THE DESIGN OF A SUPERSONIC
WIND TUNNEL

By

MYLES BARTON MITCHELL

Bachelor of Science

Oklahoma Agricultural and Mechanical College
Stillwater, Oklahoma

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and

JOHN VINCENT BIGGERS

Bachelor of Science

Oklahoma Agricultural and Mechanical College

Stillwater, Oklahoma

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THESIS AND ABSTRACT APPROVED:

Ladislaus J. Fila
Thesis Adviser

B. M. Aldrich
Faculty Representative

[signature]
Dean of the Graduate School

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LIST OF ILLUSTRATIONS

SYMBOLS AND ABBREVIATIONS

A = area

a = velocity of sound

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

c = reference velocity

d = diameter

Fig. = figure

^oF = degree fahrenheit

g = acceleration of gravity

h = enthalpy

M = Mach number

m = mass rate of flow

p = pressure

psi = pounds per square inch

psia = pounds per square inch absolute

psig = pounds per square inch gage

p_s = saturation vapor pressure

p_v = vapor pressure

R = universal gas constant

^oR = degree Rankine

RPM = revolutions per minute

T = temperature

t = time

u = velocity

w = free stream velocity

w_n = normal free stream velocity

w_t = tangential free stream velocity

Greek symbols

β = Mach angle

γ = ratio of specific heats = 1.4 for air

δ = deflection

θ = angle of turning

Superscripts

$*$ = critical (throat) condition

Subscripts

o = reservoir

1 = test section

INTRODUCTION

The development and expansion of the field of aerodynamics have been accelerated tremendously in the past few decades. Because of the recent world conflicts, the realm of supersonic flight has undergone much study and research. It is because of this intense study that flights faster than the speed of sound are now possible.

To achieve flights at supersonic speeds, the aeronautical engineer had to disregard some of the misleading basic assumptions, the most important one being that air could be considered as an incompressible fluid, and develop new equations based on advance theory and experimental results. Theodore von Karman summed up our progress in the following manner:

> "I believe we have now arrived at the stage where knowledge of supersonic aerodynamics should be considered by the aeronautical engineer as a necessary prerequisite to his art. This branch of aerodynamics should cease to be a collection of mathematical formulas and half-digested, isolated, experimental results. The aeronautical engineer should start to get the same feeling for the facts of supersonic flight as he acquired in the domain of subsonic velocities by the long process of theoretical study, experimental research, and flight experience."[1]

It is because of the necessity for aeronautical engineers interested in the field of supersonics to have a working knowledge of the method and procedure of supersonic wind tunnel testing that this project was developed. It is the desire of the authors and the faculty advisor of this thesis that the

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[1] Theodore von Karman, Jour. Aero. Sci., Vol. 14, No. 7, p. 373, July 1947.

completed tunnel will aid the student in his studies of supersonic flow. It is also felt that it will give the student some fundamental knowledge and experience in supersonic wind tunnel testing.

The scope of this thesis covers the fundamental principles and design of a supersonic wind tunnel. Included are some suggestions on the types of optical and balance systems which should be employed in this particular tunnel.

FUNDAMENTAL THEORY

Bernoulli Equation

"In the high subsonic and in the supersonic range of velocities, the air can no longer be considered to be incompressible and a more fundamental form of the Bernoulli principle must be reverted to in determining the thermodynamic relations existing between the reservoir; and any location downstream of the throat in the divergent section of the nozzle."[1]

Writing the general energy equation we get:

$$\frac{u^2}{2} + h = h_o \tag{2-1}$$

writing the enthalpy (h) in a more useable form as

$$h = CpT = Cp\,\frac{p}{\rho R} = \frac{p}{\rho}\,\frac{C_p}{C_p - C_v} = \frac{p}{\rho}\,\frac{C_p/C_v}{(C_p/C_v)-1}$$

and substituting γ for C_p/C_v the desired Bernoulli equation becomes

$$\frac{u^2}{2} + \frac{\gamma}{\gamma-1}\,\frac{p}{\rho} = \frac{\gamma}{\gamma-1}\,\frac{p_o}{\rho_o} = \text{a constant} \tag{2-2}$$

Speed of Sound

The speed of sound is defined as the speed at which an infinitesmial disturbance of the fluid is propagated to the flow surrounding the point. Writing Euler's equation as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} = -\frac{1}{\rho}\,\frac{\partial p}{\partial s} \tag{2-3}$$

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[1] E. Arthur Bonney, Engineering Supersonic Aerodynamics, p. 35.

and setting all partial derivatives with respect to time equal to 0, to satisfy the conditions of steady motion, we get

$$u\frac{\partial u}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial s} = 0 \qquad (2\text{-}4)$$

and since only one dimension is involved, equation (2-4) becomes

$$udu + \frac{1}{\rho}dp = 0 \qquad (2\text{-}5)$$

Writing the continuity equation as

$$ud\rho + \rho du = 0 \qquad (2\text{-}6)$$

and solving for the velocity u in equations (2-5) and (2-6), we obtain

$$u^2 = \frac{dp}{d\rho}$$

or

$$u = \sqrt{\frac{dp}{d\rho}}$$

It can be seen from this that in order for a stationary disturbance to exist with a given pressure and density jump, the velocity of the flow can have only one real value as shown. The accuracy of this computation is dependent upon the size of the velocity increment.[2] The speed at which the wave will propagate unchanged is denoted as

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[2] Hans Wolfgang Liepmann and Allen E. Puckett, Aerodynamics of a Compressible Fluid, p. 20.

$$a = \sqrt{\frac{dp}{d\rho}} \qquad (2\text{-}7)$$

and referred to as the local speed of sound of the fluid.

It can be seen from equation (2-7) that the speed of sound is related to the change in pressure with density, which is the reciprocal of the compressibility factor. Thus the greater the compressibility of the fluid, the lower the speed of sound.[3]

From equation (2-7) it was shown that the change of state across the wave was isentropic, the pressure and density are not independent, which allows us to evaluate the ratio $dp/d\rho$ in terms of the gas parameter since

$$p = \text{constant}\ \rho^{\gamma}$$

$$\frac{dp}{d\rho} = \frac{\gamma p}{\rho} = \gamma RTg \qquad (2\text{-}8)$$

or

$$a = \sqrt{\gamma RTg} \qquad (2\text{-}9)$$

Substituting the values for γ and R we obtain

$$a = 49.1\sqrt{T} \qquad (2\text{-}10)$$

<u>Isentropic Channel Flow</u>

Using equation (2-2), which is valid for one dimensional flow, and combining it with the equation for the speed of sound

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[3] *Ibid*., p. 21.

(2-8), we obtain

$$\frac{1}{2}u^2 + \frac{1}{\gamma-1}a^2 = \frac{1}{\gamma-1}a_o^{\ 2} \qquad (2\text{-}11)$$

where a_o is the speed of sound in the reservoir from which the flow at speed u issues.

If another useful reference velocity c is used to take the place of the fictitious reservoir, equation (2-11) can be re-written as follows:

$$\frac{1}{2}u^2 + \frac{1}{\gamma-1}a^2 = \frac{1}{2}c^2 \qquad (2\text{-}12)$$

It is interesting to note that if the flow of the gas is not coming out of a definite reservoir, but is initially at rest and the body is moving through the medium, the maximum velocity of the gas around the body can be no greater than c, as given by equation (2-12).

Another reference velocity of great importance occurs when the initial speed u, as it leaves the reservoir is low, and the speed of sound a is high; as the speed u increases, the temperature will drop, causing the speed of sound to drop. At some point the two velocities, u and a, will be equal, i.e., the free stream velocity will be exactly equal to the speed of sound. This speed will be defined as the critical speed of sound, and will be designated as a^*.

Substituting $a^* = a = u$ for the left hand side of equation (2-11), we obtain

$$\frac{1}{2}a^{*2} + \frac{1}{\gamma-1}a^{*2} = \left(\frac{1}{2} + \frac{1}{\gamma-1}\right)a^{*2}$$

reducing and simplifying, we get

$$\frac{1}{2}u^2 + \frac{1}{\gamma-1}a^2 = \frac{\gamma + 1}{2(\gamma-1)}a^{*2} \tag{2-13}$$

From this we can obtain three reference velocities, all functions of the initial enthalpy of the flow. They are:

$$\frac{a_o^2}{\gamma-1} = \frac{c^2}{2} = \frac{\gamma + 1}{2(\gamma-1)}a^{*2}$$

Substituting the value 1.4 for γ we arrive at the following constants.[4]

$$a^* = 0.913a_o \qquad a_o = 1.095a^*$$

$$c = 2.236\,a_o = 2.449a^*$$

The most important of the reference speeds given above is the local speed of sound a. The ratio of u/a is a dimensionless quantity known as the Mach number, named after the Austrian scientist Ernst Mach who pioneered in the field of compressibility. The equations for the Mach number are as follows:

$$M = \frac{u}{a} = \frac{u}{\sqrt{g\gamma RT}} \tag{2-14}$$

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[4] Ibid., p. 24.

or

$$M^2 = \frac{u^2}{a^2} = \frac{u^2}{g\gamma RT} \qquad (2\text{-}15)$$

The physical significance of the Mach number can readily be seen from equation (2-15). The ratio u^2/a^2 is a ratio of kinetic energies. Since u is the velocity of the gas, u^2 is a measurement of the kinetic energy of the directed flow. The velocity of a is dependent upon the absolute temperature of the gas, therefore, the kinetic energy of the gas corresponding to a^2 is dependent upon the thermal energy imparted to the gas. From the kinetic theory of gases it may be seen that the thermal energy is utilized in increasing the random motion of the gas molecules. The value of M^2 therefore measures the ratio of the kinetic energy of the directed flow to the kinetic energy of the random flow.[5]

It is not always convenient to use the ratio u/a because the parameter a is of local interest and varies as the velocity of the flow u varies. It was for this reason the parameters a_o, a^*, and c, which are constant for the entire flow, were introduced.

The ratio u/a^* is a very useful parameter because it defines the conditions in terms of the throat of the nozzle. It will be interesting to note from equation (2-13) that when both sides are divided by u, and the equation reduced down to

$$\left(\frac{u}{a^*}\right)^2 = \frac{\gamma + 1}{(\gamma - 1) + \frac{2}{M^2}} \qquad (2\text{-}16)$$

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[5] M. J. Zucrow, Jet Propulsion and Gas Turbines, p. 36.

we find that the maximum value of u/a^* is 2.45 for air (i.e., when the value of infinity is substituted for the Mach number.)

If the parameters a_o and a^* are substituted in equations (2-11), (2-12), and (2-13), the following useful equations can be obtained.

$$\frac{\gamma-1}{2}\left(\frac{u}{a}\right)^2 + 1 = \left(\frac{a_o}{a}\right)^2 \qquad (2\text{-}13a)$$

$$\frac{\gamma-1}{2}\left(\frac{u}{a_o}\right)^2 + \left(\frac{a}{a_o}\right)^2 = 1 \qquad (2\text{-}13b)$$

$$\frac{\gamma-1}{\gamma+1}\left(\frac{u}{a}\right)^2 + \left(\frac{a}{a_o}\right)^2 = 1 \qquad (2\text{-}13c)$$

Substituting T/T_o for $(a/a_o)^2$ the preceding equations can be written as follows

$$T_o/T = 1 + \frac{\gamma-1}{2} M^2 \qquad (2\text{-}17)$$

$$T/T_o = 1 - \frac{\gamma-1}{2}\left(\frac{u}{a_o}\right)^2 \qquad (2\text{-}18)$$

$$T/T_o = 1 - \frac{\gamma-1}{\gamma+1}\left(\frac{u}{a^*}\right)^2 \qquad (2\text{-}19)$$

All of the preceding equations are simply forms of the energy, omitting assumptions of isentropic flow, obtained from Liepmann and Puckett.[6]

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[6] Liepmann and Puckett, op. cit., pp. 20-25.

Isentropic Channel Flow Equations

If we assume that the flow in the channel is isentropic, (frictionless adiabatic), we may relate the pressure, density, and temperature to the speed ratio in the following manner: substituting

$$\left(\frac{a_o}{a}\right)^2 = \frac{T_o}{T} = \left(\frac{p_o}{p}\right)^{\frac{(\gamma-1)}{\gamma}} = \left(\frac{\rho_o}{\rho}\right)^{(\gamma-1)}$$

in equation (2-17) we get

$$\frac{p_o}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{(\gamma-1)}} \qquad (2\text{-}20)$$

Similarly,

$$\frac{\rho_o}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{(\gamma-1)}} \qquad (2\text{-}21)$$

Following the same procedure the velocity may be expressed the same way. The temperature - velocity relations are given in equations (2-18), and (2-19); substituting in the isentropic temperature - pressure relations gives,

$$\frac{p}{p_o} = \left[1 - \frac{\gamma-1}{2}\left(\frac{u}{a_o}\right)^2\right]^{\frac{\gamma}{(\gamma-1)}} \qquad (2\text{-}22)$$

and

$$\frac{\rho}{\rho_o} = \left[1 - \frac{\gamma-1}{2}\left(\frac{u}{a_o}\right)^2\right]^{\frac{1}{(\gamma-1)}} \qquad (2\text{-}23)$$

If we transform these in terms of the critical speed of sound a^* we get

$$\frac{p}{p_o} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{u}{a^*}\right)^2\right]^{\frac{\gamma}{(\gamma-1)}} \qquad (2\text{-}24)$$

and

$$\frac{\rho}{\rho_o} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{u}{a^*}\right)^2\right]^{\frac{1}{(\gamma-1)}} \qquad (2\text{-}25)$$

General Flow Relations

The continuity equation for a compressible flow along a stream tube is:

$$\rho uA = \text{constant}$$

Taking the differential, we obtain

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

If we rewrite Euler's equation, with $a^2 = \frac{dp}{d\rho}$, as follows

$$udu + a^2\frac{d\rho}{\rho} = 0$$

and eliminate $\frac{d\rho}{\rho}$ from the two equations, the following is obtained:

$$udu + a^2\left(- \frac{dA}{A} - \frac{du}{u}\right) = 0$$

or

$$\frac{dA}{A} = \frac{udu}{a^2} - \frac{du}{u}$$

which may be written as

$$\frac{dA}{A} = \frac{u^2}{a^2}\frac{du}{u} - \frac{du}{u} = - \frac{du}{u}\left(1 - \frac{u^2}{a^2}\right)$$

If we substitute the relation $M = u/a$, the equation may be simplified to read

$$\frac{dA}{A} = - (1-M^2)\frac{du}{u} \tag{2-26}$$

Some important relationships may be obtained by analyzing the above equation. If the flow enters the channel at a subsonic speed $(M<1)$, then for du to be positive it requires that dA/A must be negative, (i.e., the channel must converge). If the flow is sonic $(M=1)$, the right hand side of the equation must be equal to zero. This means that $dA/A = 0$ (i.e., the area is a maximum or minimum.) As the Mach number approaches unity from either side, the area converges, which means that it is a minimum. In the case of supersonic flow (M>1), the right hand side of the equation is always positive, which means that the area is diverging.

The foregoing analysis shows that the flow channel must be a converging section followed by a diverging section if the gas is to be accelerated from subsonic to supersonic speed. These are the general features of a DeLaval nozzle.

The critical ratios at the throat (M=1) are of great interest and importance in the solution of the nozzle design. The critical ratios are obtained by substituting M=1 into equations (2-17), (2-22), and (2-23).

$$\frac{T^*}{T_o} = \frac{2}{\gamma-1}$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{(\gamma-1)}} \qquad (2\text{-}27)$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{(\gamma-1)}}$$

In air, with $\gamma = 1.400$, these become

$$\frac{T^*}{T_o} = 0.833$$

$$\frac{p^*}{p_o} = 0.528$$

$$\frac{\rho^*}{\rho_o} = 0.634$$

DE LAVAL NOZZLE

Theory

The theory of the De Laval Nozzle (converging-diverging passage) uses the principle explained previously. The velocity of the flow is subsonic entering the converging portion of the passage; it continues to increase, and the pressure decreases up to the throat section. If the velocity remains subsonic in the throat, a flow compression occurs in the divergent section. If the flow in the throat reaches a sonic velocity, either a flow expansion with supersonic velocities or a flow compression with subsonic velocities may occur in the divergent section. The actual state of the flow in the divergent section of the channel depends upon the exit pressure.

A complete analysis of the flow in the converging-diverging passage will be covered by one of the following cases. Let u_o be the entrance velocity and u^* the velocity in the throat.

Case I. ($u_o<a$, $u^*<a$). The velocity of the flow enters at a subsonic speed and increases until it reaches the throat section. In the diverging section the velocity is still subsonic and decreases. The pressure decreases in the converging section until it reaches the throat section and then begins to increase again.

Case II. ($u_o<a$, $u^* = a$). In this case the flow enters the converging section with subsonic speed and increases up to the throat. Meanwhile the pressure is going through a corresponding decrease. When the flow reaches the throat it is at the speed of sound. In the diverging section the velocity continues

to increase while the pressure diminishes. This is a case of pure expansion where the pressure is converted into kinetic energy.

Case III. (u_o>a, u*>a). The flow enters the converging section with supersonic speed and decreases until it reaches the throat while the pressure is increasing. Since the velocity at the throat is greater than the speed of sound, the flow accelerates again in the diverging section with a decrease in pressure.

Case IV. (u_o>a, u* = a). The flow in the converging section is the same as in Case III. The velocity decreases until the speed of sound is reached in the throat. The velocity continues to decrease in the diverging section while the pressure increases. This is a case of pure flow compression as in Case I.[1]

De Laval Nozzle Equations

The equations for the throat of a nozzle are determined by the reservoir conditions. Therefore, they become useful as reference values. The equations for the acoustic velocity at the throat from equation (2-13a) becomes

$$a^* = a_o \sqrt{2/(\gamma+1)} \qquad (3\text{-}1)$$

Similarly, from equations (2-27)

$$T^* = T_o \left[2/(\gamma+1)\right] \qquad (3\text{-}2)$$

[1] M. J. Zucrow, Jet Propulsion and Gas Turbines, pp. 122-124.

$$p^* = p_o\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{(\gamma-1)}} \tag{3-3}$$

$$\rho^* = \rho_o\left(\frac{2}{\gamma+1}\right)^{\frac{1}{(\gamma-1)}} \tag{3-4}$$

Writing the continuity equation for the flow through the nozzle as

$$\rho A u = \rho^* A^* u^* = m = \text{constant}$$

and rewriting

$$\frac{A}{A^*} = \frac{\rho^* u^*}{\rho\ u} \tag{3-5}$$

substituting

$$u = Ma \quad \text{and} \quad u^* = a^*$$

we obtain

$$\frac{A}{A^*} = \frac{\rho^* a^*}{\rho Ma}$$

From equations (2-13a) and (2-21) we find

$$a = a_o/\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}}$$

$$\rho = \rho_o/\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{(\gamma-1)}}$$

substituting these values in equation (3-5)

$$\frac{A}{A^*} = \frac{\rho^* \qquad a^*}{\left[\frac{\rho_o}{(1 + \frac{\gamma-1}{2}M^2)^{1/\gamma-1}}\right] M \left[\frac{a_o}{(1 + \frac{\gamma-1}{2}M^2)^{1/2}}\right]} \qquad (3\text{-}6)$$

From equations (3-1) and (3-4) we get the relations

$$\frac{a^*}{a_o} = \left(\frac{2}{\gamma+1}\right)^{1/2}$$

and

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{(\gamma-1)}}$$

If we substitute these relations into equation (3-6), then

$$\frac{A}{A^*} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{(\gamma-1)}} \qquad \left(\frac{2}{\gamma+1}\right)^{1/2}}{M(1 + \frac{\gamma-1}{2}M^2)^{1/(\gamma-1)} \qquad (1 + \frac{\gamma-1}{2}M^2)^{1/2}}$$

and simplify into

$$\frac{A}{A^*} = \frac{1}{M} \left(\frac{1 + \frac{\gamma-1}{2} M^2}{\frac{\gamma+1}{2}} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

and for air where $\gamma = 1.400$

$$\frac{A}{A^*} = \frac{1}{M} \left(\frac{5 + M^2}{6} \right)^3 \qquad (3\text{-}7)$$

Mass Rate of Flow

The speed of sound a^* at the throat is solely a function of the free stream temperature, as given by equation

$$\left(\frac{a^*}{a_o}\right)^2 = \frac{T^*}{T_o} = \frac{T_o}{T_o(1 + \frac{\gamma-1}{2})} = \frac{2}{\gamma+1}$$

Therefore, if the pressure ratio is below the critical and the stagnation temperature T_o is known, the velocity of the flow at the throat is

$$u^* = a^* = a_o \sqrt{\frac{2}{\gamma+1}}$$

as shown in equation (3-1). If we substitute $\sqrt{\gamma R T_o}$ for a_o in the above equation, we arrive at

$$u^* = a^* = \sqrt{\frac{2\gamma R T_o}{\gamma+1}} \qquad (3\text{-}8)$$

SHOCK WAVES

Theory

If at any time in the flow of a fluid in a channel there occurs a sudden change in the pressure, density, and velocity, there must exist some boundary condition to explain this disturbance. This boundary is referred to as a shock wave. There are several different types of shock waves, each with its own particular characteristics.

> "When the density is increased through a shock wave, it is called a compression wave. Such a wave may be of either the oblique or the normal type and may occur in either two- or three-dimensional flow, i.e., where the air can change direction in only one plane normal to the free-stream direction, as with a wedge, or in all planes, as with a cone. When the density is decreased, the change is gradual rather than sudden as in the compressive case, is referred to as an expansion wave, and is always of the oblique type. Because the change is gradual, it is not a shock wave and has no normal type corresponding to the compressive case."[1]

Normal Shock Wave

In the case of subsonic flow any disturbance in the channel, such as an airfoil, will communicate itself upstream and cause the air to part before it reaches the body. If, however, the flow of the fluid is supersonic, i.e., traveling faster than the pressure surges or impulses, it is not possible to warn the oncoming flow of the disturbance. As a result of this condition, the fluid begins to pack up on the object. As the fluid decreases its velocity, the density and pressure build up in a corresponding

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[1] E. Arthur Bonney, Engineering Supersonic Aerodynamics, p. 4.

amount. As the oncoming mass rate of flow overcomes this condition, a steady equilibrium is set up. The compressed region will extend ahead of the object, depending upon the initial velocity, temperature, and size of the object. The maximum angle this strong compression wave can obtain is normal to the direction of the flow. The velocity of the flow immediately downstream from a normal wave is always subsonic.

Mach Wave

The Mach wave is a weak oblique compression wave. The flow is supersonic in front of the wave and remains supersonic behind the wave, but at a slightly lower velocity.

A convenient method of describing how the wave is formed is by the use of water-wave analogy. Consider first the case of an object dropped into a pool of still water. The disturbance will send out small waves, at a certain velocity, which are concentric, as shown in Fig. 1.

Next, suppose that the object is moving through the water at some subsonic speed. The waves generated will appear as those shown in Fig. 2. The wave sent out from station 1 will always remain outside of the waves generated at the following stations. In other words, the fluid sends out pressure waves warning the surrounding fluid of the approaching body.

In the case of supersonic speed, (i.e., the body is moving at a rate faster than the speed of the pressure waves), the waves will look like those in Fig. 3. In this case, the individual waves combine along a common front. The portions of

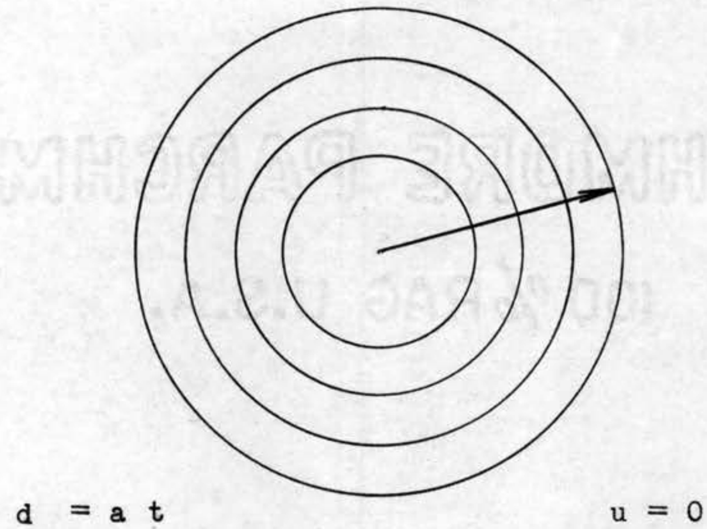

Fig. 1. Disturbance sent out by stationary object.

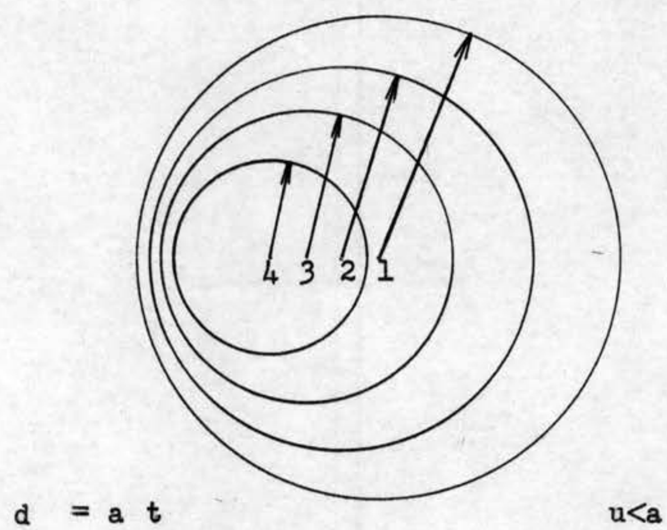

Fig. 2. Disturbances sent out by object moving at velocity less than speed of sound.

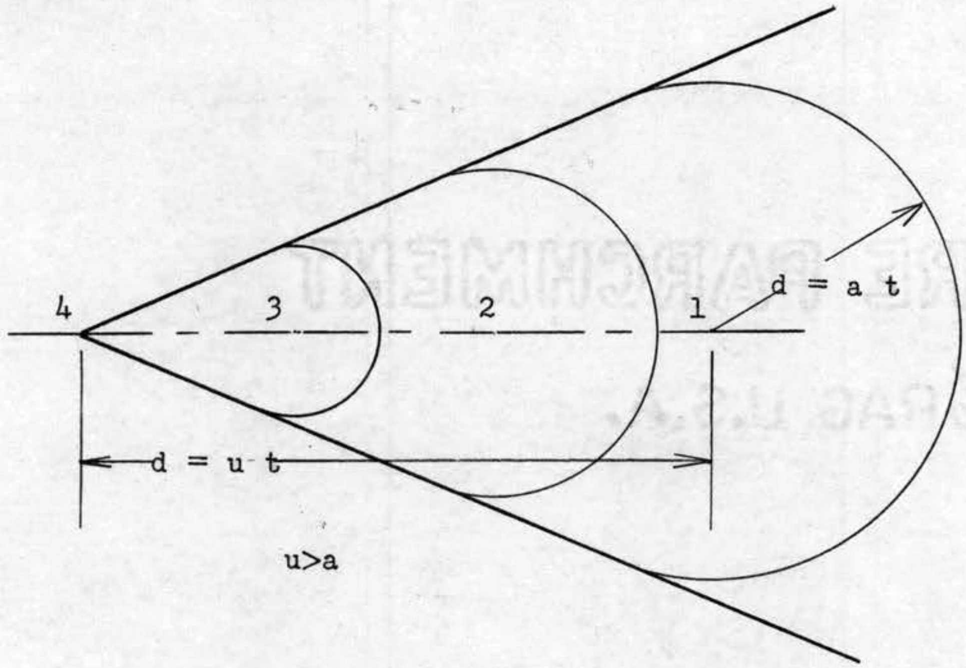

Fig. 3. Disturbance pattern of object moving at velocity greater than speed of sound.

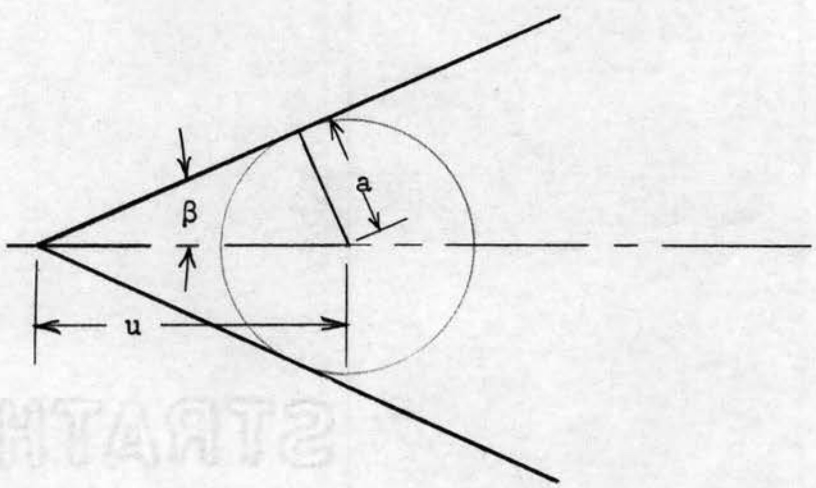

Fig. 4. The " Mach angle " .

the waves in front of the common line will lose their identity and be merged into the envelope created by the other wavelets. If the water waves are replaced by pressure waves in air, we can apply this theory to the motion of a body through the atmosphere.[2]

If the object discussed above is small and there is no change in the direction of the fluid motion, the disturbance created along the common tangent line is referred to as the "Mach line," or in the mathematical treatment of the problem, as the "characteristic line." The fluid moving across this line experiences no change in pressure, density, or temperature. It is for this reason that the Mach line is referred to as an infinitely weak shock wave.

The angle the Mach line makes with the free stream direction is called the Mach angle. The angle is defined by the relative velocities between the free stream velocity w and the speed of sound a by the following relation.

$$\beta = \sin^{-1} \frac{a}{w} \qquad (4\text{-}1)$$

Referring to Fig. 4, other trigonometric relations are

$$\cos \beta = \frac{\sqrt{w^2 - a^2}}{u^2} \qquad (4\text{-}2)$$

and

$$\tan \beta = \frac{a}{\sqrt{w^2 - a^2}} \qquad (4\text{-}3)$$

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[2] *Ibid.*, pp. 7-9.

If we combine these equations with the equation of the Mach number, they may be reduced to

$$\sin \beta = \frac{1}{M} \tag{4-4}$$

$$\cos \beta = \frac{\sqrt{M^2-1}}{M} \tag{4-5}$$

$$\tan \beta = \frac{1}{\sqrt{M^2-1}} \tag{4-6}$$

Expansion Wave

When the flow of a fluid traveling at supersonic speed turns a corner, i.e., when the wall along which the air is passing turns in a direction away from the air stream, an expansion wave is formed. Because the disturbance is so small, no compression occurs ahead of the turn. The air will remain unaffected until it crosses the line radiating from the disturbance (turn) at an angle $\beta = \sin^{-1} \frac{1}{M}$ to the free stream direction. When the flow intersects the Mach line it turns and follows a new path which is parallel to the new wall. The new Mach number will be greater than the original one. The new Mach angle β will be less than the original angle and will be measured from the direction of the new wall. If the corner is sharp, a series of Mach lines will originate and the process of turning the flow will be broken up into a series of small changes in direction.

From the above discussion, it can be seen that the expansion wave is neither a shock wave nor a single thin wave, as was the case of a compression shock wave. Therefore, the expansion is free from pressure losses.[3]

Condensation Shock Wave

Atmospheric air can, and does, hold a certain amount of moisture in suspension. The amount is dependent upon the temperature. As the temperature rises, the amount of moisture capable of being held in suspension will also rise. As the temperature drops, the moisture will form into droplets and produce fog, rain, snow, or hail, depending on other atmospheric conditions.

The point at which condensation occurs is called the saturation point of the vapor, and the partial pressure of the water vapor at this point is referred to as saturation vapor pressure p_s. The partial pressure of the unsaturated vapor is referred to as the vapor pressure p_v. The ratio of the vapor pressure to the saturation vapor pressure is called the relative humidity r.

Water vapor will condense immediately upon reaching the saturation vapor pressure, i.e., the relative humidity being equal to unity, if in the presence of a large body of water. If a large body of water is not present, the droplets will condense on particles such as dust or other foreign nuclei.

This information is of great importance in the design of a supersonic wind tunnel. The air, as it flows from the reservoir will be at some temperature T_0 and contain some water vapor.

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[3] Ibid., p. 20.

As it accelerates through the nozzle, the temperature will drop according to the isentropic law. Consequently, the saturation vapor pressure will drop very rapidly because it varies approximately exponentially with the temperature.[4] Therefore, the relative humidity will rise very rapidly as the temperature and pressure drop. As stated previously, when the relative humidity reaches unity, some type of condensation will begin. Once the condensation begins, it will continue with great rapidity, giving the occurrence the appearance of a sharp jump in the flow. This jump is referred to as a condensation shock wave. Some theoretical data, under certain assumptions, have been developed by Oswatitsch[5] showing that a supersaturated condition is possible before condensation begins. However, these assumptions lead to inaccurate results at high Mach numbers.

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[4] Hans Wolfgang Liepmann and Allen E. Puckett, Aerodynamics of a Compressible Fluid, p. 63.

[5] Ibid., p. 64.

PREVIOUS INVESTIGATION

Principal Types of Supersonic Wind Tunnels

In the past few years a great deal of time and money has been spent on different types of supersonic wind tunnels. The method and design is governed by the power available, size required, and testing to be done. At the present time, the tunnels fall into two major categories; the blow down, or intermittent - flow type, and the continuous - flow type.

Blowdown Supersonic Wind Tunnel

The blowdown (intermittent - flow) tunnel is the cheapest and simplest type of supersonic wind tunnel to design and to construct, still maintaining maximum test section size and Mach numbers. However, this type of tunnel can not be run continuously and therefore is not capable of handling tests which require long runs.

The general set-up is as follows: air is drawn through a drier, nozzle, test section, and into a large vacuum vessel. The pressure ratio that is obtained is directly a function of the amount of vacuum obtained in the vacuum vessel. The major advantage of this tunnel as compared to other types of blowdown tunnels is that the stagnation pressure, density, and temperature are fixed and will remain constant for all runs.

Another type of blowdown tunnel is like the preceding one with the addition of a pressure tank ahead of the drier. The advantage, of course, is that a higher pressure ratio is possible, allowing a higher obtainable Mach number. If an adjustable throttling valve is installed between the high pressure tank and the stilling chamber, a constant stagnation pressure will exist throughout the run.

The major advantages of the blowdown tunnels are their low construction and operating costs. The size of the power plant will depend upon the size of the vacuum vessel and the amount of vacuum desirable.

The disadvantages of this type of tunnel are: the duration of the test runs is short, which reduces the accuracy of the test because of insufficient amount of time to allow for the balancing system to stabilize; and since the time of each run is short, and the pressure, density, and temperature vary throughout the run, an expensive recording instrument is needed to take instantaneous readings if the test results are to be valid.[1]

Continuous - operation Supersonic Wind Tunnel

The continuous - operation tunnel may be either of two types: return or nonreturn. In the case of the nonreturn type of tunnel, the air is either drawn or pumped into a drier, depending upon the location of the power plant, then through the nozzle and test section, and expelled to the atmosphere. Because of the large quantity of air that is necessary, a large drier must be used. This type of tunnel is not very practical because of the large initial and operating expense.

The return type of tunnel consists of a drier, cooler, power supply, nozzle, test section, and diffuser, all linked

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[1] E. Arthur Bonney, Engineering Supersonic Aerodynamics, pp. 200-202.

together in a circuit. Axial flow compressors are the most common type of power to obtain large pressure ratios and mass flows.

Because of the large amount of heat in the air flow due to the air passing through the compressor, a cooling system is required to keep the temperature down to reasonable operating values. The maximum value of the temperature in the tunnel should not exceed 150°F.[2]

Comparison Between Blowdown and Continuous-operation Supersonic Wind Tunnels

The storage set-up is ideal for scientific research, and the continuous tunnel is better for routine testing to develop a given aircraft. The storage type of arrangement is safer than the propeller-driven type since breakage of the model does not also destroy the fan. It follows that little time is wasted on elaborate devices for the safety of the model in the tunnel driven by compressed air.

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[2] *Ibid.*, p. 203.

PRELIMINARY INVESTIGATION

The first method attempted to evaluate the possibilities of constructing a low cost supersonic wind tunnel was by the use of steam as a medium. A simply constructed nozzle was made by bolting two pieces of wood, which represented the top and bottom sides of the expanding section of the nozzle, onto a two inch diameter pipe. This method was preferred over a rigid construction because it enabled us to expand or diverge the section without much difficulty. The sides were made out of plexiglass, and were attached by the use of four clamps.

This arrangement was then hooked up to the steam line in the laboratory. The pressure in the main line was built up to 100 psi. Because the portion of the main steam line we tapped was not insulated, we obtained a very large quantity of condensation. This made it impossible to study the flow characteristics. However, it was possible to insert a large blunt body as a model into the flow and see the path the flow took around the object. Even though the moisture kept us from obtaining the desired results, the experiment was a success because we were able to obtain enough information from this crude construction to realize the possibilities of using steam as the medium.

The next step in the preliminary investigation was to design and build a nozzle of such a nature that the Mach number in the test section was 1.8. This Mach number was selected because we did not want to restrict the flow in the throat much for safety reasons. Also, the test section was kept to a minimum to eliminate the use of a large quantity of expensive glass.

A new piping arrangement, which included a by-pass line, was constructed. The reason for inserting a by-pass line was to eliminate water surges in the line. The arrangement was connected to the main steam line at a place where all of the lines, from the entrance to the building up to the connection of our nozzle, were insulated.

The tests were run after students had just finished using the laboratory. The reason for this was to be sure that all steam lines were hot and the moisture that collects in the lines was eliminated. The results of our tests showed two things. One, the water surges still existed when the valve was opened to the desired testing position. Two, the quality of the steam was found to be approximately 98 per cent. The desired degree of superheat was lost between the power plant and the building from which we were making the test. The only way to increase the quality of the steam to the desired operating range was to run the steam pipes through a boiler, an operation which would have resulted in undesirable expenses. Because of the existing conditions with which our limited budget could not deal, the idea of using steam as a medium flowing through the tunnel was abandoned.

The next method we attempted was to build an induction tunnel, still using steam. A pipe was attached to the expanding section of the nozzle and inserted into the flow of steam at right angles. The results were satisfactory at a very low mass rate of flow, but when the velocity of the steam was increased to the operating speed, the steam began to come out of the nozzle.

The conclusions drawn from this occurrence was that when the velocity of the flow reached the desired speed, a turbulent boundary condition was set up. When the flow reached the opening, instead of flowing past it, as it was expected to do, the boundary condition caused it to hug the wall and forced the steam to enter the induction pipe.

After considerable deliberation it was decided that, in any of the above mentioned methods, even if we did solve the necessary problems, the use of steam would not be practical. First, the initial cost of such items as insulated pipe, steam valves, super-heater, drier, and exhaust stack, would require the use of funds set aside for such items as the balance system and Schlieren optical apparatus. Second, the tunnel could not be operated when the other equipment, requiring the use of steam, in the laboratory was being used. Third, the tunnel could not be used in cold weather because of the overload it would throw on the power plant. Therefore, it was decided that air would be the most practical and the easiest to use.

The source of power which was used in our first investigation, using air as a medium, was the two stage reciprocating compressor, located in the Mechanical Engineering Laboratory of this Institute. The compressor was used to supply air to two forty-gallon tanks arranged in parallel. The pressure was built up to 120 psi and released into the nozzle, which was the same one we used in our investigation using steam as the medium. A sharp pointed wedge was used for a model. The results were very good and a shock wave was produced on the leading edge, plus a split tail shock wave on the trailing edge. However, it was observed that some of the oil from the compressor was suspended in the flow and a condensation shock wave soon formed, causing the compression waves to disappear. This condition was not considered to be serious because we realized that a filter and drier, which would be necessary to reduce the moisture content of the atmospheric air, would eliminate this occurrence.

The only remaining preliminary investigation left was to determine the optimum and ambient conditions. The temperature of the air leaving the storage tanks was found by allowing the compressor to run for fifteen minutes to obtain its operating temperature; then the by-pass valve to the tanks was closed and the pressure in the tanks was built up to 200 psi. The air was allowed to stand in the tanks for five minutes before being exhausted to the atmosphere through the outlet valve, which was located at the bottom of the tanks. A thermometer, which was shielded on three sides in order to obtain a stagnation temperature, was placed at the opening of the valve. This process was repeated several times and an average temperature was recorded as the reservoir temperature.

The length of the test runs was determined by assembling the tunnel and allowing the pressure in the storage tanks to diminish from 180 psi to 80 psi, which was the design pressure. It was found that it took approximately fifteen seconds for this pressure drop to occur. It was felt that this is a sufficient length of time for the balancing system to stabilize and was therefore accepted as the operating time of each run. However, this figure may be checked with great accuracy when the tunnel is completed and the balance system installed.

The final selection of the testing conditions and size of test section was based on initial cost, operating cost, and simplicity of construction. The desired Mach number was set at 3. The height of the test section selected was three-inches and the width was three fourths of an inch.

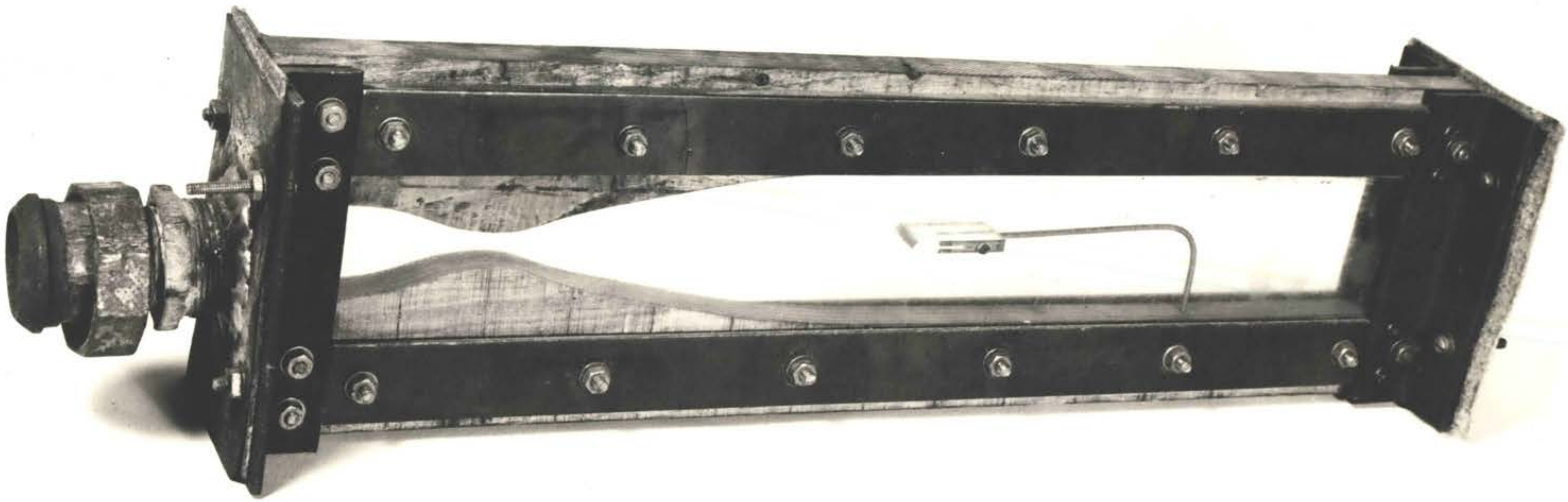

Fig. 5. Experimental Nozzle Mach. No. 3.

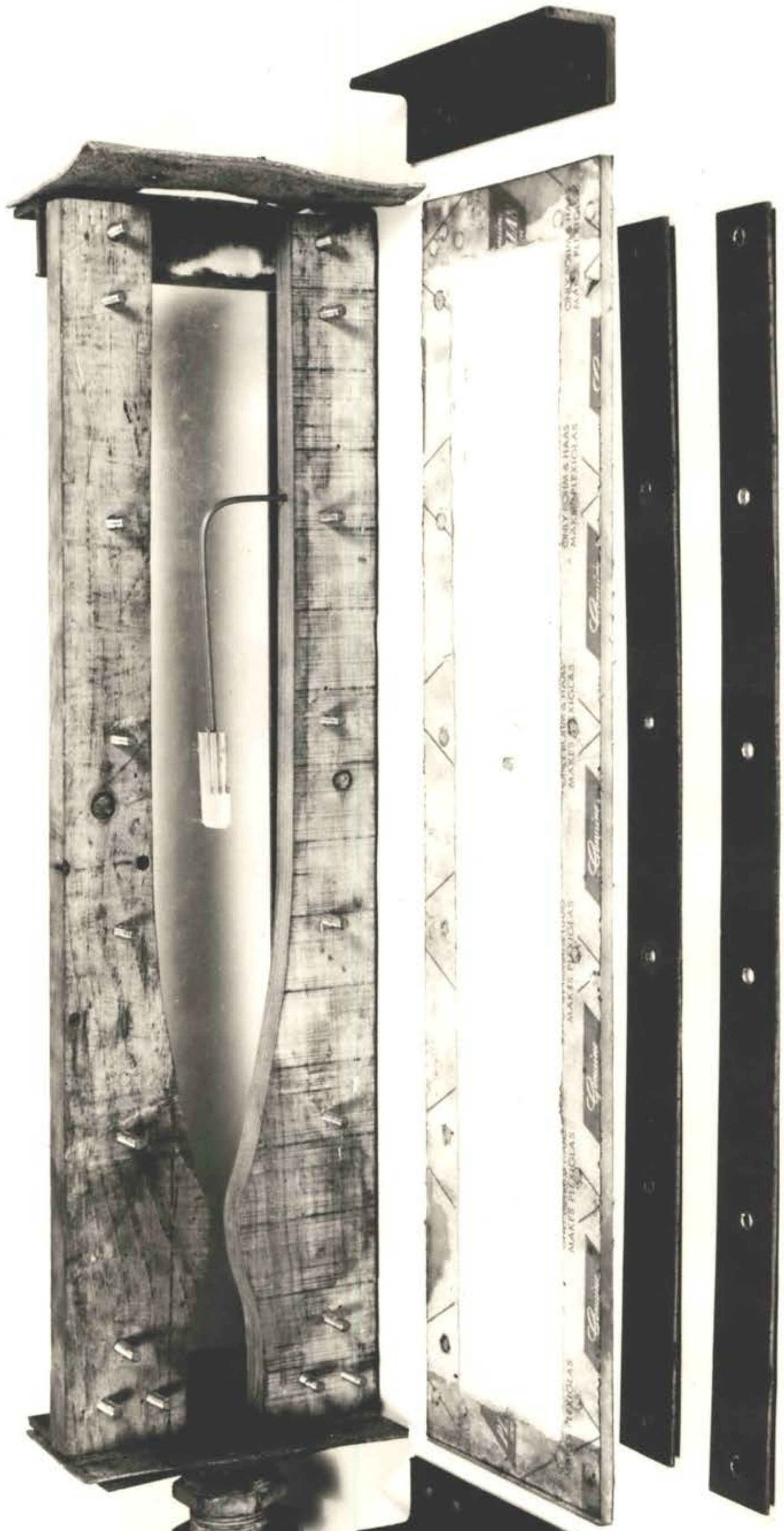

Fig. 6. Disassembled Experimental Nozzle.

APPARATUS AND EQUIPMENT

Compressor

The compressor used in this investigation has the following specifications:

Two stage, reciprocating

Water cooled

Serial No. 4724

Class 11 A T

Piston size 7 and 5 inches in diameter

Stroke 7 inches

Maximum RPM 400

Working Pressure 125 psi

Mfg. Pennsylvania Pump and Compressor Company,
Easton, Pennsylvania

Filter and Drier

The filter and drier selected for this design was the Adams aftercooler and cyclone separator. The aftercooler cools the air to within a few degrees of cooling water in order to condense the flow and to make certain that no further condensation can take place. The cyclone separator deposits all of the water and entrained oil in a special moisture chamber.

The reasons for selecting this dual purpose system instead of two separate units were: the simplicity with which the equipment can be installed, and the low amount of maintenance required during operation.

The actual size of the unit to be installed in the system should be obtained from the manufacturer after the desired range of operating Mach numbers has been determined.[1]

Storage Tank

The storage tank to be used in this design should hold approximately eighty gallons of fluid. The entrance and exit valves should be located near the top of the tank in order to eliminate the possibility of the mixing with the flow of foreign particles, which may collect in the bottom of the tank.

Piping

The piping which connects the storage tank to the nozzle should be copper tubing. Copper tubing was selected instead of some other type of material because copper tubing can be bent at right angles with a much larger radius, eliminating the turbulence which accompanies a sharp turn, which is characteristic of an elbow or tee used with rigid pipes.

Damping Screens

The flow characteristics of a supersonic wind tunnel should conform, as near as possible, with that of actual flight. In still air, (i.e., the conditions under which actual flights are performed) it has been found that there are no disturbances of sufficient quantity to produce appreciable aerodynamic effects. For this reason the design of the tunnel should include some means by which the turbulence can be reduced to a minimum.

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[1] R. P. Adams Co. Inc., Compressed Air Mag., April 1951, Vol. 56, No. 4.

The method to be used in reducing the turbulence to a minimum has been a subject of long study and investigation. As a result of these efforts, considerable information was obtained on the use of damping screens.

The use of damping screens produces a much higher turbulence immediately behind the screens as compared to that in front. However, if the mesh of the screen is fine, the scale of the turbulence is small and the eddies decay rapidly. Thus with a series of screens, each smaller than the preceding one, the turbulence can be reduced until the degree of accuracy desired is obtained.

The penalty for using these screens is a loss of energy equal to the pressure drop across the screen times the total volume of flow moving through the channel. From the above statement it can readily be seen that the screens should be located at a point where the velocity of the flow is low and the cross section large.

Data taken from an experiment made by Dryden and Schubauer[2] sugggested the use of four wire screens in our particular case. The size of the screens are 18 mesh, 20 mesh, 24 mesh, and 60 mesh, with wire diameters of 0.001 inch, 0.017 inch, 0.0075 inch, and 0.0007 inch respectively. The distance between each screen should be approximately six inches.[3]

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[2] Hugh L. Dryden and G. B. Schubauer, Jour. Aero. Sci., Vol. 14, 1947, p. 221.

[3] Dryden and Schubauer, loc. cit.

Transition Section

The transition section is located at the entrance of the nozzle and converts the flow from the circular pipe to the rectangular shape of the nozzle. This section is two inches long and has a 1.18 square inch rectangular cross section at the entrance and tapers down to a 0.834 square inch rectangular cross section at the exit.

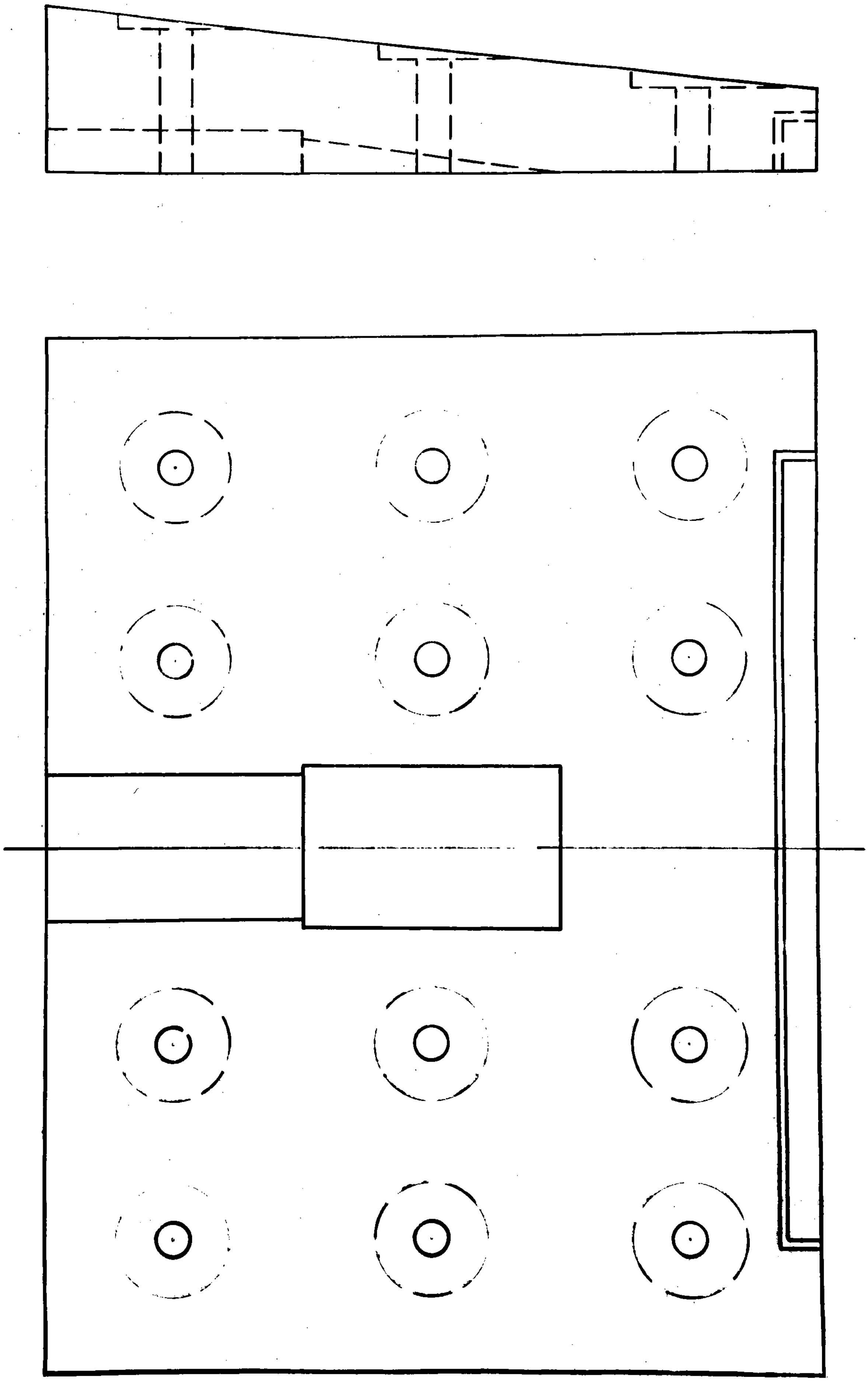

Fig. 7. Transition Section.

NOZZLE DESIGN

Calculations

The calculations used in this design were based on the formulas developed in Chapters 2 and 3. The design conditions were as follows: the Mach number in the test section should be 3, and the height and width of the test section should be 3 inches and 3/4 inches, respectively. The known conditions in the reservoir were: the pressure p_o was 94.21 psia, and the temperature T_o was 500°R. The remaining conditions in the reservoir to be calculated were: the density ρ_o from the universal gas laws is

$$\rho_o = p_o/RT_o$$

$$\rho_o = (94.21 \times 144)/(1714 \times 500)$$

$$\rho_o = 0.01582 \text{ slugs/ft}^3$$

the speed of sound a_o from equation (2-10)

$$a_o = 49.1 \sqrt{T_o}$$

$$a_o = 49.1 \sqrt{500}$$

$$a_o = 1097.9 \text{ ft/sec}$$

The solution for the conditions at the critical section (throat) is as follows: the area A* from equation (3-7)

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{5+M^2}{6}\right)^3$$

$$\frac{3 \times 3/4}{A^*} = \frac{1}{3}\left(\frac{5+3^2}{6}\right)^3$$

$$\frac{2.25}{A^*} = 4.234$$

$$A^* = 0.531 \text{ sq in.}$$

the pressure p^* from equation (3-3)

$$p^* = p_o\left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

$$p^* = 94.21\ (0.833)^{3.5}$$

$$p^* = 49.76 \text{ psia}$$

the density ρ^* from equation (3-4)

$$\rho^* = \rho_o\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)}$$

$$\rho^* = 0.01582\ (0.833)^{2.5}$$

$$\rho^* = 0.010034 \text{ slugs/ft}^3$$

the temperature T* from equation (3-2)

$$T^* = T_o\left(\frac{2}{\gamma+1}\right)$$

$$T^* = 500\ (0.833)$$

$$T^* = 416.667^\circ R$$

the speed of sound a* from equation (3-1)

$$a^* = a_o\sqrt{\frac{2}{(\gamma+1)}}$$

$$a^* = 1097.9\ \sqrt{0.833}$$

$$a^* = 1002.8 \text{ ft/sec}$$

The solution for the conditions at the exit, test section, of the nozzle is as follows:

the pressure p_1 from equation (2-20)

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

$$p_1 = p_o/(1 + 0.2M^2)^{3.5}$$

$$p_1 = 94.21/2.8^{3.5}$$

$$p_1 = 2.564 \text{ psia}$$

the density ρ_1 from equation (2-21)

$$\frac{\rho_o}{\rho_1} = (1 + \frac{\gamma-1}{2}M^2)^{1/(\gamma-1)}$$

$$\rho_1 = \rho_o/(1 + 0.2M^2)^{2.5}$$

$$\rho_1 = 0.01582/2.8^{2.5}$$

$$\rho_1 = 0.001206 \text{ slugs/ft}^3$$

the temperature T_1 from equation (2-17)

$$\frac{T_o}{T_1} = 1 + \frac{\gamma-1}{2}M^2$$

$$T_1 = T_o/1 + 0.2M^2$$

$$T_1 = 500/2.8$$

$$T_1 = 178.57^{\circ}R$$

the speed of sound a_1 from equation (2-10)

$$a_1 = 49.1 \sqrt{T_1}$$

$$a_1 = 49.1 \sqrt{178.57}$$

$$a_1 = 656.12 \text{ ft/sec}$$

the velocity u from equation (2-14)

$$M = u/a$$

$$u = Ma$$

$$u = 3 \times 656.12$$

$$u = 1968.36 \text{ ft/sec}$$

The mass rate of flow in the throat m from equation (3-10)

$$m = 76.6 \frac{p_o A^*}{\sqrt{T_o}}$$

$$m = 76.6 \times \frac{94.21 \times 0.531}{144 \sqrt{500}}$$

$$m = 1.19 \# /\text{sec}$$

checking at the test section by the use of the continuity equation, $m = \rho_1 u_1 A_1$, we obtain:

$$m = \frac{0.001206 \times 1968.36 \times 2.25 \times 32.2}{144}$$

$$m = 1.19 \# /\text{sec}$$

Contour of Nozzle

The contour of the nozzle may be broken up into two sections: the converging section, and the diverging section. The contour of the converging section of the nozzle does not have to conform to some definite pattern because the flow is at all times subsonic in this region. However, to maintain a uniform flow across the throat, the converging section must converge at a sufficiently slow rate.[1]

The contour of the diverging portion must produce uniform, parallel, and shock free flow at the test section at the desired Mach number. The method used in this thesis to obtain the conditions above is the method of characteristics. This method was developed by Prandtl and Busemann and has been put into an applicable form by Leipmann and Puckett.[2]

Method of Characteristics

The method of characteristics is a step by step summation of the straight line segments which make up the contour of the divergent section of the nozzle. The accuracy of this method will depend upon the number of segments used. The advantage of this method over the mathematical methods is that the boundary condition can be solved for readily, whereas in the mathematical methods it is extremely difficult.

From Chapter 4 we noted that the simplest flow field satisfying the equations of motion, outside of the uniform parallel flow, was the flow through a single Mach wave. It was also seen

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[1] Tsien, Jour. Aero. Sci., Vol. 10, p. 68, 1943.

[2] Hans Wolfgang Liepmann and Allen E. Puckett, Aerodynamics of a Compressible Fluid, pp. 211-214.

that the change in pressure, density, and temperature through this wave was so small that the entropy change was essentially zero. In other words, the only change which was of any importance

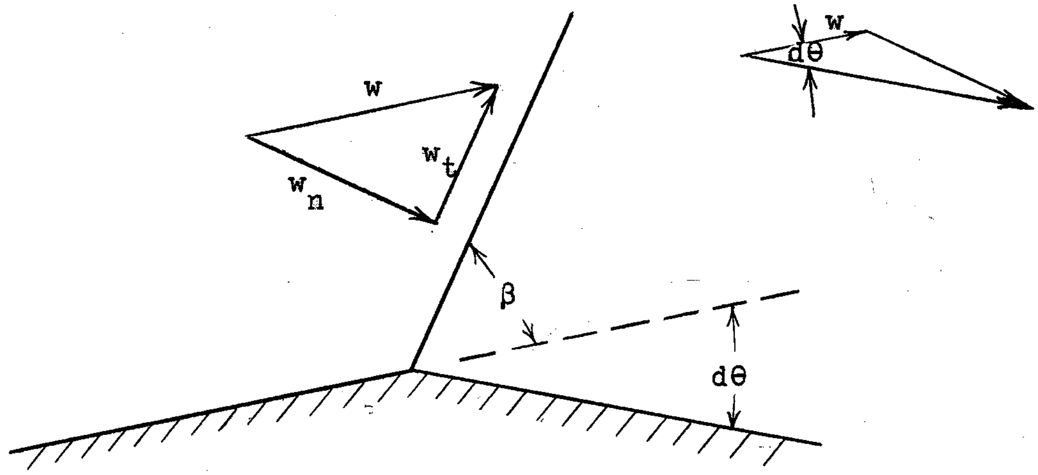

Fig. 8. Weak Expansion Wave.

was that of the change in the magnitude of the absolute velocity w . From Fig. 8 it can be seen that $w_n = w \sin \beta$. Therefore,

$$\sin \beta = \frac{a}{w} = \frac{1}{M} \tag{7-1}$$

The increase in the absolute velocity w will be

$$dw = dw_n \sin \beta \tag{7-2}$$

The change in flow direction is

$$d\theta = \frac{dw_n \cos \beta}{w} \tag{7-3}$$

If we combine equations (7-2) and (7-3), we can obtain a relation between the increase in speed and the flow direction.

$$\frac{dw}{w} = \frac{\sin \beta \, d\theta}{\cos \beta} \qquad (7\text{-}4)$$

Since the wave angle β is related to the Mach number by equation (7-1), we may write the above equation to read

$$\frac{dw}{w} = \frac{1}{\sqrt{M^2-1}} \, d\theta \qquad (7\text{-}5)$$

Equation (7-5) represents the change in flow direction between the two flow fields which are spearated by the Mach line at the angle β. The change in speed, which causes a change in the Mach number, is related to the Mach number by equation (2-16)

$$\left(\frac{w}{a^*}\right)^2 = \frac{\gamma + 1}{(\gamma-1) + \frac{2}{M^2}} \qquad (2\text{-}16)$$

For convenience, in the following computations we will use the ratio w/a^* instead of the absolute speed. This ratio will be denoted as $\bar{w}$

$$\bar{w} = \frac{w}{a^*}$$

If we solve equation (2-16) for M^2 and substitute in the new ratio, we obtain

$$M^2 = \frac{2\,\bar{w}^2}{(\gamma+1) - (\gamma-1)\,\bar{w}^2}$$

We can now substitute the above equation in equation (7-5) and get

$$d\theta = \sqrt{\frac{\bar{w}^2 - 1}{1 - \frac{\gamma-1}{\gamma+1}\bar{w}^2}} \cdot \frac{d\bar{w}}{\bar{w}} \tag{7-6}$$

From the above equation it is apparent that a second change in flow direction may occur through a second Mach wave. The velocity change being that in equation (7-6). It follows, from the above statement, that a flow field can be constructed with a series of changes in the flow direction.

If we apply the above method to the flow around a curved wall, we can integrate equation (7-6) and obtain

$$\int_{\theta_1}^{\theta} d\theta = \int_{\bar{w}_1}^{\bar{w}} \sqrt{\frac{\bar{w}^2 - 1}{1 - \frac{\gamma-1}{\gamma+1}\bar{w}^2}} \cdot \frac{d\bar{w}}{\bar{w}} \tag{7-7}$$

In this manner the speed change through each wave and the angle of each wave may be easily computed. If we relate the flow angle and velocity by referring the absolute velocity w to the critical speed of sound a^* the maximum angle of turning derived by Liepmann and Puckett[3] will be

$$\theta = \frac{1}{2}\sqrt{\frac{\gamma+1}{\gamma-1}}\left\{\frac{\pi}{2} - \sin^{-1}\left[\gamma - (\gamma-1)\left(\frac{w}{a^*}\right)^2\right]\right\} - \frac{1}{2}\left\{\frac{\pi}{2} + \sin^{-1}\left[\gamma - (\gamma-1)\left(\frac{a^*}{w}\right)^2\right]\right\} \tag{7-8}$$

[3] Ibid., p. 214.

If we plot this relation, we will get the polar diagram shown in Fig. 9. The velocity $\overline{w}$ is measured radially from the center, and the angles of the flow direction are measured as angles about this center. This curve is called the characteristic curve for the flow.

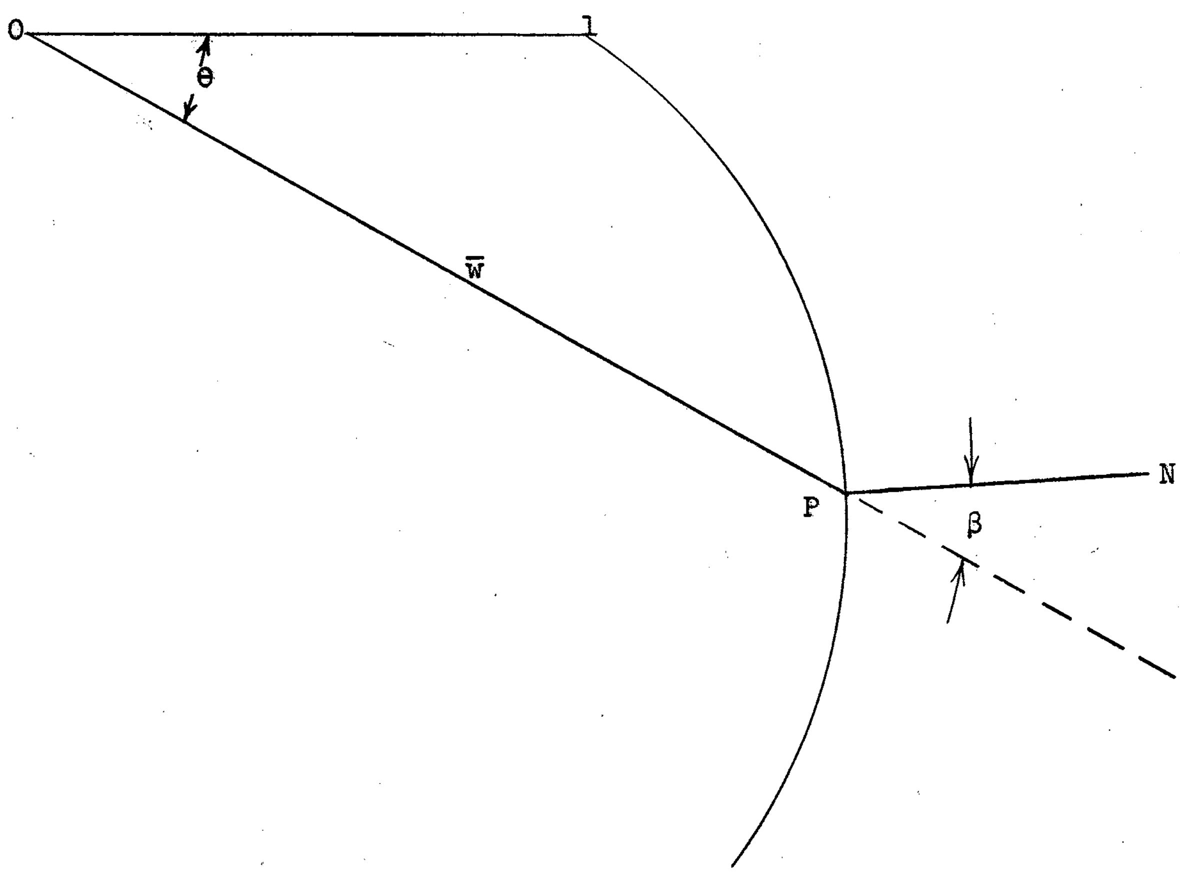

Fig. 9. Hodograph Characteristic

If we construct PN, Fig. 9, normal to the characteristic curve and let β be the angle between this normal and velocity vector, we can write

$$\tan \beta = \frac{1}{\sqrt{M^2-1}}$$

or

$$\sin \beta = \frac{1}{M}$$

This leads to the conclusion that β is the Mach angle for the flow at that point. Since the vector OP was parallel to the velocity vector in the physical plane, PN, which is normal to the hodograph characteristic curve, must coincide with the Mach line in the physical plane. These formulas and derivations were taken from Leipmann and Puckett.[4]

We may now interpret this occurrence to the flow about a single wall by taking a step-by-step construction. However, it should be noted that no restriction was placed on the direction of the wall. Therefore, if we assume that the wall may either concave upward or downward, two characteristic curves may be drawn in the hodograph diagram. The curves will be symmetrical about the $\theta = 0$ axis. If we allow these two curves to be rotated through some small angle, we may complete the characteristic diagram, as shown in Fig. 10.

If we apply the above theory to the flow between two symmet-

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[4] _Ibid._, pp. 214-216

rical concave walls, we may solve the flow conditions for the divergent portion of a supersonic nozzle. Fig. 11 shows the flow pattern between two walls. The initial uniform flow, which is at some supersonic speed u is represented by the single point F_1 in the hodograph plane. This point is shown in Fig. 12. If we compute the flow in region F_2, along the segment P_1P_2, just as we did in the case of the flow along a single wall, we will note that these two flow regions are separated by a single Mach wave, across which a velocity change occurs. This change in velocity can be computed by equation (7-5). This corresponds to the movement from the point F_1 to F_2 in the characteristic diagram. A line drawn from O to F_2 will be parallel to the segment P_1P_2. Therefore, if a line is constructed normal to the line F_1F_2 in the characteristic diagram, we will have the direction of the Mach wave, M_1. A similar process can be repeated to obtain the flow in region F_2', represented by F_2' in Fig. 12. From the discussion in Chapter 4 it can be noted that the waves M_1 and M_1' are expansion waves. For the sake of accuracy and convenience, the design of the nozzle used in this investigation was constructed six times the actual size. The straight line segments were turned two degrees every three-fourths of an inch. Therefore, these waves may rightly be designated as two degree expansion waves.

Now let us investigate the flow in the region F_3. This flow field is brought into existence by the extension of the Mach waves M_1 and M_1'. The jump from the region F_2 can satisfy the dynamic equations only if the jump lies on the characteristic through F_2. These characteristic curves passing through the point F_2 represent, graphically, equation (7-5). Similarly, a jump from region F_2' must lie along its corresponding curves. If we represent this new field as F_3, we have found the intersection of the two corresponding curves in the hodograph plane. From this point we have found the flow solution satisfying the necessary equations, with the new Mach waves M_2 and M_2' being normal to the segments F_2F_3 and $F_2'F_3$, respectively, in the hodograph plane.

The above two types of flow solutions - the wave originating from a bend in the wall and the intersection of two waves in the flow - can be applied to solve the flow conditions until we have an intersection of a wave with the wall. Such is the case at the point P_4 in Fig. 11. The field F_7 was separated from the fields F_5 and F_6 by the waves F_5F_7 and F_6F_7 as described in the above method. The flow in field F_6, by definition, is parallel to the wall segment P_3P_4. The field F_7 is separated from the field F_8 by the wave F_7F_8. Since the boundary condition requires that the flow in the field F_8 must be parallel to the wall, the point F_8 must lie along the extension of OF_6. Thus we have determined the flow in the field F_8 and also the wave F_7F_8. We have now covered all of the requirements to construct the flow pattern in the divergent portion of a supersonic nozzle.

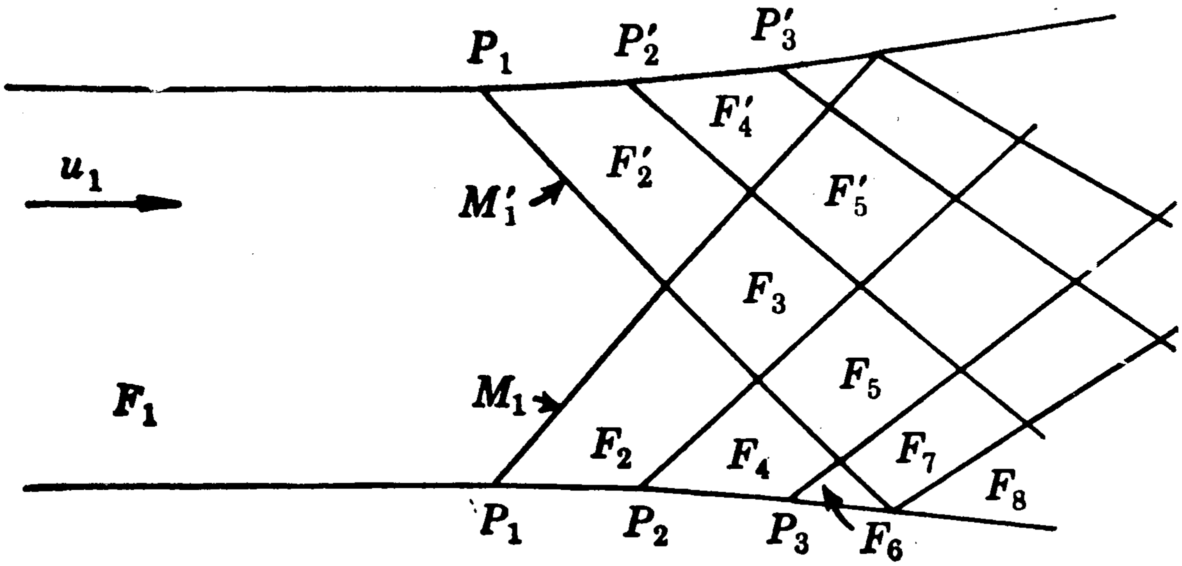


Fig. 11. Flow Between Two Walls.

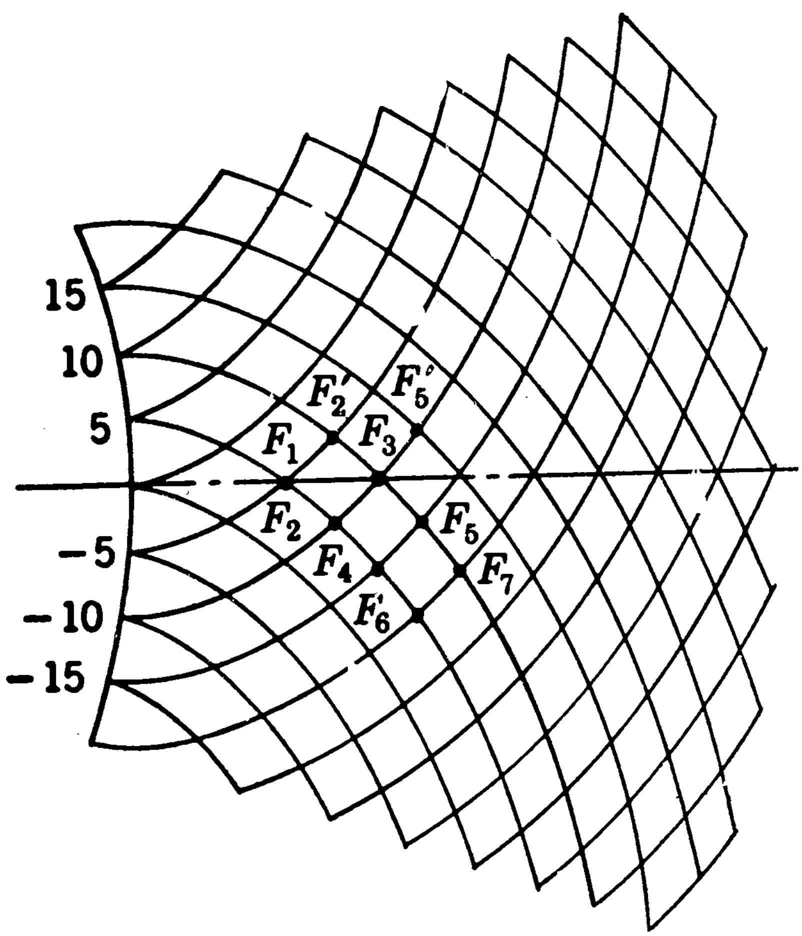
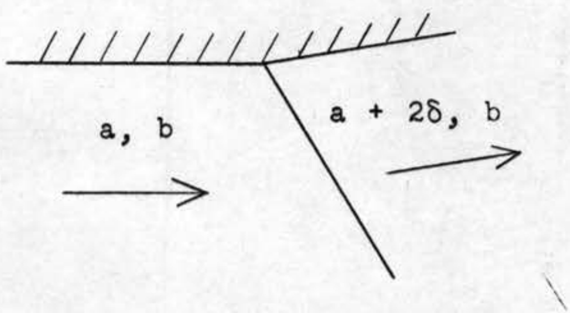


Fig. 12. Hodograph of Flow Between Two Walls.

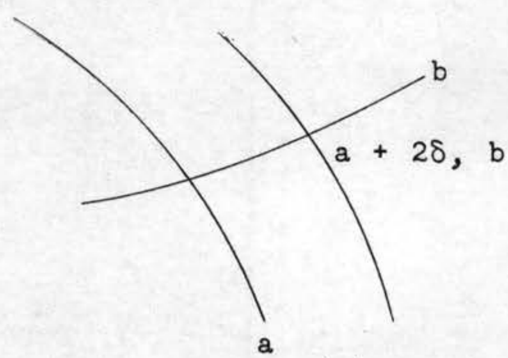
Liepmann and Puckett, op. cit.

In order to aid computations, a coordinate system was adopted for the characteristics in the hodograph plane by Liepmann and Puckett.[5] The use of this system will be introduced here and used for the remaining investigation. We will let each characteristic curve, of either family, be designated by a number, representing the angle, in degrees, from its origin on the $M = 1$ circle in the hodograph plane and the horizontal axis. Therefore, each point in the hodograph plane will have a number pair, say (a,b). The first number will represent the curves expanding downward, and the second will represent the curves expanding upward.

It is now possible to summarize the flow solutions in schematic form. Fig. 13a shows the Mach wave originating from a deflection δ in the wall. Fig. 13b shows the corresponding jump in the



(a)



(b)

Fig. 13. Origin of Expansion Wave.

characteristic diagram. Fig. 14a shows two intersecting waves and Fig. 14b shows the corresponding jump in the hodograph plane. The velocity change through this intersection is 4δ and the

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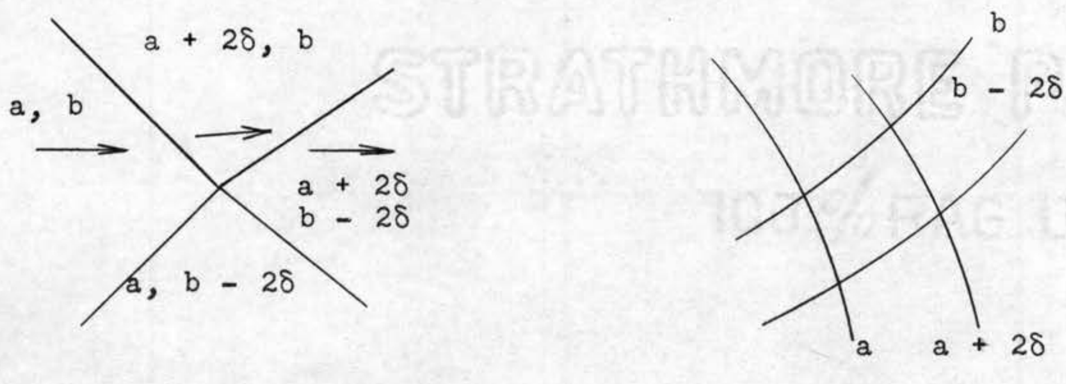
[5] Ibid., p. 221.

change in direction θ is zero.

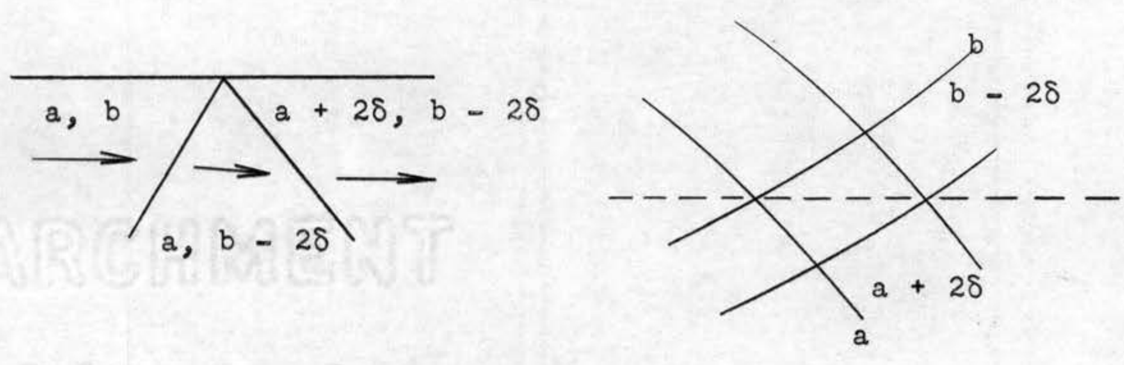
Fig. 15a shows the reflection of a wave from a straight wall, i.e., the reflection of a wave from the center line of a symmetrical



(a) (b)

Fig. 14. Intersection of Expansion Waves

nozzle. It should be noted that the final change in direction is zero, which satisfies the boundary conditions. However, the velocity of the flow has been accelerated.



(a) (b)

Fig. 15. Reflection of Expansion Wave From a Solid Boundary.

The only remaining problem in the solution of the flow between the two diverging walls of this nozzle is how to cancel these rebounding waves in order to have a shock free test section. It has been demonstrated that the expansion waves will continue to rebound off of the walls until they reach a bend in the wall, which is in the same direction and of the same magnitude as the flow deflection produced by the waves. We might think of this occurrence as the cancellation of the reflection of an expansion wave by the production of a compression wave of equal and opposite magnitude. Since all of the expansion waves were created by a two degree turning of the wall, the cancellation can also be performed by turning the wall two degrees in the opposite direction at the point of contact of one of these waves. From Fig. 11 it can be seen that ten expansion waves originated in the flow, therefore, ten reflected waves had to be canceled by this method. At the cancellation of the last rebounding expansion wave, the wall again became parallel to the center line of the nozzle. Therefore, we have completed the requirements for a uniform, parallel flow, shock free cross section at the beginning of the test section.

<u>Test Section</u>

"The test section must be effectively a constant area section. This means that the walls must actually diverge slightly to offset the effect of boundary layer growth."[6] Although the authors are aware of this occurrence, the walls of the test section have been designed with a uniform cross section. The test section was de-

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[6] *Ibid.*, p. 82.

signed in this manner because information on the exact amount of divergent is, at the best, empirical. The correction for the boundary layer in this design will be made after the tunnel is tested and the thickness of the actual boundary layer growth can be measured.

The length of the test section was selected as six inches, and as mentioned before, the height and width are three inches and three-fourths, respectively.

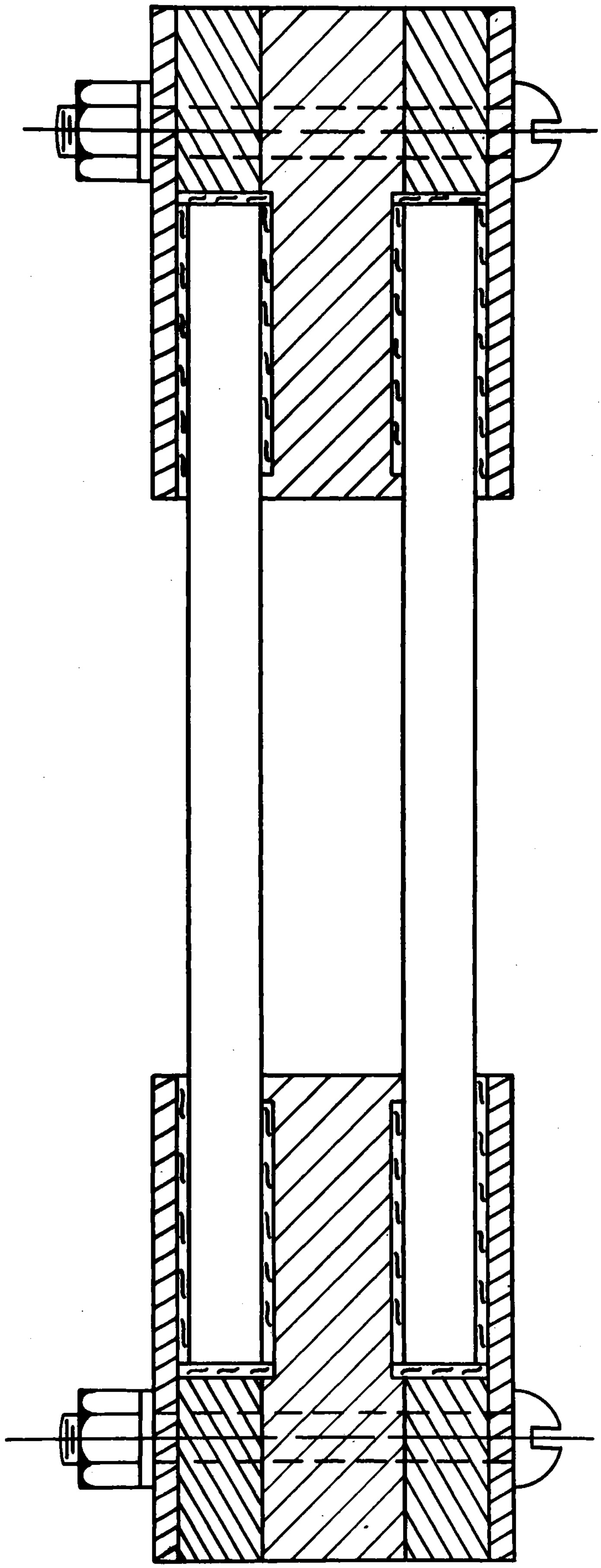


Fig. 16. Cross-section View of Test Section.

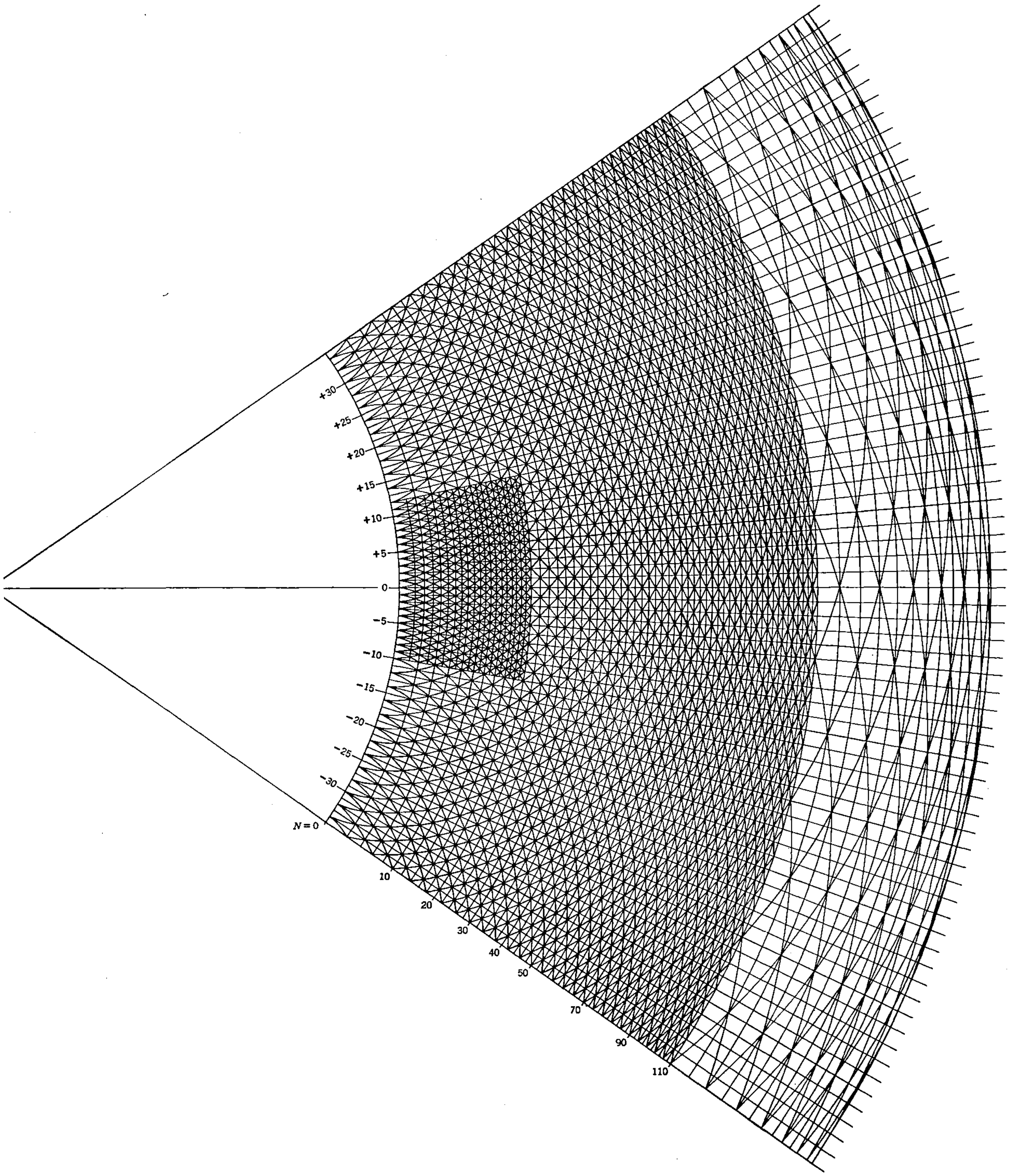


Fig. 10. Complete Characteristic Curve.

Liepmann and Puckett, Op. cit.

SUMMARY

The design of this supersonic wind tunnel was developed for the School of Mechanical Engineering in order to aid students who are interested in the field of supersonic wind tunnel operation and testing. The design has all of the paramount features, yet has kept the initial cost at a minimum.

The source of compressed air, which is to be used as the medium, will be supplied by the two-stage reciprocating compressor, located in the Mechanical Engineering Laboratory. The compressor will pump the air through a combination filter and drier to a storage tank. The storage tank will hold approximately eighty gallons of fluid at a maximum pressure of 200 psig. Although the nozzle was designed to operate at a pressure of 80 psia, the pressure in the tank will be built up to 180 psig and allowed to diminish to the design pressure. This will allow sufficient time for the balancing system to become stabilized.

Before the flow reaches the nozzle it will pass through a series of four damping screens. The sizes of these screens are 18 mesh, 20 mesh, 24 mesh, and 60 mesh respectively. The screens are placed six inches apart. The function of the damping screens is to reduce the scale of the turbulence and leave small eddies, which will decay rapidly.

As the flow enters the nozzle it passes through a two inch transition section. The transition section converts the flow from the circular section of the pipe to the rectangular section of the nozzle.

The design of the nozzle was constructed from the development of the method of characteristics by Liepmann and Puckett. The design criterion, which was satisfied, was to have a Mach number of three in the three by three-fourths inch test section. The mass rate of flow is 1.19 lbs per sec and the operating time of each run is approximately nineteen seconds.

SUGGESTIONS

In order to visualize the flow characteristics in the test section of this supersonic wind tunnel, it is recommended by the authors of this thesis that a Schlieren optical system should be incorporated.

The type of balancing system that would be the most accurate and practical in this wind tunnel is one employing the use of strain gages. The strain gages, which consist of fine wire, would be firmly attached to the surface of the model. As the model deflects due to the force of the oncoming air, the amount of deflection could be recorded and interpreted in terms of lift, drag, and moment.

Another design feature that would prove to be very beneficial is a constant pressure metering valve, which would be connected to the exit of the storage tank. This would permit the reading of each instrument individually instead of simultaneously, which is the requirement if the flow pressure is diminishing at all times.

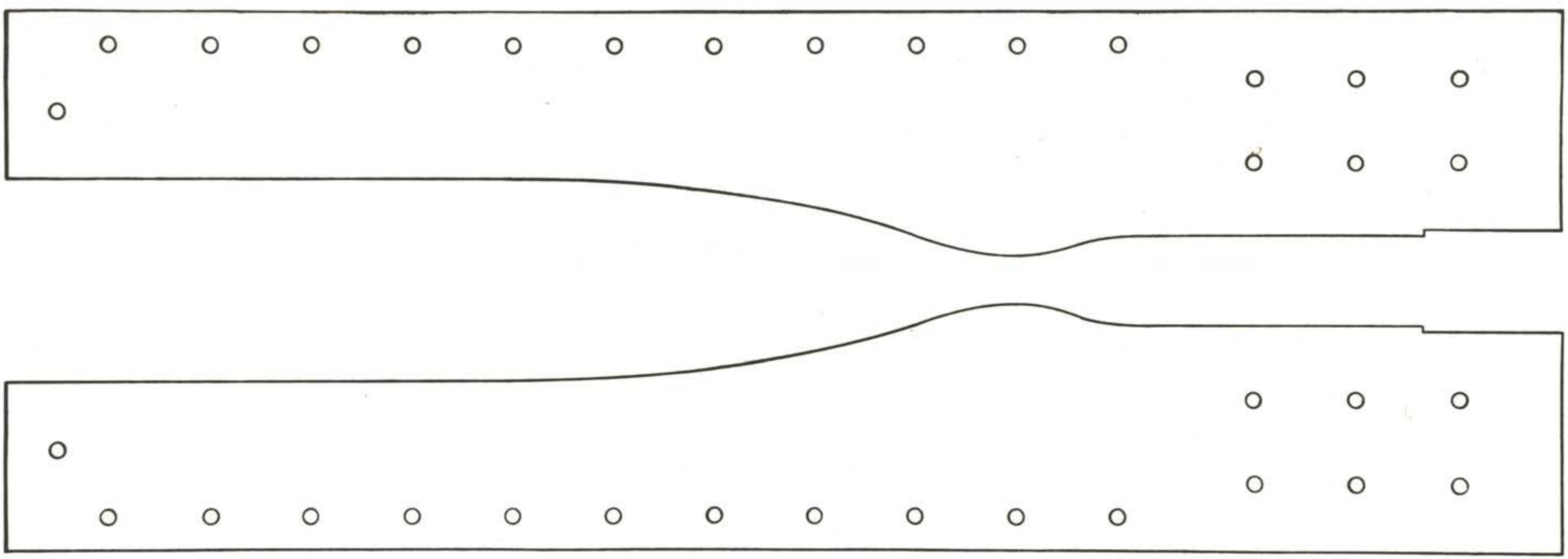


Fig. 17. Nozzle Design Omitting Shock Waves.

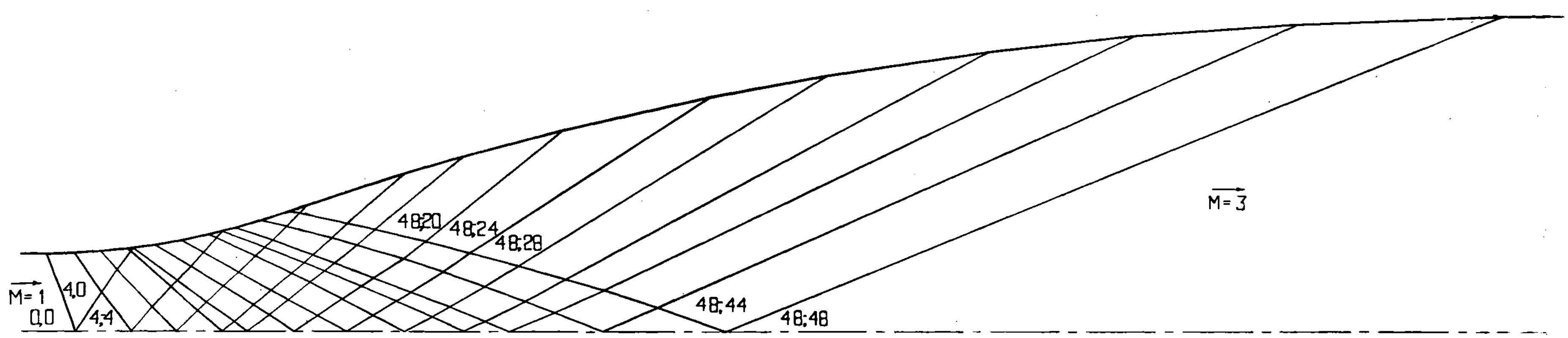
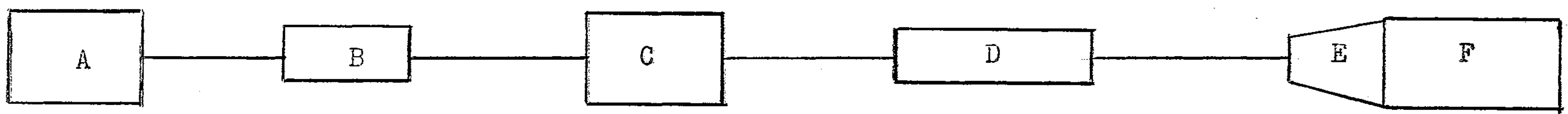


Fig. 18. Nozzle Design With Shock-Waves.



A --- Compressor

B --- Drier and Filter

C --- Tank

D --- Damping Screen

E --- Transition Section

F --- Nozzle

Fig. 19. Schematic Drawing of Supersonic Wind Tunnel.

Fig. 20. Nozzle Contour Dimensions

The ordinates listed below are measured from the center line of the nozzle to either the top or bottom curved surface. The stations are located at intervals at 1/8" along the abscissa, center line, of the nozzle.

Station	Ordinate	Station	Ordinate
1	0.6875	21	0.6250
2	0.6875	22	0.6250
3	0.6875	23	0.6250
4	0.6875	24	0.6250
5	0.6875	25	0.6250
6	0.6875	26	0.6250
7	0.6875	27	0.6250
8	0.6875	28	0.6250
9	0.6875	29	0.6250
10	0.6875	30	0.6250
11	0.6875	31	0.6250
12	0.6875	32	0.6250
13	0.6875	33	0.6250
14	0.6875	34	0.6250
15	0.6875	35	0.6250
16	0.6875	36	0.6250
17	0.6875	37	0.6250
17	0.6250	38	0.6250
18	0.6250	39	0.6250
19	0.6250	40	0.6250
20	0.6250	41	0.6250

Station	Ordinate	Station	Ordinate
42	0.6250	69	0.3650
43	0.6250	70	0.3710
44	0.6250	71	0.4000
45	0.6250	72	0.4291
46	0.6250	73	0.4583
47	0.6250	74	0.4900
48	0.6250	75	0.5258
49	0.6250	76	0.5650
50	0.6208	77	0.6083
51	0.6166	78	0.6500
52	0.6083	79	0.6900
53	0.6000	80	0.7300
54	0.5833	81	0.7633
55	0.5583	82	0.8050
56	0.5333	83	0.8400
57	0.5083	84	0.8733
58	0.4833	85	0.9066
59	0.4500	86	0.9400
60	0.4250	87	0.9666
61	0.4000	88	0.9958
62	0.3750	89	1.0250
63	0.3660	90	1.0540
64	0.3583	91	1.0830
65	0.3560	92	1.1310
66	0.3540	93	1.1560
67	0.3550	94	1.1790
68	0.3590	95	1.2030

Station	Ordinate	Station	Ordinate
96	1.2230	123	1.5000
97	1.2410	124	1.5000
98	1.2600	125	1.5000
99	1.2790	126	1.5000
100	1.2960	127	1.5000
101	1.3160	128	1.5000
102	1.3330	129	1.5000
103	1.3450	130	1.5000
104	1.3610	131	1.5000
105	1.3750	132	1.5000
106	1.3870	133	1.5000
107	1.4040	134	1.5000
108	1.4120	135	1.5000
109	1.4210	136	1.5000
110	1.4310	137	1.5000
111	1.4380	138	1.5000
112	1.4460	139	1.5000
113	1.4550	140	1.5000
114	1.4650	141	1.5000
115	1.4690	142	1.5000
116	1.4720	143	1.5000
117	1.4780	144	1.5000
118	1.4820	145	1.5000
119	1.4860	146	1.5000
120	1.4910	147	1.5000
121	1.4960	148	1.5000
122	1.5000	149	1.5000

Station	Ordinate	Station	Ordinate
150	1.5000		
151	1.5000		
152	1.5000		
153	1.5000		
154	1.5000		
155	1.5000		
156	1.5000		
157	1.5000		
158	1.5000		
159	1.5000		
160	1.5000		
161	1.5000		
162	1.5000		
163	1.5000		
164	1.5000		
165	1.5000		
166	1.5000		
167	1.5000		
168	1.5000		
169	1.5000		
170	1.5000		
171	1.5000		
172	1.5000		
173	1.5000		

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NAME OF AUTHORS: MYLES BARTON MITCHELL

and

JOHN VINCENT BIGGERS

THESIS ADVISER: PROFESSOR L. J. FILA

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