TRANSISTOR-CHOPPER STABILIZATION

OF D.C. AMPLIFIERS

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PREFACE

Although it has been known for a number of years that transistors may be used in certain connections as choppers to modulate d-c voltages, little research has been done on this subject. Replacement of mechanical choppers by transistor circuits in precise d-c measuring equipment is highly desirable since high reliability may be realized with transistor circuits. Although much work remains to be done to perfect it, the transistor chopper shows promise in excelling mechanical types in all respects.

It is the purpose of this work to present the difficulties encountered in the design of transistor chopper circuits, how they may best be overcome, and how a low-drift completely transistorized chopper-type d-c amplifier may be designed. In addition, it is necessary to consider how such an amplifier may be used to stabilize a wide-band transistor amplifier against drift, since the frequency response of the chopper amplifier is inherently limited.

The writer is indebted to Dr. Harold T. Fristoe, Professor of Electrical Engineering, for his valuable guidance and assistance, and especially for his aid in contacting a number of transistor manufacturers. Gratitude is also due to Mr. L. C. Labarthe of Labko Scientific, Inc., for his aid in running the temperature tests and in discussing much of the material.

The contribution of transistors by the Semiconductor Division of the Sperry Rand Corporation and by the General Transistor Corporation is greatly appreciated, since this made much of the experimental work possible. Finally, the writer wishes to thank the Alcoa Foundation, without whose financial assistance this study would not have been possible.

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CHAPTER I

INTRODUCTION

Since the advent of transistors, their application in d-c amplifier circuits has been a difficult task. This is especially true in low-level, high-gain amplifiers. It is possible to design circuits which may be used satisfactorily, but such designs usually specify unacceptable techniques such as extremely careful component selection, temperature compensation, and the individual adjustment of a number of controls. If these techniques are not employed, the d-c amplifier must be stabilized against drift errors. Since drift errors cannot be corrected by the application of negative feedback, some form of modulation, a-c amplification, and demodulation is necessary. The simplest method by which this may be accomplished is illustrated in Fig. 1. The d-c signal is modulated (or chopped), amplified, and then demodulated (or rectified).

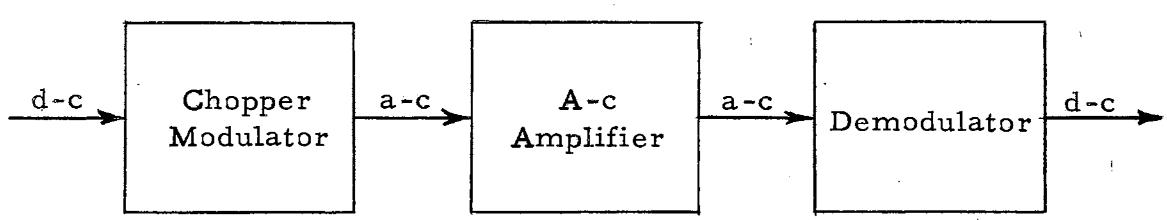

Fig. 1. Chopper Amplifier Block Diagram.

If only amplification of d-c levels is desired, the chopper amplifier may be used effectively. If, however, a wide bandwidth is required in the amplifier, the chopper amplifier can be used in a stabilizing system to stabilize a directly-coupled wide-band amplifier. Thus, the chopper amplifier may be viewed as a basic component in any low-level d-c amplification system.

In the past, and in practically all existing d-c amplifiers, mechanical choppers have been used to modulate and demodulate the d-c level to be amplified. If properly exploited, certain properties of transistors make the replacement of the mechanical chopper by a transistor circuit possible. Much greater reliability and insensitivity to the environment are only two of the obvious advantages of transistor circuits. It is surprising to note that very little work has been done in this field, although it has been known for a number of years that transistors may be used in this application. It has been found that only three major manufacturers of transistors have recently initiated research in this field.

In essence, the use of a transistor circuit (transistor chopper) to accomplish modulation is the greatest problem in designing completely transistorized low-level d-c amplifiers, since the other necessary circuits have been sufficiently developed in the literature. Hence, much of the following material will be devoted to this. In order to compare favorably with mechanical chopper circuits, the transistorized chopper amplifier should be stable and drift-free within a few microvolts.

CHAPTER II

THE TRANSISTOR AS A SWITCH

Under certain conditions, the transistor may be used as a switch. Only p-n-p transistors will be discussed here, since all transistors found to be most suitable for low-level switching applications are of this type. If n-p-n transistors with the desired properties become available, the discussion will apply to them equally, with all voltage polarities and current directions reversed.

When the base is positive with respect to the collector and the emitter in a p-n-p transistor, the emitter-to-collector impedance is very high, generally well over 1 megohm, since both p-n junctions in the transistor are reverse biased. This is the "open" or "cut-off" state of the transistor, and the transistor will block current flow in both directions. When the base is driven a sufficient amount negative of the collector and emitter, the p-n junctions are forward biased, and the emitter-to-collector impedance reduces to a few ohms in alloyed type transistors. In grown-junction units this impedance may be a few hundred ohms due to high effective emitter and collector resistances. When the transistor is biased in this manner, it is in the "on" or conducting state, and conducts current in both directions as long as the emitter and collector voltages do not exceed the base voltage.

Although the transistor may be used as a good switch when switching large currents (on the order of a few ma), a number of imperfections become apparent when low-level switching is attempted. The fact that the "open" and "closed" impedances are not infinite and zero, respectively, is not really very troublesome. Unfortunately, there are other effects which are quite undesirable when the transistor is operated as a low-level repetitive switch, or chopper. When the transistor switch is in the "open" state, and the p-n junctions are reverse biased, a small but finite leakage current flows into the external circuit. When the transistor is in the conducting state, it exhibits a small collector-to-emitter voltage drop. If the transistor is operated in the inverted connection, with the functions of the emitter and collector interchanged, this leakage current and voltage drop are reduced considerably. The factor of reduction is generally at least an order of magnitude, and is greater for more assymetric transistors. Due to this advantage, a low-level transistor chopper should always be operated in the inverted connection, and such operation will be assumed throughout the following material, unless otherwise noted.

Equivalent Circuits

A complete equivalent circuit for the transistor chopper operated in the inverted connection is shown in Fig. 2. This is a true equivalent circuit only if it is assumed that the transistor is in the saturated state when in the "on" condition and cut-off when in the "open" condition.

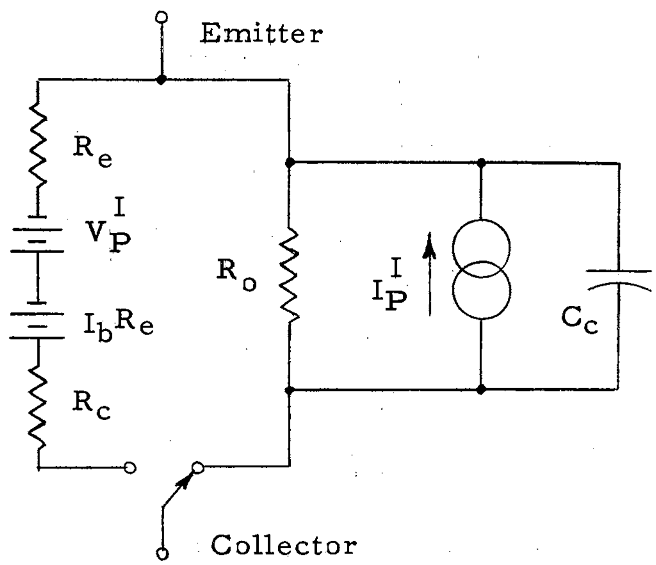

Fig. 2. A Complete Equivalent Circuit.

Actually, the emitter and collector resistances, R_e and R_c should be included in the emitter and collector leads, but since R_o, the "open" impedance is much greater than R_e and R_c, this makes no difference. Bright has derived the following equations for the values of the voltage offset and leakage current[1]:

$$V_P^I = \frac{kt}{q} \ln \frac{1}{\alpha_n} \qquad\qquad I_P^I = \frac{(1-\alpha_n) I_{ebo}}{1-\alpha_n \alpha_i}$$

Here α_n is the normal alpha, and α_i is the value of alpha in the inverted connection. The relation $\alpha_n I_{ebo} = \alpha_i I_{cbo}$ is often useful in evaluating the voltage offset and leakage current. The approximations

$$V_P^I = \frac{kt}{q} \frac{1}{\beta_n} \qquad\qquad I_P^I = \frac{\beta_i}{\beta_n} I_{cbo}$$

may be used more conveniently. The error decreases with higher β

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[1] R. L. Bright, "Junction Transistors Used As Switches," Transactions of the A.I.E.E., Part I, LXXIV (1955), 111-121.

and kt/q may be taken as 0.026 volts at 25° C.

The above equations and the equivalent circuit are useful as theoretical models, but their use in practice becomes somewhat difficult. Since the parameters β_i, β_n, α_i, α_n, I_{ebo}, I_{cbo}, R_c, and R_e vary considerably with variations in temperature and bias conditions, it is difficult to obtain good correlation between calculated and measured values for the leakage current and voltage offset. It is generally most expedient to consider a simpler equivalent circuit such as Fig. 3, and to measure its parameters. The voltage offset V_f (combining V_P^I and $I_b R_c$) is generally less than 3 millivolts, and the leakage current is on the order of 10^{-6} amperes or less.

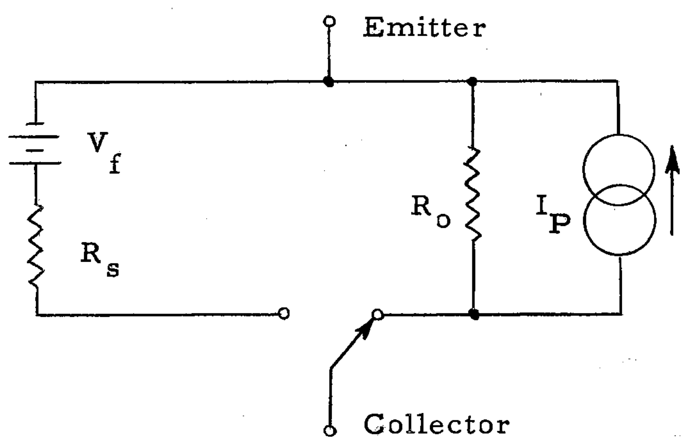

Fig. 3. A Simplified Equivalent Circuit.

Circuits such as those of Fig. 4 may be used to measure the parameters in Fig. 3. If the D.C. Amplifier input impedance in Fig. 4 (a) is made on the order of 10,000 ohms, both parameters (leakage current and voltage offset) should give on-scale readings. Care must be taken, when measuring voltage offsets with this circuit, that the amplifier lead is connected directly to the transistor socket collector lead, in order that no error is introduced by possible voltages across junction resistances. A junction resistance of 0.1 ohms is sufficient to introduce an error of 0.5 millivolts when $I_b = 5$ ma. The circuit of Fig. 4(b) may be used to measure the "on" and "off" impedances by applying the proper base-to-collector bias. The impedances can be calculated by comparison of the VTVM readings in the "Test" and "Cal." positions.

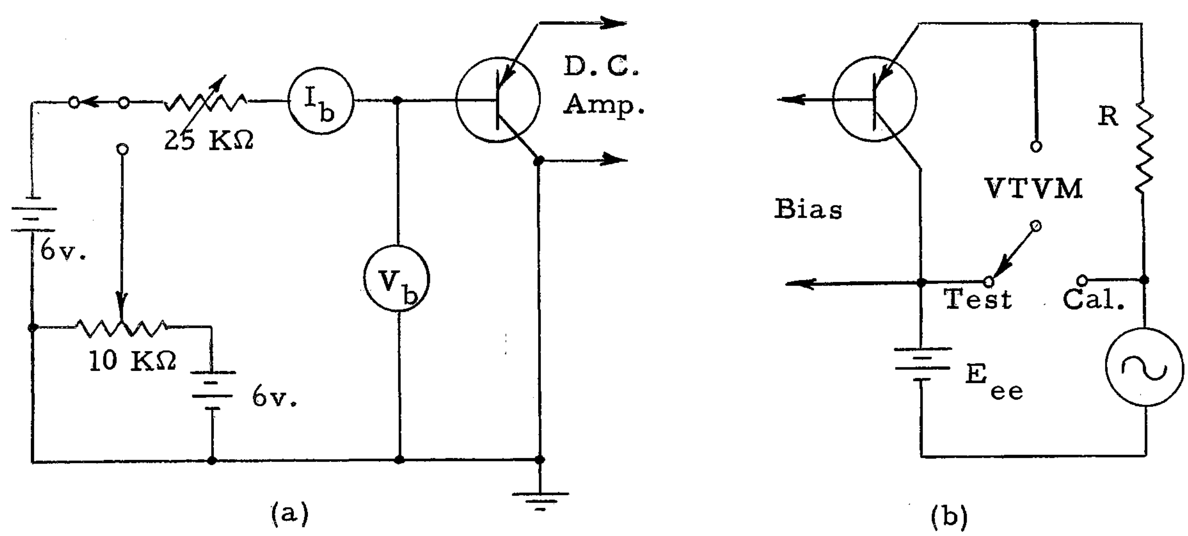

Fig. 4. Circuits for Measurement of the Parameters of Fig. 3.

In order to analyze the transistor action in switching more fully, it is desirable that the collector-base input characteristic be studied. This impedance consists, of course, simply of the collector-base p-n junction. When the transistor is in the "off" condition, this junction is reverse biased, and draws negligible current. In the "on" condition its impedance is quite low, consisting of a diode forward characteristic. Forward characteristics for this junction, which were obtained by measurements on 2N522 and 2N1027 transistors, are plotted in Fig. 5. These characteristics are not of the true diode type, since they contain the base and collector resistances.

If a transistor is to be used in a low-level chopper, its undesirable properties, as seen from Fig. 2, must be minimized. From Bright's equations for the leakage current and voltage offset it can be seen that a high value of β and a low value of I_{cbo} are highly desirable. Furthermore, a low value of collector resistance is called for to minimize the effect of the additional voltage offset introduced by the equivalent voltage source I_bR_c (see Fig. 2). A low emitter resistance is also desirable, so that the impedance of the transistor in the "on" state may be low. Since grown-junction transistors have high effective collector and emitter resistances, they are generally not applicable in low-level choppers. Other experimenters have come to the same conclusion, considering not only the collector and emitter resistances, but also higher leakage of the keying signal to the external circuit. The latter was believed to be due to the physical configuration of grown-junction transistors.[2] Since the saturation resistance of alloyed-junction transistors is, at the most, a few ohms, this type is best suited in low-level choppers.

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[2] E. A. Dorsett and J. H. Searcy, "Low-Level Electronic Switch," 1957 I.R.E. Convention Record, Part V, 57.

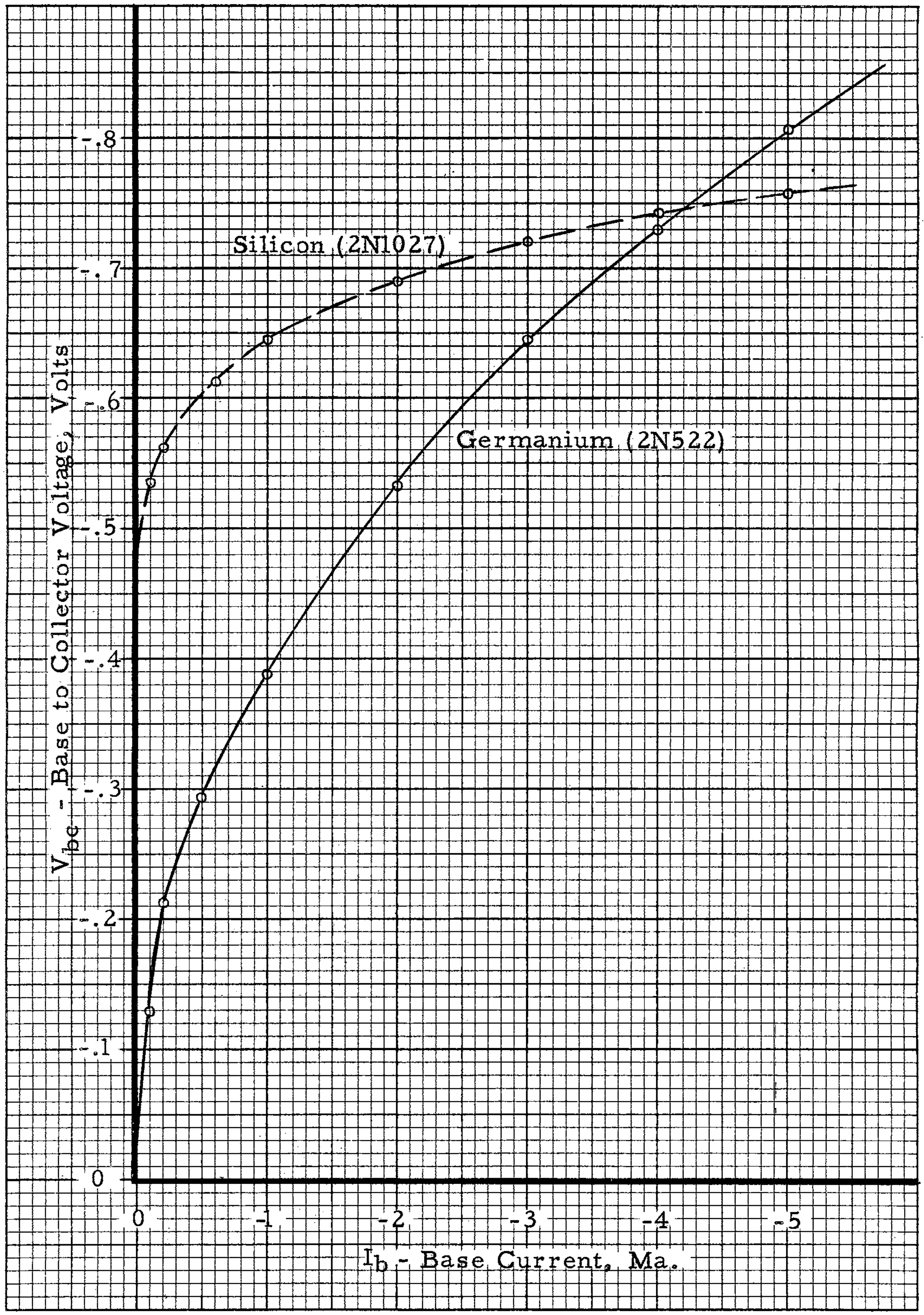

Fig. 5. Base-Collector Input Characteristic of the Transistor Switch.

The Voltage Offset

In practice, the most important parameter in the equivalent circuit of Fig. 3 is V_f. The leakage current can be reduced to a very small value by driving the base only slightly positive (a few millivolts) of the collector in the "off" condition. Variations in the "on" and "off" impedances can at most cause variations in the chopper attenuation, and hence the overall gain of the amplifier to which the chopped signal is applied. Such variations can be greatly reduced by application of negative feedback.

It is most convenient to analyze the voltage offset as a function of base current, and the leakage current as a function of the base voltage because of the diode-type input characteristic of the transistor base. The other independent variable must be taken as temperature. Fig. 6 shows the variation in V_f with base current as measured for a sample of four high-gain type 2N522 germanium transistors. The circuit of Fig. 4 (a) was used to obtain these measurements. The curves have almost equal slopes, indicating almost equal collector resistances. Displacement of the curves is due to differences in β among the four transistors. The voltage offsets lie well below 1 millivolt due to the high values of β. Fig. 7 shows the variation in V_f with temperature for transistors 2 and 4 of Fig. 6. This data was obtained by using the circuit of Fig. 4 (a) with the transistors under test housed in a closely regulated temperature oven. From these curves it can be seen that there exists an optimum value of base drive current for each transistor

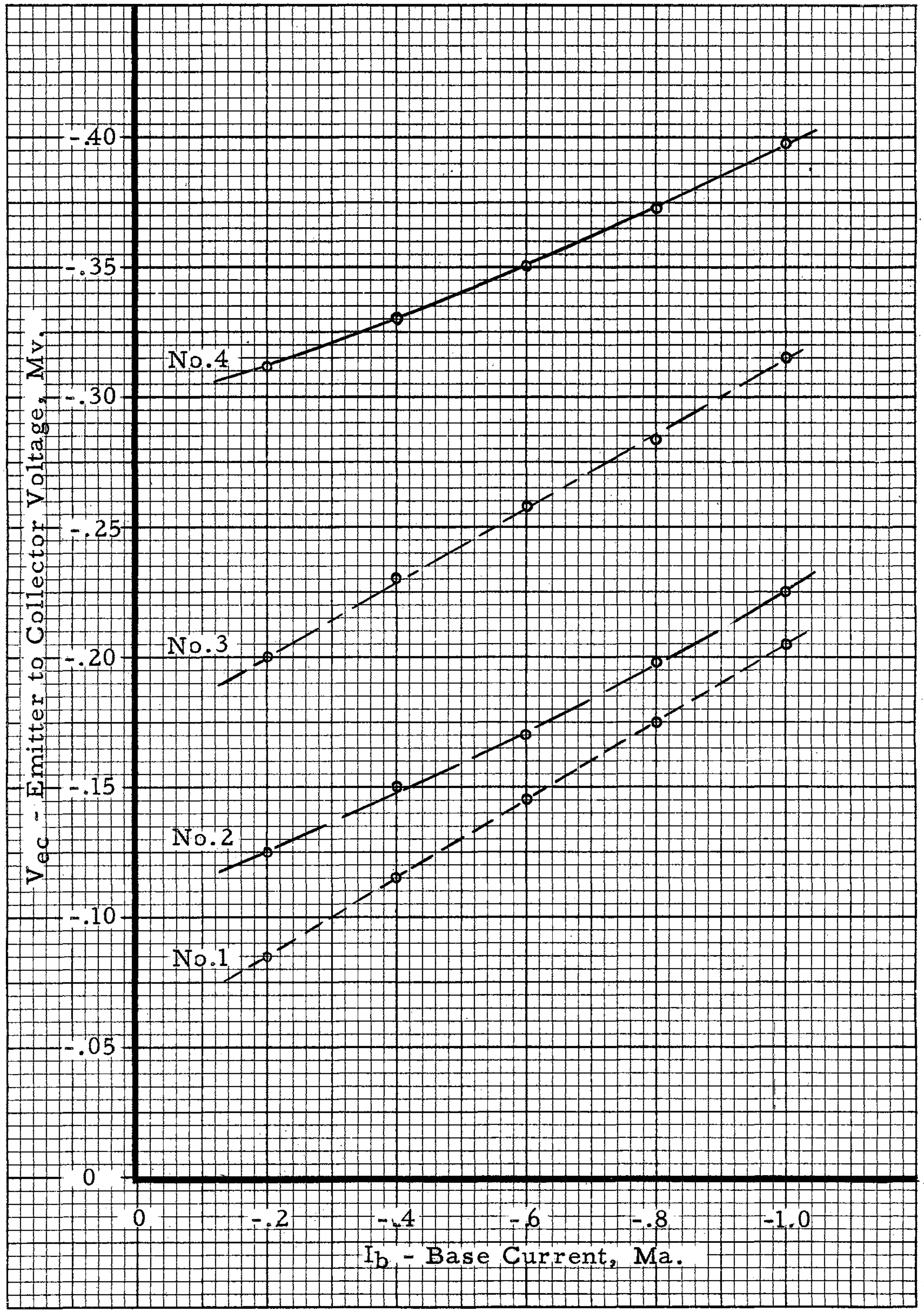

Fig. 6. Variation of the Voltage Offset with Base Current in Germanium Transistors (2N522).

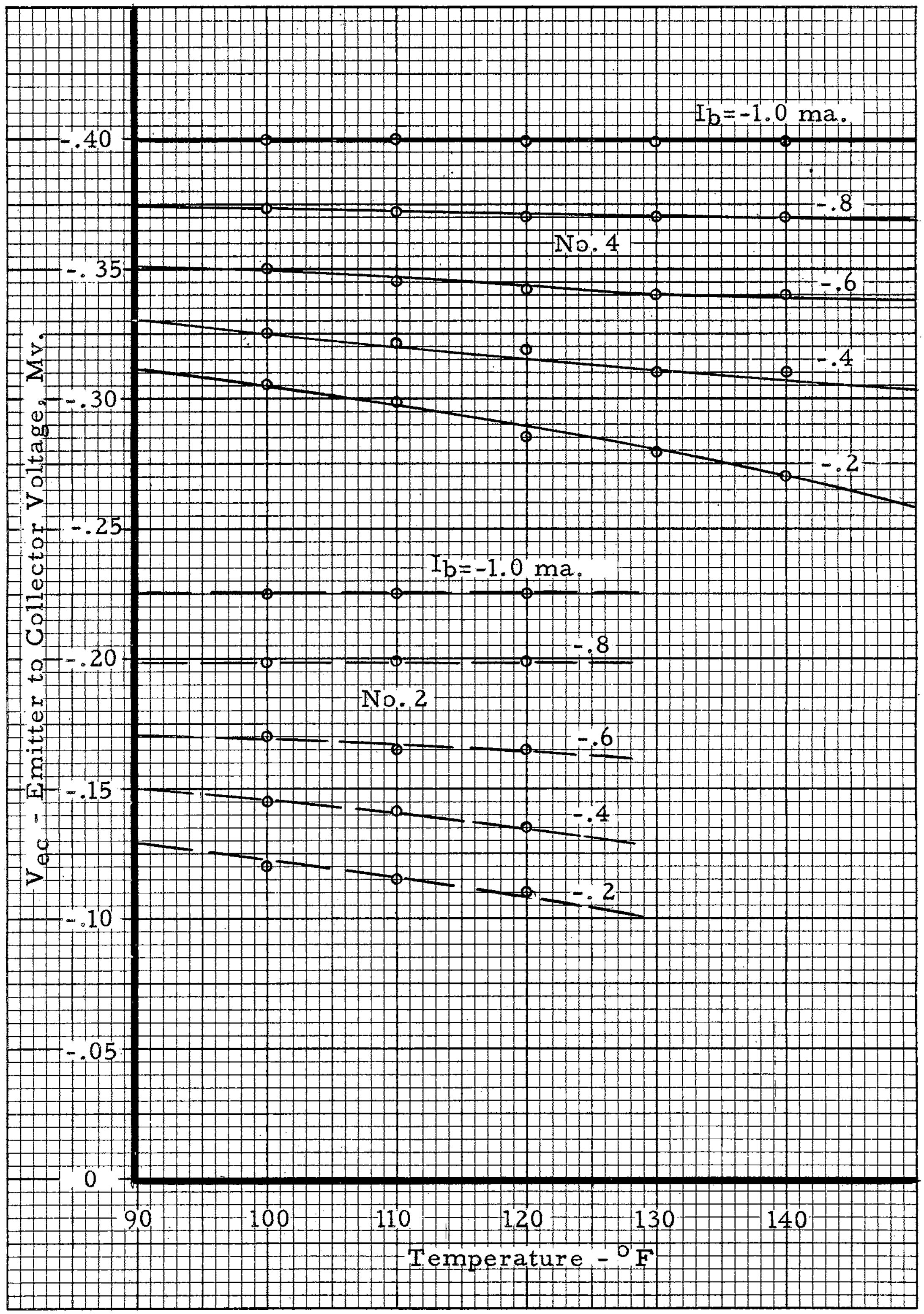

Fig. 7. Variation of the Voltage Offset with Temperature in Germanium Transistors (2N522).

of about 1 ma. With this base drive current, no variation in V_f takes place over the entire temperature range. It is interesting to note that the optimum base current for all other germanium transistors tested was found to be approximately 1 ma. Because of the additional bulk, temperature curves for all transistors are omitted here. Other experimenters have arrived at the same optimum base current for a different type germanium transistor, but no such information is available for silicon units.[3]

The variation of V_f with base current in a 2N1027 silicon transistor is shown in Fig. 8. This data was obtained by using the circuit of Fig. 4 (a) in conjunction with a high input impedance d-c millivoltmeter. It was necessary to shunt this high input impedance with a resistance of approximately 50 kilohms in order to obtain stable readings at low values of base current. The scale has been expanded in Fig. 8 for low base currents in order to show the high peak value of V_f. The same peaking effect was noticed with germanium transistors, but in that case at a much lower base current. With germanium transistors the peak was not nearly as high in proportion to the average value of V_f, and the currents and voltages in that region were so small that they could not be measured accurately. Hence the curves do not extend to zero base current in Fig. 6. It is interesting to note that there exists a region

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[3] G. B. B. Chaplin and A. R. Owens, "Some Transistor Input Stages for High-Gain D.C. Amplifiers," Proceedings of the Institution of Electrical Engineers, Part B, CV (1958), 254.

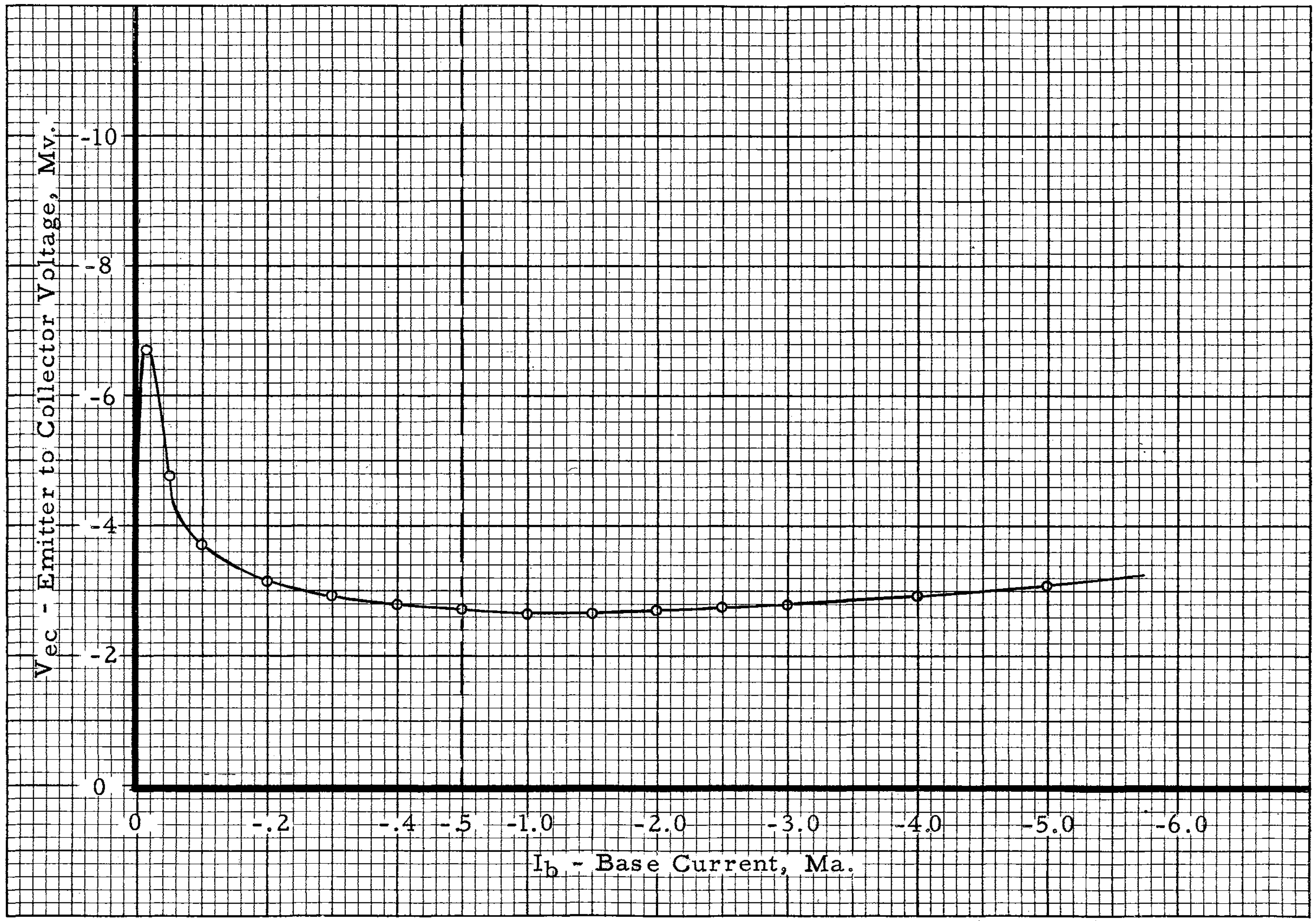

Fig. 8. Variation of the Voltage Offset with Base Current in a Silicon Transistor (2N1027).

in Fig. 8 between base currents of 1.0 and 1.5 ma in which no variation in V_f takes place. For this type of transistor, a base driving current of approximately 1.25 ma could be used without regulation, yet introducing no error voltage. Curves for the temperature variation of V_f in silicon transistors tested were somewhat erratic, and no definite conclusions could be drawn from them in regard to optimum base current. Hence they have not been included.

Transient Effects

In addition to the leakage current and voltage offset, transistors have one other undesirable property when used as low-level switches. When a transistor is switched from one state to the other by application of a voltage step at its base, a transient voltage appears across the emitter and collector terminals. Typical transients in the emitter-to-collector voltage when switching from the "off" state to the "on" state, and vice versa, are illustrated in Fig. 9. Transients of this shape were observed with all transistors tested. In low-frequency transistors these transients were quite severe, but in high-frequency types the spikes were only several times higher than V_f, the voltage offset. In all units tested, the transients were higher with greater collector capacitance. It was concluded that they are due to a combination of collector capacitance and the peaking characteristic of V_f. The width of the "on" transient was generally on the order of 1 microsecond, but the width of the "off" transient was several times that amount. The height of the "off" transient was greatly reduced when the positive swing of the base driving signal was limited to a few millivolts.

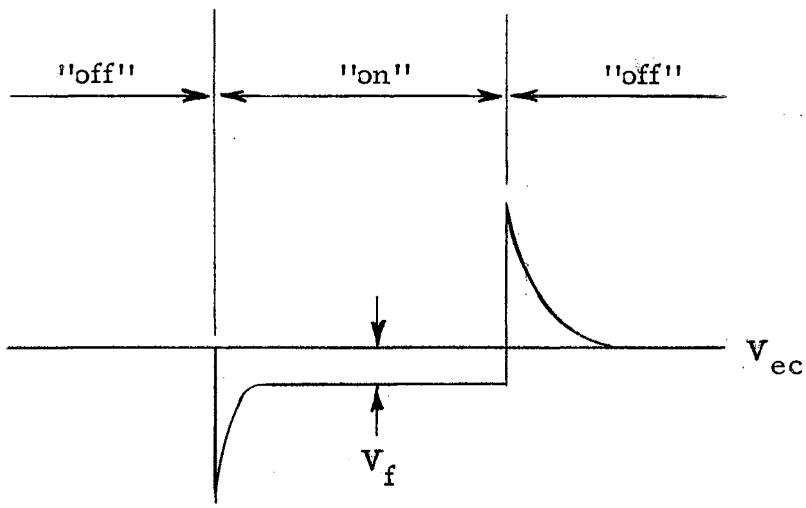

Fig. 9. Transients in the Emitter-to-Collector Voltage

To minimize the effect of transients when the transistor is used as a repetitive switch, or chopper, the base driving signal should be a square wave with a short rise-time, so that the transient region is rapidly traversed. The frequency of this driving signal should be low enough so that the transients constitute only a small portion of the chopped waveform.

Choice of Transistors

The qualities that a transistor should have in order to be used as a low-level chopper may now be summarized:

1. High d-c gain (β).
2. Low collector leakage current (I_{cbo}).
3. Low collector and emitter resistances (R_c and R_e).

4. Low inter-electrode capacitances (especially collector capacitance).

5. High frequency response.

A number of transistors presently available which most nearly satisfy the above qualifications are listed with a number of their parameters in Table I. The collector and emitter resistances in the units listed are so small (generally less than 1 ohm) that it is not necessary to list them. The Philco T1453 units are available in pairs matched for V_f within 10% for low-level chopper applications. The parameter values in Table I are typical for β, maximum for I_{cbo} and C_{ob}, and minimum for frequency response. With a moderate amount of selection it is possible to obtain units with characteristics considerably superior to those quoted.

Germanium transistors are generally characterized by higher leakage currents and lower voltage offsets than silicon types. Hence it can be expected that they are more satisfactory in low impedance circuits. By the same reasoning, superior operation may be expected with silicon transistors in high impedance circuits. If alloyed-junction silicon transistors with high frequency response and high gain become available in the future, they may displace germanium transistors in this application completely.

TABLE I

Transistors Suitable for Low-Level Chopper Operation

Transistor	Manufacturer	Type	β	I_{co} $\mu a.$	f_{osc}^{*} mc.	$f_{\alpha b}^{*}$ mc.	C_{ob} $\mu\mu f.$
S-500	Sperry	Silicon Alloyed	9	.025	8	-	7
2N1027	Sperry	Silicon Alloyed	18	.025	-	4	7
2N522	General Transistor	Germanium Alloyed	120	2	-	15	14
2N344 2N345 2N346	Philco	Germanium Surface Barrier	25	3	55	-	6
2N496	Philco	Silicon Alloyed	10	.1	11	-	12
T1453	Philco	Silicon Alloyed	25	.1	18	-	14

* f_{osc} - maximum frequency of oscillation

* $f_{\alpha b}$ - alpha-cutoff frequency

CHAPTER III

THE CHOPPER AMPLIFIER

In spite of the many imperfections of the transistor switch as discussed in the previous chapter, it is possible to attain surprisingly low drift specifications with transistor-chopper amplifiers when using proper circuit design techniques to minimize the effects of these imperfections. It should be emphasized at this point that the imperfections in the transistor switch cannot themselves produce drift in the chopper amplifier, but that their variations can produce considerable drift if not properly dealt with. Any offset in the amplifier output due to transistor leakage current, voltage offset, or the effect of transients can be reduced to zero by introduction of a corresponding extraneous input signal, but variations in the imperfections cannot be compensated in this manner. It is necessary, of course, to use transistors with minimum imperfections in the input chopper, so that the variations may be correspondingly smaller.

Input Chopper Circuits

A number of transistor chopper circuits will now be examined in order to determine which is most suitable. The circuit of Fig. 10 (b) is useful only in that it illustrates the similarity in circuit action of mechanical (Fig. 10a) and transistor choppers. When one transistor is in the "open" state in this circuit, the other conducts current with a low impedance. The current and voltage offsets in this circuit are, however, additive in the a-c output, and produce a definite output with zero input.

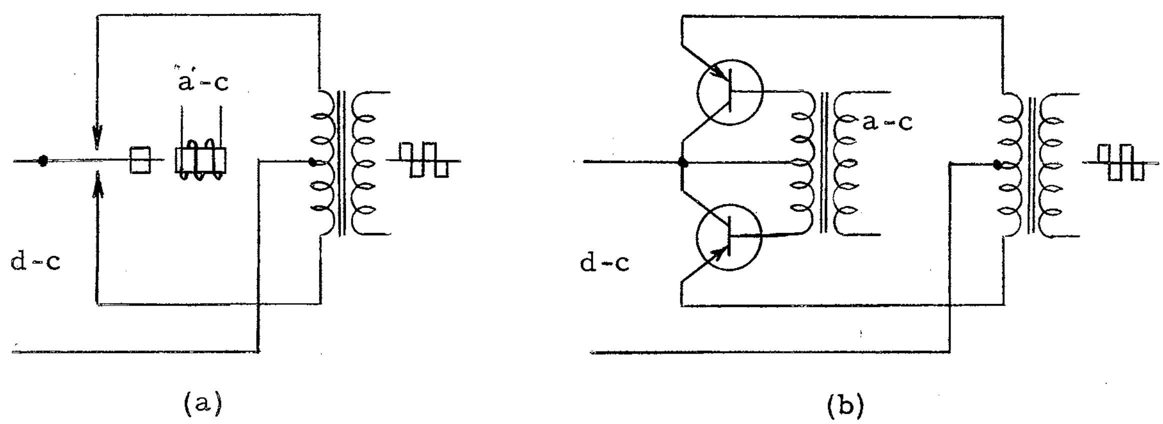

Fig. 10. Mechanical and Transistor Choppers.

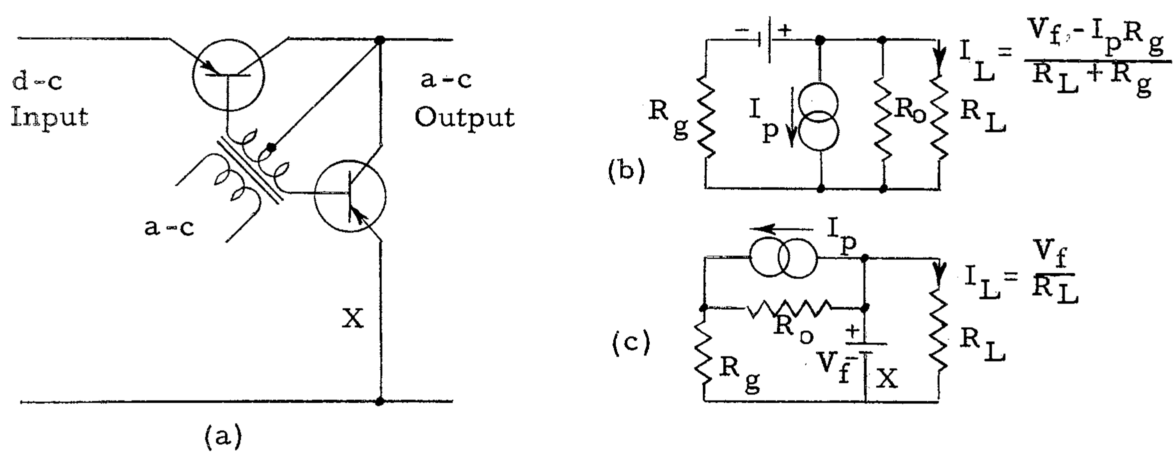

Fig. 11. The "Series-Shunt" Chopper and its Equivalent Circuits.

A chopper circuit which has often appeared in the literature is shown in Fig. 11. It is known as the "series-shunt" chopper. From the equivalent circuits of Fig. 11 (b) and (c) it can be seen that the load current I_L is the same for both states and in the same direction if R_g is small compared with R_L, and if the voltage offsets and leakage currents are identical for both transistors. The d-c current flowing in R_L then contributes no error in the a-c output when the input signal is reduced to a low value. If R_g, the d-c source impedance, is comparable to R_L, a resistor equal to R_g must be inserted in the circuit at the point "X". If this is done, the equations for I_L are the same in both states. A severe restriction on this circuit is that the base driving voltage (base-to-collector) must be greater than any d-c voltage that is to be applied at the input. If the d-c input voltage is allowed to exceed the base driving voltage, one of the blocking p-n junctions will become forward biased, and that particular junction will no longer block. Since it is highly desirable to limit the positive base voltage swing to a very small value in order to allow little leakage current, this chopper circuit is limited to very small input signal applications.

A four-transistor "series-shunt" type chopper circuit which eliminates any restrictions on the magnitudes of the input signal and source impedance is shown in Fig. 12. It consists simply of two "bilateral switches." The two transistors in each pair are always in the same state ("on" or "off"), and since they are connected back-to-back, the leakage currents and voltage offsets cancel if identical. It is actually a waste of transistors in most applications to use four of them in this manner since good transistors cannot be obtained at negligible cost. For reasonable values of load and source impedance (as encountered in transistor circuits) the series switch could be removed without changing the impedance ratio appreciably.

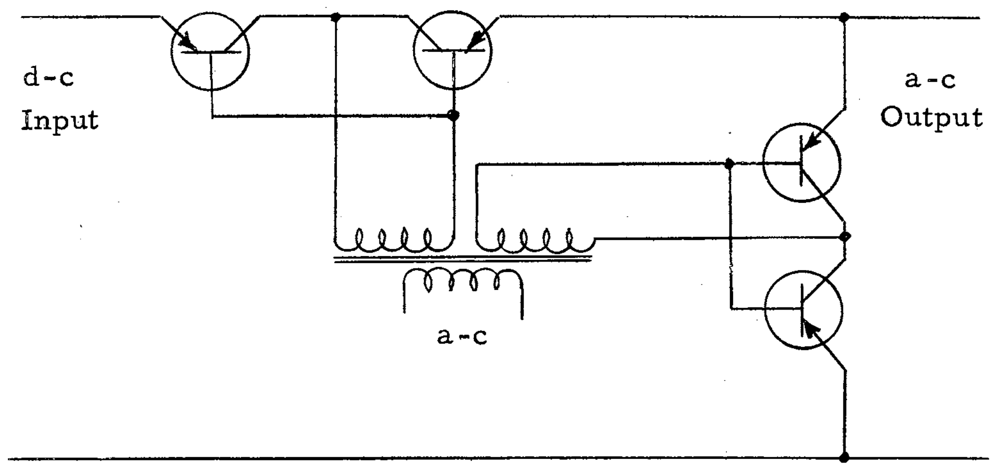

Fig. 12. A Four-Transistor "Series-Shunt" Chopper.

In all of the circuits so far considered, the input terminals for the base driving voltage are both above ground, and transformers are generally necessary. The use of transformers in low-level choppers is somewhat undesirable due to their effect on transients. The interaction of two different types of reactive elements in the circuit introduces a damped oscillation, or "ringing", in the transient portion of the collector-to-emitter voltage waveshape as shown in Fig. 13.

Although circuits have been devised to eliminate the transformer in chopper circuits, their disadvantages were found to be such that the transformer remains the best choice.[4] Since a transformer must be used, its frequency response should be well over the driving frequency in order to pass a good square wave.

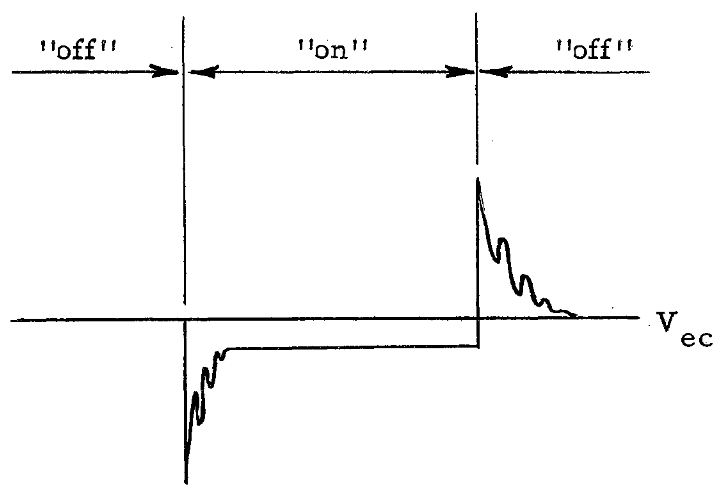

Fig. 13. Effect of a Transformer in the Driving Circuit on Transients.

Base Drive Considerations

Of the chopper circuits considered, the "bilateral switch" seems to be the most attractive proposition. In order to obtain maximum performance from this circuit, a base-drive circuit must be devised which

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[4] Wm. M. Cook and Pier L. Bargellini, "Transistor Bilateral Switches," Semiconductor Products, Sept./Oct. 1958, p. 31.

will keep the base-to-collector voltages at a very small value during the "off" period, and inject the optimum value of current in the base during the "on" period. If the two bases are connected directly to the driving waveform source, the current may divide very unevenly between the two transistors because of the input impedance diode characteristic, and the latter condition would not be satisfied. Hence, it is necessary to include a certain amount of resistance in the base leads. To satisfy the first condition, a diode may be connected across the base-driving source. This, however, is not very satisfactory since the diode forward resistance may be so high that a considerable positive voltage may appear at the chopping transistor bases. If the diode is inserted in series with the driving signal, and its anode is connected to the collectors through some resistance R, then the positive voltage appearing at the bases is the product of R and the diode reverse leakage current. If a low-leakage silicon diode is used, and R is made sufficiently small, the positive voltage at the bases may be made small indeed. A circuit incorporating this method is shown in Fig. 14. The base resistances and R can be chosen so as to provide the optimum base current in the "on" condition.

A one-transistor chopper using the same base-driving method is shown in Fig. 15 (a). This simple circuit may be used effectively in applications where only negative signals or small positive signals are encountered and little variation in V_f is expected. The V_f voltage is balanced by introducing an equal and opposite voltage at the collector

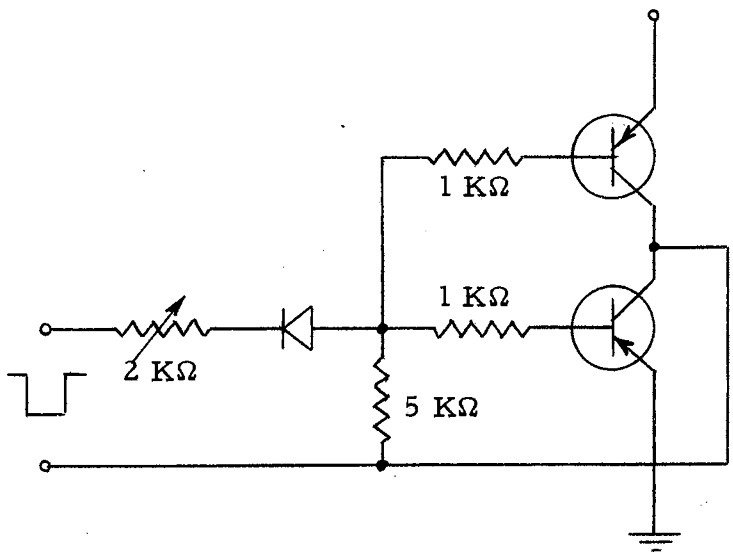

Fig. 14. A Base-Driving Circuit.

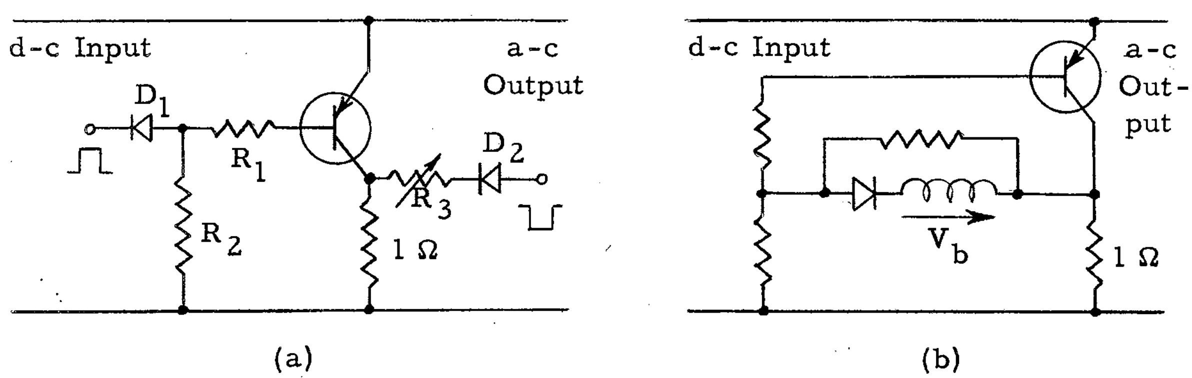

Fig. 15. Choppers Using Only One Transistor.

during the "on" period by means of the diode D_2, R_3, and the 1 ohm resistor. R_3 may be adjusted to balance the voltage offset of the transistor. To provide this compensating signal, a wave out-of-phase with the base driving signal is necessary. If such a waveform is not available, the simpler circuit of Fig. 15 (b) may be used to accomplish the same end. In this circuit, however, a transformer is necessary. The circuit of Fig. 15 (a) has been used in a transistorized d-c amplifier with a voltage drift of less than 100 microvolts, equivalent input, over a temperature range of 20 to 50 ° C.[5]

Balance of the Input Chopper

If two transistors are selected at random for use in a bilateral switch, it is quite likely that their voltage offsets will differ by a considerable amount (see Fig. 6). Hence, it is necessary that some provision be made to balance them. One method by which this could be done is adjustment of the base-drive currents so that V_f is the same in both units. This, however, would not allow the transistors to be operated at their optimum base currents in most cases. A more satisfactory method is shown in Fig. 16. A potentiometer of low resistance (on the order of 1 ohm) is inserted between the collectors. A net voltage of either polarity, equal to the difference in resistances of the two arms times base current, may be introduced. This may then be adjusted to balance the difference in V_f. A similar arrangement using a potentiometer of very high resistance to balance the leakage currents in the "off" condition is also included in this circuit. The greatest disadvantage in this method of balancing the V_f's is that variations in drive

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[5] G. B. B. Chaplin and A. R. Owens, "A Transistor High-Gain Chopper-Type D. C. Amplifier," Proceedings of the Institution of Electrical Engineers, Part B, CV (1958), 258.

current introduce offsets even if the transistor V_f's remain constant. A disadvantage in the leakage current balancing arrangement is that there are now two controls by which the chopper must be balanced. To balance it properly, this must be done under d-c conditions, rather than simple adjustment for zero output of the amplifier with no input signal. Perhaps the simplest and most effective method of balancing would be to simply introduce an extraneous input signal. This could be used to balance the total effect of voltage offsets, leakage currents, and drift due to transients.

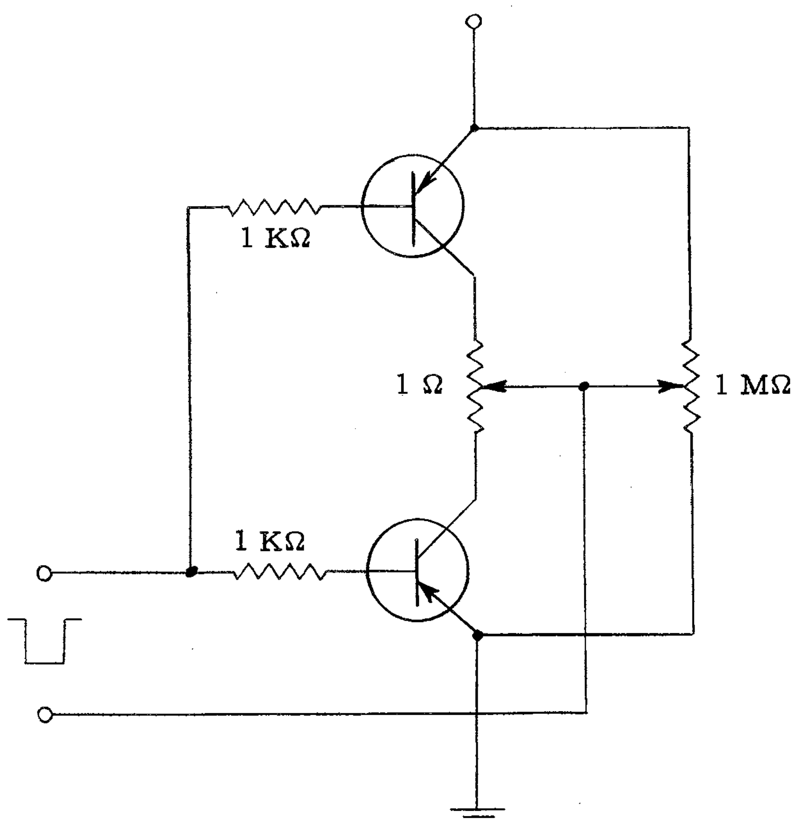

Fig. 16. One Method of Balancing the Voltage Offsets and Leakage Currents.

Connection of the Input Chopper

Probably the best method by which the time-variant impedance of the transistor chopper may be utilized is as illustrated in Fig. 17 (a). If the chopper driving signal is a pure square wave, the current through the capacitor has a waveshape as shown in Fig. 17 (b). When the switch is "open", practically all the input current, $i_s = E_{in}/(R_s + R_{in})$, flows into the amplifier input impedance, R_{in}, through the capacitor which therefore accumulates a charge. When the switch is closed, the input current is shunted to ground through r_s, the chopper conduction impedance, and the capacitor discharges through R_{in} and r_s.

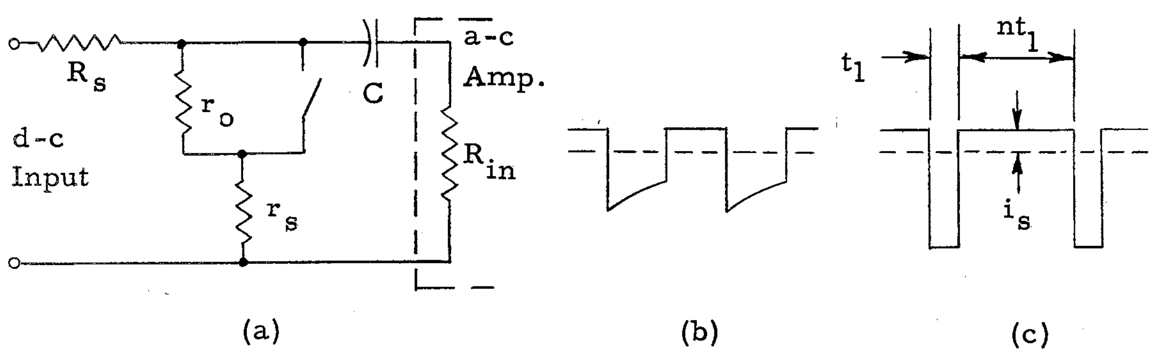

Fig. 17. Use of the Chopper Impedance Ratio in the Input Circuit.

If the base driving signal is an assymetric square wave, with short negative periods, and the time constant $C(R_{in}+r_s)$ is chosen large, the current flowing in R_{in} has a waveshape as shown in Fig. 17 (c). The capacitor discharge current is $(n)(i_s)$, where n is the ratio of "off" time to "on" time in the chopper, since the net current through C must be zero. The input chopper can thus be made to yield a useful gain of $(n + 1)$.[6]

The choice of the chopping frequency is the result of a compromise. A low frequency is desirable to minimize the effect of transients. On the other hand, a higher frequency increases the possible bandwidth of the chopper amplifier, and reduces the effect of low-frequency noise typical in transistors. A frequency of about 2 kc is a reasonable compromise when these effects are considered. A circuit which may be used to supply the required base driving signal is described in Appendix A.

The A-c Amplifier

The a-c amplifier which is to be used in amplifying the chopped waveform must possess a number of important characteristics. The first condition is that it must have high gain with phase reversal. This is necessary so that overall negative feedback may be applied in order to stabilize the gain which may otherwise vary considerably due to variations in the "open" and "closed" impedances of the input chopper, the assymetry of the driving waveform, and the input impedance and gain of the a-c amplifier itself.

The a-c amplifier input impedance must be made low enough so that proper use can be made of the time-variant impedance of the

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[6] G. B. B. Chaplin and A. R. Owens, "A Transistor High-Gain Chopper-Type D.C. Amplifier," p. 259.

chopper. At elevated temperatures the chopper "open" impedance may fall to values as low as 5 kilohms. A reasonable value for the amplifier input impedance is about 1 kilohm.

Another requirement of the a-c amplifier is that its frequency response be high so that it may pass the chopped square wave without excessive distortion. The high frequency harmonic components of an assymetric square wave may be considerable (see Appendix B). A final requirement of the a-c amplifier is that it must recover from overload conditions very rapidly. If the amplifier gain is high, the input chopper transients are sufficient to overload the amplifier, and any delay in recovery from this condition contributes to overall drift of the chopper amplifier. An a-c amplifier which was found to satisfy the requirements given here (to a sufficient extent) is derived in Appendix C.

The Complete Chopper Amplifier

In order to obtain an amplified d-c voltage, proportional to the amplifier input signal, it is necessary to demodulate the output of the a-c amplifier. A chopper circuit similar to the input chopper may be used to accomplish this, but a single transistor chopper is generally sufficient. If the amplifier gain is high, any drift caused by the output chopper is negligible when compared with that of the input chopper, since it must then be divided by the amplifier gain. Hence it is allowable to operate the output chopping transistor in the normal connection so that it mayhandlelarger signals.

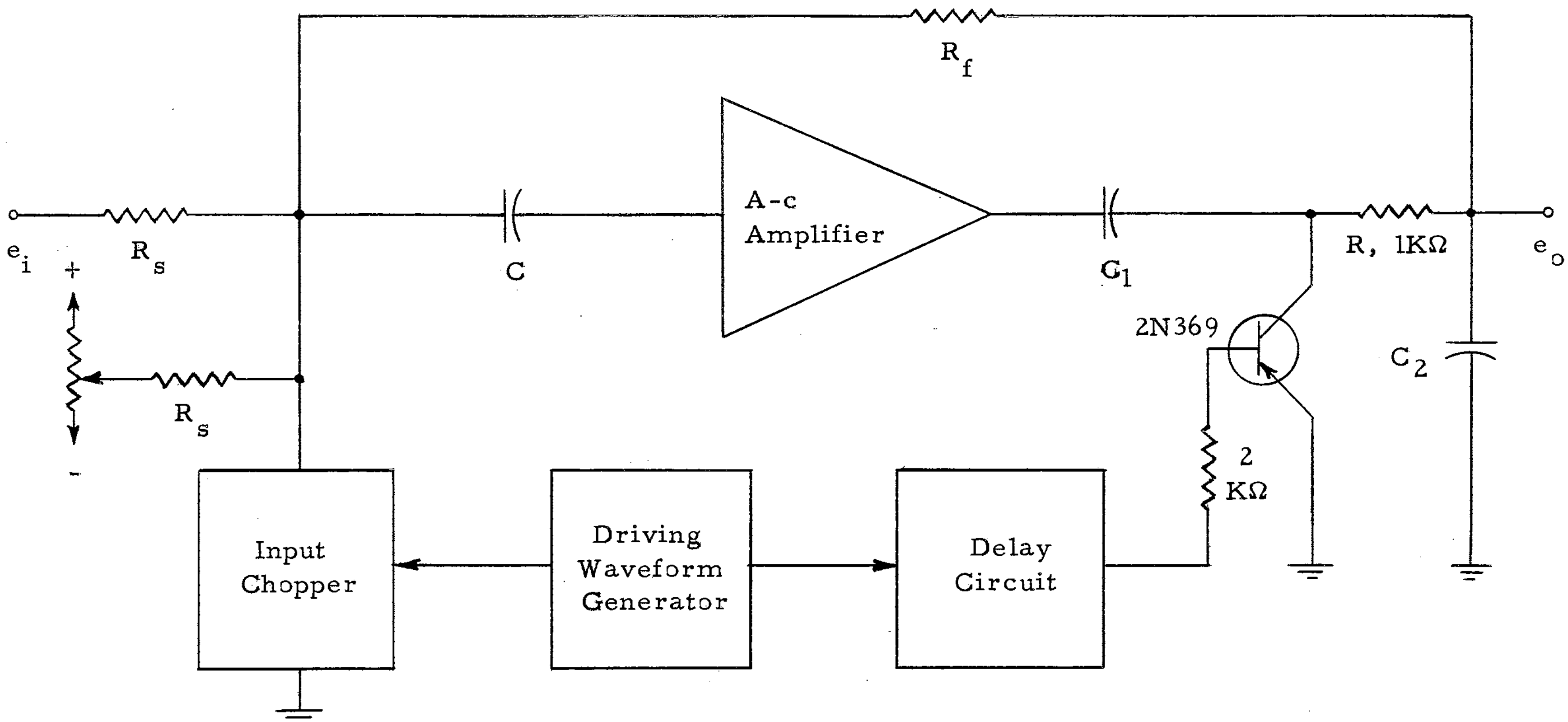

Fig. 18. The Complete Chopper Amplifier.

A complete chopper amplifier in block diagram form is shown in Fig. 18. The demodulated output signal appearing across the output chopper is integrated by the RC network following it to provide a smoothed d-c output. It is desirable that the value of R be small, so that the amplifier output impedance may be made small, but a small value of R gives rise to a large discharge current through the chopping transistor when its impedance is low. In order to limit this current, the minimum value of R must be chosen at 1 kilohm. The time constant RC_2 is instrumental in determining the frequency response of the d-c amplifier. If it is made too small, however, a considerable amount of chopping-frequency ripple occurs at the output. With an R of 1 kilohm, and a C_2 of 10 μf, a bandwidth of approximately 20 cps. is obtained with negligible ripple (assuming a chopping frequency of 2 kc).

To insure that the output chopper will operate properly at all times, it is necessary to apply a base driving signal to it which is greater than any voltage that may appear at the collector of the chopping transistor. It is also desirable to delay the clamping of the output transistor for a short period so that it will not be subjected to the amplified transients caused by the chopped input signal. A short delay period is also seen to be desirable when it is considered that the a-c amplifier may have a finite rise time. A circuit which will effect a short delay in the output chopper driving signal is included in the waveform generator of Appendix A.

The circuit of Fig. 18 includes an overall feedback resistor, R_f, which may be adjusted to give the desired gain, and an extraneous input signal which may be used to zero the amplifier. If the value of R_f is made too small, instability may occurr. In such cases the amplifier may be stabilized by inclusion of low-pass filters as in the block diagram of Fig. 19.

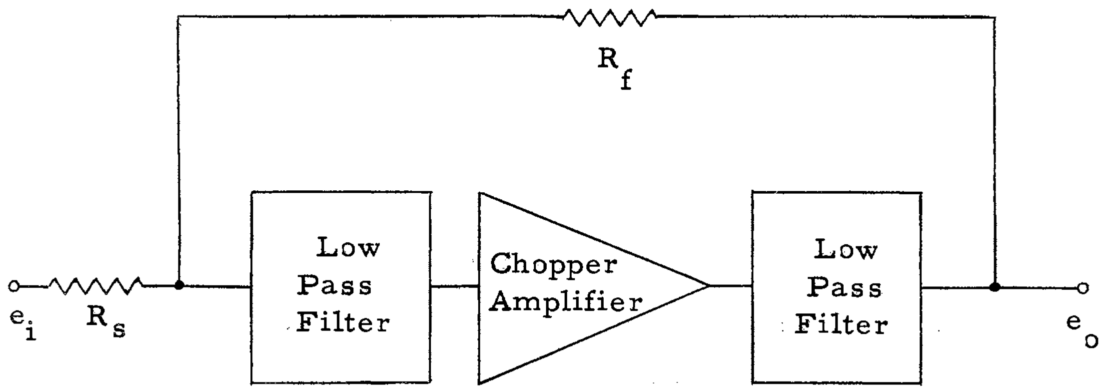

Fig. 19. Use of Low-Pass Filters to Stabilize the Amplifier.

Attainable Drift Specifications

If transistors such as the 2N522 are operated at their optimum base driving currents in the input chopper, a drift due to V_f of only a few microvolts may be expected. If variations in the transients and supply voltages occurr, drift may be expected to increase proportionally. In general, the overall drift may be limited to approximately 25 microvolts, referred to the input, if high gain transistors are used in the input chopper. If large variations in the supply voltages are expected, their effect may be reduced by using zener diodes in the driving waveform generator to regulate the chopper base drive current. Moderate efforts at selection of transistor pairs for use in the input chopper circuit can yield considerable reduction in overall drift. In this case, drift figures as low as 10 microvolts, equivalent input, may be easily achieved.

The drift figures discussed here apply at the input of the chopper amplifier, exclusive of the source impedance, R_s. Hence, they must be multiplied by a constant, $(R_s+R_{in})/R_{in}$ for any particular application. With source impedances such as those of thermocouples and strain gages, the constant is almost 1, and the chopper amplifier may be used effectively. If, however, R_s is high, a series-shunt type input chopper (Fig. 11) must be used so that R_{in} may be made correspondingly larger.

CHAPTER IV

STABILIZATION OF WIDE-BAND AMPLIFIERS

If it is necessary that a d-c amplifier have a wide bandwidth, the chopper amplifier cannot be used, since the chopping frequency must be at least several times greater than the highest frequency to be amplified. In such cases, however, the chopper amplifier can be used to stabilize a directly-coupled main amplifier as illustrated in Figs. 20 and 21.

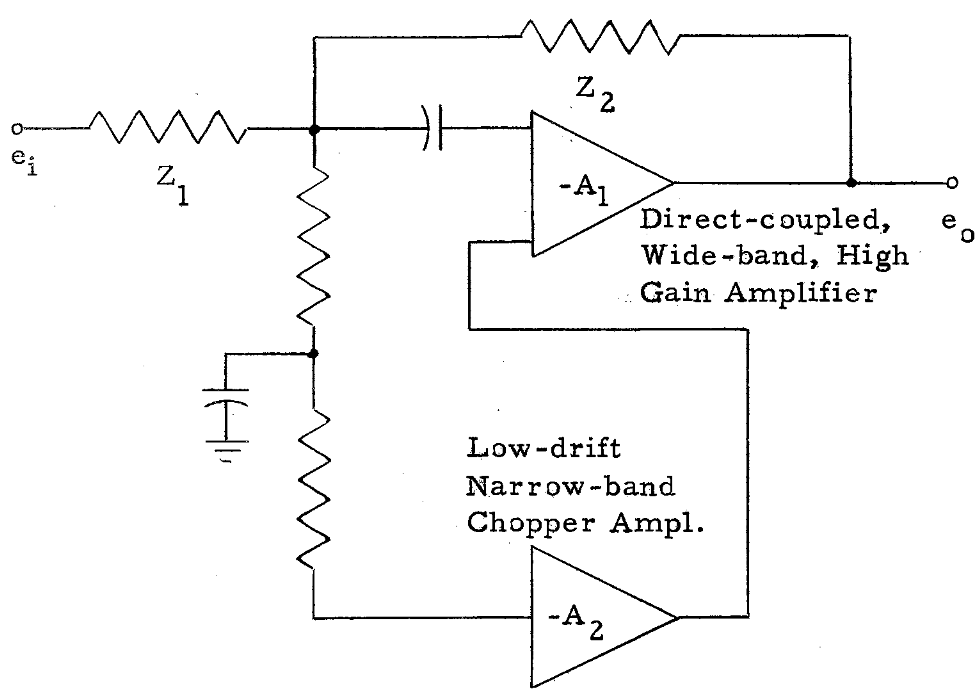

Fig. 20. Chopper-Stabilized Wide-Band Amplifier

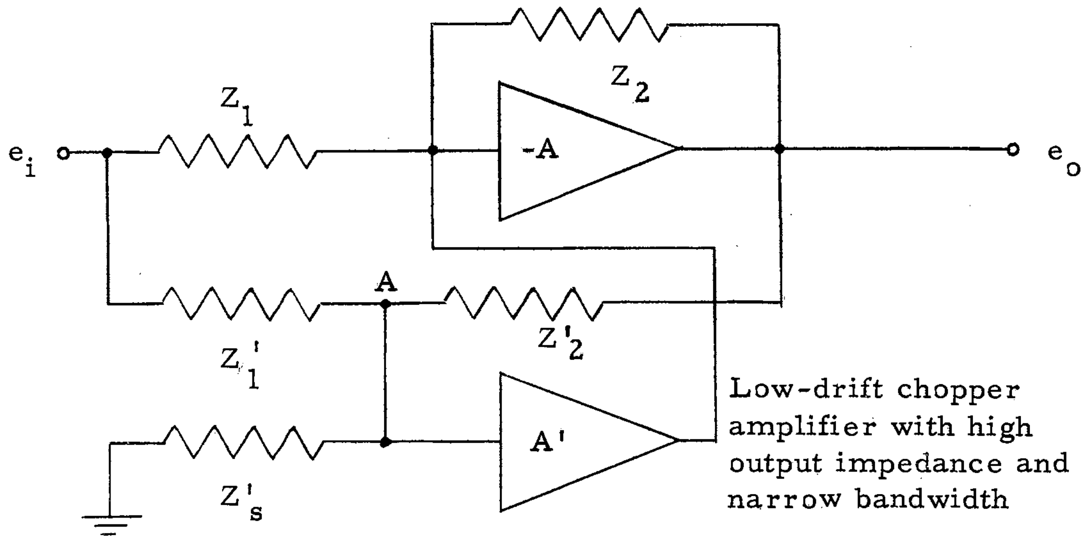

Fig. 21. Automatic Zero-Set (AZS) System.

Stabilizing Systems

In the circuit of Fig. 20 the chopper amplifier amplifies only direct and low-frequency signals, while the directly-coupled main amplifier amplifies the higher frequencies. The frequency response curves for the amplifiers must be matched, so that the entire frequency response may be flat over the desired bandwidth. In Fig. 21, the chopper amplifier does not amplify any part of the signal, but only the d-c drift signal as measured in the summing network at point A. This drift is amplified and summed into the input terminal of the main amplifier in such a phase as to cancel the original drift which caused it. This type of circuit is known as an automatic zero-set system. Its advantage lies in that matching of the frequency responses is not so critical since the stabilizing amplifier does not amplify any part of the signal. Another advantage of this system is that the input impedance of the stabilizing amplifier may be lower, and hence a transistor-chopper amplifier may be employed. It should have a high output impedance and a gain of approximately

$$A' = \frac{Z_2'}{Z_s'} \frac{1}{A} .$$

The Emitter-Coupled Differential Amplifier

A directly-coupled transistor amplifier which may be stabilized with a very simple chopper amplifier is shown in Fig. 22. This is the emitter-coupled differential amplifier. It consists of three differential stages, producing a net phase reversal, and an emitter follower output stage. The 10 ohm potentiometer in the first stage is used to balance the output of the third differential stage, so that the transistors will operate in the linear regions of their characteristics. The potentiometer in the emitter follower stage is adjusted to give zero output with no input signal. Gain can be adjusted by adjustment of R_f, the overall feedback resistance. An open loop voltage gain of over 5,000 was obtained with this circuit. Application of negative feedback to reduce the gain to 100 resulted in extremely linear gain with output levels up to 6 volts (+ or -). The input impedance of the amplifier may be increased at the expense of gain by introducing additional emitter resistances in the first stage. If high frequency response is desired, the transistors can be replaced by high frequency types.

This amplifier may be stabilized in a manner similar to the auto-

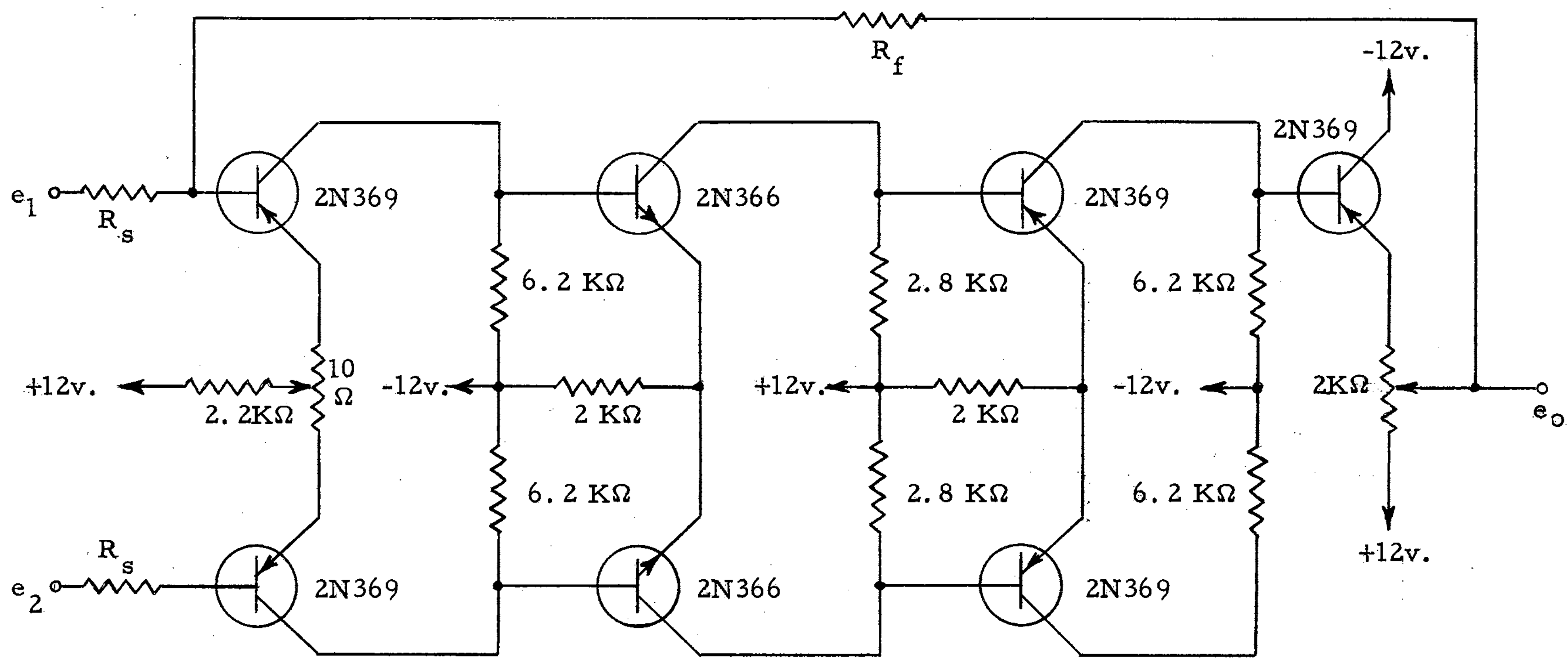

Fig. 22. An Emitter-Coupled Differential Amplifier.

matic zero-set system of Fig. 21. The difference lies in that the output of the chopper amplifier can be used as an input signal at e_2. Since e_0 is proportional to the difference between e_1 and e_2, the chopper amplifier must have no phase reversal in this case. Since the gain of the chopper amplifier is not critical in this system, its overall feedback may be removed, and the number of stages in its a-c amplifier reduced. The d-c gain of such a system was found to be extremely linear, depending only on the values of Z'_1 and Z'_2 (see Fig. 21), and the drift of the chopper amplifier. It does not depend on the linearity of the main amplifier gain due to the "nulling" characteristic of the stabilizing system.

CHAPTER V

SUMMARY AND CONCLUSIONS

Since transistor parameters vary widely with variations in ambient conditions, any transistorized low-level d-c amplifier must be designed on the basis of the chopper-modulation principle to reduce drift. If the design techniques outlined in the previous chapters are employed, it is possible to construct transistorized d-c amplifiers stable in the micro-volt region.

The transistorized chopper amplifier consists of five principal circuits. These are:

1. The input chopper.
2. The a-c amplifier.
3. The output chopper.
4. The chopper driving waveform generator.
5. The delayed output driving circuit.

In the past, mechanical choppers have been used to modulate d-c signals so that they may be amplified and demodulated. Transistor circuits may be used to effect this modulation, and this fact makes the completely transistorized d-c amplifier possible. The most effective of these transistor circuits is the "bilateral switch." In this circuit two transistors are used in such a manner that their imperfections as switches tend to cancel. Such a transistor chopper has many advantages over the mechanical types, some of which are much greater reliability, insensitivity to the environment, and higher possible chopping frequencies.

Of the many transistor types on the market, only a few are suitable for use in low-level transistor chopper circuits. These are the high-frequency alloyed-junction types. If two such transistors are used in the input chopper circuit of a chopper-type d-c amplifier, drift figures for the amplifier on the order of 25 microvolts, equivalent input, can be achieved. This figure can be greatly improved by moderate attempts to match the input chopper transistors with respect to certain parameters. Overall negative feedback may be used to stabilize the gain of the chopper amplifier. Although the chopping frequency is not limited in transistor choppers in quite the same manner as in mechanical types, extremely high chopping frequencies cannot be employed. If chopping is attempted at extremely high frequencies, transients in the chopped signal may introduce considerable drift. Hence, a wide bandwidth cannot be attained with transistor chopper-type amplifiers. When a wide bandwidth is necessary in a transistorized d-c amplifier, a conventional directly-coupled amplifier must be used in conjunction with a drift-stabilizing system including a chopper amplifier. The drift of the main amplifier may thus be reduced to that of the chopper amplifier.

Through correspondence with major manufacturers of transistors, and by judgement of the extent of literature available on the subject, it has become apparent that little research has been carried out in the field of transistor choppers. This is very surprising, since d-c amplifiers are basic components in aircraft and missile telemetering systems. Any future research in this field should include study of the basic solid-state processes in transistors when operated as choppers, with a view to developing methods whereby transistors with superior characteristics as choppers may be manufactured. It has been stated that attempts are being made to reduce the voltage offset (see Chapter II) in transistors to a few microvolts, but this statement seems to be unsubstantiated.[7]

If n-p-n transistors with good chopper characteristics become available, it may become possible to use them in conjunction with p-n-p types to eliminate the transformer necessary in the input chopper when only p-n-p types are used. The fact that transistor manufacturers supply no data on the performance of their units in chopper circuits has been a great obstacle in the design of chopper-stabilized transistor circuits. Statistical analysis of certain parameters in random batches of transistors of various types would supply much of the needed information. Such analysis may lead to discoveries whereby transistor chopper circuits would be greatly improved. In addition, statistical data of this nature would yield information useful in determining the feasibility of mass producing precise transistor-chopper type d-c amplifiers.

Exclusive of large-scale programs such as develpment of new tran-

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[7] Martin L. Klein, "Techniques for Stabilizing D.C. Transistor Amplifiers," 1958 I.R.E. Wescon Convention Record, Part 2, 97.

sistors or other solid-state devices, and statistical analysis of sample transistor batches, further investigations in the following phases of the transistor-chopper field may yield useful results:

1. Causes of the transients in the transistor chopper.
2. Selection of optimum transformers for use in the input chopper.
3. Noise in the chopper amplifier.
4. Reduction in the number of components required in the five principal circuits of the chopper amplifier.

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APPENDIX A

THE DRIVING WAVEFORM GENERATOR

A circuit which supplies the necessary driving signals to the input and output choppers is shown in Fig 23. A simple astable multivibrator generates the signal with proper frequency (2kc) and time period to pulse ratio (10). Waveforms at points throughout the circuit for no-load conditions are shown in Fig. 24. The amplitudes are decreased somewhat with loading, but circuit operation remains essentially the same.

T_3 serves to isolate the load from the multivibrator and to improve the rise and fall times of the square wave. With the circuit values shown, rise and fall times on the order of 1 microsecond were obtained. The n-p-n emitter follower stage of T_4 is necessary to supply the waveform required by the delay circuit of T_5. The resistance network at the base of T_4 is included so that the maximum voltage ratings of T_4 are not exceeded.

Circuit action of the delay circuit (T_5) may be explained as follows: If the voltage at the emitter of T_4 is low, D_1 is forward biased, and T_5 is in the saturated state. Points A and B are then at approximately the same levels (+12 v.). When the emitter of T_4 switches to a high positive value, D_1 is cut-off. The capacitor then begins to charge, tending

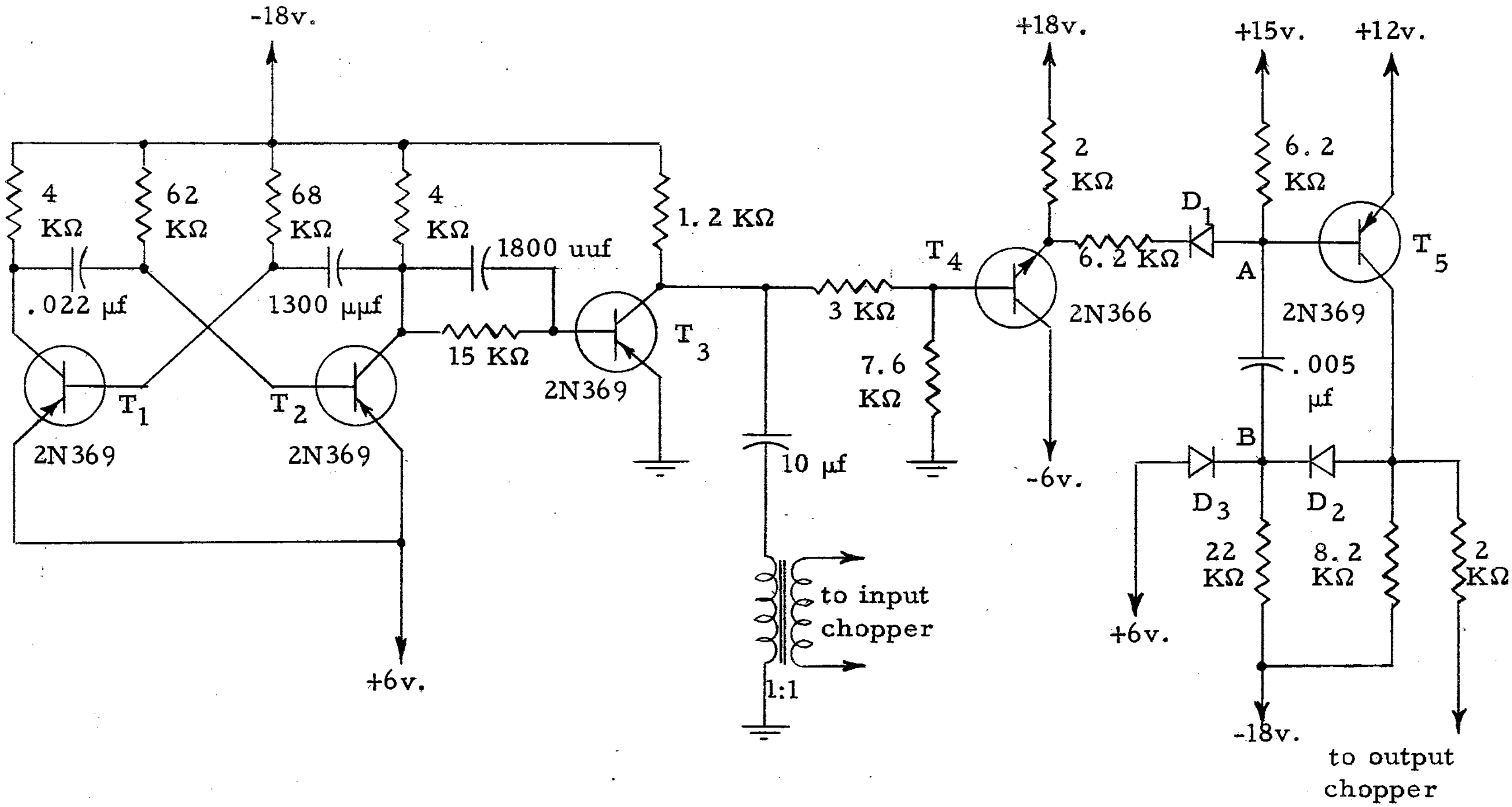

Fig. 23. The Driving-Waveform Generator.

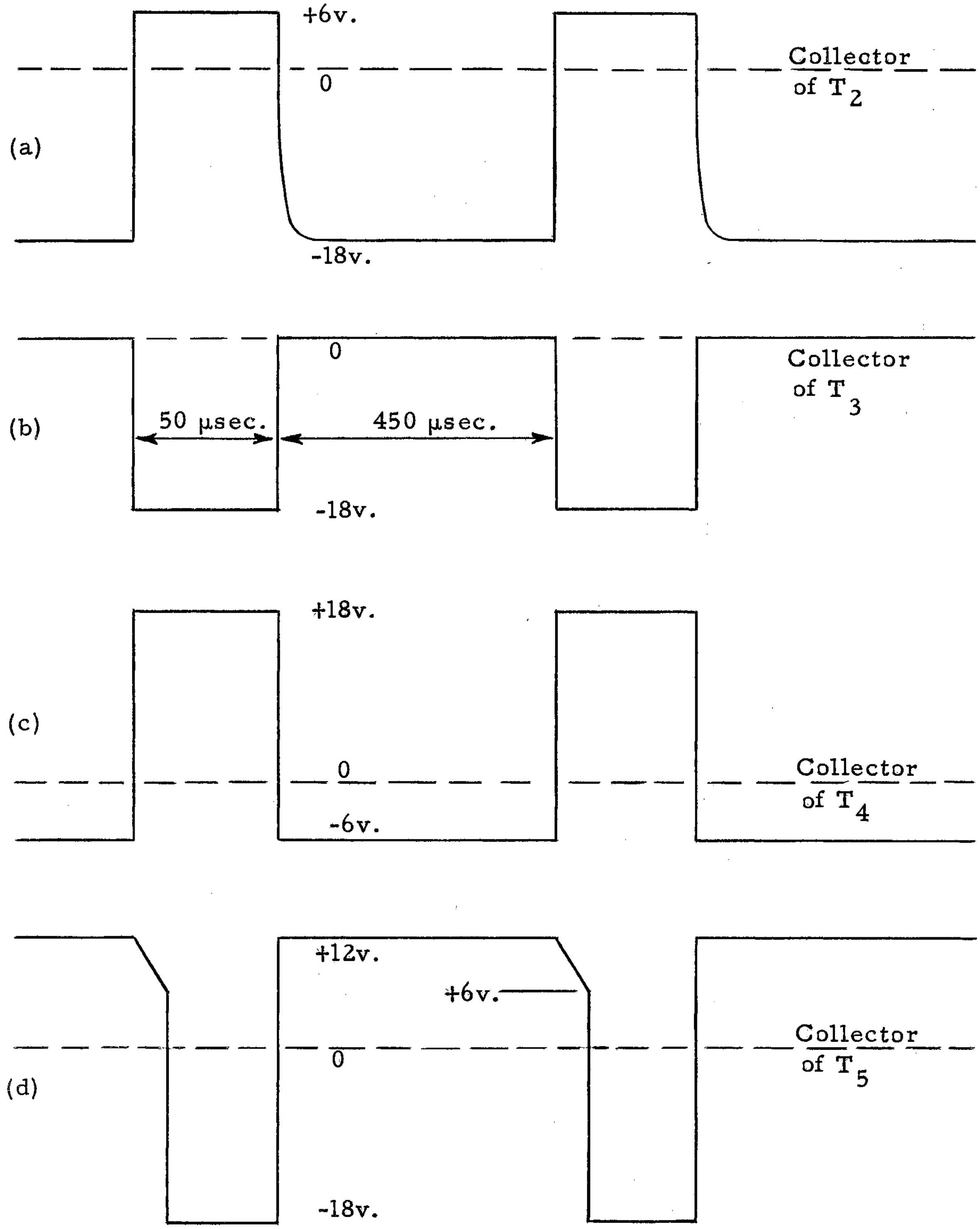

Fig. 24. Waveforms in the Circuit of Fig. 23.

to raise the voltage level at point A. As this voltage is raised, T_5 begins to cut off, thus lowering the voltage level at its collector. This lower level is reflected at point B, and hence lowers the level of point A, preventing cut-off of T_5. With this process, the level at point B lowers as the capacitor charges, until the level at the anode of D_3 is reached. The level of point B can then no longer fall, and further charging of the capacitor cuts T_5 off rapidly. The delay time can be controlled most easily by the voltage at the anode of D_3. When the level at the emitter of T_4 switches back to a low value, T_5 goes into saturation immediately, and no delay occurs.

APPENDIX B

HARMONIC ANALYSIS OF THE CHOPPED WAVEFORM

Fourier analysis may be used to determine the magnitudes of the harmonic components of the chopped waveform. The waveshape is shown as a mathematical function, f(x), in Fig. 25.

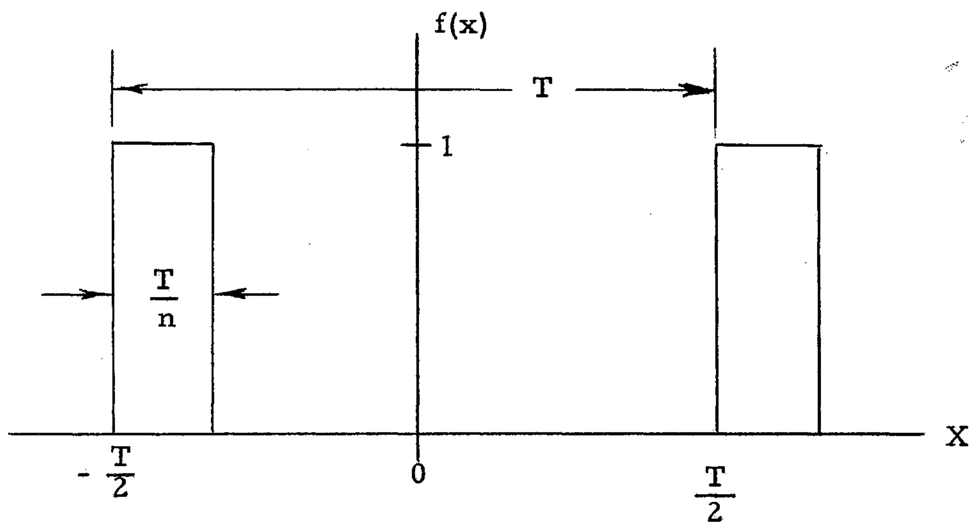

Fig. 25. The Chopped Waveform

f(x) may then be written as

$$f(x) = \frac{a_o}{2} + \sum_{i=1}^{\infty} \left(a_i \cos \frac{2i\pi x}{T} + b_i \sin \frac{2i\pi x}{T}\right) \qquad -\frac{T}{2} < x < \frac{T}{2}$$

where

$$a_i = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\cos\frac{2i\pi x}{T}\,dx$$

and

$$b_i = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\sin\frac{2i\pi x}{T}\,dx$$

If the substitution $y = x + \frac{T}{2}$ is made, a_i and b_i become

$$a_i = \frac{2}{T}\int_{0}^{\frac{T}{n}} \cos\frac{2i\pi(y - \frac{T}{2})}{T}\,dy$$

$$= \frac{1}{i\pi}\sin\frac{2i\pi(y - \frac{T}{2})}{T}\Bigg|_{0}^{\frac{T}{n}} = -\frac{1}{i\pi}\sin\left(i\pi\frac{n-2}{n}\right)$$

$$b_i = \frac{2}{T}\int_{0}^{\frac{T}{n}} \sin\frac{2i\pi(y - \frac{T}{2})}{T}\,dy$$

$$= -\frac{1}{i\pi}\cos\frac{2i\pi(y - \frac{T}{2})}{T}\Bigg|_{0}^{\frac{T}{n}} = \frac{(-1)^i}{i\pi} - \frac{1}{i\pi}\cos\left(i\pi\frac{n-2}{n}\right)$$

The harmonic components, h_i, may be found by:

$$h_i = \sqrt{a_i^2 + b_i^2}$$

$$= \frac{1}{i\pi}\sqrt{\sin^2\left(i\pi\frac{n-2}{2}\right) + \cos^2\left(i\pi\frac{n-2}{n}\right) - 2(-1)^i\cos\left(i\pi\frac{n-2}{n}\right) + 1}$$

$$= \frac{\sqrt{2}}{i\pi}\sqrt{1 - (-1)^i\cos\left(i\pi\frac{n-2}{n}\right)}$$

The harmonic components for any value of n may be calculated from this equation. Harmonic components for a wave with n = 8 are listed in Table II to illustrate the high harmonic content of this type of wave.

TABLE II

Harmonic Components of the Chopped Waveform

Harm. No.	% of Wave Height	Harm. No.	% of Wave Height
1	24.4	13	4.5
2	22.5	14	3.2
3	19.6	15	1.6
4	15.9	16	0
5	10.8	17	1.6
6	7.5	18	2.5
7	3.2	19	3.1
8	0	20	3.2
9	2.7	21	2.8
10	4.5	22	2.1
11	5.4	23	1.0
12	5.3	24	0

APPENDIX C

DESIGN OF THE A-C AMPLIFIER

An a-c amplifier consisting of three common emitter transistor stages is sufficient to provide the high gain and phase reversal necessary for use in the chopper amplifier. Use of high frequency transistors is necessary so that the amplifier may pass the chopped square wave with little distortion.

An amplifier of similar design, using low-frequency type 2N369 transistors, proved to be useless in the chopper amplifier. Its recovery time from overload conditions was so long, and distortion so severe, that it contributed excessively to overall drift of the chopper amplifier.

An emitter follower output stage is necessary so that the a-c amplifier may have a low output impedance. Because of their high gain and good frequency response, 2N522 transistors are used in the first two stages. S-500 transistors are used in the final amplification stage and the emitter follower because of their higher voltage ratings. The bias circuits are of the one-battery type, as illustrated in Fig. 26.

The following values are chosen so that the 2N522 stages are operated in their most linear regions with good stability and allowable power dissipation:

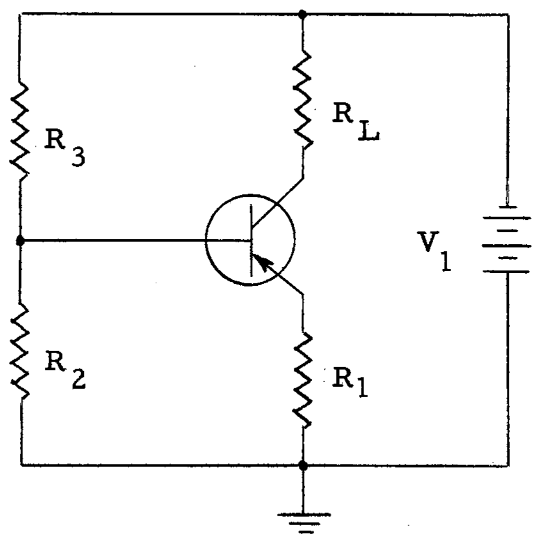

Fig. 26. Bias Circuit for One Stage.

V_1 = 6 volts

R_L = 750 Ω

I_E = 2.4 ma.

S_I = 4 (current stability)

The necessary values of R_2 and R_3 may now be calculated:[9]

$$R_3 = \frac{S_I V_1}{I_E} = \frac{4 \times 6}{2.4 \times 10^{-3}} = 10\ K\Omega$$

$$R_2 = \frac{1}{\frac{1}{S_I V_1} - \frac{1}{R_3}} = \frac{1}{\frac{1}{4 \times 500} - \frac{1}{10^4}} = 2.5\ K\Omega$$

The same procedure may be applied to the S-500 stages:

V_1 = 24 volts

[9] R. F. Shea, ed., Transistor Circuit Engineering, (New York, 1957), p. 68.

$$R_L = 1.2\ K\Omega$$

$$R_1 = 1\ K\Omega$$

$$I_E = 4\ ma.$$

$$S_I = 4$$

$$R_3 = 24\ K\Omega$$

$$R_2 = 5\ K\Omega$$

The complete a-c amplifier is shown in Fig. 27 with a mid-frequency equivalent circuit for its first three stages. The emitter follower stage will be neglected in the a-c analysis since its effect on gain is negligible. The common emitter parameters for the transistors used are:

$$\text{2N522:} \quad H_e = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} = \begin{bmatrix} 3600 & 11.34 \times 10^{-4} \\ 120 & .84 \times 10^{-4} \end{bmatrix}$$

$$|H_e| = \Delta^h_e = .169$$

$$\text{S-500:} \quad H_e = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} = \begin{bmatrix} 315 & .29 \times 10^{-4} \\ 9 & .126 \times 10^{-4} \end{bmatrix}$$

$$|H_e| = \Delta^h_e = .206 \times 10^{-2}$$

The current gain, input impedance, and transfer impedance may now be calculated:

$$K_{i3} = \frac{I_7}{I_6} = \frac{h_{fe3}}{h_{oe3}R_{L3} + 1} = \frac{9}{.0126 \times 1.2 + 1} = 8.96$$

$$R_{i3} = \frac{\Delta^h_{e3}R_{L3} + h_{ie3}}{h_{oe3}R_{L3} + 1} = \frac{2.06 \times 1.2 + 315}{1.015} = 313\Omega$$

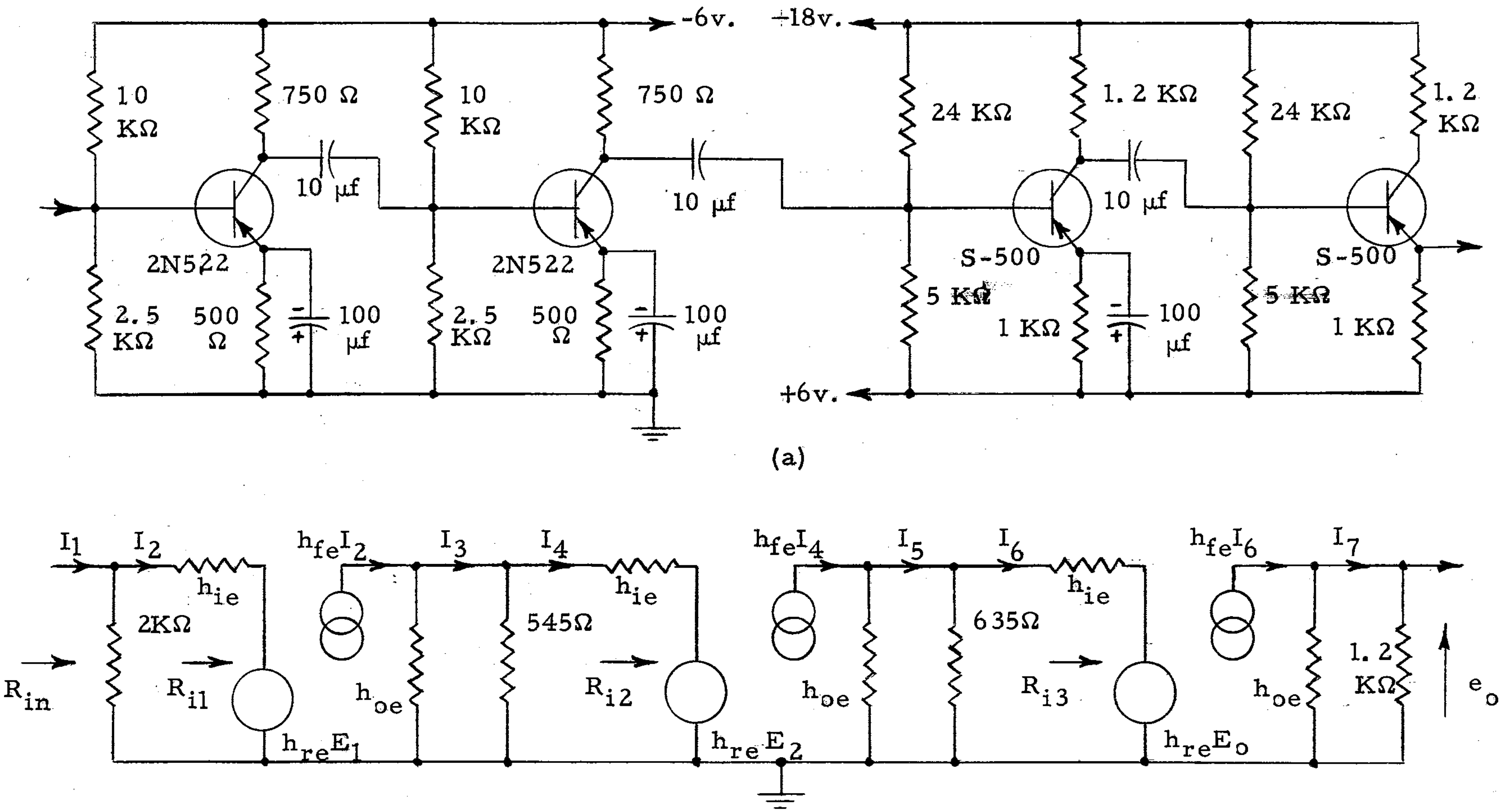

Fig. 27. Circuit of the A-c Amplifier and a Mid-Frequency Equivalent Circuit for its First Three Stages.

$$\frac{I_6}{I_5} = \frac{635}{635 + R_{i3}} = \frac{635}{635 + 313} = .670$$

$$R_{L2} = \frac{635 \times R_{i3}}{635 + R_{i3}} = 210\ \Omega$$

$$K_{i2} = \frac{I_5}{I_4} = \frac{h_{fe2}}{h_{oe2}R_{L2} + 1} = \frac{120}{.084 \times .21 + 1} = 118$$

$$R_{i2} = \frac{\Delta^{h}{}_{e2}R_{L2} + h_{ie2}}{h_{oe2}R_{L2} + 1} = \frac{1.41 \times .21 + 3600}{1.018} = 3540\ \Omega$$

$$\frac{I_4}{I_3} = \frac{545}{545 + R_{i2}} = .1333$$

$$R_{L1} = \frac{545 \times R_{i2}}{545 + R_{i2}} = 472\ \Omega$$

$$K_{i1} = \frac{I_3}{I_2} = \frac{h_{fe1}}{h_{oe1}R_{L1} + 1} = \frac{120}{.084 \times .472 + 1} = 115$$

$$R_{i1} = \frac{\Delta^{h}{}_{e1}\, R_{L1} + h_{ie1}}{h_{oe1}R_{L1} + 1} = \frac{1.41 \times .472 + 3600}{1.040} = 3530\ \Omega$$

$$\frac{I_2}{I_1} = \frac{2000}{2000 + 3530} = .371$$

$$R_{in} = \frac{2000 \times R_{i1}}{2000 + R_{i1}} = 1280\ \Omega$$

Total Current Gain:

$$K_{it} = \frac{I_7}{I_6} \cdot \frac{I_6}{I_5} \cdot \ldots \cdot \frac{I_2}{I_1}$$

$$K_{it} = 8.96 \times .67 \times 118 \times .1333 \times 115 \times .371 = 4030$$

Transfer Impedance:

$$Z_x = K_{it} \times R_{L3} = 4030 \times 1200 = 4.85\ M\Omega$$

$$= 4.85\ \text{volts}/\ \mu\ \text{amp.}$$

A slightly higher gain and transfer impedance were obtained in the actual circuit due to higher than average β in the first two stages.

VITA

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Candidate for the Degree of

Master of Science

Thesis: TRANSISTOR-CHOPPER STABILIZATION OF D.C. AMPLIFIERS

Major Field: Electrical Engineering

Biographical:

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