SUPERSYMMETRY, GRAND UNIFICATION AND FLAVOR SYMMETRY

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Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
DOCTOR OF PHILOSOPHY
July, 2006

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ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my supervisor Professor Kaladi S. Babu. Throughout my Ph. D. study, he has been the most important person for all I have learnt in my field. What I learned from him goes far beyond a Ph. D. degree. I will always remember his mentorship, scientific integrity and friendly easygoing atmosphere he, together with Dr. Satya Nandi, created among the students of HEP group during our study. I would also like to thank the other faculty members that have served on my advisory committee: Professors John Mintmire and John P. Chandler.

I want to thank the various individuals with whom I had the opportunity to collaborate during these years. In particular, I want to acknowledge the collaborations with Dr. Ilia Gogoladze and Dr. Gerhart Seidl, during their stay at High Energy Physics Theory (HEPT) Group at the Oklahoma State University.

Additionally, I want to thank all the faculty of the department of Physics for their individual contributions to my education. I would especially like to express my gratitude to Dr. Paul Westhaus for his efforts and regular guidance.

I want to thank my fellow students that I had the privilege to study with: Cyril Anoka, Abdelghafour Bachri, Marius Rujoiu, Kai Wang and Yangzhi Meng of HETP group and all other students I have befriended. Each of them has made my experience at OSU unique. Also, I want to thank the staff at the department for their assistance during my time at OSU.

I want to especially thank my father Bizya Tsedenbaljir, my mother Badamnyambuu Badam, my sister and my brothers for their unchanging moral support during this undertaking.

Lastly, I would like to acknowledge the generous financial support that I have received during my stay from the OSU Department of Physics and the US Department of Energy.

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CHAPTER 1

INTRODUCTION

The modern theory for elementary particle physics, the Standard Model (SM), has been tested in a variety of experiments since its formulation by Sheldon Glashow, Steven Weinberg and Abdus Salam [1] in the sixties. In the SM, all known interactions, except gravity, are described by a gauge theory based on the direct product group $SU(3)_c \times SU(2)_L \times U(1)_Y$, where $SU(3)_c$ is the color gauge group responsible for the strong interaction, while $SU(2)_L$ and $U(1)_Y$ are the gauge groups of the weak and hypercharge interactions. The diagonal subgroup $U(1)_{em}$ of $SU(2)_L \times U(1)_Y$ describes the electromagnetic interaction, which is left intact after spontaneous symmetry breaking. In the SM the known quarks and leptons are organized into three generations:

$$Q_{Li} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, L_{Li} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L, u_{Ri}, d_{Ri}, e_{Ri}, \; i = 1, 2, 3 \tag{1.1}$$

where $u_i\,(d_i)$ denotes the three generations of up (down)–type quarks $u, c, t\,(d, s, b)$ and $e_i\,(\nu_i)$ denotes the charged leptons (neutrinos) of the three generations, namely, the electron $e\,(\nu_e)$, muon $\mu\,(\nu_\mu)$ and the tau $\tau\,(\nu_\tau)$. The subscripts L and R denote respectively their left– and right–handed chiralities. Under $SU(2)_L$, only the left–handed fermions transform as doublets. The experimentally determined masses of the charged leptons and the quarks are

$$\begin{aligned} &m_e = 0.5109989\,\mathrm{MeV}, \quad m_\mu = 105.65836\,\mathrm{MeV}, \quad m_\tau = 1.77699\,\mathrm{GeV}, \\ &m_u(1\,\mathrm{GeV}) \simeq 5.1\,\mathrm{MeV}, \quad m_c(m_c) \simeq 1.27\,\mathrm{GeV}, \quad m_t(m_t) = 167\,\mathrm{GeV}, \\ &m_d(1\,\mathrm{GeV}) \simeq 8.9\,\mathrm{MeV}, \quad m_s(1\,\mathrm{GeV}) \simeq 130\,\mathrm{MeV}, \quad m_b(m_b) \simeq 4.25\,\mathrm{GeV} \end{aligned} \tag{1.2}$$

From this we see that there is a strong hierarchy among the masses: the fermions in the third generation are the heaviest, while those from the first generation are four to five orders of magnitude lighter. This is accommodated in the SM (but not explained) by choosing certain hierarchical "Yukawa couplings": In the SM, the mass term of a fermion arises from the Yukawa interactions:

$$\mathcal{L}_Y = Y_{ij}^f \bar{f}_{Li} f_{Rj} H. \tag{1.3}$$

Here H is the Higgs douplet scalar field. f_{Li} and f_{Rj} are left and right–handed SM fermions of i^{th} and j^{th} flavors respectively. The fermion mass matrices are given in terms of the Yukawa couplings and the vacuum expectation value (VEV) of the Higgs field: $(m^f)_{ij} = Y_{ij}^f \langle H \rangle$. Thus, once $SU(2)_L$ is broken by nonzero VEV of the Higgs field the term given in Eq. (1.3) leads to the mass term. Y_{ij}^f are to be chosen hierarchically to fit the observed fermion masses.

Due to the mismatch between the weak and the mass eigenstates of the quarks, there are inter–generation mixings in the couplings of the charged $W^{\pm}$ gauge bosons described by a unitary matrix called the CKM matrix. The magnitudes of the CKM matrix elements – deduced from experimental results of weak decays and the unitarity condition – are found to be:

$$V_{CKM} \simeq \begin{pmatrix} 0.9750 & 0.222 & 0.003 \\ 0.222 & 0.9741 & 0.04 \\ 0.009 & 0.039 & 0.999 \end{pmatrix}. \tag{1.4}$$

In the SM, the neutrinos are massless. But recent experimental results indicate otherwise. It is found that the electron neutrinos coming from the sun change or oscillate to other types of neutrinos. The same fact is found for the muon neutrinos from the atmosphere. These oscillation patterns are a strong indication that the neutrinos have small but nonzero masses. This suggests the presence of additional right-handed neutrinos, probably very heavy, to accommodate the small left–handed neutrino masses. These experiments also measured the inter–generation mixings in the lepton sector and found them to be completely different in magnitude from the quark mixings. Contrary to the small quark mixings, they are found to be large. Current experiments are sensitive only to the absolute values of the differences between the light neutrino masses. This leads to two possible scenarios: hierarchical and inverse hierarchical neutrino masses. In the hierarchical scenario, for example, one can choose the following best fit values for the neutrino masses

$$\begin{aligned} m_{\nu_e} &\simeq 2.7 \times 10^{-3}\,\text{eV}, \\ m_{\nu_\mu} &\simeq 6.4 \times 10^{-3}\,\text{eV}, \\ m_{\nu_\tau} &\simeq 8.6 \times 10^{-2}\text{eV}, \end{aligned} \tag{1.5}$$

and the leptonic mixing matrix given by

$$V_{MNS} \simeq \begin{pmatrix} 0.848 & -0.526 & \leq 0.18 \\ 0.349 & 0.619 & -0.72 \\ -0.4 & -0.59 & -0.7013 \end{pmatrix}. \tag{1.6}$$

The SM, while highly successful in explaining all experimental data, does not provide an explanation for the observed hierarchy in the masses and mixings of quarks and leptons. In particular, it leaves the following central questions unanswered:

- The origin of generations: Why are there three generations of fermions?
- The origin of masses and their hierarchy: What mechanism sets the fermion masses to the values observed in Nature? Why do they have such different masses?
- The origin of mixing: Why are the weak and mass eigenstates different, thereby causing the inter–generation mixings among quarks (small) and leptons (large)?

These observations are some of the main reasons for high energy physicists to seek a theory beyond the SM.

There is one question, however, somewhat grander in terms of energy scale, one should pose before turning to the questions listed above. A seemingly natural energy scale that could set a mass parameter is the Planck mass, $M_{Planck} = \sqrt{\hbar c/G_n} = 1.2 \times 10^{19}$ GeV, where quantum corrections from gravity becomes strong enough to be considered, at least hypothetically. Nature chooses a completely different way. The mass scale we observe in collider experiments is far below, seventeen orders of magnitude smaller than the natural mass scale M_{Planck}. In the SM, this low energy scale is chosen by the VEV of a scalar field called the Higgs particle. The fermions and $W^{\pm}$, Z bosons acquire their masses through the Higgs mechanism at the electroweak symmetry breaking scale around 10^2 GeV, which is set by the VEV of the Higgs field. The known long–range electromagnetic and the short–range weak interactions result from the symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ at this scale. The smallness of the fermion masses compared to the Planck mass is the result of the chirality dependent $SU(2)_L$ gauge symmetry, which acts only on the left–handed fermions. This chirality dependent nature of weak interactions forbids the SM fermions from acquiring a mass until this electroweak symmetry breaking occurs. On the other hand, the vital input of the SM is the mass of the Higgs field. In general, quantum corrections to the mass of a scalar field are quadratic in the energy scale (presumably M_{Planck}) which completely falsify the assumption that the mass of the Higgs particle is around 10^2 GeV, unless a severe fine–tuning is performed. This significant shortcoming of the SM is called the gauge hierarchy problem.

The most widely pursued solution to the gauge hierarchy problem is Supersymmetry (SUSY), a symmetry which puts fermions and bosons into a common multiplet, called supermultiplets. This guarantees the stability of the mass of a scalar particle against the quadratic corrections. The key result of SUSY is that the quadratic corrections from bosonic and fermionic components come with opposite sign and exactly cancel out.

If SUSY were an exact symmetry of Nature it would lead to a complete degeneracy in the masses of the fermionic and bosonic partners in a supersymmetric multiplet. Thus SUSY must be broken if it has anything to do with real world. Phenomenologically consistent SUSY breaking is the most difficult part in SUSY model building. The reason is the following. There is a powerful mass sum rule which rules out any possibility to construct a realistic model using only the SM fields. Therefore, one is forced to introduce a “hidden” sector where SUSY is broken dynamically and its effect transmitted to the visible sector by some type of interactions. The most widely studied scenario is the gravity mediated SUSY breaking or mSUGRA (minimal Supergravity) in its simplest form. The mass splittings between the fermionic and bosonic superpartners are characterized by a few parameters called soft SUSY breaking parameters. These are chosen such that quadratic divergences in the Higgs boson mass is not reintroduced. In general, non–minimal gravity mediation predicts disastrously large flavor changing neutral currents and CP–violations. This is avoided by the simple choice of soft parameters of mSUGRA referred to as universal boundary conditions. Even in this case flavor symmetries that may be present can induces FCNC effects at an interesting level which can be tested in future experiments, as we will show.

In this thesis I describe our research on the Supersymmetric flavor and Grand Unified Theories, which address the questions of fermion mass hierarchy and mixings, in particular, on their experimental implications such as flavor violation and CP–violation. This thesis contains six chapters.

In the second and third chapters, we consider a SUSY version of the SM, called Minimal Supersymmetric Standard Model (MSSM), in gravity mediated supersymmetry breaking scenario. The results are based on work done in collaboration with Kaladi S. Babu and Ilia Gogoladze [2,3]. The gauge sector is extended by an anomalous flavor $U(1)$ gauge symmetry of string origin. Under this $U(1)$ symmetry the three generations carry different charges, which leads to the observed fermion mass and mixing hierarchy upon spontaneous symmetry breaking. It has far–reaching phenomenological consequences such as flavor changing neutral currents and CP–violations. We show that the flavor violating muon decay ($\mu \to e\gamma$) and the electric dipole moments of the electron and neutron are all in an experimentally interesting range.

In the fourth chapter of the thesis, I describe our work on split supersymmetric spectrum for superpartners of the MSSM fields [4]. When the mass spectrum of the supersymmetric particles display hierarchical values between the scalar partners of the SM model fermions and the gauginos the fermionic superpartners of the SM gauge bosons it is called Slit supersymmetry. We have shown that the anomalous $U(1)$ and a hidden sector based on $SU(N_c)$ gauge group which becomes strongly coupled at some intermediate scale $\Lambda \sim 10^{12} \div 10^{15}$ GeV, naturally induces split supersymmetric spectrum. We start with the review of the previously known global version of such models. Based on that analysis we calculate the supergravity corrections to the soft parameters. These results show that the results from the global limit are not destabilized by the supergravity corrections. In an explicit models these are necessary since the sparticles neutral under the anomalous $U(1)$ obtain their soft masses only through the gravitational corrections.

The fifth chapter of the thesis describes our study on an $SU(5)$ SUSY Grand Unified Theory (GUT) [5]. We study a special class of theories called finite GUTs. A theory is finite, if the quantum corrections to the gauge and Yukawa couplings vanish to all orders of perturbation theory. Phenomenologically interesting models have been built only in supersymmetric theories. Under the criteria of finiteness, the beta–functions of the gauge and Yukawa couplings, which quantify the quantum corrections, vanish to all orders of perturbation theory. These criteria highly constrain the possible solutions leading to a single parameter in the theory, the gauge coupling. All other couplings are given in terms of the gauge coupling. Earlier attempts in this direction have achieved partial success in predicting a naturally large top mass, yet failed to explain the observed masses for lighter generations and mixings in the quark sector. We employ non–Abelian family symmetries to obtain a finite $SU(5)$ grand unified model, which in turn enables us to accommodate the quark masses and mixings.

The sixth chapter contains work done in collaboration with Gerhart Seidl on the description of the fermion mass hierarchy in a deconstructed manifold [6]. The deconstruction is an alternative ultraviolet completion of extra–dimensional theories via four–dimensional product gauge theories. We have considered a two–dimensional disk and its deconstruction by a product $U(1)^n$ gauge theory space on which the SM fermions from different generations live at different sites, in other words, transform under different $U(1)$'s. Compactifying this deconstructed space to RP^2 manifold for anomaly cancelation we were able to obtain the mass matrices for the SM fermions compatible with their experimentally known values.

Finally in chapter 7 we summarize our main results and conclude. Appendix A contains some useful formulas used in our numerical studies for lepton flavor violation and electric dipole moments in second and third chapters.

CHAPTER 2

ANOMALOUS $U(1)_A$ GAUGE SYMMETRY AS THE FLAVOR SYMMETRY AND LEPTON FLAVOR VIOLATION

2.1 Introduction

The observed hierarchy in the fermion masses and mixings is one of the most puzzling features of Nature. Extended symmetries are often speculated to address these problems. Family–dependent $U(1)$ symmetry is a widely studied extension. An attractive scenario is the Froggatt–Nielsen scheme [7] . In this scenario all the Yukawa couplings are assumed to be of order one, but the ones which generate the light fermion masses arise only as nonrenormalizable operators suppressed by powers of a small parameter $\epsilon \equiv \langle S \rangle / M$, where $\langle S \rangle$ is the flavor symmetry breaking order parameter and M is a more fundamental mass scale. With the flavor $U(1)$ charges of fermions differing only by order one, large hierarchy factors, such as $m_u/m_t \sim 10^{-6}$, are explained.

A natural origin for the flavor $U(1)$ symmetry is the anomalous $U(1)_A$ gauge symmetry of perturbative Heterotic string theory [8]. The small expansion parameter ϵ arises in a natural way in anomalous $U(1)$ models through the Fayet–Iliopoulos term induced by the gravitational anomaly [9]. Such models have been extensively studied in the literature for understanding the fermion mass hierarchy puzzle [2,10]. The purpose of this chapter is to present a class of models compatible with all low energy data, and then study their phenomenological implications in flavor changing neutral currents.

Low energy supersymmetry can potentially induce excessive flavor violation in processes such as $K^0 - \overline{K^0}$ mixing and $\mu \rightarrow e\gamma$ decay if the soft supersymmetry breaking Lagrangian takes its most general form. This potential problem is usually avoided in minimal supergravity (mSUGRA) by assuming a universal form for the soft SUSY breaking terms at the gauge unification scale. The soft SUSY breaking part of the MSSM lagrangian is given by

$$
\begin{aligned}
-\mathcal{L}_{soft} &= \left\{ A^f_{ij}\tilde{f}_i\tilde{f}^c_j H + \frac{1}{2}\sum_{i=1,2,3} M^i_{1/2}\lambda_i\lambda_i + B\mu H_u H_d + \text{h. c.} \right\} \\
&+ \sum \left(\tilde{m}^2_f\right)_{ab} \tilde{f}^\dagger_a \tilde{f}_b + \tilde{m}^2_{H_u}|H_u|^2 + \tilde{m}^2_{H_d}|H_d|^2 \,. \qquad (2.1)
\end{aligned}
$$

Here $\left(\tilde{m}^2_f\right)_{ab}$ is the scalar soft mass matrix of sfermions $\tilde{f}_a$ $(a = 1 \div 6)$, A^f_{ij} are the soft trilinear A–terms and M_i, $(i = 1, 2, 3)$ are the gaugino masses for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ gauge groups. H is the up (down) type Higgs doublet H_u (H_d) for $f = u$ $(f = d, e)$. $B\mu$ is the soft Higgs mass parameter or B–term. Then the universal initial condition for the soft parameters are

$$
\begin{aligned}
\tilde{m}^2_{f_i} &= m^2_0, \\
A^f_{ij} &= A_0 Y^f_{ij}, \\
M^i_{1/2} &= M_{1/2}. \qquad (2.2)
\end{aligned}
$$

The first two conditions guarantee the absence of SUSY flavor problem. The gaugino masses (in addition to the Higgs mass paramter μ) are chosen to be real for the absence of SUSY CP–problem. The last condition is chosen to guarantee the gauge coupling unification. There two additional parameters in mSUGRA: the sign of supersymmetric μ–term and $\tan\beta$ defined as the ratio of the vacuum expectation values of the up–type and the down–type Higgs doublets: $\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$. If there is no source of LFV at the high scale the first two conditions in Eq. (2.2) ensure the absence of LFV at low energy scale.

Even with these conditions as the initial values for the soft parameters at the fundamental scale, which we choose to be the string scale, $M_{st} = 10^{17}$ GeV, a family–dependent anomalous $U(1)_A$ symmetry will induce flavor changing processes. In the present case, such violations will be generated through the renormalization group evolution (RGE) of the SUSY breaking parameters between M_{st} and the $U(1)_A$ breaking scale quantified by the flavor gauge boson mass M_F. We derive general expressions for the evolution of these parameters in the presence of higher dimensional operators. Our results can be applied to a wide class of Froggatt–Nielsen models.

We have found several sources of flavor violation. As we will see, it is natural that the flavor $U(1)$ gauge symmetry responsible for explaining the fermion mass hierarchy breaks spontaneously at a scale M_F slightly below the fundamental Planck (or string) scale, $M_F \sim M_{st}/50$. In the momentum interval $M_F \leq \mu \leq M_{st}$, the $U(1)_A$ gaugino is active and will contribute differently to the soft masses of different families. Because $TrU(1)_A$, the sum of $U(1)_A$ charges of all matter fields, is not zero in anomalous $U(1)$ models, there are nonuniversal RGE contributions to the soft scalar masses arising from the D–term proportional to the respective flavor charges. Furthermore, the trilinear A–terms will receive vertex corrections from the $U(1)_A$ gaugino that are not proportional to the respective Yukawa couplings.

In this part of the thesis we present the result of our investigation of the combined effects of nonuniversality for lepton flavor violating (LFV) decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in an anomalous $U(1)_A$ model. Quantitative predictions for the branching ratios are presented in a class of models of fermion mass hierarchy.

We find that the branching ratio for $\mu \to e\gamma$ is around the current experimental limit. In our analysis we also include the right–handed neutrino–induced LFV effects, which have been widely studied in the literature [11–14]. These effects turn out to be significant in some but not all cases that we study.

The structure of this chapter is as follows. In 2.2 we describe the anomalous flavor $U(1)$ models of fermion mass hierarchy, in 2.3 we present our fermion mass fits for the model. In 2.4 we give the radiative corrections to the soft SUSY breaking parameters from the flavor $U(1)$ gauge symmetry. Section 2.5 is devoted to the numerical analysis of the branching ratios for the process $\mu \to e\gamma$ and $\tau \to \mu\gamma$. In 2.5.1 we outline the qualitative features of flavor violation arising from various sources. 2.5.2 has our numerical results. The conclusions of this chapter are given in Section 2.6.

2.2 Anomalous $U(1)_A$ Models

In this Section we review briefly the idea of explaining fermion mass hierarchy with a flavor dependent $U(1)$ symmetry. We focus on a specific class of anomalous $U(1)_A$ models. There are many other models which will also fall into this category and will lead to similar results [10,15]. In these models families are distinguished by their anomalous $U(1)$ charges. The $U(1)_A$ symmetry is broken spontaneously by an MSSM singlet flavon field S which acquires a vacuum expectation value (VEV) slightly below the string scale M_{st}. This provides a small expansion parameter $\epsilon = \langle S \rangle / M_{st}$ needed for explaining the fermion mass hierarchy. $U(1)$ invariance forbids renormalizable Yukawa couplings for the light families, but would allow them through effective nonrenormalizable couplings suppressed by a factor $(S/M_{st})^{n_{ij}}$ (for the fermion mass operator connecting flavors i and j) with n_{ij} being positive integers. Even with all couplings being of order one, hierarchical masses for different flavors are naturally realized [7]. Although this mechanism will work with any flavor $U(1)$, anomalous $U(1)$ models are attractive since they would also provide a natural understanding for the smallness of $\epsilon \sim 0.2$ [10], which arises from the one–loop induced Fayet–Illiopoulos D–term [9] by demanding that SUSY is left unbroken near M_{st}.

Consider the following fermion mass matrices studied in Ref. 2:

$$M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^{8-2\alpha} & \epsilon^{6-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^{6-\alpha} & \epsilon^4 & \epsilon^2 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \qquad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^{4-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix},$$
$$M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^3 & \epsilon \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \qquad M_{\nu_D} \sim \langle H_u \rangle \epsilon^s \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$
$$M_{\nu^c} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \Rightarrow \quad M_\nu^{light} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2s} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \tag{2.3}$$

Here M_u, M_d and M_e are the up–quark, down–quark, and the charged lepton mass matrices (written in the basis $uM_u u^c$, etc.). M_{ν_D} is the Dirac neutrino mass matrix, and M_{ν^c} is the right–handed neutrino Majorana mass matrix. The light neutrino mass matrix M_ν^{light} is derived from the seesaw mechanism [16]. The neutrino mass matrix is given by

$$\mathcal{L}_\nu = -\left(\nu, \nu^c\right) \begin{pmatrix} 0 & M_{\nu_D}^T \\ M_{\nu_D} & M_{\nu^c} \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}. \tag{2.4}$$

Upon integrating out the heavy right–handed neutrinos ν^c one finds the light neutrino mass matrix as follows:

$$M_\nu^{lihgt} = M_{\nu_D} \frac{1}{M_{\nu^c}} M_{\nu_D}^T. \tag{2.5}$$

When $M_R \sim 10^{14}$ GeV and $M_{\nu_D} \sim 10^2$ GeV, this mechanism leads to light neutrino masses, for exapmple, $(M_{\nu_3}^{light}) \sim 0.1$ eV for the heaviest light neutrino mass. We have not exhibited order one coefficients in the matrix elements of Eq. (2.3). The quark and lepton mass matrices arising from Eq. (2.3) are fully consistent if $\epsilon \sim 0.2$. The exponent p appearing in the overall factor ϵ^p multiplying M_d and M_e is assumed to take values 0, 1 or 2 corresponding to large (~ 20), moderate (~ 10), and small (~ 5) values of $\tan\beta$ ($\equiv \langle H_u\rangle/\langle H_d\rangle$) respectively.

The parameter α is allowed to take two values, 0 and 1, corresponding to Model 1 ($\alpha = 0$) and Model 2 ($\alpha = 1$). The two models differ only in the masses and mixings of the first family. Both models give excellent fits to the fermion masses and mixings including neutrino oscillation parameters. Their predictions for LFV are however noticeably different, which we analyze in Section 2.5.2.

A general form of the superpotential which can explain the fermion masses and mixing hierarchy through the Froggatt–Nielsen mechanism has the form

$$\begin{aligned} W &= y_{ij}^u Q_i u_j^c H_u \left(\frac{S}{M_{st}}\right)^{n_{ij}^u} + y_{ij}^d Q_i d_j^c H_d \left(\frac{S}{M_{st}}\right)^{n_{ij}^d} \\ &+ y_{ij}^e L_i e_j^c H_d \left(\frac{S}{M_{st}}\right)^{n_{ij}^e} + y_{ij}^\nu L_i \nu_j^c H_u \left(\frac{S}{M_{st}}\right)^{n_{ij}^\nu} \\ &+ \frac{1}{2} M_{Rij} \nu_i^c \nu_j^c \left(\frac{S}{M_{st}}\right)^{n_{ij}^{\nu^c}} + \mu H_u H_d \, , \end{aligned} \qquad (2.6)$$

where $i, j = (1, 2, 3)$ are family indices, n_{ij}^u, n_{ij}^d, n_{ij}^e, n_{ij}^ν and $n_{ij}^{\nu^c}$ are positive integers fixed by the choice of $U(1)_A$ charge assignment *. y_{ij}^u etc. are Yukawa coupling coefficients which are all taken to be of order one. Here μ is the Higgsino mass parameter which should be of order 10^2 GeV for consistency with electroweak symmetry breaking. The smallness of μ compared to the string scale may be explained by the Giudice–Masiero mechanism, where μ gets related to the SUSY breaking scale [17]. We assume such a mechanism explains the origin of the μ term. M_R in Eq. (2.6) is the right-handed neutrino (ν_i^c) mass scale, which is taken to be of order 10^{14} GeV.

*The definition of the Yukawa couplings here differs from that of Ref. 2 and 3 by factors $n_{ij}^f!$. The normalization in Eq. (2.6) is the consistent normalization corresponding to our RGE analysis where we adopted the effective Yukawa couplings to be $Y_{ij}^f \equiv y_{ij}^f \epsilon^{n_{ij}^f}$.

Although it is possible to explain the value of M_R through operator such as (S^4/M_{st}^3), here we assume it to be an input mass parameter.

The soft supersymmetry breaking terms which will induce LFV have the form given by

$$\begin{aligned} -\mathcal{L}_{soft} &= \Bigg\{ a_{ij}^u \tilde{Q}_i \tilde{u}_j^c H_u \left(\frac{S}{M_{st}}\right)^{n_{ij}^u} + a_{ij}^d \tilde{Q}_i \tilde{d}_j^c H_d \left(\frac{S}{M_{st}}\right)^{n_{ij}^d} \\ &+ a_{ij}^e \tilde{L}_i \tilde{e}_j^c H_d \left(\frac{S}{M_{st}}\right)^{n_{ij}^e} + a_{ij}^\nu \tilde{L}_i \tilde{\nu}_j^c H_u \left(\frac{S}{M_{st}}\right)^{n_{ij}^\nu} + \frac{1}{2} B' M_{Rij} \tilde{\nu}_i^c \tilde{\nu}_j^c \left(\frac{S}{M_{st}}\right)^{n_{ij}^{\nu^c}} \\ &+ \frac{1}{2} \sum_{i=1,2,3} M_{1/2}^i \lambda_i \lambda_i + \frac{1}{2} M_{\lambda_F} \lambda_F \lambda_F + B\mu H_u H_d + \text{h. c.} \Bigg\} \\ &+ \sum \left(\tilde{m}_f^2\right)_{ab} \tilde{f}_a^\dagger \tilde{f}_b + \tilde{m}_s{}^2 |S|^2 + \tilde{m}_{H_u}^2 |H_u|^2 + \tilde{m}_{H_d}^2 |H_d|^2 \,. \end{aligned} \tag{2.7}$$

Here a tilde stands for the scalar components of the matter superfields, and λ_i and λ_F are the MSSM gauginos and the flavor $U(1)_A$ gaugino. ($M_{1/2}^i$, M_{λ_F}) and (($\tilde{m}_f^2)_{ab}$, $\tilde{m}_s^2$) are the gaugino and scalar soft masses respectively. $\tilde{f}_a$ stands for the MSSM sfermions including the right–handed sneutrinos. Note that the generalized A–terms in Eq. (2.7) has the same structure as the corresponding superpotential terms of Eq. (2.6).

We assign flavor $U(1)_A$ charges to the MSSM fields such that the observed fermion mass and mixing hierarchies are obtained with all Yukawa couplings being order one. As we will show explicitly, the expansion parameter $\epsilon = \langle S \rangle / M_{st}$ is naturally of order 0.2 in anomalous $U(1)$ models. We use the idea of "lopsided" mass matrices for generating large neutrino mixings [18], while maintaining small quark mixings. This can be seen by examining, for example, $\theta_{23}^\nu \sim (M_\nu^{light})_{23}/(M_\nu^{light})_{33} \sim O(1)$ and $V_{cb} \sim (M_q)_{23}/(M_q)_{33} \sim O(\epsilon^2)$. W_A in Eq. (2.6) contains MSSM singlet fields X_k which would be needed for anomaly cancelation. The $U(1)$ charge assignment shown in Table 2.1 will lead to the texture of Eq. (2.3).

We use the Green–Schwarz (GS) mechanism [8] for anomaly cancellation associated with $U(1)_A$ gauge symmetry. Heterotic superstring theory when compactified to four dimensions contains the Lagrangian terms $L \supset \varphi(x) \sum_i k_i F_i^2 + i\eta(x) \sum_i k_i F_i \tilde{F}_i$, where k_i are the Kac–Moody levels, $\varphi(x)$ is the dilaton field and $\eta(x)$ is its axionic partner. The Green–Schwarz mechanism makes use of the transformation $\eta(x) \to \eta(x) - \theta(x)\delta_{GS}$, and the gauge variation for the $U(1)_A$ gauge field, $V_\mu \to V_\mu + \partial_\mu \theta(x)$.

Field	$U(1)_A$ Charge	Charge notation
Q_1, Q_2, Q_3	$4-\alpha$, 2, 0	q_i^Q
L_1, L_2, L_3	$1+s$, s, s	q_i^L
u_1^c, u_2^c, u_3^c	$4-\alpha$, 2, 0	q_i^u
d_1^c, d_2^c, d_3^c	$1+p$, p, p	q_i^d
e_1^c,e_2^c,e_3^c	$4+p-s-\alpha$, $2+p-s$, $p-s$	q_i^e
ν_1^c, ν_2^c, ν_3^c	1, 0, 0	q_i^ν
H_u, H_d, S	0, 0, -1	$(h, \bar{h}, q_s)$

TABLE 2.1. The flavor $U(1)_A$ charge assignments for the MSSM fields and the flavon field S in the normalization of $q_s = -1$. Here α is 0 (1) for Model 1 (Model 2). In the third column we list the generic notation for the charges used in the RGE analysis.

The $U(1)_A$ anomalies are cancelled by the GS mechanism [8] which requires

$$\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \frac{A_A}{3k_A} = \frac{A_{gravity}}{24}\,, \tag{2.8}$$

where A_i and A_A are the coefficients $U(1)_Y^2 \times U(1)_A$, $SU(2)_L^2 \times U(1)_A$, $SU(3)_C^2 \times U(1)_A$ and $U(1)_A^3$ gauge anomalies respectively *. k_i and k_A are the Kac–Moody levels. $A_{gravity}$ is the mixed gravitational anomaly coefficient which is given by the trace of the $U(1)_A$ charges over all fields. With the non–Abelian levels $k_2 = k_3 = 1$, which is the simplest possibility, from Table 2.1 and Eq. (2.8) one finds

$$\begin{aligned} A_2 &= \frac{19 - 3\alpha + 3s}{2}, \\ A_3 &= \frac{19 - 3\alpha + 3p}{2}\,. \end{aligned} \tag{2.9}$$

This implies that $p = s$. Furthermore,

$$A_1 = \frac{5}{6}(19 - 3\alpha + 3p)\,, \tag{2.10}$$

which fixes the level k_1 to be 5/3.

With $p = s$ the charges given in Table 2.1 are compatible with $SU(5)$ unification. In string theory gauge coupling unification can occur without a simple covering group.

*Here we include factor of 1/3 for the $U(1)_A$ cubic anomaly due to three identical external gauge boson legs which was not included in our previous discussion.[2]

The string unification condition is [19]

$$k_1 g_1^2 = k_2 g_2^2 = k_3 g_3^2 = k_A g_F^2 = 2 g_{st}^2 \,. \qquad (2.11)$$

There is subtlety in choosing the coefficient in front of the string coupling[20] which we became aware of after completing the project: according to perturbative string calculation, instead of 1 which we chose earlier[2], it may be more proper to choose 2. This and the factor 1/3 for $U(1)_A^3$ anomaly have been accommodated in our later project on EDM discussed in chapter 3. Our result $k_1 = 5/3$ is what is needed for consistency of the observed unification of gauge couplings in the MSSM. The discrepancy in the unification scale derived from low energy data versus perturbative string theory evaluation can be reconciled in the context of M–theory by making use of the radius of the eleventh dimension [21]. We assume such a scenario.

From now on we shall assume the $SU(5)$ normalization for g_1. If one assumes that the field content of the model is just the one listed in Table 2.1, the gravitational anomaly $A_{gravity}$ would not satisfy the GS condition. One simple solution is the introduction of additional MSSM singlet (hidden sector) fields X_k. Then Eq. (2.8) leads to the following result:

$$A_{gravity} = 5\,(13 - 2\alpha + 3p) + \sum_k q_k^X = 12(19 - 3\alpha + 3p)\,, \qquad (2.12)$$

where q_k^X are the $U(1)_A$ charges of the extra fields X_k. From this, one gets $\sum_k q_k^X = (163 - 26\alpha + 21p)$. We assume for simplicity that all the X_k fields have the same flavor charge equal to 1. The number n^X of X_k fields is then fixed to be

$$n^X = 163 - 26\alpha + 21p \,. \qquad (2.13)$$

We are now in a position to determine the level k_A as well as the $U(1)_A$ gauge coupling g_F at the unification scale. We renormalize the $U(1)_A$ charges by a factor $|q_s|$ so that the charge of the flavon field is now $-|q_s|$. $|q_s|$ is determined by demanding $g_F^2 = g_2^2$ at the unification scale. Eq. (2.8) and the number n^X in Eq. (2.13) then fix $|q_s|$ to be

$$|q_s| = \sqrt{\frac{19 - 3\alpha + 3p}{10(4-\alpha)^3 + 5[(1+p)^3 + 2p^3] + n^X}} \,. \qquad (2.14)$$

ϵ	$p=0$	$p=1$	$p=2$
$\alpha=0$	0.250	0.270	0.288
$\alpha=1$	0.230	0.250	0.270

TABLE 2.2. Numerical values for the small expansion parameter ϵ corresponding to different fermion mass hierarchy structure. See chapter 3 for different values.

For $p = (0,\, 1,\, 2)$ one has $|q_s| = (0.165,\, 0.172,\, 0.166)$ for Model 1 ($\alpha = 0$) and $|q_s| = (0.225,\, 0.228,\, 0.203)$ for Model 2 ($\alpha = 1$).

The Fayet–Iliopoulos term for the anomalous $U(1)_A$, generated through the gravitational anomaly, is given by [9]

$$\xi = \frac{g_{st}^2 M_{st}^2}{192\pi^2} |q_s| A_{gravity}\,, \tag{2.15}$$

where g_{st} is the unified gauge coupling at the string scale. By minimizing the potential

$$V = \frac{|q_s|^2 g_F^2}{8} \left(\frac{\xi}{|q_s|} - |S|^2 + \sum_a q_a^f |\tilde{f}_a|^2 + \sum_k q_k^X |X_k|^2 \right)^2 \tag{2.16}$$

in the unbroken supersymmetric limit we obtain

$$\epsilon = \frac{\langle S \rangle}{M_{st}} = \sqrt{\frac{g_{st}^2 A_{gravity}}{192\pi^2}}\,. \tag{2.17}$$

The numerical values of ϵ derived from Eq. (2.17) for different p and α are listed in Table 2.2 by making use of Eq. (2.14). This is the small expansion parameter appearing in the mass matrices of Eq. (2.3). Here we took $g_{st}^2/4\pi \simeq 1/24$.

The mass of the flavor $U(1)_A$ gauge boson is found to be

$$M_F = \frac{|q_s| g_F \langle S \rangle}{\sqrt{2}}\,. \tag{2.18}$$

Between the string scale M_{st} and M_F the flavor gaugino contributes to flavor violating processes. This mass can now be determined:

$$M_F = \left(\frac{M_{st}}{51.0}, \frac{M_{st}}{49.3}, \frac{M_{st}}{50.9} \right) \text{ for } p = (0,1,2)\,, \tag{2.19}$$

in the case of Model 1 and

$$M_F = \left(\frac{M_{st}}{39.7}, \frac{M_{st}}{39.2}, \frac{M_{st}}{43.0} \right) \text{ for } p = (0,1,2)\,, \tag{2.20}$$

in the case of Model 2.

2.3 Fermion Mass Fits

Here we present numerical fits to the fermion masses and mixings for Model 1 and Model 2 adopted for the calculation of the branching ratios for LFV processes. These fits will be used in our quantitative analysis of lepton flavor violation.

As input at low energy we choose the following values for the running quark masses [22]

$$m_u(1\,\mathrm{GeV}) = 5.11\,\mathrm{MeV}, \quad m_c(m_c) = 1.27\,\mathrm{GeV}, \quad m_t(m_t) = 167\,\mathrm{GeV},$$
$$m_d(1\,\mathrm{GeV}) = 8.9\,\mathrm{MeV}, \quad m_s(1\,\mathrm{GeV}) = 130\,\mathrm{MeV}, \quad m_b(m_b) = 4.25\,\mathrm{GeV}. \quad (2.21)$$

The CKM mixing matrix elements are chosen to be $|V_{us}| = 0.222$, $|V_{ub}| = 0.0035$, $|V_{cb}| = 0.04$ and $\eta = 0.33$ (the Wolfenstein parameter of CP–violation). Using two–loop QED and QCD renormalization group equations we obtain these running parameters at the SUSY breaking scale, $M_{SUSY} = 500$ GeV, with $\alpha_s(M_Z) = 0.118$, to be

$$r_f \equiv \frac{m_f(M_{SUSY})}{m_f(m_f)}\,, \quad (2.22)$$

where

$$(r_t,\, r_b,\, r_\tau,\, r_u,\, r_c,\, r_{d,s},\, r_{e,\mu}) = (0.943,\, 0.605,\, 0.991,\, 0.395,\, 0.442,\, 0.398,\, 0.989)\,. \quad (2.23)$$

Using two–loop SUSY RGE evaluation above M_{SUSY} we obtain the Yukawa couplings at the $U(1)_A$ breaking scale ($\sim 10^{15}$ GeV) to be

$$\begin{aligned}
(Y_u,\, Y_c,\, Y_t) &= \left(5.135 \times 10^{-6},\, 1.426 \times 10^{-3},\, 0.538\right), \\
(Y_d,\, Y_s,\, Y_b) &= \left(3.459 \times 10^{-5},\, 5.052 \times 10^{-4},\, 2.768 \times 10^{-2}\right), \\
(Y_e,\, Y_\mu,\, Y_\tau) &= \left(1.024 \times 10^{-5},\, 2.118 \times 10^{-3},\, 3.572 \times 10^{-2}\right), \\
(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) &= \left(3.515 \times 10^{-4},\, 8.419 \times 10^{-4},\, 1.131 \times 10^{-2}\right), \quad (2.24)
\end{aligned}$$

for $\tan\beta = 5$,

$$\begin{aligned}
(Y_u,\, Y_c,\, Y_t) &= \left(4.999 \times 10^{-6},\, 1.389 \times 10^{-3},\, 0.518\right), \\
(Y_d,\, Y_s,\, Y_b) &= \left(6.844 \times 10^{-5},\, 9.997 \times 10^{-4},\, 5.470 \times 10^{-2}\right), \\
(Y_e,\, Y_\mu,\, Y_\tau) &= \left(2.027 \times 10^{-5},\, 4.192 \times 10^{-3},\, 7.094 \times 10^{-2}\right), \\
(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) &= \left(1.708 \times 10^{-3},\, 4.105 \times 10^{-3},\, 5.519 \times 10^{-2}\right), \quad (2.25)
\end{aligned}$$

for $\tan\beta = 10$, and

$$\begin{aligned}
(Y_u,\, Y_c,\, Y_t) &= \left(4.996\times 10^{-6},\, 1.387\times 10^{-3},\, 0.518\right),\\
(Y_d,\, Y_s,\, Y_b) &= \left(1.40\times 10^{-4},\, 2.045\times 10^{-3},\, 0.113\right),\\
(Y_e,\, Y_\mu,\, Y_\tau) &= \left(4.132\times 10^{-5},\, 8.545\times 10^{-3},\, 0.147\right),\\
(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) &= \left(8.551\times 10^{-3},\, 2.059\times 10^{-2},\, 0.278\right), \qquad (2.26)
\end{aligned}$$

for $\tan\beta = 20$. $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$ and $|V_{ts}|$ are multiplicatively renormalized by an RGE factor of 0.9 in going from the low energy scale to the $U(1)_A$ breaking scale.

We have determined the Dirac neutrino Yukawa couplings as follows. First we note that the anomaly cancellation conditions in Eq. (2.8) implies $p = s$, which means that the Dirac neutrino Yukawa couplings are fixed to be of the same order as the charged lepton Yukawa couplings. Now, if one takes the right–handed Majorana neutrino mass matrix to be proportional to the transpose of the Dirac neutrino Yukawa coupling matrix for simplicity, $M_{\nu^c} = Y_\nu^T\, M_R\, \epsilon^p$, then the light neutrino mass matrix is given by

$$M_\nu^{light} = Y_\nu M_{\nu^c}^{-1} Y_\nu^T\, v^2\, sin^2\beta = Y_\nu \frac{v^2 \sin^2\beta}{M_R\, \epsilon^p}\,. \qquad (2.27)$$

This simplified choice is certainly consistent with the fermion mass structures we have chosen in Eqs. (2.3). We adopt this choice in our analysis. Y^ν is determined from a fit to the light neutrino oscillation parameters with $M_R = 10^{14}$ GeV. This fit corresponds to $m_{\nu_e} = 2.7\times 10^{-3}$ eV, $m_{\nu_\mu} = 6.4\times 10^{-3}$ eV and $m_{\nu_\tau} = 8.6\times 10^{-2}$ eV and the leptonic mixing matrix given by

$$V_{MNS} = \begin{pmatrix} 0.848 & -0.526 & -0.0409 \\ 0.349 & 0.619 & -0.72 \\ -0.4 & -0.59 & -0.7013 \end{pmatrix}. \qquad (2.28)$$

We also consider a scenario where the Dirac neutrino Yukawa couplings are maximized by choosing $M_R = 4\times 10^{14}$ GeV. In this case we have

$$\begin{aligned}
(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) &= \left(1.406\times 10^{-3},\, 3.368\times 10^{-3},\, 4.530\times 10^{-2}\right) \text{ for } \tan\beta = 5,\\
(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) &= \left(6.843\times 10^{-3},\, 1.645\times 10^{-2},\, 0.222\right) \text{ for } \tan\beta = 10,\\
(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) &= \left(3.514\times 10^{-2},\, 8.464\times 10^{-2},\, 1.237\right) \text{ for } \tan\beta = 20. \qquad (2.29)
\end{aligned}$$

There is some freedom in choosing the overall scale of Y_ν consistent with Y_{ν_3} being of order one (see Eq. (2.3)). If both M_R and Y_ν are increased by a common factor, the observable M_ν^{light} will remain unchanged (see Eq. (2.27)). Different choices of Y_ν will lead to different contributions to LFV from the ν^c sector. We illustrate this variation with two choices of Y_ν.

We now present our fits to the observables of Eqs. (2.24)–(2.29) consistent with the texture of Eq. (2.3). This cannot be done uniquely since the right–handed rotation matrices are unknown from low energy data, so we make a specific choice. In our lepton flavor violation analysis we shall make use of this specific fit. One should bear in mind that there are uncertain coefficients of order one in the Yukawa matrices of our fit, which can lead to an order of magnitude uncertainty in the branching ratios for LFV processes.

We introduce the following notation:

$$Y_{ij}^f \equiv y_{ij}^f \, \epsilon^{n_{ij}^f}. \tag{2.30}$$

This is the effective Yukawa couplings below the flavor scale M_F we have used in our fermion mass fit. In Model 1, a good fit to the Yukawa couplings matrices is found to be

$$Y^u = y_{33}^u \begin{pmatrix} 3.91\,\epsilon^8 & 0.226\,\epsilon^6 & 0.375\,\epsilon^4 \\ 0.226\,\epsilon^6 & 1.91\,\epsilon^4 & 0.499\,\epsilon^2 \\ 0.375\,\epsilon^4 & 0.499\,\epsilon^2 & 1 \end{pmatrix},$$

$$Y^d = y_{33}^d\,\epsilon^p \begin{pmatrix} (1.56+0.115i)\,\epsilon^5 & (0.909+0.054i)\epsilon^4 & (0.658+0.131i)\,\epsilon^4 \\ -2.89\,\epsilon^3 & 1.02\,\epsilon^2 & 1.22\,\epsilon^2 \\ (-0.878+0.88\times10^{-3}i)\,\epsilon & 0.412+0.11\times10^{-6}i & 1+0.73\times10^{-6}i \end{pmatrix},$$

$$Y^e = y_{33}^e\,\epsilon^p \begin{pmatrix} 1.89\,\epsilon^5 & 1.57\,\epsilon^3 & 0.812\,\epsilon \\ 0.487\,\epsilon^4 & 2.14\,\epsilon^2 & 0.316 \\ 1.10\,\epsilon^4 & 1.52\,\epsilon^2 & 1 \end{pmatrix},$$

$$Y^\nu = y_{33}^\nu\,\epsilon^p \begin{pmatrix} 1.51\,\epsilon^2 & -0.358\,\epsilon & -0.438\,\epsilon \\ -0.358\,\epsilon & 0.339 & 0.485 \\ -0.438\,\epsilon & 0.485 & 1 \end{pmatrix}. \tag{2.31}$$

Here

$$
\begin{aligned}
y^u_{33} &= (\,0.539,\, 0.523,\, 0.519)\,, \\
y^d_{33} &= (\,0.650,\, 0.257,\, 0.106)\,, \\
y^e_{33} &= (\,0.840,\, 0.333,\, 0.139)\,, \\
y^\nu_{33} &= (\,0.225,\, 0.219,\, 0.221)\,,
\end{aligned}
\tag{2.32}
$$

for ($p = 2, 1, 0$) which we shall associate with $\tan\beta = (\,5,\, 10,\, 20)$. Here we have taken $\epsilon = 0.2$. For simplicity we assumed the leptonic Yukawa couplings to be all real.

In Model 2 we have the following fit for the Yukawa coupling matrices:

$$
Y^u = y^u_{33} \begin{pmatrix} 0.876\,\epsilon^6 & 1.30\,\epsilon^5 & 0.499\,\epsilon^3 \\ 1.30\,\epsilon^5 & 2.59\,\epsilon^4 & 0.993\,\epsilon^2 \\ 0.499\,\epsilon^3 & 0.993\,\epsilon^2 & 1 \end{pmatrix},
$$

$$
Y^d = y^d_{33}\,\epsilon^p \begin{pmatrix} (3.01 + 0.13i)\,\epsilon^4 & (2.66 + 0.13i)\epsilon^3 & (1.21 + 0.13i)\,\epsilon^3 \\ 1.79\,\epsilon^3 & 2.26\,\epsilon^2 & 1.42\,\epsilon^2 \\ (1.00 + 0.33 \times 10^{-3}i)\,\epsilon & 0.987 + 0.582 \times 10^{-5}i & 1 + 0.21 \times 10^{-5}i \end{pmatrix},
$$

$$
Y^e = y^e_{33}\,\epsilon^p \begin{pmatrix} 1.19\,\epsilon^4 & 1.68\,\epsilon^3 & 0.579\,\epsilon \\ 0.892\,\epsilon^3 & 2.18\,\epsilon^2 & 0.350 \\ 1.36\,\epsilon^3 & 1.45\,\epsilon^2 & 1 \end{pmatrix},
$$

$$
Y^\nu = y^\nu_{33}\,\epsilon^p \begin{pmatrix} 1.53\,\epsilon^2 & -0.329\,\epsilon & -0.406\,\epsilon \\ -0.329\,\epsilon & 0.293 & 0.449 \\ -0.406\,\epsilon & 0.449 & 1 \end{pmatrix}. \tag{2.33}
$$

Here

$$
\begin{aligned}
y^u_{33} &= (\,0.535,\, 0.52,\, 0.515)\,, \\
y^d_{33} &= (\,0.650,\, 0.257,\, 0.107)\,, \\
y^e_{33} &= (\,0.840,\, 0.333,\, 0.139)\,, \\
y^\nu_{33} &= (\,0.233,\, 0.228,\, 0.229)\,,
\end{aligned}
\tag{2.34}
$$

for three different values of $p = (2,\, 1,\, 0)$ identified with $\tan\beta = (\,5,\, 10,\, 20)$.

2.4 Generalized RGE Analysis of Soft SUSY Breaking Parameters

In this Section we give a general RGE analysis of the soft SUSY breaking parameters including higher dimensional operators as shown in Eqs. (2.6) and (2.7). This includes the effects of the flavor $U(1)_A$ gaugino sector. Our analysis of this section should apply to a large class of Froggatt-Nielsen models.

It turns out that the generalized RGEs, although derived in the momentum range $M_F \leq \mu \leq M_{st}$, can conveniently be written in terms of the effective Yukawa couplings introduced in Eq. (2.30). For this reason, let us introduce the following notation:

$$A^f_{ij} \equiv a^f_{ij}\, \epsilon^{n^f_{ij}}. \tag{2.35}$$

The one–loop β–functions for the soft scalar masses of the sleptons are found in the momentum range $M_F \leq \mu \leq M_{st}$ to be

$$\begin{aligned} \beta\left(\tilde{m}^2_L\right)_{ij} &= \beta\left(\tilde{m}^2_L\right)^{MSSM}_{ij} + \frac{1}{16\pi^2}\Big\{\left(\tilde{m}^2_L Y^{\nu\dagger}Y^\nu + Y^{\nu\dagger}Y^\nu\tilde{m}^2_L\right)_{ij} \\ &\quad +2\left(Y^{\nu\dagger}\tilde{m}^2_\nu Y^\nu + \tilde{m}^2_{H_u}Y^{\nu\dagger}Y^\nu + A^{\nu\dagger}A^\nu\right)_{ij} \\ &\quad +2q^L_i g^2_F \delta_{ij}\left(\sigma - 4q^L_i (M_{\lambda_F})^2\right)\Big\}\ , \end{aligned} \tag{2.36}$$

$$\beta\left(\tilde{m}^2_e\right)_{ij} = \beta\left(\tilde{m}^2_e\right)^{MSSM}_{ij} + \frac{1}{16\pi^2}2q^e_i g^2_F\delta_{ij}\left(\sigma - 4q^e_i(M_{\lambda_F})^2\right), \tag{2.37}$$

$$\begin{aligned} \beta\left(\tilde{m}^2_\nu\right)_{ij} &= \frac{1}{16\pi^2}\Big\{2\left(\tilde{m}^2_\nu Y^\nu Y^{\nu\dagger} + Y^\nu Y^{\nu\dagger}\tilde{m}^2_\nu\right)_{ij} \\ &\quad +4\left(Y^\nu\tilde{m}^2_\nu Y^{\nu\dagger} + \tilde{m}^2_{H_u}Y^\nu Y^{\nu\dagger} + A^\nu A^{\nu\dagger}\right)_{ij} \\ &\quad +2q^\nu_i g^2_F\delta_{ij}\left(\sigma - 4q^\nu_i(M_{\lambda_F})^2\right)\Big\}\,. \end{aligned} \tag{2.38}$$

Similarly the β–functions for the squark soft masses are given by

$$\beta\left(\tilde{m}^2_Q\right)_{ij} = \beta\left(\tilde{m}^2_Q\right)^{MSSM}_{ij} + \frac{1}{16\pi^2}2q^Q_i g^2_F\delta_{ij}\left(\sigma - 4q^Q_i(M_{\lambda_F})^2\right), \tag{2.39}$$

$$\beta\left(\tilde{m}^2_u\right)_{ij} = \beta\left(\tilde{m}^2_u\right)^{MSSM}_{ij} + \frac{1}{16\pi^2}2q^u_i g^2_F\delta_{ij}\left(\sigma - 4q^u_i(M_{\lambda_F})^2\right), \tag{2.40}$$

$$\beta\left(\tilde{m}^2_d\right)_{ij} = \beta\left(\tilde{m}^2_d\right)^{MSSM}_{ij} + \frac{1}{16\pi^2}2q^d_i g^2_F\delta_{ij}\left(\sigma - 4q^d_i(M_{\lambda_F})^2\right)\ . \tag{2.41}$$

Here σ is defined as

$$\sigma = 3\,\mathrm{Tr}\left(2q^Q\tilde{m}_Q^2 + q^u\tilde{m}_u^2 + q^d\tilde{m}_d^2\right) + \mathrm{Tr}\left(2q^L\tilde{m}_L^2 + q^e\tilde{m}_e^2 + q^\nu\tilde{m}_\nu^2\right) + q_s\tilde{m}_s^2 + \sum_k q_k^X\tilde{m}_{X_k}^2\,, \tag{2.42}$$

where $\tilde{m}_{X_k}$ is the soft mass of the extra particles X_k and the trace is over family space. Here $\beta\,(\tilde{m}_L^2)_{ij}^{MSSM}$ stands for the MSSM β–function without the ν^c or the flavor $U(1)_A$ contributions [23].

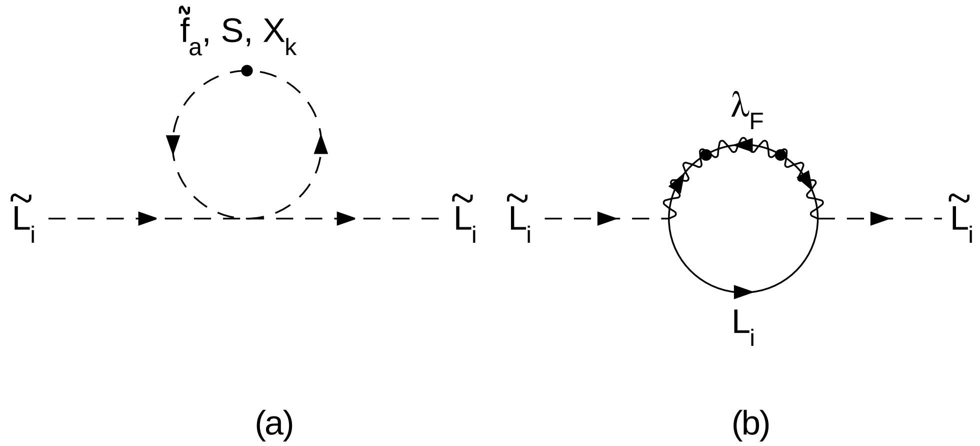

Figure 2.1. (a) Trace correction from D_A–term to the soft scalar masses and (b) $U(1)_A$ gaugino–induced corrections to the soft masses.

The contributions proportional to σ in Eqs. (2.36)–(2.41) arise from the diagram in Figure 1 (a) which has its origin from the $U(1)_A$ D–term. We call this the trace contributions. For a non–anomalous $U(1)$ gauge symmetry with universal scalar masses the trace term would vanish. However, for an anomalous $U(1)$ gauge symmetry, trace of the flavor charges is not zero, so this term will induce flavor non–universal masses. The diagram in Figure 1 (b) is the source of flavor non–universal contributions proportional to the gaugino mass M_{λ_F} in Eqs. (2.36)–(2.41).

Now we give the expressions for the one–loop contributions to the β–function of the SUSY breaking A–terms of Eq. (2.7). There are two types of contributions to the β–functions of A_{ij}^f: one from the gaugino and the other from the A–terms. The flavor gaugino contribution arises from diagrams such as the one in Figure 2.2. The A–term contribution to $\beta\left(a^f\right)$ cannot have the flavon field S propagating in the loop, so that contribution is included in the MSSM piece.

The gaugino vertex contribution to $\beta\left(A^e_{ij}\right)$ (see Figure 2.2) is

$$\beta(a^e_{ij})^V = \frac{1}{4\pi^2} M_{\lambda_F} g_F^2 y^e_{ij} \left(q_i^L q_j^e + q_i^L \bar{h} + q_j^e \bar{h} + n^e_{ij} q_s (q_i^L + q_j^e + \bar{h}) + \frac{1}{2} n^e_{ij} (n^e_{ij} - 1) q_s^2 \right). \quad (2.43)$$

Eq. (2.43) is obtained by summing all possible gaugino exchange diagrams.

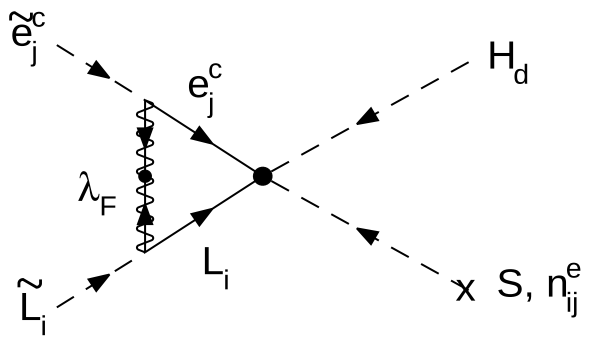

Figure 2.2. $U(1)_A$ gaugino–induced vertex correction diagram for A–terms.

We now list the full one–loop β function for each A^f_{ij}. This generalizes the results of Martin [24].

$$\beta(A^e)_{ij} = \beta(A^e)^{MSSM}_{ij} + \frac{1}{16\pi^2} \left\{ A^e \left[Y^{\nu\dagger} Y^\nu - 2\left((q_i^L)^2 + (q_j^e)^2 + \bar{h}^2 \right) g_F^2 \right] + 2 Y^e Y^{\nu\dagger} A^\nu \right\}_{ij} + \frac{1}{4\pi^2} g_F^2 \, Z^e_{ij} Y^e_{ij} M_{\lambda_F}, \quad (2.44)$$

$$\begin{aligned}\beta(A^\nu)_{ij} = \frac{1}{16\pi^2} \Big\{ & A^\nu \Big[5 Y^{\nu\dagger} Y^\nu + Y^{e\dagger} Y^e + \mathrm{T}r\left(3 Y^u Y^{u\dagger} + Y^\nu Y^{\nu\dagger} \right) \\ & - 3g_2^2 - 3g_1^2/5 - 2\left((q_i^L)^2 + (q_j^\nu)^2 + h^2 \right) g_F^2 \Big] \\ & + 2 Y^\nu \Big[2 Y^{\nu\dagger} A^\nu + Y^{e\dagger} A^e + \mathrm{T}r\left(3 A^u Y^{u\dagger} + A^\nu Y^{\nu\dagger} \right) \\ & + 3 M_1 g_1^2/5 + 3 M_2 g_2^2 \Big] \Big\}_{ij} + \frac{1}{4\pi^2} g_F^2 \, Z^\nu_{ij} Y^\nu_{ij} M_{\lambda_F}, \end{aligned} \quad (2.45)$$

$$\beta(A^u)_{ij} = \beta(A^u)^{MSSM}_{ij} + \frac{1}{16\pi^2} \left\{ A^u \left[\mathrm{T}r\left(Y^\nu Y^{\nu\dagger} \right) - 2\left((q_i^Q)^2 + (q_j^u)^2 + h^2 \right) g_F^2 \right] + 2 Y^u \mathrm{T}r\left(A^\nu Y^{\nu\dagger} \right) \right\}_{ij} + \frac{1}{4\pi^2} g_F^2 \, Z^u_{ij} Y^u_{ij} M_{\lambda_F}, \quad (2.46)$$

$$\beta(A^d)_{ij} = \beta(A^d)^{MSSM}_{ij} - \frac{1}{8\pi^2} g_F^2 A^d_{ij} \left((q_i^Q)^2 + (q_j^d)^2 + \bar{h}^2 \right) + \frac{1}{4\pi^2} g_F^2 \, Z^d_{ij} Y^d_{ij} M_{\lambda_F}. \quad (2.47)$$

Here we defined the combination of the $U(1)_A$ charges Z^f_{ij} as

$$\begin{aligned} Z^u_{ij} &= q^Q_i q^u_j + q^Q_i h + q^u_j h + n^u_{ij} q_s (q^Q_i + q^u_j + h) + \frac{1}{2} n^u_{ij} (n^u_{ij} - 1) q^2_s, \\ Z^d_{ij} &= q^Q_i q^d_j + q^Q_i \bar{h} + q^d_j \bar{h} + n^d_{ij} q_s (q^Q_i + q^d_j + \bar{h}) + \frac{1}{2} n^d_{ij} (n^d_{ij} - 1) q^2_s, \\ Z^e_{ij} &= q^L_i q^e_j + q^L_i \bar{h} + q^e_j \bar{h} + n^e_{ij} q_s (q^L_i + q^e_j + \bar{h}) + \frac{1}{2} n^e_{ij} (n^e_{ij} - 1) q^2_s, \\ Z^\nu_{ij} &= q^L_i q^\nu_j + q^L_i h + q^\nu_j h + n^\nu_{ij} q_s (q^L_i + q^\nu_j + h) + \frac{1}{2} n^\nu_{ij} (n^\nu_{ij} - 1) q^2_s. \end{aligned} \tag{2.48}$$

From the charges listed in Table 2.1 we have

$$Z^e = - \begin{pmatrix} (11,\ 13,\ 16) & (4,\ 6,\ 9) & (1,\ 3,\ 6) \\ (10,\ 11,\ 13) & (3,\ 4,\ 6) & (0,\ 1,\ 3) \\ (10,\ 11,\ 13) & (3,\ 4,\ 6) & (0,\ 1,\ 3) \end{pmatrix} \tag{2.49}$$

in Model 1 ($\alpha = 0$) and

$$Z^e = - \begin{pmatrix} (7,\ 9,\ 12) & (4,\ 6,\ 9) & (1,\ 3,\ 6) \\ (6,\ 7,\ 9) & (3,\ 4,\ 6) & (0,\ 1,\ 3) \\ (6,\ 7,\ 9) & (3,\ 4,\ 6) & (0,\ 1,\ 3) \end{pmatrix} \tag{2.50}$$

in Model 2 ($\alpha = 1$) for the three different values of $p = (0,\ 1,\ 2)$.

2.5 Lepton Flavor Violating Decays

2.5.1 Qualitative analysis

The branching ratios of the lepton flavor violating decays $l_i \to l_j \gamma$ in the SM are predicted to be extremely small and beyond the reach of any future experiments, due to suppression by a large scale such as GUT or Planck scale. Even, in the case of non–zero neutrino mass, induced by see-saw mechanism, the right–handed neutrino scale is too high to be experimentally significant. On the other hand, in the presence of low energy supersymmetry, LFV effects can be quite significant. In particular, LFV induced by the right–handed neutrino Yukawa couplings in the MSSM can lead to $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decay rates near the current experimental limits [11–14].

Here we focus on flavor violation in leptonic processes. The slepton soft masses are more sensitive to the $U(1)_A$ gaugino corrections compared to those in the squark sector. This is because flavor violation in the squark sector is diluted due to the fact that the squarks receive large gluino mass corrections which are flavor universal. This is called the gluino focusing effect[25]. This is especially so when one considers the cosmological constraints on the lightest SUSY particle (LSP) mass. Demanding that the neutralino LSP constitutes an acceptable cold dark matter imposes the condition $m_0 \simeq M_{1/2}/4.4$ in the context of supergravity models. This condition results from the coannihillation mechanism [14,26] for diluting the dark matter density which requires $\tilde{\tau}_R$ mass to be about $5 \div 15$ GeV above the LSP mass. The approximate formulae for the sfermion soft masses in terms of the universal soft scalar mass m_0 and the common gaugino mass $M_{1/2}$ (for small to medium $\tan\beta$) are [25]

$$\begin{aligned} \tilde{m}_L^2 &\simeq m_0^2 + 0.52 M_{1/2}^2, \\ \tilde{m}_e^2 &\simeq m_0^2 + 0.15 M_{1/2}^2, \\ \tilde{m}_Q^2 &\simeq m_0^2 + 6.5 M_{1/2}^2, \\ \tilde{m}_u^2 \simeq \tilde{m}_d^2 &\simeq m_0^2 + 6.1 M_{1/2}^2. \end{aligned} \tag{2.51}$$

From these expressions with $m_0 \simeq M_{1/2}/4.4$ we see that the gaugino focusing effects make the squark soft masses universal and heavier than those of sleptons so that they are much less sensitive to any flavor violating contributions.

The right–handed neutrino induced LFV effects in our models depend on the overall factor ϵ^p in Eq. (2.3). These processes will be suppressed for $p = 1, 2$ corresponding to low values of $\tan\beta$.

There are three different sources of LFV in our models: (i) RGE effects between M_{st} and the $U(1)_A$ symmetry breaking scale M_F induced by the $U(1)$ gaugino, (ii) RGE effects between M_{st} and the right–handed neutrino mass scale M_R induced by the neutrino Dirac Yukawa couplings, and (iii) the $U(1)_A$ D–term. Here we discuss only the RGE effects (i) and (ii). We call them the flavor gaugino induced LFV and ν^c–induced LFV. We give approximate formulas for these LFV processes by integrating the relevant β–functions derived in Section 3.

We adopt the minimal supergravity scenario (mSUGRA) for supersymmetry breaking given by Eq. (2.2). We assume universality of scalar masses and proportionality of the A–terms and the respective Yukawa couplings at the string scale. Gaugino mass unification is also assumed.

The various flavor violating effects are summarized below:

(1) Right-handed neutrino contributions to the scalar soft masses arising from Eq. (2.36) proportional to the Dirac neutrino Yukawa couplings:

$$\delta\left(\tilde{m}_L^2\right)_{ij}^{\nu^c} \simeq -\left(Y^{\nu\dagger}Y^{\nu}\right)_{ij}\left(3\,{m_0}^2 + A_0^2\right)\frac{\ln\left(M_{st}/M_R\right)}{8\pi^2}\,. \tag{2.52}$$

(2) Trace correction from D_A–term in Eqs. (2.36) and (2.37) from Figure 1(a):

$$\begin{aligned}
\delta\left(\tilde{m}_L^2\right)_{ij}^{A} &\simeq -q_i^L|q_s|g_F^2\delta_{ij}\left(3\,m_0^2\sum_{i=1,3}\left(n_{ii}^u + n_{ii}^d\right)\right. \\
&\quad \left. + m_0^2\sum_{i=1,3}\left(n_{ii}^e + n_{ii}^\nu\right) + n^X m_0^2 - \tilde{m}_s^2\right)\frac{\ln\left(M_{st}/M_F\right)}{8\pi^2}\,, \\
\delta\left(\tilde{m}_e^2\right)_{ij}^{A} &\simeq -q_i^e|q_s|g_F^2\delta_{ij}\left(3\,m_0^2\sum_{i=1,3}\left(n_{ii}^u + n_{ii}^d\right)\right. \\
&\quad \left. + m_0^2\sum_{i=1,3}\left(n_{ii}^e + n_{ii}^\nu\right) + n^X m_0^2 - \tilde{m}_s^2\right)\frac{\ln\left(M_{st}/M_F\right)}{8\pi^2}\,.
\end{aligned} \tag{2.53}$$

(3) Gaugino mass correction from Figure 1(b):

$$\begin{aligned}
\delta\left(\tilde{m}_L^2\right)_{ij}^{G} &\simeq \left(q_i^L\,g_F\right)^2\delta_{ij}\left(M_{\lambda_F}\right)^2\frac{\ln\left(M_{st}/M_F\right)}{2\pi^2}\,, \\
\delta\left(\tilde{m}_e^2\right)_{ij}^{G} &\simeq \left(q_i^e\,g_F\right)^2\delta_{ij}\left(M_{\lambda_F}\right)^2\frac{\ln\left(M_{st}/M_F\right)}{2\pi^2}\,.
\end{aligned} \tag{2.54}$$

(4) Right–handed neutrino induced vertex correction to the A^e–terms (see Eq. (2.44)):

$$\delta A_{ij}^e \simeq -3A_0\left(Y^e\,Y^{\nu\dagger}Y^{\nu}\right)_{ij}\frac{\ln\left(M_{st}/M_R\right)}{16\pi^2}\,. \tag{2.55}$$

(5) Flavor gaugino vertex correction to the A^e–terms arising from Figure 2 (see the last term of Eq. (2.44)):

$$\delta A_{ij}^e \simeq -M_{\lambda_F}{g_F}^2 Y_{ij}^e Z_{ij}^e\frac{\ln\left(M_{st}/M_F\right)}{4\pi^2}\,. \tag{2.56}$$

In addition, we have flavor charge dependent wave function renormalization of the A–terms as given in Eq. (2.44). These are however not significant since they are diagonalized simultaneously with the corresponding Yukawa couplings. On the other hand, the vertex corrections to the A–terms given in Eqs. (2.55) and (2.56) will induce nonproportionality in going from M_{st} to M_F.

The matrix elements Z^e_{ij} in Eq. (2.56) are given in Eqs. (2.49) and (2.50) for different values of p. The elements in the $(1,2)$ block of Z^e are rather different from each other, suggesting that the gaugino vertex contributions can be very important for the process $\mu \to e\gamma$. On the other hand, the elements in the second and the third rows are identical, hence, A^e_{23} and A^e_{33} run at the same rate as their corresponding Yukawa couplings do in the short momentum interval. Therefore, this vertex correction for the process $\tau \to \mu\gamma$ is always suppressed in models with the texture of Eq. (2.3).

For $\mu \to e\gamma$ we find that the most dominant effect is from the flavor gaugino contributions to the soft masses. This is due to the following reason. It is proportional to the flavor charge squared and to the flavor gaugino mass squared (recall that we have $m_0 \simeq M_{1/2}/4.4$), both of which are large. On the other hand, the trace contributions to the soft masses depend linearly on the flavor charges and are proportional to m_0^2, which make them relatively small although the trace of the $U(1)_A$ charges itself is large. The right–handed neutrino contributions are significant only for $p = 0$. For other values of p the ν^c–contributions to the branching ratio for $l_i \to l_j\gamma$ is suppressed by ϵ^{4p}.

We find that the gaugino contribution to the $\tau \to \mu\gamma$ decay rate is always suppressed since τ_L and μ_L have the same flavor charges and since the τ_R–μ_R mixing angle is of order ϵ^2. The only significant effect to this process is from the right–handed neutrino effects when $p = 0$.

The terms we are interested in are the ones proportional to the $U(1)_A$ gaugino mass M_{λ_F} and the term proportional to σ in the β–functions. Beside these, the A–term β–functions contain a flavor dependent piece which arises from the wave–function renormalization. Since the corresponding Yukawa β–functions contain the same terms, these are simultaneously diagonalized, and do not lead to flavor violation.

2.5.2 Numerical results for the LFV

In this Section we present our numerical results for the LFV processes $\mu \to e\gamma$. We adopt the mSUGRA scenario for the SUSY breaking parameters. At the string scale, taken to be $M_{st} = 10^{17}$ GeV, we assume a universal scalar mass m_0 and a common gaugino soft mass $M_{1/2}$. The unified gauge coupling at 2×10^{16} GeV is taken to be $\alpha_G \simeq 1/24$. We assume the $U(1)_A$ gauge coupling g_F to be equal to g_2

at the string scale. We evolve the soft SUSY breaking parameters from M_{st} to the $U(1)_A$ gaugino mass $M_F \simeq M_{st}/50$ (see Eq. (3.12). We use the numerical values of the Yukawa couplings in Section 2.3 for this evolution. For our numerical calculations we used the formulas for the soft masses and the branching ratios of LFV processes given in Appendix A.1 and A.2.

We take $m_0 = M_{1/2}/4.4$ so that the relic abundance of neutralino dark matter can be reproduced correctly. With this choice we always find the neutralino to be the LSP with the $\tilde{\tau}_R$ mass higher than the LSP mass by $5-15$ GeV. We impose radiative electroweak symmetry breaking condition. The SUSY higgs mass parameter μ is chosen to be positive which is favored by $b \rightarrow s\gamma$.

We take $M_{1/2}$ to vary in the range 250 GeV to 1 TeV. The lower value satisfies the lightest higgs boson mass limit. We present the results for three different values of $\tan\beta = (5,\ 10,\ 20)$. The corresponding values of the exponent p are taken to be $p = (2,\ 1,\ 0)$. The results are presented for two different values of $A_0 = (0,\ 300)$ GeV. When $\tan\beta = 20$, the lower limit on $M_{1/2}$ is around 300 GeV, or else the radiative electroweak symmetry breaking would fail.

In Figure 2.3 and 2.4 the combined effect for $\mu \rightarrow e\gamma$ is plotted for Model 1 and 2 respectively.

In Figure 2.5 we plot $B\,(\mu \rightarrow e\gamma)$ induced solely by the right–handed neutrino Yukawa couplings. This result is identical for Models 1 and 2 since neutrino textures are the same for the two models. In Figure 2.6 we plot the branching ratio induced by the right–handed neutrino effects and the flavor gaugino effects for Model 1. Figure 2.7 has the same plot for Model 2. In Figure 2.8 (2.9) we plot $B\,(\mu \rightarrow e\gamma)$ induced by the trace term and the right–handed neutrino for Model 1 (2). Figure 2.10 (2.11) is a plot of the branching ratio including the effects of A–terms and ν^c for Model 1 (2). Figures 2.12 and 2.13 are the branching ratios for $\tau \rightarrow \mu\gamma$ including all LFV effects for Model 1 and Model 2. Figure 2.14, which is valid for both Models 1 and 2, has the branching ratios for $\tau \rightarrow \mu\gamma$ induced only by the ν^c Yukawa coupling effects.

From these figures we see that the decay $\mu \rightarrow e\gamma$ is within the reach of forthcoming experiments. Discovery of $\tau \rightarrow \mu\gamma$ decay will strongly hint, within our framework, an origin related to the right–handed neutrino Yukawa couplings.

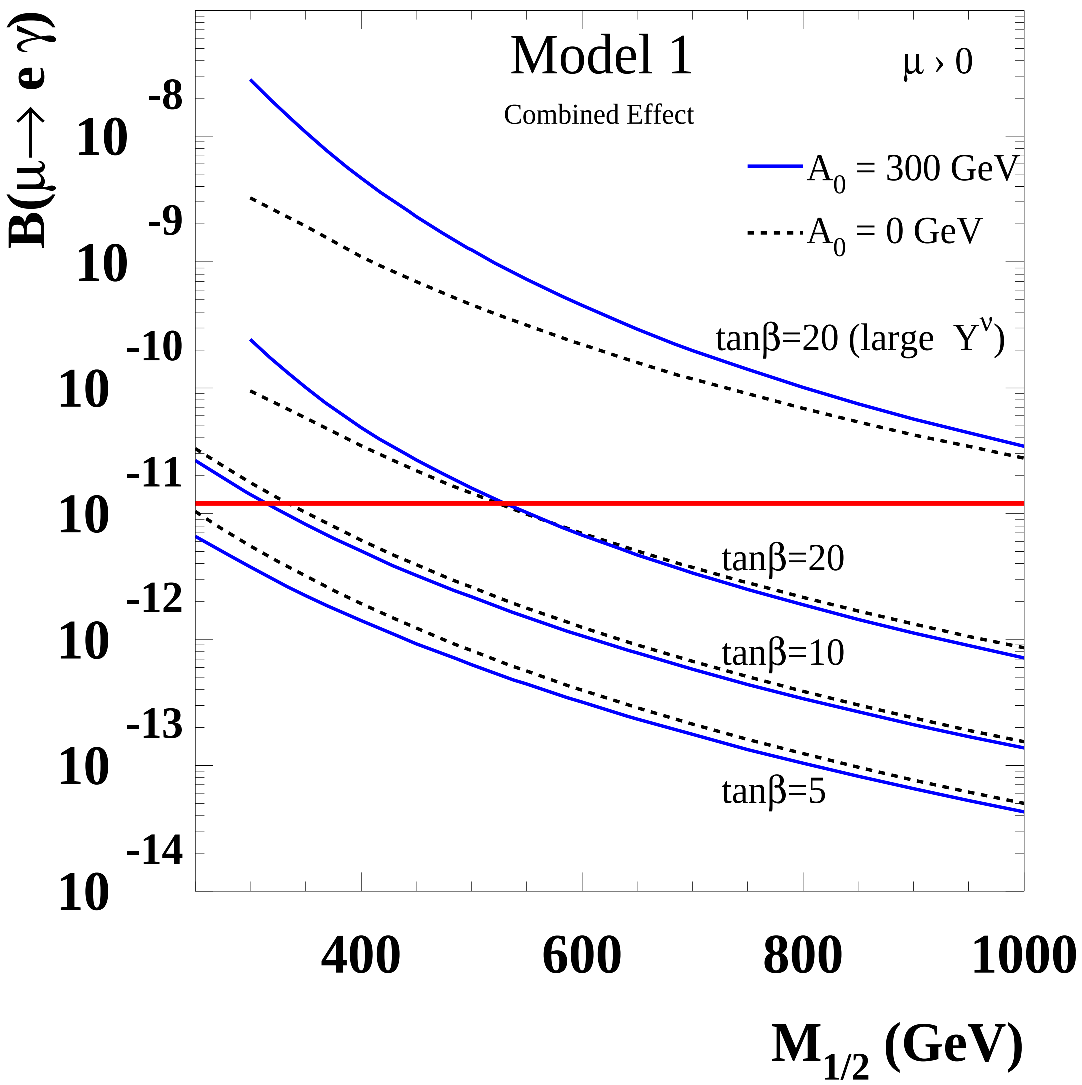

Figure 2.3. Branching ratio for the process $\mu \to e\gamma$ including all corrections for Model 1. The solid line corresponds to $A_0 = 300$ GeV and the dashed line corresponds to $A_0 = 0$ GeV. For $\tan\beta = 20$ we give two sets of curves, the upper one corresponds to the maximal value of the neutrino Yukawa coupling Y^ν. Here and in other plots, the straight horizontal line corresponds to the current experimental limit $B(\mu \to e\gamma)_{exp} < 1.2 \times 10^{-11}$ given in Particle Data Book[27].

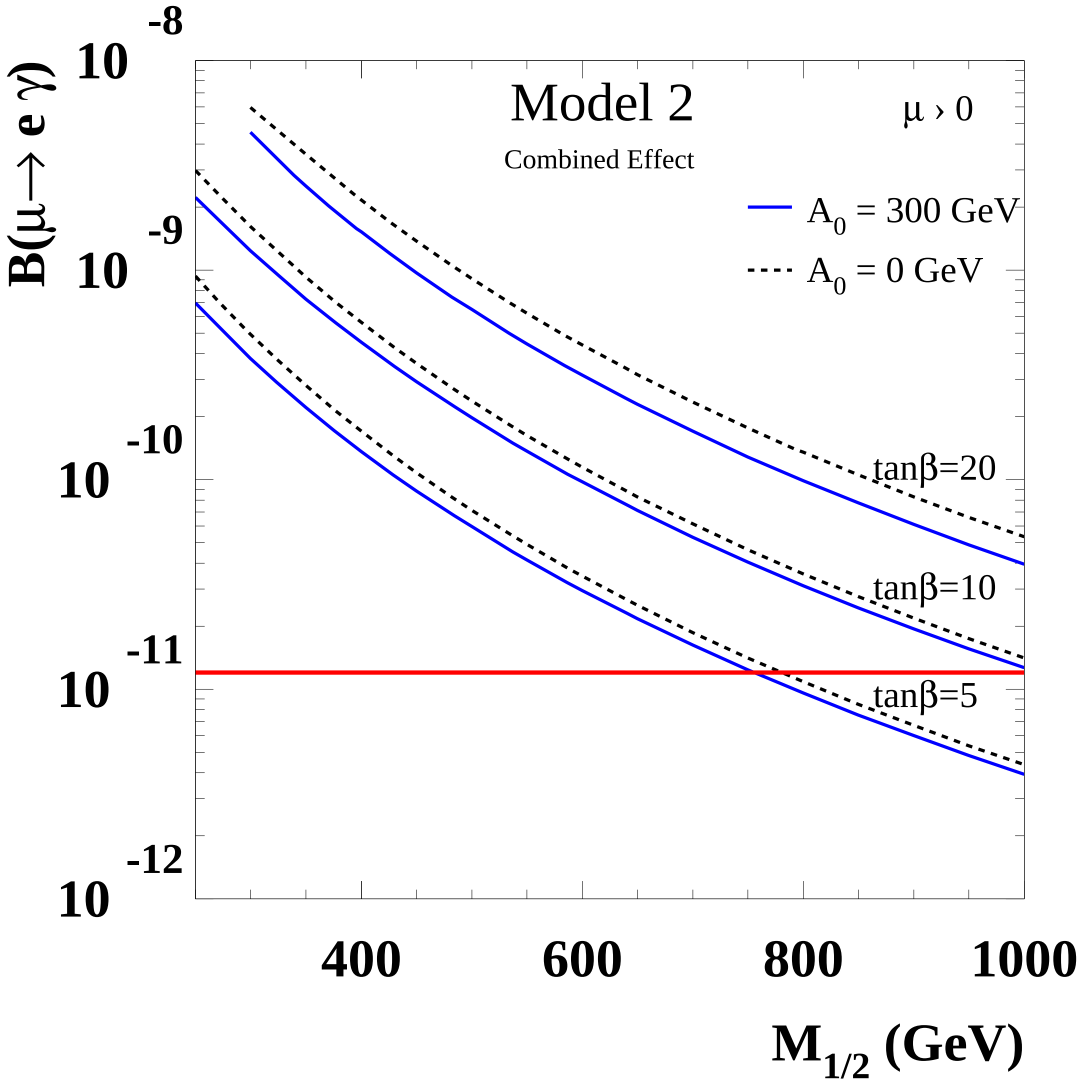

Figure 2.4. Branching ratio for the process $\mu \to e\gamma$ including all corrections for Model 2.

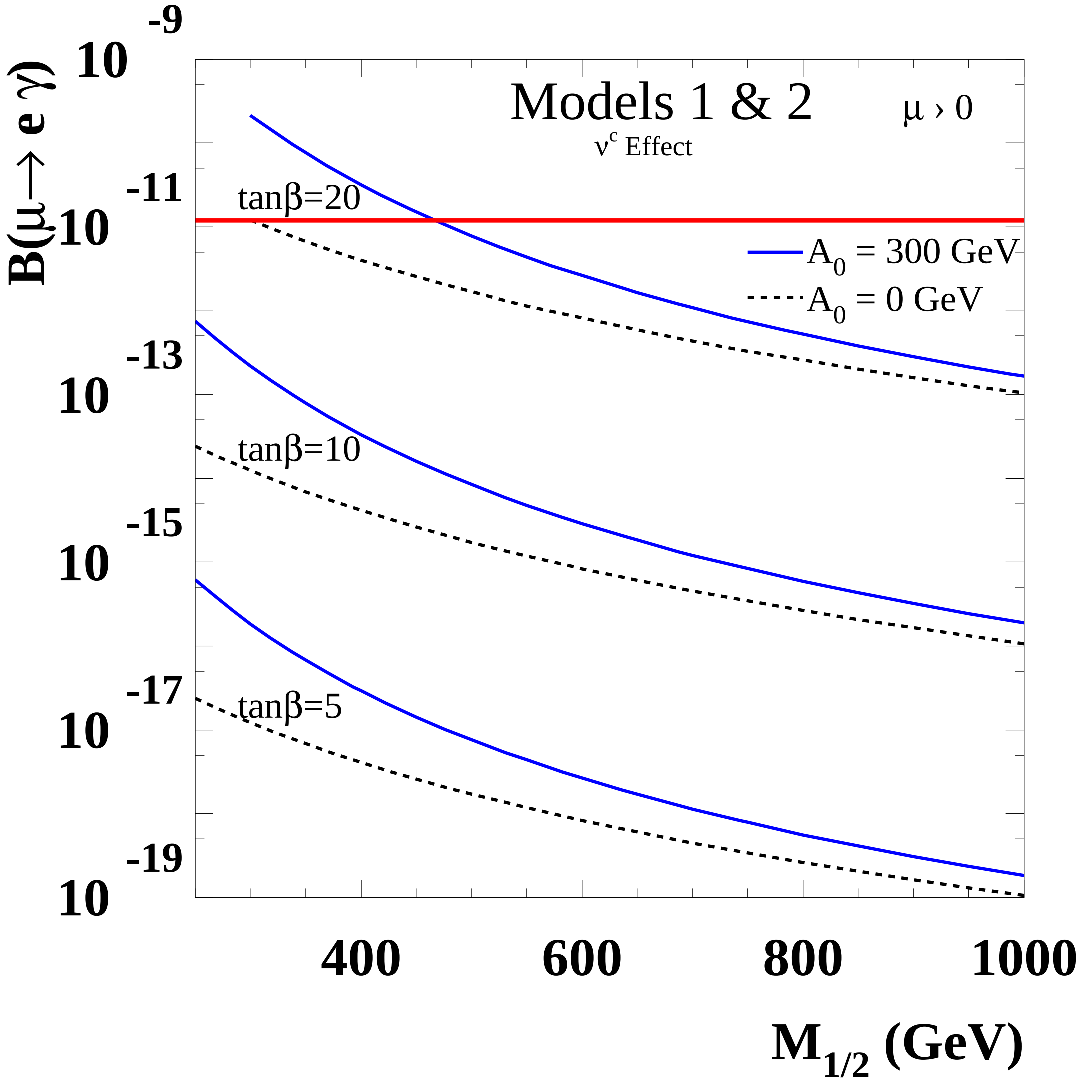

Figure 2.5. Branching ratio for the process $\mu \rightarrow e\gamma$ induced by only the right–handed neutrino Yukawa coupling effects. This result holds for both Models 1 and 2.

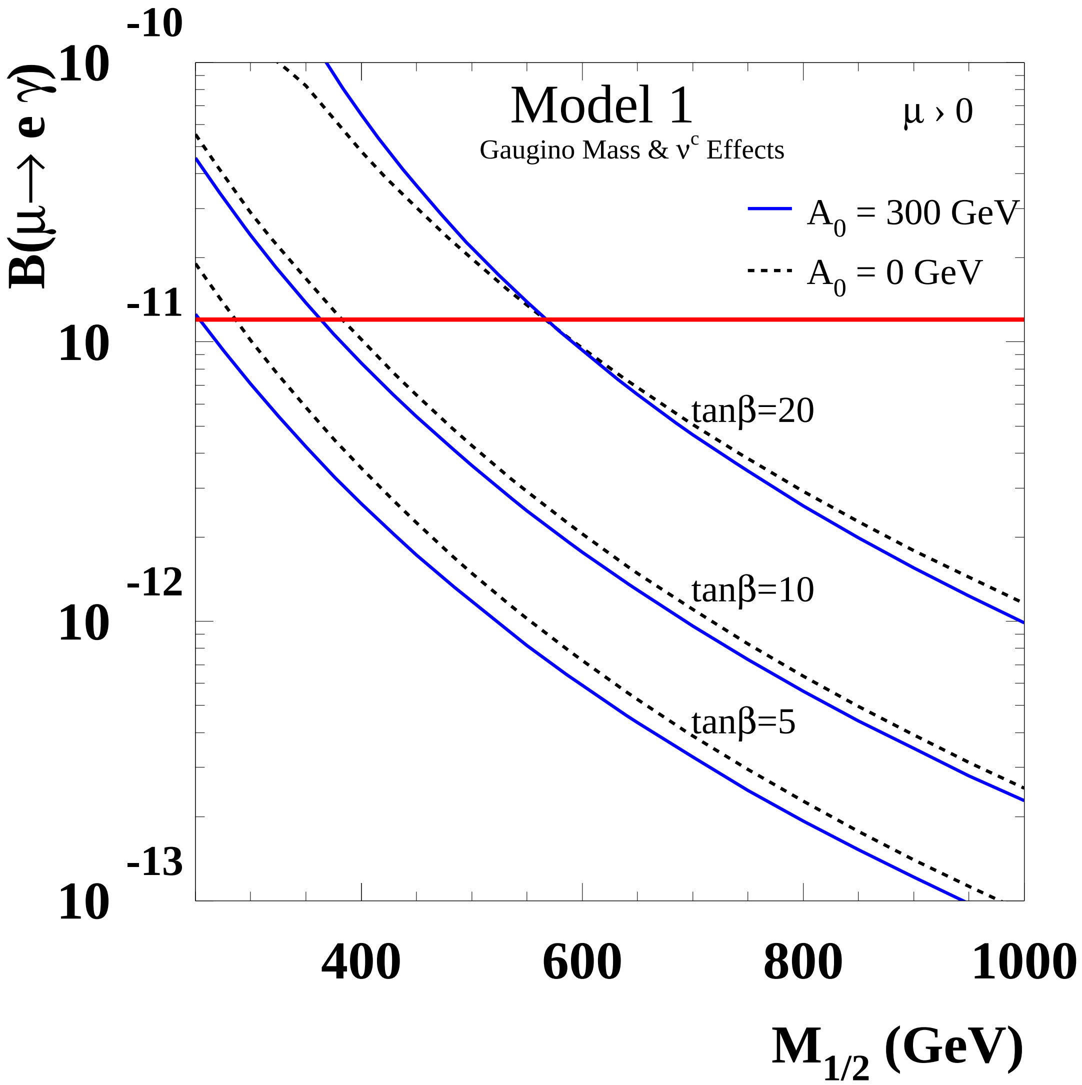

Figure 2.6. Branching ratio for the process $\mu \to e\gamma$ induced by the gaugino corrections (plus ν^c effects) for Model 1.

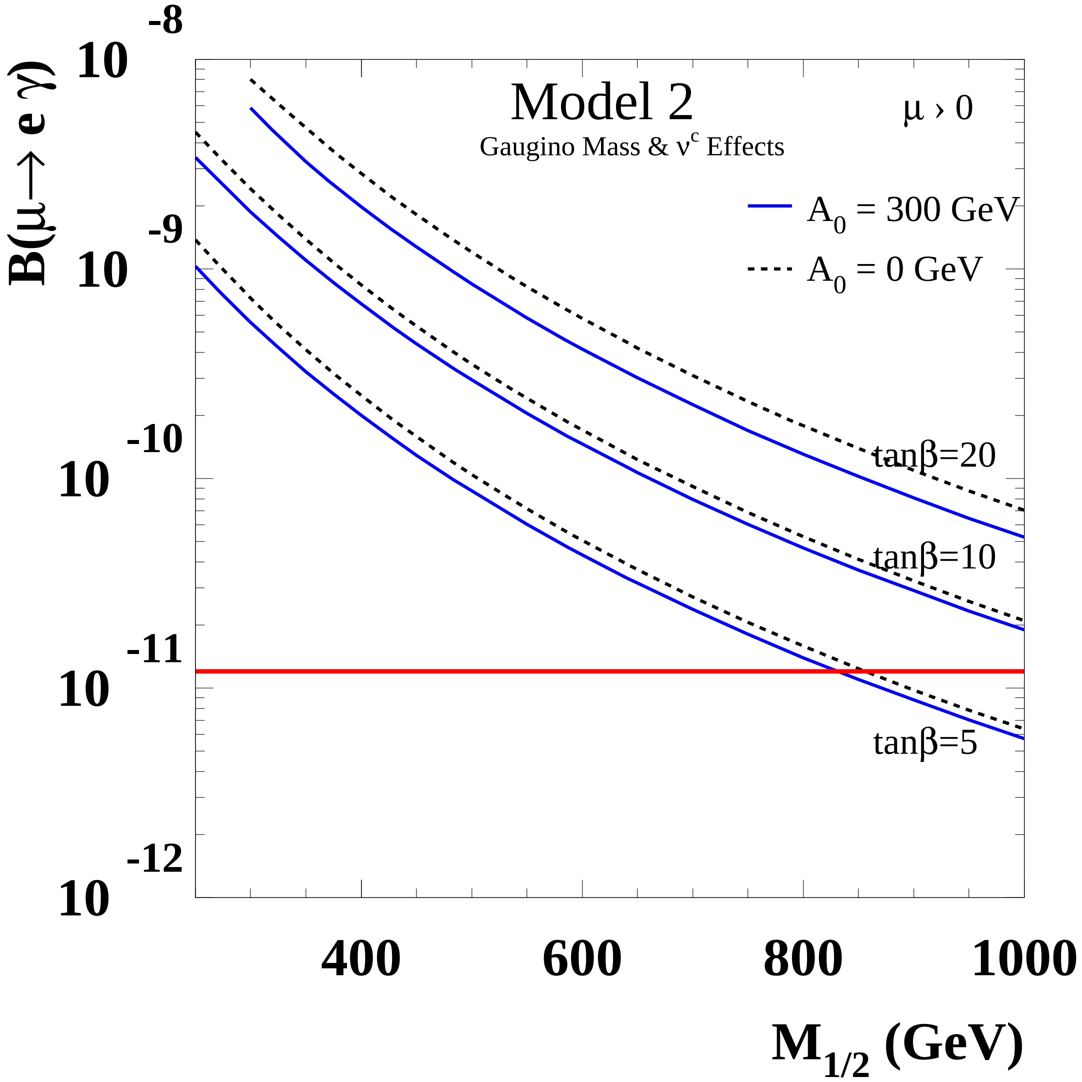

Figure 2.7. Branching ratio for the process $\mu \to e\gamma$ induced by the gaugino corrections (plus ν^c effects) for Model 2.

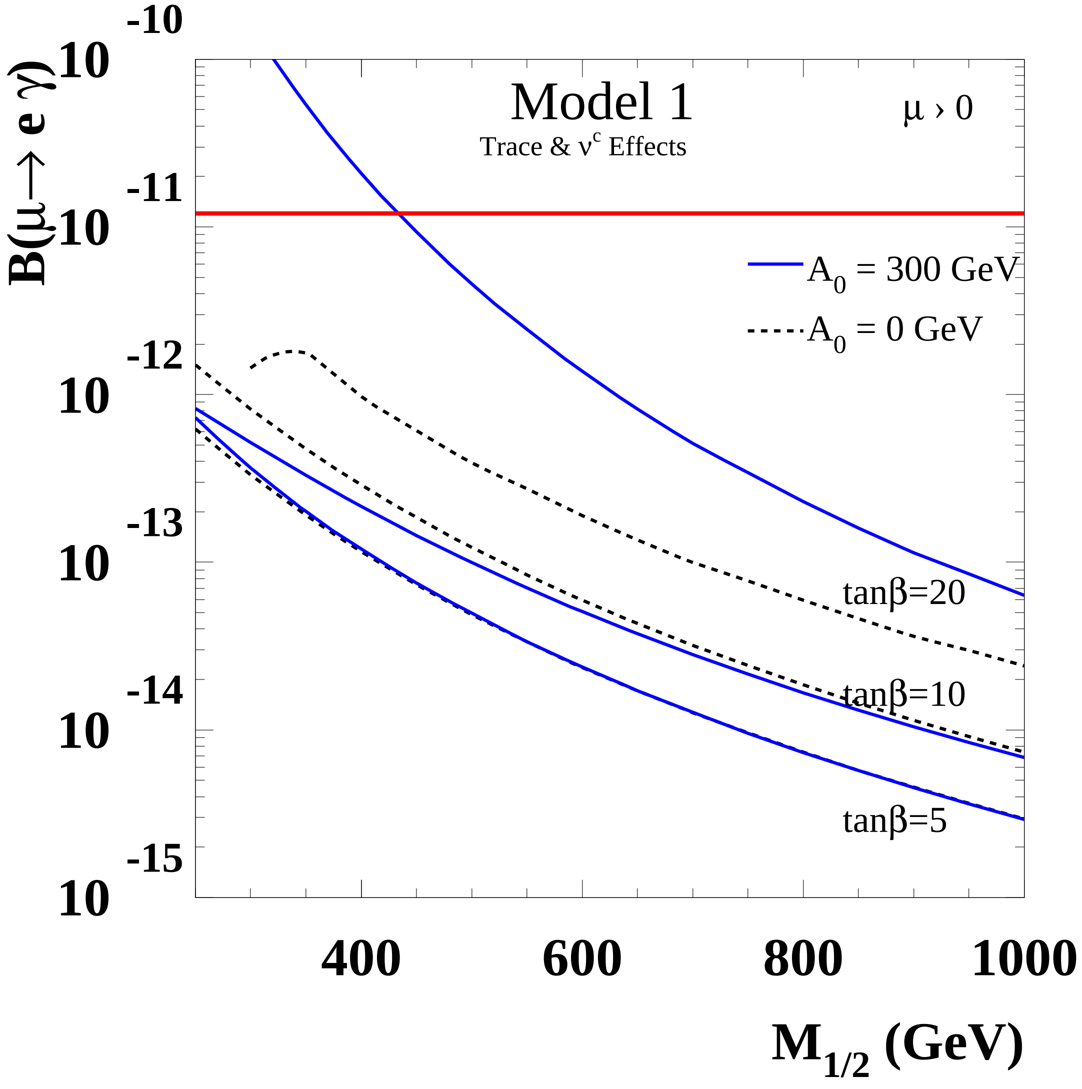

Figure 2.8. Branching ratio for the process $\mu \to e\gamma$ induced by the trace correction (plus ν^c effects) for Model 1.

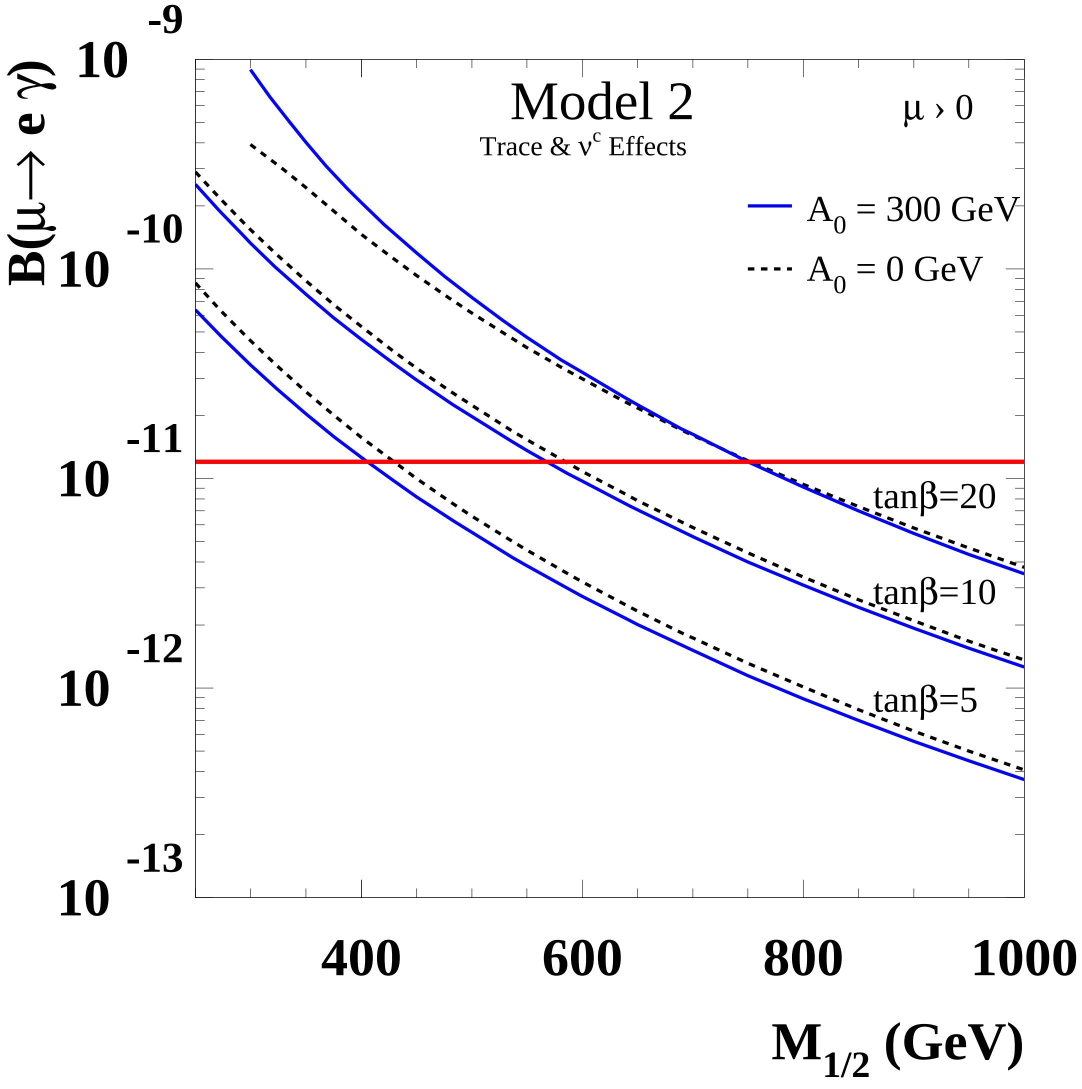

Figure 2.9. Branching ratio for the process $\mu \to e\gamma$ induced by the trace correction (plus ν^c effects) for Model 2.

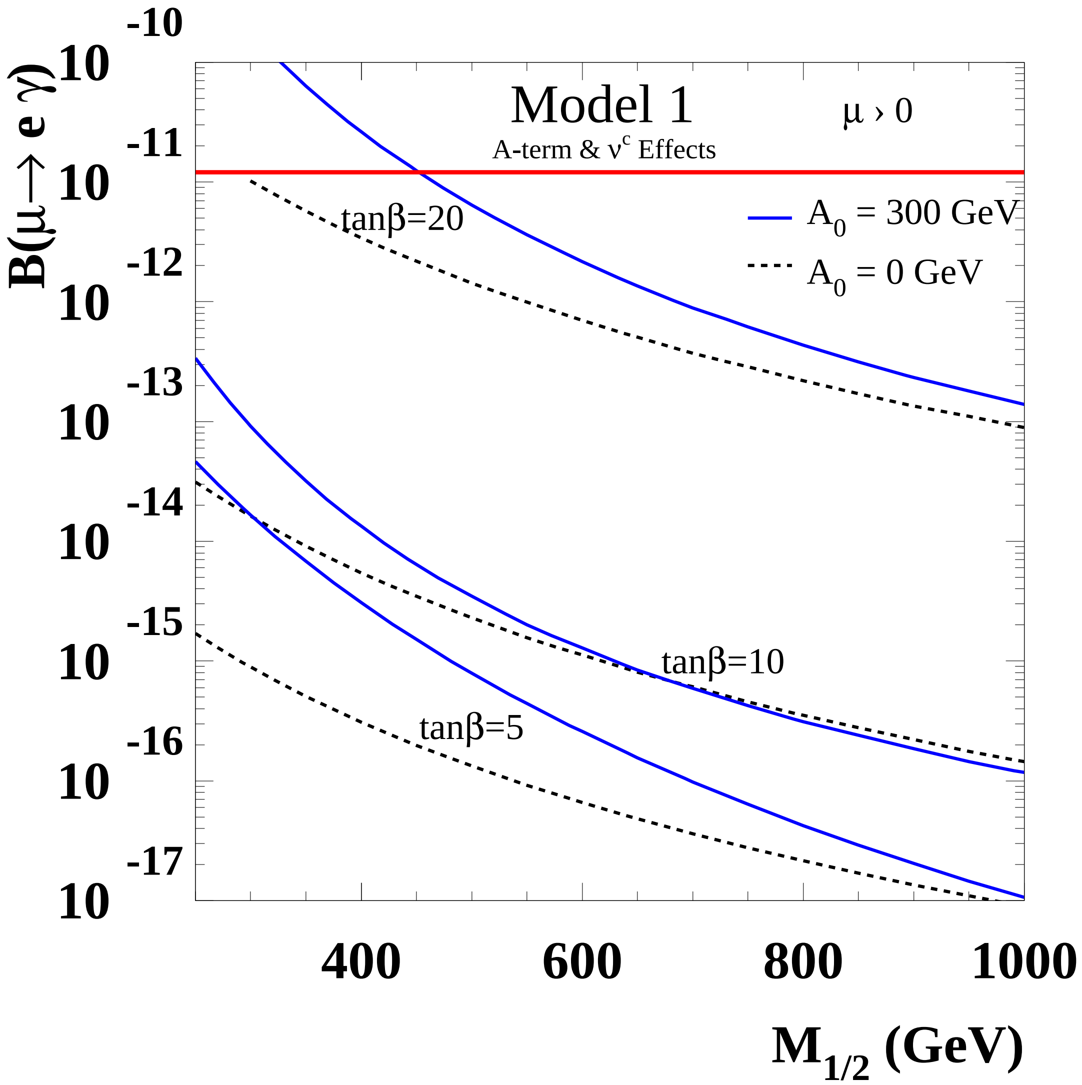

Figure 2.10. Branching ratio for the process $\mu \rightarrow e\gamma$ from the vertex corrections (plus ν^c effects) for Model 1.

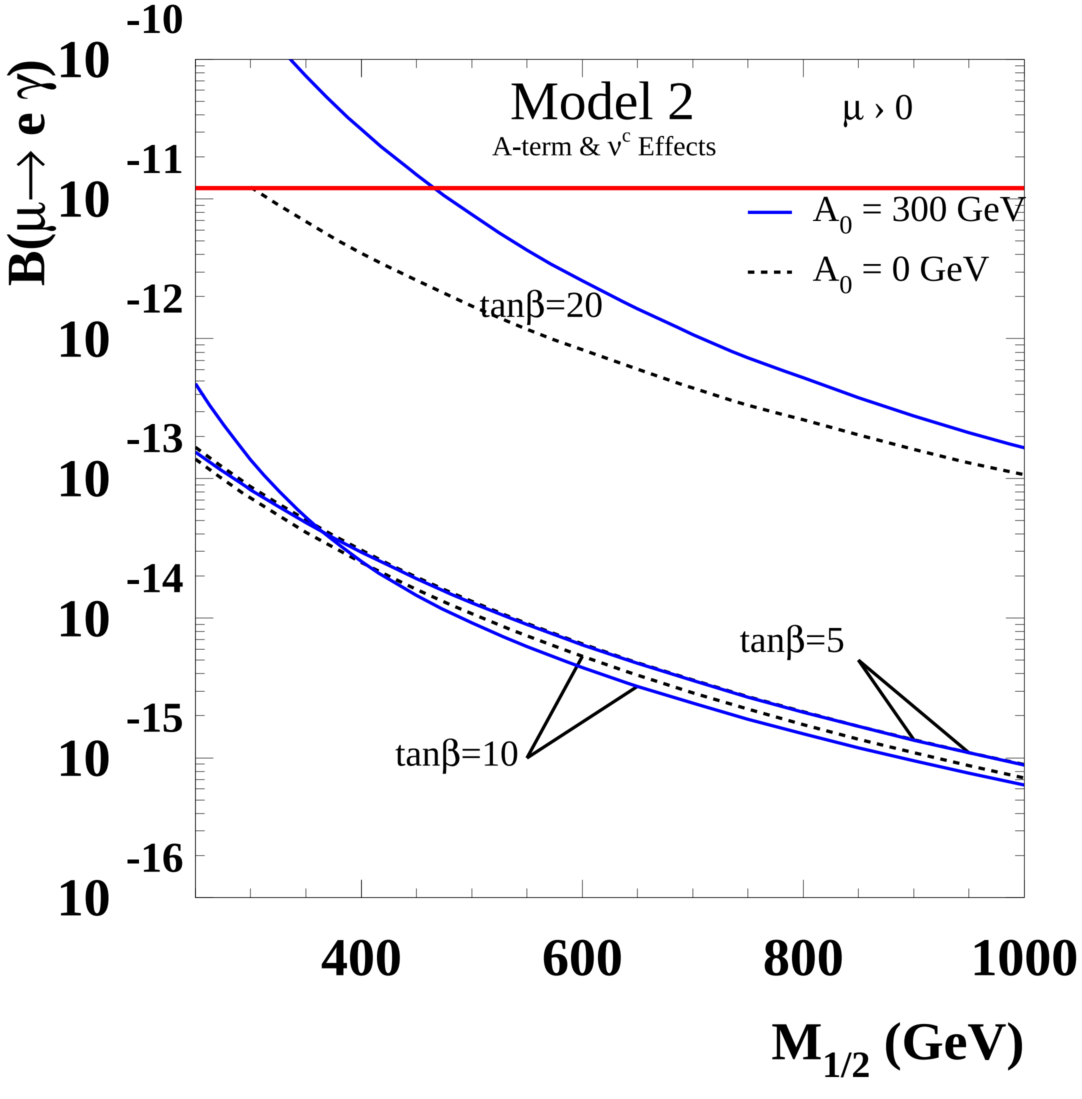

Figure 2.11. Branching ratio for the process $\mu \to e\gamma$ from the vertex corrections (plus ν^c effects) for Model 2.

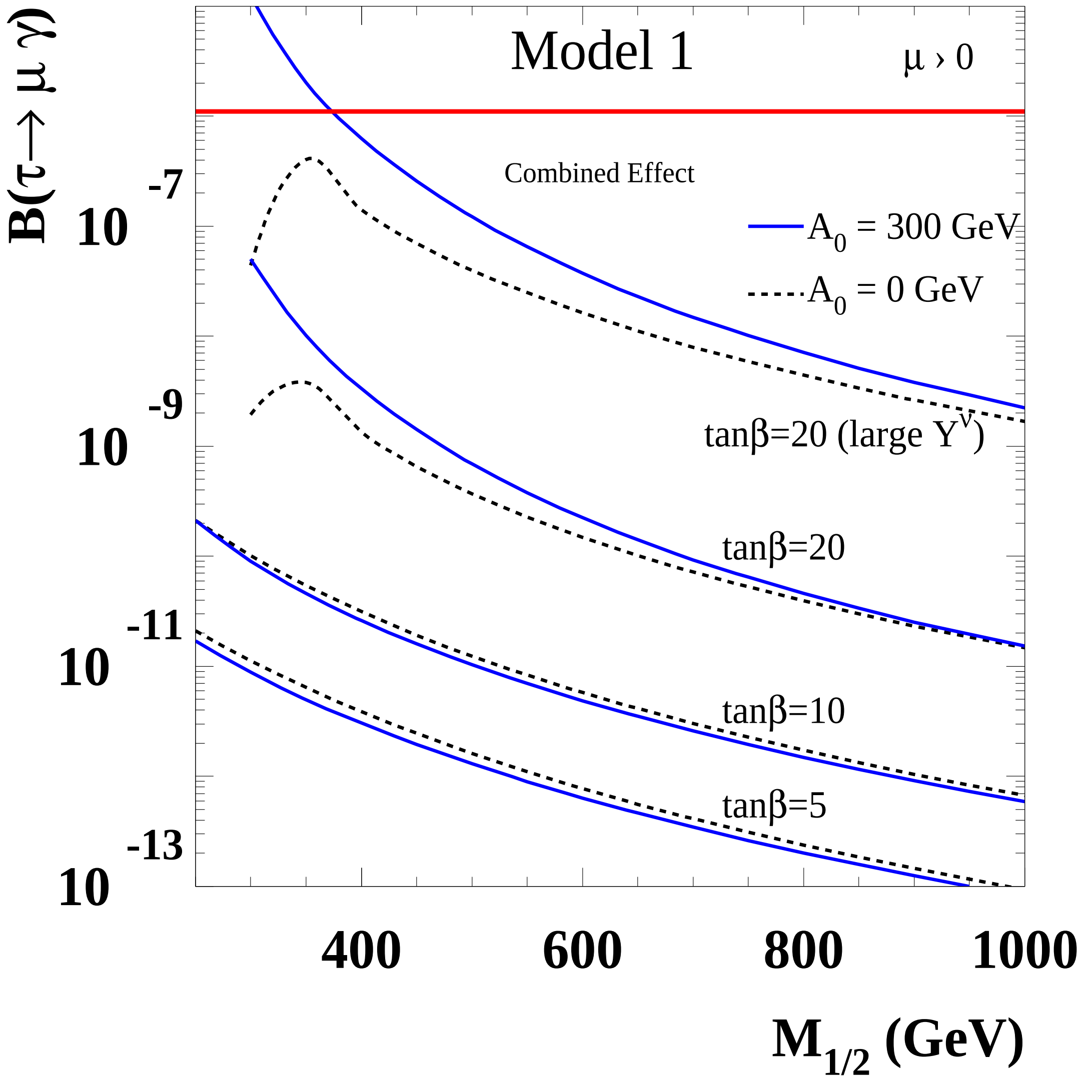

Figure 2.12. Branching ratio for the process $\tau \rightarrow \mu\gamma$ including all the effects for Model 1.

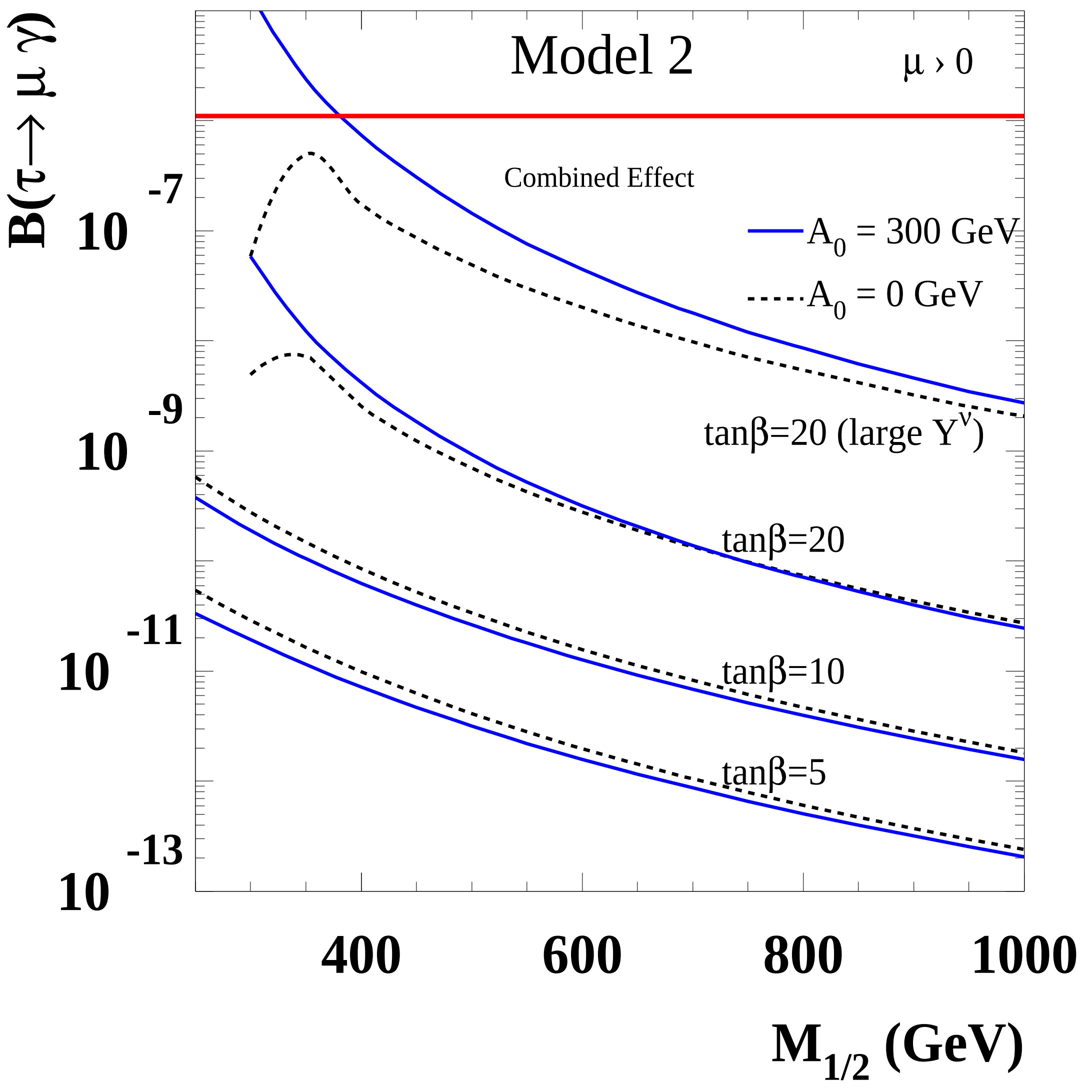

Figure 2.13. Branching ratio for the process $\tau \rightarrow \mu\gamma$ including all the effects for Model 2.

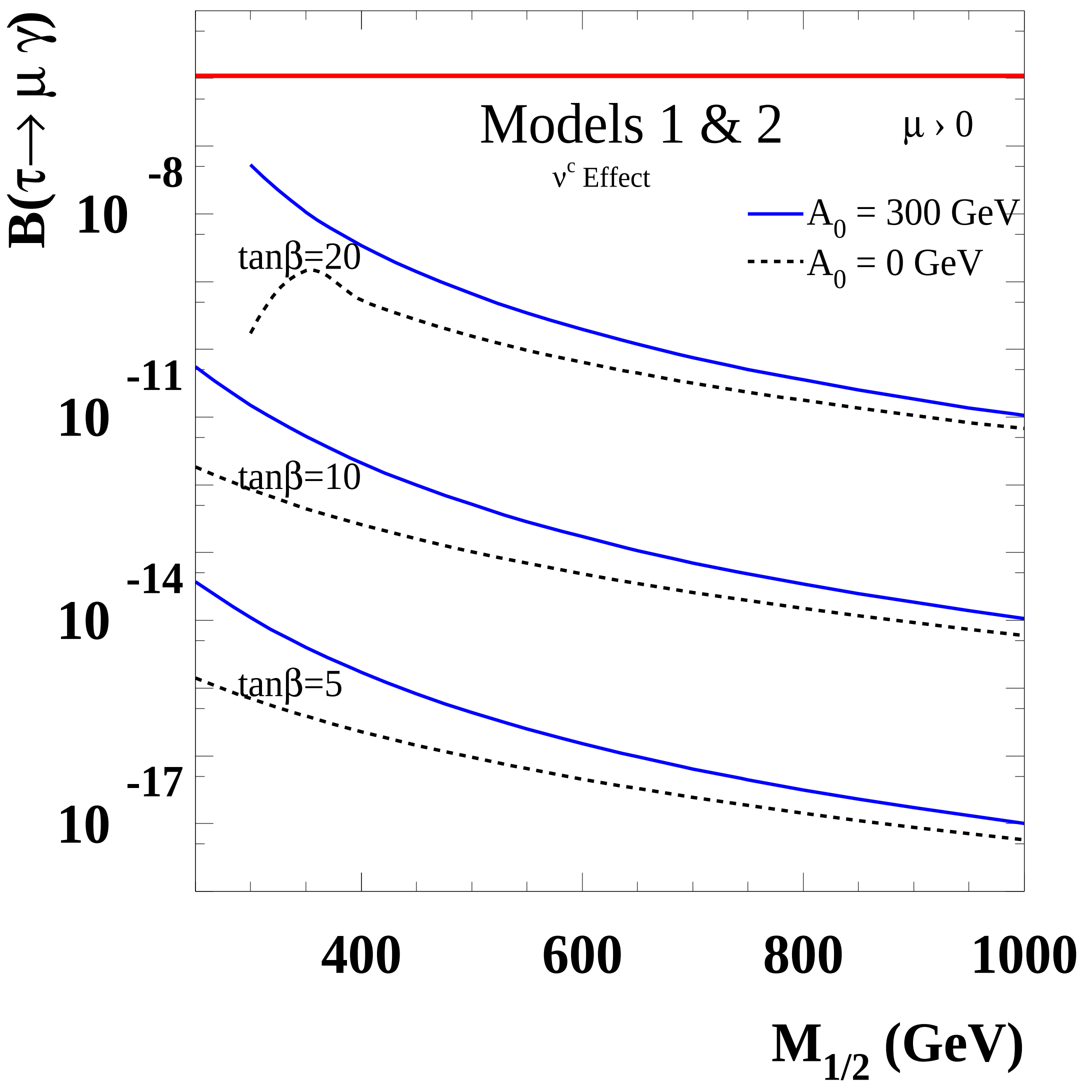

Figure 2.14. Branching ratio for the process $\tau \rightarrow \mu\gamma$ induced by only the right–handed neutrino Yukawa coupling effects for Models 1 and 2.

2.5.3 Programming on RGEs

To obtain low energy spectrum for the SUSY particles I have written a code in the programming language Fortran 90. In particular, this code accommodates the soft masses of scalar particles with flavor dependent initial values at the unification scale. As a check, we compared our code to the already existing programs such as SUSPECT 2 [27], which allows for only universal initial conditions. We have calculated the soft mass parameters at low energy beginning with the flavor independent mSUGRA initial conditions for the soft parameters and it agreed upto a few percent, which is the precision of our program. The decoupling of the right–handed neutrinos are properly taken into account at each right–handed mass eigenvalue by diagonalizing the neutrino Majorana mass matrix, which has not been accommodated in other existing codes. The RGEs are upto two–loop for the gauge couplings and the third generation Yukawa couplings. In the fermionic and sfermionic sector of the MSSM all the mass parameters and couplings are calculated in the flavor basis. This enables one to study numerically any SUSY flavor models. In the following I will briefly describe the structure of our code. The code contains several subroutines for RG evolutions which use the standard numerical fourth order Runga–Kutta method [28] for ordinary differential equations. Before full running from the UV scale down to the weak scale one needs to determine the radiative electroweak symmetry breaking scale, M_{SUSY}, taken to be the geometric mean value of the left–handed and the right–handed stop masses. The SUSY threshold, M^0_{SUSY}, is chosen to be 500 GeV as the initial trial value. The SM fermion masses are evolved from their respective low energy values to M^0_{SUSY} using the two–loop QED and QCD beta functions. From M^0_{SUSY} to UV (in our case this is the flavor symmetry breaking scale) first we use RGEs of only SUSY parameters: the gauge and Yukawa couplings. For consistent quark and lepton mixings, we take into account the CKM matrix, and the MNS mixings. The neutrino Dirac Yukawa couplings are introduced at the right handed neutrino mass scale accommodating the seesaw mechanism. The MSSM beta functions, now with the soft parameters with the universality conditions, are then used to run down to M^0_{SUSY}. After several iterations, now choosing $M_{SUSY} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ in each iteration, the value for the radiative electroweak breaking quickly converges. Once this is done we again evolve the SUSY parameters up to M_F. Here we introduced unitary rotations on the SM fermions to obtain desired structure of fermion mass matrices for our flavor model. The flavor dependent corrections from $U(1)_A$ sector are then added to the universal soft parameters. To study other flavor models one can replace this part by simply choosing flavor dependent initial conditions to the soft parameters. Then using full flavor dependent RGEs, we evolved the soft parameters down to the electroweak scale. After this, we used the Lapack package [29] for diagonalizing sfermion mass matrices. The spectrum of the sparticles are now determined and one can calculate branching ratios of LFV decays, and EDMs. For this I wrote small subroutines which calculate our numerical results and stores them in output files.

2.6 Conclusions

In this chapter I have presented our study on lepton flavor violation induced by a flavor–dependent anomalous $U(1)$ gauge symmetry of string origin in a class of models which addresses the fermion mass hierarchy problem via the Froggatt–Nielsen mechanism. We have derived a general set of renormalization group equations for the evolution of soft SUSY breaking parameters in the presence of higher dimensional operators. These results should be applicable to a large class of fermion mass models.

We have shown that the $U(1)_A$ sector induces significant flavor violation in the SUSY breaking parameters during the RGE evolution from the string scale to the flavor symmetry breaking scale, even though this momentum range is very short. We have identified several sources of flavor violation: the $U(1)_A$ gaugino contribution to the scalar masses which is flavor dependent, a contribution proportional to the trace of $U(1)_A$ charge which is also flavor dependent, non–proportional A–terms arising from the $U(1)_A$ gaugino vertex correction diagrams, and the $U(1)_A$ D–term. In addition, there are flavor violating effects in the charged lepton sector arising from the right–handed neutrino Yukawa couplings, which have also been included in our numerical analysis. The resulting flavor violation in the leptonic decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are found to be in the experimentally interesting range.

Adopting the minimal supergravity scenario for SUSY breaking, and choosing parameters such that the needed relic abundance of neutralino dark matter is realized, we have presented results for the branching ratios $B(\mu \to e\gamma)$ and $B(\tau \to \mu\gamma)$ in two specific models of fermion masses. Figures 2.3 and 2.4 are our main results for the two models for $B(\mu \to e\gamma)$, while Figures 2.13 and 2.14 are our results for $B(\tau \to \mu\gamma)$. The former should be accessible to forthcoming experiments, while the latter is also in the observable range. Although we focused on two specific fermion mass textures these effects should be significant in a large class of models.

CHAPTER 3

ELECTRIC DIPOLE MOMENTS FROM FLAVOR SYMMETRY

3.1 Introduction

In the SM the electric dipole moments of the electron, muon and the neutron are predicted to be extremely small and beyond reach of planned experiments (assuming that the QCD CP–violating θ–term is zero). In the presence of low energy supersymmetry these EDMs can exceed the current experimental limits if soft SUSY breaking parameters are complex [30–36]. We assume m_0, $M_{1/2}$ and A_0, and the Higgs mass parameters μ and $B\mu$ to be real. Thus the only source of CP–violation is in the complex Yukawa couplings. This is needed for the CKM CP–violation in the quark sector and it is natural to assume that the leptonic Yukawa couplings are complex as well.

In this chapter, we study the electric dipole moments of the electron, muon and neutron for Model 1 from chapter 2. A family–dependent anomalous $U(1)_A$ symmetry will induce permanent electric dipole moments, even when there is no additional CP violating phases beyond that of Yukawa couplings. In the present case, such violations will be generated through the renormalization group evolution (RGE) of the SUSY breaking parameters between M_{st} and the $U(1)_A$ breaking scale quantified by the flavor gauge boson mass M_F.

We have identified several sources of CP-violations from the same RGE effects which lead to LFV discussed in the previous chapter. When the supersymmetric Higgs mass term, μ, and the universal soft SUSY breaking parameters $M_{1/2}$, A_0 and B_0 are all chosen to be real, the primary sources for EDM are the trilinear A–terms which couples left and right–handed sfermions. As we discussed in great detail in the previous chapter, the trilinear A–terms will receive vertex corrections from the $U(1)_A$ gaugino that are not proportional to the respective Yukawa couplings. This means that at the $U(1)_A$ breaking scale M_F the proportionality condition for the A–terms Eq. (2.2) can no longer be satisfied. Since the Yukawa couplings are complex, upon the diagonalization of the Yukawa couplings, the diagonal entries of A–terms will be complex in general. This source of CP–violation, as we will show, lead to permanent EDMs that are in the experimentally interesting range.

We will assume in the present chapter universal SUSY breaking spectrum that is also CP–invariant so that excessive EDMs are not induced from the fundamental soft SUSY breaking parameters.

The EDMs that we find in the context of models of fermion mass hierarchy are induced purely by complex Yukawa couplings. The phases in the Yukawa couplings are believed to be the source for the observed CP–violation in the K and B meson systems (CKM CP–violation). It is thus reasonable to assume all Yukawa couplings, including the leptonic Yukawa couplings, to be complex. These effects would survive down to the SUSY breaking scale and can lead to observable phenomena. With complex Yukawa couplings, this flavor violation will also lead to EDMs for the electron (d_e), muon (d_μ), the neutron (d_n), and the deuteron (d_D) even with universal and CP–conserving soft SUSY breaking terms at the string scale. We find $d_e \sim (10^{-26} - 10^{-27})$ e cm and $d_n \sim 10^{-27}$ e cm, which are within reach of next generation experiments. There are proposals to improve the current limit on electron EDM, $|d_e| \leq 1.6 \times 10^{-27}$ e cm [37], by about two to four orders of magnitude [38]. There are also proposals which would improve the current neutron EDM limit from $|d_n| \leq 6.3 \times 10^{-26}$ e cm [39] by a factor of 5 [40]. Supersymmetry may reveal itself in these experiments before direct discovery at the Large Hadron Collider, if the current ideas of solving the fermion mass hierarchy problem are correct.

Lepton EDMs may arise even without flavor gauge symmetry from complex neutrino Yukawa couplings responsible for the seesaw mechanism in the context of low energy SUSY. This effect has received much attention recently [41,42]. We have computed such effects for d_e and d_μ, but found them to be much less significant compared to the flavor $U(1)$ induced effects. For example, we find $d_e \sim 10^{-29}$ e cm for large $\tan\beta$ from the neutrino Yukawa coupling effects, to be compared with $d_e \sim 10^{-26}$ e cm from the flavor $U(1)$ sector. There has been study of similar effects from GUT threshold[43].

The structure of this chapter is as follows. In 3.2 we briefly review EDM. In Section 3.3 we determine the small expansion parameter in Model 1 discussed in previous chapter. In 3.4.1 the qualitative analysis of the EDM is discussed. In 3.4.2 we give our fermion mass fit which use for the numerical calculations of EDMs in Section 3.4.3. In Figure 3.1 and 3.4 we show the results for the electron and the neutron EMD. The conclusions of the this chapter are given in Section 3.5.

3.2 Electric Dipole Moments: Brief Review

The expectation value of electric dipole moment is defined as

$$\vec{D} = \int \vec{x}\rho(\vec{x})d^3x, \tag{3.1}$$

where $\rho(\vec{x})$ is the charge density. From this definition one can see that in the presence of violation of both parity P and time reversal T symmetries, a stable particle, elementary and composite, can have a non zero EDM. The reason is that D is proportional to the spin of the particle. Spin is odd under T and even under P, while D has the opposite symmetry proerties: even under T and odd under P. One of the fundamental assumptions in modern particle physics is the validity of CPT theorem –CPT symmetry is unbroken. Therefore a nonvanishing EDM implies both P and CP are violated. The experimental discoveries of P and CP violation have brought a breakthrough in particle physics and the latter has been well accommodated in the SM via the CKM matrix.

For better or worse, the SM predictions for the permanent EDMs for leptons and quarks are too small to be accesible by any foreseeable experiment. Thus hand an experimental discovery of a permanent electric dipole moment would usher a new physics beyond the SM. In particular, supersymmetry has multiple sources for CP– violation. If these complex phases are chosen arbitrarily, various CP– violating processes would have grossly exceeded already existing stringent experimental bounds. This is called the SUSY CP–problem. In minimal supergravity case, the choices of soft parameters are such that these problems are avoided by fiat – viz. choosing all soft SUSY breaking parameters and μ term to be real. One should keep in mind that mSUGRA scenario is not fundamentally justified, but is suggested by the experimental constraints. In addition to these sources, supersymmetric parameter μ–term must be chosen to be real.

The EDM of a spin–$\frac{1}{2}$ particle f is defined by one of the electromagnetic form–factors in the matrix element of current J_μ as

$$\langle f(p')|J_\mu(0)|f(p)\rangle = \bar{u}(p')\Gamma_\mu(q)u(p), \tag{3.2}$$

where

$$\begin{aligned} \Gamma(q) &= F_1(q^2)\gamma_\mu + F_2(q^2)i\sigma_{\mu\nu}\gamma^\mu \\ &+ F_A(q^2)\left(\gamma_\mu\gamma_5 q^2 - 2m_f\gamma_5 q_\mu\right) \\ &+ \frac{1}{2m_f}F_3(q^2)i\sigma_{\mu\nu}\gamma_5\gamma^\mu. \end{aligned} \tag{3.3}$$

Then EDM is given by

$$d_f = -\frac{F_3(0)}{2m_f}, \tag{3.4}$$

which is expressed by the following term in the Lagrangian

$$\mathcal{L}_{EDM} = -\frac{i}{2}d_f\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}. \tag{3.5}$$

3.3 Anomaly Discussion for Model 1

Here we consider the anomaly discussion in the case of Model 1 of chapter 2. Later we consider the EDMs only for this case (similar analysis can be done for Model 2.). The $U(1)_A$ anomalies are cancelled by the Green–Schwarz mechanism [9] which requires

$$\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \frac{A_A}{3k_A} = \frac{A_{gravity}}{24}\,. \tag{3.6}$$

Observe that here we have included the proper normalization of the $U(1)_A$ cubic anomaly: the factor $1/3$ in front of the cubic anomaly A_F has a combinatorial origin owing to the three $U(1)_A$ gauge boson legs. We require string unification of all the gauge couplings including that of the $U(1)_A$, g_F, at the fundamental scale M_{st} [19]:

$$k_i g_i^2 = k_F g_F^2 = 2g_{st}^2. \tag{3.7}$$

Here we choose the string coupling unifications with factor. For a clear discussion of the coefficients in Eqs. (3.6)–(3.7) see Ref. 22. Then the last equality in Eq. (3.6) now becomes

$$A_{gravity} = \mathrm{Tr}\,(q) = 12(19 + 3p). \tag{3.8}$$

As before, this does not match Eq. (3.8). To match the anomaly we introduce MSSM singlet fields X_k obeying $\mathrm{Tr}\,(q)_X = A_{gravity} - \mathrm{Tr}\,(q)_{MSSM} = 163 + 21p$. Although this is quite different from what we found in Eq. (2.13), they lead to similar expansion parameter, which is the most inportant parameter of the model. If all the X_k fields have the same charge equal to $+1$, they will acquire masses of order $M_{st}\epsilon^2$ through the coupling $X_k X_k S^2/M_{st}$ and will decouple from low energy theory. For other choices of the charge of X_k these masss can be different. For example, if the charge is equal to $+1/2$, their masses will be of order $M_{st}\epsilon$; if the charge is $+2$ the masses will be of order $M_{st}\epsilon^4$. We will consider only the case where the X_k fields have charge $+1$.

From the Green–Schwarz anomaly cancelation condition $A_F/(3k_F) = A_2/k_2$, we have

$$\frac{\mathrm{Tr}\,(q^3)}{3k_A} = \frac{19 + 3p}{2k_2}, \tag{3.9}$$

from which we find the normalization of the $U(1)_A$ charge $|q_s| = 1/\sqrt{k_A}$ to be

$$|q_s| = (0.179,\ 0.186,\ 0.181) \ \text{ for } p = (0, 1, 2)\,. \tag{3.10}$$

For the Model 1 ($\alpha = 0$), we find the small expansion parameter to be

$$\epsilon = (0.177,\ 0.191,\ 0.204) \ \text{ for } p = (0, 1, 2)\,. \tag{3.11}$$

As we can see the value of the expansion parameter ϵ has not changed too much compared to what we have found in Chapter 2. Correspondingly the masses of the $U(1)_A$ gauge boson is found to be

$$M_F = \left(\frac{M_{st}}{54.5}, \frac{M_{st}}{52.5}, \frac{M_{st}}{53.9}\right) \ \text{for } p = (0, 1, 2)\,. \tag{3.12}$$

In the momentum range below M_{st} and above M_F, these gauge particles will be active and will induce flavor dependent corrections to the sfermion soft masses and the A–terms. It is these effects which induce EDMs for the electron, muon and the neutron at low energies.

3.4 Electric Dipole Moments from Anomalous $U(1)$ Symmetry

3.4.1 Qualitative analysis for $U(1)_A$ induced EDM:

We now give approximate expressions for the $U(1)_A$ gauge sector RGE corrections to the soft parameters between the string scale and the $U(1)_A$ breaking scale M_F. The $U(1)_A$ corrections to the soft masses for the left–handed slepton are obtained from Eq. (2.36) to be

$$\delta\left(m_{\tilde{L}}^2\right)_{ij}^A \simeq \left(4(q_i^L M_{\lambda_F})^2 - q_i^L m_0^2 \mathrm{Tr}\,(q)\right)(|q_s|g_F)^2 \delta_{ij} \frac{log\,(M_{st}/M_F)}{8\pi^2}, \quad (3.13)$$

and a similar expression for the right–handed slepton masses with the interchange $(\tilde{L},\, q^L) \rightarrow (\tilde{e},\, q^e)$. There are analogous corrections in the squark sector. The corrections to the A–terms are obtained from Eq. (2.47) as

$$\delta A_{ij}^e \simeq -M_{\lambda_F} {g_F}^2 Y_{ij}^e Z_{ij}^e \frac{log\,(M_{st}/M_F)}{4\pi^2}\,, \quad (3.14)$$

where Z_{ij} are biliear combinations of the flavor charges given by [2]

$$Z_{ij}^e \;=\; q_i^L q_j^e + q_i^L \bar{h} + q_j^e \bar{h} + n_{ij}^e q_s (q_i^L + q_j^e + \bar{h}) + \frac{1}{2} n_{ij}^e (n_{ij}^e - 1) q_s^2. \quad (3.15)$$

Numerical values of Z_{ij}^e for our model are given in Eq. (2.49). Note that these corrections in Eqs. (3.13) and (3.14) are flavor dependent. Due to the flavor dependent nature of these corrections, the fermion and the corresponding sfermion mass matrices cannot be diagonalized simultaneously. This was the source of the flavor violation discussed earlier. For the same reason, with complex Yukawa couplings Y_{ij}^f, nonzero EDMs for the fermions will be induced.

Let us now estimate the EDM of the electron arising from the corrections in Eqs. (3.13) and (3.14). There are three flavor dependent matrices in the leptonic sector, not including the neutrino Yukawa matrix Y^ν. They are the leptonic Yukawa matrix Y^e and the matrices of $U(1)_A$ charges $\hat{Q}^L = diag\,(1+p,\, p,\, p)$ and $\hat{Q}^e = diag\,(4,\, 2,\, 0)$ for the lepton doublets and singlets (see Table 2.1). In the mass eigenbasis for the charged leptons $\hat{Q}^L$ and $\hat{Q}^e$ will develop complex off diagonal entries, with the phases arising from Y^e through the unitary matrices that diagonalize Y^e. This is the basic source for the EDM. The corrections given in Eq. (3.13) will generate EDM of the electron through the product of slepton mixings in $(1i)_{LL}$, $(ii)_{LR}$ and $(i1)_{RR}$ (for

$i = 2, 3$). The induced EDM will be $d_e \propto \mathrm{Im}\left[\left(U^\dagger \hat{Q}^L Y^e \hat{Q}^e V^\dagger\right)_{11}\right]$, where U and V are unitary matrices which diagonalize Y^e, $Y^e = U Y^e_{daig} V^\dagger$. There are additional corrections which are quadratic in $\hat{Q}^L$ and $\hat{Q}^e$. The corrections to the A–terms in Eq. (3.14) will also induce EDM directly through (LR) mixings. Combining these effects we arrive at the following approximate expression for d_e:

$$\begin{aligned} d_e/e &\simeq \frac{\alpha v_d M_{\tilde{B}}}{8\pi \cos^2\theta_W} \frac{1}{m^2_{\tilde{l}}} A\left(\frac{M^2_{\tilde{B}}}{m^2_{\tilde{l}}}\right) \frac{(|q_s| g_F)^2 \log\left(M_{st}/M_F\right)}{8\pi^2} \times \\ & \sum_{i=2,3} \left[C^m_i + C^A_i\right] \mathrm{Im}\left[\frac{Y^e_{1i} Y^e_{i1}}{Y^e_{ii}}\right], \end{aligned} \quad (3.16)$$

where C^m_i and C^A_i denote the contributions from the soft masses and the A–terms respectively. They are given by

$$\begin{aligned} C^m_i &= \frac{(|q_s| g_F)^2 \log\left(M_{st}/M_F\right)}{8\pi^2} \frac{m^4_0 \left(A_0 - |\mu| \tan\beta\right)}{m^6_{\tilde{l}}} H^L_i H^R_i, \\ H^L_i &= 4\left(M_{1/2}/m_0\right)^2 \left((q^L_i)^2 - (q^L_1)^2\right) - (q^L_i - q^L_1) \mathrm{Tr} q, \\ C^A_i &= 2 \frac{M_{1/2}}{m^2_{\tilde{l}}} \left(Z^e_{i1} - Z^e_{11}\right). \end{aligned} \quad (3.17)$$

Here H^R_i is obtained from H^L_i by the replacement $q^L_i \to q^e_i$. $m_{\tilde{l}}$ is the average slepton mass and $M_{\tilde{B}}$ is the Bino mass. The function $A\left(X\right)$ is given by

$$A(X) = \frac{1 - X^2 + 2X \log X}{\left(1 - X\right)^3}. \quad (3.18)$$

We see explicitly that the complex Yukawa couplings along with nonuniversal $U(1)_A$ charges lead to nonzero EDM.

To estimate the size of this effect we choose the approximations $m_0 = M_{1/2} \simeq M_{SUSY}$. Following the mass matrices given in Eq. (refmassM01) we take $|Y^e_{ij}| \simeq \epsilon^{n^e_{ij}+p}$. We consider here only the contribution from the (13) mixing, since the $U(1)_A$ charge difference is the largest between the first and the third generations in $\hat{Q}^e$. Then we find

$$\begin{aligned} d_e/e &\sim \left(10^{-27}\mathrm{cm}\right) \times \left(\frac{500\mathrm{GeV}}{m_{\tilde{l}}}\right)^2 \\ &\times M_{\tilde{B}} \left(O(10) \frac{M^4_{SUSY} (|\mu| \tan\beta)}{m^6_{\tilde{l}}} + O(1) \frac{M_{SUSY}}{m^2_{\tilde{l}}}\right) \mathrm{Arg}\left[Y^e_{13} Y^e_{31}\right], \end{aligned} \quad (3.19)$$

where

$$B(X) \;=\; \frac{3-4X+X^2+2\log X}{(1-X)^3}. \tag{3.20}$$

From this estimate we see that the electron EDM induced by the $U(1)_A$ gauge corrections is in the experimentally interesting range and already puts constraint on the soft SUSY breaking parameters. The actual numerical result is quite sensitive to the choice of m_0 and $M_{1/2}$. In our numerical calculations we have chosen $m_0 = M_{1/2}/4.4$ for low $\tan\beta$ for cosmological reason. In this case the $O(10)$ coefficient in Eq. (3.19) will be reduced to an $O(1)$ number. For large $\tan\beta$ this coefficient will remain as $O(10)$.

Let us now compare the anomalous $U(1)$ induced EDM with the right–handed neutrino induced effects[12,41,42]. The latter effects induce EDM which are given by

$$d_i \propto [(Y^\nu)^\dagger\,\Lambda Y^\nu,(Y^\nu)^\dagger\,Y^\nu], \tag{3.21}$$

where $\Lambda_{ij} = \log(M_{GUT}/(M_{\nu^c})_i)\delta_{ij}$. Here $(M_{\nu^c})_i$ is the mass of the right–handed neutrino of flavor i. With our texture for the neutrino mass matrices dictated by $U(1)_A$ symmetry we find the right–handed neutrino induced EDM to be $d_e \sim 10^{-29}$ e cm, which is two to three orders magnitude smaller than the anomalous $U(1)_A$ induced effects. In our numerical analysis we present separately our results for the electron EDM arising from the right–handed neutrino effects.

3.4.2 Fermion mass fit for Model 1

Here we present the numerical fits to the fermion masses and mixings adopted for the calculation of the EDMs. As input at low energy for the running quark masses and their running factors r_f we choose the same values we used for the case of LFV discussion as in Eqs. (2.21) and (2.23). The CKM mixing matrix is chosen in the standard parametrization with $\theta_{12} = 0.221$, $\theta_{13} = 0.005$, $\theta_{23} = 0.043$ and the complex phase $\delta = 0.86$. Then these masses at M_{SUSY} are used to calculate the Yukawa couplings in $\overline{DR}$ scheme. Using one loop SUSY RGE evolution above M_{SUSY} we obtain the Yukawa couplings at the $U(1)_A$ breaking scale ($M_F \sim 10^{15}$ GeV) to be

$$(Y_u,\, Y_c,\, Y_t) = (5.2803\times10^{-6},\, 1.4663\times10^{-3},\, 0.55636),$$

$$(Y_d,\, Y_s,\, Y_b) = (3.4465 \times 10^{-5},\, 5.0294 \times 10^{-4},\, 2.8292 \times 10^{-2}),$$
$$(Y_e,\, Y_\mu,\, Y_\tau) = (1.0390 \times 10^{-5},\, 2.1484 \times 10^{-3},\, 3.6245 \times 10^{-2}),$$
$$(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) = (1.2107 \times 10^{-3},\, 1.9662 \times 10^{-3},\, 3.2170 \times 10^{-2}), \qquad (3.22)$$

for $\tan\beta = 5$,

$$(Y_u,\, Y_c,\, Y_t) = (5.0995 \times 10^{-6},\, 1.4161 \times 10^{-3},\, 0.53199),$$
$$(Y_d,\, Y_s,\, Y_b) = (1.3851 \times 10^{-4},\, 2.0213 \times 10^{-3},\, 0.11524),$$
$$(Y_e,\, Y_\mu,\, Y_\tau) = (4.1780 \times 10^{-5},\, 8.6437 \times 10^{-3},\, 0.14818),$$
$$(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) = (5.8781 \times 10^{-3},\, 9.5054 \times 10^{-3},\, 0.15647), \qquad (3.23)$$

for $\tan\beta = 20$ and

$$(Y_u,\, Y_c,\, Y_t) = (5.2880 \times 10^{-6},\, 1.4686 \times 10^{-3},\, 0.59327),$$
$$(Y_d,\, Y_s,\, Y_b) = (4.2275 \times 10^{-4},\, 6.1693 \times 10^{-3},\, 0.41255),$$
$$(Y_e,\, Y_\mu,\, Y_\tau) = (1.2722 \times 10^{-4},\, 2.6479 \times 10^{-2},\, 0.51817),$$
$$(Y_{\nu_1},\, Y_{\nu_2},\, Y_{\nu_3}) = (3.0041 \times 10^{-2},\, 4.8554 \times 10^{-2},\, 0.8260), \qquad (3.24)$$

for $\tan\beta = 50$. Slight differences compared to the fit values used in Section 2.3 of chapter 2 are due to our improved treatment of the SUSY threshold and different choices of neutrino mass matrices compared to a simple assumption we made in chapter 2.

We have determined the Dirac neutrino Yukawa couplings as follows. The right–handed Majorana neutrino mass matrix is taken to be proportional to the Dirac neutrino Yukawa coupling $M_{\nu^c} = Y_\nu M_R^0 \epsilon^p$. Y_ν is determined from a fit to the light neutrino oscillation parameters with $M_R^0 = 4 \times 10^{14}$ GeV. This corresponds to $m_{\nu_e} = 2.7 \times 10^{-3}$ eV, $m_{\nu_\mu} = 6.4 \times 10^{-3}$ eV and $m_{\nu_\tau} = 8.6 \times 10^{-2}$ eV and the lepton mixing matrix given by

$$V_{MNS} = \begin{pmatrix} 0.8494 & -0.5262 & -0.04 \\ 0.3915 & 0.5775 & 0.7164 \\ -0.3539 & -0.6242 & 0.6965 \end{pmatrix}. \qquad (3.25)$$

In the following, we present our fits to the texture of Eq. (refmassM01) which have been used in our numerical calculations for $\tan\beta = 5$ (We have similar fits upto an overall factor for $\tan\beta = 20$, 50). This fit is not unique, and so can lead to order one uncertainty in our EDM results. The following fit is found by applying bi-unitary transformations with complex phases on the diagonal Yukawa coupling matrices. We introduce a notation for the Yukawa couplings:

$$Y^f_{ij} \equiv y^f_{ij}\epsilon^{n^f_{ij}}. \tag{3.26}$$

At the $U(1)_A$ breaking scale we have the following fit (for $\tan\beta = 5$):

$$Y^u = \begin{pmatrix} (-0.231+0.242\,i)\,\epsilon^8 & (-1.66+0.792\,i)\,\epsilon^6 & (-0.159-0.127\,i\,)\,\epsilon^4 \\ (-1.89+2.33\,i)\,\epsilon^6 & (-0.796+2.25\,i)\,\epsilon^4 & (-0.262+2.95\times10^{-2}\,i)\,\epsilon^2 \\ (3.75-3.76\,i)\,\epsilon^4 & (3.94-3.93\,i)\,\epsilon^2 & 0.510-1.22\times10^{-5}\,i \end{pmatrix}, \tag{3.27}$$

$$Y^d = \epsilon^2 \begin{pmatrix} (2.17+0.841\,i)\,\epsilon^5 & (0.377-6.49\times10^{-2}\,i)\,\epsilon^4 & (1.36+0.425\,i)\,\epsilon^4 \\ (1.93-0.668\,i)\,\epsilon^3 & (0.354-0.944\,i)\,\epsilon^2 & (1.34-1.50\,i)\,\epsilon^2 \\ (0.225-0.131\,i)\,\epsilon & 0.223-0.119\,i & 0.635-7.12\times10^{-3}\,i \end{pmatrix}, \tag{3.28}$$

$$Y^e = \epsilon^2 \begin{pmatrix} (1.86+2.28\,i)\epsilon^5 & (-0.275-0.364\,i)\,\epsilon^3 & (0.786+0.359\,i)\,\epsilon \\ (-0.355+2.03\,i)\epsilon^4 & (0.955+7.82\times10^{-2}\,i)\,\epsilon^2 & 0.449+0.435\,i \\ (1.19+0.792\,i)\epsilon^4 & (-3.09+3.54\,i)\,\epsilon^2 & 0.632-1.49\times10^{-3}\,i \end{pmatrix}, \tag{3.29}$$

$$Y^\nu = \epsilon^2 \begin{pmatrix} (1.30-9.53\times10^{-3}\,i)\,\epsilon^2 & (-0.247-2.62\times10^{-2}\,i)\,\epsilon & (-8.42-3.26\,i)\times10^{-2}\,\epsilon \\ (-0.247-2.62\times10^{-2}\,i)\,\epsilon & 0.625+1.89\times10^{-3}\,i & 0.540+2.99\times10^{-3}\,i \\ (-8.42-3.26\,i)\times10^{-2}\,\epsilon & 0.540+2.99\times10^{-3}\,i & 0.593+2.62\times10^{-3}\,i \end{pmatrix} \tag{3.30}$$

3.4.3 Numerical results for the EDM

In this Section we present our numerical results for the electron, muon, neutron and the deuteron electric dipole moments. We choose $\mu > 0$ for all cases except in Fig. 3.1 where we also show results for $\mu < 0$. The anomalous $U(1)$ gauge coupling g_F is chosen to be $g_F^2/4\pi = 1/24$, consistent with string unification. The soft SUSY breaking parameters are evolved from M_{st} to the $U(1)_A$ breaking scale $M_F \simeq M_{st}/50$ (see Eq. (3.12)) including the $U(1)_A$ gaugino/gauge boson corrections.

We present our results for the EDM for three values of the parameter $\tan\beta$, small (5), medium (20) and large (50). As explained earlier in the LFV analysis, we take $m_0 = M_{1/2}/4.4$ for low and medium values of $\tan\beta$ consistent with the dark matter constraint [44]. For large $\tan\beta$ we also allow the choice $m_0 = M_{1/2}$, since alternative mechanisms for reproducing the right relic abundance of LSP become available in this case [45].

The electron EDMs induced by the flavor $U(1)$ gaugino/gauge boson contribution are plotted against the universal gaugino mass $M_{1/2}$ in Figure 3.1. In Figure 2.4 we plot the EDM of the neutron arising from the flavor $U(1)$ gauge boson/gaugino effects.

As input at M_{st} we choose the Yukawa coupling matrices given in Eqs. (3.27)–(3.30) (for $\tan\beta = 5$). These are obtained by extrapolating the low energy Yukawa couplings to M_{st} and applying bi-unitary transformations at M_{st} to generate the texture given in Eq. (refmassM01). As for the neutrino Dirac Yukawa couplings, we choose Y^ν to be such that in the flavor basis (after the bi–unitary rotations) it exhibits approximately the structure given in Eq. (refmassM01) with $(Y^\nu)_{33} \sim \epsilon^p$. For a given choice of hierarchical light–neutrino spectrum this would uniquely fix the right–handed neutrino mass matrix through the seesaw mechanism. M_{ν^c} will then have the form given in Eq. (refmassM01). We set $(M_{\nu^c})_{33} = M_R^0 \epsilon^{2p}$ with $M_R^0 \simeq 4 \times 10^{14}$ GeV. The eigenvalues of the right–handed neutrino mass matrix are important for the lepton EDMs induced by the right–handed neutrino threshold effects. It should be noted that the unitary rotations applied on the diagonal Yukawa matrices at M_{st} are not unique, except that they should conform to the fermion mass matrix structure shown in Eq. (refmassM01). So our fits should be taken only as indicative, and not definitive. We expect differences of order one in our numerical results on EDM arising from the arbitrariness in these unitary matrices.

In Figure 3.1 the electron EDM induced by the $U(1)_A$ gaugino/gauge boson contributions to the soft masses and A–terms are plotted as a function of $M_{1/2}$ for three values of $\tan\beta$. We see that some parts of the parameter space are already excluded by the current experimental upper bound $d_e \leq 1.6 \times 10^{-27}$ e cm and that the other parts are in the range which will be tested by next generation electron EDM experiments [38].

In Figure 3.2 we plot the muon EDM as a function of SUSY breaking parameters. We find d_μ to be in the range $(10^{-25} - 10^{-28})$ e cm for most of the parameter space. This value is somewhat smaller than than $d_e(m_\mu/m_e)$, which would be the naive expectation based on the scaling of lepton masses. This happens for the following reason. The second and third family left–handed charged sleptons have the same $U(1)_A$ charge, so the flavor gauge bosons/gaugino will not generate any mass splitting between these sleptons. The mixing in the right–handed charged slepton sector is suppressed by a factor ϵ^2 for all $\tan\beta$, compared to the suppression factor ϵ between the first and the second generations. On the other hand, we find quite an enhancement of the muon EDM for the choice $m_0 = M_{1/2}$ and $\tan\beta = 50$. For this choice, the electron EDM is well above the experimental bound. Since the two EDMs are induced by independent phases, it is possible to choose the parameters such that the electron EDM is below the experimental limit and at the same time the muon EDM is at the level of $\sim (10^{-25} - 10^{-24})\, e$ cm, although we do not attempt such an explicit solution here. It should also be pointed out that parts of the parameter space where d_μ is large is already ruled out by the experimental upper limit for the radiative decay $\mu \to e\gamma$ for the numerical fits shown [2]. The remaining regions will be put to experimental scrutiny by future experiments [46].

In Figure 3.3 we present for comparison, the electron EDM arising solely from the right–handed neutrino threshold effects [12]. With the proper decoupling of the right–handed neutrinos [42] we find our results to be in rough agreement with those found by others [12,41,42]. Nevertheless these effects, which yield at most $d_e \sim 10^{-29} e\,$cm, are much smaller compared to the $U(1)_A$ effects.

In Figure 3.4 we plot the neutron EDM versus $M_{1/2}$. In Figure 3.5 we plot the deuteron EDM. Details of the calculations are given in Appendix A.3. In both cases our numerical results are in the interesting range which should be accessible to proposed experiments in the near future. We find the contributions from the CKM phase to be of the same order as the contributions from the $U(1)_A$ gaugino/gauge boson sector. Figures 3.4–3.5 include both these effects. The flavor sector contribution to the neutron EDM is somewhat smaller compared to the leptonic EDM due to the gluino focusing effect. (The squarks receive flavor universal contributions for their masses below M_F from the gluino, which tends to suppress flavor violation and thus d_n, see Eq. (2.51))

We have also studied the constraint on the chromoelectric dipole moment for the strange quark d_s^C arising from ^{199}Hg EDM [35,36]. This bound reads as $|d_s^C| \leq$

5.8×10^{-25} $e\,$cm. This constraint is easily satisfied in our model. The down–type squark mixing in the (23) sector is suppressed by a factor ϵ^2 for the right–handed squarks, and is vanishing to leading order for the left–handed squarks, similar to the case of $\mu - \tau$ mixing. Consequently, we find the chromoelectric EDM of the strange quark to be about two to three orders of magnitude below the experimental limit.

The soft SUSY breaking bilinear B–term and the gaugino masses will develop complex phases via the one–loop and two–loop RGE corrections respectively arising from the A–term contributions. In our model we find these corrections to be negligible compared to the $U(1)_A$ flavor gaugino/gauge boson effects.

3.5 Conclusions

In this chapter I have presented the results of our study on the electric dipole moments of the electron, muon and the neutron induced by a flavor dependent $U(1)$ symmetry which explains the hierarchy of fermion masses and mixings in a natural way via the Froggatt-Nielsen mechanism. This $U(1)$ symmetry may be identified as the anomalous $U(1)$ of string theory. This symmetry is broken spontaneously at a scale M_F slightly below the string scale, $M_F \sim M_{st}/50$. In the momentum regime $M_F \leq \mu \leq M_{st}$, the flavor $U(1)_A$ gauge boson sector will be active and will contribute to the soft SUSY breaking parameters in a flavor dependent fashion. This is the main source of the EDM that we have studied here. We adopt the minimal supergravity scenario for SUSY breaking, and assume that the soft SUSY breaking parameters are universal and real. The complex Yukawa couplings will still induce phases in the soft SUSY masses and the A-parameters, leading to the generation of EDM.

We have presented our numerical results for the electron, muon and the neutron EDMs in Figures 3.1–3.5 as functions of supersymmetry breaking parameters. d_e and d_n are very close to the current experimental limits, $d_e \sim (10^{-26} - 10^{-27})$ e cm and $d_n \sim 10^{-27}$ e cm. For the case of the muon, although d_μ is rather small for low $\tan\beta$, in the case of large $\tan\beta \sim 50$, for certain choices of phases in the Yuakawa couplings, we have found the induced the EDM to be as large as $(10^{-23} - 10^{-24})$ e cm, which might be accessible to future experiments [46]. In the leptonic sector, these EDMs are much larger than the ones induced by the neutrino seesaw sector, which yields, for example, $d_e \sim 3 \times 10^{-29}$ e cm with our texture of fermion mass matrices dictated by flavor symmetries. In Figure 3.2 we present our results for the induced d_e arising from the neutrino seesaw sector. Discovery of electric dipole moments for the electron, muon and the neutron can shed light on one of the fundamental puzzles of Nature, viz., the origin mass for elementary particles.

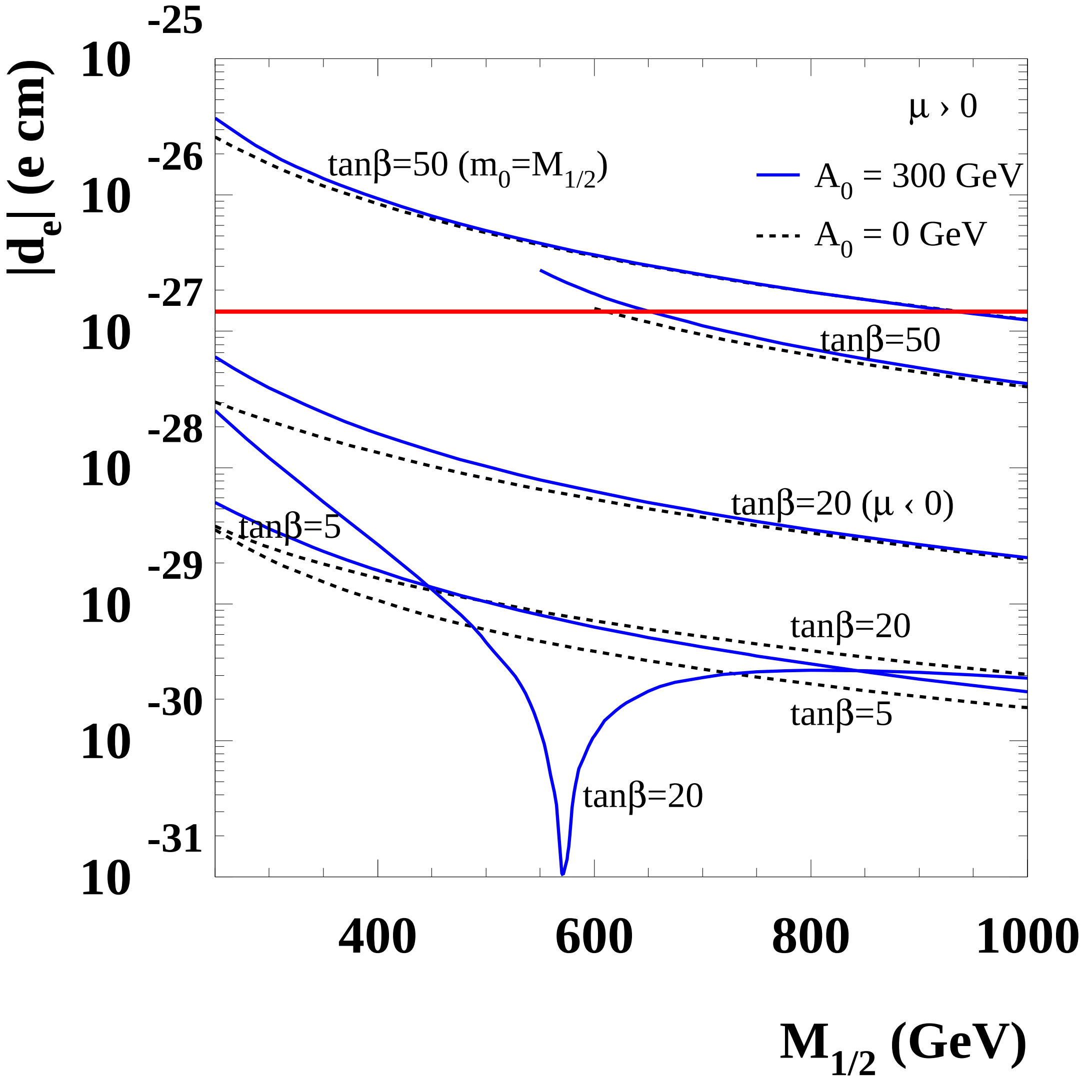

Figure 3.1. Electric Dipole Moment of the electron induced by the flavor gaugino/gauge boson. The horizontal line shows the current experimental limit on d_e. We have chosen here $m_0 = M_{1/2}/4.4$. For $\tan\beta = 50$ we show an additional case with $m_0 = M_{1/2}$ (the uppermost curve). For $\tan\beta = 20$ and $A_0 = 300$ GeV we find a cancelation between the A–term contributions given in Eq. (2.47) and the soft left/right mass contributions in Eq. (2.36) for our particular fit of the Yukawa couplings. This cancelation disappears for the choice of negative μ–term (the curve labeled by $\mu < 0$).

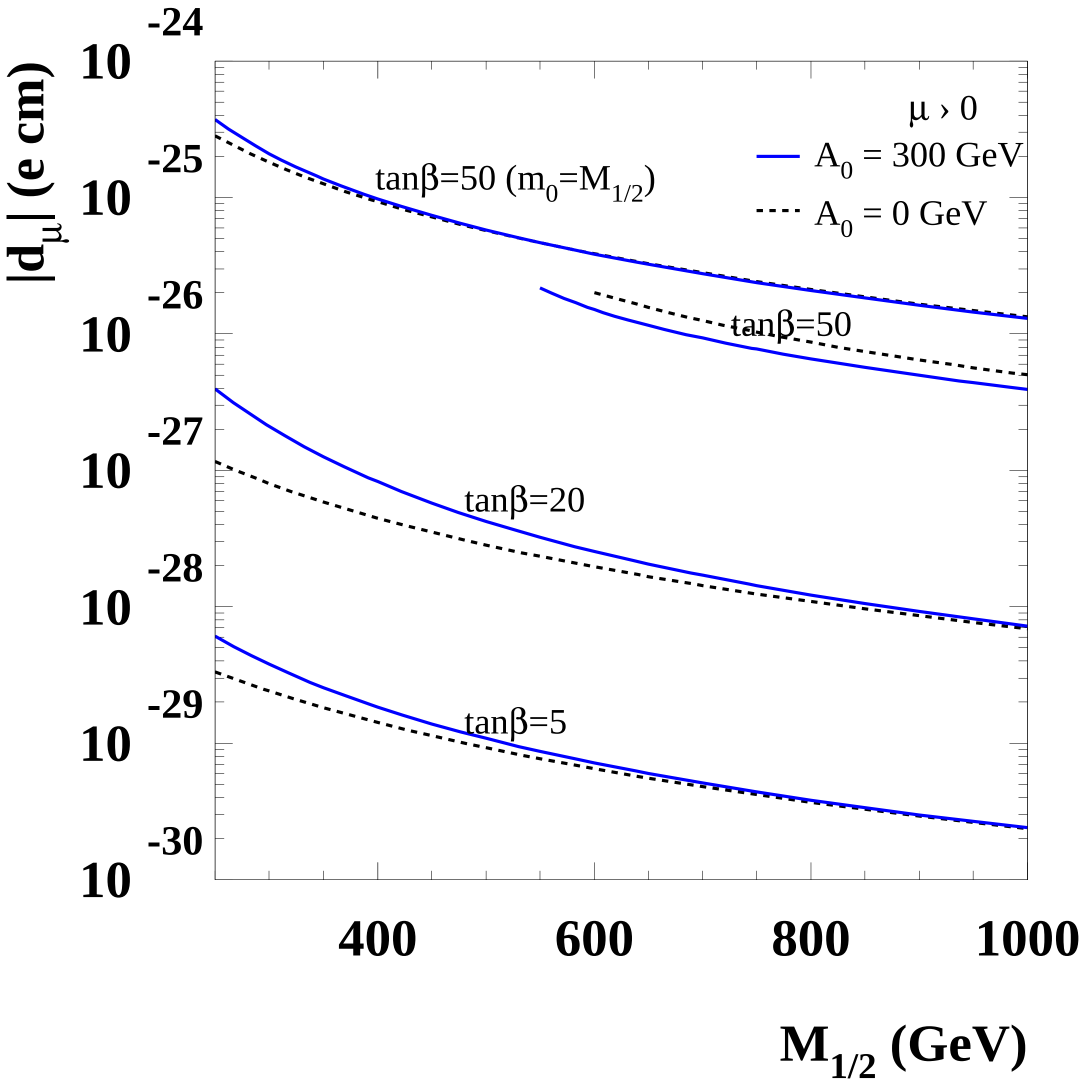

Figure 3.2. Electric Dipole Moment of the muon induced by the flavor gauge corrections. Here $m_0 = M_{1/2}/4.4$. For $\tan\beta = 50$, we also present results for the case $m_0 = M_{1/2}$.

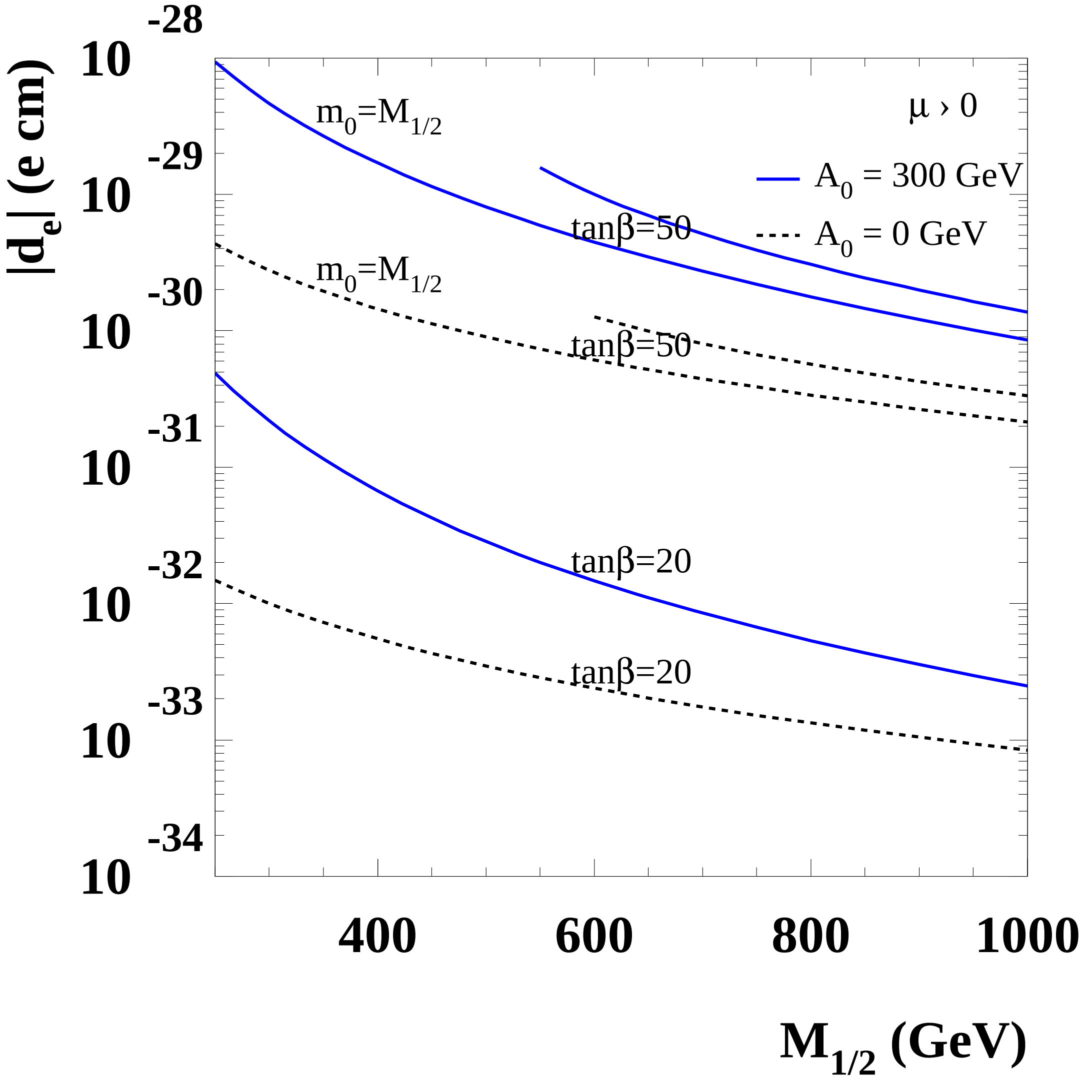

Figure 3.3. Electric Dipole Moment of the electron induced purely by the right–handed neutrino threshold corrections. The notation is the same as in Fig. 3.1.

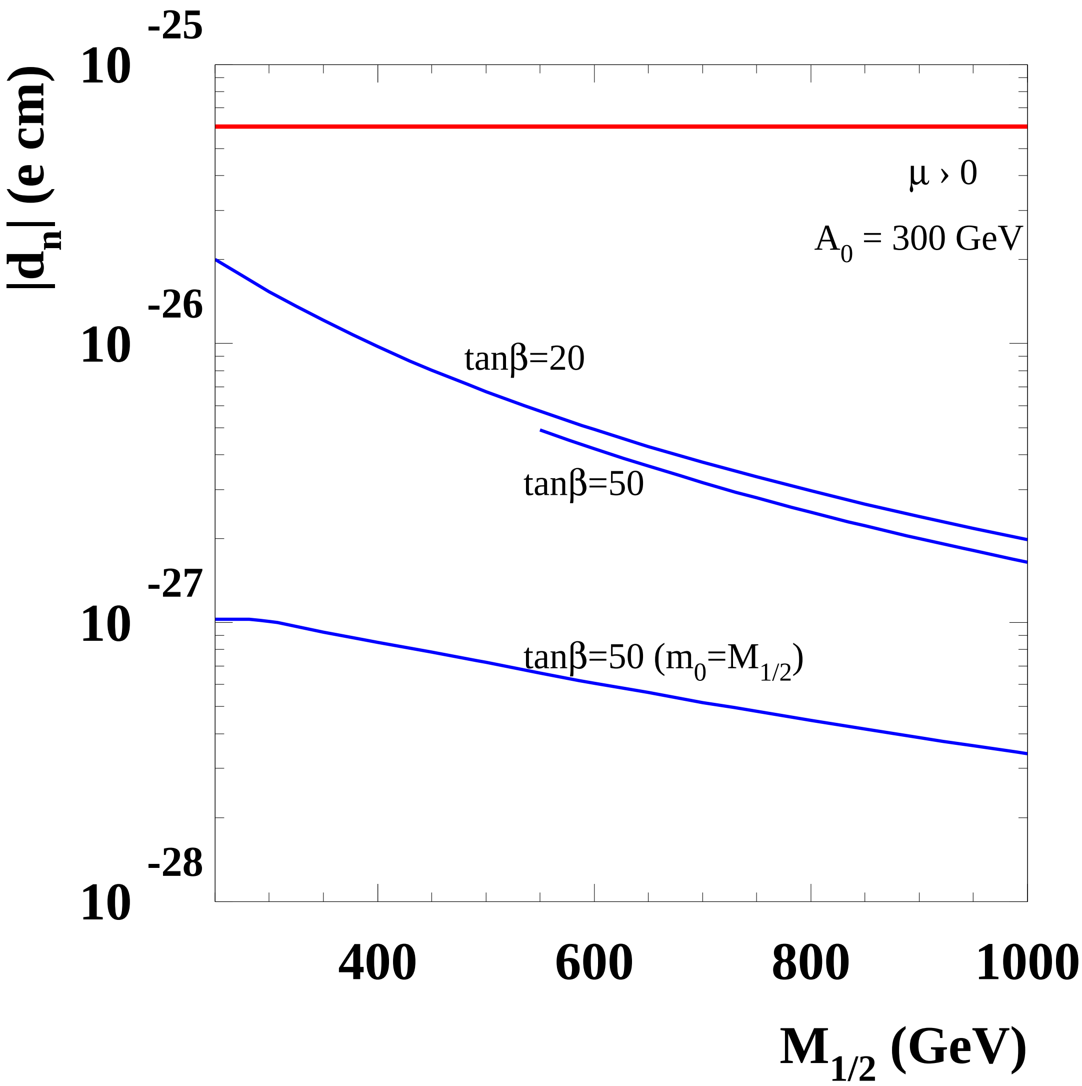

Figure 3.4. Electric Dipole Moment of the neutron induced by the flavor gaugino/gauge boson corrections. Here $m_0 = M_{1/2}/4.4$, with an additional case $m_0 = M_{1/2}$ shown for $\tan\beta = 50$. The horizontal line is the current experimental limit.

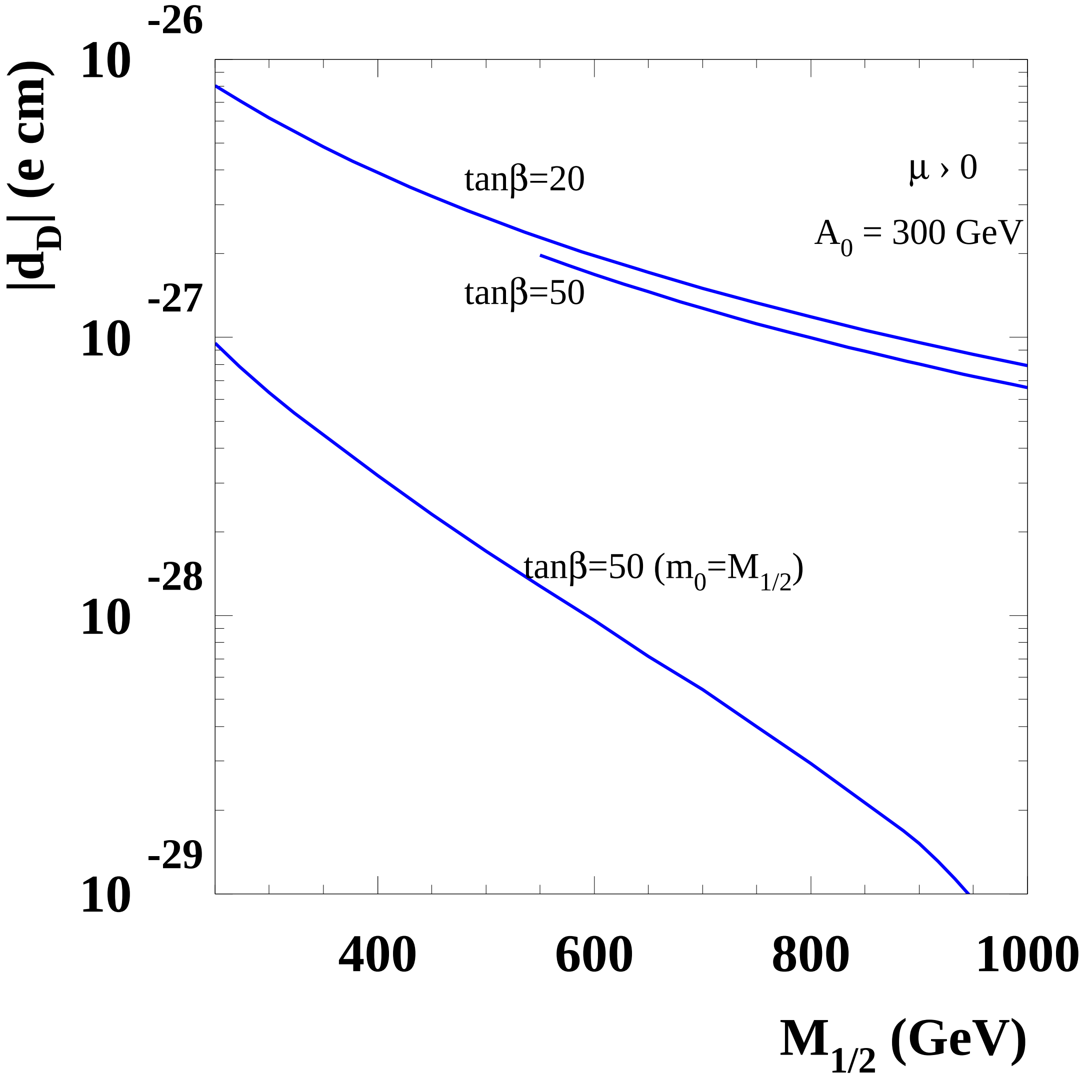

Figure 3.5. Electric Dipole Moment of the deuteron induced by the flavor gaugino/gauge boson corrections. Here $m_0 = M_{1/2}/4.4$, with an additional case $m_0 = M_{1/2}$ shown for $\tan\beta = 50$.

CHAPTER 4

SPLIT SUPERSYMMETRY FROM ANOMALOUS $U(1)$

4.1 Introduction

As noted in previous chapters, it is widely believed that supersymmetry may be relevant to Nature. There are four major observations which may justify this belief: (i) Supersymmetry (SUSY) can stabilize scales associated with spontaneous symmetry breaking. (ii) Unification of gauge couplings works well in the minimal SUSY extension of the Standard Model (SM). (iii) SUSY provides a natural candidate for cold dark matter. (iv) Supersymmetry is a necessary ingredient of superstring theory, which may eventually lead to a consistent quantum theory of gravity. Among these, reasoning (i), when applied to stabilize the electroweak scale, would suggest that all superpartners of the SM particles must have masses below or around a TeV. This is indeed what was assumed in almost all applications of supersymmetry to particle physics in the past twenty five years. The second and third observations above would only require that a subset of superpartners be lighter than a TeV, while the last one allows SUSY to be broken anywhere below the Planck scale, $M_{Planck} = 1.2 \times 10^{19}$ GeV. This is because, among the superpartners, if the split members of a unifying group ($SU(5)$, $SO(10)$, etc), namely the gauginos and the Higgsinos, are lighter than a TeV, while the complete multiplets (the scalar partners of SM fermions) are much heavier, unification of gauge couplings would work just as well. The lightest of these SUSY particles would still be a natural candidate for cold dark matter.

A scenario dubbed as "Split Supersymmetry", in which the spin 1/2 superparticles, namely, the gauginos and the Higgsinos, have masses of order TeV while the spin zero superparticles (squarks and sleptons) are much heavier, has recently been advocated [47]. This scenario gives up the conventionally employed naturalness criterion, since the light SM Higgs boson is realized only by fine–tuning. Such a finely tuned scenario, it is argued, may not be as improbable as originally thought [47]. This is because in any theory with broken SUSY one has to cope with another, even more severe, fine-tuning, in the value of the cosmological constant. A cosmic selection rule, an anthropic principle [48], may be active in this case. If so, a similar argument may also explain why the SM Higgs boson is light [49]. Supersymmetry plays no role in solving the hierarchy problem here. Recent realization of a string landscape [50], which suggests the existence of a multitude of string vacua, may justify this approach. Probabilistically, the chances of finding a vacuum with a light SM Higgs (along with a small cosmological constant) may not be infinitesimal, given the existence of a large number of string vacua [51].

Split Supersymmetry has a manifest advantage over TeV scale supersymmetry: Unacceptably large flavor changing neutral current (FCNC) processes [52], fermion electric dipole moments, and $d = 5$ proton decay rate, which generically plague TeV scale SUSY are automatically absent in Split Supersymmetry. Various aspects of this scenario have been analyzed by a number of authors [53,54].

In this chapter we address the Split Supersymmetry scenario from a theoretical point of view. Perhaps the most important question in this context is a natural realization of the split spectrum. Although it may be argued that R–symmetries would protect masses of the spin 1/2 SUSY fermions and not of the squarks and sleptons, in any specific scenario for SUSY breaking there is very little freedom in choosing the relative magnitudes of the two masses. We will focus on SUSY breaking triggered by the anomalous $U(1)$ D–term of string origin coupled to a SUSY QCD sector [55]. Each sector treated separately would preserve supersymmetry, but their cross coupling breaks it. We make extensive use of exact results known for $N = 1$ SUSY QCD [56]. In this scenario, the squarks and sleptons receive SUSY breaking masses at the leading order from the anomalous $U(1)$ D–term, while the gauginos acquire masses only at higher order. The Higgsino mass also arises at higher order and is similar in magnitude to the gaugino mass. Thus, a naturally split spectrum is realized. The anomalous $U(1)$ D–term also provides a small expansion parameter which we use to explain the mass and mixing hierarchies of quarks and leptons. We present complete models which are consistent with anomaly cancelation, and which lead to naturally Split SUSY spectrum. A somewhat similar analysis has been carried out by Nath et. al.[57], our approach is different in that we present complete models without assuming a hidden sector and address the fermion masses and mixing hierarchy problems. Our spectrum is also quite different, especially as regards the gravitino mass. We note that with flavor–dependent charges, the anomalous $U(1)$ D–term contributions to the squark and slepton masses generically lead to large FCNC processes with sub–TeV scalars [58], this problem is absent in the Split Supersymmetry scenario.

4.2 Supersymmetry Breaking by Anomalous $U(1)$ and Gaugino Condensation

In this Section we review supersymmetry breaking induced by the D–term of anomalous $U(1)$ symmetry [55,59] coupled to the strong dynamics of $N = 1$ SUSY gauge theory [56]. Each sector separately preserves supersymmetry, so an expansion parameter (the cross coupling) is available. Exact results of supersymmetric gauge theories can then be applied. Here we focus on the global supersymmetric limit, in Section 4.2.1 we extend the analysis to supergravity. In addition to the SM fields, these models contain an $SU(N_c)$ gauge sector with N_f flavors. The "quark" (Q) and "antiquark" ($\tilde{Q}$) fields of the $SU(N_c)$ sector are also charged under the $U(1)_A$. $U(1)_A$ is broken by a SM singlet field S carrying $U(1)_A$ charge of -1. The Standard Model fields carry flavor–dependent $U(1)_A$ charges so that the hierarchy in fermion masses and mixings is naturally explained. A small expansion parameter $\epsilon \sim 0.2$ is provided by the ratio $\epsilon = \langle S \rangle / M_{Pl}$ by the induced Fayet–Iliopoulos D–term for the $U(1)$. To see this, we recall that the apparent anomalies in $U(1)_A$ are canceled by the Green–Schwarz mechanism [8] we discussed in chapters 2 and 3. The anomalies are canceled if the following conditions are satisfied:

$$\frac{A_i}{k_i} = \frac{A_N}{k_N} = \frac{A_A}{3k_A} = \frac{A_{\mathrm{g}ravity}}{24} = \delta_{GS}. \tag{4.1}$$

Here we now include A_N, the anomaly coefficient for $SU(N_c)^2 \times U(1)_A$. These conditions put severe restrictions on the choice of $U(1)_A$ charges and $SU(N_c)$ sector.

String loop effects induce a nonzero Fayet–Iliopoulos D–term for the $U(1)_A$ given by [9]

$$\xi = \frac{g_{st}^2 M_{Pl}^2}{192\pi^2} A_{\mathrm{g}ravity}\,, \tag{4.2}$$

where g_{st} is the string coupling at the unification scale M_{Pl}, related to the SM gauge couplings at that scale as

$$k_i g_i^2 = 2g_{st}^2 \ . \tag{4.3}$$

The scalar potential receives a contribution from the D-term given by

$$V_D = \frac{D_A^2}{2} = \frac{g_A^2}{2}\left(\xi - |S|^2 + q_Q|Q_i|^2 + q_{\tilde{Q}}|\tilde{Q}^{\tilde{i}}|^2 + \sum_i q_i|\phi_i|^2\right)^2 . \tag{4.4}$$

Here S is the flavon field with charge -1, Q_i and $\tilde{Q}^{\tilde{i}}$ are the "quark" and "antiquark" fields belonging to the fundamental and antifundamental representaions of an $SU(N_c)$ gauge group with $U(1)$ charges q_Q and $q_{\tilde{Q}}$. ϕ_i in Eq. (4.4) stand for all the other fields, and includes the SM sector.

In our models, all fields except S, will have positive $U(1)_A$ charges, so ξ will turn out to be positive. The potential of Eq. (4.4) will minimize to preserve supersymmetry by giving the negatively charged S field a vacuum expectation value, which would break the $U(1)_A$ symmetry. To zeroth order in SUSY breaking parameters, $\langle S \rangle = S_0$, where

$$S_0 \equiv \sqrt{\xi} = \sqrt{\frac{g_{st}^2 A_{\text{gravity}}}{192\pi^2}} M_{Pl} \equiv \epsilon M_{Pl}. \tag{4.5}$$

Here $\epsilon \sim 0.2$ will provides a small expansion parameter to explain the hierarchy of quark and lepton masses and mixings.

As for the $N = 1$ SUSY QCD sector, we consider the gauge group $SU(N_c)$ with N_f flavors of quarks and antiquarks, and apply the well–known exact results[56]. For concreteness we choose $N_f < N_c$. These results have been applied to TeV scale SUSY breaking by Binetruy and Dudas [55] in the presence of anomalous $U(1)$ symmetry. These models actually lead to a Split Supersymmetry spectrum, as we will show. We also generalize the results of Binetruy and Dudas[55] to include supergravity corrections (in Section 4.2.1). In Section 4.3, we apply these results to explicit and complete models.

The effective superpotential we consider has two pieces:

$$W_{eff} = W_{tree} + W_{dynamical}, \tag{4.6}$$

where W_{tree} is the tree–level superpotential, while $W_{dynamical}$ is induced dynamically by nonperturbative effects. Since the Q and the $\tilde{Q}$ fields are charged under $U(1)_A$, a bare mass term connecting them is not allowed. A mass term will arise through the coupling

$$W_{tree} = \frac{\mathrm{Tr}\left(\lambda Q\tilde{Q}\right) S^n}{M_*^{n-1}} \tag{4.7}$$

when $\langle S\rangle = S_0$ is inserted. Here the trace is taken over the N_f flavor indices of the Q_i and $\tilde{Q}^{\tilde{i}}$ fields. M_* is a mass scale at which this term is induced. The most natural value of M_* is M_{Pl}, which is what we will use for our numerical analysis, but we allow M_* to be different from M_{Pl} for generality. We have used the definition

$$n = q_Q + q_{\tilde{Q}} \tag{4.8}$$

for the sum of the $U(1)$ charges of Q and $\tilde{Q}$. As we will see later the choice $n = 1$, which would correspond to a renormalizable superpotential will be phenomenologically unacceptable. From the exact results given by Seiberg [56], the dynamically generated superpotential is known to be (for $N_f < N_c$)

$$W_{dynamical} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det\left(Q\tilde{Q}\right)}\right)^{1/(N_c - N_f)} . \tag{4.9}$$

Here Λ is the dynamically induced scale below which the $SU(N_c)$ sector becomes strongly interacting:

$$\Lambda \sim M_{Pl} e^{-\frac{2\pi}{\alpha_{N_c}(3N_c - N_f)}}, \tag{4.10}$$

where α_{N_c} is the $SU(N_c)$ gauge coupling constant at M_{Pl}. For $N_f = N_c - 1$, the gauge symmetry is completely broken, and Eq. (4.9) is induced by instantons. For $N_f < N_c - 1$, the gauge symmetry is reduced to $SU(N_c - N_f)$ and the gaugino condensate of this symmetry induces Eq. (4.9).

Below the scale Λ the effective theory can be described in terms of $N_f \times N_f$ mesons $Z_j^{\tilde{i}}$:

$$Z_j^{\tilde{i}} = Q_j \tilde{Q}^{\tilde{i}} \quad \text{with} \quad (\tilde{i}, j = 1, .., N_f). \tag{4.11}$$

Neglecting small supersymmetry breaking effects, we can describe the theory below Λ along the D–flat directions $Q_i = \tilde{Q}_i$ in terms of the Z fields. We can make the following replacements in the D–term and the superpotential: $q_Q|Q_i|^2 + q_{\tilde{Q}}|\tilde{Q}^{\tilde{i}}|^2 \to n\mathrm{Tr}\left(Z^\dagger Z\right)^{1/2}$ and $Q_j\tilde{Q}^{\tilde{i}} \to Z^{\tilde{i}}_j$. We use the notation

$$m = \lambda \frac{S_0^n}{M_*^{n-1}}, \tag{4.12}$$

with m identified as the mass matrix of the Z field (upto small supersymmetry breaking effects). Then the F–term for the Z fields, defined as $(F_Z)^{\tilde{i}}_j = 2\left[\left(Z^\dagger Z\right)^{1/2}\right]^{\tilde{i}\tilde{k}} \partial W/\partial Z^{\tilde{k}}_j$, is found to be

$$(F_Z)^{\tilde{i}}_j = 2\left[\left(Z^\dagger Z\right)^{1/2}\left(m - \left(\frac{\Lambda^{3N_c-N_f}}{\det(Z)}\right)^{1/(N_c-N_f)}\left(\frac{1}{Z}\right)\right)^T\right]^{\tilde{i}}_j. \tag{4.13}$$

This theory preserves supersymmetry, as $F_Z = 0$ can be realized with $\langle Z\rangle \neq 0$ and given by

$$(Z_0)^{\tilde{i}}_j \equiv \left(\det(m)\,\Lambda^{3N_c-N_f}\right)^{1/N_c}\left(\frac{1}{m}\right)^{\tilde{i}}_j. \tag{4.14}$$

Note that this result holds only in the presence of a nonvanishing VEV $\langle S\rangle$, so that m is nonzero.

So far we treated the $U(1)_A$ D–term and the ensuing superpotential for the Z fields separately. The two sectors are however coupled through W_{tree} of Eq. (4.7). Owing to this coupling, supersymmetry is actually broken. This is evident by examining the F–term of the S field,

$$F_S = n\frac{Tr(mZ_0)}{S_0} \neq 0. \tag{4.15}$$

Similarly F_Z is also nonzero. The VEVs of S and Z fields will shift from the supersymmetry preserving values of Eqs. (4.5) and (4.14) when the full potential is minimized jointly. To find the soft SUSY breaking parameters we need to calculate these corrections.

The scalar potential of the model in the global limit is given by

$$V = |F_S|^2 + \frac{1}{2}\mathrm{Tr}(F_Z(Z^\dagger Z)^{-1/2}F_Z^\dagger) + \frac{1}{2}D_A^2. \tag{4.16}$$

We expand the fields around the SUSY preserving minima:

$$S = S_0 + \delta S \qquad Z^{\tilde{i}}_j = (Z_0 + \delta Z)^{\tilde{i}}_j \tag{4.17}$$

with $\delta S/S_0 \ll 1$, $\delta Z/Z_0 \ll 1$. For simplicity we assume the coupling matrix λ to be an identity matrix, $\lambda^i_j = \lambda\delta^i_j$, in which case $Z^j_i = Z\delta^j_i$ can be chosen. The VEV $\langle Z\rangle = Z_0$ arising from Eq. (4.14) in this case becomes

$$Z_0 = \frac{\Lambda^3}{m}\left(\frac{m}{\Lambda}\right)^{N_f/N_c}. \tag{4.18}$$

We make an expansion in the supersymmetry breaking parameter Δ defined as

$$\Delta \equiv Z_0/S_0^2 = \frac{\Lambda^3}{mS_0^2}\left(\frac{m}{\Lambda}\right)^{N_f/N_c} \ll 1. \tag{4.19}$$

From the minimization of the scalar potential with respect to these shifted fields, we find

$$\begin{aligned}
\langle S^\dagger S\rangle &= S_0^2\left[1+\Delta\left(nN_f\right)-\Delta^2\left(\frac{n^2N_f^2}{2N_c^2g_A^2}\right)\left\{g_A^2 n\left(N_c-N_f\right)\left(2N_c-N_f\right)\right.\right.\\
&- \left.\left.2N_c\left(N_c-nN_f\right)\frac{m^2}{S_0^2}\right\}+O(\Delta^3)\right], \\
\langle Z\rangle &= Z_0\left[1-\Delta\left\{\frac{n^2N_f\left(N_c-N_f\right)\left(2N_c-N_f\right)}{2N_c^2}\right\}+O(\Delta^2)\right].
\end{aligned} \tag{4.20}$$

This agrees with the results of Binetruy and Dudas[55], except that there are two apparent typos in Eq. (2.22) of that paper.

Now the F and the D–terms are given by

$$\begin{aligned}
\langle F_S\rangle &= mS_0\Delta\left(nN_f\right)\left(1+\Delta\frac{nN_f}{2}\left(n-1+\frac{nN_f\left(N_c-N_f\right)\left(2N_c-N_f\right)}{N_c^2}\right)\right), \\
\langle F_Z\rangle &= mZ_0\Delta\left(n^2N_f\right)\left(\frac{N_f}{N_c}-1\right), \\
\langle D_A\rangle &= m^2\Delta^2\left(nN_f\right)^2\left(\frac{nN_f}{N_c}-1\right)/g_A.
\end{aligned} \tag{4.21}$$

Consequently, the scalar soft masses induced from the D–term of anomalous $U(1)$ are

$$m^2_{\tilde{f}_i} = q_{f_i}m_0^2, \tag{4.22}$$

where

$$m_0^2 = m^2\Delta^2\left(nN_f\right)^2\left(\frac{nN_f}{N_c}-1\right). \tag{4.23}$$

There is a simple interpretation of these results in terms of the gaugino condensate (for $N_f < N_c - 1$), which is given by [60]

$$\langle\lambda_\alpha\lambda^\alpha\rangle = e^{2i\pi k/(N_c-N_f)}\Lambda^3\left(\frac{m}{\Lambda}\right)^{N_f/N_c}, \quad k = 1-(N_c-N_f). \tag{4.24}$$

The soft scalar masses are simply proportional to the gaugino condensate. We will make use of these results in Section 4.3. Note that had we chosen $n = 1$ these results would have led to negative squared masses for scalars since $N_f < N_c$, and q_{f_i} is positive, which is unacceptable. Note also that the D–term contributions are proportional to the $U(1)_A$ charges, so they are zero for particles with zero charge.

4.2.1 Gravity corrections to the soft parameters

In this Section we work out the supergravity corrections to the soft parameters found in the global SUSY limit in the previous section. Our reasons for this extension are two–fold. First, we wish to show explicitly that supergravity corrections do not destabilize the minimum of the potential that we found in the global limit. Second, the main contribution to the masses of scalars with zero $U(1)$ charge will arise from supergravity corrections. In our explicit models, we do have particles with zero charge.

It is conventional in supergravity to add a constant term to the superpotential in order to fine–tune the cosmological constant to zero:

$$W = W_{global} + \beta. \tag{4.25}$$

We separate the constant into two parts, $\beta = \beta_0 + \beta_1$, such that β_0 cancels the leading part of the superpotential in which case $\langle W \rangle = \beta_1$. The F–term contribution to the scalar potential in supergravity is given by

$$V_F = M_{Pl}^4 e^G \left(G_i \left(G^{-1}\right)^i_j G^j - 3 \right), \tag{4.26}$$

where

$$G^i \equiv \partial G / \partial \phi_i^*, \;\; G_i \equiv \partial G / \partial \phi^i, \;\; G^i_j \equiv \partial^2 G / \partial \phi_i^* \partial \phi^j. \tag{4.27}$$

We will assume for illustration the minimal form of the Kähler potential. In our model it is given by

$$G = \frac{|S|^2}{M_{Pl}^2} + 2\frac{\mathrm{Tr}(Z^\dagger Z)^{1/2}}{M_{Pl}^2} + \sum_i \frac{|\phi_i|^2}{M_{Pl}} + \ln\left(\frac{|W|^2}{M_{Pl}^6}\right). \tag{4.28}$$

Then the scalar potential is given by

$$V = V_F + V_D, \tag{4.29}$$

with

$$V_F = e^{\left(|S|^2+2\mathrm{Tr}(Z^\dagger Z)^{1/2}+\sum_i|\phi_i|^2\right)/M_{Pl}^2}\left(\left|F_S+S^*\frac{W}{M_{Pl}^2}\right|^2\right. \tag{4.30}$$

$$+\ \frac{1}{2}\mathrm{Tr}\left[\left(F_Z^\dagger+Z\frac{W}{M_{Pl}^2}\right)\left(Z^\dagger Z\right)^{-1/2}\left(F_Z+Z^\dagger\frac{W}{M_{Pl}^2}\right)\right] \tag{4.31}$$

$$+\ \left.\sum_i\left|F_{\phi_i}+\phi_i^*\frac{W}{M_{Pl}^2}\right|^2-3\frac{|W|^2}{M_{Pl}^2}\right), \tag{4.32}$$

and

$$V_D=\frac{g^2}{2}\left(G_i\left(T_a\right)_j^i\phi^j\right)^2M_{Pl}^4. \tag{4.33}$$

In our case for $G^i=\phi^i/M_{Pl}^2+\partial W/\partial\phi_i^*/W$, so $M_{Pl}^2G_i\left(T_a\right)_j^i\phi^j=\phi_i^*\left(T_a\right)_j^i\phi^j$, which is identical to the D term of global supersymmetry (Note that the term $\partial W/\partial\phi^i(T_a)_j^i\phi^j$ vanishes due to the gauge invariance of W).

Including these supergravity corrections, by minimizing the potential we find

$$\langle S^\dagger S\rangle=\langle S^\dagger S\rangle_{global}+2\Delta^2S_0^2\epsilon^2\left[-\frac{n^2N_f^2}{4g_A^2N_c^2}\left\{ng_A^2\left(N_c-N_f\right)^2+2N_c\left(nN_f+N_c\right)\frac{m^2}{S_0^2}\right\}\right.$$
$$\left.-\frac{\tilde\beta_1 nN_f}{4g_A^2N_c^2}\left\{g_A^2\left(N_c-N_f\right)^2\left(2N_c+n\left(N_c-N_f\right)\right)+2N_c\left(nN_f-4N_c\right)\frac{m^2}{S_0^2}\right\}\right], \tag{4.34}$$
$$\langle Z\rangle=\langle Z\rangle_{global}-\Delta\left(Z_0\epsilon^2\right)\frac{N_c-N_f}{2N_c^2}\left[n^2N_f\left(N_c-N_f\right)+\tilde\beta_1\left\{2N_c+n\left(N_c-N_f\right)\right\}\right],$$

where the subscript "global" denotes the contributions found in global SUSY case in Eq. (4.20). Here we have introduced a dimensionless parameter $\tilde\beta_1$ defined through the relation

$$\beta_1=\left(\tilde\beta_1 mS_0^2\right)\Delta. \tag{4.35}$$

From the condition that the vacuum energy is zero at the minimum for the vanishing of the cosmological constant, $\tilde\beta_1$ is found to be

$$\tilde\beta_1\simeq\pm\frac{nN_f}{\sqrt{3}\epsilon}\left(1\pm\frac{\epsilon}{\sqrt{3}}+\frac{2}{3}\epsilon^2\right). \tag{4.36}$$

Eq. (4.35) ensures that the cosmological constant remains zero to the scale of strong dynamics. With these corrections the soft scalar masses from the D–term are now given by

$$m_{\tilde f}^2=\left(m_{\tilde f}^2\right)_{global}+q_fm_0^2\frac{\epsilon^2}{nN_f-N_c}\left[N_c+nN_f+\tilde\beta_1\left(1-4N_c/(nN_f)\right)\right]. \tag{4.37}$$

Note that the shifts in the masses are small, suppressed by a factor of $\epsilon \simeq 0.2$.

The gravitino mass is determined to be

$$m_{3/2} \simeq m \frac{\tilde{\beta}_1 \Delta S_0^2}{M_{Pl}^2} \simeq \frac{n N_f \Lambda^3}{\sqrt{3} S_0 M_{Pl}} \left(\frac{m}{\Lambda}\right)^{N_f/N_c} . \tag{4.38}$$

In addition to the D–term corrections, all scalar fields receive a contribution to their soft masses from the term

$$\left| \phi_i^* \frac{W}{M_{Pl}^2} \right|^2 = m_{3/2}^2 |\phi_i|^2 . \tag{4.39}$$

For particles neutral under the anomalous $U(1)_A$ these are the leading source for soft masses. With the assumed minimal Kähler potential, note that these soft masses are equal to the gravitino mass.

So far we assumed the minimal form of the Kähler potential for illustration. There is no justification for this assumption. In fact, within Split Supersymmetry, since there are no excessive FCNC processes, an arbitrary form for the Kähler potential is permissible phenomenologically. The effects of such a nonminimal G can be understood in terms of higher dimensional operators suppressed by the Planck scale. Scalar fields can acquire soft SUSY breaking masses through the terms

$$\mathcal{L} \supset \int (\phi_i^* \phi^i) \frac{|S|^2}{M_{Pl}^2} d^4\theta \ . \tag{4.40}$$

The resulting masses are $m_{\tilde{f}_i}^2 = c_i m_{3/2}^2$, with c_i being order one (flavor–dependent) coefficients. We will allow for such corrections.

4.3 **Explicit Models**

In this section we consider a class of models based on flavor–dependent anomalous $U(1)$ symmetry and apply the results of the previous section. These models were developed to address the pattern of fermion masses and mixings [2,10]. As noted earlier, the anomalous $U(1)$ D–term provides a small expansion parameter $\epsilon = \langle S \rangle / M_{Pl} \sim 0.2$, which can be used to explain the mass hierarchy. We assign charge q_i to fermion f_i and charge q_j^c to fermion f_j^c, such that the mass term $f_i f_j^c H$ will arise through a higher dimensional operators with the factor $(S/M_{Pl})^{q_i + q_j^c}$ and thus suppressed by a factor $\epsilon^{q_i + q_j^c}$. By choosing the charges appropriately the observed mass and mixing hierarchy can be explained, even with all Yukawa coefficients being of order one.

With sub–TeV supersymmetry this approach to fermion mass and mixing hierarchy cannot be combined with supersymmetry breaking triggered by anomalous $U(1)$, since the D–terms will split the masses of scalars leading to unacceptable FCNC. Within Split Supersymmetry, however, these two approaches can be combined, which is what we analyze now.

The superpotential of the class of models under discussion has the following form:

$$\begin{aligned} W &= \sum_f y_{ij}^f f_i H f_j^c \left(\frac{S}{M_{Pl}}\right)^{n_{ij}^f} + \frac{M_{Rij}}{2}\nu_i^c \nu_j^c \left(\frac{S}{M_{Pl}}\right)^{n_{ij}^{\nu^c}} + \mu H_u H_d \\ &+ \frac{\mathrm{Tr}\,(\lambda Z)\, S^n}{M_{Pl}^{n-1}} + (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det(Z)}\right)^{1/(N_c-N_f)} + W_A\,(S, X_k)\ . \end{aligned} \tag{4.41}$$

Here X_k are the SM singlet fields necessary for the cancelation of gravitational anomaly. We will focus on the sub-class of such models studied in Ref. 2 and described in chapters 2 and 3. The mass matrices for the various sectors in Ref. 2 are given by:

$$\begin{aligned} &M_u \sim \langle H_u\rangle \begin{pmatrix} \epsilon^{8-2\alpha} & \epsilon^{6-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^{6-\alpha} & \epsilon^4 & \epsilon^2 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \qquad M_d \sim \langle H_d\rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^{4-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \\ &M_e \sim \langle H_d\rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^3 & \epsilon \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \qquad M_{\nu_D} \sim \langle H_u\rangle \epsilon^p \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \\ &M_{\nu^c} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \Rightarrow \quad M_\nu^{light} \sim \frac{\langle H_u\rangle^2}{M_R}\epsilon^{2p} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \end{aligned} \tag{4.42}$$

Although not unique, these mass matrices would lead to small quark mixings and large neutrino mixings. Note that the neutrino masses are hierarchical in this scheme.

The charge assignment which leads to these mass matrices is given in Table 4.1 * Here we use $SU(5)$ notation for the fields in the first column for simplicity,

*These are the same as in Table 2.1 of chapter 2 displayed in $SU(5)$ notation.

Field	Anomalous flavor charges
10_1, 10_2, 10_3	$4-\alpha$, 2, 0
$\overline{5}_1$, $\overline{5}_2$, $\overline{5}_3$	$1+p$, p, p
ν_1^c, ν_2^c, ν_3^c	1, 0, 0
H_u, H_d, S, Q, $\tilde{Q}$	0, 0, -1, $n/2$

TABLE 4.1. The flavor $U(1)_A$ charge assignment for the MSSM fields, the $SU(N_c)$ fields Q and $\tilde{Q}$ and the flavon field S in the normalization where $q_S = -1$.

although we do not explicitly assume $SU(5)$ unification. There are two parameters, p and α, which can take a set of discrete values. The parameter p takes values $p = 2$ (1, 0) corresponding to low (medium, high) value of $\tan\beta$ (the ratio of the two Higgs VEVs). Actually, in Split Supersymmetry, since $\tan\beta \sim 1$ is also permitted, $p = 3$ is also allowed. α appears in the mass of the up–quark, both $\alpha = 0$ and $\alpha = 1$ give reasonable spectrum. We also consider the case where the charge of $\bar{5}_1$ is p (rather than $1+p$) in Table 4.1. This case would have mass matrices which are very similar to those in Eq. (4.42). The main difference in this case is that all elements of M_ν^{light} will be of the same order, which would lead to larger U_{e3}. This scenario has been widely studied [61], sometimes under the name of neutrino mass anarchy [62]. The charge assignment of Table 4.1, as well as its above–mentioned variant, explain naturally the mass and mixing hierarchy of quarks and leptons, including small quark mixings and large neutrino mixings.

The Green–Schwarz anomaly cancelation conditions for these models are given by

$$\frac{A_1}{k_1} = \frac{A_i}{k_i} = \frac{A_{N_c}}{k_N} = \frac{nN_f}{2k_N} = \frac{19-3\alpha+3p}{2k_i} \text{ or } \frac{18-3\alpha+3p}{2k_i} \tag{4.43}$$

with A_i being the $(SM)^2 \times U(1)_A$ anomalies for $i = 2-3$. Their equality is automatically satisfied, due to the $SU(5)$ compatibility of charges, provided that the Kac–Moody levels k_i for the SM gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ are chosen to be, for example, 5/3, 1 and 1 respectively. For $A_{gravity}$, one needs to introduce extra heavy matter X_k (with charge +1) which decouple at or near the Planck scale (a detailed discussion has been given by Babu et. al.[2]). In Eq. (4.1) the first p–dependent factor applies to the charge assignment of Table 4.1, while the second one corresponds to the variant with $\bar{5}_1$ carrying charge p. For every choice of charge we can compute the expansion parameter ϵ from $\epsilon = \sqrt{g_{st}^2 A_{gravity}/(192\pi^2)}$. We find for $\alpha = 0$ and for the charges of Table 1, $\epsilon = 0.174\,(0.187,\, 0.199)$ for $p = 0\,(1,\, 2)$. The results are very similar for other choices.

Eq. (4.43) allows for only a finite set of choices for n, N_c and N_f. First of all, all these must be integers. Secondly, the mass parameter m of the meson fields of $SU(N_c)$ must be of order Λ or smaller, otherwise these mesons will decouple from the low energy theory, affecting its dynamics. Thirdly, the dynamical scale Λ is determined for any choice of charges, due to the string unification condition, Eq. (4.3). (We will confine to Kac–Moody level 1 for the $SU(N_c)$ as well as the SM sectors.) This should lead to an acceptable SUSY breaking spectrum. Consistent with these demands, we find four promising cases. (i) $n = 5,\ N_f = 5,\ p = 2,\ \alpha = 0$; (ii) $n = 6,\ N_f = 4,\ p = 2,\ \alpha = 0$; (iii) $n = 7,\ N_f = 3,\ p = 1,\ \alpha = 1$; and (iv) $n = 6,\ N_f = 3,\ p = 1,\ \alpha = 1$. Here (i) has $\bar{5}_1$ charge equal to $p + 1$, while the other three cases has it to be equal to p. We will see that the choices $N_c = 6$ or 7 yield reasonable spectrum.

4.3.1 The spectrum of the model

Now we turn to the spectrum of the model. We set the gaugino masses at the TeV scale. (The Higgsinos will turn out to have masses of the same order.) We then seek possible values of the scale Λ and the mass parameter m_0 (the scalar mass) that would induce the TeV scale gaugino masses. The spectrum will turn out to be that of Split Supersymmetry. The main reason for this is that the leading SUSY breaking term, the $U(1)_A$ D–term, generates squark and slepton masses, but not gaugino and Higgsino masses.

Supersymmetry breaking trilinear A terms are induced in the model by the same superpotential W (Eq. (4.41)) that generates quark and lepton masses, once the S field acquires a nonzero F component:

$$\mathcal{L} \supset y_{ij}^f \int d^2\theta f_i f_j^c H \left(\frac{S}{M_{Pl}}\right)^{n_{ij}^f} = Y_{ij}^f \left(q_i^f + q_j^{f^c}\right) \tilde{f}_i \tilde{f}_j^c H \frac{F_S}{S}. \tag{4.44}$$

Here $Y^f_{ij} \simeq y^f_{ij}\epsilon^{n^f_{ij}}$ are the effective MSSM Yukawa couplings, with $n_{ij} = q_{f_i} + q^c_{f_j}$, the sum of the anomalous charge of the SM fermions f_i and f^c_j. Substituting results from the previous section, Eqs. (4.20) and (4.21), we find

$$A^f_{ij} = Y^f_{ij}\left(q^f_i + q^{f^c}_j\right) nN_f \frac{\Lambda^3}{S_0^2}\left(\frac{m}{\Lambda}\right)^{N_f/N_c}. \tag{4.45}$$

These A–terms are induced at the scale Λ. The messengers of supersymmetry breaking are the meson fields of the $SU(N_c)$ sector, which have masses of order Λ. In the momentum range $m_0 \leq \mu \leq \Lambda$, the spectrum is that of the MSSM and there is renormalization group running of all SUSY breaking parameters as per the MSSM beta functions. This implies that once the A–terms are induced, they will generate nonzero gaugino masses through two–loop MSSM interactions. These are estimated from the two–loop MSSM beta functions to be*

$$M^i_{\tilde{g}}(m_0) \simeq -\frac{g_i^2}{(16\pi^2)^2}\left(C^b_i Y_b^2 + C^\tau_i Y_\tau^2\right)\frac{m_0}{\sqrt{nN_f/N_c - 1}}\ln\left(\Lambda^2/m_0^2\right), \tag{4.46}$$

where $C^b_i = (14/5, 6, 4)$ and $C^\tau_i = (18/5, 6, 0)$ for $i = 1-3$. Y_b and Y_τ are the MSSM Yukawa couplings of the b–quark and the τ–lepton. From the requirement that $M^i_{\tilde{g}} \sim 1$ TeV we can estimate Λ and m_0, which will enable us to obtain the full spectrum of the model. Assuming that $m \sim \Lambda$, for the Bino mass we obtain (for $p = 2$, or $\tan\beta \sim 5$):

$$M_{\tilde{B}}(m_0) \sim -10^{-5} m_0. \tag{4.47}$$

The mass of the Wino is somewhat larger than this, and that of the gluino is somewhat smaller (compare the coefficients C^b_i and C^τ_i), all at the scale m_0. There is significant running of these masses below m_0 down to the TeV scale. This running is the largest for the gluino [53] which increases its mass, while it is the smallest for the Bino, which decreases its mass. Consequently, at the TeV scale, we have the normal mass hierarchy $M_{Bino} \leq M_{Wino} \leq M_{gluino}$.

In addition to the SM gauge interactions, the gauginos receive masses from the anomaly mediated contributions [63]. These contributions may be suppressed in specific setups such as in 5 dimensional supergravity [47]. We will allow for both a

*The one–loop finite corrections arising from diagrams involving the top–quark and the stop–squark are negligible since $A_t = 0$ and $\mu \sim$ TeV $\ll m_{\tilde{t}}$.

suppressed and an unsuppressed anomaly mediated contributions to gaugino masses. These contributions are given by

$$M_{gaugino} = \frac{\beta(g)}{g} F_\phi \tag{4.48}$$

where F_ϕ is the F–component of the compensator superfield. With our setup as described in the previous section, F_ϕ is equal to the gravitino mass, so the Wino mass, for eg., will be about 3×10^{-3} of the gravitino mass, or about $10^{-3} m_0$. If we set the Wino mass at 1 TeV, m_0 will be of order 10^6 GeV in such a scenario.

As we stated in the previous section, only a limited choice of n and N_f are allowed from the mixed anomaly cancelation conditions. We have considered four cases with $nN_f = 25,\ 24,\ 21$, or 18. Our results for the spectrum are listed in Table 2. In each case we studied different values of $N_c > N_f$. $N_c = 6,\ 7$ give the correct dynamical scale Λ which leads to TeV scale gauginos. The scalar masses are found to be of order 10^6 GeV in the case of unsuppressed anomaly mediated contribution (cases 1 and 3), and of order 10^8 GeV for the suppressed case (all the other cases). Clearly this is a Split Supersymmetry spectrum. In the computation of Table 2 we assumed $g_{N_c}^2/(4\pi) = 1/28$ at $M_{Pl} = 2.4 \times 10^{18}$ GeV. The mass m for the meson fields is computed in terms of an effective coupling $\hat{\lambda} \equiv \lambda \left(\frac{M_{Pl}}{M_*}\right)^{n-1}$. We expect $\hat{\lambda}$ to be of order one from naturalness, if M_* is the same as M_{Pl}. We list the mass m in terms of $\hat{\lambda}$ in the third column in Table 2. Note that the scalar masses from anomalous $U(1)$ D–term are proportional to the $U(1)$ charges, and therefore vanish for $H_u,\ H_d$ and 10_3 fields. These fields will however acquire masses from supergravity corrections.

The $U(1)_A$ symmetry does not forbid a bare μ term in the superpotential. However, it can be banished by a discrete Z_4 R–symmetry [64]. Under this Z_4, all the SM fermion superfields (scalar components) have charge +1, the gauginos have charge +1, the Z field has charge +2 and the SM Higsses and the S fields have charge zero. This symmetry has no anomaly, as a consequence of discrete Green–Schwarz anomaly cancelation. The $SM^2 \times Z_4$ and $SU(N_c)^2 \times Z_4$ anomaly coefficients are $A_3 = 3,\ A_2 = 2 - 1 = 1$ and $A_{N_c} = N_c$. The GS condition for discrete Z_4 anomaly cancelation is that the differences $A_i - A_j$ should be an integral multiple of 2, which is automatic when N_c is odd.

$(p,\, \alpha,\, n,\, N_f,\, N_c)$	$\Lambda\,(\mathrm{GeV})$	$m\,(\mathrm{GeV})\,/\hat{\lambda}$	m_0	$M_{\tilde{B}}\,(m_0)$ (GeV)	$\mu\,(\mathrm{GeV})\,/\lambda_\mu$
(2, 0, 5, 5, 6)	3×10^{12}	8×10^{14}	$6 \times 10^5 \hat{\lambda}^{5/6}$	$5\hat{\lambda}^{5/6}$	$600/\hat{\lambda}^{1/6}$
(2, 0, 5, 5, 7)	4×10^{13}	8×10^{14}	$9 \times 10^7 \hat{\lambda}^{5/7}$	$600\hat{\lambda}^{5/7}$	$9 \times 10^4/\hat{\lambda}^{2/7}$
(2, 0, 6, 4, 6)	8×10^{12}	1×10^{14}	$7 \times 10^5 \hat{\lambda}^{2/3}$	$5\hat{\lambda}^{2/3}$	$700/\hat{\lambda}^{1/3}$
(2, 0, 6, 4, 7)	8×10^{12}	1×10^{14}	$1 \times 10^8 \hat{\lambda}^{4/7}$	$640\hat{\lambda}^{4/7}$	$10^5/\hat{\lambda}^{3/7}$
(1, 0, 7, 3, 6)	2×10^{13}	3×10^{13}	$1 \times 10^6 \hat{\lambda}^{1/2}$	$100\hat{\lambda}^{1/2}$	$1600/\hat{\lambda}^{1/2}$
(1, 0, 7, 3, 7)	1×10^{14}	3×10^{13}	$2 \times 10^8 \hat{\lambda}^{3/7}$	$10^4\hat{\lambda}^{3/7}$	$2 \times 10^5/\hat{\lambda}^{4/7}$
(1, 1, 6, 3, 6)	2×10^{13}	1×10^{14}	$2 \times 10^6 \hat{\lambda}^{1/2}$	$200\hat{\lambda}^{1/2}$	$3000/\hat{\lambda}^{1/2}$

TABLE 4.2. The spectrum of the model for different choices of $p,\ \alpha,\ n,\ N_f$ and N_c. In computing Λ, we use Eq. (10) with $\alpha_{N_c} = 1/28$ at the Planck scale. The Bino mass estimate is very rough, and includes only the two–loop MSSM induced contributions.

One can write the following effective Lagrangian for the μ term that is consistent with the Z_4 R symmetry:

$$\mathcal{L} \supset \int d^2\theta H_u H_d \frac{\mathrm{Tr}(\lambda_\mu Z) S^n}{M_{Pl}^{n+1}} = \lambda_\mu N_f \frac{\langle Z S^n \rangle}{M_{Pl}^{n+1}} H_u H_d. \tag{4.49}$$

This leads to

$$\mu = \lambda_\mu \epsilon^n N_f \frac{\Lambda^3}{m M_{Pl}} \left(\frac{m}{\Lambda}\right)^{N_f/N_c}. \tag{4.50}$$

The numerical results for μ–term are given in the last column of Table 4.2 using this relation.

The SUSY breaking bilinear Higgs coupling, the $B\mu$ term, arises from the Lagrangian

$$\begin{aligned} \mathcal{L} \ &\supset \ \int d^4\theta H_u H_d \frac{\lambda_{B1}|S|^2 + \lambda_{B2} \mathrm{Tr}\left(Z^\dagger Z\right)^{1/2}}{M_{Pl}^2} \\ &= \ \frac{\lambda_{B1}|F_S^0|^2 + \lambda_{B2} N_f |F_Z|^2/|Z_0|}{M_{Pl}^2} H_u H_d, \end{aligned} \tag{4.51}$$

leading to

$$B\mu \ = \ m_0^2 \left(\frac{N_c}{n N_f - N_c}\right)^2 \left(\lambda_{B1}\epsilon^2 + \lambda_{B2} n^2 N_f \frac{\Lambda^3}{m M_{Pl}^2} \left(\frac{m}{\Lambda}\right)^{N_f/N_c}\right). \tag{4.52}$$

The second term in Eq. (4.52) is small compared to the first. From this we see that the 2×2 Higgs boson mass matrix has its off–diagonal entry of the same order as its diagonal entries. Recall that the diagonal entries are of order $m_{3/2}^2$, since the $U(1)_A$ charges of H_u and H_d are zero. Fine–tuning can then be done consistently so that one of the Higgs doublets remain light, with mass of order 10^2 GeV.

Even when the Z_4 R symmetry is not respected by gravitational corrections, the induced μ term and gaugino masses are of order TeV. There can be a new contribution to the μ term in this case, arising from

$$\mathcal{L} \supset \int d^4\theta H_u H_d \frac{(ZS^n)^*}{M_{Pl}^{n+2}}. \tag{4.53}$$

This μ term is however smaller than that from Eq. (4.50). Similarly, gaugino masses can arise from

$$\mathcal{L} \supset \int d^4\theta W_\alpha W^\alpha \frac{ZS^n}{M_{Pl}^{n+2}} \tag{4.54}$$

which is also smaller than the SM induced corrections.

For the scalars neutral under $U(1)_A$ ($H_u,\ H_d$ and 10_3), the D–term contribution to the soft masses vanish. We should take account of the subleading supergravity corrections then. Since these corrections are suppressed by a factor of ϵ^2 in the mass–squared, we should worry about potentially large negative corrections proportional to the other soft masses arising from SM interactions through the RGE in the momentum range $m_0 \leq \mu \leq \Lambda$. We have examined this in detail and found consistency of the models.

For the masses of zero charge fields we write

$$m_{\tilde{\phi}_i}^2 = c_i m_{3/2}^2 + \delta\left(m_{\tilde{\phi}_i}^2\right) \tag{4.55}$$

with $\delta\left(m_{\tilde{\phi}_i}^2\right)$ denoting the MSSM RGE corrections. The most prominent one–loop radiative corrections are

$$\begin{aligned}
\delta\left(m_{\tilde{f}_3}^2\right)^{1-loop} &\simeq -\left(Y_b\right)^2 \frac{m_0^2}{16\pi^2}\frac{p}{(nN_f/N_c-1)}\frac{nN_f}{N_c}\ln\left(\frac{\Lambda^2}{m_{\tilde{f}}^2}\right), \\
\delta\left(m_{H_d}^2\right)^{1-loop} &\simeq -\left\{3(Y_b)^2+(Y_\tau)^2\right\}\frac{m_0^2}{16\pi^2}\frac{p}{(nN_f/N_c-1)}\frac{nN_f}{N_c}\ln\left(\frac{\Lambda^2}{m_{\tilde{f}}^2}\right)
\end{aligned} \tag{4.56}$$

where $\tilde{f}_3 = (\tilde{Q}_3,\ \tilde{e_3^c})$. Similar corrections for H_u and $\tilde{u_3^c}$ scalar components are small. Since $p = 2$, we have low $\tan\beta \sim 5$, so these corrections are not large, although not negligible. For example, for the down–type Higgs bosons we have

$$\delta\left(m_{H_d}^2\right)^{1-loop} \sim -2\times 10^{-3} m_0^2. \tag{4.57}$$

If the supergravity corrections to the mass–squared of H_d is larger than $3 \times 10^{-2} m_0$, it will remain positive down to the scale m_0.

There is an important two–loop correction to the scalar masses arising from the gauge sector:

$$\delta \left(m^2_{\tilde{\phi}} \right)^{2-loop} \simeq -g^4 \frac{m_0^2}{(16\pi^2)^2} (nN_f) K_\phi \ln \left(\frac{\Lambda^2}{m^2_{\tilde{f}}} \right) \tag{4.58}$$

where $K_\phi = (63/15,\ 16/5,\ 6/5 \text{ and } 9/5)$ for $\tilde{\phi} = (\tilde{Q},\ \tilde{u^c},\ \tilde{e^c} \text{ and } H_u)$. This correction is estimated to be

$$\left(m^2_{\tilde{f}} \right)^{2-loop} \sim -10^{-2} m_0^2. \tag{4.59}$$

We see that these corrections are, although close to the gravitino contribution, at a safe level. We conclude that Split Supersymmetry is realized consistently in these models.

4.4 Conclusions

In this chapter I presented our work where we proposed concrete models for supersymmetry breaking making use of the anomalous $U(1)$ D–term of string origin. The anomalous $U(1)$ sector is coupled to the strong dynamics of an $N = 1$ SUSY gauge theory where exact results are known. The complete models we have presented also address the mass and mixing hierarchy of quarks and leptons. We have generalized the analysis of Binetruy and Dudas[55] to include supergravity corrections, which turns out to be important for certain fields in these models which carry zero $U(1)$ charge. Table 4.2 summarizes our results on the spectrum of these models. This spectrum is that of Split Supersymmetry. The gaugino and the Higgsino masses are of the same order, when these are set at the TeV scale, the squarks and sleptons have masses in the range $(10^6 - 10^8)$ GeV. This provides an explicit realization of part of the parameter space of Split supersymmetry [47].

The experimental and cosmological implications of Split Supersymmetry have been widely studied [52–54,57]. We conclude by summarizing the salient features that apply to our framework. (i) Gauge coupling unification works well, in fact somewhat better than in the MSSM. When embedded into $SU(5)$ symmetry, proton decay via dimension six operators will result, with an estimated lifetime for $p \to e^+\pi^0$ of order $(10^{35} - 10^{36})$ yrs. There is no observable $d = 5$ proton decay in these models. (ii) The lightest neutralino, which is charge and color neutral, is a natural and consistent dark matter candidate. (iii) The gluino lifetime is estimated to be of order 10^{-7} seconds or shorter for squark masses in the range $10^6 \div 10^8$ GeV in these models. There is no cosmological difficulty with such a mass. (iv) The gravitino mass is or order 10^7 GeV, thus there is no cosmological gravitino abundance problem. (v) The low energy theory is the SM plus the neutralinos and the charginos of supersymmetry. All other particles acquire masses either near the Planck scale or through strong dynamics at a scale $\Lambda \sim 10^{14}$ GeV.

CHAPTER 5

FINITE GRAND UNIFIED THEORY AND QUARKS MASS MATRICES

5.1 **Finite Grand Unified Theory**

The evolutions of the gauge couplings of the standard model, given by their β–functions, show that they meet at a single point at a very high energy around 10^{16} GeV. This fact seems to suggest that they might have a common origin from a larger gauge group structure which contains all three SM gauge groups. Such an idea of unification of the three interactions based on a simple group $SU(5)$ was formulated first by H. Georgi and S.L. Glashow in 1974 [65].

While the number of effective parameters in the grand unified theories with higher symmetries might be smaller than the SM, because of the necessity to break the higher symmetry, the actual number of parameters are often larger than the SM. Thus it becomes natural to ask whether it is possible to have a realistic theory in which there is a smaller number of parameters.

Indeed, there exists a certain class of supersymmetric Yang–Mills theories, where one may achieve this goal. These are the so–called finite theories wherein the β functions for the gauge coupling and the Yukawa couplings vanish to all orders in perturbation theory. Certain conditions must be satisfied for a SUSY Yang–Mills theory to be finite. One of them is the vanishing of the one–loop gauge β function. This requirement constrains the spectrum of the theory essentially fixing it (upto discrete possibilities), once the gauge group is specified. A second requirement for finiteness is the vanishing of all the anomalous mass dimensions of the chiral superfields at one–loop. This would fix all the Yukawa couplings in terms of the gauge coupling, at one–loop order. This type of one–loop finiteness also implies that the theory is finite to two loops [66]. For the theory to be finite to all loops, the Yukawa couplings must have unique power series expansions in terms of the gauge coupling

[67]. With this condition satisfied, the theory would have only one coupling – the gauge coupling. The Yukawa couplings are unified with the gauge couplings. This "reduction of couplings" is one of the key ingredients of finiteness [68–71]. Certainly this makes the idea of finiteness an attractive direction to pursue in reducing the number of free parameters. One might hope that these type of theories may arise from superstring theory. Vanishing of the β functions lead to conformal invariance, which is one of the cornerstones of string theory. Indeed there have been several attempts to derive a grand unified theory from superstring theory as its low energy 4-D limit [21,72]. The approach we adopt in this thesis toward finiteness is that of Lucchesi et. al. [67]. Different approaches to finite theories have been discussed, for example, by Leigh and Strassler[73], and Ermushev et. al [74].

It will be extremely interesting to uncover finite theories that are phenomenologically viable, at least in a broad sense. Attempts have been made along this line with some success. An immediate question any finite theory should address is the consistency with the observed masses and mixings of the quarks. The Yukawa couplings are not arbitrary parameters in finite theories due to the reduction of couplings. The mass of the top–quark has been predicted within finite theories, and shown to be in good agreement with experiments [75]. The masses of the lighter generation quarks have also been consistently accommodated in this context. However, the mixing between all three generations has not been implemented successfully thus far. This is the major point we have addressed which I present in this part of the thesis.

The model based on finite SUSY $SU(5)$ theory can induce the correct pattern of quark mixing and masses. Additional flavor symmetries are necessary to meet the criterion for finiteness that the power series expansion of the Yukawa couplings in terms of the gauge coupling be unique. We find that non–Abelian discrete symmetries are extremely useful here. Abelian symmetries that we have tried were not sufficient to make the expansion coefficients of the Yukawa couplings unique, non–Abelian continuous symmetries such as $SU(2)$ and $SU(3)$ are too restrictive to allow the needed Yukawa couplings. The models that we present are based on discrete family symmetries $(Z_4)^3 \times P$, the tetrahedral symmetry A_4 and S_4 symmetry.

In Section 5.1.1, I review briefly the conditions for finiteness, starting from the RGEs for a generic supersymmetric theory. From one of the criteria, namely vanishing of the one loop gauge β function, it is not hard to see that finite models with phenomenologically favorable particle spectrum can be found more easily in $SU(5)$ than in other groups [76]. Some general results of practical interest are given for finite $SU(5)$ models. In Section 5.2 we propose three different finite models and analyze them in detail. We show that realistic quark masses and mixing angles can be generated. This enables us to address more detailed questions such as the decay rate of the proton, which is perhaps one of the thorniest problems faced by SUSY GUTs. Generically finite theories are problematic [77], we give some plausible resolutions.

5.1.1 Finite Theories: A brief review

The one loop gauge and Yukawa beta functions and the one loop anomalous dimension of the matter fields in a generic SUSY Yang–Mills theory are given by [66]:

$$\beta_g^{(1)} = \frac{g^3}{16\pi^2}\left(\sum_R T(R) - 3C_2(G)\right) \tag{5.1}$$

$$\gamma_j^{(1)i} = \lambda^{ikl}\lambda_{jkl} - 2C_2(R)g^2\delta_j^i \tag{5.2}$$

$$\beta_{ijk}^{(1)} = \frac{1}{16\pi^2}[\lambda_{ijp}\gamma_k^p + (k \leftrightarrow i) + (k \leftrightarrow j)] \tag{5.3}$$

where $T(R)$, $C_2(R)$ and $C_2(G)$ are the Dynkin indices of the matter fields and the quadratic Casimirs of the matter and gauge representations respectively. λ^{ijk} and $\beta_{ijk}^{(1)}$ are the Yukawa couplings and the one–loop Yukawa β function of λ^{ijk} defined by the following superpotential:

$$W = \frac{\lambda_{ijk}}{3!}\phi_i\phi_j\phi_k \tag{5.4}$$

The criteria of all loop finiteness for $N = 1$ supersymmetric gauge theories can be stated as follows [67]: (i) The theory should be free from gauge anomaly, (ii) the gauge β-function vanishes at one loop:

$$\beta_g^{(1)} = 0, \tag{5.5}$$

(iii) there exists solution of the form $\lambda = \lambda(g)$ to the conditions of vanishing one-loop anomalous dimensions

$$\gamma_j^{(1)i} = 0, \tag{5.6}$$

and (iv) the solution is isolated and non-degenerate when considered as a solution of vanishing one-loop Yukawa β-function:

$$\beta_{ijk}^{(1)} = 0. \tag{5.7}$$

If all four conditions are satisfied, the dimensionless parameters of the theory would depend on a single gauge coupling constant and the β functions will vanish to all orders.

The first step is to choose the gauge group. From (i), we see that the vanishing of the one loop gauge β function puts a strong constraint on the particle content, leaving only discrete set of possibilities. It seems hard to find phenomenologically viable models other than in $SU(5)$ [76]. For example, if one chooses $SO(10)$ to be the gauge group, and tries to build a finite model with necessary particle content in the traditional sense, one quickly "runs" out of the Dynkin indices: according to Eq. (5.1) the sum of the Dynkin indices over the matter fields should be equal to 24 in this case. On the other hand, field content of the traditional $SO(10)$ GUT is $3 \times \mathbf{16}$ of fermions, $\mathbf{54}$, $\mathbf{45}$, $\mathbf{10} + \mathbf{10}'$ and $\mathbf{16} + \overline{\mathbf{16}}$ of Higgs, if the gauge symmetry is to be broken by renormalizable terms in the superpotential. Then, since the sum of the Dynkin indices of these fields is equal to 32, one ends up exceeding the gauge Dynkin index. Much the same result can be reached for $SU(6)$ etc. While it will be of great interest to uncover finite models other than $SU(5)$, here we will confine ourselves to the case of finite $SU(5)$.

Beginning with the particle content of minimal SUSY $SU(5)$ theory with three families of fermions belonging to $3 \times (\mathbf{10} + \bar{\mathbf{5}})$, an adjoint $\mathbf{24}$ Higgs (Σ) and ($\mathbf{5} + \bar{\mathbf{5}}$) Higgses one sees that vanishing of the one–loop gauge β function requires the introduction of additional fields whose Dynkin indices add up to 3. This happens if there are three additional $\mathbf{5} + \overline{\mathbf{5}}$ matter fields, which may be either Higgs–like bosonic fields or vector–like fermionic fields. This is in fact the most promising case from phenomenology. There are two other possibilities, viz., adding $\mathbf{10} + \overline{\mathbf{10}}$ or adding $\mathbf{10} + \mathbf{5} + 2 \times \overline{\mathbf{5}}$. In the first case, realistic quark masses cannot arise, in the second case one would be left with a fourth family of fermions which remains light to the weak scale. For phenomenological reasons we do not pursue these two alternatives, and choose to work with $3 \times \{\mathbf{5} + \overline{\mathbf{5}}\}$ plus the minimal SUSY $SU(5)$ spectrum.

The finiteness criteria require that Eqs. (5.5)-(5.7) should give a unique set of solutions to the Yukawa couplings. The equations are linear in the square of the absolute values of the couplings. Hence one expects the solutions to the Yukawa couplings to be either zero or of order the gauge coupling. They are not free parameters anymore. In order to satisfy the hierarchy in masses of the fermions, one can choose the VEVs of the Higgs bosons to be hierarchical. Naively this would need at least three Higgs multiplets coupling to the up–quark sector, and three multiplets coupling to the down–quark sector. It is interesting that finite $SU(5)$ spectrum admits the needed Higgs, which can be as many as 4 in each sector. We will be interested in the case where at least three of the $\mathbf{5}+\overline{\mathbf{5}}$ fields are Higgs–like (viz., they develop VEVs of the order the electroweak scale). In fact, we shall see shortly that independent of this phenomenoligical requirement, the vanishing of the one–loop anomalous dimensions necessitates that at least three pairs of $\mathbf{5}+\overline{\mathbf{5}}$ have Yukawa couplings to the three families of fermions. We will focus on inducing realistic mixing among the three families of quarks, which has not been achieved in earlier analyses [75].

To search for a finite model, one has to write down a specific superpotential and try to find a set of solutions satisfying the criteria that all the Yukawa coupling wave function renormalization factors vanish at one–loop. We consider the following superpotential (assuming an unbroken R–parity):

$$
\begin{aligned}
W &= \sum_{i,j=1}^{3}\sum_{a}\left(\frac{1}{2}u^a_{ij}\mathbf{10}_i\mathbf{10}_jH_a + d^a_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{H}_a\right) \\
&+ \sum_{ab} k^{ab}H_a\Sigma\bar{H}_b + \frac{\lambda}{3}\Sigma^3 + f\,\mathbf{5}\,\Sigma\,\bar{\mathbf{5}}. \qquad (5.8)
\end{aligned}
$$

Here $i,j = (1-3)$ are family indices and a and b are Higgs indices. a and b run from 1 to either 3 or 4. If it is up to 4, the last term is absent. H and $\bar{H}$ denote the $\mathbf{5}+\overline{\mathbf{5}}$ fields and Σ the adjoint chiral matter field responsible for the GUT symmetry breaking. Note that in order to have a successful doublet–triplet mass splitting, at least one of the couplings f, k^{ab} should be non–vanishing.

From Eq. (5.8), the anomalous dimensions of Eq. (5.1) can be written in matrix form as:

$$
\gamma_{10_i10_j} = 3(u_au_a^\dagger)_{ij} + 2(d_ad_a^\dagger)_{ij} - \frac{36}{5}g^2\delta_{ij}
$$

$$\begin{aligned}
\gamma_{\bar{5}_i\bar{5}_j} &= 4(d_a^\dagger d_a)_{ij} - \frac{24}{5}g^2\delta_{ij} \\
\gamma_{H_aH_b} &= 3Tr(u_a^\dagger u_b) + \frac{24}{5}(kk^\dagger)_{ab} - \frac{24}{5}g^2\delta_{ab} \\
\gamma_{\bar{H}_a\bar{H}_b} &= 4Tr(d_a d_b^\dagger) + \frac{24}{5}(k^\dagger k)_{ab} - \frac{24}{5}g^2\delta_{ab} \\
\gamma_5 &= \gamma_{\bar{5}} = \frac{24}{5}f^2 - \frac{24}{5}g^2 \\
\gamma_{24} &= Tr(k^\dagger k) + f^2 + \frac{21}{5}\lambda^2 - 10g^2.
\end{aligned} \tag{5.9}$$

According to the third criteria, Eq. (5.6), in order to have finite theory, all these anomalous mass dimension have to be zero. Thus, the problem of finding a finite model shifts to the problem of finding a set of solutions, where all the anomalous dimensions in Eq. (5.9) vanish. Let us introduce a new notations for the matrices:

$$\begin{aligned}
U \equiv u_a u_a^\dagger, \quad D \equiv d_a d_a^\dagger, \qquad & D' \equiv d_a^\dagger d_a, \qquad \tilde{U}_{ab} \equiv Tr(u_a^\dagger u_b), \\
\tilde{D}_{ab} \equiv Tr(d_a d_b^\dagger), \qquad & K \equiv k^\dagger k, \qquad \tilde{K} \equiv kk^\dagger,
\end{aligned} \tag{5.10}$$

where the trace is over the generation indices. From Eq. (5.10), it follows that the number of H fields coupling to $\mathbf{10}_i\mathbf{10}_j$ should be equal to the number of $\overline{H}$ fields coupling to $\mathbf{10}_i\overline{\mathbf{5}}_j$ fields. Furthermore, at least 3 H fields (and 3 $\overline{H}$ fields) must have such couplings. To see this let us take the trace of the matrices of the anomalous dimensions in Eq. (5.9) over both the fermionic indices and the Higgs indices. One gets:

$$\begin{aligned}
3Tr(U) &+ 2Tr(D) = 3 \times \frac{36}{5}g^2 \\
4Tr(D') &= 3 \times \frac{24}{5}g^2 \\
3Tr(\tilde{U}) &+ \frac{24}{5}Tr(\tilde{K}) = n_H \times \frac{24}{5}g^2 \\
4Tr(\tilde{D}) &+ \frac{24}{5}Tr(K) = n_{\bar{H}} \times \frac{24}{5}g^2,
\end{aligned} \tag{5.11}$$

where n_H and $n_{\bar{H}}$ are the number of the Higgs fields coupling to the three family of fermions in the up–sector and the down–sector respectively. Subtracting the third equation from the last in Eq. (5.11), we get

$$4Tr(\tilde{D}) - 3Tr(\tilde{U}) = (n_{\bar{H}} - n_H) \times \frac{24}{5}g^2.$$

Observing the following relation

$$Tr(U) = Tr(\tilde{U}), \qquad Tr(D) = Tr(D') = Tr(\tilde{D}),$$

one finds that

$$n_H = n_{\bar{H}}. \tag{5.12}$$

One can also see that the matrices K and $\tilde{K}$ in Eq. (5.10) vanish if $n_H = n_{\bar{H}} = 3$. That is, $k_{ab} = 0$ for all (a, b). Doublet–triplet splitting can be achieved in this case since $f \neq 0$ is allowed. If $n_H = n_{\bar{H}} \leq 2$, no solution exists for Eq. (5.11). We conclude that at least three Higgs multiplets must couple to the fermion fields in finite $SU(5)$.

Vanishing of the right–hand side of Eq. (5.10), needed for finiteness, will in general lead to parametric solutions. In order to satisfy the condition for all–loop finiteness, additional symmetries are usually necessary. Under these extra symmetries different Higgs multiplets will have different charges, which would prevent them from coupling to the same set of fermion fields. If two different Higgs multiplets H_1 and H_2 coupled to the same fermion fields, say $\mathbf{10}_1\mathbf{10}_2$, then $\gamma_{H_1 H_2}$ will not vanish in general, and so the theory will not be finite. We now present a classification of the Yukawa coupling matrices which ensures in a simple way that the off–diagonal entries of the anomalous dimension matrices are all automatically zero. While this classification is not the most general, it can be applied to a wide class of models. Let us write the superpotential Eq. (5.8) in the following form:

$$W = \frac{1}{2}\mathbf{10_i}\mathbf{10_j} V^u_{ij} + \mathbf{10_i}\overline{\mathbf{5}}_j V^d_{ij} + \ldots, \tag{5.13}$$

where

$$V^u{}_{ij} = u^a_{ij} H_a, \quad V^d{}_{ij} = d^a_{ij} \bar{H}_a \ . \tag{5.14}$$

The structures of V^u matrices which automatically have all off–diagonal anomalous dimensions to be zero is obtained as follows. Consider the case where three pairs of $(H, \bar{H})$ couple to the chiral families. There are four distinct forms for the matrix V^u:

$$V^{(1)} \equiv \begin{pmatrix} u_{11}H_1 & u_{12}H_3 & u_{13}H_2 \\ u_{12}H_3 & u_{22}H_2 & u_{23}H_1 \\ u_{13}H_2 & u_{23}H_1 & u_{33}H_3 \end{pmatrix} \qquad V^{(2)} \equiv \begin{pmatrix} u_{11}H_2 & u_{12}H_1 & 0 \\ u_{12}H_1 & u_{22}H_3 & u_{23}H_2 \\ 0 & u_{23}H_2 & u_{33}H_3 \end{pmatrix}$$

$$V^{(3)} \equiv \begin{pmatrix} u_{11}H_3 & u_{12}H_1 & 0 \\ u_{12}H_1 & u_{22}H_3 & u_{23}H_2 \\ 0 & u_{23}H_2 & u_{33}H_3 \end{pmatrix} \qquad V^{(4)} \equiv \begin{pmatrix} u_{11}H_1 & 0 & 0 \\ 0 & u_{22}H_3 & u_{23}H_2 \\ 0 & u_{23}H_2 & u_{33}H_3 \end{pmatrix} \ . \tag{5.15}$$

The form of V^d in this case is identical to Eq. (5.15), except that u_{ij} is replaced by d_{ij} and H_i by $\bar{H}_i$. While V^u is a symmetric matrix, V^d is asymmetric. Any given Higgs field appears at most once in a given row or column in all the matrices of Eq. (5.15). This guarantees that all off–diagonal γ function entries are zero. It can be shown that Eq. (5.15) is the most general set of matrices that satisfy this constraint (upto relabeling of generation number and Higgs number), provided that there is no cancellation between various terms to generate a zero in the off–diagonal γ matrix. It is possible that such cancellations occur in the presence of non–Abelian flavor symmetries, but not with Abelian symmetries. Even for the case of non–Abelian symmetries, we have found the classification of Eq. (5.15) very useful.

If four pairs of $(H + \bar{H})$ couple to fermion families, the matrix V^u can have the following four structures (upto relabeling of generation and Higgs indices):

$$V^{(1)} \equiv \begin{pmatrix} u_{11}H_1 & u_{12}H_4 & u_{13}H_2 \\ u_{12}H_4 & u_{22}H_2 & u_{23}H_1 \\ u_{13}H_2 & u_{23}H_1 & u_{33}H_3 \end{pmatrix} \qquad V^{(2)} \equiv \begin{pmatrix} u_{11}H_2 & u_{12}H_1 & 0 \\ u_{12}H_1 & u_{22}H_2 & u_{23}H_4 \\ 0 & u_{23}H_4 & u_{33}H_3 \end{pmatrix}$$

$$V^{(3)} \equiv \begin{pmatrix} u_{11}H_3 & u_{12}H_1 & u_{13}H_2 \\ u_{12}H_1 & u_{22}H_2 & u_{23}H_4 \\ u_{13}H_2 & u_{23}H_4 & u_{33}H_3 \end{pmatrix} \qquad V^{(4)} \equiv \begin{pmatrix} u_{11}H_3 & u_{12}H_1 & u_{13}H_2 \\ u_{12}H_1 & u_{22}H_3 & u_{23}H_4 \\ u_{13}H_2 & u_{23}H_4 & u_{33}H_3 \end{pmatrix} . \quad (5.16)$$

V^d in this case will have similar structure, assuming that its form is similar to V^u. In all cases, one can easily verify that the off–diagonal contributions to the anomalous dimension matrices are all zero.

5.2 **The Quark Mixing in Finite GUT**

It is possible to find solutions for the vanishing of the anomalous dimensions of Eq. (5.9) with the forms of V^u and V^d given as in Eq. (5.15)-(5.16). We have examined all possible cases, including V^u taking the form of $V^{(i)}$ while V^d taking the form of $V^{(j)}$ with i and j not necessarily the same. We found parametric solutions wherein one or (typically) more parameters are not determined. That would forbid a unique expansion of the Yukawa couplings in terms of the gauge coupling, one of the requirements of finiteness. It is possible to remove this arbitrariness by imposing additional flavor symmetries. An example of this type are proposed here and analyzed in detail. In the first example, isolated non–degenerate solution to the Yukawa couplings is obtained by imposing the tetrahedral group A_4.

5.2.1 $(Z_4)^3 \times P$ model

Let us give the transformation properties of the fields under the discrete symmetry we impose. The symmetries are $(Z_4)^3$, identified as the Z_4 subgroup of generation number, and a permutation symmetry acting on both the fermion and the Higgs generations. The fields transform under $(Z_4)^3$ as:

$$\begin{array}{lll} \mathbf{10}_1 : (i,1,1), & \mathbf{10}_2 : (1,i,1), & \mathbf{10}_3 : (1,1,i), \\ \bar{\mathbf{5}}_1 : (i,1,1), & \bar{\mathbf{5}}_2 : (1,i,1), & \bar{\mathbf{5}}_3 : (1,1,i), \\ (H_1,\bar{H}_1) : (-1,1,1), & (H_2\bar{H}_2) : (1,-i,-i), & \\ (H_3,\bar{H}_3) : (1,1,-1), & (H_4,\bar{H}_4) : (1,-i,-i). & \end{array} \tag{5.17}$$

The action of the permutation symmetry P on the fields is as follows:

$$\begin{array}{llll} \mathbf{10}_1 \leftrightarrow \mathbf{10}_3, & \bar{\mathbf{5}}_1 \leftrightarrow \bar{\mathbf{5}}_3, & H_1 \leftrightarrow H_3, & \bar{H}_1 \leftrightarrow \bar{H}_3, \\ \mathbf{10}_2 \leftrightarrow \mathbf{10}_2, & \bar{\mathbf{5}}_2 \leftrightarrow \bar{\mathbf{5}}_2, & H_2 \leftrightarrow H_4, & \bar{H}_2 \leftrightarrow \bar{H}_4. \end{array}$$

The most general $SU(5) \times (Z_4)^3 \times P$ invariant superpotential is:

$$\begin{aligned} W &= a(\mathbf{10}_1\mathbf{10}_1H_1 + \mathbf{10}_3\mathbf{10}_3H_3) + b(\mathbf{10}_1\mathbf{10}_2H_4 + \mathbf{10}_2\mathbf{10}_3H_2) \\ &+ c(\mathbf{10}_1\bar{\mathbf{5}}_1\bar{H}_1 + \mathbf{10}_3\bar{\mathbf{5}}_3\bar{H}_3) + d(\mathbf{10}_1\bar{\mathbf{5}}_2\bar{H}_4 + \mathbf{10}_3\bar{\mathbf{5}}_2\bar{H}_2) \\ &+ e(\mathbf{10}_2\bar{\mathbf{5}}_1\bar{H}_4 + \mathbf{10}_2\bar{\mathbf{5}}_3\bar{H}_2) + k(H_1\bar{H}_1\Sigma + H_3\bar{H}_3\Sigma) + \frac{\lambda}{3}\Sigma^3. \end{aligned} \tag{5.18}$$

The matrices V^u and V^d (defined in Eq. (5.16)) for this model are then:

$$V^u = \begin{pmatrix} a\,H_1 & b\,H_4 & 0 \\ b\,H_4 & 0 & b\,H_2 \\ 0 & b\,H_2 & a\,H_3 \end{pmatrix} \qquad V^d = \begin{pmatrix} c\,\bar{H}_1 & d\,\bar{H}_4 & 0 \\ e\,\bar{H}_4 & 0 & e\,\bar{H}_2 \\ 0 & d\,\bar{H}_2 & c\,\bar{H}_3 \end{pmatrix}, \tag{5.19}$$

and the coupling matrix of the Higgs fields to the adjoint field is given by:

$$K = diag(k,0,k,0).$$

Note that all superpotential couplings can be made real by field redefinitions. One can then take all parameters of Eq. (5.18) to be real and positive. This is an important point for the solution to be non–degenerate.

The condition (iii) of the criteria for finiteness (vanishing of all the anomalous dimensions) leads to the following simple system of equations:

$$\begin{aligned} 3(a^2+b^2)+2(c^2+d^2) &= \frac{36}{5}g^2, & 3(2b^2)+2(2e^2) &= \frac{36}{5}g^2, \\ c^2+e^2 &= \frac{6}{5}g^2, & 2b^2 &= \frac{8}{5}g^2, \\ a^2+\frac{8}{5}k^2 &= \frac{8}{5}g^2, & d^2+e^2 &= \frac{6}{5}g^2, \\ 2d^2 &= \frac{6}{5}g^2, & a^2+\frac{6}{5}k^2 &= \frac{6}{5}g^2. \end{aligned} \tag{5.20}$$

This gives a unique solution which is isolated and non-degenerate:

$$\left(a^2, b^2, c^2, d^2, e^2, k^2, \lambda^2\right) = \left(\frac{4}{5}g^2, \frac{4}{5}g^2, \frac{3}{5}g^2, \frac{3}{5}g^2, \frac{3}{5}g^2, \frac{1}{2}g^2, \frac{15}{7}g^2\right). \tag{5.21}$$

There is no sign ambiguity for the Yukawa couplings themselves, since they have all been made real and positive.

Let us now turn to the question of comparing the predictions of Eq. (5.21) with experiments. First of all, all three families of quarks mix with one another, so realistic CKM mixings become possible, unlike earlier attempts within finite GUTs. Setting the overall factor $a\langle H_3\rangle = 1$, we can write the mass matrix M^u for the up–type quarks in the following form:

$$M^u = \begin{pmatrix} c^u_{11}\epsilon_u^4 & c^u_{12}\epsilon_u^3 & 0 \\ c^u_{21}\epsilon_u^3 & 0 & \epsilon_u \\ 0 & \epsilon_u & 1 \end{pmatrix}, \tag{5.22}$$

where $\epsilon_u \equiv \frac{\langle H_2\rangle}{\langle H_3\rangle}$, $c^u_{11}\epsilon_u^4 \equiv \frac{\langle H_1\rangle}{\langle H_3\rangle}$ and $c^u_{12}\epsilon_u^3 \equiv \frac{\langle H_4\rangle}{\langle H_3\rangle}$. The mass matrix for the down–type quarks, M^d, has a similar form, with ϵ_u replaced by ϵ_d and c^u_{ij} replaced by c^d_{ij}. These matrices are generalizations of the Fritszch form. Note that Eq. (5.22) is a special case of texture $V^{(2)}$ in Eq. (5.16). The mass eigenvalues are obtained in the approximation $\epsilon_u \ll 1$, $c^u_{ij} \sim 1$ to be:

$$\begin{aligned} m_u &\simeq \left(c^u_{11} + (c^u_{12})^2\right)\epsilon_u^4 \\ m_c &\simeq -\epsilon_u^2 + (1-c^u_{12})^2\,\epsilon_u^4 \\ m_t &\simeq 1+\epsilon_u^2-\epsilon_u^4 \end{aligned} \tag{5.23}$$

in units of $a \langle H_3 \rangle$. Similar expressions hold in the down–type quark sector. The CKM mixing elements are then given by:

$$\begin{aligned} V_{us} &= c^u_{12}\epsilon_u - c^d_{12}\epsilon_d + O(\epsilon^3) \\ V_{cb} &= \epsilon_d - \epsilon_u + O(\epsilon^3) \\ V_{ub} &= c^u_{12}\epsilon_u\epsilon_d - c^u_{12}\epsilon_u^2 + O(\epsilon^4), \end{aligned} \tag{5.24}$$

where ϵ_d and c^d_{12} correspond to the down quark sector. (For simplicity, we have assumed all parameters to be real. This assumption is not necessary, realistic CP violation can also arise from Eq. (5.22).)

Observe that the mass hierarchy between generations can be accommodated in this model by assuming a hierarchy in the VEVs of the Higgs doublets. We have in mind a scenario where only one pair of Higgs doublets survive below the GUT scale, to be identified as H_u and H_d of MSSM. These are linear combinations of all four of the original Higgs doublets. That would enable all H_i $(i = 1 - 4)$ to acquire VEVs. The H_u field of MSSM is dominantly H_3, but has small (of order ϵ_u) component of H_2 in it, and even smaller components of H_4 (of order ϵ_u^3) and H_1 (of order ϵ_u^4) in it. These amounts are dictated by the bilinear terms in the superpotential involving H_i and $\bar{H}_i$ fields ($W \sim m_{ij}H_i\bar{H}_j$). These bilinear terms are assumed to break the $(Z_4)^3 \times P$ symmetry softly. We see that the desired mass hierarchy is reproduced in this way.

Since the Yukawa couplings of the third generation quarks are fixed in this model in terms of the gauge coupling, the mass of the top quark and the parameter $\tan\beta$ are determined. Let us denote the MSSM Yukawa couplings of the top and the bottom quarks to H_u and H_d fields to be y_t and y_b respectively. To a good approximation, H_u is H_3 and H_d is $\bar{H}_3$. Thus we see from Eq. (5.21) that $y_t \simeq (\sqrt{4/5})g$ and $y_b \simeq (\sqrt{3/5})g$, both of which are fixed in terms of $\alpha_G \simeq 1/25$. We now extrapolate these Yukawa couplings to the weak scale using the MSSM renormalization group equations. The top quark mass and the parameter $\tan\beta$ can be predicted using the relations

$$\begin{aligned} m_t &= m_b\frac{y_t}{y_b}\sqrt{{y_b}^2\frac{v^2}{{m_b}^2} - 1} \simeq y_t v \\ \tan\beta &= \frac{m_t}{m_b}\frac{y_b}{y_t}, \end{aligned} \tag{5.25}$$

where $v = 174$ GeV. With $m_b(m_b)$ taken to be 4.4 GeV and with $\alpha_3(m_Z) = 0.118$ we find the numerical values to be:

$$\begin{aligned} m_t &= 174\ GeV \\ \tan\beta &= 53. \end{aligned} \tag{5.26}$$

The predicted value of m_t is nicely consistent with the experimentally determined value, $\tan\beta$ tends to be large in this class of models.

There is one other non–trivial prediction in this model, because of the zeros present in Eq. (5.22). We take it to be a prediction for the strange quark mass. From Eqs. (5.24) and (5.23) we find $m_s(1\text{ GeV}) \simeq 80$ MeV, if we take $V_{cb} \simeq 0.043, m_b(m_b) =$ 4.4 GeV, $m_c(m_c) = 1.37$ GeV, $V_{us} = 0.22$ and $V_{ub} = 0.004$. This value of m_s is on the low side, but may be consistent with recent lattice evaluations [78]. We also note that since $\tan\beta$ is predicted to be large, the finite threshold corrections to V_{cb} through chargino–stop exchange is significant [79]. This could modify V_{cb} by as much as 30%. With a 30% reduction in V_{cb} arising from this diagram, we predict $m_s(1\ GeV) \simeq 100$ MeV, which is quite acceptable.

5.2.2 A_4 finite model

Now we present a different model that leads to realistic quark mixings and masses. It is based on $SU(5) \times A_4$ symmetry. A_4 is the group of even permutations of four objects. It is the symmetry group of a regular tetrahedron. This group has irreducible representations (denoted by the dimensions) 1, $1'$, $1''$ and 3. The $1'$ and $1''$ are complex conjugate of each other. The product 3×3 decomposes as

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3. \tag{5.27}$$

If we denote the components of 3 as (a, b, c), the various terms are given by [80]:

$$\begin{aligned} 1 &= a_1a_2 + b_1b_2 + c_1c_2 \\ 1' &= a_1a_2 + \omega^2 b_1b_2 + \omega c_1c_2 \\ 1'' &= a_1a_2 + \omega b_1b_2 + \omega^2 c_1c_2, \end{aligned} \tag{5.28}$$

where $\omega = e^{2i\pi/3}$. (Note that $1 + \omega + \omega^2 = 0$.)

The transformations properties of the fields of $SU(5)$ under A_4 are:

$$
\begin{array}{rclcrclc}
\mathbf{10}_i & : & 3 & & \bar{\mathbf{5}}_i & : & 3 & \\
(H_a, H_4) & : & 3+1' & & (\bar{H}_a, \bar{H}_4) & : & 3+1' & \Sigma = 1,
\end{array}
\tag{5.29}
$$

where $i = 1 \div 3$ and $a = 1 \div 3$. Using the algebra presented in Eq. (5.28), the superpotential invariant under $SU(5) \times A_4$ symmetry is:

$$
\begin{aligned}
W &= a(\mathbf{10}_1\mathbf{10}_1 + \omega\mathbf{10}_2\mathbf{10}_2 + \omega^2\mathbf{10}_3\mathbf{10}_3)H_4 \\
&+ c(\mathbf{10}_1\bar{\mathbf{5}}_1 + \omega\mathbf{10}_2\bar{\mathbf{5}}_2 + \omega^2\bar{\mathbf{5}}_3\mathbf{10}_3)\bar{H}_4 \\
&+ b[(\mathbf{10}_1\mathbf{10}_2 + \mathbf{10}_2\mathbf{10}_1)H_1 + (\mathbf{10}_1\mathbf{10}_3 + \mathbf{10}_3\mathbf{10}_1)H_2 + (\mathbf{10}_2\mathbf{10}_3 + \mathbf{10}_3\mathbf{10}_2)H_3] \\
&+ d[(\mathbf{10}_1\bar{\mathbf{5}}_2 + \mathbf{10}_2\bar{\mathbf{5}}_1)\bar{H}_1 + d(\mathbf{10}_1\bar{\mathbf{5}}_3 + \mathbf{10}_3\bar{\mathbf{5}}_1)\bar{H}_2 + (\mathbf{10}_2\bar{\mathbf{5}}_3 + \mathbf{10}_3\bar{\mathbf{5}}_2)\bar{H}_3] \\
&+ k(\bar{H}_1H_1 + \bar{H}_2H_2 + \bar{H}_3H_3)\Sigma + \frac{\lambda}{3}\Sigma^3.
\end{aligned}
\tag{5.30}
$$

By field redefinition the ω factors can be removed from W. Actually, all the coupling constants in Eq. (5.30) can be made real and positive. The condition of vanishing anomalous dimensions for this model can be written as follows:

$$
\begin{aligned}
&3(a^2 + 2b^2) + 2(c^2 + 2d^2) = \frac{36}{5}g^2 \\
&4(c^2 + 2d^2) = \frac{24}{5}g^2 \\
&3(3a^2) = \frac{24}{5}g^2 \\
&3(2b^2) + \frac{24}{5}k^2 = \frac{24}{5}g^2 \\
&4(3c^2) = \frac{24}{5}g^2 \\
&4(2d^2) + \frac{24}{5}k^2 = \frac{24}{5}g^2.
\end{aligned}
\tag{5.31}
$$

This gives the following isolated and non-degenerate solution:

$$
a^2 = b^2 = \frac{8}{15}g^2, \;\; c^2 = d^2 = \frac{2}{5}g^2, \;\; k^2 = \frac{1}{3}g^2. \tag{5.32}
$$

The resulting up–quark mass matrix can be written as:

$$
M^u = \sqrt{\frac{8}{15}}g\langle H_4\rangle \begin{pmatrix} 1 & 1+\epsilon_1 & 1+\epsilon_2 \\ 1+\epsilon_1 & 1 & 1+\epsilon_3 \\ 1+\epsilon_2 & 1+\epsilon_3 & 1 \end{pmatrix}, \tag{5.33}
$$

where

$$\epsilon_1 = \frac{\langle H_1 \rangle}{\langle H_4 \rangle} - 1, \quad \epsilon_2 = \frac{\langle H_2 \rangle}{\langle H_4 \rangle} - 1, \quad \epsilon_3 = \frac{\langle H_3 \rangle}{\langle H_4 \rangle} - 1, \tag{5.34}$$

with a similar form for the down–type quark matrix. We can accommodate the mass hierarchy by taking $\epsilon_{1,2,3} \ll 1$. This structure has been considered by Fishbane and Kaus [81], where it has been shown to agree well with experimental data.

In the A_4 model, since all H_i have almost equal VEVs, $\langle H_4 \rangle \simeq \langle H_u \rangle /2$. Furthermore, from Eq. (5.33), we have $m_t \simeq 3\sqrt{8/15}g \langle H_4 \rangle$, so that $y_t = (\sqrt{6/5})g$ at the GUT scale. Similarly, $y_b = (\sqrt{9/10})g$ at the GUT scale. These boundary conditions lead to the predictions

$$\begin{aligned} m_t &= 177 \ GeV \\ \tan\beta &= 53. \end{aligned} \tag{5.35}$$

As shown by Fishbane and Kaus [81], all the quark mixing angles can be correctly reproduced in this model.

5.2.3 S_4 model

We now present a third example based on S_4 symmetry. This symmetry alone would lead to a one parameter family of solutions for the Yukawa couplings. Although we have not found a symmetry that will uniquely fix this parameter, we suspect that such a symmetry might actually exist. Keeping this in mind, we proceed to analyze this model. S_4 is the permutation symmetry operating on four objects. It has the following irreducible representations: $(1,\ 1',\ 2,\ 3,\ 3')$ [82]. We choose the following assignment of the chiral superfields under S_4:

$$\begin{aligned} \mathbf{10}_i &: 3, \qquad (H_a, H_4) : 3+1, \qquad \Sigma : 1, \\ \bar{\mathbf{5}}_i &: 3, \qquad (\bar{H}_a, \bar{H}_4) : 3+1, \end{aligned} \tag{5.36}$$

The superpotential invariant under this symmetry is

$$\begin{aligned} W &= a[(\mathbf{10}_1\mathbf{10}_3 + \mathbf{10}_3\mathbf{10}_1)H_1 + (\mathbf{10}_2\mathbf{10}_3 + \mathbf{10}_3\mathbf{10}_2)H_2 + (\mathbf{10}_1\mathbf{10}_1 - \mathbf{10}_2\mathbf{10}_2)H_1] \\ &+ b(\mathbf{10}_1\mathbf{10}_1 + \mathbf{10}_2\mathbf{10}_2 + \mathbf{10}_3\mathbf{10}_3)H_4 \\ &+ c[(\mathbf{10}_1\bar{\mathbf{5}}_3 + \mathbf{10}_3\bar{\mathbf{5}}_1)\bar{H}_1 + (\mathbf{10}_2\bar{\mathbf{5}}_3 + \mathbf{10}_3\bar{\mathbf{5}}_2)\bar{H}_2 + (\mathbf{10}_1\bar{\mathbf{5}}_1 - \mathbf{10}_2\bar{\mathbf{5}}_2)\bar{H}_4] \end{aligned}$$

$$\begin{aligned}
&+ \quad d(\mathbf{10}_1\bar{\mathbf{5}}_1 + \mathbf{10}_2\bar{\mathbf{5}}_2 + \mathbf{10}_3\bar{\mathbf{5}}_3)\bar{H}_4 \\
&+ \quad k(H_1\bar{H}_1 + H_2\bar{H}_2 + H_3\bar{H}_3)\Sigma \\
&+ \quad k_4 H_4\bar{H}_4\Sigma + \frac{\lambda}{3}\Sigma^3.
\end{aligned} \tag{5.37}$$

The V^u and V^d matrices that arise from this superpotential are:

$$V^u = \begin{pmatrix} a\langle H_3\rangle + b\langle H_4\rangle & 0 & a\langle H_1\rangle \\ 0 & b\langle H_4\rangle & a\langle H_2\rangle \\ a\langle H_1\rangle & a\langle H_2\rangle & -a\langle H_3\rangle + b\langle H_4\rangle \end{pmatrix}, \tag{5.38}$$

$$V^d = \begin{pmatrix} c\langle \bar{H}_3\rangle + d\langle \bar{H}_4\rangle & 0 & c\langle \bar{H}_1\rangle \\ 0 & d\langle \bar{H}_4\rangle & c\langle \bar{H}_2\rangle \\ c\langle \bar{H}_1\rangle & c\langle \bar{H}_2\rangle & -c\langle \bar{H}_3\rangle + d\langle \bar{H}_3\rangle \end{pmatrix}. \tag{5.39}$$

The coupling matrix k connecting the Higgs fields $(H, \bar{H})$ and the adjoint field Σ is:

$$k = diag(k, k, k, k_4).$$

The condition for vanishing anomalous mass dimensions is then:

$$\begin{aligned}
&3(2a^2 + b^2) + 2(2c^2 + d^2) = \frac{36}{5}g^2 \\
&4(2c^2 + 2d^2) = \frac{24}{5}g^2 \\
&4(2d^2) + \frac{24}{5}k^2 = \frac{24}{5}g^2 \\
&4(3d^2) + \frac{24}{5}{k_4}^2 = \frac{24}{5}g^2 \\
&3(2a^2) + \frac{24}{5}k^2 = \frac{24}{5}g^2 \\
&3(2b^2) + \frac{24}{5}{k_4}^2 = \frac{24}{5}g^2 \\
&3k^2 + {k_4}^2 + \frac{21}{5}\lambda^2 = \frac{24}{5}g^2.
\end{aligned} \tag{5.40}$$

The solution to this set of equations has one free parameter. We choose it be k_4, in which case the solution is:

$$\begin{aligned}
\left(a^2, b^2, c^2, d^2\right) &= \left(\frac{8}{15}g^2 + \frac{4}{15}{k_4}^2,\ \frac{8}{15}g^2 - \frac{4}{15}{k_4}^2,\ \frac{2}{15}g^2 + \frac{1}{5}{k_4}^2,\ \frac{2}{15}g^2 - \frac{1}{5}{k_4}^2\right) \\
\left(e^2, k^2, \lambda^2\right) &= \left(\frac{2}{5}g^2 - \frac{2}{5}{k_4}^2,\ \frac{1}{3}g^2 - \frac{1}{3}{k_4}^2,\ \frac{15}{7}g^2\right).
\end{aligned} \tag{5.41}$$

To eliminate this undetermined parameter k_4 one needs to introduce an additional symmetry. A Z_2 symmetry can set $k_4 = 0$, but if this Z_2 commutes with S_4, it will also set some other parameters to be zero. We suspect a Z_2 that does not commute with the S_4 symmetry might set k_4 equal to zero, while preserving the solution in Eq. (5.41). We find the model phenomenologically interesting for this case. The mass matrix for the up–type quarks is:

$$M^u = \sqrt{\frac{8}{15}}\, g \begin{pmatrix} \langle H_3\rangle + \langle H_4\rangle & 0 & \langle H_1\rangle \\ 0 & \langle H_4\rangle & \langle H_2\rangle \\ \langle H_1\rangle & \langle H_2\rangle & -\langle H_3\rangle + \langle H_4\rangle \end{pmatrix}, \tag{5.42}$$

and a similar form for the down–type quarks.

To explain the mass hierarchy, we first set the $(1,1)$ entry of the mass matrices both in the up and the down sectors to be zero by choosing $\langle H_3\rangle$ and $\langle H_4\rangle$ as:

$$\langle H_3\rangle + \langle H_4\rangle \sim 0, \qquad \langle \bar{H}_3\rangle + \langle \bar{H}_4\rangle \sim 0.$$

Furthermore, we take $\langle H_1\rangle$ and $\left\langle \bar{H}_1\right\rangle$ to be smaller than $\langle H_2\rangle \sim \langle H_4\rangle$. One immediate observation from the structure is that the rotation between the second and the third generations is large. These large rotations from the up and the down sectors will cancel out. Let us define $\frac{\langle H_2\rangle}{\langle H_4\rangle} = \sqrt{2}(1+\delta^u)$. In the limit $\epsilon^u \equiv \frac{\langle H_1\rangle}{\langle H_4\rangle} \to 0$, the rotation in the second and third generations is:

$$\begin{pmatrix} 1 & \sqrt{2}(1+\delta^u) \\ \sqrt{2}(1+\delta^u) & 2 \end{pmatrix}. \tag{5.43}$$

Form this one finds

$$\frac{m_c}{m_t} = -\frac{4}{9}\delta^u, \tag{5.44}$$

where m_c and m_t are the masses of charm and top quarks. The rotation angle is:

$$\tan(2\theta_{23}^u) = 2\sqrt{2}(1+\delta^u). \tag{5.45}$$

The large rotation angle will cancel out in V_{cb}, leaving only the smaller corrections proportional to $\delta^{u,d}$. The large rotation in the 2-3 space will induce an entry equal to $\epsilon^u \sin\theta_{23}^u \langle H_4\rangle$ in the (1,3) element. From this, we obtain the following relations for the quark mixing angles:

$$
\begin{aligned}
V_{cb} &= \frac{1}{2\sqrt{2}}\left|\frac{m_s}{m_b} \pm \frac{m_c}{m_t}\right| \\
V_{us} &= \left|\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}\right| \\
V_{ub} &= 2\sqrt{2}\left|\frac{\sqrt{m_d m_s}}{m_b} \pm \frac{\sqrt{m_u m_c}}{m_t}\right| .
\end{aligned}
\tag{5.46}
$$

These are of the right order of magnitude, although in detail, the magnitude of V_{cb} is somewhat smaller than what is needed and V_{ub} is on the larger side. We consider this general agreement with experiments to be encouraging.

5.3 Conclusions

I have presented in this chapter several models for quark masses and mixings in the context of finite $SU(5)$ GUT. These theories are attractive candidates for an underlying theory, since the β functions for the gauge and Yukawa couplings vanish to all orders in perturbation theory. The requirements on the theory to be finite also leads to Yukawa–gauge unification, leading to a single coupling constant in the theory.

The models presented are based on non–Abelian discrete symmetry, which seem to be necessary to obtain isolated and non–degenerate solutions to the Yukaw couplings when expressed as power series in terms of the gauge coupling. We find it interesting that realistic quark masses and mixings can be generated in such a framework.

There are several open questions, many of which cannot be addressed until after finding a consistent quark mixing scheme. An important question finite theories should address is how to avoid rapid proton decay. Because all the Yukawa couplings, including those of the light generations, are order of g, color triplet exchange will generate a large amplitude for proton decay through $d = 5$ operators [77]. This may simply be a technical problem associated with using $SU(5)$ as the gauge group. One can envision other groups without the color triplets, although no realistic model of this type are known to us. Within finite $SU(5)$, there are ways to suppress the troublesome proton decay operators. For example, if the SUSY particle spectrum is such that the gauginos are light (of order 100 GeV), while the squarks are very heavy (of order 100 TeV or larger), the $d = 5$ proton decay problem goes away. Although this choice may not be that attractive from the point of view of solving the gauge hierarchy problem, we emphasize that finiteness of the theory says nothing per se about the scale of SUSY breaking. A third alternative is to suppose that the masses of all the extra color triplets in the theory are much heavier than the GUT scale, even larger than the Planck scale.

In the framework of $SU(5)$ finite GUT, the following question arises naturally: Is it possible to generate small neutrino masses? If right–handed neutrinos are introduced as $SU(5)$ singlets, they can have no Yukawa couplings with the other fields, due to the demand of finiteness. We mention two possibilities to induce small neutrino masses. One is through bilinear R- parity violating terms of order the weak scale [83]. That does not contradict the requirements of finiteness. Another possibility is to make use of Planck suppressed higher dimensional operators, which can be constructed within finite $SU(5)$.

As we have shown in Section 5.1.1, within finite $SU(5)$, all four pairs of $\mathbf{5} + \overline{\mathbf{5}}$ fields present in the theory must be Higgs–like. This is needed for achieving doublet–triplet splitting. If one pair were fermionic, the bad mass relations of $SU(5)$, viz., $m_s = m_\mu$ and $m_d = m_e$ could have been corrected by terms such as $\bar{\mathbf{5}}_i\mathbf{5}$ bilinear mass terms along with $\bar{\mathbf{5}}\Sigma\mathbf{5}$ coupling. Since that is not possible, one has to rely on either Planck suppressed operators or finite gaugino diagrams to split the masses of leptons versus down type quarks [84]. Both possibilities appear to be viable.

CHAPTER 6

QUARK AND LEPTON MASS MATRIX FROM DECONSTRUCTION

6.1 **Introduction**

As we have stressed many times in earlier chapters there are many reasons for extending the Standard Model (SM) should be extended. The main questions which the SM does not provide any answer are to the gauge hierarchy problem, charge quantization, and the origin of fermion masses and mixings. Supersymmetry (SUSY) and four-dimensional (4D) Grand Unified Theories (GUT's) give partial solutions to the first two of the above problems but not to the latter. To understand the observed pattern of fermion masses and mixing angles, it seems therefore necessary that a new ingredient must be added, which allows to distinguish between the generations in a controlled way. For this purpose, one usually advocates a flavor symmetry. Generally, models based on continuous non-Abelian flavor symmetries are highly dependent on the details of the flavor symmetry breaking, without referring to deeper underlying dynamics. The models using an Abelian flavor symmetry, on the other hand, have as a common feature that the three generations carry different charges. At least from a bottom–up point of view, however, generation–dependent charges seem to be somewhat contrary to the spirit of GUT's, wherein the adhoc assignment of hypercharges to the quarks and leptons is explained.

In recent years, higher–dimensional theories opened up new possibilities for obtaining hierarchical fermion masses. [85,86]. For example, instead of assuming that the quarks and leptons carry generation–dependent charges under a flavor symmetry, the generations might be distinguished by their position in an extra dimension. A hierarchy of Yukawa couplings could then arise from the overlap of the spatial wave–functions of the matter fields in the extra dimension [87]. It would now be interesting to simulate or reproduce this higher–dimensional mechanism in a conventional 4D field theory, which is manifestly gauge–invariant and renormalizable. This can be achieved by employing the idea of dynamically generated extra dimensions, called deconstruction [88,89]. In deconstruction,[*] one considers the extra dimensions as an infrared effect of an ultraviolet complete theory described by a product of 4D gauge groups $\Pi_i \bigotimes G_i$. The deconstructed dimensions are represented in a "theory space" [90], where the gauge groups G_i correspond to "sites" that are connected by "links", like in a transverse lattice gauge theory [91]. Such a view of extra dimensions has rich theoretical and phenomenological implications covering studies in different directions and energy scales. These studies include, for example, electroweak symmetry breaking [92], GUT-type of models [93,94], supersymmetry breaking [90,95–97], and fermion masses and mixings [98–100].[†] Yet, a realistic deconstructed model, which gives all the observed fermion masses and mixing angles in the framework of a GUT, has not been proposed so far. This was the aim of the work done by Gerhart Seidl and myself which I present in the present chapter.

The 4D product GUT's which exhibit a higher-dimensional correspondence via deconstruction, have the advantage that dangerous proton decay operators can be easily suppressed by discrete symmetries. The doublet–triplet splitting problem, for example, can be solved in a model proposed by Witten [93], which is based on a 4D SUSY $SU(5)$ product GUT that is obtained from deconstruction. Here we extend this model by a $U(1)^N$ theory space. The different generations of quarks and leptons populate this space and are located at different sites in such a way, that the fermion masses and mixings emerge naturally. A simple linear structure of the product group space, corresponding to a single extra dimension, seems to be too restrictive to account for the entire fermion mass and mixing pattern of the SM. Therefore, we start instead with a deconstructed two-dimensional disk, which can be part of an even larger structure, the so called "spider web theory space" introduced by Arkani-Hamed et. al. [90]. They showed that when the spider web theory space is converted into the real projective plane RP^2, supersymmetry breaking can be viewed as arising from a

[*]For an early approach in terms of infinite arrays of gauge theories, see Halpern et. al. [90].

[†]Deconstruction has, for example, also been applied to neutrino oscillations [101], the Casimir effect [102], instantons [103], gravity [104], and calculable models of the "landscape" of string vacua [105].

topological obstruction due to a nontrivial first homology group $H^1(RP^2) = Z_2$. In spider web theory space, one can therefore simultaneously account for SUSY breaking and the generation of fermion masses and mixings.

To ensure the consistency of our model, we have to address the anomalies associated with the enlarged gauge symmetry in four dimensions. Anomaly–cancelation in theory space has been previously discussed by many authors[97,99,101,102]. The cancelation of the anomalies is generally carried out by introducing appropriate Wess–Zumino terms [103], which represent non-decoupling effects of heavy fermions in the low–energy theory. We apply this approach to our spider web theory space. In addition, we examine the continuum limit of Chern–Simons terms, which, however, do not contribute to the anomalies.

The chapter is organized as follows. In Section 6.2, we present our model. In Section 6.2.1, we review the solution to the doublet–triplet splitting problem in an $SU(5)' \times SU(5)''$ product GUT. Next, in Section 6.2.2, we introduce our model for the quark and lepton masses based on a $U(1)$ spider web theory space. We also comment in this Section on supersymmetry breaking via the nontrivial topology of RP^2. The generation of the fermion masses and mixings is described in Section 6.3. The predictions for the up–quarks, down–quarks/charged leptons, and neutrinos are presented in Sections 6.3.1, 6.3.2, and 6.3.3. The anomaly cancelation in our model is discussed in Section 6.4. Finally, we give our conclusions in Section 6.5.

6.2 Deconstructed $U(1)$

It has been proposed by Witten, that the doublet–triplet splitting problem can be solved in an $4D$ SUSY $SU(5)' \times SU(5)''$ product GUT model, which arises from deconstruction [93] (a similar approach has been given earlier by Barbieri et al. [104]). In this Section, we will build upon this setup and extend it to a model, which reproduces the observed fermion masses and mixings. We will first begin in Section 6.2.1 with a brief review of the known solution to the doublet–triplet splitting problem, which we then take in Section 6.2.2 as a starting point for introducing our model of quark and lepton masses.

6.2.1 Doublet–triplet splitting in $SU(5)' \times SU(5)''$

Following the doublet–triplet splitting mechanism proposed by Witten and Barbieri *et* al., one assumes a 4D gauge group $G = SU(5)' \times SU(5)''$, in which the SM gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ is embedded as a diagonal subgroup. The model possesses a discrete global symmetry $F = Z_N$ which commutes with G. At the GUT scale, the symmetry group $G \times F$ is spontaneously broken down to $G_{SM} \times F'$, where F' is a linear combination of F and the Z_N subgroup of the hypercharge subgroup $U(1)''_Y$ of $SU(5)''$. The MSSM Higgs doublets are contained in the $SU(5)' \times SU(5)''$ representations

$$(\mathbf{5}, \mathbf{1})^H + (\mathbf{1}, \overline{\mathbf{5}})^H, \tag{6.1}$$

i.e., the Higgs superfield that gives masses to the up quarks transforms under the fundamental representation of $SU(5)'$ and is a singlet under $SU(5)''$. The Higgs which generates the down quark and charged lepton masses, on the other hand, is in the antifundamental representation of $SU(5)''$ and is an $SU(5)'$ singlet. Under $G \supset G_{SM}$, the Higgs fields in Eq. (6.1) decompose as $\mathbf{5}^H = (Q, H)$ and $\overline{\mathbf{5}}^H = (\tilde{Q}, \tilde{H})$, in which H and $\tilde{H}$ are the MSSM Higgs doublets, whereas Q and $\tilde{Q}$ are their corresponding color triplet partners. The crucial point which now allows to solve the doublet–triplet splitting problem is here that the unbroken discrete symmetry F' commutes with $SU(5)'$ but not with $SU(5)''$. As a result, F' acts on the whole multiplet $\mathbf{5}^H$ but distinguishes in $\overline{\mathbf{5}}^H$ between the triplet and doublet components $\tilde{Q}$ and $\tilde{H}$. One can therefore have an F'-invariant coupling $\tilde{Q}Q$ in the superpotential while a μ-term-type coupling $\sim H\tilde{H}$ is (at the GUT scale) forbidden by F', which solves the doublet–triplet splitting problem.

When including quarks and leptons in this model, it is necessary that F' can forbid all dangerous baryon number violating operators, which would otherwise mediate proton decay. This becomes indeed possible [93], when we assume that under $SU(5)' \times SU(5)''$ the matter superfields transform as

$$(\mathbf{10}, \mathbf{1})_i + (\overline{\mathbf{5}}, \mathbf{1})_i + (\mathbf{1}, \mathbf{1})_i, \tag{6.2}$$

where the subscript $i = 1, 2, 3$ is the generation index. In other words, we suppose that the SM quarks and leptons are in non-trivial representations of the first factor

$SU(5)'$ and singlets under $SU(5)''$. Notice in Eq. (6.2), that we have completed each generation by one "right-handed" (*i.*e., SM singlet) neutrino required to obtain small neutrino masses via the type-I seesaw mechanism [16]. Since the down quark and charged lepton masses can thus only emerge from non–renormalizable operators, this may provide a reason why the down quarks and charged leptons are generally lighter than the up quarks. Apart from this generic property, however, it would be desirable to have in this model a more complete understanding of the observed masses and mixings of quarks and leptons. For this purpose, we will in the next Section attempt to associate the observed fermion masses and mixing angles with the coupling of the Higgs and matter fields in Eqs. (6.1) and (6.2) to the theory space of a deconstructed $U(1)$ symmetry.

6.2.2 $U(1)$ theory space

To address the fermion mass hierarchy in the model reviewed in Section 6.2.1, we will assume that the matter fields "live" in a $U(1)$ product group theory space, which describes a deconstructed manifold. The fermion mass hierarchy arises from placing the different generations in Eq. (6.2) on distinct points in the deconstructed manifold. Although there may be many possibilities, we will first confine ourselves to a theory space, which is topologically a two–dimensional disk. The reason for our choice is that a supersymmetry breaking mechanism can be made readily available in such a theory space [90]. We comment on a possible implementation of this idea in our model at the end of this Section. Our deconstructed manifold is conveniently represented by the "moose" [105] or "quiver" [106] diagram in Fig. 6.1, which describes a spider web theory space, that is topologically equivalent with a two-dimensional disk. The center of the spider web theory space is surrounded by k concentric circles. Each such circle is defined by N sites and each site i, where $i = 0, 1, 2, \ldots, kN$, symbolizes one $G_i \equiv U(1)_i$ gauge group. The total gauge group of our model is therefore $SU(5)' \times SU(5)'' \times U(1)^{kN+1}$ where $U(1)^{kN+1} \equiv \Pi_{i=0}^{kN} U(1)_i$. For definiteness, we have in Fig. 6.1 depicted the case $k = 2$ and have explicitly labeled only the sites in the inner part of the disk. When compactifying the disk later on the real projective plane RP^2, we will require that $N = 4m$, where m is some integer. In our spider web theory space, two neighboring sites are connected by a single directed link. The

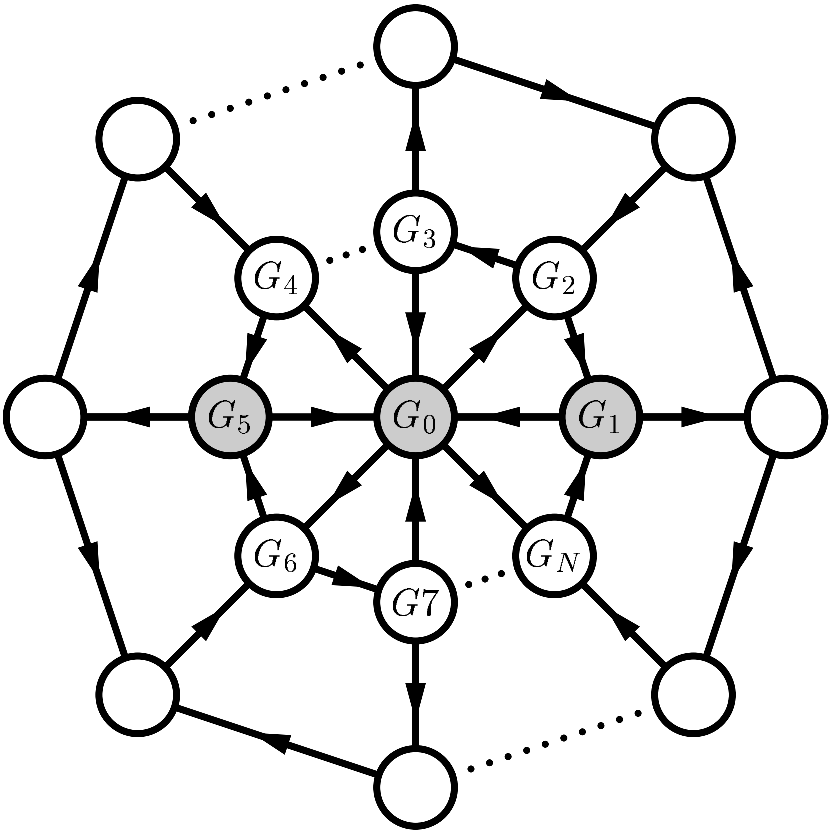

Figure 6.1. Spider web theory space for the deconstructed $U(1)$ gauge theory. Each point or site i, where $i = 0, 1, \ldots, 2N$, is associated with a gauge group $G_i \equiv U(1)_i$. An arrow which connects two groups G_i and G_j and points from i to j denotes a single chiral link superfield $\phi_{i,j}$ that is charged under $G_i \times G_j$ as $(+1, -1)$ and is a singlet under all the other gauge groups. The first, second and third generations are placed on the the sites corresponding to G_1, G_5, and G_0, respectively (gray circles). For N even, the disk is fitted together by triangular plaquettes with alternating orientations. The dotted lines represent possible insertions of extra $U(1)_i$ gauge groups.

general organization of the links and their directions is summarized in Fig. 6.1 for the example of $k = 2$. In Fig. 6.1, an arrow connecting two sites i and j that points from i to j denotes a chiral link superfield $\phi_{i,j}$, which is charged under $G_i \times G_j$ as $(+1, -1)$. Under all the other $U(1)$ factors and $SU(5)' \times SU(5)''$, however, the link fields $\phi_{i,j}$ transform only trivially.

It is important to note in our model that for even N, the directions of the link fields in the spider web theory space are such that each small triangular or quadratic plaquette has a definite orientation. Any two neighboring plaquettes have, consequently, opposite orientations. With a single directed link superfield connecting two neighboring sites, only this kind of configuration allows to have Wilson-loop-type contributions (in the sense of usual lattice gauge theory) to the superpotential from every plaquette in Fig. 6.1. As we will see later, the directions of the link fields are crucial for generating a realistic hierarchy of Yukawa couplings. In what follows, we are interested in the D–flat directions $|\phi_{0,i}| = |\phi_{i,i+1}| \equiv v$ $(i = 1, \ldots, N)$ in the classical moduli space of vacua: All scalar components of the chiral link superfields have vacuum expectation values (VEV's) with a universal magnitude v. Such a VEV v breaks the $U(1)$ product gauge group spontaneously down to the diagonal subgroup $U(1)_{diag}$. Henceforth, we will refer to the field theory defined by our spider web theory space also as the "$U(1)$ theory space" of our model.

Let us now describe how the three generations are incorporated in our theory space. We suppose that each generation in Eq. (6.2) is put on a separate site (see Fig. 6.1): the first generation "lives" on site 1, the second on site 5, and the third on site 0 in the center of the disk.* This is achieved by giving the first, second and third generations nonzero $U(1)$ charges exactly under the gauge groups $U(1)_1$, $U(1)_5$, and $U(1)_0$, respectively, while we assume that they are singlets under all the other $U(1)$ factors. Next, we have to specify on the three sites the $U(1)$ charge assignment to the matter fields within each generation. We choose the $U(1)$ charges for the fermions in each generation to be compatible with $SO(10)$ as follows

$$SO(10) \supset SU(5)' \times U(1)_1 \quad : \quad \mathbf{16}_1 = \mathbf{10}(-1)_1 + \overline{\mathbf{5}}(3)_1 + \mathbf{1}(-5)_1, \tag{6.3}$$

*Instead of putting the second generation on site 5, we could also choose any site i on the boundary as long as i is odd for the desired link direction of $\phi_{i,0}$ and $i, N - i \geq 5$.

$$SO(10) \supset SU(5)' \times U(1)_5 \quad : \quad \mathbf{16}_2 = \mathbf{10}(1)_2 + \overline{\mathbf{5}}(-3)_2 + \mathbf{1}(5)_2, \qquad (6.4)$$

$$SO(10) \supset SU(5)' \times U(1)_0 \quad : \quad \mathbf{16}_3 = \mathbf{10}(1)_3 + \overline{\mathbf{5}}(-3)_3 + \mathbf{1}(5)_3, \qquad (6.5)$$

where the parenthesis contains the the corresponding $U(1)_i$ charge of the multiplets and the subscript denotes the generation index. In Eqs. (6.3), we have, as compared to Eq. (6.2), only kept the transformation properties of the matter fields under $SU(5)'$ since they all transform trivially under $SU(5)''$. Note also in Eq. (6.3), that we have made use of an overall sign ambiguity in the branching rule and assumed that the $U(1)_1$ charges of the first generation are "flipped" with respect to the corresponding $U(1)_5$ and $U(1)_0$ quantum numbers in Eqs. (6.4) and (6.5). We emphasize that the two lighter generations are connected to the third generation by link fields $\phi_{1,0}$ and $\phi_{5,0}$, which point toward the center. This orientation is crucial for generating realistic fermion masses and mixings.

At this point, it is important to emphasize that we employ in Eqs. (6.3) the $SO(10)$ branching rules only as a mere guideline or organizing principle for the $U(1)$ charge assignment to the multiplets in Eq. (6.2). Our model does therefore not possess an actual $SO(10)$ gauge symmetry (ther have recently been studies on flavor models in $SO(10)$ framework[107]). Choosing the $SO(10)$ branching rules as a prescription for the $U(1)$ charge assignment, however, has several attractive features. One obvious advantage is, *e*.g., that the quark and lepton sectors are automatically anomaly-free, such that the discussion of anomalies is restricted to the Higgs and link fields only.

One major feature of our model is that the fermion mass hierarchy is due to the "location" of the different generations on distinct points in theory space (up to the overall sign ambiguity of the $U(1)$ generators [*c*f. Eqs. (6.3)]). This is different, *e*.g., from usual anomalous $U(1)$ models [2,10]), where the fermion mass hierarchy is understood in terms of flavor–dependent charges under a single $U(1)$. Notice, that the $U(1)$ charge assignment in Eqs. (6.3) resembles a gauged $B - L$ symmetry [108], whose deconstruction has been discussed by Skiba and Smith [99].

Next, let us consider how the Higgs fields in Eq. (6.1) couple to the $U(1)$ theory space. The $U(1)$ charge assignment to the third generation in Eq. (6.5) already fixes the transformation properties of $(\mathbf{5}, \mathbf{1})^H$. Specifically, to obtain a large top Yukawa coupling in our model, we suppose that $(\mathbf{5}, \mathbf{1})^H$ carries a $U(1)_0$ charge -2 and is a singlet under all the other $U(1)$ gauge groups. The Higgs field $(\mathbf{5}, \mathbf{1})^H$ is therefore located as a site variable together with the third generation on the center of the disk. It is interesting to note, that the $U(1)_0$ charge assignment to $\mathbf{5}^H$ becomes compatible with $SO(10)$ when considering the $\mathbf{5}^H$ as part of the decomposition $\mathbf{10}^H = \mathbf{5}^H(-2) + \overline{\mathbf{5}}^H(2)$ under $SO(10) \supset SU(5)' \times U(1)_0$, which is also in agreement with the choice of the $U(1)_0$ generator in Eq. (6.5). All matter and Higgs superfields which are located on the disk have in common, that they are singlets under the second factor $SU(5)''$. In contrast to this, we assume that $(\mathbf{1}, \overline{\mathbf{5}})^H$ in Eq. (6.1), which is the only non-trivial $SU(5)''$ representation, carries no $U(1)$ charge at all and is thus not part of the $U(1)$ theory space.

In order to break $U(1)_{diag}$, which is not observed at low energies, we assume a single vectorlike pair of chiral superfields f and $\overline{f}$, which resides on the center of the disk. The fields f and $\overline{f}$ are charged under $U(1)_0$ as $+1$ and -1, respectively, and are singlets under all the other $U(1)$ and $SU(5)$ gauge groups. In what follows, we will suppose that the scalar components of the fields f and $\overline{f}$ acquire VEV's $\langle f \rangle \simeq \langle \overline{f} \rangle \simeq \langle \phi_{i,j} \rangle \simeq v$, *i.*e., it is assumed that all $U(1)_i$ symmetries including $U(1)_{diag}$ are broken around the same scale v.

As mentioned earlier, we have an interesting possibility of supersymmetry breaking in spider web theory space. Supersymmetry breaking can be implemented in a number of different ways for our case. Among these we find the mechanism discussed by Arkani-Hamed and company [90] to be attractive and unique in deconstruction. In the remainder of this Section, we will briefly comment on this mechanism.

Arkani-Hamed et. al have shown that [90], different types of theory space preserve supersymmetry only locally, viz., the interactions on each plaquette are manifestly supersymmetric. If, however, the topology of theory space has a nontrivial first homology group, supersymmetry breaking can be seen as a topological effect. A deconstructed manifold with this property can, *e.*g., be obtained from the disk in Fig. 6.1, when we identify diametrically opposite sites and links on the boundary, which yields a real projective plane RP^2 with first homology group Z_2 (this requires in our case $N = 4m$, where m is some integer). The phase differences between the gauge couplings g_i associated with the gauge groups $U(1)_i$ and the corresponding gauge-Yukawa couplings $h_i = g_i e^{i\theta_i}$ for the interaction $\sim h_i \psi^\dagger \lambda_i \phi$ (where ψ and ϕ denote the fermionic and scalar components of a link field connected to the site i with gaugino λ_i), can be removed separately in each plaquette by field redefinitions. In this sense, supersymmetry is preserved "locally". Globally, however, there can remain one phase in the product of all the couplings h_i, which cannot be rotated away. On RP^2, this phase is either $+1$, which will lead to exact global supersymmetry, or -1 for maximal supersymmetry breaking [90]. The phase being -1 rather than arbitrary, as it would be the case on a circle, can be considered as an advantage of the spider-web theory space. The supersymmetry breaking effects are suppressed by a factor $m^2_{SUSY} \sim \Pi_i g_i^2/(4\pi)^2 v^2$ (where i runs over half of the boundary of the disk in Fig. 6.1) due to a number of N loops to account for the nontrivial global twist of RP^2, which can easily produce a TeV scale supersymmetric spectrum. With this mechanism, one can now address both the fermion mass hierarchy and supersymmetry breaking in the same theory space.

6.3 Quark and Lepton Masses

6.3.1 Up quark sector

With the representation content outlined in Section 6.2, we are now in a position to determine the fermion masses in our model. Let us first consider the up quark sector. In the notation of Eqs. (6.1) and (6.2), the up quark Yukawa couplings arise from G-invariant terms of the type $\sim (\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_i(\mathbf{10},\mathbf{1})_j$ in the superpotential. Depending on the location of the $(\mathbf{10},\mathbf{1})_i$ matter multiplets on the disk, these terms may be renormalizable or non–renormalizable. The mass of the top-quark, *e*.g., emerges from the gauge-invariant renormalizable operator $(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_3(\mathbf{10},\mathbf{1})_3$, with a top Yukawa coupling of order one. This coupling is renormalizable because the third generation is situated together with $(\mathbf{5},\mathbf{1})^H$ on the center of the disk carrying $SO(10)$ compatible $U(1)_0$ charges.

Since the first two generations are located at some distance away from the center, gauge-invariance under the deconstructed $U(1)$ requires that all other up quark mass terms come from non–renormalizable operators involving the link fields $\phi_{i,j}$, which connect the center with the first two generations. The associated effective Yukawa couplings will thus be suppressed by inverse powers of the cutoff scale Λ of the effective theory, thereby producing hierarchical mass and mixing parameters in the fermion sectors. In writing down the Yukawa couplings, it is of great importance that we work in a supersymmetric model, where the particular directions of the link fields as defined in Fig. (6.1) constrain the allowed renormalizable and non–renormalizable terms due to the holomorphicity of the superpotential. The charm quark mass, *e*.g., arises dominantly from a non–renormalizable dimension-eight operator of the type $\Lambda^{-4}\phi_{0,4}^2\phi_{4,5}^2(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_2(\mathbf{10},\mathbf{1})_2$, which involves two powers of the link fields $\phi_{0,4}$ and $\phi_{4,5}$. Here, the product of links $\phi_{0,4}\phi_{4,5}$ connects the second generation with the Higgs $(\mathbf{5},\mathbf{1})^H$ in the center along the shortest "path" on the disk consistent with the holomorphicity of the superpotential. Similarly, the second and third generations mix via the dimension-six term $\Lambda^{-2}\phi_{0,4}\phi_{4,5}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_3(\mathbf{10},\mathbf{1})_2$ associated with the same path.

Different from the two heavier generations, the mass and mixing terms of the first generation must originate from $U(1)_{diag}$ violating operators, which involve the $U(1)_{diag}$-breaking fields f or $\overline{f}$ that live in the center of the disk. This difference arises because the first generation carries, with respect to the heavier two generations, opposite charges under $U(1)_{diag}$. The up quark mass, *e*.g., is generated by the non–renormalizable term $\Lambda^{-6}f^4\phi_{1,0}^2(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_1(\mathbf{10},\mathbf{1})_1$, involving four powers of f. This operator contains also two powers of the link field $\phi_{1,0}$, which connects the first generation with the center. The link field $\phi_{1,0}$ appears therefore also in the operator $\Lambda^{-3}f^2\phi_{1,0}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_3(\mathbf{10},\mathbf{1})_1$, which mixes the up with the top quark. Correspondingly, the term $\Lambda^{-5}f^2\phi_{1,0}\phi_{0,4}\phi_{4,5}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_1(\mathbf{10},\mathbf{1})_2$ is responsible for the mixing of the up quark with the charm quark. This operator contains the product of links $\phi_{1,0}\phi_{0,4}\phi_{4,5}$, which represents on the disk the shortest connection via holomorphic couplings between the up quark and the charm quark.

In total, the most general gauge-invariant superpotential containing the renormalizable and non–renormalizable terms which are relevant for the up quark masses therefore reads

$$\begin{aligned}\mathcal{W} \supset\ & (\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_3(\mathbf{10},\mathbf{1})_3 + \frac{\phi_{0,4}^2\phi_{4,5}^2}{\Lambda^4}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_2(\mathbf{10},\mathbf{1})_2 \\ + & \frac{f^4\phi_{1,0}^2}{\Lambda^6}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_1(\mathbf{10},\mathbf{1})_1 + \frac{\phi_{0,4}\phi_{4,5}}{\Lambda^2}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_3(\mathbf{10},\mathbf{1})_2 \\ + & \frac{f^2\phi_{1,0}}{\Lambda^3}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_3(\mathbf{10},\mathbf{1})_1 + \frac{f^2\phi_{1,0}\phi_{0,4}\phi_{4,5}}{\Lambda^5}(\mathbf{5},\mathbf{1})^H(\mathbf{10},\mathbf{1})_1(\mathbf{10},\mathbf{1})_2 + \ldots\end{aligned} \quad (6.6)$$

where the dots denote negligible higher-order terms and where we have not explicitly written the different Yukawa couplings of order one. When all the link and site fields $\phi_{i,j}$ and f acquire their VEV's around the deconstruction scale v, the up quark mass matrix is then given by the well–known texture

$$M_u = \langle H \rangle \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \tag{6.7}$$

where we have introduced the small symmetry-breaking parameter $\epsilon \equiv v/\Lambda \simeq 0.2$. Since the texture in Eq. (6.7) can already fully account for the observed CKM angles, the down quark mixing should not become too large in order to avoid conflict with experiment. As we will see in Section 6.3.2, the mixing in the down sector does indeed not exceed the up quark mixing.

6.3.2 Down quark and charged lepton sectors

The construction of the down quark and charged lepton Yukawa coupling terms goes along the same lines as for the up quarks in Section 6.3.1, except for the difference that the matter fields and $(\mathbf{1},\overline{\mathbf{5}})^H$ transform in G under different $SU(5)$ factors. The down quark and charged lepton Yukawa couplings must therefore emerge from G-invariant terms $\sim (\overline{\mathbf{5}},\mathbf{5})^H(\mathbf{1},\overline{\mathbf{5}})^H(\mathbf{10},\mathbf{1})_i(\overline{\mathbf{5}},\mathbf{1})_j$, where $(\overline{\mathbf{5}},\mathbf{5})^H$ denotes in the doublet–triplet splitting mechanism reviewed in Section 6.2.1 a linear combination of Higgs superfields which transforms under $SU(5)'\times SU(5)''$ as a bifundamental representation. When $(\overline{\mathbf{5}},\mathbf{5})^H$ acquires its VEV at the GUT scale, $G\times F$ is broken down to the low-energy subgroup $G_{SM}\times F'$ (see Refs. [93,104]).

To generate the down quark and charged lepton masses, we assume two such bifundamental Higgs superfields Φ_+ and Φ_-, which are put as site variables on the center of the disk and transform under $SU(5)'\times SU(5)''\times U(1)_0$ as $\Phi_+ \sim (\overline{\mathbf{5}},\mathbf{5})^H(+2)$ and $\Phi_- \sim (\overline{\mathbf{5}},\mathbf{5})^H(-2)$, where in each of the last two expression the second parenthesis contains the value of the $U(1)_0$ charge. Under all the other $U(1)_i$ groups, Φ_+ and Φ_- transform trivially. By the same arguments as in Section 6.3.1, we then find for the relevant superpotential terms responsible for the down quark charged lepton masses

$$\mathcal{W} \supset \frac{\Phi_+}{\Lambda'}(\mathbf{1},\overline{\mathbf{5}})^H(\mathbf{10},\mathbf{1})_3(\overline{\mathbf{5}},\mathbf{1})_3 + \frac{\phi_{5,0}^2\Phi_+}{\Lambda^2\Lambda'}(\mathbf{1},\overline{\mathbf{5}})^H(\mathbf{10},\mathbf{1})_2(\overline{\mathbf{5}},\mathbf{1})_2$$

$$+ \quad \frac{\phi_{0,2}^2\phi_{2,1}^2\Phi_-}{\Lambda^4\Lambda'}(\mathbf{1},\overline{\mathbf{5}})^H(\mathbf{10},\mathbf{1})_1(\overline{\mathbf{5}},\mathbf{1})_1+\ldots, \tag{6.8}$$

where the dots denote irrelevant higher-order operators and where we have not explicitly written the Yukawa couplings of order one. In Eq. (6.8), the scale Λ' is related to the GUT–scale $M_{GUT} \simeq 10^{16}GeV$ by $\Lambda' \simeq M_{GUT}/\epsilon'$, where $\epsilon' \sim 0.1$. Observe that Λ' is a common factor to the down sector parameterizing $\tan\beta$ and thus plays no role for the flavor structure. When Φ_+ and Φ_- acquire similar VEV's $\langle\Phi_+\rangle \simeq \langle\Phi_-\rangle \sim M_{GUT}$, the mass matrix of the down quarks and charged leptons takes the form

$$M_d = \epsilon'\langle\tilde{H}\rangle \begin{pmatrix} \epsilon^4 & \epsilon^6 & \epsilon^3 \\ \epsilon^{10} & \epsilon^2 & \epsilon^2 \\ \epsilon^8 & \epsilon^3 & 1 \end{pmatrix}, \tag{6.9}$$

where the rows and columns are spanned by the $(\mathbf{10},\mathbf{1})_i$ and $(\overline{\mathbf{5}},\mathbf{1})_j$, respectively, and where we have a moderate $tan\beta \equiv \langle H\rangle/\langle\tilde{H}\rangle \sim 10$. In total, one therefore obtains for the quark and charged lepton mass ratios

$$m_u : m_c : m_t \;=\; \epsilon^6 : \epsilon^4 : 1, \tag{6.10}$$

$$m_d : m_s : m_b \;=\; \epsilon^4 : \epsilon^2 : 1, \tag{6.11}$$

$$m_e : m_\mu : m_\tau \;=\; \epsilon^4 : \epsilon^2 : 1. \tag{6.12}$$

The CKM angles are of the orders

$$V_{us} \sim \epsilon, \quad V_{cb} \sim \epsilon^2, \quad V_{ub} \sim \epsilon^3. \tag{6.13}$$

In Eq. (6.9), we observe that the charged lepton mixing angles are $\lesssim \epsilon^3$. The large leptonic mixing angles must therefore be almost entirely generated in the neutrino sector. The neutrino masses and mixing angles will be discussed now.

6.3.3 Neutrino masses

Following the generic approach [100], we shall relate the absolute neutrino mass scale to the deconstruction scale via a dynamical realization of the type-I seesaw mechanism, where the inverse lattice spacing $\sim v$ is identified with the usual $B-L$ breaking scale $v \simeq 10^{14}$ GeV. To leading order, the total effective 3×3 neutrino mass matrix M_ν can thus be written as $M_\nu = -M_D M_R^{-1} M_D^T$, where, as usually, M_D

denotes the Dirac neutrino mass matrix and M_R is the right-handed Majorana mass matrix. The qualitative difference between M_D and M_R is, of course, that M_D is protected by G_{SM} down to the electroweak scale, while M_R can already emerge at the deconstruction scale v through the Yukawa interactions between the right-handed neutrinos (which are vectorial with respect to G) and the link fields.

When determining M_D and M_R in the same way like M_u and M_d in Sections 6.3.1 and 6.3.2, however, we find that the minimal theory space introduced in Section 6.2.2 would only give small neutrino mixing angles. To arrive at a large neutrino mixing, one may deviate from minimality and add extra link fields to our $U(1)$ theory space. Specifically, we assume that each of the directed link superfields $\phi_{i,j}$ defined in Section 6.2.2 is accompanied by a pair of vectorlike chiral link superfields $\chi_{i,j}$ and $\overline{\chi}_{j,i}$ which point into opposite directions and acquire universal VEV's of the order $\langle\chi_{i,j}\rangle \simeq \langle\overline{\chi}_{j,i}\rangle \simeq v$. While $\phi_{i,j}$ carries the $G_i \times G_j$ charges $(+1,-1)$, the fields $\chi_{i,j}$ and $\overline{\chi}_{j,i}$ are charged under $G_i \times G_j$ as $(+8,-8)$ and $(-8,+8)$, respectively. One can check that the incorporation of the link fields $\chi_{i,j}$ and $\overline{\chi}_{j,i}$ has no effect on our results in Section 6.3.1 and 6.3.2 for the charged fermion mass ratios and CKM angles summarized in Eqs. (6.10). In contrast to this, the extra Yukawa interactions between the $\overline{\chi}_{j,i}$ and the right-handed neutrinos introduce a large off-diagonal term in M_R, which results in a large atmospheric neutrino mixing angle $\theta_{23} \sim 1$. This is a generalization of the scenario for soft breaking of the $L_e - L_\mu - L_\tau$ lepton number in the right-handed sector [109].

A fully realistic description of bilarge neutrino mixing with normal neutrino mass hierarchy can then be obtained by adding on the sites extra Higgs superfields known from standard realizations of the seesaw mechanism. For example, we can assume an $SU(5)' \times SU(5)''$ singlet Higgs superfield $(\mathbf{1},\mathbf{1})^H$, which is placed together with the second generation on the site 5. The $(\mathbf{1},\mathbf{1})^H$ carries a charge -10 under $U(1)_5$ and is a singlet under all the other $U(1)_i$ groups. This $U(1)_5$ charge assignment becomes compatible with $SO(10)$ on the site 5, when we identify $(\mathbf{1},\mathbf{1})^H$ with the $SU(5)'$ singlet in the decomposition

$$SO(10) \supset SU(5)' \times U(1)_5 \quad : \quad \overline{\mathbf{126}}^H = \mathbf{1}^H(-10) + \mathbf{15}^H(6) + \ldots , \qquad (6.14)$$

where we have only written the subrepresentations relevant for M_ν. The $(\mathbf{1},\mathbf{1})^H$ couples to the right-handed neutrinos via a renormalizable term $(\mathbf{1},\mathbf{1})^H(\mathbf{1},\mathbf{1})_2(\mathbf{1},\mathbf{1})_2$, thereby supplementing M_R with an additional parameter. Choosing $\langle(\mathbf{1},\mathbf{1})^H\rangle \simeq \epsilon^7 v$, the effective neutrino mass matrix comes to the familiar form

$$M_\nu = \epsilon\frac{\langle H\rangle^2}{v}\begin{pmatrix}\epsilon^4 & \epsilon^2 & \epsilon\\ \epsilon^2 & 1 & 1\\ \epsilon & 1 & 1\end{pmatrix}. \tag{6.15}$$

Taking $\sim 4\times10^{-2}\,\mathrm{e}V$ as the heaviest active neutrino mass, we find from Eq. (6.15) for the deconstruction scale a value $v \simeq 10^{14}\,\mathrm{Ge}V$, which is of the order the usual $B-L$ breaking scale. In its present form, however, the 2-3 subblock of M_ν in Eq. (6.15) has a determinant which is much smaller than ϵ, so that the solar mixing angle would be close to maximal.

In order to obtain a large but not maximal solar mixing angle, we can invoke the type-II seesaw mechanism [110], which provides a contribution of order $\sim\epsilon$ to the 2-3 subblock of M_ν in Eq. (6.15), thus suppressing θ_{23} down to values $\sim \pi/6$. The type-II seesaw mechanism may be implemented in our model by adding on the center of the disk a pair of conjugate Higgs superfields as site variables, that transform under $SU(5)'\times SU(5)''\times U(1)_0$ as $(\mathbf{15},\mathbf{1})^H(6)$ and $(\overline{\mathbf{15}},\mathbf{1})^H(-6)$, respectively, but which are singlets under all the other $U(1)_i$ gauge groups (a phenomenological implications has been discussed by, e.g., Rossi [111]). The $U(1)_0$ charges of these Higgs fields are $SO(10)$ compatible, as can be seen from the branching rule in Eq. (6.14), by replacing the gauge group $U(1)_5$ by $U(1)_0$. The superpotential couplings for the type-II seesaw mechanism involve a renormalizable term $M_{15}(\mathbf{15},\mathbf{1})^H(\overline{\mathbf{15}},\mathbf{1})^H$, where M_{15} is some high mass scale. Now, after integrating out the heavy Higgs fields, the contribution to the 3-3 element of M_ν in Eq. (6.15) is of the order $\sim\epsilon^9\langle H\rangle^2/M_{15}$. If $M_{15}\simeq 10^9\,\mathrm{Ge}V$, then the total effective neutrino mass matrix M_ν can assume a similar form like in Eq. (6.15), with the difference that the determinant of the 2-3 subblock is now of the order $\sim\epsilon$. For our choice of parameters, the model can thus lead to a normal active neutrino mass hierarchy

$$m_1 : m_2 : m_3 \;=\; \epsilon : \epsilon : 1, \tag{6.16}$$

where m_1, m_2, and m_3 are the active neutrino masses with solar and atmospheric mass squared differences of the orders $\Delta m_\odot \simeq 10^{-4}\,\mathrm{e}V$ and $\Delta m^2_{\mathrm{atm}} \simeq 10^{-3}\,\mathrm{e}V$, respectively. In this case, we then have a small reactor angle $\theta_{13}\sim\epsilon$, a large but not maximal solar angle $\theta_{12} \sim 1$ and a large atmospheric angle $\theta_{23} \sim 1$, which can be maximal. Our model can therefore accommodate current global fits to neutrino oscillation data [112].

6.4 Anomaly Cancelation

Although the $SO(10)$ compatible $U(1)$ charge assignment to the fermions in Eqs. (6.3) is anomaly-free, the Higgs field sector in its present form would contain anomalies. Note that, in our spider web theory space, any Higgs superfield with anomalous coupling is either a link field $\phi_{i,j}$ or must be situated as a site variable on a single site. The anomalies coming from these site variables may be directly canceled by simply adding in a standard fashion extra fields on the sites where they reside. In contrast to this, we shall now consider the possibility to cancel the pure and mixed anomalies associated with the link fields $\phi_{i,j}$ without introducing any new fields in the low-energy effective theory.

First, we will discuss the pure (*i*.e., non-mixed) and gauge-gravitational anomalies. When the topology of the spider web theory space in Fig. 6.1 is that of a disk, the link fields $\phi_{i,j}$ would give rise to pure and gauge-gravitational anomalies on each site of the boundary. Interestingly, these anomalies are completely eliminated, when the spider web theory space is, instead, compactified on the real projective plane RP^2. Observe that the removal of the pure and gauge-gravitational anomalies by compactifying on RP^2 relies in an essential way on our requirement to have a definite orientation for each small plaquette in Fig. 6.1. The compactification on RP^2 alone, however, does not remove the mixed anomalies induced by the link fields.

Now, let us discuss the cancelation of the mixed gauge anomalies. To this end, we add Wess–Zumino (WZ) terms [103], which can be viewed as emerging from integrating out heavy fermions with large Yukawa couplings. The mass scale of these extra fermions is one or two orders of magnitude above the inverse lattice spacing $v \sim 10^{14}\,\text{Ge}V$. In doing so, we follow Refs. [97,101], wherein the case of a deconstructed fifth dimension has been analyzed. Let us consider in Fig. 6.1 a site $i \neq 0$ which is not in the center (a similar argumentation holds for $i = 0$). The site i is connected to its four neighboring sites j_1, j_2, j_3, and j_4 by the link fields ϕ_{i,j_1}, ϕ_{i,j_2}, $\phi_{j_3,i}$, and $\phi_{j_4,i}$. Note that ϕ_{i,j_1} and ϕ_{i,j_2} point from i to j_1 and j_2, while $\phi_{j_3,i}$ and $\phi_{j_4,i}$ point from j_3 and j_4 toward the site i. The directions of the link fields are a result of the property of our theory space, that two neighboring small plaquettes have alternating orientations. Under an infinitesimal chiral gauge transformation on the site i, the vector multiplet V_i belonging to the gauge group $U(1)_i$ transforms as $V_i \to V_i + \mathrm{i}(\overline{\Lambda}_i - \Lambda_i)$, where Λ_i is the gauge parameter. Denoting by j_i^μ the chiral current associated with the gauge transformation at the site i, we can arrange in the one-loop 3-point function $\langle 0|Tj_i^\mu j_k^\nu j_l^\rho|0\rangle$ the anomalies symmetrically among the three involved currents. In a superfield language, the anomalous variation of the link field Lagrangian $\mathcal{L}_{\mathrm{l}ink}$ corresponding to the gauge transformation Λ_i can then be written as

$$\begin{aligned}\delta_{\Lambda_i}\mathcal{L}_{\mathrm{l}ink} &= -\frac{\mathrm{i}}{12\pi^2}\int d^2\theta\,\Lambda_i\left[W^\alpha_{j_1}W_{\alpha,j_1} - 2W^\alpha_i W_{\alpha,j_1} + (j_1\leftrightarrow j_2)\right.\\ &\quad \left.-W^\alpha_{j_3}W_{\alpha,j_3} + 2W^\alpha_i W_{\alpha,j_3} + (j_3\leftrightarrow j_4)\right] + \mathrm{h.}c., \qquad (6.17)\end{aligned}$$

where $W_{\alpha,i}$ denotes the supersymmetric field strength of the gauge group $U(1)_i$. An analogous expression to Eq. (6.17) holds for the mixed anomalies $\delta_{\Lambda_0}\mathcal{L}_{\mathrm{l}ink}$, associated with a chiral gauge transformation Λ_0 on the site 0 in the center of the theory space. The mixed anomalies $\delta_{\Lambda_i}\mathcal{L}_{\mathrm{l}ink}$ can be canceled in the low-energy effective theory by appropriate WZ terms, which are constructed from local polynomials in the link fields $\phi_{i,j}$ and gauge multiplets V_i. To remove the mixed anomalies $\delta_{\Lambda_i}\mathcal{L}_{\mathrm{l}ink}$ in Eq. (6.17), we add to our model the WZ terms

$$\begin{aligned}\mathcal{L}^i_{\mathrm{W}Z} &= -\frac{1}{24\pi^2}\int d^2\theta\,\{log(\phi_{i,j_1}/v)\left[(C_1-1)W^\alpha_i W_{\alpha,i} + (C_1-1)W^\alpha_{j_1}W_{\alpha,j_1}\right.\\ &\quad \left.+(C_1+2)W^\alpha_i W_{\alpha,j_1}\right] + log(\phi_{j_3,i}/v)\left[(C_1-1)W^\alpha_i W_{\alpha,i} + (C_1-1)W^\alpha_{j_3}W_{\alpha,j_3}\right.\\ &\quad \left.+(C_1+2)W^\alpha_i W_{\alpha,j_3}\right]\} - \frac{C_2}{24\pi^2}\int d^4\theta\left[(V_iD_\alpha V_{j_1} - V_{j_1}D_\alpha V_i)(W^\alpha_i + W^\alpha_{j_1})\right.\\ &\quad \left.(V_iD_\alpha V_{j_1} - V_{j_1}D_\alpha V_i)(W^\alpha_i + W^\alpha_{j_1})\right] + (j_1\leftrightarrow j_2) + (j_3\leftrightarrow j_4) + \mathrm{h.}c., \quad (6.18)\end{aligned}$$

where C_1 and C_2 are some suitable parameters. In Eq. (6.18), the terms with factors C_1 and C_2 match in the continuum limit onto six-dimensional (6D) Chern–Simons couplings, when taking the sum of these operators around a plaquette. To see this, let us consider the quadratic plaquette shown in Fig. 6.2 as a part of the spider web theory space, which is spanned by the sites i, j, k, and l. From Eq. (6.18), we find that the sum of all terms with factors C_1 and C_2, which correspond to the plaquette, is given by

$$\mathcal{L}^{ijkl}_{\mathrm{C}S} = -\frac{C_1}{24\pi^2}\int d^2\theta\,\log(\phi_{i,j})\left[W_{\alpha,i}W^\alpha_i + W_{\alpha,j}W^\alpha_j + W_{\alpha,i}W^\alpha_j\right]$$

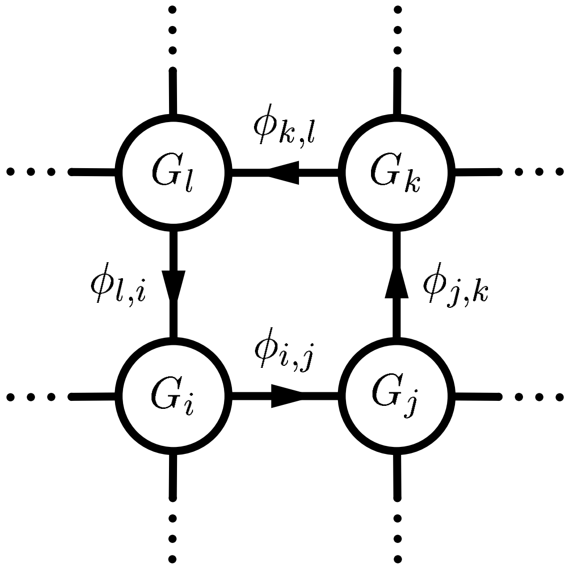

Figure 6.2. Plaquette in the spider web theory space, considered for the Chern–Simons terms.

$$-\frac{C_2}{24\pi^2}\int d^4\theta \left[(V_i D_\alpha V_j - V_j D_\alpha V_i)(W_i^\alpha + W_j^\alpha)\right]$$
$$+((i,j)\leftrightarrow(j,k)) + ((j,k)\leftrightarrow(k,l)) + ((k,l)\leftrightarrow(l,i)) + \text{h.}c., \quad (6.19)$$

where we have indicated in the last line a cyclic permutation of the four sides of the plaquette. We parameterize the link fields attached to the site i as $\phi_{i,j} = \frac{v}{\sqrt{2}}e^{(\Sigma_{i,j}+\mathrm{i}G_{i,j})/v}$ and $\phi_{l,i} = \frac{v}{\sqrt{2}}e^{(\Sigma_{l,i}+\mathrm{i}G_{l,i})/v}$. In the continuum limit, $G_{i,j}$ and $G_{l,j}$ become $G_{i,j} \to A_5$ and $G_{l,i} \to -A_6$, where A_5 and A_6 are the 5th and 6th components of the $U(1)$ gauge field of the 6D theory. Expanding around the site i, the term $\mathcal{L}_{\mathrm{C}S}^{ijkl}$ in Eq. (6.19) matches in the continuum limit onto

$$\mathcal{L}_{\mathrm{C}S}^{ijkl} \to -\frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\left[3C_1(\partial_4 A_5 - \partial_5 A_4)\partial_\mu A_\nu \partial_\rho A_\sigma - 4C_2 \partial_4 A_\mu \partial_5 A_\nu \partial_\rho A_\sigma\right], \quad (6.20)$$

which reproduces the 6D Chern–Simons term $\mathcal{L}_{\mathrm{C}S} = -(C_1/8\pi^2)\epsilon^{\alpha\beta\mu\nu\rho\sigma}\left[\partial_\alpha A_\beta \partial_\mu A_\nu \partial_\rho A_\sigma\right]$ for the choice $C_2 = (3/2)C_1$. To determine the constant C_1, note in Eq. (6.18) that the effective moduli fields $\mathrm{l}og(\phi_{i,j})$ transform under gauge transformations on the neighboring sites as $\mathrm{l}og(\phi_{i,j}) \to \mathrm{l}og(\phi_{i,j}) + 2\mathrm{i}(\Lambda_i - \Lambda_j)$. As a consequence, the anomalous variation $\delta_{\Lambda_i}\mathcal{L}_{\mathrm{W}Z}^i$ of the WZ term in Eq. (6.18) obeys $\delta_{\Lambda_i}\mathcal{L}_{\mathrm{W}Z} = -\delta_{\Lambda_i}\mathcal{L}_{\mathrm{l}ink}$ and thus cancels the mixed anomalies in Eq. (6.17) when $C_1 = 0$, *i*.e., the Chern–Simons term has to vanish.

6.5 Conclusions

In this chapter of the thesis, I have presented a model proposed by Gerhart Seidl and myself, wherein the observed fermion masses and mixing angles emerge from a deconstructed $U(1)$ theory space. We have extended a supersymmetric $SU(5)' \times SU(5)''$ product GUT, which has been previously suggested for solving the doublet–triplet splitting problem [93,104], by a deconstructed $U(1)$ theory space with disk structure. The different generations of the SM fermions live at different sites of the disk. Upon breaking the $U(1)$ product group by the link fields around the $B - L$ breaking scale $v \simeq 10^{14}$ GeV, the effective Yukawa couplings and mixing matrices of the fermions are correctly reproduced through non–renormalizable operators. The $U(1)$ charge assignment to the fermions is compatible with $SO(10)$ and, thus, free from anomalies. This is a major difference compared to usual, *e.g.*, anomalous $U(1)$ models, where the SM generations differ by flavor–dependent charges, which appears to be somewhat adhoc from a bottom–up point of view. The neutrino mass matrix receives contributions from both type–I and type–II seesaw mechanisms. Among many possibilities, we have advocated the supersymmetry breaking scenario suggested by Arkani-Hamed et. al. [90], which is unique to deconstructed models. To do so, the original disk theory space is thought to be part of a larger structure, viz., a spider web theory space. When diametrically opposite sites and links on the boundary of this space are identified, we arrive at an RP^2 manifold with nontrivial first homology group Z_2. The interactions on each plaquette are here required to be manifestly supersymmetric. The nontrivial global twist of RP^2 can be viewed as the source of supersymmetry breaking. Thus, both the fermion mass matrix structures and supersymmetry breaking can now be addressed in the same theory space, which we find interesting and economic. The choice of the charges for the link fields, which defines a direction for the links connecting two sites, is such that neighboring plaquettes have alternating orientations. As a consequence, all the sites have the same number of "ingoing" and "outgoing" link fields. This arrangement insures that the contributions to the pure and gravitational anomalies on each site vanish automatically. We cancel the mixed anomalies, along the line of Refs. [97,101], by Wess–Zumino terms, which can be considered as a result of integrating out heavy fermions with masses one or two orders of magnitude above the $B - L$ breaking scale. We have examined possible Chern–Simons terms on a rectangular plaquette, which nevertheless do not play a role in the anomaly cancelations, and have shown that they have a correct 6D continuum limit.

It would be clearly interesting to develop descriptions of our model based on gauge groups like $SO(10)$ or E_6 with a universal GUT/deconstruction scale. In possible variations of our model one could (as proposed, *e*.g., by Skiba and Smith [99]) also think of shifting anomalies between the gauge groups using a deconstructed version of the anomaly inflow mechanism known from string theory [113].

CHAPTER 7

SUMMARY AND CONCLUSIONS

In this thesis I have presented the research topics I have pursued during my Ph. D. study which primarily consist of the fermion mass hierarchy and theoretical ideas of flavor symmetry, grand unification and deconstructed theory space.

The second and third chapters of the thesis describe our study on lepton flavor violation and electric dipole moments induced by a flavor–dependent anomalous $U(1)$ gauge symmetry of string origin in models which address the fermion mass hierarchy problem via the Froggatt–Nielsen mechanism. We have derived a general set of renormalization group equations for the evolution of soft SUSY breaking parameters in the presence of higher dimensional operators appearing in these models. These results should be applicable to a large class of fermion mass models.

The anomalous $U(1)$ of string theory is broken spontaneously at a scale M_F slightly below the string scale, $M_F \sim M_{st}/50$. In the momentum regime $M_F \leq \mu \leq M_{st}$, the flavor $U(1)_A$ gauge boson sector will be active and will contribute to the evolution of the soft SUSY breaking parameters in a flavor dependent fashion. We have shown that the $U(1)_A$ sector induces significant lepton flavor violation and electric dipole moments in the SUSY breaking parameters through the RGE evolution from the string scale to the flavor symmetry breaking scale, even though this momentum range is very short. We have identified several sources of these phenomena: the $U(1)_A$ gaugino contribution to the scalar masses which is flavor dependent, a contribution proportional to the trace of $U(1)_A$ charge which is also flavor dependent, and non–proportional A–terms arising from the $U(1)_A$ gaugino vertex correction diagrams. The resulting flavor violation in the leptonic decay $\mu \to e\gamma$, and the electric dipole moments for the electron and neutron are found to be in the experimentally interesting range. Discovery of the lepton flavor violation and electric dipole moments for the electron and the neutron can shed light on one of the fundamental puzzles of Nature, viz., the origin of mass for elementary particles.

The fourth chapter of the thesis contains our work on a concrete realization of SUSY breaking using interference between anomalous $U(1)$ flavor gauge symmetry of heterotic string and strongly coupling $SU(N_c)$ sector with N_f flavors of quarks and antiquarks. We have shown that the resulting supersymmetric spectrum is that of Split SUSY. In particular, sfermions and gravitino are found to have masses of order $10^6 \div 10^8$ GeV and $10^5 \div 10^7$ GeV respectively, while gauginos and the Higgsinos are in the $10^2 \div 10^3$ GeV mass range. We have calculated the leading order supergravity corrections to the previously known results [55]. These calculations are vital to realistic models. Using these, we have presented a class of explicit models of Split SUSY. We have checked one and two–loop radiatively induced corrections to sfermion soft masses which give negative contributions –if dominant they would lead to tachyonic solutions– and have shown that they are at the safe level. We have shown that they do not lead to any conflict with current constraints. In some cases, the gluino life time is estimated to be 10^{-7} seconds, in the range where it leads to interesting collider signals, as noted by Arkani-Hamed and Dimopoulos [47].

In the fifth chapter of the thesis I have presented models for quark masses and mixings in the context of finite $SU(5)$ GUT. These theories are attractive candidates for an underlying theory, since the β functions for the gauge and Yukawa couplings vanish to all orders in perturbation theory. The requirements on the theory to be finite also leads to Yukawa–gauge unification, leading to a single coupling constant in the theory. The models presented are based on non–Abelian discrete symmetries, which seem to be necessary to obtain isolated and non–degenerate solutions to the Yukawa couplings when expressed as power series in terms of the gauge coupling. We find it interesting that realistic quark masses and mixings can be generated in such a framework. The discrete groups we have used are $(Z_4)^3 \times P$, A_4 and S_4. In the case of $(Z_4)^3 \times P$ and A_4 we have found unique nondegenerate solutions which ensure the finiteness of the models to all order of perturbation theory while in the case of S_4 we have found a model which is two–loop finite.

In the sixth chapter of the thesis, I have presented a model, wherein the observed fermion masses and mixing angles emerge from a deconstructed $U(1)$ theory space. We have extended a supersymmetric $SU(5)' \times SU(5)''$ product GUT, which has been previously suggested for solving the doublet–triplet splitting problem [93,104] , by a deconstructed $U(1)$ theory space with disk structure. The different generations of the SM fermions live at different sites of the disk. Upon breaking the $U(1)$ product group by the link fields around the $B-L$ breaking scale $v \simeq 10^{14}$ GeV, the effective Yukawa couplings and mixing matrices of the fermions are correctly reproduced through non–renormalizable operators. For the neutrino mass matrix we employ both type–I and type–II seesaw mechanisms. We have advocated the supersymmetry breaking scenario which is unique to deconstructed models. The nontrivial global twist of RP^2 can be viewed as the source of supersymmetry breaking. In our model, both the fermion mass matrix structures and supersymmetry breaking can be addressed in the same theory space. We have chosen the charge arrangement which insures the pure and gravitational anomalies on each site vanish automatically. We cancel the mixed anomalies by Wess–Zumino terms. We have examined possible Chern–Simons terms on a rectangular plaquette and have shown that they have a correct 6D continuum limit.

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APPENDICES

APPENDIX A

In this Appendix we give the formulas used in our numerical calculations. These include the definitions of the mass eigenstates and the mixings of the sfermions and the gauginos in the basis where the charged lepton and quark mass matrices are diagonal.

A.1 Sfermions, Higgsinos and Gauginos

First we give the definitions of the mass eigenstates of the SM superpartners and write the interaction part of the lagrangian in terms of these states. The fermion generations are denoted by l_i (charged leptons), ν_i (neutrinos), u_i (up–type quarks) and d_i (down–type quarks) with $i, j = 1, ..., 3$. The corresponding sfermions in two–component Weyl basis are $\tilde{f}_i$ (superpartners of the left–handed $SU(2)_L$ doublets), $\tilde{f}_i^c$ (right–handed $SU(2)$ singlets). The sfermion mass matrices are given by

$$\mathcal{L} \supset -\left(\tilde{f}^\dagger, \tilde{f}^{c\dagger}\right) \begin{pmatrix} m_L^2 & {m_{LR}^2}^\dagger \\ m_{LR}^2 & m_R^2 \end{pmatrix} \begin{pmatrix} \tilde{f} \\ \tilde{f}^c \end{pmatrix}, \tag{A.1}$$

where each entry are 3×3 mass matrix in generation space. They are given by

$$m_L^2 = \tilde{m}_f^2 + m_f^2 + M_Z^2 \cos 2\beta \left(T_{3L}^f - Q_{\text{em}}^f \sin^2\theta_W\right), \tag{A.2}$$

$$m_R^2 = \tilde{m}_{f^c}^2 + m_f^2 - M_Z^2 \cos 2\beta \left(T_{3L}^f - Q_{\text{em}}^f \sin^2\theta_W\right), \tag{A.3}$$

$$m_{LR}^2 = -A_f v \sin\beta/\sqrt{2} - m_f \mu \cot\beta \quad \text{for} \quad (f = u, \nu), \tag{A.4}$$

$$m_{LR}^2 = A_f v \cos\beta/\sqrt{2} - m_f \mu \tan\beta \quad \text{for} \quad (f = d, l). \tag{A.5}$$

Here T_3^f and Q_{em}^f are the third component of weak isospin and the electric charge of fermion f with mass m_f. Upon diagonalization

$$U^f M^2 U^{f\dagger} = (\text{diagonal}), \tag{A.6}$$

the mass eigenstates are given by

$$\tilde{f}_x = U^f_{x,i}\tilde{f}_i + U^f_{x,i+3}\tilde{f}^c_i \ \ (x = 1, ..., 6). \tag{A.7}$$

Then one obtains the following inverse transformations:

$$\tilde{f}_i = {U^f_{i,x}}^\dagger \tilde{f}_x, \tag{A.8}$$

$$\tilde{f}^c_i = {U^f_{i+3,x}}^\dagger \tilde{f}_x, \tag{A.9}$$

$$\tilde{\nu}_i = {U^\nu_{i,x}}^\dagger \tilde{\nu}_x \ \ \text{(here } x = 1, ..., 3.) \tag{A.10}$$

The mass eigenstates of the guaginos and higgsinos are admixture of both gauge and higgs sector. Depending on their electric charges they are called neutralinos and charginos. In the MSSM neutralinos are linear combinations of bino $\tilde{B}$ (superpartner of the Hypercharge gauge field B), neutral wino $\tilde{W}_3$ (superpartner of the W_3–boson), and two neutral components of higgsinos $\tilde{H}_u$ and $\tilde{H}_d$:

$$\mathcal{L} \supset -\frac{1}{2}\left(\tilde{B}, \tilde{W}_3, \tilde{H}^0_u, \tilde{N}^0_d\right) M_N \begin{pmatrix} \tilde{B} \\ \tilde{W}_3 \\ \tilde{H}^0_u \\ \tilde{N}^0_d \end{pmatrix} + \text{h.c} \tag{A.11}$$

where M_N denotes the neutralino masses, given by a 4×4 matrix:

$$M_N = \begin{pmatrix} M_{\tilde{B}} & 0 & -M_Z \sin\theta_W \cos\beta & M_Z \sin\theta_W \sin\beta \\ 0 & M_2 & M_Z \cos\theta_W \cos\beta & -M_Z \cos\theta_W \sin\beta \\ -M_Z \sin\theta_W \cos\beta & M_Z \cos\theta_W \cos\beta & 0 & -\mu \\ M_Z \sin\theta_W \sin\beta & -M_Z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix} \tag{A.12}$$

This symmetric matrix is diagonalized by an orthogonal transformation:

$$O^N M_N (O^N)^T = \text{(diagonal)}. \tag{A.13}$$

The charginos are linear combinations of wino $\tilde{W}^-$ (superpartner of W^-) and the charged component of higgsino. The mass terms for them are given by

$$\mathcal{L} \supset -\left(\bar{\tilde{W}}^-_R, \bar{\tilde{H}}^-_{dR}\right) \begin{pmatrix} M_2 & \sqrt{2}M_W \cos\beta \\ \sqrt{2}M_W \sin\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^-_L \\ \tilde{H}^-_{uL} \end{pmatrix} + \text{h.c.}. \tag{A.14}$$

This asymmetric mass matrix, which we call M_C is diagonalized by two orthogonal transformations O_L^C and O_R^C as

$$O_R^C M_C O_L^C = \text{(diagonal)}. \tag{A.15}$$

Thus the neutralinos and charginos are Majorana and Dirac spinors respectively

$$\tilde{\chi}_a^0 = \tilde{\chi}_{aL}^0 + \tilde{\chi}_{aR}^0 \text{ for } a = 1, ..., 4 \tag{A.16}$$

$$\tilde{\chi}_a^- = \tilde{\chi}_{aL}^- + \tilde{\chi}_{aR}^- \text{ for } a = 1, 2. \tag{A.17}$$

With these definitions now we write the interaction terms for the SM fermions with the neutralinos and the charginos:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= \bar{f}_i \left(N_{ixa}^{R(f)} P_R + N_{ixa}^{L(f)} P_L \right) \tilde{\chi}^0 \tilde{f}_x \\
&+ \bar{l}_i \left(C_{ixa}^{R(l)} P_R + C_{ixa}^{L(l)} P_L \right) \tilde{\chi}^- \tilde{\nu}_x \\
&+ \bar{\nu}_i \left(C_{ixa}^{R(\nu)} P_R + C_{ixa}^{L(\nu)} P_L \right) \tilde{\chi}^+ \tilde{l}_x \\
&+ \bar{d}_i \left(C_{ixa}^{R(d)} P_R + C_{ixa}^{L(d)} P_L \right) \tilde{\chi}^- \tilde{u}_x \\
&+ \bar{u}_i \left(C_{ixa}^{R(u)} P_R + C_{ixa}^{L(u)} P_L \right) \tilde{\chi}^+ \tilde{d}_x + \text{h.c.}.
\end{aligned} \tag{A.18}$$

The coefficients $N^{L(R)}$ and $C^{L(R)}$ are given as

$$\begin{aligned}
N_{ixa}^{R(l)} &= -\frac{g_2}{\sqrt{2}} \Big\{ \left(-(O^N)_{a2} - (O^N)_{a1} \tan\theta_W \right) U_{x,i}^l \\
&+ \frac{m_{l_i}}{M_W \cos\beta} (O^N)_{a3} U_{x,i+3}^l \Big\},
\end{aligned} \tag{A.19}$$

$$N_{ixa}^{L(l)} = -\frac{g_2}{\sqrt{2}} \left(\frac{m_{l_i}}{M_W \cos\beta} (O^N)_{a3} U_{x,i}^l + 2(O^N)_{a1} \tan\theta_W U_{x,i+3}^l \right), \tag{A.20}$$

$$N_{ixa}^{R(\nu)} = -\frac{g_2}{\sqrt{2}} \left((O^N)_{a2} - (O^N)_{a1} \tan\theta_W \right) U_{x,i}^\nu, \tag{A.21}$$

$$N_{ixa}^{L(\nu)} = 0, \tag{A.22}$$

$$C_{ixa}^{R(l)} = -g_2 (O_R^C)_{a1} U_{x,i}^\nu, \tag{A.23}$$

$$C_{ixa}^{L(l)} = \frac{g_2}{\sqrt{2}} \frac{m_{l_i}}{M_W \cos\beta} (O_L^C)_{a2} U_{x,i}^\nu, \tag{A.24}$$

$$C_{ixa}^{R(\nu)} = -g_2 (O_L^C)_{a1} U_{x,i}^l, \tag{A.25}$$

$$C_{ixa}^{L(\nu)} = \frac{g_2}{\sqrt{2}} \frac{m_{l_i}}{M_W \cos\beta} (O_L^C)_{a2} U_{x,i+3}^l. \tag{A.26}$$

A.2 Formulas for Lepton Flavor Violations

Here we give the formulas for the lepton flavor violations in MSSM[11], which have been used in our numerical calculations.

The LFV processes in the presence of the low energy SUSY are induced by the loop corrections from neutralinos and charginos:

$$A_a^{L,R} = A_a^{(a)L,R} + A_a^{(c)L,R} \;\; (a = 1, 2). \tag{A.27}$$

Here the amplitudes with (n) and (c) superscripts denote contributions from neutralinos and charginos respectively. The amplitudes from neutralinos are

$$A_1^{(n)L} = \frac{1}{576\pi^2} N_{ixa}^{R} N_{jxa}^{R*} \frac{1}{m_{\tilde{l}_x}^2} \mathcal{F}_1\left(\frac{M_{\tilde{\chi}_a^0}^2}{m_{\tilde{l}_x}^2}\right), \tag{A.28}$$

$$\begin{aligned} A_2^{(n)L} &= \frac{1}{32\pi^2}\frac{1}{m_{\tilde{l}_x}^2}\left(N_{ixa}^{L} N_{jxa}^{L*} \mathcal{F}_2\left(\frac{M_{\tilde{\chi}_a^0}^2}{m_{\tilde{l}_x}^2}\right)\right. \\ &+ \left. N_{ixa}^{L} N_{jxa}^{R*} \frac{M_{\tilde{\chi}_a^0}}{m_{l_j}} \mathcal{F}_3\left(\frac{M_{\tilde{\chi}_a^0}^2}{m_{\tilde{l}_x}^2}\right)\right), \end{aligned} \tag{A.29}$$

$$A_a^{(n)R} = A_a^{(n)L}|_{L\leftrightarrow R}. \tag{A.30}$$

Here $M_{\tilde{\chi}_a^0}$ $(a = 1, ..., 4)$ and $m_{\tilde{l}_x}^2$ $(x = 1, ..., 6)$ are the eigenvalues of the neutralino and the charged slepton mass matrices. The chargino contributions are

$$A_1^{(c)L} = -\frac{1}{576\pi^2} C_{iax}^{R} C_{jax}^{R*} \frac{1}{m_{\tilde{\nu}_x}^2} \mathcal{F}_4\left(\frac{M_{\tilde{\chi}_a^-}^2}{m_{\tilde{\nu}_x}^2}\right), \tag{A.31}$$

$$\begin{aligned} A_2^{(c)L} &= -\frac{1}{32\pi^2}\frac{1}{m_{\tilde{\nu}_x}^2}\left(C_{iax}^{L} C_{jax}^{L*} \mathcal{F}_5\left(\frac{M_{\tilde{\chi}_a^-}^2}{m_{\tilde{l}_x}^2}\right)\right. \\ &+ \left. C_{iax}^{L} C_{jax}^{R*} \frac{M_{\tilde{\chi}_a^-}}{m_{l_j}} \mathcal{F}_6\left(\frac{M_{\tilde{\chi}_a^-}^2}{m_{\tilde{l}_x}^2}\right)\right), \end{aligned} \tag{A.32}$$

$$A_a^{(c)R} = A_a^{(c)L}|_{L\leftrightarrow R}, \tag{A.33}$$

where $M_{\tilde{\chi}_a^-}$ and $m_{\tilde{\nu}_x}^2$ $(a = 1, 2$ and $x = 1, ..., 3)$ are the chargino and the sneutrino mass eigenvalues respectively. The functions $\mathcal{F}_i$ $(i = 1, ..., 6)$ are given by

$$\mathcal{F}_1(X) = \frac{1}{(1-X)^4}\left(2 - 9X + 19X^2 - 11X^3 + 6X^3\mathrm{Log}\,(X)\right), \tag{A.34}$$

$$\mathcal{F}_2(X) = \frac{1}{6\left(1-X\right)^4}\left(1-6X+3X^2+2X^3-6X^2\mathrm{Log}\left(X\right)\right), \quad (A.35)$$

$$\mathcal{F}_3(X) = \frac{1}{\left(1-X\right)^3}\left(1-X^2+2X\mathrm{Log}\left(X\right)\right), \quad (A.36)$$

$$\mathcal{F}_4(X) = \frac{1}{\left(1-X\right)^4}\left(16-45X+36X^2-7X^3+6\left(2-3X\right)\mathrm{Log}\left(X\right)\right), \quad (A.37)$$

$$\mathcal{F}_5(X) = \frac{1}{6\left(1-X\right)^4}\left(2+3X-6X^2+X^3+6X^2\mathrm{Log}\left(X\right)\right), \quad (A.38)$$

$$\mathcal{F}_6(X) = \frac{1}{\left(1-X\right)^3}\left(-3+4X-X^2-2\mathrm{Log}\left(X\right)\right). \quad (A.39)$$

Once amplitudes are known it is now straightforward to calculate the branching ratios for the lepton flavor violating processes:

$$\Gamma(l_j^- \to l_i^- \gamma) = \frac{\alpha_{\mathrm{em}}}{4} m_{l_j}^2 \left(|A_2^L|^2 + |A_2^R|^2\right), \quad (A.40)$$

$$B(l_j^- \to l_i^- \gamma) = \frac{\Gamma(l_j^- \to l_i^- \gamma)}{\Gamma_{\mathrm{total}}} = \Gamma(l_j^- \to l_i^- \gamma) \times \frac{\tau_{l_j}}{\hbar}. \quad (A.41)$$

Here τ_{l_j} is the lifetime of lepton l_j.

A.3 Formulas for Electric Dipole Moments

We list here the formulas for the electric dipole moments of leptons and quarks in the MSSM[32], which we have used in our numerical analysis.

The EDMs of elementary fermions are sum of neutralino, chargino and for quarks gluino contributions which we denote as d_f^N, d_f^C and d_q^G. In addition to these, the quarks receive contributions from chromoelectric and purely gluonic dimension–six operators [114]. We have not considered here the latter one, since these effects turn out to be small. The effective EDM operator d_f for a spin–$\frac{1}{2}$ particle is given by

$$\mathcal{L} = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \quad (A.42)$$

The EDM d_f in general has the following components in a supersymmetric theory:

$$d_{f_i}^N/e = \frac{\alpha}{8\pi \sin^2\theta_W} \sum_{x=1}^{6} \sum_{a=1}^{4} \mathrm{Im}\left(\mathcal{N}_{xa}^{f_i}\right) \frac{M_{\chi_a^0}}{m_{\tilde{f}_x}^2} Q_{\tilde{f}_x} A\left(\frac{M_{\chi_a^0}^2}{m_{\tilde{f}_x}^2}\right),$$

$$
\begin{aligned}
d_u^C/e &= \frac{-\alpha}{8\pi\sin^2\theta_W}\sum_{x=1}^{6}\sum_{b=1}^{2}\mathrm{Im}\left(\mathcal{C}_{xb}^{u}\right)\frac{M_{\tilde{\chi}_b^+}}{m_{\tilde{d}_x}^2}\left(B\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{d}_x}^2}\right)-\frac{1}{3}A\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{d}_x}^2}\right)\right),\\
d_d^C/e &= \frac{-\alpha}{8\pi\sin^2\theta_W}\sum_{x=1}^{6}\sum_{b=1}^{2}\mathrm{Im}\left(\mathcal{C}_{xb}^{d}\right)\frac{M_{\tilde{\chi}_b^+}}{m_{\tilde{u}_x}^2}\left(\frac{2}{3}A\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{u}_x}^2}\right)-B\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{u}_x}^2}\right)\right),\\
d_{q_i}^G/e &= -\frac{\alpha_s}{3\pi}\sum_{x=1}^{6}\mathrm{Im}\left(G_x^{q_i}\right)\frac{M_{\tilde{g}}}{m_{\tilde{q}_x}^2}Q_{\tilde{q}_i}A\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{u}_x}^2}\right), \qquad \text{(A.43)}
\end{aligned}
$$

where

$$
\begin{aligned}
A(X) &= \frac{1-X^2+2X\log X}{(1-X)^3},\\
B(X) &= \frac{3-4X+X^2+2\log X}{(1-X)^3}. \qquad \text{(A.44)}
\end{aligned}
$$

Here $M_{\tilde{\chi}_a^0}$, $M_{\tilde{\chi}_b^+}$ and $M_{\tilde{g}}$ are the neutralino, chargino and the gluino masses respectively. $m_{\tilde{f}_x}^2$ $(x = 1, ..., 6)$ are the eigenvalues of the sfermion mass matrices. The coefficients N_{xa}^f, C_{xb}^f and G_x^q are given by

$$
\begin{aligned}
\mathcal{N}_{xa}^{f_i} &= N_{xa}^{R(f_i)}N_{xa}^{L(f_i)*} = \left[\sqrt{2}\tan\theta_W Q_{f_i}\left(O^N\right)_{1a}U_{i+3,x}^f - K_f\left(O^N\right)_{a'a}U_{i,x}^f\right]\\
&\times \left[-\sqrt{2}\left\{\tan\theta_W\left(Q_{f_i}-T_{f_i}\right)\left(O^N\right)_{a1}+T_{3f_i}\left(O^N\right)_{a2}\right\}U_{i,x}^{f*}\right.\\
&- \left.K_f\left(O^N\right)_{a'a}U_{i+3,x}^{f*}\right],\\
\mathcal{C}_{xb}^u &= C_{xb}^{R(u)}C_{xb}^{L(u)*} = K_u\left(O_R^C\right)_{b2}^{*}U_{1,x}^d\left[\left(O_L^C\right)_{b1}U_{4,x}^d - K_d\left(O_L^C\right)_{b2}U_{4,x}^d\right]^*,\\
\mathcal{C}_{xb}^d &= C_{xb}^{R(d)}C_{xb}^{L(d)*} = K_d\left(O_L^C\right)_{b2}^{*}U_{i1}^u\left[\left(O_R^C\right)_{b1}U_{1,x}^u - K_u\left(O_R^C\right)_{b2}U_{4,x}^u\right]^*,\\
G_x^{q_i} &= U_{i,x}^q U_{i+3,x}^{q*}, \qquad \text{(A.45)}
\end{aligned}
$$

where $K_u = m_u/(\sqrt{2}M_W\sin\beta)$ and $K_{l,d} = m_{l,d}/(\sqrt{2}M_W\cos\beta)$. O^N and O_L^C, O_R^C matrices diagonalize the neutralino and chargino mass matrices respectively. The index a' of O^N in the neutralino contribution formula takes value of 3(4) for $T_{3f} = -\frac{1}{2}(\frac{1}{2})$. The chromoelectric dipole moments $\tilde{d}_q$ for quarks are defined as

$$
\mathcal{L}_{CEDM} = -\frac{i}{2}g_s\tilde{d}_q\bar{q}T^a\sigma_{\mu\nu}\gamma_5 qG^{\mu\nu a}. \qquad \text{(A.46)}
$$

The contributions to $\tilde{d}_q$ from neutralino, chargino and gluino are given by

$$
\tilde{d}_{q_i}^N = \frac{g^2}{32\pi^2}\sum_{x=1}^{6}\sum_{a=1}^{4}\mathrm{Im}\left(N_{xa}^{q_i}\right)\frac{M_{\chi_a^0}}{m_{\tilde{q}_x}^2}A\left(\frac{M_{\chi_a^0}^2}{m_{\tilde{q}_x}^2}\right),
$$

$$\tilde{d}_q^C = \frac{-g^2}{32\pi^2} \sum_{x=1}^{6} \sum_{b=1}^{2} \mathrm{Im}\left(C_{xb}^q\right) \frac{M_{\tilde{\chi}_b^+}}{m_{\tilde{q}_x}^2} A\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{q}_x}^2}\right),$$

$$\tilde{d}_{q_i}^G = \frac{\alpha_s}{4\pi} \sum_{x=1}^{6} \mathrm{Im}\left(G_x^{q_i}\right) \frac{M_{\tilde{g}}}{m_{\tilde{q}_x}^2} C\left(\frac{M_{\tilde{\chi}_b^+}^2}{m_{\tilde{q}_x}^2}\right), \tag{A.47}$$

where

$$C(X) = \frac{1}{6\left(1-X\right)^2}\left(10X - 26 + \frac{2X\log X}{1-X} - \frac{18\log X}{1-X}\right). \tag{A.48}$$

We use the QCD sum rule based estimate [115] to evaluate the neutron and the deuteron EDMs:

$$d_n = 0.7\left(d_d - 0.25 d_u\right) + 0.55\left(\tilde{d}_d + 0.5\tilde{d}_u\right),$$

$$d_D = 0.5\left(d_d + d_u\right) - 0.6\left(\tilde{d}_d - \tilde{d}_u + 0.3\left(\tilde{d}_d + \tilde{d}_u\right)\right). \tag{A.49}$$

Here the running factors are $\tilde{d}_q\left(1\,\mathrm{GeV}\right) \simeq 0.91\tilde{d}_q\left(\mathrm{M}_Z\right)$ and $d_q(1\,\mathrm{GeV}) \simeq 1.2 d_q(\mathrm{M}_Z)$.

VITA

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Abstract: In this thesis I have presented the findings of my research pursued during my Ph. D. study. The purpose of this thesis was to study different theoretical ideas in high energy physics model building addressed primarily towards understanding the fermion mass problem and the gauge hierarchy problem. These include: Anomalous flavor $U(1)$ symmetry and its experimental implications, finite GUT models with discrete family symmetry, and a product GUT model in a 2D deconstructed theory space. The second and third chapters of the thesis describe our study of lepton flavor violation (LFV) and electric dipole moments (EDM) induced by a flavor–dependent anomalous $U(1)$ gauge symmetry of string origin. The models considered also address the fermion mass hierarchy problem successfully. We have shown that the $U(1)$ sector induces significant LFV and EDMs through the SUSY breaking parameters. These effects arise via renormalization group evolution of the parameters in the momentum regime between the string and the anomalous $U(1)$ breaking scale. The fourth chapter of the thesis contains our work on a concrete realization of SUSY breaking using interference between the anomalous $U(1)$ flavor gauge symmetry and a strongly coupled $SU(N_c)$, leading to the so called Split SUSY spectrum where the sfermions and the gravitino acquire masses of order $10^5 \div 10^8$ GeV while the gauginos and the Higgsinos have masses of order $10^2 \div 10^3$ GeV. We have calculated the leading order supergravity corrections and have presented a class of explicit models of Split SUSY which are phenomenologically consistent. In the fifth chapter I have presented models for realistic quark masses and mixings in the context of finite $SU(5)$ GUT wherein the β functions for the gauge and the Yukawa couplings vanish to all orders in perturbation theory. The models presented are based on non–Abelian discrete symmetries. In the case of $(Z_4)^3 \times P$ and A_4 symmetries we have found models finite to all order of perturbation theory while in the case of an S_4 symmetry we have found a model which is two–loop finite. In the sixth chapter I have presented a model wherein the observed fermion masses and mixing angles emerge from a deconstructed $U(1)$ theory space with a disk structure in $SU(5)' \times SU(5)''$ product GUT. Below the $B - L$ breaking scale, the effective Yukawa couplings and mixing matrices of the fermions are correctly reproduced through non–renormalizable operators. In our model, both the fermion mass matrix structures, and supersymmetry breaking (as a global twist of RP^2) can be addressed in the same theory space consistent with phenomenology and anomaly cancelation.

ADVISOR'S APPROVAL Dr. Kaladi S. Babu