AGGREGATION APPROACHES FOR INCORPORATING E-MAIL PROCESSING HISTORY IN QUEUEING MODELS OF CUSTOMER CONTACT CENTERS

By

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Dedicated to my late father
Sri. M. Chinnaswamy
and to my mentor
Dr. Manjunath Kamath

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NOTATION

c	type of e-mail; $c = 1, 2, ..., C$ (These types may represent different classes of e-mail- product inquiry, technical support, etc.)
A	number of customer service agents
k, i, j	index for previous, current and next agent processing an e-mail; $k = 0, 1, ..., A$; $i,\ j = 1, 2, ..., A$; $k = 0$ represents a new e-mail
$\lambda(c)$	external arrival rate of (new) e-mails belonging to type c
$\lambda_i(c)$	external arrival rate of type c e-mails at agent i
λ_i	external arrival rate of e-mails at agent i
$\gamma_i(c)$	total (external plus internal) arrival rate of type c e-mails at agent i
$r_i(c)$	probability that an e-mail received by agent i is of type c
$\beta(c)$	represents the total (new plus previously processed) arrival rate of type c e-mails for Scenario 1
$\alpha_i(c)$	probability that a new type c e-mail will be routed to agent i
$w_i(c,k)$	proportion of type c e-mails received by agent i that were previously processed by agent k or new $(k = 0)$; $i = A+1$ represents the delay node
$w_{A+1}(c,k)$	probability that a type c e-mail at the delay node $(A+1)$ came from agent k
$\upsilon_i(c,k)$	average number of visits to agent i made by a type c e-mail that was previously processed by agent k
$\upsilon_{D_i}(c,k)$	average number of times a type c e-mail was processed by agent i and not resolved

$p_{i,j}(c,k)$	probability that agent i forwards a type c e-mail that was previously processed by agent k to agent j
$p_{i,j}(c)$	class-specific routing probability from agent i to agent j for a type c e-mail
$p_{i,j}$	aggregate routing probability from agent i to agent j
$p(c)$	represents the aggregate forwarding probability for type c e-mails
$q_i(c,k)$	probability that a type c e-mail currently processed by agent i and previously processed by agent k ends in problem resolution
$q(c)$	represents the aggregate resolution probability for a type c e-mail
$p_{i,A+1}(c)$	class-specific routing probability from agent i to delay node $(A+1)$ for type c e-mail
$p_{i,A+1}$	aggregate routing probability from agent i to delay node $(A+1)$
$p_{A+1,i}(c)$	class-specific routing probability from the delay node $(A+1)$ to agent i for a type c e-mail
$p_{A+1,i}$	aggregate routing probability from delay node $(A+1)$ to agent i
$P_i(c)$	random variable that represents the preprocessing time at agent i for a type c e-mail
$Z_i(c,k)$	indicator random variable that takes the value 1 if agent i processes a type c e-mail that was previously processed by agent k and the value 0 otherwise
$S_i(c,k)$	random variable that represents the processing time at agent i for a type c e-mail that was previously processed by agent k

$T_i(c,k)$	random variable that represents the overall service time at agent i for a type c e-mail that was previously processed by agent k
$\theta_i(c)$	$E[P_i(c)]$
$p_i(c,k)$	$E[Z_i(c,k)] = \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{A} p_{i,j}(c,k)\right)$
$s_i(c,k)$	$E[S_i(c,k)]$
$t_i(c,k)$	$E[T_i(c,k)]$
c^2	squared coefficient of variation (SCV) = variance/mean^2
$c_o^2(c)$	SCV of the interarrival time for new, external type c e-mails
$c_{oi}^2(c)$	SCV of the interarrival time for external type c e-mails at agent i
c_{oi}^2	SCV of the aggregate interarrival time for external e-mails at agent i
$c_{pre,i}^2(c)$	SCV of $P_i(c)$
$c_{ser,i}^2(c,k)$	SCV of $S_i(c,k)$
$c_i^2(c,k)$	SCV of $T_i(c,k)$
$c_i^2(c)$	SCV of the overall class-dependent service time at agent i
c_i^2	SCV of the overall service time at agent i

The following additional notation is needed for the Markov chain analysis presented in Section 6.5.

N	represents the "new" state - the life-cycle of a new e-mail starts in this state

R	represents the “resolution” node - the life-cycle of an e-mail ends in this state
D_i	represents the delay state linked to agent i - e-mails processed by agent i that are unresolved pass through this state
P	one-step transition probability matrix
U	unit matrix of size (1×1)
M	column matrix of size $(n\times 1)$, where $n = A^3 + A^2 + A + 1$
Q	truncated matrix associated with **P** and of size $(n\times n)$, where $n = A^3 + A^2 + A + 1$
I	identity matrix of size $(n\times n)$, where $n = A^3 + A^2 + A + 1$
F	fundamental matrix of size $(n\times n)$, where $n = A^3 + A^2 + A + 1$

CHAPTER 1

INTRODUCTION

The service sector has become a dominant part of the US economy, due in part to the e-commerce revolution of the late nineties. Better quality of service delivered virtually with very little or no waiting time is what customers frequently expect today. One notable facet of the service industry is the call center industry. A call center is any "group whose principal business is talking on the telephone to customers or prospects (Mehrotra 1997)."

Customer call centers, which represent a multi-billion dollar industry, are evolving into customer contact centers. "It is estimated about four million people in the United States- 3% of the workforce - work in contact centers, with the number growing by about 20% per year. A contact center is a collection of resources providing an interface between the service provider and its remote customers (Whitt 2002a)." The interface can be through any one or combination of media - telephone, e-mail, fax, paper, chat sessions and the Web. In the private sector, contact centers are used in various industries and are an important communication channel to acquire new customers as well as to support existing customers. In e-mail contact centers, the traffic can be inbound or outbound. In an inbound e-mail contact center, agents respond to e-mails from customers. Examples of inbound e-mail contact centers are technical product support centers and travel reservation centers. In outbound e-mail contact centers, agents initiate e-mail that is sent to customers. Examples of outbound e-mail contact centers are companies conducting surveys and market research.

In inbound e-mail contact centers the response time is flexible, i.e., customers do not expect an e-mail to be answered within minutes, whereas they could get frustrated waiting on the telephone even for a few minutes. This flexibility allows for the possibility of postponing a response. The average time an agent takes to respond to a customer's e-mail is known as the response time. This time needs to be as small as possible and is an important performance measures in both call and contact centers. The other measure of interest is the resolution time, i.e., the average time that is taken to resolve the problem represented by the e-mail. Shorter response times and resolution times can help contact centers to better serve and retain their customers. These system performance measures can be improved if the e-mail contact center employs more agents. But employing more agents leads to higher operating costs. In e-mail contact centers, more than half of the operating costs are driven by the costs of employing agents. Therefore, agent utilization is often used to indicate the economic performance measure of an inbound e-mail contact center. The number of agents is an important decision variable in designing e-mail contact centers.

Both queueing theory and simulation have been used to model the operations of call centers. These models describe the behavior of the system over time, which helps in designing call center operations. Bulk of the existing literature focuses on modeling traditional telephone call centers. As pointed out by Whitt (2002a), more research is needed in the stochastic modeling of customer contact centers, where the contact is through media other than the telephone. The focus of this thesis was on modeling an important characteristic of e-mail contact centers, namely the dependence of e-mail processing times and routing on e-mail history. A novel history-based aggregation approach was developed to handle the dependence on processing history within a multi-class open queueing network model. Through extensive numerical experimentation, this thesis shows the importance of modeling e-mail history in accurately predicting the performance of e-mail contact centers. The numerical investigations also demonstrate the accuracy and robustness of the history-based aggregation approach.

The remainder of this thesis document is organized as follows (also see Figure 1.1). Chapter 2 describes the problem statement. Chapter 3 presents an extensive literature review of the work carried to date in modeling call and contact centers. Chapter 4 presents the research statement. Also included in this chapter are the research objectives, scope and limitations, and contributions of the research conducted. Chapter 5 describes the modeling approach that was followed in developing and solving the open queueing network models of e-mail contact centers. Chapter 6 presents the nucleus of this thesis effort. It includes three analytical methods ranging from a simple averaging technique to a very detailed Markov chain based aggregation approach to model dependence on e-mail history. These analytical methods are integrated into the solution method for a multi-class, open queueing network model of e-mail contact centers. Chapter 7 extends the network model presented in Chapter 6 to include (i) multi-server nodes that represent skill-based pools of agents and (ii) random interruptions that affect agent's availability for e-mail processing. Chapter 8 presents research contributions, conclusions and directions for future research.

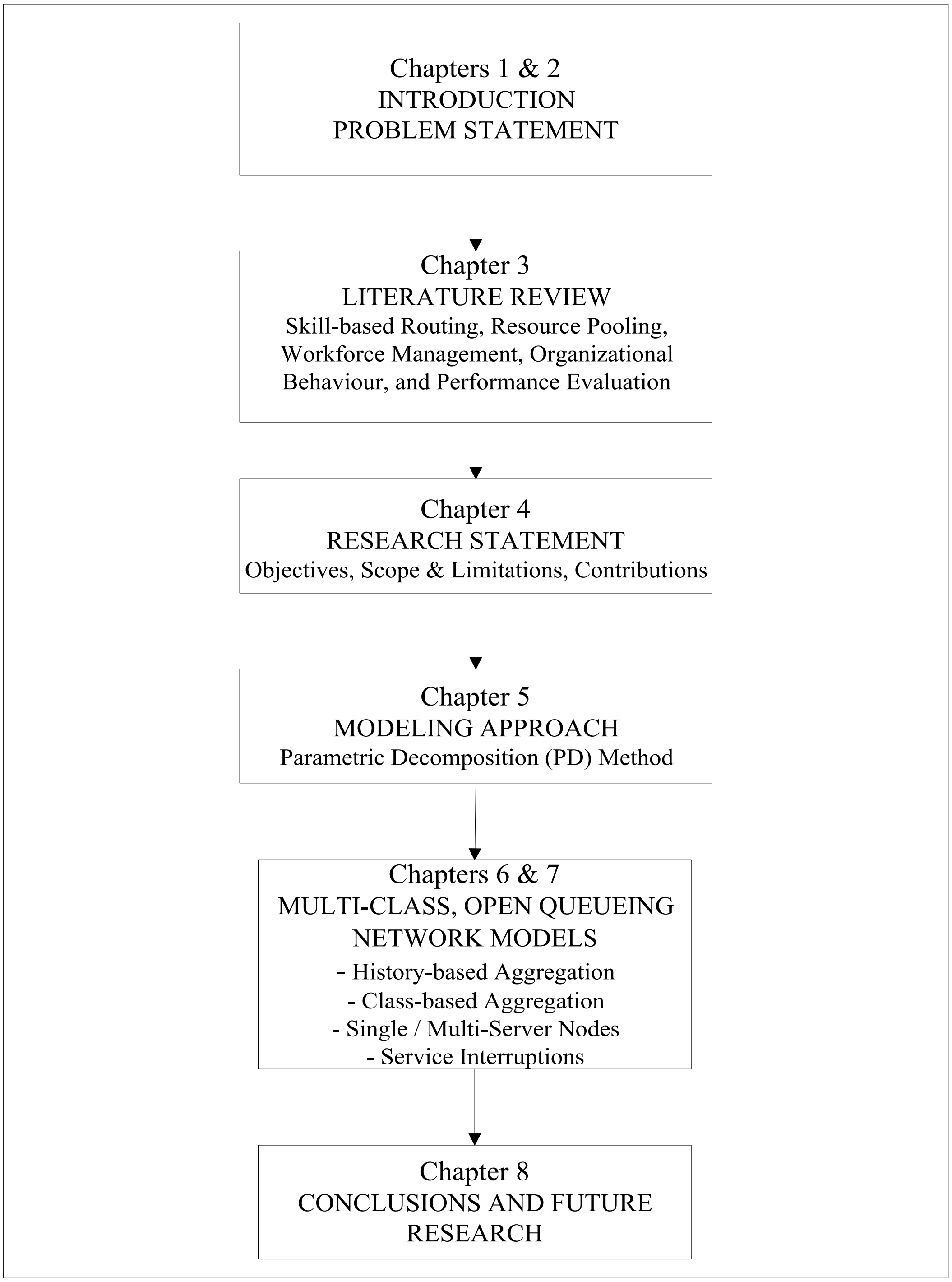

Figure 1.1: Outline of the Thesis

CHAPTER 2

STATEMENT OF THE PROBLEM

Modeling call centers using queueing theory is not new. Many researchers have modeled call centers using the standard M/M/c/∞ queues to obtain steady-state performance measures such as average number of calls in the system and average waiting time for calls. However, only very limited literature exists on queueing models of contact centers because of their recent emergence as an alternative to call centers.

In addition to analytical approaches, both call and contact centers can be modeled using simulation. Simulation models require more detailed information when compared to queueing models. The model development and model execution activities in simulation could be time consuming. On the other hand, analytical models such as queueing networks provide more insight and understanding of the system. Analytical models may require simplifying assumptions of the system, and the results obtained are generally less accurate than those estimated via simulation. But analytical models yield results quickly and "are appropriate for rapid and rough cut analysis (Suri et al. 1993)." Hence, analytical models can be used for preliminary design and simulation for fine tuning.

The evolution of e-mail contact centers has thrown light on many new modeling issues that are not typically addressed in the study of traditional call centers. Customer or problem history can be more useful with asynchronous communication tools such as e-mail and can influence routing strategies and processing times. For example, it is possible for an agent who previously attended to a customer's problem (e-mail) to also deal with the follow-up e-mail because of the asynchronous nature of e-mail processing. With regard to system performance measures, the average resolution time, i.e., the average time needed to resolve a customer's problem is not normally addressed in the call center literature. Problem resolution, while important in both call and contact centers, has much more visibility in an e-mail contact center because of the history embedded in the e-mail reply.

In a call center, because of the nature of the customer contact, the response time for a call becomes more important. In a contact center, both response and resolution times become important as the latter is a function of the former. Also, because of the asynchronous nature of e-mail, the customer service agent has more flexibility in terms of the time to generate a response to an e-mail. Also, it is possible to direct an e-mail to the appropriate agent if it contains information about previous response(s). These new modeling issues present unique challenges while developing queueing models of contact center operations. This thesis has addressed some of these issues. The problem statement can be summarized as follows: "To model the performance of an inbound e-mail contact center in which the routing of an e-mail and e-mail processing times at the various agents can depend on the e-mail history."

CHAPTER 3

LITERATURE REVIEW

The research on modeling call centers and contact centers can be broadly classified into the following categories - 1) Skill-Based Routing, 2) Resource Pooling, 3) Workforce Management, 4) Performance Evaluation and 5) Organizational Behavior. Within each category, a formal subcategory on e-mail contact centers is created only when there is a need to distinguish the work from that for the call center classification. Otherwise, the description for call centers holds good for the e-mail contact centers also. Though the scope of the literature review presented in this chapter is much broader than the performance evaluation theme of this research, it nevertheless illustrated the importance of analytical models in the analysis and design of customer call centers. This chapter also explores some new research directions and issues in the area of customer contact center modeling.

3.1 Skill - Based Routing

Modern call centers have multiple call types and multiple types of agents. One way of classifying customer calls is by language. With the globalization of many businesses, call centers receive calls in different languages from their customers throughout the world.

Another way of classifying customer calls is based on special promotions. The customer may be calling a toll-free number designated for a special promotion purpose. Training agents for a special promotion purpose is a common practice. To match the caller's need and agent's skill set, modern call centers have automatic call distributors (ACDs) that have the capability of routing calls to agents with the appropriate skills. This capability of ACDs is known as skill-based routing (SBR). In e-mail contact centers, the e-mails are routed to the appropriate agents with the help of information and communication technologies. An important issue to be addressed pertains to the optimality of e-mail/call routing policies.

3.1.1 Challenges in Skill-Based Routing

There are many challenges and issues to be addressed in performing skill-based routing well. First, with a given collection of agents, it is difficult to route multiple calls/e-mails to an agent in an optimal manner. This is due to elementary skill-based routing algorithms that are used in call/contact centers. According to "(Whitt 2002a), there remains a great opportunity for devising better routing algorithms." The routing of calls/e-mails depends on the number of agents in the center. Second, it is difficult to determine exactly the number of agents with appropriate skill sets. With multiple call/e-mail types, not only the prediction of the overall arrival rate, but also the prediction of arrival rates for individual types becomes important.

Koole et al. (2003) presented an approximation method for analyzing the performance of call centers with skill-based routing. They considered two types of call arrivals. Arriving calls abandon the system if the agents with the right skill are busy. But under these conditions, it was difficult to compute the optimal routing policies of calls because of state-space explosion. Therefore, the authors considered a different queueing system where the call gets queued if the agents are not available in any of the groups. This is equivalent to calls overflowing from the groups without available agents. This type of routing is known as overflow routing. This system is easy to characterize and optimal policies can be practically implemented, but allows less flexibility in routing policies. Koole and Talim (2000) studied a multi-skill call center as a network of queues. They approximated each queue as an M/M/r loss system, because an arriving call left if all agents in the network are busy. The inter-overflow time distribution at each node in the network was approximated by an exponential distribution, and the efficiency of the approximation was illustrated using simulation. Bhulai and Koole (2003) developed a queueing model from a call blending perspective for schedule agents either to incoming or outgoing calls in order to increase productivity and reduce the call waiting times. In addition, scheduling policies and their implementation within call center software were also discussed.

Garnett and Mandelbaum (2000) illustrated the operational complexities of skill-based routing using simulation studies. Perry and Nilsson (1992) considered simple strategies to overcome the dimensionality of call centers. They considered a two-channel agent system, where the waiting customer was assigned an aging-factor. This factor was directly proportional to the waiting time of the customer in the system. The customer with the largest aging factor (considering both queues) was chosen for service. They determined the expected waiting time for each call type, and the number of agents required for answering calls to maintain an acceptable grade of service. To conclude, there is scope for more research in developing simple and better routing algorithms, finding optimal routing policies using approximations and strategies, and implementing those in call/contact centers.

3.2 Resource Pooling

Resource pooling is overlapping of agents between groups so as to meet customer needs and to increase the efficiency of call/contact centers. In a call center, it is customary to have a large group of agents dedicated to a particular skill set. This can be viewed as a big call center with large number of smaller independent call centers. Upon call arrival, the calls can be routed to the group of agents with the appropriate skill sets. When the arrival process to the big call center is Poisson and with independent Bernoulli routing to the smaller call centers, the arrival process to the smaller call center is Poisson and tends to act independently of other smaller units.

Partitioning into subgroups tends to make call centers less efficient. While operating, some of the smaller centers tend to overloaded, while others may be underutilized. The efficiency of larger service groups is explained by Smith and Whitt (1981) and Whitt (1992). Smith and Whitt (1981) argued that if two systems were combined together into a single system, the efficiency of the combined system is higher than the efficiency of the individual systems. This seems intuitive and trivial, but becomes difficult to prove. The authors used stochastic-order relations to prove the result and concluded that the results apply to general arrival processes and general service-time distributions. The combination of systems assumes common service-time distribution, since it may be disadvantageous to combine systems with different service-time distributions. When the service time distributions are different it is advantageous to partition the system as in supermarket checkouts and reservation centers.

Whitt (1992) explained the economy of scale, and gave a quantitative characterization of a multi-server queueing system with unlimited waiting space. He showed that the increased variability in the arrival and service processes degrade server utilization with a given grade of service. He also presented a proof for finding the number of servers as a function of the arrival rate for a given service time distribution and grade of service. The proof is based on heavy-traffic limit theorems and uses infinite-server (IS) approximation to heuristically derive the result. The author developed simple approximations for the expected waiting time and the probability of delay using the infinite-server approximation, and conditional waiting time distribution using a single-server approximation. The next section throws light on the application of Stochastic-Process limits in resource pooling.

3.2.1 Stochastic-Process Limits

It is natural to study a system by allowing the number of customers and servers to grow large. This is done to gain more knowledge and insight about the behavior of the system under consideration. The mathematical results on resource pooling are based on the asymptotic regime in which the number of servers is allowed to approach infinity. This is the principle behind stochastic-process limits. Significant work on resource pooling has been done till now. The paper by Vvedenskaya et al. (1996) serves as a good example for its significance, and it considered a single stream of customers with a Poisson arrival process. Upon arrival, each customer joins one of the many identical single-server queues. The service time distribution in all queues follows an exponential distribution and all queues have an unlimited waiting space. The standard approach is joining the queue with very few customers. The number of customers is determined based on the states of all queues. Indeed, joining the shortest queue is optimal (Winston 1977). However, optimality is ceased if each queue has different service-time distribution (Whitt 1986). A difficulty in joining the shortest queue is that it may require a large amount of state information. Therefore techniques which require less amount of state information have to be adopted. For example, each arrival can be randomly assigned to one of the queues with equal probability of selecting a server. This can be modeled as an M/M/1 queue and all required performance measures can be calculated. Resource pooling has to be done very carefully. Sometimes it helps, but sometimes it hurts. The effect (good or bad) can be unbounded. The paper by Mandelbaum and Reiman (1998) clearly assesses the value of resource pooling.

Halfin and Whitt (1981) obtained limiting regimes for the Erlang C delay and GI/M/s models. This was carried out by allowing the number of servers to approach infinity, while letting the probability of delay approach a number strictly between 0 and 1. For more about Stochastic-Process limits, the reader is referred to Whitt (2002b). Heavy traffic limit regimes for the Erlang B (loss) model can be found in Srikant and Whitt (1996). Related results for queueing networks, state-dependent queues can also be found in Mandelbaum and Pats (1995). Papers about heavy traffic limit regimes on contact centers are very few. Whitt (2002a) stated that there remains a great opportunity for research in establishing some new interesting regimes.

3.3 Workforce Management

Workforce management plays a very significant role in design and maintenance of call/contact centers. Cleveland and Mayben (1997) addressed some mathematical issues in workforce management. Staffing is a challenging issue in call/contact centers. Whitt (2002a) described three types of staffing according to time scale. They are 1) real-time 2) short-term and 3) long-term.

3.3.1 Real-Time Staffing

Real-time staffing has sufficient flexibility to add agents when needed, and alternatively, pull them off to do some other work. Whitt (1999b) addressed the modeling of dynamic staffing of a call center with the aim of immediately answering the calls. The system state is exploited to obtain estimates for mean and variance of demand in near future. The staffing needs can be predicted from the information about recent demand and current calls in progress, as well as historical data. The information and telecommunication equipment makes it possible to obtain the required information. It is possible to classify the call by identifying the calling or called customer, purpose of the call and the agent who will be serving or served the call. By calculating the time length that the call has been in service before service completion, it is possible to predict the conditional probability distribution of the remaining call holding time.

By combining the information over many calls and agents, it is possible to predict the staff demands in the near future, providing a basis for real-time staffing. The paper by Whitt (1999b) "shows how stochastic models can be exploited to facilitate the process" and the author stated the idea needs be explored more thoroughly. Jennings et al. (1996) determined the number of servers as a function of time for a multi-server, time-varying demand service system based on an infinite-server (IS) approximation. The IS approximation averages the time-varying demand into an effective arrival rate which remain the same at all times. An approximate busy period server distribution is obtained by allowing the number of servers to grow large and by approximating the delay probability to a specific target value. The busy period server distribution is approximated by a time-dependent normal distribution where the mean and variance are determined by

IS approximations. Wallace and Whitt (2004) discussed a staffing algorithm for the skill-based routing call center.

Real-time staffing poses great challenges in applying queueing theory, because it requires the analysis of time-dependent behavior of queueing systems. Therefore, people seek numerical algorithms and approximations to describe the time-dependent behavior of queues. Papers in this regard include Whitt (1999a), and Abate and Whitt (1998, 1999) where they apply decomposition approximations and numerical transform inversion techniques respectively to study the time-dependent behavior of queues.

3.3.2 Short-Term Staffing

In short-term staffing, the daily staffing is carried out in response to the forecasted demand of calls and the availability agents. As the call arrivals vary significantly from day to day, one can use the steady-state behavior of queueing systems instead of the time-dependent one. This is because the call holding times are much shorter, and the time dependence can be safely ignored.

In some cases of short-term staffing, it is important to analyze the system with a time-varying arrival rate. A significant amount of effort in this area has been done by Abate and Whitt (1998), Mandelbaum and Pats (1995) and Whitt (1999b). A significant challenge in short-term staffing is scheduling agents for small breaks for example, lunch and coffee. Mathematical programming tools have been widely used in modeling this; see for example Segal (1974).

3.3.3 Long-Term Staffing

There are various challenges in long-term staffing. Training new agents is an important decision variable, which can be handled using a dynamic programming technique. On a long-term basis it is important to consider the agent’s career paths and attrition. The ideal situation is to have both satisfied customers and agents. Gans and Zhou (1999) developed a Markov decision model for call-center staffing where they address learning and turnover issues and optimal policies for long-term staffing.

3.4 Organizational Behavior

The role of human element is very important in call/contact centers. Customers are people and the service reps (agents) are people. “We can easily relate to contact centers because we ourselves often are customers of contact centers (Whitt 2002a).” The human behavior is really very important in studying the psychology of queueing systems (Gail and Scott 1997, Larson 1987). Enough time should be devoted to analyzing why customers abandon or revisit. Whitt (2002a) expressed an opinion that few papers have been published in this area.

3.5 Performance Evaluation

Considerable research has been done in performance modeling of call centers. Call center performance modeling studies have focused on (i) analyzing customer waiting times and customer impatience because of agent unavailability, (ii) finding optimal staffing to meet customer demands, and (iii) determining routing policies to serve customers at the earliest possible time. Traditional analysis techniques are typically based on the standard Erlang formula (Koole 2001). For example, the Erlang formula can be used to determine the upper and lower bounds on the number of employees needed, which are useful in employee scheduling (Koole 2001).

$$P(s,l) = \left[\frac{\frac{l^s}{s!}}{\sum_{k=0}^{s} \left(\frac{l^k}{k!} \right)} \right] \tag{1}$$

where

s	number of servers in the queueing system
l	offered load in Erlangs, given by $(l = \lambda\tau)$, where λ is the arrival rate of customers and τ is the average service time.

The Erlang formula is insensitive to the service time distribution. When the servers and the offered load become large, it becomes difficult to calculate the blocking probability. Therefore, recursive techniques are used to numerically calculate the blocking probability.

3.5.1 Queueing Models of Call Centers

According to Stolletz (2003), queueing models of an inbound call center can be described using customer profile, agent characteristics, routing policies, and limitation of waiting room. Customer profile describes the customer arrival process to the call center and the patience/impatience characteristics of customers of a particular class. The agent characteristics describe the agent skill set and the service time distribution of the agent. The routing policy defines which agent needs to serve which customer. These policies may depend on the number of busy agents and the number of waiting customers of different classes. The size of the waiting rooms defines the maximum number of customers in the system and may depend on the customer class.

3.5.1.1 Arrival Process

Customers of a call center cannot see others being served or waiting for service in the queue. Therefore, customers call independently of others and for this simple reason the arrival process in inbound call centers can be modeled as a time-inhomogeneous Poisson process (Koole and Mandelbaum 2001). As the call arrivals vary from time to time, day to day, the common approach is to approximate the time-varying arrival process by a stationary, independent period by period (SIPP) approximation (Green et al. 2001). They also discussed a situation where server staffing requirements are done based on a random cyclic demand and concluded that the SIPP approximation was not accurate for many real situations. Other than SIPP approximations, point wise approximations (PSA), simple stationary approximation (SSA) and infinite-server approximation (IS) can be found in the literature. The usage of these approximations depends on how the arrival rate varies from time to time. But all the approximations average the time-varying arrival rate into an effective arrival rate which remains constant at all times. In each time interval, arrivals occur according to a homogeneous Poisson process and it is assumed that the steady-state arrival rate does not change in each time interval. The standard steady-state approaches can then be effectively used to calculate the various performance measures of a particular queueing model (Koole and Mandelbaum 2001).

3.5.1.2 Waiting Behavior of Customers

In call centers customers can be patient or impatient. Impatient customers are of two types. Balking occurs when the arriving customer finds the server busy and leaves the system immediately without being served. Reneging is when the customer finds the server busy, waits in the queue for certain random time and leaves the system without being served. The process of reneging is described by the random waiting time distribution in the queue. Both balking and reneging may be state -dependent or constant.

Montazer-Haghighi et al. (1986) considered a multi-server queueing system with balking and reneging and obtained the average number of customers in the system under steady-state. They also presented expressions for the average loss of customers during a fixed interval of time. Abou-El-Ata and Hariri (1992) expressed the steady-state distribution for the number in the system in terms of hypergeometric function for an M/M/c/N queue with balking and reneging. Brandt and Brandt (1999) studied customer impatience in a finite-server queueing system where the arrival and service rates could depend on the number of callers in the system. Movaghar (1998) explained customer impatience for a queueing system with state-dependent Poisson arrivals, exponential service times, multiple servers and FCFS service discipline. Brandt and Brandt (2002) extended the system considered in Movaghar (1998) by assuming state-dependent exponential servers. They presented asymptotic results for the number of calls leaving the system and presented a Markovian approximation for the system. Boots and Tijms (1999) presented a simple approximation for the blocking probability in an M/G/c queue with customer impatience. Bae et al. (2001) presented limiting virtual waiting time distribution for an M/G/1 queue with impatient customers.

3.5.1.3 Service Time Distribution of Agents

Traditionally almost all papers in call center literature model service times of agents using the exponential distribution. In a majority of the models published in the call center literature, service times of agents have been typically modeled by the exponential distribution for model tractability. While some studies have shown that the exponential service time distribution is a good fit (e.g., see Koole and Mandelbaum 2001) arrivals are, other studies (e.g., Mandelbaum et al. 2001) have concluded that service time distribution cannot be approximated by the exponential distribution based on empirical data, and that the usefulness of exponential service time distribution may vary from one inbound call center to the other. Harris et al. (1987) analyzed data from a telephone taxpayer information system in which both the talk time and after-call work time (time an agent takes to fill a form or mail order) followed a Weibull distribution. They compared the performance measures obtained from the Weibull distribution to the performance measures obtained from the exponential distribution and observed a high level of insensitivity.

3.5.2 Queueing Models of Inbound E-mail Contact Centers

Very few papers have been published to date in analyzing e-mail contact centers using queueing models. This is due to the recent emergence of contact centers as an extension of traditional call centers. Most of the call center description holds good for the e-mail contact center except for the behavior of customers. This is because it is difficult to characterize customer impatience (balking and reneging). Once the customer sends an e-mail, two things can happen. The e-mail can reach the agents inbox and the agent responds to the e-mail or it can bounce back due to lack of space in agent's inbox. The second condition is of course rare, but there are some chances of its occurrence. This in a way can be thought of as balking. Once the customer's problem is not resolved after many e-mail exchanges with the agent, he/she may renege, i.e., leave the system permanently. Whitt (2002a) explained the many challenging research issues in the area of customer contact center modeling and suggested research directions related to skill-based routing, resource pooling and agent staffing.

Armony and Maglaras (2004a, 2004b) focused on a customer contact (call) center that offers two modes of service: real-time telephone service and call-back service. In Armony and Maglaras (2004b) arriving customers are informed about the delay, and the contact center is modeled as a two-class M/M/r queueing system with state-dependent arrival rates. Armony and Maglaras (2004a) proposed an estimation scheme for the anticipated delay time based on the heavy traffic regime, approximated the system performance, and presented a staffing rule that picks the minimum number of agents.

When modeling e-mail contact centers, we assume that e-mails that represent spam have been filtered and only useful e-mails arrive at the contact center. For more information about non-spam, work-related to filtering of e-mails, the reader is referred to Sharda et al. (1999). Many companies use an e-mail filtering language that can be supported in an e-mail client and client software. Greve et al. (2004) focus on e-mail response management problems in customer contact centers, where they specifically address the problem of processing e-mails in a timely manner. They use simulation to evaluate different routing policy and e-mail processing strategies that can be employed by a contact center.

In summary, work on analytical performance modeling of customer contact centers is very limited. As explained in Chapters 1 and 2, the asynchronous nature of e-mail introduces new possibilities in operating customer contact centers and consequently, new modeling challenges. This thesis effort focused on one such challenge related on modeling the dependence of routing and processing on e-mail history.

CHAPTER 4

RESEARCH STATEMENT

This chapter presents the specific objectives, scope, limitations, and contributions of the research conducted as part of this thesis effort. The overall goals of this research were (i) to develop queueing network models of inbound e-mail customer contact operations that are capable of capturing the dependence of routing and processing on e-mail history, and (ii) to support the development of rapid what-if analysis tools that can assist the decision-maker in designing and improving customer contact center operations.

4.1 Research Objectives

The specific objectives of this research were as follows.

Objective 1: To perform a thorough investigation of the literature related to the modeling of customer call and contact centers.

Objective 2: To develop queueing network models of inbound e-mail contact centers with the following characteristics.

- Multiple types of e-mail inquiries.
- Heterogeneous agents with random service interruptions: The processing of e-mails can be interrupted when agents have to handle other knowledge work or decide to take a break.
- Grouping of agents: Agents with similar skills are grouped to form an agent pool.

- Routing and service times are dependent on history of e-mail (new or previously processed).

Objective 3: To suggest extensions to the queueing network model to approximately handle daily and/or weekly schedules of agents.

4.2 Research Scope and Limitations

The scope of this thesis was limited by the following assumptions.

1. E-mails arrive continuously and the contact center agents are available 24/7. The reference to a particular agent is only with respect to a rule that requires a specific-skill set with memory augmented by customized CRM tools.
2. E-mails are selected from an in-box by an agent according to the FIFO (First in First out) service discipline.
3. Priorities of e-mails are not modeled.
4. Modeling of e-mail history is limited to capturing the identity of the previous agent in the case of a previously processed e-mail.
5. Modeling of agent schedules is limited to suggestions of potential extensions to the network models developed.

4.3 Research Contributions

The purpose of this thesis was to contribute towards the development of queueing network models of inbound e-mail customer contact center operations. The following contributions have been made by this thesis effort.

1. Development of a novel modeling and solution approach to handle routing and processing schemes that are dependent on e-mail history. The approach developed is very general and extends the power of existing queueing network models.

2. Development of queueing models that can be incorporated into rapid analysis tools that can support the analysis and design of customer contact center operations.

CHAPTER 5

MODELING METHODOLOGY

This chapter explains the methodology that was used to develop and solve the queueing network models in this thesis. It also includes a list of important contact performance measures that were addressed in this thesis effort.

The inbound e-mail customer contact center was modeled as a multi-class, open queueing network. The parametric decomposition (PD) method and its extensions presented in Whitt (1983, 1994) were used to solve the multi-class, open queueing network model. While the extensions presented in Whitt (1983, 1994) can handle multiple customer classes, the dependence of the processing times and routing probabilities on e-mail history is not addressed by the PD method and its extensions. Modeling this dependence was a key contribution of this thesis, and details are discussed in the next chapter. The PD method is briefly explained next.

5.1 The Parametric Decomposition (PD) Method

From the late 1950s to the mid 1980s, the analysis of queueing networks was dominated by the well-known product-form method (Baskett et al. 1975; Jackson 1957). The main problem with the product-form analysis method and its extensions was the assumption of Poisson arrivals and exponential service times. There was no convenient mechanism for modeling the variability present in real-world processes. A fundamental change occurred in the mid eighties. There was a paradigm shift from "exact analysis of an approximate model" to "approximate analysis of a more exact model (Whitt 1983)." The basic reason behind the distributional assumptions of the product-form analysis was the tractability of the mathematical model. In the product form analysis, the focus was more on developing a model that can be solved exactly. The method made popular by the Queueing Network Analyzer (QNA) software developed at Bell Laboratories focused on the development of a more realistic model at the cost of our ability to solve the model exactly (Whitt 1983). An analysis method known as the parametric decomposition (PD) method based on two-moment queueing approximations (using mean and SCV - Squared Coefficient of Variation = Variance/mean^2) became popular. The PD method was first proposed by Reiser and Kobayashi (1974) and subsequently extended by Kuehn (1979), Whitt (1983, 1994) and many others (see for example references in Suri et al. 1993). The PD method is the basis of many of the recent tools and techniques developed for queueing network analysis (Suri et al. 1993).

The main reason for the success of the PD method is that it does not make any distributional assumption and uses only the mean and variance information of processing times and interarrival times. Through extensions to the PD method, several features relevant to real world systems have been incorporated including multi-class networks with deterministic routing, equipment breakdown and repair, changing lot sizes, inspection and testing, batch service, and overtime (Kamath et al. 1995, Suri et al. 1993).

The PD method for a single-class, open queueing network is based on 1) analysis of interactions between the nodes to obtain the mean and SCV of the interarrival time at each node, and 2) decomposition of the network into individual nodes and calculation of node measures and network performance measures using GI/G/1 (general arrival, general service time distribution with single server) or (Whitt 1983) GI/G/m (general arrival, general service time distribution with multiple servers) approximations (Whitt 1993).

The rate and the variability parameters of the combined (external plus internal) arrival processes approximately capture interactions among the nodes. The total arrival rate at each node can be obtained by solving the traffic flow rate equations, which represent the conservation of flow. Utilizations are calculated at each node to check system stability. The system is stable if the utilization at each node is strictly less than one. Up to this point, the analysis is similar to the one carried out in the product form analysis method (Jackson 1957) for solving open networks and involves no approximations.

The SCVs of interarrival times at each node are calculated by solving the traffic variability equations which are linear. The traffic variability equations involve approximations for the basic network operations like a) flow through a node, b) merging of flow and c) splitting of flow. These approximations can be found in Whitt (1983, 1994).In calculating the performance measures, all nodes are assumed to be stochastically independent. The performance measures at each node are calculated from the GI/G/1 or GI/G/m results given in (Whitt 1983, 1993).

5.2 Aggregation Approaches

The PD method described in the previous section essentially solves a single-class network with Markovian routing probabilities. As explained in Whitt (1983, 1994), the general approach to solving multi-class networks is to aggregate the multi-class information (service and arrival) to define an aggregate single-class network; solve the single-class network using the PD method; and disaggregate to calculate class-specific performance measures. Whitt (1983, 1994) presented extensions to the PD method to handle several classes of customers. Each customer class is described by a deterministic route. Whitt's extension (1983, 1994) not only allows different service time parameters for different classes at a node, but also different service time parameters for different visits to the same node by the same class. Whit (1983) also mentioned that the routing could be probabilistic in the multi-class case. If so, then a routing probability matrix and parameters for the external arrival processes and node service times must be specified for each customer class.

Whitt's (1983) class-based aggregation method was modified to handle probabilistic routing in the case of the contact center model. This thesis effort has added a new extension to the PD method to treat an open queueing network in which routing probabilities and processing times depend on e-mail history. This extension involves a new history-based aggregation step within each customer class before the class-based aggregation step. Details of this extension are presented in Chapter 6.

5.3 Numerical Validation

The performance measures computed using the queueing network models were compared with steady-state simulation results obtained using an Arena 7.0 simulation model to evaluate the accuracy of the analytical results. The analytical results for different scenarios and different levels of service time SCVs were compare with the corresponding simulation estimates. Relative percentage error was used as an indication of the accuracy of the analytical model.

$$\text{Relative percentage error} = \frac{(\text{analytical result} - \text{simulation estimate})}{\text{simulation estimate}} \cdot 100\%$$

The performance measures that were of interest include the average response time, average resolution time, agent utilizations, and the average number of e-mails in the system. The average resolution time is the average time an e-mail spends in the system (customer and contact center) before eventual "resolution".

The parameters used for all the simulation experiments are presented in Table 5.1. The warm-up period was determined by the application of Welch's procedure (Welch 1983). Further details regarding the application of Welch's procedure are contained in Appendix A3.

Table 5.1: Simulation Parameters

Number of Replications	Warm-up period (hours)	Replication length (hours)
10	**1,680**	**18,480**

Table 5.2 presents the different levels of service time variability that were tested in all the scenarios. The details about the specific distributions used, and the procedure to calculate their parameters for the simulation model are given in Appendix A2.

Table 5.2: SCV Levels and Corresponding Distributions

SCV	Distribution
0.25	**4-stage Erlang**
1	**Exponential**
2.00	**Hyperexponential**

CHAPTER 6

A MULTI-CLASS OPEN QUEUEING NETWORK MODEL OF E-MAIL CUSTOMER CONTACT CENTERS

A multi-class open queueing network model was developed to model e-mail contact centers. The nodes of the network represent customer service agents with the exception of one special delay node that models the elapsed time at the customer end. The routing probabilities as well as the processing time parameters could depend on e-mail history. A novel history-based aggregation approach to model the dependence on e-mail history was developed. The aggregation approach extends the popular parametric decomposition (PD) method for solving multi-class open queueing networks to more general situations. A discrete-time Markov chain (DTMC) with an expanded state space was developed to model the non-Markovian routing of a new e-mail through the contact center until its eventual resolution. The analysis of this absorbing Markov chain allows the computation of the proportion of e-mails in an agent's in-box that are new, previously processed by the same agent, or previously processed by another agent. Using these proportions, a new "history-based" aggregation step for each customer class was introduced. This step precedes the existing class-based aggregation step that extends the original PD method. The resulting queueing network model was solved using the RAQS software package that implements the PD method and its extensions. The accuracy and robustness of the analytical model was demonstrated by comparing the analytical results with simulation estimates of performance measures for a variety of scenarios.

The contact center considered was similar to the example that was the subject of an extensive simulation study in Greve et al. (2004). The model developed in this chapter does not consider the pooling of agents or service interruptions. These issues are addressed in Chapter 7. The remainder of this chapter is organized as follows. Section 6.1 gives a description of the contact center that was modeled. Section 6.2 describes some additional assumptions. Section 6.3 describes the modeling of the e-mail contact center using a multi-class open queueing network. Section 6.4 presents approximate approaches to compute the weights for history-based aggregation. Section 6.5 presents the discrete-time Markov chain model of the history-based e-mail routing. The numerical experiments are presented in Section 6.6 and Section 6.7 presents a summary of the results and discussions.

6.1 Contact Center Description

A contact center with multiple types of arriving e-mails is considered. Within each type, the e-mails received are identified, by software, as new or previously processed. If e-mails are new, they are routed with equal probability to one of the agents. If an e-mail has been previously processed, then the agent who previously processed the e-mail can be identified, and the e-mail is routed to that agent. Alternatively, the e-mail could also be routed to one of the agents randomly, just like a new e-mail. Once the e-mail is routed to an agent, the agent preprocesses it. Preprocessing involves reviewing the e-mail type and its history. The history of the e-mail can be any one of the following; the e-mail can be brand new, processed by the same agent, or processed by a different agent. From this information the agent determines whether to process the e-mail or to forward the e-mail to another agent. The time required to process an e-mail is random and is influenced by both the e-mail type and history. When the e-mail response provided by an agent is sufficient to resolve the customer's problem the e-mail leaves the system permanently. If the e-mail response is not enough to address the customer's concerns, e-mail returns to the system (as another e-mail, e.g., a reply) after a random delay.

This random delay represents the time for the customer to receive the agent's response and send a reply. Without any loss of generality, we could assume for modeling purposes that "resolution" includes both the actual resolution of the customer's problem and the customer's decision to not pursue the problem resolution any further. The flowchart depicting the email handling logic is shown in Figure 6.1.

6.2 Assumptions

- An individual agent processes e-mails in his/her in-box according to a first in first out (FIFO) queueing discipline. The FIFO discipline holds only for the e-mails in an agent's inbox and not for the system.
- For an unresolved problem that will be pursued by the customer, the e-mail (response) enters the system after a random delay independent of the e-mail processing history.
- The pre-processing and processing times are independent random variables.

6.3 Modeling the E-Mail Contact Center Using an Open Queueing Network

The situation explained in Section 6.1 was modeled as an open queueing network where the nodes represent the agents and customers represent e-mails. The open network has

$(A+1)$ nodes where nodes 1 through A represent the customer service agents and node $(A+1)$ represents a delay node. The delay node was used to model the time for the customer to receive a response and send a follow-up e-mail.

The different types of e-mails were modeled using different customer classes, each having their own arrival, routing, and service characteristics. For a given e-mail type, the routing probabilities and the service time distributions depend on the history of the e-mail. This dependence on e-mail history is a feature that cannot be handled with the current extensions to the PD method. Hence, the basic idea behind the solution approach is as follows. If the history and class-based parameters were somehow aggregated into only class-based parameters, then Whitt's (1983, 1994) method could be used to solve the multi-class open queueing network. The input for the PD method includes the number of nodes, the number of servers (one in this chapter) at each node, the SCVs of the external interarrival and service time distributions at each node, and the Markovian routing probability matrix.

To perform the history-based aggregation within a customer-class, the following probability at each node or agent was needed. It is the probability that an e-mail of type c currently at agent i was previously processed by agent k ($k = 0$ represents a new e-mail). These probabilities can also be thought of as "weights" to be used within the aggregation approach. A new technique was developed for computing these probabilities and it is presented in Section 6.5. In the remainder of this section the aggregation of the detailed parameters of the contact center model to yield the single-class arrival, service, and routing parameters for solution using the PD method is discussed. The overall aggregation process is a two-level technique, where the first level takes care of the dependence on history within a class and the second level deals with the aggregation of class-specific information.

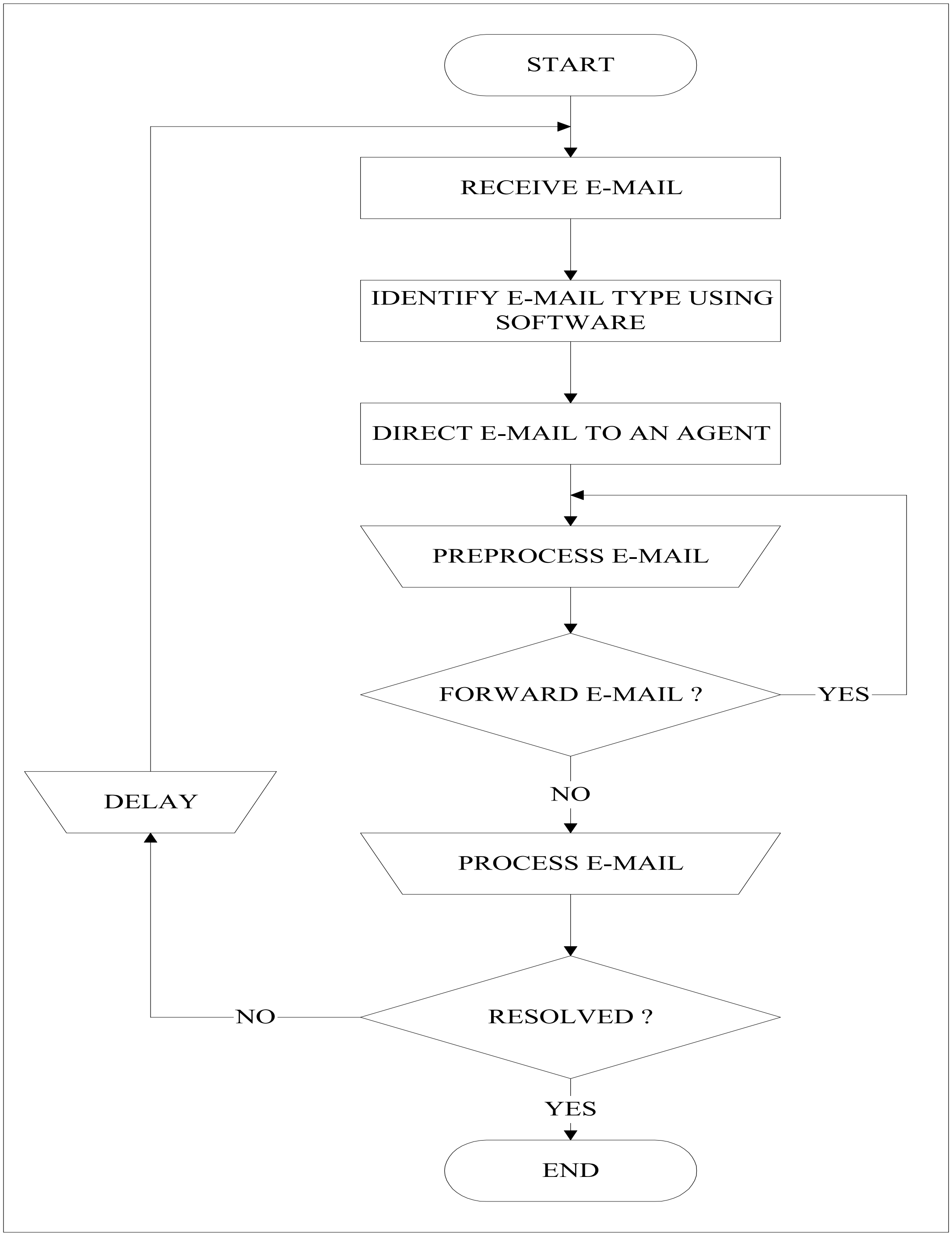

Figure 6.1: E-Mail Handling Logic

A simple aggregation scheme was first developed to convert the class and node-specific detailed routing probabilities and service time information into an approximately equivalent single-class, node-specific service time parameters and Markovian routing probabilities. The PD approach was then used to analyze the model.

6.3.1 Rate and SCV of the External Arrival Process at a Node

First, we need to compute the rate and the SCV for the external arrival process at each node that represents an agent in the single-class open queueing network model. The external arrivals correspond to the arrival of new e-mails. The delay node or node $(A + 1)$ has no external arrivals because all previously processed e-mails (or customer replies) are considered to be internal arrivals as far as the open queueing network model is concerned. If new external e-mails of type c are routed to agent i with a fixed probability distribution, say, $\{\alpha_i(c),\, i = 1, 2, ..., A\}$, then the class-specific and total external arrival rates at node i are given by

$$\lambda_i(c) = \alpha_i(c) \cdot \lambda(c)\,; \quad \lambda_i = \sum_{c=1}^{C} \lambda_i(c) \qquad i = 1, 2, ..., A \qquad (2)$$

The SCV calculations for the interarrival time for new, external e-mails at node i follow two steps. When a new e-mail of type c enters the system, it is split according to the fixed probability distribution $\alpha_i(c)$. First, we obtain the SCV of the split arrival process of new type c e-mails at node i by assuming that the external arrival process is a renewal process.

$$c_{oi}^2(c) = \alpha_i(c) \cdot c_o^2(c) + 1 - \alpha_i(c) \qquad i = 1, 2, ..., A \qquad (3)$$

At node i, the split arrival processes of new external e-mails of different types get merged. For the second step, we use results from Whitt (1983) to obtain the SCV of the external arrivals at node i

$c_{oi}^2 = \psi_i \cdot c_{hi}^2 + 1 - \psi_i$ where

$$c_{hi}^2 = \frac{\sum_{c=1}^{C} c_{oi}^2(c) \cdot \lambda_i(c)}{\sum_{c=1}^{C} \lambda_i(c)} \quad i = 1, 2, ..., A; \tag{3}$$

$\psi_i = [1 + 4(1-\rho_i)^2 \cdot (\chi_i - 1)]^{-1}$; $\chi_i = \left[\left(\frac{\lambda_i(c)}{\sum_{c=1}^{C} \lambda_i(c)} \right)^2 \right]^{-1}$; and ρ_i is the utilization of node i. It should be noted that the calculation of the SCV, c_{oi}^2, can be performed after ρ_i is available from the solution of the traffic equations for the aggregate single-class network.

6.3.2 Weights for Class-based Aggregation

Whitt (1983) presented an approach to derive the parameters for an aggregate single-class network in the case of a multi-class network with deterministic routes. We extend Whitt's (1983) approach to a multi-class network with probabilistic routing. To compute the probabilities or weights for class-based aggregation, we need, $\gamma_j(c)$, the total (internal plus external) arrival rate of type c e-mails at node j. This can be obtained by solving traffic equations for a particular class. $\gamma_j(c)$ can be obtained by solving the following system of linear equations.

$$\gamma_j(c) = \lambda_j(c) + \sum_{\substack{i=1 \\ i \neq j}}^{A+1} \gamma_i(c) \cdot p_{i,j}(c) \qquad j = 1, 2, ..., A+1 \tag{4}$$

For $i, j = 1, 2, ..., A;\ i \neq j,$ the routing probability $p_{i,j}(c)$ from agent i to agent j for a type c e-mail is given by $p_{i,j}(c) = \sum_{k=0}^{A} w_i(c,k) \cdot p_{i,j}(c,k).$ For each class, the routing probability from node i to the delay node $(A+1)$ is obtained by aggregating the product of the probability that agent i processes the e-mail and the probability that the problem is not resolved.

$$p_{i,A+1}(c) = \sum_{k=0}^{A} w_i(c,k) \cdot p_i(c,k) \cdot (1 - q_i(c,k)) \qquad i = 1, 2, ..., A \tag{5}$$

The routing probability $p_{A+1,i}(c)$ depends on the routing strategy for previously processed e-mails. If previously processed e-mails (i.e., customer responses to agents' e-mails) are routed to agent i with a fixed probability distribution $\{p_i\, ,\ i = 1, 2, ..., A\}$, then we have $p_{A+1,i}(c) = p_i$. A special case of this strategy is to set all $p_i's$ to be the same. In this case, we have $p_i = 1/A$. If previously processed e-mails are routed to the agent that processed them, then the routing probability for a type c e-mail from the delay node to agent i is equal to the probability that a type c e-mail at the delay node came from agent i, i.e., $p_{A+1,i}(c) = w_{A+1}(c,i)$.

Finally, we can compute the probabilities needed for the class-based aggregation as follows $r_i(c) = \dfrac{\gamma_i(c)}{\sum_{c=1}^{C} \gamma_i(c)} \qquad i = 1, 2, ..., A+1 \qquad (6)$

6.3.3 Markovian Routing Probabilities

We calculate the Markovian routing probabilities for the single-class open network by aggregating the class-specific probabilities computed in the previous section. The routing probabilities among the agent nodes are given by

$$p_{i,j} = \sum_{c=1}^{C} r_i(c) \cdot p_{i,j}(c) \qquad i, j = 1, 2, ..., A;\ i \neq j \tag{7}$$

Similarly, the class-independent routing probability from agent node i to the delay node $(A+1)$ is

$$p_{i,A+1} = \sum_{c=1}^{C} r_i(c) \cdot p_{i,A+1}(c) \qquad i = 1, 2, ..., A \qquad (8)$$

And the class-independent routing probability from the delay node $(A+1)$ to agent node i is

$$p_{A+1,i} = \sum_{c=1}^{C} r_i(c) \cdot p_{A+1,i}(c) \qquad i = 1, 2, ..., A \qquad (9)$$

6.3.4 Mean Service Time at a Node

The overall service time is equal to the preprocessing time plus the actual processing time needed by the agent if the agent decides to process the e-mail himself/herself. The overall service time at agent i for a new or previously processed type c e-mail is given by

$$T_i(c,k) = P_i(c) + Z_i(c,k) \cdot S_i(c,k) \qquad i = 1, 2, ..., A;\ k = 0,1,...,A \qquad (10)$$

The random variables $P_i(c)$, $Z_i(c,k)$ and $S_i(c,k)$ are assumed to be mutually independent. That is, the pre-processing time, the agent's decision to process or forward the e-mail, and the e-mail processing time are all independent random variables.

The mean service time at agent i for a new or previously processed type c e-mail is given by

$$E[T_i(c,k)] = E[P_i(c)] + E[Z_i(c,k)] \cdot E[S_i(c,k)] \qquad i = 1, 2, ..., A;\ k = 0,1,...,A \qquad (11)$$

Using the notation defined earlier we get

$$t_i(c,k) = \theta_i(c) + p_i(c,k) \cdot s_i(c,k) \qquad i = 1, 2, ..., A;\ k = 0,1,...,A \qquad (12)$$

Once again, the mean service time is calculated by using the two-level aggregation method. The mean service time for a type c e-mail at node/agent i is given by

$$t_i(c) = \sum_{k=0}^{A} w_i(c,k) \cdot t_i(c,k) \qquad i = 1, 2, ..., A \qquad (13)$$

Next, the class-specific mean service times are aggregated to obtain the mean (overall) service time at node/agent i.

$$t_i = \sum_{c=1}^{C} r_i(c) \cdot t_i(c) \qquad i = 1, 2, ..., A \qquad (14)$$

6.3.5 Squared Coefficient of Variation of the Service Time Distribution at a Node

For each class, the variance of the overall service time is first computed by using the fact that the effective processing time distribution is a mixture of processing time distributions for new and previously processed e-mails. The SCV of the overall service time is obtained by once again using the fact that it is a mixture of class-specific distributions.

By using the property that the "second moment of a mixture of distributions is the mixture of the second moments (Whitt 1983)" the first step is to obtain the service time SCV at node/agent i for a type c e-mail.

$$t_i^2(c) \cdot (c_i^2(c) + 1) = \sum_{k=0}^{A} w_i(c,k) \cdot t_i^2(c,k) \cdot (c_i^2(c,k) + 1) \quad i = 1, 2, ..., A \qquad (15)$$

where, $c_i^2(c,k)$ is obtained as follows

$$T_i(c,k) = P_i(c) + Z_i(c,k) \cdot S_i(c,k) \qquad i = 1, 2, ..., A$$

$$Var(T_i(c,k)) = Var(P_i(c) + Z_i(c,k) \cdot S_i(c,k))$$

$$Var(T_i(c,k)) = Var(P_i(c)) + Var(Z_i(c,k)) \cdot Var(S_i(c,k))$$

Using the independence assumptions stated earlier, we have

$$
\begin{aligned}
Var(T_i(c,k)) \;&= Var(P_i(c)) + Var(Z_i(c,k))\cdot Var(S_i(c,k)) + \\
&\quad E^2[Z_i(c,k)]\cdot Var(S_i(c,k)) + E^2[S_i(c,k)]\cdot Var(Z_i(c,k)) \\
&= c^2_{pre,i}(c)\cdot \theta_i^2(c) + p_i(c,k)\cdot[1-p_i(c,k)]\cdot c^2_{ser,i}(c,k)\cdot s_i^2(c,k) + \\
&\quad p^2{}_i(c,k)\cdot c^2_{ser,i}(c,k)\cdot s_i^2(c,k) + s_i^2(c,k)\cdot p_i(c,k)\cdot[1-p_i(c,k)] \\
&= c^2_{pre,i}(c)\cdot \theta_i^2(c) + p_i(c,k)\cdot s_i^2(c,k)\cdot\{[1-p_i(c,k)]\cdot c^2_{ser,i}(c,k) + \\
&\quad p_i(c,k)\cdot c^2_{ser,i}(c,k) + [1-p_i(c,k)]\} \\
&= c^2_{pre,\,i}(c)\cdot \theta_i^2(c) \,+\, p_i(c,k)\cdot s_i^2(c,k)\cdot\{c^2_{ser,\,i}(c,k) \,+\, [1-p_i(c,k)]\} \qquad (16)
\end{aligned}
$$

Using the relation, $Var(T_i(c,k)) = c_i^2(c,k)\cdot t_i^2(c,k)$ have

$$
c_i^2(c,k) = \frac{c^2_{pre,i}(c)\cdot \theta_i^2(c) + p_i(c,k)\cdot s_i^2(c,k)\cdot\{c^2_{ser,i}(c,k) + [1-p_i(c,k)]\}}{t_i^2(c,k)} \qquad (17)
$$

By substituting $c_i^2(c,k)$ in equation [15]

$$
\begin{aligned}
t_i^2(c)\cdot(c_i^2(c)+1) \;\;&=\;\; \sum_{k=0}^{A} w_i(c,k)\cdot\{c^2_{pre,i}(c)\cdot\theta_i^2(c) + p_i(c,k)\cdot s_i^2(c,k).(c^2_{ser,i}(c,k) \\
&\quad + [1-p_i(c,k)]\,) + t_i^2(c,k)\}
\end{aligned}
$$

The second step is to aggregate the class-specific service time SCV, $c_i^2(c)$ to obtain the overall service time SCV, c_i^2 at node/agent i is given by

$$
t_i^2\cdot(c_i^2+1) = \frac{\sum_{c=1}^{C}\lambda_c\cdot t_i^2(c)\cdot(c_i^2(c)+1)}{\sum_{c=1}^{C}\lambda_c} \qquad i = 1, 2, ..., A \qquad (18)
$$

6.4 Approaches to Compute the Weights for History-Based Aggregation

To demonstrate the importance of accurately modeling the dependence on the e-mail history, two additional approximate methods to compute the weights $w_i(c,k)$ are presented here and included in the numerical experimentation. M1 is a naïve method, which involves nothing but a simple average. In method M1, $w_i(c,k)$ is equal to $\left(\frac{1}{A+1}\right)$ for all i, k and c, i.e., the probability that a type c e-mail is new or previously processed by agent k remains the same irrespective of the e-mail processing history. Similarly at the delay node $(A+1)$, the probability that a type c e-mail was previously processed by agent k, i.e., $w_{A+1}(c,k)$ is equal to $\left(\frac{1}{A}\right)$. This simple method was used in Chinnaswamy et al. (2004). M2 is a method with medium complexity, where $w_i(c,k)$ are calculated using aggregate resolution and forwarding probabilities. In method M2, the processing history of e-mails is taken into account in an approximate way for calculating $w_i(c,k)$. As in method M1, the weights $w_i(c,k)$ are obtained for a type c new e-mail and previously processed e-mails. Because of aggregation, the probability that e-mail was previously processed by agent k is the same for $k=1,2,...,A.$ Again because of aggregation, the probability that a type c e-mail was previously processed by agent k, i.e., $w_{A+1}(c,k)$ is equal to $\left(\frac{1}{A}\right)$. M3 is the DTMC-based approach and it is presented in Section 6.5. In method M3, the complete history of an e-mail is indirectly captured from its entry till its resolution. This information is used in calculating the weights $w_i(c,k)$.

6.4.1 Aggregation Method M2

In this section, the M2 method is explained for the two scenarios that will be considered later in numerical experimentation. Scenario 1 is the case when a previously processed e-mail is equally likely to be routed to one of the agents. Scenario 2 is the case when a previously processed e-mail is routed to the agent that processed it. Let $p(c)$ represent the aggregate forwarding probability and $q(c)$ the aggregate resolution probability for class c. $p(c)$ and $q(c)$ are computed as follows.

$$p(c) = \begin{cases} \dfrac{\sum_{k=0}^{A}\sum_{i=1}^{A}\sum_{\substack{j=1\\ j\neq i}}^{A} p_{i,j}(c,k)}{A^3 - A} & \text{for Scenario 1} \quad (23) \\[2ex] \dfrac{\sum_{i=1}^{A}\sum_{\substack{j=1\\ j\neq i}}^{A} p_{i,j}(c,0)}{A^2 - A} & \text{for Scenario 2} \quad (24) \end{cases}$$

$$q(c) = \begin{cases} \dfrac{\sum_{k=0}^{A}\sum_{i=1}^{A} q_i(c,k)}{A^2 + A} & \text{for Scenario 1} \quad (25) \\[2ex] \dfrac{\sum_{i=1}^{A} q_i(c,i)}{A} & \text{for Scenario 2} \quad (26) \end{cases}$$

Figure 6.2 illustrates the flow of new and previously-processed e-mails through the contact center from an aggregate point of view.

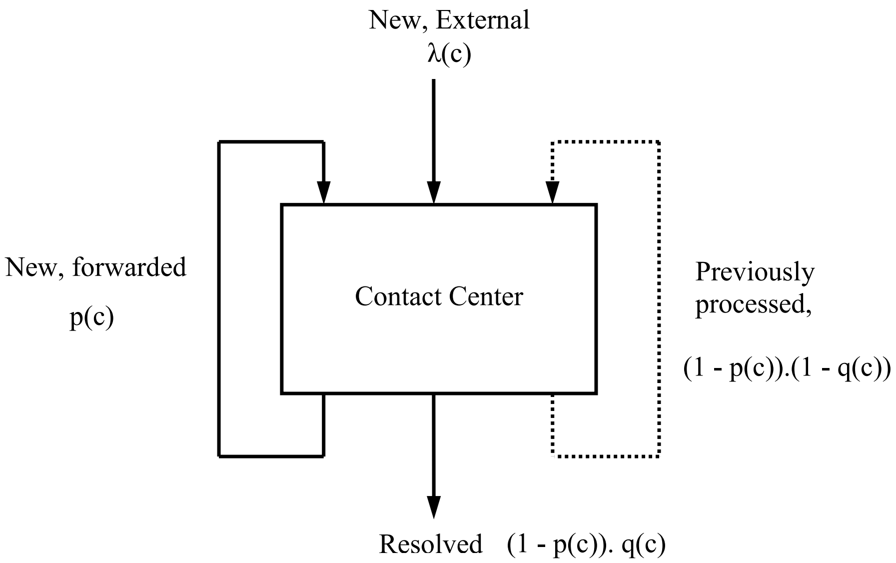

Figure 6.2: Aggregation Method M2

Let $\beta(c)$ represent the total (new plus previously processed) e-mail arrival rate. From Figure 6.2 we can see that the following flow conversation relation needs to hold.

$$\beta(c) = \lambda(c) + \beta(c) \cdot \{p(c) + (1 - p(c)) \cdot (1 - q(c))\} \tag{27}$$

This yields

$$\beta(c) = \frac{\lambda(c)}{(1 - p(c)) \cdot q(c)} \tag{28}$$

By focusing on the solid lines in Figure 6.2, we can see that

Total arrival rate of new e-mails = $\dfrac{\lambda(c)}{(1 - p(c))}$ (29)

Total arrival rate of previously processed e-mails = $\dfrac{\lambda(c) \cdot (1 - q(c))}{(1 - p(c)) \cdot q(c)}$ (30)

Hence, from (27) and (28), the proportion of new e-mails = $q(c)$ (31)

And the proportion of previously processed e-mails = $(1 - q(c))$ (32)

The weights $w_i(c,k)$ are given by

$$w_i(c,0) \quad = \quad q(c) \qquad\qquad i = 1, 2, \ldots, A \tag{33}$$

For Scenario 1, we have

$$w_i(c,k) \quad = \quad \frac{(1 - q(c))}{A} \qquad\qquad i, k = 1, 2, \ldots, A; \tag{34}$$

$$w_{A+1}(c,k) = \quad \frac{1}{A} \qquad\qquad k = 1, 2, \ldots, A \tag{35}$$

For Scenario 2, we have

$$w_i(c,k) \quad = \quad \begin{cases} 1 - q(c) & i = k = 1, 2, \ldots, A \\ 0 & i, k = 1, 2, \ldots, A; i \neq k \end{cases} \tag{36}$$

(37)

$$w_{A+1}(c,k) = \frac{1}{A} \qquad k = 1, 2, ..., A$$

6.5 Discrete-Time Markov Chain Model of History-based E-mail Routing

One of the critical steps in the successful application of the solution approach is the history-based aggregation of routing and service-time parameters within a customer class or e-mail type. As explained in Section 6.3, the key element in this aggregation step is $w_i(c,k)$, the probability that a type c e-mail currently at agent i was previously processed by agent k ($k = 0$ represents a new e-mail). This section presents the method M3, where the probability distribution $\{w_i(c,k),\ k = 0, 1, ..., A\}$ is computed by analyzing a discrete-time Markov chain (DTMC) model of the routing of an e-mail during its entire life-cycle, i.e., from a new e-mail to a series of agent responses and customer replies to eventual "resolution." It is important to note that the dependence on history is limited to only the identity of the previous agent visited in the case of a previously processed e-mail. The remainder of this section describes the definition and solution of this DTMC model.

The state-space of the DTMC can be divided into internal and boundary states. The internal states can be described by the 3-tuple $(k,\ i,\ j)$ where

k describes the e-mail history; $k = 0$ represents a new e-mail and $1 \le k \le A$ represents an e-mail previously processed by agent k.

i denotes the current location of the e-mail, i.e., with agent i; $i = 1, 2, ..., A$.

j denotes the current agent's decision regarding the processing of the e-mail; if agent i decides to process the e-mail then $j = i$ else agent i forwards the e-mail to agent j $(j \neq i)$; $j = 1, 2, ..., A$.

The boundary states are represented by the set $\{N, D_1, D_2, ..., D_A, R\}$, where

N represents the state in which all new e-mails begin their journey.

R represents the state in which all e-mails end their journey.

$D_1, D_2, ..., D_A$ represents the state in which customer receives an agent's response and sends a reply.

The entire state-space of the DTMC is

$\{N\} \cup \{(k, i, j); k = 0, 1, 2, ..., A, i = 1, 2, ..., A, j = 1, 2, ..., A,\} \cup \{D_i; i = 1, 2, ..., A\} \cup \{R\}$.

The size of the state space is equal to $(A+1) \cdot A \cdot A + A + 2$ or $A^3 + A^2 + A + 2$.

A DTMC with $(A^3 + A^2 + A + 2)$ states for each customer class or e-mail type was constructed and the possible state transitions were defined in order to construct the one-step transition probability matrix for the DTMC.

First, transitions within internal states are considered. Such transitions occur only as a result of one agent forwarding an e-mail to another. The recipient can forward it again or decide to process it.

$\forall (k, i, j)$ where $j \neq i$

$$P_{(k,i,j),(k,i^*,j^*)}(c) = \begin{cases} p_{j,j^*}(c,k) & i^* = j;\ j^* = 1,2,...,A;\ j^* \neq j \\ p_j(c,k) & i^* = j;\ j^* = j \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

Next, transitions from the internal states to the boundary states are considered. These occur when an agent decides to process an e-mail. If the e-mail is resolved, then the transition is to the resolved state R; otherwise it is to the delay node D_i.

$\forall (k, i, j)$ where $j = i$

$$P_{(k,i,i),R}(c) \quad = \quad q_i(c,k) \tag{39}$$

$$P_{(k,i,i),D_i}(c) \quad = \quad 1 - q_i(c,k) \tag{40}$$

$$P_{(k,i,i),(k,i,j^*)}(c) \quad = \quad 0\,;\; j^* = 1, 2, \ldots, A \tag{41}$$

Next, the transitions from the boundary states to the internal states are considered.

(a) From state N to an internal state

$$P_{N,(k,i,j)}(c) \quad = \quad \begin{cases} \alpha_i(c) \cdot p_{i,j}(c,k) & i, j = 1, 2, \ldots, A;\ j \neq i;\ k = 0 \\ \alpha_i(c) \cdot p_i(c,k) & i, j = 1, 2, \ldots, A;\ j = i;\ k = 0 \\ 0 & k \neq 0;\ i, j = 1, 2, \ldots, A \end{cases} \tag{42}$$

(b) From state D_i $(i = 1, 2, \ldots, A)$ to an internal state.

(i) Previously processed e-mails are routed to agent i^* with a fixed probability distribution $\{p_{i^*}, i^* = 1, 2, \ldots, A\}$ independent of processing history.

$$P_{D_i,(k,i^*,j^*)}(c) \quad = \quad \begin{cases} p_{i^*} \cdot p_{i^*,j^*}(c,k) & k = i;\ i^*, j^* = 1, 2, \ldots, A;\ j^* \neq i^* \\ p_{i^*} \cdot p_{i^*}(c,k) & k = i;\ i^*, j^* = 1, 2, \ldots, A;\ j^* = i^* \\ 0 & \text{otherwise} \end{cases} \tag{43}$$

(ii) Previously processed e-mails are routed to the agent that processed them.

$$P_{D_i,(k,i^*,j^*)}(c) \quad = \quad \begin{cases} p_{i^*,j^*}(c,k) & k = i;\ i^* = i;\ j^* = 1, 2, \ldots, A;\ j^* \neq i^* \\ p_{i^*}(c,k) & k = i;\ i^* = i;\ j^* = i \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

Finally the transitions within the boundary states are considered. The only possible transition is $P_{R,R} = 1$. All other transitions probabilities from one boundary state to the same state or any other boundary state is zero.

Using the transition probabilities, a $(n' \times n')$ one-step transition probability matrix **P**, where $n' = A^3 + A^2 + A + 2$, was defined. Because state R is an absorbing state, the DTMC represented by **P** is a reducible chain with all states except state R being transient. Next, the focus is on the analysis of this absorbing Markov chain.

6.5.1 Analysis of the Absorbing Markov Chain

The DTMC defined in the previous section consists of an absorbing state and $(A^3 + A^2 + A + 1)$ transient states. The one-step transition probability matrix, **P**, can be rearranged in such a way that the absorbing state is the first state, without any loss of generality. States are arranged in the following way - resolution node R, followed by A delay nodes numbered from $D_1, D_2, ..., D_A$, followed by the new node N and finally, the internal nodes (k, i, j).

$$\mathbf{P} = \begin{array}{cc} \left[\begin{array}{c:c} \mathbf{U} & \mathbf{0} \\ \hdashline \mathbf{M} & \mathbf{Q} \end{array}\right] & \begin{array}{l} 1 \text{ row} \\ A^3 + A^2 + A + 1 \text{ rows} \end{array} \\ \begin{array}{cc} 1 \text{ column} & \begin{array}{c} A^3 + A^2 + A + 1 \\ \text{columns} \end{array} \end{array} & \end{array}$$

Figure 6.3: One-Step Transition Probability Matrix

By analyzing this absorbing DTMC, the average number of visits an e-mail makes to each node before getting resolved or absorbed can be computed. The steps outlined in Ramakumar (1993) were followed to obtain the fundamental matrix **F** associated with the absorbing Markov chain. The fundamental matrix **F** is given by

$$\mathbf{F} = [\mathbf{I} - \mathbf{Q}]^{-1} \tag{45}$$

The row of the Fundamental matrix, **F** that corresponds to the state N gives the average number of visits an e-mail makes to the all other states before absorption. The average number of visits an e-mail that was last processed by agent k ($k = 0$ is a new e-mail) makes to agent i is given by $v_i(c,k) = \sum_{j=1}^{A} F_{N,(k,i,j)}(c)$

The probability that a type c e-mail received by agent i was previously processed by agent k ($k = 0$ is a new e-mail) is given by

$$w_i(c,k) = \frac{v_i(c,k)}{\sum_{k=0}^{A} v_i(c,k)} \qquad i = 1, 2, ..., A; \; k = 0, 1, ..., A; \tag{46}$$

The above results can be easily derived by observing that the arrival rate of type c e-mails with a processing history k at agent i is proportional to $v_i(c,k)$. Similarly, the probability that a type c e-mail received at the delay node $(A+1)$ was previously processed by agent k is given by

$$w_{A+1}(c,k) = \frac{v_{D_k}(c,k)}{\sum_{j=1}^{A} v_{D_j}(c,k)} \qquad k = 1, 2, ..., A \text{ where } v_{D_j,j}(c) = F_{N,D_j}(c) \tag{47}$$

6.5.2 Performance Evaluation of the E-Mail Contact Center

Figure 6.4 explain the step-by-step procedure followed to obtain the performance measures of an inbound e-mail contact center. It was assumed that the forwarding probabilities, resolution probabilities and service time parameters for a particular type e-mail could be obtained from the available e-mail contact center data. It was assumed that these parameters could be dependent on e-mail history, i.e., the last agent, k who previously processed the e-mail, ($k = 0$ for new e-mails). The external arrival rates and SCVs for different e-mail types, the forwarding and resolution probabilities, number of agents along with the mean service time and service-time SCV for each agent were the inputs for the analytical model. These inputs known as parameters were used for the entire aggregation procedure. For a particular e-mail type the one-step, history-embedded transition probability matrix was constructed based on possible transitions made by an e-mail between the agents' inboxes and the delay nodes representing customers. The construction of the above matrix was based on the number of agents in the contact center and history-embedded resolution and forwarding probabilities of an e-mail. A truncated portion of the one-step transition probability matrix, i.e., **Q** then was inverted using MATLAB 7.0.1 (Lipsman 2001). The truncated matrix was based on the number of absorbing nodes (one in this case). The inverted matrix gives the average number of visits an e-mail makes to a node before absorption or problem "resolution." In particular, the row corresponding to the **new e-mail** node gives the average number of visits a new e-mail makes to agents' inboxes and to the customer inbox (modeled as delay node) before getting resolved. These visits were then converted into probabilities or weights of new and previously processed e-mails as seen by an agent for a known e-mail type. These weights were used in the history-based aggregation procedure to obtain parameters that are based on only the class or e-mail type information. For the class-based aggregation, the external arrival rates and parameters obtained from the history-based aggregation were used. The class-based aggregation was done according to the method outlined in Whitt (1983) in order to obtain the routing matrix, means and SCVs of service times, and interarrival time SCVs at each agent. These were then fed into RAQS software package to obtain the performance measures for the e-mail contact center. Many steps in the overall procedure, including calculation of the one-step transition probability matrix, calculation of weights after solving the DTMC, history-based aggregation, and class-based aggregation were coded in EXCEL spread sheets.

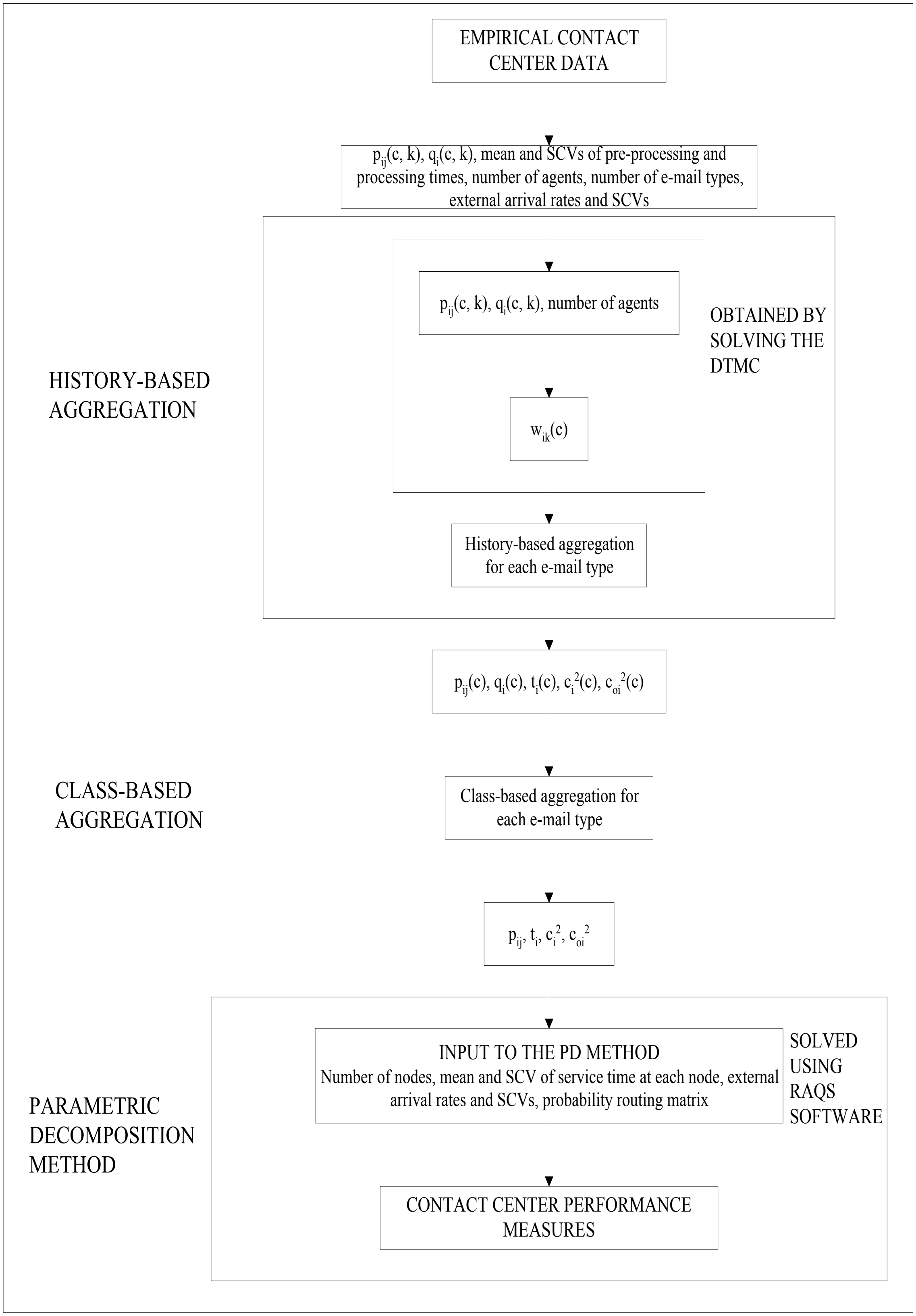

Figure 6.4: Contact Center Performance Evaluation

6.6 Numerical Experiments

One of the important tasks in the numerical evaluation of the accuracy and robustness of the approach is to construct a contact center example that is representative of real-world contact center operations. The parameter selection approach was based on e-mail contact center characteristics that are available in the public domain. Details are described in the next section.

6.6.1 E-mail Contact Center Characteristics

Survey data available at "BenchmarkPortal.com" was used to gain an initial understanding of a typical e-mail response center. Each month, "BenchmarkPortal.com" sends 1,000 surveys to randomly selected customer response centers, and typically receives a 15 to 25 % response rate. In one such survey, 53% of surveyed contact centers indicated that 80-100% of e-mail contacts were resolved with the first response (Benchmark Portal 2004b). From this statistic an estimate for the resolution probability for a typical e-mail agent was obtained. Also, 34.62% of surveyed contact centers indicated that their average e-mail response time was less than 6 hours. Another 15.38% indicated an average response time of 6 to 12 hours, and 35.38% indicated an average response time of 12 to 24 hours (Benchmark Portal 2004c). Also, the capacity of a dedicated e-mail associate is comparable to that of a telephone agent (Benchmark Portal 2004a). Given an eight-hour day, this translates to between 2.4 and 9.6 minutes of an e-mail agent's time being spent on each e-mail response. Based on the above information and the contact center example analyzed in Greve et al. (2004) and Chinnaswamy et al. (2004), the parameter values for a small contact center were defined.

6.6.2 E-mail Contact Center Example

A contact center with three agents that handles two types of e-mails was considered. We have $A = 3$ agents and $C = 2$. Although the analytical approach can handle general arrival processes, Poisson arrivals were assumed for this example. The arrival rates for the two types of e-mails were $\lambda_1 = 6.0/\text{hr}$ and $\lambda_2 = 4.5/\text{hr}$. A new e-mail of either type was assumed to be equally likely to be routed to one of the agents. Two scenarios were defined to model two different strategies with regard to the handling of previously processed e-mails. In Scenario 1, a previously-processed e-mail was routed to any one of the agents with equal probability. In Scenario 2, a previously-processed e-mail was routed to the agent that processed it. Hence, we have

$\alpha_1(c) = \alpha_2(c) = 0.33; \alpha_3(c) = 0.34$ for $c = 1,2$.

The preprocessing time distribution for both e-mail types at all three agents was assumed to be the uniform distribution. Hence, $P_i(c) \sim$ Uniform (0.01, 0.50) for $i = 1,2,3$; $c = 1,2$. With regard to forwarding probabilities, sample values were generated using uniform (0, 0.25) for the two e-mail types. While assigning the sampled values, it was assumed that an agent is more likely to forward an e-mail previously processed by another agent. The complete set of forwarding probabilities is presented in Table 6.2 for Scenario 1 and Table 6.7 for Scenario 2. In the case of resolution probabilities, samples from the following uniform distributions were used.

- Type 1, New - Uniform (0.60, 0.90)
- Type 1, Previously processed - Uniform (0.75, 0.95)
- Type 2, New - Uniform (0.50, 0.80)
- Type 2, Previously processed - Uniform (0.70, 0.90)

While assigning the sampled probabilities, it was assumed that an agent was more likely to resolve an e-mail previously processed by them. The full set of resolution probabilities are summarized in Table 6.3 for Scenario 1 and Table 6.8 for Scenario 2. To test the robustness of the approach developed, experiments with three processing-time distribution were chosen; Erlang (SCV =0.25) to represent low variability situations, exponential (SCV =1.00) to represent medium variability situations, and Hyperexponential (SCV = 2.00) to represent high variability situations. To capture the dependence on the history and class, samples for the mean processing times were drawn from the following uniform distributions.

- Type 1, New - Uniform (0.10, 0.20)
- Type 1, Previously processed - Uniform (0.05, 0.15)
- Type 2, New - Uniform (0.15, 0.25)
- Type 2, Previously processed - Uniform (0.10, 0.20)

While assigning the sampled mean processing times, it was assumed that the mean would be lower for e-mails previously processed by the same agent. The complete set of mean processing times can be viewed in Table 6.1 for Scenario1 and Table 6.6 for Scenario 2. Finally, the delay time was assumed to be exponentially distributed with a mean of 4 hours, i.e., D_i ~ Exponential (4) for $i = 1, 2, 3$. With regard to the routing of previously processed e-mails, two scenarios were considered as described earlier.

6.6.3 Probabilities for History-Based Aggregation obtained from the DTMC

In this section, the probabilities or weights used for history-based aggregation obtained from the fundamental matrix are presented for both Scenarios 1 and 2. Tables 6.4 and 6.5 give the average number of visits and probabilities for Scenario 1 obtained using the analytical method M3 for Type 1 and 2 e-mails, respectively. Tables 6.9 and 6.10 give the average number of visits and probabilities for Scenario 2 obtained using the analytical method M3 for Type 1 and 2 e-mails, respectively. The reader is referred to Appendix A1; Tables A1.1 through A1.18 for complete details related to the fundamental matrix **F**.

Table 6.1: Scenario 1 - Mean Processing Times

$s_{i,k}(c)$

(c,k) i	c = 1				c = 2			
	k=0	k=1	k=2	k=3	k=0	k=1	k=2	k=3
1	0.15	0.11	0.16	0.15	0.18	0.06	0.13	0.10
2	0.19	0.13	0.15	0.14	0.20	0.11	0.07	0.14
3	0.14	0.20	0.18	0.13	0.17	0.08	0.13	0.07

Table 6.2: Scenario 1 - Forwarding Probabilities

$p_{i,j}(c,k)$

(c,k) (i,j)	c = 1				c = 2			
	k=0	k=1	k=2	k=3	k=0	k=1	k=2	k=3
(1,2)	0.01	0.02	0.08	0.19	0.06	0.03	0.03	0.12
(1,3)	0.14	0.02	0.24	0.12	0.05	0.06	0.14	0.13
(2,1)	0.05	0.24	0.01	0.14	0.13	0.13	0.00	0.15
(2,3)	0.03	0.10	0.06	0.25	0.10	0.09	0.03	0.13
(3,1)	0.08	0.24	0.08	0.01	0.11	0.15	0.14	0.07
(3,2)	0.10	0.09	0.10	0.07	0.02	0.13	0.11	0.01

Table 6.3: Scenario 1 - Resolution Probabilities

$q_i(c,k)$

(c,k) i	c = 1				c = 2			
	k=0	k=1	k=2	k=3	k=0	k=1	k=2	k=3
1	0.71	0.88	0.79	0.74	0.75	0.94	0.76	0.78
2	0.74	0.85	0.85	0.85	0.78	0.89	0.95	0.81
3	0.72	0.87	0.77	0.88	0.79	0.78	0.83	0.86

Table 6.4: Scenario 1, Type 1 E-mails - Aggregation Probabilities for Method M3

E-mail Type 1 - Weights from the Fundamental Matrix					
011	0.32392000	021	0.01871300	031	0.03236600
012	0.00381080	022	0.34433000	032	0.04045800
013	0.05335100	023	0.01122800	033	0.33175000
v1,0(1)	**0.38108180**	**v2,0(1)**	**0.37427100**	**v3,0(1)**	**0.40457400**
111	0.05622500	121	0.01027700	131	0.01062300
112	0.00117140	122	0.02826300	132	0.00398360
113	0.00117140	123	0.00428230	133	0.02965600
v1,1(1)	**0.05856780**	**v2,1(1)**	**0.04282230**	**v3,1(1)**	**0.04426260**
211	0.02622600	221	0.00042175	231	0.00377580
212	0.00308540	222	0.03922300	232	0.00471980
213	0.00925610	223	0.00253050	233	0.03870200
v1,2(1)	**0.03856750**	**v2,2(1)**	**0.04217525**	**v3,2(1)**	**0.04719760**
311	0.03059200	321	0.00688920	331	0.00055631
312	0.00842390	322	0.03001700	332	0.00389420
313	0.00532030	323	0.01230200	333	0.05118100
v1,3(1)	**0.04433620**	**v2,3(1)**	**0.04920820**	**v3,3(1)**	**0.05563151**
overall sum	0.52255330	overall sum	0.50847675	overall sum	0.55166571
w1,0(1)	**0.72926877**	**w2,0(1)**	**0.73606315**	**w3,0(1)**	**0.73336804**
w1,1(1)	**0.11208005**	**w2,1(1)**	**0.08421683**	**w3,1(1)**	**0.08023446**
w1,2(1)	**0.07380587**	**w2,2(1)**	**0.08294430**	**w3,2(1)**	**0.08555471**
w1,3(1)	**0.08484532**	**w2,3(1)**	**0.09677571**	**w3,3(1)**	**0.10084279**

Table 6.5: Scenario 1, Type 2 E-mails - Aggregation Probabilities for Method M3

E-mail Type 2 - Weights from the Fundamental Matrix					
011	0.37462000	021	0.04721600	031	0.04371000
012	0.02525600	022	0.27967000	032	0.00794730
013	0.02104600	023	0.03632000	033	0.34571000
v1,0(2)	**0.42092200**	**v2,0(2)**	**0.36320600**	**v3,0(2)**	**0.39736730**
111	0.04334700	121	0.00554470	131	0.00650410
112	0.00142900	122	0.03326800	132	0.00563690
113	0.00285810	123	0.00383860	133	0.03122000
v1,1(2)	**0.04763410**	**v2,1(2)**	**0.04265130**	**v3,1(2)**	**0.04336100**
211	0.02284600	221	0.00000000	231	0.00404220
212	0.00082576	222	0.02666000	232	0.00317600
213	0.00385350	223	0.00082454	233	0.02165500
v1,2(2)	**0.02752526**	**v2,2(2)**	**0.02748454**	**v3,2(2)**	**0.02887320**
311	0.02770400	321	0.00508950	331	0.00274400
312	0.00443260	322	0.02442900	332	0.00039200
313	0.00480200	323	0.00441090	333	0.03606400
v1,3(2)	**0.03693860**	**v2,3(2)**	**0.03392940**	**v3,3(2)**	**0.03920000**
overall sum	0.53301996	overall sum	0.46727124	overall sum	0.50880150
w1,0(2)	**0.78969275**	**w2,0(2)**	**0.77729158**	**w3,0(2)**	**0.78098689**
w1,1(2)	**0.08936645**	**w2,1(2)**	**0.09127739**	**w3,1(2)**	**0.08522184**
w1,2(2)	**0.05164020**	**w2,2(2)**	**0.05881924**	**w3,2(2)**	**0.05674747**
w1,3(2)	**0.06930059**	**w2,3(2)**	**0.07261179**	**w3,3(2)**	**0.07704380**

Table 6.6: Scenario 2 - Mean Processing Times

$s_{i,k}(c)$

(c,k) i	c = 1				c = 2			
	k=0	k=1	k=2	k=3	k=0	k=1	k=2	k=3
1	0.15	0.11	0.00	0.00	0.18	0.06	0.00	0.00
2	0.19	0.00	0.15	0.00	0.20	0.00	0.07	0.00
3	0.14	0.00	0.00	0.13	0.17	0.00	0.00	0.07

Table 6.7: Scenario 2 - Forwarding Probabilities

$p_{i,j}(c,k)$

(c,k) (i,j)	c = 1				c = 2			
	k=0	k=1	k=2	k=3	k=0	k=1	k=2	k=3
(1,2)	0.01	0.00	0.00	0.00	0.06	0.00	0.00	0.00
(1,3)	0.14	0.00	0.00	0.00	0.05	0.00	0.00	0.00
(2,1)	0.05	0.00	0.00	0.00	0.13	0.00	0.00	0.00
(2,3)	0.03	0.00	0.00	0.00	0.10	0.00	0.00	0.00
(3,1)	0.08	0.00	0.00	0.00	0.11	0.00	0.00	0.00
(3,2)	0.10	0.00	0.00	0.00	0.02	0.00	0.00	0.00

Table 6.8: Scenario 2 - Resolution Probabilities

$q_i(c,k)$

(c,k) i	c = 1				c = 2			
	k=0	k=1	k=2	k=3	k=0	k=1	k=2	k=3
1	0.71	0.88	0.00	0.00	0.75	0.94	0.00	0.00
2	0.74	0.00	0.85	0.00	0.78	0.00	0.95	0.00
3	0.72	0.00	0.00	0.88	0.79	0.00	0.00	0.86

Table 6.9: Scenario 2, Type 1 E-mails - Aggregation Probabilities for Method M3

E-mail Type 1 - Weights from solving the Discrete-Time Markov Chain					
011	0.32392000	021	0.01871300	031	0.03236600
012	0.00381080	022	0.34433000	032	0.04045800
013	0.05335100	023	0.01122800	033	0.33175000
v1,0(1)	**0.38108180**	**v2,0(1)**	**0.37427100**	**v3,0(1)**	**0.40457400**
111	0.10675000	121	0.00000000	131	0.00000000
112	0.00000000	122	0.00000000	132	0.00000000
113	0.00000000	123	0.00000000	133	0.00000000
v1,1(1)	**0.10675000**	**v2,1(1)**	**0.00000000**	**v3,1(1)**	**0.00000000**
211	0.00000000	221	0.00000000	231	0.00000000
212	0.00000000	222	0.10532000	232	0.00000000
213	0.00000000	223	0.00000000	233	0.00000000
v1,2(1)	**0.00000000**	**v2,2(1)**	**0.10532000**	**v3,2(1)**	**0.00000000**
311	0.00000000	321	0.00000000	331	0.00000000
312	0.00000000	322	0.00000000	332	0.00000000
313	0.00000000	323	0.00000000	333	0.10556000
sum	**0.00000000**		**0.00000000**		**0.10556000**
overall sum	0.48783180	overall sum	0.47959100	overall sum	0.51013400
w1,0(1)	**0.78117458**	**w2,0(1)**	**0.78039621**	**w3,0(1)**	**0.79307398**
w1,1(1)	**0.21882542**	**w2,1(1)**	**0.00000000**	**w3,1(1)**	**0.00000000**
w1,2(1)	**0.00000000**	**w2,2(1)**	**0.21960379**	**w3,2(1)**	**0.00000000**
w1,3(1)	**0.00000000**	**w2,3(1)**	**0.00000000**	**w3,3(1)**	**0.20692602**

Table 6.10: Scenario 2, Type 2 E-mails - Aggregation Probabilities for Method M3

E-mail Type 2 - Weights from solving the Discrete-Time Markov Chain					
011	0.37462000	021	0.04721600	031	0.04371000
012	0.02525600	022	0.27967000	032	0.00794730
013	0.02104600	023	0.03632000	033	0.34571000
v1,0(2)	**0.42092200**	**v2,0(2)**	**0.36320600**	**v3,0(2)**	**0.39736730**
111	0.09963400	121	0.00000000	131	0.00000000
112	0.00000000	122	0.00000000	132	0.00000000
113	0.00000000	123	0.00000000	133	0.00000000
v1,1(2)	**0.09963400**	**v2,1(2)**	**0.00000000**	**v3,1(2)**	**0.00000000**
211	0.00000000	221	0.00000000	231	0.00000000
212	0.00000000	222	0.06476500	232	0.00000000
213	0.00000000	223	0.00000000	233	0.00000000
v1,2(2)	**0.00000000**	**v2,2(2)**	**0.06476500**	**v3,2(2)**	**0.00000000**
311	0.00000000	321	0.00000000	331	0.00000000
312	0.00000000	322	0.00000000	332	0.00000000
313	0.00000000	323	0.00000000	333	0.08441700
v1,3(2)	**0.00000000**	**v2,3(2)**	**0.00000000**	**v3,3(2)**	**0.08441700**
overall sum	0.52055600	overall sum	0.42797100	overall sum	0.48178430
w1,0(2)	**0.80860080**	**w2,0(2)**	**0.84866965**	**w3,0(2)**	**0.82478258**
w1,1(2)	**0.19139920**	**w2,1(2)**	**0.00000000**	**w3,1(2)**	**0.00000000**
w1,2(2)	**0.00000000**	**w2,2(2)**	**0.15133035**	**w3,2(2)**	**0.00000000**
w1,3(2)	**0.00000000**	**w2,3(2)**	**0.00000000**	**w3,3(2)**	**0.17521742**

The next section presents the performance evaluation of the contact center example using both analytical and simulation models and examines the accuracy of the analytical results.

6.7 Results and Discussions

The aggregated single-class open queueing network model was solved using the RAQS software package (Kamath et al. 1995). RAQS is a software package for analyzing general queueing network models using the PD method (http://www.okstate.edu/cocim/raqs/). Simulation estimates represent averages over ten independent replications. Each replication was simulated for 18,480 hours of operation with a warm up of 1,680 hours. The analytical and simulation results are shown for three processing time distributions - Erlang (SCV =0.25), exponential (SCV =1.00), and Hyperexponential (SCV = 2.00).

6.7.1 Results

The analytical and simulation results are presented in Tables 6.11 and 6.12 and in Figures 6.6 through 6.10 for Scenario 1, where a previously processed e-mail is routed to any one of the agents with equal probability. Tables 6.13 and 6.14 and Figures 6.11 through 6.15 shows the results for Scenario 2, where a previously processed e-mail is routed to the agent that processed it. The quantity in parentheses below a simulation estimate is the half-width of the 95% confidence interval. The quantity in parentheses below an analytical value is the relative percentage error. The columns labeled M1 contain analytical results obtained using the analytical method M1, i.e., the naïve method based on a simple average. The columns labeled M2 contain analytical results obtained using the analytical method M2, i.e., the approximate method based on aggregate forwarding and resolution probabilities. Finally, the column labeled M3 contain analytical results obtained using the analytical method M3, i.e., the DTMC-based method for computing the probabilities /weights for history-based aggregation.

Table 6.11: Scenario 1 - Node-level Performance Measures

Processing Time Distribution	Node (Agent)	Utilization				Average Queueing Delay (hours)				Average Number in Queue			
		M1	M2	M3	Sim	M1	M2	M3	Sim	M1	M2	M3	Sim
Erlang	1	0.735 (-17.14%)	0.910 (+2.59%)	0.890 (+0.34%)	0.887 (±0.003)	0.264 (-70.57%)	1.091 (+21.63%)	0.866 (-3.46%)	0.897 (±0.045)	1.423 (-71.32%)	6.080 (+22.53%)	4.800 (-3.26%)	4.962 (±0.258)
	2	0.728 (-20.96%)	0.951 (+3.26%)	0.923 (-0.22%)	0.921 (±0.003)	0.276 (-81.34%)	2.438 (+64.84%)	1.476 (-0.20%)	1.479 (±0.073)	1.400 (-81.62%)	12.659 (+66.17%)	7.621 (+0.04%)	7.618 (±0.391)
	3	0.793 (-8.11%)	0.876 (+1.51%)	0.865 (+0.23%)	0.863 (±0.002)	0.393 (-42.96%)	0.740 (+7.40%)	0.669 (-2.90%)	0.689 (±0.018)	2.169 (-43.82%)	4.176 (+8.16%)	3.752 (-2.82%)	3.861 (±0.108)
Exponential	1	0.735 (-17.23%)	0.910 (+2.48%)	0.890 (+0.22%)	0.888 (±0.004)	0.378 (-69.44%)	1.591 (+28.62%)	1.261 (+1.94%)	1.237 (±0.034)	2.043 (-70.18%)	8.870 (+29.49%)	6.992 (+2.07%)	6.850 (±0.196)
	2	0.728 (-20.96%)	0.951 (+3.26%)	0.923 (+0.22%)	0.921 (±0.004)	0.399 (-81.42%)	3.588 (+67.12%)	2.170 (+1.07%)	2.147 (±0.162)	2.023 (-81.75%)	18.631 (+68.12%)	11.201 (+1.07%)	11.082 (±0.872)
	3	0.793 (-8.22%)	0.876 (+1.39%)	0.865 (+0.12%)	0.864 (±0.003)	0.569 (-42.06%)	1.074 (+9.37%)	0.971 (-1.12%)	0.982 (±0.039)	3.139 (-43.00%)	6.065 (+10.13%)	5.447 (-1.09%)	5.507 (±0.224)
Hyperexponential	1	0.735 (-17.60%)	0.910 (+2.02%)	0.890 (-0.22%)	0.892 (±0.004)	0.531 (-71.00%)	2.258 (+23.32%)	1.788 (-2.35%)	1.831 (±0.104)	2.868 (-71.77%)	12.587 (+23.88%)	9.913 (-2.44%)	10.161 (±0.586)
	2	0.728 (-21.21%)	0.951 (+2.92%)	0.923 (-0.11%)	0.924 (±0.003)	0.563 (-81.61%)	5.122 (+67.33%)	3.094 (+1.08%)	3.061 (±0.223)	2.854 (-81.95%)	26.593 (+68.21%)	15.973 (+1.04%)	15.809 (±1.155)
	3	0.793 (-8.00%)	0.876 (+1.62%)	0.865 (+0.35%)	0.862 (±0.004)	0.803 (-39.03%)	1.520 (+15.41%)	1.374 (+4.33%)	1.317 (±0.059)	4.430 (-40.05%)	8.583 (+16.16%)	7.706 (+4.29%)	7.389 (±0.332)

Table 6.12: Scenario 1 - System-level Performance Measures

Processing Time Distribution	Average Response Time (hours)				Average Resolution Time (hours)				Average Number of E-mails in the System			
	M1	M2	M3	Sim	M1	M2	M3	Sim	M1	M2	M3	Sim
Erlang	0.562 (-59.80%)	1.854 (+32.62%)	1.375 (-1.64%)	1.398 (±0.029)	1.605 (-47.03%)	3.713 (+22.54%)	3.017 (-0.43%)	3.030 (±0.040)	16.858 (-46.99%)	38.990 (+22.60%)	31.681 (-0.38%)	31.803 (±0.029)
Exponential	0.733 (-61.56%)	2.624 (+37.60%)	1.920 (+0.68%)	1.907 (±0.077)	1.816 (-50.89%)	4.728 (+27.85%)	3.728 (+0.81%)	3.698 (±0.098)	19.070 (-50.93%)	49.641 (+27.74%)	39.147 (+0.74%)	38.861 (±1.097)
Hyperexponential	0.962 (-63.42%)	3.650 (+38.78%)	2.646 (+0.61%)	2.630 (±0.106)	2.097 (-54.86%)	6.080 (+30.87%)	4.676 (+0.65%)	4.646 (±0.139)	22.017 (-54.91%)	63.838 (+30.75%)	49.099 (+0.56%)	48.826 (±0.106)

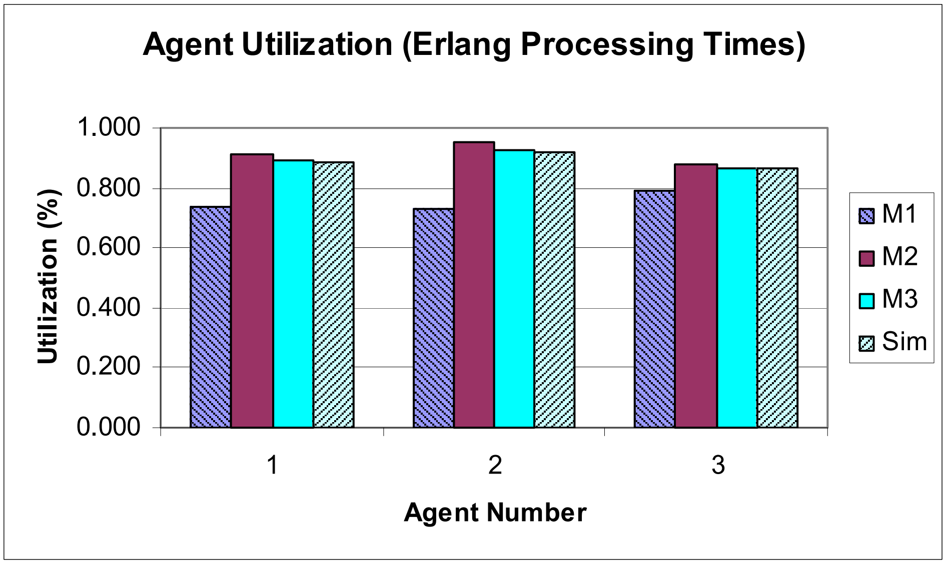

Figure 6.6: Scenario 1 - Agent Utilization - Erlang Processing Time

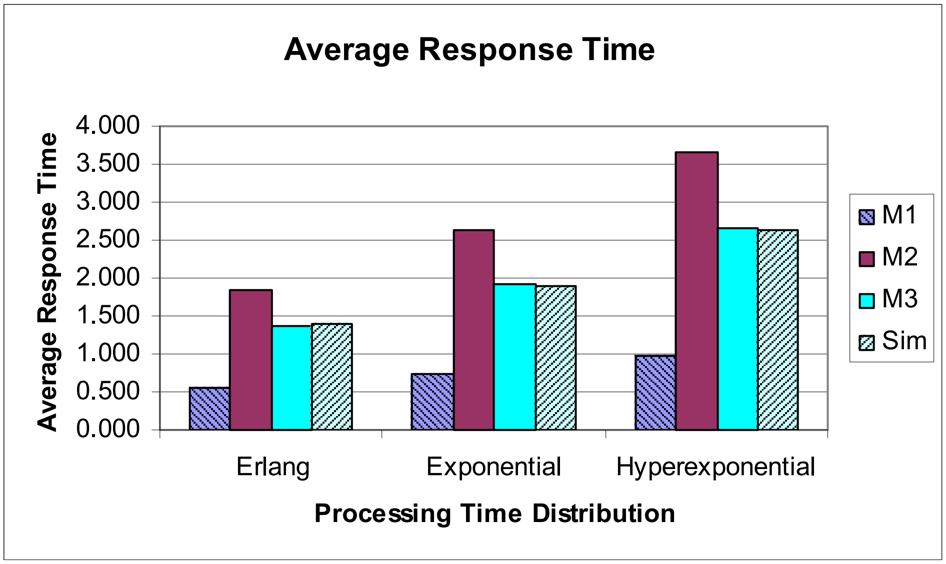

Figure 6.7: Scenario 1 - Average Response Time

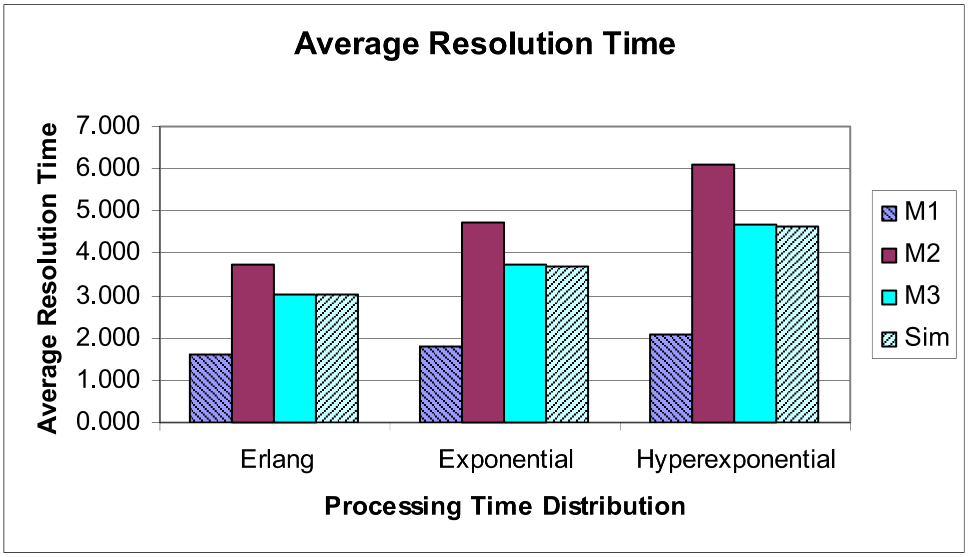

Figure 6.8: Scenario 1 - Average Resolution Time

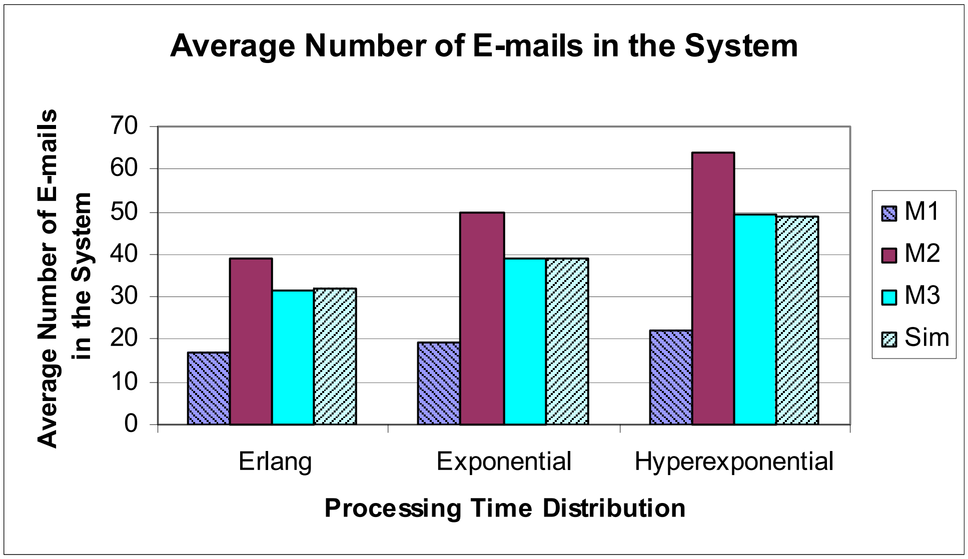

Figure 6.9: Scenario 1 - Average Number of E-mails in the System

Table 6.13: Scenario 2 - Node-level Performance Measures

Processing Time Distribution	Node (Agent)	Utilization				Average Queueing Delay (hours)				Average Number in Queue			
		M1	M2	M3	Sim	M1	M2	M3	Sim	M1	M2	M3	Sim
Erlang	1	0.670 (-21.45%)	0.864 (+1.29%)	0.853 (0.00%)	0.853 (±0.003)	0.195 (-70.18%)	0.687 (+5.05%)	0.617 (-5.66%)	0.654 (±0.024)	0.905 (-73.84%)	3.610 (+4.33%)	3.261 (-5.75%)	3.460 (±0.133)
	2	0.747 (-17.46%)	0.910 (+0.55%)	0.906 (+0.11%)	0.905 (±0.002)	0.335 (-73.41%)	1.269 (+0.71%)	1.202 (-4.60%)	1.260 (±0.053)	1.491 (-75.42%)	6.216 (+2.49%)	5.786 (-4.60%)	6.065 (±0.263)
	3	0.684 (-15.35%)	0.822 (+1.73%)	0.811 (+0.37%)	0.808 (±0.003)	0.206 (-53.18%)	0.476 (+8.18%)	0.437 (-0.68%)	0.440 (±0.014)	0.964 (-56.77%)	2.495 (+11.88%)	2.292 (+2.78%)	2.230 (±0.079)
Exponential	1	0.670 (-21.45%)	0.864 (+1.29%)	0.853 (0.00%)	0.853 (±0.003)	0.279 (-68.79%)	0.999 (+11.74%)	0.895 (+0.11%)	0.894 (±0.037)	1.295 (-72.56%)	5.248 (+11.19%)	4.731 (+0.23%)	4.720 (±0.210)
	2	0.747 (-17.28%)	0.910 (+0.77%)	0.906 (+0.33%)	0.903 (±0.004)	0.488 (-72.27%)	1.865 (+5.97%)	1.765 (+0.28%)	1.760 (±0.162)	2.168 (-74.38%)	9.133 (+7.92%)	8.499 (+0.42%)	8.463 (±0.794)
	3	0.684 (-15.35%)	0.822 (+1.73%)	0.811 (+0.37%)	0.808 (±0.003)	0.294 (-53.41%)	0.687 (+8.87%)	0.631 (0.00%)	0.631 (±0.018)	1.377 (-58.27%)	3.606 (+9.27%)	3.308 (+0.24%)	3.300 (±0.100)
Hyperexponential	1	0.670 (-21.18%)	0.864 (+1.65%)	0.853 (+0.35%)	0.850 (±0.005)	0.391 (-67.47%)	1.415 (+17.72%)	1.266 (+5.32%)	1.202 (±0.076)	1.815 (-71.36%)	7.432 (+17.26%)	6.689 (+5.54%)	6.338 (±0.421)
	2	0.747 (-17.37%)	0.910 (+0.66%)	0.906 (+0.22%)	0.904 (±0.003)	0.691 (-71.39%)	2.659 (+10.10%)	2.516 (+4.18%)	2.415 (±0.115)	3.071 (-73.53%)	13.021 (+12.24%)	12.115 (+4.43%)	11.601 (±0.561)
	3	0.684 (-15.35%)	0.822 (+1.73%)	0.811 (+0.37%)	0.808 (±0.004)	0.411 (-51.87%)	0.970 (+13.58%)	0.889 (+4.10%)	0.854 (±0.031)	1.928 (-56.79%)	5.087 (+14.00%)	4.662 (+4.48%)	4.462 (±0.167)

Table 6.14: Scenario 2 - System-level Performance Measures

Processing time Distribution	Average Response Time (hours)				Average Resolution Time (hours)				Average Number of E-mails in the System			
	M1	M2	M3	Sim	M1	M2	M3	Sim	M1	M2	M3	Sim
Erlang	0.428 (-59.70%)	1.101 (+3.67%)	1.026 (-3.39%)	1.062 (±0.022)	1.386 (-45.04%)	2.583 (+2.42%)	2.492 (-1.19%)	2.522 (±0.030)	14.548 (-45.12%)	27.121 (+2.32%)	26.161 (-1.30%)	26.507 (±0.357)
Exponential	0.544 (-61.34%)	1.519 (+7.96%)	1.409 (+0.14%)	1.407 (±0.061)	1.526 (-48.51%)	3.123 (+5.36%)	2.987 (+0.78%)	2.964 (±0.079)	16.028 (-48.54%)	32.787 (+5.27%)	31.359 (+0.68%)	31.146 (±0.853)
Hyperexponential	0.698 (-62.17%)	2.076 (+12.52%)	1.920 (+4.06%)	1.845 (±0.063)	1.714 (-51.42%)	3.842 (+8.90%)	3.646 (+3.34%)	3.528 (±0.831)	18.001 (-51.41%)	40.340 (+8.89%)	38.287 (+3.35%)	37.046 (±0.912)

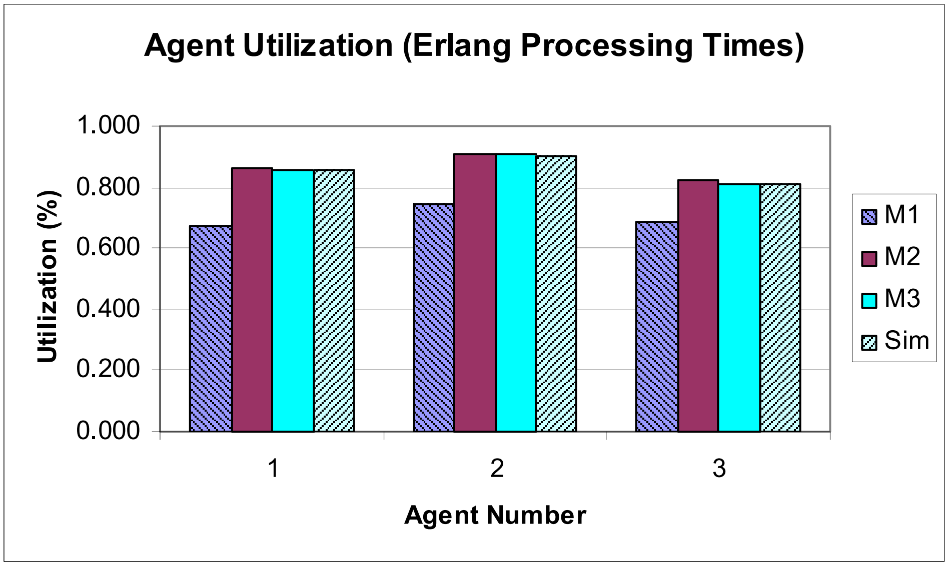

Figure 6.10: Scenario 2 - Agent Utilization - Erlang Processing Times

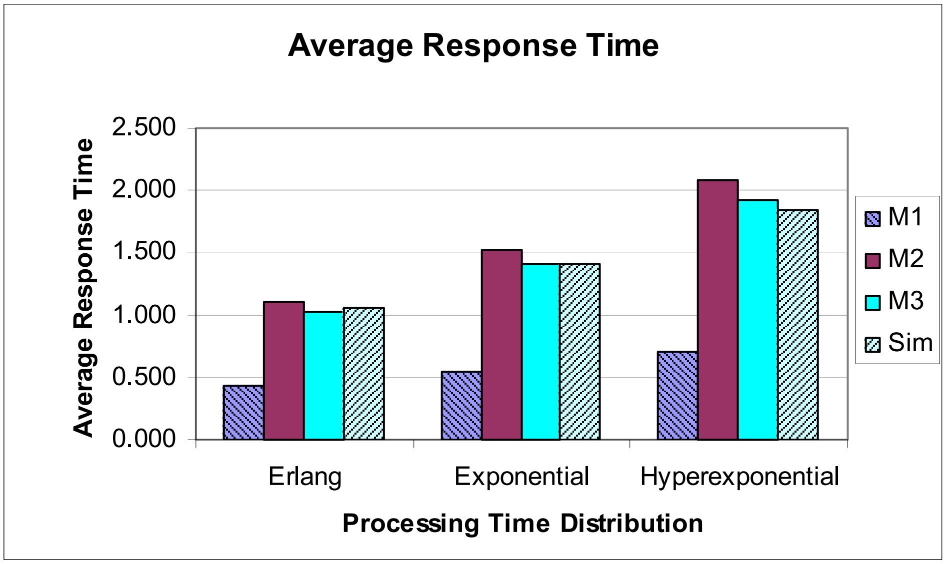

Figure 6.11: Scenario 2 - Average Response Time

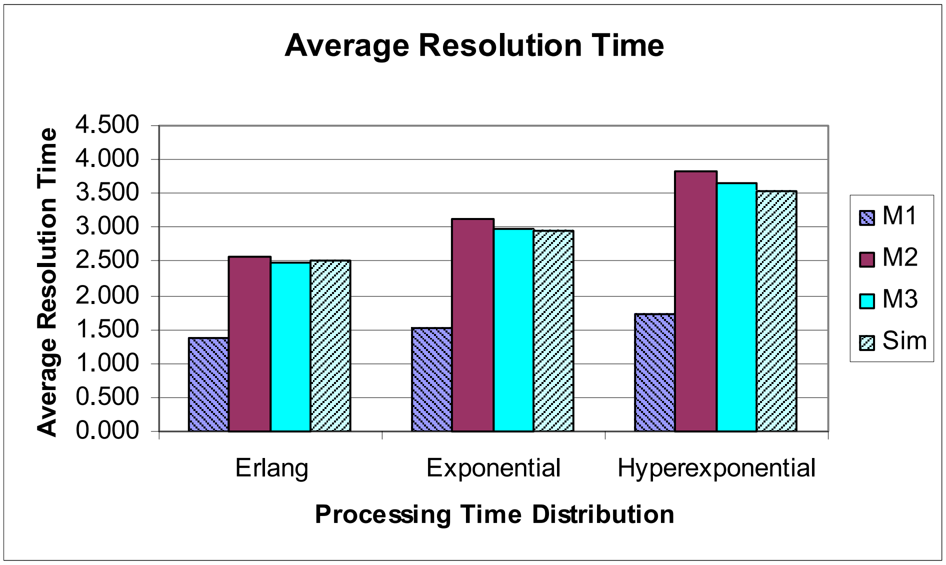

Figure 6.12: Scenario 2 - Average Resolution Time

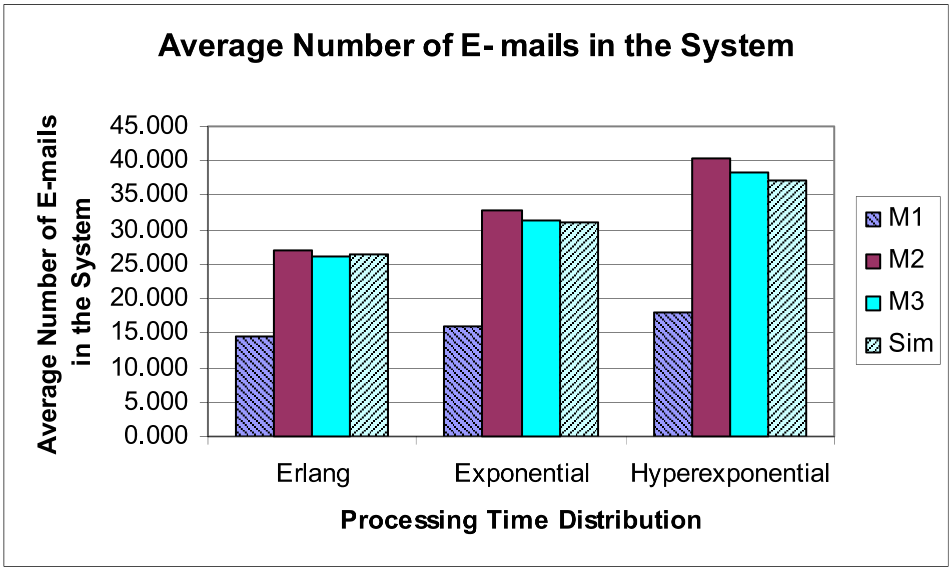

Figure 6.13: Scenario 2 - Average Number of E-mails in the System

6.7.2 Discussion

As expected the naive method M1 performed the worst because of the assumption of equal weights (=0.25) for Scenario 1 and (=0.50) for Scenario 2. By approximately capturing the forwarding and resolution probabilities, Method M2, does much better than M1, but not so good as method M3. The good performance of M2 in the case of Scenario 2 can be attributed to the high degree of homogeneity among the three agents. We would expect M2's performance to deteriorate and M3's performance to remain about the same if we were to make the agents more heterogeneous. The DTMC-based analytical method M3 performed the best and showed excellent prediction capability across the various cases examined. The relative percentage error in utilization was less than 1% in all cases and less than 6% in all cases for the other measures. In terms of prediction accuracy method M1 is the worst performer and method M3 the best. From a computational complexity viewpoint method M3 is relatively complex requiring matrix inversion. This should not be a big issue even for large matrices because of the availability of powerful software. Method M1 certainly helps in demonstrating the need to model the dependence on history for accurate performance prediction. Method M2 could be used as a rough approximation in some situations, especially when the DTMC for method M3 becomes large.

CHAPTER 7

MODEL EXTENSIONS: SKILL-BASED GROUPING OF AGENTS AND SERVICE INTERRUPTIONS

This chapter considers two extensions to the contact center model considered in Chapter 6. First, the multi-class open queueing network model described in Chapter 6 is extended to model situations where agents with similar skill sets are grouped. The group sizes can vary. Grouping of agents is quite common in contact centers and normally practiced in situations when customer arrival rates are high and where customers seek a quicker response to their problem. Section 7.1 describes the open queueing network model for skill-based grouping of agents. Section 7.2 describes the numerical experiments conducted to evaluate this multi-server extension to the queueing network model. Section 7.3 presents the results for the multi-server open queueing network model. The second extension is the random service interruptions experienced by agents. Section 7.4 considers the contact center model developed in Chapter 6 in the presence of random service time interruptions. Section 7.5 describes the numerical experiments conducted to evaluate the service-interruption extension, and results are discussed in Section 7.6.

7.1 Contact Center Model with Skill-Based Grouping

The modeling of the e-mail contact center with skill-based grouping is almost identical to the approach discussed in Section 6.3. All the descriptions and assumptions for the contact center model described in Chapter 6 hold good for the skill-based grouping model. A skill-based group can be modeled as a multi-server node within the queueing network model. The development of the discrete-time Markov chain (DTMC) based on the routing probabilities, analysis of the absorbing DTMC, and the computations of weights remain the same as in Chapter 6. The only change is the number of servers at each node of the open queueing network model. This additional input to the PD approach (RAQS software) and the specification of the number resources to match the group size in the simulation program are the only changes to be made in the analytical and simulation models.

7.2 Numerical Experiments

An e-mail contact center with three groups of agents that can handle two types of e-mails was considered. When the number of agents was increased for the parameter values chosen in Section 6.6.2 the agent utilization dropped considerably. Therefore to model realistic utilization levels, the arrival rates were increased to obtain agent utilizations that were almost the same as those in Chapter 6. In order to see how well the analytical approach performs for the multi-server case, three different configurations were considered. The three configurations were 1) same number of agents in all groups, 2) same number of agents in two groups, and 3) different number of agents. For the first two configurations the arrival rates were $\lambda_1 = 24/\text{hr}$ and $\lambda_2 = 18/\text{hr}$, and for the last configuration the arrival rates were $\lambda_1 = 18/\text{hr}$ and $\lambda_2 = 13.5/\text{hr}$.

- In the first configuration, all groups had four servers.
- In the second configuration, the number of agents in group one was four, group two was five, and group three was four.
- In the third configuration, the number of agents in group one was four, group two was five, and group three was three.

With the rates $\lambda_1 = 24/\text{hr}$ and $\lambda_2 = 18/\text{hr}$, the third configuration became unstable. Therefore for stability, the arrival rates were changed to $\lambda_1 = 18/\text{hr}$ and $\lambda_2 = 13.5/\text{hr}$. All the other parameter values were the same for the three configurations and as explained in Section 6.6.2. Both the scenarios explained in Section 6.6.2 were tested under skill-based grouping. The models were solved using the analytical method M3 only.

7.3 Results and Discussion

The analytical and simulation results for all the three configurations under two scenarios are presented in this section. The results are presented in Tables 7.1 through 7.18 and in Figures 7.1 through 7.24.

As the results almost matched for the single-server open queueing network model, the expectation was that the analytical approach will perform well for the multi-server case as well. As expected, the results for all the three processing distributions matched closely for both scenarios and all three configurations, demonstrating the robustness and accuracy of the analytical method.

7.3.1 Results

Table 7.1: Scenario 1, Configuration 1 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.456 (-1.30%)	0.462 (± 0.004)		
Average resolution time (hrs)			1.817 (+0.05%)	1.816 (± 0.006)		
Average no of e-mails in the system			76.335 (+0.06%)	76.290 (± 0.283)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.890 (+0.22%)	0.888 (± 0.001)	0.189 (-1.56%)	0.192 (± 0.004)	4.196 (-1.15%)	4.245 (± 0.094)
2	0.923 (0.00%)	0.923 (± 0.000)	0.336 (-3.17%)	0.347 (± 0.008)	6.95 (-3.03%)	7.167 (± 0.175)
3	0.865 (+0.11%)	0.864 (± 0.001)	0.142 (0.00%)	0.142 (± 0.002)	3.179 (-0.16%)	3.184 (± 0.002)

Table 7.2: Scenario 1, Configuration 1 - Performance Measures for Exponential Processing Times

Exponential Processing Times (SCV =1.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.573 (+1.78%)	0.563 (± 0.006)		
Average resolution time (hrs)			1.969 (+1.23%)	1.945 (± 0.009)		
Average no of e-mails in the system			82.718 (+1.26%)	81.691 (± 0.389)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.890 (+0.22%)	0.888 (± 0.002)	0.267 (+1.52%)	0.263 (± 0.006)	6.034 (+3.62%)	5.823 (± 0.150)
2	0.923 (+0.22%)	0.921 (± 0.002)	0.49 (+2.72%)	0.477 (± 0.017)	10.117 (+2.79%)	9.842 (± 0.370)
3	0.865 (+0.11%)	0.864 (± 0.002)	0.203 (+2.01%)	0.199 (± 0.005)	4.547 (+2.16%)	4.451 (± 0.103)

Table 7.3: Scenario 1, Configuration 1 - Performance Measures for Hyperexponential Processing Times

Hyperexponential Processing Times (SCV =2.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.722 (+2.56%)	0.704 (± 0.013)		
Average resolution time (hrs)			2.164 (+1.60%)	2.130 (± 0.019)		
Average no of e-mails in the system			90.870 (+1.57%)	89.461 (± 0.897)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.890 (+0.22%)	0.888 (± 0.002)	0.378 (+3.85%)	0.364 (± 0.007)	8.373 (+3.79%)	8.067 (± 0.174)
2	0.923 (+0.22%)	0.921 (± 0.001)	0.688 (+4.40%)	0.659 (± 0.030)	14.210 (+4.55%)	13.591 (± 0.638)
3	0.865 (+0.11%)	0.864 (± 0.000)	0.279 (+1.45%)	0.275 (± 0.006)	6.267 (+1.57%)	6.170 (± 0.128)

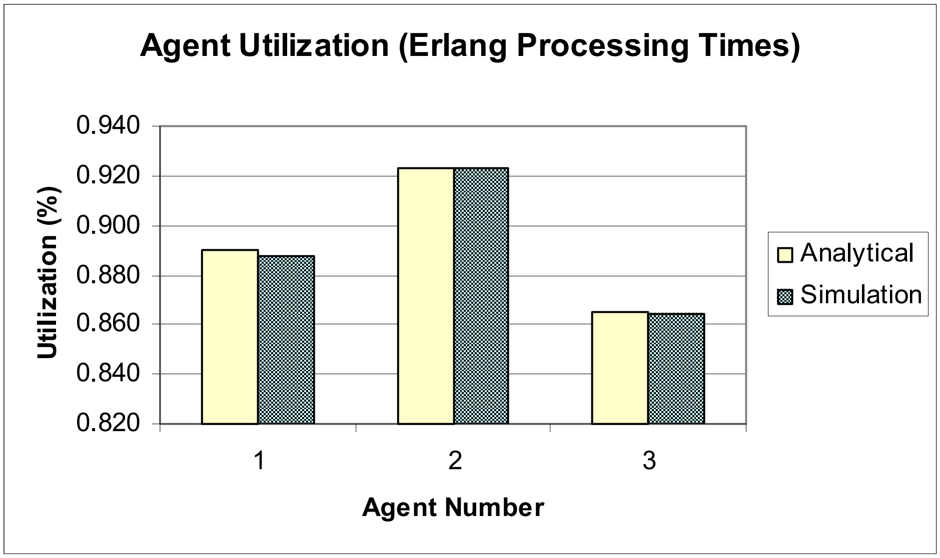

Figure 7.1: Scenario 1, Configuration 1 - Agent Utilization - Erlang Processing Times

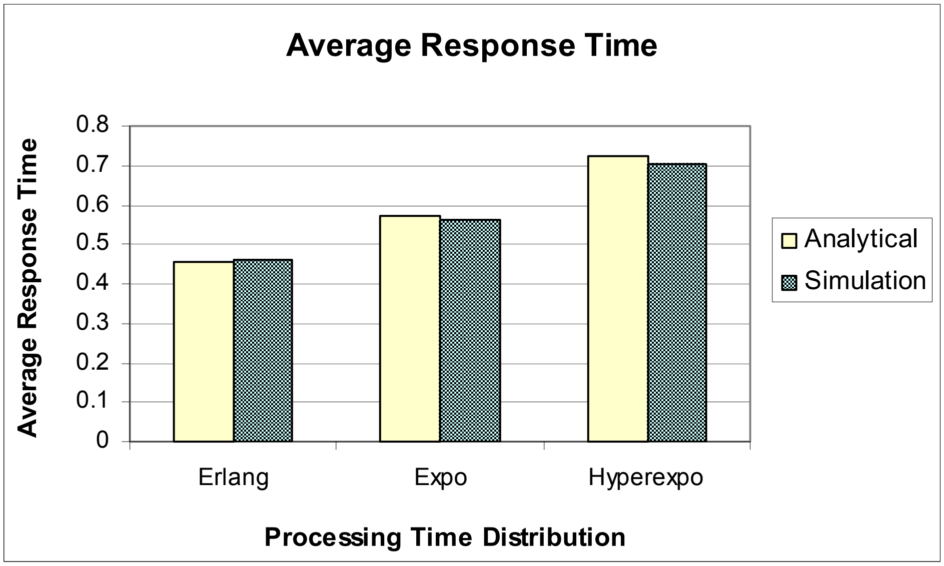

Figure 7.2: Scenario 1, Configuration 1 - Average Response Time

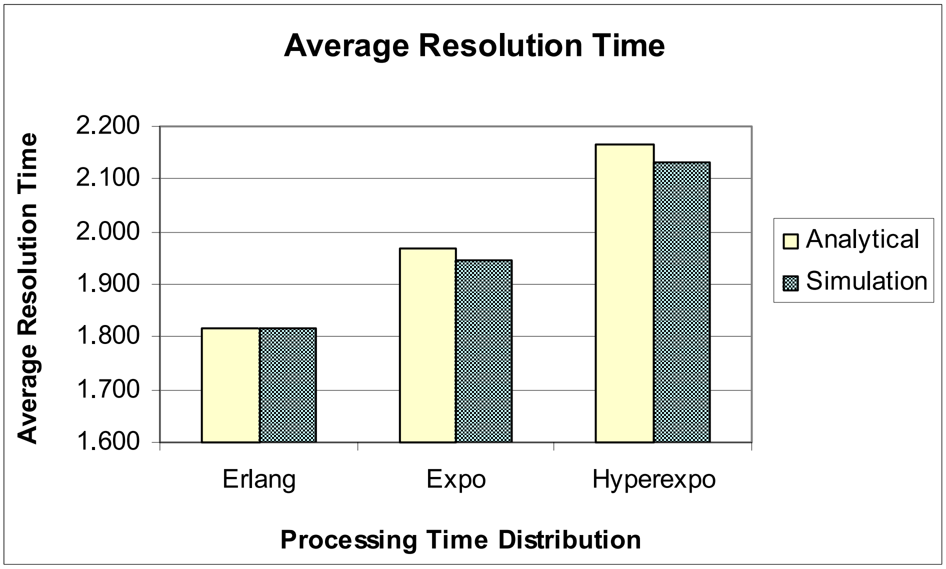

Figure 7.3: Scenario 1, Configuration 1 - Average Resolution Time

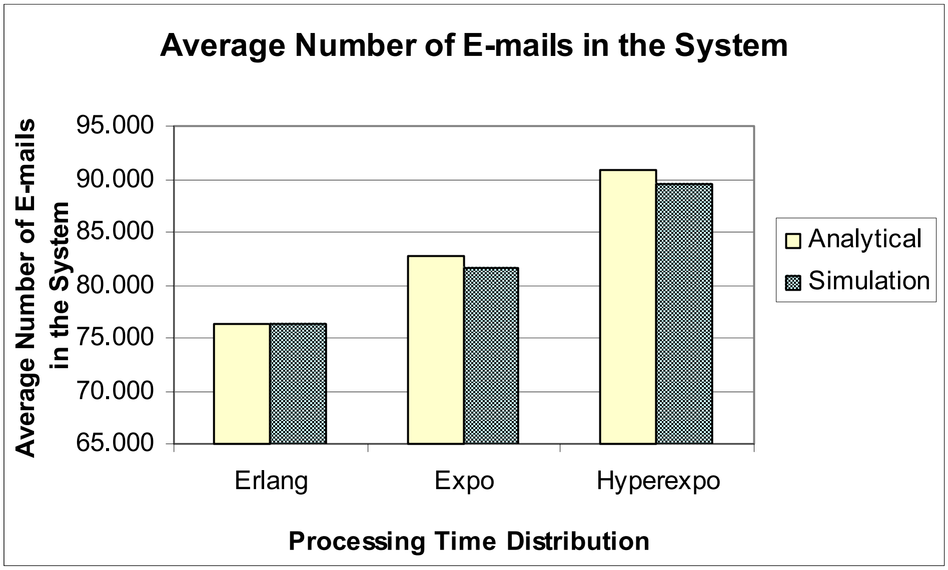

Figure 7.4: Scenario 1, Configuration 1 - Average Number of E-mails in the System

Table 7.4: Scenario 1, Configuration 2 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.346 (-0.57%)	0.348 (± 0.002)		
Average resolution time (hrs)			1.674 (+0.42%)	1.667 (± 0.005)		
Average no of e-mails in the system			70.291 (+0.42%)	69.993 (± 0.000)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.890 (+0.22%)	0.888 (± 0.001)	0.189 (-1.05%)	0.191 (± 0.004)	4.199 (-0.97%)	4.240 (± 0.103)
2	0.738 (+0.14%)	0.737 (± 0.001)	0.043 (-2.27%)	0.044 (± 0.000)	0.892 (-0.78%)	0.899 (± 0.007)
3	0.865 (+0.11%)	0.864 (± 0.001)	0.142 (-0.70%)	0.143 (± 0.000)	3.181 (-0.50%)	3.197 (± 0.021)

Table 7.5: Scenario 1, Configuration 2 - Performance Measures for Exponential Processing Times

<table>
<tr><th colspan="5">Exponential Processing Times (SCV =1.00)</th><th colspan="2" rowspan="5"></th></tr>
<tr><td colspan="3">Performance Measures</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td colspan="3">Average response time (hrs)</td><td>0.411
(+1.48%)</td><td>0.405
(± 0.004)</td></tr>
<tr><td colspan="3">Average resolution time (hrs)</td><td>1.759
(+1.03%)</td><td>1.741
(± 0.005)</td></tr>
<tr><td colspan="3">Average no of e-mails in the system</td><td>73.858
(+1.09%)</td><td>73.060
(± 0.241)</td></tr>
<tr><td rowspan="2">Node</td><td colspan="2">Utilization</td><td colspan="2">Average queueing delay (hrs)</td><td colspan="2">Average number in the queue</td></tr>
<tr><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td>1</td><td>0.890
(+0.22%)</td><td>0.888
(± 0.001)</td><td>0.272
(+3.42%)</td><td>0.263
(± 0.007)</td><td>6.033
(+3.62%)</td><td>5.822
(± 0.149)</td></tr>
<tr><td>2</td><td>0.738
(+0.27%)</td><td>0.736
(± 0.002)</td><td>0.061
(+1.67%)</td><td>0.060
(±0.001)</td><td>1.258
(+1.53%)</td><td>1.239
(± 0.029)</td></tr>
<tr><td>3</td><td>0.865
(+0.35%)</td><td>0.862
(± 0.002)</td><td>0.203
(+3.05%)</td><td>0.197
(± 0.005)</td><td>4.547
(+3.34%)</td><td>4.400
(± 0.108)</td></tr>
</table>

Table 7.6: Scenario 1, Configuration 2 - Performance Measures for Hyperexponential Processing Times

<table>
<tr><th colspan="5">Hyperexponential Processing Times (SCV =2.00)</th><th colspan="2" rowspan="5"></th></tr>
<tr><td colspan="3">Performance Measures</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td colspan="3">Average response time (hrs)</td><td>0.493
(+1.44%)</td><td>0.486
(± 0.006)</td></tr>
<tr><td colspan="3">Average resolution time (hrs)</td><td>1.865
(+1.19%)</td><td>1.843
(± 0.009)</td></tr>
<tr><td colspan="3">Average no of e-mails in the system</td><td>78.340
(+1.24%)</td><td>77.383
(± 0.390)</td></tr>
<tr><td rowspan="2">Node</td><td colspan="2">Utilization</td><td colspan="2">Average queueing delay (hrs)</td><td colspan="2">Average number in the queue</td></tr>
<tr><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td>1</td><td>0.890
(+0.11%)</td><td>0.889
(± 0.002)</td><td>0.377
(+2.73%)</td><td>0.367
(± 0.014)</td><td>8.368
(+3.05%)</td><td>8.120
(± 0.320)</td></tr>
<tr><td>2</td><td>0.738
(+0.14%)</td><td>0.737
(± 0.001)</td><td>0.082
(+2.50%)</td><td>0.080
(± 0.002)</td><td>1.687
(+1.75%)</td><td>1.658
(± 0.033)</td></tr>
<tr><td>3</td><td>0.865
(+0.12%)</td><td>0.864
(± 0.002)</td><td>0.279
(+2.20%)</td><td>0.273
(± 0.010)</td><td>6.264
(+2.66%)</td><td>6.102
(± 0.241)</td></tr>
</table>

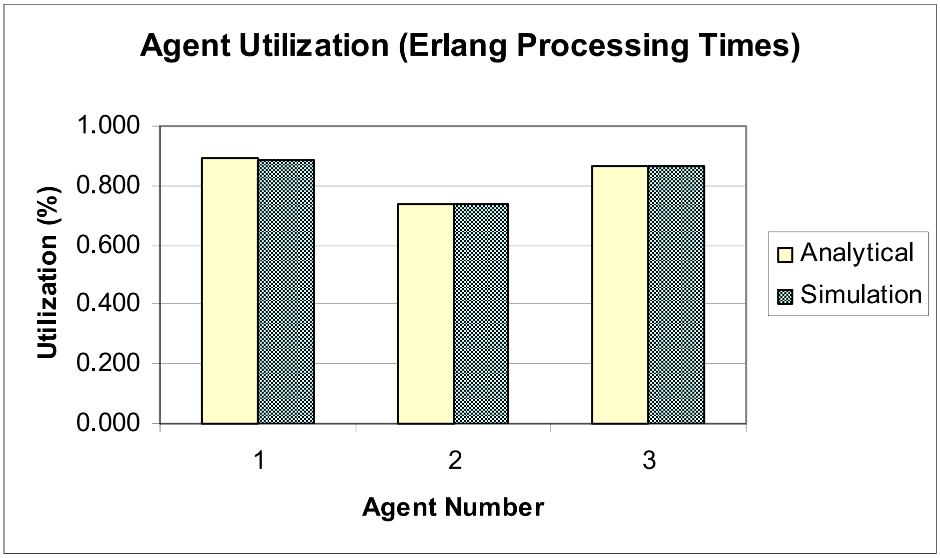

Figure 7.5: Scenario 1, Configuration 2 - Agent Utilization - Erlang Processing Times

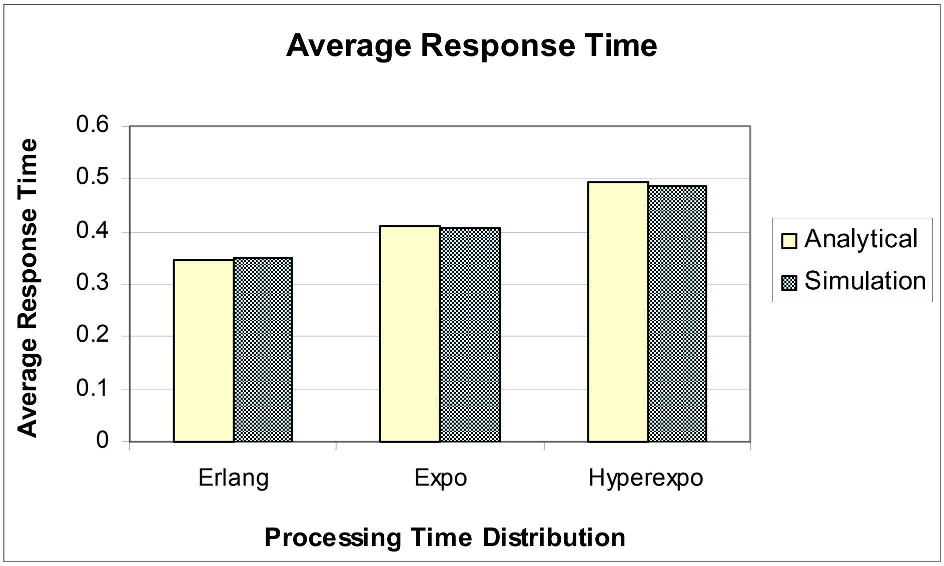

Figure 7.6: Scenario 1, Configuration 2 - Average Response Time

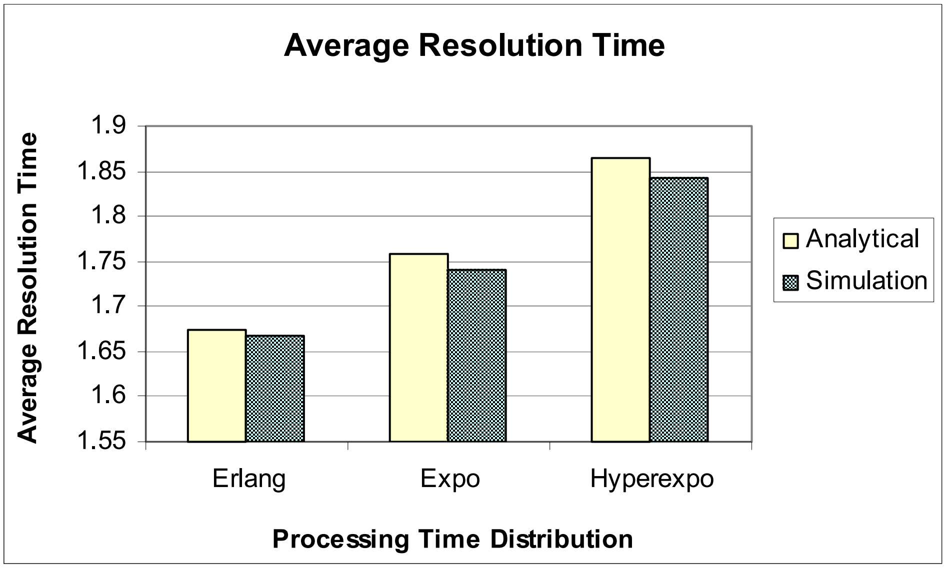

Figure 7.7: Scenario 1, Configuration 2 - Average Resolution Time

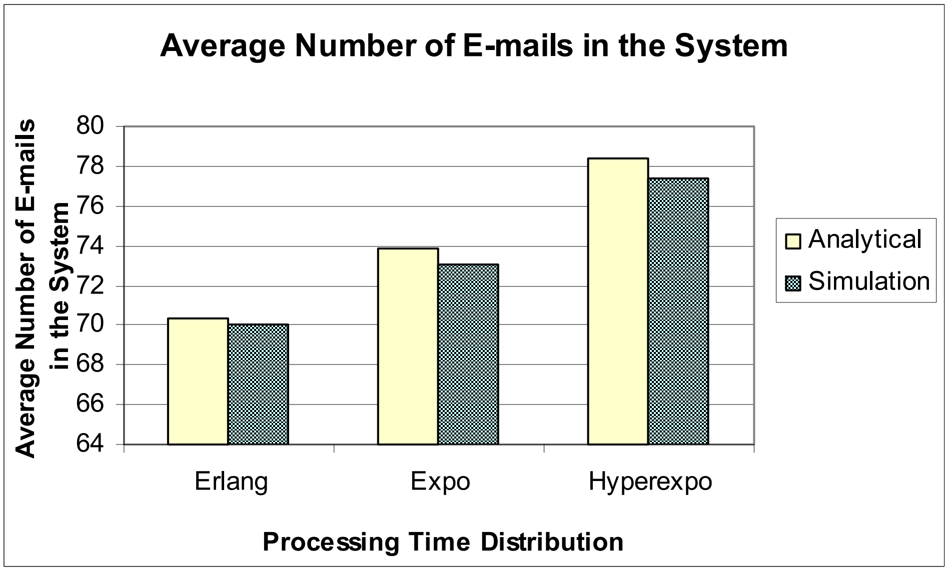

Figure 7.8: Scenario 1, Configuration 2 - Average Number of E-mails in the System

Table 7.7: Scenario 1, Configuration 3 - Performance Measures for Erlang Processing Times

<table>
<tr><th colspan="5">Erlang Processing Times (SCV =0.25)</th><th colspan="2" rowspan="5"></th></tr>
<tr><td colspan="3">Performance Measures</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td colspan="3">Average response time (hrs)</td><td>0.293
(-0.34%)</td><td>0.294
(± 0.002)</td></tr>
<tr><td colspan="3">Average resolution time (hrs)</td><td>1.605
(+0.31%)</td><td>1.600
(± 0.004)</td></tr>
<tr><td colspan="3">Average no of e-mails in the system</td><td>50.547
(+0.47%)</td><td>50.312
(± 0.105)</td></tr>
<tr><td rowspan="2">Node</td><td colspan="2">Utilization</td><td colspan="2">Average queueing delay (hrs)</td><td colspan="2">Average number in the queue</td></tr>
<tr><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td>1</td><td>0.667
(+0.15%)</td><td>0.666
(± 0.000)</td><td>0.032
(0.00%)</td><td>0.032
(± 0.000)</td><td>0.533
(-0.75%)</td><td>0.537
(± 0.007)</td></tr>
<tr><td>2</td><td>0.554
(0.00%)</td><td>0.554
(± 0.001)</td><td>0.011
(0.00%)</td><td>0.011
(± 0.000)</td><td>0.171
(+0.59%)</td><td>0.17
(± 0.002)</td></tr>
<tr><td>3</td><td>0.865
(+0.12%)</td><td>0.864
(± 0.001)</td><td>0.197
(-0.50%)</td><td>0.198
(± 0.004)</td><td>3.321
(0.00%)</td><td>3.321
(± 0.074)</td></tr>
</table>

Table 7.8: Scenario 1, Configuration 3 - Performance Measures for Exponential Processing Times

<table>
<tr><th colspan="5">Exponential Processing Times (SCV =1.00)</th><th colspan="2" rowspan="5"></th></tr>
<tr><td colspan="3">Performance Measures</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td colspan="3">Average response time (hrs)</td><td>0.335
(+1.51%)</td><td>0.330
(±0.0049)</td></tr>
<tr><td colspan="3">Average resolution time (hrs)</td><td>1.659
(+0.97%)</td><td>1.643
(±0.009)</td></tr>
<tr><td colspan="3">Average no of e-mails in the system</td><td>52.254
(+0.99%)</td><td>51.740
(±0.315)</td></tr>
<tr><td rowspan="2">Node</td><td colspan="2">Utilization</td><td colspan="2">Average queueing delay (hrs)</td><td colspan="2">Average number in the queue</td></tr>
<tr><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td>1</td><td>0.667
(+0.15%)</td><td>0.666
(± 0.001)</td><td>0.045
(+2.27%)</td><td>0.044
(± 0.001)</td><td>0.743
(+1.64%)</td><td>0.731
(± 0.011)</td></tr>
<tr><td>2</td><td>0.554
(+0.18%)</td><td>0.553
(± 0.002)</td><td>0.015
(0.00%)</td><td>0.015
(± 0.000)</td><td>0.229
(+1.33%)</td><td>0.226
(± 0.006)</td></tr>
<tr><td>3</td><td>0.865
(+0.35%)</td><td>0.862
(± 0.002)</td><td>0.283
(+4.04%)</td><td>0.272
(± 0.008)</td><td>4.760
(+4.29%)</td><td>4.564
(± 0.149)</td></tr>
</table>

Table 7.9: Scenario 1, Configuration 3 - Performance Measures for Hyperexponential Processing Times

Hyperexponential Processing Times (SCV =2.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.386 (+1.58%)	0.380 (± 0.005)		
Average resolution time (hrs)			1.726 (+0.99%)	1.709 (± 0.009)		
Average no of e-mails in the system			54.366 (+0.99%)	53.831 (± 0.289)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.667 (0.00%)	0.667 (± 0.001)	0.059 (+1.72%)	0.058 (± 0.001)	0.979 (+1.24%)	0.967 (± 0.016)
2	0.554 (+0.18%)	0.553 (± 0.001)	0.019 (+5.55%)	0.018 (± 0.000)	0.290 (+1.40%)	0.286 (± 0.004)
3	0.865 (+0.11%)	0.864 (± 0.002)	0.391 (+3.99%)	0.376 (± 0.012)	6.574 (+3.99%)	6.322 (± 0.214)

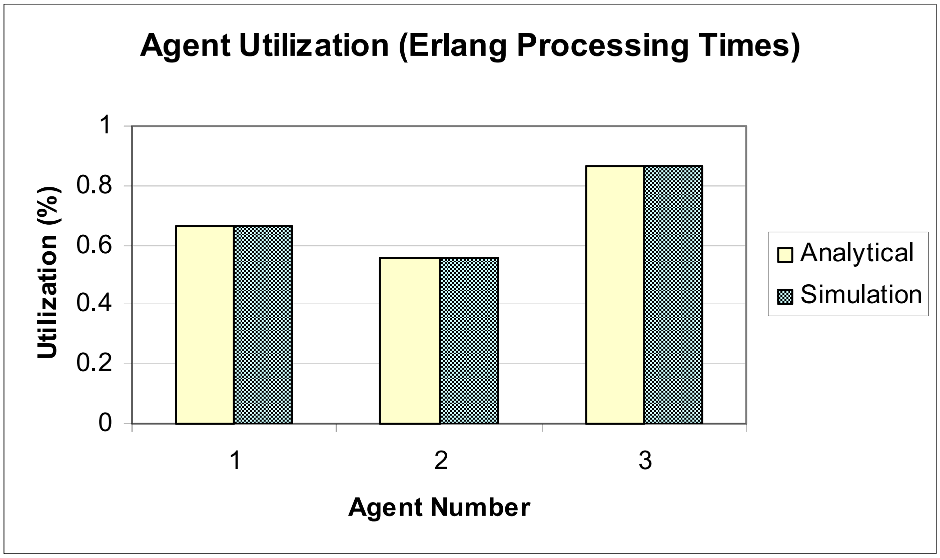

Figure 7.9: Scenario 1, Configuration 3 - Agent Utilization and Erlang Processing Times

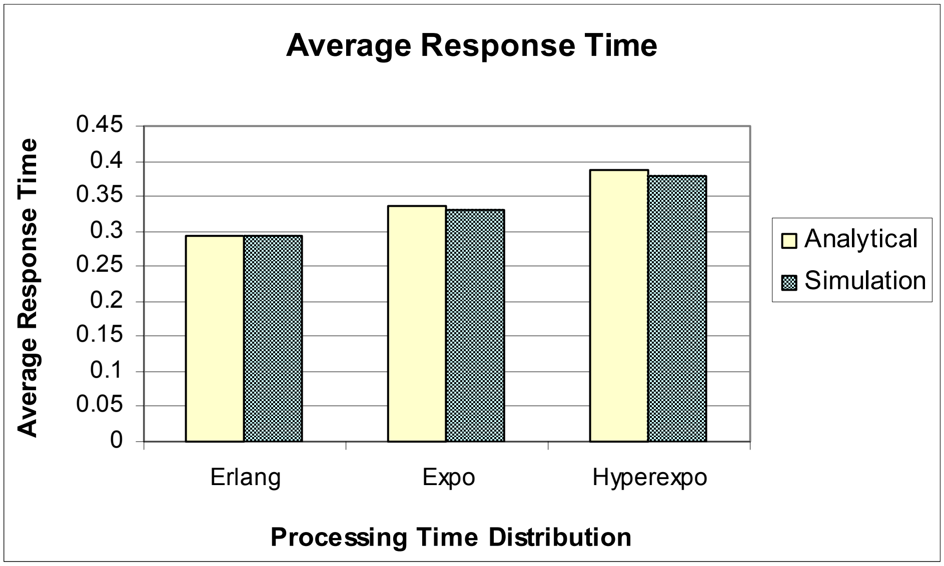

Figure 7.10: Scenario 1, Configuration 3 - Average Response Time

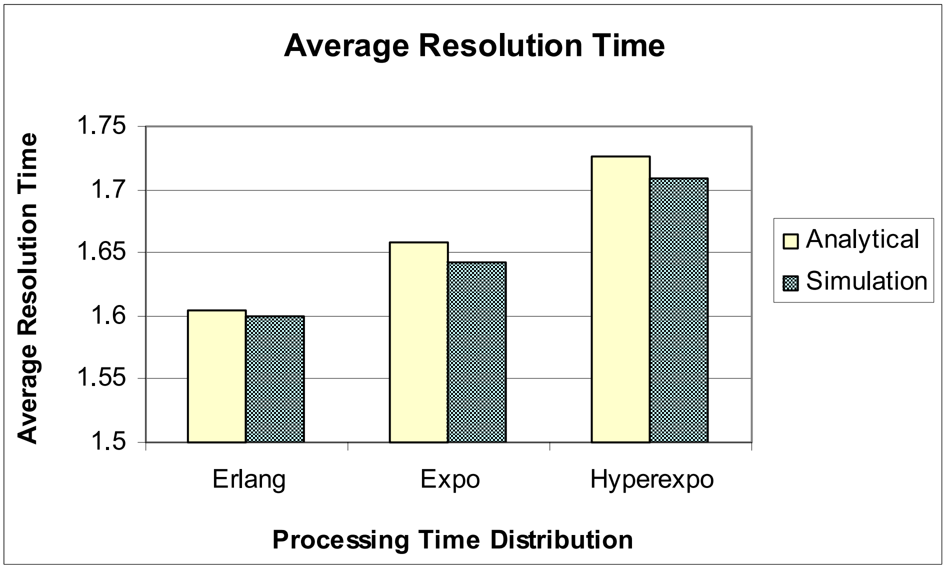

Figure 7.11: Scenario 1, Configuration 3 - Average Resolution Time

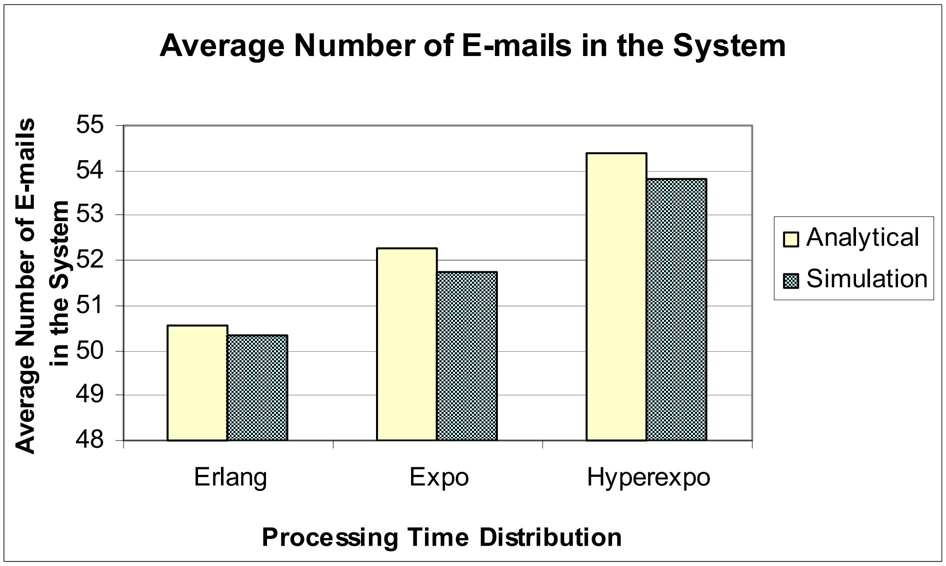

Figure 7.12: Scenario 1, Configuration 3 - Average Number of E-mails in the System

Table 7.10: Scenario 2, Configuration 1 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.367 (-0.81%)	0.370 (± 0.002)		
Average resolution time (hrs)			1.639 (+0.55%)	1.630 (± 0.005)		
Average no of e-mails in the system			68.851 (+0.51%)	68.502 (± 0.211)		
Node	Utilization		Average queueing delay (hrs)		Average number in the queue	
	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.853 (+0.23%)	0.851 (± 0.001)	0.128 (-0.77%)	0.129 (± 0.003)	2.698 (-1.06%)	2.727 (± 0.060)
2	0.906 (+0.22%)	0.904 (± 0.002)	0.267 (-1.48%)	0.271 (± 0.006)	5.139 (-1.23%)	5.203 (± 0.120)
3	0.811 (+0.12%)	0.810 (± 0.002)	0.085 (-2.30%)	0.087 (± 0.002)	1.777 (-2.42%)	1.821 (± 0.035)

Table 7.11: Scenario 2, Configuration 1 - Performance Measures for Exponential Processing Times

Exponential Processing Times (SCV =1.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.445 (+1.14%)	0.440 (± 0.004)		
Average resolution time (hrs)			1.740 (+1.04%)	1.722 (± 0.006)		
Average no of e-mails in the system			73.083 (+1.07%)	72.308 (± 0.291)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.853 (+0.35%)	0.850 (± 0.002)	0.182 (+1.68%)	0.179 (± 0.004)	3.853 (+2.31%)	3.766 (± 0.093)
2	0.906 (+0.33%)	0.903 (± 0.002)	0.388 (+3.19%)	0.376 (± 0.009)	7.478 (+3.54%)	7.222 (± 0.182)
3	0.811 (+0.12%)	0.810 (± 0.001)	0.120 (0.00%)	0.120 (± 0.002)	2.514 (-0.04%)	2.515 (± 0.055)

Table 7.12: Scenario 2, Configuration 1 - Performance Measures for Hyperexponential Processing Times

Hyperexponential Processing Times (SCV =2.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.543 (+3.43%)	0.525 (± 0.008)		
Average resolution time (hrs)			1.868 (+2.02%)	1.831 (± 0.011)		
Average no of e-mails in the system			78.439 (+1.94%)	76.943 (± 0.530)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.853 (+0.35%)	0.850 (± 0.002)	0.251 (+5.02%)	0.239 (± 0.010)	5.303 (+5.05%)	5.048 (± 0.215)
2	0.906 (+0.33%)	0.903 (± 0.002)	0.544 (+6.88%)	0.509 (± 0.021)	10.475 (+7.19%)	9.772 (± 0.422)
3	0.811 (0.00%)	0.811 (± 0.002)	0.163 (+1.87%)	0.160 (± 0.003)	3.423 (+2.33%)	3.345 (± 0.059)

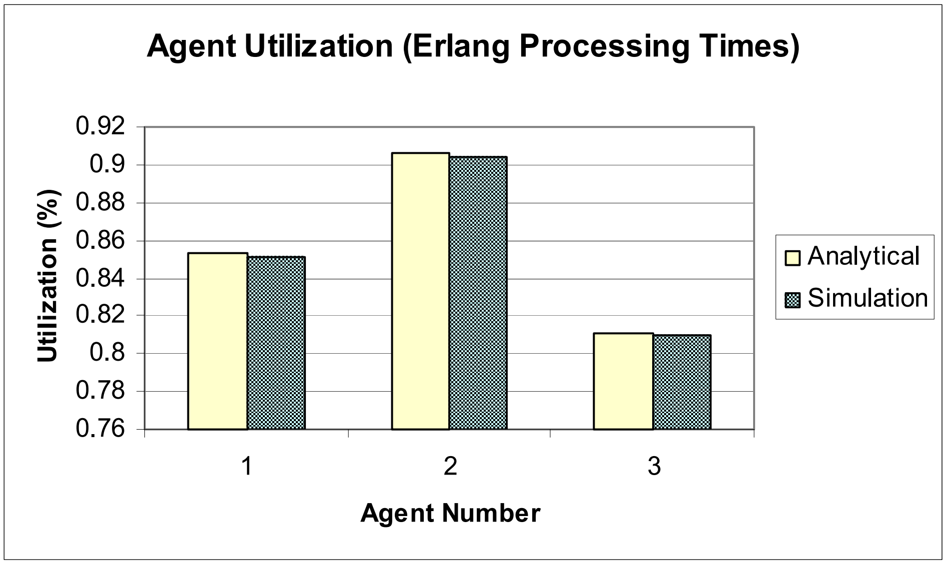

Figure 7.13: Scenario 2, Configuration 1 - Agent Utilization - Erlang Processing Times

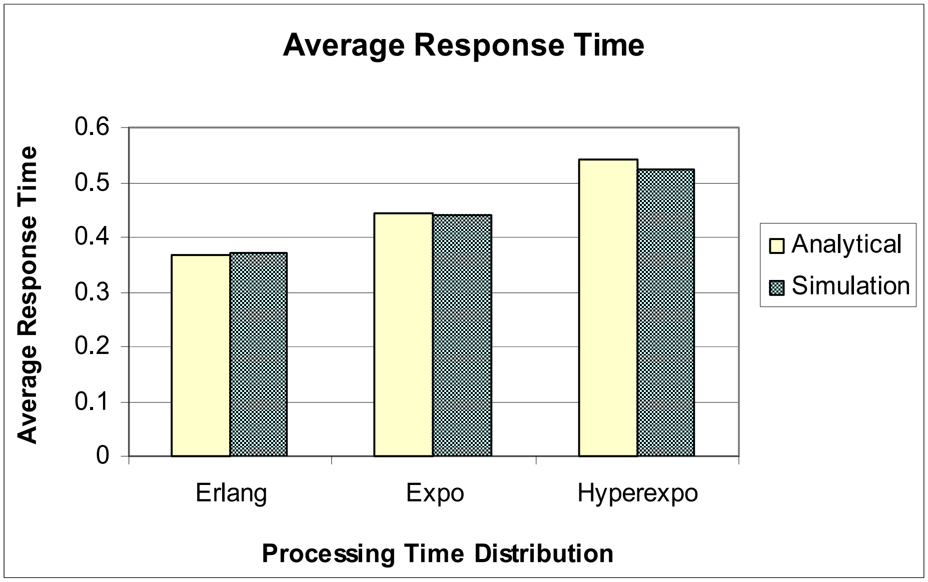

Figure 7.14: Scenario 2, Configuration 1 - Average Response Time

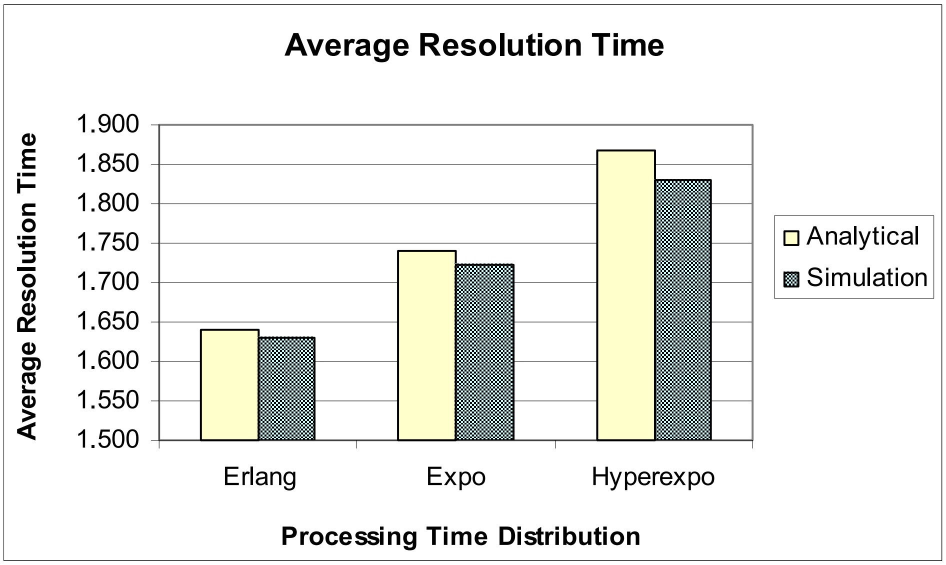

Figure 7.15: Scenario 2, Configuration 1 - Average Resolution Time

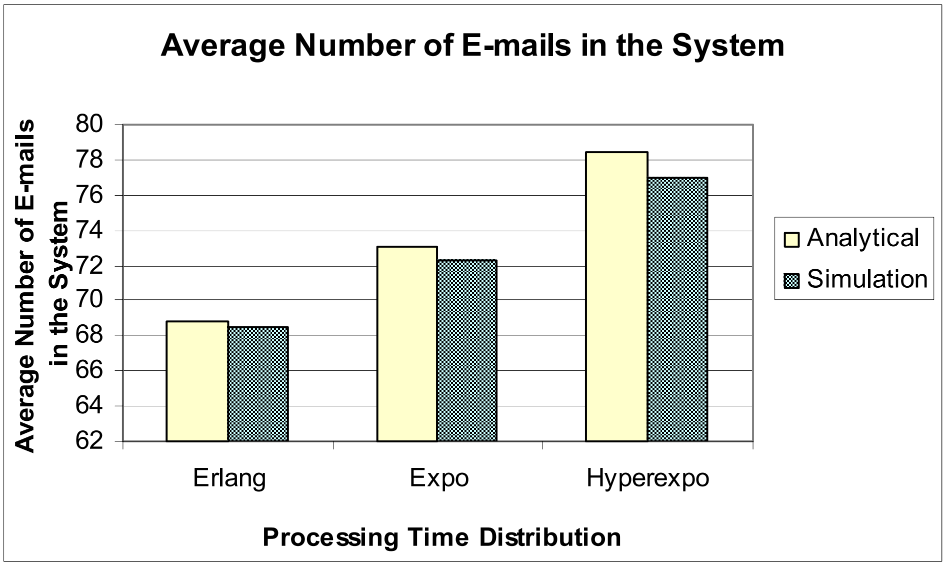

Figure 7.16: Scenario 2, Configuration 1 - Average Number of E-mails in the System

Table 7.13: Scenario 2, Configuration 2 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.286 (-0.35%)	0.287 (± 0.001)		
Average resolution time (hrs)			1.535 (+0.85%)	1.522 (± 0.004)		
Average no of e-mails in the system			64.476 (+0.90%)	63.903 (± 0.184)		
Node	Utilization		Average queueing delay (hrs)		Average number in the queue	
	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.853 (+0.23%)	0.851 (± 0.001)	0.128 (-1.54%)	0.130 (± 0.002)	2.699 (-1.53%)	2.741 (± 0.046)
2	0.724 (+0.28%)	0.722 (± 0.001)	0.040 (+2.56%)	0.039 (± 0.000)	0.762 (+0.79%)	0.756 (± 0.008)
3	0.811 (+0.25%)	0.809 (± 0.001)	0.085 (0.00%)	0.085 (± 0.001)	1.777 (-0.34%)	1.783 (± 0.031)

Table 7.14: Scenario 2, Configuration 2 - Performance Measures for Exponential Processing Times

Exponential Processing Times (SCV =1.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.327 (+0.93%)	0.324 (± 0.001)		
Average resolution time (hrs)			1.588 (+1.15%)	1.570 (± 0.003)		
Average no of e-mails in the system			66.680 (+1.10%)	65.953 (± 0.126)		
Node	Utilization		Average queueing delay (hrs)		Average number in the queue	
	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.853 (+0.35%)	0.850 (± 0.001)	0.182 (+2.25%)	0.178 (± 0.003)	3.853 (+2.93%)	3.743 (± 0.059)
2	0.724 (0.00%)	0.724 (± 0.001)	0.056 (0.00%)	0.056 (±0.001)	1.075 (0.00%)	1.075 (± 0.024)
3	0.811 (+0.25%)	0.809 (± 0.002)	0.12 (+1.69%)	0.118 (± 0.002)	2.514 (+2.02%)	2.464 (± 0.040)

Table 7.15: Scenario 2, Configuration 2 - Performance Measures for Hyperexponential Processing Times

Hyperexponential Processing Times (SCV =2.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.377 (+1.89%)	0.370 (± 0.003)		
Average resolution time (hrs)			1.652 (+1.44%)	1.629 (± 0.005)		
Average no of e-mails in the system			69.398 (+1.46%)	68.397 (± 0.192)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.853 (+0.35%)	0.850 (± 0.002)	0.251 (+5.46%)	0.238 (± 0.008)	5.302 (+5.72%)	5.015 (± 0.164)
2	0.724 (+0.14%)	0.723 (± 0.001)	0.075 (+2.74%)	0.073 (± 0.001)	1.437 (+1.84%)	1.411 (± 0.022)
3	0.811 (+0.25%)	0.809 (± 0.002)	0.163 (+1.87%)	0.160 (± 0.004)	3.422 (+2.27%)	3.346 (± 0.093)

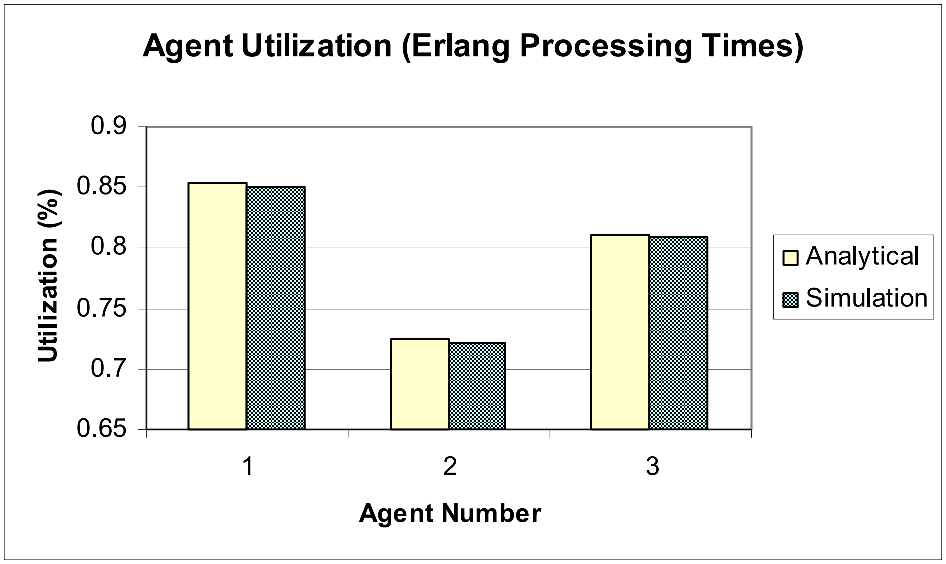

Figure 7.17: Scenario 2, Configuration 2 - Agent Utilization - Erlang Processing Times

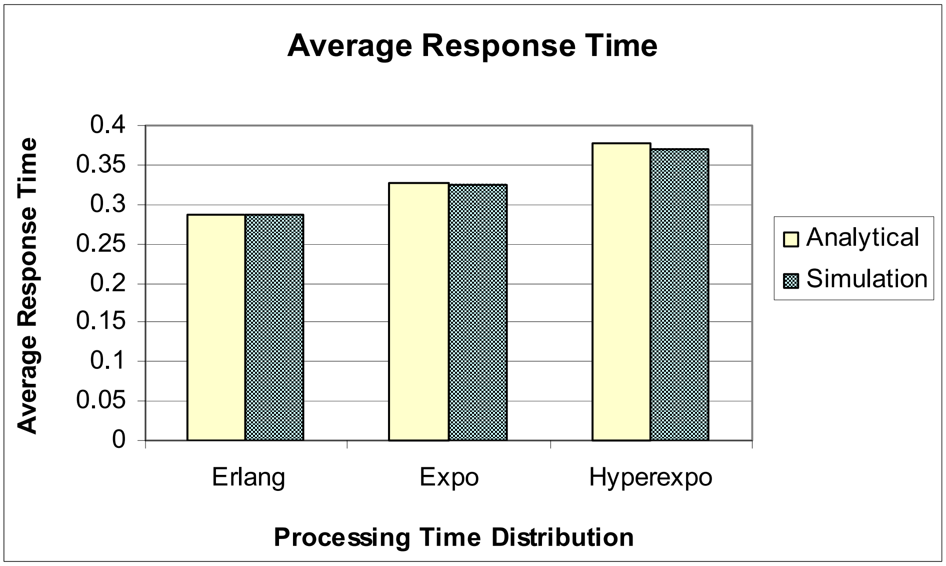

Figure 7.18: Scenario 2, Configuration 2 - Average Response Time

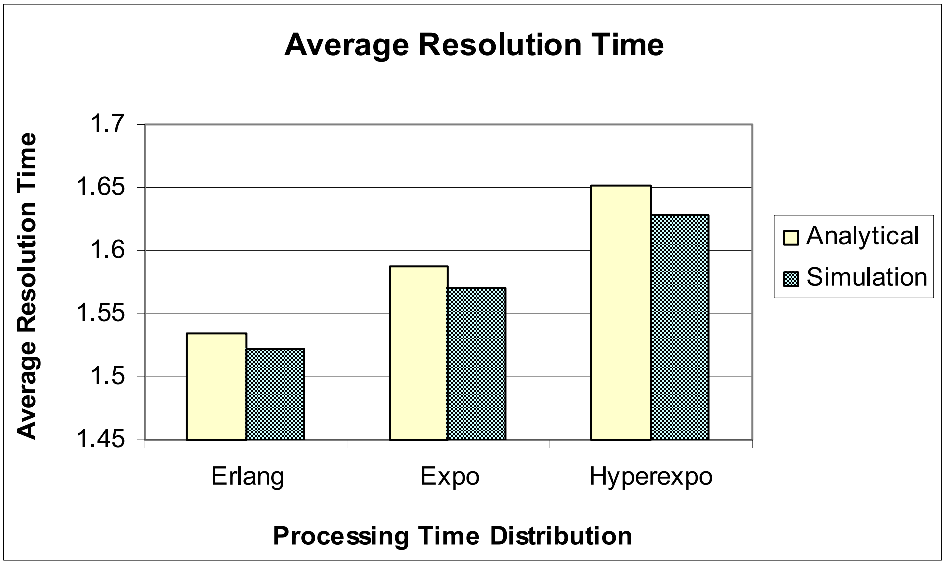

Figure 7.19: Scenario 2, Configuration 2 - Average Resolution Time

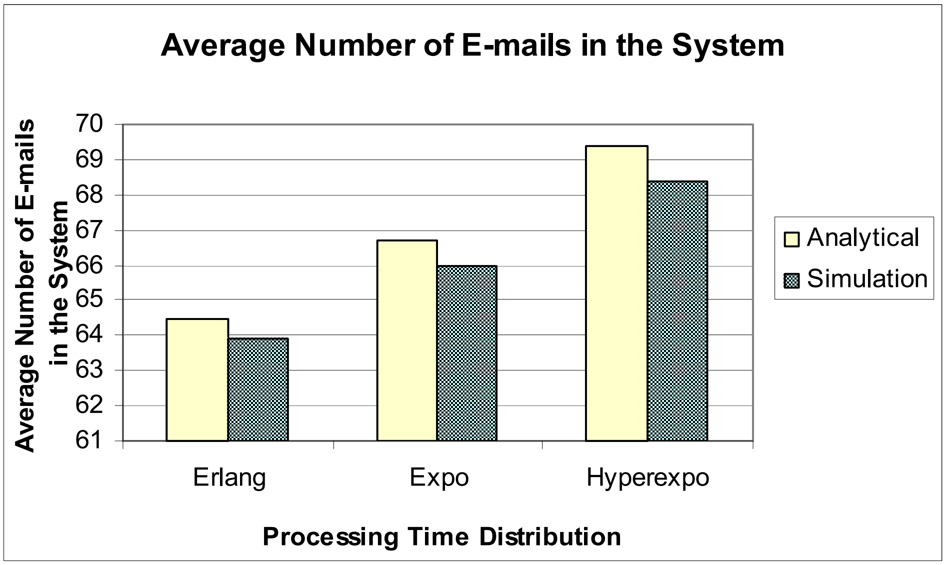

Figure 7.20: Scenario 2, Configuration 2 - Average Number of E-mails in the System

Table 7.16: Scenario 2, Configuration 3 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			0.250 (0.00%)	0.250 (± 0.001)		
Average resolution time (hrs)			1.489 (+0.88%)	1.476 (± 0.004)		
Average no of e-mails in the system			46.899 (+0.94%)	46.462 (± 0.165)		
Node	Utilization		Average queueing delay (hrs)		Average number in the queue	
	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.640 (+0.31%)	0.638 (± 0.001)	0.026 (0.00%)	0.026 (± 0.000)	0.419 (+0.48%)	0.417 (± 0.004)
2	0.543 (+0.18%)	0.542 (± 0.001)	0.010 (0.00%)	0.010 (± 0.000)	0.149 (+0.68%)	0.148 (± 0.002)
3	0.811 (+0.37%)	0.808 (± 0.001)	0.121 (-0.82%)	0.122 (± 0.002)	1.908 (+0.31%)	1.902 (± 0.027)

Table 7.17: Scenario 2, Configuration 3 - Performance Measures for Exponential Processing Times

<table>
<tr><th colspan="5">Exponential Processing Times (SCV =1.00)</th><th colspan="2" rowspan="5"></th></tr>
<tr><td colspan="3">Performance Measures</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td colspan="3">Average response time (hrs)</td><td>0.275
(0.00%)</td><td>0.275
(±0.002)</td></tr>
<tr><td colspan="3">Average resolution time (hrs)</td><td>1.521
(+1.00%)</td><td>1.506
(±0.005)</td></tr>
<tr><td colspan="3">Average no of e-mails in the system</td><td>47.913
(+0.98%)</td><td>47.450
(±0.169)</td></tr>
<tr><td rowspan="2">Node</td><td colspan="2">Utilization</td><td colspan="2">Average queueing delay (hrs)</td><td colspan="2">Average number in the queue</td></tr>
<tr><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td>1</td><td>0.640
(+0.47%)</td><td>0.637
(± 0.002)</td><td>0.037
(+2.78%)</td><td>0.036
(± 0.000)</td><td>0.580
(+2.47%)</td><td>0.566
(± 0.008)</td></tr>
<tr><td>2</td><td>0.543
(0.00%)</td><td>0.543
(± 0.001)</td><td>0.014
(0.00%)</td><td>0.014
(± 0.000)</td><td>0.201
(+2.03%)</td><td>0.197
(± 0.004)</td></tr>
<tr><td>3</td><td>0.811
(+0.25%)</td><td>0.809
(± 0.001)</td><td>0.172
(0.00%)</td><td>0.172
(± 0.004)</td><td>2.711
(+0.67%)</td><td>2.693
(± 0.062)</td></tr>
</table>

Table 7.18: Scenario 2, Configuration 3 - Performance Measures for Hyperexponential Processing Times

<table>
<tr><th colspan="5">Hyperexponential Processing Times (SCV =2.00)</th><th colspan="2" rowspan="5"></th></tr>
<tr><td colspan="3">Performance Measures</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td colspan="3">Average response time (hrs)</td><td>0.305
(+0.66%)</td><td>0.303
(± 0.003)</td></tr>
<tr><td colspan="3">Average resolution time (hrs)</td><td>1.560
(+1.16%)</td><td>1.542
(± 0.005)</td></tr>
<tr><td colspan="3">Average no of e-mails in the system</td><td>49.144
(+1.17%)</td><td>48.573
(± 0.131)</td></tr>
<tr><td rowspan="2">Node</td><td colspan="2">Utilization</td><td colspan="2">Average queueing delay (hrs)</td><td colspan="2">Average number in the queue</td></tr>
<tr><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td><td>Analytical</td><td>Simulation</td></tr>
<tr><td>1</td><td>0.640
(+0.31%)</td><td>0.638
(± 0.001)</td><td>0.048
(+2.13%)</td><td>0.047
(± 0.001)</td><td>0.759
(+2.29%)</td><td>0.742
(± 0.013)</td></tr>
<tr><td>2</td><td>0.543
(+0.18%)</td><td>0.542
(± 0.001)</td><td>0.018
(+5.88%)</td><td>0.017
(± 0.000)</td><td>0.253
(+4.54%)</td><td>0.242
(± 0.005)</td></tr>
<tr><td>3</td><td>0.811
(+0.12%)</td><td>0.810
(± 0.002)</td><td>0.236
(+3.06%)</td><td>0.229
(± 0.006)</td><td>3.710
(+3.17%)</td><td>3.596
(± 0.101)</td></tr>
</table>

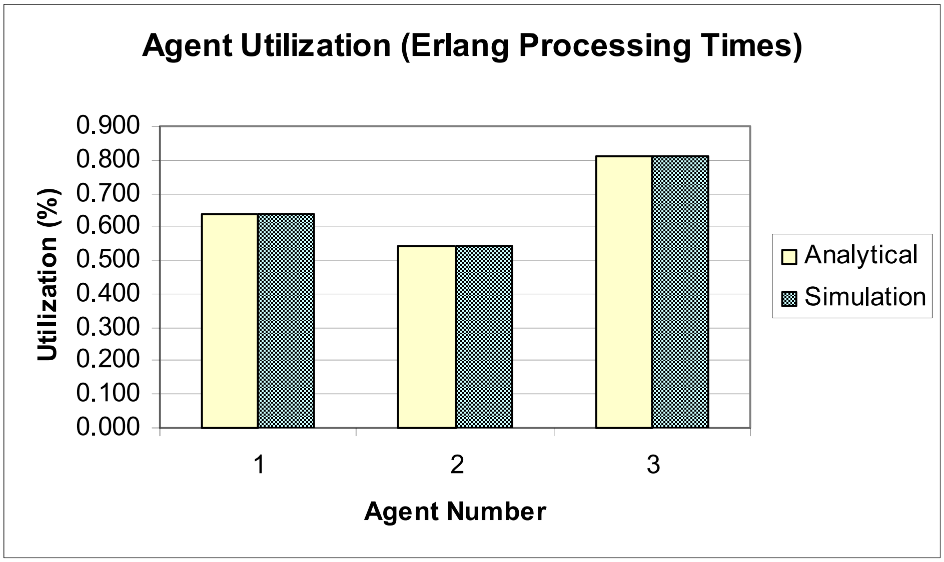

Figure 7.21: Scenario 2, Configuration 3 - Agent Utilization - Erlang Processing Times

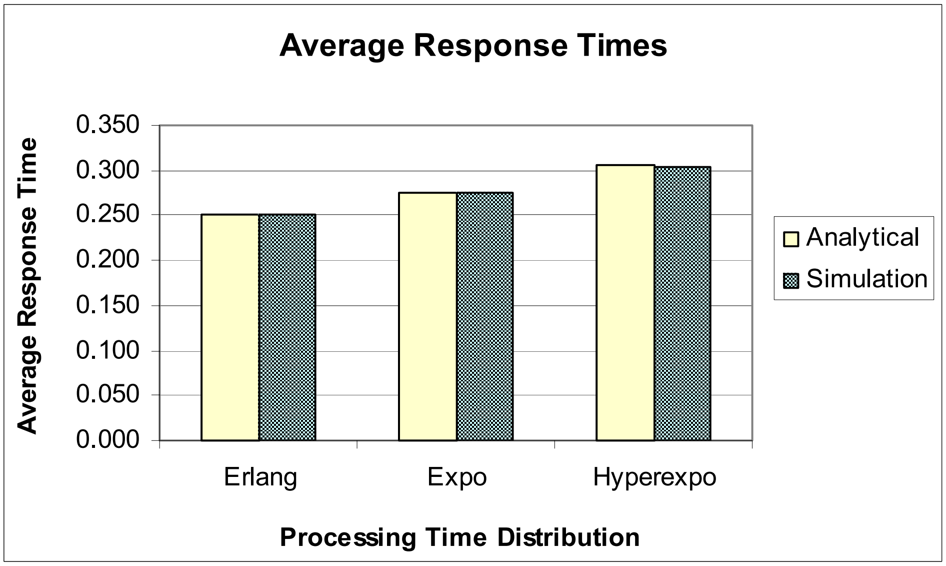

Figure 7.22: Scenario 2, Configuration 3 - Average Response Time

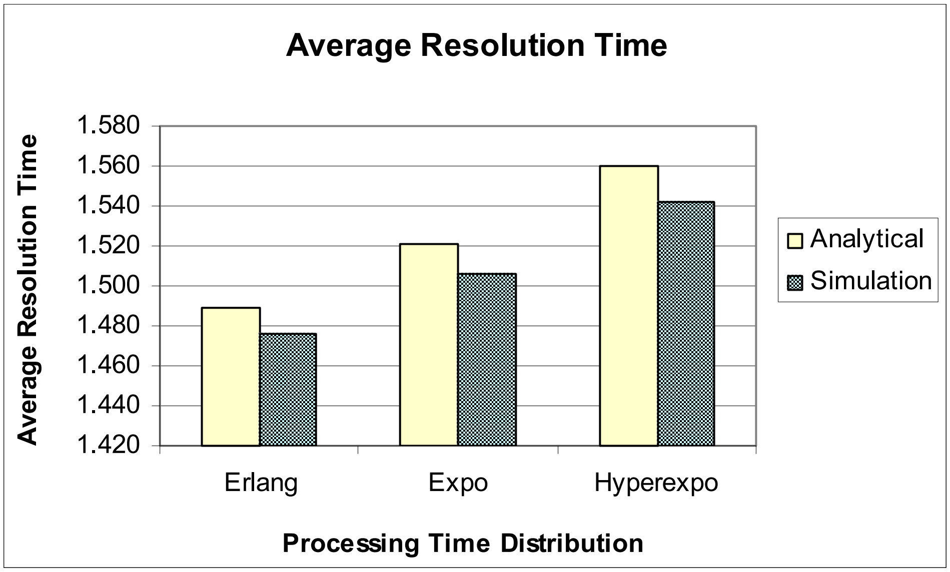

Figure 7.23: Scenario 2, Configuration 3 - Average Response Time

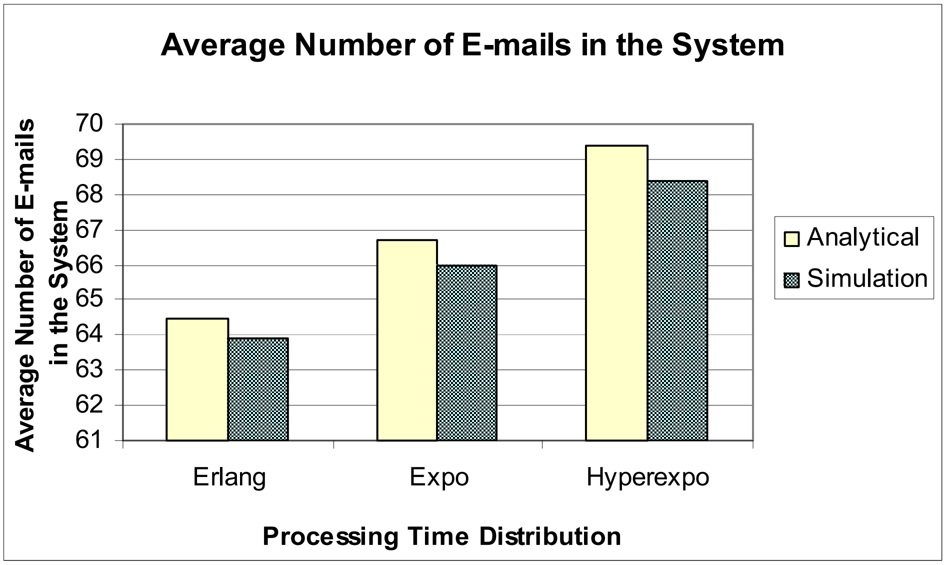

Figure 7.24: Scenario 2, Configuration 3 - Average Number of E-mails in the System

7.4 Modeling Service Interruptions

In a contact center, the agents can be interrupted by other types of work, for example, interaction with customer using telephone or the agents may decide to take a break. Although the focus is only on e-mails, but occasional work from other sources can be expected in an e-mail contact center. One cannot avoid these interruptions in real world situations. These interruptions are random in nature and have to be included in the analysis of the contact center operations to reflect the total workload and agent availability.

7.4.1 Contact Center Model with Service Interruptions

Service interruptions in a contact center can be modeled using the open queueing network model discussed in Section 6.3. Random service interruptions could be viewed as server "failures." The "repair" time is the duration of the interruption. The PD method can handle the failure and repair of servers. This feature is exploited to model service interruptions. If an agent is performing other knowledge work or taking a break, it is assumed that the agent is not available to process e-mails. The UP time can be from any one of the distributions like uniform, triangular or exponential. When the server is interrupted, the interruption time or the server DOWN time distribution should also be specified. Once the UP and DOWN time parameters are specified, they can be easily incorporated into the parametric-decomposition (PD) method. The PD method requires the number of nodes, means for the UP and DOWN time at each node and the SCV for the DOWN time at each node. The development of the discrete-time Markov chain (DTMC) based on the routing probabilities, analysis of the absorbing DTMC and the computations of weights remain the same as explained in Chapter 6. Within simulation, the resource UP and DOWN time distributions should also specified accordingly.

7.5 Numerical Experiments

The contact center example from Chapter 6 is used to test the modeling of interruptions. The external arrival rates and the parameters for the e-mails remain the same as in Section 6.6.2. In addition to the model parameters in Section 6.6.2, the UP time distribution and the DOWN time distribution of the server followed a uniform distribution with parameters (2, 4) hours and triangular distribution with parameters (5,10,15) minutes. The UP and DOWN time parameters were based on the authors engineering judgment. These additional parameters make the e-mail contact center example explained in Section 6.6.2 much more realistic. The entire situation was modeled for both the Scenarios 1 and Scenarios 2 explained in Section 6.6.2 and by method M3 only.

7.6 Results

Analytical and Simulation results for Scenario 1 are presented in Tables 7.19 through 7.21 and in Figures 7.25 through 7.28. Results for Scenario 2 are presented in Tables 7.22 through 7.24 and in Figures 7.29 through 7.32.

Table 7.19: Scenario 1 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			3.611 (+8.86%)	3.317 (±0.275)		
Average resolution time (hrs)			5.936 (+7.30%)	5.532 (±0.361)		
Average no of e-mails in the system			62.331 (+7.23%)	58.126 (±3.814)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.945 (+0.53%)	0.940 (±0.002)	1.966 (+10.82%)	1.774 (±0.108)	10.901 (+11.00%)	9.821 (±0.619)
2	0.977 (+0.20%)	0.975 (±0.003)	5.487 (+8.78%)	5.044 (±0.802)	28.326 (+8.85%)	26.023 (±4.200)
3	0.922 (+0.54%)	0.917 (±0.003)	1.325 (+10.14%)	1.203 (±0.065)	7.432 (+10.17%)	6.746 (±0.376)

Table 7.20: Scenario 1 - Performance Measures for Exponential Processing Times

Exponential Processing Times (SCV =1.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			5.009 (+12.06%)	4.470 (±0.383)		
Average resolution time (hrs)			7.761 (+10.21%)	7.042 (±0.502)		
Average no of e-mails in the system			81.487 (+10.10%)	74.013 (±5.359)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.945 (+0.21%)	0.943 (±0.003)	2.756 (+9.45%)	2.518 (±0.164)	15.279 (+9.48%)	13.956 (±0.948)
2	0.977 (+0.31%)	0.974 (±0.003)	7.782 (+16.18%)	6.698 (±0.927)	40.171 (+16.20%)	34.569 (±4.811)
3	0.922 (+0.44%)	0.918 (±0.003)	1.848 (+4.70%)	1.765 (±0.082)	10.364 (+4.65%)	9.904 (±0.463)

Table 7.21: Scenario 1 - Performance Measures for Hyperexponential Processing Times

Hyperexponential Processing Times (SCV =2.00)		
Performance Measures	Analytical	Simulation
Average response time (hrs)	6.872 (+6.76%)	6.437 (±1.004)
Average resolution time (hrs)	10.193 (+6.11%)	9.606 (±1.311)
Average no of e-mails in the system	107.024 (+6.22%)	100.76 (±13.753)

	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.945 (+0.42%)	0.941 (±0.003)	3.809 (+8.03%)	3.526 (±0.321)	21.114 (+8.40%)	19.478 (±1.788)
2	0.977 (+0.31%)	0.974 (±0.005)	10.841 (+6.11%)	10.217 (±2.569)	55.964 (+5.98%)	52.808 (±13.316)
3	0.922 (+0.65%)	0.916 (±0.003)	2.544 (+10.08%)	2.311 (±0.136)	14.273 (+10.47%)	12.921 (±0.779)

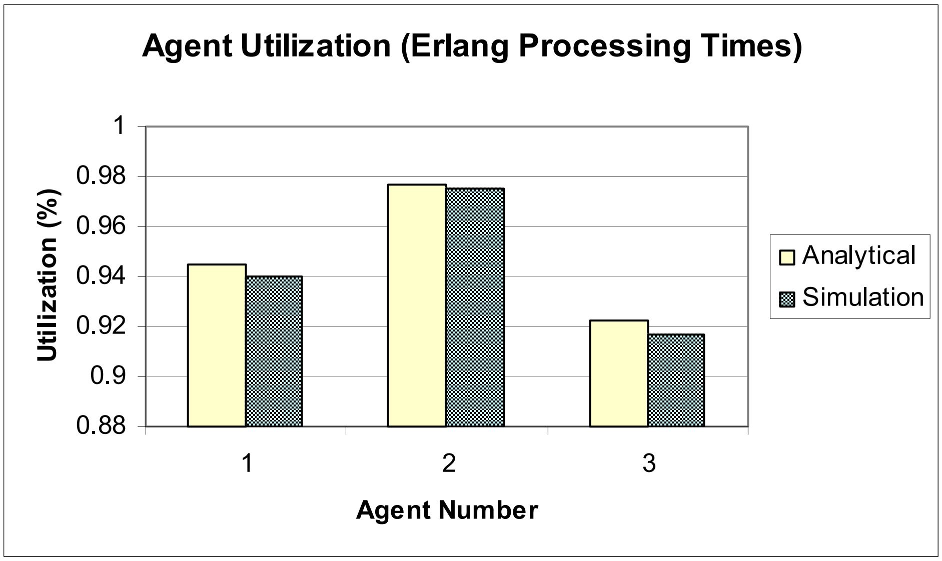

Figure 7.25: Scenario 1 - Agent Utilization - Erlang Processing Times

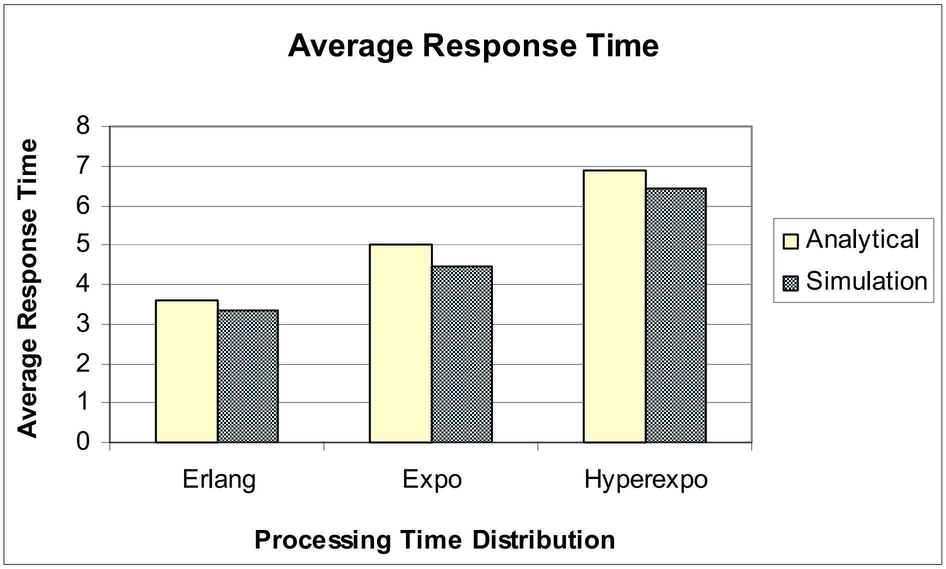

Figure 7.26: Scenario 1 - Average Response Time

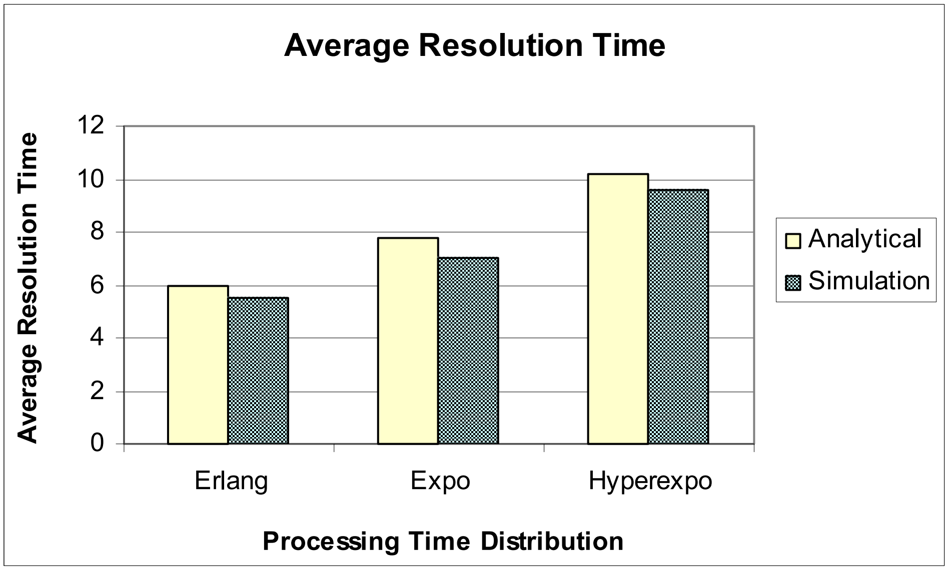

Figure 7.27: Scenario 1 - Average Resolution Time

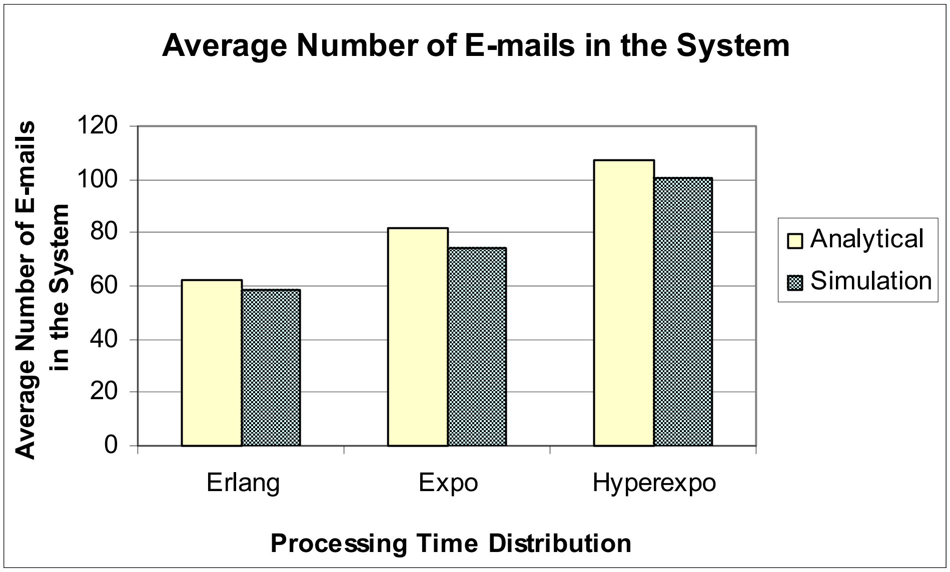

Figure 7.28: Scenario 1 - Average Number of E-mails in the System

Table 7.22: Scenario 2 - Performance Measures for Erlang Processing Times

Erlang Processing Times (SCV =0.25)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			1.984 (+0.30%)	1.978 (±0.093)		
Average resolution time (hrs)			3.73 (+0.81%)	3.700 (±0.117)		
Average no of e-mails in the system			39.168 (+0.64%)	38.920 (±1.265)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.905 (+0.11%)	0.904 (±0.003)	1.094 (0.00%)	1.094 (±0.060)	5.783 (+0.05%)	5.780 (±0.326)
2	0.958 (0.00%)	0.958 (±0.003)	3.059 (-0.16%)	3.064 (±0.210)	14.73 (-0.08%)	14.742 (±1.019)
3	0.864 (+0.11%)	0.863 (±0.003)	0.701 (+2.63%)	0.683 (±0.023)	3.676 (+2.74%)	3.578 (±0.127)

Table 7.23: Scenario 2 - Performance Measures for Exponential Processing Times

Exponential Processing Times (SCV =1.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			2.709 (+7.80%)	2.513 (±0.208)		
Average resolution time (hrs)			4.666 (+6.43%)	4.384 (±0.271)		
Average no of e-mails in the system			48.995 (+6.49%)	46.007 (±2.908)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.905 (-0.11%)	0.904 (±0.003)	1.526 (+7.54%)	1.419 (±0.062)	8.066 (+8.15%)	7.458 (±0.339)
2	0.958 (-0.94%)	0.956 (±0.003)	4.332 (+9.75%)	3.947 (±0.554)	20.860 (+10.01%)	18.961 (±2.703)
3	0.864 (0.00%)	0.862 (±0.004)	0.971 (+5.09%)	0.924 (±0.041)	5.091 (+5.38%)	4.831 (±0.222)

Table 7.19: Scenario 2 - Performance Measures for Hyperexponential Processing Times

Hyperexponential Processing Times (SCV =2.00)						
Performance Measures			Analytical	Simulation		
Average response time (hrs)			3.675 (+8.44%)	3.389 (±0.238)		
Average resolution time (hrs)			5.914 (+7.23%)	5.515 (±0.308)		
Average no of e-mails in the system			62.096 (+7.06%)	58.001 (±3.304)		
	Utilization		Average queueing delay (hrs)		Average number in the queue	
Node	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
1	0.905 (0.00%)	0.905 (±0.004)	2.102 (+6.05%)	1.982 (±0.115)	11.109 (+6.12%)	10.468 (±0.627)
2	0.958 (+0.20%)	0.956 (±0.004)	6.029 (+10.87%)	5.438 (±0.654)	29.031 (+11.03%)	26.145 (±3.213)
3	0.864 (+0.12%)	0.863 (±0.004)	1.331 (+5.80%)	1.258 (±0.067)	6.977 (+5.90%)	6.588 (±0.361)

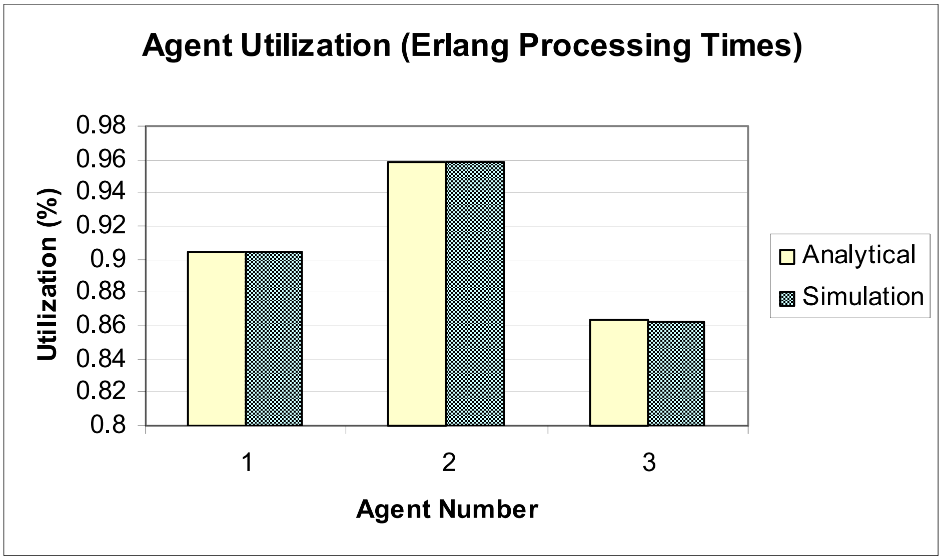

Figure 7.29: Scenario 2 - Agent Utilization - Erlang Processing Times

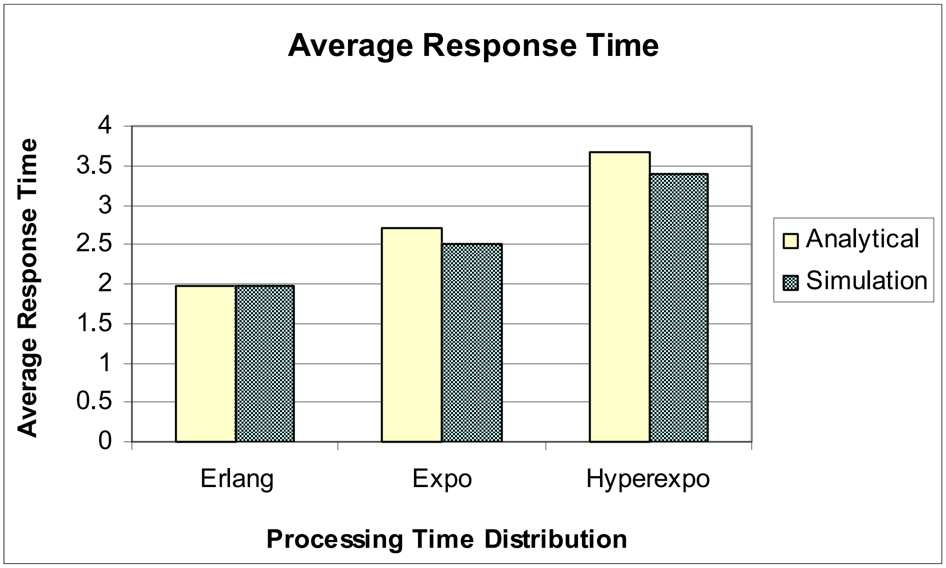

Figure 7.30: Scenario 2 - Average Response Time

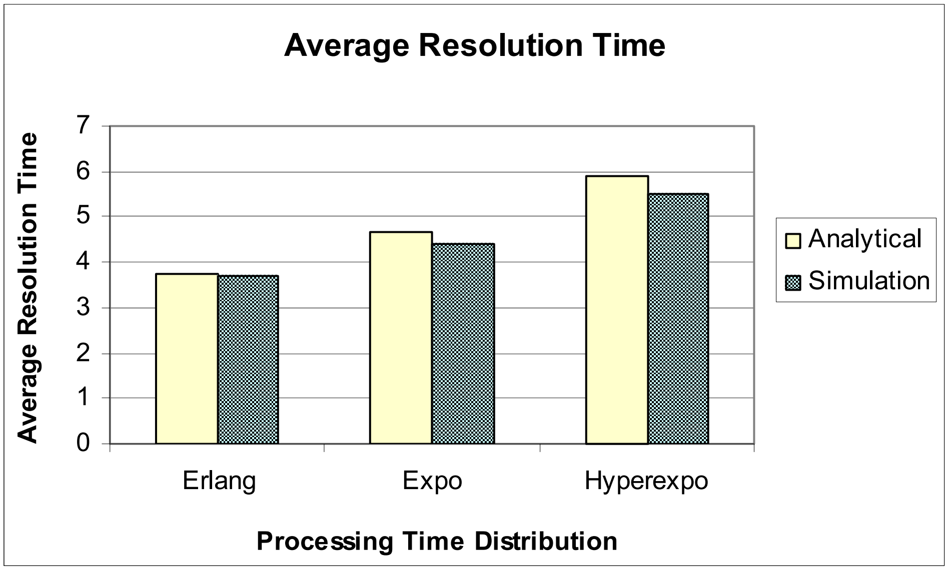

Figure 7.31: Scenario 2 - Average Resolution Time

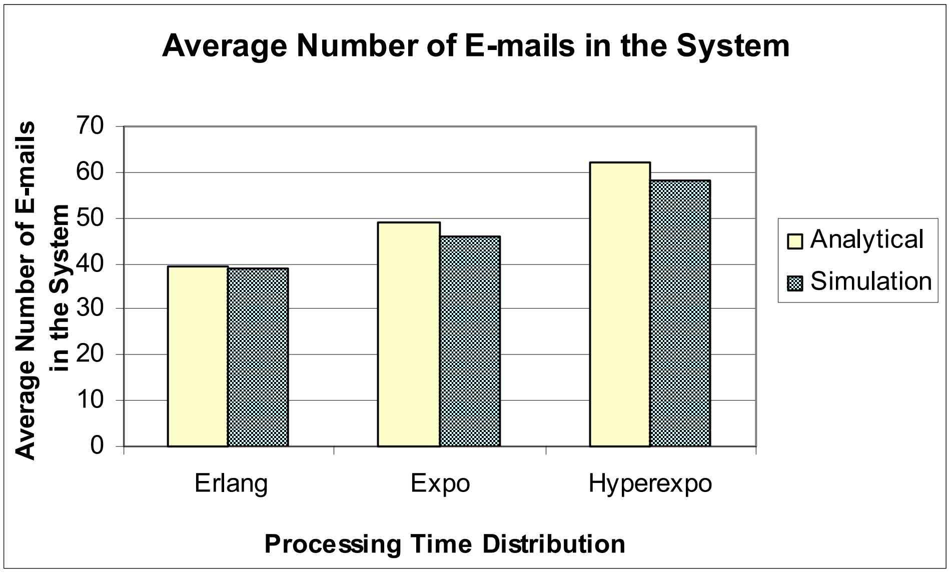

Figure 7.32: Scenario 2 - Average Number of E-mails in the System

The utilizations of the agents increased by approximately 5%. This was expected because the proportion of an agent's DOWN time was around 5% for the UP and DOWN time parameters chosen. The analytical model utilizations matched the simulation utilizations closely indicating that analytical model accurately tracks the total load on the agent. As the effective utilizations increased beyond 90% in some cases, we expected larger error rates in the performance measures because of the steep increase in waiting times for small increments in utilizations.

CHAPTER 8

CONCLUSIONS AND FUTURE RESEARCH

This chapter summarizes the research completed as part of this thesis effort and provides suggestions and extensions for future research in contact center modeling. The chapter is organized as follows. Section 8.1 provides a summary of research that has been completed. Section 8.2 summarizes the contribution made by the successful completion of the research, and Section 8.3 presents some suggestions for future research in the area of contact center modeling.

8.1 Research Summary

The objective of this research was to develop queueing-based analytical models for inbound e-mail customer contact center operations that can handle the dependence of processing time of an e-mail and it's routing on e-mail processing history. A multi-class open queueing network model was developed for an e-mail customer contact center. A novel approach to model the processing history of an e-mail was developed. This approach extended the traditional parametric decomposition (PD) method for solving multi-class open queueing networks to more general situations. The approach was based on the development of a discrete-time Markov chain (DTMC) with an appropriate state space to model the non-Markovian routing of an e-mail in the contact center until its resolution. The analysis of the discrete-time Markov chain led to computation of the proportions of e-mails in an agent's inbox that are new, previously processed by the same agent, and previously processed by others. Using these proportions a new "history-based" aggregation step was introduced for each e-mail type. This method precedes the existing class-type aggregation step in the PD method. The robustness of the approach was demonstrated by modeling different real-life scenarios of an e-mail contact center. The approach was tested for many situations including skill-based grouping of agents and random service time interruptions for a wide range of service time variability. In all the experiments conducted, the analytical model performed extremely well when compared to simulation estimates of performance measures including average response time and average resolution time.

8.2 Research Contributions

Bulk of the literature in call-center modeling is due to the research contributions by Avishai Mandelbaum (Technion - Israel Institute of Technology), Ger Koole (Vrije Universiteit), Noah Gans (Wharton School, University of Pennsylvania) and Ward Whitt (Columbia University). Their research covered almost all areas of call-center modeling including skill-based routing, staffing, call blending, analysis of call center data, resource pooling and organizational behavior. Research work on customer contact center operations is very scarce because of its recent emergence. The author's motivation in this research effort was due to the excellent contributions made by all the above researchers and especially, statements made by Whitt (2002b) with regard to the need for additional research in modeling contact centers. The specific contributions of this research effort are as follows.

1. Development of a novel modeling and solution approach to handle routing and processing schemes that are dependent on e-mail history. The approach developed is very general and extends the power of existing queueing network models.
2. Development of queueing models that can be incorporated into rapid analysis tools that can support the analysis and design of customer contact center operations.

8.3 Future Research Directions

- The history-based aggregation method provides an opportunity to model other situations where the processing time of an entity may depend on its processing history. One such example is in manufacturing operations, where the time to process a machine component and its subsequent routing could depend on the component's processing history. For example, the processing time may depend on whether the part is new or whether it has been reworked. This scenario is usually never dealt with in manufacturing operations because of the modeling complexity.
- As the proposed approach precedes the existing class-based aggregation extension to the PD method, the new approach when incorporated into existing queueing software like RAQS will increase the modeling power of queueing networks.
- Research needs to be conducted to find a way to model other queueing disciplines like priorities of e-mails in contact center operations, which would make the models developed much more realistic and useful.

8.3.1 Extensions to Model Agent Schedules

Modeling server schedules using queueing theory is difficult. For example, in contact centers e-mails arrive continuously, but agents may be scheduled for only 8 hours a day. The queueing network models considered in this thesis assume continuous arrival of e-mail and continuous (24/7) operation of the customer contact center. In reality e-mails arrive continuously throughout the day, while the agents may operate on a limited schedule basis. In such situations, either the arrival parameters or the service time parameters could be modified so that the e-mail can arrive and be processed on a continuous basis. One approach would be to increase the arrival rate to conserve the total arrival over a 24-hour period. The other approach would be to increase the service rate to clear/process the e-mail that arrived during time the server was not available. The arrival and service time SCVs could be set at their original values. Extensive numerical experimentation is needed to test the validity of these ideas. Furthermore, adjustments to the average response and resolution times and the number of e-mail need to be made to account for the "dead" time, which includes the time the contact center may be closed during a day.

Modeling agent schedules seems to offer a rich set of open problems for future research. While we can capture the agent utilizations, by adjusting arrival or service rates, it is not clear as to how well some of the suggestions outlined earlier would work in capturing the actual performance measures.

REFERENCES

1. Abate, J., W. Whitt. 1998. Calculating Transient Characteristics of the Erlang Loss Model by Numerical Transform Inversion. *Stoch. Models* **14**(3) 663-680.
2. Abate, J., W. Whitt. 1999. Computing Laplace Transforms for Numerical Transform via Continued Fractions. *INFORMS J. on Computing* **11**(4) 394-405.
3. Abou-El-Ata, M.O., A.M.A. Hariri. 1992. The M/M/c/N Queue with Balking and Reneging. *Computers and Operations Research* **19**(8) 713-716.
4. Armony, M., C. Maglaras. 2004a. On Customer Contact Centers with a Call-back Option: Customer Decisions, Routing Rules, and System Design. *Operations Research* **52**(2) 271-292.
5. Armony, M., C. Maglaras. 2004b. Contact Centers with a Call-Back Option and Real-Time Delay Information. *Operations Research* **52**(4) 527-545.
6. Bae, J., S. Kim, E.Y.Lee. 2001. The Virtual Waiting Time of the M/G/1 Queue with Impatient Customers. *Queueing Systems* **38**(4) 485-494.
7. Baskett, F., K. M. Chandy, R. R. Muntz, F. Palacios-Gomez. 1975. Open, Closed, and Mixed Networks of Queues with Different Classes of Customers. *Journal of the ACM* **22**(2) 248-260.
8. Benchmark Portal. 2004a. Ask Dr. Jon. http://www.benchmarkportal.com/newsite/article_detail.taf?topicid=14, last accessed on 01/22/05.
9. Benchmark Portal. 2004b. E-mail response time. http://www.benchmarkportal.com/newsite/article_detail.taf?topicid=241, last accessed on 01/22/05.

10. Benchmark Portal. 2004c. E-mail turnaround time. http://www.benchmarkportal.com/newsite/article_detail.taf?topicid=258, last accessed on 01/22/05.

11. Bhulai, S., G. Koole. 2003. A Queueing Model for Call Blending in Call Centers. *IEEE Transactions on Automatic Control* **48**(8) 1434 -1438.

12. Boots, M. N., H. Tijms. 1999. A Multiserver Queueing System with Impatient Customers. *Management Science* **45**(3) 444 - 448.

13. Brandt, A., M. Brandt. 1999. On the M(n)/M(n)/s Queue with Impatient Calls. *Performance Evaluation* **35**(1)1-18.

14. Brandt, A., M. Brandt. 2002. Asymptotic Results and a Markovian Approximation for the M (n)/ M(n)/s + GI System. *Queueing Systems* **41**(1) 73-94.

15. Chinnaswamy, M., M. Kamath, R. Greve. 2004. Queueing Models for Analyzing Customer Contact Center Operations. *Proceedings of the 2004 Industrial Engineering Research Conference*. Houston, TX, CD-ROM Proceedings.

16. Cleveland, B., J. Mayben. 1997. *Call Center Management on Fast Forward*. Call Center Press. MD.

17. Feinberg, M.A. 1990. Performance Characteristics of Automated Call Distribution Systems. '*GLOBE - COM*: *IEEE Global Telecommunications Conference and Exhibition. 'Communications: Connecting the Future'*. NY, 415-419.

18. Gail, T., L. Scott. 1997. A Field Study Investigating the Effect of Waiting Time on Customer Satisfaction. *Journal of Psychology* **131**(6) 655-660.

19. Gans, N., G. Koole, A. Mandelbaum. 2003. Telephone Call Centers: Tutorial, Review and Research Prospects. *Manufacturing and Service Operations Management* **5**(2) 79-141.

20. Gans, N., Y. Zhou. 1999. Managing Learning and Turnover in Employee Staffing. *Operations and Management Information Department. Financial Institutions Center. The Wharton School. University of Pennsylvania* http://knowledge.wharton.upenn.edu/papers/905.pdf, last accessed on 01/09/05.

21. Garnett, O. A. Mandelbaum. 2000. An Introduction to Skill-Based Routing and its Operational Complexities. *Faculty of Industrial Engineering and Management. Technion - Israel Institute of Technology. Haifa 32000.* http://iew3.technion.ac.il/serveng/References/SBR_Wharton_CCFORUM_Short_May03.pdf, last accessed on 01/09/05.

22. Green, L.V., P.J. Kolesar, J. Soares. 2001. Improving the SIPP Approach for Staffing Service Systems that have Cyclic Demands. *Oper. Res.* **49**(4) 549-564.

23. Greve, R., R. Sharda, M. Kamath, A. Kadam. 2004. Modeling and Analysis of Email Management for Improved Customer Relationship Management. *International Journal of Simulation and Process Modeling* (to appear).

24. Halfin, S., W. Whitt. 1981. Heavy-Traffic Limits for Queues with Many Exponential Servers. *Operations Research* **29**(3) 567-588.

25. Harris, C. M., K. L. Hoffman, P. B. Saunders. 1987. Modeling the IRS Telephone Taxpayer Information System. *Operations Research* **35**(4) 504-523.

26. Jackson, J. R. 1957. Networks of Waiting Lines. *Operations Research* **5**(4) 518-527.

27. Jennings, O. B., A. Mandelbaum, W. A. Massey, W. Whitt. 1996. Server Staffing to Meet Time-Varying Demand. *Management Science* **42**(10) 1383-1394.

28. Kamath, M., S. Sivaramakrishnan. G. Shirhatti. 1995. RAQS: A Software Package to Support Instruction and Research in Queueing Systems. *4th Industrial Engineering Research Conference Proceedings*. Nashville, TN, 944-953.

29. Kelton, W.D., R. P. Sadowski, D. A. Sadowski. 2002. Simulation with Arena. *The McGraw-Hill Companies*, Inc. NY.

30. Koole, G. 2001. The Mathematics of Call Centers. Research Highlight. *Biennual Report Stieltjes Institute. Department of Stochastics. Vrije Universiteit. Amsterdam*. 31-35. http://www.math.vu.nl/~koole/articles/highlight01/art.pdf, last accessed on 12/23/04.

31. Koole, G., A. Mandelbaum. 2001. Queueing Models of Call Centers - An Introduction. *Technical Report 2001-7. Department of Stochastics. Vrije Universiteit. Amsterdam*. http://www.math.vu.nl/~koole/articles/aor02/report.pdf, last accessed on 12/23/04.

32. Koole, G., A. Mandelbaum. 2002. Queueing Models of Call Centers - An Introduction. *Annals of Operations Research* **113**(1) 41-59.

33. Koole, G., A. Pot, J. Talim. 2003. Routing Heuristics for Multi-Skill Call Centers. *Proceedings of the Winter Simulation Conference*. LO, 1813-1816.

34. Koole, G., J. Talim. 2000. Exponential Approximation of Multi-Skill Call Centers Architecture. *Proceedings of QNETs* **23** 1-10.

35. Kuehn, P. J. 1979. Approximate Analysis of General Queueing Networks by Decomposition. *IEEE Transactions on Communication* **27** 113-126.

36. Larson, R. C. 1987. Perspective on Queues: Social Justice and the Psychology of Queueing. *Operations Research* **35**(6) 895-905.

37. Lipsman, R. 2001. *A Guide to MATLAB*. Cambridge University Press. USA.

38. Mandelbaum, A. 2001. Call Centers (Centres) Research Bibliography with Abstracts. *Faculty of Industrial Engineering and Management. Technion- Israel Institute of Technology. Haifa 32000.* http://iew3.technion.ac.il/serveng/References/ccbib.pdf, last viewed on 12/23/04.

39. Mandelbaum, A., G. Pats 1995. State-Dependent Queues: Approximations and Applications. *Stochastic Networks. IMA Volumes in Mathematics and its Applications.* F.P.Kelly and R.L.Williams. eds., *Springer*, NY, 239-282.

40. Mandelbaum, A., M. I. Reiman. 1998. On Pooling in Queueing Networks. *Management Science* **44**(7) 971-981.

41. Mandelbaum, A., A. Sakov, S. Zeltyn. 2001. Empirical Analysis of a Call Center. *Faculty of Industrial Engineering and Management. Technion-Israel Institute of Technology. Haifa 32000.* http://iew3.technion.ac.il/serveng/References/ccdata.pdf last viewed on 12/23/04.

42. Mehrotra, V. 1997. Ringing Up Big Business. *OR/MS Today* **24** (4) 18-24.

43. Montazer-Haghighi, A., J. Medi, S. G. Mohanty. 1986. On a Multiserver Markovian Queueing System with Balking and Reneging. *Computers and Operations Research* **13**(4) 421-425.

44. Movaghar A. 1998. On Queueing with Customer Impatience until the Beginning of Service. *Queueing Systems* **29**(4) 337-350.

45. Perry, M., A. Nilsson. 1992. Performance Modeling of Automatic Call Distributors: Assignable Grade of Service Staffing. *Proceedings of the 14th International Switching Symposium* 294-298.

46. Ramakumar, R. 1993. *Engineering Reliability: Fundamentals and Applications*. Prentice Hall. NJ.

47. Reiser, M., H. Kobayashi. 1974. Accuracy of the Diffusion Approximation for Some Queueing Systems. *IBM J. on Research and Development* **18** 110-124.

48. Segal, M. 1974. The Operator-Scheduling Problem: A Network-Flow Approach. *Operations Research* **22**(4) 808-823.

49. Sharda, R., G. Frankwick, O. Turetken. 1999. Group Knowledge Networks: A Framework and an Implementation. *Information Systems Frontiers* **1**(1) 221-239.

50. Smith, D. R., W. Whitt. 1981. Resource Sharing for Efficiency in Traffic Systems. *Bell System Technical Journal* **60**(13) 39-55.

51. Srikant, R., W. Whitt. 1996. Simulation Run Lengths to Estimate Blocking Probabilities. *ACM Trans. on Modeling and Computer Simulation* **6**(1) 7-52.

52. Stolletz, R. 2003. *Performance Analysis and Optimization of Inbound Call Centers*. Springer. Germany.

53. Suri, R., J. L. Sanders, M. Kamath. 1993. Performance Evaluation of Production Networks. *Handbooks in Operations Research and Management Science, Vol. 4*: *Logistics of Production and Inventory*. (S.C. Graves et al., Eds.). *Elsevier Science Publishers*. B.V, 194-286.

54. Vvedenskaya, N.D., R .L. Dobrushin, F.I. Karpelovich. 1996. Queueing Systems with Selection of the Shortest of Two Queues: An Asymptotic Approach. *Problems in Information Transmission* **32**(1) 15-29.

55. Wallace, R.B. W. Whitt. 2004. A Staffing Algorithm for Call Centers with Skill-Based Routing. Working paper. Submitted to *Manufacturing and Service Operations Management* http://www.columbia.edu/~ww2040/pooling3.pdf, last accessed on 01/06/2005.

56. Welch, P.D. 1983. The Statistical Analysis of Simulation Results. *Computer Performance Mod. Handbook*. S. S. Lavenberg. Ed. *Acad. Press*. NY, 268-328.

57. Whitt, W. 1983. The Queueing Network Analyzer. *Bell System Technical Journal* **62**(9) 2779-2815.

58. Whitt, W. 1986. Deciding which Queue to Join: Some Counter Examples. *Operations Research* **34**(1) 55-62.

59. Whitt, W. 1992. Understanding the Efficiency of Multi-Server Service Systems. *Management Science* **38**(5) 708-723.

60. Whitt, W. 1993. Approximations for the GI/G/m Queue. *Production and Operations Management* **2**(2) 114-161.

61. Whitt, W. 1994. Towards Better Multi-Class Parametric-Decomposition Approximations for Open Queueing Networks. *Annals of Oper. Res.*. **48** 221-248.

62. Whitt, W. 1999a. Decomposition Approximations for Time-Dependent Markovian Queueing Networks. *Operations Research Letters* **24**(3) 97-103.

63. Whitt, W. 1999b. Dynamic Staffing in a Telephone Call Center Aiming to Immediately Answer all Calls. *Operations Research Letters* **24**(5) 205-212.

64. Whitt, W. 2002a. Stochastic Models for the Design and Management of Customer Contact Centers: Some Research Directions. http://www.columbia.edu/~ww2040/full.pdf, last accessed on 12/23/04.

65. Whitt, W. 2002b. *Stochastic Process Limits*. Springer. NY.

66. Winston, W. 1977. Optimality of the Shortest Line Discipline. *Journal of Applied Probability* **14** 181-189.

APPENDIX A1

FUNDAMENTAL MATRIX ENTRIES FOR SCENARIOS 1 AND 2

This appendix presents the fundamental matrix that contains the average number of visits to a state in the DTMC before absorption. The matrix is presented for both Scenarios 1 and 2. There are three delay nodes (equal to the number of agents) and one new node (where new e-mails enter the agents' inbox). Tables A1.1 through A1.8 give the average visits for e-mail type 1 and Tables A1.9 through A1.16 give the average visits for the e-mail type 2. MATLAB codes for matrix inversion and hyperexponential distribution parameters calculation used in the simulation are also presented in this appendix.

Table A1.1: Scenario 1, Type 1 E-mails - Average Number of Visits by Solving the DTMC

	D1	D2	D3	New	011	012	013	021	022	023
D1	1.0685	0.04386	0.042153	0	0	0	0	0	0	0
D2	0.06743	1.0667	0.098882	0	0	0	0	0	0	0
D3	0.083316	0.048766	1.0655	0	0	0	0	0	0	0
New	**0.11414**	**0.10415**	**0.11179**	**1**	**0.32392**	**0.003811**	**0.053351**	**0.018713**	**0.34433**	**0.011228**
011	0.30986	0.012719	0.012224	0	1	0	0	0	0	0
012	0.030935	0.25726	0.033501	0	0.045238	1.0005	0.007451	0.050215	0.92395	0.030129
013	0.043809	0.038423	0.25167	0	0.073382	0.000863	1.0121	0.005119	0.094189	0.003071
021	0.26982	0.018763	0.04596	0	0.86073	0.010126	0.14177	1.0012	0.022426	0.000731
022	0.017532	0.27735	0.025709	0	0	0	0	0	1	0
023	0.043809	0.038423	0.25167	0	0.073382	0.000863	0.012086	0.005119	0.094189	1.0031
031	0.26982	0.018763	0.04596	0	0.86073	0.010126	0.14177	0.001219	0.022426	0.000731
032	0.030935	0.25726	0.033501	0	0.045238	0.000532	0.007451	0.050215	0.92395	0.030129
033	0.023328	0.013654	0.29835	0	0	0	0	0	0	0
111	0.12822	0.005263	0.005058	0	0	0	0	0	0	0
112	0.040693	0.10901	0.021163	0	0	0	0	0	0	0
113	0.040852	0.01587	0.096442	0	0	0	0	0	0	0
121	0.12472	0.00755	0.007208	0	0	0	0	0	0	0
122	0.010115	0.16001	0.014832	0	0	0	0	0	0	0
123	0.040852	0.01587	0.096442	0	0	0	0	0	0	0
131	0.12472	0.00755	0.007208	0	0	0	0	0	0	0
132	0.040693	0.10901	0.021163	0	0	0	0	0	0	0
133	0.010831	0.00634	0.13852	0	0	0	0	0	0	0
211	0.22438	0.009211	0.008852	0	0	0	0	0	0	0
212	0.012806	0.15063	0.026874	0	0	0	0	0	0	0
213	0.029855	0.026229	0.2083	0	0	0	0	0	0	0
221	0.16077	0.024609	0.058161	0	0	0	0	0	0	0
222	0.010115	0.16001	0.014832	0	0	0	0	0	0	0
223	0.029855	0.026229	0.2083	0	0	0	0	0	0	0
231	0.16077	0.024609	0.058161	0	0	0	0	0	0	0
232	0.012806	0.15063	0.026874	0	0	0	0	0	0	0
233	0.019163	0.011216	0.24507	0	0	0	0	0	0	0
311	0.2778	0.011404	0.01096	0	0	0	0	0	0	0
312	0.037702	0.10498	0.043544	0	0	0	0	0	0	0
313	0.013842	0.013026	0.12099	0	0	0	0	0	0	0
321	0.20051	0.029377	0.030354	0	0	0	0	0	0	0
322	0.010115	0.16001	0.014832	0	0	0	0	0	0	0
323	0.013842	0.013026	0.12099	0	0	0	0	0	0	0
331	0.20051	0.029377	0.030354	0	0	0	0	0	0	0
332	0.037702	0.10498	0.043544	0	0	0	0	0	0	0
333	0.009998	0.005852	0.12786	0	0	0	0	0	0	0

Table A1.2: Scenario 1, Type 1 E-mails - Average Number of Visits by Solving the DTMC (contd)

	031	032	033	111	112	113	121	122	123	131
D1	0	0	0	0.52631	0.010965	0.010965	0.096204	0.26456	0.040085	0.09944
D2	0	0	0	0.033215	0.000692	0.000692	0.006071	0.016696	0.00253	0.006276
D3	0	0	0	0.04104	0.000855	0.000855	0.007502	0.02063	0.003126	0.007754
New	**0.032366**	**0.040458**	**0.33175**	**0.056225**	**0.001171**	**0.001171**	**0.010277**	**0.028263**	**0.004282**	**0.010623**
011	0	0	0	0.15263	0.00318	0.00318	0.027899	0.076723	0.011625	0.028837
012	0.003006	0.003758	0.030815	0.015238	0.000317	0.000317	0.002785	0.00766	0.001161	0.002879
013	0.081213	0.10152	0.83243	0.021579	0.00045	0.00045	0.003945	0.010847	0.001644	0.004077
021	0.0114	0.01425	0.11685	0.13291	0.002769	0.002769	0.024294	0.06681	0.010123	0.025111
022	0	0	0	0.008636	0.00018	0.00018	0.001579	0.004341	0.000658	0.001632
023	0.081213	0.10152	0.83243	0.021579	0.00045	0.00045	0.003945	0.010847	0.001644	0.004077
031	1.0114	0.01425	0.11685	0.13291	0.002769	0.002769	0.024294	0.06681	0.010123	0.025111
032	0.003006	1.0038	0.030815	0.015238	0.000317	0.000317	0.002785	0.00766	0.001161	0.002879
033	0	0	1	0.011491	0.000239	0.000239	0.002101	0.005776	0.000875	0.002171
111	0	0	0	1.0632	0.001316	0.001316	0.011545	0.031747	0.00481	0.011933
112	0	0	0	0.27853	1.0058	0.005803	0.24727	0.67998	0.10303	0.02944
113	0	0	0	0.27626	0.005755	1.0058	0.026883	0.073928	0.011201	0.2474
121	0	0	0	1.0317	0.021494	0.021494	1.0166	0.045556	0.006902	0.016992
122	0	0	0	0.004982	0.000104	0.000104	0.000911	1.0025	0.000379	0.000941
123	0	0	0	0.27626	0.005755	0.005755	0.026883	0.073928	1.0112	0.2474
131	0	0	0	1.0317	0.021494	0.021494	0.016566	0.045556	0.006902	1.017
132	0	0	0	0.27853	0.005803	0.005803	0.24727	0.67998	0.10303	0.02944
133	0	0	0	0.005335	0.000111	0.000111	0.000975	0.002682	0.000406	0.001008
211	0	0	0	0.11053	0.002303	0.002303	0.020203	0.055558	0.008418	0.020882
212	0	0	0	0.006308	0.000131	0.000131	0.001153	0.003171	0.00048	0.001192
213	0	0	0	0.014706	0.000306	0.000306	0.002688	0.007392	0.00112	0.002779
221	0	0	0	0.079191	0.00165	0.00165	0.014475	0.039807	0.006031	0.014962
222	0	0	0	0.004982	0.000104	0.000104	0.000911	0.002504	0.000379	0.000941
223	0	0	0	0.014706	0.000306	0.000306	0.002688	0.007392	0.00112	0.002779
231	0	0	0	0.079191	0.00165	0.00165	0.014475	0.039807	0.006031	0.014962
232	0	0	0	0.006308	0.000131	0.000131	0.001153	0.003171	0.00048	0.001192
233	0	0	0	0.009439	0.000197	0.000197	0.001725	0.004745	0.000719	0.001783
311	0	0	0	0.13684	0.002851	0.002851	0.025013	0.068786	0.010422	0.025854
312	0	0	0	0.018571	0.000387	0.000387	0.003395	0.009335	0.001414	0.003509
313	0	0	0	0.006819	0.000142	0.000142	0.001246	0.003427	0.000519	0.001288
321	0	0	0	0.098767	0.002058	0.002058	0.018054	0.049647	0.007522	0.018661
322	0	0	0	0.004982	0.000104	0.000104	0.000911	0.002504	0.000379	0.000941
323	0	0	0	0.006819	0.000142	0.000142	0.001246	0.003427	0.000519	0.001288
331	0	0	0	0.098767	0.002058	0.002058	0.018054	0.049647	0.007522	0.018661
332	0	0	0	0.018571	0.000387	0.000387	0.003395	0.009335	0.001414	0.003509
333	0	0	0	0.004925	0.000103	0.000103	0.0009	0.002476	0.000375	0.00093

Table A1.3: Scenario 1, Type 1 E-mails - Average Number of Visits by Solving the DTMC (contd)

	132	**133**	**211**	**212**	**213**	**221**	**222**	**223**	**231**	**232**
D1	0.03729	0.2776	0.011044	0.001299	0.003898	0.0001776	0.016517	0.001066	0.00159	0.001988
D2	0.002353	0.017519	0.26861	0.031602	0.094805	0.0043197	0.40173	0.025918	0.038673	0.048342
D3	0.002908	0.021646	0.012279	0.001445	0.004334	0.0001975	0.018365	0.001185	0.001768	0.00221
New	**0.003984**	**0.029656**	**0.026226**	**0.003085**	**0.009256**	**0.0004218**	**0.039223**	**0.002531**	**0.003776**	**0.00472**
011	0.010814	0.080505	0.003203	0.000377	0.00113	5.15E-05	0.00479	0.000309	0.000461	0.000576
012	0.00108	0.008037	0.064779	0.007621	0.022863	0.0010417	0.096882	0.00625	0.009327	0.011658
013	0.001529	0.011382	0.009675	0.001138	0.003415	0.0001556	0.01447	0.000934	0.001393	0.001741
021	0.009417	0.070103	0.004725	0.000556	0.001668	7.60E-05	0.007066	0.000456	0.00068	0.00085
022	0.000612	0.004555	0.069839	0.008216	0.024649	0.0011231	0.10445	0.006739	0.010055	0.012569
023	0.001529	0.011382	0.009675	0.001138	0.003415	0.0001556	0.01447	0.000934	0.001393	0.001741
031	0.009417	0.070103	0.004725	0.000556	0.001668	7.60E-05	0.007066	0.000456	0.00068	0.00085
032	0.00108	0.008037	0.064779	0.007621	0.022863	0.0010417	0.096882	0.00625	0.009327	0.011658
033	0.000814	0.006061	0.003438	0.000405	0.001214	5.53E-05	0.005142	0.000332	0.000495	0.000619
111	0.004475	0.033312	0.001325	0.000156	0.000468	2.13E-05	0.001982	0.000128	0.000191	0.000239
112	0.01104	0.082186	0.027449	0.003229	0.009688	0.0004414	0.041052	0.002649	0.003952	0.00494
113	0.092776	0.69067	0.003996	0.00047	0.00141	6.43E-05	0.005977	0.000386	0.000575	0.000719
121	0.006372	0.047437	0.001901	0.000224	0.000671	3.06E-05	0.002843	0.000183	0.000274	0.000342
122	0.000353	0.002628	0.040292	0.00474	0.014221	0.000648	0.06026	0.003888	0.005801	0.007251
123	0.092776	0.69067	0.003996	0.00047	0.00141	6.43E-05	0.005977	0.000386	0.000575	0.000719
131	0.006372	0.047437	0.001901	0.000224	0.000671	3.06E-05	0.002843	0.000183	0.000274	0.000342
132	1.011	0.082186	0.027449	0.003229	0.009688	0.0004414	0.041052	0.002649	0.003952	0.00494
133	0.000378	1.0028	0.001596	0.000188	0.000563	2.57E-05	0.002387	0.000154	0.00023	0.000287
211	0.007831	0.058296	1.0023	0.000273	0.000819	3.73E-05	0.003469	0.000224	0.000334	0.000417
212	0.000447	0.003327	0.048269	1.0057	0.017036	0.010686	0.99382	0.064117	0.010589	0.013237
213	0.001042	0.007757	0.063191	0.007434	1.0223	0.0011993	0.11154	0.007196	0.083073	0.10384
221	0.005611	0.041769	0.7006	0.082424	0.24727	1.0012	0.10863	0.007009	0.021012	0.026265
222	0.000353	0.002628	0.040292	0.00474	0.014221	0.000648	1.0603	0.003888	0.005801	0.007251
223	0.001042	0.007757	0.063191	0.007434	0.022303	0.0011993	0.11154	1.0072	0.083073	0.10384
231	0.005611	0.041769	0.7006	0.082424	0.24727	0.0011681	0.10863	0.007009	1.021	0.026265
232	0.000447	0.003327	0.048269	0.005679	0.017036	0.010686	0.99382	0.064117	0.010589	1.0132
233	0.000669	0.004979	0.002824	0.000332	0.000997	4.54E-05	0.004224	0.000273	0.000407	0.000508
311	0.009695	0.072177	0.002872	0.000338	0.001014	4.62E-05	0.004295	0.000277	0.000413	0.000517
312	0.001316	0.009795	0.026434	0.00311	0.00933	0.0004251	0.039534	0.002551	0.003806	0.004757
313	0.000483	0.003596	0.00328	0.000386	0.001158	5.27E-05	0.004906	0.000316	0.000472	0.00059
321	0.006998	0.052095	0.007397	0.00087	0.002611	0.000119	0.011063	0.000714	0.001065	0.001331
322	0.000353	0.002628	0.040292	0.00474	0.014221	0.000648	0.06026	0.003888	0.005801	0.007251
323	0.000483	0.003596	0.00328	0.000386	0.001158	5.27E-05	0.004906	0.000316	0.000472	0.00059
331	0.006998	0.052095	0.007397	0.00087	0.002611	0.000119	0.011063	0.000714	0.001065	0.001331
332	0.001316	0.009795	0.026434	0.00311	0.00933	0.0004251	0.039534	0.002551	0.003806	0.004757
333	0.000349	0.002598	0.001474	0.000173	0.00052	2.37E-05	0.002204	0.000142	0.000212	0.000265

Table A1.4: Scenario 1, Type 1 E-mails - Average Number of Visits by Solving the DTMC (contd)

	233	**311**	**312**	**313**	**321**	**322**	**323**	**331**	**332**	**333**
D1	0.016298	0.011535	0.003176	0.002006	0.002598	0.011319	0.004639	0.0002098	0.001468	0.019299
D2	0.3964	0.02706	0.007451	0.004706	0.006094	0.026551	0.010882	0.0004921	0.003445	0.045271
D3	0.018121	0.29159	0.080292	0.050711	0.065664	0.28611	0.11726	0.0053025	0.037117	0.48783
New	**0.038702**	**0.030592**	**0.008424**	**0.00532**	**0.006889**	**0.030017**	**0.012302**	**0.0005563**	**0.003894**	**0.051181**
011	0.004727	0.003345	0.000921	0.000582	0.000753	0.003282	0.001345	6.08E-05	0.000426	0.005597
012	0.095596	0.009168	0.002524	0.001594	0.002065	0.008996	0.003687	0.0001667	0.001167	0.015338
013	0.014278	0.068871	0.018965	0.011978	0.01551	0.067577	0.027696	0.0012524	0.008767	0.11522
021	0.006972	0.012577	0.003463	0.002187	0.002832	0.012341	0.005058	0.0002287	0.001601	0.021042
022	0.10306	0.007036	0.001937	0.001224	0.001584	0.006903	0.002829	0.0001279	0.000896	0.011771
023	0.014278	0.068871	0.018965	0.011978	0.01551	0.067577	0.027696	0.0012524	0.008767	0.11522
031	0.006972	0.012577	0.003463	0.002187	0.002832	0.012341	0.005058	0.0002287	0.001601	0.021042
032	0.095596	0.009168	0.002524	0.001594	0.002065	0.008996	0.003687	0.0001667	0.001167	0.015338
033	0.005074	0.081644	0.022482	0.014199	0.018386	0.08011	0.032832	0.0014847	0.010393	0.13659
111	0.001956	0.001384	0.000381	0.000241	0.000312	0.001358	0.000557	2.52E-05	0.000176	0.002316
112	0.040507	0.005792	0.001595	0.001007	0.001304	0.005683	0.002329	0.0001053	0.000737	0.009689
113	0.005897	0.026392	0.007267	0.00459	0.005943	0.025896	0.010613	0.0004799	0.00336	0.044154
121	0.002806	0.001973	0.000543	0.000343	0.000444	0.001936	0.000793	3.59E-05	0.000251	0.0033
122	0.05946	0.004059	0.001118	0.000706	0.000914	0.003983	0.001632	7.38E-05	0.000517	0.006791
123	0.005897	0.026392	0.007267	0.00459	0.005943	0.025896	0.010613	0.0004799	0.00336	0.044154
131	0.002806	0.001973	0.000543	0.000343	0.000444	0.001936	0.000793	3.59E-05	0.000251	0.0033
132	0.040507	0.005792	0.001595	0.001007	0.001304	0.005683	0.002329	0.0001053	0.000737	0.009689
133	0.002356	0.037906	0.010438	0.006592	0.008536	0.037194	0.015244	0.0006893	0.004825	0.063417
211	0.003423	0.002422	0.000667	0.000421	0.000546	0.002377	0.000974	4.41E-05	0.000308	0.004053
212	0.10854	0.007354	0.002025	0.001279	0.001656	0.007216	0.002957	0.0001337	0.000936	0.012303
213	0.8515	0.057002	0.015696	0.009913	0.012837	0.055931	0.022923	0.0010366	0.007256	0.095365
221	0.21537	0.015916	0.004383	0.002768	0.003584	0.015617	0.0064	0.0002894	0.002026	0.026628
222	0.05946	0.004059	0.001118	0.000706	0.000914	0.003983	0.001632	7.38E-05	0.000517	0.006791
223	0.8515	0.057002	0.015696	0.009913	0.012837	0.055931	0.022923	0.0010366	0.007256	0.095365
231	0.21537	0.015916	0.004383	0.002768	0.003584	0.015617	0.0064	0.0002894	0.002026	0.026628
232	0.10854	0.007354	0.002025	0.001279	0.001656	0.007216	0.002957	0.0001337	0.000936	0.012303
233	1.0042	0.067065	0.018467	0.011663	0.015103	0.065805	0.026969	0.0012196	0.008537	0.1122
311	0.004238	1.003	0.000826	0.000522	0.000675	0.002943	0.001206	5.45E-05	0.000382	0.005018
312	0.039009	0.11508	1.0317	0.020015	0.1494	0.65098	0.26679	0.0030161	0.021113	0.27748
313	0.00484	0.047443	0.013064	1.0083	0.018018	0.078506	0.032175	0.010816	0.075709	0.99504
321	0.010916	0.71963	0.19816	0.12515	1.031	0.13514	0.055384	0.0019086	0.01336	0.17559
322	0.05946	0.004059	0.001118	0.000706	0.000914	1.004	0.001632	7.38E-05	0.000517	0.006791
323	0.00484	0.047443	0.013064	0.008251	0.018018	0.078506	1.0322	0.010816	0.075709	0.99504
331	0.010916	0.71963	0.19816	0.12515	0.031015	0.13514	0.055384	1.0019	0.01336	0.17559
332	0.039009	0.11508	0.03169	0.020015	0.1494	0.65098	0.26679	0.0030161	1.0211	0.27748
333	0.002175	0.03499	0.009635	0.006085	0.00788	0.034333	0.014071	0.0006363	0.004454	1.0585

Table A1.5: Scenario 1, Type 2 E-mails - Average Number of Visits by Solving the DTMC

	D1	**D2**	**D3**	**New**	**011**	**012**	**013**	**021**	**022**	**023**
D1	1.0329	0.039582	0.071958	0	0	0	0	0	0	0
D2	0.085351	1.0254	0.062031	0	0	0	0	0	0	0
D3	0.08048	0.060141	1.0695	0	0	0	0	0	0	0
New	**0.10783**	**0.071161**	**0.088197**	**1**	**0.37462**	**0.025256**	**0.021046**	**0.047216**	**0.27967**	**0.03632**
011	0.25823	0.009896	0.017989	0	1	0	0	0	0	0
012	0.049141	0.17802	0.034065	0	0.12754	1.0086	0.007165	0.1314	0.77829	0.10108
013	0.04152	0.016784	0.19915	0	0.10185	0.006867	1.0057	0.003515	0.020817	0.002704
021	0.23485	0.020327	0.028012	0	0.90275	0.060859	0.050716	1.0081	0.047738	0.0062
022	0.018777	0.22558	0.013647	0	0	0	0	0	1	0
023	0.04152	0.016784	0.19915	0	0.10185	0.006867	0.005722	0.003515	0.020817	1.0027
031	0.23485	0.020327	0.028012	0	0.90275	0.060859	0.050716	0.00806	0.047738	0.0062
032	0.049141	0.17802	0.034065	0	0.12754	0.008598	0.007165	0.1314	0.77829	0.10108
033	0.016901	0.01263	0.22459	0	0	0	0	0	0	0
111	0.061976	0.002375	0.004318	0	0	0	0	0	0	0
112	0.017041	0.090789	0.023004	0	0	0	0	0	0	0
113	0.023713	0.022262	0.17466	0	0	0	0	0	0	0
121	0.058332	0.006221	0.015098	0	0	0	0	0	0	0
122	0.009389	0.11279	0.006824	0	0	0	0	0	0	0
123	0.023713	0.022262	0.17466	0	0	0	0	0	0	0
131	0.058332	0.006221	0.015098	0	0	0	0	0	0	0
132	0.017041	0.090789	0.023004	0	0	0	0	0	0	0
133	0.017706	0.013231	0.23528	0	0	0	0	0	0	0
211	0.2479	0.0095	0.01727	0	0	0	0	0	0	0
212	0.005354	0.050174	0.007268	0	0	0	0	0	0	0
213	0.040472	0.014792	0.14198	0	0	0	0	0	0	0
221	0.21159	0.011461	0.034429	0	0	0	0	0	0	0
222	0.004268	0.051268	0.003102	0	0	0	0	0	0	0
223	0.040472	0.014792	0.14198	0	0	0	0	0	0	0
231	0.21159	0.011461	0.034429	0	0	0	0	0	0	0
232	0.005354	0.050174	0.007268	0	0	0	0	0	0	0
233	0.013682	0.010224	0.18181	0	0	0	0	0	0	0
311	0.22725	0.008708	0.015831	0	0	0	0	0	0	0
312	0.041467	0.14551	0.031836	0	0	0	0	0	0	0
313	0.023271	0.010981	0.14044	0	0	0	0	0	0	0
321	0.17844	0.02542	0.033951	0	0	0	0	0	0	0
322	0.016217	0.19482	0.011786	0	0	0	0	0	0	0
323	0.023271	0.010981	0.14044	0	0	0	0	0	0	0
331	0.17844	0.02542	0.033951	0	0	0	0	0	0	0
332	0.041467	0.14551	0.031836	0	0	0	0	0	0	0
333	0.011267	0.00842	0.14972	0	0	0	0	0	0	0

Table A1.6: Scenario 1, Type 2 E-mails - Average Number of Visits by Solving the DTMC (contd)

	031	**032**	**033**	**111**	**112**	**113**	**121**	**122**	**123**	**131**
D1	0	0	0	0.41522	0.013688	0.027377	0.053112	0.31867	0.03677	0.062302
D2	0	0	0	0.034309	0.001131	0.002262	0.004389	0.026332	0.003038	0.005148
D3	0	0	0	0.032351	0.001067	0.002133	0.004138	0.024829	0.002865	0.004854
New	**0.04371**	**0.007947**	**0.34571**	**0.043347**	**0.001429**	**0.002858**	**0.005545**	**0.033268**	**0.003839**	**0.006504**
011	0	0	0	0.1038	0.003422	0.006844	0.013278	0.079668	0.009192	0.015575
012	0.011907	0.002165	0.09417	0.019754	0.000651	0.001302	0.002527	0.015161	0.001749	0.002964
013	0.11093	0.020169	0.87733	0.01669	0.00055	0.001101	0.002135	0.012809	0.001478	0.002504
021	0.006261	0.001138	0.049517	0.094405	0.003112	0.006225	0.012076	0.072454	0.00836	0.014165
022	0	0	0	0.007548	0.000249	0.000498	0.000965	0.005793	0.000668	0.001133
023	0.11093	0.020169	0.87733	0.01669	0.00055	0.001101	0.002135	0.012809	0.001478	0.002504
031	1.0063	0.001138	0.049517	0.094405	0.003112	0.006225	0.012076	0.072454	0.00836	0.014165
032	0.011907	1.0022	0.09417	0.019754	0.000651	0.001302	0.002527	0.015161	0.001749	0.002964
033	0	0	1	0.006794	0.000224	0.000448	0.000869	0.005214	0.000602	0.001019
111	0	0	0	1.0249	0.000821	0.001643	0.003187	0.01912	0.002206	0.003738
112	0	0	0	0.14092	1.0046	0.009292	0.13315	0.79889	0.092179	0.01609
113	0	0	0	0.16547	0.005455	1.0109	0.019171	0.11503	0.013272	0.15484
121	0	0	0	0.94683	0.031214	0.062428	1.008	0.048268	0.005569	0.013175
122	0	0	0	0.003774	0.000124	0.000249	0.000483	1.0029	0.000334	0.000566
123	0	0	0	0.16547	0.005455	0.01091	0.019171	0.11503	1.0133	0.15484
131	0	0	0	0.94683	0.031214	0.062428	0.008045	0.048268	0.005569	1.0132
132	0	0	0	0.14092	0.004646	0.009292	0.13315	0.79889	0.092179	0.01609
133	0	0	0	0.007117	0.000235	0.000469	0.00091	0.005462	0.00063	0.001068
211	0	0	0	0.099652	0.003285	0.006571	0.012747	0.076481	0.008825	0.014952
212	0	0	0	0.002152	7.09E-05	0.000142	0.000275	0.001652	0.000191	0.000323
213	0	0	0	0.016269	0.000536	0.001073	0.002081	0.012486	0.001441	0.002441
221	0	0	0	0.085053	0.002804	0.005608	0.010879	0.065277	0.007532	0.012762
222	0	0	0	0.001716	5.66E-05	0.000113	0.000219	0.001317	0.000152	0.000257
223	0	0	0	0.016269	0.000536	0.001073	0.002081	0.012486	0.001441	0.002441
231	0	0	0	0.085053	0.002804	0.005608	0.010879	0.065277	0.007532	0.012762
232	0	0	0	0.002152	7.09E-05	0.000142	0.000275	0.001652	0.000191	0.000323
233	0	0	0	0.0055	0.000181	0.000363	0.000703	0.004221	0.000487	0.000825
311	0	0	0	0.091348	0.003012	0.006023	0.011685	0.070108	0.008089	0.013706
312	0	0	0	0.016669	0.00055	0.001099	0.002132	0.012793	0.001476	0.002501
313	0	0	0	0.009354	0.000308	0.000617	0.001197	0.007179	0.000828	0.001404
321	0	0	0	0.071727	0.002365	0.004729	0.009175	0.055049	0.006352	0.010762
322	0	0	0	0.006519	0.000215	0.00043	0.000834	0.005003	0.000577	0.000978
323	0	0	0	0.009354	0.000308	0.000617	0.001197	0.007179	0.000828	0.001404
331	0	0	0	0.071727	0.002365	0.004729	0.009175	0.055049	0.006352	0.010762
332	0	0	0	0.016669	0.00055	0.001099	0.002132	0.012793	0.001476	0.002501
333	0	0	0	0.004529	0.000149	0.000299	0.000579	0.003476	0.000401	0.00068

Table A1.7: Scenario 1, Type 2 E-mails - Average Number of Visits by Solving the DTMC (contd)

	132	**133**	**211**	**212**	**213**	**221**	**222**	**223**	**231**	**232**
D1	0.053995	0.29905	0.012708	0.000459	0.002144	0	0.014829	0.000459	0.002248	0.001767
D2	0.004462	0.02471	0.32919	0.011898	0.055526	0	0.38415	0.011881	0.058245	0.045764
D3	0.004207	0.0233	0.019308	0.000698	0.003257	0	0.022532	0.000697	0.003416	0.002684
New	**0.005637**	**0.03122**	**0.022846**	**0.000826**	**0.003854**	**0**	**0.02666**	**0.000825**	**0.004042**	**0.003176**
011	0.013499	0.074762	0.003177	0.000115	0.000536	0	0.003707	0.000115	0.000562	0.000442
012	0.002569	0.014227	0.057152	0.002066	0.00964	0	0.066694	0.002063	0.010112	0.007945
013	0.00217	0.012021	0.005389	0.000195	0.000909	0	0.006288	0.000194	0.000953	0.000749
021	0.012276	0.067993	0.006526	0.000236	0.001101	0	0.007616	0.000236	0.001155	0.000907
022	0.000982	0.005436	0.072422	0.002618	0.012216	0	0.084514	0.002614	0.012814	0.010068
023	0.00217	0.012021	0.005389	0.000195	0.000909	0	0.006288	0.000194	0.000953	0.000749
031	0.012276	0.067993	0.006526	0.000236	0.001101	0	0.007616	0.000236	0.001155	0.000907
032	0.002569	0.014227	0.057152	0.002066	0.00964	0	0.066694	0.002063	0.010112	0.007945
033	0.000883	0.004893	0.004055	0.000147	0.000684	0	0.004732	0.000146	0.000717	0.000564
111	0.00324	0.017943	0.000762	2.76E-05	0.000129	0	0.00089	2.75E-05	0.000135	0.000106
112	0.013944	0.077231	0.029147	0.001054	0.004916	0	0.034014	0.001052	0.005157	0.004052
113	0.13419	0.74322	0.007147	0.000258	0.001206	0	0.00834	0.000258	0.001265	0.000994
121	0.011418	0.063238	0.001997	7.22E-05	0.000337	0	0.002331	7.21E-05	0.000353	0.000278
122	0.000491	0.002718	0.036211	0.001309	0.006108	0	0.042257	0.001307	0.006407	0.005034
123	0.13419	0.74322	0.007147	0.000258	0.001206	0	0.00834	0.000258	0.001265	0.000994
131	0.011418	0.063238	0.001997	7.22E-05	0.000337	0	0.002331	7.21E-05	0.000353	0.000278
132	1.0139	0.077231	0.029147	0.001054	0.004916	0	0.034014	0.001052	0.005157	0.004052
133	0.000926	1.0051	0.004248	0.000154	0.000716	0	0.004957	0.000153	0.000752	0.000591
211	0.012959	0.071772	1.003	0.00011	0.000514	0	0.003559	0.00011	0.00054	0.000424
212	0.00028	0.00155	0.019676	1.0007	0.003319	0	0.9922	0.030687	0.007149	0.005617
213	0.002116	0.011717	0.12369	0.004471	1.0209	0	0.11893	0.003678	0.14414	0.11325
221	0.01106	0.061257	0.85044	0.030739	0.14345	1	0.04937	0.001527	0.020842	0.016376
222	0.000223	0.001236	0.01646	0.000595	0.002776	0	1.0192	0.000594	0.002912	0.002288
223	0.002116	0.011717	0.12369	0.004471	0.020863	0	0.11893	1.0037	0.14414	0.11325
231	0.01106	0.061257	0.85044	0.030739	0.14345	0	0.04937	0.001527	1.0208	0.016376
232	0.00028	0.00155	0.019676	0.000711	0.003319	0	0.9922	0.030687	0.007149	1.0056
233	0.000715	0.003961	0.003282	0.000119	0.000554	0	0.00383	0.000118	0.000581	0.000456
311	0.011879	0.065791	0.002796	0.000101	0.000472	0	0.003263	0.000101	0.000495	0.000389
312	0.002168	0.012005	0.046716	0.001689	0.00788	0	0.054515	0.001686	0.008266	0.006494
313	0.001216	0.006737	0.003525	0.000127	0.000595	0	0.004114	0.000127	0.000624	0.00049
321	0.009327	0.051659	0.008161	0.000295	0.001377	0	0.009524	0.000295	0.001444	0.001135
322	0.000848	0.004695	0.062546	0.002261	0.01055	0	0.072989	0.002257	0.011066	0.008695
323	0.001216	0.006737	0.003525	0.000127	0.000595	0	0.004114	0.000127	0.000624	0.00049
331	0.009327	0.051659	0.008161	0.000295	0.001377	0	0.009524	0.000295	0.001444	0.001135
332	0.002168	0.012005	0.046716	0.001689	0.00788	0	0.054515	0.001686	0.008266	0.006494
333	0.000589	0.003262	0.002703	9.77E-05	0.000456	0	0.003154	9.76E-05	0.000478	0.000376

Table A1.8: Scenario 1, Type 2 E-mails - Average Number of Visits by Solving the DTMC (contd)

	233	311	312	313	321	322	323	331	332	333
D1	0.012045	0.022603	0.003616	0.003918	0.004152	0.019931	0.003599	0.002239	0.00032	0.029424
D2	0.31202	0.019485	0.003118	0.003377	0.00358	0.017182	0.003102	0.00193	0.000276	0.025365
D3	0.018301	0.33593	0.053749	0.058228	0.061714	0.29623	0.053485	0.033273	0.004753	0.4373
New	**0.021655**	**0.027704**	**0.004433**	**0.004802**	**0.00509**	**0.024429**	**0.004411**	**0.002744**	**0.000392**	**0.036064**
011	0.003011	0.005651	0.000904	0.000979	0.001038	0.004983	0.0009	0.00056	8.00E-05	0.007356
012	0.054172	0.0107	0.001712	0.001855	0.001966	0.009436	0.001704	0.00106	0.000151	0.013929
013	0.005108	0.062557	0.010009	0.010843	0.011492	0.055163	0.00996	0.006196	0.000885	0.081434
021	0.006186	0.008799	0.001408	0.001525	0.001616	0.007759	0.001401	0.000872	0.000125	0.011454
022	0.068645	0.004287	0.000686	0.000743	0.000788	0.00378	0.000683	0.000425	6.07E-05	0.00558
023	0.005108	0.062557	0.010009	0.010843	0.011492	0.055163	0.00996	0.006196	0.000885	0.081434
031	0.006186	0.008799	0.001408	0.001525	0.001616	0.007759	0.001401	0.000872	0.000125	0.011454
032	0.054172	0.0107	0.001712	0.001855	0.001966	0.009436	0.001704	0.00106	0.000151	0.013929
033	0.003843	0.070546	0.011287	0.012228	0.01296	0.062207	0.011232	0.006987	0.000998	0.091834
111	0.000723	0.001356	0.000217	0.000235	0.000249	0.001196	0.000216	0.000134	1.92E-05	0.001765
112	0.027627	0.007226	0.001156	0.001253	0.001328	0.006372	0.001151	0.000716	0.000102	0.009407
113	0.006774	0.054862	0.008778	0.00951	0.010079	0.048378	0.008735	0.005434	0.000776	0.071418
121	0.001893	0.004743	0.000759	0.000822	0.000871	0.004182	0.000755	0.00047	6.71E-05	0.006174
122	0.034323	0.002143	0.000343	0.000372	0.000394	0.00189	0.000341	0.000212	3.03E-05	0.00279
123	0.006774	0.054862	0.008778	0.00951	0.010079	0.048378	0.008735	0.005434	0.000776	0.071418
131	0.001893	0.004743	0.000759	0.000822	0.000871	0.004182	0.000755	0.00047	6.71E-05	0.006174
132	0.027627	0.007226	0.001156	0.001253	0.001328	0.006372	0.001151	0.000716	0.000102	0.009407
133	0.004026	0.073905	0.011825	0.01281	0.013577	0.06517	0.011767	0.00732	0.001046	0.096207
211	0.002891	0.005425	0.000868	0.00094	0.000997	0.004784	0.000864	0.000537	7.68E-05	0.007062
212	0.038299	0.002283	0.000365	0.000396	0.000419	0.002013	0.000363	0.000226	3.23E-05	0.002972
213	0.77218	0.044596	0.007135	0.00773	0.008193	0.039325	0.0071	0.004417	0.000631	0.058054
221	0.11165	0.010814	0.00173	0.001875	0.001987	0.009536	0.001722	0.001071	0.000153	0.014078
222	0.015601	0.000974	0.000156	0.000169	0.000179	0.000859	0.000155	9.65E-05	1.38E-05	0.001268
223	0.77218	0.044596	0.007135	0.00773	0.008193	0.039325	0.0071	0.004417	0.000631	0.058054
231	0.11165	0.010814	0.00173	0.001875	0.001987	0.009536	0.001722	0.001071	0.000153	0.014078
232	0.038299	0.002283	0.000365	0.000396	0.000419	0.002013	0.000363	0.000226	3.23E-05	0.002972
233	1.0031	0.057108	0.009137	0.009899	0.010491	0.050358	0.009093	0.005656	0.000808	0.074342
311	0.00265	1.005	0.000796	0.000862	0.000914	0.004385	0.000792	0.000493	7.04E-05	0.006473
312	0.04428	0.13298	1.0213	0.023049	0.15502	0.74409	0.13435	0.011776	0.001682	0.15477
313	0.003341	0.09938	0.015901	1.0172	0.010949	0.052554	0.009489	0.075213	0.010745	0.98851
321	0.007735	0.78261	0.12522	0.13565	1.0207	0.099412	0.017949	0.01156	0.001651	0.15193
322	0.059285	0.003702	0.000592	0.000642	0.00068	1.0033	0.000589	0.000367	5.24E-05	0.004819
323	0.003341	0.09938	0.015901	0.017226	0.010949	0.052554	1.0095	0.075213	0.010745	0.98851
331	0.007735	0.78261	0.12522	0.13565	0.020711	0.099412	0.017949	1.0116	0.001651	0.15193
332	0.04428	0.13298	0.021276	0.023049	0.15502	0.74409	0.13435	0.011776	1.0017	0.15477
333	0.002562	0.04703	0.007525	0.008152	0.00864	0.041472	0.007488	0.004658	0.000665	1.0612

Table A1.9: Scenario 2, Type 1 E-mails - Average Number of Visits by Solving the DTMC

D1	1.1364	0	0	0	0	0	0	0	0	0
D2	0	1.1765	0	0	0	0	0	0	0	0
D3	0	0	1.1364	0	0	0	0	0	0	0
New	**0.10675**	**0.10532**	**0.10556**	**1**	**0.32392**	**0.003811**	**0.053351**	**0.018713**	**0.34433**	**0.011228**
011	0.32955	0	0	0	1	0	0	0	0	0
012	0.014908	0.28262	0.009805	0	0.045238	1.0005	0.007451	0.050215	0.92395	0.030129
013	0.024183	0.028811	0.26486	0	0.073382	0.000863	1.0121	0.005119	0.094189	0.003071
021	0.28365	0.00686	0.037179	0	0.86073	0.010126	0.14177	1.0012	0.022426	0.000731
022	0	0.30588	0	0	0	0	0	0	1	0
023	0.024183	0.028811	0.26486	0	0.073382	0.000863	0.012086	0.005119	0.094189	1.0031
031	0.28365	0.00686	0.037179	0	0.86073	0.010126	0.14177	0.001219	0.022426	0.000731
032	0.014908	0.28262	0.009805	0	0.045238	0.000532	0.007451	0.050215	0.92395	0.030129
033	0	0	0.31818	0	0	0	0	0	0	0
111	0.13636	0	0	0	0	0	0	0	0	0
112	0	1.1765	0	0	0	0	0	0	0	0
113	0	0	1.1364	0	0	0	0	0	0	0
121	0.13636	0	0	0	0	0	0	0	0	0
122	0	1.1765	0	0	0	0	0	0	0	0
123	0	0	1.1364	0	0	0	0	0	0	0
131	0.13636	0	0	0	0	0	0	0	0	0
132	0	1.1765	0	0	0	0	0	0	0	0
133	0	0	1.1364	0	0	0	0	0	0	0
211	1.1364	0	0	0	0	0	0	0	0	0
212	0	0.17647	0	0	0	0	0	0	0	0
213	0	0	1.1364	0	0	0	0	0	0	0
221	1.1364	0	0	0	0	0	0	0	0	0
222	0	0.17647	0	0	0	0	0	0	0	0
223	0	0	1.1364	0	0	0	0	0	0	0
231	1.1364	0	0	0	0	0	0	0	0	0
232	0	0.17647	0	0	0	0	0	0	0	0
233	0	0	1.1364	0	0	0	0	0	0	0
311	1.1364	0	0	0	0	0	0	0	0	0
312	0	1.1765	0	0	0	0	0	0	0	0
313	0	0	0.13636	0	0	0	0	0	0	0
321	1.1364	0	0	0	0	0	0	0	0	0
322	0	1.1765	0	0	0	0	0	0	0	0
323	0	0	0.13636	0	0	0	0	0	0	0
331	1.1364	0	0	0	0	0	0	0	0	0
332	0	1.1765	0	0	0	0	0	0	0	0
333	0	0	0.13636	0	0	0	0	0	0	0

Table A1.10: Scenario 2, Type 1 E-mails - Average Number of Visits by Solving the DTMC (contd)

	031	**032**	**033**	**111**	**112**	**113**	**121**	**122**	**123**	**131**
D1	0	0	0	1.1364	0	0	0	0	0	0
D2	0	0	0	0	0	0	0	0	0	0
D3	0	0	0	0	0	0	0	0	0	0
New	**0.032366**	**0.040458**	**0.33175**	**0.10675**	**0**	**0**	**0**	**0**	**0**	**0**
011	0	0	0	0.32955	0	0	0	0	0	0
012	0.003006	0.003758	0.030815	0.014908	0	0	0	0	0	0
013	0.081213	0.10152	0.83243	0.024183	0	0	0	0	0	0
021	0.0114	0.01425	0.11685	0.28365	0	0	0	0	0	0
022	0	0	0	0	0	0	0	0	0	0
023	0.081213	0.10152	0.83243	0.024183	0	0	0	0	0	0
031	1.0114	0.01425	0.11685	0.28365	0	0	0	0	0	0
032	0.003006	1.0038	0.030815	0.014908	0	0	0	0	0	0
033	0	0	1	0	0	0	0	0	0	0
111	0	0	0	1.1364	0	0	0	0	0	0
112	0	0	0	0	1	0	0	1	0	0
113	0	0	0	0	0	1	0	0	0	0
121	0	0	0	1.1364	0	0	1	0	0	0
122	0	0	0	0	0	0	0	1	0	0
123	0	0	0	0	0	0	0	0	1	0
131	0	0	0	1.1364	0	0	0	0	0	1
132	0	0	0	0	0	0	0	1	0	0
133	0	0	0	0	0	0	0	0	0	0
211	0	0	0	1.1364	0	0	0	0	0	0
212	0	0	0	0	0	0	0	0	0	0
213	0	0	0	0	0	0	0	0	0	0
221	0	0	0	1.1364	0	0	0	0	0	0
222	0	0	0	0	0	0	0	0	0	0
223	0	0	0	0	0	0	0	0	0	0
231	0	0	0	1.1364	0	0	0	0	0	0
232	0	0	0	0	0	0	0	0	0	0
233	0	0	0	0	0	0	0	0	0	0
311	0	0	0	1.1364	0	0	0	0	0	0
312	0	0	0	0	0	0	0	0	0	0
313	0	0	0	0	0	0	0	0	0	0
321	0	0	0	1.1364	0	0	0	0	0	0
322	0	0	0	0	0	0	0	0	0	0
323	0	0	0	0	0	0	0	0	0	0
331	0	0	0	1.1364	0	0	0	0	0	0
332	0	0	0	0	0	0	0	0	0	0
333	0	0	0	0	0	0	0	0	0	0

Table A1.11: Scenario 2, Type 1 E-mails - Average Number of Visits by Solving the DTMC (contd)

	132	133	211	212	213	221	222	223	231	232
D1	0	0	0	0	0	0	0	0	0	0
D2	0	0	0	0	0	0	1.1765	0	0	0
D3	0	0	0	0	0	0	0	0	0	0
New	**0**	**0**	**0**	**0**	**0**	**0**	**0.10532**	**0**	**0**	**0**
011	0	0	0	0	0	0	0	0	0	0
012	0	0	0	0	0	0	0.28262	0	0	0
013	0	0	0	0	0	0	0.028811	0	0	0
021	0	0	0	0	0	0	0.00686	0	0	0
022	0	0	0	0	0	0	0.30588	0	0	0
023	0	0	0	0	0	0	0.028811	0	0	0
031	0	0	0	0	0	0	0.00686	0	0	0
032	0	0	0	0	0	0	0.28262	0	0	0
033	0	0	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0	0	0
112	0	0	0	0	0	0	1.1765	0	0	0
113	0	1	0	0	0	0	0	0	0	0
121	0	0	0	0	0	0	0	0	0	0
122	0	0	0	0	0	0	1.1765	0	0	0
123	0	1	0	0	0	0	0	0	0	0
131	0	0	0	0	0	0	0	0	0	0
132	1	0	0	0	0	0	1.1765	0	0	0
133	0	1	0	0	0	0	0	0	0	0
211	0	0	1	0	0	0	0	0	0	0
212	0	0	0	1	0	0	1.1765	0	0	0
213	0	0	0	0	1	0	0	0	0	0
221	0	0	1	0	0	1	0	0	0	0
222	0	0	0	0	0	0	1.1765	0	0	0
223	0	0	0	0	0	0	0	1	0	0
231	0	0	1	0	0	0	0	0	1	0
232	0	0	0	0	0	0	1.1765	0	0	1
233	0	0	0	0	0	0	0	0	0	0
311	0	0	0	0	0	0	0	0	0	0
312	0	0	0	0	0	0	1.1765	0	0	0
313	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0
322	0	0	0	0	0	0	1.1765	0	0	0
323	0	0	0	0	0	0	0	0	0	0
331	0	0	0	0	0	0	0	0	0	0
332	0	0	0	0	0	0	1.1765	0	0	0
333	0	0	0	0	0	0	0	0	0	0

Table A1.12: Scenario 2, Type 1 E-mails - Average Number of Visits by Solving the DTMC (contd)

	233	**311**	**312**	**313**	**321**	**322**	**323**	**331**	**332**	**333**
D1	0	0	0	0	0	0	0	0	0	0
D2	0	0	0	0	0	0	0	0	0	0
D3	0	0	0	0	0	0	0	0	0	1.1364
New	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.10556**
011	0	0	0	0	0	0	0	0	0	0
012	0	1.73E-18	0	0	0	0	0	1.73E-18	0	0.009805
013	0	0	0	0	0	0	0	0	0	0.26486
021	0	5.55E-17	0	0	0	0	0	5.55E-17	0	0.037179
022	0	0	0	0	0	0	0	0	0	0
023	0	0	0	0	0	0	0	0	0	0.26486
031	0	5.55E-17	0	0	0	0	0	5.55E-17	0	0.037179
032	0	1.73E-18	0	0	0	0	0	1.73E-18	0	0.009805
033	0	0	0	0	0	0	0	0	0	0.31818
111	0	0	0	0	0	0	0	0	0	0
112	0	0	0	0	0	0	0	0	0	0
113	0	0	0	0	0	0	0	0	0	1.1364
121	0	0	0	0	0	0	0	0	0	0
122	0	0	0	0	0	0	0	0	0	0
123	0	0	0	0	0	0	0	0	0	1.1364
131	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	0
133	0	0	0	0	0	0	0	0	0	1.1364
211	0	0	0	0	0	0	0	0	0	0
212	0	0	0	0	0	0	0	0	0	0
213	1	0	0	0	0	0	0	0	0	1.1364
221	0	0	0	0	0	0	0	0	0	0
222	0	0	0	0	0	0	0	0	0	0
223	1	0	0	0	0	0	0	0	0	1.1364
231	0	0	0	0	0	0	0	0	0	0
232	0	0	0	0	0	0	0	0	0	0
233	1	0	0	0	0	0	0	0	0	1.1364
311	0	1	0	0	0	0	0	0	0	0
312	0	0	1	0	0	1	0	0	0	0
313	0	0	0	1	0	0	0	0	0	1.1364
321	0	1	0	0	1	0	0	0	0	0
322	0	0	0	0	0	1	0	0	0	0
323	0	0	0	0	0	0	1	0	0	1.1364
331	0	1	0	0	0	0	0	1	0	0
332	0	0	0	0	0	1	0	0	1	0
333	0	0	0	0	0	0	0	0	0	1.1364

Table A1.13: Scenario 2, Type 2 E-mails - Average Number of Visits by Solving the DTMC

	D1	D2	D3	New	011	012	013	021	022	023
D1	1.0638	0	0	0	0	0	0	0	0	0
D2	0	1.0526	0	0	0	0	0	0	0	0
D3	0	0	1.1628	0	0	0	0	0	0	0
New	**0.099634**	**0.064765**	**0.084417**	**1**	**0.37462**	**0.025256**	**0.021046**	**0.047216**	**0.27967**	**0.03632**
011	0.26596	0	0	0	1	0	0	0	0	0
012	0.033921	0.18024	0.022995	0	0.12754	1.0086	0.007165	0.1314	0.77829	0.10108
013	0.027089	0.004821	0.21423	0	0.10185	0.006867	1.0057	0.003515	0.020817	0.002704
021	0.24009	0.011055	0.012091	0	0.90275	0.060859	0.050716	1.0081	0.047738	0.0062
022	0	0.23158	0	0	0	0	0	0	1	0
023	0.027089	0.004821	0.21423	0	0.10185	0.006867	0.005722	0.003515	0.020817	1.0027
031	0.24009	0.011055	0.012091	0	0.90275	0.060859	0.050716	0.00806	0.047738	0.0062
032	0.033921	0.18024	0.022995	0	0.12754	0.008598	0.007165	0.1314	0.77829	0.10108
033	0	0	0.24419	0	0	0	0	0	0	0
111	0.06383	0	0	0	0	0	0	0	0	0
112	0	1.0526	0	0	0	0	0	0	0	0
113	0	0	1.1628	0	0	0	0	0	0	0
121	0.06383	0	0	0	0	0	0	0	0	0
122	0	1.0526	0	0	0	0	0	0	0	0
123	0	0	1.1628	0	0	0	0	0	0	0
131	0.06383	0	0	0	0	0	0	0	0	0
132	0	1.0526	0	0	0	0	0	0	0	0
133	0	0	1.1628	0	0	0	0	0	0	0
211	1.0638	0	0	0	0	0	0	0	0	0
212	0	0.052632	0	0	0	0	0	0	0	0
213	0	0	1.1628	0	0	0	0	0	0	0
221	1.0638	0	0	0	0	0	0	0	0	0
222	0	0.052632	0	0	0	0	0	0	0	0
223	0	0	1.1628	0	0	0	0	0	0	0
231	1.0638	0	0	0	0	0	0	0	0	0
232	0	0.052632	0	0	0	0	0	0	0	0
233	0	0	1.1628	0	0	0	0	0	0	0
311	1.0638	0	0	0	0	0	0	0	0	0
312	0	1.0526	0	0	0	0	0	0	0	0
313	0	0	0.16279	0	0	0	0	0	0	0
321	1.0638	0	0	0	0	0	0	0	0	0
322	0	1.0526	0	0	0	0	0	0	0	0
323	0	0	0.16279	0	0	0	0	0	0	0
331	1.0638	0	0	0	0	0	0	0	0	0
332	0	1.0526	0	0	0	0	0	0	0	0
333	0	0	0.16279	0	0	0	0	0	0	0

Table A1.14: Scenario 2, Type 2 E-mails - Average Number of Visits by Solving the DTMC (contd)

D1	**031**	**032**	**033**	**111**	**112**	**113**	**121**	**122**	**123**	**131**
D2	0	0	0	1.0638	0	0	0	0	0	0
D3	0	0	0	0	0	0	0	0	0	0
New	0	0	0	0	0	0	0	0	0	0
011	**0.04371**	**0.007947**	**0.34571**	**0.099634**	**0**	**0**	**0**	**0**	**0**	**0**
012	0	0	0	0.26596	0	0	0	0	0	0
013	0.011907	0.002165	0.09417	0.033921	0	0	0	0	0	0
021	0.11093	0.020169	0.87733	0.027089	0	0	0	0	0	0
022	0.006261	0.001138	0.049517	0.24009	0	0	0	0	0	0
023	0	0	0	0	0	0	0	0	0	0
031	0.11093	0.020169	0.87733	0.027089	0	0	0	0	0	0
032	1.0063	0.001138	0.049517	0.24009	0	0	0	0	0	0
033	0.011907	1.0022	0.09417	0.033921	0	0	0	0	0	0
111	0	0	1	0	0	0	0	0	0	0
112	0	0	0	1.0638	0	0	0	0	0	0
113	0	0	0	0	1	0	0	1	0	0
121	0	0	0	0	0	1	0	0	0	0
122	0	0	0	1.0638	0	0	1	0	0	0
123	0	0	0	0	0	0	0	1	0	0
131	0	0	0	0	0	0	0	0	1	0
132	0	0	0	1.0638	0	0	0	0	0	1
133	0	0	0	0	0	0	0	1	0	0
211	0	0	0	0	0	0	0	0	0	0
212	0	0	0	1.0638	0	0	0	0	0	0
213	0	0	0	0	0	0	0	0	0	0
221	0	0	0	0	0	0	0	0	0	0
222	0	0	0	1.0638	0	0	0	0	0	0
223	0	0	0	0	0	0	0	0	0	0
231	0	0	0	0	0	0	0	0	0	0
232	0	0	0	1.0638	0	0	0	0	0	0
233	0	0	0	0	0	0	0	0	0	0
311	0	0	0	0	0	0	0	0	0	0
312	0	0	0	1.0638	0	0	0	0	0	0
313	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0
322	0	0	0	1.0638	0	0	0	0	0	0
323	0	0	0	0	0	0	0	0	0	0
331	0	0	0	0	0	0	0	0	0	0
332	0	0	0	1.0638	0	0	0	0	0	0
333	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0

Table A1.15: Scenario 2, Type 2 E-mails - Average Number of Visits by Solving the DTMC (contd)

	132	133	211	212	213	221	222	223	231	232
D1	0	0	0	0	0	0	0	0	0	0
D2	0	0	0	0	0	0	1.0526	0	0	0
D3	0	0	0	0	0	0	0	0	0	0
New	**0**	**0**	**0**	**0**	**0**	**0**	**0.064765**	**0**	**0**	**0**
011	0	0	0	0	0	0	0	0	0	0
012	0	0	0	0	0	0	0.18024	0	0	0
013	0	0	0	0	0	0	0.004821	0	0	0
021	0	0	0	0	0	0	0.011055	0	0	0
022	0	0	0	0	0	0	0.23158	0	0	0
023	0	0	0	0	0	0	0.004821	0	0	0
031	0	0	0	0	0	0	0.011055	0	0	0
032	0	0	0	0	0	0	0.18024	0	0	0
033	0	0	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0	0	0
112	0	0	0	0	0	0	1.0526	0	0	0
113	0	1	0	0	0	0	0	0	0	0
121	0	0	0	0	0	0	0	0	0	0
122	0	0	0	0	0	0	1.0526	0	0	0
123	0	1	0	0	0	0	0	0	0	0
131	0	0	0	0	0	0	0	0	0	0
132	1	0	0	0	0	0	1.0526	0	0	0
133	0	1	0	0	0	0	0	0	0	0
211	0	0	1	0	0	0	0	0	0	0
212	0	0	0	1	0	0	1.0526	0	0	0
213	0	0	0	0	1	0	0	0	0	0
221	0	0	1	0	0	1	0	0	0	0
222	0	0	0	0	0	0	1.0526	0	0	0
223	0	0	0	0	0	0	0	1	0	0
231	0	0	1	0	0	0	0	0	1	0
232	0	0	0	0	0	0	1.0526	0	0	1
233	0	0	0	0	0	0	0	0	0	0
311	0	0	0	0	0	0	0	0	0	0
312	0	0	0	0	0	0	1.0526	0	0	0
313	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0
322	0	0	0	0	0	0	1.0526	0	0	0
323	0	0	0	0	0	0	0	0	0	0
331	0	0	0	0	0	0	0	0	0	0
332	0	0	0	0	0	0	1.0526	0	0	0
333	0	0	0	0	0	0	0	0	0	0

Table A1.16: Scenario 2, Type 2 E-mails - Average Number of Visits by Solving the DTMC (contd)

	233	**311**	**312**	**313**	**321**	**322**	**323**	**331**	**332**	**333**
D1	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
D2	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
D3	0	0	0	0	0	0	0	0	0	1.1628
New	**0**	**-1.39E-17**	**0**	**0**	**0**	**0**	**0**	**-1.39E-17**	**0**	**0.084417**
011	0	-2.78E-17	0	0	0	0	0	-2.78E-17	0	0
012	0	-6.94E-18	0	0	0	0	0	-6.94E-18	0	0.022995
013	0	0	0	0	0	-8.67E-19	0	0	-8.67E-19	0.21423
021	0	-2.78E-17	0	0	0	-1.73E-18	0	-2.78E-17	-1.73E-18	0.012091
022	0	0	0	0	0	0	0	0	0	0
023	0	0	0	0	0	-8.67E-19	0	0	-8.67E-19	0.21423
031	0	-2.78E-17	0	0	0	-1.73E-18	0	-2.78E-17	-1.73E-18	0.012091
032	0	-6.94E-18	0	0	0	0	0	-6.94E-18	0	0.022995
033	0	0	0	0	0	0	0	0	0	0.24419
111	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
112	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
113	0	0	0	0	0	0	0	0	0	1.1628
121	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
122	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
123	0	0	0	0	0	0	0	0	0	1.1628
131	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
132	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
133	0	0	0	0	0	0	0	0	0	1.1628
211	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
212	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
213	1	0	0	0	0	0	0	0	0	1.1628
221	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
222	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
223	1	0	0	0	0	0	0	0	0	1.1628
231	0	-1.11E-16	0	0	0	0	0	-1.11E-16	0	0
232	0	0	0	0	0	-1.11E-16	0	0	-1.11E-16	0
233	1	0	0	0	0	0	0	0	0	1.1628
311	0	1	0	0	0	0	0	-1.11E-16	0	0
312	0	0	1	0	0	1	0	0	-1.11E-16	0
313	0	0	0	1	0	0	0	0	0	1.1628
321	0	1	0	0	1	0	0	-1.11E-16	0	0
322	0	0	0	0	0	1	0	0	-1.11E-16	0
323	0	0	0	0	0	0	1	0	0	1.1628
331	0	1	0	0	0	0	0	1	0	0
332	0	0	0	0	0	1	0	0	1	0
333	0	0	0	0	0	0	0	0	0	1.1628

A1.1 Matlab Program for Fundamental Matrix Inversion

```
clear;
A = xlsread('sce9ci.xls');
C = eye(40);
D = C-A;
K = inv(D)
```

A1.2 Matlab Code to Obtain the Parameters for Hyperexponential Distribution Used in Simulation

```
k = 0.17;
p1 = 0.3;
p2 = 0.7;
poly = [ (p1*p2+p1*p1) (-2*k*p1) (k*k*(1-1.5*p2))];
a =  roots(poly)
b = [k-a*p1]/(p2)
```

APPENDIX A2

SIMULATION MODEL

Simulation estimates were used to test the accuracy of the analytical approximations developed. In this section, the logic of the simulation model is described. This section also provides a brief introduction to the distributions used in the study and details of how their parameters were estimated for running the simulation model.

A2.1 The Simulation Logic

The simulation logic explained in this section is for the Scenario 1 explained in section 6.6.2. The simulation model consisted of five sub models: The first sub model described the arrival process of an e-mail (Figure A2.2). The second, third and fourth sub models described inboxes of agent 1, agent 2 and agent 3, respectively. The fifth sub model was for statistics collection. The top model shows the transitions made between various sub models (Figure A2.1). A contact center with two types of e-mail and three agents was simulated. The new e-mails each with different arrival rates were combined and split into 33%, 33% and 34% to agent 1, agent 2 and agent 3 respectively. As all the three agents' sub model and the operations in the sub model are the same, we take the sub model 2 (agent 1) for our explanation. Assuming an e-mail arrives at agent 1 inbox. First the e-mail gets preprocessed. Preprocessing is determining the e-mail history and its type. The history can be any one of the following for a given e-mail type: a brand new e-mail, previously processed by agent 1, previously processed by agent 2 or previously processed by agent 3. If the e-mail is a brand new one, agent 1 can process the e-mail or forward it to agent 2 or 3 with their respective forwarding probabilities. Similarly if an e-mail is previously processed by agent 1, he/she can process it or forward the e-mail to agents 2 or 3. The only difference in both the cases is e-mail history. Similarly if an e-mail is previously processed by agent 2 or agent 3 and reaches agent 1, he/she can process the e-mail or forward it to agent 2 or agent 3. These entire operations are explained in the sub model 2 (Figure A2.3). These operations and descriptions holds remain same for sub models 3 and 4. An e-mail can enter sub model 5 only when it gets processed by agent 1 (2 or 3). Sub model 5 is shown in Figure A2.4. The timers in sub model 5 keep track of the e-mail response time as soon as it gets processed by agent 1 (by 2 or 3). The e-mail can leave the system with resolution probabilities, if agent 1 response is sufficient enough or else the e-mail once again enters the system via the delay node through sub model 1. If the e-mail gets resolved, the resolution time is calculated by the timers in sub model 5. The unresolved e-mails enter the delay nodes as internal arrivals which add up to the new e-mails coming from the outside world. The e-mail (once brand new) entering through the delay node is tagged as previously processed e-mail by agent 1 and once again split into 33%, 33% and 34% to agent 1, 2 and respectively. Simulation captures only the one step previous history and forgets the past history of the e-mail. This whole procedure gets repeated till the e-mail gets resolved.

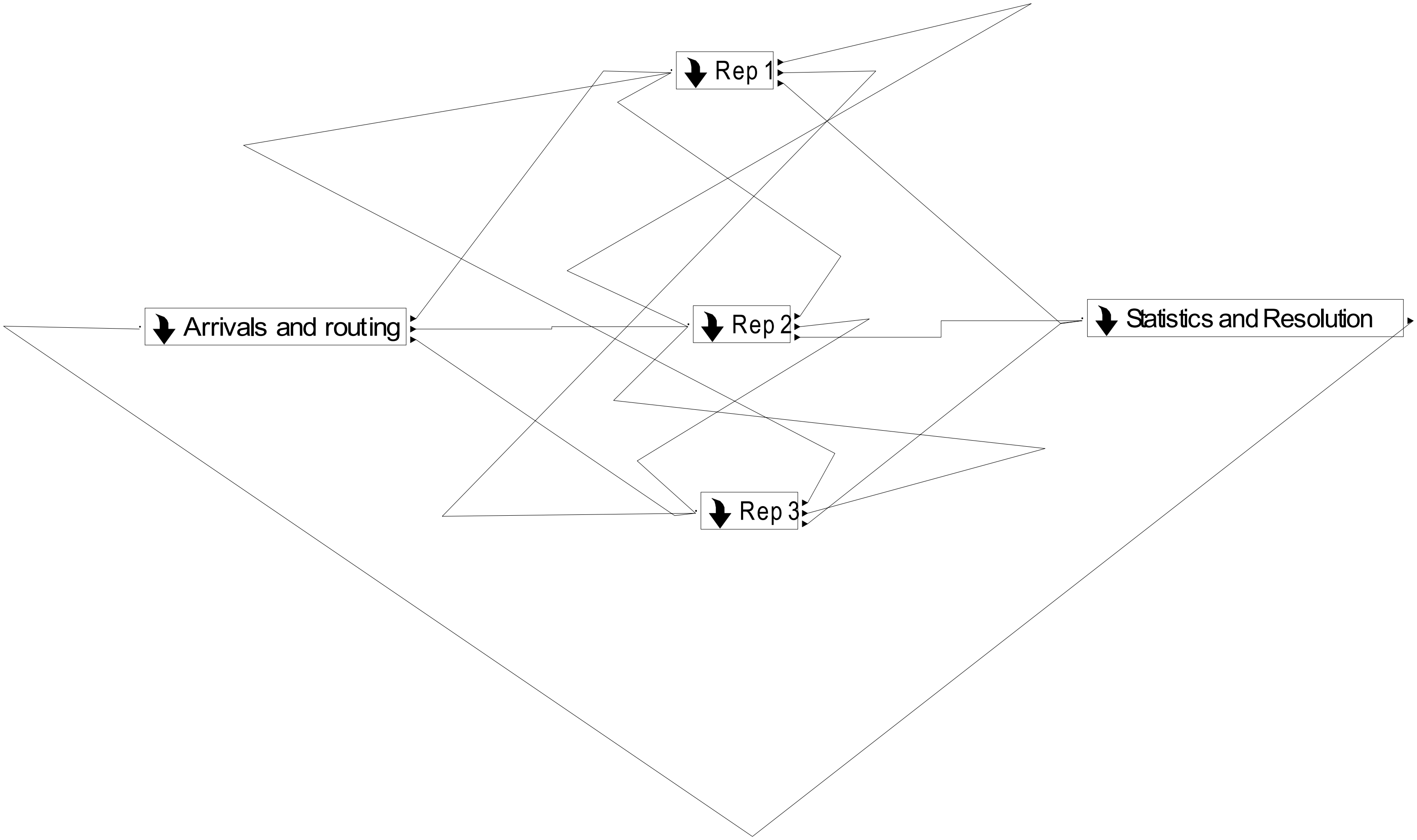

Figure A2.1: Simulation Model

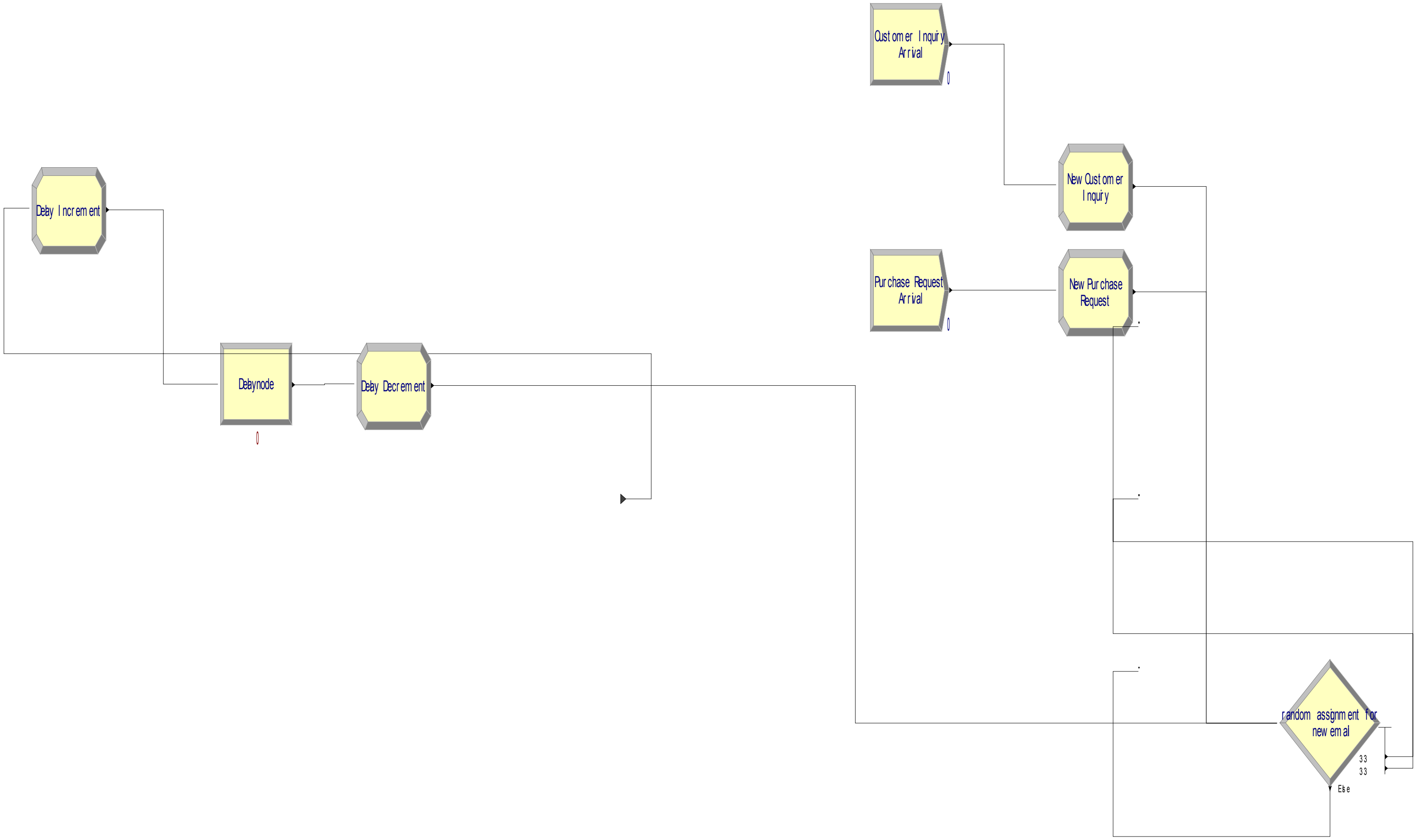

Figure A2.2: Arrival and Routing Sub Model

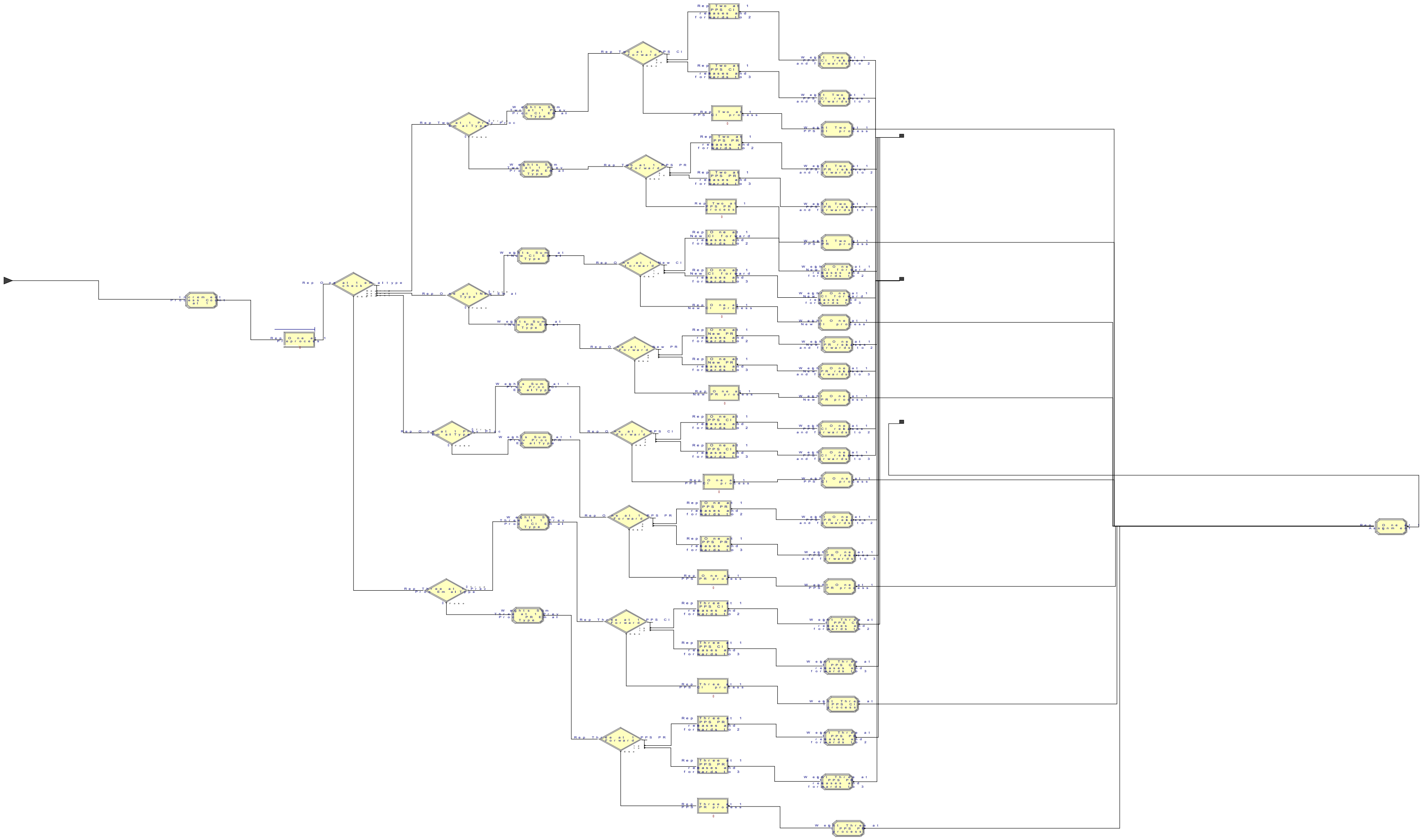

Figure A2.3: Rep i (Agent i) Sub Model i = 1, 2 and 3

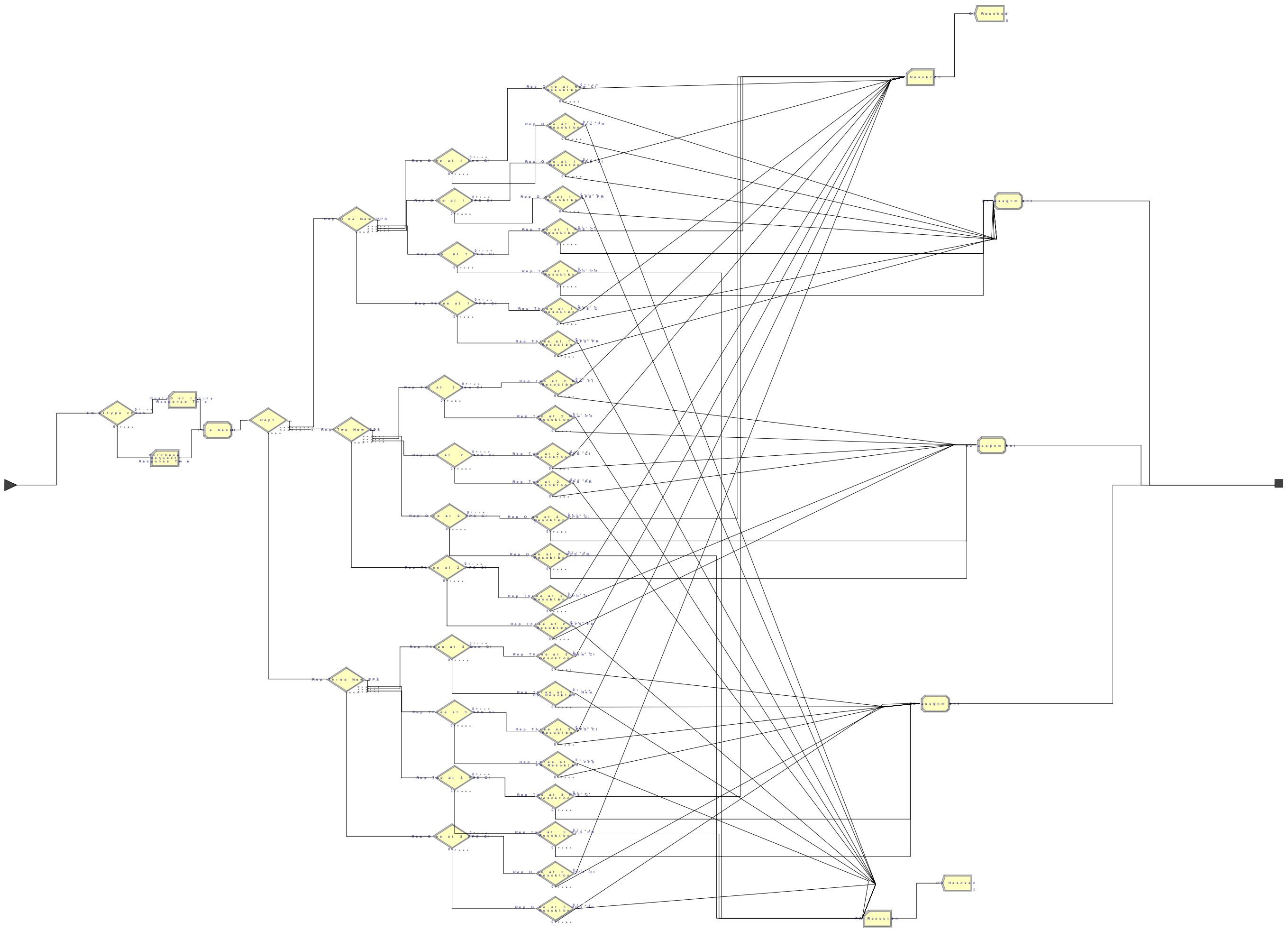

Figure A2.4: Statistics and Resolution Sub Model

A2.2 Service Time Distributions and their Parameters

A2.2.1 Erlang Distribution

When r independent sequential phases of exponential distribution with rate λ in each sequential phase are added together, then the resulting distribution gives rise to r-stage Erlang Distribution. This distribution is used in modeling ideal repair with exponentially distributed interfailure times (Ramakumar 1993).

Failure data in reliability analysis follows an Erlang distribution. The probability density function (pdf) is given by

$$f_X(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!} \qquad \forall x,\ \lambda \geq 0,\ r = 1,2...$$

The cumulative distribution function (cdf) is given by

$$F_X(x) = 1 - \sum_{u=0}^{u=r-1} \frac{(\lambda x)^u}{u!} \cdot e^{-\lambda x} \qquad \forall x,\ \lambda \geq 0,\ r = 1,2...$$

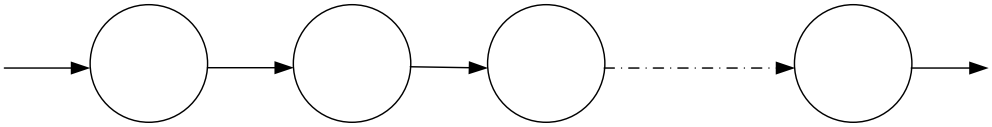

Figure A2.5: Erlang Service Time Distribution

The mean $E(X)$ and variance $V(X)$ of Erlang distribution are $\frac{r}{\lambda}$ and $\frac{r}{\lambda^2}$ respectively.

SCV for Erlang distribution is $\frac{1}{\sqrt{r}}$. As r can take only integer values, SCV is always less than one.

The parameters required for the simulation model are 1) mean of the exponential at each stage and 2) number of stages. The mean time at each stage is obtained by dividing the service time by the number of stages. For example, if the interarrival rate is 1 per unit time for a four-stage Erlang distribution, the exponential mean at each stage is 1/4 = 0.25 time unit. The distribution is declared in the simulation model as ERLA (expomean, no of stages) (Kelton et al. 2002).

A2.2.2 Exponential Distribution

Exponential distribution is one of the most widely distributions in modeling a variety of real world problems. For example in queueing theory, the service time distribution follows exponential distribution and in reliability theory, the exponential distribution is used for modeling failure data. The cumulative distribution function is given by

$$F_X(x) = 1 - e^{-\lambda x} \qquad \forall x \geq 0$$

where λ is the parameter of the distribution. The mean $E(X)$ and variance $V(X)$ of exponential distribution are $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$ respectively. The squared coefficient of variation (SCV = Variance/Mean^2) for exponential distribution is one.

The parameter required for the simulation model is the mean of the exponential distribution. The distribution is declared in the simulation model as EXPO (mean) (Kelton et al. 2002).

A2.2.3 Hyperexponential Distribution

If r parallel phases of exponential distribution occur with probability p_r in an experiment, then the overall density function of the experiment follows a r - phase hyper-exponential distribution. The probability density function (pdf) is given by

$$f_X(x) = \sum_{m=1}^{m=r} p_m \mu_m e^{-\mu_m x} \qquad \forall x \geq 0,\ \forall \mu_m \geq 0 \text{ and } \sum_{m=1}^{m=r} p_m = 1$$

The cumulative distribution function (CDF) is given by

$$F_X(x) = \sum_{m=1}^{m=r} p_m (1 - e^{-\mu_m x}) \qquad \forall x \geq 0,\ \forall \mu_m \geq 0 \text{ and } \sum_{m=1}^{m=r} p_m = 1$$

This distribution exhibits more variability than exponential and is often used for modeling CPU service time distributions in computer systems. Hyperexponential distribution is a mixture of exponential distributions and its CDF can be expressed in terms of exponential CDF, i.e.

$$F_X(x) = \sum_{m=1}^{m=r} p_m (F_m(x)) \qquad F(x) = 1 - e^{-\mu x},\ \forall x \geq 0,\ \forall \mu_m \geq 0 \text{ and } \sum_{m=1}^{m=r} p_m = 1$$

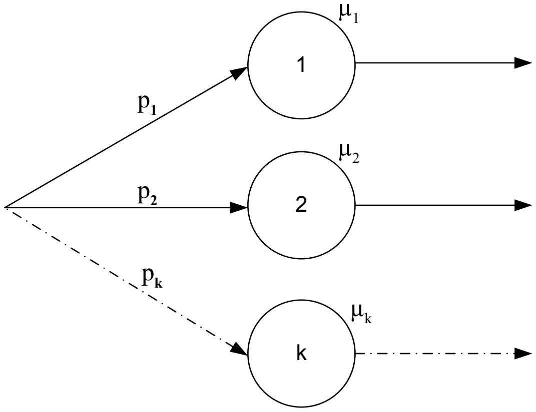

Figure A2.6: Hyperexponential Service Time Distribution

The mean $E(X)$ and variance $V(X)$ of hyper-exponential distribution are $\sum_{m=1}^{m=r} \left(\frac{p_m}{\mu_m} \right)$ and $2\sum_{m=1}^{m=r} \left(\frac{p_m}{\mu_m^2} \right) - \left[\sum_{m=1}^{m=r} \left(\frac{p_m}{\mu_m} \right) \right]^2$ respectively. SCV for hyper-exponential distribution is $2\left(\frac{\sum_{m=1}^{m=r} \left(\frac{p_m}{\mu_m^2} \right)}{\sum_{m=1}^{m=r} \left(\frac{p_m}{\mu_m} \right)^2} \right)$ - 1. This distribution is declared in the model as a discrete distribution and given by DISC (p_1, EXPO (m_1), p_2, EXPO (m_2), …, 1, EXPO(m_r)) (Kelton et al. 2002).

APPENDIX A3

DETERMINATION OF THE WARM-UP PERIOD AND RUN LENGTH FOR SIMULATION EXPERIMENTS

A3.1 Introduction

In any simulation, when the warm-up period is not considered, the results may be affected by some initialization bias. At the same time, if the simulation is not run for a sufficiently long time then any infrequent event may be missed and this may affect the resulting steady-state performance measures. Thus, it is all the more crucial and imperative that the simulation estimates must represent steady-state performance measures, especially when the accuracy of the analytical results is evaluated using these simulation estimates.

In this study, several inbound e-mail contact center configurations were modeled. Hence, each of these configurations had to be simulated to check the accuracy of the analytical model. It is indeed very important to determine the warm-up period as well as the run time as discussed earlier, but if one has to find these parameters for each system, it would be very tedious and time consuming. The key system parameters which influence these simulation parameters are the means and SCVs of the interarrival and service times and the routing probabilities of the e-mail. Thus, a system with very high variability was chosen to determine the warm-up period and run time.

The experiment to determine these simulation parameters was performed on the single-server open queueing network with three agents. The interarrival times followed an exponential distribution while the service times of all the three agents followed a Hyperexponential distribution. The simulation was run for 10 independent replications of length 2,500 time units each. This resulted in generating approximately 25,000 e-mails. The resolution time of the e-mails was considered for the analysis. Welch's method (Welch 1983) was used to determine the warm-up period. The procedure is described in the following section.

A3.2 Welch's Technique to Determine Warm-up Period

According to this procedure, n replications of the simulation are run for a length of m time units each. Let Y_{ij} represent the time in the system for the i^{th} observation from the j^{th} replication ($i = 1, 2, 3, ..., m$) and ($j = 1, 2, 3, ..., n$).

The Y_{ij} are averaged as follows.

$$\overline{Y}_i = \sum_{i=1}^{n} (Y_{ij} / n) \quad \text{for} \quad i = 1, 2, 3, ..., m. \qquad \text{A3.1}$$

The averaged processes $\overline{Y}_1, \overline{Y}_2 . ..., \overline{Y}_m$ have the same mean curves as the original process, but the plot has only $(1/n)^{\text{th}}$ of the variance of the original process.

i.e.

$$E\,[\overline{Y}_i] = E\,[Y_{ij}] \text{ and} \qquad \text{A3.2}$$

$$Var[\overline{Y}_i] = Var[Y_{ij}]/n \qquad \text{A3.3}$$

To smoothen out the high frequency oscillations in $\overline{Y}_1, \overline{Y}_2, ..., \overline{Y}_m$ the moving average $\overline{Y}_i(w)$ is calculated, where w is a positive integer.

$$\overline{Y}_i(w) = \begin{cases} \dfrac{\sum_{s=-(i-1)}^{i-1} \overline{Y_{i+s}}}{2i-1} & ; i = 1, ..., w \\ \dfrac{\sum_{s=-w}^{w} \overline{Y_{i+s}}}{2w+1} & ; i = w+1, \ \ldots, \ m-w \end{cases} \qquad \text{A3.4}$$

Thus, if i is not too close to the beginning of the replication, then $\overline{Y}_i(w)$ is just the simple average of the $(2w+1)$ observations of the averaged process centered at observation i. This is called the moving average since i moves over time.

$\overline{Y}_i(w)$ is plotted for $i=1, 2, 3, ..., (m-w)$ and the value of i beyond which $\overline{Y}_1(w)$, $\overline{Y}_2(w)$, …, $\overline{Y}_m(w)$ appear to converge is identified and that value defines the warm-up period. The above procedure was applied to the system described in the previous section and the plots of $\overline{Y}_i(w)$ are shown in several graphs (Figures A3.1 to A3.10). The value of w is 750. The X axis denotes the e-mail number and the Y axis denotes the resolution time in system for that e-mail.

A3.3 Determination of Run Length

The run length was determined using the tightness of the 95% confidence interval. Different run lengths were tried and at 18,840 time units, the half width of the 95% confidence interval was less than 3% of the mean resolution time in the system. Hence, 18,840 time units was chosen as the run length for each replication.

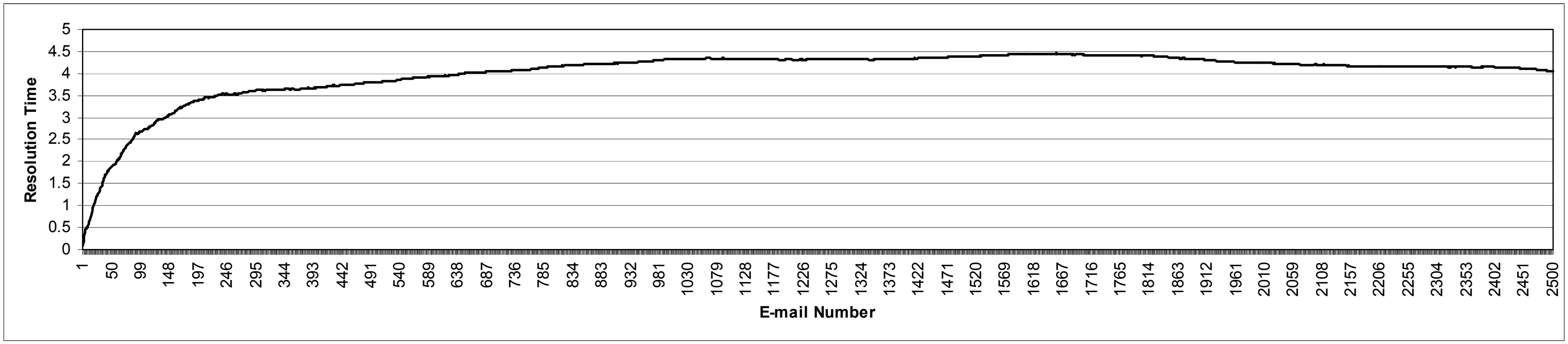

Figure A3.1: Plot of Resolution Time in System

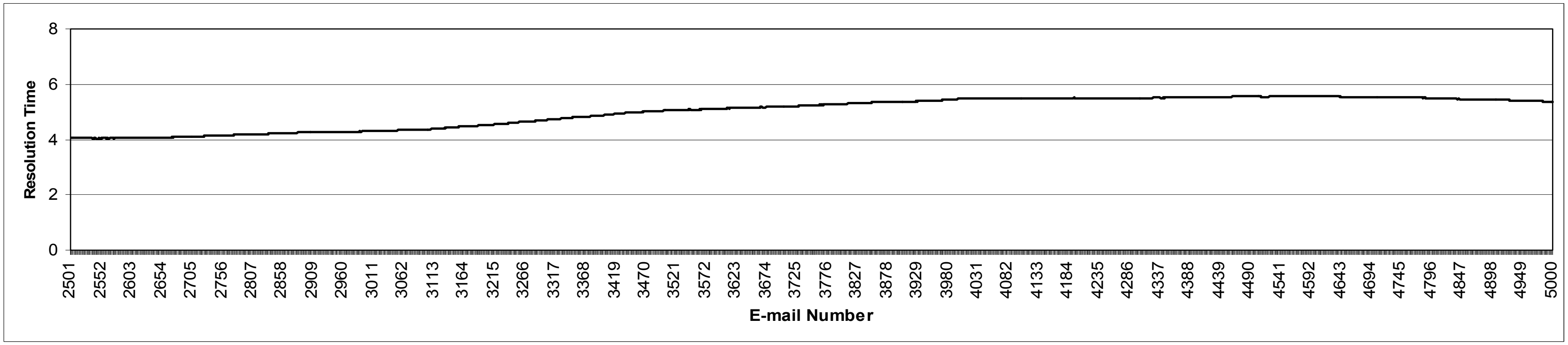

Figure A3.2: Plot of Resolution Time in System (contd)

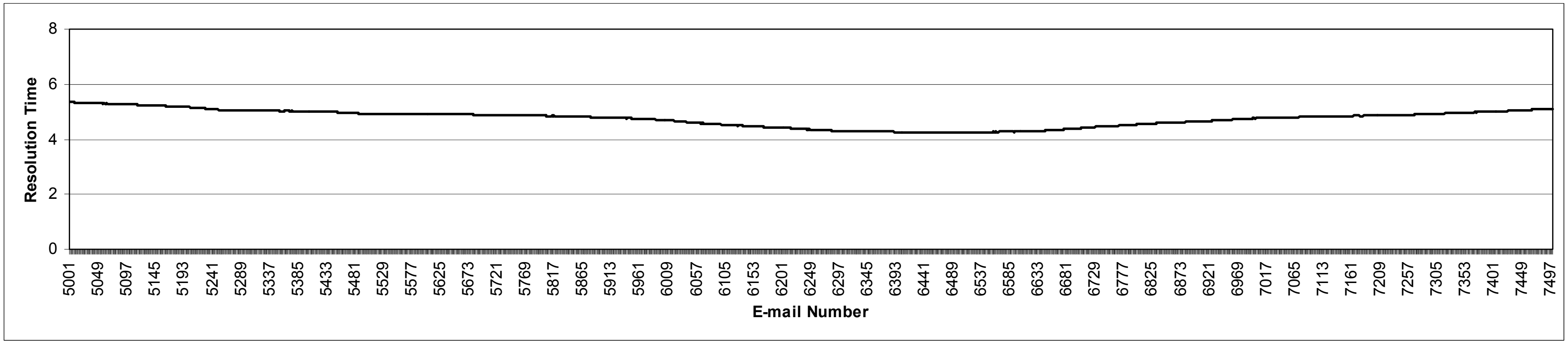

Figure A3.3: Plot of Resolution Time in System (contd)

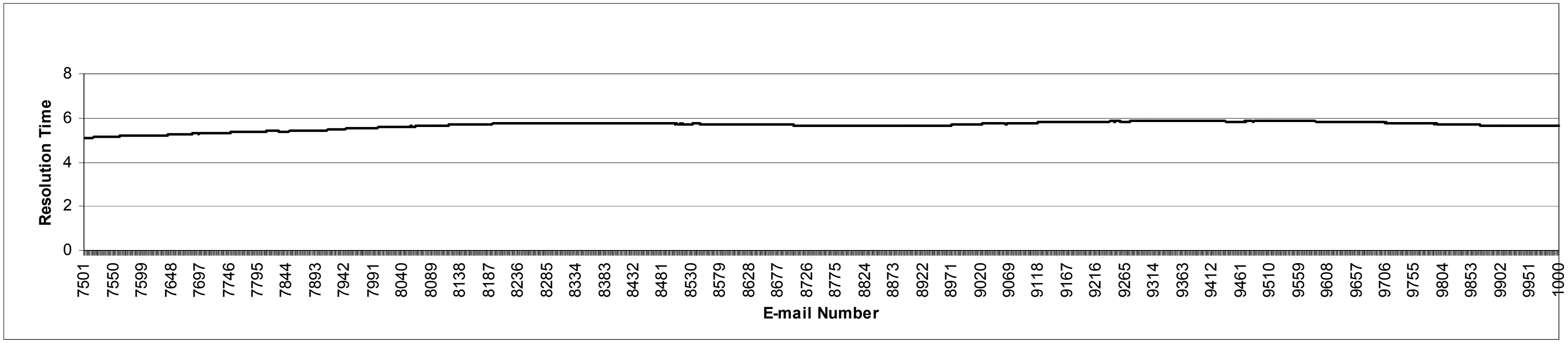

Figure A3.4: Plot of Resolution Time in System (contd)

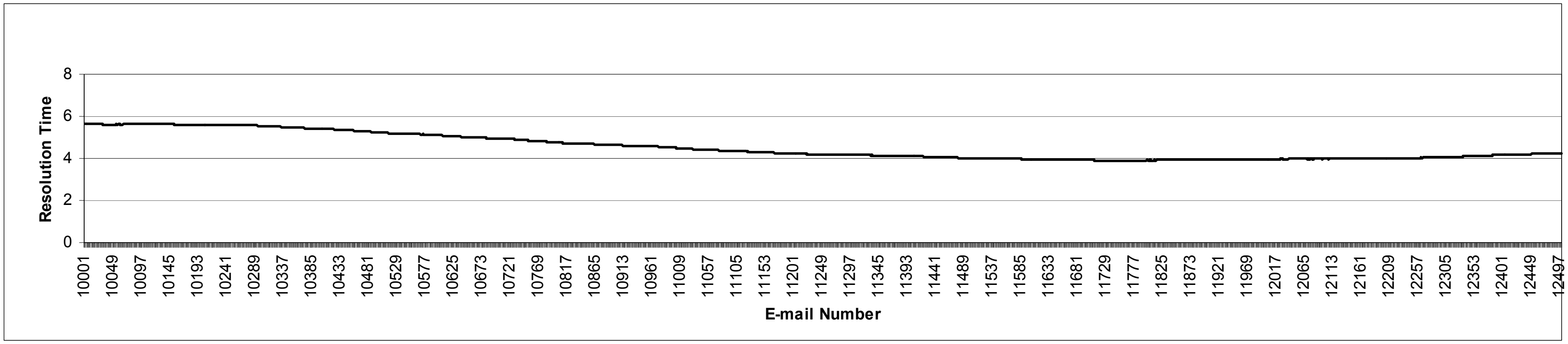

Figure A3.5: Plot of Resolution Time in System (contd)

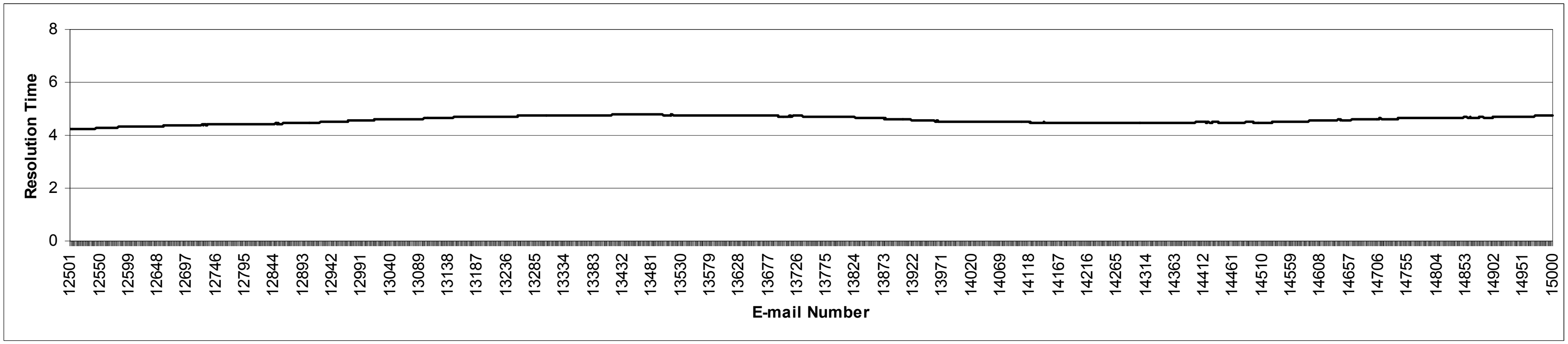

Figure A3.6: Plot of Resolution Time in System (contd)

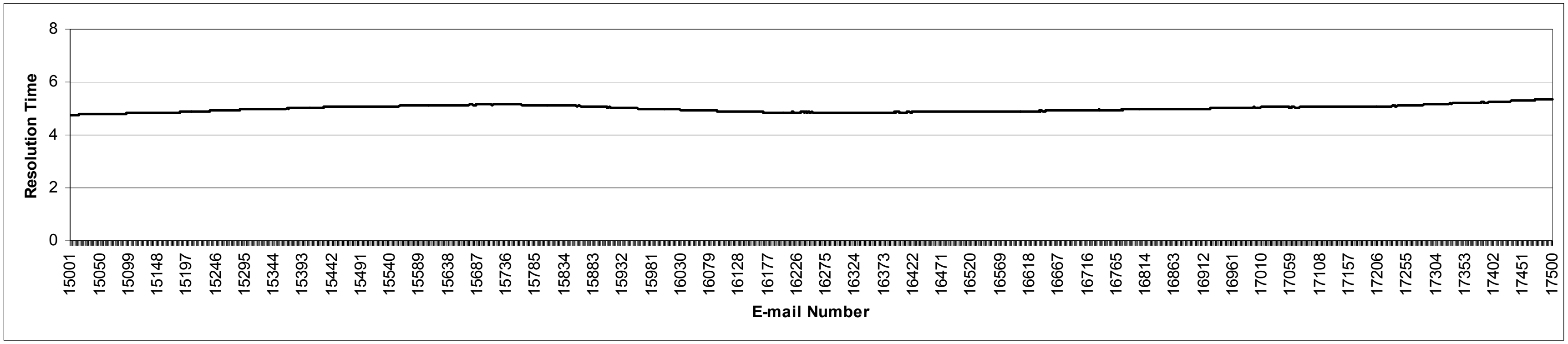

Figure A3.7: Plot of Resolution Time in System (contd)

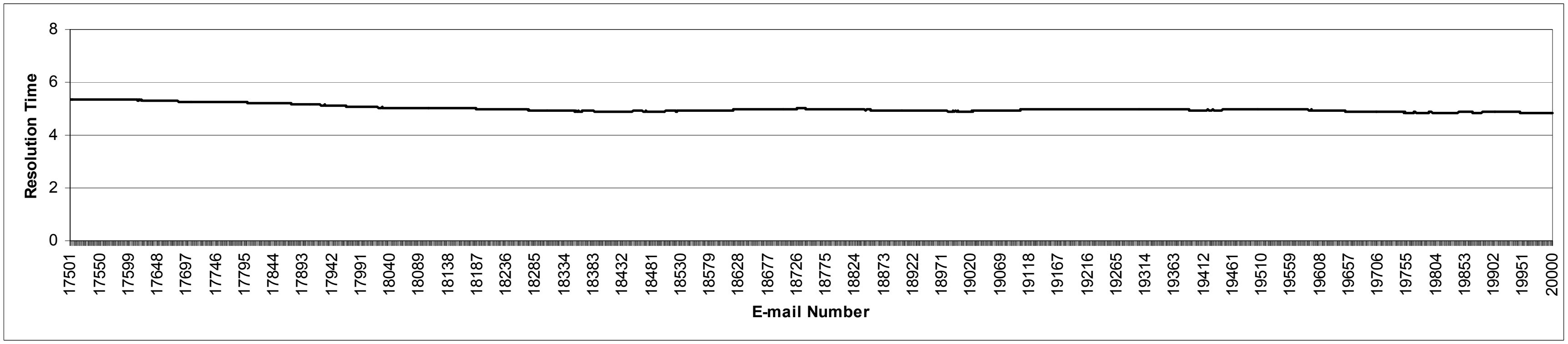

Figure A3.8: Plot of Resolution Time in System (contd)

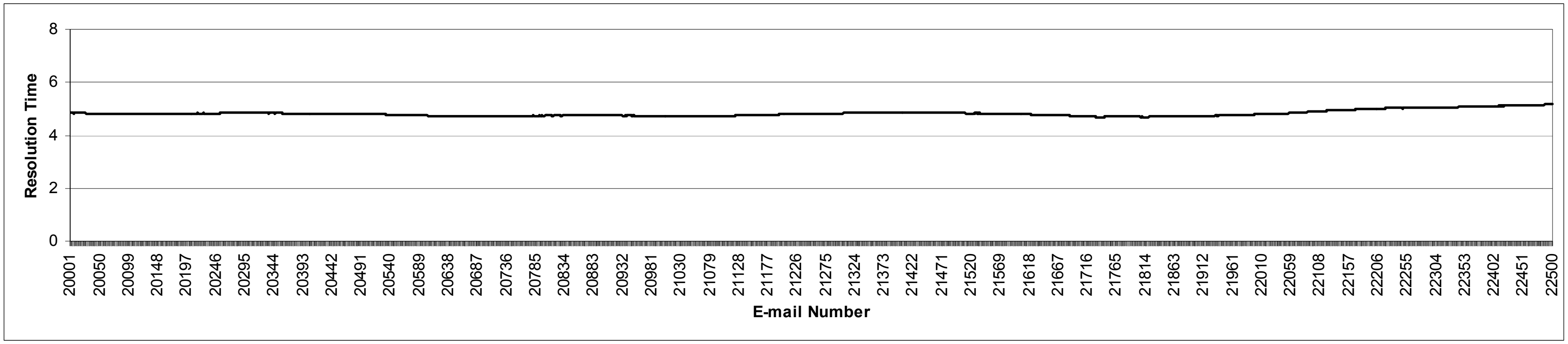

Figure A3.9: Plot of Resolution Time in System (contd)

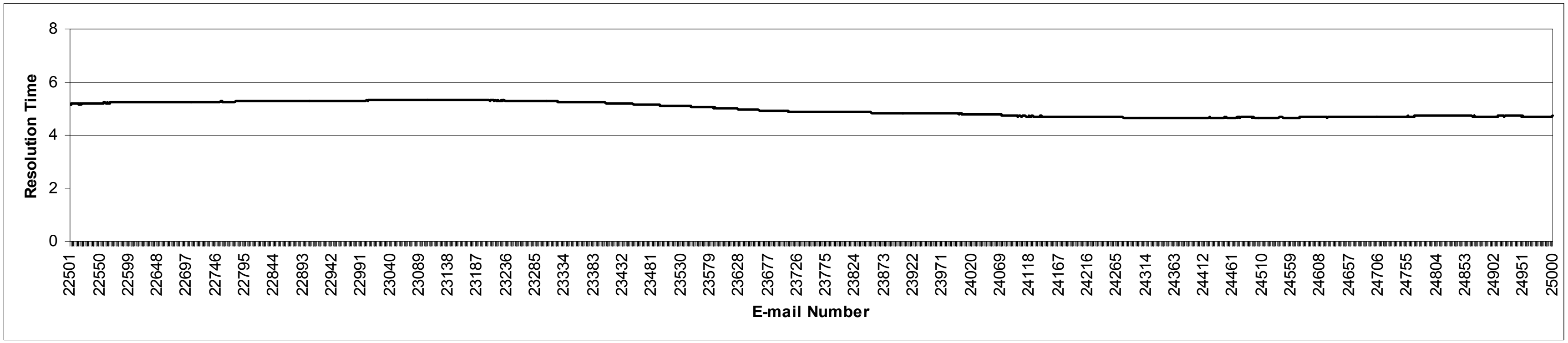

Figure A3.10: Plot of Resolution Time in System (contd)

VITA

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Scope and Method of Study: The operation of a customer contact center where the interface between the agents' and the customer happens through e-mail was modeled using a multi-class open queueing network model. A novel approach to model the dependence of processing times and routing on e-mail history called the history-based aggregation method was developed. This aggregation method extends the popular parametric decomposition (PD) method for solving multi-class open queueing networks to more general situations. A discrete-time Markov chain (DTMC) with an expanded state space was developed to model the non-Markovian routing of a new e-mail through the contact center until its eventual resolution. The analysis of this absorbing Markov chain led to the computation of the proportion of e-mails in an agent's in-box that are new, previously processed by the same agent, or previously processed by another agent. Using these proportions, a new "history-based" aggregation step for each customer class in the PD method was introduced. This step precedes the existing class-based aggregation step in the PD method. The resulting queueing network model was solved using the RAQS software package. The accuracy and robustness of the analytical approach was demonstrated by comparing the analytical results with simulation estimates of performance measures for a variety of scenarios. Typical contact center situations like grouping of agents and server interruptions were also modeled within the above approach.

Findings and Conclusions: The DTMC-based analytical method showed excellent prediction capability across the various cased examined. The relative percentage error in utilization was less than 1% in all cases and less than 17% for other performance measures. This work has laid the foundation for the development of rapid performance analysis tools for customer contact centers which can be effectively used for improving customer contact center operations. The history-based aggregation method represents a significant extension to the PD method for solving queueing network models.

Advisor's approval: Dr. Manjunath Kamath