

ESSAYS: BIOFUEL FEEDSTOCK  
PRODUCTION ECONOMICS AND  
IDENTIFYING JUMPS AND SYSTEMATIC  
RISK IN FUTURES

By

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Submitted to the Faculty of the  
Graduate College of the  
Oklahoma State University  
in partial fulfillment of  
the requirements for  
the Degree of  
DOCTOR OF PHILOSOPHY  
July, 2010

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## **ACKNOWLEDGEMENTS**

I thank almighty for giving me the opportunity and strength for successfully completing a doctoral degree in Agricultural Economics.

I wish to thank my major advisor, Dr. Francis M. Epplin, for his valuable support and encouragement throughout my research work. Special appreciation goes to other committee members Dr. Wade Brorsen, Dr. Eric Devuyst, and Dr. Yanqi Wu. I am grateful to my friends Samarth Shah, David Roberts, and Mohua Haque for the help and suggestions. I would like to extend my sincere thanks to Mrs. Gracie Teague for helping me to format my thesis.

I am also grateful to the faculty, staff especially Anna Whitney, Joyce Grizzle, and Ginny Cornelson, and fellow graduate students of the department of Agricultural Economics for their help and support.

Finally, I would like to thank my mother, Thulasi Aravind, and my brother Ajiraj Aravind, for their love support and advice throughout my college education.

### **Acknowledgements**

This project was supported by the USDA Cooperative State Research, Education and Extension Service, Hatch grant number H-2574, by USDA-CSREES Special Research Grant awards 2006-34447-16939 and 2008-34417-19201, and by the Oklahoma Bioenergy Center

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## PAPER I

# SWITCHGRASS, BERMUDAGRASS, FLACCIDGRASS, AND LOVEGRASS BIOMASS YIELD RESPONSE TO NITROGEN FOR SINGLE AND DOUBLE HARVEST

### Abstract

Switchgrass (*Panicum virgatum*) has been identified as a model dedicated energy crop species. After a perennial grass such as switchgrass is established, the major variable costs are for nitrogen (N) fertilizer and harvest. The objective of this research is to determine biomass yield response to N for four perennial grass species and to determine the species, N level, and harvest frequency that will maximize expected net returns, given the climate and soils of the U.S.A. Southern Plains. Yield data were produced in an experiment that includes four species (switchgrass, bermudagrass (*Cynodon dactylon*), weeping lovegrass (*Eragrostis curvula*), and carostan flaccidgrass (*Pennisetum flaccidum*)), four N levels, and two harvest levels. Linear response plateau (LRP), linear response stochastic plateau (LRSP), and quadratic response (QR) functions are estimated. For all combinations of biomass and N prices considered, the optimal species that maximizes net return is switchgrass. For most price situations, it is economically optimal to fertilize established stands of switchgrass with  $69 \text{ kg N ha}^{-1} \text{ yr}^{-1}$  and to harvest once yr<sup>-1</sup> after senescence.

## Introduction

Research and development is ongoing in an attempt to determine economically competitive methods to produce ethanol from cellulose. Examples of technologies under evaluation include enzymatic hydrolysis, acid hydrolysis, gasification, gasification-fermentation, liquefaction, and mixalco (Klasson et al. 1990; Wyman 1994; McKendry 2002; Aden et al. 2002; Rajagopalan, Datar, and Lewis 2002; Caputo et al. 2005; Mosier et al. 2005; Boateng, Anderson, and Phillips 2007; Service 2007). If an economically competitive business model is forthcoming based on any of these technologies, it will presumably require massive quantities of cellulosic biomass. Perlack et al. (2005) proposed that 22 million U.S.A. ha of cropland, idle cropland, and cropland pasture could be converted from current uses to the production of perennial grasses from which cellulosic feedstock could be harvested.

It is assumed that the biomass produced by any perennial grass could be used as feedstock. Research sponsored by the Bioenergy Feedstock Development Program at the Oak Ridge National Laboratory evaluated more than 30 species in research plots on a wide range of soil types at more than 30 sites across seven states (Wright 2007). Based on these trials, switchgrass (*Panicum virgatum*) has been selected as a model species for several reasons. It is an indigenous, noninvasive, widely adapted endemic species of the tall grass prairies with high water use efficiency, a large and deep root system, and a capacity for high yields on relatively poor quality sites (Wright 2007). Switchgrass also has a significant capacity to improve soil quality by sequestering carbon below ground (Lewandowski et al. 2003; Wright 2007).

While switchgrass has been identified as a model or prototype biomass species, researchers with the feedstock development program have concluded that regional and local considerations may well favor use of an herbaceous energy crop other than switchgrass (Wright 2007). Researchers in Oklahoma evaluated 14 perennial grass species and found that for the agro-climatic conditions of the state, switchgrass, bermudagrass (*Cynodon dactylon*), weeping lovegrass (*Eragrostis curvula*), and carostan flaccidgrass (*Pennisetum flaccidum*) produced more biomass than the alternative species (Rogers 2006). Prior to investing in establishing pure stands of a single species of a perennial grass on millions of hectares for intended use as a biorefinery feedstock, it would be prudent to determine the most profitable species.

Six major cost components exist in producing and delivering biomass perennial grass feedstock to a biorefinery: land rental, establishment, fertilizer, harvest, storage, and transportation. Land rental in terms of \$ ha<sup>-1</sup> could be expected to be the same across species. Three of the other cost categories (harvest, storage, and transportation) should be very similar across perennial grass species. However, establishment and fertilizer costs likely differ across species. After land rental, N fertilizer is expected to be the most costly pre-harvest input. The cost and environmental externalities associated with N use suggest that identifying biomass yield response to N for candidate perennial grass species is an essential prerequisite to determining the most cost-efficient biomass feedstock production species for an agro-climatic region (Silveria, Haby, and Leonard 2007).

For established perennial grasses, N application and harvesting are the two primary production activities. The objective of the research reported in this paper is to determine biomass yield to N response functions for four perennial grass species and to

determine the species, N level, and harvest frequency that will maximize expected net returns to a land unit, given the climate and soils of the U.S.A. Southern Plains. The species to be considered include switchgrass, bermudagrass, weeping lovegrass, and carostan flaccidgrass. These four species were selected based on their performance in yield screening trials conducted in Oklahoma (Rogers 2006).

Switchgrass is a native perennial, sod-forming grass that is adapted to all parts of the United States except California and the Pacific Northwest (USDA/NRCS 2008).

Bermudagrass is a long-lived warm season perennial that spreads by rhizome, stolon, and seed. Flaccidgrass is an upright, tall, weak bunch type perennial rhizomatous subtropical, warm-season forage grass (Belesky et al.1998; Burns et al. 1998). Weeping lovegrass is a warm-season bunchgrass characterized by quick germination, an active growth period in the summer, high drought tolerance, production of thick mass of vegetative soil cover, and a deep penetrating root system (USDA/NRCS 2008).

Studies have been conducted at several locations to determine biomass yield response to harvest frequency and harvest timing (Lee and Boe 2005; Sanderson et al. 2006; Lee, Owens, and Doolittle 2007). Regrowth characteristics of perennial grass species after harvest vary with species and soil moisture (USDA/NRCS 2008). Reynolds Walker, and Kirchner (2000) find that more N is removed under a two-cut per year system compared to a one-cut system. Also, an additional harvest is costly.

The research reported in this paper differs from previous studies in various aspects. To our knowledge, this is the first attempt to estimate biomass yield to N response functions for these four grass species from data obtained in side-by-side field trials in the Southern Plains. The agronomic experiment includes side-by-side

comparisons of four perennial grass species with four levels of N and two harvest treatments (once and twice per year). Data produced in the field trials are used to fit three functional forms including the recently introduced linear response stochastic plateau (LRSP) (Tembo et al. 2008). Statistical tests are conducted to determine the functional form that best fits the data for each species for both the single and double harvest per year systems. These response functions are used to determine the most profitable species, N level, and harvest frequency for several sets of N and biomass prices.

### Model

The farm operator is assumed to maximize expected net return  $\text{ha}^{-1}$ . The farm operator's objective can be represented as

$$(1) \quad \max_{S,N,H} E(NR) = \max\{E(P_y Y|S, N, H) - P_N N - P_{NA} H - P_{MR} H - P_B Y - AEC_S - LR - OC|S, N, H\}$$

where  $E(NR)$  is the expected net return ( $\$ \text{ha}^{-1} \text{yr}^{-1}$ ),  $P_y$  is the price of biomass ( $\$ \text{Mg}^{-1}$ ),  $Y$  is the biomass yield ( $\text{Mg ha}^{-1} \text{yr}^{-1}$ ),  $N$  is the nitrogen level applied per year to established stands ( $\text{kg ha}^{-1} \text{yr}^{-1}$ ),  $S \in \{1, 2, \dots, 4\}$ ,  $1, 2, \dots, 4$  represents the four grass species (switchgrass, bermudagrass, flaccidgrass, and lovegrass),  $H \in \{1, 2\}$ ,  $1, 2$  is the harvest frequency (once or twice per year),  $P_N$  is the price of N ( $\$ \text{kg}^{-1}$ ),  $P_{NA}$  is the cost of N application (when harvested twice, N is applied in two split doses) ( $\$ \text{ha}^{-1}$ ),  $P_{MR}$  is the cost for mowing and raking ( $\$ \text{ha}^{-1}$ ),  $P_B$  is the cost of baling ( $\$ \text{Mg}^{-1}$ ),  $AEC_S$  is the amortized establishment cost ( $\$ \text{ha}^{-1} \text{yr}^{-1}$ ),  $LR$  is the land rental ( $\$ \text{ha}^{-1} \text{yr}^{-1}$ ), and  $OC$  is the cost of operating capital ( $\$ \text{ha}^{-1} \text{yr}^{-1}$ ). The paper followed a discrete optimization procedure in which the species and harvest levels are considered as discrete choice variables and the nitrogen level as continuous choice variable. The nitrogen response

function is estimated for each combination of species and harvest level and the optimum level of nitrogen is estimated taking the first order condition. The expected net return is estimated by substituting the profit maximizing level of yield in the objective function .

To determine an estimate for cost components in equation (1), a standard enterprise budgeting procedure was used to estimate production costs for each of the four species. Budgets were prepared for each species to estimate establishment costs in the establishment (first) year. A second set of budgets was prepared to estimate maintenance and harvesting costs for established stands. The establishment budgets include the cost of field preparation, planting, weed control, fertilizer application, land rental, and operating capital. The budgeted costs of field operations were based on state average custom rates (Doye, Sahs, and Kletke 2005). The plots were prepared with conventional tillage with a moldboard plow and offset disk. Planting materials and planting constitute a major share of establishment costs that vary across species. Establishment costs are greater for flaccidgrass and bermudagrass since they require vegetative propagation. Establishment costs are lower for switchgrass and lovegrass since they can be seeded. The estimated stand life of each of the species was assumed to be ten years. The establishment costs were amortized at a rate of seven percent over a period of ten years.

The maintenance budgets include the amortized cost of stand establishment, and the cost of N, N application, harvesting (mowing, raking, and baling), operating capital, and land rental. Costs of production vary with the level and number of N applications, harvest frequency, and yield. The budgets do not include costs for fertilizer other than N because prior research has found that through the natural growth cycle of perennial grasses, near the end of the growing season, nutrients including phosphorus and

potassium translocate from the above ground parts of the plant to the below ground parts of the plant. Research has confirmed that if harvest of a perennial grass is delayed until after senescence, removal of above ground parts of the plant will not mine phosphorus and potassium from the soil (Stout 1988; Muir, et al.2001; Fuentes and Taliaferro 2002; Thomason et al. 2004; Jung et al.2005; Parrish and Fike 2005; Fike et al. 2006)

### **Field Experiment**

The field experiment was conducted on a site near Stillwater, Oklahoma on Kirkland silt loam soil. The experiment followed a randomized complete block design with a split-plot arrangement of treatment and four replications. Soil testing was conducted in April of 2002 to ensure adequate pH, phosphorous, and potassium. Tillage was used to prepare a clean seedbed, 34 kg N ha<sup>-1</sup> was applied across all plots, and the four species were planted on July 22-23. Seeds of switchgrass and lovegrass were drilled into the prepared, conventionally-tilled seedbed using a Brillion seeder. Bermudagrass sprigs and flaccidgrass sprigs were transplanted. The herbicide 2,4-D was applied at 1.68 kg ha<sup>-1</sup> across all plots to control broadleaf weeds. None of the plots were harvested in 2002. Since the grasses allocate substantial energy to root establishment during the initial growth year, agronomists recommend that they not be harvested during the establishment year in the region (McLaughlin et al. 1999; Lewandowski et al. 2003). Based on findings reported by Fuentes and Taliaferro (2002), when not harvested during the establishment year, it is assumed that in the region of the study, each of the four species achieves full yield potential in the second year. No herbicide or fertilizer other than N was applied in the second and subsequent years. Nitrogen, in the form of urea (46-0-0), was applied at levels of 34, 67, 134, and 269 kg ha<sup>-1</sup> yr<sup>-1</sup> in years after the establishment year. For the

two harvests per year sub-subplots, half of the total N was applied at the beginning of the season and half after the first harvest. The two harvest sub-subplots were harvested in July and again after senescence in October. The single harvest sub-subplots were harvested only in October. Harvesting was performed in 2003, 2004, and 2005. The experiment produced 384 yield observations over the three-year period (four species by two harvest treatments by four N levels by four replications by three years). Summary statistics of the annual biomass yield are reported in Table I-1.

**Table I-1 Summary statistics of annual yields of biomass obtained in field trials for switchgrass, bermudagrass, lovegrass, and flaccidgrass over three years (2003-2005)**

Grass Species	Nitrogen (Mg ha <sup>-1</sup> )	Single Harvest <sup>a</sup>				Double Harvest			
		Mean	SD	Min	Max	Mean	SD	Min	Max
Switch	34	8.65 <sup>b</sup>	1.57	5.60	11.40	8.31	1.77	6.07	10.60
	67	12.01	1.88	8.38	14.49	9.16	1.28	7.35	11.40
	134	12.12	1.81	8.60	14.45	11.94	2.15	8.98	16.82
	269	12.34	1.68	10.04	15.50	13.82	2.04	10.64	17.16
Bermuda	34	4.95	1.32	2.51	6.54	7.32	1.64	4.97	9.54
	67	6.68	0.87	4.95	7.75	9.07	2.11	6.14	12.21
	134	8.09	1.30	5.80	9.95	11.96	2.26	6.74	14.47
	269	10.51	2.46	6.63	13.57	14.54	2.40	11.76	18.14
Flaccid	34	8.40	1.28	6.94	11.13	8.51	1.64	5.82	11.13
	67	9.81	2.28	6.45	14.34	9.09	1.59	6.99	11.49
	134	9.07	1.43	7.21	11.92	12.77	1.50	9.81	15.37
	269	9.72	1.61	7.15	12.75	14.00	2.24	10.04	17.74
Love	34	5.98	0.90	4.32	7.82	6.36	1.25	4.55	8.60
	67	7.97	1.48	5.58	10.57	8.09	1.39	5.80	10.73
	134	8.22	1.57	5.35	10.71	11.65	1.97	7.82	14.67
	269	9.16	2.71	6.56	15.16	12.34	1.70	10.37	15.12

<sup>a</sup> The plots were planted in 2002 and harvested in 2003, 2004, and 2005. Single harvest plots were harvested once per year in October. The double harvest plots were harvested in July and October. For the double harvest plots the annual yield is the sum of the two harvests in the same calendar year.

<sup>b</sup> This is the average yield across four replications and three years in dry Mg ha<sup>-1</sup> yr<sup>-1</sup>.



## Response Function Estimation

Estimating plant yield response to N and determining economically optimal levels of N has been of interest for many decades (Tembo et al. 2008). Early attempts to fit crop yield response to N functions were inspired by agronomists who hypothesized plateau-type functional forms (Spillman 1933). Spillman, in a seminal work, developed and applied a functional form to reflect the von Liebig law of the minimum (Spillman 1933). Since that work, published in 1933, a number of researchers have used the linear response plateau (LRP) functional form to estimate crop yield response to N (Ackello-Ogutu 1985; Cerrato and Blackmer 1990; Paris 1992; Llewelyn and Featherstone 1997). Many have concluded that the LRP functional form fits N response data as well or better than polynomial specifications (Perrin 1976; Grimm, Paris, and Williams 1987; Klasson, et al. 1990; Frank, Beattie, and Embleton 1990; Chambers and Lichtenberg 1996). Tembo et al. developed a linear response model with a stochastic plateau (LRSP) applicable to experimental data collected over several years. It enables a random effect for year that can theoretically provide a better fit since yield plateaus can vary across years (Kaitibie et al. 2007; Roberts et al. 2008; Tembo et al. 2008).

Following the findings of these prior studies, three functional forms are specified: LRP; quadratic response (QR); and LRSP. Separate models are estimated for both harvest treatments for each of the four grass species. Following Tembo et al. (2008) the LRSP form is

$$(2) \quad Y_{it} = \min(\beta_0 + \beta_1 N_i, \mu_m + v_t) + u_t + \varepsilon_{it},$$

where  $Y_{it}$  is the biomass yield from N treatment  $i$  in year  $t$ ,  $N_i$  is the nitrogen level,  $\beta_s$  are the parameters to be estimated that include the intercept and slope,  $\mu_m$  is the average

plateau yield,  $v_t \sim Normal(0, \sigma_v^2)$  is the plateau year random effect,  $u_t \sim Normal(0, \sigma_u^2)$  is the year random effect, and  $\varepsilon_{it} \sim Normal(0, \sigma_e^2)$  is the random error term (Tembo et al. 2008). All three random terms are assumed to be independent. The LRP form is a special case of the LRSP form with  $\sigma_v^2 = 0$ . The LRP is

$$(3) \quad Y_{it} = \min(\beta_0 + \beta_1 N_i, \mu_m) + u_t + \varepsilon_{it}.$$

Even though many researchers have concluded that the LRP functional form provides statistical fits of N response data that is as good as or better than polynomial specifications (Perrin 1976; Lanzer and Paris 1981; Grimm, Paris, and Williams 1987; Frank, Beattie, and Embleton 1990; Chambers and Lichtenberg 1996) QR forms continue to be used. Since information is limited on perennial grass response to N and since the QR form is common (Evanylo 1991; Mjelde et al. 1991; Vanotti and Bundy 1994; Schlegel and Halvin 1995), it is also used. The QR form is

$$(4) \quad Y_{it} = \alpha_0 + \alpha_1 N_i + \alpha_2 N_i^2 + e_{it}$$

where  $\alpha_0$  is the intercept parameter,  $\alpha_1$  and  $\alpha_2$  are the slope parameters with  $\alpha_1 > 0$  and  $\alpha_2 < 0$  restrictions,  $u_t \sim Normal(0, \sigma_u^2)$  is the year random effect and  $\varepsilon_{it} \sim Normal(0, \sigma_e^2)$  is the random error term. The QR form forces symmetry relative to a unique maximum rather than a plateau (Llewelyn and Featherstone 1997).

Mixed-effects models are useful for analyzing repeated measures data (Pineiro and Bates 1995). In equations (2) and (3), the year random effects associated with the plateau, ( $v_t$ ) enter nonlinearly, and the random error term ( $\varepsilon_{it}$ ) and the year random effects associated with the intercept ( $u_t$ ) enter linearly (Fuentes and Taliaferro 2002). The SAS NLMIXED (SAS Institute 2003) procedure is used to maximize the marginal loglikelihood functions. This procedure permits both fixed and random effects to have a

nonlinear relationship to the response variable and is best suited for models with a single random effect (Wolfinger 1999). The procedure assumes that the input data set is clustered according to the year (three years), which is included in the models as a random variable.

The most suitable from among the three functional forms is selected based on the likelihood dominance criteria (LDC) and the likelihood ratio (LR) test. Likelihood dominance is an asymptotic criterion for model selection by ranking the hypotheses and does not involve a preselected level of significance (Pollak and Wales 1991). LDC ranks the hypothesis with the same number of parameters (QR and LRP) and prefers the one with higher likelihood (Pollak and Wales 1991). LDC is also used to distinguish a hypothesis with smaller parameter size (QR) with a hypothesis of larger parameter size (LRSP) based on the critical points of the LDC (Pollak and Wales 1991). The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are also used to verify the results (Wolfinger 1999; Littell et al. 2002). The LR test is used to choose between the nested models (LRP and LRSP). The LRP model is nested in the LRSP model and the null hypothesis specifies the restriction on the variance with respect to the plateau year random effect. The LR ( $\lambda$ ) is obtained as a ratio of the maximum likelihood value obtained with and without the constraint. The LR depends on the restricted and unrestricted models and under regularity, the test statistic ( $-2\ln\lambda$ ) follows a chi-squared distribution with degrees of freedom equal to the number of restrictions imposed (Greene 2003).

The objective function (equation 1) is solved for three levels of N price ( $P_n$ ) and three levels of biomass in-field price ( $P_y$ ). The average N prices in the form of urea were

\$0.77, \$0.97, and \$1.19 kg<sup>-1</sup> in the years 2006, 2007, and 2008, respectively (USDA, 2008). To incorporate the price fluctuations in the retail price of nitrogenous fertilizers at the regional level, results were obtained for N prices of \$0.66, \$1.32, and \$1.98 kg<sup>-1</sup>. Prices for mature perennial grass biomass are not available for the region. The Chariton Valley Project in Iowa procured (dry) cellulosic biomass for \$50 Mg<sup>-1</sup> (Chariton Valley Project 2008). Results were obtained for dry biomass prices of \$33, \$50, and \$66 Mg<sup>-1</sup>. Costs that do not vary with N price, N level, and yield are held constant. The species, N level, and harvest frequency that maximize expected net returns is determined for each of the nine N price-biomass price combinations.

## **Results**

Parameter estimates for the QR, LRP, and LRSP functional forms for biomass yield response to N for switchgrass are presented in Table I-2. Separate functions were estimated for the single and double harvest systems. Based on the LDC ranking, the LRP functional form provides a better statistical fit to the data than the QR functional form. The magnitude of the variance of the plateau yield was extremely small and very close to zero for both harvest levels. The LR test failed to reject the null hypothesis that the plateau is non-stochastic. By this measure for both harvest systems for switchgrass, the LRP function provides a fit at least as good as the LRSP function. This may be due to the small number of observations available. All coefficients of the LRP functions are statistically significant at the five percent level. The LRP functional form is selected for both harvest levels for switchgrass to determine the most profitable N level.

**Table I-2 Biomass yield response to nitrogen functions for switchgrass**

Statistic	Single Harvest			Double Harvest		
	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau
Intercept	7.488** (0.853) <sup>a</sup>	5.284** (1.075)	5.284** (1.075)	6.485** (0.844)	6.462** (0.609)	6.919* (0.074)
Nitrogen (kg ha <sup>-1</sup> )	0.060* (0.016)	0.100** (0.020)	0.100 (0.202)	0.052* (0.014)	0.036** (0.006)	0.037 (0.536)
Nitrogen squared	-0.00016* (0.00005)	–	–	-0.00009 (0.00005)	–	–
Plateau yield (Mg ha <sup>-1</sup> )	–	12.232** (0.340)	12.232** (0.340)	–	13.364** (0.511)	13.820** (0.600)
Variance of plateau yield			0.000 –			0.176 (0.958)
Log likelihood	-58.35	-53.95	-53.95	-56.10	-55.50	-55.80
Akaike Information Criterion	126.70	117.90	119.95	122.20	121.00	123.60
Bayesian Information Criterion	122.10	113.40	114.50	117.70	116.50	118.20

Note: The dependent variable is dry matter yield in Mg ha<sup>-1</sup> yr<sup>-1</sup> for years after establishment. Number of observations used for the estimation of each response function is 48.

\* Statistically significant at the 10% level. \*\* Statistically significant at the 5% level.

<sup>a</sup> Standard errors are in parenthesis.

The estimated plateau yield from the LRP function for a single harvest is 12.2 Mg ha<sup>-1</sup> yr<sup>-1</sup>. The spline point in the LRP single harvest function occurs at a N level of 69 kg ha<sup>-1</sup> yr<sup>-1</sup>. However, the expected yield based on the LRP double harvest function from 69 kg N ha<sup>-1</sup> yr<sup>-1</sup> is only 9.0 Mg ha<sup>-1</sup> yr<sup>-1</sup>. Based on the LRP double harvest function, 160 kg N ha<sup>-1</sup> yr<sup>-1</sup> would be required to produce 12.2 Mg ha<sup>-1</sup> yr<sup>-1</sup>. The LRP double harvest function has an estimated plateau yield of 13.4 Mg ha<sup>-1</sup> yr<sup>-1</sup> from 192 kg N ha<sup>-1</sup> yr<sup>-1</sup>. These results are consistent with those reported by others who recommend that N application rates to stands of established switchgrass fall within a range from 56 to 168 kg ha<sup>-1</sup> yr<sup>-1</sup> (Muir et al. 2001; Vogel et al. 2002; Mulkey, Owens, and Lee 2006; Fike et al. 2006). Switchgrass production systems that include a harvest during the active growing period followed by a second harvest after senescence require more N. In the region, switchgrass growth is slow to recover after a July harvest.

Table I-3 includes the regression results of biomass yield response for bermudagrass. The LR test indicates that the LRSP functional form is statistically superior to the LRP functional form for the single harvest plots. Based on the LDC ranking, the LRSP model is also preferred over the QR model when only a single harvest is conducted per year. For the double harvest plots, the statistical tests cannot distinguish among the three functional forms. Since the LRSP form was selected for the single harvest system, it was also selected for the double harvest system.

The variance identified with estimation of the LRSP plateau yield indicates that bermudagrass biomass yield is sensitive to weather conditions that vary from year to year. The variance associated with the plateau yield for a double harvest is

**Table I-3 Biomass yield response to nitrogen functions for bermudagrass**

Statistic	Single Harvest			Double Harvest		
	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau
Intercept	3.830** (0.768) <sup>a</sup>	4.019** (0.515)	4.117* (0.417)	5.237** (0.959)	5.873** (0.741)	5.871* (0.685)
Nitrogen (kg ha <sup>-1</sup> )	0.042* (0.010)	0.030** (0.004)	0.032* (0.040)	0.066* (0.016)	0.046** (0.008)	0.046 (0.790)
Nitrogen squared	-0.00005 (0.00004)	–	–	-0.00011 (0.00005)	–	–
Plateau yield (Mg ha <sup>-1</sup> )	–	10.290** (0.448)	10.732** (0.780)	–	14.529** (0.620)	14.500** (0.907)
Variance of plateau yield			4.210 (2.634)			2.052 (2.072)
Log likelihood	-43.70	-44.05	-38.30	-63.25	-63.25	-62.30
Akaike Information Criterion	97.40	98.10	88.60	136.40	136.50	136.60
Bayesian Information Criterion	92.80	93.60	83.20	131.90	132.00	131.20

Note: The dependent variable is dry matter yield in Mg ha<sup>-1</sup> yr<sup>-1</sup> for years after establishment. Number of observations used for the estimation of each response function is 48.

\* Statistically significant at 10% level. \*\* Statistically significant at 5% level.

<sup>a</sup> Standard errors are in parenthesis.

approximately half of that associated with a single harvest. Total biomass expected yield is not only greater but more stable across years from the double harvest system.

For an annual application rate of  $239 \text{ kg N ha}^{-1} \text{ yr}^{-1}$ , the estimated bermudagrass yield is  $10.3 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  for a single harvest and  $14.4 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  when N is applied in two split doses and the grass is harvested twice  $\text{yr}^{-1}$ . Harvestable bermudagrass yield increases when harvested more than once  $\text{yr}^{-1}$ . The plateau yield increases from  $10.7 \text{ Mg ha}^{-1}$  to  $14.5 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  when N is applied in split doses and the biomass is harvested twice per year. This finding is consistent with prior studies that have found that bermudagrass has high N response, a high after-harvest growth rate and a fast recovery from a July cutting (Overman, Scholtz, and Taliaferro 2003; Scarbrough et al. 2004; Silveria, Haby, and Leonard 2007; USDA-NRCS 2008;).

Response function parameter estimates for lovegrass are reported in Table I-4. Based on the LR test, the LRP functional form is statistically superior to the LRSP form for both harvest systems. The plateau variance for the LRSP was close to zero. Based on the LDC ranking, the LRP model also fits the data better than the QR functional form. The LRP functional form is selected to represent lovegrass biomass yield response to N for both harvest levels. All parameter estimates for the LRP functions are statistically significant at the five percent level. Based on the LRP functional form, the single (double) harvest lovegrass plateau yield of  $8.5$  ( $12.3$ )  $\text{Mg ha}^{-1} \text{ yr}^{-1}$  is achieved with an annual application of  $78$  ( $149$ )  $\text{kg N}$ . This finding is consistent with that reported elsewhere (McMurphy, Denman, and Tucker 1975; Taliaferro et al. 1975; Edwards 2000). Parameter estimates for flaccidgrass response functions are reported in Table I-5.



**Table I-4 Biomass yield response to nitrogen functions for lovegrass**

Statistic	Single Harvest			Double Harvest		
	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau
Intercept	4.713** (0.618) <sup>a</sup>	4.012** (0.623)	3.985 (0.641)	3.535** (0.726)	4.583** (0.549)	4.563* (0.540)
Nitrogen (kg ha <sup>-1</sup> )	0.046* (0.012)	0.058** (0.012)	0.060 (0.130)	0.086** (0.012)	0.052** (0.006)	0.053* (0.062)
Nitrogen squared	-0.00011 (0.00004)		--	-0.00018** (0.00004)		--
Plateau yield (Mg ha <sup>-1</sup> )		8.530** (0.502)	8.525* (0.533)	--	12.346** (0.452)	12.354** (0.526)
Variance of plateau yield			0.186 (0.506)			0.346 (0.622)
Log likelihood	-49.25	-46.85	-46.80	-50.40	-50.00	-49.80
Akaike Information Criterion	110.50	105.70	107.60	110.80	110.00	111.60
Bayesian Information Criterion	105.10	100.30	101.20	106.30	105.50	106.20

Note: The dependent variable is dry matter yield in Mg ha<sup>-1</sup> yr<sup>-1</sup> for years after establishment. Number of observations used for the estimation of each response function is 48.

\* Statistically significant at 10% level. \*\* Statistically significant at 5% level.

<sup>a</sup> Standard errors are in parenthesis.

**Table I-5 Biomass yield response to nitrogen functions for flaccidgrass**

Statistic	Single Harvest			Double Harvest		
	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau	Quadratic	Linear Response Plateau	Linear Response Stochastic Plateau
Intercept	8.516** (0.786) <sup>a</sup>	8.756** (0.598)	4.655*** (0.047)	6.070** (0.818)	6.673** (0.609)	6.675* (0.609)
Nitrogen (kg ha <sup>-1</sup> )	0.010 (0.014)	0.004 (0.006)	0.110* (0.123)	0.064** (0.014)	0.044** (0.006)	0.044* (0.686)
Nitrogen squared	-0.00002 (0.00005)	--	--	-0.00013 (0.00005)		--
Plateau yield (Mg ha <sup>-1</sup> )	--	9.717** (0.488)	9.528** (0.392)		13.991** (0.497)	13.988** (0.497)
Variance of plateau yield			0.246 (0.386)			0.000 –
Log likelihood	-54.50	-54.60	-52.80	-56.35	-55.55	-55.55
Akaike Information Criterion	123.10	119.20	117.60	122.70	121.10	123.10
Bayesian Information Criterion	117.70	114.70	112.20	118.20	116.60	117.70

Note: The dependent variable is dry matter yield in Mg ha<sup>-1</sup> yr<sup>-1</sup> for years after establishment. Number of observations used for the estimation of each response function is 48.

\*Statistically significant at 10% level.

\*\* Statistically significant at 5% level.

<sup>a</sup> Standard errors are in parenthesis.

For the single harvest system, the LR test identifies the LRSP model as preferred over the LRP model. Based on the LDC the LRSP is also statistically superior to the QR functional form for a single harvest system. For the double harvest system, the LR test fails to reject the null hypothesis, enabling a conclusion that the plateau is not stochastic, that the LRP is preferred over the LRSP. Based on the LDC the LRP is also statistically superior to the QR functional form for a double harvest system.

Based on the statistical tests the LRP functional form is selected to conduct economic analysis for both harvest systems for switchgrass and lovegrass and for the single harvest system for flaccidgrass. The LRSP functional form is selected for the flaccidgrass single harvest system and for both harvest systems for bermudagrass. The finding that the plateau models fit the N response data better than the QR specification is consistent with results reported by a number of others (Spillman 1933; Lanzer and Paris 1981; Ackello-Ogutu 1985; Grimm, Paris, and Williams 1987; Cerrato and Blackmer 1990; Frank, Beattie, and Embleton 1990; Chambers and Lichtenberg 1996; Tembo et al. 2008).

The profit maximizing N level and expected yields from optimum N levels for each of nine biomass in-field price and N price combinations are determined for both a single (October) harvest system (Table I-6) and double (July and October) harvest system (Table I-7). Tables I-6 and I-7 also include the expected net returns from the optimum levels of N and the estimated cost to produce a ton of biomass for these levels. For a biomass (dry) price of \$33 Mg<sup>-1</sup>, the expected net returns are negative for all price combinations and all species for both harvest systems.

**Table I-6 Estimates of profit maximizing nitrogen level, expected yield, cost and expected net returns for the selected grass species when harvested once per year in October.**

In-Field Price of Biomass (\$ Mg <sup>-1</sup> )	Price of Nitrogen (\$ kg <sup>-1</sup> )											
	Switchgrass			Bermudagrass			Lovegrass			Flaccidgrass		
	0.66	1.32	1.98	0.66	1.32	1.98	0.66	1.32	1.98	0.66	1.32	1.98
Profit maximizing N level (kg N ha <sup>-1</sup> yr <sup>-1</sup> ) <sup>a</sup>												
33	69	69	69	186	0	0	78	78	0	48	46	44
50	69	69	69	221	144	0	78	78	78	49	47	46
66	69	69	69	239	186	109	78	78	78	50	48	47
Profit maximizing expected yield (Mg ha <sup>-1</sup> yr <sup>-1</sup> ) <sup>a</sup>												
33	12.2	12.2	12.2	9.5	4.1	4.1	8.5	8.5	4.0	9.0	8.8	8.6
50	12.2	12.2	12.2	10.0	8.5	4.1	8.5	8.5	8.5	9.1	8.9	8.8
66	12.2	12.2	12.2	10.3	9.5	7.5	8.5	8.5	8.5	9.2	9.0	8.9
Profit maximizing cost of production (\$ Mg <sup>-1</sup> ) <sup>b</sup>												
33	39	43	46	56	74	74	47	54	65	50	54	57
50	39	43	46	56	69	74	47	54	61	50	54	57
66	39	43	46	57	69	79	47	54	61	49	53	57
Profit maximizing expected net returns (\$ ha <sup>-1</sup> yr <sup>-1</sup> )												
33	-69	-116	-163	-222	-165	-165	-123	-178	-131	-148	-183	-212
50	133	86	40	-69	-165	-99	17	-37	-91	2	-35	-69
66	336	289	242	94	-37	-99	158	104	49	158	116	79

<sup>a</sup> Based on LR test and LDC the suitable response functions were LRP for switchgrass and lovegrass and LSRP for bermudagrass, and flaccidgrass. These response functions are used for the estimation of optimum N and optimum yield.

<sup>b</sup> Costs are not included for collecting bales and transporting bales from the field. Charges were not assessed for overhead, risk, and management.

For a biomass price of \$50 Mg<sup>-1</sup>, and N prices of \$1.32 and \$1.98 per kg<sup>-1</sup>, expected net returns are negative for bermudagrass, lovegrass, and flaccidgrass for both harvest systems. For an in-field biomass price of \$50 Mg<sup>-1</sup>, expected net returns are positive for the single harvest switchgrass system for all three N prices. For an in-field biomass price of \$66 Mg<sup>-1</sup>, expected net returns are positive for all species, all N price levels and both harvest systems, except for the single harvest bermudagrass systems with N prices of \$1.32 and \$1.98 kg<sup>-1</sup>.

For a single harvest system, the expected net returns for switchgrass are greater than the expected net returns for bermudagrass and flaccidgrass for each of the nine biomass price and N price combinations. For eight of the nine price combinations, expected net returns are also greater for switchgrass than for lovegrass. However, for a biomass price of \$33 Mg<sup>-1</sup> and N price of \$1.98 kg<sup>-1</sup>, the expected net returns are -\$163 ha<sup>-1</sup> for switchgrass and -\$131 ha<sup>-1</sup> for lovegrass. These estimates follow from the assumption that biomass harvest is required. In low biomass price situations, if the value of the biomass is less than harvest cost, it would be optimal to not harvest.

In general, with a single harvest system, bermudagrass has the highest N requirement and the highest cost per ton of biomass followed by lovegrass. In most cases, bermudagrass records the lowest expected net returns among the four grass species. Switchgrass records the highest expected yield, lowest cost per ton, and highest expected net return per hectare. A comparison across harvest systems indicates that for most of the species, the double harvest system more than doubles the optimum N application.

**Table I-7 Estimates of profit maximizing nitrogen level, expected yield, cost per ton and expected net returns for the selected grass species when harvested two times per year, once in July and once in October.**

In-Field Price of Biomass (\$ Mg <sup>-1</sup> )	Price of Nitrogen (\$ kg <sup>-1</sup> )												
	Switchgrass			Bermudagrass			Lovegrass			Flaccidgrass			
	0.66	1.32	1.98	0.66	1.32	1.98	0.66	1.32	1.98	0.66	1.32	1.98	
Profit maximizing N level (kg N ha <sup>-1</sup> yr <sup>-1</sup> ) <sup>a</sup>													
	33	192	0	0	193	152	0	149	149	0	167	167	0
	50	192	192	0	205	181	152	149	149	149	167	167	167
	66	192	192	192	212	193	176	149	149	149	167	167	167
Profit maximizing expected yield (Mg ha <sup>-1</sup> yr <sup>-1</sup> ) <sup>a</sup>													
	33	13.4	6.5	6.5	13.7	12.6	5.9	12.4	12.4	4.6	14.0	14.0	6.7
	50	13.4	13.4	6.5	14.0	13.5	12.6	12.4	12.4	12.4	14.0	14.0	14.0
	66	13.4	13.4	13.4	14.1	13.7	13.3	12.4	12.4	12.4	14.0	14.0	14.0
Profit maximizing cost of production (\$ Mg <sup>-1</sup> ) <sup>b</sup>													
	33	46	53	53	47	56	62	45	54	66	46	55	58
	50	46	56	53	47	57	65	45	54	62	46	55	63
	66	46	56	66	47	57	66	45	54	62	46	55	63
Profit maximizing expected net returns (\$ ha <sup>-1</sup> yr <sup>-1</sup> )													
	33	-173	-128	-128	-198	-291	-168	-151	-254	-151	-193	-306	-170
	50	49	-84	-20	27	-96	-188	52	-49	-153	40	-74	-190
	66	269	138	7	262	123	5	257	156	52	272	156	42

<sup>a</sup> Based on LR test and LDC the suitable response functions were LRP for switchgrass, lovegrass and flaccidgrass and LSRP for bermudagrass. These response functions are used for the estimation of optimum N and optimum yield.

<sup>b</sup> Costs are not included for collecting bales and transporting bales from the field. Charges were not assessed for overhead, risk, and management.

For the high biomass price and low N price combinations, the increment in the yield covers the additional expenses for fertilizer, application costs, and harvesting for bermudagrass, lovegrass, and flaccidgrass. This fact is evident from the reduction in cost of biomass for these species for some of the price combinations. On the other hand, for switchgrass, the cost of biomass produced is greater for the double harvest system for all price combinations. For the highest budgeted in-field biomass price of  $\$66 \text{ Mg}^{-1}$ , the double harvest system is more profitable for all grasses except switchgrass.

Table I-8 includes the optimal species, expected net return, level of N, number of harvests, expected yield, and estimated costs for each of the nine biomass in-field price and N price combinations. For each of the price situations, switchgrass is the most profitable species. However, as noted, for an in-field biomass price of  $\$33 \text{ Mg}^{-1}$ , expected net returns are negative. Since the LRP function is used to determine the optimal level of N, it is either optimal to apply zero N or to apply  $69 \text{ kg N ha}^{-1}$  to switchgrass harvested once  $\text{yr}^{-1}$  after senescence in the region. For all but one evaluated price combination, it is optimal to apply  $69 \text{ kg N ha}^{-1} \text{ yr}^{-1}$ . For an N price of  $\$1.98 \text{ kg}^{-1}$  and an in-field biomass price of  $\$33 \text{ Mg}^{-1}$ , it is optimal to apply zero N to switchgrass and to harvest twice. The expected yield from this double harvest system of  $6.5 \text{ Mg ha}^{-1}$  would have an expected gross value of  $\$214 \text{ ha}^{-1}$  at  $\$33 \text{ Mg}^{-1}$  which exceeds the expected harvest cost for mowing and raking twice, and baling the  $6.5 \text{ Mg}$ , of  $\$170 \text{ ha}^{-1}$ .

**Table I-8 Optimal species, expected net return, optimal level of nitrogen, optimal number of harvests, expected yield, and estimated cost per ton for several sets of biomass and nitrogen prices.**

Price of Biomass (\$ Mg <sup>-1</sup> )	Price of Nitrogen (\$ kg <sup>-1</sup> )	Optimal Species	Expected Net Return (\$ ha <sup>-1</sup> yr <sup>-1</sup> )	Optimal Level of Nitrogen (kg ha <sup>-1</sup> yr <sup>-1</sup> )	Optimal Number of Harvests per Year	Expected Yield (Mg ha <sup>-1</sup> )	Estimated Cost (\$ Mg <sup>-1</sup> ) <sup>a</sup>
33	0.66	Switchgrass	-69	69	1	12.2	39
50	0.66	Switchgrass	133	69	1	12.2	39
66	0.66	Switchgrass	336	69	1	12.2	39
33	1.32	Switchgrass	-116	69	1	12.2	43
50	1.32	Switchgrass	86	69	1	12.2	43
66	1.32	Switchgrass	289	69	1	12.2	42
33	1.98	Switchgrass	-128	0	2	6.5	53
50	1.98	Switchgrass	40	69	1	12.2	46
66	1.98	Switchgrass	242	69	1	12.2	46

<sup>a</sup> Costs are not included for collecting bales and transporting bales from the field. Charges were not assessed for overhead, risk, and management.



## Conclusion and Discussion

Biomass yield to N response functions were estimated for four perennial grass species and to determine the species. These functions were used to estimate the species, N level, and harvest frequency that will maximize expected net returns to a land unit, given the climate and soils of the U.S.A. Southern Plains. For each of the four species and both harvest systems, the functional forms that include a plateau, either the LRP or LRSP, fits the data better than the QR functional form that forces a unique maximum and forces symmetry relative to the maximum point. This finding is consistent with results reported by a number of researchers.

For in-field biomass prices ranging from \$33 to \$66 Mg<sup>-1</sup> and N prices ranging from \$0.66 to \$1.98 kg<sup>-1</sup>, switchgrass is the optimal species. For a biomass price of \$50 Mg<sup>-1</sup> it is optimal to fertilize switchgrass with 69 kg ha<sup>-1</sup> in the spring and to harvest once yr<sup>-1</sup> after senescence in October. For an N price of \$1.32 kg<sup>-1</sup>, expected net returns are \$133, \$86, and \$40 ha<sup>-1</sup> yr<sup>-1</sup> for in-field biomass prices of \$50 Mg<sup>-1</sup>. For N prices of \$0.66, \$1.32, and \$1.98 kg<sup>-1</sup>, breakeven in-field prices for the optimal switchgrass production systems are \$39, \$43, and \$53 Mg<sup>-1</sup>, respectively.

Nitrogen treatment levels in the designed experiment were 34, 67, 134, and 269 kg ha<sup>-1</sup> yr<sup>-1</sup>. The estimated yield plateau for switchgrass harvested once is 69 kg ha<sup>-1</sup> yr<sup>-1</sup>. Thus, two of the points are on the slope and two are on the plateau of the LRP function and are theoretically sufficient to provide relatively precise response function parameter estimates. Field trials are costly to execute and adding N levels would add to the cost of the trials. However, if too few treatment levels are included in the field trials, resulting in parameter estimates with large standard deviation, recommendations from estimated

response functions could also be costly. For the region of the study, switchgrass would be a more economical species for biomass feedstock production than either bermudagrass, or lovegrass, or flaccidgrass. However, the assumption that each of the four species would be of equal value to a cellulosic biorefinery remains to be confirmed. Prior research has found that switchgrass does not respond to potassium and phosphorus fertilization and that if harvest of a perennial grass is delayed until after senescence, removal of above ground parts of the plant will not mine phosphorus and potassium from the soil. However, one shortcoming of the field trials was that soil tests were not conducted after the study to confirm that levels of phosphorus and potassium in the soil had not been depleted.

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**PAPER II**

**ECONOMICS OF SWITCHGRASS AND MISCANTHUS RELATIVE  
TO COAL AS FEEDSTOCK FOR GENERATING  
ELECTRICITY**

**Abstract**

Switchgrass (*Panicum virgatum*) serves as a model dedicated energy crop in the U.S.A. Miscanthus (*Miscanthus x giganteus*) has served a similar role in Europe. This study was conducted to determine the most economical species, harvest frequency, and carbon tax required for either of the two candidate feedstocks to be an economically viable alternative for cofiring with coal for electricity generation. Biomass yield and energy content data were obtained from a field experiment conducted near Stillwater, Oklahoma, U.S.A., in which both grasses were established in 2002. Plots were split to enable two harvest treatments (once and twice yr<sup>-1</sup>). The switchgrass variety ‘Alamo’, with a single annual post senescence harvest, produced more biomass (15.87 Mg ha<sup>-1</sup> yr<sup>-1</sup>) than miscanthus (12.39 Mg ha<sup>-1</sup> yr<sup>-1</sup>) and more energy (249.6 million kJ ha<sup>-1</sup> yr<sup>-1</sup> versus 199.7 million kJ ha<sup>-1</sup> yr<sup>-1</sup> for miscanthus). For the average yields obtained, the estimated cost to produce and deliver biomass an average distance of 50 km was \$43.9 Mg<sup>-1</sup> for switchgrass and \$51.7 Mg<sup>-1</sup> for miscanthus. Given a delivered coal price of \$39.76 Mg<sup>-1</sup> and average energy content, a carbon tax of \$7 Mg<sup>-1</sup> CO<sub>2</sub> would be required for

switchgrass to be economically competitive. For the location and the environmental conditions that prevailed during the experiment, switchgrass with one harvest per year produced greater yields at a lower cost than miscanthus. In the absence of government intervention such as requiring biomass use or instituting a carbon tax, biomass is not an economically competitive feedstock for electricity generation in the region studied.

## **Introduction**

A major portion of electricity in the U.S.A. is produced by burning coal and natural gas. Coal is the primary fuel used by the nation's electric power industry. It produces 36 % of the CO<sub>2</sub> emissions from energy use (DOE/EIA 2007; DOE 2009). Cofiring cellulosic biomass with coal in traditional utility boilers enables substituting fossil fuel with renewable energy sources to produce electricity and if properly executed, reducing carbon emissions. Cofiring with cellulosic biomass requires only minor modifications in the boilers and minimal investment in existing plants (Fraas and Johansson 2009). Switchgrass has been cofired with coal at the Ottumwa Generating Station near Ottumwa, Iowa, U.S.A. Technical results were promising with no slagging. However, it was determined that in the absence of subsidies, mandates, or carbon taxes, cofiring was not economically competitive (Olsen 2001).

Dedicated perennial grasses could be developed, which would be locally available, dependable, and scalable substitutes for coal. According to Perlack 22 million ha of U.S.A. land could be converted for biomass production with minimal effects on food, feed, and fiber production (Perlack, et al. 2005). A key to ensuring a long-term supply of biomass feedstock to a given power plant is selecting the most suitable perennial grass species for local soil and weather conditions.

Several studies have been conducted to screen species to identify relative suitability for biomass production. In the U.S.A., the Oak Ridge National Laboratory's (ORNL) Herbaceous Energy Crops Research Program and the Department of Energy's (DOE) Biofuel Development Program were some of the early efforts of integrated and multilocational research projects designed to select suitable species (Lewandowski et al. 2003). Most of the grass species included in these studies were chosen from the pool of native prairie grasses found on the plains of North America. The ORNL selected switchgrass from a screening trial that included 34 species conducted on 31 different sites spread over seven states in the United States (McLaughlin, and Walsh 1998; Wright 2007).

During this time period, miscanthus (*Miscanthus x giganteus*), a non-native ornamental plant, caught the attention of researchers in Europe. Several projects were conducted across Europe to develop and evaluate miscanthus hybrids (Lewandowski, Scurlock, Lindvall, and Christou 2003). Miscanthus is not native to the U.S.A. and was not included in the ORNL trials. Heaton, Voigt, and Long (2004) reviewed 13 miscanthus trials and eight switchgrass trials. Most of the miscanthus trials were conducted in Europe, and most of the switchgrass trials were conducted in the U.S.A. Heaton, Voigt, and Long (2004) reported that across the studies, miscanthus produced on average 12 Mg ha<sup>-1</sup> yr<sup>-1</sup> more biomass than switchgrass. They did not report results of any experiments in which the two species were both considered. The climate and soils varied across the trials. For comparison, for the decade from 1997-2006, the average harvested wheat yields were 5.30 Mg ha<sup>-1</sup> in the European Community and only 2.88 Mg ha<sup>-1</sup> in the U.S.A. (Vocke and Allan 2006). Clearly, climate has a major impact on yield.

Khanna used estimated yields obtained from a side-by-side trial of switchgrass (variety Cave-in-rock) and miscanthus at three locations in Illinois, U.S.A. to compute production costs for both species (Khanna, Dhungana, and Clifton-Brown 2008). Khanna budgeted an average estimated yield of 5.4 Mg ha<sup>-1</sup> for switchgrass and 18.6 Mg ha<sup>-1</sup> for miscanthus (Khanna 2008). Fuentes and Taliaferro (2002) conducted a switchgrass variety trial at two locations in Oklahoma U.S.A. They found an average yield over seven years and two locations from plots that included a mixture of Alamo and Summer varieties of 16.2 Mg ha<sup>-1</sup> compared to a yield of 9.9 Mg ha<sup>-1</sup> from the Cave-in-rock variety. The field trials confirm that switchgrass biomass yields differ substantially across variety and climate.

Other field trials have also found that yields of perennial grass species and cultivars vary with location, weather, and soil (Sladden, Bransby and Aiken 1991; Downing and Graham 1996; Heaton, Voigt and Long 2004; Fike, et al. 2006). For example, miscanthus yields were found to vary from 26.72 Mg ha<sup>-1</sup> yr<sup>-1</sup> to 0.5 Mg ha<sup>-1</sup> yr<sup>-1</sup> in the former Soviet Union and Mongolia (Fischer, Prieler, and Velthuisen 2005). A three-year study conducted with 15 miscanthus genotypes in five European countries demonstrated a strong genotype environment interaction (Fischer, Prieler, and Velthuisen 2005). For switchgrass, lowland varieties (such as Alamo and Kanlow) usually yield substantially more than upland varieties (such as Cave-in-rock). A comparison of yield performance of switchgrass at different U.S.A. locations shows a wide variation with respect to varieties and location and reinforces the need for regional trials to account for differences in climate and soil (Lewandowski, Scurlock, Lindvall, and Christou 2003).

Harvesting constitutes a major share of the cost to deliver biomass from perennial grasses. Harvest frequency not only affects the yield, but also the quality of biomass. Previous studies have found that the cost, net carbon emission, and energy content of biomass varies widely with species and stages of growth (Aravindhakshan, Epplin, and Taliaferro 2008). Lewandowski and Kicherer (1997) observed that the combustion quality of biomass is improved when harvest is delayed by three to four months and found a strong interaction between biomass yield and quality and growing conditions. Jorgensen (1997) found higher mineral concentrations in the biomass when harvests occurred in the autumn or early winter. These studies report the quality of biomass in terms of ash, K, chloride, N, and moisture content, which reduces the efficiency of power production. When grass harvest occurs once in a calendar year, it is usually performed at the senescence stage. If harvest is conducted twice  $\text{yr}^{-1}$ , the first harvest will be during the vegetative phase of growth, and the second harvest will be at the end of growing season or after frost. Nutrient and lignin content varies with the stage of growth. Lignification of biomass increases with the age of stand, and an additional harvest reduces the lignin content of biomass and thereby the energy content.

Cofiring enables using cellulosic biomass directly without converting it to other forms (such as ethanol). Cofiring biomass with coal is assumed to represent the best available control technology, and it has a comparative advantage in reducing carbon emissions relative to producing ethanol to displace gasoline (Fraas and Johansson 2009; English, Short, and Heady 1981). Since coal is the most widely used energy source in the U.S.A., in the absence of public policy incentives, cofiring biomass with coal would be profitable only if a steady supply of quality biomass could be assured at a competitive price.

The delivered cost of coal to produce electricity does not include the cost of externalities. If the external consequences of combusting coal are ignored, coal is cheap compared to cellulosic biomass. Policy makers could internalize the external costs of coal by imposing a tax based on CO<sub>2</sub> emissions.

The objective of the research reported in this paper is to determine the most economical species, harvest frequency (once or twice a yr<sup>-1</sup>), and the carbon emissions tax required for either of two candidate feedstocks (miscanthus and switchgrass) to be an economically viable alternative for cofiring with coal to generate electricity in the U.S.A. Southern Plains. Cellulosic raw material quality is measured in terms of energy content. Species selection is based on net return ha<sup>-1</sup>. The value of biomass is estimated indirectly based on the energy content in terms of the price of coal. Thus, the value of biomass is positively related to the price of its close substitute (coal) for producing electricity.

### **Theory and Estimation Procedures**

Crop selection based on the net revenue generated from a unit of land enables a comparison with other competing crops that could be grown in the same field. The objective function for the farm operator can be stated as

$$(1) \max_{S,H} E(NR) = \max \left( E\{[P_y + P_r(S, H)]Y(S, H)\} - AEC(S) - DC(Y) - FC - TC(Y) \right)$$

where  $S$  represents the species (switchgrass or miscanthus);  $H$  represents the harvest levels (once or twice yr<sup>-1</sup>);  $E(NR)$  is the expected net revenue (\$ ha<sup>-1</sup>);  $P_y$  is the biomass price (\$ Mg<sup>-1</sup>);  $P_r$  is a price premium based on biomass energy content (\$ Mg<sup>-1</sup>);  $Y$  is the biomass yield (Mg ha<sup>-1</sup>);  $AEC$  represents the amortized establishment cost (\$ ha<sup>-1</sup>);  $DC$  represents the direct cost of fertilizer, fertilizer application, and harvesting;  $FC$  represents

the fixed costs including the rental value of land;  $TC$  represents the cost to load and transport rectangular solid bales a distance of 50 km and then offload them ( $\$ \text{Mg}^{-1}$ ).

Since a market price for cellulosic biomass does not currently exist in the U.S.A., a pseudo price is estimated based on the coal price and the biomass energy content relative to the coal energy content. The equation used to calculate revenue is

$$(2) \quad R(S, H) = \left( \frac{Y(S, H) * EB(S, H)}{EC} \right) P_c,$$

where  $R$  is the revenue ( $\$ \text{ha}^{-1}$ );  $EC$  represents the energy content of coal (23.05 million  $\text{kJ Mg}^{-1}$  as per 2007 U.S consumption) supplied to U.S.A. electricity only and combined-heat-and-power plants (DOE/EIA-0035 2009);  $EB$  is the energy content of biomass (million  $\text{kJ Mg}^{-1}$ ), which depends on the selected species and harvest frequency; ( $P_c$ ) is the average market price for coal delivered to end use, which in 2007 was  $\$39.76 \text{ Mg}^{-1}$  (DOE/EIA 2009).

The energy content of coal varies across deposit (DOE/EIA 2010). In the U.S.A., the energy content of coal is greatest in the Northern Appalachia region (29.07 million  $\text{kJ Mg}^{-1}$ ) and the lowest in Powder River Basin deposit (20.47 million  $\text{kJ Mg}^{-1}$ ) (DOE/EIA 2010). The delivered cost of coal includes the cost of transportation that varies with distance between coal mine and electric plant and other handling charges. To simplify calculations, the weighted average energy content (23.05 million  $\text{kJ Mg}^{-1}$ ) of U.S.A. delivered coal and the average market price of  $\$39.76 \text{ Mg}^{-1}$  is used in the estimation of revenue (DOE/EIA 2009).

Table II-1 includes a summary of establishment and maintenance costs. Separate budgets estimate establishment year and maintenance year costs. Machinery cost estimates are based on Oklahoma farm and ranch custom rates (Doye and Sahs 2009a).

**Table II-1 Estimated establishment and maintenance budgets for switchgrass (SG) and miscanthus (MS) ha<sup>-1</sup>**

Items	Unit	Quantity	Unit Price (\$)	Value (\$ ha <sup>-1</sup> )	
				SG <sup>a</sup>	MS <sup>a</sup>
<b>Establishment year budgets</b>					
Moldboard plow	ha	1	32.1	32.1	32.1
Secondary tillage	ha	2	23.4	46.8	46.8
Fertilizer and chemical application	ha	1	19.7	19.7	19.7
Potato planter for miscanthus	ha	1	73.0		73.0
Planting using seeder	ha	1	42.7	42.7	
Switchgrass seed	kg	7	15.4	107.9	
Miscanthus rhizomes	nm <sup>-2</sup>	1			334.8
Herbicide (2,4-D)	L	2	4.0	8.0	8.0
Phosphorous (18-46-0)	kg	74	0.6	42.0	42.0
Annual operating capital	\$		0.1	20.9	38.9
Land rental	ha	1	110.0	110.0	110.0
Total establishment cost	\$ ha <sup>-1</sup>			430.2	705.3
Amortized establishment cost \$ ha <sup>-1</sup> (10 years @ 7%)			0.07	61.2	100.4
<b>Annual maintenance budgets (established stands)</b>					
Establishment cost	\$ ha <sup>-1</sup>			61.2	100.4
Fertilizer application	ha	1	9.2	9.2	9.2
Nitrogen (urea)	kg	<sup>b</sup>	0.4	45.7	46.4
Phosphorous (18-46-0)	kg	<sup>b</sup>	0.6	11.2	8.7
Annual operating capital	\$		0.1	2.7	2.6
Harvesting (mowing)	ha	1	26.2	26.2	26.2
Harvesting (raking)	ha	1	8.8	8.8	8.8
Harvesting (baling)	bale	1	14.2	331.4	258.7
Land rental	ha	1	110.0	110.0	110.0
Total production cost	\$ ha <sup>-1</sup>			606.3	571.0
Average harvested yield <sup>c</sup>	Mg ha <sup>-1</sup>			15.9	12.4
Transportation cost <sup>d</sup>	\$ ha <sup>-1</sup>			89.7	70.0
Total cost	\$ ha <sup>-1</sup>			696.0	641.0
Total delivered cost	\$ Mg <sup>-1</sup>			43.9	51.7

<sup>a</sup> SG is switchgrass, MS is miscanthus.

<sup>b</sup> Fertilizer quantity differs across species and yield.

<sup>c</sup> Estimates are for a single harvest yr<sup>-1</sup>.

<sup>d</sup> This is the estimated cost to load and transport the average number of rectangular solid bales produced ha<sup>-1</sup> a distance of 50 km.

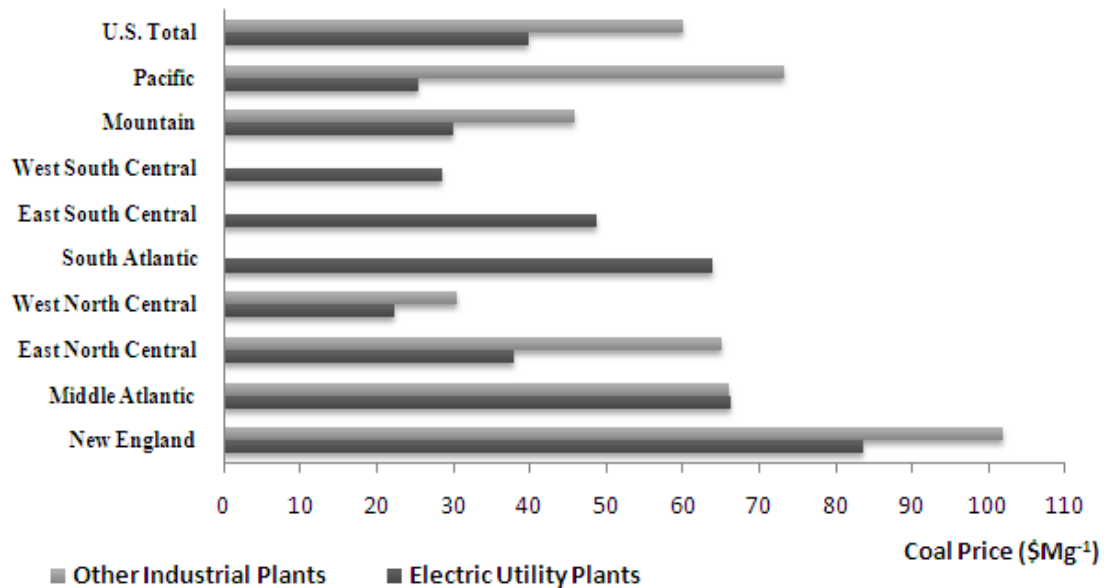
The rate for planting miscanthus rhizomes and the cost of rhizomes are based on estimates provided by Khanna, Dhungana, and Clifton-Brown (2008). Switchgrass seed rate and seed cost are obtained from Epplin (Epplin 1996). The budgeted planting density of miscanthus rhizomes is 1 m<sup>-2</sup> and the budgeted seeding rate for switchgrass is 7 kg ha<sup>-1</sup>.

Establishment costs are amortized for a period of 10 years at a rate of 7%.



For maintenance years (years 2 to 10), the only field operations are fertilizer application and harvesting. The budgeted land rental rate of \$110 ha<sup>-1</sup> is based on the average rental rate of non irrigated Oklahoma cropland of \$75 ha<sup>-1</sup> (Doye and Sahs 2009b) plus a premium of \$35 ha<sup>-1</sup> to account for the anticipated market response to competition for land in the electric plant's vicinity.

The average price of coal delivered to the end use sector by census division (2007) is given in Figure II-1. The U.S.A. price of coal varies from \$14.36 Mg<sup>-1</sup> in North Dakota to \$106.88 Mg<sup>-1</sup> in New Jersey (DOE/EIA-0584 2009). The price of coal at which the production of cellulosic biomass reaches a breakeven point is computed. A sensitivity analysis is performed on selected parameter values (land rental rates, yields, machinery costs, transportation costs, stand life, and input costs). A Mg of coal combusted to generate electricity emits 3.48 Mg CO<sub>2</sub> (Hong and Slatick 1994). This emission does not include the carbon emitted by the mining and transportation activities required to get the coal to the point of use. If the carbon emitted by machines involved in producing, harvesting, and transporting biomass is ignored, the carbon sequestered in the plant material can be assumed to be equivalent to the carbon released when biomass from perennial grasses is combusted.



**Figure II-1 Average price of coal delivered to end use sector by U.S. census division 2007.**

A carbon tax for coal that would be required to increase the cost of coal to breakeven with the cost of biomass is estimated based on the market price of coal:

$$(3) \quad TAX = \max (0, (BP - MP)/CO_2),$$

where *TAX* is the carbon tax imposed \$ Mg<sup>-1</sup> of CO<sub>2</sub> emitted to the atmosphere, *BP* is the breakeven price of coal at which biomass production is profitable (\$ Mg<sup>-1</sup>), *MP* is the market price of coal (\$ Mg<sup>-1</sup>), and *CO<sub>2</sub>* is the CO<sub>2</sub> emission from combusting coal, which is estimated to be 3.48 Mg Mg<sup>-1</sup> coal.

### Materials and Methods

A designed experiment was conducted near Stillwater, Oklahoma, U.S.A. on a Kirkland silt loam soil (fine, mixed, superactive, thermic Udertic Paleustolls). The experiment was designed as a randomized complete block with four replications and four

plots per replication. Treatments consisted of two species, miscanthus (*Miscanthus x giganteus*) and switchgrass (*Panicum virgatum* variety 'Alamo') and two harvest levels (one yr<sup>-1</sup> post senescence and two yr<sup>-1</sup>, July and post senescence). The four treatments, each consisting of a species and harvest level combination, were assigned randomly to plots with one combination of treatment in each replication. The plots were 2.44 m wide and 6.10 m in length. Four rows were planted in each plot 0.61 m apart.

The grasses were planted on June 24-25, 2002. Biomass was harvested in 2003, 2004, and 2005. Summary statistics for selected weather variables are provided in Table II-2. Annual dry-matter and gross energy (kJ gm<sup>-1</sup>) yield estimates were computed from biomass harvested from the center two rows of each plot. A calorimeter was used to determine the energy content. Summary statistics of yield and energy content are provided in Table II-3.

**Table II-2 Overview of weather during the experiment period at Stillwater, Oklahoma U.S.A. (2003-2005)**

Weather parameters	2003				2004				2005			
	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Rainfall (mm month <sup>-1</sup> )	56.1	32.0	2.03	106.4	77.7	62.2	5.8	230.6	64.3	62.2	0.0	222.8
Solar radiation (MJ m <sup>2</sup> )	15.8	6.10	7.2	25.2	15.6	5.7	6.8	22.6	16.3	6.2	7.3	25.6
Soil max. temperature (°C)	30.7	-11.6	21.1	41.1	30.0	-12.4	22.2	36.7	31.1	-12.3	22.8	37.8
Soil min. temperature (°C)	14.2	-9.0	2.8	27.2	14.4	-9.5	3.3	25.0	14.8	-9.2	3.3	26.1

Source (Mesonet 2007).

**Table II-3 Summary statistics of yield and energy content of biomass**

Species	Harvest yr <sup>-1</sup>	Biomass Yield (Mg ha <sup>-1</sup> yr <sup>-1</sup> )				Gross Energy (kJ gm <sup>-1</sup> )			
		Mean	SD	Min	Max	Mean	SD	Min	Max
Miscanthus	1	12.39	1.87	9.27	14.56	16.11	1.62	13.27	17.79
	2	13.04	1.64	11.27	15.97	15.65	2.02	11.16	17.56
Switchgrass	1	15.87	2.94	11.72	20.65	15.73	1.62	13.76	17.56
	2	15.42	2.86	10.64	20.63	15.84	2.98	8.95	17.69

## Statistical Analysis

Separate models are estimated for the treatment effects with biomass yield and energy content as dependent variables. Statistical analysis is performed using the PROC MIXED procedure of SAS (SAS 2009). In both models, the treatments (species and harvest levels) are modeled as fixed effects and replication and year as random effects. Treatment main effects and interaction effects are tested for significance. The means are compared with LSMEANS and ESTIMATE statements. The complete data generating processes to analyze the effect of treatments on biomass yield can be stated as

$$(4) \quad Y_{ijkl} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + v_t + \varepsilon_{ijkl},$$

where  $Y_{ijkl}$  is the biomass yield expressed in  $\text{Mg ha}^{-1} \text{ yr}^{-1}$ ,  $\mu$  is the intercept term,  $\alpha_i$  represents the effect of  $i^{\text{th}}$  species (miscanthus and switchgrass),  $\beta_j$  represents the effect of the  $j^{\text{th}}$  harvest level (once and twice  $\text{yr}^{-1}$ ), and  $(\alpha\beta)_{ij}$  is the species by harvest interaction effect. The error term associated with blocking (replication) is defined as  $\gamma_k \sim N(0, \sigma_\gamma^2)$ , where  $k = 1, 2, \dots, 4$  represents the replications. The experiment was continued through three harvest years ( $t = 1, 2, \text{ and } 3$  represents years 2003, 2004, and 2005). Biomass yield depends on random weather effects during a particular year, and  $v_t \sim N(0, \sigma_v^2)$  represents the year random effect. The random errors  $\varepsilon_{ijkl} \sim N(0, \sigma_\varepsilon^2)$  with each experimental unit as well as error terms associated with replication and year ( $v_t$  and  $\gamma_k$ ) are assumed to be independent and identically distributed.

The quality (energy content) of biomass is also considered as a function of species and harvest treatment. The data generating process for the energy equation can be stated as

$$(5) \quad EC_{ijklt} = \pi + \delta_i + \rho_j + (\delta\rho)_{ij} + \theta_k + \eta_t + \varpi_{ijklt},$$

where  $EC_{ijklt}$  is the energy content of the biomass,  $\pi$  is the intercept term,  $\delta_i$  is the effect of  $i^{th}$  species,  $\rho_j$  is the effect of harvest treatment,  $(\delta\rho)_{ij}$  represents the interaction effect,  $\theta_k \sim N(0, \sigma_\theta^2)$  is the random effect with respect to replication,  $\eta_t \sim N(0, \sigma_\rho^2)$  is the random effect associated with year and  $\varpi_{ijklt} \sim N(0, \sigma_\varpi^2)$  is the random error term, and the error terms are assumed to be independent and identically distributed.

## Results

Results for the biomass yield and energy content models are given in Tables II-4 and II-5. The type 3 test for mixed effects confirms that biomass production is strongly affected by species (P value < 0.001). The harvest levels and the interaction effects are not significant (P value 0.87 and 0.36 respectively). Switchgrass produced a greater biomass yield ( $15.64 \text{ Mg ha}^{-1} \text{ yr}^{-1}$ ) than miscanthus ( $12.72 \text{ Mg ha}^{-1} \text{ yr}^{-1}$ ). Across both species mean biomass yield increased slightly (but not significantly) from  $14.13 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  to  $14.23 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  with an additional harvest. Mean switchgrass yield decreased from  $15.87 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  to  $15.42 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  with an additional harvest. Switchgrass did not recover well from a July harvest (USDA/NRCS 2008). This finding is consistent with that of previous studies conducted in the region (McLaughlin and Walsh 1998; Sanderson, et al. 1999; Aravindhakshan, Eppin, and Taliaferro 2008).

**Table II-4 Results of the type III test for main effects and interactions for biomass yield and energy content as dependent variables.**

Effects	Biomass Yield		Energy Content	
	F-value	Type III P>F	F-value	Type III P>F
Species	23.39	<.0001	0.01	0.90
Harvest	0.03	0.87	0.05	0.82
Species * Harvest	0.82	0.36	0.15	0.71

**Table II-5 Least squares mean values for biomass yield and energy content of biomass**

Effects	Biomass Yield (Mg ha <sup>-1</sup> yr <sup>-1</sup> )	Energy Content (million kJ Mg <sup>-1</sup> )
Species		
Miscanthus	12.72	15.88
Switchgrass	15.64	15.79
Harvest levels		
Harvest-1	14.13	15.92
Harvest-2	14.23	15.75
Treatment Interactions		
Miscanthus * Harvest-1	12.39	16.12
Miscanthus * Harvest-2	13.04	15.65
Switchgrass * Harvest-1	15.87	15.73
Switchgrass * Harvest-2	15.42	15.85

Tests for fixed effects reveal that none of the analyzed independent variables (species and harvest levels) and their interaction effects significantly affect the energy content of biomass. The least squares mean value for energy content of biomass was higher for miscanthus (15.88 kJ gm<sup>-1</sup>). Harvesting twice rather than once yr<sup>-1</sup> reduced the mean energy content of the biomass, but not significantly. Miscanthus harvested once yr<sup>-1</sup> after senescence produced the highest energy content (16.12 kJ gm<sup>-1</sup>). However, since switchgrass produced a significantly greater biomass yield, the energy production per land unit was greater with switchgrass.

The delivered cost of biomass (Tables II-3 and II-6) was lowest for switchgrass harvested once yr<sup>-1</sup> (\$43.9 Mg<sup>-1</sup>). The delivered cost estimate is consistent with findings of previous studies (McLaughlin and Kszos (\$44 Mg<sup>-1</sup> in 2005); Epplin (\$37 Mg<sup>-1</sup> in 1996); Aravindhakshan et al. (\$39-\$46 Mg<sup>-1</sup> in 2008) (Epplin 1996; McLaughlin and Walsh 1998; Aravindhakshan, Epplin, and Taliaferro 2008).

For a U.S.A. average coal price of \$39.76 Mg<sup>-1</sup>, the value of switchgrass biomass delivered to a cofiring plant as a substitute for coal, based on energy content, is estimated to be \$27.1 Mg<sup>-1</sup>. This is an estimate of the maximum price that a profit maximizing cofiring plant manager would offer for delivered biomass. This estimate does not consider differences in external benefits and the costs of using biomass, and it does not account for additional costs incurred for modifying the electric generation facility to accommodate the biomass.

The objective of the farm operator is to obtain maximum net revenue ha<sup>-1</sup> and as per equations (1) and (2), that depends on the total energy produced ha<sup>-1</sup>. Based on results of the field trials, for the region, the best strategy would be to establish switchgrass and harvest once yr<sup>-1</sup> after senescence. Switchgrass produced 50 million kJ ha<sup>-1</sup> more energy than miscanthus when harvested once yr<sup>-1</sup>. When harvested twice yr<sup>-1</sup>, the total energy production ha<sup>-1</sup> decreased (5.2 million kJ ha<sup>-1</sup>) for switchgrass. Even though harvesting twice yr<sup>-1</sup> slightly increases the yield for miscanthus (0.6 Mg ha<sup>-1</sup>), the revenue from the second harvest is less than the additional cost of the second harvest, and the total net revenue declines by \$55 ha<sup>-1</sup>. Harvesting twice yr<sup>-1</sup> is not an economically viable cultural practice in the region for either species.



The cost of producing biomass is greater than the value based on the price of coal (Table II-6). None of the treatment combinations would produce positive net revenue if the biomass price was based on energy content relative to coal. For a coal price of \$39.76 Mg<sup>-1</sup> biomass production in the region from either switchgrass or miscanthus for cofiring with coal is not financially feasible when the value is based on the energy content of biomass.

Sensitivity of net revenue to changes in land rental rate, biomass yield, machinery cost, transportation cost, input cost, and stand life is reported in Table II-7. If the government permits the use of land currently in the Conservation Reserve Program at no cost to produce feedstock such that the rental rate assigned to the production of feedstock is assumed to be zero (Epplin 1996), the net revenue increases by 31-41% with more response for switchgrass. However, even with a land cost of zero, producing biomass from either species to substitute for coal does not generate positive returns.

**Table II-6 Results of the feasibility analysis of biomass production in the U.S.A. Southern plains**

	Harvest Level	Cost of Biomass (\$ Mg <sup>-1</sup> ) <sup>a</sup>	Total Energy (million kJ ha <sup>-1</sup> )	Value of Biomass Given a Coal Price of \$40Mg <sup>-1</sup> (\$ Mg <sup>-1</sup> ) <sup>b</sup>	Total Value (\$ ha <sup>-1</sup> )	Total Cost (\$ ha <sup>-1</sup> )	Net Value (\$ ha <sup>-1</sup> )
Switchgrass	1	43.9	249.6	27.1	430.6	696.0	-265.4
Switchgrass	2	47.3	244.4	27.3	421.6	728.3	-306.7
Miscanthus	1	51.7	199.7	27.8	344.5	641.0	-296.5
Miscanthus	2	53.9	204.1	27.0	352.0	703.1	-351.0

<sup>a</sup> This is an estimate of the costs to produce, harvest, and deliver biomass a distance of 50 km.

<sup>b</sup> The value of biomass is estimated based on its energy content relative to the price and energy content of coal.

**Table II-7 Results of sensitivity analysis of net revenue**

	Switchgrass Harvest-1		Switchgrass Harvest-2		Miscanthus Harvest-1		Miscanthus Harvest-2	
	Net Revenue (\$ ha <sup>-1</sup> )	% Chang e <sup>a</sup>	Net Revenue (\$ ha <sup>-1</sup> )	% Change	Net Revenue (\$ ha <sup>-1</sup> )	% Change	Net Revenue (\$ ha <sup>-1</sup> )	% Change
Normal	-265.4		-306.7		-296.5		-351.0	
Without Land rental value	-155.4	41%	-196.7	36%	-186.5	37%	-241.0	31%
25% Increase in rent	-292.9	-10%	-334.2	-9%	-324.0	-9%	-378.5	-8%
25% Decrease in rent	-237.9	10%	-279.2	9%	-269.0	9%	-323.5	8%
25% Increase in yield	-372.8	-40%	-411.0	-34%	-380.3	-28%	-439.3	-25%
25% Decrease in yield	-158.0	40%	-202.4	34%	-212.7	28%	-262.8	25%
25% Increase in machinery cost	-364.7	-37%	-414.8	-35%	-372.3	-26%	-441.4	-26%
25% Decrease in machinery cost	-166.0	37%	-198.6	35%	-220.7	26%	-260.7	26%
25% Increase in transportation cost	-287.8	-8%	-328.5	-7%	-314.0	-6%	-369.5	-5%
25% Decrease in transportation cost	-243.0	8%	-284.9	7%	-279.0	6%	-274.2	22%
25% Increase in input cost	-286.2	-8%	-327.5	-7%	-279.0	6%	-332.6	5%
25% Decrease in input cost	-244.6	8%	-285.9	7%	-267.5	10%	-322.0	8%
Stand life period 15 years	-251.4	5%	-292.7	5%	-273.5	8%	-328.1	7%
Breakeven price of coal	64.3		68.7		74.0		79.4	
Estimated carbon tax (\$ Mg <sup>-1</sup> CO <sub>2</sub> )								
West South Central <sup>b</sup> U.S.A.	10.2		11.5		13.0		14.6	
U.S.A.	7.0		8.3		9.8		11.4	

<sup>a</sup> The percentage change is calculated from the estimates of net revenue from Table II-6.

<sup>b</sup> The market price of coal in West South Central is \$28.67 Mg<sup>-1</sup>, and the average coal price in the U.S.A. is \$39.76 Mg<sup>-1</sup>.

The net revenue is sensitive to yield fluctuations with a wide range of 25-40% for different species harvest combinations. The cost of fertilizer application, harvest, and transportation that depends on the yield accounts for  $\$33 \text{ Mg}^{-1}$  and is greater than the price of the delivered biomass. If the value of biomass is less than  $\$33$ , an increase in yield would reduce the net revenue and increase the farm operator's losses. Under such price circumstances, if the perennial grass is established and the farmer cannot switch crops, the best strategy would be to not apply fertilizer, not harvest, and not transport the biomass. If the market price of biomass is less than variable production costs, increases in yield will not make biomass production feasible. The share of the machinery costs (custom rates) is high in the production of biomass, and the net revenue is sensitive (26-37%) to the changes in the custom rates, which in turn shows that net revenue will be sensitive to fuel and labor cost. The changes in input costs, transportation costs, and stand life period of grasses do have comparatively small effects on the net revenue. The value of biomass is estimated based on the energy content with reference to the energy and price of coal. The price of coal shows a wide range and differs between the regions and within the regions. With all other assumptions held constant, the price of coal at which the production of biomass breakeven ranges from  $\$64.3 \text{ Mg}^{-1}$  to  $\$79.4 \text{ Mg}^{-1}$  for different treatment combinations. For the U.S.A. average coal price of  $\$39.76 \text{ Mg}^{-1}$ , the carbon tax based on  $\text{CO}_2$  emission, required for cofiring switchgrass biomass with coal to breakeven with using only coal is estimated to be  $\$7 \text{ Mg}^{-1}$  of  $\text{CO}_2$ .

Table II-8 was prepared to illustrate the expected changes in cost to deliver feedstock resulting from changes in biomass yield.

**Table II-8. Estimated delivered costs for miscanthus and switchgrass for the mean yield obtained in the field trials and yields three standard deviations less than and greater than the mean yield.**

Species	Harvest	Yield (Mg ha <sup>-1</sup> yr <sup>-1</sup> )	Change from mean yield	Total estimated delivered cost (\$ Mg <sup>-1</sup> )	Change from mean delivered cost
Miscanthus	1	6.8 <sup>a</sup>	-45%	72.0	39%
		12.4 <sup>b</sup>		51.7	
		18.0 <sup>c</sup>	45%	44.1	-15%
Miscanthus	2	8.1	-38%	70.2	30%
		13.0		53.9	
		18.0	38%	46.6	-14%
Switchgrass	1	7.1	-55%	64.9	48%
		15.9		43.9	
		24.7	55%	37.9	-14%
Switchgrass	2	6.8	-56%	72.5	54%
		15.4		47.2	
		24.0	56%	40.0	-15%

<sup>a</sup> Mean yield obtained in the field trials minus three standard deviations (Table II-2).

<sup>b</sup> Mean yield obtained in the field trials (Table II-2).

<sup>c</sup> Mean yield obtained in the field trials plus three standard deviations (Table II-2).

Based on the field trials the mean annual yield of 15.9 Mg ha<sup>-1</sup> for switchgrass when harvested once per year has an estimated standard deviation of 2.94 Mg ha<sup>-1</sup>. By this measure a yield range from 7.1 to 24.7 Mg ha<sup>-1</sup> is expected to include the yield distribution from minus to plus three standard deviations. In the event of an extremely low yield of 7.1 Mg ha<sup>-1</sup>, the expected delivered cost increases by 48% from \$43.9 to \$64.9 Mg<sup>-1</sup>. For a biomass yield of 24.7 Mg ha<sup>-1</sup> (three standard deviations greater than the mean), the expected cost to deliver feedstock is decreased by 14% from \$43.9 to \$37.9 Mg<sup>-1</sup>. Cost to deliver biomass is more sensitive to lower yields ha<sup>-1</sup> than to greater yields.

## **Discussion**

Efforts have been underway for a number of years to develop energy crops for use as biorefinery feedstocks for the production of liquid fuels including ethanol. The development of technology required for economically viable conversion of cellulosic biomass feedstocks to ethanol has not progressed as rapidly as promised, however, substantial progress has been made in the development of switchgrass and miscanthus as dedicated energy crops. In addition to providing feedstock for lignocellulosic biorefineries, feedstocks such as switchgrass and miscanthus could be used to produce biomass for cofiring with coal in existing electric generating plants.

The technology for cofiring with lignocellulosic biomass is simple, mature, and requires only minor modifications of, and minimal investment in, existing plants. Biomass is more expensive than coal and if the externalities of burning coal are ignored, biomass feedstocks would be substantially more costly. A tax on carbon emissions could be used to provide an incentive for cofiring with biomass. This study was conducted to

determine the minimum carbon tax required for either of the two candidate feedstocks (miscanthus and switchgrass) to be economically viable alternatives for cofiring with coal.

Annual dry-matter yield and gross energy ( $\text{kJ gm}^{-1}$ ) data were produced in side-by-side experiment station trials conducted in Oklahoma U.S.A. Switchgrass with a single annual post-senescence harvest produced more biomass ( $15.87 \text{ Mg ha}^{-1} \text{ yr}^{-1}$  versus  $12.72 \text{ Mg ha}^{-1} \text{ yr}^{-1}$ ) and more energy ( $249.6 \text{ million kJ ha}^{-1} \text{ yr}^{-1}$  versus  $199.7 \text{ million kJ ha}^{-1} \text{ yr}^{-1}$ ) than miscanthus. For the average yields obtained, the estimated cost to produce and deliver biomass an average distance of 50 km was  $\$43.9 \text{ Mg}^{-1}$  for switchgrass and  $\$51.7 \text{ Mg}^{-1}$  for miscanthus. Based on the results of these field trials, the best strategy for producing biomass in the region would be to establish switchgrass and harvest once  $\text{yr}^{-1}$  after senescence.

For the U.S.A average coal price of  $\$39.76 \text{ Mg}^{-1}$ , the value of switchgrass biomass delivered to a cofiring plant as a partial substitute for coal, based on energy content is estimated to be  $\$27.1 \text{ Mg}^{-1}$ . Replacing one Mg of coal with switchgrass biomass reduces  $\text{CO}_2$  emission by 3.48 Mg. If a tax of  $\$7 \text{ Mg}^{-1}$  on  $\text{CO}_2$  were imposed, cofiring switchgrass with coal would breakeven with using only coal for the average coal price of  $\$39.76 \text{ Mg}^{-1}$ .

In this study, switchgrass produces significantly more biomass than miscanthus. The cost of producing a Mg of biomass is also substantially less for switchgrass than for miscanthus. However, in other studies conducted at different locations, miscanthus has produced more biomass than switchgrass. These findings confirm that the best species and variety differ across climate and region. Prior to establishing thousands of acres of a

perennial grass in a given region for use as a dedicated energy crop, side-by-side trials of competing species and varieties should be conducted to confirm the most economical species and variety for the region.

Finally, a comprehensive strategy to address greenhouse gas emissions would consider several additional issues. As noted, greenhouse gases other than CO<sub>2</sub> and greenhouse gases used to establish, produce, fertilize, and harvest the grasses, mine the coal, and transport the biomass and coal to the point of use were not considered. Since miscanthus must be propagated vegetatively whereas switchgrass may be propagated from seeds, it could be hypothesized that the greenhouse gas emissions from growing the two species are different. When perennial grasses are established on depleted cropland, they will sequester carbon in the soil. Public policy that taxes carbon emissions might also compensate land owners for sequestering carbon. This compensation could be used to offset some of the cost to deliver biomass feedstock. The quantity of carbon sequestered was not measured in the field trials and differences in the quantity of carbon sequestered across the species and harvest frequency were not determined. Additional research would be required to determine the overall net differences in greenhouse gas emissions between the two grasses and between the grasses and coal.



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## **PAPER III**

### **IDENTIFYING JUMPS AND SYSTEMATIC RISK IN FUTURES**

#### **Abstract**

A variety of multivariate jump-diffusion models have been suggested as models of asset prices. This paper extends the literature on (joint) mixed jump-diffusion processes in futures markets by using the CRB index futures to represent systematic risk in commodity prices. We derive (joint) mixed bivariate normal distributions and likelihood functions for estimating the parameters of jump-diffusion processes. Likelihood ratio tests are used to select among nested models. The empirical results show the presence of downside jumps and significant systematic risk in wheat futures returns. Amin and Ng's (1993) model with a single counter of jumps fits better than other jump-diffusion processes considered. The jump components did not have significantly more systematic risk than the continuous component. In terms of wheat prices, one standard deviation jumps are 14 cents per bushel and two standard deviation jumps are 29 cents per bushel and are within the price limits. These jumps occur once in every six business days and are mostly crashes.

## Introduction

Previous studies show that futures returns have occasional large movements that result in asymmetric and leptokurtic distributions (Hudson, Leuthold and Sarassoro 1987; Hall, Brorsen, and Irwin 1989; Koekebakker and Gudbrand 2004). In this paper futures returns is defined as the percentage change in value from the closing price on one trading day to the closing price on the next trading day of a single contract. Since the quantity of a contract is fixed, the percentage change in price is equal to the percentage change in value. Discontinuous jumps in asset prices and time-varying volatility models are the two main approaches used to model extraordinary discrete price movements (Eraker 2004). In Merton's (1976) jump diffusion (JD) process, these price changes occur at discrete points in time. The two weaknesses of the JD process that have been widely discussed in the literature are: (1) the model has only a single counter<sup>1</sup> of jumps and (2) the jump component represents only non-systematic risk and therefore a maintained hypothesis of the model is that all jump risk can be diversified. Empirical studies show high correlation between individual stock price volatility and market volatility (Jarrow and Rosenfeld 1984; Jorion 1988). If jumps in an individual asset are correlated with jumps in the overall market, then contrary to the maintained hypothesis of Merton's model, jump risk is systematic and could not be diversified. Amin and Ng (1993) derived an option pricing formula to account for stock return volatility that is both systematic and stochastic. In this model, the number of jumps in the consumption and asset price process are identical and are allowed to be correlated.

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<sup>1</sup> Mathematically a counter process defines the number of arrivals that have occurred in the interval  $(0,t)$ .

Camara's (2009) theoretical "two counters of jumps" model is a joint JD process of aggregate consumption and stock price. In this model, the jumps are separated into upside (bubbles) and downside (crashes) jumps with different intensity and distributional characteristics. Theoretically, Camara (2009) was able to generalize Merton (1976) and Amin and Ng (1993) using "two counters of jumps" model. In this extended model, Camara (2009) included additional jump parameters in stock price and aggregate consumption. Camara's model, however, has a potential estimation problem as it includes sixteen jump parameters along with other parameters that represent continuous price movements. As per Kou (2002) having so many parameters in the model makes calibration difficult. In addition, if the jump magnitudes are small, the separation of jumps from continuous co-movements and estimation of parameters becomes less precise (Todorov and Bollerslev 2010). To circumvent this empirical problem, this paper includes four JD processes with fewer parameters that are nested in Camara's (2009) two counters of jumps model. The criteria<sup>2</sup> that are used in the selection of models are: (1) the model should be able to explain the asymmetric and leptokurtic nature of returns, (2) the model should have an economic interpretation and practical implications, and (3) the likelihood function of the distribution should be mathematically tractable to compute the parameter estimates.

In Merton's (1976) model, jumps are firm specific and are not correlated with the stocks in general (*i.e.*, with the market). It is clear from the 1987 stock market crash that extreme events can influence all asset prices and market events. Similarly, commodities and commodity futures are systematically related to macroeconomic measures and the

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<sup>2</sup> For detailed description of criteria, see Kou (2002).

risk associated with jumps cannot be diversified (Hilliard and Reis 1999). The nature of the risk associated with jumps is not explicitly stated in Camara's (2009) model.

Restricting the two counters of jumps model to have no jumps in consumption and single counter of jumps in stock price results in the JD economy of Merton (1976) with non-systematic jumps. The JD process of Amin and Ng (1993) with systematic jumps can be obtained by assuming a single counter of jumps in consumption and stock price. In the capital asset pricing model (CAPM) only systematic risk is rewarded. The current paper includes a joint jump diffusion process that also represents the extended one factor model of Todorov and Bollerslev (2010) with returns associated with continuous and discontinuous price moves. The model also allows to test whether the betas associated with these price moves are same. This more recently described JD process that includes both systematic and non-systematic jumps was published subsequent to Camara's (2009) paper.

The inelastic demand and dependence on weather increases the chance of extraordinary price movements in agricultural commodities. Some discrete incidence of large price changes is confined to a single commodity. For example, freeze damage in wheat may cause an extraordinary price change in wheat yet have little influence on other commodity prices. These firm (commodity) specific events result in non-systematic jumps in the returns. On the other hand, influences of macroeconomic variables like exchange rate fluctuations affect prices of all commodities and are systematic. In addition, Todorov and Bollerslev (2010) argued that the precision of beta estimates increases with less incidence of non-systematic risk. The current paper extends Amin and Ng (1993) by adding an uncorrelated jump to futures prices. We extend the futures

literature by specifying a joint JD process with a single counter of jumps in market prices and two counters of jumps in futures prices. For the futures JD process, the jumps are defined with separate Poisson processes to represent systematic and non-systematic jumps. The model supports Todorov and Bollerslev's (2010) theoretical framework that extends the generic CAPM model to have two separate betas to represent the systematic risk attributable to continuous and discontinuous price moves.

The empirical objective of the paper is to select the most suitable JD process to model wheat futures prices and to estimate the systematic risk associated with the continuous and discontinuous components. In the current paper, we consider alternative stochastic processes that are nested in Camara's (2009) two counters of jumps model and estimate the parameters of the distributions. We contribute to the existing literature on futures by proposing a mixed bivariate normal distribution for simultaneously analyzing the price series. Both the continuous movements and the discrete movements in each price series are modeled using normal and Poisson process respectively. This joint JD process includes correlated and uncorrelated jumps and models the interaction between wheat futures returns and commodity market returns. In addition, the paper also estimates the parameters of mixed univariate normal distributions with single and two counters of jumps in futures prices.

Jump diffusion processes that model systematic risk has several practical applications in portfolio and credit risk management. As the jumps are correlated with large number of assets, the presence of systematic risk reduces the gains from diversification and substantially increases the probability to lose while holding highly levered positions (Das and Uppal 2004). According to Duffie and Pan (2001), systematic



movements also influence the credit risk as all credits exhibit correlated default risk during financial crisis. Duffie and Pan (2001) used a jump-conditional value at risk (VaR) weighted by the probability of a given number of jumps for the analytical approximation of VaR. Jumps across the assets are systematic even in the commodity markets that makes it extremely difficult for grain trading firms to hedge the risk during large correlated price movements.

Maximum likelihood is used to estimate the parameters of the diffusion process using the Kansas City Board of Trade (KCBT) wheat futures prices and the Commodity Research Bureau (CRB) index of futures prices. The Amin and Ng (1993) and Camara (2009) models are extensions of the consumption-based representative agent framework. In empirical analysis market prices are used instead of consumption growth (Amin and Ng, 2003). In this paper the CRB index of futures prices represent the ‘commodity market’. The empirical results show the presence of downside jumps and systematic jump risk in wheat futures prices. Amin and Ng’s (1993) model fits the data better than other JD processes considered. The differences in beta estimates of continuous and discontinuous price moves were not statistically significant and failed to support Todorov and Bollerslev’s (2010) extended CAPM framework.

### **Theoretical Model**

This section introduces JD processes followed by univariate and bivariate mixed normal distribution functions to model wheat futures prices, CRB index of futures prices and estimate the distribution parameters. All JD processes included in this section are restricted models of Camara’s (2009) two counters of jump process. We now outline the stochastic process of asset price followed by the JD process with a single counter and

then with two counters of jumps. Let the asset price ( $S_T$ ) be a stochastic process with no discrete jumps so it can be represented as

$$(1) \quad S_T = \exp\left(\ln(S_0) + \eta_s T - \frac{\sigma_s^2}{2} T + \sigma_s B_s(T)\right).$$

where  $S_0$  is the current futures price,  $B_s(T) \sim N(0, T)$  is standard Brownian motion,  $\eta_s$  is the instantaneous expected price without any jumps, and  $\sigma_s^2$  is the instantaneous variance of price without any jumps. The asset price is assumed to be non-negative and the  $T$  period return (logarithmic price relative)  $r_T = \ln(S_T/S_0)$  is normally distributed as  $r_T \sim N(\mu_f, \sigma_f^2)$  where  $\mu_f = \eta_f - 0.5\sigma_f^2$  is the mean (drift) and  $\sigma_f^2$  is the variance. In the return series, the jump magnitude is normally distributed with expected value of jump size  $\alpha_f$  and variance  $\gamma_f^2$ .

### **Univariate Mixed JD Process with Single Counter of Jumps**

The number of extraordinary price changes follows a Poisson counting process  $M(T)$  with mean arrival rate (intensity)  $\lambda$  and jump size  $Y_{s,i}$ . The magnitude of jumps can be either positive (upside jumps) or negative (downside jumps). The JD process that includes both continuous and discontinuous changes in prices is

$$(2) \quad S_T = \exp\left(\ln(S_0) + \eta_s T - \frac{\sigma_s^2}{2} T + \sigma_s B_s(T) + \sum_{i=1}^{M(T)} Y_{s,i}\right)$$

In equation (2), the Brownian motion is a continuous process and the Poisson process is discontinuous. It is assumed that  $B_s(T)$  and  $M(T)$ , and that  $Y_{s,i}$  and  $M(T)$ , and that  $B_s(T)$  and  $Y_{s,i}$  are independent. The model reduces to geometric Brownian motion when there

are no jumps. As per Jorion (1988), the probability density function of a mixed univariate normal distribution and a single counter of jumps is

$$(3) \quad f_r(r) = \sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \frac{1}{\sqrt{2\pi(\sigma_f^2 + i\gamma_f^2)}} \exp\left[-\frac{(r_t - \mu_f - i\alpha_f)^2}{2(\sigma_f^2 + i\gamma_f^2)}\right]$$

### **Univariate Mixed JD Process with Two Counters of Jumps**

Camara (2009) postulated a bivariate model with two separate Poisson distributed events to represent upside jump and downside jumps. A univariate version of this richer, potentially more realistic and less restricted jump-diffusion process with two counters of jumps is

$$(4) \quad S_T = \exp\left(\ln(S_0) + \eta_s T - \frac{\sigma_s^2}{2} T + \sigma_s B_s(T) + \sum_{i=1}^{M(T)} Y_{su,i} + \sum_{j=1}^{N(T)} Y_{sd,j}\right)$$

where  $Y_{su,i}$  and  $Y_{sd,j}$  represent the upside jump and downside jump sizes, respectively,  $M(T)$  is the Poisson counter process for upside jumps with intensity  $\lambda$ , and  $N(T)$  is the Poisson counter process of downside jumps with intensity  $\delta$ . In the price (logarithmic price relative) series, both jumps are normally distributed and can have different distributions each with different mean and variance. These price jumps can be represented as  $Y_{su,i} \sim N(\alpha_{fu}, \gamma_{fu}^2)$  and  $Y_{sd,i} \sim N(\alpha_{fd}, \gamma_{fd}^2)$  where,  $\alpha_{fu}$  and  $\alpha_{fd}$  are the means and  $\gamma_{fu}^2$  and  $\gamma_{fd}^2$  are the variances of upside and downside jumps respectively.

For empirical estimation, the mean of upside jumps is restricted to be positive and the mean of downside jumps is restricted to be negative. In this model, the jump magnitudes are allowed to be correlated only if  $i = j$  ( $cov(Y_{su,i}, Y_{sd,j}) \neq 0$  if  $i = j$ ) and

all other random variables are independent. The mixed bivariate normal pdf of returns with two counters of jumps is

$$(5) \quad f_r(r) = \sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \sum_{j=0}^{\infty} \frac{e^{-\delta} \delta^j}{j!} \frac{1}{\sqrt{2\pi(\sigma_f^2 + i\gamma_{fu}^2 + j\gamma_{fd}^2 + 2 \min(i, j) v_f)}} \exp \left[ -\frac{(r_t - \mu_f - (i\alpha_{fu} + j\alpha_{fd}))^2}{2(\sigma_f^2 + i\gamma_{fu}^2 + j\gamma_{fd}^2 + 2 \min(i, j) v_f)} \right]$$

where  $v_f$  represent the covariance of upside and downside jumps in returns. The covariance part in this mixed distribution merits more explanation. We assume that the continuous diffusion component (Brownian motion) is independent of all jumps<sup>3</sup>. The variance term of the mixed jump diffusion process is

$$(5a) \quad \text{var} \left( r_T + \sum_{i=1}^{\infty} Y_{su,i} + \sum_{j=1}^{\infty} Y_{sd,j} \right) = \text{var}(r) + \text{var} \left( \sum_{i=1}^{\infty} Y_{su,i} \right) + \text{var} \left( \sum_{j=1}^{\infty} Y_{sd,j} \right) + 2 \min(i, j) \text{cov} \left( \sum_{i=1}^{\infty} Y_{su,i}, \sum_{j=1}^{\infty} Y_{sd,j} \right)$$

$$(5b) \quad \text{var} \left( r_T + \sum_{i=1}^{\infty} Y_{su,i} + \sum_{j=1}^{\infty} Y_{sd,j} \right) = (\sigma_f^2 + i\gamma_{fu}^2 + j\gamma_{fd}^2 + 2 \min(i, j) v_f)$$

As per the additive rule of covariance  $\text{cov}(X + Y, Z) = \text{cov}(X, Z) + \text{cov}(Y, Z)$  and by induction it can be stated as follows:

$$\text{cov} \left[ \sum_{i=1}^{\infty} Y_{su,i}, \sum_{j=1}^{\infty} Y_{sd,j} \right] = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \text{cov} [Y_{su,i}, Y_{sd,j}].$$

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<sup>3</sup>  $B_s(T)$  and  $M(T)$ ,  $B_s(T)$  and  $N(T)$ ,  $B_s(T)$  and  $Y_{su,i}$ ,  $B_s(T)$  and  $Y_{sd,j}$ ,  $M(T)$  and  $Y_{su,i}$ ,  $N(T)$  and  $Y_{sd,j}$  are independent. In general the continuous components  $B(T)$ , the discrete components  $M(T)$  and  $N(T)$ , and the jump magnitudes are independent. The magnitudes of jumps ( $Y_{su,i}$  and  $Y_{sd,j}$ ) are allowed to be correlated. These assumptions are mentioned in subsequent models.

The minimum operator is to ensure equal number ( $i = j$ ) of jump magnitudes to carry out the estimation of covariance<sup>4</sup>. A JD process with separate upside and downside jumps is not a new concept in the financial literature. Kou (2002) proposed a double exponential jump diffusion (DEJD) model. In the DEJD model, the jumps are generated by a single Poisson process, and the upside and downside jump magnitudes are drawn from two independent exponential distributions. Later Ramezani and Zeng (2007) posited a Pareto-Beta jump-diffusion (PBJD) model in which the jumps are generated by two independent Poisson processes and the jump magnitudes are drawn from Pareto and Beta distributions respectively. In DEJD and PBJD, the distributions are univariate and the relationship with the market is not defined as in Camara's (2009) model. But, unlike Camara (2009), they do not restrict positive jumps to be positive and negative jumps to be negative.

### **Multivariate Mixed JD Process with Systematic Jumps**

The JD processes that are described in the above subsections were univariate processes of asset prices. In these models, the influence of aggregate consumption on asset price is not defined and implicitly assume non-systematic jumps. Camara's (2009) two counters of jumps model as a joint JD process of asset price and aggregate consumption is

$$(6) \quad S_T = \exp \left( \ln(S_0) + \eta_s T - \frac{\sigma_s^2}{2} T + \sigma_s B_s(T) + \sum_{i=1}^{M(T)} Y_{su,i} + \sum_{j=1}^{N(T)} Y_{sd,j} \right)$$

---

<sup>4</sup> The sample covariance of two variables X and Y each with sample size  $n$  is  $cov(X, Y) = \sum_i (x_i - \bar{x})(y_i - \bar{y})/n$ .

$$(7) \quad C_T = \exp \left( \ln(C_0) + \eta_c T - \frac{\sigma_c^2}{2} T + \sigma_c B_c(T) + \sum_{i=1}^{M(T)} Y_{cu,i} + \sum_{j=1}^{N(T)} Y_{cd,j} \right)$$

where  $M(T)$  is the Poisson counter process attributed to the incidence of upside jumps with intensity  $\lambda$ ,  $N(T)$  is the Poisson counter process attributed to the incidence of downside jumps with the intensity parameter  $\delta$ ,  $C_0$  is the current level of consumption,  $B_c(T)$  is the consumption Brownian motion,  $\eta_c$  is the instantaneous expected growth rate of consumption without any jumps,  $\sigma_c$  is the variance of consumption without any jumps, and  $Y_{cu,i}$  and  $Y_{cd,j}$  are the upside and downside jump magnitudes in the aggregate consumption. In this model, the aggregate consumption Brownian motion and asset price Brownian motion are correlated. The model also allows jump components to be correlated<sup>5</sup>.

Restricting Camara's (2009) model to a single counter of jumps achieves the JD economy of Amin and Ng (1993). In their model, the jumps in the asset price are correlated with the jumps in aggregate consumption. The arrival of information in both series is defined by a single Poisson counting process  $M(T)$  with intensity  $\lambda$  and the number of jumps in both series are the same. The joint mixed JD process with a single counter of jumps is

$$(8) \quad S_T = \exp \left( \ln(S_0) + \eta_s T - \frac{\sigma_s^2}{2} T + \sigma_s B_s(T) + \sum_{i=1}^{M(T)} Y_{s,i} \right)$$

$$(9) \quad C_T = \exp \left( \ln(C_0) + \eta_c T - \frac{\sigma_c^2}{2} T + \sigma_c B_c(T) + \sum_{i=1}^{M(T)} Y_{c,i} \right)$$

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<sup>5</sup> For more details of (auto)correlations between the jumps, see Camara (2009)

Aggregate consumption growth is not directly observable and is difficult to estimate. To bypass this, an index of market prices is used instead of consumption growth (Amin and Ng 1993). In this paper the CRB index of futures prices is used to represent the market. With two variables, and with a combination of continuous (normal) and discontinuous (Poisson) processes, a mixed bivariate normal distribution is used to examine the JD processes. The joint mixed density function, defined as an extension of univariate distribution is

$$(10) \quad f_{r,m}(r, m) = \sum_{i=1}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \frac{1}{2\pi \sqrt{(\sigma_f^2 + i\gamma_f^2) * (\sigma_m^2 + i\gamma_m^2)} \sqrt{(1-r^2)}} \left( \exp \left[ -\frac{Z}{2(1-r^2)} \right] \right)$$

$$(10a) \quad Z = \frac{(r_t - \mu_f - i\alpha_f)^2}{(\sigma_f + i\gamma_f)^2} - \frac{2r(r_t - \mu_f - i\alpha_f)(m_t - \mu_m - i\alpha_m)}{(\sigma_f + i\gamma_f) * (\sigma_m + i\gamma_m)} + \frac{(m_t - \mu_m - i\alpha_m)^2}{(\sigma_m + i\gamma_m)^2}$$

$$(10b) \quad r = \frac{(\rho_{fc}\sigma_f\sigma_m + \rho_{jp}\gamma_f\gamma_m i)}{\sqrt{(\sigma_f^2 + i\gamma_f^2)(\sigma_m^2 + i\gamma_m^2)}}$$

where  $\mu_m$  and  $\sigma_m^2$  are the instantaneous mean and variance of market returns without any jumps,  $\alpha_m$  is the mean and  $\gamma_m^2$  is the variance of jumps in the market returns,  $Z/(1-r^2)$  is the square of Mahalanobis distance<sup>6</sup> of the multivariate distribution,  $r$  is the correlation

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<sup>6</sup> If  $\mathbf{y}$  has a multivariate normal distribution with covariance matrix  $\mathbf{\Sigma}$  and mean vector  $\boldsymbol{\mu}$ , the density is given by  $g(\mathbf{y}) = \frac{1}{(2\pi)^p |\mathbf{\Sigma}|^{1/2}} e^{-(\mathbf{y}-\boldsymbol{\mu})' |\mathbf{\Sigma}|^{-1} (\mathbf{y}-\boldsymbol{\mu})/2}$ . The exponent term  $(\mathbf{y} - \boldsymbol{\mu})' |\mathbf{\Sigma}|^{-1} (\mathbf{y} - \boldsymbol{\mu})$  is the squared generalized distance from  $\mathbf{y}$  to  $\boldsymbol{\mu}$  otherwise known as Mahalanobis distance (Rencher, 2002).

coefficient between the two returns,  $\rho_{fc}$  is the correlation between returns conditional on zero jumps, and  $\rho_{jp}$  is the correlation between the jumps in both returns.

### **Multivariate Mixed JD Process with Systematic and Non-systematic Jumps**

The generic one factor model that generalizes the popular CAPM model can be stated as  $r_i = \alpha_i + \beta_i r_0 + \epsilon_i$ , where  $i=\{1, \dots, N\}$ ,  $r_i$  is the returns on the  $i^{\text{th}}$  asset,  $r_0$  is the systematic risk factor, and  $\epsilon_i$  is the idiosyncratic risk that is uncorrelated with  $r_0$ .<sup>7</sup>

Todorov and Bollerslev (2010) separated the systematic risk factor associated with continuous ( $r_0^c$ ) and discontinuous ( $r_0^d$ ) price moves. The extended one factor model can be stated as  $r_i = \alpha_i + \beta_i^c r_0^c + \beta_i^d r_0^d + \epsilon_i$ ,  $i=\{1, \dots, N\}$  where  $\beta_i^c$  and  $\beta_i^d$  represent the systematic risk attributable to continuous and discontinuous price moves, respectively.

This paper introduces a new diffusion process that separates the systematic risk and non-systematic risk associated with jumps in futures prices. In this model, the uncorrelated jump represents only non-systematic discontinuous price moves whereas in Todorov and Bollerslev's (2010) model  $\epsilon_i$  represents the idiosyncratic risk of both continuous and discontinuous price moves. The joint diffusion process that explains systematic jump risk is as follows:

$$(11) \quad S_T = \exp \left( \ln(S_0) + \eta_s T - \frac{\sigma_s^2}{2} T + \sigma_s B_s(T) + \sum_{j=1}^{M(T)} Y_{sc,j} + \sum_{k=1}^{K(T)} Y_{su,k} \right)$$

$$(12) \quad C_T = \exp \left( \ln(C_0) + \eta_c T - \frac{\sigma_c^2}{2} T + \sigma_c B_c(T) + \sum_{j=1}^{M(T)} Y_{c,j} \right)$$

---

<sup>7</sup> For more details see Todorov and Bollerslev (2009).



where  $Y_{sc,j}$  represents the jump in asset price that is correlated with the market jumps ( $Y_{c,j}$ ), the Poisson counting process that is attributed to the incidence of correlated jumps is  $M(T)$  with intensity  $\theta$ . The non-systematic jump magnitude in the asset prices ( $Y_{su,k}$ ) is modeled as the second Poisson counting process  $K(T)$  in the futures prices with an intensity parameter  $\omega$ . The correlated jump and uncorrelated jump are analogous to  $r_0^d$  and  $\epsilon_i$  respectively. Hence, in equation (11), it is assumed that the magnitude of systematic and non-systematic asset price jumps are not correlated ( $\text{cov}\left[\sum_{j=1}^{M(T)} Y_{sc,j}, \sum_{k=1}^{K(T)} Y_{su,k}\right] = 0$ ). The corresponding joint mixed bivariate normal distribution is

$$(13) \quad f_{r,m}(r, m) = \sum_{j=1}^{\infty} \frac{e^{-\theta} \theta^j}{j!} \sum_{k=1}^{\infty} \frac{e^{-\omega} \omega^k}{k!}$$

$$\frac{1}{2\pi \sqrt{(\sigma_f^2 + j\gamma_{fc}^2 + k\gamma_{fu}^2)(\sigma_m^2 + j\gamma_m^2)} \sqrt{(1-r^2)}} \exp\left[-\frac{Z}{2(1-r^2)}\right]$$

$$(13a) \quad Z = \frac{(x_f - \mu_f - j\alpha_{fc} - k\alpha_{fu})^2}{(\sigma_f + j\gamma_{fc} + k\gamma_{fu})^2} -$$

$$\frac{2r(x_f - \mu_{fc} - j\alpha_{fc} - k\alpha_{fu})(x_m - \mu_m - j\alpha_m)}{(\sigma_f + j\gamma_f + k\gamma_{fu})(\sigma_m + j\gamma_m)} + \frac{(x_m - \mu_m - j\alpha_m)^2}{(\sigma_m + j\gamma_m)^2}$$

$$(13b) \quad r = \frac{\rho_{fc}\sigma_f\sigma_m + \rho_{jp}\gamma_{fc}\gamma_m j}{\sqrt{(\sigma_f^2 + j\gamma_{fc}^2 + k\gamma_{fu}^2)} \sqrt{(\sigma_m^2 + j\gamma_m^2)}}$$

where  $\alpha_{fc}$  and  $\alpha_{fu}$  are means,  $\gamma_{fc}$  and  $\gamma_{fu}$  are the variance of correlated and uncorrelated jumps. The betas associated with continuous price moves and discontinuous price moves are  $\beta_{fc} = \rho_{fc}(\sigma_f/\sigma_m)$  and  $\beta_{jp} = \rho_{jp}(\gamma_{fc}/\gamma_m)$  respectively. The likelihood expressions for the distribution functions are given in Appendix 1.

## Data and Procedure

Summary statistics were estimated and normality tests were performed for both returns series. The parameters of four increasingly general JD processes were estimated using numerical maximization of likelihood functions. Two univariate mixed normal distributions are estimated separately for each returns series with single and two counters of jumps. Likelihood ratio (LR) tests are used to select among nested models. Finally, the parameters of a joint mixed bivariate distribution with correlated single counter of jumps in both returns, and the model with separate systematic and non-systematic jumps in wheat futures prices are also estimated. Proc NLMIXED in SAS is used to estimate the parameters (SAS 2009). To optimize the likelihood function, the infinite sum has to be truncated to provide accuracy for parameter estimates (Jorion 1998). In this paper, the infinite sum is truncated at ten. An extensive Monte Carlo simulation was performed to ensure accuracy and reliability of statistical programs and procedures. Data were simulated (100,000 observations) from the JD models known assumed parameter values. After the likelihood optimization, the resulting values are identical or sufficiently close to the assumed values.

The Kansas City Board of Trade (KCBT 2010) daily settlement wheat futures prices were used for a period of six years (January 2003 to December 2008). Five wheat futures contract maturity months (March, May, July, September, and December) are traded. The last trading day is the business day preceding the fifteenth calendar day of the liquidating month. The first delivery day is the first business day of the liquidating month. The delivery mechanism is physical by using registered warehouse receipts from elevators. A time-series dataset is constructed with the daily prices of actively traded

futures contract nearest to the twentieth calendar day of the month prior to delivery. From the twenty-first calendar day onwards, the price of the next contract closest to the delivery month is included in the dataset. This procedure reduces the maturity effect by only including contracts close to maturity. It also avoids the delivery period where position limits are removed and markets get thin. Differencing is performed before splicing the data for rollover to avoid creating outliers at the rollover.

A synchronous data set of the Thomson Reuters/Jefferies CRB index is constructed for the same time period. The CRB index is composed of four groups of nineteen components that include petroleum products, metals, and agricultural commodities. In the current study, CRB index futures represent the ‘commodity market’ (aggregate consumption) and the logarithmic price relatives as market returns (aggregate consumption growth). Wheat, with an index weight of 1%, is in group IV along with nickel, lean hogs, orange juice, and silver. The six expiration months traded are January, February, April, June, August, and November. Actively traded futures are selected based on the volume traded. The sixth calendar day of the month prior to the maturity month is selected as the rollover day. The daily closing price of index futures is used to estimate trading day to trading day changes in the value of the underlying assets. The differencing to calculate returns is performed before splicing the data. Both the CRB and wheat series include 1,380 observations of daily prices so as to provide enough degrees of freedom to use tests that are asymptotically valid. To reduce scaling problems, differences in daily prices are multiplied by 100 and expressed in percentage terms.

## Empirical Results

Table III-1 presents summary statistics and tests of normality. The positive excess kurtosis in both series indicates the presence of fat tails. The skewness of the wheat futures price differences series is close to zero while the CRB index shows considerable negative skewness.

**Table VII-1. Summary statistics and normality tests for the returns**

Parameters	Wheat Futures	CRB index
Minimum change in daily value	-8.451	-6.406
Maximum change in daily value	7.778	5.429
Mean change in daily value	0.023 <sup>a</sup>	0.009
Variance	3.636	1.049
Skewness	0.049	-0.755
Excess kurtosis	1.735	5.137
Kolmogorov-Smirnov D statistic	0.045	0.076
	(<0.01) <sup>b</sup>	(<0.01)
Shapiro-Wilk W statistic	0.981	0.932
	(<0.0001)	(<0.0001)

<sup>a</sup> The returns are expressed in percentages and the number of observations is 1,380. The beginning date of the data series is January 2003 and the ending date is December 2008.

<sup>b</sup> P-values are in the parentheses.

The normality tests (Kolmogorov-Smirnov D statistic and Shapiro-Wilk W statistic) reject the null hypotheses at conventional levels of significance and make it clear that the returns are not normally distributed. The estimates of second and third moments and results of normality tests suggest that a JD process can provide a better fit for the returns than a normal distribution.

The parameter estimates and standard errors of jump diffusion processes with a single counter of jumps (equations (2) and (3)) are displayed in table III-2. The jump intensity parameter is significant in both returns.

**Table VII-2. Parameter estimates of the distribution with single counters of jumps**

Parameter	Symbol	Wheat Futures	CRB index
Mean returns	$\mu$	0.004 <sup>a</sup> (0.060) <sup>b</sup>	0.082 <sup>**</sup> (0.024)
Volatility of returns	$\sigma^2$	2.001 <sup>**</sup> (0.235)	0.535 <sup>**</sup> (0.036)
Intensity of jumps	$\lambda$	0.283 <sup>**</sup> (0.108)	0.129 <sup>**</sup> (0.031)
Mean of jumps	$\alpha$	0.066 (0.199)	-0.564 <sup>**</sup> (0.204)
Volatility of jumps	$\gamma^2$	5.820 <sup>**</sup> (1.665)	3.710 <sup>**</sup> (0.793)
Loglikelihood value		2804.73	1854.27
Bayesian information criterion		5645.60	3744.70

<sup>a</sup> The day to day changes in value are expressed in percentages.

<sup>b</sup> Standard errors are in the parentheses

\* significant at 5% level

\*\* significant at 1% level

The parameter estimate of jump intensity ( $\lambda = 0.283$ ) indicates the presence of jumps in wheat futures returns that occur approximately once in every 4 business days ( $\lambda^{-1} = 3.53$ ). As the jump component is defined using a single counter of jumps, the jump magnitude can be either positive or negative. The sample path of wheat futures shows the presence of high intensity small-sized jumps. Even though jumps occur often in wheat futures, the upside jumps cancel with downside jumps resulting in a statistically insignificant mean (0.066%) and statistically significant variance (5.82%). The volatility associated with the jump component (5.82%) is higher than the diffusion component (2%) in wheat futures returns. In CRB index futures returns, all parameter estimates are statistically significant. The index returns shows the presence of a small number ( $\lambda = 0.129$ ) of large ( $\alpha = -0.56\%$ ) jumps. Jumps in the CRB index occur approximately once in 8 days ( $\lambda^{-1} = 7.75$ ) and are less frequent compared to wheat futures.

The parameter estimates of the normal distribution with two counters of jumps (equations (4) and (5)) are presented in Table III-3. A likelihood ratio (LR) test fails to reject the null hypothesis of a single jump versus the alternative of two counters of jumps for wheat futures returns ( $\chi^2_{(4)}$  statistic is 1.05 and p-value is 0.9) and CRB index futures returns ( $\chi^2_{(4)}$  statistic is 0.66 and p-value is 0.95). The incidences of upside jumps ( $\lambda = 0.063$  and  $\lambda^{-1} = 15.87$ ) and downside jumps ( $\delta = 0.032$  and  $\lambda^{-1} = 31.25$ ) are once in 16 days and 31 days respectively. Finally, four additional parameters associated with the jump component in the model changes the mean and variance of diffusion process in wheat futures returns.

**Table VII-3. Parameter estimates of the distribution with two counters of jumps**

Parameter	Symbol	Wheat Futures	CRB index
Mean of returns	$\mu$	-0.025 <sup>a</sup> (0.242) <sup>b</sup>	0.082** (0.024)
Volatility of returns	$\sigma^2$	2.292** (0.580)	0.402** (0.097)
Intensity of upside jumps	$\lambda$	0.063 (0.255)	0.593 (0.493)
Intensity of downside jumps	$\delta$	0.032* (0.014)	0.102** (0.034)
Mean of upside jumps	$\alpha_u$	3.015** (8.010)	0.000 -
Mean of down side jumps	$\alpha_d$	-4.467** (0.682)	-0.695* (0.277)
Volatility of upside jumps	$\gamma_u^2$	2.472 (9.019)	0.304 (0.197)
Volatility of downside jumps	$\gamma_d^2$	0.0003 (0.064)	4.142 (2.421)
Covariance of jumps	$v$	0.028 (2.856)	0.002 (2.396)
Loglikelihood value		2803.68	1853.61
Bayesian information criterion		5672.40	3772.30

<sup>a</sup> The returns are expressed in percentages.

<sup>b</sup> Standard errors are in the parentheses.

\* significant at 5% level.

\*\* significant at 1% level.

Table III-4 presents the parameter estimates of bivariate normal distribution (equations (8), (9), and (10)) with a single counter of jumps in wheat price futures and CRB index futures.

**Table VII-4. Parameter estimates of bivariate distribution with single counters of jumps**

Parameter	Symbol	Estimate
Mean of wheat futures return	$\mu_f$	0.044 <sup>a</sup> (0.052) <sup>b</sup>
Mean CRB index futures return	$\mu_m$	0.068** (0.024)
Mean of wheat futures jump	$\alpha_f$	-0.133 (0.272)
Mean of CRB index jump	$\alpha_m$	-0.370* (0.155)
Volatility of wheat futures returns	$\sigma_f^2$	2.383** (0.144)
Volatility of CRB index futures	$\sigma_m^2$	0.519** (0.032)
Intensity of jumps	$\lambda$	0.158** (0.041)
Volatility of wheat futures jump	$\gamma_f^2$	7.948** (1.500)
Volatility of CRB index futures jump	$\gamma_m^2$	3.221** (0.61)
Correlation between futures and CRB index	$\rho_{fc}$	0.335** (0.031)
Correlation between jumps	$\rho_{jp}$	0.602** (0.064)
Beta of continuous components	$\beta_{fc}$	0.718** (0.070)
Beta of discontinuous (jump)components	$\beta_{jp}$	0.945** (0.123)
Loglikelihood value		4522.11
Bayesian information criterion		9123.8

<sup>a</sup> The returns are expressed in percentages.

<sup>b</sup> Standard errors are in the parentheses.

\* significant at 5% level.

\*\* significant at 1% level.

All parameter estimates except the means of the diffusion component and the jump component are statistically significant. At first glance itself, it is possible to notice frequent jumps in returns ( $\lambda = 0.158$  and  $\lambda^{-1} = 6.33$ ) that occur once in 6 days. The intensity estimate is close to the estimate of univariate single counter of JD process in CRB index futures. During the study period the price of wheat (mean) is estimated as \$5.06 per bushel. It can be estimated that one standard deviation jumps are 14 cents bushel<sup>-1</sup> and two standard deviation jumps are 29 cents bushel<sup>-1</sup>. These estimates are reasonable and are within the price limits (30 cents bushel<sup>-1</sup>).

As per equations (8) and (9), a jump is identified only when there is a simultaneous price movement of extraordinary magnitude in both return series. It appears from the mean of jumps in wheat futures ( $\alpha_f = -0.133$ ) and CRB index futures ( $\alpha_m = -0.37$ ) that the jumps are mostly crashes. The estimate of correlation between the diffusion components of wheat futures and CRB index futures returns ( $\rho_{fc} = 0.335$ ) shows less correlation than the jump components ( $\rho_{jp} = 0.602$ ). The estimates of beta show that the estimated systematic risk associated with jump components ( $\beta_{jp} = 0.945$ ) is higher than the estimated systematic risk associated with continuous components ( $\beta_{fc} = 0.718$ ). The result of a Wald t-test, however, indicates that the difference in the estimated betas is not statistically significant (t-value = 1.46, p-value = 0.145).

Table III-5 discusses the estimated bivariate distribution with added non-systematic jumps in wheat futures returns. The magnitude of mean and variance of the continuous component changed with an additional jump in wheat futures while they remained almost the same for the CRB index futures.



**Table VII-5. Parameter estimates of bivariate distribution with systematic and non-systematic risk**

Parameter	Symbol	Estimate
Mean of wheat futures return	$\mu_f$	-0.098 <sup>a</sup> (0.132) <sup>b</sup>
Mean CRB index futures return	$\mu_m$	0.069** (0.023)
Intensity of correlated jumps	$\theta$	0.130** (0.027)
Intensity of uncorrelated jumps	$\omega$	0.709 (0.420)
Mean of correlated wheat futures jump	$\alpha_{fc}$	-0.417 (0.364)
Mean of uncorrelated wheat futures jump	$\alpha_{fu}$	0.247 (0.171)
Mean of CRB index futures jump	$\alpha_m$	-0.464* (0.190)
Volatility of wheat futures returns	$\sigma_f^2$	1.597** (0.357)
Volatility of CRB index futures returns	$\sigma_m^2$	0.534** (0.033)
Volatility of correlated wheat futures jump	$\gamma_{fc}^2$	7.067 (1.627)
Volatility of uncorrelated wheat futures jump	$\gamma_{fu}^2$	1.472** (0.614)
Volatility of CRB index futures jump	$\gamma_m^2$	3.788** (0.765)
Correlation between wheat futures and CRB index	$\rho_{fc}$	0.419** (0.056)
Correlation between jumps	$\rho_{jp}$	0.653** (0.241)
Beta of continuous components	$\beta_{fc}$	0.725 (0.069)
Beta of discontinuous (jump) components	$\beta_{jp}$	0.891 (0.126)
Loglikelihood value		4518.02
Bayesian information criterion		9137.20

<sup>a</sup> The returns are expressed in percentages

<sup>b</sup> Standard errors are in the parentheses

\* significant at 5% level

\*\* significant at 1% level

The intensity of the correlated jump ( $\theta = 0.13$  and  $\theta^{-1} = 7.69$ ) is close to the intensity of the jump in the bivariate single counter of jumps model in Table 4. The intensity of correlated jumps is highly significant and reveals that systematic jumps occur once every 8 days. The correlated jumps are mostly crashes with a mean of -0.42% in wheat futures and -0.46% in CRB index futures. There is a wide gap between the correlation coefficient of the continuous component ( $\rho_{fc} = 0.419$ ) and the jump component ( $\rho_{jp} = 0.653$ ). In terms of wheat futures prices, correlated jumps of one and two standard deviations are 13 cents bushel<sup>-1</sup> and 27 cents bushel<sup>-1</sup> respectively. In the case of uncorrelated jumps, it can be estimated that the jumps are of smaller magnitudes (6 cents bushel<sup>-1</sup> and 12 cents bushel<sup>-1</sup> respectively). The additional non-systematic jump in the wheat futures made no significant change in the magnitudes of beta estimates ( $\beta_{jp} = 0.891$  and  $\beta_{fc} = 0.725$ ). The result of t-test indicate that the difference in the estimated betas are not statistically significant (p-value = 0.145).

A likelihood ratio test is employed to select among the nested models. The LR test fails to reject ( $\chi^2_{(3)}$  statistic is 4.09 and p-value is 0.25) the null hypothesis of imposed restrictions ( $\omega = 0, \alpha_{fu} = 0, \text{ and } \gamma_{fu} = 0$ ) and showed that the bivariate distribution with a single counter of jumps (Amin and Ng, 2003) in both the wheat market and CRB index fits the data better than the model with two counters of jumps in the wheat futures market. The overall results including the summary statistics and the univariate models with single and two counters of jumps indicate that there are few jumps in the wheat futures returns to estimate several parameters of the jump component. The estimated skewness and excess kurtosis do not deviate as much from normality as does the CRB index.

## Conclusions

Camara's (2009) two counters of jumps model generalizes Merton's (1976) JD process that incorporates discontinuous jumps in asset price and the Amin and Ng (1993) model. The current paper contributes to the existing literature on the distribution of changes in futures prices by employing a mixed bivariate normal distribution and estimates the parameters using wheat futures prices and CRB index futures. The paper also extends Camara's (2009) model by defining jumps as systematic and non-systematic. The empirical analysis shows that on average crashes occur in wheat futures once every 6 days. In terms of wheat prices, one standard deviation jumps are 14 cents per bushel and two standard deviation jumps are 29 cents per bushel and are within the price limits. The high correlation between the jumps in wheat futures and CRB index futures indicates the presence of systematic risk associated with jumps. Camara's (2009) generalization of two counters of jumps based on the jump magnitudes is not suitable for wheat futures data. Camara's (2009) distinction of upside and downside jumps was implemented by imposing bounds. As the bounds are active, Camara's (2009) distinction between upside and downside jumps does not match these data. In general, the results support Amin and Ng (1993) joint JD process with a single counter of jumps. The magnitudes of the betas associated with the continuous and jump components were not statistically different in both joint JD processes. An important limitation with these models is that the jumps arrive at constant intensity. The clusters of upside and downside jumps and their temporal dependence cannot be studied using these distributions.

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**PAPER III APPENDIX**

**Maximum Likelihood Functions**

The appendix describes the maximum likelihood estimation method used in this paper. To a considerable extent the expressions of likelihood functions closely follow Jorion (1988). Five different sets of density function parameters are estimated. If  $r_1, r_2, \dots, r_T$  are continuous random variables that represent the normally distributed wheat futures returns  $r_T \sim N(\mu_f, \sigma_f^2)$  then the logarithm of the likelihood function as a function of parameter vector  $\theta = (\mu_f, \sigma_f^2)$  for a normal distribution can be written as

$$(A1) \quad l(\theta) = -\frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left[ \frac{1}{\sqrt{\sigma_f^2}} \exp \left( -\frac{(f_t - \mu_f)^2}{2\sigma_f^2} \right) \right]$$

With a single counter of jumps, the log likelihood function for the mixed jump-diffusion process (equation 3) with parameter vector  $\theta = (\mu_f, \sigma_f^2, \lambda, \alpha_f, \gamma_f^2)$  can be written as

$$(A2) \quad l_i(\theta) = -T\lambda - \frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left[ \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} \frac{1}{\sqrt{\sigma_f^2 + i\gamma_f^2}} \exp \left( -\frac{(f_t - \mu_f - i\alpha_f)^2}{2(\sigma_f^2 + i\gamma_f^2)} \right) \right]$$

With two counters of jumps the logarithm of the likelihood function for the mixed jump-diffusion process (equation 5) with parameter vector  $\theta = (\mu_f, \sigma_f^2, \lambda, \delta, \alpha_{fu}, \alpha_{fd}, \gamma_{fu}^2, \gamma_{fd}^2, \nu)$  can be written as

$$(A3) \quad l_{ij}(\theta) = -T(\lambda + \delta) - \frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left[ \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} \sum_{j=0}^{\infty} \frac{\delta^j}{j!} \frac{1}{\sqrt{\sigma_f^2 + i\gamma_{fu}^2 + j\gamma_{fd}^2 + 2 \min(i, j) v_f}} \exp \left( -\frac{(f_t - \mu_f - (i\alpha_{fu} + j\alpha_{fd}))^2}{2(\sigma_f^2 + i\gamma_{fu}^2 + j\gamma_{fd}^2 + 2 \min(i, j) v_f)} \right) \right]$$

With bivariate normal distribution logarithm of likelihood function for the mixed jump-diffusion process (equation 6) with parameter vector  $\theta = (\mu_f, \sigma_f^2, \lambda, \alpha_f, \gamma_f^2, \sigma_m^2, \gamma_m^2, r)$  can be written as

$$(A4) \quad l_j(\theta) = -T\lambda - T \ln(2\pi) + \sum_{t=1}^T \ln \left[ \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} \frac{1}{\sqrt{(\sigma_f^2 + i\gamma_f^2)(\sigma_m^2 + i\gamma_m^2)\sqrt{(1-r^2)}}} \exp \left( -\frac{Z}{2(1-r^2)} \right) \right]$$

$$Z = \frac{(x_f - \mu_f - i\alpha_f)^2}{(\sigma_f + i\gamma_f)^2} - \frac{2r(x_f - \mu_f - i\alpha_f)(x_m - \mu_m - i\alpha_m)}{(\sigma_f + i\gamma_f)(\sigma_m + i\gamma_m)} + \frac{(x_m - \mu_m - i\alpha_m)^2}{(\sigma_m + i\gamma_m)^2}$$

∞

With bivariate normal distribution with separate systematic and non-systematic jump risk, the logarithm of likelihood function for the mixed jump-diffusion process (equation 9,10, and 11) with parameter vector  $\theta = (\mu_f, \mu_c, \sigma_f^2, \sigma_m^2, \theta, \omega, \alpha_{fc}, \alpha_{fd}, \alpha_c, \gamma_{fc}^2, \gamma_{fd}^2, r)$  can be written as

$$(A5) \quad l_j(\theta) = -T(\theta + \omega) - T \ln(2\pi) + \sum_{t=1}^T \ln \left[ \sum_{i=0}^{\infty} \frac{\theta^i}{i!} \sum_{k=0}^{\infty} \frac{\omega^k}{k!} \frac{1}{2\pi \sqrt{(\sigma_f^2 + j\gamma_{fc}^2 + k\gamma_{fd}^2)(\sigma_m^2 + j\gamma_m^2)\sqrt{(1-r^2)}}} \exp \left[ -\frac{Z}{2(1-r^2)} \right] \right]$$

$$Z = \left[ \frac{(x_f - \mu_f - j\alpha_{fc} - k\alpha_{fd})^2}{(\sigma_f + j\gamma_{fc} + k\gamma_{fd})^2} - \frac{2r(x_f - \mu_f - j\alpha_{fc} - k\alpha_{fd})(x_m - \mu_m - j\alpha_m)}{(\sigma_f + j\gamma_{fc} + k\gamma_{fd})(\sigma_m + j\gamma_m)} + \frac{(x_m - \mu_m - j\alpha_m)^2}{(\sigma_m + j\gamma_m)^2} \right] \text{ and } r = \frac{\rho_{fc}\sigma_f\sigma_m + \rho_{jump}\gamma_f\gamma_m j}{\sqrt{(\sigma_f^2 + j\gamma_{fc}^2 + k\gamma_{fd}^2)*(\sigma_m^2 + j\gamma_m^2)}}$$



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