INCORPORATING PARAMETRIC UNCERTAINTY INTO FLOOD ESTIMATION METHODOLOGIES FOR UNGAGED WATERSHEDS AND FOR WATERSHEDS WITH SHORT RECORDS

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NOMENCLATURE

$\underline{X}$= The matrix named X.

tr($\underline{X}$) = The trace of $\underline{X}$.

$\underline{X}^{-1}$ = The inverse of $\underline{X}$.

$\underline{X}^{T}$ = The transpose of $\underline{X}$.

$|\underline{X}|$ = The determinant of $\underline{X}$.

$\hat{X}$ = Estimated value of X.

Y = The observed value of a response modeled as some function f(•).

$\hat{Y}$ = The value of Y as estimated by the function f(•).

Θ = Model parameter.

f($\underline{x}$) = Scalar-valued function of the model input vector $\underline{x}$.

f($\underline{x;\theta}$) = Scalar-valued function of the model input vector $\underline{x}$ and the parameter vector $\underline{\theta}$.

$\underline{f(x;\theta)}$ = Vector-valued function of the model input vector $\underline{x}$ and the parameter vector $\underline{\theta}$.

m = Number of elements of the parameter vector $\underline{\Theta}$.

n = Number of observations of Y.

k = Number of elements of the input vector $\underline{X}$.

p = Number of elements of the observed response vector $\underline{Y}$ and the estimated response vector $\underline{\hat{Y}}$.

ϵ = Residual (or error), equal to Y-$\hat{Y}$.

η = Transformed residual.

ς = Standard error of prediction.

p(x) = The probability density function of the random variable X.

$p(x|y)$ = The probability density function of the random variable X conditional upon the value of the random variable Y.

$p'(x)$ = The prior probability density function of the random variable X.

$p''(x)$ = The posterior probability density function of the random variable X.

$l(\theta)$ = The likelihood function of the random parameter Θ.

$E(X)$ = The expected value of the random variable X.

$E(X|Y)$ = The expected value of the random variable X conditional upon the value of the random variable Y.

$Var(X)$ = The variance of the random variable X.

$Corr(X,Y)$ = The correlation between random variables X and Y.

$\underline{J}$ = The Jacobian matrix.

$\prod_{i=1}^{n}$ = The product operator.

exp = The exponential operator.

$\Gamma(x)$ = The gamma function evaluated at x.

$N(\mu,\sigma^2)$ = Normally distributed with mean μ and variance σ^2.

$N_2(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$ = Bivariate normally distributed with means μ_1, μ_2, variances σ_1^2, σ_2^2, and correlation coefficient ρ.

$N_m(\underline{\mu},\underline{\Sigma})$ = m-dimensional normally distributed with mean vector $\underline{\mu}$ and covariance matrix $\underline{\Sigma}$.

$\min_{\theta}$ = Minimize with respect to the parameter θ.

CHAPTER I

INTRODUCTION

Statement of the Problem

A classical problem in hydrology is that of estimating the magnitude of the flood flow which corresponds to a given probability of exceedance. For those sites at which a sequence of observed peak annual flows is available, the traditional approach has been to fit a theoretical probability density function to the flows in order to infer the probability of occurrence of future floods. This procedure is commonly referred to as flood estimation by application of distribution theory. In many cases, however, the subject of an engineering decision is an ungaged watershed for which no data are available. It is axiomatic that the traditional approach may not be directly applied in such cases. Instead, the analysis must be conducted using one of the many methods of ungaged site estimation.

The subject of uncertainty has for some time been recognized as another of the problems of hydrology. However, it is only recently that researchers have attempted to develop procedures to analyze uncertainty and incorporate it into predictive frameworks. For a given hydrologic process, there exists a natural uncertainty which is derived from the inherent randomness of the process. When the process is to be represented by some model, there exists uncertainty in the model parameters which are determined by means of a comparison of predicted and observed outputs. This uncertainty arises because hydrologic records are typically short, leading to parameter estimates of questionable accuracy. One can not be sure, based upon the available data, of the true value of the parameter.

Procedures for estimating floods for ungaged watersheds are fraught with parametric uncertainty. However, such estimation procedures tend to beg the question of how parametric uncertainty influences the interpretation of the estimates.

Objectives

The objectives of this research were to:

1. Analyze uncertainty in the parameters of an event-based rainfall-runoff model for a set of gaged watersheds.
2. Use the rainfall-runoff model and results of Objective 1 to develop flood estimation methodologies for both ungaged watersheds and watersheds with short records which explicitly account for parametric uncertainty.
3. Evaluate the flood estimation methodologies with regard to their accuracy, their ability to demonstrate the effects of parametric uncertainty, and their practicality.

General Procedure

The SCS unit hydrograph model (1972) was selected to develop the flood estimation methodology. In order to fulfill the first objective, elements of Bayesian statistical theory were employed to determine the probability density functions of the model parameters S and Tp for a set of 15 watersheds. These probability density functions are derived using data on both peak flows and runoff volumes.

The flood estimation procedure for ungaged watersheds was developed by relating the probability density functions of S and Tp to geomorphic parameters via a set of regression-based prediction equations. The prediction equations provided a means of estimating probability density functions of S and Tp for ungaged watersheds. Flood frequency curves for the ungaged watersheds were determined by first assuming that the recurrence intervals of peak flow and associated rainfall are equal.

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Next, the stochastic behavior of the peak flow corresponding to a given recurrence interval was inferred by using the appropriate rainfall event and a sample of random pairs of values of S and Tp generated from their respective probability density functions as inputs to the SCS unit hydrograph model and computing the resulting peak flows. The point estimate of the peak flow corresponding to the recurrence interval was taken as the mean of all peak flows computed for that recurrence interval.

In order to estimate floods for watersheds with short records, site-specific data on peak flow and runoff volume were combined through Bayes' Theorem with the estimated probability density functions of S and Tp to reduce uncertainty in the two model parameters as reflected in modified probability density functions for these parameters. The remainder of this procedure of flood estimation was essentially the same as for ungaged watersheds.

The two flood estimation methodologies were evaluated using a Jackknife approach. Flood frequency curves were estimated for each of the 15 watersheds as if it were initially ungaged. The estimated flood frequency curves were compared to the observed flood frequency curves using a Kolmogorov-Smirnov goodness of fit test. The accuracy of the two methodologies relative to USGS (Tortorelli and Bergman, 1985) and SCS (1972) procedures was appraised by means of visual comparison of the resultant flood frequency curves and comparison of the Kolmogorov-Smirnov test statistics.

CHAPTER II

REVIEW OF LITERATURE

The research described in this dissertation is intimately related to two major subjects: flood estimation for ungaged watersheds and analysis of hydrologic uncertainty. Accordingly, this chapter summarizes the state of the art of both topics. Flood estimation methods are classified with a brief description of each method. Following the discussion of flood estimation procedures, three methods of analyzing uncertainty are presented. Two methods, first order analysis and non-parametric analysis, are described in moderate detail. A similar treatment of the third method presented, Bayesian analysis, is deferred until the succeeding chapter.

Flood Estimation for Ungaged Watersheds

Numerous methods have been proposed as possible solutions to the very practical problem of flood estimation for ungaged watersheds. Indeed, a modest-sized body of literature exists which merely attempts to classify, describe, and compare methods of flood estimation (Allison, 1967; Fleming and Franz, 1971; Reich and Jackson, 1971; Bowers, et al., 1972; Clarke, 1973; McCuen, et al., 1977; McCuen and Rawls, 1979). Following the classification of McCuen, et al. (1977), flood estimation procedures may be generally described as:

1. Statistical estimates of peak flows,
2. Statistical estimates of the moments of the distribution of peak flows,
3. Index flood estimation,
4. Estimation by transfer of peak flows,

5. Empirical equations,

6. Single storm event modeling with peak flow recurrence interval assumed equal to the rainfall recurrence interval,

7. Modeling of multiple discrete events, and

8. Estimation by continuous simulation modeling.

Statistical Estimates of Peak Flows

Statistical estimation approaches attempt to relate peak flows of various exceedance probabilities to measurable watershed characteristics (such as area, average slope, proportion of wooded area, etc.) via multiple regression techniques. The concept of regionalization, or delineation of hydrologically similar areas for purposes of relating watershed characteristics to other quantities of interest (see Solomon (1976) for a discussion of regionalization), is normally used in such analyses, be it implicitly used or explicitly stated. The results of a statistical estimation approach are usually a set of equations which are used to compute peak flows corresponding to recurrence intervals of interest. The equations may then be applied to ungaged watersheds in regions which are hydrologically similar to those from which the equations were developed. The methods described by Benson (1962, 1964), DeCoursey (1972), Thomas and Corley (1977), and Tortorelli and Bergman (1985) are representative of statistical estimation procedures.

Statistical Estimation of Moments

A similar approach to ungaged estimation is that of estimating the moments of the distribution of peak flows. In this method, it is the moments of the random variable annual peak flow, rather than peak flows of selected recurrence intervals, that are related to measurable watershed characteristics, again through multiple regression. Normally, the mean and variance are the moments estimated, and peak annual flow is taken as following the Log-Pearson Type III distribution. The third moment, skewness, is typically more unstable than the first two moments and is commonly determined from maps of regional skew such as those provided by the U.S. Geological Survey Hydrology Subcommittee (1981). Saah, et al. (1967) and the U.S. Army Corps of Engineers (1975) are among those who have proposed moment-estimation procedures to estimate peak flows for ungaged watersheds.

Index Flood Estimation

The index flood method is predicated on the assumption that the probability density function of the random variable peak annual flow, normalized by an index flow (commonly taken as the mean annual peak flow), is the same for all watersheds within regions defined as hydrologically similar. This assumption allows the average ratios of the index flow to flows of other recurrence intervals to be specified as constants within a given hydrologic region. These average ratios must be determined using existing information. Dalrymple (1960), Reich, et al. (1971), and Reich and Jackson (1971) have used multiple regression techniques to estimate the mean annual peak flow of ungaged watersheds on the basis of physical characteristics of the watersheds.

Estimation by Transfer of Peak Flows

This category is a rather nebulous one, and most examples given by McCuen, et al. (1977) could easily have been classified under other categories. Noteworthy exceptions include the methods advocated by Crippen and Conrad (1977), who developed envelope curves of potential maximum peak flows, and Riggs (1974, 1976) who regressed peak flow against variables such as channel dimensions, high water marks, and slope of the water surface.

Empirical Equations

Empirical equations which explicitly relate flow to rainfall and other variables have been developed by a number of researchers. Betson, et al. (1969), Chow (1962), Hewlett, et al.(1977), and Smith and Hauser (1976) describe the development of relatively simple equations suitable for use on ungaged watersheds.

Single Storm Event Modeling

The single storm event modeling procedure typically entails inputting a rainfall depth, duration, and temporal distribution to an event-based rainfall-runoff model and obtaining the resulting storm hydrograph. The peak flow is then taken as having the same recurrence interval as the rainfall event. Chu and Lytle (1972), Hawkins (1973), and Danushkodi (1979) have discussed this method using the SCS TR-20 model (SCS, 1969). Beran (1976), McSparran (1968), and Gray (1961) have used variations of other unit hydrograph methods. Others (Huggins and Monke, 1968; Judah, et al., 1975; Mein, et al., 1974) have developed their own models specifically suited to ungaged watersheds.

Multiple Discrete Event Modeling

The multiple discrete event modeling procedure uses a series of rainfall events, either actual or synthetic, as inputs to an event-based rainfall-runoff model with the end result being the flood frequency curve for the ungaged watershed of interest. This approach is very similar to that described in the previous section; the only difference is in the amount of input data used. Fogel, et al. (1974, 1975) present such an approach to flood estimation.

Continuous Simulation Modeling

The continuous simulation modeling approach to ungaged site estimation employs a continuous streamflow synthesis model, such as the Stanford Watershed

Model (Crawford and Linsley, 1966) or the USGS rainfall-runoff model (Dawdy, et al., 1972), operated using either actual or synthetic rainfall inputs. Model output is then used to construct a histogram of annual peak flows, and inferences regarding the occurrence of peak flows are drawn based upon a probability density function fit to the histogram. Lichty and Liscum (1978) discuss such an approach using the USGS rainfall-runoff model in which the model output for a gaged location is generalized to similar areas by use of multiple regression. Such an approach to ungaged site estimation appears promising due to the explicit manner in which many of the components of the hydrologic cycle are treated. However, continuous streamflow models typically have many parameters, the values of which must be determined (optimized) by comparison of predicted flows to observed flows. Magette, et al. (1976) attempted to surmount the need for a calibration data base by relating parameters of the Stanford watershed model to watershed characteristics such as area, average slope, drainage density, etc. through multiple regression. "Variable" results were reported with the regression predictions of the parameters as much as 700% in error of the optimized values of the parameters. Clearly, more reliable regression-based parameter estimation procedures are required before the simulation modeling approach may be confidently applied to ungaged watersheds.

Hydrologic Uncertainty

A factor common to any method of flood estimation, be it for gaged or ungaged watersheds, is that of hydrologic uncertainty. In a flood frequency curve developed for a gaged watershed, there exists uncertainty in both the "true" flood frequency model and in the values of the parameters of that model. Following Kuczera's (1983) definition of the true value of a parameter (that obtained from fitting the model to an arbitrarily long sequence of data), we are uncertain of the true value of the parameters because perfect information is not available. Uncertainty plays an even greater role in flood estimation for ungaged watersheds because

a. The method of estimation may not be appropriate for the site in question; it may have been developed for watersheds with different hydrologic conditions.

b. The "constant" factors which are present in most of these methods of flood estimation represent the fitting of an originally parametric model to experimental data from other watersheds. These constants should therefore be considered as uncertain parameters, owing to their being estimated from limited data.

The effects of uncertainty on flood estimation may be of considerable significance, as evidenced by the work of Wood (1976, 1978). It is generally recognized that flood estimation procedures which account for uncertainty are more conservative than those which do not account for uncertainty. Therefore, it is desirable in several respects to couple any flood estimation methodologies with an analysis and incorporation of the associated uncertainty.

Vicens, et al. (1975a) classified hydrologic uncertainty as being of two types: natural uncertainty and informational uncertainty. Natural uncertainty may be thought of as the uncertainty due to the inherent random or stochastic nature of the hydrologic process. Informational uncertainty arises from the lack of perfect information regarding the hydrologic process of interest. Informational uncertainty may be further classified as either model or parameter uncertainty. Model uncertainty refers to the fact that the model used to represent the hydrologic process may not be "correct" in some sense. It is possible, given several realizations of a hydrologic process, to infer differing models as being the correct representative mechanism of the process. Parameter uncertainty is present due to estimating the model parameters from limited data; different sets of data will generally result in different estimates of model parameters. For this reason, model parameters estimated from imperfect data may be considered random variables characterized by their probability density functions.

It has been the focus of stochastic hydrology to analyze hydrologic processes in terms of natural uncertainty. This is evidenced by the development of a multitude of models which are capable of synthesizing hydrologic processes with little consideration of informational uncertainty. Research performed during the last two decades, however, has led to an increased awareness of the existence and effects of informational uncertainty. Uncertainty in the parameters of flood frequency models has received a particularly high degree of attention.

Uncertainty in a parameter of a hydrologic model may be quantified in terms of a probability density function of the parameter, or merely in terms of the mean and variance of the parameter. Among the methods available to quantify and/or analyze the effects of parameter uncertainty are first order analysis, non-parametric statistical methods such as the Jackknife and Bootstrap methods, and Bayesian methods.

First Order Analysis of Uncertainty

First order analysis (Benjamin and Cornell, 1970) has been presented as a method of assessing the effects of uncertain model parameters on model output. To demonstrate the application of first order analysis, consider a random variable Y functionally related to a random, independent n-vector $\underline{X}$; i.e., $Y = f(\underline{X})$. The second-order Taylor series expansion of Y about $\underline{\mu}$, the mean of $\underline{X}$, is

$$Y = f(\underline{\mu}) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial X_i}\right|_{\underline{X}=\underline{\mu}} (X_i-\mu_i) +$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left.\frac{\partial^2 f}{\partial X_i \partial X_j}\right|_{\underline{X}=\underline{\mu}} (X_i-\mu_i)(X_j-\mu_j) \qquad (1)$$

Taking expectations, it follows that

$$E(Y) = f(\underline{\mu}) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left. \frac{\partial^2 f}{\partial X_i \delta X_j} \right|_{\underline{X}=\underline{\mu}} Cov(X_i, X_j) \tag{2}$$

and

$$Var(Y) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left. \frac{\partial f}{\partial X_i} \right|_{\underline{X}=\underline{\mu}} \left. \frac{\partial f}{\partial X_j} \right|_{\underline{X}=\underline{\mu}} Cov(X_i, X_j) \tag{3}$$

In the case where the X_i are uncorrelated, the variance of Y may be approximated by

$$Var(Y) = \sum_{i=1}^{n} \left(\left. \frac{\partial f}{\partial X_i} \right|_{\underline{X}=\underline{\mu}} \right)^2 Var(X_i) \tag{4}$$

In this manner, the uncertainty in Y is expressed as a function of the uncertainty in $\underline{X}$. It may be noted that use of first order analysis implies that the uncertainty in the random variable is satisfactorily described by only its variance.

Following its presentation in a hydrologic context (Cornell, 1972), first order analysis has been applied to equations describing flow in open channels (Mays, 1979; Tung and Mays, 1980, 1981a) and pipes (Clarke, et al., 1981), to flood plain mapping (Burgess, 1979), and to simulated hydrographs (Garen and Burgess, 1981).

The most obvious advantage of using first-order analysis is its relative simplicity as compared to a full probabilistic analysis, which uses probability density functions of uncertain parameters and the method of derived distributions to describe uncertainty in model output. Even when full probabilistic analyses are performed, often only the first two moments of the variable of interest are presented (Freeze, 1975); these are determined much more readily by first order analysis. Additionally, first order analysis is suited to situations in which the dependent variable is not related to the independent variables via a single equation (e.g., a simulation model). This method of analysis is limited, however, in that it is at best incomplete, it is approximate, and it may not be appropriate for some relationships of interest (e.g., Y = max(X)).

Non-Parametric Methods of Analysis

Non-parametric methods have been developed to obtain estimates of the means and variances of sample statistics. Non-parametric methods do not assume a priori the distribution of a sample statistic (e.g., a normal distribution for the mean of a normally distributed population), but instead rely on empirical methods to derive the distribution of the statistic of interest. Two non-parametric methods which have been reported in the hydrologic literature are the Jackknife and Bootstrap methods.

Jackknife Method. The Jackknife method was developed by Quenouille (1949) in order to estimate the bias of a sample statistic. Tukey (1958) proposed that the Jackknife method also be used to estimate the variance of the statistic. To illustrate the method, consider a data set $\underline{x}$, where $\underline{x} = (x_1, x_2, \ldots, x_n)$. Consider next a statistic F which is estimated as a function of the data set $\underline{x}$; i.e.,

$$\hat{F} = F(x_1, x_2, \ldots, x_n) \tag{5}$$

The Jackknife method requires that the statistic F be estimated n times, each time with the i^{th} data point deleted (i=1,2,...,n).

$$\hat{F}_i = F(x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \tag{6}$$

Efron (1983) gives the Jackknife estimate of F as

$$\hat{F}_{(\bullet)} = \frac{1}{n} \sum_{i=1}^{n} \hat{F}_i \tag{7}$$

The bias-corrected estimate of F is

$$\hat{F}_J = nF-(n-1)\hat{F}_{(\bullet)} \tag{8}$$

The Jackknife estimate of the variance of F (σ_J^2) is

$$\hat{\sigma}_J^2 = \left(\frac{n-1}{n}\right) \sum_{i=1}^{n} [\hat{F}_i-\hat{F}_{(\bullet)}]^2 \tag{9}$$

The n estimates of F provide the empirical distribution of F.

Bootstrap Method. The Bootstrap, developed by Efron (1979, 1982), is an alternative non-parametric method of estimating the mean and variance of a sample statistic. To illustrate the Bootstrap method, consider again a data set $\underline{x}$ and a statistic F which is estimated as a function of $\underline{x}$. The data set $\underline{x}$ is randomly sampled with replacement to yield R realizations of the data. Each of the new data sets, $\underline{x}_r^* = \{x_1^*, x_2^*, ..., x_n^*\}$, r=1,2,...,R, is used to estimate F, providing R estimates of F. The Bootstrap estimate of F is

$$\hat{F}_{(\bullet)} = \left(\frac{1}{R}\right) \sum_{i=1}^{R} \hat{F}_i \tag{10}$$

The bias of $\hat{F}_{(\bullet)}$ (B) is estimated by

$$\hat{B} = \hat{F}_{(\bullet)} - \hat{F} \quad (11)$$

The Bootstrap estimate of the variance of F (σ_B^2) is

$$\hat{\sigma}_B^2 = \left(\frac{1}{R-1}\right) \sum_{i=1}^{R} [\hat{F}_B - \hat{F}_{(\bullet)}]^2 \quad (12)$$

As with the Jackknife method, the empirical distribution of F is defined by the R estimates of F.

Applications of the Jackknife and Bootstrap methods in a hydrologic context have been infrequently reported in the hydrologic literature. Tung and Mays (1981b) used the Jackknife and Bootstrap methods to estimate means and variances of the parameters of the Log-Pearson Type III probability density function. Cover and Unny (1986) have used similar methods to analyze uncertainty in the parameters of an autoregressive moving average model.

Bayesian Methods

Bayesian analysis may be used to quantify parameter uncertainty in the event one has prior, or additional, information regarding the parameter. The prior information must be expressible in probabilistic terms in order to be useful in a Bayesian analysis. Bayesian techniques are appealing from a hydrologic perspective because uncertainty in the parameters of models is explicitly accounted for and translated to uncertainty in the process of interest. Following Bernier's (1967) application of Bayesian statistical theory to improve peak flow estimates of desired recurrence intervals, there have been several reports of Bayesian techniques used in a hydrologic context. Conover (1971) coupled Bayesian methods with a decision analysis in order to find the optimal estimator for the correlation coefficient of an autoregressive model. Davis, et al. (1972) present an excellent account of applying Bayesian methods in the context of decision theoretic analysis. The authors determined the optimal level of flood protection (expressed as flood levee height) to be provided and whether or not to defer the decision until more data had been collected. Vicens, et al. (1975a, 1975b) investigated reducing uncertainty in the parameters of a streamflow synthesis model via Bayesian methods. It was found that prior information from regional analyses, when appropriately expressed, significantly reduced parameter uncertainty in the presence of short periods of record (less than 25 years) with which to estimate the parameters. Wood and Rodriguez-Iturbe (1975a) applied Bayesian principles to flood frequency analysis and derived the Bayesian distribution of annual peak flows for Normal, Log-Normal, and Exceedance flood frequency models.

Kuczera (1982) proposed Empirical Bayes Theory, as discussed by Robbins (1964), as a method of combining regional and site-specific information in order to estimate peak flows of desired recurrence intervals. Regional estimates of the mean and variance of the annual flood series were used to provide the necessary prior information. This prior information was then coupled with site-specific information to derive the joint probability density function of the site mean and variance. Compound distribution theory was then used to derive the probability density function of the peak flow of the desired recurrence interval. It was noted that in general, the incorporation of regional information reduced uncertainty in the site-specific estimate of this peak flow, particularly when site records were short.

Bayesian methods have also been proposed as a method of selecting the "most correct" flood frequency model from an assortment of competing models. Wood and Rodriguez-Iturbe (1975b) used marginal likelihoods, derived using a Bayesian methodology, to find the probabilities that the correct flood frequency model was either Normal, Log-Normal, or Exceedance. The result of this procedure was the specification of a composite flood frequency model, which was the algebraic sum of weighted outputs of the competing models. Whereas Wood and Rodriguez-Iturbe

(1975b) considered only models belonging to families of natural conjugate distributions (families characterized by attractive combinative properties), Bodo and Unny (1976) broadened their scope to include some of the less tractable flood frequency models, such as the Gumbel, Gamma, and Log-Gamma models, as candidates for the "true" model and used likelihood ratios to discriminate between model candidates. The researchers encountered difficulties in the integrations required in their analysis and were forced to resort to ad hoc numerical procedures.

Kuczera (1983) presented an interesting and innovative application of Bayesian methods to the problem of reducing uncertainty in the parameters of a deterministic hydrologic model. Noting the limited success of using regression techniques to relate optimized model parameters to geomorphic parameters (see, for example, Magette, et al., 1976), the author suggested that even if there exists a useful relationship between model parameters and geomorphic parameters, poorly inferred model parameters can obfuscate this relationship. The approach to parameter inference was to first consider the model (in this case a water yield model developed by Langford, et al., 1978) as a non-linear regression model, with residuals being possibly both heteroscedastic and correlated. Initially, the ordinary least-squares criterion was used to optimize the parameters based upon available data. If the residuals were found to violate the assumptions of ordinary least-squares estimation, namely homoscedasticity and independence of the residuals, then a power transformation and/or autoregressive moving average model was fit to the residuals and the parameters redetermined. Beginning with an assumed distribution of the transformed residuals, the joint probability density function of the model parameters was analytically derived. This joint probability density function was found to be proportional to the sum of squares of the transformed residuals, raised to some power. It was then assumed that the sum of squares function could be adequately approximated as a linear function of the model parameters, and Beale's measure of nonlinearity (Beale, 1960) was used as a criterion for checking the validity of this assumption. Given a satisfactory linearization, the joint probability density function of the model parameters was found to reduce to that of a multivariate t distribution. In the presence of a large number of observations of the watershed response, the multivariate t distribution may be approximated by a multivariate normal distribution, greatly simplifying the task of inferring parameter uncertainty. Compared to independent estimates of the means and standard errors of the parameters, the Bayesian process was found to significantly reduce uncertainty in the optimized parameters.

CHAPTER III

THEORY

This research has drawn heavily upon present knowledge regarding two major subjects: Bayesian analysis of uncertainty and parameter estimation. Discussion of these two subjects comprises the totality of this chapter.

Bayesian Analysis

Bayesian analysis is a relatively straight forward method of analyzing uncertainty in model parameters. This method of analysis has as its basis Bayes' Theorem, which in itself is nothing more than a statement of relationships between conditional probabilities. It has been mentioned that Bayesian analysis requires prior information regarding the parameters of interest. This information must generally be expressible as a probability density function. Specification of this "prior" probability density function is at the heart of the controversy associated with Bayesian methods. It is intuitive that misspecification of the prior probability density function may adversely affect the quality of the results of the analysis. In addition, if one obtains the prior probability density function through questionable means, the results of the analysis are likely to be viewed by others as highly dubious. Because of the importance of how prior information is incorporated into Bayesian analysis, formulation of the prior probability density function is discussed in detail later in this chapter.

Bayes' Theorem

Consider a vector $\underline{y}=(y_1,y_2,\ldots,y_n)$ as n observations of a random variable Y, and suppose that the joint probability density function of $\underline{Y}$, $p(\underline{y|\theta})$, is dependent on the values of $\underline{\Theta}$, a k-vector of parameters. Note that the probability density function of $\underline{Y}$ is expressed as conditional upon $\underline{\Theta}$, implying that $\underline{\Theta}$ is a random rather than fixed vector. Considering $\underline{\Theta}$ thusly is a concept central to Bayesian statistical theory; that is to say, parameters of probability density functions are and should be treated as random variables. To continue, it follows from the definition of conditional probability that

$$p(\underline{y|\theta})p(\underline{\theta}) = p(\underline{y,\theta}) = p(\underline{\theta|y})p(\underline{y}) \tag{13}$$

Given $\underline{y}$, Bayes' Theorem states that

$$p''(\underline{\theta|y}) = \frac{p(\underline{y|\theta})p'(\underline{\theta})}{p(\underline{y})} \tag{14}$$

The expression $p'(\underline{\theta})$ is known as the prior probability density function of the parameter vector $\underline{\Theta}$; it represents what is known about $\underline{\Theta}$ prior to collection of $\underline{y}$. The expression $p''(\underline{\theta|y})$ is known as the posterior probability density function of $\underline{\Theta}$. It embodies knowledge of $\underline{\Theta}$ after collection of $\underline{y}$. A definition regarding conditional probability density functions may be invoked to restate Bayes' Theorem as

$$p''(\underline{\theta|y}) = \frac{p(\underline{y|\theta})p'(\underline{\theta})}{\int p(\underline{y|\theta})p'(\underline{\theta})d\underline{\theta}} \tag{15}$$

where the integration is over k-dimensional real space.

Bayes' Theorem and the Likelihood Function

After obtaining $\underline{y}$, the probability density function $p(\underline{y|\theta})$ may be considered a function of $\underline{\Theta}$ rather than $\underline{y}$. When viewed from this perspective, the function $p(\underline{y|\theta})$ is referred to as the likelihood function of $\underline{\Theta}$. When the observations $\underline{y}$ are independent and identically distributed, the likelihood function is defined as

$$l(\underline{\theta|y}) = \prod_{i=1}^{n} p(y_i|\underline{\theta}) \tag{16}$$

In terms of the likelihood function, Bayes' Theorem may now be written as:

$$p''(\underline{\theta|y}) = \frac{l(\underline{\theta|y})p'(\underline{\theta})}{\int l(\underline{\theta|y})p'(\underline{\theta})d\underline{\theta}} \tag{17}$$

where the integration is again taken over k-dimensional real space. It is apparent that the denominator of Eqn. 17 serves only as a normalizing constant; i.e., it ensures that $p''(\underline{\theta|y})$ integrates to unity. Recognition of the role of the denominator leads to an important restatement of Bayes' Theorem:

$$p''(\underline{\theta|y}) \propto l(\underline{\theta|y})p'(\underline{\theta}) \tag{18}$$

With Bayes' Theorem written thusly, the importance of the likelihood function is obvious: it is the sole means by which prior knowledge on $\underline{\Theta}$ is modified by collection of data.

Sequential Application of Bayes' Theorem

Bayes' Theorem is attractive from the standpoint that it provides a convenient algorithm for updating knowledge on $\underline{\Theta}$ as data on Y become available. To illustrate, suppose that an initial prior probability density function, $p'(\underline{\theta})$ is specified, and suppose that an initial sample of observations on Y, $\underline{y}_1$, are collected. Then, by Bayes' Theorem,

$$p''(\underline{\theta|y}_1) \propto p'(\underline{\theta})l(\underline{\theta|y}_1) \tag{19}$$

Following collection of more data, denoted by $\underline{y}_2$, information on $\underline{\Theta}$ is updated by

$$p''(\underline{\theta}|\underline{y}_1,\underline{y}_2) \propto p'(\underline{\theta})l(\underline{\theta}|\underline{y}_1)l(\underline{\theta}|\underline{y}_2) \tag{20}$$

which may be restated as

$$p''(\underline{\theta}|\underline{y}_1,\underline{y}_2) \propto p''(\underline{\theta}|\underline{y}_1)l(\underline{\theta}|\underline{y}_2) \tag{21}$$

The general forms of Eqns. 19 and 21 are identical; they differ only in which expressions serve in the roles of the prior probability density function and likelihood function of $\underline{\Theta}$. The point to be made is that the above procedure can be repeated without limit and is analogous to the fundamental process of learning from experience.

Bayesian Probability Density Function

When, as is most often the case, the interest lies in the probability density function of Y, p(y), compound distribution theory may be applied to yield

$$p(y) = \int p(y|\underline{\theta})p''(\underline{\theta}|\underline{y})d\underline{\theta} \tag{22}$$

where the integration is taken over appropriately dimensioned real space.

The probability density function of Y when derived as in Eqn. 22 is referred to as the Bayesian (Benjamin and Cornell, 1970) or predictive (Zellner, 1971) probability density function of the random variable Y. It may be thought of as the average probability density function of Y, weighted by all possible values of the parameter vector $\underline{\Theta}$.

The Bayesian probability density function of Y may be updated as more information becomes available by first updating the probability density function of $\underline{\Theta}$ via Eqn. 21 and then updating the probability density function of Y through Eqn. 22. It is incorrect to attempt to update the distribution of Y using Eqn. 22 directly.

The Prior Probability Density Function
For One Parameter

Perhaps the most contentious aspect of Bayesian analysis regards the formulation of the prior probability density function. Depending on how it is derived, the prior probability density function may be classified as being data based or non-data based (Vicens et al., 1975a). This section is devoted to discussion of the two classes of prior probability density functions with observations on their relative strengths and shortcomings.

Data Based Priors. Data based prior probability density functions are those obtained through "objective" methods (e.g., regional analysis). Provided sound methods are used in its derivation, one will seldom be criticized for using a data based prior probability density function in a Bayesian analysis.

It is convenient, though not necessary, to specify data based prior probability density functions in such a manner as to simplify the derivation of the posterior probability density function. This may be done by selecting the prior probability density function from a family of probability density functions having mathematically attractive combinative properties. Raiffa and Schlaifer (1961) suggest desirable characteristics for such families of probability density functions, which may be summarized as:

1. The posterior probability density function of a parameter should be easily determined given the prior probability density function and sample data.
2. It should be easy to find the moments of both the posterior probability density function of the parameter and functions of the parameter.
3. The family should be closed in the sense that the posterior probability density

function will be a member of the same family as the prior probability density function.

Such families of probability density functions, known as natural conjugate families, are characterized by similar kernels. Many natural conjugate families of probability density functions are developed and discussed by Raiffa and Schlaifer (1961).

In spite of the appealing characteristics of data based prior probability density functions, there are at least two reasons why one may hesitate to specify one in a Bayesian analysis. It is possible, when specifying a data based prior probability density function, for the prior probability density function to dominate the likelihood function in Eqn. 17. This is tantamount to suppression of the data in favor of the prior knowledge on the parameters. Box and Tiao (1973) forcefully argue that a dominant prior probability density function is rarely appropriate for the analysis of scientific data, commenting that it is unlikely an investigation would be undertaken unless data provided by the investigation were not of considerably greater precision than existing data. The second reason why one might opt not to use a data based prior probability density function is that such functions do not in themselves ensure robust inference; i.e., inferences may change appreciably depending on the prior probability density function specified. The subject of robust inference is magnified in importance when the Bayesian analysis is not an end in itself, but merely a component of a complex decision analysis. Berger (1985) demonstrates several cases in which use of a prior probability density function from a natural conjugate family does not lead to robust inference.

Non-Data Based Priors. Non-data based prior probability density functions include those based on subjective opinions, theoretical considerations, or other such information. Except for non-informative prior probability density functions, a special case of the non-data based class, non-data based prior probability density functions are seldom used in scientific analyses due to their vulnerability to criticism. Further treatment of such prior probability density functions is irrelevant in the context of this dissertation. The non-informative prior probability density functions, however, are deserving of further discussion.

Non-informative prior probability density functions are specified in order to reflect ignorance regarding the prior distribution of the parameters. This is not to say that nothing of the parameters is known prior to conduct of the experiment; indeed, Bodo and Unny (1976) state that complete prior ignorance is a "remarkably difficult state to achieve". Instead, use of non-informative prior probability density functions indicates merely that the prior knowledge about the parameters is minimal compared to that expected from the experiment. A traditional method of expressing prior ignorance is to invoke Bayes' Postulate; i.e., to specify a uniform prior probability density function of the form

$$p'(\theta) \propto \kappa \tag{23}$$

Obviously, if the domain of the parameter is unbounded, the uniform prior probability density function is improper in that it does not integrate to unity. Box and Tiao (1973) circumvent this theoretical difficulty by proposing that the probability density function be considered as "locally uniform" over the range of appreciably non-zero likelihood and as tailing off to zero outside this range. It may be seen that such a prior probability density function does not dominate the likelihood in Eqn. 17 by observing that if $p'(\theta) \propto \kappa$, then

$$p''(\theta|\underline{y}) = \frac{l(\theta|\underline{y})p'(\theta)}{\int l(\theta|\underline{y})p'(\theta)d\theta} \approx \frac{l(\theta|\underline{y})}{\int l(\theta|\underline{y})d\theta} \tag{24}$$

Box and Tiao (1973) advocate the use of data-translated likelihood functions to assist in the specification of non-informative prior probability density functions. A data-translated likelihood function may be defined as one for which the curve is completely specified, except for the location, prior to collection of the data. Depending on the probability density function of Y and the parameter in question, it may be necessary to transform the parameter θ in order to obtain a data-translated likelihood. Assuming such a transformation exists, Box and Tiao (1973) define a non-informative prior probability density function as one which is locally uniform on the parameter space of the data-translated likelihood. To illustrate, consider specifying a non-informative prior probability density function for the mean μ of the random variable Y, which is $N(\mu,\sigma^2)$ with known variance σ^2. The likelihood function of μ is given by

$$l(\mu|\sigma,\underline{y}) \propto \exp\left[\frac{-\sum_{i=1}^{n}\left(y_i-\mu\right)^2}{2\sigma^2}\right] \propto \exp\left[\frac{-n(\bar{y}-\mu)^2}{2\sigma^2}\right] \tag{25}$$

where $\bar{y}$ is the mean of the n observations of y. It may be shown that the likelihood function given by Eqn. 25 is data-translated. Therefore, a suitable prior probability density function is simply

$$p'(\mu) \propto \kappa \tag{26}$$

If, however, the quantity of immediate interest is not μ but rather ν, where $\nu=\mu^{-1}$, then a different prior probability density function is appropriate. The likelihood function of ν is

$$l(\nu|\sigma,\underline{y}) \propto \exp\left[\frac{-n(\bar{y}-\frac{1}{\nu})^2}{2\sigma^2}\right] \tag{27}$$

In contrast to the likelihood function defined by Eqn. 25, this likelihood function is not data-translated. A locally uniform prior probability density function for ν thus will not qualify as non-informative as defined previously. To derive a non-informative prior probability density function for ν, we recall that the uniform distribution was non-informative for μ. By the change of variables theorem, we may write

$$p'(\nu|\sigma) = p'(\mu|\sigma)\left|\frac{d\mu}{d\nu}\right| = p'(\mu|\sigma)\mu^2 \propto \frac{1}{\nu^2} \tag{28}$$

In practice, it is the parameter transformation which produces the data-translated likelihood function that is unknown, and one must derive such a transformation first and then work in the direction opposite that of the example.

The Prior Probability Density Function For Multiple Parameters

It is appropriate for most scientific work to extend the considerations involved in selection of a prior probability density function for the single parameter situation to the situation of multiple parameters. More concisely stated, non-informative prior probability density functions are, in general, desirable for the situation of multiple parameters. To aid in the specification of non-informative prior probability density functions, one may extend Jeffreys' Rule (Jeffreys, 1961) for the single parameter case to the multiple parameter case. With regard to the single parameter case, Jeffreys' Rule states that a prior probability density function is approximately non-informative if it is taken as proportional to the square root of Fisher's (1922, 1925) measure of information. Given an observation y from a population having a conditional distribution $p(y|\theta)$, Fisher's measure of information is defined as

$$\mathcal{F}(\theta) = \underset{y|\theta}{E}\left[\frac{\partial \ln\ p(y|\theta)}{\partial\theta}\right]^2 \tag{29}$$

When a random sample of size n is drawn, then Fisher's measure of information for the entire sample is given as

$$\mathcal{F}_n = n\mathcal{F}(\theta) \tag{30}$$

Suppose now that the distribution of Y is conditional upon values of $\underline{\Theta}$, a k-vector of parameters. For a random sample $\underline{y}$ drawn from this distribution, Fisher's measure of information (now contained within a kxk matrix) is given as

$$\underline{\mathcal{F}}_n(\underline{\theta}) = \underset{\underline{y}|\underline{\theta}}{E}\left[\frac{-\partial^2 \ln\ \underline{p(y|\theta)}}{\partial\theta_i\partial\theta_j}\right] \tag{31}$$

Jeffreys' Rule as extended to the multiple parameter case states that the prior probability density function of the parameters should be taken as proportional to the square root of the determinant of the information matrix; i.e.,

$$p'(\underline{\theta}) \propto |\underline{\mathcal{F}}_n(\underline{\theta})|^{1/2} \tag{32}$$

Parameter Estimation

The term "parameter estimation" refers to some process of comparing model predictions to observations with the purpose of deriving an "optimal" set of model parameters. The optimal set of parameters is that (preferably unique) set which satisfies some criterion or "objective function" specified by the model user.

The role of the objective function is central in the process of parameter estimation. In specifying an objective function, the model user commonly hopes to accomplish several things. First, the user desires to find model parameters that will lead to the best possible model predictions. The term "best" is not absolute, but will vary with the goals of the model user. For example, it may be important to one model user to accurately predict extreme events; another model user may be more interested in accuracy of model predictions over a range of magnitudes. The goals of the model user will affect the form of the objective function which is specified. A second goal in specifying an objective function is to produce unbiased model predictions. The appropriate objective function becomes, in this case, a function of the form of the model. For very simple models, the popular least-squares criterion (minimization of the sum of squared residuals) may produce unbiased model predictions. For more detailed models, a more complex objective function may be in order. A third goal in specifying an objective function is to produce residuals with statistically appealing properties. It is quite common for the model user to require that the residuals fulfill certain requirements in order for the parameters to be considered optimal for that user.

Two methods of parameter estimation, per se, are presented in the following sections of this chapter. The discussion of parameter estimation for linear and non-linear models serves as an introductory treatment of the derivation of optimal parameters using the least-squares criterion. Following is a section which describes the consequences of misapplying the least-squares criterion. The sections on Bayesian estimation of parameters are more concerned with the derivation of the probability density functions of model parameters rather than the optimal values of the parameters. However, the optimal values of parameters may be determined as a by-product of the Bayesian estimation procedure, and it is for this reason that these methods are discussed in this sequence. The chapter concludes with a discussion of specifying an objective function with the purpose of obtaining residuals with certain statistical properties.

<u>Estimation of Parameters in Linear Models</u>

If an independent variable Y is modeled as

$$\hat{Y}_i = X_1\Theta_1+X_2\Theta_2+...+X_k\Theta_k+\epsilon_i \tag{33}$$

then the model is said to be linear in the parameters. The general model of Eqn. 33 may be written in vector notation as

$$\hat{Y}_i = \underline{X}_i\underline{\Theta}+\epsilon_i \tag{34}$$

where $\underline{X}_i$ is a 1xk vector of inputs, $\underline{\Theta}$ is a kx1 vector of parameters, and ϵ_i is the residual of the i^{th} prediction. The most popular criterion used in estimation of $\underline{\Theta}$ is that $\hat{\underline{\Theta}}$ minimize the sum of squared residuals. This is referred to as the least-squares criterion. The resulting objective function is then

$$\min_{\hat{\underline{\Theta}}} (\underline{Y}-\underline{X}\hat{\underline{\Theta}})(\underline{Y}-\underline{X}\hat{\underline{\Theta}})^T \tag{35}$$

where $\underline{Y}$ is an nx1 vector of observations and the i^{th} row of $\underline{X}$ is $\underline{X}_i$. Minimization of Eqn. 35 leads to the linear normal equations, given by

$$(\underline{X}^T\underline{X})\hat{\underline{\Theta}} = \underline{X}^T\underline{Y} \tag{36}$$

The well known solution to the normal equations is given by

$$\hat{\underline{\Theta}} = (\underline{X}^T\underline{X})^{-1}\underline{X}^T\underline{Y} \tag{37}$$

No assumptions other than that of non-singular $\underline{X}^T\underline{X}$ are necessary up to this point. If, however, some assumptions about the stochastic nature of the residuals are postulated, it follows that $\hat{\underline{\Theta}}$ will possess certain optimal (from a statistical perspective) properties. These assumptions are:

1. The residuals have a mean of zero.
2. The variance of the residuals is constant.

3. The residuals are uncorrelated.

Assumptions 1-3 are referred to as the least-squares assumptions. If these assumptions are satisfied, then the least-squares estimate $\hat{\underline{\Theta}}$ is an unbiased, minimum variance estimator of $\underline{\Theta}$. Draper and Smith (1966) point out that if it may further be assumed that

4. Each residual is $N(0,\sigma^2)$

then the sampling distribution of $\hat{\underline{\Theta}}$ for fixed $\underline{X}_1$ is multivariate normal. Statistical hypothesis tests and the specification of confidence intervals may then be conducted relatively simply.

Estimation of Parameters in Non-linear Models

The non-linear class of models is much more relevant in a hydrologic context. An example of a non-linear model is

$$\hat{Y} = X_1 \exp\left[\Theta_1 (X_2+\Theta_2)^2\right] \tag{38}$$

which is not of the form specified in Eqn. 33. Even so, the least-squares criterion may be again specified for parameter estimation in non-linear models. The function to be minimized may be generally written as:

$$\min_{\hat{\underline{\Theta}}} [\underline{Y}-f(\underline{X};\underline{\Theta})][\underline{Y}-f(\underline{X};\underline{\Theta})]^T \tag{39}$$

where $\underline{Y}$ is a nx1 vector of observations, $\underline{X}$ is a nxk matrix of inputs, and $\underline{\Theta}$ is an mx1 vector of parameters. Draper and Smith (1966) give the resulting normal equations, this time non-linear, as

$$\sum_{i=1}^{n} \left(\left[Y_i - f(\underline{X}_i;\underline{\Theta}) \right] \left[\frac{\partial f(\underline{X}_i;\underline{\Theta})}{\partial \underline{\Theta}} \right] \right)_{\underline{\Theta}=\hat{\underline{\Theta}}} = 0 \tag{40}$$

where $\underline{X}_i$ is the i^{th} row vector of $\underline{X}$. The non-linear normal equations, in contrast to the linear normal equations, have in general no closed form solution. One is compelled then to use a numerical approach to estimate $\underline{\Theta}$. Beck and Arnold (1977) and Bard (1974) review methods of estimating $\underline{\Theta}$ by solution of systems of non-linear equations.

Again, some assumptions regarding the stochastic nature of the residuals are necessary in order to make statements concerning the optimality of the estimate $\hat{\underline{\Theta}}$. Draper and Smith (1966) observe that if the residuals are judged to satisfy assumptions 1-4 stated earlier, then the non-linear least-squares estimate $\hat{\underline{\Theta}}$ may be taken as identical to the maximum likelihood estimate of $\underline{\Theta}$. This implies that the least-squares estimate possesses the same optimal properties as the maximum likelihood estimate; namely, unbiasedness, minimum variance, and asymptotic efficiency.

Violation of the Least Squares Assumptions

Clarke (1973) and Sorooshian and Dracup (1980) have noted that the least-squares assumptions are particularly strong and often are not satisfied by the residuals of hydrologic models. However, as Clarke (1973) has observed, parameters of hydrologic models are most often optimized using the least-squares criterion, usually without benefit of an analysis of the residuals. In those cases in which the least-squares assumptions are not justified, the resulting parameter estimates are not statistically optimal in several respects. More specifically,

1. If the residuals have a non-zero mean, the resulting parameter estimates are biased; i.e., $E(\hat{\underline{\Theta}}) \neq \underline{\Theta}$.

2. If the residuals have a variance which is dependent upon the response, the resulting parameter estimates do not have minimum variance.

3. If the errors are correlated, both bias and non-minimum variance are induced in the parameter estimates.

Draper and Smith (1966) offer several suggestions regarding analysis of residuals, including construction of residual plots and analysis of runs of the residuals.

Bayesian Estimation of Parameters For One Model Output

Bayesian techniques may be used to provide point estimates of the parameter set $\underline{\Theta}$ as well as to determine the distribution of $\underline{\Theta}$. It should be remembered, however, that $\underline{\Theta}$ is considered in Bayesian estimation to be a random, rather than fixed, vector of parameters. The Bayes' estimator of $\underline{\Theta}$, then, carries a different connotation than the classical estimator. The Bayes' estimator of $\underline{\Theta}$, $\hat{\underline{\theta}}$, is taken as the most probable value of the random vector $\underline{\Theta}$; in other words, $\hat{\underline{\theta}}$ is the mode of the posterior joint probability density function of $\underline{\Theta}$. The assumptions necessary for the Bayesian estimation procedure are that the residuals resulting from use of $\hat{\underline{\theta}}$ satisfy the least squares assumptions. To begin the estimation procedure, an initial judgement regarding the residuals is necessary. Box and Tiao (1973) advocate use of the exponential power distribution, a family of symmetric probability density functions which includes the normal, to describe the stochastic nature of the residuals. The probability density function of a random variable X following this distribution is given by

$$p(x|\mu,\sigma,\beta) = \frac{\omega(\beta)}{\sigma} \exp\left[-c(\beta)\left|\frac{x-\mu}{\sigma}\right|^{\frac{2}{1+\beta}}\right] \qquad (41)$$

where μ = a value of the random mean of X,

σ = a value of the random standard deviation of X,

β = a value of the random measure of non-normality of X,

$$c(\beta) = \left\{ \frac{\Gamma\left[\frac{3}{2}(1+\beta)\right]}{(1+\beta)\Gamma\left[\frac{1}{2}(1+\beta)\right]} \right\}^{\frac{1}{1+\beta}} \tag{42}$$

and

$$\omega(\beta) = \frac{\left\{\Gamma\left[\frac{3}{2}(1+\beta)\right]\right\}^{\frac{1}{2}}}{(1+\beta)\left\{\Gamma\left[\frac{1}{2}(1+\beta)\right]\right\}^{\frac{3}{2}}} \tag{43}$$

Note that this is a conditional probability density function, dependent on the values (μ,σ,β). It may be found upon substitution that for β=0, the power exponential distribution simplifies to the normal distribution. For β=-1 and β=1, the resulting distributions are the uniform and double exponential distributions, respectively.

Assume now a random variable Y_i modeled as some function of a 1xk vector of inputs $\underline{X}_i$ and a mx1 vector of parameters $\underline{\Theta}$. If the residuals satisfy the least-squares assumptions and follow the power exponential distribution, then the probability density function of one of these residuals may be written as

$$p(\epsilon|\sigma,\beta) = \frac{\omega(\beta)}{\sigma} \exp\left[-c(\beta)\left|\frac{\epsilon}{\sigma}\right|^{\frac{2}{1+\beta}}\right] \tag{44}$$

Given the data $\underline{y}$, the likelihood function of the uncertain model parameters is given by

$$l(\underline{\theta},\sigma,\beta|\underline{y}) \propto \frac{[\omega(\beta)]^n}{\sigma^n} \exp\left\{-c(\beta)\sum_{i=1}^{n}\left[\left(\frac{\epsilon_i}{\sigma}\right)^{\frac{2}{1+\beta}}\right]\right\} \tag{45}$$

where n is the number of observations on Y. If the measure of non-normality is taken as fixed, then Eqn. 45 simplifies to

$$l(\underline{\theta},\sigma|\underline{y}) \propto \frac{1}{\sigma^n} \exp\left\{-c(\beta)\sum_{i=1}^{n}\left[\left(\frac{\epsilon_i}{\sigma}\right)^{\frac{2}{1+\beta}}\right]\right\} \tag{46}$$

Assuming prior independence between the vector of model parameters and the standard deviation of the residuals, an appropriate non-informative prior probability density function for these random variables is

$$p'(\underline{\theta},\sigma) \propto \frac{1}{\sigma} \tag{47}$$

By Bayes' Theorem, the posterior probability density function is proportional to the product of the prior probability density function and the likelihood function. This leads to a posterior joint probability density function of

$$p''(\underline{\theta},\sigma|\beta,\underline{y}) \propto \frac{1}{\sigma^{n+1}} \exp\left\{-c(\beta)\sum_{i=1}^{n}\left[\left(\frac{\epsilon_i}{\sigma}\right)^{\frac{2}{1+\beta}}\right]\right\} \tag{48}$$

Since we are interested only in the distribution of $\underline{\Theta}$, we may find the marginal density of $\underline{\Theta}$ by integrating Eqn. 48 with respect to σ. Noting that the probability density function of Eqn. 48 belongs to the gamma family of probability density functions, the integration may be performed to yield the joint probability density function of $\underline{\Theta}$ as

$$p''(\underline{\theta}|\beta,\underline{y}) \propto \left[\sum_{i=1}^{n} \left|\epsilon_i\right|^{\frac{2}{1+\beta}}\right]^{\frac{-n(1+\beta)}{2}} \tag{49}$$

or, equivalently,

$$p''(\underline{\theta}|\beta,\underline{y}) \propto \left[\sum_{i=1}^{n} \left|y_i - f(\underline{x}_i;\underline{\hat{\theta}})\right|^{\frac{2}{1+\beta}}\right]^{\frac{-n(1+\beta)}{2}} \tag{50}$$

The exact density of $\underline{\Theta}$ is readily seen to be

$$p''(\underline{\theta}|\beta,\underline{y}) = \frac{\left[\sum_{i=1}^{n} \left|y_i - f(\underline{x}_i;\underline{\hat{\theta}})\right|^{\frac{2}{1+\beta}}\right]^{\frac{-n(1+\beta)}{2}}}{\int \left[\sum_{i=1}^{n} \left|y_i - f(\underline{x}_i;\underline{\hat{\theta}})\right|^{\frac{2}{1+\beta}}\right]^{\frac{-n(1+\beta)}{2}} d\underline{\theta}} \tag{51}$$

where the integral is taken over appropriately dimensioned real space. The Bayes' estimator of $\underline{\Theta}$, $\underline{\hat{\theta}}$, is taken as the mode of the posterior probability density function of $\underline{\Theta}$ and is found by solving

$$\min_{\underline{\hat{\theta}}} \left[\sum_{i=1}^{n} \left|y_i - f(\underline{x}_i;\underline{\hat{\theta}})\right|^{\frac{2}{1+\beta}}\right] \tag{52}$$

It is noted that in the preceding derivation of the probability density function of $\underline{\Theta}$, it was unnecessary to assume a model linear in the parameters or a normal distribution of the residuals. If, however, the residuals should happen to be normally distributed, then the Bayes' estimator of $\underline{\Theta}$ is found as

$$\min_{\underline{\hat{\theta}}} \left[\sum_{i=1}^{n} \left| y_i - f(\underline{x_i};\underline{\hat{\theta}}) \right|^2 \right] \tag{53}$$

Equation 53 may be viewed as a justification for least-squares parameter estimation obtained in a Bayesian context.

Bayesian Estimation of Parameters For Multiple Model Outputs

The situation sometimes arises where a model produces not one, but rather several outputs. To develop the framework for parameter inference in this case, let $\underline{y}$ be an nxp matrix of observed responses, $\underline{x}$ an nxk matrix of inputs, $\underline{\theta}$ an mx1 vector of model parameters, and $\underline{\epsilon}$, an nxp matrix of residuals. No assumptions regarding whether all inputs and parameters are common to all responses are necessary. We may write

$$\underline{y} = \underline{f(x;\theta)} + \underline{\epsilon} \tag{54}$$

Equivalently,

$$\underline{\epsilon} = \underline{y} - \underline{f(x;\theta)} \tag{55}$$

Suppose now that the individual vectors of observed responses $\underline{y_i} = (y_{1i},\ldots,y_{pi})^T$ are independent and that each of the n corresponding vectors of residuals is $N_p(0,\underline{\Sigma})$, where $\underline{\Sigma}$ is the mxm covariance matrix of the residuals. The joint probability density function of the n vectors of errors is given by

$$p(\underline{\epsilon|\Sigma,\theta}) = \prod_{i=1}^{n} p(\underline{\epsilon_i|\Sigma,\theta}) \tag{56}$$

Expanding, we have

$$p(\underline{\epsilon}|\underline{\Sigma},\underline{\theta}) = (2\pi)^{-np/2} |\underline{\Sigma}|^{-n/2} \exp\left[-\frac{1}{2} \sum_{i=1}^{n} (\underline{\epsilon}_i^T \underline{\Sigma}^{-1} \underline{\epsilon}_i) \right] \tag{57}$$

Consider now a matrix $\underline{S(\theta)} = [S_{ij}(\theta_i,\theta_j)]$ where

$$S_{i,j}(\theta_i,\theta_j) = \sum_{k=1}^{n} \epsilon_{ki}\epsilon_{kj} \tag{58}$$

Box and Tiao (1973) show that

$$\sum_{i=1}^{n} \underline{\epsilon}_i^T \underline{\Sigma}^{-1} \underline{\epsilon}_i = \mathrm{tr}[\underline{\Sigma}^{-1}\underline{S(\theta)}] \tag{59}$$

Substituting this result into Eqn. 57, we have

$$p(\underline{\epsilon}|\underline{\theta},\underline{y}) = (2\pi)^{-np/2} |\underline{\Sigma}|^{-n/2} \exp\left(-\frac{1}{2}\mathrm{tr}[\underline{\Sigma}^{-1}\underline{S(\theta)}] \right) \tag{60}$$

Given the data $\underline{y}$, we may write the likelihood function of the uncertain parameters as

$$l(\underline{\theta},\underline{\Sigma}|\underline{y}) \propto |\underline{\Sigma}|^{-n/2} \exp\{-\frac{1}{2}\mathrm{tr}[\underline{\Sigma}^{-1}\underline{S(\theta)}]\} \tag{61}$$

At this point, a prior probability density function of the model parameters and the elements of the covariance matrix must be specified. If it may be assumed that $(\underline{\Theta},\underline{\Sigma})$ are independent, then we may write

$$p'(\underline{\Sigma},\underline{\theta}) = p'(\underline{\Sigma})p'(\underline{\theta}) \tag{62}$$

If $\underline{\theta}$ is not highly dimensioned, then the appropriate non-informative prior probability density function is

$$p'(\underline{\theta}) \propto \kappa \tag{63}$$

which leads to

$$p'(\underline{\Sigma},\underline{\theta}) \propto p'(\underline{\Sigma}) \tag{64}$$

Applying Jeffreys' Rule for multiple parameters leads to selection of the prior probability density function of the covariance matrix as

$$p'(\underline{\Sigma}) \propto |\underline{\Sigma}|^{-(p+1)/2} \tag{65}$$

If the residuals are uncorrelated, then Eqn. 65 reduces to

$$p'(\underline{\Sigma}) \propto \left[\prod_{i=1}^{p} \sigma_{ii}^{-1}\right]^{-(p+1)/2} \tag{66}$$

By Bayes' Theorem, the posterior joint probability density function of the model parameters and the covariance matrix is proportional to the product of their likelihood function and their prior joint probability density function. Box and Tiao (1973) provide a derivation of the marginal posterior probability density function of $\underline{\Theta}$, which is given by the remarkably simple relationship

$$p''(\underline{\theta}|\underline{y}) \propto |\underline{S(\theta)}|^{-n/2} \tag{67}$$

Upon substitution, it may be seen that for the case of p=1, Eqn. 67 reduces to Eqn. 50 for β=0. Similarly to the situation of one response, the Bayes' estimator of $\underline{\Theta}$, $\hat{\underline{\theta}}_1$ is found as

$$\min_{\hat{\underline{\theta}}} |\underline{S(\theta)}|^{-n/2} \tag{68}$$

Corrective Actions for Violations of The Least-Squares Assumptions

If the residuals are found to have a variance which is dependent on the predicted response y, then a variance-stabilizing transformation should be applied. Box and Cox (1964) present the following member of a family of parametric transformations which is commonly used to achieve constant variance:

$$y^{(\lambda)} = y^{\lambda} \quad \lambda \neq 0 \tag{69a}$$

$$= \ln(y), \ \lambda = 0 \tag{69b}$$

The goal in using such a transformation is to select the parameter λ such that the transformation induces constant variance of the transformed residuals. Sorooshian and Dracup (1980) and Kuczera (1983) discuss application of the above transformation in the context of estimating parameters of a hydrologic model.

The transformed errors are defined as

$$\eta_i = y_i^{(\lambda)} - E\left[Y_i^{(\lambda)}\right] \tag{70}$$

The probability density function of the response Y is related to that of the transformed errors by

$$p(y) = p(\eta)|\underline{J}| \tag{71}$$

where $\underline{J}$ is the Jacobian of the transformation between η and y. If λ is greater then unity, it is evident that

$$\frac{\partial \eta_i}{\partial y_j} = \frac{y_j^{\lambda-1}}{\lambda} \text{ for } i=j \tag{72a}$$

and

$$\frac{\partial \eta_i}{\partial y_j} = 0 \text{ for } i \neq j, \tag{72b}$$

The Jacobian is then given by

$$\underline{J} = \prod_{i=1}^{n} \left(\frac{y_i^{\lambda-1}}{\lambda} \right) \tag{73}$$

If the residuals are found to be serially correlated, then an appropriate corrective action is to fit an autoregressive-moving average (ARMA) model to the residuals. Box and Jenkins (1976) present the ARMA(u,v) model as

$$\epsilon_i = \phi_1 \epsilon_{i-1} + \ldots + \phi_u \epsilon_{i-u} + \alpha_i + \psi_1 \alpha_{i-1} + \ldots + \psi_v \alpha_{i-v} \tag{74}$$

where $\phi_1, \ldots, \phi_u$ are autoregressive parameters, $\psi_1, \ldots, \psi_v$ are moving average parameters, and α_i are values of a random variable A with some white noise distribution. Again, the density of the response Y is related to the random disturbance A by

$$p(y) = p(\alpha)|\underline{J}| \tag{75}$$

where $\underline{J}$ is the Jacobian of the transformation between α and y. It is readily seen that

$$\frac{\partial \alpha_i}{\partial y_j} = 1, \ i=j, \tag{76a}$$

and

$$\frac{\partial \alpha_i}{\partial y_j} = 0, \; i \neq j \tag{76b}$$

implying that the Jacobian is unity.

In the situation where correlated residuals with non-constant variance occur (as commonly arises in a continuous hydrologic simulation model), both corrective actions are appropriate. The probability density function of the response Y is then related to the twice-transformed errors by application of Eqns. 71 and 75. Upon collection of the observed model responses, the joint probability density function of the residuals becomes by definition the likelihood function of the uncertain parameters. Bayesian estimation of the parameters and the derivation of the posterior joint probability density function of the parameters then proceeds as discussed earlier.

CHAPTER IV

RAINFALL-RUNOFF MODEL AND EXPERIMENTAL DATA

Description of Rainfall-Runoff Model

The SCS unit hydrograph model (SCS, 1972) was selected as the event-based rainfall-runoff model to be used in this research. The primary reasons for the choice of this particular model were its applicability to ungaged watersheds, its widespread use, and the lack of other popular models which consider even superficially the hydrology of rainfall-runoff phenomena. At the heart of the model is the well-known relationship relating runoff volume to total depth of rainfall, given by

$$V = \frac{(P-0.2S)^2}{(P+0.8S)} \tag{77}$$

where V = runoff depth [l],

P = total precipitation [l], and

S = maximum potential soil moisture storage [l].

The parameter S (in units of inches) may be calculated from the relationship

$$S = \frac{1000}{CN} - 10 \tag{78}$$

where CN is the familiar curve number. CN may be determined from tables furnished by the SCS (1972) provided one has knowledge of the soil group, land use, and hydrologic condition of the watershed. Should the watershed be heterogeneous with respect to the previously mentioned criteria, then an areally-weighted, composite CN is appropriate.

The storm hydrograph is determined by convoluting runoff volume with a unit hydrograph, shown in Fig. 1. The peak of the unit hydrograph is computed as

$$Q_p = \frac{1290.7\ A}{Tp(1+r)} \tag{79}$$

where Q_p = flow rate (ft^3/s),

A = watershed area (mi^2),

Tp = time to peak (hr), and

r = ratio of the falling limb to the rising limb of the unit hydrograph.

The value of 1.67 is recommended for use as r in Eqn. 79. The SCS further recommends that the routing increment d be taken as 0.2Tp. The currently recommended procedure for computing Tp is discussed in detail by the SCS (1986). One relationship which has been reported by the SCS (1972) for use in determining Tp for small (less than 2000 acres) watersheds is

$$Tp = \frac{L^{0.8}(S+1)^{0.7}}{1710\ S_a^{0.5}} \tag{80}$$

where L = maximum length of the watershed (ft) and

S_a = average watershed slope (%).

Given the parameters which determine the ordinates of the unit hydrograph, the ordinates of the storm hydrograph are determined as

$$Q(n) = \sum_{i=1}^{n} \Delta V(i) UH(n - i) \tag{81}$$

where Q(n) = ordinate of the storm hydrograph for the nth

routing increment [l^3/t],

ΔV = incremental runoff depth [l], and

UH = ordinate of the unit hydrograph [l^3/t].

Incremental runoff volumes are determined by first applying Eqn. 77 to cumulative rainfall at each routing increment in order to find cumulative runoff volume at that time. Incremental runoff volumes are then found as the difference between two consecutive values of cumulative runoff volume.

Uncertain Model Parameters

The model parameters S, Tp, r, and d were taken as uncertain for the purposes of this research. Hawkins (1975) and Bondelid, et al. (1982) have investigated the sensitivity of SCS methods to the variation in the parameter S, thereby obliquely suggesting the validity of considering the parameter as uncertain. Haan and Edwards (1987), Haan and Wilson (1987), and Haan and Schulze (1986) have explicitly considered S as an uncertain parameter and have reported the effects of this approach on the results of runoff volume frequency analyses. McCuen and Bondelid (1983) recognized the inadequacy of the value of r recommended by the SCS and have reported alternative methods for selecting appropriate values of this parameter. Their analysis, however, included no acknowledgement of a stochastic nature of r. Although there are no parallel reports of considering the parameters Tp and d as uncertain, there is no compelling reason why they should not be so considered.

Experimental Data

A considerable quantity of data was necessary in order to meet the objectives of this research. The following subsections describe the nature of the data and the methods used in obtaining them. No attempt is made in this chapter to describe how the data are used as this will be addressed in the following chapters. The specific topics discussed are the selection of the watersheds used, the demarcation of appropriate periods of record for each watershed, data used as inputs to the SCS unit hydrograph model, geomorphic parameters, and flood series data.

Study Watersheds and Periods of Record

The criteria for selection of the group of study watersheds were basically the physical proximity of all watersheds within the group and the availability of sufficient experimental data for each watershed. The USDA-ARS had, for approximately 16 years, gaged several watersheds of various sizes in the Washita River basin in southern-central Oklahoma. The existence and availability of a vast amount of hydrologic data for these watersheds, which were in many cases adjacent, led to the selection of a subset of these watersheds for use in this research.

In total, 15 watersheds were chosen for use in this research. Selection criteria for specific watersheds to be used were that sufficient data exist for each watershed, no study watershed be contained within another study watershed, land usage remain relatively constant over the period of record used in this research, and that, on the whole, the watersheds exhibit a variety of sizes. Table I summarizes some of the characteristics of the study watersheds.

Fifty rainfall-runoff events for each study watershed were selected for use in the analysis of uncertainty in model parameters. In selecting these events, study periods were determined during which land usage was relatively constant. This was accomplished with the aid of the tables of land usage periodically reported for each watershed by the USDA-ARS (1962-1977). For most watersheds, changes in land usage, per se, were of relatively little consequence. More important were changes in the hydrologic regime due to the construction on several watersheds of floodwater

TABLE I

SUMMARIZED CHARACTERISTICS OF THE STUDY WATERSHEDS

ID	Area (ac)	Years of Record	Predominant Soil
111	16640.0	7	Sandy Loam
131	25660.0	16	Sandy Loam
311	15206.0	11	Silt Loam
411	34180.0	13	Silt Loam
511	38910.0	9	Silt Loam
513	12314.0	8	Loam
5142	360.3	6	Loam
5143	485.8	6	Silt Loam
5145	252.8	6	Loam
515	1620.0	5	Silt Loam
611	4845.0	5	Loam/Silt Loam
R5	23.7	11	Silt Loam
R6	27.2	11	Silt Loam
R7	19.2	11	Silt Loam
R8	27.6	11	Silt Loam

TABLE I (Continued)

ID	Percentage of Watershed In			
	Cultivation	Pasture	Wooded Pasture	Misc.
111	10	83	4	3
131	21	49	28	2
311	36	64	0	0
411	75	23	0	2
511	58	38	1	3
513	7	85	4	4
5142	0	100	0	0
5143	0	100	0	0
5145	0	100	0	0
515	31	51	0	0
611	22	72	5	1
R5	0	100	0	0
R6	0	100	0	0
R7	0	100	0	0
R8	0	100	0	0

retarding structures. Completion dates of the structures and the area controlled by each structure have been reported by USDA (1983). This knowledge allowed for the specification of study periods prior to the completion of these floodwater retarding structures. The study periods were further restricted to events occurring during the months of April through September in order that soil and vegetation characteristics might be considered relatively constant for all events. After establishing study periods for each watershed, all runoff-producing rainfall events occurring within these study periods were tabulated based upon data published by the USDA-ARS (1962-1977). From these sets of all events occurring during the study periods, 50 events per study watershed were selected. For those watersheds with more than 50 eligible events, the study events were taken as the "selected events" reported by the USDA-ARS (1962-1977) and a random sample of the remainder of the eligible events. For the watersheds with slightly less than 50 eligible events, the study periods had to be slightly expanded beyond the months of April through September, inclusive.

Model Input Data

Rainfall Data. The SCS unit hydrograph model requires knowledge of the temporal distribution of rainfall such as that obtained from the charts of recording rainfall gages. Each of the study watersheds contains at least two recording rainfall gages which will provide the necessary information on the distribution of rainfall. Rather than make direct use of all available rainfall gages, however, it was decided to use only the most centrally-located rainfall gages of each of the study watersheds as sources of information regarding the temporal distribution of rainfall. The information from other raingages in the study watersheds is used indirectly by correcting rainfall amounts recorded by the central gages on the basis of the Thiessen-weighted averages reported by the USDA-ARS (1962-1977). The original, unpublished rainfall charts for the study watersheds are stored by the USDA-ARS Watershed Research

Laboratory in Chickasha, Oklahoma. The necessary charts were copied at that location and later digitized to form a computer-readable data file for each rainfall-runoff event.

Curve Number Data. The SCS (1972, 1986) provides guidelines for determining CN for ungaged watersheds. The information required in the determination of a composite CN is, for each soil present in the watershed, the proportion of the watershed composed of the particular soil and the proportion of that soil in a particular land use. Data reported by the USDA-ARS (1962-1977) list for each study watershed the proportion of that watershed in a particular land use and the proportion of the watershed which a particular soil comprises. No further breakdown of soils and land uses is available from that source. Local soil surveys and topographic maps were of no benefit in determining the composition of a watershed by land usage and soil due to their low resolution. For seven of the study watersheds, this presented no problem as they were used solely as pasture/range. For the remaining eight watersheds, however, it was not possible to compute the composite CN and the corresponding composite S while strictly adhering to SCS guidelines. The following assumptions were made to allow for the determination of a composite CN for these watersheds:

1. The various soils comprising a particular watershed are uniformly distributed within that watershed.
2. The cropping practices used represent an amalgam of the alternatives presented by the SCS with respect to the determination of CN.
3. The CN for land reported by the USDA-ARS (1962-1977) as in a "miscellaneous" disposition is 90. This land use includes county roads, highways, airports, etc.

The values of S resulting from the composite CN's computed on the basis of these assumptions appear in Table II.

TABLE II

VALUES OF S COMPUTED FOR THE STUDY WATERSHEDS

ID	S (in)
111	4.2
131	4.0
311	3.3
411	3.2
511	3.4
513	3.6
5142	3.8
5143	4.9
5145	3.5
515	2.2
611	3.8
R5	4.1
R6	4.4
R7	3.0
R8	2.9

Observed Model Responses

The SCS unit hydrograph model may be thought of as producing two distinct outputs which are relevant to this research: runoff volume and peak flow. Estimation of the model parameters therefore requires comparisons of observed to predicted runoff volumes, observed to predicted peaks, or both. The USDA-ARS Watershed Research Laboratory in Chickasha, OK, stores data on the temporal distribution of discharge for each of the study watersheds. Unpublished printouts of discharge measurements and computations were made available to the writer and copied at the laboratory. These data made possible the determination of peak flow and runoff volume for each study event, both of which were computed under the assumption of constant base flow.

A summary of the rainfall-runoff events used in this research, in terms of storm dates, durations, depths, peak flows, and runoff volumes, is provided in Appendix A.

Geomorphic Parameters

Geomorphic parameters of the study were measured by Misra (1988) and are presented in Table III. The following abbreviations appearing in Table III are defined as follows:

A = watershed area (mi^2).

P = watershed perimeter (mi).

L_m = maximum length of the watershed (mi).

W = maximum width perpendicular to the main channel (mi).

L_c = length of the main channel (mi).

H = maximum relief of the watershed (ft).

D_d = watershed drainage density (mi^{-1}), the ratio of the total length of all identifiable streams to the watershed area.

R_r = relief ratio, the ratio of the maximum relief of the watershed to the maximum length of the watershed.

R_R = relative relief, the ratio of the maximum relief of the watershed to the perimeter of the watershed.

S_a = average slope of the watershed (%).

S_c = average slope of the main channel (%).

R_e = elongation ratio, the ratio of the diameter of a circle having the same area as the watershed to the maximum length of the watershed.

R_c = circularity ratio, the ratio of the perimeter of a circle having the same area as the watershed to the watershed perimeter.

S_f = watershed stream frequency (mi^{-2}), the ratio of the number of all identifiable streams to the watershed area.

The correlation matrix for the geomorphic parameters of Table III and the values of S shown in Table II (denoted as S_t) is presented in Table IV. The correlation matrix for the logarithmic transformations of these variables is given in Table V.

Flood Series Data

Partial duration series of peak flows were collected for each of the study watersheds. These peaks flows were obtained from the same source as the observed model responses discussed previously. The threshold value of peak flow varied with each study watershed, but was generally chosen so that each series would contain at least 50 events. Appendix B lists the partial duration series collected for each watershed.

TABLE III

GEOMORPHIC PARAMETERS OF THE STUDY WATERSHEDS

ID	A (mi^2)	P (mi)	L_c (mi)	L_m (mi)	W (mi)	H (ft)	D_d (mi^{-1})
111	26.0	23.43	6.71	6.64	4.25	341.0	2.11
131	40.0	28.44	9.73	8.56	5.52	380.0	1.95
311	23.76	24.57	10.83	9.07	2.81	252.0	2.07
411	53.4	33.77	13.40	11.54	7.77	312.0	1.62
511	60.8	39.85	16.14	15.12	5.12	272.0	1.93
513	19.24	24.34	10.19	8.44	2.32	295.0	2.52
5142	0.56	3.13	0.94	1.11	0.85	100.0	2.94
5143	0.76	4.19	1.63	1.70	0.78	150.0	3.13
5145	0.40	3.07	1.13	1.23	0.32	130.0	2.86
515	2.59	6.49	1.67	2.16	1.93	96.0	1.32
611	7.57	13.61	4.64	5.51	2.58	180.0	1.02
R5	0.04	0.84	0.14	0.32	0.15	45.8	3.86
R6	0.04	0.86	0.19	0.37	0.17	55.0	4.42
R7	0.03	0.69	0.09	0.22	0.16	40.0	3.76
R8	0.04	0.85	0.27	0.30	0.16	76.0	14.45

TABLE III (Continued)

ID	R_r	R_R	S_a (%)	S_c (%)	R_e	R_c	S_f (mi^{-2})
111	0.010	0.003	4.53	0.49	0.87	0.77	2.73
131	0.008	0.003	4.92	0.27	0.83	0.79	2.07
311	0.005	0.002	3.13	0.13	0.60	0.70	1.94
411	0.005	0.002	3.94	0.16	0.71	0.77	1.37
511	0.003	0.001	3.99	0.16	0.58	0.69	1.86
513	0.007	0.002	4.53	0.28	0.58	0.64	2.65
5142	0.017	0.006	6.55	0.81	0.76	0.85	5.33
5143	0.017	0.007	7.84	1.36	0.58	0.74	3.95
5145	0.020	0.008	6.54	0.95	0.58	0.73	2.53
515	0.008	0.003	3.03	0.46	0.84	0.88	1.93
611	0.006	0.003	5.56	0.18	0.56	0.72	0.53
R5	0.027	0.010	3.80	2.41	0.68	0.80	27.03
R6	0.028	0.012	4.98	2.71	0.63	0.85	23.53
R7	0.034	0.011	5.31	2.71	0.88	0.88	100.33
R8	0.048	0.017	7.79	4.22	0.79	0.87	207.37

TABLE IV

CORRELATION MATRIX OF THE GEOMORPHIC PARAMETERS

	A	P	L_c	L_m	W	H	D_d
A	1.000						
P	0.962	1.000					
L_c	0.949	0.987	1.000				
L_m	0.952	0.990	0.994	1.000			
W	0.929	0.914	0.870	0.877	1.000		
H	0.819	0.902	0.852	0.845	0.860	1.000	
D_d	-0.351	-0.432	-0.405	-0.430	-0.438	-0.418	1.000
R_r	-0.630	-0.751	-0.732	-0.753	-0.703	-0.727	0.841
R_R	-0.659	-0.780	-0.753	-0.773	-0.737	-0.760	0.822
S_a	-0.464	-0.507	-0.504	-0.501	-0.465	-0.324	0.553
S_c	-0.582	-0.706	-0.681	-0.702	-0.669	-0.719	0.840
R_e	-0.052	-0.132	-0.228	-0.231	0.077	-0.029	0.190
R_c	-0.458	-0.611	-0.652	-0.649	-0.372	-0.615	0.430
S_f	-0.337	-0.426	-0.405	-0.423	-0.410	-0.455	0.928
S_t	-0.081	-0.066	-0.095	-0.089	-0.092	0.127	-0.164

TABLE IV (Continued)

	R_r	R_R	S_a	S_c	R_e	R_c	S_f	S_t
R_r	1.000							
R_R	0.987	1.000						
S_a	0.552	0.565	1.000					
S_c	0.981	0.979	0.456	1.000				
R_e	0.288	0.174	-0.085	0.241	1.000			
R_c	0.649	0.612	0.157	0.632	0.731	1.000		
S_f	0.857	0.798	0.459	0.848	0.364	0.532	1.000	
S_t	-0.040	0.034	0.339	-0.006	-0.348	-0.285	-0.335	1.000

TABLE V
CORRELATION MATRIX OF LOGARITHMS OF THE GEOMORPHIC PARAMETERS

	A	P	L_c	L_m	W	H	D_d
A	1.000						
P	0.999	1.000					
L_c	0.987	0.991	1.000				
L_m	0.994	0.997	0.994	1.000			
W	0.984	0.976	0.956	0.967	1.000		
H	0.953	0.957	0.969	0.952	0.915	1.000	
D_d	-0.720	-0.716	-0.666	-0.722	-0.755	-0.555	1.000
R_r	-0.942	-0.944	-0.927	-0.955	-0.927	-0.817	0.816
R_R	-0.963	-0.962	-0.935	-0.962	-0.954	-0.840	0.803
S_a	-0.439	-0.428	-0.357	-0.418	-0.439	-0.217	0.535
S_c	-0.945	-0.947	-0.935	-0.954	-0.928	-0.842	0.816
R_e	-0.153	-0.195	-0.268	-0.263	-0.053	-0.190	0.173
R_c	-0.638	-0.676	-0.711	-0.707	-0.521	-0.687	0.421
S_f	-0.831	-0.836	-0.838	-0.855	-0.828	-0.762	0.899
S_t	-0.024	-0.003	0.014	0.019	-0.054	0.139	0.052

TABLE V (Continued)

	R_r	R_R	S_a	S_c	R_e	R_c	S_f	S_t
R_r	1.000							
R_R	0.990	1.000						
S_a	0.574	0.592	1.000					
S_c	0.977	0.971	0.527	1.000				
R_e	0.312	0.193	-0.086	0.280	1.000			
R_c	0.663	0.619	0.139	0.654	0.743	1.000		
S_f	0.867	0.835	0.352	0.880	0.392	0.635	1.000	
S_t	0.101	0.135	0.391	0.110	-0.370	-0.309	-0.091	1.000

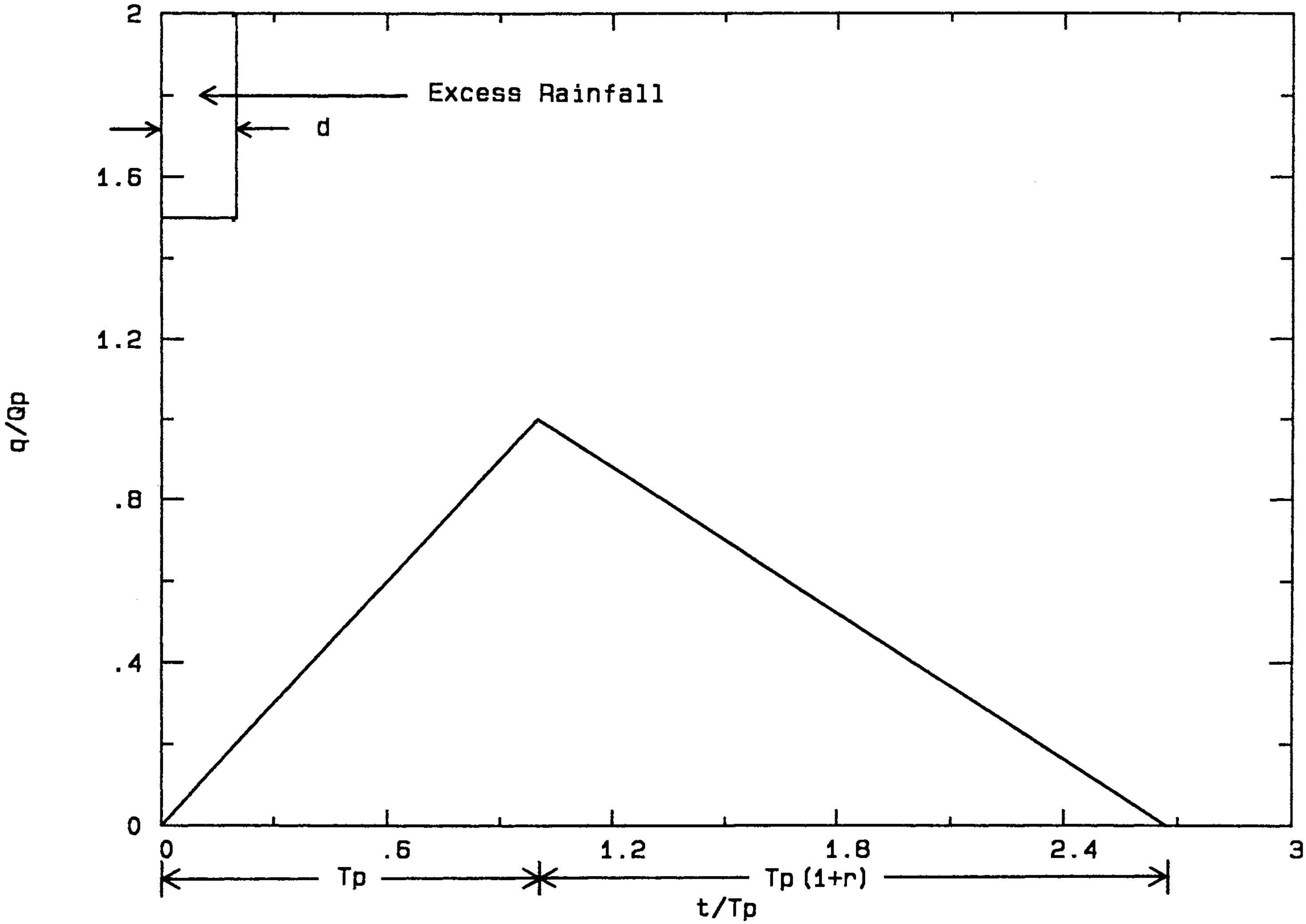


Figure 1. SCS Unit Hydrograph

CHAPTER V

ANALYSIS OF PARAMETRIC UNCERTAINTY

The ultimate goal of the analysis of parametric uncertainty was the determination of the joint and marginal probability density functions of the uncertain model parameters for each of the study watersheds. The Bayesian methodology described in Chapter III was used under the assumption of minimal prior knowledge on the model parameters. The first step in this direction was to find the optimal set of parameters for each study watershed as determined by either Eqn. 52 or Eqn. 68. This derivation of a point estimator for the set of model parameters was necessary in order to make judgments regarding the stochastic nature of the associated residuals. If the optimal parameter sets result in residuals which satisfy the necessary assumptions, then the joint probability density functions of the model parameters may be found as the solution to either Eqn. 51 or Eqn. 67, depending on how the parameter sets were determined. If, however, the optimal parameter sets do not produce residuals which satisfy the necessary assumptions, then a corrective action of some fashion is required, and the optimal values of the model parameters must be redetermined and the residuals rechecked before deriving the joint probability density functions of the parameters. Given parameter sets which, on the whole, result in satisfactory residuals, the marginal probability density functions of the model parameters may be derived upon integration of the joint probability density functions of the parameters. This chapter describes the results of this approach applied to the 15 study watersheds.

Parameters Obtained From Comparisons Of Peak Flows

The optimal values of the parameters S, Tp, r, and d of the SCS unit hydrograph model were derived for each study watershed as the solution to Eqn. 53. Model inputs for each study watershed were the area and the hyetographs for each of the 50 study events, and model outputs were taken as the corresponding peak flows. The direct search routine developed by Hooke and Jeeves (1961) and modified by Monro (1971) was used with multiple starting points in order to determine each set of optimal parameters. The only constraints placed on the optimal parameters were that d be less than Tp and that r be greater than unity. The resulting parameters are shown in Table VI. A sample of the plots of residual peak flow vs. predicted peak flow appears in Figs. 2 through 5. The four watersheds (511, 5142, 611, and R8) from which Figs. 2 through 5 were derived were selected to show representative results and to approximately span the range of watershed areas. The practice of using results particular to these four watersheds to make certain points is widespread within this dissertation. The reader should be aware that these four watersheds did not produce results generally better or generally worse than the remaining 11 watersheds.

It is apparent from Figs. 2 through 5 that the residual variance is not constant but increases with the magnitude of predicted peak flow. It should be recalled at this point that the necessary assumptions underlying Bayesian estimation of model parameters are the least squares assumptions, one of which is the assumption of homoscedasticity of the residuals. The parameter sets presented in Table VI do not, therefore, appear to satisfy this particular assumption. Little more can be said regarding whether the residuals are homoscedastic since there is no means of formally testing a hypothesis of homoscedasticity in this situation. It is also worthy of comment that several of the optimal values of the parameter S presented in Table VI

TABLE VI

OPTIMAL MODEL PARAMETERS, 1

ID	r	Tp (hr)	S (in)	d (hr)
111	8.17	7.02	1.52	1.10
131	16.42	17.50	0.25	1.75
311	3.31	11.75	1.23	0.03
411	14.61	17.77	0.51	1.97
511	2.50	6.92	3.26	0.82
513	6.04	4.00	1.53	0.50
5142	1.01	3.97	1.49	0.20
5143	6.52	2.56	1.79	0.60
5145	1.43	2.11	1.57	0.20
515	9.97	6.00	0.41	0.40
611	1.00	1.41	4.41	0.20
R5	1.00	0.29	3.36	0.06
R6	1.38	0.06	2.65	0.05
R7	2.63	0.15	0.79	0.06
R8	1.20	0.07	1.80	0.06

are remarkably low. This leads to runoff volume predictions for these study watersheds which are, in general, extremely high. Although runoff volume is not in itself a relevant quantity at this point in terms of obtaining optimal parameter sets, it is intimately related via unit hydrograph theory to peak flow. This makes it a troubling proposition to accept parameter values which are guaranteed to produce severe overestimates of this intermediate result in the SCS unit hydrograph procedure. The conclusion regarding this particular attempt to obtain optimal estimates of parameters is that the optimization criterion, the minimization of the sum of squared residual peak flows, is not strong enough to produce parameter estimates which are acceptable in the context of the SCS unit hydrograph model.

Parameters Obtained From Comparisons Of Peak Flows And Runoff Volumes

Due to the inferior characteristics associated with the optimal parameter sets presented in the preceding section, Eqn. 68 was adopted as the criterion for obtaining optimal parameter estimates. Model inputs remained the same as discussed earlier. The model was considered as producing two outputs, peak flow and runoff volume. The resulting optimal parameter sets are shown in Table VII. The problem of unrealistically low values of the parameter S has been overcome by assigning runoff volume a role in the optimization process. The values of the other model parameters also appear, on the whole, to be within the reasonable bounds which may be inferred from their use and physical significance. Figures 6 through 9 depict the relationship between residual peak flow and predicted peak flow for a subset of the study watersheds. It may be inferred from these figures that the problem of heteroscedasticity in residual peak flows has not been overcome with the adoption of the more stringent optimization criterion. Figures 10 through 13, which are plots of residual runoff volume vs. predicted runoff volume, also exhibit heteroscedasticity in the residuals.

TABLE VII

OPTIMAL MODEL PARAMETERS, 2

ID	r	Tp (hr)	S (in)	d (hr)
111	1.31	0.72	8.85	0.72
131	2.07	3.96	12.18	1.72
311	2.81	3.86	3.62	1.56
411	3.95	3.92	8.70	0.98
511	1.05	5.98	4.72	1.33
513	1.10	3.50	5.12	0.50
5142	1.34	0.18	5.27	0.14
5143	1.60	0.40	10.60	0.13
5145	1.95	0.75	3.14	0.35
515	3.34	2.38	3.84	0.34
611	1.78	0.74	5.18	0.38
R5	1.59	0.22	3.43	0.15
R6	1.84	0.06	2.73	0.04
R7	1.03	0.18	1.80	0.09
R8	1.00	0.12	1.87	0.03

The problems with the sets of optimal model parameters at this point are not their values, per se, but rather the statistical properties of the associated residuals; namely, heteroscedasticity. Transformation of the model output has been addressed earlier as a possible means of correcting this violation of the least squares assumptions and is next employed.

Parameters Obtained From Comparisons of Transformed Peak Flows and Transformed Runoff Volumes

The square root transformation, which is a member of the family of Box and Cox (1964) transformations presented in Eqns. 69a and 69b, was selected for use in order to induce homoscedasticity in peak flow and runoff volume residuals. However, the optimization criterion presented in Eqn. 68 was derived without consideration of transformations of the model responses. In order to consider transforming model responses as a possible tool in obtaining optimal parameter estimates, the posterior joint probability density function of the model parameters must be derived in terms of these transformed responses.

Consider a model $\underline{f(x_i;\theta)}$, where $\underline{x_i} = (x_{1,i},x_{2,i},\ldots,x_{j,i})$ and $\underline{\theta}=(\theta_1,\theta_2,\ldots,\theta_k)^T$, which is used to predict responses Y_1 and Y_2. The residuals may be defined as

$$\epsilon_{1,i} = y_{1,i}-f_1(\underline{x_i;\theta}) = y_{1,i}-E(Y_{1,i}) \tag{82}$$

$$\epsilon_{2,i} = y_{2,i}-f_2(\underline{x_i;\theta}) = y_{2,i}-E(Y_{2,i}) \tag{83}$$

The transformed errors $(\eta_{1,i},\eta_{2,i})$ are now defined as the differences between the square roots of the observations and the square roots of the predictions.

$$\eta_{1,i} = y_{1,i}^{1/2} - E(Y_{1,i})^{1/2} \tag{84}$$

$$\eta_{2,i} = y_{2,i}^{1/2} - E(Y_{2,i})^{1/2} \tag{85}$$

Assume now that the $\eta_{1,i}$ are $N(0,\sigma_1^2)$ and the $\eta_{2,i}$ are $N(0,\sigma_2^2)$. Then $(\eta_{1,i},\eta_{2,i})$ are $N_2(\underline{0},\underline{\Sigma})$, and their probability density function may be written as

$$p(\underline{\eta}_i) = (2\pi)^{-1}|\underline{\Sigma}|^{-1/2}\exp\left[-\frac{1}{2}(\underline{\eta}_i^T\underline{\Sigma}^{-1}\underline{\eta}_i)\right] \tag{86}$$

where $\underline{\eta}_i=(\eta_{1,i},\eta_{2,i})^T$. We may relate the probability density function of a vector of transformed errors to that corresponding vector of observations as

$$p(\underline{y}_i) = p(\underline{\eta}_i)|\underline{J}| \tag{87}$$

where $\underline{y}_i = (y_{1,i},y_{2,i})^T$, and $\underline{J}$ is the Jacobian of the transformation from $\underline{\eta}_i$ to $\underline{y}_i$, given by

$$\underline{J}= \begin{bmatrix} \frac{\partial\eta_{1,i}}{\partial y_{1,i}} & \frac{\partial\eta_{1,i}}{\partial y_{2,i}} \\ \frac{\partial\eta_{2,i}}{\partial y_{1,i}} & \frac{\partial\eta_{2,i}}{\partial y_{2,i}} \end{bmatrix} \tag{88}$$

Upon substitution, we find that

$$|\underline{J}| = \begin{vmatrix} \frac{1}{2y_{1,i}^{1/2}} & 0 \\ 0 & \frac{1}{2y_{2,i}^{1/2}} \end{vmatrix} = \frac{1}{4(y_{1,i}y_{2,i})^{1/2}} \tag{89}$$

The probability density function of $\underline{y}_i$ is then

$$p(\underline{y}_i) = |\underline{J}|(2\pi)^{-1}|\underline{\Sigma}|^{-1/2}\exp\left[-\frac{1}{2}(\underline{\eta}_i^T\underline{\Sigma}^{-1}\underline{\eta}_i)\right] \tag{90}$$

where the argument inside the exponentiation operator is indeed a function of $\underline{y}_i$ as defined by Eqns. 84 and 85, but is presented in this fashion for the sake of brevity. Now define $\underline{y}=(\underline{y}_1,\underline{y}_2,...,\underline{y}_n)^T$ and suppose that the $\underline{y}_i$ are uncorrelated for all i. Then the joint probability density function of $\underline{y}$ is found as

$$p(\underline{y}) = \prod_{i=1}^{n} p(\underline{y}_i) \tag{91}$$

This is expanded to form

$$p(\underline{y}) = g(\underline{y})(2\pi)^{-n}|\underline{\Sigma}|^{-n/2}\exp\left(\frac{-1}{2}\left[\sum_{i=1}^{n}(\underline{\eta}_i^T\underline{\Sigma}^{-1}\underline{\eta}_i)\right]\right) \tag{92}$$

$$\text{where } g(\underline{y}) = \prod_{i=1}^{n}\frac{1}{4(y_{1,i}y_{2,i})^{1/2}} \tag{93}$$

Given the data $\underline{y}$, the probability density function of $\underline{y}$ is a function of only $\underline{\Sigma}$ and $\underline{\theta}$. Therefore, the right side of Eqn. 92 may then be taken as the likelihood function of $(\underline{\Sigma},\underline{\theta})$. Discarding all constants, we are left with

$$l(\underline{\Sigma},\underline{\theta}) \propto |\underline{\Sigma}|^{-n/2}\exp\left(-\frac{1}{2}\left[\sum_{i=1}^{n}(\underline{\eta}_i^T\underline{\Sigma}^{-1}\underline{\eta}_i)\right]\right) \tag{94}$$

From this point, the derivation of the posterior probability density function of the model parameters proceeds exactly as described earlier in the section regarding parameter estimation in the case of multiple responses. The resulting probability density function is identical to Eqn. 67, but the elements of $\underline{S(\theta)}$ are derived from the transformed errors. The Bayes' estimator of $\underline{\Theta}$ is given by Eqn. 68 with the same adjustment of the elements of $\underline{S(\theta)}$.

The parameters obtained from fitting the model to transformed peak flows and runoff volumes are shown in Table VIII. Figs. 14 through 17, plots of transformed residual peak flow vs. transformed predicted peak flow, do not immediately lead one to reject a hypothesis that the transformed residuals exhibit homoscedasticity. Likewise, Figs. 18 through 21, which are plots of transformed residual runoff volume vs. predicted runoff volume, seem to indicate homoscedasticity of the residuals. These parameter sets may therefore be taken as satisfying this particular least squares assumption.

Another assumption which should be verified at this point is the assumption of normality of residuals, which was required in the derivation of the joint density of model parameters. Figs. 22 through 25 show transformed peak discharge rate residuals plotted on a normal probability scale. The distributions of the residuals are seen to be well approximated by normals. Additionally, Figs. 26 through 29, plots of transformed runoff volume residuals on a normal probability scale, suggest that the distributions of transformed runoff volume residuals may be taken as normal. Kolmogorov-Smirnov goodness of fit tests were performed on the transformed peak flow and runoff volume residuals for each of the study watersheds as a more quantitative method of classifying the residual distributions as normal or non-normal. The null hypothesis of normally distributed transformed peak flow residuals was not rejected at the 0.05 significance level for 14 of the study watersheds. The null hypothesis of normally distributed transformed runoff volume residuals was not rejected at the 0.05 significance level for 12 of the study watersheds. Lowering the power of the test to the 0.01 significance level led to the non-rejection of this null hypothesis for one more of the study watersheds.

TABLE VIII

OPTIMAL MODEL PARAMETERS, 3

ID	r	Tp (hr)	S (in)	d (hr)
111	1.15	1.71	6.85	0.44
131	1.93	4.32	6.85	0.85
311	2.36	4.54	3.31	0.95
411	2.69	4.53	6.40	1.09
511	1.86	4.36	4.57	1.00
513	1.00	3.53	3.94	1.00
5142	2.05	0.38	3.55	0.08
5143	1.40	0.80	5.20	0.14
5145	2.17	0.56	2.53	0.11
515	2.19	3.03	2.90	0.61
611	1.05	1.29	4.56	0.26
R5	2.58	0.25	3.44	0.05
R6	1.00	0.22	2.68	0.05
R7	1.10	0.16	1.30	0.03
R8	1.40	0.11	1.75	0.05

The least-squares assumption of unbiased residuals was checked by testing the null hypothesis of zero-mean residuals against the alternative hypothesis of non-zero mean residuals. For the residuals of the transformed peak flows, the null hypothesis was not rejected for 10 of the study watersheds for tests conducted at the 0.05 significance level. For the residuals of the transformed runoff volume residuals, the null hypothesis was not rejected for 11 of the study watersheds for a test of the same power.

Draper and Smith (1966) propose runs tests as a tool in determining whether there exists correlation in the residuals. However, owing to the manner in which study rainfall-runoff events were selected for use in this research, there was no reason to suppose that the residuals might be correlated. Therefore, the least-squares assumption of uncorrelated residuals was not checked.

It was concluded at this juncture that the parameter sets obtained from this optimization procedure lead to the satisfaction of the least-squares assumptions and are therefore acceptable from a statistical perspective. The joint probability density function may accordingly be taken as the solution to Eqn. 67. Furthermore, the values of the optimized parameters are not, on the whole, a source of trepidation.

Moving Toward Parsimonious Model Parameterization

It was decided that any parameters to which model performance is insensitive should not be considered as uncertain, but should be considered as fixed. This will have the effect of simplifying the SCS unit hydrograph model to one containing less than the current total of four parameters. The following subsections describe the investigation of model sensitivity to the parameters d and r and their subsequent elimination from the analysis of parametric uncertainty.

Elimination of d

The first parameter to be examined in light of its possible elimination was the routing increment, d. To examine the effect of removing d from the model, the remaining parameters were re-optimized as in the last section, but with d set to its recommended value of 0.2Tp. The resulting optimal parameter sets are shown in Table IX.

Figures 30 through 33, as well as Figs. 34 through 37, indicate no inducement of heteroscedasticity in either transformed peak flow or runoff volume residuals as a result of eliminating d as an uncertain model parameter. Figures 38 through 41 and 42 through 45 suggest that transformed peak flow and runoff volume residuals remain approximately normally distributed. This is supported by the fact that the results of the Kolmogorov-Smirnov goodness of fit tests were, for the transformed peak flow residuals, identical to those obtained when d was considered an uncertain model parameter. Furthermore, the elimination of d resulted in one additional watershed's having transformed runoff volume residuals which could be considered normally distributed. Hypothesis tests for biasedness of transformed peak flow residuals produced results identical to those for the four-parameter version of the model. Nine of the watersheds were found to have transformed runoff volume residuals which could be considered unbiased, a decrease of one as compared to the four-parameter version of the model.

In order to determine whether overall model performance suffered from the elimination of d, transformed peak flow residual variances were compared. The null hypothesis of equal transformed peak flow residual variances failed to be rejected for all of the study watersheds on the basis of a chi-square test conducted at the 0.05 significance level. It is concluded that eliminating d from consideration in this analysis has no adverse effects on the optimality of the estimates of the remaining

TABLE IX

OPTIMAL MODEL PARAMETERS, 4

ID	r	Tp (hr)	S (in)
111	1.00	1.91	6.91
131	2.26	3.93	6.93
311	2.22	4.81	3.31
411	2.67	4.68	7.15
511	2.06	4.21	4.55
513	1.00	3.86	3.98
5142	1.85	0.41	3.55
5143	1.69	0.65	5.65
5145	2.20	0.56	2.53
515	2.17	3.05	2.90
611	1.04	1.29	4.58
R5	2.60	0.25	3.44
R6	1.00	0.23	2.69
R7	1.06	0.16	1.35
R8	1.20	0.14	1.75

parameters, from the perspective of either the probabilistic nature of the residuals or the model's ability to predict peak flows.

Elimination of r

The model parameters r and Tp may be suspected as being highly interactive upon examination of the SCS unit hydrograph model. This suspicion was fueled by the behavior of the search routine in attempts to locate optimal parameter sets. Figure 46, which is a contour plot of the objective function of Eqn. 68 in the r-Tp plane for fixed, optimal S for watershed 511, confirms the presence of interaction between the two parameters. It is seen that there is a relatively long "trough" in the contour plot corresponding to pairs of values of r and Tp which may be used in the model with virtually no difference in model performance as judged by the right hand side of Eqn. 68. One may therefore choose any value of one parameter, at least within very broad limits, and choose the value of the other parameter as determined by the trough in the contour plot with the assurance of obtaining near-optimal model performance. Clearly, only one of these two parameters is deserving of consideration as uncertain. It was decided to discard the parameter r from the analysis in favor of retaining Tp. The rainfall-runoff model thus becomes a two-parameter model rather than the original four-parameter model.

The resulting optimal sets of the parameters S and Tp are presented in Table X. Figures 47 through 50 indicate no undesirable effects on the variance of the transformed peak flow residuals. Similar conclusions may be drawn regarding the runoff volume residuals upon examination of Figs. 51 through 54. Figures 55 through 62 suggest that the distribution of transformed peak flow and runoff volume residuals is still approximately normal. Kolmogorov-Smirnov tests conducted at the 0.05 significance level lead to non-rejection of the null hypothesis of normality for 14 of the watersheds in the case of transformed peak discharge rate residuals and for 12 of

TABLE X

OPTIMAL MODEL PARAMETERS, 5

ID	Tp (hr)	S (in)
111	1.51	6.83
131	4.72	6.95
311	5.87	3.34
411	6.25	6.39
511	4.77	4.55
513	2.79	3.95
5142	0.46	3.51
5143	0.67	5.60
5145	0.68	2.53
515	3.63	2.88
611	0.99	4.54
R5	0.34	3.48
R6	0.20	2.64
R7	0.12	1.32
R8	0.12	1.72

the watersheds in the case of transformed runoff volume residuals. Diminishing the power of the test to the 0.01 significance level leads to non-rejection of the null hypothesis for one additional case of transformed runoff volume residuals. Tests for biasedness of transformed residuals indicated that 10 of the study watersheds could be considered to have unbiased transformed peak flow rate residuals, and 11 could be considered to have unbiased transformed runoff volume residuals.

Transformed peak discharge residual variances resulting from the two-parameter model were compared with those from the three-parameter model by means of a chi-square test conducted at the 0.05 significance level. In no case was the transformed peak flow residual variance from the two-parameter model significantly greater than that from the three-parameter model. These residual variances resulting from the two-parameter model were also compared to those resulting from the four-parameter model, with the same result.

It is concluded that the removal of r as an uncertain parameter has, on the whole, no adverse effects on either the stochastic nature of the model residuals or the model's ability to predict peak flows. The two-parameter version of the model may thus be considered as valid a flood estimation mechanism as either the three or four-parameter version.

Marginal Probability Density Functions Of the Model Parameters S and Tp

Marginal probability density functions of the model parameters S and Tp were computed by numerically integrating Eqn. 67 for each of the study watersheds. These marginal densities were again numerically integrated in order to determine the mean, variance, and skewness of each using the respective definitions of each of these quantities. The covariance of the two model parameters was determined by integrating their joint probability density function for each study watershed.

Simpson's 3/8 rule was employed as the quadrature in each integration. Figures 63 through 66 depict the probability density function of the parameter S, while Figs. 67 through 70 depict the probability density function of the parameter Tp for a sample of the study watersheds. Table XI summarizes the relevant statistics associated with the two parameters. A key result of Table XI is that the correlation between parameters S and Tp is neither consistently high nor consistently low, suggesting that there no strong correlation structure between the two parameters which may be generally described for all 15 watersheds. For this reason, the two parameters are henceforth considered independent. Another important result that may be inferred from Table XI is that the coefficients of skewness of S and Tp are generally small in magnitude. The small coefficients of skewness, coupled with the shapes of the probability density functions in Figs. 63 through 70, lead to the assumption of normal distributions for the parameters S and Tp in all following treatments of these parameters.

TABLE XI

STATISTICS OF PARAMETERS S AND Tp, 1

ID	m_S (in)	s_S (in)	g_S (in^3)	m_T (hr)	s_T (hr)	g_T (hr^3)	ρ(S,Tp)
111	8.49	1.52	0.02	1.10	0.72	4.67	-0.17
131	7.19	1.17	5.96	4.89	0.79	8.89	0.53
311	3.43	0.27	0.16	5.87	0.19	-0.11	-0.02
411	7.09	0.96	1.25	6.64	0.99	1.22	0.10
511	4.60	0.28	0.15	4.80	0.21	1.46	0.29
513	4.02	0.29	0.56	2.77	0.14	0.23	-0.06
5142	3.55	0.33	0.38	0.46	0.12	0.59	-0.66
5143	5.74	1.10	4.52	0.71	0.32	13.68	0.90
5145	2.69	0.32	0.51	0.68	0.09	0.35	-0.21
515	2.89	0.32	0.30	3.73	0.35	0.15	-0.36
611	4.57	0.36	0.61	0.92	0.11	1.50	-0.31
R5	3.59	0.28	0.14	0.35	0.03	-0.43	-0.29
R6	2.79	0.27	0.15	0.18	0.04	-0.06	-0.58
R7	1.36	0.13	0.47	0.12	0.02	0.08	-0.04
R8	1.75	0.10	0.14	0.12	0.02	0.27	0.00

m_S = Mean of S.
s_S = Standard deviation of S.
g_S = Skewness coefficient of S.
m_T = Mean of Tp.
s_T = Standard deviation of Tp.
g_T = Skewness coefficient of Tp.
ρ(S,Tp) = Corr(S,Tp).

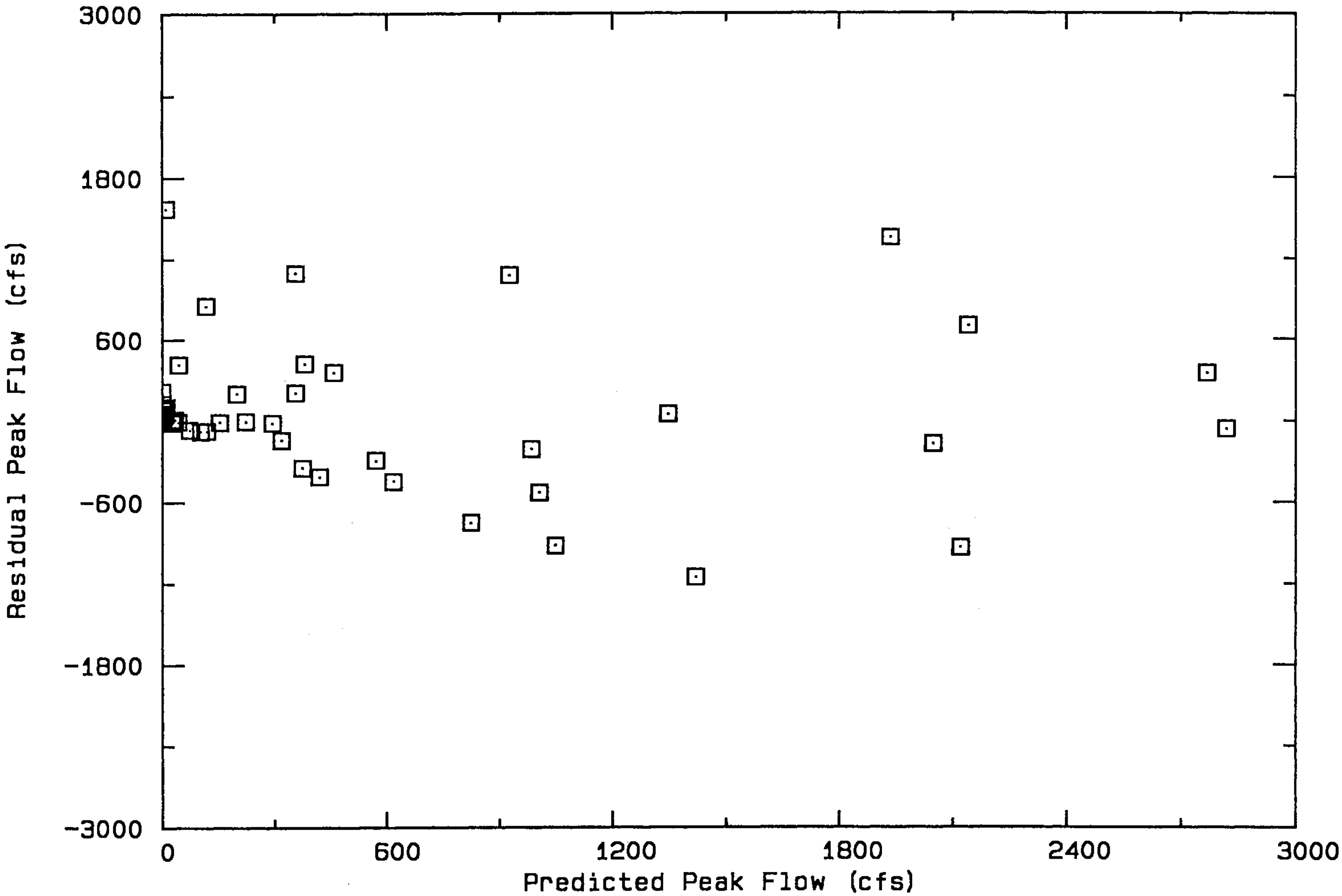


Figure 2. Peak Flow Residual Plot for Watershed 511, 1

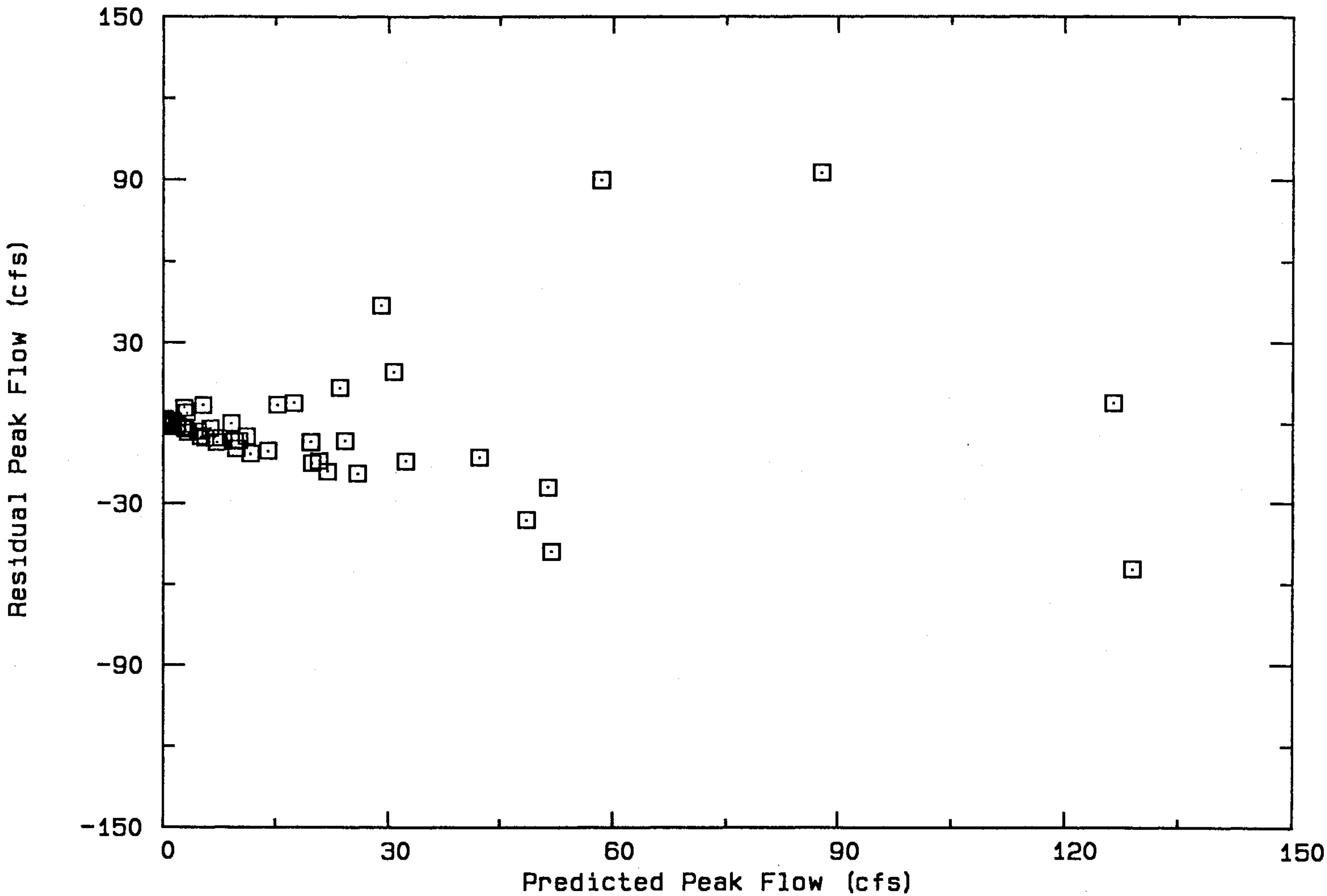


Figure 3. Peak Flow Residual Plot for Watershed 5142, 1

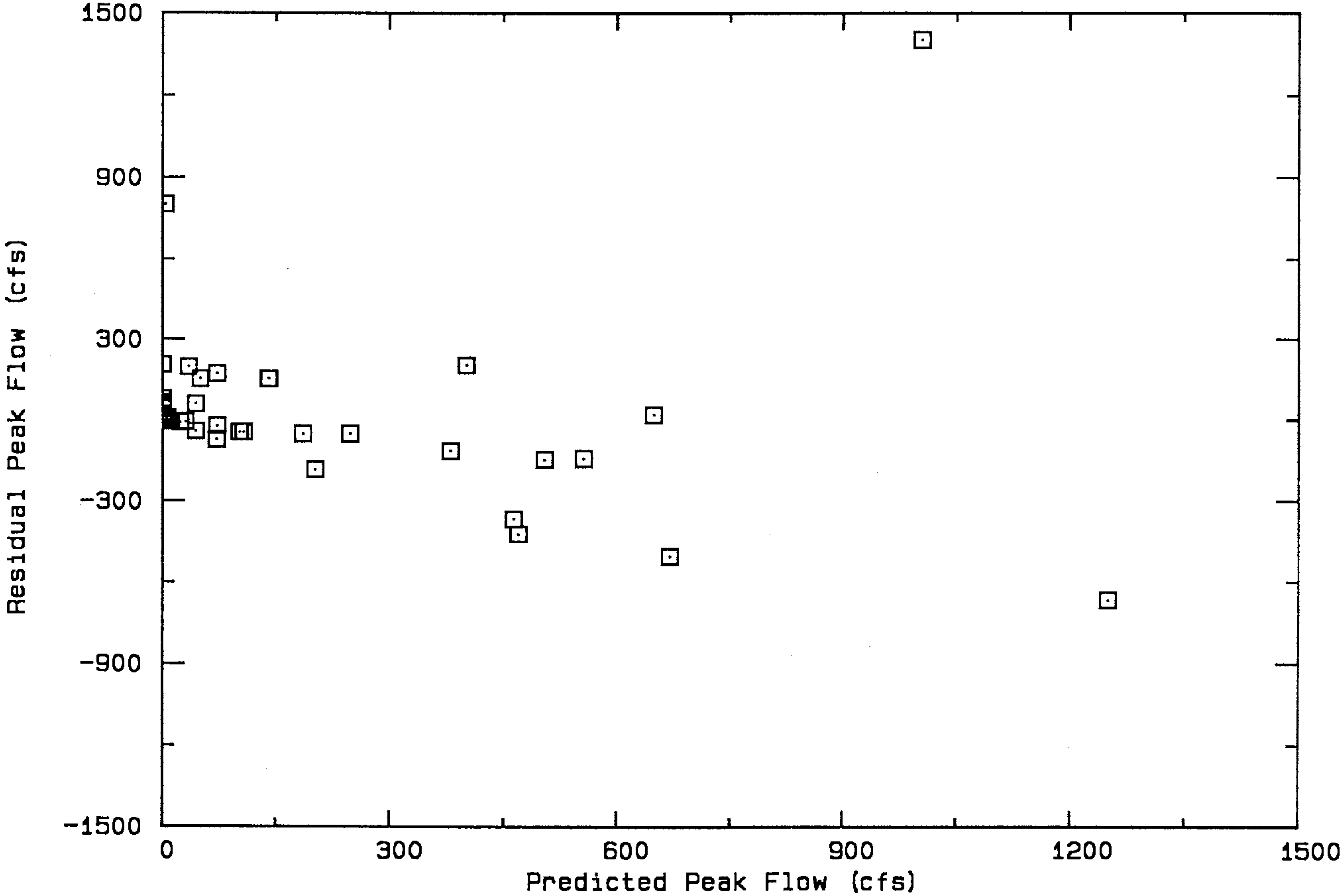


Figure 4. Peak Flow Residual Plot for Watershed 611, 1

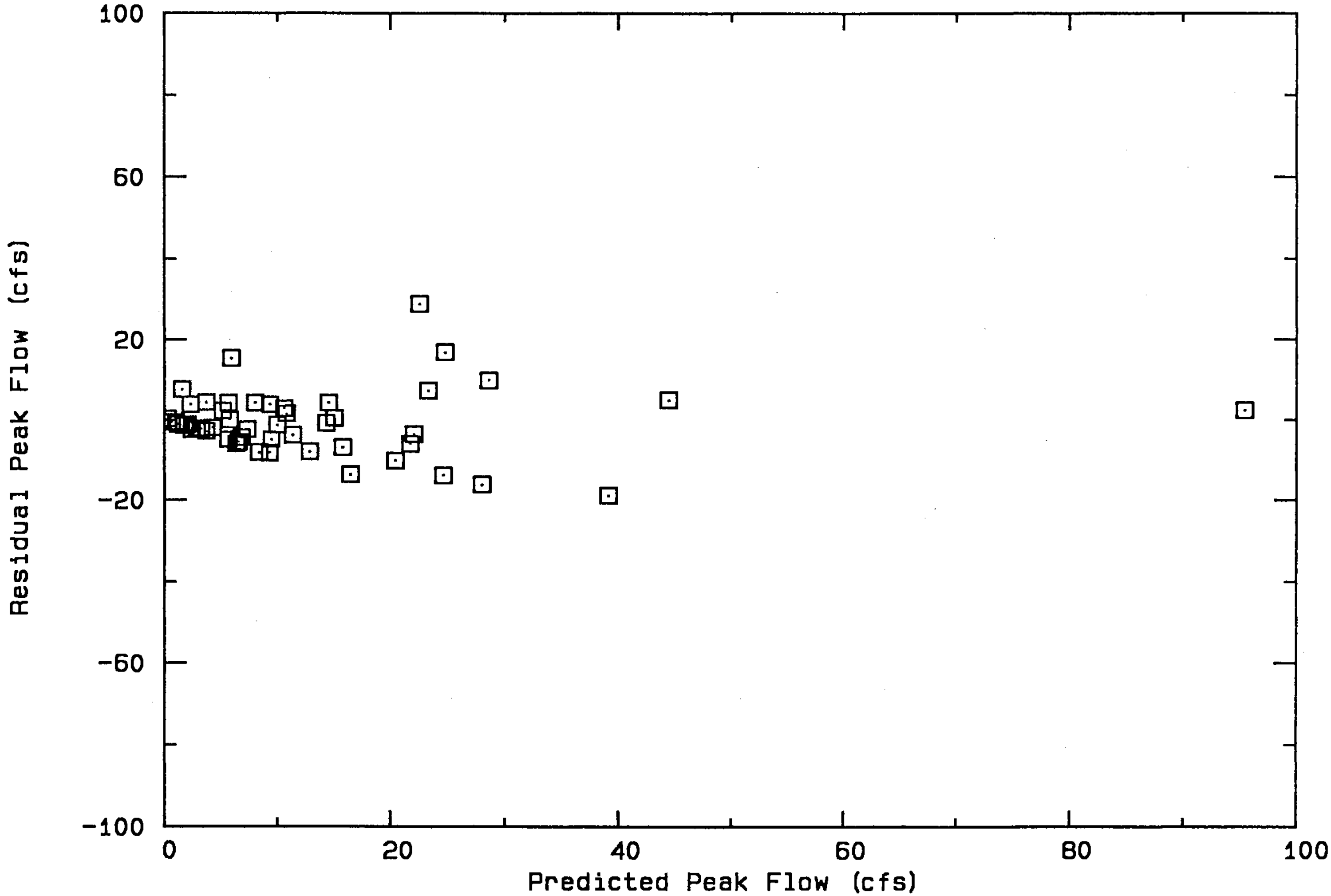


Figure 5. Peak Flow Residual Plot for Watershed R7, 1

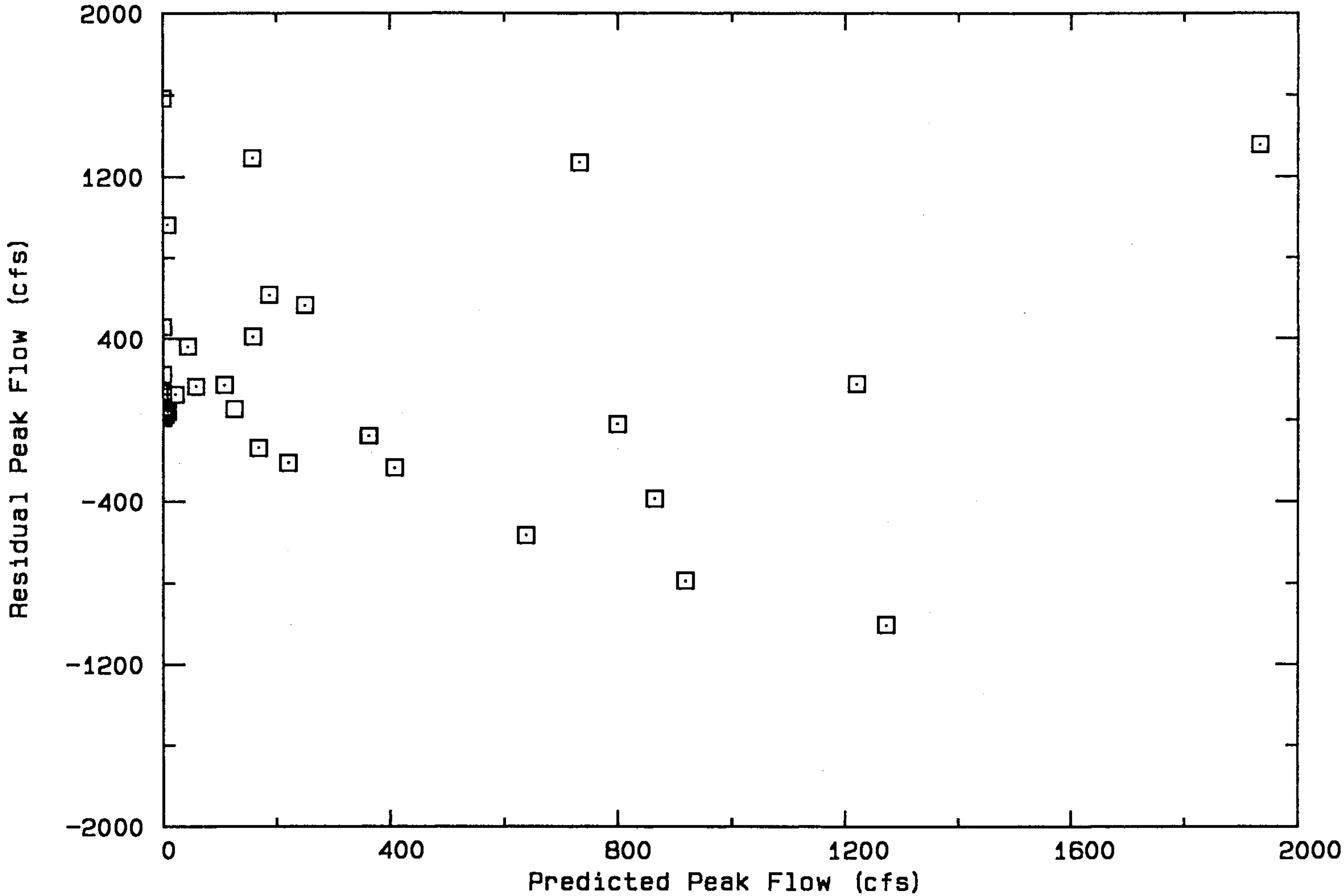


Figure 6. Peak Flow Residual Plot for Watershed 511, 2

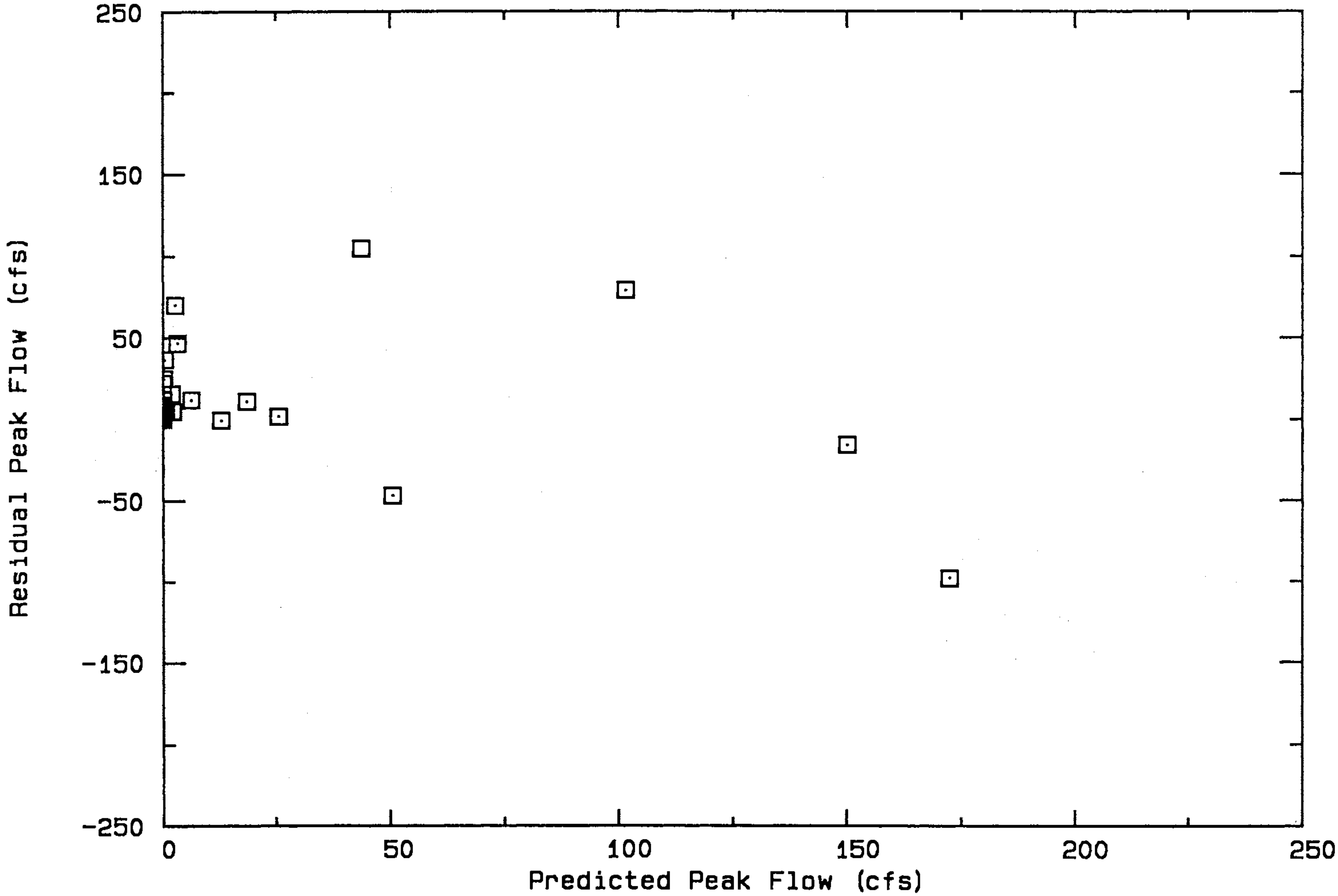


Figure 7. Peak Flow Residual Plot for Watershed 5142, 2

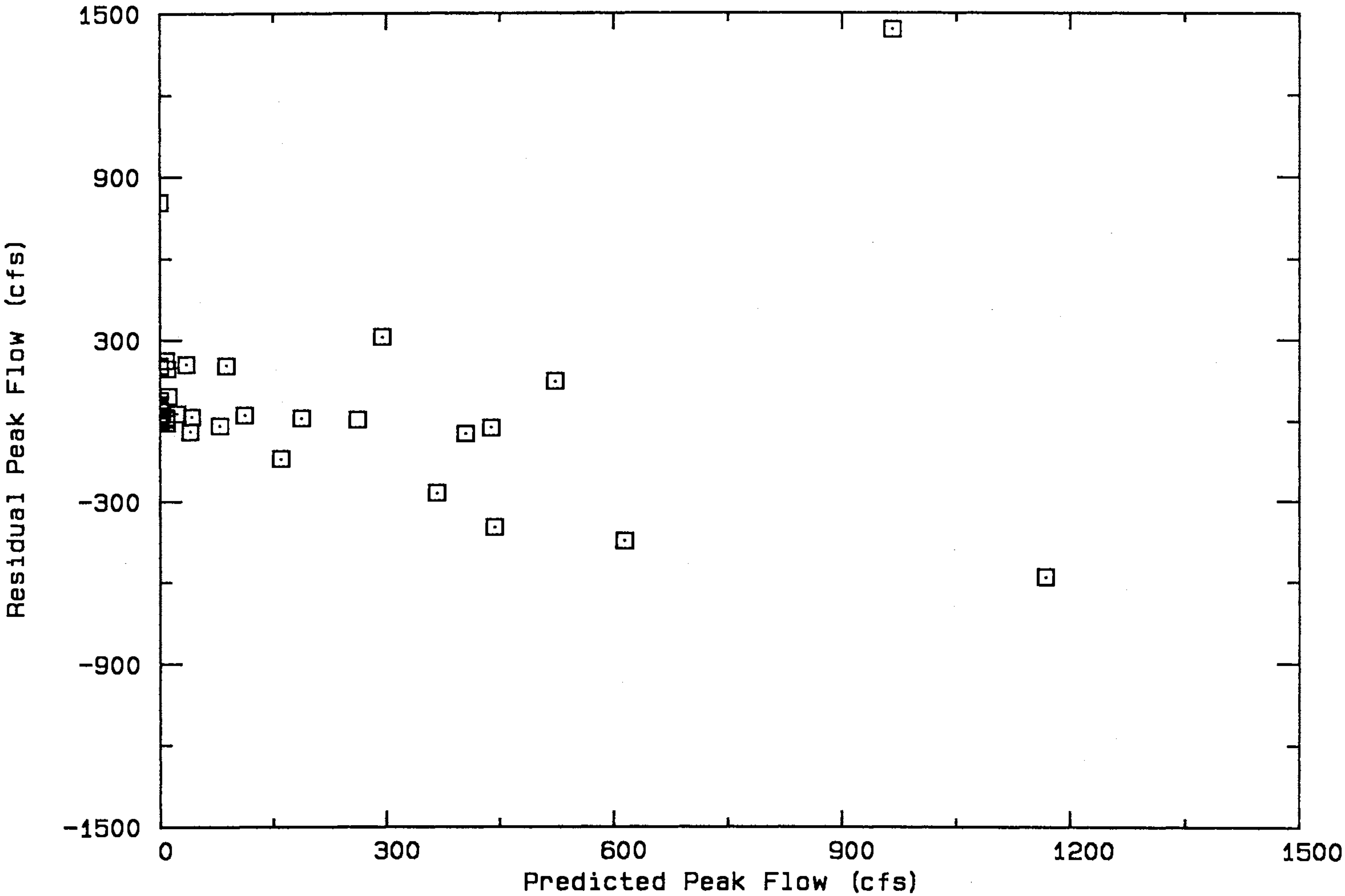


Figure 8. Peak Flow Residual Plot for Watershed 611, 2

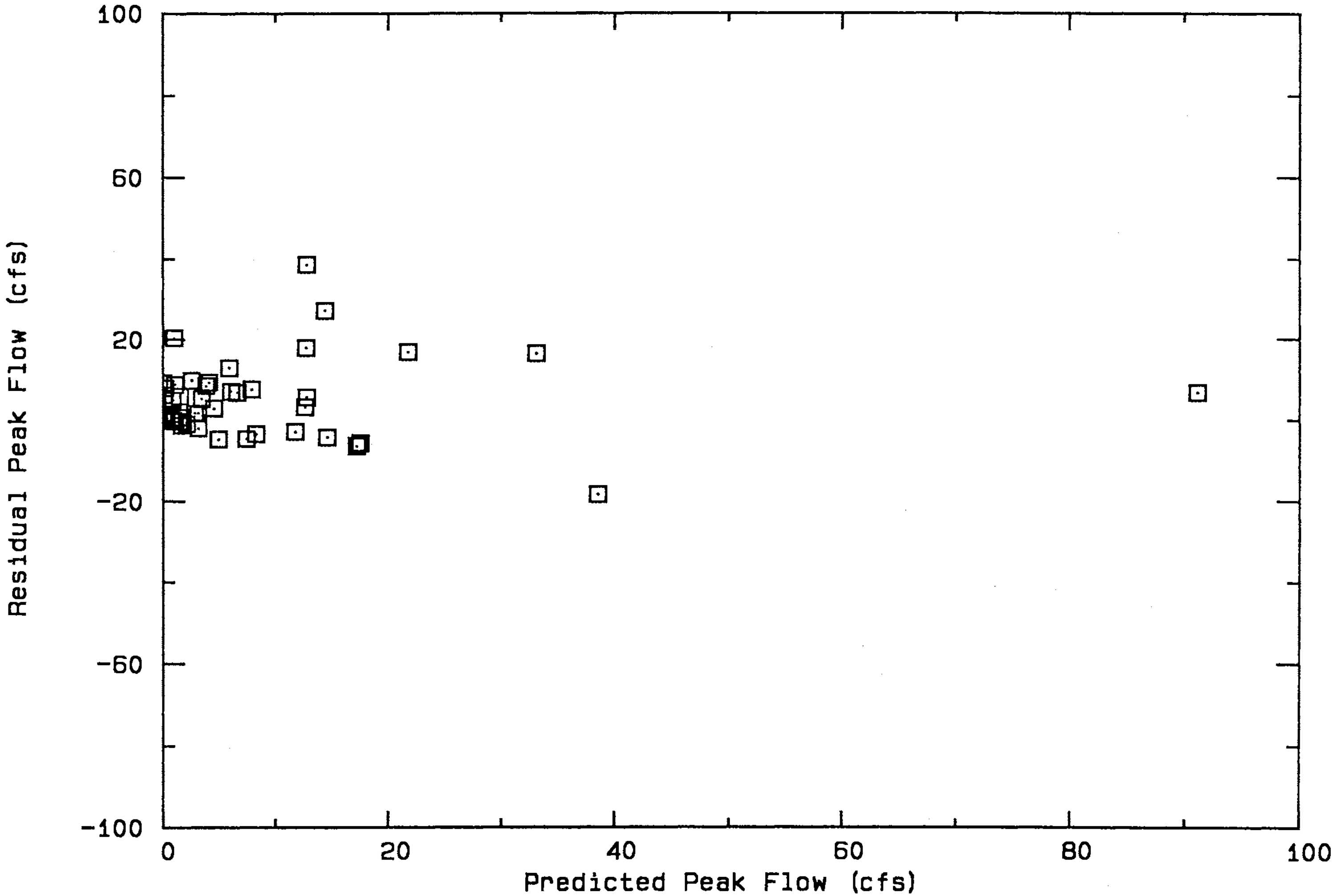


Figure 9. Peak Flow Residual Plot for Watershed R7, 2

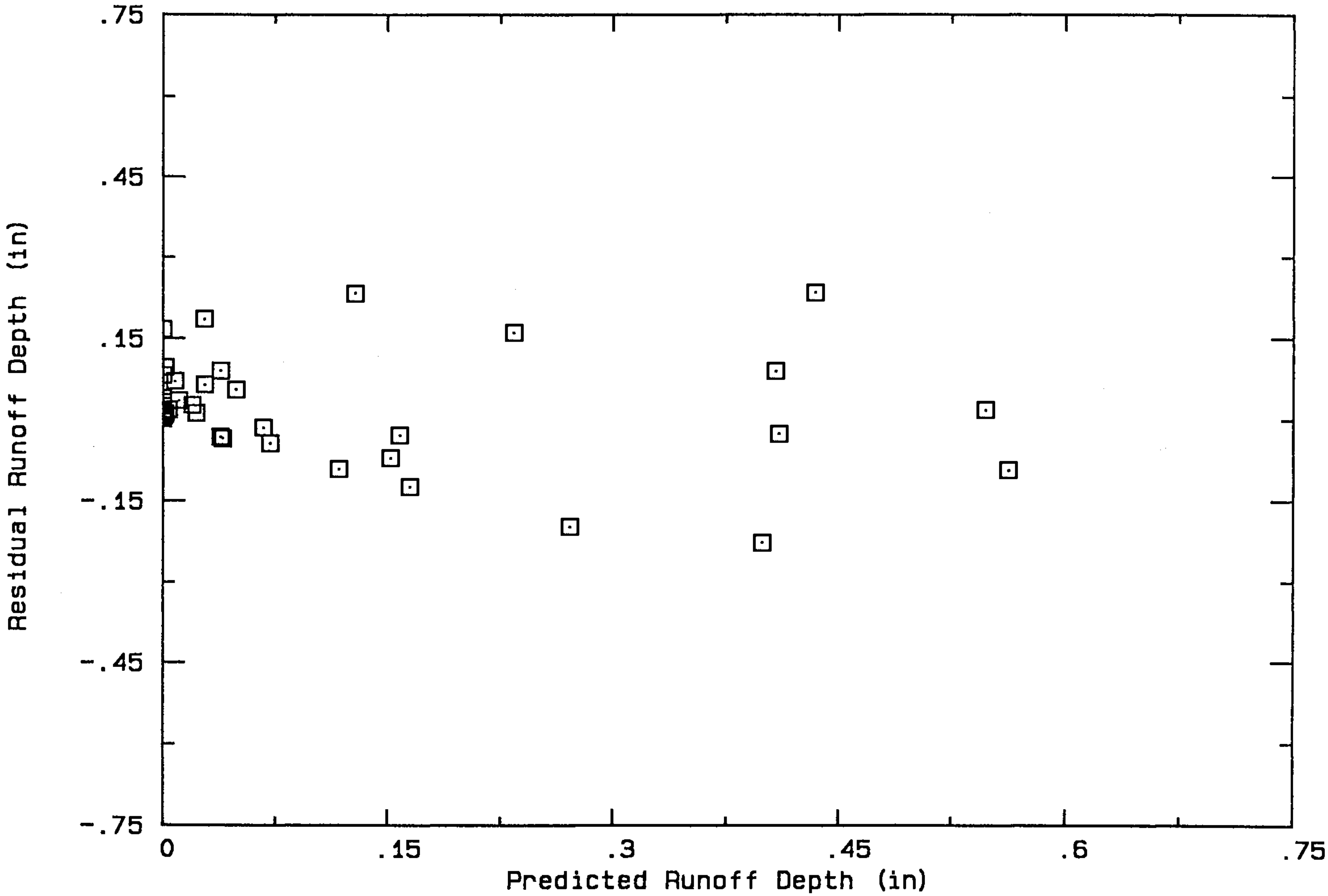


Figure 10. Runoff Depth Residual Plot for Watershed 511, 1

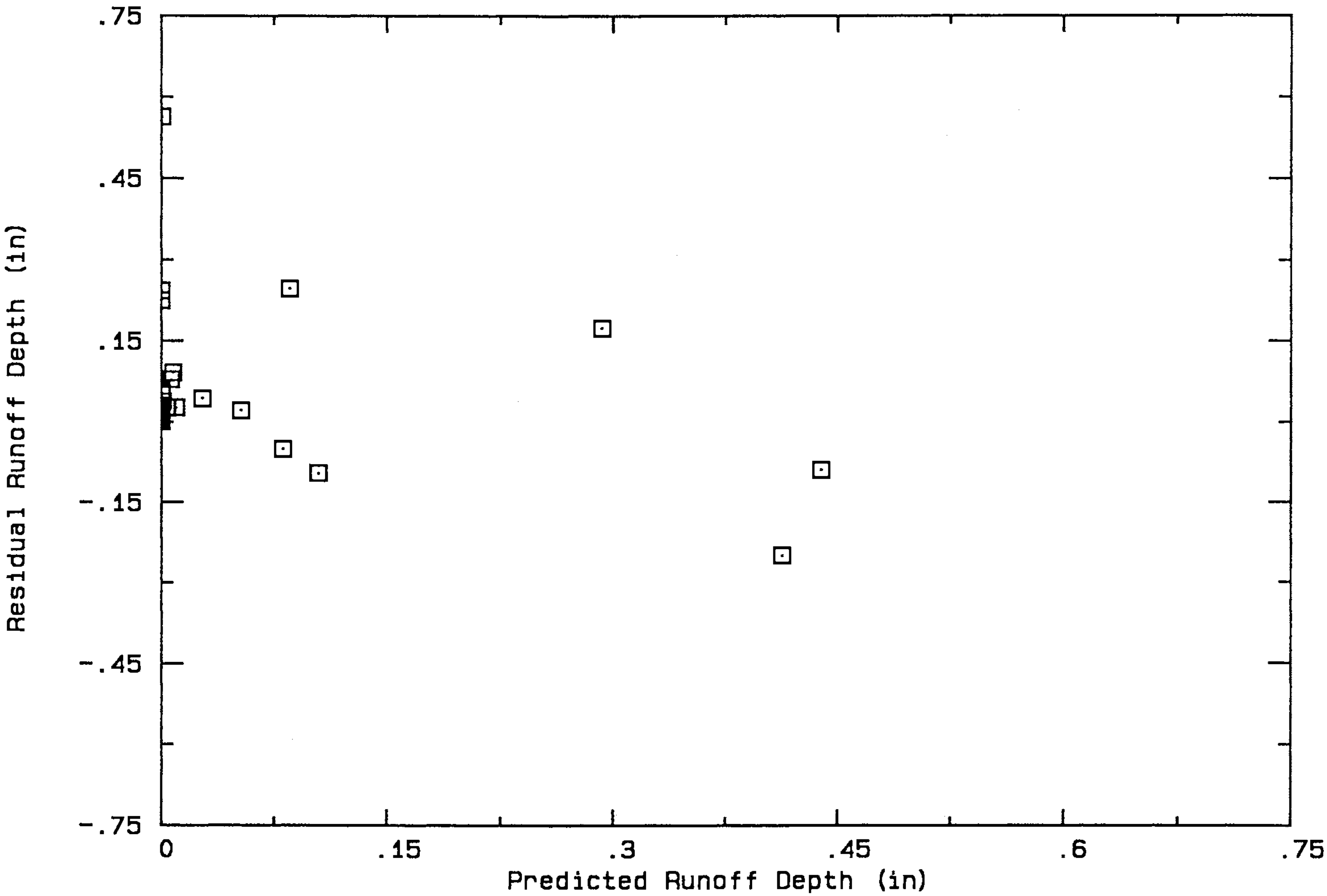


Figure 11. Runoff Depth Residual Plot for Watershed 5142, 1

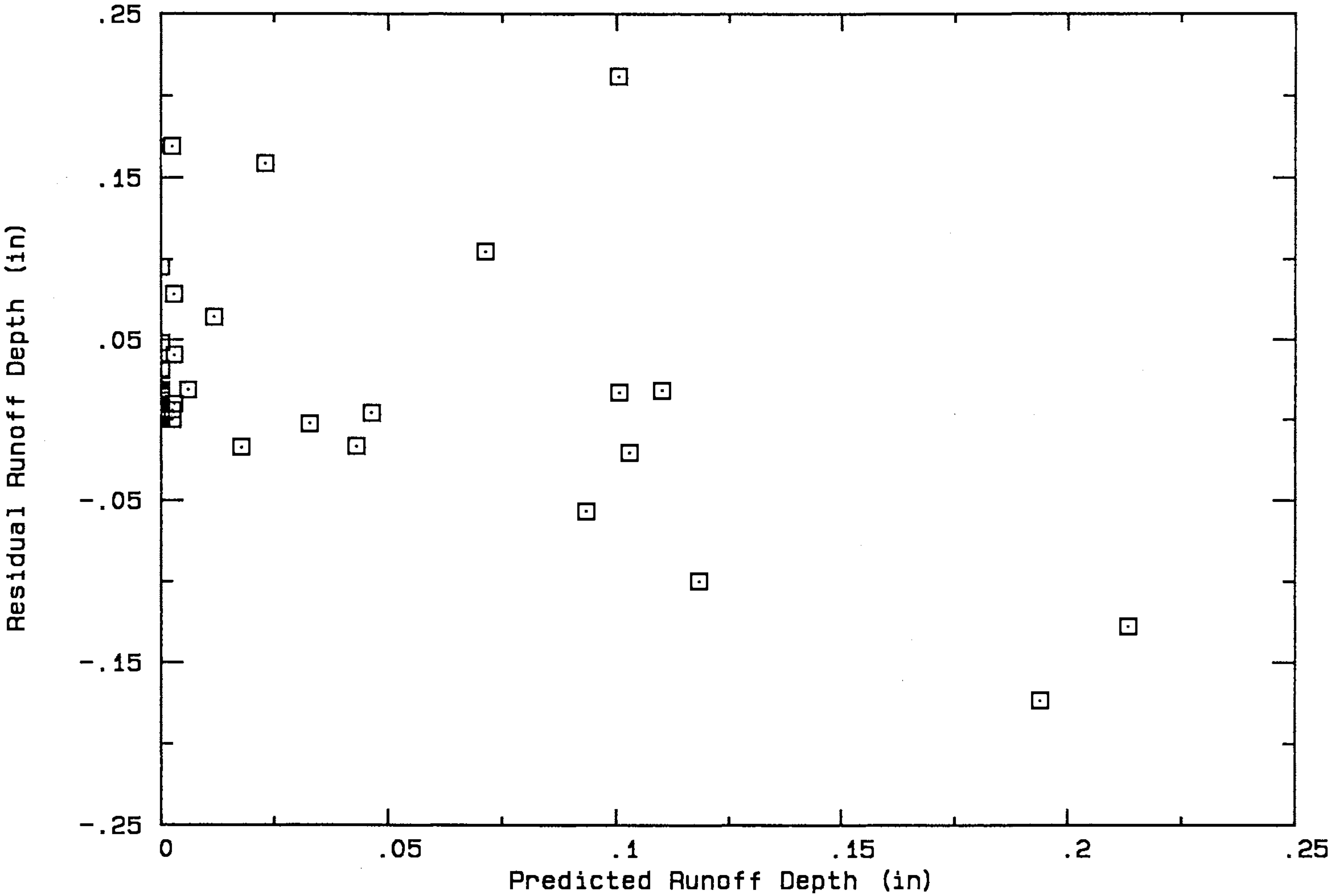


Figure 12. Runoff Depth Residual Plot for Watershed 611, 1

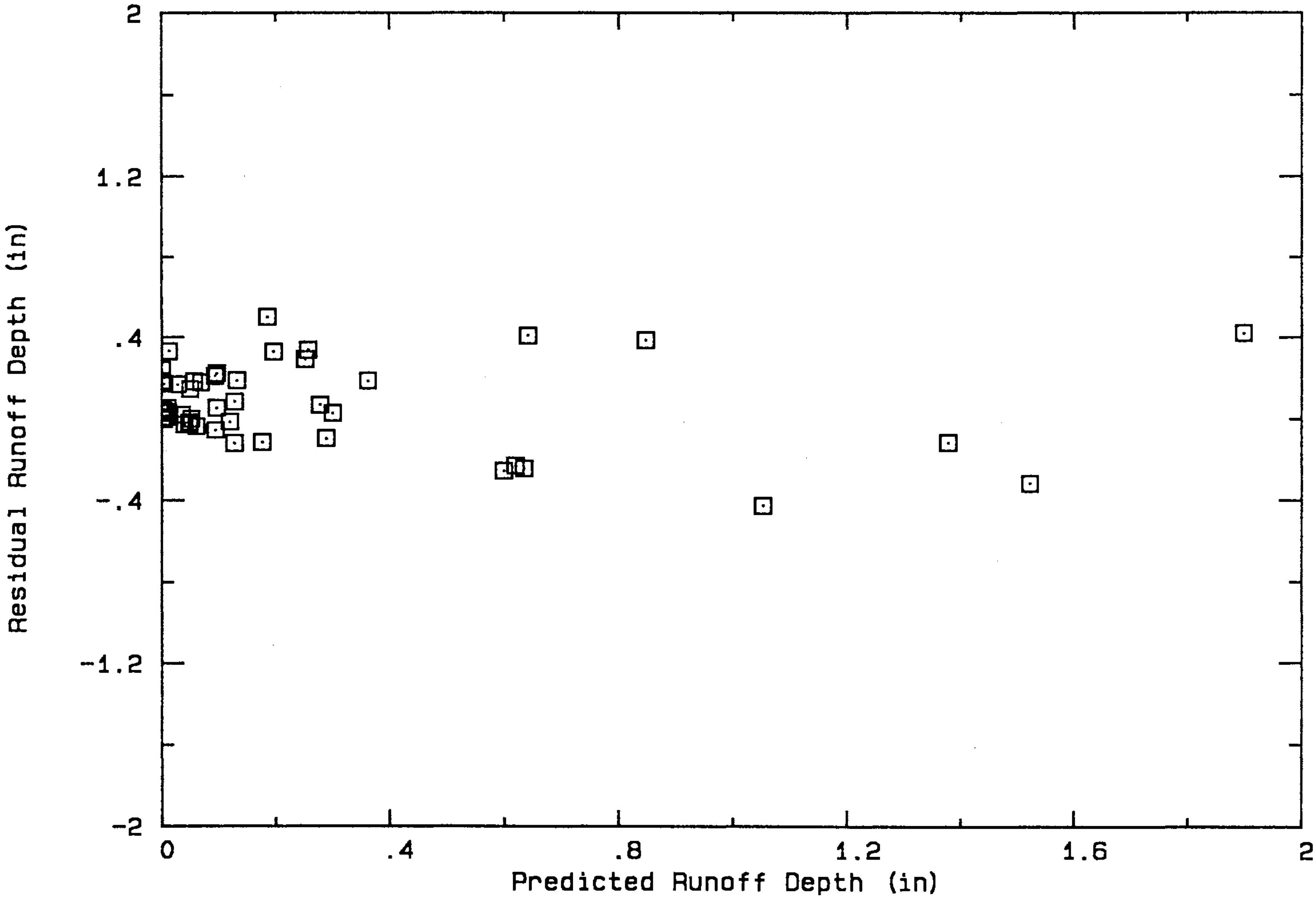


Figure 13. Runoff Depth Residual Plot for Watershed R7, 1

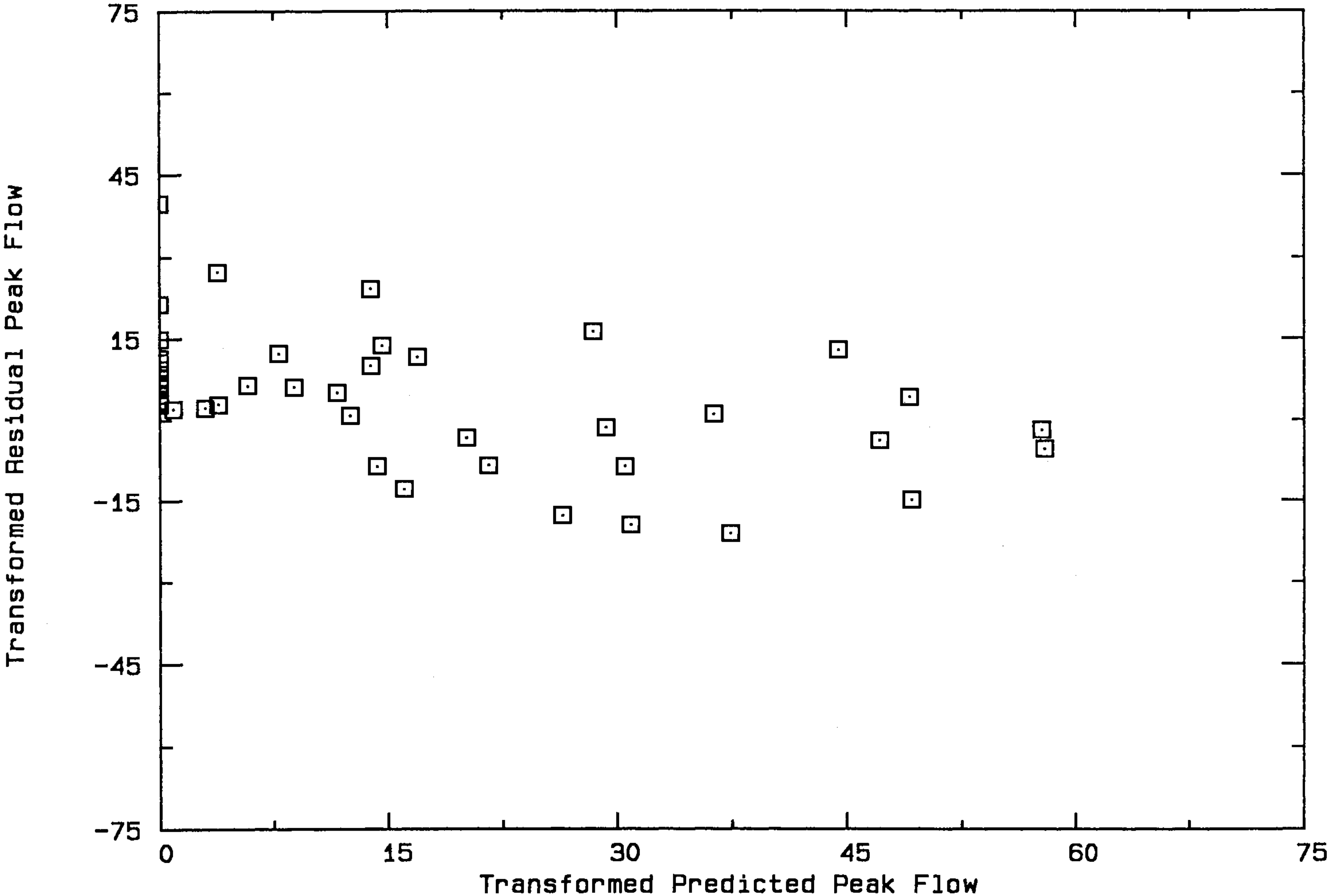


Figure 14. Transformed Peak Flow Residual Plot for Watershed 511, 1

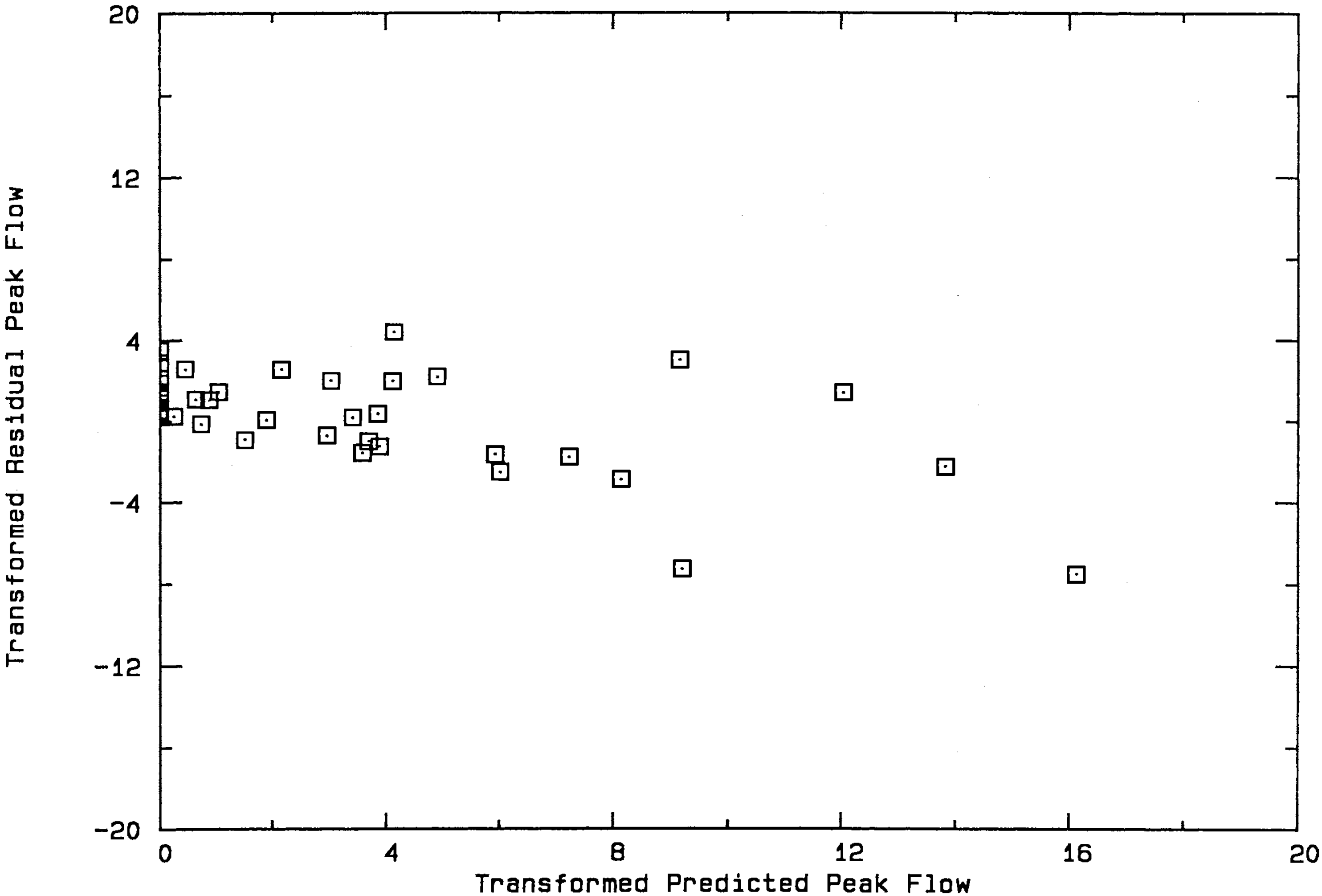


Figure 15. Transformed Peak Flow Residual Plot for Watershed 5142, 1

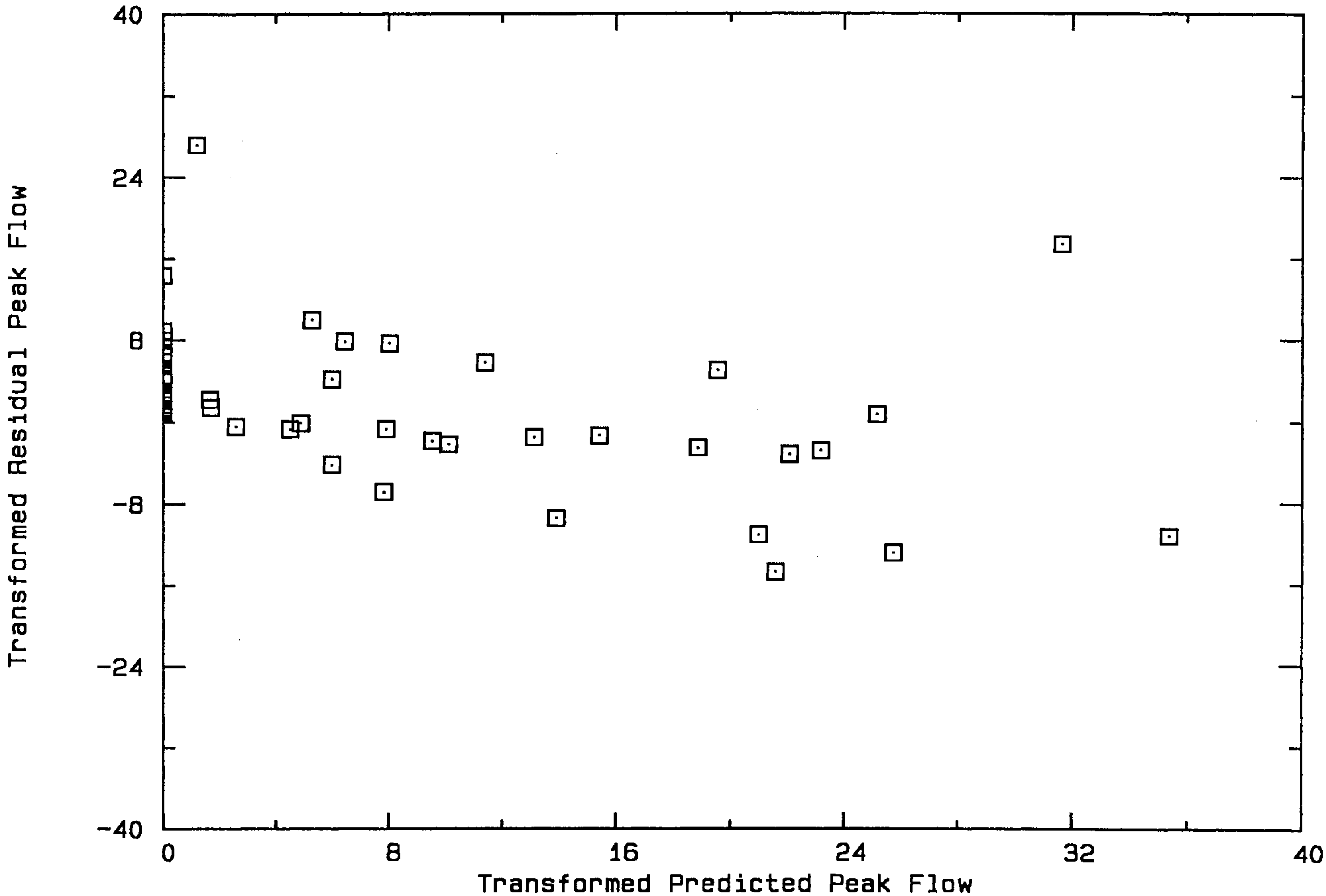


Figure 16. Transformed Peak Flow Residual Plot for Watershed 611, 1

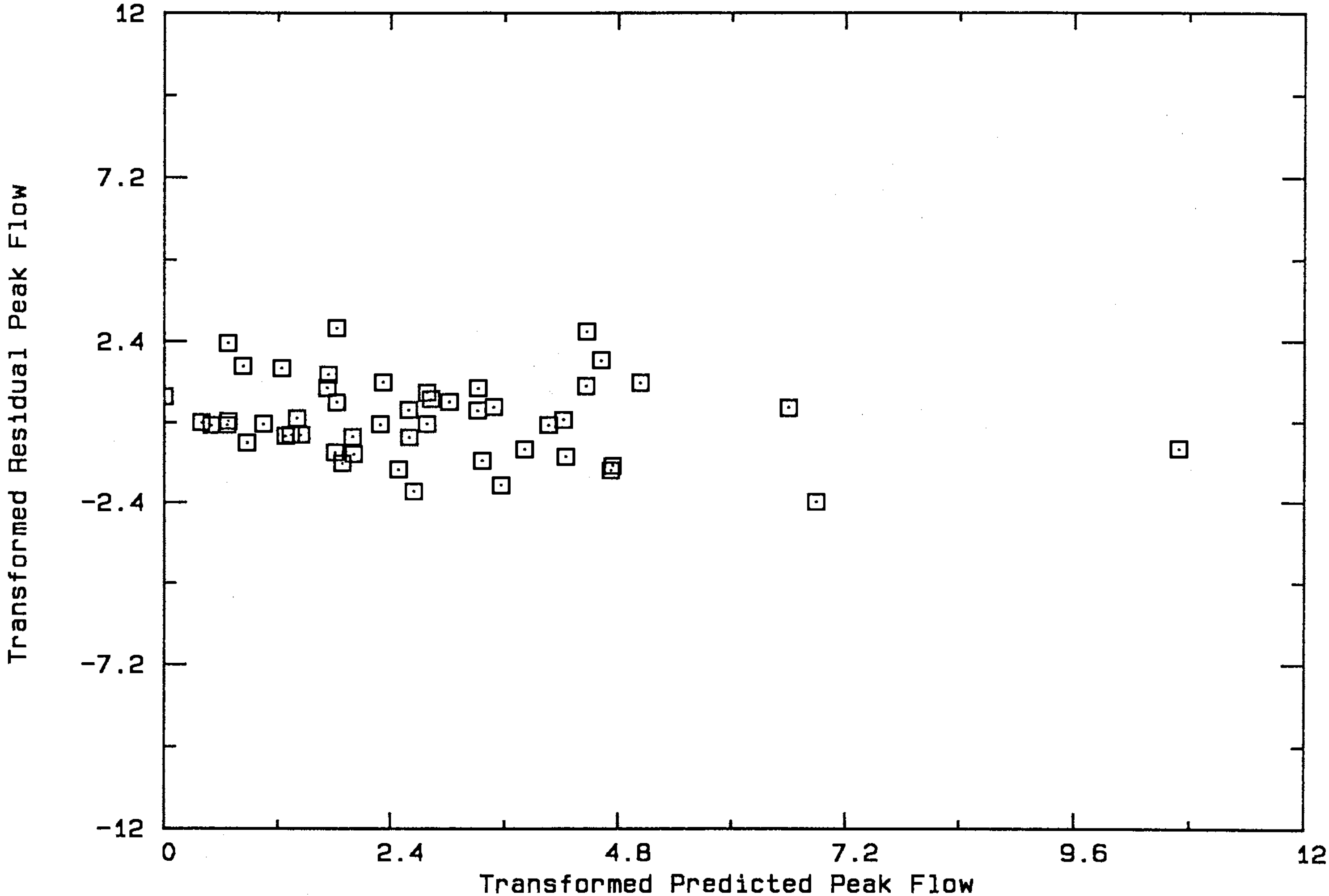


Figure 17. Transformed Peak Flow Residual Plot for Watershed R7, 1

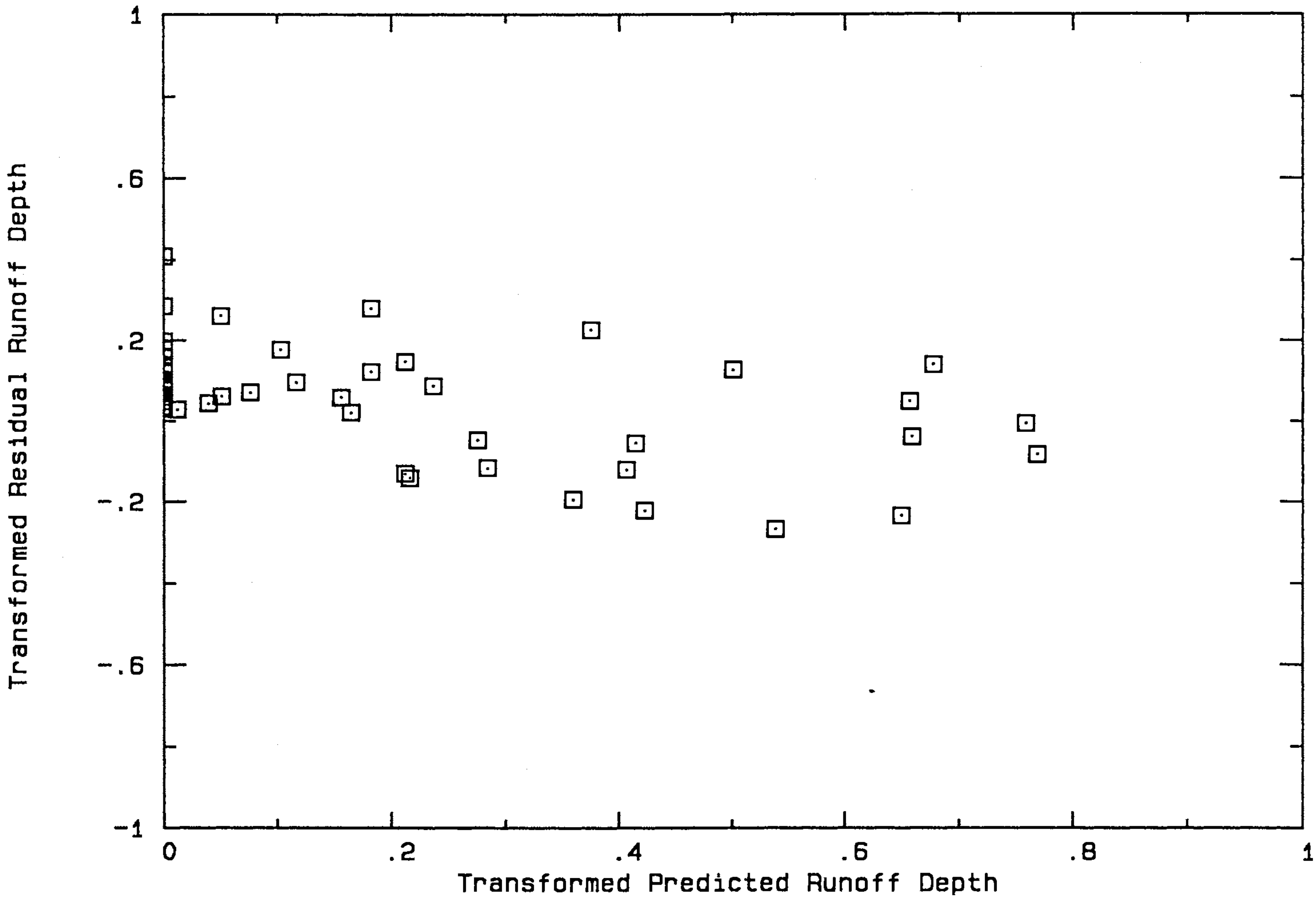


Figure 18. Transformed Runoff Depth Residual Plot for Watershed 511, 1

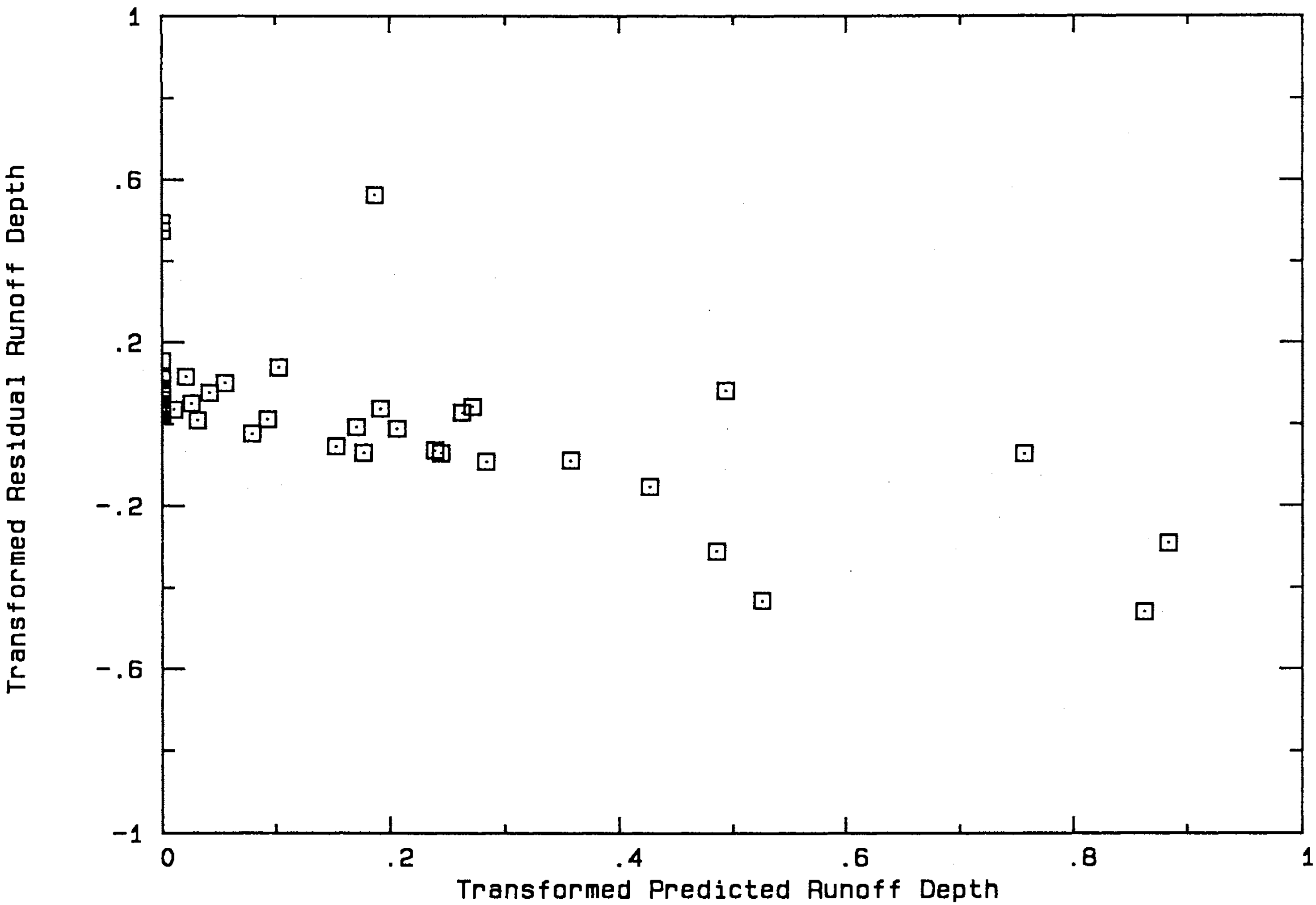


Figure 19. Transformed Runoff Depth Residual Plot for Watershed 5142, 1

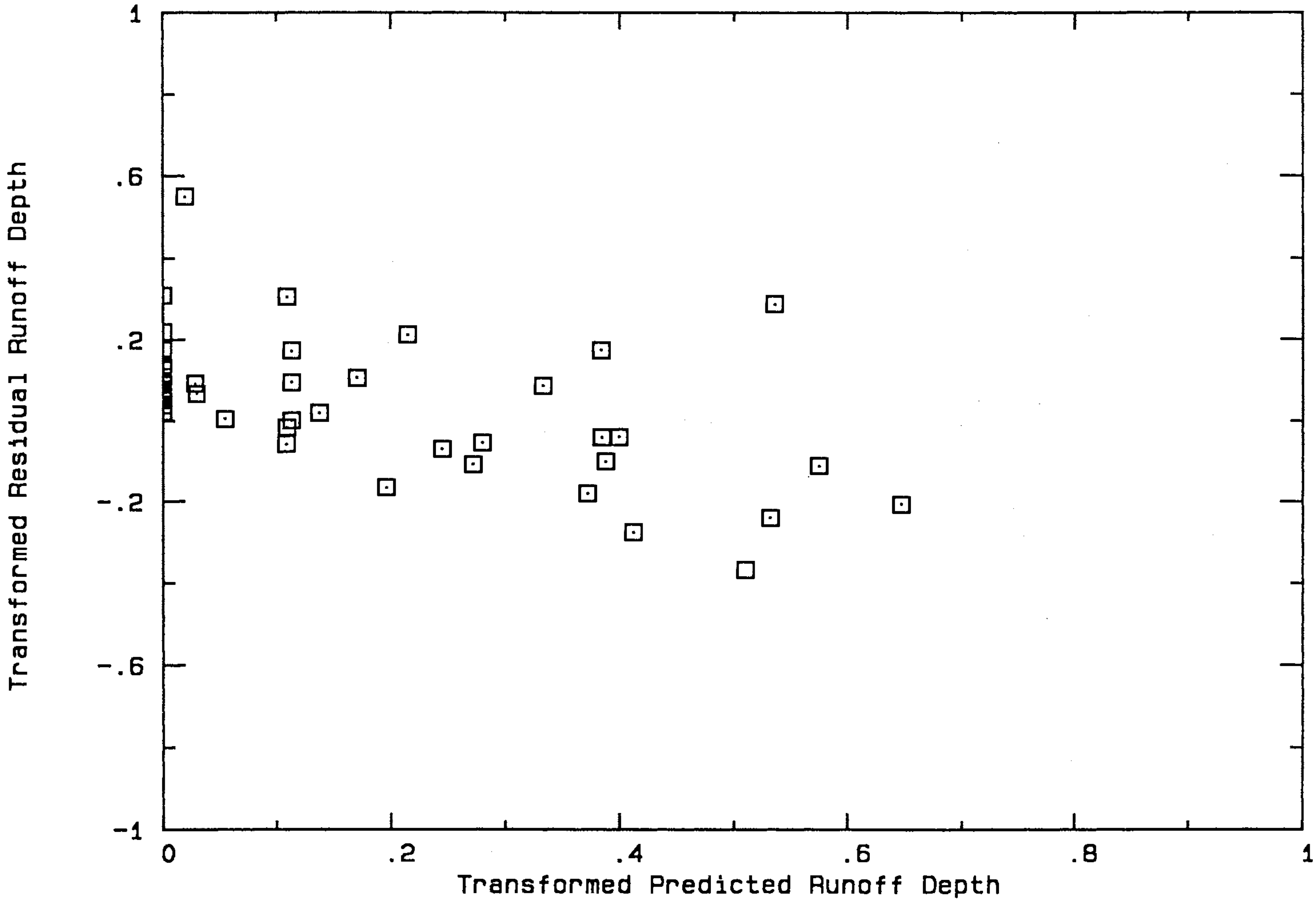


Figure 20. Transformed Runoff Depth Residual Plot for Watershed 611, 1

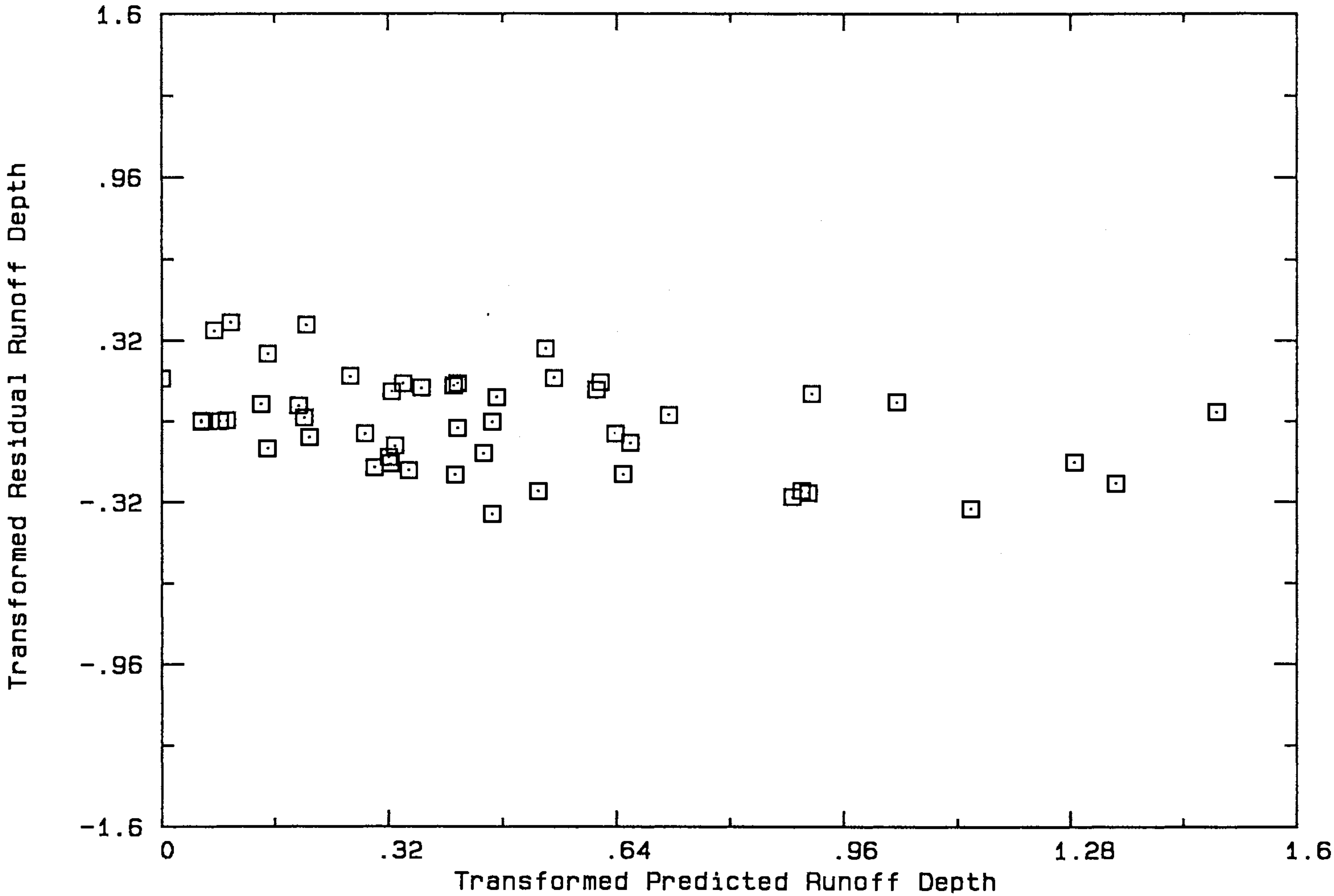


Figure 21. Transformed Runoff Depth Residual Plot for Watershed R7, 1

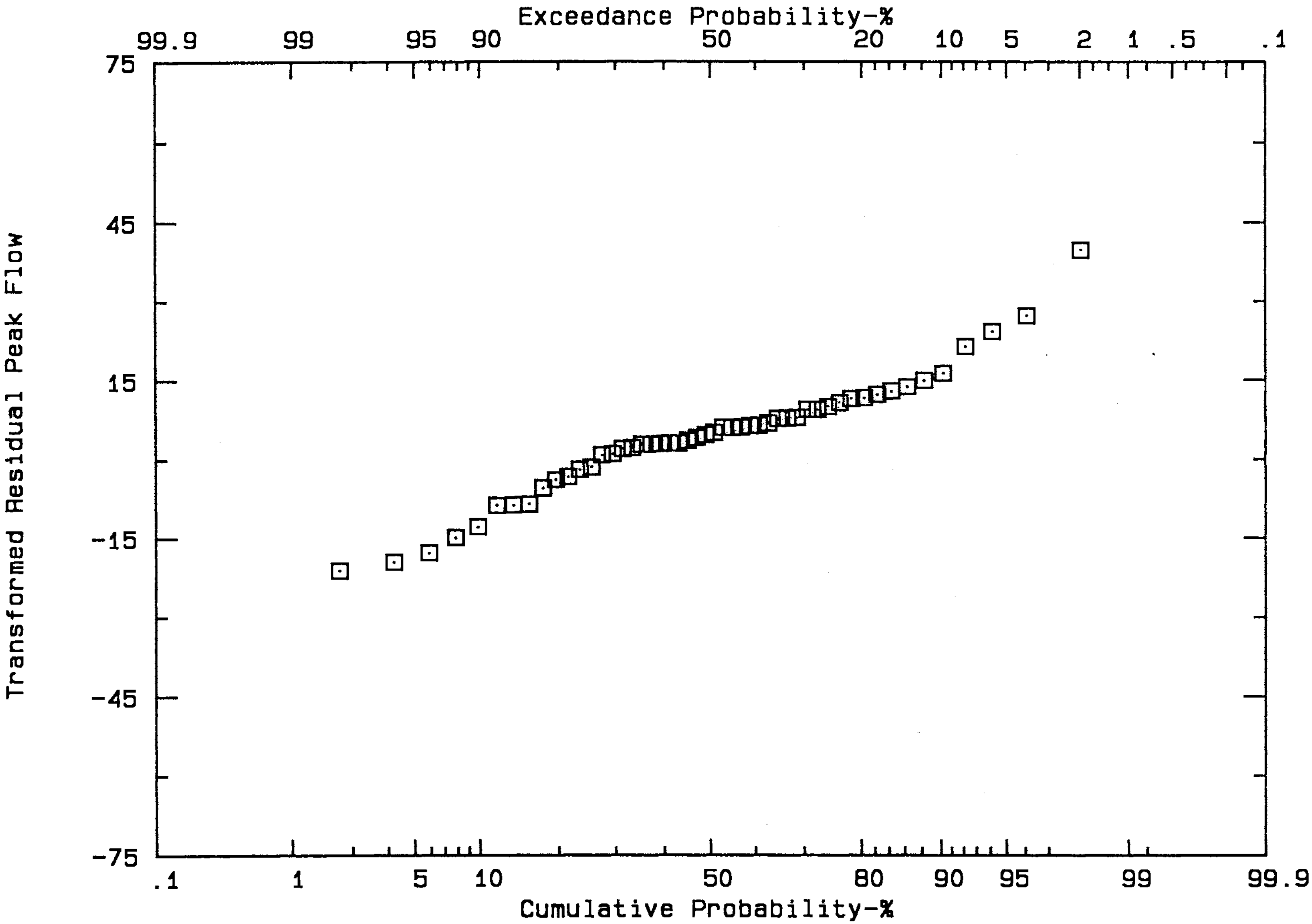


Figure 22. Probability Plot of Transformed Peak Flow Residuals for Watershed 511, 1

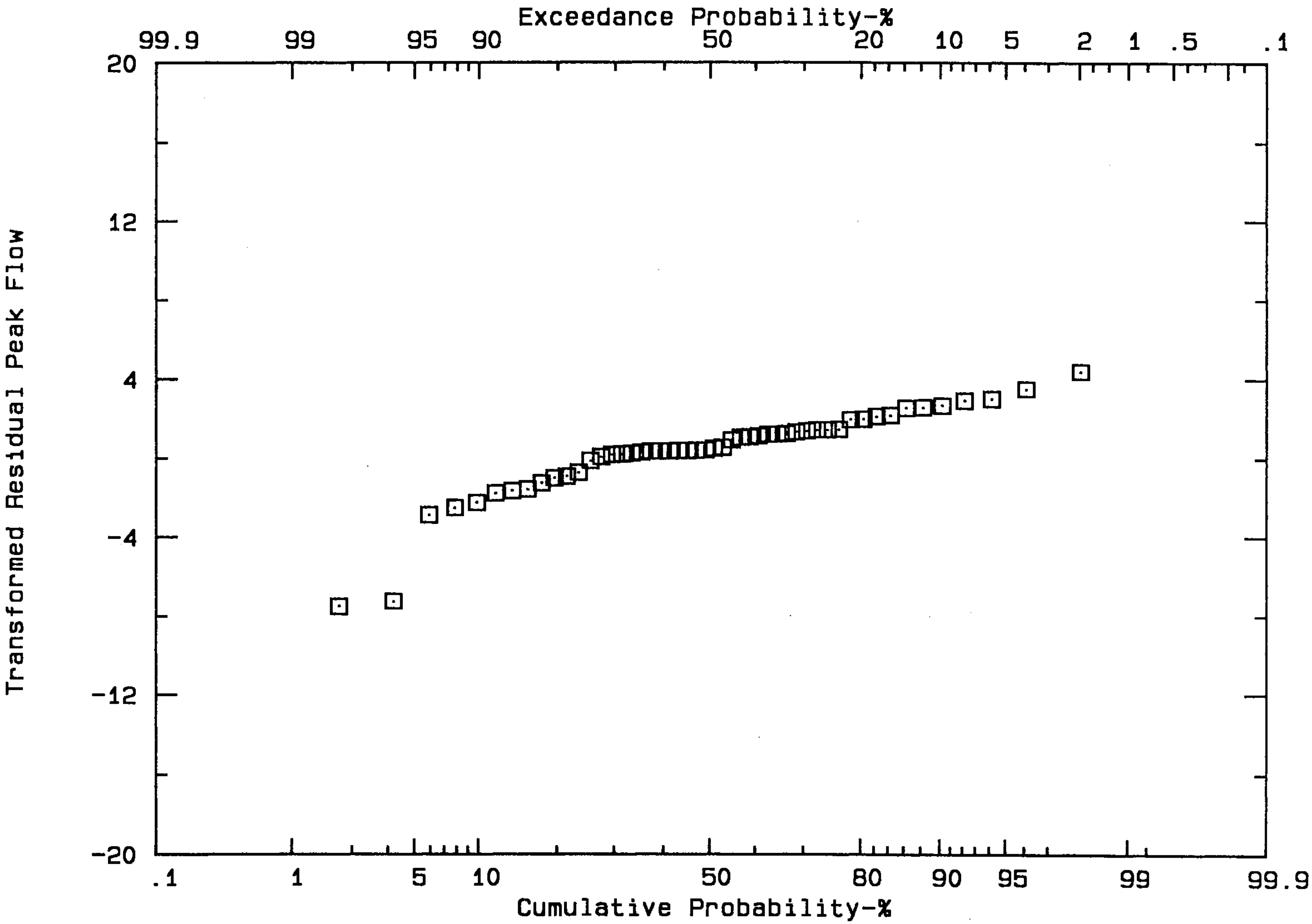


Figure 23. Probability Plot of Transformed Peak Flow Residuals for Watershed 5142, 1

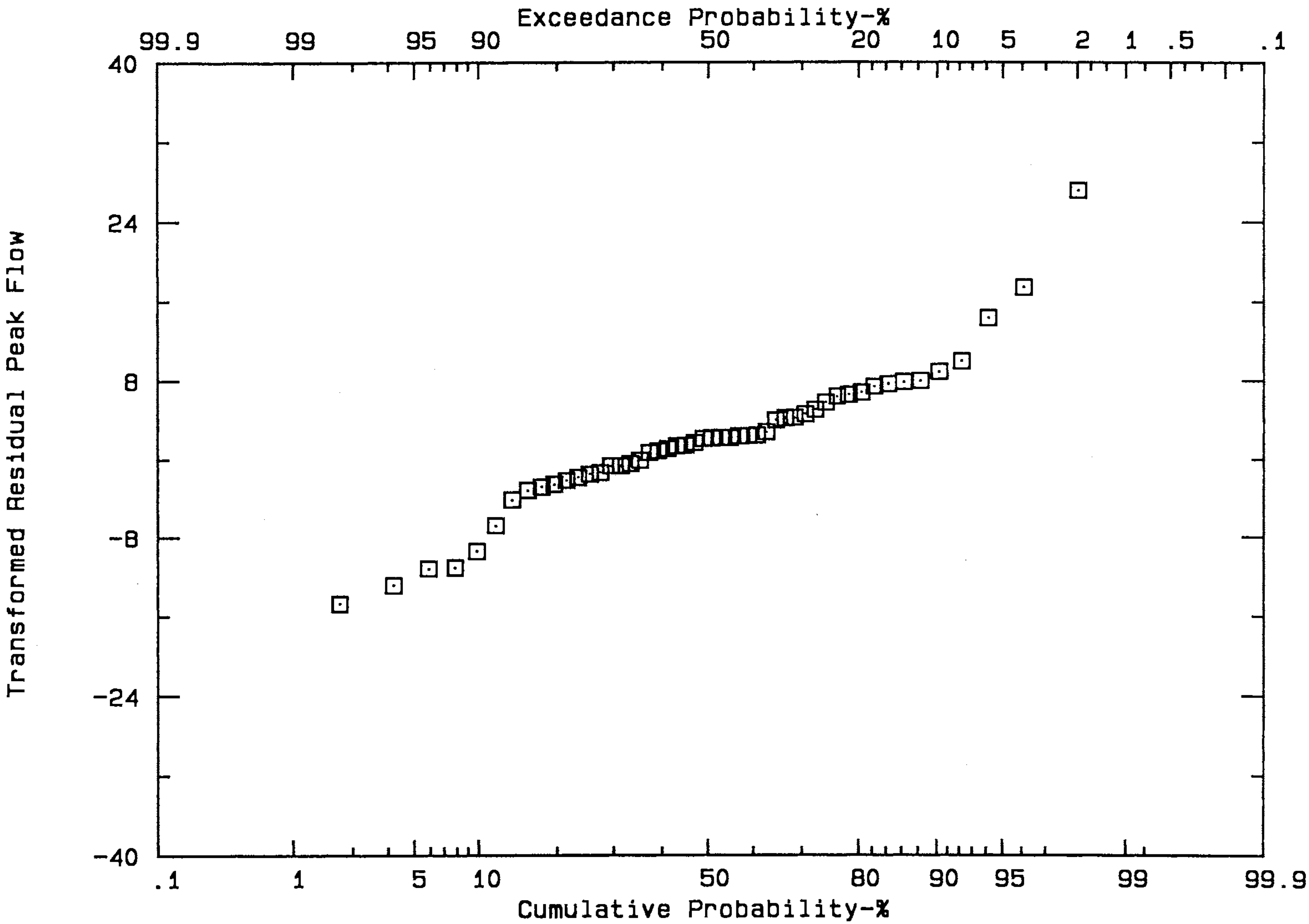


Figure 24. Probability Plot of Transformed Peak Flow Residuals for Watershed 611, 1

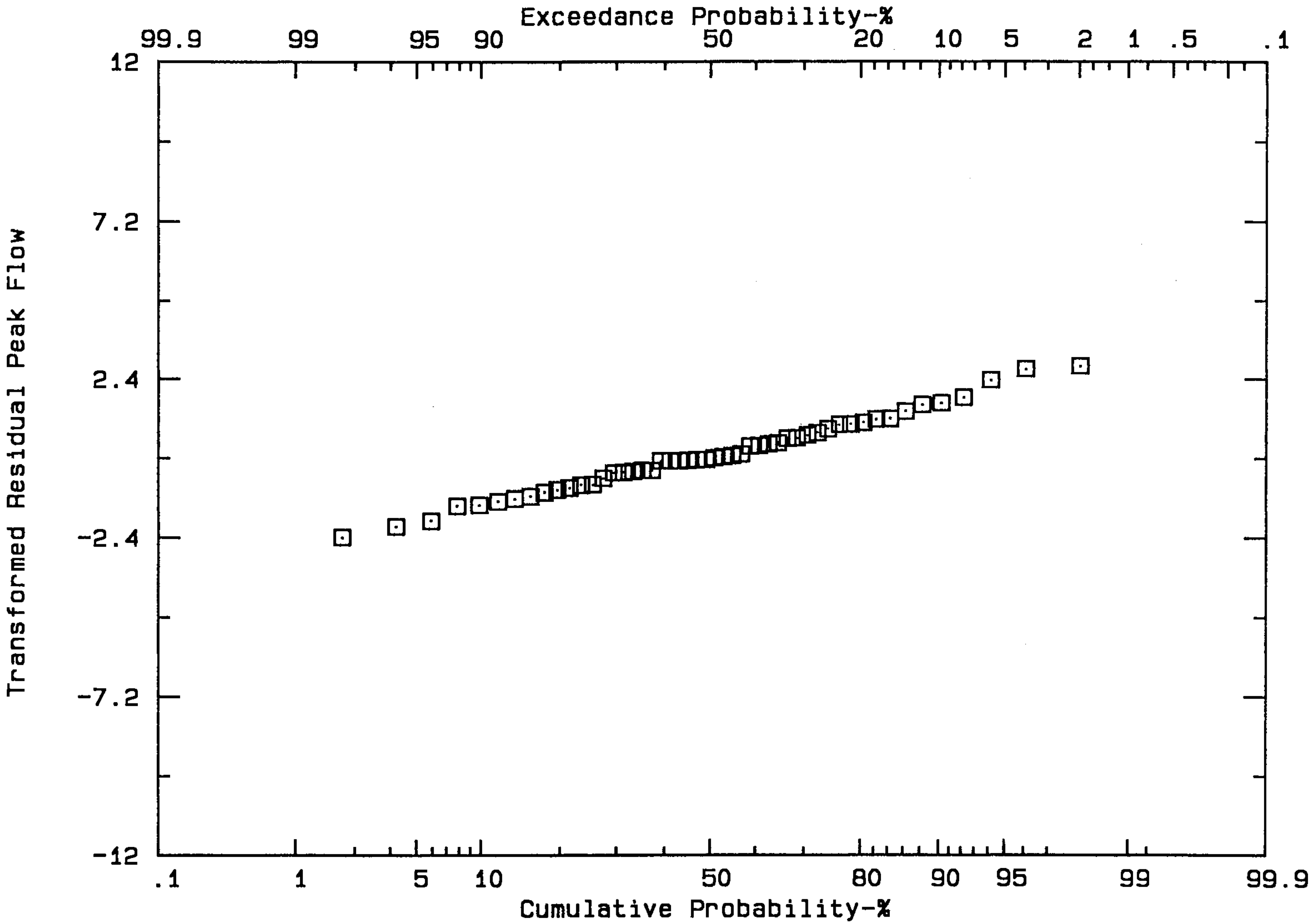


Figure 25. Probability Plot of Transformed Peak Flow Residuals for Watershed R7, 1

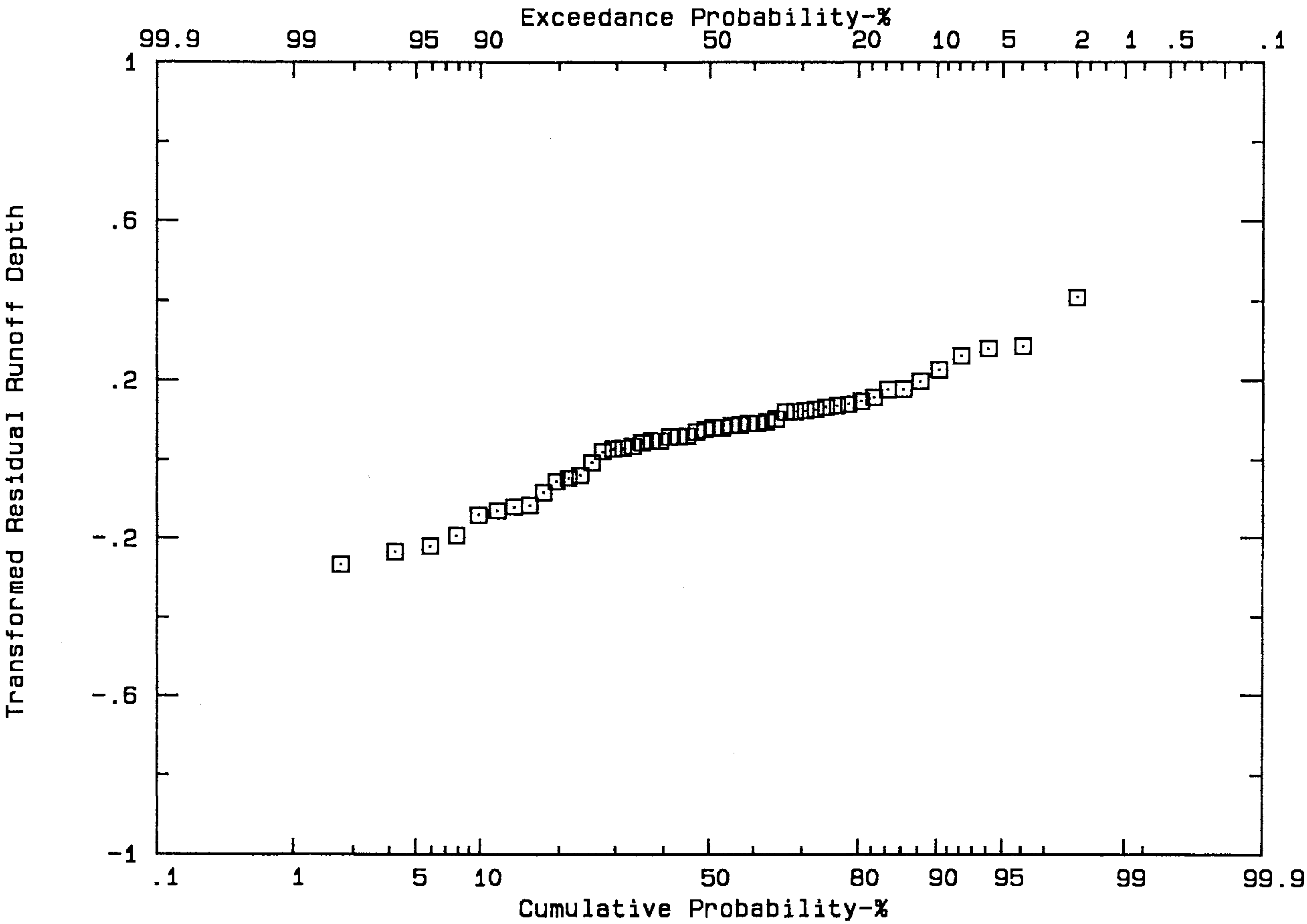


Figure 26. Probability Plot of Transformed Runoff Depth Residuals for Watershed 511, 1

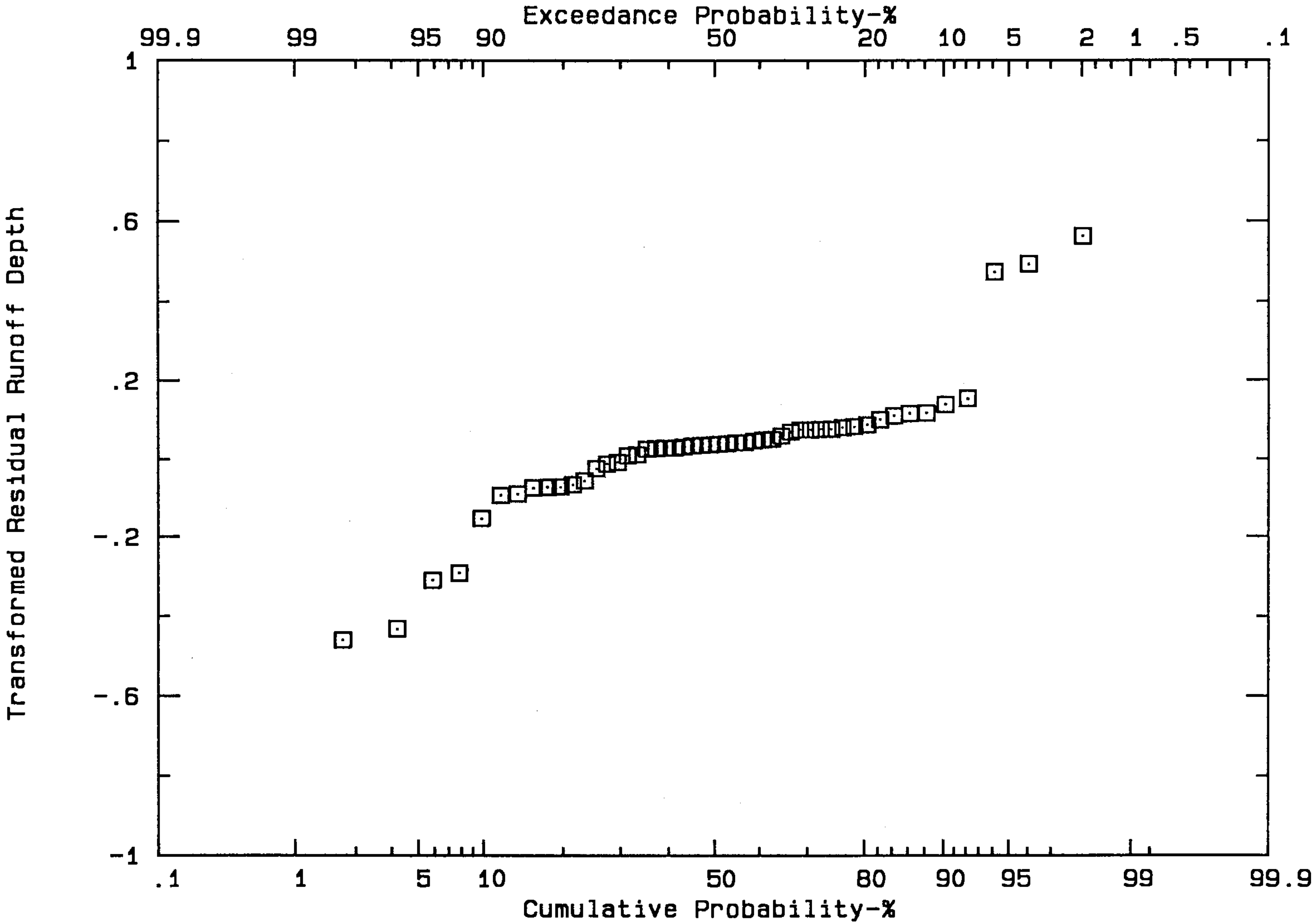


Figure 27. Probability Plot of Transformed Runoff Depth Residuals for Watershed 5142, 1

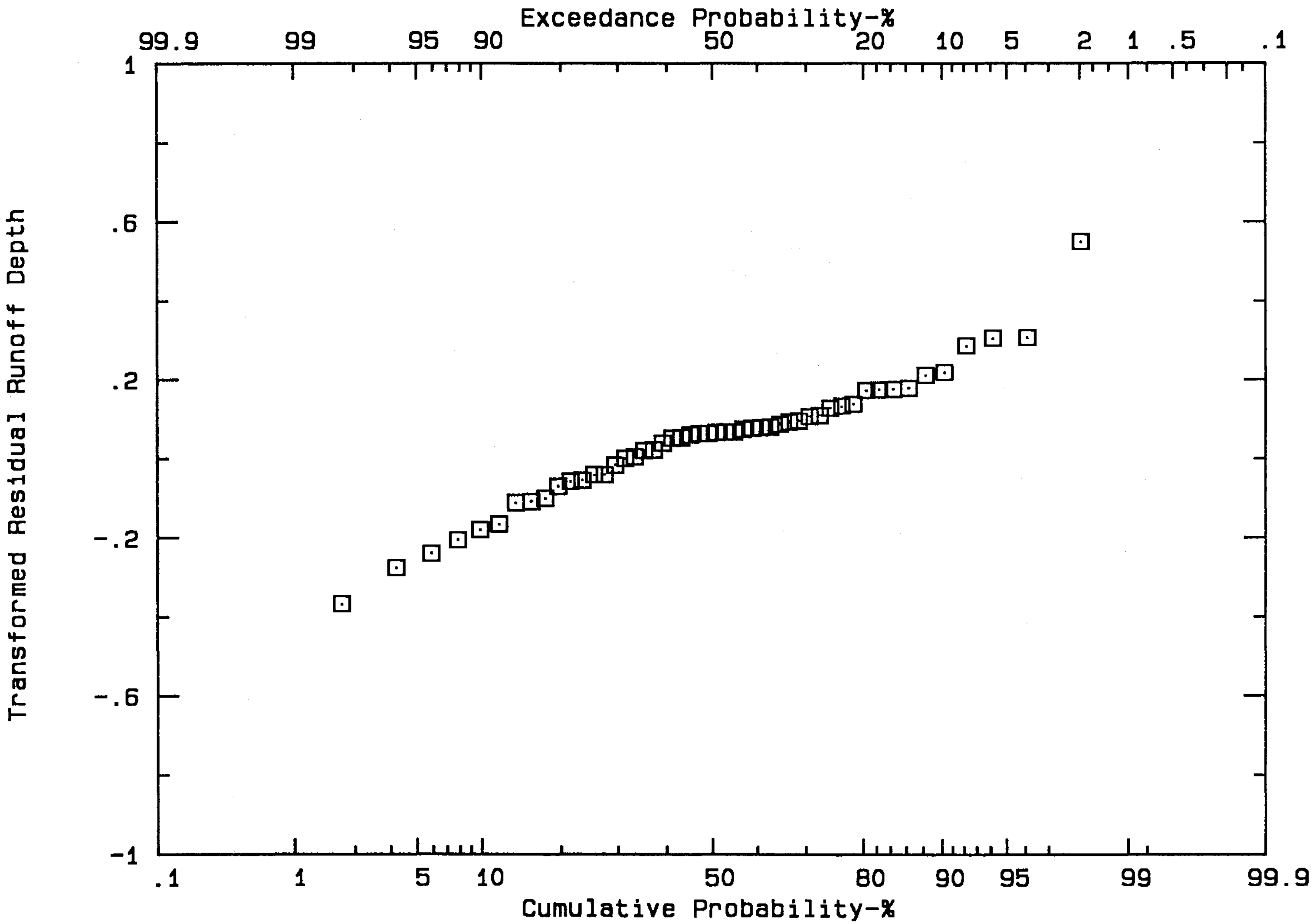


Figure 28. Probability Plot of Transformed Runoff Depth Residuals for Watershed 611, 1

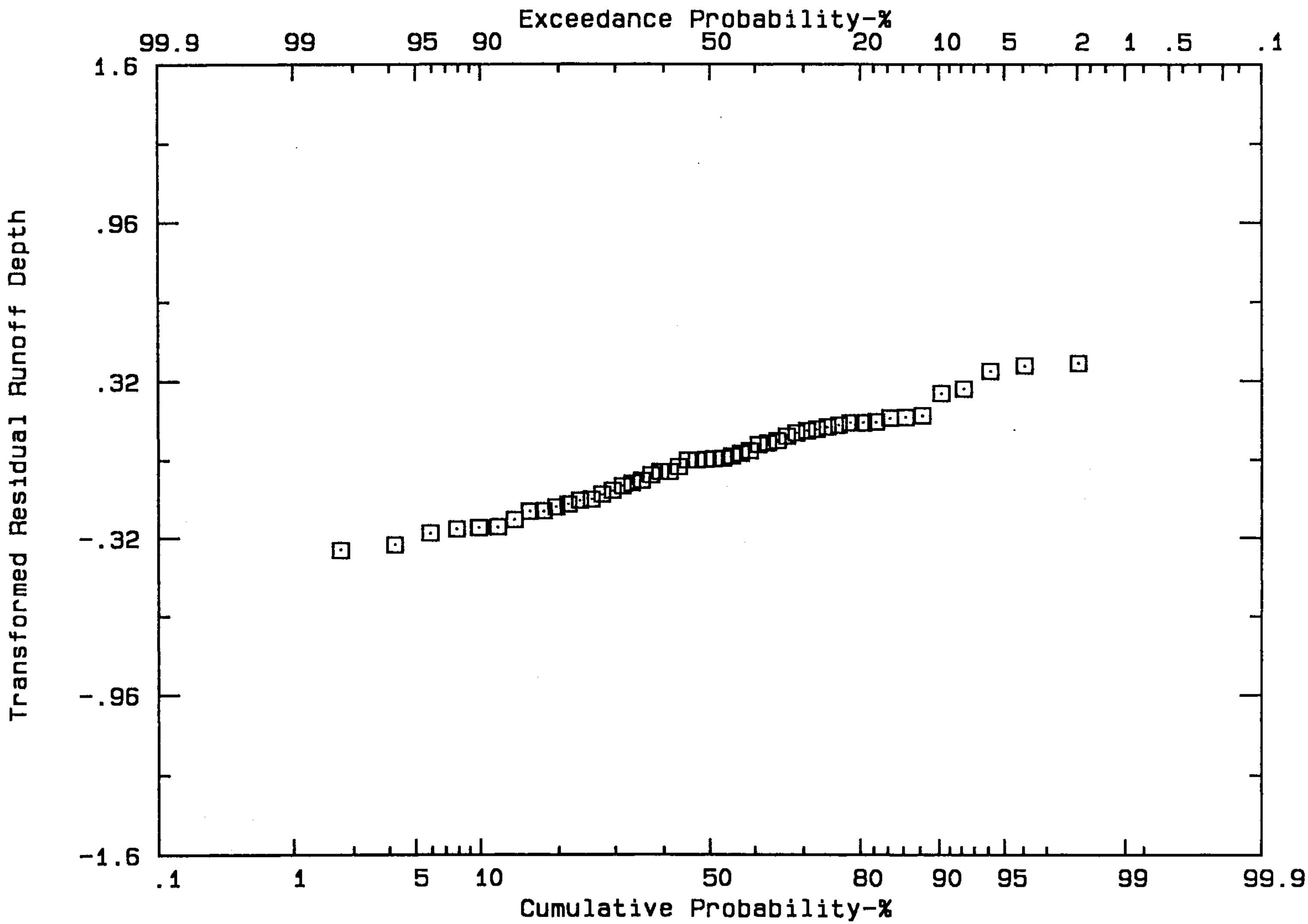


Figure 29. Probability Plot of Transformed Runoff Depth Residuals for Watershed R7, 1

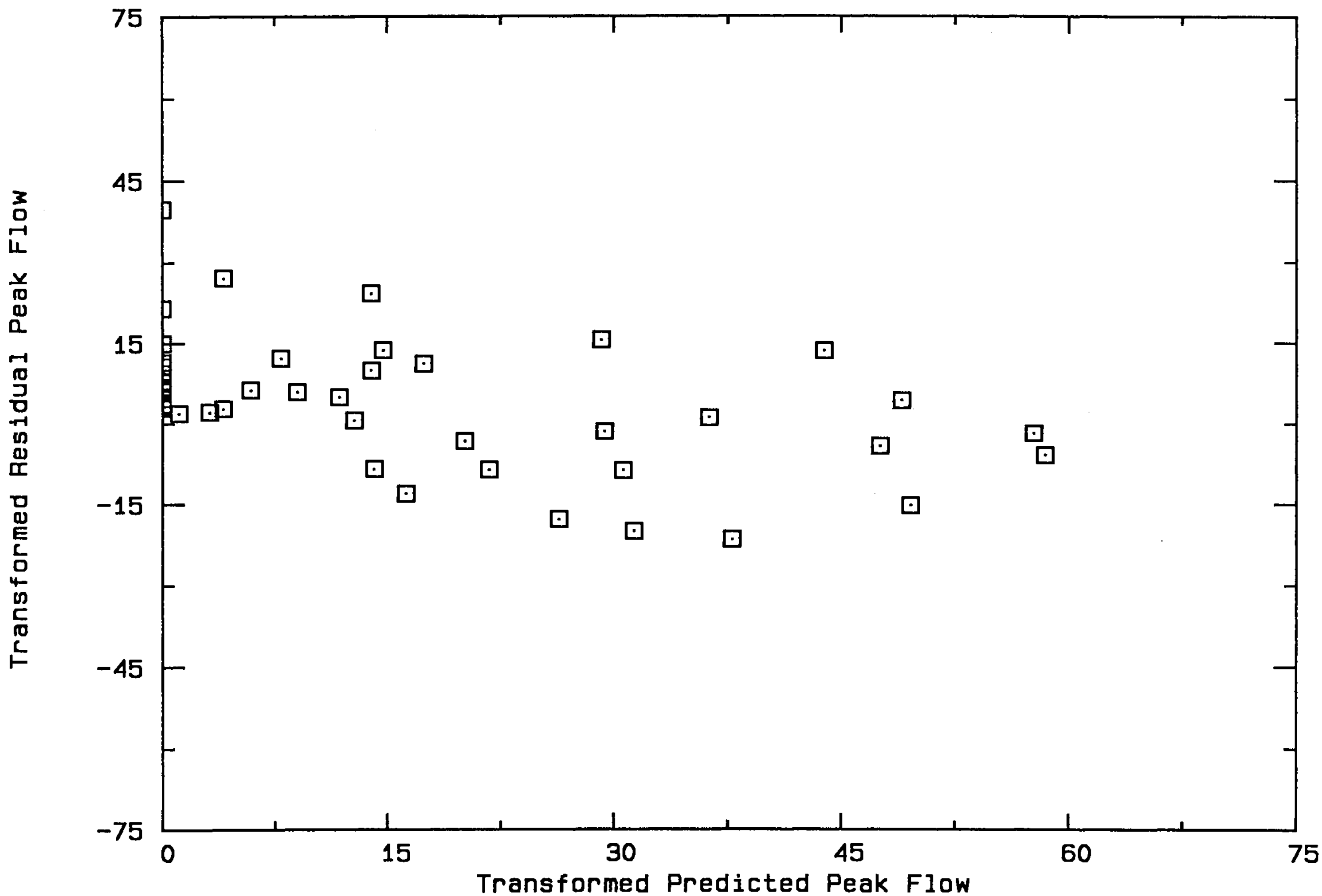


Figure 30. Transformed Peak Flow Residual Plot for Watershed 511, 2

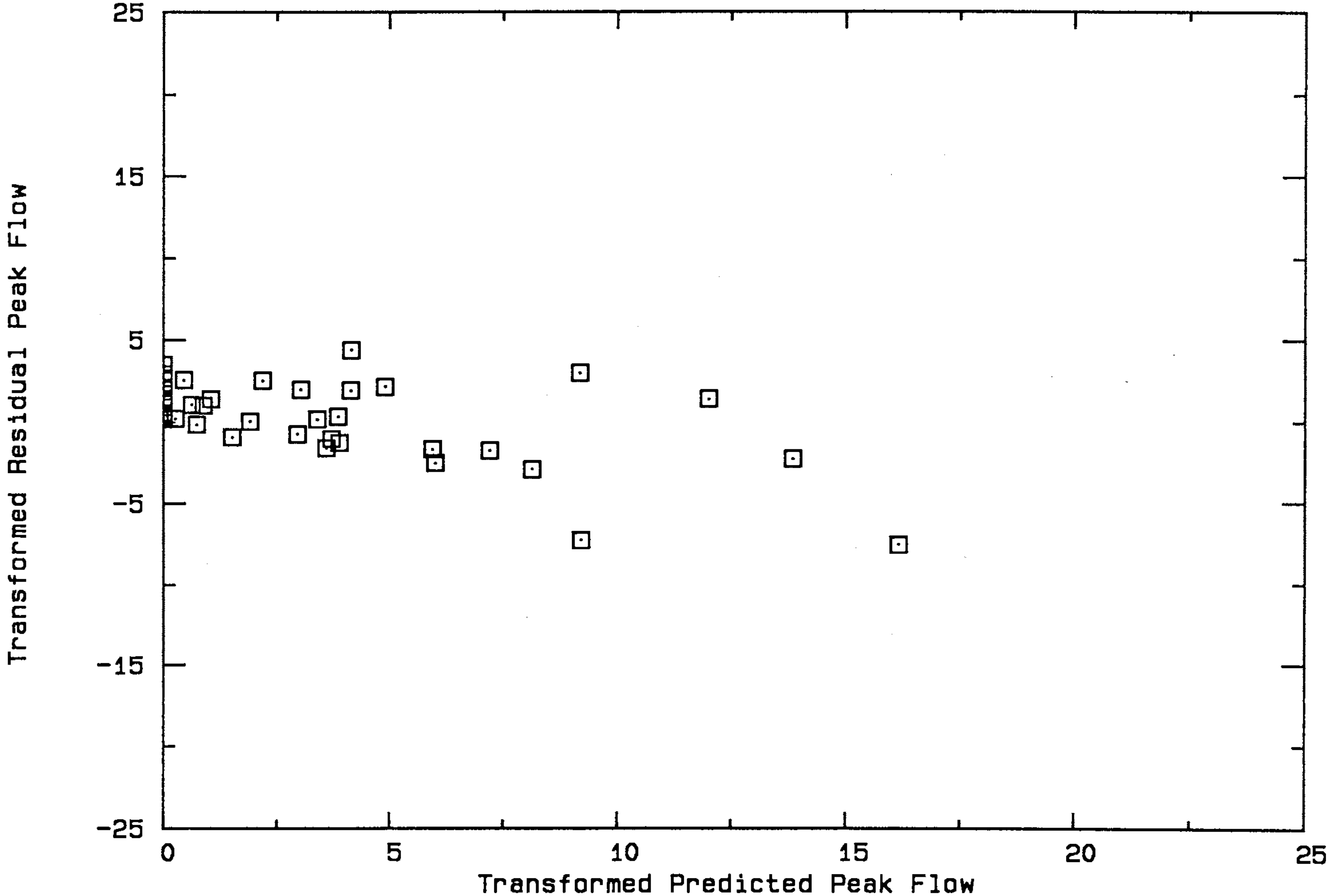
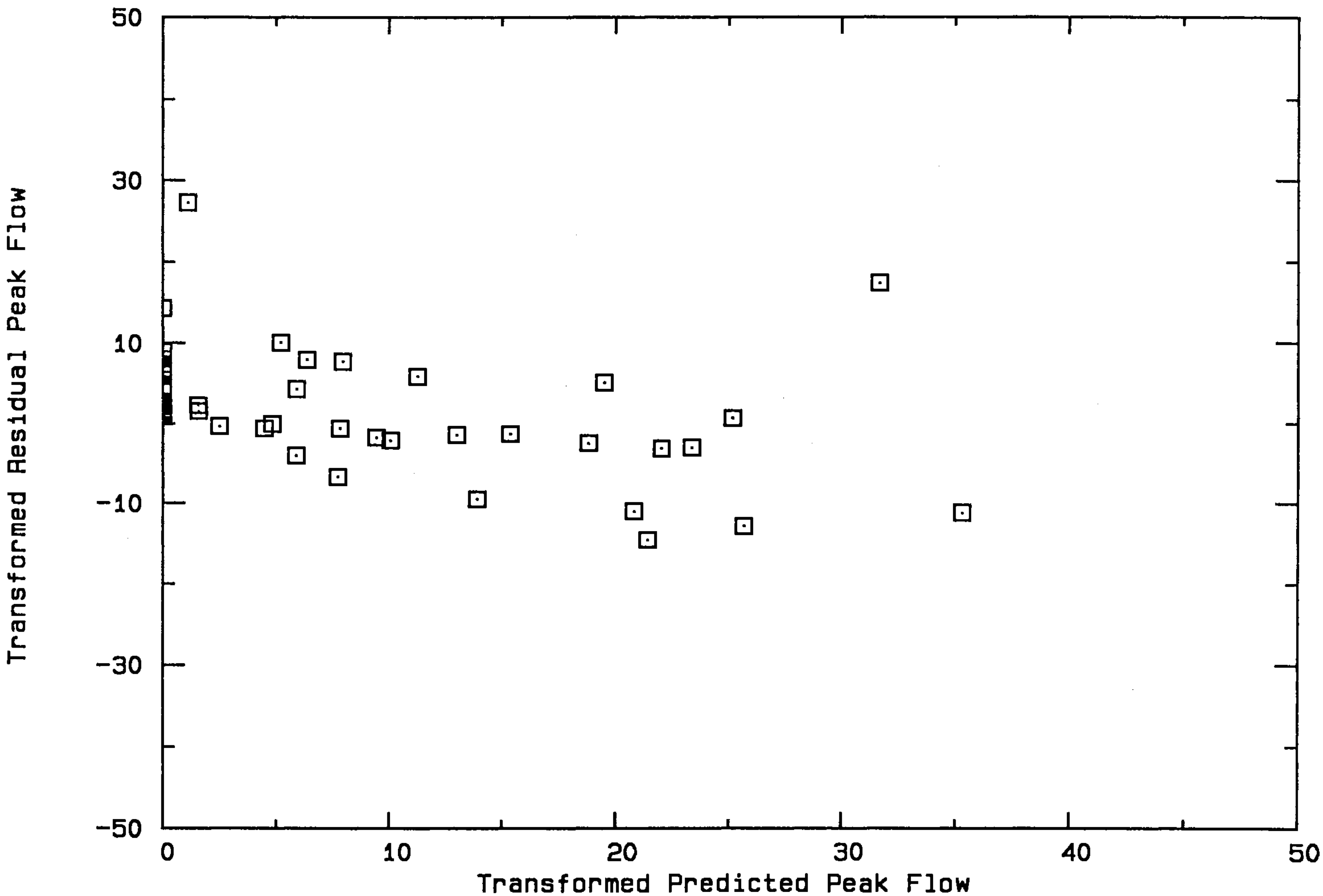


Figure 31. Transformed Peak Flow Residual Plot for Watershed 5142, 2



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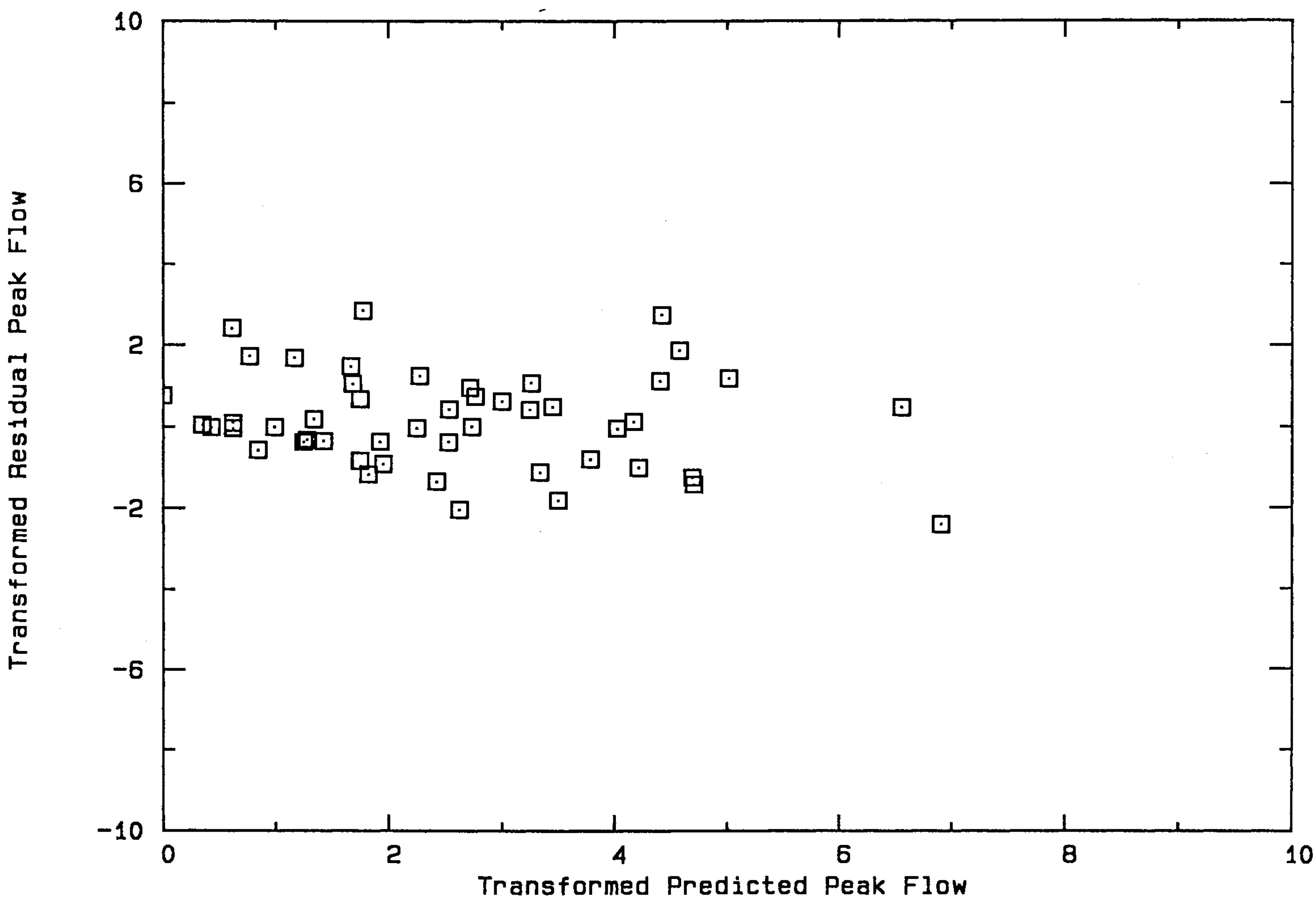

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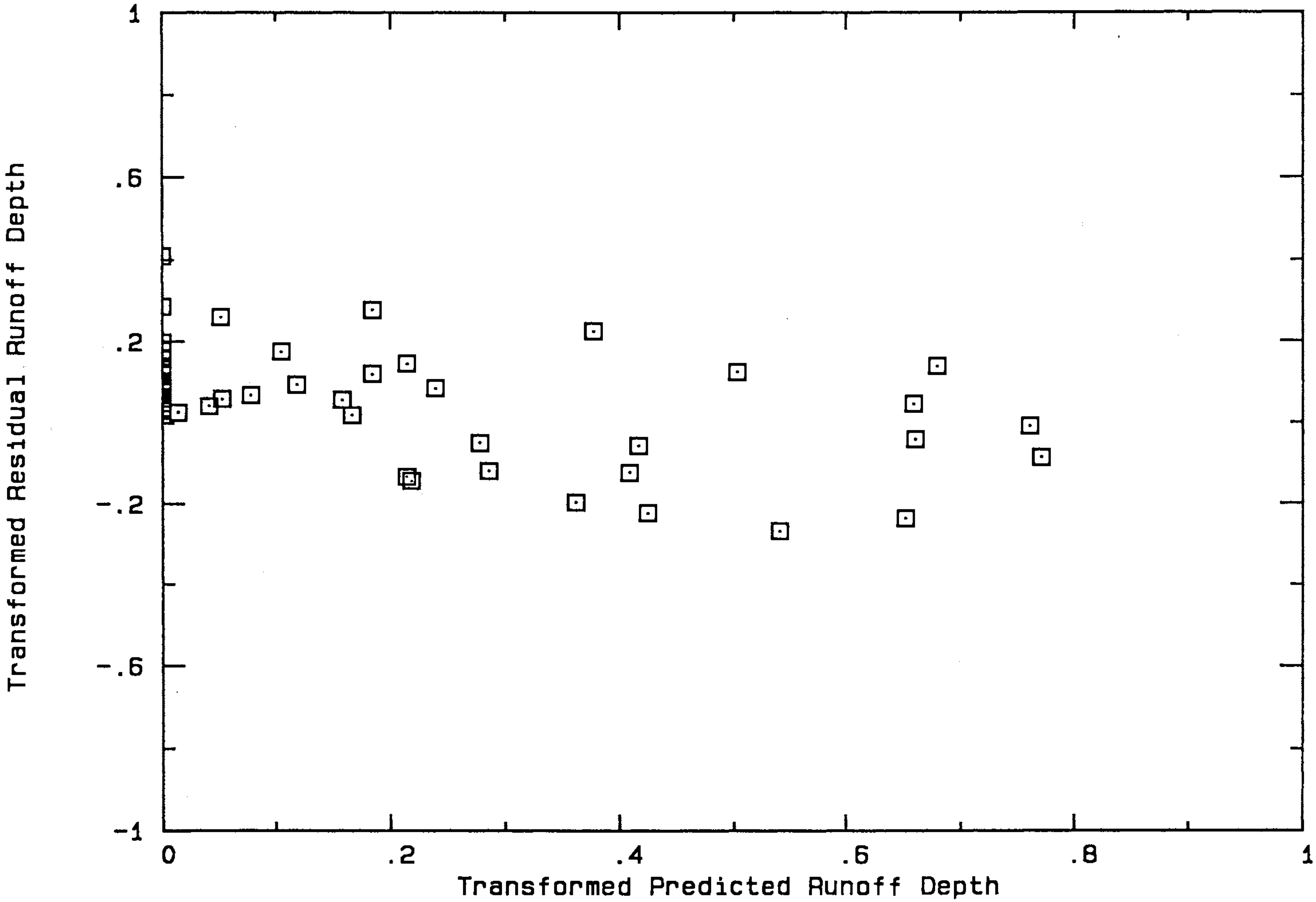

Figure 34. Transformed Runoff Depth Residual Plot for Watershed 511, 2

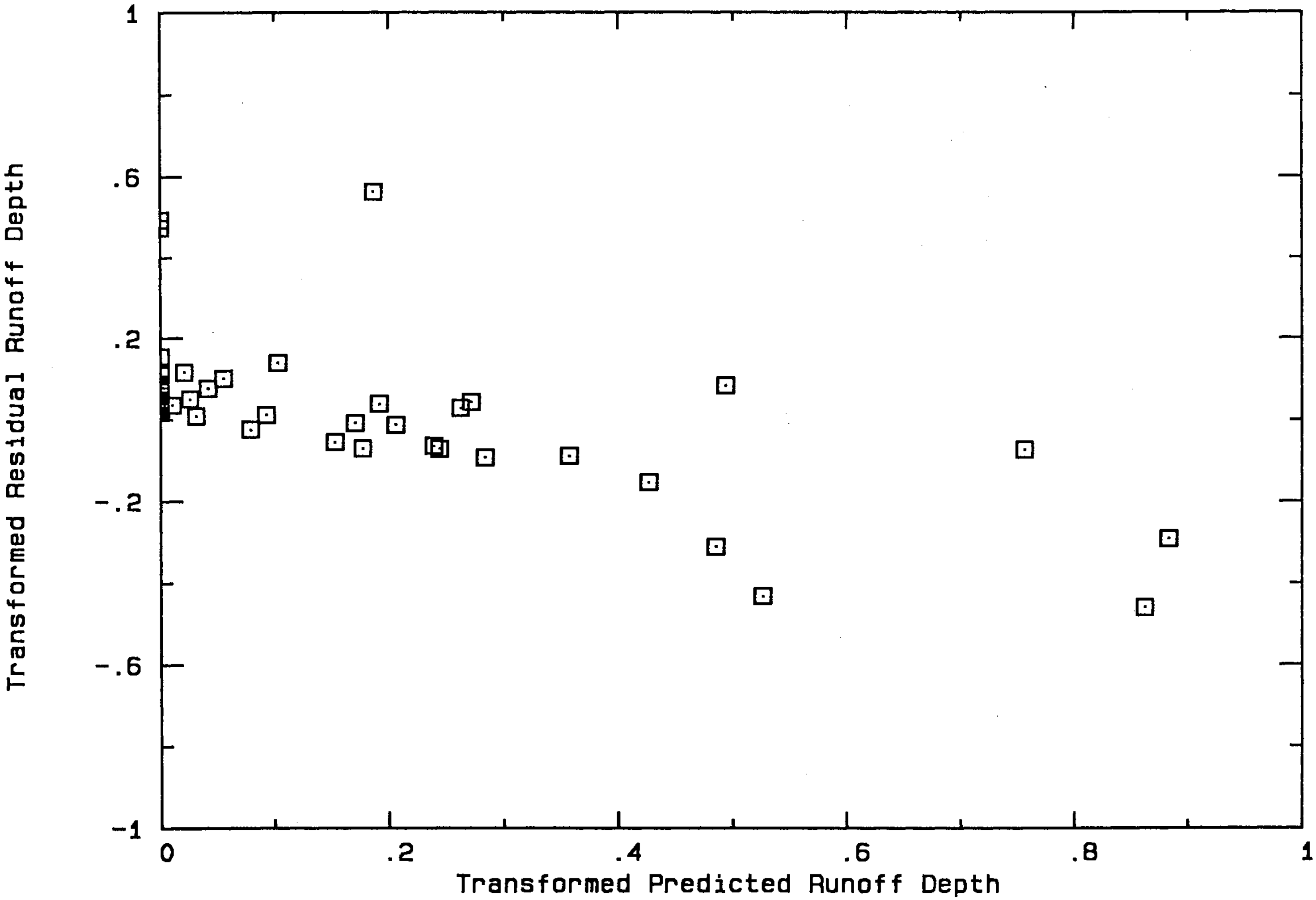

Figure 35. Transformed Runoff Depth Residual Plot for Watershed 5142, 2

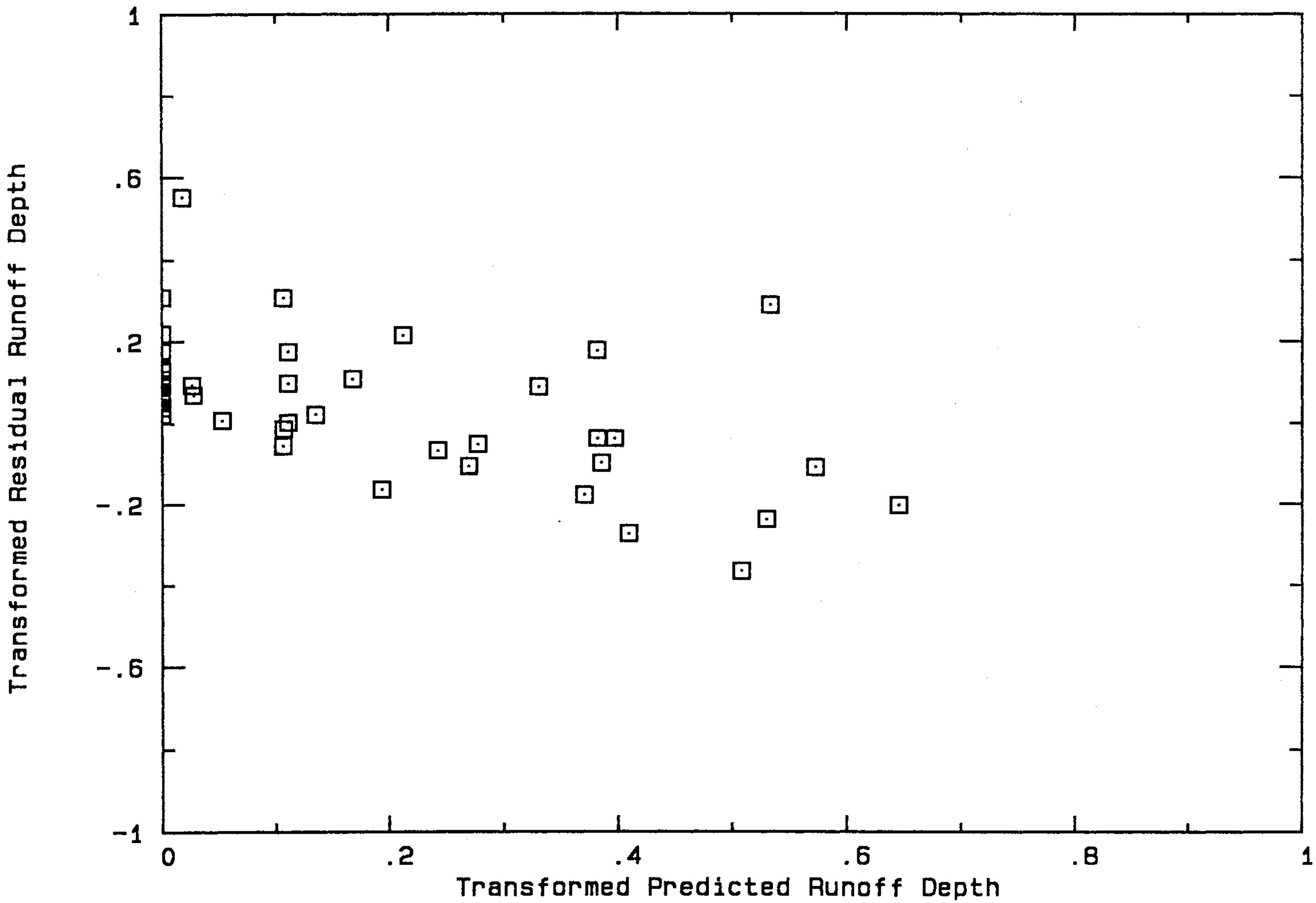

Figure 36. Transformed Runoff Depth Residual Plot for Watershed 611, 2

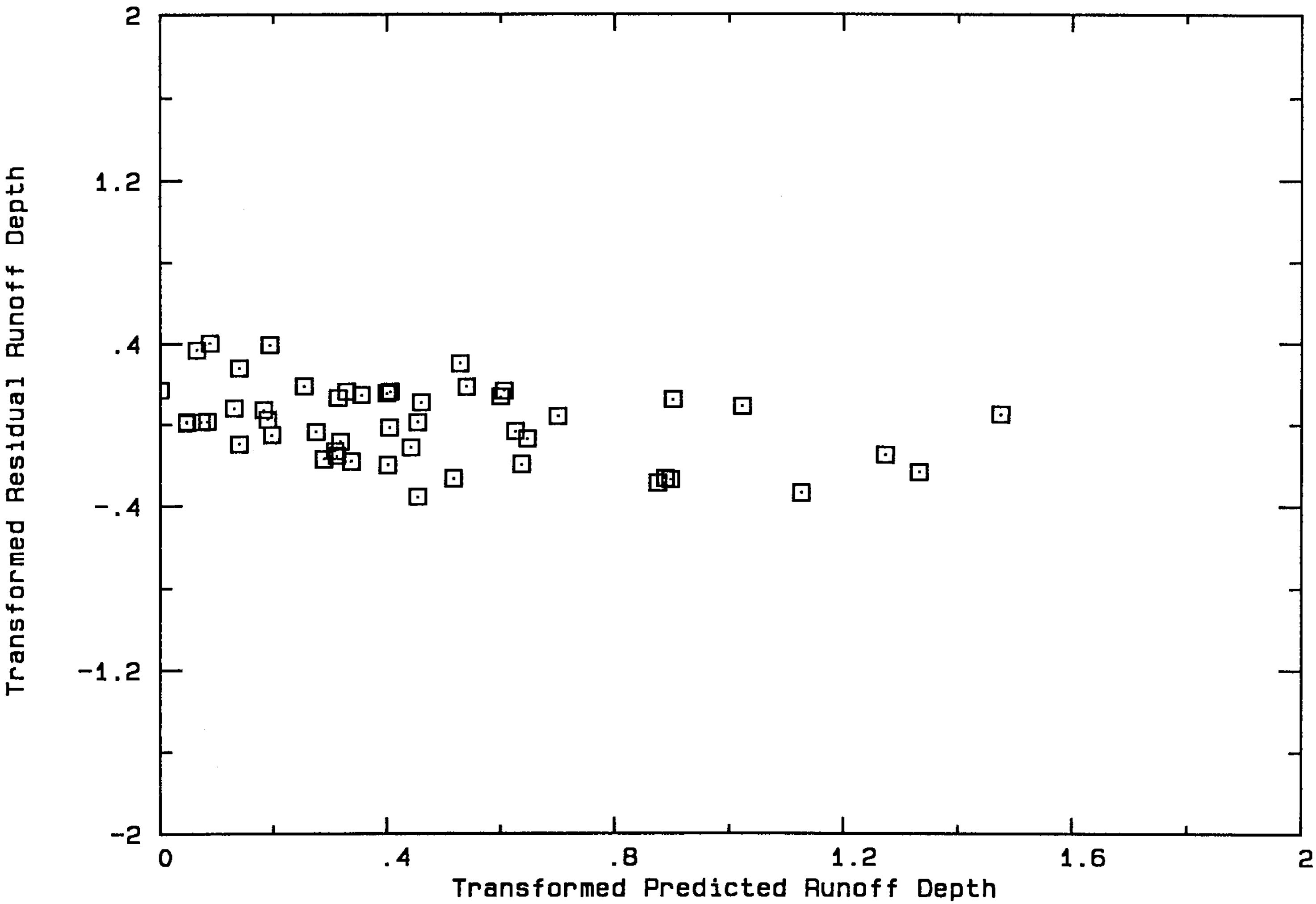

Figure 37. Transformed Runoff Depth Residual Plot for Watershed R7, 2

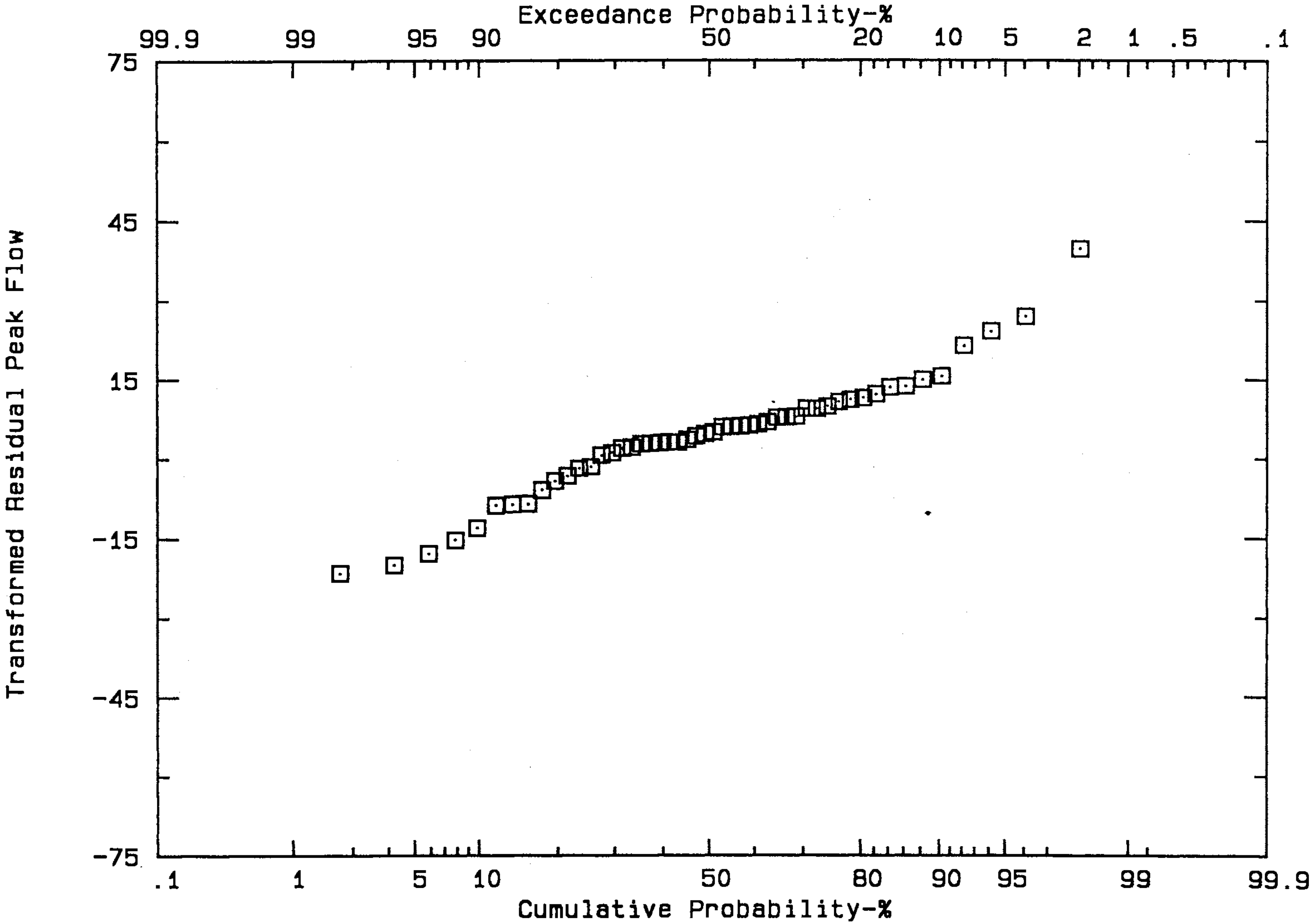

Figure 38. Probability Plot of Transformed Peak Flow Residuals for Watershed 511, 2

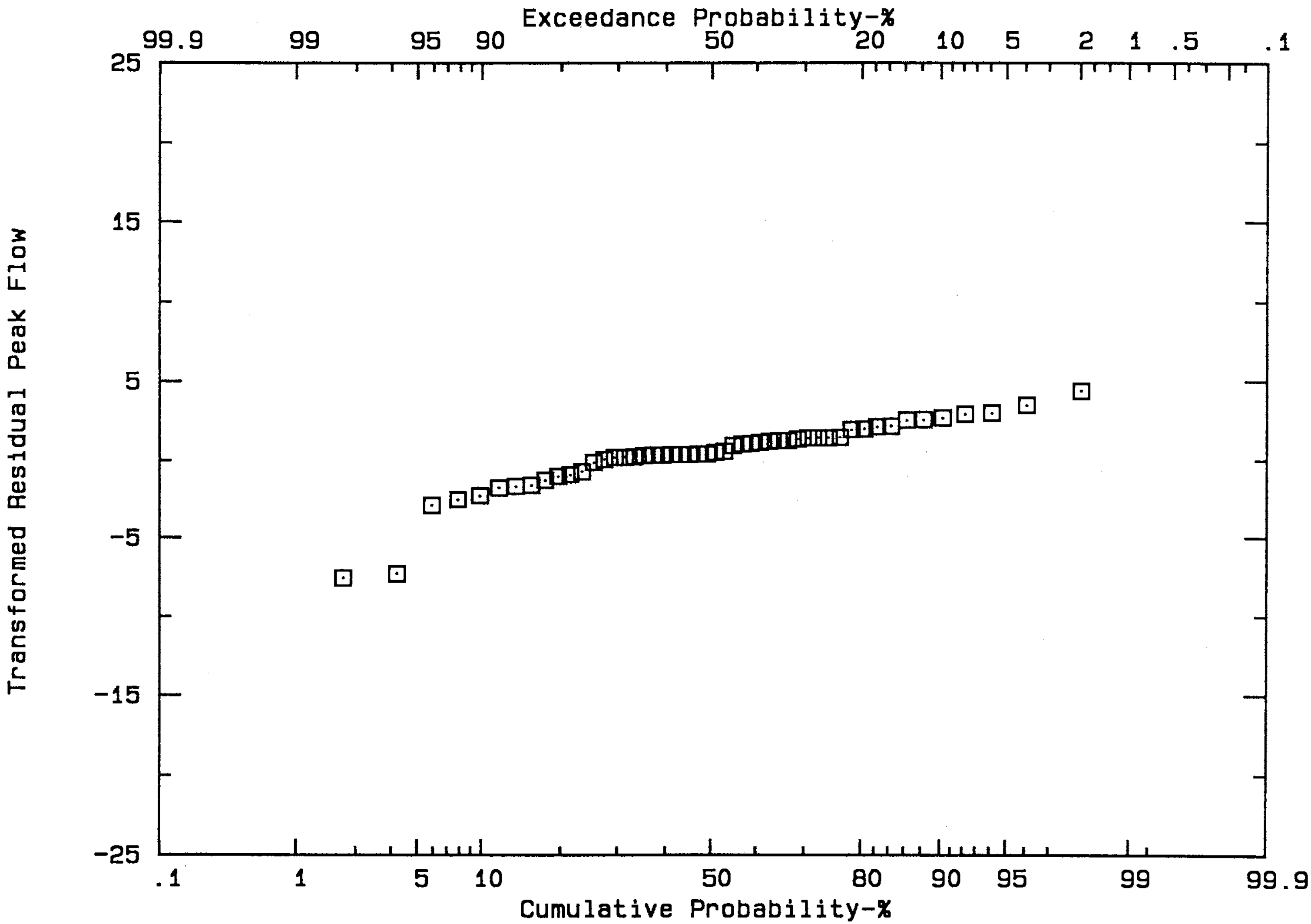

Figure 39. Probability Plot of Transformed Peak Flow Residuals for Watershed 5142, 2

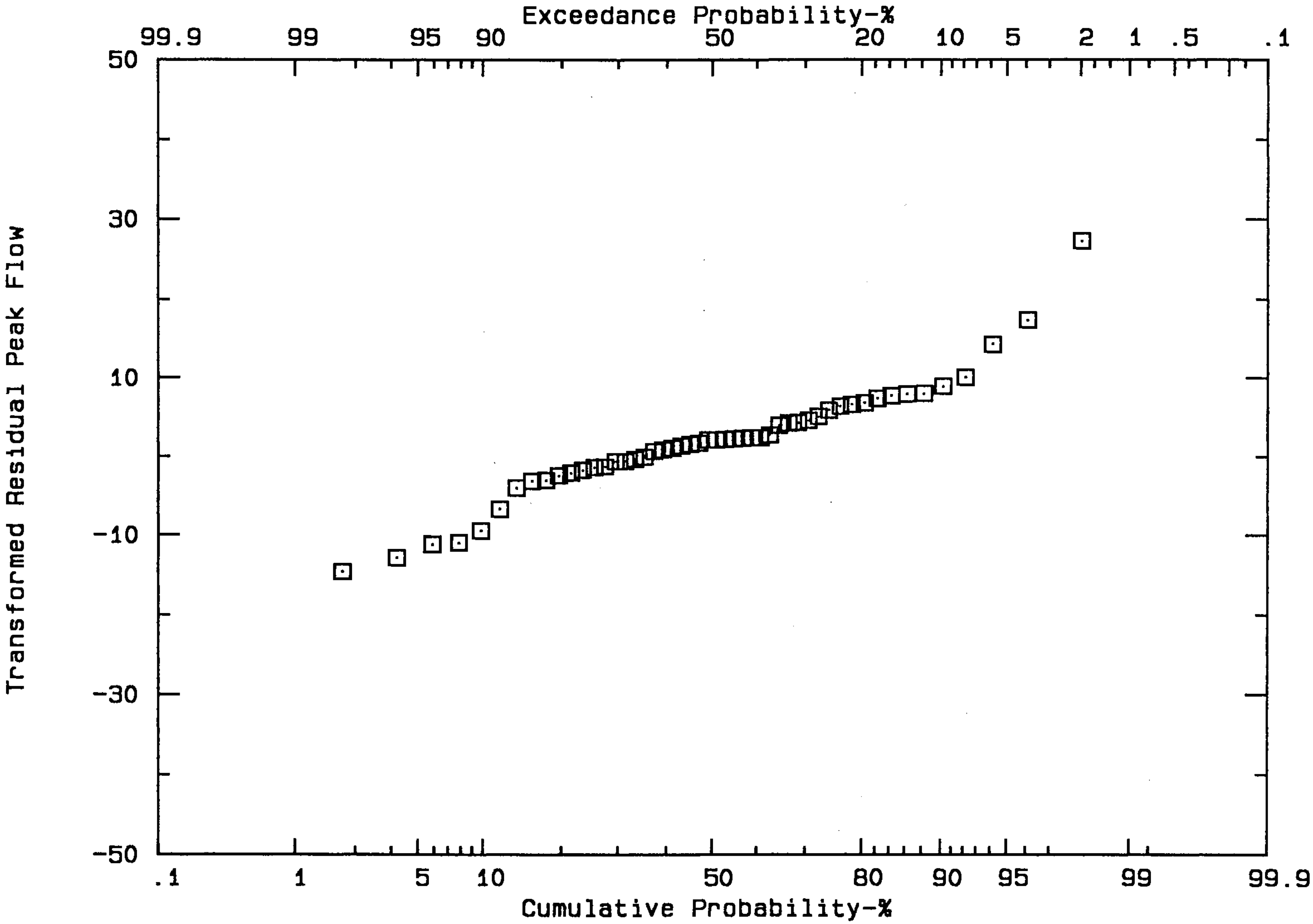

Figure 40. Probability Plot of Transformed Peak Flow Residuals for Watershed 611, 2

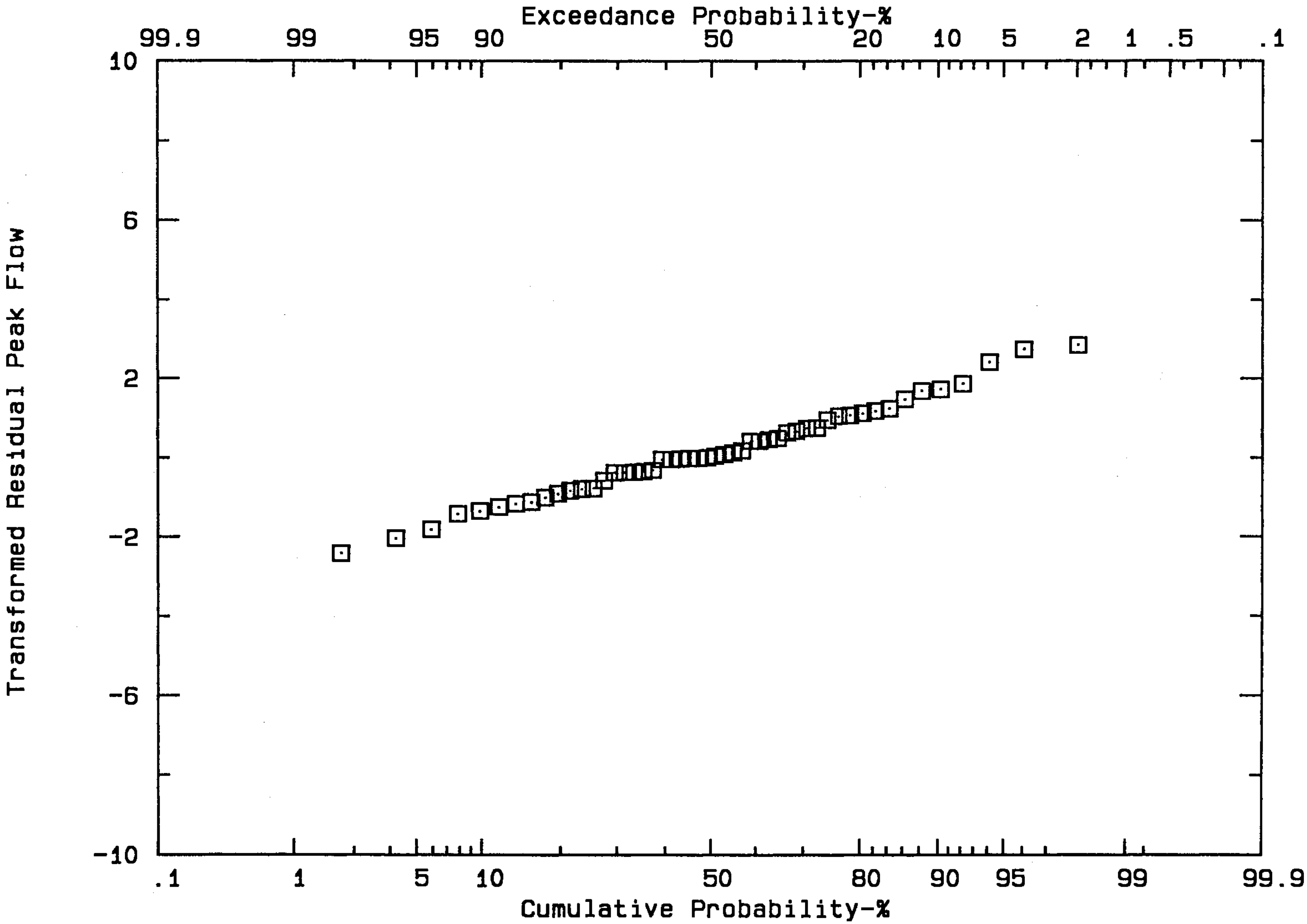

Figure 41. Probability Plot of Transformed Peak Flow Residuals for Watershed R7, 2

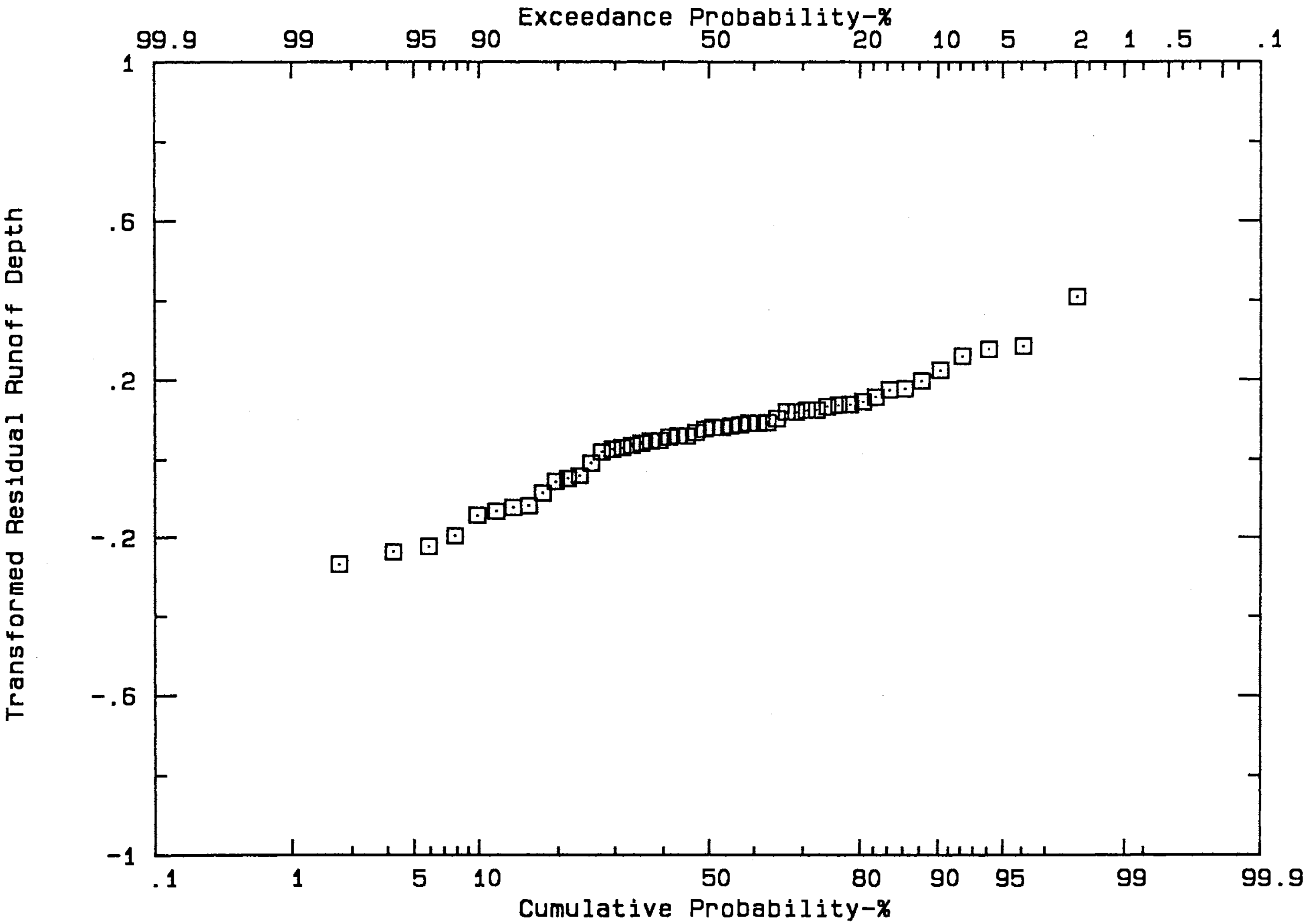

Figure 42. Probability Plot of Transformed Runoff Depth Residuals for Watershed 511, 2

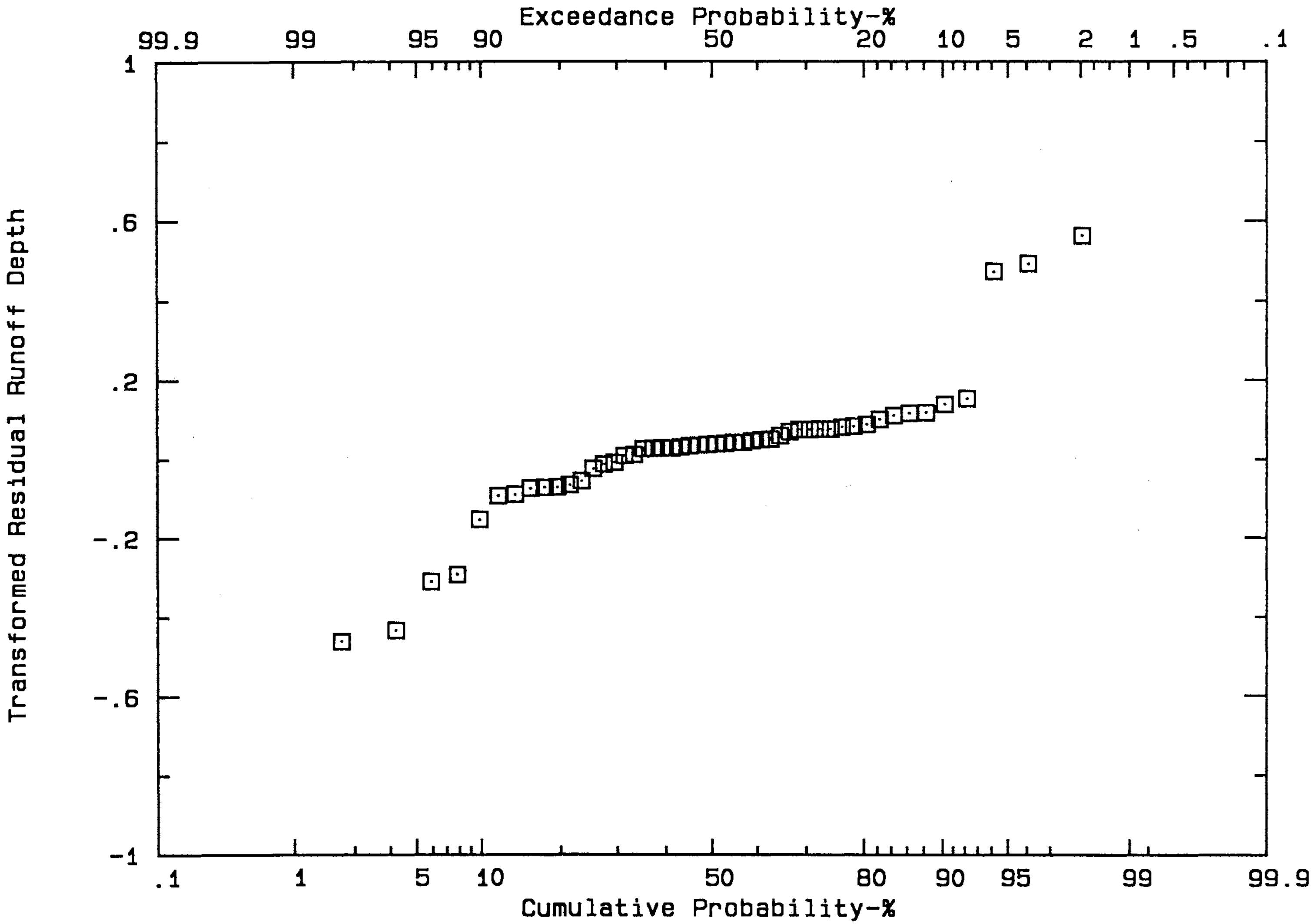

Figure 43. Probability Plot of Transformed Runoff Depth Residuals for Watershed 5142, 2

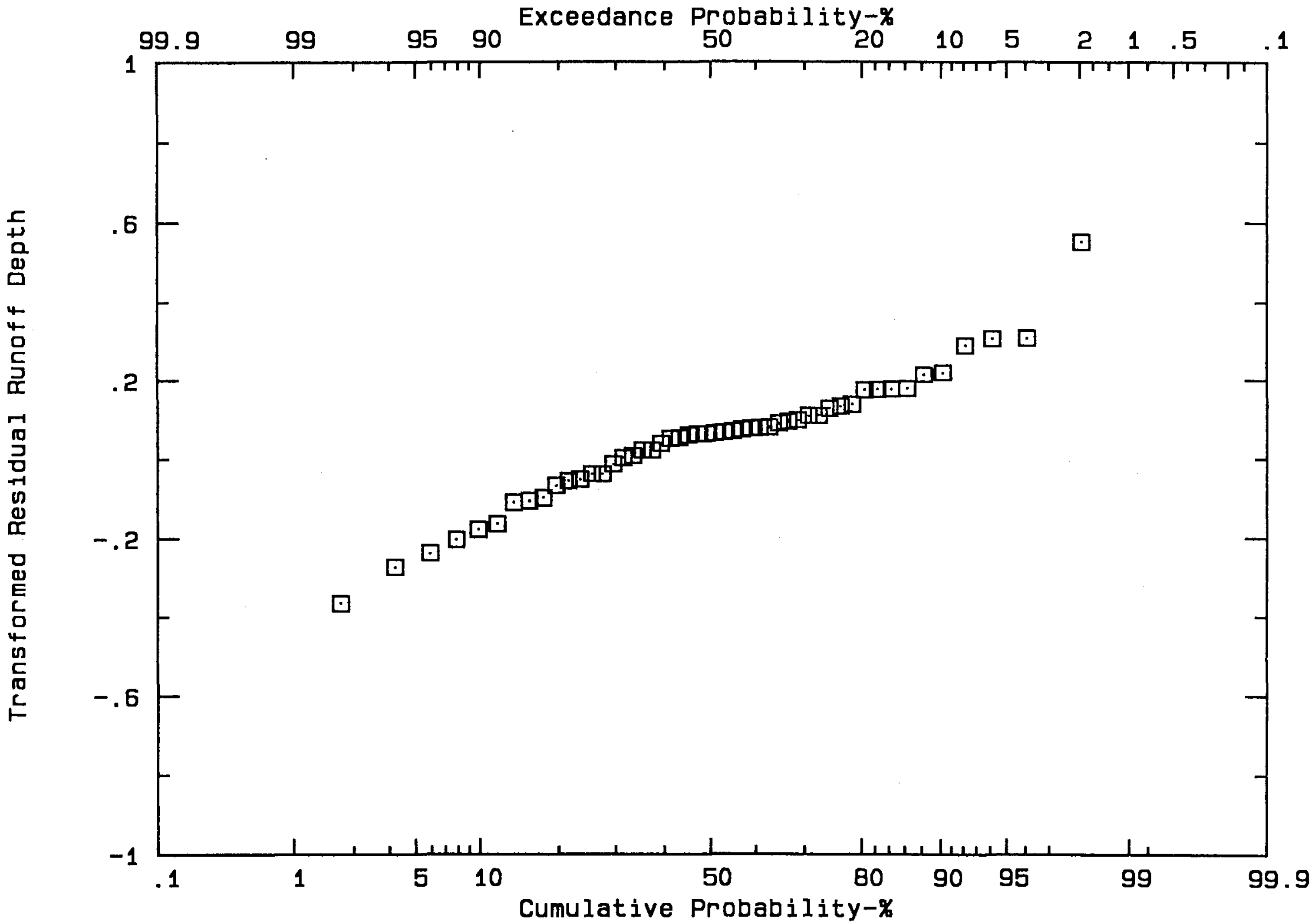

Figure 44. Probability Plot of Transformed Runoff Depth Residuals for Watershed 611, 2

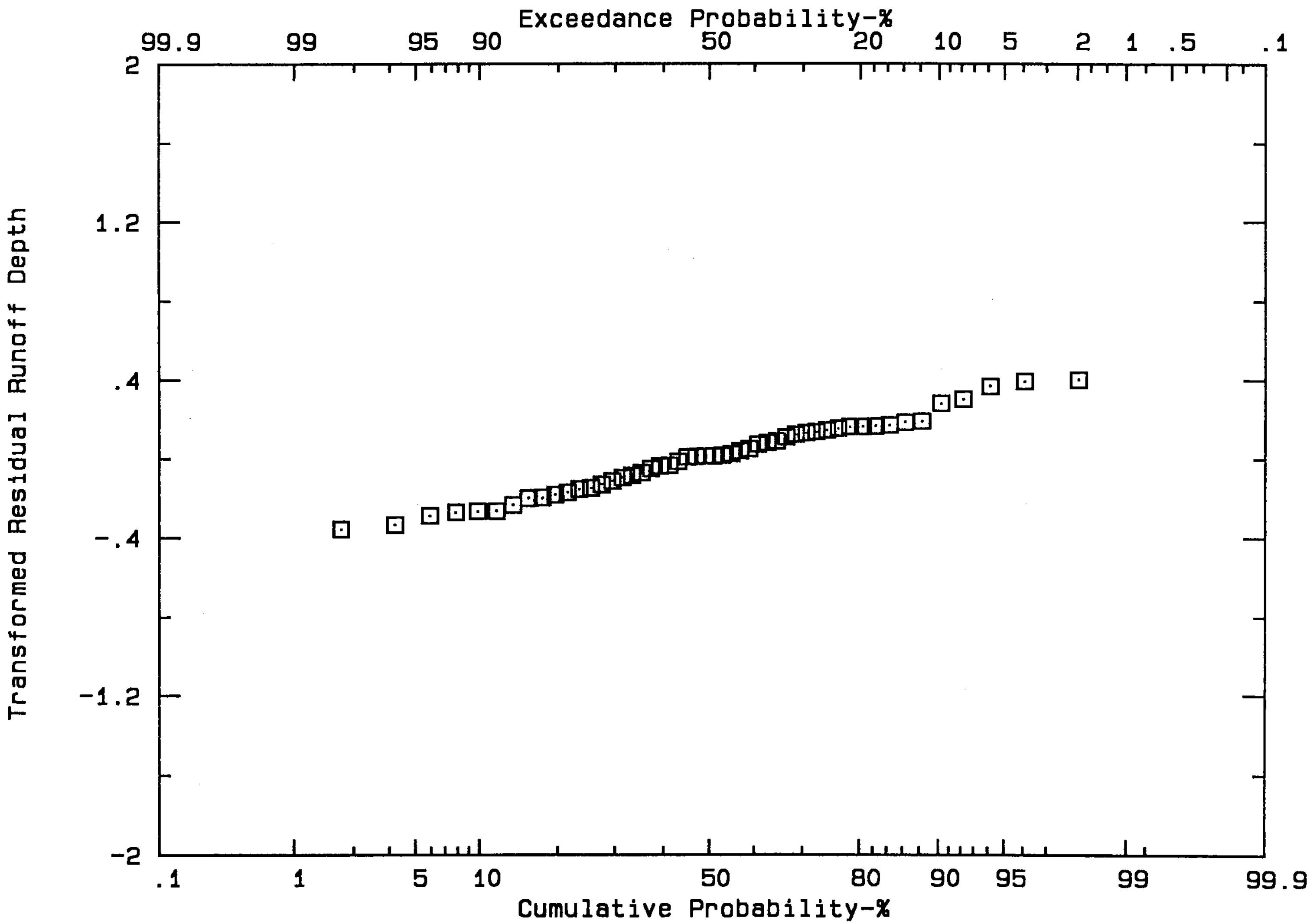

Figure 45. Probability Plot of Transformed Runoff Depth Residuals for Watershed R7, 2

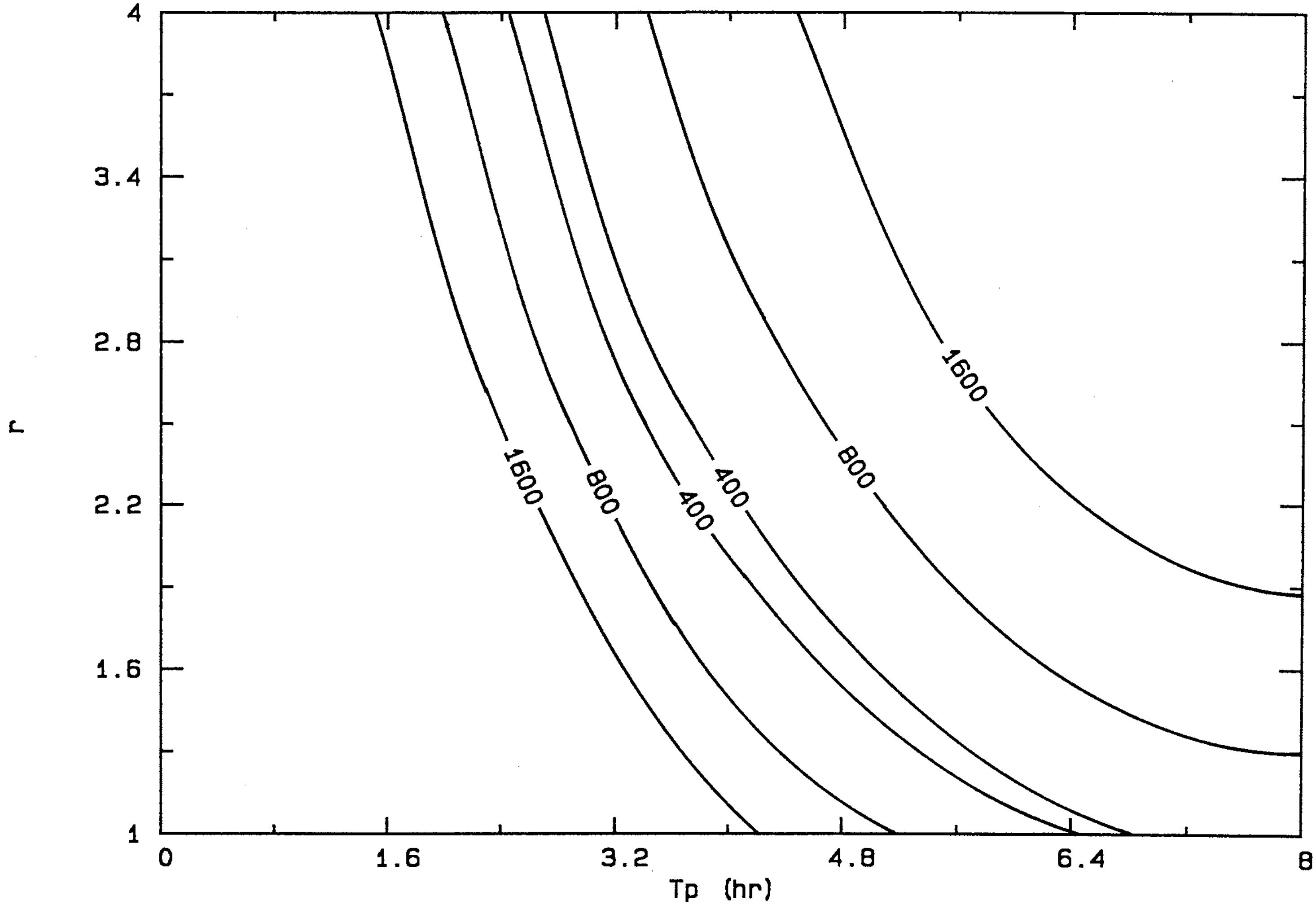

Figure 46. Optimization Criterion for Watershed 511

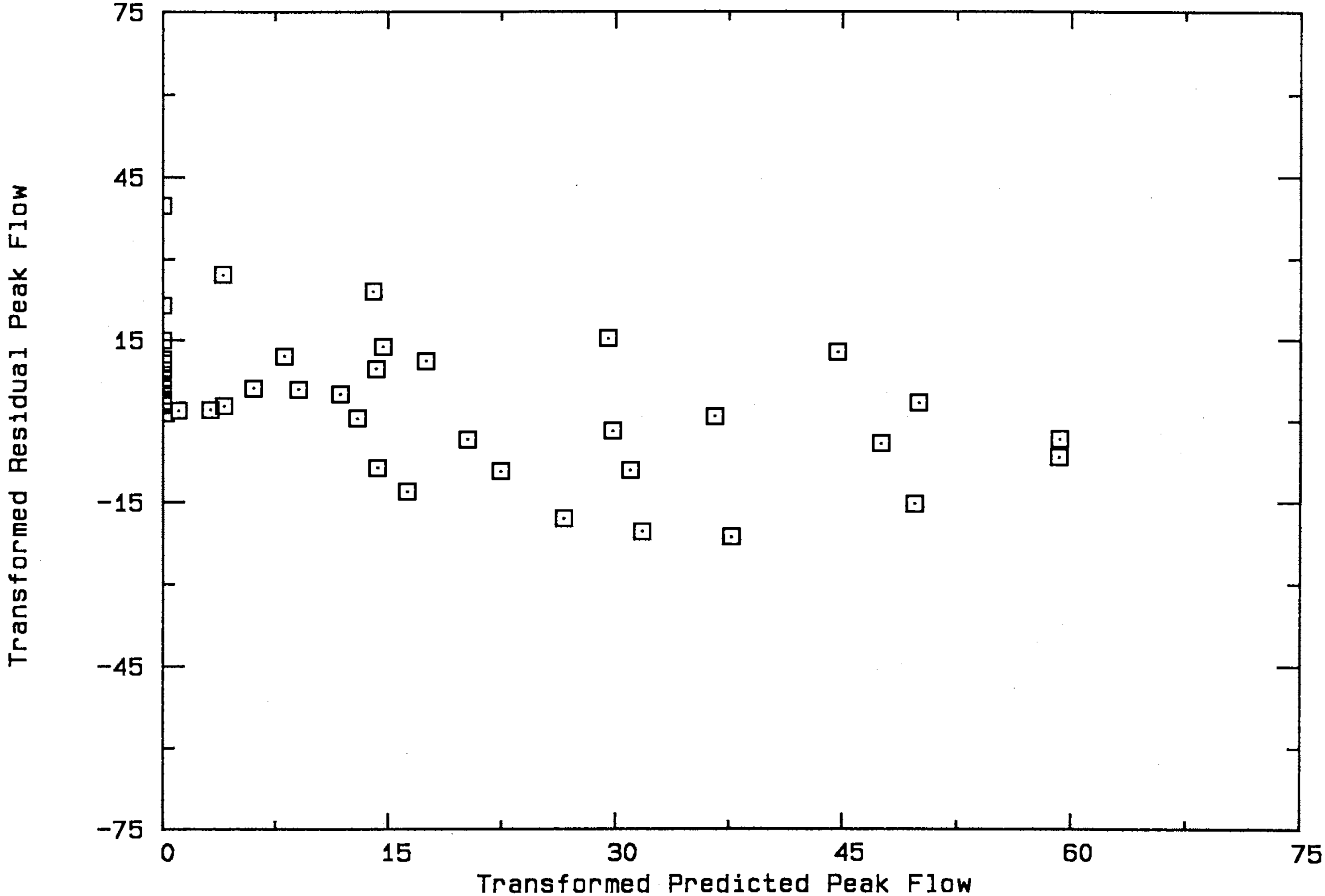

Figure 47. Transformed Peak Flow Residual Plot for Watershed 511, 3

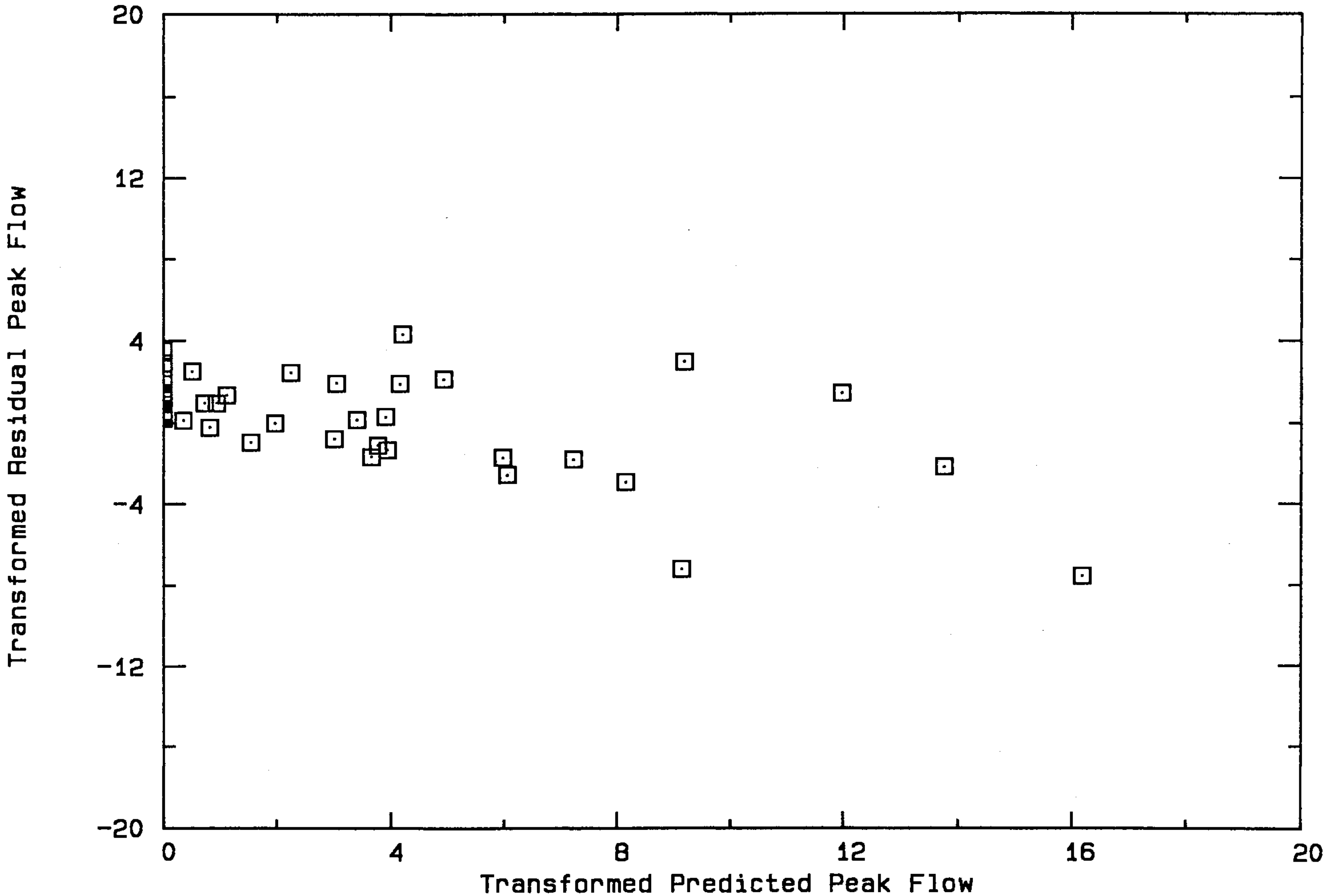

Figure 48. Transformed Peak Flow Residual Plot for Watershed 5142, 3

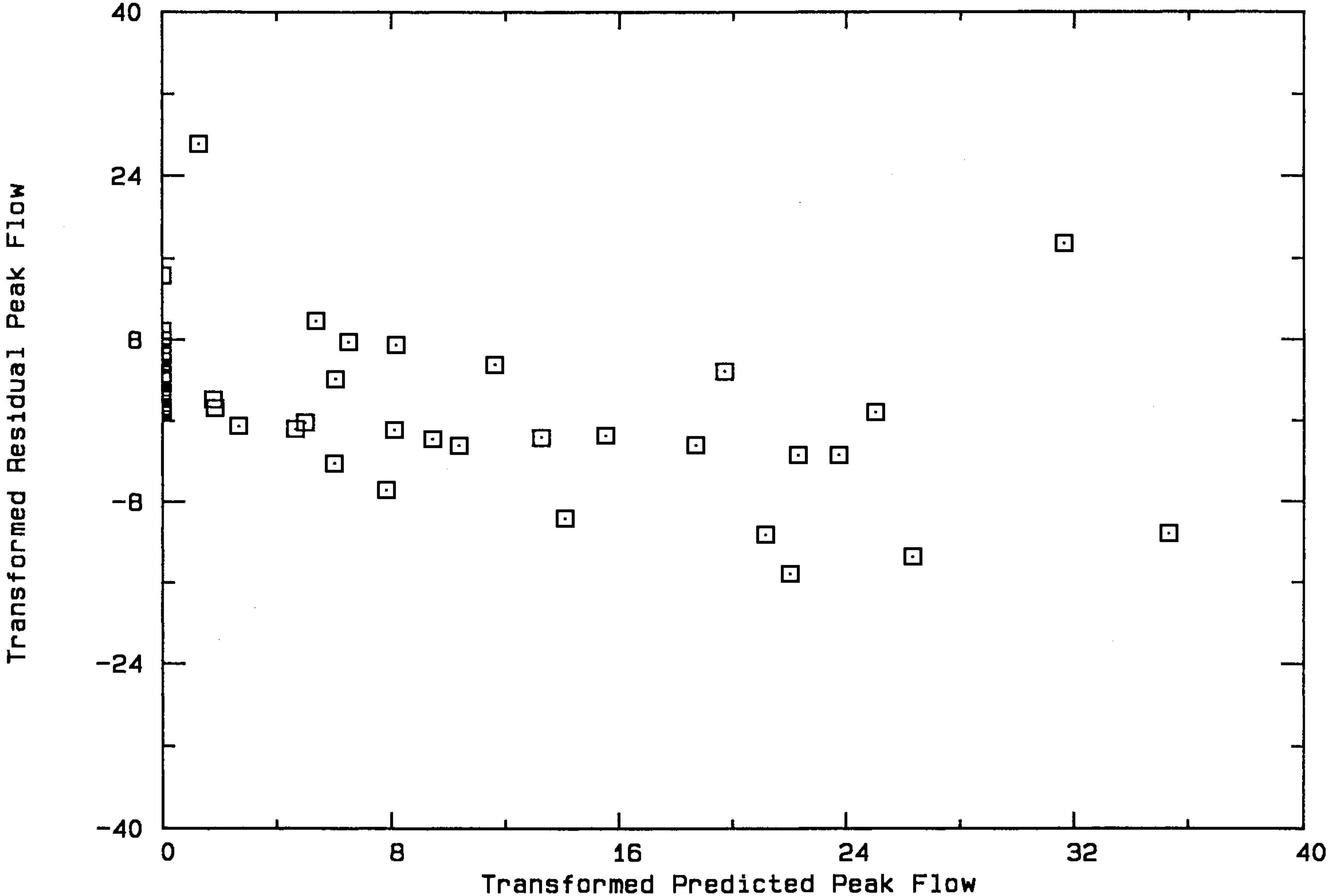

Figure 49. Transformed Peak Flow Residual Plot for Watershed 611, 3

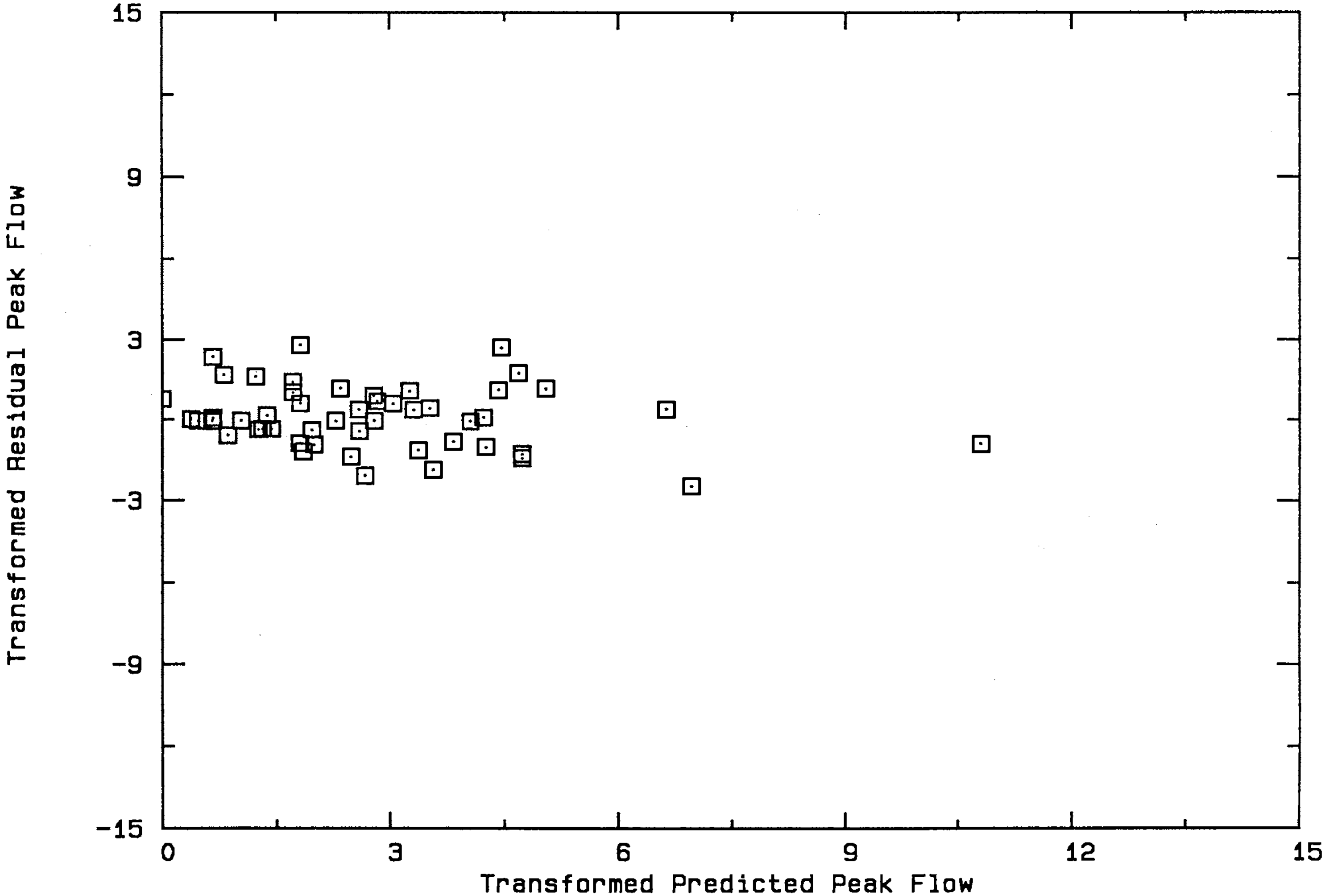

Figure 50. Transformed Peak Flow Residual Plot for Watershed R7, 3

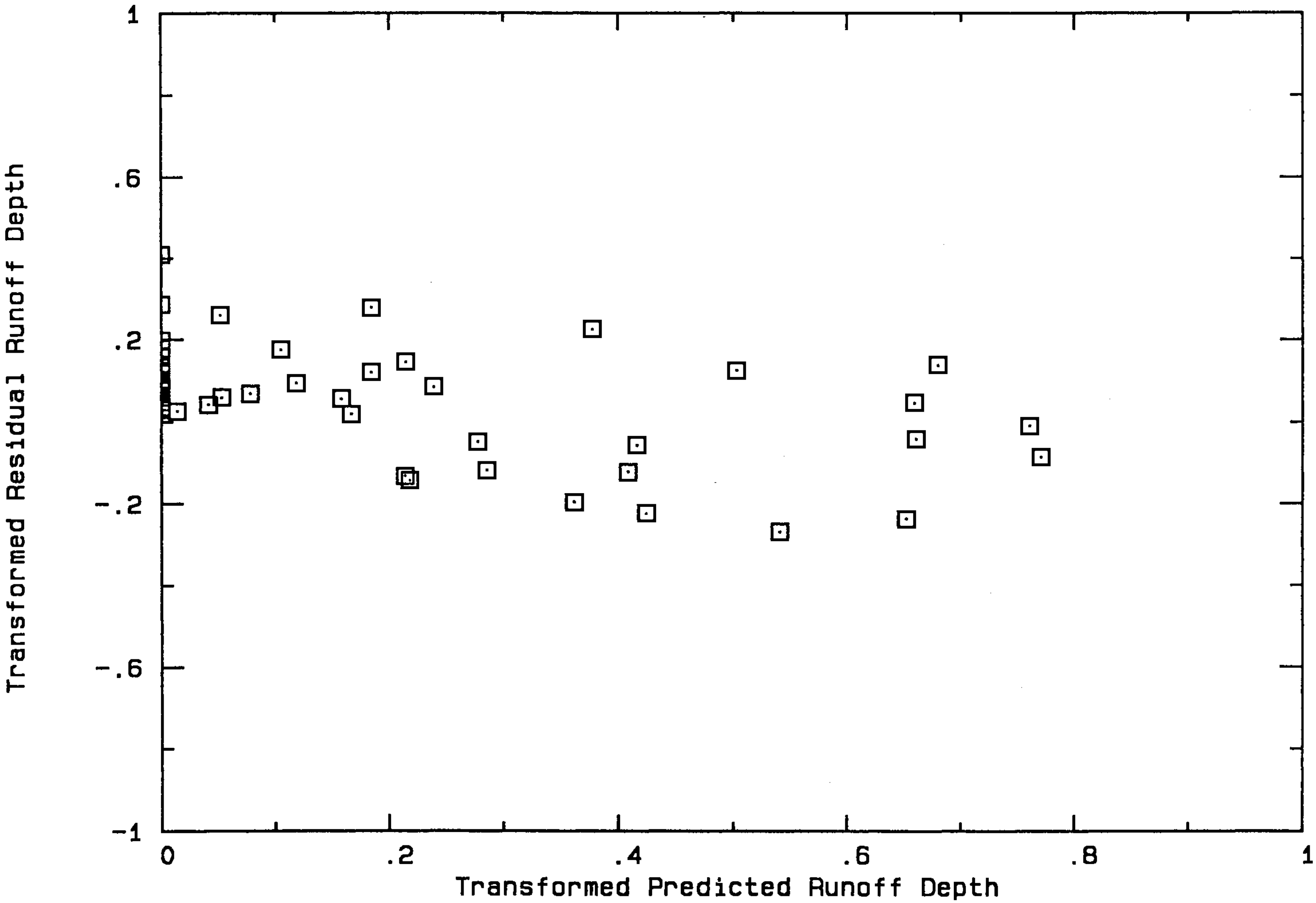

Figure 51. Transformed Runoff Depth Residual Plot for Watershed 511, 3

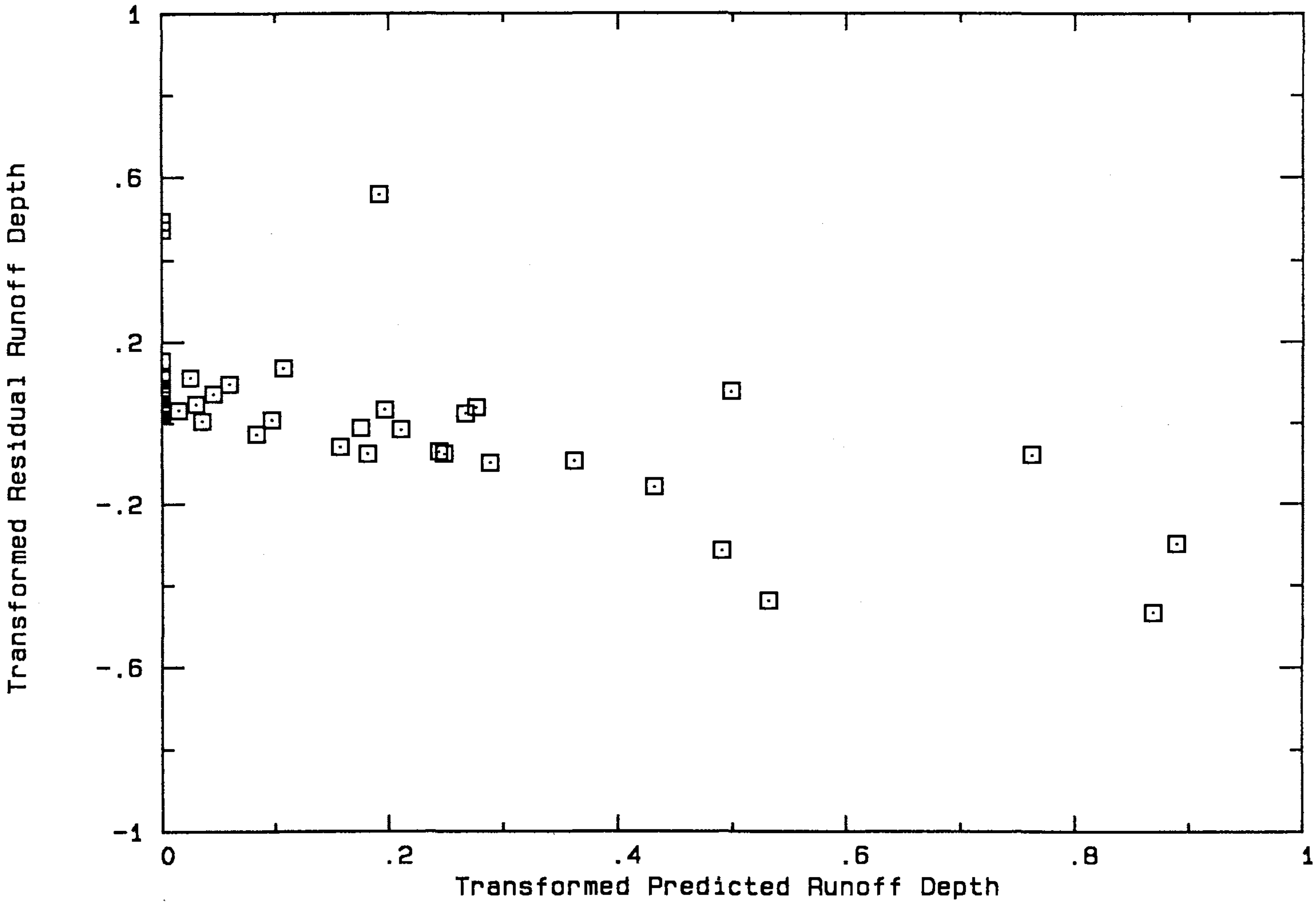

Figure 52. Transformed Runoff Depth Residual Plot for Watershed 5142, 3

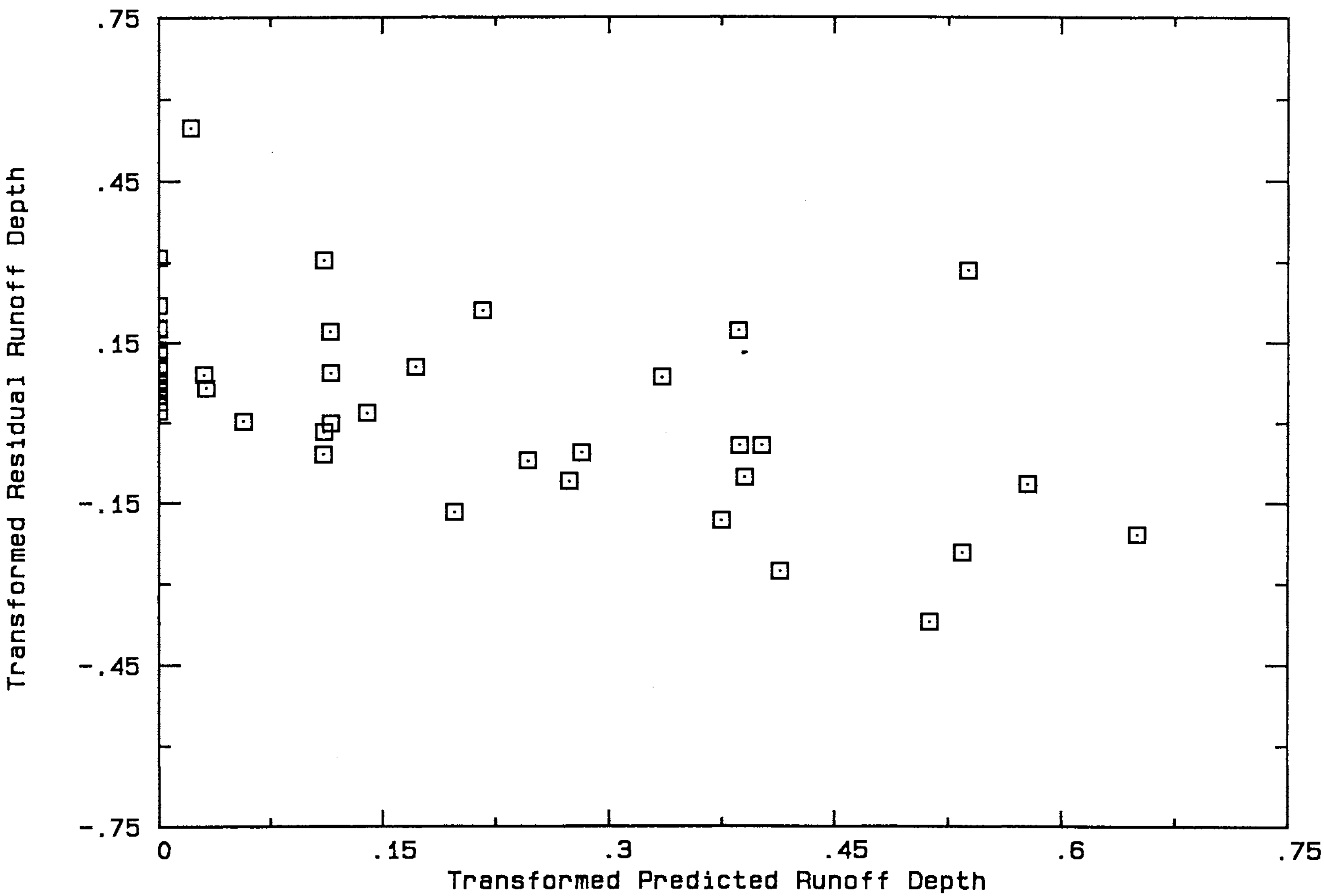

Figure 53. Transformed Runoff Depth Residual Plot for Watershed 611, 3

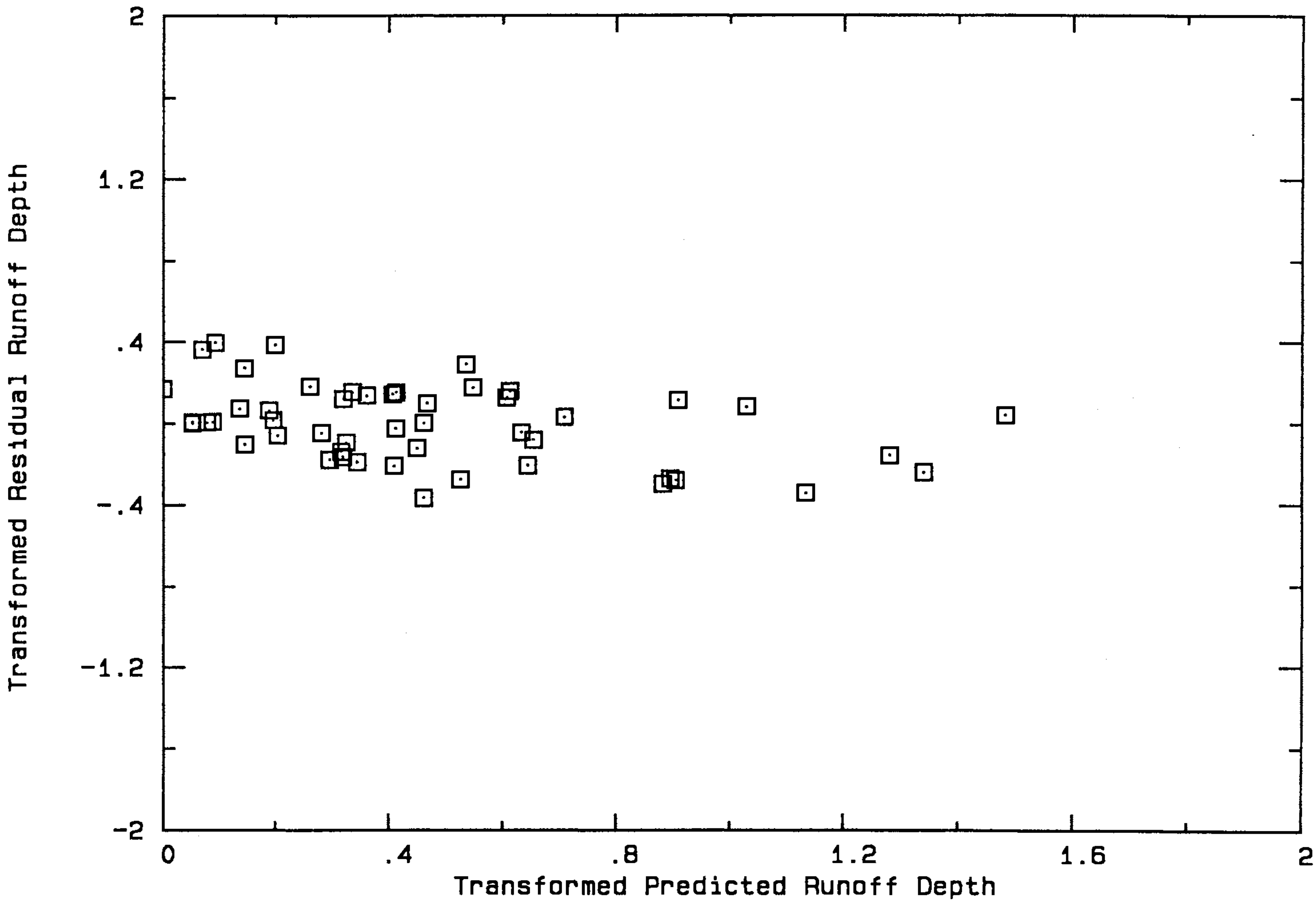

Figure 54. Transformed Runoff Depth Residual Plot for Watershed R7, 3

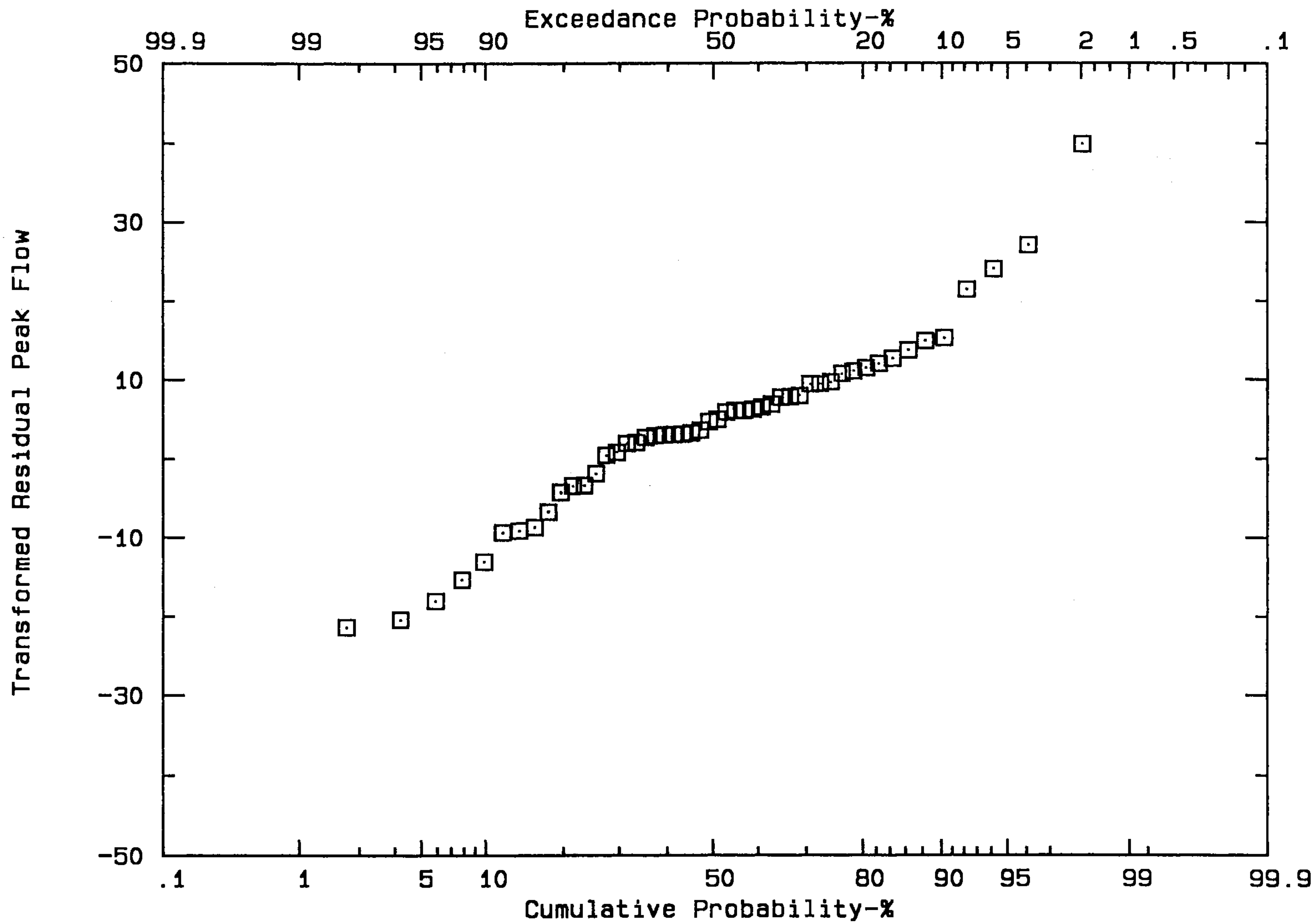

Figure 55. Probability Plot of Transformed Peak Flow Residuals for Watershed 511, 3

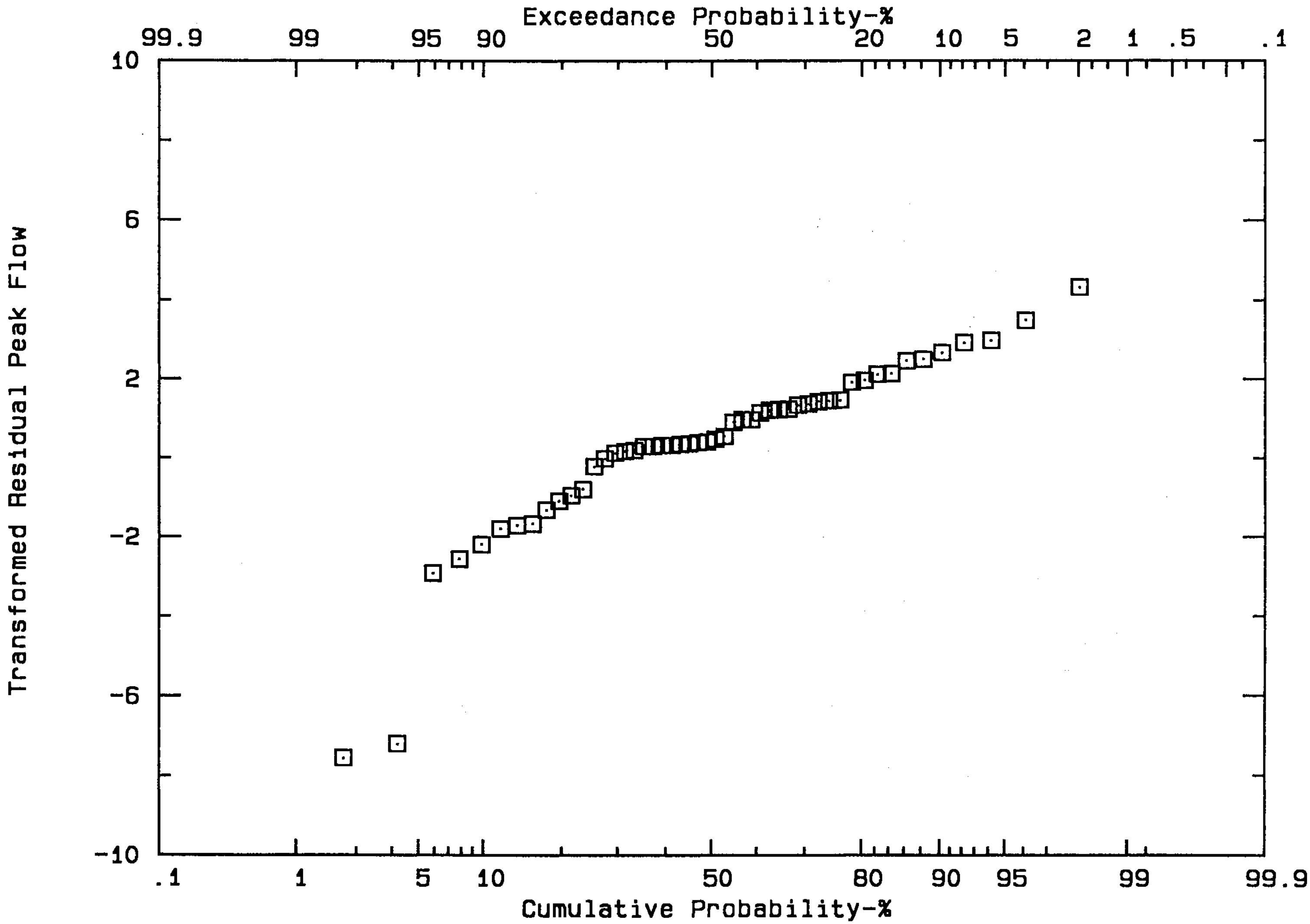

Figure 56. Probability Plot of Transformed Peak Flow Residuals for Watershed 5142, 3

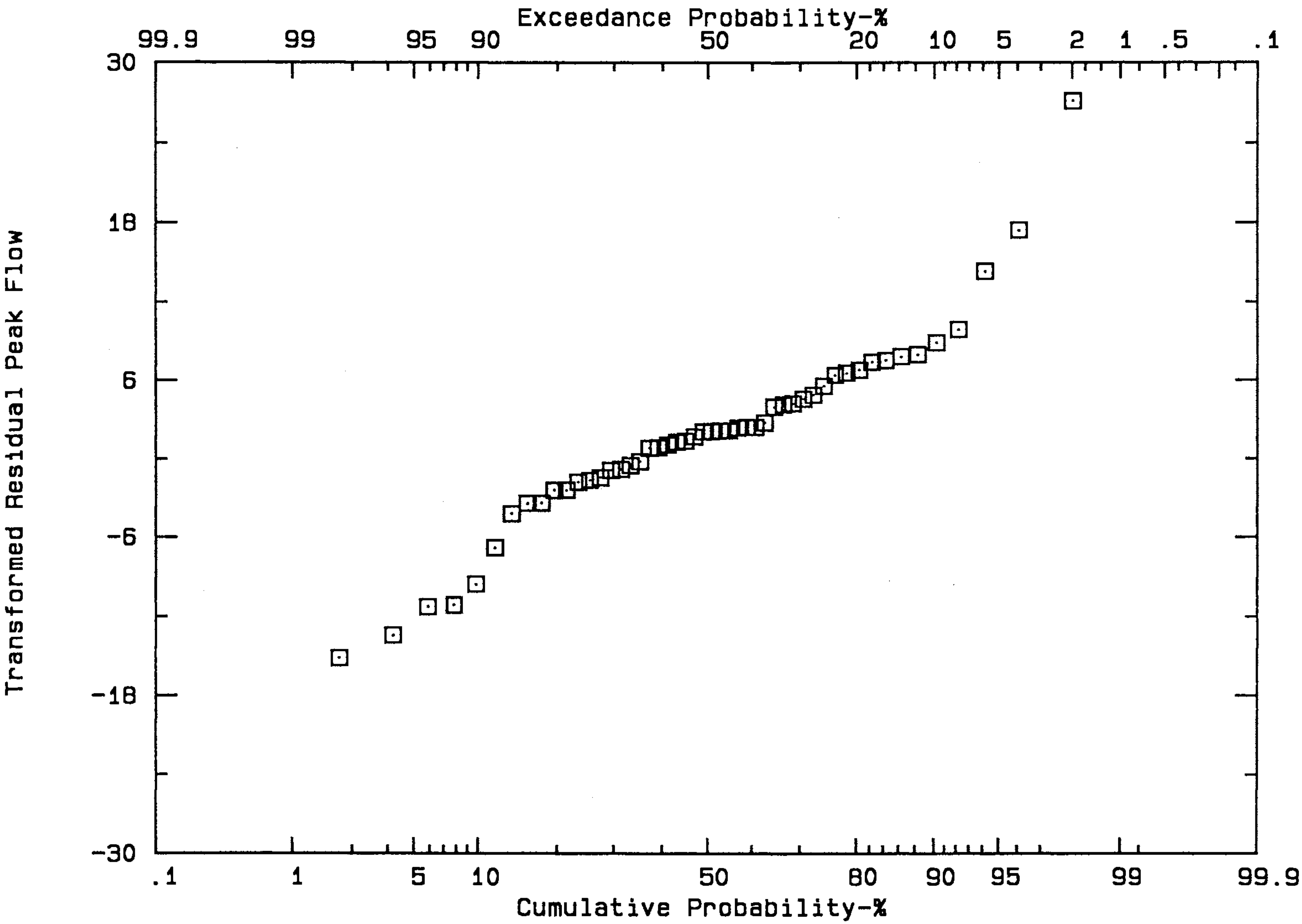

Figure 57. Probability Plot of Transformed Peak Flow Residuals for Watershed 611, 3

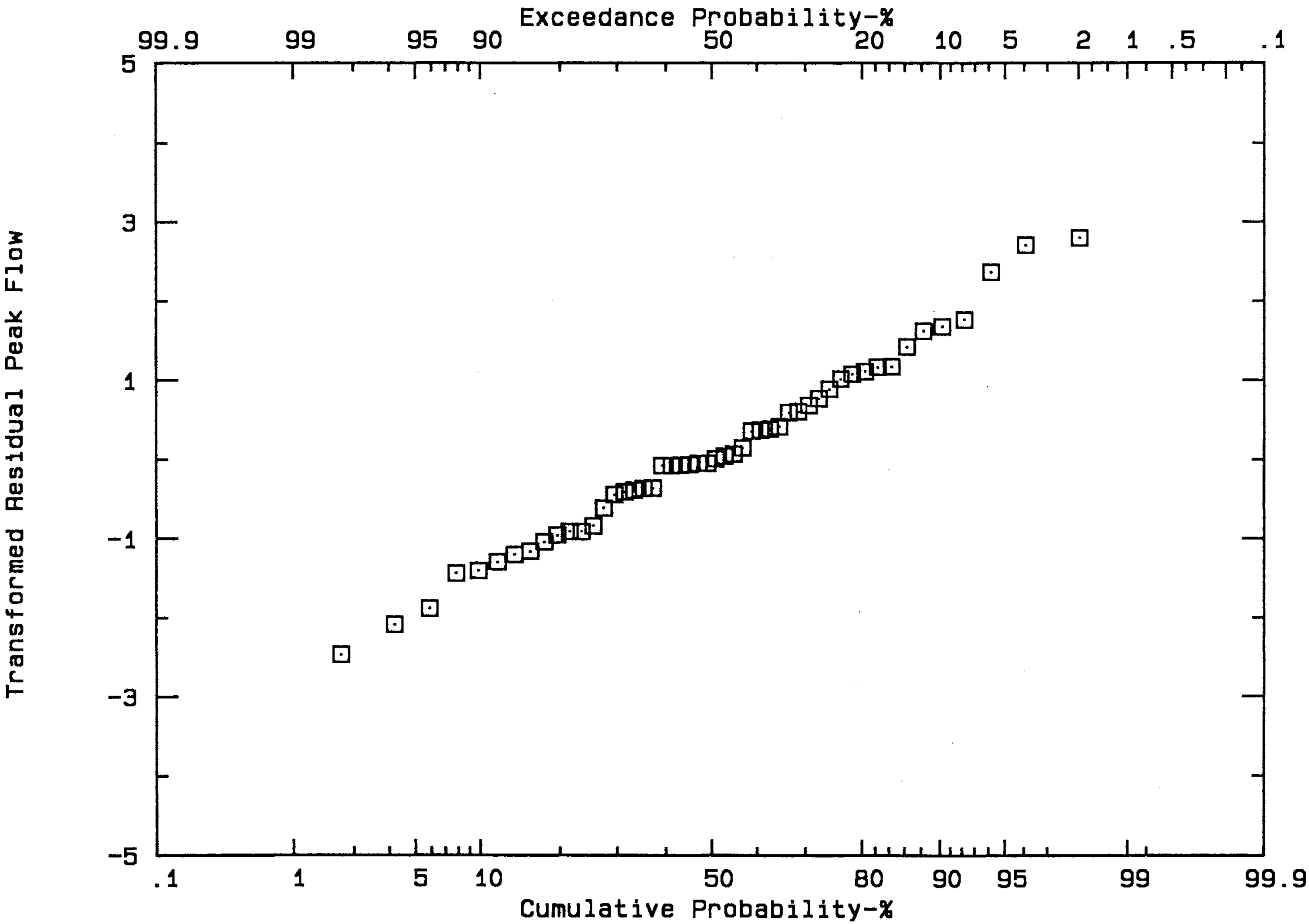

Figure 58. Probability Plot of Transformed Peak Flow Residuals for Watershed R7, 3

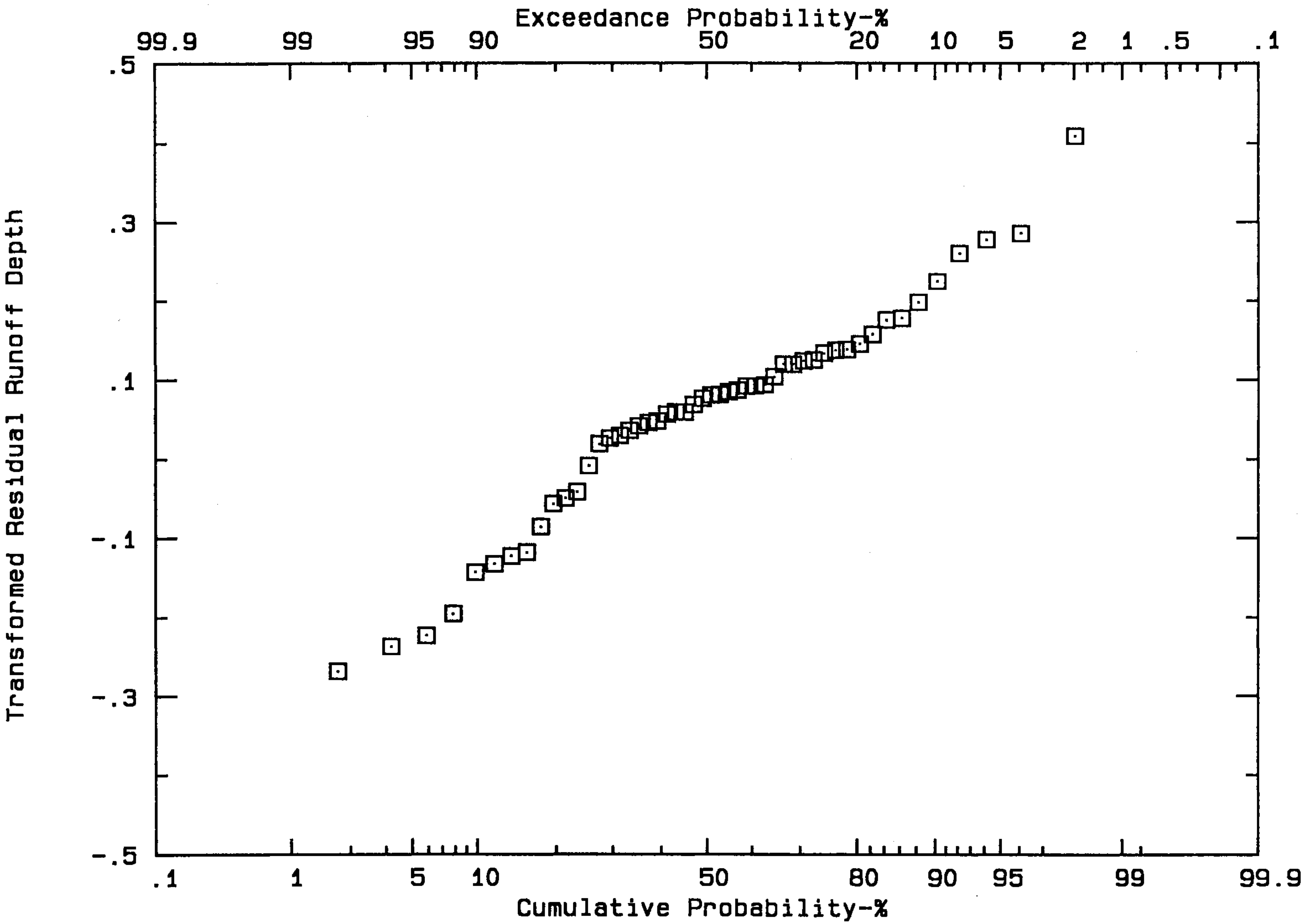

Figure 59. Probability Plot of Transformed Runoff Depth Residuals for Watershed 511, 3

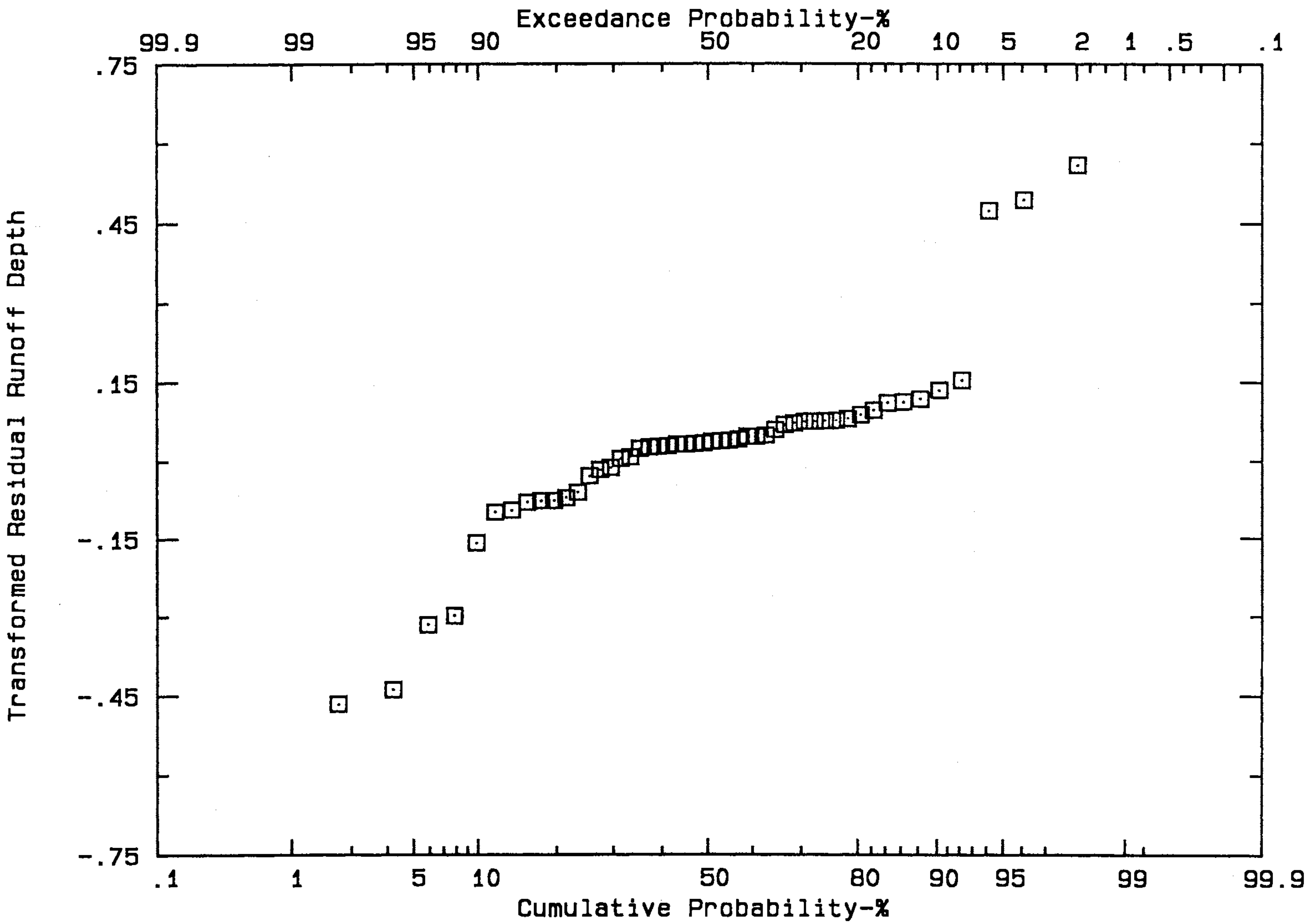

Figure 60. Probability Plot of Transformed Runoff Depth Residuals for Watershed 5142, 3

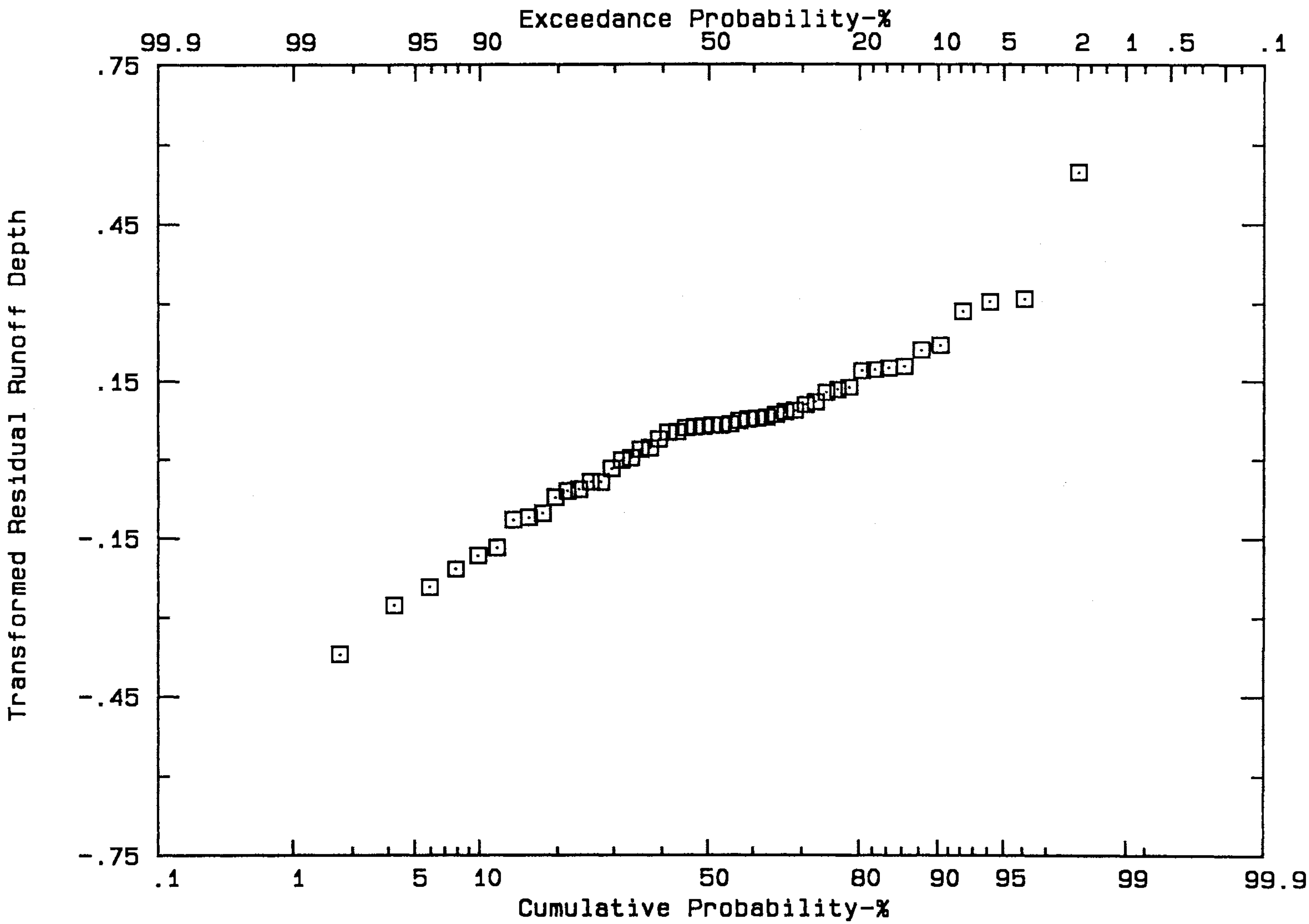

Figure 61. Probability Plot of Transformed Runoff Depth Residuals for Watershed 611, 3

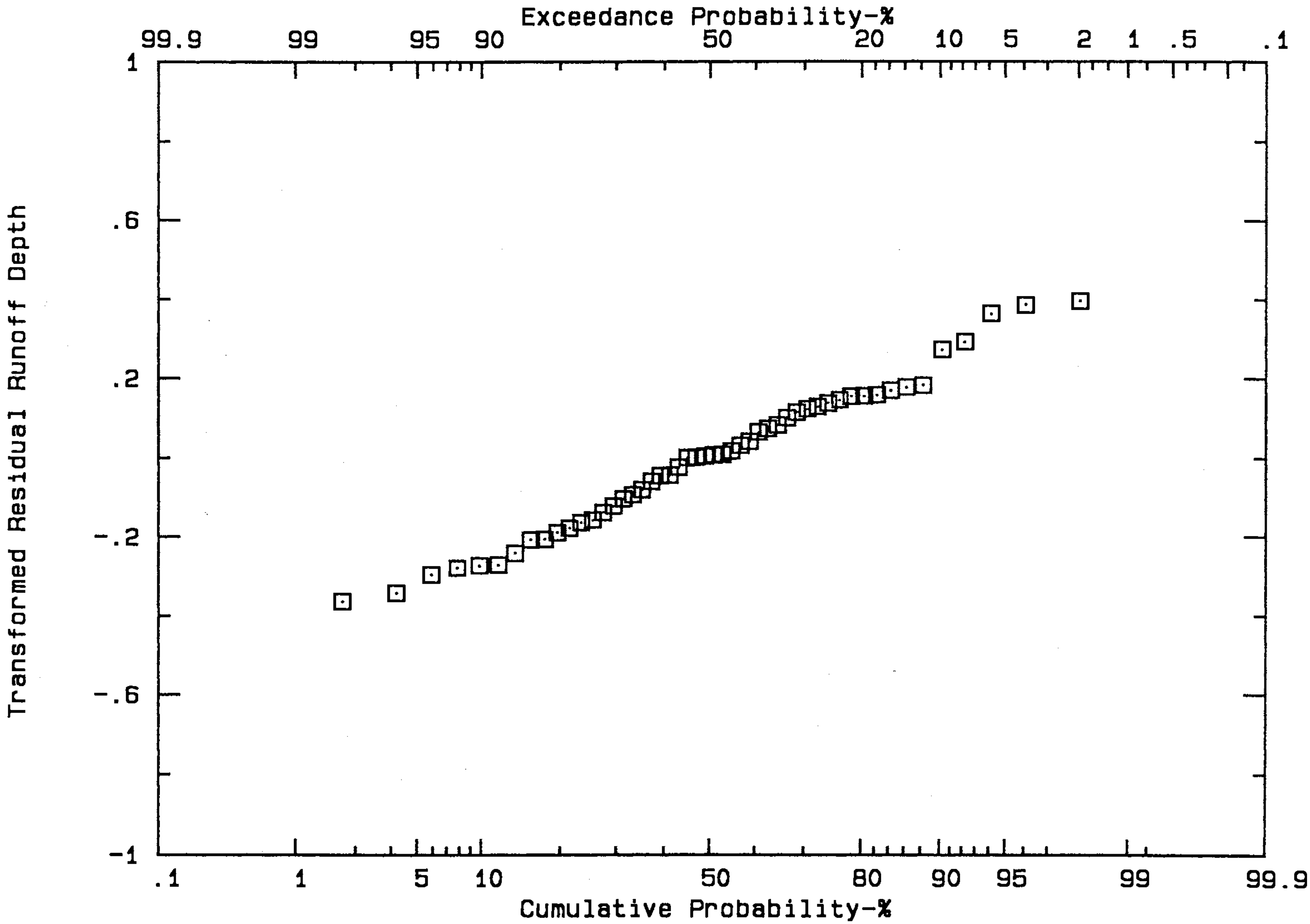

Figure 62. Probability Plot of Transformed Runoff Depth Residuals for Watershed R7, 3

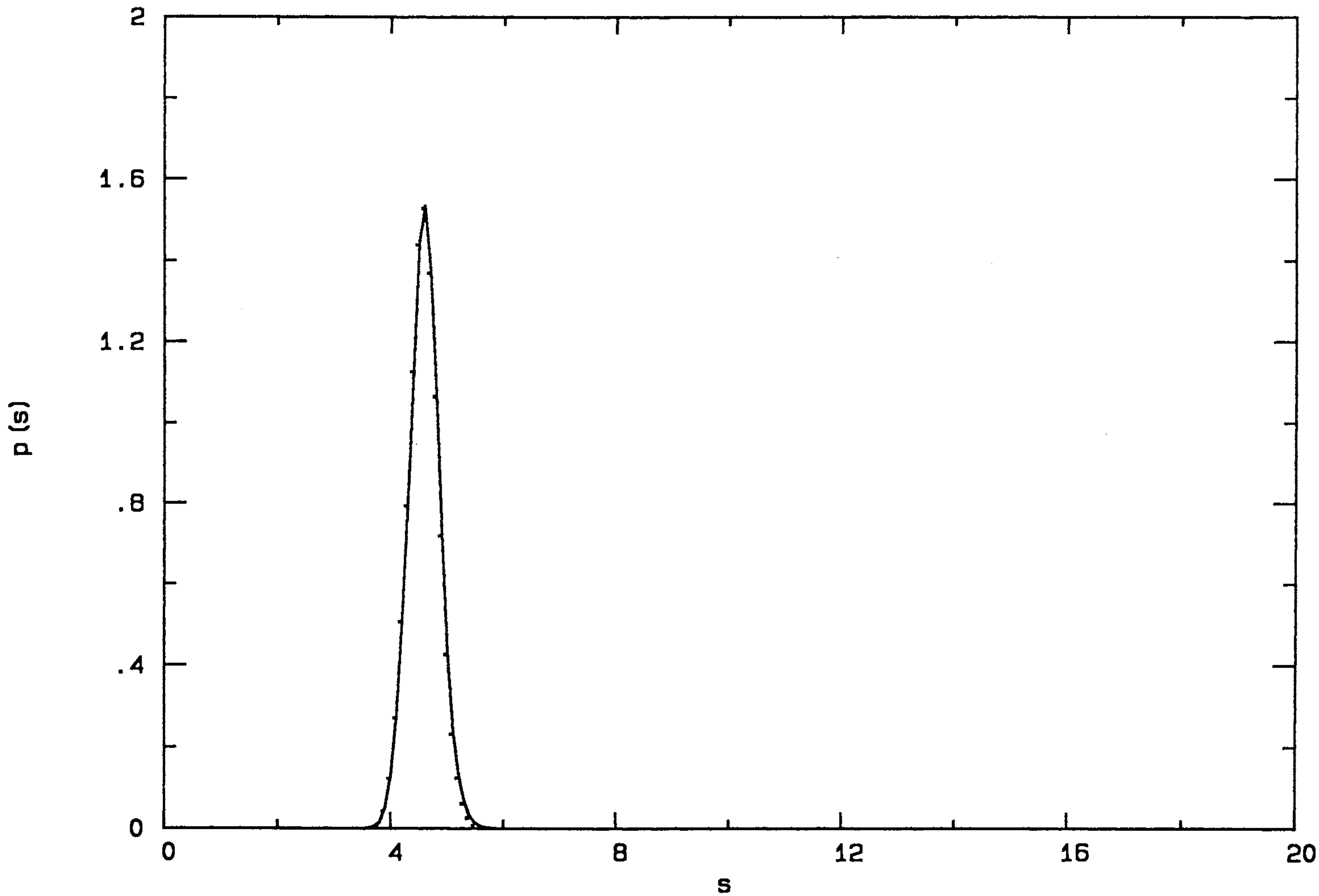

Figure 63. Probability Density Function of S for Watershed 511

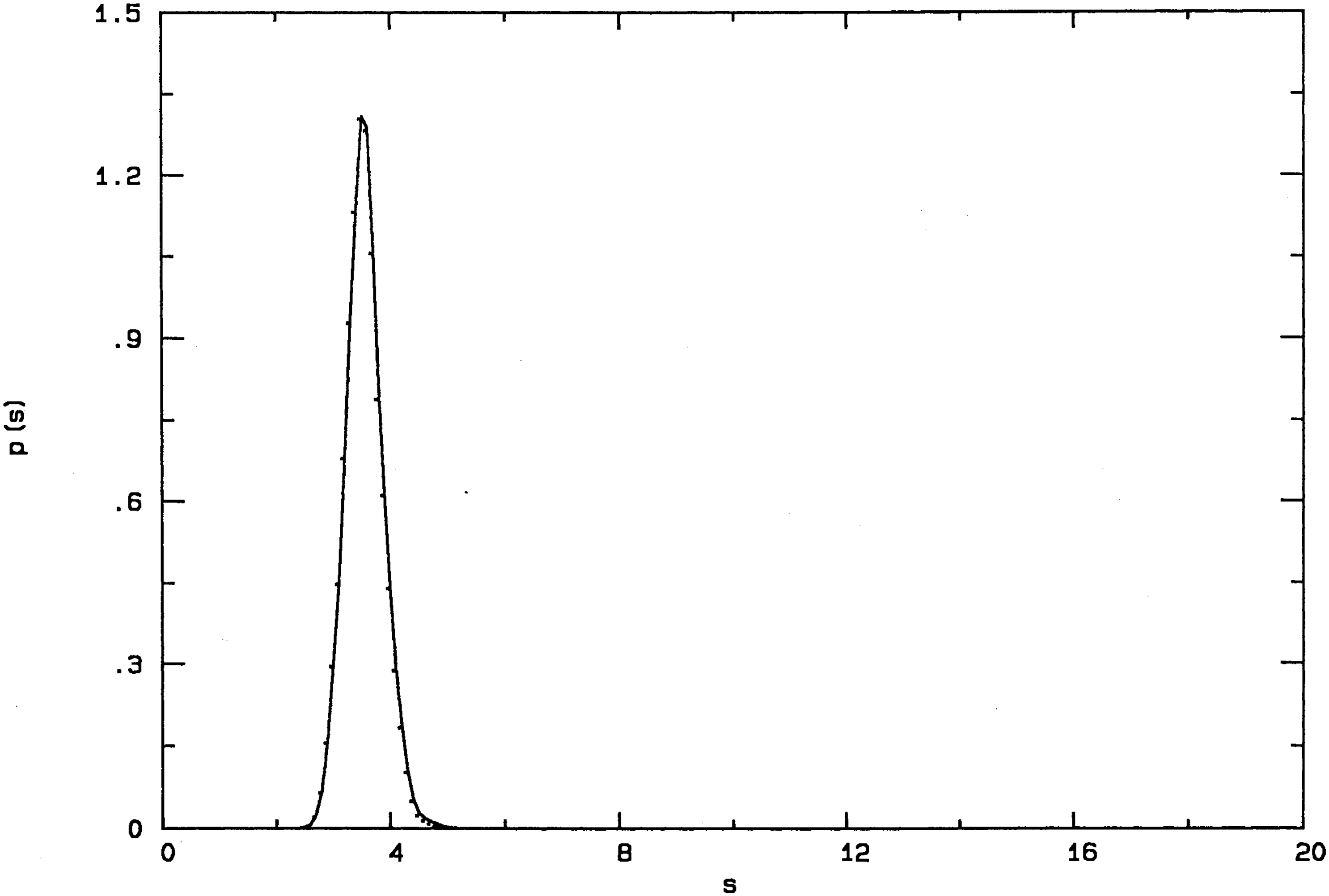

Figure 64. Probability Density Function of S for Watershed 5142

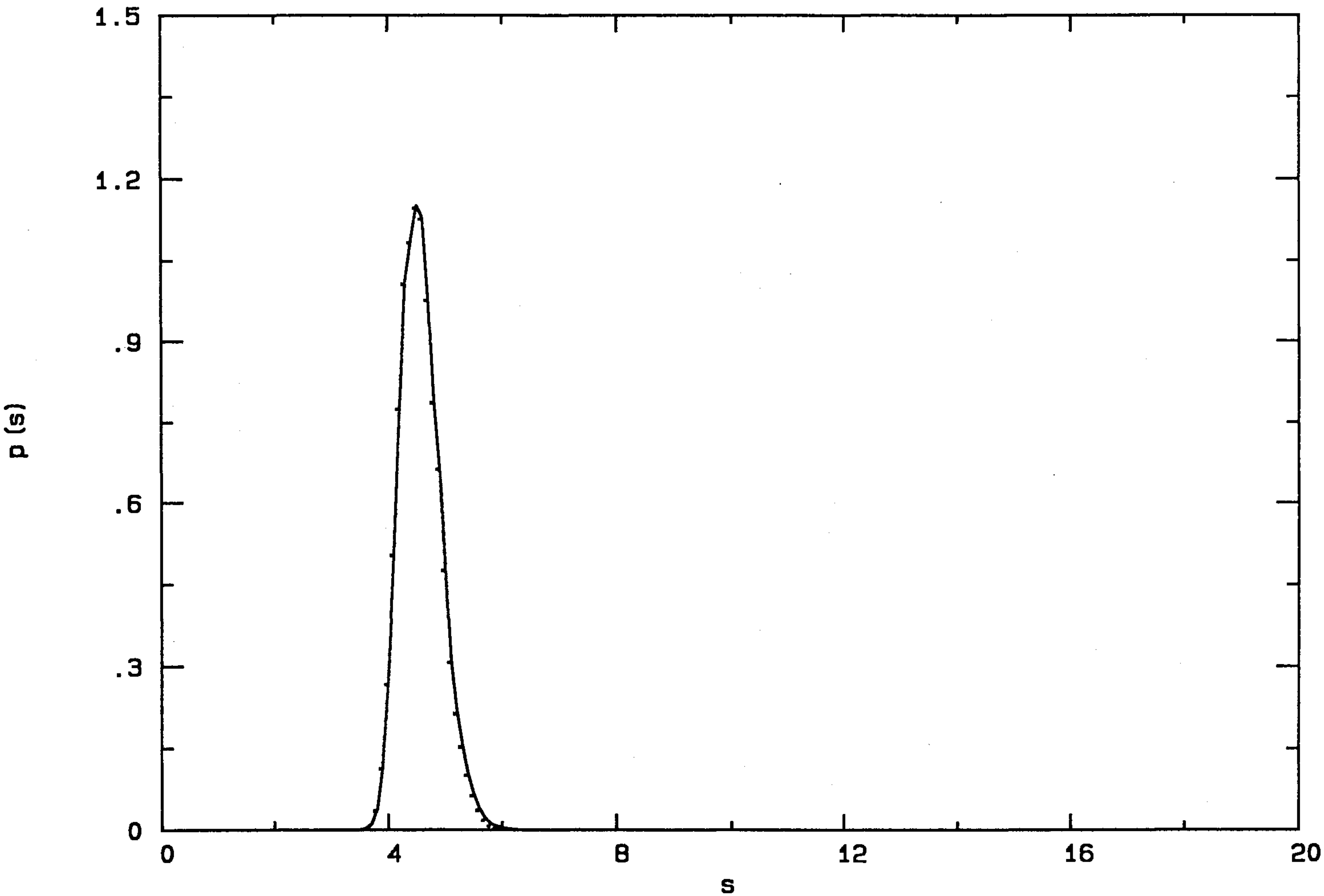

Figure 65. Probability Density Function of S for Watershed 611

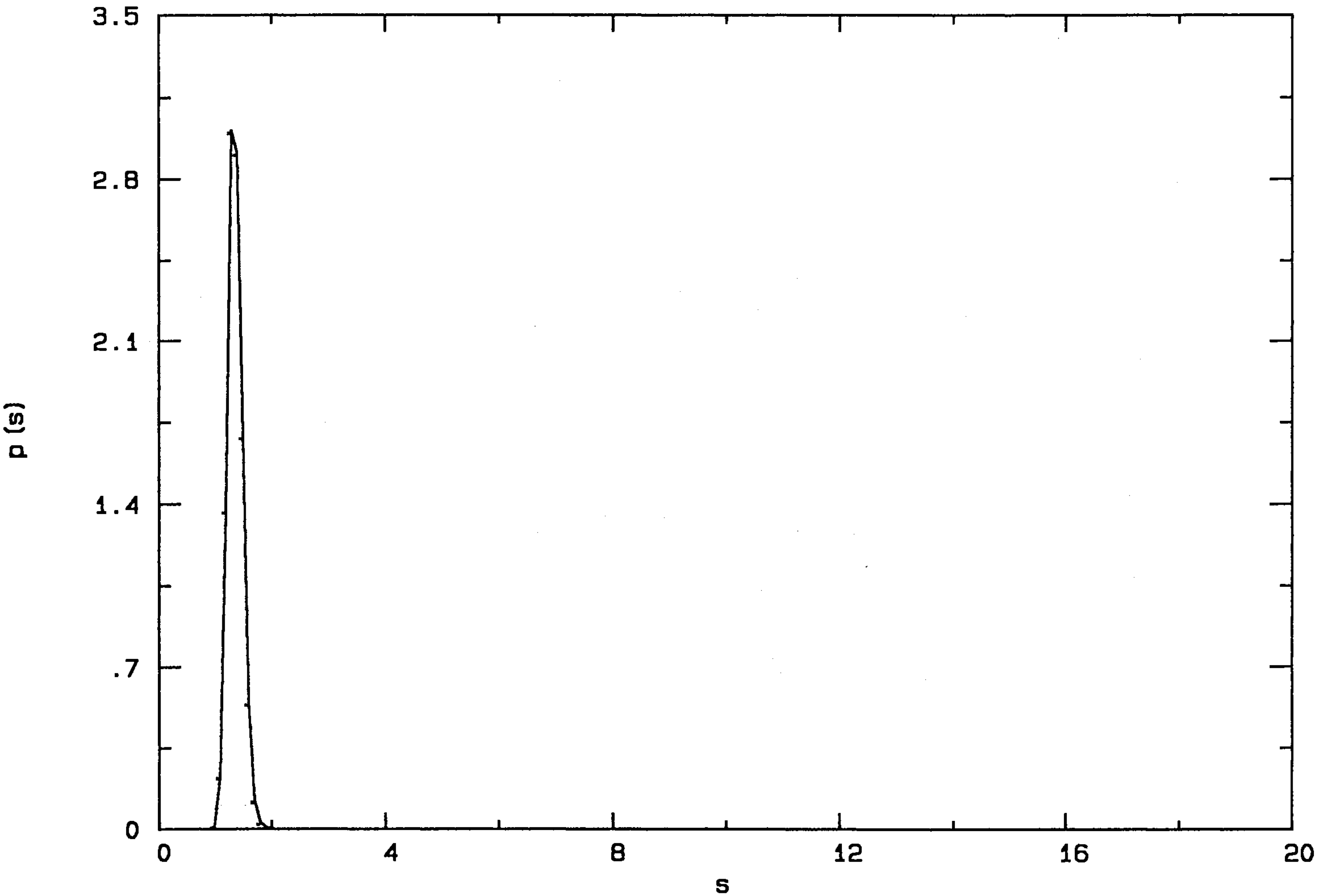

Figure 66. Probability Density Function of S for Watershed R7

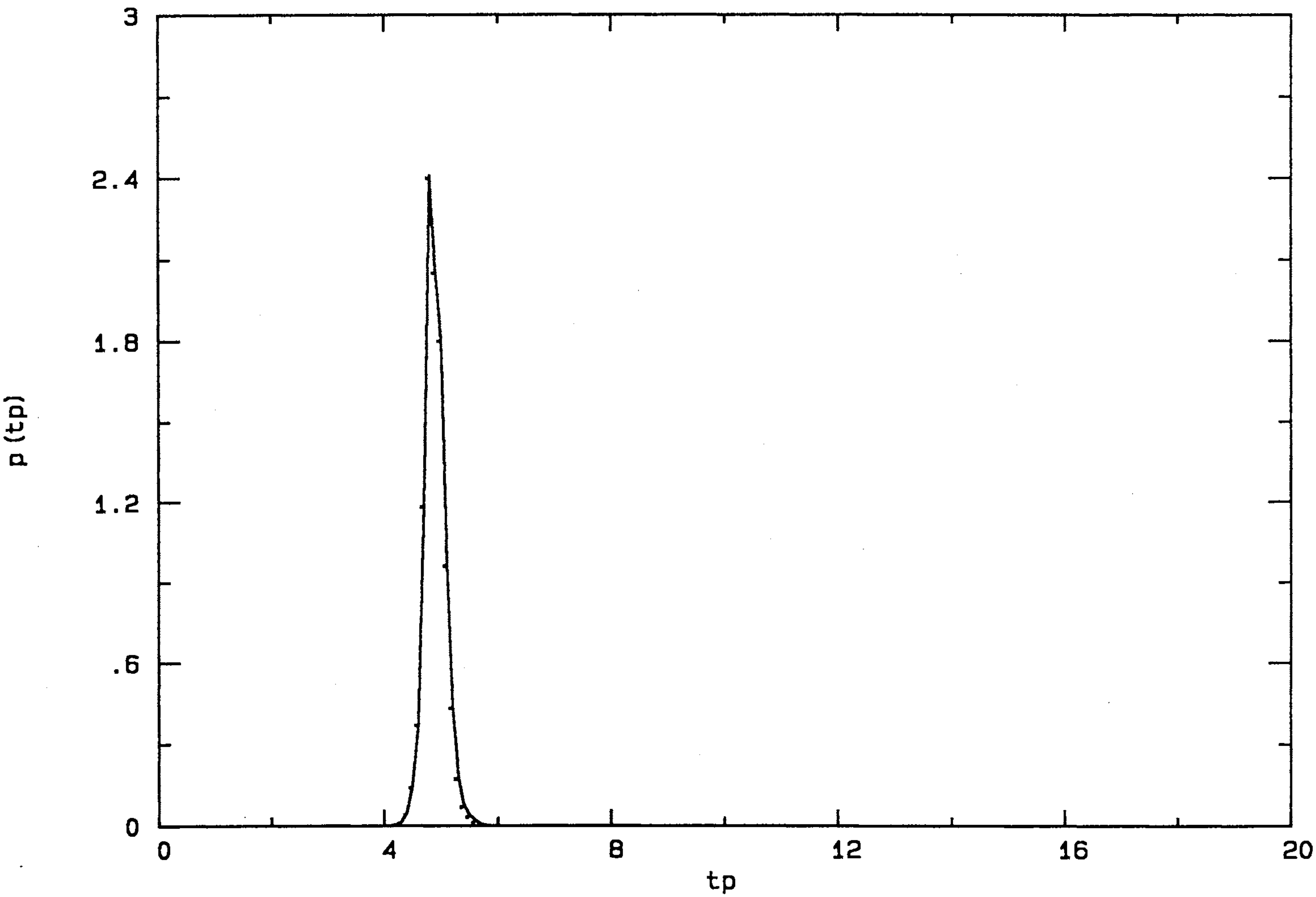

Figure 67. Probability Density Function of Tp for Watershed 511

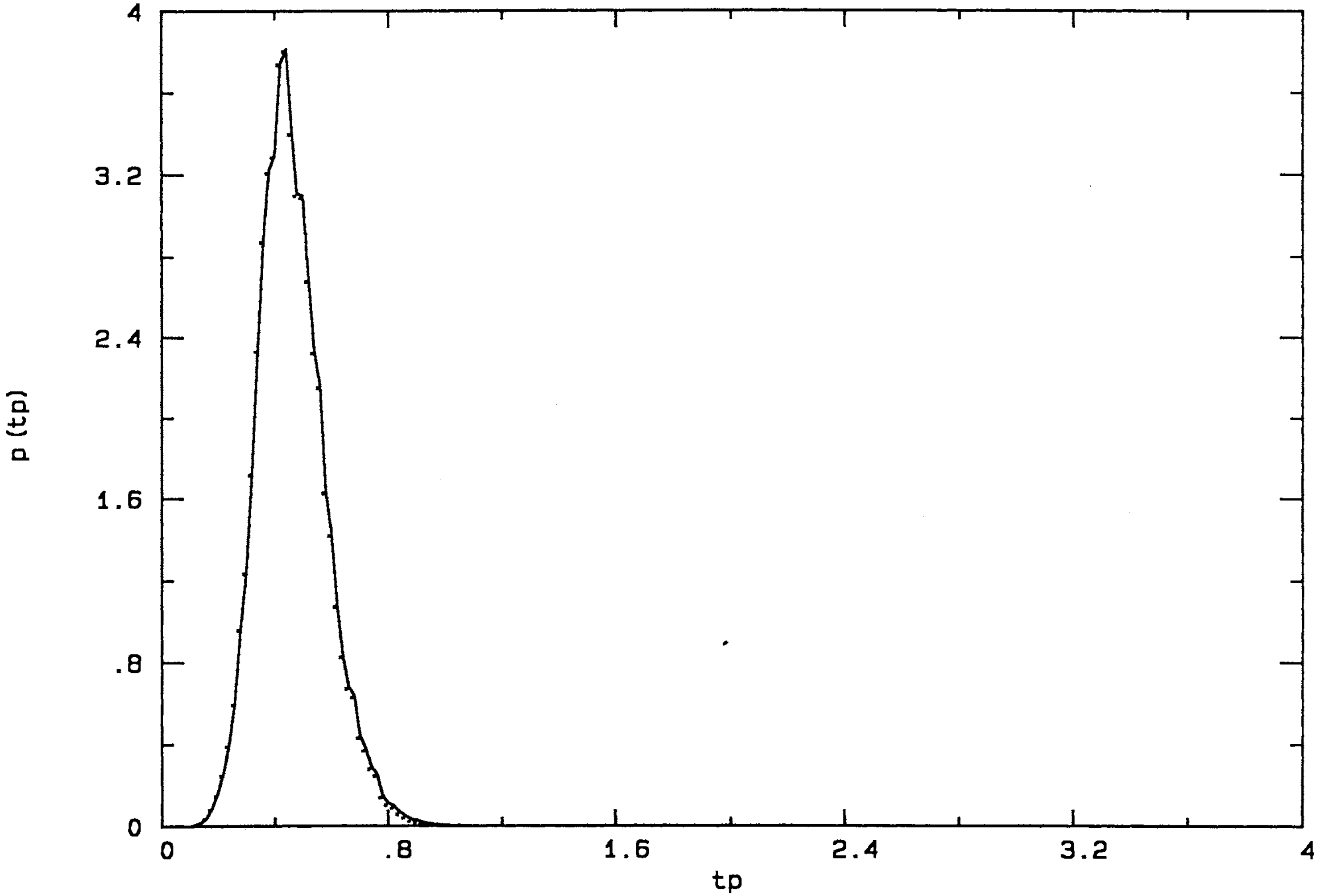

Figure 68. Probability Density Function of Tp for Watershed 5142

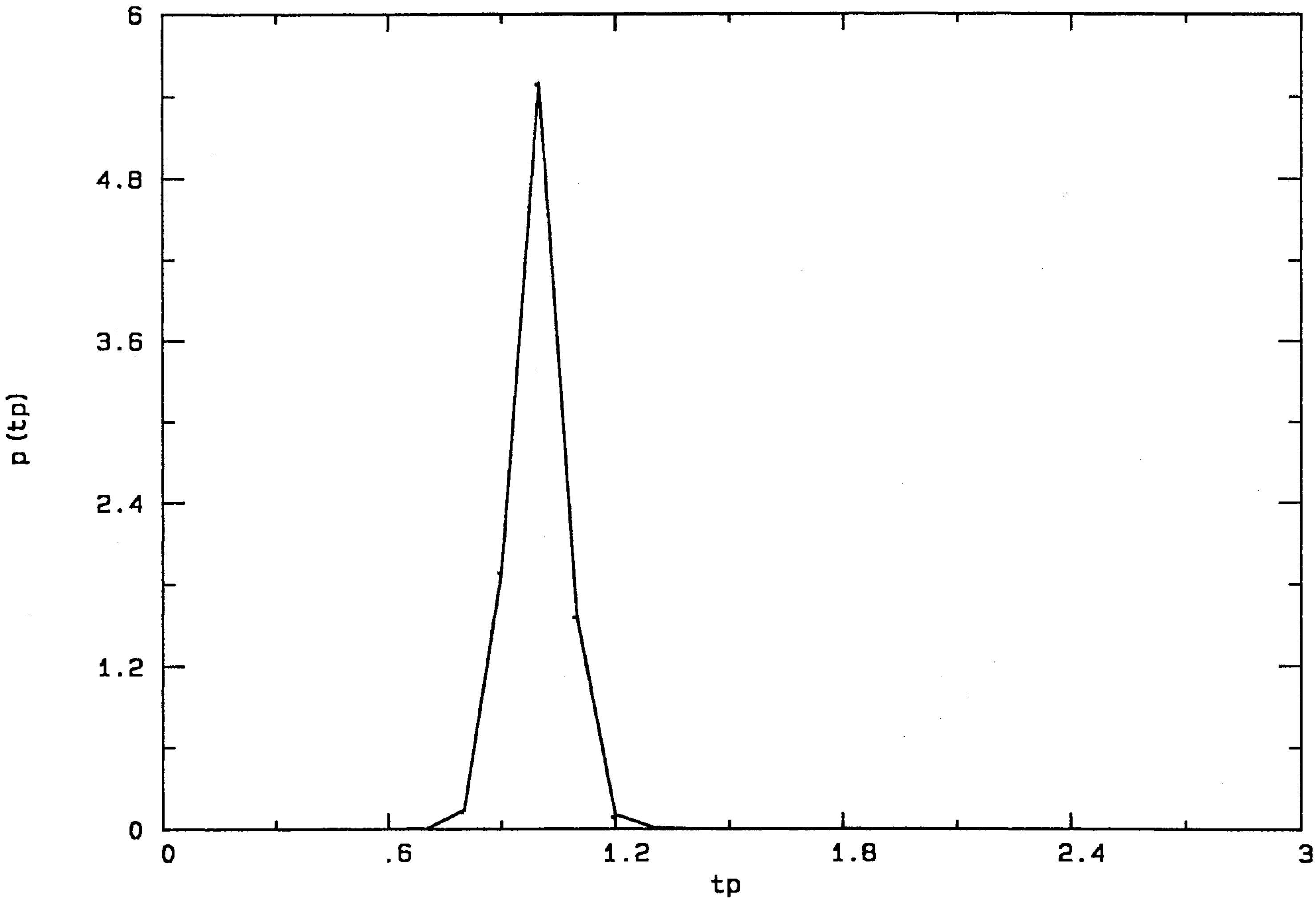

Figure 69. Probability Density Function of Tp for Watershed 611

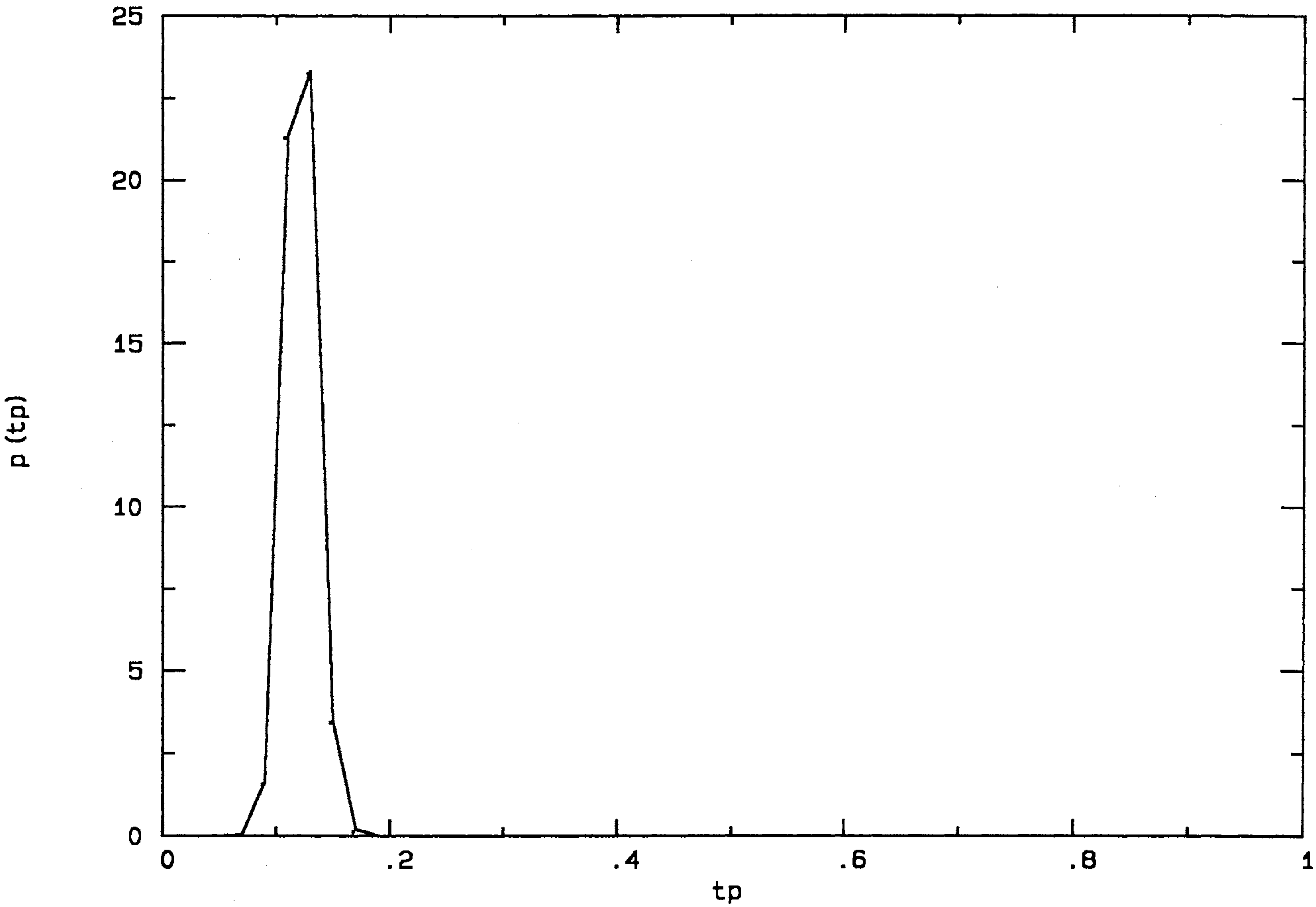

Figure 70. Probability Density Function of Tp for Watershed R7

CHAPTER VI

DEVELOPMENT OF A FLOOD ESTIMATION METHODOLOGY FOR UNGAGED WATERSHEDS

A question that should be answered prior to developing a flood estimation methodology for ungaged watersheds is "To what end is the methodology to be used?" In the context of this research, the answer lies in the following assumptions which are made:

1. The purpose of implementing the flood estimation methodology is practical; i.e., the methodology will be used as a tool in specifying design criteria for hydraulic structures such as bridges, culverts, spillways, etc.
2. The flood estimation methodology will be used to provide approximate knowledge of the hydrologic response of the watershed over the spectrum of rainfall events and antecedent moisture conditions, rather than for specific events.
3. The hydrologic quantity of interest is the magnitude of the peak flow, as opposed to the runoff volume or entire flood hydrograph.

These assumptions give rise to the form of the flood estimation methodology which is presented in this chapter as well as the procedures which are used to evaluate it.

The flood estimation procedure presented in this chapter has as its primary objective the generation of flood frequency curves for ungaged watersheds. An additional characteristic of the procedure, which stems from its probabilistic nature, is its ability to produce confidence intervals for these flood frequency curves.

Hence the procedure is concerned with the overall hydrologic response of ungaged watersheds, rather than responses due to specific events. Although hydrologic responses of ungaged watersheds to specific events may be approximated as a by-product of this methodology, such responses are not specifically considered within this dissertation.

A distinguishing feature of this flood estimation methodology, in view of its (admittedly tenuous) relationship to cause and effect hydrologic relationships, is its use of regional information to gain knowledge on the probability density functions of the model parameters for ungaged watersheds. This regional information is incorporated into a regression framework for obtaining estimates of the mean and variance of each of the model parameters. Additionally, the uncertainty of these estimates is incorporated into the flood estimation methodology.

The approach to flood estimation described in this chapter is to, in effect, regionalize the probability density functions of the model parameters S and Tp and adjust these functions to reflect the uncertainty introduced in the regionalization. Random pairs of values of S and Tp may then be generated from these adjusted probability density functions and used as inputs to the SCS unit hydrograph model. The flood frequency curve follows from the selection of appropriate rainfall amounts and durations as further inputs to the model.

Regionalization of S

The means of the model parameter S as reported in Table XI were linearly regressed against the geomorphic parameters of Table III and the computed values of S derived from Table II. Both the untransformed variables and the logarithmic transformations of the variables were used in a stepwise linear regression process (see Dillon and Goldstein, 1984, for a discussion of stepwise linear regression). The relationship obtained from the untransformed variables was the more satisfactory one, and is given by

$$\hat{\mu}_S = -3.42 - 0.06\ A + 1.2\ W + 1.61\ S_t \tag{95}$$

where $\hat{\mu}_S$ = predicted mean of S (in),

A = watershed area (mi^2),

W = maximum watershed width normal to the main channel (mi), and

S_t = the value of S derived from SCS (1986) tables (in).

The standard error of estimate of Eqn. 95 is 0.676, and the corresponding coefficient of determination is 0.92.

The next regionalization to be undertaken was of the standard deviation of the model parameter S. The same independent variables listed in the previous section were used in the linear regression analysis, with the exception that only their logarithmic-transformed values were used. The values used for the dependent variable are derived from Table XI. The stepwise linear regression procedure resulted in a six-parameter prediction equation for the standard deviation of S. In view of the fact that there are only 15 data points, this equation was judged to contain an unacceptable number of parameters. As an alternative, a prediction equation with the same independent variables as in Eqn. 95 was derived and adopted. The resulting equation is given by

$$\ln(\hat{\sigma}_S) = -3.64 - 0.431\ \ln(A) + 1.20\ \ln(W) + 2.23\ \ln(S_t) \tag{96}$$

where $\ln(\hat{\sigma}_S)$ is the predicted logarithm of the standard deviation of the model parameter S and all other variables are as previously defined. The standard error of estimate is 0.451 and the coefficient of determination is 0.86.

Regionalization of Tp

The set of means of the model parameter Tp shown in Table XI were regressed against the independent variables previously discussed, again using logarithmic transformations of all variables. The stepwise linear regression analysis resulted in a prediction equation given by

$$\ln(\hat{\mu}_T) = 2.28 + 0.62 \ln(L_c) - 1.67 \ln(S_a) \tag{97}$$

where $\ln(\hat{\mu}_T)$ = estimated logarithm of the mean of Tp (hr),

S_a = average land slope of the watershed (%), and

L_c = length of the main channel (mi).

The standard error of estimate of this equation is 1.047, and the coefficient of determination is 0.83.

The last variable to be regionalized was the standard deviation of the model parameter Tp. Again, a stepwise linear regression procedure was employed using logarithmic transformations of all variables. The resulting prediction equation is given by

$$\ln(\hat{\sigma}_t) = -0.8 + 1.31 \ln(W) + 1.23 \ln(S_c) - 0.4 \ln(S_f) \tag{98}$$

where $\ln(\hat{\sigma}_t)$ = estimated logarithm of the standard deviation of Tp (hr), and

S_f = stream frequency (mi^{-2})

The standard error of estimate for Eqn. 98 is 0.402 and the coefficient of determination is 0.96.

Upon obtaining the relationship of Eqn. 98, it was decided to minimize the number of independent variables required to estimate the means and variances of the model parameters and search for an alternative prediction equation for the standard deviation of Tp which uses only variables in Eqns. 95 through 97. The equation adopted is given by

$$\ln(\hat{\sigma}_t) = -3.09 - 0.38\ \ln(A) + 1.52\ \ln(W) + 0.98\ \ln(S_t) \tag{99}$$

where all variables are as previously defined. The standard error of prediction for this equation is 0.599, and the coefficient of determination is 0.82.

Analysis of variance tables and other information relating to Eqns. 95, 96, 97, and 99 are given in Appendix C.

A Framework for Using Regional Information

A sample of the densities of the model parameters S and Tp was presented in the previous chapter, where it was stated that these densities would be considered normal. The broad assumption now made is that the model parameters S and Tp are normally distributed for all watersheds in the general vicinity of the study watersheds. Consider now an ungaged watershed in this vicinity for which the prediction equations developed in the two previous sections apply. On this watershed, the probability density function of the model parameter S (to choose one of the two model parameters) may be written as

$$p(s)=(2\pi)^{-\frac{1}{2}}\sigma_s^{-1}\ \exp\left[-\frac{1}{2}\left(\frac{s-\mu_s}{\sigma_s}\right)^2\right] \tag{100}$$

where μ_s = mean of S (in) and

σ_s = standard deviation of S (in)

Since the watershed under consideration is ungaged, there are no data available to use in an analysis such as that performed in the previous chapter. The mean and standard deviation of S must therefore be estimated using the relationships of Eqns. 95 and 96. These estimates are uncertain, implying that the mean and variance of S should themselves be considered random variables. Equation 100 must therefore be rewritten as a conditional probability density function, given by

$$p(s|\mu_s,\sigma_s)=(2\pi)^{-\frac{1}{2}}\sigma_s^{-1}\ \exp\left[-\frac{1}{2}\left(\frac{s-\mu_s}{\sigma_s}\right)^2\right] \tag{101}$$

The unconditional probability density function of S is found as

$$p(s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(s|\mu_s,\sigma_s)p(\mu_s,\sigma_s)d\mu_s d\sigma_s \tag{102}$$

It is now necessary to specify a joint probability density function for (μ_s,σ_s). One which is convenient in terms of its ability to tractably accommodate correlation between two variables is the bivariate normal probability density function. For random variables X and Y which are $N_2(\mu_X,\mu_Y,\sigma_X,\sigma_Y,\rho)$, it may be written as

$$p(x,y) = (2\pi\sigma_x\sigma_y)^{-1}(1-\rho^2)^{-\frac{1}{2}}\exp\left[\frac{-1}{2(1-\rho^2)}g(x,y)\right] \tag{103}$$

where

$$g(x,y) = \left(\frac{x-\mu_x}{\sigma_x}\right)^2 -2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)+\left(\frac{y-\mu_y}{\sigma_y}\right)^2 \tag{104}$$

It is noted now that under the assumptions commonly employed in linear regression (particularly that of normally distributed residuals), the sampling distribution of the dependent variable, conditioned upon a fixed independent variable, is normal. This result may be used in conjunction with the results of the regionalization of the model parameter S to state that μ_s is $N(\hat{\mu}_s, \varsigma_\mu)$ where ς_μ is the standard error of estimate associated with Eqn. 95. Similarly, it may be said that $\ln(\sigma_s)$ is $N(\ln(\hat{\sigma}_s), \varsigma_{\ln(\sigma)})$, where $\varsigma_{\ln(\sigma)}$ is the standard error of Eqn. 96. If ρ_o is taken as the observed correlation between the values of the mean and logarithm of the standard deviation of S, which may be derived from Table XI, then (μ_s, σ_s) may be taken as $N_2(\hat{\mu}_s, \ln(\hat{\sigma}_s), \varsigma_\mu, \varsigma_{\ln(\sigma_s)}, \rho_o)$. The probability density function of (μ_s, σ_s) may now be determined upon an elementary application of the theory of derived distributions.

$$p\ (\mu_s, \sigma_s) \ = p\ (\mu_s, \ln(\sigma_s))|\underline{J}| \tag{105}$$

where $\underline{J}$ is the Jacobian of the transformation from $(\mu_s, \ln(\sigma_s))$ to (μ_s, σ_s) . Substituting the appropriate expressions into Eqn. 105, the probability density function of (μ_s, σ_s) is given as

$$p(\mu_s, \sigma_s) = \frac{1}{\kappa \sigma_s} \exp\left[\frac{-1}{2(1-\rho^2)} g(\mu_s, \sigma_s) \right] \tag{106}$$

$$\text{where } \kappa = (2\pi \varsigma_\mu \varsigma_{\ln(\sigma)} \sigma_s)^{-1} (1-\rho_o^2)^{-½} \tag{107}$$

$$g(\mu_s, \sigma_s) = \left(\frac{\mu_s - \hat{\mu}_s}{\varsigma_\mu} \right)^2 - 2\rho_o \left(\frac{\mu_s - \hat{\mu}_s}{\varsigma_\mu} \right) \left(\frac{\ln(\sigma_s) - \ln(\hat{\sigma}_s)}{\varsigma_{\ln(\sigma)}} \right) + \left(\frac{\ln(\sigma_s) - \ln(\hat{\sigma}_s)}{\varsigma_{\ln(\sigma)}} \right)^2 \tag{108}$$

It is unreasonable to hope for a closed-form solution to Eqn. 102, therefore a numerical approach is adopted in order to derive the probability density function of S. The mean and variance of S for the ungaged watershed are next determined by numerically integrating the empirical probability density function of S. Since S is a priori assumed normally distributed, its mean and variance completely describe the uncertainty associated with it.

A parallel approach may be taken in deriving the probability density function of the model parameter Tp. The unconditional probability density function of Tp is given by

$$p(t_p) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(t_p|\mu_t,\sigma_t)p(\mu_t,\sigma_t)d\mu_t d\sigma_t \tag{109}$$

where Tp is $N(\mu_t,\sigma_t)$, conditional upon the values of its uncertain mean and variance. Using the same reasoning employed in specifying the distribution of the uncertain mean and variance of S, it follows that

$$p(\mu_t,\sigma_t) = \frac{1}{\kappa\sigma_t} \exp\left[\frac{-1}{2(1-\rho_0^2)} g(\mu_t,\sigma_t)\right] \tag{110}$$

where $\kappa = (2\pi\varsigma_\mu\varsigma_{\ln(\sigma)}\sigma_t)^{-1}(1-\rho_0^2)^{-½}$ (111)

$$g(\mu_t,\sigma_t) = \left(\frac{\ln(\mu_t)-\ln(\hat{\mu}_t)}{\varsigma_\mu}\right)^2 - 2\rho_0\left(\frac{\ln(\mu_t)-\ln(\hat{\mu}_t)}{\varsigma_\mu}\right)\left(\frac{\ln(\sigma_t)-\ln(\hat{\sigma}_t)}{\varsigma_{\ln(\sigma)}}\right) + \\ + \left(\frac{\ln(\sigma_t)-\ln(\hat{\sigma}_t)}{\varsigma_{\ln(\sigma)}}\right)^2 \tag{112}$$

ς_μ = standard error of Eqn. 97

$\varsigma_{\ln(\sigma)}$ = standard error of Eqn. 99

ρ_0 = observed correlation between values of the logarithms of the mean and standard deviation of Tp, derived from Table XI.

Equation 109 may be numerically integrated to derive the probability density function of Tp. The result may be again integrated to find the mean and variance of Tp. Again, Tp is a priori assumed normally distributed, so the derivation of its mean and variance suffice to describe its uncertainty.

Estimation of Flood Frequency Curves

The preceding sections have described how the probability density functions of the model parameters S and Tp may be estimated for an ungaged watershed on the basis of regional information. It has been noted elsewhere that the correlation between S and Tp is generally neither strongly negative nor strongly positive, and the parameters may for this reason be considered independent. That is to say,

$$p(s,t_p) = p(s)p(t_p) \tag{113}$$

For a rainfall event of given depth, duration, and distribution, the two-parameter version of the SCS unit hydrograph model computes peak flow as a function of watershed area (which is taken as fixed), S, and Tp. The probability density function of peak flow Q and some dummy variable X may be determined as

$$p(q,x) = p(s)p(t_p)|\underline{J}| \tag{114}$$

where $\underline{J}$ is the Jacobian of the transformation between (S,Tp) and (Q,X). In this context, $\underline{J}$ accounts for the manner in which the SCS unit hydrograph model computes Q and X as a function of S and Tp. The marginal probability density function of Q is found by integrating Eqn. 114 with respect to X.

$$p(q) = \int_{-\infty}^{\infty} p(s)p(t_p)|\underline{J}|dx \tag{115}$$

Given the nature of the SCS unit hydrograph model, it is not possible to state in closed form an expression for J. This approach to deriving the probability density function of Q must therefore be modified. The alternative selected for this research was a Monte Carlo method of deriving the probability density function of Q. To use this method, one need only generate multiple pairs of values of S and Tp from their respective probability density functions and compute the resulting values of Q for the rainfall event of interest. These values of Q constitute an empirical probability density function of Q, from which may be derived E(Q), Var(Q), and desired confidence intervals on Q. It should be said regarding the generation of values of S and Tp that at this point, practice must diverge from theory. It is possible, given the probability density functions of the two model parameters, that negative values would be generated. If the two parameters are taken as having any physical significance, then negative values of the parameters are largely meaningless. The solution to the problem of negative values was to truncate the parameters' probability density functions at zero and to then rescale the probability density functions so that they integrated to unity.

It is now assumed that the recurrence interval of a particular peak flow is equal to the recurrence interval of the rainfall event which produced that peak flow. Thus derivation of p(q) for rainfall events of varying return periods allows for the construction of the flood frequency curve. The peak flow having recurrence interval T is computed as E(Q) where p(q) is derived using a rainfall event of recurrence interval T. Confidence intervals may be placed on the entire flood frequency diagram by determining the confidence intervals on Q for each recurrence interval selected.

The only point to be resolved at this stage is the specification of appropriate rainfall events in terms of their durations, depths, and temporal distributions. For the purposes of this research, the rainfall duration to be used for a particular watershed is taken as equal to its time of concentration. The SCS (1972) states that the time of concentration may be taken as 1.5Tp. The appropriate rainfall duration to be used for an ungaged watershed is thus taken as 1.5 times the Tp estimated from Eqn. 97. Rainfall depths corresponding to various durations and return periods were presented by Hershfield (1963) and are obtained from this source. The temporal distribution of rainfall is determined by adapting the SCS (1986) Type II rainfall distribution to the appropriate duration.

Summarized Flood Estimation Methodology For Ungaged Watersheds

The steps involved in this flood estimation methodology are summarized as follows:

1. Estimate μ_s using Eqn. 95.
2. Estimate $\ln(\hat{\sigma}_s)$ using Eqn. 96.
3. Derive the unconditional probability density function of S as the solution to Eqn. 102.
4. Integrate the result of Step 3 in order to find the mean and variance of S.
5. Estimate μ_t using Eqn. 97.
6. Estimate σ_t using Eqn. 99.
7. Derive the unconditional probability density function of Tp as the solution to Eqn. 109.
8. Integrate the result of Step 8 in order to find the mean and variance of Tp.
9. Compute the appropriate rainfall duration to be used as 1.5 times the result of Step 5.
10. Obtain the rainfall depth for the desired recurrence interval from Hershfield (1963) using the rainfall duration obtained in Step 9.

11. Generate multiple pairs of non-negative values for S and Tp taking S as normally distributed with mean and variance as determined in Step 4 and Tp as normally distributed with mean and variance as determined in Step 8.

12. Input each pair of values of S and Tp to the SCS unit hydrograph along with the watershed area, the rainfall depth determined in Step 10, and the temporal distribution of the rainfall (taken as SCS Type II), and compute the resulting peak flows.

13. Compute the mean resultant peak flow and assign to it the same recurrence interval as for the rainfall event.

14. Compute the upper and lower bounds of the $(1-\alpha)\%$ confidence interval for resultant peak flow from the empirical distribution of peak flow.

15. Repeat Steps 10 through 14 for all desired recurrence intervals.

16. Plotting mean resultant peak flow and the bounds of the $(1-\alpha)\%$ confidence interval vs. recurrence interval results in the estimated flood frequency curve, with confidence intervals, for the ungaged watershed.

CHAPTER VII

EVALUATION OF THE FLOOD ESTIMATION METHODOLOGY FOR UNGAGED WATERSHEDS

A problem which was encountered in this research was a relative shortage of suitable study watersheds in the region selected for use. This problem impacts most greatly upon the evaluation stage of the research. Ideally, one would develop a flood estimation methodology using data from a subset of the available study watersheds and judge the merit of the procedure by applying it to the remainder of the study watersheds. However, there exist in this situation several factors which preclude the "ideal" strategy of evaluating the flood estimation methodology.

The heart of the flood estimation methodology is the procedure for obtaining estimates of the means and variances of the model parameters S and Tp, which includes using a set of regression-based prediction equations. There are only 15 data points available for the regressions of each of these quantities. To set aside a significant number of the study watersheds for use only in the evaluation stage of the research is to omit that number of data points from the regressions. The omission of an appreciable number of data points will have the effect of increasing the standard errors of the prediction equations for the mean and variance of the model parameters and will thus diminish the amount of information gained from the regionalization process. Of course, the flood estimation methodology may be tested on only one of the study watersheds after withholding it from the regression analyses, but this would be a relatively weak evaluation.

The alternative method of evaluation employed in this research is a Jackknife approach. The flood estimation methodology will be tested on each of the 15 study watersheds as if it were the only one withheld from the regional analysis. The coefficients of the regionally-derived prediction equations (Eqns. 95, 96, 97, and 99) will be redetermined for each study watershed by omitting that watershed from the linear regression analyses. The observed correlations between the mean and logarithm of the standard deviation of S, as well as between the logarithms of the mean and standard deviation of Tp, will be redetermined for each study watershed in an analogous manner. The method of estimating flood frequency curves then proceeds exactly as discussed in Chapter VI. This strategy of evaluation will circumvent the potential problem of omitting a relatively large number of data points from the regression analyses, and it will yield more information on the performance of the flood estimation methodology than would be obtained from testing it on only one study watershed.

The specific method of testing the flood estimation methodology for ungaged watersheds is to compare the resulting flood frequency curves, referred to as the Bayesian flood frequency curves, to those resulting from fitting the partial duration series data to a Log-Pearson Type III distribution; i.e., the observed flood frequency curves derived from the partial duration series are taken as the standards. In order to make comments regarding the performance of the methodology relative to other existing methods of flood estimation, flood frequency curves derived from application of the USGS method described by Tortorelli and Bergman (1985) are presented. Also included for this purpose are flood frequency curves resulting from use of SCS (1972) unit hydrograph procedures in their unmodified context.

Data Used to Estimate Flood Frequency Curves

Data on Model Parameters S and Tp

The prediction equations for the mean and logarithm of the standard deviation of the model parameter S may be represented in a generalized fashion as

$$\hat{\mu}_S = a_0 + a_1 A + a_2 W + a_3 S_t \tag{116}$$

$$\ln(\hat{\sigma}_S) = b_0 + b_1 \ln(A) + b_2 \ln(W) + b_3 \ln(S_t) \tag{117}$$

where the a_i and b_i are coefficients determined by the linear regression procedure and all other variables are as previously defined in Eqns. 95 and 96. Fifteen sets of the coefficients a_i and b_i were determined by omitting the information derived from Tables II, III and XI for each of the study watersheds and determining the resulting least-squares estimates of the coefficients. The mean and standard deviation of S were then estimated for each study watershed using the coefficients derived by omitting its data points. The estimates of the mean and logarithm of the standard deviation of S, as well as their associated standard errors appear in Table XII. Also appearing in Table XII are the jackknived computations of the correlation between the mean of S and the logarithm of the standard deviation of S which were derived from Table XI.

Estimates of the logarithms of the mean and standard deviation of the model parameter Tp are derived from equations of the form

$$\ln(\hat{\mu}_T) = c_0 + c_1 \ln(L_c) + c_2 \ln(S_a) \tag{118}$$

$$\ln(\hat{\sigma}_T) = d_0 + d_1 \ln(A) + d_2 \ln(W) + d_3 \ln(S_t) \tag{119}$$

TABLE XII

DATA USED TO DERIVE PROBABILITY DENSITY FUNCTIONS OF S

ID	$\hat{\mu}_S$ (in)	ς_μ (in)	$\ln(\hat{\sigma}_S)$ [ln(in)]	$\varsigma_{\ln(\sigma)}$ [ln(in)]	ρ_o
111	6.45	0.53	-0.23	0.44	0.89
131	7.16	0.71	-0.15	0.46	0.90
311	3.90	0.69	-1.00	0.46	0.91
411	8.40	0.65	-0.42	0.46	0.90
511	3.81	0.70	-0.58	0.43	0.93
513	3.91	0.71	-1.00	0.47	0.92
5142	3.66	0.71	-0.50	0.44	0.92
5143	5.11	0.69	-0.44	0.45	0.92
5145	2.63	0.71	-2.13	0.40	0.92
515	1.60	0.65	-2.07	0.43	0.92
611	5.57	0.65	-0.29	0.42	0.92
R5	3.24	0.70	-1.37	0.47	0.91
R6	4.19	0.59	-1.04	0.47	0.92
R7	1.71	0.70	-1.85	0.47	0.90
R8	1.32	0.70	-2.07	0.47	0.91

$\hat{\mu}_S$ = Estimated mean of S.

ς_μ = Standard error of $\hat{\mu}$.

$\ln(\hat{\sigma})$ = Estimated logarithm of the standard deviation of S.

$\varsigma_{\ln(\sigma_S)}$ = Standard error of $\ln(\hat{\sigma})$.

ρ_o = Corr[μ_S,$\ln(\sigma_S)$].

where the c_i and d_i are generalized coefficients and the other variables have been defined in connection with Eqns. 97 and 99. A procedure analogous to that described in the previous paragraph was employed in order to obtain the coefficients c_i and d_i for each of the study watersheds. The resulting estimates of the logarithm of the mean and standard deviation of Tp appear with their respective standard errors in Table XIII. Jackknived computations of the correlation between the logarithm of the mean of Tp and the logarithm of the standard deviation of Tp, which were derived from Table XI, also appear in Table XIII.

The information appearing in Tables XII and XIII were substituted into Eqns. 102 and 109, respectively, in order to determine the unconditional probability density functions of S and Tp. The means, standard deviations, and coefficients of skewness of the two parameters were next computed by integrating these probability density functions as described in Chapter VI, and are shown in Table XIV. The means and standard deviations of Table XIV completely specify the (normal) probability density functions from which values of S and Tp were generated.

Data on Rainfall Depth and Duration

It is assumed for the purposes of this flood estimation methodology that for a given duration, rainfall depth follows the Extreme Value Type I distribution. Haan (1977) states that the parameters of this distribution may be found as functions of the 2-year and 100-year rainfall depths. Hershfield (1963) presents rainfall depths for recurrence intervals including 2 and 100 years and for durations of 0.5, 1, 2, 3, 6, 9, 12, and 24 hours. Rainfall depths for recurrence intervals of 2, 5, 10, 25, 50, and 100 years were found for each study watershed by taking storm duration as 1.5 times the estimate of Tp derived from Table XIII (rounded up to the nearest duration accommodated by Hershfield), determining the 2-year and 100-year rainfall depths reported

TABLE XIII

DATA USED TO DERIVE PROBABILITY DENSITY FUNCTIONS OF Tp

ID	$\ln(\hat{\mu}_T)$ [ln(hr)]	$\varsigma_{\ln(\mu)}$ [ln(hr)]	$\ln(\hat{\sigma}_T)$ [ln(hr)]	$\varsigma_{\ln(\sigma)}$ [ln(hr)]	ρ_o
111	1.05	0.36	-0.80	0.61	0.86
131	0.94	0.42	-0.60	0.62	0.80
311	1.90	0.45	-1.49	0.63	0.85
411	1.56	0.44	-0.46	0.62	0.79
511	1.74	0.45	-0.80	0.59	0.83
513	1.24	0.45	-1.58	0.62	0.83
5142	-0.89	0.45	-1.72	0.62	0.82
5143	-1.01	0.42	-2.06	0.58	0.84
5145	-0.82	0.44	-3.63	0.54	0.82
515	0.54	0.41	-2.62	0.54	0.81
611	0.45	0.43	-0.93	0.50	0.82
R5	-1.19	0.45	-3.28	0.63	0.81
R6	-1.34	0.44	-3.16	0.63	0.80
R7	-1.93	0.45	-3.27	0.59	0.78
R8	-1.86	0.45	-3.59	0.63	0.78

$\ln(\hat{\mu}_T)$ = Estimated logarithm of the mean of Tp.

$\varsigma_{\ln(\mu)}$ = Standard error of $\ln(\hat{\mu})$.

$\ln(\hat{\sigma}_T)$ = Estimated logarithm of the standard deviation of Tp.

$\varsigma_{\ln(\sigma)}$ = Standard error of $\ln(\hat{\sigma})$.

ρ_o = $\mathrm{Corr}[\ln(\mu_T),\ln(\sigma_T)]$.

TABLE XIV

STATISTICS OF PARAMETERS S AND Tp, 2

ID	m_S (in)	s_S (in)	g_S (in^3)	m_T (hr)	s_T (hr)	g_T (hr^3)
111	6.45	1.10	0.84	3.05	1.30	1.42
131	7.16	1.27	0.88	2.79	1.43	1.45
311	3.92	0.83	0.61	7.07	2.91	0.72
411	8.40	1.03	0.90	5.18	2.40	1.08
511	3.81	0.97	0.85	6.11	2.63	0.84
513	3.93	0.84	0.60	3.81	1.77	1.27
5142	3.66	1.02	0.89	0.46	0.34	1.73
5143	5.10	1.05	0.93	0.40	0.26	1.60
5145	2.50	0.53	1.57	0.50	0.23	1.39
515	1.48	0.53	1.37	1.82	0.80	1.44
611	5.57	1.10	0.88	1.71	0.92	1.54
R5	3.22	0.83	0.17	0.24	0.14	4.04
R6	4.21	0.73	0.68	0.30	0.16	1.27
R7	1.55	0.72	0.81	0.14	0.10	1.80
R8	1.23	0.58	1.24	0.15	0.10	1.69

m_S = mean of S.
s_S = standard deviation of S.
g_S = coefficient of skewness of S.
m_T = mean of Tp.
s_T = standard deviation of Tp.
g_T = coefficient of skewness of Tp.

by Hershfield, determining the corresponding parameters of the Extreme Value Type I distribution as per Haan, and solving the cumulative distribution function for the rainfall amounts corresponding to the appropriate exceedance probabilities. Table XV lists the rainfall duration used for each of the study watersheds with the corresponding 2-year and 100-year rainfall depths.

Flood Frequency Curves Used For Comparison Purposes

Log-Pearson Type III Flood Frequency Curves

The parameters of the Log-Pearson Type III probability density function were determined for each study watershed on the basis of the partial duration series data presented in Appendix B. The flood frequency curves result from deriving the annual peak flows corresponding to various recurrence intervals and then adjusting these recurrence intervals according to the procedure described by Chow (1964).

USGS Flood Frequency Curves

Tortorelli and Bergman (1985) present equations for computing annual peak flow, as a function of mean annual precipitation and watershed area, for recurrence intervals of 2, 5, 10, 25, 50, 100, and 500 years. These equations, with the exception of the one for estimating the 500-year peak flow, were applied to each of the study watersheds in order to derive their respective flood frequency curves resulting from the USGS procedure.

SCS Flood Frequency Curves

The SCS (1972) proposes that the peak flow corresponding to a particular recurrence interval be determined as an output of their unit hydrograph model using the 24-hour rainfall depth of the same recurrence interval as an input. Repetition of

TABLE XV

RAINFALL DURATIONS AND DEPTHS OF THE CORRESPONDING 2 AND 100-YEAR RAINFALL EVENTS, 1

ID	Duration (hr)	R_2 (in)	R_{100} (in)
111	6.0	2.54	5.95
131	6.0	2.46	5.77
311	12.0	3.10	7.28
411	12.0	2.98	6.98
511	12.0	2.98	6.98
513	6.0	2.57	6.01
5142	1.0	1.80	4.10
5143	1.0	1.80	4.10
5145	1.0	1.80	4.10
515	3.0	2.25	5.35
611	3.0	2.23	5.30
R5	0.5	1.42	3.20
R6	0.5	1.42	3.20
R7	0.5	1.42	3.20
R8	0.5	1.42	3.20

R_2 = 2-year rainfall depth.
R_{100} = 100-year rainfall depth.

this process for various rainfall recurrence intervals produces a flood frequency curve.

The SCS flood frequency curve was derived for each of the study watersheds having an area of less than 2000 acres. No SCS flood frequency curves were computed for the larger watersheds because of the inapplicability of Eqn. 80 to computation of Tp. The SCS (1986) has presented methods of determining Tp for larger watersheds, but these methods require a tremendous amount of data which were not readily available for the larger study watersheds. The SCS estimates of Tp for the eight watersheds of area less than 2000 acres were computed using Eqn. 80 and the information contained within Tables II and III. The estimates are shown in Table XVI.

The recurrence intervals of the rainfall depths used for these eight study watersheds were taken as 2, 5, 10, 25, 50, and 100 years so as to maintain uniformity with respect to recurrence intervals. The 2-year, 24-hour rainfall depth and the 100-year, 24-hour rainfall depth are, according to Hershfield (1963), the same for each of the eight watersheds and were determined as 3.7 and 8.75 inches, respectively. The 24-hour rainfall depths corresponding to the intermediate recurrence intervals were computed under the assumption of an Extreme Value Type I distribution of depths as discussed in the previous section.

Discussion of Flood Frequency Curves

Figures 71 through 85 illustrate the results of the flood estimation methodology, as modified by the Jackknived prediction equations and correlation coefficients, applied to the 15 study watersheds. Each of the six data points (and each corresponding confidence interval) from which the Bayesian flood frequency curves were constructed was derived using 2000 values of S and Tp randomly generated from their respective probability density functions. Superimposed on the Bayesian flood frequency curves of Figs. 71 through 85 are the observed (Log-Pearson Type III)

TABLE XVI

SCS ESTIMATES OF Tp FOR THE STUDY WATERSHEDS

ID	Tp (hr)
5142	0.70
5143	1.05
5145	0.74
515	1.33
R5	0.35
R6	0.37
R7	0.19
R8	0.20

curves, the USGS curves, and the SCS curves (where applicable).

The Bayesian flood frequency curves approximate the observed flood frequency curves with varying degrees of accuracy. The Bayesian curves were somewhat conservative for eleven of the study watersheds (watersheds 111, 131, 311, 411, 511, 513, 515, 5142, 5143, 5145, and R8), especially up to recurrence intervals of about 10 years. The Bayesian curves for watersheds 311, 513, 5142, 5145, R5, R7, and R8 approximated the observed flood frequency curves with considerable accuracy for recurrence intervals of 10 years and less.

The accuracy of the Bayesian flood frequency curves relative to those derived from USGS and SCS methods may be inferred from direct comparisons of the curves of Figs. 71 through 85. The curves resulting from the SCS procedure are by far the most conservative group of flood frequency curves. Indeed, they are conservative to such a degree that they may confidently be dismissed from consideration as true representatives of their respective peak annual flow generation processes. The USGS curves are in general more conservative than the Bayesian flood frequency curves. They also appear to better approximate the observed curves at higher recurrence intervals than do the Bayesian flood frequency curves. However, one should exercise a measure of common sense in drawing conclusions regarding the relative merits of the Bayesian and USGS curves on the basis of how they compare to the observed curves at high recurrence intervals. One will find upon examining Table I that the maximum length of record for the study watersheds is 16 years, and some record lengths are as short as five years. To extrapolate this historical information on peak flows to recurrence intervals of 25 years may raise serious questions on the worth of the extrapolation; to extrapolate to recurrence intervals of 100 years and greater may produce results only slightly better than what could be obtained from tarot cards or tea leaves. Thus the behavior of the observed curves in the region of high (> 25 years) recurrence intervals is suspect due to the lack of historical information, and there is no overwhelming reason for the any of the competing flood estimation methods to produce flood frequency curves which well simulate the behavior of the observed curves in this region. In the region where one may have confidence in the observed flood frequency curves (i.e., for recurrence intervals of up to about 15 years), the Bayesian curves better represent the observed curves for six watersheds (111, 131, 311, 411, 5143, and R5). For five of the watersheds (513, 5142, 5145, R7, and R8) there is little difference in the accuracy of the curves. For the remaining four study watersheds (511, 515, 611, and R6), the USGS curves more closely approximate the observed curves than do the Bayesian curves. The performance of one flood estimation method relative to another does not appear to be a function of watershed area or any other parameter.

Further inferences regarding the relative accuracy of the competing flood frequency curves may be drawn from Table XVII, which lists Kolmogorov-Smirnov statistics for testing the hypotheses that the different curves are equal to their respective observed curves. Because the critical value (that above which the null hypothesis of equal flood frequency curves is rejected) of the test statistic is 0.521 at the 0.05 significance level, each of the flood frequency curves may be taken as equal to its respective observed curve. Therefore Table XVII may not, strictly speaking, be used to discriminate between significantly better and worse curves. However, one may obtain a very general idea of the relative accuracy of the various curves by comparing the test statistics. The test statistic is taken as the maximum deviation between the cumulative distribution function (derived from the flood frequency curve) of one of the competitors and the observed cumulative distribution function, and it almost invariably takes its value as the result of a deviation in the vicinity of 50% cumulative probability. When one flood frequency curve has a lower test statistic than its competitor, one may infer that it is a better representative of the observed curve in the region of 2-5 year recurrence intervals. Since the observed

TABLE XVII

KOLMOGOROV-SMIRNOV TEST STATISTICS, 1

ID	Estimated	USGS	SCS
111	0.28	0.39	*
131	0.38	0.45	*
311	0.16	0.15	*
411	0.28	0.45	*
511	0.30	0.14	*
513	0.16	0.10	*
5142	0.20	0.19	**
5143	0.27	0.39	**
5145	0.17	0.05	**
515	0.46	0.35	**
611	0.17	0.06	*
R5	0.07	0.29	0.49
R6	0.30	0.23	0.48
R7	0.09	0.23	**
R8	0.29	0.02	**

* Not available.
** Kolmogorov-Smirnov test statistic could not be derived.

curves are probably most accurate in this region, this type of comparison is not altogether meaningless; it is only statistically inconclusive. For six of the study watersheds, the Bayesian flood frequency curves have a lower test statistic than the USGS curves. For the remainder of the study watersheds, the USGS curves have a lower test statistic. It may be inferred from this comparison that in general, the USGS curves seem to better represent the observed curves in the region of approximately 2-5 year recurrence intervals.

A striking feature of Figs. 71 through 85 is the width of the 90% confidence intervals for the modeled flood frequency curves. The 90% confidence interval on the 100-year flood for the largest study watershed (511) is from approximately 9000 to 36000 cfs; for the smallest study watershed (R7), the bounds are from approximately 80 to 400 cfs. The widths of these confidence intervals, which are indicative of the degree of uncertainty in the Bayesian flood frequency curves, are a function of informational uncertainty. At this point, the knowledge on the model parameters S and Tp is derived solely on the basis of the regression relationships. As such, it is necessarily less precise than that which would be obtained from site-specific information. The penalty for the lack of site-specific information is relatively high uncertainty in S and Tp, which is in turn passed on to the Bayesian flood frequency curves in the form of relatively wide confidence intervals.

The confidence intervals on the Bayesian flood frequency curves will have a direct impact on engineering judgments regarding hydrologic structures built upon these watersheds. To illustrate, assume that the procedure for estimating the flood frequency curves is valid and that one of these curves is to be used to design a hydrologic structure for watershed 511. If a design criterion for a hydrologic structure is that it accommodate the 100-year peak flow, then the structure must be designed for a flow of roughly 36000 cfs in order to be 90% certain that it would meet this criterion. Therefore, one must say at this point that there is a 10% risk that the structure will fail to meet the criterion if it is designed for this flow. It would be more common to design the structure for a flow of about 17000 cfs, which is the expected value of the 100-year peak flow. However, the concomitant risk that the structure will fail to accommodate the true 100-year flow will be increased to 50%. The effect of the confidence intervals is to complicate the questions of risk associated with a particular structure. One may not design a structure on watershed 511 for a flow of 17000 cfs and state that there is an annual risk of failure of 1%. The annual risk of failure is in fact somewhat greater than 1% because the value of the 100-year peak annual flow is itself uncertain, with its uncertainty described in part by its confidence intervals.

Concluding Remarks

The Bayesian flood frequency curves appear, in general, to overestimate the observed curves; however, the differences between the Bayesian and observed curves are insignificant from the standpoint of Kolmogorov-Smirnov goodness of fit tests conducted at the 0.05 significance level. By the same token, the USGS curves are not significantly different than the observed curves. In addition, the USGS curves seem, on the whole, to better represent the observed curves than do the Bayesian curves. The SCS curves appear to be greatly in error with respect to the observed curves and uniformly overestimate the T-year annual peak flow relative to either the Bayesian or USGS curves.

The 90% confidence intervals on the Bayesian flood frequency curves seem to be quite wide, indicating a high degree of uncertainty in the curves. This is not unexpected; it simply reflects the imprecision of the data available at this point on the model parameters S and Tp.

The flood estimation methodology itself has been shown practicable. Although conclusive statements regarding its accuracy are not possible, it seems to produce reasonable results. This procedure of flood estimation contains several advantages over its competitors. First, the methodology is more concerned with the factors which influence the rainfall-runoff phenomena than the USGS procedure. It allows one to isolate, to a degree, the effects of land slope, channel length, and land usage, as well as other hydrologic quantities. In contrast to unmodified SCS procedures, the flood estimation methodology produces confidence limits on the flood frequency curves, allowing one to infer the uncertainty of the curves. Also, the prediction equations regarding Tp allow one to estimate this parameter for watersheds having area greater than 2000 acres without the necessity for the impractically large amount of data required by SCS methods.

There are several possible sources of error in the procedure for estimating flood frequency curves for ungaged watersheds, all of which may contribute to differences between Bayesian and observed curves. The first source of error is the model itself. The SCS unit hydrograph model is not a physically-based model, but rather an over-simplification of the rainfall-runoff process. There is necessarily a limit to the accuracy one may reasonably expect from such a model, regardless of the precision with which its parameters are determined. A second source of error is the assumption that the T-year rainfall produces the T-year annual peak flow. This assumption is commonly made, but it is by no means universally held as true. Thirdly, the rainfall depths determined from Hershfield (1963) may not be accurate for the region in which the study watersheds are located. Haan and Edwards (1987) reported problems in some cases in resolving Hershfield's rainfall depths to observed rainfall depths. A fourth potential source of error is the regionally-derived prediction equations. The prediction equations were derived from a relatively small set of observations. The addition of more observations would certainly affect the values of the coefficients and may even change the basic forms of the equations. Even if the forms of the equations and their coefficients are correct, the random nature of the relationship between predictions and observations may lead in some cases to relatively large errors of prediction which would adversely affect the accuracy of the Bayesian flood frequency curves.

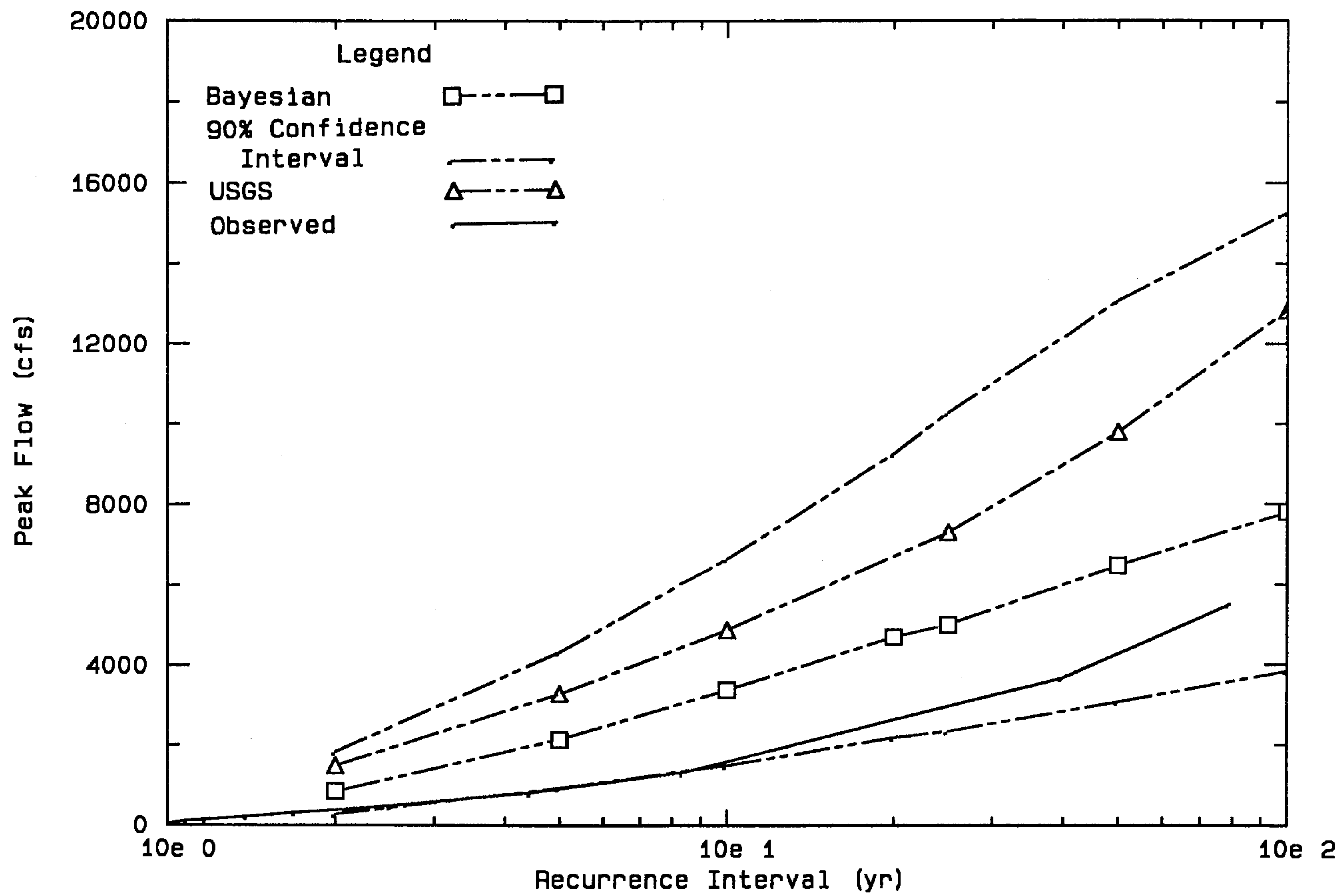

Figure 71. Flood Frequency Curves for Watershed 111, 1

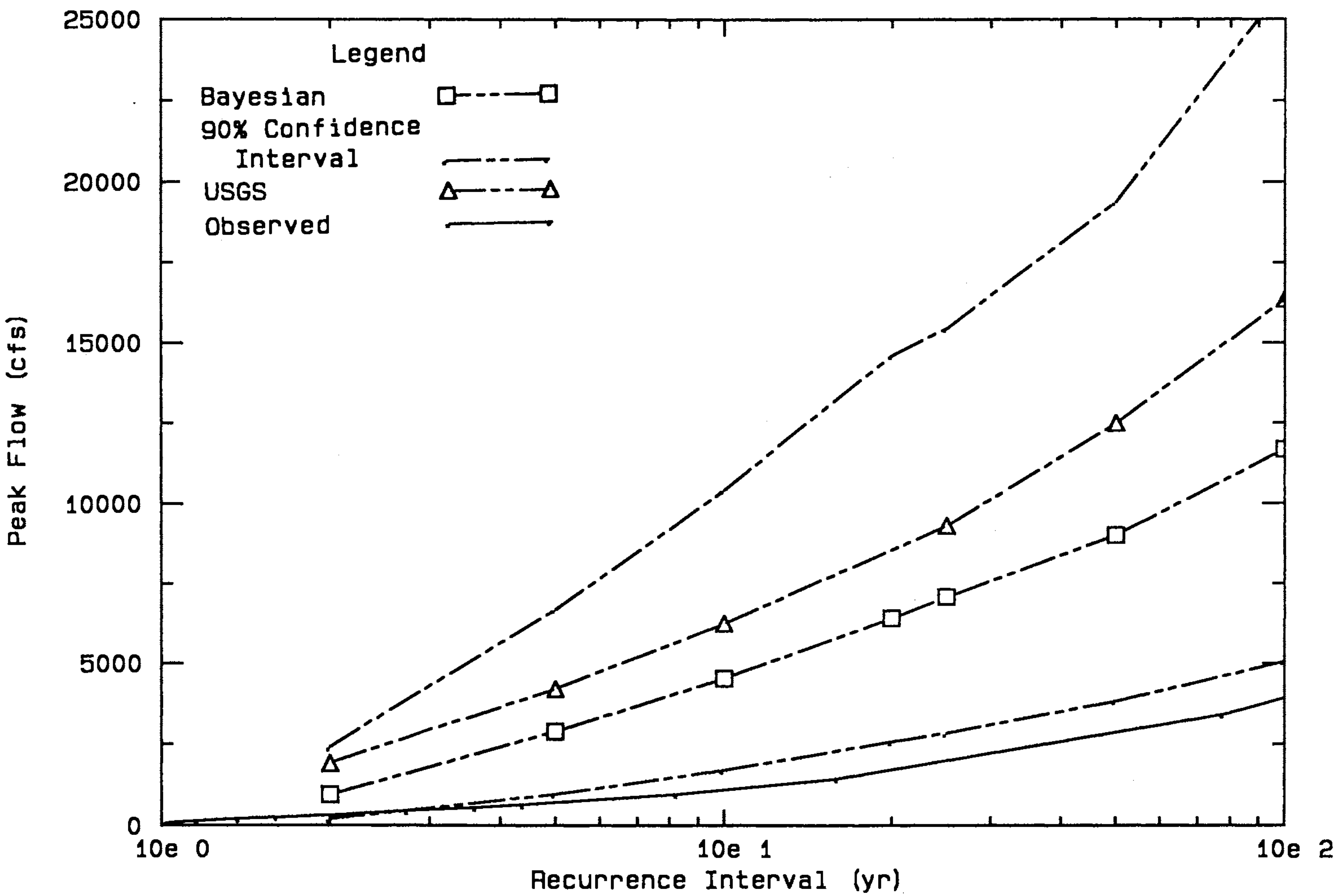

Figure 72. Flood Frequency Curves for Watershed 131, 1

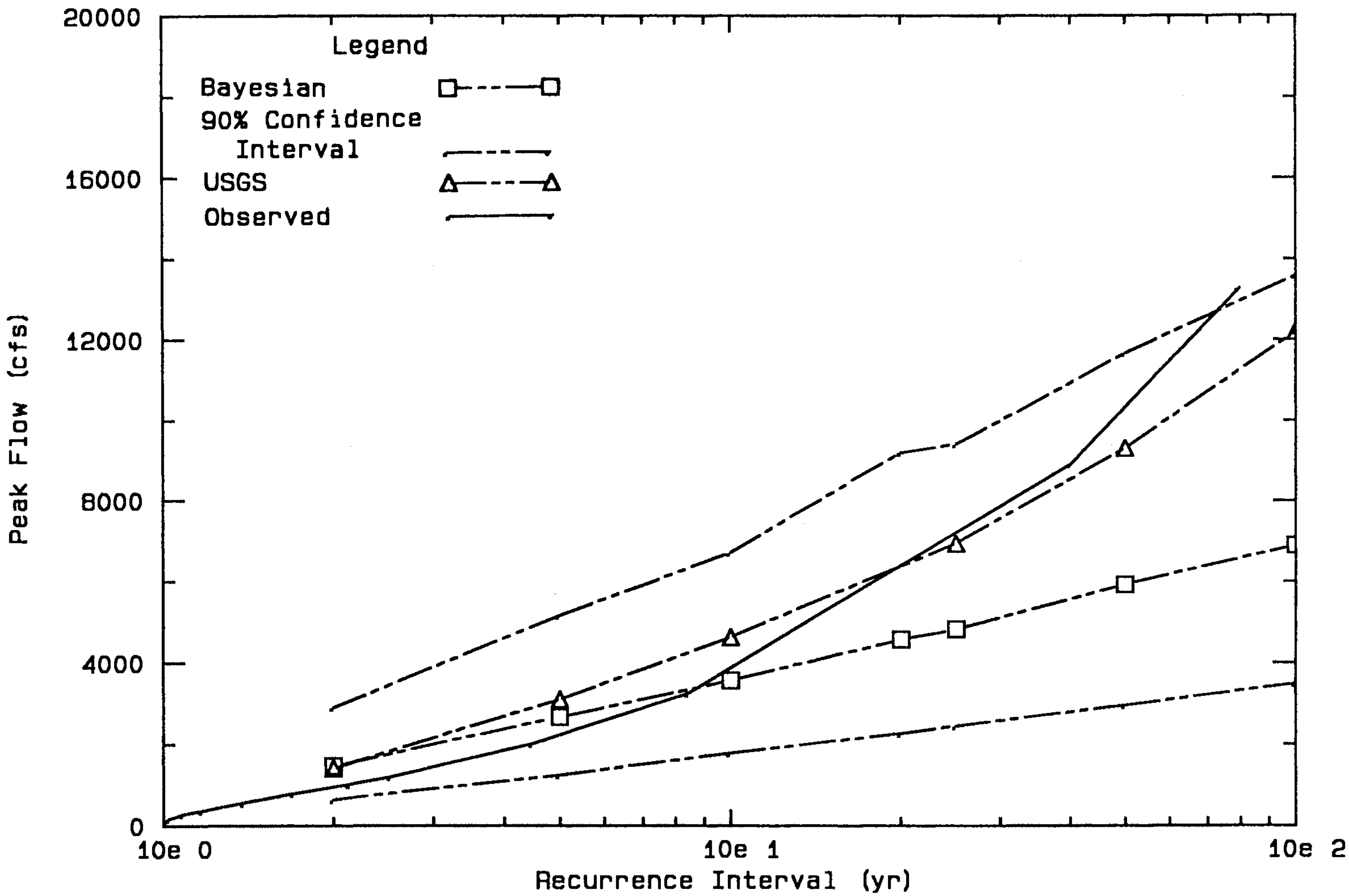

Figure 73. Flood Frequency Curves for Watershed 311, 1

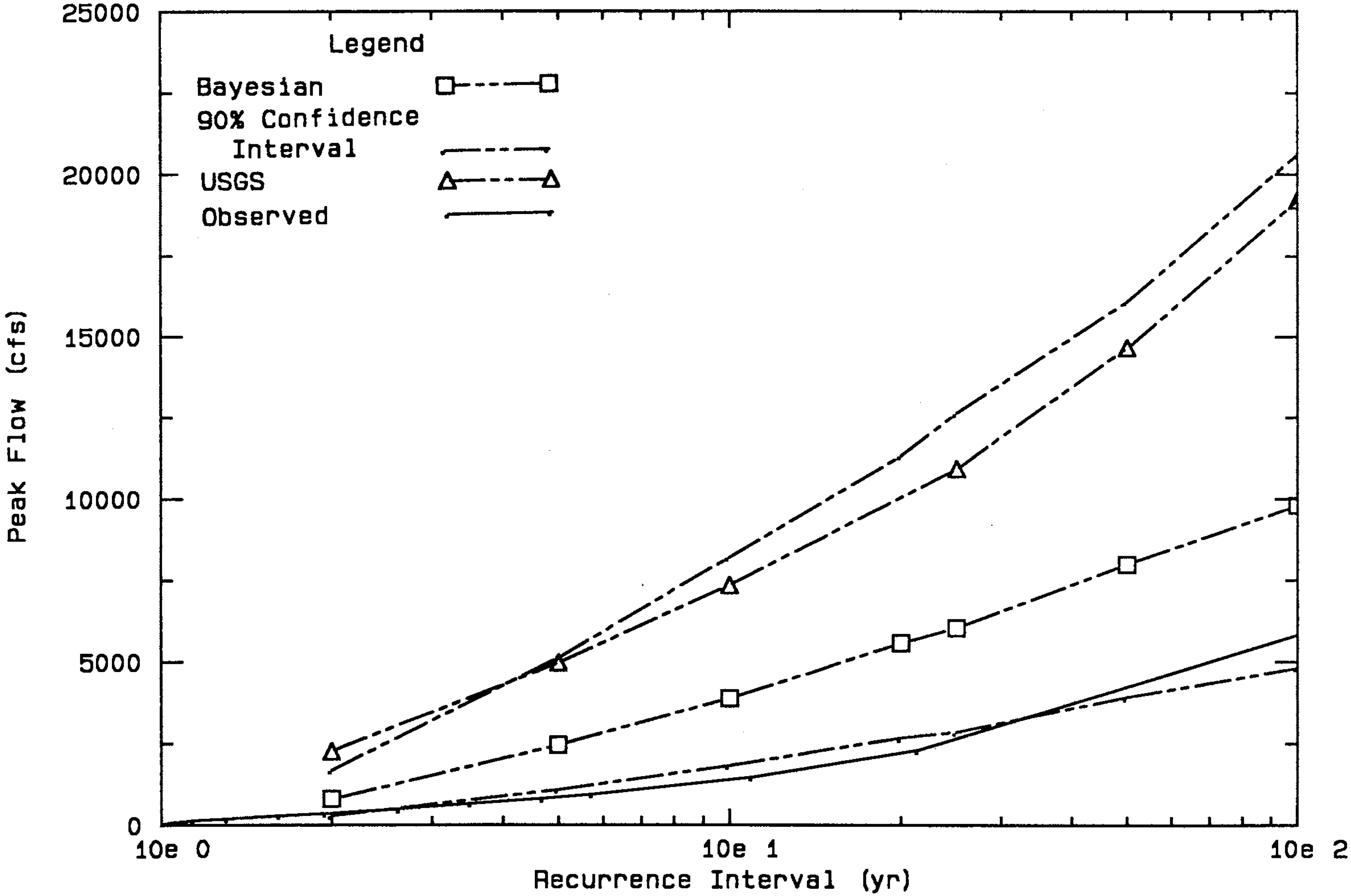

Figure 74. Flood Frequency Curves for Watershed 411, 1

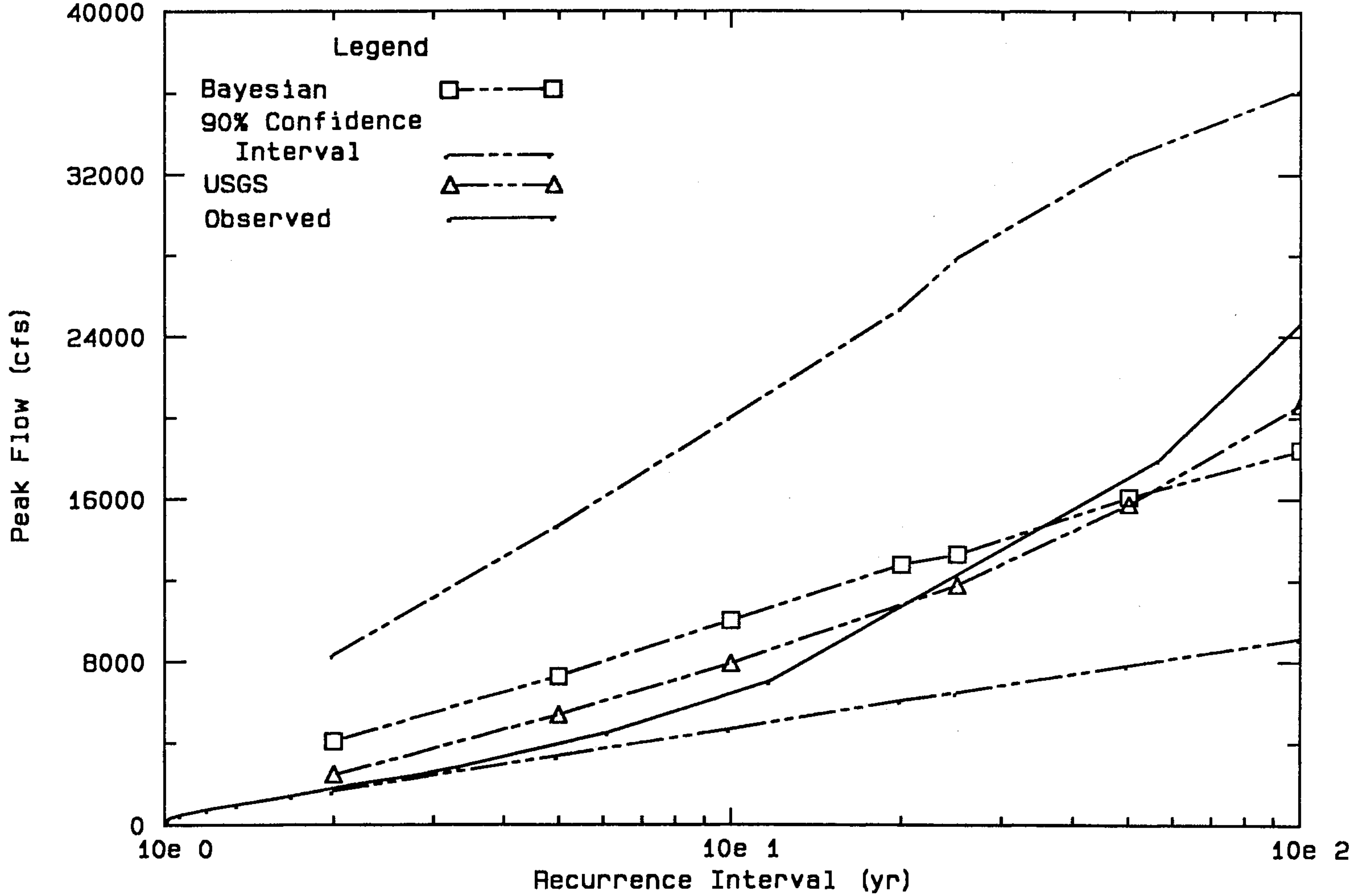

Figure 75. Flood Frequency Curves for Watershed 511, 1

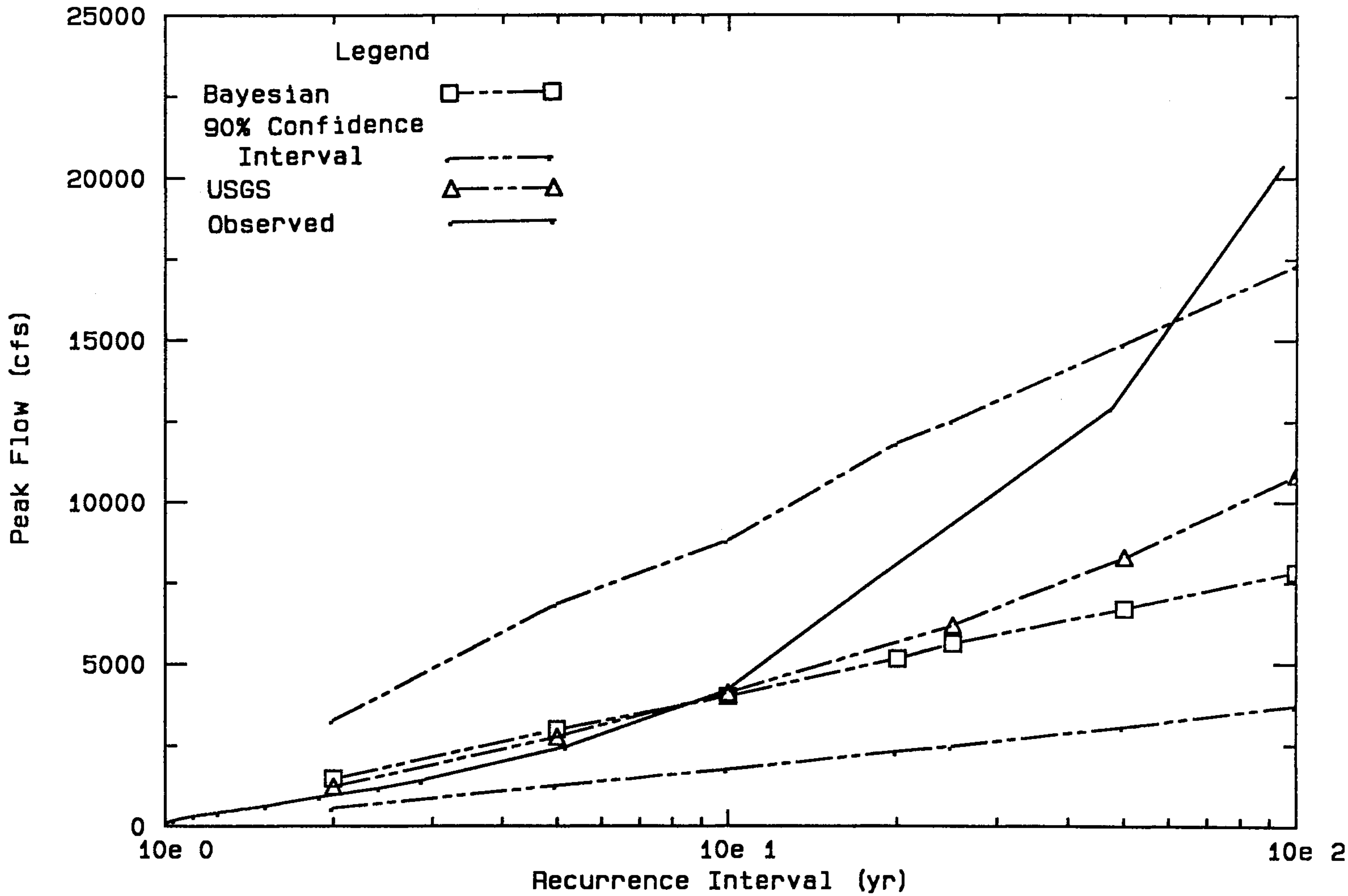

Figure 76. Flood Frequency Curves for Watershed 513, 1

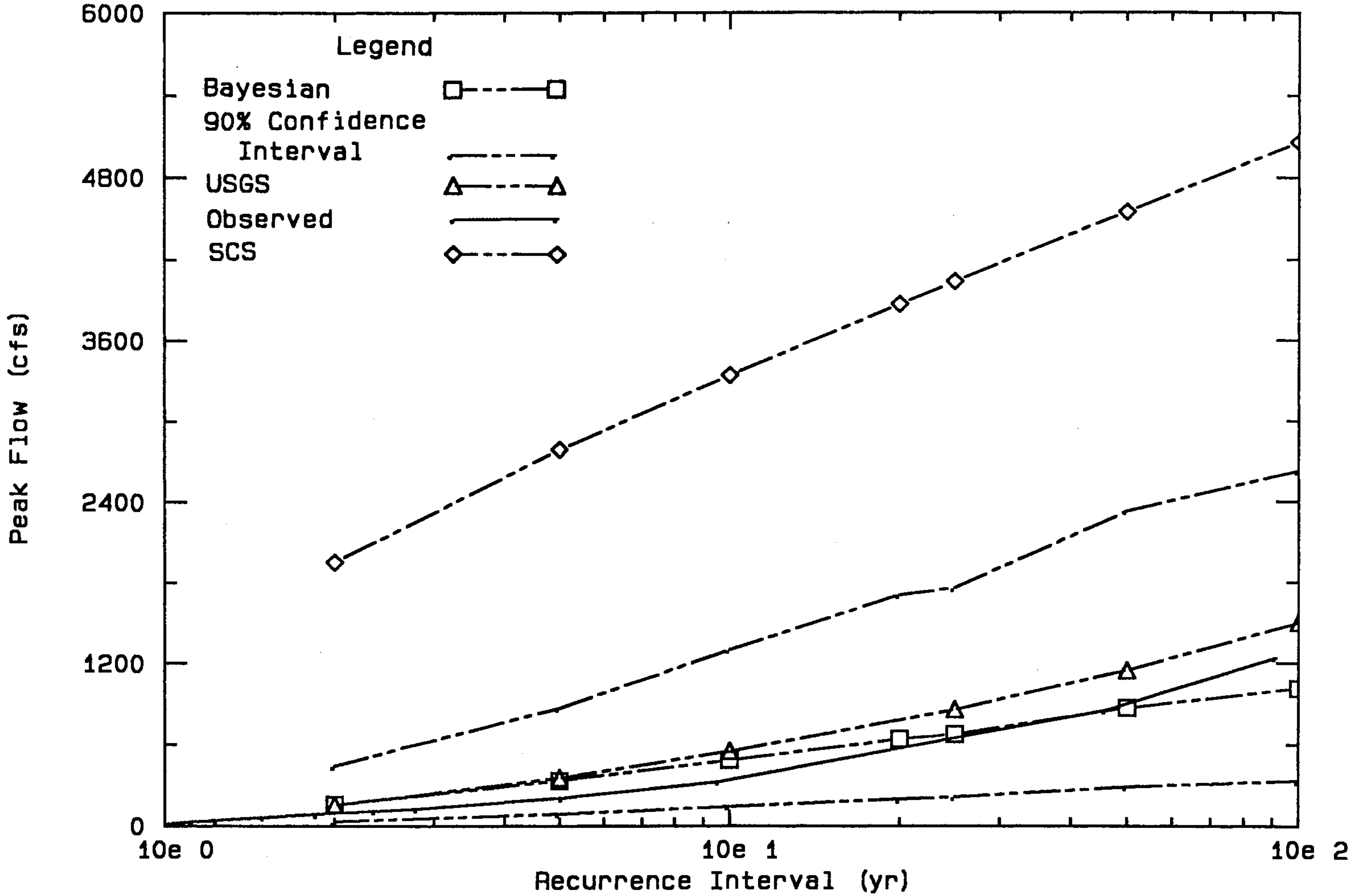

Figure 77. Flood Frequency Curves for Watershed 5142, 1

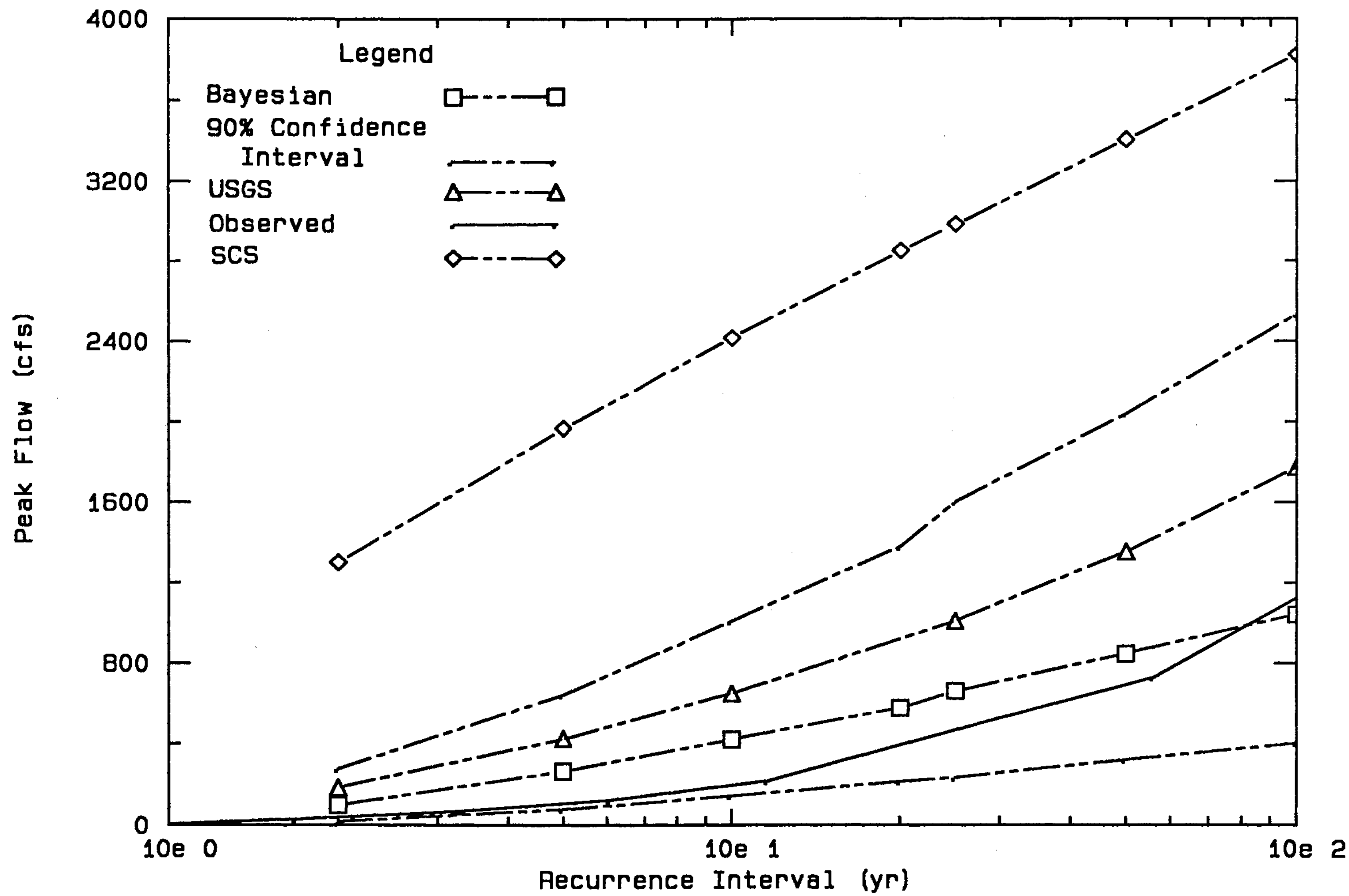

Figure 78. Flood Frequency Curves for Watershed 5143, 1

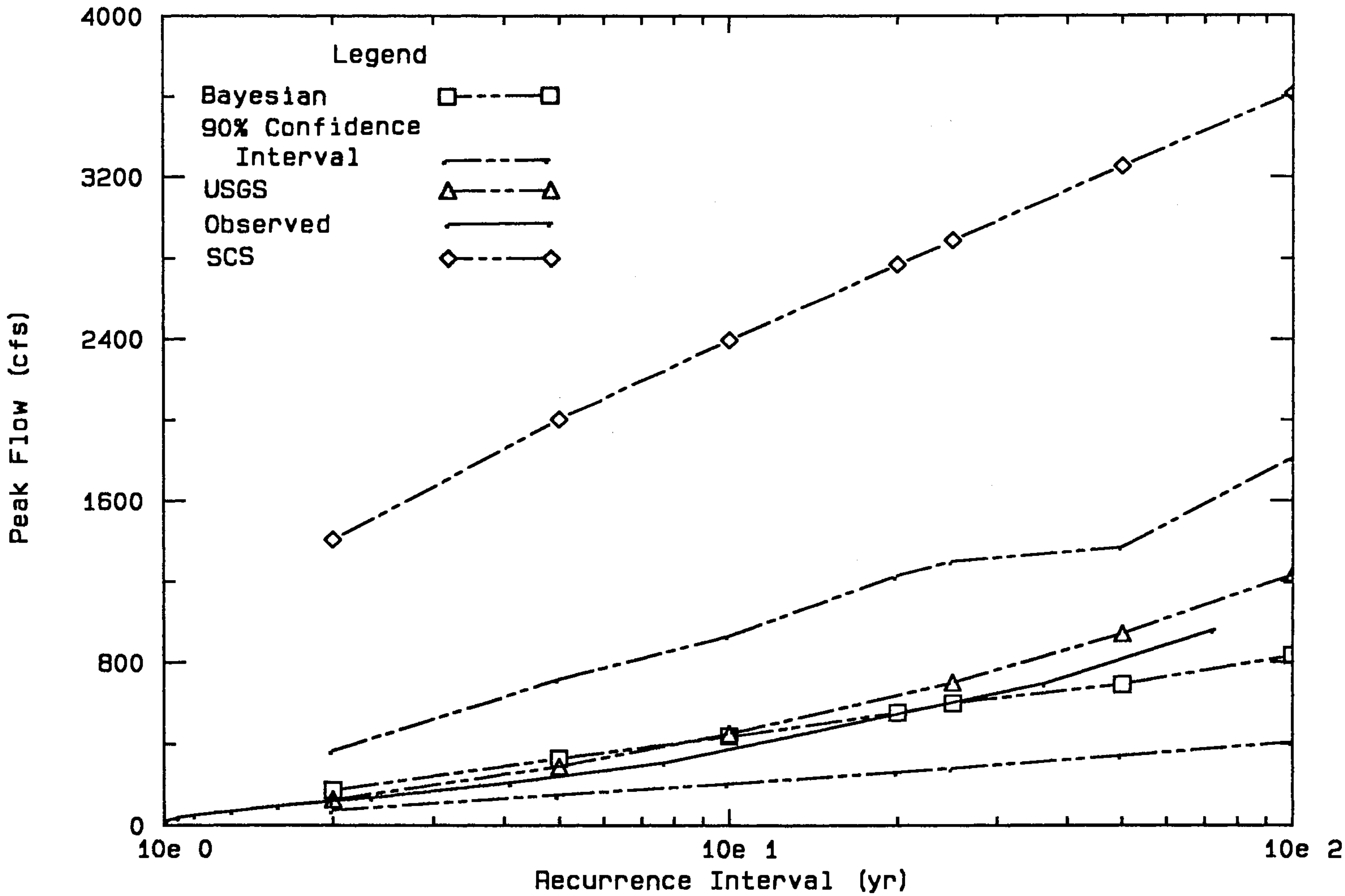

Figure 79. Flood Frequency Curves for Watershed 5145, 1

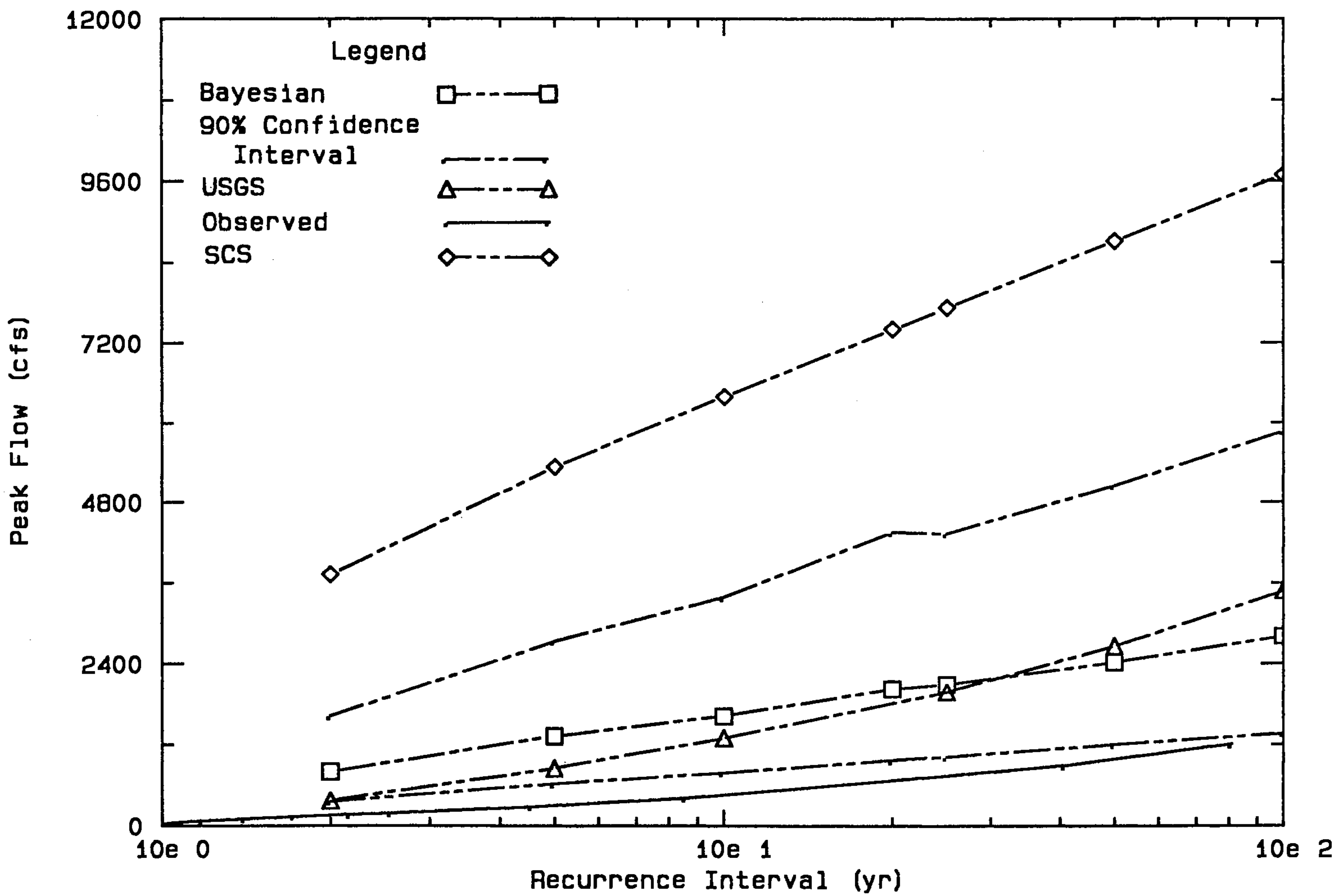

Figure 80. Flood Frequency Curves for Watershed 515, 1

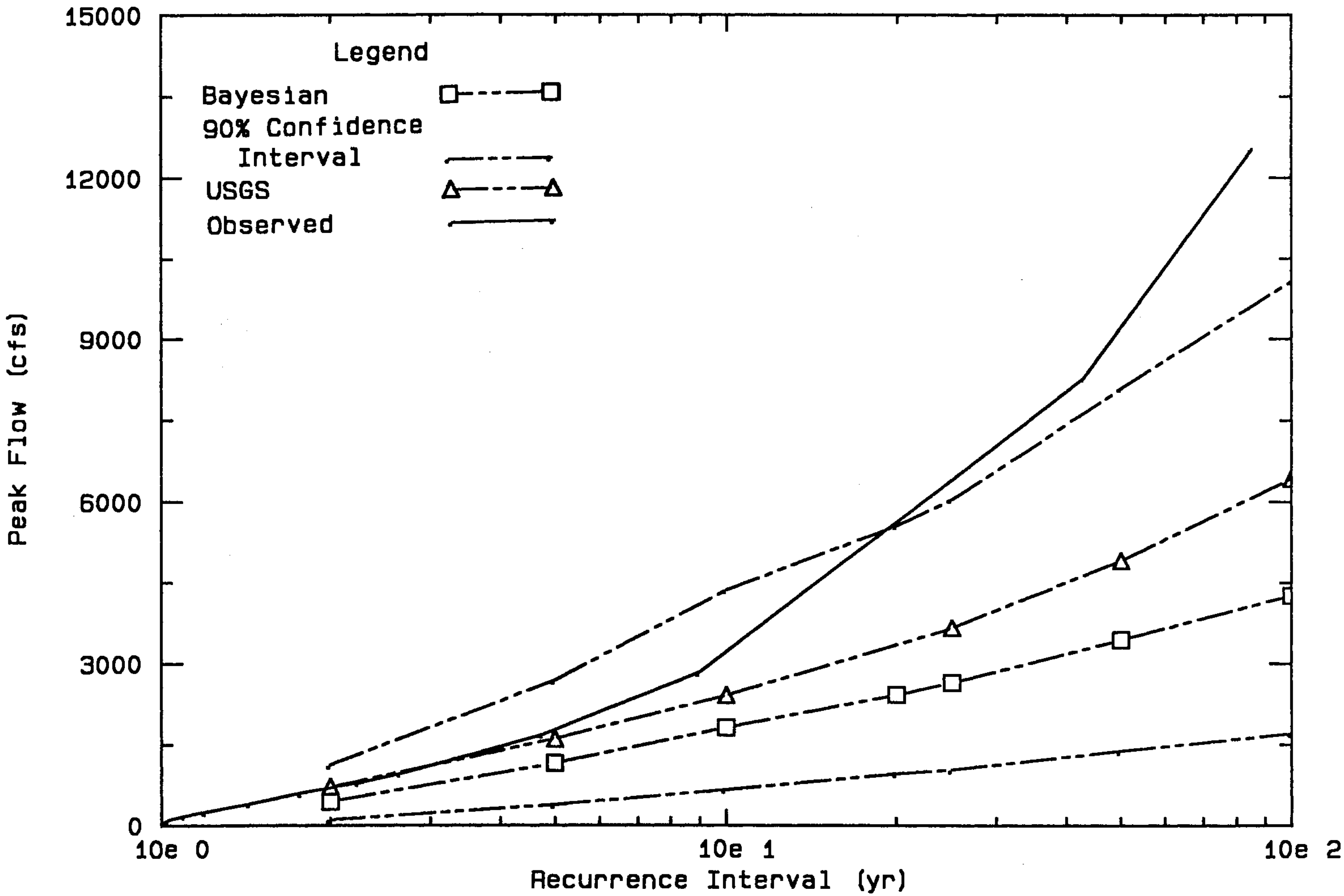

Figure 81. Flood Frequency Curves for Watershed 611, 1

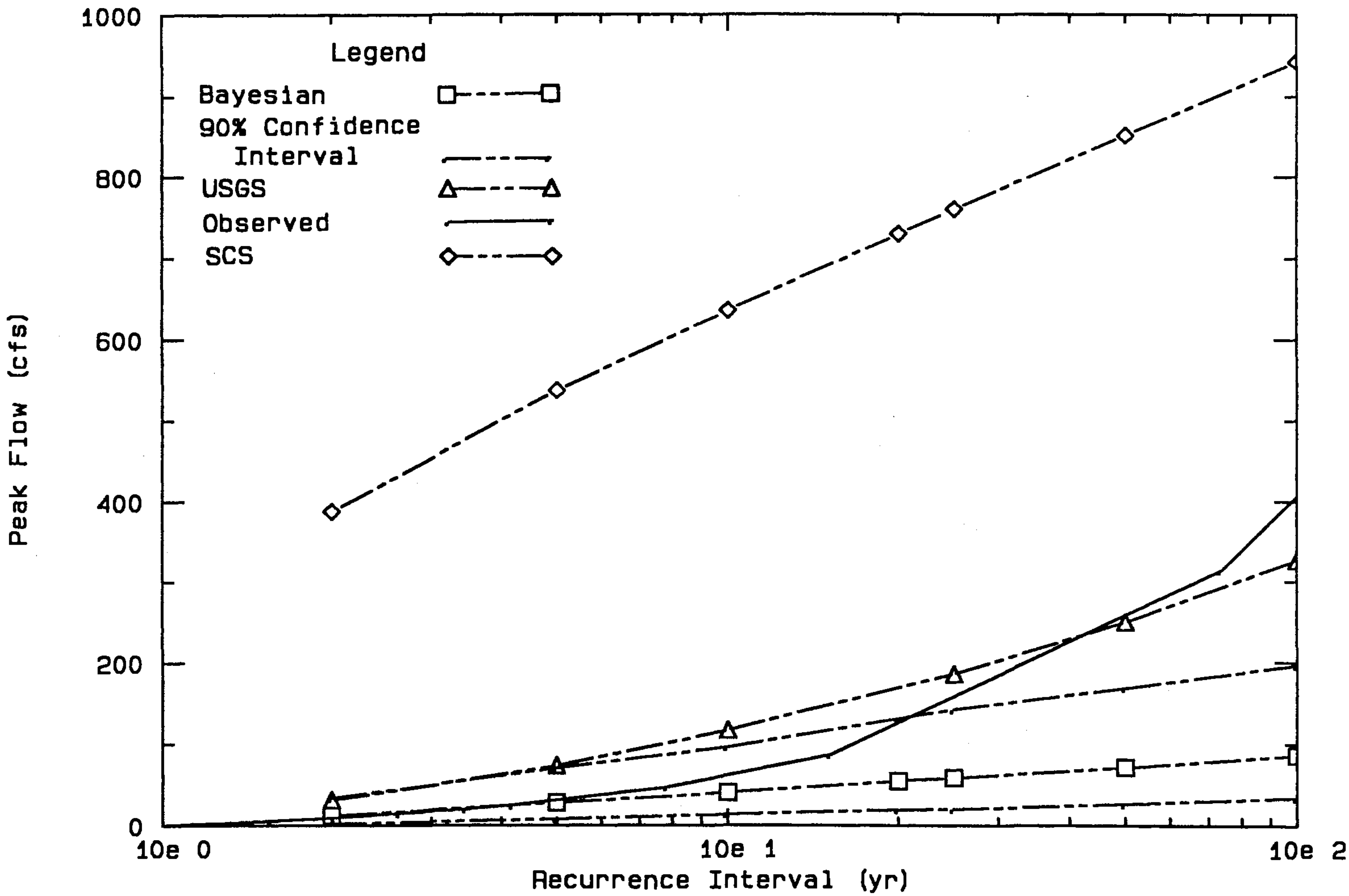

Figure 82. Flood Frequency Curves for Watershed R5, 1

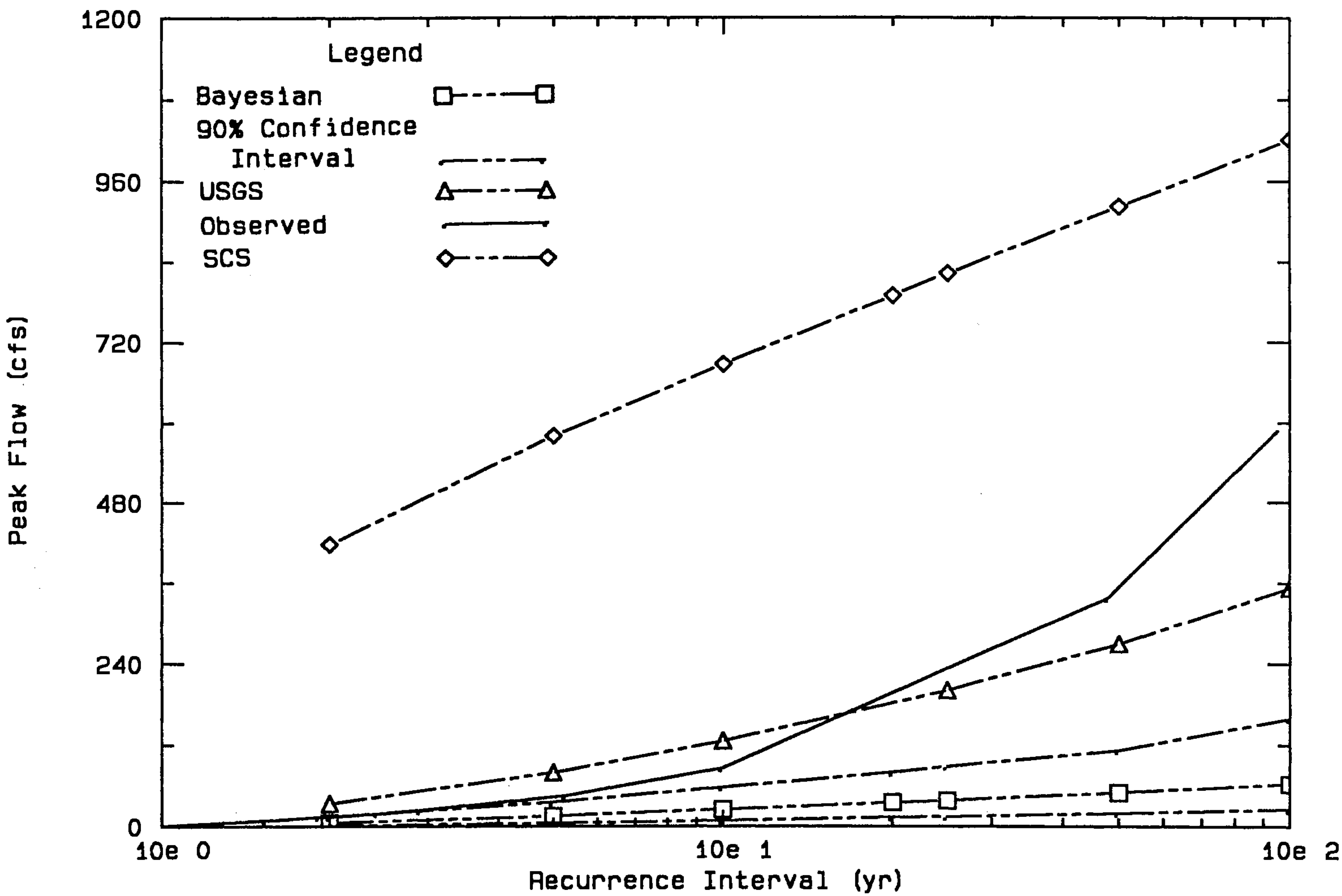

Figure 83. Flood Frequency Curves for Watershed R6, 1

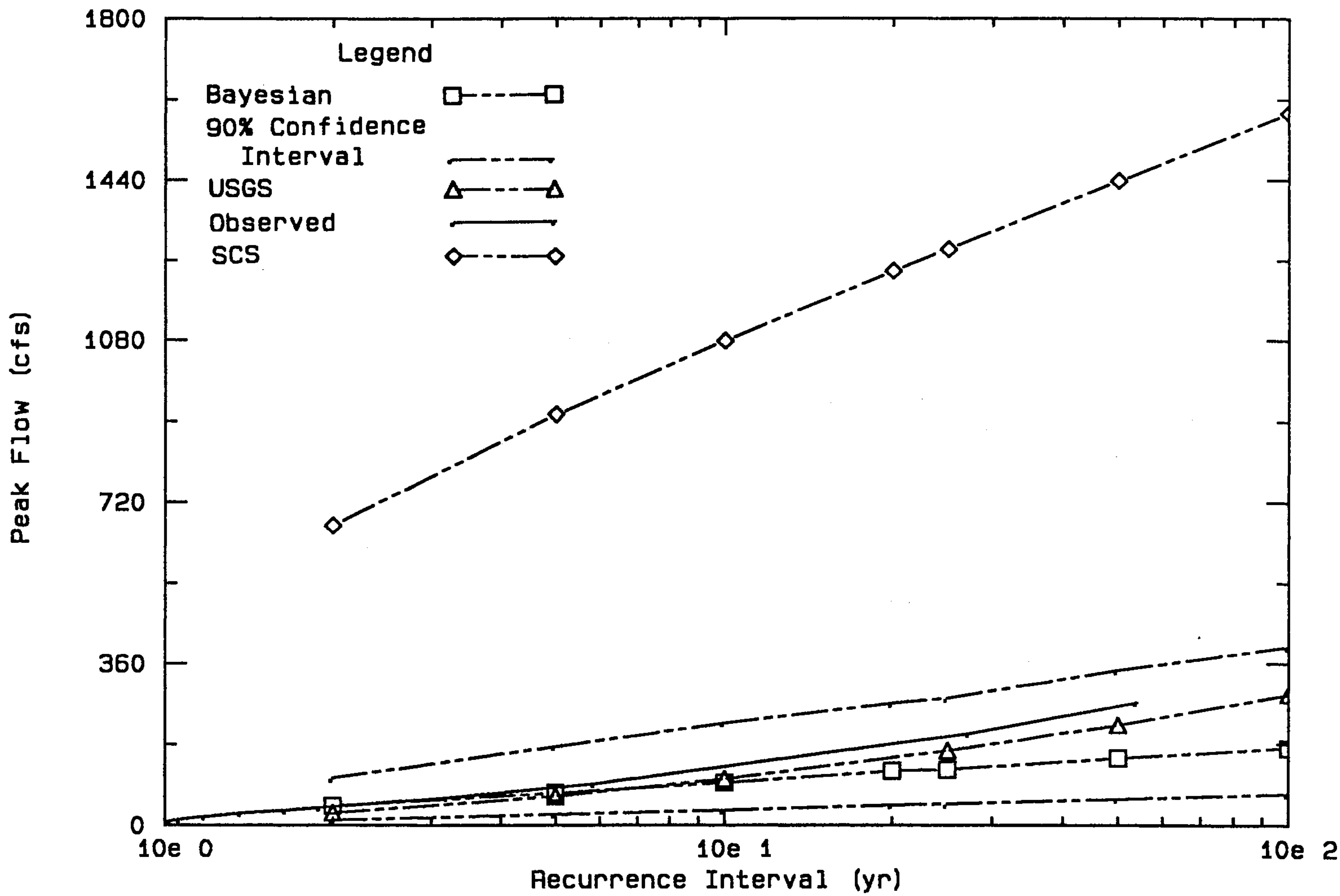

Figure 84. Flood Frequency Curves for Watershed R7, 1

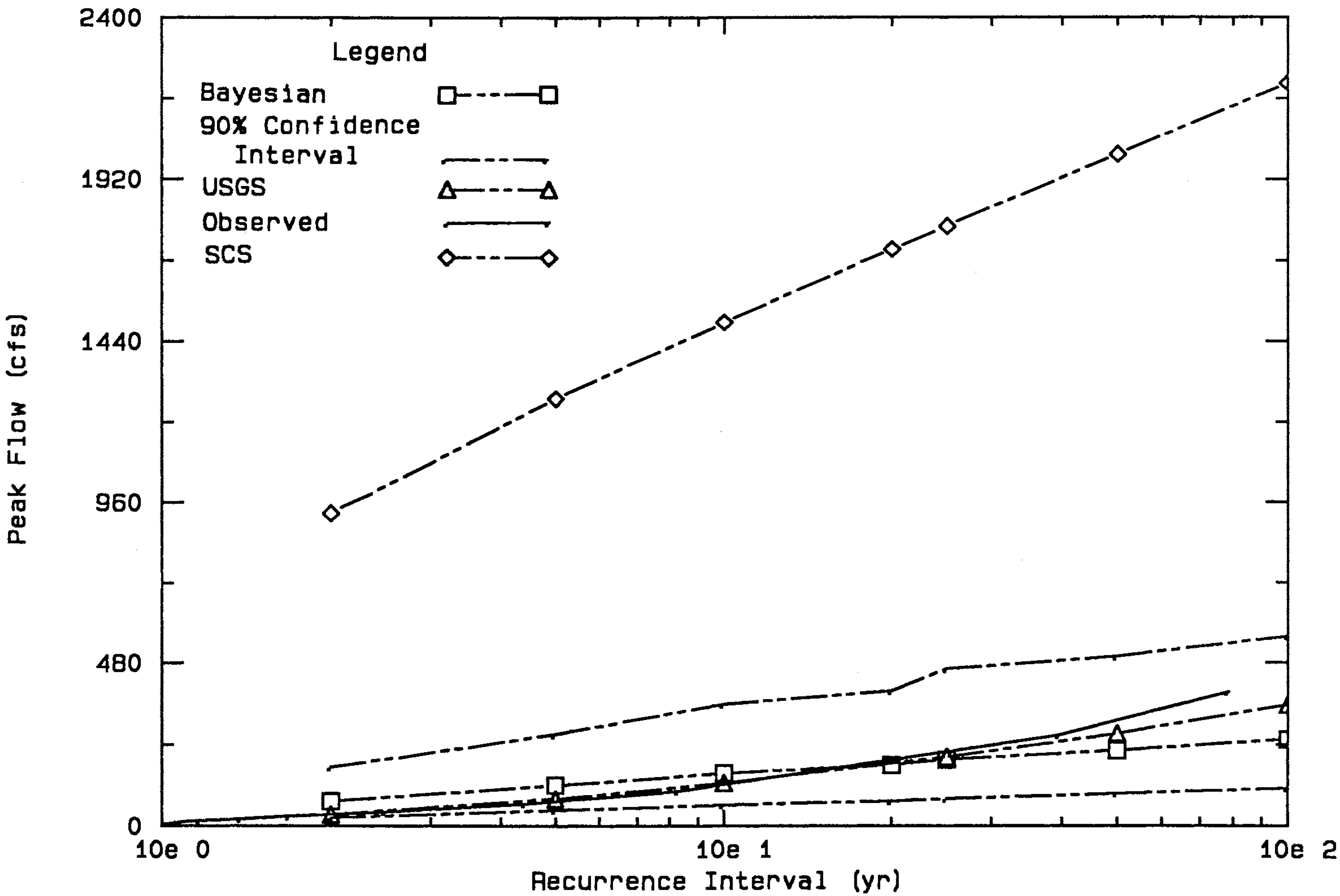

Figure 85. Flood Frequency Curves for Watershed R8, 1

CHAPTER VIII

DEVELOPMENT OF A FLOOD ESTIMATION METHODOLOGY FOR WATERSHEDS WITH SHORT RECORDS

The flood estimation methodology developed in Chapter VI is now extended to the situation of watersheds with short records. The objective is to combine site-specific information with the regionally-derived information in order to reduce informational uncertainty in the model parameters S and Tp. The reduction of uncertainty in S and Tp will be translated into a reduction of uncertainty in the estimated flood frequency curves and will effect a narrowing of the 90% confidence intervals of Figs. 71 through 85. The basic approach is to consider the joint probability density function of the model parameters S and Tp derived from regional information as the prior probability density function of Eqn. 18, which is a version of Bayes' Theorem. Such a prior probability density function is thus data based. To use this type of prior probability density function implies significant information on the parameters S and Tp prior to collecting site-specific data, and this information will augment the likelihood function of Eqn. 18 in a definite fashion. This is in contrast to the non-informative prior probability density functions used in Chapter IV, which were derived by assuming negligible prior information and contributed virtually nothing to the posterior probability density function of the model parameters. Site-specific data on peak flow and runoff volume are used to derive the likelihood function of S and Tp. Substituting this likelihood function into Eqn. 18 allows for the determination of the posterior probability density function of the model parameters. The remainder of the flood estimation methodology closely follows that presented in Chapter VI.

Derivation of the Posterior Probability Density

Function Of the Model Parameters

Given the available data on peak flow and runoff volume, the likelihood function of $\underline{\Theta}$, where $\underline{\Theta} = (S,Tp,\underline{\Sigma})$, is given by the right-hand side of Eqn. 94. The prior probability density function of (S,Tp) is derived using regional information as described in Chapter VI, which results in S being taken as $N(\mu_S,\sigma_S^2)$ and Tp being taken as $N(\mu_T,\sigma_T^2)$. Since S and Tp are considered a priori independent, their prior joint probability density function may be written as

$$p'(s,t_p) \propto |\underline{V}|^{-1/2}\exp[-\frac{1}{2}\underline{M(\theta)}^T\underline{V}^{-1}\underline{M(\theta)}] \tag{120}$$

$$\text{where } \underline{V} = \begin{bmatrix} \sigma_S^2 & 0 \\ 0 & \sigma_T^2 \end{bmatrix} \tag{121}$$

$$\underline{M(\theta)} = (s-\mu_S,t_p-\mu_T) \tag{122}$$

The prior probability density function of $\underline{\Sigma}$ is given by Eqn. 65. If it may be assumed that (S,Tp) and $\underline{\Sigma}$ are independent, then

$$p'(\underline{\theta}) \propto p'(s,t_p)p'(\underline{\Sigma}) \tag{123}$$

Bayes' Theorem may now be applied to yield the posterior probability density function of $(\underline{\Theta})$ as

$$p''(\underline{\theta}) \propto l(\underline{\theta|y})p'(s,t_p)p'(\underline{\Sigma}) \tag{124}$$

The covariance matrix of the transformed residuals, $\underline{\Sigma}$, is of no interest; it is a nuisance parameter. It should therefore be integrated out of Eqn. 124 to yield the marginal probability density function of (S,Tp) as

$$p''(s,t_p) \propto \int l(\underline{\theta}|\underline{y})p'(s,t_p)p'(\underline{\Sigma})d\underline{\Sigma} \tag{125}$$

where the integral is taken over m-dimensional real space. Box and Tiao (1973) state that if $\underline{\Sigma}$ is positive definite symmetric (as it must be in this context), then the solution to Eqn. 125 is given by

$$p''(s,t_p) \propto |\underline{V}|^{-\frac{1}{2}}\exp[-\frac{1}{2}\underline{M(\theta)}^T\underline{V}^{-1}\underline{M(\theta)}]|\ \underline{S(\theta)}|^{-n/2} \tag{126}$$

where $\underline{S(\theta)}$ is as defined in connection with Eqn. 94. The marginal posterior probability density functions of S and Tp may be found upon integrating Eqn. 126 with respect to each of the model parameters.

Marginal Probability Density Functions Of the Model Parameters

The integration of Eqn. 126 was performed for each of the study watersheds and then integrated with respect to each of the model parameters in order to derive the marginal posterior probability density functions of S and Tp. A sample of these are shown in Figs. 86 through 93. In order to illustrate the effect of site-specific information on these functions, the corresponding prior probability density functions are also plotted in these figures. It may be seen that in every case, the site-specific information has the effect of decreasing the uncertainty in the model parameters. This is evidenced by the relative peakedness of the posterior probability density functions. It may also be seen that in some cases the site-specific information has caused the peak of the posterior probability density function to be translated with respect to the peak of the prior probability density function. This illustrates that for a given watershed, the site-specific information tends to adjust the regression-based prediction of the mean of a parameter to its true (see Chapter II for a definition of "true") value.

The posterior means, variances, and coefficients of skewness of the model parameters S and Tp were computed by integrating the marginal posterior probability density functions of the two parameters. Their values appear in Table XVIII. A comparison of the variances of this table to those of Table XIV will demonstrate that the site-specific information has indeed reduced uncertainty in the model parameters. The coefficients of skewness presented in Table XVIII are generally small in magnitude. Because of these low coefficients of skewness and the shapes of the posterior probability density functions of Figs. 86 through 93, the posterior distributions of S and Tp may be considered normal.

Summarized Flood Estimation Procedure For Watersheds with Short Records

The flood estimation methodology for watersheds with short records is summarized as:

1. Derive the prior probability density functions of S and Tp as described in Chapter VI.
2. Solve Eqn. 126 to find the joint posterior probability density function of (S,Tp).
3. Compute the marginal posterior probability density functions of S and Tp by integrating the result of Step 2 with respect to each of the parameters.
4. Integrate the marginal posterior probability density functions of S and Tp in order to derive the mean of S (m_S), the variance of S (s_S^2), the mean of Tp (m_T),

TABLE XVIII

STATISTICS OF PARAMETERS S AND Tp, 3

ID	m_S (in)	s_S (in)	g_S (in^3)	m_T (hr)	s_T (hr)	g_T (hr^3)
111	6.84	0.67	0.90	1.51	0.24	-0.44
131	7.06	0.48	0.43	4.69	0.35	0.42
311	3.47	0.26	0.10	5.86	0.19	0.22
411	7.61	0.71	0.15	6.40	0.87	0.41
511	4.54	0.26	0.07	4.89	0.18	0.49
513	4.01	0.27	0.41	2.78	0.14	0.14
5142	3.56	0.31	0.30	0.46	0.11	0.46
5143	5.55	0.38	0.07	0.64	0.10	0.29
5145	2.65	0.27	0.40	0.67	0.08	-0.25
515	2.64	0.25	0.09	3.57	0.29	-0.07
611	4.66	0.34	0.55	0.97	0.08	0.86
R5	3.46	0.26	0.38	0.34	0.03	0.22
R6	2.89	0.26	0.15	0.17	0.04	-0.03
R7	1.36	0.15	0.27	0.12	0.01	-0.09
R8	1.69	0.14	0.28	0.12	0.02	0.46

m_S = mean of S.
s_S = standard deviation of S.
g_S = coefficient of skewness of S.
m_T = mean of Tp.
s_T = standard deviation of Tp.
g_T = coefficient of skewness of Tp.

and the variance of Tp (s_T^2).

5. Compute the appropriate rainfall duration to be used as $1.5m_T$.

6. Obtain the rainfall depth for the desired recurrence interval from Hershfield (1963) using the rainfall duration obtained in Step 6.

7. Generate multiple pairs of non-negative values for S and Tp taking S as $N(m_S,s_S^2)$ and Tp as $N(m_T,s_T^2)$.

8. Input each pair of values of S and Tp to the SCS unit hydrograph model along with the watershed area, the rainfall depth determined in Step 6, and the temporal distribution of the rainfall (taken as SCS Type II), and compute the resulting peak flows.

9. Compute the mean resultant peak flow and assign to it the same recurrence interval as the rainfall event.

10. Compute the upper and lower bounds of the $(1-\alpha)\%$ confidence interval for resultant peak flow.

11. Repeat Steps 5 through 10 for all desired recurrence intervals.

12. Plotting mean resultant peak flow and the bounds of the $(1-\alpha)\%$ confidence interval vs. recurrence interval results in the estimated flood frequency curve, with confidence intervals, for the watershed.

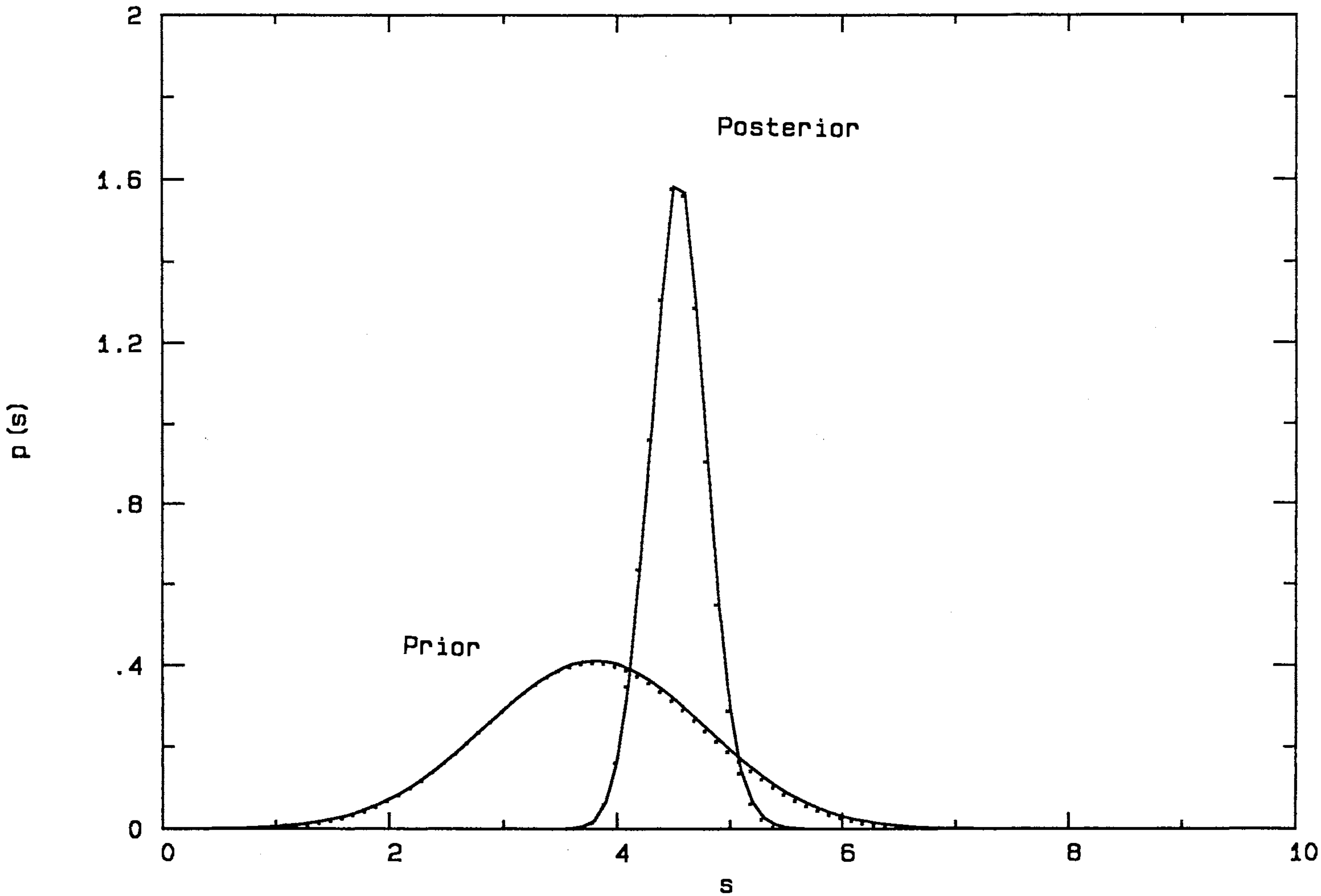

Figure 86. Prior and Posterior Probability Density Functions of S for Watershed 511

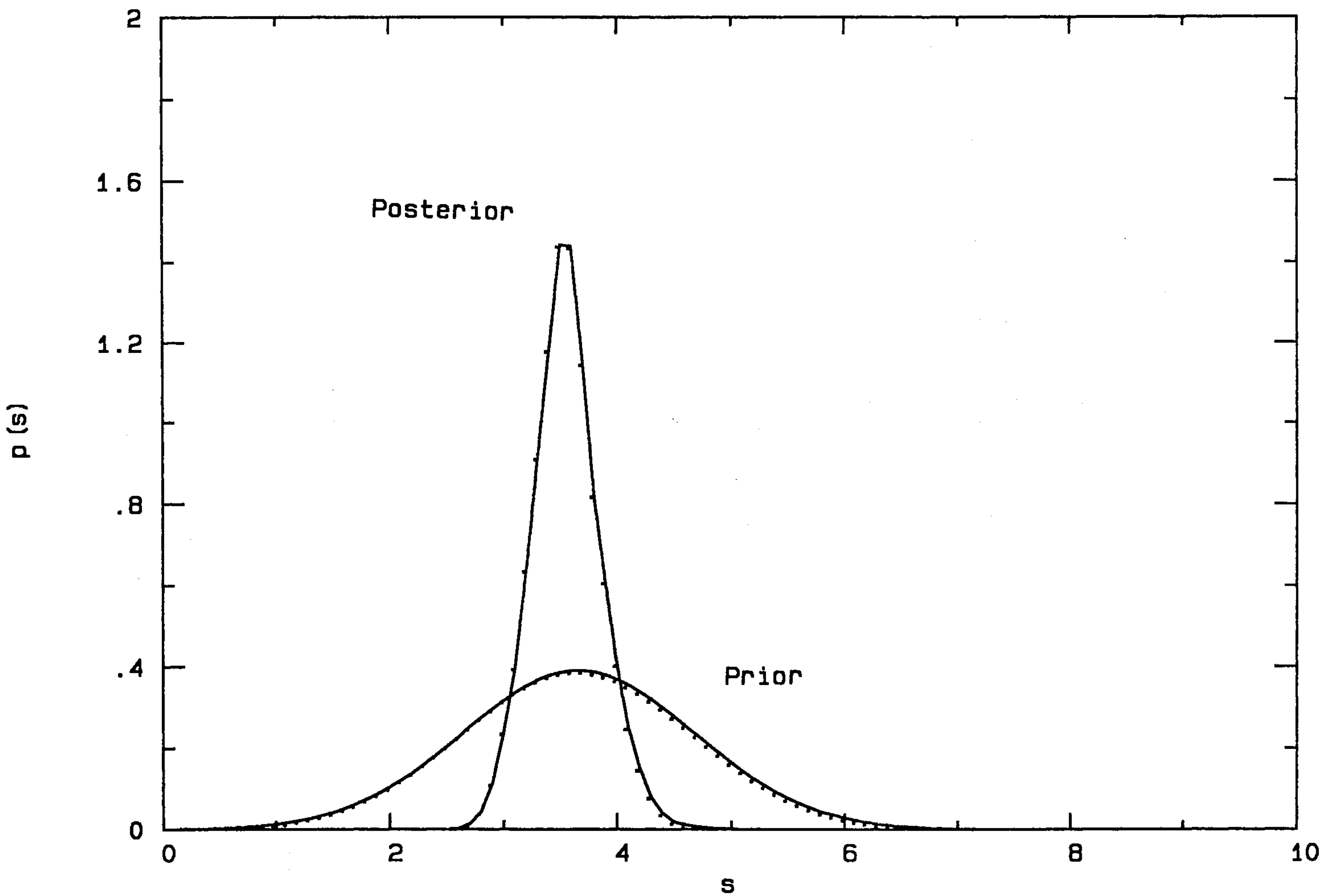

Figure 87. Prior and Posterior Probability Density Functions of S for Watershed 5142

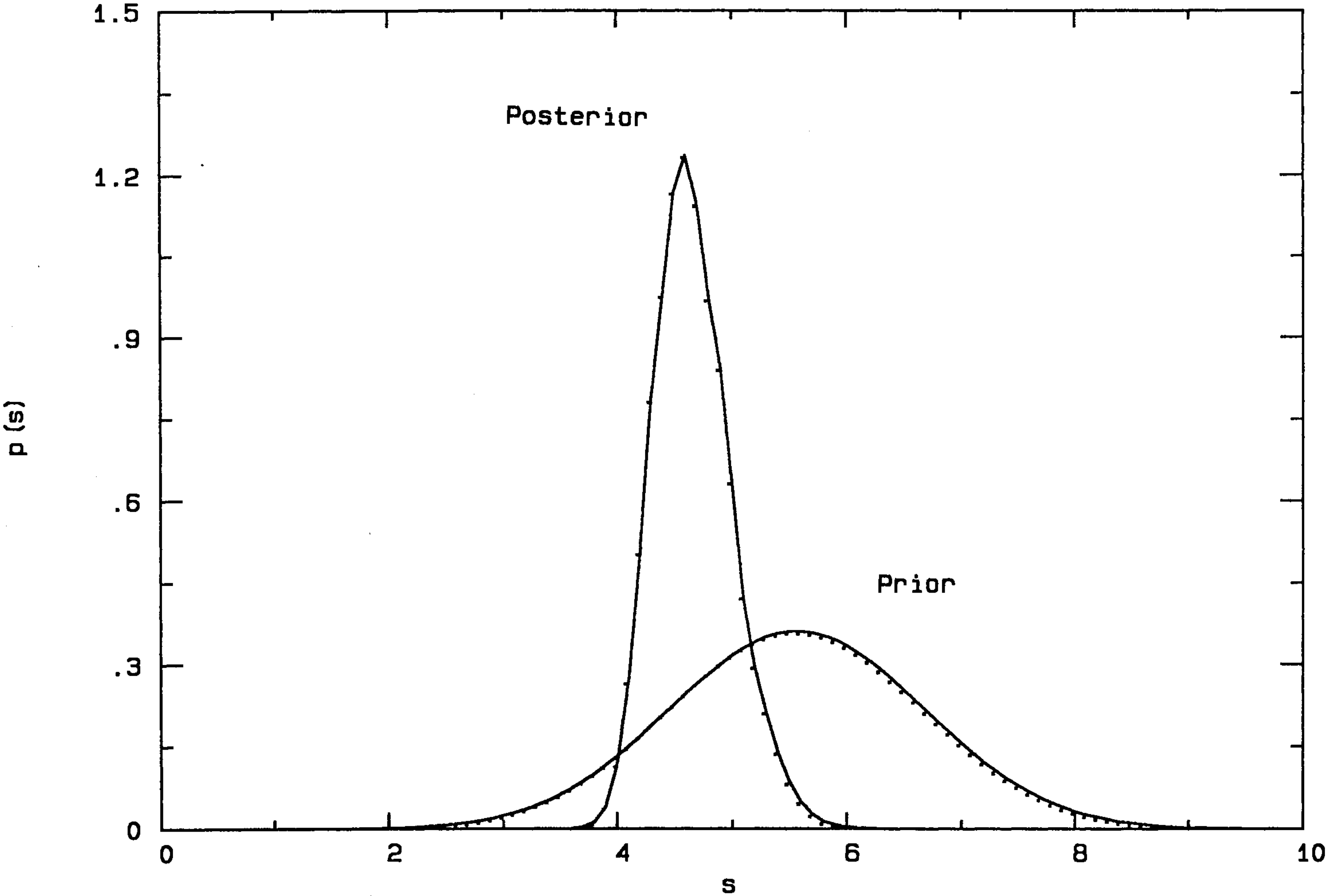

Figure 88. Prior and Posterior Probability Density Functions of S for Watershed 611

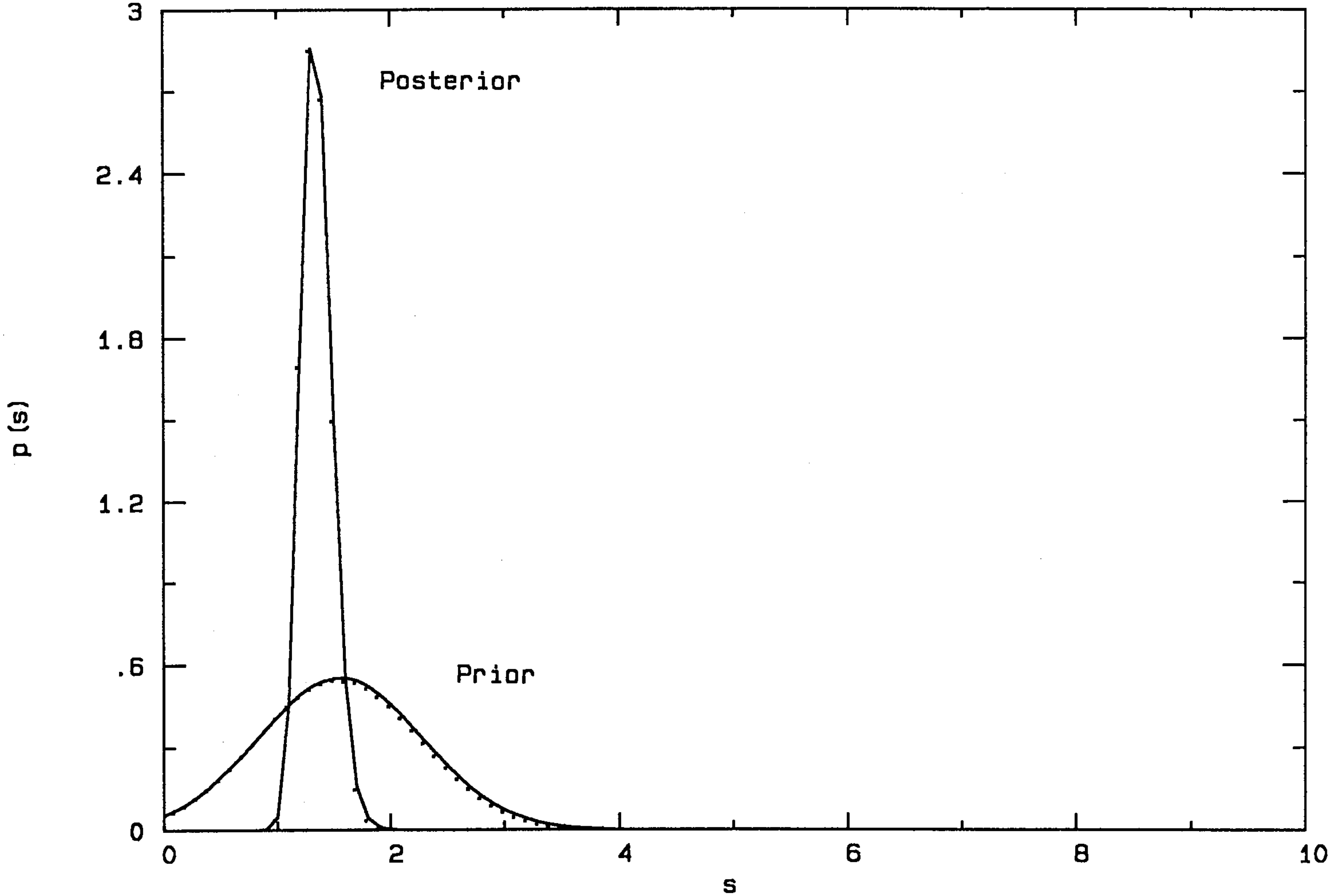

Figure 89. Prior and Posterior Probability Density Functions of S for Watershed R7

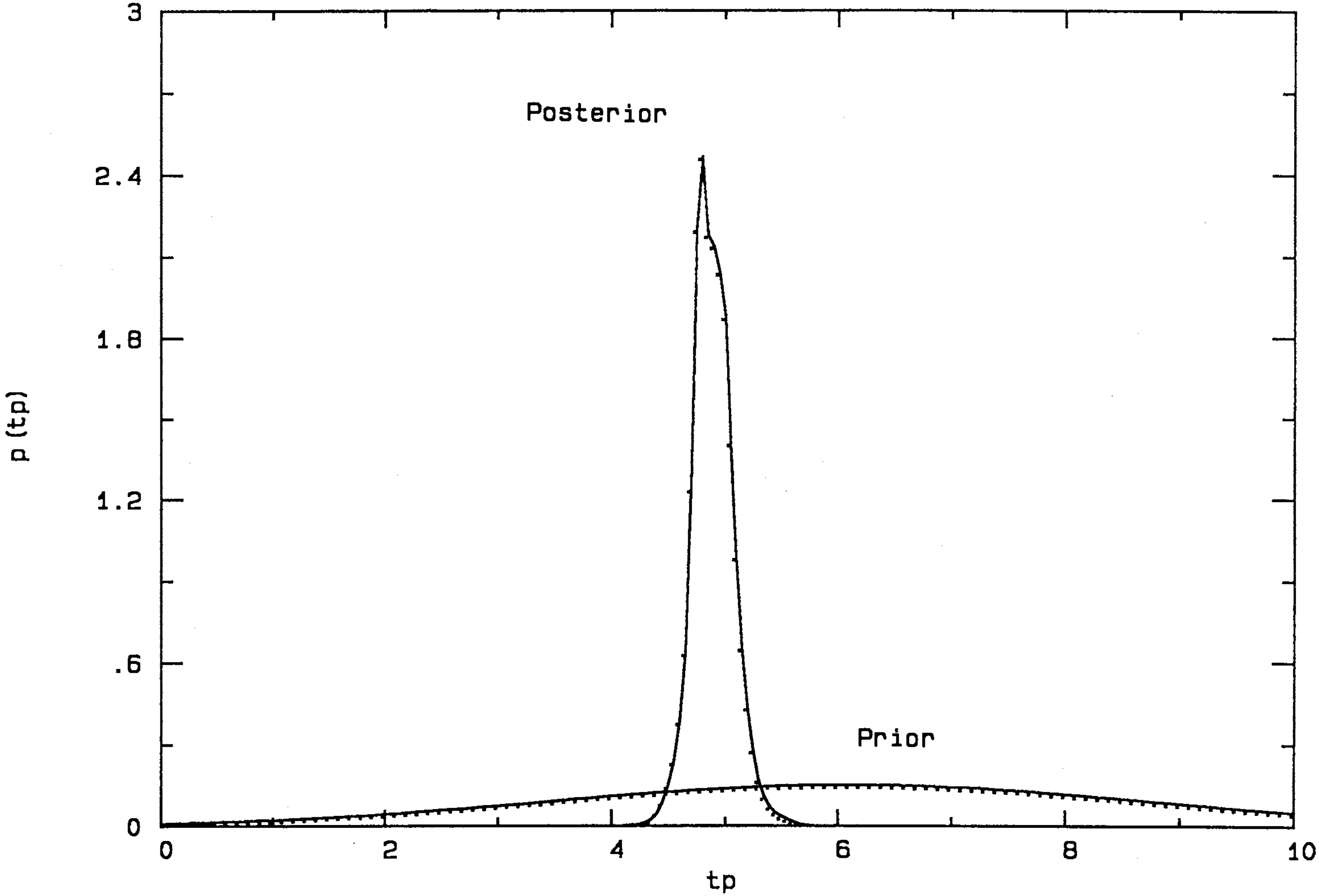

Figure 90. Prior and Posterior Probability Density Functions of Tp for Watershed 511

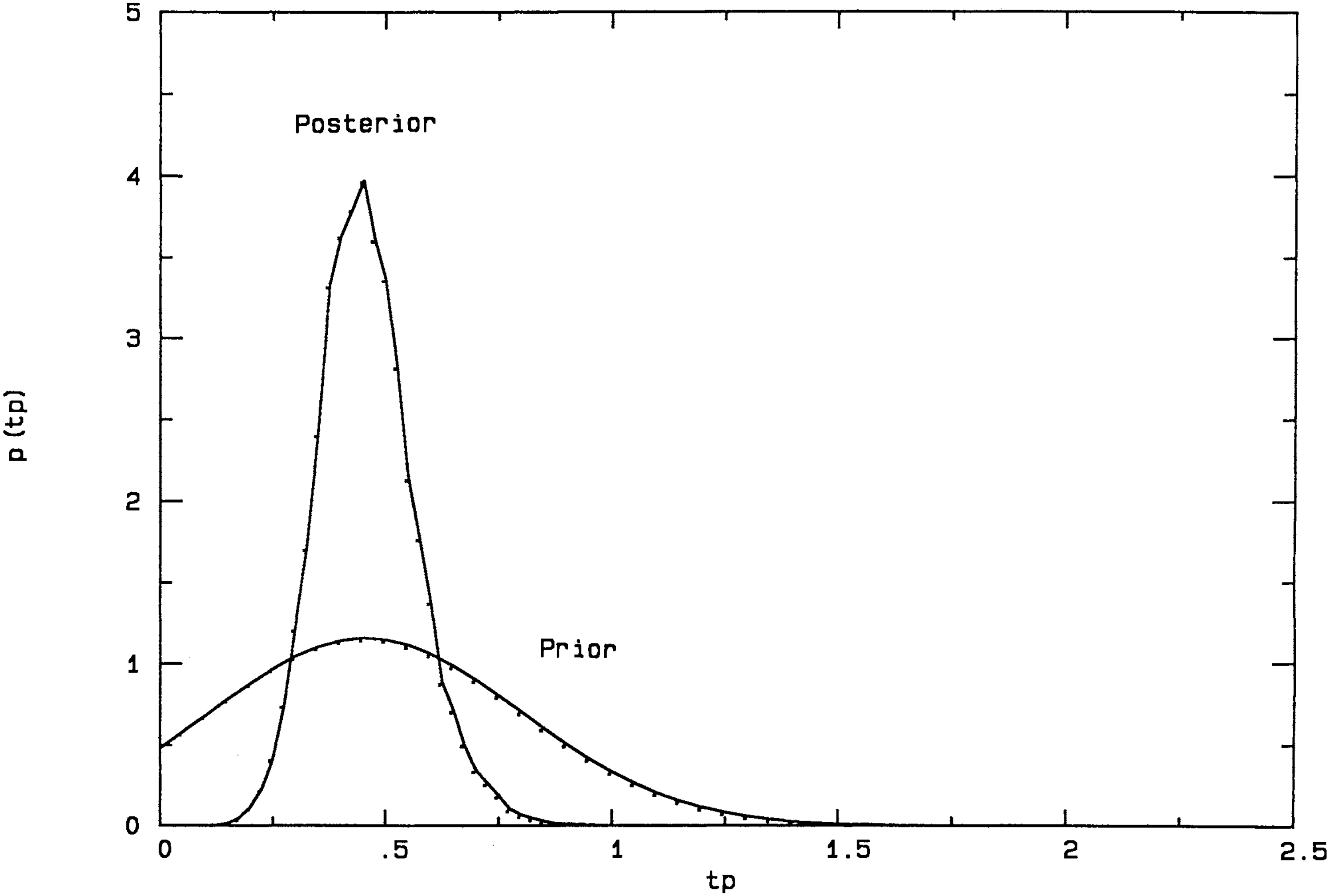

Figure 91. Prior and Posterior Probability Density Functions of Tp for Watershed 5142

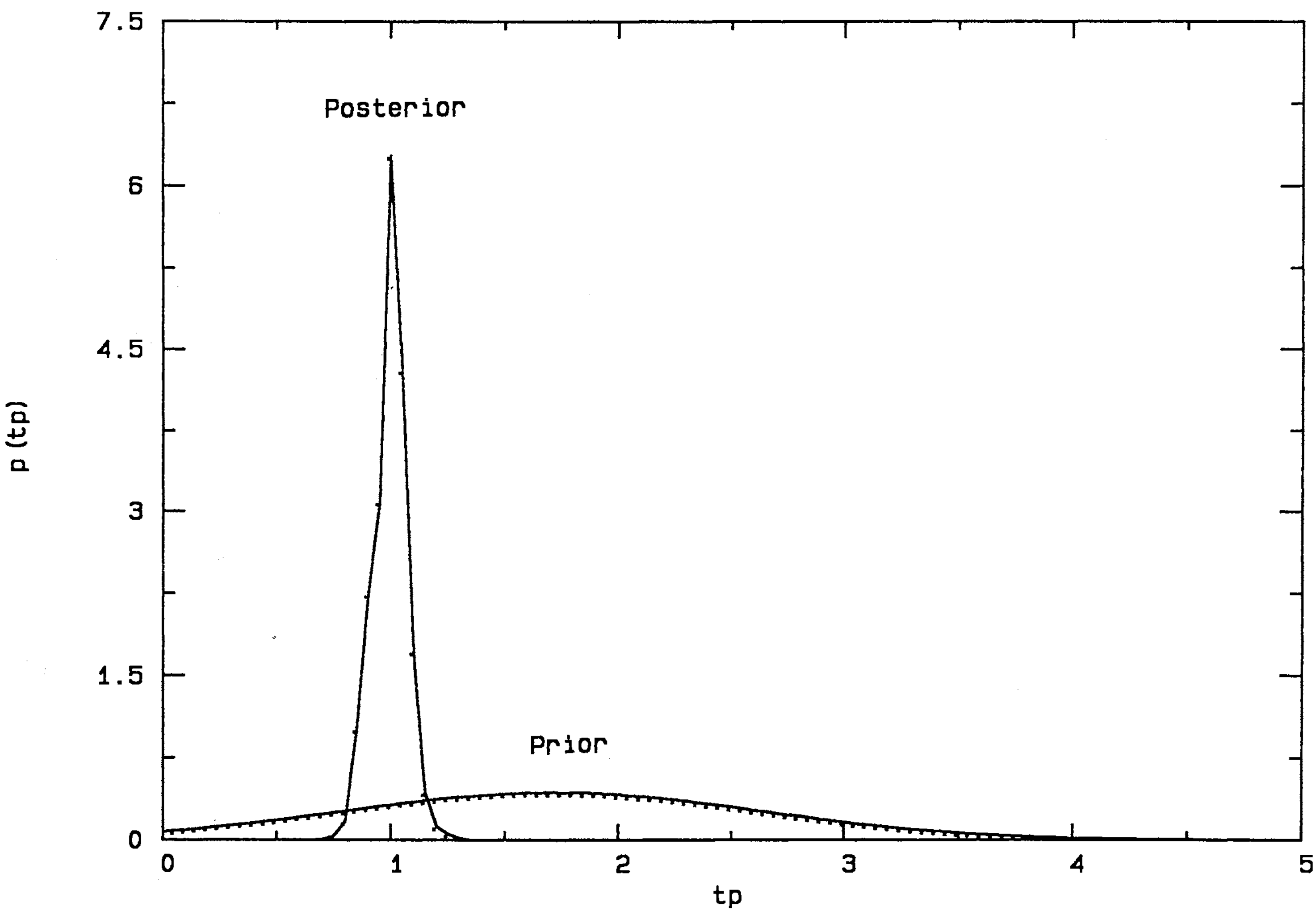

Figure 92. Prior and Posterior Probability Density Functions of Tp for Watershed 611

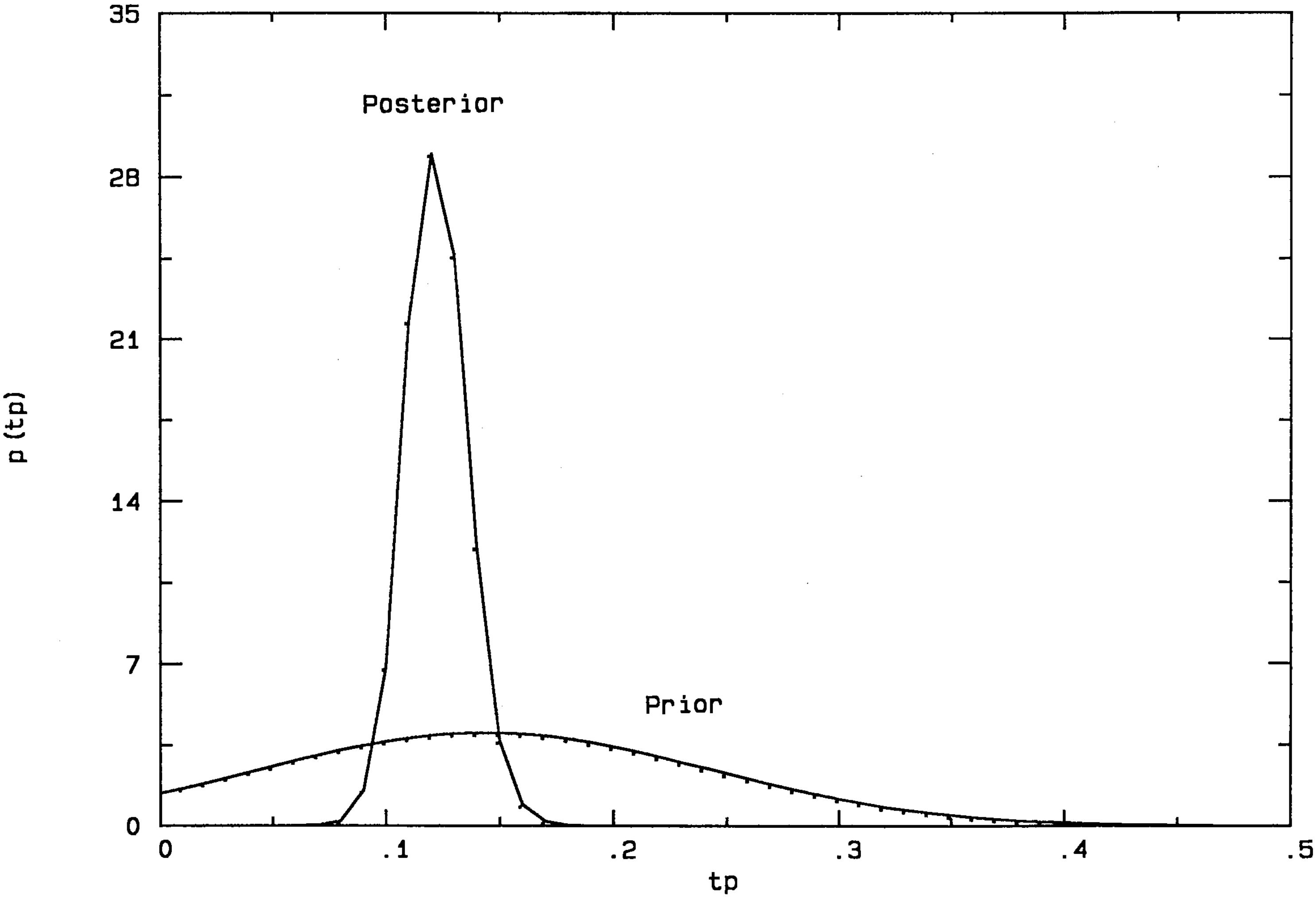

Figure 93. Prior and Posterior Probability Density Functions of Tp for Watershed R7

CHAPTER IX

EVALUATION OF THE FLOOD ESTIMATION METHODOLOGY FOR WATERSHEDS WITH SHORT RECORDS

Flood frequency curves were estimated for each of the study watersheds using the procedure described in Chapter VIII and using the information in Table XVIII to specify posterior probability density functions of the model parameters. Because the data of Tables XII and XIII were used to construct the prior probability density functions resulting in the contents of Table XVIII, this is again a Jackknife approach to the evaluation. For some of the watersheds, the mean of Tp reported in XVI was appreciably different than the value reported in Table XIII. This changed the rainfall durations and depths used for these watersheds. Table XIX lists the rainfall durations and corresponding 2-year and 100-year rainfall depths for each study watershed; depths for intermediate recurrence intervals were determined as functions of the 2-year and 100-year depths as discussed previously.

The Baysian flood frequency curves, with their associated 90% confidence intervals, appear in Figs. 94 through 108. Again, each of the six data points (and each corresponding 90% confidence interval) on the Bayesian curves was derived from 2000 values of S and Tp generated randomly from their respective probability density functions. Also shown in Figs. 94 through 108 are the the observed curves and the curves derived from the USGS and SCS (where applicable) procedures. Since the USGS flood estimation procedure contains no provisions for incorporating site-specific information, the USGS curves remain unchanged with the collection of such

TABLE XIX

RAINFALL DURATIONS AND DEPTHS OF THE CORRESPONDING 2 AND 100-YEAR RAINFALL EVENTS, 2

ID	Duration (hr)	R_2 (in)	R_{100} (in)
111	3.0	2.16	5.14
131	12.0	2.98	6.98
311	12.0	3.10	7.28
411	12.0	2.98	6.98
511	12.0	2.98	6.98
513	6.0	2.57	6.01
5142	1.0	1.80	4.10
5143	1.0	1.80	4.10
5145	1.0	1.80	4.10
515	6.0	2.65	6.20
611	2.0	2.08	4.85
R5	0.5	1.42	3.20
R6	0.5	1.42	3.20
R7	0.5	1.42	3.20
R8	0.5	1.42	3.20

R_2 = 2-year rainfall depth.

R_{100} = 100-year rainfall depth.

data. The SCS (1972) describes a procedure for modifying estimates of CN as site-specific data become available. The modified estimates of CN will lead to new estimates of Tp as computed by Eqn. 80. The changes in these two parameters will act in conjunction to effect changes in the flood frequency curves of Figs. 77 through 80 and 82 through 85. The modified CN is taken as that which leads to equal numbers of overpredictions and underpredictions of runoff volume. Modified CN's were computed for the eight watersheds having area less than 2000 acres using each of these watershed's 50 study rainfall-runoff events. The values of S resulting from the modified CN's and the SCS estimates of Tp appear in Table XX. The same procedure as described in Chapter VII was used to determine the modified SCS flood frequency curves with the exception that the values of S and Tp shown in Table XX were used rather than the values shown in Tables II and XVI.

Discussion of Flood Frequency Curves

The Baysian flood frequency curves appear to be uniformly conservative relative to the observed curves for watersheds 111, 131, and 411. For watersheds 311, 513, 5142, 5143, 5145, 515, 611, and R8, the Baysian curves closely approximate the observed curves up to recurrence intervals of roughly 10 years. For the remainder of the study watersheds (511, R5, R6, and R7), the Baysian curves generally underestimate the T-year peak flow.

The SCS flood frequency curves are again seen to greatly overestimate the T-year annual peak flow relative to the observed curves and the Baysian curves. As a group, they seem to be the worst from the standpoint of approximating the observed curves. This may be due to the input rainfall events. One will find upon examination of Tables XVIII and XIX that there is little difference between the SCS estimates of S and Tp and the means of S and Tp computed on the basis of regional/site-specific information. Therefore, the primary reason for differences between the SCS and

TABLE XX

MODIFIED SCS ESTIMATES
OF S AND Tp

ID	S (in)	Tp (hr)
5142	2.9	0.61
5143	3.1	0.81
5145	2.0	0.55
515	2.5	1.41
R5	2.5	0.27
R6	2.2	0.25
R7	1.3	0.13
R8	1.7	0.15

Baysian flood frequency curves must be that different rainfall events are used to generate the two sets of curves. It is almost certain that the SCS flood frequency curves would be less in error if the duration of the input rainfall events were allowed to vary with watershed area rather than being held at 24 hours.

The USGS flood frequency curves are very similar to the Baysian curves for watersheds 511, 513, 5142, 611, R7, and R8 for recurrence intervals up to about 10 years. For watersheds 111, 131, 411, 5143, 515, and 5145, the USGS curves are uniformly conservative relative to the Baysian curves. From this group, the USGS curve for watershed 5145 is the only one which appears to better approximate the observed curve than the Baysian curve; for the others, the USGS curves lead to consistently worse estimates of the T-year peak flow than the Baysian flood frequency curves. For watersheds 311, R5, and R6, the USGS estimates of the T-year peak flow are worse than those resulting from the Baysian curves for recurrence intervals of up to approximately 10-20 years, but better for the higher recurrence intervals. However, the significance of this is uncertain due to the short lengths of record used to construct the observed curves.

Table XXI shows the Kolmogorov-Smirnov statistics for testing the hypotheses that the Baysian and USGS flood frequency curves are equal to the observed curves for each of the study watersheds. As discussed earlier, the critical value of the test statistic is such that all curves resulting from both procedures may be taken as equal to the respective observed curves. As in Chapter VII, however, one may obtain a very general idea of the relative performance of the two procedures by comparing the two test statistics. For nine of the study watersheds, the test statistic is lower for the Baysian curves than for the USGS curves; for five of the study watersheds, the reverse is true; for one study watershed, the test statistics are the same. By no means should these results be construed as meaning that the Baysian curves are in general significantly better than the USGS curves. All that may be inferred from Table XXI

TABLE XXI

KOLMOGOROV-SMIRNOV TEST STATISTICS, 2

ID	Estimated	USGS	SCS
111	0.17	0.39	*
131	0.36	0.45	*
311	0.19	0.15	*
411	0.29	0.45	*
511	0.22	0.14	*
513	0.16	0.10	*
5142	0.13	0.19	**
5143	0.04	0.39	**
5145	0.08	0.05	**
515	0.28	0.35	**
611	0.06	0.06	*
R5	0.19	0.29	**
R6	0.08	0.23	**
R7	0.09	0.23	**
R8	0.18	0.02	**

* Not available.
** Kolmogorov-Smirnov test statistic could not be derived.

is that on the whole, the Baysian curves seem to better approximate the observed curves for recurrence intervals of roughly 2-5 years.

Effects of Site-Specific Data on the Bayesian Flood Frequency Curves

In Chapter VIII it was shown that site-specific information reduces uncertainty in the model parameters S and Tp. The effect of this reduction of informational uncertainty on the resultant Baysian flood frequency curves is illustrated in Figures 109 through 123, which are plots of the 90% confidence intervals about the Baysian curves resulting from using regional and regional/site-specific information. In every case, the inclusion of site-specific information is seen to reduce uncertainty in the Baysian flood frequency curve. The reduction of the 90% confidence bounds is in many cases quite marked. For watershed 511, the 90% confidence interval on the 100-year peak flow is approximately 9000 to 36000 cfs when derived from only regional information. Site-specific information has reduced this interval to about 16000 to 18500 cfs. Recall now the scenario presented in Chapter VII regarding the construction of a structure on this watershed. It was stated that if the structure were designed for a peak flow of 36000 cfs, there would be a 10% risk that the structure would fail to accommodate the 100-year peak flow. Site-specific information in effect allows the structure to now be designed for a peak flow of only 18500 cfs with the same risk of failure. Thus the risk one assigns to a particular design is seen to be a function of informational uncertainty.

In addition to reducing the confidence intervals of the Baysian flood frequency curves, the addition of site-specific information appears to generally improve the estimate of the flood frequency curve. Figures 124 through 138 compare the curves estimated from regional and regional/site-specific information to the observed curves. For nine of the study watersheds (131, 411, 511, 5142, 5143, 515, 611, R6, and

R8), the site-specific information has resulted in flood frequency curves which are noticeably better approximations of the observed curves (at least, up to recurrence intervals of about 10 years) than the curves derived using only regional information. For watersheds 111, 311, 513, and R7, the regional and regional/site-specific curves are largely indistinguishable. The regional curve is somewhat a better fit to the observed curve on watershed R5, and both curves seem to be equally in error up to a recurrence interval of 10 years for watershed 5145. A comparison of of Tables XIX and XXI yields the result that in 10 cases, the Kolmogorov-Smirnov test statistic is lower for the regional/site-specific flood frequency curves than for the regional curves. In two cases, the test statistics are the same. In three cases, the test statistic is lower for the regional flood frequency curves. Again, one may not infer from this type of comparison that the regional/site-specific curves are significantly better than the strictly regional curves, only that as a group they appear to perform better for recurrence intervals of about 2-5 years.

Concluding Remarks

Based upon the results of Table XXI, the Baysian curves are statistically indistinguishable from the observed curves. However, because of the relative weakness of the goodness of fit tests, little shall be made of these results. All that shall be ventured is to say that the procedure for estimating flood frequency curves for watersheds with short records seems to produce reasonable results. On basis of Table XXI, the same may be said of the USGS procedure for determining flood frequency curves. However, a pairwise comparison of the test statistics of Table XXI seems to indicate that the USGS curves are somewhat inferior as a group to the Baysian curves. The SCS flood frequency curves are ostensibly worse than either the Baysian or USGS curves, but this judgement is not supported by any statistical tests.

Uncertainty in the Baysian flood frequency curves has been shown to decrease with the collection and incorporation of site-specific information. This result was anticipated in the previous chapter and is a natural result of gaining this relatively precise data. The effects of this reduction in informational uncertainty have been alluded to in a very rudimentary manner, but it seems certain that there are a multitude of situations for which this approach may be employed to rigorously investigate relationships between risk and uncertainty.

The flood estimation methodology of Chapter VIII has, in addition to those noted in Chapter VIII, several characteristics which make it preferable to the USGS and SCS procedures. In addition to its argueably greater accuracy, this method of flood estimation has an obvious advantage over the USGS procedure: it incorporates site-specific information to improve the Baysian flood frequency curves. For a watershed with a long length of record, this advantage is trivial; flood frequency inference by means of distribution theory will produce the best results. For watersheds with short records, however, the procedure presented in Chapter IX will make maximum use of all available information via a logical framework for integrating regional and site-specific data.

The flood estimation methodology of Chapter VIII also has at least one advantage over the SCS procedures. Although SCS procedures make use of site-specific information, the only site-specific information used is data on runoff volumes. These data provide a means of modifying the estimate of S, which in turn modifies the estimate of Tp. Intuitively, it seems incredulous that runoff volume data will provide information on Tp; it would seem more logical that Tp would be better inferred from data on both runoff volumes and peak flows. The procedure of Chapter VIII makes use of both of these types of data to provide a more sound framework for inferring S and Tp. Furthermore, the Chapter VIII procedure produces flood frequency curves which seem to be more accurate than the SCS curves.

The regional/site-specific flood estimation procedure contains at this point the same potential sources of error as described in Chapter VII; namely, error due to the model, error due to the assumption of equal recurrence intervals for rainfall and peak flow, error due to the values of the rainfall depths used, and error due to the regionally-derived prediction equations. The results of this chapter indicate that effects of the fourth source of error are masked by the incorporation of site-specific data; with the collection and incorporation of increasing site-specific data, the regionally-derived information on the parameters will become less important, and the probability density functions of the model parameters will converge to their limiting form. This has already been suggested by the translation of the posterior probability density functions of the model parameters with respect to their prior probability density functions. While it is not possible, at this point, to rigorously analyze the effects of the second and third sources of error, it does not appear that they exercise a systematically adverse influence on the Baysian flood frequency curves. This statement is supported by the lack of clear patterns of overestimation or underestimation of the T-year peak annual flow up to recurrence intervals of about 10 years. It is therefore reasonable to suspect that the nature of the rainfall-runoff model is to be credited with much of the error which exists in the Baysian flood frequency curves. This is not a criticism, per se, of the SCS unit hydrograph model. It should be said in defense of the SCS that their model was formulated with very specific goals in mind and possesses several definite advantages over some of its competitors with regard to the fulfillment of these goals. However, because of the approximate nature of the model, it must be recognized that there will be a limit to the accuracy of its output. The limited accuracy of the model seems to be the largest contributor to the occasionally appreciable divergence between the Baysian and observed flood frequency curves.

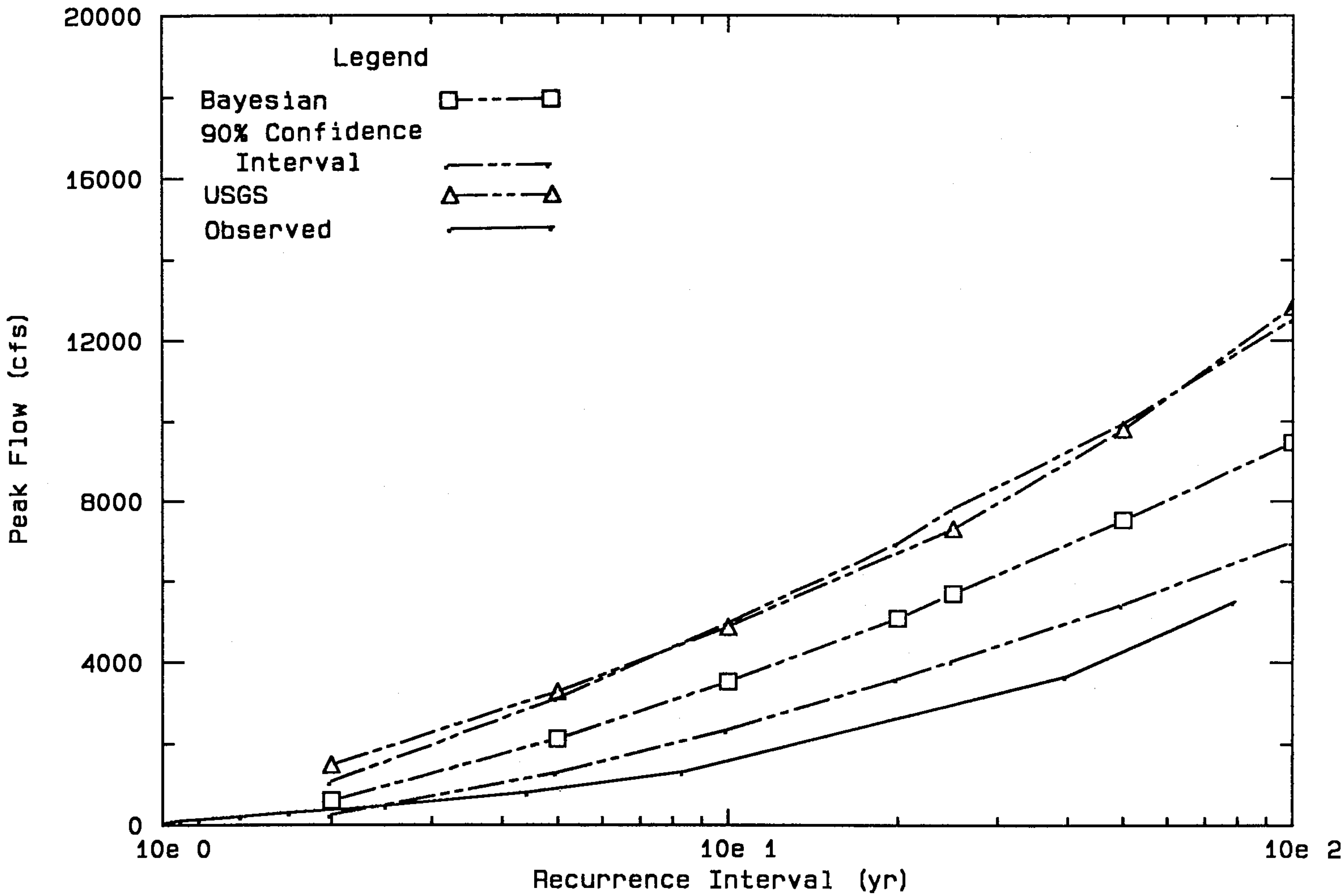

Figure 94. Flood Frequency Curves for Watershed 111, 2

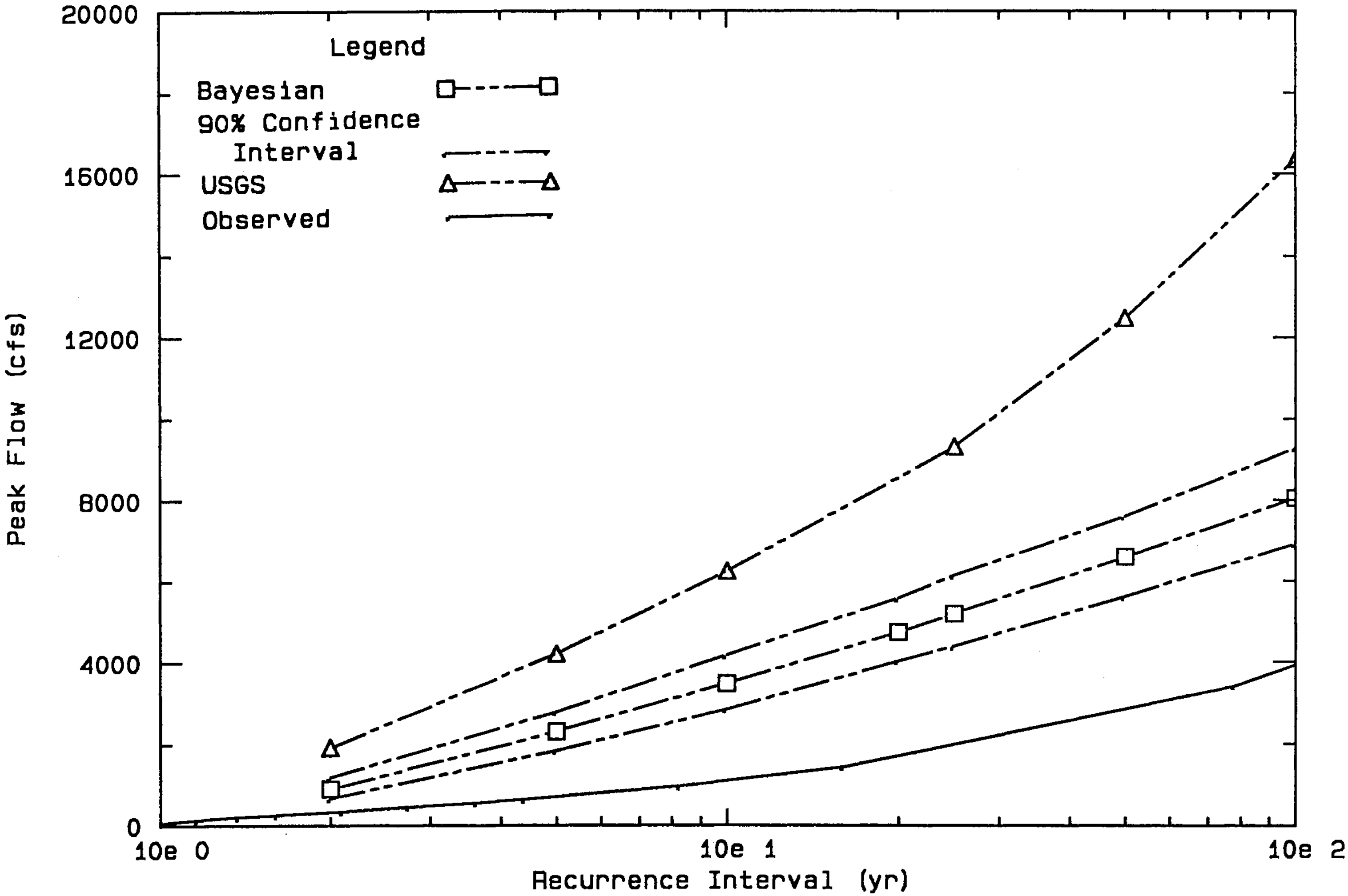

Figure 95. Flood Frequency Curves for Watershed 131, 2

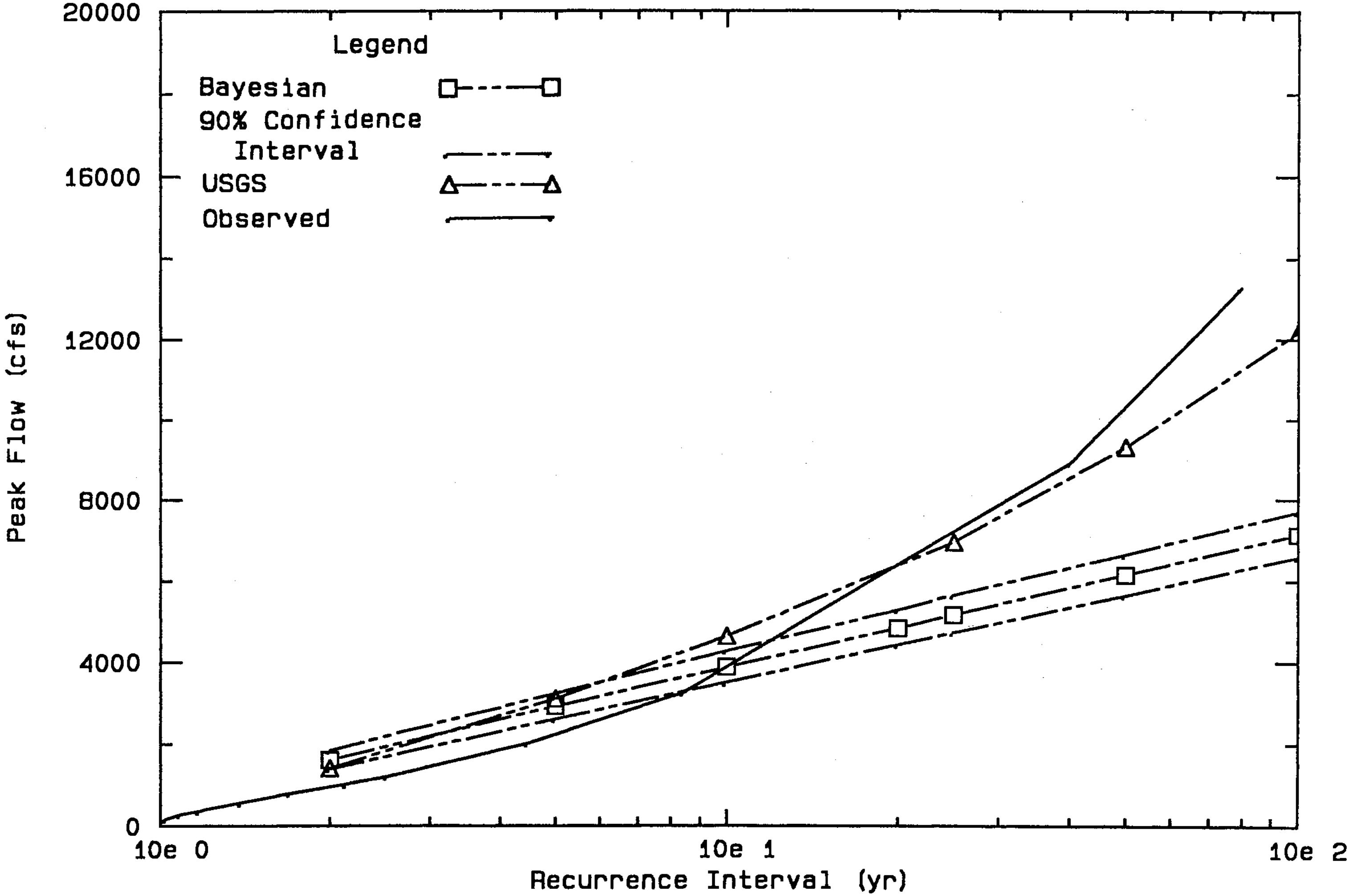

Figure 96. Flood Frequency Curves for Watershed 311, 2

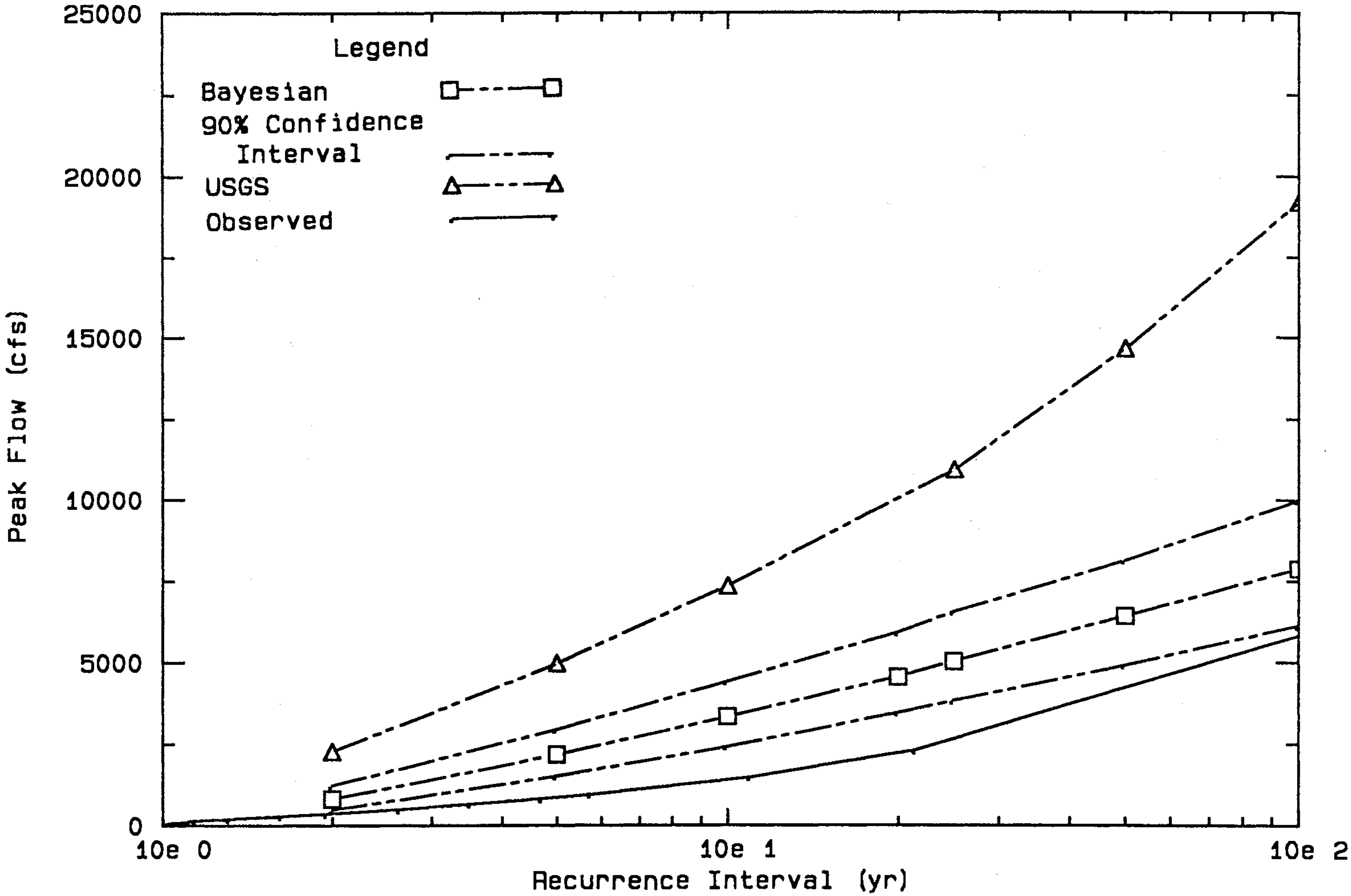

Figure 97. Flood Frequency Curves for Watershed 411, 2

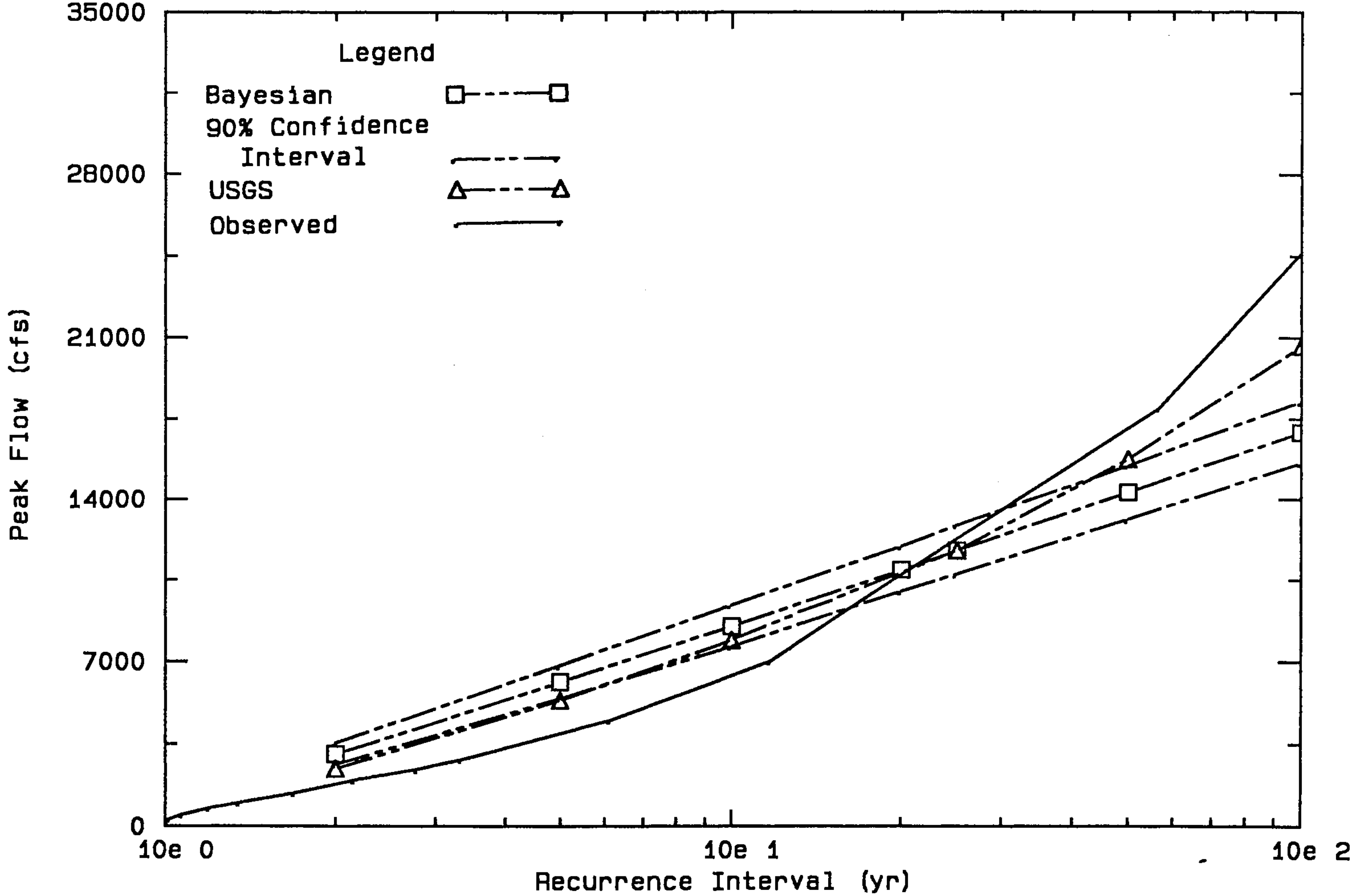

Figure 98. Flood Frequency Curves for Watershed 511, 2

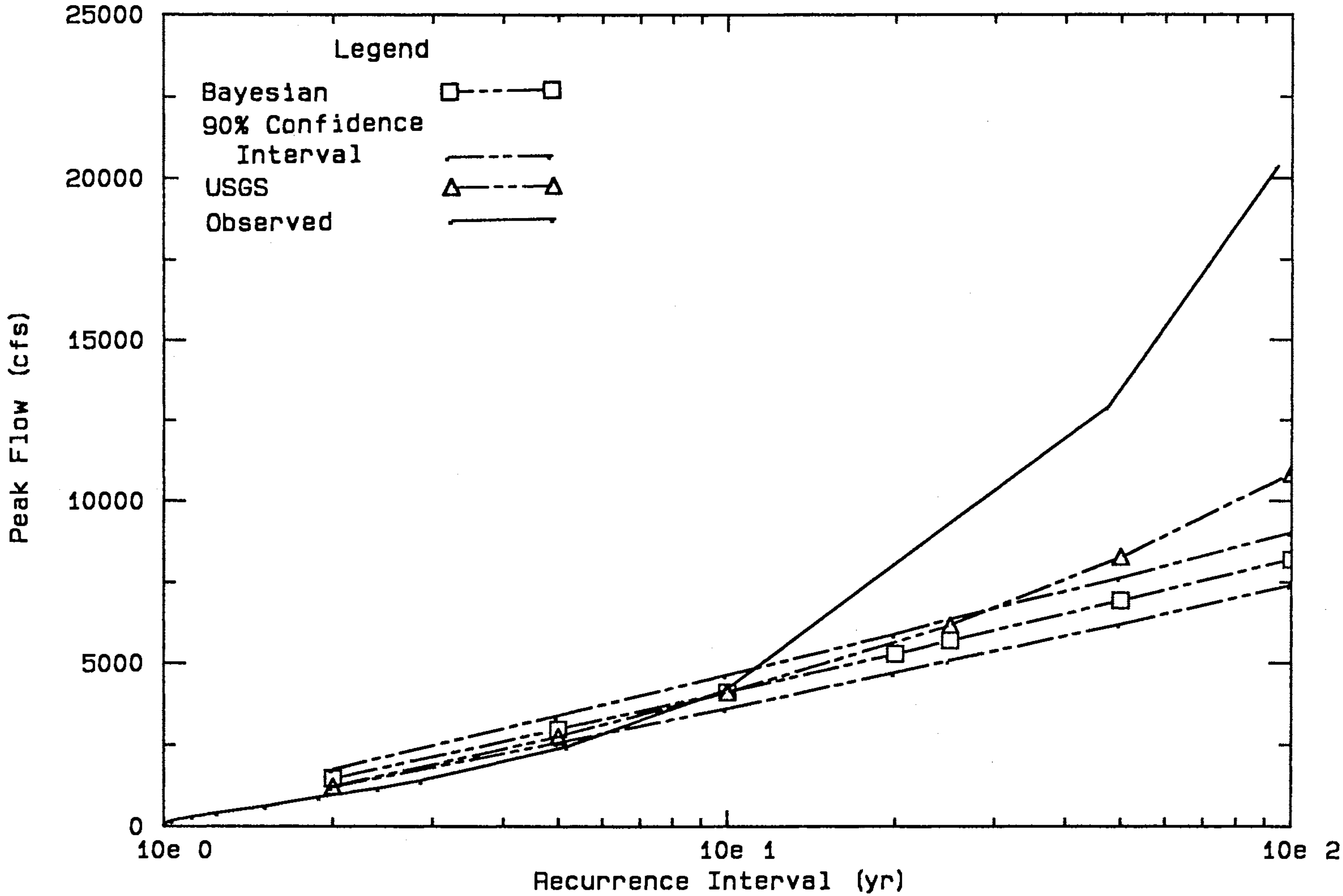

Figure 99. Flood Frequency Curves for Watershed 513, 2

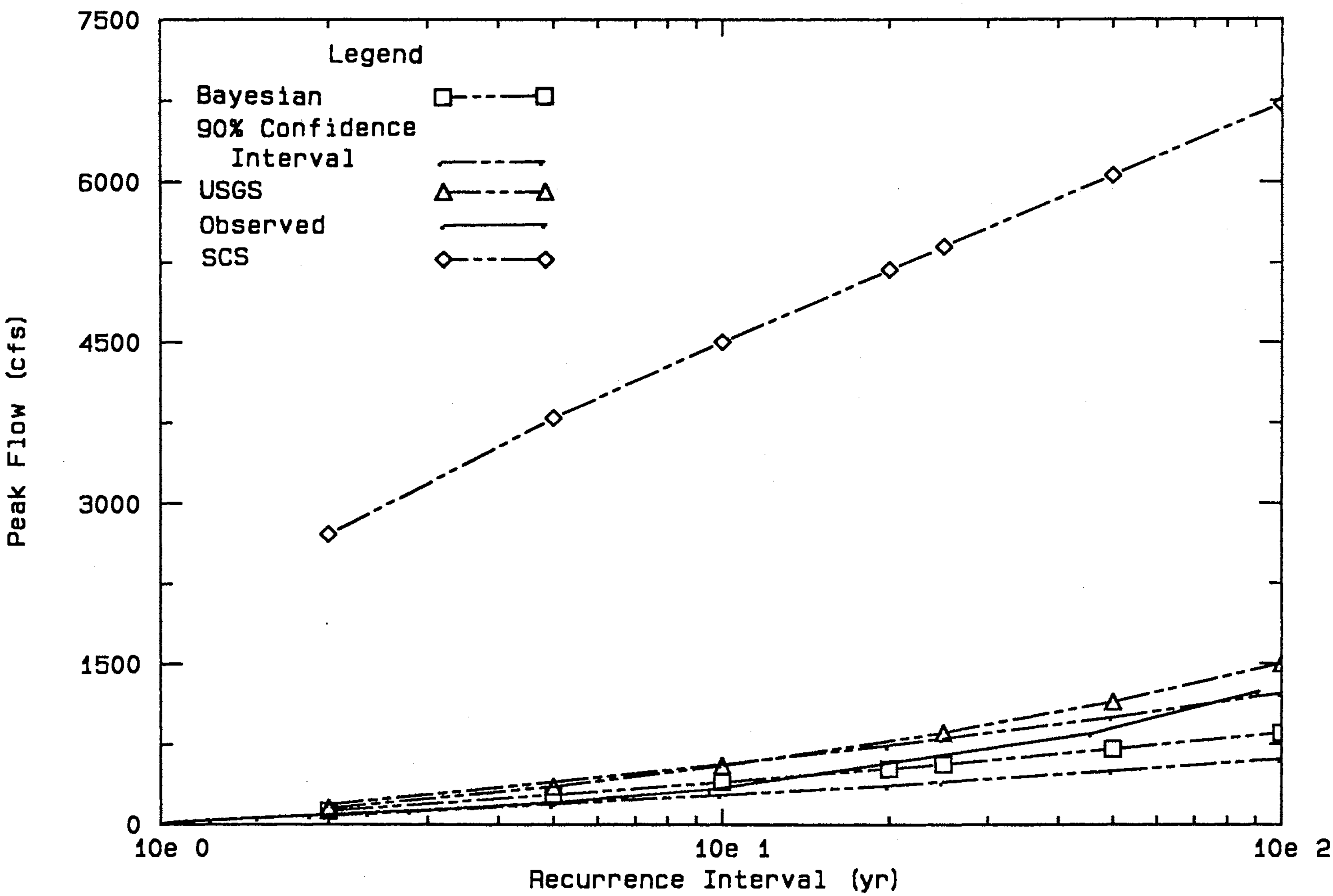

Figure 100. Flood Frequency Curves for Watershed 5142, 2

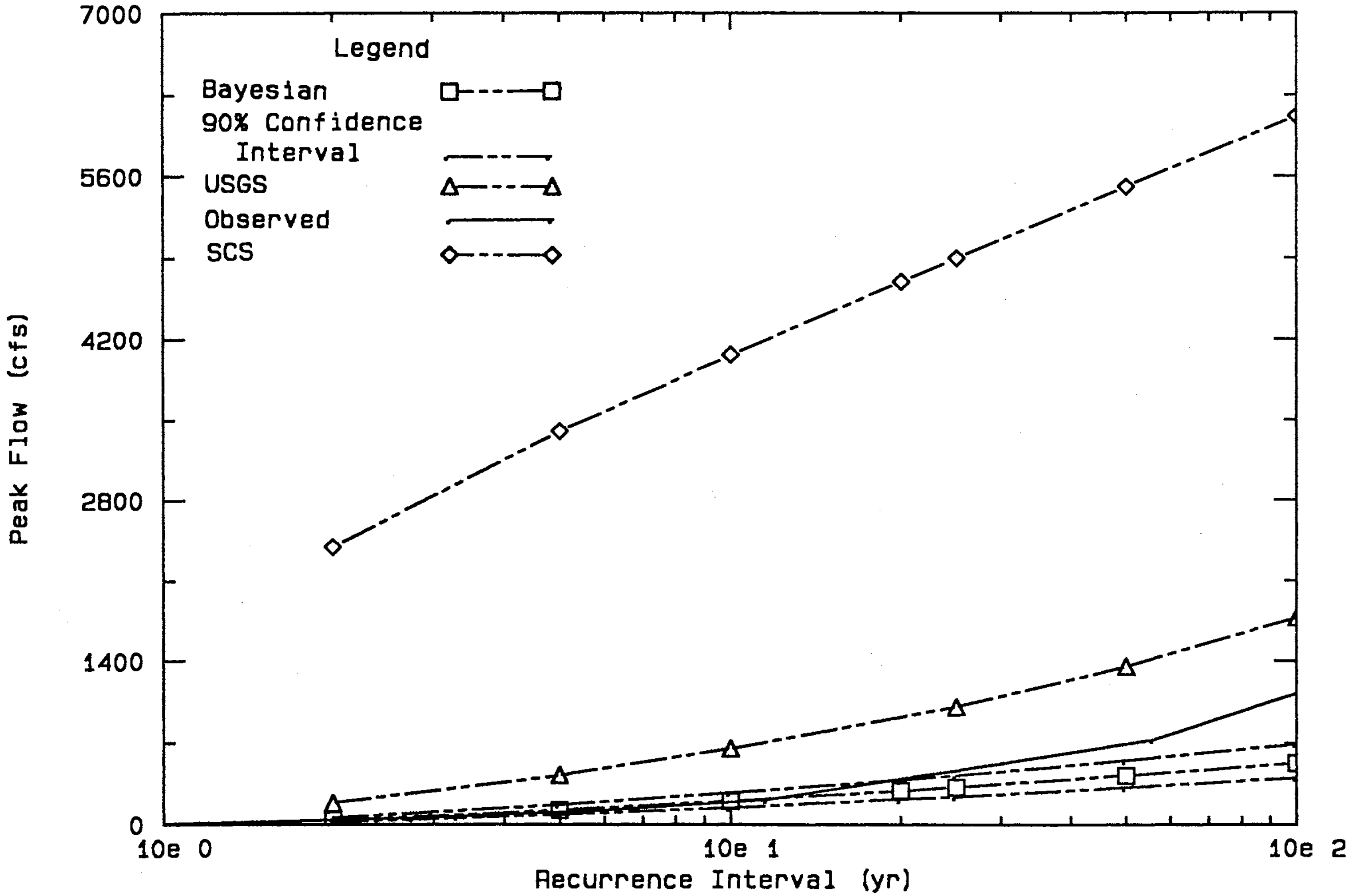

Figure 101. Flood Frequency Curves for Watershed 5143, 2

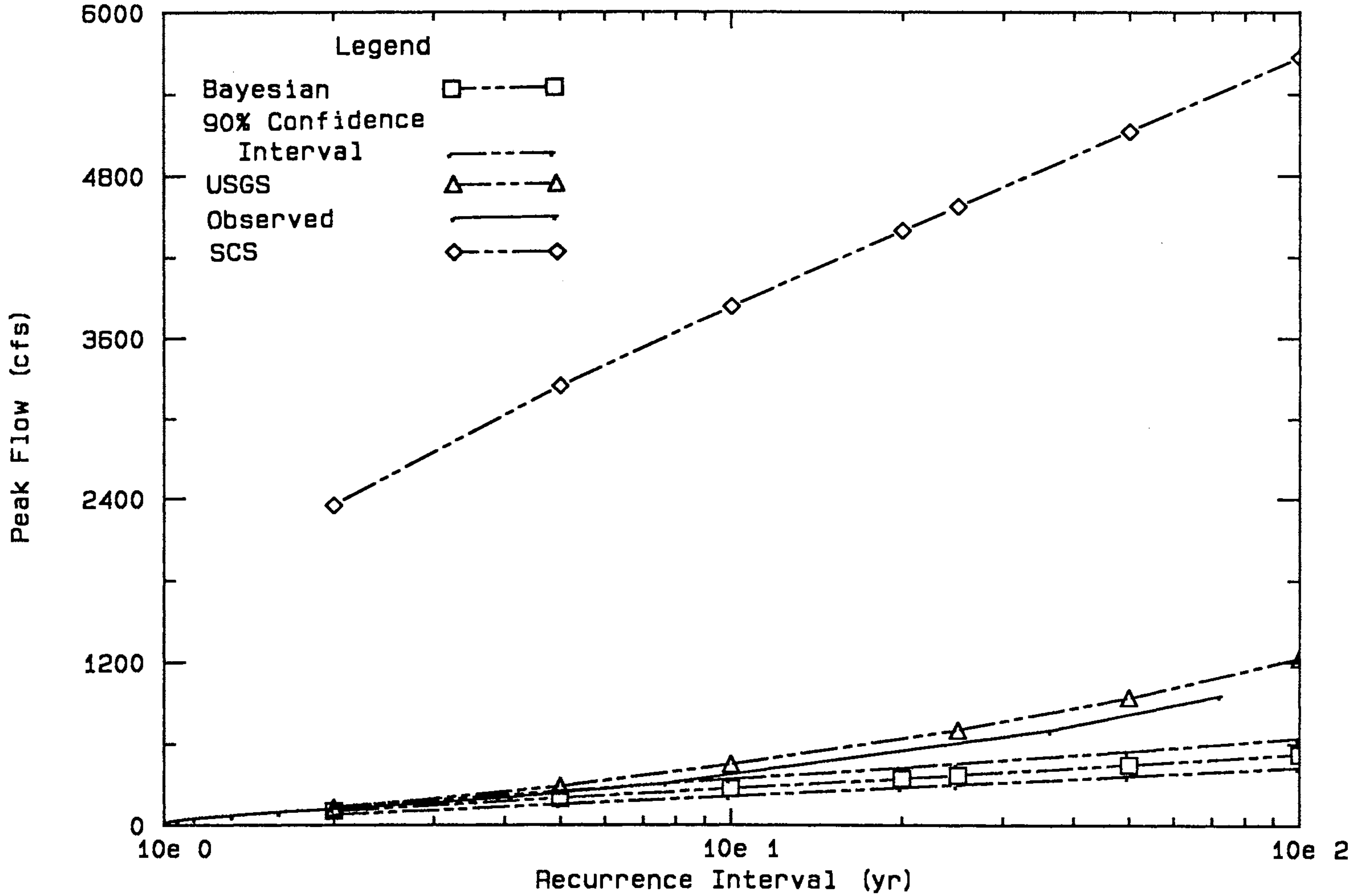

Figure 102. Flood Frequency Curves for Watershed 5145, 2

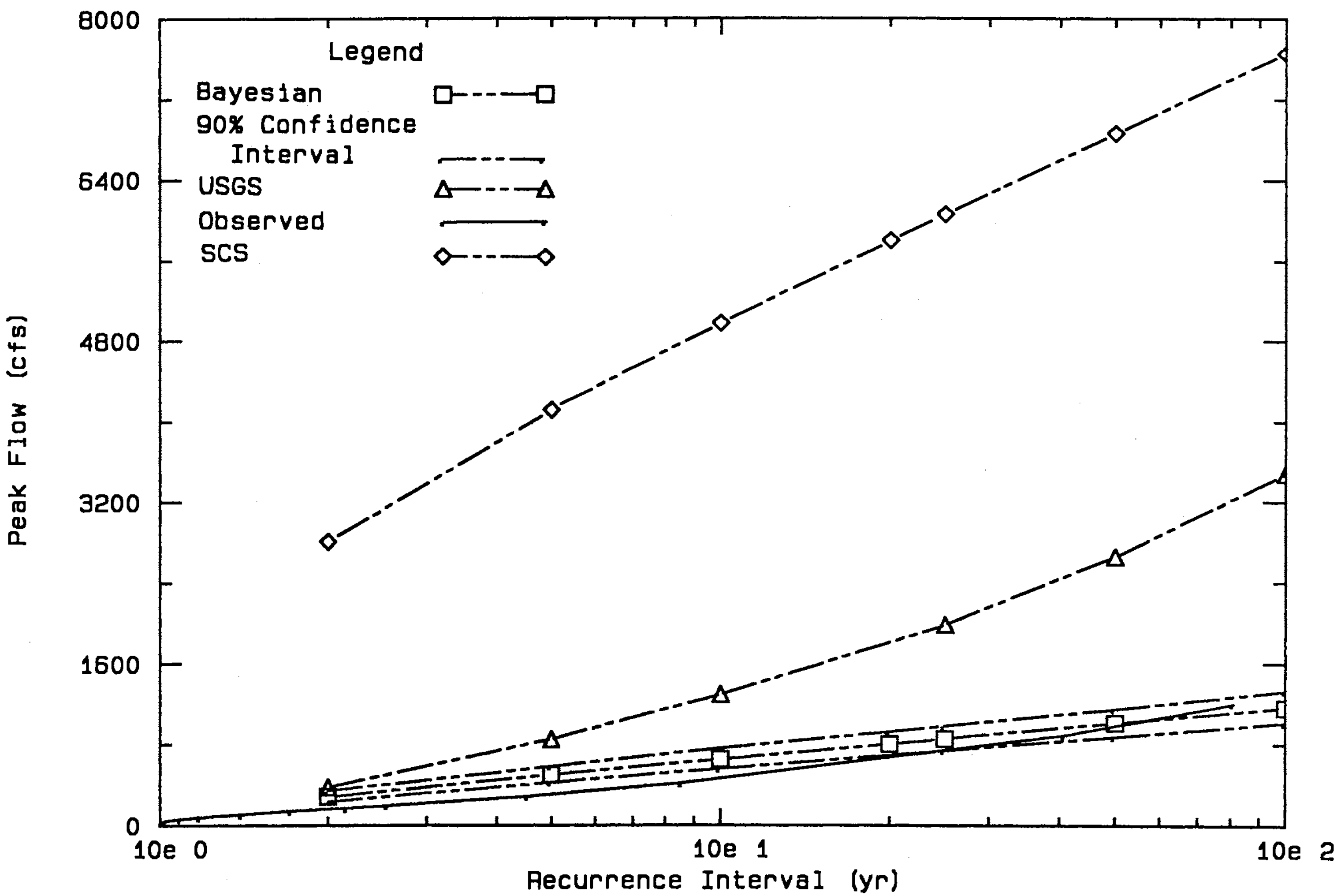

Figure 103. Flood Frequency Curves for Watershed 515, 2

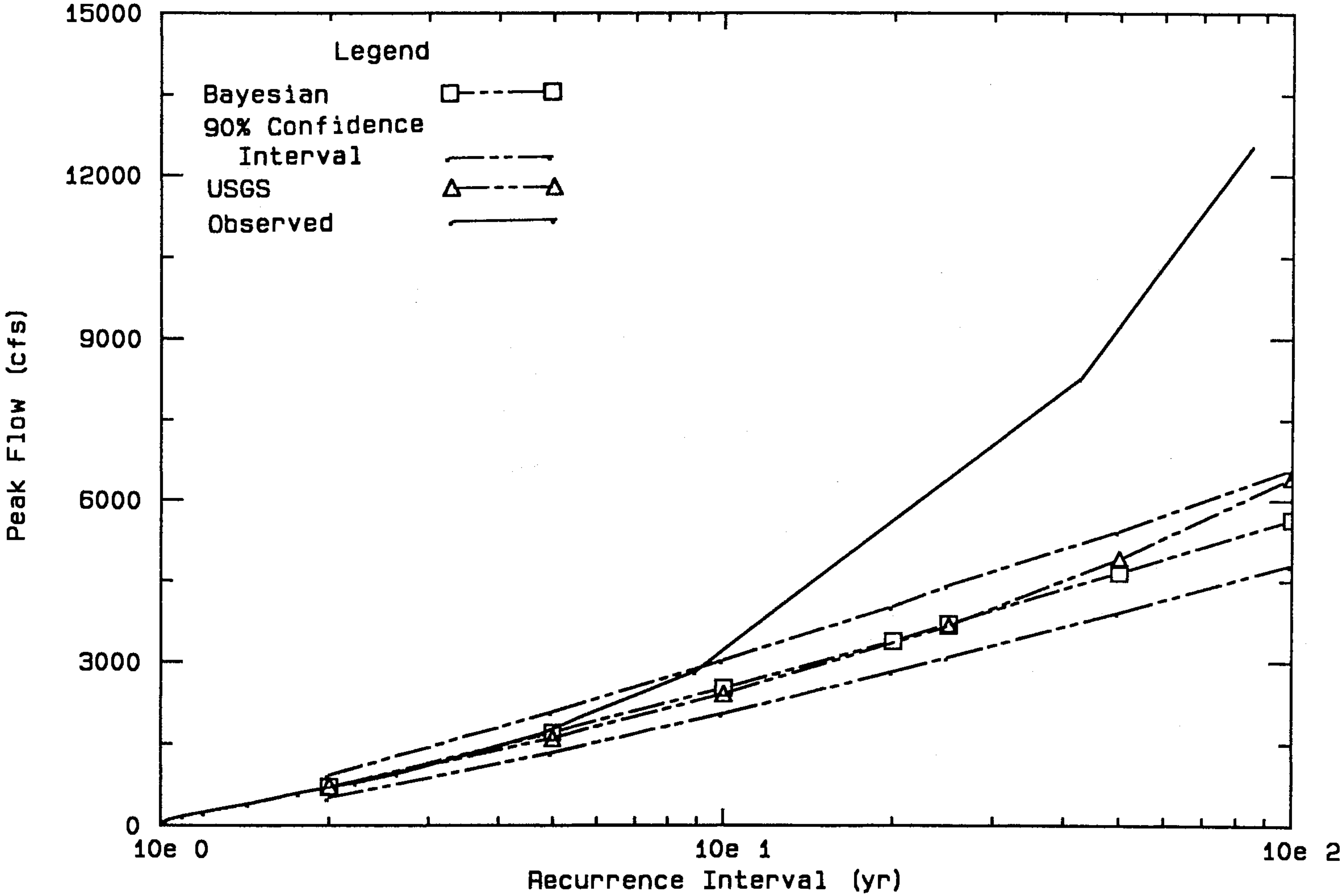

Figure 104. Flood Frequency Curves for Watershed 611, 2

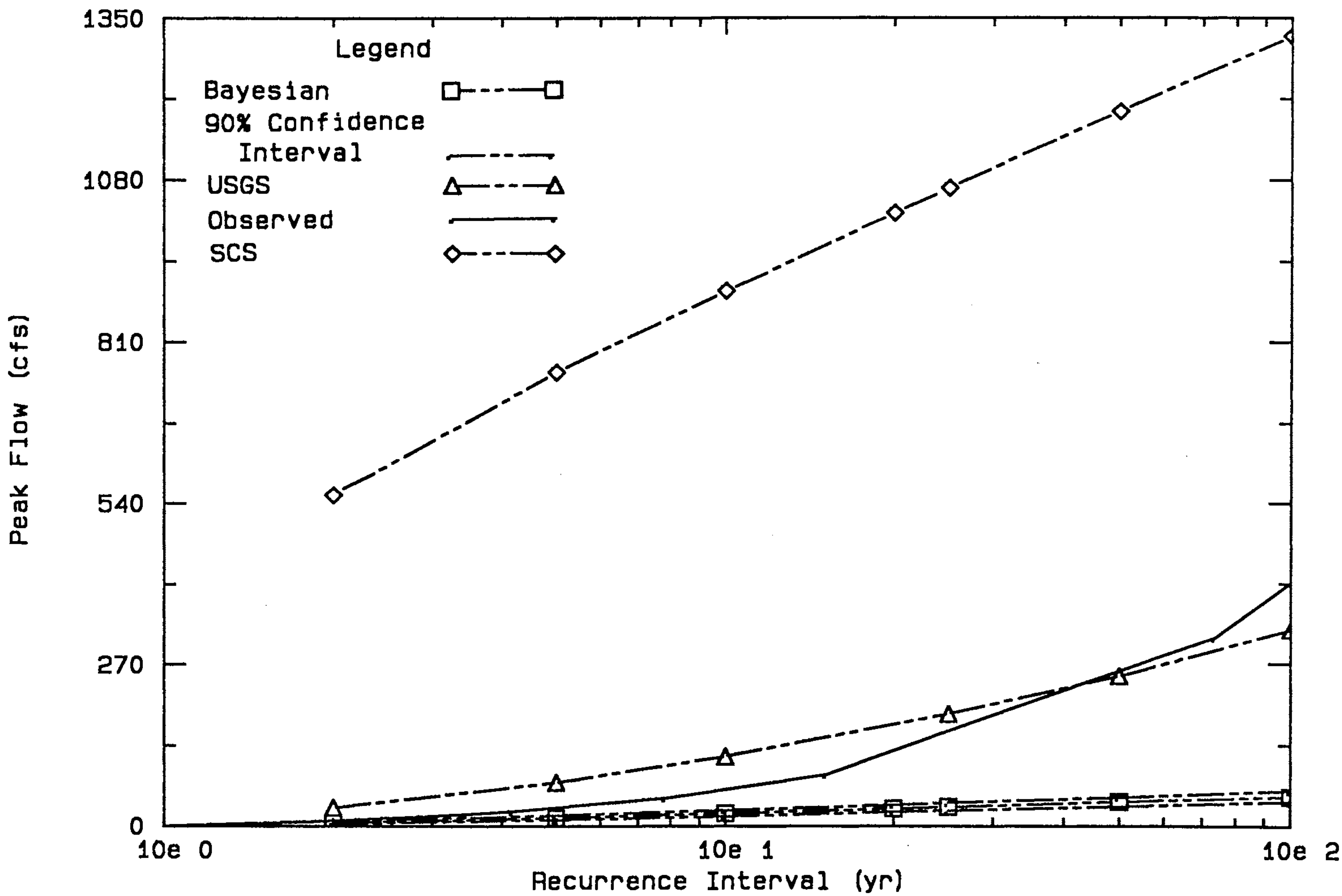

Figure 105. Flood Frequency Curves for Watershed R5, 2

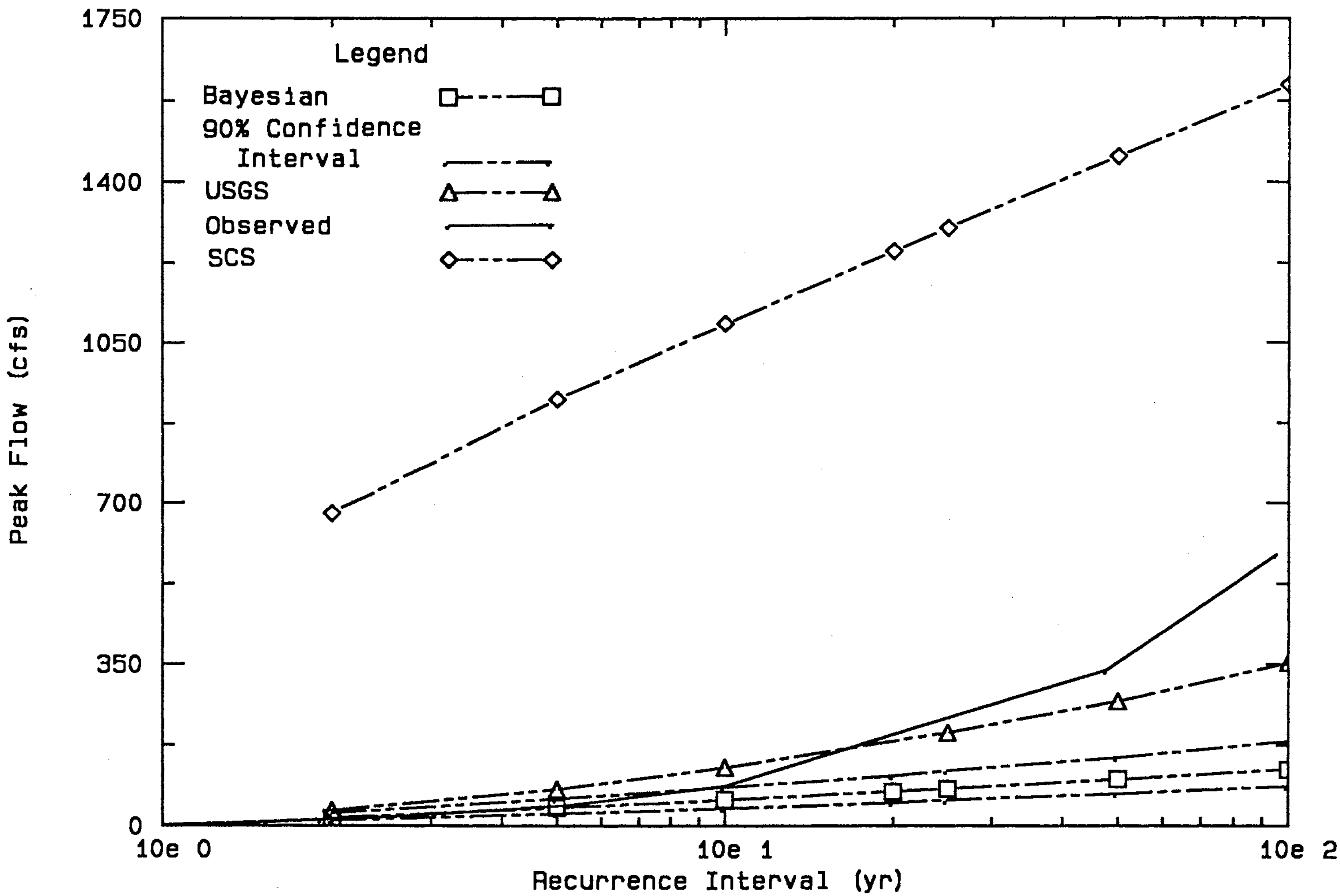

Figure 106. Flood Frequency Curves for Watershed R6, 2

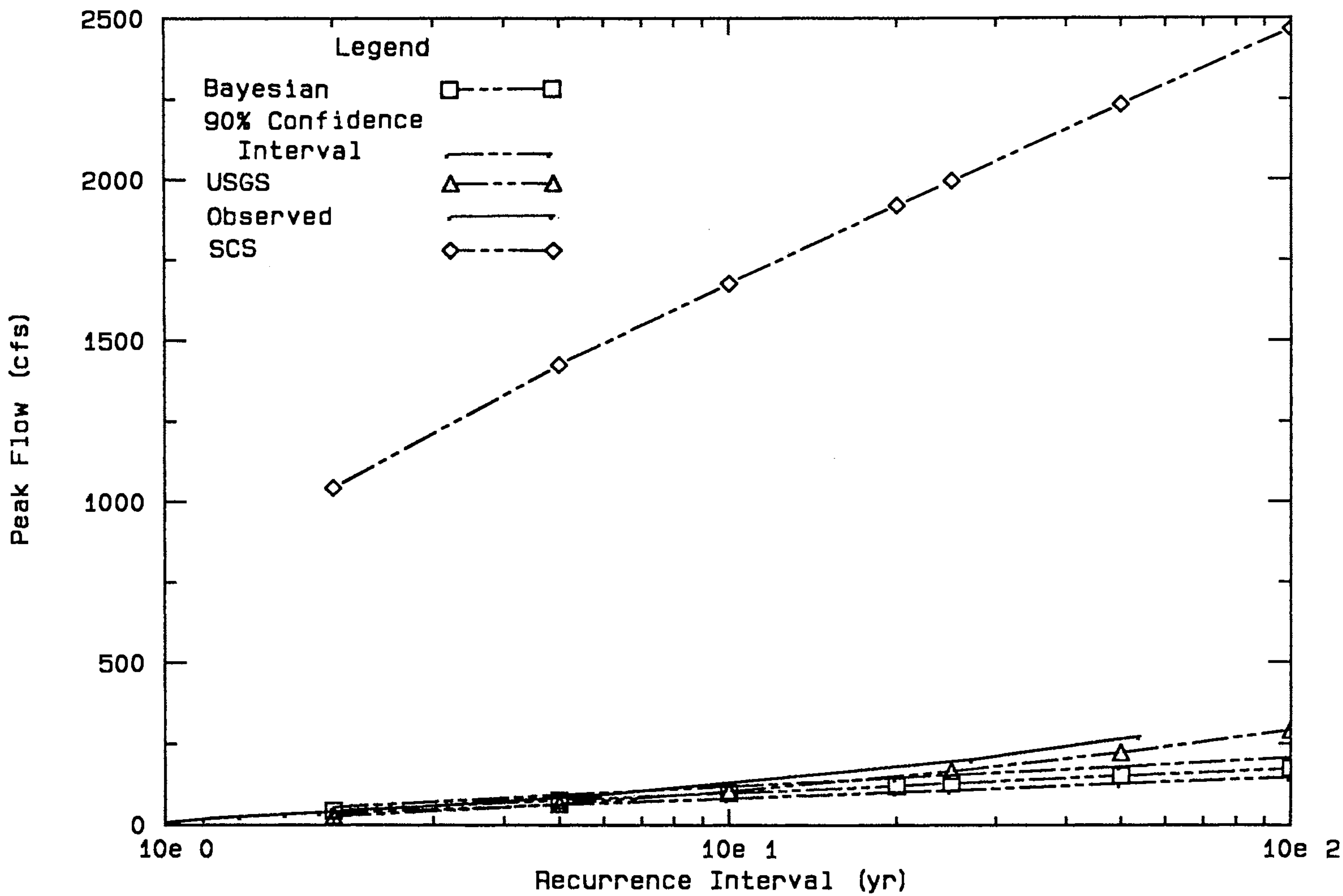

Figure 107. Flood Frequency Curves for Watershed R7, 2

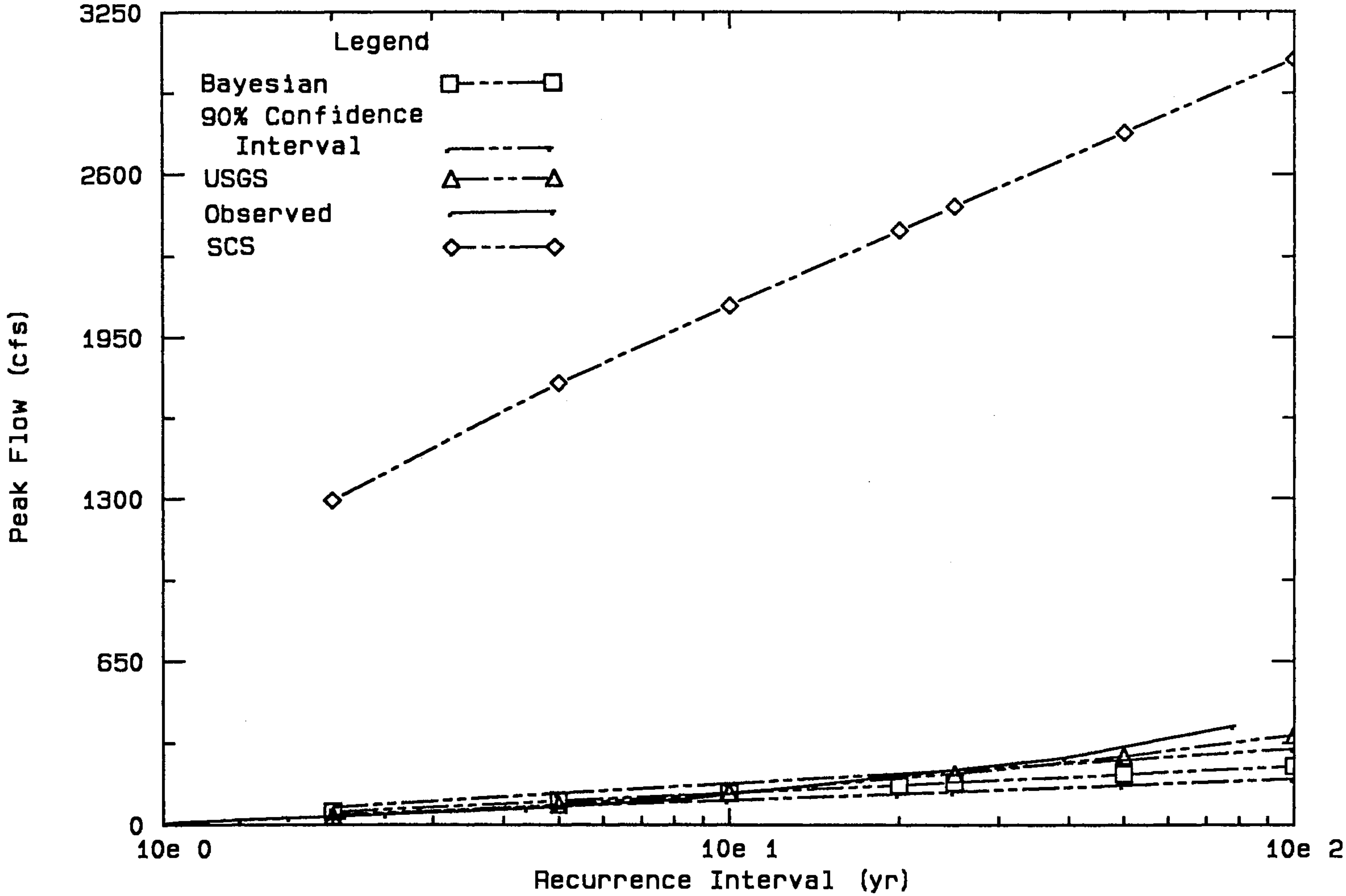

Figure 108. Flood Frequency Curves for Watershed R8, 2

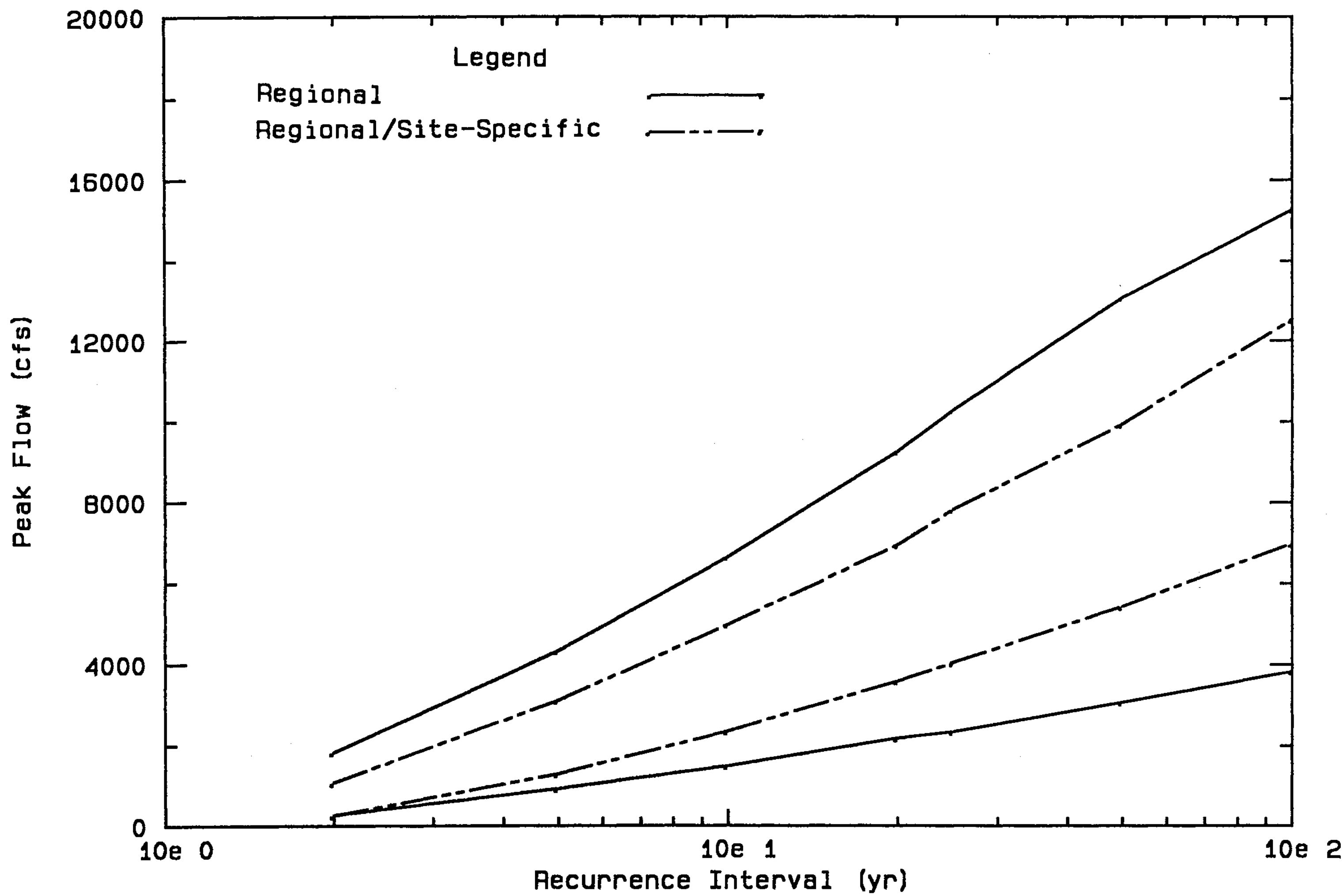

Figure 109. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 111

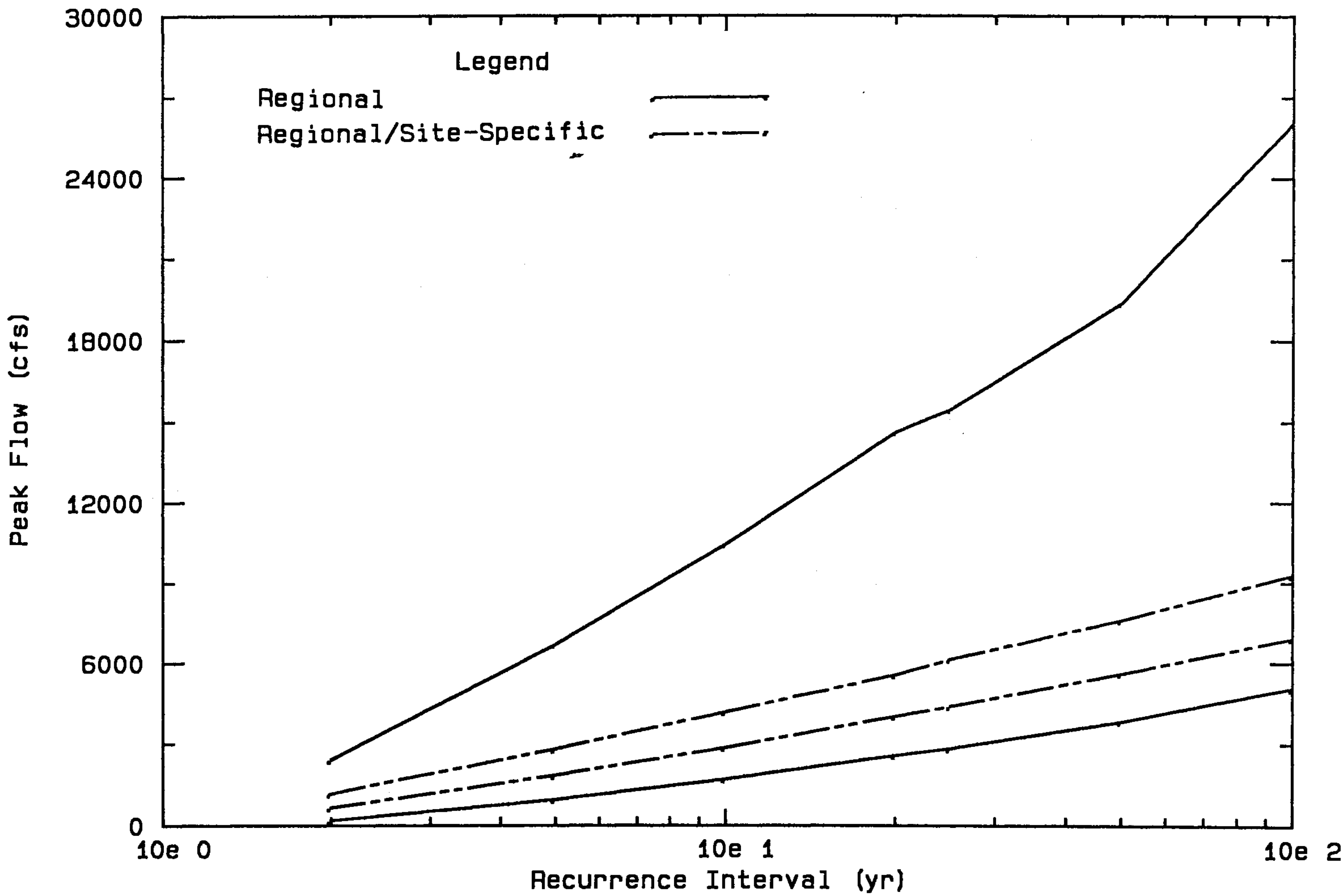

Figure 110. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 131

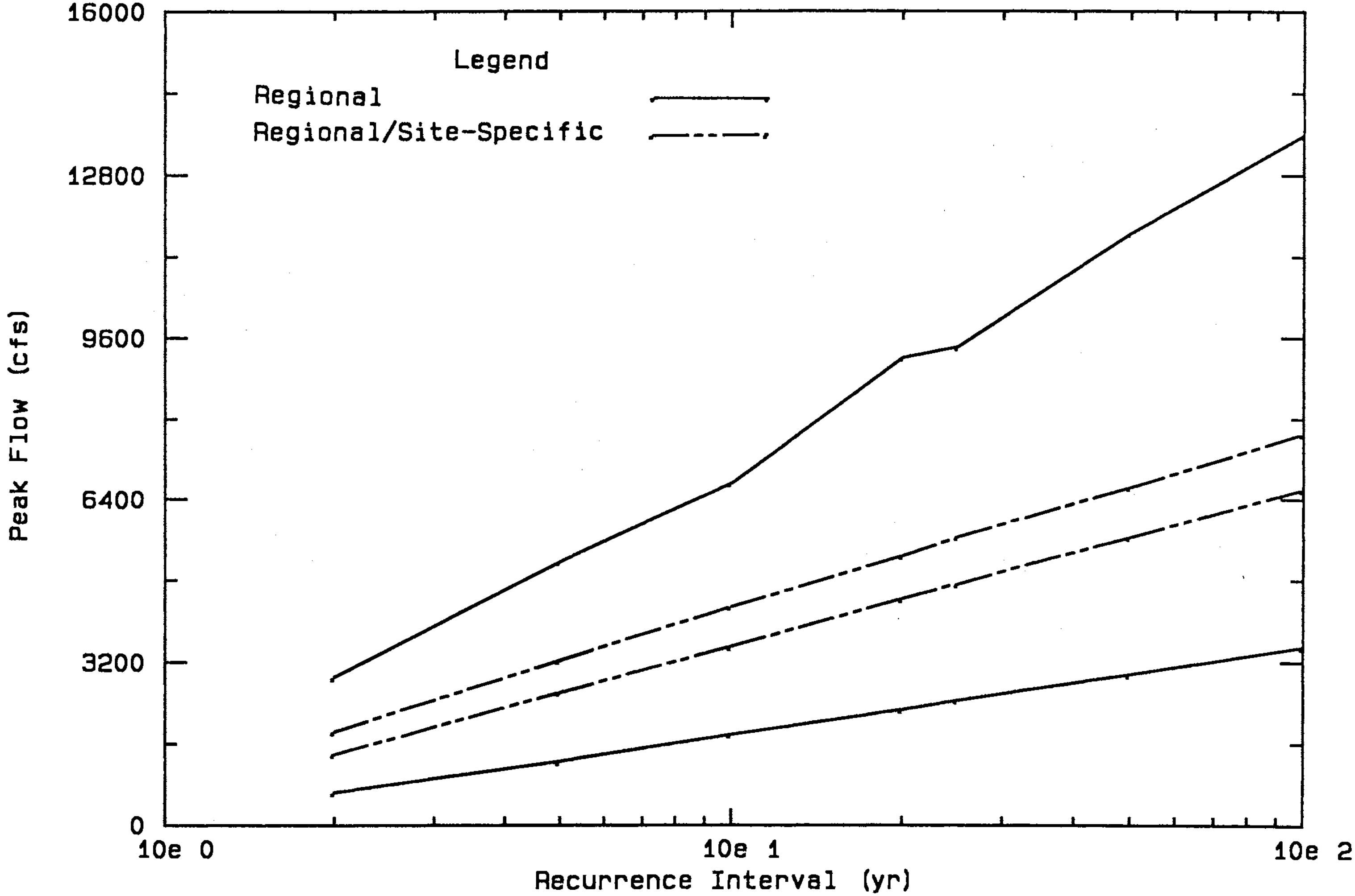

Figure 111. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 311

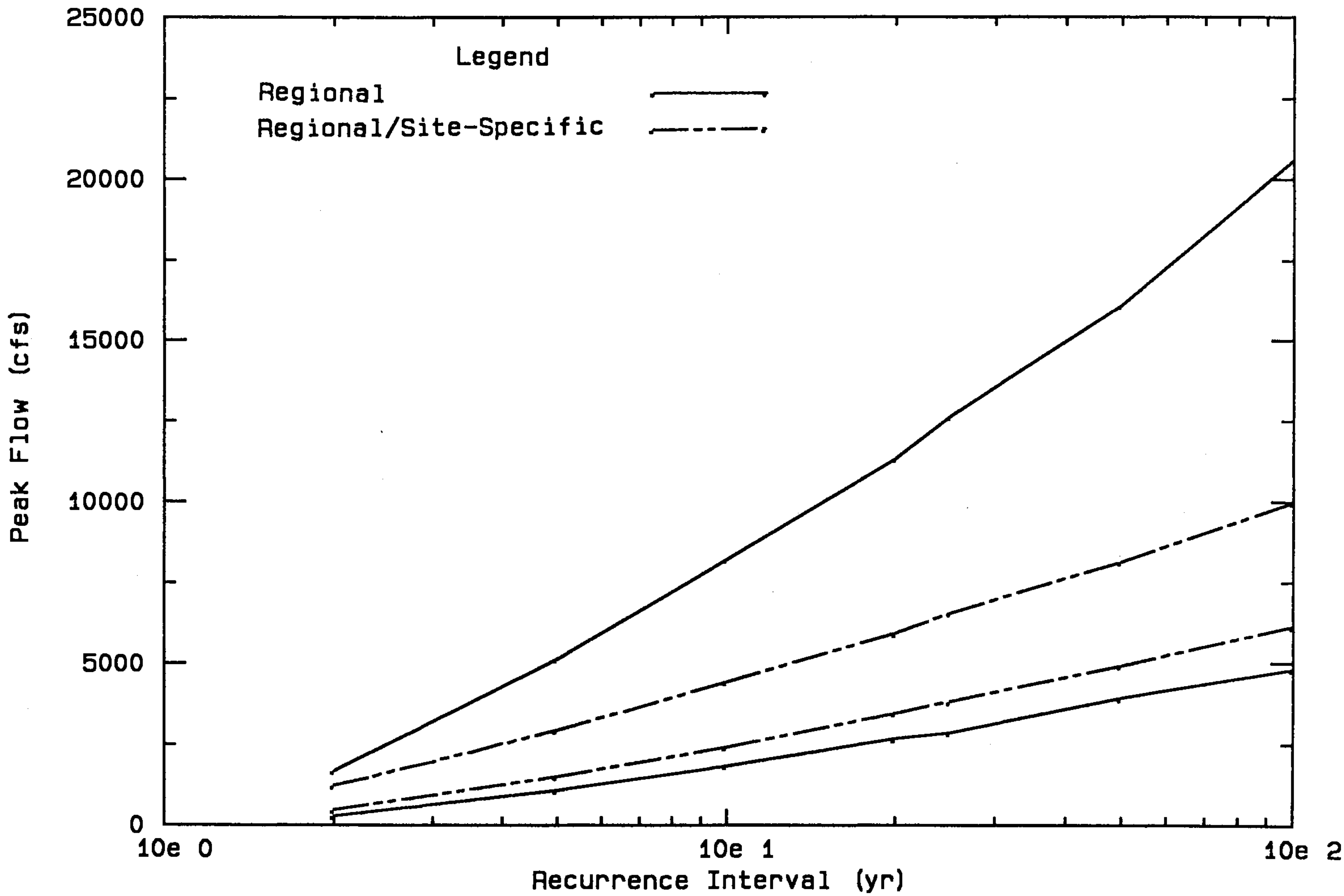

Figure 112. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 411

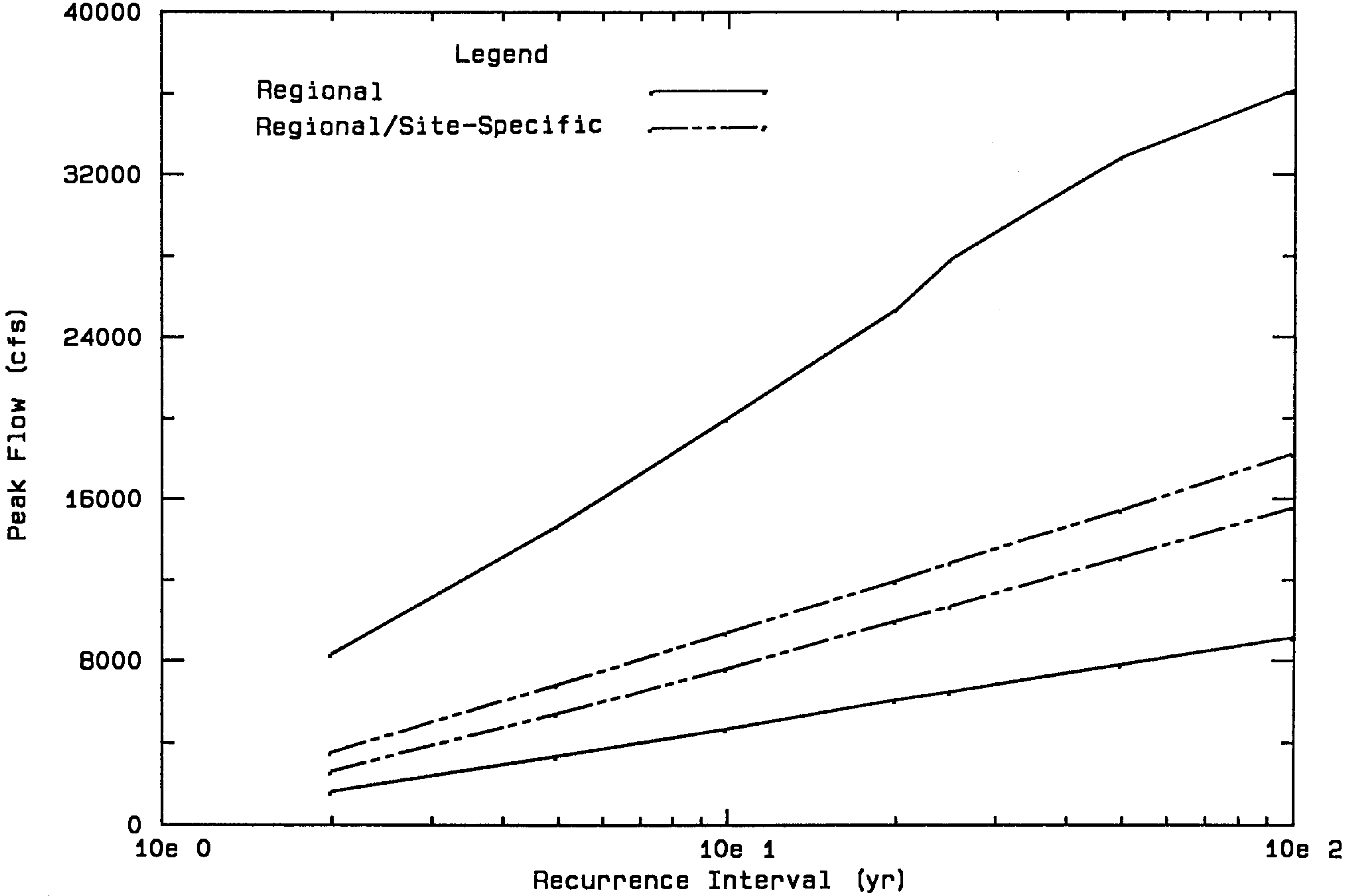

Figure 113. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 511

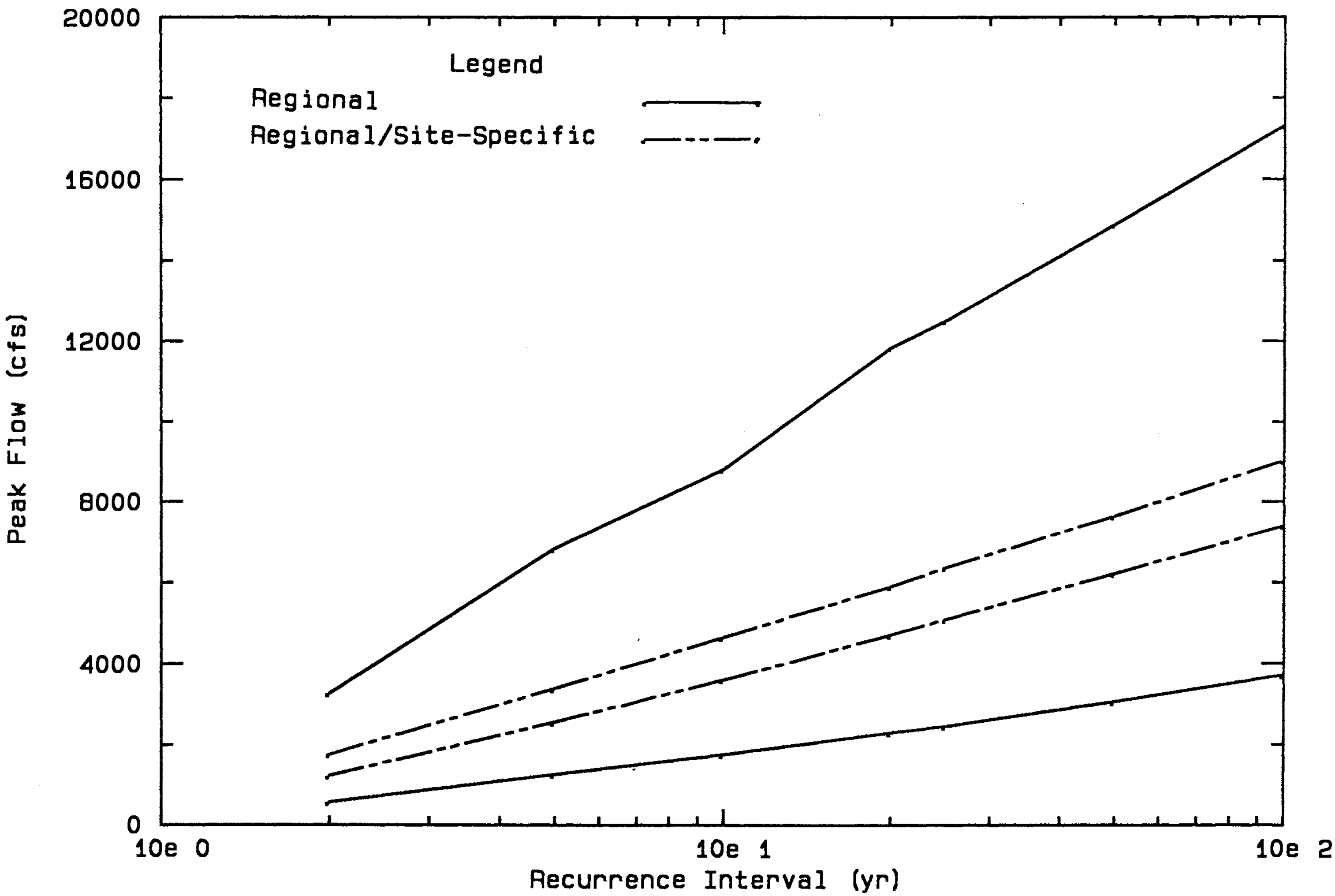

Figure 114. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 513

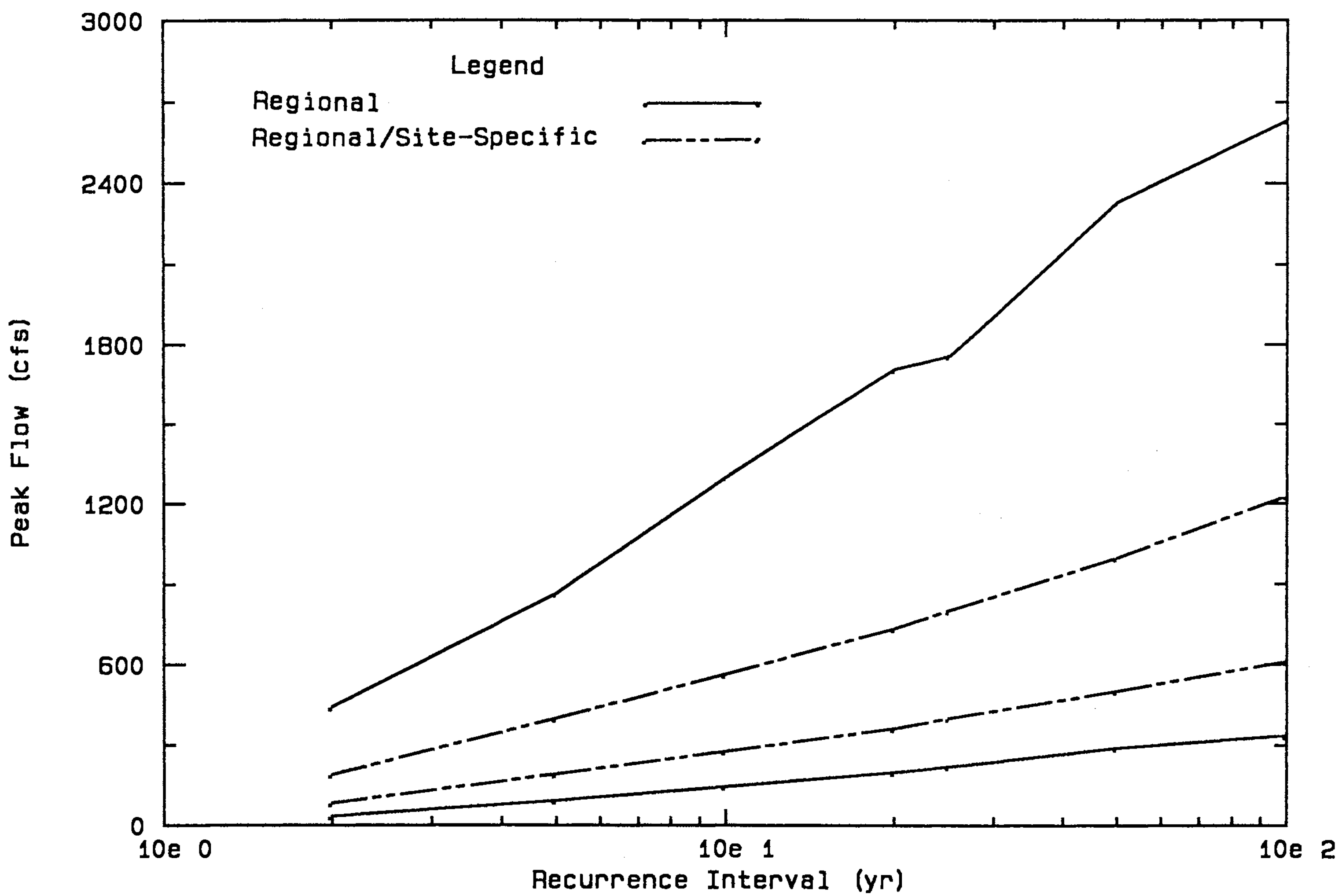

Figure 115. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 5142

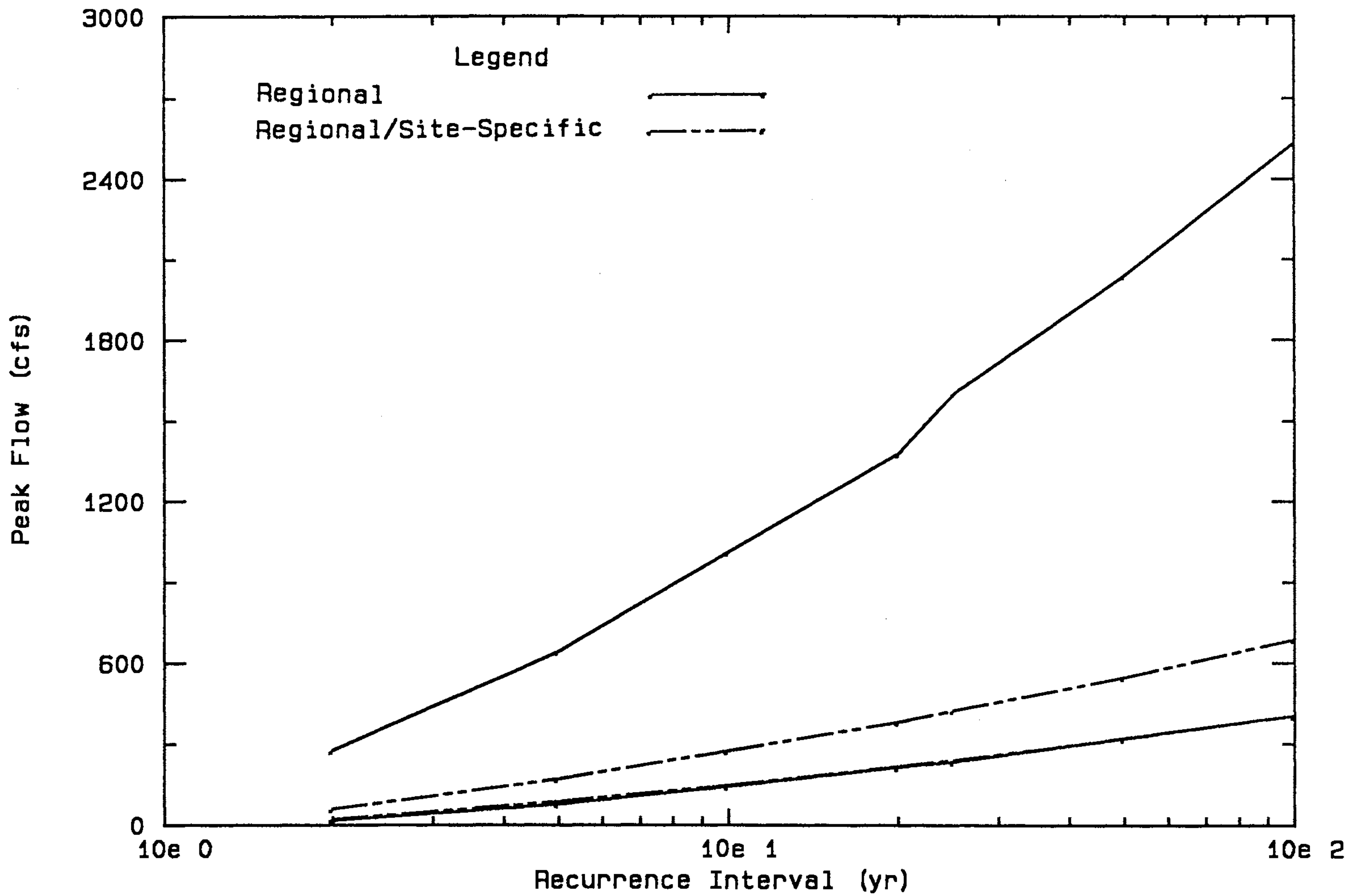

Figure 116. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 5143

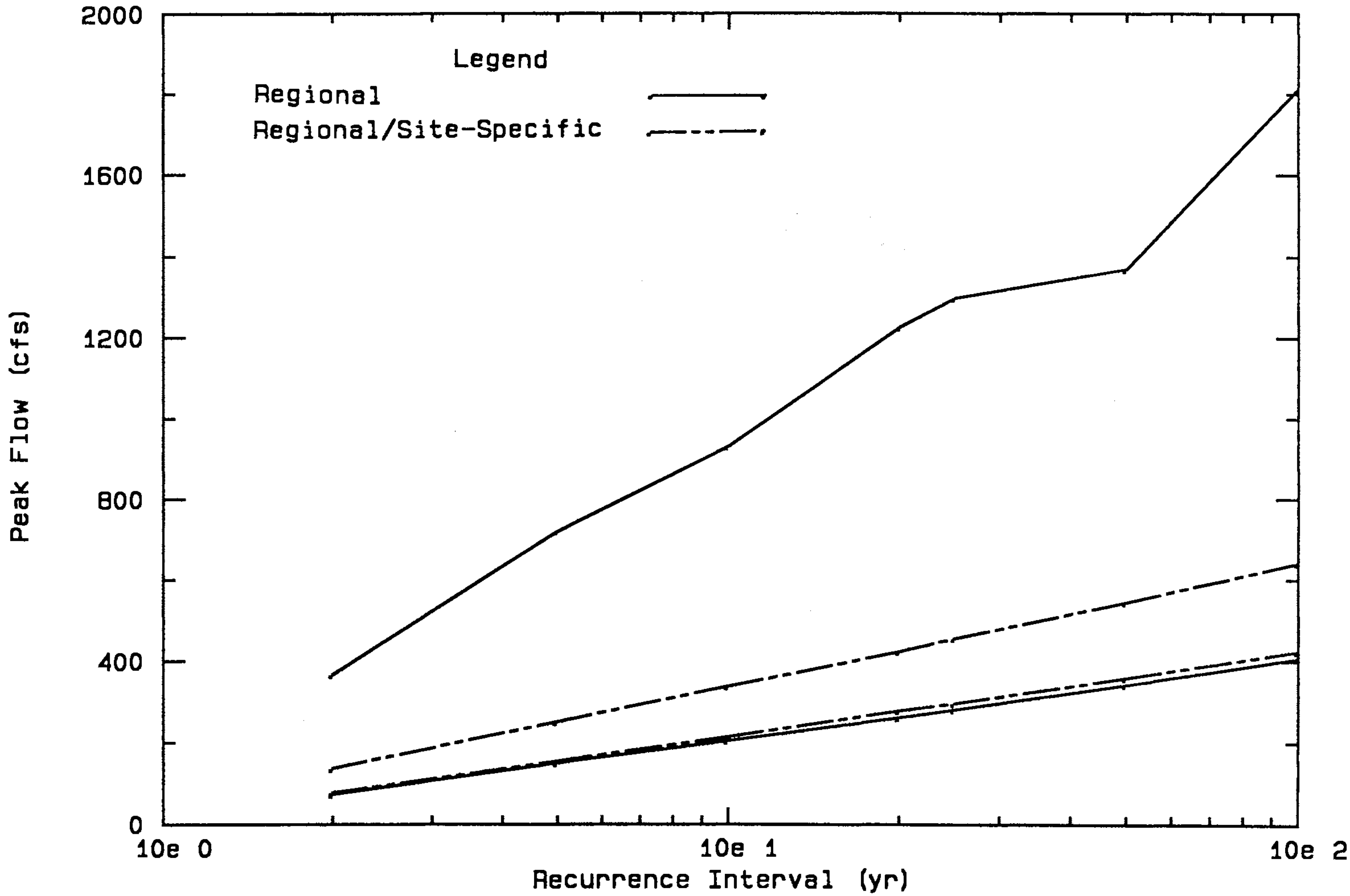

Figure 117. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 5145

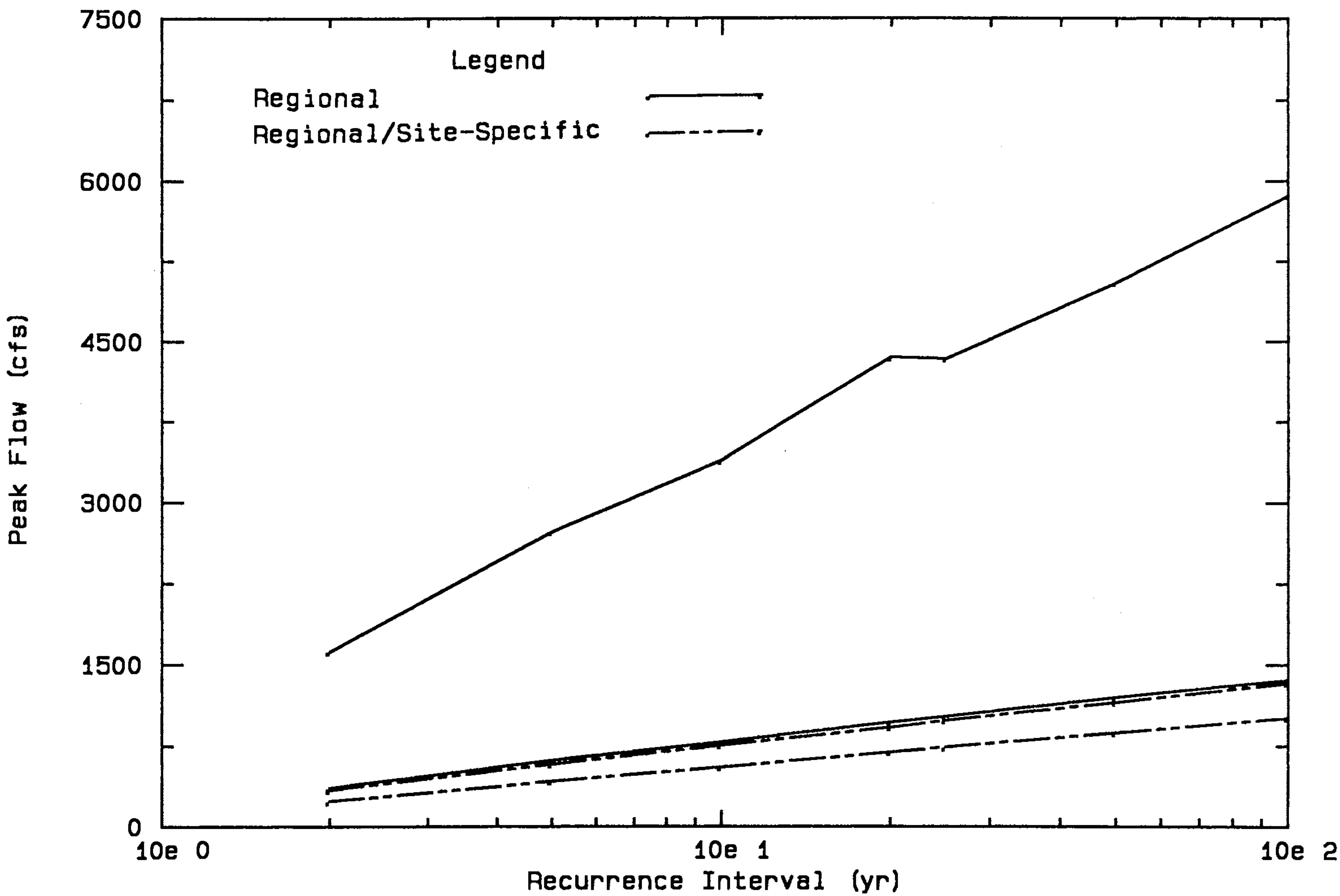

Figure 118. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 515

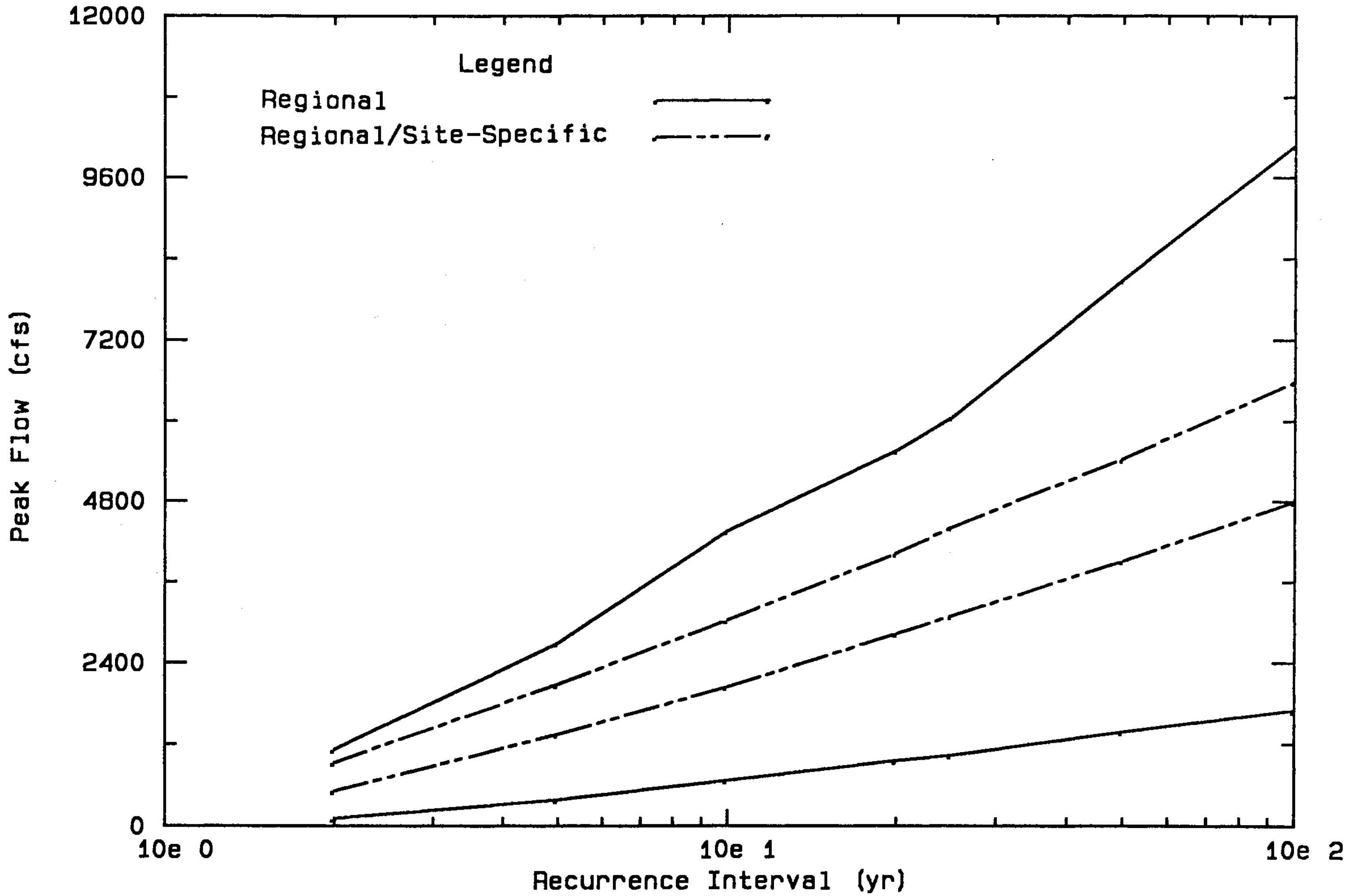

Figure 119. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed 611

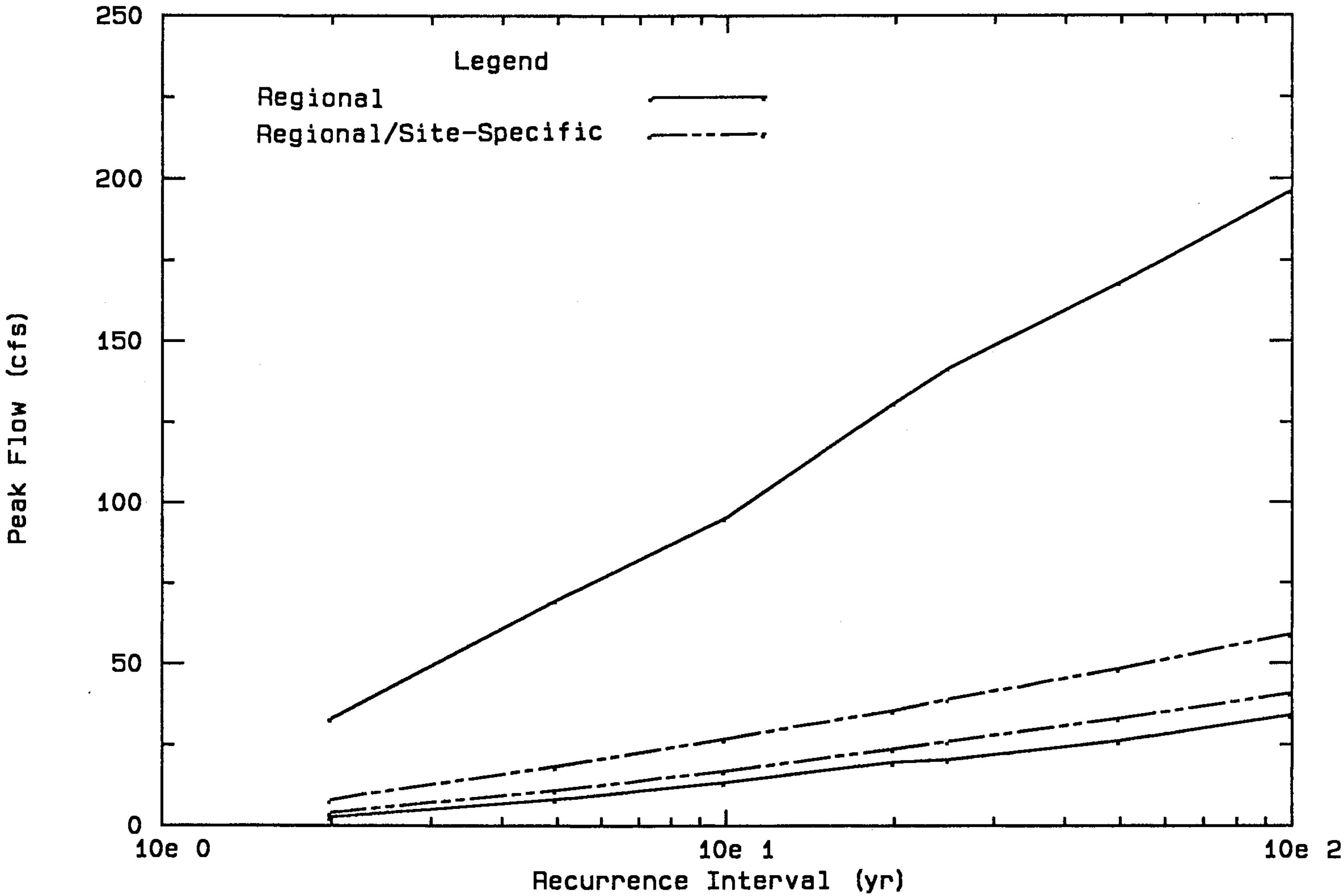

Figure 120. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed R5

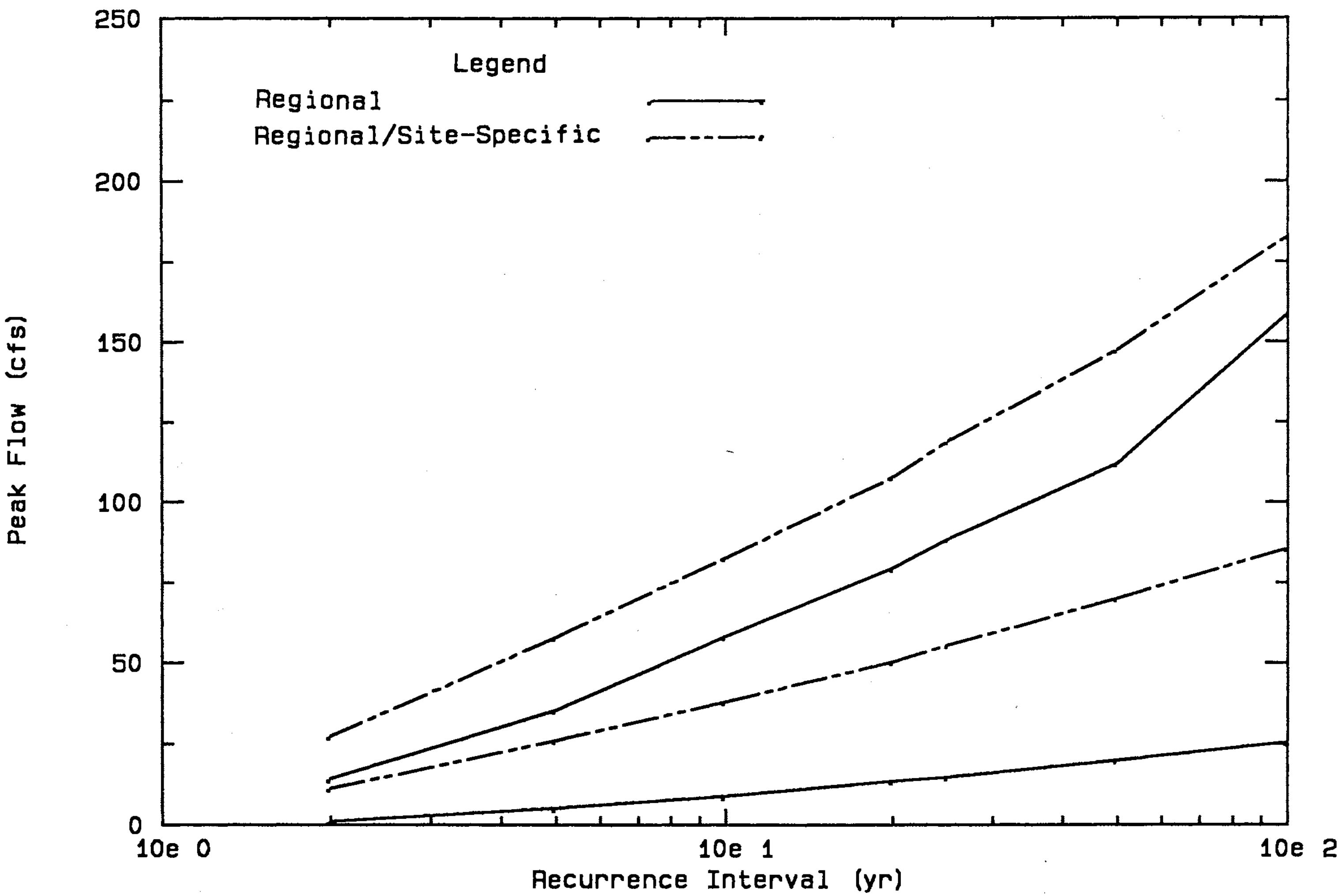

Figure 121. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed R6

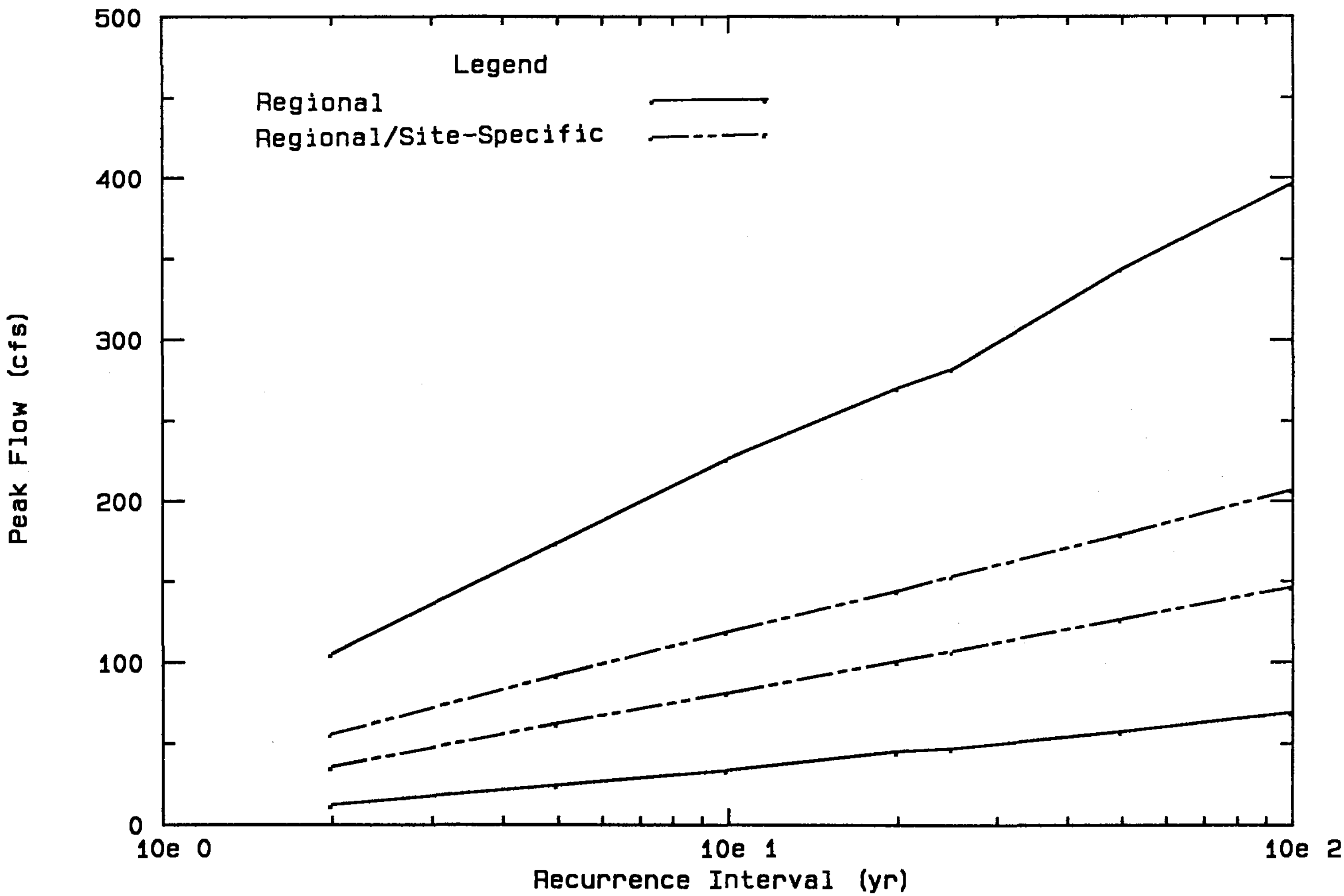

Figure 122. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed R7

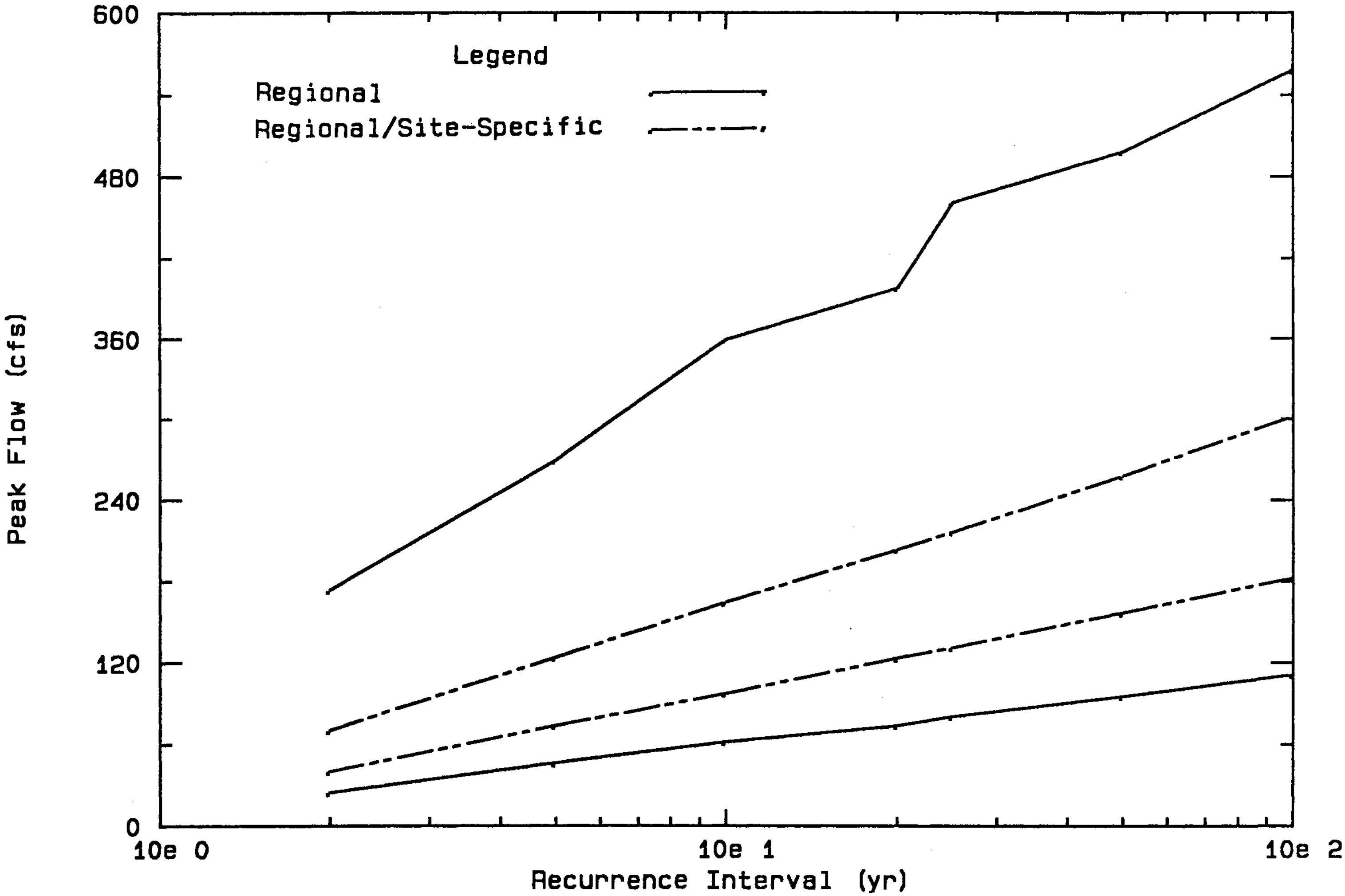

Figure 123. 90% Confidence Limits on the Bayesian Flood Frequency Curves for Watershed R8

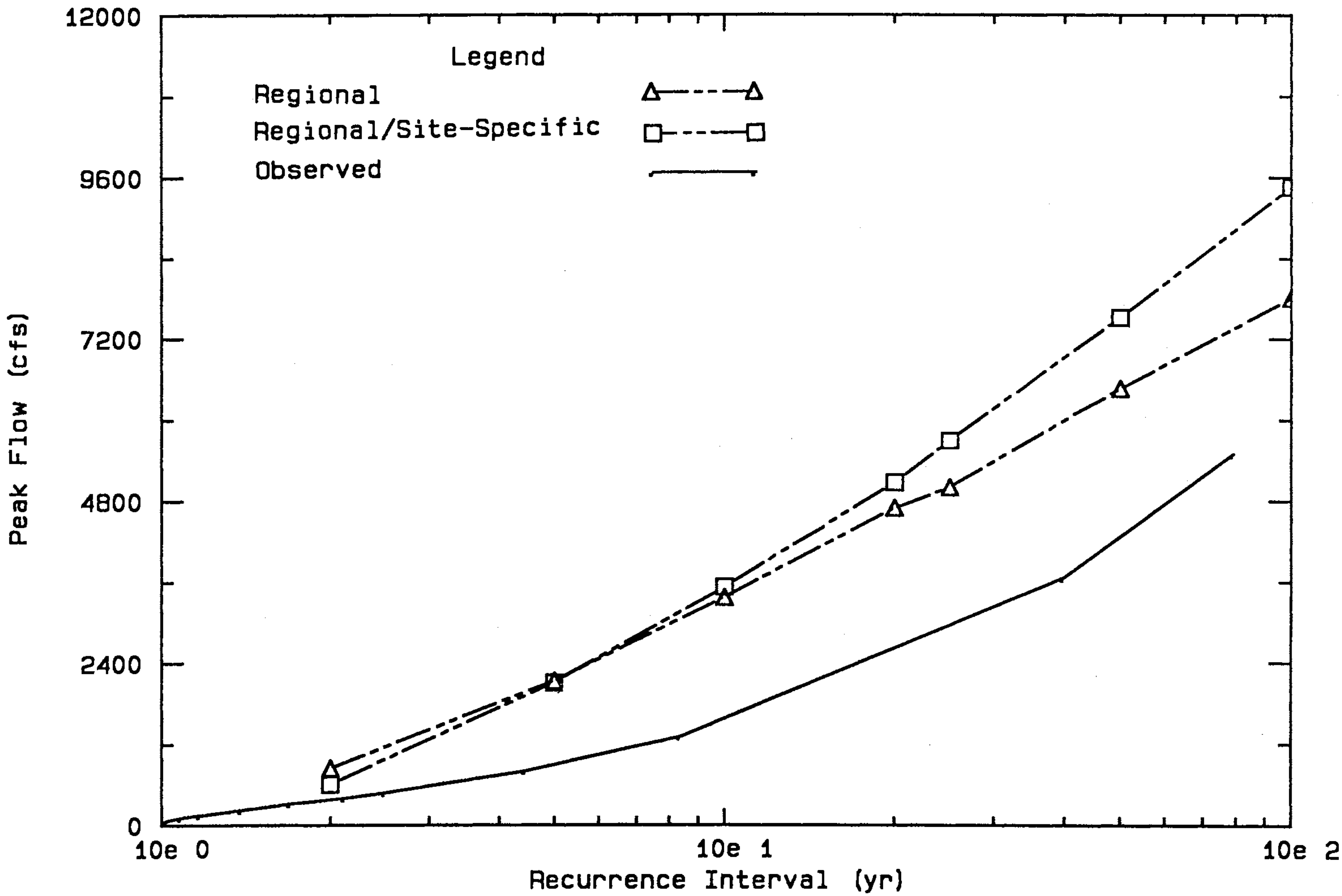

Figure 124. Comparison of Bayesian Flood Frequency Curves for Watershed 111

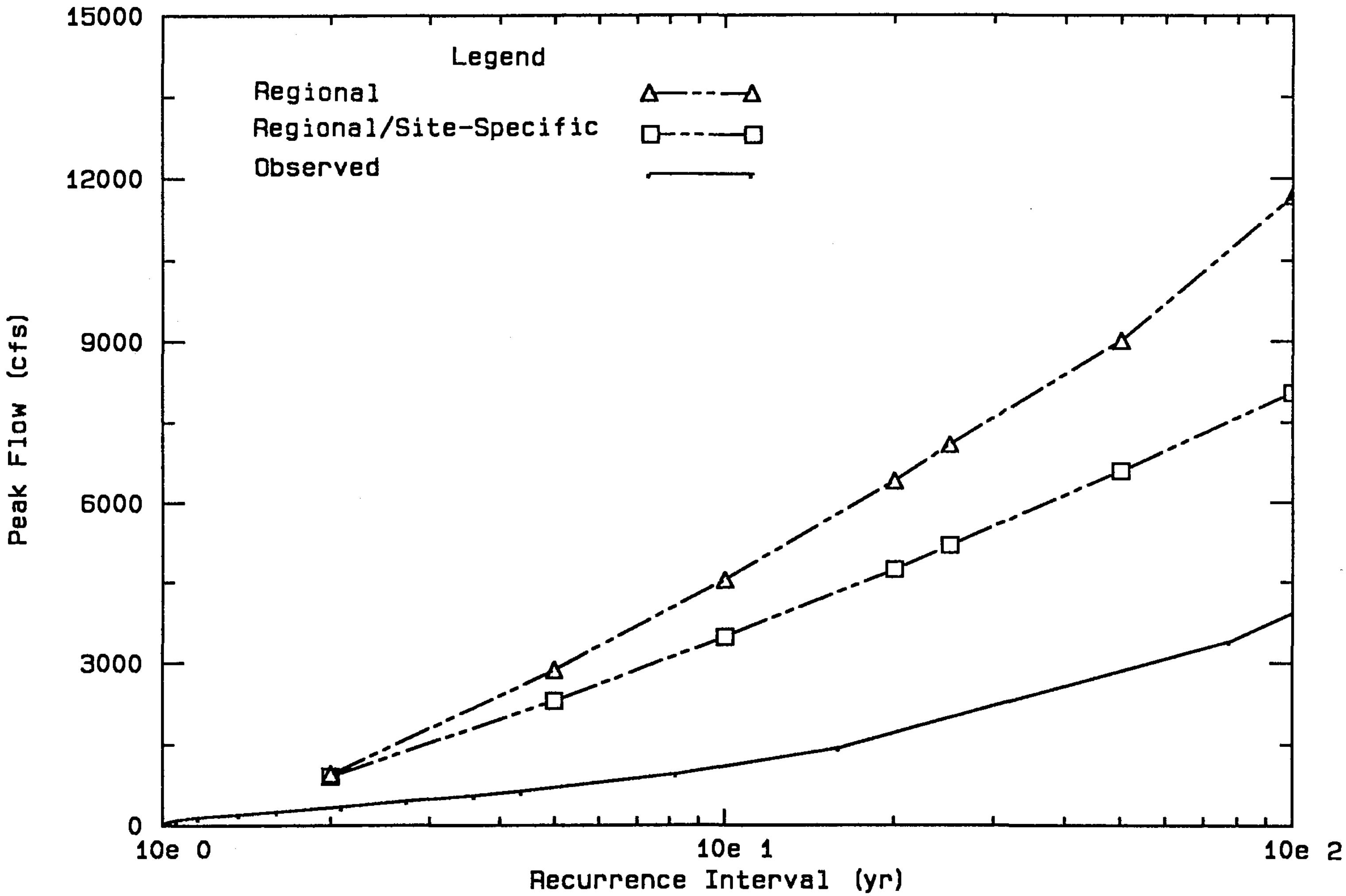

Figure 125. Comparison of Bayesian Flood Frequency Curves for Watershed 131

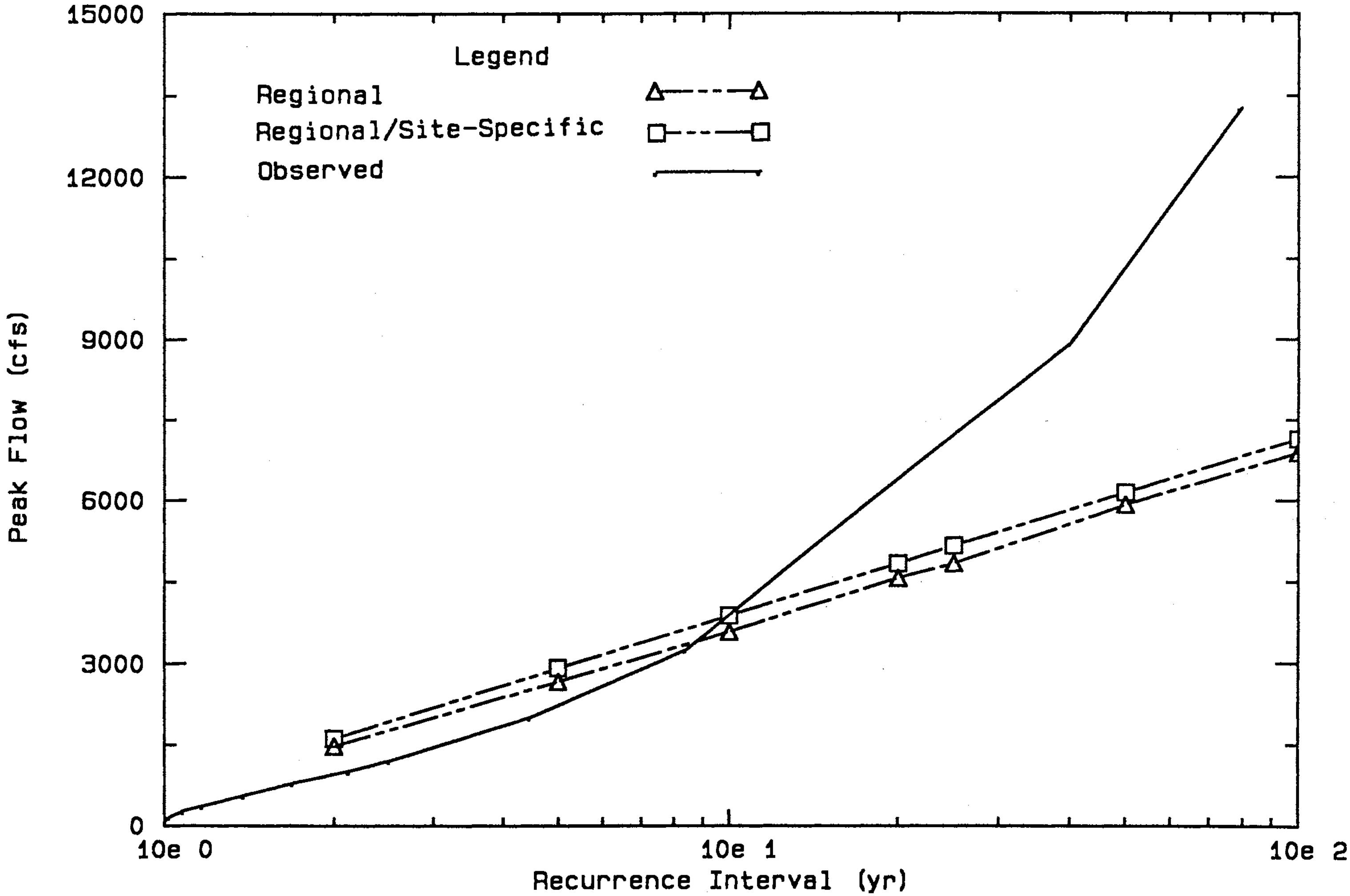

Figure 126. Comparison of Bayesian Flood Frequency Curves for Watershed 311

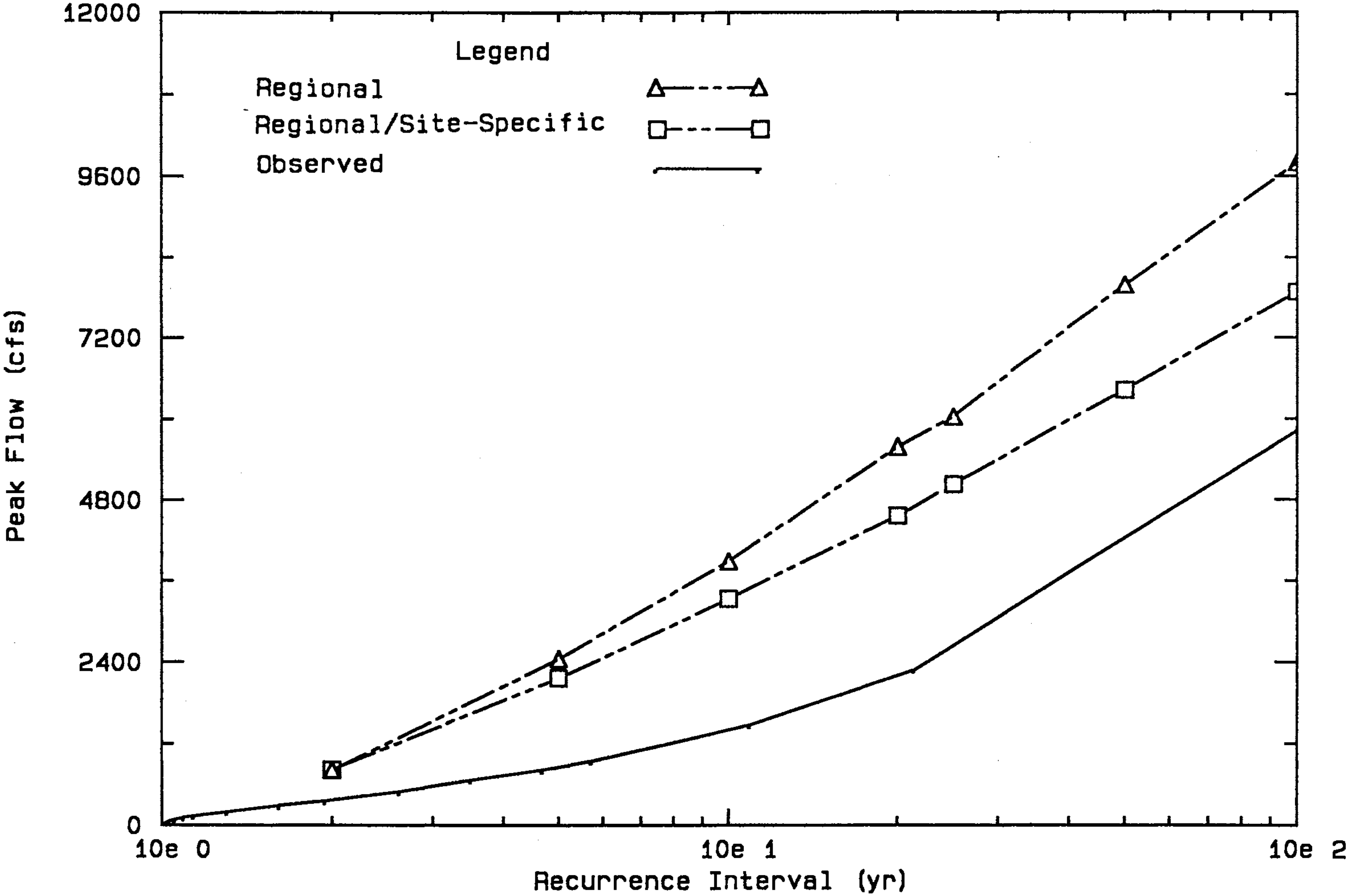

Figure 127. Comparison of Bayesian Flood Frequency Curves for Watershed 411

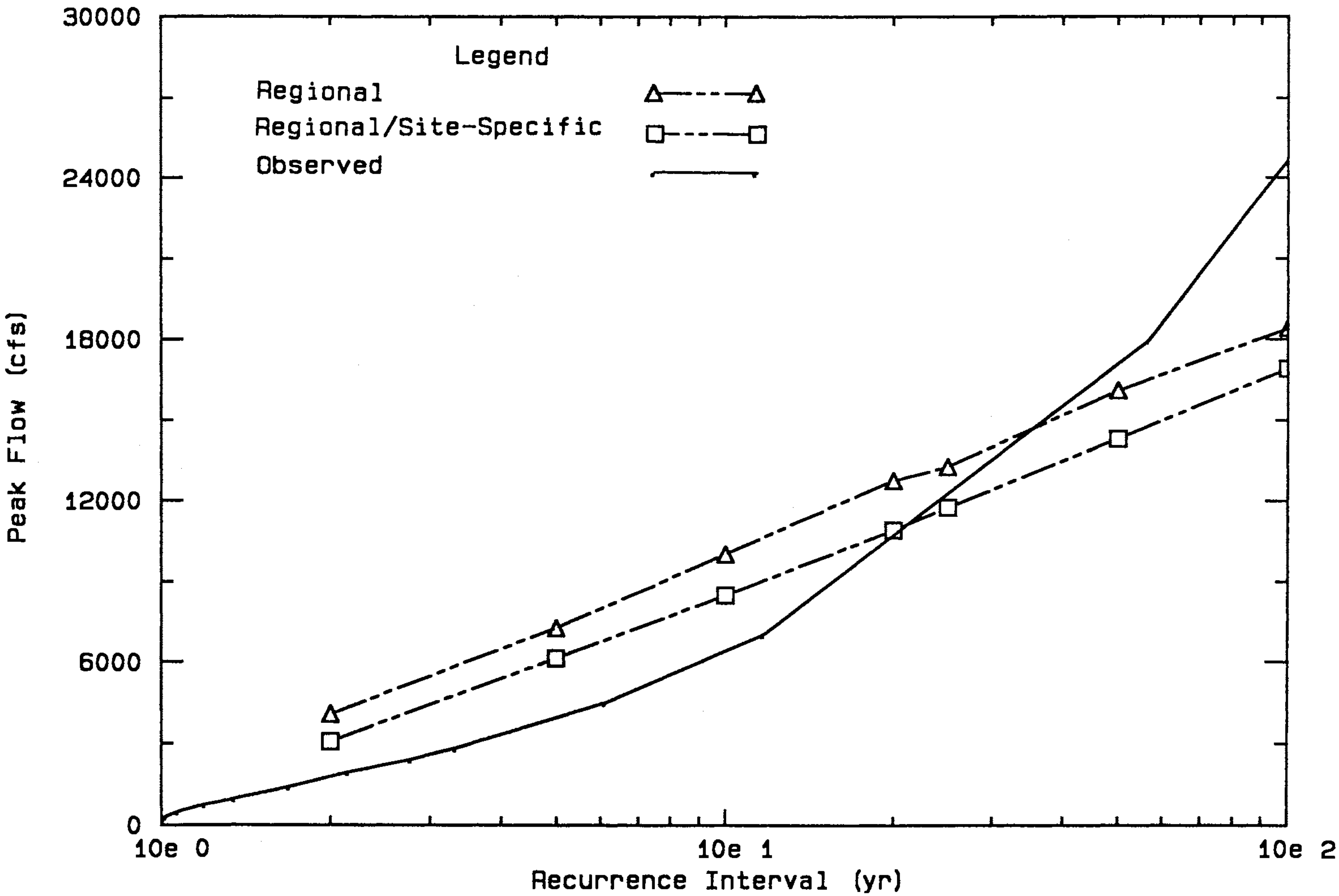

Figure 128. Comparison of Bayesian Flood Frequency Curves for Watershed 511

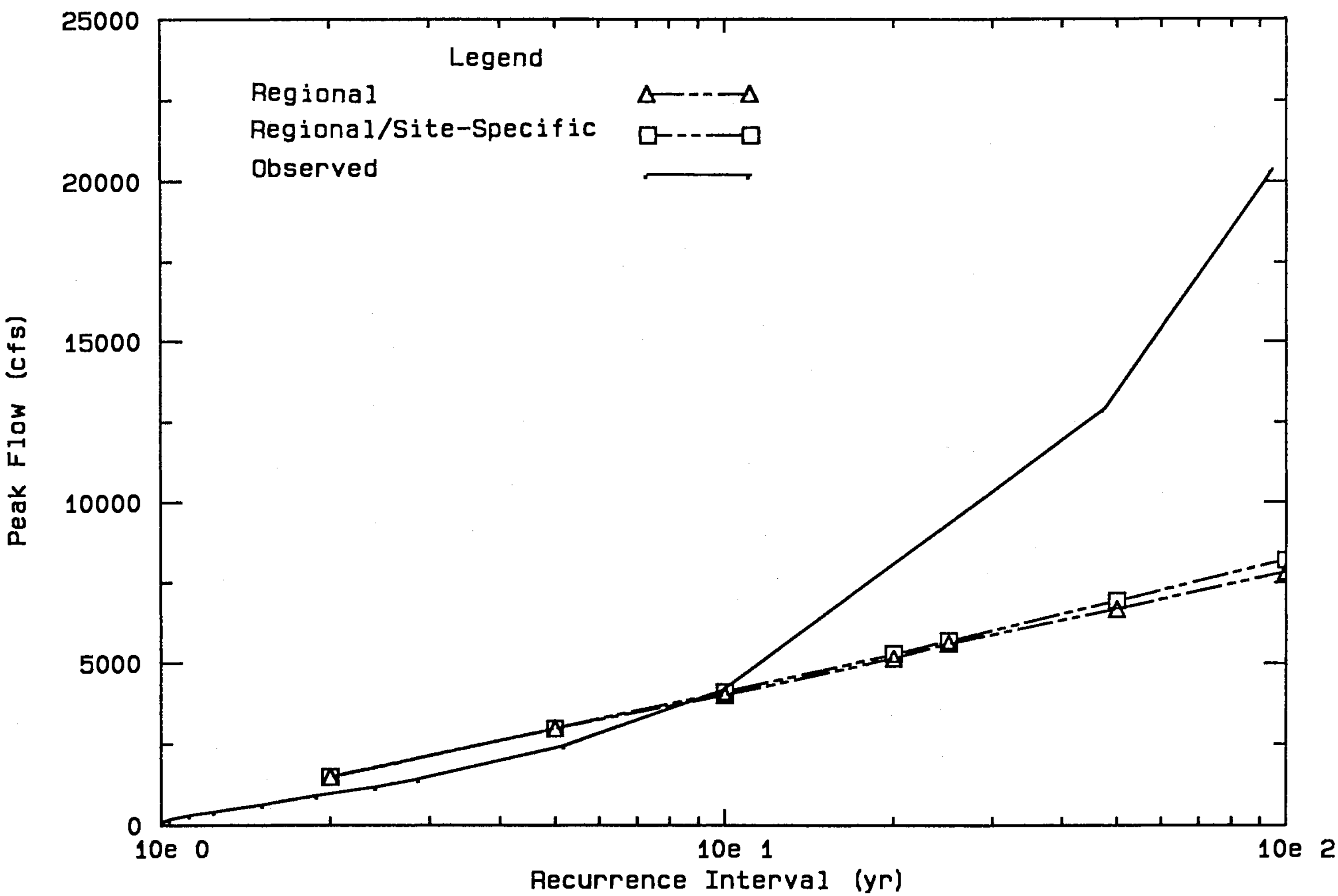

Figure 129. Comparison of Bayesian Flood Frequency Curves for Watershed 513

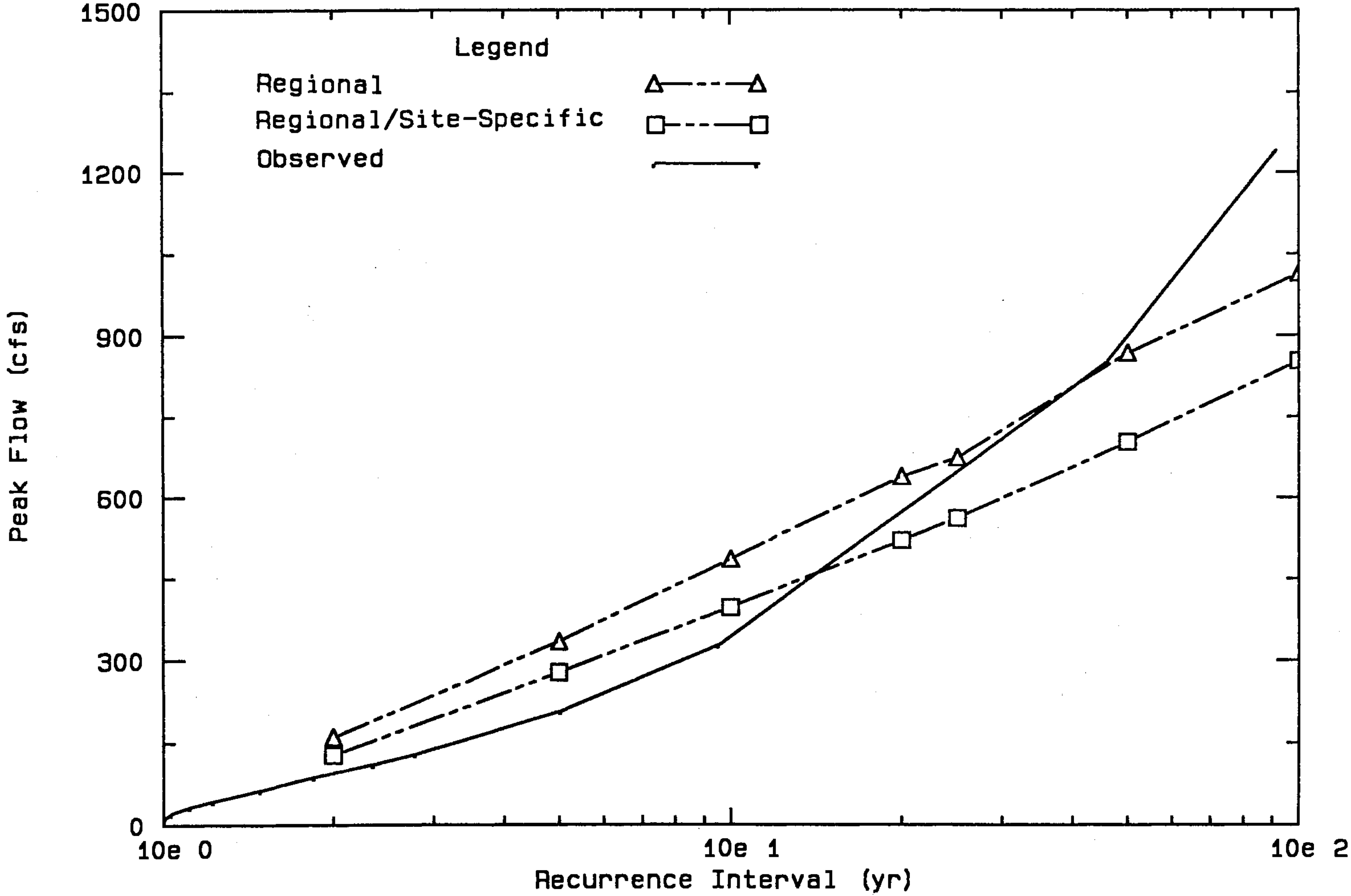

Figure 130. Comparison of Bayesian Flood Frequency Curves for Watershed 5142

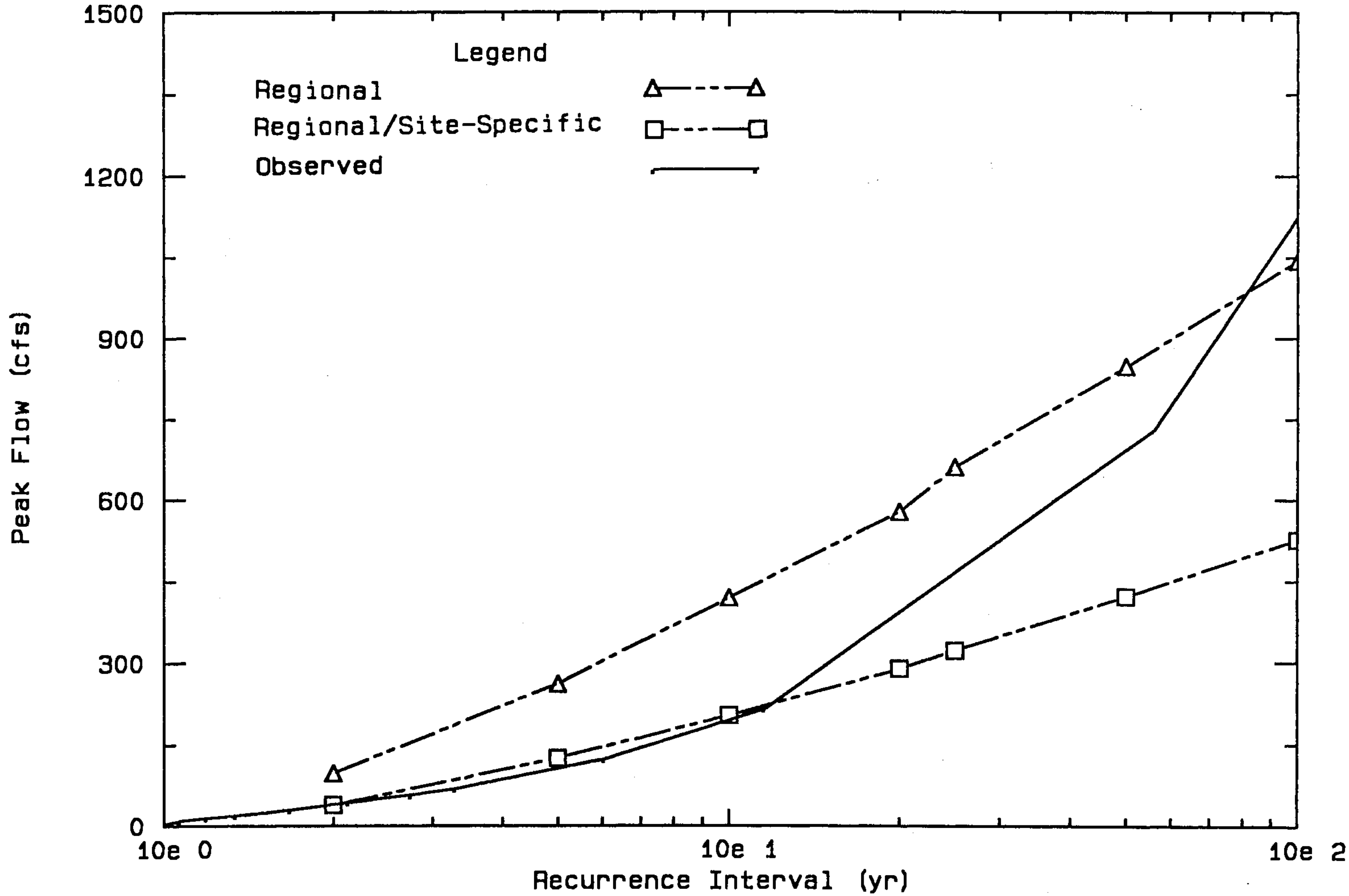

Figure 131. Comparison of Bayesian Flood Frequency Curves for Watershed 5143

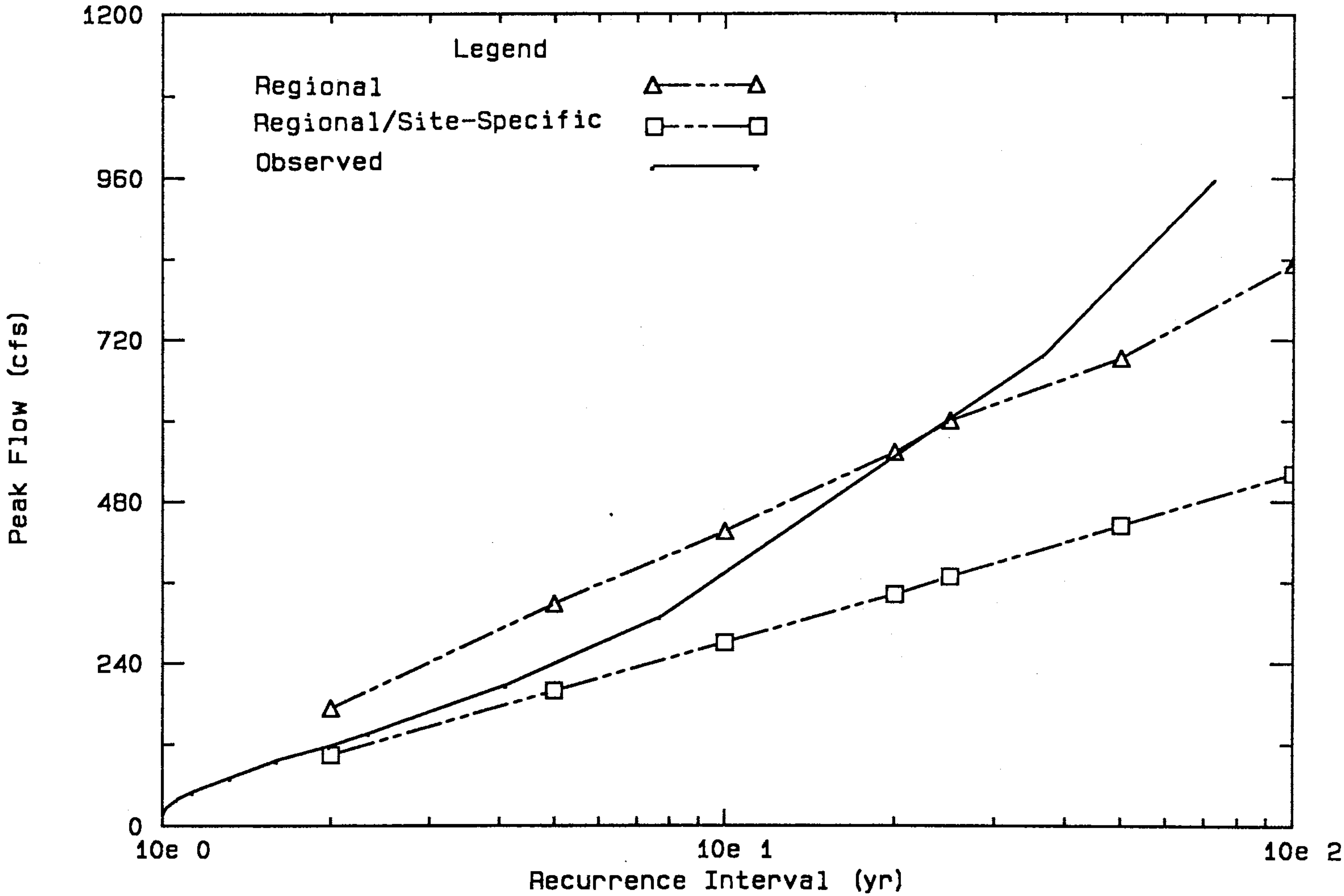

Figure 132. Comparison of Bayesian Flood Frequency Curves for Watershed 5145

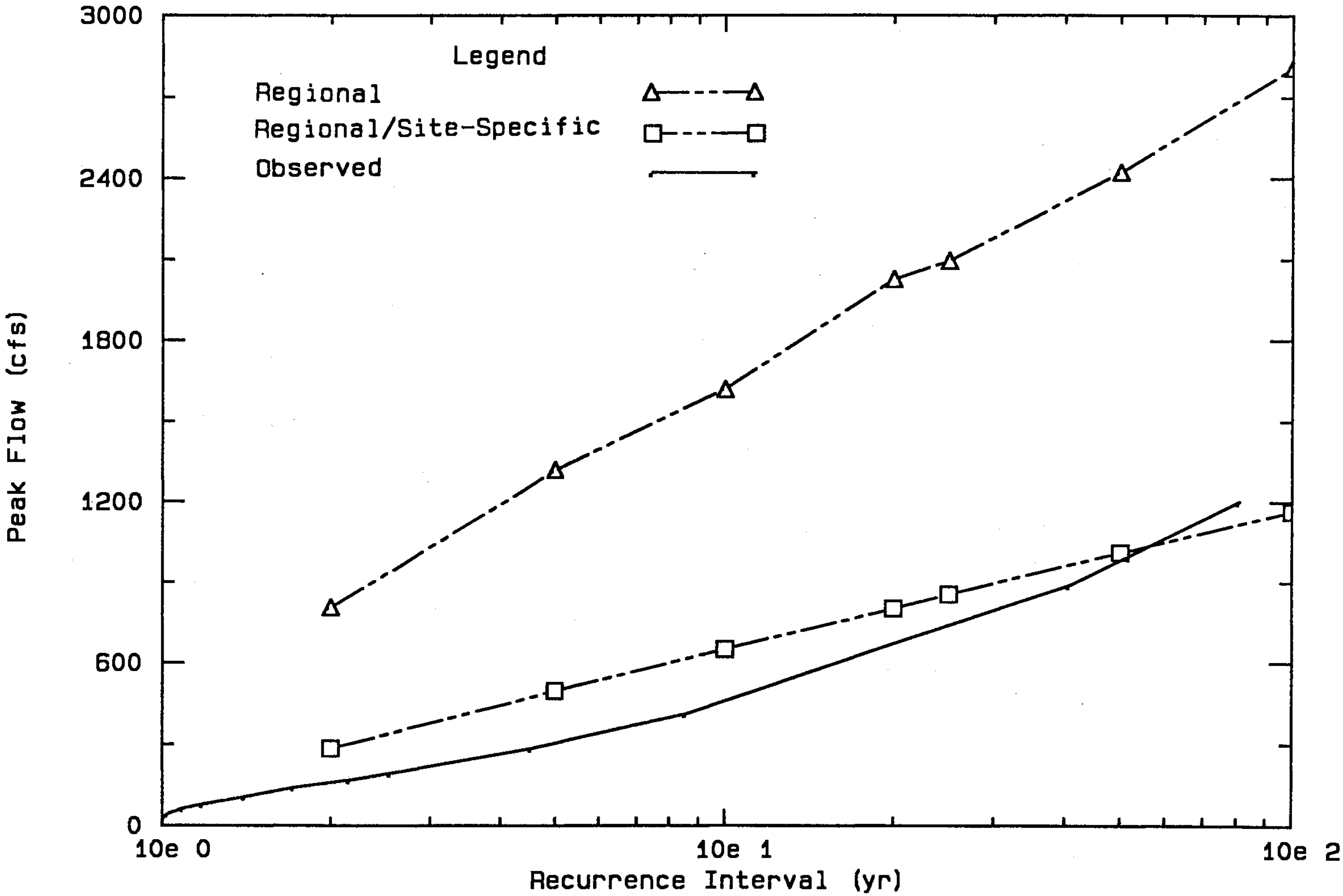

Figure 133. Comparison of Bayesian Flood Frequency Curves for Watershed 515

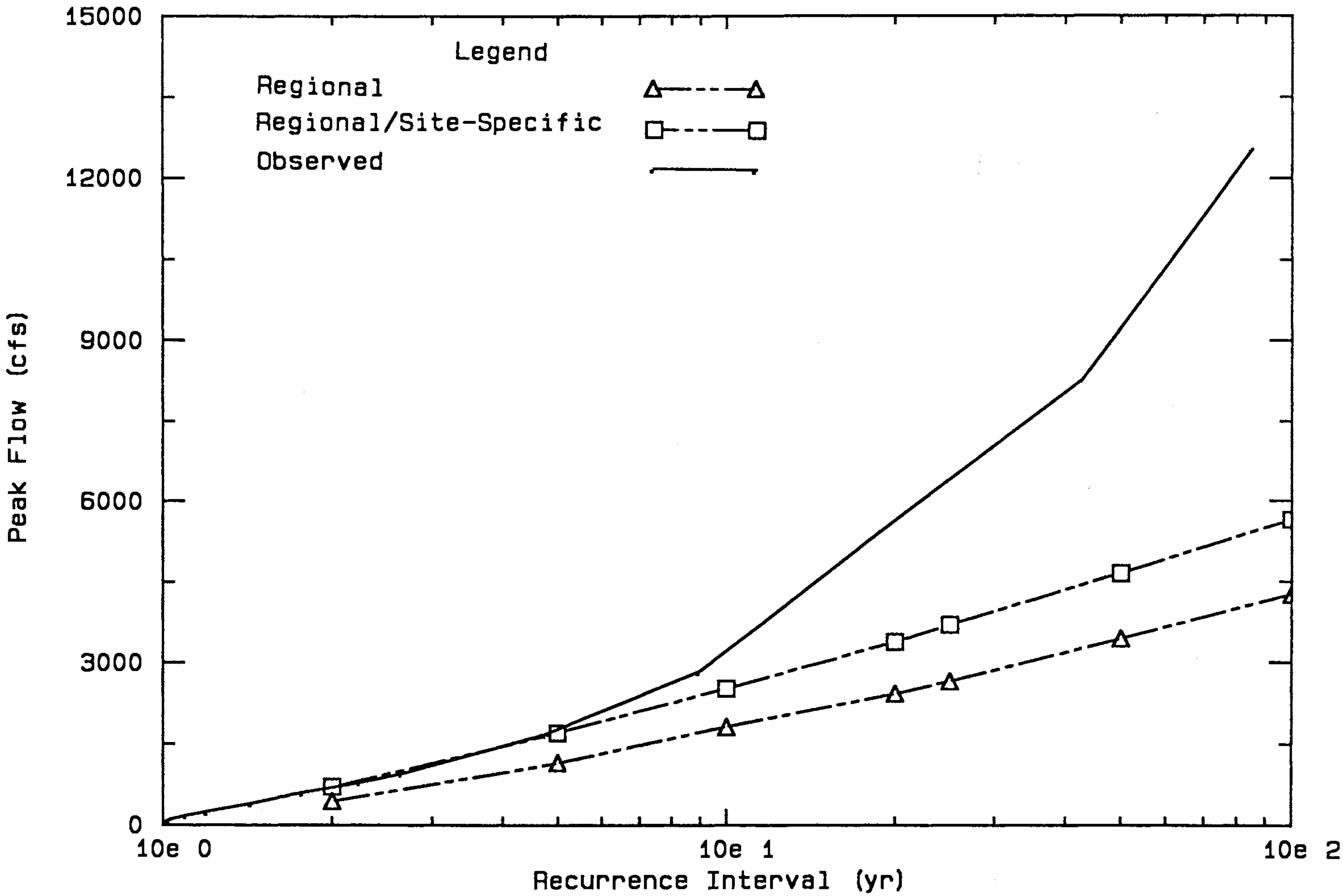

Figure 134. Comparison of Bayesian Flood Frequency Curves for Watershed 611

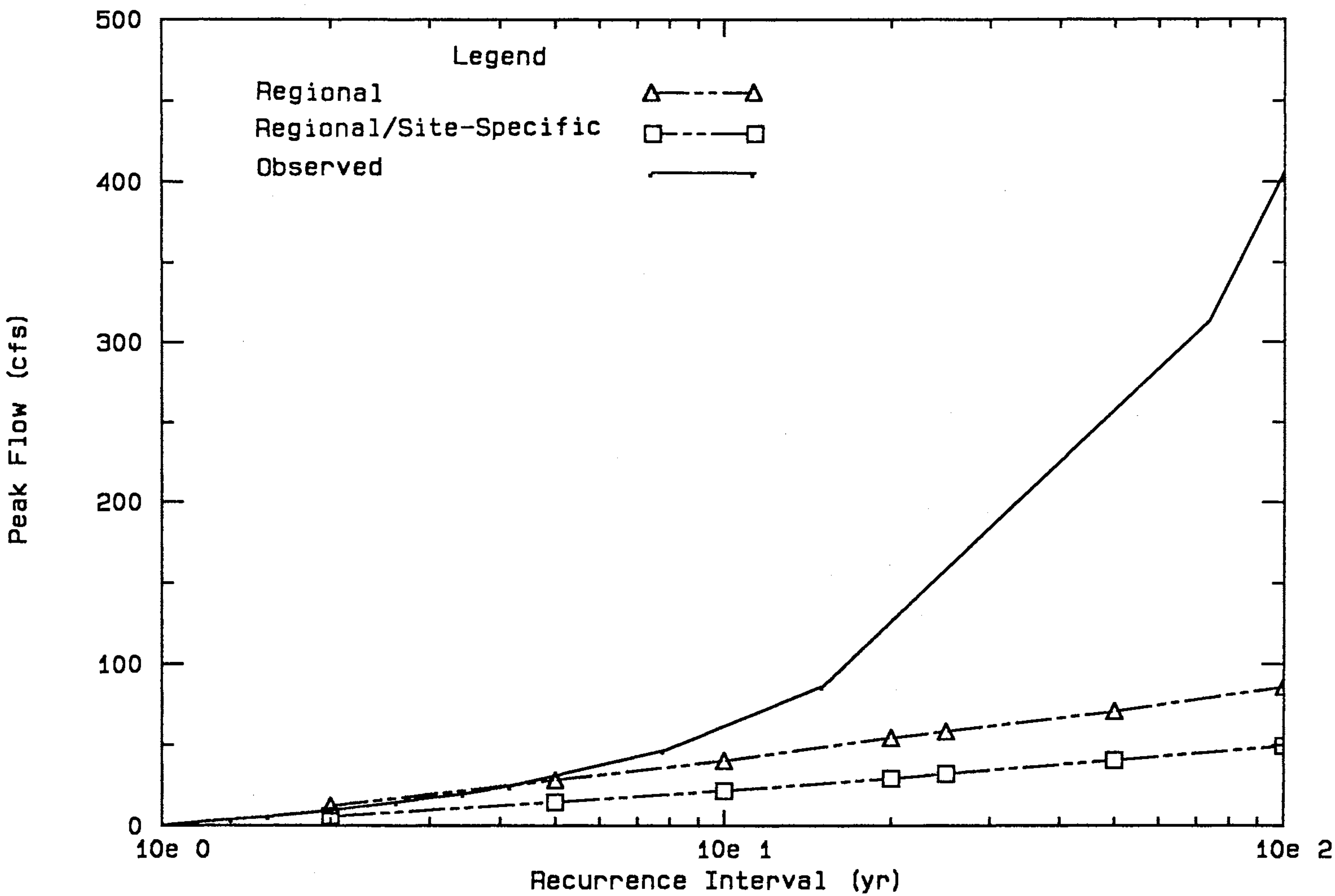

Figure 135. Comparison of Bayesian Flood Frequency Curves for Watershed R5

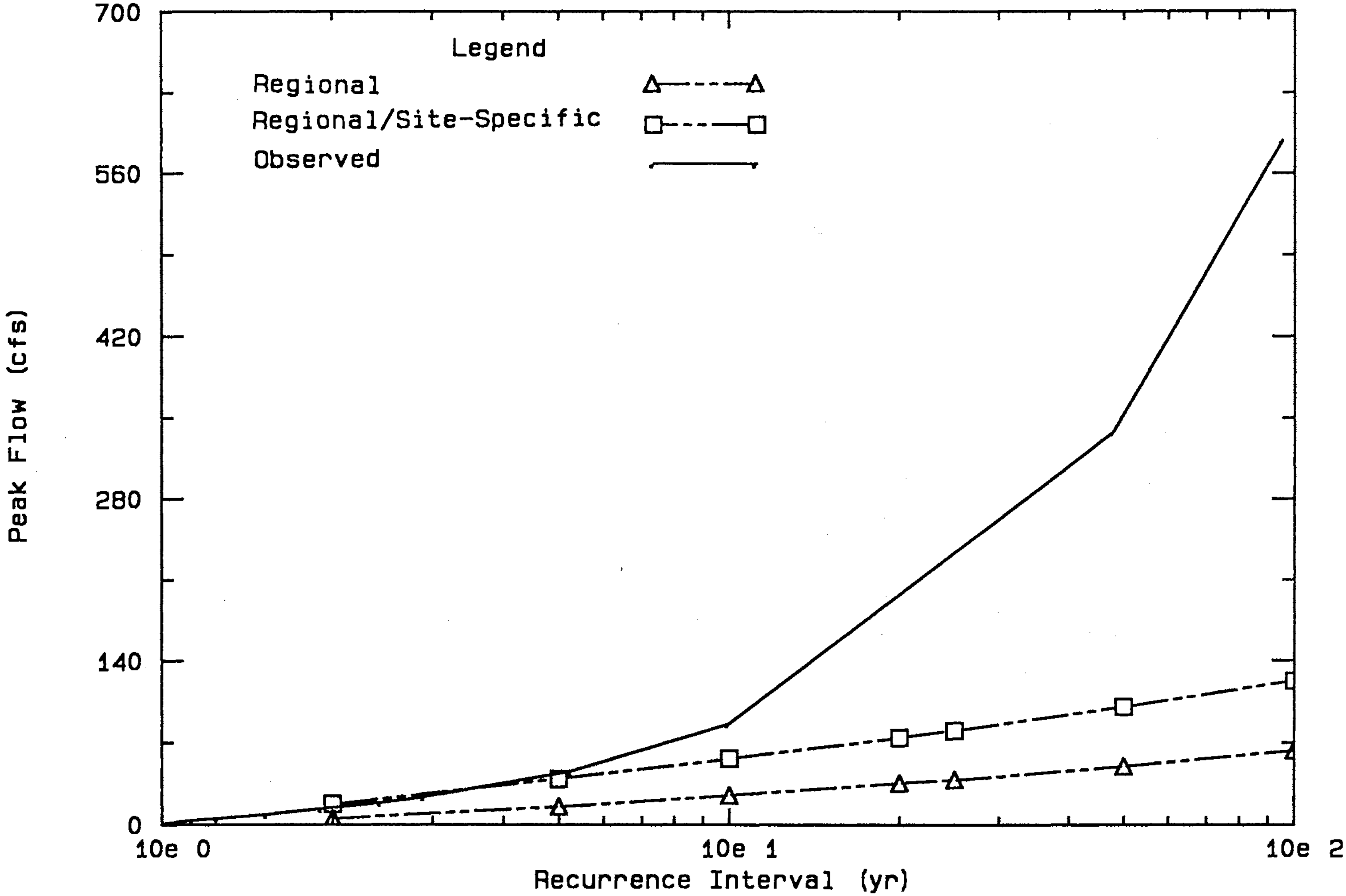

Figure 136. Comparison of Bayesian Flood Frequency Curves for Watershed R6

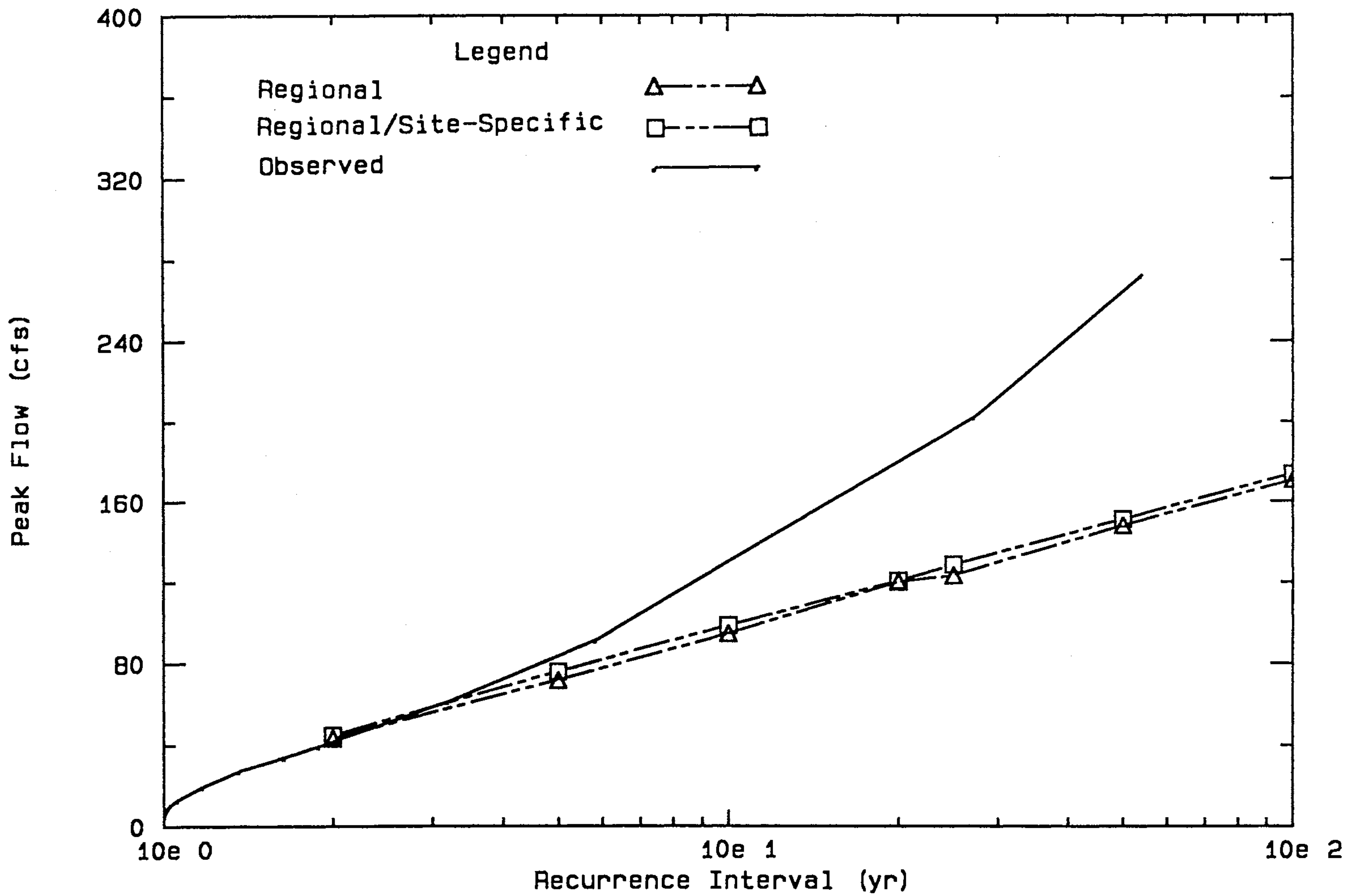

Figure 137. Comparison of Bayesian Flood Frequency Curves for Watershed R7

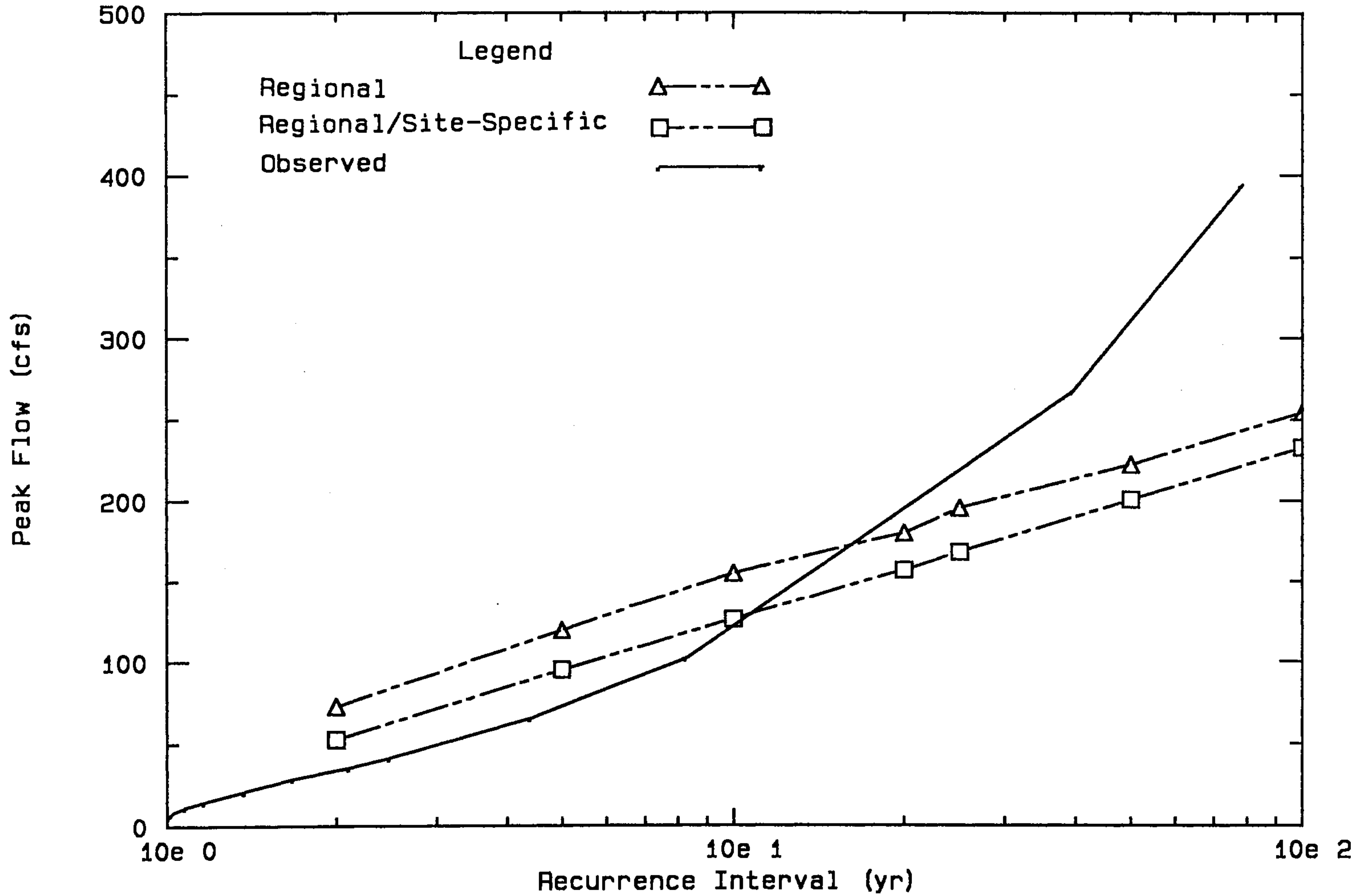

Figure 138. Comparison of Bayesian Flood Frequency Curves for Watershed R8

CHAPTER X

SUMMARY AND CONCLUSIONS

Summary

This dissertation has presented the development and evaluation of two flood estimation methodologies: one for ungaged watersheds and one for watersheds with short records. Both procedures use a variation of the SCS (1972) unit hydrograph model. The end result of both procedures is an estimated flood frequency curve, referred to as the Bayesian flood frequency curve. An additional characteristic of the procedures is the ability to specify confidence intervals on the Baysian curves, thereby yielding an indication of the uncertainty in the curves.

Probability density functions of the model parameters S and Tp were derived for 15 watersheds using methods of Bayesian statistical theory. All watersheds are contained within a relatively small region of the Washita River basin. The means and standard deviations of S and Tp computed from their respective probability density functions constitute, in effect, a pool of regional information which may be integrated into some framework for estimating floods for ungaged watersheds. Such a framework was developed, and the regional information was used in the form of prediction equations to estimate the mean and variance of S and Tp for ungaged watersheds. Based upon the probability density functions derived for the gaged watersheds, S and Tp may be taken as normally distributed. Thus for an ungaged watershed, the distributions of S and Tp may be taken as normal, and the regional information may be used to estimate the means and variances of these model parameters. Flood frequency curves are generated by taking a fixed rainfall event having duration equal to the watershed's time of concentration as an input to the SCS unit hydrograph model. Successive values of S and Tp are generated from their respective probability density functions as further model input. The mean of the resulting peak flows is taken as having a recurrence interval equal to that of the rainfall event. Confidence intervals are derived from the empirical distribution of peak flow.

The flood estimation procedure for ungaged watersheds was evaluated using a Jackknife approach. Flood frequency curves were estimated for each of the study watersheds as if it were ungaged; that is to say, the regional pool of information was adjusted to omit each study watershed as its flood frequency curve was estimated. On the basis of this type of evaluation, the procedure appeared to produce reasonable results. Kolmogorov-Smirnov goodness of fit tests indicate that one may not reject the hypothesis that the Baysian flood frequency curves are equal to the curves derived from site data. The Baysian curves were very similar to curves derived using USGS (Tortorelli and Bergman, 1985) procedures, and appeared obviously superior to curves derived using SCS (1972) procedures. The confidence bounds on the Baysian flood frequency curves indicate high uncertainty in the curves, due in great part to the imprecision of the regionally-derived data.

Flood frequency curves are estimated for watersheds with short records by augmenting, via Bayes' Theorem, regionally-derived information with site specific information as it becomes available. In proper terminology, the regionally-derived information is used to specify the prior probability density function of S and Tp. Site-specific information is used to derive the likelihood function of the two parameters. The posterior probability density function, which represents an integration of both regionally-derived and site-specific information, is determined directly upon application of Bayes' Theorem. An examination of the posterior probability density functions of S and Tp suggests that they may again be considered normal.

The remainder of the flood estimation procedure is analogous to that for ungaged watersheds.

A Jackknife approach was again used in the evaluation of the flood estimation procedure for watersheds with short records. The Baysian flood frequency curves again seemed reasonable, and Kolmogorov-Smirnov goodness of fit tests indicate that they are not significantly different from the observed curves. The addition of the site-specific data appeared to improve the accuracy of the Baysian curves relative to the USGS curves, which were unchanged with the addition of this data. The SCS flood frequency curves were modified as a result of the site-specific information, but their performance with regard to accuracy was virtually the same as without the site-specific information.

The site-specific information apparently resulted in more accurate flood frequency curves than resulted from regionally-derived information alone. It also resulted in a dramatic decrease in the 90% confidence intervals on the Baysian curves, illustrating quantitatively the effects of relatively precise data on uncertainty in the flood frequency curves.

Conclusions

Based upon the results of this research, the following conclusions are drawn:

1. The flood estimation methodologies for ungaged watersheds and for watersheds with short records are practical and yield reasonable estimates of flood frequency curves.

2. The statistical foundation of these two methodologies gives rise to the logical incorporation of all available data, both regionally-derived and site-specific, and provides an excellent means of conveying the uncertainty associated with the Baysian flood frequency curves.

3. Site-specific information reduces uncertainty in the Baysian flood frequency curves.

Recommendations for Further Research

The following topics are suggested as deserving of further investigation:

1. The statistical concepts elucidated in this dissertation should be applied to other rainfall-runoff models with a view toward improving Baysian flood frequency curves.
2. The worth of site-specific information should be investigated from a statistical and/or economic perspective.
3. The worth of regionally-derived information should be investigated from a statistical and/or economic perspective.
4. Improvements on the prediction equations should be attempted in order to improve the precision of regionally-derived information.
5. The relationship between informational uncertainty and risk should be investigated in the context of hydrologic projects.

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APPENDIXES

APPENDIX A

SUMMARIZED STUDY EVENTS

Summarized Study Events for Watershed 111

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	083162	1.50	3.25	297.38	0.03325
2	091562	2.24	4.75	775.80	0.12280
3	092062	0.42	0.75	33.70	0.00492
4	042663	1.69	7.00	136.20	0.03553
5	053163	0.93	1.00	112.52	0.01517
6	060363	0.55	1.75	8.43	0.00763
7	071363	0.80	6.75	27.69	0.00569
8	042564	0.39	0.50	13.80	0.00289
9	050964	1.61	3.00	697.10	0.14261
10	051064	0.98	1.00	759.70	0.14215
11	052964	1.35	11.50	69.50	0.01813
12	061164	1.33	2.50	276.10	0.05014
13	081864	1.24	3.75	49.60	0.00664
14	091564	0.69	2.75	12.80	0.00225
15	092064	2.47	6.00	356.00	0.05178
16	092264	0.73	7.50	18.20	0.00630
17	092664	1.28	8.00	256.20	0.04298
18	040565	1.18	1.75	343.60	0.06654
19	050965	0.98	1.75	150.70	0.02363
20	052865	0.69	8.25	8.90	0.00556
21	060165	1.20	3.25	102.20	0.02429
22	061365	1.36	8.25	33.00	0.01275
23	062565	0.63	3.25	15.60	0.00592
24	082865	1.17	3.00	32.10	0.00808
25	091965	1.51	5.50	43.80	0.00789
26	042366	0.75	5.75	9.30	0.00552
27	052166	1.35	4.00	53.00	0.01539
28	060866	0.66	3.75	6.10	0.00381
29	073066	1.04	1.75	45.40	0.00823
30	081166	0.64	2.00	5.70	0.00111
31	081966	0.97	0.75	19.30	0.00564
32	082366	0.70	7.00	5.30	0.00330
33	091466	0.61	1.00	55.20	0.01351
34	092766	1.18	5.25	79.30	0.02073
35	040967	1.29	5.50	134.50	0.02375
36	041267	1.70	10.00	636.90	0.10245
37	050567	0.87	3.75	33.00	0.00869
38	052067	0.70	8.25	9.40	0.00470
39	062667	0.60	6.50	4.80	0.00175
40	070367	1.17	4.25	24.60	0.00768

Summarized Study Events for Watershed 111 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	072867	0.70	3.25	6.20	0.00143
42	091467	0.53	2.75	2.10	0.00088
43	092667	0.95	2.50	4.60	0.00207
44	050968	1.33	13.00	107.50	0.02280
45	051568	0.90	0.75	157.10	0.03241
46	053168	1.01	3.50	84.80	0.02323
47	061568	1.48	1.00	248.00	0.04751
48	071468	1.42	5.75	87.80	0.01571
49	090368	1.79	2.00	50.20	0.01231
50	092368	0.60	1.50	17.90	0.00366

Summarized Study Events for Watershed 131

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	091562	1.75	3.75	96.80	0.02931
2	042663	1.99	6.50	180.00	0.05548
3	053163	0.90	1.00	35.15	0.00752
4	062363	1.30	2.25	31.53	0.00886
5	051064	1.15	1.00	420.90	0.08523
6	092064	1.69	5.00	41.30	0.00446
7	040565	0.61	0.50	35.00	0.00619
8	041465	0.39	0.75	25.00	0.00671
9	050965	1.95	3.25	457.60	0.11265
10	052665	0.58	5.75	7.70	0.00563
11	052865	0.95	2.50	6.30	0.00354
12	060265	1.31	2.25	29.70	0.00936
13	082865	2.03	3.75	73.30	0.01275
14	042266	1.38	8.50	13.90	0.00749
15	052166	1.46	3.75	60.00	0.01200
16	041267	1.77	9.75	336.90	0.06971
17	092667	1.02	2.50	4.80	0.00081
18	050968	2.15	2.50	731.80	0.13886
19	071468	1.03	6.00	24.50	0.00873
20	041669	1.62	1.75	552.20	0.06802
21	042669	0.90	1.00	318.10	0.04957
22	050669	1.59	6.75	471.20	0.18050
23	051470	1.70	5.50	76.90	0.01908
24	092270	3.43	10.00	117.10	0.01247
25	053171	2.41	4.00	591.47	0.15693
26	080871	1.50	6.50	21.35	0.00371
27	042772	1.59	6.50	119.71	0.04269
28	052972	1.72	5.00	51.61	0.01862
29	060273	2.30	8.25	1052.47	0.36132
30	061873	0.98	1.00	65.69	0.01616
31	092673	2.13	7.00	75.40	0.03720
32	041174	1.26	3.25	38.87	0.02215
33	050174	1.16	5.50	162.97	0.07427
34	061774	0.69	2.50	4.08	0.00314
35	070474	0.86	1.00	8.09	0.00257
36	082774	1.90	6.25	40.92	0.00747
37	050275	1.34	6.25	241.10	0.07038
38	052275	2.35	7.25	220.81	0.10867
39	062475	1.66	2.75	627.30	0.18667
40	072475	3.63	3.75	186.61	0.08180

Summarized Study Events for Watershed 131 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	072975	1.10	1.25	445.32	0.11706
42	080275	0.49	0.75	21.48	0.01093
43	041576	0.88	4.50	20.33	0.01041
44	041776	0.64	5.50	41.58	0.01666
45	091376	1.93	8.00	13.20	0.00371
46	042077	3.29	7.00	769.61	0.17621
47	053077	1.94	1.75	1196.85	0.38033
48	081077	0.38	0.75	4.86	0.00081
49	082977	0.83	2.50	36.11	0.00569
50	090577	0.73	7.25	3.22	0.00093

Summarized Study Events for Watershed 311

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	092067	0.80	2.50	8.00	0.00264
2	050567	1.01	4.25	92.10	0.04829
3	062567	1.64	4.50	137.70	0.05706
4	092667	1.01	2.75	42.20	0.02791
5	041868	1.63	2.75	347.60	0.19205
6	070168	0.86	4.75	20.60	0.00499
7	071868	0.67	1.00	5.50	0.00281
8	041669	1.65	1.75	441.50	0.25494
9	042669	1.03	1.00	229.90	0.15667
10	050469	1.65	3.50	1117.40	0.57864
11	050669	1.85	6.75	1136.00	0.82057
12	061369	2.30	10.75	244.10	0.14947
13	062569	1.11	2.25	94.60	0.05371
14	092269	0.62	2.75	8.10	0.00836
15	091669	1.62	3.00	16.80	0.01672
16	042970	3.52	8.25	908.60	0.61991
17	052970	3.09	5.50	1369.90	0.75169
18	061170	1.65	1.25	943.60	0.36930
19	052671	1.95	4.75	195.69	0.12839
20	053171	1.43	3.25	528.31	0.24135
21	060371	2.16	5.50	859.17	0.59320
22	060771	1.29	2.25	569.63	0.30309
23	060971	0.50	3.25	34.00	0.01220
24	061071	0.74	3.25	144.17	0.07824
25	072871	0.92	3.50	1.60	0.00084
26	091871	1.05	7.00	2.51	0.00108
27	092471	1.49	8.25	88.52	0.07038
28	060273	2.06	14.00	903.64	0.60450
29	080973	1.02	1.25	13.00	0.00792
30	091773	0.91	2.00	120.95	0.06400
31	042074	0.62	1.00	13.04	0.01768
32	050174	1.49	7.75	667.07	0.32814
33	053174	2.61	3.75	1145.80	0.54914
34	081074	0.82	2.00	175.36	0.09738
35	040775	0.55	3.25	149.17	0.10128
36	050275	0.93	2.00	31.94	0.04114
37	060675	0.56	1.00	19.17	0.00719
38	062275	0.97	8.25	23.58	0.01934
39	062375	0.66	2.75	56.49	0.03676
40	070375	0.50	3.50	14.53	0.00749

Summarized Study Events for Watershed 311 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	070775	0.70	1.00	420.94	0.17636
42	080175	0.64	1.75	109.61	0.04961
43	041976	0.69	1.00	112.18	0.05604
44	042876	0.85	4.50	14.17	0.02306
45	071576	1.19	2.25	6.00	0.01059
46	091476	1.93	10.25	134.85	0.03544
47	052077	1.89	9.00	898.06	0.52975
48	052677	1.74	10.00	326.60	0.22158
49	062877	0.93	2.25	35.20	0.01714
50	070177	1.69	3.00	371.02	0.18670

Summarized Study Events for Watershed 411

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	062363	3.19	2.50	416.30	0.11124
2	050964	0.94	3.25	354.01	0.04619
3	061564	0.23	0.75	70.83	0.00855
4	081864	0.73	3.25	49.31	0.00478
5	092664	0.88	6.75	155.62	0.03935
6	050965	0.98	2.50	17.70	0.00264
7	051065	0.21	1.50	33.50	0.00703
8	051365	0.57	2.00	13.20	0.00194
9	052665	0.77	4.50	12.60	0.00211
10	062465	0.62	2.50	27.80	0.00275
11	082765	0.28	0.75	17.90	0.00077
12	082865	2.42	3.75	2008.40	0.30883
13	091965	0.74	4.00	18.20	0.00230
14	092165	0.71	1.00	96.10	0.01542
15	042266	1.22	6.50	6.50	0.00112
16	061566	1.21	1.00	23.20	0.00176
17	073066	0.49	0.75	7.70	0.00023
18	092766	0.90	3.00	5.00	0.00038
19	041267	1.99	9.25	869.10	0.20783
20	042067	0.75	1.75	258.10	0.03703
21	050968	0.72	2.00	2.80	0.00021
22	051568	0.50	1.00	6.60	0.00208
23	052568	0.24	0.50	2.50	0.00111
24	061568	0.81	1.50	69.20	0.00899
25	071468	0.57	1.25	3.30	0.00012
26	092468	0.60	2.25	17.40	0.00178
27	050469	1.77	5.75	256.00	0.19365
28	061369	0.40	1.50	8.30	0.00022
29	080269	0.96	3.25	21.00	0.00079
30	091669	1.40	3.00	39.20	0.00401
31	051470	1.34	4.50	48.60	0.00536
32	052970	1.31	3.75	46.40	0.00883
33	091370	0.44	2.75	18.80	0.00136
34	092270	3.23	12.75	47.10	0.01615
35	050971	0.62	2.75	14.52	0.00111
36	053071	0.40	2.75	3.55	0.00027
37	053171	2.24	2.75	167.00	0.00943
38	072371	0.32	6.25	7.55	0.00046
39	080871	1.13	3.75	73.57	0.00652
40	100271	2.72	7.75	1208.83	0.57079

Summarized Study Events for Watershed 411 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	041572	1.19	3.00	22.31	0.00137
42	042772	1.41	6.25	54.77	0.02175
43	042972	1.01	2.75	50.72	0.01935
44	052273	2.03	3.00	57.83	0.00668
45	060273	1.89	10.50	792.35	0.45350
46	061673	0.30	0.50	15.67	0.00045
47	061873	0.71	1.25	13.67	0.00040
48	091273	0.54	0.75	41.20	0.00325
49	042974	1.32	3.75	740.08	0.16573
50	081074	3.30	12.00	42.01	0.01089

Summarized Study Events for Watershed 511

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	042663	2.12	8.25	1397.50	0.39436
2	062363	2.53	2.25	1183.80	0.17295
3	051064	1.32	1.75	568.80	0.09251
4	061564	0.44	1.50	42.40	0.00649
5	080764	1.39	7.00	31.80	0.00664
6	081564	2.22	7.50	265.80	0.07429
7	091564	0.59	3.25	8.90	0.00352
8	092064	0.76	2.75	115.70	0.01788
9	092664	0.51	7.75	61.00	0.01535
10	041465	1.02	3.00	969.40	0.09763
11	060165	0.60	2.50	21.90	0.00580
12	070965	0.72	1.00	89.20	0.01439
13	080765	2.85	1.00	3118.40	0.56559
14	082865	1.45	4.75	816.20	0.10458
15	091965	0.48	3.50	9.40	0.00087
16	092165	0.58	0.75	132.20	0.02475
17	081166	0.34	3.50	8.60	0.00131
18	041267	2.61	8.75	3291.10	0.66910
19	042067	0.76	2.75	1584.10	0.16721
20	050567	1.14	3.00	405.30	0.07865
21	052067	0.85	8.75	47.10	0.00854
22	062567	1.54	5.50	282.10	0.05219
23	090367	1.00	3.50	28.00	0.00690
24	090567	0.87	5.00	37.30	0.00850
25	092067	0.82	3.25	13.10	0.00760
26	092667	1.08	3.25	148.00	0.02156
27	041868	0.63	6.25	223.20	0.03911
28	051568	0.34	1.00	59.50	0.01079
29	052568	1.17	2.25	223.80	0.04518
30	070168	1.75	4.75	73.80	0.02745
31	071468	0.75	6.75	62.80	0.01876
32	081568	1.56	4.75	172.50	0.02819
33	090468	1.91	10.25	129.40	0.04069
34	092368	1.26	4.50	282.00	0.04602
35	041769	0.69	1.50	88.30	0.03152
36	042669	0.87	1.00	459.40	0.08166
37	050669	1.32	6.25	1451.10	0.21328
38	061469	2.88	3.25	2758.50	0.47016
39	072069	1.28	2.50	179.20	0.03444
40	082269	1.39	5.75	804.10	0.12922

Summarized Study Events for Watershed 511 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	090369	0.28	0.75	38.40	0.00664
42	091669	1.87	4.00	478.50	0.08211
43	043070	2.56	7.50	1874.40	0.38400
44	051470	1.89	6.25	777.20	0.12971
45	052970	2.55	4.00	2839.20	0.49805
46	072870	0.71	1.50	9.60	0.00235
47	052671	1.40	5.75	10.33	0.00561
48	060771	1.79	2.25	2002.38	0.36218
49	072871	0.94	3.25	8.97	0.00161
50	081471	1.02	6.00	46.53	0.01271

Summarized Study Events for Watershed 513

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	041465	0.58	2.25	121.10	0.03400
2	051365	0.72	1.75	22.80	0.01004
3	052665	0.69	4.25	5.40	0.00644
4	052865	1.01	8.75	25.90	0.02104
5	080765	3.00	1.25	2100.20	0.59117
6	082865	1.79	4.75	772.70	0.13882
7	091965	0.83	4.25	59.00	0.02065
8	042266	1.17	6.50	12.40	0.01261
9	042566	1.33	10.50	205.50	0.11594
10	051166	0.40	0.50	11.10	0.00656
11	072466	1.21	2.50	295.00	0.07355
12	082166	1.52	6.00	927.40	0.19397
13	083166	0.89	2.50	280.00	0.09608
14	091366	2.05	3.25	1143.20	0.50339
15	040967	1.05	5.75	126.30	0.03945
16	041267	2.60	8.50	1879.10	0.64283
17	042067	1.44	1.00	1861.20	0.53425
18	050567	1.07	4.50	183.70	0.06925
19	090567	0.68	5.00	28.40	0.01786
20	092067	0.91	2.50	23.30	0.00998
21	041968	0.02	0.25	4.70	0.00723
22	050968	0.36	3.00	5.10	0.00716
23	052568	1.23	5.25	166.20	0.06659
24	060168	0.32	4.50	62.70	0.03038
25	070168	2.04	5.00	156.10	0.06553
26	071368	0.80	1.50	52.80	0.01912
27	090468	2.35	10.00	257.20	0.07920
28	092168	0.51	1.75	80.00	0.02713
29	092368	0.82	2.75	40.30	0.01497
30	042669	0.65	0.75	48.70	0.02233
31	050369	1.57	6.25	403.20	0.14835
32	050669	1.72	6.00	1391.70	0.51219
33	061469	3.27	3.50	1713.40	0.62644
34	072069	3.04	2.50	846.30	0.32614
35	072569	0.41	1.25	7.90	0.00644
36	082269	0.92	5.50	10.90	0.00866
37	043070	1.18	4.25	106.30	0.05698
38	052670	1.91	3.25	623.50	0.20781
39	060370	0.41	3.00	4.10	0.00097
40	081970	2.24	7.00	91.90	0.02816

Summarized Study Events for Watershed 513 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	052671	1.11	4.25	9.14	0.00972
42	052371	0.99	1.50	102.19	0.01521
43	053171	1.04	3.00	86.31	0.03446
44	060271	1.74	5.75	550.34	0.20407
45	061271	0.92	2.00	211.22	0.11232
46	041572	1.26	3.00	57.35	0.03645
47	042772	1.23	4.75	104.26	0.05909
48	051272	1.98	6.00	278.78	0.17426
49	052972	0.66	6.00	3.93	0.00608
50	070372	0.26	2.75	8.91	0.00335

Summarized Study Events for Watershed 5142

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	040967	1.08	4.33	25.19	0.56290
2	041267	2.45	5.83	180.35	0.46659
3	042067	0.63	0.67	12.23	0.02361
4	050567	1.09	3.50	36.93	0.05298
5	052067	1.29	4.17	18.31	0.03631
6	053067	0.50	2.17	1.48	0.00654
7	062567	0.86	4.67	0.36	0.00306
8	062667	0.79	4.33	3.72	0.01408
9	070567	0.24	0.67	0.13	0.00076
10	080467	0.58	5.17	0.04	0.00349
11	090267	0.62	5.17	0.24	0.00217
12	090367	1.19	3.00	17.75	0.03010
13	090567	0.82	3.17	6.23	0.02468
14	092667	1.05	2.00	12.77	0.02649
15	041968	0.36	0.50	2.06	0.00567
16	050968	0.39	2.00	0.16	0.00138
17	052568	1.12	5.00	6.85	0.03762
18	053168	0.50	4.50	1.56	0.00569
19	060168	0.55	3.33	7.15	0.24320
20	060768	0.55	2.83	8.57	0.22440
21	061668	0.19	0.67	0.09	0.00070
22	070168	1.75	5.00	12.29	0.03070
23	081568	0.73	4.50	0.23	0.00220
24	090368	1.06	1.33	3.97	0.01121
25	090468	1.26	3.50	72.82	0.09913
26	042669	0.44	0.50	0.11	0.00083
27	050369	1.45	2.00	29.72	0.07183
28	050569	0.54	4.83	1.31	0.01224
29	050669	1.77	6.00	148.34	0.33282
30	051269	0.67	5.33	4.50	0.01388
31	061369	2.81	3.17	134.11	0.34942
32	072069	2.75	2.33	74.28	0.16241
33	092269	0.62	2.00	1.80	0.00782
34	042970	0.36	0.33	0.11	0.00102
35	043070	0.89	4.33	3.84	0.01112
36	052970	1.61	2.33	27.55	0.07502
37	081970	1.85	7.00	3.76	0.00877
38	091470	0.76	0.83	2.93	0.00594
39	091770	0.46	4.67	0.14	0.00115
40	092270	1.24	2.33	49.93	0.08459

Summarized Study Events for Watershed 5142 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	052371	1.01	1.17	4.95	0.00954
42	053171	0.75	2.00	9.11	0.01899
43	061271	0.91	1.67	22.26	0.05882
44	070171	0.45	0.33	2.15	0.00587
45	081371	0.54	0.83	0.83	0.00244
46	081471	1.20	5.67	7.17	0.02973
47	043072	0.32	1.83	0.31	0.00177
48	052972	0.65	5.17	0.17	0.00158
49	070272	0.77	1.17	0.37	0.00167
50	090872	0.68	1.17	1.52	0.00476

Summarized Study Events for Watershed 5143

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	040967	1.05	5.33	5.89	0.01333
2	041067	0.90	2.17	12.67	0.03554
3	041267	2.44	8.00	143.07	0.35755
4	041367	0.50	2.00	4.06	0.02579
5	042067	0.64	1.17	7.13	0.02244
6	050567	1.09	4.67	11.17	0.02622
7	052067	1.23	4.17	3.29	0.01637
8	062667	0.70	4.00	0.97	0.00482
9	090567	0.88	4.67	0.71	0.00250
10	092667	1.09	2.83	2.05	0.00584
11	041968	0.25	0.33	2.08	0.00613
12	050668	0.60	1.67	0.69	0.00636
13	051368	0.18	1.00	0.20	0.00144
14	052568	0.54	1.33	1.31	0.00809
15	053168	0.45	4.33	0.34	0.00702
16	060768	0.56	2.00	3.57	0.01478
17	061668	0.45	1.00	0.32	0.00242
18	070168	1.95	4.83	12.21	0.03662
19	071368	0.79	1.33	2.03	0.01328
20	071868	0.81	0.83	1.26	0.00787
21	090468	1.27	3.50	26.87	0.04146
22	041769	0.40	0.83	0.27	0.00270
23	042669	0.38	0.50	0.58	0.00386
24	050369	2.05	6.50	13.27	0.11237
25	050669	1.77	6.17	119.43	0.52889
26	051669	0.34	3.00	0.44	0.00513
27	060169	0.51	0.50	1.65	0.00652
28	072069	2.36	1.67	22.04	0.06568
29	072569	0.35	0.50	0.30	0.00379
30	091669	1.10	4.00	1.39	0.00923
31	092269	0.66	2.17	2.00	0.00941
32	043070	0.93	4.50	2.12	0.01205
33	051470	2.15	4.83	25.04	0.06097
34	052970	1.58	2.17	8.22	0.03402
35	092270	1.35	1.67	10.26	0.05625
36	052371	1.10	1.17	1.97	0.00727
37	053171	0.89	2.83	1.02	0.00608
38	060271	0.85	1.00	5.60	0.02020
39	060371	0.71	1.00	14.45	0.03454
40	060771	0.89	2.00	2.24	0.00799

Summarized Study Events for Watershed 5143 (Continued

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	061271	0.66	1.50	3.19	0.01469
42	081471	0.60	3.17	0.97	0.00632
43	091871	2.65	7.17	26.80	0.06468
44	092471	1.71	3.33	26.80	0.07662
45	041572	1.10	3.50	1.42	0.01309
46	042072	0.50	1.50	0.46	0.00786
47	042672	1.15	2.67	10.88	0.03719
48	043072	0.29	2.33	0.16	0.00171
49	051272	1.78	6.00	2.96	0.05795
50	070272	0.79	1.33	0.34	0.00197

Summarized Study Events for Watershed 5145

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	041067	0.91	2.75	31.32	0.23259
2	041267	3.01	8.25	193.18	1.30422
3	041367	0.54	0.25	7.07	0.12232
4	050567	1.21	4.50	44.95	0.15714
5	053067	0.54	4.00	4.07	0.03372
6	062667	0.67	4.17	5.31	0.02688
7	072167	0.43	0.50	3.55	0.00967
8	090467	0.96	5.50	8.63	0.06963
9	092667	1.17	3.00	34.67	0.09925
10	041868	0.25	0.67	0.47	0.00585
11	042268	0.55	2.50	3.12	0.02399
12	050968	0.57	2.50	2.92	0.02297
13	051368	0.37	4.33	2.63	0.03068
14	052568	1.23	5.17	13.68	0.12469
15	053168	0.40	4.00	3.77	0.01651
16	060168	0.57	3.17	8.22	0.04565
17	060568	0.51	2.00	1.37	0.01415
18	060768	0.57	2.50	17.46	0.07034
19	070168	1.63	5.33	11.09	0.06917
20	071368	0.73	2.33	2.19	0.00787
21	071868	0.93	1.33	18.08	0.05543
22	081568	0.95	4.67	0.85	0.00552
23	090368	1.19	3.67	2.63	0.01395
24	090468	1.36	3.50	76.39	0.17350
25	090268	0.67	2.33	0.31	0.00121
26	092368	0.70	2.17	0.35	0.00146
27	042669	0.50	0.83	1.55	0.01136
28	050469	0.57	3.33	2.17	0.02915
29	050669	2.76	6.33	111.74	1.53723
30	051269	0.49	2.83	2.46	0.01953
31	051669	0.27	2.67	0.15	0.00761
32	060169	0.55	0.50	10.01	0.05024
33	072069	3.05	2.17	87.86	0.39175
34	072569	0.35	0.67	0.15	0.00048
35	082569	0.90	4.17	0.48	0.00283
36	092269	0.64	2.17	9.63	0.05387
37	051470	1.27	4.83	13.84	0.05949
38	052970	1.99	2.83	56.80	0.33864
39	081870	2.33	7.50	23.46	0.08063
40	091470	0.40	1.17	4.08	0.01213

Summarized Study Events for Watershed 5145 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	052671	1.09	4.33	1.17	0.01422
42	053171	0.90	2.67	24.54	0.11127
43	060271	1.22	1.33	70.93	0.33942
44	072371	0.28	2.67	24.18	0.04313
45	080871	0.65	3.17	4.37	0.00936
46	081371	0.33	1.67	1.81	0.00339
47	081471	1.25	5.33	9.77	0.05645
48	091871	2.52	7.00	65.58	0.44303
49	042772	1.55	4.50	60.48	0.24989
50	051272	1.88	6.33	18.00	0.38589

Summarized Study Events for Watershed 515

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	041573	0.82	7.75	17.00	0.12760
2	041973	0.56	1.50	18.16	0.24497
3	050673	0.72	1.00	11.68	0.02156
4	052273	0.97	3.75	6.19	0.00905
5	052473	0.99	1.75	105.08	0.34291
6	053173	2.27	7.75	96.84	0.48686
7	060273	1.81	9.50	202.51	0.74909
8	060473	1.38	2.25	129.87	0.57465
9	061973	0.39	4.50	5.35	0.00656
10	080973	2.37	1.25	135.77	0.15966
11	091273	0.91	1.00	16.73	0.01222
12	092673	1.68	8.00	9.23	0.02196
13	041174	1.30	3.50	16.67	0.05806
14	042974	0.93	3.75	64.26	0.09202
15	050174	1.41	7.50	83.69	0.39590
16	053174	1.20	3.50	23.87	0.04466
17	060674	0.66	0.75	8.17	0.01382
18	072974	1.76	4.75	11.67	0.01208
19	080974	1.48	2.50	62.78	0.05756
20	081074	1.85	5.75	89.67	0.31971
21	032775	0.55	2.50	8.90	0.05538
22	040775	1.03	3.75	63.16	0.28653
23	050275	0.86	2.75	16.20	0.03592
24	051375	1.41	4.00	48.30	0.19009
25	052275	2.49	11.25	89.00	0.78729
26	052875	1.27	11.00	32.98	0.36846
27	060675	0.56	1.25	3.06	0.01069
28	061075	0.77	2.75	42.72	0.23757
29	061775	0.33	0.75	3.39	0.00381
30	062275	0.96	4.25	24.97	0.07903
31	062375	0.61	2.50	21.86	0.14039
32	070775	0.71	2.25	10.66	0.01675
33	071075	0.49	2.50	9.80	0.02312
34	072475	1.82	4.75	20.86	0.04818
35	072675	0.23	1.00	9.02	0.03323
36	080275	0.37	3.50	2.51	0.02548
37	090575	0.80	1.00	1.92	0.00166
38	032976	0.49	0.75	1.37	0.00248
39	041976	0.76	0.75	54.90	0.09011
40	042876	0.81	4.75	1.18	0.00810

Summarized Study Events for Watershed 515 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	062476	1.25	4.50	12.23	0.01480
42	080576	0.87	2.50	1.51	0.00172
43	091376	2.01	8.25	0.85	0.00238
44	042077	2.00	5.75	5.18	0.00880
45	051977	2.44	6.25	93.79	0.13453
46	052077	1.97	5.50	105.14	0.41232
47	052777	0.23	1.00	5.73	0.02301
48	053177	0.80	3.00	9.45	0.03520
49	062877	0.86	1.00	4.84	0.00436
50	070177	1.00	3.00	14.42	0.03194

Summarized Study Events for Watershed 611

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	042762	0.75	4.25	54.63	0.01891
2	052662	1.22	1.00	52.05	0.02479
3	060162	1.15	4.00	23.11	0.00867
4	060162	1.15	5.50	232.91	0.17171
5	060762	0.51	3.50	46.46	0.04807
6	060962	1.39	4.25	295.11	0.18199
7	072562	1.82	4.75	266.70	0.08272
8	091562	1.16	4.25	14.80	0.01285
9	092062	2.20	10.50	165.93	0.08552
10	102762	0.89	0.50	40.81	0.01626
11	042663	2.32	7.83	668.97	0.21462
12	062363	2.52	2.00	584.55	0.19527
13	071363	0.83	1.67	21.05	0.01201
14	111963	2.14	8.33	47.09	0.02038
15	112263	0.55	4.17	1.13	0.00145
16	050664	1.85	2.83	414.20	0.12858
17	050764	0.37	2.50	7.70	0.00607
18	050964	2.21	1.67	2406.80	0.67669
19	051064	0.95	0.50	805.60	0.32473
20	052964	0.62	2.83	4.80	0.00448
21	053064	0.67	1.50	205.10	0.09496
22	060464	0.23	0.33	5.80	0.00419
23	080764	1.53	6.75	58.64	0.02673
24	081564	1.03	4.25	4.65	0.00358
25	090464	0.69	1.00	15.79	0.00627
26	091664	1.29	2.50	245.45	0.07624
27	092064	0.25	4.00	0.70	0.00044
28	092764	0.97	5.75	15.00	0.01445
29	110364	1.46	7.25	63.13	0.03044
30	111664	1.81	3.25	605.68	0.31216
31	111864	0.37	1.75	63.42	0.03196
32	041465	0.57	8.75	5.60	0.00385
33	051365	0.69	3.25	1.86	0.00270
34	052665	1.55	4.75	134.91	0.05069
35	060265	0.88	2.00	78.84	0.03087
36	062165	0.62	0.75	2.93	0.00256
37	080665	1.78	4.00	97.97	0.03709
38	080765	0.45	0.75	17.85	0.00591
39	081065	0.45	0.75	43.32	0.01758
40	082265	0.38	1.00	5.90	0.00395

Summarized Study Events for Watershed 611 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	082865	0.98	4.75	9.82	0.00947
42	083165	0.54	1.25	4.46	0.00355
43	090365	1.16	4.25	104.06	0.04357
44	091965	0.51	4.00	4.65	0.00543
45	042266	1.88	12.00	20.11	0.01868
46	042566	1.68	10.00	198.66	0.17625
47	061566	1.81	1.25	357.26	0.11797
48	072366	1.35	3.25	1.05	0.00091
49	072466	1.16	1.50	204.22	0.08136
50	081166	1.15	3.50	3.42	0.00255

Summarized Study Events for Watershed R5

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	040967	0.89	2.83	0.03	0.00693
2	041067	1.15	2.50	3.96	0.18042
3	041267	2.50	8.00	21.02	0.80895
4	050669	1.67	5.83	16.87	0.56790
5	043070	0.98	4.33	0.02	0.00169
6	051470	1.69	4.50	0.38	0.01839
7	052970	1.42	2.00	0.35	0.01898
8	092270	1.39	1.67	2.17	0.06204
9	092270	0.39	0.50	0.52	0.02506
10	092270	0.47	0.67	0.87	0.03925
11	060271	0.70	1.00	0.22	0.01982
12	060371	0.78	0.67	4.80	0.12139
13	061271	1.14	1.17	4.09	0.16831
14	081471	0.87	2.67	0.03	0.00277
15	091871	2.08	4.67	1.43	0.08548
16	092471	1.62	3.00	1.43	0.11792
17	100271	1.54	1.17	4.51	0.13521
18	042072	0.62	4.50	0.01	0.00180
19	042772	1.53	4.33	1.82	0.07567
20	051272	1.80	5.50	0.55	0.05124
21	041573	0.53	1.50	1.11	0.01038
22	041573	0.29	1.00	0.22	0.02542
23	041973	0.58	2.00	0.83	0.06050
24	052273	1.16	2.50	0.08	0.00618
25	052473	3.73	1.50	63.07	1.80087
26	053173	2.08	8.17	3.45	0.19982
27	060173	0.57	1.33	0.26	0.03712
28	060273	1.12	2.00	13.27	0.69828
29	060473	1.28	2.17	17.86	0.65651
30	061973	0.48	0.83	0.91	0.01423
31	080973	1.26	1.50	0.15	0.00861
32	091273	0.87	0.83	0.01	0.00083
33	092673	1.64	6.17	0.17	0.02295
34	092773	0.45	5.50	0.02	0.00443
35	031074	0.69	2.50	0.99	0.06269
36	042974	1.90	2.33	7.78	0.39964
37	050174	0.50	1.17	3.09	0.23527
38	040775	0.80	1.83	0.60	0.03842
39	042975	1.13	1.00	3.33	0.11226
40	051375	1.12	4.50	0.03	0.00361

Summarized Study Events for Watershed R5 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	051475	0.32	3.50	0.02	0.00169
42	052275	2.03	1.33	20.65	0.72157
43	052975	0.42	6.33	0.17	0.02960
44	061075	0.97	2.17	0.93	0.07732
45	061775	0.45	0.50	0.02	0.00273
46	062375	1.26	1.33	11.15	0.42726
47	071576	0.87	2.67	0.32	0.01674
48	042077	1.40	2.67	2.26	0.06316
49	051977	2.68	5.50	11.66	0.42401
50	052077	1.94	3.00	10.91	0.76835

Summarized Study Events for Watershed R6

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	041067	1.18	2.33	3.93	0.13627
2	041267	2.28	3.17	29.24	0.85776
3	090468	1.26	3.33	1.28	0.02534
4	050669	1.73	6.50	23.31	0.66450
5	092270	1.24	1.67	1.89	0.03633
6	092270	0.47	1.00	0.98	0.02384
7	053171	0.76	1.50	0.06	0.00206
8	060271	0.82	1.00	1.36	0.02387
9	060371	0.80	0.83	3.93	0.08829
10	081471	0.72	3.17	0.05	0.00252
11	091871	0.82	1.17	0.47	0.01010
12	051272	1.80	6.00	0.93	0.09268
13	040273	0.29	1.83	0.08	0.00870
14	040873	0.30	2.33	0.02	0.00313
15	041973	0.55	1.83	2.15	0.07462
16	050673	0.62	1.00	1.29	0.02734
17	052273	1.14	2.17	2.34	0.05558
18	052473	3.75	1.33	105.10	2.05997
19	053173	1.23	3.50	6.41	0.01677
20	060173	0.55	1.17	1.16	0.05388
21	060273	1.10	2.83	21.01	0.69068
22	060473	1.24	2.00	27.43	0.61101
23	061673	0.52	0.67	0.24	0.00669
24	061873	0.65	2.17	1.29	0.03207
25	061973	0.50	0.83	1.36	0.04437
26	071373	0.57	0.33	0.37	0.00803
27	072273	1.01	2.33	1.65	0.04326
28	081573	0.81	1.00	0.11	0.00305
29	091273	0.88	0.83	1.10	0.02477
30	092673	0.98	2.33	0.41	0.01501
31	041174	1.27	3.00	0.55	0.02613
32	042974	1.93	4.00	9.45	0.33496
33	050174	0.55	1.17	4.78	0.16714
34	051375	1.08	3.50	0.63	0.02216
35	060675	0.68	1.17	0.68	0.03289
36	061075	1.03	2.50	4.34	0.19470
37	062375	1.01	2.33	12.14	0.28948
38	070775	0.86	1.33	1.22	0.38450
39	072475	0.69	3.67	0.29	0.01984
40	072675	0.26	2.00	0.07	0.00389

Summarized Study Events for Watershed R6 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	041976	0.81	2.83	0.63	0.01620
42	042876	0.91	4.50	0.06	0.00329
43	062476	0.94	2.00	1.16	0.03510
44	071576	0.75	2.67	1.65	0.04324
45	071576	1.95	2.67	1.22	0.04754
46	042077	1.24	0.67	11.88	0.22066
47	050377	0.51	0.83	0.82	0.02156
48	050577	0.42	1.17	0.55	0.01258
49	051977	2.77	5.67	33.54	0.79538
50	052077	2.00	2.83	33.54	0.96604

Summarized Study Events for Watershed R7

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	041067	1.06	2.83	13.20	0.52520
2	041267	2.09	2.83	38.53	1.23474
3	070168	2.36	4.83	10.35	0.62659
4	061469	2.94	3.00	20.25	1.20627
5	043070	0.91	4.33	4.91	0.21408
6	052970	1.24	2.00	4.91	0.19230
7	081970	0.69	3.50	2.43	0.05370
8	091470	0.65	0.83	0.44	0.01406
9	092270	1.37	1.67	15.86	0.54797
10	092270	0.44	0.83	8.12	0.17538
11	092270	0.44	1.00	0.07	0.00180
12	052371	1.02	1.17	2.84	0.06354
13	060771	0.68	1.83	1.09	0.03154
14	061271	1.17	1.33	15.55	0.54119
15	042072	0.38	0.83	6.22	0.24041
16	042072	0.69	1.67	8.76	0.19633
17	043072	0.43	2.33	0.98	0.04408
18	040273	0.25	1.67	0.59	0.02880
19	041973	0.60	2.17	9.89	0.19724
20	052273	1.22	2.83	13.48	0.34733
21	052473	3.39	1.33	97.90	2.31979
22	060473	1.04	0.67	51.37	0.68623
23	061673	0.51	0.67	2.33	0.04580
24	061873	0.75	1.50	13.48	0.24927
25	061973	0.35	0.50	9.20	0.18832
26	071373	0.63	0.50	4.62	0.05613
27	072073	0.36	1.83	0.51	0.00707
28	073073	0.33	2.50	0.15	0.00288
29	080973	1.26	1.50	18.48	0.32893
30	081573	0.83	1.17	7.52	0.15060
31	091273	0.92	0.83	18.83	0.32205
32	050174	0.52	0.83	21.36	0.34455
33	052574	0.33	1.83	0.17	0.00357
34	060674	0.50	0.67	5.88	0.06452
35	080974	1.77	2.00	11.85	0.38993
36	091574	0.82	5.33	0.77	0.04064
37	062375	0.83	2.33	12.38	0.32118
38	072475	0.68	3.50	0.92	0.02412
39	042876	0.89	4.50	1.15	0.10662
40	062476	1.74	4.67	8.98	0.34409

Summarized Study Events for Watershed R7 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	071576	1.79	2.67	10.84	0.39181
42	071676	0.82	2.67	7.52	0.30490
43	080576	0.91	2.50	0.34	0.00949
44	042077	1.18	0.83	30.65	0.59478
45	050377	0.71	0.67	12.38	0.24081
46	051377	0.52	0.67	0.82	0.02095
47	051577	0.37	1.83	0.34	0.00942
48	051977	2.77	5.67	49.44	1.26251
49	052077	1.80	3.00	41.58	1.04937
50	062877	0.72	0.83	1.15	0.02400

Summarized Study Events for Watershed R8

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
1	041067	0.52	0.33	5.86	0.19962
2	041267	2.17	3.33	32.38	1.33288
3	070168	2.43	4.83	10.31	0.38066
4	061469	3.22	3.50	26.83	0.77062
5	092270	1.54	1.67	17.06	0.41994
6	092270	0.54	0.83	9.61	0.21386
7	052671	1.07	4.17	0.67	0.03279
8	053171	0.70	1.67	2.52	0.05383
9	060271	0.89	1.00	12.06	0.18713
10	060371	0.80	1.00	17.05	0.31740
11	061271	0.93	1.33	6.20	0.21294
12	062171	0.78	3.67	0.02	0.00139
13	080871	0.83	1.50	0.77	0.01684
14	081471	0.73	2.67	4.75	0.10232
15	091871	2.82	7.17	19.77	0.63133
16	092471	1.62	3.33	14.28	0.58875
17	042072	0.73	1.67	4.90	0.09958
18	042772	1.39	4.33	12.33	0.31844
19	041573	0.24	0.83	2.05	0.08359
20	041573	0.51	1.83	4.04	0.10521
21	052473	3.64	1.50	133.70	2.33195
22	060273	1.19	5.50	26.84	0.68861
23	060473	1.14	2.17	41.75	0.60286
24	072273	1.12	3.83	7.49	0.18335
25	091273	0.89	0.67	8.30	0.15534
26	091773	0.53	4.17	0.09	0.00410
27	092673	1.70	7.67	4.75	0.33722
28	041174	1.28	3.00	2.84	0.16209
29	053174	0.48	2.83	0.03	0.00189
30	060674	0.45	0.33	2.14	0.02991
31	081074	1.56	2.33	9.38	0.17732
32	051375	1.02	3.33	3.17	0.08551
33	051475	0.33	3.67	0.72	0.04903
34	052275	0.45	2.83	0.26	0.00722
35	061075	1.09	2.50	14.87	0.47190
36	061775	0.53	0.67	10.79	0.16203
37	062275	0.75	5.50	2.52	0.05973
38	070775	0.73	1.17	2.33	0.03538
39	072475	0.70	3.67	1.96	0.06052
40	072675	0.22	2.33	0.59	0.01341

Summarized Study Events for Watershed R8 (Continued)

Event	Date (mmddyy)	Total Rainfall (in)	Duration (hr)	Peak Flow (cfs)	Runoff Depth (in)
41	041576	0.53	4.00	0.07	0.00301
42	041976	0.76	3.83	4.90	0.10450
43	071576	1.86	3.00	14.57	0.35725
44	080576	1.05	6.67	3.65	0.05979
45	051377	0.54	1.00	3.78	0.04771
46	051577	0.40	1.83	1.88	0.02989
47	051977	2.90	5.67	69.14	1.39128
48	052677	0.98	2.67	7.69	0.13664
49	062577	0.66	2.67	4.18	0.03503
50	062877	0.73	1.50	7.69	0.08167

APPENDIX B

PARTIAL DURATION SERIES FOR THE STUDY WATERSHEDS

Partial Duration Series for Watershed 111

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	072362	28.6	26	092664	256.5
2	072562	173.9	27	110364	276.4
3	083162	298.0	28	111664	368.0
4	090462	31.6	29	111864	57.6
5	090762	12.9	30	111964	105.8
6	091562	489.2	31	031165	8.2
7	092062	35.2	32	040565	346.3
8	102862	18.6	33	050965	152.5
9	120262	26.0	34	052865	9.8
10	042663	136.9	35	060265	103.3
11	053163	114.5	36	061365	34.0
12	060363	9.7	37	062565	16.7
13	071363	28.5	38	080665	9.8
14	111963	36.7	39	082865	32.1
15	112263	7.6	40	091965	43.8
16	042564	16.2	41	092165	22.6
17	050964	697.6	42	101865	7.5
18	051064	771.8	43	031266	9.8
19	052964	70.2	44	042266	9.6
20	053064	115.9	45	042366	12.3
21	061164	277.2	46	042566	15.2
22	081864	49.6	47	052166	53.7
23	091664	12.8	48	060966	6.1
24	092064	356.0	49	073066	45.4
25	092264	18.3	50	081166	5.7

Partial Duration Series for Watershed 111 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
51	081966	19.3	71	100767	42.4
52	082366	5.3	72	011268	15.3
53	090166	11.0	73	011868	5.1
54	091466	55.4	74	050968	108.2
55	092766	79.7	75	051568	158.6
56	032267	14.1	76	053168	85.7
57	040967	135.8	77	060168	28.1
58	041067	56.4	78	061568	248.3
59	041267	638.8	79	070168	101.6
60	041367	53.7	80	071468	88.0
61	042067	20.2	81	071568	27.0
62	050667	34.0	82	081568	7.3
63	052067	9.9	83	090468	70.5
64	062667	4.8	84	092368	8.5
65	070367	24.7	85	092468	18.2
66	072867	6.2	86	100968	284.2
67	090367	5.0	87	110268	7.8
68	090467	27.4	88	111568	54.5
69	090567	7.5	89	111268	32.9
70	092767	4.6			

Partial Duration Series for Watershed 131

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	090462	37.1	31	090468	48.3
2	091562	131.4	32	100968	87.1
3	092062	68.6	33	041669	554.5
4	042663	176.9	34	042669	320.7
5	053163	36.5	35	050569	328.3
6	062363	31.6	36	050669	480.3
7	051064	429.9	37	061469	107.2
8	053064	48.4	38	043070	49.0
9	081864	79.0	39	051570	77.3
10	092064	41.3	40	062070	171.4
11	110364	111.1	41	092270	117.1
12	111764	267.9	42	060171	601.5
13	111964	58.7	43	081471	61.2
14	040565	37.5	44	091871	44.6
15	050965	458.7	45	092471	40.4
16	060265	31.6	46	100371	437.5
17	082865	73.3	47	121471	55.3
18	042566	31.6	48	041572	32.1
19	052166	60.5	49	042772	120.5
20	040967	131.6	50	043072	107.8
21	041267	340.3	51	051272	31.0
22	041367	40.2	52	052972	51.9
23	042067	229.2	53	103172	65.3
24	090467	32.7	54	031073	144.5
25	050968	732.6	55	032473	169.8
26	051368	33.0	56	041573	40.9
27	051568	79.8	57	052373	156.5
28	060168	45.3	58	053173	135.5
29	061568	103.0	59	060173	159.2
30	070168	35.4	60	060273	1082.1

Partial Duration Series for Watershed 131 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
61	061873	68.1	83	052375	223.7
62	072173	62.6	84	052975	95.1
63	072373	208.8	85	062275	41.6
64	090673	37.6	86	062475	634.9
65	091373	38.3	87	072475	187.8
66	092673	76.7	88	072675	79.1
67	101173	32.1	89	072975	452.5
68	101273	34.5	90	030876	40.9
69	022074	154.8	91	041776	41.6
70	030874	179.7	92	041976	72.9
71	031074	272.4	93	053176	43.7
72	041174	41.6	94	062476	46.5
73	043074	123.2	95	080476	71.9
74	051074	171.6	96	042077	771.2
75	052574	110.3	97	050277	97.6
76	053174	111.5	98	050477	33.3
77	082774	40.9	99	051977	242.2
78	110274	209.8	100	052077	135.5
79	010275	40.9	101	052777	197.7
80	040875	44.4	102	053177	1201.0
81	050275	242.5	103	082977	36.3
82	051475	86.0			

Partial Duration Series for Watershed 311

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	040967	18.1	36	100670	38.4
2	041067	25.1	37	100870	33.9
3	041267	4900.0	38	042671	31.0
4	050567	93.0	39	052771	195.7
5	062567	138.8	40	060171	528.5
6	090467	49.1	41	060371	859.2
7	092667	42.2	42	060871	570.4
8	100767	707.9	43	061171	153.5
9	041968	347.6	44	062171	579.0
10	051468	18.7	45	092471	88.5
11	052568	183.9	46	100271	181.7
12	060168	54.2	47	121571	40.8
13	061568	38.6	48	042772	17.3
14	061668	45.8	49	051272	36.1
15	070168	20.7	50	103172	277.5
16	081568	12.2	51	111372	10.1
17	100968	30.3	52	111972	12.3
18	111568	59.9	53	010373	132.6
19	112768	27.8	54	011773	33.5
20	112868	23.8	55	012173	171.5
21	032369	35.1	56	012673	47.2
22	041769	427.3	57	030673	27.7
23	042769	229.9	58	031073	506.7
24	050469	307.8	59	032473	456.7
25	050569	1129.7	60	033173	35.7
26	050669	1147.2	61	040373	24.7
27	061469	244.1	62	041673	109.0
28	062669	95.0	63	041973	50.9
29	091669	16.8	64	050773	11.4
30	043070	908.7	65	052373	12.0
31	051470	198.7	66	053173	268.3
32	052970	1369.9	67	060273	924.8
33	061170	943.6	68	061973	126.7
34	081970	16.9	69	080973	13.0
35	092270	22.5	70	091373	206.5

Partial Duration Series for Watershed 311 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
71	091773	122.3	106	060775	19.2
72	092673	244.3	107	062275	24.7
73	100473	27.0	108	062375	60.5
74	101173	242.3	109	062575	63.5
75	101273	193.9	110	070475	15.5
76	112473	25.4	111	070575	14.1
77	022174	168.2	112	070775	422.5
78	030974	643.4	113	071075	65.9
79	031074	594.8	114	072475	272.4
80	041074	10.1	115	072575	455.5
81	042174	13.8	116	072875	282.7
82	043074	95.9	117	080175	119.7
83	050174	676.7	118	080275	202.9
84	053174	1145.8	119	081475	28.5
85	060374	32.6	120	030376	20.5
86	060674	72.8	121	030976	26.2
87	060874	38.4	122	041976	118.0
88	081074	174.0	123	042376	23.2
89	102674	202.8	124	042876	15.5
90	102874	25.4	125	052676	94.4
91	103174	164.0	126	062476	11.4
92	110274	600.6	127	091476	134.9
93	010275	100.4	128	042177	37.9
94	013175	141.0	129	043077	185.2
95	020475	69.0	130	050277	44.7
96	022275	42.2	131	050377	92.2
97	031175	14.4	132	051977	329.2
98	031975	27.0	133	052177	921.3
99	032775	27.3	134	052777	330.3
100	040875	154.3	135	053177	102.0
101	050375	33.9	136	061277	31.4
102	051375	87.1	137	062977	35.2
103	052375	390.6	138	070177	371.6
104	052875	58.1	139	081077	10.6
105	053075	186.0			

Partial Duration Series for Watershed 411

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	120262	155.6	32	060371	124.2
2	042663	407.9	33	062071	112.9
3	062363	373.0	34	072871	45.0
4	071363	237.3	35	080871	73.6
5	051064	354.0	36	092471	59.3
6	051164	306.9	37	100371	1208.8
7	061564	70.8	38	042772	55.5
8	081864	49.31	39	042972	55.5
9	092764	155.6	40	051272	50.3
10	111764	365.3	41	061572	80.8
11	111964	403.2	42	103172	476.8
12	080665	58.6	43	031073	43.5
13	080765	48.4	44	032473	166.2
14	082865	2008.4	45	041673	43.5
15	092165	96.1	46	052373	109.7
16	091466	91.9	47	052473	143.5
17	041067	107.7	48	053173	128.9
18	041267	869.1	49	060273	832.4
19	042067	258.1	50	071363	42.1
20	081767	44.7	51	090673	42.8
21	061568	69.2	52	091273	41.8
22	070168	60.5	53	092773	79.3
23	100968	135.4	54	101273	78.8
24	050569	256.0	55	112573	50.2
25	050669	439.8	56	022174	65.7
26	043070	109.9	57	030974	140.3
27	051470	48.6	58	031074	486.5
28	052970	46.4	59	043074	740.8
29	092270	47.1	60	050174	617.7
30	053171	167.0	61	053174	221.6
31	060171	150.4	62	080974	71.3

Partial Duration Series for Watershed 511

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	033063	322.1	41	060168	183.4
2	042463	101.5	42	060568	77.1
3	042663	953.1	43	070168	73.8
4	062363	1183.8	44	071468	62.8
5	072863	222.1	45	081568	172.5
6	051064	568.8	46	090468	129.4
7	053064	65.9	47	092468	282.0
8	081564	265.8	48	100968	381.9
9	092064	115.9	49	111568	75.5
10	092764	61.2	50	112768	56.2
11	101264	110.8	51	032369	55.5
12	110464	114.8	52	041769	92.3
13	111764	410.9	53	042769	436.6
14	111964	633.1	54	050369	292.7
15	040565	139.2	55	050469	184.7
16	041565	991.9	56	050769	1451.1
17	071065	89.2	57	051669	91.3
18	080865	3122.4	58	061469	2758.5
19	082865	816.5	59	072169	179.2
20	092165	135.7	60	082369	804.1
21	031266	495.2	61	091669	478.6
22	042366	66.5	62	092369	1005.0
23	042566	127.7	63	043070	1876.0
24	082066	195.2	64	051570	779.0
25	082166	103.4	65	052970	2740.7
26	082266	1331.4	66	081970	94.7
27	082366	928.4	67	092270	941.4
28	083166	171.8	68	100570	113.3
29	090466	544.8	69	100870	130.1
30	091466	2881.8	70	022171	97.9
31	032567	505.9	71	060171	468.6
32	041067	224.0	72	060371	1766.4
33	041267	3297.8	73	060871	2005.4
34	042067	1593.6	74	062171	144.5
35	050667	407.7	75	070271	87.4
36	062567	282.1	76	091871	151.5
37	092667	148.0	77	092471	528.4
38	041968	225.8	78	100371	3071.6
39	051568	67.4	79	120971	52.3
40	052568	224.3	80	121571	307.0

Partial Duration Series for Watershed 513

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	030165	26.6	44	112668	41.5
2	031265	23.5	45	021469	21.1
3	041465	134.9	46	032369	59.7
4	051465	23.5	47	042769	51.1
5	052865	27.5	48	050369	405.4
6	080865	2100.2	49	050569	65.1
7	082865	772.9	50	050669	1399.8
8	092065	59.8	51	051269	22.2
9	092165	81.8	52	061469	1713.4
10	020966	23.1	53	072169	846.3
11	031266	372.8	54	091669	64.3
12	042366	83.7	55	092369	79.6
13	042566	207.6	56	043070	107.6
14	060666	75.5	57	051570	344.1
15	072466	295.0	58	052970	624.2
16	081966	61.3	59	053070	20.3
17	082166	927.4	60	081970	91.9
18	082366	121.2	61	092270	842.5
19	083166	280.4	62	100570	94.9
20	090466	87.6	63	100870	38.0
21	091466	1222.2	64	022171	31.2
22	041067	133.6	65	052371	102.2
23	041267	1801.2	66	060171	87.0
24	041367	209.3	67	060371	551.1
25	042067	1861.2	68	060871	481.8
26	050667	186.4	69	061271	213.6
27	052067	40.2	70	062171	55.3
28	062567	43.3	71	081471	63.5
29	090367	24.0	72	091871	164.7
30	090567	29.0	73	092471	217.0
31	092167	24.0	74	100271	1374.6
32	092667	40.2	75	121471	293.4
33	100767	26.6	76	032172	42.1
34	031968	20.6	77	041572	59.0
35	050768	39.7	78	042072	23.9
36	052568	167.6	79	042172	22.2
37	060168	66.7	80	042772	106.5
38	070168	156.2	81	051272	281.0
39	071368	53.1	82	102272	69.2
40	090468	257.2	83	103072	220.5
41	092368	40.3	84	103172	535.5
42	100968	104.4	85	111372	21.4
43	111568	79.6			

Partial Duration Series for Watershed 5142

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	040967	25.2	34	082569	2.0
2	041067	61.7	35	043070	3.9
3	041267	180.4	36	051470	60.3
4	041367	9.3	37	052970	27.6
5	042067	12.3	38	081970	3.8
6	050667	37.0	39	091470	2.9
7	052067	18.3	40	092270	53.3
8	062667	3.8	41	100570	5.8
9	090367	17.6	42	100770	5.4
10	090567	6.2	43	100870	4.7
11	092667	12.8	44	052371	5.0
12	100767	8.4	45	053171	9.1
13	031968	4.1	46	060271	34.0
14	041968	2.1	47	060371	67.9
15	052568	6.9	48	060771	16.8
16	060168	7.7	49	061271	22.3
17	060568	3.2	50	070171	2.2
18	060768	8.6	51	081471	7.2
19	070168	12.3	52	091871	78.8
20	071368	7.2	53	092471	73.2
21	071868	2.1	54	100271	92.4
22	090368	4.0	55	101971	2.8
23	090468	72.9	56	121471	45.2
24	100568	5.7	57	032072	5.2
25	100968	16.6	58	041572	7.5
26	111568	8.2	59	042072	3.3
27	112668	5.1	60	042772	42.9
28	050369	29.7	61	051272	11.6
29	050669	148.4	62	102172	6.7
30	051269	4.6	63	103072	73.1
31	060169	2.7	64	103172	34.6
32	061469	134.2	65	110172	2.2
33	072069	74.3	66	111272	16.2

Partial Duration Series for Watershed 5143

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	041067	12.8	28	082569	1.2
2	041267	143.1	29	091669	1.4
3	041367	4.8	30	092269	2.1
4	042067	7.2	31	043070	2.2
5	050567	11.2	32	051470	25.0
6	052067	3.3	33	052970	8.2
7	092667	2.1	34	092270	30.8
8	031968	3.3	35	100570	2.5
9	041968	2.1	36	100870	1.6
10	052568	3.8	37	022171	1.3
11	060168	2.1	38	032071	1.4
12	060768	3.7	39	041571	1.4
13	070168	12.2	40	052371	2.0
14	071368	2.1	41	053171	1.0
15	071868	1.3	42	060271	5.6
16	090468	27.1	43	060371	15.0
17	100968	6.1	44	060771	2.2
18	111568	3.8	45	061271	3.2
19	032369	3.4	46	091871	26.8
20	050369	13.4	47	092471	26.8
21	050569	1.9	48	100271	60.0
22	050669	119.6	49	102171	1.2
23	051269	2.8	50	111271	2.1
24	060169	2.0	51	121472	16.5
25	061469	56.1	52	042772	11.0
26	072069	22.0	53	051272	3.0
27	082569	1.2	54	103072	18.8

Partial Duration Series for Watershed 5145

Event	Date (mmddyy)	Peak Flow (cfs)
1	040967	30.5
2	041067	29.3
3	041267	193.2
4	041367	7.2
5	042067	26.3
6	050567	45.0
7	052067	10.5
8	053067	4.1
9	062667	5.3
10	072167	3.6
11	090367	19.6
12	090567	8.6
13	092667	34.7
14	100767	2.3
15	101567	17.0
16	031968	46.4
17	042268	3.1
18	050668	3.3
19	050968	2.9
20	051368	2.6
21	052568	13.7
22	053168	3.8
23	060168	8.3
24	060768	17.5
25	061668	8.3
26	070168	11.1
27	071368	2.2
28	071868	18.1
29	090368	2.6
30	090468	76.5
31	100568	3.6
32	100968	12.5
33	111568	3.3
34	112668	4.3
35	032369	4.0
36	050369	71.5
37	050669	112.2
38	051269	2.6
39	060169	10.0
40	061469	104.3
41	072069	87.9
42	091669	2.5
43	092269	9.63
44	043070	5.3
45	051470	13.8
46	052970	56.8
47	081970	23.5
48	091470	4.1
49	092270	104.3
50	100570	22.5
51	100770	2.8
52	100870	19.6
53	102270	2.9
54	022171	8.8
55	052371	44.6
56	053171	24.5
57	060271	70.9
58	060371	48.8
59	060771	21.7
60	061271	13.9
61	070171	50.5
62	072371	24.2
63	080871	4.4
64	081471	9.8
65	091771	13.6
66	091871	65.6
67	092471	45.1
68	092571	2.3
69	100271	224.1
70	101871	13.1
71	102671	2.5
72	121471	80.9
73	122971	7.5
74	041572	15.7
75	042072	9.8
76	042772	60.5
77	050672	3.2
78	051272	23.8
79	070272	8.8
80	102172	6.2
81	103072	56.2
82	103172	73.0
83	111272	31.1

Partial Duration Series for Watershed 515

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	012173	13.8	32	072974	11.7
2	012673	8.5	33	080974	62.8
3	030673	25.3	34	081074	91.2
4	031073	71.5	35	102874	10.8
5	032373	85.4	36	103074	25.3
6	041573	17.5	37	110274	12.3
7	041973	18.6	38	010275	15.8
8	042273	8.5	39	020275	50.5
9	050673	12.0	40	032775	9.3
10	052373	6.3	41	040775	63.6
11	052473	105.2	42	050275	16.5
12	053073	5.0	43	051375	48.5
13	053173	97.1	44	052375	89.3
14	060173	23.1	45	052875	33.3
15	060273	210.3	46	052975	86.7
16	060473	132.8	47	061075	43.4
17	061973	5.7	48	062275	25.3
18	080973	135.8	49	062375	27.6
19	091273	16.8	50	070775	10.8
20	092673	9.3	51	071075	10.0
21	101173	18.3	52	072475	29.5
22	101273	11.7	53	072675	41.0
23	112473	39.8	54	030876	5.4
24	022174	12.6	55	041976	56.1
25	030874	103.2	56	062476	12.3
26	031074	106.6	57	042077	5.2
27	041174	16.8	58	051977	93.8
28	042974	64.6	59	052077	105.2
29	050174	84.2	60	052777	6.1
30	053174	23.9	61	053177	9.5
31	060674	8.3	62	070177	14.4

Partial Duration Series for Watershed 611

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	042762	56.5	31	111664	605.7
2	052662	52.1	32	111864	64.1
3	060162	223.2	33	111964	64.1
4	060862	45.6	34	041465	5.7
5	060962	296.3	35	052665	135.0
6	072562	267.1	36	060265	78.9
7	091562	14.8	37	080665	98.0
8	092062	166.0	38	080765	17.9
9	102862	465.7	39	080865	23.5
10	110762	4.4	40	081065	43.3
11	112662	9.1	41	082265	5.9
12	120262	64.8	42	082865	9.8
13	042663	669.6	43	083165	4.5
14	062363	584.6	44	090365	104.1
15	071363	21.1	45	091965	4.7
16	111963	47.1	46	092065	8.9
17	020464	3.0	47	020866	45.6
18	020564	7.6	48	031266	105.3
19	050664	414.2	49	042266	20.1
20	050864	7.8	50	042366	61.3
21	050964	2406.8	51	042566	198.7
22	051064	807.9	52	061666	357.3
23	053064	206.1	53	072466	204.2
24	060464	5.9	54	081166	3.4
25	080764	58.6	55	081966	5.3
26	081564	4.7	56	082366	35.1
27	090464	15.8	57	083166	3.8
28	091664	245.5	58	091466	123.7
29	092764	15.0	59	092766	5.7
30	110364	63.1			

Partial Duration Series for Watershed R5

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	041067	3.83	39	012673	0.38
2	041267	20.65	40	030673	1.82
3	041367	0.52	41	031073	2.56
4	050567	0.44	42	032373	5.43
5	052067	0.29	43	032473	9.49
6	031968	1.82	44	041573	0.22
7	032168	0.17	45	041973	0.83
8	052568	0.52	46	052473	63.07
9	060168	0.12	47	053173	3.45
10	060768	0.26	48	060173	0.26
11	090468	0.15	49	060273	13.27
12	021469	0.24	50	060473	17.86
13	022069	0.12	51	061973	0.19
14	022169	0.15	52	092673	0.17
15	032369	0.29	53	102773	0.26
16	050369	0.15	54	111973	19.93
17	050669	16.87	55	112473	2.26
18	061469	9.26	56	022174	0.14
19	072069	5.92	57	031074	0.99
20	051470	0.38	58	042974	6.27
21	052970	0.35	59	051074	3.09
22	092270	2.98	60	081074	0.60
23	100870	0.11	61	103074	1.30
24	060271	0.19	62	010275	0.35
25	060371	4.81	63	020375	0.32
26	061271	4.09	64	022275	0.11
27	091871	1.44	65	032775	0.17
28	092474	1.44	66	040775	0.60
29	100271	9.95	67	042975	3.33
30	121471	2.66	68	052275	20.65
31	042772	1.74	69	052975	0.17
32	051272	0.55	70	061075	0.93
33	103072	3.45	71	062375	11.15
34	103172	4.66	72	071576	0.32
35	110172	2.26	73	042077	2.26
36	111672	0.68	74	051977	11.41
37	010373	0.38	75	052077	10.91
38	012173	0.41			

Partial Duration Series for Watershed R6

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	040967	0.98	31	060371	3.94
2	041067	3.80	32	061271	2.06
3	041267	29.26	33	081471	1.29
4	041367	0.55	34	091871	4.21
5	050567	1.57	35	092471	3.31
6	052067	1.04	36	100271	12.41
7	090567	0.26	37	121471	2.44
8	092667	0.14	38	022172	0.31
9	031968	1.65	39	041572	0.47
10	032168	0.14	40	042072	0.51
11	052568	0.63	41	042772	4.78
12	060168	0.24	42	051272	0.93
13	060768	0.34	43	060272	1.29
14	070168	0.37	44	102172	0.21
15	090468	1.29	45	103072	5.89
16	100968	0.29	46	103172	4.63
17	111568	0.26	47	010373	0.34
18	021469	0.21	48	012173	0.63
19	032369	0.44	49	012673	0.21
20	050369	0.55	50	030673	1.65
21	050669	23.31	51	031073	1.73
22	051269	0.19	52	032373	5.24
23	061469	7.15	53	032473	12.95
24	072069	4.34	54	041573	1.10
25	043070	0.37	55	041973	2.15
26	051470	1.10	56	050673	1.29
27	052970	0.72	57	052373	2.34
28	092270	2.54	58	052473	105.10
29	100770	0.37	59	053073	0.63
30	100870	0.26	60	053173	6.41

Partial Duration Series for Watershed R6 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
61	060173	1.16	89	022275	0.24
62	060273	21.01	90	032775	1.43
63	060473	27.48	91	040775	2.64
64	061673	0.24	92	042975	8.36
65	061873	1.29	93	050275	0.11
66	061973	1.36	94	051375	0.63
67	071373	0.37	95	052275	29.26
68	072773	1.65	96	052875	0.34
69	080973	2.54	97	052975	0.24
70	081573	0.11	98	060675	0.68
71	091273	1.10	99	061075	4.34
72	092673	0.41	100	061775	2.15
73	101173	0.14	101	062275	0.63
74	102873	0.82	102	062375	12.14
75	111973	36.08	103	070775	1.22
76	112473	2.44	104	072475	0.29
77	022174	0.51	105	030876	0.17
78	031074	2.25	106	041976	0.63
79	041174	0.55	107	062476	1.16
80	042974	9.45	108	071576	1.22
81	050174	4.78	109	071576	1.65
82	080974	0.63	110	042077	11.88
83	081074	2.15	111	050377	0.82
84	102874	0.15	112	050577	0.55
85	103074	1.50	113	051977	33.54
86	110374	0.29	114	052077	33.54
87	010275	0.51	115	052677	0.82
88	020375	0.19	116	062877	0.31

Partial Duration Series for Watershed R7

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	040967	7.13	41	112868	0.21
2	041067	12.92	42	021469	0.40
3	041267	29.26	43	022069	0.29
4	041367	3.42	44	022169	0.47
5	042067	1.42	45	032369	1.97
6	050567	10.84	46	050369	3.07
7	052067	9.65	47	050569	0.92
8	053067	1.28	48	050669	41.58
9	053167	0.77	49	051269	2.53
10	062667	0.44	50	061469	34.57
11	070567	0.14	51	072069	15.55
12	090367	6.05	52	082569	0.77
13	090567	7.13	53	091669	2.15
14	092667	3.79	54	092169	0.37
15	100767	1.04	55	092269	2.15
16	011868	1.28	56	043070	4.91
17	011968	0.37	57	051470	10.35
18	012768	1.09	58	052970	4.91
19	013068	0.34	59	081970	2.43
20	031468	0.63	60	091470	0.44
21	031968	7.13	61	092270	15.86
22	032168	0.51	62	100570	3.42
23	041968	0.92	63	100770	3.92
24	042268	0.55	64	100870	1.64
25	050968	0.29	65	102270	0.15
26	051368	1.89	66	022171	3.30
27	052568	4.47	67	052371	2.84
28	060168	4.06	68	052671	1.49
29	060568	0.40	69	053171	4.62
30	060768	4.91	70	060271	18.48
31	061668	2.43	71	060371	18.14
32	070168	10.11	72	060771	1.04
33	071368	10.84	73	061171	0.63
34	071468	0.72	74	061271	15.55
35	071868	0.34	75	080871	0.19
36	090468	23.29	76	081471	13.20
37	100568	4.06	77	091771	6.05
38	100968	6.76	78	091871	21.73
39	111568	5.07	79	092471	13.77
40	112668	1.09	80	100271	38.25

Partial Duration Series for Watershed R7 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
81	101971	3.54	121	072073	0.51
82	102671	1.22	122	072273	17.14
83	120871	1.04	123	073073	0.15
84	121071	0.63	124	080973	18.48
85	121471	10.84	125	081573	7.52
86	122971	0.92	126	090673	1.57
87	041572	5.38	127	091273	18.83
88	042072	8.54	128	092273	0.51
89	042772	19.17	129	092673	5.38
90	043072	0.98	130	092773	1.22
91	051272	7.32	131	100473	3,42
92	102172	2.84	132	100673	0.10
93	103072	24.91	133	101173	6.05
94	103172	12.12	134	101273	2.06
95	110172	8.98	135	102773	11.09
96	111272	2.53	136	111973	56.08
97	111872	0.87	137	112473	4.91
98	010373	2.15	138	022174	4.06
99	011673	1.42	139	031074	18.14
100	012173	5.54	140	041174	6.22
101	012673	0.77	141	042974	22.12
102	030673	12.12	142	050174	21.36
103	031073	7.32	143	052574	0.17
104	032373	8.12	144	060674	5.89
105	032473	4.33	145	080974	11.85
106	040273	0.59	146	081074	12.92
107	041573	6.58	147	090274	1.89
108	041973	9.88	148	091674	0.77
109	050673	12.12	149	092474	0.77
110	052273	13.48	150	102574	0.68
111	052473	97.90	151	102874	6.94
112	053073	13.48	152	103074	14.05
113	053173	28.36	153	103174	0.55
114	060173	9.88	154	110274	1.22
115	060273	30.19	155	110374	4.06
116	060473	51.37	156	110974	0.19
117	061673	2.33	157	111074	0.24
118	061873	13.48	158	121074	0.31
119	061973	9.20	159	121174	0.26
120	071373	4.62	160	010275	2.33

Partial Duration Series for Watershed R7 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
161	013075	0.15	184	072475	0.92
162	020175	0.82	185	072675	0.77
163	020275	0.26	186	030876	1.09
164	020375	0.98	187	041976	5.22
165	021775	0.17	188	042876	1.16
166	022275	1.16	189	052676	1.89
167	032775	5.38	190	062476	8.98
168	040775	15.24	191	071576	10.84
169	042775	0.29	192	080576	0.34
170	042975	17.47	193	091376	0.55
171	050275	2.06	194	021177	0.19
172	051375	4.91	195	042077	30.65
173	051475	0.92	196	050377	12.38
174	051975	0.47	197	050577	6.22
175	052275	33.07	198	051377	0.82
176	052875	1.49	199	051577	0.34
177	052975	0.72	200	051977	49.44
178	060675	2.33	201	052077	41.58
179	060875	1.16	202	052677	4.91
180	061075	17.14	203	052777	1.28
181	061775	4.76	204	062577	0.37
182	062275	2.06	205	062877	1.16
183	062375	12.38			

Partial Duration Series for Watershed R8

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
1	040967	1.96	41	060371	17.05
2	041067	5.85	42	061271	6.20
3	041267	32.32	43	081471	4.75
4	041367	2.14	44	091771	3.41
5	050567	6.02	45	091871	19.42
6	052067	5.69	46	092471	14.28
7	090367	3.53	47	100271	36.33
8	090567	2.14	48	100371	5.20
9	031968	2.84	49	101971	2.42
10	051368	1.15	50	121471	8.51
11	052568	3.06	51	122971	1.22
12	060168	3.17	52	041572	2.94
13	060768	3.65	53	042072	4.75
14	061668	2.05	54	042772	12.33
15	070168	10.31	55	051272	3.06
16	071368	7.30	56	102172	1.15
17	090468	20.86	57	103072	22.36
18	100568	2.74	58	103172	13.14
19	100968	4.60	59	110172	1.49
20	111568	1.96	60	111272	2.05
21	050369	4.75	61	010373	1.56
22	050669	50.79	62	012173	2.62
23	051269	1.42	63	030673	3.29
24	061469	26.41	64	031073	3.91
25	072069	19.77	65	032373	9.84
26	091669	1.09	66	032473	7.30
27	092269	1.56	67	041573	4.04
28	043070	1.80	68	041973	4.18
29	051470	8.51	69	050673	5.05
30	052970	3.41	70	052273	10.07
31	081970	2.14	71	052473	133.70
32	091470	1.64	72	053073	6.92
33	092270	16.72	73	053173	25.16
34	100570	1.88	74	060173	6.20
35	100770	4.04	75	060273	26.84
36	100870	1.35	76	060473	41.75
37	022171	1.64	77	061673	1.72
38	052371	2.73	78	061873	6.20
39	053171	2.52	79	061973	3.65
40	060271	12.06	80	071373	1.96

Partial Duration Series for Watershed R8 (Continued)

Event	Date (mmddyy)	Peak Flow (cfs)	Event	Date (mmddyy)	Peak Flow (cfs)
81	072273	7.49	112	052275	39.53
82	080973	6.55	113	052875	1.42
83	081573	7.49	114	060675	2.73
84	091273	8.30	115	060875	1.28
85	092673	4.75	116	061075	14.87
86	100473	3.06	117	061775	10.79
87	101173	1.64	118	062275	2.52
88	101273	1.15	119	062375	10.31
89	102773	6.37	120	070775	2.33
90	111973	59.52	121	072475	1.96
91	112473	5.36	122	081475	2.14
92	022174	2.73	123	030876	1.15
93	031074	9.84	124	041576	1.22
94	041174	2.84	125	041976	4.90
95	042974	23.53	126	062476	14.87
96	050174	19.07	127	071576	14.57
97	060674	2.14	128	080576	3.65
98	080974	9.38	129	042077	27.71
99	081074	15.78	130	050377	8.30
100	090274	1.42	131	050577	9.61
101	091574	1.28	132	051377	3.78
102	102874	7.69	133	051577	1.88
103	103074	16.72	134	051977	69.14
104	110374	4.46	135	052077	78.81
105	010275	1.80	136	052677	7.69
106	020375	3.06	137	052777	1.42
107	032775	3.53	138	062577	4.18
108	040775	10.31	139	062877	7.69
109	042975	24.34	140	072777	1.22
110	050275	1.35	141	091377	2.05
111	051375	3.17			

APPENDIX C

ANOVA TABLES AND RELATED INFORMATION FOR PREDICTION EQUATIONS

ANOVA Table For Eqn. 95

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
Regression	54.80	3	18.26	40.03
Residual	5.02	11	0.46	
Total	59.82	14		

Standard Error of Estimate = 0.676

$r^2 = 0.916$

$r = 0.957$

Variable	Regression Coefficient	Standard Error	F(1,11)	Partial r^2
A	-0.06	0.02	7.11	0.39
W	1.20	0.21	34.22	0.76
S_t	1.61	0.26	37.13	0.77
Constant	-3.42			

ANOVA Table For Eqn. 96

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
Regression	6.26	3	2.09	10.27
Residual	2.34	11	0.20	
Total	8.50	14		

Standard Error of Estimate = 0.451

$r^2 = 0.737$

$r = 0.859$

Variable	Regression Coefficient	Standard Error	F(1,11)	Partial r^2
ln(A)	-0.43	0.24	3.32	0.23
ln(W)	1.20	0.47	6.42	0.37
$\ln(S_t)$	2.23	0.62	12.90	0.54
Constant	-3.64			

ANOVA Table For Eqn. 97

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
Regression	26.26	2	13.13	69.96
Residual	2.25	12	0.19	
Total	28.51	14		

Standard Error of Estimate = 0.433

$r^2 = 0.921$

$r = 0.960$

Variable	Regression Coefficient	Standard Error	F(1,11)	Partial r^2
$\ln(L_c)$	0.62	0.07	81.01	0.87
$\ln(S_a)$	-1.66	0.42	15.68	0.57
Constant	2.28			

ANOVA Table For Eqn. 99

Source	Sum of Squares	Degrees of Freedom	Mean Square	F
Regression	18.13	3	6.04	16.83
Residual	3.95	11	0.36	
Total	22.09	14		

Standard Error of Estimate = 0.599

$r^2 = 0.821$

$r = 0.906$

Variable	Regression Coefficient	Standard Error	F(1,11)	Partial r^2
ln(A)	-0.37	0.31	1.46	0.12
ln(W)	1.52	0.63	5.83	0.35
$\ln(S_t)$	0.98	0.83	1.42	0.11
Constant	-3.09			

VITA

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Doctor of Philosophy

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