NEURAL NETWORKS AS FORECASTING EXPERTS: TEST OF DYNAMIC MODELING OVER TIME SERIES DATA

By

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CHAPTER I

INTRODUCTION

This research tests the forecasting potential of neural networks in a problem domain where forecasting techniques have received significant attentions: time series prediction. It is observed from our empirical tests that simple neural networks are able to perform as good as conventional, complex statistical forecasting methods. The ability of neural networks to forecast in a fuzzy sense seems to be more appropriate than the other forecasting methods. Our present study was limited to univariate forecasting, but it is expected that neural networks can be used for multivariate forecasting, which is, again, a more real-world situation.

Prediction models which change their underlying structure according to the environment (adaptive) are of considerable practical interest. In many cases conventional approaches to forecasting use a linear predictive method. These linear predictive methods do not perform as desired when used to process data from a nonlinear system. Rumelhart, Hinton and Williams (1986) provide natural extension of linear methods into nonlinear domain by the backpropagation algorithm. It is shown that neural networks without hidden layers and linear input/output elements are equivalent to linear predictive models. When a non-polynomial activation function is used for the hidden layer units, the ability of neural networks to handle nonlinear data is extended. The non-linearity can be controlled by backpropagation learning algorithms, which allows for prediction of nonlinear time series better than conventional linear predictive methods.

Often time series appearing to be random are actually closer to pseudo random numbers than stochastic random numbers. Pseudo randomness has a deterministic underlying map and has a deterministic non-linearity. Such time series is observed in many microeconomic, macroeconomic, industry, and demographic observations. We have taken a large sample of such data series from M-111 Competition (Makridakis 1982) to test the forecasting ability of neural networks. As there is a deterministic underlying map in this type of pseudo random data, nonlinear neural networks can perform well because they can extract and approximate these underlying maps.

The data requirements for accurate prediction is one of the important facts which can be compared between conventional linear prediction methods and neural networks. It is seen that neural networks are parsimonious in their data requirements. This thesis provides tests over a large sample (111) of time series about the ability of nonlinear neural networks as forecasting experts. To test the feasibility of this method, real-world data sets from M-111 Competition (Makridakis 1982) are tested. Appendix-F gives the comparison of forecasting performance of neural networks and Box-Jenkins (1986) ARIMA (Auto-Regressive, Integrated, Moving-Average) models. Details about the data series are listed in Appendix-H.

Our major concern in this thesis is to see if neural networks can be used to build real-world predictive models. In this regard it is necessary to consider, how likely it is that the model is true empirically; how we can measure the parameters of the model; and if the model is true, how it can be used in real-life as the environment changes in order to get the best result. Our empirical results show that the answer to the first question is yes; in most of the cases in our test, neural networks gave an acceptable performance. The second question is answered by analyzing the underlying structure of a trained neural network, the results are discussed in Chapter III, for the third question, the trained neural network can be used to dynamically update its underlying weight structure as the environment changes.

Statement of the Problem

The problem of prediction or forecasting can be stated as:

$$\hat{y}(t+1) = \mathbf{F}(\mathbf{X}(t), \mathbf{b}) \tag{1}$$

where;

$\hat{y}(t+1)$ is simply the best prediction one could make for $y(t+1)$ from historical observations, $t = 1$ through T;

$\mathbf{F}$ is the vector valued function containing parameter $\mathbf{b}$;

$\mathbf{X}(t)$ is the vector of inputs for observation at time t.

The objective in the prediction problem is to find the vector-valued function, $\mathbf{F}$, containing parameter $\mathbf{b}$, which minimizes error in predicting $\hat{y}(t+1)$. The error measure is selected and minimized, which leads to minimum error when $\mathbf{F}$ is used to predict $\hat{y}(t+1)$ where only $\mathbf{X}(t)$ is shown.

Our objective here is to build an *adaptive* system which learns to predict or simulate its environment, as a part of a larger system of adaptive intelligence or optimization.

In our present study $\mathbf{X}(t)$ is a single time series variable (univariate modeling). The number of data points available in $\mathbf{X}(t)$ is one of the important factor for building its predictive model. Here variable $\mathbf{X}(t)$ is taken from a wide variety of domains of real-world observations. Building predictive models of $\mathbf{X}(t)$ with real-world time series and with simulated time series are two significantly different problems. When $\mathbf{X}(t)$ is over practical observations, limitations of the number of data points available is a major problem in building its predictive model. In case of simulated time series large number of observations can be generated. The point here is, the available amount of information may or may not covers the complete dynamic range of the domain, or in other words the available information may or may not be a good statistical sample of the domain. This shows that building predictive models over real-world time series observations is far more difficult than for simulated time series.

Ordinary Regression

Given the values of variable X over some observation for a particular period starting at t and ending at p;

$$\mathbf{X}_t = A_1\mathbf{X}_{t-1} + \dots + A_p\mathbf{X}_{t-p} + E_t \qquad (2)$$

A_1, A_2, ..., A_p are constants; E_t is the error.

In such case $\mathbf{X}_t$ is directly related to one or more past series values.

The above prediction formulation is completely deterministic. This type of predictive model is central to the concept of statistics. In real-life, the predictive power of a forecasting model is more important than its statistical truth. There is always a certain amount of unpredictable noise in these types of systems. However, if we want more powerful models, we will often find that we have to estimate constants which do not simply multiply an expression.

Nonlinear Signal Processing

In any nonlinear prediction problem, the core idea is to determine the vector mapping function **F** from p dimensions to q dimensions of a nonlinear process from a sample of N data points, which may or may not cover the complete dynamic range of the input domain.

$$\mathbf{F}(p) = q \qquad (3)$$

In such a situation, domain p and range q data vectors are transposed and concatenated as N rows to form the matrices X and Y. If conventional linear regression is used to represent this nonlinear association, it requires higher degree terms and causes instability.

It is well known (Lapedes & Farber 1987a,b; Sutton 1988) that nonlinear neural networks can learn arbitrary vector mapping functions from the input (domain vector, p) through the hidden (internal representation, h) layer to the output layer (range vector, q). If the domain set of data vectors used to train the network is adequate, the network generalizes or predicts from an arbitrary input space in a stable fashion.

Problem Structure Similarity with Neural Networks

In neural network training, using the delta rule (Werbos 1974, Rumelhart et. al. 1986, Hinton 1985), the target vector is a vector to be reproduced or predicted by the network. Basically the delta rule tries to estimate **b** so as to improve the prediction in (1). In that case, **F** happens to be represented as a neural network of elementary units, the weights can be interpreted as **b**, and the observations $\mathbf{X}(t)$ may be interpreted as an pattern presented to the system where the objective is to determine $\hat{y}(t+1)$. The goal is to find the values of weights **b** which best fit the historical or training data in (1). This is done by the backpropagation algorithm. Backpropagation algorithm is a natural extension of linear methods into non-linear domains. Using backpropagation learning algorithm to approximate input-output mapping is called training. In backpropagation learning algorithm, the information processing is the approximation of a mapping or function $\mathbf{F}$: $A \subset R^n \rightarrow R^m$, from a bounded subset A of n-dimensional Euclidean space to a bounded subset $\mathbf{F}[A]$ of m-dimensional Euclidean space, by means of training samples $(X_1,Y_1),(X_2,Y_2)...(X_k,Y_k),...$ of the mapping's action where $Y_k = \mathbf{F}(X_k)$. It is assumed that such samples of a mapping **F** are generated by selecting X_k vectors randomly from A in accordance with fixed probability density function P(x) (Rumelhart et.al 1986).

The auto-associative memory of certain neural network models can be tapped in prediction problems. Smolensky (1986) specifies a dynamic feed-forward network in the following way:

$$u_i(t + 1) = \mathbf{F}(\Sigma_k W_{ki} G(u_k(t))) \tag{4}$$

where,

$u_i(t)$ is the activation of unit i at time t;

F is a nonlinear sigmoid transfer function;

G is a nonlinear threshold function;

W_{ki} is the connection strength or weight from unit k to unit i.

This relationship can, in principle at least, be used for predicting future values of variables.

When ordinary regression (least squares) is used to estimate a model which forecasts variables at time $t + 1$ as a function of time t, then the forecasts for several steps ahead will tend to deteriorate, due to cumulative error effects (Werbos 1988). Looking at the time series past data, the forecasting model should be able to forecast the future values at greater accuracy; at the same time, the forecast should be consistent. The problem of steady forecast is more critical due to random noise. The ability of neural networks to "generalization" and "abstraction" can be used for generating steady forecasts for different classes (monthly, quarterly, annually, etc.) of time series and can be compared with some traditional methods of forecasting as ARIMA.

A backpropagation learning algorithm for neural networks is developed into a formalism for nonlinear signal processing. We, by empirical tests, demonstrate that prediction problems in univariate nonlinear domains can be handled well by using neural networks. We demonstrate the use of the formalism in nonlinear system modeling by providing a graphic example in which it is clear that the neural network has approximated the non-linearity of the input data set. It is interesting to note that the formalism provides an explicit, analytic, global approximation to the nonlinear maps underlying the various time series.

The major difference between neural network modeling and conventional prediction modeling is that the conventional approach to processing nonlinear signals would be to form polynomials representing non-linearity in the data and adjust the linear weight factor coefficients for each polynomial term using the least mean square algorithm. All non-polynomial non-linearities must be modeled by polynomials, which is undesirable; also the number of coefficients grows as the system size, or the order of polynomials is increased (Lapedes & Farber 1987a). Polynomial approximation is unstable under iteration. The problem is to achieve accurate predictions over long periods of time. Acceptable accuracy can be achieved by polynomial approximations over short times, which is a less interesting situation in comparison with long term prediction.

Learning

Artificial neural networks can modify their behavior response according to the environment. Shown the set of inputs (perhaps with desired output), they self-adjust (learn to approximate the input-output mapping) to produce consistent responses. This ability to modify behavior comes from the ability to learn, which is achieved by dynamically changing the underlying structure. The ability to change this underlying structure depends on network topology and network dynamics. Network topology and the learning parameters decide the final behavior of the network after training. These parameters define the surface which partitions the mapping hyperspace which finally decides the behavior of the trained network (Alexis, W., et.al 1987). Mapping hyperspace can be visualized as N-dimensional hypercube (Amit, D., 1989). If the input and output mapping needed is linear, the mapping hyperspace required to approximate this mapping can be linear. In this case a set of polynomials define this mapping space. If the input-output mapping is nonlinear the required mapping partitioning hypercube is also nonlinear. This non-linearity can be achieved by joining large number of linear segments each corresponding to a polynomial, this will need large number of polynomials (weights) and implementation and instability problems may arise. The number of dimensions and the non-linearity in the partitioning hypercube needed to achieve the required nonlinear mapping of input-output space are dependent on the degree of non-linearity and the input and output space dimensions.

In neural networks input and output space dimensions are decided by the number of input and output neurons respectively. The dimension of the mapping hypercube is decided by the number of weights in the network, which describes the polynomial and non-polynomial non-linearities of the partitioning hyperspace.

The learning process can best be visualized as curve fitting (Amit, D., 1989). While learning, the network is continuously trying to modify the hyperspace partitioning curve so that it best fits the hyperspace partitioning surface required for interpolating the unseen data to produce accurate predictions. Network topology can consist of all linear or nonlinear elements.

Learning takes place by changing the weights, which dynamically stretch or compress different axis of the mapping hyperspace. The threshold ϕ_i, in the case of thresholding units, corresponds to the minimum distance the partitioning hyperspace passes from the origin (Amit, D., 1989). If the problem is linearly separable, coming up with specific values of weights for networks with one linear element, can be used to solve the problem. Sigmoidal units are more powerful than thresholding units. In the case of thresholding units, as they are binary, the relative magnitude of the weights is significant but the absolute magnitude is not. However, Sigmoidal units are different due to the non-linearity involved. The absolute magnitude is of importance because it can be anywhere in the non-linear portion of the sigmoidal transfer function, producing different activation. Because of the non-linear, but continuous nature of the sigmoid activation function, it is possible to create nonlinear curves in partitioning the networks hyperspace. Nonlinear curves can be created by sending the output of one nonlinear unit to another nonlinear unit (Alexis, W., et.al 1987). Using thresholding units if curved partitioning of the hyperspace is required, the only way to do this is by joining large numbers of linear segments. With nonlinear units this can be achieved in a more accurate and parsimonious way. This is why learning in nonlinear neural networks is more powerful than linear neural networks.

<u>Generalization</u>

Once trained, a neural network response can be, to a degree, insensitive to minor variations in its input. This ability to see through noise and distortion into the pattern that lies within, is vital to pattern recognition in the real-world environment. It is important to note that the neural network generalizes automatically as a result of its structure, not by using human intelligence embedded in the form of ad hoc computer algorithms. This ability of linear/nonlinear interpolation in linear/nonlinear neural networks is of considerable importance when compared with other interpolation techniques.

The network is said to generalize well when the input to output relationship is nearly correct for patterns never used in training the network. The generalization process here can be thought as of interpolation. Conventional forecasting techniques like ARIMA use linear interpolation to predict. These techniques give good prediction if the input-output relationships are linear.

A nonlinear neural network can be considered simply as a nonlinear mapping of arbitrary inputs to output space. Viewing learning as curve fitting allows us to understand that generalization is simply a non-linear interpolation of data (Alexis, W., et.al 1987). In many cases it happens that, due to the properties of training data and network topology, that the network memorizes rather than generalizes. Usually, networks with too few neurons, when used for highly nonlinear mapping, may memorize the training patterns and eventually fail to generalize (Alexis, W., et.al 1987). In the memorization process the networks fail to encode the fuzzy relationship between input and output data, instead it encodes the information as a lookup table. Fuzzy interpolation between input and output data is of vital importance in forecasting problems. This is what is some times referred to as the neural networks ability to see through noise.

Abstraction

Some artificial neural networks are capable of abstracting the essence of a set of input data. This ability of extracting idealized prototypes is highly useful and is the same as the ability in humans. The abstraction process takes place during learning through a process similar to curve fitting (Amit, D., 1989). The process of abstraction is to approximate the underlying map in the training data. Fuzzy abstractions of the underlying map, rather than exact mapping, allow prediction systems to interpolate the outputs in a fuzzy way. No abstraction takes place in the process of memorization. In Chapter III, results of a simple test are discussed which demonstrate the ability of neural networks to approximate the underlying map from the training data. It should be emphasized that a major task of any theory of neural networks is to produce exceptional input-output relationships. According to Professor Amit (1989) the attractive features of neural networks are biological plausibility; associativity; parallel processing; potential for abstraction.

Network Complexity and Number of Hidden Neurons

In out tests, it was observed that the number of hidden neurons required is problem dependent. The choice of transfer function to the hidden neurons is also problem dependent. If the underlying map in the data series is simple to capture or there exists a linear relationship between the input and output mapping, simple neural network without any hidden layers and neurons with linear transfer function is sufficient to approximate this mapping. After training the network the overall transfer function of the network can be visualized as a hyperspace partitioning (Alexis, W., et.al 1987) surface which eventually does the input and output mapping (interpolation). If the underlying map in the training set is highly nonlinear, this separation surface required is also highly nonlinear, whereas if the underlying map is simple and linear then the partitioning surface required is simple. To build up the nonlinear separation surface we need to use nonlinear elements (transfer function), this is done by using hidden layer neurons with nonlinear transfer function. Each transfer function non-linearity contributes in building this partitioning surface (Alexis, W., et.al 1987). Similarly the simple and linear separation surface can be build up using linear transfer function elements, which build up the separation surface by joining linear segments. Obviously if the underlying map is highly nonlinear this linear separation surface cannot categorize the input and output mapping properly. While training is in progress this partitioning surface is continuously changing its shape in order to fit the best possible required mapping.

Complexity of the underlying map in the training set decides the complexity of the overall transfer function of the network, which is decided by the number of the nonlinear elements in the networks hidden layer, (as hidden layer is the internal representation layer). As each of these hidden layer unit contributes to the non-linearity towards building up the separation surface, the number of hidden neurons is a function of complexity of the underlying map in the training set.

$$h_n = g(C_T) \tag{5}$$

where, C_T is the complexity of the underlying map in the training data. Unfortunately, there is no easy way to define the C_T and find the exact function g, nevertheless it is observed that the number of hidden neurons needed to capture a highly nonlinear underlying map from the data are more than the number needed to model a linear map. The obvious question then is, how is the performance of highly nonlinear neural network (many nonlinear hidden neurons) over a training set with linear or simple map. Our empirical results show that using a complex network (more hidden neurons) to model a simple data series may degrade the performance. In conclusion, it very important to study the data set in order to find the neural network with an optimal number of hidden neurons for the best performance.

Network Complexity and Transfer Function

As the number of hidden neurons used is proportional to ability of neural network to capture the underlying map in the training set, and, as each element transfer function contributes to this ability, it is obvious that the ability to model complex underlying map is also proportional to the non-linearity of the transfer function. Each hidden neuron transfer function contributes to the mapping process by adding a nonlinear segment to the overall hyperspace partitioning surface. The more nonlinear the transfer function is, the more nonlinear is the surface segment contributed by each neuron. Sigmoidal non-linearity is chosen because the artificial neuron is a simplified model of real neuron, and real neurons possess a transfer function resembling to a sigmoid function. Sigmoidal units are more complex but at the same time are more powerful than thresholding units. The fact that it has a real non-zero derivative makes it useful in gradient descent learning method. If computational efficiency is not of primary importance, it is possible that a highly nonlinear transfer function can be used to model the same amount of non-linearity, with fewer number of hidden neurons as compared with the sigmoidal transfer function.

$$h_n = q(\mathbf{F}) \tag{6}$$

where, q is the function which describes the relation between the number of hidden neurons h_n, and the non-linearity of the transfer function $\mathbf{F}$.

Unfortunately, in most of the cases the number of hidden neurons are decided empirically due to the difficulty encountered in deciding the functions g and q described above.

Learning Parameters and Neural Network Performance

One of the most important aspects of the backpropagation algorithm are learning parameters. While training is in progress these parameters decide the amount of weight changes. Again, as different data sets have different underlying maps, different values of learning parameters are suitable for different problems. The problem then is how to decide the optimal learning parameters to model the required input output mapping. Each data set may define a totally different energy surface, and unfortunately due to the problems of the gradient descent method, which do not guarantee the global minimum, there is every possibility that modeling a data set needs specific learning rate and momentum parameters to achieve optimal modeling. In our study the data sets used were from real-world observations and had different properties. On an average comparison, learning rate=0.1, and momentum=0.1, was found optimal. These values were obvious, because over variety of data sets on average performance, if a networks are trained till 1000 cycles, lower values of learning parameters give a more systematic and safe way to converge. With 1000 training cycles and lower values of learning parameters, it is obvious that the final distance from the global minimum, will be higher than the distance from global minimum if higher values of learning parameters are used. As observed from Chart 5, higher values of learning parameters are not suitable in most of the cases.

Considering the energy surface with hills and valleys, higher learning rate can be visualized as the march if a dynamical process with larger steps. Marching with larger steps may skip the valley points and this may lead gradient descent approach into oscillations. If the energy surface is flat higher learning rate is harmless and may increase the speed of convergence. Higher learning rate and higher momentum is like marching at a higher speed with larger steps. Higher learning rate and lower momentum is like marching with larger steps but with slow motion. If the speed of convergence is not of primary importance small steps with slow motion approach i.e low learning rate and low momentum, gives a systematic and safe way to converge. In many cases higher learning parameter values are used to speed up the learning process. As our main interest here was to see the performance of neural networks as predictive models and not the speed of convergence, low values of learning parameters were used.

Origin, Relevant Studies and Justification

Werbos (1988) states that he laid foundations for use of backpropagation in forecasting in his doctoral dissertation in 1974 (Werbos 1974), in which he called it dynamic feedback. The algorithm is similar to the algorithm suggested by Rumelhart, Hinton, and Williams (1986).

In neural network training, using the delta rule, the target vector is a vector to be reproduced or predicted by the network. In fact, the delta rule tries to estimate **b** to improve the prediction in (1). In that case **F** happens to be represented as a neural network of elementary units, the connection weights can be interpreted as **b**, and the observations **X**(t) may be interpreted as patterns presented to the system. The goal here is to find the values of weights **b** which best fit the historical or training data in (1). This formulation describes most of the current research in neural networks. Rumelhart, Hinton and Williams (1986) take **F** as a feed-forward network. A feed-forward network is in which the interactions between the formal neurons are such that the neurons can be divided into groups (layers) and neural activities in one group can only influence the future activities of neuron in consecutive layers.

According to Werbos (1987) an adaptive intelligence system does not assume the availability of a "target vector" and would not limit itself to a single standard design. He suggests five possible network architectures--two of them supervised (use of target vector) and three of them unsupervised (only input vector)--building up to three network architectures, which he recommends as the canonical design for coping with the problems of dynamic modelling or forecasting over time (Werbos, 1988). Werbos (1987) states that a simple backpropagation algorithm and feedforward network structure with supervised learning does not lead to the best forecast over time, especially if one is concerned with prediction over more than one period into the future. He has obtained better results by using estimation methods which explicitly represent the notion of forecasting over time. Werbos (1977) describes a complete crisis-management system which exploits the wide variety of forecasting methods involving man-machine interaction, human intuition, etc. He explains the need for computer forecasting in the classical sense, in which the computer fits a quantitative model to a well-structured numerical data bank. Werbos (1987) explains how adaptive systems may be built and understood by extending control theory and statistics. Werbos (1988) summarizes a more general formulation of backpropagation, developed in 1974, which does more justice to the root of methods in numerical analysis and statistics. He discusses applications of backpropagation to forecasting over time, to optimization, to sensitivity analysis, and to brain research. The author derives a generalization of backpropagation to recurrent system, such as hybrids of perceptron-style networks and Grossberg/Hopfield networks.

Lapedes and Farber (1987) used a multilayered perceptron to predict the values of a nonlinear dynamic system with chaotic behavior. They illustrated the method by selecting two common topics in signal processing, prediction and system modelling, and showed that the nonlinear applications can be handled extremely well by using nonlinear neural networks. They reported that neural networks gave superior prediction for their dynamic system.

Sutton (1988) introduces a class of incremental learning procedures specialized for prediction--that is, for using experience with an incompletely known system to predict its future behavior. Whereas conventional prediction-learning methods assign credit by the difference between predicted and actual results, the method used by the author assigns credit by the difference between temporally successive predictions. The author also proves the convergence and optimality for special cases, relates them to supervised-learning methods, and claims that the temporal-difference method requires less memory and less peak computation than conventional methods and produces more accurate predictions. He argues that most problems to which supervised learning is now applied are really prediction problems of some sort to which temporal-difference methods can be applied to advantage.

Fishwick (1989) has compared the performance of neural networks with traditional methods for geometric data fitting such as linear regression and surface response methods. He claims that neural network models appear to be inferior in most respects and hypothesizes that accuracy problems arise, primarily because the neural network model does not capture the system structure characteristics of all physical models.

Minor (1989) suggests similarity least squares (SMILES) as an alternative to high degree polynomial regression models for analysis of nonlinear data. Local or non-parametric regression prevalent in the statistics literature are special cases of the more general SMILES estimator. The method is developed as a series of statistical projections. The results are used to construct the feed-forward neural network and prove their capability to represent vector mapping functions between two sets of vectors. A key result is how to achieve stable predictions (generalization) for highly nonlinear data.

Sharda and Patil (1990) have reported the results of an empirical test on neural networks which shows that neural networks can be used for time series forecasting at least for a single period forecast. The authors tested and compared a sample of 75 M-Competition data series over annual, quarterly and monthly observations using neural network models and traditional Box-Jenkins forecasting models. The simple neural network models tested on 75 M-Competition data series could forecast about as well as an automatic Box-Jenkins ARIMA models.

Tang, Z., et. al (1990) discuss the results of a comparative study of the performance of neural networks and conventional methods in forecasting time series. Authors experimented with three time series of different complexity using different feedforward, backpropagation models and Box-Jenkins model. Authors experiments demonstrated that for time series with long memory, both methods produced comparable results. However, for series with short memory, neural networks outperform the Box-Jenkins model. Authors conclude that neural networks are robust, parsimonious in their data requirements and provide good long-term forecasting.

Objective of the Study

Lapedes and Farber (1987) used simulated data with a nonlinear map to test the mapping ability of neural network. Tang, Z., et.al. (1990) used three real-world time series of different complexity to demonstrate the forecasting ability of neural networks at multiple forecast periods. Sharda & Patil (1990) demonstrate the single step forecasting ability of neural networks using 75 real-world time series. On the basis of these studies, the objectives of this study are: (i) To test the forecasting ability of neural networks over multiple periods with a large sample (111) of real-world time series and to observe the effect of different architectural and learning parameters (neural network architecture, # of input and hidden neurons in a particular architecture, learning rate, momentum etc.) on the performance, (ii) To compare the performance of neural network models with traditional time series forecasting (ARIMA) models, and (iii) Study the underlying model developed by the neural network. This objective involves the study of functional equivalence between neural networks and conventional ARIMA models.

CHAPTER II

MODEL DEVELOPMENT AND METHODOLOGY

Functional Problem and Neural Networks

As stated in (1) before the prediction problem is:

$$\hat{y}(t + 1) = \mathbf{F}(\mathbf{X}(t), \mathbf{b})$$

where,

$\hat{y}(t+1)$ is simply the best prediction one could make for $y(t+1)$ from historical observations $t = 1$ through T;

$\mathbf{F}$ is the vector value function containing parameter $\mathbf{b}$,

$\mathbf{X}(t)$ is the vector of inputs for observation or time number t .

The objective in the prediction problem is to find the vector-valued function, $\mathbf{F}$, containing parameter $\mathbf{b}$, which minimizes error in predicting $\hat{y}(t+1)$. The error measure is selected and minimized, which leads to minimum error when $\mathbf{F}$ is used to predict $\hat{y}(t+1)$ where only $\mathbf{X}(t)$ is shown.

The ability to learn is one of the distinguished features of neural networks. Neural networks are not programmed the same way other intelligent artificial systems are programmed. They work even when given incomplete information. The information is stored as patterns, not as series of information bits as in normal computers. Just as one cannot dissect the brain and extract the knowledge that was there, designers of neural networks cannot simply see what is stored there. The goal of neural networks is to mimic what the brain does best: associative reasoning, learning, and thoughts.

Data Series Selection and Classification

The data series selected are from the famous M-111 Competition, Makridakis (1982) to compare the performance of the various forecasting techniques. Out of 1001 series collected, only 111 series were analyzed in M-111 Competition using Box-Jenkins methodology (Makridakis et.al. 1982). Although the sample is not random, in a statistical sense, efforts were made to select time series covering a wide spectrum of possibilities. This included different sources of statistical data and different starting/ending dates. There were also data from firms, industries and nations. All the data series are graphically displayed in Appendix-H. The series in this 111 sample were every ninth entry from 1001 series starting with series 4 (a randomly selected starting point): 4, 13, 22, 31...994. The type of series (macro, micro, industry, demographic), time intervals between successive observations (monthly, quarterly, yearly), and the number of observations were recorded. Pack and Downing (1983) examined this 111 series subset and concluded that several series were not appropriate for forecasting using the Box-Jenkins technique. For our previous work, Sharda &

Patil (1990) we took a sample of 75 series after considering Pack and Dowining's recommendation and neural networks were trained on these data series. All 111 data series are used in the present study. The set contains 13 annual, 20 quarterly and 68 monthly series. Few data series have too few data points are not modeled. Series numbers ≤ 112 are annual, series numbers > 112 and ≤ 382 are quarterly and the rest are monthly.

The most convenient way to understand the behavior of a time series is to display it, not in tabular form, but in a graph. Graphs of time series provide visual insight into how the process, or activity it represents, has behaved historically (Hoff, J., 1983). All the time series data is graphically displayed in Appendix-H to give an idea about the properties of each time series used in out test. Table 33 show the ARIMA models of these series using the Box-Jenkins method.

Method of Analysis

As mentioned earlier, 111 data sets were analyzed using two approaches. For each data set, *n-k* observations were used to build the forecast model (to train the network), and then the model (the trained neural network) was used to forecast the future *k* values, where k = 6, 8, 18 for annual, quarterly and monthly series respectively. These values are well established for such comparisons in the forecasting literature. In our previous study (Sharda & Patil 1990) only single-step forecasts were generated, in present study forecasts till 18, 8, 6, step were generated for monthly, quarterly, and annually data series respectively. In case of single-step forecast, at the end of period *n-k, the n-k+1* value was forecast; at the end of *n-k+1* period, the *n-k+2* value was forecast and so on. In the case of a 2 step forecast, at the end of *n-k* period, *n-k+1* and *n-k+2* values were forecast; at the end of *n-k+1* period, *n-k+2* and *n-k+3* values are forecast and so on. Similar formulation was used in other cases of 4, 6, 8, 12 and 18-step forecasting.

These series were run in AUTOBOX using its default setting with no intervention detection. In addition to the MAPE measure, the Median APE measure was calculated which gave the most descriptive measure of the central tendency of errors in the forecasting competition.

The generated forecasts were compared with the actual values for the *k* periods, and mean absolute percent error (MAPE) was computed for each series as:

$$MAPE_i = \left(\sum_{j=1}^{k} |(A_{ij} - P_{ij})/A_{ij}| * 100\right) / k \qquad (8)$$

Box-Jenkins Models

This approach to time series forecasting is well known (Box & Jenkins 1976), and has been applied in practice. It is considered to be a 'sophisticated' approach to forecasting. This technique is well documented in Box & Jenkins (1976). Briefly the method consists of identifying and estimating, within a wide class of stochastics processes called ARIMA models, one of which best fits a time series. The application of method requires a cycle of four steps: data transformation, model identification, parameter estimation, and diagnostic checking. In terms of difficulty, it is viewed as one of the more complex times series modeling techniques.

Essentially the analyst examines both the auto and partial auto-correlations and identifies models of the form:

$$\phi(B)\Phi(B^s)\nabla^d\nabla^D_s\,(Z_t\text{-}c) = \theta(B)\Theta(B^s)a_t \qquad (7)$$

where,

B is the back-shift operator (i.e $Bx_t = x_{t-1}$)

∇ = 1-B; s=seasonality; a_t=white noise;

$\phi(B)$ and $\Phi(B^s)$ are nonseasonal and seasonal auto-regressive polynomials respectively;

$\theta(B)$ and $\Theta(B^s)$ are nonseasonal and seasonal moving average polynomials respectively;

Z_t = series (transformed if necessary) to be modeled.

After identification of several candidate models, the analyst can iterate through the process of estimation and perform diagnostic-checking. Once the final model has been selected, the forecasting process can begin.

The process of model identification, estimation and diagnostics-checking has been automated and is available in the form of a forecasting expert system. The performance of such an automatic "expert" system has been reported to be comparable to real experts, Sharda & Ireland (1987). For our tests, we used AUTOBOX (AFS Inc, 1988), an automatic Box-Jenkins modeling expert system. This program can take a data set and iterate through the model identification, estimation and diagnostics process to develop the best forecasting model.

Neural Network Models

A backpropagation rule was used to train a multilayered perceptron network. In our previous work (Sharda & Patil 1990), we used one hidden layer. The number of neurons in the input layer was decided to provide the two years of history to predict the immediate next value. In the case of monthly data series the number of input neurons was 24, for quarterly series it was 8, and for annual series models it was 2. The single hidden layer consisted of the same number of neurons as the input layer.

Different architectures, with increasing number of hidden layer neurons, were trained over different values of learning parameters to find the optimal learning parameters first and then find the optimal architecture for the given class of data series. MAPE (Mean Absolute Percent Error) and Me-APE (Median Absolute Percent Error) were used as the measure of performance. Nine combinations of three learning rates (0.1, 0.5, 0.9) and three momentum values (0.1, 0.5, 0.9) were tried.

For annual data series the forecast was limited to a maximum of 6 periods ahead, 8 for quarterly and 18 for monthly. The following description is related to specific models for annual, quarterly and annually data series. Once the optimal learning parameters and architecture was decided these parameters were used in modeling remaining data series.

Models for Annual Data series

The forecast period selected for these series were 1, 2, 4, and 6 periods ahead. With these forecasting steps different architectures used were:

for n + *f* step forecast, where *f* is the forecast step,

2-1-1, 2-2-1, 2-3-1, 2-4-1, *f* = 1;

2-1-2, 2-2-2, 2-3-2, 2-4-2, *f* = 2;

2-1-4, 2-2-4, 2-3-4, 2-4-4, *f* = 4;

2-1-6, 2-2-6, 2-3-6, 2-4-6, *f* = 6;

Neural network architecture 2-4-6 indicates, 2 input neurons, 4 hidden neurons and 6 output neurons. Neural networks with these different architectures were trained over 9 different sets of learning parameters (learning rate, momentum) given above, over all annual data series, to see the effect of learning parameters and architecture. After analyzing these results the optimal learning parameters and the architecture were decided.

Models for Quarterly Data Series

The forecast periods selected for these series were 1, 2, 4, 6, and 8 periods ahead. With these forecasting steps two configurations were selected as:

(1) To predict the next step(s), last 4 quarters of information was used, i.e 4 input neurons.

(2) To predict the next step(s), last 8 quarters of information was used, i.e 8 input neurons.

This was done to see the effect of the amount of history on the forecasting performance. Different architectures used were:

for n + *f* step forecast, where *f* is the forecast step,

4-2-1, 4-4-1, 4-6-1, 4-8-1, and 8-4-1, 8-8-1, 8-12-1, 8-16-1 for *f* = 1;

4-2-2, 4-4-2, 4-6-2, 4-8-2, and 8-4-2, 8-8-2, 8-12-2, 8-16-2 for *f* = 2;

4-2-4, 4-4-4, 4-6-4, 4-8-4, and 8-4-4, 8-8-4, 8-12-4, 8-16-4 for *f* = 4;

4-2-6, 4-4-6, 4-6-6, 4-8-6, and 8-4-6, 8-8-6, 8-12-6, 8-16-6 for *f* = 6;

4-2-8, 4-4-8, 4-6-8, 4-8-8, and 8-4-8, 8-8-8, 8-12-8, 8-16-8 for *f* = 8;

Ten data series were trained with different learning parameters. Analyzing the performance, the optimal learning and architectural parameters were selected to model remaining data series.

Models for Monthly Data Series

The forecast periods selected for these series were 1, 2, 4, 6, 8, 12 and 18 periods ahead. With these forecasting steps two configurations were selected as:

(1) To predict the next step(s), last 12 months of information was used, i.e 12 input neurons.

(2) To predict the next step(s), last 24 months of information was used, i.e 24 input neurons.

This was done to see the effect of amount of history used to forecast the variable. Different architectures used were as:

for n + *f* step forecast, where *f* is the forecast step, 12-6-1, 12-12-1, 12-16-1, 12-24-1 and 24-12-1, 24-18-1, 24-24-1 for $f = 1$;

After observing the results of single step forecasting it was observed that the increase in hidden neurons does not improve the performance. For further tests of multiple forecasts following configurations were used:

12-12-2 and 24-24-2 for $f = 2$;

12-12-4 and 24-24-4 for $f = 4$;

12-12-6 and 24-24-6 for $f = 6$;

12-12-8 and 24-24-8 for $f = 8$;

12-12-12 and 24-24-12 for $f = 12$

Ten data series were trained over all of the above architectures and all learning parameter sets.

Once the data series were trained, over different architectures and learning parameter sets, the optimal learning parameters and architecture were found. These parameters were then taken to model remaining data series. All the models discussed above were nonlinear neural network models Each hidden neuron and output neuron had a nonlinear sigmoidal transfer function.

Method of Neural Network Modeling

To build a forecasting model for a particular data series having n data points, first all n data points were used to generate another data file whose format was suitable for neural network training. This file was an unnormalized data file. If the original data file had n data points the unnormalized data file had:

$$totpat = datapts - (lead + past) + 1 \quad (9)$$

where,

totpat are the number of patterns in unnormalized data file;

datapts is the number of data points in the original data file.

lead is the lead time for forecast as 1, 2, 4, 6, for yearly; 1, 2, 4, 6, 8 for quarterly and 1, 2, 4, 6, 8, 12, 18 for monthly dat series. which equals the number of output neurons in a particular model;

past is the number of past data points used to generate the forecast which equals the number of input neurons in the model, as 2 for yearly; 4 & 8 for quarterly anf 12,.24 for monthly.

Here one pattern consists of input and output values (supervised learning). The number of input values and output values in a pattern depends on the neural network architecture being trained. The unnormalized file was then normalized by row. While normalizing the file two different data files were created; training file and testing file. The format of these two files were same as the unnormalized data file except each number was now normalized using the minimum and maximum value over the corresponding complete pattern. Each pattern had different minimum and maximum values. While normalizing the output was written to two different files. The number of patterns written to training file and testing file were:

# of test patterns = 18, 8, 6 for monthly, quarterly and yearly data series respectively.

# of training patterns = totpat - # of test patterns.

The normalizing technique over the testing file was slightly different than for the training file. In training file the minimum and maximum was found over each complete pattern (input and output part) and each number in the pattern (input and output) was normalized using:

$$normalized\ number = (number - min)/(max - min) \quad (10)$$

Note that maximum and minimum values are over all numbers of input and output part of each training pattern and each pattern was normalized by its own minimum and maximum values. In the case of test file the maximum and minimum values were only over the input part of the testing patterns, because the output part of the pattern is not supposed to be known but is needed in the pattern for calculating the pattern error and total sum of squares error (tss) while testing. The same formula was used for the normalization of test file. The maximum and minimum values were saved only over the test file for denormalization purpose.

Once the above described preprocessing was done the neural network model was trained with learning rate=0.1 and momentum=0.1. These values were found to be optimal from the tests. Maximum cycles trained were 1000. Training was stopped if total sum of squares error reached 0.04 before training reached 1000 cycles. Model which converged in less than 1000 cycles, the total sum of squares error over all the training patterns was less than 0.04. In cases where models did not converge the maximum training cycles were 1000 and the total sum of squares error over all the training pattern is given as *model error* with respective series in tables given in Appendix-B, C, D. Same seed was used in all our tests to initialize the network in order to facilitate more appropriate comparison of their performances

Once a network was trained, the test file was tested over the network and the activation values of output neurons were logged into a file and were denormalized using the normalization parameters saved during normalization of test file. The log file was the forecast generated by the trained network using the test file.

The next step was to compare the denormalized forecast with the actual values and calculate MAPE (Mean Absolute Percent Error) and Median APE (Median Absolute Percent Error). The method of calculating these errors in two different cases of 1 step forecast- (monthly data series) using 12 past values and 18 step forecast (monthly data series) are explained.

Case 1. One-step forecast (monthly data): If the original data series had *n* data points, the training file had *n-(1+12)-1* patterns. Thus the test file had all 18 values to be forecast in its output section. Note that none of these values was present in the training file.

past=1, forecast=1 step

----------input pattern---------- output pattern

Training file:

3	5	7	9	1	4	5	7	8	2	4	5	7
5	7	9	1	4	5	7	8	2	4	5	7	8
7	9	1	4	5	7	8	2	4	5	7	8	9
9	1	4	5	7	8	2	4	5	7	8	9	3
1	4	5	7	8	2	4	5	7	8	9	3	4
4	5	7	8	2	4	5	7	8	9	3	4	1
5	7	8	2	4	5	7	8	9	3	4	1	3
7	8	2	4	5	7	8	9	3	4	1	3	6
8	2	4	5	7	8	9	3	4	1	3	6	8
2	4	5	7	8	9	3	4	1	3	6	8	1

Test file:

4	5	7	8	9	3	4	1	3	6	8	1	2
5	7	8	9	3	4	1	3	6	8	1	2	4
7	8	9	3	4	1	3	6	8	1	2	4	2
8	9	3	4	1	3	6	8	1	2	4	2	1
9	3	4	1	3	6	8	1	2	4	2	1	3
3	4	1	3	6	8	1	2	4	2	1	3	7
4	1	3	6	8	1	2	4	2	1	3	7	8
1	3	6	8	1	2	4	2	1	3	7	8	5
3	6	8	1	2	4	2	1	3	7	8	5	6
6	8	1	2	4	2	1	3	7	8	5	6	2
8	1	2	4	2	1	3	7	8	5	6	2	3
1	2	4	2	1	3	7	8	5	6	2	3	9
2	4	2	1	3	7	8	5	6	2	3	9	4
4	2	1	3	7	8	5	6	2	3	9	4	5
2	1	3	7	8	5	6	2	3	9	4	5	6
1	3	7	8	5	6	2	3	9	4	5	6	1
3	7	8	5	6	2	3	9	4	5	6	1	2
7	8	5	6	2	3	9	4	5	6	1	2	8

Case 2. 18-step forecast (monthly data, multiple step forecast): If the original data series had n data points, the training file had n-(18+12)-1 patterns. The test file had 18 patterns. It is observed that in the test file output section, the upper matrix contains some values which are in the training file output section. As the training file output section is used for training it is necessary to calculate the error only over the lower part of the output section of the test file where all the values are forecasts and none of which are used in training process. The output section may not remain as MxM matrix all the time because here in the case of 18-step forecast it will be an 18x18 matrix but in the case of 6-step forecast with month data it will be an 18x6 matrix (monthly forecast). The upper matrix in this case is the upper section of the 6x6 matrix which is the top part of an 18x6 matrix has values which also appear in output section of training file. In such cases calculations of errors over the forecasts are over the values excluding the upper matrix of this 6x6 matrix from the test output. The boldface values shown are the values which appear in the test file and training file as well. While calculating the error values, only values which are real forecasts were used, as calculating forecast error over the values which appear in training file is an incorrect formulation.

past=12, forecast=18 step

Training file:

----------input pattern------------	--------------------output pattern-------------------
3 5 7 9 1 4 5 7 8 2 4 5	7 8 9 3 4 1 3 6 8 1 2 4 2 1 3 7 8 **2**
5 7 9 1 4 5 7 8 2 4 5 7	8 9 3 4 1 3 6 8 1 2 4 2 1 3 7 8 **2 2**
7 9 1 4 5 7 8 2 4 5 7 8	9 3 4 1 3 6 8 1 2 4 2 1 3 7 8 **2 2 9**
9 1 4 5 7 8 2 4 5 7 8 9	3 4 1 3 6 8 1 2 4 2 1 3 7 8 **2 2 9 3**
1 4 5 7 8 2 4 5 7 8 9 3	4 1 3 6 8 1 2 4 2 1 3 7 8 **2 2 9 3 6**
4 5 7 8 2 4 5 7 8 9 3 4	1 3 6 8 1 2 4 2 1 3 7 8 **2 2 9 3 6 1**
5 7 8 2 4 5 7 8 9 3 4 1	3 6 8 1 2 4 2 1 3 7 8 **2 2 9 3 6 1 7**
7 8 2 4 5 7 8 9 3 4 1 3	6 8 1 2 4 2 1 3 7 8 **2 2 9 3 6 1 7 1**
8 2 4 5 7 8 9 3 4 1 3 6	8 1 2 4 2 1 3 7 8 **2 2 9 3 6 1 7 1 3**
2 4 5 7 8 9 3 4 1 3 6 8	1 2 4 2 1 3 7 8 **2 2 9 3 6 1 7 1 3 7**
4 5 7 8 9 3 4 1 3 6 8 1	2 4 2 1 3 7 8 **2 2 9 3 6 1 7 1 3 7 8**
5 7 8 9 3 4 1 3 6 8 1 2	4 2 1 3 7 8 **2 2 9 3 6 1 7 1 3 7 8 1**
7 8 9 3 4 1 3 6 8 1 2 4	2 1 3 7 8 **2 2 9 3 6 1 7 1 3 7 8 1 2**
8 9 3 4 1 3 6 8 1 2 4 2	1 3 7 8 **2 2 9 3 6 1 7 1 3 7 8 1 2 6**
9 3 4 1 3 6 8 1 2 4 2 1	3 7 8 **2 2 9 3 6 1 7 1 3 7 8 1 2 6 5**
3 4 1 3 6 8 1 2 4 2 1 3	7 8 **2 2 9 3 6 1 7 1 3 7 8 1 2 6 5 3**
4 1 3 6 8 1 2 4 2 1 3 7	8 **2 2 9 3 6 1 7 1 3 7 8 1 2 6 5 3 5**
1 3 6 8 1 2 4 2 1 3 7 8	**2 2 9 3 6 1 7 1 3 7 8 1 2 6 5 3 5 8**

Test file:

3 6 8 1 2 4 2 1 3 7 8 2	**2 9 3 6 1 7 1 3 7 8 1 2 6 5 3 5 8** 1
6 8 1 2 4 2 1 3 7 8 2 2	**9 3 6 1 7 1 3 7 8 1 2 6 5 3 5 8** 1 2
8 1 2 4 2 1 3 7 8 2 2 9	**3 6 1 7 1 3 7 8 1 2 6 5 3 5 8** 1 2 2
1 2 4 2 1 3 7 8 2 2 9 3	**6 1 7 1 3 7 8 1 2 6 5 3 5 8** 1 2 2 8
2 4 2 1 3 7 8 2 2 9 3 6	**1 7 1 3 7 8 1 2 6 5 3 5 8** 1 2 2 8 3
4 2 1 3 7 8 2 2 9 3 6 1	**7 1 3 7 8 1 2 6 5 3 5 8** 1 2 2 8 3 4
2 1 3 7 8 2 2 9 3 6 1 7	**1 3 7 8 1 2 6 5 3 5 8** 1 2 2 8 3 4 2
1 3 7 8 2 2 9 3 6 1 7 1	**3 7 8 1 2 6 5 3 5 8** 1 2 2 8 3 4 2 4
3 7 8 2 2 9 3 6 1 7 1 3	**7 8 1 2 6 5 3 5 8** 1 2 2 8 3 4 2 4 9
7 8 2 2 9 3 6 1 7 1 3 7	**8 1 2 6 5 3 5 8** 1 2 2 8 3 4 2 4 9 8
8 2 2 9 3 6 1 7 1 3 7 8	**1 2 6 5 3 5 8** 1 2 2 8 3 4 2 4 9 8 2
2 2 9 3 6 1 7 1 3 7 8 1	**2 6 5 3 5 8** 1 2 2 8 3 4 2 4 9 8 2 1
2 9 3 6 1 7 1 3 7 8 1 2	**6 5 3 5 8** 1 2 2 8 3 4 2 4 9 8 2 1 2
9 3 6 1 7 1 3 7 8 1 2 6	**5 3 5 8** 1 2 2 8 3 4 2 4 9 8 2 1 2 2
3 6 1 7 1 3 7 8 1 2 6 5	**3 5 8** 1 2 2 8 3 4 2 4 9 8 2 1 2 2 3
6 1 7 1 3 7 8 1 2 6 5 3	**5 8** 1 2 2 8 3 4 2 4 9 8 2 1 2 2 3 1
1 7 1 3 7 8 1 2 6 5 3 5	**8** 1 2 2 8 3 4 2 4 9 8 2 1 2 2 3 1 8
7 1 3 7 8 1 2 6 5 3 5 8	1 2 2 8 3 4 2 4 9 8 2 1 2 2 3 1 8 9

CHAPTER III

RESULTS AND DISCUSSION

We initiated this study to examine the possibility of using neural networks for time series forecasting with real-world observations available in the M-Competition (Makridakis 1982). Our pervious study (Sharda and Patil 1990) with neural network performance as forecasting experts with M-111 data series was with 1-step forecasts, where we compared the performance of neural networks with Box-Jenkins (ARIMA) methodology over 1-step forecasts. It was observed that, over the sample of 75 data series from M-111 competition, on average performance comparison, simple neural network did as well as Box-Jenkins (ARIMA) models. In the present study we started with the objective of examining the performance of neural network models for time series forecasting over multiple step forecast. One of the objectives here was to examine the effect of different architectural and learning parameters on the performance of neural networks over time series modeling.

Different Parameter Effects

This test was carries out with monthly data. Different values of learning rate and momentum were used to train a 12-12-1, 12-12-2, 12-12-4, 12-12-6, 12-12-8, 12-12-12 architecture, generating forecasts over 1, 2, 4, 6, 8, and 12 periods ahead respectively. These architectures were trained over 9 different combinations of learning rate (0.1, 0.5, 0.9) and momentum (0.1, 0.5, 0.9) values. Observations over this test gave the optimal learning parameters. These optimal learning parameters were then used to train remaining monthly data series over different architecture as: 12-6-1, 12-12-1, 12-18-1, 12-24-1 to observe the effect of different number of hidden neurons. The architecture which performed well in average analysis was then considered as the optimal architecture. The optimal architecture test was carried out only over a 1-step forecast.

Tables 1 & 2 in Appendix-A show these results over ser400 and ser409 data series. It is interesting to observe the effect of increasing the number of output neurons. It is observed that, as the number of output neurons were increased from 1 to 12 the n+1 (single step) forecast improved. Charts 1 & 3 show the data series used for this test. Charts 2 & 4 show the average n+1 forecast using all the different networks. From this test it is seen that the learning parameters with learning rate=0.1, and momentum=0.1 gave the best results. Chart 5 shows the average MAPE values using different architectures with different learning parameters over both the series. It is noticed that the performance was poor whenever the momentum value was high. Again, the optimal parameters were, learning rate=0.1 and momentum=0.1.

Optimal learning parameters found in the above test were then used to train remaining set of data series to find the optimal architectures. This test was carried out only for single step forecast. Table 3 shows the MAPE values for this test. On

CHART 1

DATA SERIES 400

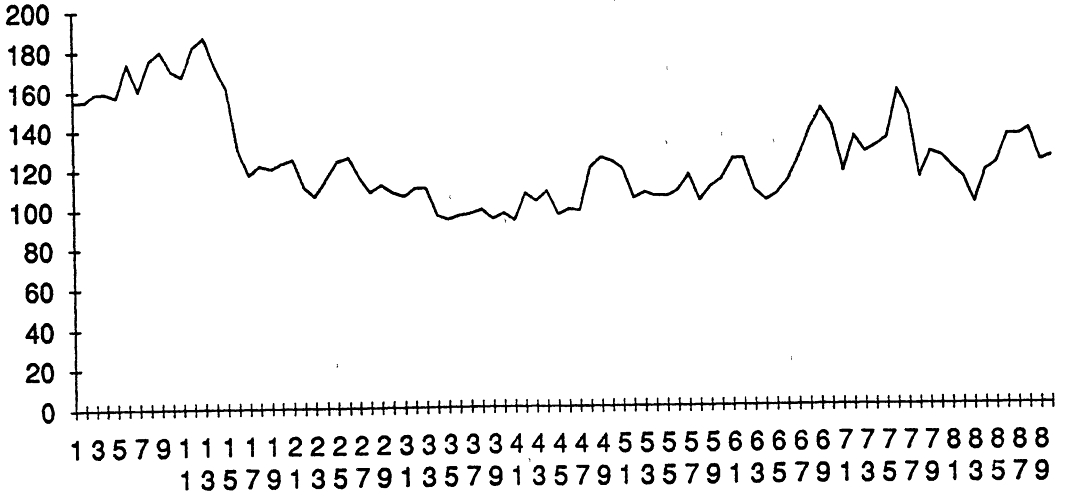


CHART 2

AVERAGE n+1 FORECAST ERROR WITH DIFFERENT NETWORK ARCHITECTURES
SER400 (LRATE=0.1, MOM=0.1)

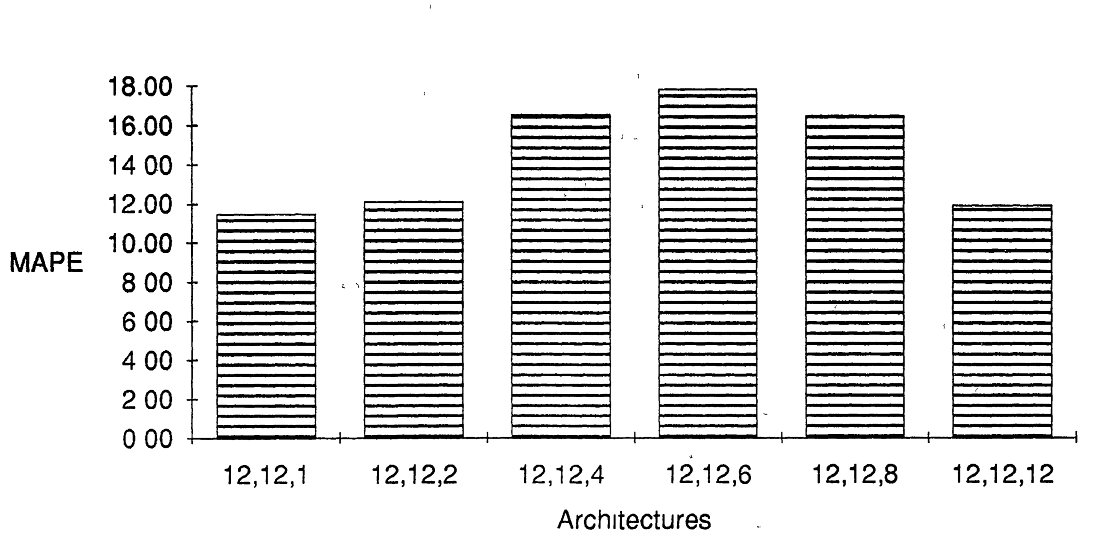


CHART 3

DATA SERIES 409

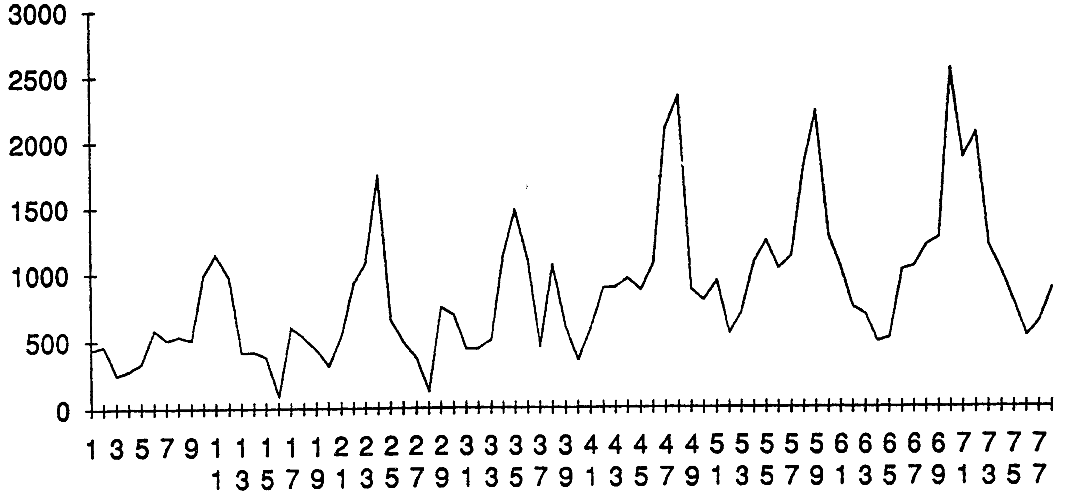


CHART 4

AVERAGE n+1 FORECAST ERROR WITH DIFFERENT NETWORK ARCHITECTURES
SER409 (LRATE=0.1, MOM=0.1)

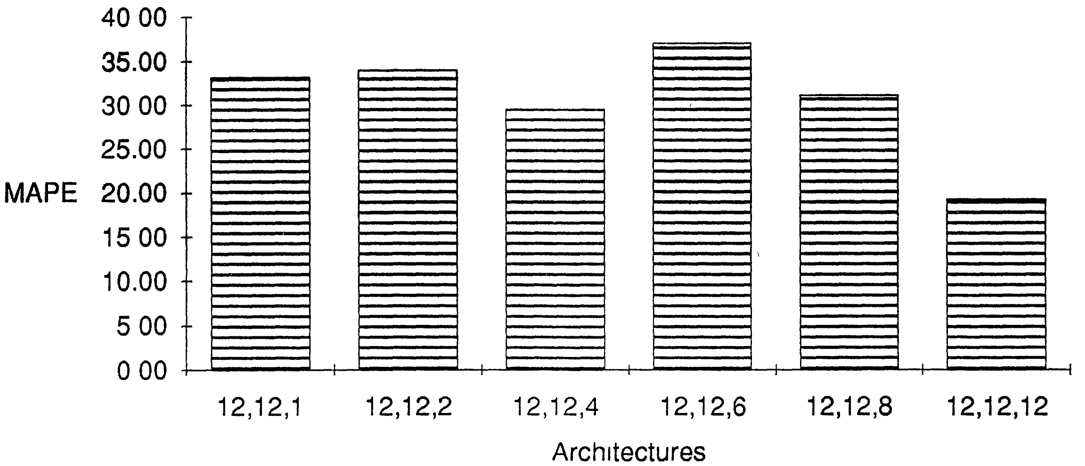
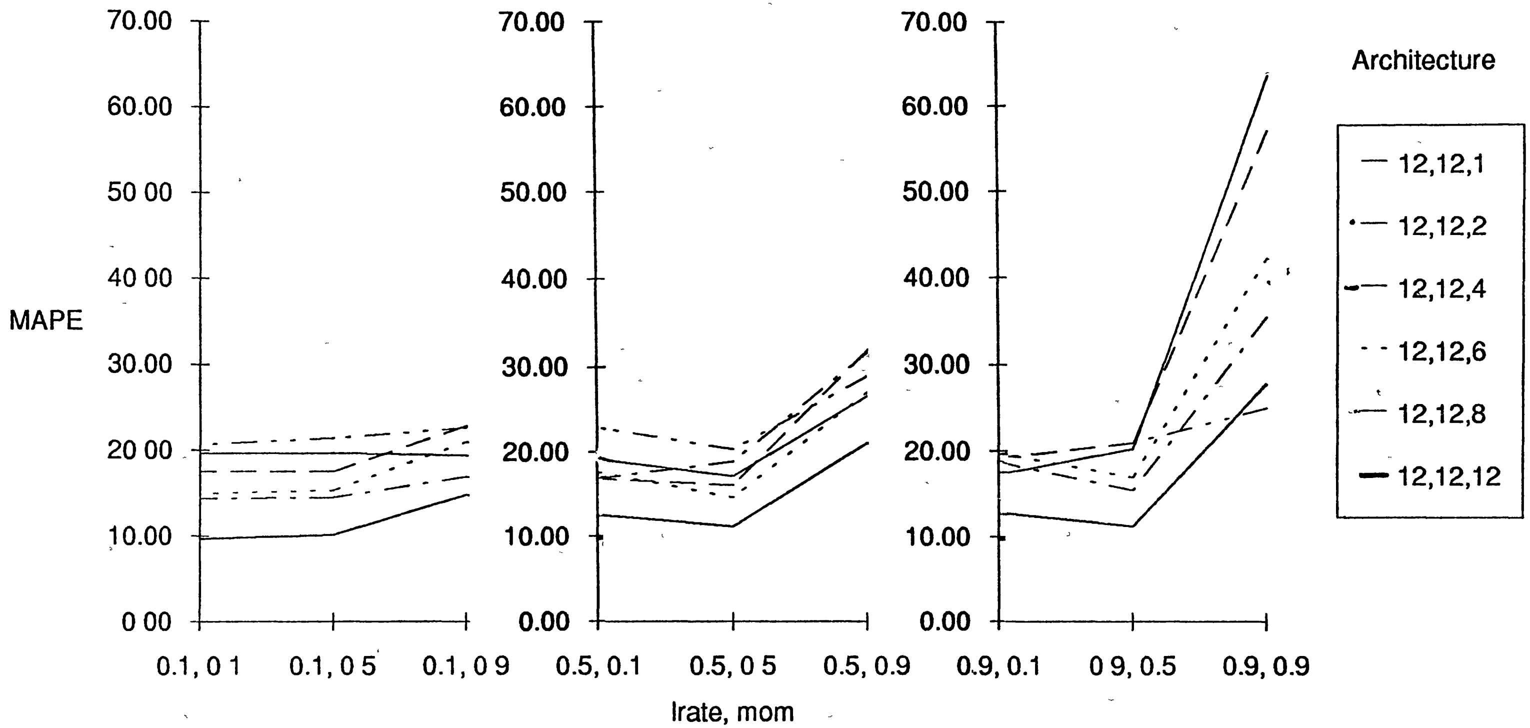


CHART 5

AVERAGE MAPE OF 1-STEP FORECAST WITH DIFFERENT ARCHITECTURES WITH VARIOUS LEARNING PARAMETERS (SER400 & SER409)



average analysis, 12-12-1 appeared to be the best architecture. This architectural parameter (equal number of input and hidden neurons) was then used in further tests.

Test Results with Monthly, Quarterly and Annual Data

Series > 382 and <=994 are monthly data series from the M-111 forecasting competition (Makridakis 1982). In this section we present the results of our empirical test over these data series using neural networks. Appendix-H gives the graphical representation of these series; the number of data points available in each data series is given in ( ) next to the series name. Appendix-G gives the Box-Jenkins (ARIMA) models.

Two different architectures were selected to train all data series. The first had 12 input and 12 hidden neurons. The second architecture had 24 input and 24 hidden neurons. This was done to observe the effect of history used on the forecasting performance. The number of output neurons in both cases depended on the forecast step. Forecasts up to 18 step were generated.

Each data series had a different underlying structure and different number of data points available to build the model. The forecasting performance of a model depends on this factor. Data series which was modeled for one-step forecast may not be modeled for multi-step forecast because the minimum number of training patterns required to build the neural network model was fixed to be at least 3 and the number of testing patterns were fixed to 18 (monthly). Due to this constraint, the data series is modeled only if it can generate at least 21 patterns (testing + training) of required size (input + target). As the forecasting step increases, the target section of the pattern increases and due to this reason, those data series which have fewer data points may appear in a single-step forecast but may not appear in a multi-step forecast. The number of data series tested in the case of the 12-12-*f* architecture (*f* is the forecast step) for 1, 2, 4, 6, 8, 12, & 18-step forecasts were 61, 61, 60, 59, 58, 56, & 48 respectively, while in the case of the 24-24-*f* architecture for 1, 2, 4, 6, 8, 12, & 18 step forecasts they were 48, 48, 48, 48, 45, 41, & 38 respectively. As the test sample in different step-forecast was not same, the average comparison of y+1 forecasts over the 1, 2, 4, 6, 8, 12 & 18-step forecast was not done. Similarly the average y+2 forecast comparison in 2, 4, 6, 8, 12 & 18-step forecast was not done. But they may be compared for individual series.

It is observed that as the forecasting step increased, the error in the forecast also increased. Tables 4-17 in Appendix-B show the MAPE (Mean Absolute Percent Error) and Median-APE (Mean Absolute Percent Error) of our tests with monthly data at 1, 2, 4, 6, 8, 12, & 18-step forecasts for both the above-mentioned cases of 12 & 24 past values as input. Charts 6 and 7 show how the MAPE and Median-APE were affected as the forecast step was increased from 1 to 18 in both the cases, using the last 12 months and the last 24 months of history.

Each model was trained only for 1000 cycles with learning parameters with learning rate =0.1 and momentum = 0.1. The total sum square error (tss) over the training patterns at the end of 1000 training cycles is given in the tables as the model error. Model error (tss) < 0.04 indicate that model converged in less than 1000 training

CHART 6

MAPE & MEDIAN APE AT 18 STEP FORECAST OVER MONTHLY DATA
ARCH. 12-12-18

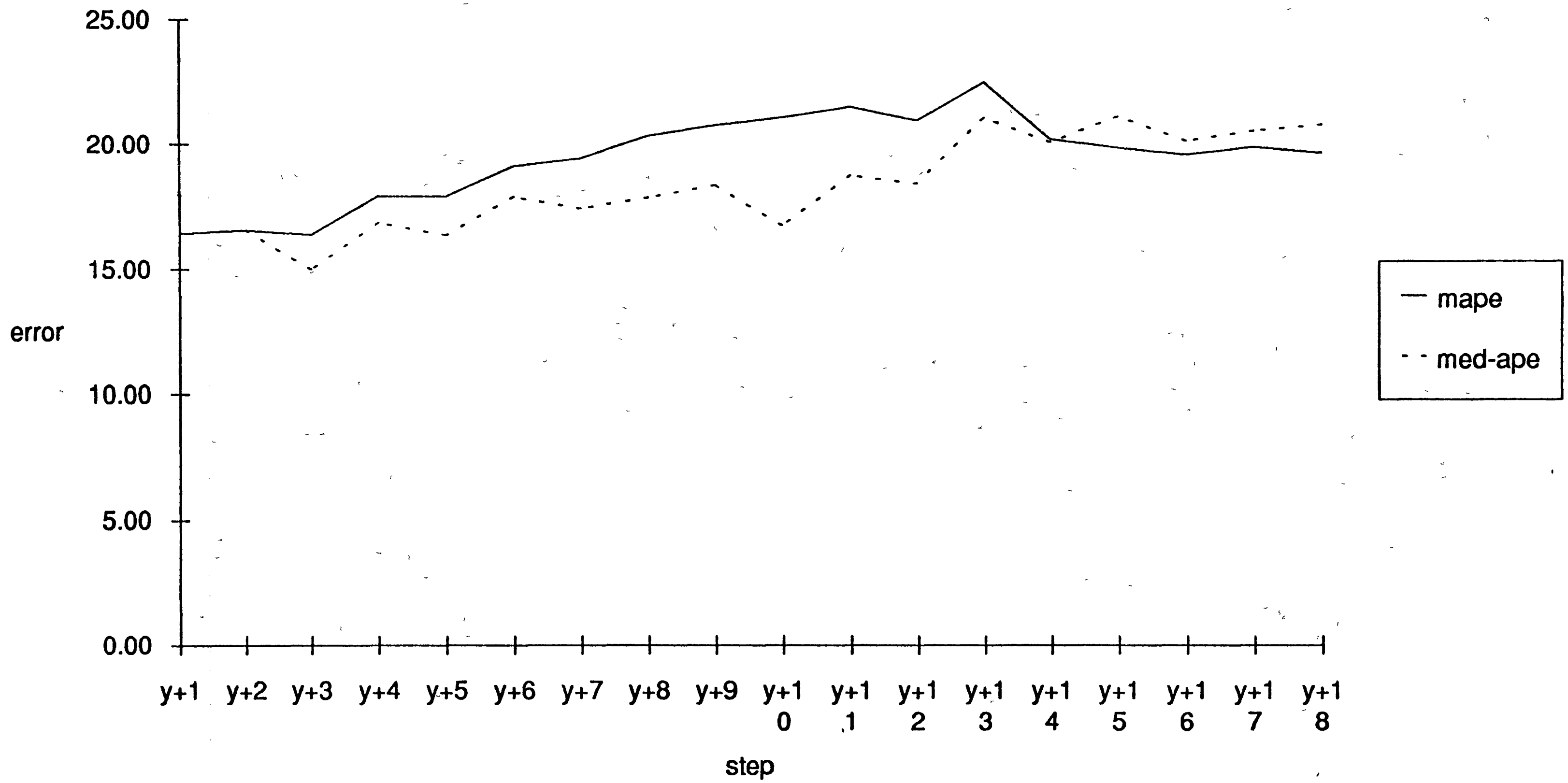
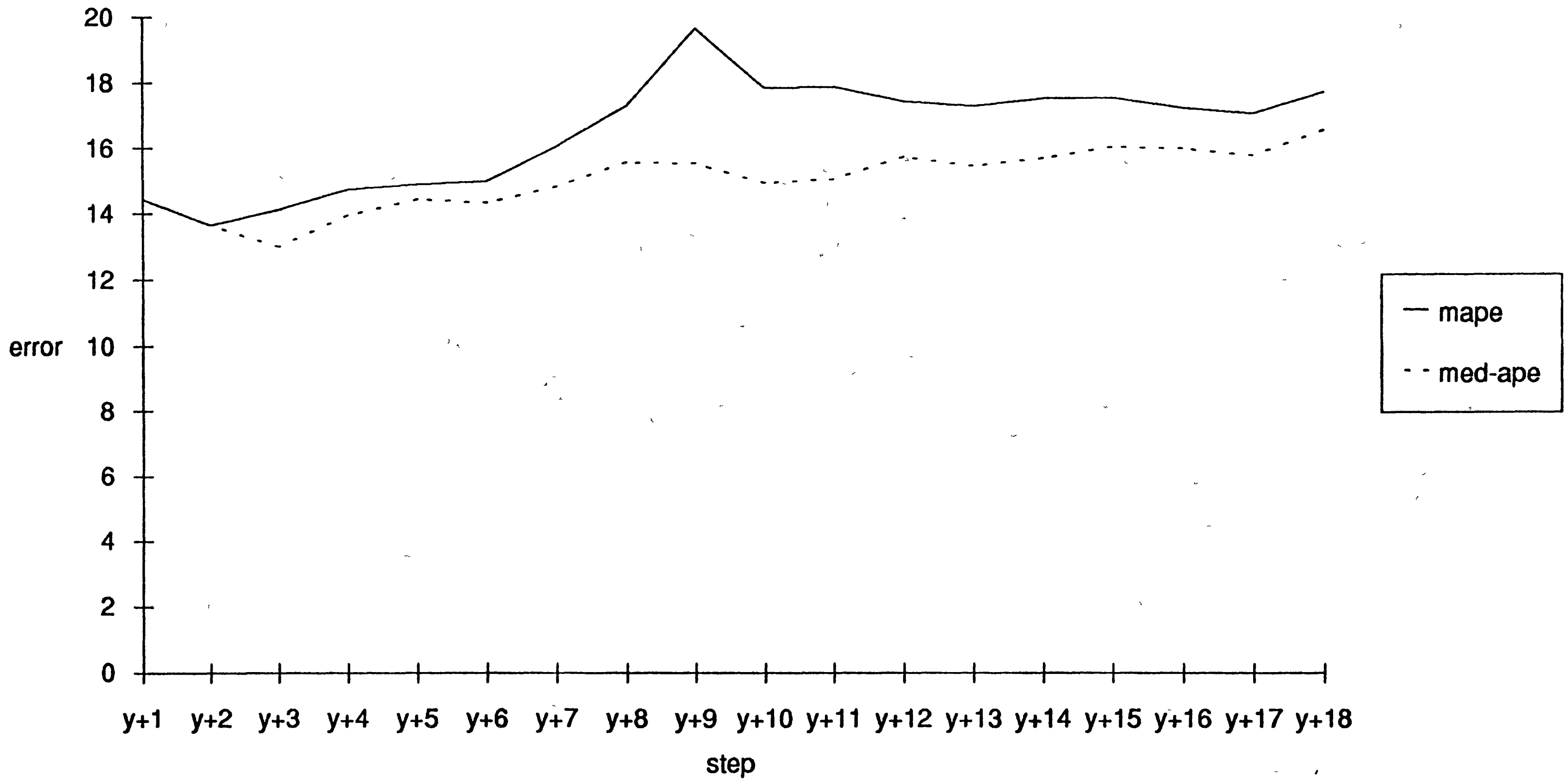


CHART 7

MAPE & MEDIAN APE AT 18 STEP FORECAST OVER MONTHLY DATA
ARCH. 24-24-18



cycles, while in cases where model error > 0.04 it is still possible to train the model further and improve the performance.

It is observed that the model error increased as the forecast step was increased. The reason may be that the difficulty encountered in the mapping process increases as the output dimension (output neurons) increases.

Data series with no seasonality were modeled by all the network architectures. In cases of seasonal data series, in many cases it was observed that, if Box-Jenkins model for the data series indicated that data series had seasonality od 12, the performance of 12-step, i.e 12-12-12 was better than other. It seems that these networks were able to capture the seasonality and were able to predict the complete future pattern (season) better than predicting a single value or the part of the future pattern. This effect was observed only for few data series and only for *y*+1 forecast.

To study the effect of the amount of history used in modeling the time series, we had 12 and 24 past values used in two different cases. It was observed that the in case of 1, 2, 4, & 6-step forecasts the performance of models using 24 input values was worse than models using 12 values. In case of 8, 12, & 18-steps forecasts the opposite was observed, models using 24 past values were better than models using 12 past values. It seems that mapping 24 values to 1, 2, 4, & 6 values was difficult for the network than to map 24 to 8, 12, 18 values.

Tables 18-27 and Charts 8, 9 in Appendix-C show the results over quarterly data series. Tables 28-31 in Appendix-D show results over annual data series. Similar observations are observed over monthly, quarterly and annual data series.

In both cases the forecasts were fairly steady, within ± 5% of MAPE. Similar observations were observed over the quarterly and yearly data.

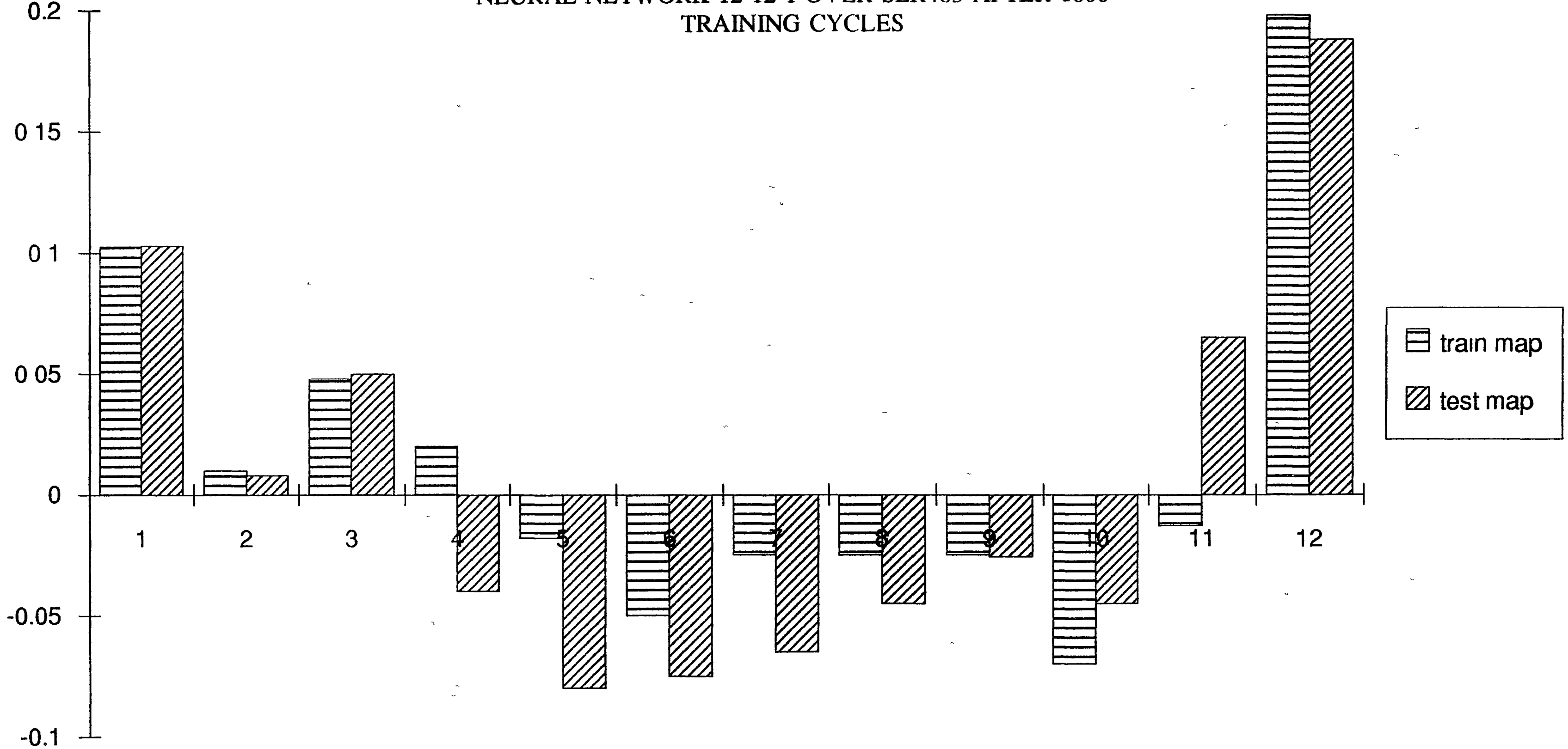
Analysis of Autocorrelations and Synaptic Weights

A data series of monthly observations was taken to perform this test. As mentioned earlier, our sample of 111 data series do not have the same number of data points. As number of data samples is one of the major factors in building forecasting models in statistical as well as neural network methods, data series having sufficient data points was chosen for the test. The neural network was trained only for 1000 cycles. As the neural network results are only over 1000 cycles, it is possible to train the model further, which may improve the performance. The data series was trained for a single-step forecast using a nonlinear 12-12-1 neural network. This network had 12 inputs, 12 hidden and 1 output neuron. The autocorrelation coefficients over the training part of the data were calculated between the actual value of $y(t+1)$ and $y(t)$, $y(t-1)$.....$y(t-11)$. Then the trained network weights were used to generate the forecast over the test data. Similar correlation factors were calculated over the test output between generated forecast $\hat{y}(t+1)$ and $y(t)$, $y(t-1)$....$y(t-11)$.

It is observed that in the training patterns $y(t+1)$ had high positive correlation with the $y(t-11)$ value. In the test output a similar high positive correlation was observed. As seen in Chart 10 the underlying map was captured by the neural network and it used similar approximations to generate the forecast. Chart 11 shows that data from series used for this test and shows the weight matrices of a trained

CHART 10

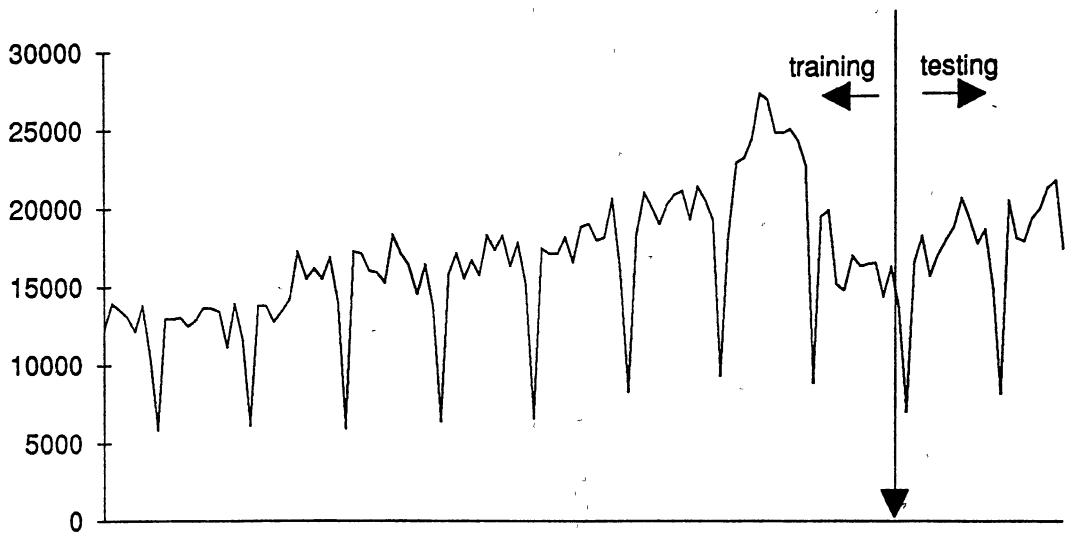
AUTOCORRELATION COEFFICIENTS BETWEEN FORECAST *t+1* AND PAST VALUES *t-1*, *t-2*, .. *t-12* OVER THE TRAINING DATA (ACTUAL MAP) AND TEST OUTPUT (CAPTURED MAP) BY NEURAL NETWORK 12-12-1 OVER SER463 AFTER 1000 TRAINING CYCLES



network. It is observed that the weights from the neuron to which *y(t-11)* input was given had positive weights leading to a high correlation between *y(t-11)* and $\hat{y}(t+1)$. The point here is to show that neural networks can learn rules. This is one very important property of nonlinear neural networks and it can be used in many control and signal processing applications.

CHART 11

DATA FROM SERIES 463



Weight matrix and biases of network 12-12-1 trained on ser463.
Wih: weights of input to hidden units; Who: weights of hidden to output units

Wih =	1.27	0.28	0.81	1.21	1.02	1.26	0.28	-0.14	5.26	-2.7	-0.87	0.3
	-0.92	-0.13	-0.31	-0.02	-0.13	-0.01	-0.18	-0.86	1.99	1.59	-0.91	0.13
	0.78	0.3	-0.67	-0.42	-0.35	0.43	-1.03	0.45	-2.67	1.78	0.12	-0.21
	0.06	-0.3	0.32	0.19	-0.4	-0.32	0.54	-0.42	-0.57	-0.35	-0.45	0.26
	-0.25	0.43	0.36	0.09	-0.25	-0.37	0.49	-0.58	0.66	0.52	0.08	-0.11
	-0.5	0.48	0.17	-0.18	-0.01	-0.44	0.01	-0.38	0.79	0.45	-0.42	-0.49
	-0.61	0.05	0.08	-0.42	0.45	-0.31	-0.01	0.27	-1.05	-0.04	0.38	-0.55
	-0.4	0.4	0.34	-0.73	0.27	-0.73	0.5	-0.26	-0.29	-0.64	0.29	0.3
	-0.83	0.47	-0.29	-0.48	0.39	-0.39	0.6	0.06	-0.47	-0.84	-0.07	0.16
	-0.8	0.54	0.66	-0.89	0.34	-1.05	1.05	-0.21	0.51	-1.54	0.09	0.19
	-0.68	-0.08	-0.36	-0.68	0.24	-0.25	0.3	-0.21	0.06	-0.63	-0.12	-0.57
	-1.04	-0.89	0.41	-0.75	0.52	-1.04	1.12	-0.31	0.77	-1.28	0.02	0.19

Who=	-2.55	-0.98	0.37	-2.03	0.37	-2.19	1.3	-1.5	5.61	-3.82	-1.42	-0.33
bias =	0.64	0.15	-0.09	0.46	-0.2	0.56	-0.82	0.25	-0.7	0.43	0	0.15

bias 25 = | -2.1 |

CHAPTER IV

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

Neural networks provide a promising alternative approach to time series forecasting. The neural networks ability to forecast in a fuzzy sense is more appropriate than the other forecasting methods. Our present study was limited to univariate forecasting, but it is expected that neural networks can be used for multivariate forecasting, which is, again, a more real-world situation. For time series with long memory, both Box-Jenkins models and neural networks perform well, with Box-Jenkins models slightly better for short term forecasting. With short memory neural networks outperform Box-Jenkins. By approximating the underlying mapping of the time series, a neural network provides robust forecasting in cases of irregular time series. Neural networks can be trained to approximate the underlying mapping of time series, albeit the accuracy of approximation depends on a number of factors such as the neural network structure, learning method, and training procedure. Without a hidden layer, the linear neural network model is functionally similar to Box-Jenkins ARIMA model.

Complexity of the network depends on the type of data series to be modeled, which decides the number of hidden neurons and the choice of transfer function. Network initialization is of importance in many cases. More systematic methods for selecting initial conditions for the neural networks are needed to achieve faster convergence.

The neural network structure and training procedure have great impact on its forecasting performance. This fact is evident from the present work that we are doing. The training algorithm is slightly different than the BrainMaker training algorithm we used in our previous study (Sharda & Patil 1990). Learning algorithms we are using in our study are by no means the best, we believe that there is still much room for improvement of neural network forecasting.

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APPENDIXES

APPENDIX A

LEARNING PARAMETERS AND ARCHITECTURE SELECTION

TABLE 1

MAPE VALUES OF SER400 AT DIFFERENT STEP FORECAST
WITH DIFFERENT VALUES OF LEARNING
RATE AND MOMENTUM

arch	step	(learning rate, momentum)								
		0.1, 0.1	0.1, 0.5	0.1, 0.9	0.5, 0.1	0.5, 0.5	0.5, 0.9	0.9, 0.1	0.9, 0.5	0.9, 0.9
12,12,1	1	10.25	11.23	12.45	12.23	9.76	16.98	9.34	9.23	11.54
12,12,2	1	12.45	13.08	13.31	11.48	8.90	15.74	9.71	8.48	15.74
	2	16.37	16.09	14.66	15.67	15.11	16.10	15.53	19.62	28.59
12.12.4	1	14.94	15.87	19.72	18.99	11.13	18.69	18.54	12.29	18.69
	2	13.24	11.68	17.74	12.88	12.78	19.25	11.09	15.38	22.84
	3	10.86	13.17	12.48	12.20	13.85	20.04	10.16	11.64	21.73
	4	9.40	9.67	13.10	14.30	10.06	20.66	8.86	20.65	20.66
12,12,6	1	14.77	16.05	26.38	19.52	15.11	14.36	21.07	18.71	14.36
	2	14.25	16.48	17.93	14.36	17.78	13.26	17.10	16.40	14.76
	3	13.07	16.75	17.20	10.23	12.73	18.72	13.59	16.10	18.72
	4	9.30	8.07	13.04	9.12	6.29	22.52	9.40	12.75	22.52
	5	2.48	3.84	11.36	7.94	9.69	10.34	10.79	9.39	19.86
	6	7.62	6.97	9.23	7.62	7.11	17.72	8.52	7.93	17.72
12,12,8	1	14.81	16.27	11.40	18.50	17.26	21.52	27.18	15.71	5.69
	2	19.45	19.02	17.95	21.06	18.85	14.77	13.40	13.76	43.36
	3	13.66	18.95	25.43	12.45	19.65	14.34	10.96	14.17	40.69
	4	4.28	5.24	4.94	1.90	8.47	7.97	4.61	4.59	7.97
	5	6.81	8.02	9.23	9.60	3.08	12.30	3.53	0.60	12.30
	6	10.36	12.70	14.05	10.65	10.49	16.17	8.54	5.23	16.17
	7	8.57	11.13	16.29	14.53	12.68	13.17	15.06	7.97	13.17
	8	10.85	11.82	23.27	8.04	17.20	5.82	10.69	8.54	10.67
12,12,12	1	6.32	23.16	14.87	16.90	11.67	11.08	11.87	8.55	2.54
	2	8.37	5.64	7.87	13.62	7.92	2.19	6.52	9.97	5.51
	3	12.08	20.30	14.58	18.87	15.98	24.71	9.59	13.59	11.24
	4	6.35	4.53	6.64	11.00	9.83	3.80	3.39	11.86	3.80
	5	8.77	3.87	8.26	7.61	10.45	12.14	3.34	7.94	3.33
	6	15.19	18.34	9.92	18.99	15.84	6.87	14.97	18.53	13.72
	7	5.03	6.02	6.04	9.57	7.05	9.49	7.18	7.03	9.49
	8	4.19	3.27	6.11	10.04	6.21	6.82	9.46	3.43	12.00
	9	1.50	4.13	5.16	0.89	1 93	8.15	7.20	3.36	8.15
	10	3.39	2.06	3.08	0 08	1.81	1.10	6.51	0.46	8.15
	11	1.47	1.97	1.01	1.70	1.54	5.26	2.59	0.26	5.26
	12	2.52	2.37	1.20	1.85	2 15	3.30	0.62	3.35	4.53

TABLE 2

MAPE VALUES OF SER409 AT DIFFERENT STEP FORECAST
WITH DIFFERENT VALUES OF LEARNING
RATE AND MOMENTUM

arch	step	(learning rate, momentum)								
		0.1, 0.1	0.1, 0.5	0.1, 0.9	0.5, 0.1	0.5, 0.5	0.5, 0.9	0.9, 0.1	0.9, 0.5	0.9, 0.9
12,12,1	1	30.99	31.54	32.67	32.98	30.43	40.45	28.70	32.57	38.12
12,12,2	1	28.25	29.83	30.78	- 31.49	24.72	51.62	23.02	34.37	51.62
	2	21.23	19.59	18.86	17.16	18.98	22.01	20.58	17.99	158.60
12.12.4	1	19.84	18.62	26.21	15.69	16.94	59.36	26.04	23.13	59.36
	2	20.46	17.77	34.50	21.99	23.94	46.00	23.46	11.07	46.00
	3	26.92	31.52	21.26	19.10	20.75	37.61	32.51	37.11	236.53
	4	24 17	21.63	37.69	18 35	18.18	32.23	22.35	35.34	32.23
12,12,6	1	23.19	23.60	41.76	36.31	24.20	69.81	26.05	18.66	69.81
	2	15.37	20 23	38.38	26.30	34.77	16 01	39.76	23.66	62.17
	3	5.72	13.87	15.61	5.06	15.02	48.90	33.78	27.65	34.54
	4	21.91	21.91	29.98	26.26	9.71	31.46	12.63	21.63	31.46
	5	31.55	27.00	16.10	19.78	10.88	33.65	34.48	10.10	177.34
	6	20.52	9 50	14.93	26.96	10.71	24.59	8.33	19.15	24.59
12,12,8	1	21.70	22.16	21.48	21.92	19.01	36.29	35.58	29.74	71.95
	2	15.03	12.67	9.62	18.14	25.22	46.81	22.73	7.58	76.96
	3	10.20	19.78	5.64	32.26	21.55	29.51	39.00	9.79	69.81
	4	17.13	15.66	23.15	24.02	33.50	62.17	25.76	30.08	62.17
	5	14.61	11.38	38.90	16.28	37.00	30.38	14.92	40.18	58.18
	6	24.66	22.68	25.62	31.53	26.75	76.10	26.67	27.72	31.46
	7	22.58	17.76	13.38	12.67	20.50	22.01	14.41	20.47	22.01
	8	14 73	6.69	9.64	14.36	7.97	93.69	23.53	9.62	24.59
12,12,12	1	8.68	8.57	43.00	1.51	9.17	27.84	14.79	3.25	56.58
	2	20.66	20.04	20.39	21.56	19.76	55.83	18.15	12.26	67.02
	3	8.43	10.72	25.66	29.07	15.02	13.21	13.66	11.06	50.09
	4	15 88	14.63	29.50	3.54	15.86	68.76	23.47	15.23	68.76
	5	24 25	17.60	12.88	32.84	22.38	44.17	22.26	30.24	57.89
	6	11.21	7.66	2.35	27.28	13.72	50.26	18.69	16.15	46.14
	7	25.03	27.24	32.18	19 63	22.32	56.38	32.64	21.93	56.38
	8	18.83	19.28	28.89	21.27	29.26	22.23	25.80	15.62	65.24
	9	11.43	9.42	21.97	14.04	8.29	25.45	15.09	10.82	25.45
	10	5.96	4.16	33.14	7.42	2.49	1.00	9.11	15.31	28.07
	11	1.78	1.95	10.56	2.53	4.68	26.64	14.96	24.66	26.64
	12	6.06	6.42	9.09	6.71	12.04	14.35	12.76	1.16	29.69

TABLE 3

EFFECT OF DIFFERENT ARCHITECTURES ON MAPE
WITH $\alpha = 0.1$ AND $\beta = 0.1$

Data: M-111-Monthly data

	MAPE			
Series #	12,6,1	12,12,1	12,16,1	12,24,1
ser400	16.98	10.25	16.98	16.98
ser409	24.12	30.99	43.31	29.76
ser418	32.39	19.65	32.39	19.24
ser427	24.78	8.16	24.78	24.78
ser436	2.27	1.92	2.27	2.27
ser445	15.22	15.27	13.88	15.85
ser454	9.66	8.27	41.96	41.96
ser463	18.37	5.7	18.37	18.37
ser472	14.85	14.54	33.94	33.94
ser481	30.42	26.05	30.30	55.04
ser490	13.27	9.14	20.98	20.98
ser499	9.75	8.89	9.75	9.75
ser508	6.84	5.91	6.84	6.84
ser526	13.44	11.65	19.03	15.81
ser544	3.19	3.05	3.10	4.53
ser562	35.38	21.26	37.75	60.45
ser571	10.07	8.14	11.97	11.39
ser580	1.35	1	1.32	1.17
ser589	3.72	3.35	4.17	3.76
ser598	9.14	11.69	15.74	15.74
ser616	17.25	5.09	20.22	17.25
ser625	20.57	21.33	18.50	17.23
ser634	30.72	21.36	30.72	30.72
ser643	21.14	18.13	28.85	28.85
ser652	17.28	16.05	22.24	36.81
ser661	24.80	19.6	40.70	40.70
ser670	11.73	18.19	11.00	52.78
ser679	22.74	25.62	24.32	69 17
ser688	8.55	7.39	37.64	37 64
ser697	2.01	1.59	6.05	2.44
ser706	8 93	6.74	8.93	8.93
ser715	50.26	48.35	59.16	85 76
ser724	23.99	19.15	26.82	26 82
ser733	25.28	19.98	34.38	34.38
ser742	2 86	2.44	3.25	13.27
ser751	7.62	5.93	16.43	16.43
ser760	9.63	8.26	10.84	28.33
ser769	12.00	9.21	19.29	19.29
ser778	4.06	4.13	4.06	4.06
ser787	5.34	2.2	5.34	5.34
ser796	41.95	16.71	41.95	41 95

TABLE 3 CONTINUED..

Series #	MAPE 12,6,1	12,12,1	12,16,1	12,24,1
ser805	1.18	1.53	1.18	1.18
ser814	0.67	0.71	0.69	0.69
ser823	0.65	0.47	1.16	1.16
ser832	1.29	1.32	1.29	1.29
ser841	9.83	9.49	21.47	9.50
ser850	6.96	6.56	6.56	6.06
ser868	10.05	9.8	10.58	12.15
ser877	4.48	4.36	14.56	19.31
ser886	31.06	29.08	31.11	31.12
ser895	14.51	12.47	14.51	14.51
ser904	5.74	3.37	5.74	5.74
ser913	31.33	34.22	57.70	34.78
ser922	26.68	5.06	26.68	5.14
ser931	20.41	23.51	21.28	22.44
ser940	26.60	25.8	32.57	29.69
ser958	7.16	6.74	6.72	6.82
ser967	7.82	7.09	7.47	8.21
ser994	9.70	9.05	32.53	10.57
Mean	14.92	12.08	19.55	21.14
Stdev	11.27	9.82	14.44	18.14

APPENDIX B

TEST RESULTS WITH MONTHLY DATA

TABLE 4

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 1 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 12-12-1

series	model error	y+1	
		mape	m-ape
SER391	0.056	19.75	9.7
SER400	0.94	10.25	7.92
SER409	1.177	30.99	26.64
SER418	1.321	19.65	15.41
SER427	1.496	8.16	6.02
SER436	1.82	1.92	1.63
SER445	0.835	15.27	14.92
SER454	0.485	8.27	6.84
SER463	0.98	5.7	5.15
SER472	1.059	14.54	11.66
SER481	0.854	26.05	25.29
SER490	1.046	9.14	8.64
SER499	3.135	8.89	8
SER508	1.724	5.91	4.9
SER526	1.022	11.65	9.78
SER544	0.226	3.05	2.37
SER562	2.657	21.26	13.17
SER571	0.898	8.14	5.22
SER580	0.349	1	0.82
SER589	0.254	3.35	2.05
SER598	0.603	11.69	6.75
SER616	1.327	5.09	4.17
SER625	0.03	21.33	16.56
SER634	2.264	21.36	16.98
SER643	0.971	18.13	9.36
SER652	0.793	16.05	11.53
SER661	0.568	19.6	19.86
SER670	0.555	18.19	14.46
SER679	0.711	25.62	13.61
SER688	0.311	7 39	4.68
SER697	1.391	1.59	0.91
SER706	0.487	6.74	6.09
SER715	1.755	48.35	28.96
SER724	1.471	19.15	20.47
SER733	0 653	19.98	13.48
SER742	0.184	2.44	2
SER751	0 878	5.93	4 05
SER760	0.582	8 26	6.17
SER769	0.989	9 21	7.02
SER778	0.046	4.13	2.21
SER787	0.256	2.2	2.16
SER796	0.447	16.71	20.01
SER805	1.151	1.53	1.47
SER814	0.038	0.71	0.89
SER823	1 063	0.47	0.35
SER832	0.001	1.32	1.24
SER841	0.04	9.49	7.82

TABLE 4 CONTINUED..

series	model error	y+1 mape	y+1 m-ape
SER850	0.04	6.56	6.83
SER868	0.04	9.8	7.67
SER877	1.039	4.36	3.59
SER886	0.038	29.08	30.73
SER895	0.072	12.47	11.6
SER904	0.172	3.37	2.87
SER913	0.342	34.22	35.91
SER922	3.716	5.06	5.15
SER931	0.04	23.51	24.03
SER940	0.04	25.8	21.89
SER958	0.294	6.74	4.88
SER967	0.548	7.09	5.89
SER985	0.072	31.16	19.44
SER994	0.04	9 05	7.66
	mean	12.52	10.12
	stdev	10	8.39

TABLE 5

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 2 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 12-12-2

series	model	y+1		y+2	
	error	mape	m-ape	mape	m-ape
SER391	0.129	19.6	7.68	21.65	10.76
SER400	2.219	10.27	8.82	11.96	11.92
SER409	2.095	31.2	27.62	35.93	26.53
SER418	2.331	23.62	22.53	19.85	18.89
SER427	2.966	8.32	5.78	9.61	8.37
SER436	2.378	1.97	1.7	2.46	2.5
SER445	1.612	15.15	12.68	16.19	15.07
SER454	0.843	7.95	5.67	9.13	9.59
SER463	1.744	4.87	4.8	8.11	6.77
SER472	1.392	14.61	12.01	15 67	12 01
SER481	2.009	27.21	21.49	27.12	24.28
SER490	1.717	11.36	8.29	11.88	8.82
SER499	6.252	9	7.4	8.95	6.66
SER508	3.438	5.33	5.95	6.37	4.72
SER526	2 037	11.63	10.22	12 08	8 6
SER544	0.69	3.06	1.93	4 64	3.8
SER562	4.333	22.06	15.67	29.1	24.31
SER571	1.783	10.6	8.45	10.98	7.91
SER580	0.645	1	0.95	1.77	1.74
SER589	0.425	3.73	2.66	5.85	3.13
SER598	1.028	9.31	5.4	11.82	7.75
SER616	2.69	5.2	5.21	5.28	3.34
SER625	0.04	20.36	13.95	21.86	16.85
SER634	4.377	20.5	15.67	18.47	15.98
SER643	1.3	18.52	9.3	20.3	12.42
SER652	1.658	18.48	13.72	14.1	8.99
SER661	0.749	15.37	14.77	18.17	14.14
SER670	1.056	18.02	13.4	22.43	14 28
SER679	1.871	45.71	43.23	36.95	38.36
SER688	0.74	7.86	5.12	9.11	5 99
SER697	2.339	1.5	0.89	1.84	1.5
SER706	1.124	5.64	5.57	7.54	6 41
SER715	2.608	56.5	24.66	55.98	37 28
SER724	2.267	19.64	19.23	21.88	20.08
SER733	1.2	20.03	13.75	20 23	18.23
SER742	0.347	2.17	1.47	2.62	2.38
SER751	1.88	6.25	5.88	6.69	6.08
SER760	0.963	7.82	6.93	9.51	10.9
SER769	1.395	9.81	9.23	9.65	6 49
SER778	0.238	3.81	2.69	4.71	3.41
SER787	0.438	2.72	2.41	2.68	2 59
SER796	0.888	18.25	19.23	16.98	19.22
SER805	1.954	2.19	2.02	2.68	2.48
SER814	0.049	0.81	0.79	0.78	0.73
SER823	2.348	0.69	0.63	0.75	0.61
SER832	0.093	2.04	1.94	2.54	2.41
SER841	0 04	10.22	7.52	10.51	7 57

TABLE 5 CONTINUED..

series	model error	y+1 mape	y+1 m-ape	y+2 mape	y+2 m-ape
SER850	0.055	6.21	4.49	12.58	10.48
SER868	0.075	10.54	7.65	10.97	9.82
SER877	2.058	4.79	3.57	4.31	2.81
SER886	0.048	28.83	29.37	31.82	33.45
SER895	0.119	11.2	10.35	13.43	12.24
SER904	0.307	3.42	3.54	3.58	3.29
SER913	0.643	27.97	26.36	44.25	41.33
SER922	7.587	6.06	5.78	7.83	7.8
SER931	0.063	21.7	18.65	26.38	23.78
SER940	0.042	17.28	8.74	30.32	25.02
SER958	0.404	7.59	4.39	9.02	6.69
SER967	0.992	12.72	12.28	11.76	8.81
SER985	0.126	33.23	14.36	36.48	19 13
SER994	0.04	11.01	9.78	9.11	7 14
	mean	13.02	10.1	14.38	11.72
	stdev	10.97	8.38	11.56	9.66

TABLE 6

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 4 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 12-12-4

series	model	y+1		y+2		y+3		y+4	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER391	0.262	18.47	6.87	26	5.98	39.85	22.12	35.97	15.99
SER400	3.344	9.22	7.21	11.08	9.84	11.19	8.35	9.78	10.37
SER409	4.157	29.62	27.29	33.95	23.25	31.77	18.65	33.66	23.03
SER418	3.758	23.85	21.4	19.52	17.44	19.7	18.47	18.64	16.33
SER427	4.978	9.49	7.98	9.37	7.74	9.26	10.98	9.42	7.58
SER436	4.385	2.09	2.3	2.65	2.56	3.04	3.12	3.57	3.07
SER445	2.753	15.18	13.19	15.29	12.85	16.11	18.54	18.89	17.85
SER454	1.631	7.78	5.6	8.91	9.66	9.85	9.67	10.19	9.74
SER463	3.61	5.7	6.37	7	6.8	8.26	8.62	8.25	9.9
SER472	3.15	17.4	15.73	18.59	16.7	17.61	15.07	16.6	14.86
SER481	2.795	23.09	15.57	25.41	17.58	25	16.83	28.97	29.85
SER490	2.85	13.53	16.4	15.39	14.94	15.36	16.78	15.49	16.35
SER499	10.42	9.47	10.21	8.73	7.32	9.79	8.75	11.7	12.08
SER508	6.302	6.67	6.5	6.28	6.21	6.6	6	7.75	8.31
SER526	4.174	11.8	11.39	12.61	9.34	13.11	12.7	14.31	13.57
SER544	1.581	3.92	2.8	5.88	5.26	6.89	5.38	6.54	6.33
SER562	7.044	22.58	15.26	33.97	27.01	42.96	29	46.76	31.98
SER571	3.249	14.74	14.04	14.83	12.35	10.54	8.11	10.55	9.29
SER580	0.94	0.86	0.76	1.53	1.45	2.26	2.45	2.93	3.14
SER589	0.585	3.67	1.33	5.51	2.64	6.78	5.88	8.09	7.44
SER598	1.719	6.69	5.55	8.26	6	10.56	6.92	13	8.11
SER616	5.214	4.28	4.36	4.48	4.13	5.62	4.26	5.91	5.5
SER625	0.045	21.79	15.25	20.21	13.5	21	14.51	21.27	16.76
SER634	8.687	16.27	11.97	16.22	13.19	17.18	15.65	16.61	14.43
SER643	2.262	19.4	11.54	20.25	13.71	18.56	10.68	16.15	8.12
SER652	2.688	18.63	18.91	17.62	15.9	18.64	16.16	15.45	12.12
SER661	1.334	15.57	12.68	16.19	13.28	15.6	17.28	15.63	13.67
SER670	1.982	17.58	16.01	23.44	11.96	24.51	15.71	22.99	21.45
SER679	3.197	41.6	19.74	63.66	47.01	45.18	47.79	31.26	29.63
SER688	1.698	8.8	7.44	10.65	9.39	11.36	10.48	11.13	10.9
SER697	4.136	1.46	0.75	1.51	0.69	1.81	1.14	1.63	1.04
SER706	1.963	4.17	3.19	5.57	4.53	6.9	6.56	9.04	8.36
SER715	4.89	57.9	27.88	59	47.02	73.91	45.71	67.83	42.75
SER724	4.213	18.75	20.87	22.74	21.53	19.06	17.74	20.91	17.07
SER733	2.226	17 06	8.29	19.06	13.38	17 55	11.82	16.07	8.46
SER742	0.634	2.11	1.54	2.46	2.02	2.65	2.32	2.46	1.85
SER751	3.18	6.72	5.76	8.62	8.79	7.26	6.27	8.16	7.05
SER760	1.13	9.52	6.92	9.36	8.32	10.46	10.32	9.68	9.2
SER769	1.86	9.48	9.06	9.03	6.09	9.94	7.09	10	10.5
SER778	0.409	4.36	4.24	4.43	3.83	5.05	4.26	5.73	4.88
SER787	1.074	2.82	2.31	3.46	2.58	2.99	2.67	2.99	2.35
SER796	1.545	17.79	20.57	15.25	16.22	15.31	14.7	13.95	12.95
SER805	3.534	2.5	2.7	3.82	3.58	4.7	5.01	5.2	5.27
SER814	0.198	1.05	0.88	1.15	1.07	1.32	1.07	1.38	1.19
SER823	5.102	0.84	0.78	1.05	0.9	1.45	1.24	1.76	1.35
SER832	0.281	2.48	2.41	3.36	3.25	4.08	3.74	4.77	4.37
SER850	0.059	6.32	4	13.02	10.58	11.24	7.7	12.32	10.09

TABLE 6 CONTINUED..

series	model	y+1		y+2		y+3		y+4	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER868	0.134	10.92	7.77	11.76	10.27	10.84	9.33	9.55	6.73
SER877	4.146	3.75	2.98	3.77	3.19	3.69	2.54	3.38	2.19
SER886	0.127	26.05	25.37	29.84	29.72	32.01	32.44	34.58	35.23
SER895	0.092	8.1	6.15	9.47	7.15	11.93	9.28	14.77	12.95
SER904	0.699	3.55	3.18	3.41	3.79	3.19	2.78	2.76	2.47
SER913	0.7	47.64	34.52	46.65	36.21	50.37	55.84	55.26	65.4
SER922	14.92	8.35	9.4	10.6	10.24	11.85	9.58	12.46	11.03
SER931	0.08	28.07	21.57	28.24	22.87	26.43	22.23	17.86	16.43
SER940	0.134	17.52	13.33	19.57	15.4	20.99	14.73	40.51	14.75
SER958	0.418	19.71	20.62	17.16	16.18	11.4	7.34	14.6	9.98
SER967	1.984	21.33	23.76	19.85	21.41	17.43	13.19	24.28	20.75
SER985	0.17	38.99	23.46	43.55	23.26	36.34	19.1	79.74	22.25
SER994	0.092	9.12	6.5	8.3	7.19	10.26	6.98	9.13	7.05
	mean	13.8	10.87	15.31	12.1	15.61	12.66	16.67	13.06
	stdev	11.79	8.21	13.34	10.06	13.66	10.96	15.59	11.06

TABLE 7

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 6 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 12-12-6

series	model	y+1		y+2		y+3		y+4		y+5		y+6	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER391	0.262	18.47	6.87	26	5.98	39.85	22.12	35.97	15.99	34.98	16.32	25.02	14.61
SER400	7 318	9.13	8.71	12.04	10.09	11.23	8.93	10.55	7.67	8.3	7.81	9.97	8.95
SER409	4 995	29.09	26.4	28.39	19.95	26.71	16.33	31.01	16.68	31.59	17.8	25.49	21.04
SER418	4.618	17.11	7.26	23.1	21.89	21.7	20.73	18.94	19.68	16 84	14.95	15.77	11.62
SER427	6 679	9.5	10.46	9.58	7.7	10.28	9.29	10.5	8.14	10.57	10.37	10.77	8.74
SER436	6.059	2.32	2.38	3.01	3.55	3.73	4.26	4.49	4.48	5.24	5.46	5.69	5.61
SER445	3 894	13.99	11.68	15.58	12.8	16.89	18 07	18.55	20.59	18.71	19.62	20.11	19.69
SER454	2.678	9 35	6.45	10.2	9.42	8 81	9.04	10.63	10.21	11.45	11.91	13.18	13.41
SER463	7 801	6.89	5.79	6.82	6.71	10.35	8.68	10.85	9.62	10.47	10 35	9.93	8.5
SER472	3 888	21.35	19.82	20.92	20 81	20.64	17.4	19.82	17.71	19.33	17.39	20.51	17.17
SER481	3 362	30 61	16.53	23.63	18.4	22.95	20.15	27.11	26 9	21 84	16 65	22.47	26.25
SER490	3.261	16.01	18 27	18.09	16.91	17.31	18.91	18.09	17.84	20.07	20.3	19.89	19.76
SER499	12 86	11.08	10 46	9.58	9.88	11.53	11 01	11.71	10.82	12.5	11.78	14.14	12.62
SER508	8.479	6.39	5.32	8.04	7.89	8.27	9.04	8.31	9.04	9.15	10.25	9.35	11.05
SER526	5 877	11.26	9.03	13.25	10.15	14.07	12.78	12.6	11.86	14.23	13.44	12.73	10.59
SER544	2.886	4.06	3.02	6.17	6.79	7.47	6.32	7.52	6.64	7.39	7.37	7.24	7.34
SER562	8.727	27.5	22.58	35.91	29.64	41.77	34.67	57.59	52.3	64.84	53	58.86	51.51
SER571	3.764	16 28	13.57	17.67	20.12	13.65	8.15	13.72	12.53	10 41	11.07	9.22	9.61
SER580	1 089	0 83	0.63	1.31	1 05	1.86	1.86	2.57	2.47	3.09	2.93	3.59	3.05
SER589	0.766	4 83	2.36	5.2	4.42	5.15	5.37	5.09	4.36	6.42	6.57	8.68	8.23
SER598	2.467	5.59	5.02	6.48	5 82	8.47	8.66	9.7	7.67	11.82	6.68	13.68	8.04
SER616	6 973	4.51	3.97	4.87	5.16	4.49	3.77	4.76	4.67	5.31	5.29	5.6	5.2
SER634	11.79	14.75	9.19	13.77	10.85	12.87	10.75	13.72	12.35	15.91	13.46	14.17	12.6
SER643	2 866	21.34	11.15	21.31	14.26	20.73	13	17.7	9.98	17.45	15.74	18.62	11.24
SER652	3 813	19.33	16.63	18.19	14.4	18.9	17.98	17.43	17.7	13.86	8.53	16.84	17.46
SER661	1.954	16.1	10.91	16.21	13.68	15.36	13.03	15	14.68	14.43	13.47	18.16	15.83
SER670	3.119	17.4	19.39	24.57	16.7	25.58	20.29	23.88	18.34	23.12	20.06	26.99	26.46
SER679	4.125	61.54	21.24	70.54	45 3	68.63	45 52	55.9	42.6	37.06	23.39	43.97	24.31
SER688	2.248	9.95	6.88	11.64	9.17	12.64	12.12	13.03	12.9	12.16	11.28	12.18	12.59
SER697	5.081	1.71	0.97	1.78	1.24	1.81	1.01	1.6	0.86	1.8	1.72	1.73	1.23

TABLE 7 CONTINUED..

series	model	y+1		y+2		y+3		y+4		y+5		y+6	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER706	2.568	3.41	1.88	3.44	1.76	4.78	3.33	6.6	6.35	8.35	8.66	9.01	9.27
SER715	6.409	36.08	21.63	63.32	25.82	59.86	29.43	77.25	47.69	54.18	46.43	53.18	31.86
SER724	4.679	19.27	18.99	19.5	14.8	18.88	13.13	20.17	18.84	19.83	19.15	21.74	21.75
SER733	4 661	16.6	7.13	13.82	8.47	12.9	6.41	17.22	9.07	16.34	8.1	14.12	4.83
SER742	0.959	2.33	1.87	2.31	1.87	2.37	1.68	2.11	1.63	2.71	2.31	3.16	2.85
SER751	4.455	6.66	6.26	8.39	9.03	8.73	8.74	9.13	8.67	8.06	6.98	6.31	5.52
SER760	1 492	10.88	12.59	9.31	7.95	7.78	4.91	10.99	10.25	11.28	9.44	11	8.66
SER769	3 459	8.52	9.91	9.19	5.48	11.11	10.32	10.31	12.55	10.99	12.39	10.7	10.35
SER778	0.426	4.5	4.85	3.69	3 39	4.42	4.27	5.34	4.71	6.06	5.49	6.51	7.22
SER787	1.734	3.95	3.27	3.89	3 46	3.46	2.88	3.48	2.59	3.64	2.82	3.07	2.18
SER796	2.126	17.5	18.99	15.14	12.44	16.97	15.84	15.29	14.04	15.45	15.75	15.4	16.49
SER805	4.857	2.11	2.41	3.5	3.28	4.83	4.72	6.07	5.96	6.98	7.48	7.47	7.9
SER814	0.274	1.47	1.42	1.49	1.43	1.7	1.64	1.87	1.96	1.98	1.86	1.93	1.78
SER823	6.661	0.67	0.61	0.96	0.7	1.38	0.98	1.88	1.37	2.53	1.9	3.11	2.34
SER832	0.38	2.36	2.34	3.31	3.15	4.24	4.02	5.12	4.86	6.06	5.79	6.82	6.53
SER850	0.67	9.13	7.96	11.13	8.62	9.8	7.49	12.36	9.08	10.99	8.23	10.22	6.01
SER868	0.297	11.03	6.43	13.38	10.2	10.77	8.97	11.26	8.93	11.3	9.27	9.57	8.28
SER877	6 143	3 82	2.97	3.44	2.72	3.88	3.12	3.35	2.39	3.53	2.93	2.8	2.32
SER886	0 127	20.53	20.41	23.8	23.95	28.64	28.67	31.36	34.5	33.47	38.28	35.14	40.18
SER895	0 087	7.7	7.8	7.49	4.92	8.69	6.04	10.85	8.47	13.64	12.38	15.91	15.27
SER904	1.052	3.21	2.75	3.11	2.84	3.54	3.5	3.31	2.97	2.53	2.79	2.69	2.53
SER913	1 025	43.3	31.37	48.17	44.79	55.57	54 84	54.75	57.06	51.8	61.61	56.16	64.04
SER922	22 6	7.11	7.52	10	9.76	12.25	11.54	14.6	14.67	18.24	16.04	18.76	15.52
SER931	0.199	15.16	16.56	41.89	27.46	44.41	36.04	42.31	29.41	15.34	9.4	51.77	39.15
SER940	0.249	28.11	31.65	26.81	15.01	20.67	18.4	18.86	18 06	30.79	16.78	81.19	58.68
SER958	0.845	24.97	27.86	28.08	30.59	15.08	9.93	15.09	10.12	17.07	14.58	17.95	14.71
SER967	2 347	20.11	21.33	22.63	23.87	32.88	38.33	35.14	38.75	34.22	33.74	29.31	28.18
SER985	0.152	31.99	18.85	37.47	30.33	51.73	35.52	95.83	19.74	72.81	14.67	68.14	15.98
SER994	0.124	13.46	11.48	24.63	12.99	15.54	11.99	15.05	10.02	10.01	9	9.44	7.8
	mean	13.8	10.88	16.05	12.4	16.62	13.49	18.06	14.27	16.79	13.72	18.26	14.96
	stdev	11.56	8.18	14.43	10.15	15.21	11.58	18.4	12.52	14.96	11.77	17.17	13.34

TABLE 8

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 8
STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 12-12-8

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER391	0.179	12.98	5.12	36.11	18.6	48.99	28.25	32.24	11.93	33.13	11.85	34.43	14.81	30.07	23.24	30.81	25.68
SER400	8.3	12 41	9 93	14.68	14.67	12.77	10.65	11.68	9.22	11.8	8.87	11.34	12.44	11.84	12.66	11.76	10.98
SER409	6.487	25.03	24.09	30.45	25.64	29.25	17.52	28.57	19.79	24 33	14.43	30.05	23.03	28.93	22.29	30.32	27.16
SER418	5.929	18.14	15.78	13.98	6	14 78	7.69	15.19	10.18	18.04	13.13	13.5	11.29	12.5	9.71	13.89	9.11
SER427	8.589	10.93	7 53	9.77	10.13	10.08	10.9	10.16	9.77	10.75	9.39	11.31	10.34	10.76	10.79	10.05	9.73
SER436	6.459	2.14	2 32	3.08	3.39	3 87	4.39	4.53	5.3	5 41	5.42	6.16	6.02	6.75	6.64	7.24	6.83
SER445	4.584	14.61	10.71	17.36	14.39	16.29	17.91	18.68	18.41	19 86	21.23	18.32	19.93	21.45	19.64	22.27	24.26
SER454	3.29	11 1	10 56	12 96	13.47	12 46	13 35	10.77	11.55	10.77	11.3	12.26	13.33	10.55	9.42	12.16	10.06
SER463	7.935	4.71	4.07	6.78	5.94	7 33	8.05	7.79	8.51	10.01	9.73	11.04	10.29	12.6	10.96	14.09	12.52
SER472	4.679	18.13	16 95	23 25	23.23	24 29	27.31	22.7	22.93	22 77	19.74	21.77	20.36	22.19	19.15	20.93	19.53
SER481	3.494	39.41	19.75	35.58	20.16	41 51	22 96	31.1	30.65	20 85	22.42	22.78	21.92	23.42	21.36	21 69	17.08
SER490	3.689	14.25	19.74	16.74	16.63	17.61	19.92	19.1	20.04	20.57	20.17	20.41	20.91	22.35	23.16	20.76	21.99
SER499	13.84	11.63	13.27	11.45	11.27	12.68	12.04	12.86	13.5	14 21	14.74	14.22	14.99	15.99	15.3	17.36	17.63
SER508	9.518	8.52	8.6	9.62	8.83	8.91	8.49	9.74	10.68	10.54	11.28	9.61	10.47	11.01	11.7	13.08	13.51
SER526	6.175	11.17	6.6	13.31	9.1	12.02	8.55	12.64	10.37	12.42	14.55	14.22	14.11	13.45	14.43	14.93	13.41
SER544	3.129	7.45	6.05	9.29	9.31	9.14	8.93	8.38	7.96	8 65	9.33	9.11	10.77	8.4	8.47	8	7.41
SER562	11.08	24.69	25.44	39.78	33.83	41.91	30.52	46.96	40.63	52.97	56.88	68.48	57.35	75.44	65.69	66.59	59
SER571	3.777	16.19	16.49	19.22	20.11	18.29	18 72	17.16	13.81	15 65	15.75	12.83	13.13	11.3	11.27	11.2	10.53
SER580	1.07	1.08	1 21	1.26	1.16	1.57	1.9	1.93	1.81	2 56	2.58	3.11	2.54	3.5	2.74	3 88	3.03
SER589	0.773	5.16	2 46	5 03	3.62	3.01	1.82	4.28	3.74	6.09	6.21	8.02	7.52	9.58	11.83	10.49	9.03
SER598	3.358	4.08	2 73	5 06	5.33	6.11	6 31	6.87	6.95	8.77	8.59	9.9	7.7	11.92	7.07	13 58	7.84
SER616	8.59	4.77	4.41	4.66	3.41	4.41	3.87	5.6	5.44	4 89	4.52	4.75	4.83	5.59	5.32	5.29	5.11
SER634	15.16	11.39	9.19	11.56	10.16	12.72	12.03	12.04	11.78	10.45	9.25	13.25	12.58	14.3	14.57	13.34	12.37
SER643	3 693	23.37	16.38	22 3	17.47	21.42	14.69	17.76	12.05	17 93	9.09	17.81	12.63	21.75	18.25	18.99	11.3
SER652	5.34	15.22	14.45	19.82	13.73	19.18	16.68	17.25	20.25	16.91	15.46	18.15	17.86	21.4	19.53	18.96	19.61
SER661	2.363	14.38	8.68	14.51	10.07	14.46	11.71	14.82	13.18	14.65	12	16.6	12.99	16.14	10.58	18.78	15.14
SER670	3.833	20.84	22.19	21.64	21.26	22.46	24.06	24.77	21.17	24.49	22.94	28.88	28.66	26.71	29.21	27.92	29.52
SER679	4.579	55.52	45.41	80.52	65.67	77.24	54	66.07	37.56	71.11	35.08	69.17	32.89	49.82	17.51	48.89	36.58
SER688	2.505	10.76	8.41	13.26	10.75	14.73	11.49	14.44	12.9	12 64	12.1	12.26	11.52	12.83	12.32	13.56	10.86
SER697	5.904	1.93	1.73	1.69	1.16	2.22	1.66	1.96	1	1.58	0.98	1.63	0.87	2.23	2.21	2.27	1.91

TABLE 8 CONTINUED..

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER706	2.954	2.93	2.23	3.1	2.06	3.39	2.51	4.42	3.43	5.73	5.81	6.56	5.72	8.08	7.18	8.91	10.14
SER715	6.171	33.42	11.74	38.54	13.68	59.35	16.37	69.66	35.38	58.89	27.97	58.44	47.78	46.88	35.4	45.2	29.28
SER724	4.883	17.25	11.49	16.76	14.01	15.89	14.58	18.88	15.33	20.2	18.85	19.71	13.92	20.66	19.71	20.49	16.49
SER733	3.991	8.55	5.28	8.84	7.44	15.02	8.18	14.89	8.6	13.44	6.71	16.61	9.25	14.69	7.57	16.29	7.26
SER742	1.023	2.36	2.06	2.3	1.8	2.62	2.11	2.77	2.28	2.77	3.04	2.46	1.49	3.58	3.69	3.59	4.01
SER751	5.298	5.68	4.08	7.68	8.88	7.63	7.34	8.76	7.02	8.87	10.01	8.38	7.44	8.93	8.74	9.01	9.19
SER760	2.101	11.69	9.75	12.06	12.56	9.41	11.12	10.88	9.57	10.71	9.27	11.58	10.44	10.12	10.12	9.28	8.78
SER769	4.798	8.2	6.85	9.59	8.14	8.55	5.08	10.03	11.56	10.4	9.38	10.47	8.27	10.09	8.85	10.76	9.74
SER778	0.196	5.59	5.96	4.85	5.11	4.26	4.45	4.42	4.35	5.41	5.08	6.08	6.54	6.66	7.99	7.27	8.69
SER787	2.419	4.28	3.62	4.45	4.77	4.65	5.32	4.05	3.56	3.67	3.01	4.06	4.35	4.27	4.35	4.14	4.69
SER796	2.657	19.71	20.75	18.37	20.35	15.93	12.74	15.19	12.53	15.22	15.1	17.65	16.38	12.82	12.48	13.72	12.66
SER805	5.871	2.57	2.87	3.81	3.62	5.02	5.12	6.18	5.93	6.97	7.17	7.79	7.78	8.79	8.72	10.17	10.74
SER814	0.123	2.38	2.18	2.2	2.11	2.53	2.44	2.77	2.74	2.98	3.16	3.14	3.16	2.75	2.8	2.6	2.59
SER823	7.221	0.69	0.51	0.91	0.52	1.27	0.85	1.62	1.02	2.07	1.34	2.57	1.66	2.94	1.64	3.28	1.96
SER832	0.652	2.51	2.51	3.37	3.24	4.23	4.07	5.13	5.03	6.07	6.03	7.02	6.95	7.86	7.82	8.6	8.38
SER868	0.297	12.14	6.88	14.69	10.65	13.78	9.7	12.52	10.34	11.12	8.43	10.82	8.9	8.71	6.85	8.51	5.79
SER877	8.283	3.7	2.98	3.55	2.25	4.35	3.14	3.8	3.55	4.52	4.22	3.25	2.03	3.89	2.84	3.31	2.17
SER886	0.117	17.59	19	20.91	21.9	23.06	23.8	26	29.11	29.36	35.84	32.02	38.49	33.66	40.02	34.9	41.87
SER895	0.079	7.85	6.65	8.31	7.86	8.61	5.06	9.28	6.12	10.71	8.14	13	12.58	15.27	15.07	17.23	19.38
SER904	1.376	3.1	2.37	3.01	3.02	3.4	2.92	4.08	3.14	4	3.23	3.22	2.98	2.29	2.7	2.78	2.7
SER913	1.2	43.54	37.36	45.06	36.58	47.77	41.62	58.02	60.82	56.29	57.99	53.86	58.4	55.79	58.78	62.53	62.47
SER922	25.18	7.22	5.62	12.02	12.8	13.76	12.68	14.8	14.87	19.22	18.38	21.8	17.76	23.6	19.66	25.61	23.14
SER931	0.1	18.46	11.27	13.46	10.53	42.61	20.84	56.2	42.3	37.26	32.84	59.35	51.06	45.6	19.49	34.47	28.03
SER940	0.194	20.12	14.82	22.19	14.93	22.11	23.17	32.09	23.86	41.83	28.05	50.83	34.89	84.97	#	82.79	86.64
SER958	0.985	31.24	34.18	34.55	34.23	23.32	25.86	18.7	14.44	16.68	7.19	16.86	6.19	18.35	6.8	19.97	12.82
SER967	1.983	23.21	26.93	27.9	32.91	33.74	40.06	44.2	50.37	47.21	52.84	48.27	54.84	47.71	52.95	46.02	49.87
SER985	0.116	27.19	14.72	36.98	26.25	36.62	27.72	55.55	21.43	86.77	15.91	84.25	17.75	68.99	13.5	51.4	15.44
SER994	0.096	14.46	12.09	14.92	13.61	23.88	18.95	35.86	27.31	25.97	17.59	24.45	23.8	20.69	19.32	12.68	9.74
	mean	13.61	11.16	15.85	13.18	17.21	13.69	18.25	14.81	18.61	14.68	19.83	16.19	19.71	16.78	19.29	16.95
	stdev	11.13	9.48	14.13	11.37	15.65	10.99	16.83	12.69	17.76	12.7	18.89	14.25	18.37	17.21	16.92	16.06

TABLE 9

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 12
STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 12-12-12

series	model	y+1		y+2		y+3		y+4		y+5		y+6
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape
SER400	9.881	7.59	3.92	8.06	5.61	5.9	3.38	10.54	8.76	11.72	6.74	10.47
SER409	7.713	22.32	20.71	26.84	22.95	29.12	19.8	29.13	28.37	30.89	24.54	29.83
SER418	6.784	20.07	12.15	17.46	14.8	14.01	8.68	14.77	9.25	14.98	11.37	10.41
SER427	10.35	9.62	7.68	8.19	7.83	8.38	10.25	9.5	10.32	12.18	11.62	12.77
SER436	10.49	1.91	2.29	2.58	2.14	3.01	3.16	3.69	3.82	4.71	5.01	5.86
SER445	5.649	20.35	18.18	19.27	16.32	18.05	21.51	19.59	20.99	21.72	24.55	22.02
SER454	4.867	14.31	16.72	18.92	19.91	11.17	10.95	9.56	9.43	13.18	14.53	14.6
SER463	19.45	5.37	4.13	6.78	6.78	7.62	6.28	9.77	10.51	11.13	12.74	10.2
SER472	5.724	24.16	17.23	25.96	25.01	20.65	14.76	17.88	12.63	20.95	18.06	24.2
SER481	3.208	32.11	18.68	33.98	24.6	41.16	24.77	47.62	25.2	35.69	18.83	34.84
SER490	5.528	19.69	19.43	21.65	24.65	20.6	20.34	18.04	19.91	19.3	19.84	22.02
SER499	15.04	13.89	12.16	13.06	10.3	14.59	12.82	17.04	18.05	18.99	18.33	20.04
SER508	10.92	11.43	13	11.81	13.02	12.25	13.06	14.13	15.59	14.35	14.71	13.29
SER526	7.247	11.65	4.24	16.77	11.28	17.28	11.09	13.21	9.37	11.35	11.3	11.52
SER544	2.82	9.91	11.13	11.34	12.26	11.9	13.26	11.28	10.46	11.37	11.01	10.69
SER562	12.3	35.19	25.2	48.24	45.65	51.88	46.76	65.88	68.56	79.52	80.57	82.29
SER571	2.612	19.32	19.35	21.67	20.45	20.02	19.76	18.74	17.21	19.54	18.51	19.07
SER580	1.13	1.28	1.47	1.48	1.71	1.49	1.49	1.4	1.15	1.39	0.99	1.49
SER589	0.765	6.6	7.52	9.16	10.93	8.22	10.46	8.89	9.02	7.91	9.97	6.25
SER598	5.625	3.85	3.58	3.38	2.72	3.43	2.76	3.52	3.05	4.64	3.19	5.08
SER616	10.75	3.27	1.27	4.28	3.16	4.05	3.4	4.52	3.7	5.12	4.69	4.95
SER634	18.19	12.66	7.68	12.79	10.1	11.44	10.77	11.38	11.01	11.39	9.95	11.32
SER643	4.414	24.15	13.4	25.56	17.62	26.38	12.34	23.73	17.77	22.16	13.39	18.57
SER652	7.478	12.56	13.75	15.82	15.72	12.63	7.32	13.82	15.6	13.39	13.58	17.97
SER661	2.935	20.13	6.39	13.46	4.45	13.33	7.15	13.9	10.26	14.59	13.06	13.67
SER670	3.7	20.92	26.21	21.65	23.51	20.58	23.77	21.35	23.29	21.83	20.94	25.43
SER679	5.31	80.76	96.65	95.85	#	#	87.13	#	81.18	95.97	72.44	98.61
SER688	2.769	6.08	4.28	12.91	10.05	17.51	17.19	16.51	14.58	15.82	10.24	14.95
SER697	6.278	2.43	2.07	2.4	2.27	1.9	1.28	2.96	2.72	2.47	2.5	2.37
SER706	2.691	2.22	2.08	1.7	1.47	2.73	2.33	2.99	2.09	3.25	2.85	3.11
SER715	10.55	45.38	18.73	34.03	33.67	34.21	31.3	49.52	34.26	43.28	46.54	46.39
SER724	5.14	17.86	15.15	18.36	15.47	19.79	15.5	21.96	20.12	23.26	22.23	21.69
SER733	4.031	9.77	5.8	10.54	7.11	10.29	6.9	8.25	5.96	9.01	5.18	8.95
SER742	1.352	3.03	2.54	2.5	2.47	3.42	3.34	2.89	2.12	3.28	2.63	2.62
SER751	5.502	6.58	5.85	5.38	3.95	5.69	3.78	7.58	6.44	8.52	6.86	7.74
SER760	2.692	11.4	11.87	13.06	13.11	13.15	12.17	10.74	10.45	10.62	10.07	11.67
SER769	4.827	5.58	5.81	7.16	6.09	8.16	5.66	11.76	11.25	10.78	11.61	11.23
SER787	4.09	5.5	5.89	5.39	4.88	5.82	6.9	5.64	5.6	6.3	6.16	6.24
SER796	3.322	17.98	17.32	19.84	21.03	16.46	17.42	20.36	18.93	15.63	13.58	13.18
SER805	7.164	2.8	2.87	3.73	3.66	4.55	4.1	5.71	5.53	6.73	7.01	7.64
SER823	9.886	0.71	0.6	0.99	0.73	1.33	0.88	1.77	1.1	2.18	1.39	2.63
SER832	1.206	2.41	2.37	3.29	3.29	4.11	4.17	4.89	5.07	5.77	6.09	6.65
SER877	10.36	3.71	3.22	3.52	2.87	2.84	2.08	3.44	2.74	3.4	3.04	3.6
SER904	2.533	2.58	1.92	3.97	4.12	3.02	3.06	3.85	3.87	4.54	4.43	3.57
SER913	0.997	69.68	67.37	51.29	25.16	48.68	32.89	47.41	45.93	48.99	58.66	57
SER922	20.1	7.5	8.27	10.13	10.77	12.56	12.36	13.78	13.29	15.66	16.04	17.22
SER958	1.18	42.56	44.2	42.55	44.58	39.14	41.35	21.71	15.95	22.56	14.1	23.68
SER967	1.792	22.91	26.91	25.8	32.16	38.56	43.74	50.98	51.55	55.38	56.76	55.85
	mean	15.7	13.73	16.55	13.24	14.96	14.49	15.91	15.71	18.07	16.51	18.45
	stdev	16.47	17.13	16.85	11.06	12.73	15.5	14.49	16.41	18.85	17.59	19.74

TABLE 9 CONTINUED..

	y+7		y+8		y+9		y+10		y+11		y+12	
m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
9.1	10.88	12.21	12	11.65	11.4	9.32	10.81	7.32	11.75	10.24	11.46	9.51
23.29	22.66	10.22	23.42	16.95	24.25	20.73	28.41	17.69	25.98	18.89	26.38	23.36
10.2	11.36	9.17	13.93	11.35	12.91	9.9	14.75	14.36	16.2	14.27	13.62	9.77
11.45	12.28	12.03	11.1	11.2	9.36	8.28	8.13	7.41	7.89	7.44	7.23	5.28
5.63	6.59	6.82	7.15	7.07	8.11	8.04	8.28	8.26	8.23	8	8	7.77
22.27	21.95	22.82	23.18	22.75	25.37	23.2	26.2	25	24.71	24.57	25.09	26.54
13.69	13.51	12.53	12.63	11.6	10.9	9.57	15.17	16.31	17.29	15.75	14.32	13.08
6.66	11.06	9.92	11.33	10.9	11.72	12.25	15.32	10.46	14.7	14.49	12.08	12.06
19.71	23.98	23.29	21.96	18.84	19.75	16.5	19.83	15.38	18.54	15.33	19.53	16.1
16.87	35.44	20.1	26.57	21.48	27.23	26.43	23.16	18.9	26.1	20.68	22.48	18.39
22.37	22.1	25.44	20.55	21.73	20.75	22.51	21.39	22.07	19.64	23.42	17.99	20.02
20.12	21.73	20.71	21.97	22	24.97	26.27	24.6	24.84	26.7	28.53	27.1	26.65
13.66	13.99	12.76	15.89	16.43	17.37	17.49	18.16	18.22	18.5	20.67	19.16	19.52
10.19	13.13	15.77	14.19	12.31	16.7	16.31	18.8	19.19	18.64	20.71	18.23	18.1
12.42	9.52	11.04	8.59	10.05	7.63	7.85	7.51	7.34	7.34	7.7	7.26	7.53
76.09	82.39	88.02	74.34	73.62	74.36	70.83	76.67	77.03	71.47	50.63	67.78	55.43
17	21.29	22.18	19.13	19.36	16.09	15.23	15.35	12.91	14.53	12.35	15.06	14.17
1.44	1.9	1.83	2.37	1.76	2.92	2.24	3.43	2.79	3.68	3	3.99	3.1
3.6	6.44	7.48	7.34	7.56	8.22	7.35	8.86	7.61	8.97	6.52	9.65	7.57
5.63	6.51	6.07	7.01	5.87	8.45	6.97	9.4	7.45	11.43	7.83	13.71	7.31
4.45	4.97	4.94	5.49	6.09	5.8	6.53	5.1	5.03	6.09	5.63	6.05	5.39
10.51	11.98	9.94	12.57	9.99	13.99	12.29	14.15	15.75	14.76	13.63	14.02	13.57
9.55	19.62	11.29	21.65	13.64	20.51	13.12	17.62	10.8	17.66	12.62	15.18	9.28
18.61	19.89	17.82	20.63	19.67	21.87	20.68	19.87	17.03	21.18	19.88	17.69	12.9
9.5	14.8	12	12.58	8.16	16.96	10.09	15.3	9.03	18.83	11.24	19.73	15.32
27.09	26.42	29.99	28.82	30.05	29.23	31.19	31.87	34.06	29.72	30.62	32.33	32.98
66.07	69.69	22.22	57.64	29.03	61.44	29.61	62.84	35.23	58.68	35.08	61.82	44.68
11.38	14.9	10.21	13.63	10.39	12.82	11.51	12.49	11.73	11.89	10.55	13.33	10.06
1.98	2.83	2.23	2.1	1.58	2.11	1.74	2.73	2.37	4.2	3.83	4.22	2.95
2.67	3.64	3.07	4.43	3.84	6.48	6.39	8.11	8.75	8.35	9.69	9.21	10.37
38.3	37.14	36.6	48.78	27.78	57.1	45.97	63.35	54.07	45.37	13.71	58.93	33.96
16.3	24.47	20.73	21.4	20.31	21.56	21.06	23.08	20.67	21.39	15.41	19.61	14.6
5.87	14.73	5.74	14.04	5.15	12.99	7.4	15.89	9.52	16.06	8.19	14.75	8.21
1.7	2.81	2.05	2.69	2.11	2.58	1.83	2.78	2.58	2.76	2.55	3.16	2.92
6.3	7.2	6.47	7.96	6.87	9.71	9.28	8.11	5.6	7.41	6	7.26	6.75
9.9	11.17	9.07	10.54	11.41	10.49	8.96	9.67	10.22	7.78	6.33	8.2	7.84
10.65	12	12.24	9.52	8.13	11.05	7.55	12.51	10.97	13.35	12.13	12.35	11.41
6.42	6.75	6.72	6.2	6.44	6.28	7.31	7.2	7.51	6.79	7.3	7.14	7.4
8.93	11.93	6.66	13.11	10.82	13.69	13.27	14.62	12.69	14.3	13.2	13.3	10.62
7.14	8.73	8.66	9.72	9.93	10.84	11.27	12.09	12.48	13.43	13.95	14.34	14.42
2.49	3.02	3.82	3.36	3.88	3.71	4.06	4.13	5.42	4.62	5.98	4.96	6.21
7.02	7.59	7.82	8.4	8.65	9.31	9.63	10.1	10.18	10.94	10.76	11.67	11.32
3.21	4.24	4.25	3.45	2.91	3.55	3.62	3.19	2.06	2.77	1.98	2.84	2.01
2.36	4.11	3.25	3.06	2.91	3.15	2.54	2.98	2.18	3.23	2.93	3.59	3.33
64.97	53.76	64.29	58.96	62.66	63.12	63.2	60.32	65.61	66.79	72.96	68.25	74.95
17.65	20.25	18.15	23.32	23.49	22.31	24.63	22.9	22.64	23.14	22.71	23.65	24.27
22.73	22.71	26.67	24.39	33.72	24.61	34.29	25.32	30.07	25.9	30.32	25.69	29.73
57.39	55.13	57	53.56	55.26	52.6	54.53	51.44	54.53	50.98	53.65	50.27	51.23
16.09	18.02	16.13	17.85	16.03	18.51	16.89	19.04	17.28	18.76	16.33	18.83	16.67
17.16	16.91	16.44	15.99	14.94	16.67	15.29	16.97	16.2	15.78	13.87	16.37	14.83

TABLE 10

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 18 STEP WITH α = 0.1, ß = 0.1 WITH ARCH. 12-12-18

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8		y+9
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape
SER391	0.04	4.60	4.60	18.10	18.10	24.70	11.66	41.44	37.55	27.21	18.74	40.19	34.63	35.31	26.34	61.52	53.20	82.52
SER400	15.98	3.34	3.34	4.79	4.79	9.99	9.06	14.00	11.90	16.96	13.69	14.15	13.44	14.60	14.56	13.49	12.44	15.81
SER409	8.79	12.41	12.41	19.66	19.66	25.17	28.87	42.55	27.86	46.28	36.82	40.54	40.72	34.78	13.53	31.23	9.34	33.49
SER418	4.64	17.43	17.43	12.78	12.78	3.91	0.65	17.80	4.38	21.50	13.69	12.49	10.01	15.49	12.69	15.15	8.07	14.11
SER427	14.01	7.25	7.25	9.66	9.66	6.77	6.09	8.43	8.78	10.87	11.27	10.86	8.84	13.05	11.19	12.29	10.67	10.93
SER436	13.03	4.90	4.90	3.48	3.48	3.67	2.50	4.52	4.29	5.33	5.85	6.24	6.74	3.97	3.89	7.04	6.68	8.45
SER445	5.76	20.61	20.61	11.80	11.80	15.35	19.90	17.39	20.02	21.72	23.94	25.68	26.12	27.18	27.74	28.46	25.10	27.42
SER454	6.07	30.58	30.58	25.63	25.63	21.63	21.00	22.31	23.51	24.48	24.18	22.39	23.26	18.98	22.93	12.18	12.87	10.04
SER463	24.44	2.68	2.68	5.23	5.23	5.42	5.94	3.42	3.23	7.83	5.61	8.30	7.43	13.49	12.98	9.73	9.83	6.31
SER472	6.86	27.10	27.10	27.76	27.76	23.29	27.96	21.63	25.56	21.42	21.00	26.17	21.69	24.10	16.43	24.05	20.29	21.92
SER481	2.08	19.07	19.07	23.86	23.86	21.44	25.36	18.57	19.44	17.47	11.02	34.52	15.19	34.32	12.86	26.97	17.05	36.51
SER490	6.33	26.93	26.93	19.58	19.58	25.21	23.49	23.30	21.86	23.37	22.12	26.30	25.74	27.33	25.23	28.09	25.31	27.87
SER499	14.29	17.51	17.51	15.31	15.31	15.71	15.92	17.32	16.81	13.03	14.29	17.62	18.78	19.58	17.96	20.17	18.86	23.60
SER508	11.46	12.55	12.55	14.15	14.15	15.22	14.53	16.33	16.27	13.86	16.44	14.94	17.09	15.62	16.97	17.92	19.58	18.94
SER526	8.68	7.95	7.95	13.47	13.47	13.52	12.88	17.59	13.36	17.00	11.73	12.46	6.12	22.42	23.80	25.04	16.37	24.31
SER544	1.55	12.38	12.38	14.25	14.25	13.66	14.28	12.14	11.96	13.31	13.00	15.16	15.28	14.09	13.59	15.21	15.20	16.77
SER562	14.63	73.86	73.86	82.46	82.46	64.40	58.80	67.99	73.47	70.27	68.89	97.55	92.07	98.01	90.22	78.94	74.42	83.93
SER571	1.90	25.92	25.92	30.65	30.65	30.15	32.02	32.24	31.49	25.40	26.70	20.22	20.03	22.54	23.84	20.12	20.58	18.95
SER580	0.56	2.76	2.76	2.24	2.24	2.22	1.99	2.17	2.10	1.63	1.53	0.89	0.70	0.90	0.64	1.48	1.28	1.80
SER589	0.86	12.15	12.15	11.02	11.02	5.65	4.63	1.85	1.93	4.88	3.81	6.93	6.46	7.39	8.16	5.55	3.43	4.15
SER598	8.54	3.62	3.62	4.54	4.54	6.81	7.55	5.64	5.91	6.55	6.38	5.21	5.55	4.61	4.87	5.35	5.27	4.42
SER616	14.39	8.73	8.73	7.04	7.04	5.07	4.17	3.82	2.93	5.61	5.02	5.80	5.51	5.77	4.03	5.09	4.36	5.66
SER625	0.04	31.68	31.68	10.22	10.22	25.50	25.42	25.70	21.65	29.55	30.73	28.78	29.79	29.21	26.63	#	20.41	#
SER634	19.79	30.74	30.74	14.08	14.08	17.95	19.13	14.42	15.78	13.45	13.28	12.84	12.83	13.11	7.82	13.76	12.33	11.96
SER643	3.76	6.85	6.85	15.47	15.47	23.98	18.77	32.38	32.28	32.16	24.78	29.75	20.64	31.01	20.67	26.57	10.72	24.43
SER652	3.84	16.13	16.13	2.66	2.66	9.61	11.92	7.57	6.86	7.27	5.12	8.20	7.53	14.50	11.96	21.91	21.51	24.77
SER661	2.64	6.30	6.30	8.66	8.66	9.08	6.33	14.16	15.06	9.32	10.80	15.63	8.74	17.39	13.68	15.97	13.41	21.54
SER670	2.67	28.68	28.68	33.13	33.13	21.06	19.94	22.42	20.59	22.82	22.65	23.73	25.90	20.57	23.74	21.77	25.77	23.53
SER688	2.49	17.84	17.84	14.86	14.86	15.63	17.46	14.12	14.57	11.27	10.63	10.87	11.05	11.02	12.90	16.57	15.01	22.30

TABLE 10 CONTINUED..

	y+10		y+11		y+12		y+13		y+14		y+15		y+16		y+17		y+18	
m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
75.85	35.84	26.09	57.29	46.85	76.21	60.22	64.82	50.30	#	31.46	#	60.01	#	46.01	#	23.60	#	72.11
14.71	15.77	14.50	12.65	7.31	11.52	8.68	12.44	9.69	11.61	9.49	11.12	8.49	10.76	6.17	9.92	7.58	7.31	3.54
6.35	36.06	29.01	27.81	20.25	26.45	21.47	24.23	15.07	20.46	12.76	23.17	18.96	29.38	23.35	28.07	16.71	28.64	21.16
10.97	16.61	13.46	16.33	16.69	11.09	10.32	15.71	15.83	17.47	14.51	16.88	16.10	14.47	11.47	14.98	12.06	12.97	8.54
9.62	9.87	7.38	10.68	9.31	9.72	7.63	9.31	6.36	8.61	7.13	7.97	6.36	6.83	5.86	7.60	7.30	6.93	6.25
7.64	9.02	8.56	9.03	8.20	9.33	9.47	8.76	10.06	8.48	8.97	9.41	11.20	9.63	10.53	9.21	9.54	7.72	8.32
25.36	32.13	32.10	31.35	33.81	30.24	28.50	26.27	26.35	27.38	28.04	28.15	33.54	29.48	30.37	30.92	34.04	33.22	38.07
10.71	12.27	16.19	13.13	12.43	16.11	18.33	16.36	17.24	15.43	14.73	13.67	11.61	17.45	15.24	20.09	17.32	19.64	19.04
6.55	16.04	12.16	8.60	6.78	7.91	7.87	28.94	27.37	32.07	32.46	29.76	34.16	33.18	35.85	34.90	38.94	33.73	30.19
16.11	22.46	16.49	21.33	14.59	20.59	16.72	24.57	23.01	25.80	25.79	23.89	17.86	22.34	20.59	19.73	16.49	18.60	15.31
19.39	34.49	13.11	26.77	11.12	26.86	16.41	29.15	17.67	28.26	21.77	24.57	21.74	24.52	17.71	27.76	27.43	30.06	27.53
23.76	27.16	27.12	25.05	21.46	25.47	25.61	24.22	24.22	23.10	24.52	23.78	24.73	23.27	24.37	19.79	19.77	21.40	23.17
26.54	25.06	26.50	26.76	26.93	28.93	28.83	29.89	28.24	31.92	32.08	36.04	35.79	36.28	37.75	41.03	43.19	41.42	41.64
19.60	21.09	23.40	21.36	23.13	22.45	24.10	25.05	26.87	26.12	26.31	25.72	26.59	24.73	26.08	24.77	26.53	25.88	27.48
16.70	19.08	15.24	18.29	13.10	21.00	16.01	18.15	15.82	17.34	11.45	17.57	11.38	14.61	9.16	12.57	7.38	15.16	10.56
16.83	16.26	16.16	12.95	13.78	9.89	11.17	9.28	10.46	9.86	11.87	11.49	13.63	12.74	14.01	12.33	12.28	11.41	14.48
79.69	91.89	#	90.60	93.57	80.51	78.81	74.64	78.77	75.07	77.77	72.89	77.67	60.82	65.91	59.45	69.17	54.19	44.69
16.12	20.59	18.24	22.01	18.04	23.70	21.50	27.21	27.87	28.71	29.68	26.76	26.50	26.87	28.69	23.18	22.85	22.61	20.21
1.51	1.84	1.52	2.04	2.03	2.19	2.37	2.33	2.59	2.53	2.50	2.72	2.51	2.77	2.04	2.86	1.59	3.17	1.29
1.21	4.20	3.76	5.12	3.00	5.11	5.88	6.14	7.06	9.30	9.39	10.74	10.24	9.72	9.23	11.37	13.19	12.90	14.52
4.17	4.04	3.54	5.14	4.96	5.11	4.92	7.00	7.77	7.79	9.08	8.56	8.25	10.20	10.99	12.10	11.21	14.10	11.59
3.56	6.36	6.54	5.99	4.89	6.60	5.52	6.38	6.15	7.42	7.37	6.78	6.15	6.64	6.34	7.96	9.08	7.08	7.55
29.84	#	33.99	#	29.18	#	43.11	#	48.88	#	#	#	#	#	#	#	#	#	#
11.89	12.35	12.90	15.65	15.32	15.68	13.48	21.62	16.25	19.68	17.55	24.02	22.31	22.17	21.88	22.58	20.36	20.42	20.22
14.45	21.05	7.74	20.91	17.67	18.42	9.27	23.99	26.99	23.45	20.86	21.72	18.58	18.51	15.46	18.28	12.73	16.01	12.91
25.68	24.33	26.42	24.24	25.44	28.08	26.73	20.78	19.66	22.35	23.52	24.69	22.79	23.23	20.14	22.47	23.44	24.03	21.04
15.72	22.36	15.03	24.55	15.91	16.00	14.80	21.06	22.66	23.66	21.81	22.09	17.33	22.14	18.97	20.22	16.39	23.05	19.30
25.22	24.30	26.35	26.16	27.51	25.27	29.78	35.02	36.51	34.41	35.81	33.20	36.18	33.73	33.18	32.99	33.29	34.04	34.17
14.55	22.46	14.68	21.98	14.58	20.33	11.58	22.79	12.82	21.83	11.64	19.06	10.64	16.12	9.88	15.60	9.87	17.29	10.62

TABLE 10 CONTINUED..

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8		y+9
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape
SER697	6.61	2.07	2.07	3.42	3.42	1.41	1.27	2.28	2.85	1.78	1.67	3.19	3.39	1.70	0.87	4.14	4.11	2.35
SER706	3.21	0.64	0.64	2.25	2.25	3.26	3.81	3.00	3.18	3.61	1.41	4.80	4.26	4.49	4.55	3.87	4.35	3.72
SER715	5.77	75.05	75.05	83.53	83.53	75.62	34.74	49.73	50.73	63.42	62.22	63.81	73.03	51.85	52.35	50.19	41.55	22.45
SER724	3.41	12.73	12.73	10.51	10.51	14.77	12.45	22.28	22.14	21.81	27.06	28.83	31.60	30.17	33.75	24.94	27.94	35.09
SER733	3.92	43.33	43.33	28.10	28.10	22.87	10.25	10.99	5.01	11.90	2.68	11.67	7.56	8.80	4.48	8.24	4.94	7.38
SER742	2.02	1.64	1.64	0.48	0.48	1.81	2.35	0.65	0.59	2.11	0.22	3.01	1.75	3.27	1.62	3.54	3.25	3.27
SER751	2.89	3.43	3.43	3.12	3.12	5.46	4.23	5.36	2.32	5.61	4.55	4.40	2.45	4.58	2.82	5.09	3.47	5.45
SER760	2.68	8.22	8.22	12.60	12.60	9.68	7.35	10.83	11.46	6.34	5.33	5.19	3.85	7.26	3.62	10.01	8.35	9.95
SER769	3.91	8.64	8.64	0.58	0.58	7.62	7.93	5.55	6.24	8.25	5.72	8.11	7.01	11.18	11.13	10.08	10.83	11.43
SER778	0.04	9.69	9.69	5.99	5.99	5.70	7.53	3.66	3.34	6.60	5.12	4.75	3.25	7.65	6.64	6.84	5.79	8.32
SER787	7.22	3.83	3.83	5.95	5.95	5.14	5.29	4.12	4.57	5.51	5.36	5.79	6.12	7.13	6.72	7.62	7.81	8.97
SER796	4.81	19.54	19.54	21.09	21.09	18.21	17.34	16.48	16.84	17.44	11.53	20.88	21.52	14.73	18.76	19.74	13.91	18.68
SER805	11.13	3.14	3.14	4.79	4.79	6.06	5.90	7.14	7.53	8.25	8.41	9.17	9.42	10.15	10.64	10.88	11.03	11.67
SER823	11.00	1.63	1.63	2.16	2.16	2.56	2.64	3.10	3.07	3.51	3.51	3.91	4.12	4.26	4.67	4.48	4.81	4.74
SER832	1.92	2.61	2.61	3.41	3.41	4.28	4.28	4.94	5.08	5.68	5.86	6.52	6.39	7.28	6.91	8.00	7.89	8.71
SER868	0.04	6.01	6.01	8.63	8.63	10.41	10.75	14.44	9.54	10.52	5.01	11.34	5.72	10.39	2.75	10.27	6.93	9.13
SER877	0.04	1.21	1.21	3.62	3.62	3.31	2.57	3.81	4.18	3.94	2.32	5.28	5.15	4.53	2.98	4.94	5.28	4.35
SER886	0.04	18.49	18.49	19.94	19.94	16.90	14.96	13.42	14.72	18.75	23.21	26.66	29.39	27.02	28.92	17.70	19.89	16.14
SER904	0.04	13.92	13.92	10.58	10.58	10.87	10.42	12.76	12.56	9.90	11.17	8.09	8.03	6.93	6.87	5.04	4.61	4.11
SER922	3.81	9.40	9.40	12.85	12.85	17.90	17.25	20.24	19.68	22.91	22.54	24.39	24.77	26.10	28.33	29.36	30.89	30.33
SER931	0.49	20.17	20.17	23.80	23.80	10.81	2.55	24.04	18.60	24.89	22.09	28.89	20.73	21.16	9.10	35.31	34.43	37.40
SER940	35.00	4.03	4.03	33.00	33.00	31.42	43.04	30.55	31.14	32.07	19.49	28.72	28.56	30.17	32.68	46.76	37.71	38.15
SER958	0.04	40.44	40.44	42.86	42.86	42.85	42.79	43.92	43.20	43.68	44.03	48.39	47.61	47.59	48.82	46.13	46.65	44.65
SER967	0.44	48.11	48.11	49.39	49.39	51.93	50.87	54.51	53.80	56.56	56.72	58.01	59.48	58.44	59.19	57.42	57.95	57.04
SER985	0.89	12.51	12.51	29.60	29.60	23.66	20.07	33.50	34.95	25.14	25.34	13.50	9.07	20.29	20.62	30.46	16.92	33.12
SER994	0.93	41.73	41.73	12.63	12.63	16.12	14.42	42.91	29.86	23.70	24.93	20.49	22.14	30.74	27.54	54.51	52.94	57.32
	mean	16.45	16.45	16.57	16.57	16.41	15.00	17.95	16.88	17.92	16.39	19.13	17.90	19.42	17.44	20.32	17.88	20.75
	stdev	16.46	16.46	16.94	16.94	14.79	12.86	15.04	14.89	15.08	14.93	17.53	17.72	16.73	16.24	16.93	15.73	18.19

TABLE 10 CONTINUED.

	y+10		y+11		y+12		y+13		y+14		y+15		y+16		y+17		y+18	
m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
3.24	2.13	2.09	4.59	4.41	4.43	4.09	5 82	5.76	6.30	6.19	5.22	4.77	5.00	3.73	4.90	4.17	4.82	4.05
2.44	3.63	2.91	4.97	4.72	5.00	4.79	5.25	4.97	5.85	5.32	6.92	6.72	8.31	8.17	8.05	8.17	7.77	8.73
10.95	34.56	24.87	35 32	13 28	33.29	10.19	23.31	8.52	54.89	12.64	42.11	16.80	42.16	22.75	59.30	36.73	48.98	27.06
31.02	30.10	26.17	27.25	19 12	33.19	25.76	27.08	23.69	26.72	20.50	27.12	22.79	25.31	26.81	21.94	14.03	24.46	18.35
4.18	9.16	5.60	7.77	4.58	7.11	4.64	17.46	8.07	17.88	8.54	13.14	7.45	15 68	8.95	16.99	10.26	16.26	10.22
3.02	3.99	4.03	4.08	4.29	4.98	4.91	4 90	4.57	5.65	4.08	6.44	4.95	5.84	3.48	6.24	4.69	6 38	4.36
1.98	4.64	4.48	5.04	3 34	7.31	6.61	10.13	7.54	11.81	11.95	10.86	10.28	11.65	9.58	11.60	10.43	11.32	11.13
9.47	9.45	9.17	8.46	6.15	9.49	8.28	13.45	12.68	11.36	12.82	10.32	11.28	10.83	9 39	9.57	5.19	8.22	5.71
10.55	15.49	15.24	15.35	14.48	12.68	12.78	12 08	11.96	10.05	9 64	13.45	10.91	13.81	14.05	13.66	14.43	11.43	10.53
9.05	8 84	10.77	8.21	10.52	#	10.48	#	11 84	#	14 65	#	13.31	#	11.23	#	13.83	#	17.10
9.23	8 91	9.49	8 56	9 67	8 85	9.46	11.33	12.12	12.14	13.30	11.81	12.89	11.97	12.83	11.97	11.91	12 22	11.95
16.37	14.68	13.09	17.42	19 27	12 97	12.10	17.00	14.13	17.25	18 88	14 88	13.51	13.59	11.05	17.65	14.74	19 60	17.30
11.82	12.56	12.94	13.77	14.16	14.77	14.85	15.77	15.88	16.74	16.71	17.59	17.68	18 48	18.47	19.52	19.62	20.36	20.39
5.25	5.01	5.43	5.53	5.63	6.16	6.15	6.67	7.00	7.35	7 64	7.93	7.72	8.37	8.17	8.59	8.71	8.67	8.47
8.68	9.36	9.58	10.08	10.52	10.79	11.15	11.51	11.76	12.19	12.28	12.94	12.88	13.65	13.54	14.33	14.17	14.98	14.69
2.93	9.16	7.53	9.28	7 28	8.72	3.60	8.48	7.18	11.71	12.06	#	7.35	#	5.71	#	4.03	#	12.58
4.44	3.63	3.48	3.65	2.45	3.87	3.48	5.59	4.10	5.23	4.37	4.59	3.18	4.82	4.53	4.99	4.71	4.69	2.90
16.61	23.84	27.82	25.40	30.96	20.69	24.69	21.80	27.21	#	29.33	#	31.21	#	24.89	#	22.64	#	23.01
3.61	4.15	3.55	4.41	3.39	4.31	2.49	5.10	2.77	5 51	3.54	5.11	4.46	6 26	6 03	5 69	4.59	5.25	4.34
32.98	31.14	34.09	31.07	30 18	31.55	31.94	32.46	36.12	32.58	37.15	33.17	38.13	33.09	36.91	31.97	38.58	33.12	36.67
18.29	47.90	29.94	43 54	24.70	37.19	36.14	33 92	33.89	#	29.77	#	38.81	#	39.34	#	61 96	#	54.80
45.37	40.01	37.28	40.42	43.54	36.19	43.00	67.75	62.23	#	50.39	#	63.92	#	54.41	#	48.01	#	#
43.78	40.49	39.04	39.42	39.38	37.95	38.91	41.44	42.27	42.48	42.17	39.16	39.48	37.86	38.17	34.87	35 88	35.07	37.76
57.29	56 46	56.44	56.64	56 31	56.14	57.00	54.15	55.59	52.56	55.54	51.15	54.77	49 16	53.16	47.02	52.80	43.43	46.99
32.19	76.44	15.29	62.40	17 80	67.44	17.32	64.69	55.21	#	31.67	#	32.54	#	32.66	#	27.69	#	20.31
64.83	27.03	24.54	65.28	72.25	32.95	28.24	38.03	30.39	#	30.23	#	38.85	#	29.44	#	51.23	#	82.65
18.36	21.07	16.72	21.51	18.73	20.92	18.40	22.48	21.02	20.14	20.03	19.79	21.07	19.53	20.09	19.82	20.49	19.57	20.71
17.88	17.79	11.61	18.22	17.46	17.64	15.74	17.03	16.65	14.49	14.44	13 64	16.39	12.56	14.43	13.23	15.63	12.43	16.75

TABLE 11

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 1 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-1

series	model	y+1	
	error	mape	m-ape
SER400	0.38	11.42	11.65
SER409	0.30	32.09	23.30
SER418	0.06	27.37	25.53
SER427	0.61	8.53	5.91
SER436	0.28	1.85	1.76
SER445	0.34	12.87	11.39
SER454	0.27	6.85	5.26
SER463	0.51	19.95	18.57
SER472	0.36	20.76	19.75
SER481	0.04	31.00	24.13
SER490	0.32	8.71	7.16
SER499	0.71	10.02	9.82
SER508	0.50	5.89	5.05
SER526	0.52	14.75	8.58
SER544	0.04	3.98	3.32
SER562	0.83	77.15	69.22
SER571	0.07	5.82	4.57
SER580	0.04	3.28	2.76
SER589	0.04	8.20	7.55
SER598	0.27	6.36	3.33
SER616	0.86	4.62	4.21
SER634	0.72	18.67	17.67
SER643	0.17	20.29	13.20
SER652	0.18	15.54	13.50

series	model	y+1	
	error	mape	m-ape
SER661	0.08	20.52	17.24
SER670	0.08	30.27	29.90
SER679	0.13	40.01	39.74
SER688	0.14	11.12	8.21
SER697	0.39	3.63	3.40
SER706	0.16	9.66	11.21
SER715	0.07	83.83	57.61
SER724	0.10	22.44	20.51
SER733	0.09	21.52	17.50
SER742	0.06	2.05	0.98
SER751	0.09	8.17	6.70
SER760	0.12	8.05	7.12
SER769	0.07	10.46	10.20
SER787	0.12	1.87	1.97
SER796	0.24	15.96	11.65
SER805	0.48	1.18	1.25
SER823	0.22	0.65	0.57
SER832	0.00	1.29	1.21
SER877	0.46	3.38	2.93
SER904	0.08	3.22	2.36
SER913	0.04	48.90	49.99
SER922	0.96	5.22	5.07
SER958	0.04	11.36	10.20
SER967	0.06	20.68	16.96
	mean	15.86	13.58
	stdev	17.35	14.68

TABLE 12

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 2 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-2

series	model	y+1		y+2	
	error	mape	m-ape	mape	m-ape
SER400	0.56	10.41	9.14	13.38	10.91
SER409	0.58	28.83	22.11	29.56	16.30
SER418	0.12	25.45	22.68	24.18	21.65
SER427	1.20	7.91	4.06	8.00	4.00
SER436	0.62	2.39	2.08	2.18	2.14
SER445	0.58	16.32	15.18	12.92	13.27
SER454	0.52	6.09	5.18	7.86	6.94
SER463	1.15	22.50	19.10	25.77	27.42
SER472	0.75	19.92	19.89	24.33	21.30
SER481	0.06	28.80	24.24	30.12	24.53
SER490	0.62	11.42	11.18	9.99	8.59
SER499	1.71	10.21	10.75	9.98	8.67
SER508	1.01	5.56	4.96	6.46	5.19
SER526	0.91	15.90	11.30	14.50	9.97
SER544	0.04	4.11	4.02	4.76	3.84
SER562	1.48	85.45	80.62	89.45	79.29
SER571	0.17	6.90	4.99	6.78	3.57
SER580	0.09	2.94	2.69	3.28	2.61
SER589	0.04	6.31	5.94	9.18	7.67
SER598	0.55	6.13	3.97	8.32	4.44
SER616	1.58	5.16	4.05	5.37	3.71
SER634	1.19	22.45	20.49	22.19	19.05
SER643	0.27	19.80	15.83	19.89	11.53
SER652	0.32	13.64	16.65	13.85	12.89
SER661	0.16	16.48	10.04	19.50	14.73
SER670	0.19	24.28	19.24	27.75	24.14
SER679	0.16	39.50	30.11	39.13	40.49
SER688	0.29	11.56	10.16	12.79	8.65
SER697	0.73	3.64	2.44	4.02	3.37
SER706	0.36	8.10	9.36	9.15	10.46
SER715	0.15	75 04	28.32	61.64	60.58
SER724	0.12	22.28	24.81	20.64	16.48
SER733	0.18	21.67	15.92	18.81	12.05
SER742	0.13	2.17	1.93	2.48	2.04
SER751	0.21	8.04	5.23	9.21	8.67
SER760	0.18	9.19	9.29	8.94	8.73
SER769	0.17	10.04	7.61	9.52	6.50
SER787	0.23	2.28	1 86	2.38	2.23
SER796	0.46	16.94	14.15	17.02	12.91
SER805	0.78	2.55	2.90	2.41	2.20
SER823	0.54	0.66	0 66	0.85	0.61
SER832	0.04	1.70	1.59	2.51	2.38
SER877	0.83	4.40	4.11	4.87	3.63
SER904	0.14	3.01	3.03	3.17	2.76
SER913	0.12	47.03	49.84	51.77	53.54
SER922	1.73	5.03	4.59	6.10	5.96
SER958	0.04	9.85	10.20	13.30	11.39
SER967	0.10	15.64	16.59	23.37	19.91
	mean	15.53	13.02	16.12	13.83
	stdev	17.00	13.90	16.63	15.73

TABLE 13

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 4 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-4

series	model	y+1		y+2		y+3		y+4	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER400	1.29	11.26	11.84	13.29	13.71	13.00	14.28	12.26	12.90
SER409	0.99	31.70	28.89	29.78	19.02	33.90	20.99	38.30	23.83
SER418	0.19	26.98	24.96	22.22	17.84	22.64	20.04	15.36	10.88
SER427	2.23	8.28	3.74	8.14	4.50	7.61	2.79	7.80	5.59
SER436	1.81	2.75	3.05	3.91	3.76	3.69	3.59	3.89	3.52
SER445	0.86	16.58	12.80	13.37	10.92	15.59	14.68	16.62	15.69
SER454	1.02	7.32	5.22	7.64	7.19	8.74	8.47	9.87	9.79
SER463	2.50	25.35	26.97	30.44	29.68	31.90	33.37	38.01	33.25
SER472	1.43	28.10	21.22	36.05	29.63	28.57	24.36	36.41	29.97
SER481	0.12	23.49	18.58	19.41	17.33	22.24	21.48	29.73	23.64
SER490	0.61	14.15	13.96	15.68	13.50	15.51	12.16	16.27	13.31
SER499	2.91	8.61	8.61	9.21	7.19	11.62	11.24	14.04	14.21
SER508	2.18	6.62	6.10	6.89	6.91	6.85	4.75	8.28	9.21
SER526	1.84	18.89	16.59	15.81	11.08	22.02	16.58	21.80	18.75
SER544	0.11	4.66	4.19	4.44	4.80	4.25	4.39	4.61	4.20
SER562	2.34	86.53	76.67	92.88	91.22	96.38	94.61	87.94	92.04
SER571	0.38	11.53	7.76	11.12	7.95	8.33	6.07	8.13	5.75
SER580	0.15	2.88	2.89	3.43	2.95	3.29	2.52	3.39	2.67
SER589	0.08	5.14	5.83	7.17	6.91	8.89	5.85	10.66	8.92
SER598	0.98	4.41	3.00	6.17	4.77	7.39	4.99	10.51	6.12
SER616	3.04	4.26	4.38	5.12	4.16	6.24	4.88	5.79	4.74
SER634	1.73	19.00	18.38	23.83	21.09	25.98	26.61	26.74	26.36
SER643	0.41	24.64	18.10	19.62	15.27	15.35	13.29	16.12	14.03
SER652	0.46	13.08	10.15	13.27	11.80	13.67	11.17	18.52	19.16
SER661	0.25	17.90	15.43	16.14	11.65	17.31	17.31	20.26	14.98
SER670	0.34	22.38	14.31	26.07	20.41	27.07	23.67	32.33	30.33
SER679	0.19	57.92	33.33	40.51	29.09	41.24	44.29	45.64	46.50
SER688	0.49	15.55	11.54	14.45	9.24	13.80	9.78	13.86	9.98
SER697	1.30	4.25	3.75	3.95	3.68	3.77	3.20	4.31	3.81
SER706	0.66	5.06	4.33	7.31	7.05	8.17	9.30	9.32	10.43
SER715	0.27	76.16	17.99	60.38	49.14	53.48	57.40	61.06	71.82
SER724	0.14	22.14	21.03	21.33	14.38	21.10	18.00	22.53	16.93
SER733	0.27	19.07	14.21	20.02	16.64	17.68	9.83	17.45	9.30
SER742	0.21	2.22	2.23	2.15	1.72	2.94	2.90	3.05	3.12
SER751	0.34	7.46	4.87	9.59	10.15	10.12	9.79	11.19	9.65
SER760	0.25	9.61	9.40	10.17	9.83	8.93	8.66	9.06	9.21
SER769	0.24	11.14	8.82	9.37	8.09	8.69	4.99	11.24	9.44
SER787	0.53	2.23	1.94	3.03	2.34	2.96	2.16	2.74	1.97
SER796	0.85	17.28	13.77	16.63	15.46	16.04	15.31	15.98	15.38
SER805	1.16	3.04	3.47	4.08	3.83	4.98	5.68	5.52	5.72
SER823	1.32	0.82	0.68	0.96	0.91	0.98	0.85	1.13	0.70
SER832	0.15	2.91	2.80	3.54	3.37	4.13	3.77	4.76	4.36
SER877	1.82	4.13	3.24	4.23	3.33	4.10	3.48	4.73	4.09
SER904	0.28	3.45	3.48	3.13	3.59	2.98	2.94	2.77	2.95
SER913	0.18	55.63	47.21	58.09	50.58	49.83	44.85	49.46	50.59
SER922	2.95	5.64	5.82	6.96	7.28	7.75	8.09	8.60	8.61
SER958	0.10	19.16	17.07	10.91	6.83	11.71	8.61	16.30	14.45
SER967	0.26	26.32	25.75	16.72	14.07	22.09	25.20	27.51	29.49
	mean	17.04	13.34	16.43	13.87	16.57	15.07	17.96	16.51
	stdev	18.32	13.45	17.25	15.63	16.84	16.84	16.95	17.76

TABLE 14

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 6 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-6

series	model	y+1		y+2		y+3		y+4		y+5		y+6	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER400	1.86	9.55	7.26	13.15	11.77	15.01	14.15	14.75	13.49	11.85	10.34	12.70	9.24
SER409	1.24	32.35	30.34	28.27	15.50	34.05	29.90	38.69	31.44	41.40	45.83	39.26	29.75
SER418	0.27	23.83	22.84	24.26	16.02	22.69	18.33	15.09	12.37	13.40	7.61	15.89	10.72
SER427	3.40	9.73	8.10	9.08	6.97	8.88	6.27	7.80	5.89	7.60	6.19	7.22	5.18
SER436	1.93	2.06	1.43	2.45	2.49	3.19	3.06	4.06	4.47	4.08	3.91	3.89	3.69
SER445	1.29	17.78	18.50	15.43	12.72	16.50	14.31	17.44	18.58	15.29	15.19	27.93	27.57
SER454	1.53	7.98	7.14	8.93	6.79	8.35	7.79	11.03	10.20	11.55	12.23	12.41	13.16
SER463	3.29	25.11	29.24	30.76	31.15	34.23	33.85	40.62	37.58	42.43	41.12	44.03	38.83
SER472	1.70	27.25	20.03	33.26	27.70	35.92	33.29	34.98	29.15	29.52	25.71	35.65	28.86
SER481	0.19	23.86	16.01	25.50	22.82	15.25	10.56	21.11	17.60	27.99	25.02	35.45	22.86
SER490	0.75	14.37	15.55	15.80	10.94	17.47	19.72	17.67	17.50	18.00	16.27	18.50	16.65
SER499	4.00	8.01	8.73	9.33	9.14	10.84	10.15	12.23	10.91	17.70	15.15	17.30	18.34
SER508	3.26	5.91	5.86	8.16	9.93	8.32	9.65	8.57	9.23	9.17	9.41	9.91	11.16
SER526	1.96	17.36	12.76	20.43	18.92	22.77	21.25	23.48	22.69	23.49	22.13	22.15	18.90
SER544	0.10	6.27	6.90	5.20	4.44	4.21	4.05	4.33	4.22	4.92	4.47	5.50	5.87
SER562	2.93	81.95	79.10	98.56	99.47	97.82	93.00	87.08	92.20	87.50	87.99	91.33	93.40
SER571	0.53	15.32	8.04	16.42	13.10	14.01	12 53	13.97	13.90	10.71	8.81	8.98	9.59
SER580	0.16	1.45	1.23	1.79	1.23	2.77	2.52	3.19	2.62	3.45	2.56	3.99	2.86
SER589	0.09	4.28	3.48	4.45	4.75	6.75	6.72	9.61	9.31	10.11	7.13	11.01	9.96
SER598	1.16	3.25	2.20	4.43	3.92	5.28	4.60	7.43	6.30	9.27	6.73	10.86	6.58
SER616	4.20	4.95	3.89	4.78	4.69	4.52	3.19	4.72	4.41	5.41	4.84	5.93	6.18
SER634	2.37	19.77	21.05	21.34	19.76	24.17	23.50	24.21	24.12	24.46	19.81	21.26	21.03
SER643	0.44	25.18	19.29	22.59	20.47	16.60	14.14	16.41	12.96	15.39	12.33	21.31	21.20
SER652	0.48	13.19	10.97	13.33	11.60	12.73	10.93	18.42	15.91	20.05	19.49	23.75	23.89
SER661	0.30	16.41	16.12	16.70	14.42	18.28	15.30	18.73	17.96	17.72	12.43	20.77	15.47
SER670	0.48	24.10	25.20	27.73	21.25	27.62	23.65	31.33	29.12	29.39	25.97	29.33	28.14
SER679	0.29	94.09	80.11	69.02	45.42	58.10	32.19	41.11	29.62	48.39	45.79	56.29	51.89
SER688	0.43	19.59	9.77	18.73	10.39	16.13	9.86	14.61	9.75	14.55	12.16	15.62	12.08
SER697	1.72	4.03	3.98	4.63	4.25	4.16	2.93	4.00	3.92	3.88	3.74	4.31	4.70
SER706	0.76	3.13	2.67	4.50	4.16	4.67	3.74	7.18	6.83	7.38	8.22	8.44	9.59
SER715	0.36	34.06	16.65	47.95	20.66	48.35	55.40	54.50	59.99	64.64	81.81	66.14	83.83
SER724	0.19	19.94	23.47	20.00	15.54	20.37	14.94	23.60	17.71	22.02	17.74	22.15	20.05
SER733	0.35	17.20	9.90	17.54	13.78	16.68	11.57	19.64	11.12	18.69	11.26	17.34	8.49
SER742	0.37	2.05	1.42	2.15	1.78	2.37	1.53	3.07	2.95	3.41	3.31	3.61	3.90
SER751	0.44	7.60	5.15	9.63	10.11	9.84	8.18	10.95	10.29	9.36	9.02	9.05	7.86
SER760	0.37	9.53	9.24	9.53	9.55	8.15	7.93	10.21	8.95	10.51	6.93	9.50	8.72
SER769	0.27	9.27	9.88	10.83	11.94	10.48	11.52	11.26	9.68	12.03	11.74	12.58	10.50
SER787	0.82	2.91	2.97	3.10	2.43	3.10	2.42	3.50	2.63	3.65	2.79	3.36	2.40
SER796	1.19	20.23	18.14	19.49	16.74	18.11	15.32	16.24	11.27	16.46	13.60	15.47	15.03
SER805	1.75	2.77	3.23	3.88	3.60	5.05	5.26	6.34	6.21	7.61	8.21	7.91	8.39
SER823	2.04	0.91	0.85	1.10	1.08	1.36	1.03	1.55	1.01	1.69	1.07	2.08	1.40
SER832	0.26	3.41	3.39	4.14	3.99	4.78	4.48	5.45	5.14	6.10	5.82	6.80	6.51
SER877	3.05	3.66	2.97	4.47	3.51	4.14	3.09	3.80	2.84	4.66	4.68	4.80	4.61
SER904	0.37	2.09	1.85	3.47	2.20	3.41	3.30	2.89	3.09	2.53	2.51	2.48	1.82
SER913	0.18	50.17	56.50	49.26	51.18	51.06	48.24	48.30	47.90	55.67	58.08	54.87	58.03
SER922	3.66	6.66	5.86	7.37	7.20	8.44	8.37	9.40	9.61	10.24	10.21	10.90	10.05
SER958	0.07	26.67	25.18	27.18	23.33	12.67	6.46	11.87	3.34	16.09	12.82	17.23	10.74
SER967	0.26	39.42	41.47	28.78	27.99	21 23	14.30	21.46	13.37	25.14	26.54	31.68	39.49
	mean	17.10	15.33	17.69	14.85	17.10	14.97	17.46	15.65	18.26	17.22	19.56	18.32
	stdev	18 70	17.60	18.29	16.45	17.68	16.48	16.22	16.54	17.34	18.81	18.23	19.39

TABLE 15

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 8
STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-8

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER400	1.27	9.85	6.51	10.72	8.43	15.30	13.42	13.24	11.77	13.95	13.19	14.69	14.73	15.29	16.90	16 02	16.95
SER409	1.24	26.07	22.09	28.12	21.35	30.77	23.70	34.36	28.89	37.15	36.53	43.15	43.42	44.13	38.23	47 77	39.89
SER418	0.36	16.10	14.49	21.43	11.94	22.60	16.74	15.08	10.02	13.27	8.41	13.72	9.36	16.47	11.32	17 01	14.50
SER427	4.45	10.48	10.34	10.34	11.29	10.37	9.28	9.52	9.96	8.65	8.44	8.03	6.31	7.87	6.10	7.97	6.24
SER436	2.56	2.61	2 44	3 39	3.60	4.69	4.65	4.95	5.32	5 06	5.29	4.58	4.07	4.35	4.55	4 24	4.43
SER445	1.22	19.10	15.70	16.52	15.35	15.59	12.67	17.38	16.18	18.18	14.97	18.49	21.33	20 08	17.14	23 18	22.90
SER454	2.15	9.12	6 32	9.64	9.33	12.92	12.26	10.84	11.64	13.08	11.58	14.43	14.46	12.47	12.35	14.82	14.73
SER463	4.85	26.05	28 77	37.65	37.22	37.75	34.60	41.59	42.73	43.57	43.88	48.62	43.21	49.78	41.22	48.93	39.63
SER472	2.32	22.38	18.20	33.30	32.67	28.22	24.57	27.67	23.67	31.44	26.74	27.73	25.01	28.23	26.49	31.83	26 23
SER481	0.18	33.47	19.98	27.07	17.18	21.66	12 33	21.40	18.73	18 69	11.66	25.13	20 95	27.85	22.67	32 22	22.73
SER490	0.86	15 48	19.05	17.16	16.67	17.61	20.10	18.07	19 44	18.32	15.77	18.98	19 77	19.33	23.61	19.71	20.16
SER499	4.96	13.15	14.68	13.79	13.56	13.25	13.35	14.28	14.45	16.46	16.57	16.39	17.41	19.59	20.01	21.44	21.71
SER508	4.05	9.54	9.52	10.50	10.90	10.33	10.26	10.75	12.26	10.63	12.02	10.58	12.35	10.64	11.12	13.38	13.94
SER526	2.04	12.91	8.00	14.28	9.90	22.09	20.35	22.44	19.76	21.13	18.30	23.06	23.00	23.59	21.85	25.42	21.98
SER544	0.11	14.49	15 29	12.31	13.45	9.83	10.55	4.86	4.14	4 65	4.54	5.51	5.60	5.85	5.39	6.01	5 84
SER571	0.52	15.39	12.71	17.38	16.97	17.04	13.57	17.83	15.27	16.74	15.49	16.30	16.50	11.77	12.58	9.22	10.03
SER580	0.12	1.07	1.07	1.19	1.01	1.41	1.01	1.52	0.84	3.12	2.79	3.51	2.81	3.74	2.86	3.06	1.93
SER589	0.08	3.98	5 35	4.24	3.22	5.59	4 69	8.45	7.26	8 71	9.35	8.73	9.49	9.83	7.94	9 56	5.46
SER598	1.38	2.63	1 80	3.47	2.90	4.06	3.72	5.27	5.10	6 70	6.38	7.58	7.00	9 66	6.71	11 29	7.02
SER616	5.10	5.80	4 87	4.68	3.36	5.83	5.55	5.97	5.89	5.44	6.00	5.47	4.78	5.60	5.58	6.06	5.72
SER634	3.16	19.60	20 88	18.71	18.29	25.05	24.89	21.47	21.37	21.15	16.92	20.34	16.09	21.44	16.90	19 38	17.62
SER643	0.47	25 38	15.06	21 52	12.05	18.82	13.41	15.35	11.43	18.22	20.67	22.49	21.19	23.15	15.94	19.33	13.04
SER652	0.62	16.66	16.36	12.15	11.55	13.82	12.13	20.91	21.31	20.14	22.18	19.95	17.73	20.53	17.84	26.17	25.80
SER661	0.42	18.97	18.06	17.53	13.31	15.15	10.85	17.66	13.65	20.42	22.27	20.34	16.22	20.36	15.52	21.22	15.63
SER670	0.69	31.21	36.77	27.81	28.16	29.54	32.06	30.49	30.96	30.01	30.83	31.81	32.93	29.93	28.71	30.74	31.88
SER688	0.49	25.11	12 57	22.56	12.26	21.32	10.12	18.71	10.72	16.64	11.39	16.33	10.55	16.50	12.55	15.98	14.89
SER697	1.81	7.58	7.43	6.06	6.05	5.58	5.43	5.08	4.61	4.79	4.72	3.74	3.50	4.16	3.80	2.31	1.87
SER706	0.82	4.09	3.01	4.63	4.55	5.31	4.78	7.32	6.02	7.95	7.58	7.91	8.47	7.30	8.48	7.55	8.69
SER715	0.33	28.07	19.77	28.15	12.50	36.52	13.72	39.19	30.53	51.81	47.73	60.20	78.17	72.21	88.07	66.13	82.38

TABLE 15 CONTINUED..

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER724	0.18	18.48	13 55	21.64	15.35	19.26	14.08	21.00	15.89	22.76	17.92	19.91	17.74	21.75	17.47	20.37	21.35
SER733	0.36	7.69	4.91	8.96	6.96	14.10	3.79	18.84	12.58	21.07	16.63	21.74	16.38	21.05	16.13	18.08	9.18
SER742	0.47	2.31	2 33	2.37	1.82	2.54	2.31	2.55	2.16	3.02	2.30	3.55	2.85	4.12	3.77	3.91	4.23
SER751	0.47	7.16	6.50	9.62	9.29	10.66	11.37	11.90	12.03	10.66	9.39	9.40	6.57	8.23	6.36	8.62	6.80
SER760	0.53	7.51	5.35	7.38	5.60	8.68	9.79	9.36	9.43	10.07	9.10	11.06	10.20	8.73	6.40	7.20	5.14
SER769	0.19	15 65	14.65	12.17	9.09	9.66	9.68	7.44	8.30	10.14	8.61	12.81	11.50	11.16	9.29	12.39	10.88
SER787	1.14	3.60	3 84	3.84	3.80	4.24	3.83	4.11	3.48	4.33	3.55	4.60	4.71	4 98	5.13	4.98	5.34
SER796	1.42	19 04	17.19	16.99	20.89	20.06	16.64	17.73	12.67	19 29	18.15	15.39	13.64	16.66	13.95	15 54	13.09
SER805	2 37	3.69	3 89	4.59	4.23	5.54	5.72	6.55	6 24	7.52	7.78	8.64	8.61	9 64	9.46	10.00	10.21
SER823	2.90	1.04	0.97	1.20	1.14	1.33	1.10	1.55	1.10	1.79	1.06	2.06	1.35	2.37	1.41	2.72	1 87
SER832	0.30	3.26	3 34	4.15	4.07	4.93	4.84	5.71	5.60	6.51	6.42	7.23	7.15	7.90	7.88	8.58	8.36
SER877	4.44	3.27	2.25	3 82	2.99	3.73	1.72	3.54	2.65	3.41	1.97	3.16	1.85	3.17	1.96	3.18	2.47
SER904	0.48	3.00	3 27	2.03	2.24	2.32	1.95	3.42	2.46	3.27	2.81	2.71	2.66	2.56	2.41	2 47	2.08
SER922	5.21	5.94	6.34	7.63	7.97	10.39	10.05	12.87	12.55	13.76	14.01	15.29	14.36	16.71	14.53	17.00	14.34
SER958	0.07	38.24	39.52	31.16	31.73	19.77	16.87	15.17	7.67	14.56	8.46	23.91	20.05	19.16	10.98	21.82	22.45
SER967	0.25	52.52	53 17	47.40	47.96	40.21	41.80	39.67	38.80	30.30	24.85	31.31	32.71	28.51	22.42	34.90	39.93
	mean	14.19	12.63	14.25	12.31	14.57	12.23	14.73	13.06	15.3	13.89	16.23	15.53	16.63	14.93	17.1	15.81
	stdev	11.26	11 26	10.88	10 83	10.16	10.13	9.03	10.40	9.70	11.03	10.54	12 35	13.83	13.45	14.34	13.49

TABLE 16

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 12 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-12

series	model	y+1		y+2		y+3		y+4		y+5		y+6
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape
SER400	2.38	8.89	3.18	10.07	6.83	9.24	10.86	7.84	5.83	10.14	10.80	14.15
SER409	1.05	19.46	8.67	20.35	13.17	21.31	11.77	36.40	20.68	30.95	27.37	35.62
SER418	0.15	17.78	10.15	20.98	16.15	22.53	13.65	12.35	4.95	9.43	6.74	12.36
SER427	6.37	9.59	11.24	7.68	6.16	6.06	4.25	7.59	6.89	8.85	8.53	8.59
SER436	3.57	1.71	2.00	2.43	1.87	3.15	3.39	4.14	4.57	5.39	5.50	6.14
SER445	1.62	19.56	18.27	17.03	15.45	11.57	4.48	14.12	14.16	19.79	23.52	23.84
SER454	2.65	14.34	13.14	11.96	12.67	7.86	4.83	8.04	8.44	11.29	11.87	11.65
SER463	12.96	23.17	18.39	39.05	39.31	43.25	44.55	44.89	47.00	47.19	47.16	46.73
SER472	2.64	20.06	10.52	20.44	21.65	22.71	22.40	18.40	16.84	18.25	18.70	26.01
SER481	0.10	32.62	20.19	34.96	15.38	24.30	13.73	24.07	16.87	32.02	22.33	28.94
SER490	0.95	21.86	22.66	23.67	22.78	23.08	21.85	23.78	25.47	21.02	24.65	20.13
SER499	6.07	17.16	16.18	18.88	17.83	20.15	18.90	21.70	21.68	24.14	23.31	25.43
SER508	5.29	13.47	14.60	12.67	14.10	13.14	14.29	13.79	15.54	13.95	15.46	14.89
SER526	1.85	18.00	9.68	23.49	19.70	15.34	9.83	15.85	10.51	14.58	6.71	15.90
SER544	0.08	16.84	16.82	17.41	17.80	16.82	16.89	10.96	10.67	7.02	6.29	5.80
SER571	0.63	21.92	19.76	23.71	24.78	19.48	19.63	19.18	16.18	20.31	18.62	21.28
SER598	1.61	1.49	1.20	2.50	2.00	2.63	2.36	3.75	3.14	4.78	3.67	4.83
SER616	7.45	6.20	5.39	6.35	5.02	5.24	3.53	5.35	3.61	6.12	5.49	5.52
SER634	4.34	20.03	23.38	21.61	22.33	19.98	20.35	23.69	28.19	18.74	16.86	21.12
SER643	0.22	26.12	11.49	21.87	10.95	22.48	14.89	15.09	14.75	20.52	13.19	20.93
SER652	0.52	14.81	18.11	12.97	12.08	12.20	12.45	16.95	14.99	22.02	21.51	19.10
SER661	0.52	25.39	30.96	24.63	26.56	15.26	16.01	14.46	11.52	13.33	9.17	14.55
SER670	0.41	28.69	29.24	26.29	25.06	25.71	25.30	26.24	27.02	26.32	30.24	26.69
SER688	0.46	10.78	9.41	17.34	13.89	24.29	16.08	26.61	15.08	24.25	11.29	23.93
SER697	1.34	6.93	6.71	6.36	6.72	6.32	6.07	6.39	7.00	5.18	6.71	4.87
SER706	0.73	1.98	1.89	1.58	1.76	1.96	1.52	2.72	2.18	6.09	4.88	5.21
SER715	0.41	33.01	22.69	27.29	23.15	30.34	12.71	29.25	12.02	35.94	21.54	43.09
SER724	0.11	27.17	23.59	17.44	14.11	24.55	20.37	26.67	19.39	22.95	17.58	20.62
SER733	0.33	12.13	12.91	12.12	10.85	10.77	12.50	4.89	4.54	6.20	3.68	7.34
SER742	0.47	4.31	3.91	3.38	3.40	4.41	3.85	4.01	3.60	3.99	3.56	3.32
SER751	0.52	5.95	4.33	7.58	4.40	4.81	4.36	7.54	7.60	7.62	7.23	8.83
SER760	0.64	9.31	9.90	9.04	7.99	9.98	14.01	8.98	7.65	8.16	5.70	7.18
SER769	0.16	10.88	11.11	13.15	12.52	17.94	19.04	10.22	8.67	6.42	6.54	7.39
SER787	2.01	4.66	3.74	5.30	4.98	5.94	5.66	6.28	4.99	6.24	5.66	6.75
SER796	1.71	22.20	23.01	22.85	21.65	19.82	18.28	21.78	20.66	20.91	18 20	14.96
SER805	3.26	3.02	3.34	3.99	3.41	4.93	4.37	5.93	5.57	7.01	7 28	8.02
SER823	4.09	1.53	1.42	1.85	1.80	2.15	1.79	2.46	2.11	2.99	2.45	3.26
SER832	0.43	3.47	3.40	4.20	4.19	4.97	5.04	5.72	5.95	6.47	6.86	7.22
SER877	6.52	4.81	5.50	3.70	3.39	3.63	2.55	3.48	2.07	3.10	2.88	3.40
SER904	0.70	3.95	3.77	3.07	3.20	3.21	3.40	3.07	2.89	2.99	3.74	2.83
SER922	6.97	10.96	13.83	10.60	11.61	12.31	11.47	13.10	13.54	15.37	15.37	15.91
	mean	14.05	12.19	14.44	12.75	14.04	12.03	14.09	12.07	14.59	12.90	15.23
	stdev	9.10	8.11	9.50	8.69	9.44	8.59	10.05	9.14	10.30	9.48	10.83

TABLE 16 CONTINUED..

	y+7		y+8		y+9		y+10		y+11		y+12	
m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
14.77	15.11	14.68	17.42	18.07	17.12	17.93	16.32	16.70	15.76	14.71	14.72	12.28
31.30	39.91	38.85	39.34	32.04	45.96	36.70	45.56	33.36	45.73	46.05	40.08	42.06
9.96	15.05	9.61	13.91	9.88	15.63	10.35	14.42	10.69	12.06	10.14	14.55	11.71
7.67	8.90	6.77	8.36	8.24	8.31	6.95	7.12	6.05	6.64	4.36	7.14	4.96
6.21	6.51	5.96	6.03	5.78	6.08	5.69	5.86	5.95	5.67	5.92	5.30	5.94
23.20	23.86	24.33	25.09	29.02	30.05	31.78	29.35	29.24	31.68	29.29	34.22	32.05
11.30	11.35	10.25	10.57	9.90	12.77	13.29	13.56	13.09	13.57	12.86	13.35	13.75
45.74	48.71	46.29	54.62	45.85	55.45	46.40	53.82	45.80	53.46	46.40	51.50	44.15
24.44	20.20	15.84	28 14	26.97	36.50	28.57	30.30	28.08	25.99	26.35	29.90	25.53
19.10	25.71	19.64	24.47	20 85	30.31	24.53	28 94	18.89	27.34	17.20	33.06	23.55
21.62	22.23	23.53	21.42	22.22	20.75	22.03	20.59	23.40	19.16	20.21	18.13	16.98
25.25	24.96	23.61	25.75	27.51	27.51	28.43	27.15	27.07	28.16	29.03	32.15	31.02
15.39	14.19	15.80	16.18	17.06	16.83	16.73	17.00	17.87	17.63	18.72	19.62	20.30
11.06	18.16	16.03	20.74	18.02	22.69	20.45	25.54	20.98	22.06	17.66	20.68	16.60
5.67	6.68	6.24	7.01	7.51	6.77	6.29	6.85	6.11	6.92	6.82	6.78	6.98
19.63	22.36	20.80	24.82	28.11	20.05	18.52	15.45	13.67	14.27	12.57	14.14	13.61
5.02	5.54	5.59	6.21	5.63	7.00	4.96	6.81	6.31	8.15	4.92	9.68	4.96
4.64	5.75	4.59	5.58	5.25	4.69	3.56	5.32	5.27	5.69	5.74	5.55	4.91
24.61	22.02	21.82	20.74	18.62	22.13	18.15	21.06	15.85	20.14	20.34	18.44	20.16
18.55	23.67	20.46	21.22	16.88	19.65	16.32	17.15	13.00	16.28	11.17	13.36	8.08
17.13	19.38	14.56	20.91	17.40	21.81	22.44	24.02	21.27	20.85	14.78	28.56	25.40
10.00	15.04	9.47	14.91	12.69	17.31	12.88	17.88	14.50	17.68	13.26	20.52	18.55
25.96	29.52	31.08	28.61	29.00	28.80	30.50	30.56	31.65	29.43	33.82	30.77	33.83
14.50	22.63	11.62	19.74	11.46	14.07	10.71	11.34	11.48	10.45	9.13	10.67	7.40
4.78	4.93	5.96	2.26	1.63	3.15	2.61	2.74	2.60	2.10	2.16	2.83	2.66
5.21	6.39	5.78	5.77	5.76	5.67	5.98	7.91	9.20	7.07	8.06	7.64	8.58
32.96	61.23	69.66	64.14	77.22	66.35	80.16	65.50	81.21	62.28	76.31	55.77	66.91
16.36	21.32	19.81	19.76	15.27	19.63	16.68	18.29	15.32	17.86	15.30	16.42	13.19
5.67	17.98	4.58	17.40	8.16	15.25	7.26	17.93	9.18	18.22	10.33	17.89	13.47
2.68	3.32	2.67	3.07	2.62	2.69	1.53	2.91	2.50	2.80	1.93	3.51	3.22
7.52	8.19	4.27	9.53	6.33	9.90	8.40	8.35	5.46	5.91	3.49	6.15	6.49
5.96	7.99	5.61	8.66	7.06	8.32	4.80	9.36	6.66	10.84	10.85	12.32	10.84
5.41	8.63	7.19	8.27	7.62	10.88	10.17	15.00	15.48	16.08	16.43	14.76	15.16
6.98	7.21	7.48	7.19	7.23	7.52	7.85	7.91	8.17	8.23	8.60	7.65	7.57
14.02	20.23	22.93	18.47	14.41	16.48	13.89	15.04	12.15	16.83	15.47	18.13	13.93
7.51	9.05	9.16	10.25	10.53	11.33	11.99	12.51	12.77	13.75	13.91	14.00	14.06
3.08	3.71	3.96	4.18	3.76	4.93	3.76	5.94	6.25	6.98	8.90	7.04	8.40
7.61	8.11	8.38	8.85	9.12	9.59	9.93	10.31	10.40	10.97	10.80	11.65	11.30
1.95	3.50	1.55	3.86	2.87	3.42	2.58	3.21	2.05	3.61	2.87	4.06	3.37
2.43	2.43	1.29	2.82	1.78	3.00	2.07	3.11	2.37	3.15	2.78	3.39	3.30
15.56	17.18	15.39	19.73	19.60	20.36	20.95	21.25	20.54	21.42	20.13	20.99	21.44
13.62	16.56	14.95	16.98	15.73	17.72	16.21	17.54	16.06	17.14	16.09	17.49	16.31
9.81	12.35	13.31	13.08	13.94	13.98	14.59	13.55	14.23	13.11	14.26	12.60	13.14

TABLE 17

MAPE AND MEDIAN APE VALUES OF MONTHLY FORECAST AT 18 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 24-24-18

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8		y+9
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape
SER400	2.03	4.12	4.12	4.80	4.80	9.37	8.66	15.19	14.33	21.56	20.38	19.71	20.50	18.25	20.82	18.49	17.26	20.37
SER409	1.56	21.53	21.53	36.91	36.91	31.38	24.98	24.63	21.50	30.68	30.54	14.27	16.57	15.34	10.70	31.61	27.30	43.29
SER418	0.04	1.95	1.95	6.95	6.95	9.18	8.11	21.29	8.93	17.64	16.89	15.47	11.47	15.48	10.28	18.55	16.28	17.58
SER427	7.91	7.48	7.48	12.23	12.23	8.73	9.43	7.49	7.10	7.42	2.88	7.50	4.43	8.38	7.84	7.07	5.74	6.83
SER436	3.76	4.50	4.50	2.63	2.63	2.10	1.14	1.55	1.02	2.28	1.80	3.17	2.42	3.24	2.31	4.17	4.22	4.96
SER445	1.21	16.18	16.18	12.52	12.52	9.81	9.17	13.31	13.80	20.71	21.96	23.94	25.44	25.84	26.46	21.18	24.27	34.30
SER454	3.11	15.67	15.67	19.26	19.26	20.21	22.29	14.87	16.62	15.21	16.04	14.13	15.31	10.63	13.16	17.34	17.57	10.54
SER463	16.12	1.95	1.95	26.90	26.90	20.61	20.42	21.42	22.06	23.47	24.36	29.14	30.19	39.37	40.98	42.70	40.21	41.41
SER472	3.41	44.02	44.02	33.68	33.68	33.18	35.21	28.42	27.67	20.96	18.71	21.20	19.57	23.32	22.49	24.68	23.63	29.21
SER490	1.20	34.73	34.73	25.87	25.87	26.08	23.47	28.18	27.11	24.91	25.24	29.42	28.97	29.76	30.34	28.97	26.76	31.85
SER499	6.46	21.57	21.57	20.21	20.21	19.64	20.38	21.50	21.83	19.97	20.75	22.11	22.51	24.32	22.10	25.83	24.27	29.77
SER508	5.99	17.62	17.62	19.14	19.14	19.03	17.85	20.51	20.25	18.05	20.50	21.68	23.19	18.27	20.59	19.68	21.00	20.98
SER526	2.45	16.69	16.69	9.08	9.08	10.66	5.69	21.01	15.52	13.94	12.30	11.74	6.31	16.85	12.87	14.22	8.69	11.76
SER571	0.12	26.68	26.68	29.13	29.13	37.33	38.55	43.85	43.65	31.41	32.77	24.92	24.56	27.93	27.64	33.90	35.18	26.76
SER598	1.57	3.75	3.75	4.38	4.38	6.12	4.51	6.77	5.78	7.62	7.86	7.57	8.11	7.17	7.41	6.83	6.48	6.75
SER616	8.99	8.76	8.76	5.39	5.39	4.28	3.17	3.17	1.75	5.69	3.03	6.85	6.87	6.09	4.27	5.89	3.50	5.93
SER634	3.30	28.17	28.17	11.41	11.41	23.49	20.25	21.22	21.22	18.23	20.36	19.33	20.89	26.68	29.15	18.07	17.07	20.25
SER643	0.06	14.49	14.49	11.36	11.36	13.96	3.55	17.66	12.42	20.07	16.13	21.41	15.81	20.39	3.87	20.61	17.50	25.05
SER652	0.13	5.46	5.46	12.68	12.68	13.92	11.08	8.63	7.11	13.95	17.96	24.33	22.39	35.04	36.06	33.56	33.85	29.03
SER661	0.27	32.86	32.86	28.56	28.56	11.50	13.75	5.81	5.96	15.10	20.81	19.55	17.36	22.58	13.26	40.48	17.29	51.88
SER670	0.19	30.38	30.38	45.85	45.85	26.44	21.64	25.01	20.86	26.32	23.10	24.54	21.32	23.76	18.46	23.95	24.35	24.40
SER688	0.24	9.68	9.68	4.99	4.99	8.48	7.53	14.84	16.06	13.16	13.53	17.05	17.25	13.73	11.21	31.06	16.72	70.84
SER697	0.95	7.15	7.15	3.72	3.72	4.69	6.28	3.65	4.18	5.36	3.77	5.57	5.48	6.31	3.86	3.16	3.48	2.54
SER706	0.52	21.82	21.82	15.18	15.18	10.03	10.78	5.85	5.97	6.40	5.64	8.07	7.93	6.47	6.70	6.77	5.92	4.36
SER724	0.04	25.14	25.14	7.62	7.62	18.86	22.75	28.93	29.31	20.43	19.13	19.73	16.20	26.06	26.50	17.22	15.27	35.44
SER733	0.14	66.47	66.47	22.78	22.78	32.80	27.09	31.25	35.10	32.26	22.70	23.44	24.03	15.67	13.07	8.39	6.75	8.22
SER742	0.29	4.48	4.48	4.68	4.68	3.82	3.95	4.64	5.13	7.83	9.74	5.92	5.80	6.88	5.93	9.97	9.69	7.26
SER751	0.20	0.38	0.38	7.47	7.47	13.18	11.99	10.50	7.45	9.08	8.41	8.19	6.07	5.91	2.19	7.74	5.65	7.68
SER760	0.41	0.49	0.49	6.52	6.52	2.26	2.71	7.63	7.16	10.66	11.25	8.81	9.21	13.79	13.55	15.87	16.33	14.08

TABLE 17 CONTINUED..

	y+10		y+11		y+12		y+13		y+14		y+15		y+16		y+17		y+18	
m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
23.83	18.49	17.11	17.63	16.25	19.94	21.78	19.52	21.18	19.47	19.99	16.72	16.26	15.86	14.74	14.31	14.75	13.56	13.72
33.40	35.07	21.44	31.60	20.89	32.47	25.66	30.31	15.26	27.35	21.01	16.73	8.54	20.78	13.73	18.24	11.39	21.19	19.54
10.74	10.16	6.66	10.02	7.29	11.44	7.86	14.98	12.64	12.98	10.04	13.98	9.07	13.38	8.91	12.48	9.48	13.74	8.13
4.75	6.18	4.78	7.50	6.07	9.07	8.35	8.82	8.12	7.73	6.34	6.78	4.49	8.97	6.91	7.78	6.60	7.48	4.36
4.79	5.58	5.73	6.46	6.72	6.99	7.03	7.57	7.63	7.46	7.06	7.54	7.32	7.42	7.14	7.87	7.55	8.34	8.80
33.61	31.62	29.92	30.91	29.27	38.71	37.69	29.25	31.15	34.56	36.03	35.01	34.89	27.32	26.44	32.90	34.09	30.87	31.47
9.58	10.44	12.78	13.62	13.65	13.70	13.42	13.09	11.13	13.94	11.15	16.17	14.37	14.65	12.32	14.20	12.35	15.93	14.08
34.77	45.09	45.25	43.38	44.23	44.39	47.19	43.16	44.32	43.79	44.37	44.26	43.23	38.77	42.86	36.23	37.62	36.99	39.86
25.20	22.02	23.32	19.94	22.67	20.17	17.41	22.79	21.32	22.88	21.13	21.55	19.05	19.88	19.42	18.88	19.01	17.94	17.33
29.42	31.39	31.08	30.40	29.30	29.53	31.44	25.60	25.11	23.35	22.93	25.27	25.89	21.76	22.50	14.57	12.37	14.92	13.62
29.87	28.84	27.38	33.74	33.20	33.08	33.77	35.67	36.66	37.15	37.83	39.02	39.54	38.61	39.35	42.20	42.75	44.55	44.88
21.28	22.71	24.50	22.12	24.30	25.59	26.95	25.64	27.03	25.23	25.75	24.19	25.51	25.58	26.90	25.19	27.33	26.42	27.84
6.38	14.21	8.31	13.12	7.46	17.46	11.61	20.23	17.17	20.06	16.93	18.85	11.54	18.82	12.75	17.14	12.81	16.49	12.47
24.66	28.41	23.28	29.14	21.80	29.20	25.23	27.19	26.48	27.09	27.09	29.26	31.58	32.07	35.79	25.16	24.47	23.38	21.21
5.59	5.69	4.56	5.35	4.47	4.49	4.61	5.25	5.01	5.68	6.28	6.36	6.71	7.36	6.27	9.26	6.25	11.48	7.64
3.40	5.96	6.09	6.23	4.44	6.02	6.57	5.58	5.17	6.11	5.23	6.34	6.34	6.23	6.17	7.04	7.59	7.28	7.91
13.66	16.12	14.22	20.44	18.66	21.31	21.69	22.77	19.45	27.00	19.95	26.35	22.01	24.21	20.14	23.96	22.38	24.20	22.72
12.15	18.11	14.00	20.49	17.15	18.52	15.13	17.51	13.23	15.95	13.23	16.88	14.20	14.05	14.30	14.01	13.71	14.41	11.63
32.38	33.29	32.92	33.42	32.22	27.39	24.66	20.79	11.87	20.16	14.75	21.57	17.53	19.08	16.39	17.59	10.36	15.19	14.82
20.66	35.50	16.44	44.02	18.09	25.32	10.31	23.40	17.73	22.38	16.77	16.94	12.14	19.61	15.59	24.00	20.08	29.87	24.22
27.39	25.66	26.20	24.12	23.48	23.98	25.62	27.60	22.06	36.27	22.86	35.84	30.77	34.98	29.56	36.07	31.99	34.14	32.62
19.59	70.04	33.07	54.36	18.47	34.86	13.13	19.15	9.64	16.45	10.67	16.42	12.53	11.73	7.82	11.46	9.46	22.62	25.96
2.08	3.37	3.92	4.15	4.48	4.09	3.90	3.83	3.44	3.63	2.80	4.29	4.63	3.38	2.67	2.78	2.17	1.94	1.97
3.75	4.67	4.45	5.05	4.96	5.81	6.00	7.75	7.41	7.70	7.31	8.24	7.73	8.76	9.11	7.72	7.98	7.28	7.60
27.53	25.23	18.86	23.69	19.40	17.08	11.65	17.39	12.30	17.29	11.66	15.54	14.74	21.48	17.46	16.81	14.26	19.18	15.45
5.55	6.46	3.92	8.41	5.82	13.19	13.56	18.13	12.47	17.81	12.69	19.82	14.98	21.48	17.29	18.41	9.07	22.08	13.49
5.90	6.25	6.52	5.10	4.75	5.68	5.94	6.29	7.01	6.94	6.88	7.10	6.09	6.50	5.11	6.96	5.82	7.41	5.44
8.10	6.82	5.95	6.14	5.46	7.62	5.63	7.56	4.78	9.50	7.44	13.25	12.45	14.35	13.61	13.53	13.02	11.36	10.61
13.62	8.56	6.14	12.88	11.51	9.44	10.63	11.17	15.57	12.01	13.37	10.96	11.29	14.90	13.39	18.89	19.10	17.17	18.53

TABLE 17 CONTINUED..

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8		y+9
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape
SER769	0.06	0.68	0.68	18.74	18.74	13.72	13.72	10.52	11.80	10.99	12.28	12.89	10.73	12.38	13.14	15.89	16.42	15.22
SER787	3.33	3.37	3.37	4.04	4.04	4.53	4.55	4.23	4.29	4.87	4.13	6.19	6.03	7.43	8.15	9.48	9.26	10.31
SER796	2.75	16.40	16.40	7.61	7.61	25.43	19.30	21.73	22.46	22.12	14.87	21.01	20.83	22.38	22.17	18.56	18.36	19.78
SER805	4.97	3.96	3.96	5.59	5.59	6.83	6.84	7.87	8.23	8.75	8.76	9.73	10.09	10.65	10.98	11.57	11.62	12.33
SER823	4.41	2.01	2.01	2.40	2.40	2.79	2.86	3.27	3.28	3.71	3.87	4.20	4.50	4.77	5.24	5.24	5.72	5.84
SER832	0.54	3.66	3.66	4.57	4.57	5.22	5.29	5.86	5.99	6.63	6.74	7.35	7.24	8.09	7.80	8.58	8.46	9.34
SER877	6.53	2.35	2.35	3.18	3.18	2.43	2.01	2.51	2.48	4.38	3.83	5.39	4.86	4.47	3.00	3.95	2.66	3.95
SER904	0.78	10.90	10.90	8.33	8.33	10.20	9.54	9.51	8.47	8.15	7.31	6.23	5.80	6.53	8.00	4.61	4.64	3.86
SER922	8.61	11.30	11.30	12.72	12.72	14.58	15.09	16.39	17.16	16.36	18.30	18.00	18.46	19.51	20.88	21.02	21.96	22.27
	mean	14.44	14.44	13.66	13.66	14.13	13.04	14.75	13.97	14.90	14.44	14.99	14.33	16.05	14.83	17.29	15.56	19.64
	stdev	14.18	14.18	10.83	10.83	9.76	9.42	9.97	10.02	8.28	8.20	7.77	7.96	9.29	9.96	10.65	9.61	15.18

TABLE 17 CONTINUED.

	y+10		y+11		y+12		y+13		y+14		y+15		y+16		y+17		y+18	
m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
12.36	10.89	11.57	6.07	3.35	9.45	9.26	7.84	3.62	6.25	3.99	8.89	5.71	10.96	9.85	11.27	11.39	12.57	11.70
9.92	10.83	10.52	10.91	11.12	11.20	11.27	10.72	10.72	11.50	12.41	11.70	13.10	12.03	13.12	11.95	12.45	12.33	12.23
16.37	14.56	6.35	13.38	13.52	14.19	12.89	25.02	22.82	22.24	21.76	23.29	23.02	15.81	14.68	22.62	22.49	22.19	20.50
12.55	13.16	13.56	14.15	14.46	14 91	14.93	15.88	16.01	16.69	16.57	17.56	17.67	18.38	18.37	19.32	19.42	19.71	19.73
6.51	6 27	7.05	7.08	7.45	7.98	8.47	8.94	9.55	9.78	10.45	10.10	10.18	10.05	10.42	9.61	10.20	9.09	9.41
9.31	9.85	10.06	10.69	11.09	11 38	11.69	12.03	12.26	12.62	12.70	13.15	13.11	13.71	13.64	14.35	14 21	15.01	14.72
3.10	3.39	2.26	4.64	4.60	5.78	4.71	5.61	5.79	4.79	4.40	5.02	4.32	6.15	5.45	5.39	5.29	5.55	5.86
3.66	3.39	3.18	3.60	3.28	3 51	1.45	4 82	2.45	4.65	1.87	5 44	4.03	5 38	3.55	5.35	4.02	5 08	4.83
22.61	23.12	23.78	25 29	26 04	27.07	28.40	27.04	30 89	28.14	31.28	28 72	31.37	28.34	31.98	30.10	31.91	31.21	29.51
15.53	17.83	14.92	17.87	15.04	17.42	15.72	17.26	15.43	17.49	15 66	17.50	16.00	17.18	15.94	16.99	15.72	17.64	16.48
10.43	14.15	10.75	12.88	10.18	10 86	10.62	9 68	9 86	10 23	10.13	10 05	10.33	9.19	9.89	9.40	9.87	9.65	9.97

APPENDIX C

TEST RESULTS WITH QUARTERLY DATA

TABLE 18

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 1 STEP WITH α = 0.1, ß = 0.1 WITH ARCH. 4-4-1

series	model error	y+1 mape	m-ape
SER157	2.17	9.05	7.83
SER184	1.18	6.64	5.19
SER193	0.58	30.08	16.58
SER202	0.42	3.92	2.71
SER211	0.98	31.94	31.24
SER220	0.38	38.69	28.99
SER229	0.84	2.01	1.41
SER238	1.12	5.73	6.01
SER265	0.73	5.08	4.68
SER283	1.6	2.86	2.54
SER292	0.9	13.3	11.4
SER301	0.64	2.51	2.16
SER310	0.04	2.69	2.67
SER319	0.95	3.42	2.88
SER328	0.61	3 58	3.48
SER337	0.6	3.02	1.9
SER346	0.85	14.67	10.11
SER355	0.64	3.55	3.23
SER364	0.5	2.37	2.76
SER382	1.35	22.94	22.16
	mean	10.4	8.5
	stdev	11.36	9.14

CHART 8

MAPE & MEDIAN APE AT 8 STEP FORECAST OVER QUARTERLY DATA ARCH. 4-4-8

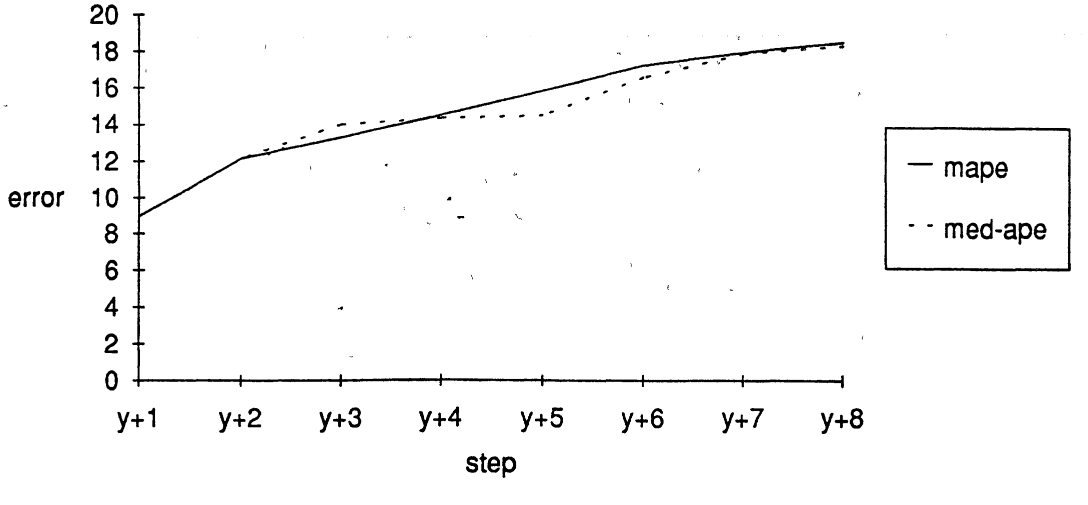


TABLE 19

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 2 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 4-4-2

series	model error	y+1 mape	y+1 m-ape	y+2 mape	y+2 m-ape
SER157	5.26	9.46	7.09	7.61	6.64
SER184	2.28	6.79	7.84	7.1	5.42
SER193	1.16	26.71	16.13	32.25	23.77
SER202	0.77	3.2	2.29	4.31	3.25
SER211	2.48	18.03	14.54	32.34	32.38
SER220	1.74	32.17	26.03	44.51	54.91
SER229	1.47	1.77	1.63	4.68	3.93
SER238	2.18	5.63	5.11	5.98	5 74
SER265	1.93	6.37	7.59	11.94	11.07
SER283	3.16	3.28	1.94	5.75	4.64
SER292	2.99	12.49	10.92	8.69	8.44
SER301	2.89	3.68	4.07	5.08	5.2
SER310	0.28	3.42	3.23	4 81	4.68
SER319	1.8	4.4	5.05	6.94	6.63
SER328	1.77	4.43	5.2	5.09	4.2
SER337	0.99	3.83	2.6	3.7	2.56
SER346	1.21	9.31	8.88	24	16.87
SER355	0.86	3.47	2.5	7.43	8.74
SER364	0.72	1.52	1.52	2.58	2.27
SER382	1.36	23.66	29.74	35.43	38.68
	mean	9.18	8.2	13.01	12.5
	stdev	8.93	7.91	12.87	14.2

TABLE 20

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 4 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 4-4-4

series	model	y+1		y+2		y+3		y+4	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER157	9.31	9.06	6.21	11.41	12.24	8.31	7.83	6.02	6.77
SER184	3.94	7.93	7.82	6.35	6.42	8.59	9.88	8.98	7.05
SER193	1.36	27.66	12.3	32.29	18.63	35.5	30.58	36.15	33.93
SER202	1.11	2.67	3.3	2.25	2.18	3.23	2.96	4.16	2.98
SER211	4.89	11.2	6.45	20.38	19.55	27.33	18.85	24.88	21.43
SER220	4.22	33.27	18.66	39	34.28	35.58	42.42	30.19	24.69
SER229	2.29	4.31	4.15	4.36	3.82	5.78	4.1	4.76	3.78
SER238	2.89	6.31	6.98	6.59	6.34	6.99	7.03	7.27	6.84
SER265	4.03	5.33	4.18	10.93	10.56	16.31	15.08	21.31	22.26
SER283	4.78	2.33	1.99	4.76	3.46	7.06	4.85	9.12	7.14
SER292	5.02	11.34	10.18	11.44	11.24	9.24	6.2	9.97	7.45
SER301	3.49	3.43	3.11	5.83	6 34	7.21	7.54	8.43	9
SER310	0.51	3.62	3.62	5.69	5.65	7 5	7.47	9.08	9.49
SER319	4.29	5.56	5.71	9.21	8.74	11.49	11.71	14.46	13.52
SER328	3.18	5.06	5.97	6.6	7.42	8.94	9	7.61	8.09
SER337	2.07	4.62	3.5	4.76	3.64	6.09	5.21	5.81	5.32
SER346	3.36	10.49	10.67	16.04	15.57	23.33	17.13	35.03	29.96
SER355	3.01	3.58	4.35	6.94	7.76	10.55	10.56	15.62	14.72
SER364	3.87	1.38	0.4	2.47	1.2	3.11	2.16	3.89	3.6
SER382	2.57	20.53	29.91	29.67	34.18	35.33	35.58	39.09	39.8
	mean	8.98	7.47	11.85	10.96	13.87	12.81	15.09	13.89
	stdev	8.6	6.71	10.49	9.44	11.08	11.16	11.69	10.97

TABLE 21

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 6 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 4-4-6

series	model	y+1		y+2		y+3		y+4		y+5		y+6	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER157	11.04	10.48	7.46	11.21	10.32	9.86	10.01	8.58	8.78	5.39	6.35	6.99	3.02
SER184	5.48	7.89	5.58	7.51	8.16	8.63	9.64	7.56	8.53	8.61	8.50	8.51	7.34
SER193	0.67	17.56	1.58	37.74	28.24	41.76	43.85	37.78	32.86	40.63	48.63	38.66	32.07
SER202	1.05	3.30	2.84	3.45	3.82	2.87	3.06	2.64	2.67	3.19	2.67	3.27	2.77
SER211	4.94	16.58	21.66	14.32	10.19	18.01	13.59	16.92	15.05	21.62	26.00	25.73	26.17
SER220	5.92	49.14	33.68	45.71	48.25	31.14	38.32	29.55	24.00	31.01	33.72	31.66	27.57
SER229	3.38	3.87	3.23	4.98	4.24	5.02	4.93	7.49	8.63	5.38	5.71	7.78	6.21
SER238	3.84	3.96	4.57	6.26	5.93	7.78	7.06	7.98	8.57	9.46	10.25	9.50	9.19
SER265	5.30	6.90	5.37	12.84	12.99	16.93	15.61	21.43	22.10	25.89	26.23	28.73	27.72
SER283	6.01	1.80	1.85	3.38	3.50	5.09	4.40	7.14	6.12	9.10	6.72	11.13	8.76
SER292	5.96	14.67	18.19	10.98	11.23	10.38	7.71	12.00	10.03	13.72	10.35	11.49	6.87
SER301	3.98	3.16	2.92	5.02	5.47	6.53	6.98	8.10	9.33	9.46	11.46	10.13	12.48
SER310	0.77	3.46	3.46	5.59	5.60	7.75	7.78	9.68	9.90	11.48	11.74	12.85	13.10
SER319	6.79	4.51	4.59	8.61	8.49	12.61	12.56	15.48	16.14	16.46	15.50	16.12	16.78
SER328	3.56	5.83	6.22	8.87	9.13	11.68	11.37	11.16	12.08	12.05	14.04	10.54	11.50
SER337	2.59	3.83	4.05	6.26	4.37	7.23	5.87	7.52	6.72	8.96	7.96	9.30	8.02
SER346	5.32	8.07	6.24	19.16	16.52	27.61	21.31	32.59	32.72	34.32	44.90	39.18	42.77
SER355	5.39	4.83	4.83	9.75	9.69	16.42	17.65	21.78	23.12	25.17	26.30	28.04	28.50
SER364	4.34	0.34	0.37	0.70	0.35	1.54	0.18	2.76	1.60	3.80	2.20	4.53	4.02
SER382	3.67	26.90	36.32	28.59	33.02	27.78	29.88	34.23	28.38	39.07	31.40	42.78	43.51
	mean	9.85	8.75	12.55	11.98	13.83	13.59	15.12	14.37	16.74	17.53	17.85	16.92
	stdev	11.33	10.37	11.82	11.70	10.65	11.63	10.82	9.56	11.96	13.76	12.67	12.96

TABLE 22

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 8 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 4-4-8

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER157	11.37	8.81	8.81	6.42	6.42	9.49	8.99	8.92	10.75	5.25	5.10	6.26	3.25	8.28	3.75	9.25	5.67
SER184	6.77	8.09	8.09	8.68	8.68	7.56	9.11	5.71	5.14	6.71	6.53	6.84	6.37	8.49	6.74	9.03	8.01
SER202	0.69	2.62	2.62	2.10	2.10	2.53	2.45	2.56	2.56	2.92	2.95	2.79	2.13	4.13	4.11	5.41	5.61
SER211	6.13	2.54	2.54	25.19	25.19	29.66	32.98	17.87	16.88	16.29	19.14	25.93	25.79	20.05	17.24	29.38	29.66
SER220	6.43	40.32	40.32	31.33	31.33	22.95	27.94	27.06	25.01	26.20	19.87	31.65	24.74	34.77	28.92	35.00	31.46
SER229	3.39	4.65	4.65	8.63	8.63	6.07	8.91	7.70	9.05	8.51	8.97	9.90	9.27	8.38	6.87	9.69	8.63
SER238	4.87	1.58	1.58	3.68	3.68	4.31	5.03	6.18	5.57	9.02	7.63	9.63	10.67	9.20	10.46	9.04	9.82
SER265	7.65	9.69	9.69	14.43	14.43	18.09	19.53	22.63	22.51	26.78	26.36	30.18	29.59	32.68	33.30	34.96	34.98
SER283	8.45	2.28	2.28	3.42	3.42	4.43	4.42	5.62	5.50	7.03	6.04	8.82	6.98	10.70	8.31	12.42	10.25
SER292	6.63	16.52	16.52	12.56	12.56	13.35	8.08	13.66	13.68	18.09	9.62	12.88	5.24	15.01	13.89	13.53	8.82
SER301	5.47	2.70	2.70	4.47	4.47	5.34	6.63	7.05	7.40	8.35	8.25	8.86	9.59	9.41	9.91	9.78	10.64
SER310	1.23	4.99	4.99	6.96	6.96	8.11	8.46	10.03	10.00	11.86	11.10	13.31	13.10	14.62	14.55	15.89	16.77
SER319	10.19	5.19	5.19	8.21	8.21	10.35	10.74	13.14	13.44	16.53	15.29	17.20	17.78	16.87	19.86	16.10	19.63
SER328	2.87	5.04	5.04	11.05	11.05	13.82	15.51	14.52	14.21	15.57	15.69	14.26	14.81	14.17	17.15	13.85	14.70
SER337	3.13	2.55	2.55	3.97	3.97	5.10	5.01	8.18	6.71	9.29	7.82	9.83	8.54	11.37	10.38	11.92	11.03
SER346	5.82	16.22	16.22	28.83	28.83	37.69	39.83	44.00	45.26	44.48	47.81	44.93	48.10	44.22	47.08	42.95	47.19
SER355	10.02	4.96	4.96	10.97	10.97	19.27	19.19	26.93	26.96	31.24	32.34	32.38	35.17	32.33	36.04	31.16	33.23
SER364	4.32	0.59	0.59	0.34	0.34	0.21	0.28	0.97	0.70	2.20	1.83	3.45	2.55	3.99	3.12	4.32	3.56
SER382	4.33	31.54	31.54	40.25	40.25	34.88	33.43	33.46	31.78	35.35	24.19	39.35	41.74	43.08	48.29	38.36	38.31
	mean	8.99	8.99	12.18	12.18	13.33	14.03	14.54	14.37	15.88	14.55	17.29	16.60	17.99	17.89	18.53	18.31
	stdev	10.58	10.58	11.12	11.12	11.04	11.64	11.46	11.40	11.81	11.62	12.72	13.78	12.85	14.18	12.35	13.19

TABLE 23

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 1 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 8-8-1

series	model	y+1	
	error	mape	m-ape
SER157	1.80	6.41	5.83
SER184	0.61	6.92	5.10
SER193	0.04	32.52	27.10
SER202	0.05	3.95	2.15
SER211	0.47	36.03	43.22
SER220	0.23	47.59	37.63
SER229	0.09	4.19	4.92
SER238	0.36	4.79	3.66
SER265	0.34	5.52	5.53
SER283	0.51	2.90	2.54
SER292	0.92	13.94	11.89
SER301	0.34	2.25	1.66
SER310	0.04	2.90	2.90
SER319	0.17	4.39	3.04
SER328	0.18	3.14	2.80
SER337	0.31	3.86	2.36
SER346	0.39	11.15	6.92
SER355	0.28	3.92	4.19
SER364	0.30	1.25	1.02
SER382	0.55	18.25	17.97
	mean	10.79	9.62
	stdev	12.98	12.29

CHART 9

MAPE & MEDIAN APE AT 8 STEP FORECAST OVER QUARTERLY DATA ARCH. 8-8-8

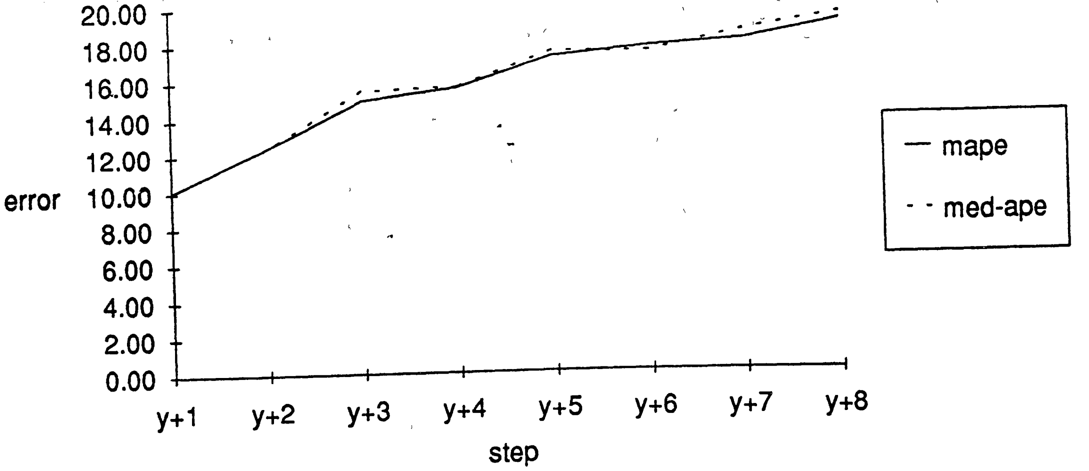


TABLE 24

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 2 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 8-8-2

series	model error	y+1 mape	y+1 m-ape	y+2 mape	y+2 m-ape
SER157	3.01	8.86	10.10	6.73	5.88
SER184	1.35	6.55	4.81	5.91	4.06
SER193	0.04	35.82	40.56	34.69	30.16
SER202	0.19	3.02	1.81	4.20	3.44
SER211	1.19	23.11	19.93	40.33	41.22
SER220	0.72	45.51	44.98	49.14	55.05
SER229	0.31	4.82	3.93	7.02	6.14
SER238	0.74	4.75	4.42	5.78	5.82
SER265	0.70	7.48	8.88	11.37	11.34
SER283	1.34	3.36	2.07	5.29	4.24
SER292	2.32	13.19	12.76	11.20	9.16
SER301	1.04	3.09	2.52	4.01	4.84
SER310	0.11	3.96	3.73	4.84	4.71
SER319	0.89	5.13	4.69	8.01	7.22
SER328	0.33	2.53	1.45	3.69	2.57
SER337	0.54	3.94	3.05	4.08	3.24
SER346	0.82	6.79	5.02	15.83	13.40
SER355	1.15	4.77	4.61	7.21	8.18
SER364	0.70	1.31	0.92	1.96	1.23
SER382	0.74	22.39	25.45	30.47	35.24
	mean	10.52	10.28	13.09	12.86
	stdev	12.01	12 79	13.86	15.05

TABLE 25

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 4 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 8-8-4

series	model error	y+1 mape	y+1 m-ape	y+2 mape	y+2 m-ape	y+3 mape	y+3 m-ape	y+4 mape	y+4 m-ape
SER157	4.65	5.64	5.46	8.48	8.47	6.01	5.54	7.06	4.90
SER184	2.13	6.41	7.00	5.40	6.89	6.37	7.62	7.33	6.24
SER202	0.36	2.14	2.26	2.18	2.14	3.39	3.17	4.33	3.19
SER211	1.90	22.86	16.16	31.85	29.61	38.73	33.81	30.36	21.73
SER220	1.60	33.84	31.24	39.96	40.42	44.58	47.36	44.62	40.84
SER229	0.93	5.75	4.79	8.28	6.89	8.02	7.51	7.81	6.96
SER238	1.37	4.64	2.63	5.64	4.98	7.66	8.36	7.77	9.03
SER265	1 07	6.42	4.99	11.41	11.34	16.17	15.70	20.74	21.40
SER283	2.64	2.66	1.99	4.54	3.02	6.65	3.85	8.94	6.93
SER292	3.42	15.15	15.31	13.04	6.52	12.26	7.72	10.42	6.53
SER301	1.80	3.48	3.09	5.12	6.10	6.05	6.46	6.80	7.54
SER310	0.30	5.33	5.01	6.83	6.75	7.95	7.95	9.10	9.52
SER319	2.65	6.88	7.64	10.64	10.85	13.74	14.79	15.62	17.70
SER328	0.56	5.13	5.53	5.30	5.28	5.68	7.18	6.07	6.26
SER337	1.09	4.59	3.29	5.41	4.38	6.05	5.34	6.38	5.87
SER346	1.56	10.28	8.89	13.65	10.65	19.67	11.58	31.10	26.46
SER355	2.47	6.27	7.49	10.47	12.01	15.00	16.80	18.92	19.91
SER364	1.35	1.30	0.18	2.29	1.21	2.94	1.86	3.70	3.27
SER382	0.69	19.08	21.88	29.76	31.93	34.19	32.38	32.78	30.98
	mean	8.83	8.15	11.59	11.02	13.74	12.90	14.73	13.43
	stdev	8.30	7.82	10.58	10.81	12.28	12.12	11.83	10.71

TABLE 26

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 6 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 8-8-6

series	model	y+1		y+2		y+3		y+4		y+5		y+6	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER157	5.77	6.49	5.37	9.10	7.48	8.05	7.55	7.05	6.72	5.38	3.42	7.39	4.22
SER184	3.14	7.03	6.93	6.76	7.37	7.21	5.62	5.21	4.32	8.46	8.69	9.03	7.81
SER202	0.18	2.80	3.83	3.63	3.59	2.58	3.08	2.56	2.57	3.28	3.13	3.47	3.04
SER211	2.99	33.99	43.64	29.18	32.99	24.19	20.47	33.54	32.32	26.65	22.80	40.21	42.75
SER220	3.17	45.94	32.30	40.75	45.18	32.66	34.34	38.33	36.63	39.44	49.28	38.28	38.36
SER229	1.21	5.55	5.53	5.71	5.45	7.30	8.66	9.16	8.32	7.93	6.26	9.56	8.26
SER238	1.84	2.76	1.51	5.29	4.13	6.57	7.57	7.47	8.52	9.05	11.09	9.08	10.77
SER265	1.66	6.66	4.83	11.89	12.16	15.74	14.81	20.25	21.45	24.85	24.62	28.09	27.42
SER283	3.37	2.19	2.31	3.26	3.09	4.96	4.25	7.33	6.28	9.50	7.43	11.21	9.11
SER292	3.35	25.52	29.49	17.27	16.83	13.98	7.36	12.73	7.97	16.04	13.50	13.70	12.35
SER301	2.07	3.57	2.98	5.72	6.14	7.31	7.69	8.12	8.60	8.20	8.84	8.79	9.26
SER310	0.47	5.22	5.32	6.88	6.95	8.74	8.82	10.12	10.28	11.65	11.78	12.86	13.11
SER319	3.88	5.34	4.18	9.57	9.74	13.28	13.12	15.27	16.34	15.27	17.96	14.57	16.22
SER328	0.63	6.76	7.98	8.71	8.53	8.47	8.85	9.26	10.98	9.71	9.71	9.41	10.61
SER337	1.37	3.65	3.40	6.91	4.94	7.09	5.99	8.03	6.95	9.46	8.84	10.07	9.18
SER346	2.38	10.79	11.99	18.17	16.61	22.59	12.42	27.38	27.01	29.82	40.04	38.55	47.45
SER355	2.19	8.67	8.22	13.89	14.65	20.87	23.05	25.17	26.95	27.68	29.48	27.94	28.04
SER364	1.57	0.24	0.21	0.78	0.42	1.60	0.29	2.68	1.41	3.46	1.99	4.22	3.49
SER382	0.84	18.53	21.74	23.10	25.00	27.31	31.52	28.63	28.14	26.63	25.33	24.88	25.86
	mean	10.62	10.62	11.93	12.17	12.66	11.87	14.65	14.30	15.39	16.01	16.91	17.23
	stdev	12.08	12.11	10.04	11.35	8.88	9.27	10.87	10.90	10.49	12.93	12.07	13.66

TABLE 27

MAPE AND MEDIAN APE VALUES OF QUARTERLY FORECAST AT 8 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 8-8-8

series	model	y+1		y+2		y+3		y+4		y+5		y+6		y+7		y+8	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER157	5.05	6.89	6.89	3.58	3.58	8.24	5.39	6.74	7.26	7.77	4.82	8.03	6.28	8.51	4.61	10.61	6.00
SER184	4.24	9.03	9.03	9.60	9.60	6.56	8.28	5.59	5.31	9.70	12.37	9.00	8.14	9.87	10.14	9.99	7.81
SER211	3.07	11.13	11.13	35.65	35.65	44.34	48.86	27.04	26.66	33.59	36.20	28.08	19.21	26.41	30.91	38.49	43.55
SER220	3.00	34.53	34.53	23.10	23.10	29.45	19.59	35.31	34.46	34.71	31.81	37.61	40.95	33.66	32.53	33.57	32.56
SER229	1.50	7.79	7.79	7.93	7.93	6.55	7.42	7.67	7.33	8.71	7.64	10.24	8.13	10.53	8.53	8.05	7.77
SER238	2.56	1.31	1.31	1.58	1.58	3.17	2.70	5.98	5.34	7.11	7.62	7.45	8.73	8.50	10.98	8.47	10.23
SER265	2.91	10.98	10.98	14.96	14.96	18.30	19.38	22.53	22.47	25.93	25.41	29.26	29.55	32.53	32.60	34.34	34.79
SER283	4.55	2.25	2.25	3.06	3.06	4.16	4.14	5.85	5.73	7.57	7.29	9.34	7.94	10.99	9.16	12.30	10.62
SER292	3.60	28.70	28.70	17.53	17.53	20.45	24.33	16.35	15.66	21.41	24.66	19.58	19.30	17.74	18.18	16.53	16.28
SER301	2.79	2.06	2.06	3.99	3.99	5.08	5.73	6.62	6.33	6.65	5.86	7.36	6.91	7.95	8.27	8.60	8.93
SER310	0.70	6.51	6.51	7.79	7.79	8.97	9.19	10.74	10.53	12.19	11.60	13.50	13.30	14.70	14.67	15.90	16.78
SER319	5.24	6.57	6.57	8.52	8.52	10.20	10.23	13.91	13.12	14.50	11.12	13.94	13.32	14.19	14.91	14.26	16.42
SER328	0.68	9.86	9.86	11.88	11.88	12.25	12.24	13.32	13.53	14.33	15.44	14.12	14.33	13.80	16.63	13.97	14.84
SER337	1.61	2.99	2.99	4.60	4.60	5.95	5.56	9.03	7.19	10.63	9.32	11.04	10.32	12.70	12.29	13.27	12.51
SER346	2.35	14.55	14.55	24.76	24.76	31.16	36.15	37.86	40.41	39.21	44.79	39.27	47.85	40.15	47.78	44.41	47.74
SER355	2.73	12.48	12.48	15.99	15.99	22.83	23.30	29.09	30.09	32.79	35.01	32.45	33.00	30.96	31.96	29.08	29.68
SER364	1.72	0.51	0.51	0.33	0.33	0.19	0.17	0.92	0.75	2.13	1.74	3.26	2.09	3.69	2.42	4.14	3.25
SER382	0.74	12.89	12.89	28.91	28.91	32.08	36.59	26.37	29.60	22.30	23.00	26.40	25.11	28.26	27.80	26.62	29.73
	mean	10.06	10.06	12.43	12.43	15.00	15.51	15.61	15.65	17.29	17.54	17.77	17.47	18.06	18.58	19.03	19.42
	stdev	8.96	8.96	10.14	10.14	12.45	13.66	11.28	11.90	11.52	12.76	11.39	12.92	10.90	12.40	12.09	13.47

APPENDIX D

TEST RESULTS WITH YEARLY DATA

TABLE 28

MAPE AND MEDIAN APE VALUES OF YEARLY FORECAST AT 1 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 2-2-1

series	model	y+1	
	error	mape	m-ape
SER4	0.48	6.43	6.89
SER13	0.43	2.59	1.44
SER22	0.04	0.04	0.03
SER31	0.04	0.55	0.52
SER40	0.46	1.03	1.25
SER49	0.42	7.65	6.67
SER58	0.39	1.34	0.68
SER67	0.72	0.93	0.75
SER76	0.16	2.27	2.09
SER85	0.04	0.29	0.32
SER94	0.09	0.76	0.58
SER103	0.04	0.41	0.41
SER112	0.18	0.1	0.11
	mean	1.88	1.67
	stdev	2.43	2.34

TABLE 29

MAPE AND MEDIAN APE VALUES OF YEARLY FORECAST AT 2 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 2-2-2

series	model	y+1		y+2	
	error	mape	m-ape	mape	m-ape
SER4	2.79	6.53	7.42	12.8	14.1
SER13	4.89	2.65	1.82	9.1	8.73
SER22	0.06	0.14	0.1	0.47	0.38
SER31	1.54	1.42	1.03	9.75	7.44
SER40	0.86	0.75	0.82	6.14	8.33
SER49	0.92	9	5.37	20.7	20.5
SER58	0.29	2.94	3.01	9.33	8.47
SER67	0.56	1.06	0.94	3.69	3.02
SER76	0.56	3.61	2.99	7.42	7.97
SER85	0.45	1.15	1.46	3.6	3.37
SER94	0.39	1.1	0.74	5.65	4.46
SER103	0.04	1.64	1.58	4.72	4.77
SER112	1.2	0.27	0.33	1.69	1.94
	mean	2.48	2.12	7.31	7.19
	stdev	2.6	2.13	5.31	5.39

TABLE 30

MAPE AND MEDIAN APE VALUES OF YEARLY FORECAST AT 4 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 2-2-4

series	model	y+1		y+2		y+3		y+4	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER4	2.89	5.29	4.88	12.8	15.4	14.7	13.5	11.4	12.2
SER13	12.4	2.23	1.55	8.96	8.8	16	13.7	21.3	22.1
SER22	0.04	0.14	0.15	0.59	0.47	0.97	0.8	1.34	1.3
SER31	2.98	1.12	1.08	9.12	5.86	10	6.44	9.3	5.75
SER40	1.6	3.61	4.04	8.29	9.86	10.7	11.5	13.2	12.4
SER49	1.82	5.44	4.5	21.4	21.2	19.3	16.1	20.4	17.3
SER58	0.32	3.44	3.14	7.75	9.65	13.1	14.7	21.4	19.8
SER85	0.93	2.89	3.49	7.09	7.69	10.3	9.47	13.9	13
SER112	2.1	0.14	0.15	1.58	1.72	3.11	3.35	4.67	4.57
	mean	2.7	2.55	8.63	8.96	10.9	9.95	13	12
	stdev	1.98	1.85	6.12	6.39	5.88	5.35	7.23	7.08

TABLE 31

MAPE AND MEDIAN APE VALUES OF YEARLY FORECAST AT 6 STEP WITH $\alpha = 0.1$, $\beta = 0.1$ WITH ARCH. 2-2-6

series	model	y+1		y+2		y+3		y+4		y+5		y+6	
	error	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape	mape	m-ape
SER4	2.94	13	13	17.1	17.1	16.6	16.4	13.1	14.5	18.7	17.1	20.9	22.6
SER13	19.2	0.6	0.6	10.6	10.6	19.6	22.1	24.2	24	31	32.4	36.6	36.1
SER31	2.66	0.15	0.15	4.65	4.65	6.2	6.47	10.1	4.74	8.79	2.71	7.5	5.71
SER40	1.18	6.49	6.49	10.5	10.5	14.6	15	16.3	15.5	17.9	17.4	20.5	21
SER49	1.98	3.23	3.23	7.58	7.58	18.8	9.37	19.1	17.4	12.8	5.08	18.9	22.2
SER58	0.14	3.51	3.51	10.9	10.9	14 9	16.9	22	21.7	28.9	24.2	36	36.2
SER85	0.71	0.86	0.86	5.5	5.5	11.9	12.9	19.3	17.2	25.5	20	29.9	26.8
SER112	2.66	1.25	1.25	2 07	2.07	3.24	3 01	4.84	4.5	6.35	5.88	8.02	7.55
	mean	3.64	3.64	8.62	8.62	13.2	12.8	16.1	14.9	18.7	15.6	22.3	22.3
	stdev	4.31	4.31	4.7	4.7	5.83	6.2	6.44	7.09	9.16	10.4	11.3	11.3

APPENDIX E

NEURAL NETWORKS AND BOX-JENKINS:

FUNCTIONAL EQUIVALENCE

NEURAL NETWORKS AND BOX-JENKINS: FUNCTIONAL EQUIVALENCE

Conventional methods used for prediction (MA, ARIMA etc.) or signal processing uses linear analysis. Using a neural network with the nonlinear elements and backpropagation, it is observed that nonlinear signals can be processed. It is the ability to control the non-linearity in the neural networks that allows prediction in nonlinear time series with accuracy.

A competing approach to processing nonlinear time series data would be to develop polynomials describing the non-linearity in the data, then adjust the coefficients using the least mean square (LSM) algorithm. With this the global minimum assured, but has disadvantage towards modeling. The Box-Jenkins approach towards time series modeling is similar and acceptable accuracy is achieved over short time periods and with data series which can be expressed with polynomials with fewer number of coefficients.

In the present research neural networks are tested for their forecasting performance over considerably large data sets to facilitate conclusions about neural networks as forecasting experts. Though the data series used in this study are from business literature the idea of time series forecasting remains the same, having major applications in control, factory automation and signal processing.

A common method of prediction in time series forecasting is the linear predictive method. Values of variable $y(t)$ in a time series at a discrete time in the past are used to predict $\hat{y}(t)$ at step(s) in future.

One of the Box-Jenkins methods (ARIMA) of modeling univariate data series which is compared here is based on the linear predictive method. Essentially, the analyst examines the partial auto correlation and identifies the model of the form given in equation (7).

If the underlying model of a trained neural network (weights and algorithm) is to be compared with the Box-Jenkins model based on linear predictive method, then the neural network model also has to be a linear model. Linear neural network models are the ones which have units with linear transfer function and do not have any hidden layers. Consider the case of a 3 input and one output neuron neural network as shown in Fig 2.

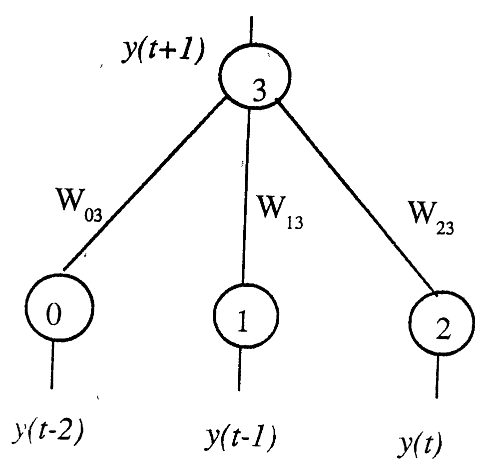


Fig. 2: Linear Neural Network

There are no hidden layers and all the units have a linear transfer function. In this network *y(t), y(t-1), y(t-2)* are used to predict *y(t+1)*. As all the units have linear transfer functions the transfer function of the complete neural network is linear and is similar to the Box-Jenkins ARIMA model. In the above model, for single step forecast:

$$\hat{y}(t+1) = W_{03}y(t-2) + W_{13}y(t-1) + W_{23}y(t) + \phi_3 \tag{11}$$

where,

W_{ij} is the weight from unit i to unit j

ϕ_3 is the bias.

In general, if n past values from a time series data *y(t)* are used to predict the *y(t+1)* value at origin *y(t)*, the best possible prediction for the values *y(t+1)* is:

$$\hat{y}(t+1) = \sum_{i=0}^{n-1} W_{in}\, y(t-i) + \phi_n \tag{12}$$

The same generalization can be extended towards the concept of multi-step forecasts. Here multi-step forecast uses a linear neural network. Multiple n-step forecasts mean all the forecasts from *y(t+1),..y(t+n)* and not just *y(t+n)*.

Linear models for multi-step forecasts of n-step ahead based on the above formulation can be written as:

$$\hat{y}(t+i) = \sum_{j=0}^{n-1} W_{ij}\, y(t-j) + \phi_i \qquad (13)$$

Which is again similar to the models used by other statistical techniques as Box-Jenkins (ARIMA). If the training set consists of N training patterns and each training pattern at a discrete time is labeled as t_p then W_{ij} may be determined by minimizing mean square error, E as:

$$E = \sum_{ip} [\, y_i - (\sum_{j=0}^{n-1} W_{ij}\, y(t-j) - \phi_i)\,]^2 \qquad (14)$$

i ranges over the output units, p indexes the discrete training patterns in the training set. This is a linear least square problem and may be solved for example by the steepest descent method. This is done by successively changing W_{ij} by an amount ΔW_{ij} where:

$$\Delta W_{ij} = -\varepsilon(\delta E/\delta W_{ij}) \qquad (15)$$

The above formulation shows that linear predictive method is a limited implementation of back propagation algorithm used in nonlinear neural networks. One clear disadvantage in the linear predictive method is that if n data items are to be put into an m^{th} order polynomial, then the number of terms grows to $(m+n)!/(m!n!)$, which has an exponential increase as m gets large.

As a functional equivalence, Box-Jenkins modeling based on the linear predictive method is functionally similar to *linear* neural networks.

Nonlinear Neural Networks

If hidden layer units have sigmoidal transfer function then the model is a nonlinear neural network. This extends the power of the neural network and is controlled by the backpropagation algorithm. Each of the neurons in this type of network not only have the continuous differentiable sigmoidal transfer function but also have a threshold $\phi_{i,}$ which controls the position of the transfer function. In such a case if y_t is the input to input layer, where i ranger over a number of input neurons, then the output of each hidden neuron h_n is:

$$h_n = \mathbf{F}\left(\sum_{i=0}^{i<I} W_{in}\, y(t-i) \; + \phi_n \right) \tag{16}$$

where, **F** is the sigmoidal transfer function usually given by;

$$f(x) = 1/(1 + e^{-x}) \tag{17}$$

ϕ_n is the threshold which shifts the sigmoidal transfer function to the left or right. Here, in the case of nonlinear neural network training is more complicated.

Nevertheless a gradient descent is used to minimize E.

Basically backpropagation can be thought as a nonlinear, least square algorithm. Consider a case of two input, two hidden and one output nonlinear neural network as shown in Fig 3.

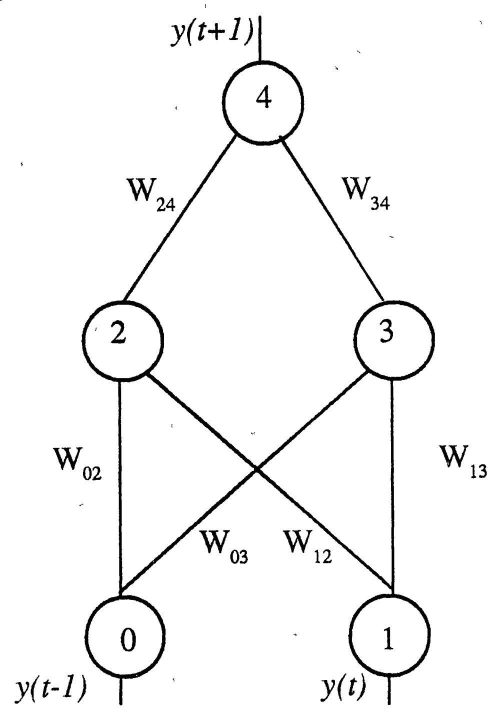


Fig 3: Nonlinear Neural Network

let $net_\iota = \sum_{\iota=0}^{\iota<I} i_n$ and a_ι = activation of unit ι.

Here input layer neurons do not have any biases.

$$net_0 = y(t\text{-}1) \tag{18}$$

$$a_0 = y(t-1) \tag{19}$$

$$net_1 = y(t) \tag{20}$$

$$a_1 = y(t) \tag{21}$$

$$net_2 = a_0\ W_{02} + a_1\ W_{12} \tag{22}$$

$$a_2 = (1/(1 + e^{net2})) + \phi_2 \tag{23}$$

$$= \frac{1}{1 + e^{-(a_0\ W_{02} + a_1\ W_{12})}} + \phi_2 \tag{24}$$

$$net_3 = a_0\ W_{03} + a_1\ W_{13} \tag{25}$$

$$a_3 = (1/(1 + e^{net3})) + \phi_3 \tag{26}$$

$$= \frac{1}{1 + e^{-(a_0\ W_{03} + a_1\ W_{13})}} + \phi_3 \tag{27}$$

$$net_4 = a_2\ W_{24} + a_3\ W_{34} \tag{28}$$

$$a_4 = (1/(1 + e^{net4})) + \phi_4 \tag{29}$$

$$= \frac{1}{1 + e^{-(a_2 W_{24} + a_3 W_{34})}} + \phi_4 \qquad (30)$$

here $a_4 = \hat{y}(t+1)$ which is the best possible prediction for $y(t+1)$.

is then;

$$\hat{y}(t+1) = \frac{1}{1 + e^{net4}} + \phi_4 \qquad (31)$$

$$\hat{y}(t+1) = \frac{1}{1 + e^{-\left(\left(\frac{1}{1 + e^{-(a_0 W_{02} + a_1 W_{12})}} + \phi_2 \right) W_{24} + \left(\frac{1}{1 + e^{-(a_0 W_{03} + a_1 W_{13})}} + \phi_3 \right) W_{34} \right)}} + \phi_4 \qquad (32)$$

In general if there are I number of input neurons, H number of hidden neurons and Y number of output neurons in a nonlinear network the single step forecast $y(t+1)$ is given by;

$$\hat{y}(t+1) = \frac{1}{1 + e^{-\sum_{h=0}^{h<H} \left(\frac{1}{1 + e^{-\sum_{n=0}^{n<I} (a_n W_{n(I+h)})}} + \phi_{(I+h)} \right) W_{(I+h)(I+H-1)}}} + \phi_{(I+H-1)} \qquad (33)$$

As in the time series forecasting problem:

$$\sum_{n=0}^{n<I} a_n = \sum_{\iota=n}^{n<I} y(t-n) \qquad (34)$$

substituting we get $y(t+1)$, one step ahead forecast in terms of the history inputs $y(t)$, $y(t-1)$....,$y(t-i)$.

$$\hat{y}(t+1) = \frac{1}{1 + e^{\left(-\sum_{h=0}^{h<H}\left(\frac{1}{1 + e^{-\sum_{n=0}^{n<I}(y(t-n)W_{n(I+h)})}} + \phi_{(I+h)}\right) W_{(I+h)(I+H-1)}\right)}} + \phi_{(I+H-1)} \qquad (35)$$

If more than one output is present (multi-step forecast) the same equation can also be used to get $\hat{y}_i(t+1)$, where i is any output neuron and indexes from 0 to $Y-1$.

For one or more outputs one minimizes E, where;

$$E = \Sigma_p \left[\Sigma_i (y_i(t+i) - \hat{y}(t+i))^2\right] \qquad (36)$$

$y_i(t+i)$ is specified output of ith output neuron for the pth input pattern and $\hat{y}(t+i)$ is the actual output of the same unit. Given pth input pattern and the present set of weights, W_{ij} and ϕ_i for *linear predictive* method discussed before, equation (2) reduces to simple form, but in this case due to nonlinear transfer functions of hidden layer units the output is a nonlinear function of the input and E in the above equation become square of a nonlinear function $\hat{y}_i(t+i)$. This clearly shows that the nonlinear neural networks are not similar to the standard statistical modeling methods where the input-output relationship is linear.

Gradient descent is performed by letting

$$\Delta W_{ij} = -\varepsilon(\delta E/\delta W_{ij}) \quad (37)$$

here a new quantity is introduced δ_i, which is given as;

$$\delta_i = - (\delta E/\delta neti) \quad (38)$$

$$(\delta E/\delta W_{ij}) = -\delta_i \hat{y}_j(t+i) \quad (39)$$

The gradient descent is implemented by making changes in W_{ij} by the amount ΔW_{ij} where;

$$\Delta W_{ij} = -\varepsilon(\delta E/\delta W_{ij}) = \varepsilon \delta y_i(t+i) \quad (40)$$

δ_i may be calculate by chain rule. If unit i is an output unit then δ_i becomes;

$$\delta_i = \Sigma_p \, (y_i(t+i) - \hat{y}_i(t+i))(\delta F/\delta(net_i)) \quad (41)$$

If y_i is the activation of the output ith neuron. If i is not the output neuron, then δ_i may be computed recursively starting at the top most layer as;

$$\delta_i = (\delta(F)/\delta(net_i)) \, \Sigma_j \, W_{ij} \, \delta_j \quad (42)$$

equations 40, 41 and 42 define the backpropagation (Rumelhart et.al 1986) rule.

APPENDIX F

NEUREAL NETWORK vs BOX-JENKINS

COMPARISON (SINGLE STEP FORECAST)

NEURAL NETWORK vs BOX-JENKINS COMPARISON (SINGLE STEP FORECAST)

This section gives the comparison of forecasting performance with the Box-Jenkins model and neural networks over single-step forecasts. Results of our previous work (Sharda and Patil 1990) are also given. The previous work was carried out using the BrainMaker neural net simulator, which uses a different version of the backpropagation algorithm than the version by PDP group used in the present study (McClelland and Rumelhart 1988).

Table 32 gives the MAPE and Median APE comparisons of 75 M-111 competition series modeled using the Box-Jenkins and neural networks. It is observed that simple neural network models are as good as the complex Box-Jenkins models at least for the single-step forecast. The comparison of the Box-Jenkins models with neural networks over multiple-step forecasts remains to be done. It is also observed that whenever Box-Jenkins model performed poorly neural networks also performed poorly. This shows that neural network modeling is indeed close to statistical modeling processes. The only difference seen is that, the neural networks' ability to interpolate the forecast in a fuzzy way is more powerful than the method of interpolation of the forecast by strict calculations. It is observed that, in all cases of seasonal data series and short memory the neural networks outperformed AUTOBOX.

TABLE 32

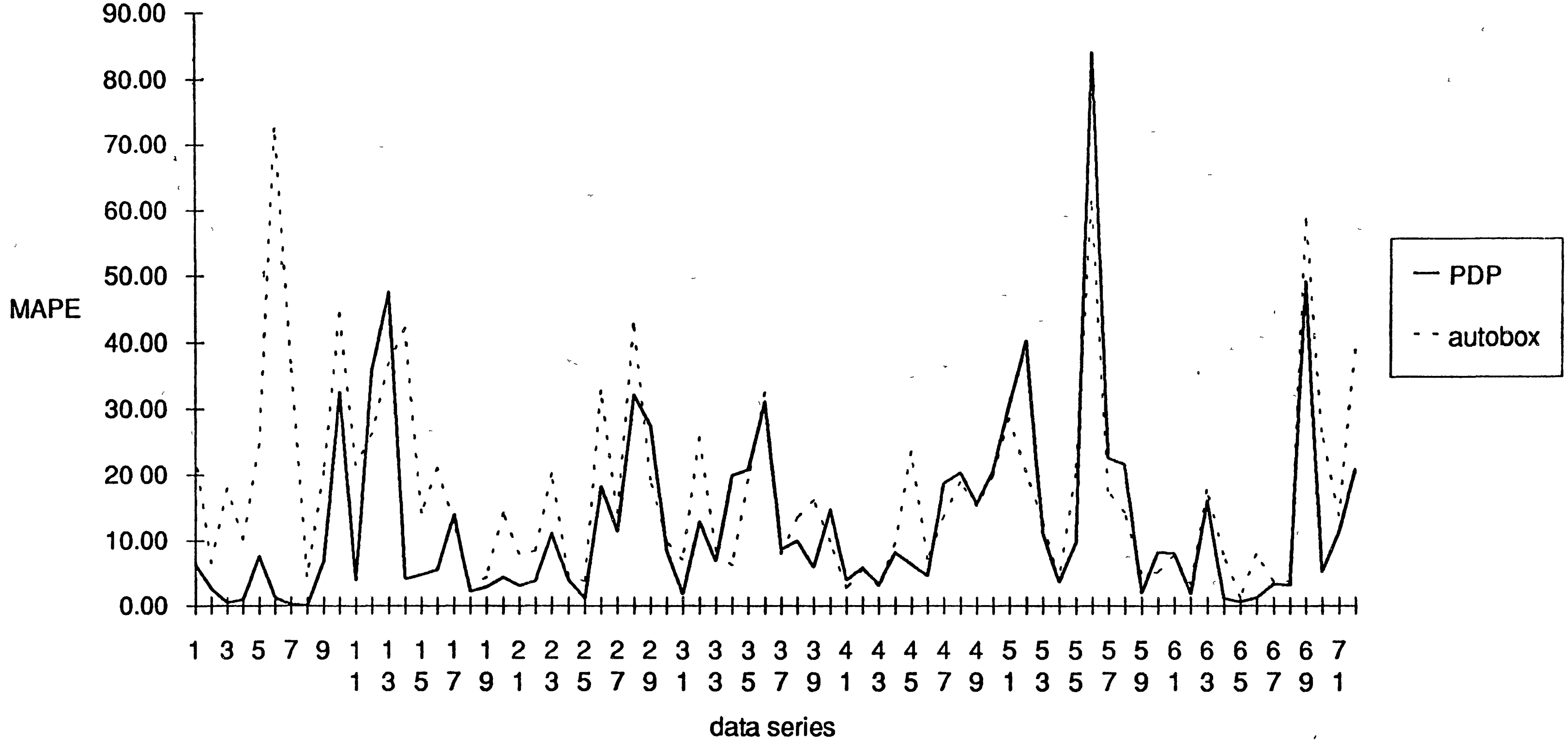
MAPE COMPARISON OF BOX-JENKINS AND NEURAL NETWORKS
AS FORECASTING EXPERTS

NN-BM: BrainMaker NN simulator used in over pervious work (Sharda & Patil [1990])
NN-PDP: Parallel Distributed Processing (McClelland and Rumelhart, [1988])

series	autobox	NN-BM	NN-PDP	series	autobox	NN-BM	NN-PDP
SER4	23.53	14.21	6.43	SER499	13.50	17.15	10.02
SER13	6.58	11.52	2.59	SER508	16.11	11.52	5.89
SER31	18.07	6.28	0.55	SER526	9.95	20.28	14.75
SER40	10.16	3.85	1.03	SER544	2.72	3.11	3.98
SER49	24.77	39.86	7.65	SER571	5.57	10.39	5.82
SER58	72.57	16.17	1.34	SER580	3.03	1.34	3.28
SER85	37.00	7.53	0.29	SER589	9.66	7.77	8.20
SER112	4.55	7.53	0.10	SER598	23.49	5.62	6.36
SER184	20.61	6.67	6.92	SER616	7.22	5.73	4.62
SER193	44.43	40.79	32.52	SER634	13.70	15.24	18.67
SER202	21.51	4.08	3.95	SER643	18.92	20.71	20.29
SER211	26.31	50.36	36.03	SER652	15.23	17.52	15.54
SER220	37.10	42.24	47.59	SER661	19.80	25.85	20.52
SER229	42.19	7.11	4.19	SER670	28.39	30.25	30.27
SER238	14.22	3.23	4.79	SER679	20.42	36.52	40.01
SER265	21.15	10.50	5.52	SER688	12.86	14.31	11.12
SER292	12.67	24.86	13.94	SER697	4.13	5.24	3.63
SER301	2.91	10.77	2.25	SER706	20.11	7.87	9.66
SER310	4.46	11.30	2.90	SER715	61.31	76.65	83.83
SER319	14.30	11.20	4.3[illegible]	SER724	17.43	22.27	22.44
SER328	8.02	1.87	3.14	SER733	14.21	20.34	21.52
SER337	8.52	3.68	3.86	SER742	5.00	4.51	2.05
SER346	20.23	15.64	11.15	SER751	5.19	8.63	8.17
SER355	4.75	9.75	3.92	SER760	7.75	8.87	8.05
SER364	3.73	1.88	1.25	SER787	3.17	7.90	1.87
SER382	32.78	17.95	18.25	SER796	17.68	15.17	15.96
SER400	14.14	9.15	11.42	SER805	7.93	7.34	1.18
SER409	42.80	97.43	32.09	SER823	1.29	2.04	0.65
SER418	19.23	22.73	27.37	SER832	7.96	5.26	1.29
SER427	9.89	9.32	8.53	SER877	3.73	5.95	3.38
SER436	7.09	5.85	1.85	SER904	3.88	3.17	3.22
SER445	25.71	13.53	12.87	SER913	58.53	57.17	48.90
SER454	8.48	16.17	6.85	SER922	26.35	17.23	5.22
SER463	6.19	8.03	19.95	SER958	13.67	10.85	11.36
SER472	19.21	15.29	20.76	SER967	38.74	10.60	20.68
SER481	32.36	26.70	31.00	mean	15.94	14.92	14.67
SER490	7.90	8.11	8.71	stdev	15.18	15.12	15.39

CHART 12

MAPE COMPARISON OF AUTOBOX AND NEURAL NETWORKS AS FORECASTING EXPERTS (SINGLE STEP FORECAST)



APPENDIX G

BOX-JENKINS MODELS

BOX-JENKINS MODELS

One good way of representing a time series is to represent the ARIMA models of a time series. Table 33 gives the ARIMA models for all the data series used in our study. The numbers in the parenthesis indicate: $(AR, I, MA)^s$, $(\text{Auto-regressive model, Difference model, Moving Average model})^{\text{seasonality}}$. The number indicates the order of the model. Non-stationary models are called ARIMA models. Once the stationary series is achieved by applying regular differences to the original series, the modeling can be done. It needs identification of basic AR, MA or ARMA models for the converted stationary series. These models are written in the same way as the basic models, except that the differenced (stationary) series Z_t is substituted for the original series X_t;

$$Z_t = A_1 Z_{t-1} + \ldots + A_p Z_{t-p} + E_t \qquad \text{(AR model for } Z_t\text{)}$$

$$Z_t = -(B_1 E_{e-1} + \ldots B_q E_{t-q}) + E_t \qquad \text{(MA model for } Z_t\text{)}$$

$$Z_t = (A_1 Z_{t-1} + \ldots + A_p Z_{t-p}) - (B_1 E_{t-1} + \ldots + B_q E_{t-q}) + E_t \quad \text{(ARMA model for } Z_t\text{)}$$

In terms of the original series, such models are called *Integrated models* and are often denoted by ARI, IMA, ARIMA. In Auto Regressive Integrated Moving Average (ARIMA) models the term *Integrated*, a synonym for *summed*, is used because the differencing process can be reversed to obtain the original series values by summing the successive values of the differenced series.

Table 33: AUTOBOX (ARIMA) models for M-111 Competition Data

(Sharda, R., et. al. 1987)

Series	coeff	ARIMA model	Series	coeff	ARIMA model
4	0	(0,0,0)	310	-.5	(0,1,0)
13	0.5	(0,1,0)	319	.5	(0,1,0)
22	not enough data		328	-	$(0,1,0)(1,0,0)^4$
31	-	(0,0,1)	337	-1	$(1,0,0)(0,0,1)^2(0,1,1)^4$
40	-1	(0,1,1)	346	0	$(0,1,2)(0,0,1)^8$
49	-1	(1,0,0)	355	-	(0,1,2)
58	0.5	(0,1,0)	364	-1	$(1,0,1)(0,0,1)^4$
67	-1	(0,1,0)	373	-	$(1,0,0)^2$
76	-	(0,1,0)	382	0	(3,0,0)
85	-	(0,1,0)	391	.5	$(2,0,0)(0,1,1)^{12}$
94	-	(0,1,1)	400	.5	$(1,0,0)(0,0,1)^3$
103	.5	(0,0,0)	409	0	$(1,0,0)^{12}$
112	0	(1,1,0)	418	0	$(1,0,0)^6(0,0,1)^{12}$
121	-.5	(0,1,0)	427	0	$(1,0,0)(0,0,1)^{12}$
130	.5	(1,1,0)	436	-.5	$(0,1,0)(0,0,1)^3(1,0,0)^{12}$
139	-.5	(0,1,0)	445	.5	$(1,0,0)(0,0,1)^3(1,0,0)^{12}$
148	0	(1,0,0)	454	.5	$(0,1,1)^{12}$
157	1	(1,0,0)	463	0	$(2,0,0)(1,0,0)^3(0,1,1)^{12}$
166	not enough data		472	0	$(2,0,0)(0,1,1)^{12}$
175	-1	(0,0,0)	481	.5	$(1,0,0)(1,0,0)^3$
184	not enough data		490	-	$(0,0,1)(1,0,0)^3(1,0,0)^{12}$
193	-	(1,0,0)	508	0	$(0,1,1)(0,0,1)^6(1,0,0)^{12}$
202	-	(0,1,0)	517	.5	$(1,0,0)(0,0,1)^6$
202	0	(1,1,0)	526	-	$(2,0,0)(1,0,0)^6$
229	-	$(0,1,2)(0,0,1)^4$	535	-	$(2,0,0)(1,0,0)^6$
238	not enough data		544	-	$(0,1,0)(1,0,0)^{12}$
247	-1	(0,1,0)	553	.5	$(0,0,1)(0,0,1)^6$
256	-1	(0,1,0)	562	-	$(2,0,0)(0,0,1)^6$
265	0	$(0,1,0)(0,0,1)^8$	571	-.5	$(1,0,0)(0,0,1)^6(0,1,0)^{12}$
274	-.5	(1,0,0)	580	-1	$(0,1,0)(0,0,1)^{12}$
283	-1	$(1,1,0)(0,0,1)^4$	589	0	$(2,0,0)(0,0,1)^{12}$
292	.5	$(1,0,0)(1,0,0)^4$	598	.5	$(0,1,2)(0,0,1)^6(0,1,0)^{12}$
301	.5	$(0,1,0)(0,0,1)^2(1,0,0)^4$	607	-	$(1,0,0)(0,0,1)^{12}$
616	-1	$(1,0,1)^{12}$	814	-	(0,1,0)
625	-	$(0,0,1)^{12}$	823	.5	(0,1,2)
634	-	$(2,0,0)(1,0,0)^{12}$	832	-.5	(2,1,0)

Series	coeff	ARIMA model
643	.5	$(1,0,0)(1,0,1)^{12}$
652	0	$(1,0,0)(0,0,1)^{2}(1,0,1)^{12}$
661	.5	$(2,0,0)(0,1,1)^{12}$
670	-	$(2,0,0)(1,0,0)^{12}$
679	.5	$(1,0,0)$
688	-	$(1,0,0)(0,1,1)^{12}$
697	-1	$(0,0,1)(1,0,1)^{12}$
706	.5	$(0,1,2)(0,0,1)^{12}$
715	not enough data	
724	.5	$(2,0,0)$
733	-	$(0,0,1)^{3}(1,0,0)^{12}$
742	0	$(0,0,1)(0,1,1)^{12}$
751	-1	$(1,0,1)(1,0,1)^{12}$
760	0	$(1,0,1)(0,1,1)^{12}$
769	0	$(1,0,0)(1,0,0)^{12}$
778	-1	$(1,0,0)$
787	-.5	$(1,0,0)(1,0,0)^{3}(0,1,1)^{12}$
805	-	$(1,0,0)(0,0,1)^{12}$

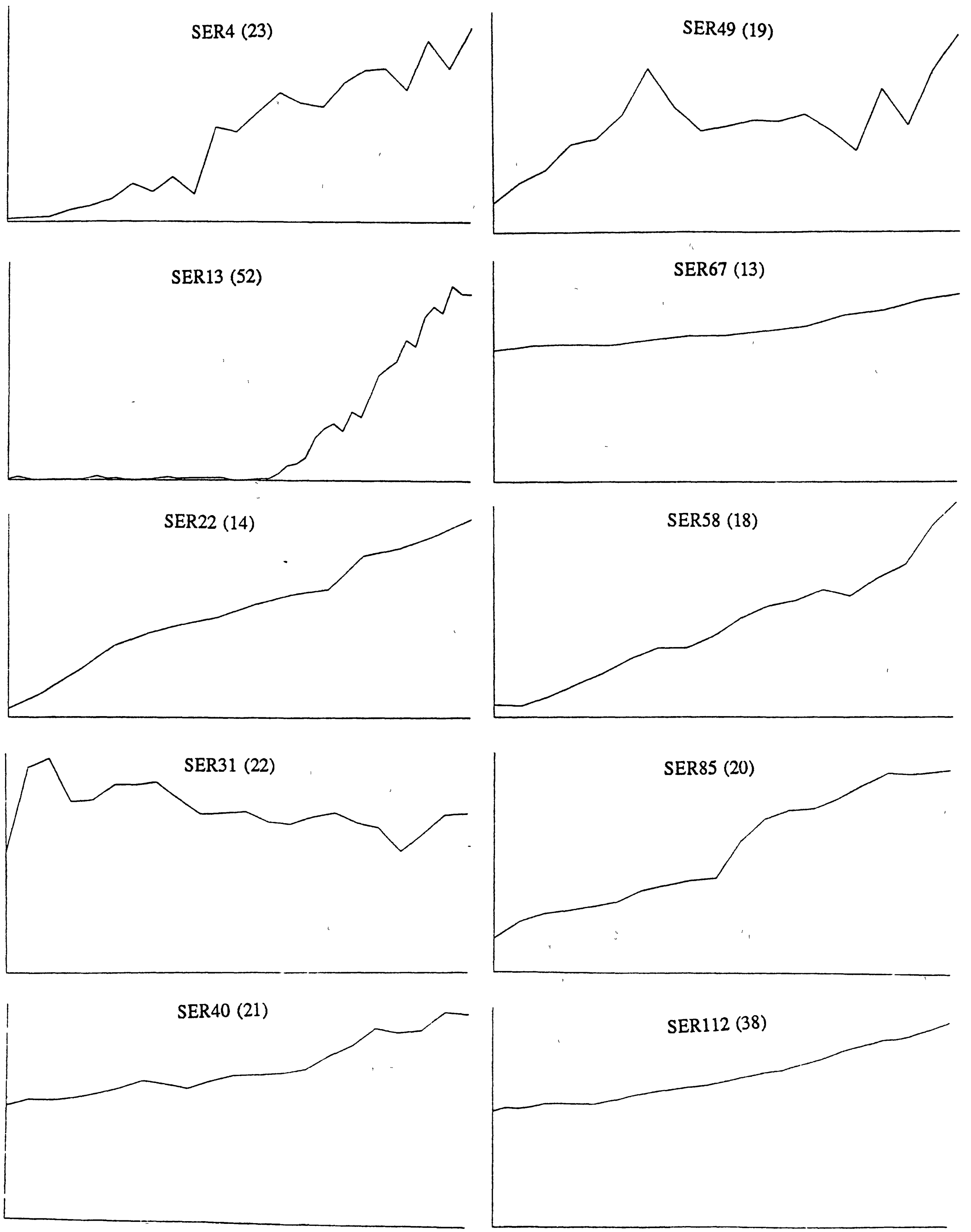
Series	coeff	ARIMA
841	-.5	$(0,0,1)^{12}$
850	-1	$(1,0,0)(1,0,0)^{12}$
859	-	$(0,0,1)^{12}$
868	not enough data	
877	-.5	$(2,0,0)(0,1,1)^{12}$
886	-	$(2,0,0)$
895	.5	$(0,1,0)$
904	0	$(0,1,1)(0,1,1)^{12}$
913	-	$(0,1,1)(0,0,1)^{3}$
922	-	$(2,1,0)$
931	-.5	$(0,0,1)^{6}(0,1,0)^{12}$
940	-.5	$(1,0,0)(1,0,0)^{6}(0,0,1)^{12}$
949	0	$(0,1,2)$
958	.5	$(1,1,0)(1,1,0)^{12}$
967	0	$(0,1,1)$
976	-	$(1,1,0)$
985	.5	$(0,0,1)(0,1,1)^{12}$
994	0	$(0,0,1)(0,1,1)^{12}$

APPENDIX H

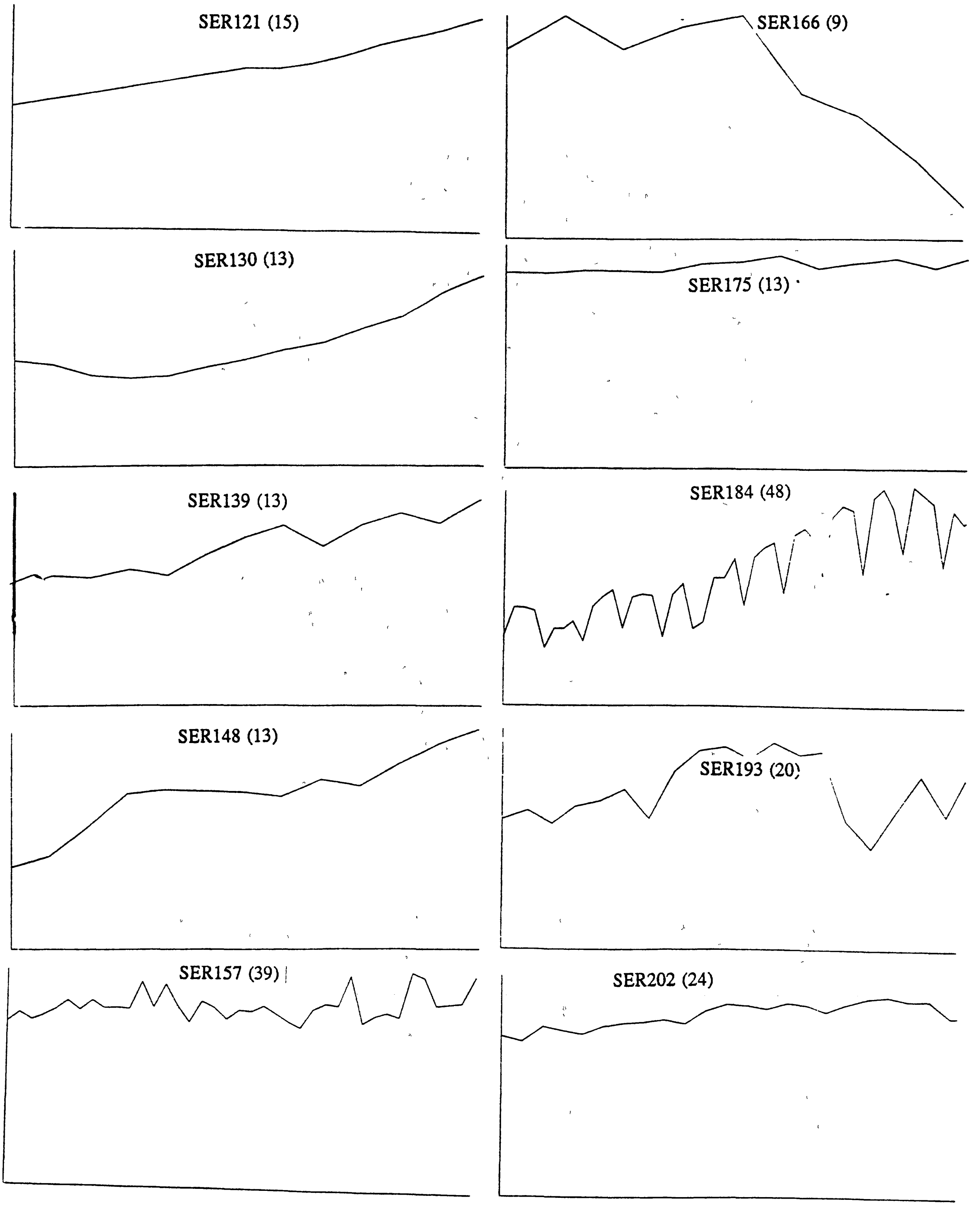
M-111 DATA

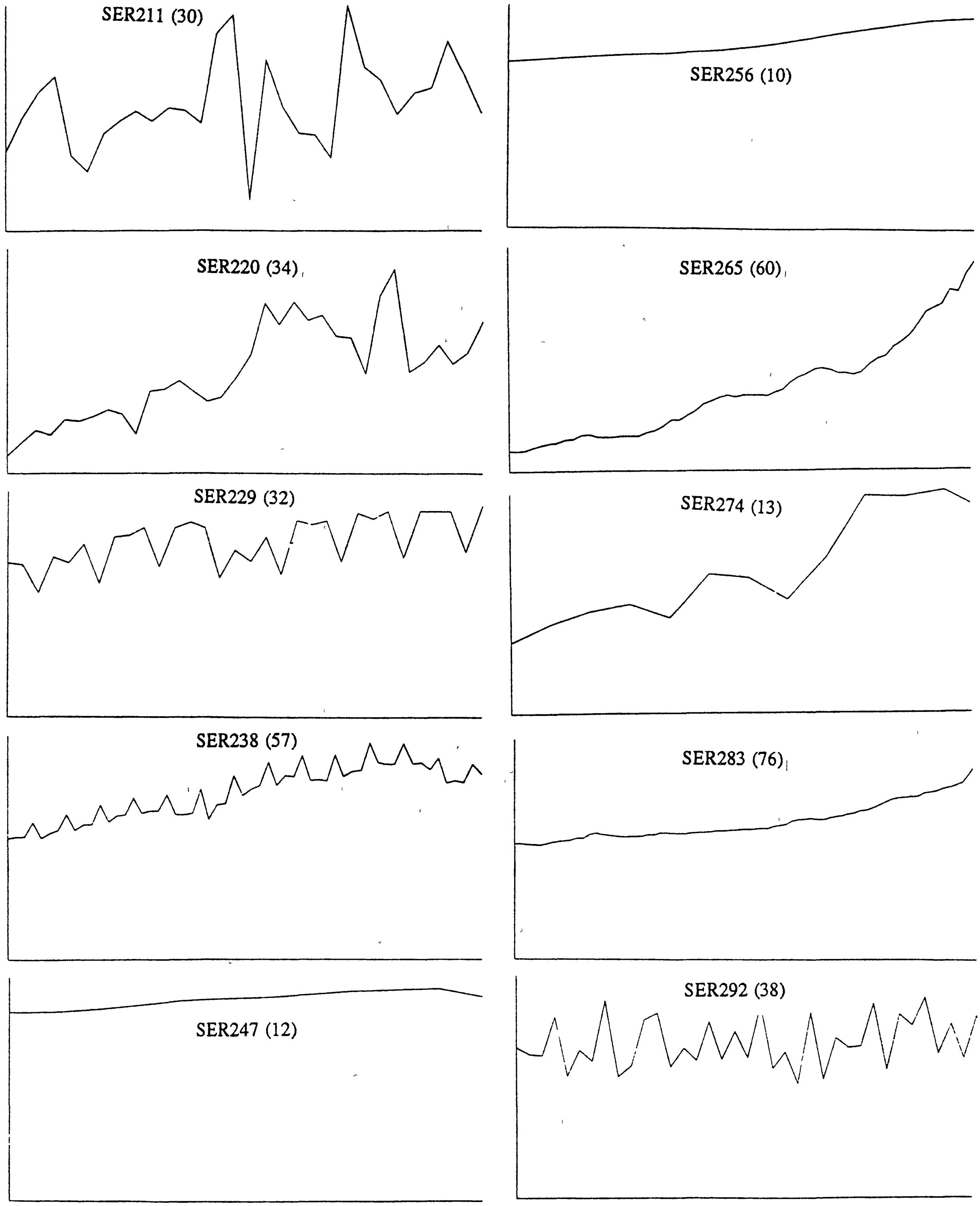
M-111 DATA

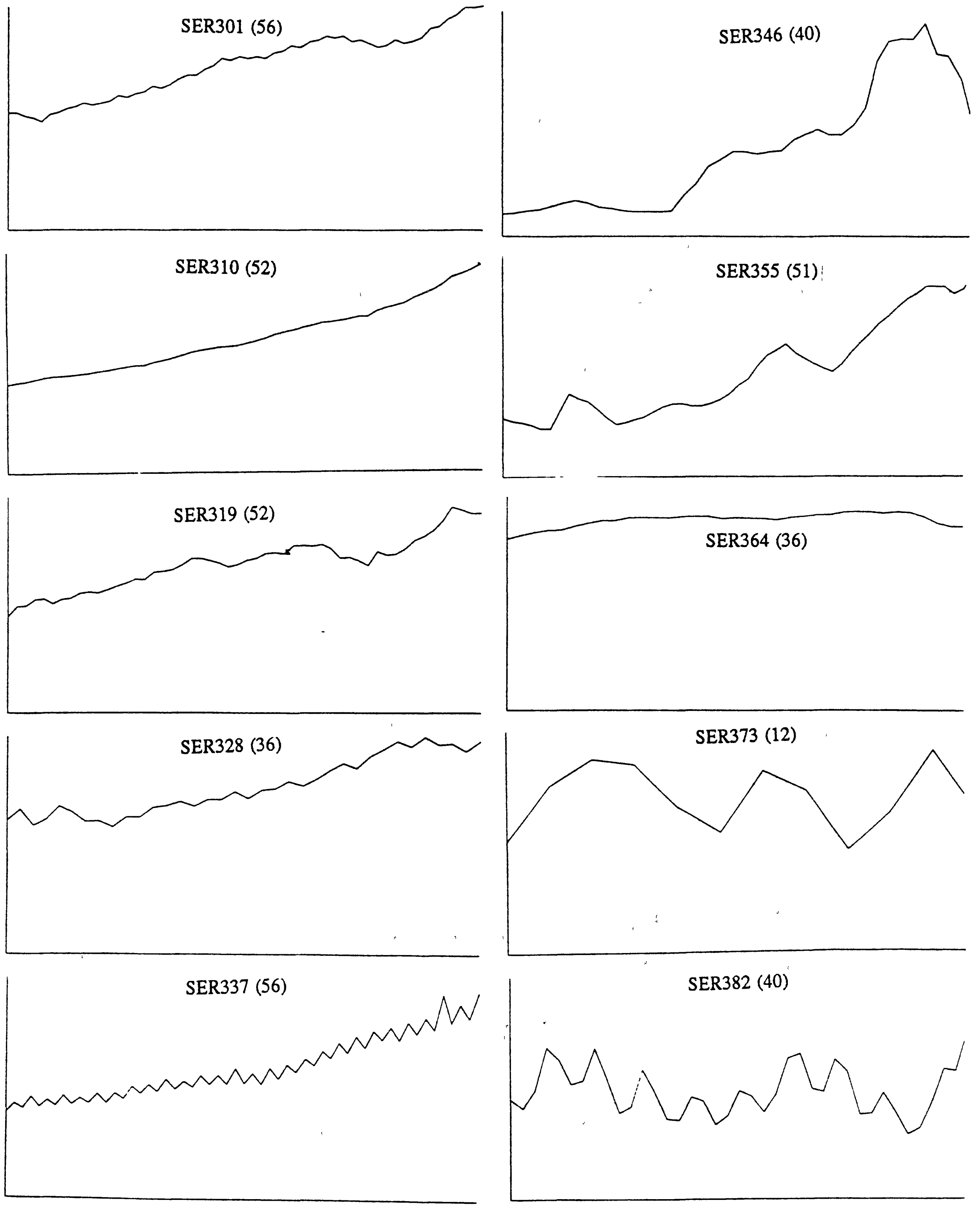
In order to give the idea about the properties of the M-111 competition data used in out present study, This section gives the graphical representation of all the data series. Graphical representation of a time series is supposed to be the best way of providing visual insight into how the process or activity it represents has behaved historically (Hoff, J., 1983).



in ( ) indicates # of data points available


SER121 (15)
SER166 (9)
SER130 (13)
SER175 (13)
SER139 (13)
SER184 (48)
SER148 (13)
SER193 (20)
SER157 (39)
SER202 (24)


SER211 (30)
SER256 (10)
SER220 (34)
SER265 (60)
SER229 (32)
SER274 (13)
SER238 (57)
SER283 (76)
SER247 (12)
SER292 (38)


SER301 (56)
SER346 (40)
SER310 (52)
SER355 (51)
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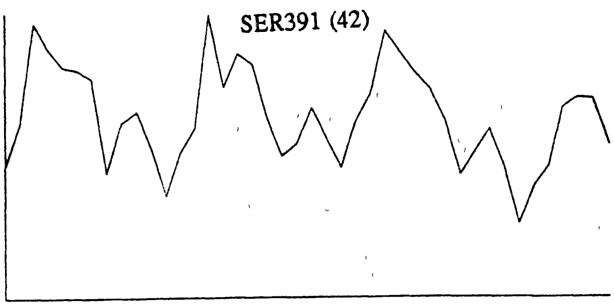
SER364 (36)
SER328 (36)
SER373 (12)
SER337 (56)
SER382 (40)

SER391 (42)



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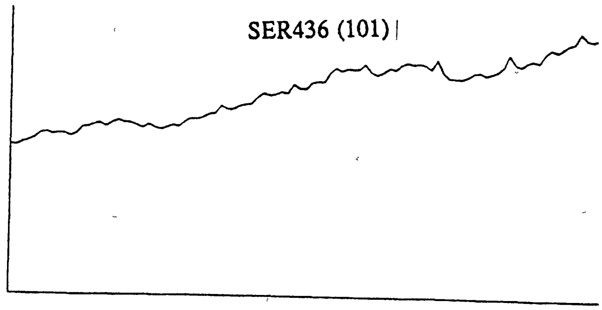
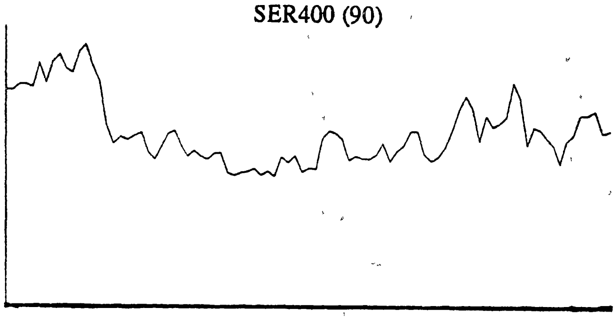
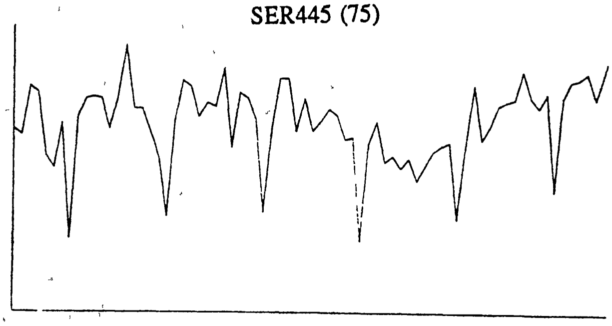


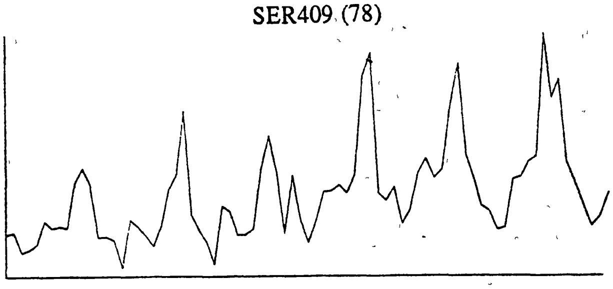
SER400 (90)



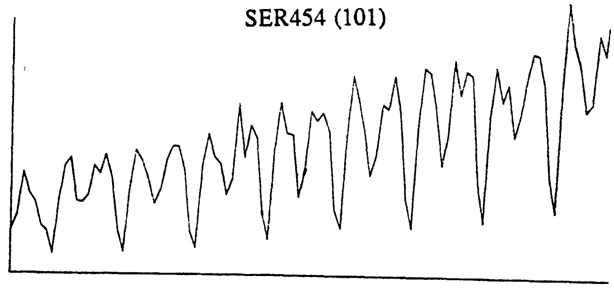
SER445 (75)



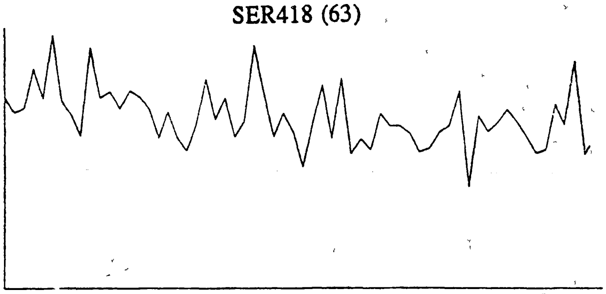
SER409 (78)



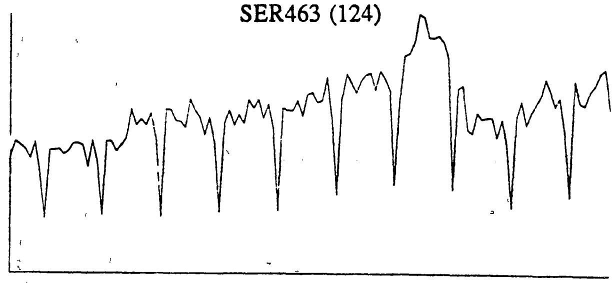
SER454 (101)



SER418 (63)



SER463 (124)



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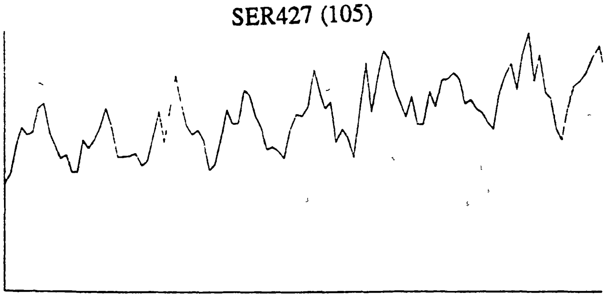

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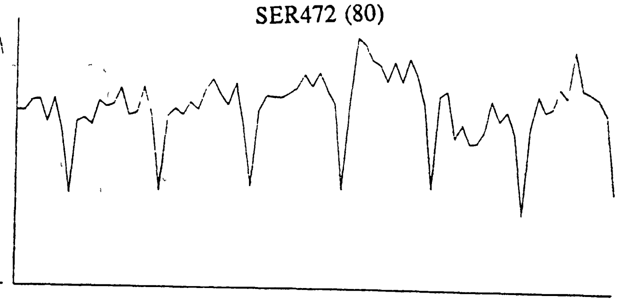

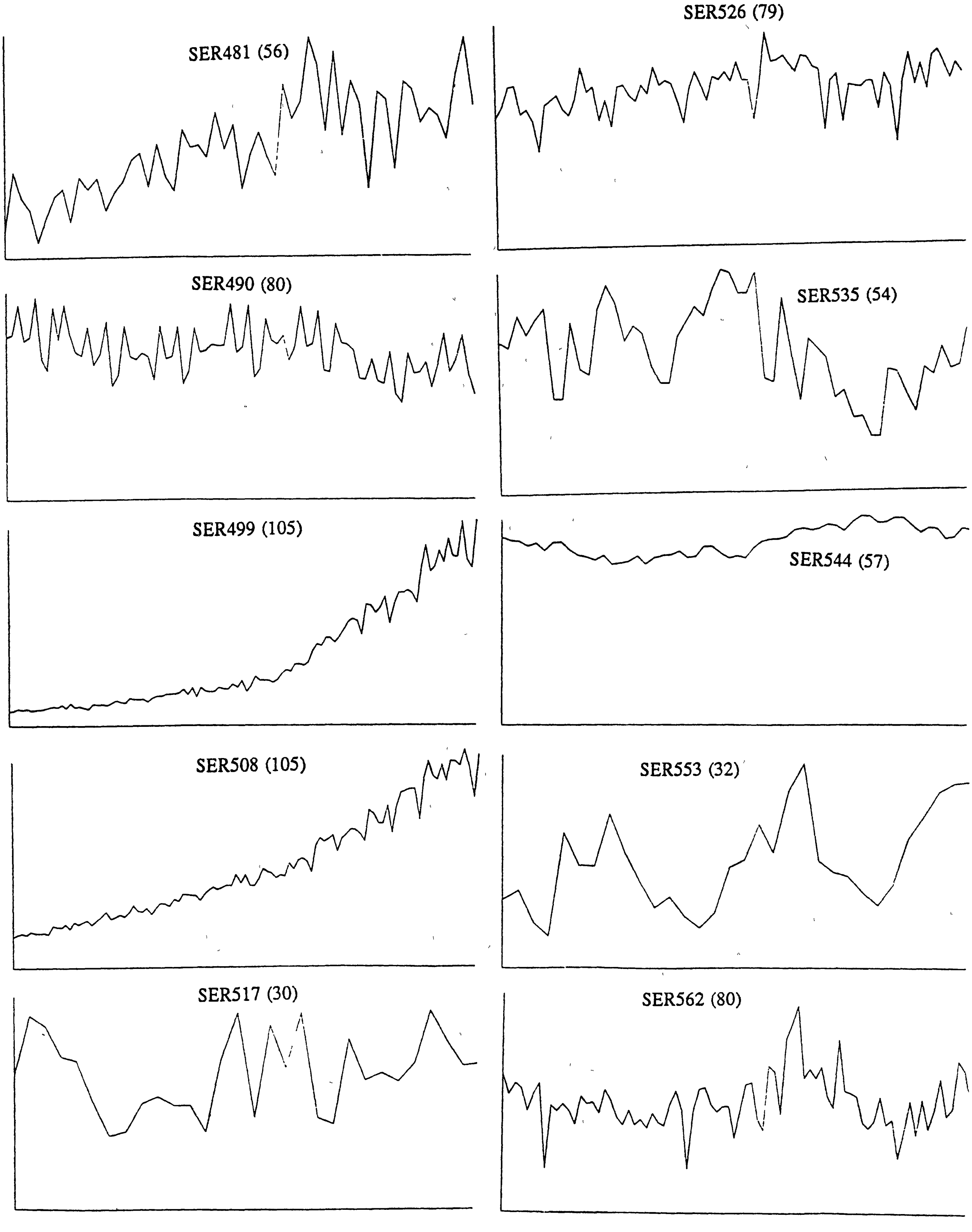
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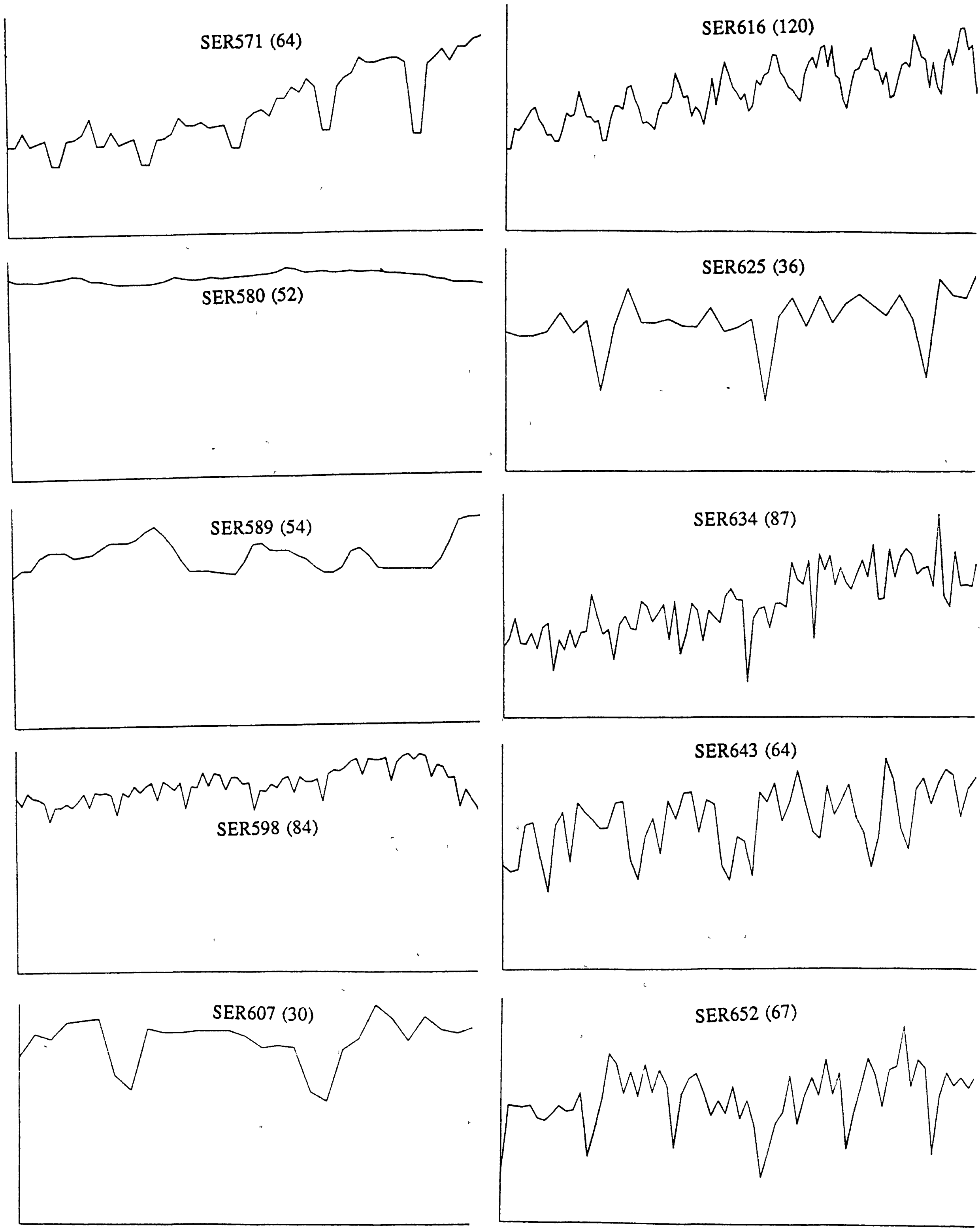
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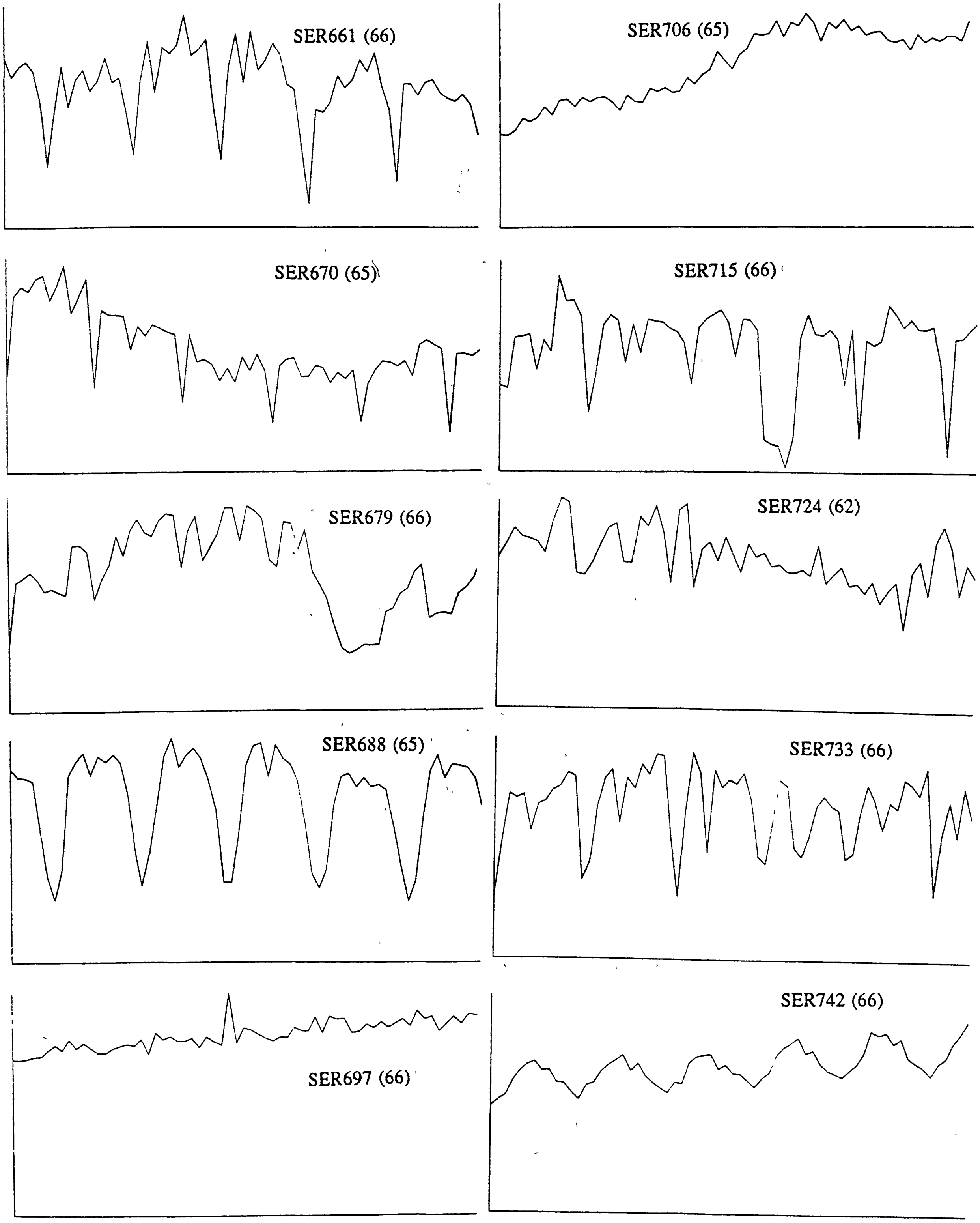
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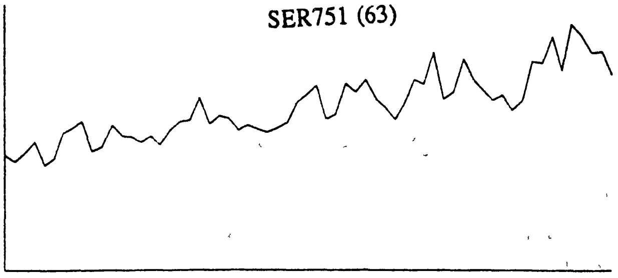
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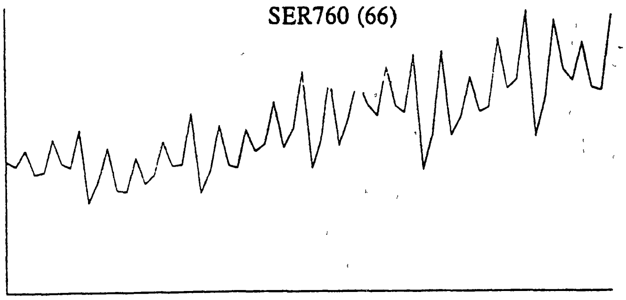
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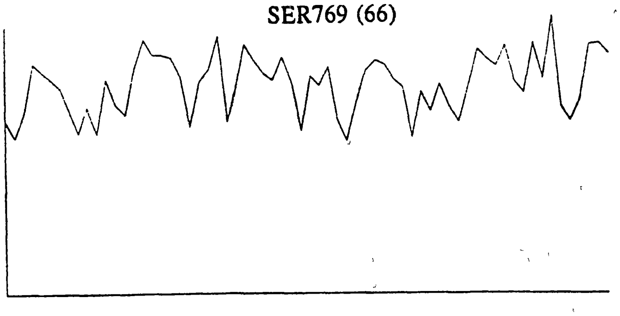
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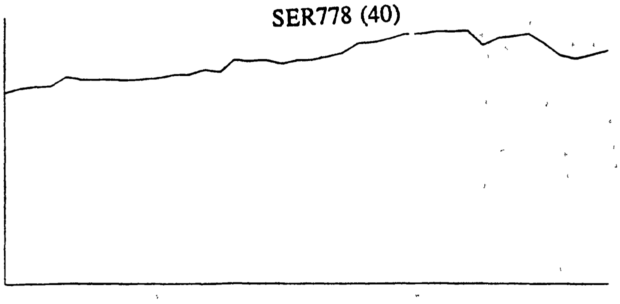
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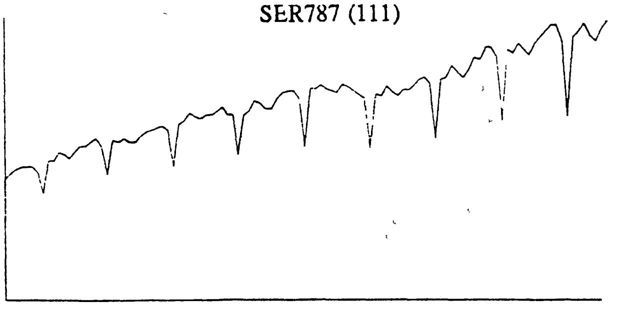
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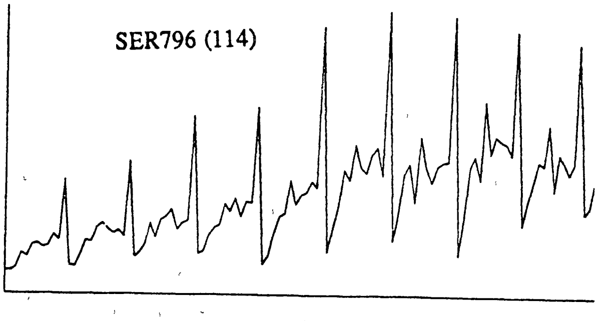
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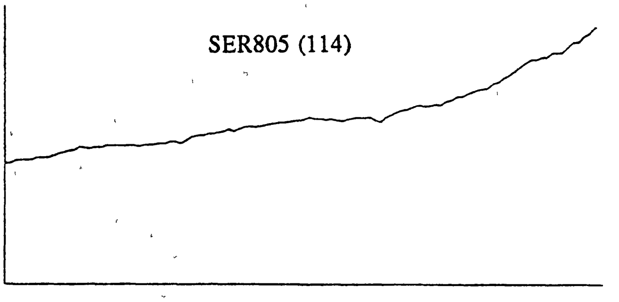
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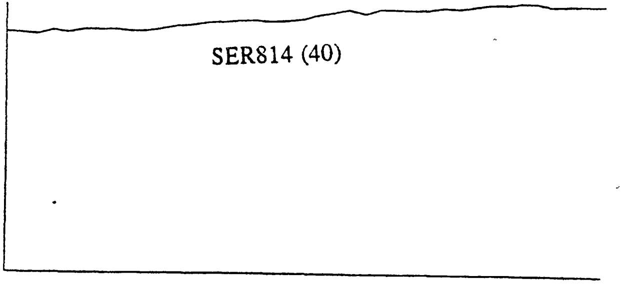
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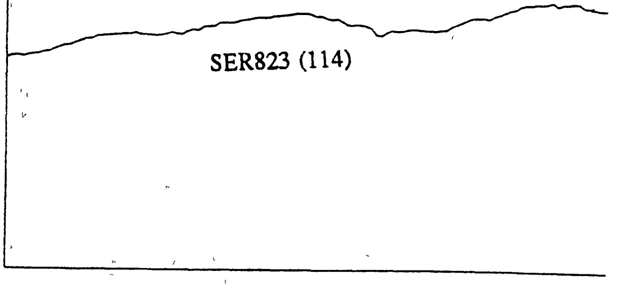
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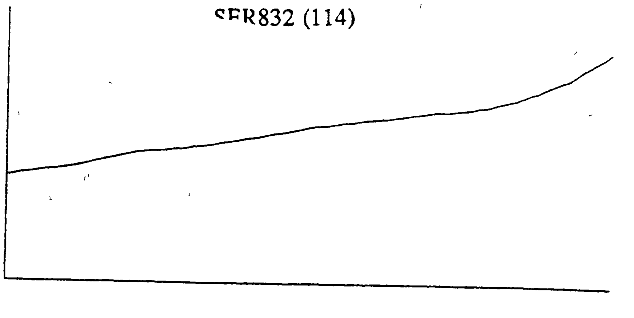
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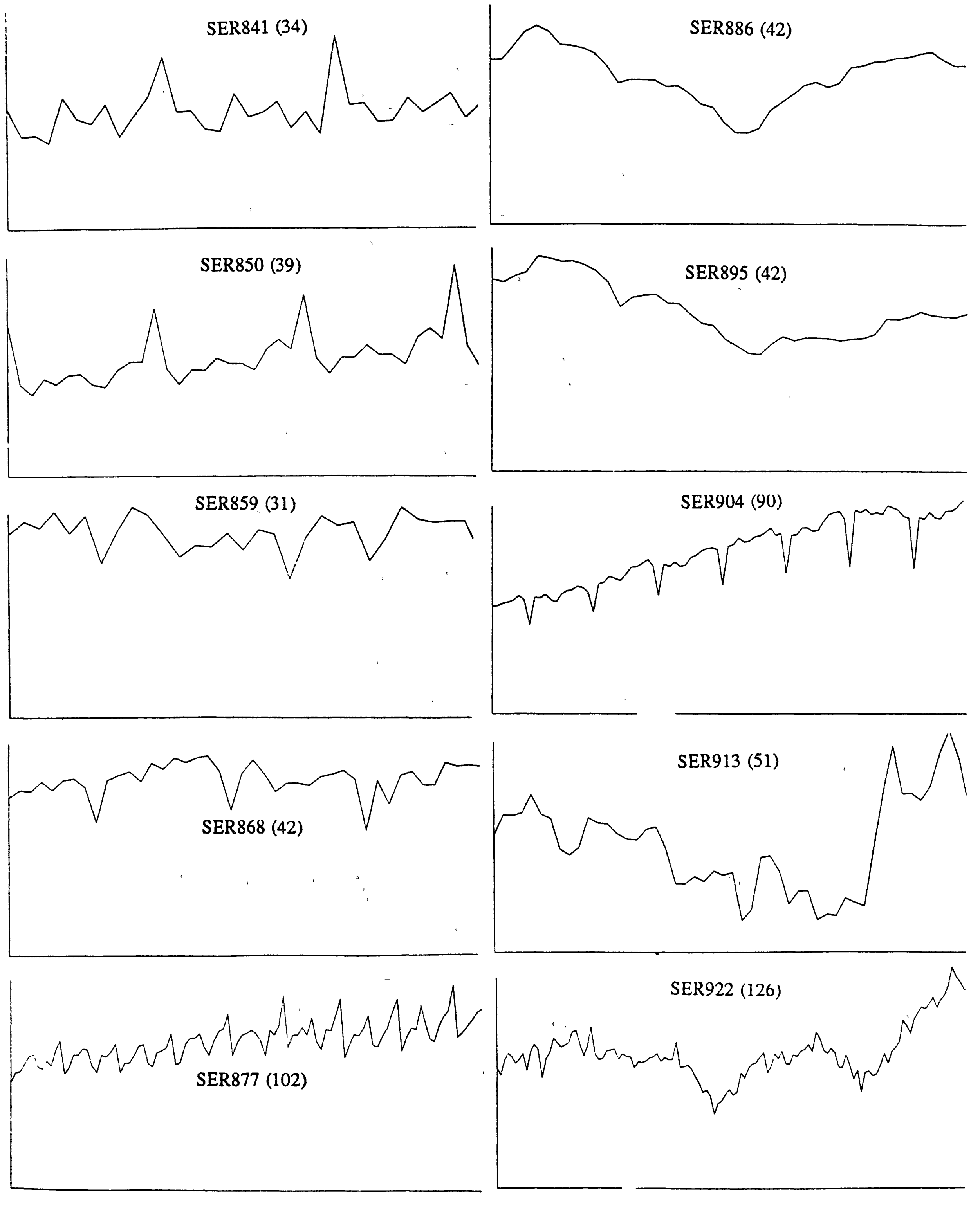
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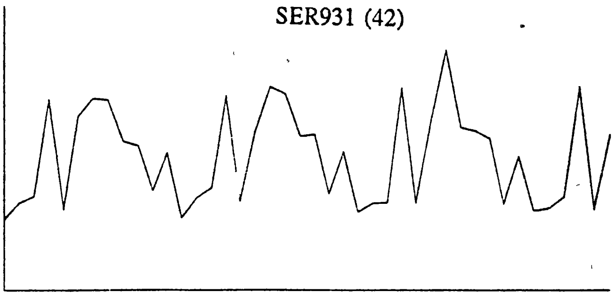
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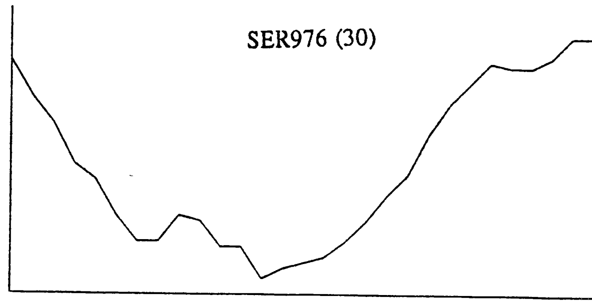
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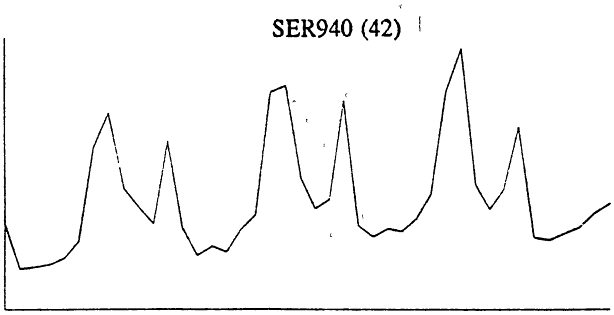
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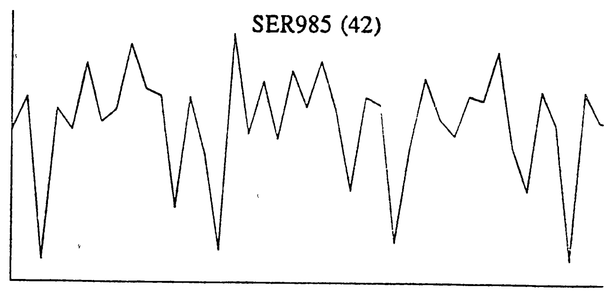
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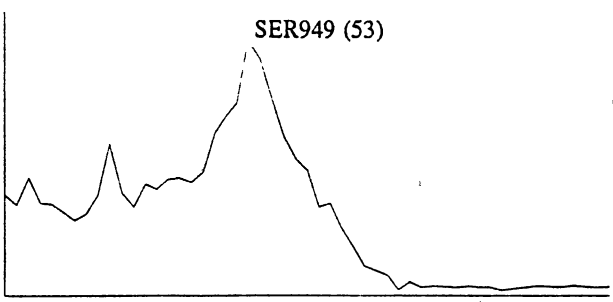
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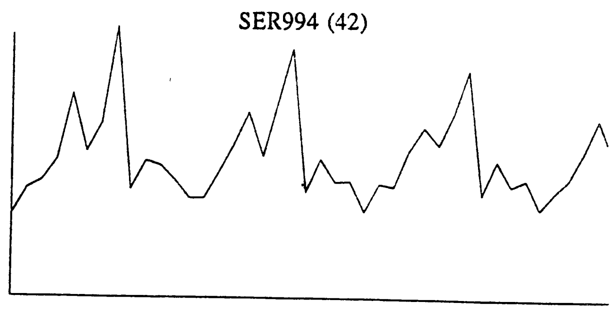
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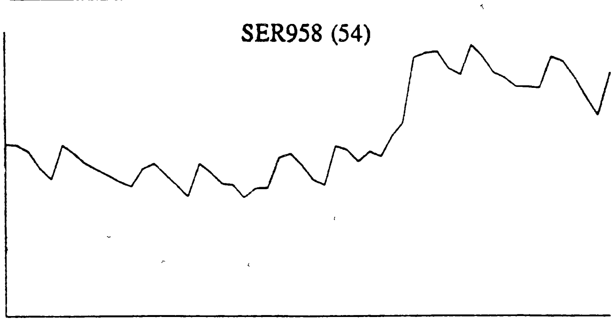
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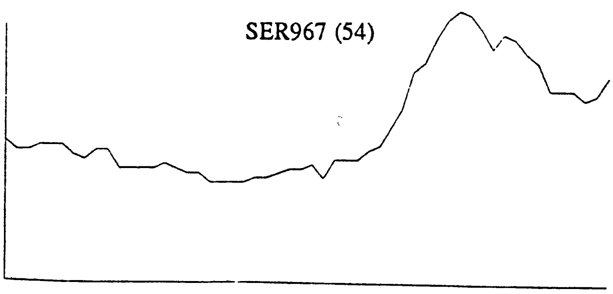
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GLOSSARY

Adaptive Coefficients: The values computed during previous training are stored in a local memory which are used for subsequent modifications.

Associative recall: Retrieval of a particular memory by a large class of initial states which are interpreted as similar stimuli.

ARIMA models: Auto Regressive Integrated Moving Average models. The term Integrated, a synonym for *summed*, is used because the differencing process can be reversed to obtain the original series values by summing the successive values of the differenced series.

Basin of attraction: The set of network states which are attracted by the dynamics to the same attractor state.

Backpropagation: The information processing is the approximation of a mapping or function **F**: $A \subset R^n \rightarrow R^m$, from a bounded subset A of n-dimensional Euclidean space to a bounded subset f[A] of m-dimensional Euclidean space, by means of training samples $(X_1,Y_1),(X_2,Y_2)...(X_k,Y_k),...$ of the mapping's action where $Y_k = \mathbf{F}(X_k)$. It is assumed that such samples of a mapping **F**, are generated by selecting X_k vectors randomly from A in accordance with fixed probability density function P(x).

BrainMaker: A commercially available neural network simulator.

Connection: The pathway formed between the neurons for signal transmission.

Feature detector: The response of the neuron when a particular pattern of stimulus is present. For example, the feature might correspond to a pixel in image processing or a particular acoustic frequency component in acoustic signals.

Content addressability: The ability to recall an item from memory using partial content of the memorized item, instead of memory address.

Deterministic process: In which each state determines uniquely its successor state.

Feed forward network: A network in which the interactions between the formal neurons are such that the neurons can be divided into groups (layers) and neural activities in one group can only influence the future activities of neurons in consecutive layers.

Fixed point: A network state which repeats itself under the dynamical process.

Hamming distance: The distance between two N-bit words which amounts to the number of positions in which they differ.

Hidden layer: The layer of neurons formed between the input and output layers. It is called hidden because it derives the input from other layers and feed its output to other layers. A neural network model may have one or more hidden layers.

Input layer: The layer of neurons to which the input pattern is applied for processing.

Input patterns: The input stimulus presented to the neural network for processing.

Layers: The neurons in the neural network are arranged into layers. The neurons in a layer have similar transfer function.

Learning algorithm: It is the equation which determines that all or some of the weights in the neuron's local memory be modified with response to the input signal and transfer function of that neuron.

Neural network: A neural network is a system in which many neurons process the information in a parallel manner. The overall function or response of the system is determined by weights present in the connection, and the processing done at the computing elements or neurons.

Neuron: The fundamental unit in the neural network. It is a nonlinear computational unit named after its counterpart in the brain.

Output layer: The layer of neurons which presents the output response of the neural network.

PDP: Parallel Distributed Processing, a synonym for neural networks, connectionist models.

Perceptron: A device comprising a set of input channels, a linear threshold calculator and an output channel. The truth values (0, 1) of elementary propositions arrive via the input channel at the calculator. Each is weighted by a channel weight, then summed by the calculator, and compared to the threshold; and the results (0, 1) are communicated along the output channel.

Supervised learning: The learning in the neural networks where the correct or expected response is provided by an external teacher at the time of training.

Stochastic process: Each state determines only the relative probabilities of its successor state.

Synaptic weights: see weights.

Synchronous dynamics: A dynamic process in which all neurons in the network determine their neural states based on the same pervious states of all the other neurons.

Time series: A Time Series is a set of numbers that measure the status of some ongoing process or activity. The measurement are assumed to be taken at equally spaced time intervals or periods.

Transfer function: The response of the neuron depends on the mathematical formula and is a function of the most recent input signals and the adaptive weights stored in memory.

Unsupervised learning: The learning in the neural networks where no external teacher provides the correct response.

Weights: The strength of the interconnection between neurons is determined by a variable, called a weight. The weights determine the intensity of the connection, which depends on the network architecture and the information it has learned.

VITA

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Master of Science

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