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GRADUATE COLLEGE

MULTI-AREA UNIT COMMITMENT VIA SEQUENTIAL METHOD AND A DC POWER FLOW NETWORK MODEL

A Dissertation

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By

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MULTI-AREA UNIT COMMITMENT VIA SEQUENTIAL METHOD AND A DC POWER FLOW NETWORK MODEL

A Dissertation APPROVED FOR THE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

By

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Abstract

In a deregulated energy industry with transmission open access, a multi-area unit commitment model is needed to simulate the competitive markets in the interconnected energy grid.

This dissertation research extends the single-area sequential bidding thermal unit commitment method to multi-area systems. In lieu of the commonly used linear flow network representation, the proposed extension employs a more accurate DC power flow model to represent the inter-area transmission network.

The sequential bidding unit commitment method employs an iterative procedure. Each iteration consists of two phases - the sequential commitment phase and the price adjustment phase. The principle of the sequential bidding unit commitment method is applicable to multi-area systems. However, in a multi-area application, the evaluation of the hourly useful-spinning-capacity contribution is much more complex than that required in a single-area application. This evaluation together with the multi-area reserve constrained generation dispatch and the estimation of multi-area hourly prices, required in the price adjustment phase, are key tasks associated with this proposed multi-area extension.

Based on the proposed multi-area sequential bidding unit commitment method, a multi-area unit commitment software is implemented and applied to an interconnected four-area system in the southwest part of U.S.A.

1 Introduction

This chapter consists of three sections. Section 1.1 provides an overview of thermal unit commitment problems. Section 1.2 provides a generic review of existing short-term thermal unit commitment methods. Section 1.3 lists development of chapters.

1.1 Thermal Unit Commitment Problems

Unit commitment function, which involves proper modeling of the hour-by-hour operation of electric generating systems, is an important short-term planning function for electric utility systems. The objective of this function is to determine unit commitment and generation scheduling decisions that minimize the operational cost over the planning period, while recognizing pertinent operational constraints. The typical unit commitment period ranges from an hour to several weeks. The system hourly demands, reserve requirements, and units' initial conditions are assumed known over the entire period.

Pertinent operating characteristics and constraints of unit commitment problems vary with simulated systems. A typical unit commitment problem has to model heat rate characteristics, high and low operating limits, minimum-up-time and minimum-down-time constraints, start-up and shutdown characteristics, plant crew constraint, fuel constraints, and emission constraints.

With increased competition, which results in transmission open access, transmission limitations on economical transfer of power from one area to another can have a significant impact on unit commitment and generation schedule. As a result, it is important for a unit commitment model to model the effect of transmission limitations among areas of operation regions.

Detailed descriptions of production cost and operational constraints of the thermal unit commitment function are provided in section 1.1.1 and 1.1.2.

1.1.1 Production Costs

The objective of unit commitment problems is to minimize system total production costs over the study period. Production costs include:

- **Fuel costs**

  For thermal generators dominated system, fuel costs constitute a major portion of the system production costs. Fuel costs are computed by determining the fuel consumption of each fuel supplied to the system multiplied by its fuel price.

- **Start-up and shutdown costs**

  Start-up and shutdown costs reflect the incurred costs when a unit is brought on-lines or off-lines. The shutdown cost is a fixed cost for each shutdown. The start-up cost has a fixed and a variable components. The fixed cost is incurred for each start-up. The variable cost takes into account the fuel required to start the unit.

- **Operating and maintenance costs**

  Labor costs for the operation and maintenance (O&M) of a unit can also be represented as a fixed cost and a variable cost. The fixed cost (S/Hour) is incurred at each on-line hour. The variable cost may be expressed as a fuel related cost ($/MBtu) and/or an output related cost ($/MWh).

- **Emission removal and allowance trade in costs**

  Electric power systems are mandated by the U.S. Clean Air Act Amendments of 1990 (CAAA) to have allowances for sulfur dioxide (SO_2) emission. In order to provide a more flexible compliance environment, the electric power utilities are permitted to trade emission allowances in the open market. Emission removal cost is the cost incurred from removing SO_2 emission by scrubbers.

1.1.2 Operating Constraints

Operating constraints associated with the unit commitment function of thermal systems consist of system constraints, regional constraints, plant constraints, and unit constraints:

- **System hourly energy requirement**

  System hourly energy requirement must be met at all time.

- **System hourly spinning and operating reserve requirements**

  The spinning reserve requirement is to cope with contingencies over a short time interval, typically ten minutes. The operating reserve requirement addresses response to contingencies over a longer time frame, typically thirty minutes. Both reserve requirements must be met at all time.

- **Unit operating limits**

  The operating limits of each unit set the range for the practical operation of a unit.

Within these operating limits, a set of economic limits establishes the normal MW dispatch range of the unit.

- **Prohibited zones**

  Prohibited zone constraints restrict generators from operating within certain specified regions.

- **Unit rate limit**

  Each unit's change in generation from one hour to the next is often restricted by a permissible rate limit. The next hour's MW generation of a unit is limited by the current MW generation level plus or minus the unit's rate limit.

- **Unit start-up and shutdown ramp**

  During the start-up or shutdown of a unit, fixed MW levels may be required to insure a smooth transition. These fixed MW profiles mandate specific generation for each hour during the start-up or shutdown of the unit.

- **Unit minimum-up-time and minimum-down-time constraints**

  Once a unit is off-line, it must remain off-line for a number of hours to meet its minimum-down-time requirement. Likewise, once a unit is on-line, it must remain on-line for a number of hours to satisfy its minimum-up-time requirement.

- **Crew constraint**

  In a power plant consisting of multiple generators, a constraint may be applied to limit the number of units being started within a certain number of hours to reflect the limited crew in the plant.

- **Combined cycle unit** [1]

  Combined cycle units consist of combustion turbines and boilers connected to stream turbines. These units, when available, can not be independently committed and dispatched.

- **Fuel constraints** [2,3,4,5,6,7]

  Fuel constraints include the maximum and minimum fuel consumption limits for each fuel. These constraints can be specified on hourly, daily, weekly or study bases. Fuel constraints may restrict fuel consumption at the unit level, plant level and system level.

- **Emission allowance** [8,9]

  The number of emission allowances consumed during the study period must be less than the sum of the allowance allotment and the traded allowances during the study.

- **CAAA phase-1 under-utilization constraint**

  The CAAA phase-1 under-utilization constraint requires that the group of phase-1 units must consume more than a specified amount of fuel during the study period. Usually the emission allowances and the phase-1 under-utilization requirements are specified on an annual basis.

- **Transmission constraints**

  Transmission constraints impose power flow limits from one area to another area in an interconnected power system.

1.2 Thermal Unit Commitment Methods

In the past, various unit commitment methods have been proposed. Among these methods, the following ones have been widely used in today's energy management systems (EMS) to guide thermal system operation:

- Heuristic methods [2,10].
- Dynamic programming (DP) based methods [10,11,12].
- Lagrangian relaxation (LR) based methods [13,14,15].
- Sequential bidding method [16,17].

1.2.1 Heuristic Methods

Heuristic methods, which reflect the dispatchers' decision processes, are simple and practical. A heuristic method consists of creating a priority list of units and a start-up/shutdown rule for units. At each hour, the commitment strategy at the previous hour will be used when the load remains unchanged, the uncommitted units may be committed according to their priority order when the load increases, and the committed units may be shut down according to their priority order when the load decreases. A simple priority list can be obtained on the basis of the AFLC (Average Full Load Cost) of each unit. Enhancement has been done to the AFLC based priority list by introducing a new index, CUF (Commitment Utilization Factor) [18]. Heuristic methods are computationally efficient, however the solution quality is not as good as more rigorous methods.

1.2.2 Dynamic Programming Based Methods

The dynamic-programming based unit commitment methods (e.g. DP-SC, DP-STC) solve the thermal unit commitment problem in its primal form. The approach used by the primal solution methods resembles decision making in a regulated environment. As a result, the advantage is the ability to maintain solution feasibility and the disadvantage is the curse of dimensionality. The dynamic-programming based unit commitment methods differ from one another in the "static" truncation (i.e. truncate the hourly state space) used to reduce the problem dimension. For instance, the DP-SC (i.e. dynamic-programming-sequential-combination) method evaluates only the sequential combined system states generated from a fixed priority list, and the DP-STC (i.e. dynamic-programming-sequential-truncated-combination) method uses an additional enumeration window to consider more states without a complete enumeration. Both methods apply "dynamic" truncation (i.e. truncate the decision paths over time) to further reduce the number of paths saved at each hour.

1.2.3 Lagrangian Relaxation Based Methods

The Lagrangian relaxation based methods solve the thermal unit commitment problem in its dual form. The approach used by the dual solution methods resembles decision making in a competitive environment. Given the hourly prices over the commitment horizon, the commitment decision of each thermal unit is made independently to maximize its profit. The decentralized commitment decisions are then iteratively coordinated by adjusting the hourly prices. The advantage of Lagrangian relaxation based methods is the problem decomposition resulting from the dual formulation. The disadvantage is that the performance of the price based coordination is often not robust.

1.2.4 Sequential Bidding Method

It is noted that the advantage of the dynamic-programming based methods corresponds with the disadvantage of the Lagrangian-relaxation based methods and vice versa. This is quite logical considering that the primal decision space resembles a regulated decision environment and the dual decision space resembles decision environment with free competition. This observation suggests an alternative unit commitment approach, decision making via "bidding", which could encompass the merits of the dynamic programming based methods (primal feasibility) and the Lagrangian relaxation based methods (dual decomposition). Based on this approach, the sequential bidding unit commitment method sequentially identifies, via "bidding", the most advantageous unit to commit until the system obligations are fulfilled.

This dissertation presents an extension of the sequential bidding unit commitment method to multi-area systems. The objective of a multi-area unit commitment problem is to determine the optimal or a near-optimal commitment strategy for generating units located in multiple areas that are interconnected via transmission lines. In the past, linear flow network model [21] has been widely applied to multi-area production simulation [19,20] and reliability analysis [22]. In a transmission network, the physical flow is governed by the Kirchoff's Current Law (KCL) and the Kirchoff's Voltage Law (KVL).

The commonly used linear flow network model observes only KCL. In order to capture the essence of physical real power flow in multi-area unit commitment and generation dispatch, the DC power flow transmission model [2] is used in this research. This improved network model adds significant complexity to the multi-area unit commitment problem.

1.3 Dissertation Overview

This dissertation is organized into six chapters.

- **Chapter 1 - Introduction**

  Presents the overview of thermal unit commitment problems and review of existing thermal unit commitment methods.

- **Chapter 2 - Problem Statement**

  Presents the mathematical formulation of the multi-area unit commitment problem. The inter-area transmission network is represented by a DC power flow model.

- **Chapter 3 - Multi-Area Sequential Bidding Method**

  Presents the formulation of the proposed extension of the single-area sequential bidding unit commitment method to a multi-area system.

- **Chapter 4 - Software Implementation**

  Describes the software implementation of a multi-area unit commitment model. The multi-area unit commitment model consists of a main program and fifteen subroutines.

- **Chapter 5 - Application**

  Presents sample results of applying the multi-area unit commitment model to a four-area test system in the southwest part of U.S.A.

- **Chapter 6 - Conclusions**

2 Problem Statement

Chapter 2 consists of two sections. Section 2.1 defines the necessary notations for the dissertation and section 2.2 presents the mathematical formulation of the multi-area unit commitment problem.

2.1 Notation

The notations used in the dissertation are defined as follows:

i,j,k	Area indices.
m,n	Unit indices.
t	Hour index.
T	Number of hours in the commitment horizon.
$F_m(.)$	Unit *m*'s input-output cost function in *$/Hr*.
MDT_m	Unit *m*'s minimum down time in *Hr*.
MUT_m	Unit *m*'s minimum up time in *Hr*.
N_a	Number of areas in the multi-area system.
$PR_i(t)$	Area *i*'s *MW* bus-bar load at hour *t*.
$P_m(\mathrm{t})$	Unit *m*'s *MW* generation at hour *t*.
$Pmax_m$	Unit *m*'s maximum permissible generation level.
$Pmin_m$	Unit *m*'s minimum permissible generation level.
$SR_i(t)$	Area *i*'s *MW* spinning reserve contribution.
$S_m(t)$	Unit *m*'s *MW* useful spinning reserve contribution at hour *t*.
$STC_m(.)$	Unit *m*'s start-up cost function in *$*.
$T_{i,j}$	MW transmission capacity from area *i* to area *j*.
$U_m(t)$	Unit *m*'s on-off status at hour t (1 - on-line and 0 - off-line).
$X_i(t)$	Area *i*'s net useful spinning capacity contribution at hour *t* in *MW*.
$Y_i(t)$	Area *i*'s net useful energy capacity contribution at hour *t* in *MW*.
$\Delta Y_i^m(t)$	The hourly useful energy capacity (*MW*) contribution from unit *m* located in area *i*.

$Z_i(t)$	Area i's net useful spinning reserve capacity contribution at hour t in *MW*.
$\Delta Z_i^m(t)$	The hourly useful spinning reserve capacity (*MW*) contribution from unit m located in area i.
Θ_i	Set of must-run units in area i.
Λ_i	Set of areas that are directly connected to area i via transmission links.
Φ_i	Set of must-off units in area i.
Π_i	Set of all units in area i.
$\tau_m(t)$	Number of consecutive hours that unit m has been on-line (if "+") or off-line (if "-") at hour t.
Ω	Set of all areas.
$\beta^{i,j}_k$	Area k's coefficient for the transmission constraint associates with the flow from area i to area j.
$\lambda_i(t)$	Area i's energy price (*$/MW-Hr*) at hour t.
$\delta_i(t)$	Area i's useful spinning reserve price (*$/MW-Hr*) at hour t.
$\gamma_i(t)$	Area i's useful energy capacity price (*$/MW-Hr*) at hour t.
$\gamma_i(t)$	Area i's useful reserve capacity price (*$/MW-Hr*) at hour t.

2.2 Mathematical formulation

A generic multi-area unit commitment problem is mathematically stated as follows:

$$\underset{\underline{U(t)},\underline{P(t)}}{Min} \sum_{i\in\Omega} \sum_{m\in\Pi_i} \sum_{t=1}^{T} \{F_m(P_m(t)) + STC_m(-\tau_m(t-1))[1-U_m(t-1)]\}U_m(t) \tag{1}$$

Subject to

<u>Multi-area energy requirement</u>

$$\sum_{i\in\Omega} \sum_{m\in\Pi_i} P_m(t)U_m(t) = \sum_{i\in\Omega} PR_i(t), \forall t = 1,\ldots,T \tag{2}$$

$$\sum_{k\in\Omega} (\beta^{i,j}_k)[\sum_{m\in\Pi_k} P_m(t)U_m(t) - PR_k(t)] \geq -T_{i,j}, \forall t = 1,\ldots,T, \forall j \in \Lambda_i, \forall i \in \Omega \tag{3}$$

Equation (2) states that the hourly energy generation equals the multi-area hourly energy requirement. Equation (3) states that the multi-area hourly energy transfer must be within the capacity of the inter-area transmission interconnections.

Multi-area spinning reserve requirement

$$\sum_{i\in\Omega}\sum_{m\in\Pi_i} S_m(t)U_m(t)=\sum_{i\in\Omega} SR_i(t),\forall t=1,\ldots,T \tag{4}$$

$$\sum_{\substack{k\in\Omega\\ k\neq i}}(\beta^{i,j}{}_k)[\sum_{m\in\Pi_k}(P_m(t)+S_m(t))U_m(t)-PR_k(t)-SR_k(t)]\geq -T_{i,j}$$
$$\forall t=1,\ldots,T,\forall j\in\Lambda_i,\forall i\in\Omega \tag{5}$$

Equation (4) states that the hourly "useful" (i.e. useful for fulfilling system requirement $\sum SR_i$) spinning reserve contribution equals the multi-area hourly reserve requirement. Equation (5) states that the combined multi-area hourly energy and reserve transfer must be within the capacity of the inter-area transmission interconnections. If Equations (3) and (5) are met simultaneously, the inter-area transmission interconnections are capable of transferring the required energy and reserve under normal and contingency conditions.

Minimum-up-time & minimum-down-time

$$\left.\begin{array}{l} U_m(t)=1,if\,0<\tau_m(t-1)<MUT_m \\ U_m(t)=0,if\,0<-\tau_m(t-1)<MDT_m \end{array}\right.,\forall t=1,\ldots,T,\forall m\in\bigcup_{i\in\Omega}\Pi_i \tag{6}$$

Must-run units & must-off units

$$\left.\begin{array}{l} U_m(t)=1,\forall m\in\bigcup_{i\in\Omega}\Theta_i \\ U_m(t)=0,\forall m\in\bigcup_{i\in\Omega}\Phi_i \end{array}\right.,\forall t=1,\ldots,T \tag{7}$$

Minimum & maximum energy generation

$$Pmin_m(t) \le P_m(t) \le Pmax_m(t), if\, U_m(t) = 1, \forall t = 1, \ldots, T, \forall m \in \bigcup_{i \in \Omega} \Pi_i \tag{8}$$

Useful spinning reserve limits

$$\begin{aligned} &S_m(t) \le Pmax_m - P_m(t) \\ &S_m(t) \le Smax_m, if\, U_m(t) = 1, \forall t = 1, \ldots, T, \forall m \in \bigcup_{i \in \Omega} \Pi_i \end{aligned} \tag{9}$$

For the simplicity of presentation, no additional constraints are included in the formulation of the multi-area unit commitment problem. However, the proposed approach can easily accommodate additional constraints.

In the proposed approach, a DC power flow model is used to represent the multi-area transmission interconnections. The DC power flow model is a significant improvement over the commonly used linear-flow network model because it reflects both the Kirchoff's Voltage Law and Current Law that govern the physical flow of electric power. Based on the DC power flow model, Equations (3) and (5) can be easily generated. Using area *i* as the swing node, area *k*'s coefficient, β_k^{i-j} associated with each transmission flow emanating from area *i*, can be determined via DC power flow equation [2]. Since β_k^{i-j} 's are computed with area *i* as the swing node. Equation (3) and (5) thus have all non-negative coefficients. This can be illustrated via a three-area system described in Table 1.

Example 1

Table 1 Three-Area System

i	j	$X_{i,j}(pu)$	$X_{j,i}(pu)$	$T_{i,j}(MW)$	$T_{j,i}(MW)$
1	2	1.0	1.0	100	100
2	3	1.0	1.0	100	100
3	1	1.0	1.0	100	100

In Table 1, $X_{i,j}$ represents the inductive reactance associated with the transmission link interconnecting area i and area j. For this three-area system, Equation (3) can be written as follows:

i=1 and j=2 (from area 1 to area 2)

$$\tfrac{2}{3}\Big(\sum_{m\in\Pi_2} P_m(t)U_m(t)-PR_2(t)\Big)+\tfrac{1}{3}\Big(\sum_{m\in\Pi_3} P_m(t)U_m(t)-PR_3(t)\Big)\geq -100,\forall t=1,\ldots,T$$

i=1 and j=3 (from area 1 to area 3)

$$\tfrac{1}{3}\Big(\sum_{m\in\Pi_2} P_m(t)U_m(t)-PR_2(t)\Big)+\tfrac{2}{3}\Big(\sum_{m\in\Pi_3} P_m(t)U_m(t)-PR_3(t)\Big)\geq -100,\forall t=1,\ldots,T$$

i=2 and j=1 (from area 2 to area 1)

$$\tfrac{2}{3}\Big(\sum_{m\in\Pi_1} P_m(t)U_m(t)-PR_1(t)\Big)+\tfrac{1}{3}\Big(\sum_{m\in\Pi_3} P_m(t)U_m(t)-PR_3(t)\Big)\geq -100,\forall t=1,\ldots,T$$

i=2 and j=3 (from area 2 to area 3)

$$\tfrac{1}{3}\Big(\sum_{m\in\Pi_1} P_m(t)U_m(t) - PR_1(t)\Big) + \tfrac{2}{3}\Big(\sum_{m\in\Pi_3} P_m(t)U_m(t) - PR_3(t)\Big) \geq -100, \forall t = 1,\ldots,T$$

<u>*i*=3 and *j*=1 (from area 3 to area 1)</u>

$$\tfrac{2}{3}\Big(\sum_{m\in\Pi_1} P_m(t)U_m(t) - PR_1(t)\Big) + \tfrac{1}{3}\Big(\sum_{m\in\Pi_2} P_m(t)U_m(t) - PR_2(t)\Big) \geq -100, \forall t = 1,\ldots,T$$

<u>*i*=3 and *j*=2 (from area 3 to area 2)</u>

$$\tfrac{1}{3}\Big(\sum_{m\in\Pi_1} P_m(t)U_m(t) - PR_1(t)\Big) + \tfrac{2}{3}\Big(\sum_{m\in\Pi_2} P_m(t)U_m(t) - PR_2(t)\Big) \geq -100, \forall t = 1,\ldots,T$$

The coefficients of the first two inequalities are computed with area 1 as the swing node. The coefficients of the third and the fourth inequalities are computed with area 2 as the swing node. The coefficients of the last two inequalities are computed with area 3 as the swing node. Equation (5) has identical coefficients as Equation (3).

3 Multi-Area Sequential Bidding Method

This chapter consists of two sections. Section 3.1 presents a brief review of the sequential bidding method followed by an overview of its application to multi-area problems. Section 3.2 presents the multi-area sequential bidding method.

3.1 Overview

The sequential bidding method has been successfully implemented in energy management systems to guide utility system operations. Its low computational requirements and excellent solution quality have demonstrated the effectiveness of this method. The sequential bidding unit commitment method employs an iterative procedure. Each iteration consists of two phases - the sequential commitment phase and the price adjustment phase. The basic function of each phase is outlined as follows:

Sequential commitment phase - Given the system hourly prices (e.g. energy price, spinning reserve price, useful capacity price), the "most advantageous" unit is sequentially identified and committed until the system obligations are fulfilled. At each sequential bidding decision point, the "most advantageous" unit is identified via a procedure that resembles "bidding". Based on the system hourly prices, the commitment value of each candidate unit is evaluated according to its estimated capacity, energy, and spinning reserve contributions. Based on the commitment values of candidate units, the "most advantageous" candidate unit is identified via equitable economic comparisons [16,17].

Price adjustment phase - Based on the feasible commitment strategy obtained from the sequential commitment phase, the various system hourly prices will be updated to reflect the system hourly marginal (for differentiable cost function) and average incremental (for non-differential cost function) costs.

The sequential bidding method is similar to the LR based methods in taking full advantage of the problem decomposition via hourly prices [16,17]. The sequential bidding method is very different from the LR based methods in their coordination approaches. The LR based methods use price based coordination. The lack of convexity in the unit commitment problems and the dynamic correlation among the hourly prices severely jeopardize the robustness of the price based coordination approach. The sequential bidding method uses a sequential "bidding" procedure to sequentially commit the most advantageous units until system obligations are met. This procedure results in robust global coordination, thus excellent convergence performance.

The principle of the sequential bidding method is applicable to multi-area systems. In the multi-area application, the candidate units are sequentially committed until the multi-area system obligations are fulfilled. The "bidding" evaluation is based on the multi-area hourly prices that reflect the operational economics of the interconnected multi-area system. However, in multi-area applications, the evaluation of useful-spinning-capacity contributions is much more complex than that required in single-area applications. This evaluation together with the multi-area reserve constrained dispatch and evaluation of multi-area hourly prices, required in the price adjustment phase, are key tasks associated with the multi-area extension. Compared to the linear flow network model, the DC power flow network model further complicates these tasks.

3.2 Description of the method

Figure 1 shows the structural flow diagram of the multi-area sequential bidding unit commitment method. In the proposed method, units at each area are organized into groups of similar units. Grouping units is practically justified, because electrical utilities tends to installed units of similar characteristics for a specific type of services (e.g. base load, cycling, peaking). All the units in a group are similar in the following aspects:

- Unit capacity
- Fuel type
- Initial conditions
- Minimum up-time and minimum down-time

Initially, a heuristic method is applied to generate a feasible multi-area unit commitment schedule. In the heuristic method, units are committed individually based on the system-wide priority list (e.g., one that is simply determined by the average-full-load-costs of available generators in the multi-area system). Based on this initial feasible commitment schedule, the multi-area hourly prices are initialized and the sequential commitment phase begins. It is shown in Figure 1 that the most advantageous units are sequentially (one unit at a time) identified via a "bidding" procedure and are committed to system operation until multi-area system obligations are satisfied. The "bidding" process proceeds as follows:

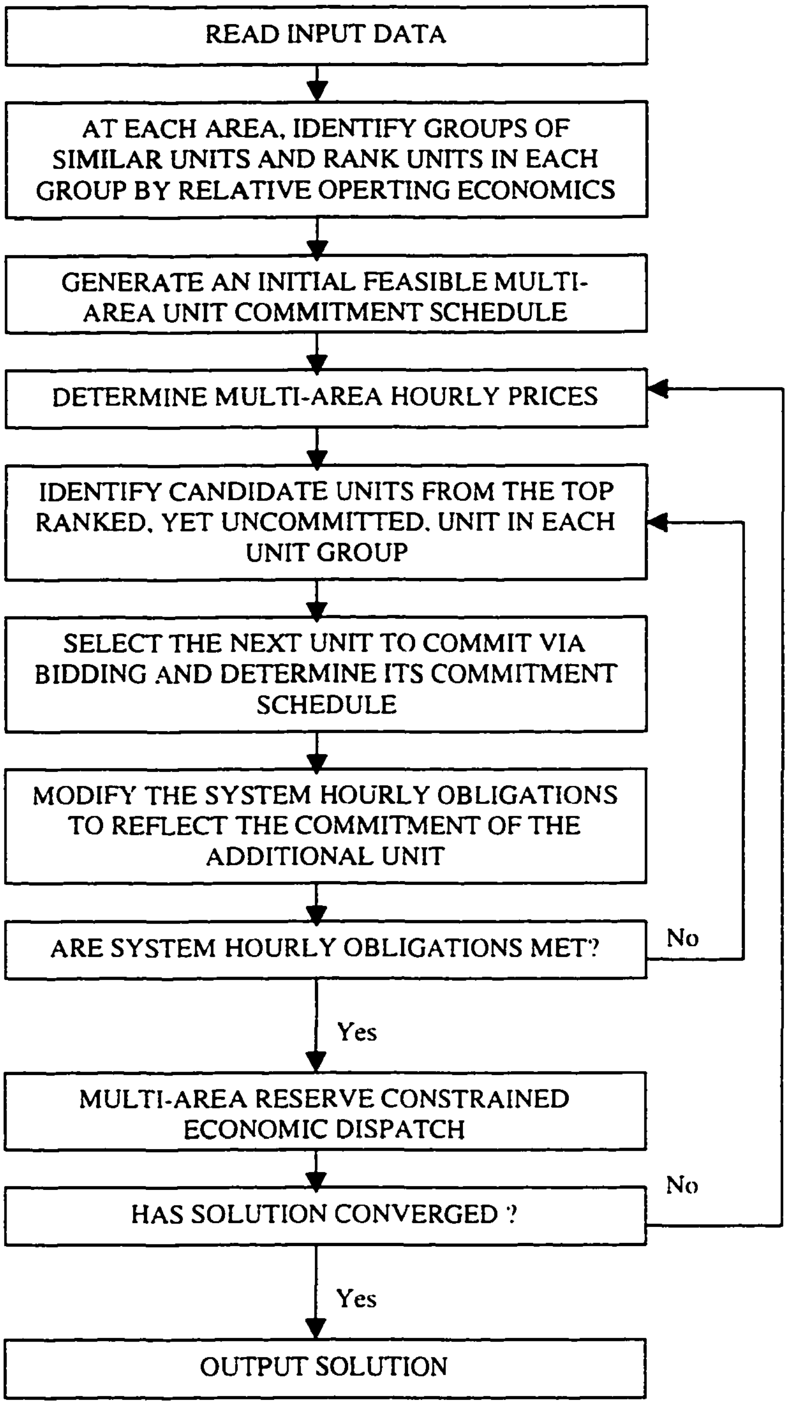

Figure 1 A Structural Flow Diagram of The Multi-Area Sequential Bidding Unit Commitment Method

Step 1. Rank the available, yet uncommitted, units in every group by their relative operating economics.

At each sequential bidding decision point, an optimization problem can be formulated to determine the optimal commitment schedule and the operating economics for each unit. Based on hourly prices, the optimization problem maximizes a unit's total profit subject to unit's local constraints and a commitment rule. The optimization problem for a unit (e.g. unit m at area i) is described as follows:

$$\begin{aligned} \underset{\underline{U_m(t)},\, \underline{P_m(t)}}{Max} \sum_{t=1}^{T} \{ & \gamma_t^e \, U\hat{E}C_m(t) + \gamma_t^s \, U\hat{R}C_m(t) + \lambda_i(t)\, \hat{P}_m(t) + \delta_i(t)\, \hat{S}_m(t) \\ & - F_m(\hat{P}_m(t)) - STC_m(-\tau_m(t-1))[1 - U_m(t-1)] \} \, U_m(t) \end{aligned} \tag{10}$$

Subject to

- Minimum-up-time and minimum-down-time constraints
- Minimum and maximum energy generation limitations
- Useful spinning reserve limits

and the following commitment rule:

- A unit is committed at hour t if it can be committed and its commitment results in useful spinning capacity contribution at hour t.

The evaluation of the hourly energy, $\hat{P}_m(t)$, and spinning reserve, $\hat{S}_m(t)$, contributions of a unit can be determined from area's hourly energy price, $\lambda_i(t)$, and hourly spinning reserve price, $\delta_i(t)$, as follows:

$$\begin{aligned}\frac{\partial F_m(\hat{P}_m(t))}{\partial \hat{P}_m(t)} &\geq \lambda_i(t), \quad \textit{if } \hat{P}_m(t) = Pmin_m \\ &= \lambda_i(t), \quad \textit{if } Pmin_m < \hat{P}_m(t) < Pmax_m - Smax_m \\ &\leq \lambda_i(t), \quad \textit{if } \hat{P}_m(t) = Pmax_m - Smax_m \\ &\geq \lambda_i(t) - \delta_i(t), \quad \textit{if } \hat{P}_m(t) = Pmax_m - Smax_m \\ &= \lambda_i(t) - \delta_i(t), \quad \textit{if } Pmax_m - Smax_m < \hat{P}_m(t) < Pmax_m \\ &\leq \lambda_i(t) - \delta_i(t), \quad \textit{if } \hat{P}_m(t) = Pmax_m \end{aligned} \qquad (11)$$

In multi-area applications, the evaluation of the useful-energy-capacity, $U\hat{E}C_m(t)$, and useful-spinning-reserve-capacity, $U\hat{R}C_m(t)$, contributions is more complex than the evaluation of the energy and spinning reserve contributions. Section 3.2.1 presents an in-depth discussion of the evaluation of the useful-energy-capacity and useful-spinning-reserve-capacity contributions.

It is noticed that, by introducing the commitment rule, the primal feasibility will be guaranteed at the end of the sequential commitment phase. Also, a reduced node dynamic programming technique [16] can be applied to determine the optimal commitment schedule for each unit. The ratio of the total profit of the optimal commitment schedule and the total useful-spinning-capacity contribution reflects the relative operating economics (ROE_m) of a unit, as shown in Equation (12) where the useful-spinning-capacity, $U\hat{S}C_m(t)$, contribution is the sum of the useful-energy-capacity and the useful-spinning-reserve-capacity contributions.

$$ROE_m = \frac{\textit{Optimal profit of unit m}}{\sum_{t=1}^{T} U\hat{S}C_m(t)} \qquad (12)$$

Step 2. Identify the candidate units from the top rank unit in each group.

It is shown in Figure 1 that the candidate units, from which the most advantageous unit is identified, are chosen from the top ranked unit in each group. A candidate unit m is dominated by a candidate unit n if either of the following is satisfied:

- $\sum_{t=1}^{T} U\hat{S}C_n(t) \geq \sum_{t=1}^{T} U\hat{S}C_m(t) \quad \& \quad ROE_n > ROE_m$
- $\sum_{t=1}^{T} U\hat{S}C_n(t) > \sum_{t=1}^{T} U\hat{S}C_m(t) \quad \& \quad ROE_n \geq ROE_m$ (13)

If a unit is dominated by other units, then it is not economical to commit the unit at the current sequential bidding decision point. Therefore, the dominated unit is eliminated from the candidate unit set.

Step 3. Select the most advantageous unit via "bidding" process.

At each sequential bidding decision point, the next unit to commit can not be determined by a direct comparison of the relative operating economics index introduced in step 1 because candidate units often have different amounts of useful spinning capacity. The further comparison is made on an equitable basis of a target amount of useful-spinning-capacity. One obvious target is the maximum of the useful spinning capacities among all the candidate units. If the maximum useful-spinning-capacity of a candidate unit is less than the target capacity, this capacity gap will be supplied by the available generating unit(s) with the highest relative operating economics. As a result, the relative operating economics of supplying the target capacity can be computed. The relative operating economics of the target capacity is defined as an equivalent economics index which directly reflects the relative operating economics of this candidate unit for supplying a target amount of useful spinning capacity. The unit with the highest equivalent economics index is the "next unit to commit". Once a unit is selected for commitment, a simple modification, if necessary, is made to place this unit at its most appropriate commitment position.

After a new unit is committed, the unfulfilled system obligations are updated accordingly. The sequential commitment process continues until the system obligations are fulfilled. Based on the new feasible unit commitment schedule determined by the sequential commitment phase, the multi-area reserve constrained economic dispatch is performed at each hour of the study horizon to compute the hourly energy/spinning reserve contributions from each committed unit and the system operational costs. Section 3.2.2 describes the multi-area reserve constrained economic dispatch calculation. If the solution has converged, the final unit commitment schedule will be summarized and reported. Otherwise, the new feasible unit commitment schedule will be used to update the multi-area hourly prices (hourly capacity prices, hourly energy price, and hourly spinning reserve price). Section 3.2.3 describes the price adjustment phase of the multi-area sequential bidding method. Based on the updated hourly prices, a new iteration will be performed.

3.2.1 Evaluation of Useful-Spinning-Capacity Contributions

At each hour, a candidate unit, if committed, may contribute spinning capacity which is useful for fulfilling capacity obligations in the multi-area system. The useful-spinning-capacity contributions consist of the following components:

- Useful-energy-capacity contribution

- Useful-spinning-reserve-capacity contribution

At each hour, $Pmax_m$ is the maximum possible useful-spinning-capacity contribution from unit m. Its useful-spinning-capacity contribution is the sum of its useful-energy-capacity contribution and its useful-spinning-reserve-capacity contribution. Unit m's hourly useful-spinning-capacity contribution must not exceed $Pmax_m$.

In order to satisfy Equations (2), (3), (4), and (5) multi-area "capacity feasibility" is required. The required capacity feasibility can be expressed in terms of the "net" hourly useful-energy-capacity contribution, $Y_k(t)$, and the "net" hourly useful-spinning-reserve capacity contribution, $Z_k(t)$, of each area. The multi-area capacity feasibility conditions are:

$$\sum_{k \in \Omega} Y_k(t) = 0, \forall t = 1, \ldots, T \tag{14}$$

$$\sum_{k \in \Omega} Z_k(t) = 0, \forall t = 1, \ldots, T \tag{15}$$

$$\sum_{\substack{k \in \Omega \\ k \neq i}} (\beta^{i,j}{}_k)(Y_k(t)) \geq -T_{i,j}, \forall t = 1, \ldots, T, \forall j \in \Lambda_i, \forall i \in \Omega \tag{16}$$

$$\sum_{\substack{k \in \Omega \\ k \neq i}} (\beta^{i,j}{}_k)(Y_k(t) + Z_k(t)) \geq -T_{i,j}, \forall t = 1, \ldots, T, \forall j \in \Lambda_i, \forall i \in \Omega \tag{17}$$

Where the "net" hourly useful-energy-capacity contribution, $Y_k(t)$, equals area k's hourly useful-energy-capacity contribution minus area k's hourly energy capacity requirement, $PR_k(t)$. The "net" hourly useful-spinning-reserve-capacity contribution, $Z_k(t)$, equals area k's hourly useful-spinning-reserve-capacity contribution minus area k's hourly spinning reserve capacity requirement, $SR_k(t)$.

At a decision point of the sequential commitment phase, the hourly useful-spinning-capacity contributions of each candidate unit, if committed, need to be evaluated according to Equations (14), (15), (16), and (17), considering the capacity that has been previously committed in the sequential commitment phase. Let Γ_k denote the set of previously committed units in area k, $\Delta Y_k^m(t)$ and $\Delta Z_k^m(t)$ denote respectively the hourly useful-energy-capacity and useful-spinning-reserve-capacity contributions of the committed unit m in area k. The task is to evaluate the hourly useful-spinning-capacity contributions of a candidate, say unit n in area j. To facilitate further discussion, a linear programming problem, **LP-MCAP** (i.e. MCAP denotes multi-area capacity), is defined:

$$\underset{\underline{\hat{Y}(t)},\ \underline{\hat{Z}(t)}}{Min} \sum_{k \in \Omega} \alpha_k [\hat{Y}_k(t) + \hat{Z}_k(t)] \qquad (18)$$

Subject to

$$\sum_{k \in \Omega} \hat{Y}_k(t) \le 0, \forall t = 1,\ldots,T \qquad (19)$$

$$\sum_{k \in \Omega} \hat{Z}_k(t) \le 0, \forall t = 1,\ldots,T \qquad (20)$$

$$\sum_{\substack{k \in \Omega \\ k \neq i}} (\beta^{i,j}{}_k)(\hat{Y}_k(t)) \ge -T_{i,j}, \forall t = 1,\ldots,T, \forall j \in \Lambda_i, \forall i \in \Omega \qquad (21)$$

$$\sum_{\substack{k \in \Omega \\ k \neq i}} (\beta^{i,j}{}_k)(\hat{Y}_k(t) + \hat{Z}_k(t)) \ge -T_{i,j}, \forall t = 1,\ldots,T, \forall j \in \Lambda_i, \forall i \in \Omega \qquad (22)$$

$$\sum_{m \in \Gamma_k} \Delta Y_k{}^m(t) - PR_k(t) \le \hat{Y}_k(t) \le \sum_{m \in \Pi_k} Pmax_m - PR_k(t), \forall t = 1,\ldots,T, \forall k \in \Omega \qquad (23)$$

$$\sum_{m \in \Gamma_k} \Delta Z_k{}^m(t) - SR_k(t) \le \hat{Z}_k(t) \le \sum_{m \in \Pi_k} Smax_m - SR_k(t), \forall t = 1,\ldots,T, \forall k \in \Omega \qquad (24)$$

Let $\hat{Y}_k(t), \hat{Z}_k(t), \forall k \in \Omega$, denote the solution to the LP-MCAP problem. It can be noted from both Equations (23) and (24) that the lower bounds will change as commitment decisions are made sequentially. As a result, the LP-MCAP solution may require adaptive update to reflect the change in commitment decisions (e.g. additional units are committed). At each sequential bidding decision point, $\hat{Y}_k(t)$ can be considered as area *k*'s "net" hourly useful-energy-capacity allowance and $\hat{Z}_k(t), \forall k \in \Omega$, can be considered as area *k*'s "net" hourly useful-spinning-reserve-capacity allowance. Obviously, these area allowances can be used to guide the allocation of useful-spinning-capacity contributions. In addition to the area allowances, the following system allowances can be defined:

$$\begin{aligned} \Delta\hat{Y}_s(t) &= -\sum_{k\in\Omega}\hat{Y}_k(t) \geq 0 \\ \Delta\hat{Z}_s(t) &= -\sum_{k\in\Omega}\hat{Z}_k(t) \geq 0 \end{aligned} \tag{25}$$

Since the transmission constraints (21) and (22) are satisfied by, $\hat{Y}_k(t)$, $\hat{Z}_k(t)$, $\forall k\in\Omega$, thus $\Delta\hat{Y}_s(t)$ represents the additional system hourly useful-energy-capacity allowance that is not constrained by transmission interconnections. As a result, this system allowance can be used by any area in the multi-area system. Similarly, $\Delta\hat{Z}_s(t)$ represents the additional system hourly useful-spinning-reserve-capacity allowance that can be used by any area in the multi-area system. Based on the area allowances $\hat{Y}_k(t), \hat{Z}_k(t)$ and the system allowances $\Delta\hat{Y}_s(t), \Delta\hat{Z}_s(t)$, the hourly useful-spinning-capacity contributions from the candidate unit "*n*", in area *j*, can be evaluated:

$$\begin{aligned} \Delta Y_j^n(t) &= (\Delta Y_j^n)_1(t) + (\Delta Y_j^n)_2(t) \\ \Delta Z_j^n(t) &= (\Delta Z_j^n)_1(t) + (\Delta Z_j^n)_2(t) \end{aligned} \tag{26}$$

Where

$$(\Delta Z_j{}^n)_1(t) = Min\ \{ Smax_n, \tilde{Z}_j(t), \Delta Z_s(t) \} \tag{27}$$

$$(\Delta Y_j{}^n)_1(t) = Min\ \{[Pmax_n - (\Delta Z_j{}^n)_1(t)], \tilde{Y}_j(t), \Delta Y_s(t) \} \tag{28}$$

$$\begin{aligned}(\Delta Z_j{}^n)_2(t) = Min\ \{&[Smax_n - (\Delta Z_j{}^n)_1(t)], [Pmax_n - (\Delta Z_j{}^n)_1(t) - (\Delta Y_j{}^n)_1(t)],\\ &[\Delta Z_s(t) - (\Delta Z_j{}^n)_1(t)],\ \Delta\tilde{Z}_s(t) \}\end{aligned} \tag{29}$$

$$\begin{aligned}(\Delta Y_j{}^n)_2(t) = Min\ \{&[Pmax_n - (\Delta Z_j{}^n)_1(t) - (\Delta Y_j{}^n)_1(t) - (\Delta Z_j{}^n)_2(t)],\\ &[\Delta Y_s(t) - (\Delta Y_j{}^n)_1(t)],\ \Delta\tilde{Y}_s(t) \}\end{aligned} \tag{30}$$

In equations (27), (28), (29), and (30), $\Delta Y_s(t)$ represents the unfulfilled system hourly energy-capacity obligation and $\Delta Z_s(t)$ represents the unfulfilled system hourly spinning-reserve-capacity obligation. Then

$$\begin{aligned}\Delta Y_s(t) &= \sum_{k\in\Omega} [\ PR_k(t) - \sum_{m\in\Gamma_k} \Delta Y_k{}^m]\\ \Delta Z_s(t) &= \sum_{k\in\Omega} [\ SR_k(t) - \sum_{m\in\Gamma_k} \Delta Z_k{}^m]\end{aligned} \tag{31}$$

In equations (27) and (28), $\tilde{Y}_j(t)$ is the "unused" hourly useful-energy-capacity allowance of area j and $\tilde{Z}_j(t)$ is the "unused" hourly useful-spinning-reserve-capacity allowance of area j. They can be defined as follows:

$$\begin{aligned}\tilde{Y}_j(t) &= Max\ \{ [\hat{Y}_j(t) - (\sum_{m\in\Gamma_j} \Delta Y_j{}^m(t) - PR_j(t))], 0 \}\\ \tilde{Z}_j(t) &= Max\ \{ [\hat{Z}_j(t) - (\sum_{m\in\Gamma_j} \Delta Z_j{}^m(t) - SR_j(t))], 0 \}\end{aligned} \tag{32}$$

The "unused" system hourly useful-energy-capacity allowance, $\Delta\tilde{Y}_s(t)$, and the "unused" system hourly useful-spinning-reserve capacity allowance, $\Delta\tilde{Z}_s(t)$, used by Equations (29) and (30), can be defined:

$$\Delta\tilde{Y}_s(t) = \Delta\hat{Y}_s(t) - \sum_{k\in\Omega} Max\ \{ [\sum_{m\in\Gamma_k} \Delta Y_k{}^m(t) - PR_k(t)] - \hat{Y}_k(t), 0\}$$

$$\Delta\tilde{Z}_s(t) = \Delta\hat{Z}_s(t) - \sum_{k\in\Omega} Max\ \{\ [\sum_{m\in\Gamma_k} \Delta Z_k{}^m(t) - SR_k(t)] - \hat{Z}_k(t), 0\} \tag{33}$$

Equations (27) and (28) are used to allocate area allowances, $\hat{Y}_j(t), \hat{Z}_j(t)$, and Equations (29) and (30) are used to allocate system allowances, $\Delta\hat{Y}_s(t)$, and $\Delta\hat{Z}_s(t)$. In either case, the allocation of hourly useful-spinning-reserve-capacity contribution, $\Delta Z_j{}^n(t)$, assumes a higher priority than the allocation of hourly useful-energy-capacity contribution, $\Delta Y_j{}^n(t)$. This allocation priority is assumed because the spinning reserve capacity is more restrictive than the energy capacity (e.g. Unit n's maximum energy capacity is $Pmax_n$ but its maximum spinning reserve capacity is only $Smax_n$ that is usually less than $Pmax_n$).

It was previously noted that the LP-MCAP solution may need to be adaptively updated to reflect the change in sequential commitment decisions (e.g. additional units are committed). From the computational point of view, we like to minimize the number of solution updates required. As a result, we need to establish a criterion for identifying the need of solution update. The need of solution update occurs when the current area allowances, $\hat{Y}_j(t)$, $\hat{Z}_j(t)$, and system allowances, $\Delta\hat{Y}_s(t)$, $\Delta\hat{Z}_s(t)$, fail to result in conclusive evaluation. The evaluation of the useful-spinning-capacity contributions of candidate unit n will be inconclusive if either of the following two cases holds:

<u>Case 1</u>

$$\begin{aligned} &\Delta Y_j{}^n(t) = \tilde{Y}_j(t) + \Delta\tilde{Y}_s(t) \quad \& \\ &\Delta Y_j{}^n(t) < Pmax_n - \Delta Z_j{}^n(t) \quad \& \\ &\Delta Y_j{}^n(t) < \Delta Y_s(t) \end{aligned} \tag{34}$$

In this case, the hourly useful-energy-capacity contribution of unit *n* is limited by the LP-MCAP solution $\hat{Y}_k(t), \forall k \in \Omega$.

Case 2

$$\begin{aligned} &\Delta Z_j{}^n(t) = \tilde{Z}_j(t) + \Delta\tilde{Z}_s(t) \quad \& \\ &\Delta Z_j{}^n(t) < Min\{Pmax_n - \Delta Y_j{}^n(t), Smax_n\} \quad \& \\ &\Delta Z_j{}^n(t) < \Delta Z_s(t) \end{aligned} \tag{35}$$

In this case, the hourly useful-spinning-reserve-capacity contribution of unit *n* is limited by the LP-MCAP solution $\hat{Z}_k(t), \forall k \in \Omega$.

In either case, the LP-MCAP solution needs to be updated. The objective of the solution update is to maximize area *j*'s allowances, $\hat{Y}_j(t)$ and $\hat{Z}_j(t)$. This objective can be accomplished by setting α_j significantly smaller than other α's in Equation (18) and solve the LP-MCAP problem.

Example 2

Consider the three-area system described in Table 1. The coefficients associated with the transmission constraints, (21) and (22), are given in Example 1. It is assumed that a set of units have already been committed in the sequential commitment phase, and the capacity contributions from these committed units result in the following:

Area 1

Area 1's unfulfilled capacity obligations at hour *t* are:

$$PR_1(t) - \sum_{m \in \Gamma_1} \Delta Y_1^m(t) = 200\ MW$$
$$SR_1(t) - \sum_{m \in \Gamma_1} \Delta Z_1^m(t) = 20\ MW$$

Area 2

Area 2's unfulfilled capacity obligations at hour *t* are:

$$PR_2(t) - \sum_{m \in \Gamma_2} \Delta Y_2^m(t) = 100\ MW$$
$$SR_2(t) - \sum_{m \in \Gamma_2} \Delta Z_2^m(t) = 10\ MW$$

Area 3

Area 3's unfulfilled capacity obligations at hour *t* are:

$$PR_3(t) - \sum_{m \in \Gamma_3} \Delta Y_3^m(t) = -180\ MW$$
$$SR_3(t) - \sum_{m \in \Gamma 3} \Delta Z_3^m(t) = 0\ MW$$

At hour *t*, the useful-energy-capacity contribution from the committed units in area 3 is 180 MW more than area 3's energy obligation, $PR_3(t)$, and the useful-spinning-reserve-capacity contribution from the committed units is area 3 equals area 3's reserve obligation, $SR_3(t)$.

At the beginning of the sequential commitment phase, the LP-MCAP problem was solved by setting $\alpha_1 = \alpha_2 = \alpha_3 = 1.0$. The solutions are:

$$\hat{Y}_k(t) = -100\ MW, \quad \forall\ k = 1, 2, 3$$
$$\hat{Z}_k(t) = 0\ MW, \quad \forall\ k = 1, 2, 3$$

Since the beginning of the sequential commitment phase, these solutions have been used to guide the allocation of useful-spinning-capacity contributions and the need for solution update has not been identified. Based on the LP solution and Equations (25), (31), (32) and (33), the following can be determined:

$\Delta\tilde{Y}_s(t) = 20\ MW;\quad \Delta\tilde{Z}_s(t) = 0\ MW$

$\Delta Y_s(t) = 120\ MW;\quad \Delta Z_s(t) = 30\ MW$

$\tilde{Y}_1(t) = 100\ MW;\quad \tilde{Z}_1(t) = 20\ MW$

$\tilde{Y}_2(t) = 0\ MW;\quad \tilde{Z}_2(t) = 10\ MW$

$\tilde{Y}_3(t) = 0\ MW;\quad \tilde{Z}_3(t) = 0\ MW$

This example considers a candidate unit *n* with $Pmax_n$ of 130 MW and $Smax_n$ of 30 MW. In order to illustrate different aspects of the evaluation procedure, two scenarios are evaluated. Scenario A assumes that the candidate unit is located in area 1 and scenario B assumes that the candidate unit is located in area 2.

<u>Scenario A - Unit *n* is located in area 1</u>

- Based on Equations (27), (28), (29), (30), determine:

$$(\Delta Z_1^{\,n})_1(t) = Min\{30, 20, 30\} = 20\ MW$$
$$(\Delta Y_1^{\,n})_1(t) = Min\{(130-20), 100, 120\} = 100\ MW$$
$$(\Delta Z_1^{\,n})_2(t) = Min\{30-20, 130-20-100, 30-20, 0\} = 0\ MW$$
$$(\Delta Y_1^{\,n})_2(t) = Min\{(130-20-100), (120-100), 20\} = 10\ MW$$

- Based on Equation (26), determine:

$$\Delta Y_1^n(t) = 100 + 10 = 110\ MW$$
$$\Delta Z_1^n(t) = 20 + 0 = 20\ MW$$

At hour t, the total useful-spinning-capacity contribution from the candidate unit is 130 MW that equals its maximum capacity. The evaluation is thus conclusive and no solution update is required.

Scenario B - Unit *n* is located in area 2

- Based on Equations (27), (28), (29), (30), determine:

$$(\Delta Z_2^n)_1(t) = Min\{30, 10, 30\} = 10\ MW$$
$$(\Delta Y_2^n)_1(t) = Min\{(130-10), 0, 120\} = 0\ MW$$
$$(\Delta Z_2^n)_2(t) = Min\{(30-10), (130-10-0), (30-10), 0\} = 0\ MW$$
$$(\Delta Y_2^n)_2(t) = Min\{(130-10-0-0), (120-0), 20\} = 20\ MW$$

- Based on Equation (26), determine:

$$\Delta Y_2^n(t) = 0 + 20 = 20MW$$
$$\Delta Z_2^n(t) = 10 + 0 = 10MW$$

At hour t, the total useful spinning capacity contribution from the candidate unit is computed to be 30 MW. It can be noted that both Equations (34) and (35) are met. As a result, the LP solution needs to be updated. The LP-MCAP problem is solved with $\alpha_1 = \alpha_3 = 1$ and $\alpha_2 = 0.1$ and the lower bounds in Equation (23) and (24) being set to reflect the capacity contributions from the previously committed units. The updated LP-MCAP solution is:

$\hat{Y}_1(t) = -120\ MW;\quad \hat{Y}_2(t) = -60\ MW;\quad \hat{Y}_3(t) = 180\ MW$

$\hat{Z}_1(t) = 0\ MW;\quad \hat{Z}_2(t) = 0\ MW;\quad \hat{Z}_3(t) = 0\ MW$

Based on the LP-MCAP solution and Equations (25), (31), (32) and (33), the following can be determined:

$\Delta\tilde{Y}_s(t) = 0\ MW;\quad \Delta\tilde{Z}_s(t) = 0\ MW$

$\Delta Y_s(t) = 120\ MW;\quad \Delta Z_s(t) = 30\ MW$

$\tilde{Y}_1(t) = 80\ MW;\quad \tilde{Z}_1(t) = 20\ MW$

$\tilde{Y}_2(t) = 40\ MW;\quad \tilde{Z}_2(t) = 10\ MW$

$\tilde{Y}_3(t) = 0\ MW;\quad \tilde{Z}_3(t) = 0\ MW$

Based on the parameters determined above, the useful-spinning-capacity contributions of unit *n* can be evaluated:

- Based on Equations (27), (28), (29), (30), determine:

  $(\Delta Z_2{}^n)_1(t) = Min\ \{30, 10, 30\} = 10\ MW$

  $(\Delta Y_2{}^n)_1(t) = Min\ \{(130-10), 40, 120\} = 40\ MW$

  $(\Delta Z_2{}^n)_2(t) = Min\ \{(30-10), (130-10-40), (30-10), 0\} = 0\ MW$

  $(\Delta Y_2{}^n)_2(t) = Min\ \{(130-50), (120-40), 0\} = 0\ MW$

- Based on Equation (26), determine:

  $\Delta Y_2{}^n(t) = 40 + 0 = 40 MW$

  $\Delta Z_2{}^n(t) = 10 + 0 = 10 MW$

At hour t, the total useful spinning capacity contribution of the candidate unit is computed to be 50 MW that is 20 MW higher than the result obtained before the LP solution is update.

It is important to note that it is not necessary to solve the LP-MCAP problem at each hour over the commitment horizon. At two different hours t_1 and t_2, the difference between the associated LP problems lies in the bounds described by Equations (23) and (24). If the bounds are not binding, the LP solution at hour t_1 will be identical as that at hour t_2. This property reduces the computational requirements a great deal.

3.2.2 Multi-Area Reserve Constrained Economic Dispatch

Based on the feasible multi-area unit commitment schedule determined in the sequential commitment phase, multi-area reserve constrained economic dispatch calculation is required at every hour to determine the following:

- Hourly energy generation of each on-line unit, $P_m(t)$.
- Hourly spinning reserve contribution of each on-line unit, $S_m(t)$.

Define the area energy generation, $G_{k,1}(t)$, $G_{k,2}(t)$, and area useful spinning reserve, $R_k(t)$, as follows:

$$\begin{aligned}
&G_{k,1}(t) = \sum_{m\in\Pi_k} Min\,\{P_m(t), [Pmax_m - Smax_m]\}\,U_m(t) \\
&G_{k,2}(t) = [\sum_{m\in\Pi_k} P_m(t) U_m(t)] - G_{k,1}(t) \quad ,\forall t = 1,\ldots,T, \forall k \in \Omega \qquad (36) \\
&R_k(t) = \sum_{m\in\Pi_k} S_m(t)\,U_m(t)
\end{aligned}$$

Area k's energy generation is the sum of $G_{k,1}(t)$ and $G_{k,2}(t)$. $G_{k,1}(t)$ represents the portion of area k's energy generation that does not impact its spinning reserve contribution and $G_{k,2}(t)$ denotes the portion of area k's energy generation that impacts its spinning reserve contribution. The multi-area reserve constrained economic dispatch problem can then be formulated as a linear programming problem, **LP-MEDC** (i.e. MEDC denotes multi-area economic dispatch calculation):

$$\underset{\underline{G(t)},\ \underline{R(t)}}{Min} \sum_{k\in\Omega} c_{k,1}G_{k,1}(t)+c_{k,2}G_{k,2}(t) \tag{37}$$

Subject to

Multi-area energy requirement

$$\sum_{k\in\Omega}(G_{k,1}(t)+G_{k,2}(t))=\sum_{k\in\Omega}PR_k(t) \tag{38}$$

$$\sum_{\substack{k\in\Omega\\ k\neq i}}(\beta^{i-j}{}_k)[\sum_{m\in\Pi_k}(G_{k,1}(t)+G_{k,2}(t))-PR_k(t)]\geq -T_{i,j},\quad \forall j\in\Lambda_i,\quad \forall i\in\Omega \tag{39}$$

Multi-area spinning reserve requirement

$$\sum_{k\in\Omega}R_k(t)=\sum_{k\in\Omega}SR_k(t) \tag{40}$$

$$\sum_{\substack{k\in\Omega\\ k\neq i}}(\beta^{i-j}{}_k)[\sum_{m\in\Pi_k}(G_{k,1}(t)+G_{k,2}(t)+R_k(t))-PR_k(t)-SR_k(t)]\geq -T_{i,j}, \tag{41}$$

$$\forall j\in\Lambda_i,\quad \forall i\in\Omega$$

Minimum & maximum energy generations of $G_{k,1}(t)$

$$\sum_{m\in\Pi_k}Pmin_m\,U_m(t)\leq G_{k,1}(t)\leq \sum_{m\in\Pi_k}Max\,\{(Pmax_m-Smax_m),\,Pmin_m\}\,U_m(t),\quad \forall k\in\Omega \tag{42}$$

Minimum & maximum energy generations of $G_{k,2}(t)$

$$0 \le G_{k,2}(t) \le \sum_{m\in\Pi_k} Min\,\{Smax_m, (Pmax_m - Pmin_m)\}\, U_m(t), \quad \forall k \in \Omega \tag{43}$$

Spinning reserve limit

$$G_{k,2}(t) + R_k(t) \le \sum_{m\in\Pi_k} Smax_m\, U_m(t), \quad \forall k \in \Omega \tag{44}$$

This formulation requires a minimum number of variables (three variables per area). Each on-line unit is represented by multiple blocks with constant incremental costs (i.e. piece wise linear input-output cost function). In each area, all the blocks associated with the on-line units are organized as follows:

- All the actively dispatchable blocks associated with $G_{k,1}(t)$ are sorted in ascending order of their effective incremental costs (incremental cost adjusted by the associated penalty factor).
- All the actively dispatchable blocks associated with $G_{k,2}(t)$ are sorted in ascending order of their effective incremental costs.

If only one variable, $G_k(t)$ (i.e. $G_{k,1}(t) + G_{k,2}(t)$), is used for each area, then the block incremental cost does not completely determine the block dispatch priority because different blocks may have different impacts on the fulfillment of area spinning reserve requirement. As a result, it may be necessary to skip some blocks with low incremental costs and dispatch higher-cost blocks to provide adequate spinning reserve. The separation into two variables, $G_{k,1}(t)$ and $G_{k,2}(t)$, can overcome this difficulty. Among all the blocks associated with either variable, the block incremental cost completely determines the block dispatch priority. Based on this formulation, the reduced-basis method [23] can be used to efficiently solve this multi-area reserve constrained dispatch problem. This method adaptively update the cost coefficients, $c_{k,1}$ and $c_{k,2}$, based on the effective incremental cost of the block being considered.

The hourly energy generation and spinning reserve contribution of each on-line unit can be determined from the solution of the LP-MEDC problem.

3.2.3 Evaluation of Multi-Area Hourly Prices

In the parameter adjustment phase, the following multi-area hourly prices need to be determined:

- Multi-area hourly energy prices, $\lambda_k(t)$, $\forall k \in \Omega$.
- Multi-area hourly spinning reserve prices, $\delta_k(t)$, $\forall k \in \Omega$.
- Multi-area hourly useful-energy-capacity prices, $\gamma_k^e(t)$, $\forall k \in \Omega$.
- Multi-area hourly useful-spinning-reserve-capacity prices, $\gamma_k^s(t)$, $\forall k \in \Omega$.

The feasible unit commitment strategy, determined in each iteration, is used to update the hourly prices. The hourly prices to used in the next iteration are convex combination of the new hourly prices and the old hourly prices. Based on the feasible commitment strategy, the new prices can be determined as follows:

Evaluation of $\lambda_k(t)$and $\delta_k(t)$.$\forall k \in \Omega$

Let $[B_{g,t}]$ denote the $3N_a \times 3N_a$ basis matrix associated with the optimal solution of the LP-MEDC problem at hour t. The elements of this matrix are the coefficients of the binding constraints. The binding constraints consist of binding system constraints and binding area constraints. Each of the area constraints, (42), (43), (44), involves only variables associated with one area, and each of the system constraints, (38), (39), (40), (41), involves variables associated with more than one areas. The matrix $[B_{g,t}]$ is arranged such that the rows associated with the binding system constraints are placed before the rows associated with the binding area constraints. Then

$$\left[B_{g,t}\right]=\begin{bmatrix}\left[B_{g,t}{}^{1,1}\right] & \left[B_{g,t}{}^{1,2}\right]\\ \left[B_{g,t}{}^{2,1}\right] & \left[B_{g,t}{}^{2,2}\right]\end{bmatrix} \tag{45}$$

In equation (45), $[B^{1,1}_{g,t}]$, $[B^{1,2}_{g,t}]$ are the sub matrices associated with the binding system constraints, $[B^{2,1}_{g,t}]$ and $[B^{2,2}_{g,t}]$ are the sub matrices associated with the binding area constraints. Let $\underline{c}_{g,t}$ represents the column vector of cost coefficients associated with the optimal solution. In the vector, $\underline{c}_{g,t}$, all the elements associated with variables, $R_k(t)$, $\forall k \in \Omega$, are zero. Then the Lagrangian multiplier vector, $\underline{\mu}_{g,t}$, associated with the binding constraints can be computed as follows:

$$(\underline{\mu}_{g,t})^T=(\underline{c}_{g,t})^T[B_{g,t}]^{-1} \tag{46}$$

In Equation (45) and the number of this section, superscript T denotes the transpose of a vector or a matrix. Let $\underline{\mu}^1_{g,t}$ denote the sub vector of $\underline{\mu}_{g,t}$ that is associated with the binding system constraints. When "third-party" wheeling charge is considered for a transmission path, the charge can be reflected by modifying the Lagrangian multiplier vector, $\underline{\mu}_{g,t}$. Based on the Lagrangian vector, $\underline{\mu}_{g,t}$, the vector of hourly marginal energy and spinning reserve prices, $\underline{v}_{g,t}$, can be determined as follows:

$$(\underline{v}_{g,t})^T=(\underline{\mu}_{g,t}{}^1)^T\left[[B_{g,t}{}^{1,1}]\ \ [B_{g,t}{}^{1,2}]\right] \tag{47}$$

In the vector, $\underline{v}_{g,t}$, the element associated with $G_{k,2}(t)$ is $\lambda_k(t)$ and the element associated with $R_k(t)$ is $\delta_k(t), \forall\, k \in \Omega$.

Evaluation of $\gamma_k^e(t)$ and $\gamma_k^s(t), \forall k \in \Omega$

Let $[B_{c,t}]$ denote the $2N_a \times 2N_a$ basis matrix associated with the useful capacity allocation solution, $\hat{Y}_k(t), \hat{Z}_k(t), \forall\, k \in \Omega$. The elements of the matrix are the coefficients of the binding constraints. The binding constraints consist of binding system constraints and binding area constraints. The system constraints of the capacity allocation problem are Equations (14), (15), (16) and (17). The area constraints contributions cannot exceed its committed capacity. The matrix $[B_{c,t}]$ is arranged such that the rows associated with the binding system constraints are placed before the rows associated with the binding area constraints.

$$[B_{c,t}] = \begin{bmatrix} [B_{c,t}^{1,1}] & [B_{c,t}^{1,2}] \\ [B_{c,t}^{2,1}] & [B_{c,t}^{2,2}] \end{bmatrix} \tag{48}$$

In Equation (48), $[B_{c,t}^{1,1}], [B_{c,t}^{1,2}]$ are the sub-matrices associated with the binding system constraints, $[B_{c,t}^{2,1}]$ and $[B_{c,t}^{2,2}]$ are the sub-matrices associated with the binding area constraints. Let $\underline{c}_{c,t}$ represents the cost vector containing the average incremental useful-energy-capacity cost and average incremental useful-spinning-reserve-capacity cost of each area at hour *t*. The average incremental useful-spinning-capacity costs of an area can be computed according to the procedure outlined in [17]. Then the Lagrangian multiplier vector, $\underline{\mu}_{c,t}$ associated with the binding constraints can be computed as follows:

$$(\underline{\mu}_{c,t})^T = (\underline{c}_{c,t})^T \, [B_{c,t}]^{-1} \tag{49}$$

Let $\underline{\mu}_{c,t}^{1}$ denote the sub-vector of $\underline{\mu}_{c,t}$ that is associated with the binding system constraints. Then the vector of average hourly incremental useful-spinning-capacity prices, $\underline{v}_{c,t}$, can be determined as follows:

$$(\underline{v}_{c,t})^T = (\underline{\mu}_{c,t}^{1})^T \left[[B_{c,t}^{1,1}][B_{c,t}^{1,2}] \right] \tag{50}$$

In the vector, $\underline{v}_{c,t}$, the element associated with $\hat{Y}_k(t)$ is $\gamma_k^r(t)$ and the element associated with $\hat{Z}_k(t)$ is $\gamma_k^s(t)$, $\forall\, k \in \Omega$.

4 Software Implementation

A comprehensive computer software has been developed based on the proposed multi-area unit commitment method. This chapter consists of two sections. Section 4.1 describes the software structure. Section 4.2 outlines each module.

4.1 Software Structure

Figure 2 illustrates the structure of the multi-area sequential bidding unit commitment software. The multi-area sequential bidding unit commitment model consists of one main program and 15 subroutines.

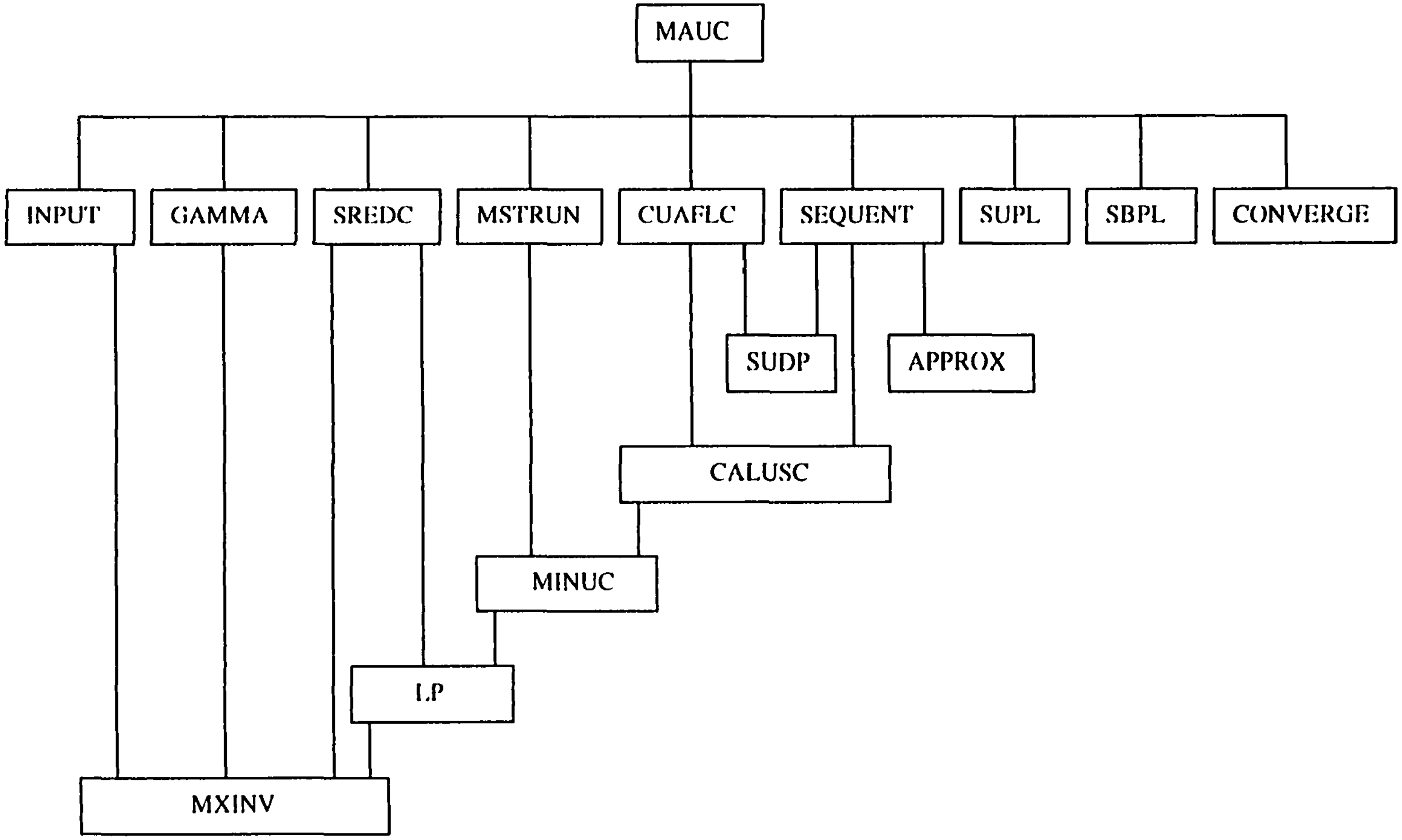

Figure 2 Multi-Area Unit Commitment Software Structure

4.2 Description of Software Modules

4.2.1 Program MAUC

The program MAUC controls the execution of the multi-area unit commitment software. The program MAUC proceeds as follows:

- Calls the subroutine INPUT to read and prepare data.
- Calls the subroutine SUPL to determine unit priority order.
- Calls the subroutine MSTRUN to determine the useful-spinning-reserve-capacity. useful-energy-capacity and useful-spinning-capacity for must-run units.
- Determines unit commitment schedule by calling either the subroutine CUAFLC or the subroutine SEQUENT. The program MAUC uses an iterative process to determine the optimal unit commitment schedule. It calls subroutine CUAFLC to determine a feasible unit commitment schedule by a fixed-priority-order based heuristic method at the first iteration and it calls the subroutine SEQUENT to determine the unit commitment schedule via sequential bidding processes at all other iterations.
- Calls the subroutine SBPL to sort unit generation blocks in the ascending order of their respective incremental costs for all the committed units.
- Calls the subroutine SREDC to determine the hourly generating levels of committed units and the system and area hourly marginal energy and spinning reserve costs.
- Calls the subroutine GAMMA to determine the hourly average incremental capacity costs for each area.

- Calls the subroutine CONVERGE to check the solution convergence and to write the optimal solution to output files. The program MAUC terminates if solution converges or the maximum allowable number of iterations has been reached.

4.2.2 Subroutine INPUT

The subroutine INPUT retrieves and prepares data for the multi-area sequential bidding unit commitment software. The subroutine INPUT proceeds as follows:

- Reads numbers of hours and numbers of areas in the study system.
- Reads transmission line data and formulates transmission line constraint equations using the DC load flow model.
- Reads numbers of units and numbers of fuels for each area.
- Reads the identification, input and output functions (in the piece wise linear format), fuel identifications, maximum spinning reserve limit, initial on-line or off-line hours, minimum-up-time and minimum-down-time constraints and start-up cost for each unit.
- Calculates the first free decision hour and the average-full-load-cost (AFLC) for each unit.
- Calculates the size and the incremental cost of each generation block. Sets a flag to a block if it can provide both energy and spinning reserve to the system.
- Reads contract price for each fuel.
- Reads hourly load data for each area.

4.2.3 Subroutine SUPL

The subroutine SUPL sorts units in the ascending order of their average-full-load-cost (AFLC) to determine their respective priority commitment order.

4.2.4 Subroutine MSTRUN

The subroutine MSTRUN calculates the hourly useful-energy-capacity, useful-spinning-reserve-capacity and useful-spinning-capacity of each must-run unit in following sequences:

- Calculates the hourly maximum useful-energy-capacity, useful-spinning-reserve capacity and the useful-spinning-capacity for each area. The hourly maximum useful-energy-capacity of an area is the sum of the hourly maximum energy capacities of units in the area. The hourly maximum useful-spinning-reserve-capacity of an area is the sum of the hourly spinning reserve capacities of units in the area.
- The hourly maximum net useful-energy-capacity of an area is the hourly maximum useful-energy-capacity minus the hourly energy requirement of the area. The hourly maximum net useful-spinning-reserve-capacity is the hourly maximum useful-spinning-reserve-capacity minus the hourly spinning reserve requirement of the area. The hourly maximum net useful-spinning-capacity is the hourly maximum useful-spinning-capacity minus the hourly energy and spinning reserve requirements of the area.
- Calls the subroutine MINUC to determine the hourly useful-energy-capacity and useful-spinning-reserve-capacity allowances for each area.
- Calculates the hourly useful-energy-capacity and useful-spinning-reserve-capacity allowances of the system by summing the hourly useful-energy-capacity and useful-spinning-reserve-capacity allowances over all areas.
- Calls the subroutine CALUSC to calculate the useful-energy-capacity and useful-spinning-reserve-capacity for each must-run unit. Updates the system hourly unfulfilled energy and spinning reserve capacity requirements and each area's

hourly net useful-energy-capacity and useful-spinning-reserve-capacity based on the useful-energy-capacity and useful-spinning-reserve-capacity contributions of must-run units.

4.2.5 Subroutine CUAFLC

The subroutine CUAFLC sequentially evaluates and commits units based on their AFLC based priority order until the system hourly energy capacity and spinning reserve capacity requirements are met. For each unit being evaluated, the subroutine CUAFLC performs the following:

- Calculates the hourly useful-energy-capacity and useful-spinning-reserve-capacity of the unit. The unit will be committed only if it can provide positive useful-energy-capacity and/or useful-spinning-reserve-capacity to the system. Otherwise, proceed to the next unit.
- Calls subroutine SUDP to determine the commitment schedule for the unit.
- Updates the system hourly unfulfilled energy and spinning reserve requirements and each area's hourly net useful-energy-capacity and useful-spinning-reserve-capacity to reflect the hourly useful-energy-capacity and useful-spinning-reserve-capacity contributions of the unit.
- Checks whether the system hourly energy and spinning reserve requirements are met or not. If the system hourly energy and/or the spinning reserve requirements have not been met, then proceed to the next unit in the priority list and continue the procedure as outlined above.

4.2.6 Subroutine SEQUENT

The subroutine SEQUENT sequentially identifies and commits the most advantageous units until the system hourly energy and spinning reserve requirements are met. The most advantageous unit at each decision point is identified via a bidding process. Given the hourly energy price, spinning reserve price, energy capacity price and spinning reserve capacity price for each area, the subroutine SEQUENT performs the following:

- Groups available, yet uncommitted, units based on their characteristics (e.g. unit capacity, minimum-up/down time).
- Sorts units in each group in descending orders of their average operating profits. The operating profit of a unit is calculated by first calling the subroutine CALUSC to determine the hourly useful-energy-capacity and useful-spinning-reserve-capacity of the unit and then calling subroutine APPROX to determine the tentative commitment schedule and the estimated operating profit of the unit. The top ranked unit in each group is a candidate for the next commitment decision.
- Eliminates dominated units from the candidate list. A unit is dominated, if both of its useful-spinning-capacity contribution and its operating profit are less than those of other candidate units.
- Determines the capacity target for the bidding evaluation. The capacity target is the maximum useful-spinning-capacity among all candidate units.
- Compares the operating profits of candidate units subject to the target capacity. If the total useful-spinning-capacity of a candidate unit is less than the target capacity, the sequential bidding logic is used to select available and uncommitted units to team up with this candidate unit to meet the target capacity. If the total useful-spinning-capacity of a team exceeds the target capacity, then prorate the useful-spinning-capacity and the operating profit of the last unit in this team based on the target capacity. The most advantageous unit is the leader of the team with the highest operating profits.

- Calls subroutine SUDP to determine commitment schedule for the most advantageous unit.
- Compares the commitment schedule of the most advantageous unit with all the previously committed units. If the most advantageous unit is less economical and more flexible than all the previously committed units, then commit the unit at this position. Otherwise identify the appropriate position to commit the subject unit and adjust the commitment schedule of the previously committed units if necessary.

4.2.7 Subroutine SBPL

The subroutine SBPL sorts generation blocks in ascending order of their respective incremental costs for all the committed units.

4.2.8 Subroutine SREDC

Based on the feasible multi-area unit commitment schedule determined by subroutines CUAFLC and SEQUENT, the subroutine SREDC determines the hourly energy generation, spinning reserve contribution, and production costs for the committed units as well as the hourly marginal energy and spinning reserve costs for each area. At each hour, the subroutine SREDC performs the following:

- Calculates the minimum generation limit and maximum spinning reserve limit for each area. An area's minimum generation limit is the sum of minimum generation limits of all the committed units in the area. Likewise an area's maximum spinning reserve limit is the sum of maximum spinning reserve limits of all the committed units in the area.
- Calculates the system hourly energy and spinning reserve requirements by summing the hourly energy and spinning reserve requirements over all areas in the

system. Initializes the system hourly unfulfilled energy requirement by subtracting the minimum generation of all the committed units from the system hourly energy requirement at the hour.

- Calculates the system hourly maximum generation limit that meets the system hourly spinning reserve requirement.
- Performs reserve constrained economic dispatch ignoring transmission constraints. The reserve constrained economic dispatch calculation dispatches system generation blocks based on their incremental costs. There are two types of generation blocks: the generation block which does not impact the spinning reserve contribution of a unit and the generation block which impacts the spinning reserve contribution of a unit. Generation of a block that impacts spinning reserve contribution is bounded by the maximum generation limit obtained from the previous step. The solution of the reserve constrained economic dispatch sets the initial conditions for a more complicate multi-area reserve constrained economic dispatch calculation (LP-MEDC).
- Based on the solution of the single-area reserve constrained economic dispatch calculation described in the previous step, identifies the system marginal generation block.
- Formulates the LP-MEDC problem. The objective function and operating constraints of the LP-MEDC problem are described in section 3.2.2.
- Calls subroutine LP to solve the LP-MEDC problem.
- Calculates the MW generation and the production cost of each unit based on the LP-MEDC solution.
- Calculates the marginal energy and spinning reserve costs for each area based on the LP-MEDC solution. This calculation is described in section 3.2.3.

4.2.9 Subroutine GAMMA

The subroutine GAMMA calculates the average hourly incremental useful-energy-capacity and useful-spinning-reserve-capacity prices for each area. The subroutine GAMMA performs the following:

- For each committed unit, the subroutine calculates the average useful-spinning-capacity cost (*$/MW-Hr*) of each on-line interval. The unit's average useful-spinning-capacity cost of an on-line interval is its operating costs minus its operating benefit divided by its useful-spinning-capacity contribution over the on-line interval.
- Calculates the average hourly incremental useful-energy-capacity cost and useful-spinning-reserve-capacity cost for each area. The average hourly incremental useful-energy-capacity cost of an area is the average hourly incremental useful-spinning-capacity cost of the last committed energy capacity increment in the area. Likewise the average hourly incremental useful-spinning-reserve-capacity cost of an area is the average hourly incremental useful-spinning-capacity cost of the last committed reserve capacity increment in the area.
- Calculates the hourly useful-energy-capacity and useful-spinning-reserve-capacity prices for each area. An area's hourly useful-energy-capacity and useful-spinning-reserve-capacity prices are those associated with binding system constraints detected in the multi-area capacity (LP-MCAP) problem. Description of this calculation is given in section 3.2.3.

4.2.10 Subroutine CONVERGE

The subroutine CONVERGE checks solution convergence and calculates the hourly energy, spinning reserve, useful-energy-capacity and useful-spinning-reserve-capacity prices at each area for the next iteration. The subroutine CONVERGE performs the following:

- Calculates the system total production costs and updates the best solution.
- Determines whether the solution process should be terminated or not. The process should be terminated if it has reached the maximum allowable number of iterations or it has met the convergence criteria. The convergence criteria of a multi-area unit commitment problem is less than 1% change in the production costs between two consecutive iterations. Upon the termination of the solution process, the subroutine CONVERGE writes the best solution to output files for user review. If the multi-area unit commitment solution process continues, the subroutine calculates the hourly energy, spinning reserve, useful-energy-capacity and useful-spinning-reserve-capacity prices of each area for the next iteration.

4.2.11 Subroutine MXINV

The subroutine MXINV performs matrix inversion by the Gauss-Jordan method.

4.2.12 Subroutine MINUC

The subroutine MINUC determines the hourly minimum net useful-energy-capacity and useful-spinning-reserve-capacity for each area by formulating and solving the multi-area capacity (LP-MCAP) program. The subroutine MINUC performs the following:

- Formulates the LP-MCAP problem. Variables of the LP-MCAP problem consist of the net useful-energy-capacity and the net useful-spinning-reserve-capacity for each area in the study system. Equations (18), (19), (20), (21), (22), (23), and (24) in chapter 3 describe the objective function and constraints of the LP-MCAP problem. Coefficients (α_k) of the objective function are set to be 1.0 except that they are set to be 0.1 for the area where the evaluated unit is located. The LP-MCAP problem has both system and local constraints. The system constraints consist of: (1) Sum of the net useful-energy-capacity should be less than or equal to zero. (2) Sum of the net useful-spinning-reserve-capacity should be less than or equal to zero. (3) Branch flows should be within transmission capabilities for both normal and contingent conditions. The local constraints consist of: (1) The net useful-energy-capacity of an area is bound by the area's maximum net useful-energy-capacity and the net useful-energy-capacity. (2) The net useful-spinning-reserve-capacity of an area is bound by the area's maximum net useful-spinning-reserve-capacity and the minimum net useful-spinning-reserve-capacity.

- Calls the subroutine LP to solve the LP-MCAP problem.

4.2.13 Subroutine CALUSC

The subroutine CALUSC calculates a candidate unit's hourly useful-energy-capacity, useful-spinning-reserve-capacity and useful-spinning-capacity in the following sequence:

- At each hour, the subroutine calculates the hourly useful-spinning-reserve-capacity allowance of the system and the hourly useful-spinning-reserve-capacity allowance of the area where the unit is located. The candidate unit's hourly useful-spinning-reserve-capacity is the minimum among its maximum spinning reserve limit, and the system hourly useful-spinning-reserve-capacity, and the area hourly useful-spinning-reserve-capacity.
- At each hour, the subroutine calculates the hourly useful-energy-capacity allowance of the system and the hourly useful-energy-capacity allowance of the

area where the unit is located. The candidate unit's hourly useful-energy-capacity is the minimum among its maximum dispatch limit minus its useful-spinning-reserve-capacity contribution, the system hourly useful-energy-capacity, and area hourly useful-energy-capacity.

- If the candidate unit's hourly useful-energy-capacity/useful-spinning-reserve-capacity calculation is inconclusive, the subroutine CALUSC calls the subroutine MINUC to resolve the system hourly and the area hourly useful-energy-capacity and useful-spinning-reserve-capacity allowances. Its hourly useful-energy-capacity and useful-spinning-reserve-capacity are then updated from the new system and area capacity allowances.

4.2.14 Subroutine SUDP

The subroutine SUDP determines the optimal unit commitment schedule for a unit in the following sequences:

- Determines the unit's tentative commitment schedule.
- Checks whether the unit's minimum-down-time constraint is satisfied or not. If an off-line interval violates its minimum-down-time constraint, the unit will be brought on-line at all hours of the off-line interval.
- Calculates the unit's hourly on-line operating profit. The hourly on-line operating profit of a unit is its hourly operating benefit minus its hourly operating costs.
- Determines the detailed commitment schedule of the unit by solving a single-unit DP problem. The objective of the DP problem is to maximize the operating profit of the unit over the study period subject to its operating constraints. The decision space of the single-unit DP problem is much smaller than that of an N-unit DP problem, and it can be further simplified by using a reduced-node formulation [16].

4.2.15 Subroutine APPROX

The subroutine APPROX determines commitment schedule and estimates the total useful-spinning-capacity contribution and the average operating profit (*$/MW-Hr*) of a unit in the following sequences:

- Determines its tentative commitment schedules.
- Checks whether its minimum-down-time constraint is satisfied. If an off-line interval violates its minimum-down-time constraint, then turn the unit on at all hours of the off-line interval.
- Calculates its hourly on-line operating profit as hourly operating benefit minus the hourly operating costs.
- Based on the tentative commitment schedule determined from the previous step, the subroutine defines the decision intervals. A decision interval consists of an on-line interval and an adjacent off-line interval.
- Determines the commitment schedule of the unit by a heuristic method. At each decision interval, it extends the on-line interval to meet the minimum-up-time constraint if necessary. The unit's operating status will be changed to on-line at all hours of the off-line interval if the minimum-down-time constraint is violated or it is more profitable for the unit to be on-line in this interval.
- Calculates the average operating profit of the unit. The average operating profit of the unit is its total operating profits over the study period divided by its total useful-spinning-capacity over the study period.

4.2.16 Subroutine LP

The subroutine LP implements the dual, reduced basis techniques proposed by B. Stott and J. Marinho [23] to solve the pertinent linear programming problems. The major advantage of the proposed LP method is that it handles multi-segment cost curves efficiently. The subroutine LP proceeds as follows:

- Initializes the basis matrix based on the initial states. The initial states are optimal solutions of the LP problem ignoring inequality system constraints.
- Generates the monitor subset. The monitor subset consists of the violated system constraints and those which are within 10% to their binding values.
- Identifies the most violated system constraint in the monitor subset. The system constraint is the incoming constraint of the LP basis. The value of the system constraint is set to be equal to the constraint's binding limit.
- Selects a binding constraint to be freed from the basis via the eligibility and ratio tests. The eligibility test checks the sensitivity between the outgoing constraint and the incoming constraint. A constraint is eligible of leaving the basis if it will back off its previously binding limit when it is freed. The constraint to be freed from the basis is the one with the minimum value from the ratio test.
- Solves linear equations. The linear equations are arranged such that the non-sparse branch constraints appear in the first few rows. This arrangement simplifies the matrix manipulation.
- If there are generators that violate their binding limits, the most violated generation will be fixed at its limit by inserting the corresponding equality constraint back into the LP process.
- Continues the process until there is no violation.

- Continues to check whether there is any violated branch or not. If there is any, repeat the process as outlined above.

5 Application

This chapter describes an application of the multi-area sequential bidding unit commitment model. Section 5.1 describes a sample test system. Section 5.2 compares the results of the multi-area sequential bidding unit commitment model with the results of a multi-area dynamic-programming based unit commitment model on the sample system.

5.1 Test System

Figure 3 shows the configuration of the sample system. Area 1 has 17 generating units with an installed capacity of 3984 MW. Area 2 has 13 generating units with an installed capacity of 3515 MW. Area 3 has 14 generating units with an installed capacity of 5008 MW. Area 4 has 11 generating units with an installed capacity of 1628 MW. It is assumed that there is a transmission link between any two areas and all transmission links have equal capacity and impedance.

In the sample four-area system, only few combustion gas turbine generators can be used for daily cycling and all other units have fairly long minimum-up-time and minimum-down-time. The unit characteristics of the four-area system are listed in Table 2, 3, 4 and 5. The decision horizon is an expanded 36-hour typical summer day. The area hourly load curves, over this decision horizon, are shown in Figure 4.

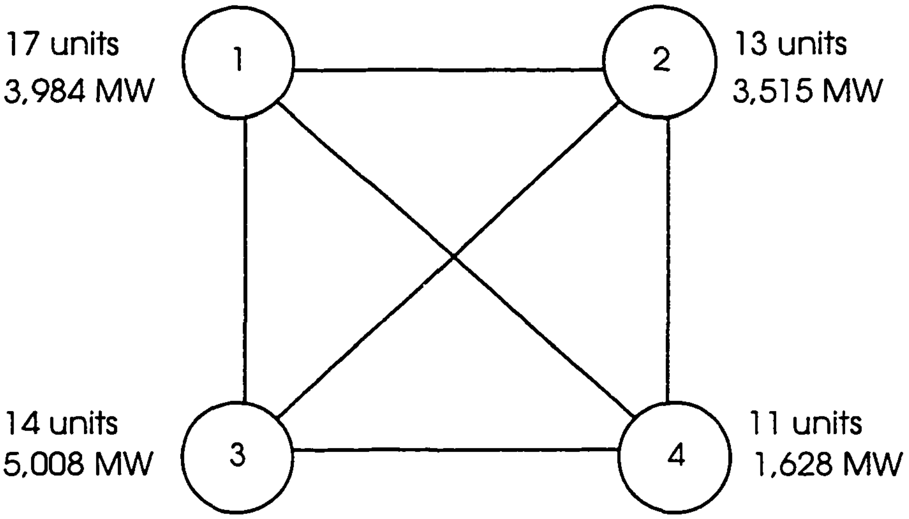

Figure 3 Configuration of A Four-Area Sample System

Table 2 Unit Characteristics of Area 1

Unit Number	AFLC ($/MW-Hr)	Minimum Dispatch Limit (MW)	Capacity (MW)	Maximum Spinning Reserve (MW)	Initial Status (Hour)	Minimum -Up-Time (Hour)	Minimum -Down-Time (Hour)	Start-up Cost ($)
1	4.45	500	504	0	1000	1000	1000	10000
2	10.35	150	630	80	168	168	168	20000
3	18.39	50	370	50	112	112	56	10000
4	19.17	40	350	50	112	112	56	10000
5	19.19	50	265	40	112	112	56	6000
6	19.23	70	370	50	112	112	56	10000
7	19.35	30	180	30	112	112	56	3000
8	19.49	80	260	40	112	112	56	6000
9	19.50	50	265	40	112	112	56	6000
10	19.87	20	170	30	112	112	56	3000
11	20.37	50	170	30	112	112	56	3000
12	20.49	20	120	30	112	112	56	2000
13	20.89	15	115	30	112	112	56	2000
14	23.13	15	75	20	12	12	12	1000
15	23.81	5	40	20	6	6	6	465
16	24.24	5	40	20	6	6	6	465
17	24.89	5	60	30	6	6	6	600

Table 3 Unit Characteristics of Area 2

Unit Number	AFLC ($/MW-Hr)	Minimum Dispatch Limit (MW)	Capacity (MW)	Maximum Spinning Reserve (MW)	Initial Status (Hour)	Minimum -Up-Time (Hour)	Minimum -Down-Time (Hour)	Start-Up Cost ($/Start)
1	17.57	180	460	80	112	112	56	15000
2	17.59	180	460	80	112	112	56	15000
3	20.73	100	310	50	112	112	56	8000
4	22.46	210	500	80	112	112	56	16000
5	22.95	180	475	70	112	112	56	15000
6	23.20	180	470	70	112	112	56	15000
7	23.58	110	310	50	112	112	56	10000
8	24.31	20	170	30	112	112	56	4000
9	27.39	20	90	40	112	112	56	800
10	27.97	20	90	40	112	112	56	800
11	31.18	5	60	30	6	6	6	300
12	31.43	10	60	30	6	6	6	300
13	31.62	10	60	30	6	6	6	300

Table 4 Unit Characteristics of Area 3

Unit Number	AFLC ($/MW-Hr)	Minimum Dispatch Limit (MW)	Capacity (MW)	Maximum Spinning Reserve (MW)	Initial Status (Hour)	Minimum -Up-Time (Hour)	Minimum -Down-Time (Hour)	Start-Up Cost ($/Start)
1	16.23	236	725	100	168	168	168	20000
2	16.73	225	580	80	168	168	168	15000
3	16.80	225	575	80	168	168	168	16000
4	16.98	225	575	80	168	168	168	16000
5	17.09	225	575	80	168	168	168	16000
6	17.37	136	358	50	112	112	56	10000
7	17.52	137	358	50	112	112	56	10000
8	17.84	120	340	50	112	112	56	10000
9	18.39	131	356	50	112	112	56	10000
10	18.50	35	113	30	112	112	56	2500
11	18.75	81	177	40	112	112	56	4000
12	19.03	35	114	30	112	112	56	2500
13	19.54	39	112	30	112	112	56	2500
14	21.75	25	50	30	6	6	6	400

Table 5 Unit Characteristics of Area 4

Unit Number	AFLC ($/MW-Hr)	Minimum Dispatch Limit (MW)	Capacity (MW)	Maximum Spinning Reserve (MW)	Initial Status (Hour)	Minimum -Up-Time (Hour)	Minimum -Down-Time (Hour)	Start-Up Cost ($/Start)
1	17.25	250	680	80	168	168	168	30000
2	22.87	38	204	50	112	112	56	8000
3	23.07	62	130	40	112	112	56	3000
4	23.23	13	85	30	12	12	12	1000
5	23.37	38	98	40	12	12	12	1100
6	23.81	28	158	30	112	112	56	3200
7	24.35	24	112	30	12	12	12	2000
8	25.62	14	41	20	6	6	6	300
9	25.74	13	54	20	6	6	6	400
10	28.12	9	33	20	6	6	6	300
11	28.12	9	33	20	6	6	6	300

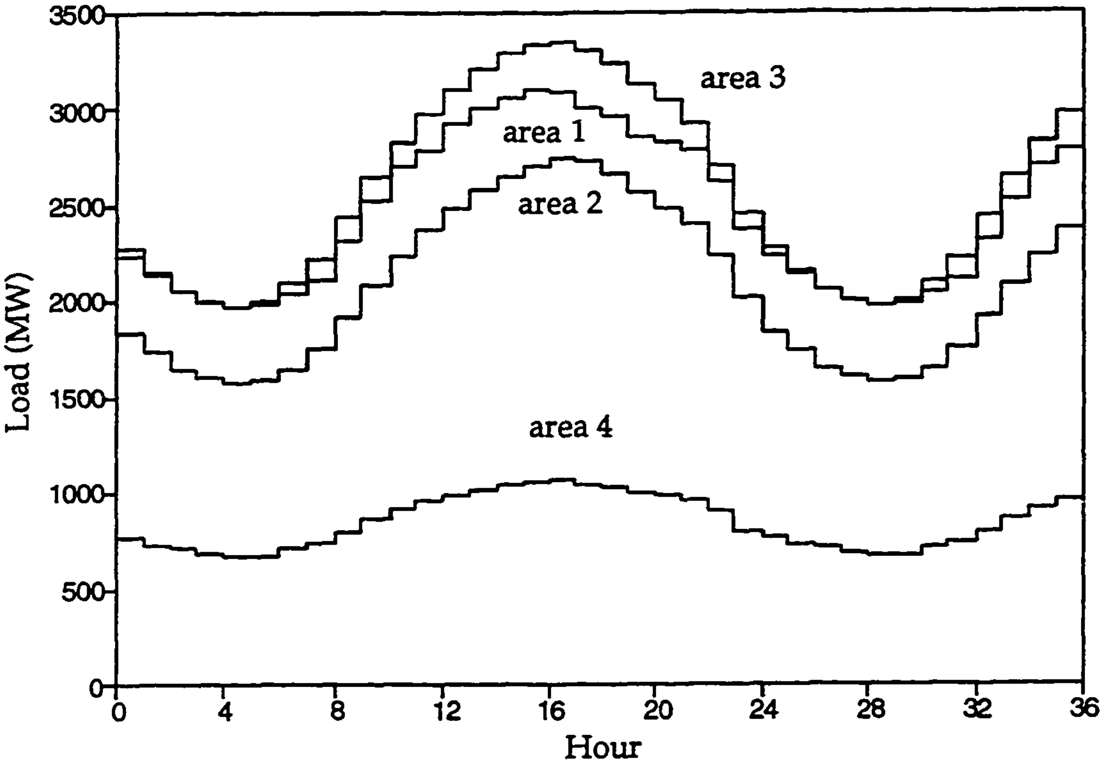

Figure 4. Area Hourly Load Curves

5.2 Simulation Results

The solutions determined by the multi-area sequential bidding unit commitment model are compared with those determined by a multi-area dynamic-programming based model. For the sample four-area, 55-unit system, there are a total of 2^{55} possible states. Although only a subset of them are feasible states, the computational requirements of applying the DP based method to evaluate all the feasible states are still prohibitive. In order to keep computational requirements manageable, the DP based multi-area method is used to evaluate only a small subset of all the possible states for each area. At each area, the state to be evaluated are those that are sequentially combined according to the area's priority commitment order (i.e. the priority order is determined by the average full load costs of all the available units located in the area). For the sample system, area 1 has 18 sequentially combined states hourly, area 2 has 14 sequentially combined states hourly, area 3 has 15 sequentially combined states hourly, and area 4 has 12 sequentially combined states hourly. As a result, there are a total of 45,360 (i.e. 18×14×15×12) sequentially combined system states at each hour. A number of these states can be eliminated by a simple feasibility test via capacity obligations and maximum possible import/export limits of each area. The remaining states require more detailed evaluation. Each of the remaining system states is evaluated by using the multi-area reserve constrained economic dispatch algorithm described in chapter 3. If a system state fails to yield a multi-area dispatch solution, then the system state will be infeasible and the infeasible state will be eliminated from further consideration. Otherwise, the energy/reserve dispatch cost will be computed if the system state is feasible. The feasible states over the commitment horizon define the various commitment paths to be evaluated and saved via the DP procedure. In addition to the hourly state space truncation (i.e. consider only the sequentially combined states at each area), the commitment paths, to be saved at each hour, are also truncated to a set of "least-cost" paths. For the sample system, it is necessary to save, at each hour, approximately 1,000 least-cost paths in order to attain a feasible solution at the end of the commitment horizon. Although these truncations reduce the computational requirements, they often jeopardize the solution quality. The sequential bidding unit commitment method and the DP based method are compared on an equitable basis by using the same input data, similarly structured computer codes, and same computational equipment.

For varying transmission capacities from 0 MW to 1,000 MW at an 100 MW increment, the multi-area system operational costs associated with the commitment strategies, determined by the multi-area sequential bidding method and the DP based method, are shown in Figure 5. It can be noted that the strategies determined by the sequential bidding method are consistently better than those determined by the DP based method. For each case, the sequential bidding method is able to reach the converged solution in four iterations. When the capacity of each transmission link is assumed to be zero MW, the commitment strategy determined by the multi-area sequential bidding method is the same as the combined four single-area strategies. When the capacity of each transmission link is assumed to be 1,000 MW, the commitment strategy determined by the multi-area sequential bidding method is the same as the single-area strategy that ignores transmission constraints. In table 6, the operational costs of the commitment strategies determined by the sequential bidding method and the DP based method are given for several different levels of transmission capacity.

The comparison of the computational time (using IBM RISC 6000 model 320 workstation) required by the sequential bidding method and the DP based method is shown in Figure 6. It can be noted that the sequential bidding method is much faster than the DP based method. The computational time, given in Figure 6, can be further reduced by optimizing the experimental computer codes.

The computational time requirement depends on the number of hours in the decision horizon and the number of areas in the interconnected system. The impact of the decision horizon on the computational time is shown in Figure 7. It can be noted that the computational time varies linearly with the length of the decision horizon. The impact of the number of areas on the computational time is shown in Figure 8. In Figure 8, the computational time required for a two-area system is used as the base and the computational time requirements for a three-area , the four-area, and a five-area systems are expressed as ratios to this base computational time. In all five cases, the number of generating units remains the same and the capacity of each transmission link is assumed to be fixed at 400 MW.

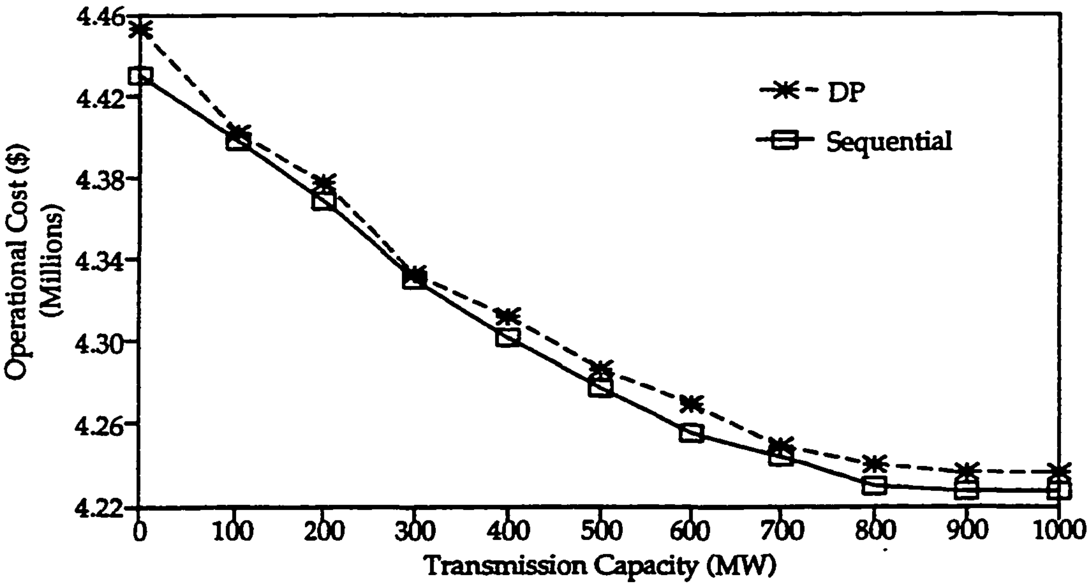

Figure 5. Multi-Area System Operational Cost

Table 6 System Operational Costs

Transmission Capacity (MW)	Operational Cost ($) Sequential Bidding	Operational Cost ($) DP
0	4430449	4454132
200	4367961	4377082
400	4302014	4312443
600	4255914	4280610
800	4229904	4240497
1000	4226662	4236130

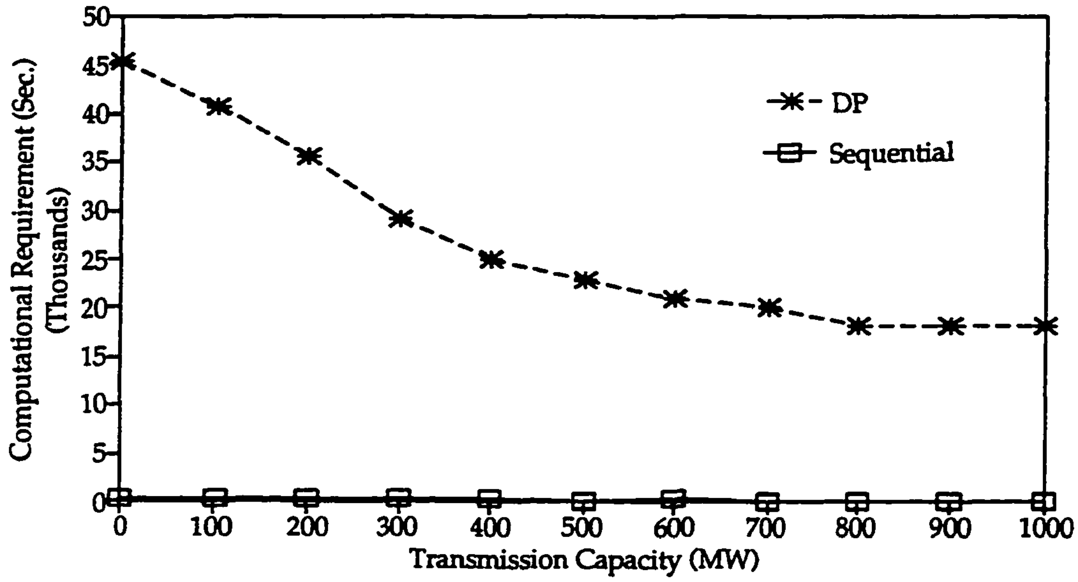

Figure 6. Comparison of Computational Time

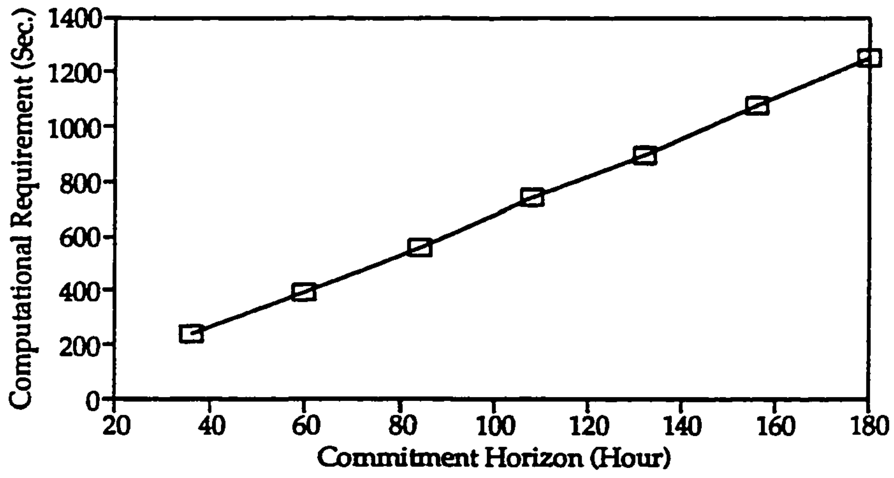

Figure 7. Computational Time vs. Length of Decision Horizon

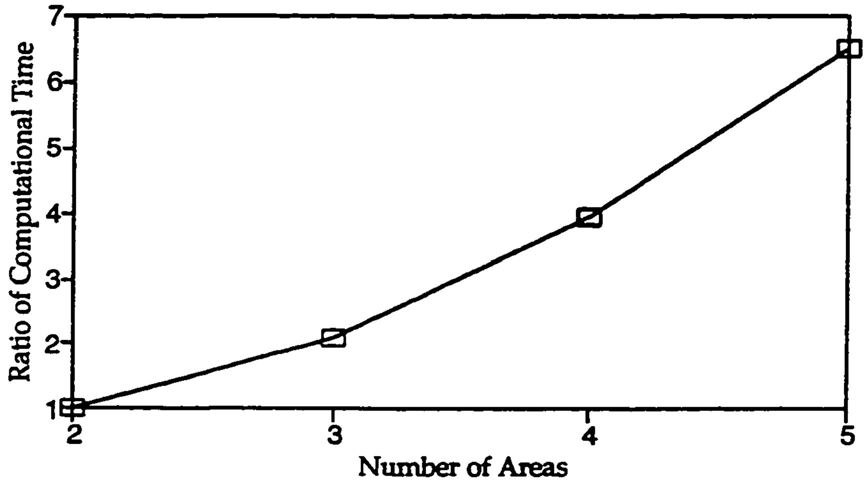

Figure 8 Computational Time vs. Number of Areas

6 Conclusions

Coping with transmission open access in the deregulated energy industry, the assessment of the impact of transmission capacity and tariff on economic operation of power systems is essential. This research has extended the single-area sequential bidding unit commitment method to a multi-area model. In this extension, the DC power flow model is used to represent the inter-area transmission network.

The multi-area sequential bidding unit commitment method employs an iterative procedure. Each iteration consists of two phases - the sequential commitment phase and the price adjustment phase. Among the available generating units in the interconnected multi-area system, the sequential commitment phase sequentially identifies, via a procedure that resembles bidding, the most advantageous units to commit until the multi-area system obligations are fulfilled. Based on the feasible unit commitment schedule determined in the sequential commitment phase, the price adjustment phase determines the multi-area hourly prices for the next iteration. The effectiveness of the proposed method is illustrated via a comparison to a dynamic programming based multi-area commitment using a four-area sample system.

The proposed multi-area unit commitment model meets the need of a tool for simulating multiple markets in this deregulated operating environment.

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