BEAM CONSTANTS BY HIGH SPEED COMPUTER

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PREFACE

The analysis of continuous structures of variable moment of inertia by moment distribution or slope deflection equations is a laborious process. The carry-over moment procedure (1) has certain advantages over these two methods in that no solutions of simultaneous equations are required and no distribution of moments is necessary. Analysis by this method, however, requires the evaluation of certain constants that are not readily available in existing tables.

In this thesis mathematical expressions and a computer program are developed to evaluate constants for beams with parabolic haunches.

The author wishes to express his indebtedness to Professor J. J. Tuma for his valuable guidance and assistance in the preparation of this thesis and for acting as the writer's major adviser. Acknowledgment is also due Professor William Granet for making the facilities of the Oklahoma State University Computing Center available; the staff of the School of Civil Engineering for their aid and instruction during the writer's years of graduate study.

The author wishes to thank Mr. J. D. Cobb, Mr. C. O. Heller, and Mrs. Willia Bernardi for their assistance in preparing the original manuscript.

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NOMENCLATURE

a, a_1 . . . . Length Coefficients

b . . . . Width of Beam

f . . . . Angular Flexibility Coefficient

g . . . . Angular Carry-Over Coefficient

h_0 . . . . Minimum Depth of Beam

h_h . . . . Depth of Haunch

h_x, h_z . . . . Depth of Beam at x and z, Respectively

n . . . . Moving Load Position Coefficient

p . . . . Maximum Intensity of Haunch Load

q . . . . Specific Weight of Material

t_{AB}, t_{BA} . . . . Angular Function Coefficients

t_x, t_z . . . . Ratio of $\frac{x}{L\alpha}$ and $\frac{z}{L\beta}$, Respectively

w . . . . Intensity of Uniform Load

x, x', z, z' . . . . Coordinates of the Cross-Section

A,B,C,D,E . . . . Letters Designating Cross-Sections of Beam

(DL). . . . . Due to Dead Load

E . . . . Modulus of Elasticity

F_{AB}, F_{BA} . . . . Angular Flexibilities

G, G_{AB}, G_{BA} . . . . Angular Carry-Over Values

(HL). . . . . Due to Haunch Load

I_0. . . . . Minimum Moment of Inertia

I_x, I_z . . . . Moments of Inertia at x and z, Respectively

L . . . . Length of Span

(LL) . . . . Due to Live Load

P . . . . Intensity of Moving Load

Q_k, R_k . . . . Integral Functions

T_z . . . . Parabolic Function of Haunch

(UL) . . . . Due to Uniformly Distributed Load

X_k . . . . Sum of Integral Functions For Beam

X_{kn} . . . . Sum of Integral Functions to n

$X_{kq}, X'_{kq}, X''_{kq}$ . . . . Integral Functions for Parabolic Haunch

$X_{kr}, X'_{kr}, X''_{kr}$ . . . . Integral Functions For Constant Depth

X'_{kn}, X'''_{kn} . . . . Integral Functions to n for Constant Depth

X''_{kn} . . . . Integral Functions to n for Parabolic Haunch

α, γ . . . . Coefficients of Length of Constant Depth

β . . . . Coefficient of Length of Haunch

ω . . . . Coefficient of Depth of Haunch

μ . . . . $\frac{n-a}{\alpha}$ or $\frac{n-a}{\gamma}$

μ_1 . . . . $\frac{n-a_1}{\beta}$

τ_{AB}, τ_{BA} . . . . Angular Load Functions

CHAPTER I

DEFINITION OF BEAM CONSTANTS

1. General.

The analysis of continuous beams by carry-over moments requires the evaluation of certain constants which are defined as either angular functions or moment functions of a simple beam. An isolated span of a continuous beam of variable cross-section is considered. The location of the cross-section is given by the ordinates x and x' (Fig. 1-1).

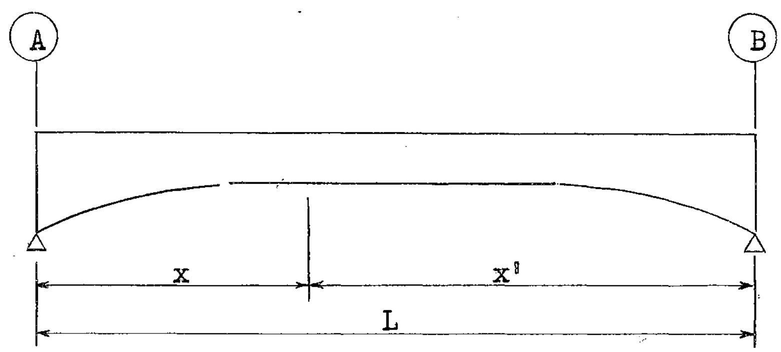


Fig. 1-1.—Isolated Span of Continuous Beam.

2. Angular Functions.

(a) Angular Flexibilities. The angular flexibility is the end slope of a simple beam due to a unit couple applied at that end. For the given beam (Fig. 1-2)

$$F_{BA} = \frac{1}{L^2}\int_0^L \frac{x^2 dx}{EI_x} \qquad (1\text{-}1a)$$

$$F_{AB} = \frac{1}{L^2}\int_0^L \frac{x'^2 dx}{EI_x} \quad . \qquad (1\text{-}1b)$$

(b) Carry-Over Values. The carry-over value is the end slope of a simple beam due to a unit couple applied at the far end (Fig. 1-2).

$$G = G_{AB} = G_{BA} = \frac{1}{L^2}\int_0^L \frac{xx' dx}{EI_x} \quad . \qquad (1\text{-}2)$$

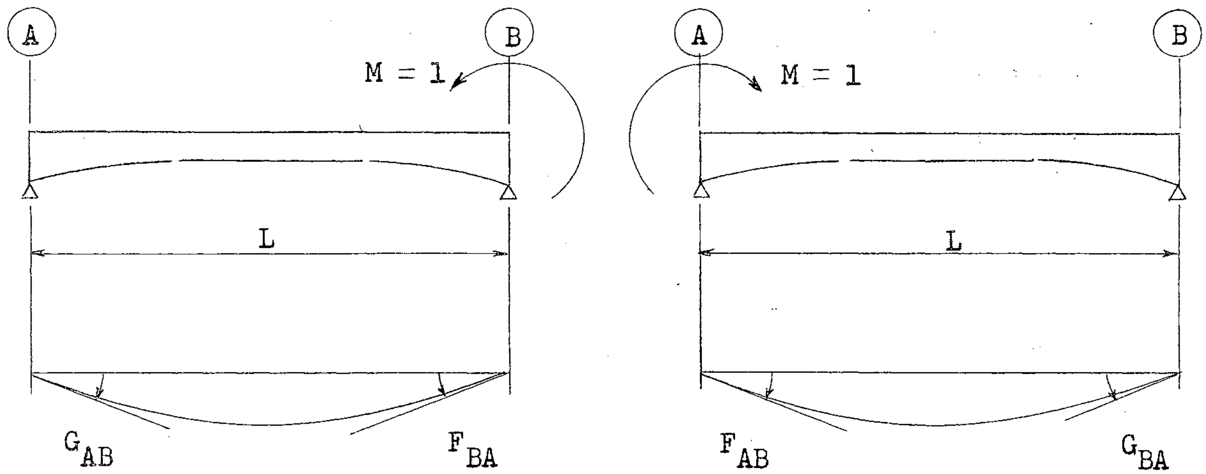


Fig. 1-2.—Angular Beam Functions.

Substituting the identity

$$x' = L - x$$

and rearranging Eqs. (1-1b, 2) gives

$$G = \frac{1}{L}\int_0^L \frac{xdx}{EI_x} - F_{BA} \qquad (1\text{-}3)$$

$$F_{AB} = \int_0^L \frac{dx}{EI_x} - 2G - F_{BA} \quad . \qquad (1\text{-}4)$$

(c) Angular Load Functions. The angular load functions are the end slopes of a simple beam due to applied transverse loads.

Live Load. If a unit load is applied to the beam (Fig. 1-3), the angular load functions are

$$\tau_{BA}^{(LL)} = nLG + \frac{1}{L}\int_0^{Ln} \frac{x^2dx}{EI_x} - n\int_0^{Ln} \frac{xdx}{EI_x} \tag{1-5}$$

$$\tau_{AB}^{(LL)} = nL\left[G + F_{AB} - \int_0^{Ln} \frac{dx}{EI_x}\right] + \int_0^{Ln} \frac{xdx}{EI_x} - \tau_{BA}^{(LL)} . \tag{1-6}$$

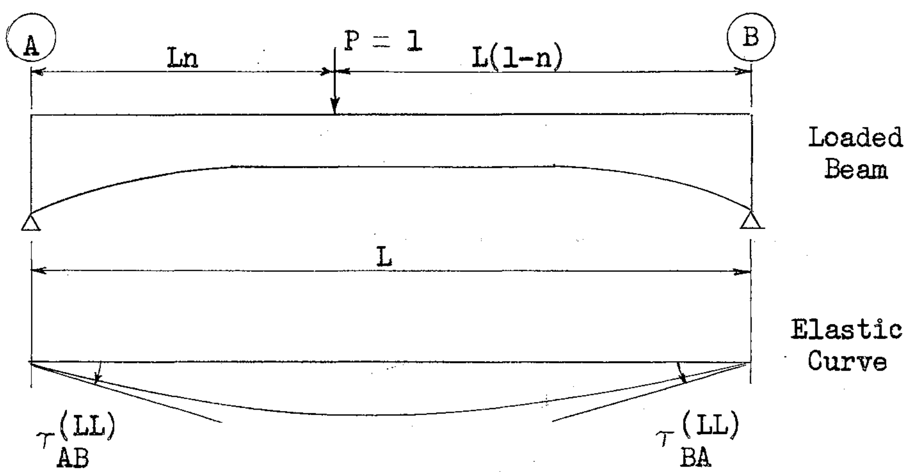


Fig. 1-3.—Angular Live Load Functions.

Uniform Load. If a uniform load of intensity w is applied to the beam (Fig. 1-4), the angular load functions are

$$\tau_{BA}^{(UL)} = \frac{w}{2}\int_0^{L} \frac{x^2dx}{EI_x} - \frac{w}{2L}\int_0^{L} \frac{x^3dx}{EI_x} \tag{1-7a}$$

$$\tau_{AB}^{(UL)} = \frac{wL}{2}\int_0^{L} \frac{xdx}{EI_x} - \frac{w}{2}\int_0^{L} \frac{x^2dx}{EI_x} - \tau_{BA}^{(UL)} . \tag{1-7b}$$

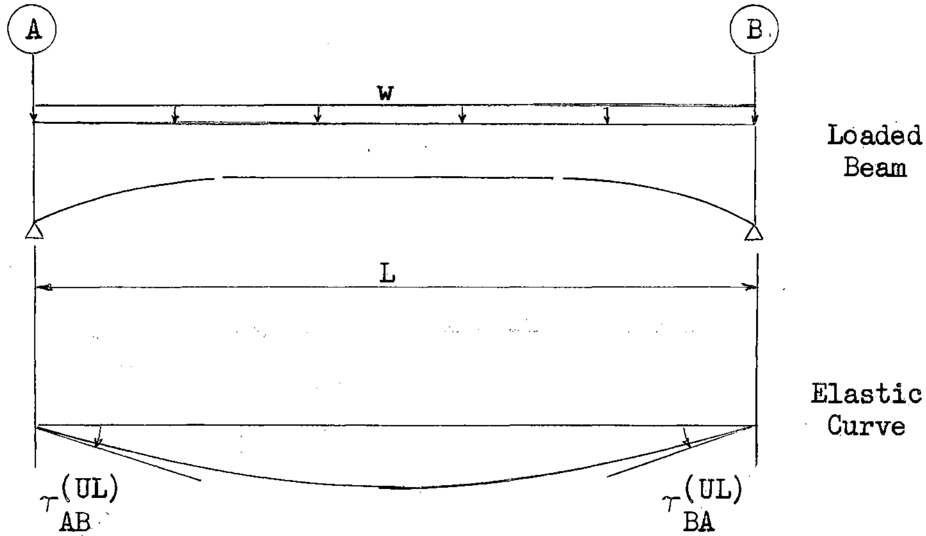


Fig. 1-4.—Angular Uniform Load Functions.

In terms of Eqs. (1-1a,2) Eqs. (1-7a,7b) become

$$\tau_{BA}^{(UL)} = \frac{wL^2}{2} F_{BA} - \frac{w}{2L}\int_0^L \frac{x^3 dx}{EI_x} \tag{1-8}$$

$$\tau_{AB}^{(UL)} = \frac{wL^2}{2} G - \tau_{BA}^{(UL)} \quad . \tag{1-9}$$

<u>Haunch Loads</u>. The angular load functions due to the dead load of the right haunch (Fig. 1-5) are

$$\tau_{BA}^{(HL)} = \frac{p\beta^2}{12}\int_0^L \frac{x^2 dx}{EI_x} - \frac{p}{12\beta^2 L^3}\int_{L(1-\beta)}^L \frac{[x-L(1-\beta)]^4 x dx}{EI_x} \tag{1-10a}$$

$$\tau_{AB}^{(HL)} = \frac{p\beta^2 L}{12}\int_0^L \frac{x dx}{EI_x} - \frac{p}{12(\beta L)^2}\int_{L(1-\beta)}^L \frac{[x-L(1-\beta)]^4 x dx}{EI_x} - \tau_{BA}^{(HL)} . \tag{1-10b}$$

where

p = The maximum intensity of the haunch load as shown in Fig. (1-5).

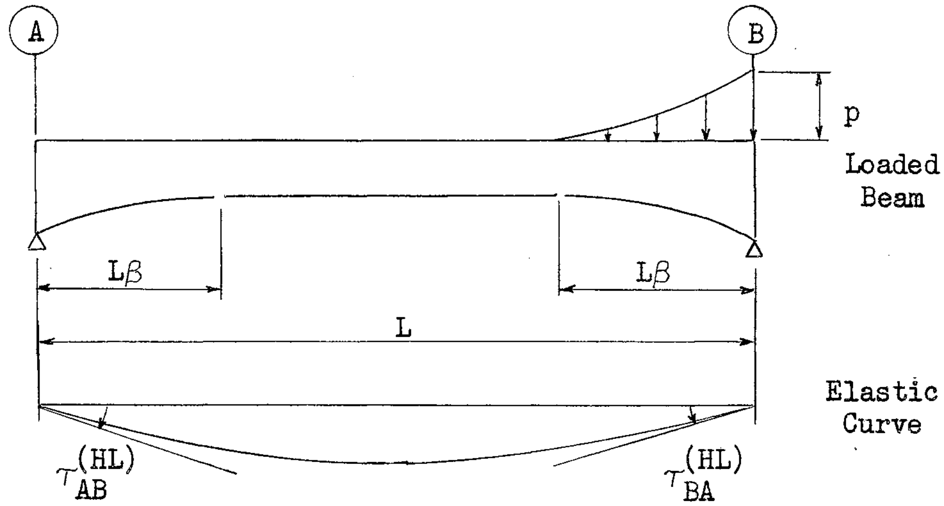


Fig. 1-5.—Angular Haunch Load Functions.

In terms of Eqs. (1-1a,3), Eqs. (1-10a,b) become

$$\tau_{BA}^{(HL)} = \frac{p\beta^2L^2}{12} F_{BA} - \frac{p}{12\beta^2L^3}\int_{L(1-\beta)}^{L} \frac{[x-L(1-\beta)]^4 x\,dx}{EI_x} \qquad (1\text{-}11)$$

$$\tau_{AB}^{(HL)} = \frac{p\beta^2L^2}{12} (G + F_{BA}) - \frac{p}{12\beta^2L^2}\int_{L(1-\beta)}^{L} \frac{[x-(L-\beta)]^4 x\,dx}{EI_x} - \tau_{BA}^{(HL)} \quad .(1\text{-}12)$$

If the beam is symmetrical with two haunches of length $L\beta$, Eqs. (1-11,12) become

$$\tau_{BA}^{(HL)} = \tau_{AB}^{(HL)} = \frac{p\beta^2L^2}{24}\int_0^L \frac{dx}{EI_x} - \frac{p}{12\beta^2L^2}\int_{L(1-\beta)}^{L} \frac{[x-L(1-\beta)]^4 dx}{EI_x} \quad . \quad (1\text{-}13)$$

Noting that for the symmetrical beam

$$F_{AB} = F_{BA}$$

and writing Eq. (1-13) in terms of Eq. (1-4)

$$\tau_{BA}^{(HL)} = \tau_{AB}^{(HL)} = \frac{p\beta^2L^2}{12} (F_{AB} + G) - \frac{p}{12\beta^2L^2}\int_{L(1-\beta)}^{L} \frac{[x-L(1-\beta)]^4 dx}{EI_x} \quad .(1\text{-}14)$$

More complete derivations of Eqs. (1-1) to (1-14) may be found in (2). The apparent difference in the limits of integration for Eqs. (1-10a) to (1-14) in this work and the reference work is due to the selection of different reference axes.

CHAPTER II

INTEGRAL FUNCTIONS

1. General.

The expressions for angular functions in Chapter 1 contain the recurring integrals

$$\int_0^L \frac{x^k dx}{I_x} \tag{2-1}$$

$$\int_0^{Ln} \frac{x^k dx}{I_x} \quad . \tag{2-2}$$

The solutions of integrals (2-1,2) may be greatly facilitated by relocating the reference axis and expressing the integrals in terms of integral functions R, Q, X_{kr}, and X_{kq}. The angular functions may then be expressed in terms of the integral functions as coefficients.

2. Integral Functions - Constant Depth.

A typical haunched beam is considered Fig. (2-1). The cross-section of the beam is constant over the length $L\gamma$. The moment of inertia

$$I_x = I_0 = \frac{1}{12} b h_0^3 \tag{2-3}$$

where

b = constant width

h_0 = reference depth .

The location of the cross-section is given by

$$x = La + t_x L\gamma \tag{2-4}$$

in which t_x varies between the limits

$$t_x = 0, \; t_x = 1 \quad .$$

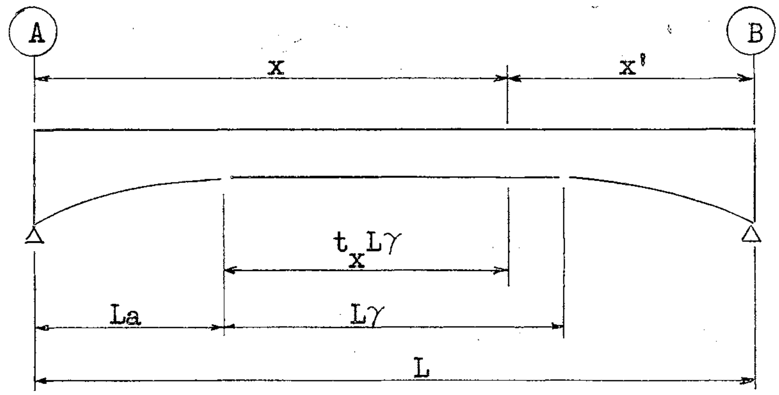


Fig. 2-1.—Typical Haunched Beam.

In terms of Eqs. (2-3,4) the integral (2-1) for constant cross-section becomes

$$\int_{La}^{L(a+\gamma)} \frac{x^k dx}{I_x} = \frac{L^{k+1}}{I_0} \int_0^1 (a+t_x\gamma)^k \gamma dt_x$$
$$= \frac{L^{k+1}}{I_0} X_{kr} \quad . \tag{2-5}$$

If the upper limit is x = Ln such that the corresponding upper limit

$$t_x = \mu = \frac{n-a}{\gamma} \quad , \tag{2-6}$$

integral (2-2) becomes in terms of Eqs. (2-3,4,6)

$$\int_{La}^{Ln} \frac{x^k dx}{I_x} = \frac{L^{k+1}}{I_0} \int_0^{\mu} (a+t_x\gamma)^k \gamma dt_x$$
$$= \frac{L^{k+1}}{I_0} X'_{kn} \quad . \tag{2-7}$$

Eqs. (2-5,7) will be evaluated in Chapters 3, 4, and 5 for values

$$k = 0,\ 1,\ 2,\ 3$$

in terms of the following integral functions and their equivalents.

$$
\begin{aligned}
R_0 &= \int_0^1 dt_x = 1 \\
R_1 &= \int_0^1 t_x dt_x = \frac{1}{2} \\
R_2 &= \int_0^1 t_x^2 dt_x = \frac{1}{3} \\
R_3 &= \int_0^1 t_x^3 dt_x = \frac{1}{4} \\
R_{0n} &= \int_0^{\mu} dt_x = \mu \\
R_{1n} &= \int_0^{\mu} t_x dt_x = \frac{\mu^2}{2} \\
R_{2n} &= \int_0^{\mu} t_x^2 dt_x \ \frac{\mu^3}{3}
\end{aligned}
\qquad (2\text{-}8)
$$

3. <u>Integral Functions - Parabolic Haunch.</u>

A typical haunch of length $L\beta$ is considered (Fig. 2-2). The depth of haunch varies as a parabola of second degree and its depth at support B is

$$h_h = \omega h_0$$

The location of the cross-section is given by

$$x = La_1 + z = La_1 + t_z L\beta \qquad (2\text{-}9)$$

in which t_z varies between the limits

$$t_z = 0,\ t_z = 1 \ .$$

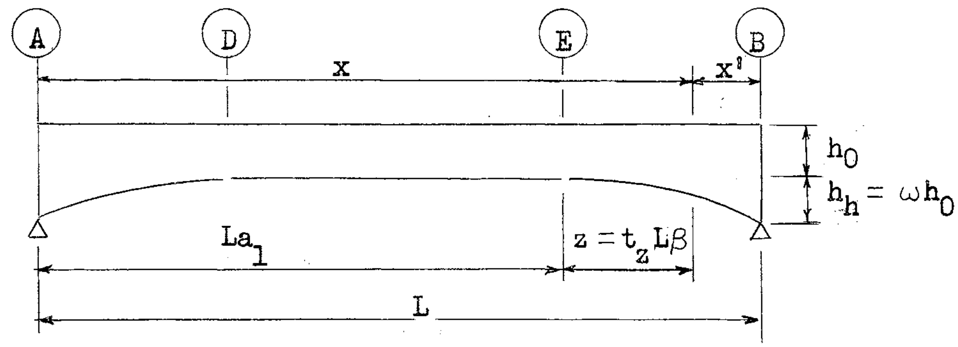


Fig. 2-2.—Typical Haunched Beam.

The depth of the cross-section at x is

$$h_x = h_z = h_0 T_z \tag{2-10}$$

where T_z is the parabolic function

$$T_z = 1 + \omega t_z^{\ 2} \ . \tag{2-11}$$

From Eq. (2-10) the moment of inertia at x is

$$I_x = I_z = I_0 T_z^{\ 3} \ . \tag{2-12}$$

In terms of Eqs. (2-9,12) the integral (2-1) for the parabolic haunch is

$$\int_{La_1}^{L(a_1+\beta)} \frac{x^k dx}{I_x} = \frac{L^{k+1}}{I_0} \int_0^1 \frac{(a_1+t_z\beta)^k \beta dt_z}{T_z^{\ 3}}$$

$$= \frac{L^{k+1}}{I_0} X_{kq} \ . \tag{2-13}$$

If the upper limit x = Ln is such that the corresponding upper limit

$$t_z = \mu_1 = \frac{n-a_1}{\beta} \ , \tag{2-14}$$

integral (2-2) becomes in terms of Eqs. (2-9,12,14)

$$\int_{La}^{Ln} \frac{x^k dx}{I_x} = \frac{L^{k+1}}{I_0} \int_0^{\mu_1} \frac{(a_1+t_z\beta)^k \beta dt_z}{T_z^{\,3}} = \frac{L^{k+1}}{I_0} X''_{kn} \quad . \tag{2-15}$$

The following algebraic equivalents are introduced for the solution of Eqs. (2-13,15).

$$B = \frac{1}{4(1+\omega)^2} \qquad B_n = \frac{\mu_1}{4(1+\omega\mu_1^{\,2})^2}$$

$$C = \frac{1}{8(1+\omega)} \qquad C_n = \frac{\mu_1}{8(1+\omega\mu_1^{\,2})} \tag{2-16a}$$

$$D = \frac{1}{8\sqrt{\omega}} \operatorname{Tan}^{-1}\sqrt{\omega} \qquad D_n = \frac{1}{8\sqrt{\omega}} \operatorname{Tan}^{-1} \mu_1\sqrt{\omega}$$

$$Lg = \frac{1}{2} \operatorname{Log} \quad (1+\omega)$$

With notations (2-16a) Eqs. (2-13,15) will be evaluated in Chapters 3, 4, and 5 in terms of the following integral functions and their equivalents.

$$Q_0 = \int_0^1 \frac{dt_z}{T_z^{\,3}} = B + 3C + 3D$$

$$Q_1 = \int_0^1 \frac{t_z dt_z}{T_z^{\,3}} = B + 2C$$

$$Q_2 = \int_0^1 \frac{t_z^{\,2} dt_z}{T_z^{\,3}} = \frac{1}{\omega}(-B + C + D) \tag{2-16b}$$

$$Q_3 = \int_0^1 \frac{t_z^{\,3} dt_z}{T_z^{\,3}} = B$$

$$Q_{0n} = \int_0^{\mu_1} \frac{dt_z}{T_z^3} = B_n + 3C_n + 3D_n$$

$$Q_{1n} = \int_0^{\mu_1} \frac{t_z dt_z}{T_z^3} = \mu_1(B_n + 2C_n) \qquad \text{(2-16b) (Cont.)}$$

$$Q_{2n} = \int_0^{\mu_1} \frac{t_z^2 dt_z}{T_z^3} = \frac{1}{\omega}(-B_n + C_n + D_n) \quad .$$

Expressed in terms of t_z and T_z, the integrals contained in the expressions for haunch load angular functions (Eqs. 1-11,12,13,14) are

$$\int_0^{L\beta} \frac{z^4 dz}{I_z} = \frac{L^5}{I_0}\int_0^1 \frac{\beta^5 t_z^4 dt_z}{T_z^3} \qquad \text{(2-17)}$$

$$= \frac{L^5}{I_0} X_{4q} \quad .$$

$$\int_0^{L\beta} \frac{z^4(La_1 + z)dz}{I_z} = \frac{L^6}{I_0}\int_0^1 \frac{\beta^5 t_z^4 (a_1 + t_z\beta)dt_z}{T_z^3} \qquad \text{(2-18)}$$

$$= \frac{L^6}{I_0} X_{5q} \quad .$$

With notations (2-16a,b) Eqs. (2-17,19) will be evaluated in Chapters 3, 4, and 5 in terms of the following integral functions and algebraic equivalents.

$$Q_4 = \int_0^1 \frac{t_z^4 dt_z}{T_z^3} = \frac{1}{\omega^2}(Q_0 - 8C) \qquad \text{(2-19)}$$

$$Q_5 = \int_0^1 \frac{t_z^5 dt_z}{T_z^3} = \frac{1}{\omega^3}(Lg - 2\omega^2 Q_3 - Q_1) \quad .$$

4. Integral Substitution And Numerical Evaluation.

Substituting the lower limits

$$t_{x,z} = 0$$

into Eqs. (2-8,16a) reduces all expressions to zero. Hence, Eqs. (2-8, 16b,19) need be evaluated only at the upper limit to obtain the desired solutions.

The integrals (2-1,2) will be replaced by the integral functions

$$\int_0^L \frac{x^k dx}{I_x} = X_k$$

$$\int_0^{Ln} \frac{x^k dx}{I_x} = X_{kn} \qquad (2\text{-}20)$$

in which the symbols X_k, X_{kn} are the sum of the X-functions as indicated by the limits.

The algebraic expressions for R-functions (Eqs. 2-8) and Q-functions (Eqs. 2-16b) will be evaluated numerically on the IBM 650 Electronic Computer. From these values, the numerical solutions for the X-functions (Eqs. 2-20) and finally the angular function coefficients to be derived in Chapters 4 and 5 will be obtained.

CHAPTER III

ANGULAR COEFFICIENTS - BEAMS WITH ONE HAUNCH

1. General.

A beam of length L with one parabolic haunch is considered Fig. (3-1). The beam is of constant depth over the length $L\alpha$ and the reference moment of inertia is Eq. (2-3)

$$I_0 = \frac{1}{12} b h_0^3 \quad . \qquad (3\text{-}1)$$

The length of the haunch is $L\beta$ and the moment of inertia at any cross-section within the haunch is Eq. (2-11)

$$I_x = I_0 T_z^3 \quad . \qquad (3\text{-}2)$$

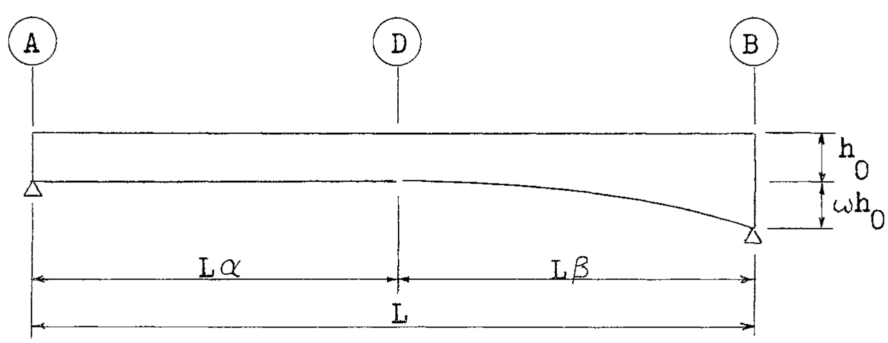


Fig. 3-1.—Beam With One Haunch.

2. Integral Functions X_{kr} $(x = 0 \rightarrow L\alpha)$.

The location of the cross-section is given by (Fig. 3-2)

$$x = t_x L\alpha \quad .$$

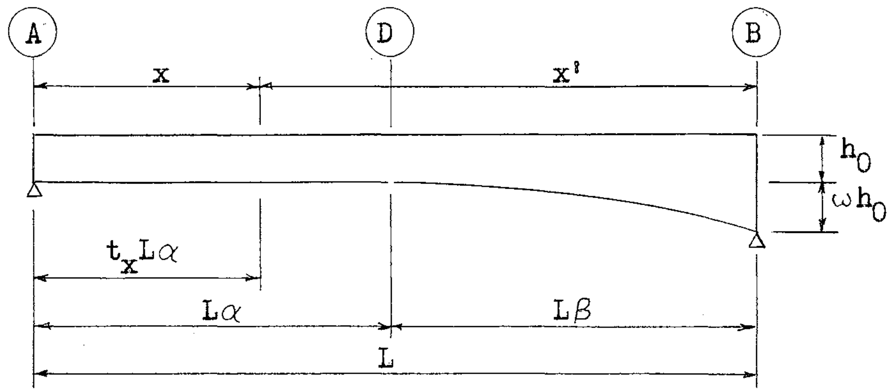


Fig. 3-2.—Beam With One Haunch For $x = 0 \rightarrow L\alpha$.

Substituting

$$a = 0$$

into Eqs. (2-5,7) gives

$$X_{kr} = \alpha^{k+1} \int_0^1 t_x{}^k dt_x \tag{3-4}$$

$$X'_{kn} = \alpha^{k+1} \int_0^{\mu} t_x{}^k dt_x \,. \tag{3-5}$$

In terms of functions R_k, Eqs. (2-7; 3-4,5) are

$$\begin{aligned}
X_{0r} &= \alpha \int_0^1 dt_x = \alpha R_0 & X'_{0n} &= \alpha \int_0^{\mu} dt_x = \alpha R_{0n} \\
X_{1r} &= \alpha^2 \int_0^1 t_x dt_x = \alpha^2 R_1 & X'_{1n} &= \alpha^2 \int_0^{\mu} t_x dt_x = \alpha^2 R_{1n} \\
X_{2r} &= \alpha^3 \int_0^1 t_x{}^2 dt_x = \alpha^3 R_2 & X'_{2n} &= \alpha^3 \int_0^{\mu} t_x{}^2 dt_x = \alpha^3 R_{2n}
\end{aligned} \tag{3-6}$$

$$X_{3r} = \alpha^4 \int_0^1 t_x^{\,3} dt_x = \alpha^4 R_3 \quad . \qquad \text{(3-6) (Cont.)}$$

If the upper limit x = Ln (Eqs. 2-6,7) is such that

$$0 \le n \le \alpha ,$$

with notation (2-20)

$$X_{kn} = X'_{kn} \quad . \qquad \text{(3-7)}$$

3. Integral Functions X_{kq} $(x = L\alpha \rightarrow L)$.

The location of the cross-section is given by (Fig. 3-3)

$$x = L\alpha + t_z L\beta \quad . \qquad \text{(3-8)}$$

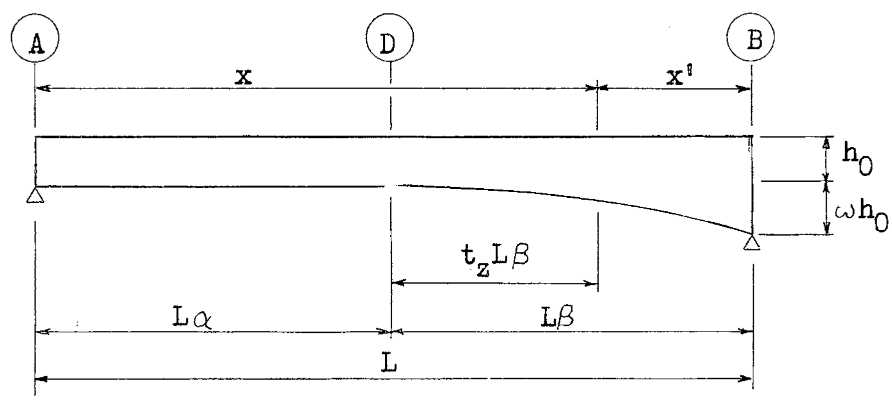


Fig. 3-3.—Beam With One Haunch For $x = L\alpha \rightarrow L$.

Substituting

$$a = \alpha = 1-\beta \qquad \text{(3-9)}$$

into Eqs. (2-12,13) yields

$$X_{kq} = \int_0^1 \frac{\left[1-\beta(1-t_z)\right]^k \beta dt_z}{T_z^{\,3}} \qquad \text{(3-10)}$$

$$X''_{kn} = \int_0^{\mu_1} \frac{\left[1-\beta(1-t_z)\right]^k \beta dt_z}{T_z^{\,3}} \quad . \qquad \text{(3-11)}$$

The bracketed terms in Eqs. (3-10,11) may be replaced by the identities

$$
\begin{aligned}
Z &= 1 - \beta(1 - t_z) \\
Z^0 &= 1 \\
Z^1 &= Z \\
Z^2 &= \beta^2(1 - t_z)^2 + 2Z - 1 \\
Z^3 &= \beta^3(1 - t_z)^3 + 3Z^2 - 3Z + 1 \quad .
\end{aligned}
\tag{3-12}
$$

Thus, each succeeding power of Z is written in terms of preceeding powers.

Expressed in terms of Q_k functions (Eqs. 2-18) and with notations (3-12) Eqs. (3-10,11) are

$$
\begin{aligned}
X_{0q} &= \int_0^1 \frac{Z^0 \beta\, dt_z}{T_z^{\,3}} = \beta Q_0 \\
X_{1q} &= \int_0^1 \frac{Z^1 \beta\, dt_z}{T_z^{\,3}} = -\beta^2(Q_0 - Q_1) + X_{0q} \\
X_{2q} &= \int_0^1 \frac{Z^2 \beta\, dt_z}{T_z^{\,3}} = \beta^3(Q_0 - 2Q_1 + Q_2) + 2X_{1q} - X_{0q} \\
X_{3q} &= \int_0^1 \frac{Z^3 \beta\, dt_z}{T_z^{\,3}} = \beta^4(-Q_0 + 3Q_1 - 3Q_2 + Q_3) + 3X_{2q} - 3X_{1q} + X_{0q} \quad .
\end{aligned}
\tag{3-13}
$$

The X_{kq} functions for uniform and dead loads (Eqs. 2-17,18) expressed in terms of notations (2-19; 3-8) are

$$
\begin{aligned}
X_{4q} &= \int_0^1 \frac{\beta^5 t_z^{\,4} dt_z}{T_z^{\,3}} = \beta^5 Q_4 \\
X_{5q} &= \int_0^1 \frac{\beta^5 t_z^{\,4} Z dt_z}{T_z^{\,3}} = X_{4q} + \beta^6(Q_5 - Q_4) \quad .
\end{aligned}
\tag{3-14}
$$

Similarly as for functions X_{kq}

$$X''_{0n} = \int_0^{\mu_1} \frac{Z^0 \beta dt_z}{T_z^3} = \beta Q_{0n}$$

$$X''_{1n} = \int_0^{\mu_1} \frac{Z^1 \beta dt_z}{T_z^3} = -\beta^2 (Q_0 - Q_1) + X''_{0n} \qquad (3\text{-}15)$$

$$X''_{2n} = \int_0^{\mu_1} \frac{Z^2 \beta dt_z}{T_z^3} = \beta^3 (Q_{0n} - 2Q_{1n} + Q_{2n}) + 2X''_{1n} - X''_{0n} \ .$$

If the upper limit $x = Ln$ (Eqs. 2-14,15) is such that

$$\alpha \leq n \leq 1,$$

$$X_{kn} = X_{kr} + X''_{kn} \ . \qquad (3\text{-}16)$$

4. Angular Functions - Coefficients.

The angular function coefficients are introduced as noted. With notations (2-5,7,13,15) the angular beam functions become:

(a) Angular Flexibility, F_{BA} (Eq. 1-1a).

$$F_{BA} = \frac{1}{L^2} \int_0^L \frac{x^2 dx}{EI_x} = \frac{L}{EI_0} X_2$$

$$= \frac{L}{EI_0} f_{BA} \qquad (3\text{-}17)$$

hence,

$$f_{BA} = X_{2r} + X_{2q} \ . \qquad (3\text{-}18)$$

(b) Angular Carry Over Value, G (Eq. 1-3).

$$G = \frac{1}{L} \int_0^L \frac{x dx}{EI_x} - F_{BA} = \frac{L}{EI_0} (X_1 - f_{BA})$$

$$= \frac{L}{EI_0} g \qquad (3\text{-}19)$$

in which

$$g = X_{1r} + X_{1q} - f_{BA} \quad . \tag{3-20}$$

(c) <u>Angular Flexibility, F_{AB} (Eq. 1-4).</u>

$$F_{AB} = \int_0^L \frac{dx}{EI_x} - 2G - F_{BA} = \frac{L}{EI_0} (X_0 - 2g - f_{BA})$$
$$= \frac{L}{EI_0} f_{AB} \tag{3-21}$$

and

$$f_{AB} = X_{0r} + X_{0q} - 2g - f_{BA} \quad . \tag{3-22}$$

Expressed in terms of X_{kq} functions (Eqs. 2-5,13) and angular beam function coefficients (Eqs. 3-18,20,22), the uniform load angular coefficients are:

(a) <u>Angular Load Function $\tau_{BA}^{(UL)}$ (Eq. 1-8)</u> .

$$\tau_{BA}^{(UL)} = \frac{wL^2}{2} F_{BA} - \frac{w}{2L} \int_0^L \frac{x^3 dx}{EI_x} = \frac{wL^3}{EI_0} \frac{1}{2} (f_{BA} - X_3)$$
$$= \frac{wL^3}{EI_0} t_{BA}^{(UL)} \tag{3-23}$$

in which

$$t_{BA}^{(UL)} = \frac{1}{2} (f_{BA} - X_{3r} - X_{3q}) \quad . \tag{3-24}$$

(b) <u>Angular Load Function $\tau_{AB}^{(UL)}$ (Eq. 1-9).</u>

$$\tau_{AB}^{(UL)} = \frac{wL^2}{2} G - \tau_{BA}^{(UL)} = \frac{wL^3}{EI_0} (\frac{1}{2} g - t_{BA}^{(UL)})$$
$$= \frac{wL^3}{EI_0} t_{AB}^{(UL)} \tag{3-25}$$

in which

$$t_{AB}^{(UL)} = \frac{1}{2} g - t_{BA}^{(UL)} \quad . \tag{3-26}$$

The haunch dead load angular coefficients expressed in terms of X_{kq} functions (Eqs. 2-17,18; 3-14) and angular beam function coefficients (Eqs. 3-18,20,22) are

(a) <u>Angular Load Function $\tau_{BA}^{(HL)}$ (Eq. 1-11).</u>

$$\tau_{BA}^{(HL)} = \frac{p\beta^2 L^2}{12} F_{BA} - \frac{p}{12\beta^2 L^3}\int_0^L \frac{z^4(La_1 + z)dz}{EI_z}$$

$$= \frac{pL^3}{EI_0} \left(\frac{\beta^2}{12} f_{BA} - \frac{1}{12\beta^2} X_{5q}\right) \tag{3-27}$$

$$= \frac{pL^3}{EI_0} t_{BA}^{(HL)}$$

since

$$p = qh_0\omega$$

$$t_{BA}^{(HL)} = \frac{\omega\beta^2}{12} f_{BA} - \frac{\omega}{12}\beta^2 X_{5q} \quad . \tag{3-28}$$

(b) <u>Angular Load Function $\tau_{AB}^{(HL)}$ (Eq. 1-12).</u>

$$\tau_{AB}^{(HL)} = \frac{p\beta^2 L^2}{12} (F_{BA} + G) - \frac{p}{12E\beta^2 L^2}\int_0^{L\beta} \frac{z^4 dz}{I_z}$$

$$= \frac{pL^3}{EI_0} \left(\frac{\beta^2}{12} f_{BA} + \frac{\beta^2}{12} g - \frac{1}{12\beta^2} X_{4q}\right) \tag{3-29}$$

$$= \frac{pL^3}{EI_0} t_{AB}^{(HL)}$$

in which

$$t_{AB}^{(HL)} = \frac{\beta^2}{12}(f_{BA} + g) - \frac{\omega}{12\beta^2} X_{4q} \quad . \tag{3-30}$$

The live load angular coefficients expressed in terms of X_{kq} functions (Eqs. 3-6,12) and angular beam function coefficients (Eqs. 3-18, 20,22) are

(a) Angular Load Function $\tau_{BA}^{(LL)}$ (Eq. 1-5).

$$\tau_{BA}^{(LL)} = nLG + \frac{1}{L}\int_0^{Ln} \frac{x^2 dx}{EI_x} - n\int_0^{Ln} \frac{xdx}{EI_x}$$

$$= \frac{L^2}{EI_0}(ng - nX_{1n} + X_{2n}) \qquad (3\text{-}31)$$

$$= \frac{L^2}{EI_0} t_{BA}^{(LL)}$$

where

$$t_{BA}^{(LL)} = n(g - X_{1n}) + X_{2n} \quad . \qquad (3\text{-}32)$$

(b) Angular Load Function $\tau_{AB}^{(LL)}$ (Eq. 1-6).

$$\tau_{AB}^{(LL)} = nL(G + F_{AB} - \frac{1}{E}\int_0^{Ln} \frac{dx}{I_x}) + \frac{1}{E}\int_0^{Ln} \frac{xdx}{I_x} - \tau_{BA}^{(LL)}$$

$$= \frac{L^2}{EI_0}(ng + nf_{AB} - nX_{0n} + X_{1n} - t_{BA}^{(LL)}) \qquad (3\text{-}33)$$

$$= \frac{L^2}{EI_0} t_{AB}^{(LL)}$$

and

$$t_{AB}^{(LL)} = n(g + f_{AB} - X_{0n}) + X_{1n} - t_{BA}^{(LL)} \quad . \qquad (3\text{-}34)$$

CHAPTER IV

ANGULAR COEFFICIENTS - SYMMETRICAL BEAMS

1. General.

A beam of length L with two symmetrical parabolic haunches is considered (Fig. 4-1). The beam is of constant depth over the length $L\gamma$ and from Eq. (2-3)

$$I_x = I_0 = \frac{1}{12} bh_0^3 \quad . \qquad (4\text{-}1)$$

The length of each haunch is $L\beta$ and for any cross-section within either haunch from Eqs. (2-11,12)

$$I_x = I_0 T_z^3 \qquad (4\text{-}2)$$

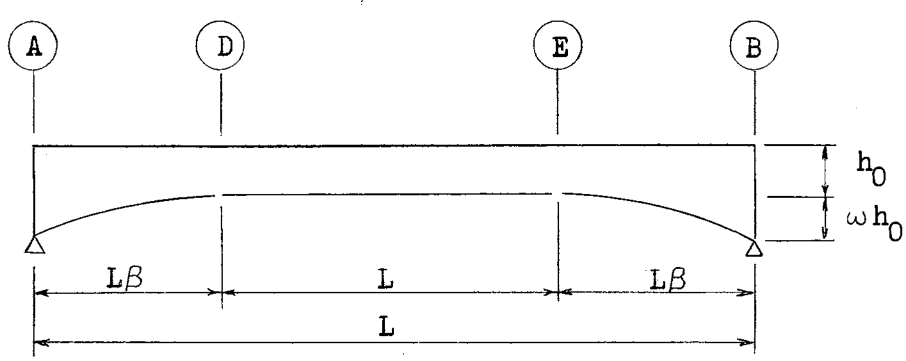


Fig. 4-1.—Symmetrical Beam.

2. Integral Functions X_{kq} $(x = 0 \rightarrow L\beta)$.

The location of the cross-section is given by (Fig. 4-2)

$$x = L\beta - t_z L\beta \quad .$$

For

$$a_1 = \beta$$

and, noting the change in sign, from Eq. (2-13)

$$X'_{kq} = \frac{L^{k+1}}{I_0} \int_0^1 \frac{\beta^k (1 - t_z)^k \beta dt_z}{T_z{}^3} \quad . \tag{4-3}$$

With notations (3-9,10) the integral functions X'_{kq} may be expressed in terms of integral functions X_{kq}. Thus,

$$\begin{aligned} X'_{kq} &= \frac{L^{k+1}}{I_0} \int_0^1 \frac{[1 - \{1 - \beta(1 - t_z)\}]^k \beta dt_z}{T_z{}^3} \\ &= \frac{L^{k+1}}{I_0} \int_0^1 \frac{(1 - Z)^k \beta dt_z}{T_z{}^3} \quad . \end{aligned} \tag{4-4}$$

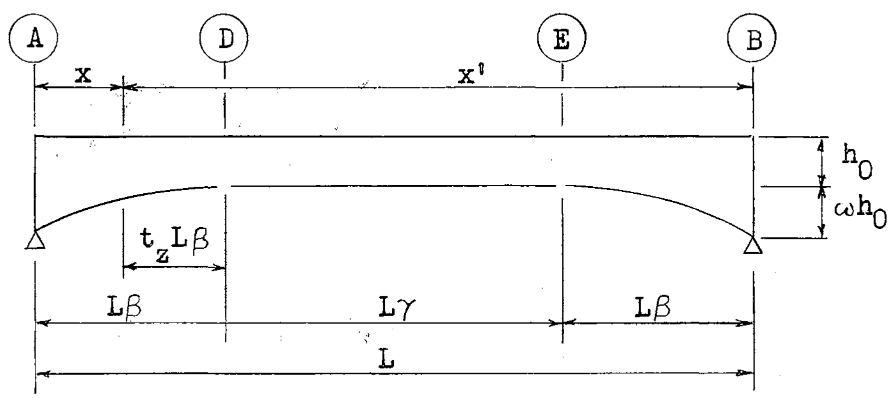


Fig. 4-2.—Symmetrical Beam For $x = 0 \rightarrow L\beta$.

For values

$$k = 0,\ 1,\ 2$$

$$X'_{0q} = \int_0^1 \frac{(1 - Z)^0 \beta dt_z}{T_z{}^3} = X_{0q} \tag{4-5}$$

$$X'_{1q} = \int_0^1 \frac{(1 - Z)^1 \beta dt_z}{T_z^{\ 3}} = X_{0q} - X_{1q}$$

$$X'_{2q} = \int_0^1 \frac{(1 - Z)^2 \beta dt_z}{T_z^{\ 3}} = X_{0q} - 2X_{1q} + X_{2q} \quad . \tag{4-5 (Cont.)}$$

Since the beam is symmetrical the angular live load coefficients need not be computed for the unit load in positions that require the computation of X_{kn} functions for the left haunch.

3. <u>Integral Functions X_{kr} $(x = L\beta \rightarrow L(\beta+\gamma))$.</u>

The location of the cross-section is given by (Fig. 4-3)

$$x = L\beta + t_x L\alpha \quad .$$

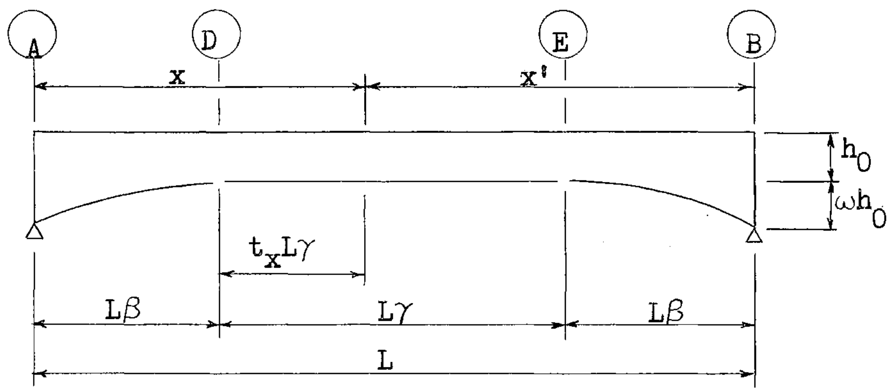


Fig. 4-3.—Symmetrical Beam For $x = L\beta \rightarrow L(\gamma+\beta)$.

Substituting into Eq. (2-4)

$$a = \beta$$

gives for functions X'_{kr} (Eq. 2-5)

$$X'_{kr} = \int_0^1 (\beta + t_x\gamma)^k \gamma dt_x \quad . \tag{4-6}$$

Expressed in terms of notations (3-6), Eq. (4-6) for values

$$k = 0, 1$$

$$X'_{0r} = \int_0^1 (\beta + \gamma t_x)^0 \gamma dt_x = X_{0r}$$

$$X'_{1r} = \int_0^1 (\beta + \gamma t_x)^1 \gamma dt_x = \beta X_{0r} + X_{1r} \quad . \tag{4-7}$$

Evaluating Eq. (4-6) in terms of Eqs. (4-7) and with notations (3-6) for $k = 2$ gives

$$X'_{2r} = \int_0^1 (\beta + \gamma t_x)^2 \gamma dt_x = -\beta^2 X'_{0r} + 2\beta X'_{1r} + X_{2r} \quad . \tag{4-8}$$

Thus, each function X'_{kr} is written in terms of previously evaluated functions X'_{kr} and notations (3-6).

For uniform load angular functions (Eqs. 1-7a,b) the integral (2-1) must be evaluated for

$$\alpha' = \frac{\gamma}{2} \quad .$$

Substituting this value into notations (3-6) gives for uniform load

$$X''_{0r} = \frac{\gamma}{2} \int_0^1 dt_x = \frac{\gamma}{2} R_0$$

$$X''_{2r} = \left(\frac{\gamma}{2}\right)^3 \int_0^1 t_x^2 dt_x = \left(\frac{\gamma}{2}\right)^3 R_2 \quad . \tag{4-9}$$

No other X''_{kr} functions are necessary.

Similarly as for functions X'_{kr}

$$X'''_{0n} = \int_0^\mu (\beta + \gamma t_x)^0 \gamma dt_x = X'_{0n}$$

$$X'''_{1n} = \int_0^\mu (\beta + \gamma t_x)^1 \gamma dt_x = X'_{0n} + X'_{1n} \tag{4-10}$$

$$X_{2n}''' = \int_0^{\mu} (\beta + \gamma t_x)^2 \gamma dt_x = -\beta^2 X_{0n}' + 2\, X_{1n}''' + X_{2n}' \quad . \qquad \begin{matrix}(4\text{-}10)\\(\text{Cont.})\end{matrix}$$

If the upper limit $x = Ln$ (Eqs. 2-6,7) is such that

$$\beta \leq n \leq \beta + \gamma$$

$$X_{kn} = X_{kq}' + X_{kn}''' \quad . \qquad (4\text{-}11)$$

4. <u>Integral Functions X_{kq} $(x = L(\beta+\gamma) \rightarrow L)$</u>.

The location of the cross-section is given by (Fig. 4-4)

$$x = L(\beta + \gamma) + t_z L\beta$$

and

$$a_1 = \beta + \gamma = 1 - \beta$$

which is identical to the value for a_1 in Eq. (3-8), therefore with the exception of uniform load (Eqs. 1-7a,b), the solutions for functions X_{kq} and X_{kn} are given by Eqs. (3-10,11,12,13,15,16).

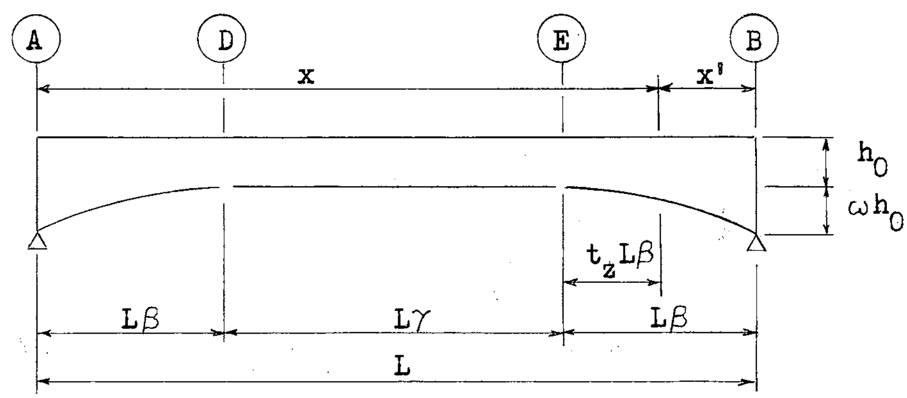


Fig. 4-4.—Symmetrical Beam For $x = L(\beta+\gamma) \rightarrow L$.

For uniform load the cross-section is measured from the center of the beam (Fig. 4-5)

$$x = L\frac{\gamma}{2} + t_z L\beta \quad .$$

Substituting

$$a_1 = \frac{\gamma}{2} = \frac{1}{2}(1 - 2\beta)$$

into Eq. (2-12) and considering notation (3-9) gives for k = 2

$$X''_{2q} = \int_0^1 \frac{\left[\frac{1}{2} - \beta(1-t_z)\right]^2 \beta dt_z}{T_z^{\,3}} = \int_0^1 \frac{(Z - \frac{1}{2})^2 \beta dt_z}{T_z^{\,3}} \tag{4-12}$$

$$= X_{2q} - X_{1q} + \frac{1}{4} X_{0q} \quad .$$

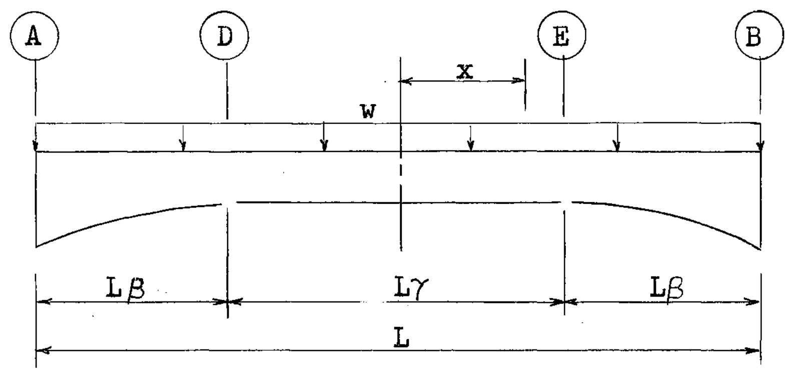


Fig. 4-5.—Symmetrical Beam For Uniform Load.

5. Angular Functions - Coefficients.

Similarly as for the unsymmetrical beam, the angular functions will be expressed in terms of coefficients as follows:

(a) Angular Flexibility Coefficient, f_{BA} (Eq. 3-17).

$$f_{BA} = X_2 = X'_{2q} + X'_{2r} + X_{2q} \quad . \tag{4-13}$$

(b) Angular Carry-Over Value Coefficient, g (Eq. 3-19).

$$g = X_1 - f_{BA} = X'_{1q} + X'_{1r} + X_{1q} \quad . \tag{4-14}$$

(c) Angular Flexibility Coefficient, f_{AB} (Eq. 3-21).

$$f_{AB} = X_0 - 2g - f_{BA} = X'_{0q} + X'_{0r} + X_{0q} - 2g - f_{BA} \quad . \tag{4-15}$$

For symmetrical beams the uniform load angular functions may be written (3)

$$\tau_{BA}^{(UL)} = \tau_{AB}^{(UL)} = \frac{wL^2}{8E}\int_0^{\frac{L}{2}} \frac{dx}{I_x} - \frac{w}{2E}\int_0^{\frac{L}{2}} \frac{x^2 dx}{I_x} \quad . \tag{4-16}$$

Proceeding as in Eqs. (3-23,24) and with notations (3-13; 4-9)

$$t_{BA}^{(UL)} = \frac{1}{8}\left(X_{0r}'' + X_{0q}\right) - \frac{1}{2}\left(X_{2r}'' + X_{2q}''\right) \quad . \tag{4-17}$$

The haunch load angular functions are (Eq. 1-13)

$$\tau_{BA}^{(HL)} = \tau_{AB}^{(HL)} = \frac{p\beta^2}{12E}\int_0^{\frac{L}{2}} \frac{dx}{I_x} - \frac{p}{12\beta^2 L^2 E}\int_0^{L\beta} \frac{z^4 dz}{I_z} \quad . \tag{4-18}$$

Proceeding as in Eqs. (3-27,28) and with notations (3-13,14; 4-9)

$$t_{BA}^{(HL)} = \frac{\beta^2\omega}{12}\left(X_{0r}'' + X_{0q}\right) - \frac{\omega}{12\beta^2} X_{4q} \quad . \tag{4-19}$$

The coefficients for the angular live load functions are given by Eqs. (3-32,34).

CHAPTER V

PROGRAM FOR THE IBM 650 ELECTRONIC COMPUTER

1. General.

Programming for the solution of any problem on a digital computer is usually accomplished in two steps. First, a drawing showing each phase of the problem and the sequence of operations is made. Second, from the schematic drawing, or flow chart, a series of instructions for the computer is established.

The program in this chapter was prepared in floating decimal arithmetic for the IBM 650 Electronic Computer at Oklahoma State University's Computing Center. The coding form used is that of IBM's Symbolic Optimum Assembly Programming, Type II. Storage locations have been re-used as soon as possible in order to take advantage of the available sixty high-speed storage locations.

2. Functional Evaluation.

The subroutines for the evaluation of the square roots, arc-tangents, and logarithms required for the solution of Eqs. (2-16a) are an integral part of the program, therefore no library subroutines are necessary.

The square roots are obtained by Newton's method which is as follows:

$$R_i = \frac{1}{2}\left(R_{i-1} + \frac{\omega}{R_{i-1}}\right) \qquad (5\text{-}1)$$

in which

ω = number for which the square root is desired,

R_i = ith approximation of the square root,

R_{i-1} = preceding approximation of the square root.

For all values of ω the initial approximation is one and fifteen successive approximations are made.

With notation (2-10) the arc-tangents are evaluated by the infinite series

$$\text{Tan}^{-1}\, t_z\sqrt{\omega} = V = \frac{R_t}{t_z\sqrt{\omega}}(1 + \frac{2}{3} R_t + \frac{2\cdot 4}{3\cdot 5} R_t^{\,2} + \frac{2\cdot 4\cdot 6}{3\cdot 5\cdot 7} R_t^{\,3} \ldots \qquad (5\text{-}2)$$

in which

$$R_t = \frac{\omega t_z^{\,2}}{1+\omega t_z^{\,2}} = \frac{T_z - 1}{T_z} \quad .$$

The series converges for all values of

$$\omega t_z^{\,2} < \infty \quad .$$

For values

$$\omega t_z^{\,2} \leq 0.1$$

computations are limited to eleven terms to avoid overflow in the computer. For larger values twenty-five terms are evaluated to obtain the desired accuracy.

With notation (2-10) the logarithms are evaluated by the infinite series

$$Lg = \frac{1}{2}\,\text{Log}_e T_z = R_\ell + \frac{1}{3}R_\ell^{\,3} + \frac{1}{5}R_\ell^{\,5} + \frac{1}{7}R_\ell^{\,7} \quad \ldots \qquad (5\text{-}3)$$

in which

$$R_\ell = \frac{T_z - 1}{T_z + 1}$$

The above series converges for all values

$$T_z > 0,$$

and nineteen terms are used.

The accuracy of Eqs. (5-1,2,3) depends upon the magnitude of the value for which the function is required. Using the number of terms indicated and rounding off the angular function coefficients to four decimal places gives results that agree with those published in other works for

$$\omega_{max} \leq 2 \quad .$$

3. Input Card Format.

The description of the beam for which constants are desired is introduced into the computer with seven words. Fig. (5-1) shows the arrangement of input data.

Word	Card Columns Inclusive	Data
1	1 - 10	ω
2	11 - 20	β
3	21 - 30	$\Delta\omega$
4	31 - 40	$\Delta\beta$
5	41 - 50	ω_{max}
6	51 - 60	β_{max}
7	61 - 70	Beam Type
8	71 - 80	zeros

Fig. 5-1.—Input Card Data.

The meanings of ω and β have already been established. The symbols

$\Delta\omega$ and $\Delta\beta$ are the increments by which the dimension coefficients are to be increased. These two words must have some positive value even though computations may be required for only one beam. The fifth and sixth words are the maximum values the dimension coefficients may attain. The beam type number is zero for unsymmetrical beams and one for symmetrical.

The first six words of the input card must be in floating decimal form. The position of the decimal is obtained by subtracting 50 from the last two digits. If the result is zero the decimal immediately precedes the first digit. If the result is negative or positive the decimal is shifted to the left or right the indicated number of places. (Fig. 5-2).

The beam type number must be entered as a fixed point number, either ten zeros for unsymmetrical beams or one preceeded by nine zeros for symmetrical beams.

Number	Floating Decimal Form
345.6	3456000053
0.3456	3456000050
0.03456	3456000049

Fig. 5-2.—Examples of Floating Decimals.

<u>Example 1</u>.

Beam constants are to be computed for the symmetrical beam in Fig. (5-3).

Since computations are required for only one beam the maximum values of dimension coefficients are equal to the initial values and the increments will be unimportant as long as they have some positive value.

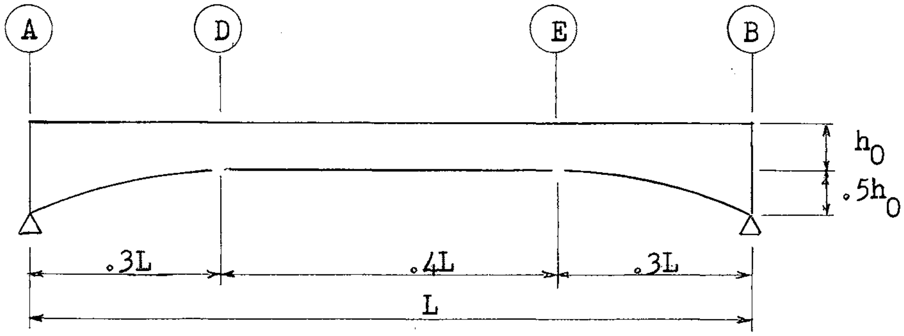


Fig. 5-3.—Symmetrical Beam.

The beam type number is one and the data is entered as in Fig. (5-4).

Word	Data Entered
1	5000000050
2	3000000050
3	1000000050
4	1000000050
5	5000000050
6	3000000050
7	0000000001
8	Not Used

Fig. 5-4.—Input Card For One Symmetrical Beam.

Example 2.

Beam constants are required for the beam shown in Fig. (5-5) for all combinations of

$\omega = 0.2 \rightarrow 1.0$ in increments of 0.2

$\beta = 0.1 \rightarrow 1.0$ in increments of 0.1 .

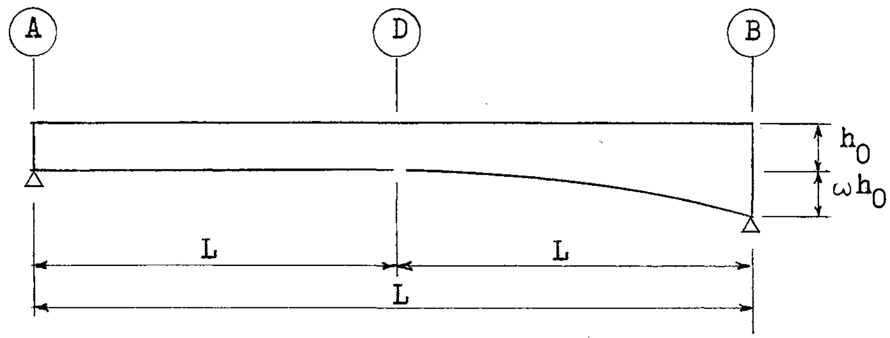


Fig. 5-5.—Unsymmetrical Beam.

The input data is entered as in Fig. (5-6) and the computer will calculate the constants for the required 50 beams.

Word	Data Entered
1	2000000050
2	1000000050
3	2000000050
4	1000000050
5	1000000051
6	1000000051
7	0000000000
8	(Not Used)

Fig. 5-6.—Input Card For Series of Unsymmetrical Beams.

4. <u>Output Card Format</u>.

The angular function coefficients will be in floating decimal form on either three or four cards depending on the type of beam. The first word of each card will be an identification number. The first two digits are ten times the value ω, the fourth and fifth digits are ten times the value β, and the last digit is the beam type number. The identification number for angular live load functions will have, in addition, ten times the last computed value of n as the seventh digit. Thus, the identification number

05 003 00 001

will appear on the first output card for the symmetrical beam for which

$$\omega = 0.5$$

$$\beta = 0.3 ,$$

and the number

05 003 09 001

will appear on the card containing influence coefficients for

$$n = 0.7,\ 0.8,\ 0.9 \quad .$$

The first output card for each beam will be arranged as follows:

Word	Information
1	Identification
2	f_{BA}
3	g
4	f_{AB}
5	$t_{BA}^{(UL)}$
6	$t_{AB}^{(UL)}$
7	$t_{BA}^{(DL)}$
8	$t_{AB}^{(DL)}$

Fig. 5-7.—First Output Card.

The angular live load coefficients will appear as in Fig. (5-8). In the case of symmetrical beams the second and third words are not used for cards bearing identification numbers of the form.

xx 0xx 06 001 .

Word	Information	Position Of Load
1	Identification	
2	$t_{BA}^{(LL)}$	n - 2
3	$t_{AB}^{(LL)}$	n - 2
4	$t_{BA}^{(LL)}$	n - 1
5	$t_{AB}^{(LL)}$	n - 1
6	$t_{BA}^{(LL)}$	n
7	$t_{AB}^{(LL)}$	n
8	(Not Used)	

Fig. 5-8.—Output Card For Live Load Function Coefficients.

5. Flow Chart.

The flow chart in Fig. (5-9) was prepared as an aid to programming the solutions for angular function coefficients for beams with one haunch and symmetrical beams.

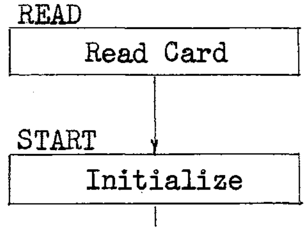


Fig. 5-9.—Flow Chart.

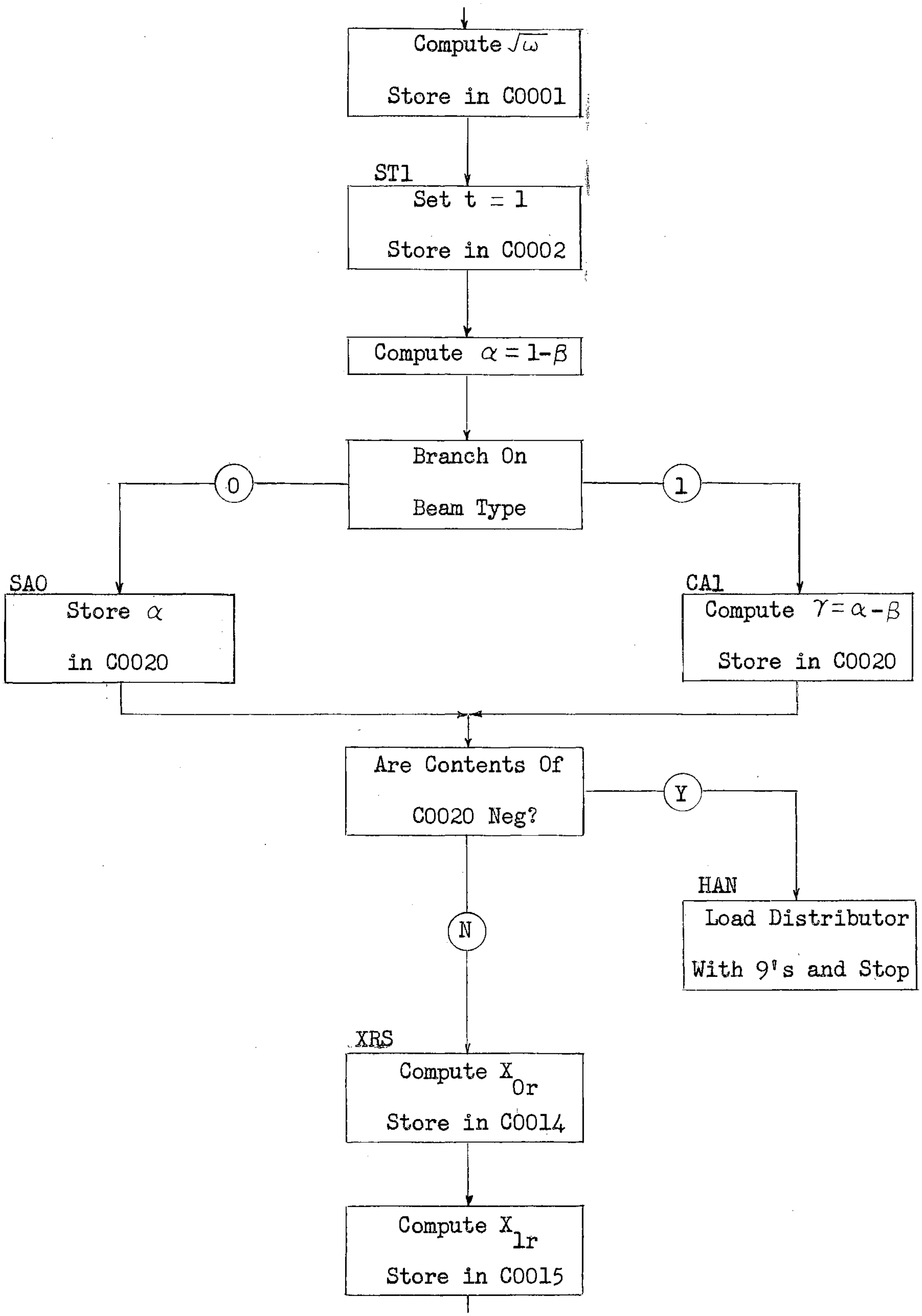

Compute $\sqrt{\omega}$
Store in C0001
ST1
Set t = 1
Store in C0002
Compute $\alpha = 1-\beta$
Branch On
Beam Type
0
1
SA0
Store α
in C0020
CA1
Compute $\gamma = \alpha - \beta$
Store in C0020
Are Contents Of
C0020 Neg?
Y
N
HAN
Load Distributor
With 9's and Stop
XRS
Compute X_{0r}
Store in C0014
Compute X_{1r}
Store in C0015

Fig. 5-9 (Cont.)

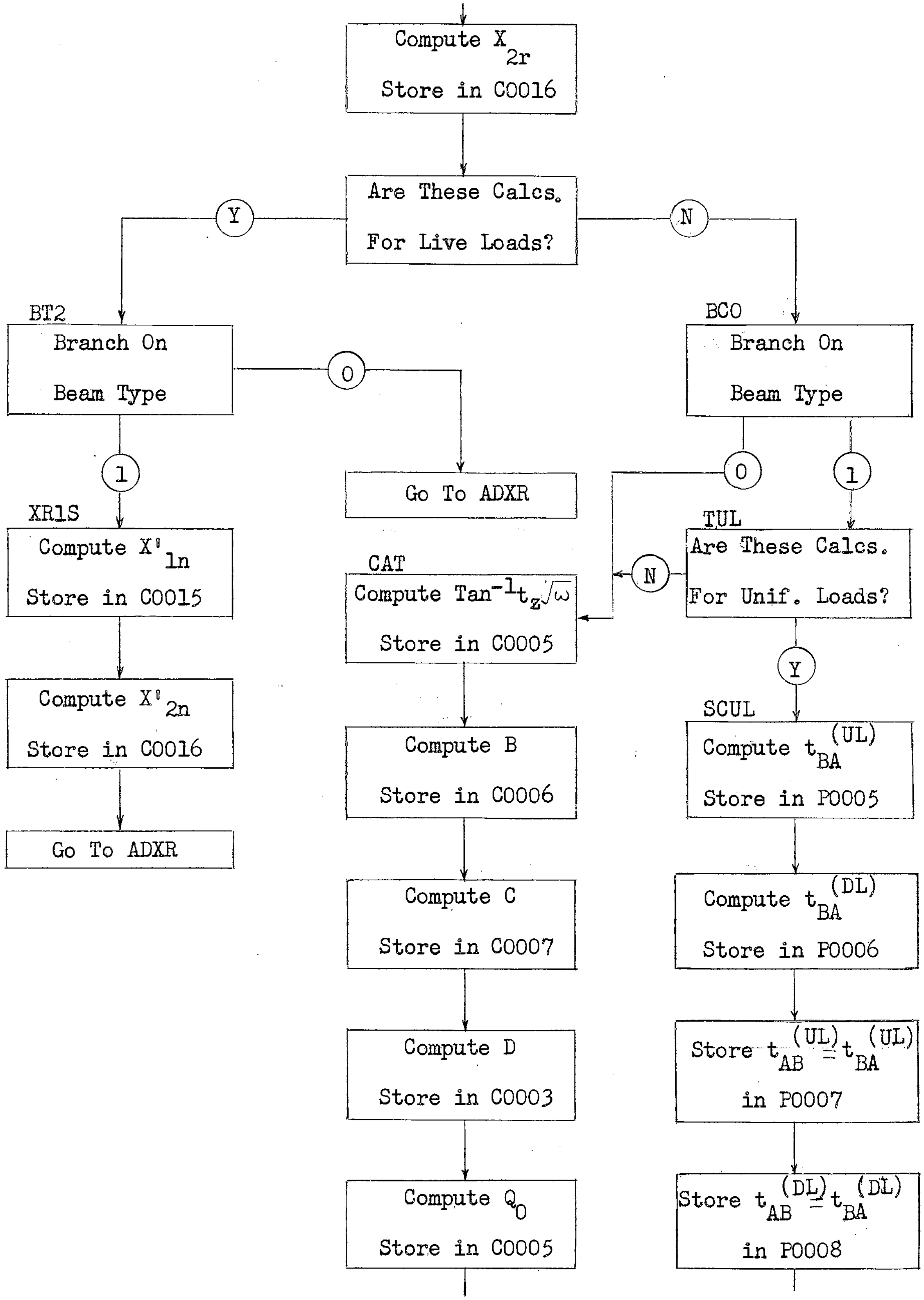

Compute X 2r
Store in C0016
Are These Calcs.
For Live Loads?
Y
N
BT2
Branch On
Beam Type
0
BCO
Branch On
Beam Type
1
Go To ADXR
0
1
XR1S
Compute X' 1n
Store in C0015
TUL
Are These Calcs.
For Unif. Loads?
CAT
Compute Tan-1tz/√ω
Store in C0005
N
Compute X' 2n
Store in C0016
Y
SCUL
Compute B
Store in C0006
Compute tBA (UL)
Store in P0005
Go To ADXR
Compute C
Store in C0007
Compute tBA (DL)
Store in P0006
Compute D
Store in C0003
Store tAB (UL) = tBA (UL)
in P0007
Compute Q0
Store in C0005
Store tAB (DL) = tBA (DL)
in P0008

Fig. 5-9 (Cont.)

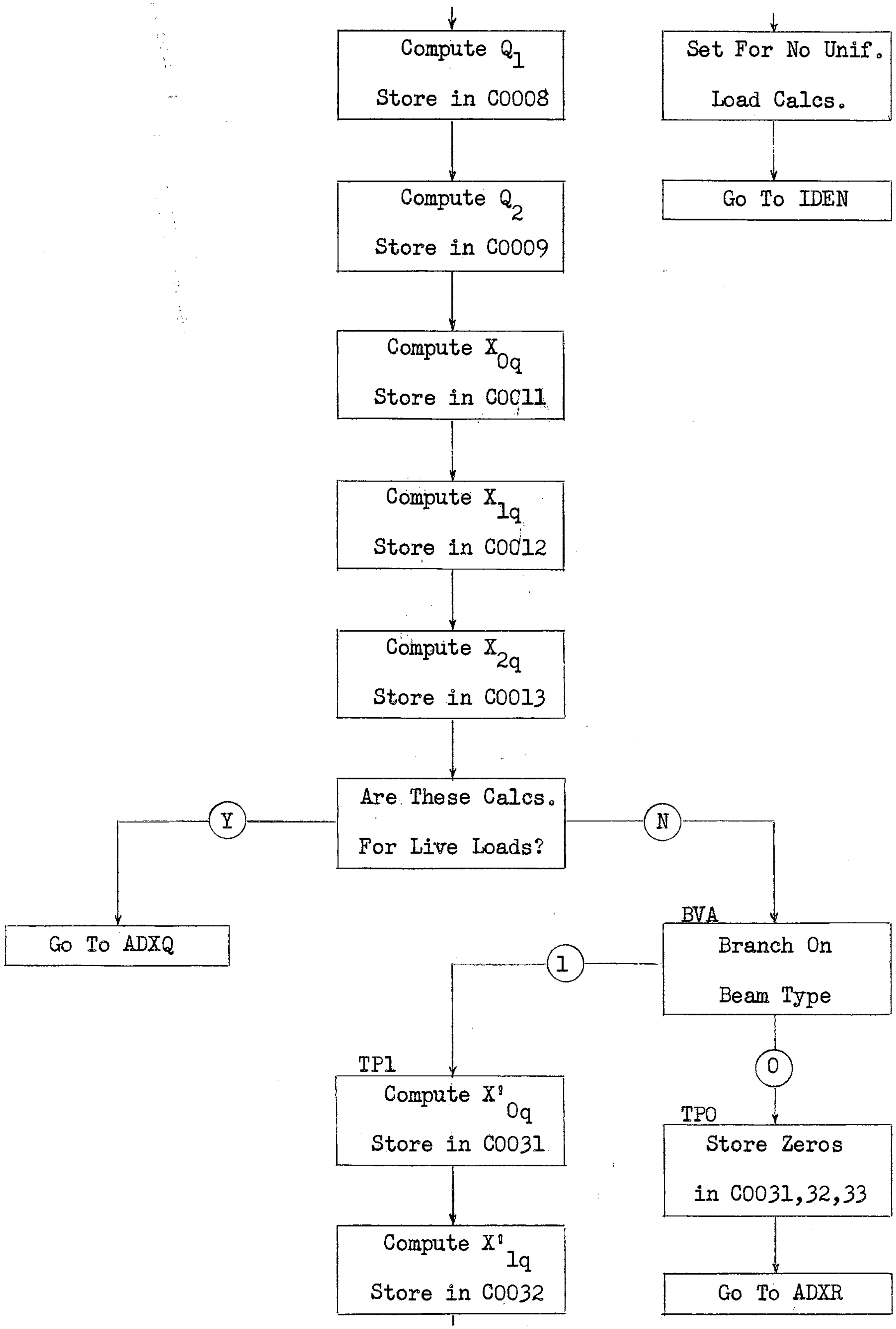

Compute Q1
Store in C0008
Set For No Unif.
Load Calcs.
Go To IDEN
Compute Q2
Store in C0009
Compute X0q
Store in C0011
Compute X1q
Store in C0012
Compute X2q
Store in C0013
Are These Calcs.
For Live Loads?
Y
N
Go To ADXQ
BVA
Branch On
Beam Type
1
0
TP1
Compute X'0q
Store in C0031
TP0
Store Zeros
in C0031,32,33
Compute X'1q
Store in C0032
Go To ADXR

Fig. 5-9 (Cont.)

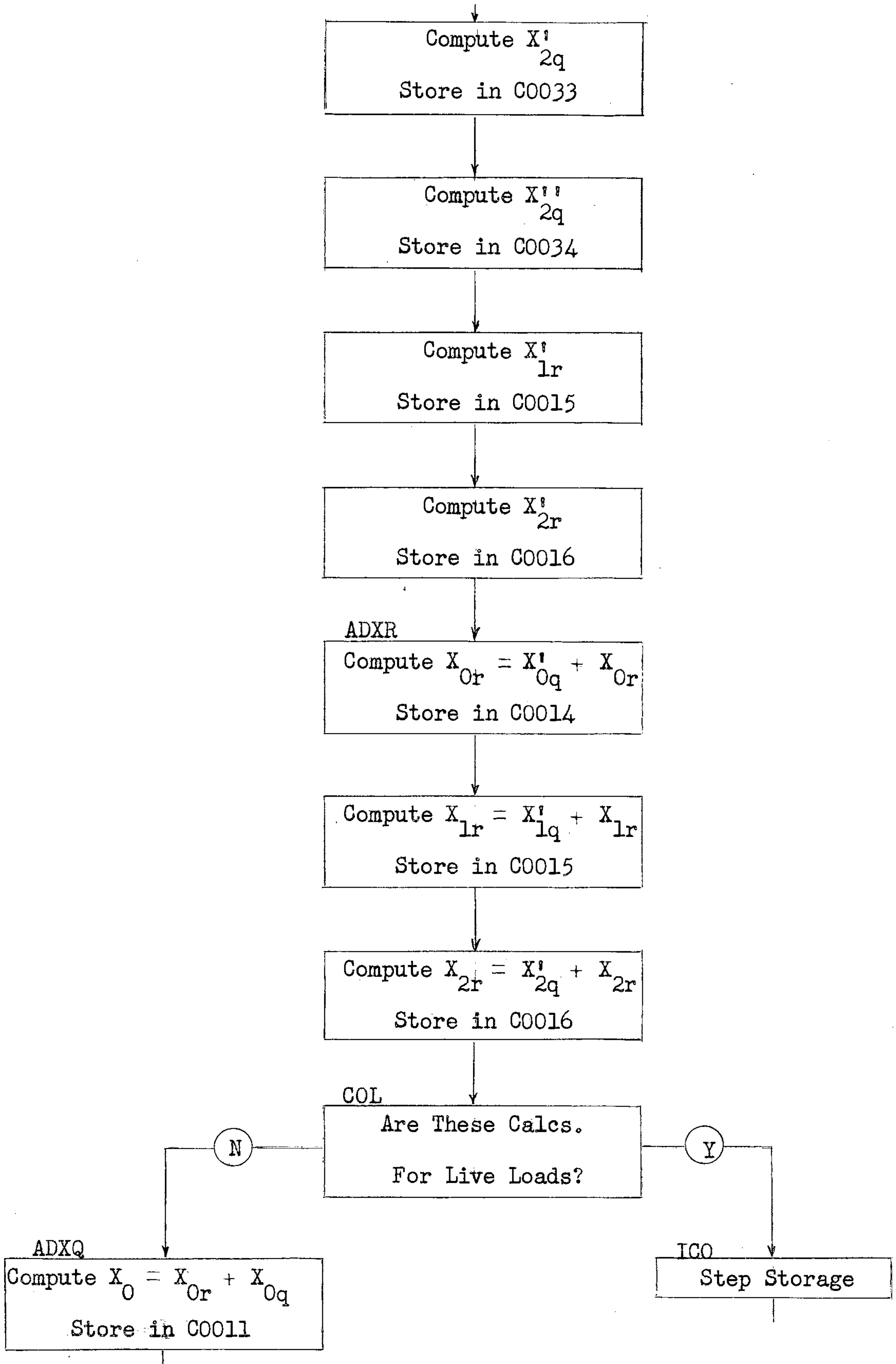
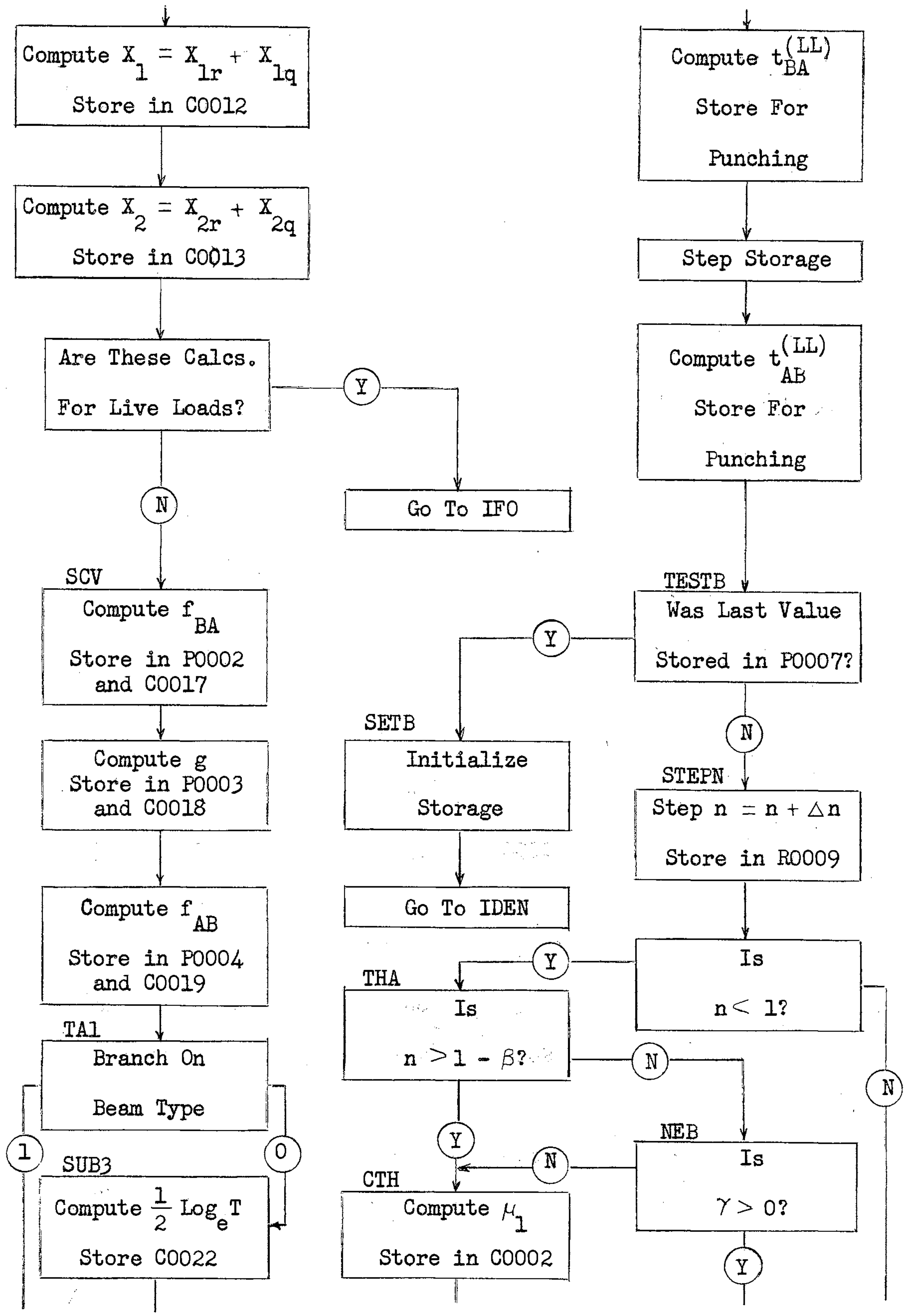

Compute X'2q
Store in C0033
Compute X''2q
Store in C0034
Compute X'1r
Store in C0015
Compute X'2r
Store in C0016
ADXR
Compute X0r = X'0q + X0r
Store in C0014
Compute X1r = X'1q + X1r
Store in C0015
Compute X2r = X'2q + X2r
Store in C0016
COL
Are These Calcs.
For Live Loads?
N
Y
ADXQ
Compute X0 = X0r + X0q
Store in C0011
ICO
Step Storage

Fig. 5-9 (Cont.)



Compute X1 = X1r + X1q
Store in C0012
Compute X2 = X2r + X2q
Store in C0013
Are These Calcs.
For Live Loads?
Y
Go To IFO
N
SCV
Compute fBA
Store in P0002
and C0017
Compute g
Store in P0003
and C0018
Compute fAB
Store in P0004
and C0019
TA1
Branch On
Beam Type
1
0
SUB3
Compute 1/2 Loge T
Store C0022
Compute t(LL)BA
Store For
Punching
Step Storage
Compute t(LL)AB
Store For
Punching
TESTB
Was Last Value
Stored in P0007?
Y
SETB
Initialize
Storage
Go To IDEN
N
STEPN
Step n = n + Δn
Store in R0009
Is
n < 1?
Y
THA
Is
n > 1 - β?
N
NEB
Is
γ > 0?
Y
N
CTH
Compute μ1
Store in C0002
Y
N

Fig. 5-9 (Cont.)

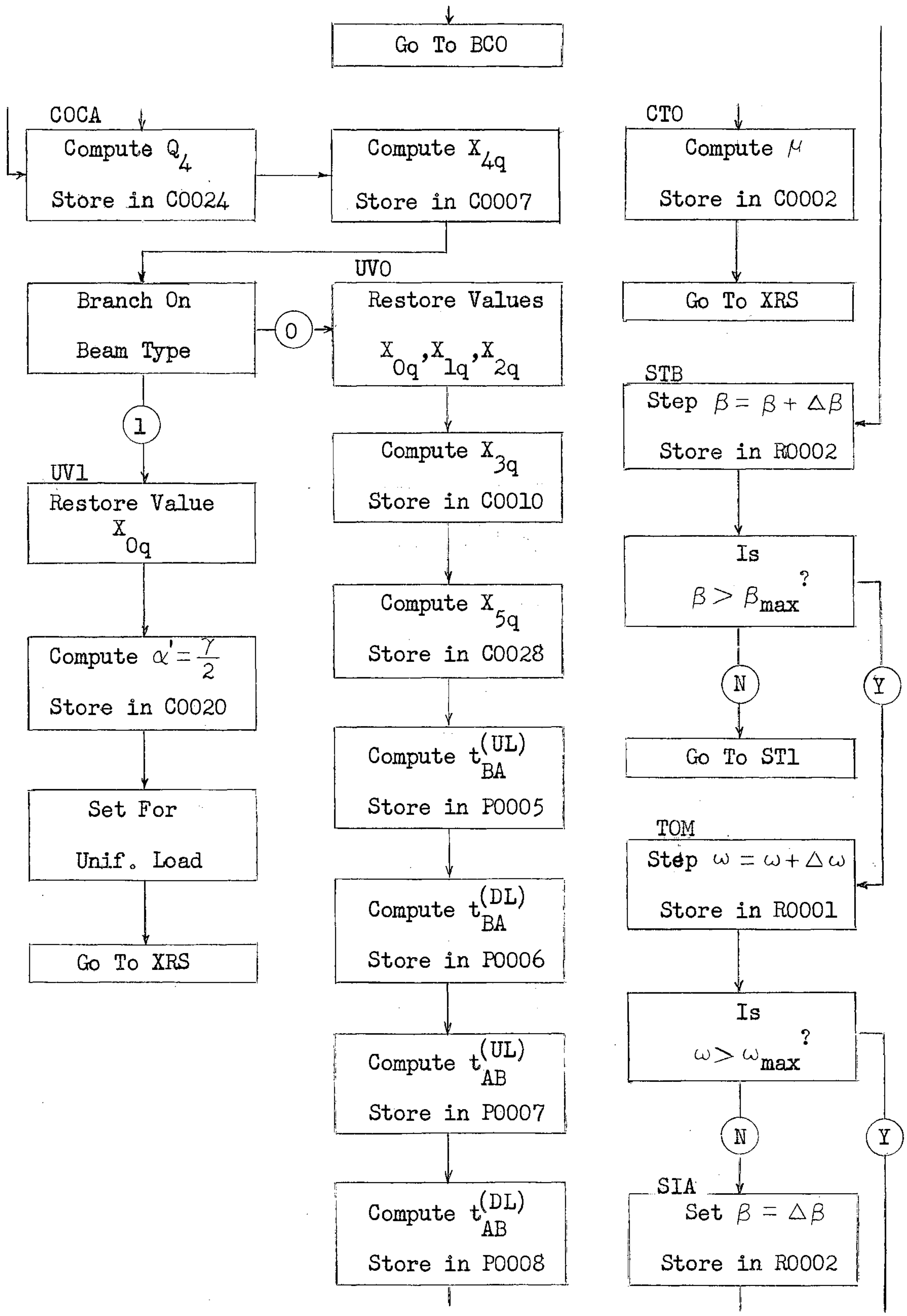

Go To BCO
COCA
Compute Q4
Store in C0024
Compute X4q
Store in C0007
CTO
Compute μ
Store in C0002
Go To XRS
UVO
Branch On
Beam Type
0
Restore Values
X0q, X1q, X2q
STB
Step β = β + Δβ
Store in R0002
1
UV1
Restore Value
X0q
Compute X3q
Store in C0010
Is
β > βmax ?
Compute X5q
Store in C0028
Compute α' = γ/2
Store in C0020
N
Y
Compute t(UL) BA
Store in P0005
Go To ST1
Set For
Unif. Load
TOM
Step ω = ω + Δω
Store in R0001
Compute t(DL) BA
Store in P0006
Go To XRS
Is
ω > ωmax ?
Compute t(UL) AB
Store in P0007
N
Y
SIA
Set β = Δβ
Store in R0002
Compute t(DL) AB
Store in P0008

Fig. 5-9 (Cont.)

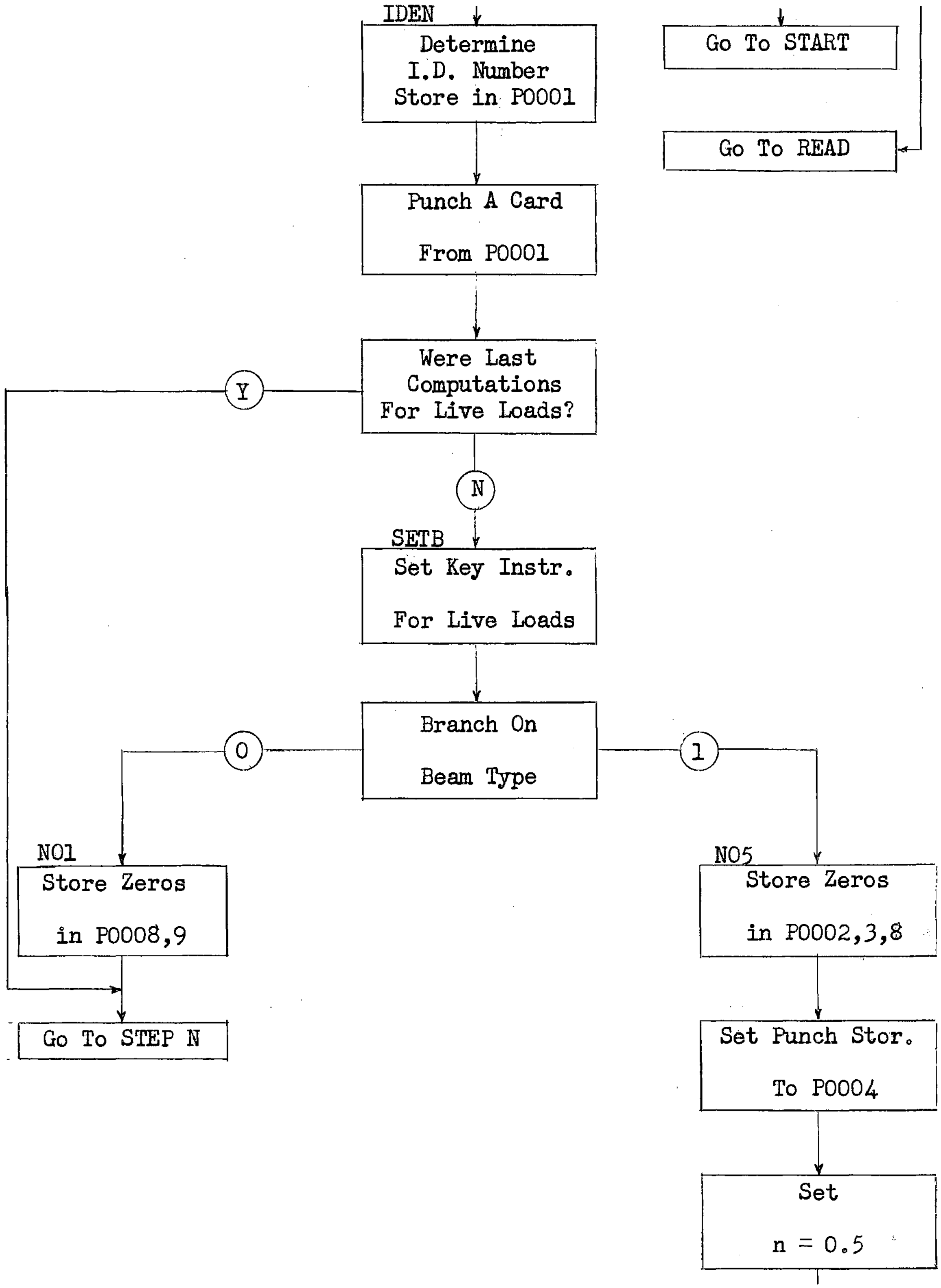

IDEN
Determine I.D. Number Store in P0001
Go To START
Go To READ
Punch A Card From P0001
Were Last Computations For Live Loads?
Y
N
SETB
Set Key Instr. For Live Loads
Branch On Beam Type
0
1
N01
Store Zeros in P0008,9
Go To STEP N
N05
Store Zeros in P0002,3,8
Set Punch Stor. To P0004
Set n = 0.5

Fig. 5-9 (Cont.)

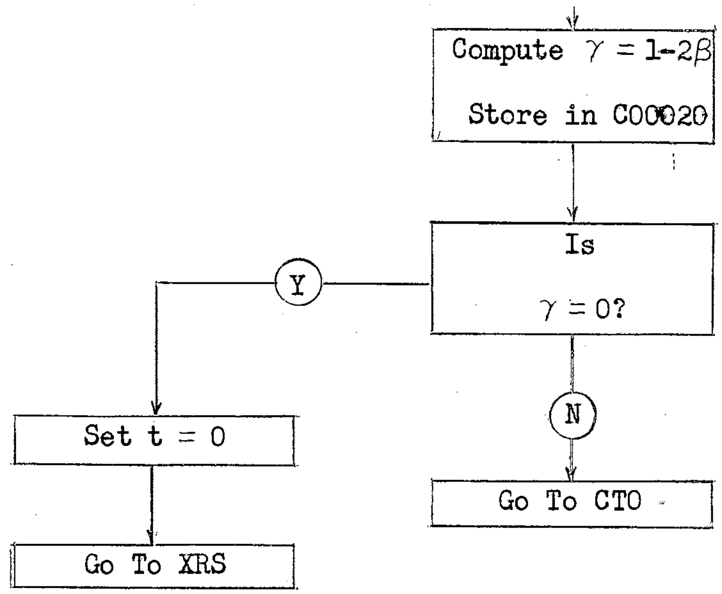


Fig. 5-9 (Cont.)

6. IBM 650 Program.

With the program in Fig. (5-10) the computer will calculate beam constants including 10-point influence coefficients for beams of either type for which is expressed as a multiple of one-tenth. Beam constants are available for beams with constant moment of inertia (2). No provision is made for them in this program and entry of ω or β equal to zero will result in an attempt to divide by zero. The computer will stop with ten nines in the distributor if α or γ become negative as would be the case if for a symmetrical beam β would be entered as 0.6. No other programmed stops are incorporated.

T P	S N	Location	Oper. Code	Data Address	T A G	Instr. Address	T A G	Remarks
1			BE	AM CO	N	STANT	S	
1			REG	R9000		9009		READ AREA

Fig. 5-10.—IBM 650 Program.

T P	S N	Location	Oper. Code	Data Address	T A G	Instr. Address	T A G	Remarks
			REG	C9010		9049		CALC AREA
			REG	P9050		9059		PCH AREA
			SYN	READ		0000		BEGIN 0000
1								
		READ	RD1	R0001		START		READ CARD
		START	LDD	ZERO				INITIALIZE
			STD	R0009				N
			STD	P0009				NO DL OP
			RAA	R0007				BM TYPE
			RAB	0000				NO LL OP
1								
			RAC	0015				TRMS SQ RT
			RAU	R0001				STORE
			FAD	ONE				INITIAL
			FMP	HALF				APPROX
			STU	C0001		SUB1		SQ RT
1								
		SUB1	RAU	R0001				COMPUTE
			FDV	C0001				AND
			FAD	C0001				STORE
			FMP	HALF				SQ RT
			STU	C0001				
			SXC	0001				
			BMC	ST1		SUB1		
1								
		ST1	RAU	ONE				SET T TO
			STU	C0002				ONE AND
			FSB	R0002				COMPUTE
			NZA	CA1		SAO		ALPHA OR
		CA1	FSB	R0002		SAO		GAMMA
		SAO	STU	C0020				AND STORE
			BMI	HAN		XRS		IF NEG
		HAN	LDD	NINES				STOP
			HLT					
1								
		XRS	RAU	C0020				COMPUTE
			FMP	C0002				AND STORE
			STU	C0014				XOR
			FMP	C0014				
			FMP	HALF				
			STU	C0015				X1R
			FMP	C0014				
			FMP	FRAC				AND
			STU	C0016				X2R
			NZB	BT2		BCO		
1								

Fig. 5-10 (Cont.)

TP	SN	Location	Oper. Code	Data Address	TAG	Instr. Address	TAG	Remarks
		BCO	NZA	TP9		CAT		IS ARC
		TP9	RAU	P0009				TANGENT
			NZU	SCUL		CAT		REQD?
1								
		CAT	RAU	C0001				COMP AND
			FMP	C0002				STORE
			STU	C0003				TANGENT
			FMP	C0003				SQUARE
			FAD	ONE				AND SET
			STU	C0004				NUMBER OF
			FSB	ONE				TERMS
			FSB	DELTN				BY MAG
			BMI	ATSV		NAT		
		ATSV	RAC	0010		ATC		
		NAT	RAC	0024		ATC		
1								
		ATC	LDD	ONE				INITIALIZE
			STD	C0022				FOR SERIES
			STD	C0023				
			STD	C0024				
			STD	C0025		SUM		
1								
		SUM	RAU	C0022				COMPUTE
			FAD	C0022				SERIES
			FAD	ONE				TERMS
			STU	C0026				
			FSB	ONE				
			FMP	C0024				
			FDV	C0026				
			STU	C0024				
			RAU	C0004				
			FSB	ONE				
			FMP	C0023				
			FDV	C0004				
			STU	C0023				
			FMP	C0024				
			FAD	C0025				SUM AND
			STU	C0025				STORE
			BMC	ARTAN		STEP		
1								
		STEP	SXC	0001				STEP FOR
			RAU	C0002				NEXT
			FAD	ONE				TERM AND
			STU	C0022		SUM		REPEAT
1								
		ARTAN	RAU	C0025				COMPUTE

Fig. 5-10 (Cont.)

T P	S N	Location	Oper. Code	Data Address	T A G	Intr. Address	T A G	Remarks
			FMP	C0003				ARC TAN
			FDV	C0004				AND
			STU	C0005				STORE
1								
			RAU	C0002				COMPUTE
			FDV	FOUR				AND STORE
			FDV	C0004				
			FDV	C0004				
			STU	C0006				B
			FMP	C0004				
			FMP	HALF				
			STU	C0007				C
			RAU	C0002				
			FMP	C0005				
			FDV	OCT				
			FDV	C0003				AND
			STU	C0003				D
1								
			FAD	C0007				COMPUTE
			FMP	THREE				AND STORE
			FAD	C0006				
			STU	C0005				QO
			RAU	C0007				
			FMP	TWO				
			FAD	C0006				
			FMP	C0002				
			STU	C0008				Q1
			RAU	C0007				
			FAD	C0003				
			FSB	C0006				
			FDV	R0001				AND
			STU	C0009				Q2
1								
			RAU	C0005				COMPUTE
			FMP	R0002				AND STORE
			STU	C0011				XOQ
			RAU	C0008				
			FSB	C0005				
			FMP	R0002				
			FMP	R0002				
			FAD	C0011				AND
			STU	C0012				X1Q
			RAU	R0002				COMPUTE
			FMP	R0002				AND STORE
			FMP	R0002				3 POWRS
			STU	P0010				OMEGA

Fig. 5-10 (Cont.)

T P	S N	Location	Oper. Code	Data Address	T A G	Instr. Address	T A G	Remarks
			RAU	C0005				COMPUTE
			FAD	C0009				AND STORE
			FSB	C0008				
			FSB	C0008				
			FMP	P0010				
			FAD	C0012				
			FAD	C0012				
			FSB	C0011				
			STU	C0013				X2Q
			NZB	ADXQ		BVA		BRN LL OP
1								
		BVA	NZA	TP1		TPO		BRN BM TP
1								
		TPO	LDD	ZERO				LT HAUNCH
			STD	C0031				XOQ
			STD	C0032				X1Q AND
			STD	C0033		ADXR		X2Q
1								
		TP1	LDD	C0011				LT HAUNCH
			STD	C0031				XOQ
			RAU	C0011				
			FSB	C0012				
			STU	C0032				X1Q
			RAU	C0013				
			FAD	C0011				
			FSB	C0012				
			FSB	C0012				
			STU	C0033				X2Q
			RAU	C0011				
			FDV	FOUR				
			FAD	C0013				
			FSB	C0012				AND MOD
			STU	C0034		XR1S		X2Q
1								
		XR1S	RAU	R0002				COMPUTE
			FMP	C0014				AND STORE
			FAD	C0015				FOR TYPE 1
			STU	C0015				X1R
			FAD	C0015				
			FDV	R0002				
			FSB	C0014				
			FMP	R0002				
			FMP	R0002				
			FAD	C0016				AND
			STU	C0016		ADXR		X2R
1								
		ADXR	RAU	C0031				SUM LT XKQ

Fig. 5-10 (Cont.)

TP	SN	Location	Oper. Code	Data Address	TAG	Instr. Address	TAG	Remarks
			FAD	C0014				AND XKR
			STU	C0014				XOR
			RAU	C0032				
			FAD	C0015				
			STU	C0015				X1R
			RAU	C0033				
			FAD	C0016				AND
			STU	C0016		COL		X2R
1								
		COL	NZB	ICO		ADXQ		BRN LL OP
1								
		ADXQ	RAU	C0011				SUM XKR
			FAD	C0014				AND XKQ
			STU	C0011				XOQ
			RAU	C0012				
			FAD	C0015				
			STU	C0012				X1Q
			RAU	C0013				
			FAD	C0016				AND
			STU	C0013				X2Q
1								
			NZB	IFO		SCV		BRN LL OP
1								
		SCV	LDD	C0013				COMPUTE
			STD	P0002				AND STORE
			STD	C0017				FBA
			RAU	C0012				
			FSB	C0017				
			STU	P0003				
			STU	C0018				G
			RAU	C0011				
			FSB	C0018				
			FSB	C0018				
			FSB	C0017				
			STU	P0004				AND
			STU	C0019		TA1		FAB
1								
		TA1	NZA	COCA		SUB3		BRN BM TP
1								
		SUB3	RAC	0018				SET TERMS
			LDD	ZERO				INITALIZE
			STD	C0022				FOR LOG
			LDD	ONE				SERIES
			STD	C0023				
			STD	C0024				
			RAU	C0004				COMPUTE

Fig. 5-10 (Cont.)

TP	SN	Location	Oper. Code	Data Address	TAG	Instr. Address	TAG	Remarks
			FAD	ONE				RATIO FOR
			STU	C0025				SERIES
			FSB	TWO				
			FDV	C0025				
			STU	C0026				
			FMP	C0026				
			STU	C0028		LOOP		
1								
		LOOP	RAU	C0024				COMPUTE
			FMP	C0026				SUM OF
			FAD	C0022				TERMS
			STU	C0022				STORE LOG
			BMC	COCA		STA		
1								
		STA	RAU	C0023				STEP FOR
			FAD	TWO				NEXT TERM
			STU	C0027				AND
			RAU	C0023				REPEAT
			FMP	C0024				
			FMP	C0028				
			FDV	C0027				
			STU	C0024				
			LDD	C0027				
			STD	C0023				
			SXC	0001		LOOP		
1								
		COCA	RAU	R0001				COMPUTE
			FMP	R0001				AND
			STU	C0023				STORE
			RAU	C0005				
			FDV	OCT				
			FSB	C0007				
			FMP	OCT				
			FDV	C0023				
			STU	C0024				Q4
			RAU	P0010				
			FMP	R0002				5
			FMP	R0002				POWRS OF
			STU	P0010				BETA
			FMP	C0024				AND
			STU	C0007				X4Q
			NZA	UV1		UVO		BRN BM TP
1								
		UVO	RAU	C0011				RESTORE
			FSB	C0014				
			STU	C0011				XOQ

Fig. 5-10 (Cont.)

T/P	S/N	Location	Oper. Code	Data Address	TAG	Instr. Address	TAG	Remarks
			RAU	C0012				
			FSB	C0015				
			STU	C0012				X1Q
			RAU	C0013				
			FSB	C0016				
			STU	C0013				X2Q
			RAU	C0008				COMPUTE
			FSB	C0009				AND
			FMP	THREE				STORE
			FAD	C0006				
			FSB	C0005				
			FMP	P0010				
			FDV	R0002				
			STU	C0010				
			RAU	C0013				
			FSB	C0012				
			FMP	THREE				
			FAD	C0010				
			FAD	C0011				
			STU	C0010				X3Q
			RAU	C0006				
			FMP	R0001				
			FMP	TWO				
			FAD	C0008				
			FMP	R0001				
			STU	C0025				
			RAU	C0022				
			FSB	C0025				
			FDV	C0023				
			FDV	R0001				
			FSB	C0024				Q5
			FMP	P0010				
			FMP	R0002				
			FAD	C0007				AND
			STU	C0028				X5Q
1								
			RAU	C0020				BRN IF
			NZU	ANO		AEQO		ALPH EQ O
1								
		AEQO	RAU	C0017		SX4R		DO NOT
1								
		ANO	RAU	C0017				DIVIDE
			FDV	C0014				BY ZERO
			FDV	DEC				IN THESE
			FSB	C0016				CALCS
			FMP	C0014				

Fig. 5-10 (Cont.)

T P	S N	Location	Oper. Code	Data Address	T A G	Instr. Address	T A G	Remarks
			FMP	DEC		SX4R		
1								
		SX4R	FSB	C0010				COMPUTE
			FMP	HALF				AND STORE
			STU	P0005				UL TBA
			RAU	P0010				
			FDV	R0002				FOUR POWRS
			STU	P0010				OF BETA
			FMP	C0017				
			FSB	C0028				
			FMP	R0001				
			FDV	TWELV				
			FDV	R0002				
			FDV	R0002				
			FAD	P0005				
			STU	P0006				DL TBA
			RAU	C0018				
			FMP	HALF				
			FSB	P0005				
			STU	P0007				UL TAB
			RAU	C0017				
			FAD	C0018				
			FMP	P0010				
			FSB	C0007				
			FMP	R0002				
			FDV	TWELV				
			FDV	R0002				
			FDV	R0002				
			FSB	P0006				
			FAD	P0007				AND
			STU	P0008		IDEN		DL TAB
1								
		UV1	RAU	C0011				RESTORE
			FSB	C0014				
			STU	C0011				XOQ
			RAU	C0020				COMPUTE
			FMP	HALF				GAMMA AND
			STU	C0020				SET FOR
			RAU	ONE				DL OP
			STU	P0009		XRS		
1								
		SCUL	RAU	C0014				COMPUTE
			FAD	C0011				AND STORE
			FDV	FOUR				
			FSB	C0016				
			FSB	C0034				

Fig. 5-10 (Cont.)

T̲ P	S̲ N	Location	Oper. Code	Data Address	T A G	Instr. Address	T A G	Remarks
			FMP	HALF				
			STU	P0005				UL TBA
			RAU	P0010				
			FDV	R0002				
			STU	P0010				
			RAU	C0014				
			FAD	C0011				
			FMP	P0010				
			FSB	C0007				
			FMP	R0001				
			FDV	TWELV				
			FDV	R0002				
			FDV	R0002				
			FAD	P0005				
			STU	P0006				DL TBA
			LDD	P0005				
			STD	P0007				UL TAB
			LDD	P0006				
			STD	P0008				DL TAB
			LDD	ZERO				SET FOR
			STD	P0009		IDEN		NO DL OP
1								
		IDEN	RAU	R0001				STORE
			FSB	ONE				FOR
			BMI	SFT1		SFT2		IDENTIF
		SFT1	RAL	R0001				
			SRT	0003				
			SLT	0002				
			STL	C0035		IDB		OMEGA
		SFT2	RAL	R0001				
			SRT	0002				
			SLT	0002				OR
			STL	C0035		IDB		OMEGA
		IDB	RAU	R0002				
			FSB	ONE				
			BMI	SFT3		SFT4		
		SFT3	RAL	R0002				
			SRT	0008				
			SLT	0004				
			STL	C0036		IDN		BETA
		SFT4	RAL	R0002				
			SRT	0008				
			SLT	0005				OR
			STL	C0036		IDN		BETA
1								
		IDN	RAL	R0009				COMPOSE

Fig. 5-10 (Cont.)

TP	SN	Location	Oper. Code	Data Address	TAG	Instr. Address	TAG	Remarks
			SRT	0006				AND STORE
			ALO	8005				
			ALO	C0035				
			ALO	C0036				
			STL	P0001				
			WR1	P0001				ID NUMBER
			NZB	STEPN		SETB		BRN LL OP
1								
		SETB	RAB	0001				INIT LL OP
			NZA	NO5		NO1		BRN BM TP
1								
		NO1	LDD	ZERO				
			STD	P0008				TYPE 0 LL
			STD	R0009		STEPN		N TO ZERO
1								
		NO5	LDD	ZERO				TYPE 1 LL
			STD	P0002				INITIAL
			STD	P0003				VALUES
			STD	P0008				STORAGE
			AXB	0002				
			LDD	HALF				AND
			STD	R0009				N IS HALF
			RAU	ONE				COMPUTE
			FSB	R0002				GAMMA
			FSB	R0002				
			STU	C0020				
			NZU	CTO		GAO		IF ZERO
		GAO	STU	C0002		XRS		LIM IS ZER
1								
		CTO	RAU	R0009				COMPUTE
			NZA	SR2		CTAL		AND STORE
		SR2	FSB	R0002		CTAL		UPPER LIM
		CTAL	FDV	C0020				FOR CONST
			STU	C0002		XRS		I
1								
		STEPN	RAU	R0009				STEP N
			FAD	DELTN				AND TEST
			STU	R0009				
			FSB	ONE				IF LESS
			BMI	THA		TAL		THAN ONE
1								
		THA	RAU	ONE				AND GRTR
			FSB	R0002				THAN ONE
			FSB	R0009				MIN BETA
			BMI	CTH		NEB		
1								

Fig. 5-10 (Cont.)

T P	S N	Location	Oper. Code	Data Address	T A G	Instr. Address	T A G	Remarks
		NEB	RAU	C0020				
			NZU	CTO		CTH		
1								
		CTH	RAU	R0002				COMPUTE
			FSB	ONE				AND STORE
			FSB	R0009				UPPER LIM
			FDV	R0002				FOR VAR
			STU	C0002		BCO		I
1								
		ICO	AXB	0001				COMPUTE
			RAU	C0018				AND
			FSB	C0015				
			FMP	R0009				STORE
			FAD	C0016				
			STU	P0000	B			LL TBA
			RAU	C0018				
			FAD	C0019				
			FSB	C0014				
			FMP	R0009				
			FAD	C0015				
			FSB	P0000	B			
			AXB	0001				AND
			STU	P0000	B	TESTB		LL TAB
1								
		TESTB	RAU	8006				IF LAST
			SUP	SVEN				VALUE IN
			BMI	STEPN		AD1		P0007
1								
		AD1	RAB	0001		IDEN		INIT B PCH
1								
		IFO	AXB	0001				COMPUTE
			RAU	C0018				FOR VAR I
			FSB	C0012				AND STORE
			FMP	R0009				
			FAD	C0013				
			STU	P0000	B			LL TBA
			RAU	C0018				
			FAD	C0019				
			FSB	C0011				
			FMP	R0009				
			FAD	C0012				
			FSB	P0000	B			
			AXB	0001				AND
			STU	P0000	B	TESTB		LL TAB
1								
		TAL	RAU	R0002				TEST IF

Fig. 5-10 (Cont.)

T P	S N	Location	Oper. Code	Data Address	TAG	Instr. Address	TAG	Remarks
			FAD	R0004				BETA IS
			STU	R0002				MAX IF NO
			RAU	R0006				STEP IF
			FSB	R0002				YES
			BMI	TOM		ST1		
1								
		TOM	RAU	R0001				TEST IF
			FAD	R0003				OMEGA IS
			STU	R0001				MAX AND
			RAU	R0005				
			FSB	R0001				
			BMI	READ		SIA		READ OR
1								
		SIA	RAU	R0004				STEP BETA
			STU	R0002		START		
1								
1			CON	STANT	S			
1								
		ZERO	00	0000		0000		
		ONE	10	0000		0051		
		HALF	50	0000		0050		
		NINES	99	9999		9999		
		FOUR	40	0000		0051		
		THREE	30	0000		0051		
		TWO	20	0000		0051		
		OCT	80	0000		0051		
		DEC	75	0000		0050		
		TWELV	12	0000		0052		
		DELTN	10	0000		0050		
		SVEN	00	0000		0007		
		FRAC	66	6666		6750		

Fig. 5-10 (Cont.)

CHAPTER VI

SUMMARY AND CONCLUSIONS

The development of a high speed computer program to evaluate beam constants for use in the carry-over moment procedure has been the purpose of this study.

The number of instructions in the program has been held to a minimum by expressing the beam constants in terms of recurring integrals which could be evaluated by supplying appropriate upper limits in equivalent algebraic expressions and by expressing succeeding beam constants in terms previously defined.

Using the program presented the computer will evaluate constants for beams with either one parabolic haunch or two symmetrical parabolic haunches for which β is expressed as a multiple of one-tenth and ω does not exceed two. If beam constants are desired for ω greater than two the number of terms in the subroutines for functional evaluation should be checked for accuracy.

The output from the computer may be listed on the IBM 402 Tabulator. Plate I shows typical results of such tabulations for a symmetrical beam for which

$$\omega = 1.0,$$

$$\beta = 0.1 \rightarrow 0.5$$

From Chapter 5 and plate I the beam for which

$$\omega = 1.0,$$

$$\beta = 0.3$$

PLATE I

Tabulated Constants
$\omega = 1.0, \beta = 0.1 \rightarrow 0.5$

1000100001	2905140250	1639384150	2905140250	4098460949	4135960949	4098460949	4135960949
1000106001	0000000000	0000000000	6178512349	6178512049	6315527449	5541497049	
1000109001	5852543049	4504481049	4689558049	3167466049	2726573049	1630451049	
1000200001	2526261050	1562787650	2526261050	3906969049	4040302349	3906969049	4040302349
1000206001	0000000000	0000000000	5964048549	5964049049	6067359749	5360737049	
1000209001	5570671049	4357426049	4373982049	3054115049	2422136049	1558226049	
1000300001	2188819950	1444752950	2188819950	3611882249	3874382249	3611882249	3874382249
1000306001	0000000000	0000000000	5606609249	5606609049	5663373649	5049844049	
1000309001	5120138049	4093080049	3896282049	2842467049	2102808049	1441250049	
1000400001	1884941050	1293156150	1884941050	3232890349	3632890349	3232890349	3632890349
1000406001	0000000000	0000000000	5106193849	5106194049	5111444549	4600943549	
1000409001	4526512049	3700761049	3371631049	2549814049	1806545049	1290088049	
1000500001	1606747650	1115873850	1606747650	2789684349	3310517649	2789684349	3310517649
1000506001	0000000000	0000000000	4462803549	4462803549	4424874449	4010423649	
1000509001	3863797249	3203876849	2857954049	2200587049	1532499049	1113038049	

is given by lines 7,8, and 9 where the identification numbers are of the form

100030n001,

and the coefficients (Fig. 6-1) are used with Eqs. (4-13) to (4-19) inclusive and related equations in chapter 3 to obtain the desired constants. The angular live load functions for

$$n = 0.1,\ 0.2,\ 0.3,\ 0.4$$

are obtained from symmetry.

Beam constants have been computed for all combinations of

$$\omega = 0.1 \rightarrow 2.0$$

$$\beta = 0.1 \rightarrow 1.0$$

for unsymmetrical beams and

$$\beta = 0.1 \rightarrow 0.5$$

for symmetrical beams. Comparisons were made with values presented in (2) and the results published in (3).

f_{BA}	g	f_{AB}	$t_{BA}^{(UL)}$	$t_{BA}^{(DL)}$	$t_{AB}^{(UL)}$	$t_{AB}^{(DL)}$
.2189	.1445	.2189	.0361	.0387	.0361	.0387

(a)

n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$t_{BA}^{(LL)}$	.0144	.0284	.0409	.0505	.0561	.0566	.0512	.0390	.0210
$t_{AB}^{(LL)}$	.0210	.0390	.0512	.0566	.0561	.0505	.0409	.0284	.0144

(b)

Fig. 6-1.—Beam Constants For $\omega = 1.0$, $\beta = 0.3$.

Computations requiring the use of these constants are normally such that four place accuracy is sufficient.

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