COMPUTER-AIDED RATING OF LARGE

DIAMETER ROTATION BEARINGS

By

URS THOMAS MOEKER
//
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Thesis Approved:

Thomas A. Cook
Thesis Adviser

P. M. Moretti

P. F. Good

Ing. L. Houg

Norman N. Durham
Dean of the Graduate College

PREFACE

An interactive computer aided analysis program is developed to assist in the rating of large diameter, special configuration, case hardened, four point contact ball bearings. The rating equations are developed from the results of two models. The finite element method is used for one model. The second model will consist of equations that are developed assuming the bearing race remain rigid when loaded. With dimensional parameters from several bearings, moment, radial, and thrust loads will be applied to both models. As needed, relationships between the two models will be obtained for the rating equations.

I would like to express my gratitude to the individuals who assisted me in my thesis and during course work at Oklahoma State University. I am very grateful to my major advisor, Mr. Thomas A. Cook, for his help, guidance and support. I would also like to thank the other committee members, Dr. Ing T. Hong, Dr. James K. Good, and Dr. Peter M. Moretti, for their help and advisement during the course of this work.

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NOMENCLATURE

A	bolt cross sectional area
a	semimajor axis of ellipse of contact
BF	Brinell factor
b	semiminor axis of ellipse of contact
c	conformity
D	ball diameter
DC	diametral clearance
d_m	ball path diameter
E	modulus of elasticity
F	normal force
F_c	basic load rating
F_D	dynamic race force
F_i	race force at θ_i
ΣF_p	bolt preload force sum
F_s	static race force
f_p	bolt preload force
f_1	rigid model race force for DC = 0
f_2	rigid model race force for DC > 0
ΣK_B	bolt stiffness sum
ΣK_I	interface stiffness sum
k_B	bolt stiffness
k_c	clearance adjustment factor
k_I	interface element stiffness

k_S	spring constant
L	fatigue life
l	bolt length
M	moment load
M_D	dynamic moment load
M_S	static moment load
N	number of balls
N_B	number of bolts
N_I	number of interface elements
PR	ball path radius
R	radial load
R_D	dynamic radial load
R_i	race force moment arm at θ_i
R_S	static radial load
r	raceway minor radius
r'	raceway major radius
r_b	ball radius
S'	von Mises stress
S_F	finite element model slope
S_{FM}	finite element model slope relating race force and moment load
S_{FR}	finite element model slope relating race force and radial load
S_R	rigid model slope
S_x	x-axis normal stress
S_y	y-axis normal stress
S_z	z-axis normal stress
S_1	maximum normal stress
S_2	middle normal stress

S_3	minimum normal stress
T	thrust load
T_D	dynamic thrust load
T_S	static thrust load
X_B	number of balls x ball path diameter
z	depth into raceway
α	contact angle
Δ	difference between spar element length and zero strain length
δ	permanent deformation
δ_i	race deformation at θ_i
δ_o	maximum race deformation
θ_i	location of ball i
θ_ℓ	load zone
ν	Poisson's ratio

CHAPTER I

INTRODUCTION

Large diameter bearings carry heavy rotating loads in applications as diverse as mobile cranes and offshore drilling rigs. Most large diameter bearings are of the four-point contact type as shown in the cross sectional view illustrated in Figure 1, where the balls contact the inner and outer raceways at four points and can accommodate combinations of moment, radial, and thrust loads, which are the rule in large bearing applications. From Pritts and Myers (1976) and Pritts (1973), Figure 2 illustrates the classical, theoretical load distribution under combined moment and thrust loading. Radial force is relatively insignificant in most applications. The front portion of the bearing carries the heaviest loading in a compressive direction, while the rear portion of the bearing transmits a lesser tensile force. Plotted in polar coordinates, this load distribution appears as a figure eight as shown in Figure 3. Advantages of four-point contact ball bearings include low cost, light weight and compact design. The contact angle, α, is formed by a line through the point of contact with the raceway and the ball center and the major plane of the bearing as illustrated in Figure 4. Larger contact angles increase a bearing's thrust capacity and decrease its radial capacity. For high thrust and/or moment loads, bearings are usually designed with a 60- to 70-degree contact angle.
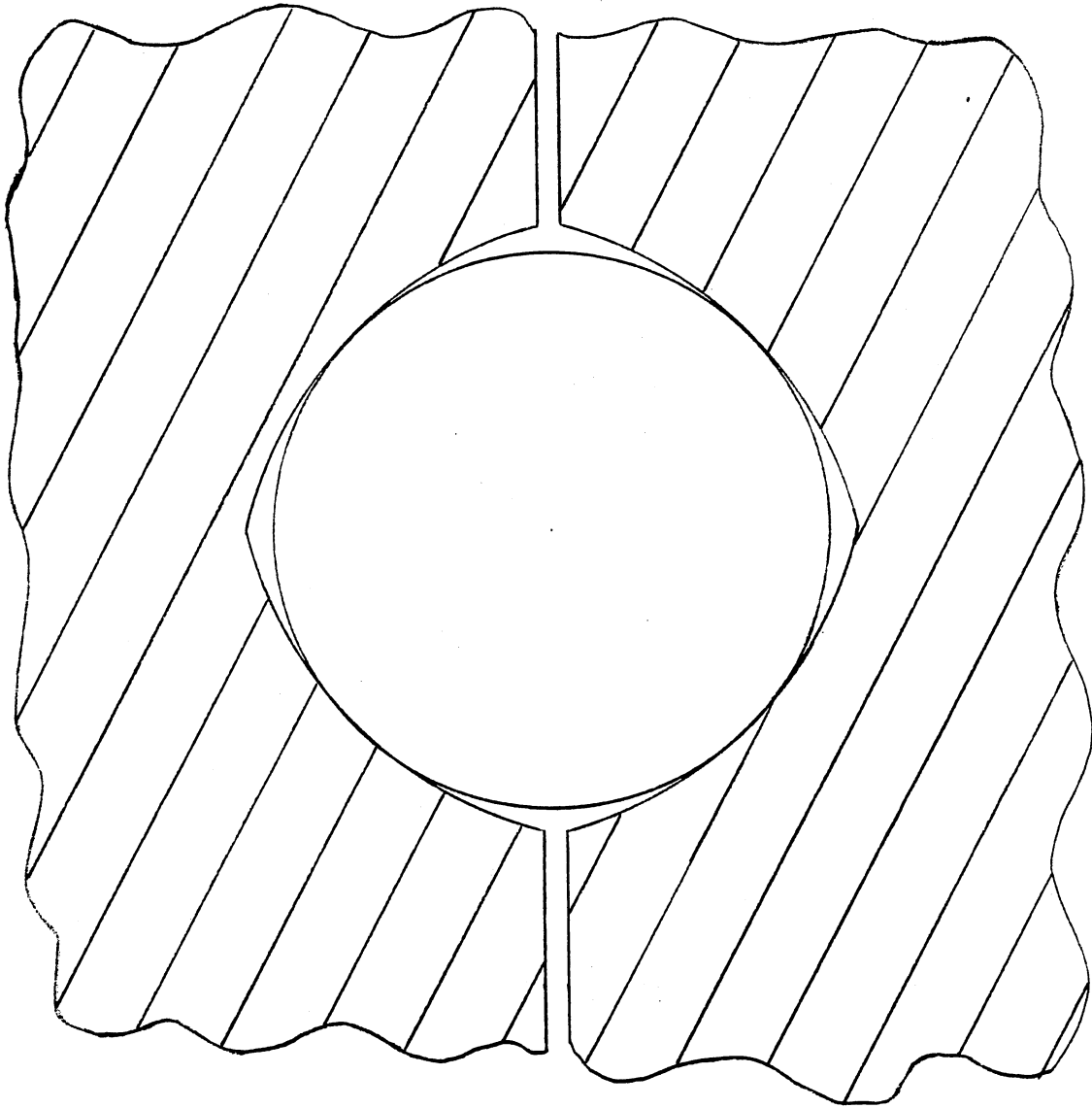
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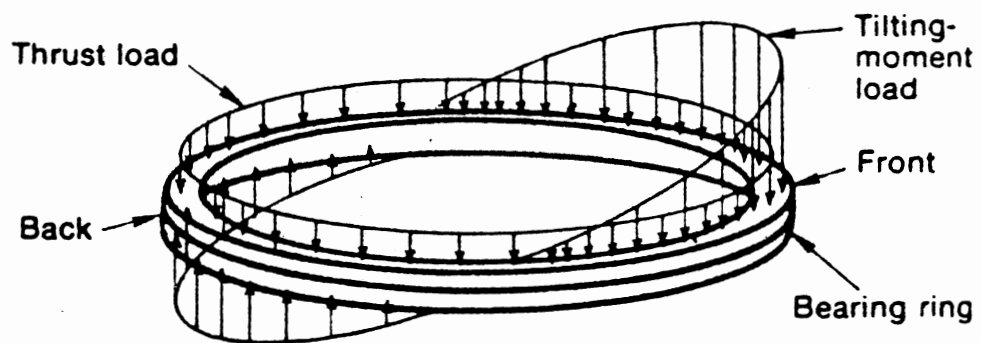
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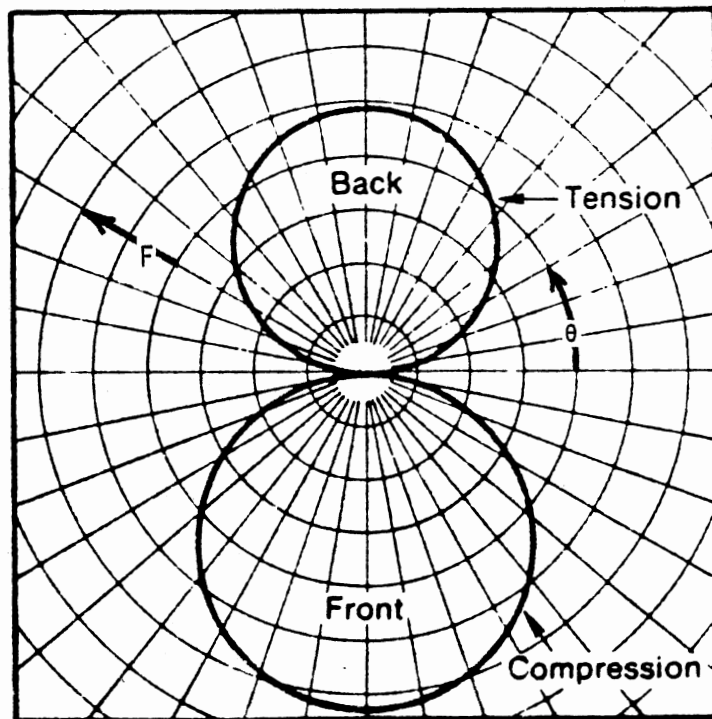
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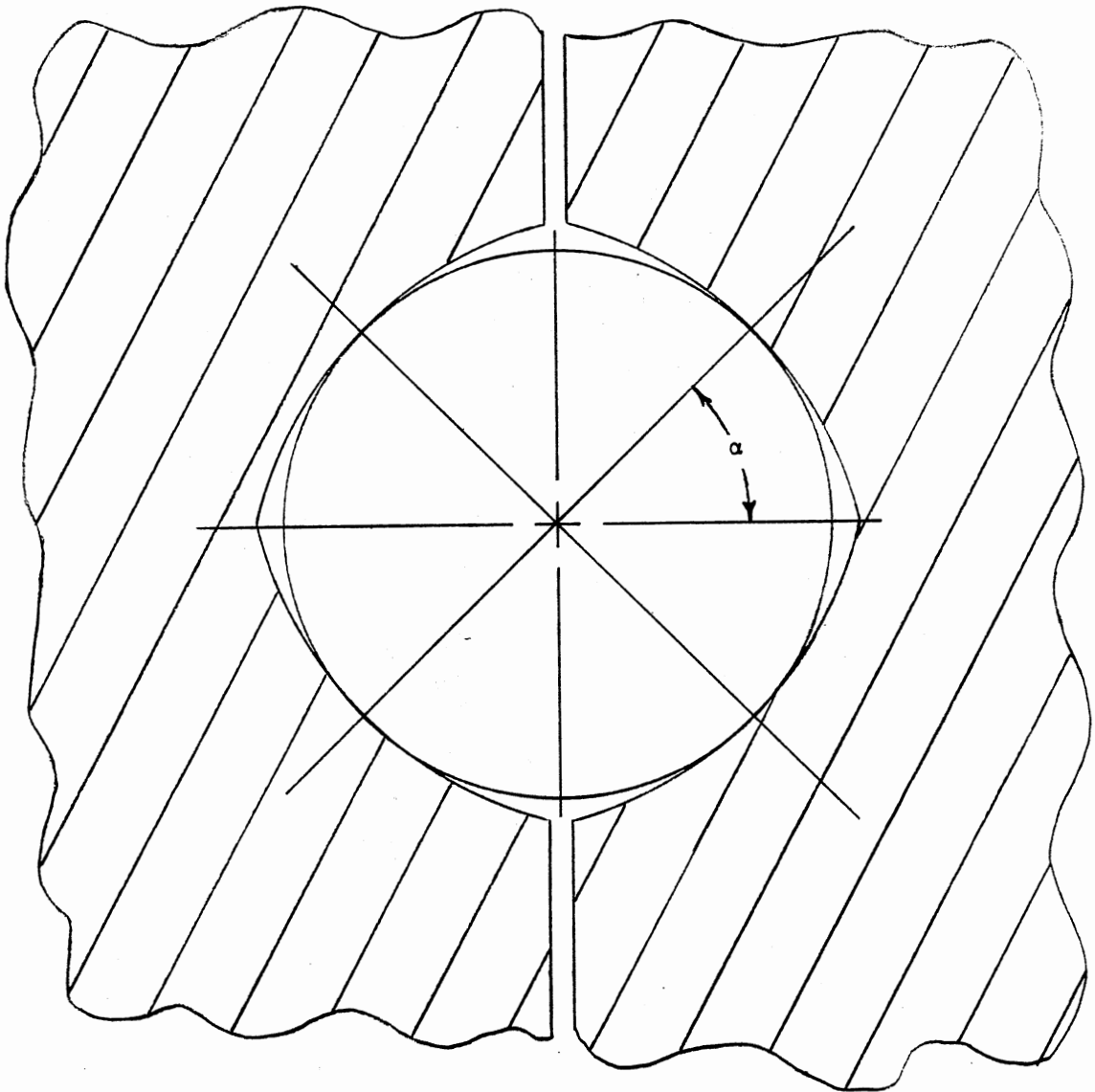
Figure 4.   Contact Angle in Four Point Contact Raceway

Several key features distinguish these bearings from their smaller high-speed counterparts. First, large diameter bearings have significantly higher values of conformity, which is the ratio of raceway minor diameter to ball diameter. Second, large diameter bearings usually are supplied with integral gearing on either the inner or the outer race. Third, the raceways are case hardened whereas they are through-hardened in small bearings. The case hardening is done by local heating through electrical induction. Finally, the procedure for determining operating loads and corresponding capacity ratings for large diameter bearings is not as straightforward as the procedure for small bearings. In the past, large bearings were generally overdesigned. The conservative design approach helped to offset some of the uncertainties involved in predicting the performance of large diameter bearings. A more reliable and less conservative rating method is needed in order to reduce cost and weight while, at the same time, improving bearing performance.

Objective

The objective of this study is to develop an interactive computer-aided analysis program to assist in the rating of large diameter, special configuration, case hardened, four point contact ball bearings. The program will be developed for use on any personal computer with an MS-DOS or PC-DOS operating system. Inputs for the program are bearing geometry, number and size of balls, and race material properties. Output will consist of static and dynamic capacity rating for moment, radial, and thrust applications, life of bearing due to static load, table of von Mises stress vs depth into raceway, and

required depth of hardened case.

Literature Survey

A review of the available literature uncovered a large amount of material available under the general subject of bearings. Concentrating on large diameter rotation bearings drastically reduced the available material. There is a broad variety of topics including; history of large diameter bearing development, manufacturing methods, optimizing mounting structures, mounting methods, fastener forces, bearing capacity and life, subsurface stresses and failure modes. Only a limited portion of this information is pertinent to this investigation.

Pritt and Jones (1967) discuss the use of large diameter bearings, with special reference to application in power cranes and shovels, gun turrets, aerial basket devices, and utility derricks. Also discussed are the features of various style bearings and guideposts for selecting bearings and for designing suitable bearing mountings. Sague and Rumbarger (1977) present the identification and illustration of core crushing in large diameter, special configuration, case hardened ball and roller bearings. Case depth and core hardness are described and a design criteria is presented to avoid core crushing failure. Pritts (1973) discusses how mounting structures, bearings, and fastners should be considered as an interdependent system to determine design requirements and applications limitations. Also, different bearing types and methods for calculating their capacity are presented. A test procedure is outlined which is used to gain additional knowledge of optimum swing bearing system design. Sague (1978) presents guidelines on how to recognize core crushing failures and their causes. Also, a

technique that determines safe operating loads based on case depth and core hardness is described. Pritts and Myers (1976) discuss a practical guide for evaluating large bearings, machine structures and fasteners with prototype testing. The AFBMA Standard (1978) presents load ratings and fatigue life calculations for ball bearings in order to estimate the suitability of a particular bearing for a given application. Several reference texts were useful to this investigation including; Harris (1966), Shigley and Mitchell (1983), Roark and Young (1975), and Boresi, et. al. (1978). These texts provide equations for calculating raceway forces, deformations, and stresses, and information on material parameters.

Organization

The rating criteria, based on the failure modes obtained from the literature survey, are discussed in Chapter II. The two bearing models that are used to describe the relationship between the race forces and the applied moment, radial and thrust loads that create them, are described in Chapter III. Chapter IV presents the results obtained from the two models and describes how the equations that represent the relationships between the race force and the applied moment, radial and thrust loads for all the bearings are determined. Chapter V describes how the interactive rating program is developed using the rating criteria from Chapter II and equations from Chapter IV. Also included in this chapter is an example of program input and output, comparison of ratings with established standards, and limitations of the program. Conclusions and recommendations for improvement are presented in Chapter VI.

CHAPTER II

RATING CRITERIA

In order to properly rate the bearings, a set of rating criteria must be established. From the literature survey it was determined that there are four failure modes that compete to cause failure of a large diameter rotation bearing. From AFBMA (1978), Sague (1978), Sague and Rumbarger (1977), Pritts and Myers (1976), Pritts (1973), and Pritts and Jones (1967) the failure modes are; 1) fatigue spalling, 2) static capacity, 3) surface failure, and 4) core crushing. The load that a bearing can withstand for an extended period of time is also of interest. Therefore, from AFBMA (1978) and Pritts and Jones (1967) the dynamic capacity of a bearing will also be included as a rating criteria. The rating criteria, how they are determined and how they relate to the performance of a bearing will be discussed in the following sections.

Fatigue Spalling

This failure mode is created by cyclic stressing of the rolling elements and racepaths causing the metal to eventually fatigue. The fatigue failure usually occurs in the raceway since the ball axis of rotation tends to change with each orbit. Furthermore, since stress is usually higher at the inner raceway contact than at the outer raceway contact, failure generally occurs on the inner raceway first. Fatigue spalling can be surface or subsurface initiated and can result from soft

spots in the racepath, grinding burns, or other manufacturing irregularities. Service life of large bearings is not ended when the first evidence of pitting occurs. Progression of pitting failure into actual service or operational failure may require many months. Fatigue life, L, is defined as the total operating life until the first microscopic evidence of pitting is seen and is expressed in millions of revolutions. Harris (1966) and AFBMA (1978) express fatigue life of a ball-raceway point contact, subjected to a normal force, F, as

$$L = (\frac{F_c}{F})^3 \tag{1}$$

in which

$$F_c = 7450 \ (\frac{2f}{2f - 1})^{0.41} \frac{(1 - \gamma)^{1.39}}{(1 + \gamma)^{1/3}} \ (\frac{\gamma}{\cos \alpha})^{0.3} \ D^{1.8} \ N^{-1/3} \tag{2}$$

where: F_c = basic load rating

f = r/D

γ = D cos α/d_m

D = ball diameter

N = number of balls

r = raceway minor radius

d_m = ball path diameter

Static Capacity

The static failure mode is characterized by Brinell marks. These marks occur when an overload is applied to a bearing resulting in plastic indentions of the rolling elements and raceways. AFBMA (1978) defines the limit for static loading of bearings as the point at which

permanent deformation equals 0.0001 of the rolling element diameter. However, because large diameter bearings are not subjected to the high rotational velocities or the continuous operation common to small bearings, this amount of deformation is not a critical problem. Pritt & Jones (1967) and Pritt & Myers (1976) suggest a deformation limit of 0.0003 of the rolling element diameter as a more realistic value. From Harris (1966) the following equation is obtained for describing permanent deformation, δ, measured normal to the surface, in ball-raceway point contact.

$$\delta = 2.5 \times 10^{-11} \frac{F_s^2}{D^3} [1 + \frac{\gamma}{(1 - \gamma)}](1 - \frac{1}{2f}) \tag{3}$$

where F_s is the limiting static raceway force. As previously defined, for large diameter bearings subjected to slow and/or intermittent operation the permanent deformation limit is

$$\delta = BF (D) \tag{4}$$

where, BF is the Brinell factor. The Brinell factor represents the deformation limit that is multiplied by the ball diameter to obtain the amount of permanent deformation. Substituting equation (4) into equation (3) and solving for the limiting static raceway force gives

$$F_s = \sqrt{\frac{4.0 \times 10^{10} BF D^4}{[1 + \frac{\gamma}{(1 - \gamma)}](1 - \frac{1}{2f})}} \tag{5}$$

Surface Failure

This mode of failure is caused by deterioration of the surface due to inadequate lubrication, excessive sliding effects, and contamination by foreign materials. Adequate seal and proper lubrication are required to prevent this mode of failure. Fretting corrosion of non-rotating bearings is also a form of surface failure. Since this failure mode is preventable with proper maintenance it will not be considered in this investigation.

Core Crushing

As previously described, large diameter bearings have a case-core boundary which is defined as the depth at which the hardness in the raceway reduces to Rockwell C50. An example of this decrease in hardness as a function of depth into the raceway is shown in Figure 5. Application of concentrated ball loading upon a hardened case will produce significant subsurface stresses which extend down into the core material. If these subsurface stresses exceed the yield strength of the core material, then deterioration of the core will result. This deterioration will reduce the support for the hardened case material. Two types of failure may then occur. The core can be crushed by physically pressing the hardened case material into the core or, the lack of core support may cause the case to crack and disintegrate. In order to prevent this type of failure it is necessary to extend the case to the depth where the subsurface stresses produced from the maximum loaded ball will not exceed the yield strength of the core material. The distortion-energy theory is used to calculate the von Mises stress for increasing depth into the bearing raceway. The stress values are
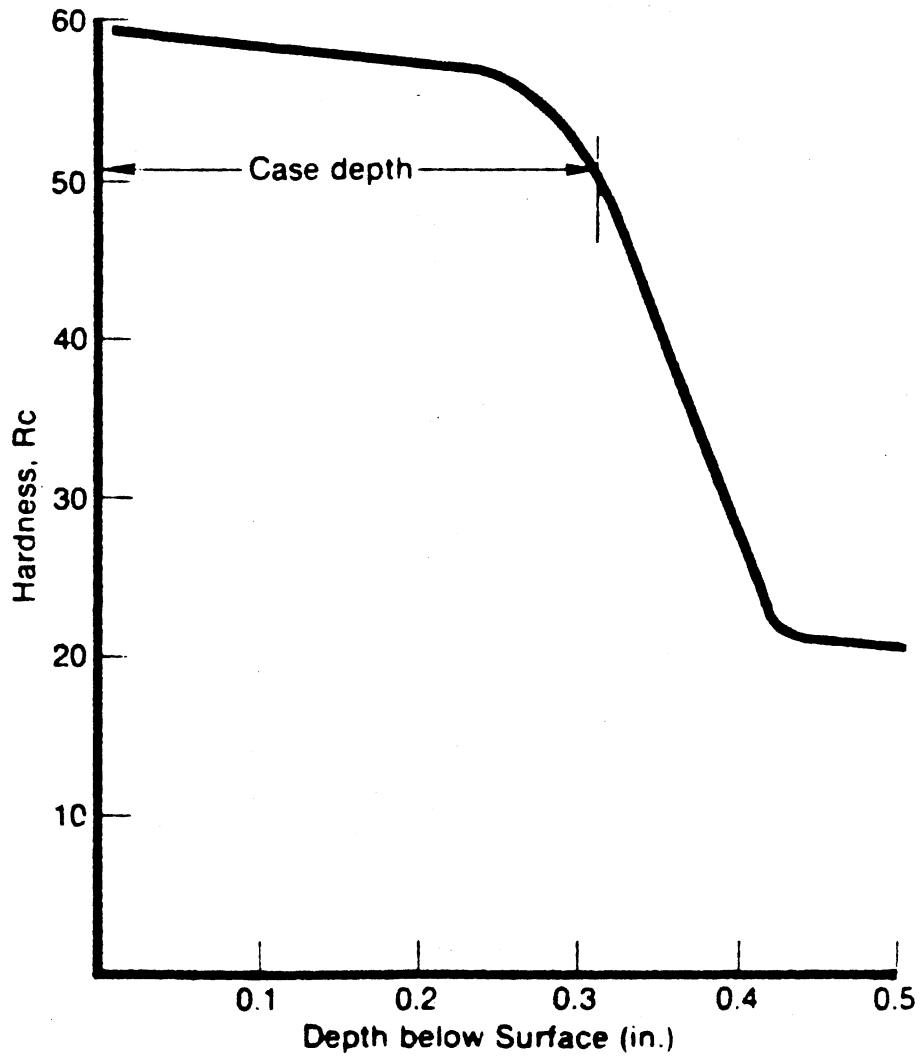
Figure 5. Hardness Traverse of Bearing Raceway

compared to the yield strength of the core material to determine the minimum case depth. From Shigley and Mitchell (1983) the equation to determine the von Mises stress, S', for a triaxial stress state is

$$S' = \sqrt{\frac{(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2}{2}} \qquad (6)$$

The three stresses S_1, S_2 and S_3, that represent the normal stresses S_x, S_y and S_z, are arranged so that $S_1 > S_2 > S_3$. The following equations, obtained from Boresi, et. al. (1978), express the normal stresses at a point on the z-axis. The point is at the distance z from the origin, which lies in the surface of contact of the ball and raceway as shown in Figure 6. The stresses act on orthogonal planes perpendicular to the x-, y-, and z-axis respectively. The normal stresses are

$$S_x = [m \ (\Omega_x + \nu\Omega_x')]\frac{b}{p}$$

$$S_y = [m \ (\Omega_y + \nu\Omega_y')]\frac{b}{p} \qquad (7)$$

$$S_z = - [\frac{m}{2} \ (\frac{1}{n} - n)]\frac{b}{p}$$

in which

$$m = \frac{2k}{k'^2 \ G(k')}$$

$$n = \sqrt{\frac{k^2 + k^2 \ (z/b)^2}{1 + k^2 \ (z/b)^2}} \qquad (8a)$$

$$p = \frac{1}{A + B} \ (\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2})$$
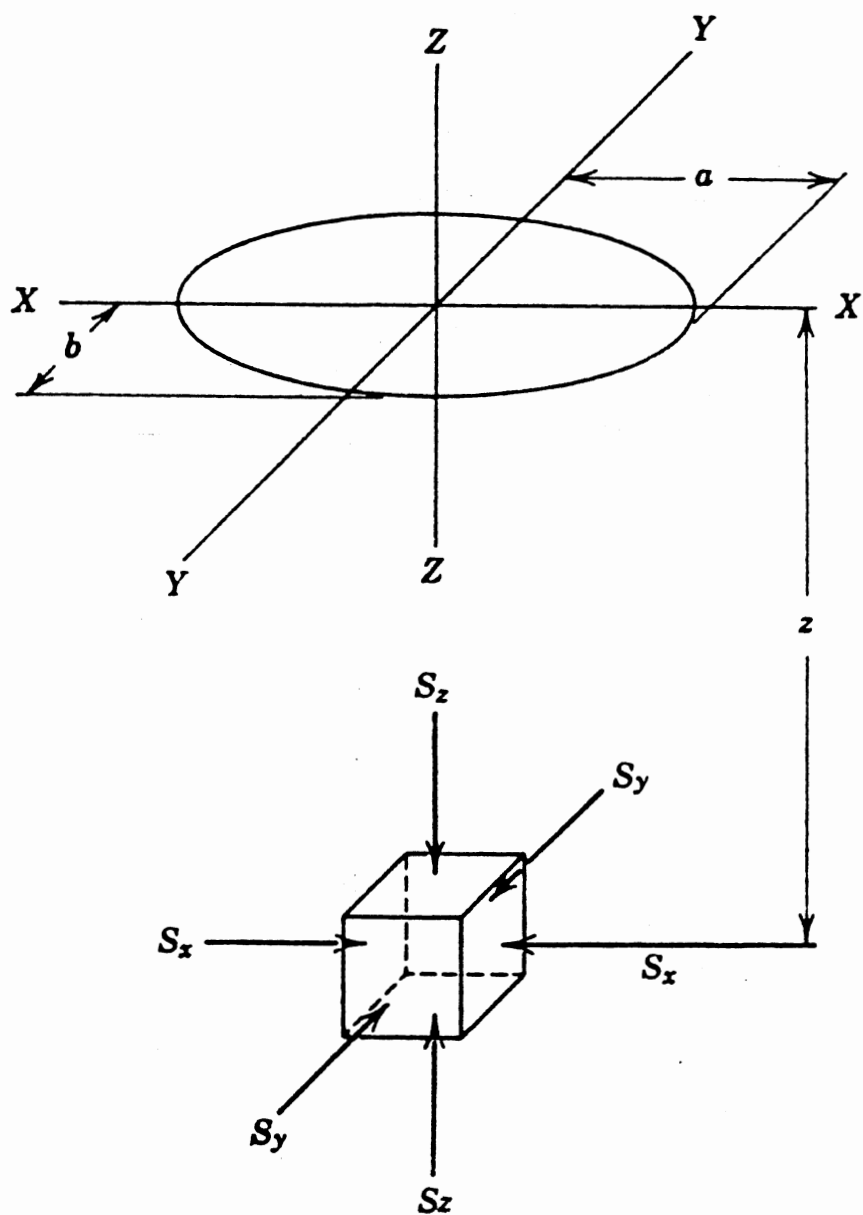
Figure 6. Normal Stresses Occuring on an Element on
the Z-Axis Below the Contact Surface

$$b = \sqrt[3]{\frac{3k\ G(k')}{2\pi}\ (pF)} = ka$$

$$\Omega_x = -\frac{1-n}{z} + k\frac{z}{b}\ [F(\phi,k') - H(\phi,k')]$$

$$\Omega'_x = -\frac{n}{k^2} + 1 + k\frac{z}{b}\ [(\frac{1}{k^2})\ H(\phi,k') - F(\phi,k')]$$

$$\Omega_y = \frac{1}{2n} + \frac{1}{2} - \frac{n}{k^2} + k\frac{z}{b}\ [(\frac{1}{k^2})\ H\ (\phi,k') - F(\phi,k')]$$

$$\Omega'_y = -1 + n + k\frac{z}{b}\ [F(\phi,k') - H(\phi,k')]$$

$$\phi = \tan^{-1}\ (\frac{b}{kz})$$

$$A = \frac{1}{2}\ (\frac{1}{r_b} - \frac{1}{r'})$$

$$B = \frac{1}{2}\ (\frac{1}{r_b} - \frac{1}{r})$$

(8b)

$$k' = 1 - k^2 \tag{9}$$

$$k = \sqrt{\frac{G(k')}{\frac{B}{A}[K(k') - G(k')] + K(k')}} \tag{10}$$

and   a = semimajor axis of ellipse of contact

   b = semiminor axis of ellipse of contact

   z = depth into raceway

   r_b = ball radius

   r = raceway minor radius

   r' = raceway major radius

   E = modulus of elasticity

ν = Poisson's ratio

$F(\phi,k')$ and $H(\phi,k')$ are incomplete elliptic integrals of the first and second kind and are expressed as,

$$F(\phi,k') = \int_0^\phi \frac{d\theta}{\sqrt{1 - k'^2 \sin^2\theta}}$$

$$H(\phi,k') = \int_0^\phi \sqrt{1 - k'^2 \sin^2\theta}\ d\theta$$

From Sokolnikoff (1939) these equations are evaluated with the use of infinite series as shown below

$$F(\phi,k') = \phi + \frac{1}{2} k'^2 \int_0^\phi \sin^2\theta d\theta + \frac{1\cdot3}{2\cdot4} k'^4 \int_0^\phi \sin^4\theta d\theta + \ldots$$

$$\ldots + \frac{1\cdot3\cdot5 \ldots (2n - 1)}{2\cdot4\cdot6 \ldots 2n} k'^{2n} \int_0^\phi \sin^{2n}\theta d\theta$$

$$(11)$$

$$H(\phi,k') = \phi - \frac{1}{2} k'^2 \int_0^\phi \sin^2\theta d\theta - \frac{1}{2\cdot4} k'^4 \int_0^\phi \sin^4\theta d\theta - \ldots$$

$$\ldots - \frac{1\cdot3\cdot5 \ldots (2n - 3)}{2\cdot4\cdot6 \ldots 2n} k'^{2n} \int_0^\phi \sin^{2n}\theta d\theta$$

$K(k')$ and $E(k')$ are complete elliptic integrals of the first and second kind and are expressed as

$$K(k') = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k'^2 \sin^2\theta}}$$

$$G(k') = \int_0^{\pi/2} \sqrt{1 - k'^2 \sin^2 \theta} \; d\theta$$

From Sokolnikoff (1939) these equations are evaluated with the use of infinite series as shown below

$$K(k') = \frac{\pi}{2} \left[1 + \left(\frac{1}{2}\right)^2 k'^2 + \left(\frac{1\cdot3}{2\cdot4}\right)^2 k'^4 + \ldots + \left(\frac{1\cdot3\cdot5 \ldots (2n-1)}{2\cdot4\cdot6 \ldots 2n}\right)^2 k'^{2n} \right.$$

$$\left. G(k') = \frac{\pi}{2} \left[1 - \left(\frac{1}{2}\right)^2 k'^2 - \left(\frac{1\cdot3}{2\cdot4}\right)^2 k'^2 \frac{k'^4}{3} - \ldots - \left(\frac{1\cdot3\cdot5 \ldots (2n-1)}{2\cdot4\cdot6 \ldots 2n}\right)^2 \frac{k'^{2n}}{(2n-1)} \right. \right]$$

(12)

These equations are solved by method of iteration. The variable k, is given an initial value and k' is calculated using equation (9). The complete and incomplete elliptic integrals of the first and second kind can then be found using equations (11) and (12). With the given bearing parameters and normal force, equations (8a) and (8b) are solved. Equation (10) is used to determine a new value of k and the process is repeated until the solution converges. Then the normal stresses are calculated from equations (7) and equation (6) is used to determine the von Mises stress.

Dynamic Capacity

Dynamic capacity is a measure of the loading which a bearing can withstand for a stated number of revolutions, resulting in a stated probability of breakdown of the raceway or ball surfaces from rolling fatigue (Pritts and Jones, 1967). Dynamic capacity is usually expressed as the loading which a bearing can withstand to provide a 90% chance of

survival for 1,000,000 revolutions. The fatigue life equation presented earlier is used to find the limiting dynamic race force. Solving equation (1) for the normal force gives.

$$F = \frac{F_c}{L^{1/3}} \tag{13}$$

The limiting dynamic race force, F_D, is obtained by substituting a value of one million revolutions for the life in equation (13).

$$F_D = F_c \tag{14}$$

CHAPTER III

DISCUSSION OF MODELS

Detailed drawings for several different sizes of bearings were obtained from a manufacturer of large diameter bearings. The ball diameters for these bearings range from 1.0 to 1.75 inches with ball path diameters ranging from 16.562 to 37.0 inches. All of the bearings have a contact angle of 60 degrees and a conformity of 1.08. A typical bearing cross section is shown in Figure 7. The dimensional constants correspond to the specific bearing values listed in Table I. Using the rating criteria described in the previous section, the race force corresponding to the maximum loaded ball can be found. A relationship between this force and the moment, radial, and thrust load that create it needs to be determined for all the bearings. In order to obtain the moment, radial, and thrust loads that create the race force, at the maximum loaded ball, two models are used. In an actual bearing application the raceways bend and flex when loaded. Therefore, one model will use the finite element method which is an ideal way to realistically and accurately model a flexible bearing. However, because of the complexity of this method, computer computation time is lengthy, which makes it impractical for repeated use. This model will be referred to as the finite element model in future discussion.

For a second model, in which we assume that the races remain rigid when loaded, a set of equations are developed to describe the race forces due to applied moment, radial and thrust loads. Because of their

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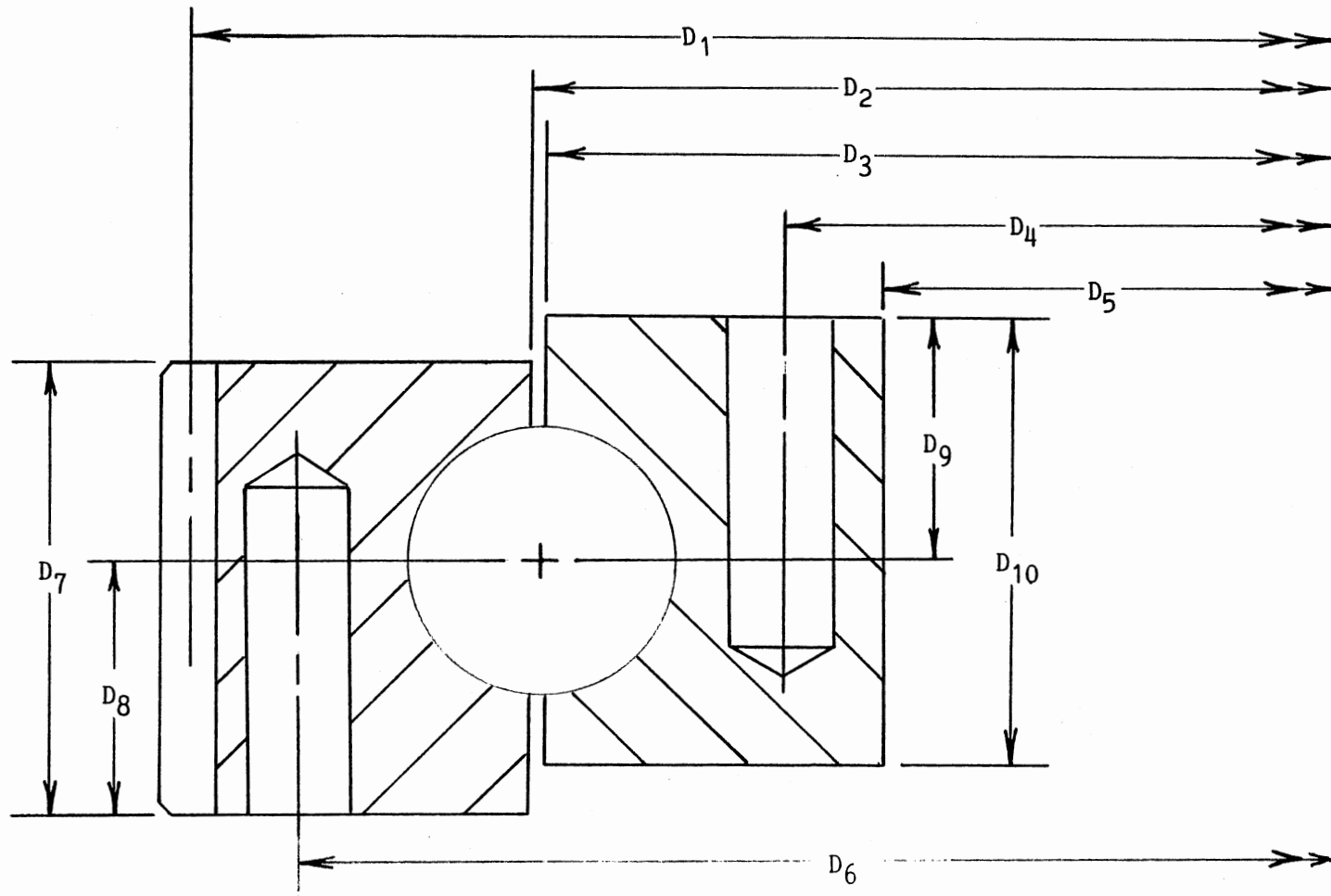
Figure 7. Typical Bearing Cross Section and Dimensional Constants Corresponding to Table I (A) & (B)

TABLE I (A)

BEARING PARAMETERS AND DIMENSIONAL CONSTANTS CORRESPONDING TO FIGURE 7

Bearing Number	D_1 (in)	D_2 (in)	D_3 (in)	D_4 (in)	D_5 (in)	D_6 (in)	D_7 (in)	D_8 (in)	D_9 (in)	D_{10} (in)
1	20.06	16.625	16.47	13.75	12.56	19.16	2.66	1.25	1.25	2.66
2	22.5	17.562	17.38	14.62	13.0	20.37	3.13	1.375	1.375	3.13
3	28.0	22.125	21.875	18.62	17.0	25.38	3.18	1.4375	1.4375	3.18
4	26.0	22.11	21.91	19.5	18.312	24.5	2.13	1.0	1.0	2.13
5	43.43	37.05	36.95	32.75	30.5	40.75	3.62	1.812	1.562	3.62
6	39.0	35.34	35.28	33.46	31.43	37.17	3.38	1.125	1.312	3.56

TABLE I (B)

BEARING PARAMETERS AND DIMENSIONAL CONSTANTS CORRESPONDING TO FIGURE 7

| Bearing Number | Number of Balls | Ball Diameter (in) | Ball Path Diameter (in) | Inner Race | | Outer Race | |
				Number of Bolts	Bolt Diameter (in)	Number of Bolts	Bolt Diameter (in)
1	34	1.5	16.652	17	0.75	16	0.75
2	36	1.5	17.5	18	0.78125	18	0.78125
3	38	1.75	22.0	19	0.875	18	0.75
4	65	1.0	22.06	13	0.625	18	0.625
5	64	1.75	37.0	32	1.0625	30	1.0625
6	72	1.5	35.312	36	0.75	30	0.75

simplicity, these equations do not require the lengthy computer computation time required by the finite element model and are practical for repeated usage. In further discussion this model will be referred to as the rigid model. As needed, a set of relationships between the two models will be determined so as to combine the speed of the rigid model with the accuracy of the finite element model. These relationships will describe the moment, radial, and thrust loads that create the race force determined from the rating criteria.

Rigid Model

Assuming a rigid body displacement of the inner race with respect to the fixed outer race, due to a moment, radial, or thrust load, equations for the determination of the maximum force on the race, due to ball contact, are developed. Figure 8 illustrates a partial top view of the ball locations in the bearing. The position of each ball is designated by θ_i. The ball path radius, PR, is the distance from the center of the bearing to the center of a ball.

Moment

Applying a moment load, M, to the inner race, the forces, F_i, that are produced from the contact of the balls and the races act along the contact angle, α, as shown in the bearing cross section illustrated in Figure 9. The sum of these forces acting on a race at each ball location multiplied by their respective moment arms, R_i, is equal to the applied moment and is expressed as

$$M = \sum_{i=1}^{N} F_i R_i \qquad (15)$$
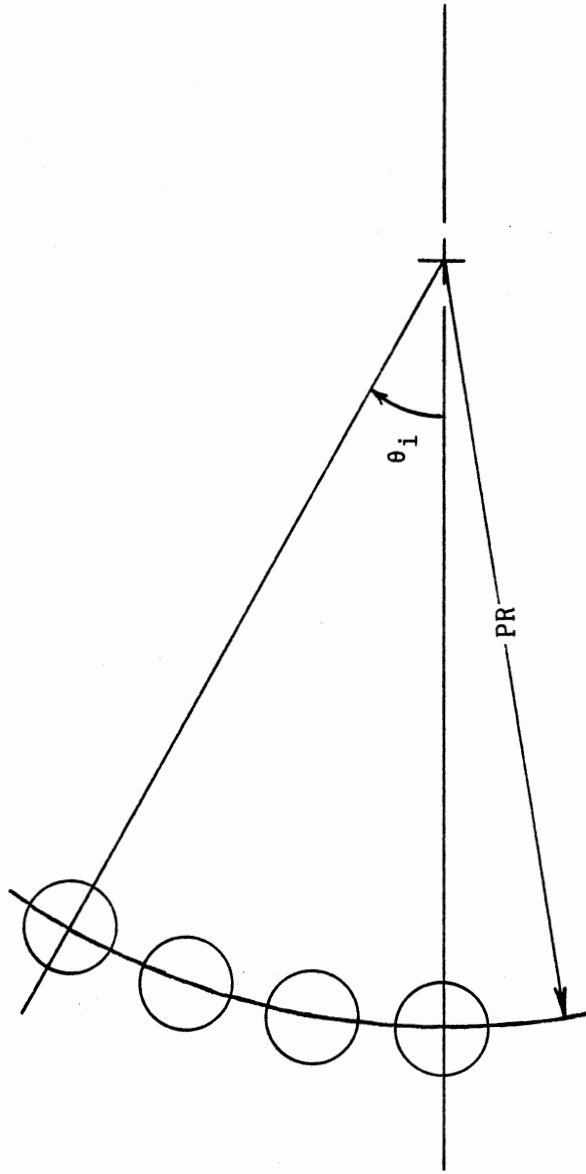
Figure 8. Partial Top View of Ball Locations in a Bearing
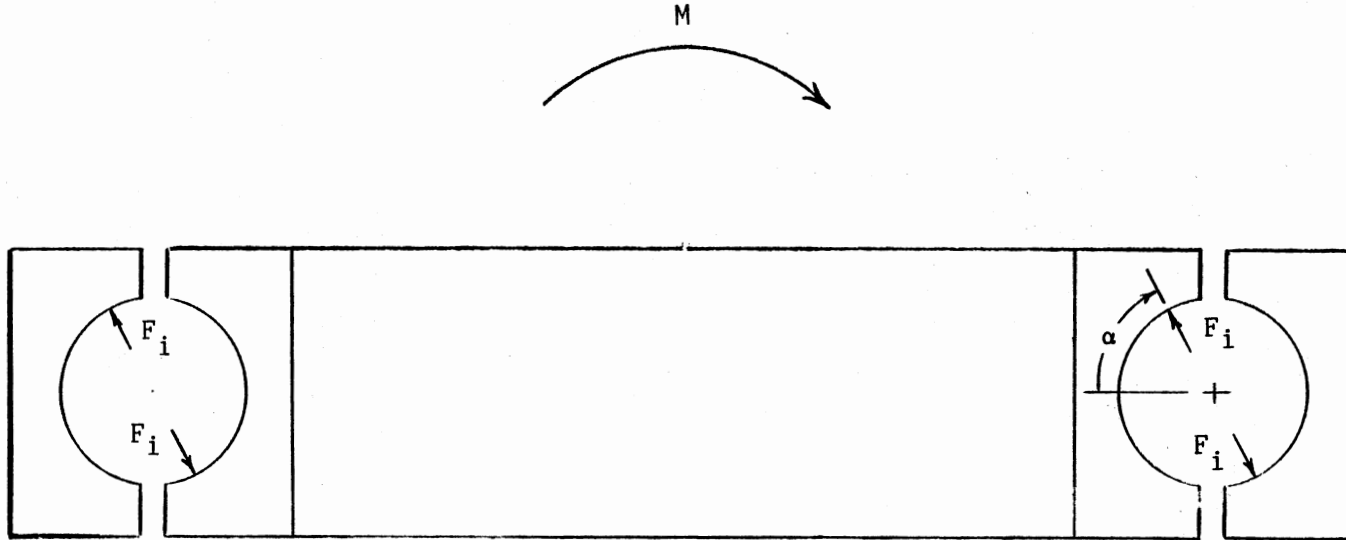
Figure 9. Bearing Cross Section Showing Race Forces due to Moment Loading

where N is the number of balls in the bearing. From Figure 10 we see that

$$R_i = PR \sin \alpha \cos \theta_i \tag{16}$$

Substituting equation (16) into equation (15) and rearranging yield

$$M = PR \sin \alpha \sum_{i=1}^{N} F_i \cos \theta_i \tag{17}$$

An expression for the force, F_i, on the race at θ_i needs to be determined. From Roark and Young (1975) the following equation, based on Hertzian contact theory is obtained,

$$\delta_i = 1.04 \sqrt[3]{\frac{F_i^2 C_E}{K_D^2}} \tag{18}$$

where, δ_i is the deformation of the race at θ_i due to F_i, and

$$C_E = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \tag{19}$$

$$K_D = \frac{2r_b r}{r - r_b} \tag{20}$$

Defining conformity as

$$c = \frac{r}{r_b} \tag{21}$$

Solving equation (21) for r and substituting into equation (20) yields

$$K_D = \frac{2cr_b}{c - 1} \tag{22}$$
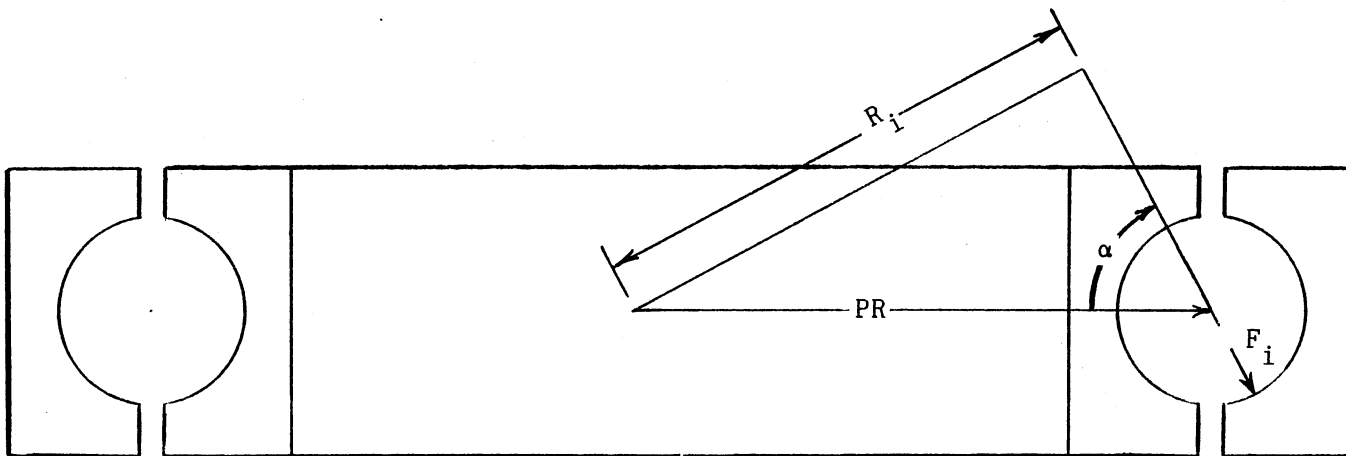
Figure 10.   Bearing Cross Section Showing Race Force Moment Arms

The values for Poisson's ratio and modulus of elasticity for steel are obtained from Shigley and Mitchell (1983)

$$E_1 = E_2 = 30 \text{ Mpsi}$$

$$\nu_1 = \nu_2 = 0.3$$

Substituting these values into equation (19) and combining with equation (22) and equation (18) yields

$$\delta_i = 1.274 \times 10^{-5} \sqrt[3]{\frac{F_i^2 (c - 1)}{cr_b}} \tag{23}$$

Solving equation (23) for F_i gives

$$F_i = 2.198 \times 10^7 \sqrt{\frac{\delta_i^3 cr_b}{(c - 1)}} \tag{24}$$

Letting

$$c_1 = 2.198 \times 10^7 \sqrt{\frac{cr_b}{(c - 1)}} \tag{25}$$

we have,

$$F_i = c_1 \delta_i^{3/2} \tag{26}$$

Assuming a rigid body displacement of the inner race with respect to the fixed outer race, the deformation of the race along the contact angle, at each ball position is

$$\delta_i = \delta_o \cos \theta_i \tag{27}$$

where, δ_o is the maximum race deformation due to the maximum loaded ball. Combining equation (26) and equation (27) gives

$$F_i = c_1 \, \delta_o^{3/2} \cos^{3/2} \theta_i \tag{28}$$

Substituting equation (28) into equation (17) yields

$$M = PR \, c_1 \, \delta_o^{3/2} \sin \alpha \sum_{i=1}^{N} \cos^{5/2} \theta_i \tag{29}$$

Solving equation (29) for δ_o yields

$$\delta_o = \left[\frac{M}{PR \, c_1 \sin \alpha \sum_{i=1}^{N} \cos^{5/2} \theta_i} \right]^{2/3} \tag{30}$$

Using equation (30) the maximum deformation of the race can be determined, then using equation (26) the maximum force on a race due to an applied moment load is calculated. An interactive computer program is developed utilizing these equations. The inputs for the program are; moment load, ball path diameter, conformity, contact angle, ball diameter, and number of balls. The outputs include the location of each ball and the race force and deformation at each of these locations. The program listing is given in Appendix A.

Radial

Applying a radial load, R, to the inner race, the forces produced from the contact of the balls and the races act along the contact angle as shown in the bearing cross section illustrated in Figure 11. Unlike the moment load application, the number of balls that carry the load is a function of the diametral clearance, DC. The equation for determining the load zone, θ_ℓ, which is obtained from Harris (1966), is
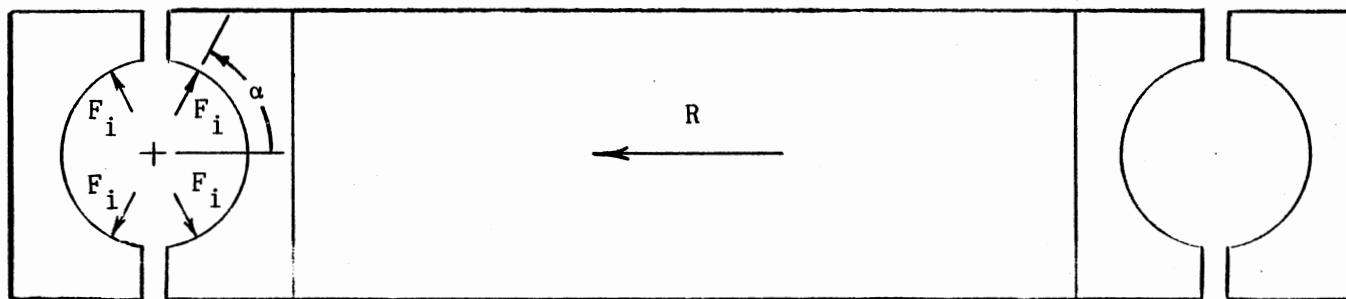
Figure 11.   Bearing Cross Section Showing Race Forces due to Radial Loading

$$\theta_\ell = \cos^{-1} [\frac{DC}{2\,\delta_o}]$$ (31)

The sum of the forces acting on a race at all the ball locations within the load zone is equivalent to the radial load and is expressed as

$$R = \sum_{i=1}^{N} 2\,F_i \cos \alpha \cos \theta_i$$ (32)

where N is the number of balls in the load zone. Substituting equation (28) into equation (32) and rearranging gives

$$R = 2\,c_1\,\delta_o^{3/2} \cos \alpha \sum_{i=1}^{N} \cos^{5/2} \theta_i$$ (33)

Solving equation (33) for δ_o yields

$$\delta_o = [\frac{R}{2\,c_1 \cos \alpha \sum_{i=1}^{N} \cos^{5/2} \theta_i}]^{2/3}$$ (34)

Using equation (34) the maximum deformation of the race can be determined, then using equation (26) the maximum force on a race due to an applied radial load is calculated. An interactive computer program is developed utilizing these equations. The inputs for the program are; radial load, ball path diameter, diametral clearance, conformity, contact angle, ball diameter, and number of balls. The outputs include the location of each ball and the race force and deformation at each of these locations. The program listing is given in Appendix B.

Thrust

Applying a thrust load, T, to the inner race, the forces produced from the contact of the balls and the races act along the contact angles as shown in the bearing cross section illustrated in Figure 12. The sum of these forces acting at all the ball locations is equivalent to the thrust force and is expressed as

$$T = \sum_{i=1}^{N} F_i \sin \alpha \qquad (35)$$

Where N is the number of balls in the bearing. All of the individual forces are equal and equation (35) reduces to

$$T = N F_i \sin \alpha \qquad (36)$$

Solving equation (36) for F_i yields

$$F_i = \frac{T}{N \sin \alpha} \qquad (37)$$

which is the maximum force on a race due to an applied thrust load. The deformation of the race due to this load can be found by solving equation (26) for the deformation

$$\delta_i = [\frac{F_i}{c_1}]^{2/3} \qquad (38)$$

An interactive computer program is developed utilizing these equations. The inputs for the program are; thrust load, contact angle, conformity, ball diameter, and number of balls. The outputs include the ball spacing and the race force and deformation for a ball. The program
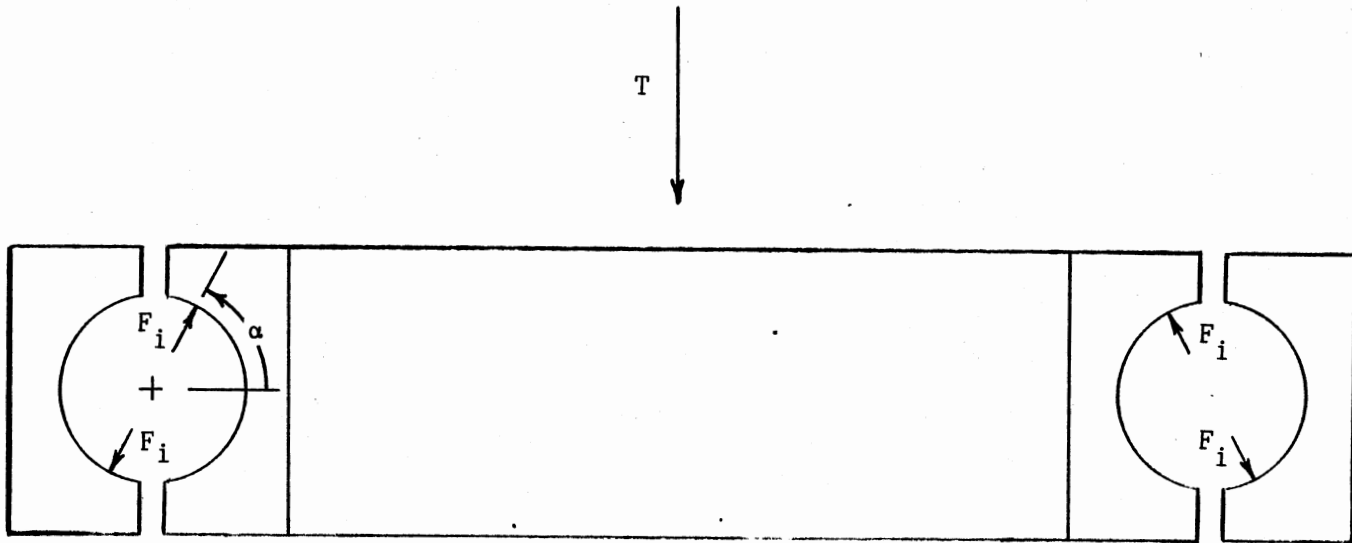
Figure 12. Bearing Cross Section Showing Race Forces due to Thrust Loading

listing is given in Appendix C.

Finite Element Model

ANSYS, a general purpose, proprietary finite element computer program, was used to model the bearing. From DeSalvo and Swanson (1985), ANSYS employs the matrix displacement method of structural analysis. The deformations of a structure are represented by the motions of a predefined number of node points. These motions are referred to as degrees of freedom. In any ANSYS linear static problem, there may be up to six degrees of freedom per node. The degree of freedom set at a given node depends upon the type of elements attached to that node. Elements are simple building blocks providing the response of pieces of the structure formulated as beams, plates, shells, solids, or springs. Most elements require the definition of material properties such as modulus of elasticity, Poisson's ratio, and density. Some elements also require real constants for geometric properties. From the element information and node definitions, ANSYS will indirectly form a set of simultaneous equations. After the displacements are obtained from the simultaneous equations, by a variation of Gaussian elimination, the stresses and reaction forces can be computed in the structure. For this analysis an academic version of ANSYS Revision 4.2B was used. The academic version has a wave front limit of 200. The wave front is determined by the size and/or the complexity of the model.

Inner Race and Supporting Structure

The inner race of the bearing is modeled since this is where the greater forces occur. In an actual bearing application the inner race is bolted to a relatively rigid plate or structure. In order to model the inner race as realistically as possible, a "rigid" plate in the form of a flat ring, and the connecting bolts are included in the model. The external moment, radial, and thrust forces are applied to this ring. The race and the rigid ring are modeled using a three-dimensional isoparametric solid element. The element is defined by eight nodal points, having three degrees of freedom at each node, and orthotropic material properties. The modulus of elasticity, E, for the steel race, obtained from Shigley and Mitchell (1983), is 30Mpsi. A modulus of elasticity of 3000Mpsi is used for the rigid ring. Side, top, and isometric views of the race and ring models are shown in Figures 13, 14, and 15, respectively. A cross section of the race and the ring is shown in Figure 16. The race dimensions for each bearing are obtained from Table 1, and the ring dimensions are generated from this information. In order to realistically translate the applied forces from the ring to the race, three dimensional interface elements are used between the race and the ring. The interface element is defined by two nodal points, having three degrees of freedom at each node, an interface stiffness, and an interface coefficient of friction. The elements represent two surfaces which may maintain or break physical contact and may slide relative to each other. They are capable of supporting only compression in the direction normal to the surfaces and Coulomb friction in the tangential direction. The interface stiffness is given a value of 10000 lb/in and the interface friction coefficient, which is obtained from
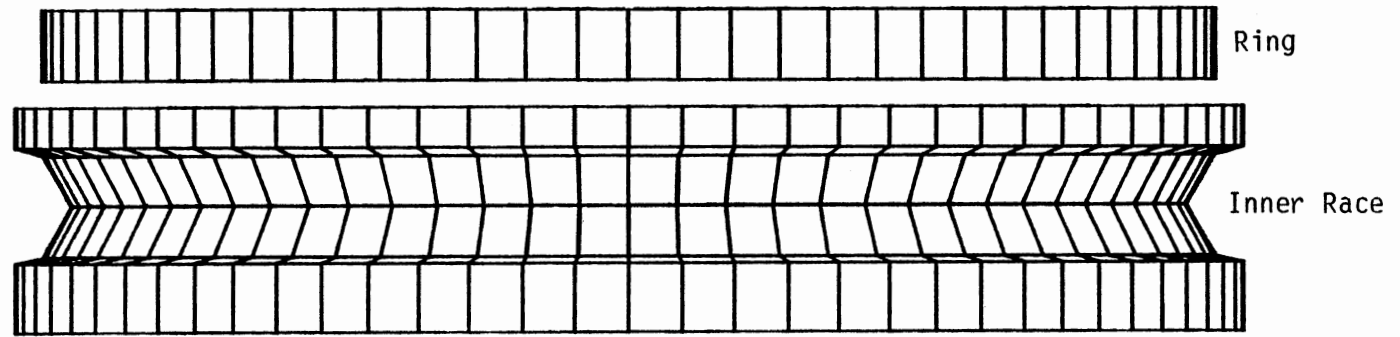
Ring

Inner Race

Figure 13.   Side View of Finite Element Inner Race and Ring Model
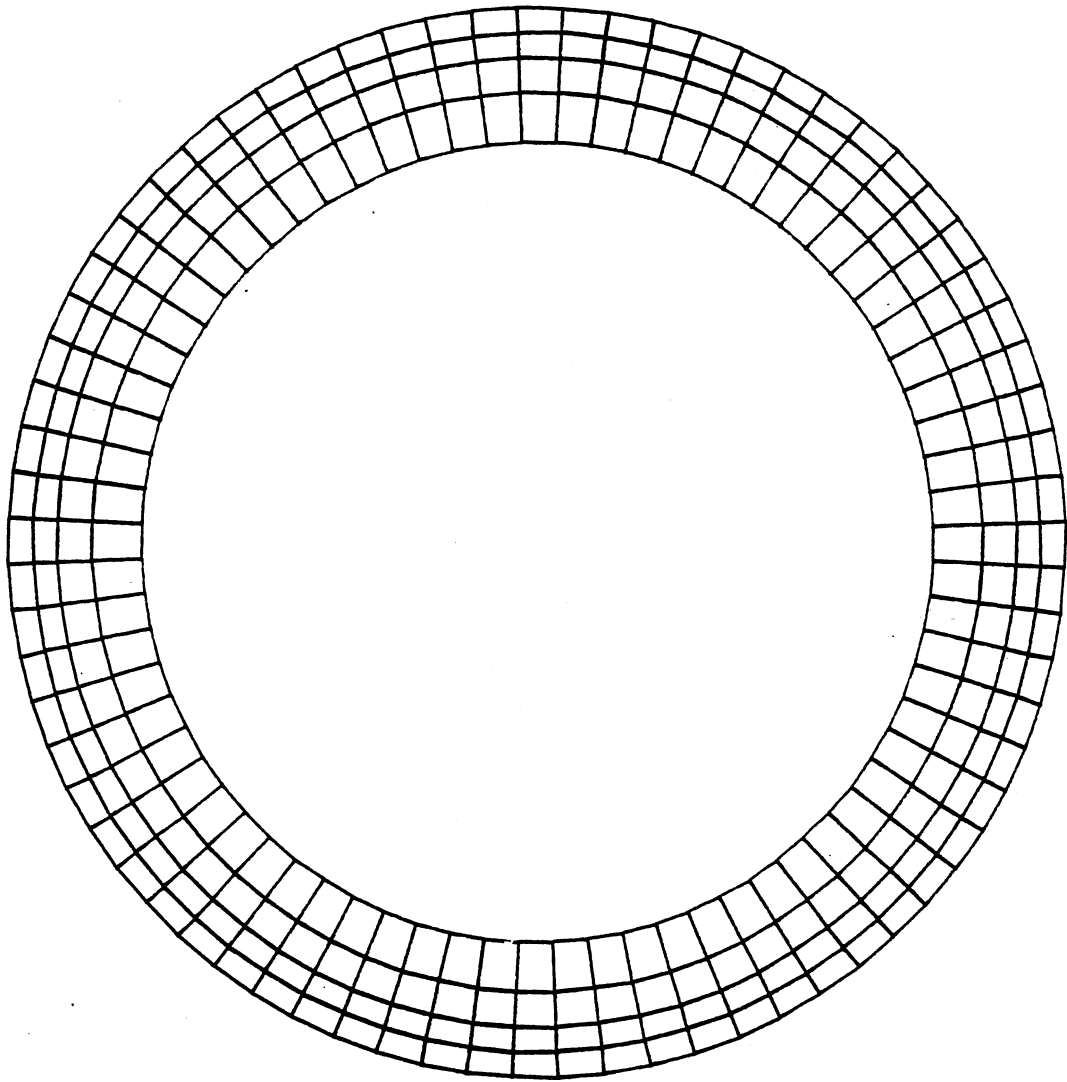
Figure 14. Top View of Finite Element Inner Race and Ring Model
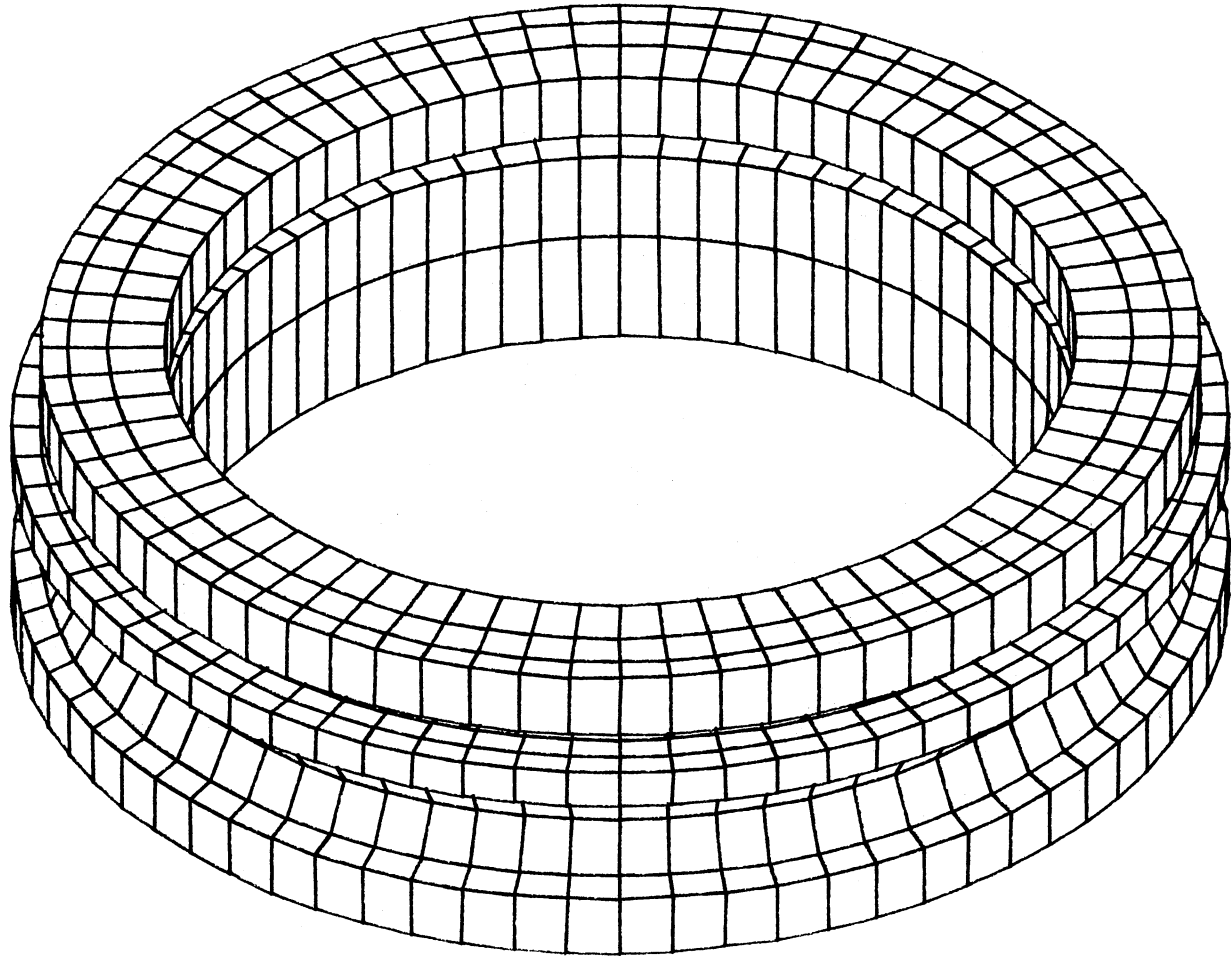
Figure 15. Isometric View of Finite Element Inner Race and Ring Model
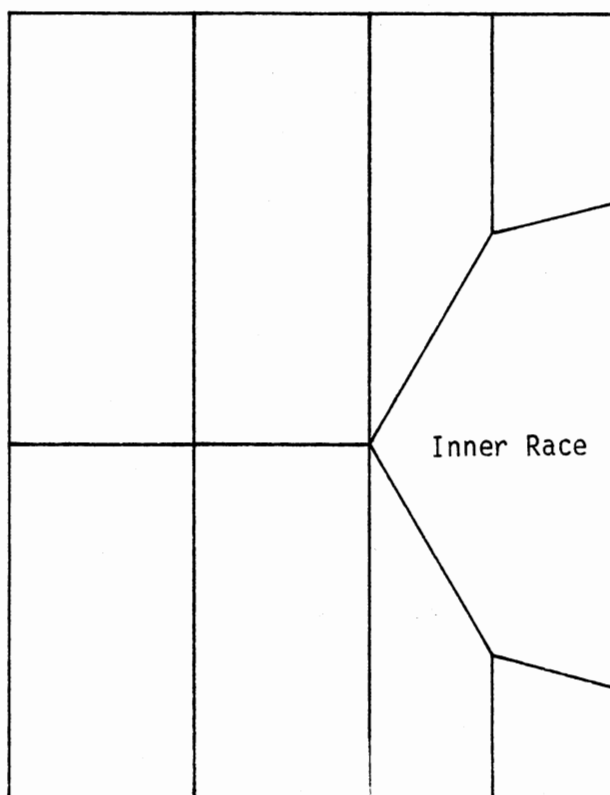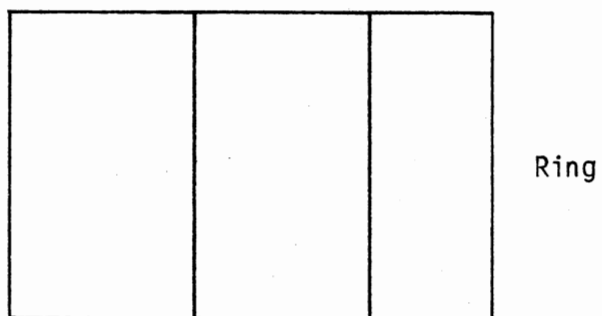
Figure 16. Cross Section of Finite
Element Inner Race
and Ring Model

Shigley and Mischke (1986), is 0.45.   The interface elements are positioned between the race and the ring as shown in the cross sectional view illustrated in Figure 17.

Inner Race Bolts

The bolts connecting the race and the ring are SAE grade 8 and are torqued to 70% of yield strength.   They are modeled using a three dimensional, tension-only, spar element.   The element is defined by two nodal points, having three degrees of freedom at each node, the cross sectional area, an initial strain, and isotropic material properties. The cross sectional area, A, is calculated from the bolt diameter.   For steel bolts a modulus of elasticity of 30Mpsi is used.   The initial strain for the element is given by Δ/l, where Δ is the difference between the element length, l, as defined by the two node locations, and the zero strain length.   For this application Δ is expressed as

$$\Delta = \Sigma \ F_p \ (\frac{1}{\Sigma K_B} + \frac{1}{\Sigma K_I}) \tag{39}$$

where

$$\Sigma F_p = N_B f_p$$

$$\Sigma K_B = N_B k_B$$

$$\Sigma K_I = N_I k_I$$

N_B = number of bolts

N_I = number of interface elements

f_p = bolt preload force

k_B = bolt stiffness

k_I = interface stiffness
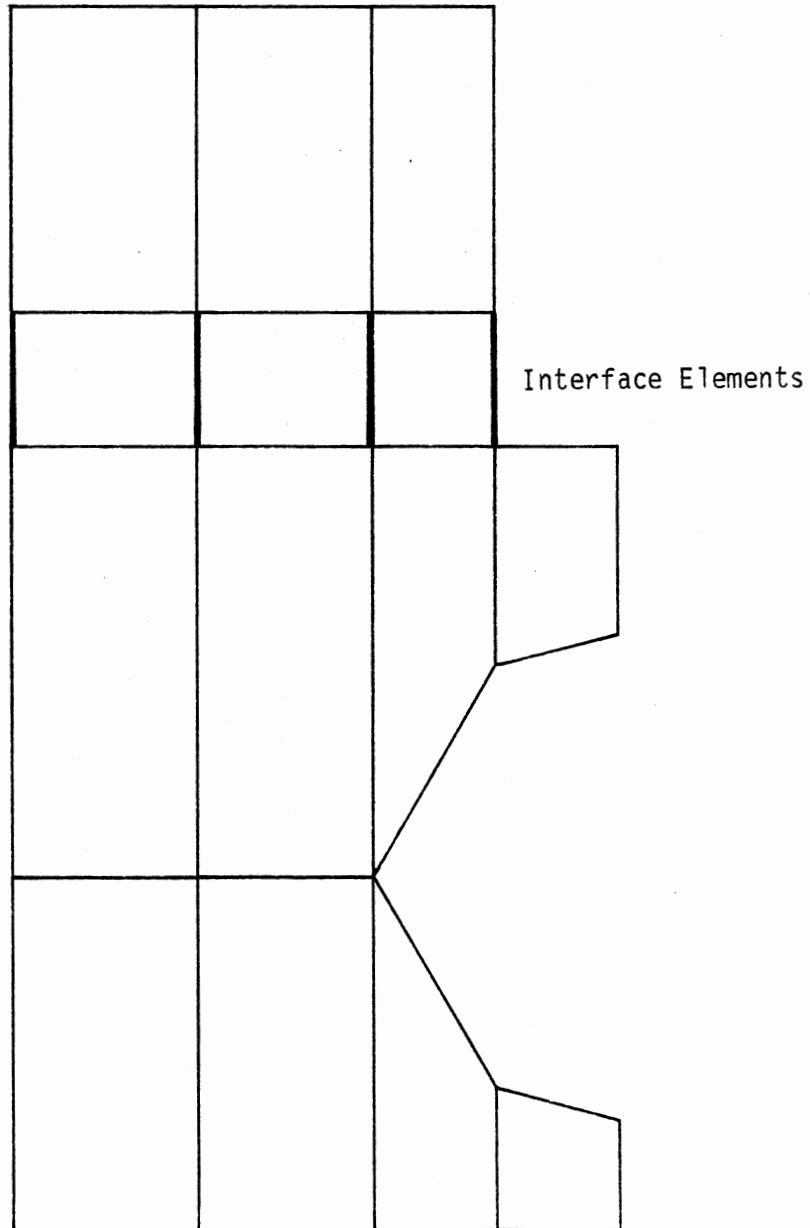
Figure 17. Locations of Interface
Elements in a Finite
Element Inner Race
and Ring Model Cross
Section

To calculate the preload force, f_p , the bolt yield strength and cross sectional area are needed. From Shigley and Mitchell (1983) the yield strength of SAE grade 8 bolts is 130 kpsi. Recalling that the bolts are torqued to 70% of their yield strength, the equation for the bolt preload force is

$$f_p = (0.7)(130 \text{ kpsi})(A) \tag{40}$$

From Shigley and Mitchell (1983) an equation for calculating bolt stiffness is obtained,

$$k_B = \frac{AE}{\ell} \tag{41}$$

The interface stiffness as noted previously, is 10000 lb/in. The bolt holes in the bearing are drilled and tapped to the middle of the race. Therefore, one node of the spar element is attached to the top of the ring and the other to the middle of the bearing race as shown in the cross section illustrated in Figure 18. The number and spacing of the bolts is given in the bearing specifications.

Ball-Race Interface

The ball contacts the inner race at two points, one at each contact angle. A nonlinear force-deflection element is used to represent these ball-race contact points. This element is a unidirectional element with a nonlinear generalized force-deflection capability. The element is defined by two node points, with one degree of freedom at each node, and a generalized force- deflection curve. The points on this curve represent force versus relative translation. The curve is generated using equation (26), which was described in the previous section. An
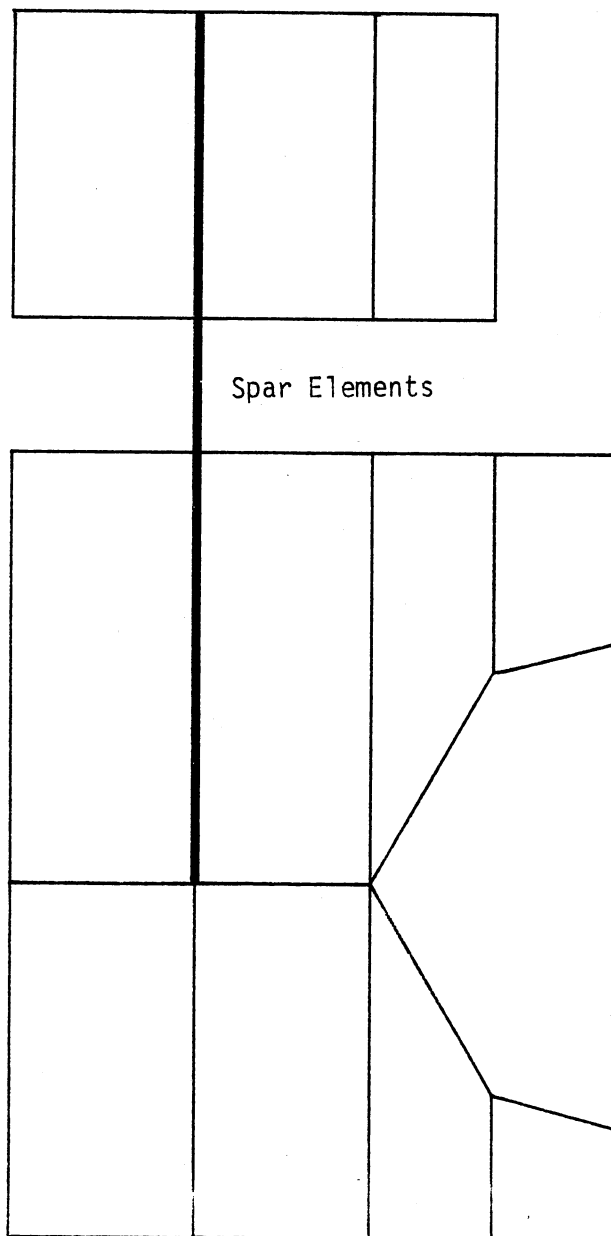
Spar Elements

Figure 18. Location of Spar Elements
in a Finite Element Inner
Race and Ring Model Cross
Section
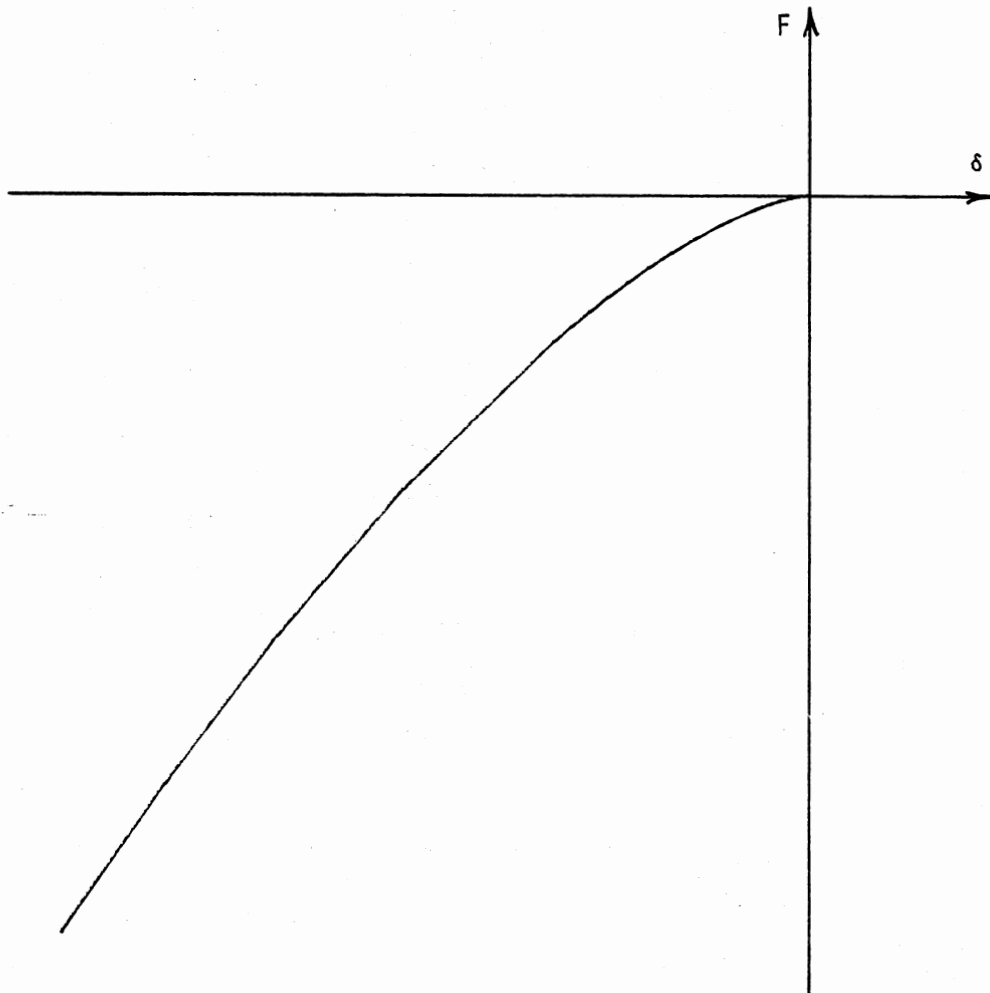
Figure 19. Force-Deflection Curve Generated From Equation (26)

example of the resulting curve is shown in Figure 19. The element's two nodal points are placed coincident at each of the two contact points and are shown as short lines for clarity in the cross section illustrated in Figure 20.

Outer Race

The ball also contacts the outer race at two points, one at each contact angle. A spring element is used to simulate the effect of the outer race. The element is defined by two nodal points, with one degree of freedom at each node, and a spring constant. As shown in Figure 21, one node of the element is attached to the nonlinear force- deflection element, representing the ball-inner race contact point, and the other node is fixed. To determine the spring constant, k_s, the outer race was modeled separately using the three dimensional isoparametric solid elements described previously. An isometric view of the outer race model is shown in Figure 22. In an actual application the outer race is bolted to a rigid structure. Therefore, the bolts need to be included in the model. Using the three dimensional, tension only, spar elements described previously, one end of the element is connected to the race and the other end is fixed in space as shown in the cross sectional view illustrated in Figure 23. A force, F, is applied to one of the ball-race contact points, at an angle corresponding to the contact angle, as shown in a cross sectional view illustrated in Figure 24. Once the displacement, y, of that node, due to the normal force, is determined a spring constant is calculated using the following equation

$$k_s = \frac{F}{y} \qquad (42)$$
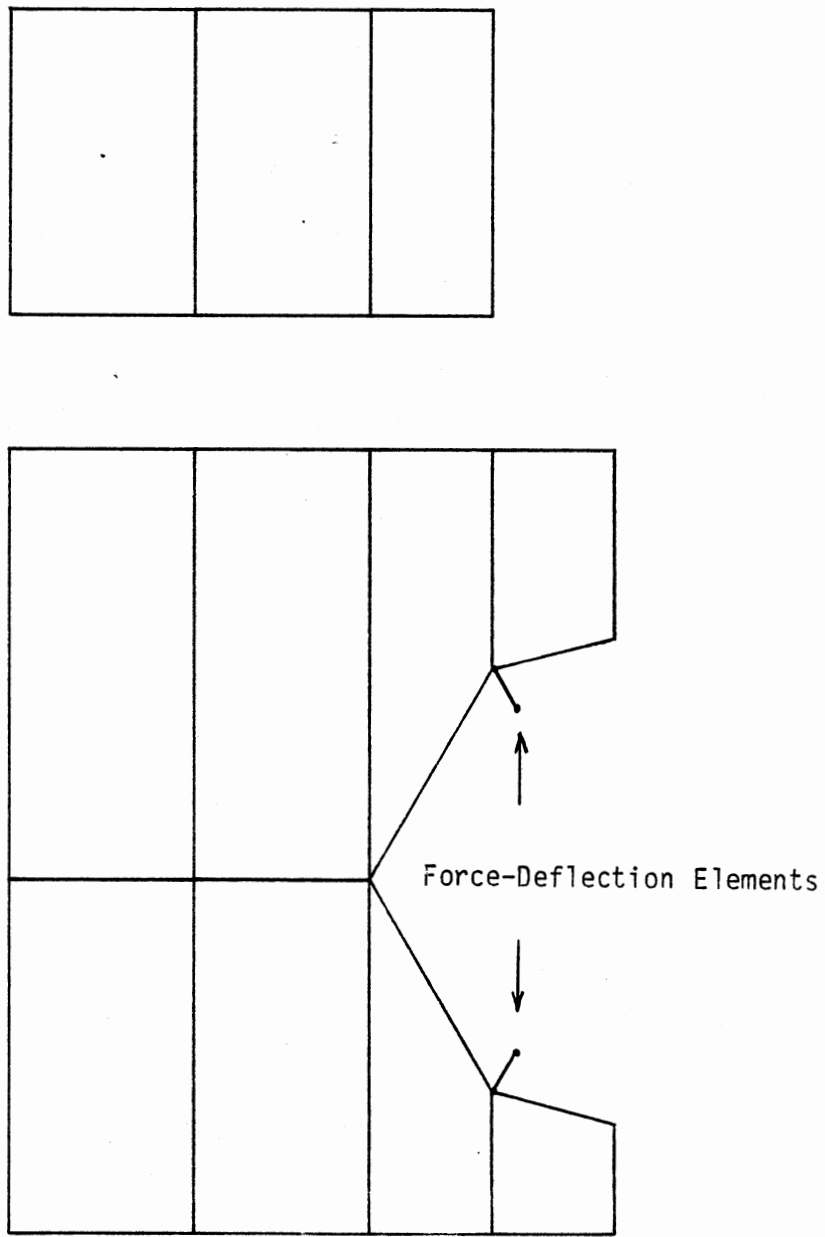
Figure 20. Locations of Force-Deflection
           Elements in a Finite Element
           Inner Race and Ring Model
           Cross Section
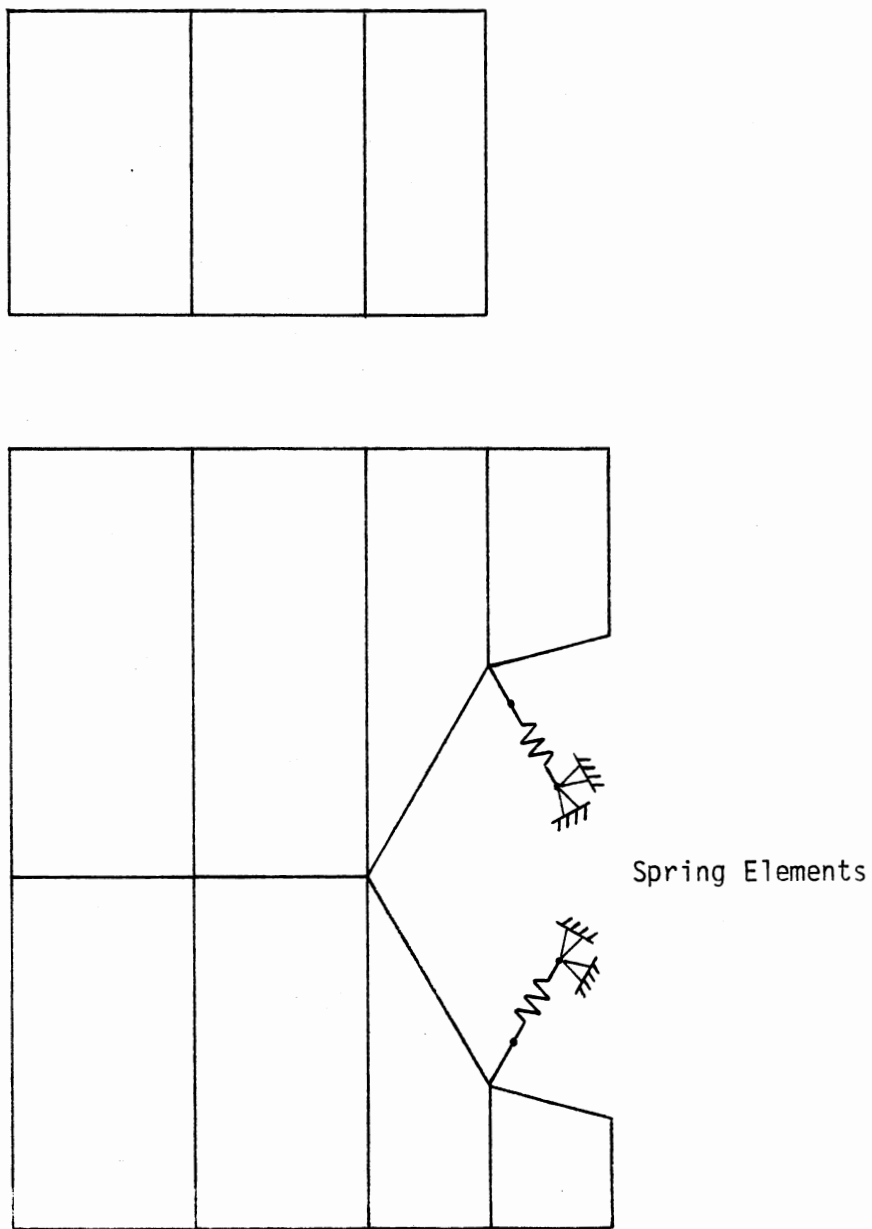
Figure 21. Locations of Spring Elements
in a Finite Element Inner
Race and Ring Model Cross
Section

Spring Elements
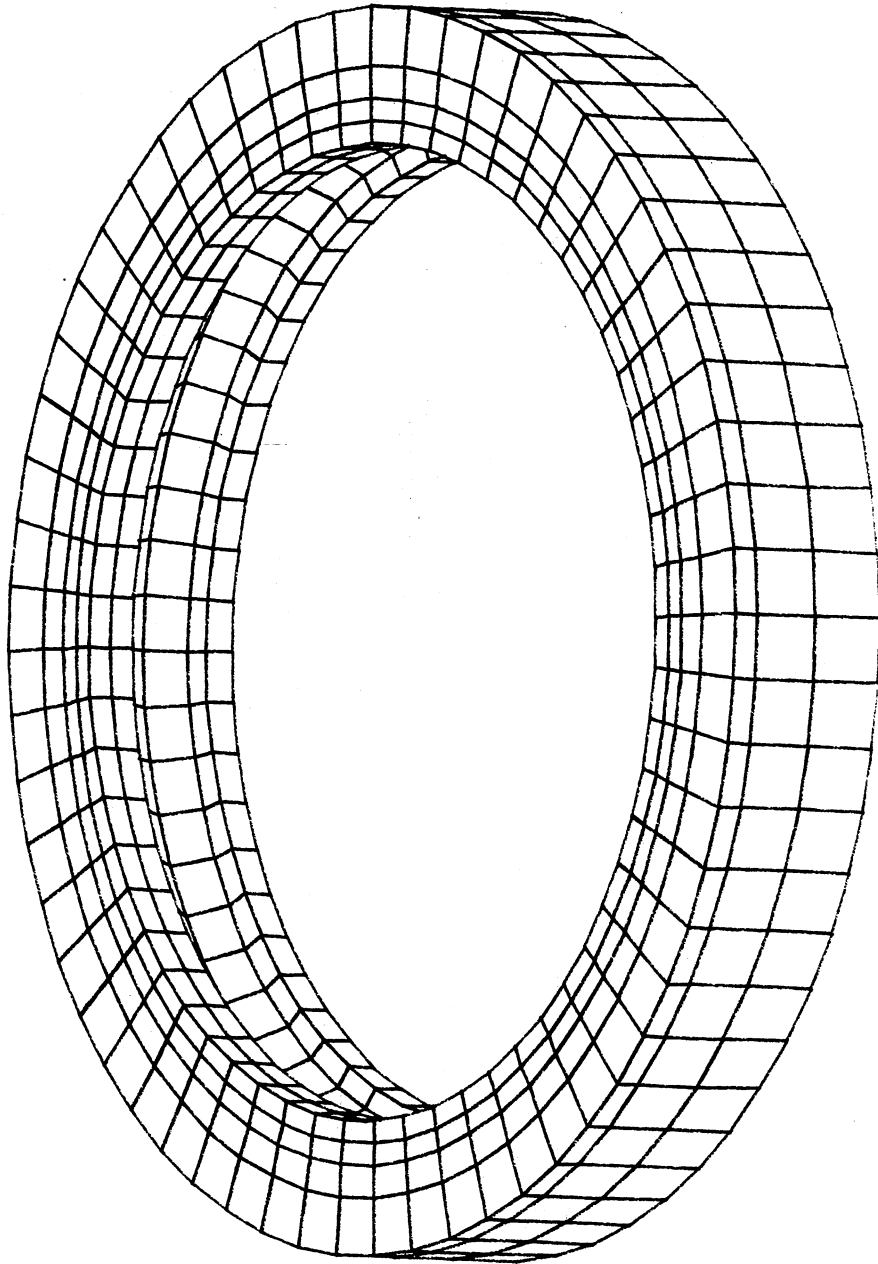
Figure 22. Isometric View of Finite Element Outer Race Model
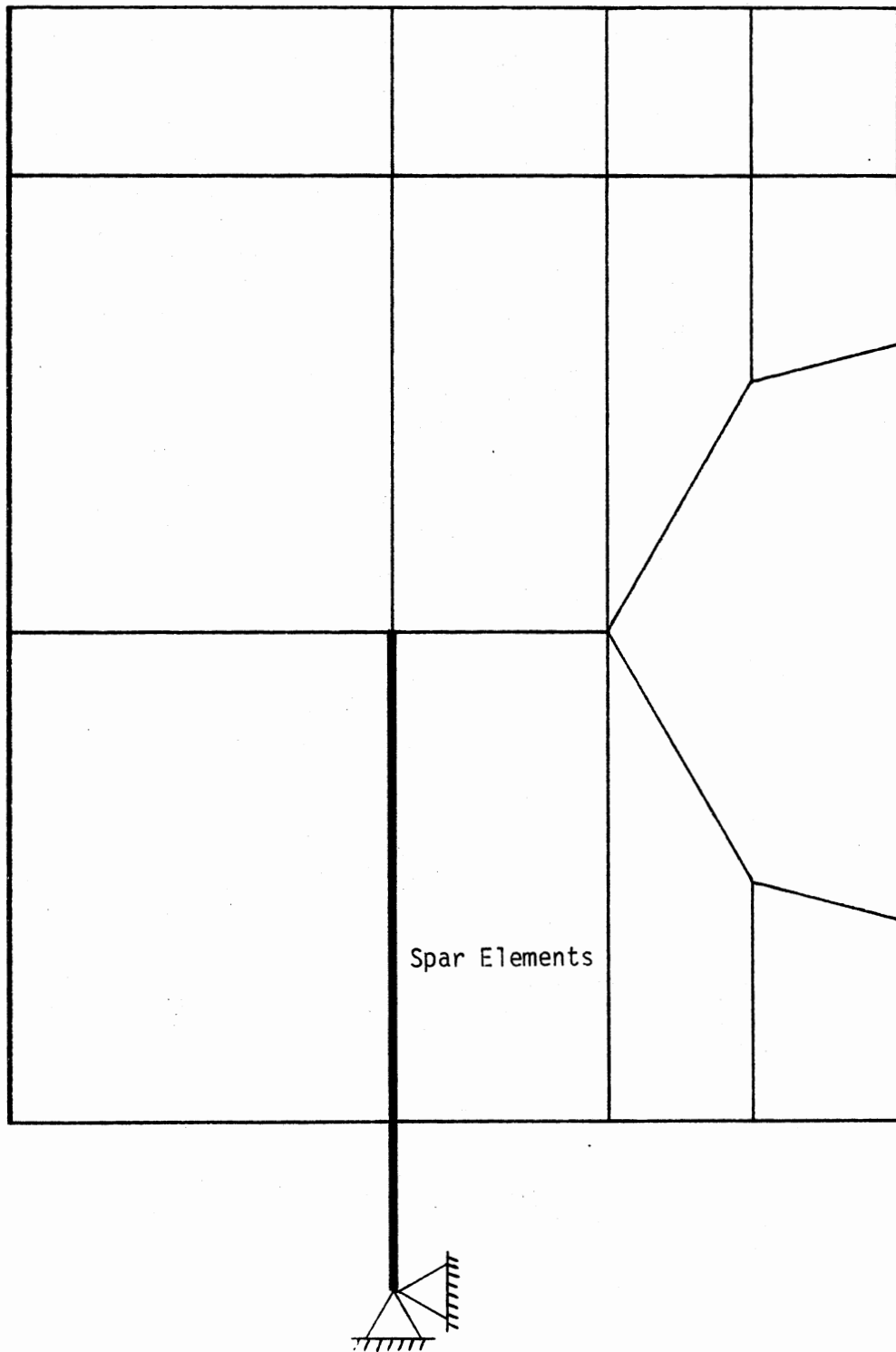
Figure 23. Location of Spar Elements in a Finite Element
Outer Race Model Cross Section
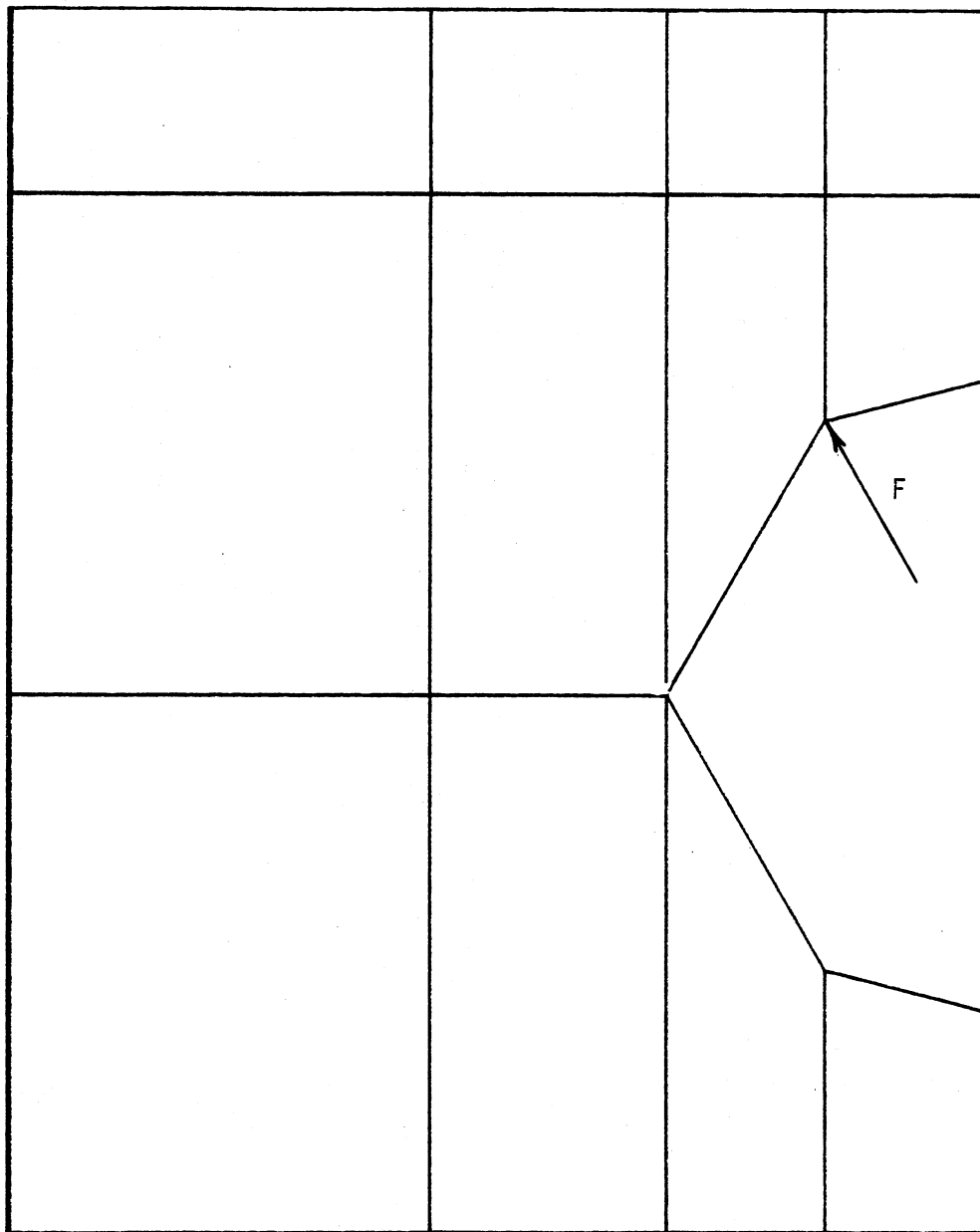
Figure 24. Location of Applied Force in a Finite Element
Outer Race Model Cross Section

Loading

To simulate an applied moment load on the bearing, two equal and opposite force are applied to the rigid ring as shown in Figure 25. An example listing of the input commands for the moment load application is given in Appendix D. Radial loads are transmitted from the structure to the bearing through the connecting bolts. The elements used to represent the bolts are tension only, spar elements and cannot transmit the required force. Figure 26 shows how the radial load is simulated. On the top of the bearing, equal radial forces are applied at each of the bolt locations simulating the way in which the bolts would transmit the forces. As described in the rigid model, the number of balls that carry the applied radial load is a function of the diametral clearance. Gap elements, which can simulate this clearance, are available in ANSYS. Unfortunately, when the extra elements are added to the model, the wave front limit of 200 is exceeded and the program will not run. To compensate for this, a clearance adjustment factor will be determined using the rigid model. The adjustment factor is described in the results section. An example listing of the input commands for the radial load is given in Appendix E. Figure 27 illustrates how a simulated thrust load is applied to the bearing. Equal forces are applied to all of the nodes on the top of the ring. Appendix F contains a listing of the input commands for the thrust application.
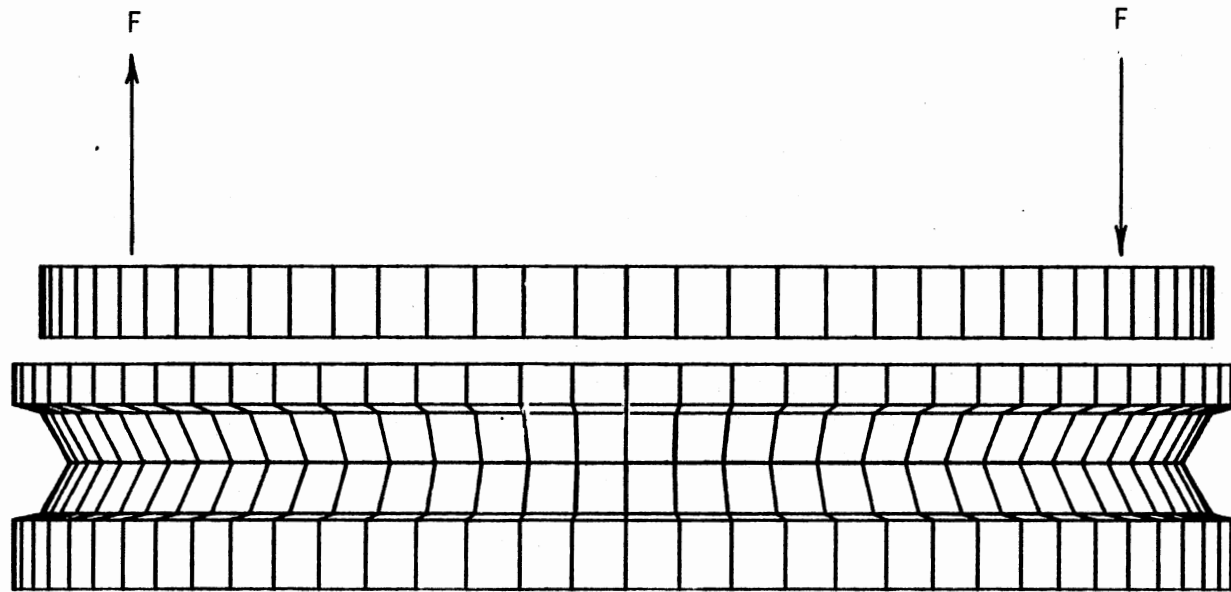
Figure 25.   Moment Load Simulation on the Finite Element Inner Race and Ring Model
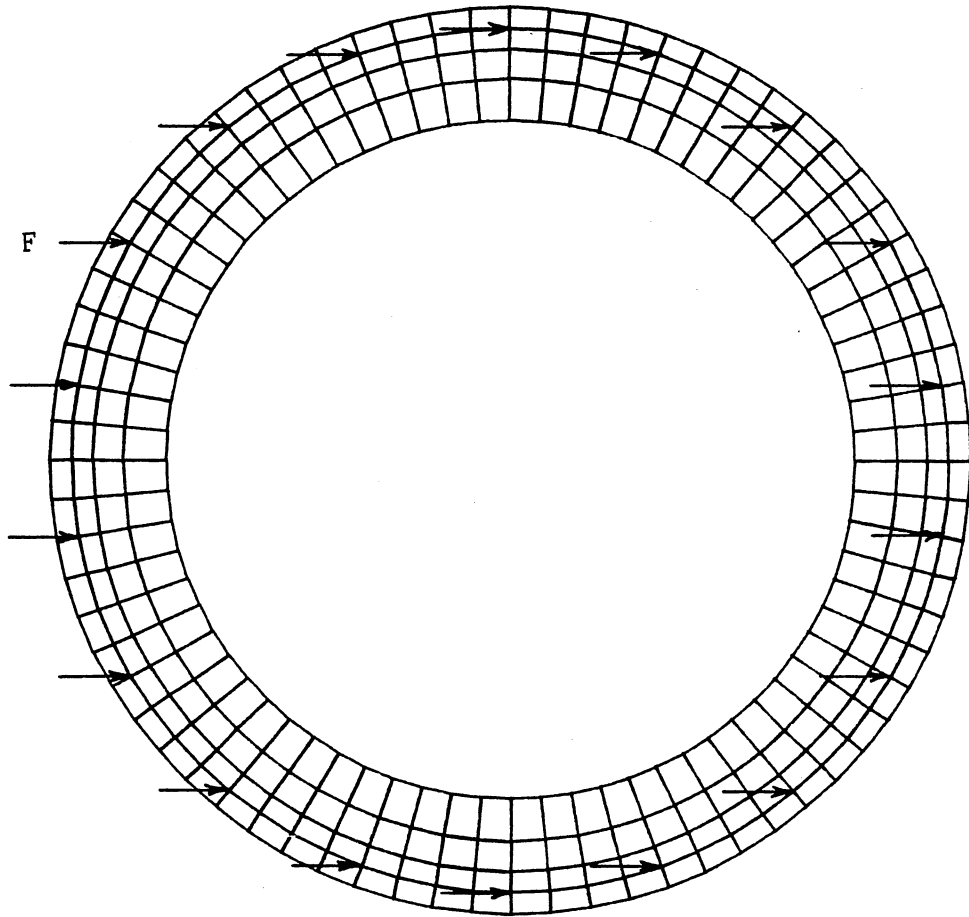
Figure 26. Radial Load Simulation on the Finite Element
Inner Race and Ring Model
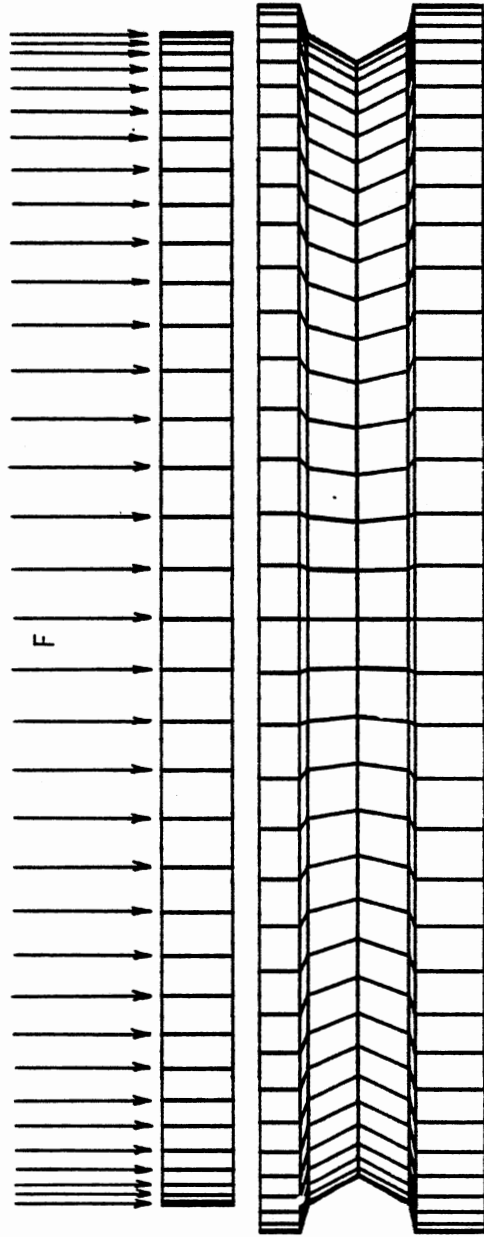
Figure 27. Thrust Load Simulation on the Finite Element Inner Race and Ring Model

CHAPTER IV

MODEL RESULTS

Moment

Using both the finite element and the rigid models, six different
moment loads are applied to each of the bearings listed in Table I. An
example of the load distribution obtained from both models is shown in
the polar plot illustrated in Figure 28. A force into the page at 90
degrees and a force out of the page at 270 degrees represent the moment
load. Recalling that the finite element model had two ball-raceway
contact points, the "+" indicates forces at the top of the raceway, and
the "*" indicates forces at the bottom of the raceway. The rigid model
forces are represented by the "O". As expected the forces a largest at
90 and 270 degrees and decrease as the moment axis of symmetry is
approached. The same bearing parameters and applied moment load were
used for both models. The moment loads and the resulting maximum race
forces from both models are listed in Tables VI-XI given in Appendix G,
for bearing numbers 1-6, respectively. For each of the bearings the
race forces are plotted against the moment loads and are shown in
Figures 29-34. As expected, when the rigid model data points, for each
bearing, are connected the result is a straight line that intersects the
origin. The finite element model data points, for each bearing, do not
intersect the origin. Connecting the data points, for each bearing,
results in an "elbow" that develops into a straight line. The "elbow"

56
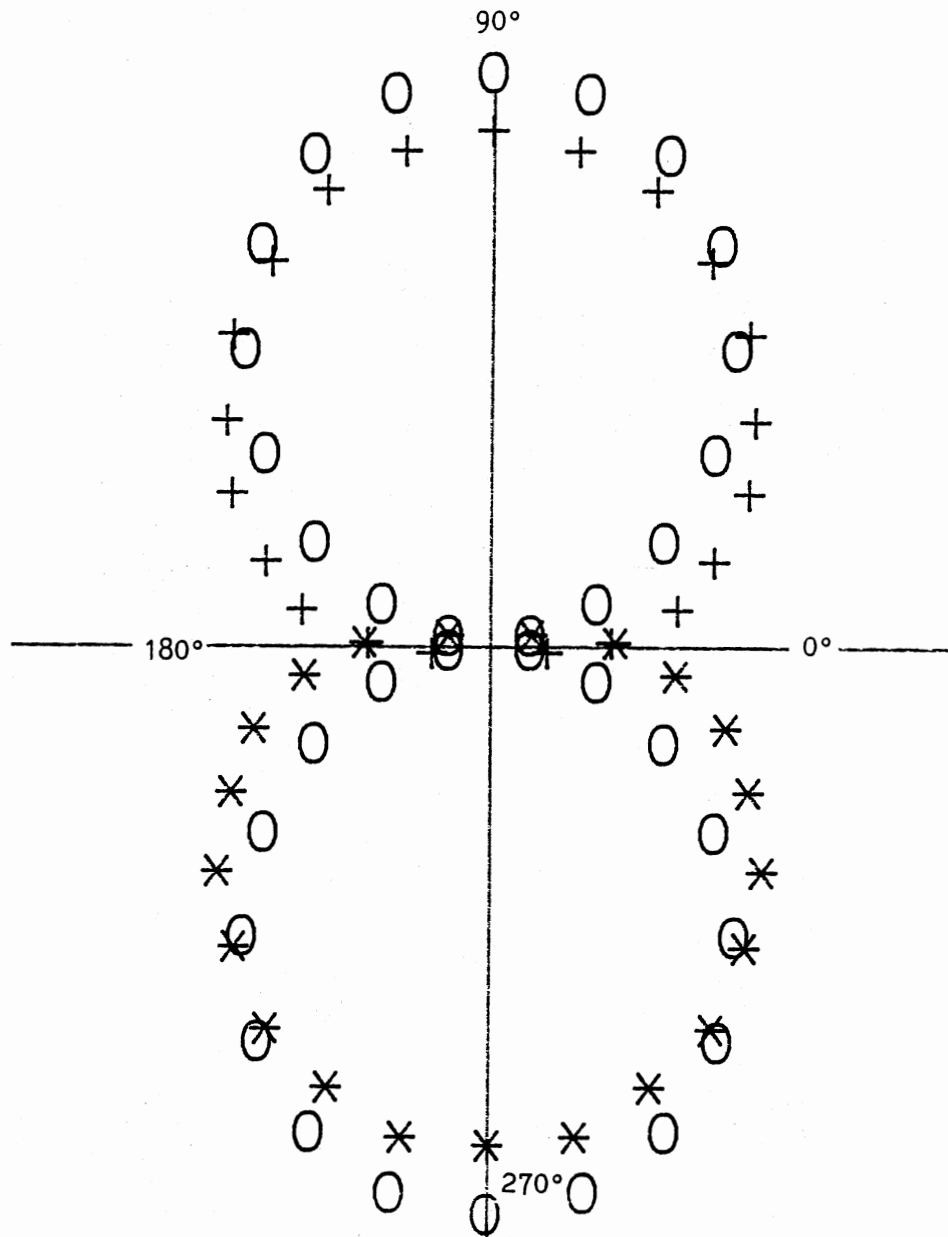
Figure 28. Polar Plot of Moment Load Distribution from
Finite Element and Rigid Model Results
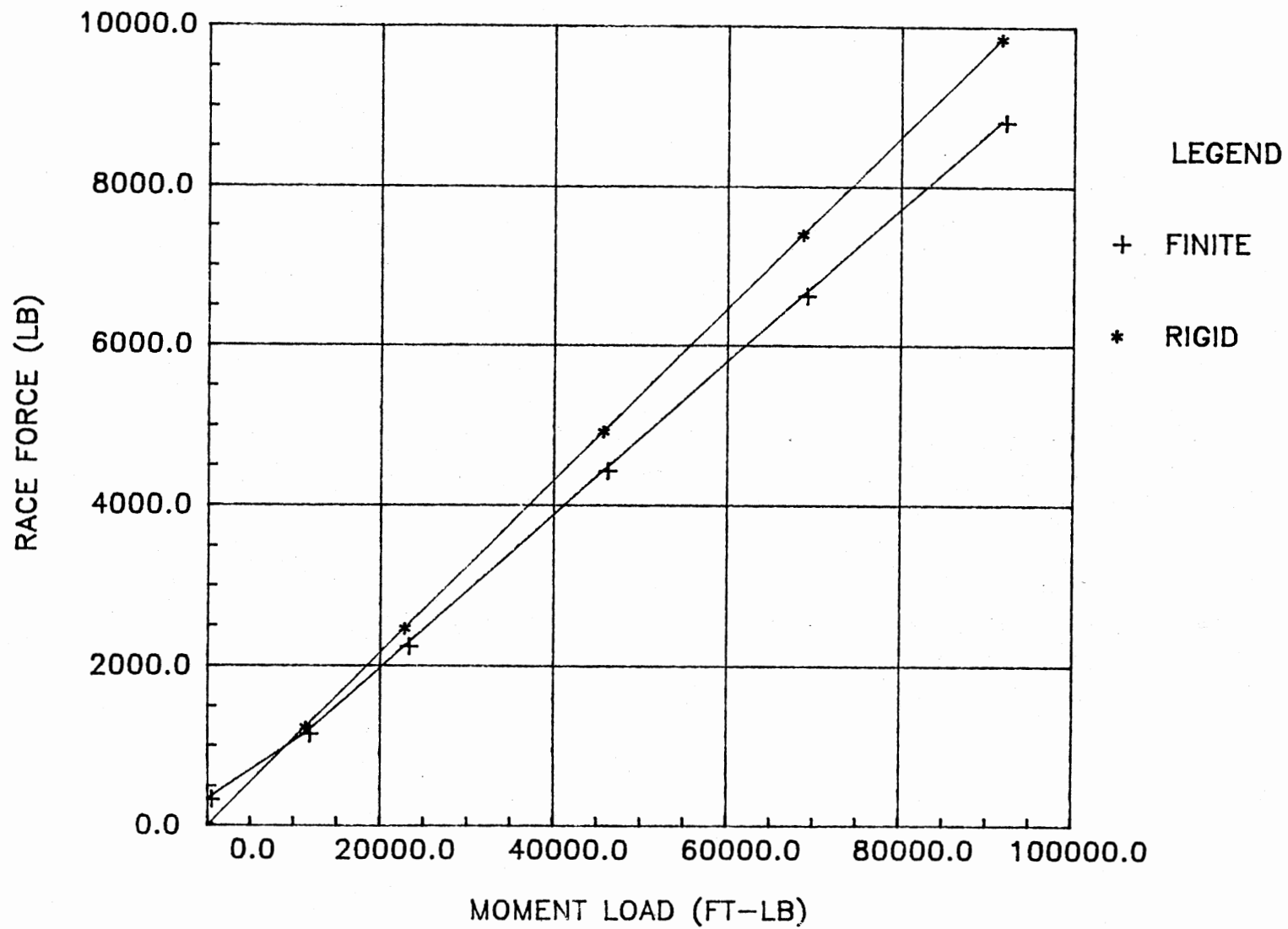
Figure 29.   Finite Element and Rigid Model Plot of Race Force vs. Moment Load for Bearing #1
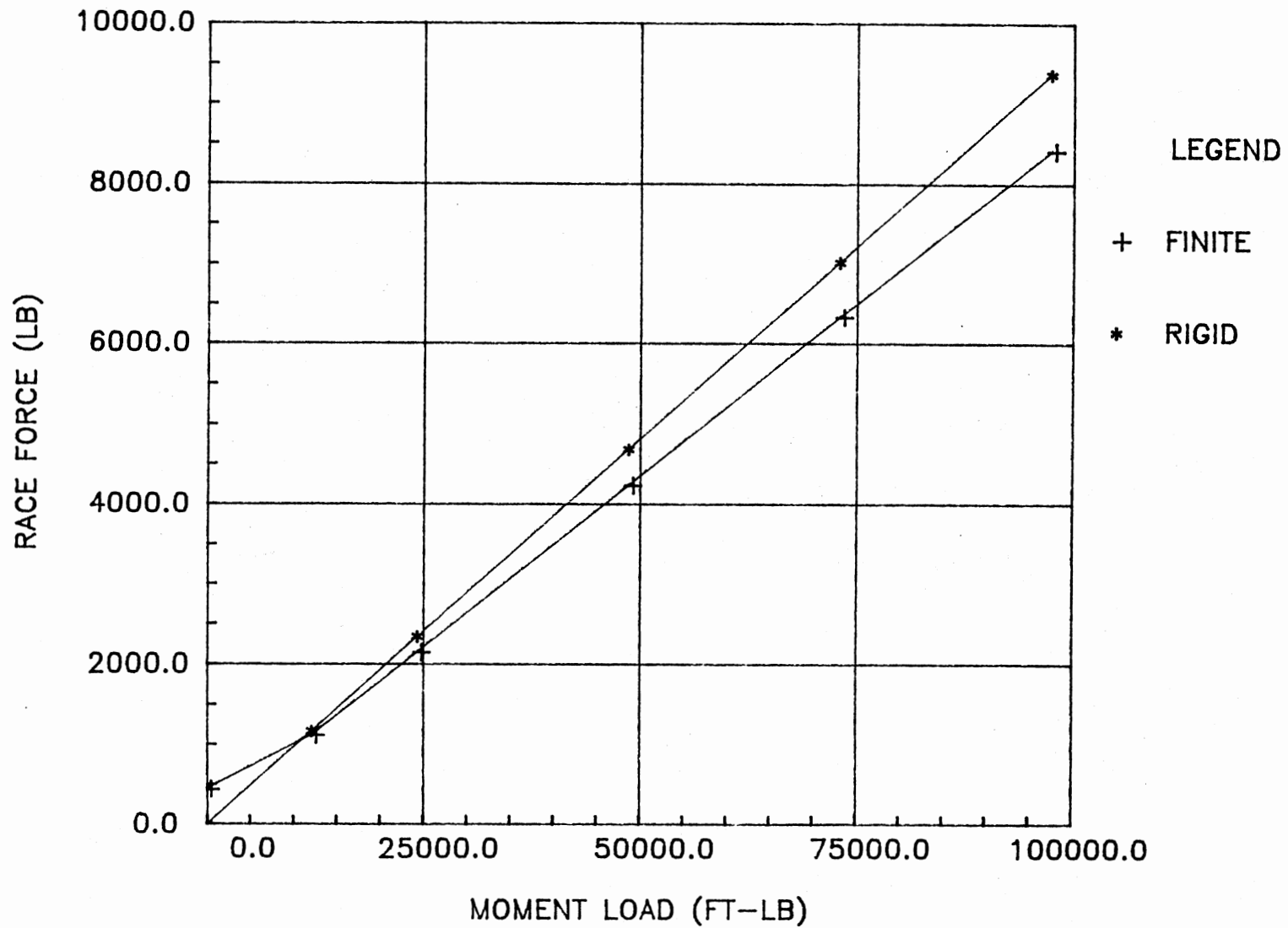
Figure 30. Finite Element and Rigid Model Plot of Race Force vs. Moment Load for Bearing #2
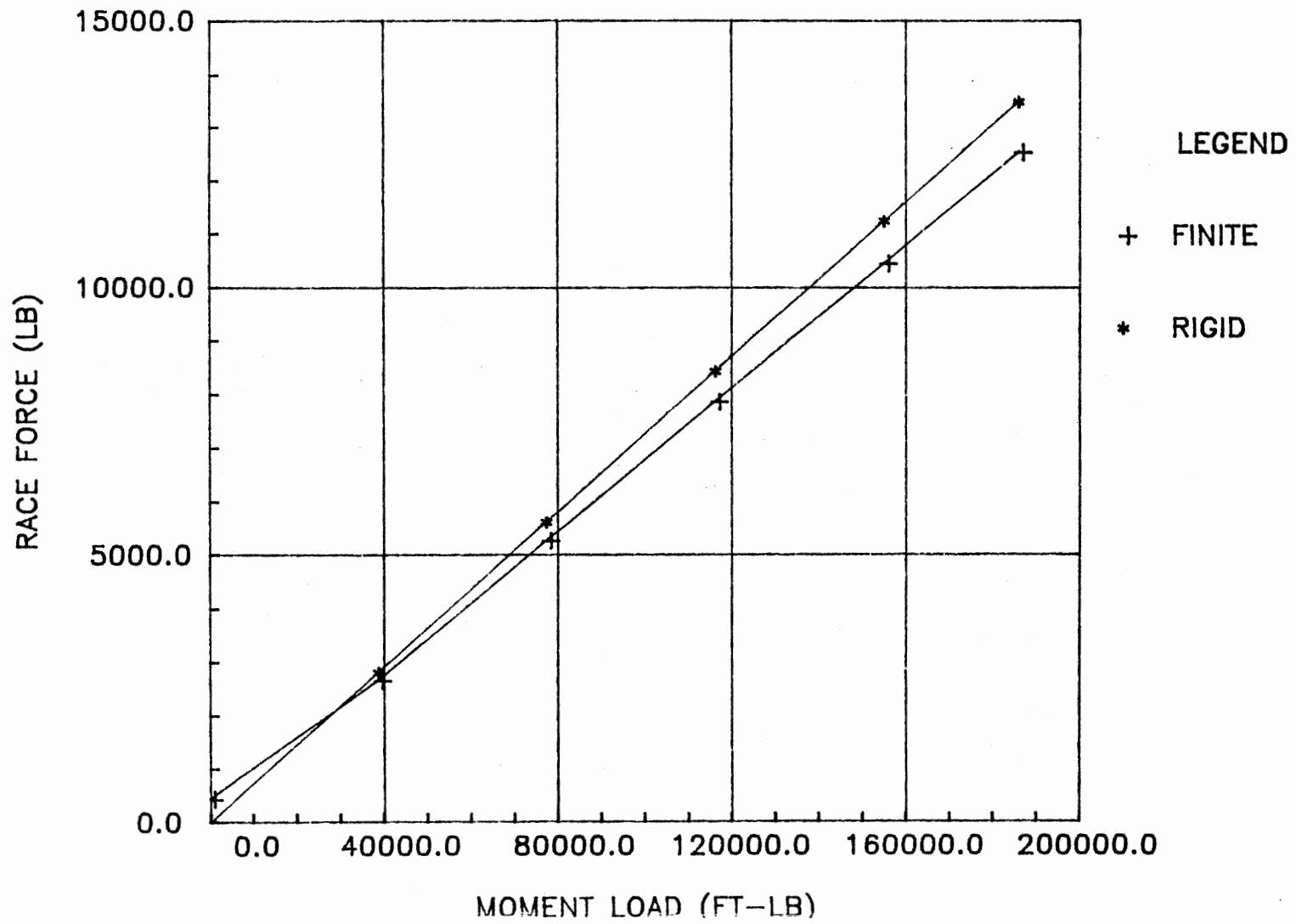
Figure 31. Finite Element and Rigid Model Plot of Race Force vs. Moment Load for Bearing #3
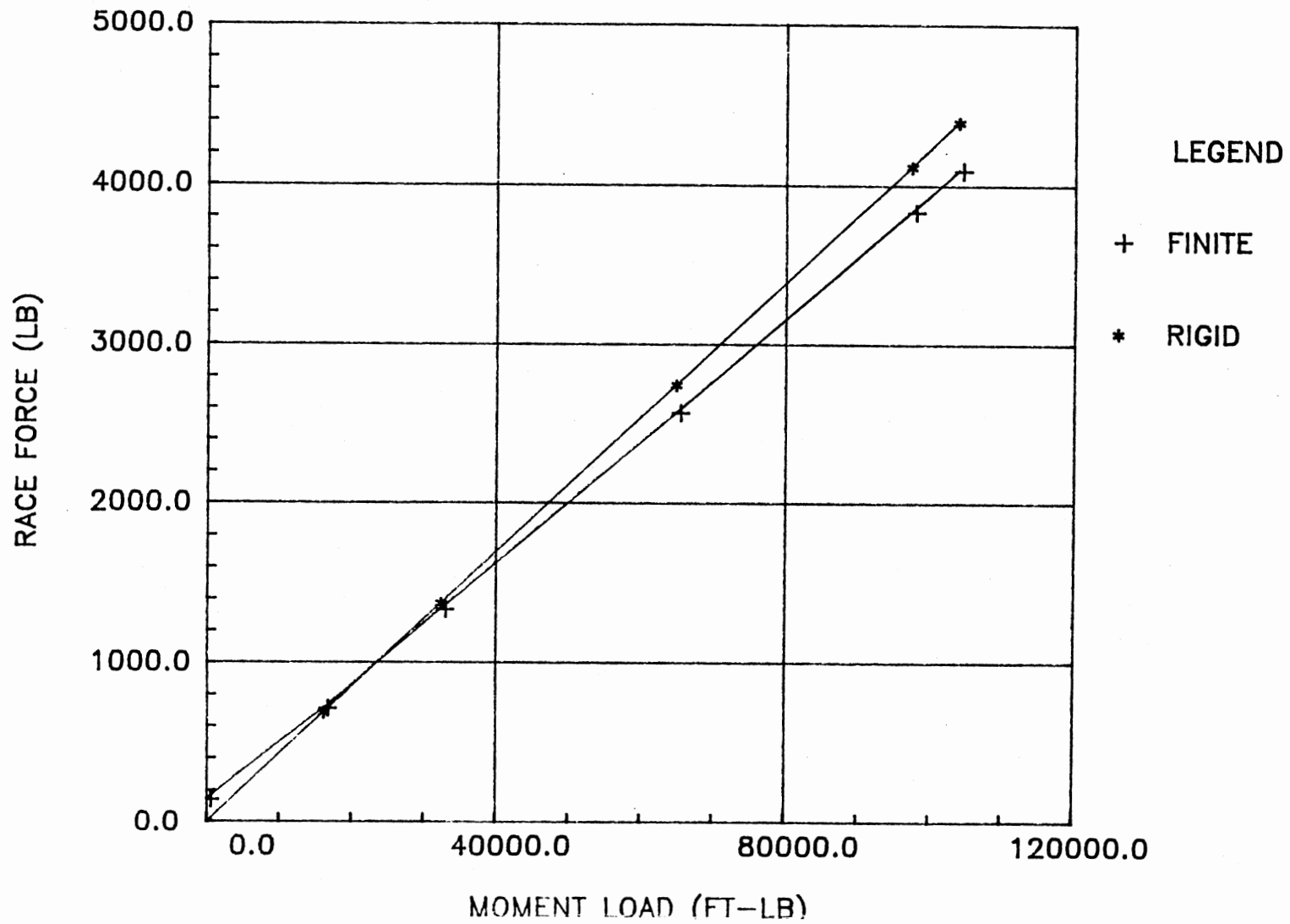
Figure 32. Finite Element and Rigid Model Plot of Race Force vs. Moment Load for Bearing #4

Figure 33. Finite Element and Rigid Model Plot of Race Force vs. Moment Load for Bearing #5

Figure 34. Finite Element and Rigid Model Plot of Race Force vs. Moment Load for Bearing #6

is caused by the preload of the bolts. The straight line develops as the increasing moment load overcomes the bolt preload. The straight line section is the area of interest and therefore the "elbow" section is neglected. For each bearing, if the straight line section is extended toward the origin, they all intersect the origin. Since all the curves for both models intersect the origin, the slopes can be directly compared. For each of the bearings, shown in Figures 29-34, the values of the slopes of the two lines, generated from the finite element and the rigid model data, are calculated. Also, for each bearing the value of the slope for the finite element model, S_F, was subtracted from the value of the slope for the rigid model, S_R, resulting in the slope difference, $S_R - S_F$. All three sets of values are listed in Table II. In order to determine a relationship between the two models for all of the bearings, these three sets of values where plotted against various parameters including; ball path diameter, number of balls, and number of balls multiplied by the ball path diameter. The best correlation was achieved when all the values of the finite element model slopes were plotted against the number of balls multiplied by the ball path diameter, X_B, as shown in Figure 35. From Chapra and Canale (1985), fitting a simple power function, by method of linear regression, to this set of data points results in the following equation.

$$S_{FM} = 18.5516 \, X_B^{-0.8361} \tag{43}$$

Where, S_{FM} is the finite element model slope of the line relating the race force and the moment load. The curve generated from this equation is shown along with the data points in Figure 35. Using equation (43) the finite element model slope of the line, relating the

TABLE II

RIGID MODEL SLOPES, FINITE ELEMENT MODEL SLOPES AND
SLOPE DIFFERENCES FOR RACE FORCE VS. MOMENT LOAD

Bearing Number	S_R ($\frac{Lb}{Ft-Lb}$)	S_F ($\frac{Lb}{Ft-Lb}$)	$S_R - S_F$ ($\frac{Lb}{Ft-Lb}$)
1	0.1075	0.0955	0.0120
2	0.0961	0.0855	0.0106
3	0.0724	0.0670	0.0054
4	0.0422	0.0385	0.0037
5	0.0256	0.0295	-0.0039
6	0.0238	0.0267	-0.0029
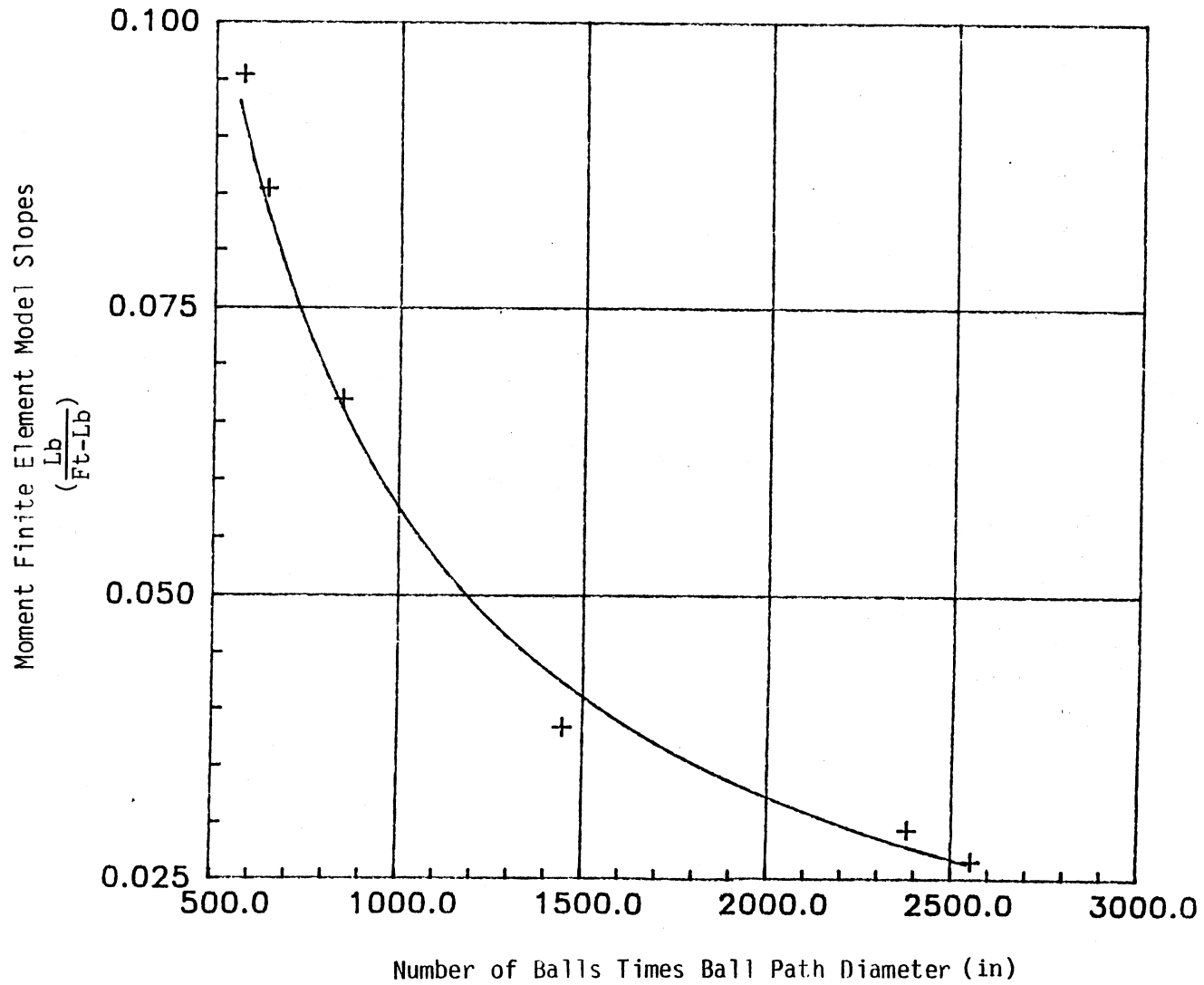
Figure 35.   Plot of Moment Finite Element Model Slopes vs. Number of Balls Times Ball
Path Diameter and Curve Generated from Equation (43)

race force and moment load for a bearing with a known ball path diameter and number of balls, can be found. Combining the value of the finite element model slope with the limiting race force determined from the rating criteria, the maximum moment load for a specific bearing can be calculated. Therefore, for the moment load application, a correlation between the two models was not found and the relationship is base soley in the finite element model. Chapter V will explain how these equations are utilized in the rating program.

Radial

For each bearing listed in Table I, four different radial loads were applied to both the finite element and rigid models. Figure 36 shows a polar plot of the load distribution obtained from both models. A force, positive to the right of the page, represents the radial load. As explained previously, "+" indicates forces at the top of the raceway, "*" indicates forces at the bottom of the raceway for the finite element model, and "O" indicates the rigid model forces. Since both models represent a zero diametral clearance, the race forces are limited to a 180 degree load zone. For both models the same bearing parameters and applied radial load were used. The radial loads and the resulting maximum race forces from both models are listed in Tables VI-XI given in Appendix G, for bearing numbers 1-6, respectively. Figures 37-42 show the race forces plotted against the radial loads for each of the bearings. The data points from the rigid model, when connected, form a straight line that intersects the origin. The data points from the finite element model are similar to those of the moment load application. The data points begin above the origin creating an
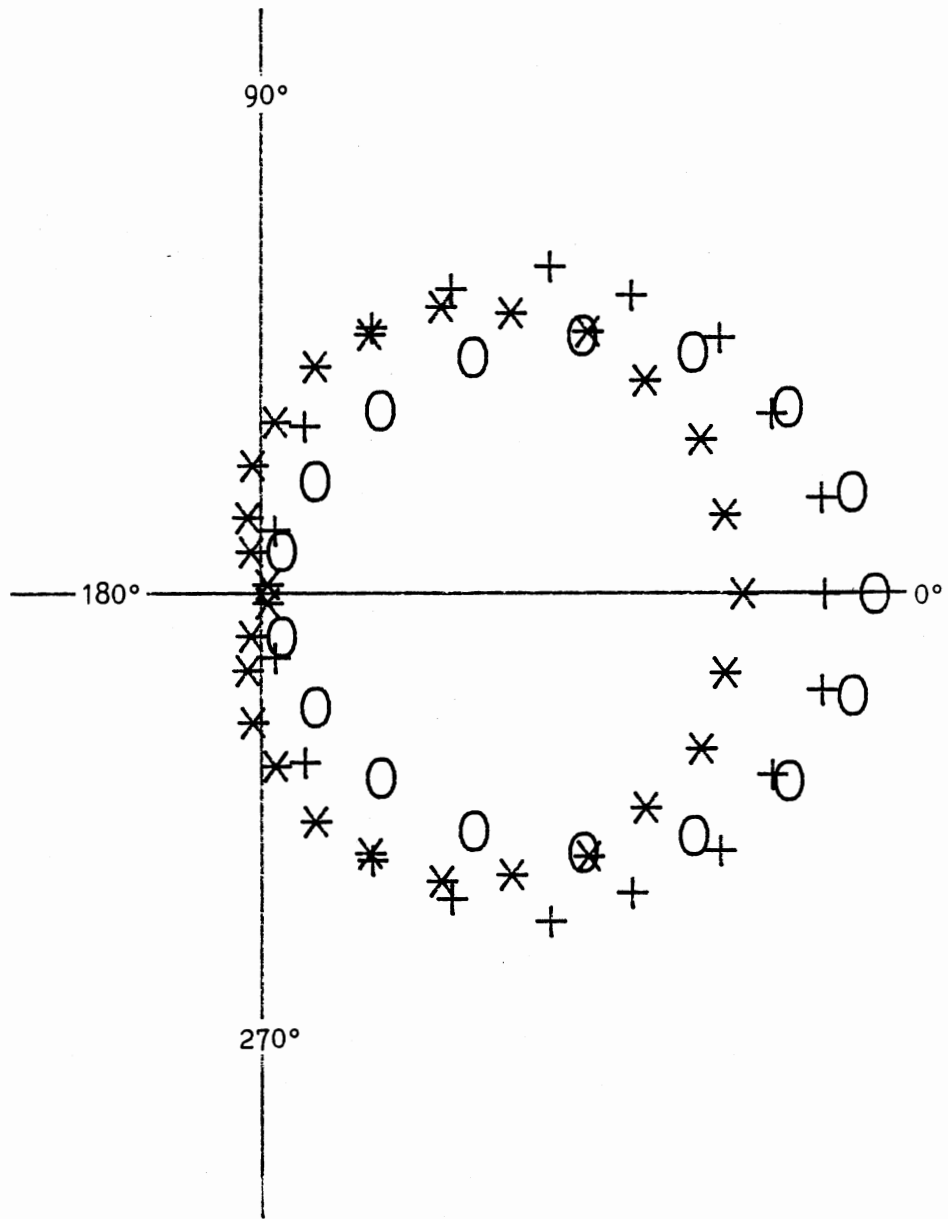
Figure 36. Polar Plot of Radial Load Distribution from
Finite Element and Rigid Model Results
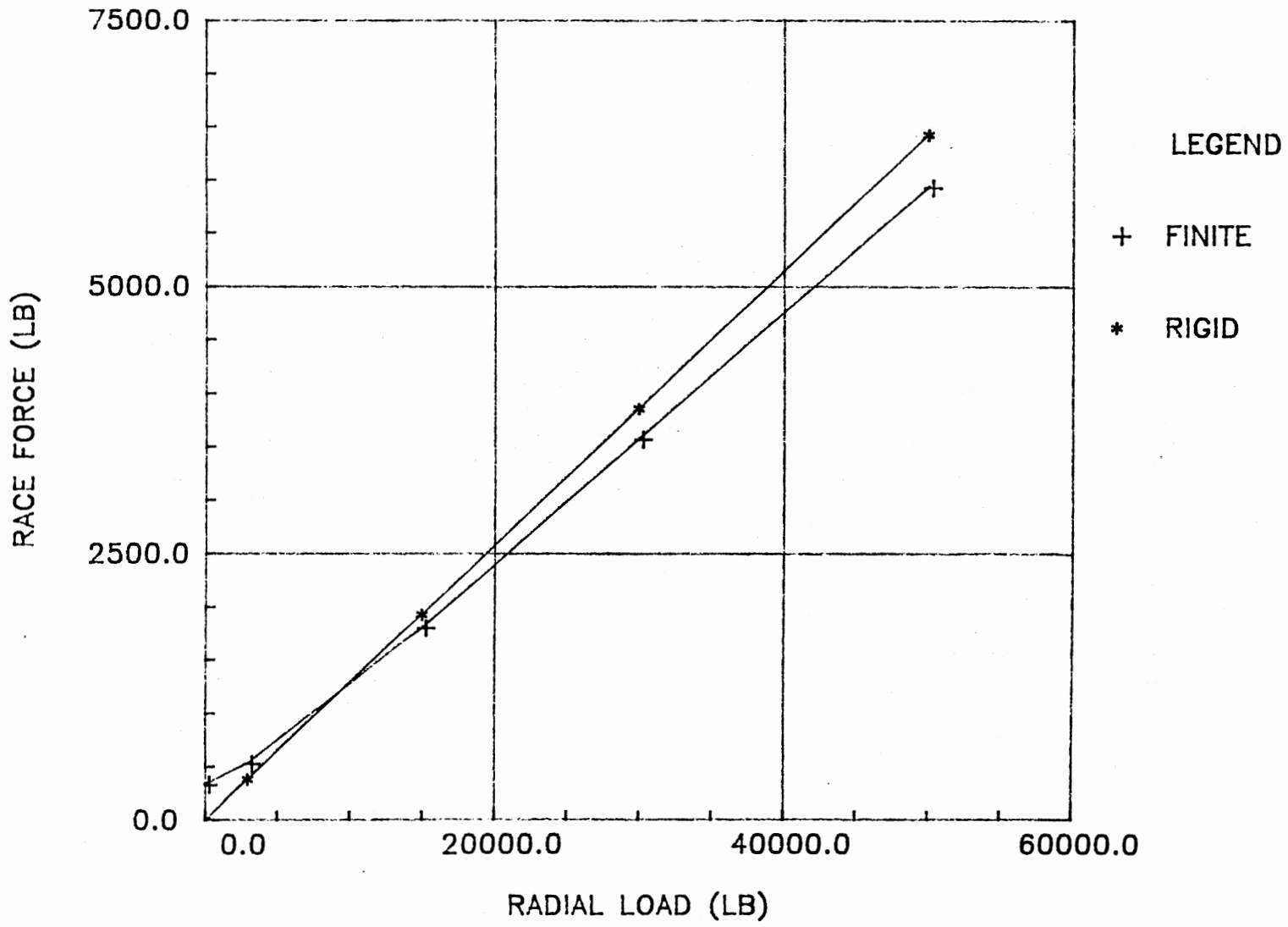
Figure 37. Finite Element and Rigid Model Plot of Race Force vs. Radial Load for Bearing #1
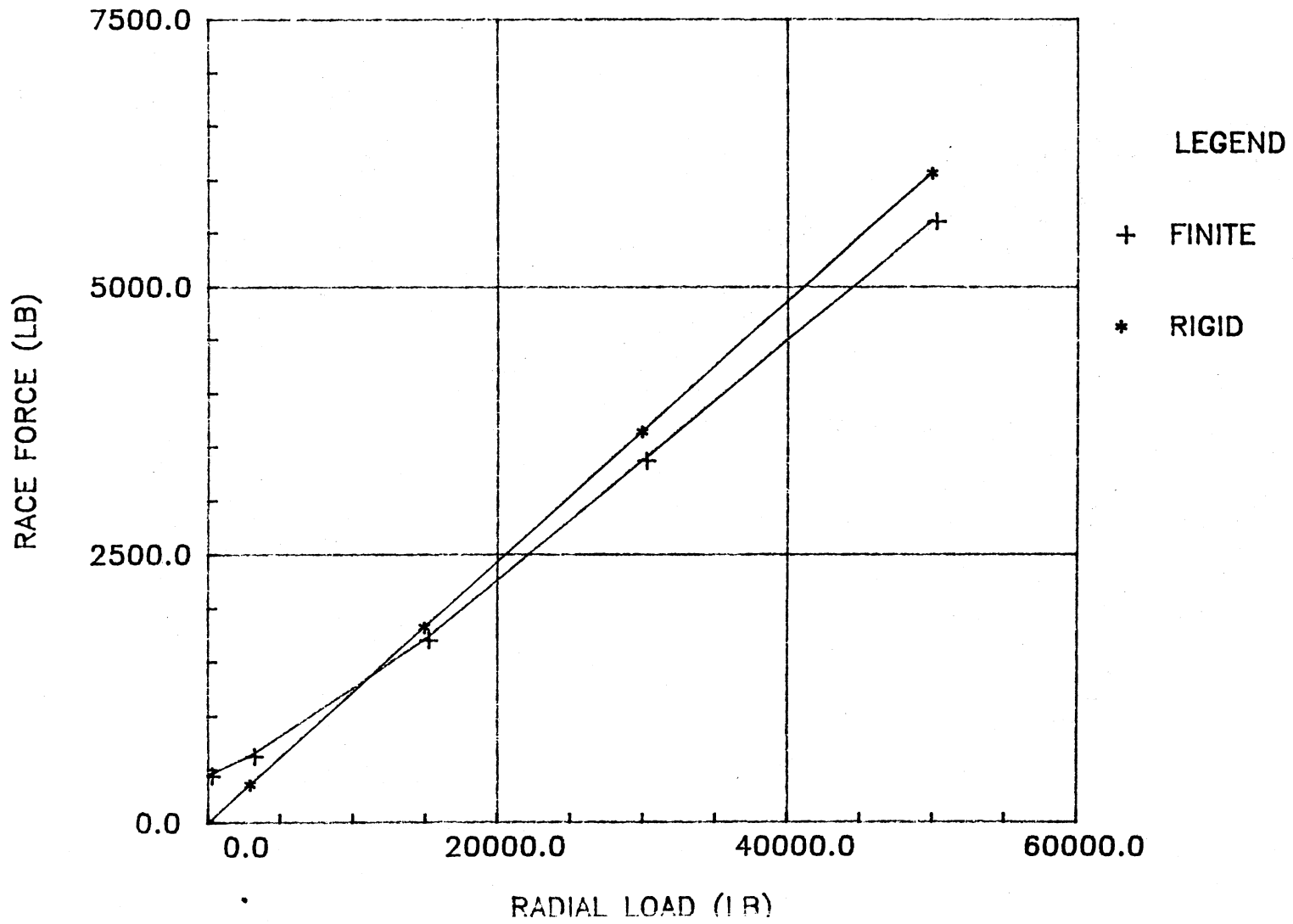
Figure 38. Finite Element and Rigid Model Plot of Race Force vs. Radial Load for Bearing #2
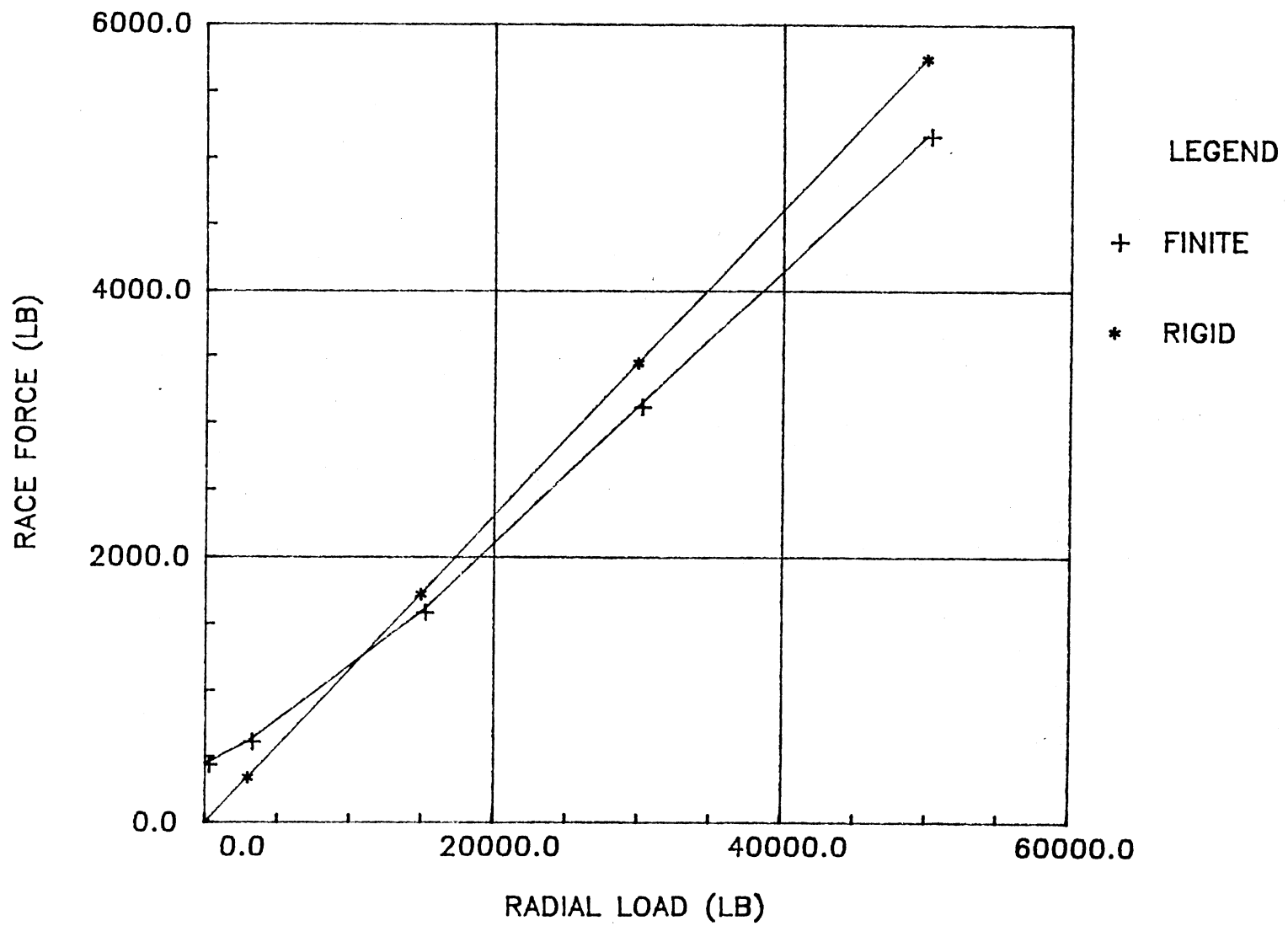
Figure 39. Finite Element and Rigid Model Plot of Race Force vs. Radial Load for Bearing #3
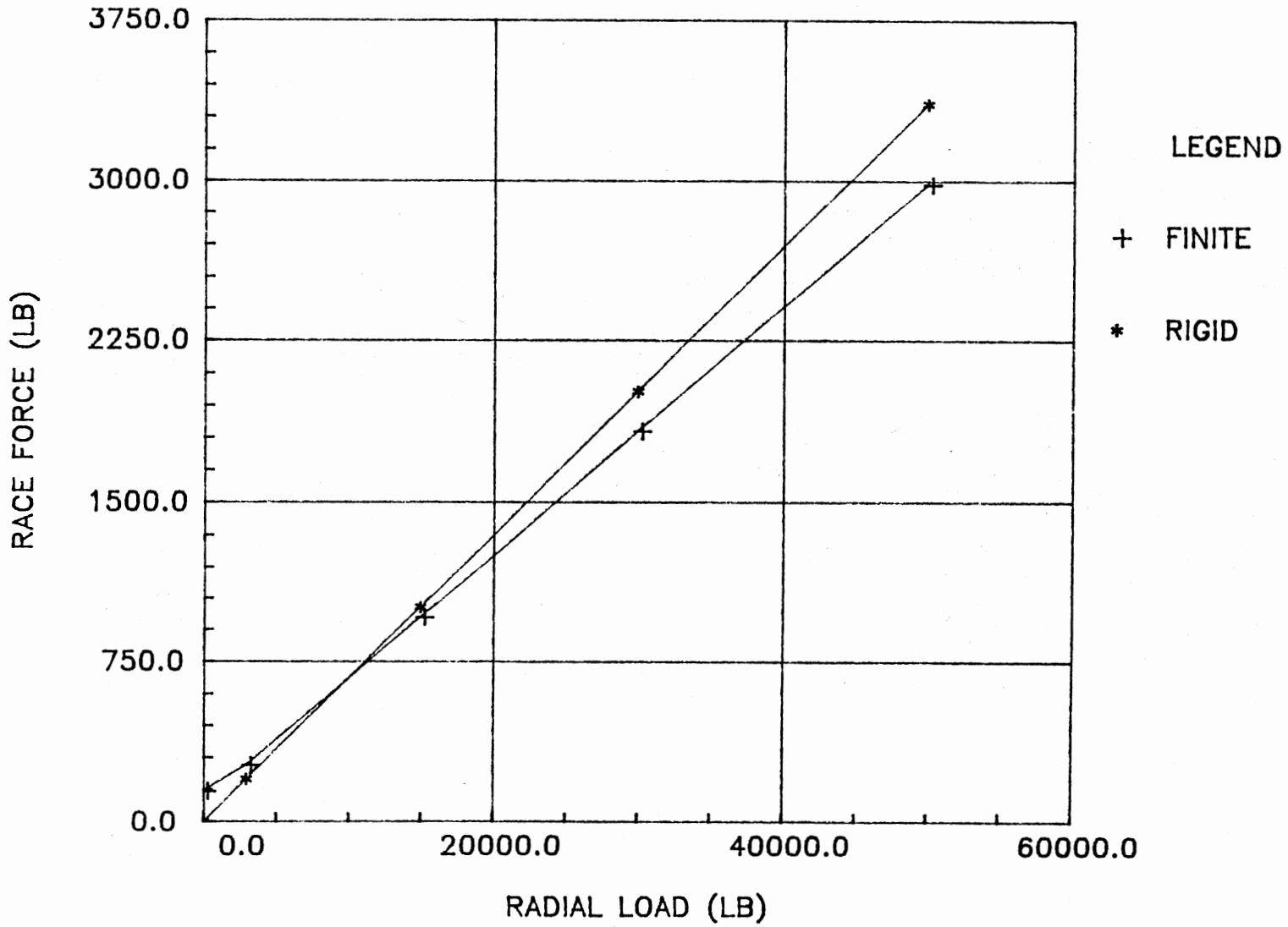
Figure 40.   Finite Element and Rigid Model Plot of Race Force vs. Radial Load for Bearing #4
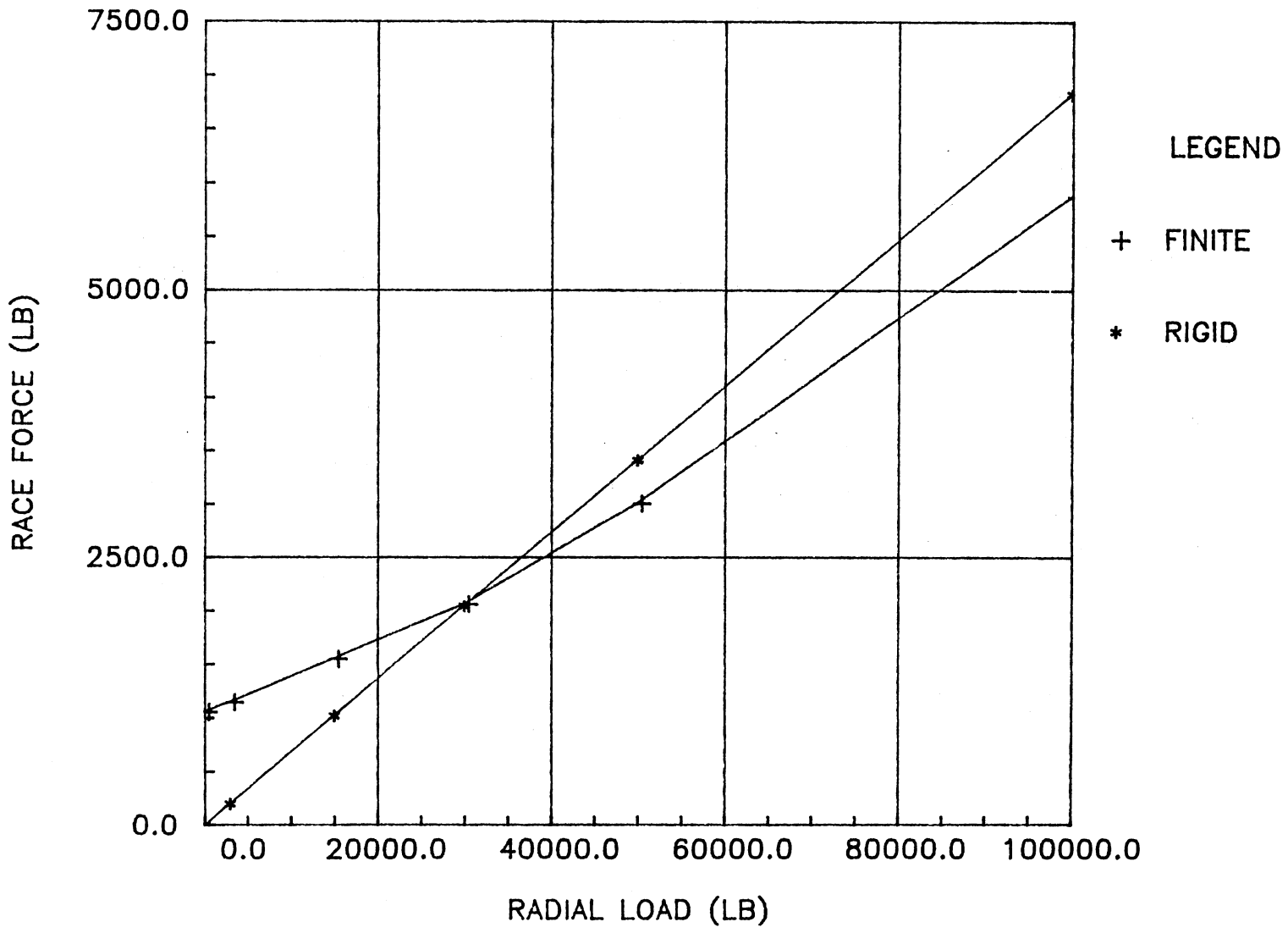
Figure 41. Finite Element and Rigid Model Plot of Race Force vs. Radial Load for Bearing #5
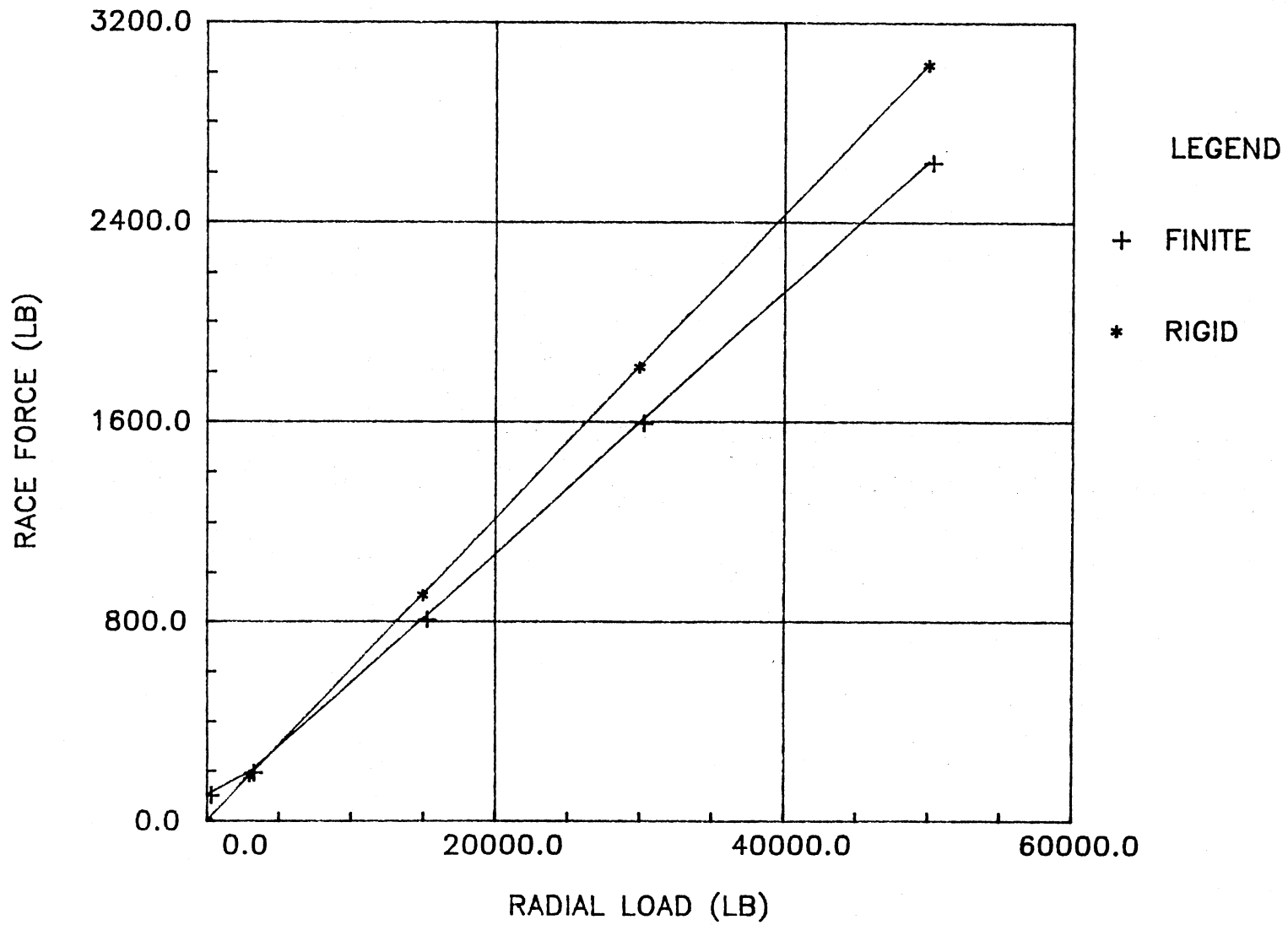
Figure 42. Finite Element and Rigid Model Plot of Race Force vs. Radial Load for Bearing #6

"elbow", due to the preload of the bolts, that develops into a straight line as the increasing radial load overcomes the bolt preload. When the straight line sections are extended they all intersect the origin. This will allow all the slopes to be directly compared. Table III shows the values of the slopes of the finite element model, S_F, the rigid model, S_R, and the slope difference, $S_R - S_F$, for each bearing. These values were plotted against various parameters including; number of balls, ball path diameter, and number of balls multiplied by the ball path diameter. As shown in Figure 43, the best correlation was obtained when the slope values from the finite element model were plotted against the number of balls multiplied by the ball path diameter, X_B. From Chapra and Canale (1985), the following equation was obtained by fitting a saturation-growth-rate function to these data points.

$$S_{FR} = \frac{(0.0464 \ X_B)}{(-368.9887 + X_B)} \tag{44}$$

Where, S_{FR} is the finite element model slope of the line relating the race force and the radial load. The curve generated from this equation is shown in Figure 43 along with the data points. Using equation (44) the value of the finite element slope of the line, relating the race force and radial load for a bearing with a known ball path diameter and number of balls, can be found. As previously mentioned, the finite element model is not able to incorporate the diametral clearance. The finite element model simulates a zero diametral clearance. Using the rigid model, a clearance adjustment factor, k_c, will be determined utilizing the same bearing parameters used in the finite element model. Two race force values will be

TABLE III

RIGID MODEL SLOPES, FINITE ELEMENT MODEL SLOPES AND
SLOPE DIFFERENCES FOR RACE FORCE VS. RADIAL LOAD

Bearing Number	S_R	S_F	$S_R - S_F$
1	0.1285	0.1181	0.0104
2	0.1214	0.1119	0.0095
3	0.1150	0.1024	0.0126
4	0.0672	0.0577	0.0095
5	0.0683	0.0574	0.0109
6	0.0607	0.0524	0.0083
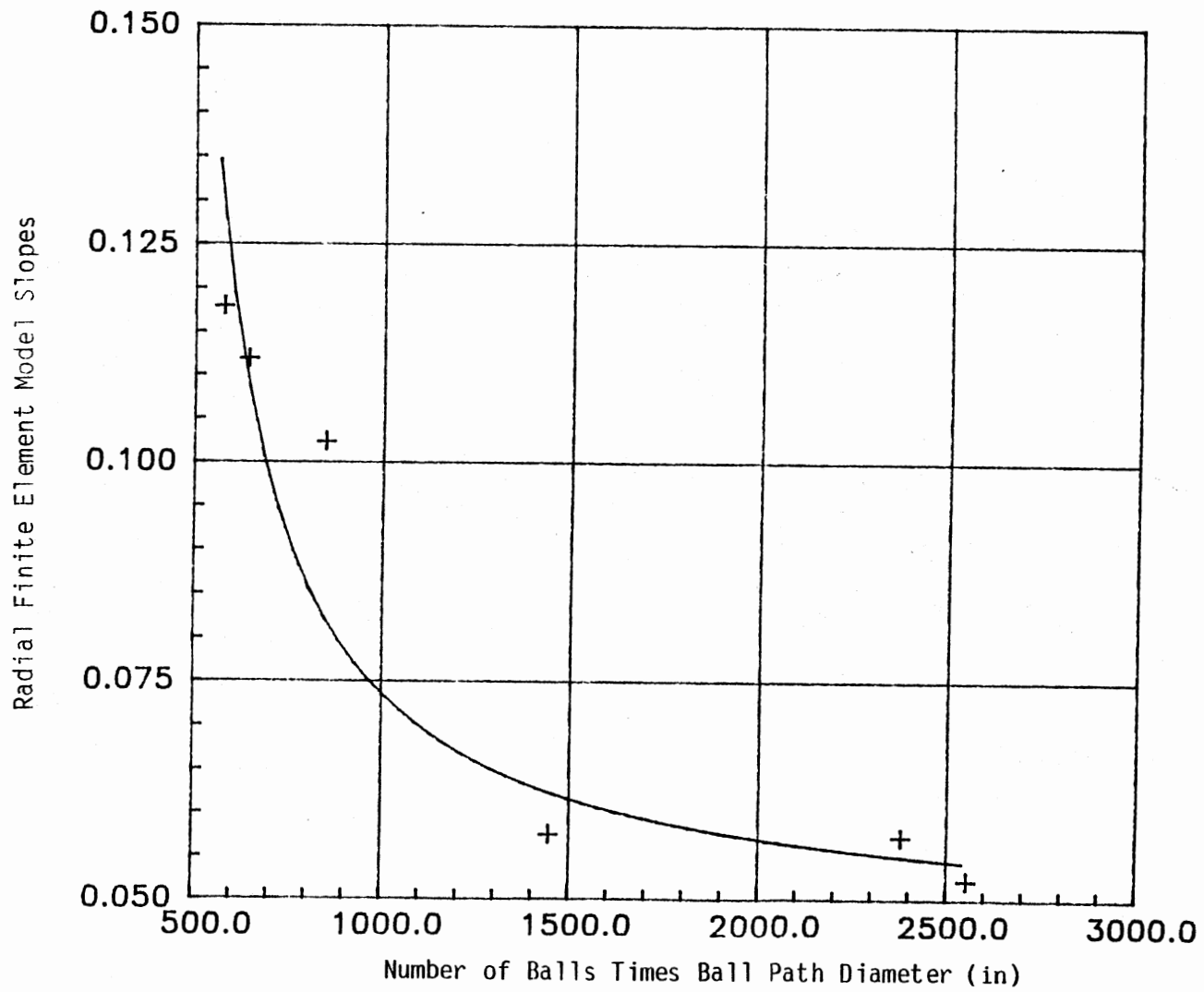
Figure 43. Plot of Radial Finite Element Model Slopes vs. Number of Balls Times Ball Path Diameter and Curve Generated from Equation (44)

calculated using the rigid model. The first race force, f_1, will be calculated using a zero diametral clearance. The second race force, f_2, will be calculated using the input diametral clearance. The clearance adjustment factor is calculated as shown below.

$$k_c = f_1/f_2 \tag{45}$$

The race force determined from the rating criteria is multiplied by the clearance adjustment factor, calculated from equation (45), to correct the finite element model. With the adjusted race force and the value of the finite element model slope, calculated from equation (44), the maximum radial load can be found for a specific bearing. An explanation of how these equations are used in the rating program will be presented in Chapter V.

Thrust

Four different thrust loads were applied to each of the bearings listed in Table I, using both the finite element and rigid models. An example of the load distribution obtained from both models is shown in the polar plot illustrated in Figure 44. The thrust load is represented by forces into the page. For the finite element model, "+" indicate forces at the top of the raceway, and the rigid model forces are indicated by "O". The forces, for each model, are equal at all the ball locations. The same applied thrust load and bearing parameters were used for both models. The maximum race forces from both models, and the thrust loads that created them, are listed in Tables VI-XI given in Appendix G, for bearing numbers 1-6, respectively. Figures 45-50 shows
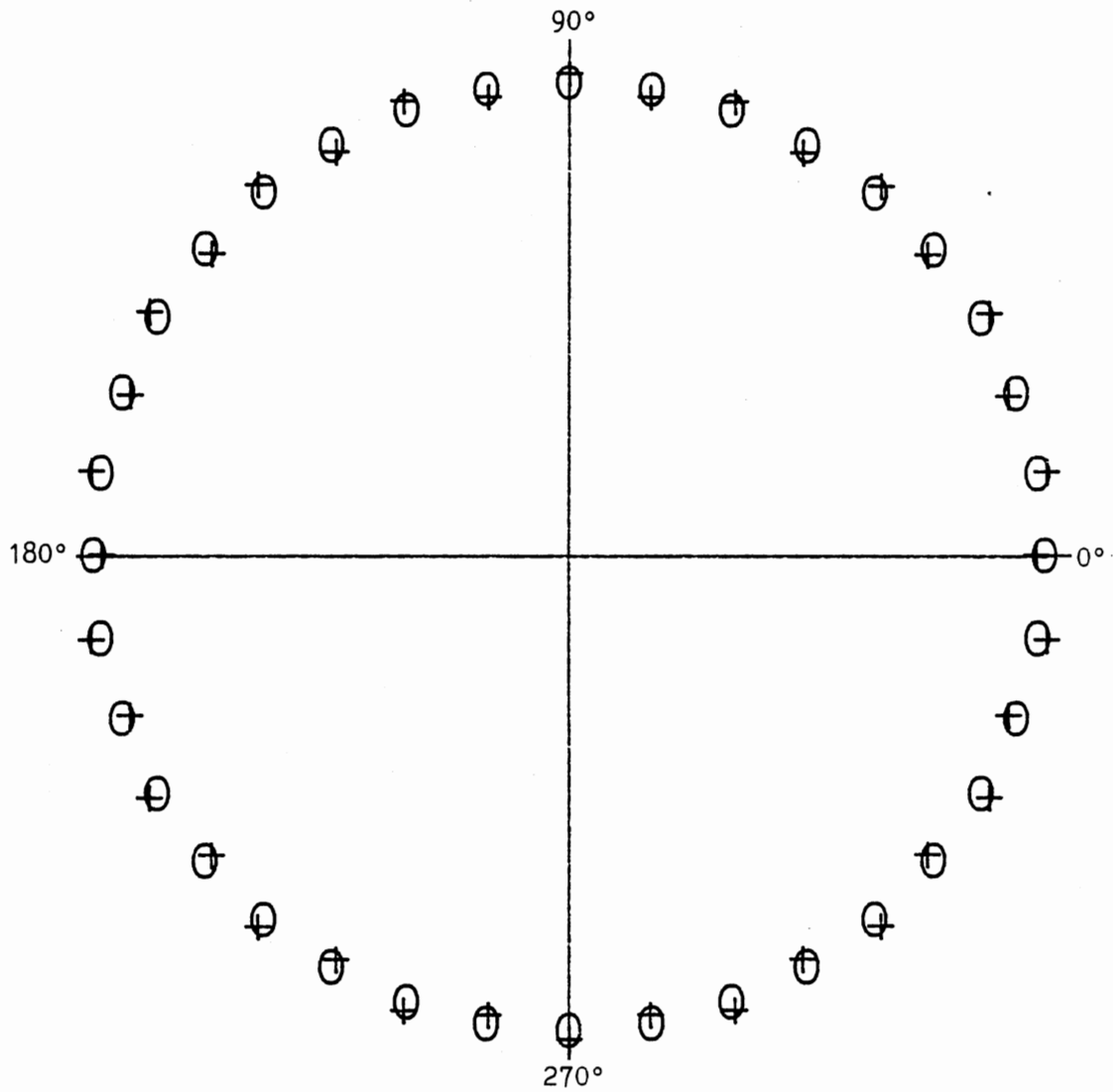
Figure 44. Polar Plot of Thrust Load Distribution from
Finite Element and Rigid Model Results

the plots of race force versus thrust load for each of the bearings. A relationship needs to be found between all the bearings. As with the moment and radial load applications, the connected rigid model data points create a straight line that intersects the origin. Also, the connected finite element model data points create an "elbow", due to the bolt preload, that begins above the origin and develops into a straight line as the applied thrust load increases. However, unlike the moment and radial load applications, when the straight line sections of the finite element model data points are extended they do not intersect the origin. Therefore, the values of the slopes of the lines generated by the two models cannot be directly compared. As can be seen in Figures 45-50, for each bearing the straight line section of the finite element model is nearly parallel with the rigid model line. For comparison the values of the slopes of the lines from the finite element model, S_F, and the rigid model, S_R, are calculated for each bearing and are shown along with the difference in slopes, $S_R - S_F$, in Table IV. The values of the slope difference indicate that the lines are parallel and therefore, the percent difference between the race forces from the two models can be used for comparison. Table IV also lists the percent difference of the race force between the finite element model and the rigid model for each bearing. In order to try to obtain a correlation between all the bearings these values were plotted against various parameters including; number of balls, ball path diameter, number of balls multiplied by ball path diameter, bearing circumference, bearing cross section moment of inertia, number of bolts, and bolt preload.
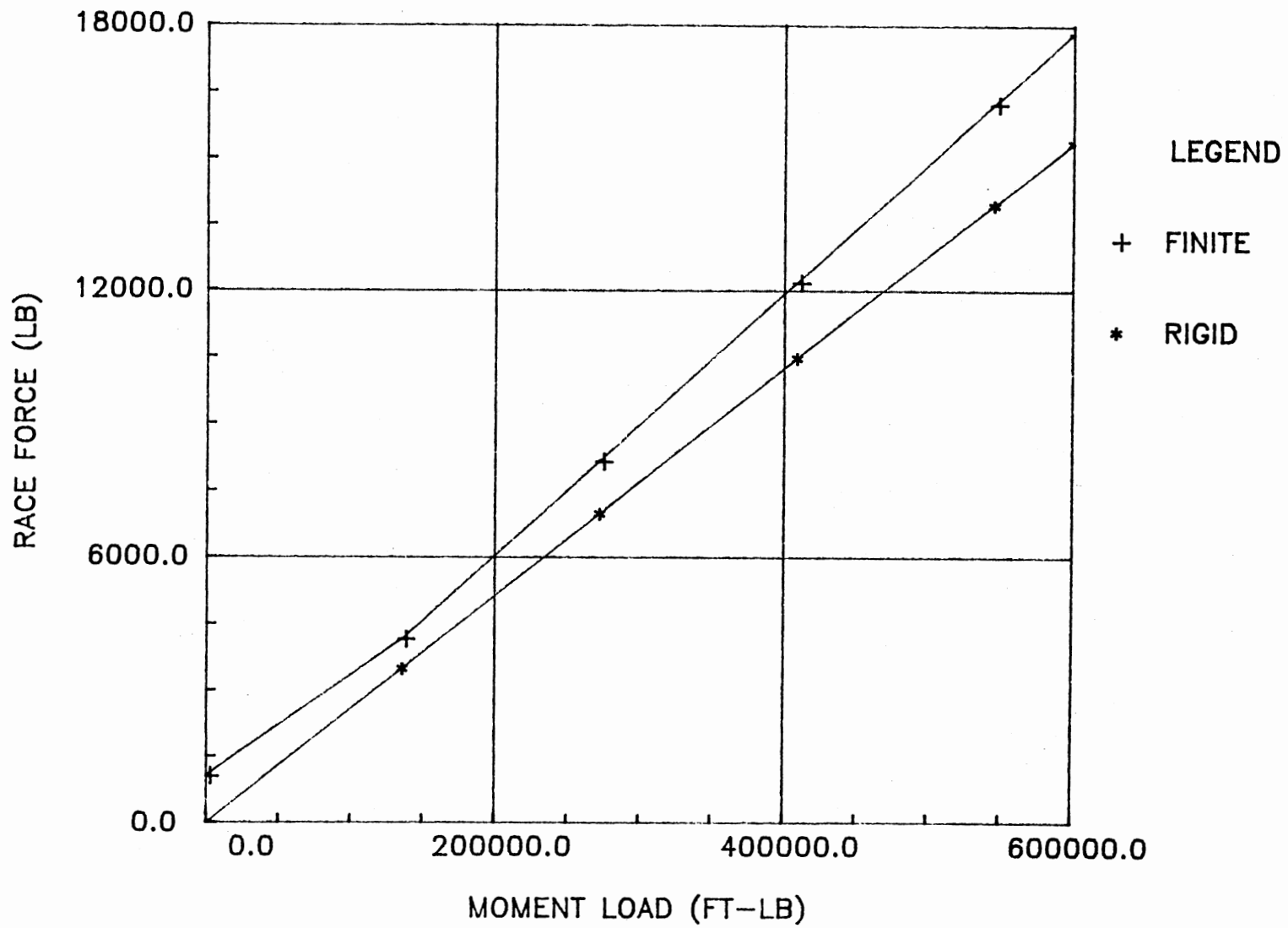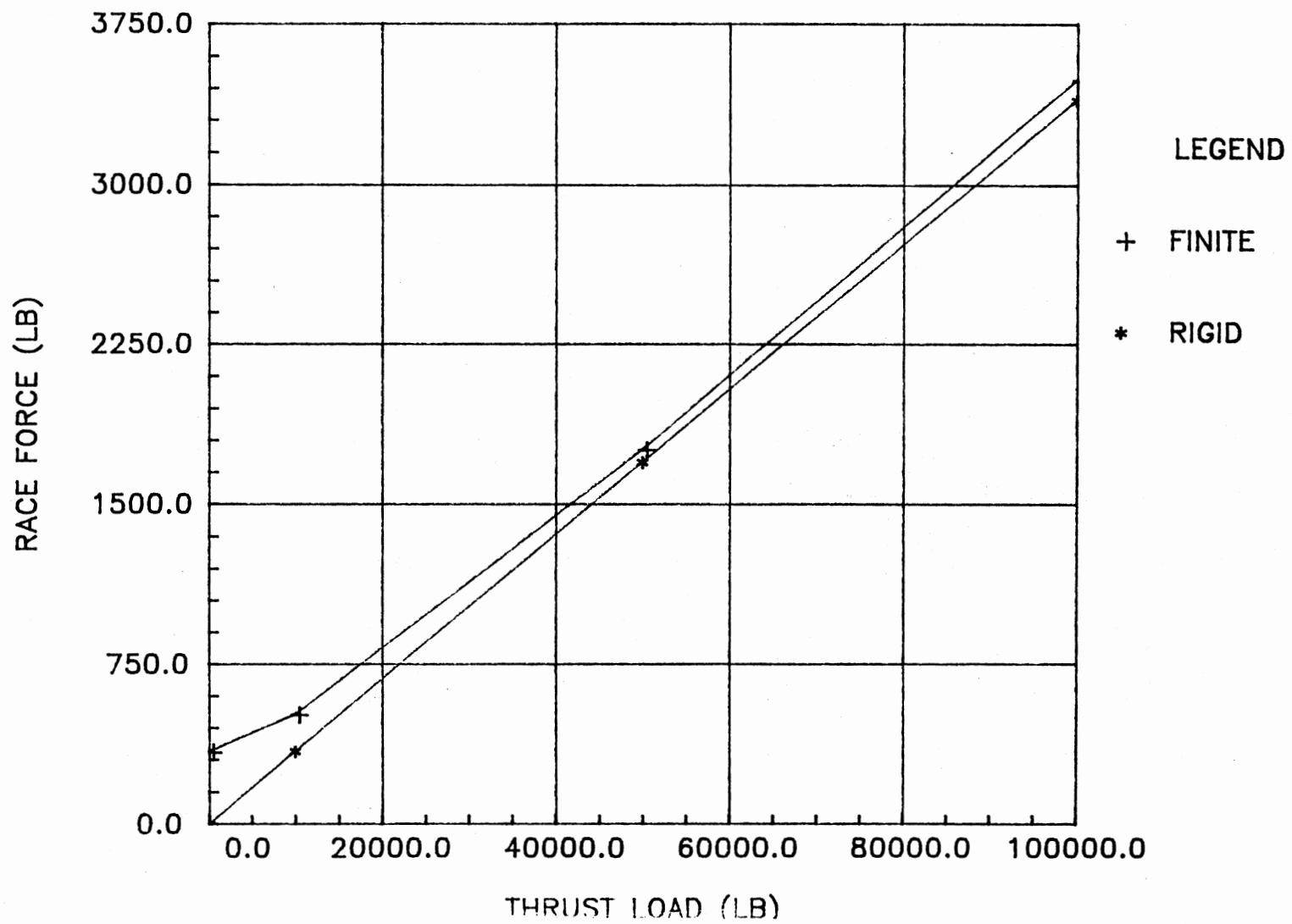
Figure 45.   Finite Element and Rigid Model Plot of Race Force vs. Thrust Load for Bearing #1

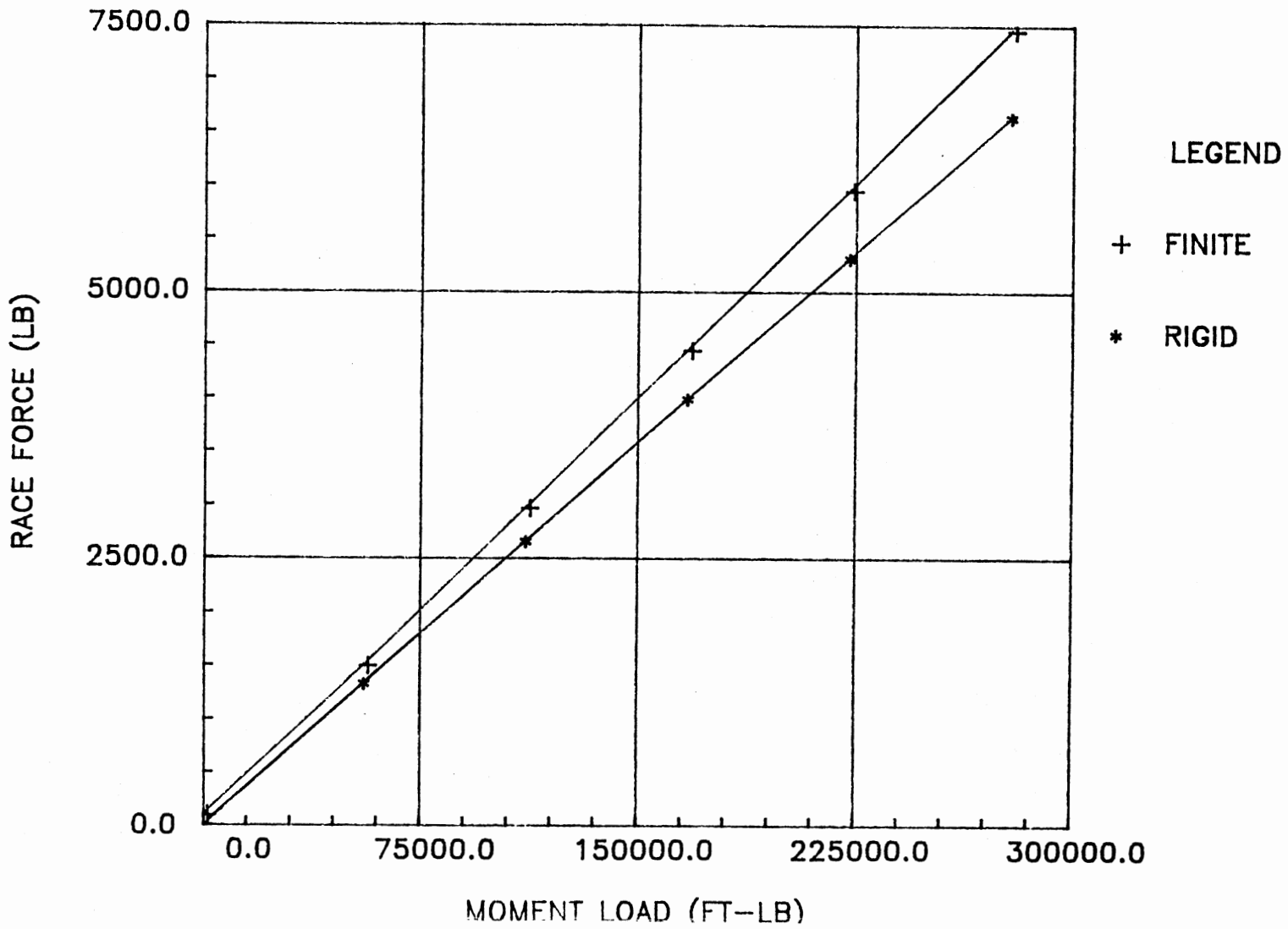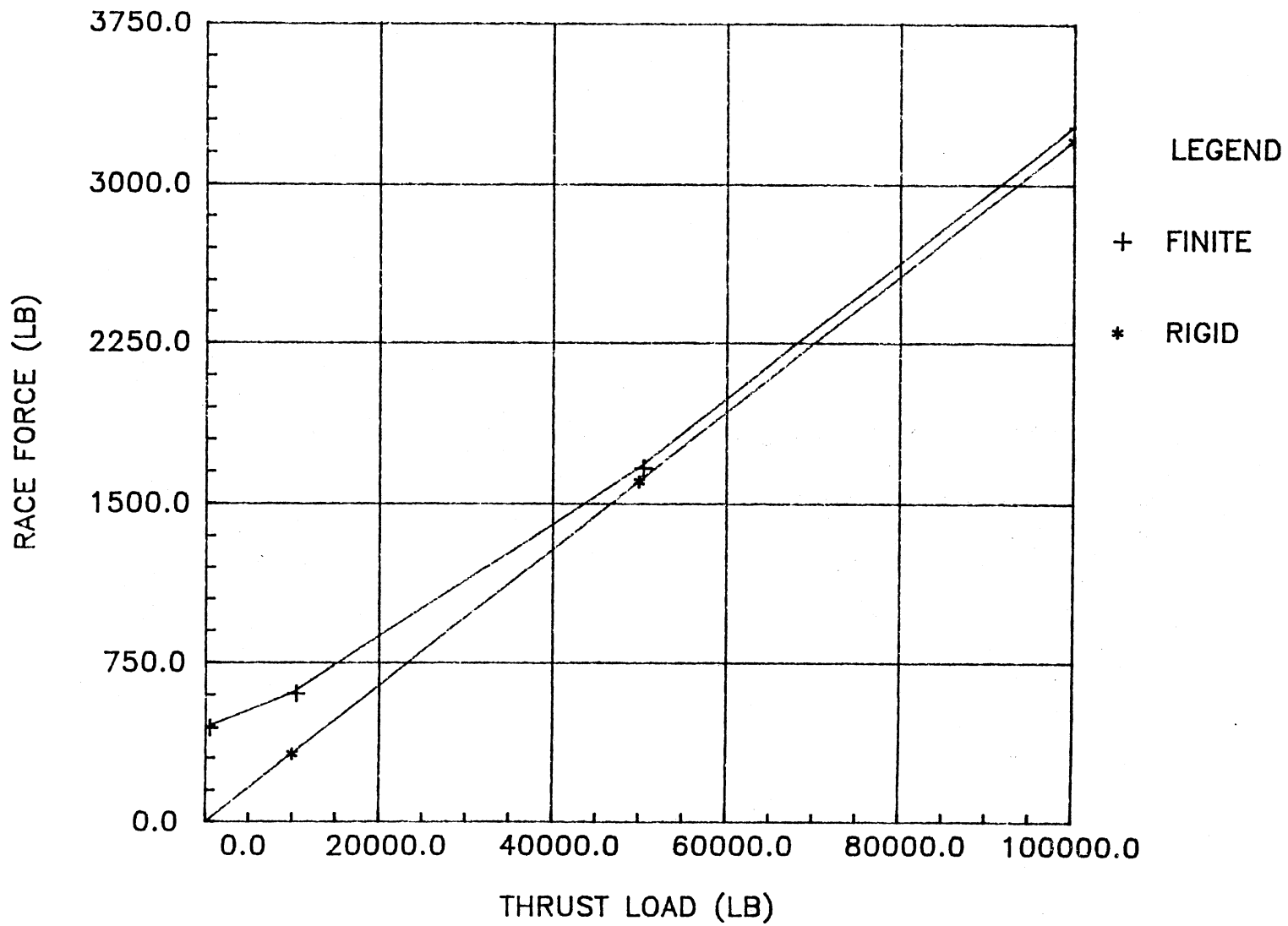Figure 46. Finite Element and Rigid Model Plot of Race Force vs. Thrust Load for Bearing #2
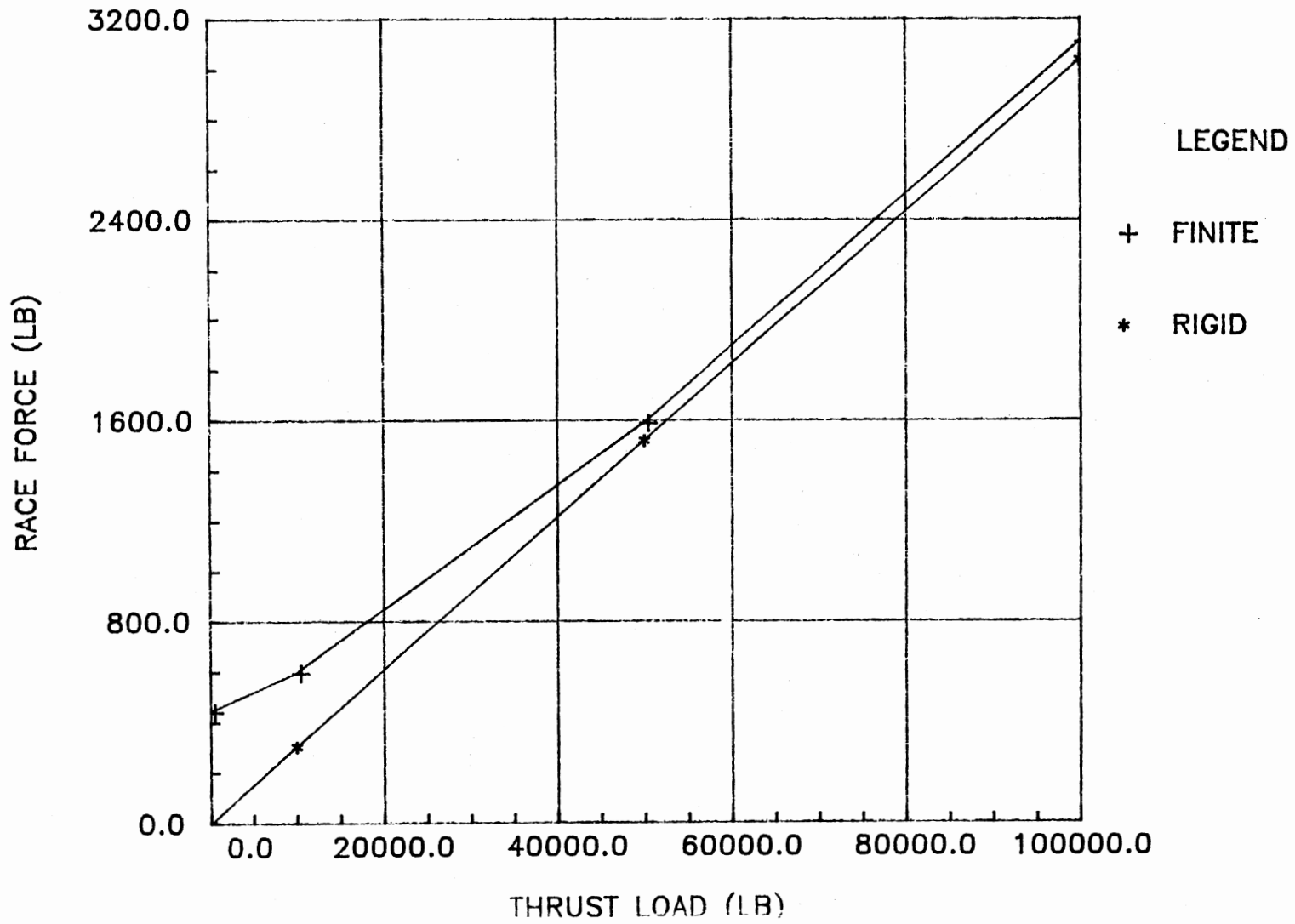
Figure 47. Finite Element and Rigid Model Plot of Race Force vs. Thrust Load for Bearing #3
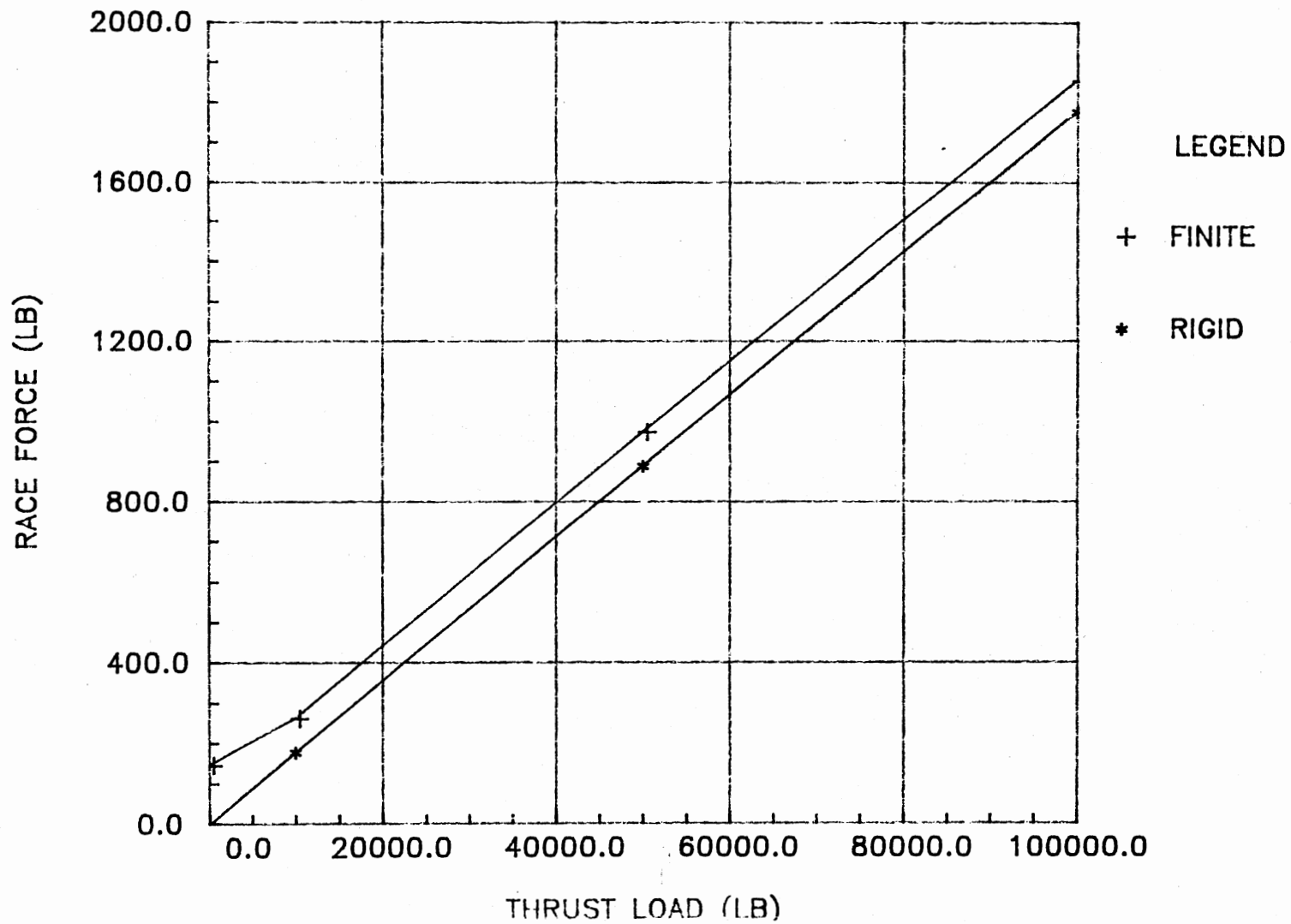
Figure 48. Finite Element and Rigid Model Plot of Race Force vs. Thrust Load for Bearing #4
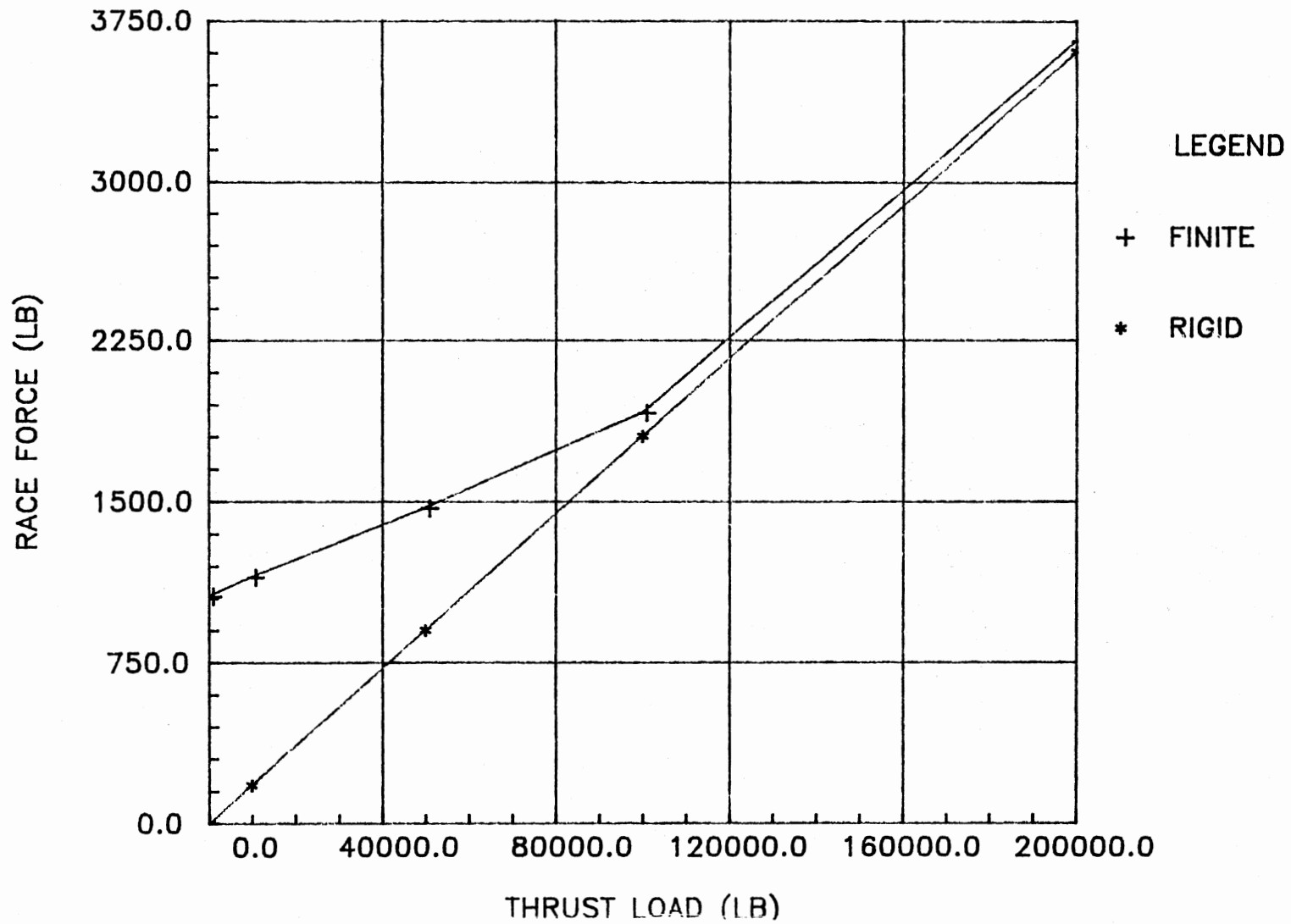
Figure 49.   Finite Element and Rigid Model Plot of Race Force vs. Thrust Load for Bearing #5
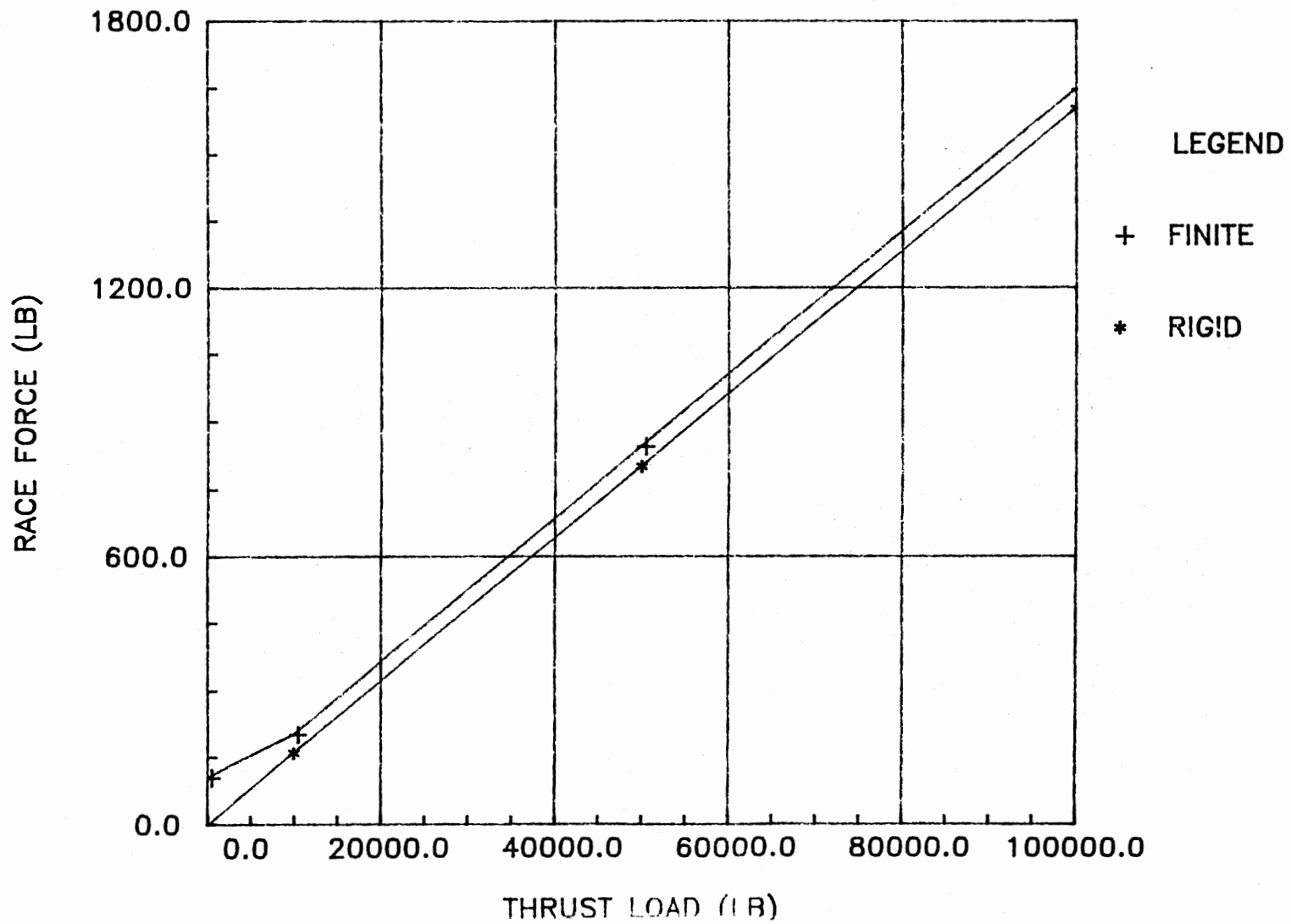
Figure 50. Finite Element and Rigid Model Plot of Race Force vs. Thrust Load for Bearing #6

TABLE IV

RIGID MODEL SLOPES, FINITE ELEMENT MODEL SLOPES,
SLOPE DIFFERENCES, AND RACE FORCE PERCENT
DIFFERENCES FOR RACE FORCE VS. THRUST LOAD

Bearing Number	S_R	S_F	$S_R - S_F$	Race Force % Difference
1	0.0340	0.0347	-0.0007	2.88
2	0.0321	0.0321	0.0	2.01
3	0.0304	0.0304	0.0	2.35
4	0.0178	0.0176	0.0002	4.48
5	0.0180	0.0175	0.0005	1.46
6	0.0160	0.0160	0.0	2.76

Unfortunately, none of these produced a useful correlation. From Table IV it can be seen that the race force percent difference between the two models is fairly small, the worst being less that five percent. This indicates that the rigid model is fairly accurate and, with a minor adjustment of five percent, can be used in rating a bearing for an applied thrust load. From the equations developed for the rigid model, equation (36) can be modified for the five percent race force difference which results in the following equation.

$$T = N(0.95\ F_i)\ \sin \alpha \qquad (46)$$

With the race force determined from the rating criteria and a bearing with a known contact angle and number of balls, the maximum thrust load can be obtained using equation (46). Chapter V will explain how this equation is utilized in the rating program.

CHAPTER V

BEARING RATING

Rating Algorithm

The objective of this study is to develop an interactive rating program for large diameter bearings. Given parameters for a certain bearing, we want to determine static and dynamic capacity ratings for moment, radial, and thrust applications; life of the bearing due to a static load; von Mises stress for increasing depth into the raceway; and required depth of the hardened case. The rating equations were developed from models that had a contact angle of sixty degrees and a conformity of 1.08, therfore these values are fixed in the program and will not be included as input variables. The program will request the input of:

> ball path diameter (inches)
>
> ball diameter (inches)
>
> number of balls
>
> diametral clearance (inches)
>
> Brinell factor
>
> safety factor
>
> raceway material yield strength (psi)

The program will also request the input of the bearing name so the

different bearings can be distinguished. The procedure for rating a bearing is as follows:

1. Calculate the limiting static race force - equation (5).

2. Calculate the limiting dynamic race force - equation (14).

3. Divide the limiting static and dynamic race forces by the safety factor.

4. Calculate the finite element model slope of the line relating race force and moment load - equation (43).

5. Calculate the static moment load.

$$M_s = \frac{F_s}{S_{FM}} \tag{47}$$

6. Calculate the dynamic moment load.

$$M_D = \frac{F_D}{S_{FM}} \tag{48}$$

7. Calculate the finite element model slope of the line relating race force and radial load - equation (44).

8. Calculate an initial static radial load.

$$R = \frac{F_s}{S_{FR}} \tag{49}$$

9. Calculate the clearance adjustment factor - equation (45). The clearance adjustment factor is found using an iteration method. The rigid model radial program is used to calculate race forces, f_1 and f_2, using the initial static radial load. The clearance

adjustment factor is calculated using equation (45). A new static radial load is calculated using the adjusted static race force and equation (48). The process is repeated until convergence is reached.

10. Calculate the static radial load.

$$R_S = \frac{F_S}{S_{FR}} \tag{50}$$

11. Calculate the dynamic radial load.

$$R_D = \frac{F_D}{S_{FR}} \tag{51}$$

12. Calculate the static thrust load - equation (46).

13. Calculate the dynamic thrust load - equation (46).

14. Calculate the fatigue life - equation (1).

15. Calculate the von Mises stress for increasing depth into the raceway based on the static race force - equations (6)-(12).

16. Determine the minimum case depth.

Appendix F contains a listing of the interactive rating program.

Example Rating

A sample bearing rating is obtained using the rating program and the following input variables;

    Bearing Name - Sample

    Ball Path Diameter - 16.5 inches

    Ball Diameter - 1.5 inches

Number of Balls - 34

Diametral Clearance - 0.0001 inches

Material Yield Strength - 130 kpsi

Brinell Factor - 0.0001

Safety Factor - 1.5

The output obtained from the rating program is shown in Figure 51. The von Mises (equivalent) stress values presented in the table require further explanation. The limiting static race force, which is based on a permanent deformation limit, is used to calculate these stresses. The calculated von Mises stresses are based on a linear elastic model. The calculated stress just below the surface may exceed the material yield limit in that region. In an actual situation the material will yield and the stresses will be lower. This will not affect the values at other points where the yielding is not exceeded such as those used for determining the minimum case depth.

Comparison With Established Standards

Established moment ratings were obtained for the six bearings listed in Table I and are shown in Table XI. These moment ratings are based on a Brinell factor of 0.0001 and a safety factor of 1.67. Corresponding moment ratings were made using the same Brinell and safety factors, the bearing data from Table I, and the rating program. The calculated moment ratings are also shown in Table V. The calculated moment ratings compare quite well with the established standards. The largest difference, of nineteen percent, is seen in bearing #6 with the calculated value being less conservative.

COMPUTER-AIDED ROTATION BEARING RATING

INPUT

```
Bearing Number - SAMPLE
Ball Path Diameter (in)          =     16.500
Ball Diameter (in)               =      1.500
Number of Balls                  =         34
Diametral Clearance (in)         =   0.000100
Material Yield Strength (kpsi)   =        130
Brinell Factor                   =   0.000100
Safety Factor                    =       1.50
```

RATINGS

```
Static Moment Load (ft-lbs)          =   115402.
Dynamic Moment Load (ft-lbs)         =    44572.
Static Radial Load (lbs)             =    79439.
Dynamic Radial Load (lbs)            =    30682.
Static Thrust Load (lbs)             =   301245.
Dynamic Thrust Load (lbs)            =   116351.
Fatigue Life For Static Load (rev)   =   194459.
```

CASE-CORE CRUSHING DATA

Depth (in)	von Mises Stress (psi)
0.000	611623.7
0.020	638109.0
0.040	597068.2
0.060	515738.1
0.080	426918.1
0.100	347652.3
0.120	282725.0
0.140	231375.1
0.160	191194.3
0.180	159713.0
0.200	134869.3
0.220	115076.9
0.240	99133.9
0.260	86151.7
0.280	75477.2
0.300	66589.7
0.320	59167.7
0.340	52822.4
0.360	47581.7
0.380	42364.8
0.400	38710.4

Suggested Minimum Case Depth (in) = 0.220

Figure 51. Sample Output of the Rating Program

TABLE V

ESTABLISHED AND CALCULATED MOMENT RATINGS FOR BEARING #1-6

Bearing Number	Moment Ratings (Ft - Lb)	
	Established	Calculated
1	93,300	104,200
2	101,500	114,600
3	188,500	197,900
4	104,000	102,400
5	533,000	476,600
6	301,000	372,100

Rating Limitations

The rating equations were developed using the dimensional constants for the the six bearings listed in Table I. Using the program for values outside of the range of these constants may produce inaccurate and unreliable results. The limiting variables include; ball path diameter (16.5 - 37.0 inches), ball diameter (1.0 - 1.75 inches), and number of balls multiplied by ball path diameter (560 - 2540 inches). The moment, radial, and thrust load ratings are calculated separately and should not be added to obtain a combined load rating. The six bearings listed in Table I all had evenly spaced bolt patterns. Bearings with unevenly spaced bolt patterns may have higher race forces and therefore should not be rated with this program.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The program developed is a versatile and reliable method of rating large diameter bearings for moment, radial, and thrust loads. The Brinell and safety factor input variables gives the user the ability to control how conservative a bearing is rated. The program allows for a wide range of ball path diameters, and can rate all but the largest bearings. The accuracy of the program was verified by comparing moment ratings against established standards.

Recommendations

Although the results from the rating program are acceptable, the usefulness, range, and accuracy can be improved. The program rating equations were developed using a limited range and number of bearing parameters. Using more bearings with a wider range of bearing parameters to develop the rating equations would increase the usefulness and range of the program's application. The affect of the diametral clearance on the bearing could not be fully investigated because of the limitations in the finite element analysis program (ANSYS). The same limitations also prevented the accuracy of the model to be checked by using a finer meshed model. A more powerful finite element analysis program would allow the diametral clearance affect and the model

accuracy to be investigated.

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APPENDIXES

APPENDIX A

RIGID MODEL: MOMENT LOAD PROGRAM LISTING

```
PROGRAM MOMENT(INPUT,OUTPUT,DATA);

TYPE NUM=ARRAY[0..100]OF REAL;

VAR
    C,                          {CONFORMITY}
    CA,                         {CONTACT ANGLE (DEGREES)}
    C1,                         {EQUATION 26 CONSTANT}
    D,                          {BALL DIAMETER (IN)}
    DM,                         {MAXIMUM BALL DEFORMATION (IN)}
    M,                          {MOMENT LOAD (FT-LB)}
    PD,                         {BALL PATH DIAMETER (IN)}
    PR,                         {BALL PATH RADIUS (IN)}
    THETA:                      {BALL SPACING (DEGREES)}
       REAL;

    N:                          {NUMBER OF BALLS}
       INTEGER;

    DI,                         {DEFORMATION OF BALL I (IN)}
    FI,                         {FORCE ON BALL I (LBS)}
    THI,                        {POSITION OF BALL I (DEGREES)}
    THIR:                       {POSITION OF BALL I (RADIANS)}
       NUM;

    DATA:                       {OUTPUT FILE}
       TEXT;

    Q:                          {YESNO PROCEDURE CONSTANT}
       BOOLEAN;


PROCEDURE YesNo(VAR Yes:BOOLEAN);

{*******************************************************************************

  Returns TRUE for an input of YES, yes, Y, or y; returns FALSE for an input
  of NO, no, N, or n.  If the user inputs anything else he is prompted to
  make a valid input.

  GLOBAL VARIABLES:  Q

*******************************************************************************}

VAR
  String : PACKED ARRAY[1..3] OF CHAR;
  Legal  : BOOLEAN;

BEGIN
  REPEAT
    Legal:=TRUE;
    READLN(String);
    IF String='YES' THEN Yes:=TRUE
    ELSE IF String='yes' THEN Yes:=TRUE
```

```
         ELSE IF String='Y  ' THEN Yes:=TRUE
         ELSE IF String='y  ' THEN Yes:=TRUE
         ELSE IF String='NO ' THEN Yes:=FALSE
         ELSE IF String='no ' THEN Yes:=FALSE
         ELSE IF String='N  ' THEN Yes:=FALSE
         ELSE IF String='n  ' THEN Yes:=FALSE
         ELSE
           BEGIN
             WRITELN('*********************************************');
             WRITELN('Response must be YES, yes, Y, y, NO, no, N, or n');
             WRITELN('Please enter a valid response ...');
             WRITELN('*********************************************');
             Legal:=FALSE;
           END;
      UNTIL Legal;
    END;  {YESNO}


    PROCEDURE CLEAR;

    {*********************************************************************

      CLEARS THE SCREEN

      GLOBAL VARIABLES:  NONE

    *********************************************************************}

    VAR B:INTEGER;

    BEGIN
      FOR B:=1 TO 24 DO WRITELN;
    END;  {CLEAR}


    FUNCTION EXPT(Y,X:REAL):REAL;

    {*********************************************************************

      RETURNS THE RESULT OF Y^X

      GLOBAL VARIABLES:  NONE

    *********************************************************************}

    VAR Z:INTEGER;

    BEGIN
      IF X=0 THEN
        EXPT:=1.0
      ELSE
        IF Y=0 THEN
          EXPT:=0.0
        ELSE
```

```
      IF Y>0 THEN
        EXPT:=EXP(X*LN(Y))
      ELSE
        IF ABS(X-ROUND(X))<1E-10 THEN
        BEGIN
        Z:=TRUNC(X);
        IF ODD(Z) THEN
          EXPT:=-EXP(X*LN(-Y))
        ELSE
          EXPT:=EXP(X*LN(-Y))
        END
END;  {EXPT}


PROCEDURE OUTDATA;

{********************************************************************

  SETS UP THE OUTPUT FILE

  GLOBAL VARIABLES:  DATA

********************************************************************}

VAR STATUS:INTEGER;
    FILENAME:PACKED ARRAY[1..17] OF CHAR;

BEGIN
  CLEAR;
  REPEAT
    WRITELN('ENTER THE NAME OF THE OUTPUT FILE - ');
    READLN(FILENAME);
    BIND(DATA,FILENAME,STATUS);
    IF STATUS=1 THEN
      WRITELN('FILE DOES NOT EXIST OR IS INACCESSIBLE, PLEASE TRY AGAIN');
    WRITELN;
  UNTIL STATUS=0;
  REWRITE(DATA);
END;  {OUTDATA}


PROCEDURE INVAR;

{********************************************************************

  INITIALIZE THE PROGRAM VARIABLES

  GLOBAL VARIABLES:  C,CA,D,DATA,M,N,PD

********************************************************************}

BEGIN
  WRITELN;
  WRITELN(DATA);
```

```
      WRITELN(DATA);
      WRITELN('ENTER THE MOMENT LOAD (FT-LB) - ');
      READLN(M);
      WRITELN;
      WRITELN('MOMENT LOAD = ',M:8:3,' FT-LB');
      WRITELN(DATA,'MOMENT LOAD = ',M:8:3,' FT-LB');
      WRITELN;
      WRITELN('ENTER THE BALL PATH DIAMETER (IN) - ');
      READLN(PD);
      WRITELN;
      WRITELN('BALL PATH DIAMETER  = ',PD:8:3,' IN');
      WRITELN(DATA,'BALL PATH DIAMETER = ',PD:8:3,' IN');
      WRITELN;
      WRITELN('ENTER THE CONFORMITY - ');
      READLN(C);
      WRITELN;
      WRITELN('CONFORMITY  = ',C:8:5);
      WRITELN(DATA,'CONFORMITY = ',C:8:5);
      WRITELN;
      WRITELN('ENTER THE CONTACT ANGLE (DEGREES) - ');
      READLN(CA);
      WRITELN;
      WRITELN('CONTACT ANGLE = ',CA:8:4,' DEGREES');
      WRITELN(DATA,'CONTACT ANGLE = ',CA:8:4,' DEGREES');
      WRITELN;
      WRITELN('ENTER THE BALL DIAMETER (IN) - ');
      READLN(D);
      WRITELN;
      WRITELN('BALL DIAMETER = ',D:6:3,' IN');
      WRITELN(DATA,'BALL DIAMETER = ',D:6:3,' IN');
      WRITELN;
      WRITELN('ENTER THE NUMBER OF BALLS - ');
      READLN(N);
      WRITELN;
      WRITELN('NUMBER OF BALLS = ',N:3);
      WRITELN(DATA,'NUMBER OF BALLS = ',N:3);
      WRITELN;
  END;  {INVAR}


PROCEDURE CONSTANTS;

{*********************************************************************

  CALCULATE THE CONSTANTS

  GLOBAL VARIABLES:  C,CA,C1,D,M,N,PD,PR,THETA

*********************************************************************}

BEGIN
  M:=M*12.0;
  PR:=PD/2.0;
  C1:=2.198E7*SQRT((C*0.5*D)/(C-1));          {EQUATION 25}
```

```
      THETA:=360.0/N;
      CA:=CA*3.1415927/180.0;
      WRITELN('THE BALL SPACING IS ',THETA:8:4,' DEGREES');
      WRITELN(DATA,'THE BALL SPACING IS = ',THETA:8:4,' DEGREES');
      WRITELN;
    END;  {CONSTANTS}


PROCEDURE MDFOR;

{*****************************************************************************

   CALCULATE THE MAXIMUM BALL DEFORMATION

   GLOBAL VARIABLES:  CA,C1,DM,M,N,PR,THETA,THI,THIR

*****************************************************************************}

VAR SUM,EXPTHIR,MAX:REAL;
    K,I:INTEGER;

BEGIN
  SUM:=0.0;
  K:=0;
  FOR I:= 1 TO N DO
  BEGIN
    THI[I]:=(THETA*K);
    THIR[I]:=THI[I]*3.1415927/180;
    EXPTHIR:=EXPT(ABS(COS(THIR[I])),2.5);
    SUM:=SUM + EXPTHIR;
    K:=K+1;
  END;
  DM:=EXPT(M/(C1*PR*SIN(CA)*SUM),2/3);     {EQUATION 30}
END;  {MDFOR}


PROCEDURE BALLDFOR;

{*****************************************************************************

   CALCULATE THE INDIVIDUAL BALL DEFORMATION

   GLOBAL VARIABLES:  DI,DM,N,THIR

*****************************************************************************}

VAR I:INTEGER;

BEGIN
  FOR I:= 1 TO N DO
  BEGIN
    DI[I]:=DM*ABS(COS(THIR[I]));           {EQUATION 27}
  END;
END;  {BALLDFOR}
```

```
PROCEDURE BALLFORCE;

{********************************************************************************

  CALCULATE THE INDIVIDUAL BALL FORCES

  GLOBAL VARIABLES:  C1,DI,FI,N

********************************************************************************}

VAR I:INTEGER;

BEGIN
  FOR I:= 1 TO N DO
    FI[I]:=C1*EXPT(DI[I],1.5);              {EQUATION 26}
END;  {BALLFORCE}


PROCEDURE WRITEDATA;

{********************************************************************************

  WRITE THE DATA TO THE SCREEN AND THE OUTPUT FILE

  GLOBAL VARIABLES:  DI,FI,N,THI

********************************************************************************}

VAR I:INTEGER;

BEGIN
  CLEAR;
  WRITELN;
  WRITELN('BALL NUMBER    POSITION (DEG)    DEFORMATION (IN)    FORCE (LBS)');
  WRITELN;
  FOR I:=1 TO N DO
    WRITELN(I,'      ',THI[I]:12:4,'      ',DI[I]:15:6,'      ',FI[I]:14:2);
  WRITELN(DATA);
  WRITELN(DATA,'BALL NUMBER   POSITION (DEG)   DEFORMATION (IN)  FORCE (LBS)');
  WRITELN(DATA);
  FOR I:=1 TO N DO
    WRITELN(DATA,I,'    ',THI[I]:14:4,'    ',DI[I]:15:9,'    ',FI[I]:14:2);
END;  {WRITEDATA}


{********************************************************************************

  BEGIN MAIN PROGRAM

********************************************************************************}

BEGIN
```

```
     OUTDATA;
     REPEAT
        INVAR;
        CONSTANTS;
        MDFOR;
        BALLDFOR;
        BALLFORCE;
        WRITEDATA;
        WRITELN('DO YOU WANT TO RUN THE PROGRAM AGAIN (Y/N) ?');
        YESNO(Q);
     UNTIL Q=FALSE;
     CLOSE(DATA);
END.  {MAIN}
```

APPENDIX B

RIGID MODEL: RADIAL LOAD PROGRAM LISTING

```
PROGRAM RADIAL(INPUT,OUTPUT,DATA);

TYPE NUM=ARRAY[1..100]OF REAL;

VAR
    C,                      {CONFORMITY}
    CA,                     {CONTACT ANGLE (DEGREES)}
    C1,                     {EQUATION (26) CONSTANT}
    D,                      {BALL DIAMETER (IN)}
    DC,                     {DIAMETERAL CLEARANCE (INCHES)}
    DM,                     {MAXIMUM BALL DEFORMATION (INCHES)}
    PD,                     {BALL PATH DIAMETER (INCHES)}
    R,                      {RADIAL LOAD (LBS)}
    RAD,                    {DEGREE TO RADIAN CONVERSION}
    THETA,                  {BALL SPACING (DEGREES)}
    THETAZONE:
      REAL;

    N:                      {NUMBER OF BALLS}
      INTEGER;

    FI,                     {FORCE ON BALL I (LBS)}
    DI,                     {DEFORMATION OF BALL I (IN)}
    THI:                    {POSITION OF BALL I (DEGREES)}
      NUM;

    DATA:                   {OUTPUT FILE}
      TEXT;

    Q:                      {YESNO PROCEDURE CONSTANT}
      BOOLEAN;


PROCEDURE YesNo(VAR Yes:BOOLEAN);

{*********************************************************************************

  Returns TRUE for an input of YES, yes, Y, or y; returns FALSE for an input
  of NO, no, N, or n.  If the user inputs anything else he is prompted to
  make a valid input.

  GLOBAL VARIABLES:  Q

*********************************************************************************}

VAR
  String : PACKED ARRAY[1..3] OF CHAR;
  Legal  : BOOLEAN;

BEGIN
  REPEAT
    Legal:=TRUE;
    READLN(String);
    IF String='YES' THEN Yes:=TRUE
```

```
      ELSE IF String='yes' THEN Yes:=TRUE
      ELSE IF String='Y ' THEN Yes:=TRUE
      ELSE IF String='y ' THEN Yes:=TRUE
      ELSE IF String='NO ' THEN Yes:=FALSE
      ELSE IF String='no ' THEN Yes:=FALSE
      ELSE IF String='N ' THEN Yes:=FALSE
      ELSE IF String='n ' THEN Yes:=FALSE
      ELSE
        BEGIN
          WRITELN('********************************************');
          WRITELN('Response must be YES, yes, Y, y, NO, no, N, or n');
          WRITELN('Please enter a valid response ...');
          WRITELN('********************************************');
          Legal:=FALSE;
        END;
    UNTIL Legal;
END;  {YESNO}


PROCEDURE CLEAR;

{*******************************************************************************

  CLEARS THE SCREEN

  GLOBAL VARIABLES:  NONE

*******************************************************************************}

VAR B:INTEGER;

BEGIN
  FOR B:=1 TO 24 DO WRITELN;
END;  {CLEAR}


FUNCTION ARCCOS(X:REAL):REAL;

{*******************************************************************************

  INVERSE COSINE FUNCTION, RETURNS ARCCOS(X), -1.0<= X <=+1.0, IN RADIANS

  GLOBAL VARIABLES:  NONE

*******************************************************************************}

CONST   PI=3.1415926536;

BEGIN
  IF (X=1) OR (X=-1) THEN
    IF X=1 THEN
      ARCCOS:=0.0
    ELSE
      ARCCOS:=PI
```

```
    ELSE
      ARCCOS:=-ARCTAN(X/SQRT(1-X*X))+PI/2.0
END;  (ARCCOS)


FUNCTION EXPT(Y,X:REAL):REAL;

(*********************************************************************

  RETURNS THE RESULT OF Y^X

  GLOBAL VARIABLES:  NONE

*********************************************************************)

VAR Z:INTEGER;

BEGIN
  IF X=0 THEN
    EXPT:=1.0
  ELSE
    IF Y=0 THEN
      EXPT:=0.0
    ELSE
      IF Y>0 THEN
        EXPT:=EXP(X*LN(Y))
      ELSE
        IF ABS(X-ROUND(X))<1E-10 THEN
        BEGIN
        Z:=TRUNC(X);
        IF ODD(Z) THEN
          EXPT:=-EXP(X*LN(-Y))
        ELSE
          EXPT:=EXP(X*LN(-Y))
        END
END;  (EXPT)


PROCEDURE OUTDATA;

(*********************************************************************

  SETS UP THE OUTPUT FILE

  GLOBAL VARIABLES:  DATA

*********************************************************************)

VAR STATUS:INTEGER;
    FILENAME:PACKED ARRAY[1..17] OF CHAR;

BEGIN
  CLEAR;
  REPEAT
```

```
      WRITELN('ENTER THE NAME OF THE OUTPUT FILE - ');
      READLN(FILENAME);
      BIND(DATA,FILENAME,STATUS);
      IF STATUS=1 THEN
        WRITELN('FILE DOES NOT EXIST OR IS INACCESSIBLE, PLEASE TRY AGAIN');
      WRITELN;
    UNTIL STATUS=0;
    REWRITE(DATA);
END;  {OUTDATA}


PROCEDURE INVAR;

{**************************************************************************

   INITIALIZE THE PROGRAM VARIABLES

   GLOBAL VARIABLES:  C,CA,D,DATA,DC,N,PD,R,THETA

*************************************************************************}

BEGIN
  WRITELN;
  WRITELN('ENTER THE RADIAL LOAD (LBS) - ');
  READLN(R);
  WRITELN;
  WRITELN('RADIAL LOAD = ',R:8:3,' LBS');
  WRITELN(DATA,'RADIAL LOAD = ',R:8:3,' LBS');
  WRITELN;
  WRITELN('ENTER THE BALL PATH DIAMETER (IN) - ');
  READLN(PD);
  WRITELN;
  WRITELN('BALL PATH DIAMETER = ',PD:8:3,' IN');
  WRITELN(DATA,'BALL PATH DIAMETER = ',PD:8:3,' IN');
  WRITELN;
  WRITELN('ENTER THE DIAMETERAL CLEARANCE (IN) - ');
  READLN(DC);
  WRITELN;
  WRITELN('DIAMETERAL CLEARANCE = ',DC:8:7,' IN');
  WRITELN(DATA,'DIAMETERAL CLEARANCE = ',DC:8:7,' IN');
  WRITELN;
  WRITELN('ENTER THE CONFORMITY - ');
  READLN(C);
  WRITELN;
  WRITELN('CONFORMITY = ',C:8:5);
  WRITELN(DATA,'CONFORMITY = ',C:8:5);
  WRITELN;
  WRITELN('ENTER THE CONTACT ANGLE (DEGREES) - ');
  READLN(CA);
  WRITELN;
  WRITELN('CONTACT ANGLE = ',CA:8:4,' DEGREES');
  WRITELN(DATA,'CONTACT ANGLE = ',CA:8:4,' DEGREES');
  WRITELN;
  WRITELN('ENTER THE BALL DIAMETER (IN) - ');
```

```
      READLN(D);
      WRITELN;
      WRITELN('BALL DIAMETER = ',D:6:3,' IN');
      WRITELN(DATA,'BALL DIAMETER = ',D:6:3,' IN');
      WRITELN;
      WRITELN('ENTER THE NUMBER OF BALLS - ');
      READLN(N);
      WRITELN;
      WRITELN('NUMBER OF BALLS = ',N:3);
      WRITELN(DATA,'NUMBER OF BALLS = ',N:3);
      THETA:=360/N;
      WRITELN;
      WRITELN('THE BALL SPACING IS ',THETA:8:4,' DEGREES');
      WRITELN(DATA,'THE BALL SPACING IS = ',THETA:8:4,' DEGREES');
    END;  {INVAR}


    PROCEDURE CONSTANTS;

    {*****************************************************************************

      CALCULATE THE CONSTANTS

      GLOBAL VARIABLES:  C,CA,C1,D,RAD,THETA,THI

    *****************************************************************************}

    VAR I:INTEGER;

    BEGIN
      FOR I:=1 TO N DO
        THI[I]:=THETA*(I-1);
      RAD:=3.1415927/180.0;
      THETA:=THETA*RAD;
      CA:=CA*RAD;
      C1:=2.198E7*SQRT((C*0.5*D)/(C-1));        {EQUATION 25}
    END;  {CONSTANTS}


    PROCEDURE ZONE(VAR THETAZ2:REAL);

    {*****************************************************************************

      CALCULATE THE LOAD ZONE

      GLOBAL VARIABLES:  DC,DM

    *****************************************************************************}

    BEGIN
      THETAZ2:=DC/(2.0*DM);                     {EQUATION 31}
      IF ABS(THETAZ2)>=1.0 THEN
        THETAZ2:=0.0
      ELSE
```

```
    THETAZ2:=ARCCOS(THETAZ2);
END;  {ZONE}


PROCEDURE MDFOR;

{***************************************************************************

  CALCULATE THE MAXIMUM BALL DEFORMATION

  GLOBAL VARIALBES:  CA,C1,DM,R,THETA,THETAZONE

***************************************************************************}

VAR THBEG,THSUM,EXPTHSUM,
    THETAZ1,THETAZ2,DTHETAZ1,DTHETAZ2,SUM:REAL;
    J:INTEGER;

BEGIN
  DTHETAZ2:=0.0;
  THETAZ2:=THETA;
  REPEAT
    THETAZ1:=THETAZ2;
    SUM:=0.0;
    THBEG:=0.0;
    WHILE THBEG >= -THETAZ1 DO
      THBEG:=THBEG - THETA;
    THBEG:=THBEG + THETA;
    THSUM:=THBEG;
    WHILE THSUM <= THETAZ1 DO
    BEGIN
      EXPTHSUM:=EXPT(COS(THSUM),2.5);
      SUM:=SUM + EXPTHSUM;
      THSUM:=THSUM + THETA;
    END;
    DM:=EXPT(R/(2.0*C1*COS(CA)*SUM),2.0/3.0);      {EQUATION 34}
    ZONE(THETAZ2);
    DTHETAZ1:=THETAZ2-THETAZ1;
    IF ABS(DTHETAZ1)=ABS(DTHETAZ2) THEN
      IF THETAZ1 > THETAZ2 THEN
      BEGIN
        THETAZ1:=THETAZ2;
        SUM:=0.0;
        THBEG:=0.0;
        WHILE THBEG >= -THETAZ1 DO
          THBEG:=THBEG - THETA;
        THBEG:=THBEG + THETA;
        THSUM:=THBEG;
        WHILE THSUM <= THETAZ1 DO
        BEGIN
          EXPTHSUM:=EXPT(COS(THSUM),2.5);
          SUM:=SUM + EXPTHSUM;
          THSUM:=THSUM + THETA;
        END;
```

```
        DM:=EXPT(R/(2.0*C1*COS(CA)*SUM),2.0/3.0);   {EQUATION 34}
          DTHETAZ1:=0.0;
        END;
      DTHETAZ2:=DTHETAZ1;
    UNTIL ABS(DTHETAZ1)<1E-3;
    THETAZONE:=THETAZ2;
END;  {MDFOR}



PROCEDURE BALLDFOR;

{************************************************************************

  CALCULATE THE INDIVIDUAL BALL DEFORMATION

  GLOBAL VARIABLES:  DC,DI,DM,N,THETA,THETAZONE

 ************************************************************************}

VAR THIR:REAL;
    I:INTEGER;

BEGIN
  I:=1;
  REPEAT
    THIR:=THETA*(I-1);
    IF THIR<=THETAZONE THEN
      BEGIN
      DI[I]:=DM*COS(THIR);            {EQUATION 27}
      I:=I+1;
      END;
  UNTIL THIR>THETAZONE;
  REPEAT
    THIR:=THETA*(I-1);
    IF THIR<(6.2831853-THETAZONE) THEN
    BEGIN
      DI[I]:=0.0;
      I:=I+1;
    END;
  UNTIL THIR>=(6.2831853-THETAZONE);
  REPEAT
    THIR:=THETA*(I-1);
    IF THIR<6.2831853 THEN
    BEGIN
      DI[I]:=DM*COS(THIR);            {EQUATION 27}
      I:=I+1;
    END;
  UNTIL THIR>=6.2831853;
END;  {BALLDFOR}



PROCEDURE BALLFORCE;

{************************************************************************
```

```
      CALCULATE THE INDIVIDUAL BALL FORCES

      GLOBAL VARIABLES:  C1,DI,FI,N

  *****************************************************************************}

  VAR I:INTEGER;

  BEGIN
    FOR I:= 1 TO N DO
      FI[I]:=C1*EXPT(DI[I],1.5);          {EQUATION 26}
  END;  {BALLFORCE}


  PROCEDURE WRITEDATA;

  {*****************************************************************************

    WRITE THE DATA TO THE SCREEN AND THE OUTPUT FILE

      GLOBAL VARIABLES:  DI,FI,N,THI

  *****************************************************************************}

  VAR I:INTEGER;

  BEGIN
    CLEAR;
    WRITELN;
    WRITELN('BALL NUMBER    POSITION (DEG)    DEFORMATION (IN)    FORCE (LBS)');
    WRITELN;
    FOR I:=1 TO N DO
    WRITELN(I,'       ',THI[I]:14:4,'   ',DI[I]:15:6,'      ',FI[I]:14:2);
    WRITELN(DATA);
    WRITELN(DATA,'BALL NUMBER   POSITION (DEG) DEFORMATION (IN)   FORCE (LBS)');
    WRITELN(DATA);
    FOR I:=1 TO N DO
    WRITELN(DATA,I,'     ',THI[I]:14:4,'   ',DI[I]:13:6,'   ',FI[I]:14:2);
  END;  {WRITEDATA}



  {*****************************************************************************

    BEGIN MAIN PROGRAM

  *****************************************************************************}

  BEGIN
    OUTDATA;
    REPEAT
      INVAR;
      CONSTANTS;
      MDFOR;
```

```
      BALLDFOR;
      BALLFORCE;
      WRITEDATA;
      WRITELN('DO YOU WANT TO RUN THE PROGRAM AGAIN (Y/N) ?');
      YESNO(G);
    UNTIL Q=FALSE;
    CLOSE(DATA);
  END.  {MAIN}
```

APPENDIX C

RIGID MODEL: THRUST LOAD PROGRAM LISTING

```
PROGRAM THRUST(INPUT,OUTPUT,DATA);

VAR
    C,                      {CONFORMITY}
    CA,                     {CONTACT ANGLE (DEGREES)}
    C1,                     {EQUATION 26 CONSTANT}
    D,                      {BALL DIAMETER (INCHES)}
    DI,                     {DEFORMATION OF BALL I (IN)}
    FI,                     {FORCE ON BALL I (LBS)}
    T,                      {THRUST FORCE (LBS)}
    THETA:                  {BALL SPACING (DEGREES)}
      REAL;

    N:                      {NUMBER OF BALLS}
      INTEGER;

    DATA:                   {OUTPUT FILE}
      TEXT;

    Q:                      {YESNO PROCEDURE CONSTANT}
      BOOLEAN;


PROCEDURE YesNo(VAR Yes:BOOLEAN);

{*************************************************************************

  Returns TRUE for an input of YES, yes, Y, or y; returns FALSE for an input
  of NO, no, N, or n.  If the user inputs anything else he is prompted to
  make a valid input.

  GLOBAL VARIABLES:  Q

*************************************************************************}

VAR
  String : PACKED ARRAY[1..3] OF CHAR;
  Legal  : BOOLEAN;

BEGIN
  REPEAT
    Legal:=TRUE;
    READLN(String);
    IF String='YES' THEN Yes:=TRUE
    ELSE IF String='yes' THEN Yes:=TRUE
    ELSE IF String='Y ' THEN Yes:=TRUE
    ELSE IF String='y ' THEN Yes:=TRUE
    ELSE IF String='NO ' THEN Yes:=FALSE
    ELSE IF String='no ' THEN Yes:=FALSE
    ELSE IF String='N ' THEN Yes:=FALSE
    ELSE IF String='n ' THEN Yes:=FALSE
    ELSE
      BEGIN
        WRITELN('**********************************************');
```

```
      WRITELN('Response must be YES, yes, Y, y, NO, no, N, or n');
      WRITELN('Please enter a valid response ...');
      WRITELN('********************************************');
      Legal:=FALSE;
    END;
  UNTIL Legal;
END; {YESNO}


FUNCTION EXPT(Y,X:REAL):REAL;

{***************************************************************************

  RETURNS THE RESULT OF Y^X

  GLOBAL VARIABLES:  NONE

***************************************************************************}

VAR Z:INTEGER;

BEGIN
  IF X=0 THEN
    EXPT:=1.0
  ELSE
    IF Y=0 THEN
      EXPT:=0.0
    ELSE
      IF Y>0 THEN
        EXPT:=EXP(X*LN(Y))
      ELSE
        IF ABS(X-ROUND(X))<=1E-10 THEN
        BEGIN
        Z:=TRUNC(X);
        IF ODD(Z) THEN
          EXPT:=-EXP(X*LN(-Y))
        ELSE
          EXPT:=EXP(X*LN(-Y))
        END
END; {EXPT}


PROCEDURE CLEAR;

{***************************************************************************

  CLEARS THE SCREEN

  GLOBAL VARIABLES:  NONE

***************************************************************************}

VAR B:INTEGER;
```

```
BEGIN
  FOR B:=1 TO 24 DO WRITELN;
END;  {CLEAR}


PROCEDURE OUTDATA;

{*************************************************************************

   SETS UP THE OUTPUT FILE

   GLOBAL VARIABLES:  DATA

*************************************************************************}

VAR STATUS:INTEGER;
    FILENAME:PACKED ARRAY[1..17] OF CHAR;

BEGIN
  CLEAR;
  REPEAT
    WRITELN('ENTER THE NAME OF THE OUTPUT FILE - ');
    READLN(FILENAME);
    BIND(DATA,FILENAME,STATUS);
    IF STATUS=1 THEN
      WRITELN('FILE DOES NOT EXIST OR IS INACCESSIBLE, PLEASE TRY AGAIN');
    WRITELN;
  UNTIL STATUS=0;
  REWRITE(DATA);
END;  {OUTDATA}


PROCEDURE INVAR;

{*************************************************************************

   INITIALIZE THE PROGRAM VARIABLES

   GLOBAL VARIABLES:  C,CA,D,N,T,THETA

*************************************************************************}

BEGIN
  WRITELN('ENTER THE THRUST LOAD (LBS) - ');
  READLN(T);
  WRITELN;
  WRITELN('THRUST LOAD = ',T:8:3,' LBS');
  WRITELN(DATA,'THRUST = ',T:8:3,' LBS');
  WRITELN;
  WRITELN('ENTER THE CONTACT ANGLE (DEGREES) -');
  READLN(CA);
  WRITELN;
  WRITELN('CONTACT ANGLE = ',CA:8:4,' DEGREES');
  WRITELN(DATA,'CONTACT ANGLE = ',CA:8:4,' DEGREES');
```

```
      WRITELN;
      WRITELN('ENTER THE CONFORMITY - ');
      READLN(C);
      WRITELN;
      WRITELN('CONFORMITY = ',C:8:5);
      WRITELN(DATA,'CONFORMITY = ',C:8:5);
      WRITELN;
      WRITELN('ENTER THE BALL DIAMETER (IN) - ');
      READLN(D);
      WRITELN;
      WRITELN('BALL DIAMETER = ',D:6:3,' IN');
      WRITELN(DATA,'BALL DIAMETER = ',D:6:3,' IN');
      WRITELN;
      WRITELN('ENTER THE NUMBER OF BALLS - ');
      READLN(N);
      WRITELN;
      WRITELN('NUMBER OF BALLS = ',N:3);
      WRITELN(DATA,'NUMBER OF BALLS = ',N:3);
      THETA:=360/N;
      WRITELN;
      WRITELN('BALL SPACING = ',THETA:8:4,' DEGREES');
      WRITELN(DATA,'BALL SPACING = ',THETA:8:4,' DEGREES');
      WRITELN;
    END;  {INVAR}


  PROCEDURE CONSTANTS;

  {*************************************************************************

    CALCULATE THE CONSTANTS

    GLOBAL VARIABLES:  C,CA,C1,D

  *************************************************************************}

  BEGIN
    C1:=2.198E7*SQRT((C*0.5*D)/(C-1));        {EQUATION 25}
    CA:=CA*3.1415927/180.0;
  END;  {CONSTANTS}


  PROCEDURE BALLFORCE;

  {*************************************************************************

    CALCULATE THE INDIVIDUAL BALL FORCES

    GLOBAL VARIABLES:  CA,FI,N,T

  *************************************************************************}

  BEGIN
    FI:=T/(N*SIN(CA));                        {EQUATION 37}
```

```
END;  {BALLFORCE}


PROCEDURE BALLDFOR;

{*********************************************************************

   CALCULATE THE INDIVIDUAL BALL DEFORMATION

   GLOBAL VARIABLES:  C1,DI,FI

*********************************************************************}

BEGIN
  DI:=EXPT(FI/C1,2/3);                    {EQUATION 38}
END;  {BALLDFOR}


PROCEDURE WRITEDATA;

{*********************************************************************

   WRITE THE DATA TO THE SCREEN AND THE OUTPUT FILE

   GLOBAL VARIABLES:  FI,DI

*********************************************************************}

BEGIN
  WRITELN('THE FORCE ON EACH BALL IS = ',FI:8:3,' LBS');
  WRITELN(DATA,'THE FORCE ON EACH BALL IS = ',FI:8:3,' LBS');
  WRITELN;
  WRITELN('THE DEFORMATION OF EACH BALL IS = ',DI:11:9,' IN');
  WRITELN(DATA,'THE DEFORMATION OF EACH BALL IS = ',DI:11:9,' IN');
  WRITELN;
END;  {WRITEDATA}



{*********************************************************************

   BEGIN MAIN PROGRAM

*********************************************************************}

BEGIN
  OUTDATA;
  REPEAT
    INVAR;
    CONSTANTS;
    BALLFORCE;
    BALLDFOR;
    WRITEDATA;
    WRITELN('DO YOU WANT TO RUN THE PROGRAM AGAIN (Y/N) ?');
    YESNO(Q);
```

```
    UNTIL Q=FALSE;
   CLOSE(DATA);
END. {MAIN}
```

APPENDIX D

FINITE ELEMENT MODEL: MOMENT LOAD INPUT COMMANDS

```
CSYS,1
N,1,6.5
N,2,6.5,,1.75
N,3,6.5,,3.13
N,4,6.5,,3.5
N,5,6.5,,4.5
N,6,7.3125
N,7,7.3125,,1.75
N,8,7.3125,,3.13
N,9,7.3125,,3.5
N,10,7.3125,,4.5
N,11,7.88
N,12,7.88,,1.75
N,13,7.88,,3.13
N,14,7.88,,3.5
N,15,7.88,,4.5
N,16,8.285
N,19,8.285,,3.13
N,20,8.285,,3.5
N,21,8.285,,4.5
N,22,8.69
N,25,8.69,,3.13
LOCAL,12,1,8.69,,1.75,,,90
N,23,0.81,270
N,17,0.81,240
N,18,0.81,120
N,24,0.81,90
CSYS,1
NGEN,72,25,1,25,1,,5
NMODIF,17,,,,,,150
NMODIF,18,,,,,,30
NMODIF,67,,,,10,,150
NMODIF,68,,,,10,,30
NMODIF,117,,,,20,,150
NMODIF,118,,,,20,,30
NMODIF,167,,,,30,,150
NMODIF,168,,,,30,,30
NMODIF,217,,,,40,,150
NMODIF,218,,,,40,,30
NMODIF,267,,,,50,,150
NMODIF,268,,,,50,,30
NMODIF,317,,,,60,,150
NMODIF,318,,,,60,,30
NMODIF,367,,,,70,,150
NMODIF,368,,,,70,,30
NMODIF,417,,,,80,,150
NMODIF,418,,,,80,,30
NMODIF,467,,,,90,,150
NMODIF,468,,,,90,,30
NMODIF,517,,,,100,,150
NMODIF,518,,,,100,,30
NMODIF,567,,,,110,,150
NMODIF,568,,,,110,,30
```

```
NMODIF,617,,,,120,,150
NMODIF,618,,,,120,,30
NMODIF,667,,,,130,,150
NMODIF,668,,,,130,,30
NMODIF,717,,,,140,,150
NMODIF,718,,,,140,,30
NMODIF,767,,,,150,,150
NMODIF,768,,,,150,,30
NMODIF,817,,,,160,,150
NMODIF,818,,,,160,,30
NMODIF,867,,,,170,,150
NMODIF,868,,,,170,,30
NMODIF,917,,,,180,,150
NMODIF,918,,,,180,,30
NMODIF,967,,,,190,,150
NMODIF,968,,,,190,,30
NMODIF,1017,,,,200,,150
NMODIF,1018,,,,200,,30
NMODIF,1067,,,,210,,150
NMODIF,1068,,,,210,,30
NMODIF,1117,,,,220,,150
NMODIF,1118,,,,220,,30
NMODIF,1167,,,,230,,150
NMODIF,1168,,,,230,,30
NMODIF,1217,,,,240,,150
NMODIF,1218,,,,240,,30
NMODIF,1267,,,,250,,150
NMODIF,1268,,,,250,,30
NMODIF,1317,,,,260,,150
NMODIF,1318,,,,260,,30
NMODIF,1367,,,,270,,150
NMODIF,1368,,,,270,,30
NMODIF,1417,,,,280,,150
NMODIF,1418,,,,280,,30
NMODIF,1467,,,,290,,150
NMODIF,1468,,,,290,,30
NMODIF,1517,,,,300,,150
NMODIF,1518,,,,300,,30
NMODIF,1567,,,,310,,150
NMODIF,1568,,,,310,,30
NMODIF,1617,,,,320,,150
NMODIF,1618,,,,320,,30
NMODIF,1667,,,,330,,150
NMODIF,1668,,,,330,,30
NMODIF,1717,,,,340,,150
NMODIF,1718,,,,340,,30
NMODIF,1767,,,,350,,150
NMODIF,1768,,,,350,,30
ET,1,45
EX,1,30E6
TYPE,1
REAL,1
MAT,1
```

```
E,1,6,7,2,26,31,32,27
E,2,7,8,3,27,32,33,28
E,6,11,12,7,31,36,37,32
E,7,12,13,8,32,37,38,33
E,11,16,17,12,36,41,42,37
E,12,18,19,13,37,43,44,38
E,16,22,23,17,41,47,48,42
E,18,24,25,19,43,49,50,44
EGEN,72,25,1,8,1
EDELE,569,576,1
E,1776,1781,1782,1777,1,6,7,2
E,1777,1782,1783,1778,2,7,8,3
E,1781,1786,1787,1782,6,11,12,7
E,1782,1787,1788,1783,7,12,13,8
E,1786,1791,1792,1787,11,16,17,12
E,1787,1793,1794,1788,12,18,19,13
E,1791,1797,1798,1792,16,22,23,17
E,1793,1799,1800,1794,18,24,25,19
ECOMPR
EX,2,30E8
MAT,2
E,4,9,10,5,29,34,35,30
E,9,14,15,10,34,39,40,35
E,14,20,21,15,39,45,46,40
EGEN,72,25,577,579,1
EDELE,790,792,1
E,1779,1784,1785,1780,4,9,10,5
E,1784,1789,1790,1785,9,14,15,10
E,1789,1795,1796,1790,14,20,21,15
ECOMPR
D,4,UY
D,454,UX
D,904,UY
D,1354,UX
F,460,FZ,-80000
F,1360,FZ,80000
ET,3,52,,,1
R,3,1.0E4,0.0
MU,3,0.45
TYPE,3
REAL,3
MAT,3
E,3,4
E,8,9
E,13,14
E,19,20
EGEN,72,25,793,796,1
ET,4,39,,,3
RSIZE,40
R,4,-0.0050367,-25000,-0.0043405,-20000,-0.003583,-15000
RMORE,-0.0027343,-10000,-0.0017225,-5000,-0.0016057,-4500
RMORE,-0.0014844,-4000,-0.001358,-3500,-0.0012254,-3000
RMORE,-0.0010851,-2500,-0.0009351,-2000,-0.0007719,-1500
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RMORE,-0.0005891,-1000,-0.0003711,-500,-0.0003198,-400
RMORE,-0.000264,-300,-0.0002015,-200,-0.0001269,-100
RMORE,0,0,1,100
LOCAL,12,1,8.69,,1.75,,,90
N,1801,0.81,240
N,1802,0.81,120
TYPE,4
REAL,4
MAT,4
E,1801,17
E,1802,18
CSYS,1
NGEN,36,50,1801,1802,1,,10
EGEN,36,50,1081,1082,1
NCOMPR
NMODIF,1801,,,,,,150
NMODIF,1802,,,,,,30
NMODIF,1803,,,,10,,150
NMODIF,1804,,,,10,,30
NMODIF,1805,,,,20,,150
NMODIF,1806,,,,20,,30
NMODIF,1807,,,,30,,150
NMODIF,1808,,,,30,,30
NMODIF,1809,,,,40,,150
NMODIF,1810,,,,40,,30
NMODIF,1811,,,,50,,150
NMODIF,1812,,,,50,,30
NMODIF,1813,,,,60,,150
NMODIF,1814,,,,60,,30
NMODIF,1815,,,,70,,150
NMODIF,1816,,,,70,,30
NMODIF,1817,,,,80,,150
NMODIF,1818,,,,80,,30
NMODIF,1819,,,,90,,150
NMODIF,1820,,,,90,,30
NMODIF,1821,,,,100,,150
NMODIF,1822,,,,100,,30
NMODIF,1823,,,,110,,150
NMODIF,1824,,,,110,,30
NMODIF,1825,,,,120,,150
NMODIF,1826,,,,120,,30
NMODIF,1827,,,,130,,150
NMODIF,1828,,,,130,,30
NMODIF,1829,,,,140,,150
NMODIF,1830,,,,140,,30
NMODIF,1831,,,,150,,150
NMODIF,1832,,,,150,,30
NMODIF,1833,,,,160,,150
NMODIF,1834,,,,160,,30
NMODIF,1835,,,,170,,150
NMODIF,1836,,,,170,,30
NMODIF,1837,,,,180,,150
NMODIF,1838,,,,180,,30
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NMODIF,1839,,,,190,,150
NMODIF,1840,,,,190,,30
NMODIF,1841,,,,200,,150
NMODIF,1842,,,,200,,30
NMODIF,1843,,,,210,,150
NMODIF,1844,,,,210,,30
NMODIF,1845,,,,220,,150
NMODIF,1846,,,,220,,30
NMODIF,1847,,,,230,,150
NMODIF,1848,,,,230,,30
NMODIF,1849,,,,240,,150
NMODIF,1850,,,,240,,30
NMODIF,1851,,,,250,,150
NMODIF,1852,,,,250,,30
NMODIF,1853,,,,260,,150
NMODIF,1854,,,,260,,30
NMODIF,1855,,,,270,,150
NMODIF,1856,,,,270,,30
NMODIF,1857,,,,280,,150
NMODIF,1858,,,,280,,30
NMODIF,1859,,,,290,,150
NMODIF,1860,,,,290,,30
NMODIF,1861,,,,300,,150
NMODIF,1862,,,,300,,30
NMODIF,1863,,,,310,,150
NMODIF,1864,,,,310,,30
NMODIF,1865,,,,320,,150
NMODIF,1866,,,,320,,30
NMODIF,1867,,,,330,,150
NMODIF,1868,,,,330,,30
NMODIF,1869,,,,340,,150
NMODIF,1870,,,,340,,30
NMODIF,1871,,,,350,,150
NMODIF,1872,,,,350,,30
ET,5,10,,1
R,5,0.479369,1.0217556E-1
EX,5,30E6
TYPE,5
REAL,5
MAT,5
E,7,10
E,107,110
E,207,210
E,307,310
E,407,410
E,507,510
E,607,610
E,707,710
E,807,810
E,907,910
E,1007,1010
E,1107,1110
E,1207,1210
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E,1307,1310
E,1407,1410
E,1507,1510
E,1607,1610
E,1707,1710
ET,6,14,,3
R,6,2.5853154E6
LOCAL,12,1,8.69,,1.75,,,90
N,1873,0.81,240
N,1874,0.81,120
TYPE,6
REAL,6
MAT,6
E,1801,1873
E,1802,1874
CSYS,1
NGEN,36,2,1873,1874,1,,10
EGEN,36,2,1171,1172,1
D,1873,ALL,,,1944,1
NMODIF,1873,,,,,,150
NMODIF,1874,,,,,,30
NMODIF,1875,,,,10,,150
NMODIF,1876,,,,10,,30
NMODIF,1877,,,,20,,150
NMODIF,1878,,,,20,,30
NMODIF,1879,,,,30,,150
NMODIF,1880,,,,30,,30
NMODIF,1881,,,,40,,150
NMODIF,1882,,,,40,,30
NMODIF,1883,,,,50,,150
NMODIF,1884,,,,50,,30
NMODIF,1885,,,,60,,150
NMODIF,1886,,,,60,,30
NMODIF,1887,,,,70,,150
NMODIF,1888,,,,70,,30
NMODIF,1889,,,,80,,150
NMODIF,1890,,,,80,,30
NMODIF,1891,,,,90,,150
NMODIF,1892,,,,90,,30
NMODIF,1893,,,,100,,150
NMODIF,1894,,,,100,,30
NMODIF,1895,,,,110,,150
NMODIF,1896,,,,110,,30
NMODIF,1897,,,,120,,150
NMODIF,1898,,,,120,,30
NMODIF,1899,,,,130,,150
NMODIF,1900,,,,130,,30
NMODIF,1901,,,,140,,150
NMODIF,1902,,,,140,,30
NMODIF,1903,,,,150,,150
NMODIF,1904,,,,150,,30
NMODIF,1905,,,,160,,150
NMODIF,1906,,,,160,,30
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NMODIF,1907,,,,170,,150
NMODIF,1908,,,,170,,30
NMODIF,1909,,,,180,,150
NMODIF,1910,,,,180,,30
NMODIF,1911,,,,190,,150
NMODIF,1912,,,,190,,30
NMODIF,1913,,,,200,,150
NMODIF,1914,,,,200,,30
NMODIF,1915,,,,210,,150
NMODIF,1916,,,,210,,30
NMODIF,1917,,,,220,,150
NMODIF,1918,,,,220,,30
NMODIF,1919,,,,230,,150
NMODIF,1920,,,,230,,30
NMODIF,1921,,,,240,,150
NMODIF,1922,,,,240,,30
NMODIF,1923,,,,250,,150
NMODIF,1924,,,,250,,30
NMODIF,1925,,,,260,,150
NMODIF,1926,,,,260,,30
NMODIF,1927,,,,270,,150
NMODIF,1928,,,,270,,30
NMODIF,1929,,,,280,,150
NMODIF,1930,,,,280,,30
NMODIF,1931,,,,290,,150
NMODIF,1932,,,,290,,30
NMODIF,1933,,,,300,,150
NMODIF,1934,,,,300,,30
NMODIF,1935,,,,310,,150
NMODIF,1936,,,,310,,30
NMODIF,1937,,,,320,,150
NMODIF,1938,,,,320,,30
NMODIF,1939,,,,330,,150
NMODIF,1940,,,,330,,30
NMODIF,1941,,,,340,,150
NMODIF,1942,,,,340,,30
NMODIF,1943,,,,350,,150
NMODIF,1944,,,,350,,30
ITER,-15
```

APPENDIX E

FINITE ELEMENT MODEL: RADIAL LOAD INPUT COMMANDS

```
CSYS,1
N,1,6.5
N,2,6.5,,1.75
N,3,6.5,,3.13
N,4,6.5,,3.5
N,5,6.5,,4.5
N,6,7.3125
N,7,7.3125,,1.75
N,8,7.3125,,3.13
N,9,7.3125,,3.5
N,10,7.3125,,4.5
N,11,7.88
N,12,7.88,,1.75
N,13,7.88,,3.13
N,14,7.88,,3.5
N,15,7.88,,4.5
N,16,8.285
N,19,8.285,,3.13
N,20,8.285,,3.5
N,21,8.285,,4.5
N,22,8.69
N,25,8.69,,3.13
LOCAL,12,1,8.69,,1.75,,,90
N,23,0.81,270
N,17,0.81,240
N,18,0.81,120
N,24,0.81,90
CSYS,1
NGEN,72,25,1,25,1,,5
NMODIF,17,,,,,,,150
NMODIF,18,,,,,,,30
NMODIF,67,,,,10,,150
NMODIF,68,,,,10,,30
NMODIF,117,,,,20,,150
NMODIF,118,,,,20,,30
NMODIF,167,,,,30,,150
NMODIF,168,,,,30,,30
NMODIF,217,,,,40,,150
NMODIF,218,,,,40,,30
NMODIF,267,,,,50,,150
NMODIF,268,,,,50,,30
NMODIF,317,,,,60,,150
NMODIF,318,,,,60,,30
NMODIF,367,,,,70,,150
NMODIF,368,,,,70,,30
NMODIF,417,,,,80,,150
NMODIF,418,,,,80,,30
NMODIF,467,,,,90,,150
NMODIF,468,,,,90,,30
NMODIF,517,,,,100,,150
NMODIF,518,,,,100,,30
NMODIF,567,,,,110,,150
NMODIF,568,,,,110,,30
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```
NMODIF,617,,,,120,,150
NMODIF,618,,,,120,,30
NMODIF,667,,,,130,,150
NMODIF,668,,,,130,,30
NMODIF,717,,,,140,,150
NMODIF,718,,,,140,,30
NMODIF,767,,,,150,,150
NMODIF,768,,,,150,,30
NMODIF,817,,,,160,,150
NMODIF,818,,,,160,,30
NMODIF,867,,,,170,,150
NMODIF,868,,,,170,,30
NMODIF,917,,,,180,,150
NMODIF,918,,,,180,,30
NMODIF,967,,,,190,,150
NMODIF,968,,,,190,,30
NMODIF,1017,,,,200,,150
NMODIF,1018,,,,200,,30
NMODIF,1067,,,,210,,150
NMODIF,1068,,,,210,,30
NMODIF,1117,,,,220,,150
NMODIF,1118,,,,220,,30
NMODIF,1167,,,,230,,150
NMODIF,1168,,,,230,,30
NMODIF,1217,,,,240,,150
NMODIF,1218,,,,240,,30
NMODIF,1267,,,,250,,150
NMODIF,1268,,,,250,,30
NMODIF,1317,,,,260,,150
NMODIF,1318,,,,260,,30
NMODIF,1367,,,,270,,150
NMODIF,1368,,,,270,,30
NMODIF,1417,,,,280,,150
NMODIF,1418,,,,280,,30
NMODIF,1467,,,,290,,150
NMODIF,1468,,,,290,,30
NMODIF,1517,,,,300,,150
NMODIF,1518,,,,300,,30
NMODIF,1567,,,,310,,150
NMODIF,1568,,,,310,,30
NMODIF,1617,,,,320,,150
NMODIF,1618,,,,320,,30
NMODIF,1667,,,,330,,150
NMODIF,1668,,,,330,,30
NMODIF,1717,,,,340,,150
NMODIF,1718,,,,340,,30
NMODIF,1767,,,,350,,150
NMODIF,1768,,,,350,,30
ET,1,45
EX,1,30E6
TYPE,1
REAL,1
MAT,1
```

```
E,1,6,7,2,26,31,32,27
E,2,7,8,3,27,32,33,28
E,6,11,12,7,31,36,37,32
E,7,12,13,8,32,37,38,33
E,11,16,17,12,36,41,42,37
E,12,18,19,13,37,43,44,38
E,16,22,23,17,41,47,48,42
E,18,24,25,19,43,49,50,44
EGEN,72,25,1,8,1
EDELE,569,576,1
E,1776,1781,1782,1777,1,6,7,2
E,1777,1782,1783,1778,2,7,8,3
E,1781,1786,1787,1782,6,11,12,7
E,1782,1787,1788,1783,7,12,13,8
E,1786,1791,1792,1787,11,16,17,12
E,1787,1793,1794,1788,12,18,19,13
E,1791,1797,1798,1792,16,22,23,17
E,1793,1799,1800,1794,18,24,25,19
ECOMPR
EX,2,30E8
MAT,2
E,4,9,10,5,29,34,35,30
E,9,14,15,10,34,39,40,35
E,14,20,21,15,39,45,46,40
EGEN,72,25,577,579,1
EDELE,790,792,1
E,1779,1784,1785,1780,4,9,10,5
E,1784,1789,1790,1785,9,14,15,10
E,1789,1795,1796,1790,14,20,21,15
ECOMPR
D,4,UY
D,904,UY
F,8,FX,2777.7778
F,108,FX,2777.7778
F,208,FX,2777.7778
F,308,FX,2777.7778
F,408,FX,2777.7778
F,508,FX,2777.7778
F,608,FX,2777.7778
F,708,FX,2777.7778
F,808,FX,2777.7778
F,908,FX,2777.7778
F,1008,FX,2777.7778
F,1108,FX,2777.7778
F,1208,FX,2777.7778
F,1308,FX,2777.7778
F,1408,FX,2777.7778
F,1508,FX,2777.7778
F,1608,FX,2777.7778
F,1708,FX,2777.7778
ET,3,52,,,1
R,3,1.0E4,0.0
MU,3.0.45
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```
TYPE,3
REAL,3
MAT,3
E,3,4
E,8,9
E,13,14
E,19,20
EGEN,72,25,793,796,1
ET,4,39,,,3
RSIZE,40
R,4,-0.0050367,-25000,-0.0043405,-20000,-0.003583,-15000
RMORE,-0.0027343,-10000,-0.0017225,-5000,-0.0016057,-4500
RMORE,-0.0014844,-4000,-0.001358,-3500,-0.0012254,-3000
RMORE,-0.0010851,-2500,-0.0009351,-2000,-0.0007719,-1500
RMORE,-0.0005891,-1000,-0.0003711,-500,-0.0003198,-400
RMORE,-0.000264,-300,-0.0002015,-200,-0.0001269,-100
RMORE,0,0,1,100
LOCAL,12,1,8.69,,1.75,,,90
N,1801,0.81,240
N,1802,0.81,120
TYPE,4
REAL,4
MAT,4
E,1801,17
E,1802,18
CSYS,1
NGEN,36,50,1801,1802,1,,10
EGEN,36,50,1081,1082,1
NCOMPR
NMODIF,1801,,,,,,150
NMODIF,1802,,,,,,30
NMODIF,1803,,,,10,,150
NMODIF,1804,,,,10,,30
NMODIF,1805,,,,20,,150
NMODIF,1806,,,,20,,30
NMODIF,1807,,,,30,,150
NMODIF,1808,,,,30,,30
NMODIF,1809,,,,40,,150
NMODIF,1810,,,,40,,30
NMODIF,1811,,,,50,,150
NMODIF,1812,,,,50,,30
NMODIF,1813,,,,60,,150
NMODIF,1814,,,,60,,30
NMODIF,1815,,,,70,,150
NMODIF,1816,,,,70,,30
NMODIF,1817,,,,80,,150
NMODIF,1818,,,,80,,30
NMODIF,1819,,,,90,,150
NMODIF,1820,,,,90,,30
NMODIF,1821,,,,100,,150
NMODIF,1822,,,,100,,30
NMODIF,1823,,,,110,,150
NMODIF,1824,,,,110,,30
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NMODIF,1825,,,,120,,150
NMODIF,1826,,,,120,,30
NMODIF,1827,,,,130,,150
NMODIF,1828,,,,130,,30
NMODIF,1829,,,,140,,150
NMODIF,1830,,,,140,,30
NMODIF,1831,,,,150,,150
NMODIF,1832,,,,150,,30
NMODIF,1833,,,,160,,150
NMODIF,1834,,,,160,,30
NMODIF,1835,,,,170,,150
NMODIF,1836,,,,170,,30
NMODIF,1837,,,,180,,150
NMODIF,1838,,,,180,,30
NMODIF,1839,,,,190,,150
NMODIF,1840,,,,190,,30
NMODIF,1841,,,,200,,150
NMODIF,1842,,,,200,,30
NMODIF,1843,,,,210,,150
NMODIF,1844,,,,210,,30
NMODIF,1845,,,,220,,150
NMODIF,1846,,,,220,,30
NMODIF,1847,,,,230,,150
NMODIF,1848,,,,230,,30
NMODIF,1849,,,,240,,150
NMODIF,1850,,,,240,,30
NMODIF,1851,,,,250,,150
NMODIF,1852,,,,250,,30
NMODIF,1853,,,,260,,150
NMODIF,1854,,,,260,,30
NMODIF,1855,,,,270,,150
NMODIF,1856,,,,270,,30
NMODIF,1857,,,,280,,150
NMODIF,1858,,,,280,,30
NMODIF,1859,,,,290,,150
NMODIF,1860,,,,290,,30
NMODIF,1861,,,,300,,150
NMODIF,1862,,,,300,,30
NMODIF,1863,,,,310,,150
NMODIF,1864,,,,310,,30
NMODIF,1865,,,,320,,150
NMODIF,1866,,,,320,,30
NMODIF,1867,,,,330,,150
NMODIF,1868,,,,330,,30
NMODIF,1869,,,,340,,150
NMODIF,1870,,,,340,,30
NMODIF,1871,,,,350,,150
NMODIF,1872,,,,350,,30
ET,5,10,,1
R,5,0.479369,1.0217556E-1
EX,5,30E6
TYPE,5
REAL,5
```

```
MAT,5
E,7,10
E,107,110
E,207,210
E,307,310
E,407,410
E,507,510
E,607,610
E,707,710
E,807,810
E,907,910
E,1007,1010
E,1107,1110
E,1207,1210
E,1307,1310
E,1407,1410
E,1507,1510
E,1607,1610
E,1707,1710
ET,6,14,,3
R,6,2.5853154E6
LOCAL,12,1,8.69,,1.75,,,90
N,1873,0.81,240
N,1874,0.81,120
TYPE,6
REAL,6
MAT,6
E,1801,1873
E,1802,1874
CSYS,1
NGEN,36,2,1873,1874,1,,10
EGEN,36,2,1171,1172,1
D,1873,ALL,,,1944,1
NMODIF,1873,,,,,,150
NMODIF,1874,,,,,,30
NMODIF,1875,,,,10,,150
NMODIF,1876,,,,10,,30
NMODIF,1877,,,,20,,150
NMODIF,1878,,,,20,,30
NMODIF,1879,,,,30,,150
NMODIF,1880,,,,30,,30
NMODIF,1881,,,,40,,150
NMODIF,1882,,,,40,,30
NMODIF,1883,,,,50,,150
NMODIF,1884,,,,50,,30
NMODIF,1885,,,,60,,150
NMODIF,1886,,,,60,,30
NMODIF,1887,,,,70,,150
NMODIF,1888,,,,70,,30
NMODIF,1889,,,,80,,150
NMODIF,1890,,,,80,,30
NMODIF,1891,,,,90,,150
NMODIF,1892,,,,90,,30
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NMODIF,1893,,,,100,,150
NMODIF,1894,,,,100,,30
NMODIF,1895,,,,110,,150
NMODIF,1896,,,,110,,30
NMODIF,1897,,,,120,,150
NMODIF,1898,,,,120,,30
NMODIF,1899,,,,130,,150
NMODIF,1900,,,,130,,30
NMODIF,1901,,,,140,,150
NMODIF,1902,,,,140,,30
NMODIF,1903,,,,150,,150
NMODIF,1904,,,,150,,30
NMODIF,1905,,,,160,,150
NMODIF,1906,,,,160,,30
NMODIF,1907,,,,170,,150
NMODIF,1908,,,,170,,30
NMODIF,1909,,,,180,,150
NMODIF,1910,,,,180,,30
NMODIF,1911,,,,190,,150
NMODIF,1912,,,,190,,30
NMODIF,1913,,,,200,,150
NMODIF,1914,,,,200,,30
NMODIF,1915,,,,210,,150
NMODIF,1916,,,,210,,30
NMODIF,1917,,,,220,,150
NMODIF,1918,,,,220,,30
NMODIF,1919,,,,230,,150
NMODIF,1920,,,,230,,30
NMODIF,1921,,,,240,,150
NMODIF,1922,,,,240,,30
NMODIF,1923,,,,250,,150
NMODIF,1924,,,,250,,30
NMODIF,1925,,,,260,,150
NMODIF,1926,,,,260,,30
NMODIF,1927,,,,270,,150
NMODIF,1928,,,,270,,30
NMODIF,1929,,,,280,,150
NMODIF,1930,,,,280,,30
NMODIF,1931,,,,290,,150
NMODIF,1932,,,,290,,30
NMODIF,1933,,,,300,,150
NMODIF,1934,,,,300,,30
NMODIF,1935,,,,310,,150
NMODIF,1936,,,,310,,30
NMODIF,1937,,,,320,,150
NMODIF,1938,,,,320,,30
NMODIF,1939,,,,330,,150
NMODIF,1940,,,,330,,30
NMODIF,1941,,,,340,,150
NMODIF,1942,,,,340,,30
NMODIF,1943,,,,350,,150
NMODIF,1944,,,,350,,30
ITER,-15
```

APPENDIX F

FINITE ELEMENT MODEL: THRUST LOAD INPUT COMMANDS

```
CSYS,1
N,1,6.5
N,2,6.5,,1.75
N,3,6.5,,3.13
N,4,6.5,,3.5
N,5,6.5,,4.5
N,6,7.3125
N,7,7.3125,,1.75
N,8,7.3125,,3.13
N,9,7.3125,,3.5
N,10,7.3125,,4.5
N,11,7.88
N,12,7.88,,1.75
N,13,7.88,,3.13
N,14,7.88,,3.5
N,15,7.88,,4.5
N,16,8.285
N,19,8.285,,3.13
N,20,8.285,,3.5
N,21,8.285,,4.5
N,22,8.69
N,25,8.69,,3.13
LOCAL,12,1,8.69,,1.75,,,90
N,23,0.81,270
N,17,0.81,240
N,18,0.81,120
N,24,0.81,90
CSYS,1
NGEN,72,25,1,25,1,,5
NMODIF,17,,,,,,150
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NMODIF,118,,,,20,,30
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NMODIF,218,,,,40,,30
NMODIF,267,,,,50,,150
NMODIF,268,,,,50,,30
NMODIF,317,,,,60,,150
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NMODIF,368,,,,70,,30
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NMODIF,567,,,,110,,150
NMODIF,568,,,,110,,30
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NMODIF,617,,,,120,,150
NMODIF,618,,,,120,,30
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NMODIF,818,,,,160,,30
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NMODIF,868,,,,170,,30
NMODIF,917,,,,180,,150
NMODIF,918,,,,180,,30
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NMODIF,1168,,,,230,,30
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NMODIF,1267,,,,250,,150
NMODIF,1268,,,,250,,30
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NMODIF,1318,,,,260,,30
NMODIF,1367,,,,270,,150
NMODIF,1368,,,,270,,30
NMODIF,1417,,,,280,,150
NMODIF,1418,,,,280,,30
NMODIF,1467,,,,290,,150
NMODIF,1468,,,,290,,30
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NMODIF,1518,,,,300,,30
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NMODIF,1568,,,,310,,30
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NMODIF,1618,,,,320,,30
NMODIF,1667,,,,330,,150
NMODIF,1668,,,,330,,30
NMODIF,1717,,,,340,,150
NMODIF,1718,,,,340,,30
NMODIF,1767,,,,350,,150
NMODIF,1768,,,,350,,30
ET,1,45
EX,1,30E6
TYPE,1
REAL,1
MAT,1
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E,1,6,7,2,26,31,32,27
E,2,7,8,3,27,32,33,28
E,6,11,12,7,31,36,37,32
E,7,12,13,8,32,37,38,33
E,11,16,17,12,36,41,42,37
E,12,18,19,13,37,43,44,38
E,16,22,23,17,41,47,48,42
E,18,24,25,19,43,49,50,44
EGEN,72,25,1,8,1
EDELE,569,576,1
E,1776,1781,1782,1777,1,6,7,2
E,1777,1782,1783,1778,2,7,8,3
E,1781,1786,1787,1782,6,11,12,7
E,1782,1787,1788,1783,7,12,13,8
E,1786,1791,1792,1787,11,16,17,12
E,1787,1793,1794,1788,12,18,19,13
E,1791,1797,1798,1792,16,22,23,17
E,1793,1799,1800,1794,18,24,25,19
ECOMPR
EX,2,30E8
MAT,2
E,4,9,10,5,29,34,35,30
E,9,14,15,10,34,39,40,35
E,14,20,21,15,39,45,46,40
EGEN,72,25,577,579,1
EDELE,790,792,1
E,1779,1784,1785,1780,4,9,10,5
E,1784,1789,1790,1785,9,14,15,10
E,1789,1795,1796,1790,14,20,21,15
ECOMPR
D,4,UY
D,454,UX
D,904,UY
D,1354,UX
NSEL,Z,4.5
F,ALL,FZ,-347.22223
NALL
ET,3,52,,,1
R,3,1.0E4,0.0
MU,3,0.45
TYPE,3
REAL,3
MAT,3
E,3,4
E,8,9
E,13,14
E,19,20
EGEN,72,25,793,796,1
ET,4,39,,,3
RSIZE,40
R,4,-0.0050367,-25000,-0.0043405,-20000,-0.003583,-15000
RMORE,-0.0027343,-10000,-0.0017225,-5000,-0.0016057,-4500
RMORE,-0.0014844,-4000,-0.001358,-3500,-0.0012254,-3000
```

```
RMORE,-0.0010851,-2500,-0.0009351,-2000,-0.0007719,-1500
RMORE,-0.0005891,-1000,-0.0003711,-500,-0.0003198,-400
RMORE,-0.000264,-300,-0.0002015,-200,-0.0001269,-100
RMORE,0,0,1,100
LOCAL,12,1,8.69,,1.75,,,90
N,1801,0.81,240
N,1802,0.81,120
TYPE,4
REAL,4
MAT,4
E,1801,17
E,1802,18
CSYS,1
NGEN,36,50,1801,1802,1,,10
EGEN,36,50,1081,1082,1
NCOMPR
NMODIF,1801,,,,,,150
NMODIF,1802,,,,,,30
NMODIF,1803,,,,10,,150
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ET,5,10,,1
R,5,0.479369,1.0217556E-1
EX,5,30E6
TYPE,5
REAL,5
MAT,5
E,7,10
E,107,110
E,207,210
E,307,310
E,407,410
E,507,510
E,607,610
E,707,710
E,807,810
E,907,910
E,1007,1010
E,1107,1110
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E,1207,1210
E,1307,1310
E,1407,1410
E,1507,1510
E,1607,1610
E,1707,1710
ET,6,14,,3
R,6,2.5853154E6
LOCAL,12,1,8.69,,1.75,,,90
N,1873,0.81,240
N,1874,0.81,120
TYPE,6
REAL,6
MAT,6
E,1801,1873
E,1802,1874
CSYS,1
NGEN,36,2,1873,1874,1,,10
EGEN,36,2,1171,1172,1
D,1873,ALL,,,1944,1
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NMODIF,1875,,,,10,,150
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NMODIF,1938,,,,320,,30
NMODIF,1939,,,,330,,150
NMODIF,1940,,,,330,,30
NMODIF,1941,,,,340,,150
NMODIF,1942,,,,340,,30
NMODIF,1943,,,,350,,150
NMODIF,1944,,,,350,,30
ITER,-15
```

APPENDIX G

BEARING #1-6 FINITE ELEMENT AND RIGID

MODEL RACE FORCES FOR MOMENT, RADIAL,

AND THRUST LOADS

TABLE VI

BEARING #1 FINITE ELEMENT AND RIGID MODEL RACE
FORCES FOR MOMENT, RADIAL, AND THRUST LOADS

| | Race Force (Lb) | |
	Finite Element Model	Rigid Model
Moment Load (Ft - Lb)		
11458.3	1153.77	1232.20
22916.7	2245.56	2464.41
45833.3	4438.36	4928.82
68750.0	6635.76	7393.23
91666.7	8816.68	9857.64
Radial Load (Lb)		
3000.0	531.47	385.61
15000.0	1801.25	1928.03
30000.0	3572.81	3856.07
50000.0	5934.27	6426.78
Thrust Load (Lb)		
10000.0	513.85	339.62
50000.0	1758.08	1698.09
100000.0	3494.00	3396.18

TABLE VII

BEARING #2 FINITE ELEMENT AND RIGID MODEL RACE
FORCES FOR MOMENT, RADIAL, AND THRUST LOADS

	Race Force (Lb)	
	Finite Element Model	Rigid Model
Moment Load (Ft - Lb)		
12187.5	1124.21	1171.40
24375.0	2151.25	2342.79
48750.0	4239.30	4685.38
73125.0	6340.14	7028.38
97500.0	8418.68	9371.18
Radial Load (Lb)		
3000.0	629.32	364.17
15000.0	1700.39	1820.83
30000.0	3372.36	3641.65
50000.0	5611.11	6069.42
Thrust Load (Lb)		
10000.0	608.31	320.75
50000.0	1667.76	1603.75
100000.0	3272.04	3207.50

TABLE VIII

BEARING #3 FINITE ELEMENT AND RIGID MODEL RACE
FORCES FOR MOMENT, RADIAL, AND THRUST LOADS

	Race Force (Lb)	
	Finite Element Model	Rigid Model
Moment Load (Ft - Lb)		
38791.67	2667.26	2809.84
77583.33	5271.19	5619.68
116375.00	7864.70	8429.52
155166.67	10458.59	11239.36
186200.00	12544.25	13487.23
Radial Load (Lb)		
3000.0	615.83	345.01
15000.0	1589.60	1725.07
30000.0	3119.66	3450.14
50000.0	5168.58	5750.23
Thrust Load (Lb)		
10000.0	597.94	303.87
50000.0	1589.72	1519.34
100000.0	3110.22	3038.69

TABLE IX

BEARING #4 FINITE ELEMENT AND RIGID MODEL RACE
FORCES FOR MOMENT, RADIAL, AND THRUST LOADS

	Race Force (Lb)	
	Finite Element Model	Rigid Model
Moment Load (Ft - Lb)		
16250.00	716.75	686.24
32500.00	1333.11	1372.48
65000.00	2570.15	2744.96
97500.00	3834.04	4117.44
104000.00	4097.72	4391.94
Radial Load (Lb)		
3000.0	267.50	201.70
15000.0	960.82	1008.49
30000.0	1832.12	2016.97
50000.0	2985.31	3361.62
Thrust Load (Lb)		
10000.0	263.34	177.65
50000.0	974.50	888.23
100000.0	1856.05	1776.46

TABLE X

BEARING #5 FINITE ELEMENT AND RIGID MODEL RACE
FORCES FOR MOMENT, RADIAL, AND THRUST LOADS

	Race Force (Lb)	
	Finite Element Model	Rigid Model
Moment Load (Ft - Lb)		
136458.33	4160.09	3489.46
272916.67	8152.81	6978.92
409375.00	12186.65	10468.92
545833.33	16228.40	13957.85
600416.67	17847.98	15353.63
Radial Load (Lb)		
3000.0	1153.11	204.85
15000.0	1554.26	1024.24
30000.0	2066.43	2048.48
50000.0	3011.67	3414.13
100000.0	5880.68	6828.25
Thrust Load (Lb)		
10000.0	1150.07	180.42
50000.0	1471.76	902.11
100000.0	1917.02	1804.22
200000.0	3661.05	3608.44

TABLE XI

BEARING #6 FINITE ELEMENT AND RIGID MODEL RACE
FORCES FOR MOMENT, RADIAL, AND THRUST LOADS

	Race Force (Lb)	
	Finite Element Model	Rigid Model
Moment Load (Ft - Lb)		
55766.67	1500.51	1328.19
111533.33	2974.42	2656.38
167300.00	4450.26	3984.57
223066.67	5958.48	5312.76
278833.33	7456.35	6640.95
Radial Load (Lb)		
3000.0	196.06	182.09
15000.0	810.62	910.43
30000.0	1596.94	1820.87
50000.0	2644.95	3034.78
Thrust Load (Lb)		
10000.0	201.91	160.38
50000.0	845.98	801.88
100000.0	1647.97	1603.75

APPENDIX H

RATING PROGRAM LISTING

```
PROGRAM RATING (INPUT,OUTPUT,DATA);

TYPE NUM=ARRAY[1..10]OF REAL;

VAR
      BF,                   {BRINELL FACTOR}
      C,                    {CONFORMITY}
      CA,                   {CONTACT ANGLE - DEGREES}
      CD,                   {CASE DEPTH - INCHES}
      C1,                   {EQUATION 26 CONSTANT}
      C2,                   {EQUATION 2 CONSTANT}
      D,                    {BALL DIAMETER - INCHES}
      DC,                   {DIAMETERAL CLEARANCE - INCHES}
      FC,                   {BASIC LOAD RATING}
      FD,                   {LIMITING DYNAMIC RACE FORCE - LBS}
      FS,                   {LIMITING STATIC RACE FORCE - LBS}
      L,                    {FATIGUE LIFE FOR STATIC LOAD - REVOLUTIONS}
      MD,                   {DYNAMIC MOMENT LOAD - LBS}
      MS,                   {STATIC MOMENT LOAD - LBS}
      PD,                   {BALL PATH DIAMETER - INCHES}
      R,                    {RACEWAY MINOR DIAMETER - INCHES}
      RB,                   {BALL RADIUS - INCHES}
      RD,                   {DYNAMIC RADIAL LOAD - LBS}
      RP,                   {RACEWAY MAJOR DIAMETER - INCHES}
      RS,                   {STATIC RADIAL LOAD - LBS}
      SF,                   {SAFETY FACTOR}
      SFM,                  {EQUATION 43 VARIABLE}
      SFR,                  {EQUATION 44 VARIABLE}
      SP,                   {VON MISES STRESS}
      TD,                   {DYNAMIC THRUST LOAD - LBS}
      THETA,                {BALL SPACING - RADIANS}
      TS,                   {STATIC THRUST LOAD - LBS}
      XB,                   {EQUATION XX AND XX CONSTANT}
      YS:                   {MATERIAL YIELD STRENGTH - PSI}
        REAL;

      N:                    {NUMBER OF BALLS}
        INTEGER;

      NAME:
        PACKED ARRAY[1..50]OF CHAR;

      DATA:                 {OUTPUT FILE}
        TEXT;

      Q:                    {YESNO PROCEDURE CONSTANT}
        BOOLEAN;


PROCEDURE CLEAR;

{*************************************************************************

  CLEARS THE SCREEN
```

```
    GLOBAL VARIABLES:  NONE

*****************************************************************************}

VAR B:INTEGER;

BEGIN
  FOR B:=1 TO 24 DO WRITELN;
END;  {CLEAR}


PROCEDURE YESNO(VAR YES:BOOLEAN);

{*****************************************************************************

    RETURNS TRUE FOR AN INPUT OF YES, yes, Y, OR y; RETURNS FALSE FOR AN INPUT
    OF NO, no, N, OR n.  IF THE USER INPUTS ANYTHING ELSE HE IS PROMPTED TO
    MAKE A VALID INPUT.

    GLOBAL VARIABLES:  Q

*****************************************************************************}

VAR
  Str : PACKED ARRAY[1..3] OF CHAR;
  Legal  : BOOLEAN;

BEGIN
  REPEAT
    Legal:=TRUE;
    READLN(Str);
    IF Str='YES' THEN Yes:=TRUE
    ELSE IF Str='yes' THEN Yes:=TRUE
    ELSE IF Str='Y  ' THEN Yes:=TRUE
    ELSE IF Str='y  ' THEN Yes:=TRUE
    ELSE IF Str='NO ' THEN Yes:=FALSE
    ELSE IF Str='no ' THEN Yes:=FALSE
    ELSE IF Str='N  ' THEN Yes:=FALSE
    ELSE IF Str='n  ' THEN Yes:=FALSE
    ELSE
      BEGIN
        WRITELN('************************************************');
        WRITELN('Response must be YES, yes, Y, y, NO, no, N, or n');
        WRITELN('Please enter a valid response ...');
        WRITELN('************************************************');
        Legal:=FALSE;
      END;
  UNTIL Legal;
END;  {PROCEDURE YESNO}


FUNCTION EXPT(Y,X:REAL):REAL;
```

```
{****************************************************************************

   RETURNS THE RESULT OF Y^X

   GLOBAL VARIABLES: NONE

****************************************************************************}

VAR Z:INTEGER;

BEGIN
  IF X=0 THEN
    EXPT:=1.0
  ELSE
    IF Y=0 THEN
      EXPT:=0.0
    ELSE
      IF Y>0 THEN
        EXPT:=EXP(X*LN(Y))
      ELSE
        IF ABS(X-ROUND(X))<1E-10 THEN
        BEGIN
        Z:=TRUNC(X);
        IF ODD(Z) THEN
          EXPT:=-EXP(X*LN(-Y))
        ELSE
          EXPT:=EXP(X*LN(-Y))
        END
END;   {EXPT}



FUNCTION ARCCOS(X:REAL):REAL;

{****************************************************************************

   INVERSE COSINE FUNCTION, RETURNS ARCCOS(X), -1.0<= X <=+1.0, IN RADIANS

   GLOBAL VARIABLES:  NONE

****************************************************************************}

CONST   PI=3.1415926536;

BEGIN
 IF (X=1) OR (X=-1) THEN
   IF X=1 THEN
     ARCCOS:=0.0
   ELSE
     ARCCOS:=PI
 ELSE
   ARCCOS:=-ARCTAN(X/SQRT(1-X*X))+PI/2.0
END;  {ARCCOS}
```

```
PROCEDURE ZONE(VAR CL,DM,THETAZ2:REAL);

{*************************************************************************

  CALCULATE THE LOAD ZONE

  GLOBAL VARIABLES:  DM

*************************************************************************}

BEGIN
  THETAZ2:=CL/(2.0*DM);                          {EQUATION 31}
  IF ABS(THETAZ2)>=1.0 THEN
    THETAZ2:=0.0
  ELSE
    THETAZ2:=ARCCOS(THETAZ2);
END;  {ZONE}



PROCEDURE MDFOR(VAR CL,RK,DM:REAL);

{*************************************************************************

  CALCULATE THE MAXIMUM BALL DEFORMATION

  GLOBAL VARIALBES:  CA,C1,THETA

*************************************************************************}

VAR THBEG,THSUM,EXPTHSUM,THETAZONE,
    THETAZ1,THETAZ2,DTHETAZ1,DTHETAZ2,SUM:REAL;

BEGIN
  DTHETAZ2:=0.0;
  THETAZ2:=THETA;
  REPEAT
    THETAZ1:=THETAZ2;
    SUM:=0.0;
    THBEG:=0.0;
    WHILE THBEG >= -THETAZ1 DO
      THBEG:=THBEG - THETA;
    THBEG:=THBEG + THETA;
    THSUM:=THBEG;
    WHILE THSUM <= THETAZ1 DO
    BEGIN
      EXPTHSUM:=EXPT(COS(THSUM),2.5);
      SUM:=SUM + EXPTHSUM;
      THSUM:=THSUM + THETA;
    END;
    DM:=EXPT(RK/(2.0*C1*COS(CA)*SUM),2.0/3.0);    {EQUATION 34}
    ZONE(CL,DM,THETAZ2);
    DTHETAZ1:=THETAZ2-THETAZ1;
    IF ABS(DTHETAZ1)=ABS(DTHETAZ2) THEN
      IF THETAZ1 > THETAZ2 THEN
```

```
      BEGIN
        THETAZ1:=THETAZ2;
        SUM:=0.0;
        THBEG:=0.0;
        WHILE THBEG >= -THETAZ1 DO
          THBEG:=THBEG - THETA;
        THBEG:=THBEG + THETA;
        THSUM:=THBEG;
        WHILE THSUM <= THETAZ1 DO
        BEGIN
          EXPTHSUM:=EXPT(COS(THSUM),2.5);
          SUM:=SUM + EXPTHSUM;
          THSUM:=THSUM + THETA;
        END;
        DM:=EXPT(RK/(2.0*C1*COS(CA)*SUM),2.0/3.0);    {EQUATION 34}
        DTHETAZ1:=0.0;
      END;
    DTHETAZ2:=DTHETAZ1;
  UNTIL ABS(DTHETAZ1)<1E-3;
END;  {MDFOR}


PROCEDURE BALLFORCE(VAR DM,F:REAL);

{ **********************************************************************

  CALCULATE THE INDIVIDUAL BALL FORCES

  GLOBAL VARIABLES:  C1

  **********************************************************************}

BEGIN
    F:=C1*EXPT(DM,1.5);                      {EQUATION 26}
END;  {BALLFORCE}


PROCEDURE ADJUST(VAR KC:REAL);

{ **********************************************************************

  CALCULATE THE CLEARANCE ADJUSTMENT FACTOR

  GLOBAL VARIABLES:  FS,SFR

  **********************************************************************}

VAR KC1,KC2,KCD,CL,DM,F1,F2,RK:REAL;

BEGIN
  KC2:=1.0;
  REPEAT
    KC1:=KC2;
    RK:=KC1*FS/SFR;
```

```
   CL:=0.0;
   MDFOR(CL,RK,DM);
   BALLFORCE(DM,F1);
   CL:=DC;
   MDFOR(CL,RK,DM);
   BALLFORCE(DM,F2);
   KC2:=F1/F2;                            {EQUATION 45}
   KCD:=KC2-KC1;
  UNTIL ABS(KCD)<1E-3;
  KC:=KC2;
END;


PROCEDURE OUTDATA;

{*************************************************************************

  SETS UP THE OUTPUT FILE

  GLOBAL VARIABLES:  DATA

*************************************************************************}

VAR FILENAME:PACKED ARRAY[1..17] OF CHAR;
    STATUS:INTEGER;

BEGIN
  CLEAR;
  WRITELN('        GEAR PRODUCTS INC., TULSA, OKLAHOMA');
  WRITELN('        COMPUTER-AIDED ROTATION BEARING RATING');
  WRITELN;
  WRITELN('ENTER THE NAME OF THE OUTPUT FILE - ');
  READLN(FILENAME);
  BIND(DATA,FILENAME,STATUS);
  REWRITE(DATA);
  WRITELN;
END;  {OUTDATA}


PROCEDURE INDATA;

{*************************************************************************

  INTIALIZE THE PROGRAM VARIABLES

  GLOBAL VARIABLES:  BF,D,DC,N,NAME,PD,SF,YS

*************************************************************************}

BEGIN
  WRITELN('ENTER THE BEARING NUMBER');
  READLN(NAME);
  WRITELN;
  WRITELN('BEARING NUMBER - ',NAME);
```

```
   WRITELN;
   WRITELN('ENTER THE BALL PATH DIAMETER IN INCHES -');
   READLN(PD);
   WRITELN;
   WRITELN('BALL PATH DIAMETER = ',PD:7:3);
   WRITELN;
   WRITELN('ENTER THE BALL DIAMETER IN INCHES -');
   READLN(D);
   WRITELN;
   WRITELN('BALL DIAMETER = ',D:6:3);
   WRITELN;
   WRITELN('ENTER THE NUMBER OF BALLS -');
   READLN(N);
   WRITELN;
   WRITELN('NUMBER OF BALLS = ',N:3);
   WRITELN;
   WRITELN('ENTER THE DIAMETERAL CLEARANCE IN INCHES -');
   READLN(DC);
   WRITELN;
   WRITELN('DIAMETERAL CLEARANCE = ',DC:8:6);
   WRITELN;
   WRITELN('ENTER THE MATERIAL YIELD STRENGTH IN KPSI -');
   READLN(YS);
   WRITELN;
   WRITELN('MATERIAL YIELD STRENGTH = ',YS:8:1);
   YS:=YS*1000.0;
   WRITELN;
   WRITELN('ENTER THE BRINELL FACTOR -');
   READLN(BF);
   WRITELN;
   WRITELN('BRINELL FACTOR = ',BF:8:6);
   WRITELN;
   WRITELN('ENTER THE SAFETY FACTOR -');
   READLN(SF);
   WRITELN;
   WRITELN('SAFETY FACTOR = ',SF:4:2);
   WRITELN;
END;  {PROCEDURE INDATA}


PROCEDURE CONSTANTS;

{*********************************************************************************

   CALCULATE THE PROGRAM CONSTANTS

   GLOBAL VARIABLES:  C,CA,C1,C2,D,N,PD,R,RP,THETA,XB

 *********************************************************************************}

BEGIN
   C:=1.08;
   CA:=1.0471976;
   C1:=2.198E7*SQRT((C*0.5*D)/(C-1));            {EQUATION 26}
```

```
      C2:=D*COS(CA)/PD;
      FC:=7450*EXPT(C/(C-1),0.41)*(EXPT(1-C2,1.39)/(EXPT(1+C2,1/3)))*
          EXPT(C2/COS(CA),0.3)*EXPT(D,1.8)*EXPT(N,-1/3);   (EQUATION 2)
      R:=C*D/2.0;
      RB:=D/2.0;
      RP:=R+(PD/2.0);
      THETA:=(360.0/N)*(3.1415927/180.0);
      XB:=N*PD;
      IF (XB<560.0) OR (XB>2370.0) THEN
      BEGIN
        WRITELN('*** WARNING: NUMBER OF BALLS TIMES BALL PATH DIAMETER OUT');
        WRITELN('              OF RANGE, RESULTS MAY BE INACCURATE');
        WRITELN(DATA,'*** WARNING: NUMBER OF BALLS TIMES BALL PATH DIAMETER OUT');
        WRITELN(DATA,'              OF RANGE, RESULTS MAY BE INACCURATE');
      END;
END;   (PROCEDURE CONSTANTS)



PROCEDURE STATIC;

(*************************************************************************

  CALCULATE THE LIMITING STATIC RACE FORCE

    GLOBAL VARIABLES:  BF,C,C2,D,FS,SF

*************************************************************************)

BEGIN
(EQUATION 5)
  FS:=SQRT((4.0E10*BF*EXPT(D,4.0))/((1+(C2/(1-C2)))*(1-1/C)));
  FS:=FS/SF;
END;   (PROCEDURE STATIC)



PROCEDURE DYNAMIC;

(*************************************************************************

  CALCULATE THE LIMITING DYNAMIC RACE FORCE

    GLOBAL VARIABLES:  FC,FD,SF

*************************************************************************)

BEGIN
  FD:=FC;                              (EQUATION 14)
  FD:=FD/SF;
END;   (PROCEDURE DYNAMIC)



PROCEDURE RATING;

(*************************************************************************
```

```
                CALCULATE THE BEARING RATING

                GLOBAL VARIABLES:  CA,FD,FS,N,SFM,SFR,XB


****************************************************************************)

VAR KC:REAL;

BEGIN
  SFM:=18.5516*EXPT(XB,-0.8361);                  {EQUATION 43}
  MS:=FS/SFM;                                      {EQUATION 47}
  MD:=FD/SFM;                                      {EQUATION 47}
  SFR:=(0.0464*XB)/(-368.9887+XB);                {EQUATION 44}
  ADJUST(KC);
  RS:=KC*FS/SFR;                                   {EQUATION 48}
  RD:=KC*FD/SFR;                                   {EQUATION 48}
  TS:=N*0.95*FS*SIN(CA);                           {EQUATION 46}
  TD:=N*0.95*FD*SIN(CA);                           {EQUATION 46}
END;  {PROCEDURE RATING}



PROCEDURE LIFE;

{****************************************************************************

    CALCULATE BEARING LIFE DUE TO LIMITING STATIC RACE FORCE

    GLOBAL VARIABLES:  FC,FS,L


****************************************************************************)

BEGIN
  L:=EXPT(FC/FS,3.0)*1E6;                          {EQUATION 1}
END;  {PROCEDURE LIFE}



PROCEDURE WRITE;

{****************************************************************************

    WRITE THE DATA TO THE OUTPUT FILE

    GLOBAL VARIABLES:  BF,D,DC,MD,MS,N,PD,RS,RD,SP,TS,TD


****************************************************************************)

VAR I:INTEGER;

BEGIN
  FOR I:=1 TO 3 DO
    WRITELN(DATA);
  WRITELN('                    GEAR PRODUCTS INC., TULSA, OKLAHOMA');
  WRITELN(DATA,'                  GEAR PRODUCTS INC., TULSA, OKLAHOMA');
```

```
      WRITELN('                        COMPUTER-AIDED ROTATION BEARING RATING');
      WRITELN(DATA,'                        COMPUTER-AIDED ROTATION BEARING RATING');
      WRITELN;
      WRITELN(DATA);
      WRITELN('           **INPUT**');
      WRITELN(DATA,'          **INPUT**');
      WRITELN;
      WRITELN(DATA);

      WRITELN('          Bearing Number - ',NAME);
      WRITELN(DATA,'          Bearing Number - ',NAME);
      WRITELN('          Ball Path Diameter (in)        = ',PD:8:3);
      WRITELN(DATA,'          Ball Path Diameter (in)       = ',PD:8:3);
      WRITELN('          Ball Diameter (in)             = ',D:8:3);
      WRITELN(DATA,'          Ball Diameter (in)            = ',D:8:3);
      WRITELN('          Number of Balls                = ',N:8);
      WRITELN(DATA,'          Number of Balls               = ',N:8);
      WRITELN('          Diameteral Clearance (in)      = ',DC:8:6);
      WRITELN(DATA,'          Diameteral Clearance (in)     = ',DC:8:6);
      WRITELN('          Material Yield Strength (kpsi) = ',ROUND(YS/1000):8);
      WRITELN(DATA,'          Material Yield Strength (kpsi) = ',ROUND(YS/1000):8);
      WRITELN('          Brinell Factor                 = ',BF:8:6);
      WRITELN(DATA,'          Brinell Factor                = ',BF:8:6);
      WRITELN('          Safety Factor                  = ',SF:8:2);
      WRITELN(DATA,'          Safety Factor                 = ',SF:8:2);
      WRITELN;
      WRITELN(DATA);
      WRITELN('          **RATINGS**');
      WRITELN(DATA,'          **RATINGS**');
      WRITELN;
      WRITELN(DATA);

      WRITELN('          Static Moment Load (ft-lbs)    = ',MS:9:0);
      WRITELN(DATA,'          Static Moment Load (ft-lbs)     = ',MS:9:0);
      WRITELN('          Dynamic Moment Load (ft-lbs)   = ',MD:9:0);
      WRITELN(DATA,'          Dynamic Moment Load (ft-lbs)    = ',MD:9:0);
      WRITELN('          Static Radial Load (lbs)       = ',RS:9:0);
      WRITELN(DATA,'          Static Radial Load (lbs)        = ',RS:9:0);
      WRITELN('          Dynamic Radial Load (lbs)      = ',RD:9:0);
      WRITELN(DATA,'          Dynamic Radial Load (lbs)       = ',RD:9:0);
      WRITELN('          Static Thrust Load (lbs)       = ',TS:9:0);
      WRITELN(DATA,'          Static Thrust Load (lbs)        = ',TS:9:0);
      WRITELN('          Dynamic Thrust Load (lbs)      = ',TD:9:0);
      WRITELN(DATA,'          Dynamic Thrust Load (lbs)       = ',TD:9:0);
      WRITELN('          Fatigue Life For Static Load (rev) = ',L:9:0);
      WRITELN(DATA,'          Fatigue Life For Static Load (rev) = ',L:9:0);
      WRITELN;
      WRITELN(DATA);
END;  {PROCEDURE OUTDATA}


PROCEDURE TABLE;

{***********************************************************************

   CALCULATE THE VON MISES STRESS AND LIMITING CASE DEPTH
```

```
      GLOBAL VARIABLES:  FS,R,RB,RP,SP

 ***********************************************************************}

VAR A,B,BE,AE,KO,KN,KP,N1,Z,PHI,FI,HI,KI,SI,OX,OXP,OY,OYP,M,SX,SY,SZ,
    S1,S2,S3,SP,P,FI1,FI2,FI3,FI4,FI5,FI6,HI1,HI2,HI3,HI4,HI5,HI6,
    TEMP,INC:REAL;
    I,J,K,L:INTEGER;
    CHECK:BOOLEAN;
    LAM2:ARRAY[1..5]OF REAL;

BEGIN
  INC:=0.02;
  CHECK:=TRUE;
  WRITELN('         **CASE-CORE CRUSHING DATA**');
  WRITELN(DATA,'          **CASE-CORE CRUSHING DATA**');
  WRITELN;
  WRITELN(DATA);
  WRITELN('         Depth (in)','  ','von Mises Stress (psi)');
  WRITELN(DATA,'         Depth (in)','  ','von Mises Stress (psi)');
  A:=0.5*((1.0/RB)-(1.0/RP));                  {EQUATION 8B}
  B:=0.5*((1.0/RB)-(1.0/R));                   {EQUATION 8B}
  P:=6.0666667E-8/(A+B);                       {EQUATION 8A}
  BE:=0.5;
  AE:=0.75;
  KN:=BE/AE;                                   {EQUATION 8B}
  FOR I:=0 TO 20 DO
    BEGIN
    Z:=INC*I;
    REPEAT
      KO:=KN;
      KP:=SQRT(1.0-SQR(KO));                   {EQUATION 9}
{EQUATION 8A}
      N1:=SQRT((SQR(KO)+SQR(KO)*SQR(Z/BE))/(1.0+(SQR(KO)*SQR(Z/BE))));
      IF Z=0.0 THEN
        PHI:=1.5707963
      ELSE
        PHI:=ARCTAN(BE/(KO*Z));                {EQUATION 8B}
{EQUATIONS 11}
      FI1:=0.5*SQR(KP)*(0.5*PHI-0.25*SIN(2.0*PHI));
      FI2:=0.375*SQR(SQR(KP))*(0.375*PHI-0.25*SIN(2.0*PHI)+
           0.03125*SIN(4.0*PHI));
      FI3:=0.3125*EXPT(KP,6.0)*(0.3125*PHI-0.2083333*SIN(2.0*PHI)+
           0.0260417*SIN(4.0*PHI)-0.1666667*COS(PHI)*EXPT(SIN(PHI),5.0));
      FI4:=0.2734375*EXPT(KP,8.0)*(0.2734375*PHI-0.1822917*SIN(2.0*PHI)+
           0.0227865*SIN(4.0*PHI)-0.1458333*COS(PHI)*EXPT(SIN(PHI),5.0)-
           0.125*COS(PHI)*EXPT(SIN(PHI),7.0));
      FI:=PHI+FI1+FI2+FI3+FI4;
      HI1:=0.5*SQR(KP)*(0.5*PHI-0.25*SIN(2.0*PHI));
      HI2:=0.125*SQR(SQR(KP))*(0.375*PHI-0.25*SIN(2.0*PHI)+
           0.03125*SIN(4.0*PHI));
      HI3:=0.0625*EXPT(KP,6.0)*(0.3125*PHI-0.2083333*SIN(2.0*PHI)+
           0.0260417*SIN(4.0*PHI)-0.1666667*COS(PHI)*EXPT(SIN(PHI),5.0));
```

```
     HI4:=0.0390625*EXPT(KP,9.0)*(0.2734375*PHI-0.1822917*SIN(2.0*PHI)+
         0.0227865*SIN(4.0*PHI)-0.1458333*COS(PHI)*EXPT(SIN(PHI),5.0)-
         0.125*COS(PHI)*EXPT(SIN(PHI),7.0));
     HI:=PHI-HI1-HI2-HI3-HI4;
{EQUATIONS 12}
     KI:=1.5707963+0.3926991*SQR(KP)+0.2208932*SQR(SQR(KP))+
         0.1533981*EXPT(KP,6.0)+0.1174454*EXPT(KP,8.0)+0.0951308*EXPT(KP,10.0);
     GI:=1.5707963-0.3926991*SQR(KP)-0.0736311*SQR(SQR(KP))-
         0.0306796*EXPT(KP,6.0)-0.0167779*EXPT(KP,8.0)-0.0105701*EXPT(KP,10.0);
     KN:=SQRT(GI/((B*(KI-GI)/A)+KI));                {EQUATION 10}
     BE:=EXPT(3.0*KN*GI*FS*P/6.2831853,1.0/3.0);     {EQUATION 9B}
     AE:=BE/KN;                                      {EQUATION 9B}
   UNTIL ABS(KN-KO)<0.001;
{EQUATIONS 8B}
   OX:=(KN*Z*(FI-HI)/BE)-0.5*(1-N1);
   OXP:=1.0+(KN*Z*((HI/SQR(KN))-FI)/BE)-(N1/SQR(KN));
   OY:=1.0/(2.0*N1)+0.5+(KN*Z*((HI/SQR(KN))-FI)/BE)-(N1/SQR(KN));
   OYP:=N1+(KN*Z*(FI-HI)/BE)-1.0;
   M:=(2.0*KN)/(SQR(KP)*GI);                         {EQUATION 8A}
{EQUATIONS 7}
   SX:=M*BE*(OX+0.3*OXP)/P;
   SY:=M*BE*(OY+0.3*OYP)/P;
   SZ:=-0.5*M*BE*(1.0/N1-N1)/P;
   LAM2[1]:=SX;
   LAM2[2]:=SY;
   LAM2[3]:=SZ;
   FOR K:=1 TO 3 DO
   BEGIN
     L:=K;
     FOR J:=L+1 TO 3 DO
     BEGIN
       IF LAM2[J] >= LAM2[L] THEN
         BEGIN
           TEMP:=LAM2[L];
           LAM2[L]:=LAM2[J];
           LAM2[J]:=TEMP;
         END;
     END;
   END;
   S1:=LAM2[1];
   S2:=LAM2[2];
   S3:=LAM2[3];
   SP:=SQRT((SQR(S1-S2)+SQR(S2-S3)+SQR(S3-S1))/2.0); {EQUATION 6}
   WRITELN('          ',Z:6:3,'          ',SP:9:1);
   WRITELN(DATA,'          ',Z:6:3,'          ',SP:9:1);
   IF CHECK=TRUE THEN
   BEGIN
     IF SP < YS THEN
     BEGIN
       CD:=Z;
       CHECK:=FALSE;
     END;
   END;
 END;
```

```
    WRITELN;
    WRITELN(DATA);
    WRITELN('          Suggested Minimum Case Depth (in) = ',CD:6:3);
     WRITELN(DATA,'          Suggested Minimum Case Depth (in) = ',CD:6:3);
    WRITELN(DATA);
    WRITELN(DATA);
END;   {PROCEDURE TABLE}



{**********************************************************************

   BEGIN MAIN PROGRAM

*********************************************************************}

BEGIN
  OUTDATA;
  REPEAT
    INDATA;
    CONSTANTS;
    STATIC;
    DYNAMIC;
    RATING;
    LIFE;
    WRITE;
    TABLE;
    WRITELN;
    WRITELN('DO YOU WANT TO RUN THE PROGRAM AGAIN (Y/N) ?');
    WRITELN;
    YESNO(Q);
  UNTIL Q=FALSE;
  CLOSE(DATA);
END.
```

VITA 2

Urs Thomas Moeker

Candidate for the Degree of

Master of Science

Thesis: COMPUTER AIDED RATING OF LARGE DIAMETER ROTATION BEARINGS

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Tulsa, Oklahoma, July 13, 1962, the son of Joachim W. and Caecilia T. Moeker.

Education: Graduated from Nathan Hale High School, Tulsa, Oklahoma, in May 1980; received Bachelor of Science Degree in Mechanical Engineering from Oklahoma State University in May, 1985; completed requirements for Master of Science degree at Oklahoma State University in May, 1988.

Professional Experience: Project Engineer, United States Gypsum Company, Southard, Oklahoma, September 1985 to August, 1986. Research Assistant, Department of Mechanical Engineering, Oklahoma State University, February, 1987, to December 1987.