72-9040

McBRIDE, Ronald Lee, 1935-
A LEARNING STRATEGY FOR CALCULUS AND
ANALYTIC GEOMETRY.

The University of Oklahoma, Ph.D., 1971
Education, theory and practice

University Microfilms, A XEROX Company, Ann Arbor, Michigan

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

A LEARNING STRATEGY FOR CALCULUS AND ANALYTIC GEOMETRY

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

RONALD L. McBRIDE

Norman, Oklahoma

1971

A LEARNING STRATEGY FOR CALCULUS

AND ANALYTIC GEOMETRY

APPROVED BY

Thomas J. Hill

Stephen D. Hungat

Harold V. Huneke

Donald Reynolds

DISSERTATION COMMITTEE

ACKNOWLEDGEMENTS

The author wishes to express appreciation to Dr. Jack Shepler, of Indiana University of Pennsylvania, for the suggestion of this research topic and for his advice during the course of the investigation.

Thanks are also due to Dr. Thomas Hill, who directed this study, and to Dr. Herbert Hengst, Dr. Harold Huneke, and Dr. Donald Reynolds for their suggestions and encouragement.

Sincere appreciation is extended to my department chairman, Dr. Melvin Woodard, for his help in appropriate scheduling for this investigation.

For their patience, sacrifice, and continual encouragement, I am deeply grateful to my wife Phyllis and my children Mark, Kathy, and Matthew.

TABLE OF CONTENTS

Chapter | Page

LIST OF TABLES

CHAPTER I

METHOD OF RESEARCH

Statement of the Problem

This study investigated the effect of an experimental learning strategy for use in the first undergraduate course in analytic geometry and calculus. This strategy employed several features designed to raise the level of expectation of the student and to provide individualized direction and opportunity for study. To accomplish this investigation, two classes were taught by the same lecture-discussion method, with the learning strategy used for one class and with the other class as a control group.

Scope of the Problem

The study was conducted at Indiana University of Pennsylvania during the first eight weeks of the 1971 spring semester. Fifty students from two sections of Mathematics 157, Analytic Geometry and Calculus 1, were involved in the study.

Procedures

The procedures developed for this learning strategy are intended as a supplement to regular classroom methods, and not as replacement for existing methods. For this reason, the researcher taught both the experimental group and the control group, using considerable care to employ the same instructional methods for both groups. To achieve this desired goal, the same daily lesson plan was used for both classes. The same instructional procedures were used in both classes for presentation of the same topics on the same days. Even the same examples were used for demonstrating concepts, and the same text problems were selected for completion by the students as homework assignments. The only way in which actual class instruction of the two groups differed was that ten to thirty minutes of class time per week was used in the experimental group for administering a formative test. The corresponding time interval in the control group was used for additional illustrative examples or for the discussion of homework problems.

Description of the Learning Strategy

The learning strategy consists of several features designed to raise the level of expectation of the members of the experimental group and also to provide additional opportunities for learning. The several ways in which this was accomplished are listed below.

1. A special effort was made to convince the members of the experimental group that simply receiving a passing grade was not an acceptable

goal for this course. Rather, it was pointed out that a high level of mastery of the concepts of this first course in calculus is necessary for success as a mathematics major. Since all students in the two classes were mathematics majors, the particular course under study is only one of many courses in mathematics which each student must complete. In particular, two subsequent courses in analytic geometry and calculus are required as well as at least one course in advanced calculus. Thus all students of both groups were aware that the concepts of this course are prerequisite for several courses which follow.

2. Each student in the experimental group received a clear and complete list of objectives to be mastered in the eight-week experimental period. These objectives for the experimental period were separated into eight units of approximately one week each. The objectives for each unit were distributed when instruction on the prior unit was complete.

3. Each of the two groups had four fifty-minute class sessions on the same four days of each week. In the class for the experimental group, ten to thirty minutes of one class period of each week wa

used for formative testing. These formative tests were brief diagnostic-progress tests, closely correlated with the list of course objectives. They were designed to determine both those objectives which had been mastered by the individual student and those which demanded additional attention. The results obtained from these formative tests were not used in the determination of course grades, and students were fully aware of this feature.

4. The results of the formative tests were used to provide each student of the experimental group with a diagnosis of his progress in achieving mastery of the stated objectives. Not only was the formative test returned to the student, but also a supplementary diagnostic sheet which offered specific suggestions for appropriate methods and materials for further study. Aids which were offered included reference to appropriate sections of the text, dealing with the development of concepts or with illustrative examples, reference to problem lists from the text which are similar to the stated objectives, attendance at group sessions directed by a graduate assistant, or short conferences with the course instructor. Individual help from

the classroom instructor for members of both the experimental group and the control group was available outside of scheduled class time on an equal basis. An additional source of aid for members of both groups was a general weekly help session directed by students of the mathematics honorary fraternity, which is offered for students of all mathematics classes.

5. For each of the formative tests, the researcher determined a particular score as the acceptable level of mastery for the objectives of that unit. This score was set at that level which the researcher estimated as representative of grade levels of A or B, based on past experience with similar classes in these same units. A recommendation was given to each student whose score was below this established level of mastery to complete a retest over these same objectives, after first completing the additional work suggested on the diagnostic sheet. Each student who completed this second attempt to achieve mastery was invited to a brief conference with the course instructor for the purpose of evaluating progress.

6. Students were not required to complete the additional work suggested on the diagnostic

sheet nor were they required to complete a second formative test on those units where mastery was not indicated by the initial test. Rather, all additional aids were available to the student who wished to participate.

Collection of Data

The data for this research was collected from a pre-test and a post-test which were given to both groups. The pre-test was given to both groups during the initial class session. The post-test was given in two parts. The first part of the post-test dealt with the first four units, and was given during the fourth week of the experimental period. The test was given to both groups on the same day, but the groups were not combined. The second part of the post-test was given at the end of the eight-week experimental period to both groups in a combined session.

An additional source of data to evaluate this experimental strategy are the records which were maintained by the researcher and by the graduate assistant. These records included the number of students who attended each of the special help sessions offered by the graduate assistant for the members of the experimental group. Also the researcher maintained a log which indicated the amount of time used outside of the classroom in helping individual students from either of the two groups.

A final source of data was an evaluation of the experimental learning strategy by the members of the experimental class.

Treatment of Data

The comparison of the achievement of the two groups was based on the data from the two-part post-test and the pre-test. Analysis of covariance was the statistical method used, employing the results of the pre-test scores as covariant. This method, referred to as the "nonequivalent control group design," is recommended by Campbell and Stanley,[1] when the control group and the experimental group do not have pre-experimental sampling equivalence.

The records maintained by the researcher and by the graduate assistant were used to determine the time used by students of the experimental group in directed problem sessions. The additional records maintained by the researcher were used to compare the amount of time used in helping individual students of the two groups, outside of scheduled class sessions.

Questions to be Answered

This study provides answers or partial answers to the questions:

1. Was the achievement of the experimental group,

---

[1]Donald T. Campbell and Julian C. Stanley, Experimental and Quasi-Experimental Designs for Research (Chicago: Rand McNally & Company, 1963), p. 47.

in which the learning strategy was used, significantly better than that of the control group, as measured by an achievement test given to both groups?

2. Did the individual members of the experimental group take advantage of the additional learning opportunities provided?
3. Did the students of the experimental group seek more individual help from the course instructor than the students of the control group?
4. Did the students of the experimental group evaluate this learning strategy as a desirable feature to be used in this course or in other courses?

Assumptions and Limitations

One assumption which must be made is that the instrument used for evaluation in this study was reasonably valid. Although an attempt was made to evaluate test items in terms of validity it is still necessary to assume the validity of this instrument.

A serious limitation of this study is that it was conducted at a single university and involved only two classes, a total of fifty students. No attempt will be made to extend the results of this study to a more general population. For the purpose of providing the reader with more information about this study, a brief description of this university is included in a later chapter.

CHAPTER II

BACKGROUND AND RELATED LITERATURE

Background of the Problem

The instructor of a typical undergraduate course begins with a preconceived notion of the expected grade distribution for that class. The most common grade in this distribution will be that of "C," indicating an acceptable grasp of concepts but not complete mastery. This expectation may lead to loss of motivation both on the part of the instructor and the student, as well as damage to the self-concept of the student. As stated by Bloom:

> Each teacher begins a new term (or course) with the expectation that about a third of his students will adequately learn what he has to teach. He expects about a third of his students to fail or to just "get by." Finally, he expects another third to learn a good deal of what he has to teach, but not enough to be regarded as "good students." This set of expectations, supported by school policies and practices on grading, becomes transmitted to the students through the grading procedures and through the methods and materials of instruction. The system creates a self-fulfilling prophecy such that the final sorting of students through the grading process becomes approximately equivalent to the original expectations.[1]

---

[1] Benjamin S. Bloom, "Learning for Mastery," UCLA CSEIP Evaluation Comment, I, No. 2 (1968), 1.

Carroll[2] has attempted to develop a conceptual model of factors effecting success in school learning and of the way in which these factors interact. He suggests that such a model should use a small number of simplifying concepts, and should suggest new and interesting research questions as well as aid the solution of some practical educational problems.

The learner's task of going from ignorance of some specified task or concept to knowledge or understanding of that concept is defined to be a learning task. Carroll's completed model then specified that the learner will succeed in learning a given task to the extent that he spends the amount of time that he <u>needs</u> to learn the task.

To explain the terms of this statement, the following definitions are given by Carroll.

> First, it should be understood that "spending time" means <u>actually spending time on the act of learning</u>. "Time" is therefore not "elapsed time" but the time during which the person is oriented to the learning task and actively engaged in learning. In common parlance, it is the time during which he is "paying attention" and "trying to learn." Second, there are certain factors which determine how much time the learner <u>spends</u> actively engaged in learning. Third, there are certain factors which determine how much time a person <u>needs to spend</u> in order to learn the task. These factors may or may not be the same as, or associated with, those which influence how much time he spends in learning.[3]

The model which is presented is not descriptive of

---

[2]John B. Carroll, "A Model of School Learning," <u>Teachers College Record</u>, LXIV (May, 1963), 723-33.

[3]<u>Ibid</u>., p. 725.

a "learning theory," but rather is a description of the variables involved in the school learning process. The study describes this complete model as a combination of five elements. Three of these elements are learning variables since they are internal to the individual. The other two elements are external to the learner and hence are instructional variables.

The three learner variables are: (1) aptitude--the amount of time needed to learn the task under optimal instructional conditions, (2) ability to understand instructions--assuming that the quality of instruction is anything less than optimal then the learner will need more time to accomplish the learning task, but some learners will be more handicapped than others. The extent of this handicap is conceived to be determined by the learner's ability to understand instructions, (3) perseverance--the amount of time that the learner is willing to actively engage in the learning task.

The instructional variables in this model, which are external to the individual, are: (4) opportunity--the time allowed for study. This factor includes both the time used during formal class sessions as well as directed and informal study time, and (5) the quality of instruction--presentation of the learning task in such a way that the learner can accomplish the task as rapidly and as efficiently as possible. This includes presentation, in words the learner can understand, of what is to be learned and how he is to learn it.

As the instructor enters the classroom for a new term he finds the first two internal factors already fixed, but the third, perseverance, will be influenced by the attitudes developed in the classroom. If the expectations of the instructor are low then this may be reflected in this third factor. Also, the two external factors are directly affected by the instructor and are to a large extent under his control.

Under ideal circumstances it could be assumed that each instructor would use those classroom methods which he feels will provide the highest quality of instruction for the group. Even with this assumption, however, few instructors make special efforts to give each student the individual opportunities needed for learning.

If appropriate opportunities for assistance could be provided by the instructor, and if the individual students could be encouraged to take advantage of these opportunities, and if further, the students were convinced that the objectives of the instructor were within their ability to attain, then perhaps the expectations of both student and instructor could be increased. The opinion of Bloom is that at the top of the aptitude scale there are one to five per cent who have a special talent for a subject, and at the bottom there is another group, probably less than five per cent, with special disabilities for a particular learning. He then states:

> In between are approximately 90 percent of the individuals where we believe that aptitudes are predictive of rate of learning rather than level (or complexity) of learning that is possible. Thus, we are expressing the view that, given sufficient time (and appropriate types of help), 95 percent of students (the top 5 percent + the next 90 percent) can learn a subject up to a high level of mastery. We are convinced that the grade of A as an index of mastery of a subject can, under appropriate conditions, be achieved by up to 95 percent of the students in a class.[4]

Bloom and a group of associates at the University of Chicago have been doing research on the variables in the learning situation. Efforts have been made to develop strategies which will make it possible for this group of 95 per cent of the students to reach a level approaching mastery in the particular subject area. Experiments have been conducted in reading at the sixth grade level, in mathematics at the eighth grade level, and in psychology and test theory at the college level. Results have indicated a dismal failure in some cases and encouraging successes in others. In one experiment in test theory classes he reports that before the experimental strategy was used, 20 per cent of the students received the grade A, which was fixed as the level of mastery. In the following year, when the experimental strategies were employed, 80 per cent of the students reached this same level of mastery as measured by parallel achievement tests. In the second year of the experimental strategies this percentage was increased

---

[4] Bloom, "Learning for Mastery," p. 3.

to 90. In developing the strategies for the experimental methods, efforts have been made to supplement the regular teaching methods of the instructor but not to have the individual teacher change his method in the classroom. This was done to avoid the necessity of extensive training of teachers and thus to develop a strategy which would receive widespread use.

The operating procedures developed by this group were designed to provide feedback to both teachers and students and thereby to suggest supplementary instructional methods as needed. The material for the course was broken down into small units, and these units were analyzed in terms of specific objectives. Brief diagnostic-progress tests, referred to as formative tests, were then prepared for each unit. These tests were used to determine whether or not an individual student had mastered the objectives of a unit, and if not, the results were used to determine which tasks the student still had to master.

For students who achieved mastery on these formative tests, the results assured the student that his present study methods were appropriate, and thus reduced anxiety about achievement. When the formative test indicated that the student had not mastered all necessary objectives then he was given a specific prescription of what he should do to improve mastery in these areas.

Justification of the Problem

The model developed by Carroll[5] suggests several ways in which an individual instructor can strive to improve the learning in a particular course area. The results achieved by Bloom and his associates suggest that control of some of the variables of learning is in the hands of the instructor, both while planning for a particular course and while the course is in progress.

Since perseverance is one internal factor which can be affected by the classroom instructor then a course should be considered in which the instructor has a good chance to use this factor to advantage. To accomplish this, a course which is a prerequisite to several other required courses in the students' major area would be appropriate.

A second factor of the model which does not always receive the attention of the classroom instructor is the provision for individual opportunities for learning. Few instructors provide these individualized learning experiences which can be designed to supplement regular classroom work.

It would appear appropriate to use Carroll's model and the suggestions made by Bloom to conduct a study dealing with the first course in calculus which is required for every mathematics major. The study would

---

[5] Carroll, "A Model of School Learning," pp. 723-33.

be designed to ascertain whether an instructor could, by making his objectives clear and reasonable to the student, and by providing appropriate opportunities for learning, expect mastery of these objectives by the majority of that 95 per cent referred to by Bloom.

Related Literature

The conceptual model of school learning developed by Carroll[6] was instrumental in the development of interest in new approaches to improvement in activities used to promote learning. The paper by Bloom[7] then provided guides for experimental strategies for implementing this conceptual model. Both of these men have presented learning concepts which have led to valuable research. Those strategies for learning which have utilized the working model developed by Bloom have become known as strategies for mastery learning.

Block[8] has summarized much of the basic research in the area of mastery learning and in areas related to Carroll's conceptual model for school learning. This book by Block, entitled Mastery Learning: Theory and Practice, consists of two major parts. Part One is a collection of six articles which focus on the theory behind mastery

---

[6] Ibid., pp. 723-33.

[7] Bloom, "Learning for Mastery," pp. 1-11.

[8] James H. Block, Mastery Learning: Theory and Practice (New York: Holt, Rinehart and Winston, Inc., 1971).

learning and some possible methods of implementing strategies. Part Two presents an annotated bibliography of research which has been accomplished in mastery learning and related areas. This book is an invaluable source since summaries are given for many studies which are not readily available in published form. The information for many of the studies discussed in this section has been obtained from these summaries presented by Block.

The attempt in this study of related literature is to analyze the results of studies which pertain to elements of the working model of school learning as developed by Carroll and by Bloom. The specific elements to be studied are: (1) the effect of the use of objective lists which are made available to the students, (2) the effect of expectation on achievement, (3) the relationship between student self-concept and achievement, (4) the use of tutorial help or special small group sessions, and (5) some mastery strategies which have been used.

Objectives

A study by Doty[9] investigated the relative effectiveness of two teaching strategies used in a unit concerning industrial arts for seventh-grade male students. The study included a comparison of two randomly

---

9 Charles R. Doty, "The Effect of Practice and Prior Knowledge of Educational Objectives on Performance" (Doctoral dissertation, The Ohio State University, 1968), Dissertation Abstracts, 1968, XXIX, 3035-A.

assigned groups, one group which had been given a list of objectives prior to instruction and the other group which did not receive this list of objectives. Both groups received the same classroom instruction. The results were that students who had prior knowledge of educational objectives scored significantly higher on the post-test which measured achievement.

Boardman[10] conducted a study which also measured the value of providing students with a list of specific behavioral objectives prior to instruction. This study was conducted with a course in remedial chemistry. The analysis of results showed no significant difference in the achievement of the two groups.

In 1969, a study was conducted by Collins,[11] on the effects of two strategies used in the teaching of freshman college mathematics courses in modern algebra and calculus. Students of the experimental groups were given a list of objectives as well as a daily problem based on the objectives for the previous session. The control groups received the same classroom instruction with the exception of the lists of objectives and the special problems. The result was that 75

---

[10] Dorris E. Boardman, "The Effect of Students' Advanced Knowledge of Behavioral Objectives on Their Achievement in Remedial Chemistry" (Doctoral dissertation, University of California, Los Angeles, 1970), Dissertation Abstracts, 1970, XXXI, 3286-A.

[11] Block, Mastery Learning, pp. 110-11.

per cent of the algebra students in the experimental group achieved a grade level of A or B as compared to 30 per cent for the control group. In the calculus groups, the achievement at this level was 65 per cent for the experimental group and 40 per cent for the control group.

These three studies indicate some conflicting results concerning the value of a list of specific objectives made available for student use prior to instruction. The study by Collins includes the use of daily problems and thus the results do not apply exclusively to the use of objectives.

Expectation

A factor which affects the learner in a given situation is the perseverance of the learner. This is an internal factor for the student but is affected by conditions which exist within the classroom and includes the instructional method. One element which can be considered as a possible source of change in the perseverance of an individual student is expectation, both of the teacher and the student. Several studies will be discussed which attempt to relate expectation and achievement.

An investigation by Feather[12] concerned the relationship between an individual's orientation toward a

---

[12] N. T. Feather, "Effects of Prior Success and Failure on Expectation of Success and Subsequent Performance," Journal of Personality and Social Psychology, III (March, 1966), 287-98.

task (seeking success or avoiding failure), his expectation of the tasks at hand, and his initial experience with the task. The sample consisted of 96 female undergraduate students of an introductory course in psychology at the University of Michigan. Four situations were investigated by changing the motivation given before experimental sessions and by changing the level of difficulty of the tasks to be performed. These four situations were designated as: (1) high expectation-initial failure, (2) low expectation-initial failure, (3) high expectation-initial success, and (4) low expectation-initial success. Results indicated the importance of prior success on the individual's expectation of later success and on actual performance. Thus this study shows that initial experiences have a significant effect on subsequent performance.

A study by Rosenthal and Jacobson[13] involved all children of a six grade elementary school. The practice in this school was to group all children of each grade level into three classrooms. One classroom was used for children performing above average, one for children of average performance, and one for those with below average levels of scholastic achievement. Twenty per cent of the children were randomly chosen from the school population. Each teacher was given the names of these selected children

---

[13]Robert Rosenthal and Lenore Jacobson, "Teacher Expectancies: Determinants of Pupils' IQ Gains," Psychological Reports, XIX (August, 1966), 115-8.

and the information that each student had been identified by a "test for intellectual blooming" as one who would show unusual intellectual gains during the academic year. The experimental treatment of this randomly selected sample consisted entirely of this identification. Those students from whom the teachers had been led to expect greater intellectual gains did show significantly more improvement than did the children of the control group.

A study of the effect of teacher expectation was investigated by Maxwell.[14] The purpose of this study was to investigate further the self-fulfilling prophecy phenomenon which has been found in several earlier studies. The intelligence test scores for one-half of the sample of 64 second and fourth grade students were falsely increased by 16 points (one standard deviation) before results were given to teachers. Two measures were used to compare the results of this group with another group in which true intelligence test scores were reported. Results indicated that members of the group with altered intelligence scores gained significantly more in intelligence during the seven months experimental period than members of the control group. The results did not indicate a significant difference in achievement measures.

---

[14] Merle L. Maxwell, "A Study of the Effects of Teacher Expectation on the IQ and Academic Performance of Children" (Doctoral dissertation, Case Western Reserve University, 1970), Dissertation Abstracts, 1970, XXXI, 3345-A.

Self-Concept

One of the important consequences of mastery learning as discussed by Bloom is the effect of such a strategy on the student's self-concept. Bloom states:

> At a deeper level is the student's self-concept. Each person searches for positive recognition of his worth and he comes to view himself as adequate in those areas where he receives assurance of his competence or success. For a student to view himself in a positive way, he must be given many opportunities to be rewarded. Mastery and its public recognition provide the necessary reassurance and reinforcement to help the student view himself as adequate. It is the opinion of this writer that one of the more positive aids to mental health is frequent and objective indications of self-development. Mastery learning can be one of the more powerful sources of mental health. We are convinced that many of the neurotic symptoms displayed by high school and college students are exacerbated by painful and frustrating experiences in school learning. If 90 percent of the students are given positive indications of adequacy in learning, one might expect such students to need less and less in the way of emotional therapy and psychological help. Contrariwise, frequent indications of failure and learning inadequacy must be accomplished by increased self-doubt on the part of the student and the search for reassurance and adequacy outside the school.[15]

A study by Brookover, Thomas and Paterson[16] supports these statements concerning self-concept, as made by Bloom. This study investigates the interaction between self-concept of students and academic achievement. Subjects were 1050 seventh grade students in the four subject matter areas of arithmetic, English, social studies, and

---

[15]Bloom, "Learning for Mastery," p. 11.

[16]Wilbur B. Brookover, Thomas Shailer, and Ann Paterson, "Self-Concept of Ability and School Achievement," Sociology of Education, XXXVII (Spring, 1964), 271-9.

science. General self-concept and academic performance were found to be positively and significantly related. It was also found that there are specific self-concepts of ability which are related to specific subject matter areas. In some cases a specific self-concept of ability was a significantly better predictor of achievement than the general self-concept of ability.

A study by Torshen[17] investigates the relationship between students' self-concept and classroom evaluation. The sample consisted of 400 fifth grade students from six school districts. Measures of students' self-concepts were obtained from a questionnaire which provided twelve distinct aspects of self-concept. Results indicated a highly significant and positive correlation between "academic" self-concept and teacher evaluations. A significant positive relationship was found between mental health and both teachers' evaluation and achievement. An additional result was that the relationships of achievement to students' self-concept and the students' mental health was not significant when the influence of teachers' evaluations was removed.

These studies suggest that a positive correlation does exist between self-concept and achievement and also between self-concept and teacher evaluation. This indicates that a proper application of a mastery learning

---

[17] Block, Mastery Learning, pp. 139-41.

strategy could result in desirable affective consequences.

Tutorial or Small Group Help

Although the use of individual tutors for each student has often been suggested as an ideal learning situation, there has been little research in recent years to measure the value of tutorial help.

A study conducted by Taylor[18] evaluated the use of a tutorial program for freshman engineering students. A sample of thirty-one students were selected and offered tutorial help in the areas of physics, mathematics, and English. A matched control group was used for comparison of academic achievement. Results show that tutored students did achieve significantly better than non-tutored students. The results also showed that the student who used the program more extensively was more likely to benefit from this tutorial help. The study indicated that even students with a college grade point average of less than 2.00 could make effective use of a tutorial program.

Schwerin[19] studied the effectiveness of small structured study groups in improving the academic achievement of one hundred college freshmen enrolled in a course

---

[18] Ronald G. Taylor, "Tutorial Services and Academic Success," The Journal of Educational Research, LXII (January, 1969), 195-7.

[19] Ursula C. Schwerin, "The Effect of Group Study on Individual Academic Achievement and Individual Study Orientation in Two-Year Higher Technical Education" (Doctoral dissertation, New York University, 1970), Dissertation Abstracts, 1970, XXXI, 1970, 3303-A.

in dental hygiene. Four sections were randomly chosen, two as experimental and two as control. Within the two experimental groups, ten study groups of five students each were formed. Each group was made up of one high, three average, and one low "prior performance level" student. Control groups received a traditional individual study approach while the experimental groups engaged in group study for all activities. Results showed that students who had participated in the study groups achieved significantly better on a test of academic achievement.

These two studies indicated that both tutorial help and small study groups can be effective elements of a learning strategy. Quite possibly these two methods can be used to increase the amount of time that the student is willing to spend in learning a given task and perhaps also can serve to reduce the total time required for a particular learning task.

Mastery Strategies

Strategies which include some of the elements suggested by Bloom have been studied in several different subject matter areas and at different levels of education. Analysis of some of these studies which have been completed can suggest the proper choice of subject area for such a strategy as well as proper choice of elements of the strategy to increase student expectation and to maximize the opportunities given for learning.

Collins[20] compared several alternate strategies for mastery learning. The investigation involved six classes of twenty-five eighth-graders each, in a modern mathematics course. Five of the sections were used as experimental, and the sixth group was used as a control. Brief descriptions of the types of mastery strategies used for the experimental groups are: (1) lists of objectives only, (2) lists of objectives, daily problem on the objectives of the previous session, and specific study prescriptions based on the results obtained from student attempts at the daily problems, (3) lists of objectives, the daily problems and prescriptions, plus alternate learning resources such as other text books, workbooks, games, and SRA instructional kits, (4) problems and prescriptions only, and (5) only the problems. Results were obtained by averaging the student scores from each group on unit tests. Results showed that 80 per cent of groups 2 and 3 attained the mastery level of A or B. Treatment 4, 1, and 5 achieved mastery at levels of 70 per cent, 60 per cent, and 50 per cent respectively. These results suggest the importance the use of the list of objectives, the problems, and the prescriptions. The use of alternate learning resources did not contribute to the strategies employing the above mentioned elements.

---

[20]Block, <u>Mastery Learning</u>, pp. 111-12.

Biehler[21] investigated the use of a mastery learning strategy in the teaching of undergraduate educational psychology. Students were given a choice of two sections, one with a traditional letter grading system and the other in which a mastery grading system was used. After each of the three unit tests, students of the mastery section who failed to achieve the level of A or B were given the opportunity to review and then take an alternate test over that unit. Term papers and other papers were used in addition to the unit tests in determining course grades. Although only anecdotal records were reported, the strategy seemed effective in both the cognitive and affective domains. When given the choice of the two instructional methods for the next course, over 90 per cent chose to learn under the mastery strategy.

Although few of the possible elements of a mastery learning strategy were employed in this study by Biehler, the results do indicate that students often appreciate additional opportunities for learning.

A study by Gentile[22] in 1970 at the State University of New York at Buffalo employed a mastery learning strategy in the teaching of introductory educational psychology. This experimental method employed student

---

[21] Robert F. Biehler, "A First Attempt at a 'Learning for Mastery' Approach," Educational Psychologist, VII, No. 3 (1970), 7-9.

[22] Block, Mastery Learning, pp. 117-18.

self-pacing over small instructional units. Students were given study questions over each unit, interviews with classmates, and unit mastery tests. Students who did not achieve mastery on a final unit mastery test were asked to review and return for retesting. Proctors and instructor were available for help in reviewing. Results were obtained by comparing the experimental group with a control group taught by a large lecture section and small discussion groups. Student evaluation of this strategy indicated that this method was rated as one of the best the students had ever used.

This strategy employed many of the elements recommended by Bloom, and had the advantage of almost unlimited opportunity for learning since students were self-paced. A major disadvantage would seem to be that individual students completed a different number of units during the standard time period for a course. This would seem to be an important disadvantage for a sequential course.

Another study which allowed student self-pacing was conducted by Keller[23] in a course in general psychology at Arizona State University. The fifteen week course was broken down into thirty instructional units. Each student was given assignments from a textbook or a program text, a list of "study questions," and the use of a supervised

---

[23] Fred S. Keller, "Good-Bye Teacher . . .," Journal of Applied Behavioral Analysis, I (Spring, 1968), 79-89.

study hall for completing each unit. A mastery test over each unit had to be successfully completed by each student. A student who failed to achieve mastery over each unit on his initial attempt was required to review and then take a similar test. Proctors were used for grading mastery tests and for providing specific suggestions for review. Proctors and graduate assistants were also available during the supervised study periods. A standard final examination was given to all students regardless of the number of units completed. Approximately 65 per cent of the students achieved course grades of A or B. Student reaction to this method was reported as quite favorable.

A study by Kim _et al._[24] investigated the effectiveness of a mastery learning strategy for a seventh grade mathematics class, in Seoul, Korea. This strategy employed the use of formative testing, remedial programmed instruction for students who scored below 80 per cent on each formative test, and review questions for each student of the experimental group. Results indicated that 74 per cent of the experimental, compared to 40 per cent of the control group, achieved mastery at the 80 per cent level as measured by an achievement test. This study also investigated the relationship between mastery and I.Q. Although the strategy was effective with both above-average

---

[24] Block, _Mastery Learning_, pp. 123-4.

and below-average students, the results show that this experimental method was most effective for the below-average I.Q. group.

This summary of related literature indicates that many different strategies have been attempted at many levels of education and in many subject areas. The results have in general indicated considerable success for strategies of mastery learning.

CHAPTER III

DESIGN OF THE STUDY

Selection of the Experimental Strategy

The review of the literature in Chapter II indicates that a variety of elements could be included in a strategy for mastery learning and also that such a strategy could be applied in many different levels of a subject matter area.

Since aptitude is one of the elements of the model of school learning which is internal to the student and therefore not under any control by the classroom instructor, a course level was chosen where the effects of this variable would be minimal. Another of the elements of this model is perseverance, the amount of time that the learner is actually willing to actively engage in the learning task. This element is to some extent under the control of the classroom instructor, since it is affected by the student's opinion of the need for mastery in the concepts presented. For this reason, the first course in calculus was chosen for this experiment. This course is a prerequisite for two subsequent courses in the calculus and also the two-semester course in advanced calculus, the first of which is required of all mathematics majors.

This selection of the fir‑‑ course in calculus for the area of experimentation then gave the researcher the opportunity to attempt to increase the perseverance of each student of the experimental group. The next step was to select activities to supplement classroom methods so that the element of the model called opportunity, could be maximized for students in the experimental group.

The use of lists of specific objectives for the experimental period was chosen as the first supplementary aid for members of the experimental group. These objectives for the eight-week period were written in the form of eight units, and each student was given a list of objectives for a unit whenever instruction on the prior unit was complete.

Formative testing, the use of short diagnostic-progress tests closely correlated with the objectives of each unit, was selected as another important feature for the learning strategy. Every member of the experimental group completed a formative test over that unit when instruction was complete. The grades from these formative tests were not used to determine course grades. After evaluation by the course instructor, each corrected formative test was returned to the student along with a personal diagnosis which indicated those objectives which had not been mastered and also listed specific suggestions for improvement. These suggestions included references to appropriate sections of the student's text, attendance at small student study groups, attendance at study groups directed by a graduate assistant, or the completion of specified problems related to the objectives yet to be mastered.

In evaluating each formative test, the course instructor used a particular score as an indication of mastery of that test at the level of A or B. This score was determined by the instructor from his experience with similar groups who had completed the work on these same concepts during prior semesters. Each formative test was scored as mastery or non-mastery. A recommendation was given to each student whose score was below the level of A or B to complete the review suggested by the diagnostic sheet and then to complete a retest over this same unit.

These elements which are described above make up the learning strategy which was employed for the experimental group. The class instruction for both groups was nearly identical, so that that variable of the model for learning called "quality of instruction" would be fixed in this study. An exception to the control of this variable was that prior knowledge of the objectives, which Carroll considers to be a necessary part of this variable, was not specifically provided for members of the control group. A daily lesson plan was prepared and used in both the experimental section and the control group. This plan was followed in both classes so that the development of concepts in both sections was as nearly identical as possible. The same examples were used in both sections for illustrating the concepts discussed. The text Calculus with Analytic Geometry, by Richard E. Johnson and Fred L. Kiokemeister[1] was used for both classes and each class was given the same daily assignment from this text. Since the formative test for each unit was given to the experimental group during one class of each week, this resulted in a difference of ten to thirty minutes in instruction time for the two groups for each unit. This corresponding unit of time was used in the control group for presentation of additional illustrative examples or for further discussion of assigned work from the text.

Preparation of Objectives

An important feature of the experimental strategy used in this research is the use of a list of specific course objectives to be given to each student of the experimental group. The process of preparing such a list of objectives is an important part in the planning of any course. When completed, this list of objectives represents a model of desired outcomes of instruction which can

---

[1]Richard E. Johnson and Fred L. Kiokemeister, Calculus with Analytic Geometry (4th ed.; Boston: Allyn and Bacon, 1969).

be valuable to both the instructor and the students.

The preparation of this list of objectives for the experimental period involved two major decisions. The first decision was the selection of appropriate objectives for this particular course and the second was to select the most appropriate method of stating these objectives.

The selection of objectives which were appropriate for the experimental period was accomplished by referring to the departmental outline for this particular course. This outline specified the concepts to be included during this time period.

Several methods of stating educational objectives have received recent attention in educational literature. As stated by Bloom, Hastings, and Madaus, "Much attention has been given to the statement of objectives in American education. There is probably no aspect of instruction about which more has been written."[2]

Many different approaches are used to define educational objectives. Some differences are due to purpose. National curriculum groups state broad objectives which could be labeled as goals while objectives for the classroom must be more specific. Even for the statement of short-range goals there are a variety of methods in use.

---

[2] Benjamin S. Bloom, J. Thomas Hastings, and George F. Madaus, Handbook on Formative and Summative Evaluation of Student Learning (New York: McGraw-Hill Book Company, 1971), p. 20.

Some statements emphasize the content of the objectives and some concentrate on the behavioral aspects. A general description of the desired method of stating objectives is as follows:

> How does one go about stating an educational objective? . . . It should be expressed in terms of desired student behavior and content area. More importantly, it must succeed in communicating the teachers' intent. Communicating is successful when any knowledgeable person in the same area can look at a student's behavior or products and decide whether or not the objective has been reached.[3]

In addition to a discussion of the use of behavioral objectives Bloom, Hastings, and Madaus state:

> Another way of giving clarity to the specifications of outcomes is to represent them in the form of the problems, questions, tasks, and the like which the student should be able to do or the kinds of reactions he should give to specific questions or situations.[4]

Since each student of the experimental section was to receive a copy of the list of objectives it was extremely important that these objectives be written in that form which would be of greatest value to the individual students. To accomplish this, each statement was written in the form of a task to be completed by each student. In this way, each student became actively involved with the list of specific objectives.

The complete list of these objectives is included in Appendix A of this report.

---

[3] _Ibid._, p. 32.

[4] _Ibid._, p. 15.

Preparation of Test

In an attempt to produce a comprehensive examination to assess the outcome of the eight-week experimental period several facets of mathematical achievement had to be considered. The examination was designed with a strategy to test objectives from all units and to select items which would evaluate a variety of levels of student behavior. Since the eight-week experimental period was the first one-half semester of work in this introductory course, approximately four weeks of this period dealt with topics which were essentially "precalculus" in nature. On this basis, and also because many students had completed some work in calculus in their secondary school course, the same instrument was used for both pre-test and the post-test.

The method used to select these items for the examination was first to prepare a list of items designed to assess mastery of the stated objectives. A panel of five professors who are familiar with the material of this course was then asked to evaluate each of these items. Each panel member rated each item as "good," "fair," or "poor," with respect to measuring concepts which were included in the list of stated objectives. A total score for each proposed test item was computed by a system of two points for a rating of "good," one point for a rating of "fair," and zero points for a rating of "poor." Only those items with a total score of eight or more points were used in the final form of the examination.

The purpose of this panel was to establish the content validity of the items for the achievement test. The use of these qualified professors insured that each test item reflected specified course objectives and that the student behavior required by each item was consistent with the behavior specified in the list of stated objectives.

To further insure that the type of student behavior demanded by the achievement test was consistent with the types of behavior specified by the list of objectives, it was necessary to analyze the objectives. As stated by Popham and Husek:

> The chief rule which guides the item writer for a criterion-referenced test is to be sure that the item is an accurate reflection of the criterion behavior. Difficult or easy, discriminating or indiscriminate, the important thing is to make the item represent the class of behaviors delimited by the criterion.[5]

Begle and Wilson[6] discuss a model for analyzing levels of behavior which are both hierarchical and ordered. The model considers order in that four categories are developed, each successive level being at a more cognitively complex

---

[5]W. James Popham and T. R. Husek, "Implications of Criterion-Referenced Measurement," Journal of Educational Measurement, VI (Spring, 1969), 4.

[6]Edward G. Begle and James W. Wilson, "Evaluation of Mathematics Programs," Mathematics Education, Sixty-Ninth Yearbook of the National Society for the Study of Education, Part I (Chicago: University of Chicago Press, 1970), pp. 371-5.

level than the former, and hierarchical in that behavior at a given level may require competence at all lower levels for mastery. This particular model was used by the National Longitudinal Studies of Mathematics Achievement[7] and is a modification of the cognitive-levels dimension developed by Bloom.[8] This modification provides a model with categories which are particularly appropriate to mathematics achievement.

The four levels of behavior which are specified by this model are described as follows:

1. Computation--Items designed to require straight-forward manipulation of problem elements according to rules the students presumably have learned. Emphasis is upon performing operations and not upon deciding which operations are appropriate.
2. Comprehension--Items designed to require either recall of concepts and generalizations, or transformation of problem elements from one mode to another. Emphasis is upon demonstrating understanding of concepts and their

---

[7]T. A. Romberg and James W. Wilson, "The Development of Tests for the National Longitudinal Study of Mathematical Abilities," The Mathematics Teacher, LXI (May, 1968), 489-95.

[8]Benjamin S. Bloom, ed., Taxonomy of Educational Objectives: Classification of Educational Goals, Cognitive Domain, Handbook I (New York: McKay, 1956).

relationships, and not upon using concepts to produce a solution.

3. Application--Items designed to require (a) recall of relevant knowledge, (b) selection of appropriate operations, and (c) performance of the operations. Items are of a routine nature. They require the student to use concepts in a specific context and in a way he has presumably practiced.

4. Analysis--Items designed to require a non-routine application of concepts. The items may require the detection of relationships, the finding of patterns, and the organization and use of concepts and operations in a non-practiced context.

An analysis of the eight units of objectives specified for the experimental period of this research is presented in Table 1. Many items in this list of objectives were written with several parts, for the convenience of the students. For the purpose of this analysis, each part of each item was treated as an individual objective.

TABLE 1

ANALYSIS OF OBJECTIVES BY LEVELS OF BEHAVIOR

Unit	I	II	III	IV	V	VI	VII	VIII	Total
Computation	0	8	9	9	3	0	4	0	33
Comprehension	10	2	13	2	8	4	4	1	44
Application	4	9	19	6	4	4	3	8	57
Analysis	8	12	0	1	16	14	4	2	57

The assignment of particular items from the list of objectives to correct levels of behavior is easily accomplished in some instances but more difficult in others. Those items requiring the student to write a proof for some generalization can usually be assigned to the highest level, namely analysis. The second category, comprehension, is appropriate for those items which require definitions, descriptions or statements of theorems. The distinction between computation and application is less evident. For the purpose of the categorization made here, items for which a theorem is available which definitely prescribes the appropriate computation are considered as level one, computation. However, quite similar items which require the combination of several theorems and the selection of appropriate methods are classified in level three, application. As an example, to write the equation of a line through two given points is considered as computational, since one theorem provides the method of solution.

A somewhat similar problem is to write the equation of the perpendicular bisector of a segment. This requires the student to compute the midpoint of the segment and the slope of the segment, and then to apply another theorem to determine the slope of the perpendicular bisector. Finally, with the slope and midpoint found, the student is expected to write the required equation. Since the behavior required for completion of this objective demands that the student select appropriate operations and perform several steps in reaching the desired conclusion, this level of behavior would be classified as application.

After all items from the list of objectives have been analyzed as shown in Table 1, items for the achievement test were selected from the list of items judged appropriate by the panel of professors. Table 2 lists the twenty test items which were selected, by referring each test item to the appropriate units and level of behavior.

TABLE 2

ANALYSIS OF TEST ITEMS BY LEVEL OF BEHAVIOR

Units	Computation	Comprehension	Application	Analysis
I		1,9		10
II	4		3	
III			2,5	
IV	7,8	1*	6	
V		2*		3*,10*
VI			4*	5*,6*
VII	7*			8*
VIII			9*	
Total	4	4	6	6

Note: The items appear as numbered in the two-part achievement test which is included in Appendix B. The items from the second part of this examination are identified with the item number and the asterisk. Thus the first item of the first part of the examination is denoted by "1" while the first item from the second part is denoted by "1*".

The analysis of the levels of behavior required by items from the list of objectives and the levels required by the questions on the achievement test indicates a reasonable amount of correspondence. The percentage of items at each level on the list of objectives is approximately the same as in the achievement test. The number of items selected for the test from each unit does not seem appropriate in all cases in terms of number. For example, Unit VII and VIII required only four class sessions, while

Units IV and V required six sessions each. Table 3 shows the correspondence in percentages of the levels of behavior as specified by both the list of objectives and by the items of the achievement test.

TABLE 3

COMPARISON OF OBJECTIVES AND TEST--
IN LEVELS OF BEHAVIOR

Levels	Percentage in Objectives	Percentage on Achievement Test
Computation	17.3	20
Comprehension	23.0	20
Application	29.8	30
Analysis	29.8	30

The use of the panel of experts in evaluating proposed test items and the analysis given above provide some evidence of validity for the prepared achievement test. As noted by Popham and Husek:

> Criterion - referenced measures are validated primarily in terms of the adequacy with which they represent the criterion. Therefore, content validity approaches are more suited to such test. A carefully made judgment, based on the test's apparent relevance to the behaviors legitimately inferable from those delimited by the criterion, is the general procedure for validating criterion - referenced measures.[9]

Another question to be considered is the reliability of the test instrument. The method used to determine

[9] Popham and Husek, "Criterion-Referenced Measurement," p. 6.

the reliability for this test was a generalization of the Kuder-Richardson formula 20.[10] The generalization was necessary since the possible scores on each test item ranged from 0 to 8, while formula 20 is suitable only for a test of dichotomously scored items. The actual calculation of this reliability coefficient was accomplished by using a library program: "Testat," which is available at the University Computer Center of Indiana University of Pennsylvania. The computed alpha coefficient of reliability was .84. Garrett states that "In order to differentiate between means of the two school grades of relatively narrow range, a reliability coefficient need be no higher than .50 or .60."[11] Thus, reliability of the test instrument was shown to be in this acceptable range.

The complete two-part achievement test is included in Appendix B.

The Sample

Due to the type of scheduling used at Indiana University of Pennsylvania, it was not possible to select students by methods of random sampling. Students are permitted to select the particular section of each class which they schedule, with the only restriction being the availability of space in the desired section at the time of

---

[10] Donald J. Veldman, Statistics in Psychology and Education (New York: Holt, Rinehart and Winston, Inc., 1967), pp. 170-81.

[11] Henry E. Garrett, Statistics in Psychology and Education (New York: Longmans, Green and Company, 1958), p. 351.

scheduling. The scheduling time is determined by college class and also by an alphabetical system. With this method, the student is usually free to make a choice of class hour and also of instructor.

As described previously, the experimental method employed required that both the experimental group and the control group receive the same classroom instruction. For this reason, the sample used for this study consisted of the two sections of Mathematics 157, Analytic Geometry and Calculus I, which were designated, prior to scheduling, to be taught by this researcher, during the spring semester of 1971.

The students of the two sections used in this experiment were nearly all second semester freshmen, and all were mathematics majors. The two sections originally consisted of twenty-three and twenty-nine students. By a toss of a coin, the class of twenty-three students was selected as the control group. Before the research was completed, two students of the control group withdrew. Thus, the control group consisted of twenty-one students and the experimental group was the other section of twenty-nine students.

Since this study was limited to this small group of students from one university, a description of the university and the student population is included so that the reader is better able to make comparisons with other situations.

Indiana University of Pennsylvania was originally founded as a state normal school in 1857, at Indiana, Pennsylvania. The role of the institution was changed in subsequent years as is reflected in the change of the name to State Teachers College, State College, and finally to University in 1965.

In the academic year 1970-71, the undergraduate enrollment was 9214 and the graduate school population was 1133. Of the 1793 freshmen for this year, 650 were enrolled in the School of Arts and Sciences and 598 in the School of Education. The remaining freshmen students were enrolled in the Schools of Business, Fine Arts, Home Economics and Health Services. Some indication of the academic ability of the students at this university is given by College Board scores. This freshman class of 1970-1971 had an average college board verbal of 543 and an average mathematics score of 579, so the average total college board score was 1122.

Survey of Student Opinion of the Learning Strategy

An important feature of the experimental strategy used in this study was to increase the perseverance of the individual members of the experimental group. This could be accomplished both in the way that the experimental strategy was originally introduced to this group and in the way the methods of the strategy were utilized throughout the course. The actual perseverance of the individual students was not assessed directly. One important factor which reflects a student's perseverance is his own evaluation of the methods which have been used. For this reason, it was very important to have some measure of student reaction to the experimental strategy.

Hartman[12] developed an instrument to reflect student attitude toward several experimental teaching techniques to be used in a course in the calculus. This instrument was used to gather opinion concerning this learning strategy, with the permission of Dr. Hartman. This form is shown in Appendix C.

A second form was also used, which was designed by the researcher to specifically reflect student opinion of the elements of the learning strategy. This form is shown in Appendix D.

The specific purpose of this second form was to point out the five major elements to the student for his evaluation. Question one requests that these elements be rated as to which was the most beneficial. The student is then asked to consider which element of the strategy could best be eliminated and also to decide whether he personally would want this change. Finally, the student was requested

---

[12] Marlin E. Hartman, "A Study of Self-Supervision in the Implementation of Innovations in Teaching Strategy in Beginning Calculus" (unpublished Ed.D. dissertation, University of Pittsburg, 1969), p. 107.

to make suggestions for improving the strategy.

The use of these two forms gave both a general student opinion of the over-all experimental strategy as well as an evaluation of the major elements of this strategy.

Method of Analysis

The comparison of the achievement of the two groups was based on the data obtained from the pre-test and the post-test. Analysis of covariance was the statistical method used with the pre-test scores as covariant. This method of analysis, referred to as the "Nonequivalent Control Group Design," is recommended by Campbell and Stanley when the control group and the experimental group do not have pre-experimental sampling equivalence.[13]

Summaries of maintained logs were reported to indicate the amount of time used for special sessions for correction of deficiencies determined by formative testing of the experimental group. These records also indicate the student attendance at these sessions.

An additional log was used to summarize the time used by members of both groups in obtaining individual assistance from the course instructor, outside of scheduled class time.

Finally, the results of the student evaluation forms were used to summarize student reaction to the experimental strategy.

---

[13] Campbell and Stanley, Experimental Design, p. 47.

CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

The Achievement Test

Pre-test scores were collected from all members of both groups during the initial class sessions. Achievement test scores were obtained from both groups at the end of four weeks and again at the end of the eight-week experimental period. The first part of this achievement test dealt with concepts from the first four units of the objectives and the second part was concerned with the remaining four units. A post-test score for the entire period was obtained for each student by combining the two scores from the two parts of the achievement test.

TABLE 4

MEANS AND STANDARD DEVIATIONS FOR SCORES OF BOTH GROUPS ON PRE-TEST AND POST-TEST

	Means		Standard Deviations	
	Control	Experimental	Control	Experimental
Pre-Test	26.86	24.00	10.92	12.11
Post-Test	99.81	126.28	25.00	24.17

The two-part achievement test is included in

Appendix B of this report. The two parts were each graded with a possible score of 80 points, for a total score of 160 points. Table 4 indicates that the mean score of the control group was higher on the pre-test than the experimental group but that the mean score of the control group on the post-test was lower than that of the experimental group.

The data from the pre-test and post-test was analyzed by the method of analysis of covariance, using the pre-test scores as covariant. The computation of this analysis was accomplished by using a library program available at the University Computer Center of Indiana University of Pennsylvania. This particular library program was designed from the reference Experimental Design in Psychological Research, by Edwards.[1] The results of this analysis are shown in Tables 5-7. Table 5 indicates a summary of an analysis of variance of the pre-test scores; Table 6 is a summary of an analysis of variance of the post-test scores; and Table 7 is a summary of the final analysis of covariance.

An assumption which is required for this analysis of covariance is that the slopes of the regression lines for the two groups not be significantly different. This hypothesis was tested for the data used in this analysis.[2]

---

[1] Allen L. Edwards, Experimental Design in Psychological Research (New York: Holt, Rinehart and Winston, Inc., 1966), pp. 281-294.

[2] Ibid., pp. 290-2.

TABLE 5

ANALYSIS OF VARIANCE--PRE-TEST

Source	df	Sum of Squares	Mean Square	F
Between	1	99.426	99.426	0.735
Within	48	6490.574	135.220	
Total	49	6590.000	134.490	

TABLE 6

ANALYSIS OF VARIANCE--POST-TEST

Source	df	Sum of Squares	Mean Square	F
Between	1	8531.750	8531.750	14.175
Within	48	28891.063	601.897	
Total	49	37422.813	763.731	

TABLE 7

ANALYSIS OF COVARIANCE

Source	df	Sum of Squares	Mean Square	F
Between	1	10071.613	10071.613	19.817
Within	47	23886.332	508.220	
Total	48	33957.945	707.457	

This calculation yields an F-ratio of .307 with 1 and 46 degrees of freedom, a nonsignificant value. Thus this requirement for an analysis of covariance is satisfied.

Table 7 indicates that the F ratio computed from this analysis is 19.817. The critical value of the F ratio at the .01 level of significance with 1 and 47 degrees of freedom is 7.20. It is therefore concluded that the use of the experimental strategy did affect achievement during the experimental period.

To emphasize the difference in the results obtained by the achievement test given at the end of the experimental period one can consider the somewhat arbitrary method often used to assign grades according to the percentage of total points received. Using the 10 per cent grade ranges of 90% for grade A, 80% for grade B, 70% for grade C, 60% for grade D, and below 60% for grade F, the results shown in Table 8 are obtained.

TABLE 8

GRADE DISTRIBUTION FOR THE ACHIEVEMENT TEST[3]

Range	Grade	Control	Experimental
160-144	A	0	7
143-128	B	4	10
127-112	C	4	7
111-96	D	6	1
Below 96	F	7	4

Clearly the grade distribution shown in Table 8 is not realistic as seen by the results of the control group which appeared to be a typical group for this course. However, the purpose of the table is to show the way in

[3]An analysis by chi square indicates that these two distributions are independent at the .05 level of significance.

which the distribution of the experimental group has been skewed by the application of the experimental strategy.

It is of interest to note that of the five members of the experimental group who scored below 112 on the achievement test, three were students who did not attempt to take advantage of the strategy. These students completed the formative tests which were given during class time but did not attend the special sessions for correction of deficiencies nor did they consult the instructor for individual help.

The Special Sessions

A summary of the special sessions offered by the graduate assistant, for correction of deficiencies indicated by the formative tests, is shown in Table 9. Each of these sessions lasted a maximum of fifty minutes. The graduate assistant was usually given two or three major points to discuss, as indicated by the results of the formative tests. Students were free to attend or not attend as they wished, but the diagnostic sheet was used to recommend attendance by those who needed further help in these areas. Quite often, students were in the classroom for the entire fifty minutes but used most of this time for working quietly in small groups. Those who needed further help in the concepts being discussed by the graduate assistant gave their attention to this discussion while the remainder of the group worked on their own problems, either individually or with classmates.

TABLE 9

SUMMARY OF SPECIAL SESSIONS FOR MEMBERS OF THE EXPERIMENTAL GROUP

Number of Special Sessions Held	12
Average number of students per session	7
Number of the 29 members of the experimental group who attended one or more special sessions	26
Total hours used by graduate assistant in special sessions	10

As shown in Table 9, the total time used by the graduate assistant in directing the special sessions for members of the experimental group was ten hours. The average number of students in those sessions was seven, but the range was from three to fifteen.

As noted above, the graduate assistant was always present at these sessions to offer help to small groups or individuals, but a great deal of work was also accomplished by small groups cooperating on common problems. A classroom was also reserved for one hour each week where students could work together on difficult concepts without any direction from the course instructor or the graduate assistant. No records were maintained to indicate attendance at these sessions, but informal observation indicated that attendance was often from four to seven and was over twenty on at least one occasion.

The groups of students who worked together during the special sessions also arranged to meet at other times as well. It is felt by the researcher that study habits of this type are an indication that the experimental strategy did have some effect in increasing the perseverance of some members of the experimental group.

Individual Help from the Instructor

Table 10 is a summary of the help given to individual students of the experimental group by the class instructor. No corresponding table is given for the control group since only three members of this group sought this type of individual help. The total time used by the instructor in giving individual help to these three members of the control group was fifty minutes. It was mentioned repeatedly in class sessions for the control group that this type of individual help was available. Unfortunately, at least at the institution where this research was accomplished, it is not unusual for students to ignore this source of individual assistance.

TABLE 10

SUMMARY OF INDIVIDUAL ASSISTANCE GIVEN TO MEMBERS OF THE EXPERIMENTAL GROUP BY THE CLASS INSTRUCTOR

Total number of student visits	45	
Total amount of instructor time used	10	hours
Total number of the 29 members who sought this type of individual help	18	
Average amount of time used for one student session	13.5	minutes
Average total time used by those students seeking this type of individual help	34	minutes

The summary in Table 10 indicates that each individual session held with members of the experimental group by the instructor averaged 13.5 minutes. For those 18 members of the experimental group of 29 students who sought this type of help the average total time used during the eight-week period was 34 minutes. The total time used by the instructor for this purpose was ten hours. This institution, like many others, requires that each professor have five office hours scheduled each week for the purpose of assisting individual students. Thus, there were a minimum of forty hours available for this purpose during the experimental period. The combination of the experimental and control group made up two-thirds of the instructor's teaching load.

It is felt by the researcher that students received a great deal of help during those individual sessions. Not only were course concepts discussed for the benefit of the student's understanding but also the instructor was given the opportunity to guide the student in his methods of study. In this way, the student's opportunities for learning were increased, which is an element of the model of student learning as described in Chapter II.

Student Evaluation of the Learning Strategy

The data from the two forms used for a student evaluation of the experimental strategy yielded information about how the student viewed this model. The first form was used to determine the attitude of the student to the experimental method as compared to other methods to which he had been exposed. The entire form used for this purpose is shown in Appendix C. A summary of student responses to the items on this instrument is shown in Table 11. Under the column entitled student responses, two rows of data are given for each of the fifteen items. The first row indicates the number of students who chose that response and the second row of data indicates the corresponding percentage of students who chose that response.

TABLE 11

ATTITUDE TOWARD THE EXPERIMENTAL STRATEGY

Responses SD	D	U	A	SA	Statements
11	16	1	0	0	1. This method of instruction is very frustrating to me as a student.
39	57	4	0	0	
0	1	3	16	8	2. I would recommend this method to mathematics teachers for use in their classes.
0	4	11	57	29	
0	0	0	13	15	3. I would not object if this method of instruction were used in this class for the entire semester.
0	0	0	46	54	
12	16	0	0	0	4. This method of instruction is really not very suitable for work at this grade level.
43	57	0	0	0	
10	15	3	0	0	5. This method did not arouse my interest.
36	54	11	0	0	
0	0	1	20	7	6. I would recommend this method of instruction for use in classes at other colleges and universities.
0	0	4	71	25	
11	17	0	0	0	7. I do not care for this method and I doubt if it appeals to other students.
39	61	0	0	0	
0	14	4	8	2	8. This method of instruction was extremely time-consuming.
0	50	14	29	7	
6	20	2	0	0	9. It is difficult to give a favorable reaction to this method of instruction.
21	71	7	0	0	
0	3	1	16	8	10. This is one of the better methods of instruction to which I have been exposed.
0	11	4	57	29	
0	2	8	16	2	11. This method proved to be quite stimulating and thought provoking
0	7	29	57	7	
0	1	5	16	6	12. I liked this method of instruction very much.
0	4	18	57	21	
9	13	5	1	0	13. I would never use this method of instruction if I were teaching.
32	46	18	4	0	

TABLE 11 (Continued)

Responses SD	D	U	A	SA	Statements
0	1	4	16	7	14. This method of instruction should be
0	4	14	57	25	used in more mathematics classes.
18	10	0	0	0	15. I thought this method was a complete
64	36	0	0	0	waste of time.

The second form used to evaluate student reaction shown in Appendix D, was designed to evaluate the specific elements of the experimental strategy. The first item on this evaluation was a request for each student to evaluate five of the major elements of the strategy. Each element was to be ranked from one to five with one indicating that element which had been of most value. The ratings received were quite varied but a summary was achieved by adding the ranks assigned to each element. This summary is shown in Table 12.

TABLE 12

SUMMARY OF THE RANKING BY STUDENTS OF FIVE ELEMENTS OF THE EXPERIMENTAL STRATEGY

Rank	Element
1	The progress-quiz (formative test) for each unit.
2	The list of objectives for each unit.
3	The help sessions which have been available
4	The diagnostic sheet returned with each quiz.
5	The second quiz for each unit.

The second item was a question of which of the five elements of the strategy could be omitted with the least effect on student mastery. Even with these instructions eight of the 28 respondents listed "none." The summary of the remaining evaluations yielded results which were consistent with those given for question 1.

The third item questioned whether that element which had been chosen as the one whose elimination would have the least effect on mastery should actually be dropped from the strategy. Every student agreed that no item should be eliminated. This was not a surprising response since each student was encouraged to complete all steps in the learning strategy, but each student was also fully aware that the decision to complete steps past the first formative test was strictly voluntary. There were several suggestions that the second formative test over each unit could be eliminated if this was replaced by a conference with the course instructor.

The final item on the evaluation of the learning strategy asked that each student recommend additional methods which would improve the strategy. Slightly more than half of the students listed at least one suggestion but many pertained to the pace of the course rather than to the learning strategy. Those suggestions which did pertain to the strategy were: 1) Use class time to discuss the items of the list of course objectives, 2) Give formative tests at more frequent intervals, and 3) Make the second formative test over each unit an oral test given by the instructor.

A summary of the student evaluation of the experimental method would indicate that with very few exceptions, the students found this method beneficial. In particular, item 6 of the summary shown in Table 11 indicates that 96 per cent of the students in the experimental group would recommend this method for use in classes at other colleges and universities.

CHAPTER V

SUMMARY, IMPLICATIONS, AND RECOMMENDATIONS

Summary

This study was conducted at Indiana University of Pennsylvania, and involved two classes of mathematics majors enrolled in the course Mathematics 157, Analytic Geometry and Calculus 1. The experimental period in which this study was conducted was the first eight weeks of the spring semester, 1971. The study was designed to provide answers to the following four questions:

1. Was the achievement of the experimental group, in which the learning strategy was used, significantly better than that of the control group, as measured by test results from the groups?
2. Did the individual members of the experimental group take advantage of the additional learning opportunities provided?
3. Did the students of the experimental group seek more individual help from the course instructor than the students of the control group?
4. Did the students of the experimental group

evaluate this learning strategy as a desirable feature to be used in this course or in other courses?

A list of specific objectives for the eight-week period was prepared from the departmental syllabus for this course. Members of the experimental group received copies of these objectives, broken down into eight separate units. Formative testing was used to indicate the progress of individuals in the experimental group in mastering stated objectives. The classroom instruction for the experimental and control group was nearly identical, with the exception of the formative tests, and special sessions directed by the graduate assistant to correct deficiencies indicated by the formative tests.

An achievement test was prepared from the list of specified course objectives and was used to compare the achievement of the two groups at the end of the experimental period.

Findings

Analysis of the results of the achievement test indicate that members of the experimental group achieved significantly better scores than the members of the control group.

Not every member of the experimental group participated fully in all of those activities designed for the group to encourage mastery of all concepts. Of the 29 members of this group, three students did not take advantage of the results of the formative tests and also did not attend sessions with the graduate assistant or seek individual help from the course instructor. Although no cause-effect relation can be inferred, these three students received the lowest three scores on the achievement test. In general, the results indicate that nearly all students did participate in the experimental strategy.

The members of the experimental group did seek much more help from the classroom instructor outside of scheduled class sessions. The graduate assistant was also available at specified hours to aid individual members of the experimental group, but no student took advantage of this source of assistance. The fact that students did seek individual tutorial help seems to be in conflict with the results obtained by Bloom. He states: "We have offered tutorial help as students desired it but so far students at the secondary or higher education level do not seek this type of help frequently."[1]

Students of the experimental group indicated a strong appreciation of the experimental method. At the end of the eight-week experimental period the students were surveyed for suggested changes in method. Every member of this experimental group reacted positively to the methods which were used, and each member requested that the

---

[1] Bloom, "Learning for Mastery," p. 10.

experimental strategy be extended to the remainder of the semester.

Although no records were maintained, it was clear to the instructor of these two sections that class attendance by members of the experimental group was much higher than by members of the control group. Class attendance is not required at the institution where this study was conducted.

Students at Indiana University of Pennsylvania have until approximately the middle of the semester to withdraw from any course without penalty. During this study, this date for official withdrawal occurred during the seventh week of the study. By that date two members of the control group had withdrawn. A check of the other six sections of this same course taught during this same semester indicated that from one to three students had withdrawn from each. No member of the experimental group even discussed this possibility with the classroom instructor.

Implications

The general implication from this study is that experimental strategies based on the model for school learning discussed in Chapter II do seem to have a place in instructional methods.

Although no specific evidence is available, the researcher found the general attitude of the experimental group to be one which is quite desirable in any classroom. The students realized that the instructional method was designed not just to present relevant material, but rather to give every student the opportunity to master the stated objectives. The experimental strategy seemed to foster a cooperative spirit among members of this group. Students were quite eager to work with groups of fellow students in mastering the course objectives.

The fact that students of the experimental group were much more concerned with seeking individual help from the classroom instructor also shows that these students were eager to take advantage of additional opportunities for learning. Since this source of help is nearly always available, but seldom used, this evidence alone suggests a distinct advantage of the experimental strategy.

An important feature of the particular learning strategy employed in this study is that little or no change is required in the classroom methods used by the individual instructor. The elements of the strategy are designed to function as supplements to regular classroom activities. For this reason, the strategy could be adopted without requirements of retraining of instructors.

The use of formative testing provides information which was not utilized in this study. This type of progress-testing yields valuable feedback information to both the teacher and the student. Since one important method of control in this investigation was the use of the same daily teaching methods in the classes for both groups, little or no advantage could be achieved from this information provided for the instructor. In subsequent use of this strategy, this information can be used to suggest classroom activities which would be appropriate for a large portion of the class.

The experimental strategy used in this study could readily be adapted for use with multiple sections of the same course. Providing appropriate learning opportunities for students of several sections could be accomplished in a more efficient manner, both from the view of the student and the instructor, than could be accomplished with the single experimental group. Increasing the number of instructors involved in this strategy should also provide additional suggestions for methods of offering even more learning opportunities to meet the needs of individual students.

Since the subjects of this study were at best representative of mathematics majors at Indiana University of Pennsylvania then generalization of these results to other subject areas and to other institutions should be considered only with some caution. However, the over-all purpose of this study was not to show that this particular strategy can be mechanically applied to other situations to produce a particular result. Rather, the results of this study would suggest the possibility of other but perhaps similar strategies which should be considered to help individual students accomplish specified learning tasks.

Recommendations for Further Study

One recommendation for further study must be that studies similar to this one be conducted in institutions which differ from this one in purpose and in student population. The review of the literature cited in this study suggests that investigations of the use of this strategy should not be restricted to a single subject area nor to higher education. A related recommendation is that a larger study of this type be conducted which would include institutions of several types so that a more comprehensive view of this method could be obtained.

Immediate attention is needed to determine the effect of this type of learning strategy on student achievement in subsequent courses. An investigation could indicate whether or not the increase in both expectation and perseverance developed by this strategy is carried over into future courses. A similar recommendation would be to investigate the possible effect that this strategy could have on other courses in which the student is enrolled during the same semester.

Finally, there is a great deal of research needed into the affective results of a strategy of mastery learning as well as possible changes in the self-concept of students involved in such a program.

BIBLIOGRAPHY

Begle, Edward G. and Wilson, James W. "Evaluation of Mathematics Programs." Mathematics Education. Sixty-Ninth Yearbook of the National Society for the Study of Education, Part 1. Chicago: University of Chicago Press, 1970.

Biehler, Robert F. "A First Attempt at a 'Learning for Mastery' Approach." Educational Psychologist, VII, No. 3, 1970.

Block, James H. Mastery Learning: Theory and Practice. New York: Holt, Rinehart and Winston, Inc., 1971.

Bloom, Benjamin S., ed. Taxonomy of Educational Objectives: Classification of Educational Goals, Cognitive Domain. Handbook I. New York: McKay, 1956.

Bloom, Benjamin S. "Learning for Mastery." UCLA CSEIP Evaluation Comment. Los Angeles: Center for the Study of Evaluation of Instructional Programs, I, No. 2, 1968.

Bloom, Benjamin S.; Hastings, J. Thomas; and Madaus, George F. Handbook on Formative and Summative Evaluation of Student Learning. New York: McGraw-Hill Book Company, 1971.

Boardman, Dorris E. "The Effect of Students' Advanced Knowledge of Behavioral Objectives on Their Achievement in Remedial Chemistry." Doctoral dissertation, University of California, Los Angeles, 1970, Dissertation Abstracts, 1970, XXXI, 3286-A.

Brookover, Wilbur B.; Shailer, Thomas; and Patterson, Ann. "Self-Concept of Ability and School Achievement." Sociology of Education, XXXVII (Spring, 1964), 271-9.

Campbell, Donald T. and Stanley, Julian C. Experimental and Quasi-Experimental Designs for Research. Chicago: Rand McNally & Company, 1963.

Carroll, John B. "A Model of School Learning." Teachers College Record, LXIV (May, 1963), 723-33.

Doty, Charles R. "The Effect of Practice and Prior Knowledge of Educational Objectives on Performance." Doctoral dissertation, The Ohio State University, 1968, Dissertation Abstracts, 1968, XXIX, 3035-A.

Edward, Allen L. Experimental Design in Psychological Research. New York: Holt, Rinehart and Winston, Inc., 1966.

Feather, N. T. "Effects on Prior Success and Failure on Expectation of Success and Subsequent Performance." Journal of Personality and Social Psychology, III (March, 1966), 287-98.

Garrett, Henry E. Statistics in Psychology and Education. New York: Longmans, Green and Company, 1958.

Hartman, Marlin E. "A Study of Self-Supervision in the Implementation of Innovations in Teaching Strategy in Beginning Calculus." Unpublished Ed.D. dissertation, University of Pittsburgh, 1969.

Johnson, Richard E., and Kiokemeister, Fred L. Calculus with Analytic Geometry. Boston: Allyn and Bacon, 1969.

Keller, Fred S. "Good-Bye Teacher. . . ." Journal of Applied Behavioral Analysis, I (Spring, 1968), 79-89.

Maxwell, Merle L. "A Study of the Effects of Teacher Expectation on the IQ and Academic Performance of Children." Doctoral dissertation, Case Western Reserve University, 1970, Dissertation Abstracts, 1970, XXXI, 3345-A.

Popham, W. James and Husek, T. R. "Implications of Criterion-Referenced Measurement." Journal of Educational Measurement, VI, No. 1 (Spring, 1969), 1-9.

Romberg, T. A. and Wilson, James W. "The Development of Tests for the National Longitudinal Study of Mathematical Abilities." The Mathematics Teacher, LXI (May, 1968), 489-95.

Rosenthal, Robert and Jacobson, Lenore. "Teachers' Expectancies: Determinants of Pupils' IQ Gains." Psychological Reports, XIX (August, 1966), 115-8.

Schwerin, Ursula C. "The Effect of Group Study on Individual Academic Achievement and Individual Study Orientation in Two-Year Higher Technical Education." Doctoral dissertation, New York University, 1970, Dissertation Abstracts, 1970, XXXI, 3303-A.

Taylor, Ronald G. "Tutorial Services and Academic Success." The Journal of Educational Research, LXII (January, 1969), 195-7.

Veldman, Donald J. Statistics in Psychology and Education. New York: Holt, Rinehart and Winston, Inc., 1967.

APPENDICES

APPENDIX A

Objectives for Mathematics 157

Weeks 1-8

The following is a list of specific objectives to be accomplished in the first eight weeks of the course Math 157. These objectives are separated into eight units and are generally of three types: definitions, theorems, and problems.

Every student is expected to be able to state each definition precisely and completely. A definition must be a statement which indicates both necessary and sufficient conditions.

All theorems which are encountered are of value in both theoretical work as well as in the solution of more practical problems. The goal for most of these theorems includes a complete and rigorous proof as well as general understanding for purposes of applications. In many instances a geometric interpretation of the theorems will aid in understanding.

The third category of objectives are those which involve application of the definitions and theorems. The specified problems are intended to be representative of possible problems from that unit. Clearly, only a sampling can be included. Emphasis must be on general understanding of problem types rather than on specific answers.

Unit I

1. Give a description of the sets of numbers called: naturals, integers, rationals, real and complex.

2. Define: field, ordered field.

3. State those properties of a field which are properties of the number system with each of the sets of numbers in #1.

4. Define: Interval (open, closed, half-open, and infinite).

5. Define the absolute value function.

6. Define: Neighborhood (including symmetric neighborhood and deleted neighborhood).

7. Define: Intersection of sets, union of sets, subtraction of sets, and complement of sets.

8. Prove the following theorems on order where a,b,c,d, are real numbers.

   i) If $a > b$ and $b > c$ then $a > c$.

   ii) If $a > b$ then $a + c > b + c$.

   iii) If $a > b$ and $c > d$ then $a + c > b + d$.

   iv) If $a > b$ and $c > 0$ then $ac > bc$.

   v) If $a > b$ and $c < 0$ then $ac < bc$.

   vi) If $0 < a < b$ and $0 < c < d$ then $ac < bd$.

   vii) If $0 < a < b$ then $1/a > 1/b$.

   viii) If $0 < a < b$ and $n \in N$ then $a^n < b^n$.

9. Find the solution set for each of the following:

   i) $2x + 3 > 4$ and $3x < 8$

   ii) $|x - 3| = 9$

   iii) $2x < 7$ or $2x > 9 - x$

   iv) $|x - 1| > 0$

Unit II

1. Prove the following theorems for all real numbers a and b:

   a) $|a| = 0$ if and only if $a = 0$.

   b) $|a| = |-a|$.

   c) $\sqrt{a^2} = |a|$.

   d) $|ab| = |a||b|$

   e) $||a| - |b|| \leq |a \pm b| \leq |a| + |b|$

2. Prove: If $c > 0$ then $|x| < c$ if and only if $-c < x < c$.
   Also, If $c > 0$ then $|x| \leq c$ if and only if $-c \leq x \leq c$.

3. Develop the formula for the distance between two points in R x R. Use the formula for several specific examples.

4. State the geometric definition of a circle. Develop the equation for a circle with center at (h,k) and with radius r.

5. State the definition of the slope of a line. Develop the formula for finding the slope of a line on the points (x_1,y_1), (x_2,y_2) where $x_1 \neq x_2$.

6. Let line L_1 have equation $y = m_1x + b_1$ and line L_2 have equation $y = m_2x + b_2$. Prove that:

   a) $L_1 \parallel L_2$ if and only if $m_1 = m_2$.

   b) $L_1 \perp L_2$ if and only if $m_1m_2 = -1$.

7. Let line L be determined by points P = (1,-3) and Q = (-2,4).

   a) Find the slope of line L.

   b) Find the length of the segment $\overline{PQ}$.

   c) Find the midpoint of the segment $\overline{PQ}$.

   d) Find the equation of the perpendicular bisector of $\overline{PQ}$.

8. Find the solution set of each of the following:

a) $|2x - 7| \leq 4$

b) $|3 - 4x| \leq 7$

c) $|5 - 2x| > 9$

d) $x^2 - 3x - 4 > 0$

e) $\sqrt{3x - 7} > 2$

f) $|x - 2| > 3 + |x - 1|$

9. Sketch the graph of each of the following in R x R.

a) $3x - 4 > 8$

b) $|x + 2| < 4$

c) $y \leq 3x - 4$

d) $|y - 2| = |x + 3|$

e) $y - |x - 4| = |3 - 2x|$

f) $|4 - 3x| \leq y$

Unit III

1. For each of the following, find the equation of a line satisfying the given conditions:

a) Slope equal -4 and on the point (-4,2).

b) Through the point (2,-3) and parallel to $x = 3y - 7$.

c) On the point (3,-7).

d) Through the points (3,-7) and (2,5).

e) Through the points (a,0) and (0,b).

f) Perpendicular to $7 - y = 3x$.

g) The perpendicular bisector of the segment from (-4,-2) to (3,9).

h) On the points (3,-7) and (3,29).

2. State the definition of a parabola. Include the definition of the focus, the directrix, the vertex, and the axis.

3. Find the equation for a parabola satisfying each of the following:

a) Vertex at the origin and focus at (0,c).

b) Directrix $y = -2$ and focus at (4,4).

c) Focus at (2,3), distance from focus to directrix equal 3, and opening to the left.

d) Focus at (h+a,k) and directrix $x = h - a$.

4. Find the focus, directrix, vertex, and axis of the parabola defined by each of the following equations:

a) $y = x^2 - x$

c) $y = 4 - x - x^2$

b) $y^2 - 4 = x + 5$

d) $5 - x = y^2 + 3y$

5. Define each of the following:

a) A relation from A into B. Include the definition of domain and range.

b) A function from A into B.

c) A 1-1 function from A into B.

d) A function from A onto B.

e) The converse of a function from A into B.

f) The inverse of a function from A into B.

6. Each of the following equations determines a function from R into R. State the domain and range of each function. Determine whether or not each function is 1-1. Also, find the equation for the inverse function if the inverse exists.

a) $f(x) = \sqrt{4 - x^2}$

c) $h(x) = x^2 - 3x + 1$

b) $g(x) = (x^2 - 4)/(x - 2)$

d) $k(x) = 2 + |3 - x|$

7. Define each of the following types of special functions:

a) Constant function.

b) Linear function.

c) A polynomial function. Include a definition of the degree.

d) A rational function.

e) An algebraic function.

f) A transcendental function.

Unit IV

1. Sketch the graph of each of the following polynomial functions:

   a) $f(x) = (x-1)^3(x+2)^2$

   b) $g(x) = x^2(2-x)$

   c) $H(x) = x(x^2-1)(x^2-4)$

   d) $K(x) = (1-x)^2(1+x)^2$

2. Let $f(x) = 3x^2 - 4$, $g(x) = 3 - 7x$, and $H(x) = \sqrt{x-1}$. Complete:

   a) $(f \cdot g)(x) =$

   b) $(f+g)(x) =$

   c) $(H^\circ g)(x) =$

   d) $(f^\circ g)(x) =$

   e) $(H^\circ f)(x) =$

   f) $(g^{-1}\circ f)(x) =$

3. Let f and g be functions from R into R. Define $f^\circ g$.

4. Sketch the graph of $y = 3x + 1/(x-1)$.

5. State the neighborhood definition of the limit.

6. Prove that the neighborhood definition of the limit and the (ε , δ) definition of the limit are equivalent.

7. Determine the following limits:

   a) $\lim_{x \to 3} (x^3 - 3x + 4)$

   b) $\lim_{x \to 2} (x-3)$

   c) $\lim_{x \to 0} 1/x$

   d) $\lim_{x \to 2} (x^2 - 4)/(x+2)$

Unit V

1. Use the definition of limit to prove: $\lim_{x \to 2} (3x-1) = 5$.

2. Prove each of the following theorems:

   a) If $f(x) = mx + b$ then $\lim_{x \to a} f(x) = ma + b$

   b) If N^* is a deleted neighborhood of a and $f(x) = g(x)$ for all x in N^* then f has a limit at a if and only if g has a limit at a. Moreover, when these limits exist they are equal.

   c) $\lim_{x \to a} 1/x = 1/a$ for $a \neq 0$

d) $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ for $a > 0$

e) If $\lim_{x \to a} f(x) = L > 0$ then there is a deleted neighborhood N^* of a such that $f(x) > 0$ for all x in N^*.

f) If $\lim_{x \to a} f(x) = L$ and $g(x) = kf(x)$ then $\lim_{x \to a} g(x) = kL$.

3. State the definition for continuity of a function at a point.

4. State the theorems for the sum, difference, product, and quotient for limits of functions. Also state the similar theorems for when one function has a finite limit and the other has an infinite limit.

5. Prove that a function which is continuous at a also has a limit there.

6. Prove that a polynomial function is continuous on all of R.

7. Prove that a rational function is continuous on its domain.

8. Find the set on which each of the following is continuous:

a) $f(x) = (x^2 - 4)/(x+2)$

b) $g(x) = x^2 - 4$

c) $H(x) = 3/\sqrt{4-x^2}$

d) $K(x) = (x-1)/(x^2 - 4)$

9. Define each of the following:

a) $\lim_{x \to a^+} f(x) = L$

b) $\lim_{x \to a^-} g(x) = B$

10. Prove that $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L$ a[n]

$\lim_{x \to a^-} f(x) = L$.

11. Prove that the following limits do not exist.

a) $\lim_{x \to 2} 1/(x-2)$

b) $\lim_{x \to 1} (x^2-1)/x-1$

12. Define each of the following:

a) $\lim_{x \to a^+} f(x) = \infty$

b) $\lim_{x \to a^-} f(x) = \infty$

c) $\lim_{x \to b^+} g(x) = -\infty$

d) $\lim_{x \to b^-} g(x) = -\infty$

13. Determine the following:

a) $\lim_{x \to 2^+} 1/(x-2)$

b) $\lim_{x \to 2^+} (x^2 - x - 2)/(x-2)$

c) $\lim_{x \to 3^-} \frac{(x^2-1)}{x-3}$

14. Use the (ϵ , δ) definition for the limit to prove:

a) $\lim_{x \to 3} (2x - 7) = -1$

b) $\lim_{x \to -2} (3 - 2x) = 7$

c) $\lim_{x \to 1} (x^2 - 3x + 5) = 3$

Unit VI

1. Define each of the following:

a) $\lim_{x \to \infty} f(x) = B$

b) $\lim_{x \to -\infty} g(x) = L$

2. State the definition of horizontal and vertical asymptotes.

3. Find the equations of the asymptotes and sketch the graph

a) $f(x) = x^2/(2 - x)$

b) $g(x) = 3x/(4 + x)$

c) $h(x) = 3x^2 - x$

d) $k(x) = 2/(x^2 - 9)$

4. Prove each of the following limit theorems:

a) If $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = B$ then $\lim_{x \to a} (f+g)(x) = A+B$

b) If $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = B$ then

$\lim_{x \to a} (f-g)(x) = A-B$

c) $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a} [f(x) - L] = 0$.

d) If $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = 0$ then

$\lim_{x \to a} (f \cdot g)(x) = 0$.

e) If $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = B$ then

$\lim_{x \to a} (f \cdot g)(x) = AB$.

f) If $\lim_{x \to a} f(x) = A$, $\lim_{x \to a} g(x) = B$ and g is nonzero

in some deleted neighborhood of a then

$\lim_{x \to a} (f/g)(x) = A/B$.

5. Prove: If f is continuous at b and $\lim_{x \to a} g(x) = b$

then $\lim_{x \to a} (f \circ g)(x) = f\left[\lim_{x \to a} g(x)\right] = f(b)$

6. Prove each of the following:

a) If f and g are continuous at a then f+g, g−f, and f•g are continuous at a.

b) If f and g are continuous at a and g is nonzero on some deleted neighborhood of a then f/g is continuous at a.

c) If f is continuous at a and g is continuous at f(a) then $g \circ f$ is continuous at a.

7. Find each of the following limits. Prove that each answer is correct by referring to the appropriate limit theorems.

a) $\lim_{x \to 2} (x^3 - 3x + 7)$

b) $\lim_{x \to 2} (x^2 - 4)/(x - 2)$

c) $\lim_{x \to 2} \sqrt{3 + x^2}$

d) $\lim_{x \to 2} \sqrt{x - 7}/(x^3 - 9 + 1/x)$

Unit VII

1. State the definition of the derivative of a function.

2. State the definition of the derivative of a function at x=a. What does it mean to say that f is differentiable at x=a?

3. State the definition of a tangent line to the graph of a function.

4. Prove: If f is differentiable at x=a then f is continuous at x=a.

5. Use the definition to determine the derivative of each of the following at the indicated point. Also find the equation of the tangent line and the normal at this point.

   a) $f(x) = 3x^2 - 2$ (1,1)

   b) $g(x) = 3 - x^3$ (1,2)

   c) $H(x) = x/(1-x)$ (2,-2)

6. Use the definition to determine the derivative of each of the following:

   a) $f(x) = x - 4$

   b) $g(x) = 7$

   c) $g(x) = 3 - x^2$

   d) $h(x) = 1/(x-3)$

7. Prove each of the following:

   a) If $f(x) = k$ then $Df(x) = 0$.

   b) If $f(x) = x$ then $Df(x) = 1$.

   c) If f and g are differentiable at a then $D(f+g)(a) = Df(a) + Dg(a)$ and $D(f-g)(a) = Df(a) - Dg(a)$.

Unit VIII

1. Prove each of the following:

   a) If f and g are differentiable at a then $D(f \cdot g)(a) = f(a)Dg(a) + g(a)Df(a)$.

   b) If f and g are differentiable at a and g is non-zero on a deleted neighborhood of a then $D(f/g)(a) = g(a)Df(a)-f(a)Dg(a) \ /g(a)^2$

2. State the chain rule for the derivative of $f^{o}g$ at a.

3. Use the differentiation formulas to find the derivative of each of the following:

   a) $f(x) = x^3 - 7x^2 + 6x + 7$

   b) $g(x) = (x^2 - 1)^2$

   c) $H(x) = (2 - x^2)(1-1/x)$

   d) $k(x) = (3 - 4x)/(2x - 1)$

   e) $F(x) = \sqrt{(x^2 - 1)^3}$

   f) $G(x) = (x+1)(2x - 1)(3 + 4x)$

   g) $h(x) = \sqrt{\pi}(x-7)$

   h) $K(x) = x^2 \, |x^2-4|$

APPENDIX B

PART I OF THE ACHIEVEMENT TEST

FIRST EXAMINATION MATH 157

CALCULUS I

1. Determine whether each of the following is true or false.

   a) Every real number can be expressed in the form p/q where p and q are integers.

   b) $(3,7)\setminus\{4\}$ is a deleted neighborhood of 4.

   c) $(4,8)$ is a symmetric neighborhood of 6.

   d) $(3,5] \cap [2,5)$ is a neighborhood of 4.

2. Let P be the point with coordinates $(-2,5)$ and Q be the point with coordinates $(3,-4)$.

   a) Find the length of segment $\overline{PQ}$.

   b) Find the coordinates of the midpoint of segment $\overline{PQ}$.

   c) Find the slope of the line through P and Q.

   d) Find the slope of a line perpendicular to the line on P and Q.

3. Solve each of the following. State your answer as a set and sketch the graph in R x R.

   a) $|3x - 7| < 5$ b) $|4 - x| \geq 3$

4. a) Find the equation of a circle with center (2,1) and radius 3.

   b) Find the center and radius of a circle with equation $x^2 + 2x + y^2 + 4y = 4$.

5. Given a parabola with equation $y - 3 = x^2 - x$, find each of the following:

   a) The focus c) The equation of the directrix

   b) The vertex d) The equation of the axis

6. State the range and domain of each of the following functions:

   a) $f(x) = \sqrt{4 - x^2}$ b) $g(x) = 2 + |3 - 2x|$

7. Sketch the graph of $f(x) = x^2(x-1)(x+2)^3$

8. Let f and g be defined by $f(x) = 3x - 7$ and $g(x) = 7 - x^2$. Find the equation which defines each of the following:

   a) $f+g$ b) f^{-1} c) $f \circ g$ d) f/g

9. Let F be an ordered field. Complete the definition: If a and b are in F then $a > b$ if and only if . . .

10. Prove: If a,b,c,d are real numbers with $a > b$ and $c > d$ then $a+c > b+d$. Do not use any previous theorems. Prove this from the basic definitions of ordered field and "$>$".

SECOND EXAMINATION CALCULUS 1 MATH 157

1. State the (ϵ , δ) definition of $\lim_{x \to c} g(x) = B$

2. Complete the definition: The function g is continuous at $x=b$ if and only if . . .

3. a) Suppose that $\lim_{x \to a^+} f(x) = -\infty$ and $\lim_{x \to a^+} g(x) = -4$.

   What does this imply about $\lim_{x \to a^+} f(x)g(x)$?

   b) Suppose that $\lim_{x \to a} [f(x)g(x)]$ exists. What does this imply about $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$?

4. Find the equations of all asymptotes of the graph of $f(x) = x/(2x-3)$.

5. Suppose that f and g are both continuous at a. Prove that the function f-g is also continuous at a. (You may use the limit theorems in this proof.)

6. Let $f(x) = \dfrac{(x-2)\sqrt{1+x}}{x^2 - 4}$. Find $\lim\limits_{x \to 2} f(x)$. Prove that your answer is correct by referring to the appropriate limit theorems.

7. Find the derivative of the function $f(x) = 1/(x-2)$ by applying the definition of the derivative. Show all steps.

8. Prove that the function $f(x) = |x-1|$ is not differentiable at $x=1$.

9. Find the derivative of each of the following:

   a) $f(x) = (x^2 - 1)^2$

   b) $h(x) = (3-4x)/(2x-1)$

   c) $F(x) = \sqrt{x^2 - 1}$

   d) $G(x) = x^2|x^2 - 4|$

10. Use the definition of limit to prove that $\lim\limits_{x \to 2} (4x-3) = 5$

APPENDIX C

ATTITUDE SCALE USED WITH EXPERIMENTAL GROUP

As part of a research study of innovations in teaching strategy, you are asked to express your real opinions about methods used in this class. Do not put your name on this survey. No one will know which responses are yours. A summary of all responses will be used by the teacher in planning future classes. Use pen or pencil to complete the following:

Date ______________________ Teacher ______________________

Course No. ________________ Course Title ________________

Attitudes toward the _______________ Method of Instruction

Circle current academic status

Freshman Sophomore Junior Senior

Directions: Each of the following statements represents a student's attitude toward the above method of instruction. If you strongly disagree with the student attitude, circle SD at the left of the item. If you disagree, circle D. If you are undecided, circle U. If you strongly agree, circle SA.

Consider each item carefully. Do not leave any item blank. Choose the response closest to your opinion.

Remember: SD means you strongly disagree; D means you disagree; U means you are undecided; A means you agree; SA means you strongly agree.

SD D U A SA 1. This method of instruction is very frustrating for me as a student.

SD D U A SA 2. I would recommend this method to mathematics teachers for use in their classes.

SD D U A SA 3. I would not object if this method of instruction were used in this class for the entire semester.

SD D U A SA 4. This method of instruction is really not very suitable for work at this grade level.

APPENDIX C (Continued)

SD D U A SA	5.	This method did not arouse my interest.
SD D U A SA	6.	I would recommend this method of instruction for use in classes at other colleges and universities.
SD D U A SA	7.	I do not care for this method and I doubt if it appeals to other students.
SD D U A SA	8.	This method of instruction was extremely time consuming.
SD D U A SA	9.	It is difficult to give a favorable reaction to this method of instruction.
SD D U A SA	10.	This is one of the better methods of instruction to which I have been exposed.
SD D U A SA	11.	This method proved to be quite stimulating and thought provoking.
SD D U A SA	12.	I liked this method of instruction very much.
SD D U A SA	13.	I would never use this method of instruction if I were teaching.
SD D U A SA	14.	This method of instruction should be used in more mathematics classes.
SD D U A SA	15.	I thought this method of instruction was a complete waste of time.

APPENDIX D

EVALUATION SHEET FOR THE EXPERIMENTAL STRATEGY

The following questions relate to the special strategy which we have been using for these first eight weeks of this course, Math 157. For the purpose of evaluation of this method I would appreciate your honest answers to the following questions.

1. Consider the following five elements which we have been using in this course. Rank these five elements 1,2,3,4,5 with "1" indicating the element which you feel has been most beneficial and "5" indicating the element which has been of least value.

   ______ a) List of objectives for each unit.

   ______ b) The progress-quiz for each unit.

   ______ c) The diagnostic sheet returned with each quiz.

   ______ d) The second quiz for each unit.

   ______ e) The help-sessions which have been available.

2. Which of the above elements do you think could be eliminated with the least effect on mastery?

3. Do you recommend the elimination of the item mentioned in #2 above? Briefly comment on why or why not.

4. What additional methods would you recommend to improve the strategy?