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CATHODE INVERTRON.

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THE UNIVERSITY OF OKLAHOMA  
GRADUATE COLLEGE

MOLECULAR LIFETIME DETERMINATION VIA COLD CATHODE  
INVERTRON

A DISSERTATION  
SUBMITTED TO THE GRADUATE FACULTY  
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BY  
WILLIAM C. PASKE  
NORMAN, OKLAHOMA  
1974

MOLECULAR LIFETIME DETERMINATION VIA COLD  
CATHODE INVERTRON

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## STATEMENT OF PURPOSE

The purpose of this research is essentially twofold. First to validate the experimental apparatus and to determine in which ways the equipment could vitiate the results; and secondly, to use the improved apparatus to determine the lifetimes of  $\text{SO}_2$ .

In 1970 Copeland<sup>1</sup> developed the cold cathode invertron for lifetime studies of molecules easily dissociated by the thermally heated cathode of the Holzberlein invertron (see the works of Holzberlein<sup>2</sup>, Johnson<sup>3</sup>, Shaeffer<sup>4</sup> and Thompson<sup>5</sup>). Copeland's studies dealt with nitric oxide, atomic and molecular oxygen, and in his work he reported lifetimes with as much as 20% uncertainty. Anderson<sup>6</sup> has used a similar device and reported lifetimes of the  $\text{C}^3\Pi_u$ , and  $\text{B}^2\Sigma_u^+$  states of  $\text{N}_2$  and  $\text{N}_2^+$ , with rather large experimental scatter. It was felt that design improvements and a better understanding of the experimental equipment could eliminate much of this scatter.

To this aim, the rather excellent data which Johnson<sup>3</sup> ( $\pm 2\%$  in most cases) presented for the  $\text{C}^3\Pi_u$  and  $\text{B}^2\Sigma_u^+$  states of  $\text{N}_2$  and  $\text{N}_2^+$  have served as a standard. The first section of this paper then will deal with the development of the experimental apparatus (where different from Copeland) and the effects of the following on lifetime measurements: data acquisition rate, discharge voltage, applied external magnetic field, cathode structure (geometry), the addition of a quenching gas and finally, the problems caused by an improperly functioning photomultiplier tube.

The remaining sections will deal first with the measurements made on nitrogen to give an overview of the system's accuracy and finally a discussion of the lifetime measurements made on sulfur dioxide will conclude this report.

## CHAPTER I

### INTRODUCTORY COMMENTS

Why measure the lifetimes or the transition probabilities of excited atomic or molecular systems? The answer to this question is intended to provide an overview for this chapter, i.e. why directly measured lifetimes are useful to various fields of physics. A few obvious examples of fields which can make good use of lifetime measurements are spectroscopy, astrophysics (similarly for environmental physics), and lasers. It should be noted that lifetimes themselves are not needed in these fields, but the Einstein transition coefficients  $A_{ik}$  are, and these are related to the measured lifetime by the relation

$$\tau_k = 1/\sum_i A_{ik} \quad 1$$

For example, the intensity of a given emission (similarly for absorption) line is related to the Einstein transition coefficient by the expression

$$I_{kn} = A_{kn} N_k \hbar \omega_{kn} \frac{V}{4\pi} \quad 2$$

where  $\omega_{kn} = (E_k - E_n)/\hbar$ ,  $E_k$  and  $E_n$  are the energies of the  $k$  and  $n$  levels,  $V/4\pi$  is the solid angle extended and  $N_k$  is the number of molecules in the state of interest  $k$ . By knowing the transition coefficient  $A_{kn}$  and measuring the relative intensities of the various emission lines, relative populations of the excited states can be determined.

In astrophysics (as well as in the environmental monitoring of the atmosphere) a knowledge of the oscillator strength  $f_{kn}$  will allow a calculation of the absorption coefficients of a gas of atoms or molecules through which the observed light is propagating. The oscillator strength is a dimensionless constant which expresses the absorptive power for each characteristic frequency as a function of the total absorptive power of the electron, i.e. it is a measure of the Fourier coefficient of the particular harmonic of the electron oscillator. It is related to the transition coefficient by the equation

$$f_{kn} = -A_{kn} mc^3 (2e^2 \omega_{kn}^2) \frac{mc}{8\pi^2 e^2} \frac{g_k}{g_n} \lambda^2 A_{kn} \quad 3$$

where  $g_k$  and  $g_n$  are the statistical weights of the levels involved; e.g.  $g_k = 2J_k + 1$ ,  $J_k$  is the total angular momentum number of the system.

In the case of lasers, it is necessary to have some knowledge of the ability to maintain a population inversion which is dependent on the decay rate of the lower laser level. This level obeys the inequality

$$dN_k/dt > A_{kn} g_k / g_n. \quad 4$$

A knowledge of these levels and their transition probabilities then can be of great help in determining the direction of laser research.

One way to obtain the transition coefficients is to calculate them. It is well known that electromagnetic radiation interacts with atomic systems, in that an atom in level  $k$  of energy  $E_k$  described by  $\psi_k$  has a non-zero probability of finding itself in level  $n$  of energy  $E_n$  described by  $\psi_n$  if the electromagnetic radiation was of

wavenumber  $\nu = (E_k - E_n)/hc$  (assuming  $k$  and  $n$  are non-degenerate). This interaction may be considered as an interaction between an electromagnetic wave having an electric vector  $\underline{E}$ , and an atomic system, which in a first approximation is an interaction with an electric dipole of moment  $\underline{M}$ . When this interaction,  $\underline{M} \cdot \underline{E}$ , is treated by quantum mechanics, it is found that the probability of a transition is proportional to the square of the matrix elements of the electric dipole moment, i.e.

$$A_{kn} = \frac{4e^2 \omega_{kn}^3}{3\hbar c^3} \frac{1}{g_k} \sum_{m_n m_k} |\underline{R}_{kn}|^2 \quad 5$$

where

$$\underline{R}_i^{kn} = \int \psi_n^* \underline{M}_i \psi_k dt \quad i = x, y, z. \quad 6$$

This assumes equal population of the  $k$  levels, a valid assumption for most gas discharges.

If  $|\underline{R}^{km}|^2 \neq 0$ , the two states have a finite probability of interaction with the emission or absorption of radiation. If  $|\underline{R}^{km}|^2 = 0$ , then the electric dipole transition is forbidden; that is to say, in the absence of external fields it must go by higher order electric multipoles or by magnetic multipole transitions. Note, however, that the magnetic dipole transitions and the electric quadrupole transitions are down by a factor of  $10^{-5}$  and  $10^{-8}$  with respect to electric dipole radiation.

For a diatomic molecular system, we may consider the molecule to have electronic configuration  $k$ , vibrational quantum number  $v'$  and rotational quantum number  $J'$ . Then, if we assume (i) separability, i.e.  $\psi = \frac{1}{r} \psi_k \psi_{v'} \psi_{J'm'}$ , and (ii) that the electronic por-



tion  $\psi_k(\underline{r}_i)$  does not depend appreciably on the internuclear separation  $\underline{r}$ , that is to say we assume that the transition occurs sufficiently rapidly that the nuclei do not have time to rearrange themselves during the interaction, we find

$$A_{kn, v'v'', J'J''} = \frac{4e^2 \omega^3}{3\hbar c^3 g_k} |R_e^{kn}|^2 |R_{vib}^{v'v''}|^2 \sum_{m', m''} |R_{rot}^{J'J''}|^2 \quad 7$$

where  $R_e^{kn} = \int \psi_{k_i} \sum_{n_i} \psi_n dr_i$

$$R_{vib}^{v'v''} = \int \psi_{v'}^* \psi_{v''} dr \quad \{ |R_{vib}^{v'v''}|^2 \equiv \text{Frank-Condon factors} \}$$

$$R_{rot}^{J'J''} = \int \psi_{J'm'}^* \psi_{J''m''} \sin\theta d\theta d\phi$$

Now, by summing over all transitions from a given vibrational level and using the sum rule

$$\sum_{J''} \sum_{m', m''} |R_{rot}^{J'J''}|^2 = 2J' + 1 \quad 8$$

we find for the transition probability  $A_{kn}$ ,

$$A_{kn, v'v''} = \frac{4e^2 \omega^3}{3\hbar c^3 g_k} |R_e^{kn}|^2 |R_{vib}^{v'v''}|^2 \quad 9$$

Using this equation, calculations can be made to determine the Einstein spontaneous transition probability (and from this the absorption probability) which may then be used in the manner sketched earlier in other branches of physics. Unfortunately many of these calculations have rather large errors associated with them, some as high as 50%<sup>7</sup>).

However, the lifetime (or mean life) of the above vibrational band is given by

$$\tau_k = 1 / \sum_{n,v} A_{kn,v} \nu$$

10

and if this lifetime can be directly measured, significant reduction in the errors of the system can be made, thereby reducing the error in the subsequent manipulations.

From a probabilistic standpoint, we could consider the time rate of change of the population of a given excited level  $N_k$  by the relation

$$\frac{dN_k}{dt} = -N_k \sum_{n,v} A_{kn,v} = -N_k / \tau_k \quad 11$$

which is simply

$$N_k(t) = N_k(0) e^{-t/\tau_k} = N_k(0) e^{-\sum_{n,v} A_{kn,v} t} \quad 12$$

This implies that a straight forward observation of the decay rate of the excited state would yield the transition probability via the mean life or the lifetime of that state.

In actuality, the time rate of change should be written

$$\begin{aligned} \frac{dN_k}{dt} = & -N_k \sum_n A_{kn} + \sum_{\ell} N_{\ell} A_{\ell k} - N_k \sum_{\ell} B_{k\ell} U(\nu_{k\ell}) + \sum_n N_n B_{nk} U(\nu_{nk}) \\ & - N_k \sum_n B_{kn} U(\nu_{kn}) + \sum_{\ell} N_{\ell} B_{\ell k} U(\nu_{\ell k}) - N_0 \nu N_k \sum_n \sigma_{kn} + N_0 \nu N_k \sum_n \sigma_{nk} \\ & - N_0 \nu \sum_{\ell} \sigma_{k\ell} N_{\ell} + N_0 \nu \sum_{\ell} \sigma_{\ell k} N_{\ell} + n_e N_0 \sigma_{ke} \nu + n_e \nu \sum_n \sigma_{kn} N_n - D_k \nu^2 N_k \\ & - \alpha_a N_e N_k - \alpha_r N_e N_k^+ \end{aligned} \quad 13$$

where:  $B_{mn}$  = Einstein transition probability: induced absorption,  $m < n$ ; induced emission,  $m > n$ .

$A_{kn}$  = Einstein spontaneous emission transition probability.

$U(\nu_{kn})$  = radiation density of photons

$\sigma_{mn}$  = cross section for radiationless collisional transition from m to n

$N_0$  = Neutral ground state molecules

$n_e v_e$  = excitation electric current density

$\alpha_a$  = attachment coefficient (electron)

$v$  = mean velocity of neutral ground state molecules

$\alpha_r$  = electron recombination coefficient

$D_k$  = diffusion coefficient

$\sigma_k$  = electron excitation cross section

$\sigma_{nk}$  = electron excitation due to metastable or other excited states

and where  $l$  refers to higher energy levels,  $n$  refers to lower energy levels with respect to the level  $k$ .

Verbally, this equation could be written as the following:

rate of change of the population of level  $k$  =

spontaneous emission from - level of interest to lower state	spontaneous cascade from + upper state to level of interest
induced absorption from - level of interest to higher level	induced absorption from + lower level to level of interest
induced emission from - level of interest to lower level	induced emission from + higher level to level of interest
collisional depopulation - from level of interest to lower level	collisional depopulation - from level of interest to higher level
collisional population + from lower level to level of interest	collisional population from + higher level to level of interest

production due to  
+ electron bombardment  
of neutrals

collisional depopula-  
- tion due to diffusion  
to walls

depopulation due to  
- recombination

production due to  
+ electron bombardment  
of metastables

depopulation by electron  
- quenching

In practice however, the following simplification is usually used, all other terms in Eq. 13 being ignored

$$\frac{dN_k}{dt} = -N_k \sum_m A_{kn} + \sum_\ell N_\ell A_{\ell k} - N_k N_0 \langle v \rangle \sum_n \sigma_{kn} \quad 14$$

and in the event that there are no cascading levels present, we have

$$N_k(t) = N_k(o) e^{-A' t} \quad 15$$

where  $A'$  now includes the pressure depopulation term, i.e.

$$A' = \sum_n A_{kn} + N_0 \langle v \rangle \sum_n \sigma_{kn} \quad 16$$

This type of system will give a pressure dependent lifetime such that  $1/\tau_k$  plotted vs the pressure will yield a slope which is proportional to the collisional depopulation cross section and the intercept will be the zero pressure lifetime of the excited state. A useful relationship between the second term of Eq. 15 and the pressure is

$$N \langle \sigma v \rangle = 1.404 p \sigma \left( \frac{m_1 + m_2}{m_1 m_2 T} \right)^{1/2} \times 10^{23} \text{ sec}^{-1}$$

where pressure  $p$  is in torr,  $\sigma$  is in  $\text{cm}^2$  and the  $m$ 's represent the masses of the respective atoms.

The following sections will deal with the development of the experimental apparatus with Eq. 12 and 13 in mind, i.e. have we left

anything out that we should not have and have we biased the results in any way by the manner in which we conduct the experiments? For example:

cathode structure: any geometrical dependence present? Also, can any new information be collected as to the depopulation of excited states by means other than neutral molecule collisions?

magnetic field: the field is necessary to initiate the discharge at low pressures...does it effect the observed lifetimes in a systematic way?

quenching or attaching gas: how big a role do recombination and attachment play?

discharge voltage: can varying the applied voltage effect the results in any manner?

count rate: does the data acquisition rate have any effect on the lifetimes?

pressure dependence: is there anything other than the usual pressure dependence?

Further discussion of the transitions probabilities may be found in ref. 8-16 and many related works.

## CHAPTER II

### EXPERIMENTAL APPARATUS

#### A

#### Vacuum System

The vacuum system was built to insure sample purity and to provide variable flow rates since the gases intended for study will dissociate rather easily. The system consists of two vacuum systems, one for purity and maintenance of the base pressure ( $10^{-5}$  torr) and the second for the removal of the waste gases. This second system allowed for special handling of the  $\text{SO}_2$ . Fig. 1 illustrates the total system and the dashed line AA' indicates the division of the two.

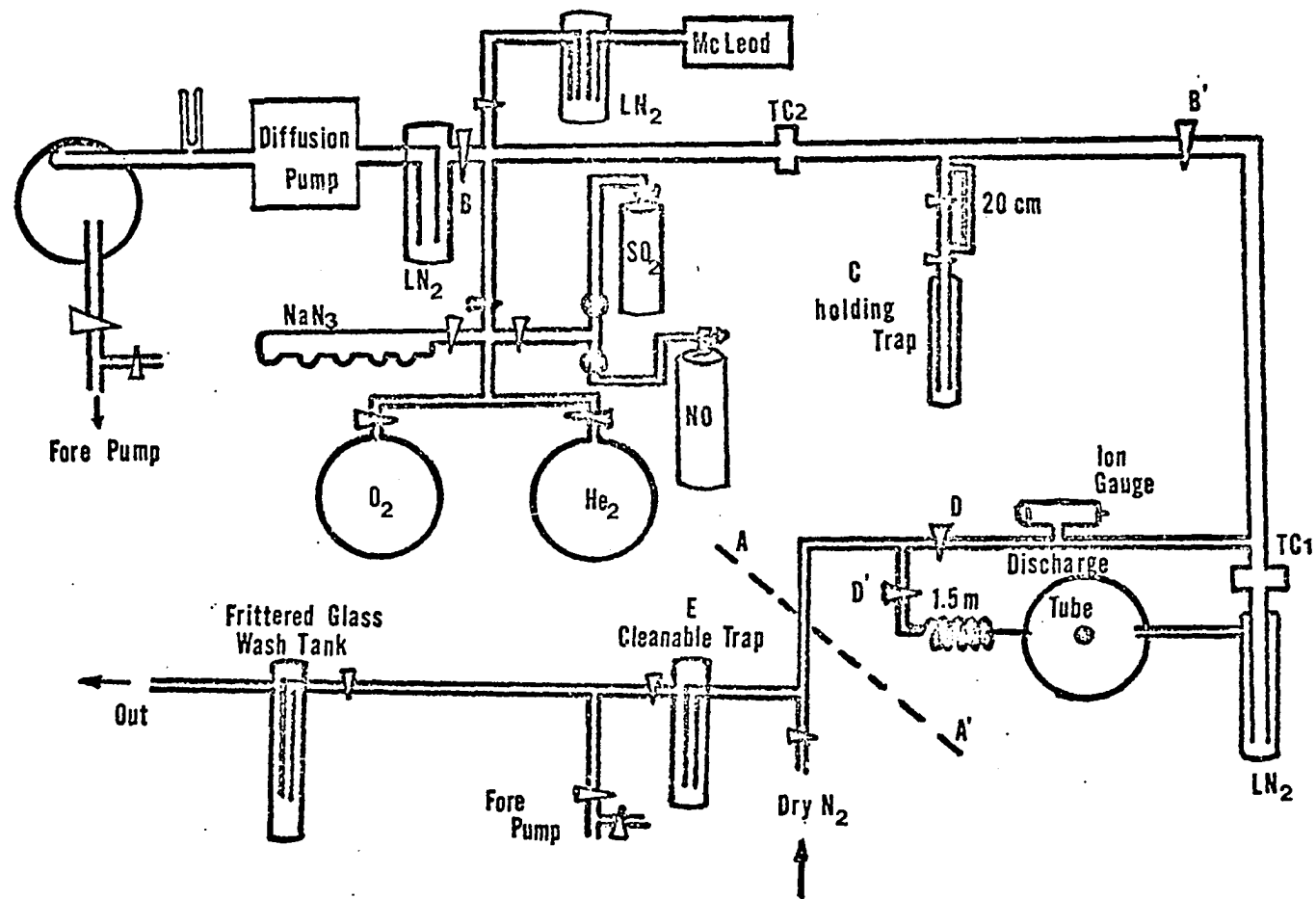


Fig. 1. Vacuum System

The main components of the system are discussed below. Both the forepumps are standard mechanical-oil roughing pumps capable of pumping 100 liters per minute and have a base pressure of approximately  $10^{-4}$  torr. These pumps were connected to the Pyrex system via Tygon tubing. All stopcocks, including the main stopcocks on the pumps were of high vacuum quality with pumpable bores. Apiezon N vacuum grease was used throughout the system and in the final version, no Black Wax or Glyptol was used.

The mercury diffusion pump was a two stage water cooled Eck and Krebs which was isolated from the main system by a liquid nitrogen trap.

Pressure in the system was monitored by two CVC type GTC-004 thermocouples (TC1, TC2) and a Bayard-Alpert type ion gauge which were maintained by a GIC-110B gauge pack. The thermocouples were calibrated for each gas with the mercury McLeod gauge in a static system.

The gas sampling units consisted of Matheson research quality Pyrex liter flasks of oxygen and helium which were used in the quenching experiments, a Matheson bottle of 95% pure NO, a lecture bottle of anhydrous sulfur dioxide and a special nipples tube (see Fig. 1) for sodium azide which provided high quality nitrogen after treatment.

The frittered glass wash tank provided a means of disposing of the used SO<sub>2</sub> safely without dumping the waste gas out the window and also prevented the gas from coming in contact with the hot mercury in the diffusion pump.



The dosed gases were handled in the usual manner with the exceptions of the  $N_2$  and the  $SO_2$ . To produce the  $N_2$ , the sodium azide tube was pumped down and gently heated to remove any moisture. The system was then closed off and heated vigorously until the  $NaN_3$  decomposed in a milky-grey cloud. This  $N_2$  charge was then used as any standard bottle of gas would be. A gram of  $NaN_3$  produces approximately one liter of nitrogen at one atmosphere.

When  $SO_2$  was used, the charge was dosed into the system up to point B' making sure that the stopcock at B was closed. Liquid nitrogen was then used to freeze the  $SO_2$  into the holding trap at C. After freezing, the valve at B was opened and the residual gases pumped off. The stopcock above C was closed and the holding trap allowed to warm up. The gas was then again cooled with liquid nitrogen and the stopcock above C opened and the residual gases were again pumped off. In this way, trace amounts of dissolved nitrogen were removed from the  $SO_2$  spectrum.

To use the  $SO_2$ , B was closed, B' opened, D closed, D' opened and the bypass valve on the holding trap was closed. The collection trap at E was cooled to liquid nitrogen temperature and the first valve above the holding trap opened. In this manner, flowing  $SO_2$  can be obtained at pressures varying from 0.015 torr to 0.500 torr for up to six hours. After the gas has been collected, the collection trap is isolated from the rest of the system by closing D' and then allowed to warm up to room temperature. Once warm, the trap is back filled with dry nitrogen to atmospheric pressure and then bled through the water trap.

In this fashion, the  $\text{SO}_2$  is dissolved by the water (3940 cc or 11.28 of  $\text{SO}_2$  will dissolve in 100 cc of water) and sulfurous acid is produced, thereby containing the toxic gas in solution. The collection trap is then roughed out by the second fore pump to ensure no water gets into the main system, after which it is pumped to base pressure by the main pumps.

The NO bottle was not used in this experiment.

B  
Excitation Electronics

The excitation electronics in this section were designed to produce essentially a square pulse of 600 to 1500 V peak and from 100 ns to a few microseconds duration at variable repetition rates from 60 Hz to 3KHz with fall times on the order of 5 to 10 ns. This has been accomplished in much the same way as described by Copeland. However, three of the circuits have been altered to allow for a more complete energy transfer and faster repetition rates. The modified circuits are: the high voltage power supply, the pulse forming generator (PFG) and the crowbar unit. All circuits, however, will be presented for completeness.

Fig. 2 shows the essential parts of this system in block form, the primary components are in double lines.

The high voltage power supply, Fig. 3, consists of a power line transformer rated at 35 KV-A, i.e. will deliver 7000 volts at 5 amps. This unit is normally only required to provide around 3 KV. The AC output of this unit is fed via high voltage standoffs and high voltage cable to a separate unit (indicated by the dashed line in the figure) where it is rectified by the silicon bridge and fed to a 14  $\mu$ f 40KV oil filled capacitor which provides the main power for the discharge when fed through the pulse forming generator. The silicon rectifiers shown in the figure consist of three diodes in series (1 amp 4KV PIV), each of these in parallel with a 0.005  $\mu$ f ceramic disc capacitor and a

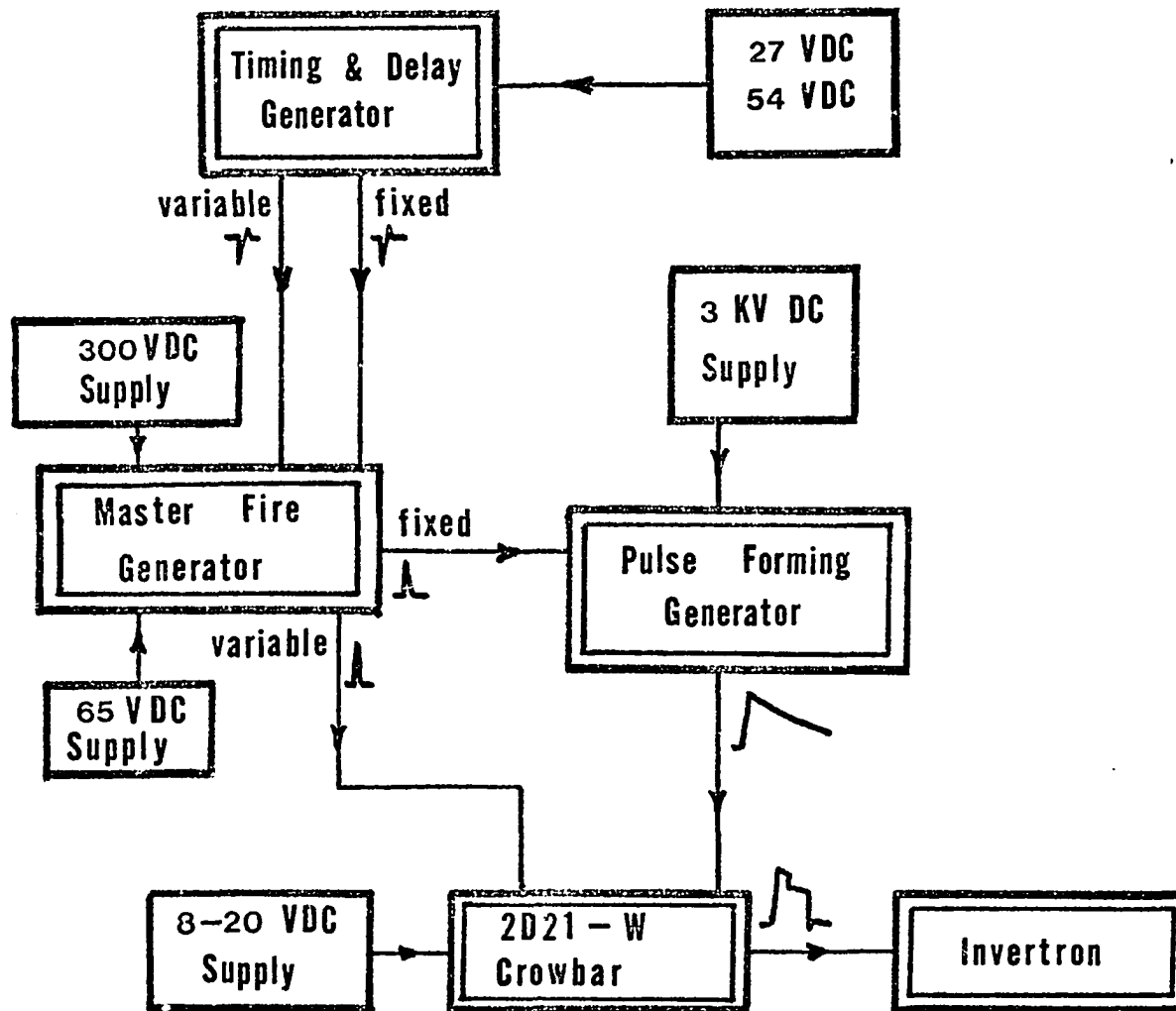


Fig.2. Excitation System

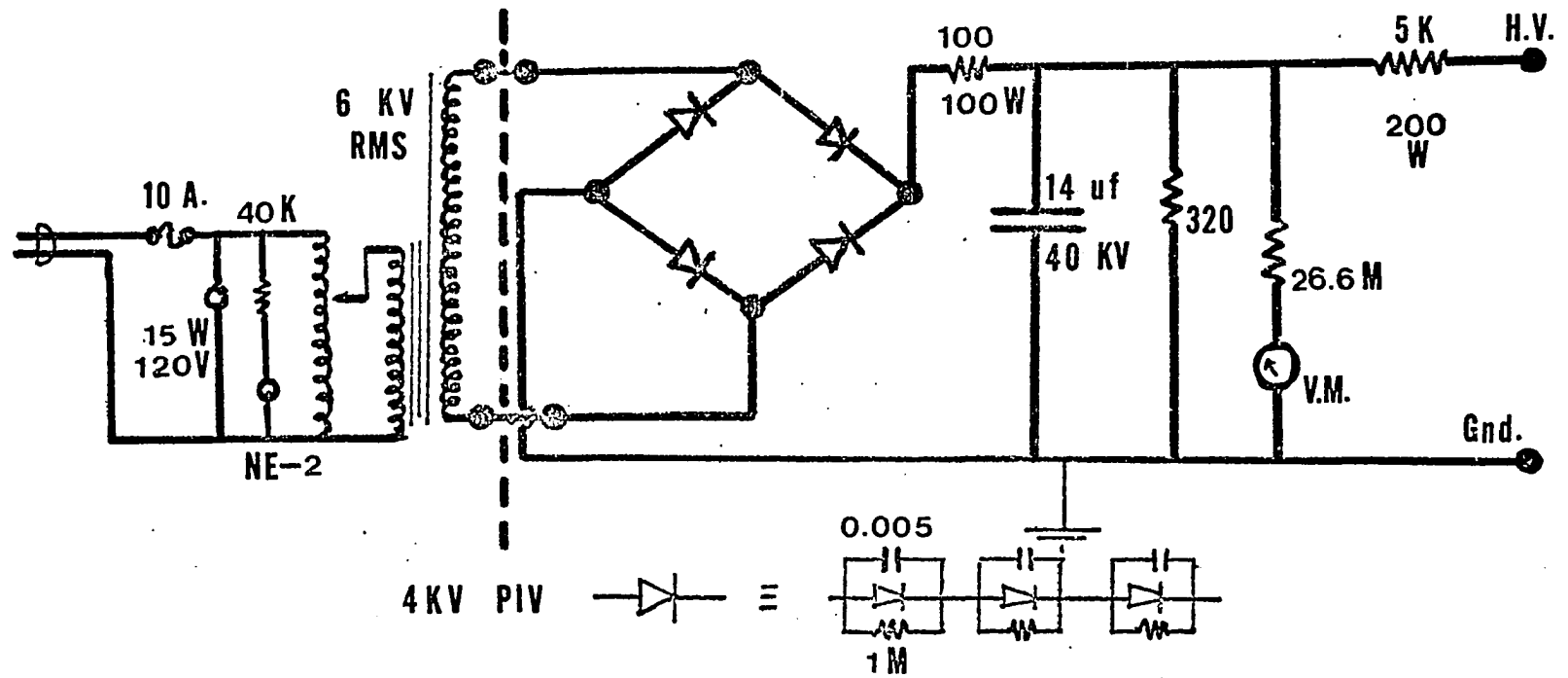


Fig. 3: High Voltage Power Supply

5W 1M resistor to reduce the spike amplitudes in the diodes and to provide a bleeder circuit to ground. The 320K resistor provides a bleed to ground on the 14  $\mu$ f capacitor for safety.

Instead of the 80K load resistor, the system now uses a 5K load resistor (200W) which allows the 14  $\mu$ f capacitor to recharge at faster repetition rates (2-3Khz). This insures that the 3C45 in the PFG will have sufficient voltage to fire everytime it is triggered by the master fire generator (MFG).

The output of this power supply is then fed to the PFG via high voltage cable to high voltage standoffs. This unit, shown in Fig. 4, was found to function much more efficiently and at much higher repetition rates if the 500 $\Omega$  developing resistor was replaced by 47 $\Omega$  (150W), the characteristic impedance of the 3C45. This system will fire, without missing, for repetition rates up to 3KHz, but this tends to reduce the usable lifetime of the hydrogen thyratron considerably. Consequently, a repetition rate of 1KHz was generally used. The 250 $\Omega$  carbon resistor was comprised of four 1K carbon resistors and was used to reduce the inductance of the system.

The third circuit modified was the crowbar unit. This was designed to clamp or short the RC decay provided by the PFG into a square pulse at variable times after the inception of the pulse. This unit, see Fig. 5, did just that with a fall time from 5 to 10ns. The crowbar was placed in direct proximity with the discharge tube via UHF connectors to reduce stray inductance. The differentiated output was used to provide a voltage trigger to the time amplitude converter (TAC) in coincidence with the cessation of the excitation pulse. It was noticed

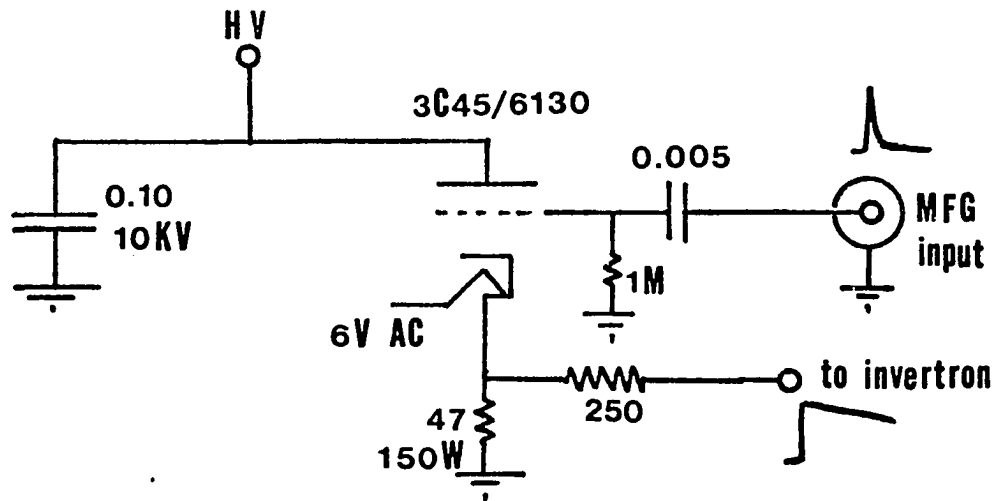


Fig. 4: Schematic for Pulse Forming Generator

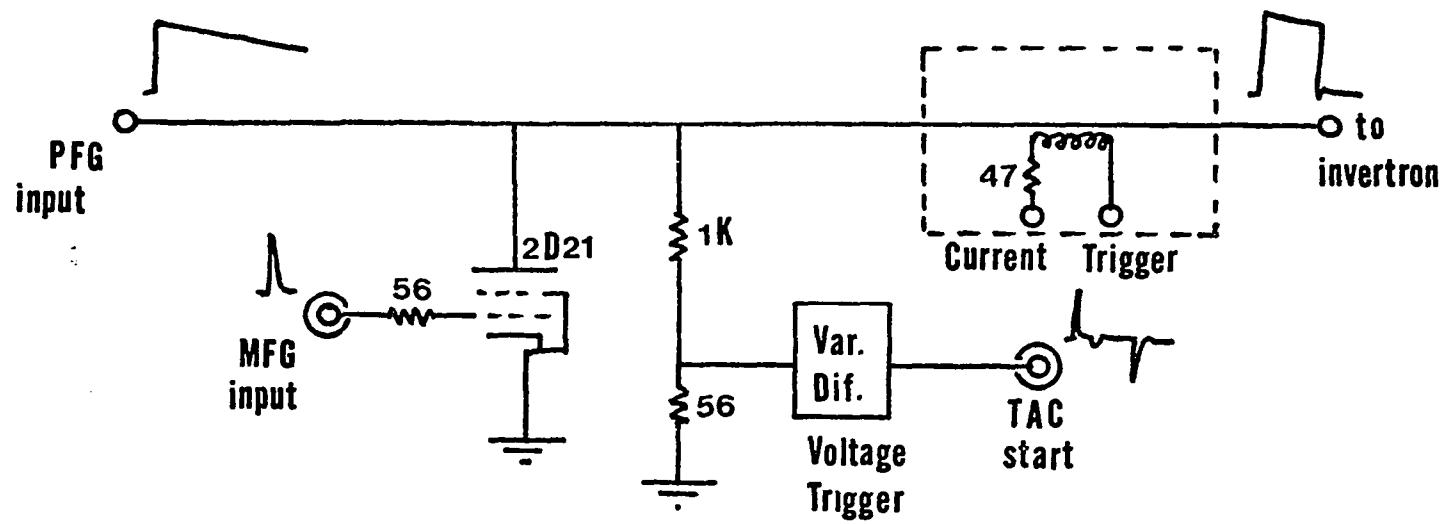


Fig. 5: Schematic for Crowbar Unit



that this 'start' pulse had some rather subtle structure as shown in Fig. 6. Further experimentation indicated that it was this second fall, not the first, which was the true cut-off of the glow discharge (note the statistical nature of this second pulse), and that in severe cases of slow cut-off, e.g.  $30+nsec$ , see Fig. 7, this fine structure could vitiate the experimental results since either of the pulses could trigger the TAC clock 'on'. This then appears as a smeared onset of the decay as shown in Fig. 8a and 8b, and can mask short lifetimes on the order of  $20nsec$ .

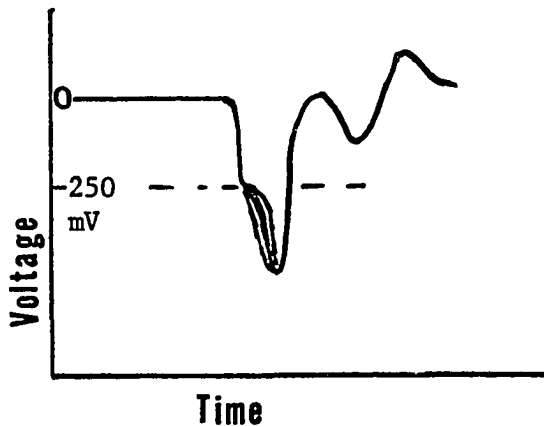


Fig. 6: fine structure of start pulse.

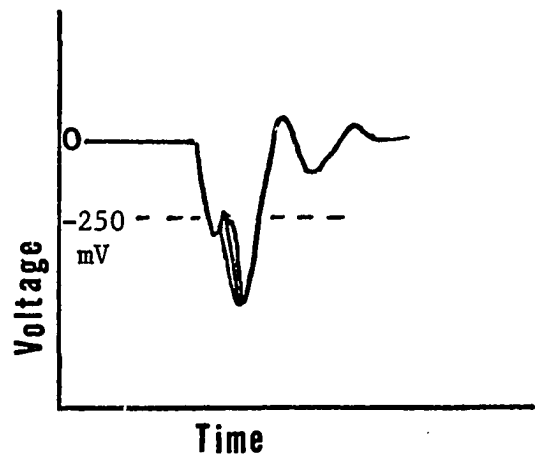


Fig. 7: start pulse with slow,  $30+nsec$ , cut-off

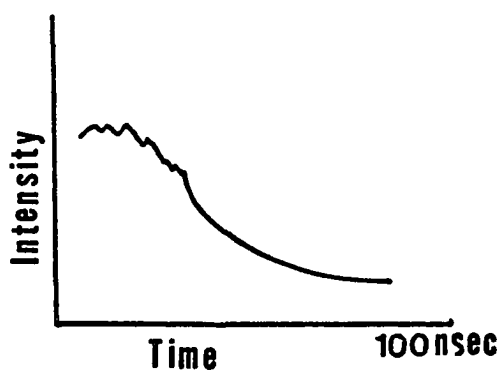


Fig. 8a: decay curve using voltage trigger

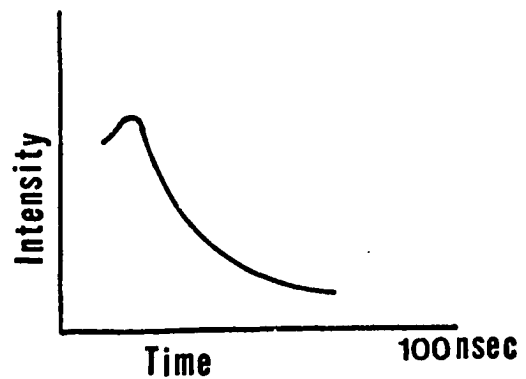


Fig. 8b: decay curve using current trigger

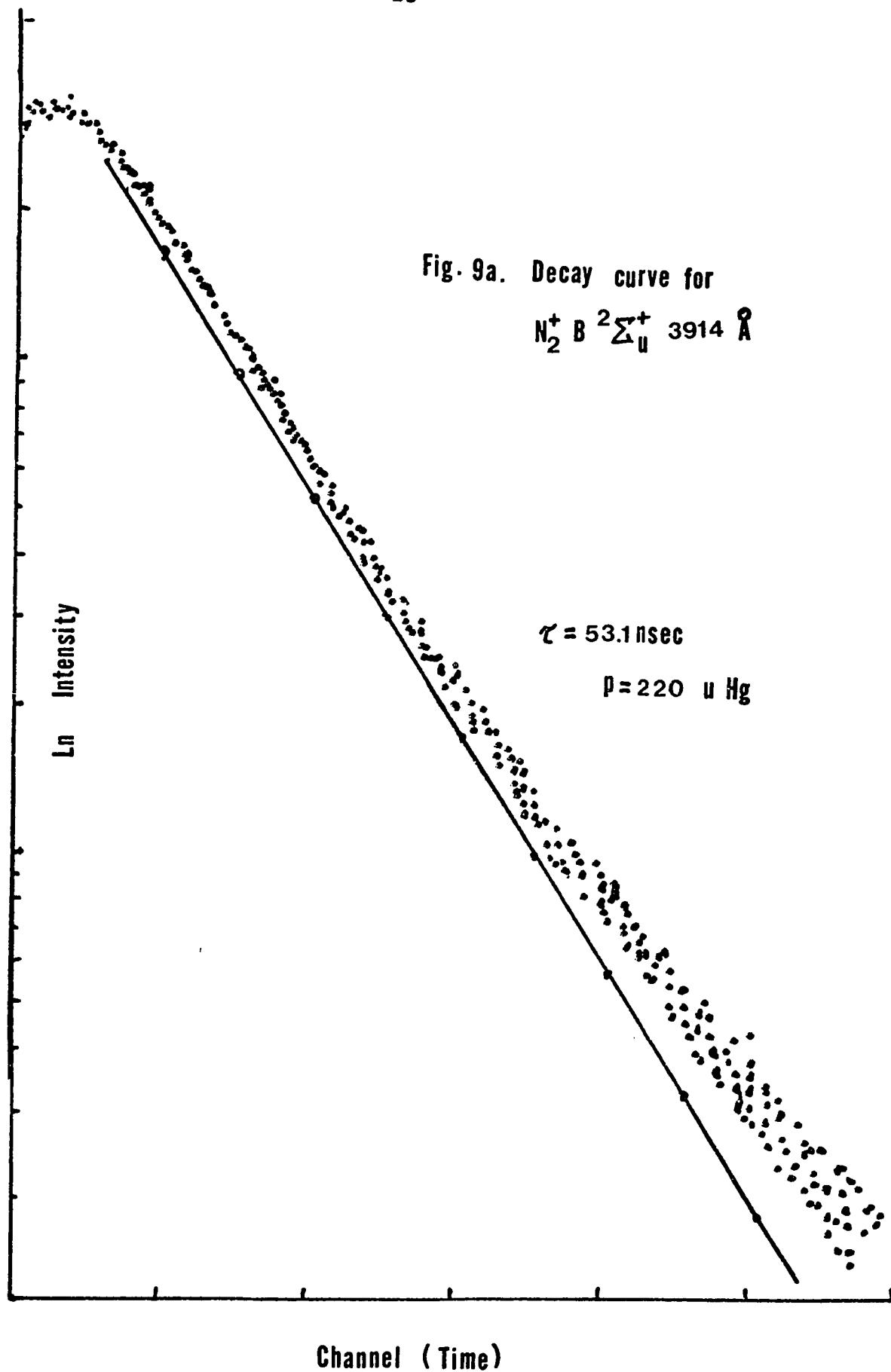
Fig. 8b was compiled using the current trigger which is shown in the dashed box in Fig. 5. This trigger only fired when the glow cut-off, and also coincided with the second pulse of the voltage trigger when superimposed. However, this current trigger could not be used with confidence as it altered the cut-off characteristics of the discharge and consequently biased the observed lifetimes for lifetimes around 30 to 40 nanoseconds. It did show legitimate lifetimes for fast states however, as we checked the  $3^1P$  (5016Å) level of He and the  $4P-4D^0$  (4650Å) level of OII and found lifetimes comparable with those reported in the literature (~10ns). It should be stressed that this problem is only important when measuring fast lifetimes, i.e. on the order of 20 nanoseconds or faster. For longer lifetimes, even those as short as 40 nanoseconds, this distortion in the lead channels due to false starts is of no consequence, adding only a small percentage of random error to the observed lifetime.

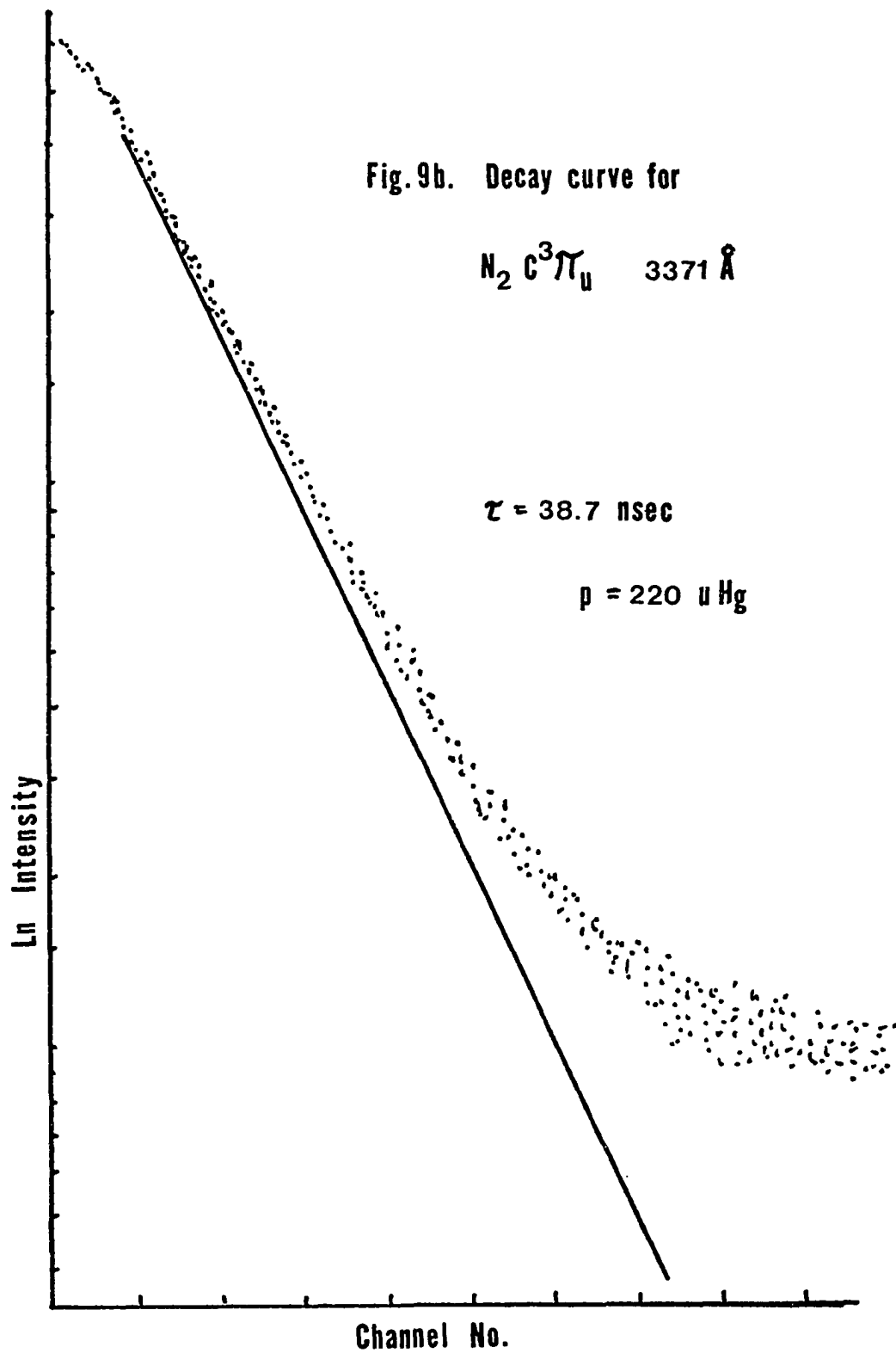
The cut-off could be altered for the above experiment by varying the filament voltage of the 2D21 in the crowbar unit from 8 to 16 volts, the higher the filament voltage, the faster the cut-off appeared. It was also found that the 2D21 filament could be increased to as high as 24 Vdc before the tube burned out.

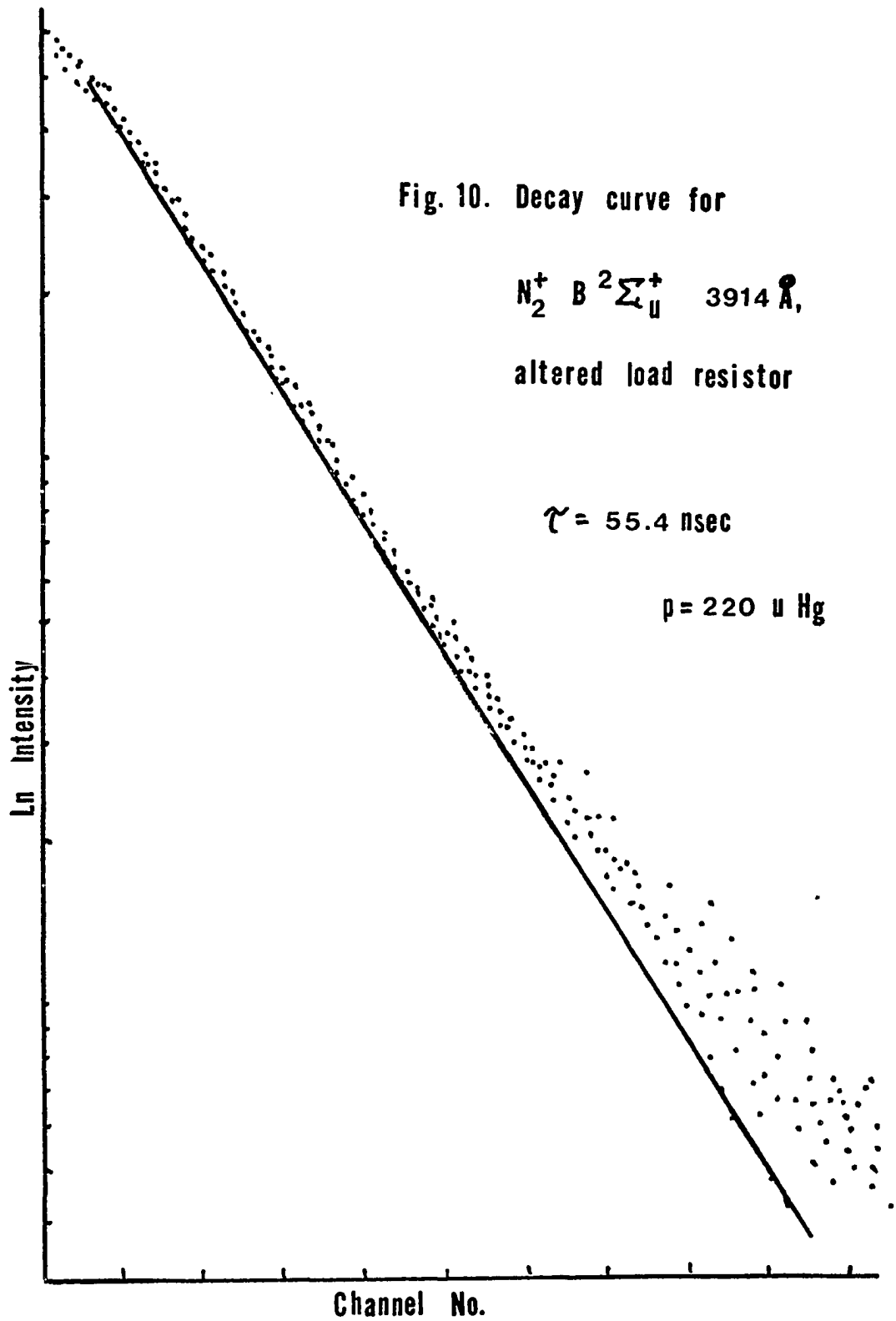
The 1K carbon load resistor was also inserted to improve cut-off characteristics. It was noticed that while the  $C^3\Pi_u$  state of  $N_2$  gave the appropriate lifetime, the  $B^2\Sigma_u^+$  state of  $N_2^+$  did not. Fig. 9a, b, show the observed decay curves for these states. Note the poor fit in the lead channels for the ionized molecule. Fig. 10 shows the same

sample except with a 500 ohm load resistor. Not only is the plateau reduced, i.e. the cut-off is truly faster, the lead channels do not appear as distorted. See also the section on cathode inserts for further discussion of this problem.

The rest of the circuitry, the master fire generator (MFG) Fig. 11, the timing and delay generator (TDG) Fig. 12a, b, the two power supplies, +27 and +54 Vdc Fig. 13, and +65 Vdc Fig. 14 are all the same as described by Copeland and are included for completeness. The  $\pm$  310 Vdc regulated power supply was a surplus unit manufactured by the U. S. Science Corporation.







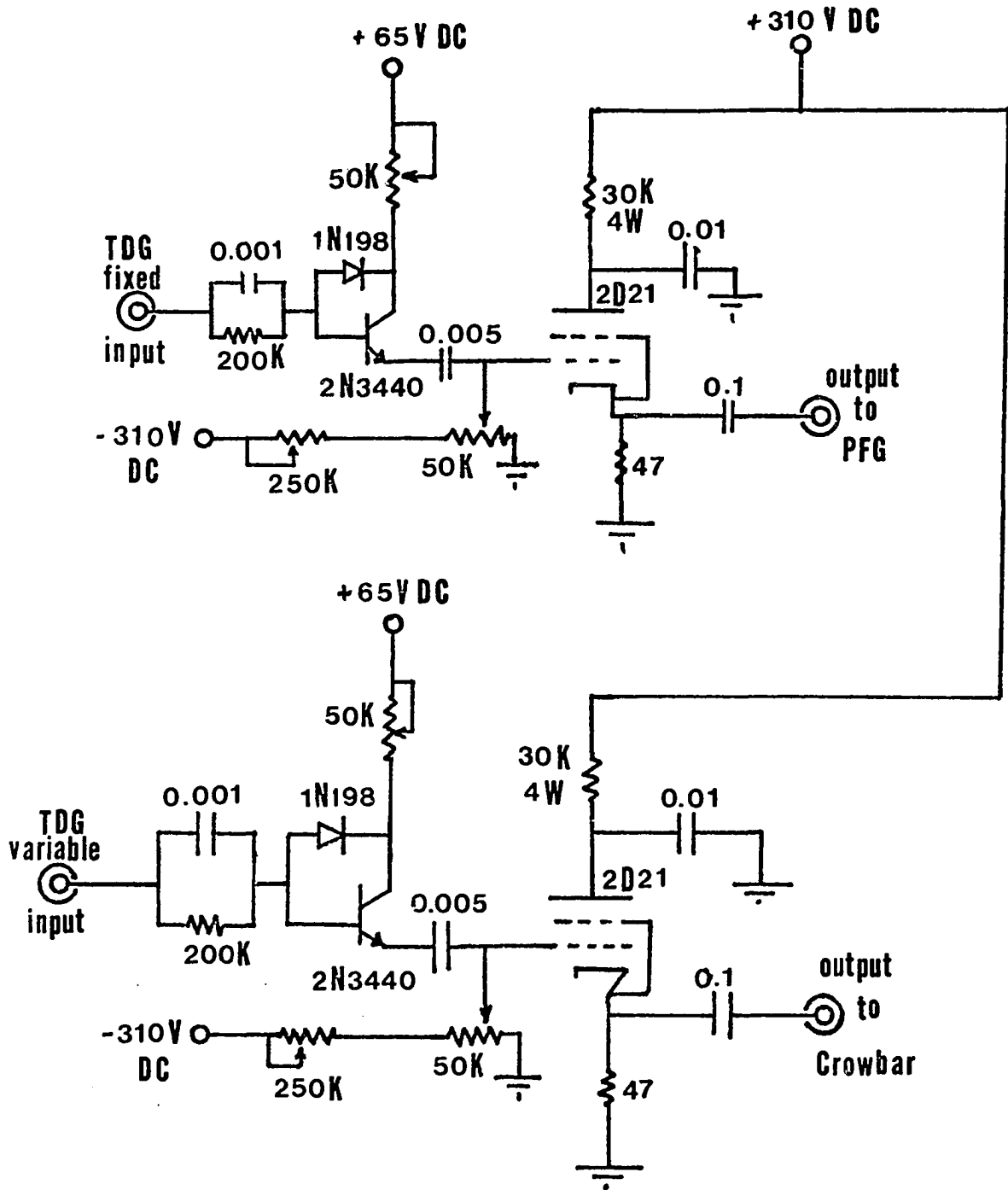


Fig. 11: Master Fire Generator

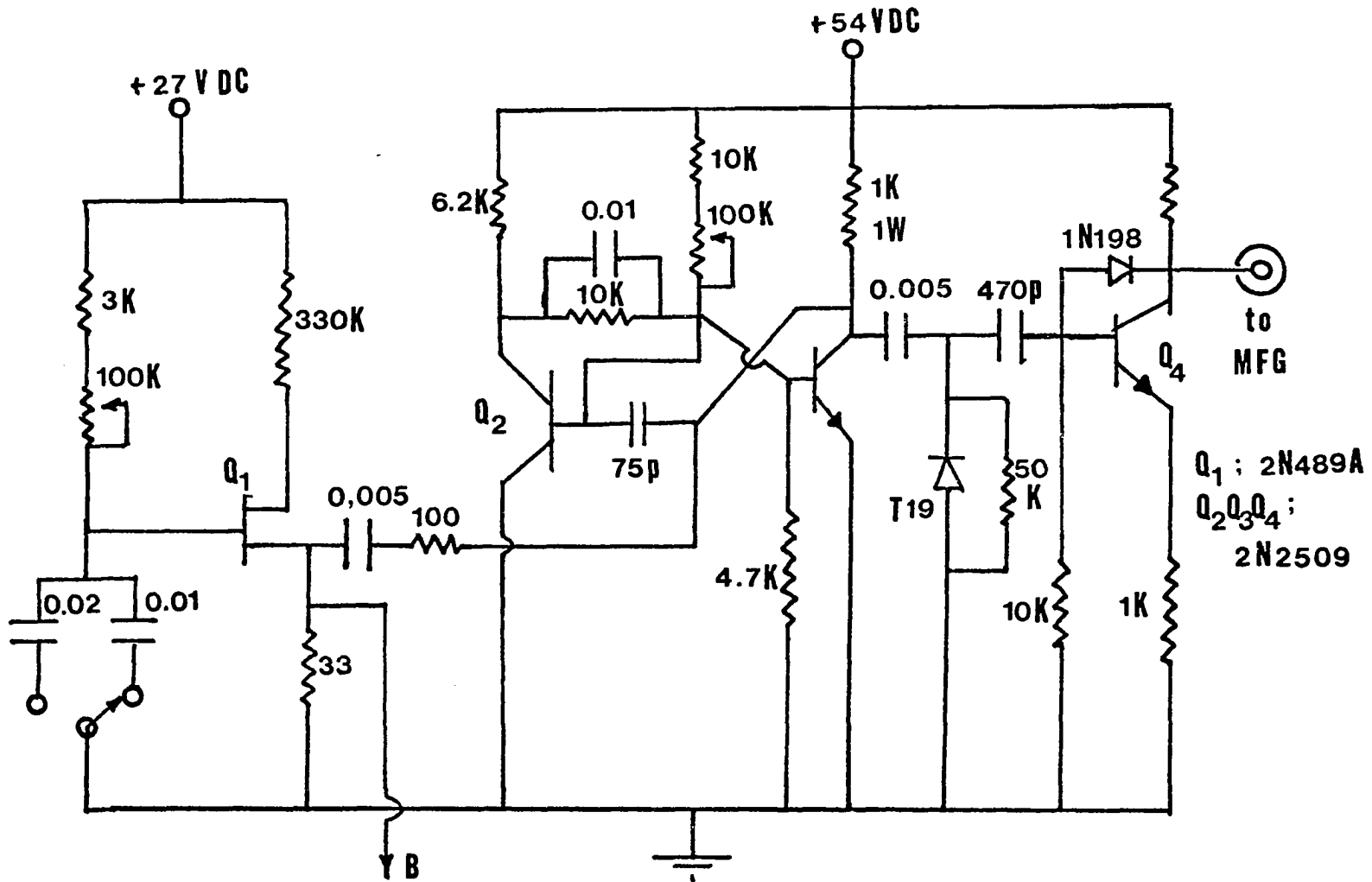


Fig. 12a: Timing and Delay Generator



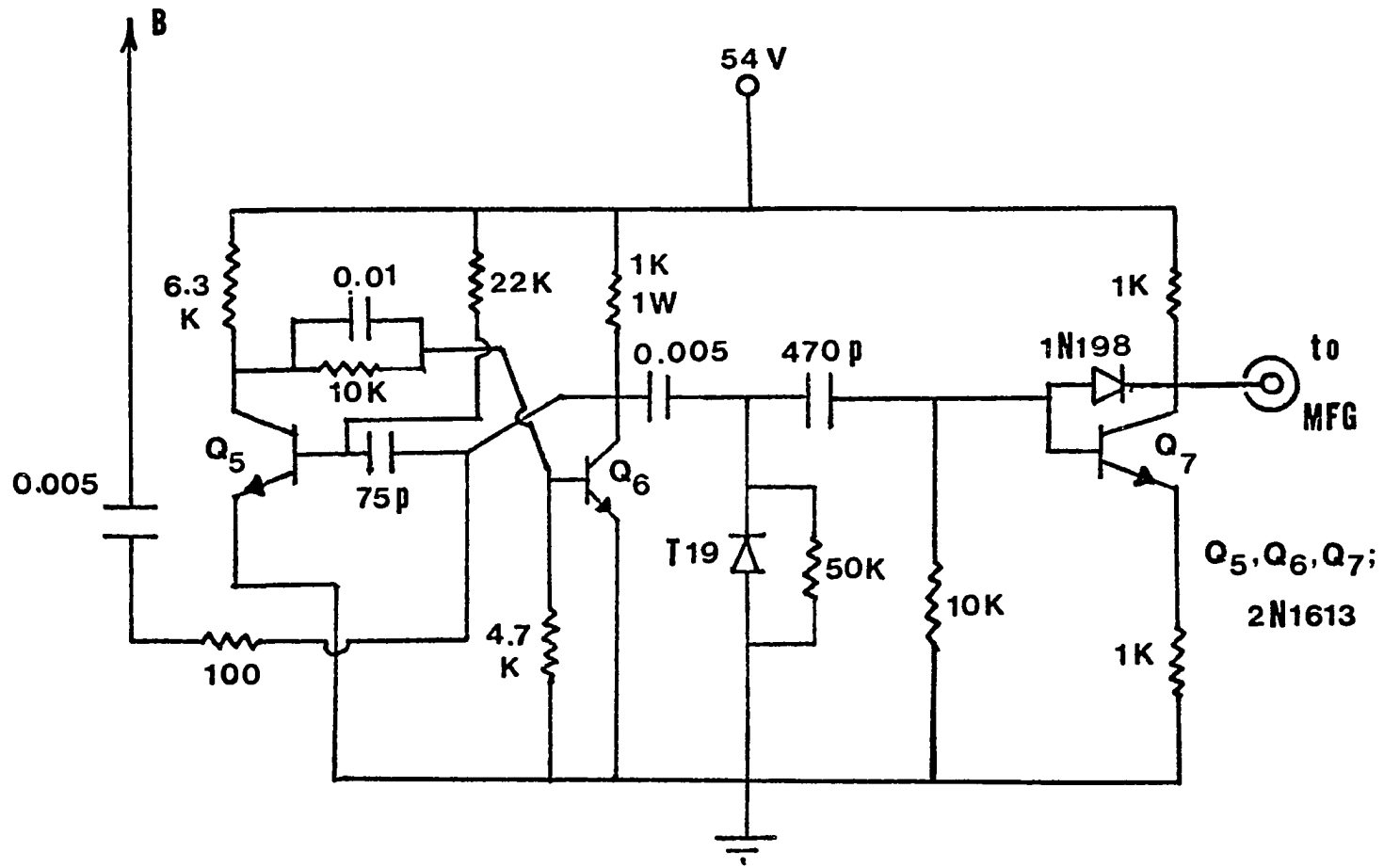


Fig. 12b: Timing and Delay Generator

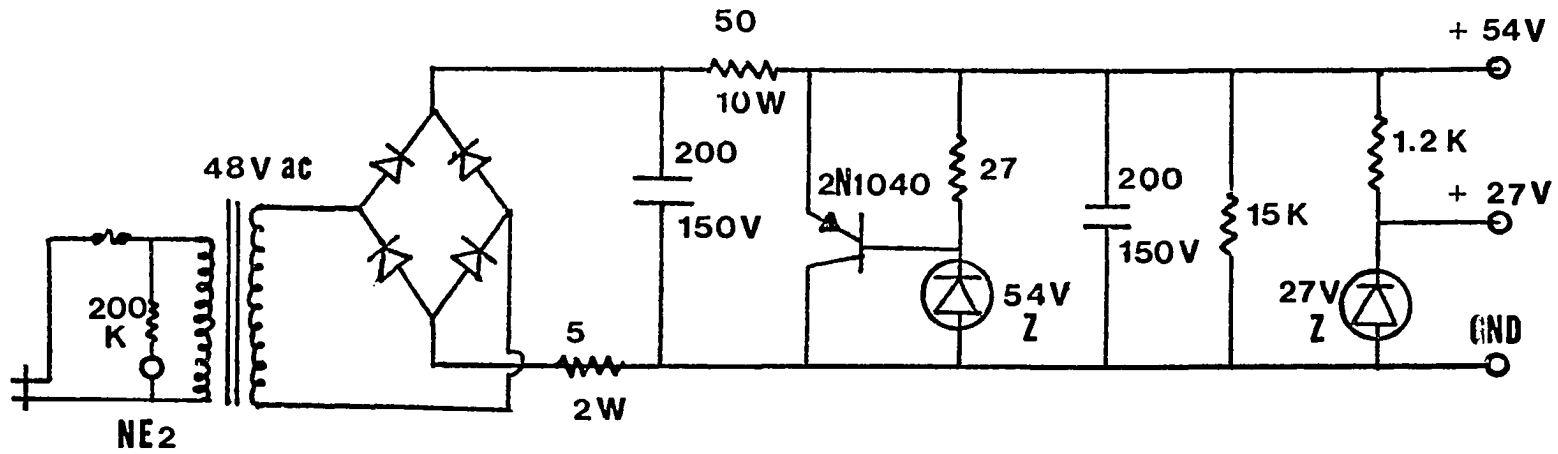


Fig. 13: +27 and +54 VDC Power Supply

## C Detection System

The experiments described in this paper are based on the probability of observing a photon of appropriate wavelength after the excitation system has been turned off; hence, the technique is known as delayed coincidence photon counting. The system is shown in block form in Fig. 15.

A typical event would proceed as follows: the excitation system clamps the discharge off, and the differentiator produces a positive and negative spike coincident with the on-set and cut-off of this pulse. The differentiated signal is sent to the ORTEC time to amplitude converter (TAC) as a 'start' signal, where only the negative spike (minimum -250 mV and 5 nsec fwhm) will start the internal clock, and to the ORTEC delay gate generator (416A) where the positive spike is used to generate a positive logic pulse (approx. 3V) which is then delayed sufficiently to coincide with the cut-off of the excitation pulse and serves as a gate to the TAC. This insures that the system will not trigger on from the differentiated portion of the discharge pulse which is due to the statistical break-down of the glow discharge as shown in Fig. 16. The first Canberra scaler provides the number of times that the system is fired in a given experimental run.

After cut-off of the excitation, the first photon of appropriate wavelength that the photo-detection system 'sees' will produce a sharp negative pulse (-800 mV, 6 nsec fwhm) which is sent to the TAC

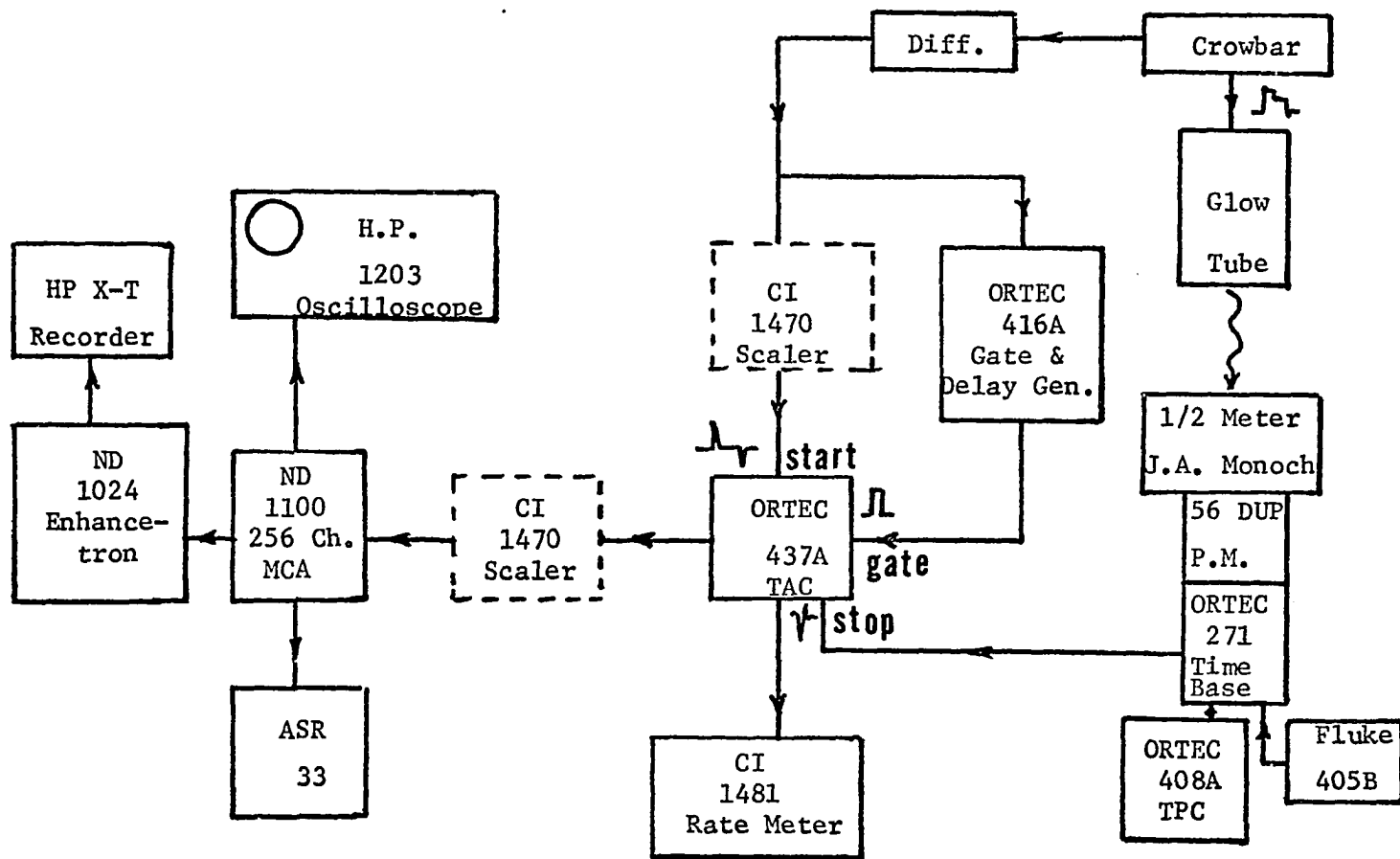


Fig. 15: Block Diagram of Detection System

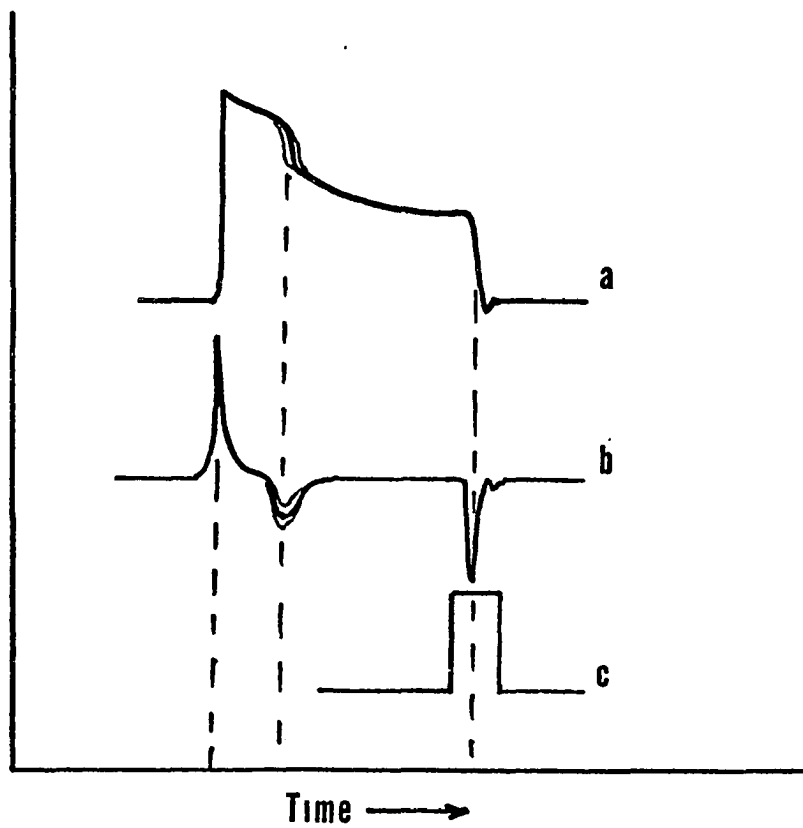


Fig. 16: Time Sequence of Gate Pulse

- a) Excitation Pulse
- b) Differentiated Pulse
- c) Logic Gate Pulse

as the 'stop' signal. The time delay from the receipt of the start pulse until the arrival of the stop pulse is then converted to a voltage pulse whose amplitude is proportional to this time delay. The output from the TAC is sent to a ND-1100 256 channel multichannel analyzer (MCA) where the amplitude of the voltage pulse determines in which channel of the memory the signal belongs. This process is repeated several hundreds of thousands of times until a probability decay curve is developed with sufficient statistics for analysis. The second Canberra scaler is used to determine the total number of events which comprise a given experiment. This is useful in determining data acquisition rate corrections. See later discussion of this problem, Chapter IV.

The Canberra rate meter (model 1481) provides an on-line acquisition rate which is useful in adjusting the experiment to a given count rate. The Hewlett-Packard model 1203 oscilloscope provides a visual monitor of the decay curves as they are acquired and the ASR-33 teletype provides printed and punched paper-tape output from the MCA memory.

The timing-calibration procedures used in these experiments are identical to those described by Copeland. The timing delay cables are listed in Table 1.

Table 1: Time delay cables

CABLE	TIME DELAY (nsec)*
1	28.0±0.30
2	44.0±0.20
3	88.09±0.90
4	88.7±0.77
5	101.4±0.50
6	183.2±1.6
7	378.5±2.2

\* errors are one standard deviation

The photo-detection system for these experiments consisted of the following various photomultipliers: three RCA 8575's, one 56TUVP Amperex, and one 56DUP Amperex. These PM's were coupled to an ORTEC 271 constant fraction timing base. It was found during the experiments that some rather subtle distortion of the data, and some not so subtle distortion, stemmed from this system. The blatant distortions were observed with the RCA 8575's which varied in age but for the most part were older than five years. These tubes would exhibit unusually high noise levels (greater than 20%) which the computer had difficulty in analyzing; It could not tell if the noise level was linear or exponential in nature and frequently failed to give any usable analysis on known molecular lifetimes. Also, when the same data was collected on a longer time scale (800 nsec instead of 400 nsec) the probability decay curves would exhibit a rather interesting plateau around 350nsec as shown in Fig. 17. This phenomenon can be explained by considering 'afterpulses' produced inside the phototubes by ionized perfused helium which drifts back to the cathode, producing a burst of secondary electrons delayed in time from the parent primary electron (delayed time due to drift velocity of the ions). This effect primarily causes an increase in the observed lifetime as well as an increase in the noise level. For example, the observed decay of the  $C^3\Pi_u$  state of  $N_2$  with an improperly functioning PM was found to be 44nsec, while the same state would be observed with a lifetime of 38.5nsec with a properly functioning PM. While this may not seem very important, being only an error of 15%, it is a systematic error and it would tend to make it very difficult to analyze properly a decay on the order of 300 to 400nsec.

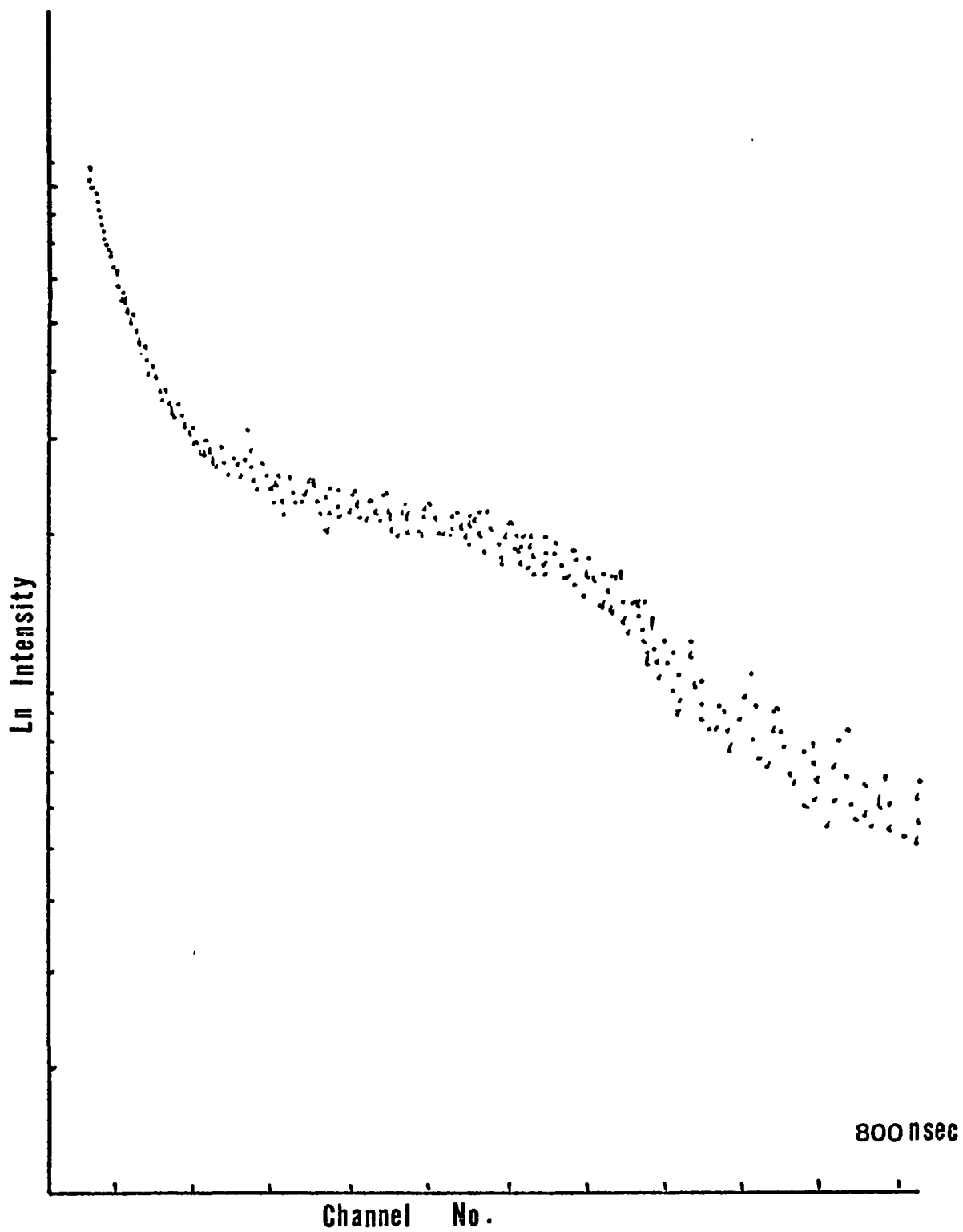


Fig. 17: Distorted Decay Curve Due to Faulty PM



This problem can reach extremes, i.e. the PM can contain so much perfused helium that dynode bursts are also seen.<sup>20</sup> These tend to vitiate the results in the opposite sense, in that they tend to produce lifetimes that are too short. One of the RCA 8575's, for example, produced invalid lifetimes for both the C  $^3\Pi_u$  and the B  $^2\Sigma_u^+$  states of  $N_2$  and  $N_2^+$  of 20nsec, independent of pressure. Both these problems, the afterpulses and the dynode bursts, can be explained with the aid of Fig. 18. Trace 'a' represents the true radiative decay of the excited source. Note the apparent RC decay of the equilibrium region before cut-off. This follows the decay of the excitation pulse and is instrumental in producing the plateau apparent in Fig. 17. Trace 'b' represents the buildup of afterpulses with a time delay of approximately 350nsec after cut-off. The decay of these pulses after this 350nsec delay is not exponential but gaussian due to the time-of-flight statistics, and makes analysis of this data utterly misleading.

Trace 'c' represents the light curve obtained for the dynode bursts and is essentially the same as that above for the afterpulses except that the time of flight to the dynodes is only on the order of 15-20nsec. These dynode bursts can be very annoying in that they provide the same 'lifetime', i.e. 20 nsec, regardless of the light source. This would of course make for easy recognition if it were not for the fact that this distortion is light sensitive, in that without the light source present, the PM appears noise free. Consequently, an experiment run on an unknown gas would produce false lifetimes which

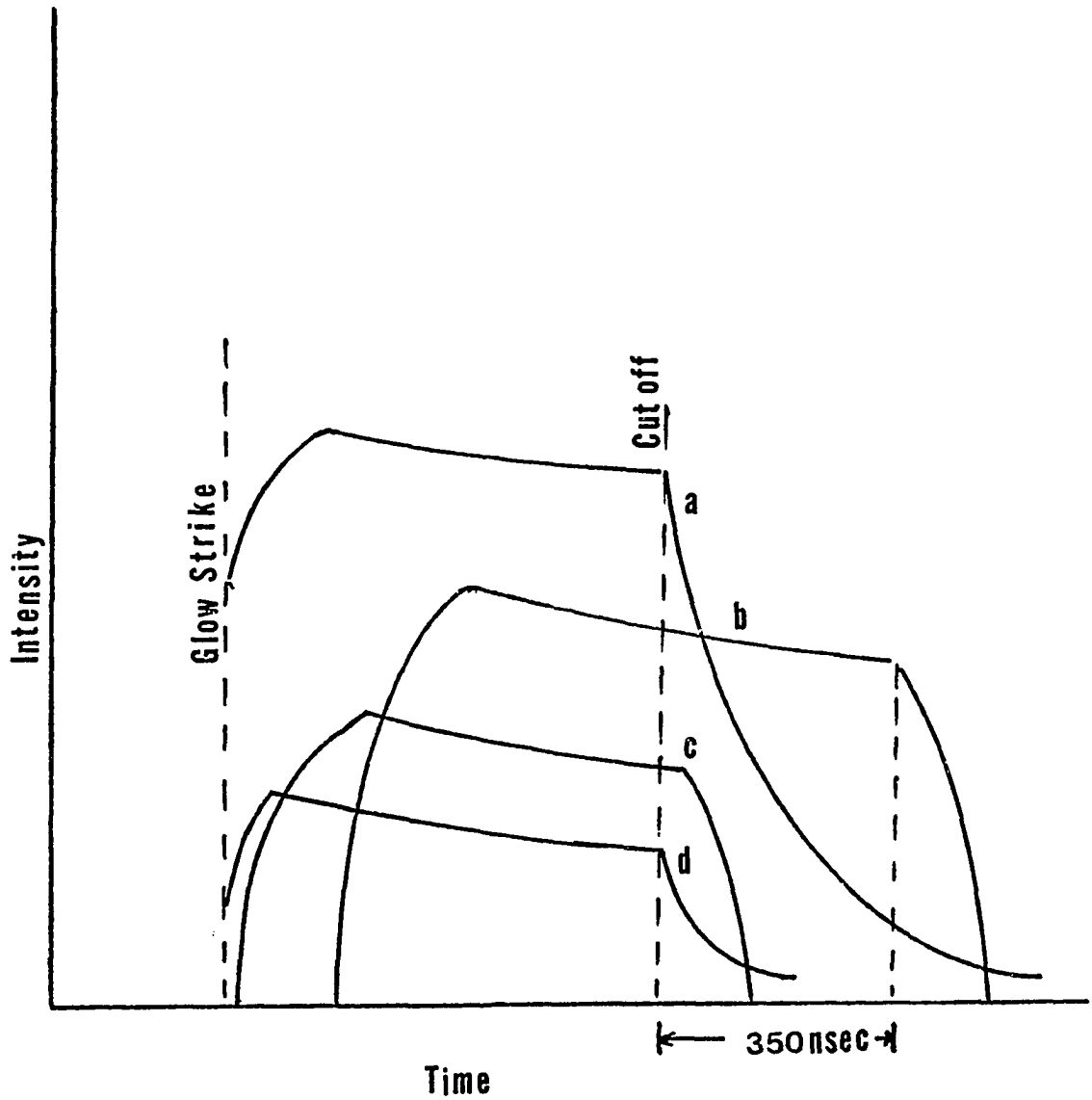


Fig. 18: Light Curve Composition During And After Excitation Pulse Cessation

would not be recognized as false. For a complete discussion of this subject, see ref. 20.

Another type of PM related distortion is caused by undervolting the PM dynode chain. Although this effect does cause shorter apparent lifetimes, it does so in a much different manner than the dynode bursts. While the dynode bursts give a constant 20nsec decay curve, regardless of the decaying source, the undervolted PM produces an apparent two-exponential decay; the 'correct' decay curve plus a fast component which appears to depend on the radiating source. Examples of two such curves are presented for the  $C^3\Pi_u$  and  $B^2\Sigma_2^+$  states of  $N_2$  and  $N_2^+$  in Fig. 19, 20. Increasing the dynode voltage eliminates this problem.

Since the Amperex 56DUP does not have a quartz window, a pilot 'B' scintillator was placed in front of the window for all UV work. This was accomplished by using Dow-Corning 20-057 Optical Coupling Compounds as a glue to affix the scintillator directly on the face of the PM window. Tests with and without it showed the coupler to be free of objectionable characteristics such as fluorescence.

This total system could be adapted to spectral investigation by using the MCA in the multiscale (MS) mode instead of the pulse height analysis (PHS) mode. The MS mode allowed a spectrum to be obtained based on the time dependent relative intensity of emission lines. The channel dwell time of the MCA in the MS mode was variable but was usually operated at 800  $\mu$ s/channel. Fig. 21 is an example of the type of spectra obtainable with this system.

One other means of obtaining spectra was available. By using a ND Enhancetron 1024 multiscaler, much more detailed spectra were

possible. This was achieved by replacing the MCA in Fig. 15 with the Enhancetron. The output was plotted via an X-T Hewlet-Packard recorder and a sample is shown in Fig. 22.

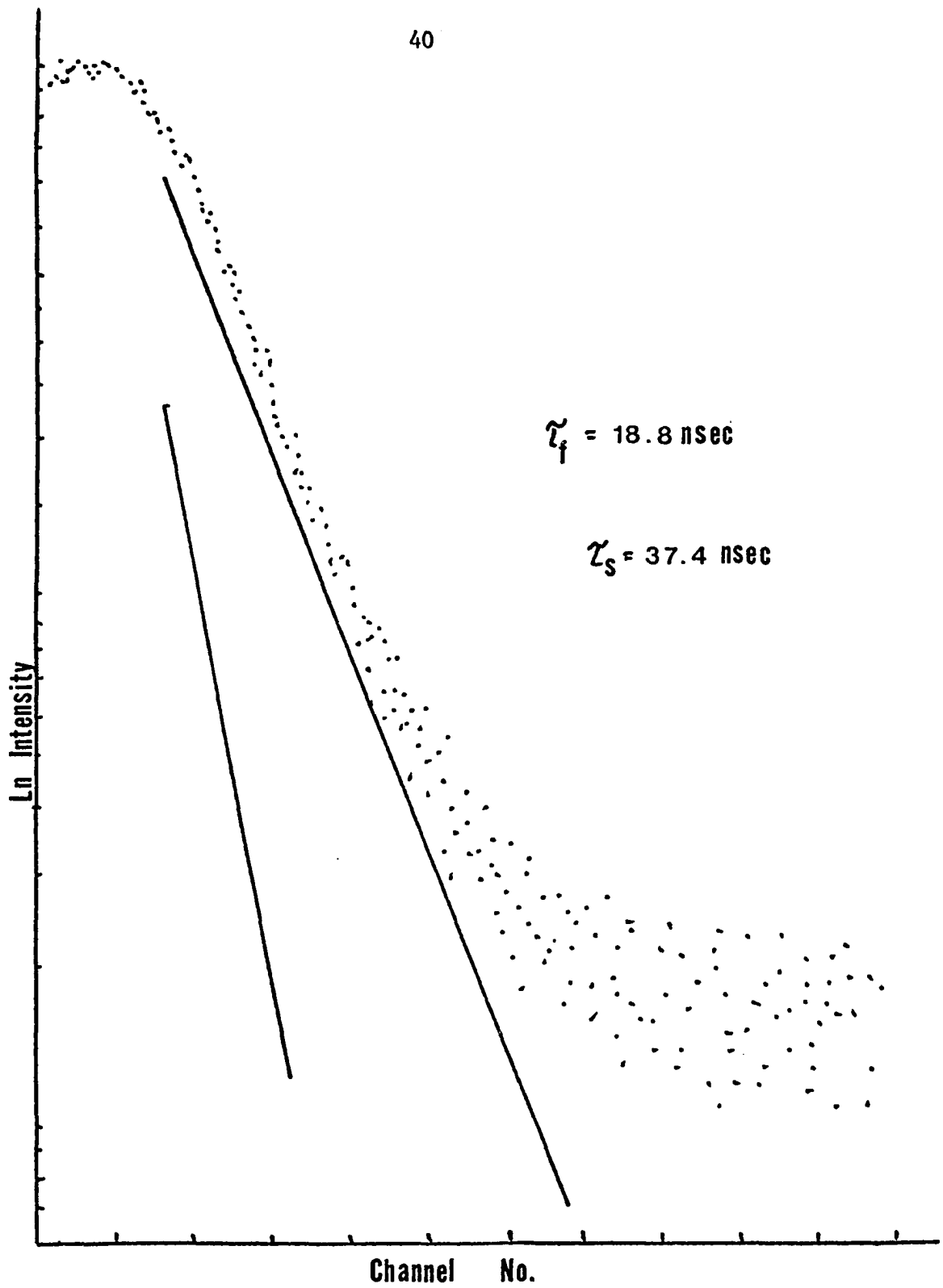


Fig. 19: Decay Curve  $N_2 C^3\Pi_u$  With Undervolter PM

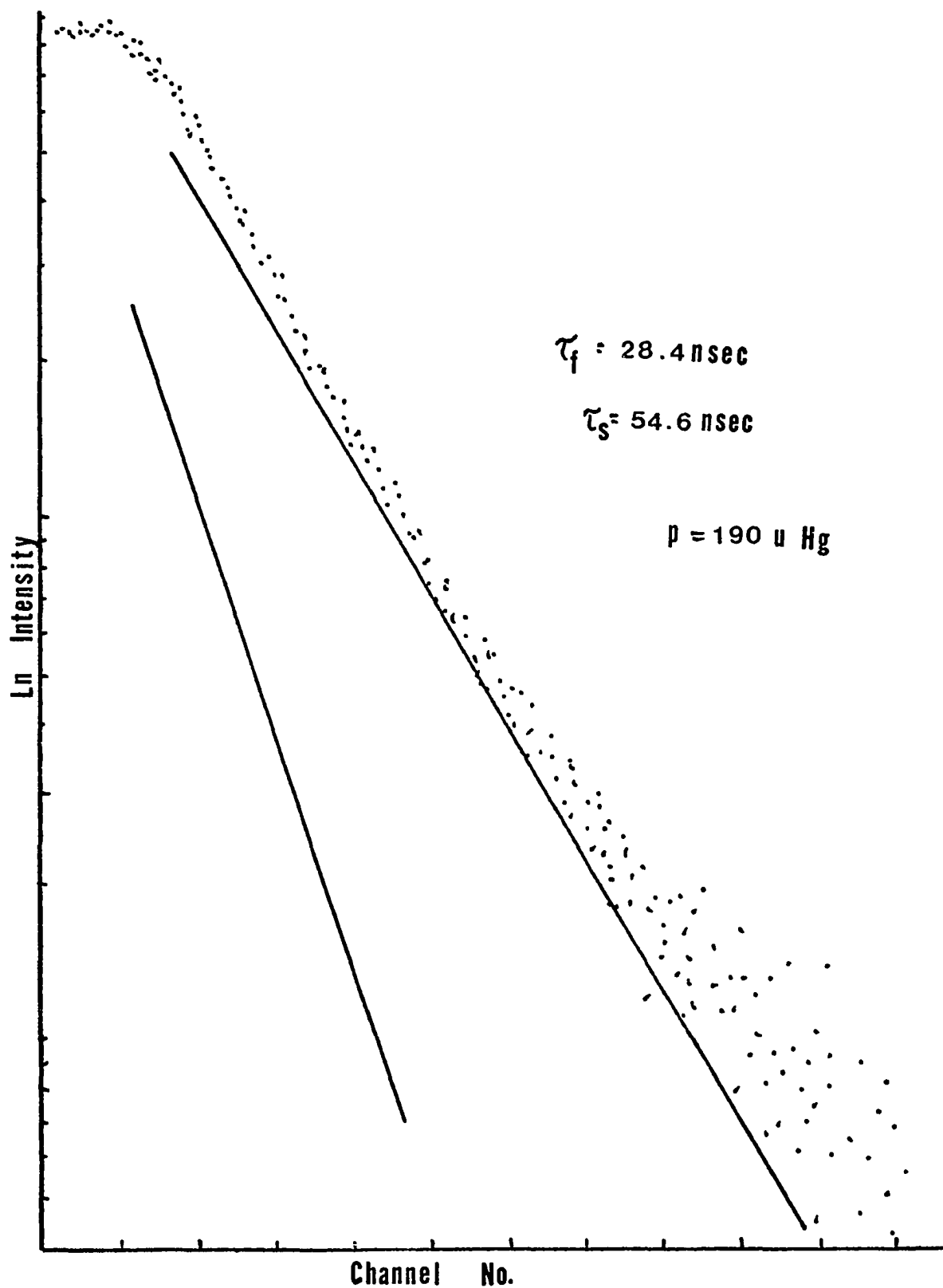


Fig. 20: Decay Curve  $N_2^+ B \ ^2\Sigma_u^+$  With Undervolted PM

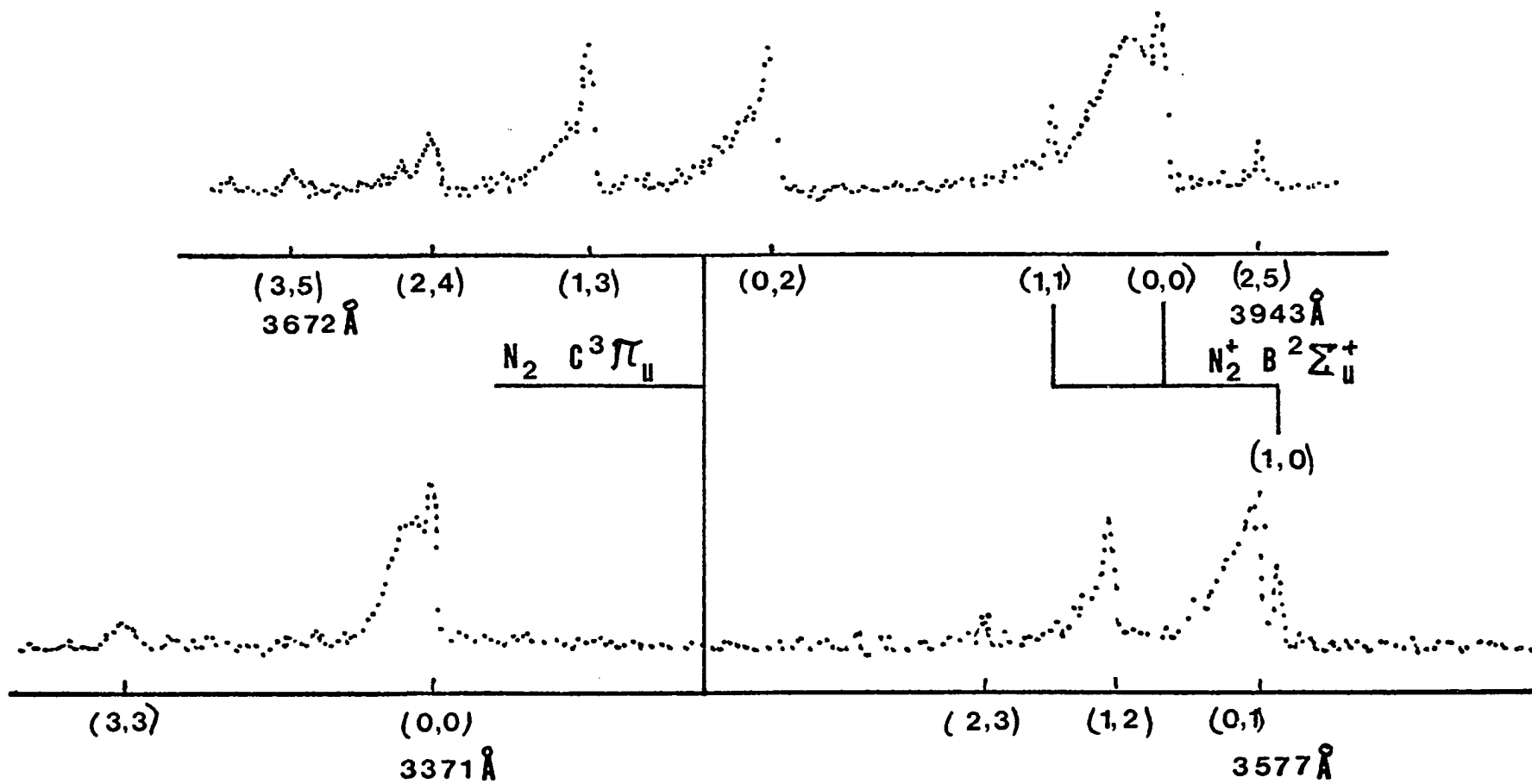


Fig. 21: Ms Spectra for  $N_2$ : 3300 - 4000 Å

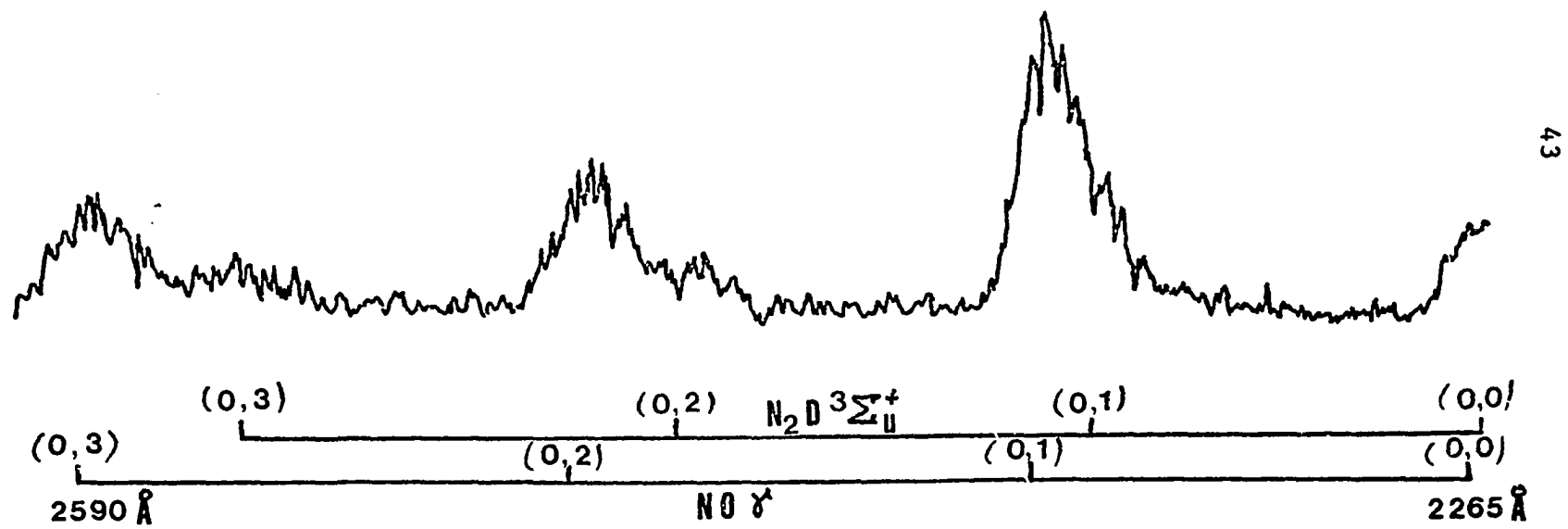


Fig. 22: 1024 ND Enhancetron Spectra For  $N_2$ : 2270 - 2590  $\text{\AA}$



D  
Discharge Tube

G. E. Copeland developed the cold cathode invertron in 1970 for the explicit application to easily dissociated polyatomic molecules and those which tend to poison oxide coated heated cathodes. A complete description of the apparatus can be found in ref. 1 and 21. The first part of this experiment was intended to validate and improve the accuracy of this instrument.

In its present form, the invertron is essentially the same as Copeland's although it has been modified to allow for experimentation to insure that geometrical factors have not biased the experimental results. Figure 23 illustrates the modified system. The dimensions of the stainless steel invertron are approximately seven inches long by three inches outer diameter and two and a half inches inner diameter at the constriction. The upper inlet serves as the main pumping port and as the flow entrance port during experimentation. This port fits 8mm pyrex tubing to facilitate primary evacuation. The lower port in the figure is for gas exhaust during the flow runs.

The window is a fused quartz disc three inches in diameter and a quarter inch thick which was sandwiched between two viton 'O' rings. The pyrex backing plate facilitated cleaning the instrument and served as an insulating plate to the back wall of the discharge tube.

The anode is a stainless steel rod eight and three quarters inches long by an eighth inch diameter. It is inserted through two

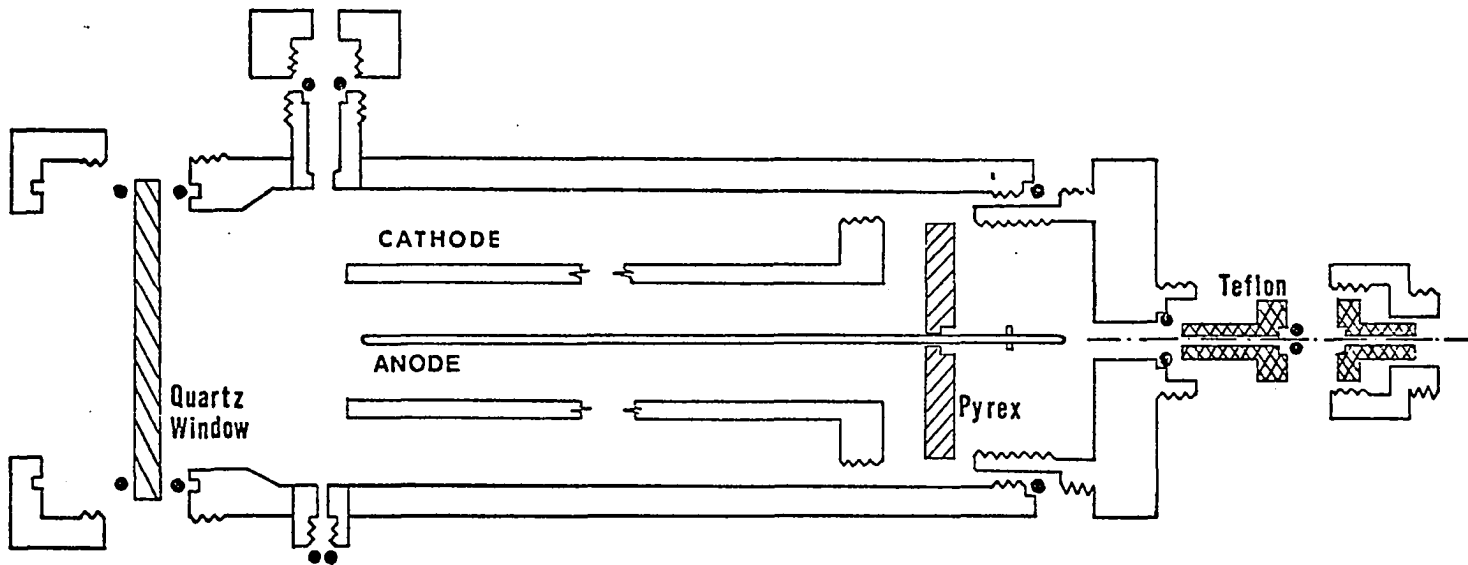


Fig. 23: Modified Invertron  
(All Viton 'O' Rings)

Teflon sleeves for electrical insulation as shown. The cathode inserts are normally aluminium cylinders which varied in diameter and configuration as will be discussed in the next section.

## E Cathode Structure

It is very important that the electronics used in a given experiment do not bias the results of that experiment, or if they do, it is important to know, at least, how and preferably why. This section is intended to cover the development of the cathode structures used in our experiments and to answer the question of experimental bias due to the glow discharge.

The cold cathode invertron is a glow discharge device which depends on electron collisions to produce the excited molecular states of interest for study. Fig. 24 illustrates the primary parts of the glow in cylindrical geometry.

The first part of this study was intended to determine if the size (diameter) or the geometry of the cathode structure effected the observed lifetimes in any way. Also, a larger diameter cathode would allow us to work at lower pressures since Paschen's law indicates that the applied voltage necessary to initiate a glow is governed by the product of pressure times diameter (pd). This increased separation of the anode and the cathode should require a slightly larger applied voltage to sustain the discharge and the positive glow should expand to fill the larger region while the negative glow should remain unaltered<sup>23</sup>.

An increase in the cathode diameter would also reduce the effect of diffusion in the system. Consider for example the following simplified rate equation

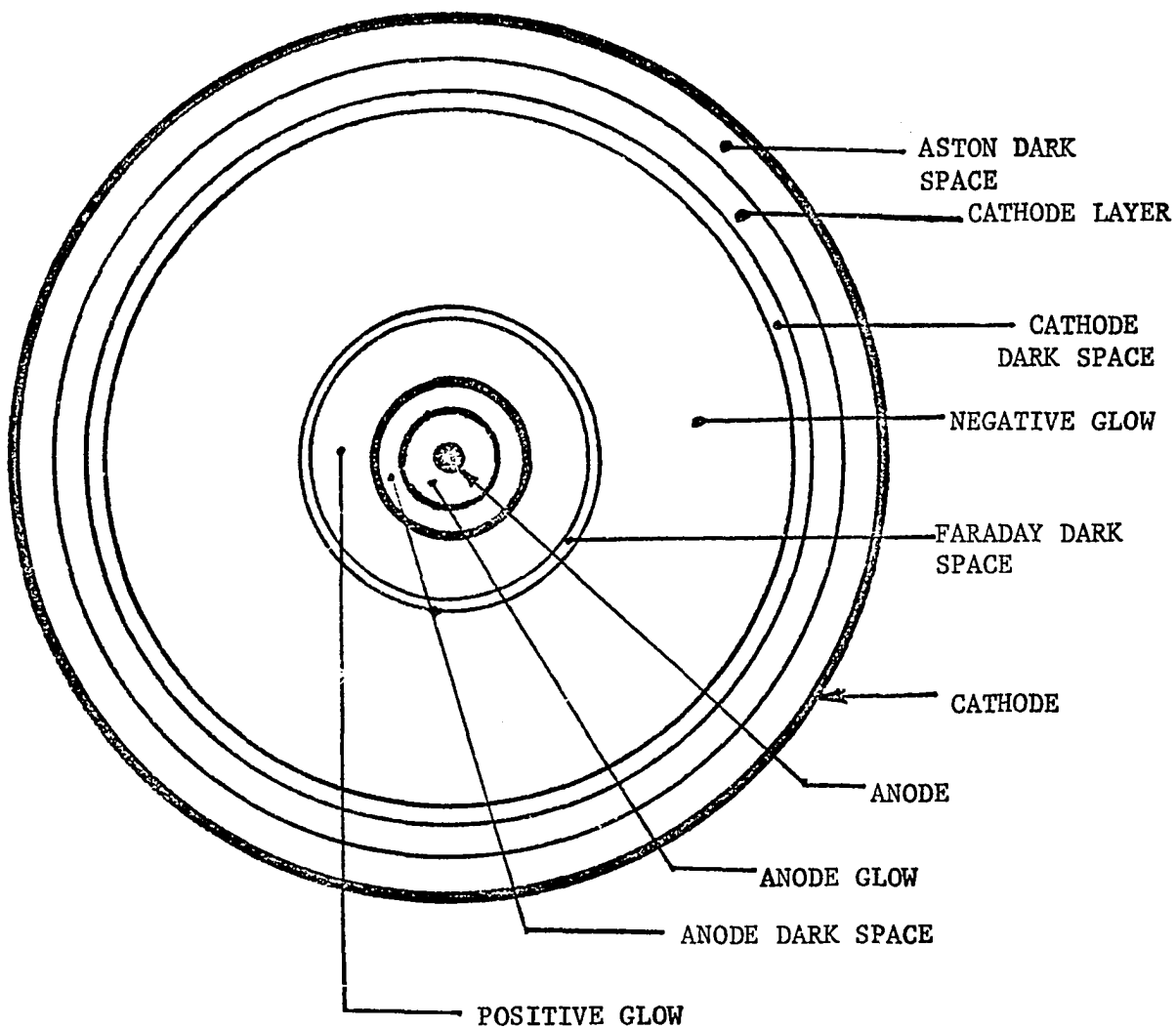


Fig. 24 . Glow region of discharge tube (cylindrical geometry).

$$\frac{dN_k}{dt} = -AN_k - N_0 \sigma \bar{v} N_k + DV^2 N_k \quad 18$$

where  $\nabla^2$  will be expressed in cylindrical coordinates and we assume no axial or angular dependence of the excited molecular density, i.e. a uniform distribution exists along the length of the discharge. We then have

$$N_k = R(r) T(t) \quad 19$$

and by using the separation constant  $-\rho^2$ , we find

$$dT/dt = -(\rho^2 + A + N_0 \sigma \bar{v})T \quad 20$$

$$r^2 \frac{d^2 R}{dr^2} + r \frac{dr}{dr} + \frac{\rho^2 r^2}{D} R = 0$$

which has the general solution

$$N_k = C \exp \{-(\rho^2 + A + N_0 \sigma \bar{v})t\} J_0(\rho r/\sqrt{D}) \quad 21$$

where  $J_0$  is the zero order Bessel function. After imposing the boundary conditions ( $a$  = radius of discharge tube)

$$N_k = N_k(0) \quad r < a$$

$$t = 0$$

$$N_k = 0 \quad r > a$$

we find that the effect of diffusion on the system is the addition of  $\Lambda^2 = (2.405/a)^2 D$  in the effective transition probability, i.e. an effective decrease in the true lifetime, as shown below.

$$A' = A + N_0 \sigma \bar{v} + \Lambda^2 \quad 22$$

This implies that as the separation of the electrodes increases, the effect of this term,  $\Lambda^2$ , decreases as  $1/r^2$ . As this effect was reportedly small for the hot cathode invertron<sup>3</sup>, it should be even smaller for the large cold cathode invertron.

It was therefore very surprising to find that not only was there an effect when the larger cathode was used, but the effect went the wrong way!!! Longer lifetimes were observed, e.g.  $B^2\Sigma_u^+$  states of  $N_2^+$  were observed at false lifetimes as long as 110nsec, which when observed at comparable pressures in the small cathode system were only 50nsec (uncorrected for pressure dependence).

This coupled with the fact that as the pressure was decreased this effect became even more pronounced led to the conclusion that this problem had to do with improper field quenching after the excitation voltage was clamped off. That is, with larger cathodes, a larger surface area was available to produce electrons while insufficient means were present to sweep these additional electrons out of the excitation region where they were continuing to produce excited molecules after the field was supposedly turned off. Consequently, while the cut-off appeared to be 10nsec on the monitor, it was probably as long as 50nsec, or longer.

This conclusion seems to be born out by the following experiments. A rod structure was inserted just inside the cathode shell, in electrical contact with the cathode as shown in Fig. 25. This structure did indeed bring the observed lifetimes back down to 50 nsec at the appropriate pressure but a very long plateau now existed as shown in Fig. 26. It was found that removal of every other rod in the cathode structure made the plateau twice as long as before. If this plateau was indeed due to insufficient electron collection, it seemed reasonable to expect an increase in the duration of the discharge pulse would produce a similar result. Fig. 27 indicates that this exactly what happens.

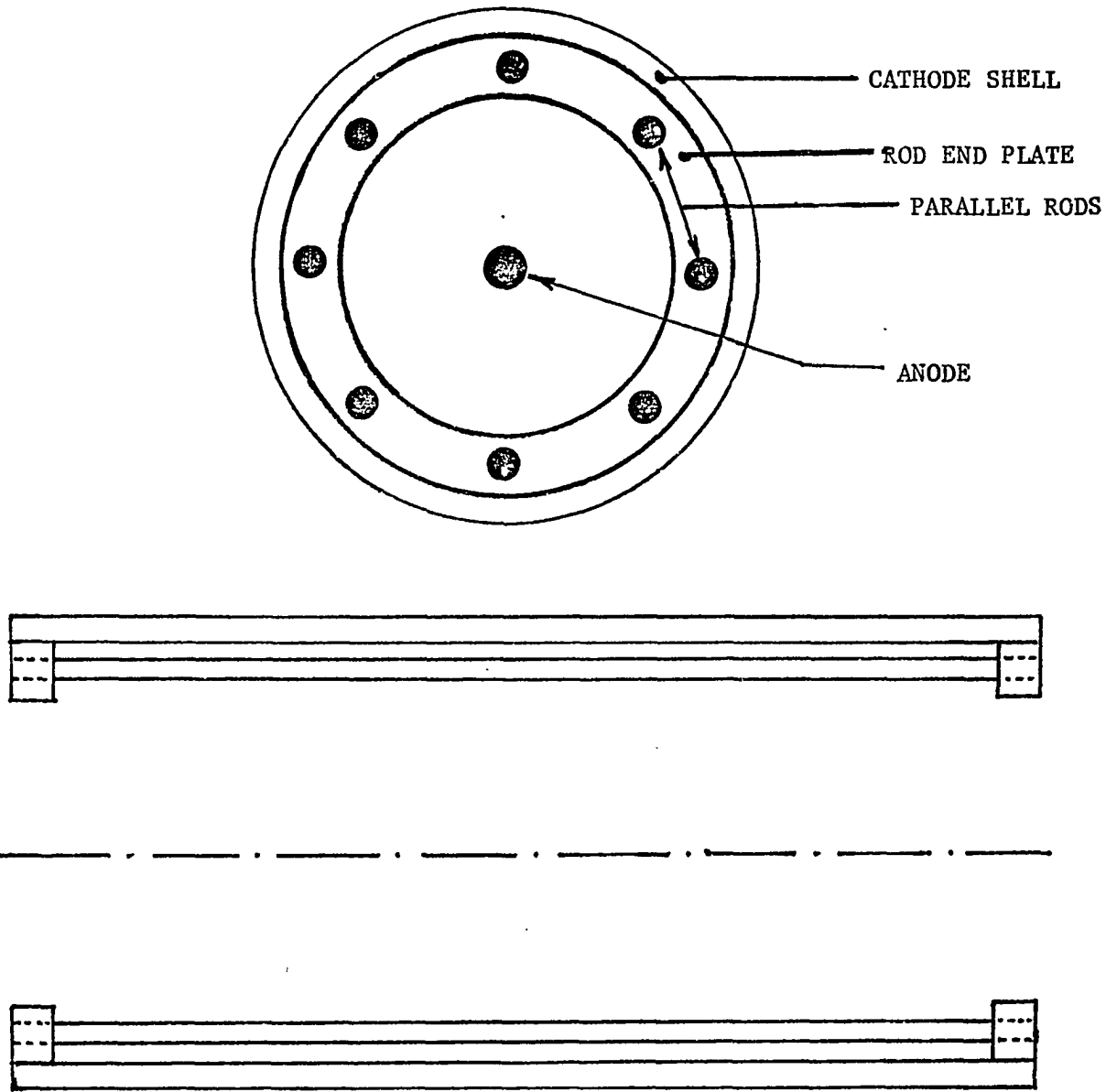


Fig. 25 Parallel rod cathode configuration



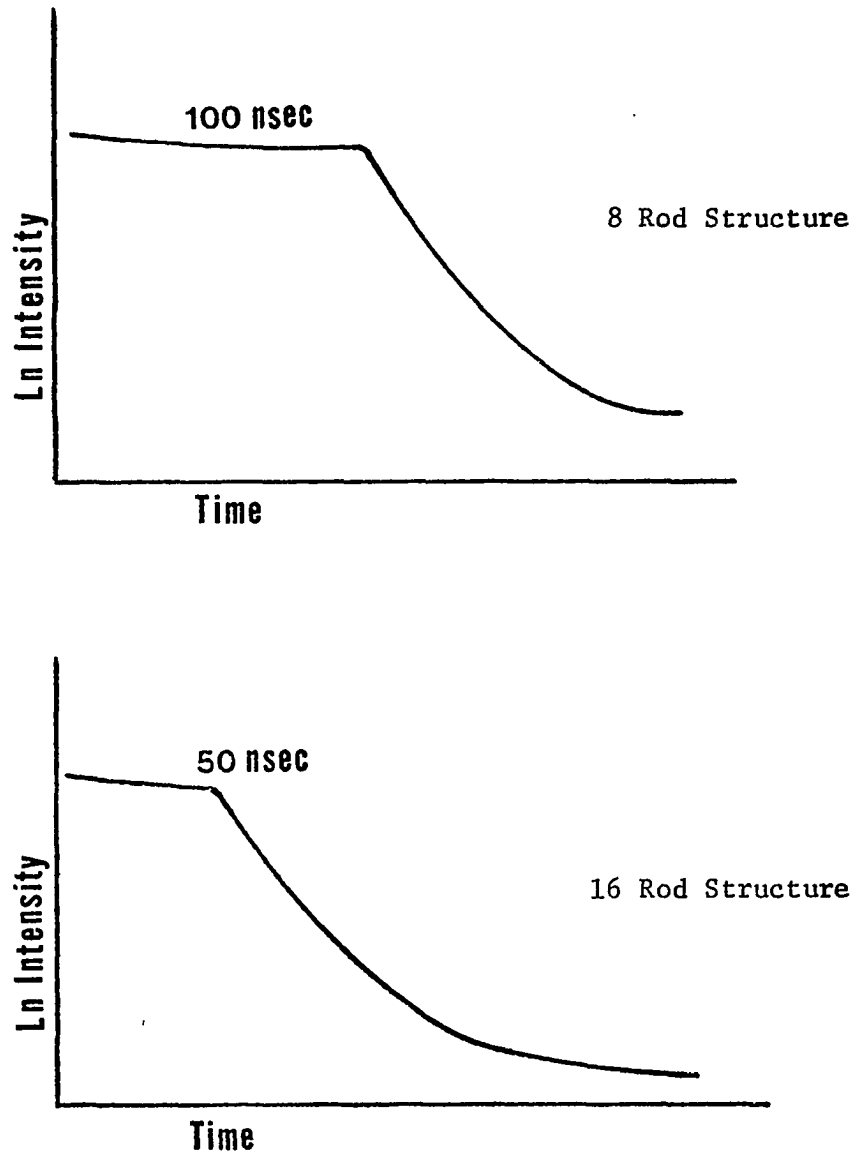


Fig. 26: Effect of the Parallel-Rod Cathode on the Apparent Cut-Off of the Decay Curve.

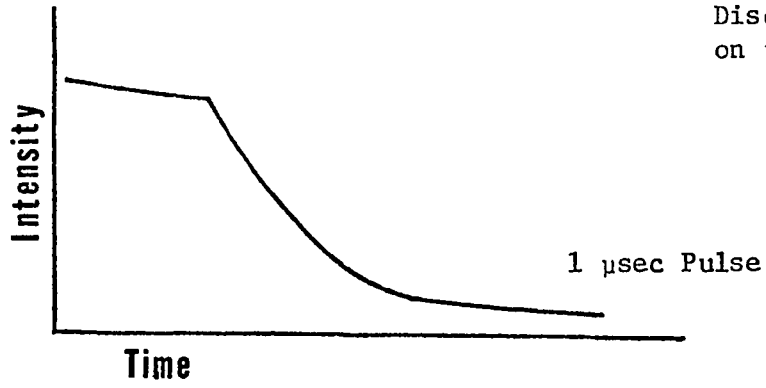
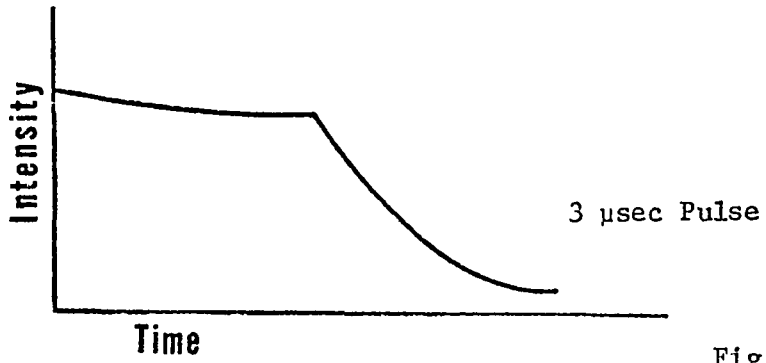
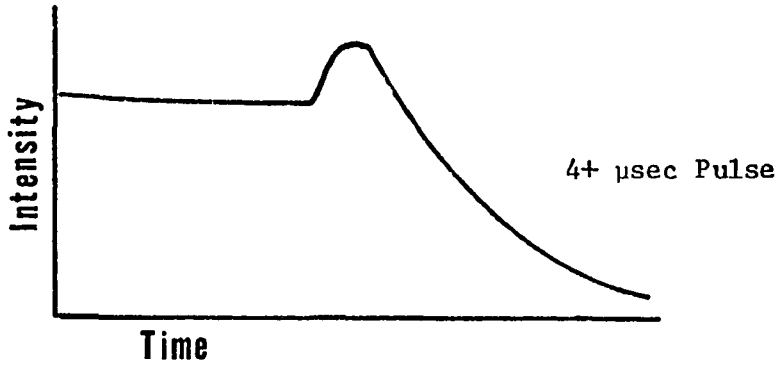
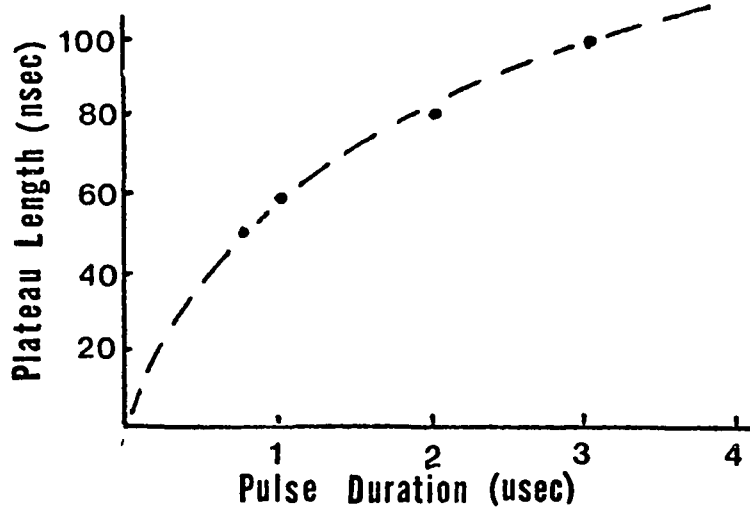


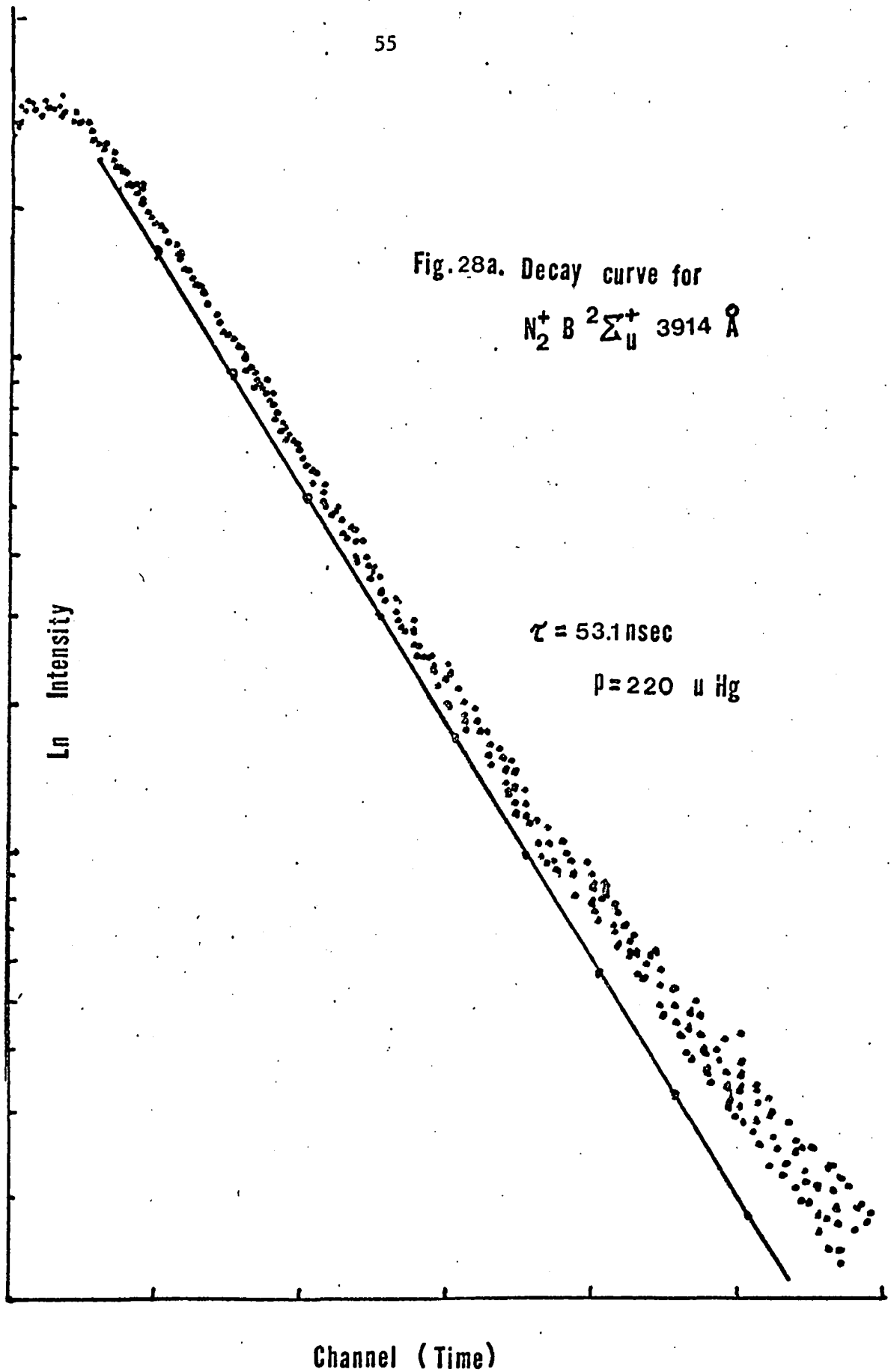
Fig. 27: Effect of the Discharge Pulse Duration - on the Decay Curve Plateau

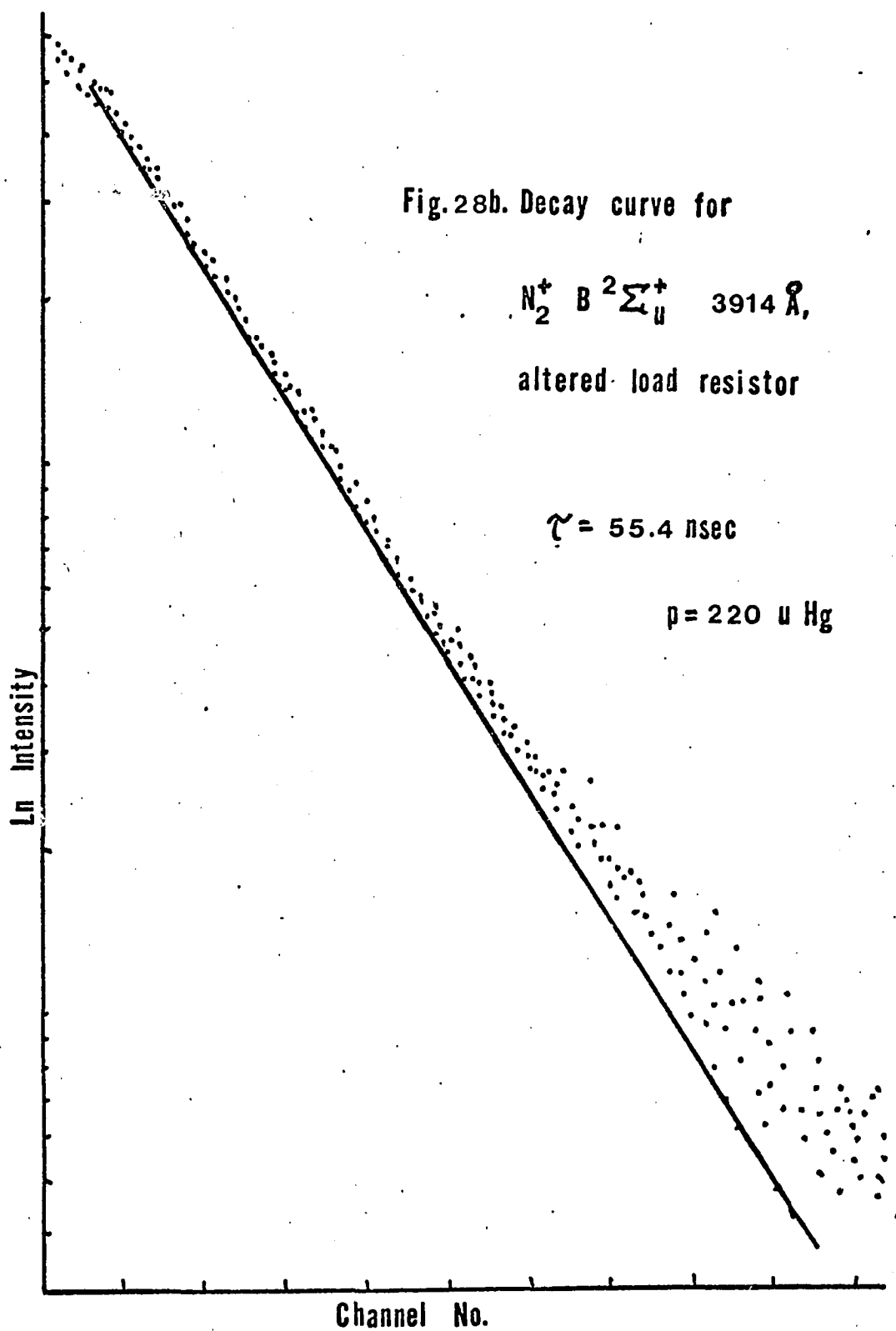


As a result of this study, it was determined that the cathode structure must accomplish two ends...first, it must be able to produce sufficient electrons to maintain the glow discharge, and second, it must be able to retrieve those electrons rapidly (10nsec or better) when clamped off. The parallel rod structure and a later developed helically-grooved structure accomplished this end independent of their large surface areas needed for electron production by the presence of short radius field points (rods or helix corners) near the cathode to provide strong retrieval field lines.

This problem is also related to the ability of the electronics to provide sufficient current on demand to give complete cut-off. This is a small problem but is probably responsible for the small amplitude oscillations present in the computer analyzed data (it was present in the hand analyzed data also, but is not as noticeable). Fig. 28a, b, are decay curves taken on  $B^2\Sigma_u^+$  where only the crowbar circuitry was altered. This may indicate an interesting phenomenon as this effect was always stronger for the B state of  $N_2^+$  than for the neutral molecular state  $C^3\Pi_u$ . This may indicate a forced diffusion (or field diffusion) of the  $N_2^+$  molecules as their lifetimes seem to be reduced very slightly in the lead channels, while the 'true' lifetime is still present in the later channels of the experiment.

Another interesting fact from these studies was the overall effect of the various cathode diameters on the observed lifetimes as illustrated in Fig. 29. This plot represents the data acquired from the original small smooth cathode, the large diameter parallel-rod cathode and the large diameter helical-groove cathode. Note that





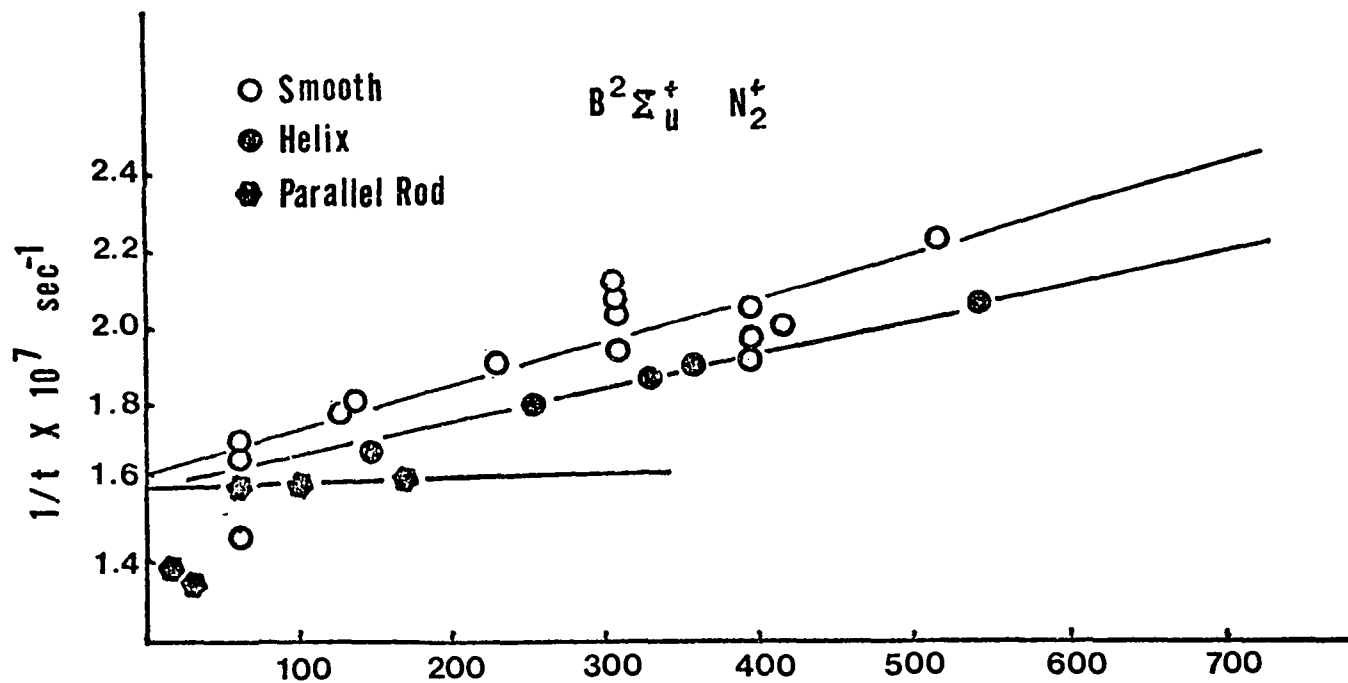


Fig. 29: Inverse Lifetime VS Pressure Curves for Different Cathode Structures

while their slopes are quite different, their zero pressure intercepts are quite similar.

This indicates that the form of Eq. 16, Chapter I, is incomplete; i.e.,

$$A' = \sum \frac{A_{kn}}{n} + N_o \langle v \rangle \sum \frac{\sigma_{kn}}{n} \quad 16$$

does not describe the curves properly. For a given pressure, excitation voltage, etc., the only factor changed in these experiments was the cathode geometry. This can only alter the electron concentration. The different cathodes have demonstrated varying efficiencies for sweeping the electrons from the active region of the excitation tube (e.g. Fig. 26) and the excitation pulse duration has also been shown to effect the decay curves (Fig. 27). These experiments point to residual electron concentrations which must be accounted for.

By considering the recombination and attachment losses given in Eq. 12, Chapter I, we have,

$$\frac{dN_k}{dt} = -N_k \sum \frac{A_{kn}}{n} - N_o N_k \sum \langle v \sigma_{kn} \rangle - \alpha_r n N_k - \alpha_a n N_k. \quad 23$$

However, we can almost discount the attachment term out of hand since pure nitrogen does not attach electrons. Air on the other hand will attach, with an attachment coefficient of  $\alpha_a = 6.3 \times 10^{-7}$  sec.<sup>22</sup> Consequently, a leaky vacuum system may require this factor to be included.

Electron recombination might appear as a questionable factor since the calculated recombination coefficient is on the order of  $10^{-14}$  cm<sup>3</sup>/sec. However, this value has been observed much larger ( $10^{-8}$  cm<sup>3</sup>/sec)<sup>23</sup> for cases when the time for the disposal of the recombination

energy is much shorter than the average life of an excited molecule. The dissociation of a molecule will occur within a time interval of one vibration ( $10^{-13}$  sec) and according to the Frank-Condon rule, the electron changes from one state to another in an excited molecule within a time which is negligible compared to this vibration. This would seem to imply that dissociative recombination might then be an important factor. Von Engel<sup>24</sup> states that this dissociative recombination coefficient is proportional to the pressure. With this in mind, and using the assumption that after cut-off of the discharge tube, with only the external magnetic field to influence them, electrons and ions should be in a fairly equal concentration, constant with respect to the nanosecond decays of the excited state, we will have

$$N_k(t) = N_k(o) \exp \left\{ - \left( \sum_n A_{kn} + N_{on} \sum \langle v \sigma_{kn} \rangle + \alpha_r \eta_e \right) \right\}. \quad 24$$

So, instead of depopulation by collision with neutrals only, we should also include the depopulation by collision with electrons in the form of dissociative recombination. The data would then imply that the smooth cathode is very inefficient in removing the electrons after cut-off, while the helix cathode and the parallel-rod cathode are much more effecient. This is an additive factor, linear in pressure and consequently it will not detract from the validity of the experiment.



## F Magnetic Field

The cold cathode invertron discharge tube is a glow discharge tube which relies on electron impact on the target gas for electrical breakdown. This is at best a statistical process and in a fixed diameter tube Paschen's Law limits the target gas pressure.

In order to promote the breakdown at lower pressures, one needs to find a means of artificially increasing the electron-target collision frequency. This can be done by applying a coaxial magnetic field to the discharge tube as Copeland did. (see also ref. 25, 26, 27) This causes the electrons to spiral around the magnetic lines of force, thus increasing their effective path length in the active region of the discharge tube. A weak field, around one hundred gauss, will allow workable discharges to gas pressures as low as 5 millitorr in this apparatus.

The field is provided by a selenoid wound around a copper sleeve to allow water cooling of the coil (typically 75 watts of power is generated by the coil). This copper sleeve fits over the discharge tube shell, as shown in Fig. 30. The current supplying the field was normally supplied by a Western Electric power supply (48 volts dc); however, for the tests to determine the effect of the magnetic field on the observed lifetimes, the current was provided by the main 100 volt dc generator in the physics building.

Since we are observing the radiation parallel to the magnetic

field, we would not really expect any effect of the field on the lifetimes. However, we do see some effect with increasing field as shown below in Figure 31. While the average value is the same, the experimental scatter increases. This data was taken on the vibrational levels of  $C \Pi_u$  ( $v'=0,1$ ) ( $3371, 3159 \text{ \AA}$ ) of nitrogen.

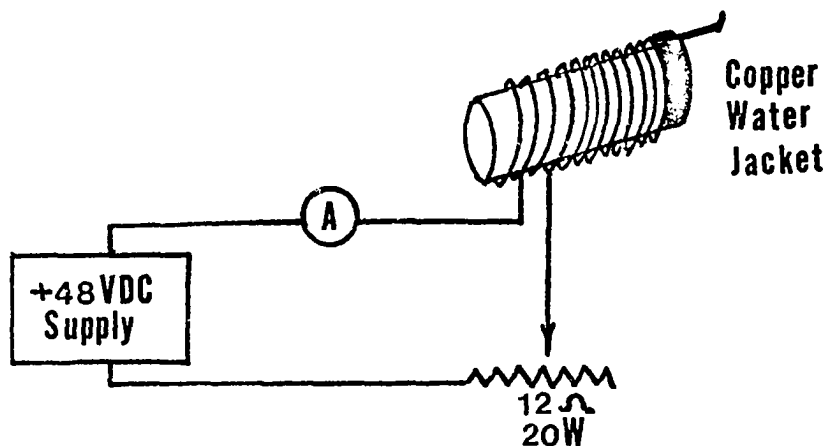


Figure 30: Magnetic field system

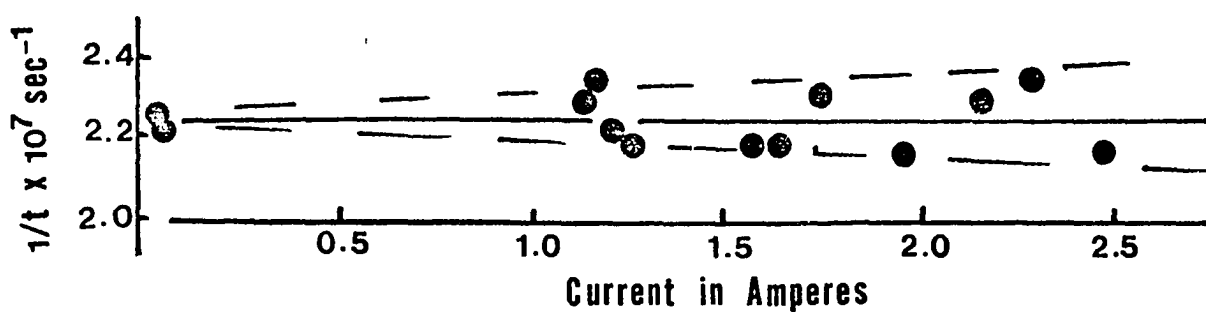


Figure 31: Effect of the magnetic field on lifetimes.

This effect is certainly not due to the interaction of the magnetic field with the decaying state, i.e. precession of the radiating dipoles around the magnetic lines of force with Larmor frequency  $\omega_L = g_J \mu_O H/h$  ( $g_J$  = Landé factor), but is probably related to interference of the increasing field with the electrical cut off of the discharge.

Evidence may be found for this in the increase of the "plateau" of the decay curves as shown in Fig. 32. This plateau represents a stable region after cut off which indicates the inability of the inverter to sweep the electrons from the field on application of the 2D21 crowbar (see pulse generation section). See also the section on cathode assemblies for similar results. This increasing plateau then increases the uncertainty of the measured lifetime, but on the average, does not bias the results.

We may then conclude the applied field offers no systematic error to the system but indicates that we should keep the field as low as possible to reduce the overall error of the system.

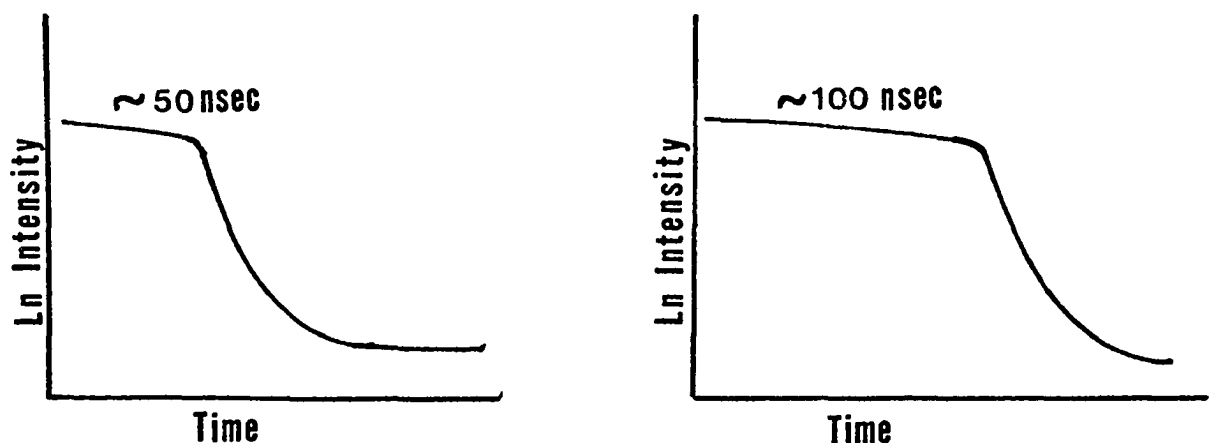


Figure 32: Effect of the magnetic field on the decay curves  
 a) low field,  $I=1.3$  amp ( $\sim 150g$ )  
 b) high field,  $I=2.5$  amp ( $\sim 300g$ )

## CHAPTER III

### DATA ANALYSIS

Within the limits discussed in the first chapter the data may be considered as consisting simply of the sum of several exponentials, the actual number determined by the number of cascading states plus an additive constant due to random noise. We then have for the time dependent intensity

$$I(t) = Be^{-A_1 t} + Ce^{-A_2 t} + \dots + K \quad 25$$

where the decay constants,  $A_1, A_2$  are the Einstein transition coefficients plus the pressure dependent quenching rates  $A_1 = \sum_i A_{ki} + N_0 \sigma \langle v \rangle$ , B and C are the respective amplitudes and K the constant background term.

Exponential curve fitting is at best an educated guess if more than two exponentials are present since in the presence of errors practically any combination of three exponentials can be made to fit a given data set involving three or more exponentials. However, if strict attention is paid to the 'clues' presented by the data, two exponential data curves can be handled with relative ease if the long lived component is much longer than the short lived one (a factor of 5 or more).

Two techniques were used in evaluating the lifetime data in this work, graphical stripping, and computer stripping. These techniques will be discussed separately below.

#### Graphical Analysis

This method can give remarkably good results if the following technique is followed. After the data is plotted on semi-log paper,

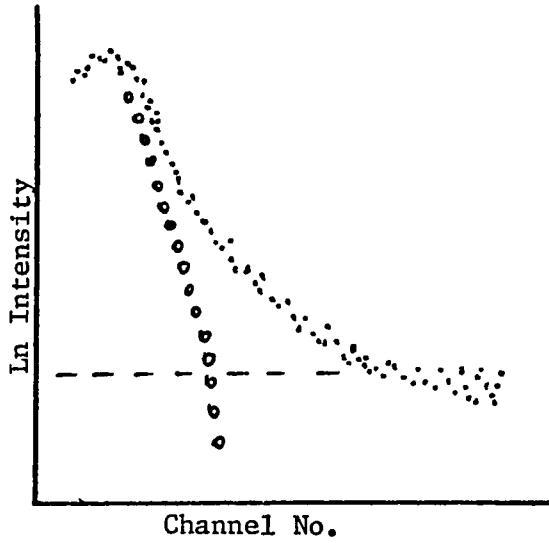
a constant amount representing the noise is subtracted from the data point by point. This is perhaps the most critical step. If too much, or too little is subtracted, the exponential overlying the background will be distorted. If this distortion is not ignored however, it will clearly indicate to the operator the proper amount to be called noise as is indicated in Fig. 33. This technique breaks down if too short a time scale is used for the data acquisition, i.e. the time scale used should be sufficiently long for at least 4 e-foldings of the decay to have occurred. For example, a 60 nsec decay should be acquired on the 400 nsec time scale and not on the 200 nsec scale. This insures also that the curve in Fig. 33b is not due in part or wholly to a real long-lived cascade component.

If two exponentials are present, where one is of rather weak amplitude, and an attempt is made to fit only one exponential to the data, a slight but rather persistent oscillation will be present as shown in Fig. 34a. This effect is obvious in the computer analysis (LASL) as the background also varies (see appendix A for sample LASL run #2).

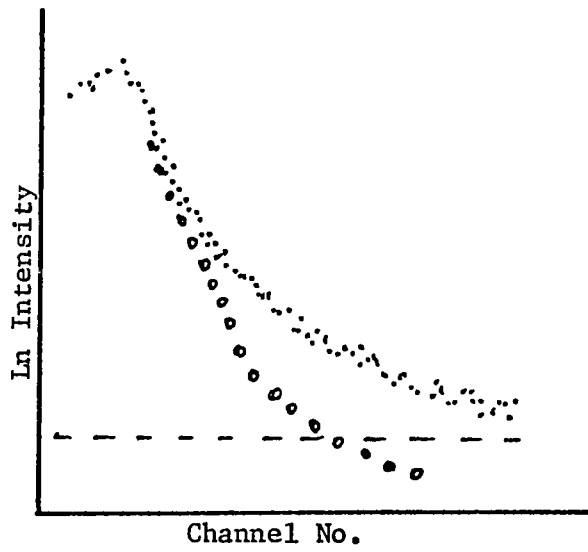
If the two exponentials are of approximately equal amplitude and only one exponent is used to fit to the data, a noticeable 'break' will occur as shown in Fig. 35. If the fast exponential is of much greater amplitude than the slow component, no problem should arise as a double exponential fit should be obvious.

One final point should be mentioned. Regardless of all else, the stripping process MUST fit in the lead channels with a high degree of accuracy. This is an additional clue as to the misinterpretation of the data in Fig. 34a, where a noticeable discrepancy exists

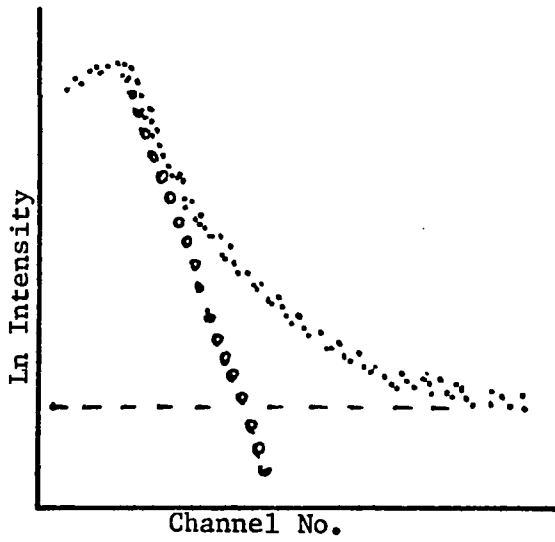
Fig. 33: Illustration of improper background subtraction. (Simulated data)



a) too much subtraction



b) too little subtraction



c) proper subtraction

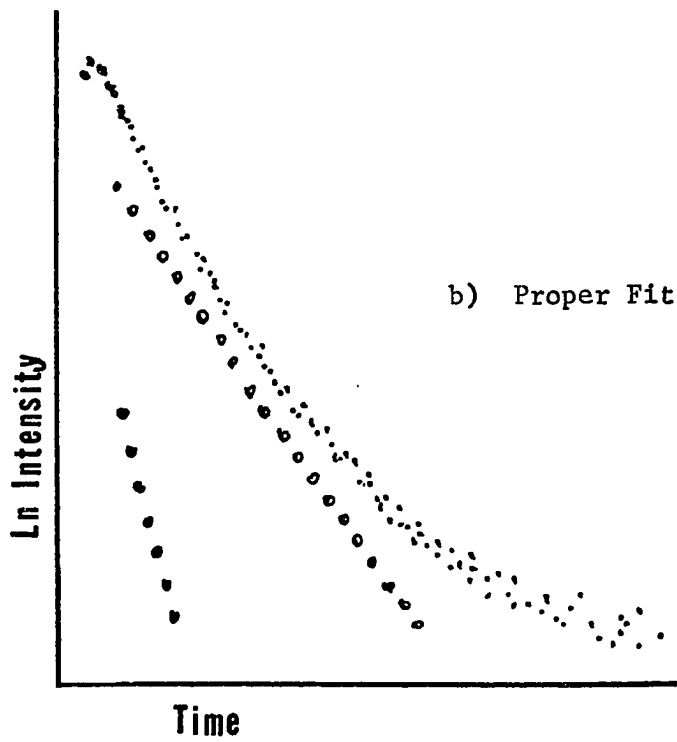
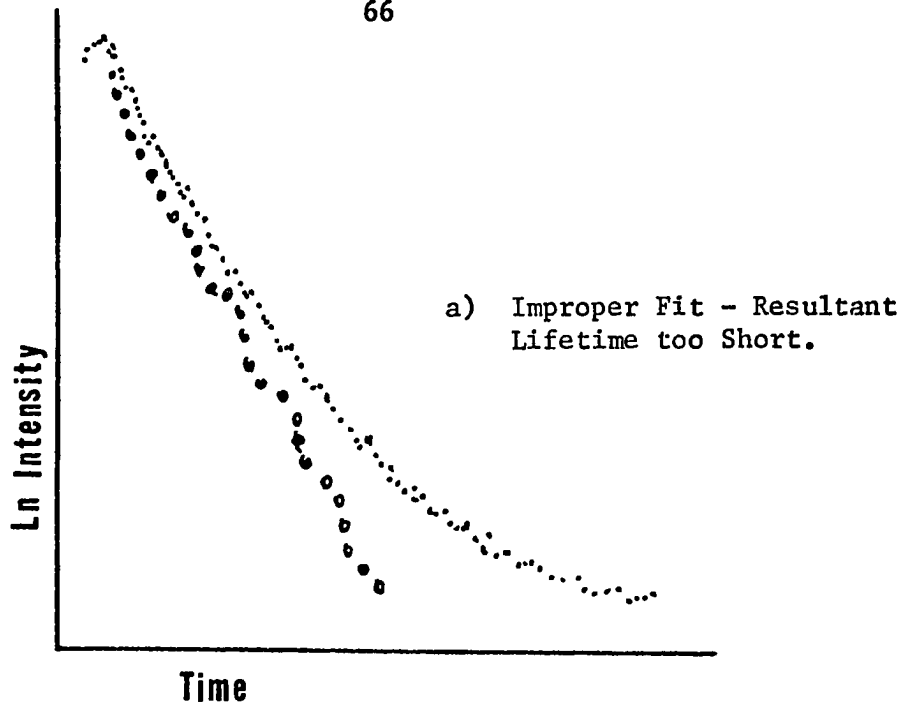


Fig. 34: Improper Fit for Small Amplitude Second Exponential

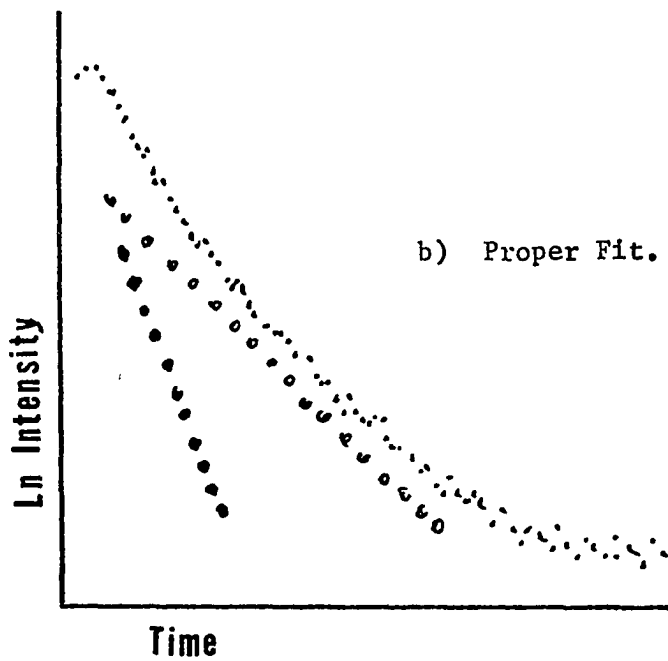
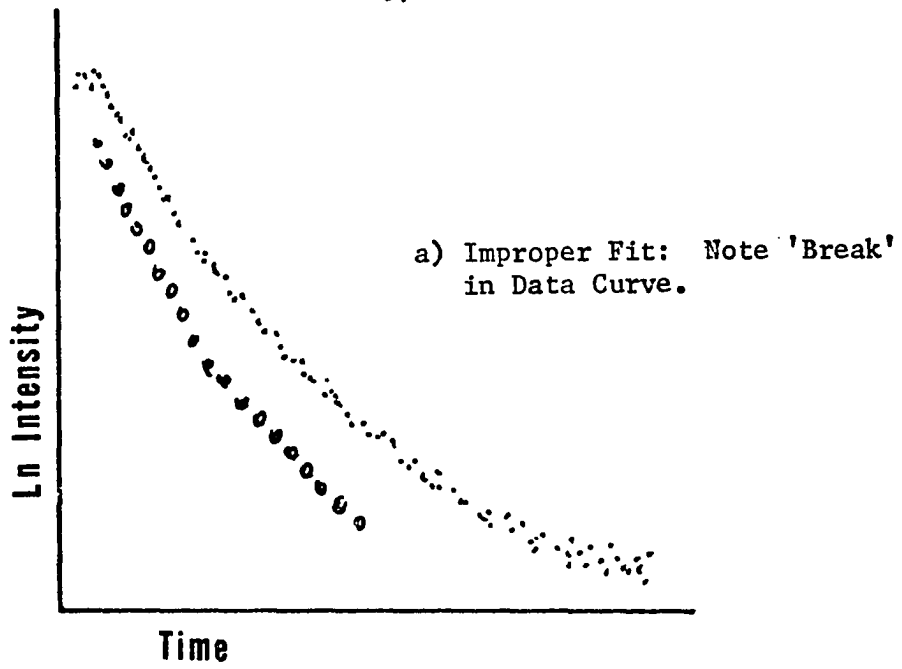


Fig. 35: Improper Fit For Equal Amplitude Second Exponential



between the data curve and the linear fit.

Computer Technique

It should be stressed at the onset that the computer cannot handle data which cannot be treated by hand graphical methods, as the computer does essentially the same thing except that instead of averaging the data by 'eye', the computer uses a linear (or non-linear) least squares fit. Therefore, while the PRECISION stated by the machine looks better, the ACCURACY cannot be any better than that achieved by an experienced individual. Also, the machine cannot discount datum errors as the eye can, and consequently a 'dropped' channel (one in which the number of counts is several standard deviations lower than the surrounding channels or even zero) might be weighted incorrectly to the point of distorting the decay curve unless a  $\chi$ -square or similar test is built into the program. This is a costly CPU procedure. However, the computer offers the advantage in that it reduces the experimental error due to the experimenter's bias about his preferred answer. However, since he can select weight factors, etc., for the program, bias still may not be entirely eliminated.

Two computer programs are used in these experiments in addition to the above mentioned 'hand' techniques. These are the programs LPLOT and LASL (both Fortran G-level).

LPLOT is a semi-logarithmic plotting routine which will also reproduce the data file in punched deck form for later re-use and permanent storage of the data. This program is used to provide information as to how many exponentials are present; where to start the fitting routine; where to stop the routine (channel number); how

much experimental error to expect in the reduced data. Appendix A contains a program listing for LPLOT followed by a sample data run.

LASL is a non-linear least squares program which through iteration techniques adjust the parameters (see equation III-1) until a best fit is reached. The program was initially developed by the Los Alamos Scientific Laboratory and has been extensively modified for use in the IBM 360-40 (currently being run on the IBM 370-158). The least squares technique used is that of Gauss-Newton which has been modified by Thompson<sup>5</sup> to provide a steepest path of descent as described by Marquardt<sup>28</sup>. Proofs of the technique may be found in ref. 29, 30 and 31. A thorough description of this program and its limitations may be found in R. T. Thompson's dissertation<sup>5</sup>. A program listing is provided in Appendix A along with two sample data runs.

## CHAPTER IV

### COUNT RATE DISTORTION

It was noticed by Johnson<sup>3</sup> that the rate at which data is acquired in pulse height analysis (PHA) could cause distortions in the decay probability curves. This is illustrated in Fig. 36 a-d, which represent the decay curves observed for the  $B^2\Sigma_2^+$  state of  $N_2^+$  which were obtained at count rates of 4, 11, 17 and 25%.

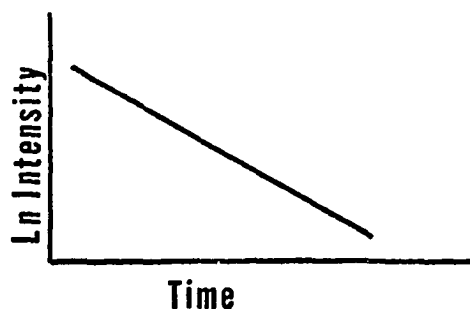
In order to better understand why large count rates distort the data taken in delayed coincidence photon counting, it would be useful to look at why data acquired in the MS mode used for nuclear half-lives is not distorted handling the same effective count rates. Consider the time rate of change of the population  $N(t)$  of a given radionuclide

$$dN(t)/dt = -AN \quad 26$$

or the number of nuclei at any given time  $t$  is

$$N(t) = N(o) e^{-At} \quad 27$$

which gives the time dependent distribution as follows:



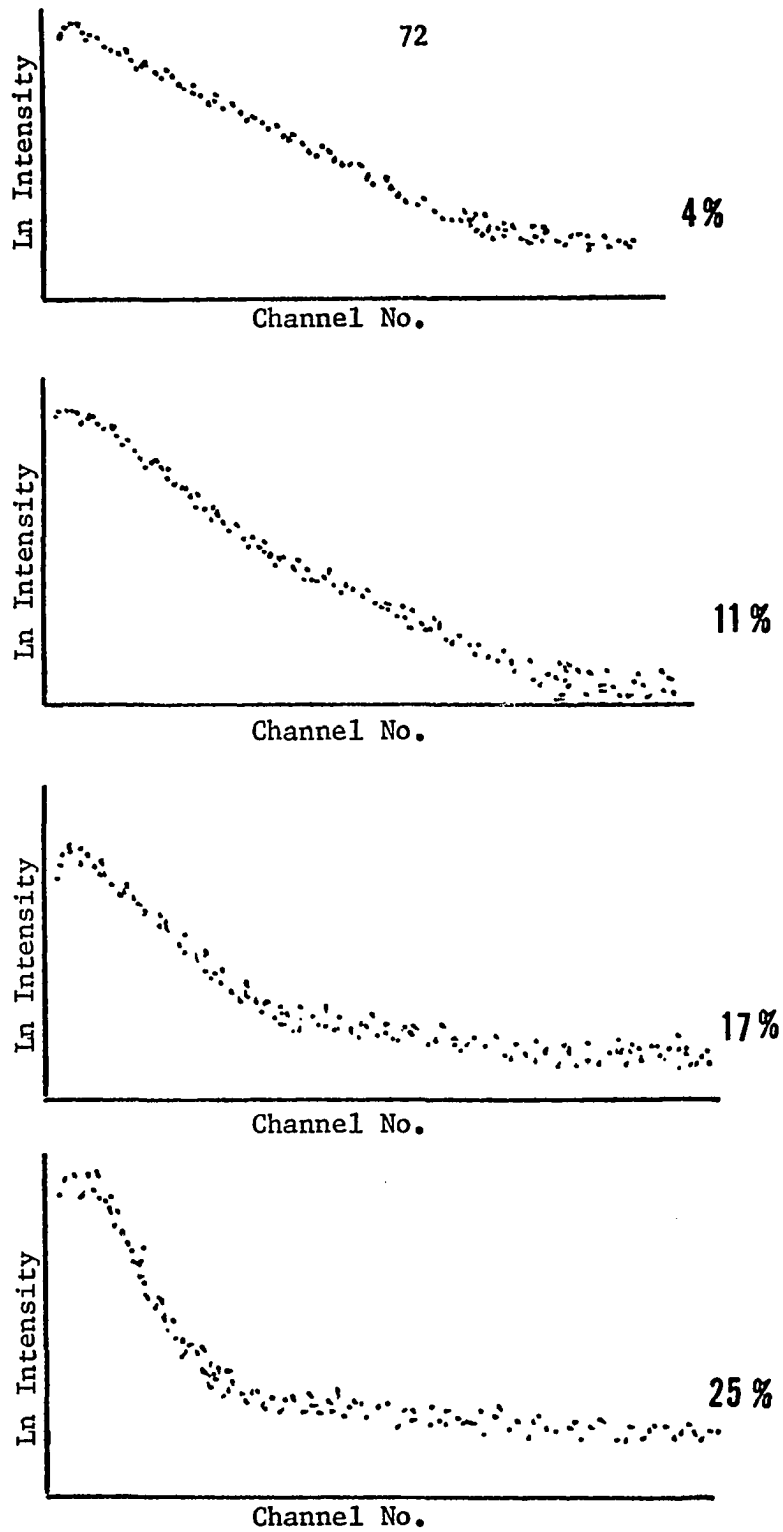


Fig. 36. Decay curves at various count rates.

Now, consider the probability distribution obtained by a MCA in the MS mode, i.e. all counts delivered in a prescribed time  $t$  to  $t + \tau_\omega$ , are accepted (ignoring MCA dead time which is a different type of distortion) and the analyzer goes on to the next channel  $\tau_\omega$ . We need to know if the distribution is indeed that which is predicted by Eq. 27.

Since intensity  $I(t)$ , is the number of events per unit time we should have

$$I(t) = N_0 \frac{dp}{dt} \quad 28$$

where  $N_0$  is the population of possible events (number of radionuclei),  $dp/dt$  is the probability an event will occur per unit time (or  $dp/d\tau_\omega$  per unit channel) and  $dp = p_e p_s dp_d$  where  $p_e$  is the probability that the nuclei exist  $\exp(-At) = (e^{-At})$ ,  $p_s$  is the probability that we detect it (efficiency, etc.) and  $dp_d$  is the probability of decay  $A dt$  (or  $A d\tau_\omega$ ).

Therefore

$$I(t) = N_0 p_s A e^{-At} \quad 29$$

and at  $t=0$  we have  $I(0) = N_0 p_s A$  so we can write

$$I(t) = I(0) e^{-At} \quad 30$$

This is of the same form as Eq. 27 and therefore indicates that the data acquisition technique does not interfere with the experimental results.

Now, whereas the above technique uses one initial population to determine the half-life, the mean lifetimes of molecules must be determined from many different initial systems, where only one decay per initial system is observed, all others are ignored. The probability decay curve is then the collection of many hundreds of thou-

sands of separate events, each contributing at most only one count.

Therefore, we will have for the intensity

$$I(t) = N dp/dt \quad 31$$

where  $N$  is the number of events sampled (invertron pulses),  $dp/dt$  is the probability that the event will occur per unit time (channel) and now

$$dp = P_e P_s P_m^d P_d \quad 32$$

where  $p_e$  is the probability that the molecule exists  $\exp(-At)$ ,  $p_s$  is the probability that it is detected (efficiency etc.),  $p_m$  is the probability that the event was missed, and  $dp_d$  is the probability that the molecule will decay in  $t \rightarrow t+dt$ .

The factor  $p_m$  is required since we are basing our measurements on the time a pulse is seen after the excitation is effected, all subsequent pulses from that excitation being ignored. This is not done in the MS mode and consequently no correction is required as missed events are assumed accountable in the detection efficiency of the equipment. The question then is how much bias does this factor cause.

Consider the probability of seeing an event

$$p_s (1 - e^{-At}) \quad 33$$

Then the probability of missing the event is simply

$$1 - p_s (1 - e^{-At}). \quad 34$$

For  $n$  possible decay events (in the direction of the detection apparatus) we see only 1 decay event before the TAC turns the system off, so we miss

$$\{1 - p_s (1 - e^{-At})\}^{n-1} \quad 35$$

The intensity is thus given by

$$I(t) = NAp_s e^{-At} \{1-p_s (1-e^{-At})\}^{n-1} \quad 36$$

and for  $t=0$ , we have  $I(0) = NAp_s$  or

$$I(t) = I(0) e^{-At} \{1-p_s (1-e^{-At})\}^{n-1} \quad 37$$

which indicates a deviation from the curve of interest by the term in braces. But note if we consider only one possible event and we detect it, i.e.  $n-1=0$ , we get the undistorted curve.

Here  $n$  represents the number of events which did get through the apparatus and could have been detected but were not recorded by the method of counting employed, since all but the first were ignored. The value of  $n$  can be artificially kept low by keeping the count rate low. The count rate is the number of measured counts divided by the number of times the system (or invertron) is cycled. By keeping the count rate around 10%, we imply that the probability of two simultaneous events is 1%, that of three events 0.1% etc. Therefore we could write for a "double event":

$$I(t) = I(0) \{(1-p_s)e^{-At} + p_s e^{-2At}\} \quad 38$$

and for a "triple event":

$$I(t) = I(0) \{(1-p_s)^2 e^{-At} + 2p_s (1-p_s) e^{-2At} + p_s^2 e^{-3At}\} \quad 39$$

To reiterate, corrections are required not because events are missed by the system, but because one event is detected and another selectively ignored. This biases the data to the earlier channels and appears as a fast exponential in the data curve as illustrated in Fig. 36

A much more convenient analytical form may be obtained by re-writing the probability of lost events as follows.



$$\begin{aligned}
 p_m &= \{1 - p_s (1 - e^{-At})\}^{n-1} \\
 &= 1 - (n-1) [1 - e^{-At}] p_s - \frac{(n-1)(n-2)}{2!} p_s^2 [1 - e^{-At}]^2 + \dots
 \end{aligned} \tag{40}$$

which is approximately

$$\sum_{i=0}^{\infty} \frac{(np_s)^i [1 - e^{-At}]^i (-1)^i}{i!} \tag{41}$$

since  $\frac{np_s}{i!} \ll 1$ ,  $i$  an integer, is correct for much smaller values of  $i$  than is the expression  $(n-i) \ll n$ . See ref. 3, 32.

Now, by defining  $m = np_s$ , we have

$$p_m = \sum_{i=0}^{\infty} (-1)^i \frac{m^i [1 - e^{-At}]^i}{i!} = \exp \{-m [1 - e^{-At}]\}.$$

Consequently Eq. 37 becomes

$$I(t) = I(o) e^{-At} e^{-m(1 - e^{-At})} \tag{42}$$

where  $m = \ln \left(1 - \frac{C_m}{C_p}\right)$  and  $C_p$  is the number of counts measured (total number of events seen by the MCA), and  $C_m$  is the total number of counts possible (number of times the system fires since only one count is allowed per excitation).

Fig. 37 shows how Eq. 42 fits the experimental data compiled on the  $C^3\Pi_u$  state of nitrogen. Currently, additional work is being conducted on this problem and will be reported later by others.

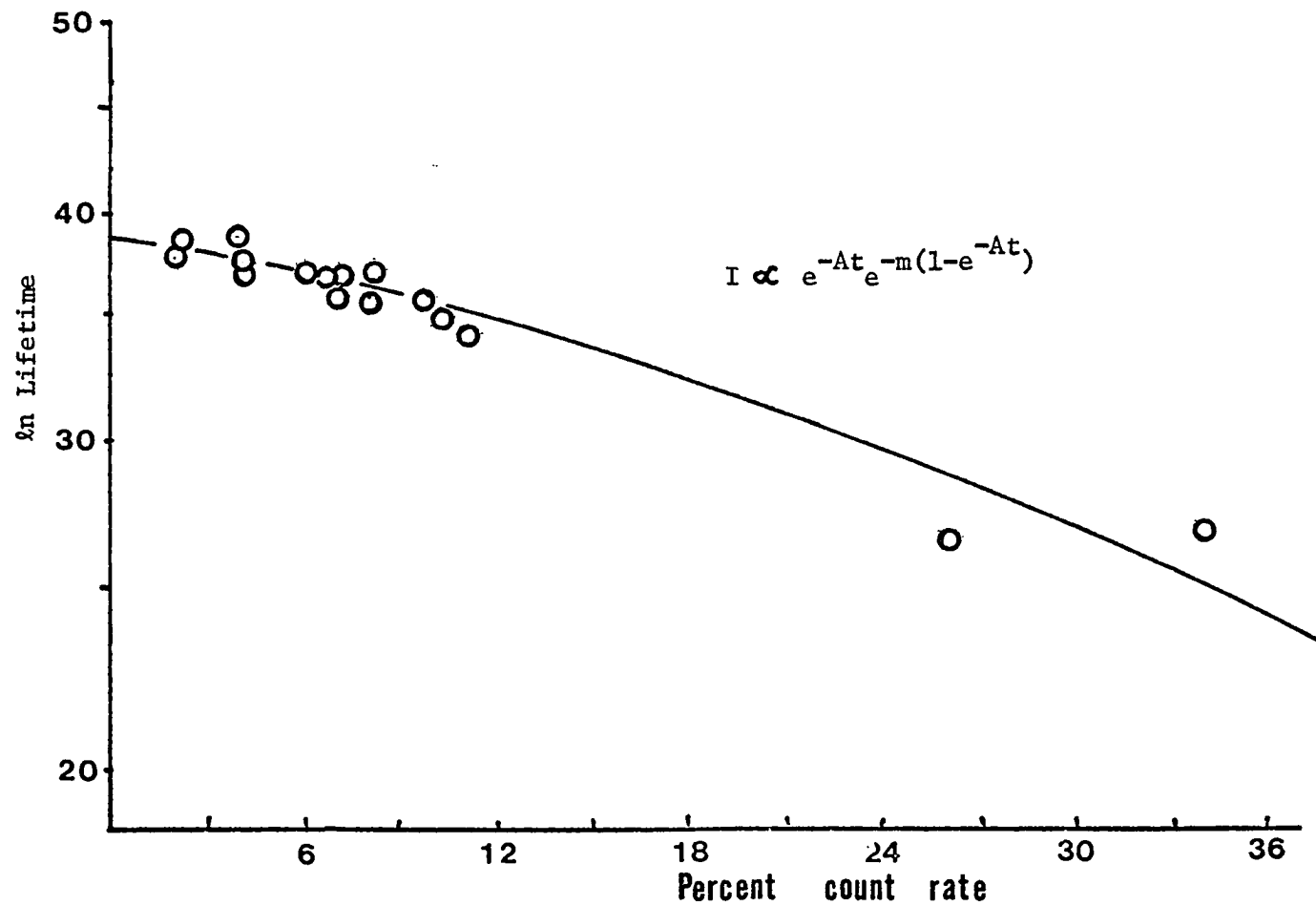


Fig. 37: Theoretical and Observed Dependence of the Lifetime on the Count Rate

## CHAPTER V

### Nitrogen Lifetimes

Nitrogen was chosen for this series of experiments because the  $B^2\Sigma_u^+$  state of  $N_2^+$  and the  $C^3\Pi_u$  state of  $N_2$  have been extensively studied by many different techniques and Johnson<sup>3</sup> completed some very precise work using the hot cathode invertron with which this work was compared. Also, the  $C^3\Pi_u$  state appears to be pressure independent and does not possess a cascade component. This state then provides a clean single exponential of 38.4 nsec mean life with which to calibrate the cold cathode system and to determine any equipment dependences. Likewise, the  $B^2\Sigma_u^+$  state of  $N_2^+$  provides a single exponential (59 nsec), but exhibits a pressure dependence which should be useful in determining depopulation mechanisms.

It is felt that an adequate understanding now exists such that experimental results are reliable and reproducible. A recapitulation of the major experimental problems will be presented in this section followed by tables of this work compared with that previously reported in the literature.

"Typical" data in the form of pressure vs the transition probability is presented in Fig. 38 which represents the quality of early data acquired on the unmodified system for the  $B^2\Sigma_u^+$  state of  $N_2^+$ . Suffice to say that this quality of data could not be tolerated.

Most of this problem was cleared up by recognizing the severity of the count rate distortion discussed earlier. This distortion

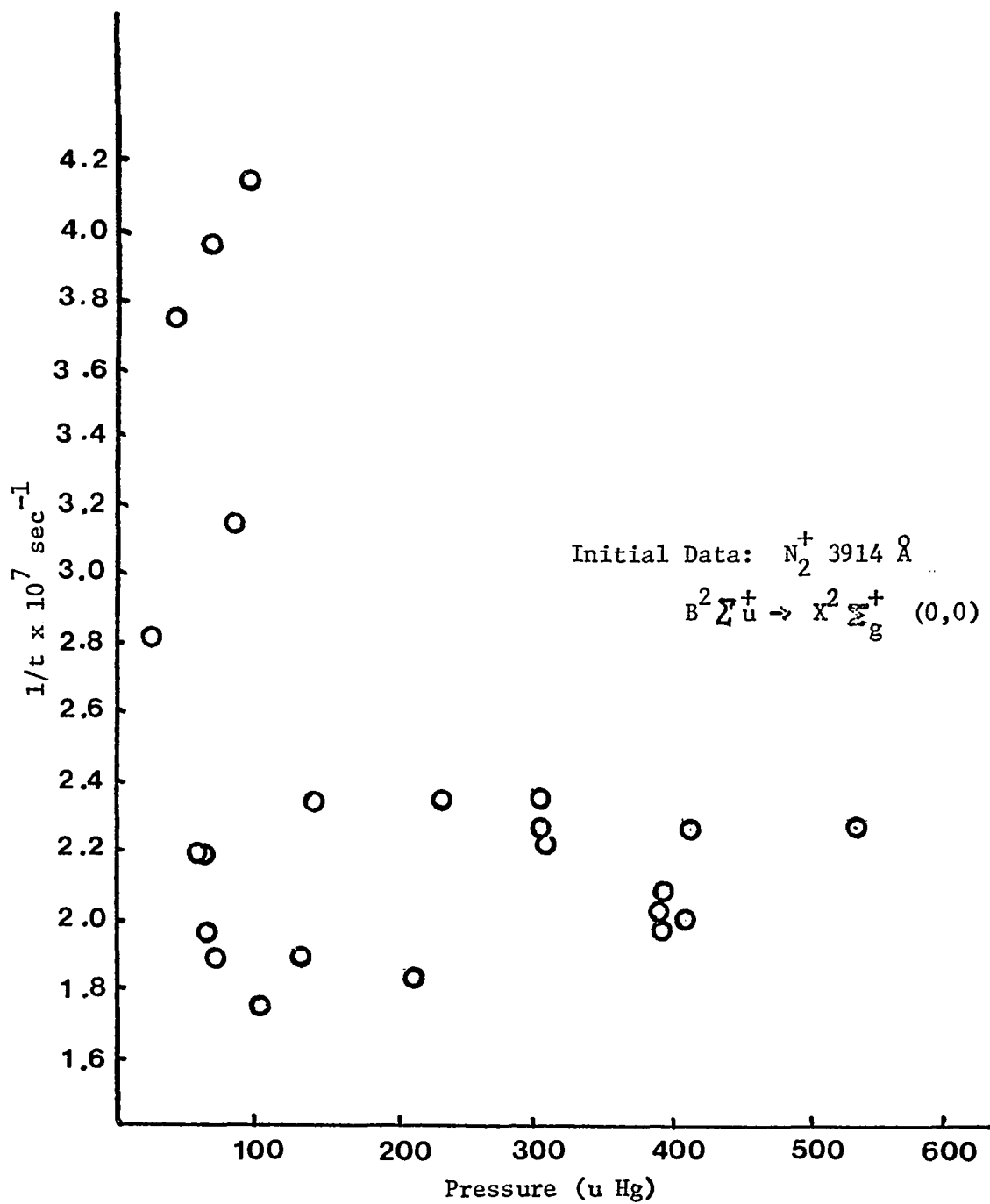


Fig. 38 Initial lifetime data for  $B^2 \Sigma_u^+ N_2^+$  ( $v'=0$ ).

was especially rampant in the earlier stages of these experiments since the excitation electronics system did not pulse at the repetition rates indicated by the monitor, i.e. the discharge tube did not fire every time it was pulsed. For example, if the repetition rate was set at 500 Hz, the discharge tube might fire anywhere from 200-300 Hz regularly, and at any other value sporadically. Also, the vacuum system, primarily the discharge tube, leaked. The boron nitride spacers were found to be quite porous. It was not clear if this was a problem of age, or if the spacers were worn out due to removal and reinsertion of the anode for cleaning. The leaking system was the easiest to remedy, the boron spacers were replaced with Teflon sleeves and subsequently the whole discharge tube was redesigned as described in Chapter II-D. The problem of the discharge tube misfiring was at first tolerated by inserting the Canberra scalers as shown in Chapter II-C and directly measuring the events counted vs the number of times the system fired (i.e. the count rate) for a given experiment. Later the difficulty was removed by properly modifying the excitation electronics to give dependable firing rates. (See Chapter II-B)

With these two corrections, our data were now much improved as Fig. 39 indicates. It was then determined that the data were of sufficient quality for the more subtle problems of the glow discharge to be approached. With the help of Fig. 40 a rather interesting problem was recognized. At low pressures it was found that both the  $B^2\Sigma_u^+$  and the  $C^3\Pi_u$  states of  $N_2^+$  and  $N_2$  exhibited an increase in their observed lifetimes which appeared at first glance to be equipment

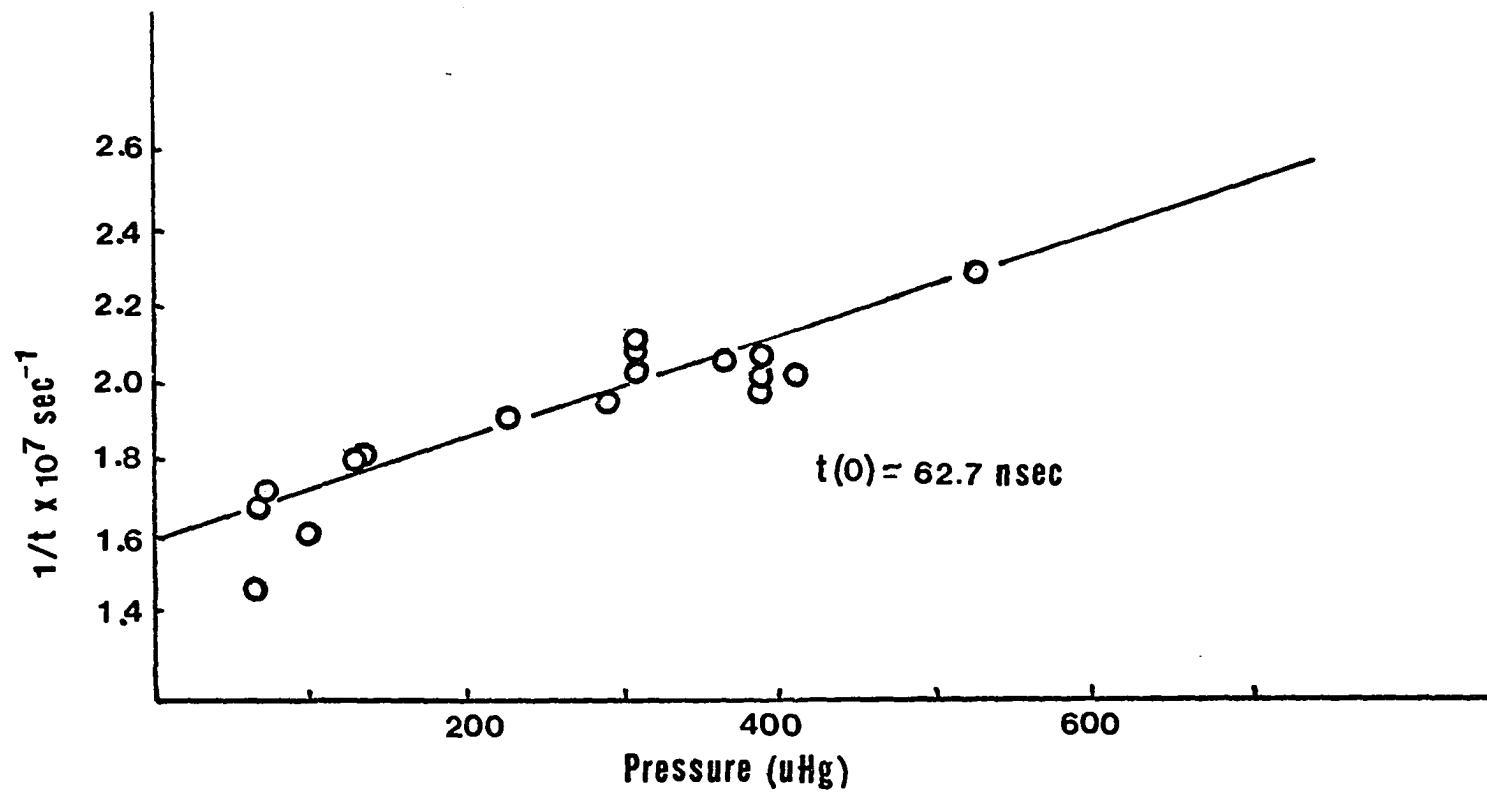


Fig. 39: Improved Lifetime Data for  $B \ ^2\Sigma_u^+ N_2^+ (v'=0)$

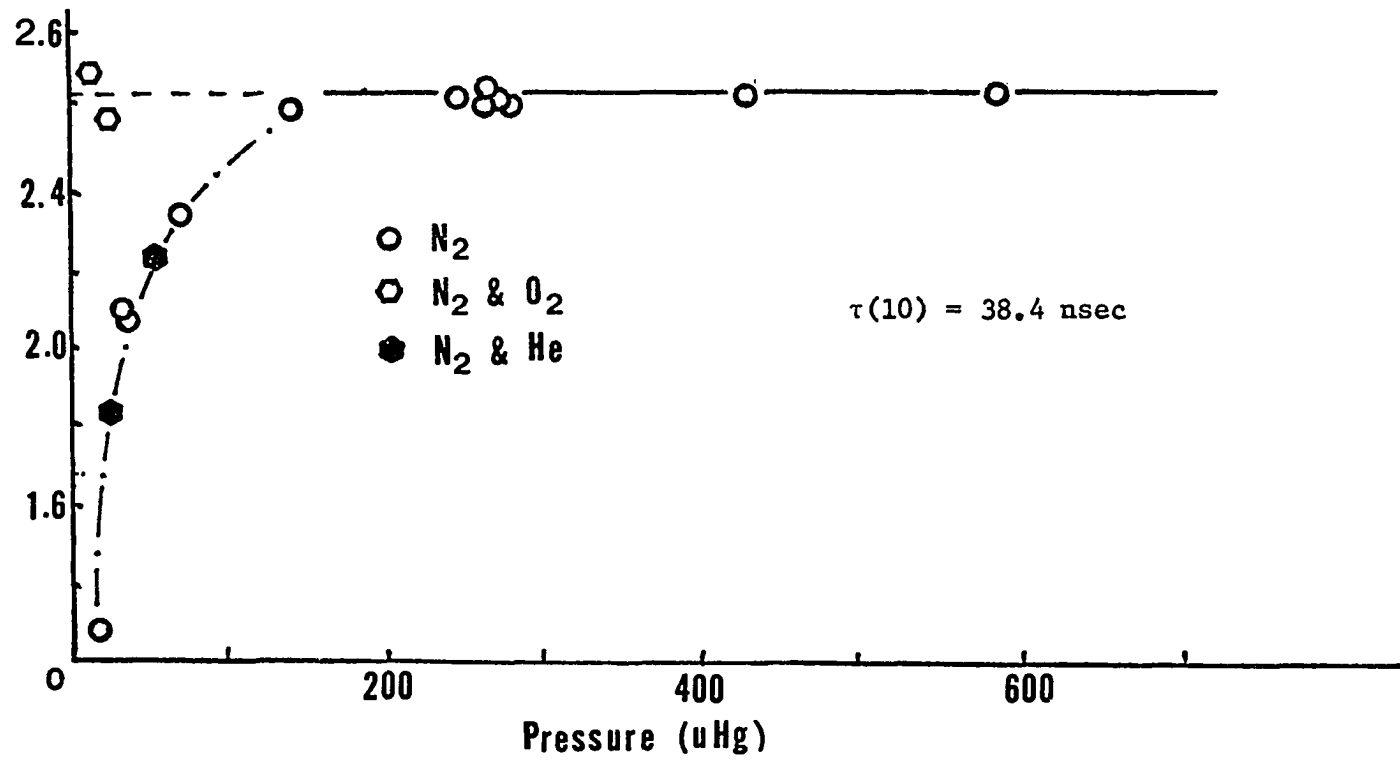


Fig. 40: Low Pressure Distortion of Lifetime Data of the C  $^3\Pi_u$  N<sub>2</sub> (v'=0)

oriented. As Fig. 40 indicates, this problem could be solved by the addition of small amounts of oxygen to the system, while the addition of helium did absolutely nothing. It was determined therefore that the pressure was not really the controlling variable, but rather the presence or absence of an electronegative gas capable of clearing the electrons out of the system after cut-off, in conjunction with an inability of the cathode structure to sweep these electrons.

This inability to sweep the field properly was also indicated by the probability decay curves which exhibited a "hump", and in severe cases, a peak in the lead to middle channels depending on the diameter and structure of the cathodes used as shown in Fig. 41. This was corrected after extensive cathode design studies were completed and cathode inserts of helix-grove or parallel-rod design were used (see Chapter II-E). These improvements then make possible the data shown in Fig. 42. Note however, that the problem at low pressures is still not completely solved. It probably cannot be solved while working with pure nitrogen as the attachment coefficient for pure nitrogen matches that of the noble gases...zero. However, studies in oxygen indicate that work at these lower pressures is indeed possible if the gas under study is electronegative.

Our attention was then turned to that of the magnetic field's influence on the observed results. The only effect observed with increasing field was an increasingly poor electrical cut-off of the discharge, and therefore an increase in the errors of data reduction. This effect was found to be random and it was therefore concluded merely that the magnetic field should be kept as low as necessary to sustain the discharge at low pressures (and off when possible)



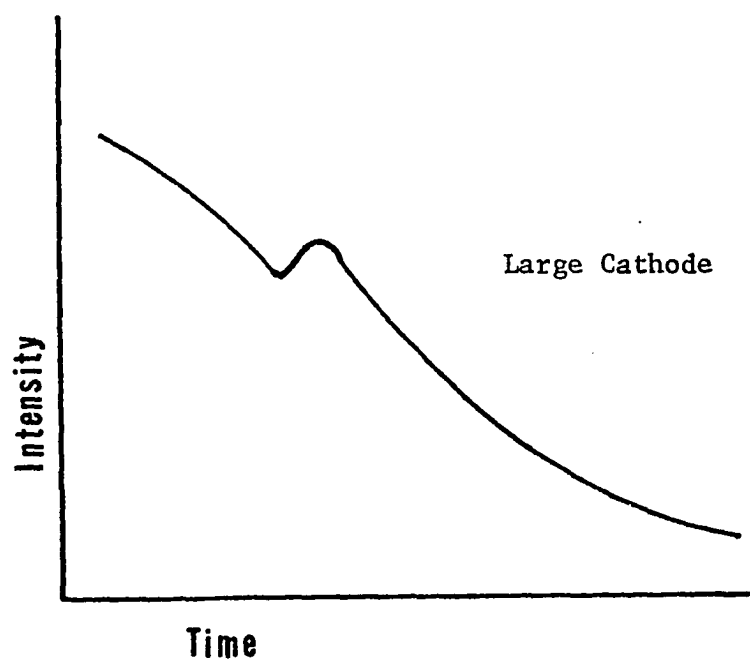
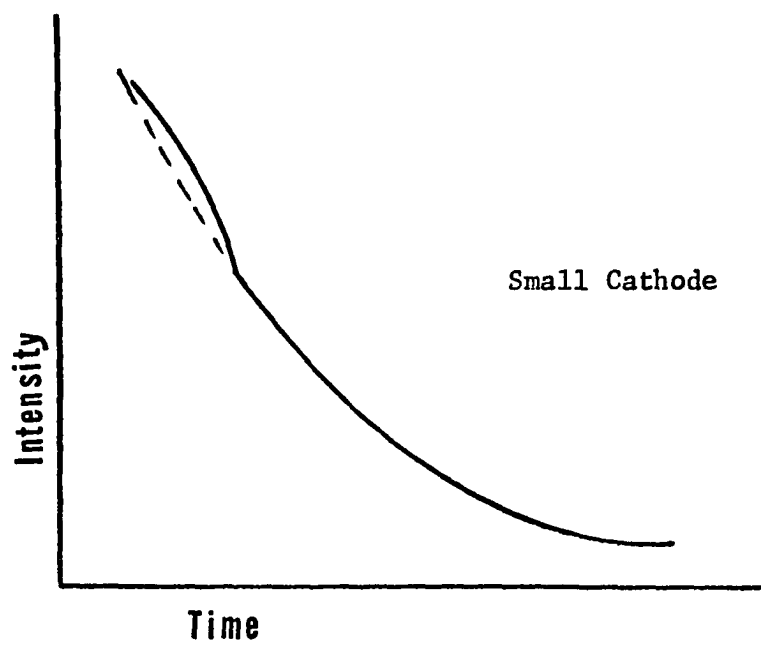


Fig. 41: Distorted Decay Curves Due to Improper Electron Quenching

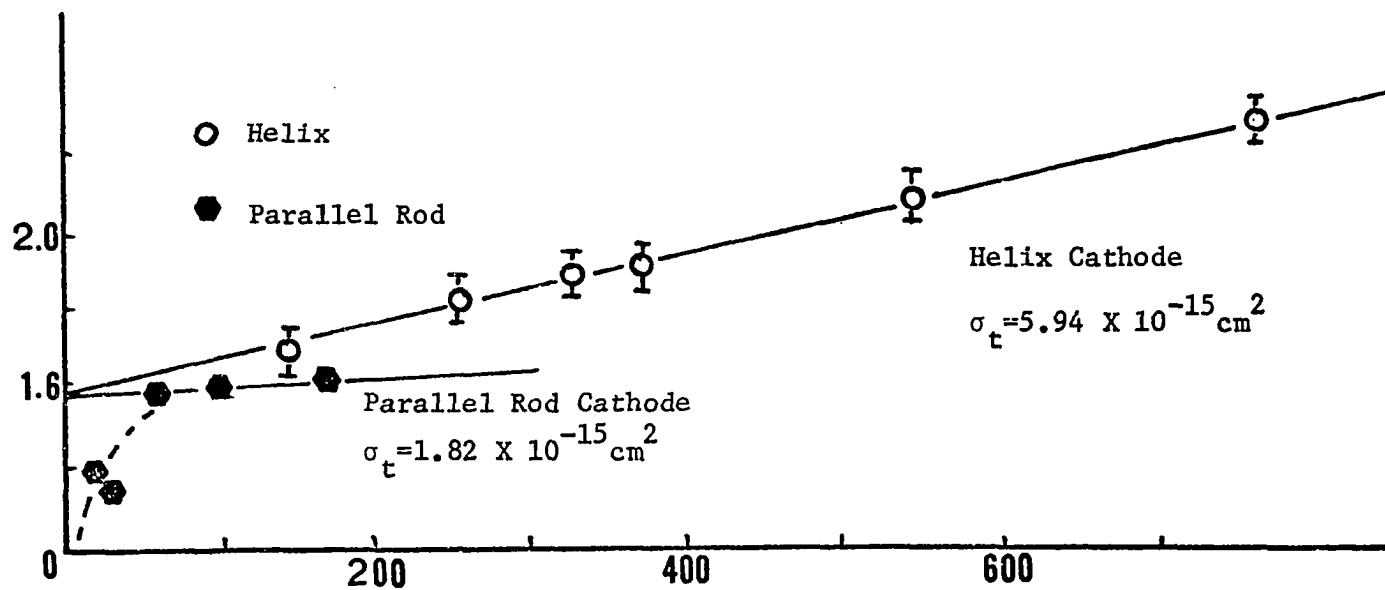


Fig. 42: Improved Lifetime Data Using the Helix and Parallel-Rod Cathodes

to reduce the errors in the data reduction.

A high applied voltage affected the results adversely in a manner similar to that of the magnetic field. At higher applied voltages larger numbers of electrons made cut-off difficult to achieve and consequently, the data reduction errors were increased. This occurred only when the applied voltages were sufficient to cause the discharge to exist in the abnormal glow region. If this was not the case, i.e. the glow remained in the normal region, the increased voltage did not effect the results appreciably.

In conclusion, Table II and Table III list the reported lifetimes for the  $B^2\Sigma_u^+$  and  $C^3\Pi_u$  states of  $N_2^+$  and  $N_2$  as reported by other workers for a comparison of results.

<u>Experimentalist</u>	<u><math>\tau</math> (Nanoseconds)</u>
Bennett & Dalby <sup>33</sup>	65.8 $\pm$ 3.5
Fowler & Holzberlein <sup>34</sup>	70 $\pm$ 15
Jeunehomme <sup>35</sup>	71.5 $\pm$ 5
Johnson <sup>3</sup>	59.2 $\pm$ 4
Hesser & Dressler <sup>36</sup>	59 $\pm$ 6
Nichols & Wilson <sup>37</sup>	65.9 $\pm$ 1
Sebacher <sup>38</sup>	65 $\pm$ 2
<u>This Work</u>	<u>62.7 <math>\pm</math> 2.5</u>

<u>Experimentalist</u>	<u>(sec.)</u>
Bennett & Dalby <sup>33</sup>	44.5 $\pm$ 6
Jeunehomme <sup>35</sup>	49 $\pm$ 5
Johnson <sup>3</sup>	39 $\pm$ 2.5
Nichols & Wilson <sup>37</sup>	45.4
<u>This Work</u>	<u>38.4 <math>\pm</math> 2.5</u>

## CHAPTER VI

### SO<sub>2</sub>

#### Introduction

Sulfur dioxide is a tri-atomic planar non-linear molecule belonging to the symmetry group  $C_{2v}$  and classed as an asymmetric top. This implies that the molecule SO<sub>2</sub> has two vertical planes of reflective symmetry,  $\sigma_v$  and  $\sigma_v'$ , and has a two-fold axis of cyclic or rotational symmetry,  $C_2$ . The planes of symmetry are referred to as vertical planes since by convention the main axis of symmetry is assumed to be the vertical (z) axis, and  $C_2$  refers to a rotation by 180° about the z-axis. The vertical planes  $\sigma_v$  are sometimes written  $\sigma_v(x,y)$  and  $\sigma_v(y,z)$  to denote in which plane the reflection is considered.

Since there are four different ways of assigning plus and minus signs to the two reflections, the point group is represented by the four characters  $A_1$ ,  $A_2$ ,  $B_1$ ,  $B_2$  which denote symmetry or anti-symmetry with respect to the operation  $C_2$ . The characters of this group are given below in Table IV.

Table IV:  $C_{2v}$  Character table

$C_{2v}$	I (or E)	$C_2(z)$	$\sigma_v(x,y)$	$\sigma_v(y,z)$
$A_1$	+1	+1	+1	+1
$A_2$	+1	+1	-1	-1
$B_1$	+1	-1	+1	-1
$B_2$	+1	-1	-1	+1

The notation I and E are interchangeable depending on the reference used and represent the identity element (E=Einheit), i.e.  $C_2^2 = I$  or E. The plus and minus signs refer to the preservation of the symmetry in a given operation.

As a matter of caution, the following notation will be observed to prevent confusion.  $A_1, A_2$  etc. will refer as mentioned to symmetry or antisymmetry operations with respect to the cyclic rotation  $C_n$ . This is not to be confused with  $A_u, A_g, B_u, B_g$ , (u = ungerade, g = gerade) which denote symmetry or antisymmetry with respect to the center of symmetry (as in the planar  $X_2Y_2$  of the group  $C_{2h}$ ), nor should these be confused with  $A'_1, A''_1$ , etc. which refer to symmetry or antisymmetry with respect to the plane  $\sigma_h$  (reflection in a plane perpendicular to the principle axis, i.e., horizontal reflection) as in a planar  $XY_3$  molecule from the group  $D_{3h}$ . Also, the symbol I should be used for the identity operator to avoid confusion with the degenerate species  $E_1, E_2$ , etc. found in higher order degenerate groups. One last word on notation, the symbol  $\tilde{A}$  or  $\tilde{X}$  refers to a particular molecular level and not to the symmetry species. Consequently, a state may be referred to as  $\tilde{X}^1A_1$  or  $\tilde{A}^3B_1$

without confusion. For further clarification, see ref. 39, 40.

Since a non-linear molecule does not have an axis about which the moment of inertia vanishes (as in linear molecules), there are three mutually perpendicular directions about which the moment of inertia has a maximum or a minimum.<sup>41</sup> These are termed the principle axes (or principle moments of inertia) and are designated  $\ell_A$ ,  $\ell_B$ ,  $\ell_C$  where generally  $\ell_A < \ell_B < \ell_C$ . If the molecule has an axis of symmetry then this axis will be a principle axis. Likewise, any plane of symmetry the molecule possess must have a principle axis perpendicular to it. If two of the principle moments are equal, the molecule is referred to as a symmetric top. If  $\ell_A = \ell_B$ , it is called an oblate top and if  $\ell_B = \ell_C$ , it is referred to as a prolate top. If all three are equal, it is called a spherical top. In the case of  $\text{SO}_2$ , where none of the three moments are equal, the molecule is termed an asymmetric top.

The vibration of tri-atomic molecules can be broken down into three normal modes of vibration  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  as shown in Fig. 43. These modes are termed symmetrical valance ( $\nu_1$ ), deformation ( $\nu_2$ ), and anti-symmetrical ( $\nu_3$ ) by Metropolis<sup>42</sup>. These vibrations cause what are termed vibronic species. These vibronic eigenfunctions  $\psi_{ev}$  are to a first approximation

$$\psi_{ev} = \psi_e(q,0)\psi_v(Q) \quad 43$$

where  $\psi_e(q,0)$  refers to the electronic wave function for the equilibrium position  $Q=0$ . This implies that the vibronic species are simply the direct products of the electronic species with the vibrational species. As an example consider an  $\text{SO}_2$  molecule ( $C_{2v}$  symmetry) which is in the  $B_1$  electronic state. If a  $B_2$  vibration is excited (i.e.)

similar to  $\nu_3$  in Fig. 43) the resultant vibronic state is

$$B_1 \times B_2 = A_2.$$

44

Degenerate vibrations will not be considered as they do not occur except by accident for molecules with only two-fold symmetry or less<sup>43</sup>.

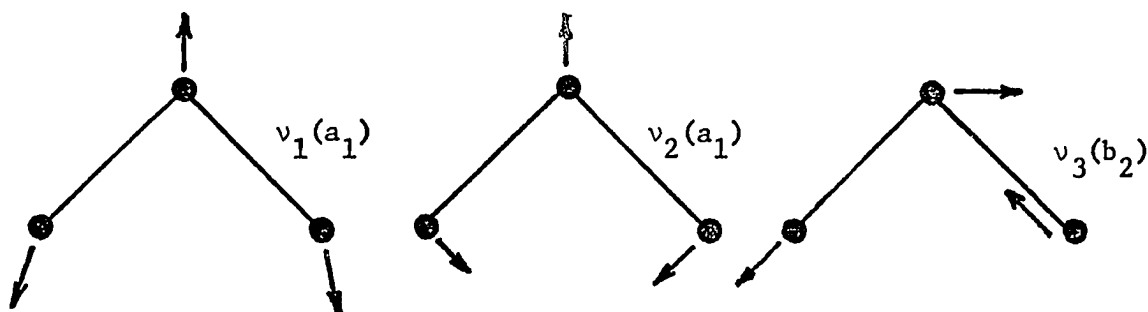


Fig. 43: Normal modes of vibration for  $\text{SO}_2$

### $\text{SO}_2$ SPECTRAL ANALYSIS

Pearse<sup>44</sup> has grouped the emission spectra of  $\text{SO}_2$  into three categories, all of which are degraded to the red. These are: group A, 4340 to 2700 Å; group B, 2640 to 2350 Å; and group C 2343 to 2170 Å. The work presented here has dealt exclusively with the second and third groups.

With the MCA used in the MS mode, the spectra illustrated in Fig's. 44-46 were obtained. Fig. 44 shows how important it is to keep out trace quantities of nitrogen. A multiple distillation using liquid nitrogen traps as described earlier (see Chapter II Sec. A)

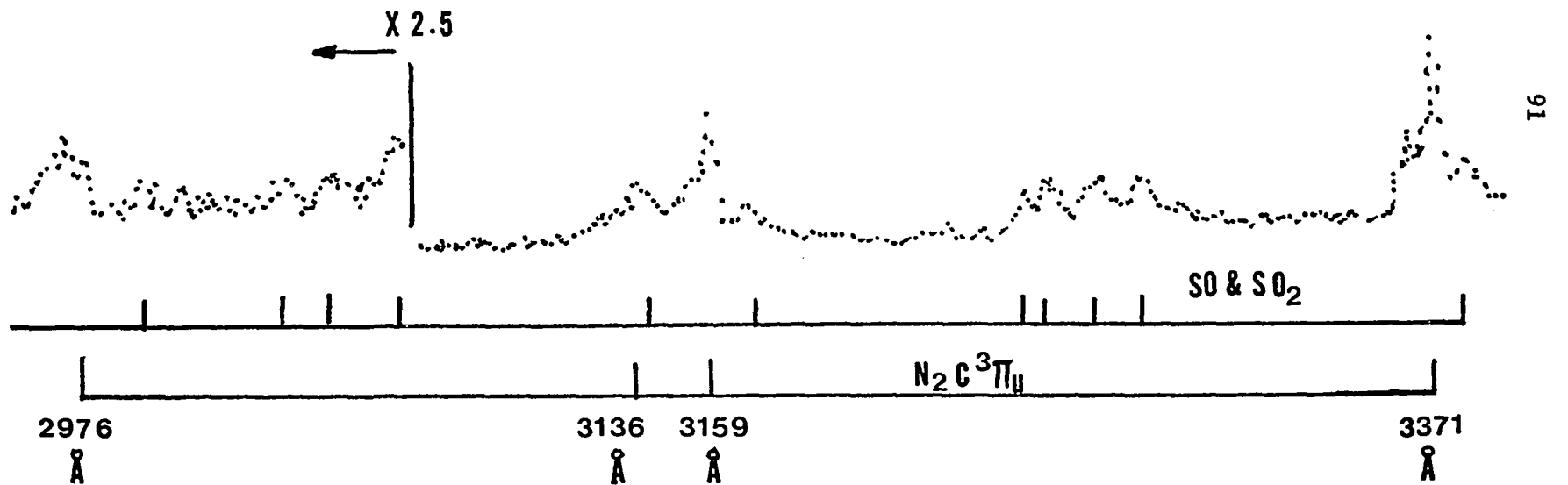


Fig. 44: SO<sub>2</sub> Spectra With Trace Nitrogen Contamination



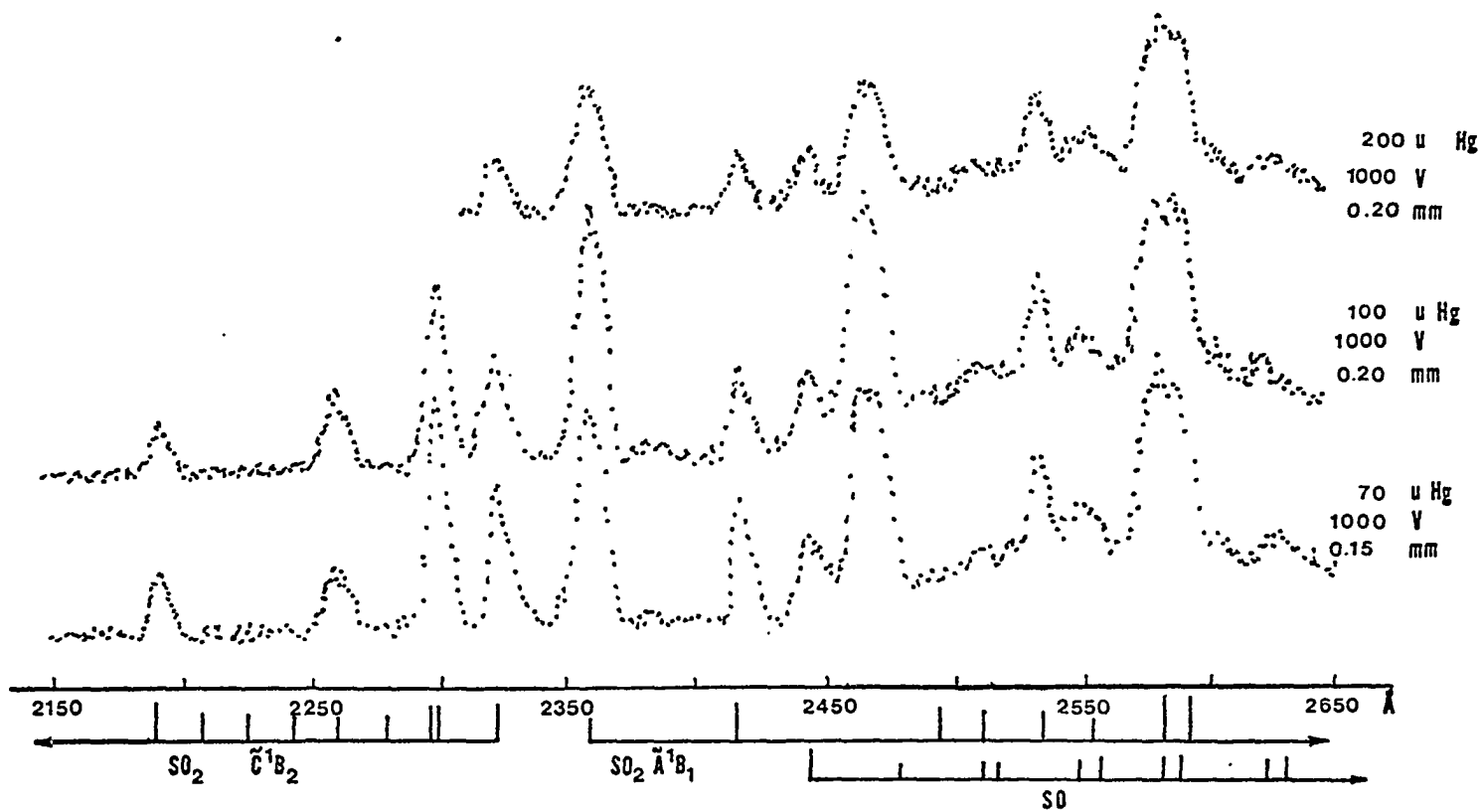


Fig. 45: SO<sub>2</sub> Spectra Showing Pressure Dependence

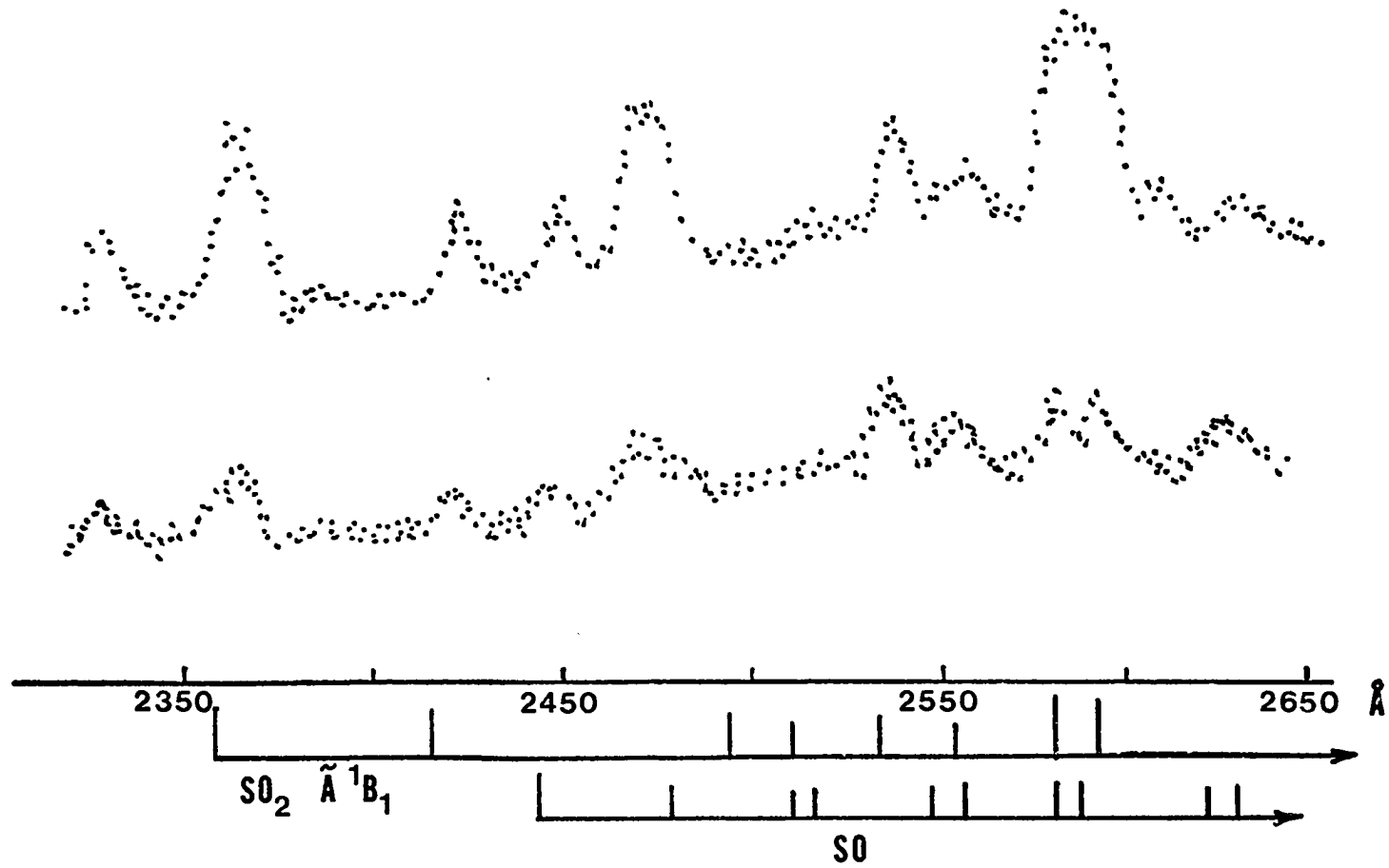
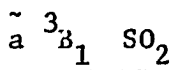


Fig. 46: SO<sub>2</sub> Spectra Illustrating Voltage Dependence

was used to eliminate these unwanted lines and subsequent spectra were free of  $N_2$  emission spectra. Fig. 45 and 46 illustrate the pressure and applied voltage dependence of the spectra. Please note the slit width change. It appears from Fig. 45 that at least two of the peaks, i.e. those centered around 2365 and 2263 Å, are not due to the emission of  $SO_2$  as they gain relative intensity with pressure with respect to the other peaks. Also, in Fig. 46, these peaks suffer severe attenuation with a drop in applied voltage of 200 volts. It is felt that these peaks are due to the dissociation of  $SO_2$  and the subsequent decay of excited atomic oxygen. This idea will be discussed later.

All of these spectra were obtained using a static gas sample, operating from 800 to 1000 volts. Very little sulfur deposition was observed except at high pressures (400+  $\mu$ Hg). The MCA was operated in the MS mode with a dwell time of 0.8 seconds per channel and 256 channels were used on each sweep.

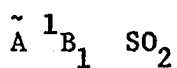
PREVIOUS LIFETIME WORK



Previous lifetime determinations<sup>45-47</sup> have associated the region of group A with the first excited state, designated  $\tilde{a} \ ^3B_1$ . I determined this level to be very long lived (>200 $\mu$ sec) and consequently could not study it further. However, for completeness and to help illustrate the inconsistencies in the reported lifetimes for this molecule, the known work for this level is presented below in Table V.

Table V: Reported lifetimes for  $\tilde{A}^3B_1$  SO<sub>2</sub>.

Worker	Pressure	Lifetime
Caton & Duncan <sup>45</sup>	30 $\mu$ - 1.5 mm (no pressure dep. found)	7 $\pm$ 1 msec
Levitt & Sheen <sup>46</sup>	not stated in work (a few torr?)	0.2 msec.
Greenough & Duncan <sup>47</sup>	solid at 77 <sup>o</sup> K.	0.5 msec.



The second excited state designated  $\tilde{A}^1B_1$  from the electron configuration  $(1a_2)^2(3b_2)^2(4a_1)(2b_1)^1,^3B_1$ , has been reported to have very diverse lifetimes and even to possess a non-exponential decay. Mettee<sup>48</sup> alone presents a lifetime varying from 12  $\mu$ sec to 128  $\mu$ sec. Greenough and Duncan give a lifetime of 45  $\mu$ sec, while Levitt and Sheen<sup>46</sup> list a value of only 7  $\mu$ sec. In his work, Sidebottom<sup>49</sup> found a nonexponential decay which had a time dependent decay varying from 14 to 33  $\mu$ sec, and Hui and Price<sup>50</sup> simply state that the decay is non-exponential for the  $^1B_1$  state. Several arguments have been put forth by the collective authors as to why their data do, or do not, appear to agree. Mettee for example feels that perhaps his frequency dependent lifetimes should all have the value of 64  $\mu$ sec and the variations are due to a frequency dependent excitation cross section. However, a correlation does exist which these authors seem to overlook. Table VI presents their data and Fig. 47 is a lifetime versus wavelength plot of this data on semi-log paper.

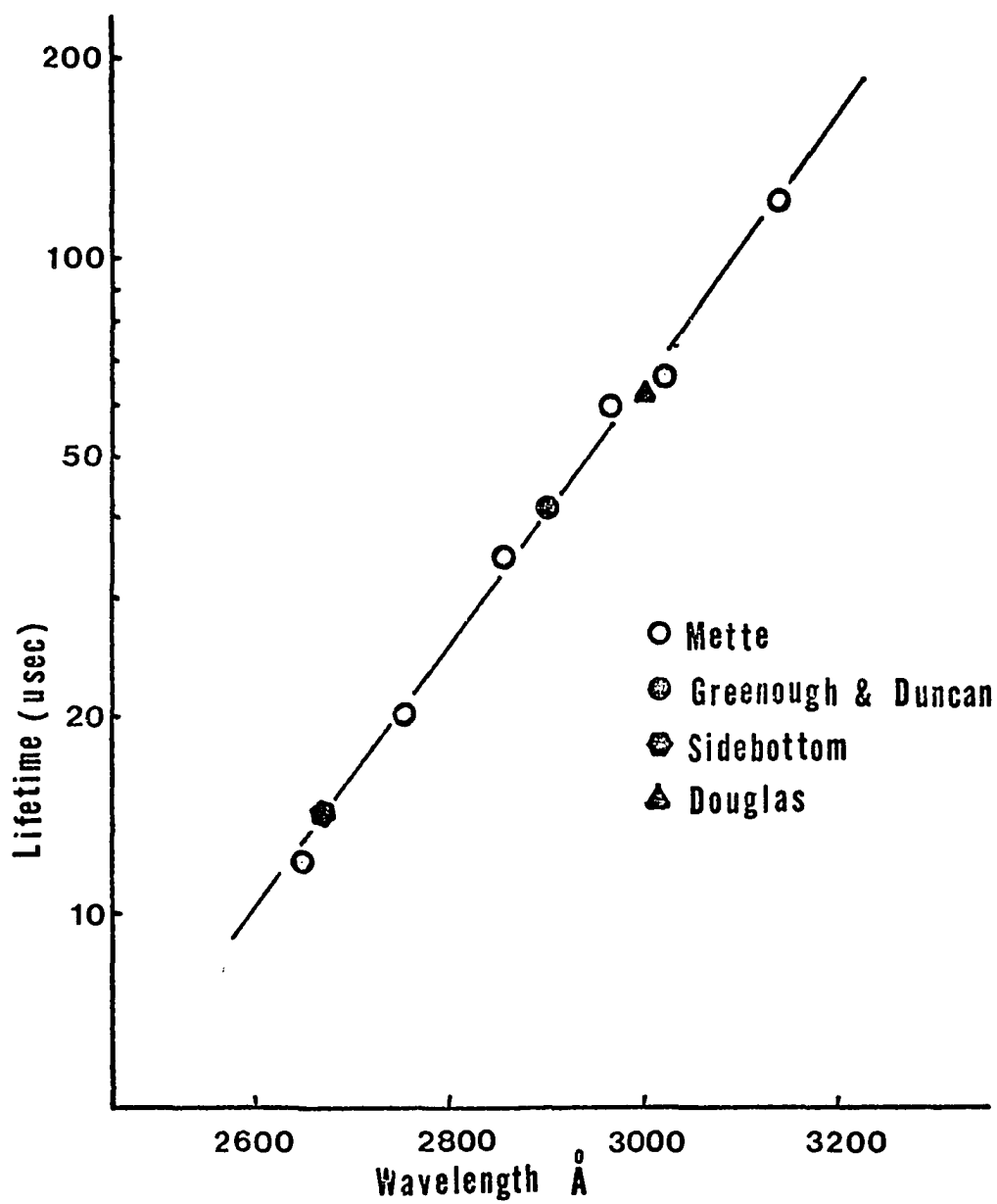


Fig. 47: Lifetime VS Wavelength for the  $\tilde{A}^1B_1$   $SO_2$

Table VI: Reported lifetimes for  $\tilde{A}^1B_1 SO_2$ 

Worker	Wavelength	Pressure	Lifetime
Levitt & Sheen <sup>46</sup>	4360(?)	not stated	7 usec
Greenough & Duncan <sup>47</sup>	2700-3100	solid at 77°K	42 usec
Mettee <sup>48</sup>	3130	1--1000 $\mu$ Hg	128 usec
	3020		68
	2960		60
	2850		35
	2750		20
	2650		12
Sidebottom <sup>49</sup> (Laser)	2662	2- 80 $\mu$ Hg	33 usec
			14 usec
			36 $\pm$ 4 usec 18 $\pm$ 6 usec
		calculated values	
Douglas <sup>57</sup>	3000	?	63 usec

The double value given by Sidebottom refers to the non-exponential nature of the observed decay and is an attempt to analyze the apparent time dependency of the decay. His calculated values correspond to levels which are partially and totally vibrationally equilibrated. The values stated for Douglas is based on his statement that his lifetime was approximately 50% higher than that of Greenough and Duncan.

Fig. 47 would seem to be more than a coincidence. This is obviously not due to the  $v^3$  factor and can perhaps be explained by considering interelectronic level mixing. Douglas<sup>51</sup> has stated four requirements for this type of effect to occur which are:

- 1) A strong electronic transition moment existing between levels  $\tilde{B}$  and  $\tilde{X}$  (see Fig. 48).

- 2) A weak or no electronic transition moment existing between levels  $\tilde{A}$  and  $\tilde{X}$ ,
- 3) A strong interaction existing between  $\tilde{A}$  and  $\tilde{B}$ , i.e., vibronic or Coriolis, or if they have different multiplicities, strong spin-orbit interactions,
- 4) Density of vibrational levels of  $\tilde{A} \gg \tilde{B}$ .

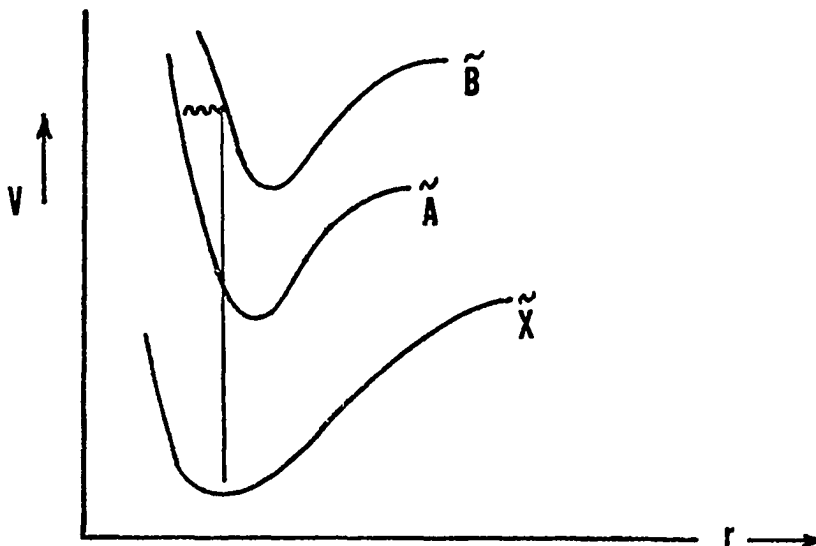


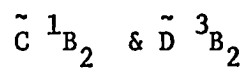
Fig. 48: Schematic potential wells for interelectronic level mixing.

This type of system will add large numbers of extra vibrational levels which can be reached by absorbing radiation from the ground state, i.e. several levels from  $\tilde{A}$  may be reached by one level from  $\tilde{B}$ . This will produce a spectrum in which a large number of weak lines are seen instead of a few strong lines. This in turn increases the apparent degeneracy, alters the line strengths and consequently increases the observed lifetime. The degree of mixing will obviously depend on the extent of the interaction and on the type of interaction

present. In the limit of large interaction, Douglas states that the increase in the observed lifetimes will vary as the density of the levels  $\tilde{A}$  and  $\tilde{B}$ .

This criterion seems applicable to  $\text{SO}_2$ . The  $\tilde{B} (\tilde{A}^1B_1) \rightarrow X(X^1A_1)$  transition is quite strong while the  $\tilde{A} (\tilde{a}^3B_1) \rightarrow \tilde{X}(\tilde{X}^1A_1)$  transition involves a different multiplicity and are reportedly very weak. This multiplicity difference could also provide the interaction necessary (spin-orbit for example) between  $\tilde{a}$  and  $\tilde{A}$ .

One other effect is mentioned by Douglas. If the interaction is fairly weak and if it varies with the rotational and vibrational quantum numbers of the excited state, then even closely spaced levels may differ in their observed lifetimes. This implies that the decay of fluorescence excited with even a narrow band of radiation (Sidebottom's LASER) will be a mixture of the various lifetimes and will be observed as non-exponential. Sidebottom<sup>49</sup> and Hui<sup>50</sup> report such decays for the  $\tilde{A}^1B_1$  level of  $\text{SO}_2$ . Perhaps the graph in Fig. 47 will be useful in determining the extent and type of interaction between the levels  $\tilde{a}^3B_1$  and  $\tilde{A}^1B_1$  of  $\text{SO}_2$  in the future.



The last reported states of  $\text{SO}_2$ , the  $\tilde{C}$  and  $\tilde{D}$  states have been reported on by Hui and Price<sup>50</sup>, who used single photon counting techniques and concluded that these states were not as strongly coupled to any other states as were the lower levels. An extrapolation of their data reveals a zero pressure lifetime for the composite system of  $55 \pm 8$  nsec.



PRESENT LIFETIME WORK

The first excited level of  $SO_2$ , the  $\tilde{a} \ ^3B_1$  state was not measured in this work as the levels are too long lived to allow accurate work with the present equipment. It is hoped that the level can be studied in the future after modification of the apparatus and will be reported on in the future elsewhere.



The  $\tilde{A} \ ^1B_1$  state has been observed in a flowing gas system at several frequencies to determine if the same frequency dependence was present as was reported earlier. It was not. However a reciprocal transfer or close coupling of near states was observed as is apparent in Fig. 49. A complete theoretical description of this process may be found in Morton<sup>7</sup> and in Thompson<sup>5</sup>. Essentially, the process is as follows (see Fig. 50).

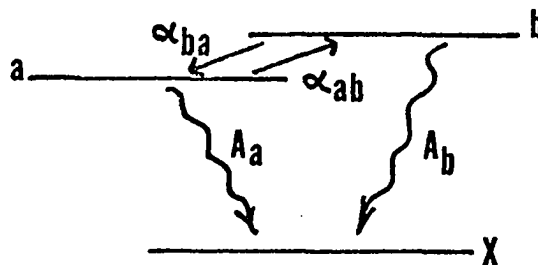


Fig. 50: Energy level diagram for close coupling of states.

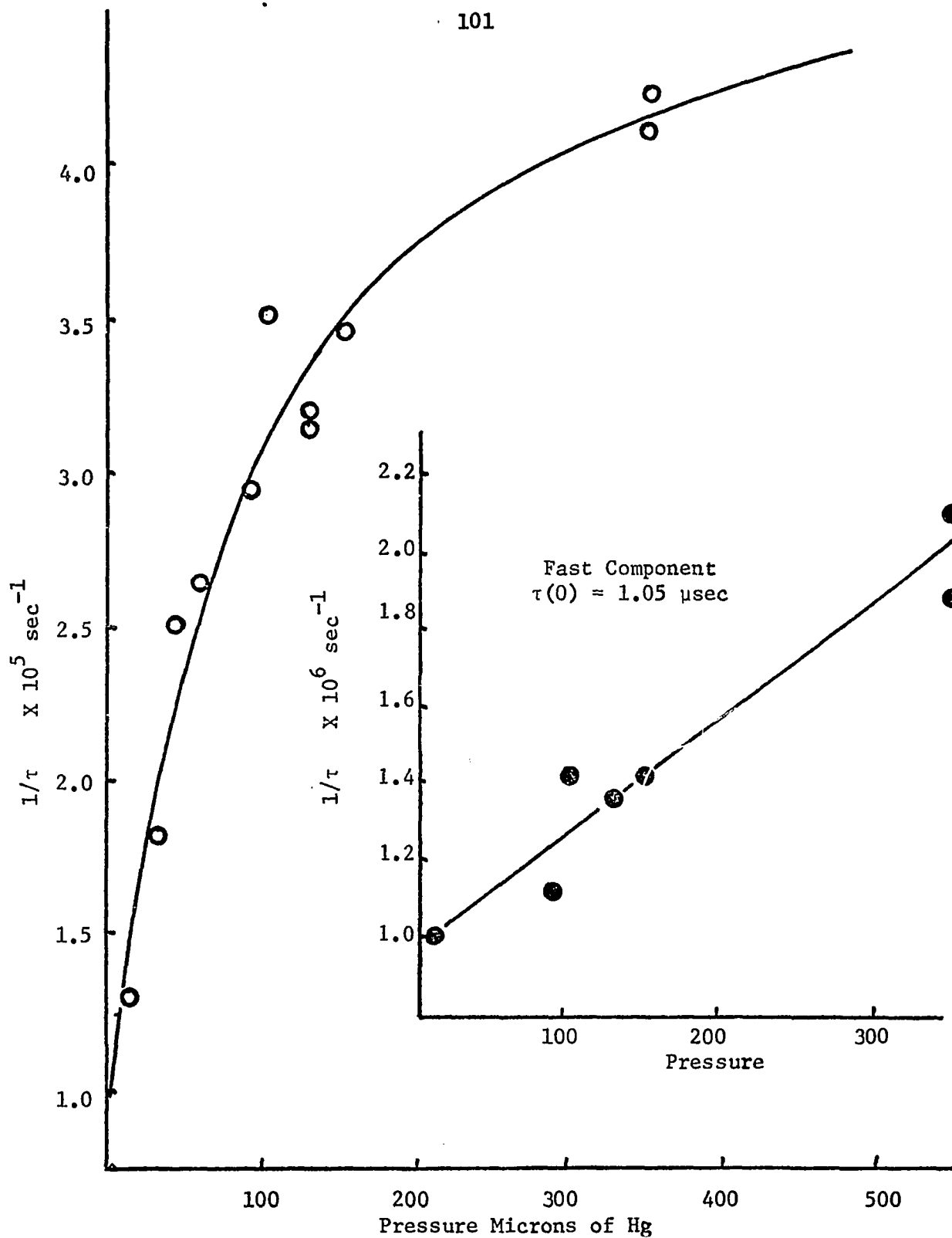


Fig. 49: Inverse Lifetime VS Pressure for  $\tilde{A}^1B_1 \text{SO}_2$

Levels a and b are nearly the same energy above the ground state and allow a collisional resonance, i.e.,

$$\frac{dN_a}{dt} = -\alpha_a N_a + \alpha_{ba} N_b + P_a \quad 45a$$

$$\frac{dN_b}{dt} = -\alpha_b N_b + \alpha_{ab} N_a + P_b \quad 45b$$

where  $P_a$  and  $P_b$  are production rates important only during the discharge on-time, and where

$$\alpha_a = A_a + \sigma_a \bar{v}N + \sigma_{ab} \bar{v}N, \quad 46a$$

$$\alpha_b = A_b + \sigma_b \bar{v}N + \sigma_{ba} \bar{v}N, \quad 46b$$

$$\alpha_{ab} = \sigma_{ab} \bar{v}N. \quad 46c$$

The basic solution to these equations is a double exponential (where after cut-off we have  $P_a = P_b = 0$ ), i.e.,

$$N_a = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} \quad 47$$

and

$$N_b = -\frac{\alpha_a + \alpha_b}{\alpha_{ba}} C_1 e^{-\lambda_1 t} + \frac{\alpha_a - \alpha_b}{\alpha_{ba}} C_2 e^{-\lambda_2 t} \quad 48$$

where

$$\alpha^1 = (\alpha_b^2 - 2\alpha_b \alpha_a + \alpha_a^2 + 4\alpha_{ab} \alpha_{ba})^{1/2}$$

As we are interested in the population of  $N_a$  only ( $N_b$  will decay at a different frequency) and we expect  $\lambda_1 \gg \lambda_2$  such that  $\lambda_1$  will be an unobservably rapid buildup in the amplitude of the state of interest, we may consider the exponential form of  $\lambda_2$ ;

$$\lambda_2 = \frac{1}{2} \{ A_a + A_b + (\sigma_a + \sigma_b + \sigma_{ab} + \sigma_{ba}) \bar{v}N \} - \frac{1}{2} \{ [A_a - A_b + (\sigma_a - \sigma_b + \sigma_{ab} - \sigma_{ba}) \bar{v}N]^2 + 4\sigma_{ab} \sigma_{ba} \bar{v}^2 N^2 \}^{1/2} \quad 49$$

as being responsible for the observed curve in Fig. 49. The solid line in Fig. 51 represents the analytic fit of  $\lambda_2$  to the observed data. The parameters used in the fit were:

$$\begin{aligned} A_a &= 5.98 \times 10^5 \text{ sec}^{-1} \\ A_b &= 0.8 \times 10^5 \text{ sec}^{-1} \\ \sigma_a &= \sigma_b = 9 \times 10^{-16} \text{ cm}^2 \\ \sigma_{ab} &= 7.5 \times 10^{-15} \text{ cm}^2; \sigma_{ba} = 7.0 \times 10^{-15} \text{ cm}^2 \\ v &= 3.18 \times 10^4 \text{ cm/sec (T = 300}^\circ \text{ K)} \\ N &= 8.05 \times 10^{15} \text{ p (torr)/cc} \end{aligned}$$

It is felt that this process is similar to the interelectronic mixing apparently responsible for the frequency dependency observed by previous workers working in the absorption region.

This level did exhibit a double exponential decay; however, the fast component ( $\sim 1 \mu\text{sec}$  decay) observed is believed<sup>53</sup> to be due to the underlying recombination continuum caused by  $\text{SO} + \text{O} \longrightarrow \text{SO}_2 + h\nu$ , and not to be the level in collisional resonance with the  ${}^1\text{B}_1$  level since the zero pressure lifetime for such a collisional state should be around  $1.7 \mu\text{sec}$ . Fig. 52 shows a schematic representation for such a double exponential system and illustrates why the mixed state should be around  $1.7 \mu\text{sec}$ .

The intercept of the dashed lines should be half the sum of the decay probabilities (high pressure extrapolation of the terms  $\lambda_1$  and  $\lambda_2$ , where  $\lambda_1$  differs from  $\lambda_2$  only in the sign of the second term).

Table VII lists the wavelengths of the observed levels of the  $\tilde{\text{A}} {}^1\text{B}_1$  state of  $\text{SO}_2$ . The measurements made in this work would then indicate a zero pressure lifetime of  $12.5 \pm 2.5 \mu\text{sec}$  which is in good

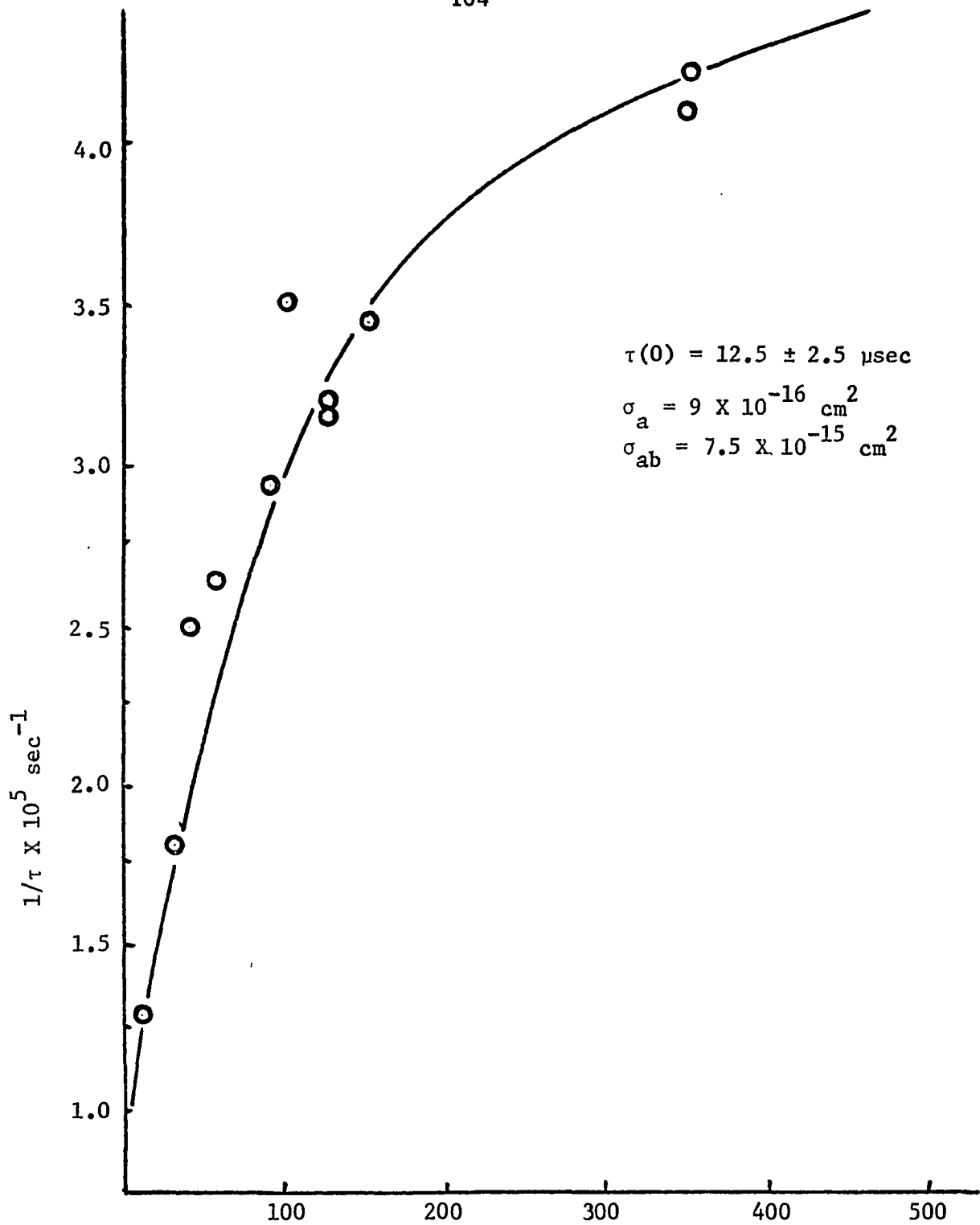


Fig. 51: Analytic Fit to inverse Lifetime VS Pressure Curve  
 $\tilde{A}^1B_1 \text{ SO}_2$

agreement with the lowest level observed by Mettee<sup>48</sup> and with the faster value (14  $\mu\text{sec}$ ) listed by Sidebottom. Recall that Sidebottom calculated a value for the lifetime of 18  $\mu\text{sec}$  for a vibrationally equilibrated system. It would perhaps be safe to assume that the level measured by Mettee at 2650  $\text{\AA}$  was also in vibrational equilibrium, and by the nature of the excitation used in these experiments, our levels should also be in vibrational equilibrium.

Table VII:  $A^1B_1$  levels of  $\text{SO}_2$

WAVELENGTH ( $\text{\AA}$ )	PRESSURE	LIFETIME ( $\mu\text{sec}$ )
2593		
2561		
2493	15 -400 u Hg	12.5 $\pm$ 2.5
2415	flowing	
2383		
2263		

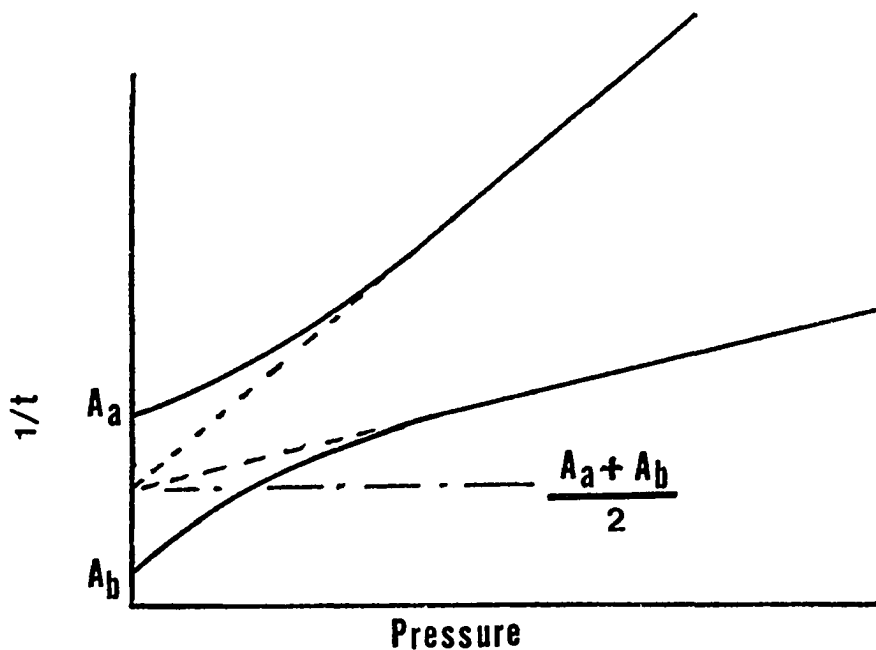


Fig. 52: Decay of collisionally resonance states.

$\tilde{C}$  and  $\tilde{D} \ ^1,3B_2$

The  $\tilde{C}$  and  $\tilde{D}$  states of  $SO_2$  ( $\ ^1,3B_2$ ) proved to be a straight forward measurement with results in good agreement with those of Hui and Price<sup>50</sup>. A fast single exponential was observed with a linear pressure dependence indicating a total collisional depopulation cross section of  $\sigma_t = 2.72 \times 10^{-15} \text{ cm}^2$ . Fig. 53 is a reciprocal lifetime vs pressure curve for the  $\tilde{C}$  state which indicates a zero pressure lifetime of  $60.6 \pm 1.8 \text{ nsec}$ . This is a count rate corrected curve. This value compares well to that extrapolated for Hui and Price of  $55 \pm 8 \text{ nsec}$ . This level did not exhibit any frequency dependence similar to that seen for the  $\tilde{A}$  state.

#### OII LINES

The oxygen lines observed in this work have been studied elsewhere and are mentioned here only because of the quadratic pressure dependence they seem to possess. Three ionized oxygen lines have been observed, the  $3p \ ^2D_{3/2}^o - 3d \ ^1P_{1/2}$  (2365.15 Å) the  $3p \ ^2D_{3/2}^o - 3d \ ^1P_{3/2}$  (2365.03 Å) and the  $3p \ ^2P_{3/2}^o - 5s \ ^2P_{1/2}$  (2263 Å) with lifetimes ranging from 5.3  $\mu\text{sec}$  at 600  $\mu \text{ Hg}$  to 1.18 msec at 30  $\mu \text{ Hg}$ . This data is presented in Fig. 54. Note that the data is quadratic in pressure. This would tend to indicate a three body collision, the molecule  $SO_2$  and two slow electrons with excited ionized atomic oxygen as one of the products. We then observe the decay of the OII atom not at its characteristic decay rate, but at the rate of production of the ions.

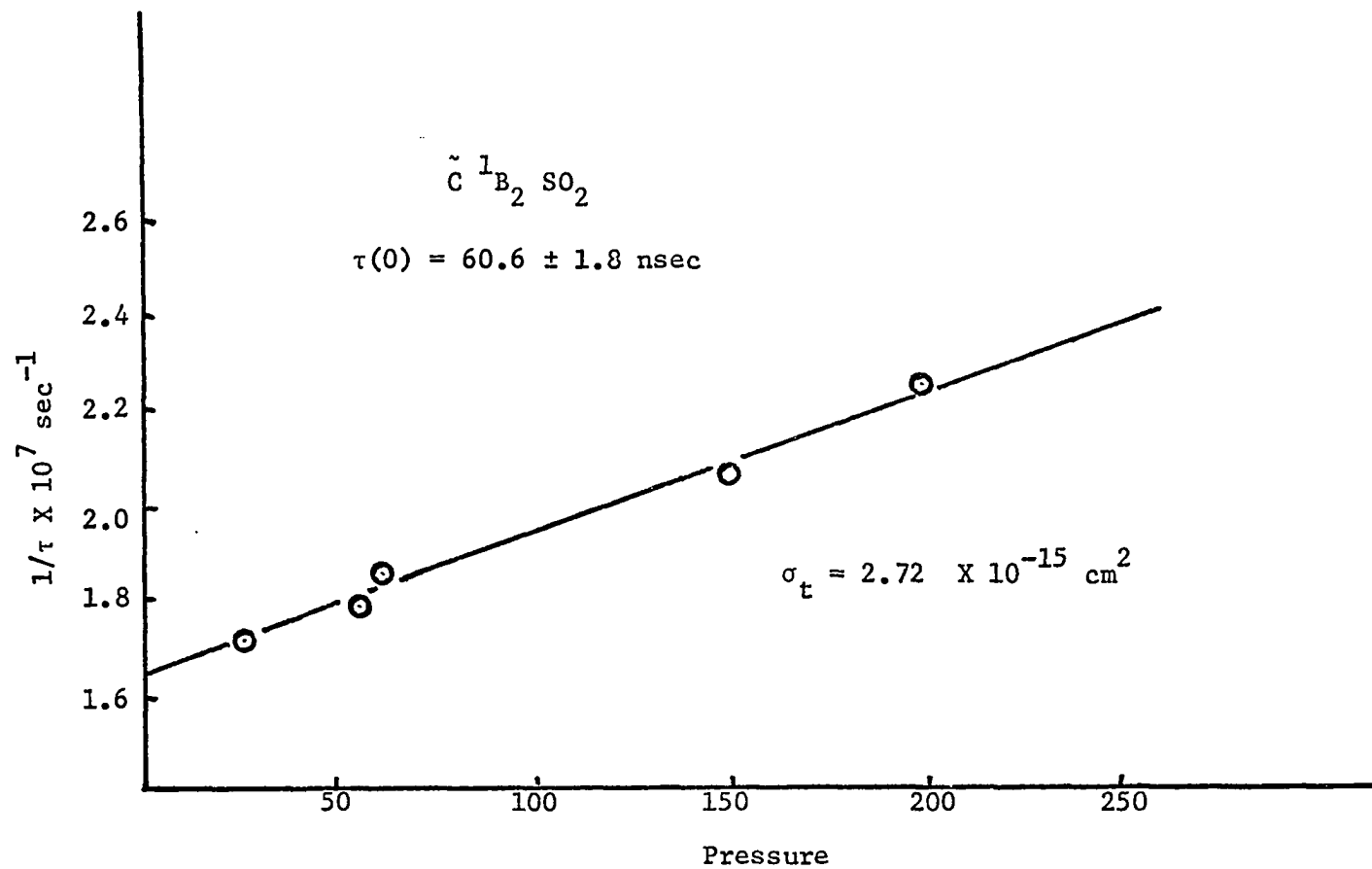


Fig. 53: Inverse Lifetime VS Pressure Curve for  $\tilde{C}^1B_2$  of  $\text{SO}_2$



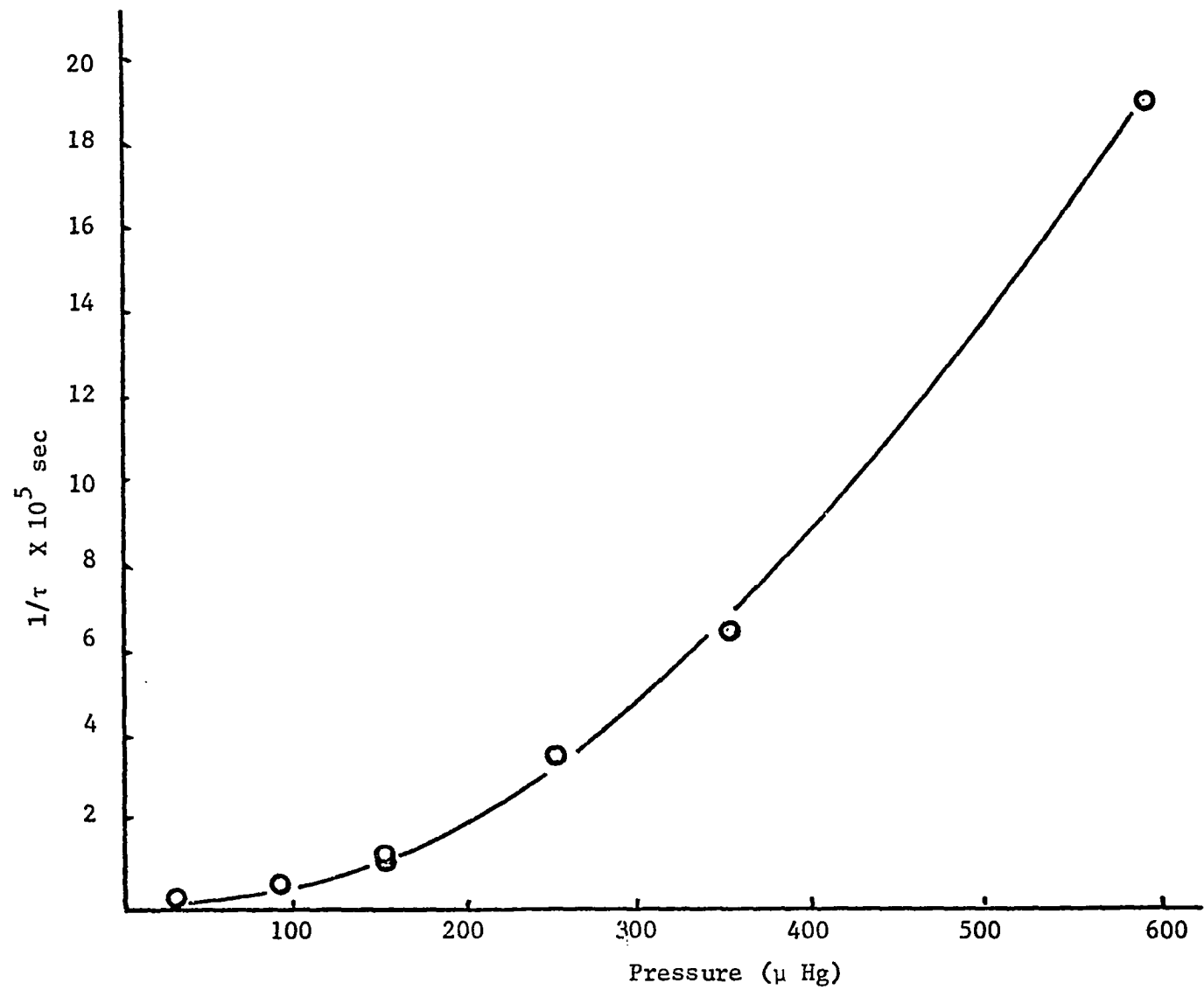


Fig. 54. Quadratic Pressure Dependence Of Recombination OII (2365 and 2265 Å )

## CHAPTER VII

### CONCLUSIONS

It is the opinion of the author that with a little care and patience, the cold cathode invertron can be a very useful tool in examining the lifetimes and related parameters of polyatomic molecules. It has been shown to be accurate in reproducing the known lifetimes of He, OI and N<sub>2</sub> which were initially determined on entirely different equipment, some determinations even being made on equipment using entirely different concepts of physics.

The observed lifetimes of SO<sub>2</sub> are well within the experimental errors reported by some (i.e., the  $\tilde{C}$  and  $\tilde{D}^{1,3}B_2$ ) and are consistent with the reported values of theirs ( $\tilde{A}^1B_1$ ), while improving the precision of both considerably ( $\pm 3\%$  as apposed to their  $\pm 15\%$ ). By combining the results obtained through the absorption spectra and that obtained in emission, it is felt that a better understanding of the molecule has been achieved. Also, data has been collected which perhaps in the future will help determine the extent and the type of vibrational and interelectronic level mixing.

Future work is still required in the visible region of SO<sub>2</sub> and will be completed after modification of the apparatus to allow millisecond investigations. At present the lifetime of the  $\tilde{a}^3B_1$  excited level of SO<sub>2</sub> is too long for accurate measurement on the existing apparatus. Finally a thorough investigation of the visible and perhaps the infrared regions will reveal the state which appears to

be close coupled with the  $\tilde{A}^1B_1$  level (assuming that it is not the excited state  $\tilde{a}^3B_1$ ).

This equipment should now be useful in measuring lifetimes for other easily dissociated polyatomic molecules.

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## APPENDIX A

## LISTING OF COMPUTER PROGRAMS AND SUBROUTINES

LFLOT 20:08 THU. FEB 05, 1974

```
DIMENSION Y(512),A(5),YP(512)
DIMENSION CHAN(10),TIM(10)
REAL INCR,L1,L2
DATA A/"X", "+", ".", " ", "*"/
C DATA INPUT
CALL INPAK(N,NTC,CHAN,TIM,TIME,Y)
PRINT1234,
1234 FORMAT(1H,"ENTER PLOT INCREMENT")
1 INPUT ,K
IF (K)2,2,3
2 PRINT999
CALL EXIT
C HEADING
3 CONTINUE
C Y VALUES TO BE PLOTTED
9201 FORMAT(1H,2A6,3X,"DATE-",A2,A3,A4,3X,"RUN #",I2)
C SEARCH FOR MAXIMUM AND MINIMUM VALUES
B=Y(1)
S=10000.
DO 20 I=2,N
IF (Y(I)-B)15,15,11
11 B=Y(I)
GO TO 20
15 IF (Y(I)-S)16,20,20
16 IF(Y(I).EQ.0.0)GO T020
S=Y(I)
20 CONTINUE
C CALCULATION OF SCALING FACTOR AND RANGE
PRINT 910,S,B
INPUT,S,B
PRINT 201,S,B
B=ALOG(B)
S =ALOG(S)
SC=(B-S)/63.0
C SET UP BORDER AND PRINT SCALE
PRINT903
PRINT 902,(A(5),I=1,65)
C PLOTTING OF DATA
DO 30 I=1,N,K
I1=I-1
NY=1
IF(Y(I)-S.LE.0.0)GO TO 30
NY=(ALOG(Y(I))-S)/SC+1
DO 21 J=1,NY
21 YP(J)=A(4)
```

L PLOT 20:08 THU. FEB 05, 1974

```
      YP(NY)=A(2)
      30 PRINT 293,I1,(YP(J),J=1,NY)
C FINAL BORDER
      PRINT 902,(A(5),I=1,65)
      PRINT 202,SC
      GO TO 1
      902 FORMAT(1H ,3X,65A1)
      201 FORMAT(1H1,F8.0,52X,F8.0)
      202 FORMAT(1H0,"SCALE=",F8.5,"LN(COUNTS)/PRINT CHANNEL")
      293 FORMAT(I4,"*",64A1)
      903 FORMAT(1H ,3X,"*",63X,"*")
      910 FORMAT("ENTER SCALE LIMITS,DEFAULT VALUES ARE ",I6,I8)
      999 FORMAT(1H0,"END OF JOB")
      END
```

INPAK 20:08 THU. FEB 05, 1974

```
      SUBROUTINE INPAK(N1,NTC,CHAN,TIM,TIME,Y)
      DIMENSION CHAN(10),TIM(10),Y(256)
      I,DATE(3)
      INTEGER CHAN
      REAL L1,L2
      CALL DATIME(TIMED,TIMEB,DATE)
      CALL OPENF(1,"INPUT")
      READ(1,9101)L1,L2,DA,XMO,YR,RN,SA,VL,P
      READ(1,9102)N1,NTC,(CHAN(I),I=1,NTC)
      READ(1,9103)(TIM(I),I=1,NTC)
      9101 FORMAT(2A6,A2,A3,A4,I2,A6,F5.0,F5.1)
      9102 FORMAT(I3,I1,8I3)
      9103 FORMAT(8F7.2)
      N=N1
      READ(1,7000)TIME,(Y(I),I=1,N)
      4000 FORMAT(15F4.0)
      5000 FORMAT(14F5.0)
      6000 FORMAT(10F7.0)
      7000 FORMAT(I6,9I7)
      CALL CLOSEF(1,"INPUT")
      PRINT9201,L1,L2,DA,XMO,YR,RN
      PRINT9202,SA,VL,P,TIMED,DATE
      9201 FORMAT(1H0,/////,2A6,3X,"DATE-",A2,A3,A4,3X,"RUN #",I2)
      9202 FORMAT(1H , "SAMPLE-",A6,3X,"WAVELENGTH=",F5.0," A",
      13X,"PRESSURE=",F5.1," MICRONS"//" CALCULATED AT",
      21X,A6,1X,3A6/)
      RETURN
      END
```



RGRES 20:08 THU. FEB 05, 1974

```

SUBROUTINE RGRES (V,U,A,B,N1,N2,R2,SWTCH)
DIMENSION U(512),V(512)
REAL M1,M2
120 FORMAT("ZERO (STD ERROR OF ESTIMATE EXCEEDS STD DEV
10F Y ")
N=N2-N1+1
100 FORMAT(1X,"SIMPLE LINEAR REGRESSION
1 EQUATION: LOG(Y-F(X))=A+B*X",//1X,"INDX R*R VALUE 95%
2CONF LIMITS BEGIN END DIFF")
150 FORMAT (1H1)
200 FORMAT(F9.5,4H A=,3F12.8,14,16,16/,9X,4
1H B=,3F12.8)
IF(SWTCH.EQ.2)GO TO 11
IF(A) 8,10,8
8 IF (ICNT-25) 11,11,9
9 PRINT 150
10 ICNT=1
IF(SWTCH.EQ.1)GO TO 101
102 PRINT110,
110 FORMAT(1H,"ITERATIONS NOT DISPLAYED")
GO TO 11
101 PRINT 100,
11 CONTINUE
S1=0
S2=0
S3=0
S4=0
S5=0
DO 20 I=N1,N2
S1=S1+U(I)
S2=S2+U(I)*U(I)
S3=S3+V(I)
S4=S4+V(I)*V(I)
20 S5=S5+U(I)*V(I)
M1=S1/N
M2=S3/N
D1=S2/N-M1*M1
D2=S4/N-M2*M2
D3=S5/N-M1*M2
C1=N*D1
R8=0
B=D3/D1
A=M2-B*M1
D4=D2-B*D3
D44=D4
IF(D4-D2) 2070,2040,2040
```

RGRES 20:08 THU. FEB 05, 1974

```
2040 R8=1
      R2=0
      GO TO 2080
2070 R2=1-(D4/D2)
2080 CONTINUE
      IF(R8) 2110,2130,2110
2110 PRINT 120
      GO TO 2160
2130 D2M4=D2-D4
      D4SCR=SQRT(D4)
2160 D4=N*D4/(N-2)
      T=1.95996+2.37226/(N-2)+2.82250/(N-2)*(N-2)
      D5=SQRT(D4/C1)
      D6=SQRT(D4/N)
      B1=B-T*D5
      B2=B+T*D5
      A1=A-T*D6
      A2=A+T*D6
      IF(SW TCH.NE.1)GO TO 56
      PRINT 200,R2,A,A1,A2,N1,N2,N,B,B1,B2
56 CONTINUE
      IF(SW TCH.NE.2.0)GO TO 57
      R2=D5
      SW TCH=D6
57 CONTINUE
      RETURN
      END
```

DSPAK 20:08 THU. FEB 05, 1974

```
      SUBROUTINE DSPAK
C PLOT ROUTINE
      COMMON N,IK,IW,M,IB,ITEST,I DUM,NDUM,I PR,IFG,IM,YT,TEST,
      1WVAR,SSQ,IDF,DET,ISW,IPLT,ISC,Y(300),X(1,300),W(300),IX(10),
      2FG(10),P(10),SP(10),YC(300),DY(300),BM(10,11),ALAB(10),INTT,
      3PART(10),NSETS
      EQUIVALENCE (I DUM,IN1),(NDUM,NN2)
      DIMENSION ICH(10),APLT(65)
      INTEGER APLT
      DATA IBCH,ISCH,I CH/" ", "* ", "A", "B", "C", "D", "E", "T", "U", "V"
      1, "W", "X"/
      DATA IOCH/"0"/
500  FORMAT (1X,I3,65A1)
901  FORMAT(1H0)
905  FORMAT (2X,F7.3,53X,F7.0)
      K=IK-1
      KK=IK/2-1
      YQ=Y(1)
      DO 5 I=1,N
5    YQ=AMAX1(YQ,Y(I))
      SCALE=63./ALOG(1000.)
      CHKZ=EXP(1./SCALE)*YQ/1000.
      J= -IPLT
      PRINT 901
      YQLN1=(YQ/1000.)
      PRINT 905,YQLN1,YQ
170  IXPLT=0
      N1=65
200  DO 210 I=1,65
210  APLT(I)=ISCH
      APLT(32)=IOCH
      J=J+1
      GO TO 250
215  J=J+IPLT
      IF(J-N) 216,216,170
216  CONTINUE
      IXPLT=IN1+J-1
      DO 220 I=2,64
220  APLT(I)=IBCH
      DO 240 I=1,KK
      DUM=Y(J)-P(K)
      DO 230 IJ=1,KK
      IF(IJ-1)225,230,225
225  DUM=DUM-P(2*IJ-1)*EXP(-X(1,J)/P(2*IJ))
230  CONTINUE
      IF(DUM-CHKZ)237,237,235
235  NMBR =(ALOG(DUM)-ALOG(YQ/1000.))*SCALE+1.
      IF(NMBR-64)238,238,236
236  NMBR=64
      GO TO 238
```

DSPAK 20:08 THU. FEB 05, 1974

```
237 NMBR = 2
238 APLT(NMBR) = ICH(I)
    IF (N1.LT.NMBR) N1=NMBR
240 CONTINUE
    NMBR=(DUM-P(K-2)*EXP(-X(1,J)/P(2*KK)))/(Y(J)**.5)*2.+32.
    IF(NMBR-1      )247,247,245
245 CONTINUE
    IF(NMBR-64)248,248,246
246 NMBR=64
    GO TO 248
247 NMBR = 2
248 APLT(NMBR) = ICH(KK+1)
    IF(N1.LT.NMBR)N1=NMBR
250 PRINT 500,IXPLT,(APLT(I),I=1,N1)
    N1=2
    IF (J-N)215,170,251
251 CONTINUE
    RETURN
    END
```

W C PASKE      DATE-JUN 21.73    RUN    1  
SAMPLE=N2 2PS    WAVELENGTH=3159.A    PRESSURE=325.0MICRONS

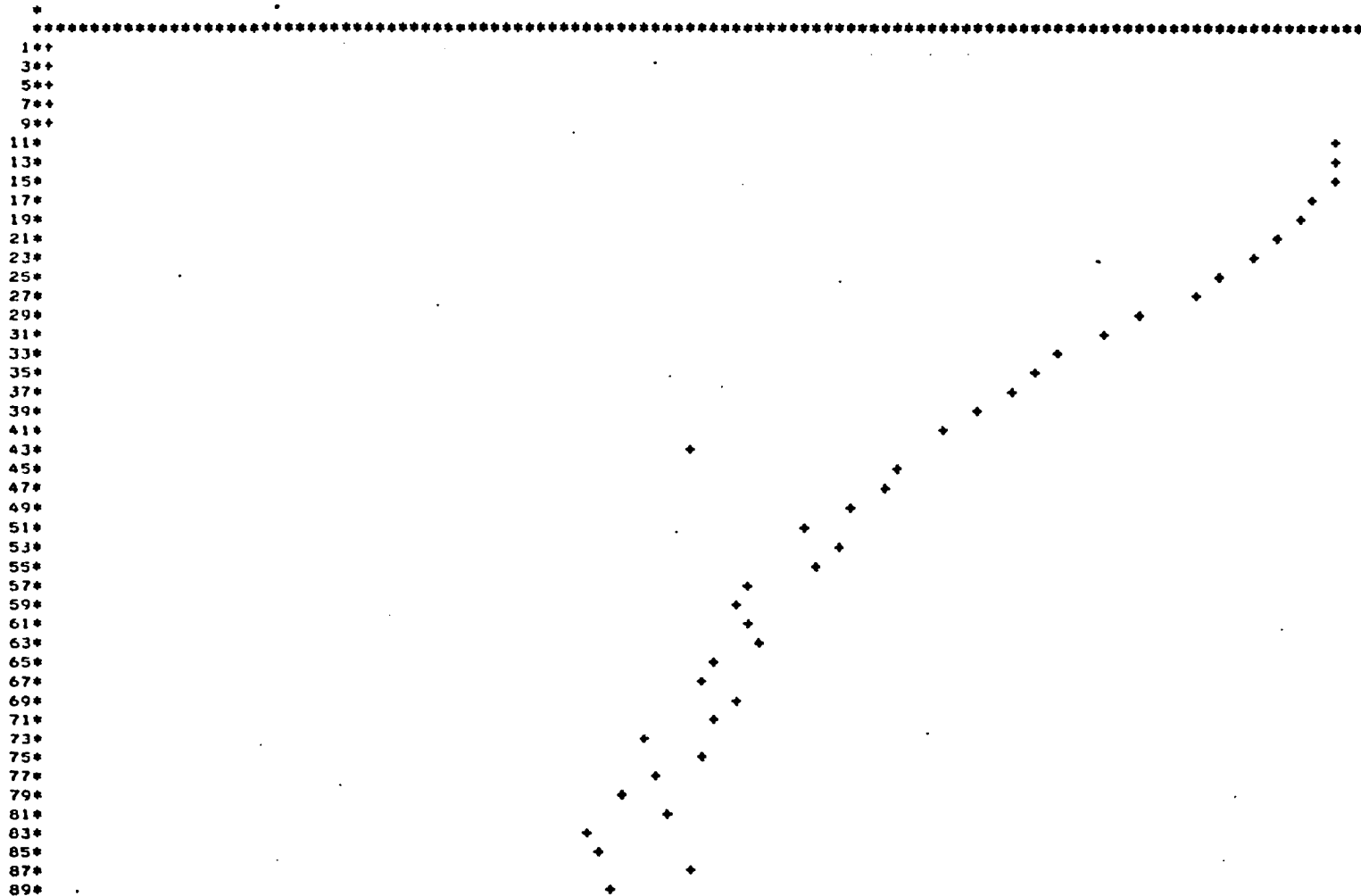
CALCULATED AT 08 00    THU. JUN. 21.1973

ENTER PLCT INCREMENT,LOWER SCALE LIMIT, UPPER SCALE LIMIT,BACKGROUND.

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```
// EXEC FORTGCLG.PARM.FORT='NAME=LASL'
//FORT.SYSIN DD *
C MAIN PROGRAM FOR NONLINEAR LEAST SQ. EXP. FIT. FROM LASL
  DOUBLE PRECISION YT,WVAR
  DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB
  COMMON N,IK,IW,M,IB,ITEST,IDUM,NDUM,IPR,IFG,IM,TEST,YT,WVAR,SSQ,
  IIDF,DET,ISW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
  210),SP(10),YC(300),DY(300), BM(10,11),ALAB(10),PART(10),NSETS,
  SDUM(1),Z(1)
  EQUIVALENCE (IDUM,IN1),(NDUM,NN2)
  READ(5,930) NRD
930 FORMAT(2I4)
  READ(NRD,920)(ALAB(I),I=1,4)
  IPLT=0
C CLEAR DUM, PART
  1 DUM(1)=0.
  DD 20 K=1,10
  20 PART(K)=0.0
C CALL INPUT ROUTINE
  ALAB(10)=NRD
  READ(NRD,920)
  READ(NRD,920)
  IN1=0
  NN2=5
  CALL ISPAK
C CALL CALCULATION ROUTINE
```

```

      CALL PSPAK
C     CALL OUTPUT ROUTINE
      CALL RSPAK
C     CALL PLOT ROUTINE
900 FORMAT(* ENTER PLOT INCR IF PLOT IS DESIRED THEN 0 FOR NEXT CASE,1
      1 TO RERUN WITH NEW PARAMETERS,-1 TO CALL EXIT*)
      WRITE(6,900)
      READ(NRD,910)IPLT,JPLT
      WRITE(6,910)IPLT,JPLT
910 FORMAT(2I3)
920 FORMAT(4A6)
      IF(IPLT.GT.0)CALL DSPAK
      IPLT=JPLT
      IF(IPLT.LT.0)CALL EXIT
      GO TO 1
      END
      SUBROUTINE ISPAK
C     INPUT ROUTINE
      DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB
      DOUBLE PRECISION YT,WVAR
      COMMON N,IK,IM,M,IB,ITEST,IDUM,NDUM,IPR,IFG,IM,TEST,YT,WVAR,SSQ,
      1IDF,DET,ISW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
      210),SP(10),YC(300),DY(300), BM(10,11),ALAB(10),PART(10),NSETS,
      3DUM(1),Z(1)
      EQUIVALENCE (IDUM,INI),(NDUM,NN2)
      NRD=ALAB(10)

```



```
IF(IPLT.EQ.1) GO TO 4
CALL INPAK(N,IB,IX,P,TIME,Y,ALAB,NRD)
NSAV=N
IF(P(1)) 350,355,360
350 CLBRTR=-P(1)
GO TO 4
355 CLBRTR=1.0
GO TO 4
360 DET=2.0
DO 3 I=1,IB
DY(I)=P(I)
3 W(I)=IX(I)
CALL RGRES(DY,W,TEST,CLBRTR,1,IB,AI,DET)
940 FORMAT(' ENTER NUMBER OF PARAMS. FIRST POINT, LAST POINT, WEIGHT POW
6ER. IPR=0,1,2 OR 3 FOR INCREASED OUTPUT. IM=I HOLD CONSTANT.*/.' A
6ND ENTER REP. RATE OF PULSING SYSTEM IF PILE UP CORRECTION IS DESI
6RED')
4 WRITE(6,940)
N=NSAV
HEAD(NRD,950)IK,IN1,IN2,WEIT,IPR,IM,TDCR
950 FORMAT(3I4,F4.1,2I4,F10.2)
WRITE(6,950)IK,IN1,IN2,WEIT,IPR,IM,TDCR
IX(2)=IK
NSETS=0
IB=0
ITEST=0
```

```
IK=IK+1
ISW=0
M=1
IF(IM.NE.2)IM=1
IX(1)=IK
IFG=2
KK=IK-2
KI=IK-1
960 FORMAT(' ENTER LIFETIMES(SHORT, LONG), COEFFS(SHORT, LONG), CONSTANT')
WRITE(6,960)
READ(NRD,970)(PG(K),K=2, KK,2),(PG(J),J=1, KI,2)
970 FORMAT(1P10E8,1)
971 FORMAT(1X,1P10E10,1)
WRITE(6,971)(PG(K),K=2, KK,2),(PG(J),J=1, KI,2)
PG(IK)=0.0
PG(IK-1)=PG(IK-1)+.5
980 FORMAT(' CALIBRATION IS',F10.5,' NS/CHAN +OR-',F10.7)
WRITE(6,980)CLBRTR,AI
DO 11 I=1,N
AI=1-1
11 X(1,I)=AI*CLBRTR
C PHOTON PILE-UP CORRECTION ROUTINE
IDCR=TDCR
IF(IDCR.EQ.0)GO TO 80
YSUM=Y(1)
SOCR=TDCR
```

```
TDCR=SDCR*TIME
DO 81 I=2,NSAV
  YTEMP=Y(I)/(1.0-(YSUM/TDCR))
  YSUM=YSUM+Y(I)
  Y(I)=YTEMP
81 CONTINUE
  CNRT=(YSUM/TDCR)*100.
  YMAX=Y(1)
DO 83 J=2,NSAV
  IF(YMAX-Y(J))82,83,83
82 YMAX=Y(J)
83 CONTINUE
  PMAX=YMAX/TDCR
  WRITE(6,981)CNRT,PMAX
981 FORMAT(' COUNTRATE FOR THIS RUN IS ',F5.2,' PER CENT',/, ' MAXIMUM P
  ROBILITY OF A COUNT OCCURRING IN A CHANNEL, IN ONE CYCLE, IS ',F8.
  66)
80 N=IN2-IN1+1
C   SET UP WEIGHTS
  WRITE(6,950)IN1,NN2
  WRITE(6,971)Y
  IF(WEIT.NE.0)GO TO 6
  DO 40 I=1,N
40 W(I)=1.0
  GO TO 8
6 DO 50 I=1,N
```

```
J=INI-1+I
50 W(I)=Y(J)/Y(INI)
   WRITE(6,950)I,J,INI
   IF(WEIT.E0.1.0) GO TO 8
405 FORMAT (' WEIGHT RANGES FROM 1.0 TO',F7.3,' FOR CHAN ',I3)
7 DO 60 I=2,N
   IF(W(I).E0.0.0)GO TO 59
   W(I)=W(I)**WEIT
   GO TO 60
59 W(I)=0.0
60 CONTINUE
8 TEST=1.0E-08
   WRITE(6,405)W(N),IN2
   DO 55 I=1,N
   INIM1=I+INI-1
55 Y(I)=Y(INIM1)
   RETURN
1 CALL EXIT
   END
   SUBROUTINE RSPAK
C   OUTPUT ROUTINE
   REAL*8 DSORT
   DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB
   DOUBLE PRECISION YT,WVAR
   COMMON N,IK,IW,M,IB,ITEST,IDUM,NDUM,IPR,IFG,IN,TEST,YT,WVAR,SSQ,
1IDF,DET,ISW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
```

```

210).SP(10).YC(300).DY(300).    BM(110).ALAB(10).PART(10).NSETS.
. 3DUM(1).Z(1)
  DIMENSION TS(40)
C   PAGE 1 OF THE STANDARD OUTPUT
  I=-1
  CALL YPS(1)
  WRITE (6.200)WVAR,TEST,SSQ
200 FORMAT(/26H THE WEIGHTED VARIANCE IS 1PE14.7./
1.' THE UNWEIGHTED SIGMA IS *.1PE14.7
2.' AND THE UNWEIGHTED SUM OF SQUARES OF THE DEVS IS *.1PE14.7)
300 FORMAT(*      GUESS OF FINAL VAL OF  S.D. OF      EXACT LST S
1QRS EQNS'./.' K K-TH PARAM K-TH PARAM K-TH PARAM    FITTED FC
2TN  INPUT DATA*)
  WRITE(6.300)
  KFREE=IK-IM
  DO 40 K=1,KFREE
2  A=0.0
  B=0.0
  DO 30 I=1,N
  A=A+W(I)*YC(I)*PART(K)
30 B=B+W(I)*Y(I)*PART(K)
  WRITE(6.500)K,PG(K),P(K),SP(K),A,B
500 FORMAT(13,1X,1P3E12.4,3X,1P2E12.4)
40 CONTINUE
C   CAL AND RITE CORR MATRIX
  IF(IPR.LT.1) GO TO 70

```

```

WRITE(6,700)
700 FORMAT(/////' MATRIX OF CORRELATIONS BETWEEN FREE PARAMS')
DO 60 K1=1,KFREE
DO 50 K2=1,KFREE
IJ=K2*KFREE+K1
JJ=K1*KFREE+K1
KJ=K2*KFREE+K2
50 TS(K2)=BM(IJ)/DSQRT(BM(JJ)*BM(KJ))
851 FORMAT(13,10F8.3 )
56 WRITE (6,851)K1,(TS(K),K=1,KFREE)
60 CONTINUE
70 RETURN
END
SUBROUTINE LAMBDA (NM,PC,LBDA,DP,IT,AN)
REAL LRDA,NU
DOUBLE PRECISION YT,WVAR,PHI
DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB,PC,DP,AN,AM
DIMENSION PC(10),AM(110),DP(10)
DIMENSION AN(100)
COMMON N,IK,IW,M,IB,ITEST,IDUM,NDUM,IPR,IFG,IM,TEST,YT,WVAR,SSQ,
1IDF,DET,ISW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
210),SP(10),YC(300),DY(300), BM(110),ALAB(10),PART(10),NSETS,
3DUM(1)
KFREE=IK-IM
NU=10
NI=KFREE*(KFREE+1)

```

```
PHI=0.0
H=1.0
GO TO (10,100),NM
10 DO 20 K=1,KFREE
  IJ=(K-1)*KFREE+K
  BM(K)=BM(K)/DSORT(AN(IJ))
  DO 20 KK=1,KFREE
    KJ=(KK-1)*KFREE+KK
    I=K*KFREE+KK
    BM(I)=BM(I)/DSORT(AN(IJ)*AN(KJ))
  20 AM(I)=BM(I)
  IF(IPR.GT.2)WRITE(6,940)(BM(J),J=1,KFREE)
  IF(IPR.GT.2)WRITE(6,950)(AN(J),J=1,IJ,IK)
  GO TO 500
100 D=0.
  GD=0.
  G=0.
  DO 120 J=1,KFREE
    G=BM(J)**2+G
    D=SP(J)**2+D
  120 GD=BM(J)*SP(J)+GD
  GD=GD/SORT(D*G)
  IF(IPR.NE.0) WRITE(6,900)GD
160 PHI=0.0
  DO 200 J=1,N
    YT=PC(IK-1)
```

```
IN=IK-3
DO 150 K=1,IN,2
  JJ=K
150 YI=YI+PC(K)*DEXP(-X(I,J)/PC(K+1))
200 PHI=b(J)*(Y(J)-YI)**2+PHI
  IF(PHI.LE.WVAR) GO TO 600
  IF(GD.LT..707.AND.LBDA.GT.0)GO TO 400
  H=H/2.0
  K=0
  IF(IPR.NE.0) WRITE(6,920)H,PHI
  DO 220 I=1,KFREE
    DP(I)=H*DP(I)
    PC(I)=P(I)+DP(I)
    CHK=DABS((PC(I)-P(I))/P(I))
    IF(CHK.LT.TEST)K=K+1
220 CONTINUE
  IF(KFREE-K)600,600,160
400 LBDA=LBDA*NU
  IF(IPR.NE.0) WRITE(6,910)LBDA,IT,PHI
  IJ=KFREE+1
  DO 300 K=IJ,NI
300 BM(K)=AM(K)
  GO TO 520
500 IF(LBDA.LE..00001) GO TO 520
  LBDA=LBDA/NU
520 IJ=KFREE+1
```



```

      DD 530 K=IJ,NI,IJ
530 BM(K)=AM(K)+LBDA
      RETURN
600 NM=1
      RETURN
900 FORMAT(' COS(GAMMA)= ',1PE11.4)
910 FORMAT(' LAMBDA= ',1PE7.1,' ITER NO. ',I2,' PHI= ',1PE15.9)
920 FORMAT(' H= ',1PE8.2,' PHI= ',1PE15.9)
930 FORMAT(2I4,1PE10.3)
940 FORMAT(' BM(J)= ',1P7E17.8)
950 FORMAT(' AN(J,J)= ',1P7E17.8)
      END
      SUBROUTINE YPS(I)
C      FUNCTION AND PARTIAL DERIVATIVE ROUTINE
      DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB
      DOUBLE PRECISION YT,WVAR
      COMMON N,IK,IW,M,IB,ITEST,IDUM,NDUM,IPR,IFG,IM,TEST,YT,WVAR,SSQ,
1IDF,DET,ISW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
210),SP(10),YC(300),DY(300), BM(10,11),ALAB(10),PART(10),NSETS,
3DUM(1),Z(1)
900 FORMAT(' SUM OF EXPONENTIALS. Y(I)=P(1)*EXP(-X(I)/P(2))+...+P(',I1
1,')')
      IF (I)1,3,3
1 K=IK-1
      WRITE(6,900)K
      GO TO 4

```

```

3 YT=0.0
  KI=IK-2
  DO 10 K=2,KI,2
    PART(K-1)=DEXP(-Z(1)/P(K))
    PART(K)=P(K-1)*PART(K-1)
    YT=YT+PART(K)
10 PART(K)=PART(K)*Z(1)/P(K)**2
  YT=YT+P(1K-1)
  PART(1K-1)=1.0
4 RETURN
  END
  SUBROUTINE PSPAK
C  CALCULATION ROUTINE
  DIMENSION AM(100),DP(10),PC(10),AN(10)
  DIMENSION MA(10),MB(10),IIW(200),JIW(100)
  EQUIVALENCE(IIW(101),JIW(1))
  REAL*8 DSGRT
  DOUBLE PRECISION YT,WVAR
  DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB,AM,DP,PC,AN
  COMMON N,IK,IW,M,IB,ITEST,1DUM,NDUM,IPR,IFG,IM,TEST,YT,WVAR,SSQ,
11DF,DET,1SW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
210),SP(10),YC(300),DY(300), BM(110),ALAB(10),PART(10),NSETS,
3DUM(1),Z(1)
  REAL LBDA
  EQUIVALENCE(ITEST,1ASTIT)
C  INITIAL OUTPUT OPTION

```

```
      IF(IPR.EQ.0) GO TO 1
      WRITE(6,400) TEST
400 FORMAT(/BH TEST = ,1PE15.7/)
C     INITIALIZATION FOR MAIN ITERATION LOOP
1  KFREE=IK-IM
      KP=KFREE+1
      IDF=N-KFREE
      DF=IDF
      IT=0
      LRDA=.0001
      XSVAR=1.0E+15
      IREJ=0.
      DO 10 K=1,IK
      DP(K)=0.0
      SP(K)=0.0
      PC(K)=PG(K)
10  P(K)=PG(K)
      LASTIT=0
      M25C=0
      IF(KFREE.EQ.0)GO TO 3
C     MAIN ITERATION LOOP
2  IT=IT+1
      H=1
      DO 30 K=1,KFREE
      DO 20 KK=1,KFREE
      JJ=K+(KK-1)*KFREE
```

```
      AM(JJ)=0.
20  BM(JJ)=0.
      JJ=K+(KP-1)*KFREE
      BM(JJ)=0.
      JJ=K+K*KFREE
30  BM(JJ)=0.
      3  VAR=0.0
      SSQ=0.0
C    LOOP TO SET UP NORMAL EQUATIONS
      DO 90 I=1,N
C    CALL YP ROUTINE, CALCULATE SUM OF SQUARES
      DO 40 J=1,M
40  Z(J)=X(J,I)
      CALL YPS(I)
      YC(I)=YT
      DY(I)=Y(I)-YC(I)
      DEL=DY(I)**2*W(I)
      IF(DEL.LT.X5VAR)GO TO 35
      W(I)=0.0
      IREJ=IREJ+1
      IIW(IREJ)=I+IDUM-1
      IIW(IREJ+100)=Y(I)
35  VAR=VAR+W(I)*DY(I)**2
      SSQ=SSQ+DY(I)**2
      IF(KFREE.EQ.0) GO TO 90
C    SET UP AN AS VECTOR OF PARTIAL DERIVATIVES
```

```
K1=0
DO 60 K=1,IK
IF(IM.EQ.0) GO TO 4
DO 50 KK=1,IM
IF(K.EQ.IX(KK)) GO TO 5
50 CONTINUE
4 K2=K-K1
AN(K2)=PART(K)
GO TO 60
5 K1=K1+1
60 CONTINUE
C FORM A AND B MATRICES
DO 80 K=1,KFREE
DO 70 KK=1,KFREE
JJ=K+(KK-1)*KFREE
AM(JJ)=AM(JJ)+AN(K)*AN(KK)*W(I)
IJ=JJ+KFREE
70 BM(IJ)=AM(JJ)
80 BM(K)=BM(K)+AN(K)*DY(I)*W(I)
90 CONTINUE
WVAR=VAR
XSVAR=12.*WVAR/DF
IF(KFREE.EQ.0) GO TO 23
IF(LASTIT.EQ.1)LBDA=0.
CALL LAMBDA(1,PC,LBDA,DP,IT,AM)
NM=2
```

```
C      OPTIONAL PRINTOUT OF A AND B MATRICES ON LAST ITERATION
      IF(LASTIT.EQ.0) GO TO 6
      IF(IPR.LT.3) GO TO 6
      WRITE(6,500)
500  FORMAT(///,3X,1HK,19X,6HA(K,L),28X,4HB(K)///)
      DO 110 K=1,KFREE
      WRITE(6,600)DM(K)
600  FORMAT(//1H+,1P1E67.5)
      JJ=(K-1)*KFREE+1
      IJ=K*KFREE
110  WRITE(6,700)K,(AM(II),II=JJ,IJ)
700  FORMAT(14,1P10E12.4)
C      SOLVE THE NORMAL EQUATIONS
6      IF(KFREE.GT.1) GO TO 7
      DET=AM(1)
      BM(2)=1.0/AM(1)
      GO TO 8
7      CONTINUE
      CALL MINV(BM(KP),KFREE,DET,HA,MB)
      IF(DET.EQ.0)LASTIT=1
      IF(DET.EQ.0) WRITE(6,750)IT
750  FORMAT(* SINGULAR SYSTEM ITER NO *,12)
      CALL GMPRD(BM(KP),BM,DP,KFREE,KFREE,1)
      DO 115 I=1,KFREE
      IJ=(I-1)*KFREE+1
      SP(1)=DP(I)
```

```

DP(I)=DP(I)/DSORT(AM(I,J))
115 PC(I)=P(I)+DP(I)
C   WRITE THE VALUE OF THE DETERMINANT, A INVERSE, AND NO. OF ITERATIONS
CALL LAMBDA(NM,PC,LBDA,DP,IT,AM)
IF(NM.EQ.2) GO TO 7
8 IF (LASTIT.EQ.0) GO TO 11
WRITE(6,800)IT,DET
800 FORMAT(I6,' ITERATIONS, DET. OF PART. DERIV. MATRIX =',1PE14.6)
C   CALCULATE NEW PARAMETER VALUES AND CHECK FOR SIGN CHANGES IF NECESSARY
11 K1=0
DO 140 K=1,IK
IF(IM.EQ.0) GO TO 12
DO 130 KK=1,IM
IF(K.EQ.IX(KK)) GO TO 16
130 CONTINUE
12 K2=K-K1
13 PC(K)=P(K)+H*DP(K)
IF(LASTIT.NE.0) GO TO 140
IF(IFG-1)14,140,15
14 IF(IT.GT.5) GO TO 140
15 IF(P(K)*PC(K).GE.0) GO TO 140
H=H/2
IF(H.GE.1.0E-10) GO TO 11
WRITE(6,1500)
1500 FORMAT(' PROG. QUIT ITERATING DUE TO PARAMETER',I3,' CHANGED SIGN'
1)

```

```
H=0
LASTIT=1
GO TO 8
16 K1=K1+1
140 CONTINUE
C   OPTIONAL PRINTOUT FOR EACH ITERATION
    IF(LASTIT.NE.0) GO TO 19
    IF(IPR.LT.1) GO TO 17
    WRITE(6,1300) IT,H,VAR
1300 FORMAT( 1H0,13,1P2E17.7)
1400 FORMAT( 1P6E11.3)
    IF(IPR.LT.2) GO TO 17
    WRITE(6,1400) (PC(K),K=1,IK)
C   TEST FOR CONVERGENCE
17 KK=0
   DO 160 K=1,IK
     IF(P(K).EQ.0) GO TO 18
     IF(DABS((PC(K)-P(K))/P(K))-TEST)160.160.19
18 KK=KK+1
160 CONTINUE
   IF(KK.EQ.IK) GO TO 19
   M25C=1
C   SET PARAMETER VALUES FOR THE NEXT ITERATION
19 DO 170 K=1,IK
170 P(K)=PC(K)
C   AFTER LAST ITERATION GO BACK FOR FINAL CALCULATION OF YC,DY,ETC.
```



```
IF(LASTIT.EQ.0) GO TO 21
KFREE=0
GO TO 3
C TEST WHETHER 25 ITERATIONS HAVE BEEN TAKEN
21 IF(M25C.EQ.1) GO TO 22
IF(IT.LT.25) GO TO 2
C GO BACK FOR LAST ITERATION
22 LASTIT=1
GO TO 2
C CALCULATE WEIGHTED VARIANCE, STANDARD DEV. OF THE PARAMS.
23 WVAR=VAR/DF
TEST = SORT(SSQ/DF)
K1=0
DO 190 K=1,IK
IF(IM.EQ.0) GO TO 24
DO 180 KK=1,IM
IF(K.EQ.IX(KK)) GO TO 25
180 CONTINUE
24 K2=K-K1
K3=K2+K2*(IK-IM)
K4=K3-1K+1M
SP(K)=DSORT(BM(K3)+WVAR/AM(K4))
GO TO 190
25 KI=KI+1
190 CONTINUE
IF(IREJ.EQ.0)GO TO 200
```

```

WRITE(6,1600)IREJ
WRITE(6,1901) (I*W(J),JIW(J) ,J=1,IREJ)
1901 FORMAT(10(I4,I8))
1600 FORMAT(20X,'THE FOLLOWING',I4,' POINTS ARE OUTSIDE 3.46 SIGMA')
200 CONTINUE
RETURN
END
SUBROUTINE MINV(A,N,D,L,M)
C
C .....MINV 002
C
C SUBROUTINE MINV
C
C PURPOSE
C
C INVERT A MATRIX
C
C USAGE
C
C CALL MINV(A,N,D,L,M)
C
C DESCRIPTION OF PARAMETERS
C
C A - INPUT MATRIX. DESTROYED IN COMPUTATION AND REPLACED BY
C RESULTANT INVERSE.
C
C N - ORDER OF MATRIX A
C
C D - RESULTANT DETERMINANT
C
C L - WORK VECTOR OF LENGTH N
C
C M - WORK VECTOR OF LENGTH N
MINV 033
MINV 001
MINV 003
MINV 004
MINV 005
MINV 006
MINV 007
MINV 008
MINV 009
MINV 010
MINV 011
MINV 012
MINV 013
MINV 014
MINV 015
MINV 016
MINV 017
MINV 018

```

C		MINV 019
C	REMARKS	MINV 020
C	MATRIX A MUST BE A GENERAL MATRIX	MINV 021
C		MINV 022
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	MINV 023
C	NONE	MINV 024
C		MINV 025
C	METHOD	MINV 026
C	THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT	MINV 027
C	IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT	MINV 028
C	THE MATRIX IS SINGULAR.	MINV 029
	DIMENSION A(1),L(1),M(1)	MINV 034
	REAL*8 DABS	
	DOUBLE PRECISION A,D,RIGA,HOLD	MINV 042
C		MINV 030
C	.....	MINV 031
C		MINV 032
C		MINV 035
C	.....	MINV 036
C		MINV 037
C	IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE	MINV 038
C	C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION	MINV 039
C	STATEMENT WHICH FOLLOWS.	MINV 040
C		MINV 041
C		MI-V 043
C	THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS	MINV 044

C	APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS	MINV 045
C	ROUTINE.	MINV 046
C		MINV 047
C	THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO	MINV 048
C	CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS. ABS IN STATEMENT	MINV 049
C	10 MUST BE CHANGED TO DABS.	MINV 050
C		MINV 051
C	.....	MINV 052
C		MINV 053
C	SEARCH FOR LARGEST ELEMENT	MINV 054
C		MINV 055
	D=1.0	MINV 056
	NK=-N	MINV 057
	DO 80 K=1,N	MINV 058
	NK=NK+N	MINV 059
	L(K)=K	MINV 060
	M(K)=K	MINV 061
	KK=NK+K	MINV 062
	BIGA=A(KK)	MINV 063
	DO 20 J=K,N	MINV 064
	IZ=N+(J-1)	MINV 065
	DO 20 I=K,N	MINV 066
	IJ=IZ+I	MINV 067
	10 IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20	MINV 068
	15 BIGA=A(IJ)	MINV 069
	L(K)=I	MINV 070

M(K)=J	MINV 071
20 CONTINUE	MINV 072
C	MINV 073
C INTERCHANGE ROWS	MINV 074
C	MINV 075
J=L(K)	MINV 076
IF(J-K) 35,35,25	MINV 077
25 KI=K-N	MINV 078
DO 30 I=1,N	MINV 079
KI=KI+N	MINV 080
HOLD=-A(KI)	MINV 081
JI=KI-K+J	MINV 082
A(KI)=A(JI)	MINV 083
30 A(JI) =HOLD	MINV 084
C	MINV 085
C INTERCHANGE COLUMNS	MINV 086
C	MINV 087
35 I=M(K)	MINV 088
IF(I-K) 45,45,38	MINV 089
38 JP=N*(I-1)	MINV 090
DO 40 J=1,N	MINV 091
JK=NK+J	MINV 092
JJ=JP+J	MINV 093
HOLD=-A(JK)	MINV 094
A(JK)=A(JI)	MINV 095
40 A(JI) =HOLD	MINV 096

C		MINV 097
C	DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS	MINV 098
C	CONTAINED IN BIGA)	MINV 099
C		MINV 100
	45 IF(BIGA) 48,46,48	MINV 101
	46 D=0.0	MINV 102
	RETURN	MINV 103
	48 DO 55 I=1,N	MINV 104
	IF(I-K) 50,55,50	MINV 105
	50 IK=NK+I	MINV 106
	A(IK)=A(IK)/(-BIGA)	MINV 107
	55 CONTINUE	MINV 108
C		MINV 109
C	REDUCE MATRIX	MINV 110
C		MINV 111
	DO 65 I=1,N	MINV 112
	IK=NK+I	MINV 113
	HOLD=A(IK)	MINV M01
	IJ=I-N	MINV 114
	DO 65 J=1,N	MINV 115
	IJ=IJ+N	MINV 116
	IF(I-K) 60,65,60	MINV 117
	60 IF(J-K) 62,65,62	MINV 118
	62 KJ=IJ-I+K	MINV 119
	A(IJ)=HOLD*A(KJ)+A(IJ)	MINV M02
	65 CONTINUE	MINV 121

C		MINV 122
C	DIVIDE ROW BY PIVOT	MINV 123
C		MINV 124
	KJ=K-N	MINV 125
	00 75 J=1.N	MINV 126
	KJ=KJ+N	MINV 127
	IF(J-K) 70.75.70	MINV 128
	70 A(KJ)=A(KJ)/BIGA	MINV 129
	75 CONTINUE	MINV 130
C		MINV 131
C	PRODUCT OF PIVOTS	MINV 132
C		MINV 133
	D=D*BIG A	MINV 134
C		MINV 135
C	REPLACE PIVOT BY RECIPROCAL	MINV 136
C		MINV 137
	A(KK)=1.0/BIG A	MINV 138
	80 CONTINUE	MINV 139
C		MINV 140
C	FINAL ROW AND COLUMN INTERCHANGE	MINV 141
C		MINV 142
	K=N	MINV 143
	100 K=(K-1)	MINV 144
	IF(K) 150.150.105	MINV 145
	105 I=L(K)	MINV 146
	IF(I-K) 120.120.108	MINV 147

108 JO=N*(K-1)	MINV 148
JR=N*(I-1)	MINV 149
DO 110 J=1..N	MINV 150
JK=JO+J	MINV 151
HOLD=A(JK)	MINV 152
JI=JR+J	MINV 153
A(JK)=-A(JI)	MINV 154
110 A(JI) =HOLD	MINV 155
120 J=M(K)	MINV 156
IF(J-K) 100,100,125	MINV 157
125 KI=K-N	MINV 158
DO 130 I=1..N	MINV 159
KI=KI+N	MINV 160
HOLD=A(KI)	MINV 161
JI=KI-K+J	MINV 162
A(KI)=-A(JI)	MINV 163
130 A(JI) =HOLD	MINV 164
GO TO 100	MINV 165
150 RETURN	MINV 166
END	MINV 167
SUBROUTINE GMPRD(A,B,R,N,M,L)	GMPRD037
C	GMPRD001
C .....	GMPRD002
C	GMPRD003
C     SUBROUTINE GMPRD	GMPRD004
C	GMPRD005



C	PURPOSE	GMPRD006
C	MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL	GMPRD007
C	MATRIX	GMPRD008
C		GMPRD009
C	USAGE	GMPRD010
C	CALL GMPRD(A,B,R,N,M,L)	GMPRD011
C		GMPRD012
C	DESCRIPTION OF PARAMETERS	GMPRD013
C	A - NAME OF FIRST INPUT MATRIX	GMPRD014
C	B - NAME OF SECOND INPUT MATRIX	GMPRD015
C	R - NAME OF OUTPUT MATRIX	GMPRD016
C	N - NUMBER OF ROWS IN A	GMPRD017
C	M - NUMBER OF COLUMNS IN A AND ROWS IN B	GMPRD018
C	L - NUMBER OF COLUMNS IN B	GMPRD019
C		GMPRD020
C	REMARKS	GMPRD021
C	ALL MATRICES MUST BE STORED AS GENERAL MATRICES	GMPRD022
C	MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A	GMPRD023
C	MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B	GMPRD024
C	NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWS	GMPRD025
C	OF MATRIX B	GMPRD026
C		GMPRD027
	DOUBLE PRECISION A,B,R	
	DIMENSION A(1),B(1),R(1)	GMPRD038
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	GMPRD028
C	NONE	GMPRD029

C		GMPRD030
C	METHOD	GMPRD031
C	THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A	GMPRD032
C	AND THE RESULT IS STORED IN THE N BY L MATRIX R.	GMPRD033
C		GMPRD034
C	.....	GMPRD035
C		GMPRD036
C		GMPRD039
	IR=0	GMPRD040
	IK=-M	GMPRD041
	DO 10 K=1,L	GMPRD042
	IK=IK+M	GMPRD043
	DO 10 J=1,N	GMPRD044
	IR=IR+1	GMPRD045
	JI=J-N	GMPRD046
	IB=IK	GMPRD047
	R(IR)=0	GMPRD048
	DO 10 I=1,M	GMPRD049
	JI=JI+N	GMPRD050
	IB=IB+1	GMPRD051
	10 R(IR)=R(IR)+A(JI)*B(IB)	GMPRD052
	RETURN	GMPRD053
	END	GMPRD054
	SUBROUTINE PGRES (V,U,A,B,N1,N2,R2,SWTCH)	
	DOUBLE PRECISION V,U,SWTCH	
	DIMENSION U(255),V(255)	

```

REAL M1,M2
120 FORMAT(' ZERO          (STD ERROR OF ESTIMATE EXCEEDS STD DEV OF Y
      I)')
      N=N2-N1+1
100 FORMAT ( 3X,'S I M P L E L I N E A R R E G R E S S I O N
      1EQUATION LOG(Y-F(X)) = A + B*X' ,//1X,'INDX R*R EXPL VAR UNEX
      2P VAR STD ERR          VALUE          95 PCT CONF LIMITS BEGIN END
      3DIFF' )
150 FORMAT (1H1)
200 FORMAT (F9.5,2F10.6,F9.6,4H A=,3F12.8,14,16,16,/,3BX,4H B=,3F12.
      18)
      IF(SWCH.EQ.2.0)GO TO 11
      IF (A) 8,10,8
      8 IF (ICNT-25) 11,11,9
      9 WRITE (6,150)
      10 ICNT = 1
      IF(SWCH.EQ.1)GO TO 101
102 WRITE(6,110)
110 FORMAT(1H ,'ITERATIONS NOT DISPLAYED')
      GO TO 11
101 WRITE(6,100)
      11 ICNT = ICNT+1
      S1=0
      S2=0
      S3=0
      S4=0

```

```
S5=0
DU 20 I=N1.N2
S1=S1+U(I)
S2=S2+U(I)*U(I)
S3=S3+V(I)
S4=S4+V(I)*V(I)
20 S5=S5+U(I)*V(I)
M1=S1/N
M2=S3/N
D1=S2/N-M1*M1
D2=S4/N-M2*M2
D3=S5/N-M1*M2
C1=N*D1
R8=0
R=D3/D1
A=M2-R*M1
D4=D2-R*D3
D44 = D4
IF (D4-D2)2070,2040,2040
2040 R8=1
R2=0
GO TO 2080
2070 R2=1-(D4/D2)
2080 CONTINUE
IF(R8)2110,2130,2110
2110 WRITE (6,120)
```

```

      GO TO 2160
2130 D2M4=D2-D4
      D4SQR=SQRT(D4)
2160 D4=N*D4/(N-2)
      T=1.95996+2.37226/(N-2)+2.82250/(N-2)*(N-2)
      D5=SQRT(D4/C1)
      D6=SQRT(D4/N)
      B1=B-T*D5
      R2=R+T*D5
      A1=A-T*D6
      A2=A+T*D6
      IF(SWCH.NE.1)GO TO 56
      WRITE (6,200)R2,D2M4,D44,D4SQR,A,A1,A2,N1,N2,N,B,B1,B2
56 IF(SWCH.NE.2.0)GO TO 57
      R2=D5
      SWCH=D6
57 CONTINUE
      RETURN
      END
      SUBROUTINE DSPAK
C      PLOT ROUTINE
      DOUBLE PRECISION W,P,SP,YC,DY,PART,BM,DET,ALAB
      DOUBLE PRECISION YT,WVAR
      COMMON N,IK,IW,M,IB,ITEST,IDUM,NDUM,IPR,IFG,IM,TEST,YT,WVAR,SSQ,
      1IDF,DET,ISW,IPLT,ISC,INTT,Y(300),X(1,300),W(300),IX(10),PG(10),P(
      210),SP(10),YC(300),DY(300), BM(10,11),ALAB(10),PART(10),NSETS

```

```

EQUIVALENCE (IDUM,IN1),(NDUM,NN2)
DIMENSION ICH(10),APLT(128)
INTEGER APLT
DATA IBCH,ISCH,ICH/' ','A','B','C','D','E','T','U','V','W','X'
1/
DATA IOCH/'0'/
500 FORMAT (1X,I3,128A1)
901 FORMAT(1H1)
905 FORMAT(2X,F7.3,116X,F7.0)
      K=IK-1
      KK=IK/2-1
      IJ=1
      J=1
      WVAR=DSORT(WVAR)
      DO 5 I=1,N
        IF(Y(J).LT.Y(I))J=I
5     CONTINUE
      DO 7 I=1,KK
        SP(I)=P(2*I-1)*DEXP(-X(1,J)/P(2*I))
        IF(SP(I).GT.SP(IJ))IJ=I
7     CONTINUE
      YQ=Y(J)-P(K)
      DO 8 I=1,KK
        IF(I.EQ.IJ)GO TO 8
        YQ=YQ-SP(I)
8     CONTINUE

```

```
SCALE=115./ALOG(1000.)
CHKZ=EXP(1./SCALE)*Y0/1000.
J=-IPLT
WRITE(6,901)
YOLN1=(Y0/1000.)
WRITE(6,905)YOLN1,Y0
170 IXPLT=0
    NI=116
200 DO 210 I=1,116
210 APLT(I)=ISCH
    APLT(58)=IOCH
    J=J+1
    GO TO 250
215 J=J+IPLT
    IF(J-N) 216,216,170
216 CONTINUE
    IXPLT=IN1+J-1
    DO 220 I=2,115
220 APLT(I)=IBCH
    DO 240 I=1,KK
    DUM=Y(J)-P(K)
    DO 230 IJ=1,KK
    IF(IJ-1)225,230,225
225 DUM=DUM-P(2*IJ-1)*DEXP(-X(1,J)/P(2*IJ))
230 CONTINUE
    IF(DUM-CHKZ)237,237,235
```

```
235 NMBR=(ALOG(DUM)-ALOG(YG/1000.))*SCALE+1.  
    IF(NMBR-116)238,238,236  
236 NMBR=116  
    GO TO 238  
237 NMBR=2  
238 APLT(NMBR)=ICH(I)  
    IF(N1.LT.NMBR) N1=NMBR  
240 CONTINUE  
    NMBR=((DUM-P(K-2)*DEXP(-X(1,J)/P(2*KK)))*W(J)**.5*.1)+64  
    IF(NMBR-1)247,247,245  
245 CONTINUE  
    IF(NMBR-115)248,248,246  
246 NMBR=115  
    GO TO 248  
247 NMBR=2  
248 APLT(NMBR)=ICH(KK+1)  
    IF(N1.LT.NMBR)N1=NMBR  
250 WRITE(6,500)IXPLT.(APLT(I),I=1,N1)  
    N1=2  
    IF(J-N)215,170,251  
251 CONTINUE  
    RETURN  
    END  
    SUBROUTINE INPAK(N1,NTC,CHAN,TIM,TIME,Y,DATE,NRD)  
    DIMENSION CHAN(10),TIM(10),Y(512),DATE(4)  
    INTEGER RN
```





9202 FORMAT(1H ,\*SAMPLE=\*,A6.3X,\*WAVELENGTH=\*,F5.0,\*A\*,3X,\*PRESSURE=\*,F

15.1,\*MICRONS\*///\* CALCULATED AT\*,1X,A6.1X,3A6/)

RETURN

END

/\*

//GU.SYSIN DD \*

W C PASKE DATE-JUN 30.73 RUN 3  
 SAMPLE-N2 2PS WAVELENGTH=3371.A PRESSURE=100.0MICRONS

CALCULATED AT 14 00 MON, JUL. 02,1973

ENTER NUMBER OF PARAMS. FIRST POINT, LAST POINT, WEIGHT POWER. IPR=0.1, 0.2 OR 3 FOR INCREASED OUTPUT. IM=1 HOLD CONSTANT

3 30 250-1.0 0 0  
 ENTER LIFETIMES(SHORT, LONG), COEFFS(SHORT, LONG), CONSTANT  
 4.0E 01 6.0E 02 7.0E 01  
 CALIBRATION IS 1.66512 NS/CHAN +OR- 0.0078660  
 30 250

0.0	0.0	0.0	0.0	0.0	7.0E 02	8.6E 02	8.8E 02	8.3E 02	9.0E 02
9.1E 02	8.8E 02	8.7E 02	8.7E 02	8.2E 02	8.8E 02	8.1E 02	8.3E 02	8.3E 02	7.7E 02
8.5E 02	7.7E 02	8.2E 02	8.0E 02	8.6E 02	7.9E 02	7.4E 02	8.0E 02	7.1E 02	7.5E 02
7.0E 02	6.5E 02	6.6E 02	4.0E 02	6.0E 02	5.7E 02	6.0E 02	5.8E 02	5.8E 02	5.6E 02
5.0E 02	5.5E 02	1.2E 02	4.8E 02	4.8E 02	4.6E 02	4.6E 02	4.2E 02	4.8E 02	4.3E 02
4.1E 02	3.4E 02	3.7E 02	3.6E 02	3.6E 02	3.7E 02	3.2E 02	3.5E 02	3.3E 02	3.3E 02
3.0E 02	2.9E 02	2.8E 02	2.7E 02	2.9E 02	2.9E 02	2.7E 02	2.5E 02	2.8E 02	2.6E 02
2.4E 02	2.4E 02	2.5E 02	2.4E 02	1.9E 02	2.1E 02	1.9E 02	2.1E 02	1.9E 02	1.9E 02
1.9E 02	1.8E 02	1.9E 02	1.7E 02	1.7E 02	1.9E 02	1.7E 02	1.7E 02	1.6E 02	1.7E 02
1.6E 02	1.6E 02	1.4E 02	1.4E 02	1.4E 02	1.7E 02	1.6E 02	1.3E 02	1.3E 02	1.5E 02
1.5E 02	1.6E 02	1.3E 02	1.3E 02	1.3E 02	1.6E 02	1.3E 02	1.3E 02	1.1E 02	1.1E 02
1.3E 02	1.2E 02	1.0E 02	1.1E 02	1.2E 02	1.3E 02	1.3E 02	1.3E 02	1.2E 02	1.1E 02
1.1E 02	1.0E 02	1.1E 02	1.2E 02	1.2E 02	9.4E 01	1.0E 02	1.1E 02	1.1E 02	9.5E 01
1.1E 02	1.1E 02	9.6E 01	1.1E 02	1.2E 02	9.2E 01	1.2E 02	1.0E 02	1.0E 02	9.5E 01
1.1E 02	9.9E 01	1.1E 02	1.0E 02	9.9E 01	1.2E 02	9.3E 01	1.0E 02	9.9E 01	9.7E 01
1.1E 02	8.9E 01	1.1E 02	8.7E 01	1.1E 02	8.7E 01	1.0E 02	1.0E 02	1.2E 02	1.0E 02
1.2E 02	1.1E 02	8.9E 01	7.6E 01	9.3E 01	8.0E 01	1.0E 02	1.0E 02	1.0E 02	1.0E 02
8.1E 01	7.9E 01	1.1E 02	6.4E 01	9.8E 01	9.5E 01	9.6E 01	9.3E 01	6.8E 01	8.5E 01
7.2E 01	7.8E 01	1.0E 02	8.6E 01	8.1E 01	8.2E 01	9.7E 01	9.8E 01	8.8E 01	1.0E 02
8.3E 01	9.3E 01	1.0E 02	9.0E 01	8.3E 01	8.2E 01	8.2E 01	8.6E 01	8.7E 01	7.7E 01
6.0E 01	8.1E 01	8.4E 01	7.3E 01	8.2E 01	8.3E 01	9.3E 01	8.5E 01	7.5E 01	7.3E 01
7.9E 01	6.7E 01	8.4E 01	7.7E 01	8.7E 01	9.7E 01	7.3E 01	8.7E 01	9.3E 01	6.7E 01
7.9E 01	6.7E 01	6.9E 01	7.4E 01	7.7E 01	7.7E 01	7.7E 01	7.9E 01	7.1E 01	9.4E 01
7.5E 01	9.4E 01	8.8E 01	7.5E 01	7.3E 01	6.9E 01	7.0E 01	6.4E 01	7.6E 01	7.0E 01
9.1E 01	7.1E 01	9.8E 01	8.4E 01	6.9E 01	6.8E 01	6.0E 01	5.9E 01	8.9E 01	7.3E 01
8.1E 01	6.0E 01	7.1E 01	9.6E 01	6.9E 01	-3.4E 03	-8.9E 05	8.6E 37	-5.8E 22	-1.9E 22
2.5E-01	2.5E-01	2.5E-01	-1.4E 01	-1.1E 42	9.5E-07	2.0E-79	2.8E-72	5.1E-85	3.0E 53
-9.8E-03	5.1E-85	5.1E-85	3.0E 53	-9.8E-03	5.1E-85	5.1E-85	3.0E 53	-9.8E-03	5.1E-85
5.1E-85	3.0E 53	-9.8E-03	5.1E-85	1.3E-79	3.0E 52	-6.6E-80	5.1E-85	1.3E-79	5.0E 52
-6.4E 02	5.1E-85	1.3E-79	3.0E 52	-6.8E-80	5.1E-85	5.1E-85	-3.0E 53	-9.8E-03	5.1E-85

221 250 30  
 WEIGHT RANGES FROM 1.0 TO 10.329 FOR CHAN 250

6 ITERATIONS, DET. OF PART. DERIV. MATRIX = 2.546128D-01

THE FOLLOWING 2 POINTS ARE OUTSIDE J.46 SIGMA

43 124 34 403

SUM OF EXPONENTIALS, Y(I)=P(1)\*EXP(-X(I)/P(2))+...+P(3)

THE WEIGHTED VARIANCE IS 9.44718311D 02

THE UNWEIGHTED SIGMA IS 3.2957916E 01 AND THE UNWEIGHTED SUM OF SQUARES OF THE DEVS IS 2.3679681E 05

GUES OF FINAL VAL OF S.D. OF EXACT LST SQRS EQNS

K K-T H PARAM K-T H PARAM K-T H PARAM FITTED FC TN INPUT DATA

1 6.0000E 02 6.4175D 02 9.3628D 00 1.0430E 02 1.0430E 02

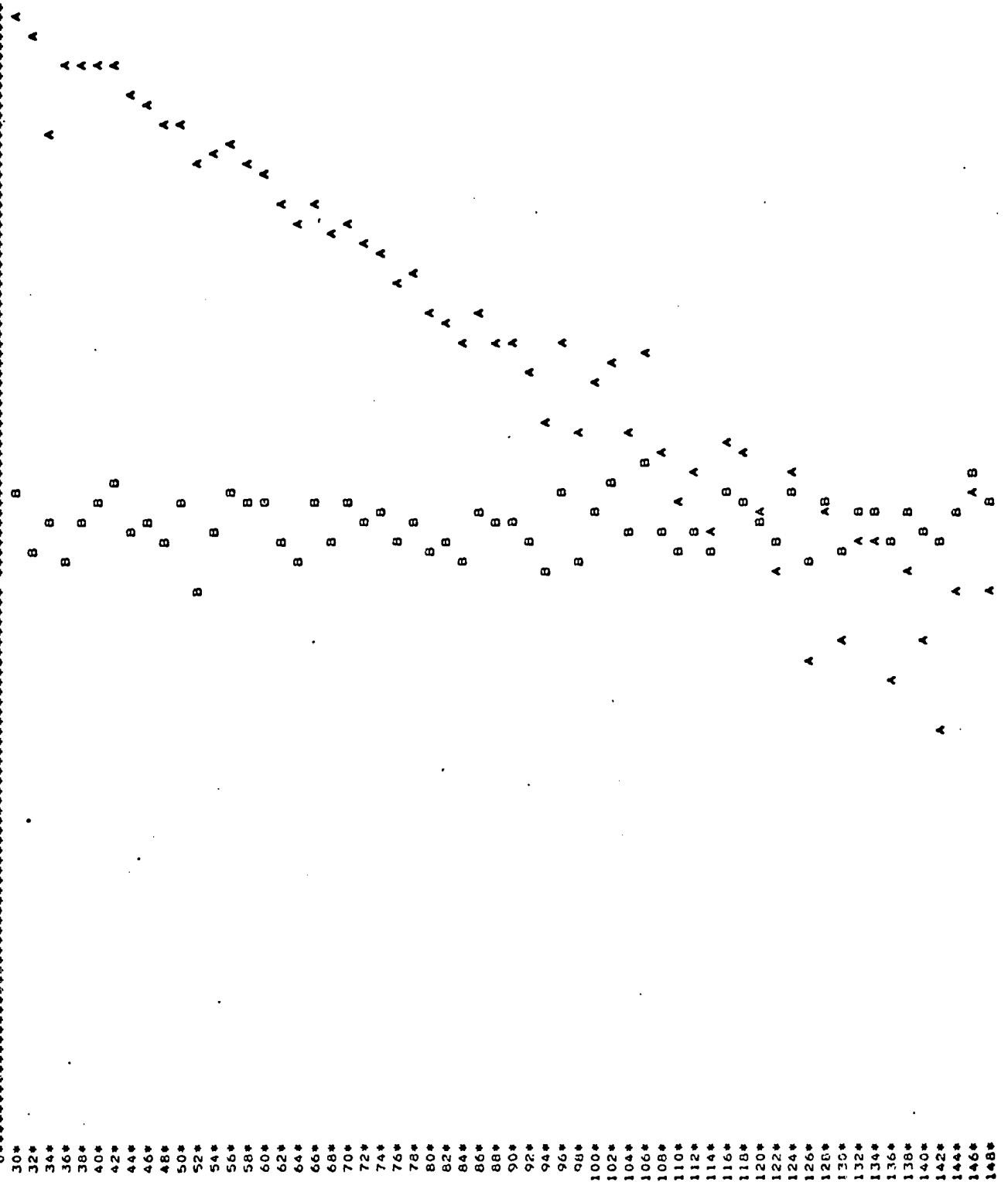
2 4.0000E 01 4.9724D 01 8.5087D-01 9.9167E 03 9.9168E 03

3 7.0500E 01 8.0461D 01 1.0786D 00 1.6512E 05 1.6513E 05

ENTER PLOT INCR IF PLOT IS DESIRED THEN 0 FOR NEXT CASE, 1 TO RERUN WITH NEW PARAMETERS, -1 TO CALL EXIT

2 0

0.674



- 30\*
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- 34\*
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- 142\*
- 144\*
- 146\*
- 148\*

W C PASKE DATE-JUN 30.73 RUN 4  
SAMPLE-N2 ZPS WAVELENGTH=3371.A PRESSURE=400.OMICRONS

CALCULATED AT 14 00 MON. JUL. 02,1973

ENTER NUMBER OF PARAMS. FIRST POINT, LAST POINT, WEIGHT POWER, IPR=0,1,2 OR 3 FOR INCREASED OUTPUT, IM=1 HOLD CONSTANT

3 35 250-1.0 0 0  
ENTER LIFETIMES(SHORT, LONG), COEFFS(SHORT, LONG), CONSTANT  
4.0E 01 1.5E 03 2.0E 02  
CALIBRATION IS 1.66512 NS/CHAN +DR- 0.0078660

35 250  
0.0 0.0 0.0 0.0 1.0E 00 1.3E 03 1.6E 03 1.7E 03 1.6E 03 1.7E 03  
1.7E 03 1.8E 03 1.7E 03 1.7E 03 1.6E 03 1.7E 03 1.6E 03 1.7E 03 1.6E 03 1.7E 03  
1.7E 03 1.6E 03 1.7E 03 1.7E 03 1.5E 03 1.6E 03 1.5E 03 1.5E 03 1.5E 03 1.5E 03  
1.5E 03 1.5E 03 1.4E 03 1.4E 03 1.4E 03 1.3E 03 1.3E 03 1.3E 03 1.3E 03 1.2E 03  
1.2E 03 1.1E 03 1.1E 03 1.1E 03 1.1E 03 1.0E 03 9.9E 02 9.4E 02 9.7E 02 9.0E 02  
8.9E 02 7.8E 02 8.1E 02 8.3E 02 7.4E 02 8.7E 02 7.2E 02 7.9E 02 6.5E 02 6.6E 02  
7.3E 02 6.8E 02 6.7E 02 6.7E 02 6.7E 02 6.7E 02 5.8E 02 5.6E 02 5.9E 02 5.6E 02  
5.5E 02 5.3E 02 4.8E 02 5.4E 02 5.1E 02 4.8E 02 5.0E 02 4.5E 02 4.5E 02 4.7E 02  
4.4E 02 4.5E 02 4.3E 02 4.3E 02 4.5E 02 3.8E 02 4.2E 02 4.0E 02 3.9E 02 4.0E 02  
3.7E 02 3.7E 02 3.4E 02 3.7E 02 3.4E 02 3.3E 02 1.1E 02 3.5E 02 3.0E 02 3.1E 02  
3.7E 02 3.3E 02 3.3E 02 3.3E 02 3.1E 02 3.2E 02 3.1E 02 3.1E 02 2.9E 02 2.7E 02  
8.4E 01 3.0E 02 3.1E 02 2.9E 02 2.7E 02 2.9E 02 2.9E 02 3.0E 02 2.6E 02 2.6E 02  
2.8E 02 2.7E 02 2.8E 02 2.8E 02 2.9E 02 2.6E 02 2.6E 02 2.6E 02 2.4E 02 2.8E 02  
2.5E 02 2.2E 02 2.5E 02 2.7E 02 2.4E 02 2.7E 02 2.6E 02 2.5E 02 2.6E 02 2.4E 02  
2.6E 02 2.5E 02 2.4E 02 2.4E 02 2.7E 02 2.5E 02 2.4E 02 2.7E 02 2.1E 02 2.2E 02  
2.3E 02 2.6E 02 2.4E 02 2.2E 02 2.4E 02 2.4E 02 2.3E 02 2.3E 02 1.2E 02 2.2E 02  
2.5E 02 2.5E 02 2.4E 02 2.2E 02 2.3E 02 2.1E 02 2.4E 02 2.3E 02 2.3E 02 2.5E 02  
2.3E 02 2.2E 02 2.3E 02 2.4E 02 2.3E 02 2.1E 02 2.3E 02 2.0E 02 2.2E 02 2.3E 02  
2.4E 02 2.3E 02 2.3E 02 2.6E 02 2.2E 02 2.6E 02 2.3E 02 2.1E 02 2.3E 02 2.0E 02  
2.0E 02 2.5E 02 2.4E 02 2.3E 02 2.4E 02 2.3E 02 2.0E 02 2.3E 02 2.1E 02 2.0E 02  
1.9E 02 2.1E 02 2.3E 02 2.3E 02 2.0E 02 2.3E 02 2.2E 02 2.1E 02 2.2E 02 2.2E 02  
2.2E 02 2.3E 02 1.9E 02 1.9E 02 2.4E 02 1.9E 02 1.9E 02 2.1E 02 2.4E 02 1.9E 02  
2.2E 02 2.4E 02 2.3E 02 2.0E 02 2.0E 02 1.9E 02 2.3E 02 2.1E 02 2.1E 02 1.9E 02  
2.0E 02 1.8E 02 2.0E 02 2.0E 02 2.0E 02 2.1E 02 1.9E 02 2.1E 02 1.6E 02 1.9E 02  
2.0E 02 1.9E 02 2.0E 02 2.0E 02 2.3E 02 1.9E 02 1.8E 02 2.1E 02 2.0E 02 2.0E 02  
1.9E 02 1.9E 02 1.9E 02 2.1E 02 1.9E 02 -3.4E 03 -8.9E 05 6.6E 37 -5.8E 22 -1.9E 22  
2.5E-01 2.5E-01 2.5E-01 -1.4E 01 -1.1E 42 9.5E-07 2.0E-79 2.8E-72 5.1E-85 3.0E 53  
-9.8E-03 5.1E-85 5.1E-85 3.0E 53 -9.8E-03 5.1E-85 5.1E-85 3.0E 53 -9.8E-03 5.1E-85  
5.1E-85 3.0E 53 -9.8E-03 5.1E-85 1.3E-79 3.0E 52 -6.8E-80 5.1E-85 1.3E-79 5.0E 52  
-6.4E 02 5.1E-85 1.3E-79 3.0E 52 -6.8E-80 5.1E-85 5.1E-85 3.0E 53 -9.8E-03 5.1E-85

216 250 35  
WEIGHT RANGES FROM 1.0 TO 6.613 FOR CHAN 250  
5 ITERATIONS, DET. OF PART. DERIV. MATRIX = 2.5676150-01  
THE FOLLOWING 3 POINTS ARE OUTSIDE 3.46 SIGMA  
97 112 111 R4 159 115  
SUM OF EXPONENTIALS.  $Y(I)=P(1)*EXP(-X(I)/P(2))+...+P(3)$

THE WEIGHTED VARIANCE IS 1.7355181D 03  
THE UNWEIGHTED SIGMA IS 3.2627060E 01 AND THE UNWEIGHTED SUM OF SQUARES OF THE DEVS IS 2.2674394E 05  
GUESS OF FINAL VAL OF S.D. OF EXACT LST SORS EQNS  
K K-TH PARAM K-TH PARAM K-TH PARAM FITTED FCYN INPUT DATA  
1 1.5000E 03 1.1572D 03 1.2953D 01 1.8370E 02 1.8369E 02  
2 4.0000E 01 4.8838D 01 7.0280D-01 3.1906E 04 3.1906E 04  
3 2.0050E 02 2.1023D 02 1.7897D 00 2.8030E 05 2.8031E 05  
ENTER PLOT INCR IF PLOT IS DESIRED THEN 0 FOR NEXT CASE, 1 TO RERUN WITH NEW PARAMETERS, -1 TO CALL EXIT

1.106

