PREDICTING NUTRIENT INTAKE AND PRODUCTION RESPONSES OF LAYING HENS

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CHAPTER I

GENERAL INFORMATION

Introduction

It is felt by many research workers in poultry nutrition that the best way to express nutrient requirement standards for poultry is on a daily nutrient intake basis. In order for a daily nutrient intake standard to be of any value, a method must be developed by which the intake of feed and nutrients can be controlled. In larger animals such as dairy cattle, feed intake can be controlled by weighing out a specific amount of feed for each cow every day. By doing this each animal can receive the exact amount of nutrients prescribed by the standard. This type of individual feeding works very well for dairy cattle; however, for poultry the equipment and labor that would be required for a feeding system of this type would make the cost prohibitive. Therefore, poultry must be fed on an *ad libitum* basis.

The obvious questions which must be answered in order to establish accurately and to utilize fully the daily nutrient intake standards for poultry are: (1) can feed and nutrient intake be regulated with *ad libitum* feeding, and if so (2) what factors are involved and to what extent do they affect feed intake and production responses; (3) how can the knowledge of these regulating factors be used to control nutrient intake and production responses; and (4) what is the direct effect of nutrient intake upon the productive performance of the birds? The first question was answered by Gleaves et al., 1963a, when dietary volume for laying hens was controlled in order to regulate the intake of diets with various nutrient concentrations. This work clearly established the fact that feed intake can be controlled, and that there are four primary dietary factors involved in regulating feed intake. These factors are dietary protein, dietary energy, dietary weight and dietary volume. In a review by Anand, 1961, evidence was cited which establishes these four dietary factors as definite physiological food intake regulators.

It was further established by Gleaves, 1965, that there are many interactions among the dietary factors in their effects upon feed and nutrient intake in laying hens. The interaction effects appeared to be decreased in intensity among hens that were laying at approximately the same rate of production. This establishes the fact that production characteristics constitute additional factors which influence feed intake. In the same report, Gleaves pointed out some of the effects of the dietary factors upon the production responses.

To the author's knowledge there has been no report involving the problems in questions 3 and 4. For this reason the purpose of this thesis is to attack the problems which are involved with these two questions. The specific objectives are as follows: (1) to develop prediction equations for protein intake, energy intake, body weight change, number of eggs and average egg weight of laying hens with dietary protein, dietary energy, dietary weight and dietary volume as the independent variables, (2) to study the effect of egg production and body weight change upon the intake of protein and energy, (3) to study the effects of protein and energy intake upon egg production, body weight change and egg weight, and (4) to describe conditions under which a multiple response analysis can be performed, and to perform multiple response analyses where the necessary conditions are satisfied.

Data

A 3^4 factorial arrangement of dietary protein, energy, weight and volume was used to formulate 81 rations which were fed to laying hens. In order to give a clear perspective of the factorial arrangement of the treatments, Table I is presented with the actual levels of each factor and coded numbers representing each value. There were seven replications of each treatment and the replicates were completely randomized. The hens were housed in individual cages with individual feed and water. Therefore, each hen was an experimental unit. The hens were allowed to consume feed and water ad libitum. The duration of the experiment was eight four-week periods.

At the end of each four-week period the hens were weighed, feed consumption calculated, and the number of eggs and average egg weights recorded. The data for each period were summarized, and at the end of the eight four-week periods the data for the last seven periods were cumulated to give one overall summary. This overall summary provides the data for this report. The specific data used are average daily protein consumption, average daily energy consumption, number of eggs laid, average egg weight and average body weight change. These data are presented in the Appendix.

The data were obtained from the same experiment as that which was described by Gleaves, 1965. Therefore, the analyses and results of this dissertation are tied directly to that study. In order to prevent

TABLE I

FACTORIAL ARRANGEMENT OF TREATMENTS

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13 — Milliliters 180	13 — 230	13 — 280	16 — Milliliters 180	16 — 230	16 — 280	19 — Milliliters 180	19 — 230	19 — 280
260	127	-1 -1*	-1 -1	-1 -1	0 -1	0 -1	0 -1	1 -1	1 -1	1 -1
		-1 -1	-1 0	-1 1	-1 -1	-1 0	1 1	-1 -1	-1 0	-1 1
	137	-1 -1	-1 -1	-1 -1	0 -1	0 -1	0 -1	1 -1	1 -1	1 -1
		0 -1	0 0	0 1	0 -1	0 0	0 1	0 -1	0 0	0 1
	147	-1 -1	-1 -1	-1 -1	0 -1	0 -1	0 -1	1 -1	1 -1	1 -1
		1 -1	1 0	1 1	1 -1	1 0	1 1	1 -1	1 0	1 1
300	127	-1 0	-1 0	-1 0	0 0	0 0	0 0	1 0	1 0	1 0
		-1 -1	-1 0	-1 1	-1 -1	-1 0	-1 1	-1 -1	-1 0	-1 1
	137	-1 0	-1 0	-1 0	0 0	0 0	0 0	1 0	1 0	1 0
		0 -1	0 0	0 1	0 -1	0 0	0 1	0 -1	0 0	0 1
	147	-1 0	-1 0	-1 0	0 0	0 0	0 0	1 0	1 0	1 0
		1 -1	1 0	1 1	1 -1	1 0	1 1	1 -1	1 0	1 1
340	127	-1 1	-1 1	-1 1	0 1	0 1	0 1	1 1	1 1	1 1
		-1 -1	-1 0	-1 1	-1 -1	-1 0	-1 1	-1 -1	-1 0	-1 1
	137	-1 1	-1 1	-1 1	0 1	0 1	0 1	1 1	1 1	1 1
		0 -1	0 0	0 1	0 -1	0 0	0 1	0 -1	0 0	0 1
	147	-1 1	-1 1	-1 1	0 1	0 1	0 1	1 1	1 1	1 1
		1 -1	1 0	1 1	1 -1	1 0	1 1	1 -1	1 0	1 1

*The four numbers in the square are the coded representation of the dietary level combination of protein, energy, weight and volume. The upper two numbers represent protein and energy, respectively and the lower two numbers represent weight and volume, respectively.

duplication of material, there are many places throughout the present study where the reader is referred to Gleaves. For this reason, to be familiar with the former study of the experiment will greatly enhance the understandability of this thesis. Hereafter, unless otherwise stated, when reference is made to Gleaves, it will mean Gleaves, 1965.

In the report by Gleaves, analyses of variance were performed on the following responses; feed weight intake, protein intake, energy intake, feed volume intake, egg production, body weight change, and egg weight. The main effects and interactions of the four dietary factors upon these responses were pointed out and discussed in detail. The effects of egg production upon the intake of dietary factors were given special attention. Due to the type of analyses, the results were mostly qualitative in nature. That is, estimates for future responses would need to be based largely upon judgment rather than upon mathematical procedures. The purpose of all of the analyses in this thesis is to obtain mathematical functions to estimate future responses.

CHAPTER II

PREDICTION OF NUTRIENT INTAKE AND PRODUCTION RESPONSES BASED UPON DIETARY FACTORS

In cases where moderate environmental conditions can be maintained, it should be possible to predict the nutrient intake of laying hens based strictly upon dietary factors. Therefore, in this study equations were developed to predict protein and energy intakes, body weight change, egg production and egg weight. The general procedure for obtaining these prediction equations is as follows:

(1) There are 81 possible dietary effects which could go into each of these equations. Therefore, with each response it is necessary to select only the effects which appear to have a significant influence. This was done by an analysis of variance in which the sums of squares were partitioned for single degrees of freedom by orthogonal comparisons. In the work by Gleaves, the analyses of variance for the same responses were presented but the sums of squares were not completely partitioned.

(2) From this analysis of variance, the effects which appeared to have some influence upon the response being studied were used to form the model for the response prediction.

(3) The parameters in the model were estimated by the method of least squares as described by Graybill, 1961.

(4) The validity of each prediction equation was challenged by testing the residual sum of squares, by finding the predicted values, $\hat{Y}$, and making a half-normal plot of the treatment residual deviations and by testing the hypothesis that each parameter is equal to zero.

(5) The parameters which were not significantly different from zero are eliminated to simplify the model and the remaining parameters are re-estimated.

(6) $\hat{Y}$ was calculated and a half-normal plot was made of the treatment residuals for the simplified model.

The models used to derive these prediction equations are of the general form

$$Y = \Sigma\Sigma\Sigma\Sigma\, a_{jklm}\, x_1^{\,j} x_2^{\,k} x_3^{\,l} x_4^{\,m} + e ,$$

where

Y = observation (nutrient intake or production response)

a_{jklm} = unknown parameter

$j = 0, 1, 2$

$k = 0, 1, 2$

$l = 0, 1, 2$

$m = 0, 1, 2$

x_1 = dietary protein level
x_2 = dietary energy level
x_3 = dietary weight
x_4 = dietary volume
} $= x_i ,\ i = 1, 2, 3, 4$

e = random error that has a normal distribution with zero mean and variance = σ^2.

The x_i are coded numbers which represent the actual levels of dietary factors. These coded numbers are used to make the calculations as simple as possible. The coded numbers (X_i) which correspond to the actual levels are given in Table I. The five observations represented by Y are denoted as follows:

Y_1 = protein intake per bird per day

Y_2 = energy intake per bird per day

Y_3 = body weight change

Y_4 = number of eggs produced

Y_5 = average egg weight.

The X_i along with the observations for each response variable are given in Appendix Table I. This is the summary data for the entire experiment, or the accumulated summary of periods 2 through 8.

All six steps in the procedure will be performed on the data for each response, one at a time, and the results will be discussed after each step. The general model as expressed above can be expanded into 81 possible terms, but only those terms which represent the effects obtained in step one will be included in the models for the specific observations.

Nutrient Intake

Protein Intake: The analysis of variance of protein consumption as affected by dietary protein, dietary energy, dietary weight and dietary volume is presented in Table II. From this table it can be seen that many of the interaction effects which could not be seen in the analysis presented by Gleaves are significant at the 5 percent level of probability. As an example, protein x energy with 4 degrees of freedom is not significant, but protein linear x energy linear is

TABLE II

ANALYSIS OF VARIANCE OF PROTEIN CONSUMPTION

Source	DF	SS	MS	F[1]
Total	566	6,585.32		
Treatment	80			
Protein(P)	(2)	3,597.69		
P_L	1	3,596.69	3,596.69	1,499.87*
P_Q	1	1.00	1.00	.42
Energy(E)	(2)	1,556.30		
E_L	1	1,550.65	1,550.65	646.64*
E_Q	1	5.65	5.65	2.35
Weight(W)	(2)	3.44		
W_L	1	.20	.20	.08
W_Q	1	3.24	3.24	1.35
Volume(V)	(2)	29.30		
V_L	1	29.08	29.08	12.12*
V_Q	1	.22	.22	.09
Interaction	(72)			
P x E	(4)	17.92		
P_L E_L	1	9.95	9.95	4.14*
P_Q E_L	1	1.76	1.76	
P_L E_Q	1	5.40	5.40	2.25
P_Q E_Q	1	.81	.81	
P x W	(4)	6.25	1.56	
P x V	(4)	2.01	.50	
E x W	(4)	20.51		
E_L W_L	1	9.91	9.91	4.12*
E_Q W_L	1	1.42	1.42	
E_L W_Q	1	.25	.25	
E_Q W_Q	1	8.93	8.93	3.72*
E x V	(4)	7.91	1.97	
W x V	(4)	11.12		
W_L V_L	1	1.56	1.56	
W_Q V_L	1	.10	.10	
W_L V_Q	1	9.34	9.34	3.89*
W_Q V_Q	1	.12	.12	
P x E x W	(8)	38.04		
P_L E_L W_L	1	8.47	8.47	3.52*
P_Q E_L W_L	1	1.14	1.14	
P_L E_Q W_L	1	.61	.61	
P_L E_L W_Q	1	1.90	1.90	
P_Q E_Q W_L	1	8.34	8.34	3.47*
P_Q E_L W_Q	1	5.68	5.68	2.36
P_L E_Q W_Q	1	.36	.36	
P_Q E_Q W_Q	1	11.54	11.54	4.80*

TABLE II (CONTINUED)

Source	DF	SS	MS	F[1]
P x E x V	(8)	28.01		
P_L E_L V_L	1	1.11	1.11	
P_Q E_L V_L	1	9.30	9.30	3.87*
P_L E_Q V_L	1	1.79	1.79	
P_L E_L V_Q	1	4.36	4.36	1.81
P_Q E_Q V_L	1	.32	.32	
P_Q E_L V_Q	1	9.77	9.77	4.07*
P_L E_Q V_Q	1	.39	.39	
P_Q E_Q V_Q	1	.97	.97	
P x W x V	(8)	13.94	1.74	
E x W x V	(8)	14.21	1.77	
P x E x W x V	(16)	73.11		
P_L E_L W_L V_L	1	8.23	8.23	3.42*
P_Q E_L W_L V_L	1	2.99	2.99	
P_L E_Q W_L V_L	1	.83	.83	
P_L E_L W_Q V_L	1	2.66	2.66	
P_L E_L W_L V_Q	1	.83	.83	
P_Q E_Q W_L V_L	1	13.51	13.51	5.62*
P_Q E_L W_Q V_L	1	5.83	5.83	2.42
P_Q E_L W_L V_Q	1	3.46	3.46	
P_L E_Q W_Q V_L	1	8.06	8.06	3.35*
P_L E_Q W_L V_Q	1	1.86	1.86	
P_L E_L W_Q V_Q	1	16.07	16.07	6.69*
P_Q E_Q W_Q V_L	1	3.63	3.63	
P_Q E_Q W_L V_Q	1	3.26	3.26	
P_Q E_L W_Q V_Q	1	.13	.13	
P_L E_Q W_Q V_Q	1	.70	.70	
P_Q E_Q W_Q V_Q	1	1.06	1.06	
Error	486	1,165.53	2.4	

[1]With 1 and 486 degrees of freedom the following probabilities hold:

F = 6.70 P < .01

F = 3.86 P < .05

F = 2.75 P < .10

*Effect is included in the first model for prediction purposes.

significant at the 5 percent level of probability. Whether this level of significance is meaningful will be seen from the analysis which follows.

The effects which have an asterisk by the F values were included in the model for prediction purposes. The specific model for protein intake is

$$Y_1 = \begin{bmatrix} 1 \\ X_1 \\ X_2 \\ X_4 \\ X_1 X_2 \\ X_2 X_3 \\ X_2^2 X_3^2 \\ X_3 X_4^2 \\ X_1 X_2 X_3 \\ X_1^2 X_2^2 X_3 \\ X_1^2 X_2^2 X_3^2 \\ X_1^2 X_2 X_4 \\ X_1^2 X_2 X_4^2 \\ X_1 X_2 X_3 X_4 \\ X_1^2 X_2^2 X_3 X_4 \\ X_1 X_2^2 X_3^2 X_4 \\ X_1 X_2 X_3^2 X_4^2 \end{bmatrix} \quad \begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{0001} \\ a_{1100} \\ a_{0110} \\ a_{0220} \\ a_{0012} \\ a_{1110} \\ a_{2210} \\ a_{2220} \\ a_{2101} \\ a_{2102} \\ a_{1111} \\ a_{2211} \\ a_{1221} \\ a_{1122} \end{bmatrix}$$

This will be referred to as Protein Intake Model 1. Note: Where $_j = _k = _l = _m = 0$ a_{0000} is denoted as μ.

From the data in Appendix Table I, the least squares estimate of the parameters are calculated and are given in the following expression:

$$\begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{0001} \\ a_{1100} \\ a_{0110} \\ a_{0220} \\ a_{0012} \\ a_{1110} \\ a_{2210} \\ a_{2220} \\ a_{2101} \\ a_{2102} \\ a_{1111} \\ a_{2211} \\ a_{1221} \\ a_{1122} \end{bmatrix} = \begin{bmatrix} 13.20 \\ 3.08 \\ -2.08 \\ -.277 \\ .176 \\ .198 \\ .340 \\ -.115 \\ .225 \\ -.0436 \\ -.387 \\ -.0948 \\ .128 \\ -.271 \\ -.0349 \\ -.0938 \\ .0499 \end{bmatrix}$$

Two questions about this prediction should be considered: (1) how well will this equation predict protein intake in laying hens? and (2) can this equation be simplified without reducing the prediction quality? Tests will now be made to study these questions.

The sum of squares removed by the parameters in the model and the residual sum of squares are of importance when deciding how well a model fits the data. These are given in Table III. It can be seen from the data in this table that almost all of the variation among treatments is removed by parameters in the model. The total corrected mean square is

326.97 as compared to 2.94 for the residual. With this model the sum of squares due to variation within treatments can be separated from the residual sum of squares and can be used to test the residual. In this case the residual does not appear to be different from the error. This indicates that the parameters of the model probably account for all the variation due to treatment except for random error.

TABLE III

ANALYSIS OF VARIANCE OF PROTEIN INTAKE MODEL 1

Source	DF	SS	MS	F
Total	567	105,880.84		
$R(\mu)$[1]	1	99,295.53		
$R(\mu,a)$[2]	17	104,527.06		
R(a) adj.	16	5,231.53	326.97	136.24($P < .01$)
Residual	64	188.25	2.94	1.22
Error	486	1,165.53	2.40	

[1] $R(\mu)$ - Reduction in sum of squares due to μ.

[2] $R(\mu,a)$ - Reduction in sum of squares due to μ, and a. In this, a includes all the a_{jkl} in the model.

In order to test further the residual variation, the predicted value, $\hat{Y}_1$, is calculated. This is the predicted set of values for future observations of protein intake for the treatments, if the experiment were to be repeated. These values are presented in Table IV. When these values are compared to the treatment means which were presented by Gleaves, the general trends are the same. The treatment

TABLE IV

PREDICTED VALUES, $\hat{Y}_1$, FOR PROTEIN INTAKE MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	12.17	12.35	12.60	16.21	15.82	15.66	19.05	18.60	18.03
	137	12.43	12.38	12.07	15.56	15.28	15.00	18.24	18.18	17.87
	147	12.51	12.31	11.73	15.58	15.42	15.03	17.42	17.67	17.34
300	127	10.51	10.12	9.96	13.59	13.20	13.04	16.67	16.28	16.12
	137	10.40	10.12	9.84	13.48	13.20	12.92	16.56	16.28	16.00
	147	10.28	10.12	9.73	13.36	13.20	12.81	16.44	16.28	15.89
340	127	8.59	7.89	7.57	11.65	11.26	11.10	14.40	13.95	14.08
	137	8.36	7.86	7.62	11.40	11.12	10.84	14.88	14.38	14.13
	147	7.75	7.74	7.67	11.82	11.66	11.27	15.54	14.71	14.00

means are denoted by $\bar{\bar{Y}}$. In order to make a more direct comparison of the $\hat{Y}_i$ to $\bar{\bar{Y}}_i$, the residual deviations, $\bar{\bar{Y}}_i - \hat{Y}_i$, are calculated and presented in Table V.

According to the general model, the assumption is made that the random error, e, is distributed normally with a mean of zero. The residuals, $\bar{\bar{Y}} - \hat{Y}$, estimate this random error. $\bar{\bar{Y}}$ is the set of treatment means which was presented in the report by Gleaves. It is admitted that $Y - \hat{Y}$ is a better estimate of the random error than $\bar{\bar{Y}} - \hat{Y}$; however, in this case, with 567 Y values, it would be extremely difficult to calculate and to present $Y - \hat{Y}$. Therefore, $\bar{\bar{Y}} - \hat{Y}$ is used as a substitute.

If there are any definite trends of either positive or negative numbers in the residual deviations, it is probable that they are not distributed normally. However, there is no obvious indication that there is any such trend in the results presented here.

The test used to check for zero mean and normal distribution is a half-normal plot of the residuals. They are ranked according to absolute size (that is, without regard to sign). Then the rank, expressed as a percentage of sample size, is plotted against absolute value on normal graph paper. The percentage of sample size contains a continuity correction of .5 and is calculated by $P = (i - .5)/N \times 100$. In this case $N = 81$. If the points on the graph form a straight line which passes through the origin, then it is assumed that the residual deviation is a result of random error.

For Protein Intake Model 1 the half-normal plot is presented in Figure 1. It appears from this figure that the residual variation for this model is very close to random error. Although the points do not exactly form a straight line, it is very close and the line does pass

TABLE V

THE RESIDUAL DEVIATIONS, $\bar{Y}_1 - \hat{Y}_1$, FOR PROTEIN INTAKE MODEL 1

		Protein (Grams)								
		13			16			19		
		Milliliters			Milliliters			Milliliters		
Metabolizable Energy (Kilocalories)	Weight (Grams)	180	230	280	180	230	280	180	230	280
260	127	-.12	.53	-.61	-.29	-.53	-.23	.31	-.30	.25
	137	.17	-1.55	.08	-.19	-.50	-.06	-.87	.65	-.43
	147	.14	.01	-.08	.98	-.11	-.22	-.20	.10	.66
300	127	.08	.20	.32	1.10	-.45	-.22	-.60	-.09	.88
	137	.90	-.08	.21	.79	.60	.35	.88	-.19	-.59
	147	1.04	.92	-.46	-.78	.02	0	-.27	.15	0
340	127	-.37	-.38	.17	-1.50	.38	-.12	.82	-.65	.31
	137	-.29	.71	-.08	-1.20	-.04	-1.03	-.67	-.26	.71
	147	-.24	-.02	-.61	-.23	.88	.95	.17	.70	-.65

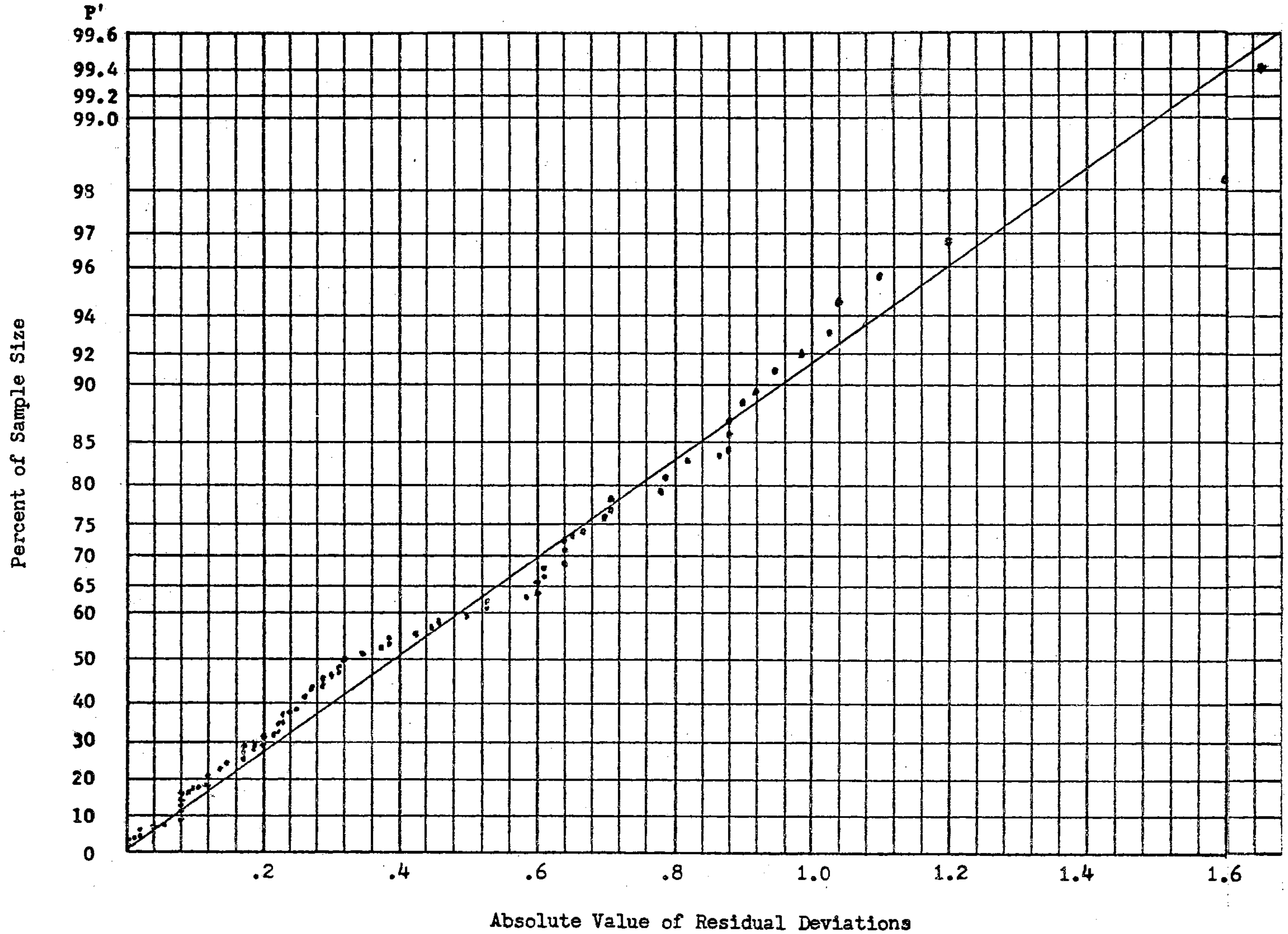

Figure 1. Half-Normal Plot of the Residual Deviations for Protein Intake Model 1

through the origin. The fact that the line passes through the origin indicates that the mean is zero.

It appears from the preceding tests that the Protein Intake Model 1 does a reasonably good job of predicting. However, the only true test would be to perform actual feeding trials and compare the observed outcome to the predicted outcome.

Protein Intake Model 1 has a large number of terms and it is difficult to calculate predicted values with such a complicated equation. It would be advantageous to have a simpler prediction equation if a simpler one would give results comparable to Model 1. The hypothesis that each parameter is equal to zero is tested in order to eliminate some of the parameters and still have a good prediction equation. The t test is used for this purpose. The calculations are made using

$$t = \frac{\hat{a}_{jklm} - a_{jklm}}{\sqrt{\hat{\sigma}^2 \quad c_{ii}}} \quad \text{with } N - p \text{ degrees of freedom}$$

where c_{ij} is the ijth element of $(X'X)^{-1}$ (Graybill, 1961). Since the hypothesis is that $a_{jklm} = 0$ the formula becomes

$$t = \frac{\hat{a}_{jklm}}{\sqrt{\hat{\sigma}^2 \quad c_{ii}}}$$

In this case N = 81 and P = 17 so there are 64 degrees of freedom. From Table III, $\hat{\sigma}^2 = 2.94$ is obtained.

The calculated t values for the estimated parameters are in Table VI. It can be seen from the data in this table that only μ, a_{1000}, a_{0100}, and a_{0001} are significantly different from zero at the 5 percent level of probability. Based upon this, the model

$$Y_i = \mu + a_{1000}\ X_1 + a_{0100}\ X_2 + a_{0001}\ X_4$$

TABLE VI

CALCULATED t FOR THE ESTIMATED PARAMETERS OF
PROTEIN INTAKE MODEL 1

Parameter	Estimate	c_{ii}	$\hat{\sigma}^2 c_{ii}$ [1]	$\sqrt{\hat{\sigma}^2 c_{ii}}$	t*
μ	13.20	.00317	.00932	.09654	136.73
a_{1000}	3.08	.00264	.00776	.08809	34.96
a_{0100}	-2.08	.00476	.0140	.1183	17.58
a_{0001}	- .28	.00264	.00776	.08809	3.18
a_{1100}	.18	.00714	.0210	.1449	1.24
a_{0110}	.20	.00397	.0117	.1082	1.85
a_{0220}	.34	.0151	.0444	.2107	1.61
a_{0012}	- .11	.00564	.0166	.1288	.85
a_{1110}	.22	.00595	.0175	.1323	1.66
a_{2210}	- .044	.00846	.0249	.1577	.27
a_{2220}	- .39	.0179	.0579	.2406	1.62
a_{2101}	- .095	.00595	.0175	.1323	.72
a_{2102}	.13	.0107	.0314	.1772	.73
a_{1111}	- .27	.00893	.0262	.1618	1.66
a_{2211}	- .035	.00893	.0262	.1618	.21
a_{1221}	- .094	.00893	.0262	.1618	.58
a_{1122}	.050	.0161	.0473	.2175	.23

$*t_{.05} = 2.00$

[1] $\hat{\sigma}^2 = 2.94$

is used for the prediction of protein intake. This model is called Protein Intake Model 2. The parameters are re-estimated and the resulting prediction equation is

$$\hat{Y}_1 = 13.23 + 3.08x_1 - 2.03x_2 - .28x_4 .$$

The residual sum of squares is tested in the same way as for Protein Intake Model 1. The analysis of variance is presented in Table VII. The results of this test indicate essentially the same thing as the corresponding test for Protein Intake Model 1. This means that, based upon the residual sum of squares, the simpler equation will predict future observations of protein intake as accurately as the one with 17 terms.

TABLE VII

ANALYSIS OF VARIANCE OF PROTEIN INTAKE MODEL 2

Source	DF	SS	MS	F
Total	567	105,880.84		
$R(\mu)$	1	99,295.56		
$R(\mu,a)$	4	104,472.06		
$R(a)$	3	5,176.53	1725.51	546.04($P < .01$)
Residual	77	243.25	3.16	1.31
Error	486	1,165.53	2.40	

In order to check further the prediction ability of Protein Intake Model 2, the predicted values, $\hat{Y}_1$, and the residual deviations, $\bar{Y}_1 - \hat{Y}_1$, were calculated. They are presented in Tables VIII and IX, respectively. There is nothing in these two tables to indicate that Protein Intake

TABLE VIII

THE PREDICTED VALUES, $\hat{Y}_1$, FOR PROTEIN INTAKE MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	12.46	12.46	12.46	15.54	15.54	15.54	18.62	18.62	18.62
	137	12.18	12.18	12.18	15.26	15.26	15.26	18.34	18.34	18.34
	147	11.90	11.90	11.90	14.98	14.98	14.98	18.06	18.06	18.06
300	127	10.43	10.43	10.43	13.51	13.51	13.51	16.59	16.59	16.59
	137	10.15	10.15	10.15	13.23	13.23	13.23	16.31	16.31	16.31
	147	9.87	9.87	9.87	12.95	12.95	12.95	16.03	16.03	16.03
340	127	8.40	8.40	8.40	11.48	11.48	11.48	14.56	14.56	14.56
	137	8.12	8.12	8.12	11.20	11.20	11.20	14.28	14.28	14.28
	147	7.84	7.84	7.84	10.92	10.92	10.92	14.00	14.00	14.00

TABLE IX

THE RESIDUAL DEVIATIONS, $\bar{Y}_1 - \hat{Y}_1$, FOR PROTEIN INTAKE MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein 13 (Grams), Milliliters 180	13: 230	13: 280	16: 180	16: 230	16: 280	19: 180	19: 230	19: 280
260	127	-.41	.42	-.47	.38	-.25	-.11	.74	-.32	-.34
	137	.42	-.95	-.03	.11	-.48	-.32	-.97	.49	-.90
	147	.75	.42	-.25	1.58	.33	-.17	-.84	-.29	-.06
300	127	.16	-.11	-.15	1.18	-.76	-.69	-.52	-.40	.41
	137	1.15	-.11	-.10	1.04	.57	.04	1.13	.22	-.90
	147	1.45	1.17	-.60	-.37	.27	-.14	.14	.40	-.14
340	127	-.18	-.89	-.66	-1.33	.16	-.50	.80	-1.26	-.17
	137	-.05	.45	-.58	-1.00	-.10	-1.39	-.07	-.16	.56
	147	-.33	-.12	-.78	.67	1.62	1.30	1.71	1.41	-.65

Model 2 would not accurately predict future observations. The trends in the $\hat{Y}_1$ are the same as in $\bar{\bar{Y}}_1$ and there is no indication that the residual deviations follow any certain trend. The half-normal plot of the residual deviations is presented in Figure 2. The result is almost identical to that for Protein Intake Model 1. The residual deviations appear to approach the normal distribution with a mean of zero.

It can be concluded from the preceding tests that the less complicated model, Protein Intake Model 2, is superior to Protein Intake Model 1 by the fact that Model 2 is the simpler. There are no indications that there are any differences in the ability of the two models to predict future responses. The analysis of variance for protein intake presented by Gleaves shows significant ($P < .05$) 3-way and 4-way interactions. However, the results here indicate that these effects need not be considered when estimating future responses.

Energy Intake: From the analysis of variance of energy consumption which is presented in Table X, 15 terms were picked to go into Energy Intake Model 1. This was done in the same manner as for protein intake. In order to include every effect that may be of importance insofar as prediction is concerned, the effects that have F values with $P < .1$ were included in the model. The model is

$$\begin{aligned} Y_2 = \mu &+ a_{1000} X_1 + a_{0001} X_4 + a_{0200} X_2^2 + a_{1100} X_1 X_2 + a_{1200} X_1 X_2^2 \\ &+ a_{0110} X_2 X_3 + a_{0220} X_2^2 X_3^2 + a_{0012} X_3 X_4^2 + a_{2210} X_1^2 X_2^2 X_3 \\ &+ a_{2120} X_1^2 X_2 X_3^2 + a_{2220} X_1^2 X_2^2 X_3^2 + a_{2101} X_1^2 X_2 X_4 \\ &+ a_{2102} X_1^2 X_2 X_4^2 + a_{2211} X_1^2 X_2^2 X_3 X_4 + a_{1122} X_1 X_2 X_3^2 X_4^2 \end{aligned}$$

The least squares estimate of the parameters is as follows:

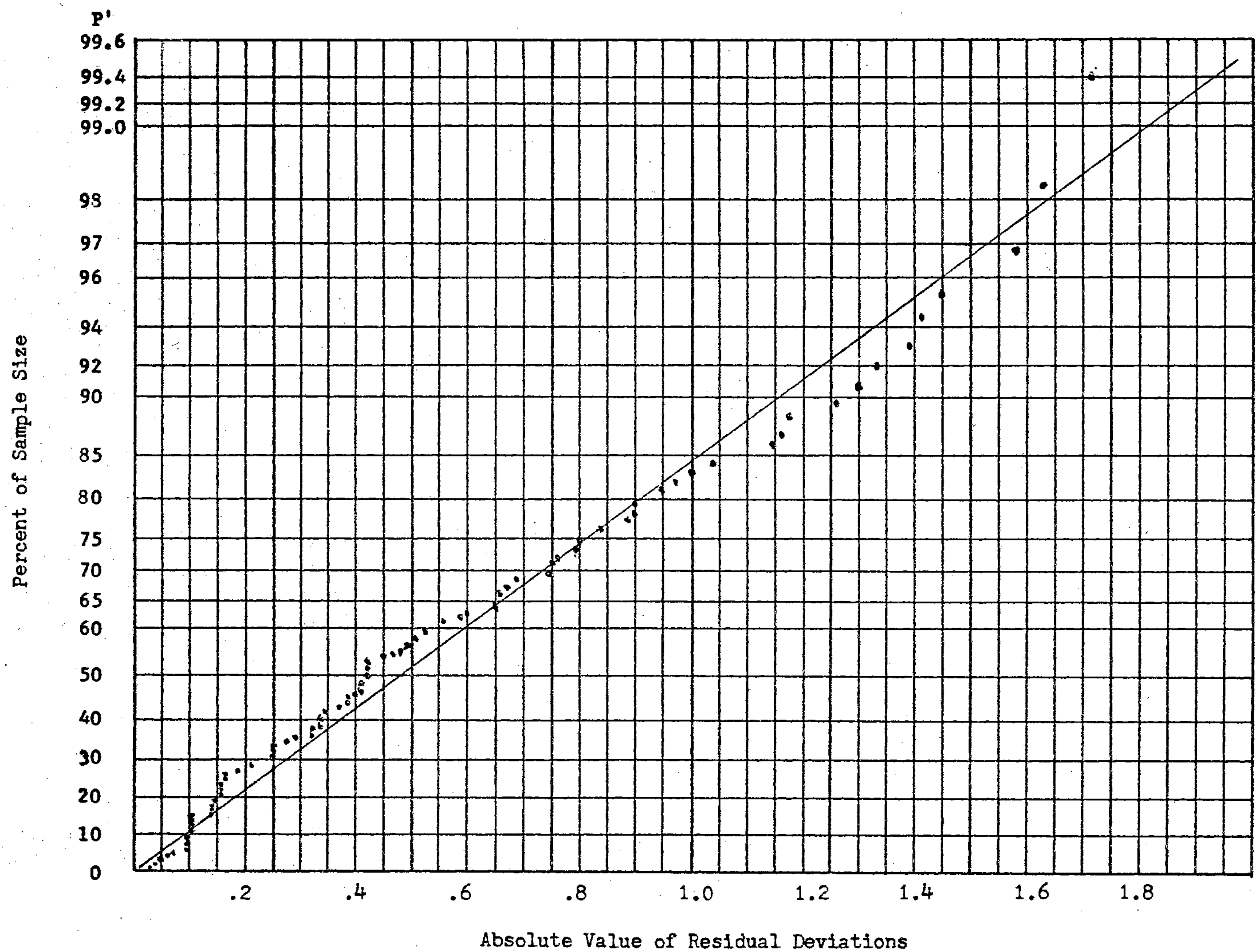

Figure 2. Half-Normal Plot of the Residual Deviations for Protein Intake Model 2

TABLE X

ANALYSIS OF VARIANCE OF
ENERGY CONSUMPTION

Source	DF	SS	MS	F^2
Total	566	709,973.08		
Treatment	[80]			
Protein (P)	(2)	75,576.71		
P_L	1	73,982.20	73,928.20	77.11*
P_Q	1	1,594.51	1,594.51	1.66
Energy(E)	(2)	29,470.38		
E_L	1	2,390.94	2,390.94	2.49
E_Q	1	27,079.14	27,079.44	28.22*
Weight(W)	(2)	976.04		
W_L	1	4.17	4.17	.004
W_Q	1	971.87	971.87	1.012
Volume(V)	(2)	11,401.34		
V_L	1	11,206.95	11,206.95	11.68*
V_Q	1	194.39	194.39	.20
Interaction	(72)			
P x E	(4)	45,578.81		
P_L E_L	1	41,811.57	41,811.57	43.58*
P_Q E_L	1	22.23	22.23	
P_L E_Q	1	3,454.30	3,454.30	3.60*
P_Q E_Q	1	290.71	290.71	
P x W	4	2,736.74	684.19	
P x V	4	2,084.23	521.06	
E x W	(4)	6,212.15		
E_L W_L	1	2,707.44	2,707.44	2.82*
E_Q W_L	1	562.20	562.20	
E_L W_Q	1	34.71	34.71	
E_Q W_Q	1	2,907.68	2,907.68	3.03*
E x V	4	3,806.59	951.65	
W x V	(4)	4,288.23		
W_L V_L	1	764.76	764.76	
W_Q V_L	1	39.58	39.58	
W_L V_Q	1	3,365.11	3,365.11	3.50*
W_Q V_Q	1	118.78	118.78	
P x E x W	(8)	16,932.43		
P_L E_L W_L	1	2,600.72	2,600.72	2.71
P_Q E_L W_L	1	1,113.09	1,113.09	
P_L E_Q W_L	1	778.33	778.33	
P_L E_L W_Q	1	1,247.74	1,247.74	
P_Q E_Q W_L	1	3,399.00	3,399.00	3.54*
P_Q E_L W_Q	1	3,215.62	3,215.62	3.35*
P_L E_Q W_Q	1	141.77	141.77	
P_Q E_Q W_Q	1	4,736.16	4,736.16	4.93*

TABLE X (CONTINUED)

Source	DF	SS	MS	F^2
P x E x V	(8)	10,094.91		
P_L E_L V_L	1	360.21	360.21	
P_Q E_L V_L	1	3,281.34	3,281.34	3.42*
P_L E_Q V_L	1	1,068.96	1,068.96	
P_L E_L V_Q	1	1,536.50	1,536.50	
P_Q E_Q V_L	1	143.62	143.62	
P_Q E_L V_Q	1	3,259.52	3,259.52	3.39*
P_L E_Q V_Q	1	2.72	2.72	
P_Q E_Q V_Q	1	442.03	442.03	
P x W x V	8	4,821.68	602.71	
E x W x V	8	5,396.53	674.57	
P x E x W x V	(16)	23,701.14		
P_L E_L W_L V_L	1	2,304.14	2,304.14	2.40
P_Q E_L W_L V_L	1	651.85	651.85	
P_L E_Q W_L V_L	1	90.10	90.10	
P_L E_L W_Q V_L	1	946.71	946.71	
P_L E_L W_L V_Q	1	10.01	10.01	
P_Q E_Q W_L V_L	1	4,637.14	4,637.14	5.88*
P_Q E_L W_Q V_L	1	2,114.68	2,114.68	2.20
P_Q E_L W_L V_Q	1	1,580.00	1,580.00	
P_L E_Q W_Q V_L	1	1,989.14	1,989.14	2.07
P_L E_Q W_L V_Q	1	667.06	667.06	
P_L E_L W_Q V_Q	1	5,723.81	5,723.81	5.97*
P_Q E_Q W_Q V_L	1	1,156.38	1,156.38	
P_Q E_Q W_L V_Q	1	1,213.97	1,213.97	
P_Q E_L W_Q V_Q	1	223.44	223.44	
P_L E_Q W_Q V_Q	1	646.78	646.78	
P_Q E_Q W_Q V_Q	1	366.73	366.73	
Error	486	466,274.27	959.41	

[2]With 1 and 486 degrees of freedom the following probabilities hold:

$F = 6.70 \quad P < .01$

$F = 3.86 \quad P < .05$

$F = 2.75 \quad P < .10$

*Effect is included in the model for prediction purposes.

$$\begin{bmatrix} \mu \\ a_{1000} \\ a_{0001} \\ a_{0200} \\ a_{1100} \\ a_{1200} \\ a_{0110} \\ a_{0220} \\ a_{0012} \\ a_{2210} \\ a_{2120} \\ a_{2220} \\ a_{2101} \\ a_{2102} \\ a_{2211} \\ a_{1122} \end{bmatrix} = \begin{bmatrix} 250.99 \\ 9.71 \\ -5.45 \\ -13.40 \\ 12.14 \\ 6.41 \\ 3.28 \\ 12.78 \\ -1.84 \\ -.84 \\ -7.32 \\ -9.91 \\ -2.07 \\ -2.05 \\ -1.23 \\ 1.69 \end{bmatrix}$$

The tests that were made on Protein Intake Model 1 were made on the parameters of this model. The analysis of variance to test the residual sum of squares is in Table XI. The F test indicates that the residual sum of squares is probably a result of random error. However, further tests must be made before a conclusion can be drawn.

The predicted values, $\hat{Y}_2$, for Energy Intake Model 1 are presented in Table XII, and the residual deviations, $\bar{\bar{Y}}_2 - \hat{Y}_2$, are in Table XIII. Visual observations of the numbers in these two tables do not reveal anything that would indicate that this prediction equation would give skewed results. However, the half-normal plot of the residual deviations

TABLE XI

ANALYSIS OF VARIANCE FOR ENERGY INTAKE MODEL 1

Source	DF	SS	MS	F
Total	567	34,688,311		
$R(\mu)$	1	33,978,338		
$R(\mu, a)$	16	34,146,719		
$R(a)$	15	168,381	11,225.40	11.70($P < .01$)
Residual	65	75,316	1,158.72	1.20
Error	486	466,274	959.41	

in Figure 3 shows that something other than random error is affecting the residuals. It can be seen from this graph that the residual deviations probably do not have a normal distribution. This indicates that the prediction equation contains error other than random error. It would be extremely difficult to determine the cause of the non-normal distribution of the residuals.

It is possible that a simplification of Energy Intake Model 1, such as was done with Protein Intake Model 1, could result in alteration of the distribution of the residual deviations. The hypothesis that each parameter is equal to zero is tested in the same way as for Protein Intake Model 1. This information is tabulated in Table XIV. By taking only the terms for which the parameters differ from zero with a 95 percent level of confidence, the simplified model becomes

TABLE XII

PREDICATED VALUES, $\hat{Y}_2$, FOR ENERGY INTAKE MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	255.65	247.92	251.35	260.94	253.65	250.04	260.23	255.88	255.93
	137	239.04	233.61	232.28	243.04	237.59	232.14	247.00	241.57	240.24
	147	246.19	239.68	236.97	250.70	247.09	239.80	250.77	247.64	241.55
300	127	248.57	241.28	237.67	258.28	250.99	247.38	267.99	260.70	257.09
	137	246.73	241.28	235.83	256.44	250.99	245.54	266.15	260.70	255.25
	147	244.89	241.28	233.99	254.60	250.99	243.70	264.31	260.70	253.41
340	127	206.83	202.44	194.25	254.38	247.09	243.48	266.73	258.96	254.15
	137	214.80	209.33	199.76	243.04	237.59	232.14	271.32	265.85	256.28
	147	210.49	207.32	192.99	257.26	253.65	246.36	270.39	263.84	252.89

TABLE XIII

THE RESIDUAL DEVIATIONS, $\bar{Y}_2 - \hat{Y}_2$, FOR ENERGY INTAKE MODEL 1

		Protein (Grams)								
		13			16			19		
		Milliliters			Milliliters			Milliliters		
Metabolizable Energy (Kilocalories)	Weight (Grams)	180	230	280	180	230	280	180	230	280
260	127	-13.22	11.22	-10.35	-.38	-3.22	2.53	8.34	-2.31	-2.64
260	137	14.39	-16.04	12.15	8.82	4.55	12.19	-6.00	19.29	1.62
260	147	8.38	8.46	-2.40	20.59	3.48	2.91	-11.91	-1.35	7.74
300	127	-5.00	-3.85	-1.53	19.29	-10.56	-5.67	-11.42	-1.99	14.34
300	137	13.13	-10.42	-4.54	12.85	9.30	4.60	12.28	-3.99	-9.11
300	147	15.54	12.86	-20.56	-17.60	-1.42	-1.84	-6.31	1.59	.30
340	127	9.03	-5.15	9.18	-37.52	1.48	-8.77	8.56	-18.67	5.99
340	137	-3.09	15.81	-1.62	-25.18	-.88	-22.85	-14.61	-10.56	11.86
340	147	-13.35	-4.32	-7.56	-9.83	13.92	14.78	13.47	14.59	-11.75

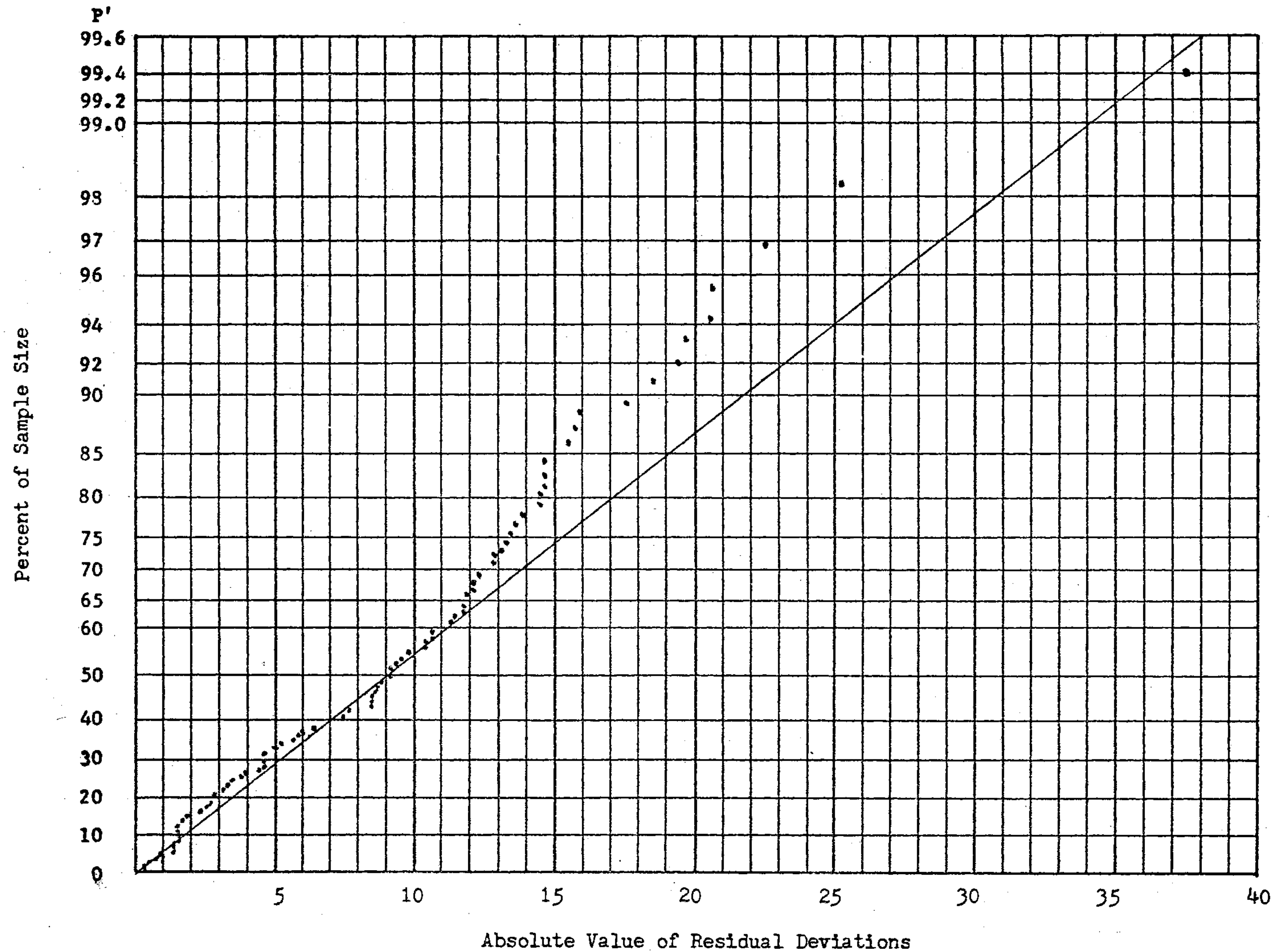

Figure 3. Half-Normal Plot of the Residual Deviations for Energy Intake Model 1

TABLE XIV

CALCULATED t FOR THE ESTIMATED PARAMETERS OF ENERGY INTAKE MODEL 1

Parameter	Estimate	c_{ii}	$\hat{\sigma}^2 c_{ii}$ [1]	$\sqrt{\hat{\sigma}^2 c_{ii}}$	t*
μ	250.99	.00529	6.13	2.476	101.37
a_{1000}	9.71	.00794	9.20	3.033	3.20
a_{0001}	-5.45	.00265	3.07	1.752	3.11
a_{0200}	-13.40	.01323	15.33	3.915	3.42
a_{1100}	12.14	.00714	8.27	2.876	4.22
a_{1200}	6.41	.01190	13.78	3.728	1.72
a_{0110}	3.28	.00397	4.60	2.144	1.53
a_{0220}	12.78	.01984	22.99	4.795	2.66
a_{0012}	-1.84	.00564	6.54	2.557	.72
a_{2210}	-.84	.00846	9.80	3.130	.27
a_{2120}	-7.32	.01071	12.41	3.521	2.08
a_{2220}	-9.91	.01786	20.69	4.561	2.17
a_{2101}	-2.07	.00595	6.89	2.625	.79
a_{2102}	-2.05	.01071	12.41	3.521	.58
a_{2211}	-1.23	.00893	10.40	3.225	.38
a_{1122}	1.69	.01607	18.62	4.313	.39

[1] $\hat{\sigma}^2$ = 1,158.72

*$t_{.05}$ = 2.00

$$Y_2 = \begin{bmatrix} 1 \\ X_1 \\ X_4 \\ X_2^2 \\ X_1 X_2 \\ X_2^2 X_3^2 \\ X_1^2 X_2 X_3^2 \\ X_1^2 X_2^2 X_3^2 \end{bmatrix}' \begin{bmatrix} \mu \\ a_{1000} \\ a_{0001} \\ a_{0200} \\ a_{1100} \\ a_{0220} \\ a_{2120} \\ a_{2220} \end{bmatrix}$$

This model is called Energy Intake Model 2. When the parameters are re-estimated the prediction equation becomes

$$\hat{Y}_2 = \begin{bmatrix} 1 \\ X_1 \\ X_4 \\ X_2^2 \\ X_1 X_2 \\ X_2^2 X_3^2 \\ X_1^2 X_2 X_3^2 \\ X_1^2 X_2^2 X_3^2 \end{bmatrix}' \begin{bmatrix} 250.99 \\ 13.99 \\ -5.45 \\ -13.40 \\ 12.88 \\ 12.78 \\ -8.68 \\ -9.91 \end{bmatrix}$$

The analysis of variance for this equation is presented in Table XV. From the standpoint of total residual sum of squares, this equation has equal predicting ability to that of Energy Intake Model 1. From predicted values, $\hat{Y}_2$, in Table XVI and the deviations from the means, $\bar{Y}_2 - \hat{Y}_2$, in Table XVIII, there are no indications that Energy Intake Model 2 gives a skewed prediction pattern. The half-normal plot in Figure 4 shows that the distribution of the residual deviations is closer to normal

TABLE XV

ANALYSIS OF VARIANCE OF ENERGY INTAKE MODEL 2

Source	DF	SS	MS	F
Total	567	34,688,311		
$R(\mu)$	1	33,978,338		
$R(\mu,a)$	9	34,137,567		
$R(a)$	8	159,238	19,904.75	16.96($P < .01$)
Residual	72	84,461	1,173.06	1.22
Error	486	466,274	959.41	

than that for Energy Intake Model 1. This indicates that the simpler equation is not only easier to use but it will probably do a better job of predicting.

It was stated by Gleaves that there must be an interrelationship between energy and weight on energy intake, but he was not able to show it. In the prediction equations for energy intake, the coefficient of the term $x_2^2x_3^2$ is significant at the one percent level of probability. Since x_2 and x_3 represent the levels of dietary energy and dietary weight, this substantiates the statement made by Gleaves with positive evidence.

A 3-way interaction of protein x energy x weight on energy intake was pointed out and is discussed at length by Gleaves. The analysis here more clearly defines where the interaction occurs. In the prediction equations for energy intake, the coefficients for the terms $x_1^2x_2x_3^2$ and $x_1^2x_2^2x_3^2$ are significant at the 95 percent confidence level.

TABLE XVI

PREDICTED VALUES, $\hat{Y}_2$, FOR ENERGY INTAKE MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	253.48	248.03	242.58	255.82	250.37	244.92	255.70	250.25	244.80
	137	241.93	236.48	231.03	243.04	237.59	232.14	244.15	238.70	233.25
	147	253.48	248.03	242.58	255.82	250.37	244.92	255.70	250.25	244.80
300	127	242.45	237.00	231.55	256.44	250.99	245.54	270.43	264.98	259.53
	137	242.45	237.00	231.55	256.44	250.99	245.54	270.43	264.98	259.53
	147	242.45	237.00	231.55	256.44	250.99	245.54	270.43	264.98	259.53
340	127	210.36	204.91	199.46	255.82	250.37	244.92	264.10	258.65	253.20
	137	216.17	210.72	205.27	243.04	237.59	232.14	269.91	264.46	259.01
	147	210.36	204.91	199.46	255.82	250.37	244.92	264.10	258.65	253.20

TABLE XVII

THE RESIDUAL DEVIATIONS, $\bar{Y}_2 - \hat{Y}_2$, FOR ENERGY INTAKE MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	-11.05	11.11	-1.58	4.75	.06	7.65	12.87	3.32	8.49
	137	11.50	-18.91	13.40	8.82	4.55	12.29	-3.15	22.16	8.61
	147	1.09	.11	-8.01	15.47	.20	-2.21	-16.84	-3.96	4.49
300	127	1.12	.43	4.59	20.85	-10.56	-3.83	-13.86	-6.27	11.90
	137	17.41	-6.14	-.26	12.85	9.30	4.60	8.00	-8.27	-13.39
	147	17.98	17.14	-18.12	-19.44	-1.42	3.68	-12.43	-2.69	-5.82
340	127	5.50	-7.62	3.97	-38.96	-1.80	-10.21	11.19	-18.35	6.94
	137	-4.46	14.42	-7.13	-25.18	-.88	-22.85	-13.20	-9.17	9.13
	147	13.22	-1.91	14.03	-8.39	17.20	16.22	19.76	19.78	-12.06

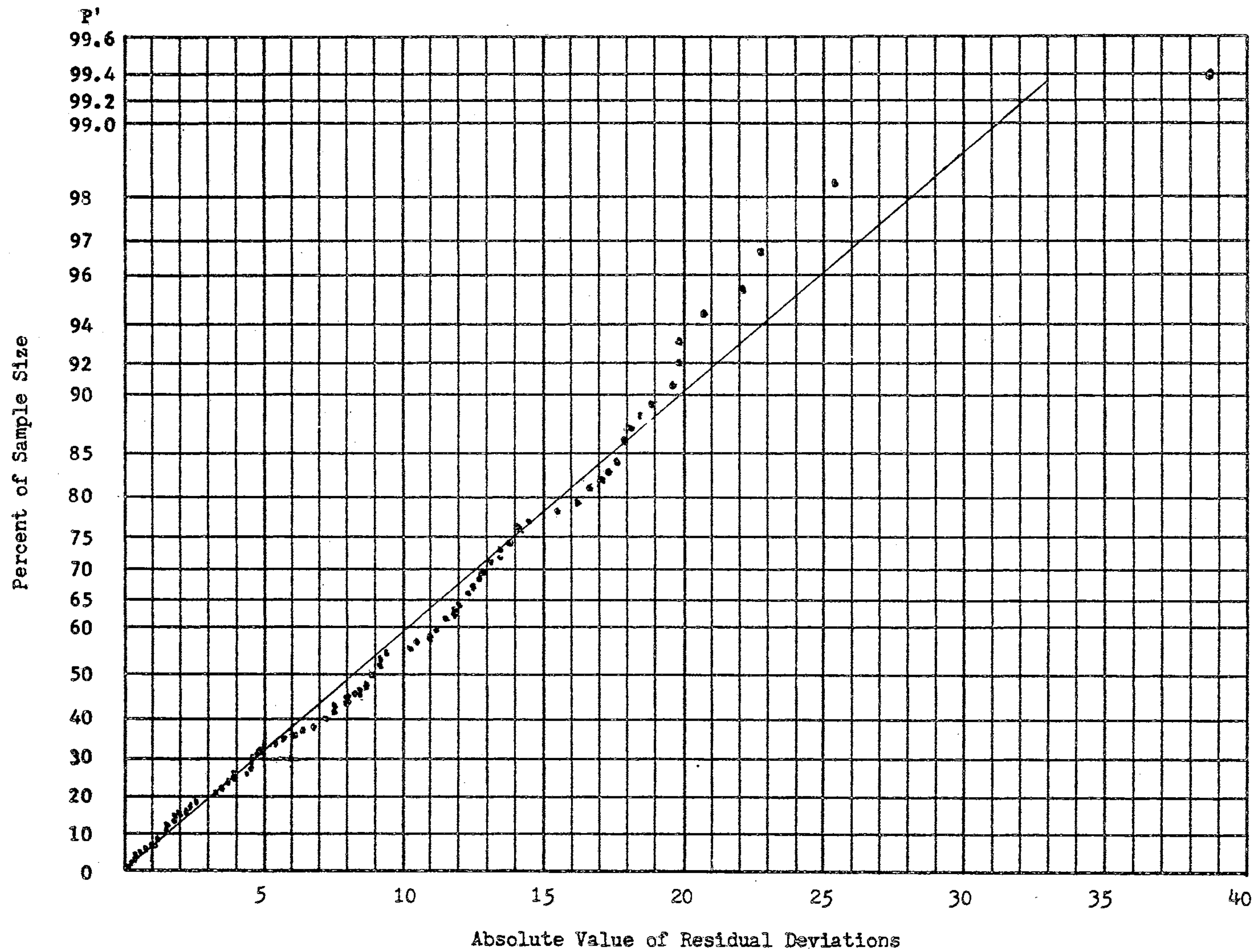

Figure 4. Half-Normal Plot of the Residual Deviations for Energy Intake Model 2

When the prediction equations for protein intake are compared to those for energy intake, it is apparent that the control of energy intake requires the consideration of more factors than for the control of protein intake. The results of the prediction equation from Protein Intake Model 2 show that the linear effects of only three dietary factors are necessary for reasonable prediction of protein intake. However, the main effects of 2 dietary factors and 4 interaction effects were included in Energy Intake Model 2. It is well known that energy is the dominant dietary factor in the control of feed intake. Therefore, dietary energy largely determines the intake of protein. The intake of protein can be changed simply by changing the calorie:protein ratio, and dietary factors other than protein and energy have only a small influence upon protein intake. On the other hand, changes in energy intake are affected more readily by certain non-dietary factors than by dietary factors. Egg production, as pointed out by Gleaves, is a good example of a non-dietary factor which strongly affects energy intake. For these reasons it is much more difficult to estimate future responses of energy intake by dietary factors only. These ideas are the subject of the next chapter.

Production Responses

Body Weight Change: Prediction equations for the production responses are developed in the same way as for the nutrient intake responses. The analysis of variance for body weight change is in Table XVIII. From this table the terms are picked for the model in the same way as was done before. Therefore, Body Weight Change Model 1 is:

TABLE XVIII

ANALYSIS OF VARIANCE OF
BODY WEIGHT CHANGE

Source	DF	SS	MS	F[1]
Total	566	42,501,182.86		
Treatment	[80]			
Protein(P)	(2)	5,725,360.65		
P_L	1	5,715,507.88	5,715,507.88	99.20*
P_Q	1	9,852.77	9,852.77	
Energy(E)	(2)	2,131,659.24		
E_L	1	1,732,739.54	1,732,739.54	30.07*
E_Q	1	398,909.70	398,909.70	6.92*
Weight(W)	(2)	131,514.68		
W_L	1	70,584.78	70,584.78	
W_Q	1	60,929.90	60,929.90	
Volume(V)	(2)	302,698.95		
V_L	1	301,667.63	301,667.63	5.24*
V_Q	1	1,031.32	1,031.32	
Interaction	(72)			
P x E	(4)	1,371,978.69		
P_L E_L	1	1,267,493.53	1,267,493.53	21.99*
P_Q E_L	1	91,344.04	91,344.04	
P_L E_Q	1	11,464.46	11,464.46	
P_Q E_Q	1	1,676.66	1,676.66	
P x W	4	205,968.96	51,492.24	
E x V	4	182,289.74	45,572.44	
E x W	(4)	525,474.56		
E_L W_L	1	363,356.19	363,356.19	6.30*
E_Q W_L	1	74,384.92	74,384.92	
E_L W_Q	1	36,333.44	36,333.44	
E_Q W_Q	1	51,449.99	51,449.99	
E x V	4	136,670.06	34,167.52	
W x V	(4)	653,682.88		
W_L V_L	1	49,028.67	49,028.67	
W_Q V_L	1	90,095.25	90,095.25	
W_L V_Q	1	512,044.47	512,044.47	8.88*
W_Q V_Q	1	2,520.49	2,520.49	
P x E x W	(8)	644,273.23		
P_L E_L W_L	1	5,497.14	5,497.14	
P_Q E_L W_L	1	62,200.00	62,200.00	
P_L E_Q W_L	1	64,080.33	64,080.33	
P_L E_L W_Q	1	24,402.87	24,402.87	
P_Q E_Q W_L	1	232,043.88	232,043.88	4.02*
P_Q E_L W_Q	1	4,123.68	4,123.68	
P_L E_Q W_Q	1	258.37	258.37	
P_Q E_Q W_Q	1	251,666.96	251,666.96	4.36*

TABLE XVIII (CONTINUED)

Source	DF	SS	MS	F[1]
P x E x V	(8)	495,490.77		
P_L E_L V_L	1	179,470.72	179,470.72	3.11*
P_Q E_L V_L	1	177,825.57	177,825.57	3.08*
P_L E_Q V_L	1	19,800.16	19,800.16	
P_L E_L V_Q	1	674.38	674.38	
P_Q E_Q V_L	1	60,888.90	60,888.90	
P_Q E_L V_Q	1	37,590.29	37,590.29	
P_L E_Q V_Q	1	4,280.38	4,280.38	
P_Q E_Q V_Q	1	14,960.37	14,960.37	
P x W x V	(8)	475,836.72		
P_L W_L V_L	1	212,077.14	212,077.14	3.68*
P_Q W_L V_L	1	10,359.57	10,359.57	
P_L W_Q V_L	1	26,796.87	26,796.87	
P_L W_L V_Q	1	6,223.09	6,223.09	
P_Q W_Q V_L	1	204,053.72	204,053.72	3.54*
P_Q W_L V_Q	1	439.30	439.30	
P_L W_Q V_Q	1	15,435.55	15,435.55	
P_Q W_Q V_Q	1	452.35	452.35	
E x W x V	(8)	642,337.25		
E_L W_L V_L	1	3,268.33	3,268.33	
E_Q W_L V_L	1	10,215.00	10,255.00	
E_L W_Q V_L	1	53,177.24	53,177.24	
E_L W_L V_Q	1	60,479.04	60,479.04	
E_Q W_Q V_L	1	87,817.14	87,817.14	
E_Q W_L V_Q	1	23,564.12	23,564.12	
E_L W_Q V_Q	1	25,023.04	25,023.04	
E_Q W_Q V_Q	1	378,793.37	378,793.37	6.57*
P x E x W x V	(16)	874,501.06		
P_L E_L W_L V_L	1	7,540.72	7,540.72	
P_Q E_L W_L V_L	1	2,524.52	2,524.52	
P_L E_Q W_L V_L	1	87,979.07	87,979.07	
P_L E_L W_Q V_L	1	12,373.57	12,373.57	
P_L E_L W_L V_Q	1	117,413.57	117,413.57	2.03
P_Q E_Q W_L V_L	1	148,361.54	148,361.54	2.57
P_Q E_L W_Q V_L	1	4,991.12	4,991.12	
P_Q E_L W_L V_Q	1	82,532.38	82,532.38	
P_L E_Q W_Q V_L	1	2,083.01	2,083.01	
P_L E_Q W_L V_Q	1	3,799.45	3,799.45	
P_L E_L W_Q V_Q	1	242,327.02	242,327.02	4.20*
P_Q E_Q W_Q V_L	1	58,645.00	58,645.00	
P_Q E_Q W_L V_Q	1	50,282.26	50,282.26	
P_Q E_L W_Q V_Q	1	21,456.02	21,456.02	
P_L E_Q W_Q V_Q	1	31,739.81	31,739.81	
P_Q E_Q W_Q V_Q	1	452.00	452.00	
Error	486	28,001,455.44	57,616.16	

FOOTNOTES FOR TABLE XVIII

[1]With 1 and 486 degrees of freedom the following probabilities hold:

$F = 6.70 \quad P < .01$

$F = 3.86 \quad P < .05$

$F = 2.75 \quad P < .10$

*Effect is included in the model for prediction purposes.

$$Y_3 = \mu + a_{1000}x_1 + a_{0100}x_2 + a_{0001}x_4 + a_{0200}x_2^2$$
$$+ a_{1100}x_1x_2 + a_{0110}x_2x_3 + a_{0012}x_3x_4^2 + a_{2210}x_1^2x_2^2x_3$$
$$+ a_{2220}x_1^2x_2^2x_3^2 + a_{1101}x_1x_2x_4 + a_{2101}x_1^2x_2x_4$$
$$+ a_{1011}x_1x_3x_4 + a_{2021}x_1^2x_3^2x_4 + a_{0222}x_2^2x_3^2x_4^2$$
$$+ a_{1122}x_1x_2x_3^2x_4^2$$

The least squares estimate of the parameters of this model was calculated and is presented as follows:

$$\begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{0001} \\ a_{0200} \\ a_{1100} \\ a_{0110} \\ a_{0012} \\ a_{2210} \\ a_{2220} \\ a_{1101} \\ a_{2101} \\ a_{1011} \\ a_{2021} \\ a_{0222} \\ a_{1122} \end{bmatrix} = \begin{bmatrix} 80.27 \\ 122.97 \\ -67.71 \\ -30.29 \\ -73.36 \\ 87.81 \\ 37.97 \\ -15.66 \\ 7.42 \\ -5.76 \\ -32.68 \\ -1.24 \\ 35.53 \\ 4.61 \\ 44.24 \\ -37.99 \end{bmatrix}$$

The sum of squares removed by these parameters is given in Table XIX. Here again the size of the residual sum of squares indicates that the variation not accounted for by the model is probably due to random error. To test this further, the predicted values, $\hat{Y}_3$, are calculated and are presented in Table XX. The residual deviations, $\bar{\bar{Y}}_3 - \hat{Y}_3$, are given in Table XXI. The general trends for the $\hat{Y}_3$ are similar to that of $\bar{\bar{Y}}_3$. There are no definite trends for $\bar{\bar{Y}}_3 - \hat{Y}_3$ that would indicate that the residual is not distributed normally. The half-normal plot of the residual deviations is presented in Figure 5. This graph shows the residuals are not too far from a normal distribution; however, the mean is not zero. If the mean is zero, the line must pass through the origin.

TABLE XIX

ANALYSIS OF VARIANCE OF BODY
WEIGHT CHANGE MODEL 1

Source	DF	SS	MS	F
Total	567	43,537,988		
$R(\mu)$	1	1,036,806		
$R(\mu, a)$	16	11,512,129		
$R(a)$	15	10,475,323	698,354.86	12.12($P < .01$)
Residual	65	4,024,403	61,913.89	1.07
Error	486	28,001,455	57,616.16	

The estimated parameters of Body Weight Change Model 1 are tested in the same way as the parameters in the nutrient intake Models. The t value is calculated for the parameter to test the hypothesis that each one is equal to zero. These are listed in Table XXII. The 5 percent level of probability is used to select terms for Body Weight Change

TABLE XX

THE PREDICTED VALUES, $\hat{Y}_3$, FOR BODY WEIGHT CHANGE MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	107.75	64.25	64.57	202.78	112.59	142.20	259.75	134.57	205.17
	137	101.19	39.46	-22.27	104.91	74.62	44.33	106.15	109.78	113.41
	147	86.39	3.15	-98.81	95.52	36.65	34.94	96.27	73.47	183.81
300	127	-36.89	-42.70	-17.19	126.22	80.27	65.64	280.11	203.24	157.69
	137	-12.41	-42.70	-72.99	110.56	80.27	49.98	233.53	203.24	172.95
	147	2.85	-42.70	-119.57	94.90	80.27	34.32	177.73	203.24	197.43
340	127	-266.13	-322.73	-183.55	-8.58	-98.77	-69.16	215.87	98.83	25.61
	137	-272.73	-271.58	-270.43	-30.51	-60.80	-91.09	214.19	149.98	85.77
	147	-135.61	-231.95	-195.15	36.04	-22.83	-24.54	204.27	189.61	156.13

TABLE XXI

THE RESIDUAL DEVIATIONS, $\bar{Y}_3 - \hat{Y}_3$, FOR BODY WEIGHT CHANGE MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	-34.89	81.46	13.00	-91.35	-86.88	-42.20	45.96	-24.57	-28.03
	137	-16.90	-73.54	-3.44	106.52	-44.62	-142.70	-59.01	101.65	-56.27
	147	-33.53	-33.15	57.38	146.00	200.49	92.20	-23.41	47.96	-108.10
300	127	-135.97	-67.30	-88.52	18.07	-25.98	-65.64	-17.25	72.47	40.60
	137	32.84	97.30	68.44	173.73	19.73	-31.41	27.03	-86.10	22.76
	147	80.01	102.70	2.43	-176.33	125.44	-121.46	-47.73	121.05	-67.43
340	127	94.70	-124.44	64.12	-204.28	-72.66	37.73	18.42	-85.97	-19.90
	137	82.98	98.29	79.00	-169.08	30.80	-90.34	38.67	2.88	92.80
	147	-42.96	90.52	-11.99	-33.18	67.12	114.57	75.73	67.53	-96.13

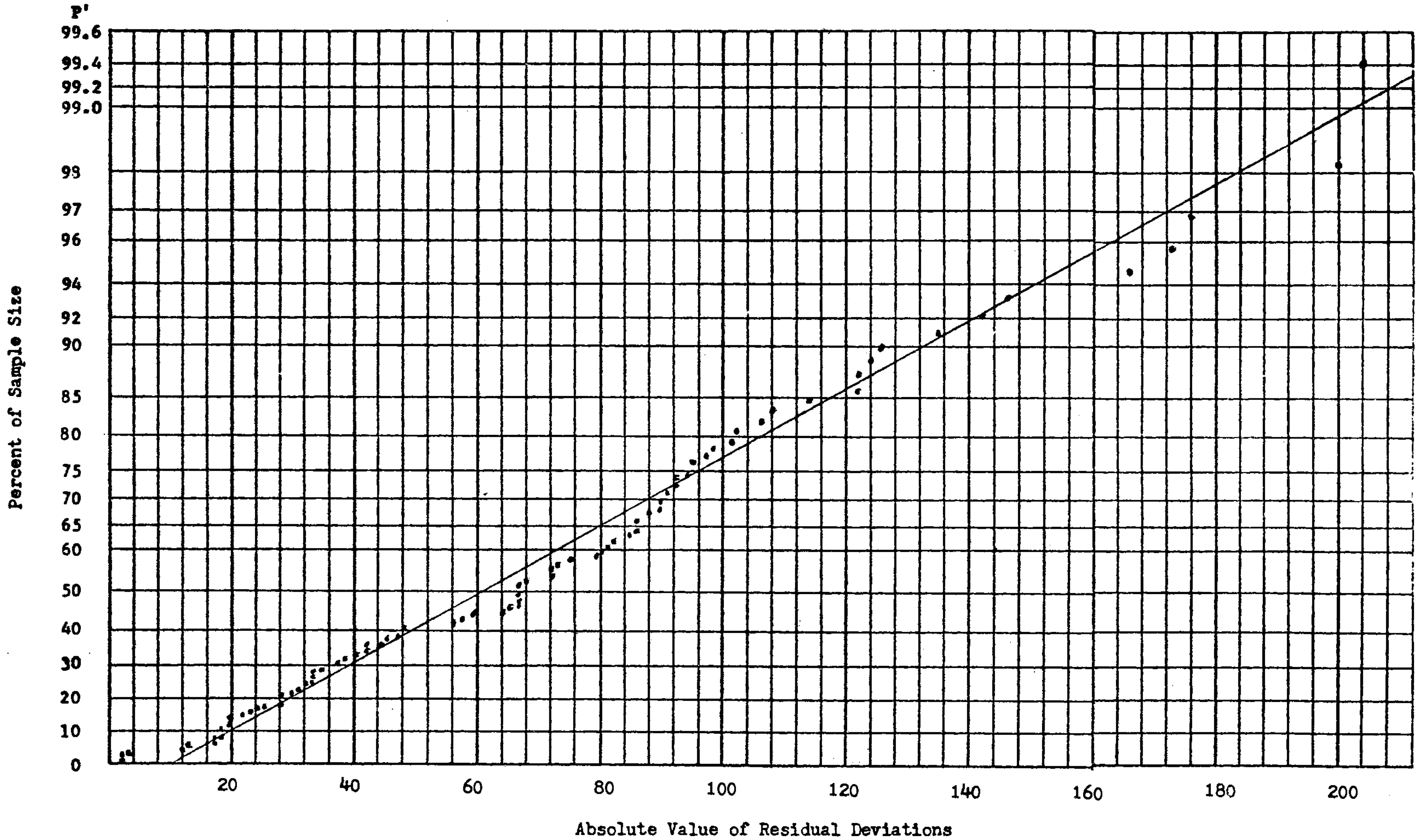

Figure 5. Half-Normal Plot of the Residual Deviations for Body Weight Change Model 1

TABLE XXII

CALCULATED t FOR THE ESTIMATED PARAMETERS OF BODY WEIGHT CHANGE MODEL 1

Parameter	Estimate	c_{ii}	$\hat{\sigma}^2 c_{ii}$ [1]	$\sqrt{\hat{\sigma}^2 c_{ii}}$	t*
μ	80.27	.00529	327.52	18.08	4.44
a_{1000}	122.97	.00265	164.07	12.81	9.60
a_{0100}	-67.71	.00265	164.07	12.81	5.28
a_{0001}	-30.29	.00476	294.71	17.14	1.76
a_{0200}	-73.36	.01096	678.57	26.04	2.81
a_{1100}	87.81	.00714	442.06	21.02	4.17
a_{0110}	37.97	.00397	245.79	15.65	2.42
a_{0012}	-15.66	.00564	349.19	18.68	.84
a_{2210}	7.42	.00846	523.79	22.87	.32
a_{2220}	- 5.76	.01275	789.40	28.08	.21
a_{1101}	-32.68	.00595	368.38	19.18	1.70
a_{2101}	- 1.24	.00595	368.38	19.18	.06
a_{1011}	35.53	.00595	368.38	19.18	1.85
a_{2021}	4.61	.01071	663.09	25.75	.18
a_{0222}	44.24	.01275	789.40	28.08	1.57
a_{1122}	-37.99	.01607	994.95	31.53	1.20

[1] $\hat{\sigma}_2$ = 61,913,89

$^*t_{.05}$ = 2.00

Model 2. This model is:

$$Y_3 = \mu + a_{1000}x_1 + a_{0100}x_2^2 + a_{0200}x_2^2 + a_{1100}x_1x_2 + a_{0110}x_2x_3$$

When the parameters are re-estimated, the new prediction equation becomes

$$\hat{Y}_3 = 80.27 + 122.97x_1 - 67.71x_2 - 56.26x_2^2 + 70.92x_1x_2 + 37.97x_2x_3$$

The analysis of variance for this model is presented in Table XXIII. Here again the residual sum of squares is not statistically significant, so from this test it appears that this Model accounts for as much treatment variation as the previous model. The $\hat{Y}_2$ and $\bar{Y}_3 - \hat{Y}_3$ are calculated and listed in Tables XXIV and XXV, respectively. As would be expected, the general trends for the values in these tables give no indication that the residual variation is caused by something other than random error. The residual deviations plotted in Figure 6 show that the distribution is not as close to normal as it was for Body Weight Change Model 1. However, the mean seems to be nearer to zero. Considering all tests made on the body weight change prediction models, it is difficult to pick one over the other.

TABLE XXIII

ANALYSIS OF VARIANCE OF BODY WEIGHT CHANGE MODEL 2

Source	DF	SS	MS	F
Total	567	43,537,988		
$R(\mu)$	1	1,036,806		
$R(\mu, a)$	6	10,514,869		
$R(a)$	5	9,478,063	1,895,612	28.31($P < .01$)
Residual	75	5,021,664	66,955	1.16
Error	486	28,001,455	57,616	

TABLE XXIV

PREDICTED VALUES, $\hat{Y}_3$, FOR BODY WEIGHT CHANGE MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein 13 (Grams), Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	77.64	77.64	77.64	129.69	129.69	129.69	181.75	181.75	181.75
	137	39.67	39.67	39.67	91.72	91.72	91.72	143.77	143.77	143.77
	147	1.70	1.70	1.70	53.75	53.75	53.75	105.80	105.80	105.80
300	127	-42.70	-42.70	-42.70	80.27	80.27	80.27	203.24	203.24	203.24
	137	-42.70	-42.70	-42.70	80.27	80.27	80.27	203.24	203.24	203.24
	147	-42.70	-42.70	-42.70	80.27	80.27	80.27	203.24	203.24	203.24
340	127	-275.56	-275.56	-275.56	-81.67	-81.67	-81.67	112.22	112.22	112.22
	137	-237.59	-237.59	-237.59	-43.70	-43.70	-43.70	150.20	150.20	150.20
	147	-199.62	-199.62	-199.62	-5.73	-5.73	-5.73	188.16	188.16	188.16

TABLE XXV

RESIDUAL DEVIATIONS, $\bar{Y}_3 - \hat{Y}_3$, FOR BODY WEIGHT CHANGE MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	-4.78	68.07	-.07	-18.26	-103.98	-29.69	123.96	-71.75	-4.61
	137	44.62	-73.96	-65.38	119.71	-61.72	-190.15	-96.63	67.66	-86.63
	147	51.16	-31.70	-43.13	187.68	183.39	73.39	-32.94	15.63	-30.09
300	127	-130.16	-67.30	147.41	64.02	-25.98	-80.27	59.62	72.47	-4.95
	137	63.13	-97.30	-98.73	204.02	19.73	-61.70	103.90	-86.10	-7.53
	147	125.56	102.70	-74.44	-161.70	125.44	-167.41	-73.24	121.05	-73.24
340	127	104.13	-171.61	156.13	-131.19	-89.76	50.24	122.07	-99.36	-106.51
	137	-118.12	64.30	46.16	-94.87	13.70	-137.73	102.86	2.66	28.37
	147	21.05	58.19	-7.52	8.59	50.02	95.73	91.84	68.98	-128.16

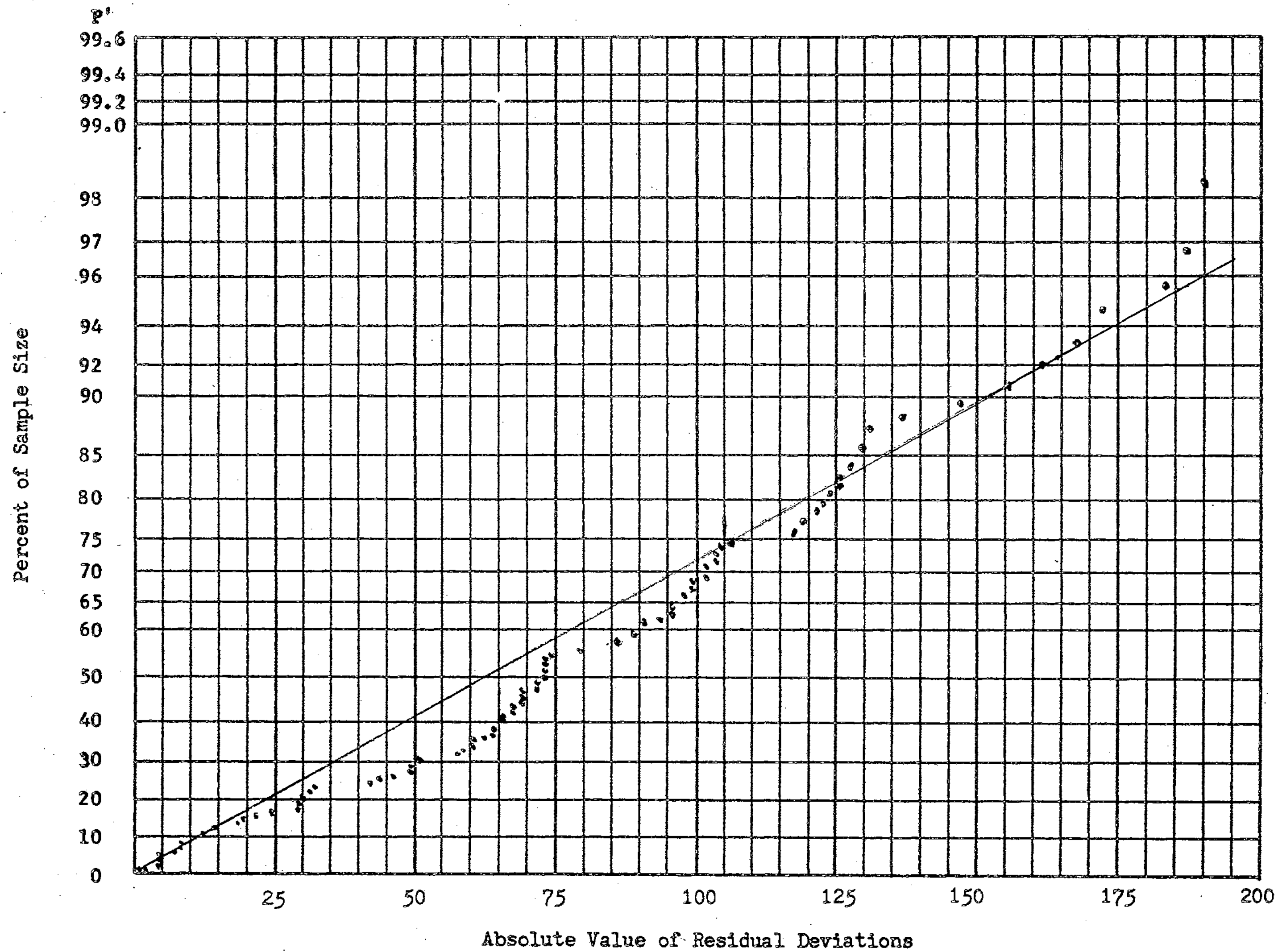

Figure 6. Half-Normal Plot of the Residual Deviations for Body Weight Change Model 2

In the analysis of these data by Gleaves, there occurred a weight x volume interaction on body weight change. It was stated in his discussion that there was no consistent pattern of response to this effect which was significant at the five percent level of probability and that the interpretation was very difficult. In the present analysis, there is a weight linear x volume quadratic interaction which is significant at the one percent level of probability. This can be seen in the analysis of variance (Table XVIII). This effect was included in Body Weight Change Model 1. The t test on the coefficient to this term shows that it is not significantly different from zero. Therefore, it was not included in the Body Weight Change Model 2. With no consistent pattern of response, as reported by Gleaves, it could well be that those effects are nothing more than random error.

Egg Production: The analysis of variance for egg production is given in Table XXVI. By choosing the effects which are significant at the ten percent level of probability, the Egg Production Model 1 is

$$\begin{aligned} Y_4 = \mu &+ a_{1000} x_1 + a_{0100} x_2 + a_{2000} x_1 x_2 + a_{1100} x_1 x_2 + a_{2100} x_1^2 x_2 \\ &+ a_{1010} x_1 x_3 + a_{2210} x_1^2 x_2^2 x_3 + a_{2101} x_1^2 x_2 x_4 + a_{1201} x_1 x_2^2 x_4 \\ &+ a_{2011} x_1^2 x_3 x_4 + a_{2211} x_1^2 x_2^2 x_3 x_4 + a_{2121} x_1^2 x_2 x_3^2 x_4 \\ &+ a_{1221} x_1 x_2^2 x_3^2 x_4 + a_{2221} x_1^2 x_2^2 x_3^2 x_4 \, . \end{aligned}$$

The least squares estimate of the parameters of this model are as follows:

TABLE XXVI

ANALYSIS OF VARIANCE FOR EGG PRODUCTION

Source	DF	SS	MS	F[1]
Total	566	1,021,487.43		
Treatment	[80]			
Protein(P)	(2)	171,553.31		
P_L	1	160,485.95	160,485.95	137.58*
P_Q	1	11,067.36	11,067.36	9.49*
Energy(E)	(2)	112,259.57		
E_L	1	111,372.87	111,372.87	95.48*
E_Q	1	886.70	886.70	.76
Weight(W)	(2)	1,752.51		
W_L	1	1,635.38	1,635.38	1.40
W_Q	1	117.13	117.13	.10
Volume(V)	(2)	3,027.31		
V_L	1	1,190.93	1,190.93	1.02
V_Q	1	1,836.38	1,836.38	1.57
Interaction	(72)			
P x E	(4)	78,366.65		
P_L E_L	1	69,235.43	69,235.43	59.35*
P_Q E_L	1	5,905.78	5,905.78	5.06*
P_L E_Q	1	2,515.40	2,515.40	
P_Q E_Q	1	710.04	710.04	
P x W	4	2,016.40	504.10	
P x V	4	3,768.84	942.21	
E x W	(4)	5,409.74		
E_L W_L	1	4,792.86	4,792.86	4.11*
E_Q W_L	1	520.01	520.01	
E_L W_Q	1	95.72	95.72	
E_Q W_Q	1	1.15	1.15	
E x V	4	4,055.69	1,013.92	
W x V	4	3,492.78	873.20	
P x E x W	(8)	9,822.97		
P_L E_L W_L	1	238.10	238.10	
P_Q E_L W_L	1	2,220.96	2,220.96	
P_L E_Q W_L	1	828.01	828.01	
P_L E_L W_Q	1	40.01	40.01	
P_Q E_Q W_L	1	4,160.10	4,160.10	7.474*
P_Q E_L W_Q	1	1,288.90	1,288.90	
P_L E_Q W_Q	1	57.95	57.95	
P_Q E_Q W_Q	1	988.96	988.96	
P x E x V	(8)	16,332.44		
P_L E_L V_L	1	2,530.38	2,530.38	
P_Q E_L V_L	1	4,803.84	4,803.84	4.118*
P_L E_Q V_L	1	5,856.20	5,856.20	5.021*
P_L E_L V_Q	1	2,008.01	2,008.01	

TABLE XXVI (CONTINUED)

Source	DF	SS	MS	F[1]
P_Q E_Q V_L	1	200.07	200.07	
P_Q E_L V_Q	1	333.40	333.40	
P_L E_Q V_Q	1	128.04	128.04	
P_Q E_Q V_Q	1	472.51	472.51	
P x W x V	(8)	12,448.28		
P_L W_L V_L	1	375.01	375.01	
P_Q W_L V_L	1	6,407.16	6,407.16	5.493*
P_L W_Q V_L	1	2,133.67	2,133.67	
P_L W_L V_Q	1	830.34	830.34	
P_Q W_Q V_L	1	1,657.33	1,657.33	
P_Q W_L V_Q	1	226.34	226.34	
P_L W_Q V_Q	1	744.52	744.52	
P_Q W_Q V_Q	1	73.91	73.91	
E x W x V	8	8,785.16	1,098.15	
P x E x W x V	(16)	21,514.06		
P_L E_L W_L V_L	1	670.32	670.32	
P_Q E_L W_L V_L	1	165.76	165.76	
P_L E_Q W_L V_L	1	1,500.29	1,500.29	
P_L E_L W_Q V_L	1	210.58	210.58	
P_L E_L W_L V_Q	1	3.05	3.05	
P_Q E_Q W_L V_L	1	3,795.57	3,795.57	3.25*
P_Q E_L W_Q V_L	1	4,259.11	4,259.11	3.65*
P_Q E_L W_L V_Q	1	764.76	764.76	
P_L E_Q W_Q V_L	1	5,619.44	5,619.44	4.82*
P_L E_Q W_L V_Q	1	30.73	30.73	
P_L E_L W_Q V_Q	1	2,826.73	2,826.73	
P_Q E_Q W_Q V_L	1	3,235.57	3,235.57	2.77*
P_Q E_Q W_L V_Q	1	36.01	36.01	
P_Q E_L W_Q V_Q	1	0.82	0.82	
P_L E_Q W_Q V_Q	1	1,486.24	1,486.24	
P_Q E_Q W_Q V_Q	1	6.67	6.67	
Error	486	566,891.72	1,166.44	

[1]With 1 and 486 degrees of freedom the following probabilities hold:

$F = 6.70 \quad P < .01$

$F = 3.86 \quad P < .05$

$F = 2.75 \quad P < .10$

*Effect is included in the model for prediction purposes.

$$\begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{2000} \\ a_{1100} \\ a_{2100} \\ a_{1010} \\ a_{2210} \\ a_{2101} \\ a_{1201} \\ a_{2011} \\ a_{2211} \\ a_{2121} \\ a_{1221} \\ a_{2221} \end{bmatrix} = \begin{bmatrix} 136.53 \\ 20.61 \\ -11.57 \\ -9.35 \\ 16.57 \\ -8.38 \\ 2.08 \\ .92 \\ -1.04 \\ 2.71 \\ -2.70 \\ 11.57 \\ -.89 \\ 1.57 \\ -5.95 \end{bmatrix}$$

The analysis of variance for this model is presented in Table XXVII. From this table it appears that the parameters in the model have removed all variation except random error. The $\hat{Y}_4$ and $\bar{Y}_4 - \hat{Y}_4$ are calculated and listed in Tables XXVIII and XXIX, respectively. These two tables of data are in agreement with the results of the analysis of variance for this model. There are no obvious trends in these results which would indicate that Egg Production Model 1 would give predictions different than the expected values. The half-normal plot of the residual deviations is in Figure 7. This shows that variation not accounted for by the model is approaching a normal distribution.

TABLE XXVII

ANALYSIS OF VARIANCE OF EGG PRODUCTION MODEL 1

Source	DF	SS	MS	F
Total	567	10,648,299		
$R(\mu)$	1	9,626,812		
$R(\mu, a)$	15	10,001,787		
R(a)	14	374,975	26,783.92	22.96($P < .01$)
Residual	66	79,620	1,206.36	1.03
Error	486	566,891	1,166.44	

In order to eliminate some of the terms in the model, the hypothesis that each parameter is equal to zero was tested. The results of this test are given in Table XXX. As before, the effects for which the parameters are different from zero, with 95 percent confidence, are included in Egg Production Model 2. The new model is

TABLE XXVIII

PREDICTED VALUES, $\hat{Y}_4$, FOR EGG PRODUCTION MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13 Milliliters 180	13 230	13 280	16 Milliliters 180	16 230	16 280	19 Milliliters 180	19 230	19 280
260	127	161.36	144.19	127.02	148.10	148.10	148.10	156.72	148.11	139.50
	137	144.70	143.03	141.36	148.10	148.10	148.10	147.36	151.11	154.86
	147	141.30	141.87	142.44	148.10	148.10	148.10	144.98	154.11	163.24
300	127	105.95	108.65	111.35	136.53	136.53	136.53	143.01	145.71	148.41
	137	106.57	106.57	106.57	136.53	136.53	136.53	147.79	147.79	147.79
	147	107.19	104.49	101.79	136.53	136.53	136.53	152.57	149.87	147.17
340	127	91.30	71.27	50.24	124.96	124.96	124.96	153.94	141.47	129.00
	137	73.86	70.11	66.36	124.96	124.96	124.96	142.80	144.47	146.14
	147	72.24	68.95	65.66	124.96	124.96	124.96	142.20	147.47	152.74

TABLE XXIX

RESIDUAL DEVIATION, $\bar{Y}_4 - \hat{Y}_4$, FOR EGG PRODUCTION
MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13			16			19		
		Milliliters			Milliliters			Milliliters		
		180	230	280	180	230	280	180	230	280
260	127	-1.79	9.10	-9.88	4.04	-6.24	-1.24	3.28	.89	12.21
	137	8.44	-23.03	-.36	3.90	-3.67	1.53	-7.93	12.32	5.14
	147	.84	1.27	-3.44	7.04	.76	-21.24	-4.12	-7.25	-.67
300	127	-.81	18.35	.94	4.18	9.76	10.90	-5.15	-2.57	19.70
	137	-16.71	8.00	5.86	-4.39	10.90	13.61	18.92	-1.08	-25.36
	147	8.95	14.80	-.22	-7.67	7.18	-7.96	-2.57	17.13	-6.17
340	127	2.27	-10.70	-5.10	-39.25	-19.39	17.04	6.20	-24.33	11.71
	137	-14.72	11.18	-1.65	-4.10	2.33	-25.96	9.91	1.53	1.15
	147	-15.10	11.04	3.63	3.75	25.47	21.90	9.66	5.10	-8.17

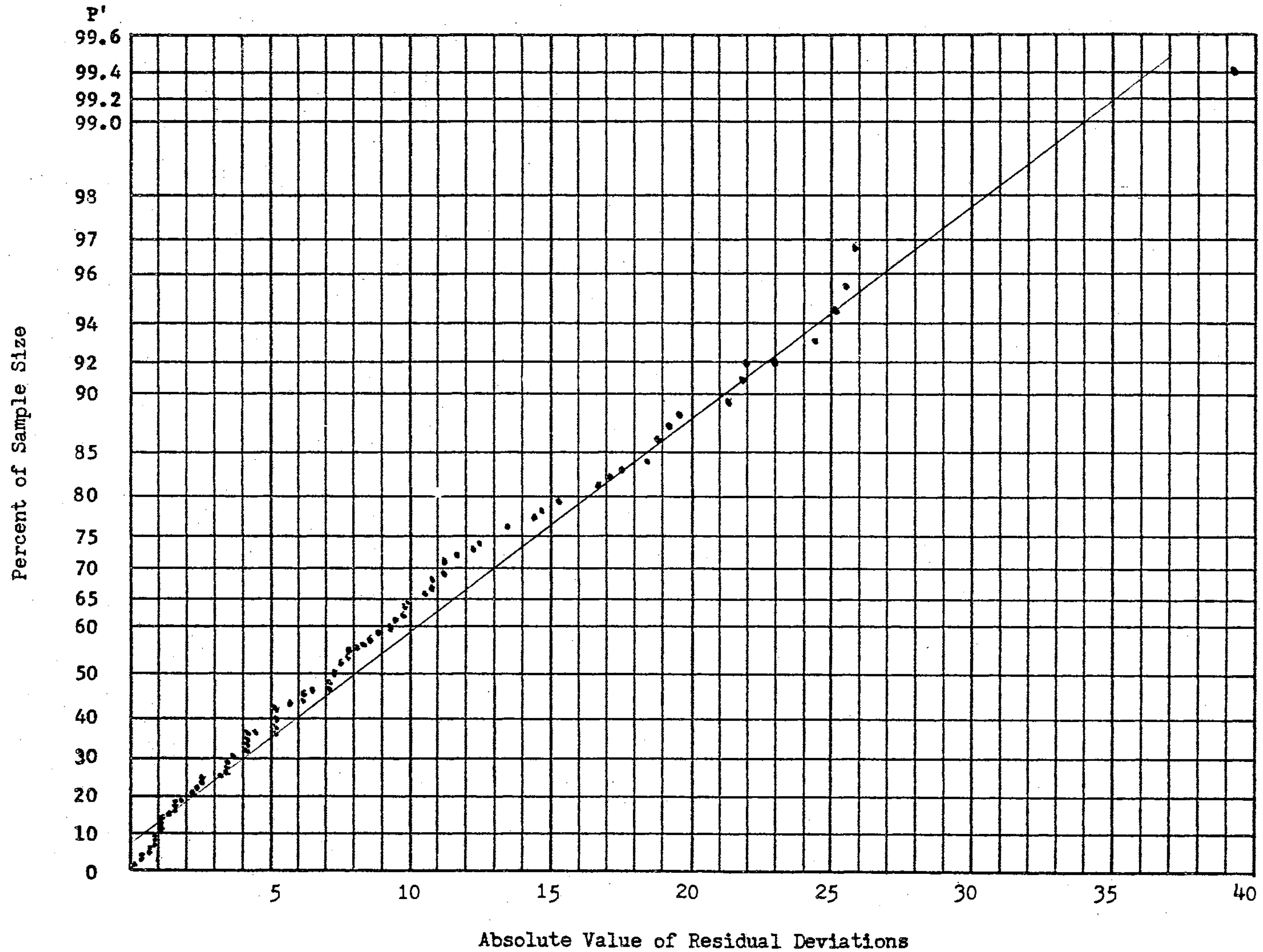

Figure 7. Half-Normal Plot of the Residual Deviations for Egg Production Model 1

TABLE XXX

CALCULATED t FOR THE ESTIMATED PARAMETERS OF EGG PRODUCTION MODEL 1

Parameter	Estimate	c_{ii}	$\hat{\sigma}^2 c_{ii}$[1]	$\sqrt{\hat{\sigma}^2 c_{ii}}$	t*
μ	136.53	.00529	6.38	2.526	54.04
a_{1000}	20.61	.00265	3.20	1.789	11.52
a_{0100}	-11.57	.00794	9.58	3.095	3.74
a_{2000}	- 9.35	.00794	9.58	3.095	3.02
a_{1100}	- 8.38	.01190	14.35	3.781	2.21
a_{1010}	2.08	.00397	4.79	2.188	.95
a_{2210}	.92	.00595	7.18	2.679	.34
a_{2101}	- 1.04	.01786	21.54	4.637	.22
a_{1201}	2.71	.01786	21.54	4.637	.58
a_{2011}	2.70	.01786	21.54	4.637	.58
a_{2211}	11.57	.02678	32.31	5.683	2.03
a_{2121}	-.89	.02678	32.31	5.683	.16
a_{1221}	1.57	.02678	32.31	5.683	.27
a_{2221}	- 5.95	.00893	10.77	3.286	1.81

[1] $\hat{\sigma}^2 = 1206.36$

$^*t_{.05} = 2.00$

$$Y_4 = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_1^2 x_2 \\ x_1^2 x_2^2 x_3 x_4 \end{bmatrix}' \begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{2000} \\ a_{1100} \\ a_{2100} \\ a_{2211} \end{bmatrix}$$

When the parameters are re-estimated, the simpler prediction equation is

$$\hat{Y}_4 = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_1^2 x_2 \\ x_1^2 x_2^2 x_3 x_4 \end{bmatrix}' \begin{bmatrix} 136.53 \\ 20.61 \\ -11.57 \\ -9.35 \\ 16.57 \\ -8.38 \\ 8.88 \end{bmatrix}$$

The analysis of variance for this model, which is presented in Table XXXI, indicates that for Egg Production Model 2 the residual sum of squares appears to be entirely random error.

The predicted values, $\hat{Y}_4$, and the residual deviations, $\bar{Y}_4 - \hat{Y}_4$, are calculated and presented in Tables XXXII and XXXIII, respectively. The values in these tables indicate that Egg Production Model 2 should do a reasonable job of prediction. The half-normal plot of the residual deviations is in Figure 8. The result of this test shows that Egg Production Model 2 is probably better for prediction purposes than Model 1. The residuals in Model 2 appear more nearly to approximate a normal distribution with a zero mean than those for Model 1.

TABLE XXXI

ANALYSIS OF VARIANCE OF EGG PRODUCTION MODEL 2

Source	DF	SS	MS	F
Total	567	10,648,299		
$R(\mu)$	1	9,626,812		
$R(\mu,a)$	7	9,993,654		
$R(a)$	6	366,815	61,135.83	51.67($P < .01$)
Residual	74	87,541	1,182.98	1.01
Error	486	566,891	1,166.44	

In the analysis of egg production, Gleaves was not able to demonstrate any interaction effect except for protein x energy. In the present analysis, the 4-way interaction $x_1^2 x_2^2 x_3 x_4$ was significant at the 5 percent level of probability in each test. Since the protein x energy interaction is so strong, as demonstrated both by Gleaves and by the present analysis, it is the author's opinion that this 4-way interaction is primarily protein quadratic x energy quadratic. Just why volume and weight show up in the same term is unexplainable with the data now available. Future experiments will be necessary in order to understand these results more fully.

Egg Weight: The analysis of variance for egg weight is in Table XXXIV. According to the procedure with the previous models, the effects which were significant at the 10 percent level of probability are used to make up Egg Weight Model 1. The prediction equation model is therefore,

TABLE XXXII

PREDICTED VALUES, $\hat{Y}_4$, FOR EGG PRODUCTION MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	151.97	143.09	134.21	148.10	148.10	148.10	159.52	150.64	141.76
	137	143.09	143.09	143.09	148.10	148.10	148.10	150.64	150.64	150.64
	147	134.21	143.09	151.97	148.10	148.10	148.10	141.76	150.64	159.52
300	127	106.57	106.57	106.57	136.53	136.53	136.53	147.26	147.26	147.26
	137	106.57	106.57	106.57	136.53	136.53	136.53	147.26	147.26	147.26
	147	106.57	106.57	106.57	136.53	136.53	136.53	147.26	147.26	147.26
340	127	78.93	70.05	61.17	124.96	124.96	124.96	153.29	144.41	135.53
	137	70.05	70.05	70.05	124.96	124.96	124.96	144.41	144.41	144.41
	147	61.17	70.05	78.93	124.96	124.96	124.96	135.53	144.41	153.29

TABLE XXXIII

RESIDUAL DEVIATION, $\bar{Y}_4 - \hat{Y}_4$, FOR EGG PRODUCTION
MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	7.60	10.20	-17.07	4.04	-6.24	-1.24	.48	-1.64	9.95
	137	10.05	-23.09	-2.09	3.90	-3.67	-1.53	-11.21	12.79	9.36
	147	7.93	.05	-12.97	7.04	.76	-21.24	-.90	-3.78	3.05
300	127	-1.43	20.43	5.79	4.18	9.76	10.90	-9.40	-4.12	18.55
	137	-16.71	8.00	5.86	-4.39	10.90	13.61	19.45	-.55	24.83
	147	9.57	12.72	-5.00	-7.67	7.18	-7.96	2.74	19.74	-6.26
340	127	14.64	-9.48	-16.03	-39.25	-20.39	17.04	6.85	-27.27	5.18
	137	-10.91	11.24	5.90	-4.10	2.33	-25.96	8.30	1.59	2.88
	147	-4.03	9.95	-9.64	3.75	25.47	21.90	16.33	8.16	-8.72

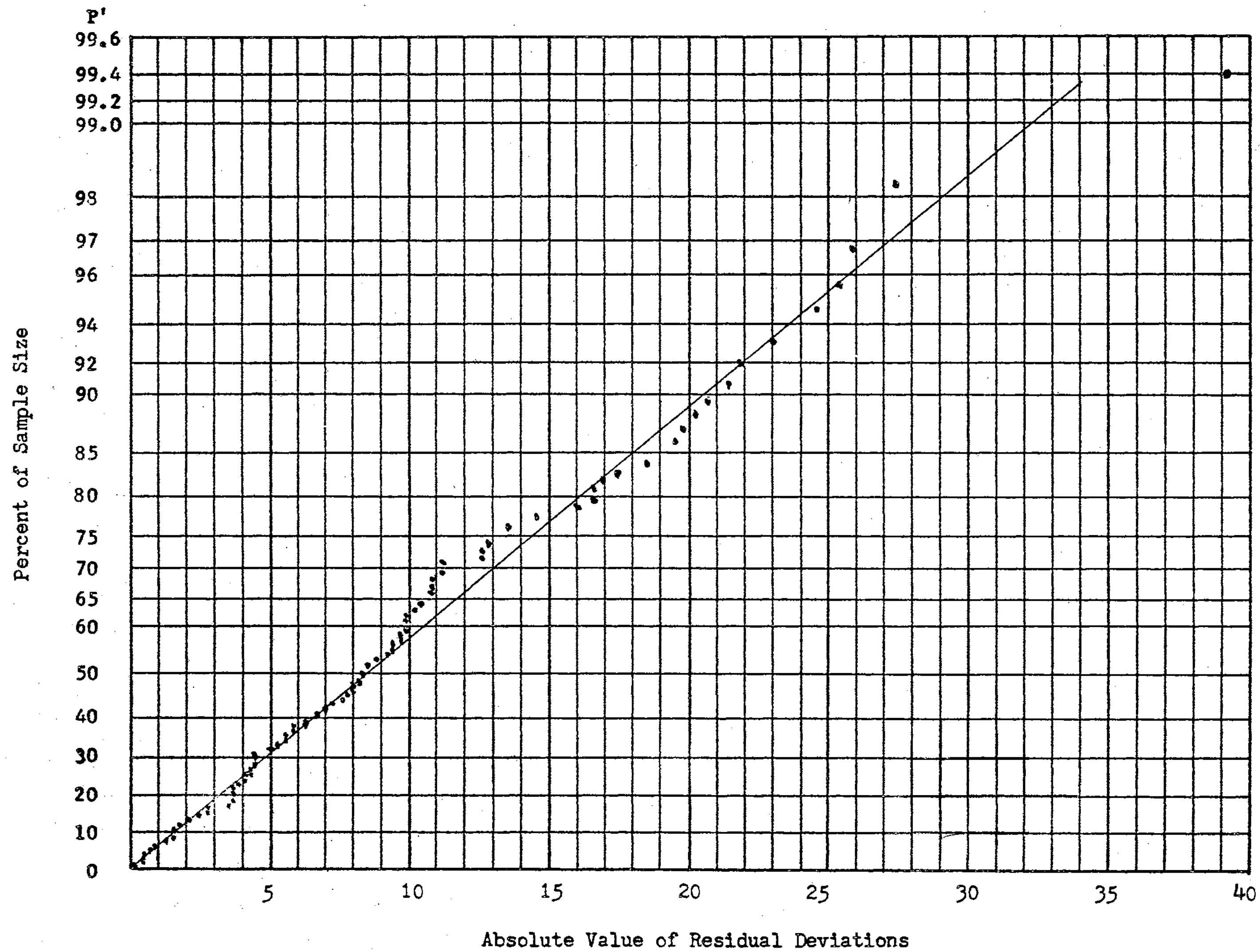

Figure 8. Half-Normal Plot of the Residual Deviations for Egg Production Model 2

TABLE XXXIV

ANALYSIS OF VARIANCE OF EGG WEIGHT

Source	DF	SS	MS	F[1]
Total	566	5,061.40		
Treatment	[80]			
Protein(P)	(2)	473.01		
P_L	1	431.96	431.96	56.18*
P_Q	1	41.04	41.04	5.34*
Energy(E)	(2)	177.00		
E_L	1	175.04	175.04	22.76*
E_Q	1	1.96	1.96	.25
Weight(W)	(2)	32.13		
W_L	1	17.55	17.55	2.28
W_Q	1	14.57	14.57	1.89
Volume(V)	(2)	19.47		
V_L	1	1.08	1.08	0.14
V_Q	1	18.38	18.38	2.39
Interaction	(72)			
P x E	(4)	150.56		
P_L E_L	1	115.14	115.14	14.97*
P_Q E_L	1	7.24	7.24	
P_L E_Q	1	27.04	27.04	3.51*
P_Q E_Q	1	1.14	1.14	
P x W	4	2.31	.58	
P x V	(4)	30.56		
P_L V_L	1	3.47	3.47	
P_Q V_L	1	24.60	24.60	3.19*
P_L V_Q	1	2.05	2.05	
P_Q V_Q	1	.64	.64	
E x W	4	7.94	1.98	
E x V	(4)	34.50		
E_L V_L	1	15.70	15.70	2.09
E_Q V_L	1	9.62	9.62	
E_L V_Q	1	3.92	3.92	
E_Q V_Q	1	5.26	5.26	
W x V	4	20.27	5.07	
P x E x W	(8)	66.65		
P_L E_L W_L	1	49.40	49.40	6.42*
P_Q E_L W_L	1	.22	.22	
P_L E_Q W_L	1	6.81	6.81	
P_L E_L W_Q	1	1.17	1.17	
P_Q E_Q W_L	1	2.18	2.18	
P_Q E_L W_Q	1	4.07	4.07	
P_L E_Q W_Q	1	.81	.81	
P_Q E_Q W_Q	1	1.99	1.99	

TABLE XXXIV (CONTINUED)

Source	DF	SS	MS	F[1]
P x E x V	(8)	63.61		
P_L E_L V_L	1	19.47	19.47	
P_Q E_L V_L	1	1.42	1.42	
P_L E_Q V_L	1	6.13	6.13	
P_L E_L V_Q	1	3.53	3.53	
P_Q E_Q V_L	1	20.20	20.20	2.62
P_Q E_L V_Q	1	4.68	4.68	
P_L E_Q V_Q	1	.29	.29	
P_Q E_Q V_Q	1	7.89	7.89	
P x W x V	8	16.44	2.05	
E x W x V	(8)	104.00		
E_L W_L V_L	1	13.48	13.48	
E_Q W_L V_L	1	.16	.16	
E_L W_Q V_L	1	10.86	10.86	
E_L W_L V_Q	1	14.67	14.67	
E_Q W_Q V_L	1	54.32	54.32	7.06*
E_Q W_L V_Q	1	7.57	7.57	
E_L W_Q V_Q	1	2.84	2.84	
E_Q W_Q V_Q	1	.10	.10	
P x E x W x V	(16)	126.78		
P_L E_L W_L V_L	1	8.25	8.25	
P_Q E_L W_L V_L	1	7.50	7.50	
P_L E_Q W_L V_L	1	9.53	9.53	
P_L E_L W_Q V_L	1	3.68	3.68	
P_L E_L W_L V_Q	1	.10	.10	
P_Q E_Q W_L V_L	1	8.47	8.47	
P_Q E_L W_Q V_L	1	2.38	2.38	
P_Q E_L W_L V_Q	1	23.65	23.65	3.07*
P_L E_Q W_Q V_L	1	.30	.30	
P_L E_Q W_L V_Q	1	5.37	5.37	
P_L E_L W_Q V_Q	1	45.34	45.34	5.89*
P_Q E_Q W_Q V_L	1	8.63	8.63	
P_Q E_Q W_L V_Q	1	.49	.49	
P_Q E_L W_Q V_Q	1	.05	.05	
P_L E_Q W_Q V_Q	1	.48	.48	
P_Q E_Q W_Q V_Q	1	2.06	2.06	
Error	486	3,736.46	7.69	

[1]With 1 and 486 degrees of freedom, the following probabilities hold:

$F = 6.70 \quad P < .01$

$F = 3.86 \quad P < .05$

$F = 2.75 \quad P < .10$

*Effect is included in the model for prediction purposes.

$$Y_5 = \mu + a_{1000}x_1 + a_{0100}x_2 + a_{2000}x_1^2 + a_{1100}x_1x_2 + a_{1200}x_1x_2^2 + a_{2001}x_1^2x_4 + a_{1110}x_1x_2x_3 + a_{0221}x_2^2x_3^2x_4 + a_{2112}x_1^2x_2x_3x_4^2 + a_{1122}x_1x_2x_3^2x_4^2 .$$

The least squares estimates of the parameters are

$$\begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{2000} \\ a_{1100} \\ a_{1200} \\ a_{2001} \\ a_{1110} \\ a_{0221} \\ a_{2112} \\ a_{1122} \end{bmatrix} = \begin{bmatrix} 53.98 \\ .69 \\ -.68 \\ -.57 \\ .95 \\ .57 \\ .33 \\ .54 \\ -.45 \\ -.36 \\ -.62 \end{bmatrix}$$

The analysis of variance of Egg Weight Model 1 is in Table XXXV. The results of this analysis show that the residual sum of squares is probably nothing more than random error, and that the parameters of the model account for all of the significant variation in egg weight.

The predicted values for Egg Weight Model 1 are listed in Table XXXVI. The trends of these values correspond closely to those of the means. This can be seen in the residual deviations, $\bar{Y}_5 - \hat{Y}_5$, which are in Table XXXVII. There is no indication from either of these two tables that, for Egg Weight Model 1, the predicted values would not be similar to the observed values in a future experiment. The half-normal plot of the residual deviations in Figure 9 indicates that the residuals have a distribution similar to that of a normal distribution.

TABLE XXXV

ANALYSIS OF VARIANCE OF EGG WEIGHT MODEL 1

Source	DF	SS	MS	F
Total	567	1,634,297.73		
$R(\mu)$	1	1,629,236.34		
$R(\mu, b, a)$	11	1,630,141.39		
$R(b, a)$	10	905.05	90.5	15.08($P < .01$)
Residual	70	419.88	6.00	.78
Error	486	3,736.46	7.69	

The calculated t for each of the parameters of Egg Weight Model 1 are listed in Table XXXVIII. By choosing the parameters which are significant at the 5 percent level of probability, Egg Weight Model 2 becomes

$$Y_s = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_1 x_2 x_3 \end{bmatrix}' \begin{bmatrix} \mu \\ a_{1000} \\ a_{0100} \\ a_{2000} \\ a_{1100} \\ a_{1110} \end{bmatrix}$$

This model is similar to that of Egg Production Model 2. This is to be expected, since the egg weight means followed the same pattern as

TABLE XXXVI

PREDICTED VALUES, $\hat{Y}_s$, FOR EGG WEIGHT MODEL 1

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein 13 (Grams), Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	52.38	53.24	52.14	55.11	54.66	54.21	55.32	54.94	55.08
	137	53.45	53.78	54.11	54.66	54.66	54.66	54.07	54.40	54.73
	147	54.18	54.32	53.94	55.11	54.66	54.21	54.96	53.86	54.72
300	127	52.39	52.72	53.05	53.98	53.98	53.98	53.77	54.10	54.43
	137	52.39	52.72	53.05	53.98	53.98	53.98	53.77	54.10	54.43
	147	52.39	52.72	53.05	53.98	53.98	53.98	53.77	54.10	54.43
340	127	52.16	51.06	51.92	53.75	53.30	52.85	54.26	54.40	54.02
	137	50.19	50.52	50.85	53.30	53.30	53.30	54.61	54.94	55.27
	147	50.36	49.98	50.12	53.75	53.30	52.85	54.62	55.48	54.38

TABLE XXXVII

RESIDUAL DEVIATIONS, $\bar{Y}_s - \hat{Y}_s$, FOR EGG WEIGHT
MODEL 1

		Protein (Grams)								
		13			16			19		
		Milliliters			Milliliters			Milliliters		
Metabolizable Energy (Kilocalories)	Weight (Grams)	180	230	280	180	230	280	180	230	280
260	127	-.74	1.30	.32	-1.01	.11	.95	-1.29	.45	.82
	137	-.56	-1.70	.28	.73	-.53	-.33	-.10	.50	-1.69
	147	-.72	-.26	1.25	1.09	.31	-.42	.87	-.55	.04
300	127	-.85	-1.36	1.28	-.82	-.95	.51	.30	-.16	.31
	137	.46	-.51	-.48	2.29	-.92	.01	1.32	.39	-1.80
	147	.67	.78	.25	-.38	-.04	2.22	-.26	-.46	.08
340	127	.45	1.00	-.06	.82	.43	-1.95	.10	-.94	.35
	137	.71	.24	-.65	-.53	-.89	-1.06	-.38	-.91	-.23
	147	1.07	-.28	.37	1.24	-.36	-.35	.57	.65	1.12

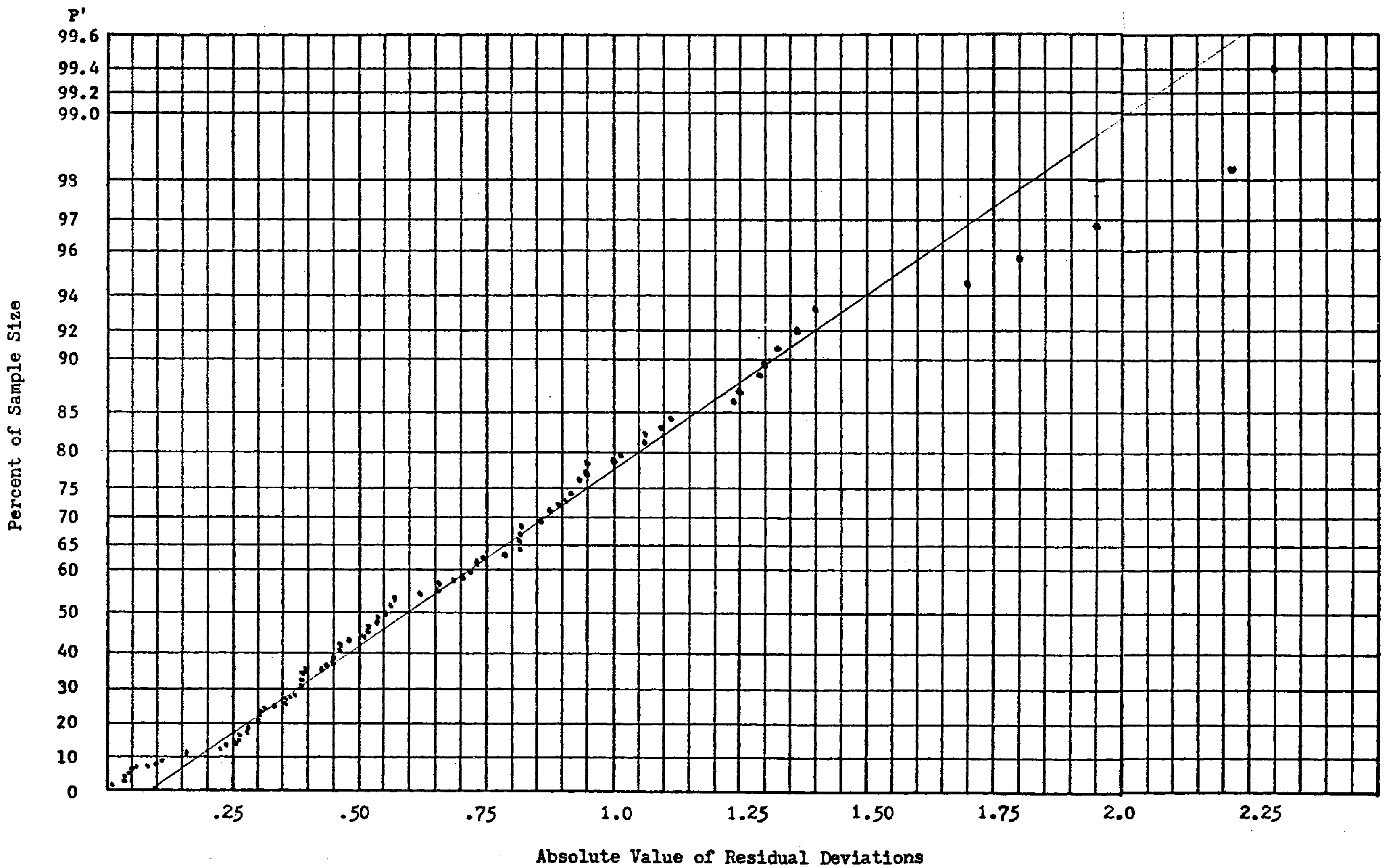

Figure 9. Half-Normal Plot of the Residual Deviations for Egg Weight Model 1

TABLE XXXVIII

CALCULATED t FOR ESTIMATED PARAMETERS OF EGG WEIGHT MODEL 1

Parameter	Estimated	c_{ii}	$\hat{\sigma}^2 c_{ii}$ [1]	$\sqrt{\hat{\sigma}^2 c_{ii}}$	t*
μ	53.98	.00529	.0418	.2044	264.09
a_{1000}	.69	.00794	.0627	.2504	2.75
a_{0100}	-.68	.00265	.0209	.1445	4.70
a_{2000}	-.57	.00794	.0627	.2504	2.27
a_{1100}	.95	.00714	.0564	.2374	4.00
a_{1200}	.57	.01190	.0940	.3066	1.85
a_{2001}	.33	.00564	.0445	.2109	1.56
a_{1110}	.54	.00595	.0470	.2168	2.49
a_{0221}	-.45	.00846	.0668	.2584	1.74
a_{2112}	-.36	.00893	.0705	.2655	1.35
a_{1122}	-.62	.01607	.127	.3563	1.74

[1] $\hat{\sigma}^2 = 7.90$

*$t_{.01} = 2.648$

$t_{.05} = 1.994$

egg production. The least squares estimate of the new egg weight model results in the following equation:

$$\hat{Y}_5 = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_1 x_2 x_3 \end{bmatrix}' \begin{bmatrix} 53.98 \\ 1.07 \\ -.68 \\ -.57 \\ .67 \\ .54 \end{bmatrix}$$

The analysis of variance of this model is in Table XXXIX. The results in this table indicate that Egg Weight Model 2 is just as good for prediction purposes as Model 1. The residual sum of squares appears to be due to random error. The predicted values for the equation from this model are listed in Table XL and the residual deviations are listed in Table XLI. The data in these two tables indicate that the prediction equation is reasonably good. The half-normal plot in Figure 10 shows that the residual is not distributed as close to normal as in the case of Egg Weight Model 1.

TABLE XXXIX

ANALYSIS OF VARIANCE OF EGG WEIGHT MODEL 2

Source	DF	SS	MS	F
Total	567	1,634,297		
R(μ)	1	1,629,236		
R(μ,a)	6	1,630,048		
R(a)	5	812	162.31	23.73 ($P < .01$)
Residual	75	513	6.84	.89
Error	486	3,736	7.69	

TABLE XL

PREDICTED VALUES, $\hat{Y}_s$, FOR EGG WEIGHT MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	53.15	53.15	53.15	54.66	54.66	54.66	55.03	55.03	55.03
	137	53.69	53.69	53.69	54.66	54.66	54.66	54.49	54.49	54.49
	147	54.23	54.23	54.23	54.66	54.66	54.66	53.95	53.95	53.95
300	127	52.34	52.34	52.34	53.98	53.98	53.98	54.48	54.48	54.48
	137	52.34	52.34	52.34	53.98	53.98	53.98	54.48	54.48	54.48
	147	52.34	52.34	52.34	53.98	53.98	53.98	54.48	54.48	54.48
340	127	51.53	51.53	51.53	53.30	53.30	53.30	53.93	53.93	53.93
	137	50.99	50.99	50.99	53.30	53.30	53.30	54.47	54.47	54.47
	147	50.45	50.45	50.45	53.30	53.30	53.30	55.01	55.01	55.01

TABLE XLI

RESIDUAL DEVIATIONS, $\bar{Y}_5 - \hat{Y}_5$, FOR EGG WEIGHT
MODEL 2

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	-1.53	1.39	-.69	-.56	.11	.50	-1.00	.36	.87
	137	-.80	-1.70	.70	.73	.53	-.33	-.52	.41	-1.45
	147	-.77	-.17	.96	1.54	.31	-.87	1.88	-.64	.81
300	127	-.80	-.98	1.99	-.82	-.95	.51	-.41	-.54	.26
	137	.50	-.13	.23	2.29	-.92	.01	.61	.01	-1.85
	147	.72	1.16	.96	-.38	-.04	2.22	-.45	-.84	.03
340	127	1.08	-1.47	.33	1.27	.43	-2.40	.43	-.47	.44
	137	-.09	-.23	-.79	-.53	-.89	-1.06	-.24	-.44	1.03
	147	.98	-.75	.04	1.69	-.36	.80	.18	1.12	.49

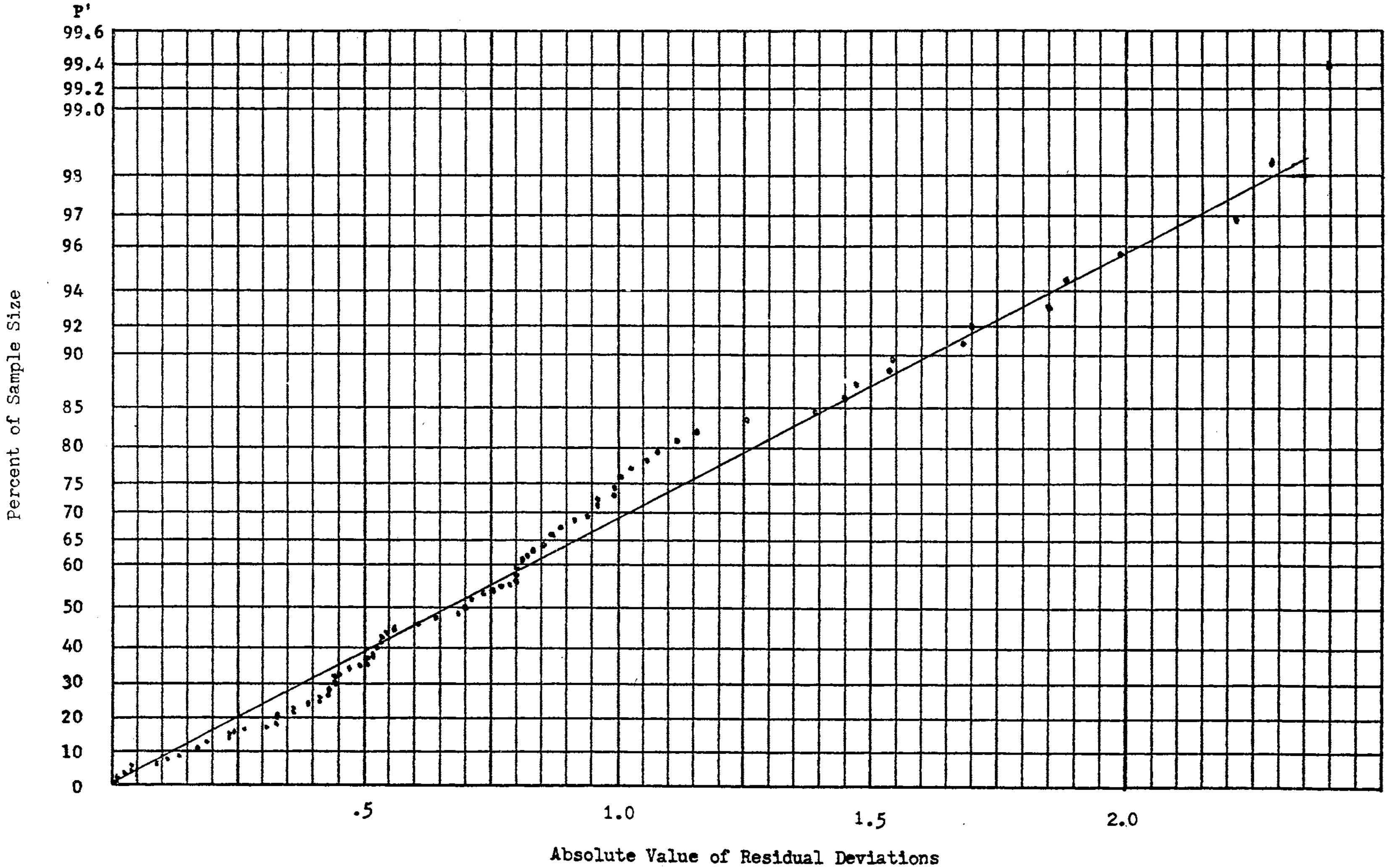

Figure 10. Half-Normal Plot of the Residual Deviations for Egg Weight Model 2

Quadratic Model

The prediction equations to this point in the chapter were developed with the idea of using any or all of the effects of the dietary factors which were included in the experiment plus any of the interaction effects which could be tested by orthogonal comparisons. For prediction purposes only, these equations are valid. But from the standpoint of functional relationships they are invalid. In order to make any statements concerning functional relationships from equations of the type in this chapter, the degree of any term in the functions must not be greater than one less than the number of levels of each factor in the experiment. In the experiment from which these data came, there were three levels of each factor. Therefore, the highest degree that any term can be in the equations and still be valid from a functional relationship standpoint is two. This means that, in a prediction equation developed in this thesis, if the exponents of any term sum to three or greater, the equation is invalid from the standpoint of function relationships. Since the models thus far were designed for prediction purposes only, the equations with terms which are greater than the second degree present no problem.

With the data used in this study, the model which will account for the maximum number of dietary effects and still be considered valid from the standpoint of functional relationships is a full quadratic. The general quadratic model is

$$Y = \mu + \Sigma b_i x_i + \sum_{i \leq j}\Sigma a_{ij} x_i x_j + e$$

In order to understand fully the action and interaction of physiological food intake regulators in laying hens, it will be necessary to understand the functional relationships between and among the various dietary factors, environmental factors and production responses. The full quadratic model, due to the nature of it, is not only suited to the study of various functional relationships in this case but it is the most convenient model which will allow for a multiple response analysis. For this reason, four of the five responses (protein intake, energy intake, egg production, and egg weight) studied previously in this chapter are fitted to the quadratic model.

It is assumed, in the model, that $e \sim N(0, \sigma^2)$; therefore, the same tests for goodness of fit that were performed for the previous models can be performed on the quadratic response functions. The parameters for the specific quadratic model,

$$Y = \mu + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + a_{12} x_1 x_2 + a_{13} x_1 x_3$$
$$+ a_{14} x_1 x_4 + a_{25} x_2 x_3 + a_{24} x_2 x_4 + a_{34} x_3 x_4 + a_{11} x_1^2$$
$$+ a_{22} x_2^2 + a_{33} x_3^2 + a_{44} x_4^2 ,$$

are estimated for each response. After each response equation the residual sum of squares is calculated and tested. Then the predicted values and residual deviations are calculated and a half-normal plot is made of the residual deviations. The responses will be taken in the same order as before; however, body weight change is omitted.

Protein Intake: The quadratic response equation for protein intake is

$$\hat{Y}_1 = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_1 x_2 \\ x_1 x_3 \\ x_1 x_4 \\ x_2 x_3 \\ x_2 x_4 \\ x_3 x_4 \\ x_1^2 \\ x_2^2 \\ x_3^2 \\ x_4^2 \end{bmatrix}' \begin{bmatrix} 13.35 \\ 3.08 \\ -2.03 \\ -.023 \\ -.277 \\ .199 \\ -.031 \\ .067 \\ .198 \\ .041 \\ -.079 \\ -.086 \\ -.211 \\ .160 \\ -.042 \end{bmatrix}$$

The analysis of variance for this equation is in Table XLII. This analysis indicates that the quadratic model accounts for the majority of the variance not due to random error.

The predicted values and the residual deviations are given in Tables XLIII and XLIV, respectively. From the data in these two tables, like that from the previous tables of similar data, one can tell only that the predicted values follow the same trends as the observed values. The half-normal plot of the residual deviations in Figure 11 shows considerable deviation from the normal distribution. For this reason it is concluded that the quadratic model does not adequately describe the response of protein intake as obtained in this experiment.

TABLE XLII

ANALYSIS OF VARIANCE OF PROTEIN INTAKE WITH QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	105,880.84		
$R(\mu)$	1	99,295.53		
$R(\mu, b, a)$	15	104,505.49		
$R(b, a)$	14	5,209.96	372.14	117.02
Residual	66	209.82	3.18	1.33
Error	486	1,165.53	2.40	

Energy Intake: The quadratic response equation for energy intake is

$$\hat{Y}_2 = \begin{bmatrix} 1 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_1 X_2 \\ X_1 X_3 \\ X_1 X_4 \\ X_2 X_3 \\ X_2 X_4 \\ X_3 X_4 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix}' \begin{bmatrix} 252.34 \\ 13.99 \\ -7.02 \\ -.101 \\ -5.45 \\ 12.88 \\ -.019 \\ 2.19 \\ 3.28 \\ .480 \\ -1.74 \\ -3.59 \\ -9.28 \\ 2.78 \\ -1.21 \end{bmatrix}$$

TABLE XLIII

PREDICTED VALUES, $\hat{Y}_1$, FOR PROTEIN INTAKE WITH QUADRATIC MODEL

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	12.81	12.54	12.20	15.74	15.55	15.27	18.51	18.37	18.16
	137	12.53	12.19	11.76	15.44	15.17	14.81	18.17	17.97	17.67
	147	12.59	12.16	11.66	15.46	15.10	14.67	18.16	17.87	17.50
300	127	10.56	10.33	10.02	13.69	13.53	13.29	16.66	16.56	16.39
	137	10.48	10.18	9.79	13.59	13.35	13.03	16.52	16.35	16.10
	147	10.73	10.35	9.88	13.80	13.48	13.09	16.70	16,45	16.13
340	127	7.88	7.70	7.43	11.21	11.10	10.90	14.37	14.32	14.20
	137	8.01	7.75	7.40	11.31	11.11	10.84	14.44	14.31	14.10
	147	8.45	8.11	7.69	11.72	11.45	11.09	14.82	14.62	14.33

TABLE XLIV

RESIDUAL DEVIATIONS, $\bar{Y}_1 - \hat{Y}_1$, FOR PROTEIN INTAKE WITH QUADRATIC MODEL

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	-.76	.34	-.21	.18	.26	.16	.85	-.07	.12
	137	.07	-1.38	.39	-.07	-.39	.13	-.80	.86	.23
	147	.06	.16	-.01	1.10	.21	.14	-.94	-.10	.50
300	127	.03	-.01	.26	1.00	-.78	-.47	-.59	-.37	.61
	137	.82	-.14	.26	.68	.45	.24	.92	-.26	.69
	147	.59	.69	-.61	-1.22	-.26	-.28	-.53	-.02	-.24
340	127	.37	-.19	.31	-1.06	.54	.08	.85	-1.02	.19
	137	.06	.81	.14	-1.11	-.03	-1.03	-.23	-.19	.74
	147	-.94	-.39	-.63	-.13	1.09	1.13	.89	.79	-.98

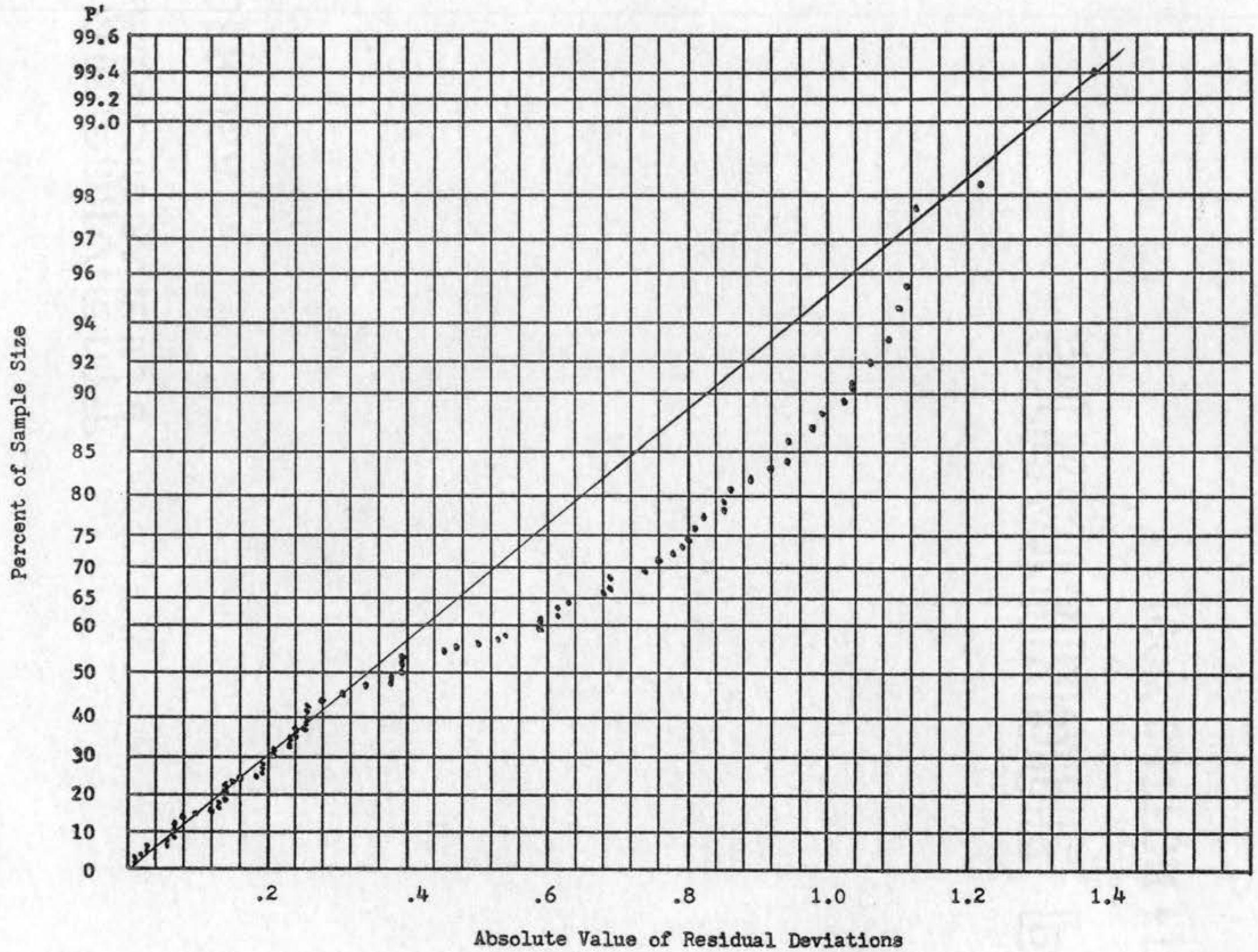

Figure 11. Half-Normal Plot of the Residual Deviations for Protein Intake with the Quadratic Model

The analysis of variance for this function is in Table XLV. The sizes of the mean squares in this Table show that the quadratic model removes the greater portions of the variance due to treatment.

TABLE XLV

ANALYSIS OF VARIANCE OF ENERGY INTAKE
WITH THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	34,688,311		
$R(\mu)$	1	33,978,338		
$R(\mu, b, a)$	15	34,142,343		
$R(b, a)$	14	164,005	11,714.64	9.70
Residual	66	79,694	1,207.48	1.26
Error	486	466,274	959.41	

The predicted values, $\hat{Y}_2$, and the residual deviations, $\bar{Y}_2 - \hat{Y}_2$, are listed in Tables XLVI and XLVII, respectively. Although these values have the same general trend as the treatment means, the half-normal plot (Figure 12) of the residuals gives a similar picture as that for protein intake with the quadratic model. Part of the residuals seem to follow the normal distribution, but this trend is not consistent. This is an indication that the quadratic model does not adequately describe the distribution of the energy intake response.

Egg Production: The response equation for egg production with the quadratic model is

TABLE XLVI

PREDICTED VALUES, $\hat{Y}_2$, FOR ENERGY INTAKE WITH THE QUADRATIC MODEL

		Protein (Grams)								
		13			16			19		
		Milliliters			Milliliters			Milliliters		
Metabolizable Energy (Kilocalories)	Weight (Grams)	180	230	280	180	230	280	180	230	280
260	127	256.68	251.52	243.92	259.20	256.23	250.83	254.53	253.75	250.55
	137	252.28	245.37	236.04	254.78	250.07	242.93	250.10	247.58	242.63
	147	253.44	244.79	233.71	255.92	249.47	240.58	251.22	246.96	240.27
300	127	242.31	237.62	230.51	257.71	255.22	250.30	265.92	265.63	262.90
	137	241.19	234.76	225.90	256.57	252.34	245.68	264.77	262.73	258.27
	147	245.62	237.45	226.85	260.99	255.02	246.61	269.18	265.39	259.19
340	127	209.36	205.16	198.52	237.65	235.64	231.20	258.75	258.93	256.69
	137	211.52	205.57	197.20	239.79	236.04	229.86	260.87	259.32	255.33
	147	219.23	211.54	201.42	247.49	241.99	234.07	268.56	265.25	259.53

TABLE XLVII

RESIDUAL DEVIATIONS, $\bar{Y}_2-\hat{Y}_2$, FOR ENERGY INTAKE WITH THE QUADRATIC MODEL

		Protein (Grams)								
		13			16			19		
		Milliliters			Milliliters			Milliliters		
Metabolizable Energy (Kilocalories)	Weight (Grams)	180	230	280	180	230	280	180	230	280
260	127	-14.25	7.62	-2.92	1.37	-5.8	1.74	14.04	-.18	3.02
	137	1.15	-27.80	8.39	-2.92	-7.93	1.50	-9.10	13.28	-.77
	147	1.13	3.35	.88	15.37	1.10	2.13	-12.36	-.67	9.02
300	127	1.26	.19	5.63	19.58	-14.79	-8.59	-9.35	-6.92	8.53
	137	18.67	-3.9	5.39	12.72	7.95	4.46	13.66	-6.02	-12.13
	147	14.81	16.69	-13.42	-23.99	-5.45	-4.75	-11.18	-3.10	-5.48
340	127	6.5	-7.87	4.91	-20.79	12.93	3.51	16.52	-18.64	3.45
	137	.19	19.57	.94	-21.93	.67	-20.57	-4.16	-4.03	12.81
	147	-22.09	-8.54	-15.99	-.06	25.58	27.07	-15.30	-13.18	18.39

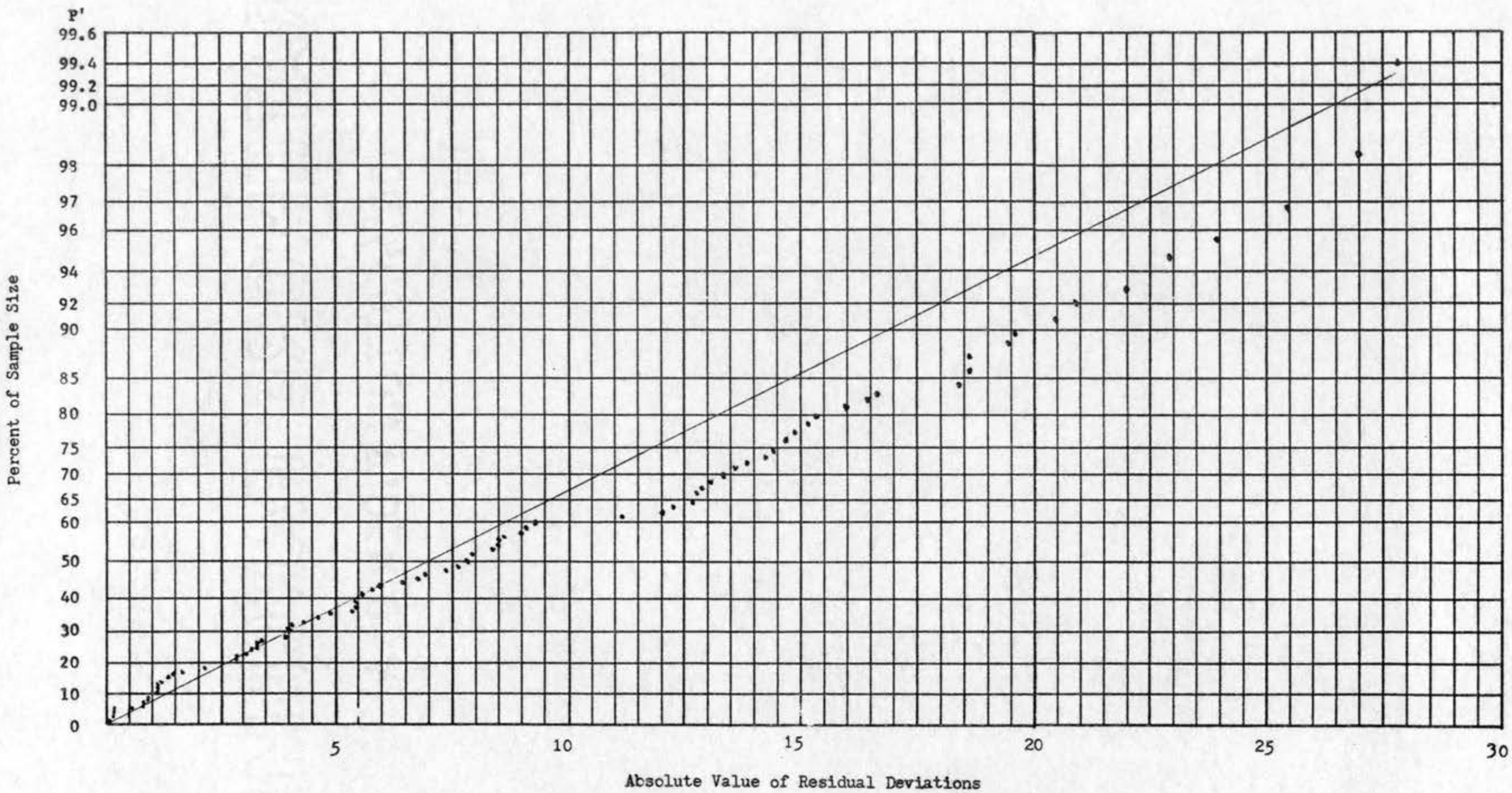

Figure 12. Half-Normal Plot of the Residual Deviations for Energy Intake with the Quadratic Model

$$\hat{Y}_4 = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_1 x_2 \\ x_1 x_3 \\ x_1 x_4 \\ x_2 x_3 \\ x_2 x_4 \\ x_3 x_4 \\ x_1^2 \\ x_2^2 \\ x_3^2 \\ x_4^2 \end{bmatrix}' \begin{bmatrix} 140.25 \\ 20.61 \\ -17.16 \\ 2.08 \\ -1.77 \\ 16.57 \\ 2.08 \\ .353 \\ 4.36 \\ 1.46 \\ 1.45 \\ -9.35 \\ -2.72 \\ .968 \\ -3.82 \end{bmatrix}$$

The analysis of variance for this model is in Table XLVIII. The residual sum of squares in this analysis appears to be no different than the error sum of squares.

The predicted values and residual deviations for egg production with the quadratic model are given in Tables XLIX and L, respectively. The half-normal plot of the residuals is in Figure 13. It appears from this figure that the residuals are not distributed normally. The picture is somewhat like that of protein and energy intake with the quadratic model. Therefore, it is assumed that the quadratic model does not describe the distribution of the egg production responses in this experiment.

TABLE XLVIII

ANALYSIS OF VARIANCE OF EGG PRODUCTION
WITH THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	10,648,299		
$R(\mu)$	1	9,626,812		
$R(\mu,b,a)$	15	9,991,619		
$R(b,a)$	14	364,807	26,057.64	19.15
Residual	66	89,789	1,360.44	1.17
Error	486	566,891	1,166.44	

Egg Weight: The response function for egg weight with the quadratic model is

$$Y_5 = \begin{bmatrix} 1 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_1X_2 \\ X_1X_3 \\ X_1X_4 \\ X_2X_3 \\ X_2X_4 \\ X_3X_4 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix}' \begin{bmatrix} 53.59 \\ 1.07 \\ -.680 \\ .215 \\ -.053 \\ .673 \\ -.031 \\ -.117 \\ -.017 \\ -.249 \\ -.157 \\ -.572 \\ -.126 \\ .340 \\ .382 \end{bmatrix}$$

TABLE XLIX

PREDICTED VALUES, $\hat{Y}_4$, FOR EGG PRODUCTION WITH THE QUADRATIC MODEL

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein 13 (Grams), Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	147.85	146.63	137.78	158.80	157.94	149.44	151.05	150.54	142.39
260	137	141.07	141.30	133.90	154.10	154.68	147.64	148.43	149.37	142.67
260	147	136.23	137.91	131.96	151.33	153.37	147.78	147.75	150.14	144.89
300	127	111.02	111.26	103.86	133.55	139.14	132.09	147.37	148.32	141.63
300	137	108.60	110.29	104.34	138.26	140.25	134.65	149.11	151.51	146.26
300	147	108.11	111.26	106.76	139.80	143.29	139.16	152.78	156.63	152.85
340	127	68.74	70.44	64.50	112.85	114.89	109.30	138.25	140.65	135.41
340	137	70.69	73.83	69.34	116.87	120.36	116.23	144.35	148.20	144.42
340	147	74.56	79.16	76.12	122.82	127.77	125.09	152.38	157.69	155.35

TABLE L

RESIDUAL DEVIATIONS, $\bar{Y}_4 - \hat{Y}_4$, FOR EGG PRODUCTION WITH THE QUADRATIC MODEL

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	11.72	6.66	-20.64	-6.66	-16.08	-2.58	8.95	-1.54	10.86
	137	12.07	-21.30	7.10	-2.10	-10.25	-1.07	-9.00	14.06	17.33
	147	5.91	5.23	7.04	3.81	-6.51	-20.92	-6.89	-3.28	17.68
300	127	-5.88	15.74	8.43	2.16	7.15	15.34	-9.51	-5.18	-12.92
	137	-18.74	4.28	8.09	-6.12	7.18	15.49	17.6	-4.80	-23.83
	147	8.03	8.03	-5.19	-10.94	.42	10.58	-2.78	10.37	-11.85
340	127	24.83	-9.87	-19.36	-27.14	-9.32	32.70	21.89	-23.51	5.30
	137	-11.55	7.46	-4.63	3.99	6.93	-17.23	8.36	-2.20	2.87
	147	-17.42	.84	-6.83	5.89	-22.66	-21.77	-.52	-5.12	-10.78

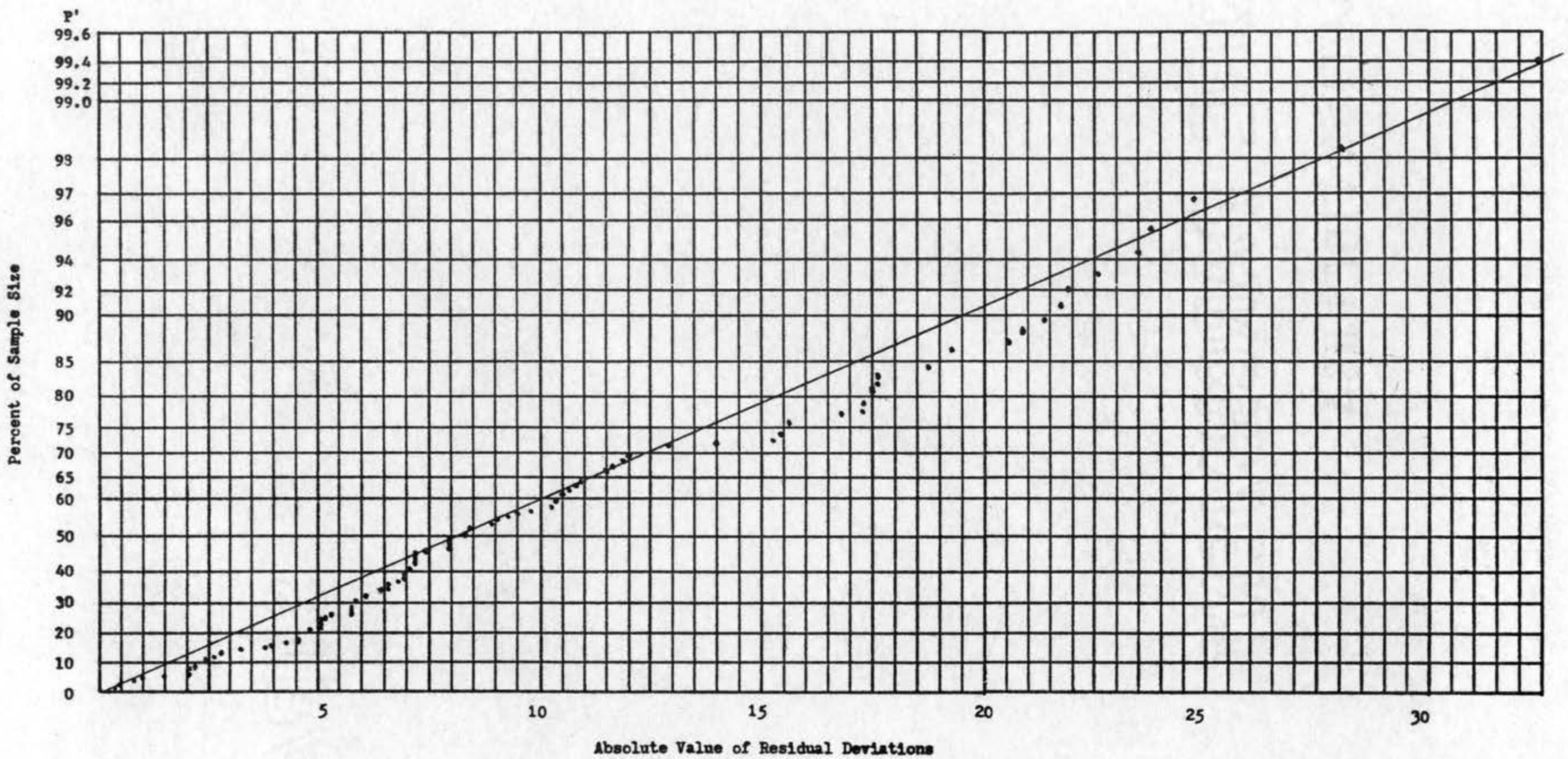

Figure 13. Half-Normal Plot of the Residual Deviations for Egg Production with the Quadratic Model

The analysis of variance for this function is in Table LI. This analysis shows that the treatment variation removed by the model is highly significant ($P < .01$), and that the residual variation is non-significant.

TABLE LI

ANALYSIS OF VARIANCE OF EGG WEIGHT WITH THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	1,634,297		
$R(\mu)$	1	1,629,236		
$R(\mu,b,a)$	15	1,630,077		
$R(b,a)$	14	841	60.07	8.19
Residual	66	484	7.33	.95
Error	486	3,736	7.69	

The predicted values and the residual deviations for this equation are listed in Tables LII and LIII, respectively. The half-normal plot of the residuals is in Figure 14. The data in this graph show that the quadratic probably fits the data for egg weight better than any of the responses tested. The residual deviations appear to follow a normal curve very well.

It was mentioned at the beginning of this section that the quadratic model was the best to use if a multiple response analysis was anticipated. A condition necessary for this type of analysis of the quadratic distribution is that at least one of the quadratic forms be positive definite

TABLE LII

PREDICTED VALUES, $\hat{Y}_5$, FOR EGG WEIGHT WITH THE QUADRATIC MODEL

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, Milliliters 180	16, 230	16, 280	19, Milliliters 180	19, 230	19, 280
260	127	53.16	53.25	54.10	54.27	54.25	54.98	54.25	54.10	54.72
	137	53.24	53.17	53.87	54.32	54.14	54.72	54.27	53.97	54.43
	147	54.00	53.78	54.32	55.06	54.71	55.14	54.97	54.51	54.81
300	127	52.20	52.04	52.64	53.99	53.71	54.20	54.64	54.24	54.61
	137	52.26	51.94	52.39	54.02	53.59	53.92	54.64	54.08	54.30
	147	53.01	52.53	52.82	54.74	54.14	54.31	55.32	54.61	54.66
340	127	50.98	50.57	50.93	53.45	52.92	53.16	54.77	54.12	54.24
	137	51.03	50.46	50.66	53.47	52.78	52.86	54.75	53.95	53.91
	147	51.76	51.04	51.07	54.16	53.32	53.24	55.42	54.46	54.26

TABLE LIII

RESIDUAL DEVIATIONS, $\bar{Y}_s - \hat{Y}_s$, FOR EGG WEIGHT WITH THE QUADRATIC MODEL

Metabolizable Energy (Kilocalories)	Weight (Grams)	Protein (Grams) 13, Milliliters 180	13, 230	13, 280	16, 180	16, 230	16, 280	19, 180	19, 230	19, 280
260	127	-1.52	1.29	-1.64	-.17	.52	.18	-.22	1.29	1.18
	137	-.35	-1.18	.52	1.07	-.01	-.39	-.30	.93	-1.39
	147	-.54	.28	.87	1.14	.26	-.93	.86	-1.20	-.05
300	127	-.66	-.68	1.69	-.83	-.68	.29	-.57	-.30	.13
	137	.58	.27	.18	2.25	-.53	.07	.45	.41	-1.67
	147	.05	.97	.48	-.80	-.20	1.89	-1.29	-.97	-.15
340	127	1.63	-.51	.93	1.12	.81	2.26	-.41	-.66	.23
	137	-.13	.30	-.46	-.70	-.37	-.54	-.52	.08	1.60
	147	-.33	-1.34	.58	.83	-.38	-1.12	-.23	1.67	1.24

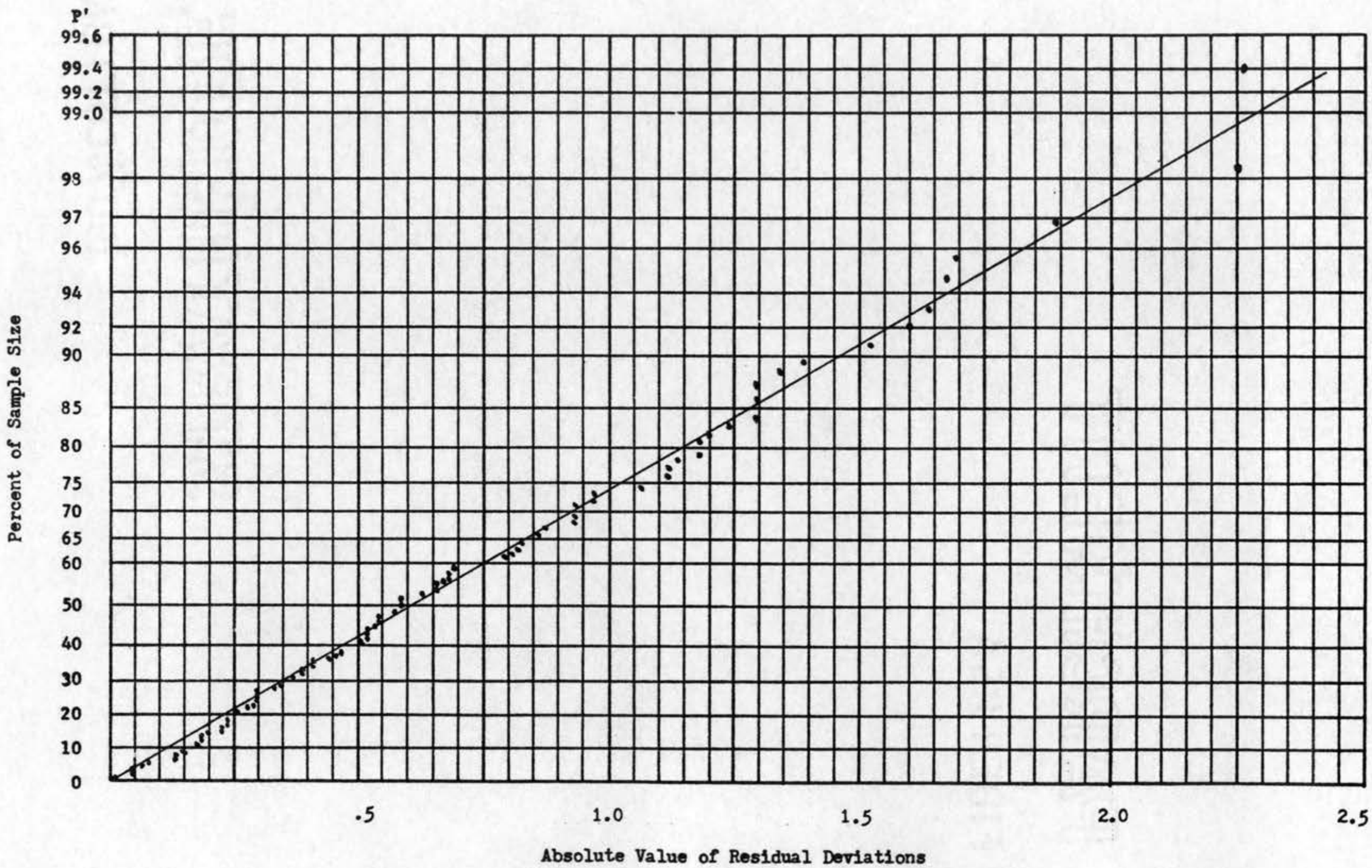

Figure 14. Half-Normal Plot of the Residual Deviations for Egg Weight with the Quadratic Model

(Antle, 1962). An inspection of the quadratic equations in this section reveals that none of them is positive definite. This was not unexpected after the tests for goodness of fit showed that the quadratic model did not fit the data to any great extent.

CHAPTER III

PREDICTION OF NUTRIENT INTAKE BASED UPON DIETARY AND PRODUCTION FACTORS

In Chapter II, only dietary factors were considered when response functions were developed. However, it is known that there are many factors other than dietary factors which can and do affect feed intake and production responses in laying hens. It was well established by Gleaves that there is a close relationship between egg production and feed intake. This was determined by selecting hens from each treatment which had high egg production and hens which had low egg production. Then egg production was used as another factor in the analysis of variance. It is very likely that other production factors are of great importance also in the study of feed intake. The purpose of this chapter is to study the relationship of egg production and body weight change to the intake of protein and energy from least squares estimates of prediction equations.

"Covariance Model"

The term covariance is used here only because the model which is used looks like a covariance model. The model is as follows:

$$Y = \mu + \beta_1 E + \beta_2 W + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4$$
$$+ a_{12} x_1 x_2 + a_{13} x_1 x_3 + a_{14} x_1 x_4 + a_{23} x_2 x_3 + a_{24} x_2 x_4$$
$$+ a_{34} x_3 x_4 + a_{11} x_1^2 + a_{22} x_2^2 + a_{33} x_3^2 + a_{44} x_4^2$$

where E is the observed number of eggs and W is observed body weight change. The model contains egg production and body weight change along with dietary protein, dietary energy, dietary weight and dietary volume in the quadratic form. If the production factors were independent of the dietary factors, this would be a true covariance model with the production factors as the covariables. However, for obvious reasons they are not independent, and a covariance analysis in the strict sense cannot be made.

In order to design intelligently a future experiment to study the relationship between production factors and nutrient intake, it would be advantageous to obtain as much information as possible from data presently available. Therefore, this so-called "covariance model" is used by considering the production factors as predicting variables instead of covariables. The quadratic model is used with the anticipation of obtaining some clue as to why it did not fit the response distributions as presented in Chapter II. The predicting variables, egg production and body weight change, will be considered separately and in combination.

Egg Production Effect: When Y is protein intake per bird per day and egg production is used as the predicting variable, the least squares estimates of the parameters in the model results in the following equation:

$$\begin{aligned}\hat{Y}_1 = 9.66 &+ .026E + 2.54x_1 - 1.57x_2 - .078x_3 - .231x_4 \\ &- .288x_1x_2 - .086x_1x_3 + .057x_1x_4 + .083x_2x_3 \\ &+ .0026x_2x_4 - .117x_3x_4 + .161x_1^2 - .139x_2^2 \\ &+ .135x_3^2 + .059x_4^2\end{aligned}$$

The analysis of variance of protein intake with this model is presented in Table LIV. As expected, the sum of squares for egg production is highly significant ($P < .01$). An important thing here is the effect that the variable, egg production, had upon the parameters in the quadratic portion of the model. The parameters of this model, the straight quadratic model, for protein intake are compared in Table LV. It can readily be seen that some of the parameters are quite different as a result of egg production. One major difference can be seen as a result of the t tests in Table LV. The coefficient for protein x energy interaction has a reversal in sign and is significant ($P < .01$) in the "covariance model", whereas it is not significant in the straight quadratic.

TABLE LIV

ANALYSIS OF VARIANCE OF PROTEIN INTAKE WITH EGG PRODUCTION AS A PREDICTING VARIABLE IN THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	105,880.84		
$R(\mu)$	1	99,295.53		
$R(b,a/\mu,\beta_1)$[1]	14	5,209.96		
$R(\beta_1/\mu,b,a)$	1	456.10	456.10	273.11
Residual	551	919.25	1.67	

[1] $R(b,a/\mu,\beta_1)$ - Reduction in sum of squares due to b and a, adjusted for μ and β_1.

With egg production as the predicting variable with the quadratic

TABLE LV

CALCULATED t FOR THE PARAMETERS IN THE QUADRATIC MODEL AND IN THE QUADRATIC MODEL WITH EGG PRODUCTION AS A PREDICTING VARIABLE FOR PROTEIN INTAKE

Parameter	Estimate Quadratic Model	t^1	Estimate "Covariance Model"	t^2
μ	13.35	59.33	9.66	34.93
β_1	-----	-----	.026	16.35
b_1	3.08	33.62	2.54	33.91
b_2	-2.03	22.16	-1.57	21.86
b_3	- .023	.25	- .078	1.17
b_4	- .277	3.02	- .231	3.47
a_{12}	.199	1.77	- .284	3.37
a_{13}	- .031	.27	- .086	1.05
a_{14}	.067	.61	.057	.70
a_{23}	.198	1.76	.083	1.01
a_{24}	.041	.36	.0026	.03
a_{34}	- .079	.70	- .117	1.44
a_{11}	- .086	.56	.161	1.38
a_{22}	- .211	1.33	- .139	1.20
a_{33}	.160	1.01	.135	1.17
a_{44}	- .042	.26	.059	.51

[1] $t_{.05}$ with 66 degrees of freedom = 2.00

[2] $t_{.05}$ with 551 degrees of freedom = 1.96

model on energy intake, the following equation is obtained:

$$\hat{Y}_2 = \begin{bmatrix} 1 \\ E \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_1 X_2 \\ X_1 X_3 \\ X_1 X_4 \\ X_2 X_3 \\ X_2 X_4 \\ X_3 X_4 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix}' \begin{bmatrix} 175.96 \\ .545 \\ 2.76 \\ 2.33 \\ -1.23 \\ -4.48 \\ 3.86 \\ -1.14 \\ 2.00 \\ .903 \\ -.313 \\ -2.53 \\ 1.49 \\ -7.80 \\ 2.25 \\ .864 \end{bmatrix}$$

The analysis of variance for energy intake from this model is in Table LVI. This shows egg production to be a highly significant ($P < .01$) factor in the control of energy intake. In fact, the reduction in sum of squares due to egg production is greater than all the effects in the quadratic model.

The parameters for this model and the straight quadratic model are compared in Table LVII. It is obvious from the data in this table that egg production must be closely related to feed and energy intake. With egg production variation accounted for in the model, both protein linear,

x_1, and energy linear, x_2, effects were reduced to nonsignificance. Gleaves found that when two levels of egg production were used as another factor, the quadratic effect of dietary energy on energy intake was reduced. In this analysis, when egg production was included in the model, the quadratic effect of dietary energy remained approximately the same.

TABLE LVI

ANALYSIS OF VARIANCE OF ENERGY INTAKE WITH EGG PRODUCTION AS A PREDICTING VARIABLE IN THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	34,688,311		
$R(\mu)$	1	33,978,338		
$R(b,a/\mu,\beta_1)$	14	164,005		
$R(\beta_1/\mu,b,a)$	1	194,754	194,754	305.74
Residual	551	351,214	637	

<u>Body Weight Change Effect</u>: With body weight change as the predicting variable in the quadratic model, the estimates for the parameters when protein intake is the response are in the equation

$$\hat{Y}_1 = 13.15 + .0034W + 2.67x_1 - 1.79x_2 - .069x_3 - .183x_4 - .039x_1x_2 - .009x_1x_3 + .135x_1x_4 + .071x_2x_3 - .018x_2x_4 - .023x_3x_4 - .115x_1^2 - .023x_2^2 + .087x_3^2 - .032x_4^2 .$$

The analysis of variance in Table LVIII shows that body weight change is a very important factor associated with protein intake. The

TABLE LVII

CALCULATED t FOR THE PARAMETERS IN THE QUADRATIC MODEL AND IN THE QUADRATIC MODEL WITH EGG PRODUCTION AS A PREDICTING VARIABLE FOR ENERGY INTAKE

Parameter	Estimate Quadratic Model	t^1	Estimate "Covariance Model"	t^2
μ	252.34	57.58	175.97	32.56
β_1	------	-----	.545	14.69
b_1	13.99	7.85	2.76	1.91
b_2	-7.02	3.94	2.33	1.66
b_3	- .101	.05	-1.23	.95
b_4	-5.45	3.05	-4.48	3.44
a_{12}	12.88	5.88	3.86	2.31
a_{13}	- .019	.00	-1.15	.72
a_{14}	2.19	.96	2.00	1.26
a_{23}	3.28	1.49	.903	.58
a_{24}	.480	.21	- .313	.20
a_{34}	-1.74	.79	-2.53	1.59
a_{11}	-3.59	1.16	1.50	.66
a_{22}	-9.28	3.00	-7.80	3.47
a_{33}	2.78	.89	2.25	1.00
a_{44}	-1.21	.39	.865	.38

[1] $t_{.05}$ with 66 degrees of freedom = 2.00

[2] $t_{.05}$ with 551 degrees of freedom = 1.96

parameters in the model are tested for significance in Table LIX. This test shows that there are no important differences in the parameters of the quadratic portion of the "covariance model" as compared to the straight quadratic.

TABLE LVIII

ANALYSIS OF VARIANCE OF PROTEIN INTAKE WITH BODY WEIGHT CHANGE AS A PREDICTING VARIABLE IN THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	105,880.84		
$R(\mu)$	1	99,295.53		
$R(b,a/\mu,\beta_2)$	14	5,209.96		
$R(\beta_2/\mu,b,a)$	1	363.32	363.32	197.45
Residual	551	1,012.03	1.84	

The parameters for the same model with energy intake as the response are listed in Table LX. The analysis of variance for this response is in Table LXI. It can be seen from the t test and analysis of variance that body weight change is associated with energy intake to a highly significant ($P < .01$) degree. In Table LX it can be seen that, when the body weight change is included in the model, there are two obvious changes in the parameters of the quadratic model. The magnitude of the coefficient (b_2) of dietary energy on energy intake was decreased from $|-7.02|$ to $|-2.55|$, and there appears to be a significant ($P < .05$) interaction between dietary protein and dietary volume (a_{14}) on energy intake.

TABLE LIX

CALCULATED t FOR THE PARAMETERS IN THE QUADRATIC MODEL AND IN THE QUADRATIC MODEL WITH BODY WEIGHT CHANGE AS A PREDICTING VARIABLE FOR PROTEIN INTAKE

Parameter	Estimate Quadratic Model	t[1]	Estimate "Covariance Model"	t[2]
μ	13.35	59.33	13.15	76.72
β_2	-----	-----	.0034	14.16
b_1	3.08	33.62	2.67	35.31
b_2	-2.03	22.16	-1.79	25.00
b_3	- .023	.25	- .069	.99
b_4	- .277	3.02	- .183	2.61
a_{12}	.199	1.77	- .039	.45
a_{13}	- .031	.27	- .009	.10
a_{14}	.067	.61	.135	1.58
a_{23}	.198	1.76	.071	.83
a_{24}	.041	.36	- .018	.21
a_{34}	- .079	.70	- .023	.27
a_{11}	- .086	.56	- .115	.95
a_{22}	- .211	1.33	- .023	.19
a_{33}	.160	1.01	.087	.72
a_{44}	- .042	.26	- .032	.26

[1] $t_{.05}$ with 66 degrees of freedom = 2.00

[2] $t_{.05}$ with 551 degrees of freedom = 1.96

TABLE LX

CALCULATED t FOR THE PARAMETERS IN THE QUADRATIC MODEL AND IN THE QUADRATIC MODEL WITH BODY WEIGHT CHANGE AS A PREDICTING VARIABLE FOR ENERGY INTAKE

Parameter	Estimate Quadratic Model	t^1	Estimate "Covariance Model"	t^2
μ	252.34	57.58	248.27	72.57
β_2	------	-----	.066	13.92
b_1	13.99	7.85	5.88	3.89
b_2	-7.02	3.94	-2.55	1.78
b_3	- .101	.05	-1.00	.71
b_4	-5.45	3.05	-3.58	2.56
a_{12}	12.88	5.88	8.21	4.71
a_{13}	- .019	.00	.424	.24
a_{14}	2.19	.96	3.53	2.06
a_{23}	3.28	1.49	.773	.45
a_{24}	.480	.21	- .676	.39
a_{34}	-1.74	.79	- .822	.48
a_{11}	-3.59	1.16	-4.18	1.73
a_{22}	-9.28	3.00	-5.57	2.29
a_{33}	2.78	.89	1.33	.55
a_{44}	-1.21	.39	-1.02	.42

[1] $t_{.05}$ with 66 degrees of freedom = 2.00

[2] $t_{.05}$ with 551 degrees of freedom = 1.96

TABLE LXI

ANALYSIS OF VARIANCE OF ENERGY INTAKE WITH BODY WEIGHT CHANGE AS A PREDICTING VARIABLE IN THE QUADRATIC MODEL

Source	DF	SS	MS	F
Total	567	34,688,311		
$R(\mu)$	1	33,978,338		
$R(b,a/\mu,\beta_2)$	14	164,005		
$R(\beta_2/\mu,b,a)$	1	140,651	140,651	191.36
Residual	551	405,317	735	

The estimates of the parameters for the "covariance model", with egg production and body weight change as prediction variables in the quadratic model for protein intake and energy intake, are in Tables LXII and LXIII, respectively. For protein intake, the results are very similar to the results obtained when egg production was the single prediction variable. This indicates that the egg production effect is probably much stronger than the body weight change effect on protein intake. However, with energy intake as the response (Table LXIII), there is one difference with the two prediction variables, as compared to the model with egg production as the only prediction variable. That is, dietary energy has a significant ($P < .01$) coefficient in the model with the two prediction variables. An explanation for this difference is not obvious.

The "covariance models" which have been used in this chapter help a great deal in locating the sources of variation in nutrient intake.

TABLE LXII

CALCULATED t FOR THE PARAMETERS IN THE QUADRATIC MODEL AND IN THE QUADRATIC MODEL WITH EGG PRODUCTION AND BODY WEIGHT CHANGE AS PREDICTING VARIABLES FOR PROTEIN INTAKE

Parameter	Estimate Quadratic Model	t^1	Estimate "Covariance Model"	t^2
μ	13.35	59.33	10.38	38.87
β_1	-----	-----	.0202	12.62
β_2	-----	-----	.0022	9.56
b_1	3.08	33.62	2.40	34.18
b_2	-2.03	22.16	-1.53	23.93
b_3	- .023	.25	- .095	1.54
b_4	- .277	3.02	- .179	2.91
a_{12}	.199	1.77	- .293	3.69
a_{13}	- .031	.27	- .059	.78
a_{14}	.067	.61	.104	1.38
a_{23}	.198	1.76	.026	.34
a_{24}	.041	.36	- .027	.36
a_{34}	- .079	.70	- .077	1.02
a_{11}	- .086	.56	.084	.77
a_{22}	- .211	1.33	- .031	.28
a_{33}	.160	1.01	.092	.86
a_{44}	- .042	.26	.042	.39

[1] $t_{.05}$ with 66 degrees of freedom = 2.00

[2] $t_{.05}$ with 550 degrees of freedom = 1.96

TABLE LXIII

CALCULATED t FOR THE PARAMETERS IN THE QUADRATIC MODEL AND IN THE QUADRATIC MODEL WITH EGG PRODUCTION AND BODY WEIGHT CHANGE AS PREDICTING VARIABLES FOR ENERGY INTAKE

Parameter	Estimate Quadratic Model	t^1	Estimate "Covariance Model"	t^2
μ	252.34	57.58	189.62	36.49
β_1	------	-----	.429	13.62
β_2	------	-----	.0418	9.32
b_1	13.99	7.85	.0032	.00
b_2	-7.02	3.94	3.18	2.43
b_3	- .101	.05	-1.56	1.29
b_4	-5.45	3.05	-3.50	2.89
a_{12}	12.88	5.88	2.81	1.80
a_{13}	- .019	.00	- .628	.42
a_{14}	2.19	.96	2.89	1.95
a_{23}	3.28	1.49	- .181	.12
a_{24}	.480	.21	- .878	.59
a_{34}	-1.74	.79	-1.78	1.20
a_{11}	-3.59	1.16	.044	.02
a_{22}	-9.28	3.00	-5.76	2.74
a_{33}	2.78	.89	1.44	.68
a_{44}	-1.21	.39	.547	.26

[1] $t_{.05}$ with 66 degrees of freedom = 2.00

[2] $t_{.05}$ with 550 degrees of freedom = 1.96

By the use of these models it has been possible to show that, when production variation is removed, some dietary effects are reduced and others are increased.

With the response functions to this point in this chapter, it may be possible to predict feed and nutrient intake of laying hens in various levels of egg production and in various stages of body weight change. If the quadratic model should not seem desirable, then other covariance models could be used. It would be possible to estimate maintenance requirements for laying hens with response functions of this type by assuming egg production and body weight change to be zero and then estimating nutrient consumption.

The prediction variables in the models used in this study were observed variables. For this reason the equations cannot be used to consider functional distributions. If egg production could be a mathematical variable instead of an observed variable, then functional distributions could be considered. The method employed by Gleaves to hold egg production constant, in effect, converts an observed variable into a mathematical variable. Therefore, a response equation developed from the high egg production hens or the low egg production hens used by Gleaves would be valid from a functional distribution standpoint. At the same time, the variation due to variation in egg production would be minimized. If a multiple response analysis is desired after the effect of egg production is removed, it would be necessary to have response surfaces valid from a functional distribution standpoint.

Quadratic Model With Constant Egg Production

In order to observe the response functions with constant egg production, and to check for conditions necessary for multiple response analysis, the quadratic response surfaces for nutrient intakes are now estimated from the low egg production and high egg production data used by Gleaves. The data for this are listed in Appendix Tables II and III. The only test made on these functions will be to check for conditions necessary for a multiple response analysis. Comments will be made on any obvious differences in the parameters compared to those in the "covariance model".

The prediction response for protein intake of the hens in low egg production (approximately 15 eggs per 28-day period) is

$$\hat{Y}_{1\,1} = \begin{bmatrix} 1 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_1 X_2 \\ X_1 X_3 \\ X_1 X_4 \\ X_2 X_3 \\ X_2 X_4 \\ X_3 X_4 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix} \cdot \begin{bmatrix} 12.44 \\ 3.17 \\ -2.36 \\ -.172 \\ -.108 \\ .161 \\ -.299 \\ .294 \\ .156 \\ -.121 \\ .184 \\ -.121 \\ -.019 \\ .113 \\ .271 \end{bmatrix}$$

The quadratic prediction response for energy intake by the same hens is

$$\hat{Y}_{2\,1} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_1 x_2 \\ x_1 x_3 \\ x_1 x_4 \\ x_2 x_3 \\ x_2 x_4 \\ x_3 x_4 \\ x_1^2 \\ x_2^2 \\ x_3^2 \\ x_4^2 \end{bmatrix}' \begin{bmatrix} 234.15 \\ 17.83 \\ -14.79 \\ -2.55 \\ -2.92 \\ 13.04 \\ -3.97 \\ 5.53 \\ .972 \\ -.218 \\ 2.00 \\ -4.24 \\ -5.41 \\ 2.23 \\ 5.18 \end{bmatrix}$$

In this equation the coefficients for x_1, x_2 and $x_1 x_2$ are quite large as compared to the corresponding coefficients in the covariance models. This is probably an indication that, at lower egg production levels, feed intake is more dependent upon dietary factors than at higher egg production levels.

The estimate for the quadratic response of protein intake for the high egg producing hens (25 eggs in a 28-day period) is

$$\hat{Y}_{1h} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_1 x_2 \\ x_1 x_3 \\ x_1 x_4 \\ x_2 x_3 \\ x_2 x_4 \\ x_3 x_4 \\ x_1^2 \\ x_2^2 \\ x_3^2 \\ x_4^2 \end{bmatrix}' \begin{bmatrix} 14.23 \\ 2.40 \\ -1.73 \\ .086 \\ -.375 \\ -.291 \\ -.233 \\ .190 \\ -.141 \\ .098 \\ -.132 \\ .053 \\ -.079 \\ -.104 \\ .140 \end{bmatrix}$$

The only obvious difference between this function and the comparable one for low egg production is that the protein intake is higher for the high egg producing hens.

The quadratic response function for energy intake for the high egg producing hens is

$$\hat{Y}_{2h} = 268.92 - 4.09x_1 + 2.77x_2 + 2.04x_3 - 7.61x_4 + .139x_1 x_2$$
$$- 5.26x_1 x_3 + 5.49x_1 x_4 - 1.57x_2 x_3 + .930x_2 x_4 - 2.37x_3 x_4$$
$$+ 2.59x_1^2 - 6.87x_2^2 - 2.07x_3^2 + 2.59x_4^2 .$$

The parameters in this function are smaller than those of the comparable function for low egg production. This probably indicates that dietary factors are not as effective in controlling feed intake when the hens are producing at a high rate as when they are producing at a low rate. However, the factor of dietary volume, x_4, appears to have a greater influence at the high egg production level than at the low egg production level. As expected, these observations are similar to those which were made by Gleaves using a different type of analysis.

It was hoped that by holding egg production constant the quadratic response function would be positive definite for at least one of the responses. This is the necessary condition for estimating efficient points of two responses at the same time. However, an examination of the parameters shows that none of the functions satisfies the necessary condition.

CHAPTER IV

PRODUCTION AS AFFECTED BY NUTRIENT INTAKE

Most research workers in poultry nutrition would agree that actual nutrient intake and balance of nutrients are the most important nutritional factors which determine egg production in laying hens. However, to the knowledge of the author, no work has been done in which variations in actual nutrient intake were studied in relation to variations in egg production.

To study the relationship of nutrient intake to production factors (body weight gain, egg production and egg weight), two difficult problems must be overcome. The first of these is the problem of obtaining graded levels of nutrient intake with ad libitum fed birds. The data for this thesis came from a feeding trial in which the experimental diets were formulated in such a way as to give various combinations of protein and energy intakes. The intake levels of protein and energy which were obtained are listed in Appendix Table I under Y_1 and Y_2, respectively. It can be seen in these data that there was a great deal of variation in the intake of protein and in the intake of energy.

The second problem is to use protein and energy intake data effectively in a study of the relationship of these intake variables to production variables (body weight gain, egg production and average egg weight). The intake of protein and energy is affected by the dietary factors (protein, energy, weight and volume), as was established by

Gleaves. The dietary factors also affect the production responses. Prediction equations in Chapter II were developed based on these facts. However, there is a question as to whether the dietary factors have any direct effect upon the production factors, or whether the effect is indirect. In other words, is the variation in the production factors entirely due to the variation in nutrient intake? This is an important question from the standpoint of statistics. If the effects of dietary factors upon production are indirect and are actually a result of nutrient intake, then it would be valid from a statistical standpoint to study the relationship of nutrient intake to production, independently of the dietary factors. But, if there is some direct effect of the dietary factors upon production, or if the intake of the non-nutrient dietary factors have an effect upon production, then it would be invalid from a statistical standpoint to study nutrient intake in relation to production, independently of the dietary factors.

It seems logical that the dietary factors and the intake of non-nutrient dietary factors may have some direct effect upon production. It also seems logical, since the nutrients are the necessary ingredients for production, that the direct effect of the dietary factors and the intake of non-nutrient dietary factors would be small in relation to the effects due to nutrient intake. There are several reasons to think that the dietary factors and the intake of non-nutrient dietary factors could affect production by means other than through nutrient intake. Variations in digestibility and variations in energy expended in digestion could possibly be affected by these factors. If digestibility is altered, then utilization of the nutrients digested may be altered. All of these things would tend to cause variation in production.

From the preceding discussion it is obvious that dietary factors are related to production by some means other than through nutrient intake. Nevertheless, it is the opinion of the author that a study of the relationship of protein intake and energy intake to production, independently of all other factors, would be a definite contribution in the field of poultry nutrition. This information concerning the relationship of nutrient intake to production is vitally needed in the poultry industry today.

In the analysis to follow, the assumption is made that variation in production (body weight gain, egg production and egg weight) is due to variations in protein intake and energy intake, independently of the dietary factors. The data in Appendix Table I are fitted to the following model:

$$Y = a_0 + b_1 x_1 + b_2 x_2 + a_{12} x_1 x_2 + a_{11} x_1^2 + a_{22} x_2^2$$

where

Y = production response

Y_3 = body weight gain
Y_4 = number of eggs
Y_5 = average egg weight

x_1 = grams of protein intake per bird per day

x_2 = (Calories of energy intake per bird per day)/10
Energy is divided by 10 to facilitate computations.

a_{ii} and b_i = unknown parameters .

The parameters are estimated by the method of least squares and the following response functions are obtained:

$$\hat{Y}_3 = -1134.91 + 22.89x_1 + 41.47x_2 - .197x_1 x_2 + .235x_1^2 - .203x_2^2$$

$$\hat{Y}_4 = -205.67 + 22.49x_1 + 10.15x_2 + .033x_1 x_2 - .673x_1^2 - .156x_2^2$$

$$\hat{Y}_5 = 49.42 + 1.079x_1 - .47x_2 + .0009x_1 x_2 - .029x_1^2 + .011x_2^2 .$$

The parameters in the equation for body weight gain, $\hat{Y}_3$, are tested for significance by an analysis of variance in Table LXIV. The data in this table show that energy linear is the only factor which has a significant (P < .05) sum of squares. The coefficient of this factor is positive, indicating that increases in energy intake cause an increase in body weight gain.

TABLE LXIV

ANALYSIS OF VARIANCE OF BODY WEIGHT GAIN AS AFFECTED BY PROTEIN AND ENERGY INTAKE

Source	DF	SS	MS	F
Total	567	43,537,988		
Regression	(6)	18,801,614		
Protein linear, b_1, adj.	1	44,087	44,087	
Energy linear, b_2, adj.	1	190,459	190,459	4.31(P < .05)
Protein x Energy, a_{12}, adj.	1	20,206	20,206	
Protein Quadratic, a_{11}, adj.	1	3,433	3,433	
Energy Quadratic, a_{22}, adj.	1	11,621	11,621	
Residual	561	24,736,374	44,093	

It is surprising that protein intake does not remove a significant sum of squares. The work by Gleaves indicated that protein intake was probably as important as energy intake in affecting body weight gain. The parameter a_{12}, which is the coefficient to protein intake x energy intake, is not significant. This fact was not surprising. Apparently the relationship of dietary protein and dietary energy to body weight gain, which was observed by Gleaves, resulted from the action of these two dietary factors upon feed intake. Although the parameters in this equation other than for energy linear do not remove sums of squares greater than the residual, they will play an important part in a simultaneous analysis of body weight gain and egg production.

The parameters in the function for egg production are tested by the analysis of variance which is presented in Table LXV. The results of this analysis indicate that protein linear, energy linear, protein quadratic, and energy quadratic are all significant at the one percent level of probability. In this analysis of variance, as was the case for body weight gain, there is no interaction between protein intake and energy intake. However, the work by Gleaves showed that there was a highly significant interaction between dietary protein and dietary energy upon egg production. In the light of these observations, it can be seen that many wrong conclusions could be drawn from a study of the effect of dietary factors upon production characteristics of laying hens, without considering nutrient intake.

The analysis of variance for egg weight is presented in Table LXVI. The results of this analysis indicate that sums of squares are removed in the same pattern for both egg weight and egg production. Protein linear and protein quadratic are significant at the one percent level of probability. Energy quadratic is significant at the 5 percent level of probability, and energy linear approaches the 5 percent probability level. Here again, there is no interaction between protein intake and energy intake.

TABLE LXV

ANALYSIS OF VARIANCE OF EGG PRODUCTION AS AFFECTED BY PROTEIN AND ENERGY INTAKE

Source	DF	SS	MS	F
Total	567	10,648,299		
Regression	(6)	10,235,102		
Protein linear, b_1, adj.	1	42,551	42,551	57.81($P < .01$)
Energy linear, b_2, adj.	1	11,412	11,412	15.50($P < .01$)
Protein x Energy, a_{12}, adj.	1	677	677	.91
Protein quadratic, a_{11}, adj.	1	28,174	28,174	38.27($P < .01$)
Energy quadratic, a_{22}, adj.	1	6,862	6,862	9.32($P < .01$)
Residual	561	413,197	736	

TABLE LXVI

ANALYSIS OF VARIANCE OF AVERAGE EGG WEIGHT AS AFFECTED BY PROTEIN AND ENERGY INTAKE

Source	DF	SS	MS	F
Total	567	1,634,297		
Regression	(6)	1,630,263		
Protein linear, b_1, adj.	1	98	98	13.61($P < .01$)
Energy linear, b_2, adj.	1	25	25	3.47($P < .10$)
Protein x Energy, a_{12}, adj.	1	1	1	
Protein quadratic, a_{11}, adj.	1	53	53	7.36($P < .01$)
Energy quadratic, a_{22}, adj.	1	33	33	4.58($P < .05$)
Residual	561	4,034	7.2	

In order to evaluate further the relationship of protein and energy intake to body weight gain, egg production and average egg weight, the simultaneous responses of egg production and egg weight and the simultaneous responses of body weight gain and egg production will be studied. A necessary condition under which these studies can be made is for at least one of the quadratic response functions to be positive definite. An examination of the equation for egg production reveals that it is positive definite.

The procedure used for the egg production-egg weight study is outlined by Antle, 1962. The two functions for this analysis are

$$\hat{Y}_4 = -205.67 + 22.49x_1 + 10.15x_2 + .033x_1x_2 - .673x_1^2 - .156x_2^2$$

and

$$\hat{Y}_5 = 49.42 + 1.079x_1 - .47x_2 + .0009x_1x_2 - .029x_1^2 + .011x_2^2$$

These equations can be put into matrix form as follows:

$$\hat{Y}_4 = \hat{a}_4 - X'\hat{A}_4X + X'\hat{B}_4$$

where

$$\hat{a}_4 = -205.67$$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \hat{A}_4 = \begin{pmatrix} .673 & -.016 \\ -.016 & .156 \end{pmatrix}, \quad \hat{B}_4 = \begin{pmatrix} 22.49 \\ 10.15 \end{pmatrix}$$

and

$$\hat{Y}_5 = \hat{a}_5 - X'\hat{A}_5X + X'\hat{B}_5$$

where

$$\hat{a}_5 = 49.42$$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \hat{A}_5 = \begin{pmatrix} -.029 & .00045 \\ -.00045 & .011 \end{pmatrix}, \quad \hat{B}_5 = \begin{pmatrix} 1.079 \\ -.47 \end{pmatrix}.$$

Since A_4 is positive definite, the complete set of efficient points for $Y_4(X)$ and $Y_5(X)$ is given by

$$\left\{X \middle| X = .5\left[\alpha\hat{A}_4 + (1-\alpha)\hat{A}_5\right]^{-1}\left[\alpha\hat{B}_4 + (1-\alpha)\hat{B}_5\right];\ (0 \le \alpha \le 1)\right\} .$$

When the matrix values are substituted into this expression, and the necessary operations have been completed, the following set of parametric equations of the complete set of efficient points is obtained:

$$x_1 = \frac{1.87057\alpha^2 - .11017\alpha - .00603}{.10730\alpha^2 - .00219\alpha + .00031}$$

$$x_2 = \frac{3.58663\alpha^2 + .01535\alpha - .00659}{.10730\alpha^2 - .00219\alpha + .00031}$$

where

$$0 \le \alpha \le 1 .$$

The predicted responses at the efficient points obtained from these parametric equations are given in Figure 15. It can be shown that maximum egg size is obtained with the same combination of protein intake and energy intake that gives maximum egg production. This result was not unexpected.

When $\alpha = 1.0$, maximum egg production and maximum egg weight are predicted. The levels of protein intake and energy intake corresponding to $\alpha = 1.0$ are 16.64 grams of protein and 341 Calories of energy intake per hen per day. This particular combination of protein and energy intake was not obtained as a treatment average in the present experiment. However, the values for protein and energy intake for the predicted maximums correspond very closely to the nutrient intake standard set up for laying hens by Gleaves et al., 1963b.

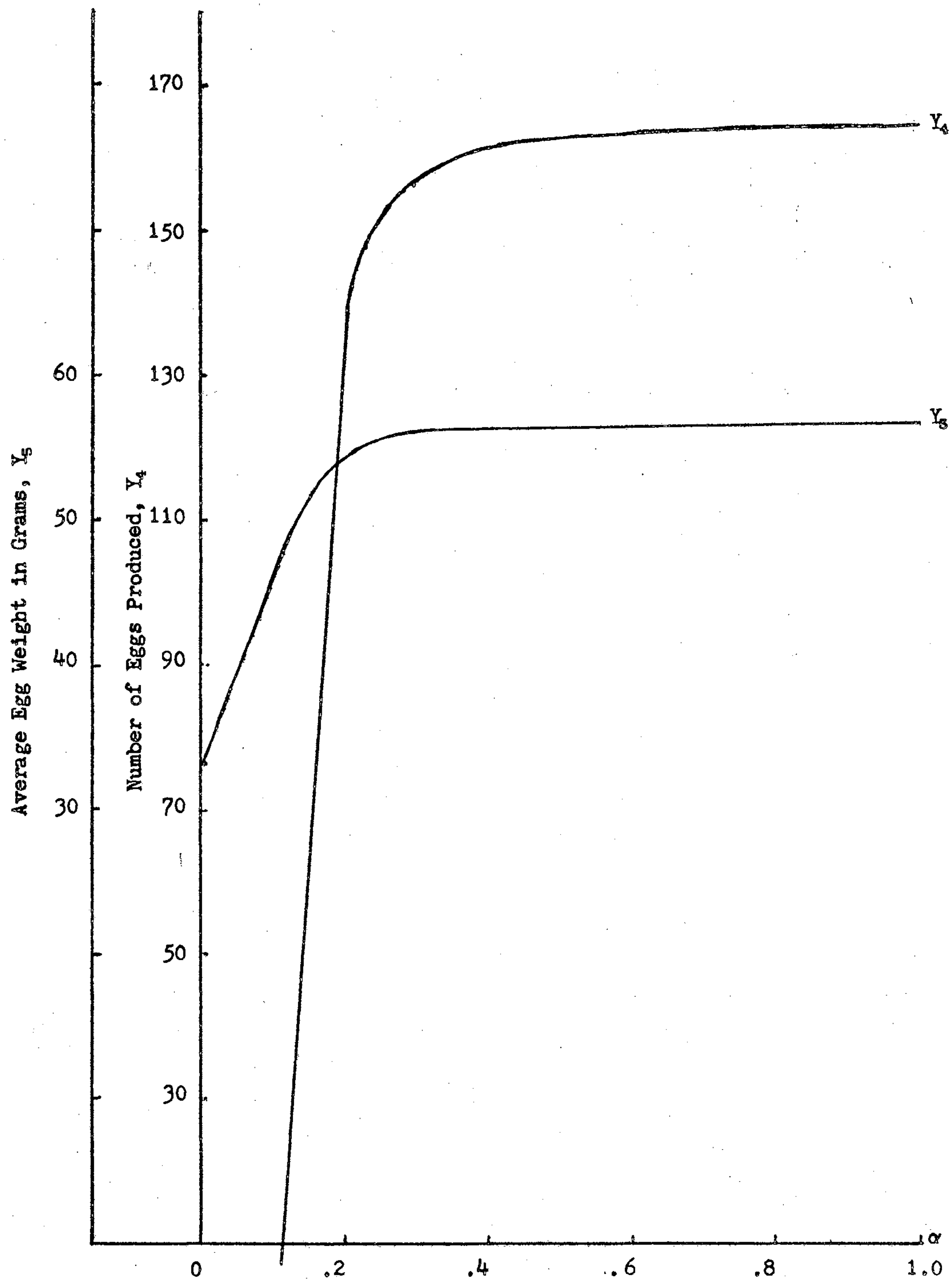

Figure 15. Predicted Responses at the Efficient Points

A simultaneous analysis of body weight gain and egg production, in response to protein intake and energy intake, was attempted using the method outlined by Antle, 1962. However, sufficient conditions to obtain a complete set of efficient points were not present. Although the condition that one of the two response functions be positive definite is a necessary condition, it is not a sufficient condition. The sufficient conditions are not exactly known, but there is a discussion on the subject in the paper by Antle, 1962. Since sufficient conditions are not present to use the method described by Antle for the simultaneous analysis, another method was employed. This method is not as exact as that of Antle's, but it offers an overall perspective of the response surface.

In Figure 16a the two responses were plotted on the same graph with x_1 and x_2 as the coordinates. The portion of this figure within the dotted lines is plotted in Figure 16b, to give a more detailed picture of the egg production surface. The quadratic response for body weight gain is indefinite, and the coefficients of the quadratic terms have opposite signs. Therefore, the surface formed by Y_3 is a hyperbolic paraboloid. When Y_3 is set equal to a constant, a hyperbola can be plotted from the function. Since the desirable response for body weight gain in laying hens is zero, only the contour which corresponds to zero body weight gain is plotted.

The surface formed by Y_4 is an elliptic paraboloid. This is evidenced by the fact that the quadratic response is positive definite. When Y_4 is set equal to a constant, the function will form an ellipse. The elliptic contours for 0, 50, 100, 150, 157 and the predicted maximum egg production are plotted in Figure 16b. The experimental period for

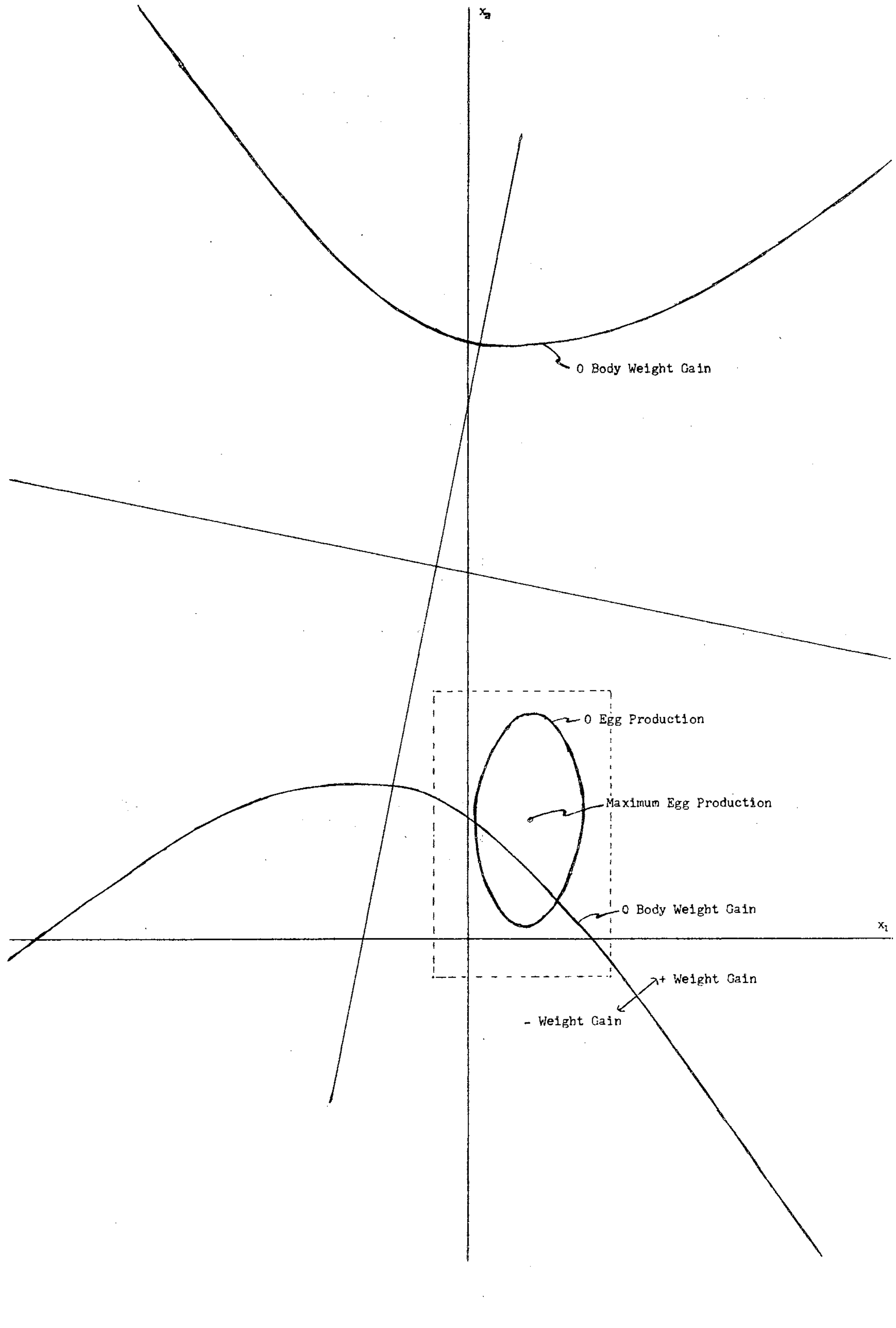

Figure 16a. The Response Surface of Egg Production in Relation to the Line of Zero Body Weight Gain

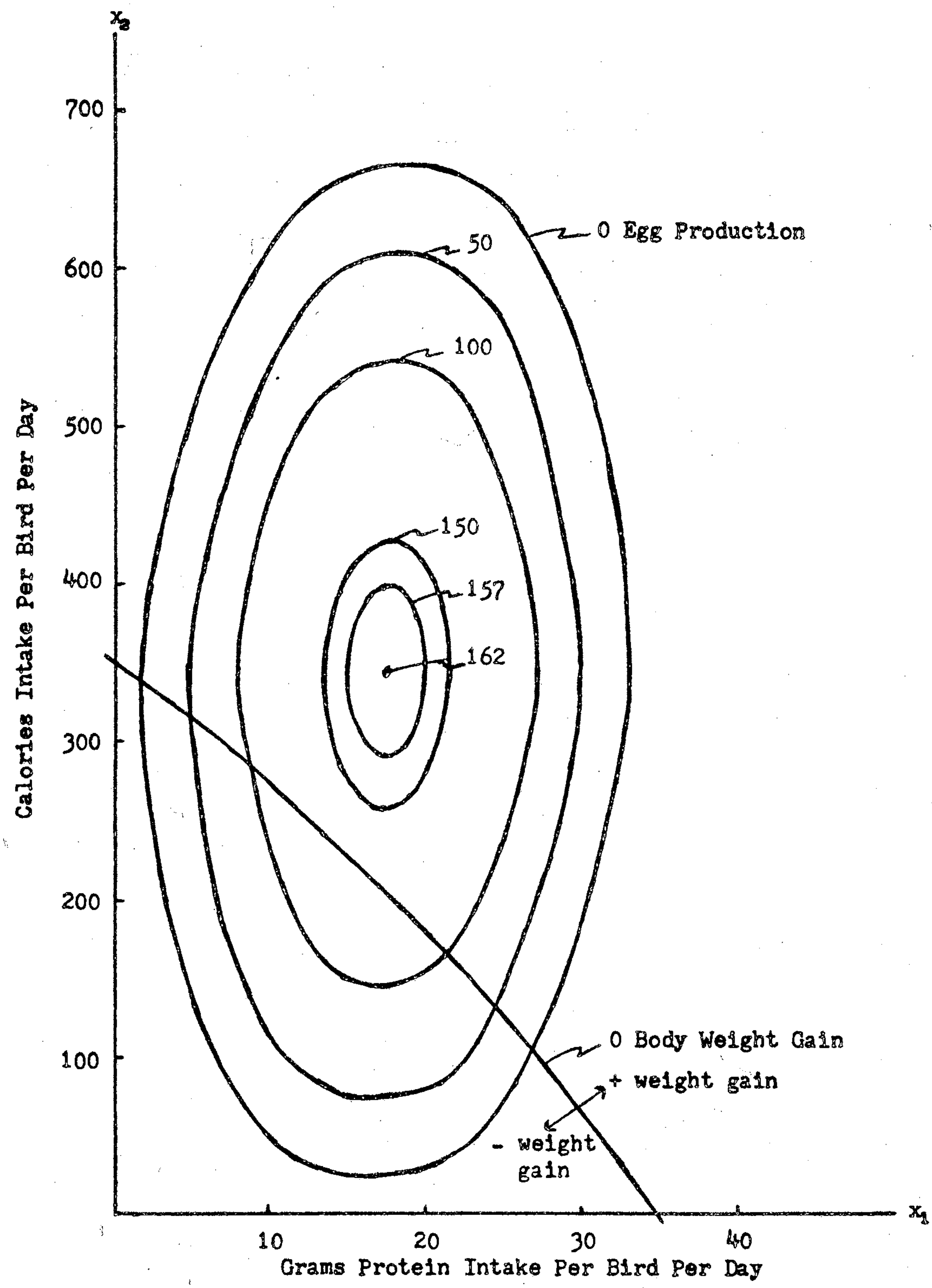

Figure 16b. The Response Surface of Egg Production in Relation to the Line of Zero Body Weight Gain

the number of eggs predicted is 196 days.

The elliptic contour for zero egg production is quite small compared to the hyperbolic paraboloid which represents body weight gain. For this reason, only the small section of the line for zero body weight gain which intersects the ellipse for egg production is shown.

There will be some body weight gain when the hens are receiving the protein and energy intake necessary for maximum egg production. The body weight gain which corresponds to the maximum predicted egg production (162 eggs in 196 days) is 415 grams. The daily protein intake and energy intake values which are calculated to give maximum egg production are 17.5 grams and 343 Calories, respectively. These values correspond closely to the nutrient intake standard developed by Gleaves _et al._, 1963b.

One of the most important observations that can be made from the data in Figure 16b is the very narrow range of daily protein intake and energy intake which the laying hens can have and still maintain a high level of egg production. In order for a hen to maintain 80 percent production (157 eggs in the graph), the daily protein intake must be between 15 and 20 grams and the daily intake of energy must be between 290 and 400 Calories. It is unlikely that a hen would consume an average as high as 400 Calories of intake per day in actual practice, because of the limitations that energy intake places upon energy consumption. Therefore, the range for energy is actually smaller than the graph shows. In practice, too much protein intake would probably never occur, but too little protein intake probably occurs frequently.

CHAPTER V

SUMMARY

Prediction equations were developed to estimate responses of laying hens to protein intake, energy intake, body weight change, egg production and average egg weight directly from the dietary factors of dietary protein, dietary energy, dietary weight and dietary volume. The data for making the estimates were obtained from an experiment with laying hens in which a 3^4 factorial arrangement of dietary protein, dietary energy, dietary weight and dietary volume constituted the treatments.

For each response, two general prediction equations were estimated. The first model for each response contained all effects which appeared to have some influence upon the response, as determined from an analysis of variance. The second model contained only the effects which were significant ($P < .05$) in the first model, as determined by the t test. The prediction ability of each equation was checked by the residual sum of squares and by a half-normal plot of the residual deviations.

Protein intake appeared to be more dependent upon dietary factors than energy intake. An equation which contained only the linear effects of dietary protein, dietary energy and dietary volume appeared to be just as capable of predicting protein intake as one which contained eleven interaction effects in addition to the linear effects.

The prediction of the intake of energy, based solely upon dietary effects, required several interaction effects in addition to the linear

effects of dietary protein and dietary energy. The results of the tests for goodness of fit to the general model show that dietary effects alone are not enough to predict energy intake satisfactorily. It was evident that non-dietary factors have a great influence upon energy intake.

The tests on the prediction equations for egg production and egg weight showed that these two responses could be predicted from the dietary factors with the same accuracy as protein intake. However, the prediction of body weight change appeared to be influenced by factors other than dietary factors.

The quadratic model was used as a response function for protein intake, energy intake, egg production, and egg weight as affected by dietary factors. The tests for goodness of fit for these functions showed that only egg weight could be predicted with confidence from the dietary factors by the quadratic model.

The quadratic model is the most convenient model to use if a multiple response analysis is desired. If the quadratic response is positive definite for one of the responses, then a necessary condition for the multiple response analysis is satisfied. In the case of the response variables which were fitted to the quadratic model involving only dietary factors, none of the functions was positive definite.

An attempt was made to determine why the quadratic model did not fit the data for protein intake and energy intake. This was done by including in the model the effect of egg production and body weight change. It was found that egg production has a definite influence upon protein intake and energy intake. Body weight change did not affect the parameters of the model with protein intake as the response; but with energy intake as the response, body weight change did have an effect.

The relationship of protein intake and energy intake to body weight change, egg production and egg weight was studied. It was found that energy intake is the primary factor affecting body weight change. Protein and energy intake appeared to be equal in their effect in influencing rate of egg production. Egg weight was affected by protein intake to a somewhat greater degree than by energy intake.

A simultaneous response analysis of egg production and egg weight showed that egg weight increases with each increase in egg production. The optimum daily protein intake and energy intake for maximum egg production and maximum egg weight are 16.64 grams and 341 Calories, respectively.

It was found by a simultaneous analysis of body weight change and egg production that, at maximum egg production, the hens would gain approximately 415 grams of body weight during a 7-month laying period. Protein intake and energy intake for maximum egg production were predicted to be 17.5 grams and 343 Calories per day, respectively.

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TABLE I

DATA FOR ENTIRE EXPERIMENT

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
1	-1	-1	-1	-1	11.64	234	+ 80	149	47.3
1	-1	-1	-1	-1	12.04	242	+ 80	169	52.2
1	-1	-1	-1	-1	10.07	203	+ 70	176	47.4
1	-1	-1	-1	-1	13.63	274	+ 240	155	53.0
1	-1	-1	-1	-1	11.62	234	+ 20	151	54.1
1	-1	-1	-1	-1	13.78	277	+ 60	182	54.7
1	-1	-1	-1	-1	11.60	233	- 40	135	52.8
2	-1	-1	0	-1	14.23	286	+ 380	173	55.1
2	-1	-1	0	-1	12.25	247	- 280	122	54.5
2	-1	-1	0	-1	11.39	229	+ 90	142	52.4
2	-1	-1	0	-1	12.09	243	- 200	153	53.4
2	-1	-1	0	-1	11.59	233	+ 120	150	50.6
2	-1	-1	0	-1	13.11	264	+ 380	159	53.7
2	-1	-1	0	-1	13.51	272	+ 100	173	50.5
3	-1	-1	1	-1	13.57	273	+ 290	162	55.8
3	-1	-1	1	-1	12.19	245	+ 60	161	50.3
3	-1	-1	1	-1	10.91	220	- 420	57	49.4
3	-1	-1	1	-1	12.33	248	+ 100	146	56.0
3	-1	-1	1	-1	12.38	249	+ 200	137	54.0
3	-1	-1	1	-1	14.07	283	+ 160	160	56.8
3	-1	-1	1	-1	13.09	264	- 20	172	51.9

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
4	-1	-1	-1	0	11.45	230	+ 80	154	52.0
4	-1	-1	-1	0	13.02	262	+ 160	146	55.0
4	-1	-1	-1	0	14.09	284	- 40	149	59.5
4	-1	-1	-1	0	12.49	251	+ 230	157	52.0
4	-1	-1	-1	0	15.49	312	+ 220	167	60.5
4	-1	-1	-1	0	12.64	254	+ 170	148	55.5
4	-1	-1	-1	0	10.98	221	+ 200	152	47.3
5	-1	-1	0	0	9.94	200	+ 20	100	50.8
5	-1	-1	0	0	11.09	223	- 140	144	50.0
5	-1	-1	0	0	13.89	280	+ 80	153	53.2
5	-1	-1	0	0	12.34	248	- 20	147	51.5
5	-1	-1	0	0	7.04	142	- 400	53	52.9
5	-1	-1	0	0	13.02	262	- 10	166	50.6
5	-1	-1	0	0	8.36	168	+ 230	77	54.9
6	-1	-1	1	0	11.86	239	+ 100	157	54.3
6	-1	-1	1	0	11.39	229	- 710	75	58.5
6	-1	-1	1	0	12.73	256	+ 220	162	54.6
6	-1	-1	1	0	12.69	256		155	51.7
6	-1	-1	1	0	13.15	265	+ 140	159	54.4
6	-1	-1	1	0	11.53	232	- 60	144	50.7
6	-1	-1	1	0	12.90	260	+ 100	150	54.2

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
7	-1	-1	-1	1	12.59	253	+ 240	143	55.2
7	-1	-1	-1	1	13.24	266	+ 150	148	55.1
7	-1	-1	-1	1	11.74	236	+ 120	157	48.5
7	-1	-1	-1	1	11.35	228	+ 30	141	55.4
7	-1	-1	-1	1	11.24	226	- 120	86	49.7
7	-1	-1	-1	1	12.04	242	+ 83		50.1
7	-1	-1	-1	1	11.72	236	+ 40	145	53.2
8	-1	-1	0	1	12.35	249	+ 160	169	52.2
8	-1	-1	0	1	12.47	251	- 90	125	57.6
8	-1	-1	0	1	12.58	253	- 30	94	54.1
8	-1	-1	0	1	11.70	235	- 30	149	55.2
8	-1	-1	0	1	11.68	235	- 110	144	53.0
8	-1	-1	0	1	11.35	228	- 100	139	56.2
8	-1	-1	0	1	12.92	260	+ 20	167	52.4
9	-1	-1	1	1	9.50	191	- 100	96	50.7
9	-1	-1	1	1	11.64	234	- 80	159	50.6
9	-1	-1	1	1	12.26	247	+ 190	155	52.6
9	-1	-1	1	1	12.26	247	- 220	161	56.2
9	-1	-1	1	1	12.26	247	+ 50	138	57.9
9	-1	-1	1	1	11.36	229		116	60.3
9	-1	-1	1	1	12.29	247	- 60	148	58.0

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
10	-1	0	-1	-1	10.66	245	- 40	144	49.8
10	-1	0	-1	-1	7.34	169	- 910	74	50.2
10	-1	0	-1	-1	12.24	281	- 30	69	50.1
10	-1	0	-1	-1	15.17	349	+ 280	155	50.5
10	-1	0	-1	-1	10.84	249	+ 40	127	52.7
10	-1	0	-1	-1	5.99	138	- 640	19	52.1
10	-1	0	-1	-1	11.90	274	+ 90	148	55.4
11	-1	0	0	-1	10.83	249	+ 23		48.9
11	-1	0	0	-1	6.98	161	- 110	56	50.0
11	-1	0	0	-1	11.62	267	+ 20	130	58.7
11	-1	0	0	-1	11.05	254	+ 20	133	53.2
11	-1	0	0	-1	11.52	265	+ 100	145	54.9
11	-1	0	0	-1	14.69	338	- 40	24	48.8
11	-1	0	0	-1	12.41	285	+ 130	141	55.4
12	-1	0	1	-1	11.25	259	+ 80	133	51.7
12	-1	0	1	-1	9.65	222	+ 60	84	54.4
12	-1	0	1	-1	10.61	244	+ 300	105	52.4
12	-1	0	1	-1	12.16	280	+ 50	143	54.6
12	-1	0	1	-1	14.00	322	+ 90	165	52.5
12	-1	0	1	-1	12.51	288	+ 320	147	54.6
12	-1	0	1	-1	9.03	208	- 320	36	51.2

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
13	-1	0	-1	0	11.61	267	- 120	152	54.3
13	-1	0	-1	0	10.38	239	+ 80	133	51.6
13	-1	0	-1	0	10.40	239	+ 60	164	48.3
13	-1	0	-1	0	10.21	235	- 270	98	52.4
13	-1	0	-1	0	7.93	182	- 510	43	47.6
13	-1	0	-1	0	11.33	261	+ 80	154	52.5
13	-1	0	-1	0	10.37	239	- 90	145	52.8
14	-1	0	0	0	9.09	209	- 730	70	52.9
14	-1	0	0	0	9.05	208	- 100	136	52.1
14	-1	0	0	0	11.65	268	+ 140	141	53.1
14	-1	0	0	0	8.17	188	+ 30	41	48.8
14	-1	0	0	0	9.90	228	- 180	127	55.2
14	-1	0	0	0	11.78	271	- 40	143	52.7
14	-1	0	0	0	10.63	244	- 100	144	50.7
15	-1	0	1	0	8.99	207	- 140	65	55.7
15	-1	0	1	0	12.21	281	+ 180	154	55.1
15	-1	0	1	0	12.09	278		71	51.1
15	-1	0	1	0	9.66	222	+ 60	120	52.4
15	-1	0	1	0	12.43	286	+ 160	154	54.2
15	-1	0	1	0	11.51	265	+ 160	126	51.9
15	-1	0	1	0	10.42	240		145	54.1

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
16	-1	0	-1	1	9.45	217	+ 50	99	56.6
16	-1	0	-1	1	10.15	233	+ 50	111	52.5
16	-1	0	-1	1	8.63	198	- 460	100	51.6
16	-1	0	-1	1	10.40	239	+1000	115	55.9
16	-1	0	-1	1	10.39	239	+ 60	100	58.0
16	-1	0	-1	1	11.56	266		152	52.3
16	-1	0	-1	1	11.36	261	+ 40	109	53.4
17	-1	0	0	1	10.08	232	- 140	128	51.3
17	-1	0	0	1	10.91	251	- 100	140	52.4
17	-1	0	0	1	10.07	232	- 30	124	53.5
17	-1	0	0	1	10.18	234	- 180	120	50.5
17	-1	0	0	1	11.11	256	- 130	141	54.4
17	-1	0	0	1	8.44	194	- 400	45	53.1
17	-1	0	0	1	9.57	220	- 10	89	52.8
18	-1	0	1	1	9.47	218	- 10	116	53.1
18	-1	0	1	1	10.25	236	+ 40	145	49.6
18	-1	0	1	1	10.11	233	- 120	134	54.8
18	-1	0	1	1	7.25	167	- 560	11	55.8
18	-1	0	1	1	8.61	198	- 160	67	53.7
18	-1	0	1	1	9.15	211	+ 170	115	52.0
18	-1	0	1	1	10.06	231	- 180	123	54.1

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
19	-1	1	-1	-1	7.34	193	- 200	70	49.7
19	-1	1	-1	-1	10.57	277	- 140	137	53.7
19	-1	1	-1	-1	10.12	266	- 70	125	54.6
19	-1	1	-1	-1	9.15	240	- 80	153	48.0
19	-1	1	-1	-1	4.53	119	- 640	12	52.2
19	-1	1	-1	-1	8.56	225	+ 140	76	55.5
19	-1	1	-1	-1	7.28	191	- 210	82	54.6
20	-1	1	0	-1	7.48	196	- 890	39	54.2
20	-1	1	0	-1	7.72	203	- 470	31	51.5
20	-1	1	0	-1	12.08	317	+ 350	150	52.5
20	-1	1	0	-1	7.95	209	- 440	52	52.3
20	-1	1	0	-1	7.21	189	- 280	72	45.4
20	-1	1	0	-1	8.02	210	- 360	45	52.6
20	-1	1	0	-1	6.01	158	- 400	25	47.8
21	-1	1	1	-1	7.29	192	- 30	79	50.2
21	-1	1	1	-1	5.71	150	- 70	32	53.3
21	-1	1	1	-1	7.77	204	- 340	40	51.6
21	-1	1	1	-1	10.04	264		127	53.4
21	-1	1	1	-1	8.73	229	- 500	63	53.3
21	-1	1	1	-1	7.21	189	- 150	26	48.4
21	-1	1	1	-1	5.80	152	- 160	33	49.8

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
22	-1	1	-1	0	8.10	213	- 230	43	54.9
22	-1	1	-1	0	6.31	166	- 960	50	50.0
22	-1	1	-1	0	6.89	181	- 160	31	50.5
22	-1	1	-1	0	6.35	167	- 390	28	47.9
22	-1	1	-1	0	6.95	183	- 820	76	50.8
22	-1	1	-1	0	10.53	276	- 120	139	50.6
22	-1	1	-1	0	7.42	195	- 450	57	45.7
23	-1	1	0	0	8.18	215	- 570	50	48.9
23	-1	1	0	0	7.66	201	- 370	75	49.7
23	-1	1	0	0	10.35	272	+ 20	125	54.8
23	-1	1	0	0	8.57	225	- 173		49.9
23	-1	1	0	0	9.16	241	- 80	120	50.8
23	-1	1	0	0	8.89	233	+ 100	121	55.0
23	-1	1	0	0	7.19	189	- 140	78	46.2
24	-1	1	1	0	7.34	193	- 300	57	50.0
24	-1	1	1	0	6.04	159	- 460	81	48.4
24	-1	1	1	0	7.72	203	+ 100	83	49.9
24	-1	1	1	0	6.35	167	- 320	47	46.7
24	-1	1	1	0	6.27	165	- 410	40	50.7
24	-1	1	1	0	11.24	295	+ 440	119	52.2
24	-1	1	1	0	9.08	239	- 40	133	50.0

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
25	-1	1	-1	1	10.63	279	- 290	56	47.6
25	-1	1	-1	1	8.00	210	- 146		45.0
25	-1	1	-1	1	5.58	147	- 340	20	50.8
25	-1	1	-1	1	5.22	137	- 80	19	51.3
25	-1	1	-1	1	9.69	255	- 240	93	56.5
25	-1	1	-1	1	7.12	187	+ 80	65	54.1
25	-1	1	-1	1	7.95	209	+ 180	63	57.7
26	-1	1	0	1	6.16	162	- 470	47	50.5
26	-1	1	0	1	8.62	226	- 110	106	54.0
26	-1	1	0	1	7.74	203	+ 50	81	49.3
26	-1	1	0	1	6.84	180	- 130	59	52.7
26	-1	1	0	1	6.77	178	- 200	7	45.6
26	-1	1	0	1	10.72	282	- 80	138	53.7
26	-1	1	0	1	5.95	156	- 400	15	45.6
27	-1	1	1	1	8.85	233		122	50.1
27	-1	1	1	1	7.51	197	- 160	96	49.9
27	-1	1	1	1	5.70	150	- 100	25	45.9
27	-1	1	1	1	6.71	176	- 400	54	50.8
27	-1	1	1	1	5.59	147	- 480	30	57.1
27	-1	1	1	1	7.54	198	- 20	92	52.4
27	-1	1	1	1	7.50	197	- 290	66	47.2

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
28	0	-1	-1	-1	14.23	233	+ 100	162	55.8
28	0	-1	-1	-1	13.28	217	- 540	89	54.1
28	0	-1	-1	-1	14.65	240	+ 60	156	53.0
28	0	-1	-1	-1	17.37	284	+ 600	150	53.6
28	0	-1	-1	-1	16.63	272	+ 240	176	53.6
28	0	-1	-1	-1	19.75	323	+ 210	168	53.9
28	0	-1	-1	-1	15.55	255	+ 110	164	54.7
29	0	-1	0	-1	15.92	261	+ 200	144	55.2
29	0	-1	0	-1	16.54	271	+ 660	153	53.7
29	0	-1	0	-1	14.41	236	+ 100	134	56.7
29	0	-1	0	-1	15.77	258	+ 120	160	57.7
29	0	-1	0	-1	14.03	230	- 20	169	51.7
29	0	-1	0	-1	15.02	246	+ 240	147	53.2
29	0	-1	0	-1	15.93	261	+ 180	157	59.5
30	0	-1	1	-1	17.14	281	+ 240	166	54.5
30	0	-1	1	-1	14.84	243	+ 10	148	57.8
30	0	-1	1	-1	17.86	292	+ 410	152	56.7
30	0	-1	1	-1	15.13	248	+ 150	150	55.0
30	0	-1	1	-1	17.62	289	+ 160	169	58.7
30	0	-1	1	-1	16.93	277	+ 240	147	54.1
30	0	-1	1	-1	16.41	269	+ 480	154	56.6

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
31	0	-1	-1	0	14.92	244	- 440	98	53.6
31	0	-1	-1	0	16.72	274	+ 260	152	54.4
31	0	-1	-1	0	15.83	259	+ 200	157	57.8
31	0	-1	-1	0	15.92	261	- 340	167	57.6
31	0	-1	-1	0	14.95	245	+ 160	159	51.0
31	0	-1	-1	0	17.08	280	+ 200	175	57.4
31	0	-1	-1	0	11.58	190	+ 140	85	51.6
32	0	-1	0	0	15.39	252	+ 390	131	55.0
32	0	-1	0	0	11.48	188	- 680	60	55.9
32	0	-1	0	0	14.70	241	+ 60	156	52.6
32	0	-1	0	0	14.83	243	+ 140	162	56.3
32	0	-1	0	0	13.91	228	+ 100	162	49.8
32	0	-1	0	0	17.34	284	+ 120	176	52.9
32	0	-1	0	0	15.79	259	+ 80	164	56.4
33	0	-1	1	0	16.88	276	+ 410	152	57.0
33	0	-1	1	0	15.36	252	- 10	164	55.2
33	0	-1	1	0	13.20	216	+ 80	149	54.1
33	0	-1	1	0	16.26	266	+ 500	164	59.5
33	0	-1	1	0	14.11	231	+ 440	107	51.0
33	0	-1	1	0	16.82	275	+ 140	145	58.0
33	0	-1	1	0	14.54	238	+ 100	161	50.0

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
34	0	-1	-1	1	15.01	246	+ 90	161	55.4
34	0	-1	-1	1	15.48	253	+ 150	169	56.1
34	0	-1	-1	1	15.83	259	+ 100	178	53.4
34	0	-1	-1	1	14.55	238	+ 80	37	53.8
34	0	-1	-1	1	15.74	258	+ 140	165	53.2
34	0	-1	-1	1	15.06	247	- 80	153	58.4
34	0	-1	-1	1	16.33	267	+ 220	165	55.8
35	0	-1	0	1	14.87	243	+ 20	170	51.3
35	0	-1	0	1	16.45	269	- 90	171	54.8
35	0	-1	0	1	13.89	227	- 240	144	57.1
35	0	-1	0	1	15.94	261	+ 140	146	51.7
35	0	-1	0	1	10.35	169	-1039	60	60.3
35	0	-1	0	1	16.11	264	+ 440	165	51.0
35	0	-1	0	1	16.97	278	+ 80	170	54.1
36	0	-1	1	1	13.09	214	+ 80	132	52.4
36	0	-1	1	1	11.02	181	+ 90	75	52.3
36	0	-1	1	1	15.24	250	+ 240	132	57.8
36	0	-1	1	1	14.00	229	- 50	130	56.1
36	0	-1	1	1	15.99	262	- 20	90	49.5
36	0	-1	1	1	18.14	297	+ 300	173	56.0
36	0	-1	1	1	16.22	266	+ 250	156	52.4

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
37	0	0	-1	-1	15.79	298	+ 150	161	50.7
37	0	0	-1	-1	16.66	314	+ 370	157	55.1
37	0	0	-1	-1	12.97	245	+ 140	153	51.9
37	0	0	-1	-1	18.11	342	+ 660	175	53.5
37	0	0	-1	-1	12.40	234	- 480	51	55.2
37	0	0	-1	-1	13.10	247	- 230	134	55.9
37	0	0	-1	-1	13.83	261	+ 400	154	49.8
38	0	0	0	-1	15.30	289	+ 480	121	56.8
38	0	0	0	-1	16.53	312	+ 280	163	60.0
38	0	0	0	-1	14.16	267	+ 100	117	57.4
38	0	0	0	-1	14.20	268	+ 450	122	55.0
38	0	0	0	-1	13.26	250	+ 20	125	55.9
38	0	0	0	-1	13.57	256	+ 510	116	55.7
38	0	0	0	-1	12.87	243	+ 150	161	53.1
39	0	0	1	-1	13.58	256	+ 80	157	50.3
39	0	0	1	-1	12.58	237	+ 40	135	58.3
39	0	0	1	-1	12.36	233	- 360	155	50.3
39	0	0	1	-1	13.48	254	+ 40	146	56.1
39	0	0	1	-1	12.85	242	+ 50	159	51.0
39	0	0	1	-1	15.11	285	- 80	129	57.2
39	0	0	1	-1	8.08	152	- 340	21	52.0

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
40	0	0	-1	0	12.29	232		157	53.5
40	0	0	-1	0	12.36	233	- 40	138	54.0
40	0	0	-1	0	12.21	230	- 20	110	54.9
40	0	0	-1	0	13.97	263	+ 60	154	53.6
40	0	0	-1	0	11.85	224	+ 100	155	52.1
40	0	0	-1	0	13.14	248	+ 200	157	49.7
40	0	0	-1	0	13.42	253	+ 80	153	53.4
41	0	0	0	0	11.86	224	- 10	143	52.0
41	0	0	0	0	13.85	261	+ 40	166	51.2
41	0	0	0	0	14.24	269	+ 130	176	52.5
41	0	0	0	0	13.96	263	+ 60	166	53.1
41	0	0	0	0	14.93	282	+ 160	163	55.9
41	0	0	0	0	14.27	269	+ 20	76	49.3
41	0	0	0	0	13.50	254	+ 300	142	57.4
42	0	0	1	0	10.91	206	- 10	151	53.4
42	0	0	1	0	14.04	265	+ 160	166	53.5
42	0	0	1	0	14.28	269	+ 360	153	54.8
42	0	0	1	0	11.88	224	+ 20	134	52.2
42	0	0	1	0	12.70	240	+ 180	150	54.3
42	0	0	1	0	13.12	248	+ 560	116	55.7
42	0	0	1	0	15.64	295	+ 170	136	53.7

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
43	0	0	-1	1	13.62	257	- 120	150	58.2
43	0	0	-1	1	12.84	242	- 120	156	53.0
43	0	0	-1	1	12.79	241	- 20	149	53.1
43	0	0	-1	1	12.90	243	+ 40	162	52.4
43	0	0	-1	1	13.92	263	+ 60	168	53.4
43	0	0	-1	1	10.57	199	+ 240	79	60.5
43	0	0	-1	1	13.07	247	- 80	168	50.8
44	0	0	0	1	12.57	237	+ 80	156	55.3
44	0	0	0	1	13.26	250		148	52.9
44	0	0	0	1	14.00	264	+ 160	145	53.3
44	0	0	0	1	14.26	269		166	53.4
44	0	0	0	1	12.91	244	- 250	140	55.7
44	0	0	0	1	13.85	261	+ 240	149	54.1
44	0	0	0	1	12.01	226	- 100	147	53.2
45	0	0	1	1	10.10	191	- 760	61	50.9
45	0	0	1	1	9.92	187	- 330	73	55.1
45	0	0	1	1	14.73	278	+ 190	156	56.7
45	0	0	1	1	14.73	278	+ 40	155	60.6
45	0	0	1	1	13.09	247	- 50	146	59.1
45	0	0	1	1	14.02	265	+ 180	147	56.9
45	0	0	1	1	13.09	247	+ 120	162	54.1

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
46	0	1	-1	-1	10.26	219	+ 20	119	54.8
46	0	1	-1	-1	6.23	133	- 230	11	50.0
46	0	1	-1	-1	11.93	255	- 260	83	58.7
46	0	1	-1	-1	12.37	264	+ 300	165	56.9
46	0	1	-1	-1	9.63	206	- 250	61	53.9
46	0	1	-1	-1	10.21	218	- 390	63	54.5
46	0	1	-1	-1	10.43	223	- 680	98	53.2
47	0	1	0	-1	10.71	229	- 540	130	50.1
47	0	1	0	-1	11.81	252	+ 10	150	53.5
47	0	1	0	-1	11.67	249	+ 260	156	53.2
47	0	1	0	-1	10.86	232		146	52.6
47	0	1	0	-1	8.18	175	+ 120	77	51.3
47	0	1	0	-1	10.59	226	- 540	109	56.1
47	0	1	0	-1	7.60	162	- 280	78	52.6
48	0	1	1	-1	10.75	229	- 140	125	53.5
48	0	1	1	-1	11.22	240	+ 100	120	57.8
48	0	1	1	-1	15.12	323	+ 460	158	58.2
48	0	1	1	-1	10.26	219	+ 60	131	52.9
48	0	1	1	-1	8.57	183	- 280	92	48.3
48	0	1	1	-1	13.42	287	+ 30	137	59.1
48	0	1	1	-1	11.77	251	- 210	138	55.1

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
49	0	1	−1	0	13.95	298		147	56.6
49	0	1	−1	0	12.80	273	− 130	73	50.9
49	0	1	−1	0	13.14	281	− 130	150	56.1
49	0	1	−1	0	10.28	219	+ 490	78	53.6
49	0	1	−1	0	10.12	216	− 670	61	51.5
49	0	1	−1	0	8.01	171	− 740	73	52.1
49	0	1	−1	0	13.19	282	− 20	157	55.3
50	0	1	0	0	11.84	253	− 70	141	54.9
50	0	1	0	0	7.29	156	− 140	47	50.1
50	0	1	0	0	11.01	235	+ 160	136	49.4
50	0	1	0	0	11.84	253	+ 80	111	55.9
50	0	1	0	0	12.96	277	− 140	159	54.5
50	0	1	0	0	11.59	247	− 40	145	51.6
50	0	1	0	0	11.05	236	− 60	152	50.5
51	0	1	1	0	13.73	293	+ 250	162	50.7
51	0	1	1	0	10.26	219	− 170	123	50.4
51	0	1	1	0	12.09	258	− 40	161	53.0
51	0	1	1	0	12.52	267	− 20	151	53.3
51	0	1	1	0	12.97	277	− 50	148	56.1
51	0	1	1	0	13.48	288	+ 120	174	53.8
51	0	1	1	0	12.70	271	+ 220	134	53.3

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
52	0	1	-1	1	10.35	221	+ 110	150	47.3
52	0	1	-1	1	10.86	232	- 70	127	53.2
52	0	1	-1	1	11.60	248	- 40	151	52.2
52	0	1	-1	1	9.72	208	- 40	118	49.4
52	0	1	-1	1	11.15	238	+ 100	145	55.8
52	0	1	-1	1	11.37	243	- 200	146	49.0
52	0	1	-1	1	11.84	253	- 80	157	49.4
53	0	1	0	1	9.40	201	- 150	56	53.6
53	0	1	0	1	11.07	236	- 40	118	55.9
53	0	1	0	1	11.68	249	+ 100	152	55.3
53	0	1	0	1	7.08	151	- 430	46	51.2
53	0	1	0	1	12.51	267	+ 170	163	49.8
53	0	1	0	1	7.87	168	- 450	83	49.2
53	0	1	0	1	9.06	193	- 470	75	50.7
54	0	1	1	1	10.91	233	+ 30	140	48.9
54	0	1	1	1	11.88	254	+ 200	154	49.5
54	0	1	1	1	12.57	268	+ 90	145	55.4
54	0	1	1	1	12.26	262	+ 180	158	53.2
54	0	1	1	1	11.97	256	- 180	153	55.6
54	0	1	1	1	12.79	273	+ 210	142	52.0
54	0	1	1	1	13.19	282	+ 100	136	52.9

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
55	1	-1	-1	-1	17.78	247	+ 30	169	52.4
55	1	-1	-1	-1	19.31	268	+ 400	155	54.6
55	1	-1	-1	-1	19.18	266	+ 290	170	51.8
55	1	-1	-1	-1	19.08	265	+ 400	158	53.4
55	1	-1	-1	-1	21.52	298	+ 430	157	57.3
55	1	-1	-1	-1	18.32	254	+ 20	156	52.9
55	1	-1	-1	-1	20.35	282	+ 570	155	55.8
56	1	-1	0	-1	16.94	235	- 250	42	51.6
56	1	-1	0	-1	18.02	250	+ 370	163	48.2
56	1	-1	0	-1	18.38	255	+ 240	164	52.2
56	1	-1	0	-1	16.42	228	+ 90	155	53.4
56	1	-1	0	-1	18.24	253	- 90	175	57.9
56	1	-1	0	-1	16.92	235	- 50	124	57.9
56	1	-1	0	-1	16.66	231	+ 20	153	56.6
57	1	-1	1	-1	16.56	230	+ 130	132	56.9
57	1	-1	1	-1	19.75	274	+ 220	160	56.8
57	1	-1	1	-1	19.08	264	+ 110	165	55.3
57	1	-1	1	-1	19.54	271	+ 40	178	55.5
57	1	-1	1	-1	15.75	218	- 60	140	56.0
57	1	-1	1	-1	14.54	202	- 240	134	55.0
57	1	-1	1	-1	15.34	213	+ 310	77	55.3

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
58	1	-1	-1	0	19.24	267	+ 180	155	52.6
58	1	-1	-1	0	18.06	250	+ 140	116	58.7
58	1	-1	-1	0	18.49	256	+ 30	159	56.8
58	1	-1	-1	0	17.82	247		135	59.0
58	1	-1	-1	0	19.36	268	+ 300	163	55.2
58	1	-1	-1	0	16.03	222	- 140	140	53.7
58	1	-1	-1	0	19.08	265	+ 260	175	51.7
59	1	-1	0	0	19.26	267	+ 280	168	54.0
59	1	-1	0	0	18.28	253	+ 200	176	51.9
59	1	-1	0	0	18.00	249	+ 20	154	53.3
59	1	-1	0	0	18.70	259	+ 80	173	53.5
59	1	-1	0	0	20.51	284	+ 320	154	57.4
59	1	-1	0	0	17.83	247	+ 300	160	56.8
59	1	-1	0	0	19.24	267	+ 280	159	57.4
60	1	-1	1	0	15.52	215	+ 140	134	52.0
60	1	-1	1	0	18.33	254	- 50	159	56.1
60	1	-1	1	0	17.07	237	- 20	75	49.4
60	1	-1	1	0	17.76	246	+ 40	177	52.0
60	1	-1	1	0	18.55	257	+ 460	166	50.4
60	1	-1	1	0	17.52	243	+ 60	160	53.3
60	1	-1	1	0	19.61	272	+ 220	157	60.0

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
61	1	-1	-1	1	18.26	253	+ 400	154	53.9
61	1	-1	-1	1	16.68	231	- 80	135	56.6
61	1	-1	-1	1	18.17	252	+ 190	145	56.9
61	1	-1	-1	1	19.02	264	+ 60	164	56.5
61	1	-1	-1	1	19.06	264	+ 120	160	58.3
61	1	-1	-1	1	18.43	255	+ 270	158	52.5
61	1	-1	-1	1	18.34	254	+ 280	146	56.6
62	1	-1	0	1	17.75	246	+ 270	172	50.1
62	1	-1	0	1	15.62	217	- 40	149	53.1
62	1	-1	0	1	17.74	246	- 280	147	56.9
62	1	-1	0	1	19.24	267	+ 240	169	54.4
62	1	-1	0	1	17.08	237	+ 30	169	53.9
62	1	-1	0	1	16.33	226	+ 180	140	54.1
62	1	-1	0	1	18.35	254		174	48.8
63	1	-1	1	1	18.53	257	+ 60	173	55.0
63	1	-1	1	1	17.42	241	+ 80	164	50.3
63	1	-1	1	1	17.69	245	+ 20	132	59.6
63	1	-1	1	1	15.89	220	+ 120	161	50.4
63	1	-1	1	1	17.90	248	+ 140	165	58.9
63	1	-1	1	1	21.24	294	+ 350	169	56.3
63	1	-1	1	1	17.33	240	- 240	174	52.8

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
64	1	0	-1	-1	16.28	260	+ 420	153	54.8
64	1	0	-1	-1	16.54	264	+ 340	175	54.4
64	1	0	-1	-1	16.35	261	- 80	169	50.9
64	1	0	-1	-1	16.83	269	+ 330	158	57.3
64	1	0	-1	-1	16.46	263	+ 280	144	54.9
64	1	0	-1	-1	14.04	224	+ 190	51	49.3
64	1	0	-1	-1	15.96	255	+ 360	115	56.9
65	1	0	0	-1	16.25	259	+ 310	164	52.0
65	1	0	0	-1	17.66	282	+ 300	161	58.3
65	1	0	0	-1	16.63	266	+ 350	171	56.8
65	1	0	0	-1	16.96	271	+ 460	167	56.3
65	1	0	0	-1	18.43	294	+ 320	177	48.8
65	1	0	0	-1	18.68	298	+ 300	159	56.8
65	1	0	0	-1	17.49	279	+ 110	168	56.6
66	1	0	1	-1	15.96	255	+ 560	162	55.7
66	1	0	1	-1	12.03	192	- 270	97	51.3
66	1	0	1	-1	18.24	291	- 500	159	54.3
66	1	0	1	-1	15.83	253	+ 100	162	54.0
66	1	0	1	-1	16.50	263	+ 280	160	52.4
66	1	0	1	-1	19.05	304	+ 600	160	57.9
66	1	0	1	-1	15.56	248	+ 140	150	52.6

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
67	1	0	−1	0	14.50	232	+ 90	151	53.6
67	1	0	−1	0	15.97	255	+ 80	178	53.9
67	1	0	−1	0	14.50	232	+ 60	136	52.1
67	1	0	−1	0	19.66	314	+ 400	157	60.2
67	1	0	−1	0	17.33	277	+ 580	158	53.2
67	1	0	−1	0	14.83	237		140	52.9
67	1	0	−1	0	16.56	264	+ 720	82	51.7
68	1	0	0	0	14.97	239	+ 60	170	51.8
68	1	0	0	0	16.02	256	+ 280	161	52.3
68	1	0	0	0	13.28	212	− 30	110	55.4
68	1	0	0	0	14.37	229	+ 40	152	56.8
68	1	0	0	0	17.93	286	− 40	104	53.8
68	1	0	0	0	16.78	268	+ 40	160	56.0
68	1	0	0	0	19.25	307	+ 470	170	55.3
69	1	0	1	0	15.17	242	+ 290	170	48.7
69	1	0	1	0	16.55	264	+ 310	162	56.8
69	1	0	1	0	15.07	241	+ 160	164	55.1
69	1	0	1	0	16.89	270	+ 380	174	54.6
69	1	0	1	0	18.31	292	+ 280	150	49.4
69	1	0	1	0	16.78	268	+ 590	168	58.7
69	1	0	1	0	16.24	259	+ 260	181	52.2

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
70	1	0	-1	1	14.66	234	- 140	134	58.0
70	1	0	-1	1	16.93	270	+ 280	156	54.2
70	1	0	-1	1	17.00	272	+ 198		54.0
70	1	0	-1	1	17.35	277	+ 220	172	54.7
70	1	0	-1	1	16.55	264	+ 200	161	55.0
70	1	0	-1	1	18.67	298	+ 240	121	57.5
70	1	0	-1	1	17.85	285	+ 390	157	49.8
71	1	0	0	1	15.45	247	+ 240	141	46.6
71	1	0	0	1	15.66	250	+ 90	172	52.8
71	1	0	0	1	15.21	243	+ 200	160	54.7
71	1	0	0	1	16.83	269	+ 70	174	49.8
71	1	0	0	1	16.50	263	- 40	26	53.2
71	1	0	0	1	12.65	202	+ 790	36	55.0
71	1	0	0	1	15.57	249	+ 20	148	56.3
72	1	0	1	1	15.53	248	- 120	132	58.1
72	1	0	1	1	17.31	276	+ 510	105	56.3
72	1	0	1	1	16.06	256	+ 180	146	54.8
72	1	0	1	1	14.33	229	+ 80	158	50.7
72	1	0	1	1	15.70	251	+ 60	132	50.6
72	1	0	1	1	14.53	232	- 60	148	56.9
72	1	0	1	1	17.77	284	+ 260	166	54.2

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
73	1	1	-1	-1	16.14	292	+ 340	171	53.4
73	1	1	-1	-1	14.26	258	+ 120	169	51.0
73	1	1	-1	-1	13.55	245		137	58.1
73	1	1	-1	-1	15.99	289	+ 420	148	54.3
73	1	1	-1	-1	15.76	285	+ 240	161	55.1
73	1	1	-1	-1	15.88	287	+ 240	167	55.7
73	1	1	-1	-1	14.99	271	+ 280	168	52.9
74	1	1	0	-1	14.63	264	+ 140	169	53.3
74	1	1	0	-1	14.29	258	+ 760	138	50.9
74	1	1	0	-1	14.46	261	+ 140	178	52.3
74	1	1	0	-1	13.74	248	+ 440	157	52.5
74	1	1	0	-1	13.15	238	- 550	119	59.4
74	1	1	0	-1	14.30	258	+ 480	150	53.2
74	1	1	0	-1	14.93	270	+ 360	158	58.0
75	1	1	1	-1	14.99	271	+ 150	169	54.5
75	1	1	1	-1	16.15	292	+ 520	173	53.3
75	1	1	1	-1	16.87	305	+ 260	164	58.5
75	1	1	1	-1	12.95	234	- 240	92	52.7
75	1	1	1	-1	16.48	298	+ 400	143	57.8
75	1	1	1	-1	17.54	317	+ 410	167	57.7
75	1	1	1	-1	14.96	270	+ 460	155	51.8

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
76	1	1	-1	0	14.07	254	+ 260	121	56.7
76	1	1	-1	0	11.43	207	- 10	82	58.0
76	1	1	-1	0	14.62	264	+ 120	150	52.7
76	1	1	-1	0	12.23	221	- 120	142	56.3
76	1	1	-1	0	12.45	225	- 470	87	47.0
76	1	1	-1	0	13.45	243	- 80	67	49.5
76	1	1	-1	0	14.84	268	+ 390	171	54.0
77	1	1	0	0	12.16	220	+ 310	130	48.7
77	1	1	0	0	15.34	277	- 160	94	57.8
77	1	1	0	0	15.60	282	+ 320	168	56.0
77	1	1	0	0	12.75	230	+ 60	155	51.9
77	1	1	0	0	14.58	264	+ 360	168	56.2
77	1	1	0	0	14.64	265	+ 220	132	53.2
77	1	1	0	0	13.80	249	- 40	175	54.4
78	1	1	1	0	15.91	287	+ 60	170	55.8
78	1	1	1	0	15.54	281	+ 470	152	56.5
78	1	1	1	0	17.48	316	+ 500	160	56.1
78	1	1	1	0	16.03	290	+ 130	149	54.5
78	1	1	1	0	17.26	312	+ 560	153	60.2
78	1	1	1	0	12.75	230	+ 40	139	55.2
78	1	1	1	0	12.89	233	+ 40	145	54.6

TABLE I (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2	Y_3	Y_4	Y_5
79	1	1	-1	1	11.88	215	- 790	71	52.4
79	1	1	-1	1	14.42	261	+ 190	148	56.1
79	1	1	-1	1	13.88	251	+ 240	150	53.7
79	1	1	-1	1	14.56	263	+ 40	155	54.0
79	1	1	-1	1	16.80	304	+ 10	156	54.2
79	1	1	-1	1	14.68	265	+ 180	151	57.5
79	1	1	-1	1	14.51	262	+ 170	154	52.7
80	1	1	0	1	12.83	232	- 190	142	54.0
80	1	1	0	1	15.70	284	+ 120	169	54.5
80	1	1	0	1	12.55	227	+ 290	85	54.0
80	1	1	0	1	15.40	278	+ 20	162	55.7
80	1	1	0	1	17.46	315	+ 620	146	58.4
80	1	1	0	1	14.73	266	+ 140	165	55.0
80	1	1	0	1	15.23	275	+ 250	162	56.9
81	1	1	1	1	13.13	237	- 150	135	60.6
81	1	1	1	1	10.26	185	- 30	72	54.4
81	1	1	1	1	13.09	237	+ 240	177	49.1
81	1	1	1	1	15.08	272	+ 300	158	56.7
81	1	1	1	1	14.87	269	+ 180	166	59.4
81	1	1	1	1	12.59	227	- 60	148	56.5
81	1	1	1	1	14.45	261	- 60	156	51.8

TABLE II

DATA FOR LOW EGG PRODUCING HENS

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
1	-1	-1	-1	-1	12.62	254
1	-1	-1	-1	-1	10.67	215
2	-1	-1	0	-1	12.51	252
2	-1	-1	0	-1	11.70	235
3	-1	-1	1	-1	12.54	253
3	-1	-1	1	-1	11.91	240
4	-1	-1	-1	0	13.35	269
4	-1	-1	-1	0	11.72	236
5	-1	-1	0	0	9.28	187
5	-1	-1	0	0	10.12	204
6	-1	-1	1	0	13.43	270
6	-1	-1	1	0	11.72	236
7	-1	-1	-1	1	11.55	233
7	-1	-1	-1	1	11.63	234
8	-1	-1	0	1	11.78	237
8	-1	-1	0	1	11.49	231
9	-1	-1	1	1	13.29	268
9	-1	-1	1	1	11.44	230

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
10	-1	0	-1	-1	7.70	177
10	-1	0	-1	-1	9.39	216
11	-1	0	0	-1	11.49	264
11	-1	0	0	-1	10.15	233
12	-1	0	1	-1	9.12	210
12	-1	0	1	-1	9.41	216
13	-1	0	-1	0	9.13	210
13	-1	0	-1	0	10.40	239
14	-1	0	0	0	7.84	180
14	-1	0	0	0	8.42	194
15	-1	0	1	0	8.27	190
15	-1	0	1	0	11.50	265
16	-1	0	-1	1	8.77	202
16	-1	0	-1	1	9.28	213
17	-1	0	0	1	8.94	206
17	-1	0	0	1	9.30	214
18	-1	0	1	1	9.49	218
18	-1	0	1	1	9.50	219

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
19	-1	1	-1	-1	6.97	183
19	-1	1	-1	-1	8.16	214
20	-1	1	0	-1	9.45	248
20	-1	1	0	-1	8.06	212
21	-1	1	1	-1	6.76	178
21	-1	1	1	-1	7.62	200
22	-1	1	-1	0	6.51	171
22	-1	1	-1	0	8.55	224
23	-1	1	0	0	7.87	207
23	-1	1	0	0	6.19	163
24	-1	1	1	0	6.84	180
24	-1	1	1	0	5.39	141
25	-1	1	-1	1	5.61	147
25	-1	1	-1	1	9.86	259
26	-1	1	0	1	6.82	179
26	-1	1	0	1	5.78	152
27	-1	1	1	1	5.67	149
27	-1	1	1	1	6.37	167

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
28	0	-1	-1	-1	16.99	278
28	0	-1	-1	-1	21.82	357
29	0	-1	0	-1	16.23	266
29	0	-1	0	-1	10.64	174
30	0	-1	1	-1	16.71	274
30	0	-1	1	-1	14.79	242
31	0	-1	-1	0	16.22	266
31	0	-1	-1	0	12.72	208
32	0	-1	0	0	15.44	253
32	0	-1	0	0	13.15	215
33	0	-1	1	0	18.78	307
33	0	-1	1	0	13.21	216
34	0	-1	-1	1	14.55	238
34	0	-1	-1	1	16.67	273
35	0	-1	0	1	14.91	244
35	0	-1	0	1	17.53	287
36	0	-1	1	1	12.24	200
36	0	-1	1	1	15.61	256

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
37	0	0	-1	-1	14.64	276
37	0	0	-1	-1	10.68	202
38	0	0	0	-1	15.79	298
38	0	0	0	-1	12.36	233
39	0	0	1	-1	14.23	268
39	0	0	1	-1	15.07	284
40	0	0	-1	0	12.42	234
40	0	0	-1	0	9.85	186
41	0	0	0	0	13.57	256
41	0	0	0	0	14.18	267
42	0	0	1	0	12.76	241
42	0	0	1	0	14.29	270
43	0	0	-1	1	11.61	219
43	0	0	-1	1	9.69	183
44	0	0	0	1	12.92	244
44	0	0	0	1	11.69	220
45	0	0	1	1	11.01	208
45	0	0	1	1	12.53	236

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
46	0	1	-1	-1	10.84	232
46	0	1	-1	-1	7.82	167
47	0	1	0	-1	9.97	213
47	0	1	0	-1	7.65	163
48	0	1	1	-1	11.13	238
48	0	1	1	-1	8.98	192
49	0	1	-1	0	12.20	260
49	0	1	-1	0	9.08	194
50	0	1	0	0	9.17	196
50	0	1	0	0	11.98	256
51	0	1	1	0	7.94	170
51	0	1	1	0	11.05	236
52	0	1	-1	1	11.16	238
52	0	1	-1	1	9.33	199
53	0	1	0	1	9.47	202
53	0	1	0	1	7.79	166
54	0	1	1	1	10.85	232
54	0	1	1	1	13.23	283

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
55	1	-1	-1	-1	19.77	274
55	1	-1	-1	-1	21.00	291
56	1	-1	0	-1	15.50	215
56	1	-1	0	-1	17.44	242
57	1	-1	1	-1	10.83	150
57	1	-1	1	-1	15.17	210
58	1	-1	-1	0	18.73	260
58	1	-1	-1	0	17.07	237
59	1	-1	0	0	18.11	251
59	1	-1	0	0	20.61	286
60	1	-1	1	0	15.01	208
60	1	-1	1	0	18.61	258
61	1	-1	-1	1	19.02	264
61	1	-1	-1	1	18.51	257
62	1	-1	0	1	18.90	262
62	1	-1	0	1	17.09	237
63	1	-1	1	1	21.61	300
63	1	-1	1	1	16.25	225

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
64	1	0	-1	-1	18.54	296
64	1	0	-1	-1	14.24	227
65	1	0	0	-1	15.50	248
65	1	0	0	-1	18.11	289
66	1	0	1	-1	12.79	204
66	1	0	1	-1	15.84	253
67	1	0	-1	0	15.07	241
67	1	0	-1	0	14.48	231
68	1	0	0	0	14.46	231
68	1	0	0	0	16.35	261
69	1	0	1	0	16.67	266
69	1	0	1	0	17.18	274
70	1	0	-1	1	14.48	231
70	1	0	-1	1	18.51	296
71	1	0	0	1	16.05	256
71	1	0	0	1	15.60	249
72	1	0	1	1	17.50	279
72	1	0	1	1	14.73	235

TABLE II (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
73	1	1	-1	-1	12.72	230
73	1	1	-1	-1	15.57	282
74	1	1	0	-1	13.23	239
74	1	1	0	-1	13.52	244
75	1	1	1	-1	16.62	300
75	1	1	1	-1	14.55	263
76	1	1	-1	0	13.94	252
76	1	1	-1	0	13.97	253
77	1	1	0	0	12.09	218
77	1	1	0	0	13.13	237
78	1	1	1	0	12.05	218
78	1	1	1	0	10.97	198
79	1	1	-1	1	15.21	275
79	1	1	-1	1	13.16	238
80	1	1	0	1	12.63	228
80	1	1	0	1	18.75	339
81	1	1	1	1	12.91	233
81	1	1	1	1	11.54	209

TABLE III

DATA FOR HIGH EGG PRODUCING HENS

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
1	-1	-1	-1	-1	12.58	253
1	-1	-1	-1	-1	11.63	234
2	-1	-1	0	-1	13.48	271
2	-1	-1	0	-1	13.70	276
3	-1	-1	1	-1	13.83	278
3	-1	-1	1	-1	13.83	278
4	-1	-1	-1	0	12.36	249
4	-1	-1	-1	0	15.17	305
5	-1	-1	0	0	13.35	269
5	-1	-1	0	0	12.56	253
6	-1	-1	1	0	11.91	240
6	-1	-1	1	0	12.70	256
7	-1	-1	-1	1	11.98	241
7	-1	-1	-1	1	11.65	234
8	-1	-1	0	1	11.92	240
8	-1	-1	0	1	12.33	248
9	-1	-1	1	1	12.29	247
9	-1	-1	1	1	13.88	279

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
10	-1	0	-1	-1	16.43	378
10	-1	0	-1	-1	13.33	307
11	-1	0	0	-1	11.51	265
11	-1	0	0	-1	13.72	316
12	-1	0	1	-1	14.37	331
12	-1	0	1	-1	14.29	329
13	-1	0	-1	0	12.73	293
13	-1	0	-1	0	9.92	228
14	-1	0	0	0	13.62	313
14	-1	0	0	0	12.12	279
15	-1	0	1	0	12.73	293
15	-1	0	1	0	13.43	309
16	-1	0	-1	1	11.59	267
16	-1	0	-1	1	12.03	277
17	-1	0	0	1	10.52	242
17	-1	0	0	1	11.82	272
18	-1	0	1	1	11.64	268
18	-1	0	1	1	10.17	234

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
19	-1	1	-1	-1	11.08	291
19	-1	1	-1	-1	7.57	199
20	-1	1	0	-1	12.90	339
20	-1	1	0	-1	8.72	229
21	-1	1	1	-1	10.55	277
21	-1	1	1	-1	12.21	321
22	-1	1	-1	0	8.34	219
22	-1	1	-1	0	9.98	262
23	-1	1	0	0	11.82	310
23	-1	1	0	0	10.29	270
24	-1	1	1	0	9.74	256
24	-1	1	1	0	9.95	261
25	-1	1	-1	1	9.45	248
25	-1	1	-1	1	9.95	261
26	-1	1	0	1	8.98	236
26	-1	1	0	1	10.98	288
27	-1	1	1	1	10.23	269
27	-1	1	1	1	10.14	266

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
28	0	-1	-1	-1	15.25	250
28	0	-1	-1	-1	15.65	256
29	0	-1	0	-1	18.68	306
29	0	-1	0	-1	17.78	291
30	0	-1	1	-1	17.43	285
30	0	-1	1	-1	15.81	259
31	0	-1	-1	0	16.83	276
31	0	-1	-1	0	17.30	283
32	0	-1	0	0	13.70	224
32	0	-1	0	0	16.67	273
33	0	-1	1	0	16.40	269
33	0	-1	1	0	16.38	268
34	0	-1	-1	1	15.47	253
34	0	-1	-1	1	15.88	260
35	0	-1	0	1	14.91	244
35	0	-1	0	1	15.48	253
36	0	-1	1	1	15.65	256
36	0	-1	1	1	18.00	295

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
37	0	0	−1	−1	10.91	206
37	0	0	−1	−1	17.01	321
38	0	0	0	−1	15.71	296
38	0	0	0	−1	12.36	233
39	0	0	1	−1	13.62	257
39	0	0	1	−1	13.08	247
40	0	0	−1	0	12.02	227
40	0	0	−1	0	15.68	296
41	0	0	0	0	13.26	250
41	0	0	0	0	13.93	263
42	0	0	1	0	12.12	229
42	0	0	1	0	14.07	265
43	0	0	−1	1	13.21	249
43	0	0	−1	1	12.49	236
44	0	0	0	1	14.77	278
44	0	0	0	1	15.46	292
45	0	0	1	1	15.32	289
45	0	0	1	1	14.31	270

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
46	0	1	-1	-1	12.58	269
46	0	1	-1	-1	12.60	269
47	0	1	0	-1	12.25	262
47	0	1	0	-1	12.55	268
48	0	1	1	-1	14.99	320
48	0	1	1	-1	11.34	242
49	0	1	-1	0	12.85	274
49	0	1	-1	0	14.07	300
50	0	1	0	0	13.03	278
50	0	1	0	0	10.89	233
51	0	1	1	0	12.92	276
51	0	1	1	0	11.32	242
52	0	1	-1	1	11.75	251
52	0	1	-1	1	11.02	235
53	0	1	0	1	11.25	240
53	0	1	0	1	13.11	280
54	0	1	1	1	12.12	259
54	0	1	1	1	13.58	290

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
55	1	-1	-1	-1	15.57	216
55	1	-1	-1	-1	19.69	273
56	1	-1	0	-1	17.88	248
56	1	-1	0	-1	21.80	302
57	1	-1	1	-1	19.40	269
57	1	-1	1	-1	19.65	272
58	1	-1	-1	0	18.46	256
58	1	-1	-1	0	20.33	282
59	1	-1	0	0	19.76	274
59	1	-1	0	0	17.59	244
60	1	-1	1	0	18.19	252
60	1	-1	1	0	18.75	260
61	1	-1	-1	1	18.22	253
61	1	-1	-1	1	18.51	257
62	1	-1	0	1	18.01	250
62	1	-1	0	1	17.26	239
63	1	-1	1	1	19.14	265
63	1	-1	1	1	17.96	249

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
64	1	0	-1	-1	16.06	256
64	1	0	-1	-1	17.12	273
65	1	0	0	-1	17.81	284
65	1	0	0	-1	16.84	269
66	1	0	1	-1	16.49	263
66	1	0	1	-1	18.73	299
67	1	0	-1	0	14.35	229
67	1	0	-1	0	16.40	262
68	1	0	0	0	15.85	253
68	1	0	0	0	17.44	278
69	1	0	1	0	14.32	229
69	1	0	1	0	16.21	259
70	1	0	-1	1	16.83	269
70	1	0	-1	1	17.42	278
71	1	0	0	1	15.75	252
71	1	0	0	1	15.93	254
72	1	0	1	1	15.12	241
72	1	0	1	1	17.62	281

TABLE III (CONTINUED)

Treat. No.	x_1	x_2	x_3	x_4	Y_1	Y_2
73	1	1	-1	-1	16.72	302
73	1	1	-1	-1	14.11	255
74	1	1	0	-1	15.98	289
74	1	1	0	-1	14.61	264
75	1	1	1	-1	13.85	250
75	1	1	1	-1	13.88	251
76	1	1	-1	0	15.23	275
76	1	1	-1	0	14.93	270
77	1	1	0	0	14.76	267
77	1	1	0	0	13.25	239
78	1	1	1	0	14.45	261
78	1	1	1	0	17.25	312
79	1	1	-1	1	16.52	298
79	1	1	-1	1	15.40	278
80	1	1	0	1	15.98	289
80	1	1	0	1	15.85	286
81	1	1	1	1	10.64	192
81	1	1	1	1	13.53	244

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