GAME THEORETIC SOLUTIONS APPLIED TO ACCOUNTING COST ALLOCATION: A LABORATORY EXPERIMENT

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INTRODUCTION

Background

The allocation problem, arbitrary allocations, and incorrigible allocations are all commonly used phrases in the accounting literature. Thomas is credited with the current concern for cost allocations.[1]

Allocation is defined as the partitioning or assigning of costs or revenues to items such as time periods, long-term assets, or activities (e.g., divisions) within an entity. For example, depreciable assets, inventories, prepayments, labor services, research and development, and advertising are inputs that are partitioned to time periods in the form of expense or cost of goods manufactured. The purchase price of a basket purchase, the purchase of a group of long-term assets as one unit, is partitioned to the individual long-term assets. Joint or common costs such as income taxes, financing costs, and general and administrative expenses are assigned to activities within an entity. This study will address the allocation of joint costs to activities within an entity.

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[1]Arthur L. Thomas, "The Allocation Problem in Financial Accounting Theory," Studies in Accounting Research No. 3, 1969.

Statement of the Problem

Introduction

A decision required of many accountants is the selection and justification of particular allocation methods employed in cost allocation situations. The allocation problem in accounting is the inability of accountants to justify the particular allocation methods selected. The allocation problem circumscribes the problems of cost accumulation and of matching costs with revenues. According to Thomas, a solution to the allocation problem requires identification of allocation methods which satisfy three requirements:

1. The method should be unambiguous.
2. It should be possible to defend the method.
3. The method should divide up what is available to be allocated, no more and no less. The allocation should be additive.[2]

To be unambiguous an allocation scheme should result in a unique solution. In other words, there should be but one solution.

Thomas stated that the defense of an allocation method requires some type of theoretical justification. Justification can be in the form of assumptions or axioms which are not subject to conclusive demonstration or proof. Further justification would be the general acceptance of an allocation method by the parties involved (mutual satisfaction) and a demonstration that the allocation of cost is a consequent of the assumptions or axioms.

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[2]Arthur L. Thomas, "The Allocation Problem in Financial Accounting Theory," Studies in Accounting Research No. 3, 1969, p. 7.

The final requirement additivity, is very basic. An allocation scheme must divide up exactly, the amount of total cost.

Scope Limitation

While Thomas asserts that the allocation problem is applicable to all classes of accounting allocation, in Studies in Accounting Research No. 3, he addresses only those allocations that deal with financial accounting. Thomas' research study primarily discusses the class of cost allocations that have a direct effect on income determiniation and asset valuation (balance sheet numbers).

A class of allocations not discussed by Thomas, but also related to the allocation problem, is an allocation that involves the assignment of joint or common costs to different activities (divisions) within an entity. Joint costs are accumulated in a number of ways; for example, income taxes are incurred due to the legal process; financing costs are incurred generally through borrowing capital; and general and administrative expenses are incurred due to a need for the service function within an organization.

Income taxes, financing costs, and general and administrative expenses are not generally traceable directly to a specific activity within an entity. The aforementioned costs are incurred by the entity to assist the different activities of its business for the purpose of, or as a result of, earning revenue. For example, income taxes are incurred by the entity as a result of the total revenue earned due to the combined effort of the different activities. Financing costs, and general and administrative costs are incurred by the entity as a result of the combined effort of the different activities that precede the earning of revenue. Thus, the entity incurs costs of this nature in relationship to the revenue earning process. Since the revenue earning process is a combined effort of the various activities within an entity, the aforementioned costs are traceable indirectly to the various activities and are allocated to each activity.

Some joint costs are incurred because a benefit (e.g., cost savings) is perceived by decision makers. An example would be the cost two or more managers incur by jointly leasing a copy machine, rather than using the more costly copy machine service supplied by their employer (a corporation). The allocation of joint costs incurred where a benefit is perceived by decision makers will be addressed in this research study.

Thomas issued a challenge to accounting researchers to develop theoretical justification for allocation methods or to avoid allocations completely. Since abandonment is infeasible given the current state of the accounting art, a solution is essential.

Purpose of the Study

Thomas stated that the three aforementioned requirements that serve to justify a financial accounting allocation scheme, could apply to the class of allocations involving the assignment of joint costs. He suggested that further research was required in the area of joint cost allocation. Therefore, the purpose of the research reported in this dissertation was to evaluate the behavior of subjects, acting as surrogate division managers, in a joint cost allocation setting where the cost savings were available to each division manager. The allocation of joint costs reported by the groups was analyzed to ascertain if some game theoretic allocation scheme was approximated.

Cooperative game theory[3] is applicable in allocation situations where a benefit (e.g., synergy, arbitrage, or cost savings) is perceived by the players[4] for forming a coalition(s);[5] thus, game theory might serve as a theoretical justification for allocation schemes of this class. The subject of the reported research study was a laboratory experiment of a cost allocation situation involving the allocation of joint costs. The observed partitioning of cost savings was analyzed to make inferences about group behavor as viewed from a game theoretic perspective. Joint costs were allocated by the groups. Cost savings allocations were analyzed because game theoretic allocation schemes are generally formulated from the viewpoint of cost savings, arbitrage, or synergy. Identical statistical results occurred whether analyzing the joint cost allocations or the cost savings allocations.

Game Theory

Historical Background

In an attempt to solve the allocation problem in accounting, accounting researchers have introduced the use of game theory. Social scientists, mathematicians and economists have applied the theory of games in many decision making contexts.

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[3]Cooperative game theory is a game theoretic allocation where the participants are allowed to communicate with one another and bargain or negotiate for a solution that is acceptable to all the participants.

[4]Player is the term used for each participant in a game.

[5]A coalition is a group formed by some or all of the players.

Game theory is a branch of mathematics and is built on assertions which can be proved to be true if certain other assertions are true. Thus, game theory is basically a collection of theorems derived from axioms. The axioms defend a chosen allocation scheme against competing alternatives.

The solutions that result from game theoretic techniques are unique. Also, each technique is additive. Therefore, allocation methods using a game theoretic approach are theoretically justified because they meet the three minimum requirements proposed by Thomas.

Definition of Game Theory

A working definition of game theory is the theory of interest conflict. From the viewpoint of one of the players, the ultimate outcome depends on the actions of the other players. Each player attempts to select that move[6] which will benefit him most based on his partial influence over the game solution.

Game theory is comprised of three levels of abstraction; in ascending order they are extensive, normal and the characteristic function. Extensive form, the least abstract, involves the expression of each player's outcome in terms of utility. The normal form is a reduction of every game in extensive form that limits every player to one move and one move only. The characteristic function form, the most abstract, involves the assignment of a value of a game to each subset of players forming a coalition. The value is assigned as a consequence of

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[6]A move is the set of choices a player has at a particular decision point.

the rules of the specific game. The characteristic function form of the game is applicable to cost allocation and is the primary form employed in this study.

Development of Game Theory Applications to Management Accounting

The study of game theory seems to have evolved from mathematics, to the behavioral sciences, and finally to business applications in economics and accounting. Even though Shubik's[7] 1962 study set the tone in accounting, it did not attract much interest until the early 1970's. Thomas's allocation problem research study in 1969 seems to have instigated the current interest in game theory as applied to cost allocation.

The application of game theory to accounting cost allocation can be traced to Shubik in 1962. He described, through the use of axioms, how the managers of a decentralized firm could make choices which were best for the individual decision maker and the overall organization. The allocation scheme he used was the Shapley value.

Prior to Shubik, game theory literature was primarily a phenomena of the behavioral sciences. von Neumann and Morgenstern[8] and Luce and Raiffa,[9] wrote texts that seem to have pioneered the application of game theory to laboratory situations.

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[7]Martin Shubik, "Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing," Management Science, 8 (April, 1962), pp. 325-343.

[8]John von Neumann and Oskar Morgenstern, Theory of Games and Economic Behavior (New York, 1953).

[9]R. Duncan Luce and Howard Raiffa, Games and Decisions (New York, 1967).

Research Methodology

A laboratory experiment was conducted using 75 business students as surrogate division managers. The subjects were randomly partitioned into groups of three, each of which simulated a division managers' meeting. The problem each group confronted was how to get computer printed weekly reports to corporate headquarters at a prescribed time. Two decisions had to be made by each group. The first decision they had to make was whether to use an outside computer facility or a computer facility made available within the corporation. The second decision required was the partitioning of any joint costs to each division.

Data needed to test the hypotheses of this study were gathered from the two decisions required of each group. The surrogate division managers had the choice of using an outside computer facility, alone or as a group,[10] or using the computer facility within the corporation, alone or as a group. Group formation data were required to evaluate the subjects' coalition formation.

Group behavior was compared with three elements of game theory: coalition formation, core theory, and game theoretic cost allocation schemes. The concept of coalition (collusion) formation involves the separation of the participants of a game into a group or a number of groups. The formation of coalitions is a common phenomenon of conflict situations, and such is the case in game theory.[11] Thus, coalition formation is a vital step in the determination of the results of a game.

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[10]A group of this case could only be two division, because that was the maximum the outside computer facility could increase its capacity and still promise timely delivery.

[11]R. Duncan Luce and Howard Raiffa, Games and Decisions (New York, 1967), p. 8.

Core is defined as the set of payoffs that are required for players to remain in a coalition. Each participant must receive at least the amount he could command regardless of how the participants who are not members of the coalition behave.

The game theoretic cost allocation schemes employed in this research study were the Shapley, Bargaining Theory--Initial Trial and Bargaining Theory--Asymptote. The Shapley cost allocation scheme is based on a marginal cost concept. As the grand coalition, the coalition that includes all of the participants of a game, forms by the sequential addition of players, each player is charged an amount equal to the expected marginal cost incurred when the player enters the grand coalition. Bargaining theory--Initial Trial is based on parity, a player's percentage contribution to a coalition, and equality. Thus, the payoff to each player in a coalition is expected to be midway between parity and equality. Bargaining Theory--Asymptote assumes that the members of a coalition will use their best payoff in other coalitions as an element of threat to increase their payoff.

Two joint cost allocation schemes that are not game theoretic were also employed in this research study: the Activity Level and Moriarity joint cost allocation schemes. The Activity Level joint cost allocation scheme partitions the total joint cost as a percentage of each participant's hours contributed towards the incurrence of the joint cost. The Moriarity joint cost allocation schemes partitions the total joint cost savings as a percentage of each participant's cost of acting alone.

For testing purposes, hypotheses were constructed based on the aforementioned elements of game theory. The hypotheses were as follows:

1. Each group would form a grand coalition. A grand coalition is composed of all the participants in a game. Refer to Appendix C for the details of grand coalition formation.

2. The observed allocations of cost savings would be core solutions. Appendix D discloses the core calculations.

3. A game theoretic allocation scheme would be approximated by the subjects. Refer to Appendix D for solutions for the allocation schemes employed in this research study.

4. Each allocation scheme that was compared with the observed allocation schemes had an equal probability of occurrence, .20 (1/5).

Based on the information provided to the subjects, bargaining theory of coalition formation predicted grand coalitions would form. The observed percentage of groups that formed grand coalitions was used to compute confidence intervals as an estimate for a population proportion.

The cost savings allocations reported by the groups were used three ways. First, the observed allocations were individually evaluated to determine if they were core soluations. An allocation scheme was considered in core if three rationality conditions were met. The rationality conditions were as follows:

1. Each player to remain in the grand coalition must receive a payoff that is at least as great as the payoff he could receive acting alone.

2. The grand coalition must allocate the total benefit of cooperation to the players.

3. Every possible coalition must behave rationally. That is, the payoffs to each player must be as great as the benefit of their cooperation.[12]

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[12]S. S. Hamlen, W. A. Hamlen, and J. T. Tschinhart, "The Use of Core Theory in Evaluating Joint Cost Allocation Schemes," The Accounting Review, 52 (July, 1977), p. 618.

Second, the observed allocations were compared with five allocation schemes. Three of the allocation schemes, the Shapley, Bargaining Theory--Initial Trial, and Bargaining Theory--Asymptote, were game theoretic models. The remaining two schemes, Activity Level and Moriarity, although often employed in game theoretic research, were not clearly defined as game theoretic models. Each of the five allocation schemes was compared with the mean of the observed allocations. A Hotelling's T^2 statistic was computed for each of the five comparisons. The most closely approximated allocation scheme(s) was indicated by the lowest T^2 statistic.

Finally, a χ^2 goodness of fit statistic was utilized to lend support to the Hotelling's T^2 statistic. Since Hotelling's T^2 used mean data, the statistic could be influenced by observations that were extremely diverse from one of the five known allocations.

χ^2 is a summary statistic, thus extreme observations do not influence (lead to possible incorrect inferences) its results. Five allocation schemes were being compared with the observed allocations, therefore each of the five allocation schemes had an equal probability (20 percent) of occurrence. The allocation scheme that contributed the highest amount toward the overall χ^2 statistic was also an indication of the allocation scheme most closely approximated by the groups.

Preview of Remaining Chapters

Chapter II contains a detailed specification of the research methodology. Also encompassed in Chapter II is a literature review supporting each hypothesis and the limitations of the research methodology.

The results of the research study are disclosed in Chapter III. Group behavior is evaluated with regard to three elements of game theory: coalition formation, core theory, and game theoretic solutions. The test statistics employed for evaluative purposes are defined and their results are analyzed.

Chapter IV contains a summary of the research study and the results of the experiment. Inferences are made concerning group behavior and implications of the research study are formulated.

A selected bibliography and appendixes follow the summary. The appendixes are developed to assist the reader with basic game theory terminology and to disclose the pertinent computations used in data analyses.

CHAPTER II

DEVELOPMENT OF HYPOTHESES AND RESEARCH METHODOLOGY

Introduction

A vast majority of game theory research has been either descriptive or laboratory in nature. Very little real data based research has been done to date, and in accounting, the only research study of this type was a manuscript drafted by Boatsman and Hansen.[1] The primary purpose of the Boatsman and Hansen study was to investigate the predictive ability of game theory with regard to exchange ratios in stock for stock mergers.

The shortage of real data based game theoretic research in accounting is due to the unavailability of data. Most gaming situations are of an internal (within the firm) nature (i.e., cost allocations). Internal (not requiring disclosure) information is not generally available to the public, thus there is a lack of data required for such studies. Since real data are not available, researchers have had to generate their own data in the laboratory.

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[1] James Boatsman and Don R. Hansen, "Game Theoretic Approaches to Allocations: Evidence from Business Combinations," (unpublished manuscript, 1980), pp. 1-36.

Instrumentation and Subjects

Game theory is applicable in allocation situations where a benefit (e.g., synergy or cost savings) is perceived by the players for forming a coalition(s). Therefore, this study evaluated the decisions that subjects, acting as dividion managers, made regarding the partitioning of cost savings generated by forming a coalition and using an internal (within the firm) computer facility rather than an outside facility.

The situation that was presented to the subjects is as follows:[2]

> You are the manager of division (A, B, or C). The corporation has recently developed a new dividion, D, which is strictly a computer service division. Dividion D leases its computer at a fixed rate of $5,010 per week.
>
> In recent weeks the divisional weekly accounting and production reports, which are handwritten, have been received well past the due date (Monday morning, 9:00 a.m.). Also, the corporate staff has had a difficult time integrating the divisional reports because the reports were not uniform. Therefore, corporate headquarters issued to each division manager a memo stating that their quarterly bonuses would be reduced <u>accordingly</u> for any more tardy weekly reports. Also, a uniform format for the weekly reports was attached. It was also made clear in the memo that computer printed reports would be required.
>
> Today is Friday, the day division managers have their weekly meeting. One week from the following Monday is the day penalties will be assessed for tardy reports.
>
> You and the other division managers have just heard presentations from outside computer firms and from Division D, the new computer division. (Cost estimates and each division's computer requirements are attached.)[3] Each presenter stressed that they could guarantee timely reports, one week from Monday, only if they could get started this afternoon. Therefore, time is of the essence.

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[2]A sample of the test instrument employed in the laboratory experiment is available in Appendix E.

[3]See Appendix B.

You as a division manager now face this dilemma: How to get weekly reports, computer printed, to the corporate office every Monday by 9:00 a.m.

Here are some additional considerations:

1. You may decide to act alone, or form a group with one, or both, of the other division managers.
2. One of the criteria used to evaluate your performance for determining your quarterly bonus is cost savings you can verify when making investment decisions. (i.e., Cost to make a raw material \$100; cost to buy the raw material, \$75. If the decision maker buys the raw material, his cost savings would be \$25.)

The decision you must make is:

A. What computer facility should your division use (either Division D or an outside facility)?
B. If you and another, or both other divisions, decide to form a group, how should the cost (see attached cost figures) be allocated or partitioned to the using divisions?

The subjects were informed orally, just prior to the beginning of the experiment, that the amount of their payoff would be based on the quality (soundness) of their decisions. Seventy-five (75) subjects were employed in the experiment and each subject was randomly assigned the role of a division manager (Division A, B, or C). Groups composed of one division manager from each of the three divisions were formed and the decision making process commenced.

Senior business students were used as the surrogates for division managers. They were selected because they should have completed a majority of the basic business college requirements and thus, should be somewhat familiar with the role and responsibilities of a division manager.

Development of Hypotheses

Introduction

The behavior of the subjects, acting as surrogate division managers, was evaluated. The evaluation involved a comparison of the subjects' behavior with three elements of game theory: coalition formation, core theory, and game theoretic allocation schemes.

Coalition Formation

Eight possible coalitions[4] can be formed in a three-person game.[5] Coalition formation is important because it has implications concerning players' behavior, and game theoretic solutions and their axioms.

Gameson[6] in 1961 developed a theory of coalition formation. He sought to predict who would join with whom to form coalitions in specific instances. His model required the following information:

1. The initial distribution of resources.
2. The payoff for each coalition.
3. The non-utilitarian strategy preferences.
4. The effective decision point.

He assumed that all the players had the same information about the initial distribution of resources and the payoffs to all coalitions,

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[4]The eight possible coalitions are: (1), (2), (3), (1,2), (1,3), (2,3), (1,2,3), and the empty set.

[5]The experiment employed in this research study was a three-person game. The three-persons were the three surrogate division managers assigned to each of the 25 groups.

[6]William A. Gameson, "A Theory of Coalition Formation," American Sociological Review (October, 1961), pp. 373-382.

that all the players had a ranking of non-utilitarian preferences for joining with the other players, and that the players did not distinguish between payoffs in the same payoff class.

Based on the above assumptions, Gameson hypothesized a minimum resource theory[7] of coalition formation. The theory was compared with Caplow's prediction theory.[8] Gameson found that two theories to be identical when the payoffs and non-utilitarian strategy preferences were constant.

In the early 1970's Conrath,[9] Chertkoff,[10] and Komorita[11] introduced laboratory studies that sought to answer questions about the formation of coalitions. Conrath's results indicated that not only was experience a determining variable in subject behavior, but also communication and size and dominance[12] of the payoffs were important variables.

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[7]A player will expect the other players to demand from a coalition a share of the payoff proportional to the amount of resources they contribute to the coalition.

[8]The initial distribution of resources is the primary element in the prediction of coalition formation. (See Appendix A.)

[9]David W. Conrath, "Experience as a Factor in Experimental Gaming Behavior," Journal of Conflict Resolution, 14 (June, 1970), pp. 195-202.

[10]Jerome M. Chertkoff, "Coalition Formation as a Function of Differences in Resources," Journal of Conflict Resolution, 15 (September, 1971), pp. 371-383.

[11]S. S. Komorita and Jerome Chertkoff, "Psychological Bargaining Theory of Coalition Formation," Psychological Review, 80 (May, 1973), pp. 149-162.

[12]A payoff is better than another payoff if it is feasible and preferred. (See Appendix A.)

Chertkoff tested various theories and found that Gameson's minimum resource theory was most often accurate. Komorita and Chertkoff proposed a predictive theory, bargaining theory, that they compared with minimum resources and pivotal power[13] theories. Their theory opposed the other two because they assumed that differences in resources would cause people to adopt different orientations toward the reward division (minimum resource and pivotal power assume people have the same orientation toward the reward division). Based on this difference and other subtle differences, the data gathered supported their proposed theory.

Bargaining theory of coalition formation postulates that the most likely coalition to form is the one that minimizes coalition members' temptation to leave a coalition. The temptation to defect is defined by the factor $(O_{ij} - E_{ij})$.[14] The smaller the factor, the less likely that members will defect. In other words, the coalition with the smallest factor is the coalition that is most likely to form. Within the context of the present research the coalition with the smallest factor, -.71,[15] was the grand coalition, therefore the first hypothesis of this dissertation research was that each group would form a grand coalition.

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[13]A player's resources are pivotal when their inclusion in a losing coalition can convert it into a winning coalition. (See Appendix A.)

[14]J. Keith Murnigham, "Strength and Weakness in Four Coalition Situations," Behavioral Science (May, 1978), p. 197. O_{ij} = predicted reward of player i in coalition j; and E_{ij} = maximum expectation in alternative coalitions.

[15]The computations of the temptation factors are in Appendix C.

Confidence intervals were calculated to test the above hypothesis. A binomial sample distribution[16] existed because there were only two possible outcomes associated with each group of subjects. Confidence intervals are based on the normal distribution, but they do serve as an approximation to a binomial distribution. Therefore, confidence intervals were utilized in this research study to allow inferences to be made about the coalition formation behavior of the subjects.

Core Theory

The cost savings allocations reported by the groups were evaluated in terms of core theory. As discussed in Chapter I and Appendix A, core theory is the set of payoffs that are required for players to remain in a coalition. Core theory was most appropriate for this study because in situations where the marginal cost function for an allocated cost is decreasing (as it was in this study), suboptimal (at the corporate level) decisions on the divisional level can be avoided.[17] That is, core theory only recognizes as solutions those allocations that are stable.

Stability of outcomes should be important to the accountant. If one of the objectives of game theory, in an accounting sense, is to save the cost of time-consuming-negotiation-processes in gaming

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[16] Binomial distribution is defined as a sampling situation that allows only two possible outcomes. The hypothesis concerning coalition formation only allows these two possible outcomes: (1) a grand coalition will form; or (2) a grand coalition will not form.

[17] S. S. Hamlen, W. A. Hamlen, and J. T. Tschirhart, "The Use of Core Theory in Evaluating Joint Cost Allocation Schemes," The Accounting Review, 52 (July, 1977), pp. 616-627.

situations, then the accountant (acting as an advisor) would have a more convincing case suggesting a stable allocation scheme.

Jensen[18] explored the negotiation process and how accountants could assist the negotiators (manager) in reaching a mutually satisfactory allocation[19] of costs in a joint cost setting. He did not prescribe any specific allocation scheme. Rather, he strongly urged accountants to become familiar with game theoretic approaches for allocating joint costs. This knowledge, he suggests, could serve as valuable input to a group(s) of collaborators in attempting to reach a mutually satisfactory allocation of cost.

Hughes and Scheiner[20] questioned what they perceived as the current development of game theory in accounting research. Their opinion was that the current state of the art had ignored the efficiency properties[21] of an allocation scheme. In direct response to Jensen's research,[22] Hughes and Scheiner implied that a mutually satisfactory allocation scheme is not a surrogate for the efficiency properties of that scheme. Their basic argument was that accounting researchers employing game theory must go a step further. They must evaluate allocation schemes,

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[18] Daniel L. Jensen, "A Class of Mutually Satisfactory Allocations," The Accounting Review, 52 (October, 1977), pp. 842-856.

[19] Those allocations whereby individuals or organizations agree to distributions of resources.

[20] John S. Hughes and James H. Scheiner, "Efficiency Properties of Mutually Satisfactory Cost Allocations," The Accounting Review, 55 (January, 1980), pp. 85-95.

[21] Efficiency properties, as defined by Hughes and Scheiner, require that decisions made on a division level be optimal in the sense of the overall corporation.

[22] Jensen, pp. 842-856.

not just with the divisions (players) of a corporation in mind, but also with the overall corporation in mind.

In games involving more than two players a possibility exists of more than one acceptable solution. Berl, McKelvey, Ordeshook, and Winer[23] developed a theory to reduce the number of acceptable solutions. They made two critical assumptions that they felt would accomplish their purpose. The assumptions were:

1. A coalition's value can be represented by a single number that corresponds to the amount of some divisible transferable commodity (e.g., money) the coalition can secure for its members.

2. Each player's utility for the commodity is linear and hence transferable as well.[24]

These assumptions, added to the general game theory framework, developed a solution named the core. Berl, McKelvey, Ordeshook, and Winer's laboratory experiment, which involved 17 games and 31 players, strongly supported the core as a solution. The results of their experiment were so supportive of core that even when players did not understand the theoretical properties of the game they were playing, they tended to end up in or near core anyhow. For games in which core does exist--especially for those games which correspond to a unique outcome--the theory offers a highly predictable outcome.

Hamlen, Hamlen, and Tschirhart[25] used core theory to describe joint cost allocation schemes. Their primary purpose was to determine those

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[23]Janet E. Berl et al., "An Experimental Test of the Core in a Simple N-Person Cooperative nonside-Payment Game," Journal of Conflict Resolution, 20 (September, 1976), pp. 453-479.

[24]Berl et al., p. 454.

[25]Hamlen, pp. 616-627.

payoffs to the players that would be satisfactory enough to keep them in a particular coalition. The authors applied this theory to four joint cost allocation schemes: Activity Level,[26] Shapley value, Nucleolus, and Moriarity.[27]

For a solution to be considered in core, three rationality conditions had to be met:

1. Each player to remain in the grand coalition must receive a payoff that is at least as great as the payoff he could receive acting alone.

2. The grand coalition must allocate the total benefit of cooperation to the players.

3. Every possible coalition must behave rationally. That is, the payoffs to each player must be at least as great as the benefit of their cooperation.[28]

The avoidance of cost allocation schemes that would result in sub-optimal decisions on the corporate level was the primary objective of the authors. They evaluated each cost allocation scheme on the basis of rationality and fairness, and neutrality. An allocation scheme was judged to be acceptable if the following criteria were met:

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[26]Cost is allocated to each player in direct proportion to the player's activity level. (See Appendix A.)

[27]The cost allocated to each player is equal to the difference between his independent cost and a specific fraction of the total value obtained by forming the grand coalition. (See Appendix A.)

[28]Hamlen, p. 618, and Karl Henrik Borch, The Economics of Uncertainty (New Jersey, 1972), pp. 154-155. Core conditions in notation are:

1) $X_i \geq V(\{i\})$ for all i, X_i = payoff to i.

2) $\sum_{i=1}^{n} X_i = V(N)$, V(N) = payoff to grand coalition.

3) $\sum_{i \varepsilon s} X_i \geq V(S)$ for all S in N.

1. Each division received a charge which was less than the cost it would incur by acting alone.

2. The total sum of all costs to all divisions had to be equal to the total joint cost to the corporation.

3. The allocation scheme resulted in a set of payoffs which were in core.

Three of the four allocation schemes did result in core solutions. Only the Moriarity scheme's solution was outside of the core. The authors also introduced some useful information about the other cost allocation schemes which they examined. Both the Shapley value and nucleolus schemes tended to favor the smaller divisions. Even though the larger divisions contributed a proportionately larger part of the joint cost savings, the two schemes divided the savings evenly among all the divisions. The Activity Level scheme favored the smaller divisions even more.

Thus, a second hypothesis of this dissertation research was that the solution observed from each group would be in core. The solutions derived by each group might not be comparable to any game theoretic solution, but they could still be in core. The inference in such cases must be that some of the axioms underlying game theoretic solutions are too restrictive.

Cost Allocation Schemes

Finally, the most important aspect of the subjects' behavior that was evaluated was the game theoretic solution, if any, that was employed by the groups. To date, the accounting literature has not furnished enough evidence to gain a consensus regarding a game theoretic approach to joint cost allocations as a possible solution to the allocation problem. As discussed in Chapter I, the allocation problem is the inability of accountants to justify the particular allocation methods selected. The accounting literature does seem to be gathering evidence that supports the use of certain allocation schemes (e.g., Shapley and Shapley related solutions, Minimum Resource Theory, and Bargaining Theory).

The empirical study by Boatsman and Hansen[29] revealed some interesting evidence in support of game theory in an accounting setting. They applied a game theory framework (Shapley value, Minimum Resource Theory, and Bargaining Theory)[30] to predict exchange ratios associated with mergers (stock for stock only) during the period 1974-1976. They discovered that the Shapley solution did not incorporate the threat powers of the players.[31] Boatsman and Hansen modified the Shapley solution by integrating Minimum Resource Theory and Bargaining Theory. The weight normally applicable to each player in a Shapley, two-person game solution is 1/2. Boatsman and Hansen altered these weights so as to reflect the threat strength of each player. They used earnings the year prior to the merger as the relative threat strength[32] of the players.

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[29]Boatsman, pp. 1-36.

[30]This tenet suggests that players use their best payoff in another coalition as a threat to improve their payoff in a current coalition. (See Appendix A.)

[31]The players in this setting were the stockholder groups of each firm.

[32]Assume two firms A and B. The relative strength of each firm was calculated as follows:

$$A = \frac{\text{Income Year Prior to Merger for A}}{\text{Income Year Prior to Merger for A+B}}$$

$$B = \frac{\text{Income Year Prior to Merger for B}}{\text{Income Year Prior to Merger for A+B}}$$

Results after applying the modified model provided strong evidence in support of Minimum Resource and Bargaining Theories.

Roth and Verrecchia[33] extended the current research in game theory as it applies to the allocation of cost. Previous accounting research tended to favor the Shapley solution, thus the authors' purpose was to further refine the Shapley technique for accounting application.

The authors added three assumptions to the general game theory framework for the Shapley solution and deduced a theorem. The assumptions served as surrogates for the qualitative factors fairness, equity, and neutrality. Roth and Verrecchia theorized that "a manager's expected utility for playing in a game is equal to this Shapley value, if and only if, his preferences obey the above assumptions."[34]

Roth and Verrecchia's study was not data based, thus they proved their theorem mathematically. They concluded by saying that their refined model depends entirely on managers behaving as prescribed by the three assumptions. The implication was that laboratory and real data research were needed to support their theorem.

One of the few studies using real data and applying game theory was done by Littlechild and Thompson[35] for the Birmingham Airport in England. They investigated various pricing policies where the cost of runway construction was to be shared by the different aircraft models

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[33]Alvin E. Roth and Robert E. Verrecchia, "The Shapley Value as Applied to Cost Allocation: A Reinterpretation," Journal of Accounting Research (April, 1979), pp. 295-303.

[34]Roth, p. 301.

[35]S. C. Littlechild and G. F. Thompson, "Aircraft Landing Fees: A Game Theory Approach," The Bell Journal of Economics (Spring, 1977), pp. 186-206.

(types). Game theory was employed along with linear programming to insure optimality, efficiency, and fairness in the currently used pricing structure and to derive some rules of thumb for allocating costs. The currently used pricing structure was compared with the results suggested by linear programming and three game theory approaches, the Shapley value, nucleolus and anti-nucleolus.[36] In general, the currently used pricing structure was consistent with the linear programming results and the Shapley value.

The application of game theory to the relationship between an information evaluator and a decision maker was the intent of a recent study by Sundem.[37] Sundem asserted that a gaming situation existed because each person (information evaluator and decision maker) could influence the payoffs to the other. Each person had the ability to increase his payoff by gaining knowledge of the other's alternatives and payoffs.

By varying the level of communication, the amount of information (complete or incomplete), and side payments (allowed or disallowed), Sundem developed game theoretic models for six (6) situations. His results were not conclusive, but interestingly he found that ambiguous solution concepts resulted in the cooperative game settings.

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[36]This solution minimizes the maximum surplus. (See Appendix A, Nucleolus.)

[37]Gary L. Sundem, "A Game Theory Model of the Information Evaluator and the Decision Maker," Journal of Accounting Research, 17 (Spring, 1979), pp. 243-261.

Spinetto[38] in 1975 reintroduced the application of game theory to the business setting (e.g., cost allocation within and between firms). His primary purpose was to illustrate how cooperative games could assist arbitrators in selecting fair solutions in the resolution of conflicts among several parties. He described how an arbitrator could propose one solution that would be satisfactory to all the players. Applying the Shapley value he was able to illustrate that a single solution, termed fair to all parties was possible.

Accounting researchers have yet to gather substantial evidence that supports one particular game theoretic cost allocation scheme. The Shapley solution has been the most utilized game theoretic technique in accounting research involving the allocation problem. The results of research utilizing the Shapley solution have been inconclusive and this lack of conclusive evidence could imply that some other game theoretic allocation scheme might be appropriate for accounting cost allocations.

Therefore, five allocation schemes[39] were utilized in this research study. Each of the five allocation schemes was compared with the allocation schemes disclosed by the subjects. A Hotelling's T^2 test statistic was employed to test the results of the five comparisons. For testing purposes, the third hypothesis of this research study was:

$$H_o: \quad \underset{\sim}{\mu} = \underset{\sim}{\mu}_o$$

$$H_a: \quad \underset{\sim}{\mu} \neq \underset{\sim}{\mu}_o$$

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[38] Richard D. Spinetto, "Fairness in Cost Allocation and Cooperative Games," Decision Sciences, 6 (July, 1975), pp. 482-491.

[39] The five allocation schemes employed were the Shapley, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level, and Moriarity. The latter two schemes are not game theoretic models. See Appendix D for the solution to each method.

μ_o was the vector of cost savings allocated to each division as calculated by one of five allocation schemes: the Shapley, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level or Moriarity. μ was the mean vector of cost savings allocations disclosed by the subjects.

The test statistic allowed the researcher to make inferences about allocation scheme preference and group behavior. The allocation schemes, $(\bar{X}_A, \bar{X}_B, \bar{X}_C)$, reported by the subjects were compared with each of the aforementioned allocation schemes, μ_o (X_A, X_B, X_C). Each comparison resulted in a Hotelling's T^2. The level of significance associated with each T^2 indicated how closely the observed allocation scheme approximated the allocation scheme with which it was being compared. The comparison resulting in the lowest level of significance connoted the most closely approximated allocation scheme.

A summary statistic was also used to evaluate the results. Since five allocation schemes were tested, each scheme had an equal probability (20 percent) of being approximated by the subjects. Thus, each allocation scheme should have been the most closely approximated scheme 20 percent of the time.

Since each of the five allocation schemes had an equal probability of being most closely approximated by the subjects, a uniform distribution existed. A fourth hypothesis of this research study was formulated to determine whether or not the observed allocation schemes fit (approximated) a uniform distribution. Thus the fourth hypothesis of this research study was:

$$H_o: \quad \pi = 20\%$$

$$H_a: \quad \pi \neq 20\%$$

π was the probability of occurrence of each of the five allocation schemes. The hypothesis was tested using a χ^2 goodness of fit statistic. The χ^2 statistic was employed to gather knowledge about whether or not the observed data approximated the given distribution, i.e., each of the five allocation schemes tested had a probability of 20 percent occurrence.

χ^2 lent support to the inferences that resulted from the Hotelling's T^2 test. The most closely approximated allocation scheme, as inferred by Hotelling's T^2, was also disclosed by the allocation scheme contributing the most to the χ^2 statistic.

Limitations

Typically in laboratory experiments the independent variables lack strength and may induce only weak responses. Also, the realism forfeited makes generalization of the results a problem.

However, laboratory studies have the advantage of allowing the researcher nearly complete control over the setting and the experiment by eliminating or controlling the influences of a large number of extraneous variables. Complete randomization is possible, and the researcher can manipulate numerous independent variables. Due to the unavailability of real data, this study employed a laboratory setting with the understanding that disadvantages (limitations) did exist.

In addition, this particular research study was limited by the sample size and by the fact that students were used as the subjects. The small sample size, 23 observations, had an effect on the binomial distribution test that was approximated by confidence intervals and the χ^2 statistic employed in this research study.

Even though the subjects employed in this research study were senior business students, their lack of experience in the business sector and lack of negotiation ability or experience posed additional limitations. Inferences made concerning subjects', acting as surrogate division managers, decision making behavior may not be applicable to business sector division managers.

Summary

The research methodology was designed to enable the researcher to observe and evaluate subjects' behavior in a simulated business environment. Students, acting as surrogate division managers, were required to make decisions that are encountered by real world division managers.

Predictions were made concerning the subjects' behavior. The subjects' observed behavior was evaluated in terms of three elements of game theory: coalition formation, core theory, and allocation scheme preference.

Four hypotheses were formulated for the evaluation process. They were as follows:

1. Each group would form a grand coalition.
2. Each group would report a core solution.
3. The mean of the observed solutions would approximate one of five joint cost allocation schemes.
4. Each allocation scheme would be approximated 20 percent of the time.

Confidence intervals were computed to test the first hypothesis. An inference could be made about the proportion of groups that formed grand coalitions was the reason confidence intervals were utilized.

The second hypothesis was tested in terms of the three rationality conditions required for a solution to be considered a core solution. The rationality conditions are as follows:

1. Each player to remain in the grand coalition must receive a payoff that is at least as great as the payoff he could receive acting alone.

2. The grand coalition must allocate the total benefit of cooperation to the players.

3. Every possible coalition must behave rationally. That is, the payoffs to each player must be at least as great as the benefit of their cooperation.[40]

Each observed solution (cost allocation scheme) was individually evaluated to determine if it met the three rationality conditions stated above. A percentage of observed solutions meeting the three requirements would be reported.

Hypotheses three and four were formulated to evaluate the allocation scheme preference of the subjects. The third hypothesis utilized Hotelling's T^2 statistic, which was calculated with the mean of the observation allocation schemes. The mean data were compared with the five known solutions; the Shapley, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level, and Moriarity. The comparison that resulted in the lowest Hotelling's T^2 was inferred to be the most closely approximated allocation scheme.

The fourth hypothesis utilized a summary statistic, χ^2, for the purpose of evaluating the five comparisons mentioned above. Each observed allocation scheme was individually compared with the five known solutions. The numerical differences were squared and summed.

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[40]Hamlen, p. 618.

The known solution that resulted in the lowest summed squared difference was judged the most closely approximated solution.

A χ^2 statistic was calculated to determine if the uniform distribution (each of the five known solutions had an equal probability of occurrence, 20 percent) was approximated by the sample distribution. The known solution that contributed the most (highest numerical value) to the χ^2 statistic was inferred to be the most closely approximated known solution (allocation scheme).

The researcher anticipated that the results of this research study would make a contribution towards the resolution of the joint cost allocation problem in accounting when a benefit results in forming a coalition. The intent of this research study was to supply more evidence in support of game theoretic solutions, bargaining theory in regards to coalition formation, and the theory of core. Conclusive evidence would be a positive step in the direction of developing some theoretical justification of a cost allocation method for situations where a benefit is perceived by the players from forming a coalition(s).

CHAPTER III

ANALYSIS OF RESULTS

Introduction

Twenty-five groups of three subjects participated in the experiment which served as the basis for data collection. Senior business students were utilized as the subjects because they were considered to be somewhat familiar with the role and responsibilities of a division manager. The experiment was conducted outside of the course requirements for the students' respective classes, and the data were collected during a two-day period at Oklahoma State University in Stillwater, Oklahoma.

The subjects were told prior to the experiment that they would be rewarded in dollars for their participation. As no specified amount was mentioned, the subjects were led to believe that their payoffs were dependent on the quality (as determined by the experimenter) of their decisions.

The above approach was utilized to create a competitive atmosphere. Game theory literature asserts that a benefit (in this experiment, a monetary payoff) must be perceived by the players for them to form a coalition(s). Thus, the monetary payoff was used as an incentive to induce the subjects to behave in a game theoretic manner. After all of the data were collected, each subject was paid five dollars and debriefed about the experiment.

The remainder of Chapter III includes the data collected from the experiment and an analysis of the results. Initially, coalition formation data were summarized and statistically tested with confidence intervals. The observed cost allocation schemes were then evaluated in terms of core theory.

In addition, the observed allocation schemes were compared with five known allocation schemes. The comparisons were performed to determine which of the five known allocation schemes was most closely approximated by the subjects, acting as surrogate division managers. Hotelling's T^2 and χ^2 test statistics were used to evaluate the comparisons. Finally, the results of the research were summarized and implications of this research study were proposed.

Participants' Cost Allocations

The participants were instructed that they were expected to act as division managers. Each of the 25 consecutively numbered groups of three surrogate division managers was to assume that they were all a part of the same company.

The division managers were asked to place themselves in a hypothetical situation. The setting was a weekly division managers meeting, where one of the topics discussed was the utilization of a computer facility for the production of weekly accounting and production reports. The division managers had previously heard presentations from Division D, a new computer division organized within their company, and from various external computer firms. The amounts disclosed in Figure 1 are the lowest cost estimates and the cost savings available to each division and coalition. They were derived from the cost estimates submitted by

Cost Estimates Per Week:

Division A - Acting Alone	$2,610
Division B - Acting Alone	$2,030
Division C - Acting Alone	$1,070
Divisions A and B - Coalition AB	$4,260
Divisions A and C - Coalition AC	$3,390
Divisions B and C - Coalition BC	$2,835
Divisions A, B, and C - Grand Coalition	$5,010

Cost Savings Per Week:

Division A - Acting Alone	$ 0
Division B - Acting Alone	$ 0
Division C - Acting Alone	$ 0
Divisions A and B - Coalition AB	$380
Divisions A and C - Coalition AC	$290
Divisions B and C - Coalition BC	$265
Divisions A, B, and C - Grand Coalition	$700

Figure 1. Weekly Cost Estimates and Cost Savings Per Division and Coalition

Division D and the external computer firms. The amounts in Figure 1 are used extensively throughout the remainder of Chapter III.

Due to limited capacity, no single outside computer firm could accommodate all three divisions. Division D did have the excess capacity to accommodate the three divisions. The participants were given a summary of the lowest cost estimates and the division requirements for each of the three divisions, A, B, and C. The summary and division requirements appear in Appendix E. Using the cost estimates and division requirements, each division manager had to decide whether to utilize an outside computer facility, either alone or together with another division manager; or, to utilize Division D together with the other division managers. The division managers were allowed to discuss the situation in arriving at their decisions.

If a group[1] (e.g., Divisions A and B together using an outside computer facility and incurring a joint cost) was formed, an additional decision was required. The group members had to also decide how the joint cost was to be partitioned to the participating divisions (e.g., 50% to each division for a coalition of two).

The decisions made by the 25 groups of subjects are summarized in Table I. Column one identifies the group number. The remaining columns signify the three divisions A, B, and C. Where a number other than zero appears in a division column, that symbolizes a coalition formation. For example, group 23 formed a grand coalition (group of three) as indicated by a number, other than zero, in the column for each division.

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[1]A group in game theory terminology is defined as a coalition.

Group 25 did not form any type of group as symbolized by the zeroes in each division column.

TABLE I

COALITION FORMATION AND THE ALLOCATION OF JOINT COST BY DIVISION

Group Number	Division A Fraction	Division A Dollars	Division B Fraction	Division B Dollars	Division C Fraction	Division C Dollars
1-20	.48	$2,405	.35	$1,753	.17	$ 852
21	.45	2,255	.36	1,804	.19	951
22	.45	2,255	.35	1,753	.20	1,002
23	.47	2,354	.36	1,804	.17	853
24	.56	2,386	.44	1,874	0	0
25	0	0	0	0	0	0

The decimal fractions in Table I indicate the portion of the joint cost allocated to a division as a result of forming a group. For example, the joint cost of all the divisions forming a grand coalition was $5,010. Group 23, which formed a grand coalition, partitioned the joint cost to each division in the following manner: Division A, .47 $(\frac{\$2,354}{\$5,010})$, Division B, .36 $(\frac{\$1,804}{\$5,010})$, and Division C, .17 $(\frac{\$\ 852}{\$5,010})$. Group 24 on the other hand, formed a coalition of divisions A and B as indicated by the decimal franctions .56 and .44. The joint cost of divisions A and B forming a coalition was $4,260. Group 24 partitioned the joint cost in the following manner: Division A, .56 ($2,386) and Division B, .44 ($1,874). Since Division C was not a member of a coalition, Division C was not involved in a joint cost situation; therefore Division C incurred $1,070 of cost, which was Division C's cost of acting alone. Groups 1-20 made identical decisions, therefore they were combined in Table I and throughout the remainder of the analysis process.

Testing of Hypotheses

Coalition Formation

Bargaining theory of coalition formation predicts that in a game theoretic situation the coalition that minimizes members' temptation to leave a coalition will prevail. The subjects reinforced this theory in that 23 of the 25 groups, 92 percent, did form grand coalitions.

The first hypothesis was that each group would form a grand coalition. The fact that 92 percent of the groups behaved as hypothesized reflects substantial support for the bargaining theory of coalition formation.

Additional support of the first hypothesis might also be gained by confidence intervals which were computed to estimate a population proportion. Since there were only two possible outcomes, to form a grand coalition or not to form a grand coalition, a binomial sampling distribution existed. That is, each possible outcome had a 50 percent probability of occurrence. The parameter that had to be estimated by a confidence interval was the proportion of grand coalitions that would form or the probability that a randomly selected group would form a grand coalition.

Even though confidence intervals are based on the normal distribution they can serve as an approximation to a binomial distribution.[2] The 95 percent confidence interval is defined as follows:

$$\hat{p} \pm z_{.05}(\text{normal})\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \qquad (1)$$

where $\hat{p}$ is the observed proportion of groups that formed grand coalitions (estimate of the mean, p); $z_{.05}$ is the table value required for significance at the 5 percent level; n is the sample size; and $\hat{p}(1 - \hat{p})/n$ is an estimate of the variance. The 95 percent confidence interval was calculated as follows:

$$\frac{23}{25} \pm 1.96 \sqrt{\frac{\frac{23}{25}}{25} \times \frac{1 - \frac{23}{25}}{25}}$$

$$(.814 < p < 1.00)^{3}$$

The binomial distribution test above was affected by the sample size. Statisticians generally suggest a sample size of 30, with a minimum of 15 in the smaller class, when confidence intervals for normal approximation are applied to a binomial distribution that has a mean probability of .5 in the larger class.[4] A table of Confidence Belts for Proportions[5] was also utilized to increase the credence of the confidence intervals. The use of confidence belts did not eliminate

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[2]Robert G. D. Steel and James H. Torrie, Principles and Procedures of Statistics (New York, 1960), p. 353.

[3]The calculated value of the upper limit was 1.026. Since the probability could not be greater than 1.00, that was used as the upper limit.

[4]Steel, p. 354.

[5]Ibid, pp. 354 and 458.

the limitation created by the small sample size. They did, however, serve as support for any inferences made based on the binomial distribution test.

The Table of Confidence Belts for Proportions can be utilized for sample sizes as small as eight. For the 95 percent confidence interval, $(.74 < p < 1.00)$ was obtained from the table. Thus the two methods used to determine confidence intervals, normal distribution as an approximation to a binomial distribution and confidence belts for proportions, yielded similar results.

The confidence intervals indicated that the population proportion, the probability of a grand coalition forming among all possible participants given a gaming environment identical to the gaming environment of this research study, would lie between .74 and 1.0 with 95 percent confidence. Since the unknown population proportion was apparently greater than .50 (the probability of a grand coalition forming), the behavior of the subjects with respect to the bargaining theory of coalition formation was confirmed.

Core Theory

The second hypothesis of this dissertation study was that the solution observed from each group would be in core. Ninety-two percent of the solutions supported this hypothesis.

Each observation was evaluated in association with the three rationality conditions required for a core solution. The three rationality conditions were:

1. Each division, to remain in the grand coalition, must receive a payoff (cost savings in this study) that is at least as great as the cost savings he could receive acting alone.

2. The grand coalition must allocate the total benefit (cost savings) of cooperation to the divisions.

3. Every possible coalition must behave rationally. That is, the cost savings allocated to each division must be at least as great as the benefit of their cooperation.

The participants in the experiment reported the allocation of joint costs. Core theory is addressed from the viewpoint of the partitioning of a payoff or benefit, which was cost savings in this research study. Cost savings were the dollars a division, or divisions, saved by selecting a less costly alternative rather than a more costly alternative. For example, the cost to Division A for acting alone was $2,610. If another alternative were available that was less than $2,610 and the manager of Division A selected that alternative, the cost savings would be the difference between the $2,610 and the cost to Division A had the alternative been selected. Thus, the observed joint cost allocations were converted to cost savings allocation for the core evaluation process.

The conversion of the observed joint cost allocations to cost savings allocations involved three steps. The first step required a calculation of the cost allocated to each division based on the cost of the alternative each division manager selected. For example, in group number 21, the manager of Division A selected the alternative of forming a grand coalition. The division manager agreed to incur 45 percent (.45 from Table I) of the total joint costs, $5,010, for forming a grand coalition. Thus, the joint cost allocation to Division A in group 21 was $2,255 ($5,010 x .45).

Step two involved the calculation of the cost of Division A in group 21 acting alone. In this case the cost was $2,610.

The final step was the calculation of the cost savings for Division A in group 21. The cost of forming a grand coalition (the alternative selected) was $2,255; the cost of acting alone was $2,610 (the foregone alternative). Thus, the cost savings to Division A in group 21 was the difference in the cost of the two alternatives, $355 ($2,610 - $2,255).

The conversion process was performed for each division in the first 23 groups. Groups 24 and 25 were not involved in the conversion process nor were they evaluated for the three core conditions. Since both groups, 24 and 25, did not form grand coalitions, the division managers made decisions that were suboptimal at the company level. The decisions were suboptimal from a company viewpoint, because the division managers in groups 24 and 25 failed to save the company the maximum amount possible. The cost of forming a grand coalition was $5,010; the cost of each division acting alone totaled $5,710 ($2,610 + $2,030 + $1,070). Thus, the division managers in groups 24 and 25 neglected the opportunity to save the company $700 ($5,710 - $5,010). The other possible coalition formations afforded the company cost savings which were all less than $700. Therefore, the division managers of groups 24 and 25 behaved in a manner, other than rational, and neither group was employed in the remainder of the data analyses.

Table II contains a summary of the conversion of the joint cost allocations to cost savings allocations by division. All 23 observed allocation schemes evaluated met the first condition for a core solution. The cost savings received by each division, within a group, exceeded the cost savings each division could receive acting alone.

No cost savings were available if a division acted alone. Thus, to meet the first condition for a core solution required a division to

TABLE II

CONVERSION OF JOINT COST ALLOCATIONS TO COST SAVINGS ALLOCATIONS BY DIVISION

Group Number	Observed Joint Cost Allocation Percentage	X	Total Joint Cost to Grand Coalition	=	Joint Cost Allocation	-	Cost of Acting Alone	=	Cost Savings	Cost Savings Converted to Decimal*
					Division A					
1-20	.48		$5,010		$2,405		$2,610		$205	.29
21	.45		$5,010		$2,255		$2,610		$355	.51
22	.45		$5,010		$2,255		$2,610		$355	.51
23	.47		$5,010		$2,354		$2,610		$256	.37
					Division B					
1-20	.35		$5,010		$1,753		$2,030		$277	.40
21	.36		$5,010		$1,804		$2,030		$226	.32
22	.35		$5,010		$1,753		$2,030		$277	.40
23	.36		$5,010		$1,804		$2,030		$226	.32
					Division C					
1-20	.17		$5,010		$ 852		$1,070		$218	.31
21	.19		$5,010		$ 951		$1,070		$119	.17
22	.20		$5,010		$1,002		$1,070		$ 68	.09
23	.17		$5,010		$ 852		$1,070		$218	.31

*This column represents the division's cost savings as a percentage of the total cost savings available ($700) to each group.

receive one dollar, or more, of cost savings. Division C in group 23 received $218 of cost savings (from Table II). The $218 exceeded the zero dollars of cost savings from acting alone. Table III contains a summary of the evaluation of the cost savings allocations in regards to the first condition for a core solution.

TABLE III

INDIVIDUAL RATIONALITY CONDITION

Group Number	Division A Grand Coalition		Division A Alone	Division B Grand Coalition		Division B Alone	Division C Grand Coalition		Division C Alone
1-20	$205	>	0	$277	>	0	$218	>	0
21	$355	>	0	$226	>	0	$119	>	0
22	$355	>	0	$277	>	0	$ 68	>	0
23	$256	>	0	$226	>	0	$218	>	0

The second core condition required the grand coalition to allocate the total cost savings. Cost savings were the difference between the cost of forming a grand coalition, $5,010, and the cost of each division acting alone, $5,710, or $700.

To meet the second core condition, each group had to allocate $700 of cost savings to the three divisions within each group. The cost savings allocations disclosed in Table II were totaled for each group to determine whether $700 of cost savings were allocated. Group 21 did allocate $700: Division A, $355; Division B, $226; and Division C, $119. Table IV contains the results of the summation process.

TABLE IV

TOTAL COST SAVINGS BY GROUP

Group Number	Division A	Division B	Division C	Total
1-20	$205	$277	$218	$700
21	$355	$226	$119	$700
22	$355	$277	$ 68	$700
23	$256	$226	$218	$700

The third, and final, condition for an allocation scheme to be in core is that each feasible coalition (excluding the grand coalition) behave rationally. That is, the payoffs (cost savings) allocated to each division within a coalition, must be as great as the benefit (cost savings) of their cooperation. Three coalitions were feasible, A and B, A and C, and B and C. A division acting alone was not feasible because no cost savings were available when doing so.

Table V reflects the results of the group rationality comparisons. The total cost savings from forming a grand coalition received by each division of a feasible coalition. Group rationality required that the cost savings from the grand coalition be equal to or greater than the cost savings that accrued from the feasible coalition. In each case, the grand coalition cost savings were greater than those from the feasible coalition.

Divisions A and B would have received $380 in cost savings if they had formed a coalition. The $380 was the difference between the cost of

TABLE V

COALITION RATIONALITY CONDITION

Group Number	Coalition AB			Coalition AC			Coalition BC		
	Cost Savings Allocated to Division A and B from the Grand Coalition		Cost Savings from Coalition AB	Cost Savings Allocated to Division A and C from the Grand Coalition		Cost Savings from Coalition AC	Cost Savings Allocated to Division A and C from the Grand Coalition		Cost Savings from Coalition BC
1-20	$482	≥	$380	$423	≥	$290	$495	≥	$265
21	581	≥	380	474	≥	290	345	≥	265
22	632	≥	380	423	≥	290	345	≥	265
23	482	≥	380	474	≥	290	444	≥	265

both divisions acting alone $4,640 (Division A, $2,610 and Division B, $2,030) and the joint cost of forming a coalition which was $4,260.

The cost savings from forming coalition AB were compared with the cost savings Divisions A and B received from forming the grand coalition. For example, in group 22 Division A received #355 of cost savings from forming a grand coalition, while Division B received $277 of cost savings. The total cost savings for Division A and B, $632 exceeded the $380 of cost savings they would have received from forming coalition AB. Therefore the coalition AB in group 22 behaved rationally and thus met the third condition required for a core solution.

The cost savings allocations reported by the subjects met the three requirements for a core solution. Each surrogate division manager behaved rationally because each division received cost savings that exceeded the cost savings a division would receive acting alone. The total cost savings from forming a grand coalition, $700, was allocated to each division within every group. Finally, each feasible coalition (coalitions AB, AC, and BC) behaved rationally. That is, each feasible coalition received a greater amount of cost savings from the grand coalition than they would have received from another feasible coalition. Thus, the second hypothesis of this research study that the observed allocation schemes would be core solutions, was strongly supported.

Cost Allocation Schemes

The third hypothesis of this research study was that the cost savings allocation reported by the students would approximate a game theoretic solution. The results of the experiment did not support the hypothesis that a game theoretic allocation scheme would be followed by the subjects. The results did however support the hypothesis that one of five joint cost allocation schemes would be approximated by the subjects. One allocation scheme, Activity Level, appeared to be used by the subjects. Twenty of the groups that formed grand coalitions reported allocations that were exact Activity Level solutions.

The evaluation process which led to this conclusion involved a comparison of the observed cost savings allocations and five known allocation schemes. The five known allocation schemes were the Shapley, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level and Moriarity. Only the first three known solutions, the Shapley and the two Bargaining Theory models, were game theoretic allocation schemes.

Each of the known allocation schemes was applied to the cost data employed in the experiment. Solutions were derived, which resulted in a cost savings allocation to each of the three divisions, A, B, and C. For example, the solution for the Moriarity model was: Division A, .46 of the total cost savings;[6] Division B, .36; and Division C, .18. The solution vector was (.46, .36, .18), which was compared with a vector of the mean observed cost savings allocations (.31, .39, .30). The solution vectors of the observed cost savings allocations. (Refer to Appendix D for the calculation of solutions for the known allocation schemes.) Table VI discloses the solution vectors for the known allocation schemes and the mean vector of the observed cost savings allocations.

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[6]All 23 groups of subjects formed a grand coalition, thus the total cost savings that was allocated was $700. The total joint cost for forming a grand coalition was $5,010; the total cost of each division acting alone was $5,710 (Division A, $2,630; Division B, $2,030; and Division C, $1,070). The difference between the $5,710 and the $5,010 was $700, the total cost savings.

TABLE VI

SOLUTION VECTORS AND OBSERVED MEAN VECTOR OF COST SAVINGS ALLOCATIONS

Allocation Scheme	Division A	B	C
Shapley	.37	.35	.28
Bargaining Theory - Initial Trial	.40	.35	.25
Bargaining Theory - Asymptote	.42	.38	.20
Activity Level	.29	.40	.31
Moriarity	.46	.36	.18
Observed Mean	.31	.39	.30

Hotelling's T^2, which is approximated by an F value, was used to test the results of five comparisons. The mean vector of the observed cost savings allocations was compared with the five known solution vectors. For testing purposes, the mean vector of the observed cost savings (.31, .39, .30) was compared with the Shapley solution vector (.37, .35, .28). A Hotelling's T^2 statistic was computed and transformed into an F value. The calculated F value was compared to the tabular F values at the generally accepted levels of significance, .005, .010, .025, .050, and .100. When the calculated F value exceeded the tabular F value, the inference was made that a significant difference existed between the mean vector of the observed cost savings and the Shapley solution vector. For example, the calculated F value for the comparison of the mean vector of the observed cost savings and the Shapley solution vector was 425.225. The calculated F value exceeded the tabular F values[7] at .005 (5.82), .010 (4.94), .025 (3.86), .050 (3.10), and .100 (2.38), thus the two vectors were significantly different at all the generally accepted significance levels. The inference was that the subjects' reported cost savings allocation schemes did not approximate a Shapley allocation scheme.

The preceding procedure was utilized for the other four allocation schemes, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level, and Moriarity. For the purpose of testing whether the observed cost savings allocation schemes approximated a game theoretic allocation scheme, the following hypothesis was formulated:

$$H_o: \utilde{\mu} = \utilde{\mu}_o$$

$$H_a: \utilde{\mu} \neq \utilde{\mu}_o$$

μ was the mean vector of the observed cost saving allocations and μ_o was the vector of one of the five known solutions. Thus, μ was the same for the five comparisons, while μ_o was changed for each comparison to the solution vector that was associated with the known solutions; the Shapley, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level, and Moriarity. Table VII contains a summary of the statistical results of the five comparisons at the .100 level of significance.

The results of the Hotelling's T^2 statistical test were quite clear. Only the Activity Level allocation scheme was not significant at the .100 level of significance. Even at the .005 level of significance, the Activity Level allocation scheme was not significant. Therefore, the inference as a result of the Hotelling's T^2 statistic

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[7] Steel, p. 438.

was that the Activity Level allocation was most closely approximated by the subjects. The hypothesis that a game theoretic allocation would be approximated by the subjects of the experiment was not supported.

TABLE VII

STATISTICAL RESULTS

Allocation Scheme	Hotelling's T^2	F	d.f.	Significant	Significance Level
Shapley	1,403.242	425.225	3,20	yes	.100
Bargaining Theory--Initial Trial	1,210.100	366.697	3,20	yes	.100
Bargaining Theory--Asymptote	106.405	32.244	3,20	yes	.100
Activity Level	3.300	1.000	3,20	no	.100
Moriarity	500.256	151.593	3,20	yes	.100

Since the Hotelling's T^2 test statistic used mean data, a possibility existed that observations that were extremely different from the mean could have had a distorting impact on the test statistic. Therefore, a χ^2 test was also employed to evaluate the observed cost savings allocations. χ^2 was defined by the following equation:

$$\chi^2 = \Sigma \frac{(\text{observed frequency} - \text{expected frequency})^2}{\text{expected frequency}} \qquad (2)$$

The observed frequency was the number of observed cost savings allocation schemes that most closely approximated one of the known allocation schemes. The expected frequency was the probability of a known allocation scheme's occurrence, 20 percent,[8] times the sample size, 23. Thus, the expected frequency was 4.6 (.20 x 23).

The determination of the observed frequency required an involved process. Each observed cost savings allocation scheme was compared with the five known allocation schemes on the basis of, squared and summed, numerical differences. Table II contains a summary of the observed cost savings allocations. These cost savings allocations were then compared with the five known allocation schemes for the Shapley, Bargaining Theory--Initial Trial, Bargaining Theory--Asymptote, Activity Level and Moriarity from Table VI.

For example, group 21 reported the following cost savings allocation: Division A, .51, Division B, .32, and Division 3, .17. This cost savings allocation was compared with each of the five known solutions from Table VI. Thus, the comparison with the Shapley solution, Division A, .37, Division B, .35, Division C, .28, was:

$$(.51 - .37)^2 + (.32 - .35)^2 + (.17 - .28)^2 = .0326$$

The same procedure was repeated using the other four known solutions. The entire process was completed for each group and is represented in Table VIII.

The most closely approximated known allocation scheme was judged as the scheme with the lowest sum of squared differences. For example,

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[8]The probability of occurrence for each of the five known allocation schemes was 1/5 or 20 percent. In other words, each known allocation scheme had an equal probability of occurrence.

group 22 most closely approximated a Moriarity allocation scheme because Moriarity had the lowest sum of squared differences, .0122.

TABLE VIII

SUM OF SQUARED DIFFERENCES

Group Number	Allocation Scheme				
	Shapley	Bargaining Theory-Initial Trial	Bargaining Theory-Asymptote	Activity Level	Moriarity
1-20	.0098	.0182	.0294	.0000*	.0474
21	.0326	.0194	.0126	.0744	.0042*
22	.0582	.0402	.0206	.0968	.0122*
23	.0018*	.0054	.0182	.0128	.0266
Total*	1.0	0.0	0.0	20	2.0

*Indicates the allocation scheme most closely approximated by each group.

An asterisk was used to reflect the most closely approximated known allocation scheme for each group. The number of asterisks was totaled for each known allocation scheme (e.g., Shapley, 1.0). The total asterisks per known allocation scheme served as the observed frequencies for the χ^2 test.

For the purpose of testing the sampling distribution, which was hypothesized as being a uniform distribution, the following hypothesis was formulated:

$$H_o: \pi = 20\%$$

$$H_a: \pi \neq 20\%$$

π was the probability of a known allocation scheme being most closely approximated. Since five known allocation schemes were involved in the experiment each scheme had a 1/5 (20 percent) probability of occurrence.

χ^2 was calculated as follows:

$$\chi^2 = \frac{(1.0 - 4.6)^2}{4.6} + \frac{(20.0 - 4.6)^2}{4.6} + \frac{(2.0 - 4.6)^2}{4.6}$$

$$\chi^2 = 55.843 \qquad \text{d.f.} = 4$$

The calculation of χ^2 only involved the Shapley, Activity Level, and Moriarity because only those three known allocation schemes had observed frequencies greater than zero.

The χ^2 statistic was significant at the .10 level of significance. The null hypothesis was rejected. It was concluded that the distribution was not a uniform distribution.

The small sample size, 23 observations, had an effect on the power of the χ^2 statistical test employed in this research study. The power of a statistical test is the probability of accepting the alternative hypothesis (rejecting the null hypothesis) when the alternative hypothesis is true. Statisticians assert that the power of the test increases as the sample size increases. Since the sample size in this research study was small, the power of the χ^2 statistical test employed was apparently low.

The calculation of the χ^2 statistic required the actual number of observations, 23, and an estimate of the expected number of observations per allocation scheme. The expected number of observations per allocation scheme was estimated by the product of the actual number of observations and the probability of occurrence of each allocation scheme.

Five allocation schemes were employed in this research study and each scheme had an equal probability of occurrence. Thus, each scheme had a 20 percent probability of occurrence. Therefore, the expected number of observations for each allocation scheme was 4.6 (23 x 20%).

Cochran stated that if any expected number of observations was less than one, or if more than 20 percent of the expected number of observations was less than five, the χ^2 statistical test could be poor at low levels of significance.[9] However, he further stated that there was little disturbance to the 5 percent level of significance when a single expectation was as low as .5 and two expectations were as low as 1 for fewer degrees of freedom than 11.[10]

The degrees of freedom for the χ^2 statistical test employed in this research study was 4 and the expected number of observations per allocation scheme was 4.6. The 10 percent level of significance was used for the χ^2 test as a means of precaution due to the apparently low power of the test, the limited number of expected observations per allocation scheme and the low number of degrees of freedom.

The Activity Level allocation scheme contributed 92 percent $(\frac{51.556}{55.843})$ of the χ^2 test statistic.[11] Thus, the results of the χ^2 test provided additional evidence supporting the inference that the

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[9]W. J. Conover, Practical Nonparametric Statistics (New York, 1971), p. 152.

[10]Robert G. D. Steel and James H. Torrie, Principals and Procedures of Statistics (New York, 1960), p. 350.

[11]The χ^2 test statistic was calculated by summarizing $\frac{(\text{observed frequency} - \text{expected frequency})^2}{\text{expected frequency}}$ for each of the five known allocation schemes. The Activity Level portion was $\frac{(20.0 - 4.6)^2}{4.6}$ or 51.556.

Activity Level allocation scheme was most closely approximated by the subjects.

Summary

Bargaining theory of coalition formation, which predicted that grand coalitions would be formed by the subjects, received strong support based on the results of the experiment. Twenty-three of 25 groups, 92 percent, employed in the laboratory experiment formed grand coalitions. Confidence intervals were determined using both the normal distribution as an approximation to a binomial distribution and confidence belts for proportions, to evaluate the 92 percent response[12] rate of the subjects. The probability that a randomly selected group would form a grand coalition was contained within each confidence interval at the .95 level of confidence. Thus the apparent population proportion was substantially greater than the probability of a grand coalition forming, .50. Since 92 percent of the groups did form grand coalitions, the prediction that grand coalitions would form was substantiated.

Twenty-three of the observed cost savings allocation schemes were schemes which were in core. All 23 observed solutions met the three rationality conditions required for a solution to be in the core.

The observed cost savings allocation schemes most closely approximated the Activity Level allocation scheme. Two statistical tests, a Hotelling's T^2 and a χ^2 test of goodness of fit, were utilized to evaluate the data. Both test results clearly implied that the

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[12]Response in this sense represents the subjects' selecting to form grand coalitions rather than selecting other available alternatives.

Activity Level allocation scheme was most closely approximated by the subjects. Therefore, no apparent support was observed for the hypothesis that a game theoretic allocation scheme would be approximated by the subjects.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Introduction

The purpose of this research was to evaluate the behavior of subjects, acting as surrogate division managers, in a joint cost allocation setting where cost savings were available to each division manager. The subjects' observed behavior was evaluated in association with three elements of game theory: coalition formation, core theory, and game theoretic cost allocation schemes.

The results of the research study were mixed; which coalition formation and core theory were strongly supported, evidence did not support the hypothesis that a game theoretic allocation scheme would be approximated by the groups. Even though the results of the research study were not as expected, some useful conclusions and implications surfaced as a consequence.

Summary of Findings

Coalition Formation

Twenty-three groups behaved as anticipated with respect to the bargaining theory of coalition formation. Bargaining theory of coalition formation predicted that the most likely coalition to form was a grand coalition. Ninety-two percent of the groups behaved as predicted; 23 of the 25 groups sampled, formed grand coalitions.

Coalition formation is the initial action in a game theory setting. If the players had not formed the predicted coalitions, their remaining actions would not have been within the rules of game theory, and thus no further evaluation of the data would have been worthwhile. The results were supportive of bargaining theory, thus inferring that positive evidence was provided by the experiment.

Core Theory

The experiment results also supported core theory. All of the groups that formed grand coalitions provided solutions that met the three rationality conditions for a core solution.

Core solutions are important in a game theoretic framework because such solutions indicate rationality and fairness.[1] Also core solutions are allocations that avoid suboptimal decisions on the corporate level due to decisions made on the division level. These criteria provide a sense of stability to a solution. If a stable solution were not available, it is doubtful if players would participate in a game. Thus, if no stable solution exists, a unique solution would not be available.

If game theory is to provide a solution to the allocation problem in joint cost settings, unique solutions are required. Twenty-three groups provided core solutions which implied that the subjects acted in a rational manner. The experiment, then, provided positive evidence of

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[1]Rationality and fairness are measures of acceptance of an allocation scheme as viewed by external users of data and decision makers (the players).

another necessary element of game theory, core theory, as it applies to joint cost allocation situations.

Cost Allocation Schemes

Finally, after having evaluated the subjects' behavior in terms of coalition formation and core theory, their behavior was evaluated for compatibility with five joint cost allocation schemes. The observed cost savings allocation schemes were compared with three game theoretic cost allocation schemes, the Shapley, Bargaining Theory--Initial Trial, and Bargaining Theory--Asymptote; and two non-game theoretic cost allocation schemes, Activity Level and Moriarity. The anticipated behavior of the subjects was that their reported cost savings allocations would most closely approximate a game theoretic cost allocation scheme.

Hotelling's T^2 and χ^2 test statistics were employed to evaluate the observed cost savings allocations for cost allocation scheme preference. The results indicated that the groups of subjects allocated costs in a manner which most closely approximated the Activity Level technique. Thus, on average no evidence that a game theoretic cost allocation was approximated by the subjects was observed.

Conclusions and Recommendations

The subjects behaved in the anticipated manner in terms of coalition formation and core theory. Even more interestingly, the subjects behaved as anticipated having had no knowledge of bargaining theory and the criteria for a core solution. Thus, the subjects' behavior observed in this research study, evaluated in regards to coalition formation and core theory, was viewed as a positive step in the direction of game theory and its applicability to joint cost allocation situations where a benefit is perceived by the players.

Even though game theoretic cost allocation schemes were not used by the subjects, the results of the evaluation of the observed cost savings allocations provided an implication about game theory. The implication was that possible users of game theory in joint cost allocation situations similar to that present in this research study may require exposure to game theory and game theoretic solutions before these types of results can be expected.

The researcher recommends that further research be done with game theory in joint cost allocation situations where a benefit is perceived by the players. Future research could proceed along any of the following lines. First, replications of this research study could be undertaken. Second, researchers might use business sector division managers, instead of students, as subjects. Third, other types of game theoretic cost allocation schemes such as Nash, Nucleolus, and Minimum Resource Theory as measures of evaluation, would be useful. Further research in these areas might provide empirical support for the use of game theoretic allocation schemes as a partial solution to the allocation problem in joint cost allocation situations, the selection and justification of the particular allocation method employed.

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APPENDIXES

APPENDIX A

GAME THEORY TERMINOLOGY

Game Theory Terminology

1. Activity Level Allocation Scheme--Cost is allocated to each player (usually a division of a corporation) in direct proportion to the player's activity level.

2. Bargaining Theory--This tenet suggests that players in a gaming situation use their best payoff in another coalition as a threat to improve their payoff in a current coalition.

3. Caplow's Prediction Theory--The initial distribution of resources is the primary element in the prediction of coalition formation. His major assumption is that each player will choose the coalition that maximizes the number of players he controls.

4. Core Theory--The set of imputations that require for players to be content in a coalition, they must receive at least the amount they can command regardless of what players outside of the coalition can do.

5. Dominance--One imputation dominates another imputation if it is both feasible and preferred.

   A. Feasible

   $$v(T) \geq \sum_{i \text{ in } T} yi$$

   B. Preferred

   $y_i > X_i$ for all i in T

   V(T) - coalition T

   y - imputation $y = (y_1, y_2, \ldots, y_n)$

   x - imputation $x = (x_1, x_2, \ldots, x_n)$

6. Grand Coalition--It is the group (coalition) composed of all the players in a game.

7. Imputation--Any payoff that satisfies the following two rationality conditions is an imputation.

   A. Individual rationality--No player will accept a final payoff less than the least he can receive if he were to play alone against a coalition of all the other players.

   B. Group rationality--No group of players will accept a total payoff that is less than the total of each of their individual payoffs.

8. Minimum Resource Theory--A player will expect the other players to demand from a coalition a share of the payoff proportional to the amount of resources which they contribute to a coalition.

9. Moriarity Allocation Scheme--The cost allocated to each player is equal to the difference between his independent cost and a specific fraction of the total value obtained by forming the grand coalition.

10. Move--A move is the set of choices a player has at a particular decision point.

11. Nash Solution--This allocation scheme requires that the following properties be met:

    A. Invariance with respect to utility transformations.
    B. The bargained value must be at least as good as the players acting alone, it must be feasible, and it must be better than any other feasible point.
    C. Independence of irrelevant alternatives.
    D. The game must be symmetric (i.e., equal utility payoffs).

12. Non-zero-sum game--A game in which the payoffs to the players does not sum to zero is a non-zero-sum game.

13. Nucleolus Allocation Scheme--This is a solution that is based on the criterion of Pareto Optimality. The minimum surplus (i.e., synergy or cost savings) over all coalitions is maximized. An antinucleolus solution minimizes the maximum surplus.

14. Payoff matrix--It is a set of associate outcomes.

$$\text{Player 1} \quad \begin{array}{c} \\ a_1 \\ a_2 \end{array} \overset{\text{Player 2}}{\begin{array}{cc} b_1 & b_2 \\ \end{array}} \begin{bmatrix} 0_{11} & 0_{12} \\ 0_{21} & 0_{22} \end{bmatrix}$$

15. Pivotal Power Theory--A player's resources are pivotal when their inclusion in a losing coalition can convert it into a winning coalition. The theory predicts that all coalitions are equally likely and the payoffs divided equally when one player cannot win by himself.

16. Player--A player is a participant in a game.

17. Shapley Allocation Scheme--This solution is based on pivotal power theory. Thus, as the grand coalition forms by the sequential addition of players, each player is charged an amount equal to the expected marginal cost incurred when the player enters.

18. von Neumann-Morgenstern--Their book, Theory of Games and Economic Behavior serves as a prelude to the theory of interest conflict in the behavior sciences. They developed the basic game theory reasoning (i.e., basically rationality axioms) that has led to the development of the various allocation schemes.

19. Zero-sum game--A game where the sum of payoffs to the players is zero is a zero-sum game.

APPENDIX B

COST DATA AND DIVISION REQUIREMENTS

Cost Data and Division Requirements

Computer Time Requirements:

Division	Total Computer Hours Required Per Week
A	20
B	15
C	7
	42

Cost Estimates Per Week:

Division A--Using an outside computer firm	$2,610
Division B--Using an outside computer firm	$2,030
Division C--Using an outside computer firm	$1,070
Divisions A and B--Using an outside computer firm	$4,260
Divisions A and C--Using an outside computer firm	$3,390
Divisions B and C--Using an outside computer firm	$2,835
Divisions A, B, and C--Using Division D	$5,010

Cost Savings Per Week:

Division A	$ 0
Division B	$ 0
Division C	$ 0
Divisions A and B together	$380
Divisions A and C together	$290
Divisions B and C together	$265
Divisions A, B, and C together	$700

APPENDIX C

COALITION FORMATION

Coalition Formation

Coalition formation is predicted by bargaining theory. Bargaining theory postulates that the most likely coalition to form is the one that minimizes members' temptation to defect.

Expected Payoffs (Based on Computer Hours Required Per Week):

Division	Hours	Coalition AB	Coalition AC	Coalition BC	Coalition ABC
A	20	20/35 = 56%	20/27 = 71%		20/42 = 46%
B	15	15/35 = 44%		15/22 = 65%	15/42 = 36%
C	7		7/27 = 29%	7/22 = 35%	7/42 = 18%

Temptation Factors:

$$\sum_j (O_{ij} - E_{ij})$$

O_{ij} = predicted payoff to division i in coalition j.

E_{ij} = maximum expectation in alternative coalitions.

AB = (.56 - .71) + (.44 - .65) = -.36

AC = (.71 - .56) + (.29 - .35) = .09

BC = (.65 - .44) + (.35 - .29) = .27

ABC = (.46 - .71) + (.36 - .65) + (.18 - .35) = -.71*

*ABC is the most likely coalition to form.

APPENDIX D

COST ALLOCATION SOLUTIONS AND CORE THEORY

Characteristic Function:

V(A) = \$2,610	V(AB) = \$4,260
V(B) = \$2,030	V(AC) = \$3,390
V(C) = \$1,070	V(BC) = \$2,835

V(ABC) = \$5,010

Hours Per Week:

Division A =	20
Division B =	15
Division C =	7
Total	42

Cost Allocation Solutions and Core Theory

A. Shapley Solution (Game Theoretic)

axioms:

1. The value should be determined by the characteristic function and be independent of how the players are labelled.

2. The set of values to the n players should be an imputation.

3. If two games are merged into one, the value of the new game should be the sum of the two original games.

There is only one vector of payoffs that satisfies the above axioms, thus the Shapley Solution results in a unique value to the game.

$$\varepsilon(A) = \frac{s!(n-s-1)!}{n!}[V(A)] + \frac{s!(n-s-1)!}{n!}$$

$$[V(AB) - V(B)] + \frac{s!(n-s-1)!}{n!}$$

$$[V(AC) - V(C)] + \frac{s!(n-s-1)!}{n!}$$

$$[V(ABC) - V(BC)]$$

$\varepsilon(A)$ = expected payoff to Division A.

$\frac{s!(n-s-1)!}{n!}$ = the probability that Division A will join the coalition.

$s!$ = the possible arrangements of divisions already in the coalition before Division A.

$(n-s-1)!$ = the possible arrangements of divisions who join the coalition after Division A.

$n!$ = number of different ways the coalition can be formed.

$\varepsilon(A)$ = 1/3(\$2,610) + 1/6(\$2,230) + 1/6(\$2,320) + 1/3(\$2,175)

$\varepsilon(A)$ = \$2,354

$\varepsilon(B) = 1/3[V(B)] + 1/6[V(AB) - V(B)] + 1/6[V(BC) - V(C)] +$

$1/3[V(ABC) - V(AC)]$

$\varepsilon(B) = 1/3[\$2,030] + 1/6[\$1,650] + 1/6[\$1,756] + 1/3[\$1,620]$

$\varepsilon(B) = \$1,786$

$\varepsilon(C) = 1/3[V(C)] + 1/6[V(AC) - V(A)] + 1/6[V(BC) - V(B)] +$

$1/3[V(ABC) - V(AB)]$

$\varepsilon(C) = 1/3[\$1,070] + 1/6[\$780] + 1/6[\$805] + 1/3[\$750]$

$\varepsilon(C) = \$871$

Cost Savings (Xi) Allocation

$X_A = V(A) - \varepsilon(A)$

$X_A = \$2,610 - \$2,354$

$X_A = \$256$ 37%

$X_B = V(B) - \varepsilon(B)$

$X_B = \$2,030 - \$1,786$

$X_B = \$244$ 35%

$X_C = V(C) - \varepsilon(C)$

$X_C = \$1,070 - \$871$

$X_C = \$200$ 28%

Core Calculation

$$\sum_{s} Xi \geq V(S) \qquad S \subseteq N$$

where

$$V(S) = \sum_{s} (Qi) - (\sum_{s} Qi)$$

where

$\sum_{s} (Qi)$ = the i^{th} division's cost acting alone.

$(\sum_{s} Qi)$ = the cost of acting jointly.

V(S) = the joint cost savings.

AB

V(S) = \$4,640 - \$4,260
V(S) = \$380

AC

V(S) = \$3,680 - \$3,390
V(S) = \$290

BC

V(S) = \$3,100 - \$2,835
V(S) = \$265

ABC

V(S) = \$5,710 - \$5,010
V(S) = \$700

$$\sum_{s} Xi \geq V(S) \qquad S \subseteq N$$

AB

\$500 > \$380

AC

\$456 > \$290

BC

\$444 > \$265

ABC

\$700 = \$700

Thus, the Shapley Solution is in core.

B. Moriarity Allocation

$$\varepsilon(i) = V(i) - \frac{V(i)}{\sum_{i=A \text{ to } C}^{i} V(i)} V(N)$$

$$\varepsilon(A) = \$2,610 - \frac{\$2,610}{\$5,710} (\$700)$$

$$\varepsilon(A) = \$2,610 - .46(\$700)$$

$$\varepsilon(A) = \$2,610 - \$320$$

$$\varepsilon(A) = \$2,290$$

$$\varepsilon(B) = \$2,030 - \frac{\$2,030}{\$5,710} (\$700)$$

$$\varepsilon(B) = \$2,030 - .35(\$700)$$

$$\varepsilon(B) = \$2,030 - \$249$$

$$\varepsilon(B) = \$1,781$$

$$\varepsilon(C) = \$1,070 - \frac{\$1,070}{\$5,710} (\$700)$$

$$\varepsilon(C) = \$1,070 - .19(\$700)$$

$$\varepsilon(C) = \$1,070 - \$131$$

$$\varepsilon(C) = \$939$$

Cost Savings (Xi) Allocation

from above:

$X_A = \$320$	46%
$X_B = \$249$	36%
$X_C = \$131$	18%

Core Calculation

$$\sum_{s} \varepsilon(i) \leq \left(\sum_{i=A \text{ to } C}^{i} V(i) \right) \quad \text{for all S in N}$$

$$\$5,010 < \$5,710$$

Thus, the Moriarity solution is in core.

C. Activity Level

$$\varepsilon(i) = \frac{Qi}{\sum_{n} Qi} V(ABC) \qquad Qi = \text{hours per week for division i.}$$

$$\varepsilon(A) = \frac{20}{42} (\$5{,}010)$$

$$\varepsilon(A) = .48(\$5{,}010)$$

$$\varepsilon(A) = \$2{,}405$$

$$\varepsilon(B) = \frac{15}{42} (\$5{,}010)$$

$$\varepsilon(B) = .35(\$5{,}010)$$

$$\varepsilon(B) = \$1{,}753$$

$$\varepsilon(C) = \frac{7}{42} (\$5{,}010)$$

$$\varepsilon(C) = .17(\$5{,}010)$$

$$\varepsilon(C) = \$852$$

Cost Savings (Xi) Allocation

$$X_A = V(A) - \varepsilon(A)$$

$$X_A = \$2{,}610 - \$2{,}405$$

$$X_A = \$205 \qquad 29\%$$

$$X_B = V(B) - \varepsilon(B)$$

$$X_B = \$2{,}030 - \$1{,}753$$

$$X_B = \$277 \qquad 40\%$$

$$X_C = V(C) - \varepsilon(C)$$

$$X_C = \$1{,}070 - \$852$$

$$X_C = \$218 \qquad 31\%$$

Core Calculation

$\sum_{s} \varepsilon(i) < (\sum_{s} Qi)$ $\qquad (\sum_{s} Qi)$ = the cost of acting jointly

\$5,010 = \$5,010

Thus, the Activity Level solution is in core.

D. Bargaining Theory--Initial Trial (Game Theoretic)

Parity and equality norms are followed by the players. Therefore, the payoff to each player will be midway between parity (players percentage contribution) and equality.

$$ABC = \left(\frac{\$2,610}{\$5,010}, \frac{\$2,030}{\$5,010}, \frac{\$1,070}{\$5,010}\right)$$

ABC = (.52, .41, .21)

adjusted to 100%

ABC = (.46, .36, .18)

Application of parity and equality norms:

parity equality

A: .46 - .333 = .127 ÷ 2 = .06

B: .36 - .333 = .027 ÷ 2 = .01

C: .18 - .333 = -.153 ÷ 2 = -.08

adjusted

A: .46 - .06 = .40

B: .36 - .01 = .35

C: .18 + .08 = .25

Cost Savings (Xi) Allocation

X_A = \$700 x .40 = \$280 40%

X_B = \$700 x .35 = \$245 35%

X_C = \$700 x .25 = \$175 25%

Core Calculation

$Xi \geq V(\{i\})$ for all i

$X_A = \$280 > 0$

$X_B = \$245 > 0$

$X_C = \$175 > 0$

$$\sum_{i=1}^{n} Xi = V(N)$$

\$700 = \$700

$\sum_{s} Xi \geq V(S)$ for all $S \subseteq N$

AB

\$525 > \$380

AC

\$455 > \$290

BC

\$420 > \$265

Thus, Bargaining Theory--Initial Trial is in core.

E. Bargaining Theory--Asymptote (Game Theoretic)

Players use their best payoff in other coalitions as an element of threat.

A = .71 in (AB)

B = .65 in (BC)

C = .35 in (BC)

Adjusted to 100%

$$\frac{.71}{1.71} + \frac{.65}{1.71} + \frac{.35}{1.71} = 1.00$$

$$.42 + .38 + .20 = 1.00$$

Cost Savings (Xi) Allocation

$X_A = .42 \times \$700 = \$294$		42%
$X_B = .38 \times \$700 = \$266$		38%
$X_C = .20 \times \$700 = \$140$		20%

Core Calculation

$Xi \geq (\{i\})$ for all i

$X_A = \$294 > 0$

$X_B = \$266 > 0$

$X_C = \$175 > 0$

$$\sum_{i=1}^{n} Xi = V(N)$$

$700 = $700

$\Sigma Xi \geq V(S)$ for all $S \subseteq N$

<u>AB</u>

$560 > $380

<u>AC</u>

$469 > $290

<u>BC</u>

$441 > $265

Thus, Bargaining Theory--Asymptote is in core.

APPENDIX E

TEST INSTRUMENT

BACKGROUND INFORMATION

You are the manager of division (A, B, or C). The Corporation has recently developed a new division, D, which is strictly a computer service division. Division D leases its computer at a fixed rate of $5,010 per week.

In recent weeks the divisional weekly accounting and production reports, which are handwritten, have been received well past the due date (Monday morning, 9:00 a.m.). Also, the corporate staff has had a difficult time integrating the divisional reports because the reports were not uniform. Therefore, corporate headquarters issued to each division manager a memo stating that their quarterly bonuses would be reduced accordingly for any more tardy weekly reports. Also, a uniform format for the weekly reports was attached. It was also made clear in the memo that computer printed reports would be required.

Today is Friday, the day division managers have just heard presentations from outside computer firms and from division D, the new computer division. (Cost estimates and each division's computer requirements are on page 2.) Each presented stressed that they could guarantee timely reports, one week from Monday, only if they could get started this afternoon. Therefore, time is of the essence.

STATEMENT OF THE PROBLEM

You as a division manager now face this dilemma: How to get weekly reports, computer printed, to the corporate office every Monday by 9:00 a.m.

Here are some additional considerations:

1. You may decide to act alone, or form a group with one, or both, of the other division managers.
2. One of the criteria used to evaluate your performance for determining your quarterly bonus is cost savings you can verify when making investment decisions. (i.e., Cost to make a raw material, $100; cost to buy the raw material, $75. If the decision maker buys the raw material, his cost savings would be $25.)

The decision you must make is:

1. What computer facility should your division use (either division D or an outside facility)?
2. If you and another, or both other divisions, decide to form a group, how should the cost (see attached cost figures) be allocated or partitioned to the using divisions?

Cost Data and Division Requirements

Computer Time Requirements:

Division	Total Computer Hours Required Per Week
A	20
B	15
C	7
	42

Cost Estimates Per Week:

Division A - using an outside computer firm	$2,610
Division B - using an outside computer firm	$2,030
Division C - using an outside computer firm	$1,070
Divisions A and B - using an outside computer firm	$4,260
Divisions A and C - using an outside computer firm	$3,390
Divisions B and C - using an outside computer firm	$2,835
Divisions A, B, and C - using Division D	$5,010

Cost Savings Per Week:

Division A	$ 0
Division B	$ 0
Division C	$ 0
Divisions A and B together	$380
Divisions A and C together	$290
Divisions B and C together	$265
Divisions A, B, and C together	$700

Please answer the following questions by circling the correct response.

1. What was your decision?

   A. To act alone.

   B. To form a group.

If your answer to number 1 was A, stop.

If your answer to number 1 was B, please answer the next two questions.

2. What group was formed?

   A. A + B

   B. A + C

   C. B + C

   D. A + B + C

3. How are the weekly costs to be allocated (divided up) to the members of the group? (Give answer in either percentages or fractions.)

   A =

   B =

   C =

Group Number ______________________________

Briefly describe how you arrived at your decision.

Informed Consent by Subjects in Experiments

I, ____, have carefully read-
(Print Name)

listened to (circle one) and fully understand the instructions for this experiment on Cost Allocation. I give my consent to serve as a subject in this experiment on __________________________. I am aware
(Date)

that I can ask questions or terminate the experiment at any point.

____
Signature

VITA

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