THE ANALYSIS OF FIXED-END FRAMES WITH BENT MEMBERS BY THE STRING POLYGON METHOD

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PREFACE

The material presented in this thesis is the outgrowth of seminar lectures delivered by Professor Jan J. Tuma in the spring of 1959 and 1960. The literature survey and the general theory of the string polygon for straight and bent members were prepared by Professor Tuma (1).

The application of the string polygon to the calculation of beam constants was reported by Chu (2). Maydayag (3) developed the string polygon calculations for deflections of airplane wings and programmed this procedure for the IBM 650 Electronic Computer. Harvey (4) developed a string polygon application to the analysis of column beams.

The application of the String Polygon Method to the analysis of single span rigid frames with members of variable cross-section is presented in this thesis. Two other graduate students are extending the application of this theory to frames with members of linear variation in cross-section. (5, 6).

Finally, the general theory of the string polygon in terms of the energy due to shearing forces, normal forces, and bending moments is being developed by Wu (7).

I wish to express my indebtedness to Professor Tuma, not only for introducing to me the possibilities of the application of the String Polygon Method to the analysis of fixed-end frames, but also for his valuable assistance and guidance throughout the preparation of this thesis. To Dr. K. S. Havner I would like to give my sincere thanks for reading the final manuscript and making many helpful suggestions.

I would like to thank Mr. T. I. Lassley, Instructor in the School of Civil Engineering, who prepared the program for the IBM 650 Digital Computer from which data was obtained and used to compile the tables of beam constants in the fifth chapter of this thesis.

To Mrs. Glenna Banks and Mrs. Dorothy Messenter I wish to express my gratitude. They have given renewed proof of their exceptional skill in their careful typing of the manuscript.

Thanks are also due Mrs. Judy Sadler, who helped type the tables of beam constants.

J. T. O.

TABLE OF CONTENTS

Chapter | Page

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

b	Width of Beam.
d_j	Length of Bent Member ij.
d_{jx}	Horizontal Projection of d_j.
d_{jy}	Vertical Projection of d_j.
e_o, f_o	Coordinates of Elastic Center.
f	Angular Flexibility Coefficient.
g	Angular Carry-Over Value Coefficient.
h_o	Minimum Depth of Beam.
$h_o \delta$	Maximum Depth of Haunch.
q	Specific Weight of Beam.
t	End Slope Coefficient.
u, u', v, v', x, y, $\bar{x}$, $\bar{y}$	Coordinates of Cross-Sections.
w	Intensity of Load.
BM	Bending Moment Due to Loads.
F_{ij}, F_{ji}	Angular Flexibilities.
G_{ij}, G_{ji}	Carry-Over Values.
I_o^*	Minimum Moment of Inertia of Beam.
I_o	Minimum Moment of Inertia of Structure.
$\bar{I}_x$, $\bar{I}_y$	Moments of Inertia of the Elastic Area with Respect to the $\bar{x}$ and $\bar{y}$ Axes, Respectively.

I_{xx}, I_{yy} . . . . Moments of Inertia of the Elastic Area with Respect to the x and y Axes, Respectively.

$\overline{I}_{xy}$ . . . . Product of Inertia of the Elastic Area with Respect to the $\overline{x}$ and $\overline{y}$ Axes, Respectively.

I_{xy} . . . . Product of Inertia of the Elastic Area with Respect to the x and y Axes, Respectively.

$L\beta$. . . . . Length of Haunch.

L_i, L_j, L_k . . . . Lengths of Spans i, j, k, Respectively.

M_i, M_j, M_k . . . . Bending Moments at Points i, j, k, Respectively.

$\overline{M}_x$, $\overline{M}_y$ . . . . Moments of the Elastic Weights with Respect to the $\overline{x}$ and $\overline{y}$ Axes, Respectively.

M_{xx}, M_{yy} . . . . Moments of the Elastic Weights with Respect to the x and y Axes, Respectively.

$\overline{P}$ . . . . Elastic Weight.

$\overline{P}_j^L$ . . . . Elastic Weight at j Due to Loads.

$\overline{P}_j^o$ . . . . Elastic Weight at j Due to a Unit Couple at the Elastic Center.

$\overline{P}_j^x$ . . . . Elastic Weight at j Due to a Unit Horizontal Force at the Elastic Center.

$\overline{P}_j^y$ . . . . Elastic Weight at j Due to a Unit Vertical Force at the Elastic Center.

$\overline{R}$ . . . . Reaction of Conjugate Beam.

R_{ok}, R_{oy}, M_o. . . . . . Reactive Redundants.

U_{ijk} . . . . Strain Energy of Member ijk.

π . . . . Angle that a Bent Member Makes with the Horizontal.

ρ_i . . . . Angle Between Extensions of Bent Members ij and jk.

τ_{ij}, τ_{ji} . . . . Angular Load Functions.

ϕ_{ji} . . . . Angle Between String Line ij and Tangent to Elastic Curve at j.

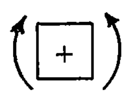
ϕ_j . . . . Change in Angle of String Line at j.

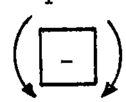
ω . . . . Angle Between Horizontal and String Line.

SIGN CONVENTION

Axial Forces . . . . + tension - compression

Moment . . . .





CHAPTER I

INTRODUCTION

The representation of the elastic curve of a straight beam as a differential string polygon was introduced by Mohr (8) in connection with his concept of elastic weights and of conjugate beams.

Müller-Breslau developed the idea of joint loads (knoten lasten) for straight members (9) and bent members (10). In his definition of joint loads the influence of the load on the element was neglected and only the effects of the moments, shears, and normal forces were considered.

The restatement of the formulation of joint loads may be found in recent publications. (11, 12, 13, 14).

The idea of angle change traverses is discussed by Cross (15) and Michalos (16), and is essentially the idea of the string polygon as formulated by Mohr and Müller-Breslau.

By adding the angular load function "τ", Tuma (1) generalized the String Polygon Method and related it to the three moment equation, thus making possible the application of beam constants now available. (17). This function accounts for loads at the intermediate points between vertices of the polygon and yields exact results.

The analysis of fixed end panels has become a common problem in modern structural engineering. Frames with members of variable moments of inertia are frequently used since they are not only functional but also are often more economical and eye-pleasing than those of constant cross-section. Difficulty arises, however, in the analysis of such structures, as the methods of slope deflection, moment distribution, column analogy, or elastic centers usually employed lead to laborious and time consuming procedures.

The String Polygon Method has become a useful and efficient tool for many structural problems and may be applied in the analysis of fixed-end frames. The approach incorporates the elastic center of the structure and the evaluation of elastic weights. From this standpoint, the method is similar to the column analogy, but a more thorough investigation shows that an entirely different philosophy, a more direct approach, and a less tedious procedure is afforded. Due to this similarity no distribution procedures or sets of simultaneous equations are needed, and with the aid of beam constant tables little difficulty arises in the evaluation of elastic weights.

In the second chapter of this thesis the basic theory of the string polygon is presented along with the philosophy of the analogy of elastic weights. The third chapter is dedicated to the extension of this theory to bent members and to the derivation of relationships needed to compute the elastic constants and load functions necessary in the evaluation of elastic weights. These elastic weights, it is shown, are found by means of a three moment equation giving the change in angle of two adjacent string lines of the polygon, and thus are developed at the intersection of these string lines.

The fourth chapter deals with the application of the String Polygon Method to the analysis of fixed-end frames. Completely general cases are considered and expressions are developed to fit any type of variation in the cross-section. Tables of beam constants for members with the common parabolic haunches comprise the fifth chapter, and the sixth chapter demonstrates the procedure of analysis by the String Polygon Method through numerical examples. In the final chapter of the thesis the study is summarized and conclusions are drawn.

CHAPTER II

THE THEORY OF THE STRING POLYGON

2-1. Basic Derivation

Analysis by the string polygon method falls within the limits of the simple theory of bending and assumptions common to general structural analysis apply.

First, considering the general case, the elastic curve of a simple beam AB (Fig. 2-1) of variable cross-section loaded by a general system of loads will be examined. The elastic curve of AB may be simulated by dividing the beam into n separate portions; (0, 1), (1, 2)...(i, j), (j,k)...(n-1, n), each being simply supported at their ends with reactions due solely to the external loading.

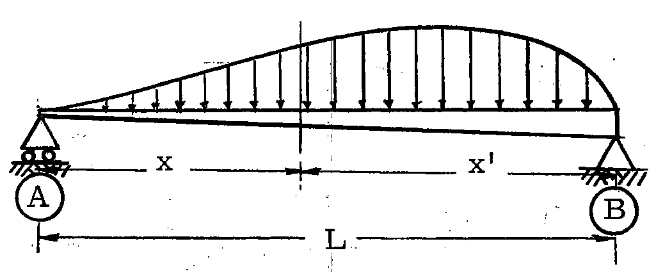


Fig. 2-1. Simple Beam

Without loss of generality, two adjacent portions of the elastic curve, ij and jk (Fig. 2-2 a,b), may be considered.

By inspection of Fig. 2-2b it may be seen that ϕ_{ji} and ϕ_{jk} are the angles between a tangent drawn to the elastic curve at j and the segments of the polygon $\overline{ij}$ and $\overline{jk}$ respectively. It is also obvious that

$$\phi_j = \phi_{ji} + \phi_{jk}$$

... and from Fig. 2- 3 it may further be deduced that

$$M_u = BM_u - \frac{M_i - M_j}{L_j} u + M_i$$

$$M_u = BM_u + M_i \frac{u'}{L_j} + M_j \frac{u}{L_j} \qquad (2\text{-}1a)$$

... and similarly,

$$M_v = BM_v + M_j \frac{v'}{L_k} + M_k \frac{v}{L_k} \qquad (2\text{-}1b)$$

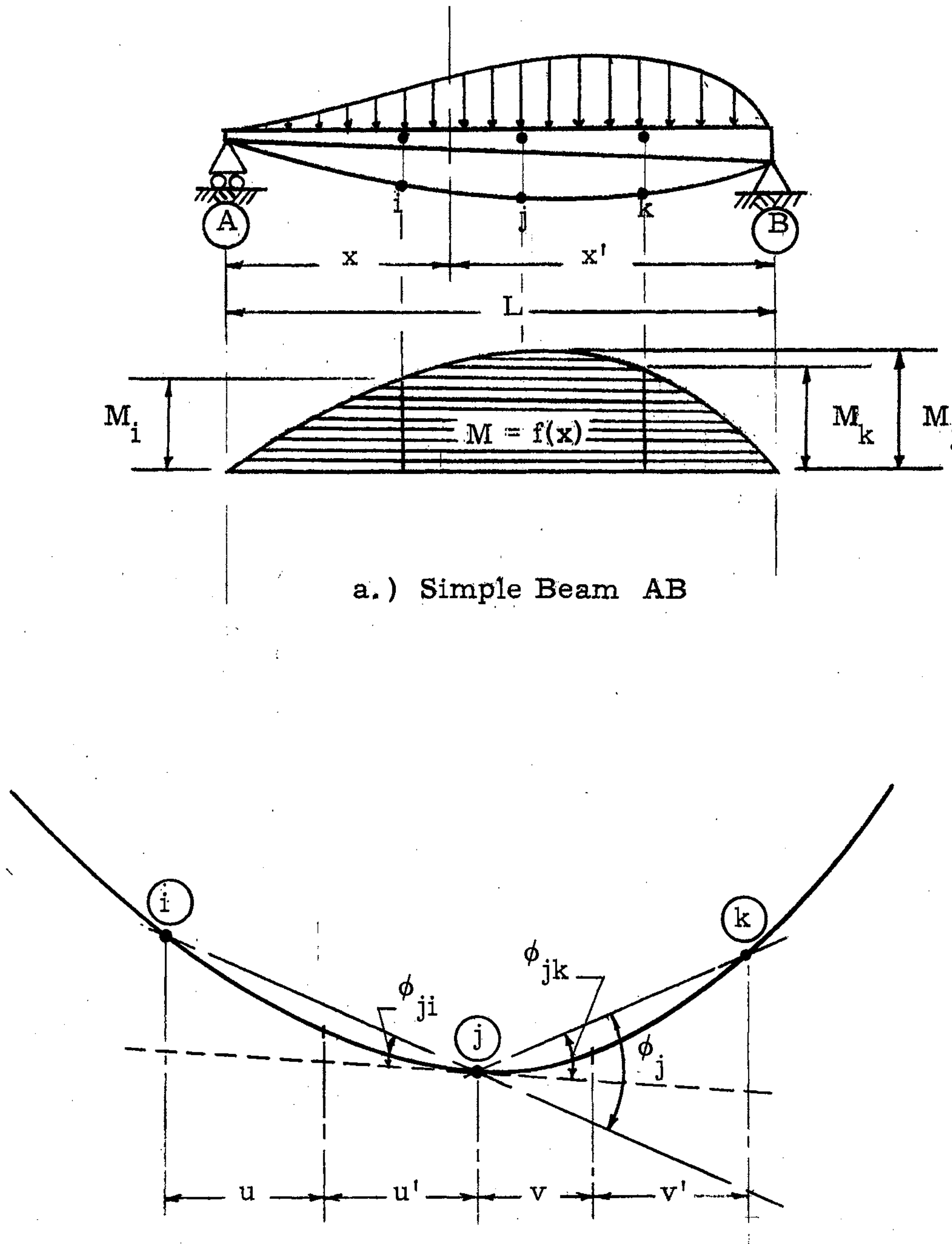
..., where M_u and M_v are the bending moments of segments $\overline{ij}$ and $\overline{jk}$ respectively.

The strain energy due to bending of $\overline{ijk}$ is:

$$U_{ijk} = \int_i^j \frac{M_u^2 du}{2EI_u} + \int_j^k \frac{M_v^2 dv}{2EI_v} . \qquad (2\text{-}2)$$

From Castigliano's first theorem

$$\frac{\partial U_{ijk}}{\partial M_j} = \frac{\partial (U_{ij} + U_{jk})}{\partial M_j} = \phi_{ji} + \phi_{jk} = \phi_j \qquad (2\text{-}3a)$$



b.) Segment ijk of the Elastic Curve

Fig. 2-2. Three Point Polygon ijk .

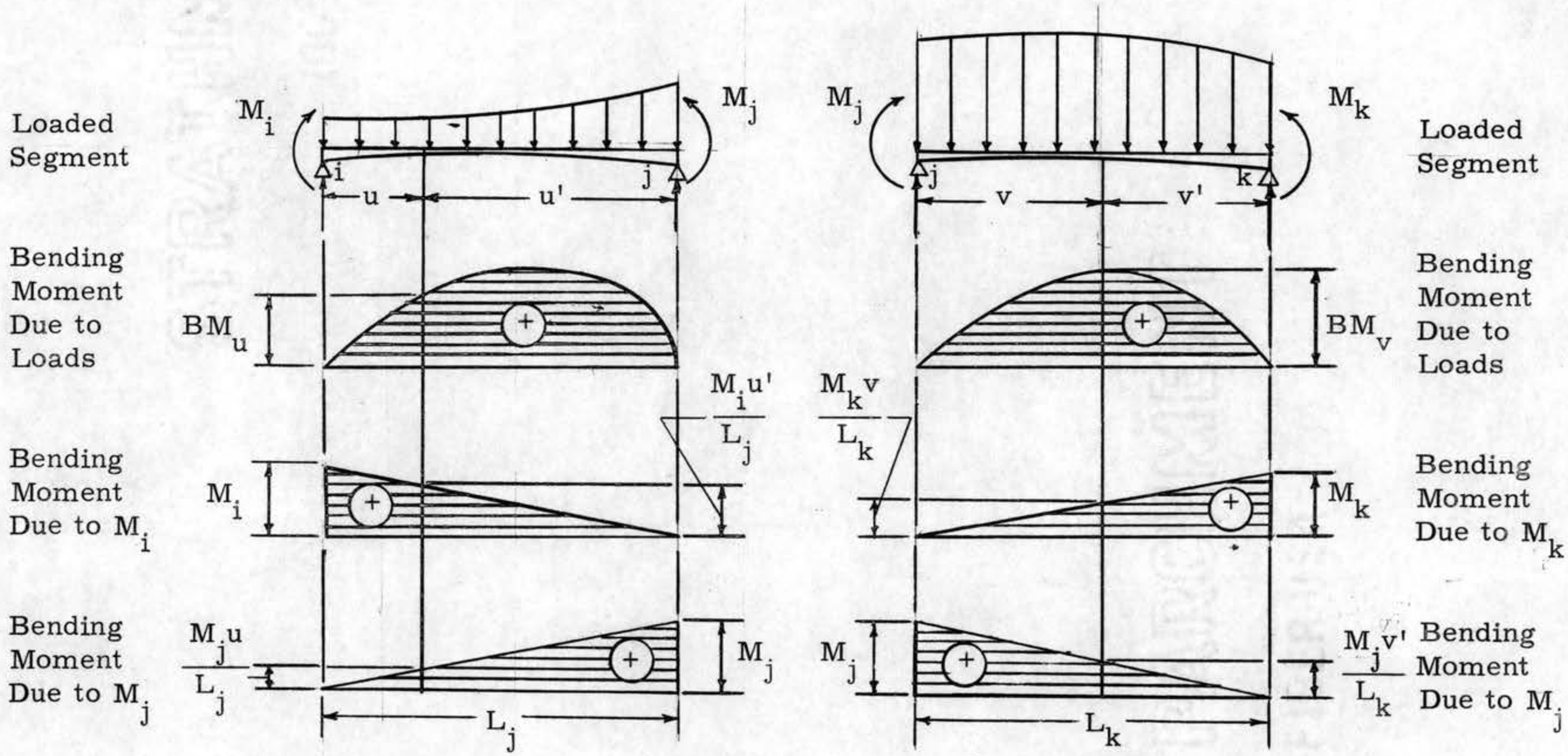


Fig. 2-3. Free Bodies $\overline{ij}$ and $\overline{jk}$

... or

$$\phi_j = \int_i^j M_u \frac{\partial M_u}{\partial M_j} \frac{du}{EI_u} + \int_j^k M_v \frac{\partial M_v}{\partial M_j} \frac{dv}{EI_v} \quad . \qquad (2\text{-}3b)$$

Substituting (2-1a, b) into (2-3b) gives:

$$\phi_j = \int_i^j \frac{BM_u udu}{L_j EI_u} + M_i \int_i^j \frac{uu' du}{L_j^2 EI_u} + M_j \int_i^j \frac{u^2 du}{L_j^2 EI_u} +$$

$$\int_j^k \frac{BM_v v' dv}{L_k EI_v} + M_j \int_j^k \frac{v'^2 dv}{L_k^2 EI_v} + M_k \int_j^k \frac{vv' dv}{L_k^2 EI_v}$$

$$= \tau_{ji} + M_i G_{ij} + M_j F_{ji} + \tau_{jk} + M_j F_{jk} + M_k G_{jk}$$

... or finally, denoting $F_{ji} + F_{jk} = \Sigma F_j$, and $\tau_{ji} + \tau_{jk} = \Sigma\tau_j$,

$$\phi_j = M_i G_{ij} + M_j \Sigma F_j + M_k G_{jk} + \Sigma \tau_j \qquad (2\text{-}4)$$

which is the familiar Clapeyron's three moment equation.

The quantities τ, G, and F in Equation (2-4) have important physical interpretations and will be defined in the following section.

2-2. Angular Functions

a.) The angular flexibility F_{ij} is the end slope of a simple beam $\overline{ij}$ at j due to a unit moment applied at that end (Fig. 2-3a).

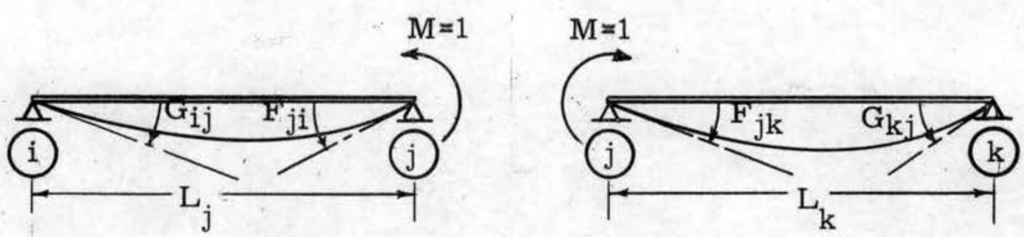
$$F_{ij} = \int_i^j \frac{u^2 du}{L_j^2 EI_u} \qquad F_{jk} = \int_j^k \frac{v'^2 dv}{L_k^2 EI_v} \quad . \qquad (2\text{-}5a)$$

b.) The carry-over value G_{ij} is the end slope of a simple beam $\overline{ij}$ at i due to a unit moment applied at the far end j (Fig. 2-3a).

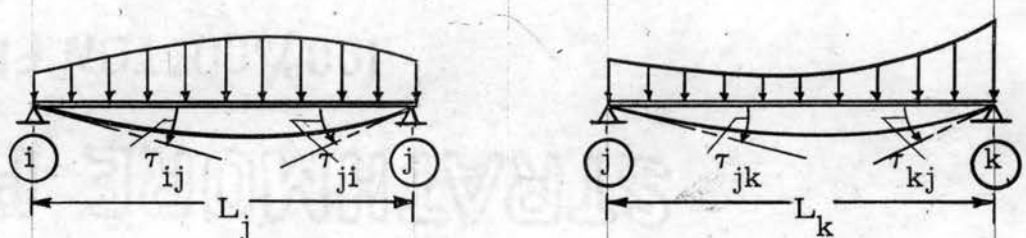
$$G_{ij} = \int_i^j \frac{uu' \, du}{L_j^2 EI_u} \qquad G_{kj} = \int_j^k \frac{vv' \, dv}{L_k^2 EI_v} . \qquad (2\text{-}5b)$$

c.) The angular load function τ_{ji} is the end slope of a simple beam $\overline{ij}$ at j due to loads (Fig. 2-3b).

$$\tau_{ji} = \int_i^j \frac{BM_u \, u du}{L_j EI_u} \qquad \tau_{jk} = \int_j^k \frac{BM_v \, v' dv}{L_k EI_v} . \qquad (2\text{-}5c)$$



a.) Angular Flexibilities and Carry-Over Values



b.) Angular Load Functions

Fig. 2-4. Angular Functions

It will now be shown that knowing ϕ_i, $\phi_j \ldots . \phi_n$, the deflection of any point on the beam AB (Fig. 2-1, 2-2a) which is a vertex of the string polygon may be determined; and from resulting relationships it may be concluded that ϕ_j, Equation (2-4), can be considered as an elastic weight of the conjugate structure.

2-3. Elastic Weights

Suppose that the string polygon for the beam AB has the points i, j, and k for vertices, as shown in Fig. 2-5. After the angular functions for the segments $\overline{Ai}$, $\overline{ij}$, $\overline{jk}$, and $\overline{kB}$ have been determined from Equation (2-5), the quantities ϕ_i, ϕ_j and ϕ_k may be obtained by substituting the appropriate F's, G's, and τ's into Equation (2-4).

Figure 2-5 provides the following simple geometric relationships:

$$\omega_i = \frac{y_j - y_i}{L_j} \qquad \omega_j = -\frac{y_j - y_k}{L_k}$$

$$\omega_A = \frac{y_i}{L_i} \qquad \omega_B = \omega_k = \frac{y_k}{L_m} \tag{2-6}$$

... where ω_A, ω_i, ω_j, and ω_k are the angles that the "string lines" Ai, ij, jk, and kB make with the horizontal, respectively; y_i, y_j, and y_k are the actual vertical displacements of points i, j, and k respectively.

It may also be shown that

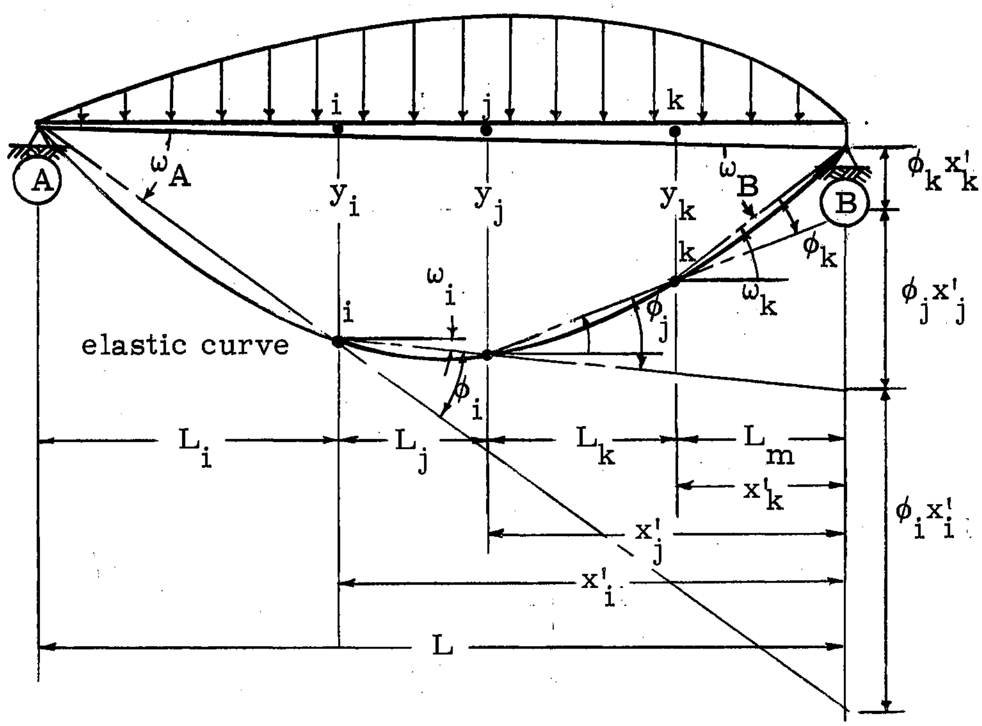


Fig. 2-5. The String Polygon

$$\left.\begin{aligned} \omega_A - \phi_i &= \omega_i \\ \omega_A - \phi_j - \phi_i &= \omega_j \\ \omega_A - \phi_i - \phi_j - \phi_i &= \omega_k = \omega_B \end{aligned}\right\} \quad (2\text{-}7a)$$

. . . and also

$$\omega_A = \frac{\phi_k x'_k + \phi_j x'_j + \phi_i x'_i}{L} \quad . \qquad (2\text{-}7b)$$

The deflections y_i, y_j, y_k are

$$\left.\begin{aligned} y_i &= \omega_A L_i \\ y_j &= \omega_A (L_i + L_j) - \phi_i L_j \\ y_k &= \omega_B L_m = \omega_A (L_i + L_j + L_k) - \phi_i (L_i + L_j) - \phi_j (L_k) \ . \end{aligned}\right\} \quad (2\text{-}7c)$$

Equation (2-7) yields an important analogy. If a conjugate beam A'B' (Fig. 2-6) is loaded with the forces $\overline{P}_i$, $\overline{P}_j$, and $\overline{P}_k$ at points on the conjugate beam corresponding to i, j, and k respectively, it is seen that the reaction $\overline{R}_A$ at A' is

$$\overline{R}_A = \frac{\overline{P}_i x'_i + \overline{P}_j x'_j + \overline{P}_k x'_k}{L} \tag{2-8a}$$

. . ., or, summing forces,

$$\overline{R}_A = \overline{P}_i + \overline{P}_j + \overline{P}_k - \overline{R}_B \, . \tag{2-8b}$$

The bending moments $\overline{M}_i$, $\overline{M}_j$, and $\overline{M}_k$ at sections i, j, and k respectively are

$$\begin{aligned} \overline{M}_i &= \overline{R}_A L_i \\ \overline{M}_j &= \overline{R}_A (L_i + L_j) - \overline{P}_i L_j \\ \overline{M}_k &= \overline{R}_A (L_i + L_j + L_k) - \overline{P}_i (L_j + L_i) - \overline{P}_j L_k \, . \end{aligned} \tag{2-8c}$$

Now denoting . .

ω_A = $\overline{R}_A$ = reaction of the conjugate beam at A

ϕ_i = $\overline{P}_i$ = elastic weight acting at point i

ϕ_j = $\overline{P}_j$ = elastic weight acting at point j

ϕ_k = $\overline{P}_k$ = elastic weight acting at point k

ω_B = $\overline{R}_B$ = reaction of the conjugate beam at B

the analogy of elastic weights is obtained, as Equation (2.7) is identical in form to (2-8).

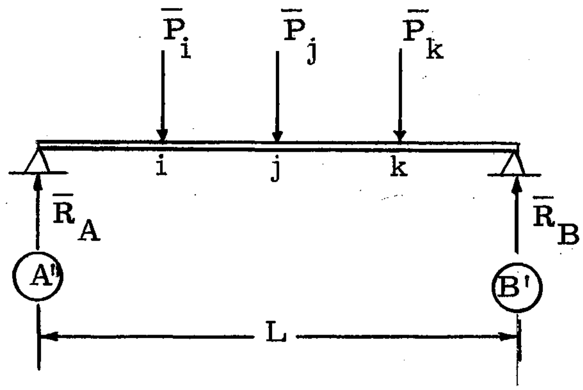


Fig. 2-6. Conjugate Beam

Hence, computing the ϕ' s, the exterior angles between the legs of the string polygon, and regarding them as elastic weights on the conjugate structure, enables the determination of the deflections at the points at which the loads were applied.

End-slopes of beams may also be determined by merely adding to the reaction of the conjugate beam the end slopes of the adjacent section. This may be expressed by the equation:

$$\tau_{AB} = \omega_A + M_i G_{iA} + \tau_{Ai} \quad . \qquad (2\text{-}9)$$

CHAPTER III

THE STRING POLYGON FOR BENT MEMBERS

3-1. Elastic Weights for Bent Members

It was shown in Chapter II that the string polygon method can be applied successfully to straight beams of variable cross-section. The question of its applicability to structures with bent members automatically arises.

Consider the members $\overline{ij}$ and $\overline{jk}$ of a structure with bent members with variable moments of inertia loaded with a general system of loads (Fig. 3-1). Again it is seen that the bending moments M_u and M_v of members $\overline{ij}$ and $\overline{jk}$ respectively can be written:

$$\left.\begin{aligned} M_u &= M_i \frac{u'}{d_j} + M_j \frac{u}{d_j} + BM_u \\ M_v &= M_j \frac{v'}{d_k} + M_k \frac{v}{d_k} + BM_v \quad . \end{aligned}\right\} \quad (3\text{-}1)$$

The usual assumption of rigid joints is made and no difficulty in visualizing the angle ϕ_j occurs if it is recalled that the string lines of the string polygon are drawn on the deformed structure, and by definition ϕ_j is the sum of the angles that the lines $\overline{ij}$ and $\overline{jk}$ on the

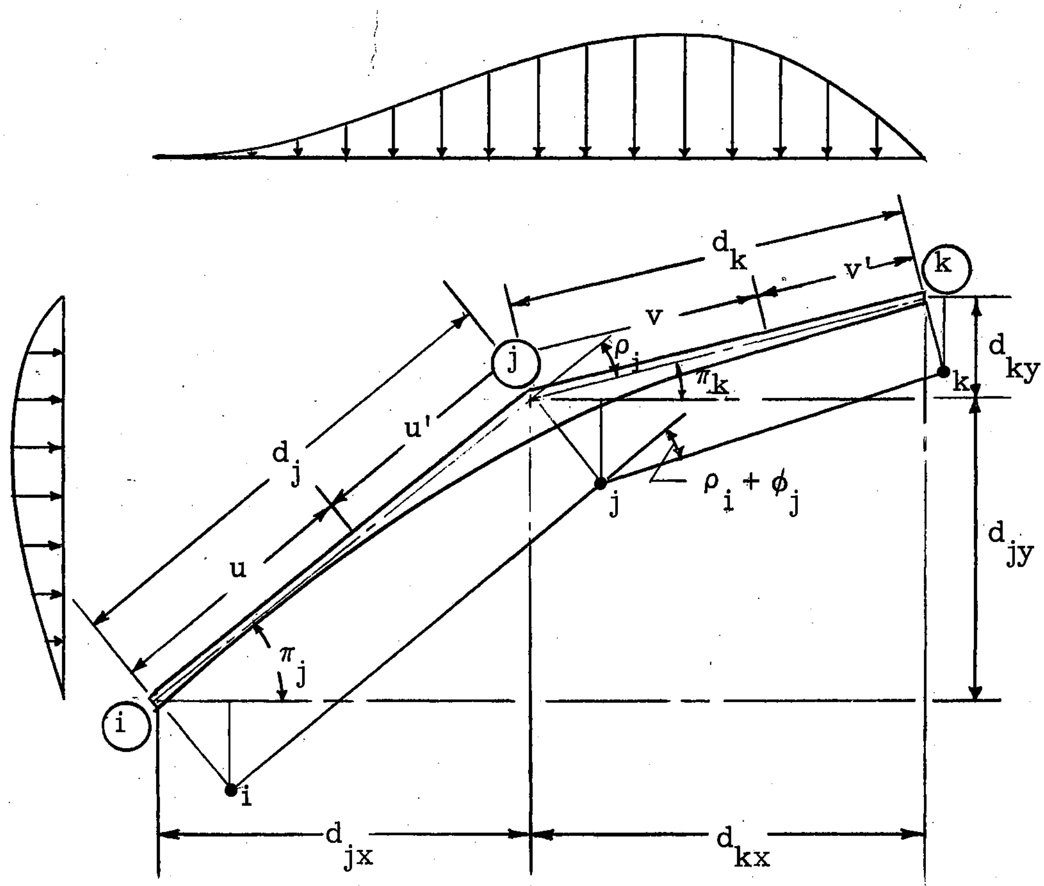


Fig. 3-1. Bent Member ijk

polygon make with their respective undeformed positions, i.e., the slopes due to deformation of segments $\overline{ij}$ and $\overline{jk}$ at j (see Fig. 3-2).

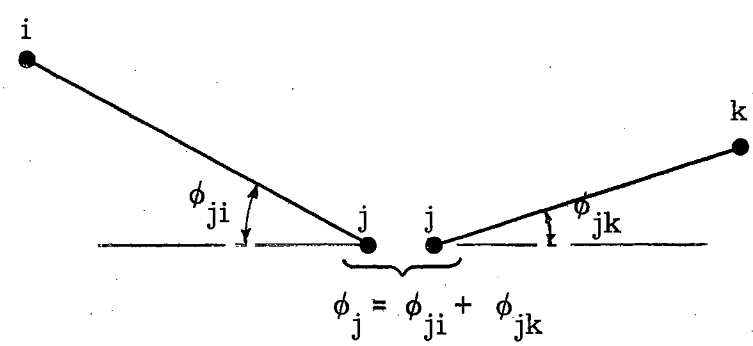


Fig. 3-2. Diagram of Displacement

The strain energy of ijk is:

$$U_{ijk} = \int_i^j \frac{M_u^2 du}{2EI_u} + \int_j^k \frac{M_v^2 dv}{2EI_v}$$

... and again using Castigliano's first theorem, the following relationships evolve:

$$\phi_j = \frac{\partial U_{ijk}}{\partial M_j} = \int_i^j \frac{BM_u udu}{d_j EI_u} + M_i \int_i^j \frac{uu'du}{d_j^2 EI_u} + M_j \int_i^j \frac{u^2 du}{d_j^2 EI_u}$$

$$M_j \int_j^k \frac{v'^2 dv}{d_k^2 EI_v} + M_k \int_j^k \frac{vv'dv}{d_k^2 EI_v} + \int_j^k \frac{BM_v v'dv}{d_k EI_v}$$

$$\phi_j = M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} + \Sigma \tau_j \qquad (3\text{-}2)$$

... which is, again, the general three moment equation.

The initial inclination of $\overline{ij}$ and $\overline{jk}$ necessitates a more thorough interpretation of the angular load functions τ_{ji} and τ_{jk}.

3-2. Angular Load Functions for Bent Members

Consider the segment $\overline{ij}$ of the member ijk of Fig. 3-1 loaded only by a system of vertical loads (Fig. 3-3). It is desirable to evaluate τ_{ji} in terms of horizontal or vertical coordinates since loads are usually applied in these directions. It is necessary to imagine the horizontal projection of member ij (i'j' in Fig. 3-3).

τ_{jix} may be defined as the slope of the simple beam i'j' at j' due to loads, i'j' being the horizontal projection of ij.

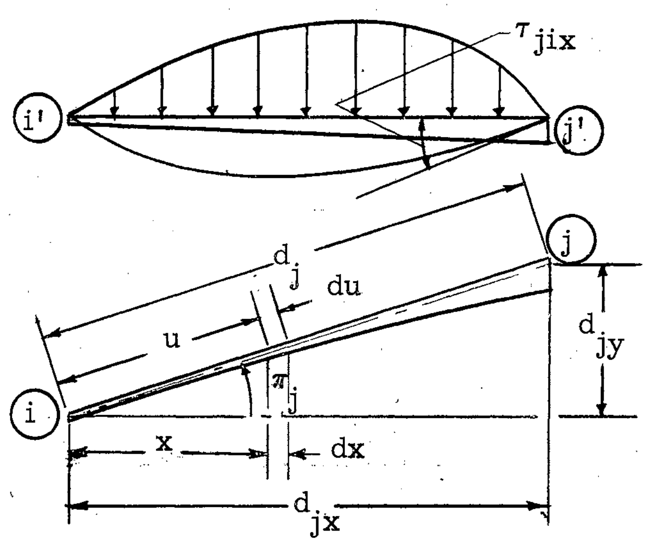


Fig. 3-3. Interpretation of τ_{jix}

If the unloaded member ij is naturally inclined at an angle π_j as shown in Figs. 3-2 and 3-3, it follows that

$$du = \frac{dx}{\cos \pi_j}$$

...., and defining τ_{jix} by

$$\tau_{jix} = \int_i^j \frac{BM_x \, xdx}{d_{jx} EI_u} ,$$

it is seen that

$$\tau_{ji} = \int_i^j \frac{BM_u \, udu}{d_j EI_u} = \frac{1}{\cos \pi_j} \int_i^j \frac{BM_x \, xdx}{d_{jx} EI_u}$$

or

$$\tau_{ji} = \frac{1}{\cos\pi_j} \tau_{jix} . \qquad (3\text{-}3a)$$

The elastic weight for these vertical loads becomes

$$\overline{P}_j = M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} + \frac{1}{\cos\pi_j} \tau_{jix} + \frac{1}{\cos\pi_k} \tau_{jkx} . \qquad (3\text{-}3b)$$

In a similar manner the angular load-functions for $\overline{ij}$ under the action of horizontal loads only may be evaluated. Defining τ_{ijy} as the slope of the simple beam i"j" at j" due to loads, where i"j" is the vertical projection of ij (Fig. 3-4), it is seen that

$$\tau_{jiy} = \int_i^j \frac{BM_y ydy}{d_{jy} EI_u} .$$

Since $du = dy/\sin\pi_j$ it follows that

$$\tau_{ji} = \frac{1}{\sin\pi_j} \int_i^j \frac{BM_y ydy}{d_{jy} EI_u}$$

or

$$\tau_{ji} = \frac{1}{\sin\pi_j} \tau_{jiy} . \qquad (3\text{-}4a)$$

The equation for the elastic weight due to the horizontal loading is

$$\left.\begin{aligned}\dot{\vec{P}}_j &= M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} \\ &+ \frac{1}{\sin \pi_j} \tau_{jiy} + \frac{1}{\sin \pi_j} \tau_{jiy} .\end{aligned}\right\} \quad (3\text{-}4b)$$

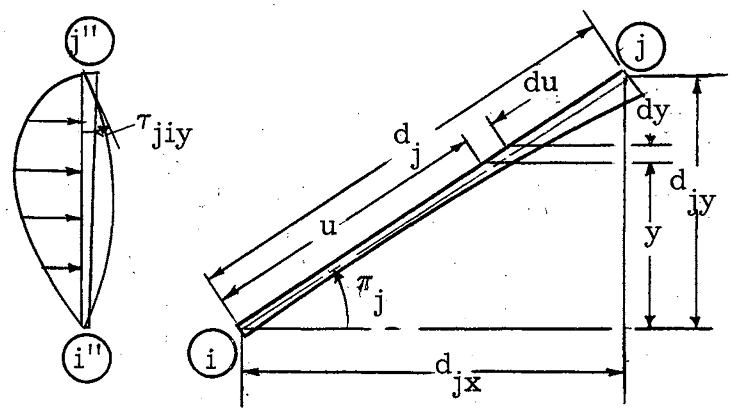


Fig. 3-4. Interpretation of τ_{jiy}

CHAPTER IV

THE STRING POLYGON METHOD FOR FIXED-END PANELS

4-1. Theory

The computation of elastic weights for a structure brings to mind the familiar column analogy or the method of elastic centers often used in the analysis of irregularly shaped structures. It is to be noted that simplified solutions by these methods call for the selection of a basic system and for the calculation of the elastic center of the system. This suggests that the string polygon method might be used to analyze such systems; and it will be shown that this can be easily done and that the procedure retains considerable advantages over its more tedious predecessors.

Of the many basic systems used in the analysis of structures, those of the form shown in Fig. 4-1 are probably the most common; that in Fig. 4-1a usually finds applications to symmetrical structures with symmetrical loading. The following treatment applies to all basic systems, and the structure shown in Fig. 4-2 may be considered as being of general form.

Imagine a rigid arm extending from the left support of a structure with bent members (Fig. 4-2) to the elastic center of the system. The

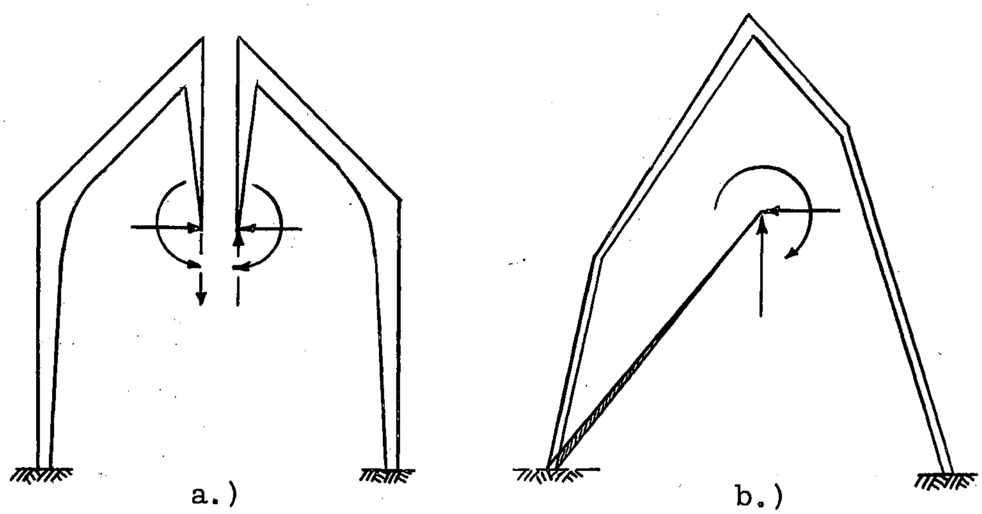


Fig. 4-1. Common Basic Structures

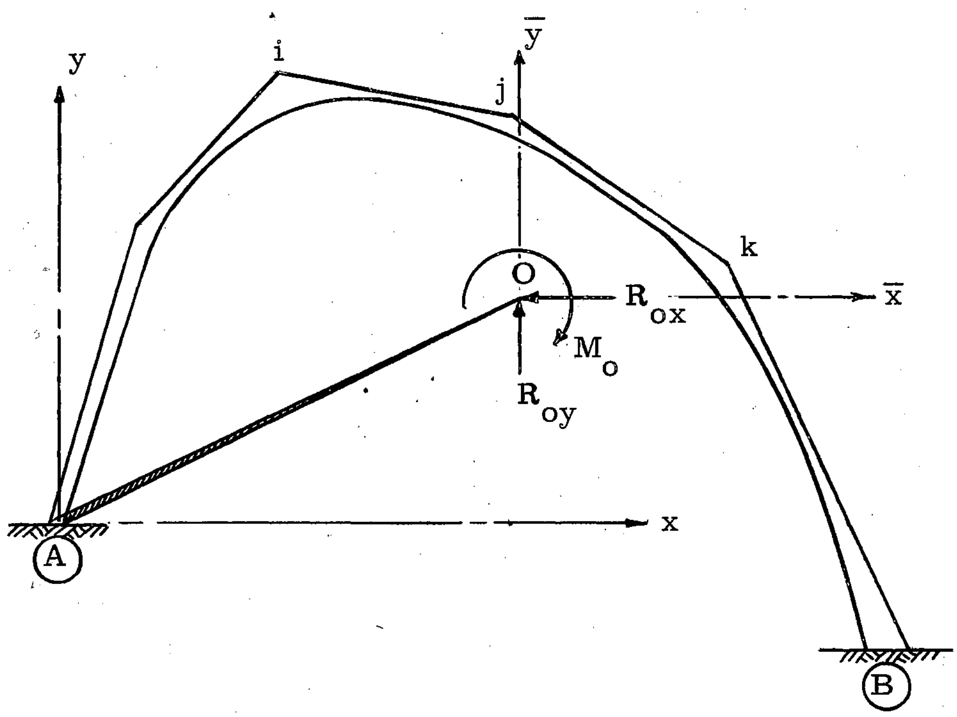


Fig. 4-2. Structure with Bent Members

locations of points in this system are specified by the coordinates x and y when the origin is taken at A, or by $\bar{x}$ and $\bar{y}$ with the origin at O, the elastic center. If the reactive redundants R_{ox}, R_{oy}, and M_o are placed at the elastic center, the bending moment at any point is given by

$$M_s = M_o + R_{ox}\bar{y} + R_{oy}\bar{x} + BM_s \tag{4-1}$$

... where BM_s is the bending moment due to loads. If a segment ijk is examined, it will be noted from Equation (4-1) that the bending moment at these points may be considered to be induced by four separate effects, the four terms in the equation. Hence, it is necessary to find elastic weights corresponding to each of these effects.

Denoting the bending moment due to loads at i, j, and k as BM_i, BM_j, and BM_k respectively, the elastic weight due to loads at joint j may be defined as

$$\overline{P}_j^L = BM_i G_{ij} + BM_j \Sigma F_j + BM_k G_{kj} + \Sigma \tau_j . \tag{4-2a}$$

To find the elastic weight at j due to the redundants R_{ox}, R_{oy}, and M_o, merely place unit horizontal and vertical forces at O along with a unit couple, all in directions of the respective redundants. The moments at any point due to the unit horizontal and vertical forces are $\bar{y}$ and $\bar{x}$ respectively. A unit moment at O produces a unit moment throughout the structure. Hence, the elastic weight at a typical joint j due to a unit horizontal force applied at O is

$$\overline{P}_j^x = \bar{y}_i G_{ij} + \bar{y}_j \Sigma F_j + \bar{y}_k G_{kj} \tag{4-2b}$$

. . . and that due to a unit vertical force applied at the elastic center is

$$\overline{P}_j^y = \overline{x}_i G_{kj} + \overline{x}_j \Sigma F_j + \overline{x}_k G_{kj} . \qquad (4\text{-}2c)$$

Similarly, the elastic weight due to a unit couple applied at the elastic center is

$$\overline{P}_j^o = G_{ij} + \Sigma F_j + G_{kj} . \qquad (4\text{-}2d)$$

By superposition, the elastic weights at joint j due to the redundants R_{ox}, R_{oy}, and M_o, respectively, are

$$R_{ox}\overline{P}_j^x = R_{ox}\overline{y}_i G_{ij} + R_{ox}\overline{y}_i \Sigma F_j + R_{ox}\overline{y}_k G_{kj}$$

$$R_{oy}\overline{P}_j^y = R_{oy}\overline{x}_i G_{ij} + R_{oy}\overline{x}_j \Sigma F_j + R_{oy}\overline{x}_k G_{kj} \qquad (4\text{-}3)$$

$$M_o\overline{P}_j^o = M_o G_{ij} + M_o \Sigma F_j + M_o G_{kj} .$$

The total elastic weight becomes

$$\overline{P}_j = \overline{P}_j^L + R_{ox}\overline{P}_j^x + R_{oy}\overline{P}_j^y + M_o\overline{P}_j^o . \qquad (4\text{-}4)$$

Recalling from Chapter II that the moment of the elastic weights about a point on the conjugate structure represents the actual deflection of that point, it is seen that the algebraic sum of the elastic loads to one side of the point represents the angle between the member and the string line drawn on the deflected structure. By definition, the member AO in Fig. 4-2 may be regarded as a rigid arm that neither deflects nor rotates. Hence, the sum of the respective elastic weights is zero, or . .

$$\sum \bar{P}_j = 0$$

$$\sum \bar{P}_j \bar{y}_j = 0 \tag{4-5}$$

$$\sum \bar{P}_j \bar{x}_j = 0$$

which may be written

$$\sum \bar{P}_j^L + R_{ox} \sum \bar{P}_j^x + R_{oy} \sum \bar{P}_j^y + M_o \sum \bar{P}_j^o = 0 \tag{4-6a}$$

$$\sum \bar{P}_j^L \bar{y}_j + R_{ox} \sum \bar{P}_j^x \bar{y}_j + R_{oy} \sum \bar{P}_j^y \bar{y}_j + M_o \sum \bar{P}_j^o \bar{y}_j = 0 \tag{4-6b}$$

$$\sum \bar{P}_j^L \bar{x}_j + R_{ox} \sum \bar{P}_j^x \bar{x}_j + R_{oy} \sum \bar{P}_j^y \bar{x}_j + M_o \sum \bar{P}_j^o \bar{x}_j = 0 . \tag{4-6c}$$

By the definition of the elastic center,

$$\left.\begin{aligned} \sum \bar{P}_j^o \bar{y}_j &= 0 & \qquad \sum \bar{P}_j^x &= 0 \\ \sum \bar{P}_j^o \bar{x}_j &= 0 & \qquad \sum \bar{P}_j^y &= 0 . \end{aligned}\right\} \tag{4-7a}$$

Denoting

$$\left.\begin{aligned} \sum \bar{P}_j^o &= \bar{A} & \qquad \sum \bar{P}_j^L &= \bar{P} \\ \sum \bar{P}_j^x \bar{y}_j &= \bar{I}_x & \qquad \sum \bar{P}_j^L \bar{y}_j &= \bar{M}_x \\ \sum \bar{P}_j^x \bar{x}_j = \sum \bar{P}_j^y \bar{y}_j &= \bar{I}_{xy} & \qquad \sum \bar{P}_j^L \bar{x}_j &= \bar{M}_y , \end{aligned}\right\} \tag{4-7b}$$

$$\sum \bar{P}_j^y \bar{x}_j = \bar{I}_y .$$

Equations (4-6) become

$$\bar{P} + M_o \bar{A} = 0 \qquad \bar{M}_x + R_{ox} \bar{I}_x + R_{oy} \bar{I}_{xy} = 0$$

$$\overline{M}_y + R_{ox}\overline{I}_{xy} + R_{oy}\overline{I}_y = 0 .$$

Solving the above equations gives the familiar column analogy equations in terms of the elastic weights of the string polygon:

$$
\left.
\begin{aligned}
M_o &= - \frac{\sum \overline{P}_j^L}{\sum \overline{P}_j^o} = - \frac{\overline{P}}{\overline{A}} \\
R_{oy} &= \frac{\left(\sum \overline{P}_j^x \overline{x}_j\right)\left(\sum \overline{P}_j^L \overline{y}_j\right) - \left(\sum \overline{P}_j^x \overline{y}_j\right)\left(\sum \overline{P}_j^L \overline{x}_j\right)}{\left(\sum \overline{P}_j^x \overline{y}_j\right)\left(\sum \overline{P}_j^y \overline{x}_j\right) - \left(\sum P_j^x \overline{x}_j\right)^2} \\
&= \frac{\overline{I}_{xy}\overline{M}_x - \overline{I}_x \overline{M}_y}{\overline{I}_y \overline{I}_x - I_{xy} I_{xy}} \\
R_{ox} &= \frac{\left(\sum \overline{P}_j^x \overline{x}_j\right)\left(\sum \overline{P}_j^L \overline{x}_j\right) - \left(\sum \overline{P}_j^y \overline{x}_j\right)\left(\sum \overline{P}_j^L \overline{y}_j\right)}{\left(\sum \overline{P}_j^x \overline{y}_j\right)\left(\sum \overline{P}_j^y \overline{x}_j\right) - \left(\sum \overline{P}_j^x \overline{x}_j\right)^2} \\
&= \frac{\overline{I}_{xy}\overline{M}_y - \overline{I}_y \overline{M}_x}{\overline{I}_y \overline{I}_x - \overline{I}_{xy} \overline{I}_{xy}} .
\end{aligned}
\right\} \quad (4\text{-}8)
$$

In the case of a symmetrical structure $\overline{I}_{xy}$ is zero and Equations (4-8) become:

$$\left.\begin{aligned} M_o &= -\frac{\sum \bar{P}_j^L}{\sum \bar{P}_j^o} = -\frac{\bar{P}}{\bar{A}} \\ R_{oy} &= -\frac{\sum \bar{P}_j^L \bar{x}_j}{\sum \bar{P}_j^y \bar{x}_j} = -\frac{\bar{M}_y}{\bar{I}_y} \\ R_{ox} &= -\frac{\sum \bar{P}_j^L \bar{y}_j}{\sum \bar{P}_j^x \bar{y}_j} = -\frac{\bar{M}_x}{\bar{I}_x} \quad . \end{aligned}\right\} \quad (4\text{-}9)$$

4-2. Location of the Elastic Center

Before the theory developed in the preceding section can be applied to a particular structure, the "elastic" properties of the structure must be determined. Again, the string polygon method provides ample tools for this. Equation (4-7) gives

$$\sum \bar{P}_j^o \bar{y}_j = 0$$

$$\sum \bar{P}_j^o \bar{x}_j = 0 \quad .$$

If the origin of the system is temporarily taken as point A (Fig. 4-3), the coordinates of the elastic center O may be designated by (e_o, f_o) as shown. It is seen that

$$x_j = e_o + \bar{x}_j$$

$$y_j = f_o + \bar{y}_j \quad .$$

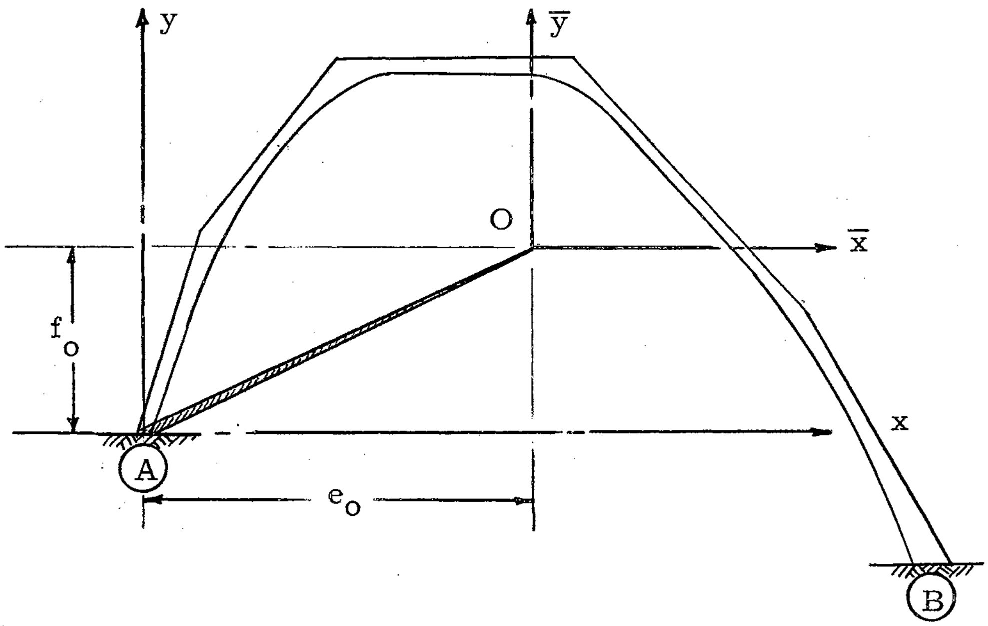


Fig. 4-3. Geometry of the Elastic Center

Substituting gives

$$\sum \bar{P}_j^o (y_j - f_o) = O$$

$$\sum \bar{P}_j^o (x_j - e_o) = O \quad .$$

Hence, the coordinates of the elastic center are:

$$\left.\begin{aligned} f_o &= \frac{\sum \bar{P}^o_j y_j}{\sum \bar{P}^o_j} \\ e_o &= \frac{\sum \bar{P}^o_j x_j}{\sum \bar{P}^o_j} \end{aligned}\right\} \quad (4\text{-}10)$$

It may be desirable in some instances to refer the elastic constants and load functions to the x and y - axes. From Equation (4-7b) the following relationships may be written:

$$\begin{aligned} \bar{M}_x &= \sum \bar{P}^L_j \bar{y}_j \\ &= \sum \bar{P}^L_j (y_j - f_o) \\ &= \sum \bar{P}^L_j y_j - f_o \sum \bar{P}^L_j \\ &= M_{xx} - f_o \bar{P} \end{aligned}$$

... where M_{xx} is the moment of the elastic weights with respect to the x-axis. Similarly,

$$\begin{aligned} \bar{M}_y &= \sum \bar{P}^L_j \bar{x}_j \\ &= \sum P^L_j (x_j - e_o) \\ &= \sum \bar{P}^L_j x_j - e_o \sum \bar{P}^L_j \\ &= M_{yy} - e_o \bar{P} \end{aligned}$$

...where M_{yy} is the moment of the elastic weights with respect to the y-axis.

Similarly,

$$\overline{I}_x = \sum \overline{P}_j^x \overline{y}_j$$

$$= \sum (\overline{y}_i G_{ij} + \overline{y}_j \Sigma F_j + \overline{y}_k G_{kj})(y_j - f_o)$$

$$= \sum \left[(y_i - f_o) G_{ij} + (y_i - f_o) \Sigma F_j + (y_k - f_o) G_{kj} \right] (y_j - f_o)$$

$$= \sum \left[(y_i G_{ij} + y_j \Sigma F_j + y_k G_{kj}) - f_o (G_{ij} + \Sigma F_j + G_{kj}) \right] (y_j - f_o)$$

$$= \sum \left[\overline{P}_j^x - f_o \overline{P}_j^o \right] (y_j - f_o)$$

$$= \sum \overline{P}_j^x y_j - f_o \sum \overline{P}_j^o y_j - f_o \sum \overline{P}_j^x + f_o^2 \sum \overline{P}_j^o$$

...but

$$\sum \overline{P}_j^o y_j = \sum \overline{P}_j^x$$

.. and from Equation (4-10)

$$f_o = \frac{\sum \overline{P}_j^o y_j}{\sum \overline{P}_j^o} \; .$$

Substituting these expressions into the above equations gives

$$\overline{I}_x = \sum \overline{P}_j^x y_j - 2 \frac{\left(\sum \overline{P}_j^o y_j\right)\left(\sum \overline{P}_j^o y_j\right)}{\left(\sum \overline{P}_j^o\right)} \cdot \left(\frac{\sum \overline{P}_j^o}{\sum \overline{P}_j^o}\right)$$

$$+ \left(\frac{\sum \overline{P}_j^o y_j}{\sum \overline{P}_j^o}\right)^2 \cdot \sum \overline{P}_j^o$$

$$= \sum \overline{P}_j^x y_j - f_o^2 \sum \overline{P}_j^o$$

...which may be written

$$\overline{I}_x = I_{xx} - f_o^2 \overline{A}$$

... where I_{xx} may be defined as the elastic moment of inertia referred to the x-axis. If I_{yy} is defined as the elastic moment of inertia with respect to the y-axis, it can be shown in a similar manner that

$$\overline{I}_y = \sum \overline{P}_j^y x_j - e_o^2 \sum \overline{P}_j^o$$

$$= I_{yy} - e_o^2 \overline{A} \quad .$$

In the case of an unsymmetrical structure a transfer equation for the product of inertia is needed. Defining I_{xy} as the product of inertia of the elastic area with respect to the x and y axes, it is seen from Equation (4-7b) that

$$\overline{I}_{xy} = \sum \overline{P}_j^x \overline{x}_j$$

$$= \sum (\overline{y}_i G_{ij} + \overline{y}_j \Sigma F_j + \overline{y}_k G_{kj})(x_j - e_o)$$

$$= \sum \Big[(y_i - f_o) G_{ij} + (y_j - f_o) \Sigma F_j + (y_k - f_o) G_{kj} \Big] (x_j - e_o)$$

$$= \sum \Big[(y_i G_{ij} + y_j \Sigma F_j + y_k G_{kj})(x_j - e_o) \Big]$$

$$- f_o (G_{ij} + \Sigma F_j + G_{kj})(x_j - e_o)$$

$$= \sum \overline{P}_j^x x_j - e_o \sum \overline{P}_j^o y_j - f_o \sum \overline{P}_j^o x_j + e_o f_o \sum \overline{P}_j^o \quad .$$

Substituting Equation (4-10) into the above expression gives

$$\overline{I}_{xy} = \sum \overline{P}_j^x x_j - 2 \frac{\left(\sum \overline{P}_j^o x_j\right)\left(\sum \overline{P}_j^o y_j\right)}{\left(\sum \overline{P}_j^o\right)\left(\sum \overline{P}_j^o\right)} \cdot \sum \overline{P}_j^o$$

$$+ \frac{\left(\sum \overline{P}_j^o x_j\right)\left(\sum \overline{P}_j^o y_j\right)\left(\sum \overline{P}_j^o\right)}{\left(\sum \overline{P}_j^o\right)^2}$$

which becomes

$$\overline{I}_{xy} = I_{xy} - e_o f_o \overline{A} .$$

Summarizing, the transfer equations are

$$\left.\begin{aligned} \overline{M}_x &= M_{xx} - f_o \overline{P} \\ \overline{M}_y &= M_{yy} - e_o \overline{P} \\ \overline{I}_x &= I_{xx} - f_o^2 \overline{A} \\ \overline{I}_y &= I_{yy} - e_o^2 \overline{A} \\ \overline{I}_{xy} &= I_{xy} - e_o f_o \overline{A} . \end{aligned}\right\} \quad (4\text{-}11)$$

CHAPTER V

BEAM CONSTANTS

5-1. General Notes

In order to calculate the elastic load for points on the string polygon of a structure with members of variable moment of inertia, the elastic constants in the elastic-weight expressions of the preceding chapters must be evaluated. Without the use of tables this leads to complex and often tedious approximate integration procedures.

Tuma, Lassley, and French (17) have compiled tables of beam constants for continuous haunched beams which may be used in computing the elastic weights of the string polygon. Data from the IBM 650 Digital Computer that were not used in the above paper were translated, recorded, and added to existing material to give the more complete collection of beam constants found in this chapter.

The meaning and organization of the tables is as follows.

5-2. Table A-O

Beam constants for prismatic beams of constant cross-section are recorded in Table A-O. The geometry of the beam is given by the sketch found at the top of the table. Constants listed in the table are:

a.) Angular Flexibilities (Equation 2-5a)

$$F_{AB} = F_{BA} = F = \frac{L}{3EI_o}$$

b.) Angular Carry-Over Values (Equation 2-5b)

$$G_{AB} = G_{BA} = G = \frac{L}{6EI_o}$$

c.) Angular Load Functions (Equation 2-5c)

1.) Due to a Unit Load

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o} \qquad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to a unit load at L_n.

t_2 = right end slope coefficient due to a unit load at L_n.

2.) Due to a Dead Load

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \frac{bh_o qL^3}{24EI_o}$$

...where

b = width of the beam

h_o = constant depth of the beam

q = specific weight of the beam.

3.) Due to a Uniformly Distributed Load

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{wL^3}{24EI_o}$$

w = intensity of the load.

5-3. Tables A-1, 2, ...9, 10

Beam constants for a prismatic beam with one parabolic haunch are recorded in Tables A-1, 2, ..., 9, 10. The geometry of the beam, as before, is given by the sketch at the top of each table and the parameters,

L_β = length of the haunch,

$h_A = h_o$ = minimum depth,

$h_B = h_o \delta$ = maximum depth.

The beam constant formulas listed in each table are:

a.) Angular Flexibilities (Equation 2-5a)

$$F_{AB} = f_1 \frac{L}{EI_o} \qquad F_{BA} = f_2 \frac{L}{EI_o}$$

f = angular flexibility coefficient.

b.) Carry-Over Values (Equation 2-5b)

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

g = angular carry-over value coefficient.

c.) Angular Load Functions (Equation 2-5c)

1.) Due to a Unit Load

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o} \qquad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to a unit load at L_n.

t_2 = right end slope coefficient due to a unit load at L_n.

2.) Due to Dead Load

$$\tau_{AB}^{(DL)} = t_3 \frac{bh_o q L^3}{EI_o} \qquad \tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

t_3 = left end slope coefficient due to the dead load of the beam.

t_4 = right end slope coefficient due to the dead load of the beam.

3.) Due to a Uniformly Distributed Load

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o} \qquad \tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

t_5 = left end slope coefficient due to a uniformly distributed load.

t_6 = right end slope coefficient due to a uniformly distributed load.

5-4. Tables B-1, 2, 3, 4, 5

Beam constants for a prismatic beam with two symmetrical parabolic haunches are recorded in Tables B-1, 2, 3, 4, 5. The geometry is defined by the sketch and parameters

L_β = length of the haunch,

$h_A = h_B = h_o\delta$ = maximum depth,

$h_c = h_o$ = minimum depth.

The beam constant formulas listed in each table are:

a.) Angular Flexibilities (Equation 2-5a)

$$F_{AB} = F_{BA} = F = f\frac{L}{EI_o}$$

f = angular flexibility coefficient.

b.) Carry-Over Values (Equation 2-5b)

$$G_{AB} = G_{BA} = G = g\frac{L}{EI_o}$$

g = angular carry-over value coefficient.

c.) Angular Load Functions (Equation 2-5c)

1.) Due to a Unit Load

$$\tau_{AB}^{(LL)} = t_1\frac{L^2}{EI_o} \qquad \tau_{BA}^{(LL)} = t_2\frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to a unit load at L_n.

t_2 = right end slope coefficient due to a unit load at L_n.

From the symmetry of the beam,

t_1 due to a unit load at L_n = t_2 due to a unit load at $L(1-n)$.

2.) Due to Dead Load

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3\frac{bh_o qL^3}{EI_o}$$

t_3 = end slope coefficient due to the dead load of the beam.

3.) Due to a Uniformly Distributed Load

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_5\frac{wL^3}{EI_o}$$

t_5 = end slope coefficient due to a uniformly distributed load.

5-5. Members with Unsymmetrical Haunches

Lengths of haunched members of rigid frames are defined as the distances between the intersection of the axes of the members. To obtain the effective depth of a haunched member at its haunched end extend the haunch to a line drawn perpendicular to the axis of the member at its point of intersection with the axis of the adjacent member (see Fig. 5-1).

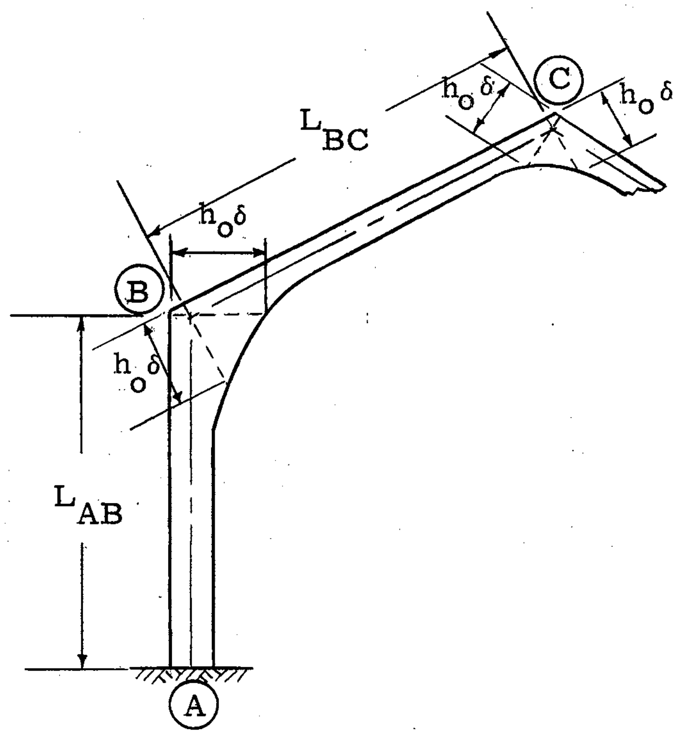


Fig. 5-1. Effective Dimensions of Haunched Members

Occasionally the structural engineer finds it necessary to analyze members with unsymmetrical haunches. The beam constants tabulated in this chapter may be used for such a problem.

Leontovich (18) and Guldan (19) are among those who have shown that beam constants for unsymmetrically haunched beams may be found by simply superimposing the constants of component parts of the member. The procedure involved may best be described by an example.

Consider a structural member with unsymmetrical haunches (Fig. 5-2).

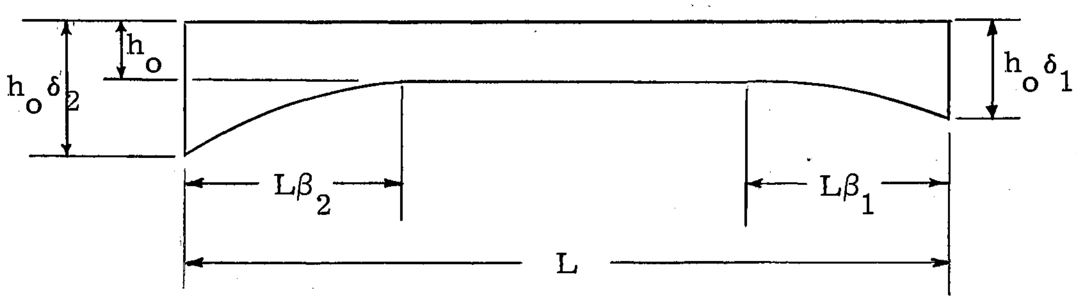


Fig. 5-2. Unsymmetrical Haunched Beam

To find, for instance, the angular flexibilities F_{AB}, F_{BA}, and the angular carry-over values G_{AB}, G_{BA} of the beam shown, obtain from the tables constants for beam number one and two (Fig. 5-3) and for a beam of constant cross-section and with an equal minimum depth, h_o. The desired constants are found by simply adding those of beams numbered one and two, and substracting from this sum the appropriate constants of the beam of constant cross-section, as shown in Fig. 5-3.

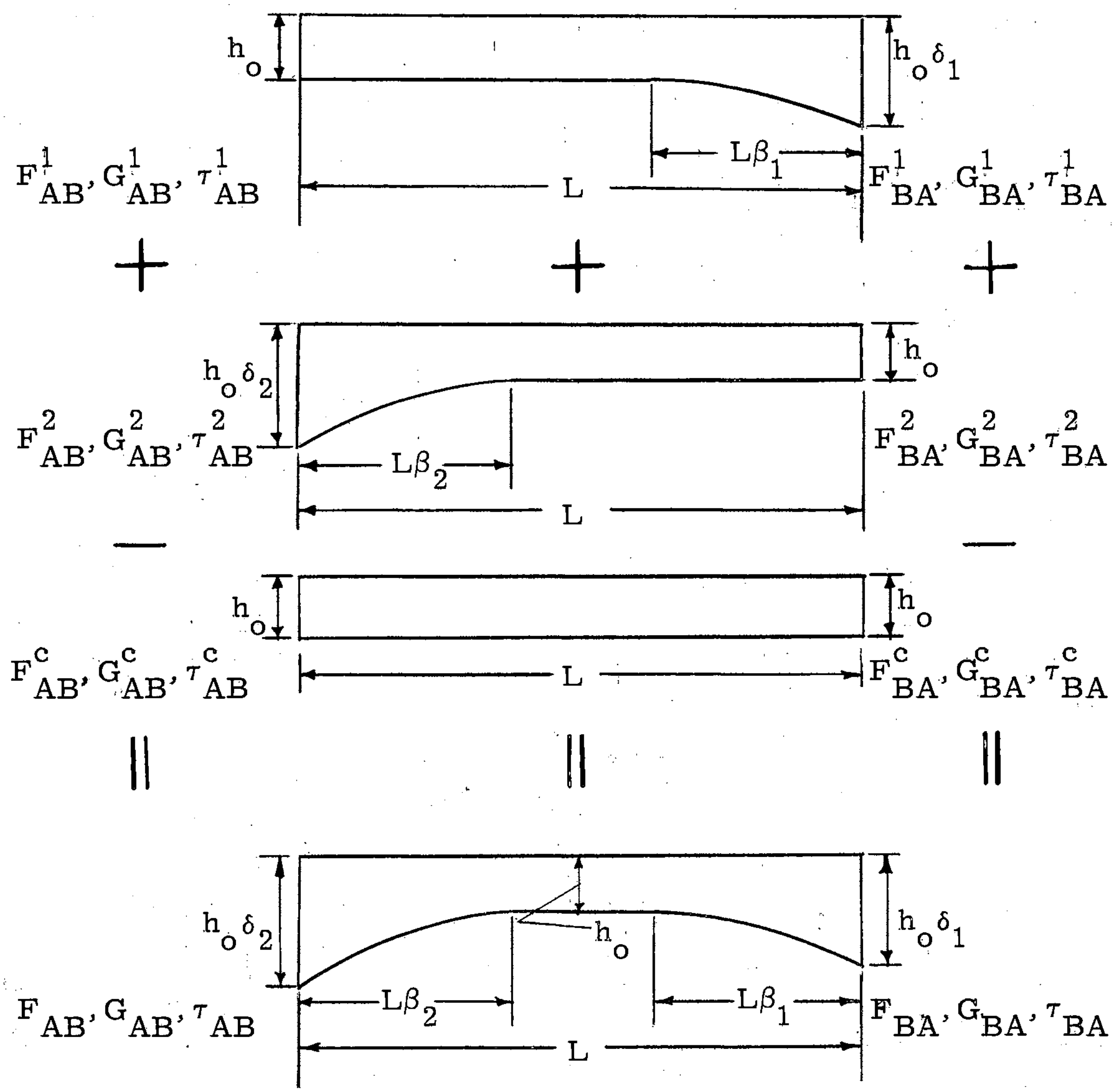
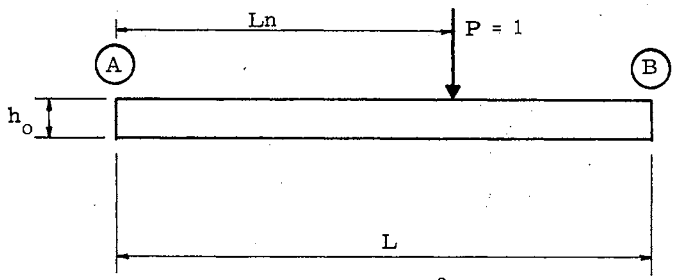


Fig. 5-3. Superposition of Beam Constants

5-6. Beam Constant Tables

TABLE A-0

$\beta = 0.0$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = \frac{L}{3\,EI_o}$$

$$G_{AB} = G_{BA} = \frac{L}{6\,EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \frac{bh_o q L^3}{24\,EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{wL^3}{24\,EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficients t_1

n	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0033	.0065	.0096	.0125	.0154	.0182	.0209	.0236	.0261
0.1	.0285	.0309	.0331	.0353	.0373	.0393	.0412	.0430	.0448	.0464
0.2	.0480	.0495	.0509	.0522	.0535	.0547	.0558	.0568	.0578	.0587
0.3	.0595	.0603	.0609	.0615	.0621	.0626	.0630	.0633	.0636	.0638
0.4	.0640	.0641	.0642	.0641	.0641	.0639	.0638	.0635	.0632	.0629
0.5	.0625	.0621	.0616	.0610	.0604	.0598	.0591	.0584	.0577	.0569
0.6	.0560	.0551	.0542	.0532	.0522	.0512	.0501	.0490	.0479	.0467
0.7	.0455	.0443	.0430	.0417	.0404	.0391	.0377	.0363	.0349	.0335
0.8	.0320	.0305	.0290	.0275	.0260	.0244	.0229	.0213	.0197	.0181
0.9	.0165	.0149	.0133	.0116	.0100	.0083	.0067	.0050	.0033	.0017

Influence Coefficients t_2

n	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0017	.0033	.0050	.0067	.0083	.0100	.0116	.0133	.0149
0.1	.0165	.0181	.0197	.0213	.0229	.0244	.0260	.0275	.0290	.0305
0.2	.0320	.0335	.0349	.0363	.0377	.0391	.0404	.0417	.0430	.0443
0.3	.0455	.0467	.0479	.0490	.0501	.0512	.0522	.0532	.0542	.0551
0.4	.0560	.0569	.0577	.0584	.0591	.0598	.0604	.0610	.0616	.0621
0.5	.0625	.0629	.0632	.0635	.0638	.0639	.0641	.0641	.0642	.0641
0.6	.0640	.0638	.0636	.0633	.0630	.0626	.0621	.0615	.0609	.0603
0.7	.0595	.0587	.0578	.0568	.0558	.0547	.0535	.0522	.0509	.0495
0.8	.0480	.0464	.0448	.0430	.0412	.0393	.0373	.0353	.0331	.0309
0.9	.0285	.0261	.0236	.0209	.0182	.0154	.0125	.0096	.0065	.0033

TABLE A-1
$\beta = 0.1$

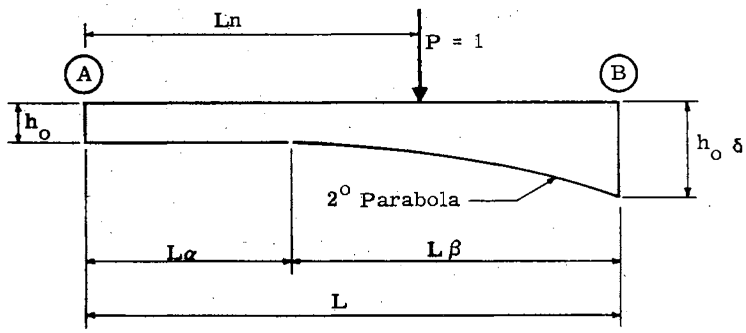
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$



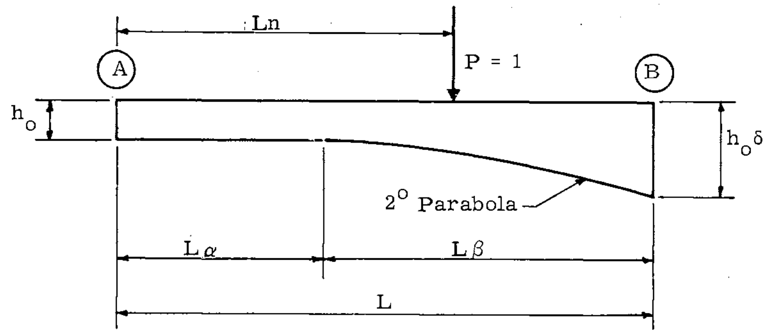
$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1664	.0417	.0417
1.2	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1663	.0417	.0417
1.3	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1661	.0417	.0417
1.4	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1659	.0417	.0417
1.5	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1658	.0417	.0416
1.6	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1657	.0417	.0416
1.7	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1656	.0417	.0416
1.8	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1655	.0418	.0416
1.9	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1654	.0418	.0416
2.0	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0164	.3333	.1653	.0418	.0416
2.1	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0319	.0164	.3333	.1652	.0418	.0416
2.2	.0285	.0480	.0595	.0640	.0625	.0560	.0454	.0319	.0164	.3333	.1652	.0418	.0416
2.3	.0285	.0480	.0595	.0640	.0625	.0560	.0454	.0319	.0164	.3333	.1650	.0418	.0416
2.4	.0285	.0480	.0595	.0640	.0625	.0560	.0454	.0319	.0164	.3333	.1650	.0418	.0416
2.5	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1650	.0418	.0416
2.6	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1649	.0418	.0416
2.7	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1649	.0419	.0416
2.8	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1648	.0419	.0416
2.9	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1648	.0419	.0416
3.0	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1647	.0419	.0416

TABLE A-1

β = 0.1



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0165	.0320	.0454	.0559	.0624	.0639	.0593	.0478	.0283	.3249	.1664	.0416	.0416
1.2	.0165	.0319	.0454	.0558	.0623	.0638	.0592	.0477	.0281	.3181	.1663	.0415	.0415
1.3	.0164	.0319	.0453	.0558	.0622	.0637	.0591	.0475	.0280	.3125	.1661	.0415	.0414
1.4	.0164	.0319	.0453	.0557	.0621	.0636	.0590	.0474	.0278	.3078	.1659	.0414	.0413
1.5	.0164	.0318	.0452	.0557	.0621	.0635	.0589	.0473	.0277	.3039	.1658	.0414	.0413
1.6	.0164	.0318	.0452	.0556	.0620	.0634	.0588	.0472	.0276	.3005	.1657	.0413	.0412
1.7	.0164	.0318	.0452	.0556	.0620	.0633	.0588	.0471	.0275	.2975	.1655	.0413	.0411
1.8	.0164	.0318	.0451	.0555	.0619	.0633	.0587	.0470	.0274	.2949	.1655	.0413	.0411
1.9	.0164	.0317	.0451	.0555	.0619	.0632	.0586	.0470	.0273	.2926	.1654	.0413	.0411
2.0	.0164	.0317	.0451	.0555	.0618	.0632	.0586	.0469	.0273	.2906	.1653	.0413	.0410
2.1	.0164	.0317	.0451	.0554	.0618	.0631	.0585	.0468	.0272	.2887	.1652	.0412	.0410
2.2	.0164	.0317	.0450	.0554	.0617	.0631	.0584	.0468	.0271	.2871	.1652	.0412	.0409
2.3	.0163	.0317	.0450	.0554	.0617	.0631	.0584	.0467	.0271	.2856	.1651	.0412	.0409
2.4	.0163	.0317	.0450	.0553	.0617	.0630	.0583	.0467	.0270	.2842	.1650	.0412	.0409
2.5	.0163	.0317	.0450	.0553	.0616	.0630	.0583	.0466	.0270	.2830	.1650	.0412	.0409
2.6	.0163	.0316	.0450	.0553	.0616	.0629	.0583	.0466	.0269	.2818	.1649	.0412	.0409
2.7	.0163	.0316	.0450	.0553	.0616	.0629	.0582	.0466	.0269	.2807	.1649	.0412	.0408
2.8	.0163	.0316	.0449	.0553	.0616	.0629	.0582	.0465	.0268	.2797	.1648	.0412	.0408
2.9	.0163	.0316	.0449	.0552	.0615	.0629	.0582	.0465	.0268	.2788	.1648	.0412	.0408
3.0	.0163	.0316	.0449	.0552	.0615	.0628	.0581	.0464	.0267	.2778	.1647	.0412	.0407

TABLE A-2
β=0.2

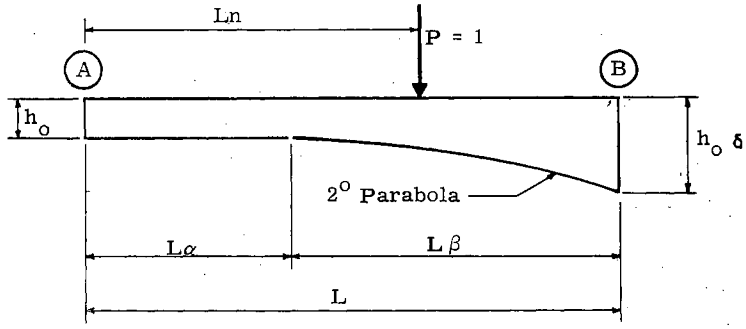
$F_{AB} = f_1 \frac{L}{E I_o}$

$G_{AB} = g \frac{L}{E I_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$

$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$

$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$



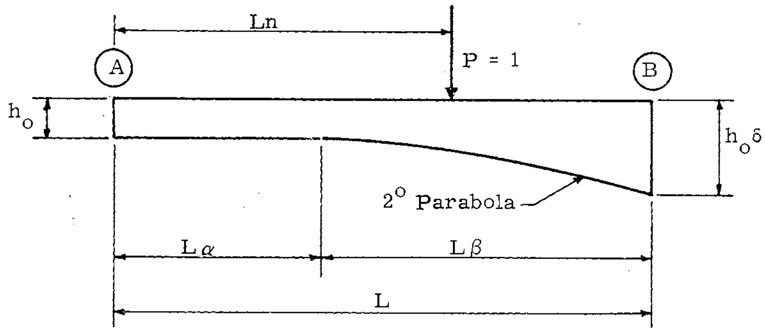
$I_o = \frac{b h_o^3}{12}$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0285	.0480	.0595	.0640	.0625	.0560	.0454	.0319	.0164	.3333	.1658	.0416	.0417
1.2	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1651	.0416	.0417
1.3	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0318	.0163	.3331	.1644	.0417	.0416
1.4	.0285	.0480	.0594	.0639	.0624	.0558	.0453	.0318	.0163	.3331	.1639	.0418	.0415
1.5	.0285	.0480	.0594	.0639	.0623	.0558	.0453	.0317	.0163	.3330	.1634	.0418	.0415
1.6	.0285	.0480	.0594	.0639	.0623	.0558	.0452	.0317	.0162	.3330	.1629	.0418	.0415
1.7	.0285	.0479	.0594	.0638	.0623	.0558	.0452	.0317	.0162	.3329	.1625	.0419	.0415
1.8	.0285	.0479	.0594	.0638	.0623	.0557	.0452	.0316	.0162	.3329	.1621	.0419	.0415
1.9	.0285	.0479	.0594	.0638	.0623	.0557	.0452	.0316	.0161	.3328	.1618	.0419	.0414
2.0	.0284	.0479	.0593	.0638	.0622	.0557	.0451	.0316	.0161	.3328	.1614	.0420	.0414
2.1	.0284	.0479	.0593	.0638	.0622	.0557	.0451	.0316	.0161	.3328	.1612	.0420	.0414
2.2	.0284	.0479	.0593	.0638	.0622	.0556	.0451	.0315	.0161	.3327	.1609	.0420	.0414
2.3	.0284	.0479	.0593	.0638	.0622	.0556	.0451	.0315	.0160	.3327	.1607	.0421	.0414
2.4	.0284	.0479	.0593	.0637	.0622	.0556	.0450	.0315	.0160	.3327	.1604	.0421	.0414
2.5	.0284	.0479	.0593	.0637	.0622	.0556	.0450	.0315	.0160	.3327	.1602	.0422	.0414
2.6	.0284	.0479	.0593	.0637	.0621	.0556	.0450	.0314	.0160	.3326	.1600	.0422	.0414
2.7	.0284	.0479	.0593	.0637	.0621	.0556	.0450	.0314	.0160	.3326	.1598	.0422	.0413
2.8	.0284	.0478	.0593	.0637	.0621	.0555	.0450	.0314	.0159	.3326	.1596	.0423	.0413
2.9	.0284	.0478	.0593	.0637	.0621	.0555	.0450	.0314	.0159	.3326	.1594	.0423	.0413
3.0	.0284	.0478	.0593	.0637	.0621	.0555	.0449	.0314	.0159	.3325	.1593	.0424	.0413

TABLE A-2

β = 0.2



$$I_o = \frac{b h_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{E I_o}$$

$$G_{BA} = g \frac{L}{E I_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{E I_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{w L^3}{E I_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0164	.0318	.0452	.0556	.0621	.0635	.0589	.0473	.0278	.3173	.1658	.0414	.0413
1.2	.0163	.0317	.0450	.0554	.0617	.0630	.0584	.0467	.0272	.3044	.1651	.0411	.0409
1.3	.0163	.0316	.0448	.0551	.0614	.0627	.0579	.0462	.0266	.2939	.1644	.0409	.0406
1.4	.0162	.0314	.0447	.0549	.0611	.0623	.0575	.0458	.0262	.2851	.1639	.0407	.0404
1.5	.0162	.0313	.0445	.0547	.0609	.0620	.0572	.0454	.0258	.2777	.1634	.0406	.0402
1.6	.0161	.0313	.0444	.0545	.0606	.0618	.0569	.0450	.0254	.2714	.1629	.0405	.0400
1.7	.0161	.0312	.0443	.0543	.0604	.0615	.0566	.0447	.0251	.2659	.1625	.0404	.0398
1.8	.0160	.0311	.0441	.0542	.0602	.0613	.0563	.0444	.0248	.2611	.1621	.0403	.0396
1.9	.0160	.0310	.0440	.0540	.0601	.0611	.0561	.0441	.0245	.2569	.1618	.0402	.0394
2.0	.0160	.0310	.0439	.0539	.0599	.0609	.0559	.0438	.9243	.2532	.1615	.0401	.0393
2.1	.0160	.0309	.0439	.0538	.0598	.0607	.0557	.0436	.0240	.2498	.1612	.0401	.0392
2.2	.0159	.0308	.0438	.0537	.0596	.0605	.0555	.0434	.0238	.2468	.1609	.0400	.0391
2.3	.0159	.0308	.0437	.0536	.0595	.0604	.0553	.0432	.0236	.2440	.1607	.0400	.0389
2.4	.0159	.0308	.0436	.0535	.0594	.0603	.0551	.0430	.0235	.2415	.1604	.0399	.0388
2.5	.0159	.0307	.0436	.0534	.0593	.0601	.0550	.0428	.0233	.2392	.1602	.0399	.0387
2.6	.0158	.0307	.0435	.0533	.0592	.0600	.0548	.0427	.0231	.2371	.1600	.0399	.0386
2.7	.0158	.0306	.0434	.0533	.0591	.0599	.0547	.0425	.0230	.2352	.1598	.0399	.0386
2.8	.0158	.0306	.0434	.0532	.0590	.0598	.0546	.0424	.0229	.2334	.1596	.0398	.0385
2.9	.0158	.0306	.0433	.0531	.0589	.0597	.0544	.0422	.0227	.2317	.1594	.0398	.0384
3.0	.0158	.0305	.0433	.0530	.0588	.0596	.0543	.0421	.0226	.2302	.1593	.0398	.0383

TABLE A-3
β = 0.3

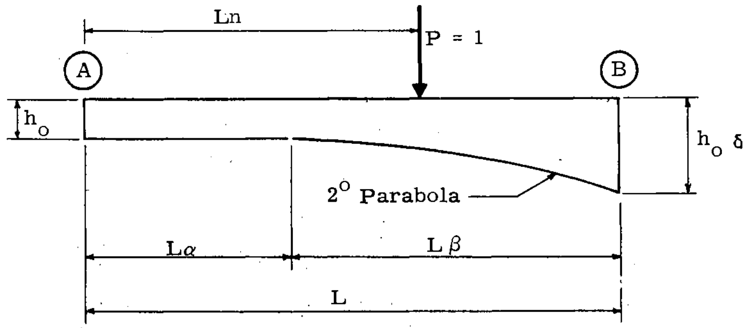
$F_{AB} = f_1 \frac{L}{E I_o}$

$G_{AB} = g \frac{L}{E I_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$

$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$

$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$



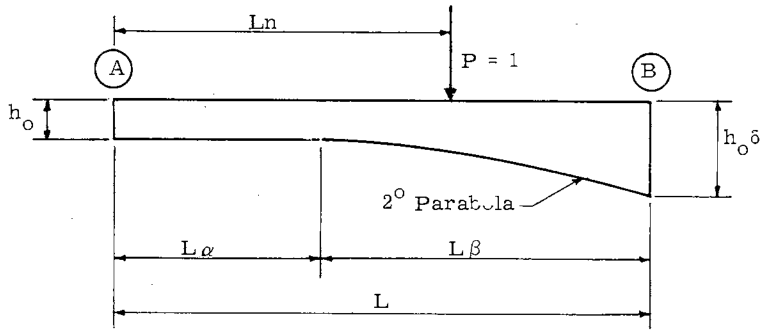
$I_o = \frac{b h_o^3}{12}$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0285	.0479	.0594	.0639	.0624	.0558	.0453	.0318	.0163	.3331	.1648	.0417	.0416
1.2	.0285	.0479	.0594	.0638	.0623	.0557	.0452	.0316	.0162	.3328	.1633	.0417	.0415
1.3	.0284	.0479	.0593	.0637	.0622	.0556	.0450	.0315	.0161	.3326	.1619	.0417	.0414
1.4	.0284	.0478	.0592	.0636	.0621	.0555	.0449	.0313	.0160	.3324	.1607	.0418	.0413
1.5	.0284	.0478	.0592	.0636	.0620	.0554	.0448	.0312	.0159	.3323	.1596	.0418	.0412
1.6	.0284	.0478	.0591	.0635	.0619	.0553	.0446	.0311	.0158	.3321	.1586	.0419	.0412
1.7	.0284	.0477	.0591	.0634	.0618	.0552	.0445	.0309	.0157	.3320	.1578	.0419	.0411
1.8	.0283	.0477	.0590	.0634	.0617	.0551	.0444	.0308	.0157	.3318	.1570	.0420	.0410
1.9	.0283	.0477	.0590	.0633	.0617	.0550	.0443	.0307	.0156	.3317	.1562	.0420	.0410
2.0	.0283	.0476	.0590	.0633	.0616	.0549	.0443	.0306	.0155	.3316	.1556	.0421	.0409
2.1	.0283	.0476	.0589	.0632	.0616	.0549	.0442	.0306	.0155	.3314	.1550	.0421	.0409
2.2	.0283	.0476	.0589	.0632	.0615	.0548	.0441	.0305	.0154	.3313	.1544	.0422	.0408
2.3	.0283	.0476	.0589	.0632	.0614	.0547	.0440	.0304	.0154	.3312	.1538	.0423	.0408
2.4	.0283	.0476	.0588	.0631	.0614	.0547	.0440	.0303	.0153	.3311	.1533	.0424	.0408
2.5	.0283	.0475	.0588	.0631	.0614	.0546	.0439	.0303	.0153	.3310	.1529	.0424	.0407
2.6	.0283	.0475	.0588	.0630	.0613	.0546	.0438	.0302	.0152	.3309	.1524	.0425	.0407
2.7	.0283	.0475	.0588	.0630	.0613	.0545	.0438	.0301	.0152	.3309	.1520	.0426	.0406
2.8	.0282	.0475	.0587	.0630	.0612	.0545	.0437	.0301	.0151	.3308	.1516	.0426	.0406
2.9	.0282	.0475	.0587	.0630	.0612	.0544	.0437	.0300	.0151	.3307	.1513	.0427	.0406
3.0	.0282	.0475	.0587	.0629	.0611	.0544	.0436	.0299	.0151	.3306	.1509	.0428	.0405

TABLE A-3

$\beta = 0.3$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0163	.0316	.0450	.0553	.0616	.0629	.0582	.0466	.0272	.3105	.1648	.0411	.0409
1.2	.0162	.0313	.0445	.0546	.0608	.0620	.0571	.0453	.0261	.2922	1633	0406	.0402
1.3	.0160	.0310	.0441	.0541	.0601	.0611	.0562	.0442	.0252	.2774	.1619	.0401	.0396
1.4	.0159	.0308	.0437	.0536	.0595	.0604	.0553	.0433	.0244	.2651	.1607	.0398	.0390
1.5	.0158	.0306	.0434	.0532	.0590	.0598	.0546	.0425	.0237	.2548	.1596	.0394	.0386
1.6	.0157	.0304	.0431	.0528	.0585	.0592	.0539	.0417	.0231	.2459	.1586	.0392	.0382
1.7	.0156	.0302	.0428	.0524	.0580	.0587	.0533	.0410	.0225	.2383	.1578	.0389	.0378
1.8	.0155	.0301	.0426	.0521	.0577	.0582	.0527	.0404	.0220	.2317	.1570	.0387	.0375
1.9	.0155	.0299	.0424	.0518	.0573	.0577	.0522	.0398	.0216	.2258	.1562	.0386	.0371
2.0	.0154	.0298	.0422	.0516	.0570	.0573	.0517	.0393	.0212	.2207	.1556	.0384	.0369
2.1	.0153	.0297	.0420	.0513	.0566	.0570	.0513	.0388	.0208	.2160	.1550	.0383	.0366
2.2	.0153	.0295	.0418	.0511	.0564	.0566	.0509	.0384	.0205	.2119	.1544	.0382	.0364
2.3	.0152	.0294	.0417	.0509	.0561	.0563	.0505	.0380	.0202	.2081	.1538	.0381	.0361
2.4	.0152	.0293	.0415	.0507	.0558	.0560	.0502	.0376	.0199	.2047	.1533	.0380	.0359
2.5	.0151	.0292	.0414	.0505	.0556	.0557	.0499	.0373	.0197	.2016	.1529	.0379	.0357
2.6	.0151	.0292	.0412	.0503	.0554	.0555	.0495	.0369	.0194	.1987	.1524	.0378	.0355
2.7	.0150	.0291	.0411	.0501	.0552	.0552	.0493	.0366	.0192	.1961	.1520	.0378	.0354
2.8	.0150	.0290	.0410	.0500	.0550	.0550	.0490	.0363	.0190	.1936	.1516	.0377	.0352
2.9	.0150	.0289	.0409	.0498	.0548	.0548	.0487	.0360	.0188	.1914	.1513	.0377	.0351
3.0	.0149	.0289	.0408	.0497	.0546	.0546	.0485	.0358	.0186	.1894	.1509	.0377	.0349

TABLE A-4
β = 0.4

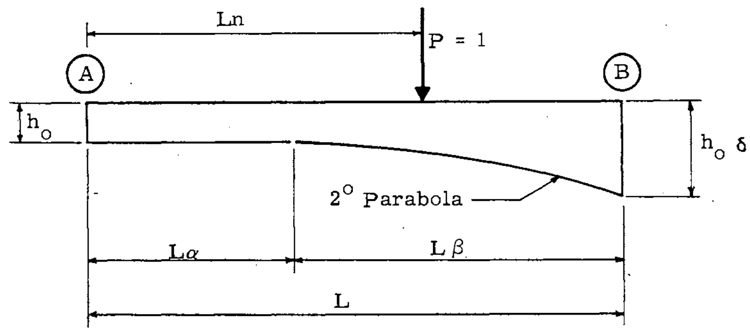
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$



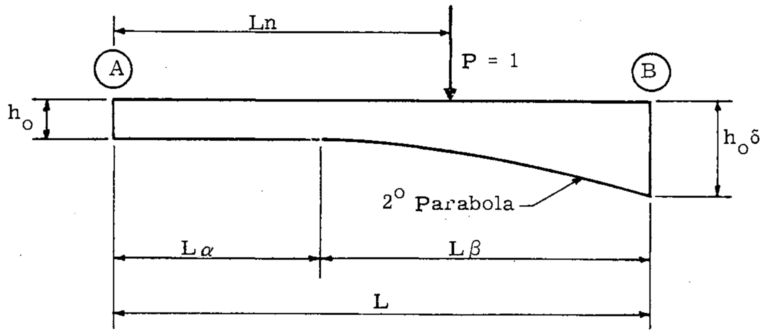
$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0284	.0479	.0593	.0638	.0622	.0556	.0451	.0316	.0162	.3327	.1636	.0416	.0414
1.2	.0284	.0478	.0592	.0635	.0619	.0553	.0447	.0312	.0160	.3322	.1609	.0416	.0412
1.3	.0283	.0477	.0590	.0633	.0617	.0550	.0444	.0309	.0158	.3317	.1586	.0416	.0410
1.4	.0283	.0476	.0589	.0632	.0615	.0547	.0441	.0306	.0156	.3312	.1566	.0417	.0408
1.5	.0282	.0475	.0587	.0630	.0612	.0545	.0438	.0303	.0154	.3308	.1547	.0417	.0407
1.6	.0282	.0474	.0586	.0628	.0610	.0543	.0435	.0301	.0153	.3304	.1531	.0417	.0405
1.7	.0282	.0473	.0585	.0627	.0609	.0540	.0433	.0298	.0151	.3301	.1517	.0418	.0404
1.8	.0281	.0473	.0584	.0626	.0607	.0538	.0430	.0296	.0150	.3297	.1503	.0418	.0403
1.9	.0281	.0472	.0583	.0624	.0605	.0537	.0428	.0294	.0149	.3294	.1491	.0419	.0401
2.0	.0281	.0472	.0582	.0623	.0604	.0535	.0426	.0292	.0148	.3291	.1480	.0420	.0400
2.1	.0281	.0471	.0582	.0622	.0603	.0533	.0424	.0291	.0147	.3289	.1470	.0421	.0399
2.2	.0280	.0471	.0581	.0621	.0601	.0532	.0422	.0289	.0146	.3286	.1460	.0421	.0398
2.3	.0280	.0470	.0580	.0620	.0600	.0530	.0421	.0288	.0145	.3283	.1451	.0422	.0397
2.4	.0280	.0470	.0579	.0619	.0599	.0529	.0419	.0286	.0144	.3281	.1443	.0423	.0396
2.5	.0280	.0469	.0579	.0618	.0598	.0527	.0418	.0285	.0143	.3279	.1435	.0424	.0396
2.6	.0279	.0469	.0578	.0617	.0597	.0526	.0416	.0283	.0143	.3277	.1428	.0425	.0395
2.7	.0279	.0468	.0577	.0617	.0596	.0525	.0415	.0282	.0142	.3275	.1421	.0426	.0394
2.8	.0279	.0468	.0577	.0616	.0595	.0524	.0413	.0281	.0141	.3273	.1415	.0427	.0393
2.9	.0279	.0468	.0576	.0615	.0594	.0523	.0412	.0280	.0141	.3271	.1409	.0428	.0393
3.0	.0279	.0467	.0576	.0614	.0593	.0521	.0411	.0279	.0140	.3269	.1403	.0429	.0392

TABLE A-4

$\beta = 0.4$



$$I_o = \frac{b h_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{E I_o}$$

$$G_{BA} = g \frac{L}{E I_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{E I_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{w L^3}{E I_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0162	.0314	.0446	.0548	.0609	.0621	.0573	.0457	.0267	.3044	.1636	.0407	.0404
1.2	.0159	.0308	.0438	.0537	.0596	.0605	.0555	.0437	.0251	.2815	.1609	.0399	.0392
1.3	.0157	.0304	.0431	.0528	.0585	.0592	.0539	.0420	.0238	.2629	.1586	.0392	.0383
1.4	.0155	.0300	.0425	.0520	.0574	.0579	.0525	.0406	.0227	.2476	.1566	.0386	.0374
1.5	.0153	.0296	.0419	.0512	.0565	.0568	.0512	.0393	.0218	.2347	.1547	.0381	.0367
1.6	.0151	.0293	.0414	.0506	.0557	.0559	.0501	.0381	.0210	.2238	.1531	.0376	.0360
1.7	.0150	.0290	.0410	.0500	.0550	.0550	.0491	.0371	.0202	.2144	.1517	.0372	.0354
1.8	.0149	.0287	.0406	.0495	.0543	.0542	.0481	.0362	.0196	.2062	.1503	.0369	.0349
1.9	.0147	.0285	.0402	.0490	.0537	.0535	.0473	.0353	.0190	.1990	.1491	.0366	.0344
2.0	.0146	.0283	.0399	.0485	.0532	.0528	.0465	.0346	.0185	.1927	.1480	.0363	.0340
2.1	.0145	.0281	.0396	.0481	.0526	.0522	.0458	.0339	.0180	.1871	.1470	.0361	.0336
2.2	.0144	.0279	.0393	.0477	.0522	.0516	.0452	.0332	.0176	.1820	.1460	.0359	.0332
2.3	.0143	.0277	.0390	.0474	.0517	.0511	.0445	.0326	.0172	.1775	.1451	.0357	.0328
2.4	.0143	.0275	.0388	.0471	.0513	.0506	.0440	.0321	.0169	.1733	.1443	.0355	.0325
2.5	.0142	.0274	.0386	.0467	.0509	.0501	.0434	.0316	.0165	.1696	.1435	.0354	.0322
2.6	.0141	.0272	.0383	.0465	.0506	.0497	.0429	.0311	.0162	.1662	.1428	.0353	.0319
2.7	.0140	.0271	.0381	.0462	.0502	.0493	.0425	.0306	.0160	.1630	.1421	.0352	.0317
2.8	.0140	.0270	.0379	.0459	.0499	.0489	.0420	.0302	.0157	.1601	.1415	.0351	.0314
2.9	.0139	.0268	.0378	.0457	.0496	.0485	.0416	.0298	.0155	.1574	.1409	.0350	.0312
3.0	.0139	.0267	.0376	.0454	.0493	.0482	.0412	.0295	.0152	.1550	.1403	.0349	.0310

TABLE A-5
β=0.5

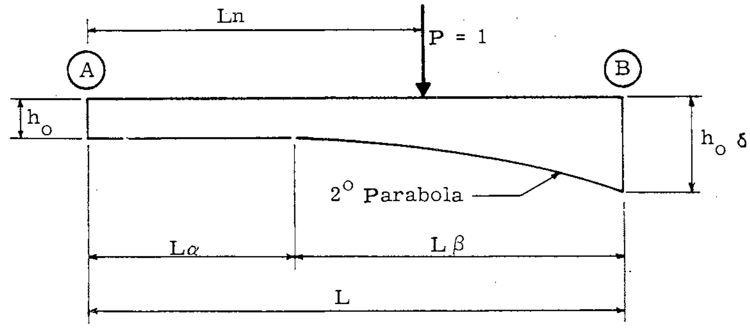
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$



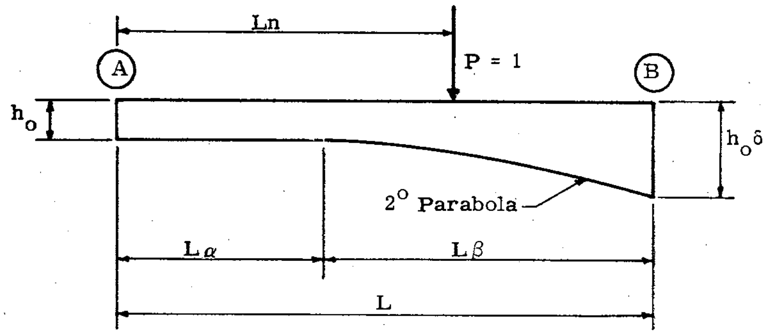
$$I_o = \frac{bh_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0284	.0478	.0591	.0635	.0619	.0552	.0447	.0313	.0161	.3321	.1621	.0415	.0412
1.2	.0283	.0476	.0588	.0631	.0614	.0547	.0440	.0307	.0157	.3311	.1581	.0415	.0408
1.3	.0282	.0474	.0585	.0627	.0609	.0541	.0434	.0301	.0154	.3301	.1547	.0414	.0405
1.4	.0281	.0472	.0583	.0624	.0604	.0536	.0429	.0297	.0151	.3292	.1517	.0414	.0402
1.5	.0280	.0470	.0580	.0620	.0600	.0531	.0423	.0292	.0148	.3284	.1490	.0414	.0399
1.6	.0279	.0469	.0578	.0617	.0597	.0526	.0419	.0288	.0146	.3277	.1466	.0414	.0396
1.7	.0279	.0467	.0576	.0615	.0593	.0522	.0414	.0285	.0144	.3270	.1445	.0414	.0393
1.8	.0278	.0466	.0574	.0612	.0590	.0518	.0410	.0281	.0142	.3263	.1425	.0414	.0391
1.9	.0277	.0465	.0572	.0609	.0587	.0515	.0407	.0278	.0140	.3257	.1408	.0415	.0389
2.0	.0277	.0464	.0570	.0607	.0584	.0511	.0403	.0275	.0139	.3251	.1391	.0415	.0387
2.1	.0276	.0463	.0569	.0605	.0581	.0508	.0400	.0272	.0137	.3246	.1376	.0416	.0385
2.2	.0276	.0461	.0567	.0603	.0579	.0505	.0397	.0270	.0136	.3241	.1362	.0416	.0383
2.3	.0275	.0461	.0566	.0601	.0576	.0502	.0394	.0268	.0135	.3236	.1349	.0418	.0382
2.4	.0275	.0460	.0564	.0599	.0574	.0499	.0391	.0265	.0134	.3231	.1337	.0418	.0380
2.5	.0274	.0459	.0563	.0597	.0572	.0497	.0388	.0263	.0132	.3227	.1326	.0419	.0378
2.6	.0274	.0458	.0562	.0596	.0570	.0494	.0386	.0261	.0131	.3223	.1316	.0420	.0377
2.7	.0274	.0457	.0561	.0594	.0568	.0492	.0383	.0260	.0130	.3219	.1306	.0421	.0376
2.8	.0273	.0456	.0560	.0593	.0566	.0490	.0381	.0258	.0130	.3215	.1297	.0422	.0374
2.9	.0273	.0456	.0558	.0591	.0564	.0488	.0379	.0256	.0129	.3211	.1288	.0424	.0373
3.0	.0272	.0455	.0557	.0590	.0562	.0486	.0377	.0255	.0128	.3208	.1279	.0425	.0372

TABLE A-5

$\beta = 0.5$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0160	.0311	.0441	.0542	.0602	.0612	.0564	.0448	.0262	.2991	.1621	.0403	.0398
1.2	.0156	.0303	.0429	.0526	.0582	.0589	.0537	.0422	.0242	.2721	.1581	.0391	.0382
1.3	.0153	.0296	.0419	.0512	.0565	.0568	.0514	.0399	.0226	.2502	.1547	.0381	.0369
1.4	.0150	.0290	.0410	.0500	.0550	.0550	.0494	.0380	.0213	.2323	.1517	.0373	.0357
1.5	.0147	.0285	.0402	.0489	.0537	.0534	.0476	.0363	.0201	.2173	.1490	.0365	.0347
1.6	.0145	.0280	.0395	.0480	.0525	.0520	.0460	.0348	.0191	.2046	.1466	.0359	.0337
1.7	.0143	.0276	.0388	.0471	.0514	.0507	.0446	.0334	.0182	.1937	.1445	.0353	.0329
1.8	.0141	.0272	.0383	.0463	.0504	.0496	.0433	.0323	.0174	.1843	.1425	.0348	.0322
1.9	.0139	.0268	.0377	.0456	.0495	.0485	.0422	.0312	.0168	.1761	.1408	.0344	.0315
2.0	.0137	.0265	.0372	.0450	.0487	.0475	.0411	.0302	.0161	.1689	.1391	.0340	.0309
2.1	.0136	.0262	.0368	.0444	.0480	.0466	.0401	.0293	.0156	.1625	.1376	.0336	.0303
2.2	.0135	.0259	.0364	.0438	.0473	.0458	.0392	.0285	.0151	.1567	.1362	.0333	.0298
2.3	.0133	.0257	.0360	.0433	.0466	.0450	.0384	.0278	.0147	.1516	.1349	.0330	.0293
2.4	.0132	.0254	.0356	.0428	.0460	.0443	.0376	.0271	.0142	.1469	.1337	.0328	.0289
2.5	.0131	.0252	.0353	.0424	.0455	.0437	.0369	.0265	.0139	.1427	.1326	.0326	.0285
2.6	.0130	.0250	.0350	.0420	.0450	.0430	.0362	.0259	.0135	.1389	.1316	.0324	.0281
2.7	.0129	.0248	.0347	.0416	.0445	.0424	.0356	.0254	.0132	.1353	.1306	.0322	.0277
2.8	.0128	.0246	.0344	.0412	.0440	.0419	.0350	.0249	.0129	.1321	.1297	.0320	.0274
2.9	.0127	.0244	.0341	.0408	.0436	.0414	.0345	.0244	.0126	.1291	.1288	.0319	.0271
3.0	.0126	.0243	.0339	.0405	.0431	.0409	.0340	.0240	.0124	.1263	.1279	.0317	.0268

TABLE A-6
β=0.6

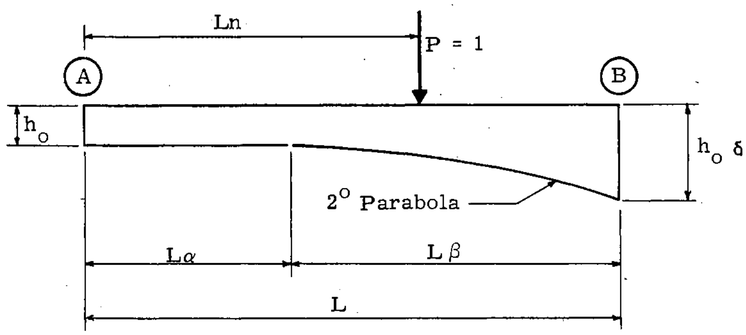
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$



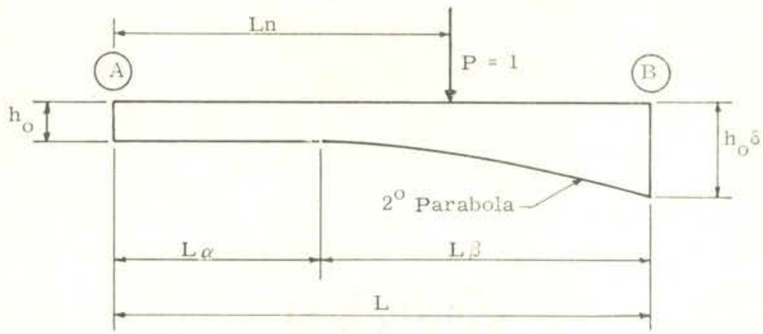
$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

δ \ n	Influence Coefficients t_1 .1	.2	.3	.4	.5	.6	.7	.8	.9	f_1	g	t_3	t_5
1.1	.0283	.0476	.0589	.0632	.0615	.0548	.0443	.0310	.0159	.3313	.1604	.0414	.0410
1.2	.0281	.0472	.0583	.0624	.0606	.0538	.0432	.0301	.0154	.3294	.1550	.0412	.0403
1.3	.0279	.0469	.0578	.0618	.0597	.0528	.0423	.0293	.0149	.3278	.1503	.0410	.0398
1.4	.0278	.0466	.0574	.0612	.0590	.0520	.0414	.0286	.0146	.3262	.1463	.0409	.0392
1.5	.0277	.0463	.0570	.0606	.0583	.0512	.0406	.0280	.0142	.3248	.1427	.0408	.0388
1.6	.0275	.0460	.0566	.0601	.0576	.0504	.0399	.0274	.0139	.3235	.1394	.0407	.0383
1.7	.0274	.0458	.0562	.0596	.0570	.0498	.0393	.0269	.0136	.3223	.1366	.0407	.0379
1.8	.0273	.0456	.0559	.0591	.0565	.0491	.0387	.0264	.0134	.3212	.1339	.0407	.0376
1.9	.0272	.0454	.0555	.0587	.0559	.0485	.0381	.0260	.0131	.3201	.1316	.0407	.0372
2.0	.0271	.0452	.0552	.0583	.0554	.0480	.0376	.0256	.0129	.3192	.1294	.0407	.0369
2.1	.0270	.0450	.0550	.0580	.0550	.0475	.0371	.0252	.0127	.3182	.1274	.0407	.0366
2.2	.0269	.0448	.0547	.0576	.0545	.0470	.0366	.0249	.0125	.3173	.1255	.0407	.0363
2.3	.0268	.0446	.0545	.0573	.0541	.0465	.0362	.0246	.0124	.3165	.1238	.0408	.0361
2.4	.0267	.0445	.0542	.0570	.0537	.0461	.0358	.0243	.0122	.3157	.1222	.0408	.0358
2.5	.0267	.0443	.0540	.0566	.0534	.0457	.0354	.0240	.0121	.3150	.1207	.0409	.0356
2.6	.0266	.0442	.0538	.0564	.0530	.0453	.0351	.0237	.0119	.3142	.1193	.0410	.0354
2.7	.0265	.0440	.0536	.0561	.0527	.0449	.0347	.0235	.0118	.3136	.1180	.0410	.0351
2.8	.0265	.0439	.0534	.0558	.0523	.0446	.0344	.0232	.0117	.3129	.1168	.0411	.0349
2.9	.0264	.0438	.0532	.0556	.0520	.0442	.0341	.0230	.0116	.3123	.1156	.0412	.0347
3.0	.0263	.0437	.0530	.0553	.0517	.0439	.0338	.0228	.0114	.3117	.1145	.0413	.0346

TABLE A-6

β = 0.6



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0159	.0307	.0436	.0535	.0594	.0603	.0554	.0441	.0257	.2944	.1604	.0399	.0392
1.2	.0153	.0297	.0420	.0513	.0567	.0571	.0520	.0408	.0234	.2638	.1550	.0383	.0372
1.3	.0149	.0287	.0406	.0495	.0543	.0544	.0490	.0380	.0215	.2392	.1503	.0370	.0354
1.4	.0145	.0279	.0394	.0478	.0523	.0520	.0464	.0356	.0200	.2191	.1463	.0359	.0339
1.5	.0141	.0272	.0383	.0464	.0505	.0499	.0442	.0336	.0186	.2023	.1427	.0349	.0326
1.6	.0138	.0266	.0373	.0451	.0489	.0480	.0422	.0318	.0175	.1882	.1394	.0341	.0314
1.7	.0135	.0260	.0365	.0440	.0475	.0463	.0404	.0302	.0165	.1762	.1366	.0333	.0303
1.8	.0132	.0255	.0357	.0429	.0462	.0448	.0388	.0288	.0156	.1658	.1339	.0327	.0294
1.9	.0130	.0250	.0350	.0420	.0450	.0434	.0374	.0276	.0148	.1567	.1316	.0321	.0285
2.0	.0128	.0345	.0343	.0411	.0439	.0422	.0361	.0265	.0142	.1488	.1294	.0316	.0278
2.1	.0126	.0241	.0337	.0403	.0429	.0410	.0349	.0255	.0136	.1418	.1274	.0311	.0271
2.2	.0124	.0238	.0332	.0395	.0420	.0399	.0338	.0245	.0130	.1356	.1255	.0306	.0264
2.3	.0122	.0234	.0326	.0389	.0411	.0389	.0328	.0237	.0125	.1300	.1238	.0303	.0258
2.4	.0121	.0231	.0322	.0382	.0403	.0380	.0319	.0229	.0121	.1250	.1222	.0299	.0253
2.5	.0119	.0228	.0317	.0376	.0396	.0372	.0311	.0222	.0117	.1204	.1207	.0296	.0248
2.6	.0118	.0225	.0313	.0371	.0389	.0364	.0303	.0216	.0113	.1163	.1193	.0293	.0243
2.7	.0116	.0223	.0309	.0365	.0382	.0357	.0295	.0210	.0109	.1125	.1180	.0290	.0239
2.8	.0115	.0220	.0305	.0360	.0376	.0350	.0289	.0204	.0106	.1090	.1168	.0288	.0235
2.9	.0114	.0218	.0302	.0356	.0370	.0343	.0282	.0199	.0103	.1059	.1156	.0285	.0231
3.0	.0113	.0216	.0299	.0351	.0365	.0337	.0276	.0195	.0101	.1029	.1145	.0283	.0227

TABLE A-7

β= 0.7

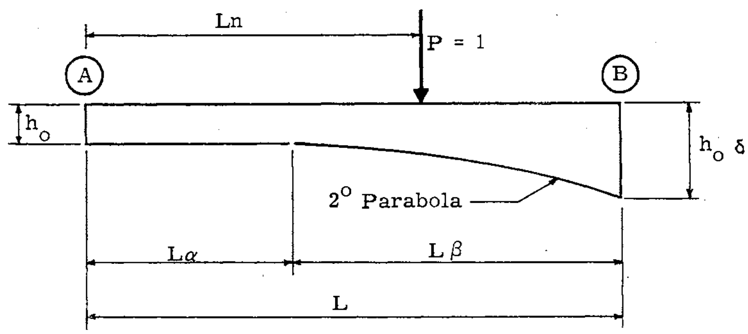
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

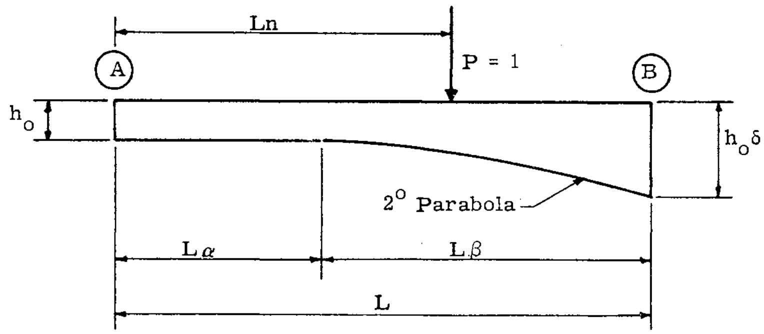


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0282	.0474	.0585	.0627	.0609	.0543	.0438	.0306	.0157	.3301	.1586	.0412	.0406
1.2	.0279	.0468	.0576	.0615	.0595	.0527	.0423	.0294	.0151	.3272	.1517	.0408	.0397
1.3	.0276	.0462	.0569	.0605	.0582	.0513	.0410	.0284	.0145	.3245	.1457	.0405	.0389
1.4	.0274	.0457	.0561	.0595	.0571	.0501	.0398	.0275	.0140	.3221	.1405	.0402	.0381
1.5	.0272	.0453	.0555	.0586	.0560	.0489	.0388	.0267	.0135	.3198	.1359	.0400	.0374
1.6	.0269	.0449	.0548	.0578	.0550	.0479	.0378	.0259	.0131	.3178	.1318	.0398	.0368
1.7	.0268	.0445	.0543	.0570	.0541	.0469	.0369	.0252	.0128	.3159	.1282	.0397	.0362
1.8	.0266	.0441	.0537	.0563	.0532	.0460	.0361	.0246	.0125	.3141	.1249	.0396	.0357
1.9	.0264	.0438	.0532	.0557	.0524	.0452	.0353	.0241	.0121	.3124	.1219	.0395	.0352
2.0	.0262	.0435	.0527	.0550	.0517	.0444	.0346	.0236	.0119	.3108	.1191	.0394	.0347
2.1	.0261	.0432	.0523	.0544	.0510	.0437	.0340	.0231	.0116	.3093	.1166	.0393	.0343
2.2	.0260	.0429	.0519	.0539	.0503	.0430	.0334	.0227	.0114	.3079	.1143	.0393	.0339
2.3	.0258	.0427	.0515	.0534	.0497	.0423	.0328	.0222	.0112	.3066	.1121	.0393	.0335
2.4	.0257	.0424	.0511	.0529	.0491	.0417	.0323	.0219	.0110	.3054	.1101	.0392	.0332
2.5	.0256	.0422	.0507	.0524	.0486	.0412	.0318	.0215	.0108	.3042	.1083	.0392	.0328
2.6	.0255	.0419	.0504	.0519	.0480	.0406	.0314	.0212	.0106	.3030	.1066	.0392	.0325
2.7	.0254	.0417	.0501	.0515	.0475	.0401	.0309	.0209	.0105	.3019	.1049	.0393	.0322
2.8	.0253	.0415	.0498	.0511	.0470	.0397	.0305	.0206	.0103	.3009	.1034	.0393	.0319
2.9	.0252	.0413	.0495	.0507	.0466	.0392	.0301	.0203	.0102	.2999	.1020	.0393	.0316
3.0	.0251	.0411	.0492	.0503	.0461	.0388	.0297	.0200	.0101	.2989	.1006	.0394	.0314

TABLE A-7
β=0.7



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_oqL^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0157	.0304	.0431	.0528	.0585	.0593	.0545	.0433	.0253	.2903	.1586	.0394	.0387
1.2	.0150	.0290	.0410	.0500	.0551	.0554	.0503	.0395	.0227	.2566	.1517	.0376	.0361
1.3	.0144	.0278	.0392	.0476	.0521	.0520	.0468	.0363	.0206	.2297	.1457	.0360	.0340
1.4	.0139	.0268	.0377	.0455	.0496	.0491	.0437	.0336	.0189	.2077	.1405	.0346	.0322
1.5	.0134	.0259	.0363	.0437	.0473	.0465	.0411	.0312	.0174	.1896	.1359	.0334	.0305
1.6	.0130	.0250	.0351	.0421	.0453	.0442	.0387	.0292	.0161	.1743	.1318	.0323	.0291
1.7	.0127	.0243	.0340	.0406	.0435	.0422	.0367	.0275	.0150	.1613	.1282	.0314	.0279
1.8	.0123	.0236	.0330	.0393	.0419	.0403	.0349	.0259	.0141	.1502	.1249	.0306	.0268
1.9	.0120	.0230	.0321	.0381	.0404	.0387	.0332	.0245	.0132	.1405	.1219	.0298	.0257
2.0	.0117	.0225	.0312	.0370	.0390	.0372	.0317	.0233	.0125	.1321	.1191	.0292	.0248
2.1	.0115	.0220	.0305	.0360	.0378	.0358	.0304	.0222	.0119	.1247	.1166	.0286	.0240
2.2	.0113	.0215	.0298	.0351	.0367	.0346	.0292	.0212	.0113	.1181	.1143	.0280	.0232
2.3	.0110	.0211	.0291	.0342	.0356	.0334	.0281	.0203	.0108	.1122	.1121	.0275	.0226
2.4	.0108	.0207	.0285	.0334	.0347	.0324	.0271	.0195	.0103	.1069	.1101	.0270	.0219
2.5	.0107	.0203	.0280	.0327	.0338	.0314	.0261	.0187	.0098	.1022	.1083	.0266	.0213
2.6	.0105	.0200	.0275	.0320	.0329	.0305	.0253	.0180	.0095	.0979	.1066	.0262	.0208
2.7	.0103	.0197	.0270	.0313	.0322	.0296	.0245	.0174	.0091	.0940	.1049	.0259	.0203
2.8	.0102	.0193	.0265	.0307	.0314	.0288	.0237	.0168	.0088	.0904	.1034	.0256	.0198
2.9	.0100	.0191	.0261	.0302	.0307	.0281	.0230	.0163	.0085	.0871	.1020	.0252	.0193
3.0	.0099	.0188	.0257	.0296	.0301	.0274	.0224	.0158	.0082	.0841	.1006	.0250	.0189

TABLE A-8
β = 0.8

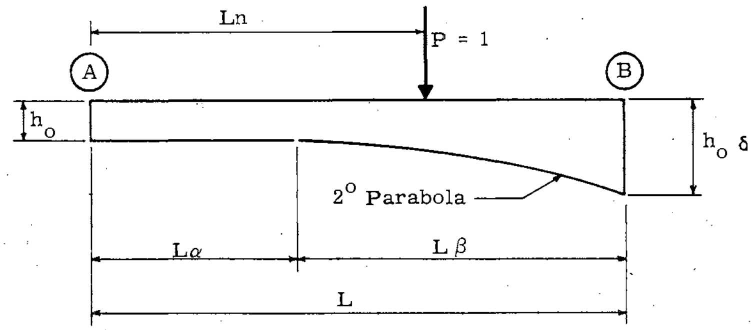
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$



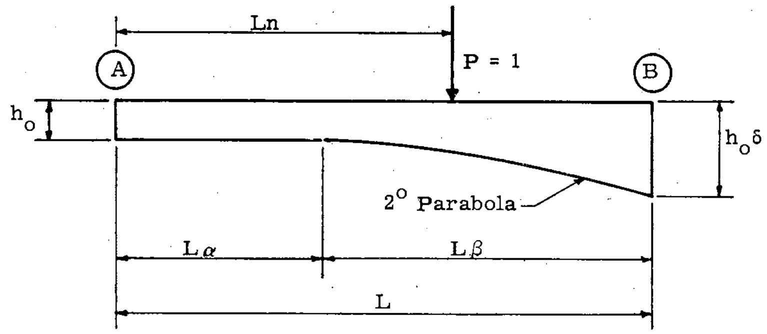
$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0282	.0470	.0580	.0621	.0603	.0536	.0433	.0303	.0155	.3285	.1567	.0410	.0402
1.2	.0276	.0462	.0567	.0604	.0583	.0516	.0414	.0288	.0147	.3241	.1482	.0404	.0390
1.3	.0272	.0454	.0556	.0589	.0565	.0497	.0397	.0275	.0140	.3202	.1409	.0398	.0378
1.4	.0268	.0446	.0545	.0574	.0548	.0480	.0381	.0263	.0134	.3165	.1346	.0394	.0368
1.5	.0265	.0440	.0535	.0562	.0534	.0465	.0368	.0253	.0128	.3132	.1290	.0390	.0359
1.6	.0262	.0434	.0526	.0550	.0520	.0451	.0356	.0244	.0124	.3101	.1241	.0387	.0350
1.7	.0259	.0428	.0517	.0539	.0507	.0438	.0344	.0236	.0119	.3073	.1197	.0384	.0342
1.8	.0256	.0422	.0509	.0528	.0496	.0427	.0334	.0228	.0115	.3046	.1157	.0381	.0335
1.9	.0254	.0417	.0501	.0519	.0485	.0416	.0325	.0221	.0112	.3021	.1121	.0379	.0328
2.0	.0251	.0413	.0494	.0510	.0475	.0406	.0316	.0215	.0109	.2997	.1088	.0377	.0322
2.1	.0249	.0408	.0488	.0501	.0465	.0397	.0308	.0209	.0106	.2975	.1058	.0375	.0317
2.2	.0247	.0404	.0482	.0493	.0456	.0388	.0301	.0204	.0103	.2954	.1030	.0373	.0311
2.3	.0245	.0400	.0476	.0486	.0448	.0380	.0294	.0199	.0100	.2934	.1004	.0372	.0306
2.4	.0243	.0396	.0470	.0478	.0440	.0372	.0288	.0195	.0098	.2916	.0981	.0371	.0301
2.5	.0241	.0393	.0465	.0472	.0433	.0365	.0282	.0190	.0096	.2898	.0959	.0370	.0297
2.6	.0240	.0389	.0460	.0465	.0426	.0358	.0276	.0186	.0094	.2881	.0938	.0369	.0293
2.7	.0238	.0386	.0455	.0459	.0419	.0352	.0271	.0183	.0092	.2864	.0919	.0368	.0289
2.8	.0237	.0383	.0450	.0453	.0413	.0346	.0266	.0179	.0090	.2849	.0901	.0368	.0285
2.9	.0235	.0380	.0446	.0448	.0407	.0340	.0261	.0176	.0088	.2834	.0884	.0367	.0281
3.0	.0234	.0377	.0442	.0442	.0401	.0335	.0257	.0173	.0087	.2820	.0868	.0367	.0278

TABLE A-8

$\beta = 0.8$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0155	.0300	.0425	.0520	.0576	.0584	.0537	.0427	.0250	.2867	.1567	.0390	.0381
1.2	.0147	.0283	.0400	.0487	.0535	.0538	.0489	.0384	.0221	.2504	.1482	.0368	.0352
1.3	.0139	.0269	.0378	.0458	.0500	.0498	.0448	.0348	.0198	.2215	.1409	.0350	.0327
1.4	.0133	.0256	.0359	.0433	.0469	.0464	.0413	.0318	.0179	.1981	.1346	.0334	.0305
1.5	.0127	.0245	.0342	.0410	.0442	.0434	.0384	.0292	.0163	.1787	.1290	.0319	.0287
1.6	.0122	.0235	.0327	.0391	.0419	.0408	.0358	.0270	.0150	.1626	.1241	.0307	.0270
1.7	.0118	.0226	.0314	.0373	.0398	.0385	.0335	.0251	.0138	.1489	.1197	.0296	.0256
1.8	.0114	.0218	.0302	.0358	.0379	.0364	.0315	.0234	.0128	.1372	.1157	.0286	.0243
1.9	.0110	.0211	.0291	.0343	.0362	.0345	.0297	.0220	.0119	.1271	.1121	.0277	.0232
2.0	.0107	.0204	.0281	.0330	.0346	.0329	.0281	.0207	.0111	.1184	.1088	.0269	.0222
2.1	.0104	.0198	.0272	.0318	.0332	.0313	.0266	.0195	.0105	.1107	.1058	.0262	.0212
2.2	.0101	.0193	.0264	.0308	.0319	.0300	.0253	.0184	.0099	.1038	.1030	.0255	.0204
2.3	.0099	.0188	.0256	.0297	.0307	.0287	.0241	.0175	.0093	.0978	.1004	.0249	.0196
2.4	.0096	.0183	.0249	.0288	.0296	.0276	.0231	.0166	.0088	.0924	.0981	.0244	.0189
2.5	.0094	.0178	.0243	.0280	.0286	.0265	.0221	.0159	.0084	.0876	.0959	.0239	.0182
2.6	.0092	.0174	.0237	.0272	.0277	.0255	.0212	.0152	.0080	.0832	.0938	.0234	.0176
2.7	.0090	.0170	.0231	.0264	.0268	.0246	.0203	.0145	.0076	.0792	.0919	.0230	.0171
2.8	.0088	.0167	.0225	.0257	.0260	.0238	.0196	.0139	.0073	.0756	.0901	.0225	.0166
2.9	.0087	.0163	.0220	.0250	.0252	.0230	.0188	.0134	.0070	.0723	.0884	.0222	.0161
3.0	.0085	.0160	.0216	.0244	.0245	.0222	.0182	.0129	.0067	.0693	.0868	.0218	.0156

TABLE A-9
β = 0.9

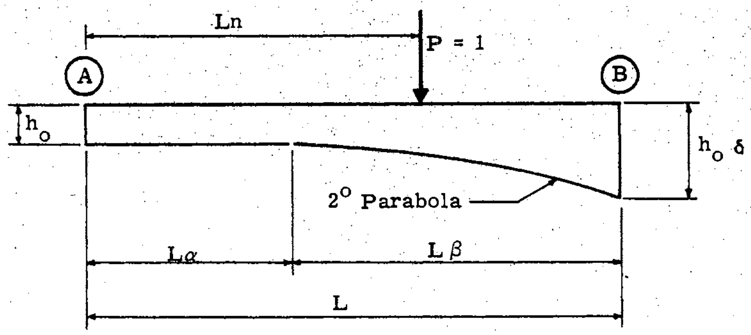
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$



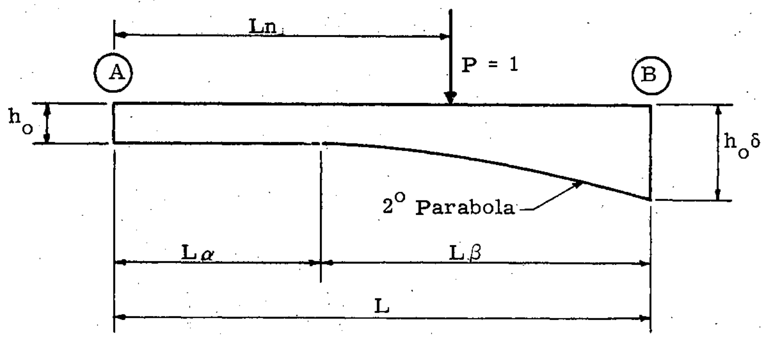
$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0278	.0466	.0575	.0614	.0596	.0530	.0428	.0299	.0154	.3264	.1548	.0407	.0398
1.2	.0272	.0454	.0556	.0591	.0570	.0503	.0404	.0281	.0144	.3202	.1448	.0398	.0381
1.3	.0267	.0443	.0540	.0570	.0546	.0480	.0383	.0265	.0135	.3146	.1362	.0391	.0367
1.4	.0261	.0432	.0525	.0551	.0525	.0459	.0365	.0252	.0128	.3094	.1288	.0384	.0353
1.5	.0256	.0423	.0511	.0534	.0506	.0440	.0348	.0240	.0122	.3047	.1222	.0378	.0342
1.6	.0252	.0414	.0498	.0516	.0489	.0423	.0333	.0229	.0116	.3003	.1164	.0373	.0331
1.7	.0248	.0406	.0486	.0503	.0475	.0407	.0320	.0219	.0111	.2962	.1113	.0368	.0321
1.8	.0244	.0398	.0475	.0490	.0458	.0393	.0308	.0210	.0106	.2924	.1067	.0364	.0312
1.9	.0240	.0391	.0465	.0477	.0444	.0380	.0297	.0202	.0102	.2888	.1025	.0360	.0303
2.0	.0237	.0385	.0455	.0465	.0431	.0368	.0287	.0195	.0098	.2855	.0987	.0356	.0295
2.1	.0234	.0378	.0446	.0454	.0420	.0357	.0277	.0188	.0095	.2823	.0952	.0353	.0288
2.2	.0231	.0372	.0437	.0444	.0408	.0347	.0269	.0182	.0092	.2794	.0921	.0350	.0281
2.3	.0228	.0367	.0429	.0434	.0398	.0337	.0261	.0177	.0089	.2765	.0891	.0347	.0275
2.4	.0226	.0361	.0422	.0425	.0389	.0328	.0254	.0171	.0086	.2739	.0864	.0345	.0269
2.5	.0223	.0356	.0414	.0416	.0379	.0319	.0246	.0167	.0084	.2713	.0839	.0343	.0263
2.6	.0221	.0352	.0407	.0408	.0371	.0311	.0240	.0162	.0081	.2689	.0816	.0341	.0258
2.7	.0218	.0347	.0401	.0400	.0363	.0304	.0234	.0158	.0079	.2666	.0794	.0339	.0253
2.8	.0216	.0343	.0395	.0392	.0355	.0297	.0228	.0154	.0077	.2644	.0774	.0337	.0249
2.9	.0214	.0338	.0389	.0385	.0348	.0290	.0223	.0150	.0075	.2622	.0755	.0335	.0244
3.0	.0212	.0334	.0383	.0378	.0341	.0284	.0218	.0147	.0074	.2602	.0737	.0334	.0240

TABLE A-9

β = 0.9



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0153	.0296	.0420	.0513	.0568	.0576	.0530	.0422	.0247	.2836	.1548	.0387	.0376
1.2	.0143	.0276	.0390	.0474	.0521	.0523	.0476	.0374	.0216	.2451	.1448	.0362	.0343
1.3	.0135	.0259	.0364	.0440	.0480	.0479	.0431	.0335	.0191	.2145	.1362	.0341	.0314
1.4	.0127	.0244	.0342	.0411	.0445	.0440	.0393	.0303	.0171	.1899	.1288	.0322	.0290
1.5	.0121	.0231	.0322	.0386	.0415	.0407	.0360	.0275	.0154	.1697	.1222	.0306	.0270
1.6	.0115	.0220	.0305	.0363	.0388	.0378	.0332	.0252	.0140	.1528	.1164	.0292	.0252
1.7	.0110	.0209	.0290	.0343	.0365	.0353	.0308	.0232	.0128	.1386	.1113	.0280	.0236
1.8	.0105	.0200	.0276	.0325	.0344	.0330	.0286	.0214	.0117	.1266	.1067	.0268	.0222
1.9	.0101	.0192	.0263	.0309	.0325	.0310	.0267	.0199	.0108	.1162	.1025	.0258	.0209
2.0	.0097	.0184	.0252	.0295	.0308	.0293	.0250	.0185	.0100	.1072	.0987	.0249	.0198
2.1	.0094	.0177	.0242	.0281	.0293	.0276	.0235	.0173	.0093	.0993	.0952	.0241	.0188
2.2	.0090	.0171	.0232	.0269	.0279	.0262	.0222	.0162	.0087	.0924	.0921	.0233	.0179
2.3	.0087	.0165	.0224	.0258	.0266	.0249	.0210	.0153	.0082	.0863	.0891	.0226	.0171
2.4	.0085	.0160	.0216	.0248	.0254	.0236	.0198	.0144	.0077	.0808	.0864	.0220	.0163
2.5	.0082	.0155	.0208	.0238	.0243	.0225	.0188	.0136	.0072	.0760	.0839	.0214	.0156
2.6	.0080	.0150	.0201	.0229	.0233	.0215	.0179	.0129	.0068	.0716	.0816	.0209	.0150
2.7	.0078	.0146	.0195	.0221	.0224	.0206	.0171	.0123	.0065	.0676	.0794	.0204	.0144
2.8	.0076	.0142	.0189	.0214	.0215	.0197	.0163	.0117	.0062	.0641	.0774	.0199	.0139
2.9	.0074	.0138	.0183	.0207	.0207	.0189	.0156	.0111	.0059	.0608	.0755	.0194	.0134
3.0	.0072	.0134	.0178	.0200	.0200	.0182	.0149	.0106	.0056	.0578	.0737	.0190	.0129

TABLE A-10
β = 1.0

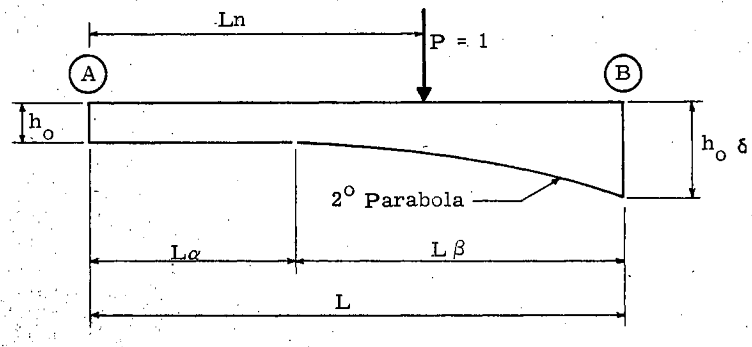
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$



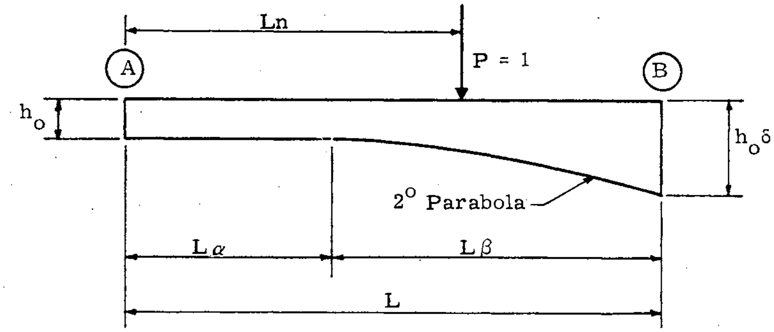
$$I_o = \frac{bh_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0276	.0461	.0568	.0607	.0588	.0523	.0422	.0295	.0152	.3239	.1530	.0404	.0393
1.2	.0267	.0445	.0544	.0578	.0556	.0492	.0394	.0274	.0140	.3153	.1415	.0392	.0373
1.3	.0259	.0430	.0523	.0551	.0527	.0463	.0370	.0256	.0130	.3076	.1316	.0382	.0355
1.4	.0252	.0416	.0503	.0527	.0502	.0438	.0348	.0241	.0122	.3005	.1231	.0373	.0339
1.5	.0246	.0403	.0485	.0506	.0479	.0416	.0329	.0227	.0115	.2904	.1151	.0365	.0324
1.6	.0240	.0391	.0469	.0486	.0457	.0396	.0312	.0214	.0109	.2880	.1092	.0358	.0311
1.7	.0234	.0381	.0453	.0468	.0438	.0378	.0297	.0203	.0103	.2824	.1034	.0351	.0299
1.8	.0229	.0370	.0439	.0451	.0421	.0361	.0283	.0193	.0098	.2772	.0984	.0345	.0288
1.9	.0224	.0361	.0426	.0435	.0405	.0346	.0270	.0184	.0093	.2723	.0935	.0339	.0278
2.0	.0220	.0352	.0414	.0421	.0390	.0332	.0259	.0176	.0089	.2677	.0893	.0334	.0268
2.1	.0215	.0344	.0402	.0407	.0376	.0320	.0249	.0169	.0085	.2634	.0855	.0329	.0260
2.2	.0211	.0336	.0391	.0395	.0363	.0308	.0239	.0162	.0082	.2593	.0820	.0325	.0252
2.3	.0207	.0329	.0381	.0383	.0351	.0297	.0230	.0156	.0078	.2554	.0788	.0320	.0244
2.4	.0204	.0322	.0371	.0372	.0340	.0287	.0222	.0150	.0076	.2518	.0758	.0317	.0237
2.5	.0200	.0315	.0362	.0362	.0329	.0277	.0214	.0145	.0073	.2483	.0730	.0313	.0230
2.6	.0197	.0309	.0354	.0352	.0319	.0268	.0207	.0140	.0070	.2449	.0705	.0309	.0224
2.7	.0194	.0303	.0346	.0343	.0310	.0260	.0200	.0135	.0068	.2417	.0681	.0306	.0218
2.8	.0191	.0297	.0338	.0334	.0301	.0252	.0194	.0131	.0066	.2387	.0659	.0303	.0213
2.9	.0187	.0291	.0330	.0326	.0293	.0245	.0188	.0127	.0064	.2358	.0639	.0300	.0208
3.0	.0185	.0286	.0323	.0318	.0285	.0238	.0182	.0123	.0062	.2330	.0619	.0298	.0203

TABLE A-10

β = 1.0



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0151	.0293	.0414	.0507	.0561	.0569	.0523	.0417	.0244	.2809	.1530	.0384	.0372
1.2	.0140	.0270	.0380	.0462	.0508	.0511	.0465	.0366	.0216	.2405	.1415	.0356	.0334
1.3	.0130	.0250	.0351	.0424	.0463	.0461	.0416	.0324	.0186	.2086	.1316	.0333	.0303
1.4	.0121	.0233	.0326	.0392	.0424	.0420	.0375	.0290	.0164	.1830	.1231	.0312	.0277
1.5	.0114	.0218	.0304	.0363	.0391	.0384	.0341	.0261	.0147	.1620	.1157	.0295	.0255
1.6	.0107	.0205	.0285	.0339	.0362	.0353	.0311	.0237	.0132	.1447	.1092	.0279	.0235
1.7	.0102	.0194	.0268	.0317	.0337	.0326	.0285	.0216	.0119	.1302	.1034	.0265	.0218
1.8	.0097	.0183	.0252	.0297	.0314	.0303	.0263	.0197	.0109	.1178	.0982	.0253	.0203
1.9	.0092	.0174	.0239	.0280	.0294	.0282	.0243	.0182	.0100	.1073	.0935	.0242	.0190
2.0	.0088	.0166	.0226	.0264	.0276	.0263	.0226	.0168	.0091	.0982	.0893	.0232	.0178
2.1	.0084	.0158	.0215	.0250	.0260	.0246	.0210	.0156	.0084	.0903	.0855	.0223	.0168
2.2	.0080	.0151	.0205	.0237	.0245	.0231	.0197	.0145	.0078	.0833	.0820	.0214	.0158
2.3	.0077	.0145	.0196	.0225	.0232	.0218	.0184	.0135	.0073	.0772	.0788	.0207	.0150
2.4	.0074	.0139	.0187	.0214	.0220	.0205	.0173	.0126	.0068	.0718	.0758	.0200	.0142
2.5	.0071	.0133	.0179	.0204	.0209	.0194	.0163	.0119	.0063	.0670	.0730	.0193	.0135
2.6	.0069	.0128	.0172	.0195	.0199	.0184	.0154	.0112	.0060	.0626	.0705	.0187	.0128
2.7	.0066	.0124	.0165	.0187	.0189	.0175	.0146	.0105	.0056	.0587	.0681	.0182	.0122
2.8	.0064	.0119	.0158	.0179	.0181	.0166	.0138	.0099	.0053	.0552	.0659	.0176	.0117
2.9	.0062	.0115	.0153	.0171	.0173	.0158	.0131	.0094	.0050	.0521	.0639	.0172	.0112
3.0	.0060	.0111	.0147	.0165	.0165	.0151	.0124	.0089	.0047	.0492	.0619	.0167	.0107

TABLE B-1
β=0.1

$F_{AB} = F_{BA} = f \dfrac{L}{EI_o}$

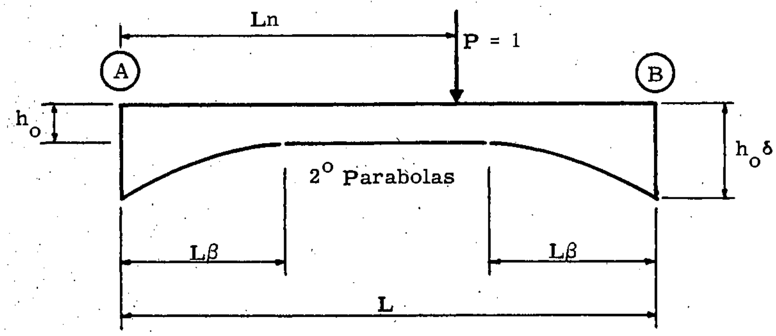
$G_{AB} = G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \dfrac{bh_o qL^3}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \dfrac{wL^3}{EI_o}$

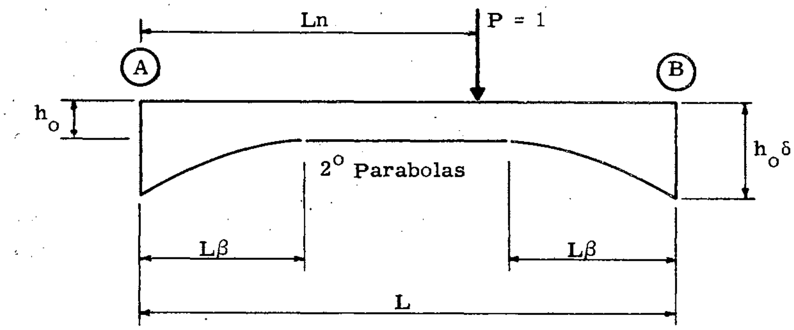


$I_o = \dfrac{bh_o^3}{12}$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficient t_1										f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0283	.0478	.0593	.0639	.0624	.0559	.0454	.0319	.0165	.3248	.1662	.0416	.0416
1.2	.0281	.0477	.0592	.0637	.0623	.0558	.0454	.0319	.0164	.3180	.1658	.0415	.0415
1.3	.0280	.0475	.0591	.0636	.0622	.0558	.0453	.0319	.0164	.3124	.1655	.0415	.0414
1.4	.0278	.0474	.0590	.0635	.0621	.0557	.0453	.0318	.0164	.3078	.1652	.0415	.0413
1.5	.0277	.0473	.0589	.0635	.0620	.0556	.0452	.0318	.0164	.3038	.1649	.0414	.0412
1.6	.0276	.0472	.0588	.0634	.0620	.0556	.0452	.0318	.0164	.3004	.1647	.0414	.0412
1.7	.0275	.0471	.0587	.0633	.0619	.0555	.0451	.0317	.0163	.2975	.1645	.0414	.0411
1.8	.0274	.0470	.0587	.0633	.0619	.0555	.0451	.0317	.0163	.2949	.1643	.0414	.0411
1.9	.0273	.0470	.0586	.0632	.0618	.0555	.0451	.0317	.0163	.2926	.1641	.0414	.0410
2.0	.0273	.0469	.0585	.0632	.0618	.0554	.0450	.0317	.0163	.2905	.1639	.0414	.0410
2.1	.0272	.0468	.0585	.0631	.0617	.0554	.0450	.0317	.0163	.2887	.1638	.0414	.0409
2.2	.0271	.0468	.0584	.0631	.0617	.0554	.0450	.0316	.0163	.2870	.1636	.0414	.0409
2.3	.0271	.0467	.0584	.0630	.0617	.0553	.0450	.0316	.0163	.2855	.1635	.0414	.0409
2.4	.0270	.0467	.0583	.0630	.0616	.0553	.0449	.0316	.0163	.2841	.1634	.0414	.0408
2.5	.0270	.0466	.0583	.0629	.0616	.0553	.0449	.0316	.0163	.2829	.1633	.0414	.0408
2.6	.0269	.0466	.0582	.0629	.0616	.0552	.0449	.0316	.0162	.2817	.1631	.0414	.0408
2.7	.0269	.0465	.0582	.0628	.0615	.0552	.0449	.0316	.0162	.2806	.1630	.0414	.0408
2.8	.0268	.0465	.0582	.0628	.0615	.0552	.0449	.0316	.0162	.2796	.1629	.0414	.0407
2.9	.0268	.0465	.0581	.0628	.0615	.0552	.0449	.0315	.0162	.2787	.1629	.0414	.0407
3.0	.0267	.0464	.0581	.0628	.0615	.0552	.0448	.0315	.0162	.2778	.1628	.0414	.0407
δ \ n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
Influence Coefficients t_2													

TABLE B-2
$\beta = 0.2$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficient t_1									f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0278	.0473	.0589	.0635	.0620	.0556	.0452	.0318	.0164	.3172	.1650	.0414	.0412
1.2	.0272	.0467	.0583	.0630	.0616	.0553	.0449	.0316	.0162	.3042	.1635	.0412	.0409
1.3	.0266	.0462	.0579	.0626	.0613	.0550	.0447	.0314	.0161	.2937	.1622	.0410	.0406
1.4	.0262	.0457	.0575	.0622	.0610	.0547	.0445	.0312	.0160	.2849	.1611	.0408	.0403
1.5	.0257	.0453	.0571	.0619	.0607	.0545	.0443	.0311	.0159	.2774	.1601	.0407	.0400
1.6	.0254	.0449	.0568	.0616	.0604	.0543	.0441	.0310	.0158	.2711	.1592	.0406	.0398
1.7	.0250	.0446	.0565	.0613	.0602	.0541	.0440	.0308	.0158	.2655	.1583	.0405	.0396
1.8	.0247	.0443	.0562	.0611	.0600	.0539	.0438	.0307	.0157	.2607	.1576	.0405	.0394
1.9	.0245	.0440	.0559	.0609	.0598	.0538	.0437	.0306	.0156	.2564	.1569	.0404	.0392
2.0	.0242	.0437	.0557	.0607	.0596	.0536	.0436	.0305	.0156	.2526	.1563	.0404	.0391
2.1	.0240	.0435	.0555	.0605	.0595	.0535	.0435	.0305	.0155	.2492	.1557	.0404	.0389
2.2	.0238	.0433	.0553	.0603	.0593	.0533	.0434	.0304	.0155	.2462	.1552	.0404	.0388
2.3	.0236	.0431	.0551	.0601	.0592	.0532	.0433	.0303	.0154	.2434	.1546	.0404	.0387
2.4	.0234	.0429	.0549	.0600	.0591	.0531	.0432	.0302	.0154	.2408	.1542	.0404	.0385
2.5	.0232	.0427	.0548	.0598	.0589	.0530	.0431	.0302	.0153	.2385	.1537	.0404	.0384
2.6	.0231	.0425	.0546	.0597	.0588	.0529	.0430	.0301	.0153	.2364	.1533	.0404	.0383
2.7	.0229	.0424	.0545	.0596	.0587	.0528	.0429	.0300	.0153	.2344	.1529	.0404	.0382
2.8	.0228	.0422	.0543	.0595	.0586	.0527	.0429	.0300	.0152	.2326	.1526	.0404	.0381
2.9	.0227	.0421	.0542	.0593	.0585	.0526	.0428	.0299	.0152	.2309	.1522	.0405	.0381
3.0	.0225	.0419	.0541	.0592	.0584	.0526	.0427	.0299	.0152	.2293	.1519	.0405	.0380
δ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
	Influence Coefficients t_2												

TABLE B-3
$\beta = 0.3$

$F_{AB} = F_{BA} = f \frac{L}{EI_o}$

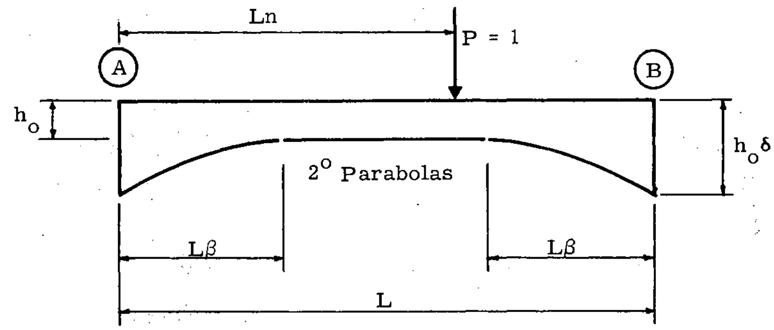
$G_{AB} = G_{BA} = g \frac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$



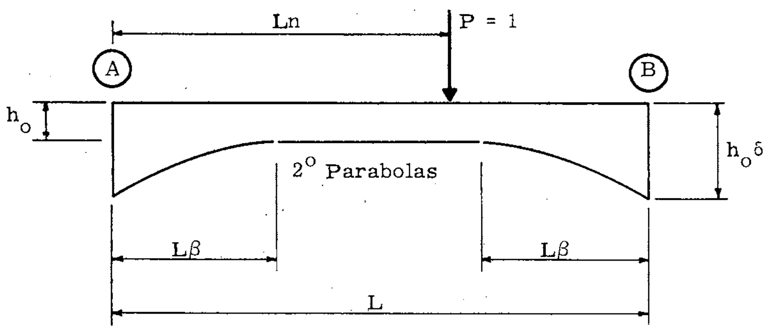
$I_o = \frac{bh_o^3}{12}$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficient t_1									f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0272	.0465	.0581	.0628	.0615	.0551	.0448	.0314	.0162	.3102	.1630	.0411	.0408
1.2	.0261	.0452	.0570	.0618	.0606	.0543	.0441	.0309	.0159	.2918	.1599	.0406	.0400
1.3	.0251	.0441	.0559	.0609	.0598	.0537	.0436	.0305	.0156	.2767	.1571	.0402	.0393
1.4	.0243	.0431	.0550	.0601	.0591	.0531	.0431	.0301	.0154	.2642	.1547	.0398	.0387
1.5	.0236	.0422	.0542	.0593	.0584	.0525	.0426	.0298	.0152	.2537	.1525	.0396	.0381
1.6	.0230	.0415	.0535	.0587	.0579	.0521	.0422	.0295	.0150	.2447	.1506	.0393	.0377
1.7	.0224	.0407	.0529	.0581	.0574	.0516	.0419	.0292	.0148	.2369	.1489	.0391	.0372
1.8	.0219	.0401	.0523	.0576	.0569	.0512	.0415	.0289	.0147	.2302	.1473	.0390	.0368
1.9	.0214	.0395	.0517	.0571	.0565	.0508	.0412	.0287	.0145	.2242	.1458	.0389	.0365
2.0	.0210	.0390	.0512	.0566	.0561	.0505	.0409	.0284	.0144	.2189	.1445	.0387	.0361
2.1	.0207	.0385	.0507	.0562	.0557	.0502	.0407	.0282	.0143	.2141	.1432	.0387	.0358
2.2	.0203	.0380	.0503	.0558	.0554	.0499	.0404	.0280	.0142	.2099	.1421	.0386	.0355
2.3	.0200	.0376	.0499	.0555	.0550	.0496	.0402	.0278	.0141	.2060	.1410	.0385	.0353
2.4	.0197	.0372	.0495	.0551	.0547	.0494	.0400	.0277	.0140	.2025	.1400	.0385	.0350
2.5	.0194	.0368	.0492	.0548	.0545	.0491	.0398	.0275	.0139	.1993	.1391	.0385	.0348
2.6	.0192	.0364	.0488	.0545	.0542	.0489	.0396	.0273	.0138	.1964	.1382	.0385	.0346
2.7	.0190	.0361	.0485	.0542	.0539	.0487	.0394	.0272	.0137	.1936	.1374	.0385	.0344
2.8	.0188	.0358	.0482	.0540	.0537	.0485	.0392	.0271	.0136	.1911	.1366	.0385	.0342
2.9	.0186	.0355	.0479	.0537	.0535	.0483	.0390	.0269	.0136	.1888	.1359	.0385	.0340
3.0	.0184	.0352	.0477	.0535	.0533	.0481	.0389	.0268	.0135	.1866	.1352	.0385	.0338
δ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
	Influence Coefficients t_2												

TABLE B-4

$\beta = 0.4$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficient t_1										f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0266	.0456	.0572	.0619	.0606	.0544	.0442	.0310	.0159	.3038	.1605	.0406	.0401
1.2	.0250	.0435	.0551	.0601	.0590	.0530	.0430	.0301	.0154	.2804	.1551	.0398	.0388
1.3	.0237	.0417	.0534	.0585	.0576	.0518	.0419	.0293	.0150	.2613	.1505	.0391	.0376
1.4	.0225	.0402	.0518	.0571	.0564	.0507	.0410	.0286	.0146	.2455	.1464	.0385	.0366
1.5	.0215	.0388	.0504	.0558	.0553	.0497	.0402	.0279	.0142	.2322	.1428	.0380	.0357
1.6	.0207	.0375	.0492	.0547	.0543	.0488	.0394	.0273	.0139	.2209	.1396	.0375	.0349
1.7	.0199	.0364	.0481	.0537	.0534	.0480	.0388	.0268	.0136	.2111	.1366	.0372	.0342
1.8	.0192	.0354	.0471	.0528	.0525	.0473	.0381	.0263	.0134	.2026	.1340	.0368	.0335
1.9	.0186	.0345	.0461	.0519	.0518	.0466	.0375	.0259	.0131	.1951	.1315	.0366	.0329
2.0	.0181	.0337	.0453	.0511	.0511	.0460	.0370	.0255	.0129	.1885	.1293	.0363	.0323
2.1	.0176	.0330	.0448	.0504	.0504	.0454	.0365	.0251	.0127	.1826	.1273	.0361	.0318
2.2	.0171	.0323	.0437	.0497	.0498	.0449	.0360	.0248	.0125	.1773	.1253	.0359	.0313
2.3	.0167	.0316	.0430	.0491	.0492	.0444	.0356	.0244	.0123	.1725	.1236	.0358	.0309
2.4	.0163	.0310	.0424	.0485	.0487	.0439	.0352	.0241	.0122	.1681	.1219	.0357	.0305
2.5	.0160	.0305	.0418	.0479	.0482	.0435	.0348	.0238	.0120	.1641	.1204	.0356	.0301
2.6	.0157	.0300	.0412	.0474	.0477	.0431	.0345	.0236	.0119	.1605	.1189	.0355	.0297
2.7	.0154	.0295	.0407	.0469	.0473	.0427	.0341	.0233	.0117	.1572	.1176	.0354	.0294
2.8	.0151	.0290	.0402	.0465	.0469	.0423	.0338	.0231	.0116	.1541	.1163	.0354	.0291
2.9	.0148	.0286	.0397	.0460	.0465	.0419	.0335	.0228	.0115	.1512	.1151	.0353	.0288
3.0	.0146	.0282	.0393	.0456	.0461	.0416	.0332	.0226	.0114	.1485	.1139	.0353	.0285
δ \ n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
Influence Coefficients t_2													

TABLE B-5

$\beta = 0.5$

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

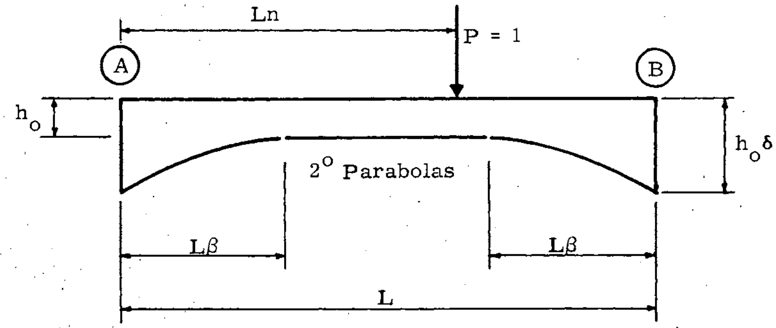
$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$



$$I_o = \frac{bh_o^3}{12}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficient t_1									f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0260	.0446	.0560	.0608	.0596	.0535	.0433	.0304	.0156	.2979	.1575	.0401	.0394
1.2	.0240	.0417	.0530	.0580	.0571	.0512	.0415	.0290	.0148	.2698	.1496	.0389	.0374
1.3	.0223	.0393	.0504	.0555	.0549	.0493	.0398	.0278	.0142	.2470	.1427	.0378	.0357
1.4	.0208	.0372	.0482	.0534	.0530	.0476	.0384	.0267	.0136	.2282	.1367	.0368	.0342
1.5	.0196	.0353	.0461	.0515	.0512	.0460	.0371	.0257	.0131	.2124	.1314	.0360	.0328
1.6	.0185	.0336	.0443	.0497	.0496	.0446	.0359	.0248	.0126	.1990	.1266	.0353	.0316
1.7	.0176	.0322	.0427	.0482	.0482	.0433	.0348	.0240	.0122	.1874	.1223	.0346	.0306
1.8	.0167	.0309	.0412	.0468	.0469	.0422	.0338	.0233	.0118	.1773	.1184	.0341	.0296
1.9	.0160	.0297	.0399	.0455	.0457	.0411	.0329	.0226	.0115	.1685	.1148	.0336	.0287
2.0	.0153	.0286	.0386	.0442	.0446	.0401	.0320	.0220	.0111	.1607	.1116	.0331	.0279
2.1	.0147	.0276	.0375	.0431	.0436	.0392	.0312	.0214	.0108	.1537	.1086	.0327	.0271
2.2	.0142	.0267	.0365	.0421	.0427	.0383	.0305	.0209	.0106	.1475	.1058	.0323	.0265
2.3	.0137	.0259	.0355	.0411	.0418	.0375	.0299	.0204	.0103	.1418	.1032	.0320	.0258
2.4	.0132	.0251	.0346	.0402	.0409	.0368	.0292	.0200	.0101	.1367	.1008	.0317	.0252
2.5	.0128	.0244	.0337	.0394	.0402	.0361	.0286	.0195	.0098	.1321	.0986	.0314	.0246
2.6	.0124	.0237	.0329	.0386	.0394	.0354	.0281	.0191	.0096	.1278	.0965	.0312	.0241
2.7	.0121	.0231	.0322	.0379	.0387	.0348	.0275	.0187	.0094	.1239	.0945	.0310	.0236
2.8	.0117	.0225	.0315	.0372	.0381	.0342	.0270	.0184	.0093	.1203	.0926	.0307	.0232
2.9	.0114	.0220	.0308	.0365	.0375	.0336	.0265	.0180	.0091	.1169	.0909	.0306	.0227
3.0	.0111	.0215	.0302	.0359	.0369	.0331	.0261	.0177	.0089	.1138	.0892	.0304	.0223
δ \ n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
	Influence Coefficients t_2												

The tables on the preceding pages may be used to evaluate constants for the analysis of pinned-end frames, continuous beams, and continuous frames of variable cross-section. The use of these beam constants is not restricted solely to analyses by the String Polygon Method as there exists a series of simple relationships between the angular functions of a structural system and the so-called moment functions.

The moment functions are

(a) Stiffness Factor

(b) Carry-Over Stiffness Factor

(c) Fixed-End Moments.

The relationship between these functions and the angular functions is given by:

(a) Stiffness Factor - The stiffness factor K_{ij} is defined as the moment required to produce unit rotation at i, end j being fixed.

$$K_{ij} = \frac{F_{ji}}{N}$$

$$K_{ji} = \frac{F_{ij}}{N}$$

where $N = F_{ij} F_{ji} - G_{ij}^2$.

(b) Carry-Over Stiffness Factor - The carry-over stiffness factor C_{ij} is defined as the ratio of the moment produced at the fixed end j to the moment applied at i .

$$C_{ij} = \frac{G_{ij}}{F_{ji}}$$

$$C_{ji} = \frac{G_{ji}}{F_{ij}} \quad .$$

(c) Fixed-End Moments - The fixed-end moment FM_{ij} is defined as the moment developed at i of the member ij if ends i and j are held fixed.

$$FM_{ij} = \frac{G_{ij}\tau_{ji} - F_{ji}\tau_{ij}}{N}$$

$$FM_{ji} = \frac{F_{ij}\tau_{ji} - G_{ij}\tau_{ij}}{N} \quad .$$

CHAPTER VI

NUMERICAL EXAMPLES

6-1. General Notes

The following illustrative examples comprise this chapter of the thesis and describe the numerical procedure of analysis by the string polygon method. References are made in each example to the equations, and tables are used. The width of members of the structures analyzed is taken as one foot for simplicity, and constants for particular members are expressed in terms of the minimum flexural rigidity of the structure rather than that of each member to permit a less cumbersome evaluation of the elastic weights. Dimensions are chosen to simplify calculations in illustrating the use of the beam constant tables of Chapter V rather than to try to simulate those typical of similar existing structures.

Units for various values are in terms of kips, feet, or kip-feet.

The tabulation of elastic constants and load functions, as is done in each example, is suggested as an efficient approach in the solution of problems of this nature.

6-2. Example No. 1

The symmetrical gabled-frame shown is analyzed and results are compared with those found by Leontovich (18). (See Fig. 6-1.)

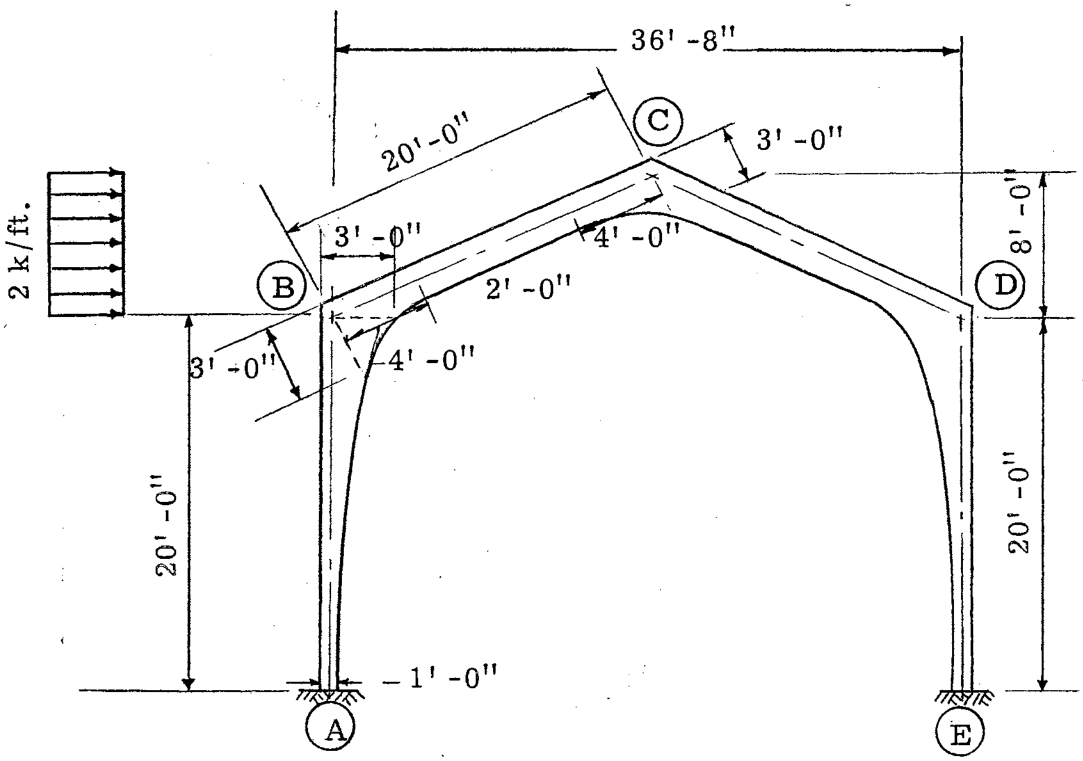


Fig. 6-1. Symmetrical Gabled Frame

a.) Elastic Constants and Load Functions (Eqs. 1-5a, b, c)(Table 6-1)

TABLE 6-1a. ELASTIC CONSTANTS AND LOAD FUNCTIONS

Member	β	Table	h_o ft.	$h_o \delta$ ft.	Flex. f_1	Coeff. f_2	C.O.V. Coeff. (g)
AB	1.0	A-10	1.00	3.00	.2330	.0492	0.0619
BC	0.2	B-2	2.00	3.00	.2774	.2774	0.1601

I_o^* = minimum moment of inertia of a particular member,

I_o = minimum moment of inertia of structure.

TABLE 6-1b. FINAL ELASTIC CONSTANTS						
Member	I_o^*/I_o	$F = f^L/EI_o$		GEI_o	Angular L'd. Funct's.	
		$F_{ij}EI_o$	$F_{ji}EI_o$		$\tau_{ij}^{LL}EI_o$	$\tau_{ji}^{LL}EI_o$
AB	1.000	4.660	0.984	1.238		
BC	8.000	0.694	0.694	0.400	12.800	12.800

b.) Location of Elastic Center (Eqs. 4-2d, 4-10) (Table 6-2)

The basic structure used in the analysis is shown in Fig. 6-2.

TABLE 6-2. LOCATION OF ELASTIC CENTER						
Joint	$G_{ij}EI_o$	$\Sigma F_j EI_o$	$G_{kj}EI_o$	$\overline{P}_j^o EI_o$	y_j(ft.)	$\overline{P}_j^o y_j EI_o$
A		4.660	1.238	5.90	0.00	0.00
B	1.238	1.678	0.400	3.32	20.00	66.40
C	0.400	1.388	0.400	2.19	28.00	61.32
D	0.400	1.678	1.238	3.32	20.00	66.40
E	1.238	4.660		5.90	0.00	0.00
$\sum$				20.63		194.12

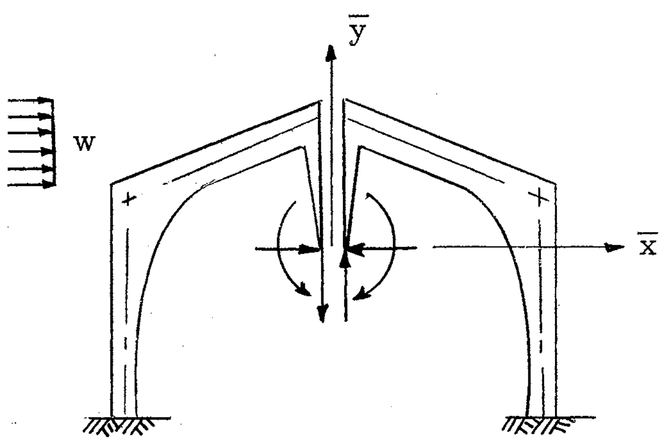


Fig. 6-2. Basic Structure

$$e_o = \frac{\sum \overline{P}_j^o x_j}{\sum \overline{P}_j^o} = 18.33 \text{ ft. (symmetry)}$$

$$f_o = \frac{\sum \overline{P}_j^o y_j}{\sum \overline{P}_j^o} = \frac{194.12}{20.63} = 9.41 \text{ ft.}$$

$$\overline{A} = \sum \overline{P}_j^o = \frac{20.63}{EI_o}$$

c.) Calculation of Elastic Weights (Eqs. 4-2a, b, c, 4-7a, b) (Table 6-3)

TABLE 6-3. CALCULATION OF ELASTIC WEIGHTS

Joint	$\bar{x}_j$(ft.)	$\bar{y}_j$(ft.)	BM_j	$\bar{P}_j^L EI_o$	$\bar{P}_j^L \bar{x}_j EI_o$	$\bar{P}_j^L \bar{y}_j EI_o$	$\bar{P}_j^x EI_o$	$\bar{P}_j^y EI_o$	$\bar{P}_j^x \bar{y}_j EI_o$	$\bar{P}_j^y \bar{x}_j EI_o$
A	-18.33	-9.41	-384.00	-1868.7	34,253	17,584	-30.717	-108.110	288.9	1981.7
B	-18.33	+10.59	-64.00	-570.0	10,448	-6,036	13.570	-53.450	143.8	979.7
C	0.00	18.59	0.00	-12.8		-238	34.285	0.000	637.5	0.0
D	18.33	10.59	0.00	0.0			13.570	53.450	143.8	979.7
E	18.33	-9.41	0.00	0.0			-30.717	108.110	+288.9	1981.7
$\sum$				-2451.5	44,701	11,310			1502.9	5922.8

From Equation 4-7b,

$$\bar{A} = \sum \bar{P}_j^o = \frac{20.63}{EI_o} \qquad \bar{P} = \sum \bar{P}_j^L = -\frac{2451.5}{EI_o}$$

$$\bar{I}_x = \sum \bar{P}_j^x \bar{y}_j = \frac{1502.9}{EI_o} \qquad \bar{M}_x = \sum \bar{P}_j^L \bar{y}_j = \frac{11,310}{EI_o}$$

$$\bar{I}_y = \sum \bar{P}_j^y \bar{x}_j = \frac{5922.8}{EI_o} \qquad \bar{M}_y = \sum \bar{P}_j^L \bar{x}_j = \frac{44,701}{EI_o}$$

$$\bar{I}_{xy} = \sum \bar{P}_j^x x_j = 0 \text{ (symmetry)}$$

d.) Reactive Redundants (Equation 4-9)

$$M_o = -\frac{\bar{P}}{\bar{A}}$$

$$= \frac{2451.5}{20.63}$$

$$= 118.83 \text{ kip-feet.}$$

$$R_{ox} = -\frac{\bar{M}_x}{\bar{I}_x}$$

$$= -\frac{11,310}{1502.9}$$

$$= -7.52 \text{ kips.}$$

$$R_{oy} = -\frac{\bar{M}_y}{\bar{I}_y}$$

$$= -\frac{44,701}{5922.8}$$

$$= -7.55 \text{ kips.}$$

e.) Final Reactions

Applying the equations of statics to the right half of the basic structure gives:

$\sum F_x = O$ $R_{Ex} = 7.52$ kips ←

$\sum F_y = O$ $R_{Ey} = 7.55$ kips ↑

$\sum M_E = O$ $M_{ED} = 51.20$ kip-feet ↻

Similarly, the reactions at A are;

$R_{Ax} = 8.48$ kips ← $M_{AB} = 56.02$ kip-feet ↺

$R_{Ay} = 7.55$ kips ↓

f.) Comparison of Results

	String Polygon	Leontovich
R_{Ex}	7.52 kips	7.50 kips
R_{Ey}	7.55 kips	7.59 kips
M_{ED}	51.20 kip-ft.	50.21 kip-ft.
R_{Ax}	8.48 kips	8.50 kips
R_{Ay}	7.55 kips	7.59 kips
M_{AB}	56.02 kip-ft.	55.50 kip-ft.

The small discrepancies arise because initial values for beam constants in the procedure outlined by Leontovich must be taken from graphs.

6-3. Example No. 2

The unsymmetrical gabled frame shown will be analyzed for live-load. (See Fig. 6-3.)

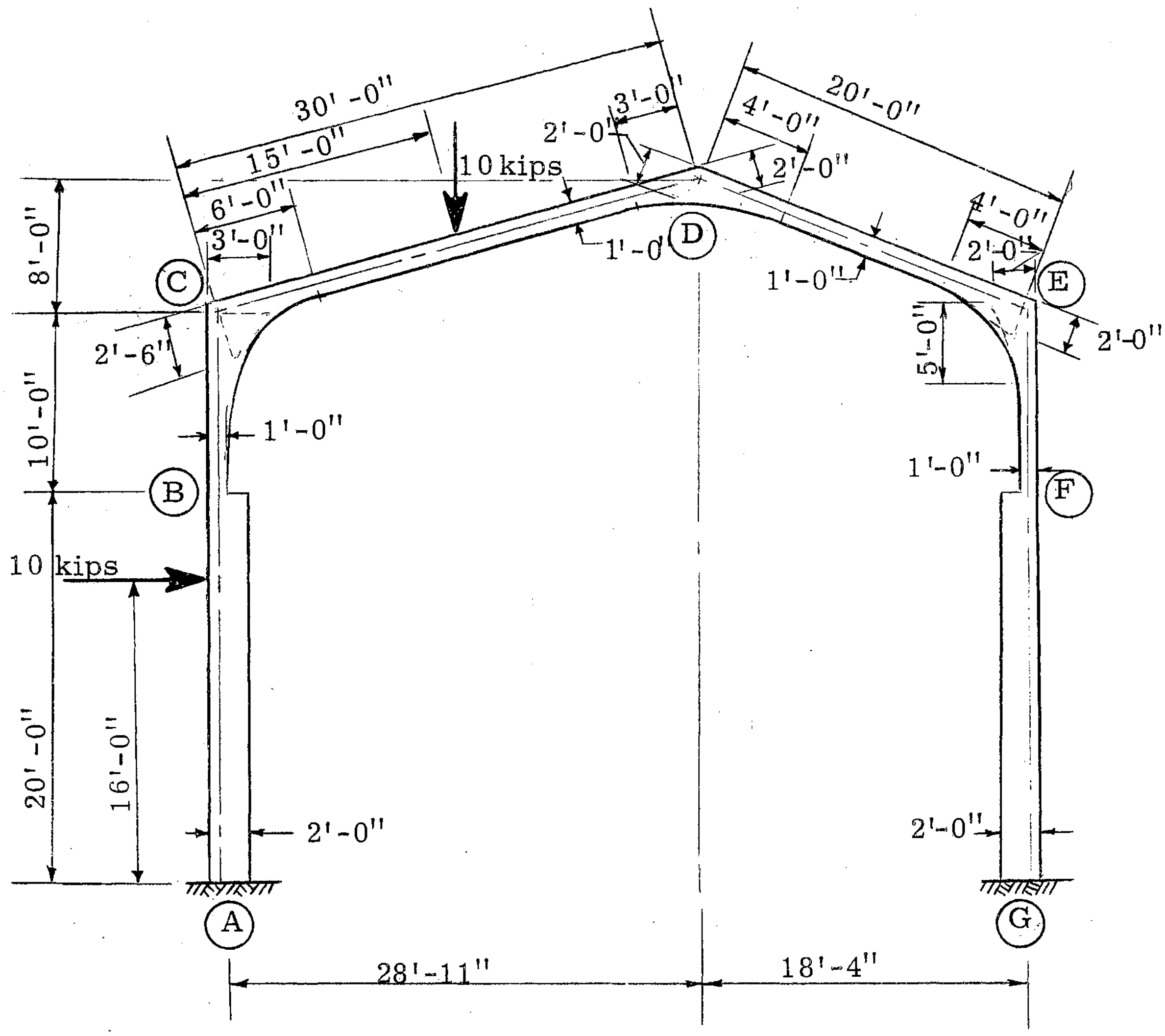


Fig. 6-3. Unsymmetrical Gabled Frame

a.) Elastic Constants and Load Functions (Equation 2-5a, b, c) (Table 6-4)

TABLE 6-4. ELASTIC CONSTANTS AND LOAD FUNCTIONS

Member	β	h_o ft.	$h_o \delta$ ft.	Table Used	I^*_o / I_o	Flex. Coeff. f_1	Flex. Coeff. f_2	$F = f^L/EI_o$: $F_{ij}EI_o$	$F = f^L/EI_o$: $F_{ji}EI_o$	COV g	Angular L'd Funct. $EI_o \tau_{ij}^{LL}$	Angular L'd Funct. $EI_o \tau_{ji}^{LL}$	GEI_o
AB	0.0	2.0	2.00	A-0	8.004			0.833	0.833		15.992	23.988	0.416
BC	1.0	1.0	3.00	A-10	1.000	0.2330	0.0492	2.330	0.492	0.0619			0.619
CD	0.2 / 0.1	1.0	2.50 / 2.00	A-0, 1, 2	1.000			7.176	8.699		51.432	53.340	4.765
DE	0.2	1.0	2.00	B-2	1.000	0.2526	0.2526	5.052	5.052	0.1563			3.126
EF	0.5	1.0	2.00	A-5	1.000	0.3251	0.1689	1.689	3.251	0.1391			1.391
FG	0.0	2.0	2.00	A-0	8.004			0.833	0.833	0.416			0.416

I^*_o = minimum moment of inertia of a particular member.

I_o = minimum moment of inertia of structure = 0.0833 ft.^{4}.

b.) Location of Elastic Center (eqs. 4-2d, 4-10) (Table 6-5)

TABLE 6-5. LOCATION OF ELASTIC CENTER

Joint	$G_{ij}EI_o$	$\Sigma F_j EI_o$	$G_{kj}EI_o$	$\overline{P}_j^o EI_o$	x_j(ft.)	y_j(ft.)	$\overline{P}_j^o x_j EI_o$	$\overline{P}_j^o y_j EI_o$
A		0.833	0.416	1.25	0.00	0.00		
B	0.416	3.163	0.619	4.20	0.00	20.00		84.00
C	0.619	7.668	4.765	13.05	0.00	30.00		391.50
D	4.765	13.751	3.126	21.64	28.91	38.00	625.61	822.30
E	3.126	6.741	1.391	11.26	47.24	30.00	531.92	337.80
F	1.391	4.084	0.416	5.89	47.24	20.00	278.24	117.80
G	0.416	0.833		1.25	47.24	0.00	59.05	
$\sum$				58.54			1494.82	1753.40

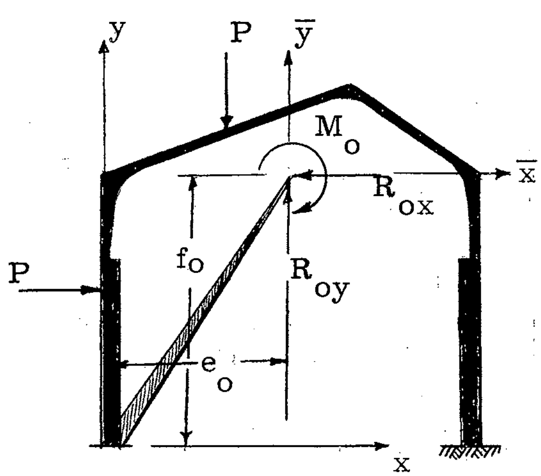


Fig. 6-4. Basic Structure

$$f_o = \frac{\sum \overline{P}_j^o y_j}{\sum \overline{P}_j^o} = \frac{1753.40}{58.54} = 29.95 \text{ ft.}$$

$$e_o = \frac{\sum \overline{P}_j^o x_j}{\sum \overline{P}_j^o} = \frac{1494.82}{58.54} = 25.54 \text{ ft.}$$

$$\overline{A} = \sum \overline{P}_j^o = \frac{58.54}{EI_o}$$

c.) Calculation of Elastic Weights and Load Functions (Eq. 4-2a, b, c, 4-7a, b) (Table 6-6)

TABLE 6-6. CALCULATION OF ELASTIC WEIGHTS AND LOAD FUNCTIONS

Joint	$\bar{x}_j$ (ft.)	$\bar{y}_j$ (ft.)	BM_j	$\bar{P}^L_j EI_o$	$\bar{P}^x_j EI_o$	$\bar{P}^y_j EI_o$	$\bar{P}^x_j \bar{x}_j EI_o$	$\bar{P}^x_j \bar{y}_j EI_o$	$\bar{P}^y_j \bar{x}_j EI_o$	$\bar{P}^L_j \bar{x}_j EI_o$	$\bar{P}^L_j \bar{y}_j EI_o$
A	-25.54	-29.95	0.00	- 1	-29.097	- 31.899	743.07	872	815	17	19
B	-25.54	- 9.95	- 40.00	- 189	-43.928	-107.209	1,121.82	437	2,738	4,832	1,883
C	-25.54	0.05	-140.00	- 2,784	32.539	-195.546	- 830.98	2	4,994	71,098	- 131
D	3.38	8.05	-364.57	- 7,089	111.018	- 7.413	374.80	893	- 25	- 23,934	-57,049
E	21.71	0.05	-467.87	- 4,805	11.623	187.066	252.29		4,060	-104,303	- 226
F	21.71	- 9.95	-367.87	- 2,206	-53.058	+127.880	-1,151.68	528	2,776	- 47,892	21,960
G	21.71	-29.95	-127.87	- 260	-29.097	27.121	- 631.58	872	589	- 5,634	7,774
$\sum$				-17,334	0.000 check	0.000 check	- 122.26	3,604	15,947	-105,816	-25,770

From Equation (4-7b)

$$\bar{A} = \sum \bar{P}_j^o = \frac{58.54}{EI_o} \qquad \bar{P} = \sum \bar{P}_j^L = -\frac{17,334}{EI_o}$$

$$\bar{I}_x = \sum \bar{P}_j^x \bar{y}_j = \frac{3604}{EI_o} \qquad \bar{M}_x = \sum \bar{P}_j^L \bar{y}_j = -\frac{25,770}{EI_o}$$

$$\bar{I}_y = \sum \bar{P}_j^y \bar{x}_j = \frac{15947}{EI_o} \qquad \bar{M}_y = \sum \bar{P}_j^L \bar{x}_j = -\frac{105,816}{EI_o}$$

$$\bar{I}_{xy} = \sum \bar{P}_j^x \bar{x}_j = \frac{-122.26}{EI_o}$$

d.) Reactive Redundants (Eq. 4-8)

$$M_o = -\frac{\bar{P}}{\bar{A}}$$

$$= \frac{17,334}{58.54}$$

$$= 296.11 \text{ kip-feet.}$$

$$R_{ox} = \frac{\bar{I}_{xy}\bar{M}_y - \bar{I}_y\bar{M}_x}{\bar{I}_y\bar{I}_x - \bar{I}_{xy}\bar{I}_{xy}}$$

$$= \frac{(-122.26)(-105,816) - (15,947)(-25,770)}{(3604)(15,947) - (-122.26)^2}$$

$$= 7.38 \text{ kips.}$$

$$R_{oy} = \frac{\bar{I}_{xy}\bar{M}_x - \bar{I}_x\bar{M}_y}{\bar{I}_y\bar{I}_x - \bar{I}_{xy}\bar{I}_{xy}}$$

$$= \frac{(-122.26)(-25,770) - (3604)(-105,816)}{(3604)(15,947) - (-122.26)^2}$$

$$= 6.69 \text{ kips.}$$

e.) Final Reactions

Applying the equations of statics to the basic structure gives

$$\sum F_x = 0, \quad R_{Gx} + 10 - 7.38 = 0, \quad R_{Gx} = 2.62 \text{ kips} \leftarrow$$

$$\sum F_y = 0, \quad R_{Gy} + 6.69 - 10 = 0, \quad R_{Gy} = 3.31 \text{ kips} \uparrow$$

$$\sum M_G = 0, \quad M_G + 10(16.00 - 32.78) + (21.71)(6.69)$$

$$+ \; 296.11 - 7.38(29.95) = 0$$

$$M_G = 52.43 \text{ kip feet} \circlearrowleft .$$

Again, by statics, it is easily shown that

$$R_{Ax} = 7.38 \text{ kips} \leftarrow$$

$$R_{Ay} = 6.69 \text{ kips} \uparrow$$

$$M_A = 95.75 \text{ kip-feet} \circlearrowleft .$$

f.) Alternate Procedure

For structures of this type the engineer may find it advantageous to compute the elastic constants and load functions of part c.) of this example by the alternate procedure suggested by Equation (4-11), in which elastic properties are referred to the x and y axes rather than those through the elastic center. This is done as follows in Table 6-7.

TABLE 6-7. CONSTANTS REFERRED TO A SET OF PARALLEL AXES

Joint	x(ft.)	y(ft.)	$\bar{P}_j^L x_j EI_o$	$\bar{P}_j^L y_j EI_o$	$\bar{P}_j^y EI_o$	$\bar{P}_j^y x_j EI_o$	$\bar{P}_j^x EI_o$	$\bar{P}_j^x y_j EI_o$	$\bar{P}_j^x x_j EI_o$
A	0.00	0.00			0.00		8.32		
B	0.00	20.00		-3,784	0.00		81.83	1,637	
C	0.00	30.00		-83,521	137.78		423.49	12,705	
D	28.91	38.00	-204,985	-269,400	545.28	15,766	759.27	28,852	21,953
E	47.24	30.00	-227,020	-144,158	474.57	22,421	348.84	10,465	16,481
F	47.24	20.00	-104,238	-44,128	278.31	13,149	123.41	2,468	5,830
G	47.24	0.00	-12,262		59.01	2,788	8.32		393
$\sum$			-548,505	-544,991		54,124		56,127	44,657

$$M_{xx} = \sum \bar{P}_j^L y_j = -\frac{544,991}{EI_o} \qquad I_{yy} = \sum \bar{P}_j^y x_j = \frac{54,124}{EI_o}$$

$$M_{yy} = \sum \bar{P}_j^L x_j = -\frac{548,505}{EI_o} \qquad I_{xy} = \sum \bar{P}_j^x x_j = \frac{44,657}{EI_o}$$

$$I_{xx} = \sum \bar{P}_j^x y_j = \frac{56,127}{EI_o}$$

It should be noted that the values of x and y used to compute the above constants are taken from vertical and horizontal axes through A and are not measured from the elastic center.

Equation (4-11) gives

$$\overline{M}_x = M_{xx} - f_o\overline{P}$$

$$= \frac{1}{EI_o}\left[-544,991 - (29.95)(-17,334)\right]$$

$$= \frac{-25,770}{EI_o}$$

$$\overline{M}_y = M_{yy} - e_o\overline{P}$$

$$= \frac{1}{EI_o}\left[-548,505 - (25.54)(-17,334)\right]$$

$$= \frac{-105,816}{EI_o}$$

$$\overline{I}_x = I_{xx} - f_o^2\overline{A}$$

$$= \frac{1}{EI_o}\left[56,127 - (29.95)^2(58.54)\right]$$

$$= \frac{3604}{EI_o}$$

$$\overline{I}_y = I_{yy} - e_o^2\overline{A}$$

$$= \frac{1}{EI_o}\left[54,124 - (25.54)^2(58.54)\right]$$

$$= \frac{15,947}{EI_o}$$

$$\overline{I}_{xy} = I_{xy} - e_of_o\overline{A}$$

$$= \frac{1}{EI_o}\left[44,657 - (29.95)(25.54)(58.54)\right]$$

$$\bar{I}_{xy} = -\frac{122.26}{EI_o} \quad .$$

The above values agree with those calculated in the first procedure illustrated; hence, substituting them into Equation (4-8) yields identical results.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The analysis of fixed-end frames by the String Polygon Method is presented in this thesis. Points of major significance found in the study may be summarized as follows:

1. The elastic curve of straight as well as bent members of constant or variable cross-section may be divided into any number of segments or string lines of any desired length. Each may be considered as a simple beam.

2. Using the angular functions F, G, and τ, the change in angle of the string lines at the ends of these segments may be expressed in terms of the general three moment equation and used as an elastic weight acting on the conjugate structure. Exact deflections and slopes at these points may be calculated.

3. Employing the concept of the elastic center of a structure, elastic weights of the String Polygon may be expressed in terms of reactive redundants and loads and placed at the joints of rigid frames. Deflections of the real structure are moments of the conjugate structure and their directions depend upon the axes about which these moments are taken. Using this principle, relationships may be obtained from which the redundants of the structure can be found with little

difficulty.

4. Tables of beam constants included in this thesis for members with parabolic haunches greatly shorten the task of computing elastic weights. These constants may be used in procedures other than the String Polygon Method to analyze many types of structures.

The String Polygon Method involves a complete physical analogy, it yields exact solutions, and it saves a considerable amount of computing time compared to the more common methods of analysis of fixed-end frames.

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