SYNTHESIS OF A GEARED SPHERICAL
FIVE-LINK MECHANISM

By

L. MICHAEL MCKEE

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

1973

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
MASTER OF SCIENCE
July, 1975

SYNTHESIS OF A GEARED SPHERICAL

FIVE-LINK MECHANISM

Thesis Approved:

Thesis Adviser

Dean of the Graduate College

PREFACE

In this thesis, the analysis and synthesis equations (Rigid Body Guidance, Path Point Generation and Function Generation) for a Geared Spherical Five-Link Mechanism are derived. Generalized solutions were formed for computer solution. Graphical results are presented for an analysis solution, and computer results are given for the synthesis problems.

I would like to express my gratitude to my advisor Dr. A. H. Soni, for providing opportunities to grow in the area of mechanisms science and for providing the encouragement to complete this work. I am obliged to an active group of "mechanisms researchers" for their stimulating discussions and friendship. My sincerest thanks are extended to Professor L. E. Torfason, "The Mechanisms Man," and to Dr. Dilip Kohli for their continued encouragement, assistance, and friendship. Also, I would like to thank the other mechanisms men, Mr. Brad Grant, Mr. Jack Lee, Mr. Siddhanty, and Mr. John Vadasz.

I would like to express very special thanks to my parents for their encouragement and assistance and to my wife, Vici, for her patience and encouragement.

TABLE OF CONTENTS

Chapter Page

LIST OF FIGURES

NOMENCLATURE

α_1	twist angle of input link MA
α_2	twist angle of input link AB
α_3	twist angle of coupler link BC
α_4	twist angle of output link CQ
α_5	twist angle of ground link MQ
θ_2	displacement angle of input link MA
θ_3	displacement angle of input link AB
θ_4	displacement angle of coupler link BC
θ_5	displacement angle of output link CQ
G_1	gear fixed to the ground link MQ
G_2	gear fixed to input link AB
R_1	radius of G_1
R_2	radius of G_2
N	gear ratio
β	initial displacement angle of input link AB

CHAPTER I

INTRODUCTION

Industrial linkage problems are planar, spherical, and spatial. Beyer (1) states that spherical mechanisms are just as important in machine design as planer mechanisms. This indicates that the majority of industrial linkage problems can be satisfied by planar or spherical mechanisms. Synthesis of planer linkages has reached a high level of sophistication and completeness. However, the development of synthesis procedures for spherical mechanisms is incomplete. The objectives of the present study is to develop closed form equations for the analysis and synthesis of a geared spherical five-link mechanism. This will complete to a large extent the synthesis problems for spherical mechanisms.

A number of studies have been made on the analysis and synthesis of spherical mechanisms. Soni (2) developed the design procedures for a spherical drag-link (four bar) mechanism. Suh (3) synthesized the spherical four-link mechanism with the use of the displacement matrix. Spherical six link mechanisms were synthesized for path generation by Hamid (4). And Kohli (5) designed spherical four-link and six-link mechanisms for multiple separated positions of a rigid body. Other significant contributions in the designing of spherical mechanisms have been made by Huang (6), Hartenburg and Denavit (7) and Yang (8,9).

Displacement, velocity and acceleration analyses are considered. The synthesis problems included in the present study are:

1. Rigid Body Guidance and Coordination of Input

2. Path Point Generation and Coordination of Input

3. Function Generation

Chapter II presents a description of the geared spherical five-link mechanism, and, the development of analysis is given. Chapter III presents the synthesis procedures for the mechanism. Finally, in Chapter IV a summary of this study is given.

CHAPTER II

KINEMATIC ANALYSIS OF A GEARED SPHERICAL FIVE-LINK MECHANISM

2.1 Introduction

A geared spherical five-link mechanis is shown in Figure 1.

Where M, A, B, C, and Q are points on the center of the revolute pairs. The vectors $\overline{M}$, $\overline{A}$, $\overline{B}$, $\overline{C}$, and $\overline{Q}$ are unit vectors passing from the center of the sphere to the respective points, M, A, B, C, and Q. These vectors are the axes of rotation of the revolute pairs.

These vectors are also labeled as the kink-links of the spherical mechanism. The twist angles α_i's are the angles between two vectors denoting links where

α_1 = twist angle of input link MA

α_2 = twist angle of input link AB

α_3 = twist angle of coupler link BC

α_4 = twist angle of output link QC

α_5 = twist angle of ground link MQ

The Gear Ratio N is equal to the Ration R1/R2 where R1 = Radius of the ground gear, G_1, and R2 = Radius of the moving gear, G_2. This gives a displacement relationship of

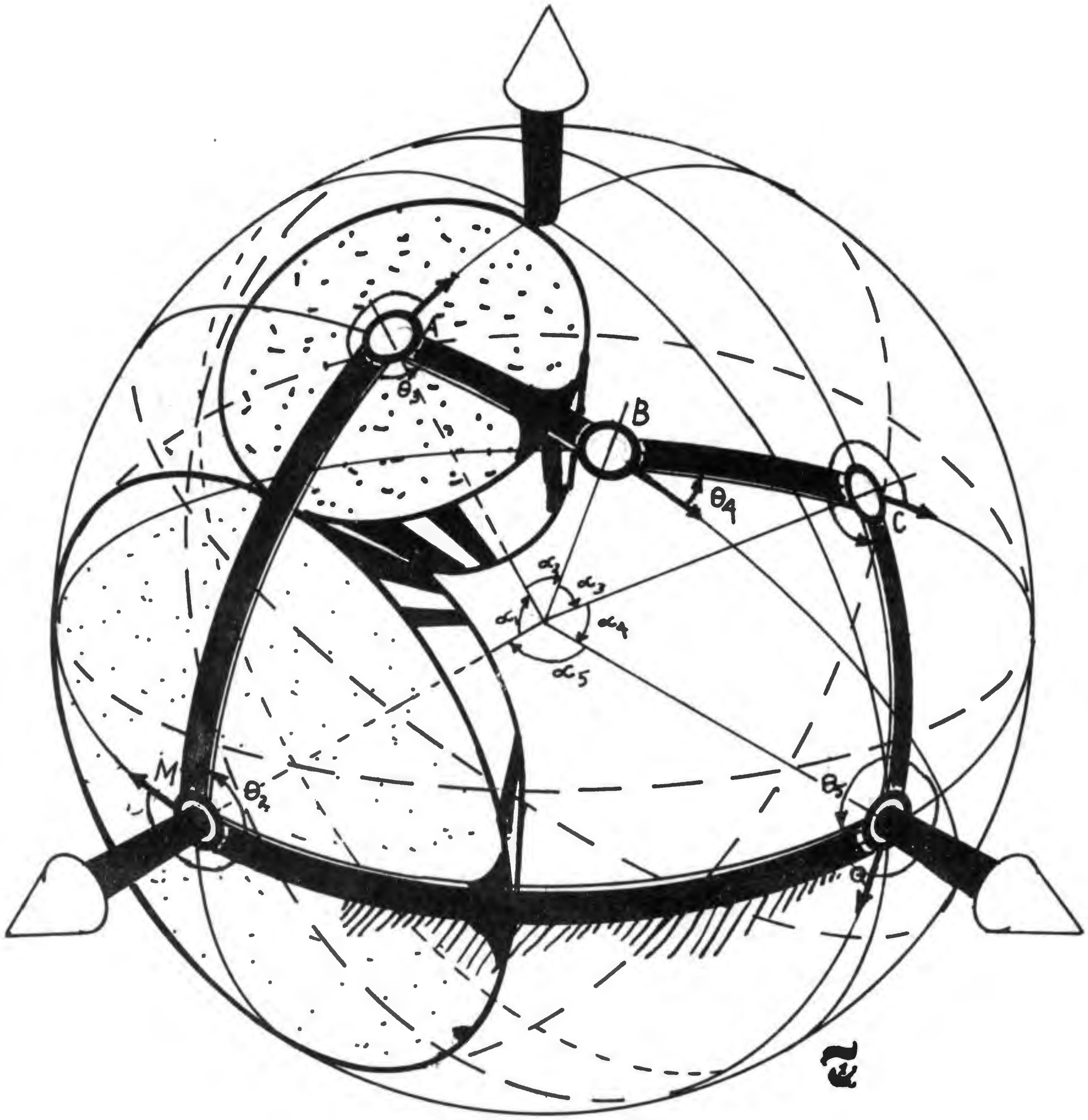

Figure 1. Nomenclature of a Geared Spherical Five-Link Mechanism

$$\theta_3 = N\theta_2 + \beta$$

where β is the initial position of link AB.

The rotations of the revolute pairs are measured relative to an extension of the previous link. All rotations are measured using the right-hand rule, about the unit vector from the center of the sphere through the revolute pair.

2.2 Displacement Analysis of the Geared Spherical Five-Link Mechanism

In kinimatic analysis the position of the components of the mechanism must be computed for a given mechanism. Closed-form displacement relationships are required to obtain all the possible geometric inversions of the mechanism. These relationships allow the rotations of the links to be calculated for positions of the input link, MA. By computing the infinitesimal motion of the various links in terms of the infinitesimal motion of the input link MA, velocity and acceleration relationships may be obtained for the mechanism.

2.3 Discussion of Analysis Technique

The approach used for this analysis is screw motion. Various works have previously been developed by Roth (10), Chen and Roth (11, 12), and Tsai and Roth (13, 14). In particular, the methods of successive screw displacements, Kohli (15), are used to perform the mechanism analysis.

The mechanism is separated in two separate open chains by "disconnecting" the mechanism at one of the revolute pairs. In this study, the separation was made at revolute pair C. The chains are now rotated successively where all rotation angles θ_i $i = 1, \cdots, 5$ are zero. This in effect stretches the links of the two open chains along a common axis. However, in spherical mechanisms the link lengths are zero, as described by Denavit and Hartenberg (7). The result is that all kink lengths (the vectors $\overline{M}$, $\overline{A}$, $\overline{B}$, $\overline{C}$, and $\overline{Q}$) lie on a common plane. In the analysis presented, the mechanism was stretched along the Z axis forcing the kinks to lie in the X-Y plane.

2.4 Loop Closure Equation

By specifying $\overline{M} = 1\ i + 0\ j + 0\ k$, and specifying MQ as the fixed link, the positions of $\overline{A}$, $\overline{B}$, $\overline{C}_1$, and $\overline{C}_2$ can be found (see Figure 2). The vectors are

$$\overline{M} = \overline{i}$$

$$\overline{Q} = \cos(\alpha_5)\ \overline{i} + \sin(\alpha_5)\ \overline{j}$$

$$\overline{C}_2 = \cos(\alpha_4 + \alpha_5)\ \overline{i} + \sin(\alpha_4 + \alpha_5)\ \overline{j}$$

$$\overline{A} = \cos(\alpha_1)\ \overline{i} - \sin(\alpha_1)\ \overline{j}$$

$$\overline{B} = \cos(\alpha_1 + \alpha_2)\ \overline{i} - \sin(\alpha_1 + \alpha_2)\ \overline{j}$$

$$\overline{C}_1 = \cos(\alpha_1 + \alpha_2 + \alpha_3)\ \overline{i} - \sin(\alpha_1 + \alpha_2 + \alpha_3)\ \overline{j}$$

For the loop closure equation, the unit vectors in each open chain are successively rotated: e.g. rotate $\overline{C}_1$ about $\overline{B}$ resulting in $\overline{C}_1'$, then rotate $\overline{C}_1'$ about $\overline{A}$ resulting in $\overline{C}_1''$, and rotate $\overline{C}_1''$ about $\overline{M}$ to produce $\overline{C}_1'''$. Rotating $\overline{C}_2$ about $\overline{Q}$ yields $\overline{C}_2'$. The mechanism was previously broken at pair C resulting in two vectors, $\overline{C}_1$ and $\overline{C}_2$. These vectors are the same vector in the closed chain. By equating the rotated vectors $\overline{C}_1'''$ and $\overline{C}_2'$, the loop closure equation is obtained. The loop closure equation is:

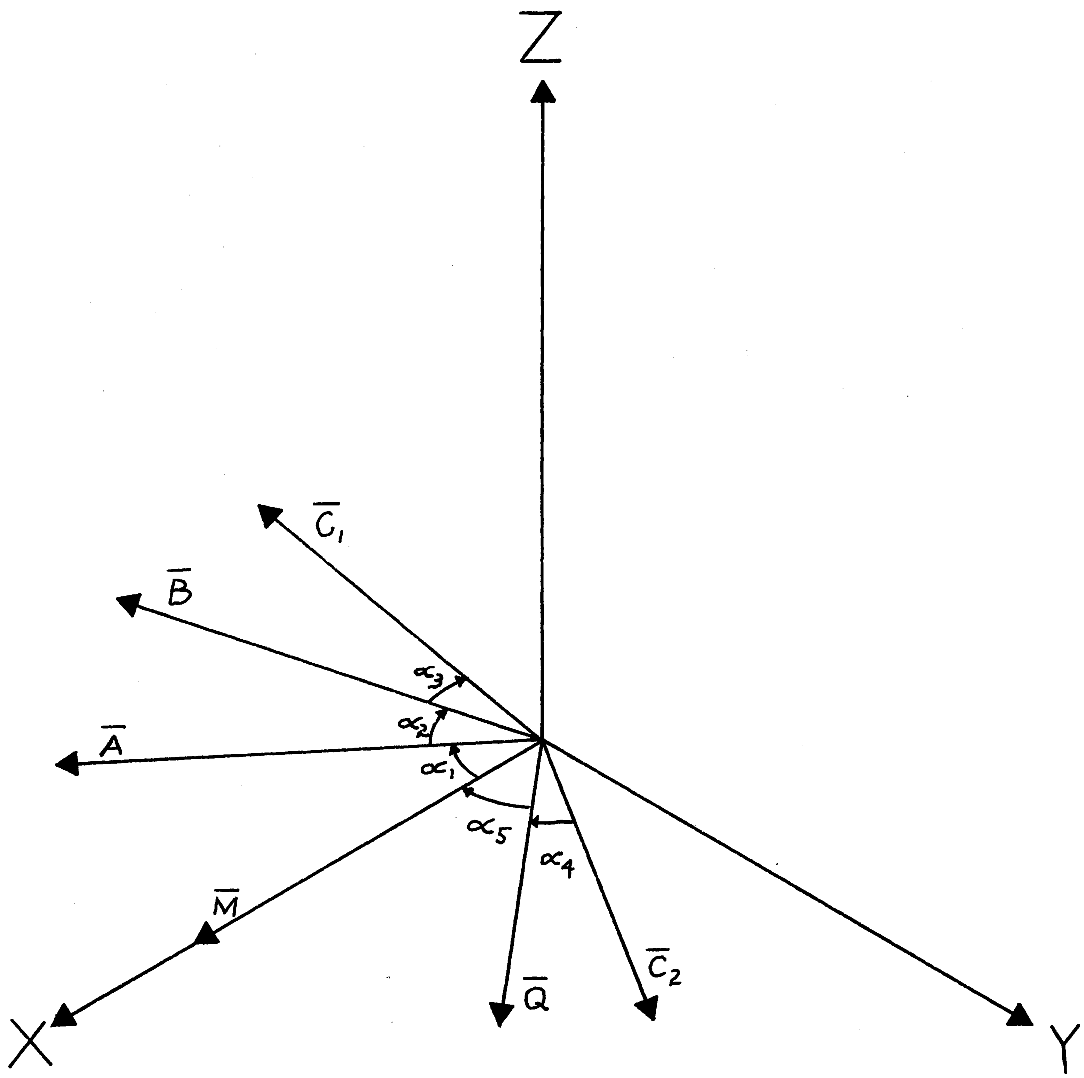

Figure 2. Mechanism Unfolded Onto the X-Y Plane

$$\begin{aligned}
&\cos\theta_4 \Big[\cos\theta_2 (\cos\theta_3\, \overline{S}_1 \; -\sin\theta_3\, \overline{S}_4 + \overline{S}_7) \\
&\qquad -\sin\theta_2 (\cos\theta_3\, \overline{T}_1 \; -\sin\theta_3\, \overline{T}_4 + \overline{T}_7) \\
&\qquad\qquad + (\cos\theta_3\, \overline{U}_1 \; -\sin\theta_3\, \overline{U}_4 + \overline{U}_7) \Big] \\
&-\sin\theta_4 \Big[\cos\theta_2 (\cos\theta_3\, \overline{S}_2 \; -\sin\theta_3\, \overline{S}_5 + \overline{S}_8) \\
&\qquad -\sin\theta_2 (\cos\theta_3\, \overline{T}_2 \; -\sin\theta_3\, \overline{T}_5 + \overline{T}_8) \\
&\qquad\qquad + (\cos\theta_3\, \overline{U}_2 \; -\sin\theta_3\, \overline{U}_5 + \overline{U}_8) \Big] \\
&\quad + \cos\theta_2 (\cos\theta_3\, \overline{S}_3 \; -\sin\theta_3\, \overline{S}_6 + \overline{S}_9) \\
&\qquad -\sin\theta_2 (\cos\theta_3\, \overline{T}_3 \; -\sin\theta_3\, \overline{T}_6 + \overline{T}_7) \\
&\qquad\qquad + (\cos\theta_3\, \overline{U}_3 \; -\sin\theta_3\, \overline{U}_6 + \overline{U}_9) = \\
&\qquad \cos\theta_5\, \overline{V}_1 + \sin\theta_5\, \overline{V}_2 + \overline{V}_3
\end{aligned} \tag{2.1}$$

The derivation of the loop closure equation and the constants are found in Appendix A.1.

2.5 Displacement Analysis

Freudenstein's displacement analysis is obtained by rearranging the loop closure equation so that the rotations of the other links are functions of the input rotation for a given mechanism. It is seen that the rotation of the input link AB is a function of the rotation of input line MA. The relationship is

$$\theta_3 = N\,\theta_2 + \beta \tag{2.2}$$

Equation 2.1 is now in two unknowns, θ_4 and θ_5 for specified rotation angles of input link MA. To obtain an equation to compute the output displacement, θ_4 must be eliminated. This may be accomplished by the following procedures.

Let,

$$\overline{X}_1 = f(\theta_2, \theta_3, \overline{S}_i, \overline{T}_i, \overline{U}_i) \quad i = 1, 4, 7$$

$$\overline{X}_2 = f(\theta_2, \theta_3, \overline{S}_i, \overline{T}_i, \overline{U}_i) \quad i = 2, 5, 8$$

$$\overline{X}_3 = f(\theta_2, \theta_3, \overline{S}_i, \overline{T}_i, \overline{U}_i) \quad i = 3, 6, 9$$

$$\overline{X}_4 = f(\theta_5, \overline{V}_i) \quad i = 1, 2, 3$$

so that equation 2.1 becomes

$$\cos\theta_4 \overline{X}_i - \sin\theta_4 \overline{X}_2 + \overline{X}_3 = \overline{X}_4 \tag{2.3}$$

The angle θ_4 can now be easily eliminated by taking the dot product of $(\overline{X}_1 \times \overline{X}_2)$ and equation (2.3). This produces the displacement equation for θ_5:

$$\theta_5 = 2{*}\mathrm{TAN}^{-1} \frac{AA \pm\sqrt{AA^2 + BB^2 - CC^2}}{BB + CC} \tag{2.4}$$

where

$$AA = (\overline{X}_1 \times \overline{X}_2) \cdot \overline{V}_2$$

$$BB = (\overline{X}_1 \times \overline{X}_2) \cdot \overline{V}_1$$

$$CC = (\overline{X}_1 \times \overline{X}_2) \cdot (\overline{X}_3 - \overline{V}_3)$$

This will product two possible positions of θ_5. By substituting the values of θ_5 into equation (2.3), θ_4 may be computed

$$\theta_4 = \cos^{-1}\left[\frac{FF - EE}{DD}\right]$$

where,

$$DD = (\overline{X}_2 \times \overline{X}_1) \cdot (\overline{i} + \overline{j} + \overline{k})$$

$$EE = (\overline{X}_2 \times \overline{X}_3) \cdot (\overline{i} + \overline{j} + \overline{k})$$

$$FF = (\overline{X}_2 \times \overline{X}_4) \cdot (\overline{i} + \overline{j} + \overline{k})$$

This will product two possible positions of θ_4. The link BC can assume these two positions (one for each θ_5 from the preceeding analysis). Complete derivations and constants are found in Appendix A.2.

2.6 Velocity Analysis

The velocity analysis is obtained by taking the first derivative with respect to time of equations (2.1) and (2.2). This gives equations,

$$\dot{\theta}_5 \overline{W}_1 = \dot{\theta}_4 \overline{W}_2 + \overline{W}_3 \tag{2.5}$$

and

$$\dot{\theta}_3 = N \dot{\theta}_2 \tag{2.6}$$

By specifying $\dot{\theta}_2$ equation (2.5) contains two unknowns, $\dot{\theta}_5$ and $\dot{\theta}_4$. Taking the cross product of $\overline{W}_2$ and equation (2.5) eliminates $\dot{\theta}_4$ and produces the equation (in one unknown $\dot{\theta}_5$).

$$\dot{\theta}_5 = \frac{(\overline{W}_2 \times \overline{W}_3) \cdot \overline{i}}{(\overline{W}_2 \times \overline{W}_1) \cdot \overline{i}} \tag{2.7}$$

Then, the values of $\dot{\theta}_5$ may be substituted in equation (2.5) to compute $\dot{\theta}_4$.

$$\dot{\theta}_4 = (\dot{\theta}_5 \overline{W}_1 - \overline{W}_3) \cdot \overline{i} \tag{2.8}$$

The complete derivation and constants may be found in Appendix A.3.

2.7 Acceleration Analysis

Taking the second derivative with respect to time of equations (2.1) and (2.2) provide

$$\ddot{\theta}_5 \overline{Z}_1 + \overline{Z}_2 = \ddot{\theta}_4 \overline{Z}_3 + \overline{Z}_4 \tag{2.9}$$

$$\ddot{\theta}_3 = N \ddot{\theta}_2 \tag{2.10}$$

By taking the cross product of $\overline{Z}_3$ and equation (2.9), $\ddot{\theta}_4$ is eliminated and the acceleration relationship as a function of $\ddot{\theta}_2$ is obtained.

$$\ddot{\theta}_5 = \frac{(\overline{Z}_3 \times \overline{Z}_4) \cdot \overline{i} - (\overline{Z}_3 \times \overline{Z}_2) \cdot \overline{i}}{(\overline{Z}_3 \times \overline{Z}_1) \cdot i} \tag{2.11}$$

After computing the values of $\ddot{\theta}_5$, a substitution into equation (2.9) provides a relationship for $\ddot{\theta}_4$.

$$\ddot{\theta}_4 = \frac{(\theta_5 \overline{Z}_1 + \overline{Z}_2 - \overline{Z}_4) \cdot \overline{i}}{\overline{Z}_3 \cdot \overline{i}} \tag{2.12}$$

Appendix A.4 gives the complete derivation and constants.

2.8 Sample Computations for a Given Spherical Five-Link Mechanism

The derivations in the previous sections are used to compute the displacements, velocities and accelerations of each component in order to provide an Analysis.

The input data was

$$\alpha_1 = 45^\circ$$
$$\alpha_2 = 45^\circ$$
$$\alpha_3 = 90^\circ$$
$$\alpha_4 = 90^\circ$$
$$\alpha_5 = 45^\circ$$
$$N = 2.0$$
$$\beta = 0^\circ$$
$$\dot{\theta}_2 = 1.0$$
$$\ddot{\theta}_2 = 1.0$$

Computations were made for increments of 5° taken from 5° to 360° of rotation for the input link MA. The results are plotted in Figures 3, 4, and 5.

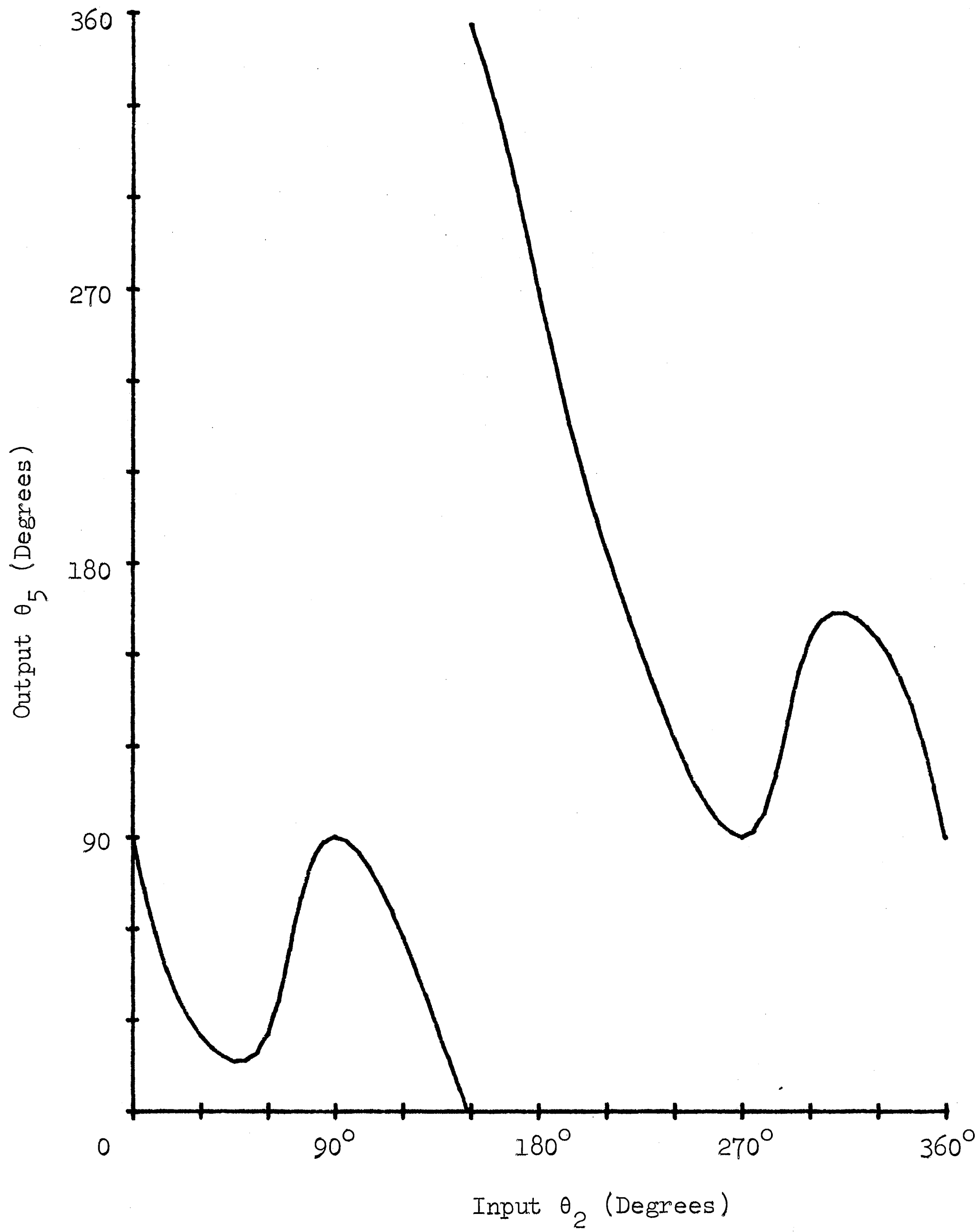

Figure 3. Displacement Analysis

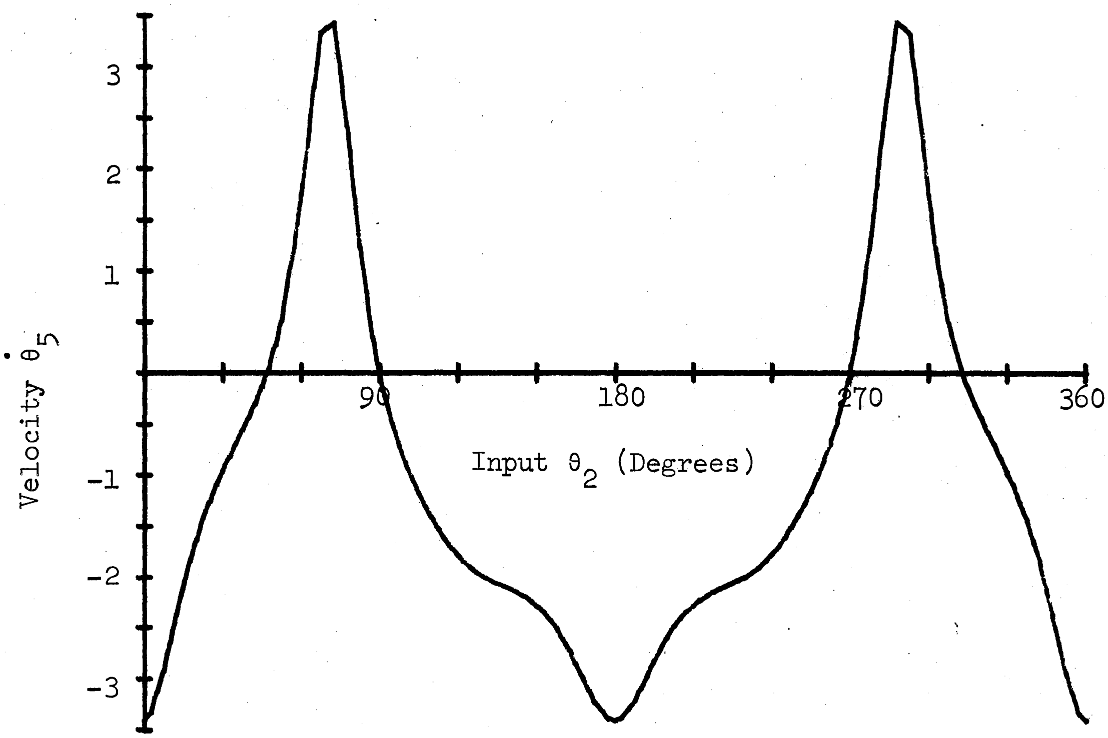

Figure 4. Velocity Analysis

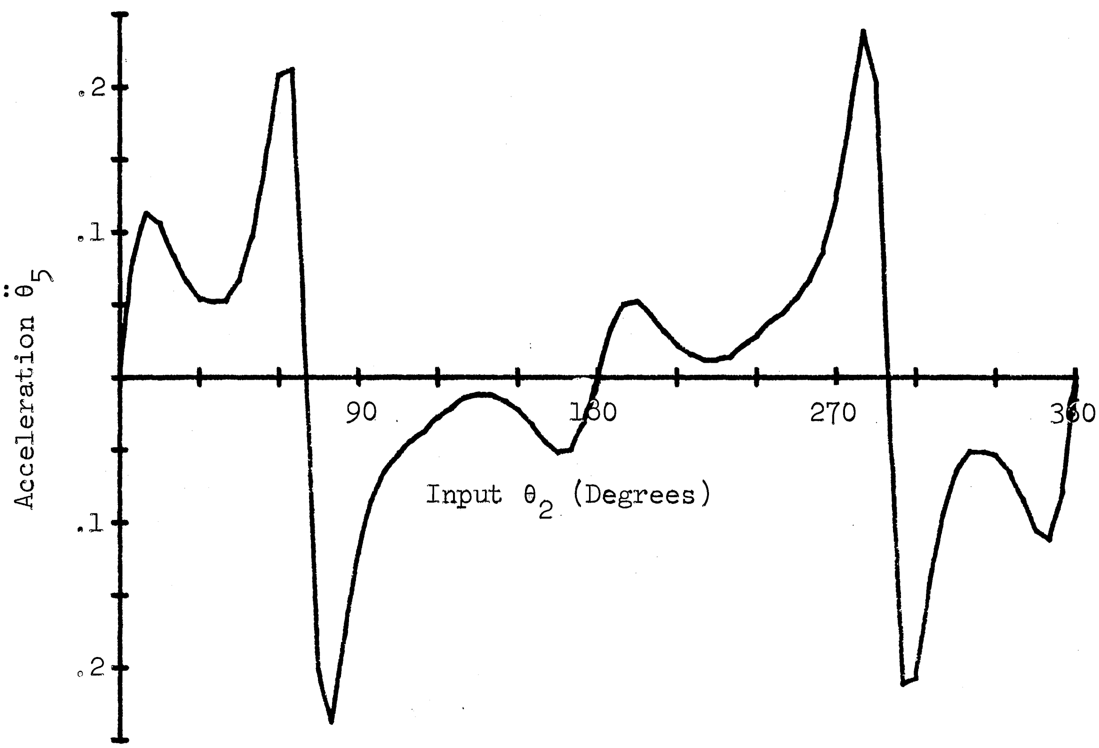

Figure 5. Acceleration Analysis

CHAPTER III

SYNTHESIS OF A GEARED SPHERICAL FIVE-LINK MECHANISM

3.1 Introduction

Kinematic Synthesis is the inverse of Kinematic Analysis. That is, the dimensions of the mechanism components must be found, so that the mechanism will provide a specified motion. In this chapter, the Geared Spherical Five-Link Mechanism is synthesised for rigid body guidance, point-path generation, and function generation.

The synthesis of the mechanism was achieved through the use of the displacement matrix. This method provides a convenient step-by-step solution. For problems of two and three positions, the solution can be simplified while still in matrix form. Mathematical procedures which Suh (9, 16) developed to design a spherical four-link mechanism are extended to derive synthesis equations for the geared spherical five-link mechanism.

Suh's approach states that a point P_1 (x_1, y_1, z_1) can be displaced to a point P_2 (x_2, y_2, z_2) by rotating P_1 about an axis $\overline{U}$ through θ degrees to point P_2, by the equation,

$$\begin{bmatrix} x_2 \\ Y_2 \\ z_2 \end{bmatrix} = \left[D_{12}\right] \overline{U}, \theta_{12} \begin{bmatrix} x_1 \\ Y_1 \\ z_1 \end{bmatrix} \qquad (3.1)$$

Where,

$$[D_{12}]_{\bar{U},\theta_{12}} = \begin{bmatrix} U_x^2 \text{ vers } \theta_{12} + \cos\theta_{12}, & U_x U_y \text{ vers } \theta_{12} - U_z \sin\theta_{12}, & U_x U_z \text{ vers } \theta_{12} + U_y \sin\theta_{12} \\ U_x U_y \text{ vers } \theta_{12} + U_z \sin\theta_{12}, & U_y^2 \text{ vers } \theta_{12} + \cos\theta_{12}, & U_y U_z \text{ vers } \theta_{12} - U_x \sin\theta_{12} \\ U_x U_z \text{ vers } \theta_{12} - U_y \sin\theta_{12}, & U_y U_z \text{ vers } \theta_{12} + U_x \sin\theta_{12}, & U_z^2 \text{ vers } \theta_{12} + \cos\theta_{12} \end{bmatrix} \quad (3.2)$$

and θ_{12} is the rotation difference $(\theta_2 - \theta_1)$ from position 1 to position 2. By employing Suh's method and variation of it, the geared spherical five-line mechanism can be designed for rigid body guidance, path-point generation, and function generation.

3.2 Rigid Body Guidance

The problem of synthesis for Rigid Body Guidance is one of dimensioning a mechanism so that it will move a rigid body connected to the coupler link BC through a number of specified positions. The maximum number of positions of Rigid Body Guidance for a geared spherical five-link mechanism is limited to five by the CQ side of the mechanism.

The positions of a rigid body can be specified by rotating the rigid body from position 1 to position n about a unique axis $\bar{S}_{1N}$ through an angle Φ_{1N}. A displacement matrix, previously solved by Suh (9), may be found that will describe this rotation by using the equation

$$[D_{1N}] = \begin{bmatrix} P_{1,i} \\ P_{2,i} \\ P_{3,i} \end{bmatrix} \begin{bmatrix} P'_{1,N} \\ P'_{2,N} \\ P'_{3,N} \end{bmatrix}^{-1} \quad (3.3)$$

where,

D_{1N} - is the displacement matrix which rotates the rigid body from Position 1 to Position n

$P_{i}, i = 1,2,3$ - a point on the rigid body in the initial position. Two points will uniquely describe a rigid body

$P'_{i}, i = 1,2,3$ - designates the point in the nth position.

The displacement matrix D_{1N} describes the motion of any point on the rigid body. Thus, any point on the rigid body may be computed in the nth position as a function of the initial position.

3.3 Derivation of Design Equations for Rigid Body Guidance and Coordination of the Rotation of Input Link MA

The mechanism is designed in two parts for rigid body guidance. The positions of the points C and Q are determined on the CQ side, which is identical to the problem with a four-link spherical mechanism. The MAB side of the mechanism can be defined by two equation sets in twelve unknowns, which would indicate seven positions. However, the mechanism is constrained to five positions by the CQ link. Thus, this allows for a solution to the rigid body guidance with input coordination for four positions by specifying the input link rotations.

3.3.1 General Equations for CQ Side

Point C lies on the rigid body, therefore the nth position of C can be found by using Equation 3.1,

$$\begin{bmatrix} C_{xN} \\ C_{yN} \\ C_{zN} \end{bmatrix} = \left[D_{1N}\right] \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}$$

This will provide $\overline{C}_N$ in terms of $\overline{C}$ such that

$$\begin{aligned} C_{xN} &= a_{11} C_x + a_{12} C_y + a_{13} C_z \\ C_{yN} &= a_{21} C_x + a_{22} C_y + a_{23} C_z \\ C_{zN} &= a_{31} C_x + a_{32} C_y + a_{33} C_z \end{aligned} \tag{3.4}$$

The a's represent the elements of the displacement matrix D_{1N}.

The link CQ has a constant length. Therefore, the condition,

$$\begin{aligned} &(C_{xN} - Q_x)^2 + (C_{yN} - Q_y)^2 + (C_{zN} - Q_z)^2 = \\ &(C_x - Q_x)^2 + (C_y - Q_y)^2 + (C_z - Q_z)^2 \end{aligned} \tag{3.5}$$

will insure that the link CQ will be of constant length for all n positions. By using the equations,

$$C_{xN}^2 + C_{yN}^2 + C_{zN}^2 = 1$$

and

$$Q_x^2 + Q_y^2 + Q_z^2 = 1 \tag{3.6}$$

which constrain the points to lie on the unit sphere and by using equation (3.5) the equation

$$(C_{xN} - C_x)Q_x + (C_{yN} - C_y)Q_y + (C_{zN} - C_z)Q_z = 0 \tag{3.7}$$

can be derived. By substituting the values of C_{xN}, C_{yN}, and C_{zN} into equation (3.7) and by simplifying the general rigid body guidance equation for the CQ side is obtained

$$\begin{aligned} &(a_{11}-1)\frac{C_x}{C_z}\,\frac{Q_x}{Q_z} + a_{12}\frac{C_y}{C_z}\,\frac{Q_x}{Q_z} + a_{13}\frac{Q_x}{Q_z} + a_{21}\frac{C_x}{C_z}\,\frac{Q_y}{Q_z} \\ &(a_{22}-1)\frac{C_y}{C_z}\,\frac{Q_y}{Q_z} + a_{23}\frac{Q_y}{Q_z} + a_{31}\frac{C_x}{C_z} + a_{32}\frac{C_y}{C_z} + \\ &(a_{33}-1) = 0 \end{aligned} \tag{3.8}$$

3.3.2 General Equations for MAB Side

The value for the unknown gear ratio, N, may be specified for the MAB side of the mechanism. This will leave 6 + (n-1) unknowns in A_x, A_y, B_x, B_y, M_x, M_y, and θ_{2i}, i = 2, ... n.

By using the displacement matrix mathematics, the point B can be rotated about A and then about M to obtain B in its nth position. This is found by,

$$\overline{B}_N = \left[D_{1N}\right] \overline{M}, \theta_{2N} \left[D_{1N}\right] \overline{A}, \theta_{3N} \overline{B} \tag{3.9}$$

where,

$\left[D_{1N}\right] \overline{A}, \theta_{3N}$ is the displacement matrix for rotating a point about $\overline{A}$ by θ_{3N}

and

$\left[D_{1N}\right] \overline{M}, \theta_{2N}$ is the displacement matrix for rotating a point about $\overline{M}$ by θ_{2N}.

The point B_N on the rigid body can also be found in terms of B from equation (3.1). Since the mechanism is assumed to lie on a unit sphere only B_{xN} and B_{yN} are necessary to define the point. Setting the two resulting values of B_{xN} and B_{yN} equal produces the general rigid body guidance equations for side MAB. These are in the form of:

$$A_x^2M_x^2B_xT1 + A_x^2B_xT3 + M_x^2B_xT2 + B_xT4$$

$$+ A_xA_yM_x^2B_yT1 + A_xA_yB_yT3 - A_zM_x^2B_yT5 - A_zB_yT6$$

$$+ A_xA_zM_x^2B_zT1 + A_xA_zB_zT3 + A_yM_x^2B_zT5 + A_yB_zT6$$

$$+A_xA_yM_xM_yB_xT1 - A_xA_yM_zB_xT7 + A_zM_xM_yB_xT5 - A_zM_zB_xT8$$

$$+ A_y^2M_xM_yB_yT1 - A_y^2M_zB_yT7 + M_xM_yB_yT2 - M_zB_yT9$$

$$+ A_y A_z M_x M_y B_z T1 - A_y A_z M_z B_z T7 - A_x M_x M_y B_z T5 + A_x M_z B_z T8$$

$$+ A_x A_z M_x M_z B_x T1 - A_x A_z M_y B_x T7 - A_y M_x M_z B_x T5 - A_y M_y B_x T8$$

$$+ A_y A_z M_x M_z B_y T1 - A_y A_z M_y B_y T7 + A_x M_x M_z B_y T5 + A_x M_y B_y T8$$

$$+ A_z^2 M_x M_z B_z T1 + A_x^2 M_y B_z T7 + M_x M_z B_z T2 + M_y B_z T9$$

$$- a_{11} B_x - a_{12} B_y - a_{13} B_z = 0 \qquad (3.10a)$$

and

$$A_x^2 M_x M_y B_x T1 + A_x^2 M_z B_x T7 + M_x M_y B_x T2 + M_z B_x T9$$

$$+ A_x A_y M_x M_y B_y T1 + A_x A_y M_z B_y T7 - A_z M_x M_y B_y T5 - A_z M_z B_y T8$$

$$+ A_x A_z M_x M_y B_z T1 + A_x A_z M_z B_z T7 + A_y M_x M_y B_z T5 + A_y M_z B_z T8$$

$$+ A_x A_y M_y^2 B_x T1 + A_x A_y B_x T3 + A_z M_y^2 B_x T5 + A_z B_x T6$$

$$+ A_y^2 M_y^2 B_y T1 + A_y^2 B_y T3 + M_y^2 B_y T2 + B_y T4$$

$$+ A_y A_z M_y^2 B_z T1 - A_y A_z B_z T3 - A_x M_y^2 B_z T5 - A_x B_z T6$$

$$+ A_x A_z M_y M_z B_x T1 - A_x A_z M_x B_x T7 - A_y M_y M_z B_x T5 + A_y M_x B_x T8$$

$$+ A_y A_z M_y M_z B_y T1 - A_y A_z M_x B_y T7 + A_x M_y M_z B_y T5 - A_x M_x B_y T8$$

$$+ A_z^2 M_y M_z B_z T1 - A_z^2 M_x B_z T7 + M_y M_z B_z T2 - M_x B_z T9$$

$$- a_{21} B_z - a_{22} B_y - a_{23} B_z = 0 \qquad (3.10b)$$

T's are functions of θ_2 and θ_3 in the nth position, and a's are elements in the displacement matrix for the nth position. (See Appendix B.1 for the expansion of the T's).

3.4 Rigid Body Guidance and Coordination of Input Link MA

3.4.1 Two Positions of a Rigid Body

For two positions of a rigid body the general equations 3.8, 3.10a, and 3.10b are written once. This gives the difference of rotations from position 1 to 2. By specifying all of the unknowns except one in 3.8 and two in 3.10a and 3.10b, the design may be computed.

Side CQ

Specify: Q_x/Q_z, Q_y/Q_z, and C_y/C_z

Substituting these values into the general equation provides the solution,

$$\frac{C_x}{C_z} = -\left[a_{12}\frac{C_y}{C_z}\frac{Q_x}{Q_z} + a_{13}\frac{Q_x}{Q_z} + (a_{22}-1)\frac{C_y}{C_z}\frac{Q_y}{Q_z} + a_{23}\frac{Q_y}{Q_z} + a_{32}\frac{C_y}{C_z} + (a_{33}-1)\right] \Big/ \left[(a_{11}-1)\frac{Q_z}{Q_z} + a_{21}\frac{Q_y}{Q_z} + a_{31}\right] \qquad (3.11)$$

where

$$C_z = \frac{1}{\sqrt{\dfrac{C_x^2}{C_z^2} + \dfrac{C_y^2}{C_z^2} + 1}}$$

Side MAB

Specify: N(Gear Ratio),

θ_{22}(Rotation of input θ_2 from position 1 to 2)

A_x, A_y, M_x, and M_y

Substituting these values into the general equation and reducing to two unknowns by dividing by B_z provides the solutions:

$$\frac{B_y}{B_z} = \frac{E_1 E_6 - E_3 E_4}{E_2 E_4 - E_1 E_5} = E_7$$

$$\frac{B_x}{B_z} = \frac{-E_2 E_7 - E_3}{E_1} = E_8$$

Where the E's are constants obtained from substitution. Appendix B.2 contains the values of these constants.

3.4.2 Three Positions of a Rigid Body

In the three positions rigid body guidance problem, each of the general equations must be written twice. The displacement matrix D_{1N} is used twice (once for a position change from 1 to 2 and then for a position change from 1 to 3). With this data, we may specify the needed values for each equation and compute the coordinates of the remaining unknowns.

Side CQ

Specify: C_x, C_y

C_z may be computed from the constraint condition:

$$C_x^2 + C_y^2 + C_z^2 = 1$$

By substituting the specified values into the general equation, the equation may be expressed as:

$$D_{1N} \frac{Q_x}{Q_z} + D_{2N} \frac{Q_y}{Q_z} + D_{3N} = 0 \qquad (3.12)$$

The D_{1N}'s (i = 1,2,3) are the coefficients for the position change from 1 to n. Writing equation (3.12) two times, once for each position change, enables the solution to be found by simultaneous equations, so that

$$\frac{Q_x}{Q_z} = \frac{D_{32}\,D_{23} - D_{33}\,D_{23}}{D_{12}\,D_{23} - D_{13}\,D_{22}} = D_1$$

$$\frac{Q_y}{Q_z} = \frac{-D_{12}\,D_1 - D_{32}}{D_{22}}$$

By rearranging the constraint equation for a unit sphere the x, y, and z coordinates are found,

$$Q_z = \frac{1}{\sqrt{D_1^2 + D_2^2 + 1}}$$

$$Q_x = D_1 Q_z$$

$$Q_y = D_2 Q_y$$

Appendix B.3 contains the values of O.

Side MAB

Specify: M_x, M_y, θ_{22} and θ_{23}

Substituting the known values into the general equations (3.10a) and (3.10b) results in four non-linear equations having four unknowns B_x/B_z, B_y/B_z, A_x, and A_y. This class of problems may be solved with the Newton-Raphson Iteration Technique (17,18). In using this technique, initial estimates are made for the unknowns. These estimates are continually corrected during the solution process until the error is minimized. Appendix C has a description of the Newton-Raphson Technique.

3.4.3 Four Positions of a Rigid Body

Four positions is the maximum number of positions for Rigid Body Guidance with coordination of input link rotations. The solution for the MAB side is simplified if the angular displacements of the input link are specified. However, three other values may be chosen if desirable.

Side CQ

Specify: $\frac{C_x}{C_z}$

Substitute this value into equation (3.8) in the form

$$d_{1N} \frac{Q_x}{Q_z} + d_{2N} \frac{C_y}{C_z} \frac{Q_x}{Q_z} + d_{3N} \frac{Q_y}{Q_z} + d_{4N} \frac{C_y}{C_z} \frac{Q_y}{Q_z}$$

$$+ d_{5N} \frac{C_y}{C_z} + d_{6N} = 0 \tag{3.13}$$

The d_{in}'s are coefficients of the general equation. (See Appendix B.4 for the definition of all d_{in}'s.

The general equation (3.13) may be solved by the method of linear superposition.

Let,

$$\lambda_1 = \frac{C_y}{C_z} \frac{Q_x}{Q_z} \tag{3.14a}$$

$$\lambda_2 = \frac{C_y}{C_z} \frac{Q_y}{Q_z} \tag{3.14b}$$

$$\frac{Q_x}{Q_z} = L_1 + M_1 \lambda_1 + N_1 \lambda_2 \tag{3.15a}$$

$$\frac{Q_y}{Q_z} = L_2 + M_2 \lambda_1 + N_2 \lambda_2 \tag{3.15b}$$

and

$$\frac{C_y}{C_z} = L_3 + M_3 \lambda_1 + N_3 \lambda_2 \qquad (3.15c)$$

The general equation now takes the form of:

$$d_{1N}L_1 + d_{1N}M_1\lambda_1 + d_{1N}N_1\lambda_2 + d_{3N}L_2 + d_{3N}M_2\lambda_1$$

$$+ d_{3N}N_2\lambda_2 + d_{5N}L_3 + d_{5N}M_3\lambda_1 + d_{5N}N_3\lambda_2$$

$$= -d_{6N} - d_{2N}\lambda_1 - d_{4N}\lambda_2 \qquad (3.16)$$

This may be broken into three sets of equations:

$$a_{1N}L_1 + a_{3N}L_2 + a_{5N}L_3 = -a_{6N} \qquad n = 2,4$$

$$a_{1N}M_1 + a_{3N}M_2 + a_{5N}M_3 = -a_{2N} \qquad n = 2,4$$

$$a_{1N}N_1 + a_{3N}N_2 + a_{5N}N_3 = -a_{4N} \qquad n = 2,4$$

By solving each set of simultaneous equations the values of L_i, M_i, and N_i i=1,2,3 are found. Substituting these values into the compatability equations (3.14a) and (3.14b) and expanding results in:

$$t_1\lambda_2^2 + t_2\lambda_2 + t_3 = 0 \qquad (3.17a)$$

and

$$t_4\lambda_2^2 + t_6\lambda_2 + t_6 = 0 \qquad (3.17b)$$

The t_i's are functions of λ_1. Appendix B.5 contains the values of t_i (i = 1, ... 6).

By using Sylvesters dialytic eliminate technique, λ_2 may be eliminated, and a solution of λ_1 may be found. This is obtained by solving the determinant:

$$\begin{vmatrix} t_1 & t_2 & t_3 & 0 \\ 0 & t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 & 0 \\ 0 & t_4 & t_5 & t_6 \end{vmatrix} = 0$$

for λ_1. This will result in a fourth order polynomial in λ_1 with 0, 2 or 4 real roots. Substituting each real answer of λ_1 into equation (3.17a) and (3.17b) gives a solution for λ_2. By substituting the solutions of λ_1 and λ_2 into equations (3.15a), (3.15b), and (3.15b), Q_x/Q_z, Q_y/Q_z, and C_y/C_z may be found.

Side MAB

Specify: θ_{22}, θ_{23}, and θ_{24}

The solutions of A_x, A_y, B_x, B_y, M_x and M_y are found by using the Newton-Raphson Iteration Technique for non-linear equations. The procedure is the same as in part 3.4.2.

3.4.4. Five Positions of a Rigid Body

The MAB side of the Geared Spherical Five-Link Mechanism is solved by the method used for three and four positions of a rigid body. However, M_x and M_y are the only specified variables. A solution (for five positions) may also be obtained with two input rotations specified.

The CQ side of the mechanism is solved by using the techniques of four position synthesis of a rigid body. However, the problem is solved in two parts. First, solutions for positions 12, 13 and 14 are obtained. By varying the value of C_x/C_z, a curve representing the solutions of this part may be drawn. Then, solutions for positions 12, 13 and 15 are obtained and graphed in the same manner. The intersections of these two curves is the solution for all five positions. Appendix D.1 contains solutions of rigid body guidance problems.

3.5 Path Point Generation

In path point generation, the problem is to design a mechanism such that a point on the rigid body of the coupler link will trace a path through a number of specified points. A procedure often used for point path generation is to extend the rigid body guidance problem. Suh (9) has previously developed this technique.

On the sphere, the displacement of the rigid body with a point tracing a path may be described as follows. A point on the rigid body is rotated about an axis $\overline{S}_{1N}$ by an angle Φ_{1N} from position 1 to position n. This may be achieved by taking the cross product $(\overline{P}_1 \times \overline{P}_N)$. ($\overline{P}_1$ and $\overline{P}_N$ are unit vectors from the sphere center to points P_1 and P_N respectively.) The cross product provides the screw axis:

$$\overline{S}_{1N} = \overline{P}_1 \times \overline{P}_N \tag{3.18}$$

and

$$\Phi_{1N} = \cos^{-1} (\overline{P}_1 \cdot \overline{P}_N) \tag{3.19}$$

Thus, a displacement matrix may be found to rotate point P_1 to P_N, which is

$$\left[D_{1N} \right]_{\overline{S}_{1N}\ \Phi_{1N}} \tag{3.20}$$

The rigid body may also experience a rotation β_{1N} about $\overline{P}_1$ from position P_1 to P_N. This rotation may be placed in the displacement matrix form:

$$\left[D_{1N} \right]_{\overline{P}_1,\ \beta_{1N}} \tag{3.21}$$

This matrix will result in elements having cos (β_{1N}) and sin (β_{1N}) terms. By multiplying these two displacement matrices, the displacement of the rigid body may be described by a displacement matrix with β_{1N} as an unknown.

3.6 Development of the General Equations for Path Point Generation for the Geared Spherical Five-Link Mechanism

From the development in Section 3.5, the equation for the displacement equation of the path generation rigid body is:

$$\left[D_{1N} \right] = \left[D_{1N} \right]_{\overline{S}_{1N},\ \Phi_{1N}} \left[D_{1N} \right]_{\overline{P}_{1N},\ \beta_{1N}} \qquad (3.22)$$

Equations for point B_N on the rigid body may be found from the rigid body displacement equation. Equations are also found from rotating $\overline{B}$ about $\overline{A}$ by θ_3 and then rotating the displacement $\overline{B}$ about $\overline{M}$ by θ_2. These equations appear as:

$$\overline{B}_N = \left[D_{1N}\right] \overline{B} \qquad (3.23)$$

and

$$\overline{B}_N = \left[D_{1N}\right]_{\overline{M},\ \theta_{2N}} \left[D_{1N}\right]_{A,\theta_{3N}} \overline{B} \qquad (3.24)$$

The point C on the rigid body may also be described by two equations:

$$\overline{C}_N = \left[D_{1N}\right] \overline{C} \qquad (3.25)$$

and

$$(C_{xN} - C_x)\ Q_x + (C_{yN} - C_y)\ Q_y + (C_{zN} - C_z)\ Q_z = 0 \qquad (3.26)$$

Together these general equations produce 3(n-1) equations in 2(n-1) + 10 unknowns. Thus, a maximum of eleven points may be traced by the coupler link. There are two methods by which these path generating problems may be solved. One method is for two-five positions, and the other is for six-eleven positions.

3.7 Path Point Generation for Two-Five Points

Equations (3.18) and (3.19) are used to calculate the displacement matrix:

$$\left|D_{1N}\right| \; \overline{S}_{1N},\ \Phi_{1N}$$

By specifying B_x, B_y, A_x, A_y, M_x, M_y, and θ_{2N} in equation (3.24), the values of B_{xN}, B_{yN}, and B_{zN} may be calculated. Then, by substituting these values into equation (3.23) and (3.22), the β_{1N}'s may be computed. This provides the displacement matrix for the rigid body. The solution may be obtained by solving for the CQ side. This is exactly the same solution as obtained in rigid body guidance.

3.8 Path Point Generation of Six-Eleven Points

Setting equation (3.23) equal to (3.24) produces three equations of which any two are unique general equations. Another unique equation may be obtained by substituting values of $\overline{C}_N$ obtained in equation (3.25) into equation (3.26). These three general equations may be written (n - 1) times and solved using the Newton-Raphson Iteration Technique described in Appendix C. Appendix D.2 contains a solution of a five-point generation problem.

3.9 Function Generation for the Geared Spherical Five-Link Mechanism

Function Generation is the cooperation of input and output rotational displacements for multiply-separated positions. The mechanism may be designed for function generation by kinematic inverstion (9). If the output link CQ is fixed, as the ground link, then successive rotations of the links about vectors $\overline{A}$, $\overline{M}$, and $\overline{Q}$ are made respectively. This may be expressed as,

$$\overline{B}_N = \left[D_{1N}\right] \overline{Q}, -\theta_{5N} \left[D_{1N}\right] \overline{M}, \theta_{2N} \left[D_{1N}\right] \overline{A}, \theta_{3N} \overline{B} \qquad (3.27)$$

The mechanism must be rotated by $-\theta_5$ about $\overline{Q}$ due to the inversion.

3.10 Derivation of the General Design Equation for Function Generation of the Geared Spherical Five-Link Mechanism

There is a constraint imposed on the coordinates of M and Q because M and Q lie on a great circle. Since the choice of that great circle will not help specify an additional position of function generation, the following simplifications may be made:

$$Q_y = 1$$

$$Q_x = 0$$

$$Q_z = 0$$

$$M_z = 0$$

and

$$M_x^2 + M_y^2 = 1$$

Making these substitutions in equation (3.27) produces B as a function of B_x, B_y, B_z, A_x, A_y, A_z, M_x and M_y. Knowing that the link BC is of constant length allows the use of the equation

$$(B_{xN} - B_x)C_x + (B_{yN} - B_y)C_y + (B_{zN} - B_z)C_z = 0 \tag{3.28}$$

Substituting B_N into this equation will produce (n-1) equations in seven independent unknowns (B_x, B_y, A_x, A_y, C_x, C_y, M_x). Thus, eight positions of function generation may be solved. The gear ratio, N, and a sphere of unknown radius would allow for two more positions; however, these would produce highly complex equations.

3.10.1 Function Generation for Two Positions

Specify: M_x, A_x, A_y, B_x, B_y, C_x/C_z, θ_2, θ_5, N

Compute: M_y, A_z, B_z from unit sphere constraing equations

By computing B_N from equation (3.27) values may be found for B_{xN}, B_{yN}, and B_{zN}. Substituting these values into equation (3.28) results in the equation:

$$C_x/C_z = -|C_y/C_z\ (B_{yN} - B_y) + (B_{zN} - B_z)| \quad |(B_{xN} - B_x)| \tag{3.29}$$

3.10.2 Function Generation for Three Position Synthesis

Specify: M_x, A_x, A_y, B_x, B_y, N

Compute: M_y, A_z, B_z

Solving for $\overline{B}_2$ and $\overline{B}_3$ with equation (3.27) provides two equations in two unknowns:

$$D_{12} \frac{C_x}{C_z} + D_{22} \frac{C_y}{C_z} + D_{32} = 0 \tag{3.30a}$$

and

$$D_{13} \frac{C_x}{C_z} + D_{23} \frac{C_y}{C_z} + D_{33} = 0 \tag{3.30b}$$

Solving these equations by simultaneous equations produces:

$$\frac{C_x}{C_z} = \frac{D_{32}D_{23} - D_{33}D_{22}}{D_{12}D_{23} - D_{13}D_{22}} = D_1$$

and

$$\frac{C_y}{C_z} = \frac{-(D_{12}D_1 + D_{32})}{D_{22}} = D_2$$

The values of C_x, C_y, and C_z may now be computed to be:

$$C_z = \frac{1}{\sqrt{D_1^2 + D_2^2 + 1}}$$

$$C_x = D_1 C_z$$

$$C_y = D_2 C_z$$

The D's are constants obtained from the displacement equation (3.28).

3.10.3 Function Generation for Four Positions

Specify: M_x, A_x, A_y, B_x/B_z, N

Compute: M_y, A_z

The solution for this problem is similar to the solution for four positions of a rigid body for the CQ side. The solution procedures are identical. If an exact answer is not required, then the Newton-Raphson Iteration Technique may be used.

3.10.4 Function Generation for Five Positions

Specify: M_x, A_x, A_y, N

Compute: M_y, A_z

Solve for $\overline{B}_N$ in terms of $\overline{B}$ using equation (3.27). By substituting $\overline{B}_N$ into equation (3.28), the problem is identical to a five position rigid body guidance problem for the CQ side of the geared spherical five-link mechanism. By iterating the value of B_x/B_y for positions 12, 13, and 14, the solution curve may be obtained for positions 12, 13, and 14. Then, by iterating B_x/B_z for position changes 12, 13, and 15, the solution set for these positions is obtained. The intersection of the two curves is the solution to the five position problem.

3.10.5 Function Generation for Six to Eight Positions

By applying the Newton-Raphson Iteration Technique for sets of non-linear equations (Appendix C) this set of problems may be solved. There will be (n-1) equations in (n-1) unknowns for an n position function generation problem. The Newton-Raphson Iteration Technique may be used to solve all the function generation problems. This would simplify programming for the set of problems.

CHAPTER IV

SUMMARY

As a result of the research, a unified approach for the analysis and synthesis of the Geared Spherical Five-Link Mechanism has been developed. The successive screw displacement method (15) was used for the analysis of the mechanism, and the displacement matrix method (9, 16) were applied for synthesis of rigid body guidance, path-point generation, and function generation.

The screw displacement method proved to be very adaptable to spherical mechanisms. By "unfolding" the linkage onto a plane and successively rotating the kink-links the motion may be easily visualized. Equating the two parts of the disconnected joint results in a closed form solution. Any gearing arrangement may be readily incorporated in the analysis. This allows gearing changes after the general analysis equations have been derived.

The use of the displacement matrix for synthesis provides a generalized approach to rigid body guidance, path-point generation, and function generation. The initial matrix equations are produced by rotational matrices in a successive order. This allows the simplification of equations (while still in matrix form) for problems of less than maximum synthesis positions. By arranging the synthesis equations for path-point generation and function generation, the solutions may be obtained through the use of the rigid body guidance equations for the

CQ side of the mechanism for a maximum of five positions. Through performing the proper matrix multiplications and substitutions, path-point and function generation problems may be simplified in an equation form identical to those of rigid body guidance equations for the CQ side.

Since the equations for synthesis are developed by employing the displacement method, the general computer program which uses the Newton-Raphson Iteration Technique for non-linear sets of equations would make solutions available to all the synthesis problems. Changes in gearing ratio or arrangement can be made after the general equations have been developed. This may be accomplished either by substitution into the present equations and/or by inversion of the mechanism.

The present work is concerned with only one gearing arrangement of the Geared Spherical Five-Link Mechanism. However, the developed equations are very general and proper substitution into these equations will define all of the gearing combinations. Thus, this study provides the general equations and methods for their solution for analysis and synthesis of the Geared Spherical Five-Link Mechanism.

SELECTED BIBLIOGRAPHY

(1) Beyer, R. The Kinematic Synthesis of Mechanisms. Translated from the German by H. Kuenzel. New York: McGraw-Hill, 1963.

(2) Soni, A. H., and C. Harrisberger. "The Design of the Spherical Drag Link Mechanis." Journal of Engineering for Industry, No. 89B (1967), pp. 177-181.

(3) Suh, C. H., and C. W. Radcliffe. "Synthesis of Spherical Linkages with the use of the Displacement Matrix." Journal of Engineering for Industry, No. 89B (1967), pp. 215-222.

(4) Hamid, S., and A. H. Soni. "Synthesis of Spherical Six-Link Mechanisms for Path Generation." ASME Publication No. 72-Mech-83, New York, 1972.

(5) Kohli, D. D., and A. H. Soni. "Synthesis of Spherical Mechanisms for Multiply-Separated Positions of a Rigid Body." Proceedings of IFTOMM International Symposium on Linkages, Bucharest, Romania, June 6-13, 1973.

(6) Huang, M., R. P. Pamidi, and A. H. Soni. "Design of a Spherical Four-Link Crank-Rocker Mechanism." Journal of Mechanism, Vol. 5 (1970), pp. 5-10.

(7) Hartenberg, R. S., and J. Denavit. Kinematic Synthesis of Linkages. New York: McGraw-Hill, 1964.

(8) Yang, A. T. "Harmonic Analysis of Spherical Four-Bar Mechanisms." Journal of Applied Mechanics, Vol. 29, No. 84E (1962), pp. 683-688

(9) Yang, A. T. "Static Force and Torque Analysis of Spherical Four-Bar Mechanisms." Journal of Engineering for Industry, No. 87B (1965), pp. 221-227.

(10) Roth, B. "On the Screw Axis and Other Special Lines Associated with Spatial Displacements of a Rigid Body." Journal of Engineering for Industry, Trans. ASME, Series B, Vol. 89, No. 1 (1967), pp. 102-110.

(11) Chen, P., and B. Roth. "A Unified Theory for the Finitely and Infinitesimally Separated Position Problems of Kinematic Synthesis." Journal of Engineering for Industry, Trans. ASME, Series B, Vol. 91 (1969), pp. 203-208.

(12) Chen, P., and B. Roth. "Design Equations for the Finitely and Infinitesimally Separated Position Synthesis of Binary Links and Combined Link Chains." Journal of Engineering for Industry, Trans. ASME, Series B, Vol. 19 (1969), pp. 209-219.

(13) Tsai, L. W., and B. Roth. "Design of Dyads with Helical, Cylindrical, Spherical, Revolute, and Prismatic Joints." Mechanism and Machine Theory, Vol 7 (1972), pp. 85-102.

(14) Tsai, L. W., and B. Roth. "Design of Triads Using the Screw Triangle Chain," Proceedings of the Third World Congress for the Theory of Machines and Mechanisms, Kupari, Yugoslavia, September 13-20, 1971. Vol. D, Paper D-19, pp. 273-286.

(15) Kohli, D., "Synthesis and Analysis of Spatial, Two-Loop Six-Link Mechanisms." (Ph. D. Dissertation, Oklahoma State University, Stillwater, Oklahoma, 1973).

(16) Suh, C. H., and C. W. Radcliffe. "Synthesis of Plane Linkages with Use of the Displacement Matrix." Journal of Engineering for Industry, Trans. ASME, No. 89B (1967) pp. 206-214.

(17) Gerald, C. F. Applied Numerical Analysis. Massachusetts: Addison-Wesley, 1970.

(18) Carnahan, B., H. A. Luther, and J. O. Wilkes. Applied Numerical Methods. New York: John Wiley and Sons, 1969.

APPENDIX A

ANALYSIS OF A GEARED SPHERICAL FIVE-LINK MECHANISM

A.1 Loop Closure Equation

$$\overline{M} = 1$$

$$\overline{Q} = \cos(\alpha_5)i + \sin(\alpha_5)j$$

$$\overline{C}_2 = \cos(\alpha_4 + \alpha_5)i + \sin(\alpha_4 + \alpha_5)j$$

$$\overline{A} = \cos(\alpha_1)i - \sin(\alpha_1)j$$

$$\overline{B} = \cos(\alpha_1 + \alpha_2)i - \sin(\alpha_1 + \alpha_2)j$$

$$\overline{C}_1 = \cos(\alpha_1 + \alpha_2 + \alpha_3)i - \sin(\alpha_1 + \alpha_2 + \alpha_3)j$$

Rotations of vectors on the right hand side of the X-axis are right-hand positive screw sense. Rotations of vectors on the left hand side of the X-axis are negative right hand screw sense.

$$\overline{C}'_{2N} = \cos\theta_5 \left[\overline{C}_2 - (\overline{C}_2 \cdot \overline{Q})Q\right] + \sin\theta_5 (\overline{Q} \times \overline{C}_2) + (\overline{C}_2 \cdot \overline{Q})\overline{Q}$$

$$\overline{C}'_{1N} = \cos\theta_4 \left[\overline{C}_1 - (\overline{C}_1 \cdot \overline{B})B\right] - \sin\theta_4 (\overline{B} \times \overline{C}_1) + (\overline{C}_1 \cdot \overline{B})B$$

$$\overline{C}''_{1N} = \cos\theta_3 \left[\overline{C}'_1 - (\overline{C}'_1 \cdot \overline{A})\overline{A}\right] - \sin\theta_3 (\overline{A} \times \overline{C}'_1) + (\overline{C}'_1 \cdot \overline{A})\overline{A}$$

$$\overline{C}_{1N} = \cos\theta_2 \left[\overline{C}''_1 - (\overline{C}''_1 \cdot \overline{M})\overline{M}\right] - \sin\theta_2 (\overline{M} \times \overline{C}''_1) + (\overline{C}''_1 \cdot \overline{M})\overline{M}$$

By substituting $\overline{C}''_{1N}$ into $\overline{C}''_{1N}$ and $\overline{C}''_{1N}$ into $\overline{C}'''_{1N}$, and by setting $\overline{C}'''_{1N} = \overline{C}'_{2N}$, the loop closure equation is obtained. The simplified equation is obtained by letting:

$$\overline{L}_1 = \overline{C}_1 - (\overline{C}_1 \cdot \overline{B})\overline{B}$$

$$\overline{L}_2 = \overline{B} \times \overline{C}_1$$

$$\overline{L}_3 = (\overline{C}_1 \cdot \overline{B})\overline{B}$$

$$\overline{R}_1 = \overline{L}_1 - (\overline{L}_1 \cdot \overline{A})\overline{A}$$

$$\overline{R}_2 = \overline{L}_2 - (\overline{L}_2 \cdot \overline{A})\overline{A}$$

$$\overline{R}_3 = \overline{L}_3 - (\overline{L}_3 \cdot \overline{A})\overline{A}$$

$$\overline{R}_4 = \overline{A} \times \overline{L}_1$$

$$\overline{R}_5 = \overline{A} \times \overline{L}_2$$

$$\overline{R}_6 = \overline{A} \times \overline{L}_3$$

$$\overline{R}_7 = (\overline{L}_1 \cdot \overline{A})\overline{A}$$

$$\overline{R}_8 = (\overline{L}_2 \cdot \overline{A})\overline{A}$$

$$\overline{R}_9 = (\overline{L}_3 \cdot \overline{A})\overline{A}$$

$$\overline{S}_i = \overline{R}_i - (\overline{R}_i \cdot \overline{M})\overline{M} \qquad i = 1, \ldots, 9$$

$$\overline{T}_i = \overline{M} \times \overline{R}_i \qquad i = 1, \ldots, 9$$

$$\overline{U}_i = (\overline{R}_i \cdot \overline{M})\overline{M} \qquad i = 1, \ldots, 9$$

$$\overline{V}_1 = \overline{C}_2 - (\overline{C}_2 \cdot \overline{Q})\overline{Q}$$

$$\overline{V}_2 = \overline{Q} \times \overline{C}_2$$

$$\overline{V}_3 = (\overline{C}_2 \cdot \overline{Q})\overline{Q}$$

This produces the equation found on page 7.

A.2 Constants for Displacement Analysis

$$\begin{aligned}\overline{X}_1 = {} & \cos\theta_2 (\cos\theta_3\overline{S}_1 - \sin\theta_3\overline{S}_4 + \overline{S}_7) \\ & - \sin\theta_2 (\cos\theta_3\overline{T}_1 - \sin\theta_3\overline{T}_4 + \overline{T}_7) \\ & + (\cos\theta_3\overline{U}_2 - \sin\theta_3\overline{U}_5 + \overline{U}_8)\end{aligned}$$

$$\begin{aligned}\overline{X}_2 = {} & \cos\theta_2 (\cos\theta_3\overline{S}_2 - \sin\theta_5\overline{S}_5 + \overline{S}_8) \\ & - \sin\theta_2 (\cos\theta_3\overline{T}_2 - \sin\theta_3\overline{T}_5 + \overline{T}_8) \\ & + (\cos\theta_3\overline{U}_2 - \sin\theta_3\overline{U}_5 + \overline{U}_8)\end{aligned}$$

$$\overline{X}_3 = \cos\theta_2 (\cos\theta_3\overline{S}_3 - \sin\theta_3\overline{S}_6 + \overline{S}_9)$$
$$- \sin\theta_2 (\cos\theta_3\overline{T}_3 - \sin\theta_3\overline{T}_6 + \overline{T}_9)$$
$$+ (\cos\theta_3\overline{U}_3 - \sin\theta_3\overline{U}_6 + \overline{U}_9)$$
$$\overline{X}_4 = \cos\theta_5 (\overline{V}_1) + \sin\theta_5 (\overline{V}_2) + (\overline{V}_3)$$

A.3 Constants for Velocity Analysis

$$\overline{W}_1 = \overline{V}_2 \cos\theta_5 - \overline{V}_1 \sin\theta_5$$
$$\overline{W}_2 = -\sin\theta_4 [\cos\theta_2 (\cos\theta_3\overline{S}_1 - \sin\theta_3\overline{S}_4 + \overline{S}_7)$$
$$-\sin\theta_2 (\cos\theta_3\overline{T}_1 - \sin\theta_3\overline{T}_4 + \overline{T}_7)$$
$$+ (\cos\theta_3\overline{U}_1 - \sin\theta_3\overline{U}_4 + \overline{U}_7)]$$
$$-\cos\theta_2 [\cos\theta_2(\cos\theta_3\overline{S}_2 - \sin\theta_3\overline{S}_5 + \overline{S}_8$$
$$-\sin\theta_2(\cos\theta_3\overline{T}_2 - \sin\theta_3\overline{T}_5 + \overline{T}_8)$$
$$+ (\cos\theta_3\overline{U}_2 - \sin\theta_3\overline{U}_5 + \overline{U}_8)]$$
$$\overline{W}_3 = \cos\theta_4 [-\dot{\theta}_2 \sin\theta_2 (\cos\theta_3\overline{S}_1 - \sin\theta_3\overline{S}_4 + \overline{S}_7)$$
$$-\dot{\theta}_2 \cos\theta_2 (\cos\theta_3\overline{T}_1 - \sin\theta_3\overline{T}_4 + \overline{T}_7)$$
$$+ \cos\theta_2 (-\dot{\theta}_3 \sin\theta_3\overline{S}_1 - \dot{\theta}_3 \cos\theta_3\overline{S}_4)$$
$$-\sin\theta_2 (-\dot{\theta}_3 \sin\theta_3\overline{T}_1 - \dot{\theta}_3 \cos\theta_3\overline{T}_4)$$
$$+ (-\dot{\theta}_3 \sin\theta_3\overline{U}_1 - \dot{\theta}_3 \cos\theta_3\overline{U}_4)]$$
$$\sin\theta_4 [-\dot{\theta}_2 \sin\theta_2 (\cos\theta_3\overline{S}_2 - \sin\theta_3\overline{S}_5 + \overline{S}_8)$$
$$-\dot{\theta}_2 \cos\theta_2 (\cos\theta_3\overline{T}_2 - \sin\theta_3\overline{T}_5 + \overline{T}_8)$$
$$+ \cos\theta_2 (-\dot{\theta}_3 \sin\theta_3\overline{S}_2 - \dot{\theta}_3 \cos\theta_3\overline{S}_5)$$
$$- \sin\theta_2 (-\dot{\theta}_3 \sin\theta_3\overline{T}_2 - \dot{\theta}_3 \cos\theta_3\overline{T}_5)$$
$$+ (-\dot{\theta}_3 \sin\theta_3\overline{U}_2 - \dot{\theta}_3 \cos\theta_3\overline{U}_5)]$$
$$+ -\dot{\theta}_2 \sin\theta_2 (\cos\theta_3\overline{S}_3 - \sin\theta_3\overline{S}_6 + \overline{S}_9)$$
$$-\dot{\theta}_2 \cos\theta_2 (\cos\theta_3\overline{T}_3 - \sin\theta_3\overline{T}_6 + \overline{T}_9)$$

$$+ \cos\theta_2 \, (-\dot{\theta}_3 \sin\theta_3 \overline{S}_3 - \dot{\theta}_3 \cos\theta_3 \overline{S}_6)$$

$$- \sin\theta_2 \, (-\dot{\theta}_3 \sin\theta_3 \overline{T}_3 - \dot{\theta}_3 \cos\theta_3 \overline{T}_6)$$

$$+ (-\dot{\theta}_3 \sin\theta_3 \overline{U}_3 - \dot{\theta}_3 \cos\theta_3 \overline{U}_6)$$

A.4 Constants for Acceleration Analysis

$$\overline{Z}_1 = \cos\theta_5 \overline{V}_2 - \sin\theta_5 \overline{V}_1$$

$$\overline{Z}_2 = -\dot{\theta}_5^2 \cos\theta_5 \overline{V}_1 - \dot{\theta}_5^2 \sin\theta_5 \overline{V}_2$$

$$\overline{Z}_3 = \sin\theta_4 \, [\cos\theta_2 \, (\cos\theta_3 \overline{S}_1 - \sin\theta_3 \overline{S}_4 + \overline{S}_7)$$

$$- \sin\theta_2 \, (\cos\theta_3 \overline{T}_1 - \sin\theta_3 \overline{T}_4 + \overline{T}_7)$$

$$+ (\cos\theta_3 \overline{U}_1 - \sin\theta_3 \overline{U}_4 + \overline{U}_7)]$$

$$-\cos\theta_4 \, [\cos\theta_2 \, (\cos\theta_3 \overline{S}_2 - \sin\theta_3 \overline{S}_5 + \overline{S}_8)$$

$$-\sin\theta_s \, (\cos\theta_3 \overline{T}_2 - \sin\theta_3 \overline{T}_5 + \overline{T}_8)$$

$$+ (\cos\theta_3 \overline{U}_2 - \sin\theta_3 \overline{U}_5 + \overline{U}_8)]$$

$$\overline{Z}_4 = -\dot{\theta}_4^2 \cos\theta_4 \, [\cos\theta_2 \, (\cos\theta_3 \overline{S}_1 - \sin\theta_3 \overline{S}_4 + \overline{S}_7)$$

$$-\sin\theta_2 \, (\cos\theta_3 \overline{T}_1 - \sin\theta_3 \overline{T}_4 + \overline{T}_7)$$

$$+ (\cos\theta_3 \overline{U}_1 - \sin\theta_3 \overline{U}_4 + \overline{U}_7)]$$

$$-\dot{\theta}_4^2 \sin\theta_4 \, [\cos\theta_2 \, (\cos\theta_3 \overline{S}_2 - \sin\theta_3 \overline{S}_5 + \overline{S}_8)$$

$$-\sin\theta_2 \, (\cos\theta_3 \overline{T}_2 - \sin\theta_3 \overline{T}_5 + \overline{T}_8)$$

$$+ (\cos\theta_3 \overline{U}_2 - \sin\theta_3 \overline{U}_5 + \overline{U}_8)]$$

$$-2\dot{\theta}_4 \sin\theta_4 \, [-\dot{\theta}_2 \sin\theta_2 (\cos\theta_3 \overline{S}_1 - \sin\theta_3 \overline{S}_4 + \overline{S}_7)$$

$$-\dot{\theta}_2 \cos\theta_2 (\cos\theta_3 \overline{T}_1 - \sin\theta_3 \overline{T}_4 + \overline{T}_7)$$

$$+ \cos\theta_2 (-\dot{\theta}_3 \sin\theta_3 \overline{S}_1 - \dot{\theta}_3 \cos\theta_3 \overline{S}_4$$

$$-\sin\theta_2 (-\dot{\theta}_3 \sin\theta_3 \overline{T}_1 - \theta_3 \cos\theta_3 \overline{T}_9)$$

$$+ (-\dot{\theta}_3 \sin\theta_3 \overline{U}_1 - \theta_3 \cos\theta_3 \overline{U}_4)]$$

$$-2\dot{\theta}_4 \cos\theta_4 \, [-\dot{\theta}_2 \sin\theta_2 (\cos\theta_3 \overline{S}_2 - \sin\theta_3 \overline{S}_5 + \overline{S}_8)$$

$$-\dot{\theta}_2 \cos\theta_2 (\cos\theta_3 \overline{T}_2 - \sin\theta_3 \overline{T}_5 + \overline{T}_8)$$

$$+ \cos \theta_2(-\dot{\theta}_3 \sin \theta_3\overline{S}_2 - \dot{\theta}_3 \cos \theta_3\overline{S}_5)$$

$$-\sin \theta_2(-\dot{\theta}_3 \sin \theta_3\overline{T}_2 - \dot{\theta}_3 \cos \theta_3\overline{T}_5)$$

$$+ (-\dot{\theta}_3 \sin \theta_3\overline{U}_2 - \dot{\theta}_3 \cos \theta_3\overline{U}_5)]$$

$$+ \cos \theta_4 \; [-\ddot{\theta}_2 \sin \theta_2 \; (\cos \theta_3\overline{S}_1 - \sin \theta_3\overline{S}_4 + \overline{S}_7)$$

$$-\ddot{\theta}_2 \cos \theta_2 \; (\cos \theta_3\overline{T}_1 - \sin \theta_3\overline{T}_4 + \overline{T}_7)$$

$$-\dot{\theta}_2^2 \cos \theta_2 \; (\cos \theta_3\overline{S}_1 - \sin \theta_3\overline{S}_4 + \overline{S}_7)$$

$$+\dot{\theta}_2^2 \sin \theta_2 \; (\cos \theta_3\overline{T}_1 - \sin \theta_3\overline{T}_4 + \overline{T}_7)$$

$$-2\dot{\theta}_2 \cos \theta_2 \; (-\dot{\theta}_3 \sin \theta_3\overline{S}_1 - \dot{\theta}_3 \cos \theta_3\overline{S}_4)$$

$$-2\dot{\theta}_2 \cos \theta_2 \; (-\dot{\theta} \;\; \sin \theta_3\overline{T}_1 - \dot{\theta}_3 \cos \theta_3\overline{T}_4)$$

$$+ \cos \theta_2 \; (-\ddot{\theta}_3 \sin \theta_3\overline{S}_1 - \ddot{\theta}_3 \cos \theta_3\overline{S}_4)$$

$$-\sin \theta_2 \; (-\ddot{\theta}_3 \sin \theta_3\overline{T}_1 - \ddot{\theta}_3 \cos \theta_3\overline{T}_4)$$

$$+ (-\ddot{\theta}_3 \sin \theta_3\overline{U}_1 - \ddot{\theta}_3 \cos \theta_3\overline{U}_4)$$

$$+ \cos \theta_2 \; (-\dot{\theta}_3^2 \cos \theta_3\overline{S}_1 + \dot{\theta}_3^2 \sin \theta_3\overline{S}_4)$$

$$-\sin \theta_2 \; (-\dot{\theta}_3^2 \cos \theta_3\overline{T}_1 + \dot{\theta}_3^2 \sin \theta_3\overline{T}_4)$$

$$+ (-\dot{\theta}_3^2 \cos \theta_3\overline{U}_1 + \dot{\theta}_3^2 \sin \theta_3\overline{U}_4)]$$

$$-\sin \theta_4 \; [-\ddot{\theta}_2 \sin \theta_2 \; (\cos \theta_3\overline{S}_2 - \sin \theta_3\overline{S}_5 + \overline{S}_8)$$

$$-\ddot{\theta}_2 \cos \theta_2 \; (\cos \theta_3\overline{T}_2 - \sin \theta_3\overline{T}5 + \overline{T}_8)$$

$$-\dot{\theta}_2^2 \cos \theta_2 \; (\cos \theta_3\overline{S}_2 - \sin \theta_3\overline{S}_5 + \overline{S}_8)$$

$$+\dot{\theta}_2^2 \sin \theta_2 \; (\cos \theta_3\overline{T}_2 - \sin \theta_3\overline{T}_5 + \overline{T}_8)$$

$$-2\dot{\theta}_2 \sin \theta_2 \; (-\dot{\theta}_3 \sin \theta_3\overline{S}_2 - \dot{\theta}_3 \cos \theta_3\overline{S}_5)$$

$$-2\dot{\theta}_2 \cos \theta_2 \; (-\dot{\theta}_3 \sin \theta_3\overline{T}_2 - \dot{\theta}_3 \cos \theta_3\overline{T}_5)$$

$$+ \cos \theta_2 \; (-\ddot{\theta}_3 \sin \theta_3\overline{S}_2 - \ddot{\theta}_3 \cos \theta_3\overline{S}_5)$$

$$-\sin \theta_2 \; (-\ddot{\theta}_3 \sin \theta_3\overline{T}_2 - \ddot{\theta}_3 \cos \theta_3\overline{T}_5)$$

$$+ (-\ddot{\theta}_3 \sin \theta_3\overline{U}_2 - \ddot{\theta}_3 \cos \theta_3\overline{U}_5)$$

$$+ \cos \theta_2 \; (-\dot{\theta}_3^2 \cos \theta_3\overline{S}_2 + \dot{\theta}_3^2 \sin \theta_3\overline{S}_5)$$

$$+ \sin \theta_2 \; (-\dot{\theta}_3^2 \cos \theta_3\overline{T}_2 + \dot{\theta}_3^2 \sin \theta_3\overline{T}_5)$$

$$+ (-\dot{\theta}_3^2 \cos \theta_3\overline{U}_2 + \dot{\theta}_3^2 \sin \theta_3\overline{U}_5)]$$

$$+ -\ddot{\theta}_2 \sin\theta_2 \, (\cos\theta_3 \bar{S}_3 - \sin\theta_3 \bar{S}_6 + \bar{S}_9)$$
$$-\ddot{\theta}_2 \cos\theta_2 \, (\cos\theta_3 \bar{T}_3 - \sin\theta_3 \bar{T}_6 + \bar{T}_9)$$
$$-\dot{\theta}_2^2 \cos\theta_2 \, (\cos\theta_3 \bar{S}_3 - \sin\theta_3 \bar{T}_6 + \bar{T}_9)$$
$$+\dot{\theta}_2^2 \sin\theta_2 \, (\cos\theta_3 \bar{T}_3 - \sin\theta_3 \bar{T}_6 + \bar{T}_9)$$
$$-2\dot{\theta}_2 \sin\theta_2 \, (-\dot{\theta}_3 \sin\theta_3 \bar{S}_3 - \dot{\theta}_3 \cos\theta_3 \bar{S}_6)$$
$$-2\dot{\theta}_2 \sin\theta_2 \, (-\dot{\theta}_3 \sin\theta_3 \bar{T}_3 - \dot{\theta}_3 \cos\theta_3 \bar{T}_6)$$
$$+ \cos\theta_2 \, (-\ddot{\theta}_3 \sin\theta_3 \bar{S}_3 - \ddot{\theta}_3 \cos\theta_3 \bar{S}_6)$$
$$-\sin\theta_2 \, (-\ddot{\theta}_3 \sin\theta_3 \bar{T}_3 - \theta_3 \cos\theta_3 \bar{T}_6)$$
$$+ (-\ddot{\theta}_3 \sin\theta_3 \bar{U}_3 - \ddot{\theta}_3 \cos\theta_3 \bar{U}_6)$$
$$+ \cos\theta_2 \, (-\dot{\theta}_3^2 \cos\theta_3 \bar{S}_3 + \dot{\theta}_3^2 \sin\theta_3 \bar{S}_6)$$
$$-\sin\theta_2 \, (-\dot{\theta}_3^2 \cos\theta_3 \bar{T}_3 + \dot{\theta}_3^2 \sin\theta_3 \bar{T}_6)$$
$$+ (-\dot{\theta}_3^2 \cos\theta_3 \bar{U}_3 + \dot{\theta}_3^2 \sin\theta_3 \bar{U}_6)$$

APPENDIX B

CONSTANTS FOR RIGID

BODY GUIDANCE

B.1 Definition of T's for General

Equation, MAB Side

$$T_1 = (1 - \cos\theta_2)(1 - \cos\theta_3)$$
$$T_2 = (1 - \cos\theta_2)(\cos\theta_3)$$
$$T_3 = (\cos\theta_2)(1 - \cos\theta_3)$$
$$T_4 = (\cos\theta_2)(\cos\theta_3)$$
$$T_5 = (1 - \cos\theta_2)(\sin\theta_3)$$
$$T_6 = (\cos\theta_2)(\sin\theta_3)$$
$$T_7 = (1 - \cos\theta_3)(\sin\theta_2)$$
$$T_8 = (\sin\theta_2)(\sin\theta_3)$$
$$T_9 = (\sin\theta_2)(\cos\theta_3)$$

B.2 Definition of E's for Two Position

Synthesis of Rigid Body Motion

for MAB Side

$$E_1 = A_x^2 M_x^2 T_1 + A_x^2 T_3 + M_x^2 T_2 + T_4$$
$$+ A_x A_y M_x M_y T_1 - A_x A_y M_z T_7 + A_z M_x M_y T_5 - A_z M_z T_8$$
$$+ A_x A_z M_x M_z T_1 + A_x A_z M_y T_7 - A_y M_x M_z T_5 - A_y M_y T_8$$
$$- a_{11}$$

$$E_2 = A_xA_yM_x^2T_1 + A_xA_yT_3 - A_zM_x^2T_5 - A_zT_6$$
$$+ A_y^2M_xM_yT_1 - A_y^2M_zT_y + M_xM_yT_2 - M_zT_9$$
$$+ A_yA_zM_xM_zT_1 + A_yA_zM_yT_7 + A_xM_xM_zT_5 + A_xM_yT_8$$
$$- a_{12}$$
$$E_3 = A_xA_zM_x^2T_1 + A_xA_zT_3 + A_yM_x^2T_5 + A_yT_6$$
$$+ A_yA_zM_xM_yT_1 - A_yA_zM_zT_7 - A_xM_xM_yT_5 + A_xM_zT_8$$
$$+ A_z^2M_xM_zT_1 + A_z^2M_yT_7 + M_xM_zT_2 + M_yT_9$$
$$- a_{13} = 0$$
$$E_4 = A_x^2M_xM_yT_1 + A_x^2M_zT_7 + M_xM_yT_2 + M_zT_9$$
$$+ A_xA_yM_y^2T_1 + A_xA_yT_3 + A_zM_y^2T_5 + A_zT_6$$
$$+ A_xA_zM_yM_zT_1 - A_xA_zM_xT_7 - A_yM_yM_zT_5 + A_yM_xT_8$$
$$- a_{21}$$
$$E_5 = A_xA_yM_xM_yT_1 + A_xA_yM_zT_7 - A_zM_xM_yT_5 - A_zM_zT_8$$
$$+ A_y^2M_y^2T_1 + A_y^2T_3 + M_y^2T_2 + T_4$$
$$+ A_yA_zM_yM_zT_1 - A_yA_zM_xT_7 + A_xM_yM_zT_5 - A_xM_xT_8$$
$$- a_{22}$$
$$E_6 = A_xA_zM_xM_yT_1 + A_xA_zT_7 + A_yM_xM_yT_5 + A_yM_zT_8$$
$$+ A_yA_zM_y^2T_1 + A_yA_zT_3 - A_xM_y^2T_5 - A_xT_6$$
$$+ A_z^2M_yM_zT_1 - A_z^2M_xT_7 + M_yM_zT_z - M_xT_9$$
$$- a_{23}$$

B.3 Definition of the Constants D

$$D_{1N} = (a_{11} - 1)C_x/C_z + a_{12}C_y/C_z + a_{13}$$
$$D_{2N} = a_{21}C_x/C_z + (a_{22} - 1)C_y/C_z + a_{23}$$
$$D_{3N} = a_{31}C_x/C_z + a_{32}C_y/C_z + (a_{33} - 1) \quad \text{(for n = 2,3)}$$

where:

$$a_{11} = S_{xN}^2 \ (1 - \cos \Phi_{1N}) + \cos \Phi_{1N}$$

$$a_{12} = S_{xN}S_{yN} \ (1 - \cos \Phi_{1N}) - S_{zN}S_{1N}\Phi_{1N}$$

$$a_{13} = S_{xN}S_{zN} \ (1 - \cos \Phi_{1N}) + S_{yN}S_{1N}\Phi_{1N}$$

$$a_{21} = S_{xN}S_{yN} \ (1 - \cos \Phi_{1N}) + S_{zN}s_{1N}\Phi_{1N}$$

$$a_{22} = S_{yN}^2 \ (1 - \cos \Phi_{1N}) + \cos$$

$$a_{23} = S_{yN}S_{zN} \ (1 - \cos \Phi_{1N}) + S_{xN}S_{1N}\Phi_{1N}$$

$$a_{31} = S_{xN}S_{zN} \ (1 - \cos \Phi_{1N}) \ - S_{yN}S_{1N}\Phi_{1N}$$

$$a_{32} = S_{yN}S_{zN} \ (1 - \cos \Phi_{1N}) + S_{xN}S_{1N}\Phi_{1N}$$

$$a_{33} = S_{zN}^2 \ (1 - \cos \Phi_{1N}) + \cos \Phi_{1N}$$

B.4 Definition of d's for Rigid Body Motion of the QC Side

$$d_{1N} = (a_{11} - 1)C_x/C_z + a_{13}$$

$$d_{2N} = a_{12}$$

$$d_{3N} = a_{21}C_x/C_z + a_{23}$$

$$d_{4N} = a_{22} - 1$$

$$d_{5N} = a_{32}$$

$$d_{6N} = a_{31}C_x/C_z + (a_{33} - 1)$$

The a_{1N}'s are elements of the displacement matrix describing the rigid body motion.

B.5 Definition of t's for Linear Superposition

$$t_1 = N_1N_3$$

$$t_2 = \lambda_1(M_3N_1 + M_1N_3) + (L_3N_1 + L_1N_3)$$

$$t_3 = \lambda_1^2 M_1 M_3 + \lambda_1 (L_3 M_1 + L_1 M_3 - 1) + L_1 L_3$$

$$t_4 = N_2 N_3$$

$$t_5 = \lambda_1 (M_3 N_2 + M_2 N_3) + (L_3 N_2 + L_2 N_3 - 1)$$

$$t_6 = \lambda_1^2 M_2 M_3 + \lambda_1 (L_3 M_2 + L_2 M_3) + L_2 L_3$$

APPENDIX C

NEWTON-RAPHSON ITERATION TECHNIQUE FOR SETS OF NON-LINEAR EQUATIONS

Given two non-linear equations,

$$f(x,y) = 0$$

and $$g(x,y) = 0$$

in two unknowns x and y. An iterative solution for x and y may be obtained by using the Newton-Raphson Technique. Let,

$$\partial f/\partial x = f_x$$

$$\partial f/\partial y = f_y$$

$$\partial g/\partial x = g_x$$

$$\partial g/\partial y = g_y$$

Let x = r and y = s be roots, and expand both functions in Taylor Series form about point (x, y) in terms of (r - x) and (s - y). Where (x, y) is a point in the neighborhood of the root (r, s).

Then,

$$f(r,s)=0=f(x_1,y_1)+f_x(x_1,y_1)(r-x_1)+f_y(x_1,y_1)(s-y_1)+ \ldots$$

and

$$g(r,s)=0=g(x_1,y_1)+g_x(y_1,y_1+(r-x_1)+g_y(x_1,y_1)(s-y_1)+\ldots$$

Let:

$$r-x_1 = \Delta_x$$

and

$$s-y_1 = \Delta_y$$

So that the Taylor Series expansion ending with the first partials is represented in matrix form:

$$\begin{bmatrix} f_x(\Delta x) & f_y(\Delta y) \\ g_x(\Delta x) & g_y(\Delta y) \end{bmatrix} = \begin{bmatrix} -f \\ -g \end{bmatrix}$$

This provides the corrected solution for (x_2, y_2):

$$x_2 = x_1 + \Delta x$$

$$y_2 = y_1 + \Delta y$$

Solving for Δx and Δy:

$$\Delta x = \frac{\begin{vmatrix} -f & f_y \\ -g & y_y \end{vmatrix}}{\begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}}$$

and

$$\Delta y = \frac{\begin{vmatrix} f_x & -f \\ g_x & -g \end{vmatrix}}{\begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}}$$

will provide a correction to the initial estimates, resulting in an answer closer to the real root. By repeating this procedure several times, an answer may be determined sufficiently close to the real root

APPENDIX D

COMPUTER SOLUTIONS FOR SYNTHESIS PROBLEMS OF A GEARED SPHERICAL FIVE-LINK MECHANISM

D.1 Rigid Body Guidance

Displacement Matrix for Position 1-2

0.848235274	-0.418857144	0.324122778
-0.140815306	-0.768326496	-0.624375963
0.510556527	0.483976328	-0.710703157

Displacement Matrix for Position 1-3

0.922058397	0.251532581	-0.294176319
0.350445872	-0.865189829	0.358656169
-0.164304593	-0.433794788	-0.885904025

Displacement Matrix for Position 1-4

0.49202976	-0.376829005	-0.78479127
0.692799582	0.715384221	0.090852029
0.527195799	-0.588409099	0.613057282

Displacement Matrix for Position 1-5

0.163691592	-0.077171637	-0.993488332
0.984524696	-0.050461101	0.167822524
-0.06247914	-0.995739933	0.06771754

RIGID BODY GUIDANCE FOR THE GEARED SIDE OF A

GEARED SPHERICAL FIVE-LINK MECHANISM

USING THE NEWTON-RAPHSON TECHNIQUE FOR SETS OF NON-LINEAR EQUATIONS

FOR 4 POSITIONS OF A RIGID BODY

HAVING A GEAR RATIO OF 2

****

INITIAL VALUES AND ESTIMATES OF POINTS IN THE INITIAL POSITION

```
B1= Ø     B2=-Ø.7Ø71Ø6781   B3=-Ø7Ø71Ø6781
A1= Ø.7Ø71Ø6781   A2=-Ø.5  A3=-Ø.5
M1= 1     M2= Ø     M3= Ø
```

INITIAL VALUES OR ESTIMATES FOR INPUT ROTATIONS (DEG) ABOUT M

```
THETA 12= 165
THETA 13= 190
THETA 14= 245
```

****

VALUES FOR THE 1   TH CALCULATION

FOR THE FUNCTION F

```
F( 1    )= 1.17ØØ8E-Ø6
F( 2    )=-2.Ø8ØØ1E-Ø6
F( 3    )=-4.92474E-Ø6
F( 4    )= 2.18516E-Ø6
F( 5    )= 1.2813ØE-Ø5
F( 6    )=-7.73942E-Ø6
F( 7    )=-5.3ØØØØE-1Ø
F( 8    )=-2.6ØØØØE-1Ø
F( 9    )= Ø
```

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

```
C( 1    )= 2.92279E-Ø4
C( 2    )= 8.74616E-Ø5
C( 3    )=-8.74725E-Ø5
C( 4    )=-8.82444E-Ø5
C( 5    )=-2.Ø3938E-Ø4
C( 6    )= 7.91441E-Ø5
C( 7    )= Ø
C( 8    )=-6.31Ø59E-Ø5
C( 9    )= 5.Ø5Ø51E-Ø5
```

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

O(1)= 164.9998129   O(2)= 189.9996803   O(3)= 244.9996180

B1= 2.92279E-04   B2=-0.707019319   B3=-0.707194254
A1= 0.707018537   A2=-0.500203938   A3=-0.499920856
M1= 1   M2=-6.31059E-05   M3= 5.05051E-05

VECTOR B= 1.000000116
VECTOR A= 1.000000053
VECTOR M= 1.000000007

****

CQ Side

END OF RUN
ERROR CODE = 3
FOR EQUATION
696883 +-808041.5   *Q+ 1390885.62   *Q+2+ 1633917.529   *Q+3+
-1.30000E-05   Q+4

REAL ROOTS	IMAGIMARY ROOTS
0.2814738l0	-0.471551323
0.281475810	0.471551323
-1.414209775	0

CX/CZ = 0

LAMDA1=-1.414209775   LAMDA22=-1.000002217

O1= 0
O2=-0.707110853
O3= 0.707102709

Q1= 0.707103781
Q2= 0.499999718
Q3= 0.500004525

RIGID BODY GUIDANCE FOR THE GEARED SIDE OF A

GEARED SPHERICAL FIVE-LINK MECHANISM

USING THE NEWTON-RAPHSON TECHNIQUE FOR SETS OF NON-LINEAR EQUATIONS

FOR 5 POSITIONS OF A RIGID BODY

HAVING A GEAR RATIO OF 2

****

INITIAL VALUES AND ESTIMATES OF POINTS IN THE INITIAL POSITION

```
B1= Ø     B2=-Ø.7Ø71Ø6781     B3=-Ø.7Ø71Ø6781
A1= Ø.7Ø71Ø6781     A2=-Ø.45     A3=-Ø.55
M1= 1     M2= Ø     M3= Ø

------------------------------------------------------------------------

INITIAL VALUES OR ESTIMATES FOR INPUT ROTATIONS (DEG) ABOUT M

THETA 12= 165
THETA 13= 19Ø
THETA 14= 24Ø
THETA 15= 3Ø5

************************************************************************

VALUES FOR THE 1    TH CALCULATION

FOR THE FUNCTION F

F( 1   )= Ø.Ø353569
F( 2   )=-Ø.Ø241858
F( 3   )=-Ø.132631
F( 4   )=-Ø.Ø125394
F( 5   )=-3.34724E-Ø3
F( 6   )=-2.457Ø6E-Ø3
F( 7   )=-Ø.15856
F( 8   )=-0.137267
F( 9   )=-5.3ØØØØE-1Ø
F( 1Ø  )= 4.99999E-Ø3

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

C( 1   )=-Ø.Ø968723
C( 2   )=-Ø.Ø144711
C( 3   )= Ø.Ø1447Ø7
C( 4   )=-Ø.Ø7123Ø3
C( 5   )=-Ø.Ø98Ø293
C( 6   )=-6.Ø9Ø31E-Ø3
C( 7   )=-Ø.Ø145326
C( 8   )= Ø.Ø16Ø144
C( 9   )= Ø.Ø61147
C( 1Ø  )=-Ø.Ø695924

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

O(1)= 164.1667322    O(2)= 190.9169858    O(3)= 243.5Ø3Ø522    O(4)=
 3Ø1.Ø11972

B1=-Ø.Ø968723    B2=-Ø.721577881    B3=-Ø.692636Ø81
A1= 0.635876481     A2=-Ø.5489293  A3=-Ø.556Ø9Ø31
M1= 1     M2= Ø      M3= Ø
```

VECTOR B= 1.ØØ98Ø3622
VECTOR A= 1.Ø148987Ø8
VECTOR M= 1

NORMALIZED INPUT VALUES AND CORRECTED ESTIMATES

B1=-Ø.Ø964ØØ913 B2=-Ø.718Ø66639 B3=-Ø.689265671
A1= Ø.631191893 A2=-Ø.544885264 A3=-Ø.551993518
M1= 1 M2= Ø M3= Ø

VECTOR B= 1
VECTOR A= 1
VECTOR M= 1

VALUES FOR THE 2 TH CALCULATION

FOR THE FUNCTION F

F(1)= 8.3423ØE-Ø5
F(2)= 1.7Ø567E-Ø3
F(3)=-3.89242E-Ø3
F(4)= 5.265Ø6E-Ø3
F(5)=-2.Ø285ØE-Ø3
F(6)= 1.Ø6672E-Ø3
F(7)= 4.Ø719ØE-Ø4
F(8)=-1.6Ø4Ø2E-Ø3
F(9)=-3.ØØØØØE-11
F(1Ø)=-2.ØØØØØE-11

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

C(1)=-Ø.149Ø91
C(2)=-Ø.Ø583194
C(3)= Ø.Ø815969
C(4)=-Ø.Ø413965
C(5)=-3.84855E-Ø3
C(6)=-Ø.Ø435365
C(7)=-Ø.Ø114792
C(8)= Ø.Ø136387
C(9)= 1.48476E-Ø3
C(1Ø)= Ø.1692

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

O(1)= 163.5Ø89767 O(2)= 191.6979273 O(3)=243.58785ØØ O(4)=
31Ø.7Ø64179

B1=-Ø.245491913 B2=-Ø.776386Ø39 B3=-Ø.6Ø7668771
A1= Ø.589795393 A2=-Ø.548733814 A3=-Ø.595530Ø18
M1= 1 M2= Ø M3= Ø

VECTOR B= 1.Ø323Ø2897
VECTOR A= 1.ØØ36234Ø6
VECTOR M= 1

NORMALIZED INPUT VALUES AND CORRECTED ESTIMATES

B1=-Ø.24162Ø411 B2=-Ø.764142129 B3=-Ø.598Ø85599
A1= Ø.588729754 A2=-Ø.547742365 A3=-Ø.594454Ø17
M1= 1 M2= Ø M3= Ø

VECTOR B= 1
VECTOR A= 1
VECTOR M= 1

VALUES FOR THE 3 TH CALCULATION

FOR THE FUNCTION F

F(1)= 4.94987E-Ø3
F(2)=-2.41Ø86E-Ø3
F(3)= 3.4Ø228E-Ø3
F(4)=-Ø.Ø187969
F(5)= 1.93948E-Ø3
F(6)=-4.522ØØE-Ø3
F(7)= 2.79Ø96E-Ø3
F(8)=-Ø.Ø2813Ø8
F(9)= 8.ØØØØØE-11
F(1Ø)= 8.ØØØØØE-11

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

C(1)= Ø.Ø426769
C(2)= 2.24733E-Ø3
C(3)=-Ø.Ø2Ø11Ø7
C(4)= Ø.Ø2853Ø4
C(5)= 9.Ø9581E-Ø3
C(6)= Ø.Ø198762
C(7)= 4.36545E-Ø3
C(8)=-6.5619ØE-Ø3
C(9)= 1.97477E-Ø3
C(1Ø)=-0.0391762

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

```
O(1)= 163.7587863   O(2)= 101.3214940   O(3)= 243.7007227   9(4)=
 308.4615693

B1=-0.198943511   B2=-0.761894799   B3=-0.618196299
A1= 0.617260154   A2=-0.538646555   A3=-0.574577817
M1= 1     M2= 0     M3= 0

VECTOR B= 1.002228869
VECTOR A= 1.001289877
VECTOR B= 1
```

NORMALIZED INPUT VALUES AND CORRECTED ESTIMATES

```
B1=-0.198722171   B2=-0.761047134   B3=-0.617508509
A1= 0.616862444   A2=-0.538299496   A3=-0.574207608
M1= 1     M2= 0     M3= 0

VECTOR B= 1
VECTOR A= 1
VECTOR M= 1
```

VALUES FOR THE 4   TH CALCULATION

FOR THE FUNCTION F

```
F( 1    )= 4.02949E-04
F( 2    )=-1.58632E-04
F( 3    )= 2.24839E-04
F( 4    )=-1.29345E-03
F( 5    )= 4.41853E-04
F( 6    )=-4.03972E-04
F( 7    )=-4.48451E-05
F( 8    )=-1.10785E-04
F( 9    )= 7.00000E-11
F( 10   )= 1.00000E-11
```

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

```
C( 1    )= 0.0171664
C( 2    )= 6.44831E-03
C( 3    )=-0.0134702
C( 4    )= 2.68357E-03
C( 5    )=-4.96640E-04
C( 6    )= 3.34847E-03
C( 7    )= 1.79375E-03
C( 8    )=-1.21257E-03
C( 9    )= 4.94062E-04
C( 10   )=-0.0202752
```

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

O(1)= 163.8613457   O(2)= 191.2515931   O(3)= 243.7287976   O(4)=
307.2996109

B1=-0.181555771   B2=-0.754598824   B3=-0.630978709
A1= 0.619546014   A2=-0.538796136   A3=-0.570859138
M1= 1   M2= 0   M3= 0

VECTOR B= 1.000516014
VECTOR A= 1.000018696
VECTOR M= 1

NORMALIZED INPUT VALUES AND CORRECTED ESTIMATES

B1=-0.181508946   B2=-0.754404207   B3=-0.630815975
A1= 0.619540223   A2=-0.538791100   A3=-0.570853802
M1= 1   M2= 0   M3= 0

VECTOR B= 1
VECTOR A= 1
VECTOR M= 1

VALUES FOR THE 5 TH CALCULATION

FOR THE FUNCTION F

F(1)= 7.36091E-05
F(2)=-2.75780E-05
F(3)= 1.04460E-05
F(4)=-3.12216E-04
F(5)=-2.30757E-05
F(6)=-2.59281E-05
F(7)= 5.89722E-05
F(8)=-4.53012E-04
F(9)=-5.00000E-11
F(10)= 0

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

C(1)=-3.86642E-04
C(2)=-5.79916E-04
C(3)= 8.04780E-04
C(4)= 5.24423E-04
C(5)= 3.77287E-04
C(6)= 2.13051E-04
C(7)=-1.62613E-05
C(8)=-5.59040E-05
C(9)= 1.58510E-04
C(10)= 1.05252E-03

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

O(1)= 163.8601998 O(2)= 191.2481554 O(3)= 243.7373920 O(4)= 307.3597715

B1=-0.181895588 B2=-0.754984123 B3=-0.630011195
A1= 0.620064646 A2=-0.538413813 A3=-0.570640751
M1= 1 M2= 0 M3= 0

VECTOR B= 1.000001137
VECTOR A= 1.000000465
VECTOR M= 1

NORMALIZED INPUT VALUES AND CORRECTED ESTIMATES

B1=-0.181895485 B2=-0.754983694 B3=-0.630010837
A1= 0.620065401 A2=-0.538413688 A3=-0.570640618
M1= 1 M2= 0 M3= 0

VECTOR B= 1
VECTOR A= 1
VECTOR M= 1

VALUES FOR THE 6 TH CALCULATION

FOR THE FUNCTION F

F(1)= 2.58089E-06
F(2)=-9.50508E-07
F(3)=-0.65782E-06
F(4)= 8.95462E-07
F(5)=-1.54680E-07
F(6)= 9.62521E-06
F(7)=-7.28057E-06
F(8)= 2.04944E-06
F(9)=-2.00000E-11
F(10)=-5.00000E-11

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

C(1)= 1.01663E-06
C(2)=-9.49546E-06
C(3)= 1.10861E-05
C(4)= 1.55446E-05
C(5)= 1.14327E-05
C(6)= 6.10396E-06
C(7)= 3.22449E-06
C(8)= 2.23591E-06
C(9)= 1.55902E-05
C(10)= 2.26013E-05

INPUT VALUES AND CORRECTED ESTIMATES FOR NEXT CALCULATION

O(1)= 163.86Ø1998 O(2)= 191.2481554 O(3)= 243.7379649 O(4)= 3Ø7.36Ø9174

B1=-Ø.181894468 B2=-Ø.75499319Ø B3=-Ø.62999975Ø
A1= Ø.62ØØ8ØØ46 A2=-Ø.5384Ø2255 A3=-Ø.57Ø634514
M1= 1 M2= Ø M3= Ø

VECTOR B= Ø.999999999
VECTOR A= 1
VECTOR M= 1

****

CQ Side

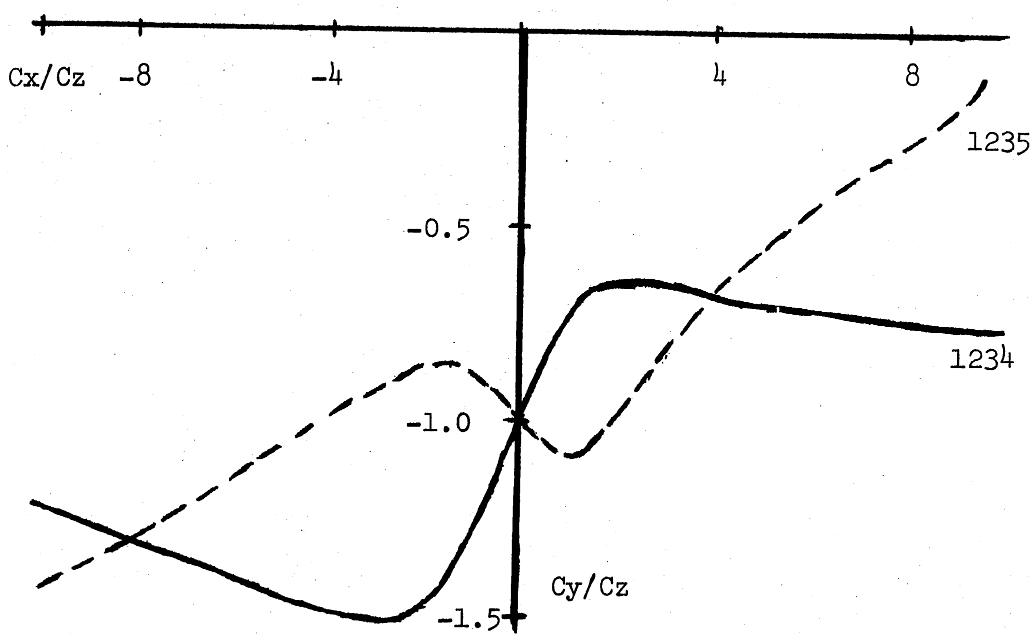

Figure 6. Circle Point Curves for Positions 12, 13, 14 and 12, 13, 15

D.2 PATH POINT GENERATION

P1=(.577350269,-.816496581,0)
P2=(.831724289,.546036202,-.100395069)
P3=(.326975171,.908754556,.259330660)
P4=(.591753109,-.184120746,.784810654)
P5=(.157517762,.609616916,.776888168)

S_{12}=(.081972231,.057963120,.994354186)
S_{13}=(.-.211742597,-.149724626,.791643797)
S_{14}=(-.640795216,-.453110642,.37686228)
S_{15}=(-.634326533,-.448536593,.480575204)

Q_{12}=88.03075945)
Q_{13}=123.5879102)
Q_{14}=60.52901277)
Q_{15}=114.0044255)

LET,

β_{12}=130.02
β_{13}=128.12
β_{14}=27.07
β_{15}=7.76

The resulting displacement matrices are identical to those obtained in the five position rigid body guidance problem. Therefore, the solutions are the same.

D.3 FUNCTION GENERATION

FUNCTION GENERATION FOR EIGHT POSITIONS

OF THE INPUT AND OUTPUT LINKS OF A

GEARED SPHERICAL FIVE-LINK MECHANISM

HAVING A GEAR RATIO OF 2

USING THE NEWTON-RAPHSON TECHNIQUE FOR SETS OF NON-LINEAR EQUATIONS

****

ESTIMATES OF POINTS IN THE INITIAL POSITION

M1= 0.707106781 M2= 0.707106781 M3= 0.000000000

A1= 1.000000000 A2= 0.000000000 A3= 0.000000000

B1= Ø.7Ø71Ø6781   B2=-Ø.7Ø71Ø6781   B3= Ø.ØØØØØØØØØ

C1= Ø.ØØØØØØØØØ   C2= Ø.ØØØØØØØØØ   C3= 1.ØØØØØØØØØ

Q1= Ø.ØØØØØØØØØ   Q2= 1.ØØØØØØØØØ   Q3= Ø.ØØØØØØØØØ

INPUT-OUTPUT ROTATIONS

POSITION	INPUT	OUTPUT
1- 2	1Ø.ØØØ	31.365
1- 3	2Ø.ØØØ	52.497
1- 4	3Ø.ØØØ	65.135
1- 5	4Ø.ØØØ	72.135
1- 6	5Ø.ØØØ	73.555
1- 7	6Ø.ØØØ	65.ØØ1
1- 8	7Ø.ØØØ	37.117

****

VALUES FOR THE 1   TH CALCULATION

FOR THE FUNCTIONS

F(1)=-6.55394E-Ø6
F(2)=-4.64776E-Ø6
F(3)=-1.58377E-Ø5
F(4)=-0.88581E-Ø7
F(5)=-5.83188E-Ø6
F(6)=-5.445Ø4E-Ø6
F(7)=-3.73264E-Ø6
F(8)=-5.3ØØØØE-1Ø
F(9)= Ø
F(1Ø)=-5.3ØØØØE-1Ø
F(11)= Ø

CORRECTION FACTORS FOR VARIABLES IN NEXT CALCULATION

C(1)=-Ø.Ø8Ø591148
C(2)= Ø.Ø8Ø591149
C(3)= Ø
C(4)= Ø.Ø68Ø92834
C(5)= Ø.171Ø44Ø88
C(6)=-Ø.Ø19366894
C(7)=-Ø.Ø19366894
C(8)= Ø.13Ø9Ø7895
C(9)=-Ø.Ø44537Ø71
C(1Ø)= Ø.Ø54182563
C(11)= Ø

CORRECTED ESTIMATES FOR NEXT CALCULATIONS

M1= Ø.626515633	M2= Ø.78769793Ø	M3= Ø.ØØØØØØØØØ
A1= 1.ØØØØØØØØØ	A2= Ø.Ø68Ø92834	A3= Ø.171Ø44Ø88
B1= Ø.687739887	B2=-Ø.726473675	B3= Ø.13Ø9Ø7895
C1=-Ø.Ø44537Ø71	C2= 0.054182563	C3= 1.ØØØØØØØØØ
Q1= Ø.ØØØØØØØØØ	Q2= 1.ØØØØØØØØØ	Q3= Ø.ØØØØØØØØØ

VECTOR Q= 1
VECTOR M= 1.Ø12989867
VECTOR A= 1.Ø33892714
VECTOR B= 1.Ø17887Ø3
VECTOR C= 1.ØØ49193

NORMALIZED INPUT VALUES AND CORRECTED ESTIMATES

M1= Ø.622485675	M2= Ø.782631193	M3= Ø.ØØØØØØØØØ
A1= Ø.983472595	A2= 0.066967436	A3= Ø.168217173
B1= Ø.68167Ø379	B2=-Ø.72ØØ6233Ø	B3= Ø.129752594
C1=-Ø.Ø44427928	C2= Ø.Ø54Ø49782	C3= Ø.997549387
Q1= Ø.ØØØØØØØØØ	Q2= 1.ØØØØØØØØØ	Q3= Ø.ØØØØØØØØØ

VITA

L. Michael McKee

Candidate for the Degree of

Master of Science

Thesis: SYNTHESIS OF A GEARED SPHERICAL FIVE-LINK MECHANISM

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Borger, Texas, October 31, 1951, the son of Mr. and Mrs. M. L. McKee.

Education: Graduated from Shidler High School, Shidler, Oklahoma, in May, 1969; received Bachelor of Science degree in Mechanical Engineering from Oklahoma State University in May, 1973; completed requirements for the Master of Science degree in July, 1975.

Professional Experience: Student Assistant in Mechanical Engineering Labs at Oklahoma State University, Summer and Fall 1971 and Spring 1972. Research Assistant, Oklahoma State University School of Mechanical Engineering, Fall 1973 and Spring 1974. Member of Technical Staff, Ingersoll-Rand Research, Inc., 1975.

Professional Organizations: Associate Member of the American Society of Mechanical Engineers.