residual HPR

The statistical tests follow the same procedure as described in Chapter III. Differences in means are evaluated using one tailed paired sample t-tests, which are as previously described. Recall that the subscripts i and j are used to denote strategies being compared. In all matched pairs, i subscripts represent the strategy with fewer constraints.

The following hypotheses are evaluated at the 0.01 significance level.

H_0: MAD(Q_i) - MAD(Q_j) = 0 (i=1,...,6); (j=2,...,7)

H_1: MAD(Q_i) - MAD(Q_j) > 0

H_0: MAD(Q_1) - MAD(Q_j) = 0 (j=3,...,7)

H_1: MAD(Q_1) - MAD(Q_j) > 0

H_0: $\mu(TV_i) - \mu(TV_j) = 0$ (i=1,...,6); (j=2,...,7)

H_1: $\mu(TV_i) - \mu(TV_j) < 0$

H_0: $\mu(TV_1) - \mu(TV_j) = 0$ (j=3,...,7)

H_1: $\mu(TV_1) - \mu(TV_j) < 0$

H_0: MAD(TV_i) - MAD(TV_j) = 0 (i=1,...,6); (j=2,...,7)

H_1: MAD(TV_i) - MAD(TV_j) > 0

H_0: MAD(TV_1) - MAD(TV_j) = 0 (j=3,...,7)

H_1: MAD(TV_1) - MAD(TV_j) > 0

H_0: $\mu(R/V_i) - \mu(R/V_j) = 0$ (i=1,...,6); (j=2,...,7)

H_1: $\mu(R/V_i) - \mu(R/V_j) < 0$

H_0: $\mu(R/V_1) - \mu(R/V_j) = 0$ (j=3,...,7)

H_1: $\mu(R/V_1) - \mu(R/V_j) < 0$

This format results in fifty five hypotheses to be tested (eleven for each summary statistic) on each sample. Superior performance by the more complex strategy, is indicated by the direction of the inequality sign in each alternate hypothesis. Rejection of the null hypothesis in either test would support the conclusion that the more complex strategy outperforms the simpler one to which it is compared. Conversely, failure to reject either null would indicate no apparent advantage to the more complex strategy. Strongest support for the CCM (1988) conclusions would be indicated by rejection of each null hypothesis in all of the tests.

Difference in Proportions Tests

It might be argued that the most critical indicator of immunization efficacy is the frequency by which a particular strategy generates returns less than promised returns. To evaluate this variable, we construct a test of the difference in the proportions of returns below the target value. To derive a large number of samples from which differences in proportions are calculated, the samples described in Chapter IV above are combined to get a grand total of 240 observations on holding period returns. From this total, 20 samples of size 12 are selected by randomly assigning observations.

The percentage of returns below the target is computed on each alternative strategy from each sample. This statistic is denoted as p. A number of differences in proportions equal the number of samples are computed for each matched pair of strategies. Pairwise tests of the difference in proportions are performed using the t-statistic. Under the null hypothesis of no difference in proportions, the test statistic is $t = d\sqrt{n'} / S_d$ with n'-1 degrees of freedom. Where,

n' is the number of samples from which matched pair proportions are derived,

$d = \Sigma d_i / n'$ is the mean of the matched pair samples differences in proportions,

$d_i = p_{i1} - p_{i2}$ is the difference in proportion for the matched pair from sample i (i=1,...,n'),

p_i=[the number of observed values below target return]/12,

$S_d = \sqrt{[E(d_i^2) - n'd^2 / n'-1]}$ is the standard deviation of the matched pairs differences.

We rely on the central limit theorem (the normal approximation to the binomial distribution) and the similarity between the normal and the t sampling distributions for large samples as justification for this test. The statistical hypotheses are summarized below.

H_0: $\mu_i - \mu_j = 0$ (i=1,...,6); (j=2,...,7); (iØj)

H_1: $\mu_i - \mu_j > 0$

H_0: μ_1-μ_j = 0 (j=3,...,7)

H_1: μ_1-μ_j > 0

Again, rejection of the null in either case would indicate that the more complicated strategy outperforms the simpler one to which it is compared. Failure to reject either null would be an indication of no advantage for the more complex strategy. If all of the CCM (1988) conclusions are to be supported, each null hypothesis should be rejected.

The Test Procedures

The following series of steps are carried out to generate observations on the variables of interest:

1. The software package, @Risk, is used to generate 60 sample observations of prices and yields on each of thirty bonds at the ends of years 1-5. These represent maturities of one year through thirty years.[6]

2. Duration, M^2, and D(1)-D(7) are computed for each bond included in the initial sample at time s=0. This results in computation of 270 values (30 bonds X 9 measures).

3. These values are computed again from the Monte Carlo prices and yields at times s=1 through s=5. Because a one year bond matures at each time s, the size of the initial array declines by one to a total of 25 bonds at time s=5. A total of 15660 values are computed at time s=1 (29 bonds X 9 measures X 60 observations). This number declines to 13500 values at time s=5.

4. At time s=0, initial portfolios are selected for each of the alternative strategies by solving the optimization models with the GAMS software. This yields seven initial portfolios. To maintain a constant number of bonds from which portfolios are constructed, only the first twenty five bonds are included at each time s.

[6]@Risk is a risk analysis and modeling add-in for spreadsheets such as Lotus 123. It is copyrighted by the Palisade Corporation, Newfield, New York.

5. At one year intervals at times s=1 through s=4, the portfolios are rebalanced to maintain the horizon constraints by solving the seven updated optimization models for each of the sixty observations.

6. The time s=1 investment value is computed for each strategy on each of the sixty sample observations. It is assumed that the initial investment is $100.

7. The investment value computations are repeated for times s=2 through s=5. The s=5 value is terminal portfolio value and represents a datum on each alternative strategy. There are sixty such values for each alternative strategy. These data are then used to complete the tests outlined above.

Data Analysis and Results

Data Analysis for Sample 1

The initial sample of bond prices and yields was taken from the November 16, 1981, issue of the Wall Street Journal. A single bond for each year from 1982 through 2011 that matures in November is included. For years where there are no bonds maturing in November, bonds maturing in the next closest month is used. for purposes of initiating the Monte Carlo procedure, all bonds are assumed to be selling at par value. Therefore, only the yields on the selected bonds are taken from the Journal. For any years where there are no maturing bonds, hypothetical data are inserted.

The Monte Carlo procedure described in Chapter IV, with the maturity independent and uncorrelated N(0,.01) distribution, is implemented to generate price and yield data from which observations on the variables of interest are derived for H=5 years. The target holding period yield is 12.5 percent, and the target terminal portfolio value, assuming an initial investment of $100, is $183.35. For the samples of size 60, the critical value of t (df=59) for rejecting the null hypothesis is 2.39 at the 0.01 significance level.

Terminal values resulting from the portfolio simulations are presented in Table 5 for

TABLE 5

Sample 1 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	200.85	190.08	192.14	191.33	195.54	193.77	194.34
2	204.03	190.76	192.87	193.36	196.21	193.83	194.29
3	166.86	179.87	191.18	188.35	183.81	182.81	185.09
4	186.44	188.96	182.39	183.24	187.14	186.58	188.12
5	211.20	190.30	191.65	192.06	194.81	195.04	195.08
6	182.43	185.73	174.48	175.19	182.57	181.38	182.39
7	188.75	190.86	175.01	177.44	189.08	189.67	188.10
8	198.37	188.09	185.55	185.98	187.33	187.59	187.55
9	200.85	190.20	185.62	187.51	189.56	190.82	188.49
10	241.20	189.42	200.84	201.39	189.07	189.33	189.97
11	201.27	188.24	186.84	186.83	189.69	189.71	186.99
12	198.90	187.49	189.71	190.37	187.79	188.62	187.12
13	182.48	186.52	185.95	185.75	189.80	189.61	191.47
14	193.95	189.78	185.12	186.36	191.11	190.51	192.04
15	171.74	179.24	188.95	187.04	179.97	180.62	180.05
16	176.22	188.94	167.91	169.88	185.63	185.66	184.82
17	164.48	182.49	178.67	178.72	181.60	182.00	180.94
18	148.93	182.47	181.84	180.45	190.60	190.37	184.12
19	173.70	185.67	178.40	178.84	186.31	187.01	185.52
20	195.56	190.50	179.67	179.25	193.47	191.28	195.35
21	161.96	182.17	176.70	176.26	180.26	180.34	179.87
22	199.78	185.43	189.25	190.98	184.41	185.43	183.21
23	205.27	182.22	197.06	195.84	183.21	183.46	182.65
24	180.72	182.04	183.64	182.72	180.24	179.33	180.16
25	169.07	180.50	182.13	182.13	177.38	178.25	176.69
26	182.00	185.01	177.98	177.82	184.32	184.12	183.99
27	192.16	190.10	171.95	173.66	187.66	186.88	188.05
28	174.84	190.50	172.31	174.16	189.41	189.44	189.34
29	187.04	184.17	191.31	190.75	185.34	185.98	184.78
30	185.52	184.30	192.29	191.75	187.90	186.40	187.32
31	185.99	182.40	189.96	188.94	182.05	182.06	183.31
32	175.31	180.02	174.86	172.89	179.98	179.09	180.13
33	215.60	191.86	180.97	183.66	187.51	187.85	186.66
34	181.97	184.39	174.23	173.02	182.24	180.72	181.70
35	180.50	181.57	178.33	179.32	177.09	177.95	177.16
36	175.56	184.70	184.93	184.94	186.28	186.08	186.87
37	205.82	185.82	188.78	189.69	183.95	184.38	184.25
38	223.17	191.14	190.50	191.23	191.26	190.05	193.42
39	232.57	192.02	188.55	191.24	188.96	189.95	189.06
40	177.84	180.46	176.64	175.87	178.17	178.54	176.71
41	174.16	184.20	179.23	180.41	182.57	183.96	180.83
42	200.38	190.41	179.81	180.58	188.92	188.91	189.02
43	166.60	182.24	182.63	181.04	180.74	184.24	183.83
44	205.36	194.80	183.63	186.78	195.49	195.73	197.13
45	194.27	185.06	186.46	185.95	183.86	183.65	184.88
46	176.85	181.72	183.84	180.69	183.29	181.83	184.74
47	178.88	184.88	183.10	183.32	186.16	185.32	186.25
48	181.85	185.51	177.60	178.68	183.47	184.32	183.02
49	171.92	183.16	185.50	185.58	183.89	184.41	183.88
50	176.06	179.80	183.33	183.02	177.99	178.16	178.07
51	210.46	188.66	187.44	188.13	188.81	188.42	189.49
52	159.18	180.98	192.71	186.63	191.19	188.61	192.25
53	196.86	184.45	185.09	184.96	180.50	180.44	180.01
54	215.07	190.06	183.80	184.71	187.33	186.36	187.83
55	193.06	187.20	193.16	192.73	189.67	189.48	190.26
56	185.90	186.24	189.50	189.08	188.49	188.26	190.08
57	178.02	184.91	178.90	181.03	181.56	183.07	179.86
58	166.64	181.97	190.30	189.07	185.45	185.28	185.68
59	176.46	181.78	195.80	195.39	181.63	182.16	181.80
60	199.12	188.29	181.03	182.08	185.36	186.82	185.15

the alternative strategies[7]. Summary statistics for the four difference in means tests are presented in Table 6. Underlined t values in this table denote significance at the 0.01 level.

Panel 1 of Table 6 summarizes statistics on the difference in mean terminal values for each pairing of strategies. To reiterate, relative immunization efficacy does not necessarily depend on the magnitude of terminal portfolio values, but higher values are preferred, ceteris paribus. Therefore, superior performance of a strategy over the one to which it is compared is indicated by higher terminal value. The critical t value for rejection of the null hypothesis is -2.39 at the 0.01 significance level. We are unable to reject the null for any pairing of strategies in this test. We conclude that the duration vector strategies do not outperform simple Macaulay duration matching with respect to expected holding period return.

Panel 2 of Table 6 summarizes the differences in mean residual returns for the paired samples. Superior performance of an alternative strategy is indicated by smaller residual values. Since the more complex strategies are expected to generate smaller residuals, the critical t-value for rejecting the null hypothesis for any pair is +2.39 at the 0.01 level. Based on the results in Panel 2, we are, again, unable to reject either null. We conclude, therefore, that the more complex duration vector strategies do not outperform simple duration with regard to size of expected residual holding period returns. Neither of the three CCM conclusions is supported by this test.

Panel 3 of Table 6 summarizes the difference in mean absolute deviation (MAD) of returns for the paired samples. This measure is employed to evaluate the volatility of the returns from each strategy. Superior performance of an immunization strategy is indicated by smaller values representing less risk. The more complex strategies are expected to generate smaller MADs. Therefore the critical value for rejecting the null hypothesis for

[7]Values for residual returns, absolute deviation of returns, and return to volatility measures from Sample 1 are included in Appendix III.

TABLE 6
Sample 1 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	2.253	1.545	–0.10	–1.48	0.052	0.011	3.799	3.698	2.215	2.267	2.279
STD	15.42	7.921	1.539	6.52	1.041	1.776	16.59	15.62	16.31	16.26	16.08
t	1.131	1.511	–0.50	–1.76	0.389	0.050	1.773	1.833	1.052	1.080	1.097

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	2.253	1.545	–0.10	–1.48	0.052	0.011	3.799	3.698	2.215	2.267	2.279
STD	15.42	7.921	1.539	6.52	1.041	1.776	16.59	15.62	16.31	16.26	16.08
t	1.131	1.511	–0.50	–1.76	0.389	0.050	1.773	1.833	1.052	1.080	1.097

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	11.12	–2.21	0.206	1.460	0.263	–0.36	8.913	9.12	10.58	10.84	10.47
STD	9.947	7.921	1.539	6.52	1.041	1.776	16.59	15.62	16.31	16.26	16.08
t	8.665	–3.93	1.075	2.549	2.074	–1.82	6.328	6.623	7.749	7.936	7.580

PANEL 4: Difference in Return to Volatility

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	–0.43	0.590	–0.02	–0.46	–0.03	0.068	0.152	0.126	–0.33	–0.37	–0.30
STD	0.915	1.731	0.279	1.389	0.269	0.454	1.380	1.217	1.274	1.247	1.245
t	–3.70	2.643	–0.71	–2.59	–0.99	1.163	0.857	0.807	–2.05	–2.31	–1.89

any pair is again +2.39 at the 0.01 level. Based on the results in Panel 3, the null is rejected for the pairing of strategy 1 with each of the strategies 2 through 7. For the pairings of duration vectors of differing lengths, the null is rejected only for strategy 4 minus strategy 5. A fair interpretation of these results is that (1) each of the duration vector strategies result in smaller holding period returns than does simple duration. They, therefore, result in superior performance when measured in this manner; (2) there does not appear to be substantial differences between successive pairs of duration vector strategies.

The results of this test provide strong support for the CCM conclusion that the duration vector strategies outperform simple Macaulay duration with regard to the variation of holding period returns. However, it does not support the conclusion that immunization performance improves incrementally with the order of the vector employed, or that a duration vector of 5 to 7 terms is optimal. It appears that, at least on this measure of performance, a two term duration vector is optimal.

Finally, panel 4 of Table 6 summarizes the differences in return to volatility of the paired strategies. Superior performance is indicated by higher values of this measure. Since more complex strategies are expected to generate higher risk adjusted returns, the critical value of t for rejection of the null hypothesis of no difference in means is -2.39 at the 0.01 level. The null is rejected for the pairings of strategies 1 and 2, and of strategies 3 and 4. We can not reject the null for any pairings of strategy 1 with either of the duration vector strategies 3 through 7. The superiority of the two term duration vector model is supported by the return to volatility tests. However, the superiority of the more complex duration vector strategies is not.

The composite results of the four tests from Monte Carlo Sample 1 provide moderate support for the CCM conclusion 1; that a duration vector strategy employing D1 and D2 outperforms Macaulay duration matching. However, the CCM conclusions 2 and 3 are not supported.

The inferences to be drawn from these results are that: (1) as a risk minimizing (minimax) strategy, either of the duration vector strategies probably outperform duration matching with min M^2. This result supports the CCM (1988) conclusions regarding the superiority of the duration vector model. (2) There does not appear to be any marginal risk reduction advantage of duration vectors above two terms. This result is contrary to the CCM conclusions. (3) A two term duration vector appears to be optimal. This is contrary to the CCM conclusion that a five to seven term duration vector is optimal.

Data Analysis for Sample 2

This second Monte Carlo sample was generated under the following set of assumptions concerning the distribution of unanticipated term structure changes:

1. The distribution is N(0,0.025) for maturities of two years or less.
2. The distribution is N(0,0.01) for maturities between two and five years.
3. The distribution is N(0,0.005) for maturities greater than five years.
4. The unanticipated term structure changes are perfectly correlated across maturities at each observation date.

It is noted that the perfect correlation assumption here, like the independence assumption for sample 1, is not an assertion about the true nature of unanticipated term structure changes. Neither assumption is likely to precisely reflect actual term structure innovations. Because lending and borrowing in different maturity ranges are, to some extent, substitutes for each other, there is likely to be some degree of positive correlation across maturities. Less than perfect positive correlations are assumed in generating samples 3 and 4.

Price and yield data for a sample of size 60 was generated under the assumptions above. The initial bond data is the same as for sample 1. Again, the target holding period return is 12.5 percent, and the target horizon value is \$183.35. Terminal values for the

Sample 2 portfolio simulations are presented in Table 7.[8] Summary statistics on the difference in means data are presented in Table 8. Underlined t values denote significance at the 0.01 level.

Panel 1 of Table 8 summarizes statistics on the difference in mean terminal values. Superior performance of a strategy over the one with which it is paired is indicated by a greater terminal value. The critical value of the t-statistic for rejection of the null hypothesis is -2.39 at the 0.01 significance level. We reject the null for all pairings of strategy 1 with the duration vectors 2 through 6. The null is not rejected for the pairing of strategy 1 and strategy 7. For the pairings of duration vector strategies,

we reject the null hypothesis of no difference in means for the pairs 3-4 and 5-6. The results from this sample suggest that the duration vector model outperforms simple duration with regard to expected holding period portfolio return. This supports the first CCM conclusion. There is not significant support for CCM conclusions two and three.

Panel 2 of Table 8 summarizes the differences in mean residual returns for the paired samples. Superior performance of a strategy is indicated by smaller values. Since the more complex strategies are expected to generate smaller residuals, the critical value of t for rejecting the null hypothesis in any pairing is +2.39 at the 0.01 significance level. Based on the results in Panel 2, we are unable to reject either null in the pairings of strategy 1 with the duration vectors. The null is rejected for the duration vector pairs 2-3, 4-5, and 6-7. We conclude that the more complex duration vector strategies do not outperform simple duration with regard to size of expected residual holding period returns. Neither of the CCM conclusions is supported by this test.

Panel 3 of Table 8 summarizes the difference in mean absolute deviation (MAD) of returns for the paired samples. Superior performance of a strategy is indicated by smaller

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[8]Sample 2 values of residual returns, absolute deviations, and return to volatility measures are provided in Appendix IV.

TABLE 7

Sample 2 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	192.88	191.18	189.31	190.54	188.20	191.88	191.07
2	184.89	186.68	184.17	184.56	185.26	187.80	182.98
3	184.14	186.59	183.86	184.52	183.03	187.56	179.18
4	191.32	189.36	187.00	189.19	186.36	189.26	186.64
5	192.50	187.46	186.56	187.06	186.16	189.18	184.62
6	180.16	183.55	181.34	181.59	182.01	187.91	183.91
7	169.28	180.07	175.68	177.20	174.27	176.54	174.20
8	189.37	188.17	185.33	186.35	183.18	186.59	183.89
9	181.18	183.98	183.34	183.93	184.34	184.94	182.32
10	176.35	182.73	178.11	179.96	178.81	179.55	178.05
11	186.65	186.56	183.98	184.75	183.42	185.97	182.08
12	190.36	189.23	185.08	188.13	184.59	189.10	186.41
13	170.47	181.29	178.49	178.01	177.04	179.24	176.07
14	178.07	183.59	183.64	183.96	180.73	185.01	181.53
15	190.82	190.09	187.95	189.35	187.47	188.67	184.07
16	174.14	182.36	179.70	179.66	178.51	180.80	175.39
17	185.38	187.15	184.87	185.56	182.42	186.66	179.78
18	176.25	181.42	180.75	179.22	179.64	181.94	178.21
19	173.27	182.38	179.62	179.48	178.42	180.62	176.18
20	188.01	186.46	185.80	187.22	185.30	188.00	184.91
21	181.00	184.38	181.46	182.87	180.82	182.98	178.73
22	167.89	180.51	178.20	178.96	178.22	178.93	175.37
23	185.51	183.78	181.03	185.04	182.38	183.64	181.09
24	187.32	187.76	184.50	188.46	184.73	186.88	181.06
25	181.98	185.91	183.23	185.36	182.14	184.51	177.87
26	181.45	186.10	183.20	185.82	184.70	184.30	181.13
27	178.29	185.10	184.96	185.44	182.49	185.98	184.19
28	204.30	196.41	195.52	199.25	198.54	198.94	195.01
29	163.38	176.38	176.08	177.46	176.33	177.81	174.51
30	189.82	189.71	188.80	191.35	189.87	192.34	188.53
31	187.89	189.64	189.65	192.13	189.70	191.82	189.50
32	174.50	181.30	182.37	184.20	182.31	183.64	182.53
33	184.53	187.29	186.65	187.54	187.25	190.27	186.53
34	169.03	180.50	180.82	181.16	178.93	181.76	176.30
35	176.05	183.39	183.07	188.26	183.50	185.30	182.49
36	183.37	186.72	186.41	187.40	185.41	190.05	185.93
37	190.47	190.64	190.60	193.21	190.65	195.83	189.46
38	180.37	186.22	186.85	187.82	186.46	189.45	184.51
39	174.06	184.12	183.58	185.00	183.13	185.72	181.03
40	185.66	190.22	189.59	193.06	190.41	195.59	190.19
41	183.22	188.79	190.08	192.05	189.78	191.26	187.95
42	181.96	187.87	180.90	191.40	188.38	192.35	186.50
43	178.69	185.19	184.87	187.69	183.12	188.22	183.78
44	177.92	185.94	185.92	188.38	185.95	192.98	137.38
45	189.81	191.00	192.53	195.30	193.06	196.14	124.75
46	173.37	184.57	185.77	183.69	185.70	187.97	182.55
47	179.18	176.00	177.77	177.40	175.09	178.52	176.12
48	180.89	187.77	189.19	188.40	187.99	190.98	188.86
49	168.14	180.12	179.41	180.29	179.01	181.84	177.77
50	187.17	190.84	190.23	193.91	191.87	196.40	189.01
51	185.59	189.03	190.22	191.92	189.14	194.29	186.67
52	182.39	187.57	189.51	191.15	188.38	191.43	186.32
53	181.40	186.76	186.57	188.46	186.58	188.63	184.69
54	184.11	186.07	184.88	184.14	184.82	186.50	183.14
55	191.86	190.17	187.52	190.55	188.65	193.51	187.98
56	188.44	188.22	184.86	185.20	186.59	189.68	187.01
57	175.87	182.82	180.56	178.56	179.54	183.22	179.25
58	178.56	183.85	181.56	179.69	181.17	182.60	178.30
59	182.99	185.40	182.69	183.96	182.60	186.82	181.37
60	174.05	177.62	175.26	175.42	174.94	176.03	173.14

TABLE 8
Sample 2 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	–3.89	1.341	–1.43	1.634	–2.88	6.039	–2.55	–3.99	–2.35	–5.23	0.8
STD	4.547	1.781	1.928	1.449	1.471	10.97	5.037	4.873	4.850	4.943	11.07
t	–6.64	5.832	–5.76	8.733	–15.1	4.264	–3.93	–6.34	–3.76	–8.20	0.559

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	–3.89	1.341	–1.43	1.634	–2.88	6.039	–2.55	–3.99	–2.35	–5.23	0.8
STD	4.547	1.781	1.928	1.449	1.471	10.97	5.037	4.873	4.850	4.943	11.07
t	–6.64	5.832	–5.76	8.733	–15.1	4.264	–3.93	–6.34	–3.76	–8.20	0.559

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	2.904	–0.35	–0.69	0.332	–0.43	–1.57	2.548	1.852	2.185	1.754	0.183
STD	3.363	1.507	1.180	1.096	1.252	8.011	3.929	4.007	3.851	4.077	9.291
t	6.690	–1.83	–4.56	2.349	–2.66	–1.51	5.023	3.581	4.395	3.331	0.153

PANEL 4: DIFFERENCE IN RETURN TO VOLATILITY

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	–0.61	0.383	–0.42	0.436	–0.73	1.114	–0.22	–0.65	–0.22	–0.95	0.160
STD	0.622	0.535	0.465	0.357	0.348	2.030	0.849	0.851	0.835	0.872	1.898
t	–7.61	5.550	–7.12	9.446	–16.2	4.250	–2.08	–5.97	–2.04	–8.46	0.654

values reflecting less risk. The more complex strategies are expected to generate smaller MADs. Therefore, the critical value for rejecting the null hypothesis for any pair is again +2.39 at the 0.01 level. Based on the results in Panel 3, we reject the null hypothesis of no difference in means between strategy 1 and strategies 2 through 6. For pairings of duration vectors of different lengths, the null is rejected only for strategy 4 minus strategy 5. The results from this sample support the conclusion that the duration vector model outperforms simple duration matching in minimizing variability of returns. There is, however, no substantial support for strategies employing more than two terms, D1 and D2.

Panel 4 of Table 8 summarizes the differences in return to volatility of the paired strategies. Superior performance is indicated by higher values of this measure. Since more complex strategies are expected to generate higher risk adjusted returns, the critical t-value for rejecting the null hypothesis is -2.39 at the 0.01 level. The null is rejected for the pairings of strategies 1 and 2, 1 and 4, and 1 and 6. Consistent with these results, the null is rejected for the pairs of strategies 3 minus 4 and 5 minus 6. The superiority of the two term duration vector model is supported by the return to volatility tests.

The composite results of the four tests from Monte Carlo Sample 2, like sample 1, provide moderate support for the conclusion that a duration vector strategy employing D1 and D2 outperforms Macaulay duration matching. However, the CCM conclusions 2 and 3, once again, are not supported.

Data Analysis for Sample 3

The third Monte Carlo sample was generated under the following set of assumptions concerning the distribution of unanticipated term structure changes:

1. The distribution is N(0,0.025) for maturities of two years or less.

2. The distribution is N(0,0.01) for maturities between two and five years.

3. The distribution is N(0,0.005) for maturities greater than five years.

4. The unanticipated term structure changes are partially correlated across maturities

with a 0.80 correlation coefficient.

Price and yield data for a sample of size 60 was generated under the assumptions above. The initial bond data is similar to that of samples 1 and 2. An important exception is that the target holding period return is 12.8 percent, and the target horizon value is $185.96. Terminal values for the Sample 3 portfolio simulations are presented in Table 9.[9] Summary statistics on the difference in means data are presented in Table 10. Underlined t values denote significance at the 0.01 level.

Panel 1 of Table 10 summarizes statistics on the difference in mean terminal values for sample 3. The critical value of the t-statistic for rejection of the null hypothesis is -2.39 at the 0.01 significance level. We reject the null for all pairings of strategy 1 with duration vectors 2 through 7. For the pairings of duration vector strategies, we reject the null for strategies 2 and 3 only. The results from this sample suggest that the duration vector model outperforms simple duration with regard to expected holding period portfolio return. This supports the first CCM conclusion. There is not significant support for CCM conclusions two and three.

Panel 2 of Table 10 summarizes the differences in mean residual returns for the paired samples. Superior performance of a strategy is indicated by smaller values. Since the more complex strategies are expected to generate smaller residuals, the critical value for rejection of the null hypothesis for any pair is +2.39 at the 0.01 level. Based on the results in Panel 2, the null is rejected only for duration vector pairs 3-4, 5-6, and 6-7. We are unable to reject the null in the pairing of strategy 1 with either duration vector. We conclude, therefore, that the more complex duration vector strategies do not outperform simple Macaulay duration with regard to size of expected residual holding period returns. Neither of the three CCM (1988) hypotheses is supported by the results of this test.

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[9]Sample 3 values of residual returns, absolute deviations, and return to volatility are provided in Appendix V.

TABLE 9

Sample 3 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	181.23	186.67	187.47	186.93	186.12	185.39	184.42
2	181.15	187.02	188.18	187.87	186.80	186.17	185.03
3	182.19	187.57	188.22	186.09	186.56	186.15	186.45
4	182.42	187.55	188.51	186.29	186.98	186.42	186.96
5	183.43	188.24	188.77	186.67	185.43	186.79	185.12
6	182.07	187.81	187.88	185.80	186.33	185.87	184.41
7	181.71	187.88	188.38	188.43	186.97	188.40	183.60
8	181.23	187.78	188.42	186.48	186.94	186.51	185.00
9	181.60	188.29	188.83	186.91	185.28	186.94	184.94
10	178.73	186.88	187.36	187.09	185.56	187.08	182.05
11	176.49	186.18	186.87	186.61	185.20	186.66	180.53
12	182.20	188.13	188.58	186.62	185.12	186.64	185.28
13	181.32	187.09	189.93	188.51	187.07	184.86	181.33
14	180.36	187.49	189.55	189.62	187.87	185.73	184.17
15	180.28	186.88	188.63	186.60	186.96	184.75	183.85
16	177.51	186.82	188.98	186.95	187.17	185.04	184.05
17	182.34	188.05	188.44	186.50	188.92	186.57	189.39
18	181.02	188.30	190.78	191.03	188.93	186.96	180.67
19	177.90	186.51	186.60	189.09	187.63	185.42	182.08
20	179.07	187.33	189.96	187.54	187.61	187.36	180.68
21	180.45	187.51	188.88	186.15	188.46	186.17	184.87
22	178.25	187.94	191.59	187.23	186.84	188.73	187.24
23	179.76	187.42	189.80	189.35	187.67	185.48	185.38
24	179.71	186.91	189.84	186.88	185.46	185.10	182.20
25	177.64	187.96	190.77	190.43	187.95	186.20	183.37
26	181.43	186.75	190.57	189.19	188.42	187.63	188.46
27	186.50	189.05	191.96	190.88	190.35	187.48	190.70
28	181.80	187.89	191.35	188.17	189.64	186.72	188.17
29	181.12	185.72	188.70	185.65	187.03	186.05	181.94
30	189.76	188.40	192.10	186.85	190.35	187.39	185.16
31	192.27	187.97	191.18	187.95	189.54	186.58	188.06
32	180.32	186.30	189.51	188.16	187.96	186.81	186.71
33	183.38	186.13	189.41	186.18	187.65	186.62	184.46
34	182.87	187.03	189.88	188.59	188.30	185.35	187.01
35	184.54	187.53	190.75	187.40	189.22	186.18	186.01
36	188.86	187.72	191.45	188.42	189.69	186.90	186.16
37	187.33	187.21	191.00	187.90	189.29	188.28	185.51
38	186.46	186.87	190.73	187.28	188.86	185.98	185.59
39	185.10	187.32	190.36	187.12	188.81	185.81	185.79
40	189.82	187.49	191.05	187.75	189.32	188.26	182.34
41	188.43	188.53	192.02	189.45	190.38	187.39	187.08
42	188.06	187.66	191.22	190.77	189.60	186.69	184.49
43	179.18	185.19	188.47	185.75	186.71	183.73	181.81
44	190.76	188.29	192.17	188.77	190.19	187.37	186.92
45	185.75	187.01	189.96	188.57	188.19	185.32	182.95
46	185.72	187.72	190.92	187.58	189.28	186.29	188.14
47	179.83	186.66	191.15	188.42	188.23	185.28	181.47
48	186.51	187.54	190.86	187.58	189.22	188.13	184.25
49	175.98	184.80	185.95	182.43	186.04	183.10	184.82
50	189.91	188.07	191.61	188.34	189.83	186.93	190.33
51	190.31	188.95	192.73	191.67	191.18	188.27	188.00
52	185.12	188.53	190.71	187.23	188.91	185.95	187.54
53	187.61	188.77	190.45	189.42	188.95	186.03	184.03
54	183.87	188.07	189.92	186.57	188.29	185.30	185.46
55	192.61	188.92	192.46	189.34	190.76	189.78	189.46
56	185.38	187.56	189.03	185.69	189.34	186.29	190.04
57	183.16	186.89	190.15	186.85	188.64	185.60	185.61
58	183.24	186.98	189.25	188.03	189.57	188.46	183.80
59	186.55	187.37	189.18	186.07	187.52	186.50	186.05
60	184.79	187.13	187.61	188.88	188.77	185.73	183.90

TABLE 10
Sample 3 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	-4.03	-2.34	2.074	-0.38	1.637	1.271	-6.37	-4.30	-4.69	-3.05	-1.78
STD	3.639	1.248	1.434	1.502	1.494	2.404	3.221	3.839	3.099	3.645	3.533
t	-8.57	-14.5	11.20	-1.99	8.487	4.094	-15.3	-8.68	-11.7	-6.48	-3.90

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	4.664	6.419	4.872	4.882	3.978	3.325	5.243	5.074	3.225	1.650	2.325
STD	2.764	3.136	3.072	2.783	2.582	2.114	4.547	3.463	2.239	1.295	1.638
t	-8.57	-14.5	11.20	-1.99	8.487	4.094	-15.3	-8.68	-11.7	-6.48	-3.90

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	2.779	0.829	0.870	0.980	0.928	1.413	2.201	2.336	2.344	2.588	2.129
STD	2.033	0.532	0.574	0.655	0.598	1.337	1.768	2.048	1.829	1.933	1.716
t	10.58	12.04	11.74	11.59	12.02	8.183	9.641	8.837	9.927	10.37	9.609

PANEL 4: DIFFERENCE IN RETURN TO VOLATILITY

Pair	1-2	2-3	3-4	4-5	5-6	6-7	1-3	1-4	1-5	1-6	1-7
Mean	-3.00	-0.73	1.565	-0.25	1.119	0.951	-3.74	-2.17	-2.43	-1.31	-0.36
STD	1.165	1.134	1.140	1.197	1.363	1.536	0.972	1.447	0.864	1.301	1.242
t	-19.9	-5.02	10.63	-1.67	6.362	4.794	-29.8	-11.6	-21.8	-7.84	-2.28

Panel 3 of Table 10 summarizes the differences in mean absolute deviation (MAD) of returns for the paired samples. The more complex strategies are expected to generate smaller MADs. Therefore, the critical value for rejection of the null hypothesis for any pair is again +2.39 at the 0.01 level. Based on the results in Panel 3, we are able to reject every null hypothesis of no difference in means. The results of this sample, unlike the prior ones, support all three of the CCM conclusions.

Panel 4 of Table 10 summarizes the differences in return to volatility of the paired strategies. Since more complex strategies are expected to generate higher risk adjusted returns, the critical t-value for rejection of the null hypotheses of no difference in means is -2.39 at the 0.01 level. The null is rejected for the pairings of strategy 1 with all duration vectors except for strategy 7. Contrary to these results, the null is rejected only for the duration vector pair 2 minus 3. The superiority of the duration vector model is supported by the return to volatility tests. The three term duration vector model is supported as optimal.

The composite results of the four tests from Monte Carlo Sample 3 provide stronger support than the prior samples for the conclusion that the duration vector model outperforms Macaulay duration matching. These results, unlike in samples 1 and 2, provide moderate support for CCM conclusions two and three.

Data Analysis for Sample 4

The final Monte Carlo sample was generated under the following set of assumptions concerning the distribution of unanticipated term structure changes:

1. The distribution is N(0,0.025) for maturities of two years or less.

2. The distribution is N(0,0.01) for maturities between two and five years.

3. The distribution is N(0,0.005) for maturities greater than five years.

4. The unanticipated term structure changes are partially correlated across maturities with a 0.60 correlation coefficient.

Price and yield data for a sample of size 60 were generated under the assumptions above. The initial bond data are the same as for Sample 3. The target holding period return is 12.8 percent, and the target horizon value is $185.96. Terminal values for the Sample 4 portfolio simulations are presented in Table 11.[10] Summary statistics on the difference in means data are presented in Table 12. Underlined t-values denote significance at the 0.01 level.

Panel 1 of Table 12 summarizes statistics on the difference in mean terminal values for sample 4. The critical value of the t-statistic for rejection of the null hypothesis is -2.39. We reject the null for all pairings of strategy 1 with all duration vectors except the 7 term strategy. The null is rejected only for the duration vector pairing of strategies 3 and 4. The results from this sample suggest that the duration vector model outperforms simple duration with regard to expected holding period portfolio return. This supports the first CCM conclusion. These data do not support CCM conclusions 2 and 3.

Panel 2 of Table 12 summarizes the differences in mean residual returns for the paired samples. The critical value for rejection of the null hypothesis for any pair is +2.39 at the 0.01 level. Based on the results in Panel 2, we are unable to reject either null except for the pairs 2-3 and 5-6. We conclude that the more complex duration strategies do not outperform simple Macaulay duration with regard to size of expected residual holding period returns. Neither of the CCM conclusions is supported by this test.

Panel 3 of Table 12 summarizes the differences in mean absolute deviation (MAD) of returns for the paired samples. The critical value for rejection of the null hypothesis for any pair is again +2.39 at the 0.01 level. Based on the results in Panel 3, we reject every null pairing Macaulay duration matching with either of the duration vectors. We fail to reject the null for all duration vector pairings.

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[10]Sample 4 values of residual returns, absolute deviations, and return to volatility are provided in Appendix VI.

TABLE 11

Sample 4 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	193.24	188.49	187.93	189.30	189.04	187.39	187.46
2	193.38	188.80	188.24	189.78	189.43	187.83	187.90
3	188.46	185.76	185.21	185.86	185.88	184.13	184.14
4	183.34	184.95	184.25	184.58	185.03	183.06	183.32
5	181.39	186.09	185.21	185.99	186.06	184.25	182.79
6	183.08	185.59	183.97	184.85	184.58	182.96	182.84
7	175.70	183.22	181.99	182.33	182.80	180.75	182.94
8	189.07	190.11	189.22	190.99	190.57	188.91	188.84
9	185.66	186.57	185.02	186.60	185.80	184.39	183.80
10	183.12	185.86	184.49	185.75	185.61	183.87	184.10
11	192.27	189.10	187.34	189.45	188.53	187.16	187.06
12	187.57	187.33	187.83	188.80	188.54	188.82	186.66
13	186.47	186.56	185.31	199.43	185.89	184.43	182.36
14	182.56	186.56	185.67	186.95	186.44	184.86	184.93
15	182.40	185.49	184.29	185.31	184.62	183.28	182.85
16	185.02	186.74	185.19	186.53	185.97	184.48	184.37
17	186.53	187.52	185.51	187.04	186.84	185.07	185.43
18	191.54	189.00	187.85	189.33	189.05	187.34	186.11
19	184.65	186.84	185.94	187.35	186.33	185.15	182.59
20	184.25	187.86	188.46	189.65	189.57	187.82	186.40
21	184.89	191.28	190.34	193.11	191.54	190.55	189.98
22	180.29	183.13	181.42	182.50	180.34	180.56	180.49
23	188.73	188.14	187.59	188.55	188.97	186.91	187.57
24	190.24	189.42	189.10	190.64	190.32	188.63	188.81
25	183.90	186.13	185.24	186.04	186.11	186.19	184.77
26	182.53	190.32	188.55	192.13	191.40	189.99	188.08
27	177.33	183.12	182.08	182.54	182.34	180.73	178.83
28	185.40	188.30	185.87	187.90	187.24	185.69	185.78
29	196.73	188.85	187.85	189.67	189.51	187.66	188.17
30	187.89	187.38	187.40	186.25	188.34	186.51	186.88
31	188.48	186.42	185.80	186.60	186.44	184.76	184.53
32	180.69	185.06	183.20	184.70	183.98	182.48	182.34
33	184.39	186.85	186.25	187.60	187.50	185.63	188.19
34	191.74	191.39	190.60	193.11	192.05	190.68	188.65
35	188.71	186.96	186.05	185.66	186.84	185.38	183.53
36	184.64	187.42	187.44	188.34	188.12	186.48	186.40
37	182.18	186.47	185.44	187.74	187.95	186.10	186.15
38	187.13	187.10	185.50	186.47	186.68	184.76	187.17
39	188.25	184.34	181.75	182.57	184.42	180.78	180.88
40	192.01	191.72	192.42	192.47	193.95	192.33	192.57
41	184.12	187.43	187.59	188.35	188.57	186.70	187.36
42	181.21	184.90	184.19	184.87	184.62	183.00	184.68
43	182.73	186.23	184.09	185.23	186.87	183.32	183.37
44	181.50	186.05	185.78	186.55	188.51	184.81	183.31
45	184.92	188.75	188.42	189.31	189.35	187.54	189.49
46	181.40	185.40	184.63	185.26	185.15	183.50	183.29
47	188.13	187.90	186.09	187.55	187.51	185.72	187.95
48	189.49	189.43	190.29	191.26	191.45	189.57	188.88
49	174.29	182.83	182.33	180.79	182.55	180.87	182.41
50	183.66	189.16	188.75	190.63	190.76	188.88	189.67
51	186.46	188.38	187.59	187.29	188.60	187.08	187.30
52	179.55	183.00	180.89	179.51	181.27	179.54	179.41
53	186.61	188.64	187.42	189.18	187.88	188.57	182.37
54	186.32	188.49	187.77	189.31	188.59	189.07	186.82
55	180.69	186.32	185.44	186.88	185.87	184.65	183.50
56	188.83	187.54	186.81	187.65	187.48	185.82	186.04
57	185.38	187.77	188.88	189.36	189.35	187.63	187.68
58	182.41	186.91	186.67	185.80	187.23	185.67	185.71
59	185.99	187.70	186.71	185.91	187.37	185.81	185.69
60	176.56	184.28	184.05	182.85	184.53	182.86	179.50

TABLE 12
Sample 4 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	−1.82	0.87	−1.18	0.165	1.579	0.304	−0.95	−2.13	−1.96	−0.38	−0.08
STD	3.304	0.778	1.961	1.986	0.758	1.46	3.401	3.638	3.368	3.414	3.502
t	−4.26	8.659	−4.66	0.644	16.12	1.615	−2.16	−4.54	−4.52	−0.87	−0.18

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	−1.82	0.87	−1.18	0.165	1.579	0.304	−0.95	−2.13	−1.96	−0.38	−0.08
STD	3.304	0.778	1.961	1.986	0.758	1.46	3.401	3.638	3.368	3.414	3.502
t	−4.26	8.659	−4.66	0.644	16.12	1.615	−2.16	−4.54	−4.52	−0.37	−0.18

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	1.862	−0.30	−0.54	0.355	−0.09	−0.19	1.558	1.017	1.372	1.273	1.076
STD	2.291	0.669	1.605	1.592	0.672	1.168	2.508	2.839	2.437	2.53	2.625
t	6.294	−3.50	−2.61	1.729	−1.14	−1.30	4.813	2.774	4.361	3.898	3.176

PANEL 4: DIFFERENCE IN RETURN TO VOLATILITY

Pair	1–2	2–3	3–4	4–5	5–6	6–7	1–3	1–4	1–5	1–6	1–7
Mean	−0.90	0.568	−0.45	−0.02	0.748	0.116	−0.33	−0.78	−0.80	−0.06	0.055
STD	0.973	0.405	0.818	0.844	0.355	0.645	1.062	1.196	1.073	1.087	1.099
t	−7.20	10.86	−4.28	−0.19	16.32	1.394	−2.44	−5.10	−5.84	−0.43	0.388

Panel 4 of Table 12 summarizes the differences in return to volatility of the paired strategies. The critical t-value for rejection of the null hypotheses of no difference in means is -2.39 at the 0.01 level. The null is rejected for the pairings of strategy 1 with duration vector strategies 2 through 5. For the pairings of different duration vectors, the null is rejected only for the difference, 3 minus 4. The superiority of the two term duration vector model is supported by the return to volatility tests.

The composite results of the four tests from Monte Carlo Sample 4 provide fairly strong support for the conclusion that a duration vector strategy employing D1 and D2 outperforms Macaulay duration matching. The CCM conclusions 2 and 3 are not supported.

Data analysis for Difference in Proportions Test

The terminal values from the four Monte Carlo samples were combined into 240 total observations. Members of this group were randomly assigned to 20 subgroups of size 12. The residual value, Q =Terminal Value-Target Value, was computed for each observation. Each sample proportion, p, was computed by dividing the sum of observations with $Q<0$, by 12 (the number of total observations). A summary of the values of p is included as Table 13 below. The differences in average proportion of below target values are included as Table 14.

As an indicator of relative immunization efficacy, smaller expected proportions of outcomes below the target are desirable; higher proportions are less desirable. We test the following hypothesis of no difference in average proportions below target value at a 0.05 significance level:

H_0: μ(less complex strategy)- μ(more complex strategy)=0

H_1: μ(less complex strategy)- μ(more complex strategy)>0

Since the more complex strategies are expected to be superior to less complex ones, the critical t-value for rejecting the null hypothesis is +1.74 at the 0.05 level. Significant values

Table 13
Proportions of Terminal Values Below Target

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	0.5833	0.3333	0.3333	0.5000	0.3333	0.5833	0.5833
2	0.6667	0.3333	0.6667	0.4167	0.5000	0.4167	0.7500
3	0.7500	0.2500	0.4167	0.3333	0.1667	0.4167	0.5833
4	0.5833	0.2500	0.3333	0.1667	0.1667	0.4167	0.4167
5	0.5000	0.3333	0.3333	0.3333	0.3333	0.2500	0.4167
6	0.5833	0.4167	0.5000	0.4167	0.5833	0.5000	0.6667
7	0.6667	0.2500	0.5833	0.5000	0.4167	0.3333	0.5833
8	0.5833	0.1667	0.2500	0.1667	0.2500	0.3333	0.4167
9	0.7500	0.4167	0.3333	0.5000	0.3333	0.5833	0.7500
10	0.5000	0.2500	0.5833	0.5000	0.5833	0.4167	0.6667
11	0.5833	0.2500	0.5000	0.4167	0.3333	0.4167	0.6667
12	0.5833	0.2500	0.5000	0.5000	0.4167	0.5000	0.5833
13	0.5833	0.1667	0.1667	0.1667	0.2500	0.1667	0.5833
14	0.7500	0.0833	0.2500	0.0833	0.2500	0.3333	0.5000
15	0.6667	0.2500	0.5833	0.4167	0.4167	0.3333	0.4167
16	0.6667	0.2500	0.2500	0.2500	0.2500	0.3333	0.5000
17	0.6667	0.4167	0.3333	0.1667	0.3333	0.4167	0.5000
18	0.5833	0.3333	0.3333	0.3333	0.3333	0.4167	0.4167
19	0.5000	0.1667	0.4167	0.3333	0.2500	0.2500	0.5833
20	0.7500	0.2500	0.4167	0.4167	0.3333	0.3333	0.5000
MEAN	0.6250	0.2708	0.4042	0.3458	0.3417	0.3875	0.5542
STD	0.0833	0.0892	0.1359	0.1359	0.1175	0.1057	0.1091

Table 14

Differences in Proportions Below Target Value

OBS	S1–S2	S2–S3	S3–S4	S4–S5	S5–S6	S6–S7	S1–S3	S1–S4	S1–S5	S1–S6	S1–S7
1	0.250	0	–0.17	0.166	–0.25	0	0.25	0.083	0.25	0	0
2	0.333	–0.33	0.25	–0.08	0.083	–0.33	0	0.25	0.166	0.25	–0.08
3	0.500	–0.17	0.083	0.166	–0.25	–0.17	0.333	0.416	0.583	0.333	0.166
4	0.333	–0.08	0.166	0	–0.25	0	0.25	0.416	0.416	0.166	0.166
5	0.167	0.00	0	0	0.083	–0.17	0.166	0.166	0.166	0.25	0.083
6	0.167	–0.08	0.083	–0.17	0.083	–0.17	0.083	0.166	0	0.083	–0.08
7	0.417	–0.33	0.083	0.083	0.083	–0.25	0.083	0.166	0.25	0.333	0.083
8	0.417	–0.08	0.083	–0.08	–0.08	–0.08	0.333	0.416	0.333	0.25	0.166
9	0.333	0.0833	–0.17	0.166	–0.25	–0.17	0.416	0.25	0.416	0.166	0
10	0.250	–0.33	0.083	–0.08	0.166	–0.25	–0.08	0	–0.08	0.083	–0.17
11	0.333	–0.25	0.083	0.083	–0.08	–0.25	0.083	0.166	0.25	0.166	–0.08
12	0.333	–0.25	0	0.083	–0.08	–0.08	0.083	0.083	0.166	0.083	0
13	0.417	0	0	–0.08	0.083	–0.42	0.416	0.416	0.333	0.416	0
14	0.667	–0.17	0.166	–0.17	–0.08	–0.17	0.5	0.666	0.5	0.416	0.25
15	0.417	–0.33	0.166	0	0.083	–0.08	0.083	0.25	0.25	0.333	0.25
16	0.417	0	0	0	–0.08	–0.17	0.416	0.416	0.416	0.333	0.166
17	0.250	0.0833	0.166	–0.17	–0.08	–0.08	0.333	0.5	0.333	0.25	0.166
18	0.250	0	0	0	–0.08	0	0.25	0.25	0.25	0.166	0.166
19	0.333	–0.25	0.083	0.083	0	–0.33	0.083	0.166	0.25	0.25	–0.08
20	0.500	–0.17	0	0.083	0	–0.17	0.333	0.333	0.416	0.416	0.25
MEAN	0.354	–0.133	0.058	0.004	–0.05	–0.17	0.221	0.279	0.283	0.238	0.071
STD	0.121	0.144	0.105	0.110	0.131	0.115	0.163	0.165	0.159	0.122	0.130
T	13.14	–4.138	2.483	0.169	–1.56	–6.49	6.064	7.563	7.990	8.724	2.428

of t are underlined in Table 14.

The null hypothesis is rejected for all pairings of Strategy 1 (Macaulay duration) with the duration vectors. In addition, the null is rejected for pairings of successive duration vectors, 3-4. The null is not rejected for pairs, 2-3, 4-5, 5-6, and 6-7. Interpretation of these results is that, (1) either duration vector strategy is more likely to generate returns that are at least equal to target returns than is simple Macaulay duration matching with an optimal objective function, and (2) duration vectors of more than 2 terms are not likely to be superior to the 2-term vector. Based on this measure of performance, these results strongly support CCM (1988) conclusion number 1. They do not support the CCM conclusions 2 and 3.

Chapter Summary

This chapter has provided paired t-tests on four Monte Carlo samples of simulated performances on the alternative strategies. The performance measures examined are (1) Terminal portfolio value, a surrogate for holding period return, (2) Residual holding period value, (3) Mean absolute deviation of return, and (4) Return to volatility, a risk adjusted return measure. It also included a test of the difference in proportion of returns below target value for the paired strategies. The results of these tests are summarized below for each sample.

In sample 1, the duration vector strategies are supported as generating less volatile returns than the min M^2 duration strategy. However, they are not supported as generating smaller expected residuals. The results in sample 2 are similar to those of sample 1. They differ in that the hypothesis of no difference between the duration vectors and Strategy 1 is rejected for both Terminal Value and Return to Volatility. Sample 3 results provide the strongest support for the CCM conclusions. Sample 4 results parallel those of sample 2. Finally, the proportions tests unequivocally support the superiority of the duration vectors over duration matching.

Sample 1

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	no diff	no diff	S2 favored	S2 favored
S1-S3	no diff	no diff	S3 favored	no diff
S1-S4	no diff	no diff	S4 favored	no diff
S1-S5	no diff	no diff	S5 favored	no diff
S1-S6	no diff	no diff	S6 favored	no diff
S1-S7	no diff	no diff	S7 favored	no diff
S2-S3	no diff	no diff	no diff	no diff
S3-S4	no diff	no diff	no diff	no diff
S4-S5	no diff	no diff	S5 favored	S5 favored
S5-S6	no diff	no diff	no diff	no diff
S6-S7	no diff	no diff	no diff	no diff

Sample 2

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S2 favored	no diff	S2 favored	S2 favored
S1-S3	S3 favored	no diff	S3 favored	no diff
S1-S4	S4 favored	no diff	S4 favored	S4 favored
S1-S5	S5 favored	no diff	S5 favored	no diff
S1-S6	S6 favored	no diff	S6 favored	S6 favored
S1-S7	no diff	no diff	no diff	no diff
S2-S3	no diff	S3 favored	no diff	no diff
S3-S4	S4 favored	no diff	no diff	S4 favored
S4-S5	no diff	S5 favored	no diff	no diff
S5-S6	S6 favored	no diff	no diff	S6 favored
S6-S7	no diff	S7 favored	no diff	no diff

Sample 3

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S2 favored	no diff	S2 favored	S2 favored
S1-S3	S3 favored	no diff	S3 favored	S3 favored
S1-S4	S4 favored	no diff	S4 favored	S4 favored
S1-S5	S5 favored	no diff	S5 favored	S5 favored
S1-S6	S6 favored	no diff	S6 favored	S6 favored
S1-S7	S7 favored	no diff	S7 favored	no diff
S2-S3	S3 favored	no diff	S3 favored	S3 favored
S3-S4	no diff	S4 favored	S4 favored	no diff
S4-S5	no diff	no diff	S5 favored	no diff
S5-S6	no diff	S6 favored	S6 favored	no diff
S6-S7	no diff	S7 favored	S7 favored	no diff

Sample 4

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S2 favored	no diff	S2 favored	S2 favored
S1-S3	no diff	no diff	S3 favored	S3 favored
S1-S4	S4 favored	no diff	S4 favored	S4 favored
S1-S5	S5 favored	no diff	S5 favored	S5 favored
S1-S6	no diff	no diff	S6 favored	no diff
S1-S7	no diff	no diff	S7 favored	no diff
S2-S3	no diff	S3 favored	no diff	no diff
S3-S4	S4 favored	no diff	no diff	S4 favored
S4-S5	no diff	no diff	no diff	no diff
S5-S6	no diff	S6 favored	no diff	no diff
S6-S7	no diff	no diff	no diff	no diff

Proportions Test

Paired Strategies	1 vs 2	1 vs 3	1 vs 4	1 vs 5	1 vs 6	1 vs 7	2 vs 3	3 vs 4	4 vs 5	5 vs 6	6 vs 7
More Complex Strategy Favored	yes	yes	yes	yes	yes	yes	no	yes	no	no	no

Tests similar to these in this chapter are executed to assess the optimality of the min M^2 objective function relative to extant alternatives in Chapter VI.

PART TWO

The Impact of Alternative Objective Functions on the Immunization Performance of Macaulay Duration Matching

The purpose in Part Two is to design and implement an experiment to investigate the efficacy of min M^2 as an optimal objective function for selecting portfolios immunized via Macaulay duration. Multiple comparisons tests are performed on the Monte Carlo samples described in Part One. The objective functions to be evaluated are summarized in Table 15 below. In addition to min M^2 and the quadratic minimization objective of CCM, convexity maximization and M^2 maximization are included in the investigation. Two additional portfolio selection criteria, suggested by Fooladi and Roberts (1992), are also included.

Table 15

Alternative Objective Functions Evaluated

Strategy Number	Objective Function	Constraint
1	Minimize weighted average M^2	Duration=h
2	Minimize sum of portfolio weights	Duration=h
3	Maximize weighted average M^2	Duration=h
4	Maximize average price convexity	Duration=h
5	Maturity barbell duration matching	Duration=h
6	Maturity bullet duration matching	Duration=h

Notes: h = time remaining to the end of the planned holding period.
Strategies 4, 5, and 6 are discussed in Chapter VI.

CHAPTER VI

THE OBJECTIVE FUNCTIONS

Introduction

Rationales for the CCM quadratic minimization and the min M^2 objective functions have been discussed. In addition to these two objectives, the performances of bond price convexity maximization, M^2 maximization, the two Fooladi and Roberts (1992) strategies are also examined. They are described and discussed below.

Bond Price Convexity

The convexity index, Con, is derived from the second term of a Taylor series expansion for discrete time, or from the second derivative of the bond price function in continuous time.[10] It is used along with duration to provide a more complete measure of bond price volatility. For bonds with equal duration, the one with the greater convexity will exhibit the smallest price decline when interest rates rise and the largest price increase when interest rates fall. The index is normally defined as

$$Con=(1/1+r)^2[\Sigma(t^2+t)C_t/(1+r)^t](1/P) \qquad (1)$$

where Con indicates convexity and P is the current bond price. All other variables are as previously defined.

A comprehensive discussion of convexity, its alternative specifications, and its properties is found in Fabozzi and Fabozzi (1989). Most of the discussion of convexity maximization as an optimization strategy in the literature addresses its potential as a stand alone price risk management strategy or as a selection criterion in immunizing interest rate sensitive liabilities. A sampling of these studies include Toevs (1985), Dunetz and

[10]The derivation of Convexity is shown in Appendix VII.

Mahoney (1988), Grantier (1988), and Kahn and Lochoff (1990). The implications of convexity as an objective function in an immunization model is suggested by Christensen and Sorensen (1994).

The potential of convexity maximization as an optimal selection strategy for holding period return immunization is related to the requirement of periodic portfolio rebalancing. Since this procedure will entail the selling of some securities at the rebalancing dates, it is desirable that selling prices be as high as possible. Given the uncertainty surrounding the direction of interim interest rate changes, this objective function will ensure maximum price increases when rates fall and minimum price declines when rates rise. This objective function is expected to perform particularly well when there are large, near parallel, shifts in the term structure.

M^2 Maximization

This objective function is included to provide a stringent test of the efficacy of M^2 minimization. Since min M^2 portfolios are compressed closely around the horizon, max M^2 portfolios should, conversely, be widely dispersed. This will produce maximum contrast between outcomes when compared to the min M^2 selection strategy. As such, it can be viewed as providing a control sample.

Of additional interest, M^2 can be shown to represent an index of terminal portfolio value convexity under a single term structure shift assumption. Since the return function of duration matched portfolios is strictly convex with a minimum when there is no change in yields, the greatest terminal value would be realized when there is a large yield shift and when weighted terminal value convexity is maximized. To the extent that immunization can be achieved with near precision without periodic rebalancing, maximum terminal value convexity, thus maximum M^2, may be an optimal portfolio selection criterion.

Maturity Constrained Bullets and Barbells

Fooladi and Roberts (1992) observed characteristics of extant tests of single factor immunization models. They found that researchers who achieved favorable immunization results, relative to maturity matching, all employ portfolio selection criteria that include the security with maturity equal to the horizon in every portfolio. To the contrary, Ingersoll (1983), who concluded that maturity matched portfolios perform as well as duration constrained ones, did not require the portfolio to include the maturity bond. This prompted Fooladi and Roberts (1992) to investigate the issue of alternative portfolio design criteria. This issue was investigated within the context of a broader study of single factor immunization strategies, but is similar in spirit to the tests in this chapter.

Fooladi and Roberts (1992) compared the alternative strategies in Table 16 below. They found that either the maturity bullet or the maturity barbell, but not necessarily both, always outperformed the other alternatives in their tests. They paid particular attention to the bullet strategy because, as one would expect, it demonstrated the smallest M^2 when compared to the other alternatives. They surmised that the min M^2 objective function, like the other strategies investigated, is inferior to their maturity constrained duration matching strategies. Of course, that conclusion can be questioned since in some of their tests the bullet portfolio (their proxy for min M^2) outperformed either the maturity bullet or the maturity barbell. In addition, they permitted short selling in the maturity constrained portfolios, but not in the straight bullets and barbells. It is also noted that,

TABLE 16

Fooladi and Roberts Alternative Strategies

1. Maturity matching - select and hold a bond with time zero maturity equal to holding period, H.

2. Bullet duration matching - select a one or two bond duration matched portfolio at time zero and at rebalancing dates with maturity closest to the horizon, h.

3. Barbell duration matching - select two-bond duration matched portfolios with the shortest and longest maturities available.

4. Ladder duration matching - select duration matched portfolios that included roughly equal percentages of all available bonds. Fooladi and Roberts selected portfolios from an eight bond universe.

5. Maturity barbell duration matching - select a two-bond portfolio like the straight barbell above with the short bond constrained to equal time to horizon, h.

6. Maturity bullet duration matching - select a two-bond portfolio like the straight bullet above with the shorter bond constrained to equal time to horizon, h.

because of the sensitivity of M^2 to coupons, yields, and prices, as well as time to maturity, the most compressed bullet portfolio will not necessarily result in its minimum value. Multiple comparisons analyses of the six alternative selection criteria discussed above are performed. The procedure entails pairwise examinations of the differences between the performances of min M^2 and each of the five alternative objective functions on each of the variables of interest. The target returns, the return generating model, the simulation procedures, and the method of inference are identical to those employed in **Part One** of this dissertation.

Difference in Means Tests

Differences in the following variables are investigated using paired t-tests. Derivation of the test statistic has been previously described.

VARIABLE	DESCRIPTION OF VARIABLE
$\mu(TV)$	Mean terminal holding period value
$\mu(Q)$	Mean residual holding period terminal value
MAD(TV)	Mean absolute deviation of terminal value
$\mu(R/V)$	Excess Terminal Value divided by the mean absolute deviation of residual value

The following hypotheses are evaluated at the 0.01 level of significance. Tests of the hypotheses are performed on each variable for each of the four samples generated by the Monte Carlo simulation procedures described in Part One.

H_0: $\mu(Q_1) - \mu(Q_j) = 0$ (j=2,..,6)

H_1: $\mu(Q_1) - \mu(Q_j) < 0$

H_0: $\mu(TV_1) - \mu(TV_j) = 0$ (j=2,..,6)

$H_1: \mu(TV_1) - \mu(TV_j) > 0$

$H_0: MAD(TV_1) - MAD(TV_j) = 0 \qquad (j=2,..,6)$

$H_1: MAD(TV_1) - MAD(TV_j) < 0$

$H_0: \mu(R/V_1) - \mu(R/V_j) = 0 \qquad (j=2,..,6)$

$H_1: \mu(R/V_1) - \mu(R/V_j) > 0$

This results in twenty hypotheses to be tested (five for each summary statistic) on each sample. Superior performance is indicated by the direction of the inequality sign in each alternate hypothesis. Rejection of the null hypothesis in either test would support the conclusion that min M^2 outperforms the alternative to which it is being compared. Failure to reject a null would indicate no apparent advantage of min M^2 over the paired alternative. If min M^2 is optimal, we would expect to reject each null hypothesis in favor of its alternate.

<u>Difference in Proportions Tests</u>

Again, the differences in the percentage of holding period returns below target returns are evaluated. The percentage of returns below the target is computed on each alternative strategy from the 20 observations on samples of size 12 described in Part One. The population proportion is denoted as π. Pairwise tests of the difference in proportions are performed using the test statistic, $t=d\sqrt{20} / S_d$ with 19 degrees of freedom. All variables are as previously defined.

The statistical hypotheses are summarized below.

$H_0: \pi_1-\pi_j = 0 \qquad (j=2,..,6)$

$H_1: \pi_1-\pi_j < 0$

Because of the small sample sizes, these hypotheses are evaluated at a 0.05 level of significance. The critical value of t (df=19) is 1.729. Again, rejection of the null in either case would indicate that the min M^2 objective function outperforms the paired alternative. Failure to reject either null would be an indication of no advantage for min M^2. If the optimality of M^2 minimization is to be supported, each null hypothesis should be rejected.

The Optimization Models

The alternative strategies will be subsequently referred to as Strategy 1 through Strategy 6. The descriptions of the alternative strategies are provided below.

Strategy 1 Min M^2 Duration Matching

Minimize $\Sigma y_i M_i^2$

Subject to $\Sigma y_i Dur_i = h$

$\Sigma y_i = 1$, $y_i \geq 0$

Strategy 2 Min Sum of Squared Weights Duration Matching

Minimize $\Sigma y_i M_i^2$

Subject to $\Sigma y_i Dur_i = h$

$\Sigma y_i = 1$, $y_i \geq 0$

Strategy 3 Max M^2 Duration Matching

Maximize $\Sigma y_i M_i^2$

Subject to $\Sigma y_i Dur_i = h$

$\Sigma y_i = 1$, $y_i \geq 0$

Strategy 4 Max Convexity Duration Matching

Maximize $\Sigma y_i Con_i$

Subject to $\Sigma y_i Dur_i = h$

$\Sigma y_i=1$, $y_i \geq 0$

Strategy 5 Maturity Constrained Barbell Portfolio

Two bond portfolio with maturity of Bond1=h and Bond2=the longest maturity.

Subject to $\Sigma y_i Dur_i=h$

$\Sigma y_i=1$, $y_i \geq 0$

Strategy 6 Maturity Constrained Bullet Portfolio

Two bond portfolio with maturity of Bond1=h and Bond2=the shortest maturity longer than h.

Subject to $\Sigma y_i Dur_i=h$

$\Sigma y_i=1$, $y_i \geq 0$

The Test Procedure

The following series of steps are carried out to generate observations on the variables of interest:

1. The software package, @Risk, is used to generate 60 sample observations of prices and yields on each of thirty bonds at the ends of years 1-5. These represent maturities of one year through thirty years.
2. Duration, M^2, and Con are computed for each bond included in the initial sample at time s=0. This results in computation of 90 values (30 bonds X 3 measures).
3. These values are computed again from the Monte Carlo prices and yields at times s=1 through s=5. Because a one year bond matures at each time s, the size of the initial array declines by one to a total of 25 bonds at time

s=5. A total of 5220 values are computed at time s=1 (29 bonds X 3 measures X 60 observations). This number declines to 4500 values at time s=5.

4. At time s=0, initial portfolios are selected for each of the alternative strategies by solving the optimization models with the GAMS software. This yields six initial portfolios. To maintain a constant number of bonds from which portfolios are constructed, only the first twenty five bonds are included at each time s.

5. At one year intervals at times s=1 through s=4, the portfolios are rebalanced to maintain the horizon constraints by solving the six updated optimization models for each of the sixty observations.

6. The time s=1 investment value is computed for each strategy on each of the sixty sample observations. It is assumed that the initial investment is $100.

7. The investment value computations are repeated for times s=2 through s=5. The s=5 value is terminal portfolio value and represents a datum on each alternative strategy. There are sixty such values for each alternative strategy. These data are then used to complete the tests outlined above.

8. The entire process is repeated for each Monte Carlo sample of size 60.

Data Analysis and Results

Data Analysis for Sample 1

The Monte Carlo procedure described in Chapter IV, with the uncorrelated

N(0,0.01) distribution, is implemented to generate price and yield data from which observations on the variables of interest are derived for sample 1. The target holding period yield is 12.5 percent, and the target terminal portfolio value, assuming an initial investment of $100, is $183.35. For the samples of size 60, the critical value of t (df=59) for rejecting the null hypothesis is 2.39 at the 0.01 significance level.

Terminal values resulting from the portfolio simulations are presented in Table 17 for the alternative strategies[11]. Summary statistics for the four difference in means tests are presented in Table 18. Underlined t values in this table denote significance at the 0.01 level.

Panel 1 of Table 18 summarizes statistics on the difference in mean terminal values for each pairing of strategies. The critical t value for rejection of the null hypothesis is +2.39 at the 0.01 significance level. We are unable to reject the null for any pairing of strategies in this test. The data suggest that both the convexity and M^2 maximization objectives generate higher expected returns than does M^2 minimization.

Panel 2 of Table 18 summarizes the differences in mean residual returns for the paired samples. Superior performance of an alternative strategy is indicated by smaller residual values. The critical t-value, therefore, is -2.39. The null is rejected for the pairings of strategy 1 with both strategy 2 and strategy 3.

Panel 3 of Table 18 summarizes the difference in mean absolute deviation tests. Smaller values indicate lower risk. The critical value for rejecting the null is again -2.39. The null is rejected for the pairing of strategy 1 with strategies 3, 5, and 6.

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[11]Values for residual returns, absolute deviation of returns, and return to volatility measure m Sample 1 are included in Appendix VIII.

TABLE 17

Sample 1 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	200.85	208.39	196.32	203.03	187.53	187.48
2	204.03	230.30	218.78	193.21	188.60	206.60
3	166.86	210.22	200.62	184.87	159.20	170.55
4	186.44	225.03	211.01	195.61	187.84	205.09
5	211.20	216.58	186.30	204.01	196.47	201.91
6	182.43	228.42	190.66	187.55	184.39	196.33
7	188.75	200.82	184.28	180.81	199.41	222.88
8	198.37	211.55	196.35	203.42	199.83	200.56
9	200.55	212.02	196.63	197.99	208.88	204.96
10	241.20	194.49	193.32	210.85	221.55	195.53
11	201.27	217.14	198.31	196.05	199.18	183.98
12	198.90	194.61	178.81	190.58	186.18	193.00
13	182.48	210.29	203.28	179.87	181.70	193.81
14	193.95	228.60	183.55	190.52	175.86	187.10
15	171.74	197.00	195.77	188.50	173.39	164.18
16	176.22	195.57	174.18	184.26	197.04	222.82
17	164.48	218.87	210.35	177.03	171.19	177.24
18	148.93	197.78	181.29	168.65	159.00	172.63
19	173.70	182.60	169.69	185.88	196.52	201.36
20	195.56	234.09	180.29	194.48	185.73	197.42
21	161.96	196.04	191.02	188.47	197.21	185.99
22	199.78	198.07	188.78	183.62	192.70	191.75
23	205.27	225.29	221.10	202.34	197.07	171.45
24	180.72	207.30	211.08	182.08	175.52	174.06
25	169.07	230.49	205.33	178.80	176.63	181.20
26	182.00	189.86	178.72	179.58	205.08	200.73
27	192.16	214.89	185.83	195.85	189.65	218.71
28	174.84	210.13	186.61	184.23	194.01	202.20
29	187.04	220.49	213.52	188.60	184.64	179.19
30	185.52	222.28	208.56	184.69	170.52	168.55
31	185.99	213.96	202.00	187.64	178.80	167.45
32	175.31	193.76	191.74	181.15	204.01	191.28
33	215.60	213.29	184.94	193.98	205.33	217.72
34	181.97	221.98	220.97	195.64	172.22	191.57
35	180.50	199.33	195.00	190.20	196.91	201.21
36	175.56	249.67	260.50	180.80	169.04	177.92
37	205.82	233.38	230.62	188.39	184.91	195.67
38	223.17	210.33	200.29	199.76	203.93	210.25
39	232.57	211.02	187.49	200.36	225.78	220.17
40	177.84	224.19	188.25	194.08	191.20	173.47
41	174.16	186.22	176.00	184.54	200.47	202.48
42	200.38	215.19	197.10	201.09	208.11	209.47
43	166.60	218.53	197.22	179.82	177.22	181.73
44	205.36	216.42	200.04	195.66	208.53	225.47
45	194.27	206.63	180.96	183.29	194.48	178.88
46	176.85	206.27	189.60	183.39	175.37	169.63
47	178.88	207.29	188.52	186.98	186.15	188.58
48	181.85	210.92	180.99	188.16	182.22	196.51
49	171.92	196.74	174.28	185.33	175.79	173.26
50	176.06	197.64	181.25	180.46	177.02	180.57
51	210.46	233.06	231.92	188.90	195.20	192.60
52	159.18	198.00	185.04	184.54	171.57	148.02
53	196.86	184.67	194.24	196.41	202.09	200.23
54	215.07	222.38	217.22	203.54	200.15	213.33
55	193.06	223.29	219.69	194.21	184.43	173.08
56	185.90	192.02	178.64	186.48	182.22	191.95
57	178.02	207.75	198.28	181.44	198.72	194.40
58	166.64	177.29	178.87	178.36	168.77	161.32
59	176.46	213.59	186.94	192.01	168.68	166.92
60	199.12	188.17	180.21	202.49	195.41	209.19

TABLE 18

Sample 1 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-21.9031	-7.5186	-1.4425	-0.6542	-2.9265
STD	20.7161	22.356	12.5499	13.09	16.995
t	-8.1898	-2.6051	-0.8903	-0.3871	-1.3338

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-21.9031	-7.5186	-1.4425	-0.6542	-2.9265
STD	20.7161	22.356	12.5499	13.09	16.995
t	-8.1898	-2.6051	-0.8903	-0.3871	-1.3338

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	2.525	-9.3895	-1.4472	-6.3762	-10.2708
STD	13.924	11.7552	6.1065	8.7328	10.3564
t	1.4047	-6.1871	-1.8357	-5.6557	-7.6819

PANEL 4: Difference in Return to Volatility

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-1.9128	-0.2007	-0.7172	-0.261	0.1233
STD	1.5653	1.7769	1.4405	1.7023	1.4097
t	-9.4656	-0.8749	-3.8566	-1.1876	0.6775

Panel 4 of Table 18 summarizes the differences in return to volatility measure of performance. Superior performance is indicated by higher values, and the critical value of t is +2.39. We are unable to reject the null for any pairing.

Data Analysis for Sample 2

This second Monte Carlo sample was generated under the following set of assumptions concerning the distribution of unanticipated term structure changes:

1. The distribution is N(0,0.025) for maturities of two years or less.
2. The distribution is N(0,0.01) for maturities between two and five years.
3. The distribution is N(0,0.005) for maturities greater than five years.
4. The unanticipated term structure changes are perfectly correlated across maturities at each observation date.

Terminal values for the Sample 2 portfolio simulations are presented in Table 19.[12] The target terminal value is $183.35. Summary statistics on the difference in means data are presented in Table 20. Underlined t values denote significance at the 0.01 level.

Panel 1 of Table 20 summarizes statistics on the difference in mean terminal values. The critical value of the t-statistic for rejection of the null hypothesis is +2.39 at the 0.01 significance level. The null, of no difference in means, is rejected for the pairing of strategy 1 with strategy 6, the maturity barbell, only.

Panel 2 of Table 20 summarizes the differences in mean residual returns for the paired samples. The critical value of t is -2.39 at the 0.01 significance level. The null is rejected again for the pairings of strategy 1 with both strategy 2 and strategy 3.

[12]Sample 2 values of residual returns, absolute deviations, and return to volatility measures are provided in Appendix IX.

TABLE 19

Sample 2 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	192.88	200.60	194.34	200.53	194.65	190.55
2	184.89	190.85	189.74	187.71	184.91	181.10
3	184.14	191.63	188.99	179.54	180.09	178.85
4	191.32	196.21	190.34	186.23	194.23	187.21
5	192.50	200.13	192.83	192.86	186.17	184.25
6	180.16	188.42	188.71	177.35	179.67	177.29
7	169.28	186.22	184.27	172.17	168.76	166.30
8	189.37	205.26	195.67	193.70	189.38	182.87
9	181.18	189.35	187.09	179.67	180.38	178.08
10	176.35	185.17	187.77	163.50	168.46	170.46
11	186.65	195.24	191.27	187.19	182.38	178.87
12	190.36	197.43	192.91	192.98	192.30	185.55
13	170.47	185.91	186.61	169.55	167.65	167.23
14	178.07	191.06	189.86	191.60	183.38	177.47
15	190.82	199.49	197.83	200.26	194.58	187.87
16	174.14	183.79	186.04	164.29	168.02	169.42
17	185.38	190.77	190.26	184.34	186.50	181.53
18	176.25	188.33	190.06	174.85	172.07	170.92
19	173.27	184.49	186.97	169.42	170.46	169.13
20	188.01	190.04	191.62	184.87	185.95	182.51
21	181.00	186.12	186.84	186.66	176.97	174.35
22	167.89	169.69	179.13	164.26	161.24	160.96
23	185.51	188.73	190.88	185.54	182.29	178.73
24	187.32	202.90	199.50	190.70	191.28	185.55
25	181.98	186.53	190.71	181.16	179.29	176.32
26	181.45	188.20	187.86	187.22	181.92	181.82
27	178.29	195.22	198.26	186.11	185.15	182.63
28	204.30	199.40	199.74	204.20	205.94	200.49
29	163.38	173.05	178.72	166.67	157.84	157.46
30	189.82	194.90	194.67	193.68	198.39	188.14
31	187.89	195.45	195.25	189.21	196.06	184.35
32	174.50	182.72	184.03	173.32	166.14	163.22
33	184.53	188.24	189.56	189.08	182.75	176.17
34	169.03	176.83	171.42	167.06	160.89	157.39
35	176.05	182.72	184.53	174.92	171.63	165.27
36	183.37	186.32	183.72	187.09	183.42	174.47
37	190.47	206.87	202.50	204.14	201.16	188.41
38	180.37	188.72	188.58	181.05	182.41	172.55
39	174.06	180.07	185.64	175.59	170.18	165.09
40	185.66	201.61	197.66	201.88	195.62	182.69
41	183.22	191.27	193.03	189.95	183.41	175.39
42	181.96	187.81	191.13	186.35	186.31	176.19
43	178.69	186.83	185.04	181.53	170.92	163.95
44	177.92	187.40	187.79	182.13	182.12	171.46
45	189.81	197.35	190.15	197.71	188.16	175.97
46	173.37	183.75	187.13	178.04	170.29	164.72
47	179.18	165.59	168.57	156.09	147.94	146.70
48	180.89	194.50	192.80	187.78	184.20	174.38
49	168.14	171.65	173.43	165.51	156.47	150.13
50	187.17	202.21	202.70	193.94	196.77	185.82
51	185.59	195.72	193.96	192.28	188.66	177.76
52	182.39	190.51	190.89	185.56	182.58	172.97
53	181.40	192.60	191.52	185.45	180.57	172.80
54	184.11	186.82	186.95	185.88	181.96	179.66
55	191.86	194.55	194.95	193.48	193.77	193.42
56	188.44	184.27	186.47	186.84	182.41	180.93
57	175.87	180.65	182.85	173.69	172.25	172.16
58	178.56	186.66	185.00	173.06	176.47	176.88
59	182.99	188.42	188.78	181.04	183.40	182.82
60	174.05	172.34	173.06	161.97	155.97	155.31

TABLE 20
Sample 2 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-7.4603	-7.1778	-1.2077	1.3795	6.2498
STD	5.5017	5.5079	6.3492	6.6219	5.4645
t	-10.5035	-10.0944	-1.4734	1.6137	8.8591

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-7.4603	-7.1778	-1.2077	1.3795	6.2498
STD	5.5017	5.5079	6.3492	6.6219	5.4645
t	-10.5035	-10.0944	-1.4734	1.6137	8.8591

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-0.4967	-1.8853	-5.6086	-5.2217	-4.223
STD	4.9338	3.509	5.0173	5.9352	5.8337
t	-0.7798	-4.1617	-8.6588	-6.8148	-5.6073

PANEL 4: DIFFERENCE IN RETURN TO VOLATILITY

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-1.2082	-0.6953	-0.1418	0.553	0.8243
STD	0.8727	0.7812	0.6725	0.7158	0.9763
t	-10.7238	-6.8942	-1.6333	5.9842	6.54

Panel 3 of Table 20 summarizes the difference in mean absolute deviation tests. Smaller values indicate lower risk. The critical value for rejecting the null is again -2.39. The null is rejected for the pairing of strategy 1 with strategies 3, 4, 5, and 6.

Panel 4 of Table 20 summarizes the differences in return to volatility measure of performance. The critical value of t is +2.39. The null is rejected for the pairing of strategy 1 with both strategies 5 and 6. The results of this sample indicate that the min M^2 objective outperforms both the maturity bullet and barbell strategies on this measure.

Data Analysis for Sample 3

The third Monte Carlo sample was generated under the following set of assumptions concerning the distribution of unanticipated term structure changes:

1. The distribution is N(0,0.025) for maturities of two years or less.
2. The distribution is N(0,0.01) for maturities between two and five years.
3. The distribution is N(0,0.005) for maturities greater than five years.
4. The unanticipated term structure changes are partially correlated across maturities with a 0.80 correlation coefficient.

Terminal values for the Sample 3 portfolio simulations are presented in Table 21.[13] Summary statistics on the difference in means data are presented in Table 22. Underlined t values denote significance at the 0.01 level.

Panel 1 of Table 22 summarizes statistics on the difference in mean terminal values for sample 3. The critical value of the t-statistic for rejection of the null hypothesis is +2.39. Neither null is rejected.

[13]Sample 3 values of residual returns, absolute deviations, and return to volatility are provided in Appendix X.

TABLE 21

Sample 3 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	181.23	186.21	186.54	185.96	182.67	182.53
2	181.15	187.02	188.16	184.62	182.31	182.54
3	182.19	189.26	189.26	186.48	184.36	185.56
4	182.42	186.94	191.56	187.10	182.41	184.46
5	183.43	188.64	190.76	187.12	184.86	186.61
6	182.07	191.18	191.45	188.58	184.50	186.00
7	181.71	189.81	189.63	184.60	182.75	184.19
8	181.23	189.80	190.06	187.49	183.31	183.15
9	181.60	187.95	188.76	186.34	183.67	184.99
10	178.73	187.53	187.98	187.40	182.78	183.29
11	176.49	184.67	185.43	181.29	180.41	179.84
12	182.20	187.02	187.50	182.37	185.05	185.75
13	181.32	186.95	187.35	184.18	184.11	184.76
14	180.36	188.06	188.06	185.07	182.80	183.52
15	180.28	187.02	187.02	184.96	182.89	184.65
16	177.51	187.93	187.93	183.98	181.81	183.25
17	182.34	186.52	186.52	187.34	185.89	186.82
18	181.02	190.51	192.71	185.56	185.13	185.22
19	177.90	189.93	189.02	184.94	179.81	183.04
20	179.07	187.05	187.19	182.86	183.75	184.46
21	180.45	186.83	187.65	187.81	183.70	183.37
22	178.25	186.95	187.46	185.20	182.51	181.59
23	179.76	184.80	186.08	188.15	182.93	183.59
24	179.71	189.79	190.27	186.01	182.82	182.42
25	177.64	189.47	191.18	181.30	181.22	182.63
26	181.43	189.07	188.25	184.09	182.05	183.32
27	186.50	189.97	189.97	189.41	188.94	189.81
28	181.80	187.72	187.34	188.88	185.47	187.30
29	181.12	185.92	185.92	186.60	182.07	178.65
30	189.76	188.22	188.22	187.01	188.63	189.35
31	192.27	187.35	187.35	190.00	192.05	190.26
32	180.32	187.64	186.15	181.30	179.58	181.02
33	183.38	187.13	187.13	187.02	181.71	180.14
34	182.87	187.22	188.42	184.87	182.52	182.55
35	184.54	184.84	187.22	184.97	183.91	184.75
36	188.86	187.03	187.03	190.71	188.54	184.72
37	187.33	183.65	184.16	185.83	186.83	184.35
38	186.46	186.61	186.61	186.86	185.60	184.52
39	185.10	186.02	186.02	186.47	183.60	185.09
40	189.82	187.36	185.66	192.36	189.08	187.00
41	188.43	217.63	189.11	189.39	187.65	190.42
42	188.06	182.84	183.63	185.95	187.06	185.49
43	179.18	186.98	186.98	180.55	178.07	178.66
44	190.76	191.72	191.44	191.54	189.85	187.85
45	185.75	186.18	187.67	184.59	185.34	185.68
46	185.72	192.06	190.73	188.43	184.65	185.56
47	179.83	187.40	186.87	177.44	180.85	181.96
48	186.51	186.14	186.36	184.54	187.25	185.48
49	175.98	178.32	178.71	176.12	176.08	175.61
50	189.91	192.51	192.51	191.65	189.52	189.32
51	190.31	191.10	191.10	191.35	189.96	190.27
52	185.12	187.59	187.59	186.94	184.32	185.05
53	187.61	187.01	187.12	188.85	186.77	185.36
54	183.87	186.47	186.08	184.57	183.25	184.14
55	192.61	192.72	192.72	191.59	192.12	191.97
56	185.38	185.29	185.29	186.75	184.57	185.27
57	183.16	183.42	183.11	184.09	182.33	183.90
58	183.24	184.02	184.88	181.96	182.47	182.00
59	186.55	189.00	187.89	188.44	186.14	185.80
60	184.79	184.00	183.23	182.41	184.01	183.90

TABLE 22
Sample 3 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-4.5268	-4.3272	-2.598	-0.9482	-1.1719
STD	5.2729	4.3748	2.9547	1.8798	2.6049
t	-6.6499	-7.6617	-6.8109	-3.9072	-3.4848

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-4.5268	-4.3272	-2.598	-0.9482	-1.1719
STD	5.2729	4.3748	2.9547	1.8798	2.6049
t	-6.6499	-7.6617	-6.8109	-3.9072	-3.4848

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	0.95	-1.2112	-1.181	-1.1817	-0.8825
STD	4.1763	1.6501	1.9337	1.9603	1.887
t	1.762	-5.6857	-4.7308	-4.6694	-3.6226

PANEL 4: Difference in Return to Volatility

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-1.5881	1.2949	2.9616	2.0779	2.3137
STD	1.9889	1.2531	1.2937	1.387	1.1652
t	-6.185	8.0044	17.7324	11.6044	15.3809

Panel 2 of Table 22 summarizes the differences in mean residual returns for the paired samples. The critical value for rejection of the null hypothesis for any pair is -2.39 at the 0.01 level. Based on the results in Panel 2, we are able to reject the null for each pairing of strategy 1. This strongly supports the hypothesis that the *min* M^2 outperforms its counterparts with regard to minimizing expected residual returns.

Panel 3 of Table 22 summarizes the differences in mean absolute deviation (MAD) of returns for the paired samples. The critical value for rejection of the null hypothesis for any pair is again -2.39 at the 0.01 level. Based on the results in Panel 3, we are able to reject every null except in the pairing with strategy 2, convexity maximization.

Panel 4 of Table 22 summarizes the differences in return to volatility of the paired strategies. The critical t-value for rejection of the null hypotheses of no difference in means is +2.39 at the 0.01 level. Again, we are able to reject the null for all pairs except for strategy 2. The data in this panel, along with that in panels 2 and 3, strongly support strategy 1 as optimal.

Data Analysis for Sample 4

The final Monte Carlo sample was generated under the following set of assumptions concerning the distribution of unanticipated term structure changes:

1. The distribution is N(0,0.025) for maturities of two years or less.
2. The distribution is N(0,0.01) for maturities between two and five years.
3. The distribution is N(0,0.005) for maturities greater than five years.
4. The unanticipated term structure changes are partially correlated across maturities with a 0.60 correlation coefficient.

Terminal values for the Sample 4 portfolio simulations are presented in Table 23.[14] Summary statistics on the difference in means data are presented in Table 24. Underlined t values denote significance at the 0.01 level.

Panel 1 of Table 24 summarizes statistics on the difference in mean terminal values for sample 4. The critical value of the t-statistic for rejection of the null hypothesis is +2.39. Again we fail to reject the null for any pairing of strategy 1 with either alternative. The composite results of all four samples do not support the hypothesis that the min M^2 objective function outperforms either alternative with regard to expected return over the holding period.

Panel 2 of Table 24 summarizes the differences in mean residual returns for the paired samples. The critical value for rejection of the null hypothesis for any pair is -2.39 at the 0.01 level. Based on the results in Panel 2, we are unable to reject either null. This is inconsistent with the corresponding results in the three prior samples. The null was rejected for, at least, the pairing of strategy 1 with both 2 and 3 with those data. The composite results suggest that the min M^2 strategy can be expected to generate smaller residual values than the alternatives. This conclusion is strongest with respect to pairings with the convexity and M^2 maximization strategies.

Panel 3 of Table 24 summarizes the differences in mean absolute deviation (MAD) of returns for the paired samples. The critical value for rejection of the null hypothesis for any pair is again -2.39 at the 0.01 level. Based on the results in Panel 3, we are able to reject the null for the maturity bullet and barbell strategies only. The composite results strongly support the superiority of strategy 1 over all alternatives with

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[14]Sample 4 values of residual returns, absolute deviations, and return to volatility are provided in Appendix XI.

TABLE 23
Sample 4 Terminal Portfolio Values

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	193.24	183.71	184.38	186.93	190.74	187.06
2	193.38	183.86	184.54	187.07	190.90	187.21
3	188.46	185.15	184.52	184.75	186.01	181.14
4	183.34	184.70	187.07	184.00	183.55	182.24
5	181.39	188.25	187.96	183.96	183.33	185.12
6	183.08	185.63	186.11	184.77	184.45	185.58
7	175.70	183.31	185.32	179.85	177.99	178.43
8	189.07	184.64	186.44	188.09	191.82	197.03
9	185.66	187.12	187.72	184.72	184.25	183.51
10	183.12	186.98	185.90	183.12	182.02	182.28
11	192.27	187.64	187.48	184.88	190.72	193.34
12	187.57	184.07	184.07	184.12	186.65	183.51
13	186.47	186.60	186.79	184.78	186.97	187.14
14	182.56	187.48	187.48	184.34	183.49	184.15
15	182.40	184.82	185.66	185.17	181.82	181.25
16	185.02	186.41	186.97	184.75	185.84	187.08
17	186.53	187.40	187.16	184.65	186.49	188.90
18	191.54	185.00	187.04	190.18	191.47	192.91
19	184.65	185.54	186.50	184.75	183.46	181.83
20	184.25	185.50	186.57	185.18	183.91	182.96
21	184.89	189.36	187.67	186.16	186.93	191.01
22	180.29	184.49	185.86	179.35	178.16	174.33
23	188.73	184.34	184.34	186.92	187.78	190.42
24	190.24	186.33	184.94	187.54	189.46	190.46
25	183.90	183.98	185.25	185.97	183.87	185.20
26	182.53	187.57	187.57	188.57	190.66	189.06
27	177.33	187.41	188.33	183.78	177.96	175.94
28	185.40	182.80	184.94	185.39	187.45	193.97
29	196.73	183.46	185.16	189.36	193.11	190.05
30	187.89	184.23	186.28	187.20	186.91	184.22
31	188.48	186.09	187.69	187.29	189.13	184.29
32	180.69	186.19	186.24	182.25	180.05	183.36
33	184.39	185.75	184.89	186.06	183.86	184.51
34	191.74	184.00	186.22	187.75	191.83	193.75
35	188.71	184.54	189.28	185.48	187.34	183.43
36	184.64	183.52	186.91	186.34	185.01	186.22
37	182.18	183.65	184.86	185.25	181.59	181.35
38	187.13	186.64	187.33	185.00	187.94	192.12
39	188.25	183.69	185.30	184.28	187.08	185.26
40	192.01	190.22	190.22	188.09	190.20	191.54
41	184.12	181.95	183.18	184.90	185.66	182.63
42	181.21	186.90	186.14	183.39	181.06	180.31
43	182.73	183.96	183.96	183.90	186.05	189.70
44	181.50	185.49	185.60	182.63	181.30	184.18
45	184.92	186.55	187.09	187.88	187.76	193.64
46	181.40	183.32	184.99	181.87	181.81	181.88
47	188.13	184.30	185.12	186.66	187.39	190.17
48	189.49	185.26	188.51	191.34	189.14	187.35
49	174.29	184.79	185.08	179.16	174.50	173.09
50	183.66	185.35	185.22	185.55	185.44	188.45
51	186.46	183.68	182.65	185.87	188.31	187.12
52	179.55	184.48	185.73	180.30	179.20	178.26
53	186.61	184.77	187.76	187.52	186.67	190.18
54	186.32	185.40	187.12	187.45	187.46	187.73
55	180.69	183.72	183.72	181.78	179.81	179.13
56	188.83	185.50	185.50	188.26	190.14	189.08
57	185.38	185.00	185.17	187.81	186.52	186.47
58	182.41	185.91	185.79	184.96	183.34	182.56
59	185.99	187.70	190.84	187.12	186.05	189.16
60	176.56	186.93	186.88	181.47	176.56	174.86

TABLE 24
Sample 4 Difference in Means Tests

PANEL 1: Mean Terminal Portfolio Values

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-0.1159	-0.9153	-0.0301	-0.1712	-0.3835
STD	4.8303	4.6343	3.0031	1.7668	3.7077
t	-0.1859	-1.5299	-0.0776	-0.7506	-0.8012

PANEL 2: Mean Residual Portfolio Values (Q)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	-0.1159	-0.9153	-0.0301	-0.1712	-0.3835
STD	4.8303	4.6343	3.0031	1.7668	3.7077
t	-0.1859	-1.5299	-0.0776	-0.7506	-0.8012

PANEL 3: Mean Absolute Deviations (MADs)

Pair	1-2	1-3	1-4	1-5	1-6
Mean	2.1389	0.3419	-0.0113	-0.8742	-2.0558
STD	2.825	1.6472	1.4779	2.538	2.6846
t	5.8647	1.6078	-0.0592	-2.6681	-5.9317

PANEL 4: Difference in Return to Volatility

Pair	1-2	1-3	1-4	1-5	1-6
Mean	0.2349	0.5275	0.4814	0.7455	0.6842
STD	1.8356	1.6381	0.8316	1.0203	0.9197
t	0.9912	2.4944	4.484	5.6597	5.7625

the exception of convexity maximization. Neither sample has provided evidence of difference between the performance of these two objective functions with respect to minimizing the volatility of returns.

Panel 4 of Table 24 summarizes the differences in return to volatility of the paired strategies. The critical t-value for rejection of the null hypotheses of no difference in means is +2.39 at the 0.01 level. Again the null is rejected for the pairing of strategy 1 with all alternatives except strategy 2.

Data Analysis for Difference in Proportions Test

The terminal values from the four Monte Carlo samples were combined into 240 total observations. Members of this group were randomly assigned to 20 subgroups of size 12. The residual value, Q = Terminal Value-Target Value, was computed for each observation. Each sample proportion, p, was computed by dividing the sum of observations with $Q<0$, by 12 (the number of total observations). A summary of the values of p is included as Table 25 below. The differences in average proportion of below target values are included as Table 26.

As an indicator of relative immunization efficacy, smaller expected proportions of outcomes below the target are desirable; higher proportions are less desirable. We test for differences between strategy 1, min M^2, and each of the five alternatives. Recall that the following hypotheses are examined:

$$H_0: \pi_1-\pi_j=0 \quad (j=2,\ldots,6)$$

$$H_1: \pi_1-\pi_j<0$$

Since the superiority of a strategy is indicated by smaller proportions of outcomes below the target value, the critical t-value for rejecting the null hypothesis is -1.729 at the 0.05

Table 25
Proportions of Terminal Values Below Target

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	0.833333	0.416666	0.25	0.75	0.833333	0.833333
2	0.583333	0.083333	0.166666	0.583333	0.583333	0.5
3	0.5	0.166666	0.333333	0.333333	0.416666	0.75
4	0.666666	0.416666	0.5	0.666666	0.75	0.833333
5	0.666666	0.25	0.083333	0.416666	0.5	0.416666
6	0.5	0.333333	0.25	0.333333	0.333333	0.333333
7	0.416666	0.25	0.416666	0.5	0.333333	0.666666
8	0.5	0.166666	0.166666	0.333333	0.25	0.5
9	0.5	0.25	0.166666	0.333333	0.583333	0.833333
10	0.833333	0.333333	0.416666	0.333333	0.916666	0.833333
11	0.583333	0.416666	0.25	0.583333	0.583333	0.666666
12	0.75	0.25	0.166666	0.583333	0.583333	0.5
13	0.5	0.416666	0.583333	0.583333	0.5	0.666666
14	0.583333	0.333333	0.333333	0.583333	0.583333	0.666666
15	0.583333	0.333333	0.166666	0.416666	0.5	0.5
16	0.5	0.083333	0	0.333333	0.5	0.583333
17	0.666666	0.25	0.333333	0.25	0.333333	0.416666
18	0.833333	0.416666	0.333333	0.5	0.833333	0.75
19	0.75	0.166666	0.333333	0.416666	0.75	0.666666
20	0.75	0.416666	0.333333	0.416666	0.666666	0.75
Mean	0.625	0.2875	0.279166	0.4625	0.566666	0.633333
STD	0.131066	0.113022	0.141201	0.136462	0.184565	0.156253

Table 26
Difference in Proportions Below Target Value

OBS	S1-S2	S1-S3	S1-S4	S1-S5	S1-S6
1	0.416666	0.583333	0.083333	0	0
2	0.5	0.416666	0	0	0.083333
3	0.333333	0.166666	0.166666	0.083333	-0.25
4	0.25	0.166666	0	-0.08333	-0.16666
5	0.416666	0.583333	0.25	0.166666	0.25
6	0.166666	0.25	0.166666	0.166666	0.166666
7	0.166666	0	-0.08333	0.083333	-0.25
8	0.333333	0.333333	0.166666	0.25	0
9	0.25	0.333333	0.166666	-0.08333	-0.33333
10	0.5	0.416666	0.5	-0.08333	0
11	0.166666	0.333333	0	0	-0.08333
12	0.5	0.583333	0.166666	0.166666	0.25
13	0.083333	-0.08333	-0.08333	0	-0.16666
14	0.25	0.25	0	0	-0.08333
15	0.25	0.416666	0.166666	0.083333	0.083333
16	0.416666	0.5	0.166666	0	-0.08333
17	0.416666	0.333333	0.416666	0.333333	0.25
18	0.416666	0.5	0.333333	0	0.083333
19	0.583333	0.416666	0.333333	0	0.083333
20	0.333333	0.416666	0.333333	0.083333	0
Mean	0.3375	0.345833	0.1625	0.058333	-0.00833
STD	0.136462	0.181922	0.163288	0.111803	0.170782
t	11.06049	8.501502	4.450550	2.333333	-0.21821

level. Significant values of t are underlined in Table 26. We are unable to reject the null for any pair of strategies. Based on this test, we can not conclude that the min M^2 objective results in lower percentage of outcomes below target than do either of the five alternatives evaluated.

Chapter Summary

This chapter has provided paired t-tests on four Monte Carlo samples of simulated performances on the alternative strategies. The performance measures examined are (1) Terminal portfolio value, a surrogate for holding period return, (2) Residual holding period value, (3) Mean absolute deviation of return, and (4) Return to volatility, a risk adjusted return measure. It also included a test of the difference in proportion of returns below target value for the paired strategies. The results of these tests are summarized below for each sample.

The min M^2 objective function (Strategy 1) is favored over strategies 2 and 3 on the size of residual returns by the results of Sample 1. It is favored over strategies 3,5, and 6 on MADs. In Sample 2, Strategy 1 is favored over Strategy 6 on terminal value, strategies 2 and 3 on residual returns, strategies 3 through 6 on the size of MADs, and over strategies 5 and 6 on return to volatility. In Sample 3, Strategy 1 is favored over all five alternatives on size of residuals, and all except Strategy 2 on both MADs and return to volatility. In Sample 4, Strategy 1 is favored over strategies 5 and 6 on MADs, and over all except strategy 2 on return to volatility. Finally, it is favored over all alternatives except Strategy 6 on frequency of returns at least equal to target return.

to volatility. In Sample 4, Strategy 1 is favored over strategies 5 and 6 on MADs, and over all except strategy 2 on return to volatility. Finally, it is favored over all alternatives

except Strategy 6 on frequency of returns at least equal to target return.

Sample 1

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S1 not	S1 favored	S1 not	S1 not
S1-S3	S1 not	S1 favored	S1 favored	S1 not
S1-S4	S1 not	S1 not	S1 not	S1 not
S1-S5	S1 not	S1 not	S1 favored	S1 not
S1-S6	S1 not	S1 not	S1 favored	S1 not

Sample 2

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S1 not	S1 favored	S1 not	S1 not
S1-S3	S1 not	S1 favored	S1 favored	S1 not
S1-S4	S1 not	S1 not	S1 favored	S1 not
S1-S5	S1 not	S1 not	S1 favored	S1 favored
S1-S6	S1 favored	S1 not	S1 favored	S1 favored

Sample 3

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S1 not	S1 favored	S1 not	S1 not
S1-S3	S1 not	S1 favored	S1 favored	S1 favored
S1-S4	S1 not	S1 favored	S1 favored	S1 favored
S1-S5	S1 not	S1 favored	S1 favored	S1 favored
S1-S6	S1 not	S1 favored	S1 favored	S1 favored

Sample 4

Pairing	Terminal Value	Residual Value	MADs	Return to Volatility
S1-S2	S1 not	S1 not	S1 not	S1 not
S1-S3	S1 not	S1 not	S1 not	S1 favored
S1-S4	S1 not	S1 not	S1 not	S1 favored
S1-S5	S1 not	S1 not	S1 favored	S1 favored
S1-S6	S1 not	S1 not	S1 favored	S1 favored

Proportions Test

Paired Strategies	S1-S2	S1-S3	S1-S4	S1-S5	S1-S6
min M^2 Favored	yes	yes	yes	yes	no

CHAPTER VII

SUMMARY AND SUGGESTIONS FOR ADDITIONAL RESEARCH

Summary

The primary purpose of this study is to design and execute a comprehensive reexamination of the Chambers, Carleton, and McEnally CCM (1988) duration vector bond portfolio immunization model. Such a study is warranted because the CCM methodology left their conclusions open to question. The specific problems were addressed in Chapter I. A second objective in this study is to rigorously examine the Fong and Vasicek (1984) M^2 minimization objective function for implementing the simple Macaulay duration immunization strategy. No comprehensive study of this issue has been forthcoming to date.

This study is presented in two parts. Part One addresses the primary objective. This is accomplished in two separate tests that are embodied in chapters III, IV, and V. Chapter III designs and executes a replication of the CCM (1988) duration vector tests. This replication is carried out with the min M^2 objective function, instead of the CCM quadratic minimization criterion, to implement the single factor duration model. In addition, portfolios were constrained to disallow negative weights, the behavioral equivalent of short selling.

A limitation on both the CCM test and the replication in Chapter III is that the models are evaluated over holding periods of only nine months. This is in contrast to the immunization tradition of five years or more. These tests are limited because of the paucity of quality time series data that can be accessed for samples of sufficient size. To overcome this problem, the second test in Part One employs Monte Carlo sampling procedures to generate large numbers of observations on the variables of interest and to execute repeat tests. The sample generation procedure and the return generating model are described and developed in Chapter IV. Four independent samples of size sixty are generated and evaluated in Chapter V. In addition, a test of the differences in the frequency of returns below the immunization target is performed on results generated by Monte Carlo sampling.

The second objective of this study is addressed in Part Two. It is embodied in Chapter VI. To assess its optimality, five alternative duration matched portfolio selection strategies are evaluated against the M^2 minimization model. The differences in performance are assessed by evaluating paired samples generated by the Monte Carlo method. A test of the proportion of returns less than target return is also executed.

Results: Part One

Differences in mean returns, residual returns, absolute deviation of returns, and return to volatility are investigated using paired t-tests. Hypotheses are evaluated at the 0.05 level of significance for both the CCM replication and the proportions test: they are evaluated at the 0.01 level for the Monte Carlo samples. The hypotheses are constructed such that rejection of the null indicates preference for the more complex strategy of the pair being investigated. The results of the CCM replication test, of the tests of the four Monte Carlo samples, and of the differences in proportions test are summarized in order below.

The CCM Replication

The null hypothesis of no difference in means is not rejected for the pairing of simple duration matching with any of the more complex duration vectors. In the pairings of duration vectors of successively higher order, we fail to reject the null in every case except two - terminal value between strategies 4 and 5, and return to volatility between strategies 3 and 4. The results of this test do not support the CCM (1988) conclusions.

Monte Carlo Sample 1

The null hypothesis of no difference in means is rejected for the pairing of simple duration (strategy 1) with each duration vector on the measurement variable, mean absolute deviation of return. It is rejected for pairs 1 and 2 only for the return to volatility measure. There is no support for the duration vectors on the important minimax measure, size of residual returns. For pairings of duration vectors, strategy 5 is favored over strategy 4 on both the mean absolute deviation variable, and on the return to volatility variable.

Monte Carlo Sample 2

The null hypothesis of no difference in means is not rejected for the pairing of strategy 1 with either duration vector on the residual return variable. It is rejected for the pairings of strategy 1 with strategies 2 through 6 on the terminal value and the mean absolute deviation variables. The null is rejected for pairings of strategy 1 with strategies 2, 4, and 6 on the return to volatility measure. For duration vector pairings, the null is rejected only for pairs 3-4 and 5-6 on the terminal value and on the return to volatility measures. The null is not rejected for any pairing of strategies on the size of residuals measure. In spite of this, the results of this test provide some support for the CCM conclusion that the duration vector outperforms simple duration matching.

Monte Carlo Sample 3

The null hypothesis of no difference in means is rejected in every pairing of strategy 1 with the duration vectors on the terminal value and the mean absolute deviation measures. It is rejected for all pairings of strategy 1 on the return to volatility measure with the exception of strategy 7. In addition, the results favor strategy 3 over strategy 2 on the terminal value, MADs, and return to volatility measures. Strategy 4 is favored over strategy 3 on the residual returns and the MADs measures. In every paired test, the more complex strategy is favored on the MADs measure of dispersion.

Monte Carlo Sample 4

In pairings of strategy 1 with alternative duration vectors, strategies 2, 4, and 5 are favored on the terminal value variable; strategies 2 through 5 are favored on the return to volatility measure, and each of the duration vectors is favored on the absolute deviation measure. We are unable to reject the null for any pairing of strategy 1 on the residual return measure. The results favor strategy 4 over strategy 3 on the terminal value and the return to volatility measures. Strategy 3 is favored over strategy 2, and strategy 6 over strategy 5 on the residual returns measure of performance.

Difference in Proportions Test

The hypothesis of no difference in proportion of returns below target is rejected in every test pairing strategy 1 with the duration vectors. The superiority of the duration vector model is, therefore, supported. In the comparisons of successive duration vectors, only strategy 4 is favored over strategy 3. The results of this test do not support the CCM conclusions 2 and 3.

Results: Part Two

The efficacy of the min M^2 objective function (strategy 1) for implementing the simple Macaulay duration model is evaluated against five alternatives. Differences in mean returns, residual returns, absolute deviation of returns, and return to volatility are investigated using paired t-tests. Hypotheses comparing strategy 1 with each of the five alternatives are evaluated at the 0.01 level of significance. Differences of returns below target are evaluated at the 0.05 level. The hypotheses are constructed such that rejection of the null indicates preference for the min M^2 objective function. Results of the tests of the four Monte Carlo samples, and of the differences in proportions are summarized in order below.

Monte Carlo Sample 1

The null hypothesis of no difference in means is rejected for the pairing of strategy 1 with strategy 2 (convexity maximization) and strategy 3 (M^2 maximization) on the residual return measure. It is rejected for the pairing with strategy 5 (maturity bullet) and strategy 6 (maturity barbell), as well as strategy 3, on the absolute deviation measure. The null is not rejected in any other comparisons.

Monte Carlo Sample 2

The null hypothesis of no difference in means is rejected for the pairing of strategy 1 with strategy 6 on the terminal value measure, with strategies 2 and 3 on the residual returns measure, with strategies 3, 4 (quadratic minimization), 5. and 6 on the mean absolute deviation measure, and with 5 and 6 on the return to volatility measure. Strategy 1 is favored over each alternative on at least one measure of performance. The evidence in support of strategy 1 is least convincing against the CCM (1988) quadratic minimization strategy where it is favored only on the absolute deviation variable.

Monte Carlo Sample 3

The null hypothesis of no difference in means is rejected in every pairing of strategy 1 on the important residual return measure. With the exception of convexity maximization, the null is rejected for each pairing of strategy 1 on the absolute deviation and the return to volatility measures. The results of this sample strongly support the hypothesis concerning the optimality of M^2 minimization for implementing simple duration matching.

Monte Carlo Sample 4

In pairings of strategy 1 with alternative objective functions, we are unable to reject the null for any test of the terminal value and residual return measures. The null is rejected for strategies 5 and 6 on the absolute deviation measure, and for strategies 3 through 6 on the return to volatility measure.

Difference in Proportions Test

The hypothesis of no difference in proportion of returns below target is rejected for the comparison of strategy 1 with strategies 2 through 5. The results of this test indicate no significant difference between strategy 1 and strategy 6 (maturity barbell).

Conclusions

Based on the composite results of the CCM replication and the Monte Carlo sampling tests of Part One, the following conclusions are justified:

1. With performance of the alternative strategies measured by size of expected residual returns, mean absolute deviation of returns, and return to volatility, the evidence from the quarterly data does not support the hypothesis that a two constraint duration vector model outperforms simple Macaulay duration matching implemented with the min M^2 objective function. Neither does that evidence support the hypothesis that duration vector strategies with successively higher numbers of constraints outperform immediately lower ones. Finally, this evidence does not support the hypothesis that a five to seven constraint duration vector strategy is optimal.

2. The evidence from the Monte Carlo samples on the five-year portfolios is mixed. With regard to the mean absolute deviation measure, superiority of the duration vectors over simple duration is supported by the evidence from each of the four samples. The return to variation tests also support, though not quite as strongly, the duration vector strategies. Conversely, the evidence from the residual returns tests fails to support the duration vectors in each sample. However, the proportion of returns below target tests strongly support the superiority of each duration vector strategy over simple duration. All of this taken together causes us to conclude that a two constraint duration vector outperforms simple duration matching implemented with the min M^2 objective function.

3. The results from the various tests do not support the CCM conclusion that immunization is incrementally improved by sequentially increasing the duration vector over two terms. Therefore, the hypothesis that the optimal duration vector strategy includes 5 to 7 terms is rejected. We conclude that the optimal duration vector strategy includes D1 and D2 only.

Based on the results of the tests in Part Two, we conclude that the min M^2 objective function is clearly superior to all strategies evaluated. Strategy 2, convexity maximization, appears to be the next most desirable objective function.

Implications and Suggestions for Further Research

The implications of this study for the implementation of immunization strategies are clear. Based on the results of Part Two, the optimal procedure for implementing a Macaulay duration matching strategy is selection with the min M^2 objective function. Approximate results can probably be achieved with the max Convexity objective. The least desirable objective functions appear to be the maturity bullet and the maturity barbell duration strategies advanced by Fooladi and Roberts (1992). Either the min M^2 or the max Convexity objective function should be used in empirical studies that compare more complex immunization strategies with simple duration matching.

The results of all the tests in Part One of this study clearly point to a two constraint duration vector including D1 and D2 as the best of extant multiple factor, deterministic, immunization strategies. There is no apparent advantage to employing duration vector models of any higher order. The two constraint duration vector, as claimed by CCM (1988), appears to be marginally superior to the best Macaulay duration matching strategy for the purpose of minimizing the magnitude of portfolio results below the immunization target (minimax strategy). Its expected benefits should be weighed against the cost of its greater complexity of implementation.

This study suggests a number of areas of additional inquiry. The most apparent starting point is a comprehensive study of the nature of the actual distribution of term structure innovations. This can be accomplished by fitting actual interest rate data to a variety of distributions for different time series and over the entire period for which data are available.

Another important opportunity for future research is to attempt to replicate the results of this study. It is preferable that such replication attempts follow the identification of the distribution of best fit to the term structure data. However, studies to achieve this are likely to take a long time. In the interim, it might be worthwhile to attempt replication under a wider range of distributional assumptions for additional Monte Carlo sampling. A wider variety of initial conditions from which to generate the Monte Carlo simulations is also desirable in future studies. Finally, the difference in proportions tests should be revisited using a larger number of paired observations and samples.

The Monte Carlo, or Latin Hypercube, sampling tools available with @Risk and other risk analysis packages have opened unlimited horizons for the empirical study of risk control procedures. While the focus of this dissertation has been on the relative efficacy of alternative immunization strategies, the methods employed here can be easily adapted to the study of alternative portfolio selection or optimization strategies under uncertainty for equities and derivative securities in addition to the fixed payment securities examined herein.

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APPENDIX I

Duration Specifications Examined in the BKST (1981) Study

Definition of Variables

$h^*(0,t)$= new term structure after an instantaneous shock.

$h(0,t)$= term structure before shock.

Φ= magnitude of term structure shock.

C_t= cash flow from bond at time t (t=1,...,n).

n= the number of cash payments received on bond.

P_0= initial bond price.

D= duration index

Alternative Duration Measures

1. Fisher and Weil additive shock (discrete compounding)

$h^*(0,t)=h(0,t)+\Phi$ $D=\Sigma C_t t[1+h(0,t)]^{-t-1} / P_0$; (t=1,...,n)

2. Bierwag multiplicative shock (discrete compounding)

$1+h^*(0,t)= \Phi[1+h(0,t)]$ $D=\Sigma C_t t[1+h(0,t)]^{-t} / P_0$

3. Khang additive term dependent shock (discrete compounding)

$h^*(0,t)= h(0,t) + [\Phi \ln(1+\alpha t)] / \alpha t$

$D= \Sigma C_t \ln(1+\alpha t)[1+h(0,t)]^{-t-1} / P_0$

α = an index representing the change in short term rates relative to long term rates given a shock, Φ.

4. Khang multiplicative term dependent shock (discrete compounding)

$h^*(0,t) = [1+\Phi \ln(1+\alpha t) / \alpha t][1+h(0,t)]$

$D= \Sigma C_t \ln(1+\alpha t)[1+h(0,t)]^{-t}$

APPENDIX II

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for the quarterly data in Chapter III.

II.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	1.568640	0.798126	-0.03195	-0.03563	-0.03563	-0.06474	-0.03563
2	-0.03370	-0.52145	-0.91580	-0.76777	-0.76777	-0.67245	-0.66413
3	-0.18138	-1.77501	-1.96058	-1.54154	-1.54643	-1.55264	-1.54154
4	0.039488	0.168197	0.501140	0.568134	0.538826	0.567735	0.550381
5	-0.91169	-0.80607	-0.80902	-0.81901	-0.82909	-0.81528	-0.81356
6	0.003651	-0.38316	-0.21881	-0.35218	-0.36035	-0.33311	-0.33572
7	-0.00933	-0.30538	-0.32148	-0.32751	-0.34354	-0.34679	-0.34842
8	-0.15893	0.460874	0.572665	0.536103	0.584267	0.553047	0.549704
9	0.023288	0.259342	0.367810	0.297832	0.293725	0.292984	0.293188
10	0.138649	-0.09048	-0.03774	-0.08862	-0.08583	-0.08417	-0.08545
11	-0.00341	0.035153	-0.14858	-0.06767	-0.06709	-0.06802	-0.06802
12	-0.04150	0.047166	-0.00239	-0.00052	-0.01292	-0.00113	-0.00113
13	0.702285	-0.30537	-1.62457	-0.55273	-1.62457	0.513373	-0.55882
14	0.176444	0.736671	0.561321	0.563549	0.561321	0.541593	0.541593
15	0.040961	0.440299	0.635415	0.635415	0.635415	0.635415	0.635415
16	-0.10494	-0.34130	0.154060	0.187509	0.190844	0.190844	0.187509
17	-2.68612	-0.23064	-0.47449	-0.44106	-0.44443	-0.44443	-0.44443
18	0.264646	-0.58628	-1.30427	-1.19926	-1.31475	-1.32291	-1.33129
19	-0.00036	-0.60168	-0.71891	-0.71994	-0.37545	-0.69089	-0.70277
20	-0.28217	0.250524	0.504929	0.540624	0.554671	0.518806	0.505494
21	-0.03073	0.767708	0.326802	0.339061	0.326802	0.305075	0.305075
22	0.011084	1.268900	1.201148	1.075074	1.022831	0.974511	0.989580
23	0.995215	1.093507	3.491711	3.470122	3.456992	3.432880	3.430464
24	-0.40721	0.801948	1.214731	1.225999	1.214731	1.268628	1.214731
25	2.396810	0.549455	1.158514	1.213042	1.106422	1.261535	1.110840
26	0.157438	1.973015	4.658815	4.600720	4.635127	4.600720	4.635127
27	-0.35731	2.257982	3.759388	3.788241	3.778528	3.789635	3.787876
28	0.860444	-0.62824	-1.45109	-1.46035	-1.45109	-1.44455	-1.43263
29	-0.00301	0.476166	0.167422	0.122207	0.129836	0.167689	0.169318
30	0.133861	0.009098	0.071818	0.120731	0.105084	0.046000	0.068697
31	0.031245	0.027706	0.088375	0.083397	0.102501	0.083137	0.083137
MEAN	0.075235	0.188603	0.303752	0.354641	0.305771	0.383949	0.344984
STD	0.807155	0.827669	1.484626	1.415383	1.459621	1.404751	1.413027

II.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	1.173749	0.289867	0.655362	0.709930	0.661060	0.768349	0.700273
2	1.855714	2.456839	2.966334	2.869198	2.820328	2.803185	2.755893
3	2.158057	3.865056	4.165777	3.797630	3.753645	3.838037	3.787973
4	2.718694	2.703353	2.485559	2.469454	2.449892	2.499161	2.477550
5	2.629175	2.636918	2.755019	2.815900	2.777114	2.841477	2.800798
6	0.171533	0.671722	0.622512	0.806778	0.766074	0.817009	0.780658
7	0.835083	0.425663	0.294415	0.237499	0.270337	0.188912	0.226243
8	1.662833	1.156395	1.159753	1.247204	1.150170	1.259568	1.223946
9	1.863003	1.740318	1.746998	1.867866	1.823102	1.902021	1.862851
10	2.064118	2.406620	2.469034	2.570801	2.519140	2.595660	2.557968
11	1.296292	1.371097	1.669986	1.639960	1.590511	1.669626	1.630661
12	0.360324	0.385018	0.549728	0.598742	0.562274	0.628667	0.589701
13	2.270357	1.149326	0.285016	0.735929	0.287035	1.772731	0.739498
14	3.178450	3.625309	3.334809	3.286149	3.332790	3.234885	3.273850
15	1.146430	1.432400	1.512367	1.461477	1.510347	1.432170	1.471135
16	6.278685	5.928963	6.309178	6.291738	6.343943	6.265765	6.301396
17	2.534001	4.876113	4.517115	4.499654	4.545155	4.466977	4.505942
18	0.993329	0.029031	0.804111	0.749989	0.816605	0.902945	0.872361
19	1.794594	1.079898	0.847524	0.795599	0.688961	0.795349	0.822430
20	2.375895	2.795230	2.934486	2.919291	2.982209	2.868165	2.893819
21	0.815648	1.500725	0.944669	0.906039	0.942650	0.842746	0.881711
22	0.169162	0.975285	0.792384	0.615420	0.612048	0.485550	0.539584
23	0.673413	0.688490	1.594565	1.522086	1.557826	1.455536	1.492086
24	1.257507	0.161711	0.135922	0.096301	0.133902	0.109622	0.094690
25	1.327865	0.632857	0.138947	0.135308	0.193058	0.116123	0.227853
26	0.233650	1.935858	4.506509	4.397525	4.480801	4.368217	4.441589
27	0.005964	2.495960	3.882217	3.860181	3.899338	3.832267	3.869473
28	1.072751	0.529310	1.467301	1.527456	1.469320	1.540960	1.490074
29	1.344265	0.978453	1.402345	1.498450	1.441951	1.482276	1.441682
30	2.292941	2.531072	2.583500	2.585477	2.552254	2.689516	2.627853
31	3.507490	3.624397	3.678877	3.734745	3.666770	3.764312	3.725346
MEAN	1.679386	1.841266	2.039107	2.040315	2.019375	2.072187	2.035706
STD	1.268510	1.465201	1.585665	1.547960	1.573871	1.525840	1.547435

II.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	3.660587	1.303019	-0.03300	-0.03858	-0.03745	-0.07142	-0.03901
2	-0.07864	-0.85133	-0.94597	-0.83128	-0.80700	-0.74183	-0.72716
3	-0.42327	-2.89788	-2.02518	-1.66904	-1.62543	-1.71282	-1.68786
4	0.092149	0.274599	0.517651	0.615123	0.566351	0.626306	0.602619
5	-2.12753	-1.31598	-0.83567	-0.88675	-0.87145	-0.89939	-0.89078
6	0.008520	-0.62556	-0.22601	-0.38131	-0.37876	-0.36747	-0.36758
7	-0.02178	-0.49857	-0.33207	-0.35459	-0.36109	-0.38256	-0.38149
8	-0.37088	0.752423	0.591532	0.580443	0.614114	0.610102	0.601878
9	0.054345	0.423401	0.379928	0.322464	0.308730	0.323210	0.321016
10	0.323553	-0.14772	-0.03899	-0.09595	-0.09022	-0.09286	-0.09356
11	-0.00795	0.057392	-0.15348	-0.07326	-0.07051	-0.07504	-0.07448
12	-0.09686	0.077003	-0.00247	-0.00056	-0.01358	-0.00125	-0.00124
13	1.638857	-0.49855	-1.67809	-0.59845	-1.70756	0.566335	-0.61186
14	0.411751	1.202689	0.579815	0.610159	0.589995	0.597467	0.592998
15	0.095587	0.718831	0.656350	0.687968	0.667874	0.700967	0.695724
16	-0.24491	-0.55721	0.159136	0.203018	0.200593	0.210532	0.205306
17	-6.26835	-0.37654	-0.49012	-0.47754	-0.46713	-0.49028	-0.48661
18	0.617579	-0.95716	-1.34724	-1.29845	-1.38191	-1.45939	-1.45765
19	-0.00084	-0.98231	-0.74259	-0.77949	-0.92017	-0.76216	-0.76947
20	-0.65849	0.409006	0.521565	0.585337	0.583006	0.572328	0.553472
21	-0.07172	1.253360	0.337569	0.367104	0.343497	0.336548	0.334031
22	0.025866	2.071604	1.240723	1.163990	1.075082	1.075046	1.083503
23	2.322441	1.785257	3.606753	3.757126	3.633590	3.787031	3.756059
24	-0.95028	1.309259	1.254752	1.327398	1.276784	1.399506	1.330024
25	5.593208	0.897040	1.196684	1.313369	1.162943	1.391681	1.216273
26	0.367400	3.221141	4.812309	4.981233	4.871909	5.075351	5.075060
27	-0.83382	3.686377	3.883248	4.101556	3.971552	4.180591	4.147393
28	2.007936	-1.02567	-1.49890	-1.58113	-1.52521	-1.59358	-1.56860
29	-0.00703	0.777387	0.172938	0.132314	0.136468	0.184988	0.185388
30	0.312379	0.014854	0.074185	0.130717	0.110453	0.050745	0.075217
31	0.072914	0.045233	0.091287	0.090294	0.107737	0.091714	0.091028
MEAN	0.358447	0.808595	1.091575	1.404407	1.735642	0.999126	0.970046
STD	1.004438	1.917054	2.874232	4.120316	5.197692	1.654977	1.991287

APPENDIX III

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 1 data in Chapter V.

III.a. Residual Values (O)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	17.49558	6.726919	8.792790	7.975561	12.18634	10.42423	10.98903
2	20.68408	7.407388	9.518045	10.01028	12.86244	10.47905	10.93710
3	-16.4880	-3.47806	7.831803	5.004412	0.462080	-0.54001	1.739915
4	3.088823	5.612250	-0.95624	-0.10710	3.786001	3.231300	4.773347
5	27.84791	6.950454	8.301688	8.708659	11.45648	11.68839	11.73398
6	-0.92399	2.377586	-8.86556	-8.15666	-0.78137	-1.96509	-0.96446
7	5.398364	7.512177	-8.34007	-5.90548	5.727367	6.319234	4.754084
8	15.01986	4.739819	2.196338	2.633335	3.981128	4.244420	4.204517
9	17.50116	6.848297	2.267497	4.164164	6.205867	7.469696	5.138460
10	57.84925	6.074427	17.48648	18.03960	5.716888	5.977744	6.623189
11	17.91941	4.888446	3.488060	3.484373	6.344952	6.358286	3.644944
12	15.54675	4.140405	6.359026	7.024848	4.436409	5.274109	3.767205
13	-0.87225	3.174919	2.602960	2.402803	6.454240	6.261844	8.119667
14	10.59840	6.432562	1.770242	3.008137	7.755368	7.156415	8.694197
15	-11.6086	-4.11145	5.600570	3.694249	-3.37905	-2.73179	-3.30320
16	-7.13444	5.590078	-15.4384	-13.4712	2.278922	2.314413	1.469235
17	-18.8660	-0.86219	-4.68176	-4.62711	-1.74803	-1.34763	-2.40577
18	-34.4168	-0.87867	-1.50922	-2.89860	7.250418	7.017865	0.767061
19	-9.64691	2.318316	-4.94973	-4.50891	2.963887	3.655725	2.168354
20	12.21288	7.147541	-3.68423	-4.10132	10.11755	7.925744	12.00025
21	-21.3919	-1.17613	-6.64558	-7.08974	-3.08799	-3.00750	-3.47914
22	16.42862	2.083387	5.895140	7.634241	1.056951	2.075380	-0.14247
23	21.92120	-1.13491	13.71288	12.48670	-0.14462	0.110811	-0.70475
24	-2.62622	-1.30916	0.288875	-0.62816	-3.11442	-4.01867	-3.18746
25	-14.2780	-2.85202	-1.22256	-1.21808	-5.96716	-5.10329	-6.66228
26	-1.35132	1.655877	-5.37332	-5.53450	0.968256	0.769685	0.640259
27	8.810341	6.749150	-11.3989	-9.69408	4.313209	3.533855	4.702014
28	-8.50781	7.145394	-11.0356	-9.19358	6.057678	6.088812	5.990342
29	3.689067	0.819675	7.963379	7.396700	1.989292	2.632523	1.426565
30	2.169800	0.951972	8.942982	8.404256	4.550440	3.050947	3.971210
31	2.636949	-0.94531	6.605780	5.590037	-1.29769	-1.29368	-0.03834
32	-8.03810	-3.33168	-8.48687	-10.4614	-3.37246	-4.26242	-3.21690
33	32.25029	8.505911	-2.38295	0.311871	4.155337	4.497099	3.306431
34	-1.37595	1.043480	-9.12212	-10.3316	-1.11171	-2.62518	-1.64976
35	-2.85307	-1.77676	-5.01606	-4.02770	-6.26030	-5.39603	-6.18963
36	-7.79362	1.354076	1.583027	1.594374	2.934052	2.730772	3.519846
37	22.46647	2.466101	5.432893	6.340438	0.597108	1.031747	0.904440
38	39.82262	7.794859	7.152721	7.883500	7.909214	6.701586	10.07343
39	49.22092	8.672385	5.195309	7.886515	5.611387	6.599261	5.707410
40	-5.50525	-2.89185	-6.70971	-7.47542	-5.17528	-4.80797	-6.64041
41	-9.18772	0.849082	-4.11774	-2.93878	-0.78306	0.613624	-2.51960
42	17.02839	7.063654	-3.54295	-2.77220	5.567487	5.558423	5.671595
43	-16.7481	-1.11090	-0.72460	-2.31296	-2.61133	0.888069	0.480593
44	22.00758	11.45352	0.284420	3.432709	12.13857	12.38364	13.77527
45	10.92496	1.710814	3.110357	2.602760	0.506339	0.29657	1.530112
46	-6.49730	-1.62862	0.494854	-2.66451	-0.05608	-1.52259	1.387401
47	-4.46935	1.525733	-0.24702	-0.02776	2.813827	1.973043	2.899690
48	-1.49839	2.162454	-5.74933	-4.67168	0.122182	0.974830	-0.32981
49	-11.4343	-0.18623	2.150466	2.226608	0.541962	1.061837	0.534032
50	-7.29077	-3.55495	-0.02089	-0.32893	-5.35794	-5.19369	-5.27543
51	27.11079	5.311922	4.088619	4.781999	5.462733	5.070181	6.137816
52	-24.1711	-2.36713	9.362891	3.281624	7.840670	5.255506	8.904409
53	13.50721	1.102679	1.742679	1.611316	-2.84572	-2.91294	-3.34220
54	31.72308	6.713144	0.450463	1.359013	3.983529	3.006550	4.483343
55	9.714827	3.854597	9.814008	9.377752	6.320602	6.126455	6.912025
56	2.549541	2.892729	6.148583	5.730506	5.144423	4.910176	6.725187
57	-5.33268	1.555531	-4.44656	-2.32071	-1.79184	-0.27986	-3.48919
58	-16.7051	-1.37666	6.953645	5.719995	2.104901	1.933354	2.332599
59	-6.89493	-1.56826	12.45051	12.04147	-1.71821	-1.19118	-1.54954

III.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	12.71213	4.197327	7.808541	6.890495	9.618226	7.908509	8.484800
2	15.90063	4.877796	8.533797	8.925218	10.29432	7.963336	8.432871
3	21.27147	6.007652	6.847555	3.919346	2.106034	3.055737	0.764315
4	1.694631	3.082658	1.940489	1.192166	1.217887	0.715578	2.269117
5	23.06446	4.420862	7.317439	7.623593	8.888371	9.172675	9.229754
6	5.707445	0.152005	9.849811	9.241735	3.349488	4.480818	3.468692
7	0.614910	4.982585	9.324327	6.990551	3.159252	3.803511	2.249854
8	10.23640	2.210227	1.212089	1.548269	1.413014	1.728698	1.700286
9	12.71770	4.318704	1.283248	3.079098	3.637753	4.953973	2.634230
10	53.06580	3.544835	16.50223	16.95454	3.148773	3.462022	4.118959
11	13.13595	2.358854	2.503811	2.399307	3.776838	3.842563	1.140714
12	10.76329	1.610813	5.374777	5.939782	1.868295	2.758386	1.262975
13	5.655704	0.645327	1.618712	1.317737	3.886125	3.746121	5.615436
14	5.814951	3.902970	0.785993	1.923071	5.187253	4.640692	6.189966
15	16.39209	6.641044	4.616321	2.609183	5.947170	5.247522	5.807434
16	11.91790	3.060486	16.42270	14.55628	0.289191	0.201308	1.034994
17	23.64946	3.391788	5.666013	5.712177	4.316150	3.863361	4.910006
18	39.20030	3.408270	2.493472	3.983673	4.682304	4.502142	1.737168
19	14.43037	0.211275	5.933982	5.593980	0.395773	1.140002	0.335876
20	7.429429	4.617949	4.668485	5.186392	7.549437	5.410021	9.496025
21	26.17535	3.705730	7.629838	8.174814	5.656104	5.523227	5.983379
22	11.64516	0.446205	4.910891	6.549175	1.511162	0.440342	2.646703
23	17.13775	3.664510	12.72863	11.40163	2.712743	2.404911	3.208980
24	7.409682	3.838756	0.695373	1.713230	5.682536	6.534402	5.691696
25	19.06148	5.381614	2.206817	2.303152	8.535276	7.619019	9.166518
26	6.134784	0.873714	6.357570	6.619570	1.599857	1.746037	1.863971
27	4.026886	4.219557	12.38318	10.77915	1.745095	1.018133	2.197784
28	13.29127	4.615802	12.01992	10.27865	3.489564	3.573089	3.486112
29	1.094387	1.709916	6.979131	6.311634	0.578822	0.116801	1.077664
30	2.613654	1.577619	7.958733	7.319190	1.982326	0.535225	1.466979
31	2.146505	3.474908	5.621531	4.504971	3.865807	3.809405	2.542572
32	12.82156	5.861282	9.471125	11.54650	5.940576	6.778148	5.721138
33	27.40684	5.976319	3.367203	0.773194	1.587222	1.981377	0.802201
34	6.159410	1.486112	10.10637	11.41673	3.679825	5.140910	4.153996
35	7.636533	4.306354	6.000312	5.112773	8.828418	7.911761	8.693862
36	12.57708	1.175515	0.598778	0.509308	0.365938	0.215049	1.015616
37	17.68301	0.063490	4.448644	5.255372	1.971006	1.483975	1.599789
38	35.03917	5.265267	6.168473	6.798434	5.341099	4.185864	7.569209
39	44.43747	6.142793	4.211060	6.801449	3.043273	4.083538	3.203180
40	10.28871	5.421448	7.693967	8.560489	7.743401	7.323693	9.144641
41	13.97118	1.680509	5.101994	4.023854	3.351175	1.902098	5.023833
42	12.24493	4.534062	4.527199	3.857273	2.999372	3.042700	3.167365
43	21.53164	3.640492	1.708849	3.398033	5.179449	1.627652	2.023637
44	17.22413	8.923930	0.699828	2.347643	9.570461	9.867926	11.27104
45	6.141513	0.818777	2.126109	1.517694	2.061774	2.219152	0.974117
46	11.28076	4.158216	0.489393	3.749581	2.624203	4.038321	1.116828
47	9.252814	1.003858	1.231272	1.112829	0.245713	0.542679	0.395460
48	6.281853	0.367137	6.733580	5.756748	2.445931	1.540892	2.834044
49	16.21782	2.715831	1.166217	1.141542	2.026152	1.453884	1.970198
50	12.07422	6.084546	1.005143	1.413998	7.926056	7.709418	7.779663
51	22.32733	2.782329	3.104370	3.696933	2.894618	2.554458	3.633586
52	28.95457	4.896725	8.378643	2.196558	5.272556	2.739783	6.400179
53	8.723759	1.426912	0.758430	0.526250	5.413837	5.428663	5.846439
54	26.93962	4.183552	0.533784	0.273947	1.415415	0.490827	1.979113
55	4.931372	1.325005	8.829760	8.292686	3.752488	3.610732	4.407795
56	2.233913	0.363137	5.164334	4.645440	2.576309	2.394453	4.220956
57	10.11614	0.974060	5.430816	3.405780	4.359956	2.795588	5.993427
58	21.48855	3.906254	5.969396	4.634929	0.463213	0.582368	0.171631
59	11.67838	4.097861	11.46626	10.95640	4.286327	3.706902	4.053773
60	10.98702	2.407245	3.301087	2.357529	0.553442	0.954007	0.700580

III.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	1.213814	2.047344	1.598747	1.506615	3.178904	2.919620	2.792837
2	1.435027	2.254446	1.730617	1.890982	3.355271	2.934976	2.779639
3	-1.14391	-1.05855	1.424016	0.945353	0.120537	-0.15124	0.442195
4	0.214297	1.708094	-0.17386	-0.02023	0.987608	0.905023	1.213135
5	1.932041	2.115377	1.509453	1.645100	2.988516	3.273687	2.982165
6	-0.06410	0.723620	-1.61197	-1.54082	-0.20382	-0.55038	-0.24511
7	0.374529	2.286338	-1.51643	-1.11556	1.494029	1.769891	1.208239
8	1.042053	1.442568	0.399348	0.497447	1.038509	1.188777	1.068568
9	1.214201	2.084285	0.412287	0.786627	1.618850	2.092113	1.305928
10	4.013484	1.848757	3.179478	3.407753	1.491296	1.674247	1.683268
11	1.243218	1.487803	0.634216	0.658212	1.655131	1.780829	0.926354
12	1.078607	1.260136	1.156229	1.327021	1.157272	1.477173	0.957426
13	-0.06051	0.966289	0.473282	0.453899	1.683640	1.753818	2.063594
14	0.735299	1.957756	0.321874	0.568249	2.023049	2.004369	2.209610
15	-0.80538	-1.25132	1.018322	0.697858	-0.88145	-0.76512	-0.83950
16	-0.49497	1.701345	-2.80709	-2.54476	0.594475	0.648221	0.373402
17	-1.30889	-0.26241	-0.85126	-0.87407	-0.45598	-0.37744	-0.61142
18	-2.38778	-0.26742	-0.27441	-0.54755	1.891329	1.965564	0.194947
19	-0.66928	0.705581	-0.89998	-0.85175	0.773153	1.023895	0.551082
20	0.847309	2.175361	-0.66988	-0.77475	2.639244	2.219843	3.049837
21	-1.48413	-0.35795	-1.20833	-1.33928	-0.80552	-0.84234	-0.88421
22	1.139790	0.634080	1.071883	1.442138	0.275714	0.581272	-0.03620
23	1.520856	-0.34541	2.493343	2.358787	-0.03772	0.031036	-0.17911
24	-0.18220	-0.39844	0.052524	-0.11866	-0.81242	-1.12555	-0.81008
25	-0.99058	-0.86801	-0.22229	-0.23010	-1.55658	-1.42933	-1.69320
26	-0.09375	0.503967	-0.97700	-1.04548	0.252577	0.215573	0.162720
27	0.611246	2.054110	-2.07261	-1.83125	1.125135	0.989762	1.195006
28	-0.59025	2.174707	-2.00656	-1.73670	1.580194	1.705355	1.522432
29	0.255941	0.249468	1.447940	1.397266	0.518922	0.737317	0.362558
30	0.150537	0.289733	1.626056	1.587597	1.187018	0.854509	1.009274
31	0.182947	-0.28770	1.201095	1.055980	-0.33851	-0.36233	-0.00974
32	-0.55767	-1.01400	-1.54312	-1.97620	-0.87973	-1.19382	-0.81756
33	2.237471	2.588782	-0.43328	0.058913	1.083952	1.259548	0.840322
34	-0.09546	0.317584	-1.65862	-1.95169	-0.28999	-0.73526	-0.41928
35	-0.19794	-0.54075	-0.91204	-0.76084	-1.63305	-1.51132	-1.57308
36	-0.54070	0.412114	0.287833	0.301183	0.765371	0.764835	0.894560
37	1.558686	0.750560	0.987835	1.197733	0.155760	0.288971	0.229861
38	2.762826	2.372373	1.300542	1.489224	2.063181	1.876981	2.560141
39	3.414865	2.639449	0.944636	1.489794	1.463775	1.848321	1.450525
40	-0.38194	-0.88013	-1.21999	-1.41213	-1.35001	-1.34661	-1.68764
41	-0.63742	0.258419	-0.74870	-0.55514	-0.20426	0.171864	-0.64035
42	1.181401	2.149830	-0.64419	-0.52368	1.452323	1.556803	1.441423
43	-1.16196	-0.33810	-0.13175	-0.43692	-0.68118	0.248730	0.122141
44	1.526849	3.485890	0.051714	0.648452	3.166444	3.468413	3.500955
45	0.757956	0.520688	0.565540	0.491671	0.132082	0.083063	0.388874
46	-0.45077	-0.49567	0.089976	-0.50333	-0.01463	-0.42644	0.352605
47	-0.31007	0.464358	-0.04491	-0.00524	0.734009	0.552610	0.736949
48	-0.10395	0.658145	-1.04537	-0.88249	0.031872	0.273030	-0.08382
49	-0.79329	-0.05668	0.391008	0.420615	0.141375	0.297399	0.135723
50	-0.50582	-1.08195	-0.00379	-0.06213	-1.39766	-1.45465	-1.34073
51	1.880901	1.616688	0.743412	0.903338	1.424997	1.420057	1.559912
52	-1.67695	-0.72043	1.702406	0.619911	2.045301	1.471962	2.263035
53	0.937107	0.335601	0.316862	0.304384	-0.74232	-0.81585	-0.84941
54	2.200894	2.043152	0.081905	0.256723	1.039135	0.842074	1.139431
55	0.673998	1.173150	1.784430	1.771494	1.648779	1.715898	1.756675
56	0.176882	0.880405	1.117965	1.082515	1.341964	1.375242	1.709191
57	-0.36997	0.473427	-0.80849	-0.43839	-0.46741	-0.07838	-0.88677
58	-1.15897	-0.41898	1.264345	1.080529	0.549080	0.541494	0.592824
59	-0.47835	-0.47730	2.263812	2.274682	-0.44820	-0.33362	-0.39381
60	1.094129	1.502531	-0.42125	-0.24037	0.525543	0.971802	0.458393

APPENDIX IV

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 2 data in Chapter V.

IV.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	6.917331	5.222730	3.354247	4.581497	2.238647	5.923981	5.109875
2	-1.06744	0.721846	-1.79041	-1.40303	-0.70064	1.837109	-2.97714
3	-1.81721	0.632680	-2.09680	-1.44070	-2.92559	1.596519	-6.78292
4	5.364254	3.399376	1.035970	3.230649	0.398494	3.302894	0.675031
5	6.542536	1.497511	0.598010	1.104182	0.204516	3.221487	-1.34079
6	-5.79927	-2.41052	-4.62275	-4.36857	-3.95109	1.948984	-2.05082
7	-16.6774	-5.89406	-10.2775	-8.76264	-11.6888	-9.41878	-11.7608
8	3.414616	2.205241	-0.63037	0.393794	-2.78051	0.633045	-2.07469
9	-4.78379	-1.97592	-2.62372	-2.03475	-1.61737	-1.01754	-3.64147
10	-9.61127	-3.22763	-7.84537	-6.00017	-7.15458	-6.40951	-7.90531
11	0.686928	0.601959	-1.98310	-1.21282	-2.54437	0.011178	-3.88364
12	4.398069	3.274767	-0.88093	2.166374	-1.37046	3.136491	0.449722
13	-15.4940	-4.67316	-7.46626	-7.94712	-8.92490	-6.71726	-9.88572
14	-7.88878	-2.37401	-2.32099	-1.99567	-5.22770	-0.94791	-4.43337
15	4.862140	4.134973	1.989169	3.389351	1.506262	2.711710	-1.88768
16	-11.8189	-3.59977	-6.26440	-6.30159	-7.44697	-5.15639	-10.5681
17	-0.57692	1.185943	-1.09245	-0.39523	-3.53831	0.702863	-6.18082
18	-9.70932	-4.54129	-5.20719	-6.74313	-6.31551	-4.02109	-7.75272
19	-12.6949	-3.58330	-6.33765	-6.47519	-7.53956	-5.34020	-9.78346
20	2.048792	0.504460	-0.16377	1.264982	-0.66487	2.042367	-1.05056
21	-4.96269	-1.58083	-4.50151	-3.08827	-5.13647	-2.98380	-7.22763
22	-18.0731	-5.45035	-7.75768	-7.00420	-7.73903	-7.02903	-10.5927
23	-0.44906	-2.18356	-4.92896	-0.91960	-3.58085	-2.31895	-4.86787
24	1.361128	1.801797	-1.46021	2.499305	-1.23208	0.916564	-4.90414
25	-3.98165	-0.05235	-2.72863	-0.59942	-3.81798	-1.45225	-8.09170
26	-4.50623	0.135922	-2.76346	-0.13682	-1.25966	-1.66161	-4.83138
27	-7.67427	-0.86236	-0.99754	-0.52272	-3.47188	0.017833	-1.77441
28	18.34189	10.45028	9.563438	13.28876	12.58247	12.97995	9.050284
29	-22.5779	-9.57899	-9.87716	-8.49537	-9.63035	-8.14885	-11.4503
30	3.856705	3.745861	2.839979	5.390419	3.913314	6.378785	2.569472
31	1.929327	3.678341	3.692734	6.167779	3.737671	5.856096	3.535167
32	-11.4561	-4.66077	-3.59149	-1.75702	-3.64606	-2.31781	-3.42990
33	-1.42509	1.328342	0.694561	1.582679	1.287728	4.305973	0.568349
34	-16.9282	-5.45660	-5.14214	-4.80404	-7.03171	-4.20266	-9.65713
35	-9.91084	-2.57278	-2.88878	2.301831	-2.46000	-0.65944	-3.47486
36	-2.59098	0.761026	0.445986	1.440145	-0.55259	4.092142	-0.02669
37	4.507094	4.676178	4.644837	7.252544	4.692066	9.865882	3.498082
38	-5.58584	0.260809	0.893278	1.864980	0.503799	3.492451	-1.45250
39	-11.9024	-1.84163	-2.38378	-0.96466	-2.82830	-0.23548	-4.92809
40	-0.29608	4.261813	3.628462	7.100010	4.450264	9.628638	4.229546
41	-2.74410	2.832959	4.116676	6.090606	3.816714	5.299307	1.985932
42	-3.99533	1.914772	-5.05841	5.436719	2.423418	6.387224	0.538426
43	-7.26970	-0.77053	-1.08599	1.730519	-2.84124	2.261846	-2.17653
44	-8.04176	-0.01708	-0.03984	2.420537	-0.00645	7.023526	-48.5750
45	3.849422	5.039201	6.571882	9.335678	7.100347	10.18086	-61.2066
46	-12.5900	-1.38556	-0.19175	-2.27114	-0.26445	2.011080	-3.41012
47	-6.78498	-9.95980	-8.18765	-8.55951	-10.8698	-7.43570	-9.84349
48	-5.07197	1.806752	3.234564	2.439735	2.025185	5.015147	2.900542
49	-17.8173	-5.84019	-6.54842	-5.67236	-6.94552	-4.12100	-8.18961
50	1.205273	4.877825	4.266443	7.948823	5.914993	10.44292	3.054212
51	-0.37241	3.068469	4.262623	5.960564	3.184659	8.331799	0.706065
52	-3.56997	1.606305	3.547114	5.185566	2.417566	5.468437	0.361697
53	-4.56225	0.796879	0.608294	2.495206	0.624717	2.669979	-1.26802
54	-1.84528	0.110294	-1.08369	-1.82266	-1.14000	0.535412	-2.82383
55	5.900170	4.207781	1.560327	4.590603	2.689633	7.548416	2.024705
56	2.478703	2.257227	-1.09835	-0.75615	0.633683	3.722450	1.048268
57	-10.0940	-3.14235	-5.39548	-7.39569	-6.42446	-2.73998	-6.71217
58	-7.40256	-2.10563	-4.39534	-6.27102	-4.78748	-3.35824	-7.65831
59	-2.97242	-0.55733	-3.26821	-2.00303	-3.35516	0.857036	-4.59071
60	-11.9074	-8.34229	-10.7000	-10.5428	-11.0182	-9.92883	-12.8169

IV.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	11.07795	5.483404	4.956412	4.748389	4.040066	4.845048	10.07046
2	3.093173	0.982520	0.188252	1.236145	1.100771	0.758176	1.983450
3	2.343410	0.893354	0.494642	1.273813	1.124175	0.517586	1.822328
4	9.524876	3.660050	2.638134	3.397541	2.199913	2.223961	5.635626
5	10.70315	1.758185	2.200174	1.271074	2.005935	2.142554	3.619797
6	1.638656	2.149854	3.020589	4.201687	2.149677	0.870051	2.909765
7	12.51679	5.633395	8.675424	8.595753	9.887385	10.49772	6.800234
8	7.575238	2.465915	0.971788	0.560686	0.979097	0.445887	2.885899
9	0.623169	1.715247	1.021555	1.867867	0.184043	2.096477	1.319116
10	5.450651	2.966956	6.243213	5.833285	5.353166	7.488448	2.944722
11	4.847550	0.862633	0.380942	1.045933	0.742959	1.067754	1.076948
12	8.558691	3.535441	0.721232	2.333267	0.430950	2.057558	5.410316
13	11.33346	4.412490	5.864102	7.780233	7.123486	7.796201	4.925126
14	3.728168	2.113337	0.718830	1.828786	3.426285	2.026845	0.527222
15	9.022762	4.395647	3.591333	3.556243	3.307681	1.632777	3.072904
16	7.658376	3.339102	4.662244	6.134705	5.645556	6.235325	5.607598
17	3.583701	1.446617	0.509706	0.228338	1.736894	0.376069	1.220227
18	5.548707	4.280618	3.605026	6.576243	4.514097	5.100027	2.792129
19	8.534282	3.322631	4.735494	6.308299	5.738146	6.419138	4.822872
20	6.209413	0.765134	1.438388	1.431874	1.136539	0.963434	3.910031
21	0.802076	1.320165	2.899355	2.921383	3.335053	4.062737	2.267037
22	13.9124[illegible]	5.1[illegible]9685	6.155515	6.837316	5.937615	8.107968	5.632124
23	3.711555	1.922888	3.326802	0.752716	1.779440	3.397891	0.092720
24	5.521750	2.062471	0.141952	2.666198	0.569331	0.162369	0.056449
25	0.178970	0.208318	1.126465	0.432531	2.016569	2.531185	3.131105
26	0.345610	0.396596	1.161296	0.030065	0.541751	2.740549	0.129206
27	3.513656	0.601687	0.604617	0.355834	1.670468	1.061099	3.186174
28	22.50251	10.71095	11.16560	13.45565	14.38389	11.90101	14.01087
29	18.41729	9.318316	8.275001	8.328485	7.828936	9.227789	6.489751
30	8.017327	4.006535	4.442143	5.557311	5.714732	5.299852	7.530066
31	6.089948	3.939015	5.294898	6.334671	5.539089	4.777163	8.495762
32	7.295484	4.400096	1.989334	1.590128	1.844647	3.396746	1.530685
33	2.735526	1.589016	2.296725	1.749571	3.089147	3.227040	5.528943
34	12.76759	5.195931	3.539982	4.637155	5.230292	5.281602	4.696542
35	5.750227	2.312112	1.286622	2.468723	0.658587	1.738375	1.485727
36	1.569637	1.021700	2.048150	1.607037	1.248821	3.013209	4.933896
37	8.667716	4.936852	6.247001	7.419436	6.493484	8.786949	8.458677
38	1.425226	0.521483	2.495442	2.031872	2.305218	2.413518	3.508088
39	7.741788	1.580965	0.781624	0.797776	1.026882	1.314418	0.032502
40	3.864537	4.522487	5.230626	7.266902	6.251683	8.549705	9.190140
41	1.416514	3.093633	5.718840	6.257498	5.618133	4.220374	6.946527
42	0.165290	2.175446	3.456255	5.603611	4.224837	5.308291	5.499021
43	3.109084	0.509860	0.516165	1.897411	1.039830	1.182913	2.784056
44	3.881145	0.243587	1.562320	2.587429	1.794961	5.944592	43.61444
45	8.010043	5.299875	8.174046	9.502570	8.901766	9.101931	56.24600
46	8.429396	1.124891	1.410408	2.104257	1.536968	0.932147	1.550469
47	2.624365	9.699130	6.585491	8.392623	9.068437	8.514634	4.882902
48	0.911356	2.067426	4.836728	2.606627	3.826604	3.936214	7.861136
49	13.65669	5.579516	4.946256	5.505469	5.144110	5.199937	3.229019
50	5.365895	5.138499	5.868607	8.115715	7.716412	9.363991	8.014806
51	3.788205	3.329143	5.864787	6.127456	4.986078	7.252866	5.666660
52	0.590650	1.866979	5.149278	5.352458	4.218984	4.389504	5.322292
53	0.401633	1.057553	2.210459	2.662098	2.426135	1.591046	3.692571
54	2.315338	0.370963	0.518470	1.655777	0.661410	0.543521	2.136759
55	10.06079	4.468455	3.162491	4.757495	4.491052	6.469483	6.985299
56	6.639325	2.517901	0.503808	0.589266	2.435102	2.643517	6.008863
57	5.933460	2.881677	3.793316	7.228807	4.623050	3.818921	1.751578
58	3.241939	1.844958	2.793184	6.104134	2.986070	4.437174	2.697719
59	1.188196	0.296664	1.666050	1.836144	1.553741	0.221896	0.369877
60	7.746859	8.081625	9.097874	10.37599	9.216840	11.00776	7.856334

IV.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	1.159720	1.706889	0.981819	1.114205	0.592331	1.406937	0.883909
2	-0.17896	0.235913	-0.52407	-0.34121	-0.18538	0.436311	-0.51498
3	-0.30466	0.206772	-0.61375	-0.35037	-0.77409	0.379171	-1.17331
4	0.899340	1.110982	0.303238	0.785683	0.105438	0.784433	0.116767
5	1.096884	0.489415	0.175043	0.268533	0.054113	0.765099	-0.23193
6	-0.97227	-0.76780	-1.35312	-1.06242	-1.04543	0.462881	-0.35475
7	-2.79604	-1.92629	-3.00834	-2.13104	-3.09278	-2.23694	-2.03439
8	0.572475	0.720715	-0.18451	0.095769	-0.73570	0.150347	-0.35888
9	-0.80202	-0.64576	-0.76798	-0.49484	-0.42794	-0.24166	-0.62990
10	-1.61137	-1.05485	-2.29641	-1.45922	-1.89305	-1.52225	-1.36746
11	0.115166	0.196731	-0.58047	-0.29495	-0.67322	0.002654	-0.67179
12	0.737355	1.070257	-0.25785	0.526855	-0.36261	0.744912	0.077793
13	-2.59764	-1.52728	-2.18544	-1.93271	-2.36147	-1.59534	-1.71003
14	-1.32258	-0.77587	-0.67937	-0.48534	-1.38321	-0.22512	-0.76688
15	0.815158	1.351389	0.582248	0.824279	0.398547	0.644027	-0.32653
16	-1.98150	-1.17647	-1.83365	-1.53252	-1.97042	-1.22463	-1.82809
17	-0.09672	0.387589	-0.31977	-0.09611	-0.93621	0.166929	-1.06916
18	-1.62781	-1.48418	-1.52419	-1.63990	-1.67104	-0.95500	-1.34107
19	-2.12835	-1.17109	-1.85509	-1.57474	-1.99491	-1.26829	-1.69235
20	0.343488	0.164867	-0.04793	0.307639	-0.17592	0.485059	-0.18172
21	-0.83201	-0.51664	-1.31763	-0.75105	-1.35907	-0.70864	-1.25024
22	-3.03003	-1.78128	-2.27074	-1.70340	-2.04769	-1.66938	-1.83233
23	-0.07528	-0.71363	-1.44275	-0.22364	-0.94747	-0.55074	-0.84204
24	0.228198	0.588862	-0.42741	0.607823	-0.32600	0.217682	-0.84832
25	-0.66754	-0.01711	-0.79869	-0.14577	-1.01021	-0.34490	-1.39970
26	-0.75548	0.044421	-0.80889	-0.03327	-0.33329	-0.39463	-0.83573
27	-1.28662	-0.28183	-0.29199	-0.12712	-0.91863	0.004235	-0.30694
28	3.075096	3.415355	2.799307	3.231784	3.329239	3.082721	1.565524
29	-3.78528	-3.13060	-2.89113	-2.06604	-2.54812	-1.93534	-1.98068
30	0.646593	1.224220	0.831288	1.310932	1.035437	1.514953	0.444468
31	0.323459	1.202153	1.080897	1.499983	0.988963	1.390815	0.611515
32	-1.92066	-1.52323	-1.05126	-0.42730	-0.96472	-0.55047	-0.59330
33	-0.23892	0.434128	0.203304	0.384902	0.340724	1.022663	0.098313
34	-2.83808	-1.78332	-1.50515	-1.16832	-1.86054	-0.99812	-1.67049
35	-1.66159	-0.84083	-0.84557	0.559797	-0.65090	-0.15661	-0.60108
36	-0.43438	0.248718	0.130544	0.350238	-0.14621	0.971878	-0.00461
37	0.755633	1.528266	1.359587	1.763795	1.241489	2.343134	0.605100
38	-0.93649	0.085237	0.261470	0.453557	0.133301	0.829452	-0.25125
39	-1.99548	-0.60188	-0.69775	-0.23460	-0.74834	-0.05592	-0.85246
40	-0.04963	1.392843	1.062084	1.726699	1.177510	2.286789	0.731629
41	-0.46006	0.925866	1.204989	1.481215	1.009877	1.258578	0.343527
42	-0.66983	0.625784	-1.48064	1.322192	0.641220	1.516957	0.093137
43	-1.21879	-0.25182	-0.31788	0.420856	-0.75177	0.537185	-0.37649
44	-1.34823	-0.00558	-0.01166	0.588666	-0.00170	1.668078	-8.40254
45	0.645372	1.646908	1.923651	2.270406	1.878705	2.417942	-10.5875
46	-2.11077	-0.45282	-0.05612	-0.55233	-0.08997	0.477628	-0.58988
47	-1.13753	-3.25505	-2.39660	-2.08164	-2.87609	-1.76596	-1.70273
48	-0.85033	0.590481	0.946787	0.593335	0.535850	1.191090	0.501737
49	-2.98714	-1.90868	-1.91678	-1.37950	-1.83774	-0.97873	-1.41664
50	0.202069	1.594168	1.248827	1.933128	1.565068	2.480181	0.528319
51	-0.06243	1.002835	1.247709	1.449590	0.842639	1.978791	0.122135
52	-0.59852	0.524971	1.038273	1.261113	0.639672	1.298746	0.062566
53	-0.76488	0.260435	0.178053	0.606826	0.165296	0.634116	-0.21934
54	-0.30936	0.036046	-0.31720	-0.44326	-0.30163	0.127159	-0.48846
55	0.989188	1.375184	0.456722	1.116419	0.711659	1.792739	0.350234
56	0.415565	0.737705	-0.32149	-0.18389	0.167668	0.884077	0.181330
57	-1.69231	-1.02698	-1.57930	-1.79861	-1.69987	-0.65074	-1.16107
58	-1.24107	-0.68816	-1.28655	-1.52509	-1.26673	-0.79757	-1.32473
59	-0.49833	-0.18214	-0.95663	-0.48713	-0.88775	0.203544	-0.79410
60	-1.99634	-2.72642	-3.13200	-2.56399	-2.91535	-2.35808	-2.21708

APPENDIX V

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 3 data in Chapter V.

V.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	-4.72870	0.709058	1.512966	0.974348	0.157825	-0.57218	-1.53620
2	-4.80794	1.061238	2.216953	1.907228	0.839930	0.209591	-0.92524
3	-3.77382	1.608691	2.264047	0.126363	0.601910	0.185206	0.490931
4	-3.53622	1.593360	2.550103	0.329118	1.016686	0.457912	1.000207
5	-2.52905	2.282341	2.806983	0.708329	-0.53060	0.833326	-0.83654
6	-3.89033	1.845647	1.924561	-0.16464	0.367435	-0.08518	-1.54607
7	-4.25485	1.915738	2.420366	2.470239	1.010885	2.442373	-2.35665
8	-4.73455	1.819455	2.461623	0.520247	0.980723	0.554700	-0.95754
9	-4.36255	2.329170	2.870095	0.947630	-0.68238	0.976561	-1.01586
10	-7.22922	0.916643	1.401767	1.127449	-0.40428	1.122900	-3.91477
11	-9.46526	0.220064	0.909185	0.654485	-0.75820	0.695012	-5.43232
12	-3.75711	2.166749	2.619963	0.656243	-0.84188	0.683235	-0.67829
13	-4.64401	1.125997	3.971917	2.546471	1.105339	-1.10211	-4.62565
14	-5.60002	1.532308	3.585784	3.661681	1.911834	-0.22595	-1.78575
15	-5.68053	0.922031	2.674923	0.637764	0.998177	-1.21053	-2.10985
16	-8.44761	0.857762	3.023329	0.992813	1.213446	-0.92077	-1.91195
17	-3.62239	2.090904	2.476165	0.542997	2.963801	0.605279	3.428477
18	-4.93532	2.341044	4.822861	5.071290	2.971699	1.000840	-5.28589
19	-8.05870	0.547480	0.635497	3.133876	1.665039	-0.54452	-3.88483
20	-6.89330	1.368962	4.001587	1.582493	1.647787	1.401242	-5.27978
21	-5.51360	1.546587	2.922291	0.185881	2.503081	0.209434	-1.09293
22	-7.71464	1.978078	5.629496	1.273805	0.879461	2.766552	1.279452
23	-6.20316	1.462658	3.844922	3.389884	1.714723	-0.48046	-0.57712
24	-6.24982	0.949252	3.879757	0.920808	-0.50197	-0.85630	-3.75687
25	-8.31950	1.998919	4.806827	4.472504	1.992709	0.235146	-2.58930
26	-4.53034	0.785006	4.610227	3.227405	2.464778	1.668741	2.499413
27	0.542501	3.087406	6.002994	4.919763	4.389266	1.522032	4.738155
28	-4.16130	1.934519	5.389527	2.210440	3.676478	0.764250	2.207090
29	-4.84327	-0.24325	2.744137	-0.30560	1.072726	0.088129	-4.02033
30	3.795195	2.441243	6.136245	0.890694	4.385728	1.431376	-0.79900
31	6.310616	2.014297	5.217965	1.990535	3.582209	0.617080	2.097800
32	-5.64407	0.340872	3.550626	2.204662	1.996561	0.850107	0.751884
33	-2.58018	0.174873	3.448655	0.222762	1.694577	0.664260	-1.49976
34	-3.09058	1.072516	3.922209	2.630627	2.335493	-0.60666	1.053509
35	-1.42496	1.573082	4.792452	1.439832	3.258579	0.218489	0.047814
36	2.899137	1.759931	5.487159	2.459654	3.730379	0.937929	0.199206
37	1.372939	1.250706	5.040280	1.943421	3.325029	2.324486	-0.45094
38	0.497783	0.908827	4.771270	1.324888	2.899200	0.016933	-0.37402
39	-0.86169	1.357051	4.403928	1.157823	2.846770	-0.14763	-0.16752
40	3.862621	1.531031	5.091517	1.793349	3.358557	2.303563	-3.62467
41	2.472953	2.565674	6.061461	3.485651	4.415118	1.434104	1.115253
42	2.104841	1.704124	5.256247	4.809512	3.635155	0.730352	-1.46547
43	-6.78124	-0.77292	2.512040	-0.21100	0.746727	-2.22838	-4.14613
44	4.804330	2.331609	6.209999	2.811765	4.231468	1.411395	0.957214
45	-0.21425	1.047324	3.998805	2.611102	2.228212	-0.63735	-3.00541
46	-0.23782	1.757366	4.962950	1.621291	3.323551	0.326884	2.181278
47	-6.13165	0.695153	5.188922	2.459287	2.273904	-0.68134	-4.48586
48	0.553052	1.579231	4.897650	1.622889	3.263420	2.171279	-1.71290
49	-9.98443	-1.16374	-0.00916	-3.53240	0.078169	-2.85945	-1.14263
50	3.954143	2.107621	5.648614	2.377948	3.873850	0.974521	4.373816
51	4.351011	2.985294	6.766741	5.710132	5.221586	2.310705	2.042524
52	-0.83854	2.569747	4.754631	1.271965	2.952885	-0.01248	1.579649
53	1.645392	2.809436	4.491249	3.457498	2.986731	0.073920	-1.92799
54	-2.09444	2.106574	3.959078	0.614585	2.326427	-0.66417	-0.49626
55	6.649548	2.962468	6.495443	3.376352	4.799897	3.822826	3.501491
56	-0.57606	1.598023	3.067185	-0.26758	3.375427	0.329877	4.084185
57	-2.79813	0.928968	4.185718	0.894557	2.677905	-0.36070	-0.35433
58	-2.72015	1.021269	3.291322	2.068822	3.606866	2.504544	-2.16327
59	0.590268	1.412713	3.218493	0.113327	1.559259	0.539703	0.094913
60	-1.16688	1.166365	1.647026	2.917466	2.810572	-0.23041	-2.05872

V.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	2.174942	0.767450	2.311160	0.775530	1.979284	1.072016	0.765029
2	2.254175	0.415270	1.607172	0.157349	1.297179	0.290244	0.154074
3	1.220060	0.132182	1.560079	1.623515	1.535200	0.314629	1.262107
4	0.982453	0.116851	1.274023	1.420760	1.120423	0.041923	1.771382
5	0.024716	0.805832	1.017143	1.041549	2.667718	0.333490	0.065368
6	1.336569	0.369138	1.899565	1.914519	1.769675	0.585019	0.774902
7	1.701092	0.439228	1.403759	0.720360	1.126225	1.942536	1.585480
8	2.180790	0.342946	1.362503	1.229631	1.156387	0.054863	0.186368
9	1.808785	0.852661	0.954030	0.802248	2.819492	0.476725	0.244690
10	4.675461	0.559865	2.422359	0.622429	2.541393	0.623064	3.143601
11	6.911494	1.256444	2.914940	1.095393	2.895316	0.195176	4.661152
12	1.203343	0.690240	1.204162	1.093635	2.978998	0.183399	0.092879
13	2.090249	0.350511	0.147790	0.796592	1.031770	1.601950	3.854479
14	3.046254	0.055798	0.238342	1.911802	0.225276	0.725794	1.014577
15	3.126764	0.554477	1.149203	1.112114	1.138933	1.710367	1.338677
16	5.893843	0.618747	0.800797	0.757064	0.923663	1.420611	1.140775
17	1.068627	0.614395	1.347961	1.206881	0.826690	0.105443	4.199652
18	2.381555	0.864535	0.998734	3.321411	0.834589	0.501004	4.514721
19	5.504938	0.929028	3.188628	1.383997	0.472071	1.044365	3.113663
20	4.339541	0.107546	0.177461	0.167385	0.489322	0.901406	4.508606
21	2.959839	0.070077	0.901835	1.563997	0.365970	0.290401	0.321761
22	5.160879	0.501568	1.805370	0.476073	1.257649	2.266716	2.050627
23	3.649395	0.013850	0.020796	1.640005	0.422386	0.980296	0.194046
24	3.696055	0.527256	0.055631	0.829070	2.639080	1.356138	2.985700
25	5.765733	0.522410	0.982701	2.722626	0.144401	0.264690	1.818128
26	1.976578	0.691502	0.786100	1.477526	0.327667	1.168905	3.270589
27	3.096269	1.610897	2.178867	3.169884	2.252156	1.022196	5.509330
28	1.607540	0.458010	1.565401	0.460561	1.539367	0.264414	2.978265
29	2.289503	1.719767	1.079989	2.055479	1.064383	0.411706	3.249156
30	6.348962	0.964733	2.312118	0.859184	2.248617	0.931540	0.027833
31	8.864383	0.537788	1.393838	0.240656	1.445098	0.117244	2.868976
32	3.090304	1.135636	0.273500	0.454783	0.140549	0.350271	1.523060
33	0.026418	1.301635	0.375470	1.527116	0.442532	0.164424	0.728589
34	0.536817	0.403992	0.098083	0.880748	0.198382	1.106504	1.824684
35	1.128804	0.096573	0.968325	0.310046	1.121468	0.281346	0.818990
36	5.452904	0.283422	1.663032	0.709775	1.593268	0.438093	0.970382
37	3.926706	0.225802	1.216154	0.193542	1.187918	1.824650	0.320232
38	3.051551	0.567681	0.947144	0.424990	0.762090	0.482902	0.397153
39	1.692073	0.119457	0.579802	0.592055	0.709659	0.647472	0.603651
40	6.416388	0.054522	1.267390	0.043470	1.221446	1.803726	2.853495
41	5.026720	1.089165	2.237334	1.735772	2.278007	0.934268	1.886429
42	4.658608	0.227615	1.432120	3.059633	1.498044	0.230516	0.694301
43	4.227476	2.249438	1.312085	1.960886	1.390383	2.728217	3.374960
44	7.358097	0.855100	2.385873	1.061886	2.094357	0.911558	1.728389
45	2.339515	0.429184	0.174678	0.861223	0.091102	1.137188	2.234241
46	2.315945	0.280857	1.138823	0.128587	1.186440	0.172952	2.952454
47	3.577884	0.781355	1.364795	0.709408	0.136793	1.181177	3.714688
48	3.106819	0.102722	1.073523	0.126989	1.126309	1.671442	0.941724
49	7.430669	2.640254	3.833291	5.282288	2.058941	3.359289	0.371456
50	6.507910	0.631112	1.824488	0.628069	1.736740	0.474685	5.144991
51	6.904778	1.508785	2.942615	3.960253	3.084475	1.810869	2.813700
52	1.715223	1.093238	0.930504	0.477913	0.815774	0.512321	2.350825
53	4.199160	1.332927	0.667122	1.707619	0.849620	0.425915	1.156821
54	0.459319	0.630065	0.134951	1.135293	0.189317	1.164006	0.274914
55	9.203315	1.485959	2.671317	1.626473	2.662786	3.322990	4.272666
56	1.977701	0.121513	0.756941	2.017461	1.238316	0.169958	4.855360
57	0.244366	0.547540	0.361591	0.855321	0.540794	0.860543	0.416842
58	0.166390	0.455239	0.532804	0.318943	1.469755	2.004708	1.392102
59	3.144035	0.063795	0.605633	1.636551	0.577851	0.039867	0.866088
60	1.386886	0.310143	2.177099	1.167587	0.673462	0.730249	1.287546

V.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	-1.41427	1.077438	1.178797	0.787416	0.123602	-0.63410	-0.79160
2	-1.43797	1.612589	1.727295	1.541320	0.657797	0.232275	-0.47677
3	-1.12868	2.444463	1.763986	0.102120	0.471390	0.205251	0.252976
4	-1.05762	2.421167	1.986861	0.265975	0.796225	0.507472	0.515405
5	-0.75639	3.468098	2.187004	0.572433	-0.41554	0.923516	-0.43106
6	-1.16353	2.804525	1.499483	-0.13305	0.287759	-0.09440	-0.79669
7	-1.27255	2.911031	1.885780	1.996316	0.791681	2.706709	-1.21438
8	-1.41602	2.764726	1.917924	0.420436	0.768060	0.614735	-0.49342
9	-1.30476	3.539256	2.236177	0.765824	-0.53441	1.082254	-0.52347
10	-2.16213	1.392872	1.092158	0.911144	-0.31661	1.244432	-2.01727
11	-2.83089	0.334396	0.708373	0.528920	-0.59379	0.770233	-2.79927
12	-1.12368	3.292451	2.041292	0.530341	-0.65933	0.757181	-0.34952
13	-1.38894	1.710992	3.094639	2.057922	0.865654	-1.22139	-2.38359
14	-1.67486	2.328396	2.793792	2.959176	1.497266	-0.25041	-0.92019
15	-1.69894	1.401059	2.084112	0.515407	0.781729	-1.34154	-1.08720
16	-2.52653	1.303399	2.355566	0.802339	0.950319	-1.02043	-0.98522
17	-1.08339	3.177202	1.929254	0.438821	2.321121	0.670788	1.766689
18	-1.47606	3.557299	3.757635	4.098347	2.327307	1.109160	-2.72381
19	-2.41021	0.831915	0.495135	2.532632	1.303987	-0.60346	-2.00185
20	-2.06166	2.080187	3.117756	1.278886	1.290476	1.552898	-2.72066
21	-1.64902	2.350093	2.276844	0.150219	1.960305	0.232101	-0.56318
22	-2.30731	3.005759	4.386109	1.029421	0.688756	3.065975	0.659299
23	-1.85525	2.222560	2.995694	2.739523	1.342897	-0.53246	-0.29739
24	-1.86921	1.442422	3.022835	0.744148	-0.39312	-0.94897	-1.93591
25	-2.48821	3.037428	3.745143	3.614440	1.560604	0.260595	-1.33426
26	-1.35494	1.192844	3.591966	2.608217	1.930308	1.849348	1.287944
27	0.162252	4.691422	4.677112	3.975891	3.437485	1.686761	2.441564
28	-1.24457	2.939569	4.199142	1.786360	2.879259	0.846964	1.137310
29	-1.44853	-0.36963	2.138039	-0.24697	0.840113	0.097667	-2.07167
30	1.135076	3.709554	4.780932	0.719811	3.434714	1.586294	-0.41172
31	1.887394	3.060795	4.065473	1.608644	2.805432	0.683867	1.080993
32	-1.68804	0.517967	2.766399	1.781690	1.563620	0.942114	0.387445
33	-0.77168	0.265726	2.686951	0.180024	1.327120	0.736152	-0.77282
34	-0.92433	1.629726	3.055911	2.125933	1.829058	-0.67232	0.542871
35	-0.42618	2.390353	3.733943	1.163595	2.551979	0.242136	0.024638
36	0.867081	2.674277	4.275210	1.987761	2.921472	1.039441	0.102650
37	0.410622	1.900492	3.927033	1.570570	2.604020	2.576065	-0.23237
38	0.148878	1.380994	3.717440	1.070704	2.270529	0.018766	-0.19273
39	-0.25771	2.062087	3.431232	0.935691	2.229467	-0.16361	-0.08632
40	1.155242	2.326456	3.966953	1.449289	2.630277	2.552876	-1.86778
41	0.739616	3.898632	4.722666	2.816918	3.457730	1.589317	0.574688
42	0.629521	2.589477	4.095299	3.886792	2.846897	0.809398	-0.75515
43	-2.02815	-1.17449	1.957206	-0.17052	0.584804	-2.46955	-2.13649
44	1.436890	3.542963	4.838396	2.272319	3.313904	1.564149	0.493251
45	-0.06407	1.591445	3.115589	2.110153	1.745040	-0.70633	-1.54868
46	-0.07112	2.870380	3.866783	1.310241	2.602862	0.362262	1.124009
47	-1.83386	1.056309	4.042844	1.987465	1.780823	-0.75508	-2.31155
48	0.165408	2.399697	3.815906	1.311532	2.555770	2.406275	-0.88265
49	-2.98616	-1.76835	-0.00714	-2.85470	0.061218	-3.16893	-0.58879
50	1.182614	3.202605	4.401005	1.921731	3.033833	1.079993	2.253821
51	1.301310	4.536259	5.272171	4.614625	4.089321	2.560792	1.052510
52	-0.25079	3.904821	3.704475	1.027934	2.312572	-0.01383	0.813991
53	0.492108	4.269037	3.499266	2.794166	2.339079	0.081920	-0.99349
54	-0.62641	3.201014	3.084636	0.496674	1.821958	-0.73605	-0.25572
55	1.988762	4.501575	5.060795	2.728588	3.759073	4.236569	1.804313
56	-0.17229	2.428252	2.389736	-0.21624	2.643489	0.365580	2.104574
57	-0.83687	1.411599	3.261218	0.722933	2.097220	-0.39974	-0.18258
58	-0.81355	1.551854	2.564367	1.671912	2.824742	2.775610	-1.11473
59	0.176538	2.146668	2.507624	0.091585	1.221144	0.598115	0.048908
60	-0.34899	1.772333	1.283247	2.357741	2.201119	-0.25535	-1.06085

APPENDIX VI

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 4 data in Chapter V.

VI.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	7.280418	2.533399	1.971569	3.340739	3.080998	1.427824	1.502270
2	7.417998	2.841507	2.278759	3.823860	3.469379	1.870084	1.941971
3	2.498229	-0.19915	-0.75284	-0.09823	-0.08423	-1.82952	-1.81901
4	-2.61626	-1.01447	-1.70899	-1.37722	-0.92680	-2.89841	-2.63696
5	-4.56508	0.126374	-0.74846	0.034493	0.102129	-1.71053	-3.16992
6	-2.88301	-0.36857	-1.99286	-1.11326	-1.37842	-2.99917	-3.12043
7	-10.2610	-2.73558	-3.96875	-3.62664	-3.15704	-5.21377	-3.02270
8	3.109378	4.150646	3.261590	5.027257	4.605614	2.952582	2.876028
9	-0.29890	0.608008	-0.94429	0.643978	-0.16295	-1.56743	-2.16383
10	-2.83814	-0.09986	-1.47148	-0.20865	-0.35014	-2.09315	-1.86441
11	6.306359	3.140234	1.375275	3.488856	2.570253	1.201751	1.101341
12	1.608964	1.373317	1.866083	2.837667	2.583850	2.857120	0.704762
13	0.514036	0.603424	-0.64763	13.46779	-0.07240	-1.52765	-3.59732
14	-3.39746	0.601204	-0.29174	0.988160	0.479841	-1.10354	-1.03331
15	-3.55842	-0.47266	-1.67434	-0.64788	-1.34121	-2.68153	-3.11129
16	-0.94422	0.777147	-0.76823	0.574402	0.014356	-1.47665	-1.59187
17	0.565284	1.561826	-0.45047	1.078778	0.881026	-0.89318	-0.53064
18	5.575388	3.039474	1.890383	3.374645	3.086437	1.381183	0.145967
19	-1.30806	0.882762	-0.02462	1.387853	0.368068	-0.80713	-3.36781
20	-1.70828	1.900690	2.500825	3.690681	3.605776	1.857545	0.435678
21	-1.06762	5.318454	4.378348	7.152385	5.581029	4.588888	4.021357
22	-5.66843	-2.83159	-4.54326	-3.46332	-5.61553	-5.40318	-5.46839
23	2.769461	2.175727	1.628263	2.594914	3.010586	0.952878	1.614797
24	4.284526	3.463702	3.143831	4.684593	4.363100	2.671877	2.852438
25	-2.05719	0.167802	-0.72485	0.078186	0.148268	0.225657	-1.18685
26	-3.42630	4.361073	2.585159	6.173278	5.437165	4.028179	2.121723
27	-8.62908	-2.83515	-3.88157	-3.42237	-3.61899	-5.23152	-7.12510
28	-0.55941	2.338917	-0.09431	1.940494	1.280721	-0.26635	-0.17589
29	10.77361	2.890460	1.889115	3.707582	3.546063	1.703811	2.209245
30	1.925525	1.424148	1.436424	0.288775	2.382295	0.552451	0.919915
31	2.521532	0.458401	-0.16066	0.639908	0.480371	-1.20371	-1.43129
32	-5.27380	-0.89804	-2.76098	-1.26171	-1.97878	-3.47735	-3.62282
33	-1.57097	0.889108	0.291648	1.644933	1.538811	-0.32500	2.230698
34	5.778924	5.434341	4.643474	7.146230	6.089534	4.718178	2.693795
35	2.747139	0.998965	0.094461	-0.30202	0.884009	-0.58067	-2.42634
36	-1.32053	1.455181	1.475955	2.383591	2.158347	0.516937	0.440837
37	-3.78245	0.510830	-0.51971	1.784634	1.994359	0.141161	0.186570
38	1.168248	1.140384	-0.45717	0.513132	0.715637	-1.20134	1.207044
39	2.293316	-1.62456	-4.20599	-3.39330	-1.54126	-5.18094	-5.07622
40	6.052976	5.762772	6.462719	6.508995	7.989597	6.372588	6.611423
41	-1.83996	1.472457	1.628254	2.392667	2.614013	0.740135	1.398379
42	-4.75422	-1.06259	-1.77189	-1.08756	-1.34072	-2.96046	-1.27879
43	-3.23364	0.267992	-1.86983	-0.72586	0.906097	-2.64067	-2.58775
44	-4.45781	0.085843	-0.17599	0.590183	2.548241	-1.14693	-2.65459
45	-1.04464	2.793447	2.461444	3.349024	3.386905	1.584813	3.534959
46	-4.56337	-0.55517	-1.32782	-0.69539	-0.81094	-2.46499	-2.66945
47	2.165623	1.942961	0.125976	1.592265	1.554550	-0.23784	1.985846
48	3.529007	3.471269	4.328775	5.300991	5.490057	3.605056	2.921713
49	-11.6702	-3.12904	-3.63241	-5.17465	-3.41013	-5.09170	-3.54566
50	-2.29507	3.196517	2.787749	4.667687	4.798988	2.924523	3.713607
51	0.497366	2.419480	1.632792	1.326519	2.640623	1.124167	1.338712
52	-6.41429	-2.96498	-5.06846	-6.45404	-4.69208	-6.41610	-6.55475
53	0.645078	2.675048	1.457304	3.218550	1.920825	2.609969	-3.59477
54	0.362477	2.526687	1.808079	3.350008	2.632122	3.113539	0.860530
55	-5.26506	0.363691	-0.52284	0.924055	-0.09138	-1.30640	-2.46178
56	2.869300	1.579448	0.846009	1.692749	1.516071	-0.13888	0.082070
57	-0.57674	1.814305	2.918506	3.404710	3.393180	1.665367	1.719064
58	-3.54558	0.950564	0.712752	-0.16473	1.266001	-0.29051	-0.25014
59	0.031911	1.736679	0.750629	-0.04643	1.405589	-0.15449	-0.27496
60	-9.39832	-1.67754	-1.90940	-3.11173	-1.42638	-3.10181	-6.46013

VI.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	7.972111	1.404105	1.712233	1.900153	1.805641	1.731730	2.110646
2	8.109692	1.712212	2.019423	2.383274	2.194021	2.173990	2.550348
3	3.189923	1.328454	1.012180	1.538825	1.359592	1.525614	1.210642
4	1.924573	2.143770	1.968332	2.817807	2.202164	2.594504	2.028589
5	3.873395	1.002919	1.007796	1.406091	1.173227	1.408628	2.561552
6	2.191322	1.497870	2.252198	2.553849	2.653779	2.695268	2.512058
7	9.569312	3.864879	4.228093	5.067232	4.432402	4.909867	2.414324
8	3.801071	3.021352	3.002254	3.586671	3.330257	3.256487	3.484405
9	0.392792	0.521285	1.203628	0.796607	1.438307	1.263527	1.555458
10	2.146448	1.229158	1.730820	1.649241	1.625505	1.789244	1.256039
11	6.998052	2.010939	1.115939	2.048271	1.294896	1.505657	1.709718
12	2.300657	0.244023	1.606747	1.397081	1.308493	3.161025	1.313138
13	1.205730	0.525869	0.906975	12.02721	1.347768	1.223749	2.988945
14	2.705773	0.528090	0.551080	0.452425	0.795515	0.799635	0.424934
15	2.866731	1.601954	1.933684	2.088473	2.616571	2.377630	2.502915
16	0.252528	0.352146	1.027571	0.866182	1.261000	1.172745	0.983498
17	1.256978	0.432531	0.709807	0.361806	0.394330	0.589279	0.077727
18	6.267082	1.910180	1.631047	1.934060	1.811080	1.685089	0.754343
19	0.616372	0.246531	0.283957	0.052731	0.907288	0.503225	2.759435
20	1.016590	0.771396	2.241489	2.250095	2.330419	2.161451	1.044054
21	0.375926	4.189160	4.119012	5.711799	4.305672	4.892794	4.629733
22	4.976741	3.960891	4.802600	4.903905	6.890889	5.099277	4.860019
23	3.461155	1.046432	1.368926	1.154329	1.735229	1.256784	2.223173
24	4.976220	2.334408	2.884495	3.244007	3.087743	2.975782	3.460814
25	1.365498	0.961492	0.984188	1.362398	1.127088	0.529563	0.578481
26	2.734607	3.231779	2.325823	4.732693	4.161808	4.332084	2.730099
27	7.937387	3.964446	4.140915	4.862962	4.894348	4.927616	6.516733
28	0.132281	1.209623	0.353650	0.499909	0.005364	0.037554	0.432480
29	11.46530	1.761166	1.629779	2.266997	2.270706	2.007717	2.817622
30	2.617218	0.294853	1.177087	1.151809	1.106938	0.856356	1.528291
31	3.213225	0.670892	0.420001	0.800676	0.794985	0.899805	0.822915
32	4.582110	2.027341	3.020323	2.702300	3.254138	3.173450	3.014449
33	0.879277	0.240186	0.032312	0.204347	0.263454	0.021095	2.839074
34	6.470618	4.305046	4.384138	5.705644	4.814177	5.022084	3.302171
35	3.438832	0.130328	0.164874	1.742607	0.391347	0.276769	1.817967
36	0.628843	0.325887	1.216619	0.943005	0.882990	0.820842	1.049213
37	3.090756	0.618463	0.779053	0.344049	0.719002	0.445066	0.794946
38	1.859942	0.011090	0.716513	0.927452	0.559719	0.897435	1.815420
39	2.985010	2.753863	4.465327	4.833891	2.816620	4.877034	4.467846
40	6.744670	4.633478	6.203383	5.058410	6.714240	6.676494	7.219800
41	1.148274	0.343162	1.368918	0.952082	1.338656	1.044041	2.006755
42	4.062531	2.191884	2.031228	2.528150	2.616086	2.656559	0.670414
43	2.541947	0.861302	2.129175	2.166451	0.369260	2.336765	1.979381
44	3.768123	1.043450	0.435331	0.850402	1.272884	0.843025	2.046218
45	0.352950	1.664153	2.202108	1.908438	2.111547	1.888719	4.143336
46	3.871679	1.684468	1.587159	2.135981	2.086301	2.161093	2.061075
47	2.857316	0.813667	0.133359	0.151679	0.279193	0.066064	2.594222
48	4.220700	2.341975	4.069438	3.860406	4.214700	3.908961	3.530089
49	10.97857	4.258334	3.891747	6.615244	4.685493	4.787803	2.937292
50	1.603382	2.067222	2.528412	3.227101	3.523631	3.228429	4.321983
51	1.189060	1.290185	1.373456	0.114065	1.365266	1.428072	1.947088
52	5.722597	4.094279	5.327804	7.894634	5.967446	6.112198	5.946374
53	1.336771	1.545754	1.197967	1.777964	0.645468	2.913875	2.986398
54	1.054170	1.397393	1.548743	1.909422	1.356765	3.417445	1.468906
55	4.573368	0.765602	0.782183	0.516529	1.366742	1.002495	1.853407
56	3.560993	0.450154	0.586673	0.252164	0.240714	0.165031	0.690446
57	0.114949	0.685010	2.659169	1.964125	2.117823	1.969272	2.327440
58	2.853887	0.178730	0.453416	1.605320	0.009355	0.013389	0.358226
59	0.723604	0.607385	0.491292	1.487017	0.130232	0.149405	0.333409
60	8.706629	2.806841	2.168743	4.552323	2.701744	2.797909	5.851755

VI.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6	STRAT7
1	2.101815	1.581538	1.034939	1.365324	1.473244	0.651763	0.629362
2	2.141534	1.773882	1.196192	1.562770	1.658957	0.853643	0.813571
3	0.721224	-0.12433	-0.39519	-0.04014	-0.04027	-0.83512	-0.76206
4	-0.75530	-0.63331	-0.89710	-0.56285	-0.44317	-1.32304	-1.10473
5	-1.31791	0.078892	-0.39289	0.014097	0.048835	-0.78081	-1.32801
6	-0.83231	-0.23009	-1.04611	-0.45497	-0.65912	-1.36904	-1.30727
7	-2.96229	-1.70775	-2.08332	-1.48217	-1.50960	-2.37994	-1.26633
8	0.897659	2.591145	1.712111	2.054586	2.202272	1.347774	1.204886
9	-0.08629	0.379564	-0.49568	0.263187	-0.07791	-0.71549	-0.90651
10	-0.81935	-0.06234	-0.77242	-0.08527	-0.16743	-0.95546	-0.78108
11	1.820610	1.960370	0.721925	1.425858	1.229021	0.548567	0.461397
12	0.464498	0.857328	0.979566	1.159724	1.235523	1.304199	0.295253
13	0.148399	0.376703	-0.33996	5.504144	-0.03462	-0.69733	-1.50706
14	-0.98082	0.375316	-0.15314	0.403850	0.229446	-0.50373	-0.43289
15	-1.02729	-0.29507	-0.87891	-0.26478	-0.64133	-1.22404	-1.30344
16	-0.27259	0.485154	-0.40327	0.234752	0.006864	-0.67405	-0.66690
17	0.163194	0.975009	-0.23646	0.440885	0.421281	-0.40771	-0.22231
18	1.609583	1.897468	0.992321	1.379181	1.475845	0.630473	0.061151
19	-0.37763	0.551086	-0.01292	0.567200	0.175999	-0.36843	-1.41091
20	-0.49317	1.186554	1.312762	1.508341	1.724178	0.847919	0.182523
21	-0.30821	3.320179	2.298333	2.923103	2.668688	2.094705	1.684711
22	-1.63644	-1.76769	-2.38490	-1.41542	-2.68518	-2.46640	-2.29093
23	0.799527	1.358252	0.854726	1.060513	1.439576	0.434963	0.676504
24	1.236918	2.162303	1.650296	1.914542	2.086309	1.219640	1.195003
25	-0.59389	0.104754	-0.38049	0.031954	0.070897	0.103006	-0.49722
26	-0.98915	2.722510	1.357031	2.522952	2.599896	1.838756	0.888877
27	-2.49116	-1.76991	-2.03756	-1.39868	-1.73049	-2.38805	-2.98500
28	-0.16149	1.460128	-0.04950	0.793059	0.612404	-0.12158	-0.07368
29	3.110280	1.804442	0.991656	1.515249	1.695625	0.777744	0.925543
30	0.555888	0.889060	0.754024	0.118019	1.139145	0.252179	0.385390
31	0.727952	0.286168	-0.08433	0.261523	0.229699	-0.54946	-0.59962
32	-1.52251	-0.56062	-1.44932	-0.51564	-0.94619	-1.58732	-1.51775
33	-0.45353	0.555048	0.153095	0.672266	0.735815	-0.14835	0.934530
34	1.668342	3.392524	2.437506	2.920587	2.911840	2.153722	1.128541
35	0.793083	0.623629	0.049585	-0.12343	0.422708	-0.26506	-1.01649
36	-0.38123	0.908433	0.774775	0.974148	1.032059	0.235967	0.184684
37	-1.09197	0.318898	-0.27281	0.729361	0.953645	0.064436	0.078161
38	0.337266	0.711913	-0.23998	0.209711	0.342197	-0.54838	0.505680
39	0.662067	-1.01417	-2.20785	-1.38680	-0.73598	-2.36496	-2.12653
40	1.747460	3.597555	3.392485	2.660156	3.820396	2.908916	2.769796
41	-0.53118	0.919218	0.854722	0.977857	1.249946	0.337852	0.585838
42	-1.37251	-0.66334	-0.93012	-0.44447	-0.64109	-1.35137	-0.53573
43	-0.93353	0.167300	-0.98153	-0.29665	0.433269	-1.20539	-1.08411
44	-1.28694	0.053590	-0.09238	0.241201	1.218495	-0.52354	-1.11211
45	-0.30158	1.743880	1.292089	1.368710	1.619520	0.723425	1.480939
46	-1.31741	-0.34658	-0.69701	-0.28420	-0.38777	-1.12520	-1.11834
47	0.625203	1.212942	0.066129	0.650741	0.743341	-0.10856	0.831952
48	1.018804	2.167027	2.272310	2.166458	2.625188	1.645611	1.224025
49	-3.36913	-1.95338	-1.90676	-2.11482	-1.63062	-2.32422	-1.48542
50	-0.66257	1.995506	1.463377	1.907633	2.294738	1.334967	1.555782
51	0.143587	1.510421	0.857104	0.542134	1.262670	0.513152	0.560841
52	-1.85176	-1.85096	-2.66059	-2.63770	-2.24362	-2.92878	-2.74605
53	0.186230	1.669966	0.764984	1.315387	0.918483	1.191381	-1.50599
54	0.104645	1.577348	0.949117	1.369112	1.258605	1.421247	0.360511
55	-1.51999	0.227043	-0.27445	0.377651	-0.04369	-0.59633	-1.03134
56	0.828350	0.986010	0.444097	0.691808	0.724941	-0.06339	0.034382
57	-0.16650	1.132625	1.532015	1.391468	1.622521	0.760195	0.720186
58	-1.02358	0.593413	0.374146	-0.06732	0.605365	-0.13261	-0.10479
59	0.009212	1.084165	0.394028	-0.01897	0.672112	-0.07052	-0.11519
60	-2.71324	-1.04725	-1.00230	-1.27173	-0.68205	-1.41589	-2.70641

APPENDIX VII

Derivation of Bond Price Convexity Index

Let the price of a bond be expressed as

$$P = \Sigma C_t(1/1+r)^t = \Sigma C_t(1+r)^{-t} (1)$$

where, C_t = the periodic bond cash flow at time t (t=1,...,N),

N = number of periodic payments on the bond,

r = the periodic yield to maturity on the bond.

For small instantaneous changes in yield, the change in price can be approximated by a Taylor Series expansion,

$$dP \approx dP/dr + \frac{1}{2} d^2P/dr^2 (dr)^2 + ... + 1/n! d^nP/dr^n (dr)^n \quad (2)$$

The convexity index, Con, is derived from the second term of the price change function. The first derivative is

$$dP/dr = -\Sigma t C_t(1+r)^{-(t+1)} \quad (3)$$

The second derivative is

$$d^2P/dr^2 = \Sigma t(t+1)C_t(1+r)^{-(t+2)} \quad (4)$$

or, $$d^2P/dr^2 = \Sigma[t(t+1)C_t/ (1+r)^{t+2}] \quad (5)$$

This can be simplified to

$$d^2P/dr^2 = (1/1+r)^2 \Sigma[t(t+1)C_t / (1+r)^t] \quad (6)$$

Con is defined as the second derivative divided by price;

Therefore,

$$Con = (1/1+r)^2 [\Sigma t(t+1)C_t / (1+r)^t] (1/P) \quad (7)$$

The percent change in price due to convexity is

$$dP/P = (Con) dr^2. \quad (8)$$

APPENDIX VIII

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 1 data in Chapter VI.

VIII.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	17.49558	25.04392	12.96953	19.68245	4.175460	4.128031
2	20.68408	46.94584	35.43440	9.860830	5.247090	23.24500
3	−16.4880	26.87099	17.27381	1.517961	−24.1466	−12.8034
4	3.088[illegible]23	41.68387	27.65728	12.25918	4.491078	21.73790
5	27.84[illegible]91	33.22613	2.950949	20.65748	13.12227	18.56125
6	−0.92[illegible]99	45.07332	7.305748	4.204143	1.038508	12.98166
7	5.3983[illegible]	17.48938	0.930661	−2.54101	16.06482	39.53190
8	15.01986	28.19609	13.00182	20.07156	16.48059	17.21247
9	17.50116	28.67298	13.28017	14.63729	25.52766	21.61158
10	57.84925	11.13829	9.969207	27.50080	38.19774	12.17566
11	17.91941	33.79265	14.95907	12.70463	15.82815	0.629785
12	15.54675	11.25735	−4.54388	7.233096	2.834345	9.653203
13	−0.87225	26.94407	19.92609	−3.47909	−1.65066	10.45948
14	10.59840	45.25063	0.199136	7.173393	−7.48848	3.747185
15	−11.6086	13.65296	12.41875	5.147373	−9.95647	−19.1650
16	−7.13444	12.21816	−9.16960	0.913722	13.68720	39.47155
17	−18.8660	35.51709	27.00117	−6.31961	−12.1589	−6.10814
18	−34.4168	14.43290	−2.06474	−14.6952	−24.3530	−10.7218
19	−9.64691	−0.75128	−13.6630	2.526896	13.16859	18.01061
20	12.21288	50.74274	−3.05680	11.13091	2.382449	14.06916
21	−21.3919	12.68566	7.671831	5.118373	13.86441	2.642661
22	16.42862	14.71894	5.428012	0.265688	9.350354	8.402329
23	21.92120	41.94137	37.74844	18.99032	13.72265	−11.9001
24	−2.62622	23.95368	27.72527	−1.26636	−7.83159	−9.28713
25	−14.2780	47.14185	21.98153	−4.54908	−6.720[illegible]	−2.15246
26	−1.35132	8.508089	−4.63368	−3.76653	21.72570	17.37506
27	8.810341	31.53772	2.477506	12.49599	6.300547	35.36416
28	−8.50781	26.78351	3.257931	0.880492	10.66154	18.84535
29	3.689067	37.14356	30.17238	5.247084	1.291216	−4.15684
30	2.169800	38.93443	25.21275	1.344968	−12.8333	−14.8002
31	2.636949	30.61306	18.65259	4.287479	−4.55171	−15.9042
32	−8.03810	10.40637	8.385797	−2.19948	20.65937	7.931146
33	32.25029	29.93780	1.585922	10.62896	21.98263	34.37432
34	−1.37595	38.63433	37.61803	12.28827	−11.1343	8.222308
35	−2.85307	15.97554	11.64522	6.848018	13.56151	17.85987
36	−7.79362	66.31741	77.14632	−2.55332	−14.3066	−5.42850
37	22.46647	50.03362	47.27057	5.035214	1.563798	12.32236
38	39.82262	26.98418	16.93805	16.41464	20.58188	26.89677
39	49.22092	27.67135	4.135726	17.00747	42.42906	36.82215
40	−5.50525	40.84005	4.899851	10.72932	7.846758	−9.87591
41	−9.18772	2.868831	−7.34733	1.193225	17.11629	19.13034
42	17.02839	31.83705	13.74724	17.74374	24.76217	26.11805
43	−16.7481	35.17911	13.86879	−3.53281	−6.13439	−1.62401
44	22.00758	33.07303	16.69315	12.31456	25.18329	42.11743
45	10.92496	23.27710	−2.38556	−0.05662	11.13147	−4.46664
46	−6.49730	22.91721	6.245829	0.043871	−7.97742	−13.7188
47	−4.46935	23.93725	5.172552	3.632629	2.800279	5.232667
48	−1.49839	27.56907	−2.36031	4.813635	−1.13238	13.15692
49	−11.4343	13.39283	−9.06654	1.975291	−7.55742	−10.0882
50	−7.29077	14.29095	−2.09838	−2.88726	−6.33357	−2.78328
51	27.11079	49.70961	48.57073	5.553386	11.85300	9.248542
52	−24.1711	14.64620	1.686921	1.188924	−11.7814	−35.3345
53	13.50721	1.318972	10.89496	13.05768	18.74346	16.88135
54	31.72308	39.03141	33.86846	20.19384	16.80367	29.98381
55	9.714827	39.93539	36.34074	10.85908	1.082525	−10.2712
56	2.549541	8.673406	−4.71487	3.126226	−1.13147	8.599563
57	−5.33268	24.39655	14.93390	−1.91002	15.37020	11.05159
58	−16.7051	−6.06349	−4.47884	−4.98787	−14.5834	−22.0289
59	−6.89493	30.23845	3.594876	8.658774	−14.6713	−16.4330
60	15.77047	4.824843	−3.14164	19.14045	12.06093	25.84292

VIII.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	12.71213	1.642649	0.667460	13.45653	1.262195	3.581891
2	15.90063	20.25927	23.13232	3.634913	0.190564	15.53507
3	21.27147	0.184420	4.971738	4.707956	29.58425	20.51339
4	1.694631	14.99729	15.35520	6.033265	0.946577	14.02797
5	23.06446	6.539557	9.351124	14.43156	7.684621	10.85132
6	5.707445	18.38674	4.996326	2.021773	4.399147	5.271743
7	0.614910	9.217193	11.37141	8.766935	10.62717	31.82198
8	10.23640	1.509520	0.699746	13.84564	11.04294	9.502547
9	12.71770	1.986407	0.978104	8.411380	20.09001	13.90166
10	53.06580	15.54827	2.332867	21.27488	32.76008	4.465745
11	13.13595	7.106079	2.657000	6.478721	10.39049	7.080137
12	10.76329	15.42922	16.84596	1.007178	2.603310	1.943280
13	5.655704	0.257500	7.624024	9.705008	7.088320	2.749559
14	5.814951	18.56406	12.10293	0.947475	12.92614	3.962736
15	16.39209	13.03360	0.116681	1.078544	15.39412	26.87492
16	11.91790	14.46841	21.47167	5.312195	8.249552	31.76163
17	23.64946	8.830519	14.69909	12.54553	17.59655	13.81806
18	39.20030	12.25367	14.36681	20.92117	29.79067	18.43180
19	14.43037	27.43786	25.96516	3.699021	7.730936	10.30068
20	7.429429	24.05616	15.35888	4.904997	3.055206	6.359245
21	26.17535	14.00091	4.630242	1.107544	8.426759	5.067261
22	11.64516	11.96763	6.874062	5.960229	3.912699	0.692406
23	17.13775	15.25480	25.44637	12.76441	8.285003	19.61003
24	7.409682	2.732897	15.42319	7.492287	13.26924	16.99706
25	19.06148	20.45527	9.679456	10.77500	12.15827	9.862389
26	6.134784	20.17848	16.93576	9.992450	16.28805	9.665139
27	4.026886	4.851150	9.824568	6.270078	0.862891	27.65423
28	13.29127	0.096933	9.044142	5.345424	5.223888	11.13543
29	1.094387	10.45698	17.87030	0.978833	4.146438	11.86676
30	2.613654	12.24785	12.91068	4.880949	18.27104	22.51021
31	2.146505	3.926485	6.350521	1.938437	9.989375	23.61419
32	12.82156	16.28019	3.916277	8.425398	15.22171	0.221223
33	27.46684	3.251226	10.71615	4.403044	16.54498	26.66439
34	6.159410	11.94775	25.31596	6.062354	16.57204	0.512385
35	7.636533	10.71103	0.656851	0.622100	8.123863	10.14995
36	12.57708	39.63084	64.84425	8.779246	19.74434	13.13843
37	17.68301	23.34704	34.96849	1.190703	3.873857	4.612446
38	35.03917	0.297604	4.635982	10.18873	15.14423	19.18685
39	44.43747	0.984781	8.166348	10.78155	36.99141	29.11223
40	10.28871	14.15348	7.402222	4.503404	2.409102	17.58583
41	13.97118	23.81774	19.64940	5.032691	11.67863	11.42042
42	12.24493	5.150474	1.445171	11.51782	19.32452	18.40813
43	21.53164	8.492541	1.566717	9.758730	11.57205	9.333941
44	17.22413	6.386457	4.391082	6.088645	19.74564	34.40751
45	6.141513	3.409475	14.68764	6.282544	5.693822	12.17657
46	11.28076	3.769357	6.056244	6.182045	13.41508	21.42880
47	9.252814	2.749319	7.129522	2.593288	2.637376	2.477255
48	6.281853	0.882495	14.66239	1.412282	6.570038	5.447004
49	16.21782	13.29374	21.36861	4.250626	12.99507	17.79815
50	12.07422	12.39562	14.40046	9.113184	11.77123	10.49321
51	22.32733	23.02303	36.26865	0.672530	6.415353	1.538619
52	28.95457	12.04037	10.61515	5.036992	17.21907	43.04450
53	8.723759	25.36760	1.407108	6.831765	13.30580	9.171428
54	26.93962	12.34483	21.56638	13.96792	11.36601	22.27389
55	4.931372	13.24881	24.03867	4.633165	4.355130	17.98114
56	2.233913	18.01317	17.01694	3.099691	6.569128	0.889640
57	10.11614	2.290017	2.631828	8.135937	9.932549	3.341673
58	21.48855	32.75007	16.78091	11.21378	20.02113	29.73888
59	11.67838	3.551876	8.707197	2.432856	20.10901	24.14292
60	10.98702	21.86173	15.44372	12.91454	6.623281	18.13300

VIII.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	1.213814	2.106536	1.023223	2.833245	0.357798	0.292677
2	1.435027	3.948786	2.795575	1.419444	0.449627	1.648073
3	-1.14391	2.260217	1.362807	0.218507	-2.06914	-0.90776
4	0.214297	3.506182	2.182004	1.764681	0.384843	1.541219
5	1.932041	2.794771	0.232813	2.973598	1.124457	1.315994
6	-0.06410	3.791281	0.576382	0.605176	0.088990	0.920401
7	0.374529	1.469413	0.073423	-0.36577	1.376607	2.802816
8	1.042053	2.371676	1.025770	2.889256	1.412235	1.220365
9	1.214201	2.411789	1.047731	2.107005	2.187485	1.532263
10	4.013484	0.936882	0.786514	3.958677	3.273193	0.863256
11	1.243218	2.842423	1.180187	1.828803	1.356326	0.044651
12	1.078607	0.946897	-0.35848	1.041187	0.242877	0.684413
13	-0.06051	2.266364	1.572057	-0.50080	-0.14144	0.741578
14	0.735299	3.806196	0.015710	1.032593	-0.64169	0.265675
15	-0.80538	1.148401	0.979770	0.740952	-0.85317	-1.35880
16	-0.49497	1.027714	-0.72343	0.131528	1.172867	2.798537
17	-1.30889	2.987472	2.130241	-0.90969	-1.04190	-0.43306
18	-2.38778	1.214004	-0.16289	-2.11534	-2.08682	-0.76018
19	-0.66928	-0.06319	-1.07794	0.363741	1.128426	1.276954
20	0.847309	4.268157	-0.24116	1.602270	0.204153	0.997505
21	-1.48413	1.067037	0.605264	0.736778	1.188052	0.187364
22	1.139790	1.238063	0.428239	0.038245	0.801239	0.595726
23	1.520856	3.527842	2.978140	2.733614	1.175905	-0.84371
24	-0.18220	2.014831	2.187368	-0.18229	-0.67109	-0.65845
25	-0.99058	3.965273	1.734219	-0.65483	-0.57589	-0.15261
26	-0.09375	0.547419	-0.36557	-0.54218	1.861692	1.231893
27	0.611246	2.652753	0.195461	1.798770	0.539898	2.507322
28	-0.59025	2.252858	0.257032	0.128744	0.913595	1.336137
29	0.255941	3.124280	2.380431	0.755305	0.110645	-0.29472
30	0.150537	3.274917	1.989145	0.193605	-1.09970	-1.04934
31	0.182947	2.574976	1.471585	0.617172	-0.39004	-1.12761
32	-0.55767	0.875318	0.661592	-0.31661	1.770317	0.562319
33	2.237471	2.518177	0.125120	1.530015	1.883709	2.437143
34	-0.09546	3.249674	2.967852	1.768868	-0.95411	0.582962
35	-0.19794	1.343761	0.918742	0.985756	1.162096	1.266267
36	-0.54070	5.578200	6.086413	-0.36754	-1.22595	-0.38488
37	1.558686	4.208510	3.729383	0.724807	0.134003	0.873657
38	2.762826	2.269737	1.336317	2.362851	1.763677	1.906984
39	3.414865	2.327538	0.326285	2.448187	3.635779	2.610694
40	-0.38194	3.435206	0.386570	1.544461	0.672394	-0.70020
41	-0.63742	0.241307	-0.57966	0.171762	1.466708	1.356343
42	1.181401	2.677930	1.084580	2.554171	2.121890	1.851772
43	-1.16196	2.959044	1.094169	-0.50854	-0.52566	-0.11514
44	1.526849	2.781893	1.316996	1.772653	2.157976	2.986130
45	0.757956	1.957921	-0.18820	-0.00815	0.953864	-0.31668
46	-0.45077	1.927650	0.492760	0.006315	-0.68359	-0.97267
47	-0.31007	2.013450	0.408085	0.522908	0.239958	0.370996
48	-0.10395	2.318935	-0.18621	0.692911	-0.09703	0.932827
49	-0.79329	1.126520	-0.71529	0.284338	-0.64760	-0.71525
50	-0.50582	1.202064	-0.16555	-0.41561	-0.54272	-0.19733
51	1.880901	4.181256	3.831958	0.799397	1.015693	0.655722
52	-1.67695	1.231945	0.133088	0.171143	-1.00955	-2.50522
53	0.937107	0.110943	0.859551	1.879623	1.606141	1.196889
54	2.200894	3.283074	2.672031	2.906857	1.439919	2.125855
55	0.673998	3.359111	2.867081	1.563140	0.092762	-0.72823
56	0.176882	0.729551	-0.37197	0.450013	-0.09695	0.609709
57	-0.36997	2.052083	1.178201	-0.27494	1.317084	0.783559
58	-1.15897	-0.51002	-0.35335	-0.71799	-1.24966	-1.56185
59	-0.47835	2.543466	0.283615	1.246410	-1.25720	-1.16510
60	1.094129	0.405835	-0.24785	2.755225	1.033510	1.832266

APPENDIX IX

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 2 data in Chapter VI.

IX.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	6.917331	14.64086	8.384997	14.57244	8.688593	4.594394
2	-1.06744	4.889919	3.781911	1.750282	-1.04688	-4.85702
3	-1.81721	5.670634	3.029225	-6.41821	-5.87270	-7.10702
4	5.364254	10.24671	4.375699	0.265721	8.269576	1.249105
5	6.542536	14.17372	6.869066	6.900530	0.212578	-1.71169
6	-5.79927	2.463877	2.746606	-8.60671	-6.28710	-8.66957
7	-16.6774	0.262505	-1.68825	-13.7869	-17.1956	-19.6571
8	3.414616	19.29602	9.712374	7.742882	3.417411	-3.09280
9	-4.78379	3.386898	1.129111	-6.28598	-5.57745	-7.87588
10	-9.61127	-0.79052	1.812193	-22.4613	-17.4984	-15.5003
11	0.686928	9.277903	5.313559	1.228566	-3.57982	-7.08730
12	4.398069	11.46844	6.948377	7.018577	6.343319	-0.41050
13	-15.4940	-0.04827	0.650513	-16.4102	-18.3099	-18.7312
14	-7.88878	5.104073	3.899631	5.643157	-2.58055	-8.49379
15	4.862140	13.52866	11.87476	14.29817	8.621971	1.907954
16	-11.8189	-2.17229	0.077895	-21.6701	-17.9362	-16.5422
17	-0.57692	4.810901	4.300668	-1.62012	0.535842	-4.42869
18	-9.70932	2.372125	4.103037	-11.1103	-13.8858	-15.0371
19	-12.6949	-1.46975	1.006581	-16.5404	-15.5020	-16.8299
20	2.048792	4.075359	5.663253	-1.09115	-0.00593	-3.45128
21	-4.96269	0.156116	0.881534	0.701897	-8.99498	-11.6142
22	-18.0731	-16.2670	-6.82887	-21.6999	-24.7155	-24.9981
23	-0.44906	2.766882	4.922093	-0.41944	-3.67176	-7.22516
24	1.361128	16.93819	13.54403	4.737116	5.316388	-0.40925
25	-3.98165	0.571245	4.754171	-4.79924	-6.67458	-9.64368
26	-4.50623	2.238864	1.902934	1.258531	-4.04267	-4.13601
27	-7.67427	9.262108	12.29597	0.152461	-0.81468	-3.33217
28	18.34189	13.43811	13.78446	18.24445	19.97847	14.53493
29	-22.5779	-12.9121	-7.23959	-19.2913	-28.1234	-28.4986
30	3.856705	8.944751	8.712046	7.721573	12.42941	2.177477
31	1.929327	9.490112	9.292023	3.247184	10.09953	-1.60791
32	-11.4561	-3.23621	-1.93359	-12.6408	-19.8179	-22.7360
33	-1.42509	2.275143	3.599216	3.123220	-3.21419	-9.79071
34	-16.9282	-9.12746	-14.5371	-18.8985	-25.0747	-28.5651
35	-9.91084	-3.23861	-1.42642	-11.0449	-14.3292	-20.6928
36	-2.59098	0.362351	-2.24344	1.127878	-2.53858	-11.4891
37	4.507094	20.91124	16.54400	18.17692	15.19833	2.452417
38	-5.58584	2.758884	2.623129	-4.91117	-3.55481	-13.4070
39	-11.9024	-5.89175	-0.31696	-10.3654	-15.7781	-20.8675
40	-0.29608	15.64737	11.70306	15.91564	9.655601	-3.27219
41	-2.74410	5.310462	7.069469	3.989545	-2.54808	-10.5701
42	-3.99533	1.853382	5.172941	0.389430	0.352724	-9.76904
43	-7.26970	0.872714	-0.91929	-4.42989	-15.0393	-22.0069
44	-8.04176	1.437973	1.830132	-3.82985	-3.84222	-14.4995
45	3.849422	11.38856	4.189782	11.74523	2.202336	-9.98880
46	-12.5900	-2.20791	1.170248	-7.92481	-15.6658	-21.2427
47	-6.78498	-20.3662	-17.3865	-29.8693	-38.0165	-39.2566
48	-5.07197	8.542048	6.836257	1.821332	-1.75793	-11.5781
49	-17.8173	-14.3134	-12.5255	-20.4491	-29.4922	-35.8269
50	1.205273	16.25021	16.74051	7.983013	10.80706	-0.13989
51	-0.37241	9.756500	8.000640	6.316426	2.703158	-8.19771
52	-3.56997	4.547907	4.934157	-0.40094	-3.37574	-12.9870
53	-4.56225	6.639183	5.562552	-0.51294	-5.39275	-13.1580
54	-1.84528	0.860966	0.992223	-0.07609	-4.00316	-6.30276
55	5.900170	8.593946	8.987821	7.515779	7.808635	7.455382
56	2.478703	-1.68597	0.508767	0.683585	-3.54694	-5.03046
57	-10.0940	-5.31262	-3.11465	-12.2681	-13.7053	-13.8035
58	-7.40256	0.696940	-0.95698	-12.9006	-9.48684	-9.08269
59	-2.97242	2.456672	2.815094	-4.91780	-2.56082	-3.13770
60	-11.9074	-13.6151	-12.9035	-23.9940	-29.9897	-30.6460

IX.b. <u>Mean Absolute Deviations (MAD)</u>

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	11.07795	11.34117	5.367866	17.52536	14.22870	15.00477
2	3.093173	1.590219	0.764780	4.703199	4.493224	5.553355
3	2.343410	2.370934	0.012095	3.465295	0.332591	3.303355
4	9.524876	6.947019	1.358569	3.218637	13.80968	11.65949
5	10.70315	10.87402	3.851936	9.853447	5.752688	8.698688
6	1.638656	0.835822	0.270524	5.653799	0.746994	1.740808
7	12.51679	3.037194	4.705380	10.83404	11.65550	9.246763
8	7.575238	15.99632	6.695243	10.69579	8.957521	7.317576
9	0.623169	0.087198	1.888019	3.333064	0.037342	2.534499
10	5.450651	4.090225	1.204937	19.50839	11.95830	5.089931
11	4.847550	5.978203	2.296429	4.181483	1.960280	3.323079
12	8.558691	8.168744	3.931246	9.971494	11.88342	9.999876
13	11.33346	3.347978	2.366617	13.45737	12.76981	8.320843
14	3.728168	1.804373	0.882500	8.596074	2.959556	1.916591
15	9.022762	10.22896	8.857632	17.25108	14.16208	12.31833
16	7.658376	5.471994	2.939235	18.71719	12.39610	6.131819
17	3.583701	1.511201	1.283538	1.332791	6.075953	5.981690
18	5.548707	0.927574	1.085906	8.157453	8.345755	4.626752
19	8.534282	4.769453	2.010549	13.58756	9.961892	6.419568
20	6.209413	0.775659	2.646122	1.861761	5.534178	6.959104
21	0.802076	3.143583	2.135595	3.654813	3.454875	1.203894
22	13.91248	19.56672	9.846005	18.74706	19.17543	14.58778
23	3.711555	0.532817	1.904963	2.533474	1.868346	3.185224
24	5.521750	13.63849	10.52690	7.690033	10.85649	10.00113
25	0.178970	2.728454	1.737040	1.846329	1.134478	0.766701
26	0.345610	1.060835	1.114195	4.211447	1.497432	6.274371
27	3.513656	5.962408	9.278846	3.105378	4.725425	7.078207
28	22.50251	10.13841	10.76733	21.19736	25.51858	24.94532
29	18.41729	16.21188	10.25672	16.33842	22.58338	16.08826
30	8.017327	5.645051	5.694916	10.67448	17.96952	12.58786
31	6.089948	6.190412	6.274893	6.200100	15.63964	8.802468
32	7.295484	6.535911	4.950727	9.687979	14.27785	12.32569
33	2.735526	1.024556	0.582086	6.076136	2.325919	0.619668
34	12.76759	12.42716	17.55425	15.94563	19.53463	18.15480
35	5.750227	6.538312	4.443559	8.092067	8.789187	10.28242
36	1.569637	2.937347	5.260575	4.080794	3.001520	1.078812
37	8.667716	17.61154	13.52687	21.12984	20.73844	12.86280
38	1.425226	0.540815	0.394001	1.958255	1.985295	2.996689
39	7.741788	9.191457	3.334092	7.412547	10.23802	10.45719
40	3.864537	12.34767	8.685936	18.86856	15.19571	7.138192
41	1.416514	2.010762	4.052338	6.942462	2.992028	0.159767
42	0.165290	1.446317	2.155811	3.342346	5.892834	0.641341
43	3.109084	2.426985	3.936423	1.476977	9.499212	11.59659
44	3.881145	1.861728	1.186998	0.876942	1.697889	4.089187
45	8.010043	8.088864	1.172651	14.69815	7.742447	0.421581
46	8.429396	5.507619	1.846881	4.971893	10.12574	10.83237
47	2.624365	23.66595	20.40372	26.91647	32.47639	28.84630
48	0.911356	5.242349	3.819127	4.774249	3.782174	1.167809
49	13.65669	17.61318	15.54266	17.49627	23.95216	25.41660
50	5.365895	12.95051	13.72338	10.93593	16.34717	10.27048
51	3.788205	6.456801	4.983510	9.269343	8.243268	2.212666
52	0.590650	1.248207	1.917026	2.551967	2.164365	2.576643
53	0.401633	3.339483	2.545421	2.439975	0.147355	2.747635
54	2.315338	2.438733	2.024906	2.876819	1.536943	4.107617
55	10.06079	5.294247	5.970691	10.46869	13.34874	17.86576
56	6.639325	4.985675	2.508363	3.836502	1.993162	5.379916
57	5.933460	8.612321	6.131789	9.315259	8.165208	3.393124
58	3.241939	2.602759	3.974111	9.947688	3.946735	1.327685
59	1.188196	0.843027	0.202036	1.964892	2.979282	7.272682
60	7.746859	16.91486	15.92072	21.04113	24.44966	20.23564

IX.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	1.159720	2.265929	1.695610	1.614675	0.930896	0.574124
2	-0.17896	0.756800	0.764776	0.193937	-0.11216	-0.60694
3	-0.30466	0.877629	0.612568	-0.71115	-0.62920	-0.88810
4	0.899340	1.585858	0.884851	0.029442	0.886002	0.156090
5	1.096884	2.193629	1.389059	0.764601	0.022775	-0.21389
6	-0.97227	0.381327	0.555417	-0.95365	-0.67360	-1.08336
7	-2.79604	0.040627	-0.34139	-1.52764	-1.84233	-2.45639
8	0.572475	2.986395	1.964031	0.857937	0.366141	-0.32648
9	-0.80202	0.524181	0.228328	-0.69650	-0.59756	-0.98418
10	-1.61137	-0.12234	0.366460	-2.48878	-1.87478	-1.93695
11	0.115166	1.435916	1.074505	0.136129	-0.38354	-0.88564
12	0.737355	1.774941	1.405097	0.777681	0.679623	-0.05129
13	-2.59764	-0.00747	0.131546	-1.81831	-1.96172	-2.34069
14	-1.32258	0.789944	0.788581	0.625280	-0.27648	-1.06140
15	0.815158	2.093796	2.401309	1.584284	0.923758	0.238421
16	-1.98150	-0.33620	0.015751	-2.40112	-1.92168	-2.06714
17	-0.09672	0.744570	0.869679	-0.17951	0.057410	-0.55341
18	-1.62781	0.367127	0.829714	-1.23106	-1.48773	-1.87907
19	-2.12835	-0.22746	0.203550	-1.83273	-1.66088	-2.10310
20	0.343488	0.630732	1.145220	-0.12090	-0.00063	-0.43127
21	-0.83201	0.024161	0.178263	0.077772	-0.96372	-1.45134
22	-3.03003	-2.51760	-1.38093	-2.40442	-2.64802	-3.12382
23	-0.07528	0.428223	0.995343	-0.04647	-0.39339	-0.90287
24	0.228198	2.621480	2.738867	0.524888	0.569598	-0.05114
25	-0.66754	0.088410	0.961386	-0.53177	-0.71511	-1.20509
26	-0.75548	0.346503	0.384810	0.139449	-0.43313	-0.51684
27	-1.28662	1.433472	2.486486	0.016893	-0.08728	-0.41639
28	3.075096	2.079782	2.787488	2.021545	2.140494	1.816315
29	-3.78528	-1.99838	-1.46398	-2.13754	-3.01315	-3.56124
30	0.646593	1.384355	1.761746	0.855575	1.331688	0.272101
31	0.323459	1.468759	1.879028	0.359798	1.082064	-0.20092
32	-1.92066	-0.50086	-0.39101	-1.40065	-2.12329	-2.84114
33	-0.23892	0.352118	0.727831	0.346063	-0.34436	-1.22346
34	-2.83808	-1.41263	-2.93969	-2.09402	-2.68650	-3.56956
35	-1.66159	-0.50123	-0.28845	-1.22382	-1.53524	-2.58581
36	-0.43438	0.056080	-0.45366	0.124972	-0.27198	-1.43571
37	0.755633	3.236378	3.345520	2.014063	1.628349	0.306459
38	-0.93649	0.426985	0.530448	-0.54417	-0.38086	-1.67537
39	-1.99548	-0.91185	-0.06409	-1.14852	-1.69047	-2.60765
40	-0.04963	2.421703	2.366588	1.763506	1.034501	-0.40889
41	-0.46006	0.821886	1.429584	0.442054	-0.27300	-1.32086
42	-0.66983	0.286843	1.046069	0.043150	0.037790	-1.22075
43	-1.21879	0.135067	-0.18589	-0.49084	-1.61131	-2.75003
44	-1.34823	0.222551	0.370088	-0.42436	-0.41165	-1.81189
45	0.645372	1.762578	0.847255	1.301410	0.235958	-1.24822
46	-2.11077	-0.34171	0.236647	-0.87809	-1.67844	-2.65453
47	-1.13753	-3.15203	-3.51590	-3.30962	-4.07309	-4.90559
48	-0.85033	1.322030	1.382424	0.201809	-0.18834	-1.44683
49	-2.98714	-2.21526	-2.53290	-2.26583	-3.15980	-4.47701
50	0.202069	2.515004	3.385258	0.884544	1.157869	-0.01748
51	-0.06243	1.509988	1.617885	0.699881	0.289616	-1.02440
52	-0.59852	0.703867	0.997782	-0.04442	-0.36167	-1.62288
53	-0.76488	1.027529	1.124856	-0.05683	-0.57778	-1.64425
54	-0.30936	0.133249	0.200647	-0.00843	-0.42889	-0.78760
55	0.989188	1.330062	1.817513	0.832773	0.836617	0.931639
56	0.415565	-0.26093	0.102882	0.097904	-0.38002	-0.62861
57	-1.69231	-0.82222	-0.62984	-1.35935	-1.46838	-1.72491
58	-1.24107	0.107863	-0.19352	-1.42942	-1.01642	-1.13499
59	-0.49833	0.380212	0.569266	-0.54490	-0.27436	-0.39209
60	-1.99634	-2.10718	-2.60935	-2.65862	-3.21310	-3.82958

APPENDIX X

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 3 data in Chapter VI.

X.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	-4.72870	0.246821	0.577697	-0.00183	-3.28990	-3.42967
2	-4.80794	1.060562	2.203789	-1.33924	-3.65293	-3.41666
3	-3.77382	3.301186	3.301186	0.515677	-1.59706	-0.40384
4	-3.53622	0.980694	5.604189	1.136387	-3.55314	-1.50303
5	-2.52905	2.681094	4.803734	1.157520	-1.10014	0.650288
6	-3.89033	5.215874	5.492029	2.619090	-1.45544	0.040171
7	-4.25485	3.850664	3.674379	-1.36472	-3.20720	-1.76673
8	-4.73455	3.840690	4.097138	1.533674	-2.64894	-2.80621
9	-4.36255	1.989823	2.803361	0.376895	-2.29272	-0.97260
10	-7.22922	1.574534	2.016793	1.436391	-3.17978	-2.66899
11	-9.46526	-1.28948	-0.52512	-4.67124	-5.54914	-6.12276
12	-3.75711	1.064806	1.544254	-3.58564	-0.91096	-0.20524
13	-4.64401	0.994168	1.391344	-1.78327	-1.84970	-1.19746
14	-5.60002	2.098082	2.098082	-0.88734	-3.15576	-2.44302
15	-5.68053	1.059399	1.059399	-0.99734	-3.06618	-1.31189
16	-8.44761	1.969715	1.969715	-1.97595	-4.15272	-2.71466
17	-3.62239	0.556470	0.556470	1.377236	-0.07245	0.857453
18	-4.93532	4.549583	6.746474	-0.39749	-0.83376	-0.73753
19	-8.05870	3.973345	3.058106	-1.01725	-6.15457	-2.92323
20	-6.89330	1.085637	1.227821	-3.09610	-2.20727	-1.50330
21	-5.51360	0.868820	1.693418	1.854951	-2.25770	-2.58816
22	-7.71464	0.994227	1.503457	-0.75768	-3.45387	-4.37241
23	-6.20316	-1.15743	0.122414	2.185139	-3.02685	-2.36735
24	-6.24982	3.826722	4.308674	0.050663	-3.13742	-3.53838
25	-8.31950	3.513824	5.219445	-4.65759	-4.74350	-3.33138
26	-4.53034	3.109663	2.291018	-1.86576	-3.91328	-2.64453
27	0.542501	4.014350	4.014350	3.453208	2.977562	3.853929
28	-4.16130	1.757819	1.379345	2.918710	-0.48570	1.340063
29	-4.84327	-0.04367	-0.04367	0.642471	-3.89435	-7.30787
30	3.795195	2.256288	2.256288	1.046505	2.666797	3.387978
31	6.310616	1.394391	1.394391	4.041202	6.092054	4.301922
32	-5.64407	1.676352	0.188963	-4.66076	-6.37933	-4.94317
33	-2.58018	1.167037	1.167037	1.057429	-4.24960	-5.82273
34	-3.09058	1.258449	2.457388	-1.09400	-3.43811	-3.41271
35	-1.42496	-1.12428	1.263537	-0.98940	-2.04657	-1.21364
36	2.899137	1.069688	1.069688	4.754019	2.577558	-1.23922
37	1.372939	-2.31385	-1.80041	-0.12821	0.866464	-1.61279
38	0.497783	0.645206	0.645206	0.898324	-0.35670	-1.44472
39	-0.86169	0.059751	0.059751	0.508820	-2.36464	-0.87454
40	3.862621	1.401549	-0.30131	6.404465	3.116907	1.036126
41	2.472953	31.67402	3.153359	3.434183	1.688728	4.458198
42	2.104841	-3.12419	-2.33062	-0.00652	1.096347	-0.46501
43	-6.78124	1.019069	1.019069	-5.41087	-7.88517	-7.30496
44	4.804330	5.755227	5.476783	5.582475	3.887118	1.885731
45	-0.21425	0.222495	1.710184	-1.36529	-0.62340	-0.28049
46	-0.23782	6.102166	4.769618	2.470518	-1.31480	-0.40329
47	-6.13165	1.435695	0.909832	-8.52037	-5.11420	-3.99841
48	0.553052	0.179394	0.402596	-1.41662	1.288220	-0.48362
49	-9.98443	-7.63881	-7.24877	-9.84190	-9.87739	-10.3453
50	3.954143	6.548815	6.548815	5.689882	3.555151	3.355721
51	4.351011	5.136120	5.136120	5.393987	3.999735	4.305075
52	-0.83854	1.633805	1.633805	0.978076	-1.64353	-0.91060
53	1.645392	1.054303	1.157008	2.890393	0.812990	-0.60328
54	-2.09444	0.510933	0.119942	-1.39276	-2.70873	-1.82360
55	6.649548	6.758836	6.758836	5.627624	6.159762	6.006249
56	-0.57606	-0.66927	-0.66927	0.790869	-1.39111	-0.69416
57	-2.79813	-2.54172	-2.85049	-1.87271	-3.62785	-2.06112
58	-2.72015	-1.93786	-1.07831	-4.00207	-3.48861	-3.96073
59	0.590268	3.042392	1.929399	2.477681	0.184930	-0.15890
60	-1.16688	-1.96100	-2.73128	-3.55295	-1.95270	-2.05911

X.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	2.174942	1.726160	1.195743	0.046027	1.684328	2.047767
2	2.254175	0.912420	0.430349	1.383437	2.047359	2.034756
3	1.220060	1.328203	1.527745	0.471486	0.008513	0.978063
4	0.982453	0.992288	3.830748	1.092196	1.947567	0.121127
5	0.024716	0.708111	3.030294	1.113329	0.505436	2.032194
6	1.336569	3.242891	3.718588	2.574899	0.150138	1.422077
7	1.701092	1.877681	1.900939	1.408918	1.601626	0.384825
8	2.180790	1.867707	2.323698	1.489482	1.043363	1.424313
9	1.808785	0.016840	1.029920	0.332704	0.687149	0.409300
10	4.675461	0.398448	0.243352	1.392200	1.574202	1.287090
11	6.911494	3.262462	2.298563	4.715438	3.943562	4.740860
12	1.203343	0.908176	0.229185	3.629839	0.694618	1.176664
13	2.090249	0.978813	0.382095	1.827467	0.244121	0.184437
14	3.046254	0.125099	0.324642	0.931536	1.550185	1.061121
15	3.126764	0.913583	0.714041	1.041538	1.460608	0.070010
16	5.893843	0.003266	0.196275	2.020146	2.547144	1.332759
17	1.068627	1.416511	1.216969	1.333045	1.533123	2.239359
18	2.381555	2.576600	4.973033	0.441688	0.771812	0.644374
19	5.504938	2.000362	1.284665	1.061446	4.548994	1.541333
20	4.339541	0.887344	0.545619	3.140293	0.601694	0.121396
21	2.959839	1.104162	0.080022	1.810760	0.652126	1.206261
22	5.160879	0.978755	0.269982	0.801877	1.848299	2.990510
23	3.649395	3.130419	1.651025	2.140948	1.421276	0.985448
24	3.696055	1.853739	2.535234	0.006472	1.531846	2.156477
25	5.765733	1.540842	3.446005	4.701783	3.137922	1.949477
26	1.976578	1.136681	0.517577	1.909952	2.307705	1.262623
27	3.096269	2.041367	2.240909	3.409017	4.583141	5.235835
28	1.607540	0.215163	0.394095	2.874519	1.119872	2.721969
29	2.289503	2.016660	1.817117	0.596280	2.288773	5.925970
30	6.348962	0.283305	0.482848	1.002314	4.272377	4.769885
31	8.864383	0.578590	0.379048	3.997011	7.697634	5.683829
32	3.090304	0.296630	1.584476	4.704952	4.773752	3.561264
33	0.026418	0.805944	0.606402	1.013238	2.644029	4.440829
34	0.536817	0.714532	0.683947	1.138199	1.832539	2.030810
35	1.128804	3.097264	0.509902	1.033594	0.440993	0.168258
36	5.452904	0.903294	0.703752	4.709828	4.183138	0.142680
37	3.926706	4.286834	3.573852	0.172407	2.472044	0.230892
38	3.051551	1.327776	1.128234	0.854132	1.248874	0.062816
39	1.692073	1.913231	1.713688	0.464629	0.759067	0.507357
40	6.416388	0.571433	2.074753	6.360273	4.722487	2.418033
41	5.026720	29.70104	1.379919	3.389992	3.294308	5.840104
42	4.658608	5.097178	4.104061	0.050717	2.701927	0.916894
43	4.227476	0.953913	0.754371	5.455062	6.279599	5.923060
44	7.358097	3.782245	3.703343	5.538283	5.492698	3.267638
45	2.339515	1.750486	0.063256	1.409483	0.982174	1.101407
46	2.315945	4.129184	2.996177	2.426327	0.290779	0.978613
47	3.577884	0.537287	0.863607	8.564563	3.508628	2.616510
48	3.106819	1.793588	1.370844	1.460818	2.893800	0.898281
49	7.430669	9.611795	9.022211	9.886093	8.271816	8.963437
50	6.507910	4.575832	4.775375	5.645691	5.160730	4.737627
51	6.904778	3.163138	3.362680	5.349796	5.605315	5.686982
52	1.715223	0.339176	0.139634	0.933885	0.037956	0.471301
53	4.199160	0.918679	0.616432	2.846202	2.418570	0.778621
54	0.459319	1.462049	1.653497	1.436954	1.103157	0.441695
55	9.203315	4.785853	4.985395	5.583432	7.765342	7.388155
56	1.977701	2.642262	2.442719	0.746678	0.214463	0.687745
57	0.244366	4.514707	4.623932	1.916906	2.022279	0.679223
58	0.166390	3.910850	2.851756	4.046268	1.883036	2.578833
59	3.144035	1.069409	0.155958	2.433490	1.790510	1.223000
60	1.386886	3.933992	4.504724	3.597142	0.347120	0.677211

X.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	-1.41427	0.103120	0.309041	-0.00074	-1.35995	-1.58828
2	-1.43797	0.443094	1.178925	-0.54341	-1.51002	-1.58225
3	-1.12868	1.379207	1.765981	0.209243	-0.66018	-0.18701
4	-1.05762	0.409725	2.997981	0.461105	-1.46877	-0.69605
5	-0.75639	1.120138	2.569775	0.469680	-0.45476	0.301149
6	-1.16353	2.179148	2.937980	1.062733	-0.60163	0.018603
7	-1.27255	1.608775	1.965622	-0.55375	-1.32576	-0.81817
8	-1.41602	1.604607	2.191779	0.622310	-1.09499	-1.29956
9	-1.30476	0.831331	1.499668	0.152930	-0.94775	-0.45041
10	-2.16213	0.657827	1.078890	0.582836	-1.31443	-1.23601
11	-2.83089	-0.53873	-0.28091	-1.89542	-2.29386	-2.83545
12	-1.12368	0.444867	0.826104	-1.45492	-0.37656	-0.09504
13	-1.38894	0.415355	0.744304	-0.72358	-0.76461	-0.55454
14	-1.67486	0.876561	1.122376	-0.36005	-1.30450	-1.13136
15	-1.69894	0.442607	0.566729	-0.40468	-1.26747	-0.60753
16	-2.52653	0.822930	1.053706	-0.80177	-1.71662	-1.25716
17	-1.08339	0.232488	0.297686	0.558833	-0.02995	0.397087
18	-1.47606	1.900777	3.609050	-0.16129	-0.34465	-0.34155
19	-2.41021	1.660030	1.635945	-0.41276	-2.54412	-1.35375
20	-2.06166	0.453570	0.656827	-1.25628	-0.91242	-0.69618
21	-1.64902	0.362985	0.905900	0.752673	-0.93327	-1.19858
22	-2.30731	0.415379	0.804280	-0.30744	-1.42773	-2.02486
23	-1.85525	-0.48356	0.065486	0.886651	-1.25121	-1.09632
24	-1.86921	1.598772	2.304941	0.020557	-1.29692	-1.63862
25	-2.48821	1.468046	2.792161	-1.88988	-1.96083	-1.54276
26	-1.35494	1.299191	1.225588	-0.75705	-1.61764	-1.22468
27	0.162252	1.677161	2.147491	1.401188	1.230841	1.784757
28	-1.24457	0.734402	0.737885	1.184308	-0.20077	0.620584
29	-1.44853	-0.01824	-0.02336	0.260692	-1.60981	-3.38428
30	1.135076	0.942658	1.207009	0.424634	1.102379	1.568975
31	1.887394	0.582565	0.745934	1.639775	2.518285	1.992223
32	-1.68804	0.700365	0.101086	-1.89117	-2.63703	-2.28918
33	-0.77168	0.487578	0.624311	0.429067	-1.75667	-2.69651
34	-0.92433	0.525769	1.314588	-0.44390	-1.42122	-1.58043
35	-0.42618	-0.46971	0.675933	-0.40146	-0.84599	-0.56204
36	0.867081	0.446906	0.572233	1.929011	1.065490	-0.57388
37	0.410622	-0.96670	-0.96313	-0.05202	0.358172	-0.74688
38	0.148878	0.269561	0.345155	0.364507	-0.14745	-0.66905
39	-0.25771	0.024963	0.031964	0.206461	-0.97747	-0.40500
40	1.155242	0.585555	-0.16118	2.598702	1.288442	0.479831
41	0.739616	13.23314	1.686901	1.393469	0.698073	2.064595
42	0.629521	-1.30526	-1.24677	-0.00264	0.453199	-0.21534
43	-2.02815	0.425758	0.545154	-2.19553	-3.25951	-3.38293
44	1.436890	2.404485	2.929825	2.265168	1.606826	0.873284
45	-0.06407	0.092956	0.914869	-0.55398	-0.25769	-0.12989
46	-0.07112	2.549433	2.551524	1.002448	-0.54350	-0.18676
47	-1.83386	0.599821	0.486718	-3.45726	-2.11407	-1.85167
48	0.165408	0.074949	0.215370	-0.57481	0.532514	-0.22396
49	-2.98616	-3.19143	-3.87775	-3.99349	-4.08304	-4.79093
50	1.182614	2.736039	3.503312	2.308751	1.469600	1.554037
51	1.301310	2.145827	2.747586	2.188687	1.653379	1.993684
52	-0.25079	0.682590	0.874010	0.396868	-0.67939	-0.42170
53	0.492108	0.440478	0.618945	1.172818	0.336067	-0.27938
54	-0.62641	0.213463	0.064163	-0.56513	-1.11971	-0.84451
55	1.988762	2.823784	3.615663	2.283488	2.546274	2.781499
56	-0.17229	-0.27961	-0.35803	0.320906	-0.57504	-0.32146
57	-0.83687	-1.06191	-1.52488	-0.75988	-1.49965	-0.95451
58	-0.81355	-0.80962	-0.57684	-1.62390	-1.44209	-1.83422
59	0.176538	1.271085	1.032139	1.005354	0.076445	-0.07358
60	-0.34899	-0.81929	-1.46111	-1.44166	-0.80719	-0.95357

APPENDIX XI

Residual Terminal Value (Q), Mean Absolute Deviations of Returns (MADs), and Return to Volatility (R/V) values for Sample 4 data in Chapter VI.

XI.a. Residual Values (Q)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	7.280418	-2.25082	-1.57659	0.973346	4.775325	1.099749
2	7.417998	-2.09617	-1.42137	1.106436	4.935970	1.252668
3	2.498229	-0.80551	-1.44413	-1.21439	0.054903	-4.82185
4	-2.61626	-1.25783	1.114065	-1.96441	-2.41028	-3.72017
5	-4.56508	2.289415	2.004186	-2.00226	-2.63134	-0.83506
6	-2.88301	-0.32680	0.153830	-1.18763	-1.50515	-0.38395
7	-10.2610	-2.65295	-0.63881	-6.10543	-7.98567	-7.53359
8	3.109378	-1.32489	0.475571	2.134189	5.856066	11.07184
9	-0.29890	1.157593	1.761703	-1.24277	-1.71120	-2.45463
10	-2.83814	1.024385	-0.05943	-2.84165	-3.93507	-3.68138
11	6.306359	1.682503	1.522585	-1.07722	4.756217	7.382567
12	1.608964	-1.88791	-1.88791	-1.84026	0.686415	-2.44892
13	0.514036	0.642706	0.825393	-1.18483	1.007360	1.176389
14	-3.39746	1.520302	1.520302	-1.61695	-2.47125	-1.80662
15	-3.55842	-1.14086	-0.30029	-0.79462	-4.13562	-4.70712
16	-0.94422	0.453049	1.011635	-1.20548	-0.11549	1.118420
17	0.565284	1.438414	1.204460	-1.30708	0.529582	2.940923
18	5.575388	-0.95620	1.083218	4.221238	5.505882	6.948102
19	-1.30806	-0.41899	0.540989	-1.20886	-2.50192	-4.13464
20	-1.70828	-0.46053	0.607374	-0.77979	-2.04917	-3.00299
21	-1.06762	3.400269	1.706244	0.197865	0.974308	5.046084
22	-5.66843	-1.47130	-0.10165	-6.61139	-7.80437	-11.6331
23	2.769461	-1.62377	-1.62377	0.955327	1.822150	4.455156
24	4.284526	0.372406	-1.02373	1.577303	3.500944	4.503116
25	-2.05719	-1.98178	-0.71254	0.009333	-2.09498	-0.76135
26	-3.42630	1.610046	1.610046	2.607603	4.703811	3.096859
27	-8.62908	1.446491	2.365694	-2.18112	-8.00054	-10.0205
28	-0.55941	-3.15857	-1.01851	-0.56882	1.493928	8.007134
29	10.77361	-2.49831	-0.79747	3.404988	7.152461	4.094244
30	1.925525	-1.72702	0.318973	1.240554	0.947465	-1.74091
31	2.521532	0.128601	1.726644	1.325204	3.166820	-1.67155
32	-5.27380	0.225982	0.280262	-3.71231	-5.91312	-2.60109
33	-1.57097	-0.20575	-1.06750	0.096092	-2.10227	-1.44589
34	5.778924	-1.95736	0.256225	1.788074	5.868967	7.786995
35	2.747139	-1.41682	3.322023	-0.48338	1.378968	-2.53091
36	-1.32053	-2.44108	0.946222	0.380244	-0.94608	0.260015
37	-3.78245	-2.30995	-1.09552	-0.70825	-4.36884	-4.61113
38	1.168248	0.680468	1.367952	-0.95717	1.977664	6.155197
39	2.293316	-2.26875	-0.66421	-1.67751	1.122835	-0.69506
40	6.052976	4.262742	4.262742	2.127159	4.241990	5.579712
41	-1.83996	-4.00617	-2.78149	-1.06367	-0.30464	-3.33031
42	-4.75422	0.941514	0.182230	-2.57355	-4.90163	-5.65413
43	-3.23364	-2.00312	-2.00312	-2.05613	0.090422	3.741969
44	-4.45781	-0.46562	-0.35883	-3.32665	-4.66450	-1.77586
45	-1.04464	0.586834	1.130961	1.924513	1.803202	7.682225
46	-4.56337	-2.63909	-0.96826	-4.08520	-4.14586	-4.07908
47	2.165623	-1.65987	-0.83962	0.697853	1.432494	4.214508
48	3.529007	-0.69905	2.552209	5.379104	3.176649	1.386361
49	-11.6702	-1.17325	-0.87529	-6.80128	-11.4556	-12.8689
50	-2.29507	-0.60928	-0.74325	-0.40738	-0.52358	2.491419
51	0.497366	-2.28028	-3.30639	-0.09380	2.346739	1.164509
52	-6.41429	-1.48381	-0.23118	-5.65760	-6.75693	-7.69676
53	0.645078	-1.19348	1.802374	1.563401	0.707403	4.216742
54	0.362477	-0.56043	1.162308	1.487579	1.499416	1.774258
55	-5.26506	-2.24329	-2.24329	-4.18158	-6.14947	-6.82838
56	2.869300	-0.45894	-0.45894	2.303106	4.184347	3.116173
57	-0.57674	-0.95821	-0.78906	1.853272	0.562187	0.509729
58	-3.54558	-0.04589	-0.17041	-1.00115	-2.62283	-3.39510
59	0.031911	1.735290	4.881021	1.164673	0.089846	3.201721
60	-9.39832	0.971420	0.920589	-4.48939	-9.39650	-11.0977

XI.b. Mean Absolute Deviations (MAD)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	7.972111	1.675003	1.800221	1.634891	5.295848	1.407987
2	8.109692	1.520349	1.644999	1.767981	5.456492	1.560905
3	3.189923	0.229694	1.667756	0.552854	0.575425	4.513616
4	1.924573	0.682008	0.890443	1.302874	1.889761	3.411935
5	3.873395	2.865239	1.780564	1.340716	2.110820	0.526831
6	2.191322	0.249023	0.069790	0.526094	0.984636	0.075718
7	9.569312	2.077134	0.862435	5.443889	7.445153	7.225356
8	3.801071	0.749068	0.251949	2.795734	6.376588	11.38008
9	0.392792	1.733417	1.538081	0.581229	1.190679	2.146399
10	2.146448	1.600209	0.283060	2.180112	3.414554	3.373149
11	6.998052	2.258327	1.298964	0.415682	5.276739	7.690804
12	2.300657	1.312089	2.111535	1.178722	1.206937	2.140686
13	1.205730	1.218530	0.601772	0.523285	1.527882	1.484627
14	2.705773	2.096126	1.296680	0.955405	1.950735	1.498388
15	2.866731	0.565037	0.523919	0.133080	3.615100	4.398891
16	0.252528	1.028873	0.788014	0.543943	0.405024	1.426658
17	1.256978	2.014238	0.980838	0.645542	1.050104	3.249161
18	6.267082	0.380377	0.859596	4.882783	6.026404	7.256339
19	0.616372	0.156829	0.317367	0.547324	1.981406	3.826405
20	1.016590	0.115286	0.383752	0.118249	1.528657	2.694755
21	0.375926	3.976093	1.482623	0.859410	1.494831	5.354321
22	4.976741	0.895484	0.325279	5.949850	7.283855	11.32489
23	3.461155	1.047951	1.847397	1.616872	2.342672	4.763393
24	4.976220	0.948230	1.247361	2.238848	4.021466	4.811353
25	1.365498	1.405955	0.936163	0.670878	1.574462	0.453118
26	2.734607	2.185871	1.386425	3.269148	5.224333	3.405096
27	7.937387	2.022316	2.142072	1.519579	7.480017	9.712356
28	0.132281	2.582748	1.242135	0.092721	2.014451	8.315372
29	11.46530	1.922486	1.021093	4.066533	7.672983	4.402482
30	2.617218	1.151199	0.095351	1.902099	1.467987	1.432674
31	3.213225	0.704425	1.503022	1.986749	3.687342	1.363316
32	4.582110	0.801806	0.056641	3.050765	5.392598	2.292859
33	0.879277	0.370071	1.291131	0.757637	1.581755	1.137660
34	6.470618	1.381543	0.032603	2.449619	6.389490	8.095233
35	3.438832	0.840998	3.098402	0.178164	1.899490	2.222673
36	0.628843	1.865258	0.722600	1.041789	0.425558	0.568252
37	3.090756	1.734133	1.319146	0.046714	3.848322	4.302893
38	1.859942	1.256293	1.144330	0.295631	2.498186	6.463434
39	2.985010	1.692932	0.887840	1.015974	1.643357	0.386822
40	6.744670	4.838567	4.039121	2.788704	4.762512	5.887950
41	1.148274	3.430350	3.005116	0.402128	0.215880	3.022081
42	4.062531	1.517338	0.041391	1.912011	4.381111	5.345901
43	2.541947	1.427305	2.226751	1.394593	0.610944	4.050206
44	3.766123	0.110194	0.582460	2.665112	4.143979	1.467625
45	0.352950	1.162659	0.907339	2.586058	2.323724	7.990463
46	3.871679	2.063268	1.191888	3.423656	3.625339	3.770843
47	2.857316	1.084048	1.063248	1.359398	1.953016	4.522746
48	4.220700	0.123229	2.328588	6.040649	3.697171	1.694598
49	10.97857	0.597430	1.098913	6.139742	10.93513	12.56073
50	1.603382	0.033462	0.966880	0.254162	0.003059	2.799656
51	1.169060	1.704465	3.530015	0.567735	2.867262	1.472747
52	5.722597	0.907994	0.454807	4.996064	6.236414	7.388530
53	1.336771	0.617661	1.578752	2.224946	1.227925	4.524980
54	1.054170	0.015393	0.938687	2.149124	2.019938	2.082496
55	4.573368	1.667473	2.466919	3.520044	5.628947	6.520152
56	3.560993	0.116881	0.682564	2.964651	4.704870	3.424410
57	0.114949	0.382389	1.012688	2.514817	1.082709	0.817966
58	2.853887	0.529933	0.394041	0.339605	2.102317	3.086870
59	0.723604	2.311114	4.657400	1.826218	0.610368	3.509959
60	8.706629	1.547244	0.696967	3.827848	8.875983	10.78954

XI.c. Return to Volatility (R/V)

OBS	STRAT1	STRAT2	STRAT3	STRAT4	STRAT5	STRAT6
1	2.101815	-1.69871	-1.25130	0.507936	1.437912	0.265183
2	2.141534	-1.58199	-1.12810	0.577388	1.486284	0.302057
3	0.721224	-0.60793	-1.14617	-0.63372	0.016532	-1.16269
4	-0.75530	-0.94929	0.884203	-1.02512	-0.72576	-0.89704
5	-1.31791	1.727836	1.590668	-1.04487	-0.79233	-0.20136
6	-0.83231	-0.24663	0.122091	-0.61976	-0.45322	-0.09258
7	-2.96229	-2.00220	-0.50700	-3.18609	-2.39856	-1.81658
8	0.897659	-0.99990	0.377448	1.113716	1.763337	2.669765
9	-0.08629	0.873643	1.398215	-0.64853	-0.51526	-0.59188
10	-0.81935	0.773110	-0.04717	-1.48290	-1.18490	-0.88769
11	1.820610	1.269796	1.208434	-0.56214	1.432159	1.780166
12	0.464498	-1.42482	-1.49838	-0.96033	0.206688	-0.59051
13	0.148399	0.485054	0.655092	-0.61829	0.303329	0.283664
14	-0.98082	1.147382	1.206622	-0.84379	-0.74412	-0.43563
15	-1.02729	-0.86101	-0.23833	-0.41467	-1.24528	-1.13503
16	-0.27259	0.341919	0.802907	-0.62907	-0.03477	0.269685
17	0.163194	1.085581	0.955947	-0.68209	0.159464	0.709147
18	1.609583	-0.72165	0.859721	2.202832	1.657892	1.675402
19	-0.37763	-0.31621	0.429368	-0.63084	-0.75336	-0.99699
20	-0.49317	-0.34757	0.482056	-0.40693	-0.61703	-0.72411
21	-0.30821	2.566206	1.354199	0.103255	0.293377	1.216767
22	-1.63644	-1.11040	-0.08068	-3.45012	-2.34999	-2.80510
23	0.799527	-1.22547	-1.28874	0.498532	0.548673	1.074276
24	1.236918	0.281057	-0.81251	0.823107	1.054179	1.085841
25	-0.59389	-1.49566	-0.56552	0.004870	-0.63082	-0.18358
26	-0.98915	1.215113	1.277850	1.360764	1.416378	0.746748
27	-2.49116	1.091677	1.877587	-1.13820	-2.40906	-2.41627
28	-0.16149	-2.38379	-0.80836	-0.29683	0.449841	1.930768
29	3.110280	-1.88549	-0.63293	1.776876	2.153699	0.987249
30	0.555888	-1.30339	0.253160	0.647377	0.285294	-0.41978
31	0.727952	0.097056	1.370390	0.691551	0.953570	-0.40306
32	-1.52251	0.170550	0.222436	-1.93725	-1.78051	-0.62720
33	-0.45353	-0.15528	-0.84725	0.050145	-0.63302	-0.34865
34	1.668342	-1.47723	0.203359	0.933097	1.767222	1.877686
35	0.793083	-1.06928	2.636599	-0.25224	0.415225	-0.61028
36	-0.38123	-1.84230	0.750990	0.198428	-0.28487	0.062697
37	-1.09197	-1.74334	-0.86948	-0.36960	-1.31551	-1.11188
38	0.337266	0.513554	1.085706	-0.49949	0.595500	1.484209
39	0.662067	-1.71224	-0.52717	-0.87540	0.338100	-0.16760
40	1.747460	3.217120	3.383222	1.110047	1.277318	1.345441
41	-0.53118	-3.02348	-2.20759	-0.55507	-0.09173	-0.80304
42	-1.37251	0.710567	0.144631	-1.34299	-1.47594	-1.36338
43	-0.93353	-1.51177	-1.58982	-1.07298	0.027227	0.902304
44	-1.28694	-0.35141	-0.28480	-1.73599	-1.40454	-0.42821
45	-0.30158	0.442888	0.897613	1.004297	0.542967	1.852422
46	-1.31741	-1.99174	-0.76848	-2.13184	-1.24837	-0.98359
47	0.625203	-1.25271	-0.66638	0.364171	0.431342	1.016248
48	1.018804	-0.52758	2.025619	2.807058	0.956530	0.334294
49	-3.36913	-0.88546	-0.69469	-3.54921	-3.44944	-3.10310
50	-0.66257	-0.45983	-0.58990	-0.21259	-0.15765	0.600758
51	0.143587	-1.72095	-2.62419	-0.04895	0.706633	0.280799
52	-1.85176	-1.11984	-0.18348	-2.95239	-2.03460	-1.85592
53	0.186230	-0.90073	1.430495	0.815853	0.213008	1.016787
54	0.104645	-0.42296	0.922492	0.776286	0.451493	0.427828
55	-1.51999	-1.69303	-1.78044	-2.18214	-1.85168	-1.64653
56	0.828350	-0.34636	-0.36425	1.201864	1.259961	0.751405
57	-0.16650	-0.72317	-0.62626	0.967120	0.169281	0.122911
58	-1.02358	-0.03463	-0.13525	-0.52244	-0.78977	-0.81866
59	0.009212	1.309635	3.873933	0.607779	0.027053	0.772034
60	-2.71324	0.733137	0.730646	-2.34276	-2.82941	-2.67601