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USING TECHNOLOGY TO FACILITATE VISUALIZATION IN MULTIVARIABLE CALCULUS

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USING TECHNOLOGY TO FACILITATE VISUALIZATION
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A Dissertation APPROVED FOR THE
DEPARTMENT OF MATHEMATICS

BY

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Introduction

Technology is quickly transforming the way mathematics is taught and learned. Furthermore it has begun to produce radical changes in *which* mathematics is taught and learned, as some of the tedious tasks that formerly constituted a large portion of the curriculum come to be delegated to calculators or computers, leaving higher-order tasks to humans. Many of these changes are happening quietly, without organized study of their nature or consequences. This study proposes to document and analyze one small portion of this larger movement.

This research examines some of the effects of the use of a computer algebra system (CAS), *Mathematica*, by students in multivariable calculus classes. In particular it examines some effects of CAS use on ability to visualize, especially visualization useful in multivariable calculus. This study has developed in stages, beginning with broad data gathering, and subsequently continuing with more focused collection of data intended to explore specific patterns which arose in earlier stages. The intention of this sort of cyclical design is to allow research to evolve over time to focus on salient developments as the project develops (Confrey & Lachance, 2000).

This research was carried out in Calculus IV (and in some cases Calculus III, for the sake of contrast) classes at the University of Oklahoma (subsequently abbreviated OU) between the Spring 1999 and Summer 2002 terms. OU has a four semester, three credit per semester calculus sequence. Calculus I is primarily differential calculus, Calculus II is mostly integral calculus, and Calculus III is an assortment of miscellaneous topics including series, parametric functions, and polar coordinates. Calculus IV is the culmination, treating functions of more than one variable, with topics including their derivatives and integrals, as well as the standard generalizations of the Fundamental Theorem of Calculus such as Stokes' and Green's Theorems. Specifically, during the period of this research, the course covered Chapters 12-14 (Partial Derivatives, Multiple Integrals, and Vector Calculus) of Stewart (1995) which was in later years renumbered as Chapters 14-16 of Stewart (1999), although the actual material remained essentially unchanged.

The largest constituency of students in the Calculus sequence at OU is consistently engineering majors, with considerably smaller numbers of geoscience, physics, and mathematics majors, and with nearly all students taking the course to satisfy requirements for their majors. Fuller description of the actual participants will be given in the Conclusions chapter. Sections of Calculus IV at OU generally have enrollment limited to around 35 students, and are taught predominantly by faculty but occasionally by adjuncts and experienced graduate teaching assistants. Instructors are given considerable freedom to follow their individual tastes, so the textbook is perhaps the only truly common factor among the various sections.

The central focus of this dissertation was investigating a phenomenon which was noticed from the earliest stages of this work. Students who worked with a CAS showed marked, but very selective, improvements in their abilities to perform certain visual tasks. Exploring this improvement, particularly isolating what was improving and what caused it to improve, was the primary goal of this work.

It should be recognized that this research comes more than a decade into what is known as the calculus reform movement, with its intentions to update and improve the calculus curriculum in a variety of ways. Principal among those ways is the integration of technology, and while this has generally meant graphing calculators, this naturally extends to computers for the more advanced classes. Visualization has also been a prominent concern in the reform movement, so this research addresses several current themes.

In addition, I used the large body of data now at my disposal to investigate the interaction of factors such as gender with the effects of technology. The intention here was not to duplicate the large body of existing research on these matters. However, little of the existing research addresses the particular population or outcomes examined here, so the question of whether patterns noted elsewhere extend to undergraduates taking multivariable calculus is a legitimate one. Furthermore, if this research is to have any value in deciding whether CAS use is beneficial or detrimental, it seems only diligent to consider the possibility that the answer depends upon whose benefit is being considered.

Some particular questions that this work seeks to address:

- **Visualization**: To what extent can previously observed differences between CAS and non-CAS students' visualization skills be replicated? With the large data set available, what other differences in visualization skills can be detected?
- **Gender**: What gender differences exist in these visualization skills? How does gender influence attitudes about computation and visualization?

- **Technology**: Does degree of CAS use, as opposed to simple presence in a classroom, affect these visualization skills? Are there differences between CAS and non-CAS students' attitudes?

I also note that this research is timely: These technologies have only been available for a few years now, and may soon be widespread enough to make access to "control" populations highly problematic. Now is the best time to explore these issues.

Literature

There is a considerable body of literature on visualization in mathematics, as well as a considerable body of literature on the use of technology in the learning of mathematics, not to mention literature on the role of gender in mathematics. The intersection of these sets, however, is relatively small. In addition, the amount of research dealing with the learning of mathematics at the undergraduate level is, as usual, much smaller than that at the K-12 levels. The net effect of all this is that while there is much literature which touches on the material involved in this project, there is little which could be considered highly related.

I note from the outset that the term “visual ability” and variations on it are problematic. Some researchers use the term “ability” to refer to an underlying faculty, whereas “skill” is used for the actual operational performance. However, precise definitions of these terms are rarely agreed upon, and there are difficulties of a philosophical nature involved which are simply beyond the scope of this dissertation. The construct of a monolithic “ability” is sufficiently suspect that I prefer to avoid it, particularly in phenomenon-driven work such as this. I avoid the distinction and use the terms interchangeably, following Sorby (1999) among others.

Reform

Much of what follows should be understood in light of a larger context. In 1985 a Calculus Workshop, funded by the Sloan Foundation, was held at Tulane University, and resulted in the publication of *Toward a Lean and Lively Calculus* (Douglas, 1986). The intention was an overhaul of the existing Calculus courses, although what sort of overhaul was in order remained unclear even among those who were certain that one was necessary. In January of 1987 the National Science Foundation (NSF) announced the Calculus Program, the funding impetus behind much of what came generally to be known as the Calculus Reform movement (Haver, 1998). The deliberate and prolonged support of the NSF is frequently credited for much of the extent of the Reform movement. This movement had several goals, including:

> (a) extensive recognition of technology, both as a tool for learning and as the context in which mathematics is currently used, (b) substantially more applications of Calculus, both as a way of understanding a variety of everyday phenomena and as a tool in other academic disciplines, (c) more explicit expectations that students work and study as members of teams with other students, and that they work on long-term, demanding projects and problems in addition to short exercises, and (d) restructured courses of study that are designed to assure that students achieve a deeper understanding of Calculus from a geometric and numerical as well as analytical point of view. (Haver, p. 7)

Thus the integration of technology into the Calculus curriculum has been an at least somewhat systematic effort, but as one aspect of a larger program. The current study, while recognizing that this context is significant in many ways, attempts to focus on selected issues associated with technology, leaving connections with the larger picture aside.

Visualization

Bishop (1989), summarizing research on visualization in mathematics education, wrote that "An interactive computer environment, particularly when dynamic visual images are employed, can encourage and to some extent develop the pupils' visualization abilities." (p. 13) Although the evidence for this assertion is on the whole rather mixed, the positive indications are substantial enough to merit serious consideration.

Much foundational work on visualization in mathematics was done by Krutetskii (1969) in the Soviet Union through the 1950s and 60s. Working within a framework that mandated an understanding of ability as something which any individual could acquire, his research focused on habits of mind and tendencies among students considered particularly mathematically apt or inept. His "genetic" research methods focused heavily on observations and interviews with schoolchildren, and to some extent anticipated recent trends toward qualitative research in mathematics education.

One noteworthy and especially relevant piece of research on visual ability is Ferrini-Mundy's attempt at spatial training for college calculus students (1987). Students in beginning calculus classes were randomly assigned to either an audiovisual spatial training program, an audiovisual-tactual version of the same program, or a control group which received no special training. Performance on subsequent calculus exams, including a subscore on volumes of solids of revolution, was then compared for the three different treatments. An ANCOVA revealed no main effects, but there were some interaction effects with gender. Specifically, total calculus scores showed treatment by gender interaction, and scores on volumes of solids of revolution also showed treatment by gender interaction. These results are far from clear, but at least suggest that gender should be carefully considered when evaluating effectiveness of interventions on visual abilities.

Presmeg has published a considerable body of material (1986, 1992) pertaining to visualization and imagery, particularly in the high school classroom. Much of her work builds on the foundation of Krutetskii's writing on visualization, particularly his interviews with gifted mathematics students. In particular, she has provided a definition of a *visual image* as "a mental scheme depicting visual or spatial information" (1986, p. 297) which has been adopted by many other researchers in part because its generality allows for inclusion of phenomena such as symmetry which might be excluded by more restrictive formulations. Other influential contributions include definitions of *visual* and *nonvisual* methods of solution to mathematical problems as:

> A visual method of solution is one which involves visual imagery, with or without a diagram, as an essential part of the method of solution, even if reasoning or algebraic methods are also employed.
> A nonvisual method of solution is one which involves no visual imagery as an essential part of the method of solution. (p. 298)

She also provides working definitions of *visualizers*, *nonvisualizers* and *mathematical visuality* which have been followed by other researchers. In more recent writings Presmeg has also begun (1992) to explore the role of imagery in mathematical thinking, including mathematical processes which might be considered highly analytic. Much of Presmeg's work draws heavily from student interviews to illustrate her contentions that much, if not all, of students' mathematical knowledge is tightly linked to mental images.

Murphy has published several previous works on the earlier stages of the research involved in this dissertation (Murphy, Goodman & White, 1999; Murphy, et al.,1999; Murphy, 1999a; Murphy 1999b, Murphy & White, 2001). While some of this work was directed at the particular visualization issues that are the focus of this dissertation, more dealt with students' use and perceptions about various approaches to learning multivariable calculus. This dissertation to some extent represents the culmination of this research program.

There are also some cautionary notes regarding visualization. Among others, Aspinwall, Shaw, and Presmeg (1997), have discussed instances of students whose use of mental images becomes an obstacle. They point out that they are not saying something new, quoting Galton's 1880 comment that "An over-readiness to perceive clear mental pictures is antagonistic to the acquirement of habits of highly generalized and abstract thought and if the faculty of producing them was ever possessed by men who think hard, it is very apt to be lost by disuse" (302).

Vinner (1989) has argued against the conventional division of students into visual and non-visual, or at least against the conventional means of so dividing them. He instead suggests that traditional instruction, particularly in conjunction with traditional testing methods, values algebraic approaches more highly, so that students respond with these privileged approaches in formal settings. An emphasis on routine problems only exacerbates this pattern. Vinner suggests that when faced with more novel problems, students are more apt to employ visual strategies, and that existing but sublimated visual tendencies should be recognized and encouraged.

Gender

Gender is often thought to affect spatial abilities and, perhaps as an indirect effect, mathematical achievement. However, evidence for these contentions is mixed. Many studies suggest that gender differences in attitudes toward math, and notions of what mathematics is and what constitutes success in math, underlie apparent differences in performance (Pedersen, 1990). Perhaps in part because females at the upper levels of the mathematics curriculum are not representative of the general population, and perhaps in part because of ongoing social trends, research has revealed few clear patterns. Similarly, other under-represented groups are often taken to perform differently, but the evidence resists simple generalizations.

Friedman (1995) undertook a meta-analysis involving 75 previous studies on correlations of gender differences in spatial, verbal, and mathematical skills. As she says:

> Whether targeted on gender differences or not, research reports often rely on correlational evidence in one form or another: factor analysis, path analysis, regression equations, as well as simple correlations. The results are frequently contrasting, both in the approximate size of correlations found and the conclusions drawn from them. (p. 23)

Friedman finds only one overall pattern in the data:

> For students under high school age, the average differences between verbal-mathematical and spatial-mathematical correlations were often larger for females than for males, though the statistical significance of differences was roughly the same. (p. 35)

However, when restricting attention to only the more academically selective data, Friedman found math-space correlations were significantly higher for females than males. Thus although the connection might not be strong in the general population, it might be precisely among the population dealt with in the present study that it is most pronounced.

Sorby, Leopold, and Górska (1999) have compared performance on a variety of widely-used tests of visual ability by first-year engineering students in the United States, Germany, and Poland . They found several significant gender differences on several of the tests, with the differences being largest in Germany, smaller in Poland, and least pronounced in the U.S. They also detected correlations with several background factors, including play with construction toys and drafting experience, and credit these background factors with at least some portion of the differences in visual performance. In other work, Sorby and Baartmans (1998) found that participation in a course intended to improve spatial skills increased retention for women with weak visualization skills. Thus, background and training may be important factors in determining visual skills and thus academic success in mathematical fields of study, particularly for women.

Seymour and Hewitt (1997) have made a remarkably extensive study of retention issues in science, mathematics, and engineering fields with particular attention to women and minorities. With interviews and focus groups at a large variety of institutions, they have uncovered several factors described by many students who fail to persist in mathematics and related areas. They liken these factors to icebergs, in the sense that only a small portion is apparent – they see the actual attrition as the visible feature of a larger phenomenon affecting most or all students in such majors. As they put it in their conclusion:

> It is also clear from our data that the most effective way to improve retention among women and students of color, and to build their numbers over the longer-term, is to improve the quality of the learning experience for all students – including those non-science majors who wish to study science and mathematics as part of their overall education. (Seymour & Hewitt, p. 394)

They recommend curriculum changes aimed at actual student comprehension rather than content, and suggest that attention to how students actually learn has been the key missing component in many previous attempts to promote retention.

One researcher whose work should certainly be noted is Treisman (1990). In an attempt to replicate the academic success frequently found among Asian-American students, he founded what is generally known as the "Workshop" approach originating at the University of California, Berkeley, which emphasizes cooperative group learning rather than remediation. The success of such programs has been striking, and has focused some attention on social forces, rather than simple ability and preparation, as important factors in success for minorities in mathematics. In a followup study, Murphy, Stafford, and McCreary (1998) followed subsequent course enrollment and degree paths of students who had participated in a Treisman-style workshop calculus program and noted particular effectiveness for women and Hispanic students.

Before proceeding to survey the literature on technology, it might be fitting to consider one other theme of the research on gender and mathematics: According to *Gender Gaps: Where Schools Still Fail Our Children* (1998), a report of the American Association of University Women, there remain serious differences in the style and quantity of computer use by males and females. While the topics of gender, and technology are divided in the structure of this chapter, that division should not be taken to indicate any lack of interactions.

Technology

In a recent editorial surveying research on technology in mathematics education , Olson and Sakshaug say

> Addressing the role technology plays or should be playing in mathematics ... is akin to the blind mice describing the elephant. The overall beast is huge, and each point where one stops deserves further examination, exploration and discussion. In any one area, the body of research relating to using technology in the teaching and learning of mathematics can be addressed in terms of what has been done, what is being done, and what still needs to be done. (p. 8)

As a consequence of this, the following notes about the existing research should be taken as an extremely brief and incomplete synopsis. No effort has been made here to address every aspect of the literature. Only work which is especially relevant to this dissertation, or which in some sense reflects broader currents, has been mentioned here. This is not out of any intention to discount particular work, but rather a necessary consequence of the hugeness of the "beast."

The nature of the changes wrought on mathematics education by the advent of technology has been the subject of considerable discussion. Borba (1995), surveying research on computers in mathematics education, grouped the effects as "quasi-empirical studies in the classroom" (wherein the power of graphing utilities made possible exploratory activities which would have been impractical with pencil and paper alone), "use of multiple representations" (wherein the former hegemony of algebraic methods was lost as graphs, tables, and other representations became available), "emphasis on visualization" and "emphasis on tables" (due again to the ease of producing these with technology).

Dunham (1999) claims that one of the most profound effects of graphing calculators in mathematics classrooms is a change in the learning environment, wherein students become more active and engaged learners, and also summarizes a wide and varied array of research pointing to other benefits of technology use. In particular she notes that there appears to be a "leveling" effect whereby various traditionally disadvantaged groups benefit most from technology use.

One especially relevant piece of research is the work of Habre (2001) on visualization of three-dimensional surfaces by multivariable calculus students. Students who had undergone a multivariable calculus course which specifically attempted to emphasize visualization and computer use answered a questionnaire and were interviewed to assess their abilities and tendencies to visualize three-dimensional surfaces. The students were scored for their Mathematical Visuality (subsequently abbreviated MV) as defined by Presmeg (1986, p. 298). Among other interview tasks, students were asked to compute the double integral of the function $f(x,y) = x + y$ over the region $[-2,2] \times [-2,2]$ and interpret their answers (the value of the integral is zero, since there are matching volumes above and below the *xy*-plane). It was precisely the three students whose MV scores were above the median who were successfully able to explain the answer of zero. The other students tended to question their algebraic accuracy and express surprise upon obtaining zero. Habre notes that nearly all of the students had on the questionnaire expressed the belief that visualization was a necessary component in the teaching and learning of multivariable calculus, but that this seldom accorded with students' actual tendencies to choose or avoid visual approaches. According to Habre, "This means that, at the rhetorical level, the students' attitudes have changed but in practice they have not." (p. 43)

Some work has been directed specifically to the consequences of computer use on attitudes toward mathematics. Ganguli (1992) used graphing software which is quite crude by today's standards in experimental sections of an intermediate algebra class at a large Midwestern state university, and found significant improvement on attitudes toward their teachers, decrease in anxiety toward mathematics, and increase in "self-concept, enjoyment, and motivation regarding mathematics." (615)

Tall and Thomas (1989) have suggested on the basis of several studies that computer use in what they term "the enhanced Socratic mode" makes students more versatile learners, able to transition fruitfully between serialist/analytic strategies and global/holistic strategies. They found that high school students scored higher, especially in delayed tests, when they had undergone instruction featuring computers than in a conventional curriculum. They also found a particular increase in "higher order skills" for computer-using students than for control groups.

Although much research into the consequences of technology for mathematics instruction is simply addressed to showing that technological interventions are superior to existing practices, there has been some more discerning research into the nature of the changes wrought. Schwarz and Hershkowitz (1999) investigate the effects of a curriculum redesign on students' understandings of the concept of function. They approach these questions by way of the idea of "prototypes," i.e. particularly exemplary instances used as representatives of entire classes. Thus the common student mistake of treating all functions as linear is understood as a faulty extension to all functions of a quality particular to a prototype. The research concluded that use of multirepresentational software made students' use of prototypes more judicious, although this did not always correspond to correctly completing problems.

It is perhaps also necessary to mention the considerable body of publication by researchers attempting to apply Dubinsky's Action-Process-Object-Schema (APOS) ideas to undergraduate mathematics students, and much of this effort touches on the use of technology. Asiala, Cottrill, Dubinsky, and Schwingendorf (1997), for instance, attempts to understand the ways students conceive functions graphically and how this influences their understandings of calculus concepts such as limits and derivatives. The fruitfulness of these efforts has been questioned by Schoenfeld (2000), among others.

It could be noted that many of the studies referred to here might be subject to criticisms regarding their methods. Comparisons to "control groups" are problematic, since the novelty alone of a new approach can energize both teachers and students without producing sustained differences. Additionally, the suitedness of individual instructors for particular instructional approaches is a factor which cannot be truly controlled for. With these considerations in mind, however, it is still broadly true that educational researchers seem to find many possibilities for the use of technology in enhancing instruction in mathematics.

There also exists, it should be mentioned, a considerable body of literature warning of potential dangers associated with the use of technology in the mathematics classroom. Some of this is research-based, and a great deal of it is not. One instance, perhaps as representative of the genre as it is possible to be, is Goldenberg (1998), who warns of a variety of potential problems which offset the virtues of graphing calculators. Many things could be said regarding such writings, but for the present purposes it might suffice to point out that they indicate considerable passion from the mathematical community on topics relating to instructional technology, and that at least some of the efforts currently underway are taking place in a reflective manner.

Methods

Prior to any other discussion of the methods used in this research, it is important to recognize that mixing the roles of researcher and instructor is a threat to validity. In the spirit of McKnight, Magid, Murphy, & McKnight (2000):

> Threats to validity should be discussed explicitly in any research report. It is better to acknowledge the threats, even when no solutions were found, than to ignore them... The special threats to the validity of teaching experiments should be recognized and care taken before choosing this approach. (p. 88)

Thus a teaching experiment, that is, research in which specific teaching practices are employed, often by the researcher as instructor, to determine their effects, presents particular difficulties. Specifically, the instructor-researcher combination is a factor in the possibility of generalizing any conclusions of the present work

In the case of the present study the nature of the research questions made an instructor-researcher combination expedient. Because many of the faculty in the department were hostile to the use of technology, instructors willing to participate wholeheartedly were not readily available. Furthermore, the involvement of multiple instructor-researchers over the course of the full project may help to ameliorate concerns somewhat. If a particular instructor was biasing observations, it could be hoped that pooling with another instructor could at least dampen that effect, or better yet the effect might be revealed by contrast within the experimental groups.

There is also a case to be made for researchers who are fully immersed in the teaching they are examining. As Steffe and Thompson (2000) have written, “A primary purpose for using teaching experiment methodology is for researchers to experience, firsthand, students' mathematical learning and reasoning." (p. 267) Just as a researcher who is "too close" to the objects of study might be blinded to obvious explanations, a researcher who is "too far" from the objects of study might be seduced by explanations which are obviously unreasonable to someone more familiar with the realities of the full classroom experience, or overlook factors which are obviously important to those in the classroom. Neither stance has any automatic claim to objectivity, and in fact the construct of an "objective researcher" is itself suspect. Rather than presume or pursue objectivity, researchers might more fruitfully accept the limitations of their viewpoints and embrace any advantages of those particular perspectives.

It is in this general spirit that this project has been pursued. There are inevitable limitations inherent in the role of instructor-researcher, and those must be acknowledged. Simultaneously, there may be advantages to the role. In the end, it would be best to sharply limit overly broad claims based on this work – perhaps suggesting only that these results might be expected with other similar instructors. More will be said regarding the possibilities of generalizing this work in the Conclusions chapter.

Overview

The project has proceeded in a cyclical manner. Each iteration has involved planning, some form or forms of data collection, and analysis of these data. The results of each iteration have then guided the following work, so that promising approaches could be followed. When the project was initiated, there was no preconceived roadmap beyond a general intention to explore the effects of technology use on calculus students.

The description that follows will be basically chronological, detailing the work done in each semester. It is not entirely possible to separate the results in each previous step from the questions investigated in the following iteration, but to the greatest extent possible this chapter will focus on the approaches undertaken and specifics regarding the results will be delayed until the following chapter.

The following Table 3.1 provides a summary of the data collected. The numbers in the second column include all students taking at least one of the questionnaires.

Table 3.1
Summary of Data Collected

Term	Sample	Major Changes
Spring 1999	3 CAS Calculus IV sections (n = 81) and 3 non-CAS Calculus IV sections (n = 66)	▸ Initial Iteration ▸ Single Questionnaire
Fall 1999	2 CAS sections of Calculus IV (n = 71) and 3 non-CAS sections of Calculus IV (n = 72) Interviews (n = 7)	▸ First & Second Questionnaires ▸ Mesh/Color graphs used ▸ Task-based Interviews conducted with select students ▸ Added Rolling Box Item
Spring 2000	6 non-CAS sections of Calculus IV (n = 205), 1 section of Calculus III (n = 37), and 3 sections of Calculus III (n = 101) taught by Murphy and White	▸ Expanded items for use and usefulness
Summer 2000	1 CAS section of Calculus IV (n = 34)	▸ Reverted to core items
Summer 2002	1 CAS section of Calculus IV (n = 20) and 2 non-CAS sections of Calculus IV (n = 39)	▸ Added "How Hard" and "Which Approaches" items

Treatment

Traditionally, a great deal of attention would be paid to specifying details of the exact ways in which the "experimental" groups were treated differently from the "control" groups. While this approach has undergone some serious scrutiny and revision (Kelly & Lesh, 2000), still most of the work here is sufficiently quantitative that it might be reasonably subjected to conventional expectations. However, since in the earliest iterations significant differences were detected without rigorously applied treatments, attempts to replicate that pattern faced some problems.

One difficulty in educational research with repeating precisely the same curriculum is that once material has been used, it will to some extent pass into general circulation among students. "Test files" and the notes of previous students make it impractical to repeat problems which are too closely related. There is a substantial danger of students learning how to perform frequently-appearing types of problems in a rote manner, and this can undermine both research and pedagogical validity.

Another issue is that quality teaching often must adapt to each group of students, and sometimes to individual students, in drastic ways. At the least, one student's questions in the classroom can seriously alter the impressions with which everyone leaves, and this could hardly be replicated perfectly in later semesters – even if such lasting impressions could be properly identified. Further, an instructor attempting to follow a strict program when it seems ill-suited to the circumstances at hand is unlikely to be completely comfortable. Aside from the ethical issues of requiring instructors to follow an educational program that seems inappropriate to them, it is unlikely that such discomfort will lead to an optimal experience for students.

Beyond this, groups of students with different abilities, backgrounds, and interests will certainly have very different experiences even in the exact same class. Educational phenomena are complex things, with interactions that defy easy simplification. Without indulging overly in post-modern theorizing about the subjectivity of experience, it is still necessary to recognize that attempts to replicate educational experiences face fundamental difficulties.

For all of these reasons, we constrained teaching practices as little as possible throughout this work. The CAS sections made extensive use of *Mathematica* in a variety of ways, as outlined below, but only as seemed appropriate to the instructors at the time. Instead of mandating CAS use, either by ourselves or our students, we attempted to encourage and support CAS use. For purposes of analysis, our questionnaires included items to measure actual CAS use, so that at least in a rudimentary way we could verify that results for CAS sections coincided with actual CAS use.

Another traditional feature found in work such as this is random assignment of experimental subjects to treatment or control groups. There are important advantages to such randomization, particularly with regard to satisfying certain assumptions of many common statistical tools. No such attempt has been made for the present work, and it would have been difficult to do such a thing under these circumstances – the institutional structure at OU inhibits such things, and students' schedules are subject to numerous pressures which would make randomization a problem. Apart from these practical considerations, there are further issues at stake. Measures appropriate to laboratory science do not necessarily carry over to education in straightforward ways. Random assignment might, for instance, split up a cohort of friends who had taken other classes together and worked well together. In such a case the randomization itself – rather than the intended experimental treatment – might produce marked effects, both on educational and personal levels. Although it might be hoped that such effects would be evenly distributed between experimental and control groups, still it should be recognized that the results could be drastically different than what would have occurred in a situation without artificial intervention. For the present study, effort was made to keep unnecessary intervention to a minimum, both for the sake of examining an educational situation as natural as possible and out of respect for the student participants. I acknowledge that this has repercussions where statistical matters and generalization are concerned, but judge those preferable to the alternatives.

CAS-Section Features

One of the standard features of CAS sections was a session very early in the semester devoted to introducing students to the software. This generally took place during approximately half of a class period, with the entire class session held in a classroom/lab (it was not possible, due to limited facilities at OU, to have most class meetings in computer-equipped spaces). In most cases a relatively easy assignment was given which either required or benefitted heavily from CAS use, and that assignment served to structure the students' introduction. A small but carefully chosen collection of examples was given, and a reference sheet with key examples of *Mathematica* syntax was provided. In most cases this introductory session took place during the second half of a class session devoted to limits of functions of several variables. This is a topic for which graphical representations can be especially effective, especially for discontinuous functions of two variables, so it serves as a natural demonstration of the power of a CAS.

The assignments which accompanied this introduction to *Mathematica* were somewhat varied, but had common features. Most included at least a few problems involving Calculus I or II material of the most tedious variety in order to demonstrate the power of a CAS in dealing with such things (see Figure 3.1 for some examples from Spring 1999):

> 1. Some Calc I problems are ugly enough that even though you know perfectly well how to do them, it's much more reasonable to let Mathematica do the messy part. Use Mathematica to find the derivative of f(x) = $\sqrt{1-\sqrt{2-\sqrt{3-x}}}$. [Stewart, Problems Plus p. 180 #13b].
>
> 2. You probably don't need to be told that the same goes for some Calc II problems. Use Mathematica to find $\int \frac{x \ln x}{\sqrt{x^2-1}}\,dx$ [Stewart, Section 7.6 p.472 #66]. The most natural way to do this one by hand involves using a substitution, a property of logs, integration by parts, long division, and finally a trig substitution.

Figure 3.1. Sample problems from assignment intended to illustrate CAS power.

Other problems on these CAS-introductory assignments were intended to require students to get used to viewing surfaces from different viewpoints and paying attention to particular features on the graphs of functions of two variables such as vertical traces and level curves. Another goal was to lead students to describe these graphs verbally,

4. So are these 3D graphs always the easy way to answer Calc IV problems?

Graphs alone can be deceptive, and learning to read them carefully is itself a serious skill. Investigate the graphs of $g_1(x,y) = \cos x \cos y$ and $g_2(x,y) = \cos x + \cos y$. At a casual glance someone might say they're pretty much the same thing. Find three different ways of demonstrating to someone (using coherent English and possibly pictures) how you know the surfaces are different. You'll probably have to do more than just look at the standard graph to do this well.

Figure 3.2. Example problem involving CAS use.

whether with technical or figurative language. Figure 3.2 gives an example of such a problem. Another feature carefully included in these problems, and to a limited extent present in Figure 3.2, is the possibly deceptive ways that a CAS can present graphs. A deliberate attempt was made to show that a CAS is not a panacea, and that judicious use is important.

Figure 3.3 shows one additional problem used on these CAS-introductory assignments. This problem is noteworthy in part because without CAS use, many

5. In a study of frost penetration it was found that the temperature T at time t (measured in days) at a depth x (measured in feet) can be modeled by the function

$$T(x, t) = T_0 + T_1 e^{-\lambda t} \sin(\omega t - \lambda x)$$

where $\omega = 2\pi/365$ and λ is a positive constant.
(a) Find $\partial T/\partial x$. What is its physical significance?
(b) Find $\partial T/\partial t$. What is its physical significance?
[Stewart 3rd p.786 # 92]

Figure 3.3. Example problem facilitated by CAS use.

students find the problem utterly impenetrable, but after looking closely at a variety of computer-generated graphs most find it very easy to answer the questions. The suggestion here that CAS use can empower students to deal with harder problems, and in particular problems which incorporate some of the complications of modeling, is a potent one.

It would be remiss to move on without acknowledging some drawbacks to *Mathematica* which present themselves quite boldly at this stage. The software is extremely sensitive to syntax, and students often have difficulty adapting to this. Furthermore, the error messages with which the software responds to errors tend to be unhelpful. Left to themselves students almost invariably find their first encounter with *Mathematica* to be frustrating. As mentioned above, a reference sheet with examples and tips provides some help. Having a knowledgeable instructor available and roving the lab during the first session is essential. Having students paired up, with two students to each machine, seems to provide a much more thorough and comfortable experience for both – contrary to what some might expect, both the student typing and the student reading/recording tend to grasp the basics of the syntax much more quickly and thoroughly than individuals working alone.

Spring 1999

As part of a project funded by OU's College of Arts and Sciences, a questionnaire (see Appendix) was administered to 147 students in six sections of Calculus IV taught at OU during the Spring semester of 1999. The project was an attempt to integrate technology into the calculus sequence at OU, beginning with multivariable calculus and working backwards through the sequence. The questionnaires were intended to provide some measure of the effects of these changes by the end of the semester. They included demographic and attitudinal items, as well as some items directly pertaining to multivariable calculus content and some items pertaining to spatial visualization and amount of experience with technology. The visualization items are certainly not central to the traditional calculus curriculum, but it seemed reasonable to test the conjecture that experience with computer-generated images might have some consequences. The content items, on the other hand, were chosen to represent some of the central material common to virtually all multivariable calculus courses.

Three of the sections involved were CAS sections, where the use of *Mathematica* was emphasized, and three were not. Murphy taught two of the CAS sections and White the other, while the non-CAS sections were taught by two faculty members and one senior graduate student. 81 questionnaires were collected from the CAS sections and 66 from the non-CAS sections. The questionnaires were completed in-class, during either the first or last (at the convenience of each instructor) 15-20 minutes of class time at a point most of the way through the semester. The questionnaires were administered during class time to make the return rates as high as possible. Since the investigators were teaching some of the sections in question, all analysis of the data was delayed until the semester was over and grades had been determined. Data were entered into a spreadsheet with only code numbers to identify individuals so that data analysis was effectively anonymous.

Since at this stage all work was being done on an informal basis, IRB approval was not sought. Obviously no students were subjected to risks beyond those of everyday life, participation was voluntary, and privacy was strictly guarded. However, due to the lack of IRB approval, no specific data from this phase will be included in this dissertation.

Fall 1999

To further investigate developments from the initial phase, two questionnaires were administered to students enrolled in Calculus IV during the Fall term of 1999. Five different sections and a total of 142 students participated. Questionnaires were given first during the first two weeks of the semester and then after approximately two-thirds of the semester, and during either the first or last ten to fifteen minutes of the class period according to the preference of the individual instructors. These questionnaires (see Appendix) consisted mainly of items used previously, including demographic, technology use, attitudinal, Calculus IV content, and spatial visualization items. They also included several additional items intended to measure actual student practices and perceived usefulness of various practices.

Some differences had emerged between the performance of CAS and non-CAS students on the Spring 1999 questionnaires, specifically in response to an item measuring skill at mentally rotating a particular three-dimensional graph. To address the possibility that these differences might be due strictly to CAS students' familiarity with wire-frame graphics, half of the second questionnaires were administered with color graphics for the graphs item. Questionnaires were randomly distributed so that approximately half of the students in each section saw the graphs with the standard wire-frame graphics and the other half saw color images without grids.

Again, once the semester was over individuals' first and second questionnaires were matched by signatures on Informed Consent Forms and responses were entered into a spreadsheet identified only by code numbers.

In addition, seven task-based interviews were conducted jointly by Murphy and White with Calculus IV students. Participants were chosen particularly for their ability to articulate their thought processes, and focused on explaining their efforts on the Graphs Item and Rolling Box Item from the questionnaires. Audio recordings of these sessions were then transcribed and analyzed for recurrent themes which might help to account for patterns in the questionnaire data.

Spring 2000

We administered two questionnaires to all willing students (n = 205) in six non-CAS sections of Calculus IV during the Spring term of 2000. These questionnaires included demographic, technology use, attitudinal, and visualization items previously used. In addition, questionnaires were administered to four sections of Calculus III in hopes of obtaining a contrast pool against which the Calculus IV results could be gauged. Murphy taught two of these Calculus III sections and White one of the other Calculus III sections, so these data can also be used to control for instructor effect. Again the first questionnaires were administered during the first two weeks of class meetings during either the first or last 10 to 15 minutes of class time, and approximately two thirds of the way through the term we administered the second questionnaire (see Appendix). This second questionnaire had the same cover sheet attached to facilitate matching of first and second questionnaires. The second questionnaire included some attitudinal items, items addressing degree of technology experience, and repeats of the visualization items from the previous questionnaire for comparison.

Summer 2000

During the Summer 2000 term two questionnaires were administered in a single section of Calculus IV taught by White. The questionnaires and procedures used were identical to those from the Spring of 2000.

Summer 2002

The final phase of this project was focused on questionnaires given in all three sections of multivariable calculus during the Summer term of 2002. One section was taught by White as a CAS section and one of the other sections, independently of this research, was taught with an emphasis on *Mathematica* use as well. The sections were smaller than any in previous iteration due to new enrollment caps.

The questionnaires consisted mainly of items used previously, in order to check that previous patterns were replicated, but also included a few new items intended to measure two hypotheses which had arisen during the interviews in Spring 2000 to account for the observed differences. Again, the first questionnaires were administered early in the Summer session during the first week of class meetings. According to the preferences of the instructors, we gave the questionnaires during either the first or last 10 to 15 minutes of class time. This questionnaire (see Appendix) included demographic information, technology use, attitudinal, and visualization items used in previous iterations.

In accordance with the mandate of the IRB, no names were collected during this iteration. In order to match participants' first and second questionnaires while protecting anonymity, a cover sheet was attached to each questionnaire. The cover sheet requested several pieces of information (eye color, day and month of birth, last digit of home phone number) so that each individual's first and second questionnaires could be matched, and then an identification code assigned to each. Once the identification codes were assigned, the cover sheets were removed and destroyed to prevent identification of participants.

Since White taught one of the sections of Calculus IV, we performed no analysis whatsoever until the Summer term was over and grades had been assigned. This of course also sharply restricted the possibilities for other channels of investigation – the restrictions imposed by the Institutional Review Board on research conducted by an instructor eliminated many options, such as follow-up interviews, from the array of tools available.

After completing the portion of the course dealing heavily with three-dimensional objects, and hence use of computers for generating their graphical representations (approximately two thirds of the way through the Summer term, depending on the pace of each section), we administered the second questionnaire (see Appendix). This second questionnaire had the same cover sheet attached to facilitate matching of first and second questionnaires. The second questionnaire included some attitudinal items, items involving degree of technology experience, repeats of the visualization items from the previous questionnaire for comparison, and a few additional items probing how the students arrived at their answers.

Again, we performed no immediate analysis. Once the term was completed and grades assigned, data from the questionnaires was entered into a spreadsheet identified only by identifier codes, and analyses were then carried out. Participants were not be identifiable from the questionnaire forms themselves or from the computer files used in the analysis. No project publications identify individual participants.

The following table summarizes the main questions addressed by the final round of data collection and which data are intended to address each question. In some instances data were gathered on both the first and second questionnaires, generally for purposes of before-and-after comparisons. In other cases data were collected only once, as in the case of gender. Presumably these responses would change little between the first and second questionnaires, and in cases where an individual did not participate in both the first and second questionnaires these data would not be useful anyway. It could be noted that this approach is susceptible to some difficulties – self-reporting of race, for instance, is not unproblematic – but a fuller treatment of those issues is simply beyond the scope of the current study.

Some other issues with these methods should also be acknowledged. Among other things, self-reporting of some data might be subject to some biases. Presumably gender and major are relatively reliable in a large majority of cases (although see *Visualization Results from the Full Data Set* in the next chapter for some limitations to this), and race and ethnicity can be accepted as they are reported, but other items on the questionnaires were more subject to judgement. Ratings of the importance of computation and visualization are of course intended to be subjective, but it should also be kept in mind that what the terms "computation" and "visualization" refer to might be subject to some variation among individuals, and that this might furthermore be subject to influence by instructors which has little or nothing to do with CAS use. Many items might be understood differently by foreign students, especially those for whom English is not their primary language. While little can be done to address these possibilities, at least they should be mentioned.

Beyond this, there are some systematic considerations. The data gathered include Spring, Fall, and Summer term classes. There is no reason to assume these are all comparable, and several good reasons to suspect that they are not. In particular, students taking summer classes might be doing so because they have fallen behind or wish to get ahead of the standard track, and in both cases thus represent atypical cases. Furthermore, since the summer schedule proceeds approximately twice as quickly, and students generally are taking at most two classes during the summer, the experience is unusual in some important ways. The fact that a particular curricular change has some effect under these circumstances does not automatically ensure that it will have the same effect during a regular term. Once these factors have been acknowledged, some of them can be to some extent analyzed with the data at hand, but others must simply be kept in mind as further limitations to the generalization of any findings.

Coding

All the first and second questionnaires were matched and data entered into a spreadsheet identified by code numbers. In some cases individuals were also categorized in other ways, for instance students who indicated themselves to be "Hispanic or Latino/a," or who indicated race other than "White", were classified as "Minority" in the computer records for ease of later analysis. Students were also classified as CAS or non-CAS, and as Calculus III or IV. Once all coding and data entry were complete, analysis began.

The following table summarizes some fo the questions this research attempted to address, and which data were gathered for each question:

Table 3.2

Summary of Questions and Questionnaire Items Addressing Each

Effect to be Investigated	Items to be used	First Questionnaire	Second Questionnaire
Replicate difference on Graph C	Graphs Item	✓	✓
Check for other visualization effects	Rolling Box Item	✓	✓
Check Gender Effect	Gender Item	✓	
Check Minority Effect	Minority Item	✓	
Check CAS Effect	CAS Item		✓
Check other attitudinal differences	"Visualization Important" and "Computation Important" Items		✓
"Extra effort" hypothesis	"How Hard" Item		✓
"Extra tools" hypothesis	"Which Approaches" Item		✓

Results

Demographics from Fall 1999

A demographic summary of the respondents from the first questionnaires in the Fall of 1999 is provided in Tables 4.1 and 4.2:

Table 4.1
Gender of CAS and non-CAS students, Fall 1999

	non-CAS Calculus IV	CAS Calculus IV	Combined
Male	48/68 (70.6%)	48/67 (71.6%)	96/135 (71.1%)
Female	20/68 (29.4%)	19/67 (28.4%)	39/135 (28.9%)

Table 4.2
Race and Ethnicity of CAS and non-CAS students, Fall 1999

	non-CAS Calculus IV	CAS Calculus IV	Combined
American Indian or Alaska Native	0/68 (0%)	1/67 (1.5%)	1/135 (0.7%)
Asian	15/68 (22.4%)	8/67 (11.9%)	23/135 (17.0%)
Black or African-American	9/68 (13.4%)	6/67 (9.0%)	15/135 (11.1%)
Native Hawaiian or Other Pac. Isl.	0/68 (0%)	0/67 (0%)	0/135 (0%)
White, non-Hispanic or Latino/a	38/68 (56.7%)	48/67 (71.6%)	86/135 (63.7%)
Hispanic or Latino/a	4/68 (6.0%)	3/67 (4.5%)	7/135 (5.2%)

None of the proportions are significantly different between the CAS and non-CAS sections, although Asians and non-Hispanic or Latino/a Whites approach significance ($z \approx 1.56, p \approx 0.12$, and $z \approx 1.90, p \approx 0.06$, respectively).

The following table 4.3 summarizes the majors of respondents to the first questionnaire. Note that in a few cases totals exceed the number of students due to double majors, which were included in the counts for both majors.

Table 4.3
Majors of CAS and non-CAS students, Fall 1999

	non-CAS Calculus IV	CAS Calculus IV	Combined
Engineering	49/68 (72.1%)	43/67 (64.2%)	92/135 (68.1%)
Computer Science	11/68 (16.2%)	9/67 (13.4%)	20/135 (14.8%)
Geoscience	3/68 (4.4%)	7/67 (10.4%)	10/135 (7.4%)
Mathematics	4/68 (5.9%)	6/67 (9.0%)	10/135 (7.4%)
Physics	1/68 (1.5%)	3/67 (4.5%)	4/135 (3.0%)
Education	1/68 (1.5%)	0/67 (0%)	1/135 (0.7%)
Fine/Applied Arts	0/68 (0%)	3/67 (4.5%)	3/135 (2.2%)

Again, there are no significantly different proportions (cells with counts of four or less were not tested, since the normal distribution used in the standard comparison of proportions test is a poor approximation for the binomial distribution in these cases).

Visualization Results from Fall 1999

On the visualization items from the Fall semester of 1999, several differences show up between the CAS and non-CAS sections, shown in Table 4.4:

Table 4.4
CAS and non-CAS students on Visualization Items, Fall 1999

	Non-CAS Calculus IV			CAS Calculus IV		
	Right on First Questionnaire	Right on Second Questionnaire	Change	Right on First Questionnaire	Right on Second Questionnaire	Change
Graph A	36/41 (87.8%)	36/41 (87.8%)	0/41 (0%)	38/50 (76.0%)	44/50 (88.0%)	6/50 (12.0%)
Graph B	37/41 (90.2%)	38/41 (92.7%)	1/41 (2.5%)	46/50 (92.0%)	49/50 (98.0%)	3/50 (6.0%)
Graph C	33/41 (80.5%)	33/41 (80.5%)	0/41 (0%)	31/50 (62.0%)	39/50 (78.0%)	8/50 (16.0%)
Graph D	40/41 (97.6%)	38/41 (92.7%)	-2/41 (4.9%)	47/50 (94.0%)	48/50 (96.0%)	1/50 (2.0%)
Graph E	29/41 (70.7%)	33/41 (80.5%)	4/41 (9.8%)	39/50 (78.0%)	45/50 (90.0%)	6/50 (12.0%)
Graph F	37/41 (90.2%)	41/41 (100%)	4/41 (9.8%)	46/50 (92.0%)	49/50 (98.0%)	3/50 (6.0%)
Rolling Box	20/40 (50.0%)	24/40 (60.0%)	4/40 (10.0%)	28/48 (58.3%)	32/48 (66.7%)	4/48 (8.4%)

Almost across the board correct response rates increased from the first to the second questionnaires (although for the most part these increases do not represent statistically different proportions of the population in a one-tailed test of proportions). The significant improvements are indicated by the diagonally shaded cells in Table 4.4, with significant improvement on Graph C for the CAS students ($z \approx 1.75, p \approx 0.04$; note that cells with four or fewer students were not tested).

However, the improvements are generally higher for the CAS students than the non-CAS students. There were significantly greater proportions improving in the CAS sections over the non-CAS sections on Graph A ($z \approx 2.30, p \approx 0.01$) and Graph C ($z \approx 2.68, p \approx 0.004$). Thus Calculus IV students who were in a section which emphasized working with a CAS through the semester seem to show greater increases in ability to perform certain visual tasks than students in traditional sections of the same class.

Multivariable Calculus Results from Fall 1999

The data collected on mastery of multivariable calculus content revealed some differences between CAS and non-CAS students. Table 4.5 summarizes the performances of both groups on several items: An item on the first questionnaire asking students to compute a second derivative, an item on the second questionnaire involving a second order partial derivative, and an item involving setup of a triple integral.

Table 4.5
Multivariable Calculus Items, Fall 1999

	non-CAS	CAS
Derivative	18/44 (40.9%)	22/51 (43.1%)
Partial Derivative	26/45 (57.8%)	33/55 (60.0%)
Integral (completely correct)	2/48 (4.2%)	10/54 (18.5%)
Integral (nearly or completely correct)	7/48 (14.6%)	17/54 (31.5%)

For the integral, counts are included both for students providing completely correct and at least nearly correct answers. Answers were judged nearly correct if, for instance, symmetry of the region in question was used inappropriately but otherwise the answer indicated good grasp of the matter at hand. In both cases for integrals, the proportion of CAS students answering correctly is significantly higher than the proportion of non-CAS students ($z \approx 2.25, p \approx 0.02$, for completely correct, $z \approx 1.99, p \approx 0.05$, for nearly or completely correct). The proportions for derivatives are not significantly different on either the first or second questionnaire.

Demographics for the Full Data Set

Tables 4.6, 4.7, and 4.8 summarize the gender, race/ethnicity, and majors of students who completed the first questionnaire in the Fall of 1999:

Table 4.6
Gender of CAS and non-CAS Students, All Calculus IV Data

	non-CAS Calculus IV	CAS Calculus IV	Combined
Male	184/252 (73.0%)	85/116 (73.3%)	269/368 (73.1%)
Female	68/252 (27.0%)	31/116 (26.7%)	99/368 (26.9%)

Table 4.7
Race and Ethnicity, All Calculus IV Data

	non-CAS Calculus IV	CAS Calculus IV	Combined
American Indian or Alaska Nat.	3/253 (1.2%)	3/116 (2.6%)	6/369 (1.6%)
Asian	30/253 (11.9%)	13/116 (11.2%)	43/369 (11.7%)
Black or African-American	28/253 (11.1%)	8/116 (6.9%)	36/369 (9/8%)
Nat. Hawaiian or Other Pac. Isl.	1/253 (0.4%)	0/116 (0%)	1/369 (0.3%)
White, non-Hisp. or Latino/a	163/253 (64.4%)	84/116 (72.4%)	247/369 (66.9%)
Hispanic or Latino/a	23/253 (9.1%)	6/116 (5.2%)	29/369 (10.6%)
Other	4/253 (1.6%)	1/116 (0.9%)	5/369 (1.4%)

None of the proportions listed in the table are significantly different.

Table 4.8
Majors of CAS and non-CAS students, Fall 1999

	non-CAS Calculus IV	CAS Calculus IV	Combined
Engineering	181/287 (63.1%)	68/116 (58.6%)	249/403 (61.8%)
Computer Science	41/287 (14.3%)	11/116 (9.5%)	52/403 (12.9%)
Geoscience	32/287 (11.1%)	20/116 (17.2%)	52/403 (12.9%)
Mathematics	13/287 (4.5%)	14/116 (12.1%)	27/403 (6.7%)
Physics	13/287 (4.5%)	4/116 (3.4%)	17/403 (4.2%)
Chemistry	2/287 (0.7%)	1/116 (0.9%)	3/403 (0.7%)
Education	4/287 (1.4%)	2/116 (1.7%)	6/403 (1.5%)
Fine/Applied Arts	1/287 (0.3%)	4/116 (3.4%)	5/403 (1.2%)
Com. or Business	1/287 (0.3%)	1/116 (0.9%)	2/403 (0.5%)
Hum., Lib. Arts, Soc. Sci.	2/287 (0.7%)	2/116 (1.7%)	4/403 (1.0%)
Undecided	1/287 (0.3%)	0/116 (0%)	1/403 (0.2%)
Other	5/287 (1.7%)	1/116 (0.9%)	6/403 (1.5%)

The only proportions that are significantly different are the mathematics majors in CAS and non-CAS sections ($z \approx 2.74, p \approx 0.006$; as usual, cells with counts of 4 or less were not tested).

Did Treatment Occur?

Given the imprecise nature of the difference between CAS and non-CAS Calculus IV sections, it is important to determine whether there were in fact substantial variations from the traditional curriculum in the CAS sections. The data were coded on a 1 through 5 scale and a Wilcoxon two-sample test was performed to compare the increase from first questionnaire to second questionnaire response. The test showed a significant difference between CAS and non-CAS sections ($z \approx 6.49, p < 0.0001$). The mean increase for non-CAS students was 0.1675, whereas the mean increase for CAS students was 0.8152. Thus the CAS section students appear to have substantially increased their use of computer algebra systems during the term. It should be noted that all Calculus IV students might well gain experience with such software during the term due to other classes or experiences, so it is the relatively larger increase for CAS section students which is relevant.

Visualization Results from the Full Data Set

As described previously, in the early iterations of this work, one particular difference was noted between the CAS and non-CAS students. Between their performance on the first and second questionnaires, CAS students showed a greater improvement in correctly identifying Graph C as a non-match for the original graph.

This apparent difference in improvement was replicated in all subsequent iterations. However, with more data another pattern became more apparent. Table 4.9 below gives the numbers of Multivariable Calculus students who answered each portion of the Graphs Item correctly on both the first and second questionnaires, or correctly on the first and then incorrectly on the second, and so forth:

Table 4.9

Visualization Items, First to Second Questionnaire by CAS use, All Calculus IV Data

	Non-CAS Calculus IV				CAS Calculus IV			
	Right - Right	Right - Wrong	Wrong - Right	Wrong - Wrong	Right - Right	Right - Wrong	Wrong - Right	Wrong - Wrong
Graph A	134/159	25/159 (15.7%)	20/32 (62.5%)	12/32	56/66	10/66 (15.2%)	14/24 (58.3%)	10/24
Graph B	164/172	8/172 (4.7%)	13/19 (68.4%)	6/19	82/83	1/83 (1.2%)	5/7 (71.4%)	2/7
Graph C	118/138	20/138 (14.5%)	32/53 (60.4%)	21/53	46/53	7/53 (13.2%)	23/37 (62.2%)	14/37
Graph D	177/184	7/184 (3.8%)	6/7 (85.7%)	1/7	82/86	4/86 (4.7%)	3/4 (75.0%)	1/4
Graph E	127/143	16/143 (11.2%)	24/48 (50.0%)	24/48	70/74	4/74 (5.4%)	10/16 (62.5%)	6/16
Graph F	178/182	4/182 (2.2%)	9/9 (100.0%)	0/9	79/81	2/81 (2.5%)	7/9 (77.8%)	2/9
Rolling Box	57/72	15/72 (20.8%)	26/72 (36.1%)	46/72	18/21	3/21 (14.3%)	9/19 (47.4%)	10/19

Note. Percentages and denominators are of the non-CAS students answering right or wrong on the first questionnaire, and CAS students answering right or wrong on the first questionnaire, respectively.

Essentially the table allows us to compare, out of those who could improve, how many did improve. If we restrict our attention to students who mis-identified Graph C on the first questionnaire (the highlighted cells in the table), we see that very nearly the same proportion of them correct their mistake on the second questionnaire. Of the 53 Non-CAS Calculus IV students who misidentified Graph C on the first questionnaire, there were 32 (or 60.4%) who correctly identified it on the second questionnaire. Of the 37 CAS Calculus IV students who misidentified Graph C on the first questionnaire, there were 23 (or 62.2%) who misidentified it on the second questionnaire as well. These proportions are not significantly different ($z \approx 0.17, p \approx 0.87$). The table shows the corresponding counts and proportions for the other visualization items, and it can readily be seen that all proportions are reasonably well matched between the CAS and non-CAS students. None of the differences is statistically significant.

Table 4.10 is a contingency table repeating the counts from Table 4.9 for Graphs Item C:

Table 4.10
Graph Item C, First to Second Questionnaire, All Calculus IV Data

	Right - Right	Right - Wrong	Wrong - Right	Wrong - Wrong	Total
Non-CAS Calculus IV	118	20	32	21	191
CAS Calculus IV	46	7	23	14	90

A homogeneity of proportions test does not reveal significant differences between the proportions of CAS and non-CAS students in each category ($\chi^2 \approx 5.10, p \approx 0.17$).

The question naturally arises, then: Where did the previously observed differences come from? They do not appear to have been a fluke. Although CAS students show more improvement than non-CAS students (when measured by proportion improving), this difference arises almost entirely because a larger share of the CAS students misidentified Graph C on the first questionnaire. Of 191 non-CAS Calculus IV students, 138 (or 72.3%) correctly identified Graph C on the first questionnaire. For the CAS Calculus IV students, only 53 of 91 (or 58.9%) correctly identified Graph C on the first questionnaire. This difference is significant ($z \approx 2.24, p \approx 0.03$), although with the smaller sample available from the Fall 1999 data alone it was originally not significant. So although CAS and non-CAS students improved at nearly-equal rates, since more CAS students had scored poorly in the first place they had more room for improvement.

The following Table 4.11 gives the corresponding counts for the other visualization items. In addition to Graph C ($z \approx 2.24, p \approx 0.03$), the differences also approach significance in the proportions of students who initially misidentified Graph A ($z \approx 1.94, p \approx 0.05$) and F ($z \approx 1.69, p \approx 0.09$).

Table 4.11
First Questionnaire Visualization Items, All Calculus IV Data

	Non-CAS Calculus IV	CAS Calculus IV
	Right first try	Right first try
Graph A	159/191 (83.2%)	66/90 (73.3%)
Graph B	172/191 (90.1%)	83/90 (92.2%)
Graph C	138/191 (72.3%)	53/90 (58.9%)
Graph D	184/191 (96.3%)	86/90 (95.6%)
Graph E	143/191 (74.9%)	74/90 (82.2%)
Graph F	182/191 (95.3%)	81/90 (90.0%)
Rolling Box	72/144 (50.0%)	21/40 (52.5%)

The next natural question is, why is there a difference in the first questionnaire scores? Since the questionnaires were administered so early in the semester, it is difficult to attribute them to differences in the treatment of students in the different sections. In all cases the first questionnaires were administered before introducing students in the CAS sections to *Mathematica*, so that does not appear to be a source of difference. The apparent explanation is that the students who registered for Murphy's and White's Calculus IV sections were simply different than the general population.

Although demographically the sections initially appear quite similar, closer inspection reveals at least two further patterns, essentially representing lurking variables. One involves majors. In the non-CAS Calculus IV sections, 32 out of 317 (or 10.1%) described themselves as geoscience majors. In the CAS Calculus IV sections, 20 out of 124 (or 16.1%), described themselves as geoscience majors. The geoscience majors at

OU include several groups, particularly geology majors and meteorology majors, both of which require at least Calculus IV. It is possible that these numbers under-report the true situation somewhat, since the questionnaires listed "Geosciences" as a possible major and some students marked "Other" and wrote in "Meteorology," apparently not realizing that within the organization of the University that was classified as a geoscience major. In cases where the students' intent could be determined, they were coded as geoscience majors, but this confusion may not have resulted in identifying all meteorology majors. The reason this particular major is worth such emphasis is that of the 20 geoscience majors in the CAS sections, only 10 (or 50%) correctly identified Graph C on the first attempt.

The other pattern within the CAS sections which seems to be heavily involved with the different initial success on the visualization items involves gender. Although the proportion of females in the CAS and non-CAS sections is similar (31 of 85, or 26.7% in the CAS sections and 68 of 252, or 27.0% in the non-CAS sections), and females on the whole did not score significantly lower than males on Graph C, (see *Gender* below), this does not tell the whole story. Within the non-CAS Calculus IV sections, 46 out of 66 females (or 69.7%) correctly identified Graph C, reasonably in line with the general population proportion. However, in the CAS Calculus IV sections, only 15 out of 31 females (or 48.4%) correctly identified Graph C.

Thus two particular sub-populations of the CAS sections, females and meteorology majors, account for nearly all of the difference between initial scores on

Graph C. Obviously there are dangers in post-hoc analysis of this sort, but a fuller treatment will be deferred to the Conclusions chapter.

Gender

Table 4.12 summarizes the proportions of males and females who correctly answered each visualization item on the first questionnaires. While the table includes data from all semesters, only students who completed both first and second questionnaires have been included in order to remain consistent with other analyses which compare success rates on first and second questionnaires.

Table 4.12
First Questionnaire Visualization Item Results by Gender, All Calculus IV Data

	Male	Female
	Right first try	Right first try
Graph A	151/188 (80.3%)	56/70 (80.0%)
Graph B	174/188 (92.6%)	60/70 (85.7%)
Graph C	131/188 (69.7%)	42/70 (60%)
Graph D	181/188 (96.3%)	66/70 (94.3%)
Graph E	149/188 (79.3%)	49/70 (70.0%)
Graph F	179/188 (95.2%)	61/70 (87.1%)
Rolling Box	101/183 (55.2%)	26/66 (39.4%)

The proportions are significantly different on Graph F ($z \approx 2.26, p \approx 0.01$) and the Rolling Box item ($z \approx 2.20, p \approx 0.01$). It is also striking that in every case the males were more successful than the females. These results were unforseen, since none of these proportions had been significantly different in the Fall 1999 data shown in Table 4.13 below.

Table 4.13
First Questionnaire Visualization Item Results by Gender, Fall 1999 Both Questionnaire Responders

	Male	Female
	Right first try	Right first try
Graph A	49/64 (76.6%)	25/27 (92.6%)
Graph B	60/64 (93.8%)	23/27 (85.2%)
Graph C	48/64 (75.0%)	16/27 (59.3%)
Graph D	61/64 (95.3%)	26/27 (96.3%)
Graph E	48/64 (75.0%)	20/27 (74.1%)
Graph F	59/64 (92.2%)	24/27 (88.9%)
Rolling Box	33/62 (53.2%)	14/27 (51.9%)

There is no obvious explanation for the differences between these results from Fall 1999 and the entire data set, and the possibility of simple random variation should be kept in mind.

Table 4.14 below gives the numbers of males and females who got each visualization item right on both the first and second questionnaires, right on the first but wrong on the second, and so forth:

Table 4.14

Visualization Items, First to Second Questionnaire by Gender, All Calculus IV Data

	Male				Female			
	Right - Right	Right - Wrong	Wrong - Right	Wrong - Wrong	Right - Right	Right - Wrong	Wrong - Right	Wrong - Wrong
Graph A	120/151	31/151 (20.5%)	22/37 (59.5%)	15/37	46/56	10/56 (17.9%)	7/14 (50.0%)	7/14
Graph B	169/174	5/174 (2.9%)	11/14 (78.6%)	3/14	56/60	4/60 (6.7%)	5/10 (50%)	5/10
Graph C	114/131	17/131 (13.0%)	37/57 (64.9%)	20/57	33/42	9/42 (21.4%)	16/28 (57.1%)	12/28
Graph D	171/181	10/181 (5.5%)	7/7 (100%)	0/7	65/66	1/66 (1.5%)	2/4 (50%)	2/4
Graph E	137/149	12/149 (8.1%)	21/39 (53.8%)	18/39	40/49	9/49 (18.4%)	13/21 (61.9%)	8/21
Graph F	174/179	5/179 (2.8%)	8/9 (88.9%)	1/9	61/61	0/61 (0%)	8/9 (88.9%)	1/9
Rolling Box	84/101	17/101 (16.8%)	35/82 (42.7%)	47/82	16/26	10/26 (38.5%)	16/40 (40%)	24/40

Note. Percentages and denominators are of the males answering right or wrong on the first questionnaire, and females answering right or wrong on the first questionnaire, respectively.

It is apparent that males had greater improvement rates than females on almost every item, but the counts involved are too small to make statistical tests appropriate.

Attitudes Toward Computation and Visualization

Students' responses to the questions about importance of computation and visualization (see Figure 4.1 below) were coded on a 1 to 5 scale and the results were analyzed. CAS and non-CAS students were not significantly different in their attitudes

3. Computation is an important skill in multivariable calculus (circle one response).
Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree
4. Visualization is an important skill in multivariable calculus (circle one response).
Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

Figure 4.1. "Importance" items from Summer 2002 second questionnaire.

regarding the importance of computation in multivariable calculus (a Wilcoxon two-sample test resulted in $z \approx 0.8554$, $p \approx 0.40$). The mean rating for CAS students was 4.12, while the mean rating for non-CAS students was 4.15. On the importance of visualization in multivariable calculus, however, CAS students were somewhat higher, with a mean rating of 4.71 compared with 4.56 for the non-CAS students (the Wilcoxon two-sample test resulted in $z \approx 1.85$, significant for a one-tailed test, $p \approx 0.03$).

The "Extra Effort" Hypothesis

Regarding the possibility that CAS students were improving on the Graphs Items due to some sort of extra effort, the data fail to support such a contention. In response to the questionnaire item shown in Figure 4.2 below (administered on the second Summer 2002 questionnaire), with responses coded as 1 for "Not very hard", 2 for "Fairly hard",

> 8. Honestly, how hard would you say you tried in answering item 6, the one with the graphs?
> ____ Not very hard.
> ____ Fairly hard.
> ____ Very hard.
> ____ As hard as I possibly could.

Figure 4.2. "How hard" graphs item from Summer 2002 second questionnaire.

etc., students in the CAS sections had a mean response of 1.67, whereas students in non-CAS sections had a mean response of 1.89. This difference is not statistically significant according to a Wilcoxon Rank Sum test ($z \approx 0.81, p \approx 0.42$), and furthermore is not even in the direction suggested by the "Extra Effort" hypothesis.

Similarly, for the analogous "How Hard" Box Item, students in the CAS sections had a mean response of 2.07 whereas non-CAS students had a mean response of 2.22, also not significant according to the Wilcoxon test ($z \approx 0.56, p \approx 0.58$). The sample sizes were of course small for this iteration, with 15 CAS students and 27 non-CAS students, and thus there was not an especially great probability of detecting a difference should one exist. However, given the direction of the differences observed, future possibilities do not seem promising.

The "Extra Tools" Hypothesis

The possibility that CAS students were improving more on Graphs Items because their work with computers had equipped them with extra ideas of what things to look for when comparing graphs also finds no support in the data. The second questionnaires in the Summer 2002 term included the item shown in Figure 4.3 below in a preliminary attempt to measure this possibility. The choices provided on this item are a distillation

7. Thinking back to item 6, the one you just finished with the graphs, which of these approaches did you consider in deciding for or against at least one of the candidates?
Please check all that apply:

_____ I mentally pictured the original graph spinning around.
_____ I compared particular vertical traces (cross sections) in the graphs.
_____ I compared particular horizontal traces (cross sections) in the graphs.
_____ I compared how the surfaces lay relative to the x, y, or z-axes.
_____ I counted or compared positions of high or low points in the graphs.
_____ I looked for features in the lower six graphs which appeared in the original graph, particularly:
    _____ The "wings" at the back of the original graph.
    _____ The "hump" at the front of the original graph.
    _____ Other - please describe briefly: ____________________________
    ____
_____ I used some other tactic - please describe briefly: ________________
____

Figure 4.3. "Tools" item from Summer 2002 second questionnaire.

based on techniques students described during the interviews conducted during the Spring

2000 term. Table 4.15 below summarizes the numbers of students who indicated using particular approaches in answering the Graphs Item:

Table 4.15
Use of Particular Approaches to Graphs Item, Summer 2002

	Non-CAS Calculus IV	CAS Calculus IV
Spinning	19/27 (70.4%)	12/15 (80.0%)
Vertical Traces	13/27 (48.1%)	6/15 (40.0%)
Horizontal Traces	12/27 (44.4%)	4/15 (26.7%)
Axes	25/27 (92.6%)	12/15 (80.0%)
High/Low Points	12/27 (44.4%)	7/15 (46.7%)
Features: "Wings"	17/27 (63.0%)	11/15 (73.3%)
Features: "Hump"	19/27 (70.4%)	9/15 (60.0%)
Features: Other	4/27 (14.8%)	0/15 (0%)
Other Tactics	0/27 (0%)	0/15 (0%)

It should certainly be kept in mind that the validity of this sort of approach is uncertain. Whether direct questions with prompts such as those used here produce accurate reflections of actual student practices is simply not known, and not readily established within the scope of this study. In particular, providing figurative language such as "wings" and "hump" might constitute a significant imposition, and which terms such as these were frequently used by the students in interviews, the question of whether all students would resort to such terms without provocation remains open (and mathematical language like "traces" is perhaps subject to the same caveat). However, what must be concluded here is that there is no support here for any contention of systematic differences between CAS and non-CAS students on these items.

There is some indication that students' answers to these items were accurate. For instance, of the 5 students who did not indicate that they used the axes, 3 also incorrectly identified Graph A as a match for the original. Considering that this was a relatively uncommon mistake (overall 13 out of 281, or 4.6%, misidentified Graph A on the second questionnaire), which would be made predominantly by those students who failed to note the labels on the axes, this seems to suggest there was some reliability to the students' self-reporting of approaches. However, the sample size here is simply too small to allow for serious inferences.

Are Summer Data Comparable?

An important question mentioned in the Methods chapter is whether data from Summer terms can reasonably be pooled with data from other terms. Since Summer terms formed a considerable share of the CAS data used for this study, many of the other analyses here are contingent upon this question. Since the rate at which students correctly answered Graph Item C on the first questionnaire took on particular importance as analysis of these data progressed, it seemed suitable to compare these proportions for summer and non-summer data. As it turns out, 54 of 83 Summer term students (or 65.1%) answered Graphs Item C correctly on the first questionnaire, compared to 217 of 313 Calculus IV students from Fall and Spring terms (or 69.33%). These proportions are not significantly different ($z \approx 0.74, p \approx 0.46$). While this single comparison makes no pretense of being a complete examination of differences between Summer term and other terms, it does address concerns regarding the data where pooling with Summer terms was most important.

Calculus III as a Control Group

Questionnaires were administered to four sections of Calculus III during the Spring 2000 term (one of these sections taught by White and two taught by Murphy) in hopes of providing an additional comparison group. One possibility for these data would be to pool with the non-CAS Calculus IV sections to provide a larger control group for the CAS sections, and especially to establish the typical improvement of individuals who see the visualization items twice. The performance of Calculus III students, however, turned out to be sufficiently distinct from that of Calculus IV students to make this pooling seem unjustified. Another possibility would be comparing the improvement of Calculus III students taught by Murphy and White to that of Calculus IV students taught by Murphy and White in order to explore instructor effect. Since the apparently greater improvement of CAS Calculus IV students has already been accounted for in other analysis (see *Visualization Results from the Full Data Set* above), the need for this is less pressing. Still, the possibility of interesting contrasts exists.

The performance of the Calculus III students on the first questionnaire is summarized below in Table 4.16 against the non-CAS and CAS Calculus IV students:

Table 4.16
Correct First Questionnaire Visualization Responses, All Data

	Calculus III	CAS Calculus IV	Non-CAS Calculus IV
Graph A	80/102 (78.4%)	66/90 (73.3%)	159/191 (83.2%)
Graph B	95/102 (93.1%)	83/90 (92.2%)	172/191 (90.1%)
Graph C	87/102 (85.3%)	53/90 (58.9%)	138/191 (72.3%)
Graph D	100/102 (98.0%)	86/90 (95.6%)	184/191 (96.3%)
Graph E	63/102 (61.8%)	74/90 (82.2%)	143/191 (74.9%)
Graph F	100/102 (98.0%)	81/90 (90.0%)	182/191 (94.8%)
Rolling Box	49/99 (49.5%)	21/40 (52.5%)	72/144 (50.0%)

Performing comparisons of the proportions of Calculus III and CAS Calculus IV students who gave correct responses the visualization items on the first questionnaire reveals several significant differences. On Graph Item C the Calculus III students scored substantially higher than the CAS Calculus IV students, 85.3% compared to 58.9% ($z \approx 4.11, p \approx 0.0001$). This difference is especially interesting considering that among Calculus IV students it appeared that weaker students were self-selecting into the sections taught by Murphy and White.

On Graph Item E the Calculus III students scored substantially worse than the CAS Calculus IV students, 61.8% compared to 82.2% ($z \approx 3.13, p \approx 0.0009$). This difference is probably accounted for by the lack of previous exposure Calculus III students have had to three-dimensional coordinate systems, since success on Graph E depends heavily on correctly distinguishing the different axes. On Graph Item F the Calculus III students also scored higher than the CAS Calculus IV students, 98.0%

compared to 90.0% ($z \approx 2.39, p \approx 0.008$, however the normal approximation to the binomial distribution used here is poor for proportions as large as these).

Thus the self-selecting enrollment patterns which appeared to account for the poor initial success of CAS Calculus IV students (see *Visualization Results from the Full Data Set* above) do not appear to have created similar effects in Calculus III classes. In fact, females in White and Murphy's Calculus III classes were relatively successful on Graph Item C on the first questionnaire, with 16 of 19, or 84.2% correctly identifying it as a non-match, compared to 85.3% in the overall Calculus III group. Similarly geoscience majors in White and Murphy's Calculus III classes correctly identified Graph C in 11 of 13, or 84.6%, of the cases.

The following Table 4.17 summarizes the success of Calculus III students through the second questionnaire, along with the non-CAS and CAS Calculus IV students for comparison.

Table 4.17
Graphs Items from First to Second Questionnaire, All Data

	Calculus III				Non-CAS Calculus IV				CAS Calculus IV			
	Right - Right	Right - Wrong	Wrong - Right	Wrong-Wrong	Right - Right	Right - Wrong	Wrong - Right	Wrong-Wrong	Right - Right	Right - Wrong	Wrong - Right	Wrong-Wrong
Graph A	68	12 (15.0)	15 (68.2)	7	134	25 (15.7)	20 (62.5)	12	56	10 (15.2)	14 (58.3)	10
Graph B	95	0 (0)	6 (85.7)	1	164	8 (4.7)	13 (68.4)	6	82	1 (1.2)	5 (71.4)	2
Graph C	78	9 (10.3)	11 (73.3)	4	118	20 (14.5)	32 (60.4)	21	46	7 (13.2)	23 (62.2)	14
Graph D	98	2 (2.0)	2 (100)	0	177	7 (3.8)	6 (85.7)	1	82	4 (4.7)	3 (75.0)	1
Graph E	53	10 (15.9)	27 (69.2)	12	127	16 (11.2)	24 (50.0)	24	70	4 (5.4)	10 (62.5)	6
Graph F	96	4 (4.0)	2 (100)	0	178	4 (2.2)	9 (100)	0	79	2 (2.5)	7 (77.8)	2
Rolling Box	41	8 (16.3)	23 (46.0)	27	57	15 (20.8)	26 (36.1)	46	18	3 (14.3)	9 (47.4)	10

None of the proportions switching from right to wrong or wrong to right are significantly different between the Calculus III and CAS Calculus IV students (as elsewhere, cells representing four or fewer individuals were not tested).

Conclusions

In the first phases of this research, we administered questionnaires to a large number of students enrolled in Calculus IV and Calculus III at the University of Oklahoma (OU) during the Fall 1999, Spring 2000, and Summer 2000 semesters (IRB FY00-25). Our intent was to provide a large bank of data on the current status of technology use and effects in mathematics at OU. Among the patterns we recognized in the first iteration of data gathering (in the Fall 1999 term) was a considerable difference in the students' success on a particular questionnaire item measuring visualization abilities (referred to as the Graphs Item, see Appendix). Students who were in sections of Calculus IV that emphasized use of the computer software *Mathematica* performed markedly better when asked, given a particular three-dimensional surface, to determine which other graphs from a gallery of six possibilities represented the same surface, but seen from a different viewpoint.

Subsequent investigations consistently replicated this difference between CAS section students and all others. The effect seems to be quite limited, however: The difference between CAS students and non-CAS students on the total number of graphs in the gallery correctly identified as matches or non-matches is not statistically significant. Only on response to Graph C is there a difference. Another item intended to measure development of visualization ability, referred to as the Rolling Box Item, consistently shows little improvement for the CAS students and less improvement for non-CAS students, with the differences being well below the level of statistical significance.

Data gathered in the Spring of 2000 also ruled out some possible extraneous sources for this difference, particularly regarding the CAS-section students' presumably greater familiarity with the style of graphics produced by *Mathematica*.

Items involving computation of second-order partial derivatives showed no statistically significant differences between CAS students and those in more conventional sections. On the other hand, students in CAS sections were significantly more successful in setting up a triple integral than non-CAS students. It appears that work with computers had no discernable effect on success in some traditional tasks, while having certain beneficial effects on visualization ability and other traditional tasks.

It should also be briefly noted that the exposure to computer algebra systems in the CAS sections was kept to fairly low levels. The goal was to discover effects of technology use in actual classrooms, rather than effects which could only be replicated under artificially constrained laboratory conditions. While students were aware that the emphasis on computer use in their sections was unusual, and that they were filling out two questionnaires to measure its effects, otherwise the conduct of research was fairly discrete. The intention was to keep the intervention minimal, with the hope of measuring as natural a situation as possible.

The final round of data collection aimed to explore further the difference between students exposed to technology and those with less exposure. The primary questions were whether the previous differences could be replicated, and to understand more precisely what effect the exposure to technology is having on these students' visualization abilities. In particular, two conjectures emerged in the course of interviews conducted with selected students during the Spring of 2000. The interviews focused on the Graphs Item and Rolling Box Item, with students encouraged to explain their thought processes as they attempted the items. Through the course of the interviews, it became apparent that in an interview situation students seemed to perform substantially better on both items than was generally the case for questionnaires administered in class. This difference was not readily susceptible to exact measurement for several reasons, including the impracticality of a large number of interviews and the fact that interview participants were carefully selected for ability to communicate effectively, rather than as a representative sample. However, the strong impression left on the interviewers was that as students continued to think about the Graphs Item, their likelihood of successfully identifying graphs grew considerably. In an interview situation where there was encouragement to continue discussing an item, not to mention desire to please the interviewers, sustained effort seemed to lead to much greater successes. Thus the hypothesis emerged that greater effort was partly responsible for the different performances of CAS and non-CAS students.

A second conjecture which emerged from the interviews was equally unsuited to precise measurement in an interview setting, but also involved a possible explanation for the higher success levels of students in CAS sections. Interview participants used a surprisingly large variety of tactics and terms in explaining how they decided which graphs matched or did not match the original. No single tactic was sufficient to rule out all possibilities, so participants tended to proceed through a list of characteristics they could identify in each graph and compare to others. Some characteristics were extremely figurative, whereas others were more mathematically rigorous, but it became apparent that success came not through examining any single characteristic but rather through persisting with examinations of many different characteristics. This of course partly returns to the previous conjecture regarding degree of effort, but it also became apparent that the number of significant traits a student could think of to examine played a significant role. It is possible that students in CAS sections, after spending a considerable amount of time with computer-generated graphics, were familiar with a larger number of traits that could be readily examined than were non-CAS students. This conjecture, then, is not strictly an alternative to the claim that CAS students have a greater facility to visualize, but rather a refinement of that claim. Perhaps exposure to a CAS provides students with more tools with which they can approach tasks that involve visualization. Furthermore, we consistently observed in the interviews that students decided on an answer to graph (c) last of all. The other decisions came more quickly, and it was graph (c) which presented the greatest difficulty. It is then particularly interesting that it was on this item that the largest difference between CAS and non-CAS students shows up.

Unfortunately, the data gathered did not support these conjectures. Although the number of students involved in the final round was extremely small and would have made acquiring statistically significant results fairly unlikely, in fact the data gathered tended to support the opposite contention: That CAS students tried less hard than non-CAS students, and perhaps used fewer tools in coming to their conclusions. It should be noted that the validity of self-reported measures of effort is uncertain, so it is possible that further examination by other methods might reveal something more, but the methods used here probably do not merit further work.

On the whole, though, inquiry into the relatively greater improvement of CAS students on some of the visualization items is moot. With the early data, the rate of improvement of CAS students was significantly higher than that of the non-CAS students, while their initial success rates were not significantly different. However, with the full data set, the difference between the CAS and non-CAS students' initial successes became significant. It is now apparent that underlying the previous observations is a small but consistent difference between the students entering the CAS and non-CAS sections. Women and geoscience majors in the CAS sections performed relatively poorly on the visualization items, enough so that these subgroups alone account for the observed differences between CAS and non-CAS sections. It is also true that women generally performed less well than men on these items, although not necessarily significantly so, but the difficulties were especially acute with the individuals who opted to enroll in CAS sections.

It appears that quite a number of weaker students sought out Murphy and White's sections of Calculus IV. The geoscience majors in particular form a close-knit community, and often students recommend certain instructors to one another. The most likely explanation for the phenomena observed in this study is that this self-selection into Murphy and White's sections is behind the differences observed in visualization ability. While there is no way of further testing this explanation with the data at hand, there is also little left to motivate further investigation. Since it is fairly clear that the lack of random assignment to treatment or control groups is responsible for the observed differences, further investigation would be better directed toward other avenues entirely.

It goes without saying at this point that issues of generalization make broader application of any of the findings of this study highly problematic. In fact, the lesson to be drawn is probably that self-selection into treatment or non-treatment groups presents a challenge to other research on curriculum innovations as well. Although this is a difficult challenge to address in most institutional settings, the present study demonstrates that it can be a genuine confounding factor. Finding ways at least to measure homogeneity of treatment and control groups is essential.

One further issue which has not been seriously discussed to this point is attrition. This study for the most part focused on students who completed both the first and second questionnaires, but it is not clear that this reflects all students who originally enrolled for the class. Students who did not complete the second questionnaire had a different success rate on the first questionnaire's visualization items, and this suggests that ability to visualize might be involved in choices to drop Calculus IV. While this might be a fruitful question to pursue, it goes beyond the means of the present study. The unpredictable factors involved in questionnaire return rates make any judgements based on them alone uncertain. While enrollment records could perhaps be used, these would not likely provide a complete picture of which students abandoned the class either, since in many cases students who give up on a class do not formally drop it – whether for financial aid status or other reasons. Probably the most effective means of determining who abandoned the class and why would be some form of exit interviews with all students who withdrew or failed the class, and that would be a serious endeavor in its own right.

Significant differences also emerged between the males and females taking Calculus IV. Although examination of results for individual semesters showed little pattern, the numbers for the entire data set are strongly tilted. It should be noted that the Rolling Box Item was one where the difference reached the level of statistical significance – since this item is of a sort frequently used to measure innate capacity to visualize, it may point to an underlying cause. Among other possibilities for further study are more serious tests of spatial visualization abilities. The Purdue Spatial Visualization Test: Rotations and the Mental Rotations Test are two frequently used standardized exams with well-researched characteristics. There is a substantial literature on gender differences on these tests (e.g. Sorby et al., 1999), and it might be interesting to see if these more complete assessments of visual ability would reveal further interactions with CAS use.

One final cautionary note is also in order: The number of statistical comparisons made here is considerable, and the possibility of at least some type I error (that is, incorrectly concluding that the difference between two groups is due to something other than random factors) is high. This is not an issue for several of the conclusions here: The students in CAS and non-CAS sections were genuinely different, and those differences were most acute among women and geoscience majors. However, that is a statement about the specific individuals who participated in this research, and all of the usual limitations of statistical methods apply to efforts to generalize those conclusions to larger populations.

In conclusion, I diverge from most dissertations which end with recommendations for further study of the questions at hand. The main issues which prompted this research have been resolved. The differences between CAS and non-CAS students on visualization items have been accounted for, although not by means anticipated at the outset. Certainly other phenomena have arisen along the way which might merit more investigation, but those are concerned with gender differences and attrition, already areas of active study. The specific questions this work set out to address have been answered.

References

American Association of University Women, (1998). *Gender gaps: Where schools still fail our children.* Washington, D.C.: American Association of University Women.

Asiala, M., Cottrill, J., & Dubinsky, E. (1997). The development of students' graphical understanding of the derivative. *Journal of Mathematical Behavior, 16*, 399-431.

Aspinwall, L., Shaw, K., & Presmeg, N. (1997). Uncontrollable mental imagery: Graphical connections between a function and its derivative. *Educational Studies in Mathematics, 33*, 301-317.

Bishop, A. (1989). Review of research on visualization in mathematics education. *Focus on Learning Problems in Mathematics, 11*, 7-16.

Borba, M. C. (1995). Teaching mathematics: Computers in the classroom. *The Clearing House, 68*, 333-334.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education.* (pp. 231-265) Mahwah, New Jersey: Lawrence Erlbaum Associates.

Douglas, R. G. (Ed.). (1986). *Toward a lean and lively calculus.* Washington, D.C.: Mathematical Association of America.

Dunham, P. H. (1999). *"Hand-held calculators in mathematics education: A reflection on theories of learning and education."* http://mathforum.org/technology/papers/papers/dunham.html

Ferrini-Mundy, J. (1987). Spatial training for calculus students: Sex differences in achievement and in visualization ability. *Journal for Research in Mathematics Education, 18*, 126-140.

Friedman, L. (1995). The space factor in mathematics: Gender differences. *Review of Educational Research, 65*, 22-50.

Ganguli, A. (1992). The effect on students' attitudes of the computer as a teaching aid. *Educational Studies in Mathematics, 23*, 611-618.

Goldenberg, E. P. (1998). *"Chipping away at mathematics: A long-time technophile's worries about computers and calculators in the classroom."* http://mathforum.org/technology/papers/papers/goldenberg/goldenberg.html

Habre, S. (2001). Visualization in multivariable calculus: The case of 3D-surfaces. *Focus on Learning Problems in Mathematics, 23*, 30-48.

Haver, W. (Ed.). (1998). *Calculus: Catalyzing a national community for reform.* Washington, D.C.: Mathematical Association of America.

Kelly, A., & Lesh, R. (2000). Trends and Shifts in Research Methods. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education.* (pp. 35-44) Mahwah, New Jersey: Lawrence Erlbaum Associates.

Krutetskii, V. A. (1969). An investigation of mathematical abilities in schoolchildren. In J. Kilpatrick and I. Wirszup (Eds.), *Soviet Studies in the Psychology of Learning and Teaching Mathematics, Volume 2. (pp. 5-58)* Chicago: The University of Chicago.

McKnight, C., Magid, A., Murphy, T., & McKnight, M. (2000). *Mathematics education research: A guide for the research mathematician.* Providence, RI: American Mathematical Society.

Murphy, T. J. (1999a). Using computers to facilitate visualization in multivariable calculus: A variety of options. *Proceedings of the Twenty First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Cuernavaca, Morelos, Mexico, p. 410.

Murphy, T. J. (1999b). Recognizing a surface from different viewpoints: Results from a questionnaire item and task-based interviews. *Electronic Proceedings of the Twelfth Annual International Conference on Technology in Collegiate Mathematics, San Francisco.*

Murphy, T. J., Goodman, R. E., & White, J. J. (1999). Using the WWW in multivariable calculus to enhance visualization. *International Journal of Engineering Education, 15(6)*, pp. 425-431.

Murphy, T. J., Stafford, K. L., & McCreary, P. (1998). Subsequent course and degree paths of students in a Treisman-style workshop calculus program. *Journal of Women and Minorities in Science and Engineering, 4*, pp. 381-396.

Murphy, T. J. & White, J. J. (2001) Student Use and Perceived Usefulness of Technology Resources. *Proceedings of the Fourteenth Annual International Conference on Technology in Collegiate Mathematics, Baltimore*, pp. 202-206.

Murphy, T. J., White, J. J., Kline, B. J., Black, E., Goodman, R., & Hofer, M. (1999). Using Mathematica with multivariable calculus. *Proceedings of the 1999 American Society for Engineering Education Annual Conference and Exposition, Charlotte.*

Olson, J. & Sakshaug, L. (2000). Editorial. *Focus on Learning Problems in Mathematics, 22*, 1-9.

Pedersen, K. (1990). Combating the stereotype of women in mathematics and women's stereotype of mathematics. In N. Fisher, H. Keynes, & P. Wagreich (Eds.), *Mathematics and education reform: Proceedings of the July 6-8, 1988 workshop.* (pp. 205-211) AMS.

Presmeg, N. (1986). Visualization and mathematical giftedness. *Educational Studies in Mathematics, 17*, 297-311.

Presmeg, N. (1992). Prototypes, metaphors, metonymies and imaginative rationality in high school mathematics. *Educational Studies in Mathematics, 23*, 595-610.

Schoenfeld, A. H. (2000). Purposes and methods of research in mathematics education. *Notices of the American Mathematical Society, 47*, 641.

Schwartz, B. B., & Hershkowitz, R. (1999). Prototypes: brakes or levers in learning the function concept? The role of computer tools. *Journal for Research in Mathematics Education, 30*, 362-389.

Seymour, E. & Hewitt, N. (1997). *Talking About Leaving: Why Undergraduates Leave the Sciences.* Boulder, CO: Westview Press.

Sorby, S., Leopold, C., & Górska, R. (1999). Cross-cultural comparisons of gender differences in the spatial skills of engineering students. *Journal of Women and Minorities in Science and Engineering, 5*, 279-291.

Sorby, S. A., & Baartmans, B. J. (1998). A longitudinal study of a pre-graphics course designed to improve the 3-D spatial skills of low visualizers. *Proceedings of the 8th International Conference on Engineering Design Graphics and Descriptive Geometry*, Austin, TX, 247-251.

Steen, L. A. (Ed.). (1988). *Calculus for a new century: A pump, not a filter*. Washington, D.C.: Mathematical Association of America.

Steffe, L. & Thompson, P. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education*. (pp. 267-306) Mahwah, New Jersey: Lawrence Erlbaum Associates.

Stewart, J. (1995). *Calculus: Early transcendentals, 3rd edition*. Pacific Grove, CA: Brooks/Cole Publishing Company.

Stewart, J. (1999). *Calculus: Early transcendentals, 4th edition*. Pacific Grove, CA: Brooks/Cole Publishing Company.

Tall, D. & Thomas, M. (1989). Versatile learning and the computer. *Focus on Learning Problems in Mathematics, 11*, 117-125.

Treisman, U. (1990). A study of the mathematics performance of black students at the University of California, Berkeley. In N. Fisher, H. Keynes, and P. Wagreich (Eds.), *Mathematics and education reform: Proceedings of the July 6-8, 1988 workshop*. (pp. 33-46) AMS.

Vinner, S. (1989). The avoidance of visual considerations in calculus students. *Focus on Learning Problems in Mathematics, 11*, 149-156.

Appendix

Data Collection Instruments

As part of an ongoing project in the College of Arts and Sciences to monitor the use of technology in our courses, we are requesting the following information. Please respond to each item in the context of the Calculus IV class that you are taking this semester. The first few items will allow us to analyze whether technology is being used differently by different groups. This survey is adapted from the NSF-funded project: Developing Statistical Indicators to Monitor the Condition of Undergraduate Mathematics Education.

DO NOT PUT YOUR NAME ON THIS SURVEY.

1. Please indicate your gender: F M

2. Please indicate your ethnicity:
   ________ Hispanic or Latino/a ________ Non-Hispanic or Latino/a

3. Please indicate your race:
   ________ American Indian or Alaska Native ________ Asian ________ Black or African-American
   ________ Native Hawaiian or Other Pacific Islander ________ White ________ Other

4. Please indicate the student status that best describes you:
   ________ Freshman ________ Sophomore ________ Junior ________ Senior ________ Other

   Are you a transfer student this semester? Yes No

5. Please indicate the enrollment status that best describes you:
   ________ Full-Time ________ Part-Time (more than one course) ________ Single Course-Taker

6. Please indicate your intended major:
   ________ Engineering (not CS) Specify: ____________________
   ________ Computer Science
   ________ Geoscience
   ________ Mathematics
   ________ Physics
   ________ Chemistry
   ________ Life Science Specify: ____________________
   ________ Education
   ________ Fine or Applied Arts
   ________ Commerce or Business-Related Majors
   ________ Humanities, Liberal Arts, or Social Science
   (English, History, Psychology, Sociology, etc.)
   ________ Undecided
   ________ Other

7. Please indicate whether you intend or expect to teach at the K-12 level at some point your career: Yes No Maybe

8. Is this the first time you are taking Calculus IV? Yes No

   If you are repeating Calculus IV this semester, why?

   ________ Failed the course the first time.

   ________ Dropped the course due to a failing grade.

   ________ Dropped the course for other reasons.

   ________ Did not fail the course, but am repeating it for other reasons.

9. Please indicate which calculator(s) you use for Calculus IV (check all that apply):

   ________ TI-92, TI-89, or other graphing calculator that can draw 3D graphs.

   ________ TI-82, TI-83, TI-85, TI-86, or other graphing calculator that can't draw 3D graphs.

   ________ Non-graphing calculator.

10. Before you took Calculus IV, how much experience did you have with computer algebra systems (e.g., *Mathematica*, MathCad, DERIVE)?

    None Knew They Exist Have Used Once Have Used Several Times Have Used Often

11. Outside of your Calculus IV class (e.g., in other classes), how much do you work with computer algebra systems (e.g., *Mathematica*, MathCad, DERIVE)?

    Not at All Less Than Once a Month At Least Once a Month At Least Once a Week

12. Computation is an important Calculus IV skill.

    Strongly Disagree Disagree Neutral Agree Strongly Agree

13. Visualization is an important Calculus IV skill.

    Strongly Disagree Disagree Neutral Agree Strongly Agree

14. Calculate f_{xx} (i.e., $\frac{\partial^2 f}{\partial x^2}$, the second order partial derivative of f with respect to x both times) for the function $f(x, y) = e^{x^2+y}$. Show your work.

15. If you were asked to compute the integral

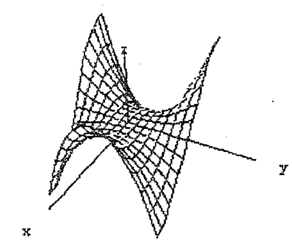
$$\int_0^1 \int_e^{\pi} x \sin(x + y)dxdy$$

how would you do it?

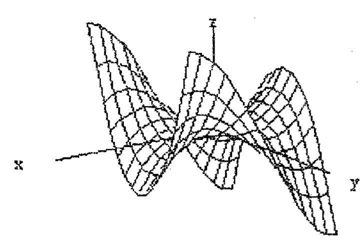
(a) By hand.
(b) Using a table.
(c) Using a calculator.
(d) With a computer.
(e) I wouldn't do it at all.

Please use a few sentences to explain your response.

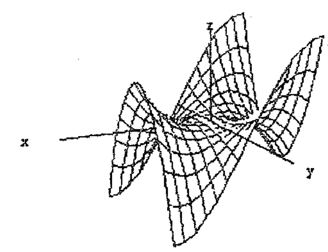
16. Consider the graph at the right. Identify which of the graphs below depict the same surface but from a different viewpoint. Circle all that apply. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.



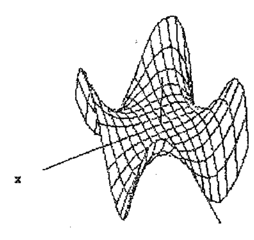
(a)



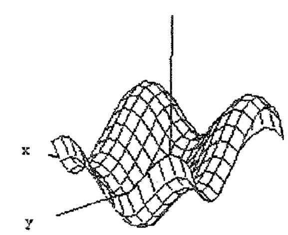
(b)



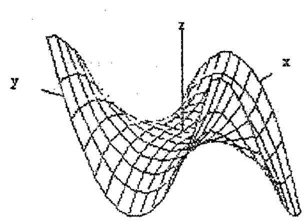
(c)



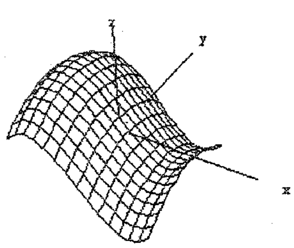
(d)



(e)



(f)



For one of the graphs that you believe does not represent the given surface, briefly explain why you think that.

17. Indicate how often each activity was used for your Calculus IV class. (Note: "computer algebra system" refers to software such as *Mathematica*, MathCad, DERIVE, etc.)

For my Calculus IV class I	Not Available	Available, Not Used	Used Less Than Once a Month	Used At Least Once a Month	Used At Least Once a Week
Listen to lecture without taking notes					
Take notes from lecture					
Watch the instructor demonstrate how to construct and develop proofs					
Construct and develop proofs myself					
Watch the instructor demonstrate how to create mathematical models of everyday situations (e.g., population growth, pressure of gas in a container)					
Create mathematical models myself					
Work in a small group (in-class or out-of-class)					
Ask homework questions (in-class)					
Ask homework questions (out-of-class)					
Watch the instructor draw graphics by hand					
See graphics that were generated by a computer (on TV monitor or overhead projector)					
Draw graphics by hand (in- or out-of class)					
Use a calculator to generate graphics (in-class or out-of-class)					
Use a computer algebra system to generate graphics (in-class or out-of-class)					
Do computations by hand (in-class or out-of-class)					
Do computations with a calculator (in-class or out-of-class)					
Do computations with a computer algebra system (in-class or out-of-class)					
Use e-mail (individual, discussion groups, etc)					
Use the internet (class web site, other web sites, downloadable software, etc)					
Use spreadsheets or other software (Excel, word processing, presentation software, etc.)					

18. Indicate how helpful each of the following has been for you in learning Calculus IV material.

Instructional Device	Does Not Apply	Not Helpful	Somewhat Helpful	Very Helpful
Calculator (graphing or other)				
Computer Algebra System (e.g., *Mathematica*, MathCad)				
E-mail (individual, discussion groups, etc)				
Internet (class web site, other web sites, downloadable software, etc)				
Spreadsheets or other software (Excel, word processing, presentation software, etc.)				
Lectures				
Office Hours				
Homework from the textbook				
Other assignments (e.g., projects)				
The textbook (other than homework)				
Working alone				
Working with a partner or group				
Studying for tests				

19. If you feel strongly about any issues related to Calculus IV that we have neglected in this survey, please write your comments below.

1. Please indicate your gender (circle one): F M

2. Please indicate your ethnicity (check one):

   ________ Hispanic or Latino/a ______ Non-Hispanic or Latino/a

3. Please indicate your race (check one):

   ______ American Indian or Alaska Native ______ Native Hawaiian or Other Pacific Islander
   ______ Asian ______ White
   ______ Black or African-American ______ Other

4. Please indicate the student status that best describes you (circle one):

   Freshman Sophomore Junior Senior Other

   Are you a transfer student this semester (circle one)? Yes No

5. Please indicate the enrollment status that best describes you (circle one):

   Full-Time Part-Time (more than one course) Single Course-Taker

6. Please indicate your intended major (check all that apply):

   ________ Engineering (not CS) Specify: ______________________________
   ________ Computer Science
   ________ Geoscience
   ________ Mathematics
   ________ Physics
   ________ Chemistry
   ________ Life Science Specify: ______________________________
   ________ Education
   ________ Fine or Applied Arts
   ________ Commerce or Business-Related Majors
   ________ Humanities, Liberal Arts, or Social Science (English, History, Psychology, Sociology, etc.)
   ________ Undecided
   ________ Other Specify: ______________________________

7. Indicate which calculator(s) you used for this calculus course (check all that apply).

_______ None. (If you did not use a calculator at all, skip to item 9.)
_______ Non-graphing calculator.
_______ TI-82, TI-83, TI-85, TI-86, or other graphing calculator that can't draw 3D graphs.
_______ TI-92, TI-89, or other graphing calculator that can draw 3D graphs.

8. Circle T(rue) or F(alse) to indicate your calculator use for your calculus class this semester.

As a habit, I used a calculator to …	
to do calculations for out-of-class assignments.	T F
to generate graphics for out-of-class assignments.	T F
to do calculations for quizzes and/or tests.	T F
to generate graphics for quizzes and/or tests.	T F
other (please specify):	

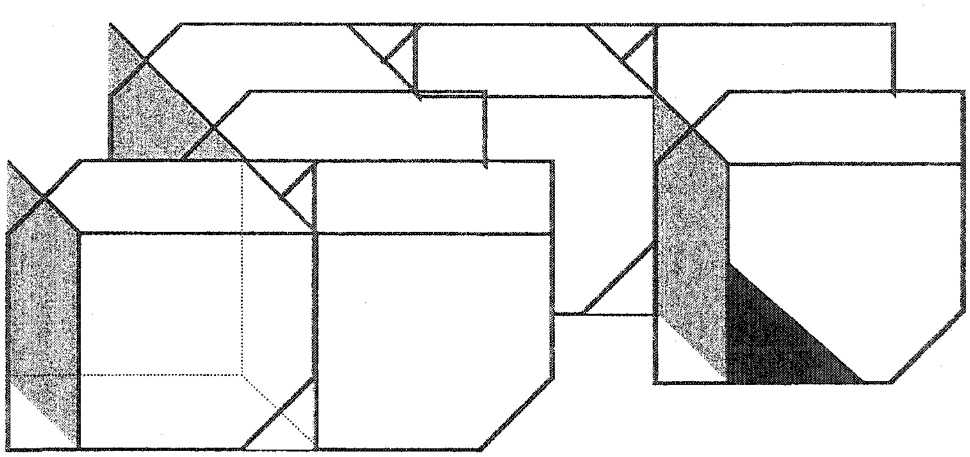
9. Computation is an important skill to succeed in calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

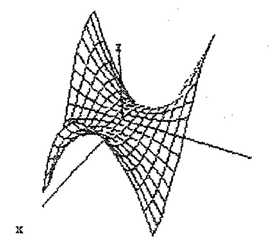
10. Visualization is an important skill to succeed in calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

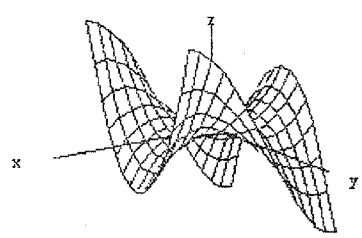
11. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.



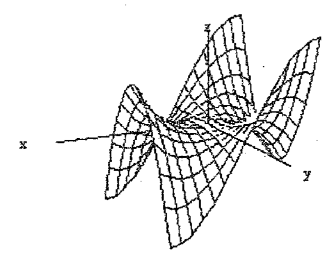
12. Consider the graph at the right. Identify which of the graphs below depict the same surface but from a different viewpoint. Circle all that apply. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.



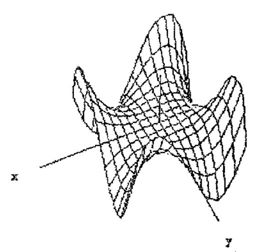
(a)



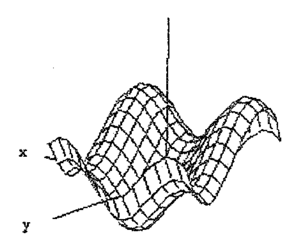
(b)



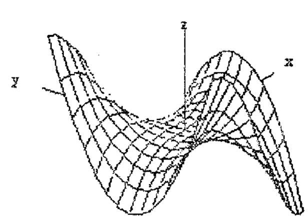
(c)



(d)



(e)



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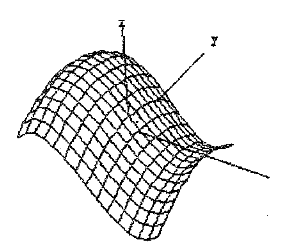


13. For your answers to the previous item (with all the graphs): please explain why you accepted/rejected the choice as a match to the original graph. Be specific. (e.g., "It is the same/a different surface." is not a helpful answer. Explain *what* is the same/different.) Do NOT change your answers on the previous page. If you want to change an answer, do it at the bottom of this page.

(a)

(b)

(c)

(d)

(e)

(f)

15. If you want to change any of your decisions, explain here which ones and why.

1. Indicate which calculator(s) you used for this calculus course (check all that apply):

________ Non-graphing calculator.
________ TI-82, TI-83, TI-85, TI-86, or other graphing calculator that can't draw 3D graphs.
________ TI-92, TI-89, or other graphing calculator that can draw 3D graphs.

2. For each technology below, please check the amount of experience you have.

	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g., *Mathematica*, MathCad, DERIVE)					
e-mail					
internet					

3. Indicate how often each activity was used for your calculus class this semester. (Note: "computer algebra system" refers to software such as *Mathematica*, MathCad, DERIVE, etc.)

For this calculus class I (in- or out-of-class)	not at all	a few times	more than a few times	as a habit
listened to/take notes from lecture				
worked in a small group				
did computations by hand				
did computations with a calculator				
did computations with a computer algebra system				
drew graphics by hand				
used a calculator to generate graphics				
used a computer algebra system to generate graphics				
other (please specify:)				

4. Circle T(rue) or F(alse) to indicate your resource use for your calculus class this semester.

As a habit, I used (calculator/computer) to …	calculator		computer	
to do calculations for out-of-class assignments.	T	F	T	F
to generate graphics for out-of-class assignments.	T	F	T	F
to do calculations for quizzes and/or tests.	T	F	T	F
to generate graphics for quizzes and/or tests.	T	F	T	F
not at all because I didn't want/need to.	T	F	T	F
not at all because I didn't know how to.	T	F	T	F
not at all because my instructor wouldn't let me.	T	F	T	F
other (please specify):				

5. Circle T(rue) or F(alse) to indicate your resource use for your calculus class this semester.

At least sometimes, I used (website/CourseNet) to get …	my instructor's website		the Calculus at OU website		OU library's electronic reserve website		CourseNet at OU	
announcements.	T	F	T	F	T	F	T	F
assignments.	T	F	T	F	T	F	T	F
solutions to assignments and/or tests.	T	F	T	F	T	F	T	F
supplemental graphics or other math content.	T	F	T	F	T	F	T	F
general information about Calculus at OU.	T	F	T	F	T	F	T	F
other (please specify):								

6. Indicate how helpful each of the following resources has been for you in learning calculus this semester.

	I did not use …			I used it and …			
	… and don't know what it is.	… and probably would not use if available.	but would probably use if available.	it was a waste of time.	it was somewhat helpful.	it was very helpful.	please don't take it away.
calculator (graphing or other)							
computer algebra system (e.g., *Mathematica*, MathCad, DERIVE, etc.)							
e-mail							
instructor's website							
the Calculus at OU website							
CourseNet at OU							
spreadsheets or other software (excel, word processing, presentation software, etc.)							
lectures							
office hours							
homework from the textbook							
other assignments (e.g., projects)							
the textbook (other than homework)							
working alone							
working with a partner or group							
studying for tests							
other (please specify:)							

7. Computation is an important skill in multivariable calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

8. Visualization is an important skill in multivariable calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

9. Calculate f_{xx} (i.e., $\frac{\partial^2 f}{\partial x^2}$, the second order partial derivative of f with respect to x both times) for the function $f(x,y) = e^{x^2+y}$. Show your work.

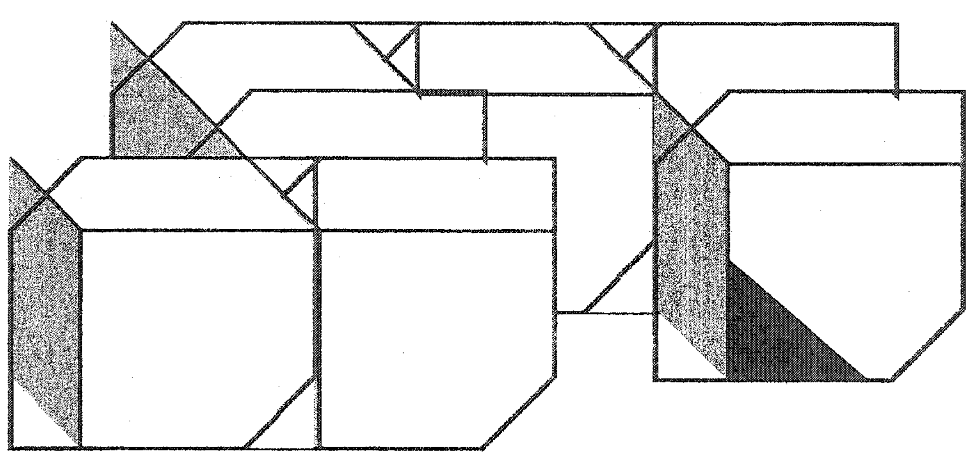
10. If you were asked to compute the integral $\int_0^1 \int_c^{\pi} x \sin(x+y)dxdy$ how would you do it? Rank each of the following strategies in the order that you would try them (1 is what you would try first, etc.). If you wouldn't use a strategy at all, mark a "0".

________ By hand.
________ Using a table.
________ Using a calculator.
________ With a computer.

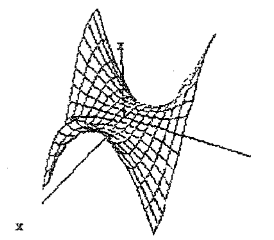
11. Set up a triple integral to find the volume of the solid in the first octant ($x > 0$, $y > 0$, $z > 0$) bounded by the elliptic cylinder $y^2 + 4z^2 = 4$ and the plane $y = x$. (You need NOT evaluate the integral, just set it up.)

$$\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \square \; d\square \; d\square \; d\square$$

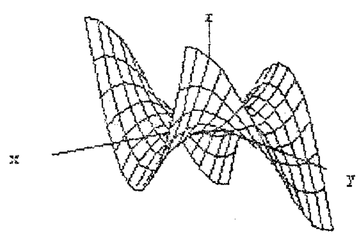
12. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.



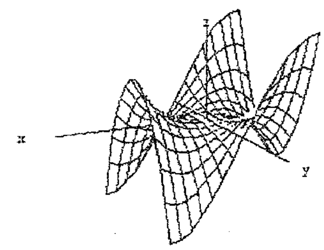
13. Consider the graph at the right. Identify which of the graphs below depict the same surface but from a different viewpoint. Circle all that apply. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.



(a)



(b)



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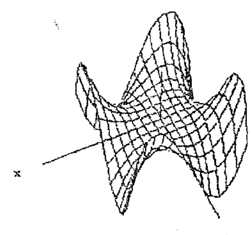
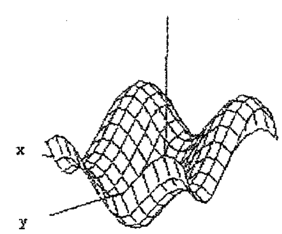
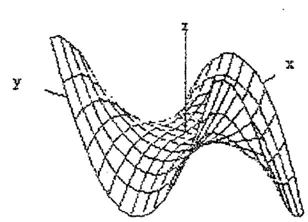


(d)



(e)



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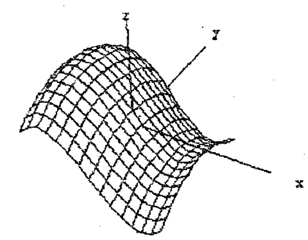


Hungry? If you are willing to stay after class today for 15 more minutes to answer some additional questions, please follow the administrator, who will provide a free snack (e.g., candy, fruit).

14. For the answers that you turned in: please explain why you accepted/rejected the choice as a match to the original graph. Be specific. (e.g., "It is the same/a different surface." is not a helpful answer. Explain *what* is the same/different.) Do NOT write on the sheet that you turned in during class.

(a)

(b)

(c)

(d)

(e)

(f)

15. If you want to change any of your decisions, explain here which ones and why.

Circle T(rue) or F(alse) to indicate your resource use for your calculus class this semester.

1. As a habit, I used (notes/textbook) to ...	lecture notes		textbook sections (not the homework part)	
to find examples to help me do the homework	T	F	T	F
to help me understand mathematics theory and concepts.	T	F	T	F
to prepare for quizzes and/or tests.	T	F	T	F
other (please specify):				

2. As a habit, I worked (alone/with someone) to ...	alone		with a partner or group	
do out-of-class assignments that were graded.	T	F	T	F
do out-of-class assignments that were NOT graded.	T	F	T	F
prepare for quizzes and/or tests.	T	F	T	F
other (please specify):				

3. As a habit, I used (calculator/computer) to ...	calculator		computer	
to do calculations for out-of-class assignments.	T	F	T	F
to generate graphics for out-of-class assignments.	T	F	T	F
to do calculations for quizzes and/or tests.	T	F	T	F
to generate graphics for quizzes and/or tests.	T	F	T	F
not at all because I didn't want/need to.	T	F	T	F
not at all because I didn't know how to.	T	F	T	F
not at all because my instructor wouldn't let me.	T	F	T	F
other (please specify):				

4. As a habit, I used (e-mail, CourseNet) to ...	e-mail		CourseNet	
make appointments with my instructor.	T	F	T	F
have math conversations with my instructor (e.g., how do you #7?)	T	F	T	F
have non-math conversations with my instructor. (e.g., what's on the test?)	T	F	T	F
arrange study meetings with my classmates.	T	F	T	F
have math conversations with my classmates	T	F	T	F
have non-math conversations with my classmates.	T	F	T	F
other (please specify):				

5. At least sometimes, I used (website/CourseNet) to get ...	my instructor's website		the Calculus at OU website		OU library's electronic reserve website		CourseNet at OU	
announcements.	T	F	T	F	T	F	T	F
assignments.	T	F	T	F	T	F	T	F
solutions to assignments and/or tests.	T	F	T	F	T	F	T	F
supplemental graphics or other math content.	T	F	T	F	T	F	T	F
general information about Calculus at OU.	T	F	T	F	T	F	T	F
other (please specify):								

1. Please indicate your gender (circle one): F M

2. Please indicate your ethnicity (check one):

    _______ Hispanic or Latino/a ______ Non-Hispanic or Latino/a

3. Please indicate your race (check one):

    ______ American Indian or Alaska Native ______ Native Hawaiian or Other Pacific Islander
    ______ Asian ______ White
    ______ Black or African-American ______ Other

4. Please indicate the student status that best describes you (circle one):

    Freshman Sophomore Junior Senior Other

    Are you a transfer student this semester (circle one)? Yes No

5. Please indicate the enrollment status that best describes you (circle one):

    Full-Time Part-Time (more than one course) Single Course-Taker

6. Please indicate your intended major (check all that apply):

    _______ Engineering (not CS) Specify: ____________________________
    _______ Computer Science
    _______ Geoscience
    _______ Mathematics
    _______ Physics
    _______ Chemistry
    _______ Life Science Specify: ____________________________
    _______ Education
    _______ Fine or Applied Arts
    _______ Commerce or Business-Related Majors
    _______ Humanities, Liberal Arts, or Social Science (English, History, Psychology, Sociology, etc.)
    _______ Undecided
    _______ Other Specify: ____________________________

7. Indicate which calculator(s) you used for this calculus course (check all that apply).

________ None. (If you did not use a calculator at all, skip to item 9.)
________ Non-graphing calculator.
________ TI-82, TI-83, TI-85, TI-86, or other graphing calculator that can't draw 3D graphs.
________ TI-92, TI-89, or other graphing calculator that can draw 3D graphs.

8. Circle T(rue) or F(alse) to indicate your calculator use for your calculus class this semester.

As a habit, I used a calculator to …	
to do calculations for out-of-class assignments.	T F
to generate graphics for out-of-class assignments.	T F
to do calculations for quizzes and/or tests.	T F
to generate graphics for quizzes and/or tests.	T F
other (please specify):	

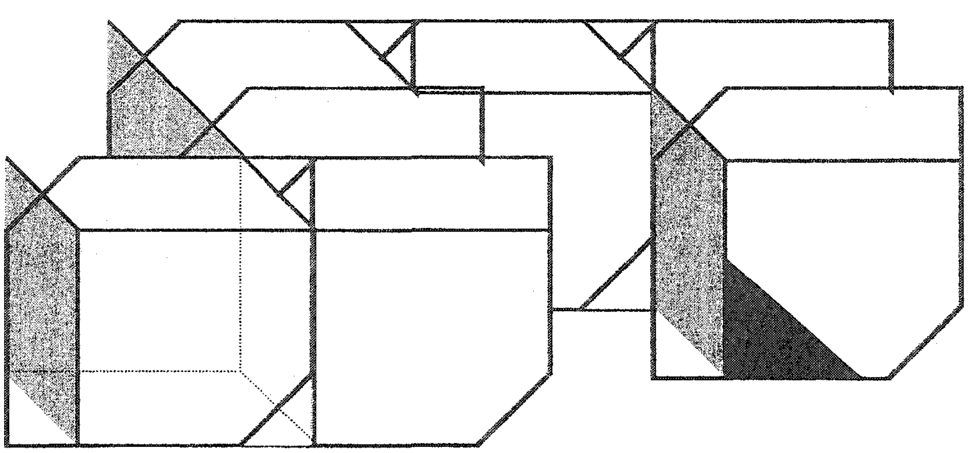
9. Computation is an important skill to succeed in calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

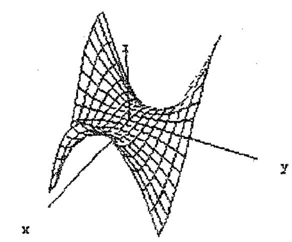
10. Visualization is an important skill to succeed in calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

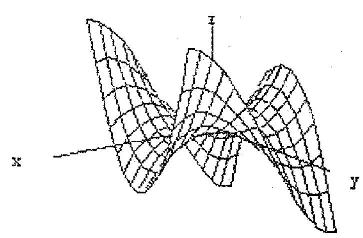
11. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.



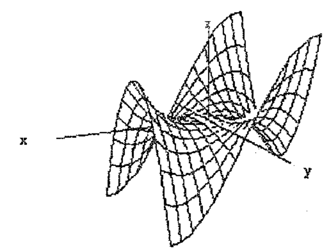
12. Consider the graph at the right. Identify which of the graphs below depict the same surface but from a different viewpoint. Circle all that apply. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.



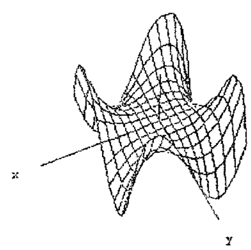
(a)



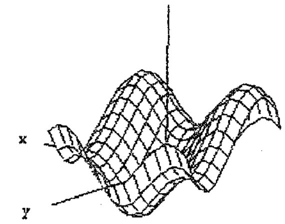
(b)



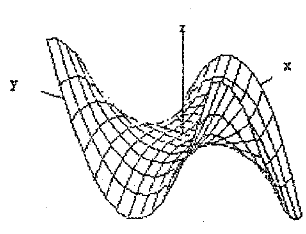
(c)



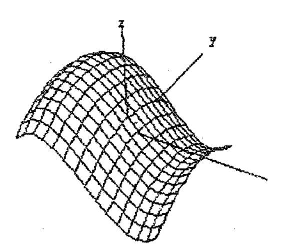
(d)



(e)



(f)



13. For your answers to the previous item (with all the graphs): please explain why you accepted/rejected the choice as a match to the original graph. Be specific. (e.g., "It is the same/a different surface." is not a helpful answer. Explain *what* is the same/different.) Do NOT change your answers on the previous page. If you want to change an answer, do it at the bottom of this page.

(a)

(b)

(c)

(d)

(e)

(f)

15. If you want to change any of your decisions, explain here which ones and why.

1. Indicate which calculator(s) you used for this calculus course (check all that apply):

_______ Non-graphing calculator.
_______ TI-82, TI-83, TI-85, TI-86, or other graphing calculator that can't draw 3D graphs.
_______ TI-92, TI-89, or other graphing calculator that can draw 3D graphs.

2. For each technology below, please check the amount of experience you have.

	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g., *Mathematica*, MathCad, DERIVE)					
e-mail					
internet					

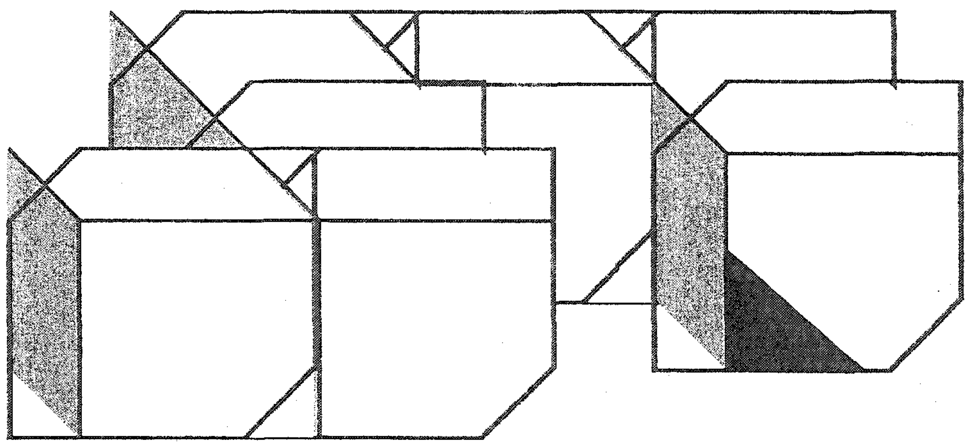
7. Computation is an important skill in multivariable calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

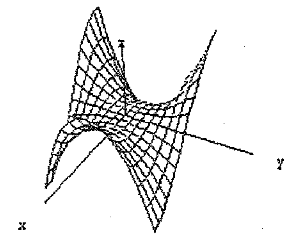
8. Visualization is an important skill in multivariable calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

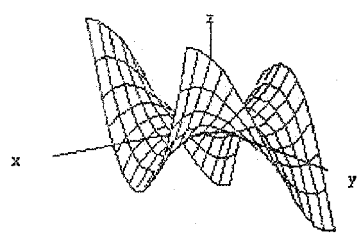
12. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.



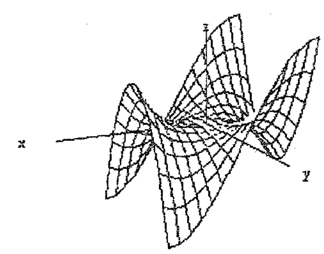
13. Consider the graph at the right. Identify which of the graphs below depict the same surface but from a different viewpoint. Circle all that apply. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.



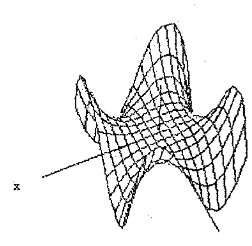
(a)



(b)



(c)



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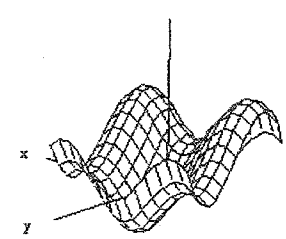

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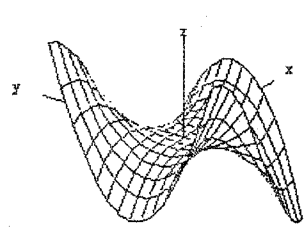

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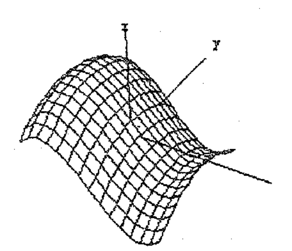


1. Please indicate your gender (circle one): F M

2. Please indicate your ethnicity (check one):

   _______ Hispanic or Latino/a ______ Non-Hispanic or Latino/a

3. Please indicate your race (check one):

   ______ American Indian or Alaska Native ______ Native Hawaiian or Other Pacific Islander
   ______ Asian ______ White
   ______ Black or African-American ______ Other

4. Please indicate the student status that best describes you (circle one):

   Freshman Sophomore Junior Senior Other

   Are you a transfer student this semester (circle one)? Yes No

5. Please indicate the enrollment status that best describes you (circle one):

   Full-Time Part-Time (more than one course) Single Course-Taker

6. Please indicate your intended major (check all that apply):

   _______ Engineering (not CS) Specify: ____________________________
   _______ Computer Science
   _______ Geoscience
   _______ Mathematics
   _______ Physics
   _______ Chemistry
   _______ Life Science Specify: ____________________________
   _______ Education
   _______ Fine or Applied Arts
   _______ Commerce or Business-Related Majors
   _______ Humanities, Liberal Arts, or Social Science (English, History, Psychology, Sociology, etc.)
   _______ Undecided
   _______ Other Specify: ____________________________

7. Do you intend or expect to teach at the K-12 level at some point your career (circle one)?

Yes No Maybe

8. Is this the first time you are taking Calc III? Yes No

If you are repeating Calc III this semester, why?

________ Failed the course the first time.

________ Dropped the course due to a failing grade.

________ Dropped the course for other reasons.

________ Did not fail the course, but am repeating it for other reasons.

9. For each technology below, please check the amount of experience you have.

	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g., *Mathematica*, MathCad, DERIVE)					
e-mail					
internet					

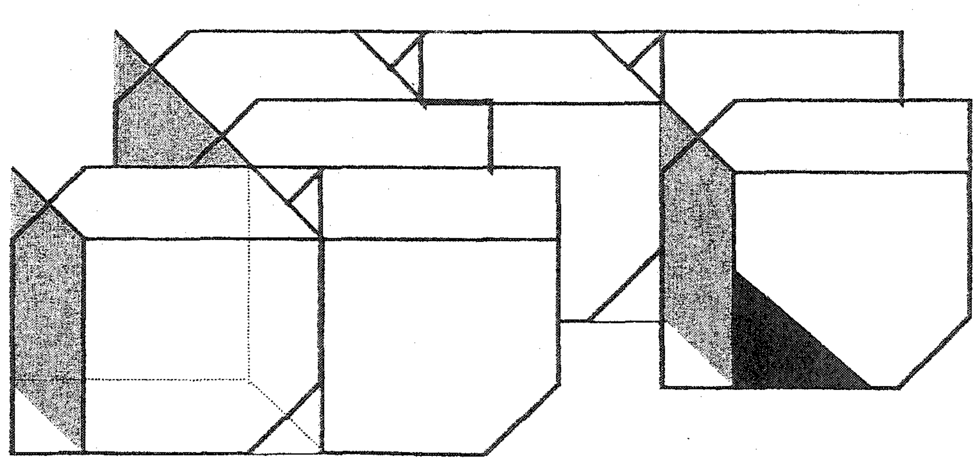
10. Computation is an important skill to succeed in calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

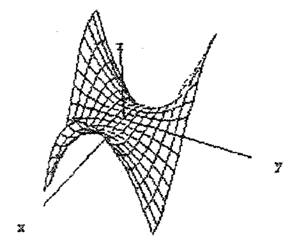
11. Visualization is an important skill to succeed in calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

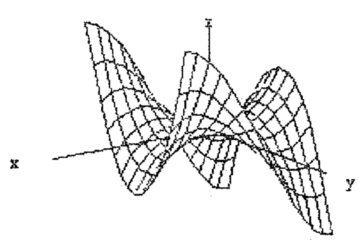
12. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.



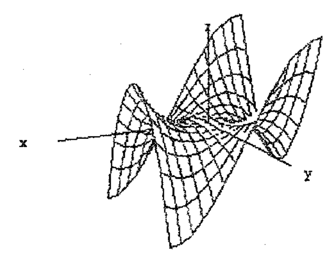
13. Consider the graph at the right. Identify which of the graphs below depict the same surface but from a different viewpoint. Circle all that apply. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.



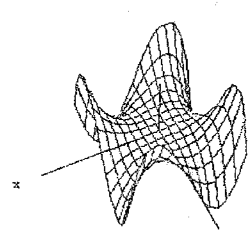
(a)



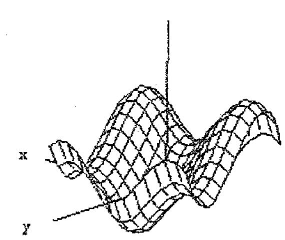
(b)



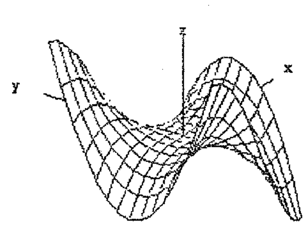
(c)



(d)



(e)



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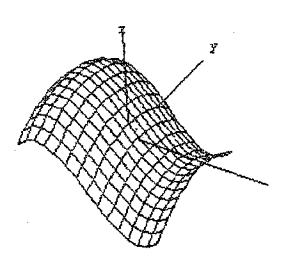

1. Indicate which calculator(s) you used for this calculus course (check all that apply):

_______ Non-graphing calculator.
_______ TI-82, TI-83, TI-85, TI-86, or other graphing calculator that can't draw 3D graphs.
_______ TI-92, TI-89, or other graphing calculator that can draw 3D graphs.

2. For each technology below, please check the amount of experience you have.

	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
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computer algebra systems (e.g., *Mathematica*, MathCad, DERIVE)					
e-mail					
internet					

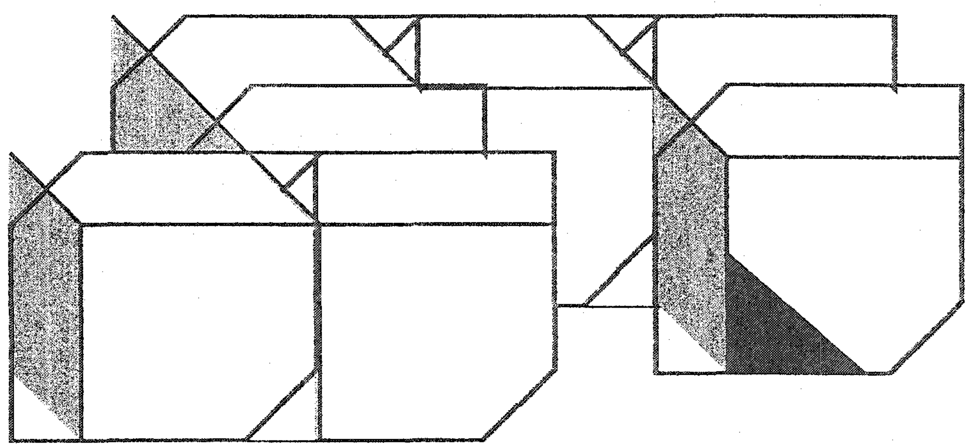
7. Computation is an important skill in multivariable calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

8. Visualization is an important skill in multivariable calculus (circle one response).

Strongly Disagree Disagree Neutral Agree Strongly Agree

12. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.



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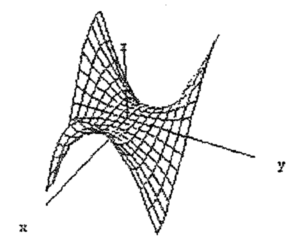
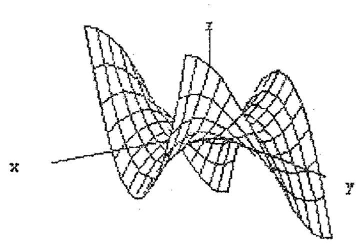


(a)



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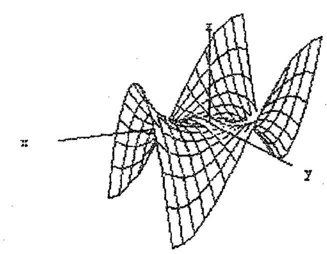
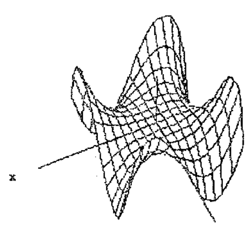
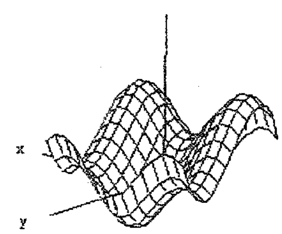


(c)



(d)



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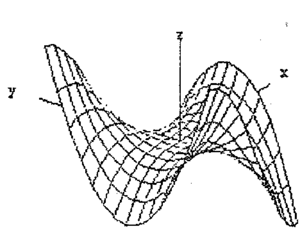

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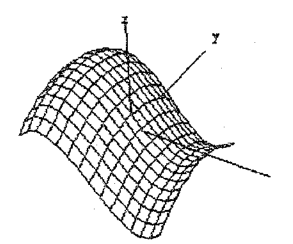


Calculus IV First Questionnaire (Summer 2000)

1. Please indicate your gender (circle one): F M

2. Please indicate your ethnicity (check one):

_______Hispanic or Latino/a _______Non-Hispanic or Latino/a

3. Please indicate your race (check one):

_______American Indian or Alaska Native _______Native Hawaiian or Other Pacific Islander

_______Asian _______White

_______Black or African-American _______Other

4. Please indicate the student status that best describes you (circle one):

Freshman Sophomore Junior Senior Other

Are you a transfer student this semester? (circle one): Yes No

5. Please indicate the enrollment status that best describes you (circle one):

Full-Time Part-Time Single Course Taker

6. Do you intend or expect to teach at the K-12 level at some point in your career? (circle one)

Yes No Maybe

7. Is this the first time you are taking Calc 3? Yes No

If you are repeating Calc 3 this semester, why?

_______Failed the course the first time.

_______Dropped the course due to a failing grade.

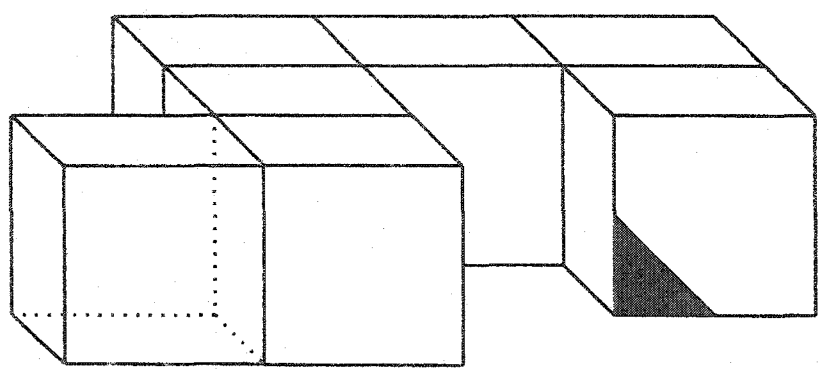
_______Dropped the course for other reasons.

_______Did not fail the course, but am repeating it for other reasons.

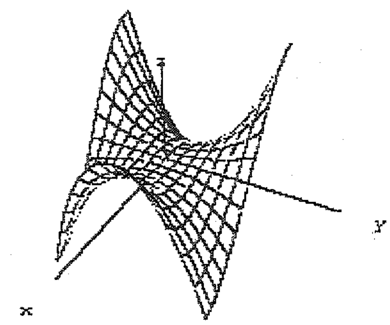
8. For each technology below, please check the amount of experience you have.

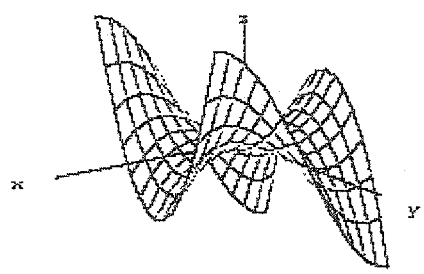
	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g. *Maple, Mathematica*, MathCad, DERIVE)					
e-mail					
internet					

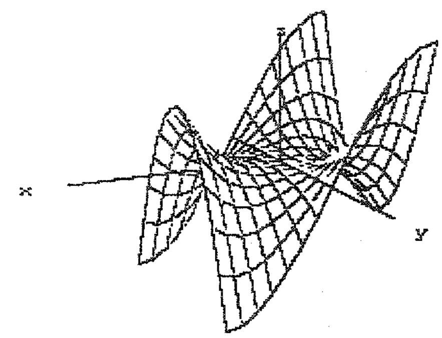
9. The cube shown at the far right is rolled several times via one of its edges, following the path indicated. Please draw in the position of the small black triangle on the final resting place of the cube at the far left.

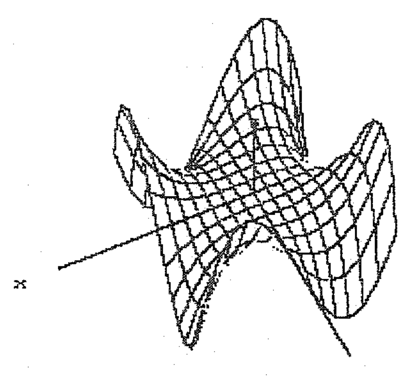


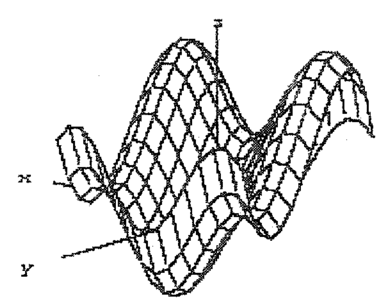
10. Consider the original graph at right. Identify which of the candidate graphs below depict the same surface but from a different viewpoint. **Circle all that apply**. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.

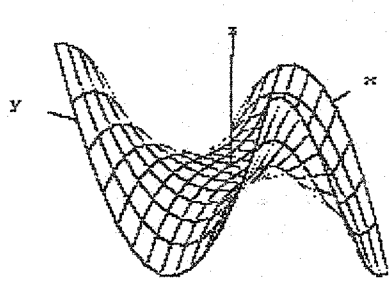


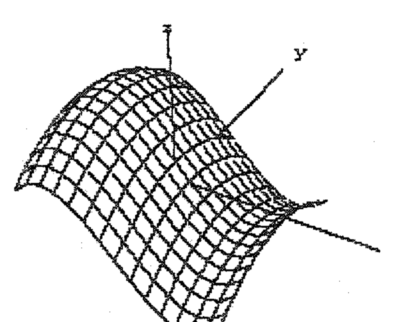












1. Indicate which calculator(s) you used for this calculus course (check all that apply):

   ________Non-graphing calculator.

   ________TI-82, -83, -85, -86, or other graphing calculator that can't draw 3D graphs.

   ________TI-89, TI-92, or other graphing calculator that can draw 3D graphs.

2. For each technology below, please check the amount of experience you have.

	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g. *Maple, Mathematica,* MathCad, DERIVE)					
e-mail					
internet					

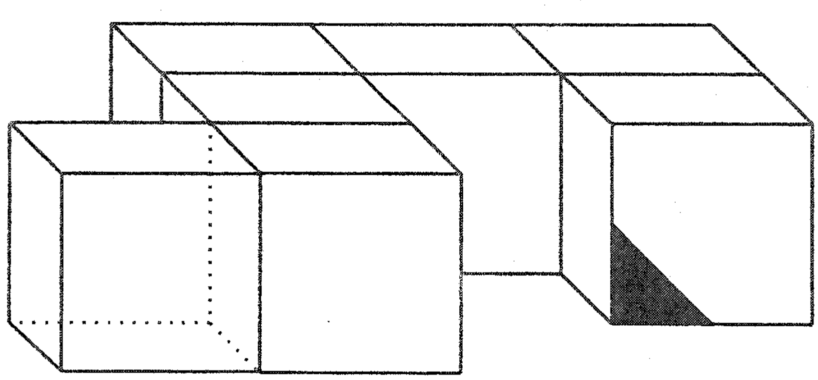
3. Computation is an important skill in calculus (circle one response).

   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

4. Visualization is an important skill in calculus (circle one response).

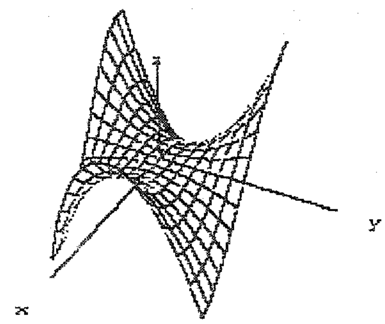
   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

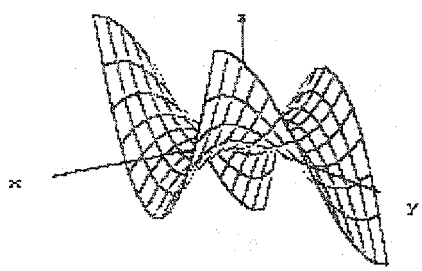
5. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.

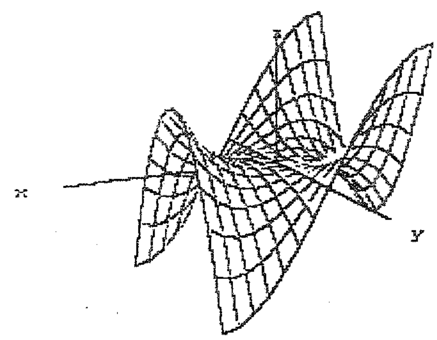


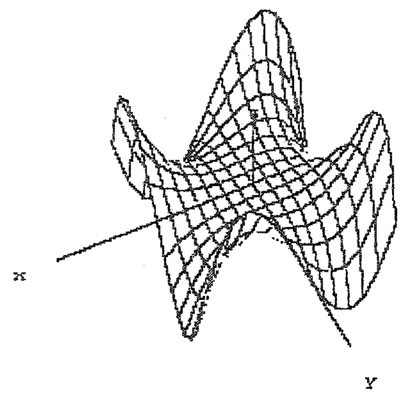
(please continue to the back side of this page once you have completed item 5)

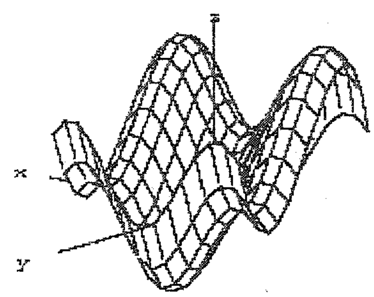
6. Consider the original graph at right. Identify which of the candidate graphs below depict the same surface but from a different viewpoint. **Circle all that apply**. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.

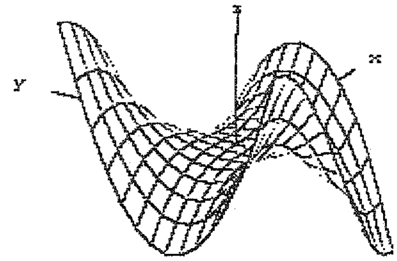


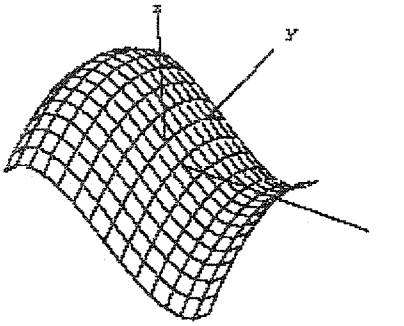












Calculus IV First Questionnaire (Summer 2002)

1. Please indicate your gender (circle one): F M

2. Please indicate your ethnicity (check one):

   ________Hispanic or Latino/a ________Non-Hispanic or Latino/a

3. Please indicate your race (check one):

   ________American Indian or Alaska Native ________Native Hawaiian or Other Pacific Islander
   ________Asian ________White
   ________Black or African-American ________Other

4. Please indicate the student status that best describes you (circle one):

   Freshman Sophomore Junior Senior Other

   Are you a transfer student this semester? (circle one): Yes No

5. Please indicate the enrollment status that best describes you (circle one):

   Full-Time Part-Time Single Course Taker

6. Do you intend or expect to teach at the K-12 level at some point in your career? (circle one)

   Yes No Maybe

7. Is this the first time you are taking Calc 3? Yes No

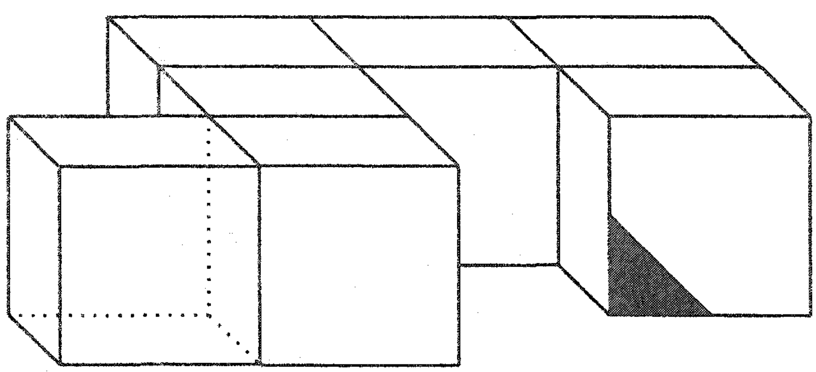
   If you are repeating Calc 3 this semester, why?

   ________Failed the course the first time.
   ________Dropped the course due to a failing grade.
   ________Dropped the course for other reasons.
   ________Did not fail the course, but am repeating it for other reasons.

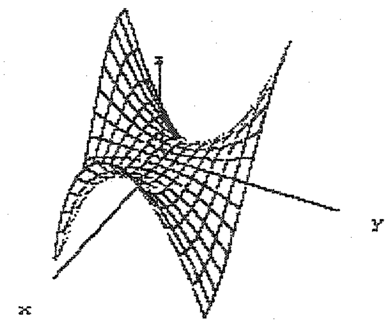
8. For each technology below, please check the amount of experience you have.

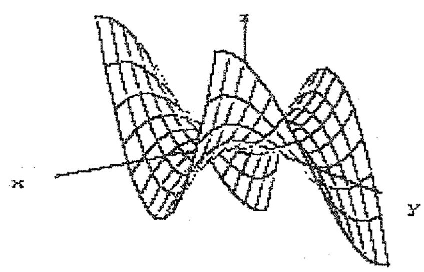
	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g. *Maple, Mathematica*, MathCad, DERIVE)					
e-mail					
internet					

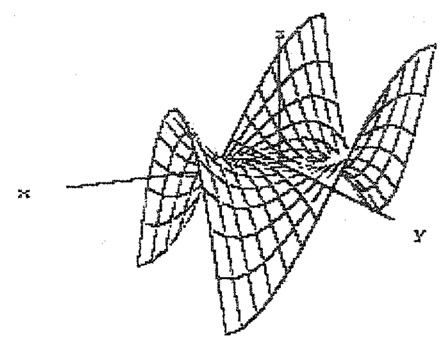
9. The cube shown at the far right is rolled several times via one of its edges, following the path indicated. Please draw in the position of the small black triangle on the final resting place of the cube at the far left.

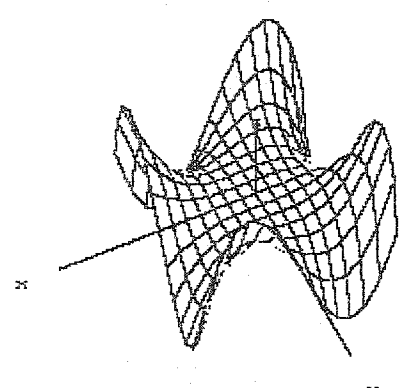


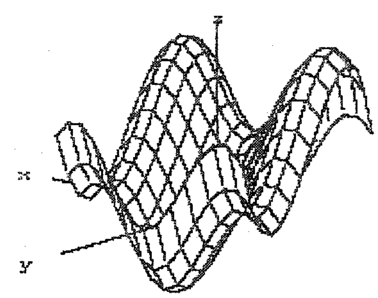
10. Consider the original graph at right. Identify which of the candidate graphs below depict the same surface but from a different viewpoint. **Circle all that apply**. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.

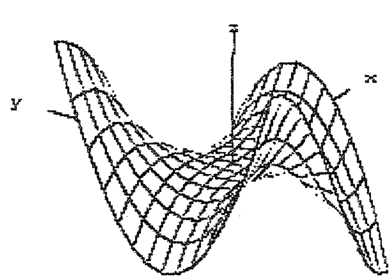


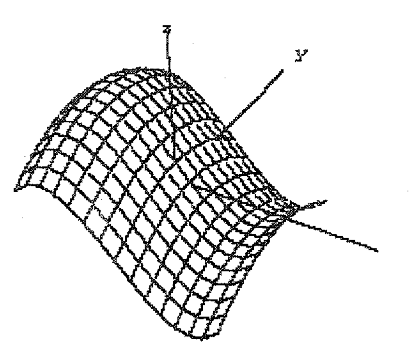












1. Indicate which calculator(s) you used for this calculus course (check all that apply):

 ________Non-graphing calculator.

 ________TI-82, -83, -85, -86, or other graphing calculator that can't draw 3D graphs.

 ________TI-89, TI-92, or other graphing calculator that can draw 3D graphs.

2. For each technology below, please check the amount of experience you have.

	What is that?	I have heard of it but not used it.	I have used it a few times.	I have used it more than a few times.	I have used it as a habit.
graphing calculators					
computer algebra systems (e.g. *Maple, Mathematica,* MathCad, DERIVE)					
e-mail					
internet					

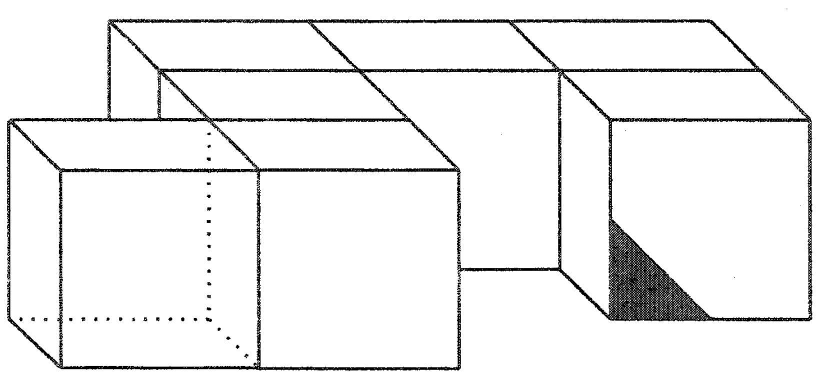
3. Computation is an important skill in calculus (circle one response).

 Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

4. Visualization is an important skill in calculus (circle one response).

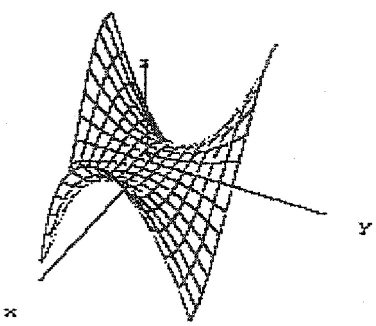
 Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

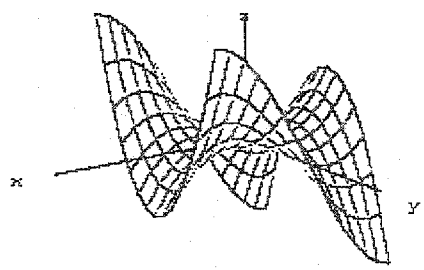
5. The given cube is rolled several times via one of its edges, following the path indicated. Draw in the position of the small black triangle on the final resting place of the cube.

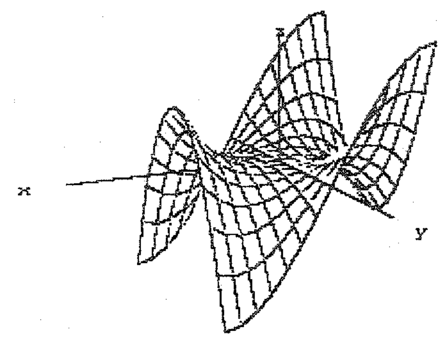


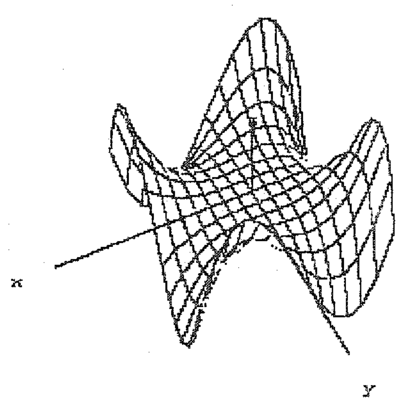
(please continue to the back side of this page once you have completed item 5)

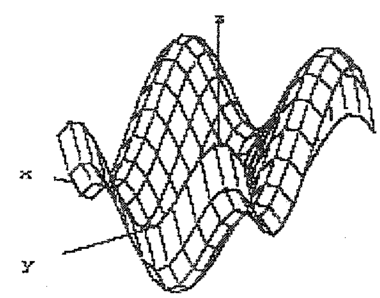
6. Consider the original graph at right. Identify which of the candidate graphs below depict the same surface but from a different viewpoint. **Circle all that apply**. Note: "x" marks the positive x-axis, "y" marks the positive y-axis, and "z" marks the positive z-axis. The same domain was used for all graphs.

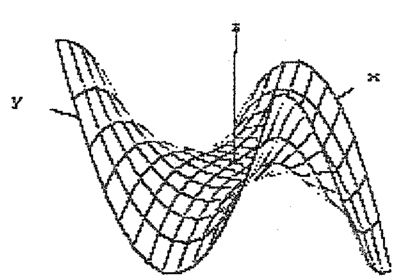


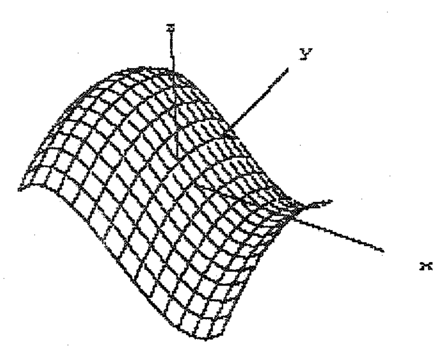












Please do not continue to this page until you have completed items 1-6.

Please do not change your responses to items 1-6 once you reach this point. Feel free to look back to help your memory if necessary, but please don't make any further marks on the previous pages.

7. Thinking back to item 6, the one you just finished with the graphs, which of these approaches did you consider in deciding for or against at least one of the candidates? **Please check all that apply:**

_____ I mentally pictured the original graph spinning around.
_____ I compared particular vertical traces (cross sections) in the graphs.
_____ I compared particular horizontal traces (cross sections) in the graphs.
_____ I compared how the surfaces lay relative to the x, y, or z-axes.
_____ I counted or compared positions of high or low points in the graphs.
_____ I looked for features in the lower six graphs which appeared in the original graph, particularly:

_____ The "wings" at the back of the original graph.
_____ The "hump" at the front of the original graph.

_____ Other - please describe briefly: ____________________

____

_____ I used some other tactic - please describe briefly: ____________________

____

8. Honestly, how hard would you say you tried in answering item 6, the one with the graphs?

_____ Not very hard.
_____ Fairly hard.
_____ Very hard.
_____ As hard as I possibly could.

9. Honestly, how hard would you say you tried in answering item 5, the one with the rolling box?

_____ Not very hard.
_____ Fairly hard.
_____ Very hard.
_____ As hard as I possibly could.

Thank you very much for your time in completing this questionnaire!