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SHORT-TERM AND LONG-TERM PLANNING OF THE ALLOCATION

OF EDUCATIONAL RESOURCES IN A SECONDARY SCHOOL SYSTEM

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ABSTRACT

This research deals with the allocation of scarce economic resources in a secondary educational system. It is also concerned with a methodology for the analysis of the effects that resource allocation have on the variables used to measure the operation of a secondary school.

The first portion of the research deals with the similarities and differences of resource allocation in the educational environment and the traditional mercantile environment. This portion of the investigation identifies three basic improvements needed by educational resource management. They are 1) a quantitative process formula relating inputs to outputs, 2) better organization and analysis of existing data, and 3) a resource planning model for the local school.

The review of past research in the area of educational resources indicates that the development of a planning model with these improvements is feasible; however, a critical factor in the development will be the formulation of the weighting coefficients that are used in analyzing individual factors measuring educational operations. The formulation of the weighting coefficients in previous research did not appear to be satisfactory because the techniques involved making subjective judgments by the principal.

A model is then developed that eliminated the arbitrary determination of the weighting coefficients. This is done by assuming that

the present operating policy is optimal. In addition, the weighting coefficients are assumed to be given by ratios of the various costs associated with the problem's variables. This allows the application of certain mathematical techniques to the problem, such that, the weighting coefficients are found analytically without involving any subjective judgment.

The model development in this research would provide the following results to a secondary school: 1) enable the secondary school to quantitatively identify program costs for its present resource allocation, 2) furnish the administrator with important information concerning the economic requirements for implementation of future change or to realize future goals, 3) indicate what areas might be strengthened in the present system by identifying the manner in which present resources were allocated and 4) provide the state educational department with additional information to aid them in their allocative decisions.

The feasibility of the model is demonstrated by applying it to hypothetical data for a typical secondary school. The methods to plan quantitatively for future targets and/or changes are also described.

Finally the thesis discusses procedures for data gathering and the various types of sources that could be utilized to obtain these data. This data collection phase would be critical for the actual application of the planning model.

CHAPTER I

I. INTRODUCTION

Education is the largest industry in the United States and its total expenditures are exceeded only by national defense. Public awareness of the needs for and the opportunities through education has resulted in education receiving a larger share of the available resources each year. Today nearly ten per cent of the gross national product is being spent for education (36). Public secondary schools alone annually spent more than $11.4 billion dollars and "employ" more than seven per cent of the population (21). While considering the magnitude of school expenditures, Benson (3 , p. 15) raised several profound questions. Why does formal education cost represent ten per cent of the GNP, and how is this money distributed? Benson states that these matters are a topic of great interest to economists and are part of the body of knowledge relevant to the "allocation of resources".

II. The General Problem

The central topic of educational economics concerns the formulation of the most advantageous organization of the traditional scarce resources, which are labor, land, and capital.* In other words, the

*Viner (47) adds a fourth factor of production, technology, to the three traditional ones mentioned. This factor has been severely limited in its application to education and may be considered to be included in the capital component for the purpose of this research.

students, teachers, school facilities, curriculum, and time are incorporated into an operational master schedule. The limited school's financial resources are not exceeded, and the educational needs of the social and economic environments are satisfied.

There are a number of problems, however, that prevent the administrator from dealing with resource allocation in the usual manner.

1. The outputs of an educational system are not easily visible or readily measured.
2. The precise formula of the educational process, relating the inputs to the outputs, does not exist.
3. A data base of many of the variables associated with the educational process is lacking.
4. The organizational structure, size, clientele, staff expertise, and fiscal resources, differ for local schools across the nation.

The basic needs for the improvement of educational resource management appear to be:

1. A quantitative process formula that would indicate the effectiveness of various systems of educational resource allocation in satisfying the social and economic educational needs;
2. Improved methods of organizing and making detail analysis of the available data;
3. A model that would be realistic in applicability and feasibility for the local school.

III. Past Research

The absence of an adequate quantitative learning theory makes it most difficult to devise educational strategies for resource allocation. This, however, does not create an insurmountable barrier. Careful assesments of both the economics and the educational aspects of alternative educational programs are feasible. In recent years considerable progress has been made in estimating school inputs (11, p. 167).

Burkhead (6 , p. 27) started with the classic economic definition of economic inputs, land, labor, and capital. These were broken into the following:

1. Student Time
2. Personnel Time
    a. administration
    b. teaching
    c. clerical
    d. maintenance and operation
3. Materials and Supplies
    a. instructional
    b. other
4. Buildings and Equipment.

Various schemes have been formulated that attempted to give an indication of the distribution of resources and then used for comparative purposes. Some of the independent variables used by Burkhead in the Chicago Public High Schools study (6 , p. 49) were:

Age of school building,

Textbook expenditure per pupil,

Material and supplies expenditure per pupil,

Teacher salary,

Teacher man-years per pupil,

Administration man-years per pupil,

Auxiliary man-years per pupil.

This study indicated that the primary contribution by the school to student performance was the quality of the teacher, which was usually correlated with the teacher salary. Burkhead, (6 , p. 104) however, cautions that cost-effectiveness measures should not determine policy. These measures should, however, provide educational policy makers with a knowledge of the probable outcomes.

The findings of a study of the Michigan State Department of Education (37) agreed with the work of Burkhead. This study demonstrated that the independent variables having the strongest correlation with pupil performance were of the non-school nature. These were the variables representing the student's social and economic background. This study also showed that money made a difference only because the teachers' salaries level appeared to be related to the system's expenditure.

The above appears to represent the typical application of input-output techniques to education. Cohn commented that,

> the techniques could not, as yet, be used due to the inherent flaws in the analysis. This is not to say that such efforts are useless; nothing is farther from the truth. But for our purposes here, such a tool cannot be, as yet, used. (11, p. 167)

The inherent flaws that Cohn referred to are the inability to enumerate and quanitify the following:

> 1. A comprehensive list of all inputs entering the process; 2. A comprehensive list of all outputs (or outcomes) resulting from the process; and
> 3. The relationship between inputs and outputs, that is, the manner by which inputs are transferred into outputs. (10, p. 453)

An alternate to the input-output techniques is PPBS (Planning, Programming, and Budgeting System).

PPBS is currently being considered for educational resource allocation and recent attempts appear promising. Sisson and others (39), however, point out a number of difficulties. Two of the most serious ones were relating the planning process to the day-to-day operation and accounting for the interaction between projects. As in the input-output studies, PPBS application has had the difficulty of identifying the factors, called indicators, which will be used to judge the benefits of the educational activities (39, p. 240).

A more general approach, dealing with the allocation of resources, attempts to integrate an educational model into macro-economic growth models. These models are divided into four categories dealing with: 1. student flows; 2. teachers and class-rooms; 3. costs and finances; and 4. educational personnel needed for social development (12, p. 23). Such models have not been applied to individual schools or school districts. The reader interested in the large macro educational models should consult the references mentioned in Correa's paper (12) or Forrester's book (16).*

> *Forrester argues, though not so much in this book, that econometricians act in an almost anal-compulsive manner, being afraid to try to model anything that they cannot measure with considerable accuracy. He himself adopts the courageous attitude that it is better to guess at the values of important parameters that are hard to measure than to leave them out of the model. (38, p. 1014)

Lyle, in a survey, (29) listed the following six classical variables as achievement determinants in an education system:

1. Male teacher starting salary;
2. Number of books in the library:
3. Average number of years of teaching experience;
4. Average class size;
5. Teacher/Student ratio; and
6. Per cent of graduates going to college.

The above determinants do not appear to be independent, however. Obviously average class size and teacher/student ratio would be correlated. Because starting salary, library books, and compensation for teaching experience all require financial resources, there would presumably be some interaction between them.

One report that suggested a mathematical formulation of the interaction between teachers, students, and classrooms was given by Correa (12, p. 52). Referring to a large macro model, dealing with the flow of students and manpower requirements, Correa used the interaction concept to study the equilibrium between the supply and demands of teachers, and the unit cost per student to arrive at the national educational targets.

On a micro level, Stankard and Sisson (42) used interaction formulas in order to model the relationships between student performances and resource allocation. They hypothesized a number of process functions and combined them to arrive at an overall process function. Their primary difficulty was obtaining a reliable estimate of the large number of constants used in the final process function.* Actual costs were not considered in their model.

O'Brien (32) constructed a cost model for an urban school. O'Brien's model included cost of school construction, fixed and variable land cost, salaries and current operating expense, fixed and variable equipment cost, plus transportation cost. He did not relate these to, or incorporate the possibility of, varying teacher work load, teacher quality, or any curriculum variables. O'Brien's cost model would offer some interesting possibilities for finding the desired number and location of schools in a large municipal school district, such that, construction and transportation cost would be minimized.

Cohn (11), an economist at Pennsylvania State University, used the interaction concept to propose the following production function:

(Total product of the educational system) = f [Number of units taught,
Average teacher salary
Number of units per teacher,
Number of different subject matter assignments per teacher,
Average class size,
Random variation.]

This was similar to Stankard's and Sisson's work (42) except Cohn did not actually attempt to define the production function. Cohn, instead, proposed "barter terms of trade" which were the partial derivatives of one input with respect to another. As the sum of these marginal productions approached zero, the product of the educational

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*A lack of a sufficient data base has been previously mentioned.

system increased. As an example, he assumed that all inputs were constant except one, teachers. A larger number of teachers was negatively related to the total product through its effect on the average teacher salary, but positively related through its effect on both the number of subjects per teacher and the number of units per teacher. The amount of change each term would produce in the production function had to be weighted. By varying the number of teachers, while the other variables were held fixed, a suboptimal point was obtained. Then another variable was varied until another suboptimal was reached. It was required to return to the previous variables, and make readjustments. This was repeated until no change in any variable could improve the product. This procedure did not guarantee a global optimal and the only costs considered were teachers' salaries.

The main difficulty in Cohn's model would appear in trying to establish the various weighting coefficients. In order to explain this difficulty the example given by Cohn will be used. First it is necessary to give Cohn's notation (11, p. 168).

A = Number of subject matter assignments;

S = Number of sections per unit taught;

T = Number of teachers in the school;

U = Number of units taught;

F = Total amount of funds for teachers salaries;

A/T = Number of subject assignments per teacher;

F/T = Average teacher salary;

S·U/T = Number of courses per teacher;

Q = Total product of the educational system.

The example, as given by Cohn (11, Footnote 12, p. 174), is the following:

> Suppose we have initially the following T = 100, F/T = \$10,000, S•U = 200 (S•U/T = 2), and A = 50 (A/T = 0.5). All that we require of the principal, at this point, is to weight the possibilities of increasing Q by changing T <u>alone</u>. We might ask him the following question: If F/T were to be reduced to \$9,000, so that we can now hire 11 more teachers, would the reduction in Q due to the supposedly reduced quality of the <u>average</u> teacher be more or less than compensated for by the reduction in the teaching load (the new S•U/T is now only 1.8) and the increase in specialization (A/T is reduced to only 0.45)? If he is able to provide answers to such questions, marginal changes in T would then be made until a small change in T would result in no appreciable increase in Q.

In the evaluation of applicability of Cohn's model one must examine several of his assumptions caustiously. Although Professor Cohn does not specify the process function, if it is assumed that Q is the summation of each term, an elementary check of the units of each term will show them to be inconsistent. Another difficulty is that costs have not been considered and therefore would appear to leave unanswered the question: How can educational expenditures be allocated more efficiently? Moreover, because of the arbitrariness and personal judgment in the determination of the weighting coefficients, in addition to the lack of common units, it would seem that actual application of this model would be improbable. However, the concept of the existence of an equilibrium appears interesting. If expenditures could be incorporated and the above mentioned difficulties reduced, a school model that would be useful and applicable for a local school would appear feasible.

This study will develop a model that should provide the decision maker with the data and information concerning other alternatives. To provide the educational decision-maker with such a model this study will develop an analytical model which would transform economic data into operational alternatives. These alternatives should lead toward a more effective allocation of the limited educational resources.

CHAPTER II

PLANNING INDEX MODEL

I. Introduction

Cohn's (11) concept of a state of system equilibrium in a school is an interesting adaption of a concept that has been used in some of the large macro educational models (12, p. 52).

Barnard (2 , p. 240) states that a successful organization is one that can maintain a state of equilibrium such that its activities satisfy the individual needs sufficiently that they will be induced to continue their corporative activities. In other words, an equilibrium between the benefits and burdens exist. This same type of phenomenon should exist in an educational organization.

II. The Study

An equilibrium model of the secondary school was developed using Cohn's work (11) as an initial point of departure. This model did not attempt, however, to optimize any of the variables.

> Our understanding of the underlying structure of most complex systems is incomplete, and we are often unable to understand the interrelationships of all the factors bearing on the decision problem in question. To expect optimization in such a state of knowledge would be utter folly. (8 , p. 23)

The model tried to help explain how resources are presently allocated. This type of model should provide four kinds of results:

1. It should enable a secondary school to quantitatively appraise its present program;
2. It should furnish the administrator the necessary information to implement plans to realize future goals or accommodate expected changes;
3. It should provide an indication of what might be done to strengthen the present program;
4. It should provide state education departments with operating standards to aid in allocative decisions.

III. Development of the Model

In order to simulate adequately a secondary school educational system, the following variables were included in the model formulation.

x_1 = Number of classes taught;

x_2 = Number of teachers in the school;

x_3 = Number of different subjects;

x_4 = Number of enrollments;

x_4/x_3 = Average number of enrollments per subject;

x_1/x_2 = Average number of classes per teacher;

x_3/x_2 = Average number of subjects per teacher;

x_4/x_1 = Average number of enrollments per class.

Classrooms and space variables were not included at this point in order to simplify the presentation of the proposed model.

> Most school systems are considerably more sophisticated in their planning for building programs than they are in planning for operating programs. Both are important, and there are some evident interrelationships between the two. A program-performance approach, together with conventional estimates of

> future enrollment, will provide an important part of the data necessary for planning both long range capital and operating programs. (6 , p. 104)

In order to describe the above terms, an example was used. A secondary school that teaches in three areas, English, Math, and Social Studies was employed. There were four levels of English, three levels of Math, and two levels of Social Studies. Each level of English and Social Studies have two identical sections, and each level of Math has one section. A diagram of this is given below:

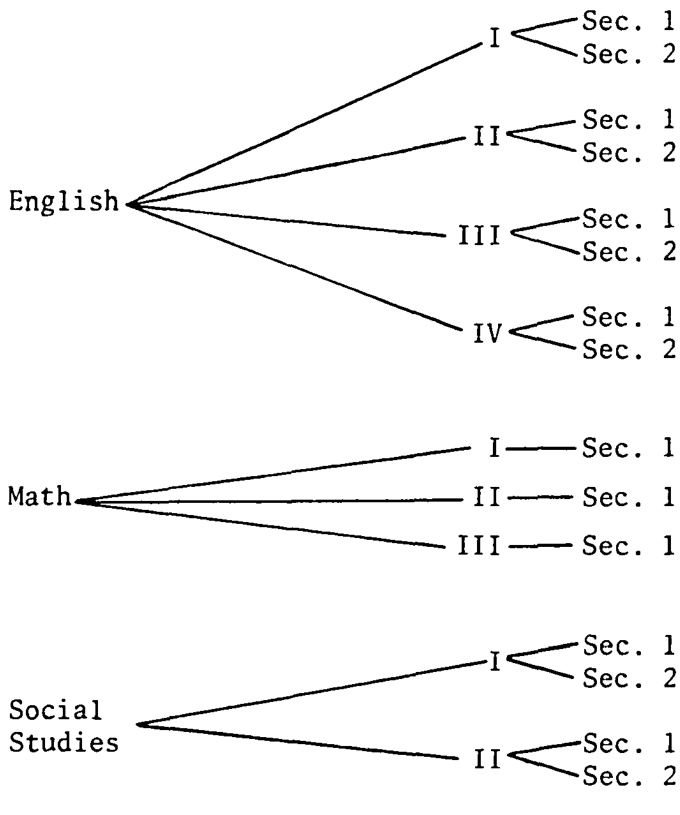

Figure 2-1. Diagram of Subjects, Courses, and Sections.

Using this basic course structure, one can assign typical values to the variables so that x_1 = 15 classes.

If it is assumed that the courses in each area were similar in content and required approximately the same skill to teach them then,

$$x_3 = 3 \text{ subjects.*}$$

If there were three teachers in this school and the average student enrolled in four classes, where the average daily attendance is sixty, then,

$$x_2 = 3 \text{ teachers, and}$$

$$x_4 = 240 \text{ enrollments.}$$

Also, it would follow that

$$x_4/x_3 = 80 \text{ enrollment per subject,}$$

$$x_1/x_2 = 15/3 = 5 \text{ classes per teacher,}$$

$$x_3/x_2 = 3/3 = 1 \text{ subject per teacher,** and}$$

$$x_4/x_1 = 240/15 = 16 \text{ enrollments per class.}$$

The above terms could now be combined to demonstrate the interaction, and to form a planning index function. This function would be given by the following:

$$f(x) = w_1 \frac{x_4}{x_3} + w_2 \frac{x_3}{x_2} - w_3 \frac{x_1}{x_2} - w_4 \frac{x_4}{x_1} \qquad (1)$$

where $w_i (i = 1,\ldots,4)$ is a weighting coefficient.

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*This would probably not be the case in an actual situation.

**If this were actually true, the English teacher would have 8 classes, the Math teacher 3 classes, and the Social Studies teacher 4 classes.

The above function combines the basic elements of the school in such a manner that certain mathematical techniques can be used to improve the analysis of available data. These techniques will be discussed in detail in Chapter III.

The terms used in the above function will now be explained. x_4/x_3 has been used to indicate the curriculum depth. In a study of the Iowa High Schools (9) a significant correlation (.8007) was demonstrated between the average daily attendance and the number of credit-units offered. It would appear then, that a ratio of total enrollment to the number of different subjects would be an indication of the depth of the curriculum. Curriculum depth is contrasted with curriculum breadth. The principal might choose to sacrifice breadth by offering fewer subjects while introducing added depth (11) by offering advanced courses (perhaps equivalent to college freshman courses) in a limited number of subject matters.

This balancing between curriculum breadth and depth is indicated by comparing the first two terms of the above function, x_4/x_3 and x_3/x_2. Increasing the number of subjects (x_3) would decrease the first term and increase the second term. Hence the second term might be used to directly indicate curriculum breadth.

The second term also might be used as an indicator of curriculum continuity. If one wants to assume that a smaller faculty (x_2) would be able to do more cooperative planning among themselves, then this planning could organize the curriculum in such a manner that it would enhance the students' understanding of the relationships among different subjects and activities encountered throughout the school day (34, p. 114). This relationship among subjects and activities would be reflected as a measure of curriculum continuity. Then the larger the ratio x_3/x_2 the greater the continuity. While curriculum continuity would tend to keep the number of teachers (x_2) small, the third term (x_1/x_2) would oppose this reduction. x_1/x_2 tends to indicate the teaching load of the average teacher. Decreasing the number of teachers (x_2) would cause an increase in the work load thus opposing the increase in curriculum continuity because of the difference in the signs of the two terms.

Since this second term is a ratio of number of subjects per activity its inverse is an indicator of the degree in which teachers are assigned to their areas of specialization and training. In an extremely small school, teachers may have to teach outside their area of specialization which would increase the teacher's load (14, p. 79). However, in schools with a graduating class of at least 100 students, practically every teacher should be able to teach in the area of his major specialization (34, p. 204) and hence would not appear to be of as significant a factor as the previously mentioned factor of curriculum continuity.

The last term x_4/x_1 indicates a measure of the class size. Increasing the number of classes (x_1) would tend to decrease the average class size but would also tend to increase the teacher work load as indicated by the third term (x_1/x_2). Hence there exists a balancing effect between excessive teacher loads and the preference of smaller class sizes to larger class sizes. Decreasing the total enrollment (x_4) would tend to decrease the class size; however, this would tend to limit the curriculum depth (x_4/x_3) which would also tend to influence costs.* Hence the above function appears to relate the basic elements of the school, classes (x_1), teachers (x_2), subjects (x_3), and enrollments (x_4), in such a manner that the interactions and tradeoffs the principal must contend with are evident.** Figure (2-2) depicts these relationships in a closed loop network.

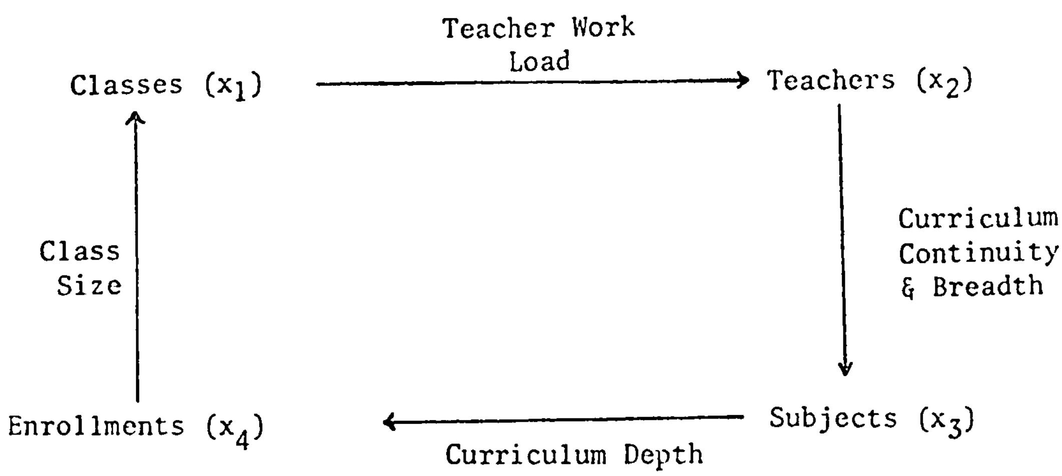

Figure 2-2. Relationships between the basic elements of the school.

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*Several studies (9), (24) have been made to determine the economy of scale with regards to average daily attendance. Cohn estimated the optimal to be about 1,500 pupils for the Iowa State Secondary Schools, while Hansen estimated the most efficient size of a school district to be approximately 50,000 students for the Boston area.

**There are a number of other terms that might be included in the planning index function. A number of these were mentioned in Chapter I. For the purposes of this study, however, the terms that have been defined are sufficient to demonstrate the effects due to interaction and the different choices in resource allocation.

The directions of flow are determined by the terms in the planning index function. The first term relates the enrollment to subjects (x_4/x_3). Increasing the enrollment tends to increase f(x); hence the flow direction would be positive from subjects to enrollments. Increasing the enrollments, however, effects the last term (x_4/x_1). The flow direction points from the enrollment toward the classes because of the negative sign on (x_4/x_1). Changes in any term or terms has a chain reaction on the rest of the system and affects the entire system. Thus changes in teachers will affect class size, curriculum, and work load. Either a new operating level will result from these changes or policy changes will be in order.

While the formulation just described provides interesting insights into interactions of the components in this educational system, it still does not yield much information concerning costs and expenditures. In addition, there is no available information concerning the values of the weighting coefficients (w_1, w_2, w_3, w_4). If costs could be related to the weighting coefficients then the two above mentioned difficulties would be minimized.

One possibility of relating the cost to the weighting coefficients would be to determine the cost associated with each of the variables (x_1, x_2, x_3, x_4) and let each weighting coefficient be equal to ratio of the costs of the corresponding variables. The following terms might be used:

C_1 = Average cost per class,

C_2 = Average cost per teacher,

C_3 = Average cost per subject, and

C_4 = Average cost per enrollment.

The sum of these costs is given by:

$$C_t = C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 \ ,$$

where C_t is the total expenditure of the school less the transportation and building cost.

Then the weighting coefficients would be given by the following:

$$w_1 = C_4/C_3$$

$$w_2 = C_3/C_2$$

$$w_3 = C_1/C_2$$

$$w_4 = C_4/C_1$$

Substituting the above cost ratios for the weighting coefficients into equation 1 the planning index function would then be given by the following:

$$f(X) = \frac{C_4}{C_3} \cdot \frac{x_4}{x_3} + \frac{C_3}{C_2} \cdot \frac{x_3}{x_2} - \frac{C_1}{C_2} \cdot \frac{x_1}{x_2} - \frac{C_3}{C_1} \cdot \frac{x_4}{x_1} \ .$$

C_2 and C_t appear to be the only costs that could be determined without a great deal of difficulty. An estimate of C_2 might be the average teacher salary plus 15 per cent for fringe benefits and overhead costs. C_t could probably be determined from the budget.* Unfortunately the accurate cost data required to establish a consistent and complete set of weighting coefficients does not exist and would be

*See Appendix A for an example of a budget and its typical organizational structure. Appendix B demonstrates how the typical budget might be reorganized into the above proposed categories. This cannot be done in practice however, because the needed cost data does not exist.

very difficult to obtain, in actual practice. In fact, since most districts do not budget for the individual schools (3 , p. 262). The determination of the weighting coefficients would be a serious problem.

A procedure for determining these unknown costs needed for the model will be developed in the next chapter. These procedures will later be used for planning purposes and will be demonstrated in Chapter IV.

Most states require some minimum number of classes in specified subjects matters; however, these minimum requirements rarely serve as constraints except for perhaps the very small school. In addition to state board of education requirements there are a number of other constraints with which the principal must contend. Some of these other constraints are quite formal while others are informal and difficult to quantify. Teacher contacts, Federal standards, and P.T.A. requests are some of the less visible environmental pressures that influences the principal's choices. A lower constraint on the number of classes might be determined by the requirements in the teacher contracts that class size must be less than thirty. Thus given the enrollment, the number of classes needed could be determined. The enrollment is usually determined by the board of education by fixing the school boundaries. A lower constraint on enrollment would be all the pupils that were within the school boundaries that did not seek other forms of education. The minimum number of subjects might be the result of a combination of P.T.A. wishes for certain educational emphasis, and requirements for eligibility for federal funds.

Of course there are also financial constraints. Financial constraints result from the competition for public funds. The financial level of economic support for education directly reflects the educational system's ability to compete effectively with other public institutions, such as, welfare, highways, and medical care. These constraints must be included in the planning model if it is to reflect these additional pressures.

IV. Summary

Summarizing the model presented in this chapter,

Planning Index Function = f[Curriculum Depth, Curriculum Continuity & Breadth, Teacher Work Load, Class Size]

or

$$f(X) = \frac{C_4x_4}{C_3x_3} + \frac{C_3x_3}{C_2x_2} - \frac{C_1x_1}{C_2x_2} - \frac{C_4x_4}{C_1x_1} .$$

Subject to:

$$x_i \geq b_i \quad i = 1,\ldots,4$$

$$\sum_{i=1}^{4} C_ix_i \leq C_t$$

and

$$x_i \geq 0, \quad b_i \geq 0, \quad C_i \geq 0, \qquad i = 1,\ldots,4$$

Where

x_1 = Number of classes

x_2 = Number of teachers

x_3 = Number of subjects

x_4 = Number of enrollments

and

C_1 = Average cost per class,

C_2 = Average cost per teacher,

C_3 = Average cost per subject,

C_4 = Average cost per enrollment,

b_1 = Lower constraint on classes,

b_2 = Lower constraint on teachers,

b_3 = Lower constraint on subjects,

b_4 = Lower constraint on enrollment, and

C_t = Total expenditure of the school less transportation and building costs.

CHAPTER III

SOLUTION TECHNIQUES

I. Introduction

It has been asserted by educators and economists alike, that there exists, in education, a serious misallocation of resources. A considerable number of researchers have investigated the problem; however, at the present time their solutions have not been fully implemented so that they have not achieved significant results for education. Many of the problems that have been encountered are traceable to the weaknesses of the traditional budgeting and accounting procedures that limit the amount of data that can be obtained.

Successful model application has been found to be difficult in a number of areas. Formulating the general structure of most models is straightforward, but as Wagner (48) warns "an application may be standard, yet it need not be routine." Wagner also stated that a common element of successful Operations Research (OR) model application has been,

> ....a willingness on the part of the operations researcher to devise a model that plays down the emphasis on producing rational decisions. The guiding idea has been to devise models that can inform an executive as to the likely effects of decision strategies that he himself has formulated. This approach must be contrasted with the usual models that yield their own recommended decisions: in those cases, the proposed solutions are based on a limited amount of data and a restricted internal logic. ...there is a need for OR models that permit a manager to evaluate decisions that satisfy his personalized rationality. (p. 1271)

The model developed in this study attempted to provide the manager of an educational system with an OR model that will assist in decision making as described by Wagner.

II. The Nonlinear Problem

It may be recalled that in the development of the objective function for the model in Chapter II and summarized on pages 21-22, that the objective function was composed of selected indicators of the educational operation. These indicators were curriculum depth (x_4/x_3), curriculum breadth and continuity (x_3/x_2), teacher work load (x_1/x_2), and class size (x_4/x_1). The need for curriculum breadth was based on two premises: one, the needs of the student, and two, the needs of society. The needs of society require numerous mandatory courses and the needs of the student call for a diversified program. A wide range of elective sourses is basic in meeting the needs of the individual students (34, p.116). In addition to the desire for curriculum breadth is the desire for curriculum depth and continuity. A school should provide sufficient depth in its subjects so that each student has an opportunity to pursue his programs of special interest and develop his full potential (34, p.118). It is also desirable to achieve a curriculum organizational structure that integrates the subject matter into a program that gives each student maximum experiences that facilitate the students' seeing relationships among the different subjects and activities (34, p.115). Hence it would be desirable for the indicators of curriculum depth, breadth, and continuity to show evidence of high levels.

Even the best teacher can not be expected to perform well when his teaching load is excessive (11, p.169). A teacher that has an excessive work load is placed under a double handicap. He will lack sufficient time for the individual student and will be able to do only minimal preparation for his classes (34, p.204). Therefore excessive work loads should be discouraged. There is a general feeling among educators that class size is also a crucial variable and that education can be improved as class size is reduced (6, p.32). Thus reducing the number of pupils per class should be encouraged.

Using the above rational, the model may be classified as a maximization problem for the normal ranges encountered for the x_i's. The model has been rewritten below in order to put it into the standard Operations Research form as described by Zangwill (51).

$$\max.\ f(X) = \frac{C_4x_4}{C_3x_3} + \frac{C_3x_3}{C_2x_2} - \frac{C_1x_1}{C_2x_2} - \frac{C_4x_4}{C_1x_1}$$

Subject to:

$$x_1 - b_1 \geq 0$$

$$x_2 - b_2 \geq 0$$

$$x_3 - b_3 \geq 0$$

$$x_4 - b_4 \geq 0$$

$$C_t - (C_1x_1+C_2x_2+C_3x_3+C_4x_4) \geq 0$$

and

$$x_i \geq 0, \quad b_i \geq 0, \quad C_i \geq 0, \quad i = 1,\ldots,4\ .$$

III. Present Level of Operation

It was assumed that the present values of the x_i's with a particular secondary school were the optimal operating policy for the initial planning period. This meant that the policy makers for a particular school were reacting to the total environment, both quantified and nonquantified factors. The present level of operation may or may not have appeared to be rational. This was due to the previously mentioned factors of limited data, incomplete or limited understanding of the process, or internal logic, and a complex personalized rationality.

Hence the assumption that the present level of operations was the optimal policy basically assumed that the particular school was operated by professionals who adapted their decision process to an individual school's environment. The assumption took into account the particular school, the particular policy maker, and the particular forces acting on both. This optimal operating level vector is designated by X°.

IV. Statement of the Problem

The value of the optimal operating level X° is known for the function f(x), subject to the given constraints. While X° is given, the C_i's and b_i's are not known. C_t is known. Thus, given X° and C_t it is necessary to find the values of the C_i's and test the effects of the b_i's.

V. Solution Procedure

The Kuhn-Tucker conditions are necessarily satisfied if X° is the optimal point for the nonlinear programming problem.* The Kuhn-Tucker (K-T) conditions are defined in the following:

Consider the NLP (nonlinear programming problem)

$$\max.\ f(x)$$

$$\text{subject to } g_i(x) \geq 0 \qquad i = 1,\ldots,m$$

where all functions are differentiable. Let X° be an optimal solution, and assume the constraint qualifications hold. Then the following three conditions also hold:

*Throughout the remainder of the development all functions are assumed differentiable.

(1) X° is feasible.

(2) There exist multipliers $\lambda_i \geq 0$, $i = 1,\ldots,m$, such that,

$$\lambda_i g_i(x^{\circ}) = 0 \quad i = 1,\ldots,m, \text{ and}$$

(3)
$$\nabla f(x^{\circ}) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x^{\circ}) = 0$$

Conditions 1, 2, and 3 collectively are called the Kuhn-Tucker (K-T) conditions.

First, K-T condition 1 holds trivially because of the assumed nature of the problem. In other words, the values in the model are assumed feasible because they are being used. The optimal values are given by:

$$x_1^{\circ} = x_1 \qquad x_3^{\circ} = x_3$$

$$x_2^{\circ} = x_2 \qquad x_4^{\circ} = x_4 \ .$$

Next, K-T condition 2 is given by the following:

(4) $\lambda_1(x_1^{\circ}-b_1) = 0$

(5) $\lambda_2(x_2^{\circ}-b_2) = 0$

(6) $\lambda_3(x_3^{\circ}-b_3) = 0$

(7) $\lambda_4(x_4^{\circ}-b_4) = 0$

(8) $\lambda_5(C_t-C_1x_1^{\circ}-C_2x_2^{\circ}-C_3x_3^{\circ}-C_4x_4^{\circ}) = 0$.

Finally, K-T condition 3 is given by the following:

(9) $$-\frac{C_1}{C_2x_2^{\circ}} + \frac{C_4x_4^{\circ}}{C_1x_1^{\circ 2}} + \lambda_1 - \lambda_5C_1 = 0$$

(10) $$-\frac{C_3x_3^{\circ}}{C_2x_2^{\circ 2}} + \frac{C_1x_1^{\circ}}{C_2x_2^{\circ 2}} + \lambda_2 - \lambda_5C_2 = 0$$

$$(11)\quad -\frac{C_4x_4^\circ}{C_3x_3^{\circ 2}} + \frac{C_3}{C_2x_2^\circ} + \lambda_3 - \lambda_5C_3 = 0$$

$$(12)\quad \frac{C_4}{C_3x_3^\circ} - \frac{C_4}{C_1x_1^\circ} + \lambda_4 - \lambda_5C_4 = 0$$

The (K-T) conditions generated nine equations and introduced an additional five unknown variables, the λ_i's. Thus, additional information concerning the λ_i's is desired. The λ_i's, on an intuitive basis, indicate the approximate increase in the objective function of a per unit increase in the b_i's. If $x(b)$ is the optimal point and is expressed as a function of the resource availability b then,

$$\frac{\partial f[x(b)]}{\partial b_i} = \lambda_i \quad (51,\ p.\ 66)$$

This can be rewritten, using the chain rule, as the following:

$$(13)\quad \frac{\partial f[x(b)]}{\partial b_k} = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \cdot \frac{\partial x_i}{\partial b_k} = \lambda_k \quad k = 1,\ldots,m$$

For the model, in the case at hand, Equation 13 becomes the following:

$$(14)\quad \lambda_1 = \sum_{i=1}^{4} \frac{\partial f}{\partial x_i}\cdot\frac{\partial x_i}{\partial b_1} = \frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial b_1} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial b_1} + \frac{\partial f}{\partial x_3}\cdot\frac{\partial x_3}{\partial b_1} + \frac{\partial f}{\partial x_4}\cdot\frac{\partial x_4}{\partial b_1}$$

$$(15)\quad \lambda_2 = \sum_{i=1}^{4} \frac{\partial f}{\partial x_i}\cdot\frac{\partial x_i}{\partial b_2} = \frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial b_2} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial b_2} + \frac{\partial f}{\partial x_3}\cdot\frac{\partial x_3}{\partial b_2} + \frac{\partial f}{\partial x_4}\cdot\frac{\partial x_4}{\partial b_2}$$

$$(16)\quad \lambda_3 = \sum_{i=1}^{4} \frac{\partial f}{\partial x_i}\cdot\frac{\partial x_i}{\partial b_3} = \frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial b_3} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial b_3} + \frac{\partial f}{\partial x_3}\cdot\frac{\partial x_3}{\partial b_3} + \frac{\partial f}{\partial x_4}\cdot\frac{\partial x_4}{\partial b_3}$$

$$(17)\quad \lambda_4 = \sum_{i=1}^{4} \frac{\partial f}{\partial x_i}\cdot\frac{\partial x_i}{\partial b_4} = \frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial b_4} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial b_4} + \frac{\partial f}{\partial x_3}\cdot\frac{\partial x_3}{\partial b_4} + \frac{\partial f}{\partial x_4}\cdot\frac{\partial x_4}{\partial b_4}$$

$$(18) \quad \lambda_5 = \sum_{i=1}^{4} \frac{\partial f}{\partial x_i} \cdot \frac{\partial x_i}{\partial C_t} = \frac{\partial f}{\partial x_1} \cdot \frac{\partial x_1}{\partial C_t} + \frac{\partial f}{\partial x_2} \cdot \frac{\partial x_2}{\partial C_t} + \frac{\partial f}{\partial x_3} \cdot \frac{\partial x_3}{\partial C_t} + \frac{\partial f}{\partial x_4} \cdot \frac{\partial x_4}{\partial C_t}$$

This gives some additional expressions concerning the λ_i's but there has been a number of partial derivatives introduced.

The evaluation of the partial of $f(\bar{x})$ with respect to the partial of x_i is straightforward. The evaluation of the partial of x_i with respect to the partial of b_i must be analyzed for two cases. Case 1 is where the constraint j is active so that,

$$g_j[x(b)] = b_j \ .$$

Case 2 is where constraint j is not active so that,

$$g_j[x(b)] > b_j \ .$$

For Case 1, where all the constraints are assumed active, it is assumed that any constraint j will remain active in some small neighborhood by b_j as b_j is varied, hence

$$(19) \qquad \frac{\partial g_j}{\partial b_k} = \delta_{jk}$$

where

$$\delta_{jk} = \begin{cases} 0 \text{ if } j \neq k \\ 1 \text{ if } j = k \end{cases} \qquad (51, \text{ p. } 68).$$

Rewriting the constraints for Case 1, the following is obtained.

$$x_1 = b_1$$

$$x_2 = b_2$$

$$x_3 = b_3$$

$$x_4 = b_4$$

$$C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 = C_t$$

Equation (14) is then given by the following:

$$\text{(20)} \qquad \lambda_1 = \frac{\partial f}{\partial x_1} \cdot 1 + \frac{\partial f}{\partial x_2} \cdot 0 + \frac{\partial f}{\partial x_3} \cdot 0 + \frac{\partial f}{\partial x_4} \cdot 0$$

$$= (-\frac{C_1}{C_2 x_2} + \frac{C_4 x_4}{C_1 x_1^2})$$

because

$$\frac{\partial x_2}{\partial b_1} = \frac{\partial x_3}{\partial b_1} = \frac{\partial x_4}{\partial b_1} = 0$$

Similarily, Equations 15, 16 and 17 become:

$$\text{(21)} \qquad \lambda_2 = (-\frac{C_3 x_3}{C_2 x_2^2} + \frac{C_1 x_1}{C_2 x_2^2})$$

$$\text{(22)} \qquad \lambda_3 = (-\frac{C_4 x_4}{C_3 x_3^2} + \frac{C_3}{C_2 x_2})$$

$$\text{(23)} \qquad \lambda_4 = (-\frac{C_4}{C_1 x_1} + \frac{C_4}{C_3 x_3})$$

From the budget constraint

$$C_1 x_1 + C_2 x_2 + C_3 x_3 + C_4 x_4 = C_t$$

the following are obtained

$$\frac{\partial x_1}{\partial C_t} = 1/C_1$$

$$\frac{\partial x_2}{\partial C_t} = 1/C_2$$

$$\frac{\partial x_3}{\partial C_t} = 1/C_3$$

$$\frac{\partial x_4}{\partial C_t} = 1/C_4$$

Here Equation 18 is written as:

$$\lambda_5 = 1/C_1(-\frac{C_1}{C_2x_2} + \frac{C_4x_4}{C_1x_1^2}) + 1/C_2(-\frac{C_3x_3}{C_2x_2^2} + \frac{C_1x_1}{C_2x_2^2})$$

$$+ 1/C_3(-\frac{C_4x_4}{C_3x_3^2} + \frac{C_3}{C_2x_2}) + 1/C_4(\frac{C_4}{C_3x_3} - \frac{C_4}{C_1x_1})$$

or

$$(24) \quad \lambda_5 = (\frac{C_1x_1 - C_3x_3}{(C_2x_2)^2}) + C_4x_4(\frac{1}{(C_1x_1)^2} - \frac{1}{(C_3x_3)^2}) + (\frac{1}{C_3x_3} - \frac{1}{C_1x_1})$$

This gives the additional equations desired for Case 1. Next, Case 2 will be investigated in order to see if an information can be obtained when all the constraints are not active.

For Case 2, where none of the constraints are active, it is assumed that they will remain inactive as b_i is varied in a sufficiently small neighborhood. Consequently,

$$\frac{\partial x_i}{\partial b_j} = \lambda_j = 0 , \quad i = 1,\dots,n .$$

(51, p. 68).

This essentially states that if a constraint is not a binding constraint, it can be moved around in a small neighborhood without changing the solution.

Case 2, did not yield new information about the λ_i's; however, perhaps some information can be obtained concerning the C_i's. From Equations 9 through 12, where the constraints were not active (Case 2), the following are obtained:

$$(25) \qquad \frac{C_4x_4}{C_1x_1^2} = \frac{C_1}{C_2x_2}$$

(26) $$\frac{C_1x_1}{C_2x_2^2} = \frac{C_3x_3}{C_2x_2^2}$$

(27) $$\frac{C_3}{C_2x_2} = \frac{C_4x_4}{C_3x_3^2}$$

(28) $$\frac{C_4}{C_3x_3} = \frac{C_4}{C_1x_1}$$

From both Equation 26 and 28 the following is obtained:

(29) $$C_1x_1 = C_3x_3\text{*}$$

This indicated that for Case 2, where the constraints were inactive, the total expenditure for subjects must be equal to the total expenditure for classes. From Equations 25 and 27 the following relationship is indicated.

(30) $$C_4x_4 = \frac{(C_1x_1)^2}{C_2x_2} = \frac{(C_3x_3)^2}{C_2x_2}$$

Substituting Equations 29 and 30 into the budget constraint

$$C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 < C_t$$

yields,

$$(C_1x_1)^2 + 2\ C_2x_2(C_1x_1) + (C_2x_2)^2 < C_t(C_2x_2)$$

or

$$(C_1x_1)^2 + 2\ C_2x_2(C_1x_1) + (C_2x_2)^2 - C_t(C_2x_2) < 0$$

---

*This implied that C_4 was strictly positive.

Thus for Case 2 only ranges for the costs values, C_1x_1, C_3x_3, and C_4x_4, can be found.

It is desirable to obtain a unique solution, if possible, for the C_i's in order that the planning periods in the future could be determined with as small a variation as possible. Hence Case 1, where the constraints were active, will be investigated in more detail to see if a unique solution for the C_i's can be obtained.

By definition, the K-T multipliers (λ_i's) must be zero or positive. In the original formulation of the problem the x_i's and the C_i's were also required to be zero or positive. This might be expressed as,

$$\lambda_i x_i \geq 0 \qquad \text{and}$$

$$C_i x_i \geq 0 \qquad i = 1,\ldots,4.$$

Thus, returning to Case 1, Equations 20, 21, 22 and 23 are rewritten by multiplying Equation 20 by x_1, Equation 21 by x_2, Equation 22 by x_3, Equation 23 by x_4 and transferring the negative term to the other side of the inequality. Hence,

$$\frac{C_4x_4}{C_1x_1} \geq \frac{C_1x_1}{C_2x_2} \tag{31}$$

$$\frac{C_1x_1}{C_2x_2} \geq \frac{C_3x_3}{C_2x_2} \tag{32}$$

$$\frac{C_3x_3}{C_2x_2} \geq \frac{C_4x_4}{C_3x_3} \tag{33}$$

$$\frac{C_4x_4}{C_3x_3} \geq \frac{C_4x_4}{C_1x_1} \tag{34}$$

When 31, 33 and 34 are added, the following results,

$$\frac{C_4x_4}{C_1x_1} + \frac{C_3x_3}{C_2x_2} + \frac{C_4x_4}{C_3x_3} \geq \frac{C_1x_1}{C_2x_2} + \frac{C_4x_4}{C_3x_3} + \frac{C_4x_4}{C_1x_1}$$

When the redundant terms C_4x_4/C_1x_1 and $C_4x\ /C_3x_3$ are subtracted from each side of the above inequality, then

(35) $$\frac{C_3x_3}{C_2x_2} \geq \frac{C_1x_1}{C_2x_2}\ *.$$

In order to satisfy expressions 32 and 35 simultaneously the following equality must hold

(36) $$C_1x_1 = C_3x_3\ **\ .$$

This reduces expressions 32, 34 and 35 to identities. Now from 31:

$$(C_4x_4)(C_2x_2) \geq (C_1x_1)^2$$

and from 33

$$(C_4x_4)(C_2x_2) \leq (C_3x_3)^2\ .$$

But from 36, $C_1x_1 = C_3x_3$, thus in order to satisfy both 31 and 33 equality must hold. Consequently,

(37) $$(C_1x_1)^2 = (C_3x_3)^2 = C_2x_2 \cdot C_4x_4\ .$$

The budget constraint, for Case 1, requires that

---

*It was assumed that $C_1x_1 > 0$. In the applied case this restriction would be appropriate. The equality conclusion could have also been reached by adding 25, 26 and 28 and subtracting out the redundant terms. A similar operation would show the necessity for equality for 25. This equality in turn, would demonstrate the necessity of $C_ix_i > 0$, $i = 1,\ldots,4$.

**This was the same result as obtained for Case 2 and given by Equation 29.

$$C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 = C_t.*$$

Substituting 36 and 37 into the budget constraint

$$C_1x_1 + C_2x_2 + C_1x_1 + \frac{(C_1x_1)^2}{C_2x_2} = C_t$$

or

$$(38) \qquad (C_1x_1)^2 + 2C_2x_2(C_1x_1) + (C_2x_2)^2 - C_tC_2x_2 = 0 .$$

From the quadratic formula,

$$(39) \qquad C_1x_1 = - C_2x_2 + \sqrt{C_t \cdot C_2x_2}$$

If $C_2x_2 = \phi C_t$ where,

$$0 \leq \phi \leq 1$$

then

$$C_1x_1 = \sqrt{\phi}\, C_t - \phi C_t \ . **$$

Equation 40 results from substituting Equation 37 into Equation 38 and applying the quadratic formula again,*** and

$$(40) \qquad C_4x_4 = C_t + C_2x_2 - 2\sqrt{C_t \cdot C_2x_2}$$

---

*This assumes that money was one of the limiting scarce resources.

**ϕ is the fraction of the total budget being allocated for teachers ($\phi = C_2x_2/C_t$).

***In both cases the infeasible roots of the quadratic are to be disregarded. The infeasible roots are the ones that cause $C_ix_i < 0$, $i = 1,\ldots,4$.

If $C_2x_2 = \phi C_t$, as before, then Equation 40 can be rewritten as the following:

$$(41) \qquad C_4x_4 = C_t(1+\phi-2\sqrt{\phi})$$

The above will now be briefly summarized. The K-T conditions yielded a number of necessary conditions for $X°$ to be an optimal. However they also introduced five additional variables, the λ_i's. Additional information was obtained for the λ_i's by breaking the problem into two cases. Case 1, where all the constraints were active, yielded five equations for the λ_i's. Case 2, where no constraint was active, simply yielded that when a constraint is inactive it cannot affect the value of the objective function f(x) as the constraint is moved in a small neighborhood. However, some information was obtained about the C_i's but it was not sufficient to result in a unique solution for the C_i's. When Case 1 was investigated further, using the previously obtained information about the λ_i's, a unique solution was obtained.

Equations 39 and 40 yielded the desired unique solution.* This was obtained for Case 1, where all the constraints were active. It was of interest to note that Equation 29 was the same as Equation 36. In other words, both Case 1 and Case 2 yielded $C_1x_1 = C_3x_3$. It was also interesting to note that for the value of C_4x_4 the difference between the results for Case 1 and Case 2 where none of the constraints were active was an inequality instead of an equality as in Equation 40.

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*A unique solution was possible when C_2x_2 and C_t were known. In practice the fraction of the total budget spent for teachers could be obtained without a great deal of difficulty and the total expenditure (C_t) would be obtained from the budget and would normally be considered a constraining scarce resource (3 , p. 14).

$$C_4x_4 < C_t + C_2x_2 - 2\sqrt{C_t \cdot C_2x_2}$$

This inequality was due to the budget constraint. Hence C_4x_4 would be given by a range in Case 2 instead of a unique value as in Case 1. The range of C_4x_4 was dependent upon the slack in the budget constraint. As the slack in the budget constraint decreased, so did the range of C_4x_4 until the constraint became active, in which case, C_4x_4 was fixed.

Because of the dependency of C_1x_1, C_3x_3, and C_4x_4 values upon the value of the fraction of the total budget spent for teachers (ϕ), this relationship was investigated further.

Figure 3-1 shows the variation in the fraction of the budget spent for classes (C_1x_1/C_t) as the fraction spent for teachers (ϕ) varied from zero to one. It might be noted that the total expenditure for classes cannot exceed twenty-five per cent of the total budget.

Figure 3-2 shows the variation in the fraction of the budget spent for the subjects as varied from zero to one. It might be noted that Fig. 3-2 and Fig. 3-1 are identical. Thus the total expenditure for subjects cannot exceed twenty-five per cent of the total budget. These two curves are identical because of the identity $C_1x_1 = C_3x_3$.

Figure 3-3 shows the variation in the enrollment expenditure fraction as ϕ is varied for zero to one.

Figure 3-4 demonstrates the variation in the fraction of the budget spent for classes as the expenditure for enrollment was changed. Of course, since $C_1x_1 = C_3x_3$, the curve for the subject expenditure

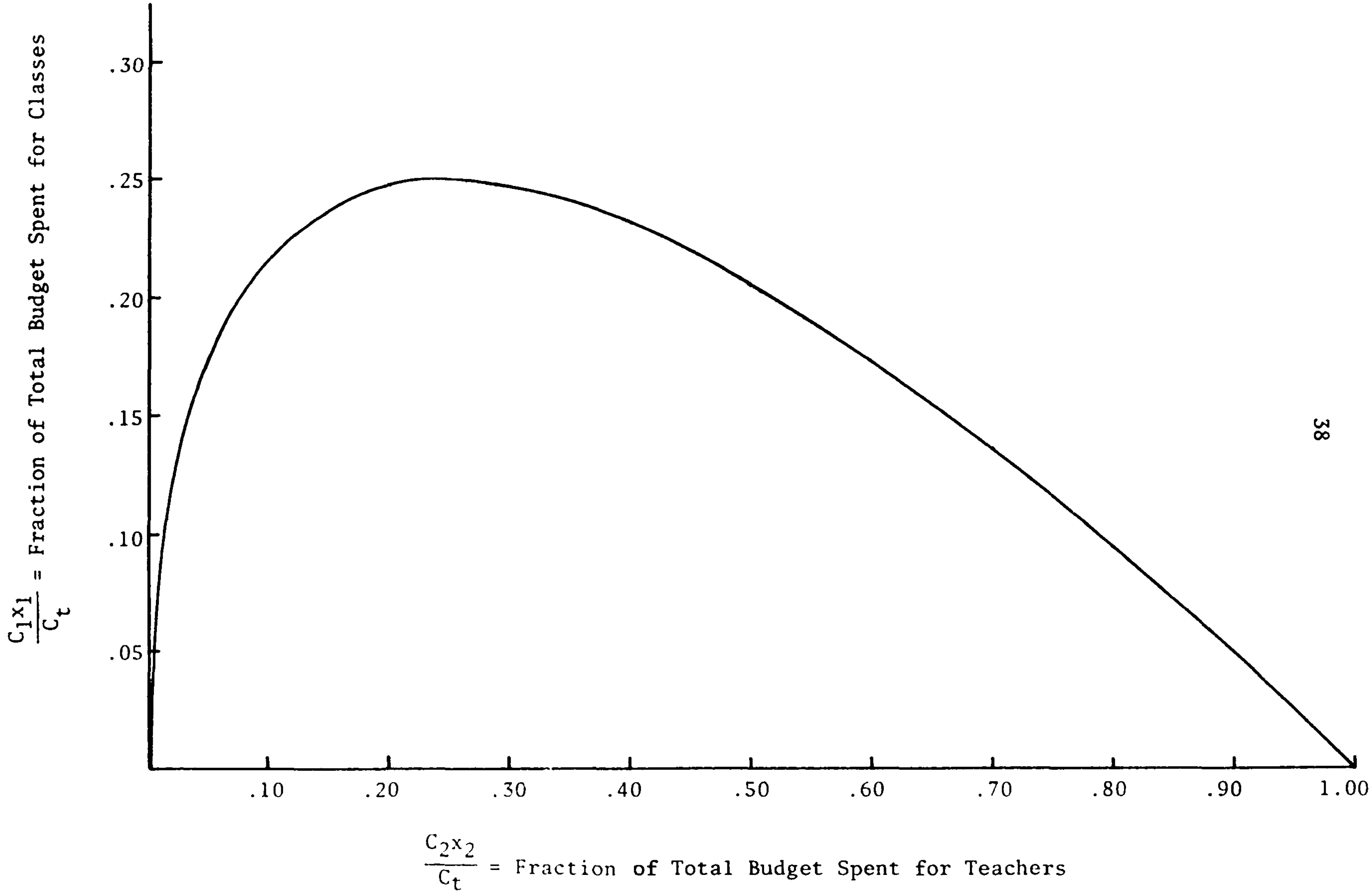

Figure 3-1. Teacher and Class Costs Curve.

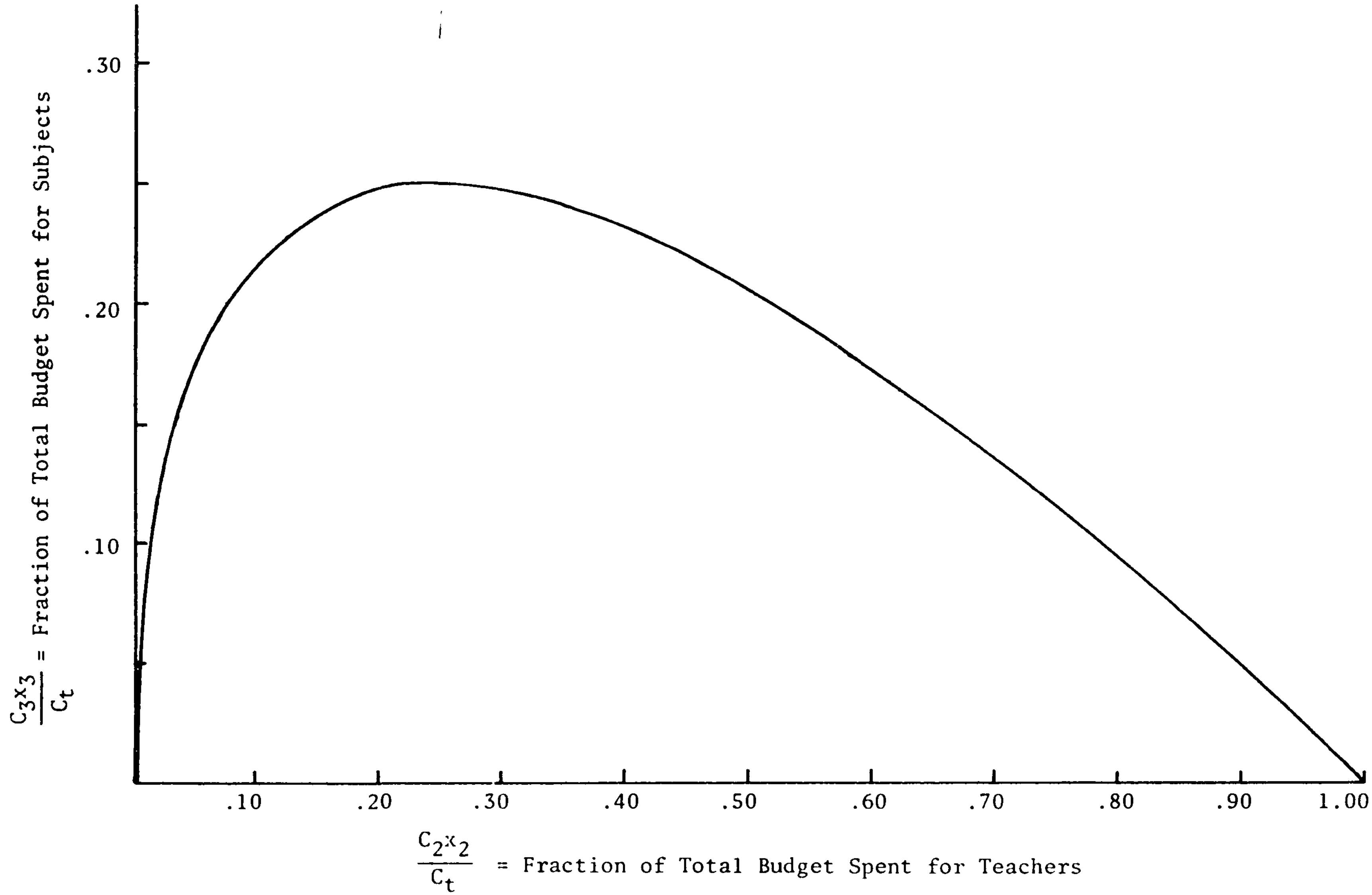

Figure 3-2. Teacher and Subject Costs Curve.

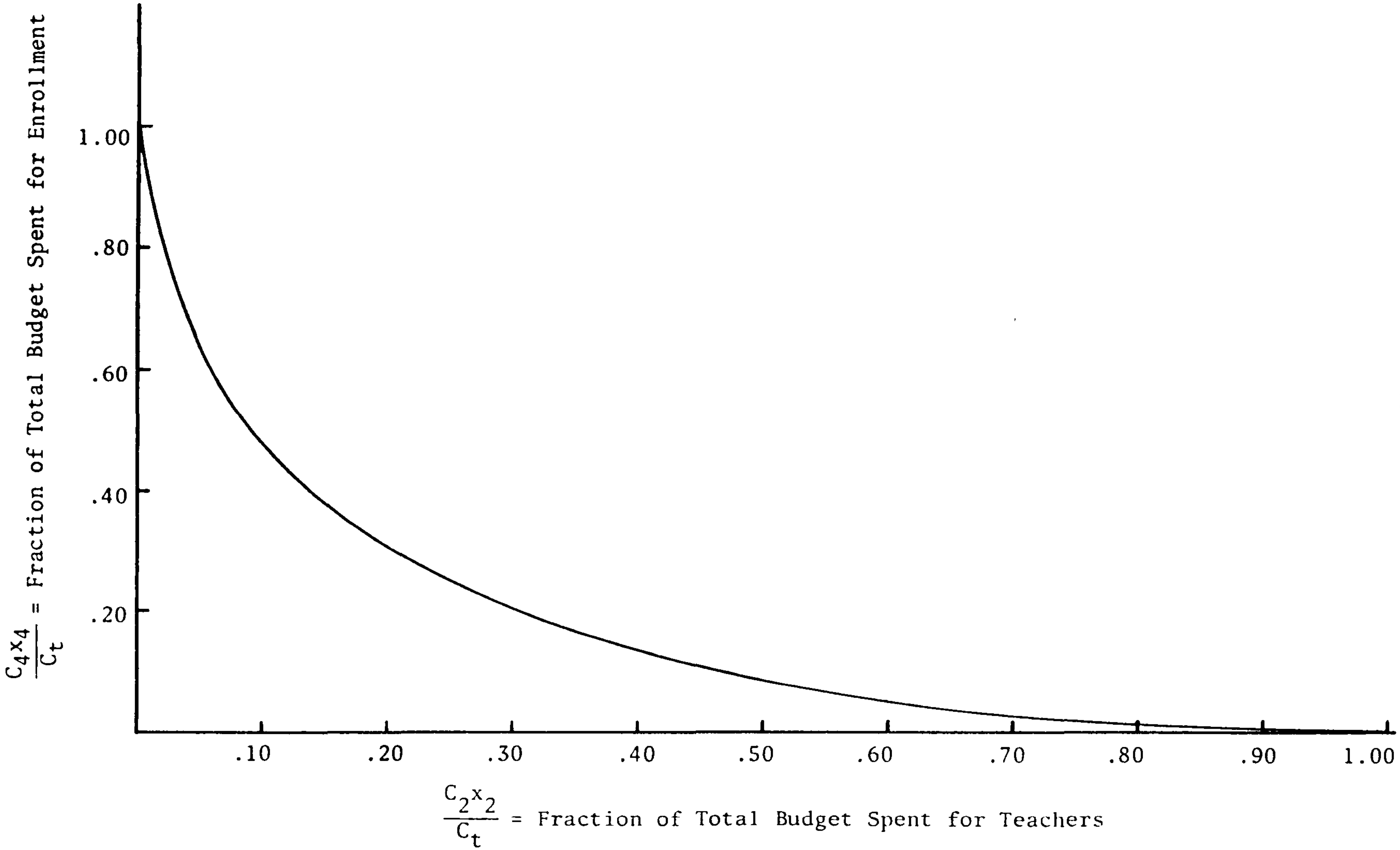

Figure 3-3. Teacher and Enrollment Costs Curve.

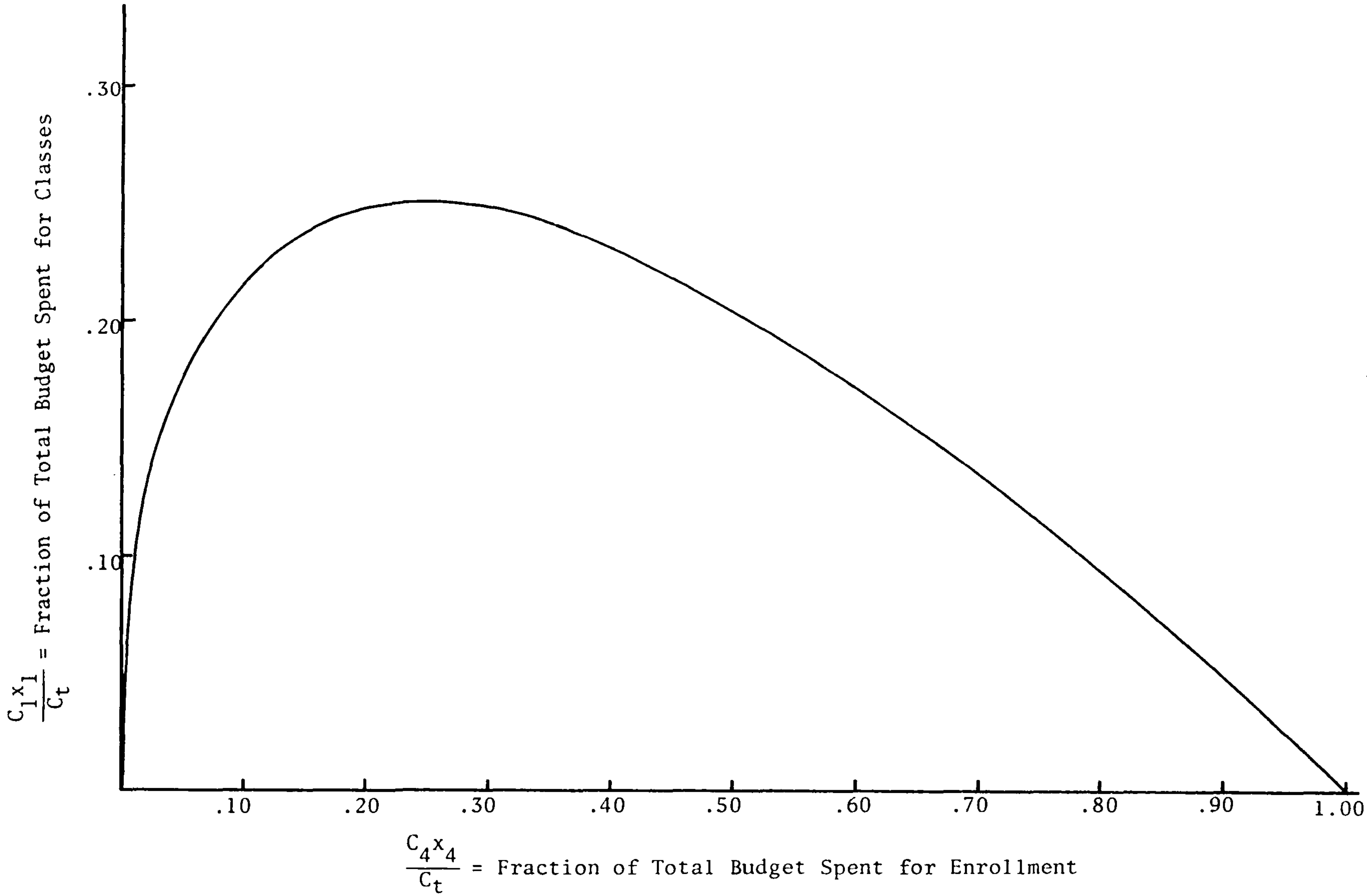

Figure 3-4. Enrollment and Class Costs Curves.

and enrollment expenditure would be the same as Fig. 3-4.*

Returning to the expressions 20, 21, 22, 23 and 24, the reader may note that the values of the λ_i's would be zero at the optimal point when the constraints were active because of the equalities for the expressions 25, 26, 27 and 28. A closer inspection of the objective function would show it to be zero also. This was due primarily to Equation 26.

Of significant interest was the value of the objective function and whether it was the maximum value. Optimality was not guaranteed by only satisfying the Kuhn-Tucker conditions. The K-T conditions are, by themselves, not sufficient as was illustrated by an example given by Zangwill (51, p. 43).

In Case 1, where all the constraints were active, the solution space was limited to a point; hence, the solution had to be the maximum. It must also be remembered that this point was assumed to be an maximum. When the lower constraints on the x_i's were relaxed, one was able to demonstrate that the solution was in general not the maximum solution.**

For example, when the only active constraint was the budget constraint, increasing x_3 at the expense of x_4, (increasing subjects while decreasing enrollments) increased the value of the objective function when the other values were held constant. Decreasing x_3 while increasing x_4 also increased f(x). Also any changes in x_1

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*See Appendix D for the data for Figs. 3-1 through 3-4.

**This was demonstrated by finding the values for the C_i's and then, while holding the other variables fixed, varying one variable at a time for both increasing and decreasing changes.

(classes) caused decreases in f(x). The reason for this was that the solution was located at a saddle point for x_1 and x_3. The point was the maximum for x_1 (classes) and the minimum for x_3 (subjects). This solution established a relationship between x_1 and x_3, namely $C_1x_1 = C_3x_3$. The addition of this relationship to the nonlinear programming problem would now cause the given operating level to be the maximum in the general case.

This relationship of $C_1x_1 = C_3x_3$ was found to have several practical interpretations in the actual case. For example, x_1/x_3 would be the average number of classes per subject. This ratio would normally be constant or at least the average value for the ratio for a school would tend to remain the same from one planning period to the next. There are several reasons for this. First, most state school systems are administered by a state department of public instruction which develops courses of study, and provides uniform leadership and general supervision for the various curriculum programs (20, p. 113). This tends to result in uniform curriculums. Next, the local boards of education may regulate requirements beyond those prescribed by the state agency. Finally, the National Science Foundation and other groups have developed national curriculum programs that are being made available to state and local education boards. Thus the maximum solution, when the relationship $C_1x_1 = C_3x_3$ held, was found to be $f(x) = 0$.

The next part of the problem was to solve for the x_i's given the C_i's. This problem would arise when the model would be used for planning to accommodate certain expected changes, such as increases or decreases in the total enrollment. The factors that were to be determined in this case were 1) the level of the economic resource input, and 2) the manner in which the economic resources were allocated within the organization.

There were several approaches available to determine these factors. The first approach described below offers some interesting insight into the internal workings of the model. The solution of the model was found to be independent of the values of the b_i's. This was assuming that the values of the b_i's did not result in the x_i's being infeasible (K-T condition 1).* It was convenient and reasonable then to assume that $x_i = b_i$, $i = 1,\ldots,4$. This permitted the λ_i's ($i = 1,\ldots,4$) to be given by Equations 20, 21, 22 and 23,** which are repeated below.

(20) $$\lambda_1 = \frac{C_4x_4}{C_1x_1^2} - \frac{C_1}{C_2x_2}$$

(21) $$\lambda_2 = \frac{1}{C_2x_2^2} \cdot (C_1x_1 - C_3x_3)$$

(22) $$\lambda_3 = \frac{C_3}{C_2x_2} - \frac{C_4x_4}{C_3x_3^2}$$

(23) $$\lambda_4 = C_4\left(\frac{1}{C_3x_3} - \frac{1}{C_1x_1}\right)$$

(24) $$\lambda_5 = \frac{(C_1x_1 - C_3x_3)}{(C_2x_2)^2} + C_4x_4\left(\frac{1}{(C_1x_1)^2} - \frac{1}{(C_3x_3)^2}\right) + \left(\frac{1}{C_3x_3} - \frac{1}{C_1x_1}\right)$$

---

*It should be pointed out that the b_i's are only for $i = 1,\ldots,4$ and do not include C_t.

**The principal difference between the K-T multipliers (λ_i's) for inequality and equality constraints is that the multipliers for the equality constraints are permitted to be negative in addition to positive or zero, while the multipliers for inequality constraints are restricted to be positive or zero.

Prior to considering the interpretation of the λ_i's, it was necessary to examine the effect of possible variations in the values of the costs. It may not be realistic in the applied case to assume that the values of the costs would remain constant over a planning period of approximately five years. If one were to consider variations due to a known inflation rate, then period-by-period variations in the costs could be readily found. The simplest case would be where the variations in the rate remain constant; i.e., the variations in the inflation rate would be so small between periods as to be considered insignificant (36, p. 373).* The primary effect of a positive inflation rate is to decrease the present worth of future expenditures. This effect becomes more pronounced as the number of planning periods increase.

Once the costs were determined for a particular planning period it was a straightforward task to find the various x_i's required to reestablish the system's equilibrium. For example, if x_4 (enrollment) was expected to change in some future period then the expected results could be found by examining Equations 20 through 24. An increase in x_4 caused λ_1 to become positive and λ_3 to become negative. As mentioned earlier, this would indicate that increasing b_1 (lower bound on classes) would increase f(x) while increasing b_3 (lower bound on subjects) would decrease f(x). There were several ways of obtaining equilibrium

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*Inflation rates rarely, if ever, remain constant. Reisman presents a generalized model for the case where inflation rates are different during different periods (36, p. 379). A considerable simplification of the calculations, however, can be accomplished by finding the mean inflation rate and using this value to determine the variations in the costs (36, p. 390).

in the system. To obtain a value of zero for λ_1, for example, one might increase the value of x_1 (classes) or decrease the value of x_2 (teachers). Similarly λ_3 could become zero by increasing x_3 (subjects) or decreasing x_2 (teachers). Decreasing the number of teachers however, violates the constraint, $x_2 \geq b_2$. Increasing the number of classes and subjects would result in the budget constraint being violated. Thus, unless C_t was increased or the lower limit on the number of teachers was decreased, equilibrium was not possible. Of course there was always the possibility of changing some of the C_i's. In the actual case, C_t is usually increased and hence, the number of classes could be increased. This, however, causes λ_2 and λ_5 to become positive and λ_4 to be negative. b_2 (lower limit on the number of teachers) would normally be increased when the number of classes and the enrollment were increased and this would tend to increase f(x). Increasing b_2 would also cause λ_1 to become more positive and λ_3 to become more negative. In addition to the λ_i requirements for optimality it must be remembered that the model will not be optimal unless Equation 36 is satisifed.

The above discussion demonstrates the interaction of the variables and the dynamic nature of the system. This type of exercise would enable a practitioner to gain the seasoning needed for implementation which inherently requires adhering to systematic procedures and paying careful attention to detail (49, p. 927). This type of exercise would eventually lead to the new equilibrium state in a manner similar to that which produces an asymptotically stable condition in the large for systems based on the Liapunov theory.*

The most straightforward method of finding the new values for the x_i's, was using the same equations used to compute the C_i's.**

In Equations 39 and 40, it was noted that C_t must be known before the new values for the x_i's can be determined. The projection of the availability of funds to finance an educational plan, (C_t) could be made by studying the characteristics of the sources of the funds in the past (13, p. 205); for a trend, observations in the ten most recent years could probably be justified as could extrapolations for ten years into the future. Rapid changes in economic and social conditions are likely to prevent valid projections beyond the ten-year period (20, p. 327). Such a model may be economically naive since possible discontinuities are rarely considered.***

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*See Lasalle and Lefschetz (28) for more detail. Zangwill (51, p. 225) summarizes asymptotic stability with the following comments. "A ... system represented by A is called asymptotically stable in the large if, given any initial point z^1,

$$\lim_{k \to \infty} z^k = 0$$

where

$$z^{k+1} = A(z^k)$$

Asymptotic stability in the large means that, given any initial state of the system, as time progresses the system eventually evolves to the equilibrium position." (51, p. 225-226)

**See Equations 39 through 41.

***While the projection of these values is an important topic, it is not within the scope of this study for a typical projection methodology, see Garvue (20).

Garvue (20, p. 356) pointed out that cost projections for educational budgeting were straight-line in form. Benson (3) indicates the reason for this was that "The comfortable position for a school board is to maintain a habitual pattern of expenditure." In so doing, the school board avoids facing the taxpayers with any sharp increases in tax rate except for those that can be clearly justified in terms of physical growth, i.e., growth in size of pupil population. (3 , p. 302)*

Once the long-term total educational expenditure projection was complete, the x_i's were obtained from Equations (39) and (40), which are repeated below.

$$C_1x_1 = C_3x_3 = \sqrt{C_t \cdot C_2x_2} - C_2x_2 \tag{39}$$

$$C_4x_4 = C_t + C_2x_2 - 2\sqrt{C_t \cdot C_2x_2} \tag{40}$$

If x_2 were already known in addition to x_4;** then the above equations could be used for long-term budgeting such as attempted in planning programming and budgeting systems.

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*This pattern was not expected to change in the near future. Garvue states, "It is likely, that budgeting will remain on a crisis to crisis, short-term basis, and emphasis will continue to be on determining 'what the traffic will bear' ... Thus, the tail (revenue) will continue to wag the dog (program)." (20, p. 357)

**This may be due to contractual agreements with the teachers, legal constraints imposed by the State Board of Education, or perhaps educational targets to be strived for.

> If estimated values for funds needed and funds available are in harmony or if the difference between them is not too large, it is likely that it will be possible to finance the educational plan as it is. If this is not the case, it will be necessary to study the possibility of reducing expenditures on education while still attaining provisional targets. If this were not possible, the targets themselves might need to be revised. (13, p. 205)

VI. Summary

This chapter described the techniques used to find a solution to the resource allocation model that was developed in Chapter II. Several assumptions were necessary. First, the present level of operation was assumed to be the optimal operating policy. The next assumption was that all costs were greater than zero. This was followed by the assumption that all the constraints were active. Then it was assumed that the total expenditure (C_t) and the cost for teachers (C_2) were known. These resulted in unique values for the remainder costs. Once these equations (39) and (40) for the costs were established to give the optimal solution; the next step was to indicate how the model could be used for planning future operating policies. To do this either the total expenditure (C_t) and one variable (x_i) or two variables must be known for the future period. Finally, effects of inflation on the values of the costs found in the first part were discussed.

CHAPTER IV

THE APPLIED MODEL

I. Introduction

After developing the resource allocation model and solution techniques, the model's use was then demonstrated with data from a typical school found in Appendix A for two cases. The first case used the expected projection of enrollment and a variation in revenues through a fixed rate of inflation. In the second case a revised set of educational targets of classes and teachers were substituted into the model to determine the resulting projected educational expenditure needs. The two cases were compared to see if the estimated values for funds needed and funds available were in harmony. Where these fund flows were not equal, various revisions were discussed that would make it possible to bring the two variables into harmony. Finally, the data gathering was discussed along with the sources and various types of data needed.

II. Case 1

In Case 1 an expected projection of the future enrollments was assumed along with the expected revenues and a fixed rate in inflation. The expected revenue was given by expenditure per pupil plus an increment for inflation. The projection of the enrollment is given in

Fig. 4-1.*

Figure 4-2 shows the expected revenue for the planning period.** This projection was based upon the current expenditure per pupil plus an allowance for inflation. Thus the level of community support for the educational system was not expected to change. Therefore, one would not expect radical change or experimentation within the school system. Also shown in Fig. 4-2 is a plot of the expected revenue if inflation was ignored. It was interesting to note the amplification in the slope of the revenue curve when an increase in enrollment was coupled with an increasing inflationary environment. This is of particular importance when the long-term educational planning horizon extends past a few years. Thus inflation had to be taken in account in the preparation and utilization of a complete long-term education plan for the school.***

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*Planeville expected a 50% increase in the enrollment over the next five years. The reason for this increase was due to a campaign by the local Chamber of Commerce to attract new industry. The data for the increase in the enrollment were based upon the schedule that the Chamber of Commerce was working on to attract industry and thus population.

**The data for the figures and discussion in this chapter are given in Appendix D as computer output. The program listing of the planning index model is given in Appendix C.

***Most of the present methodologies proposed for long-term micro educational planning models fail to include inflation in their estimates. Since the inflation rate is an uncertain factor, reliability decreases as one moves further into the future and thus, any assumptions based on the model are more likely to be invalidated as the planning horizon increases. (51, p. 413) However, ignoring inflation only amplifies the problem.

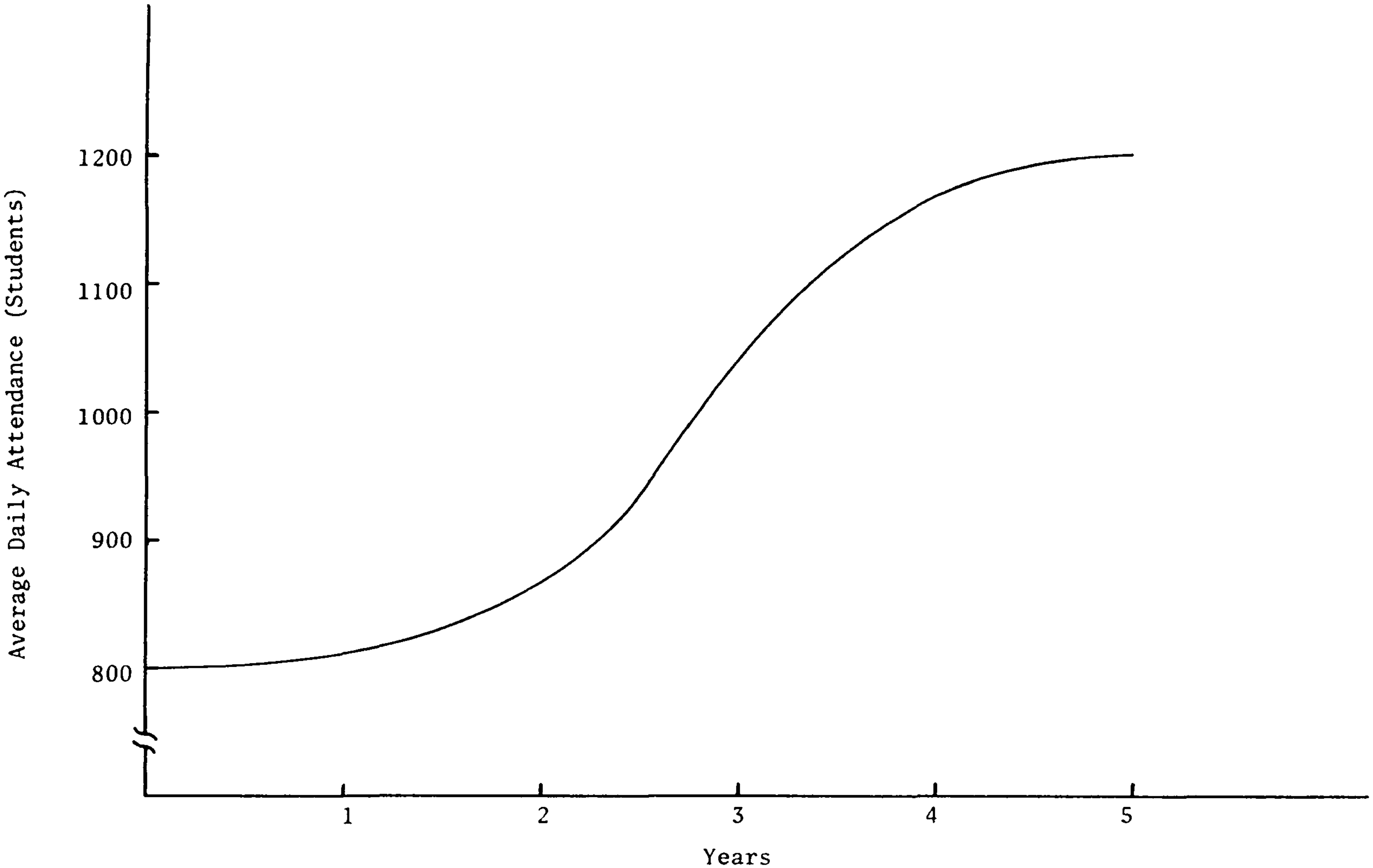

Figure 4-1. Projection of Expected Average Daily Attendance.

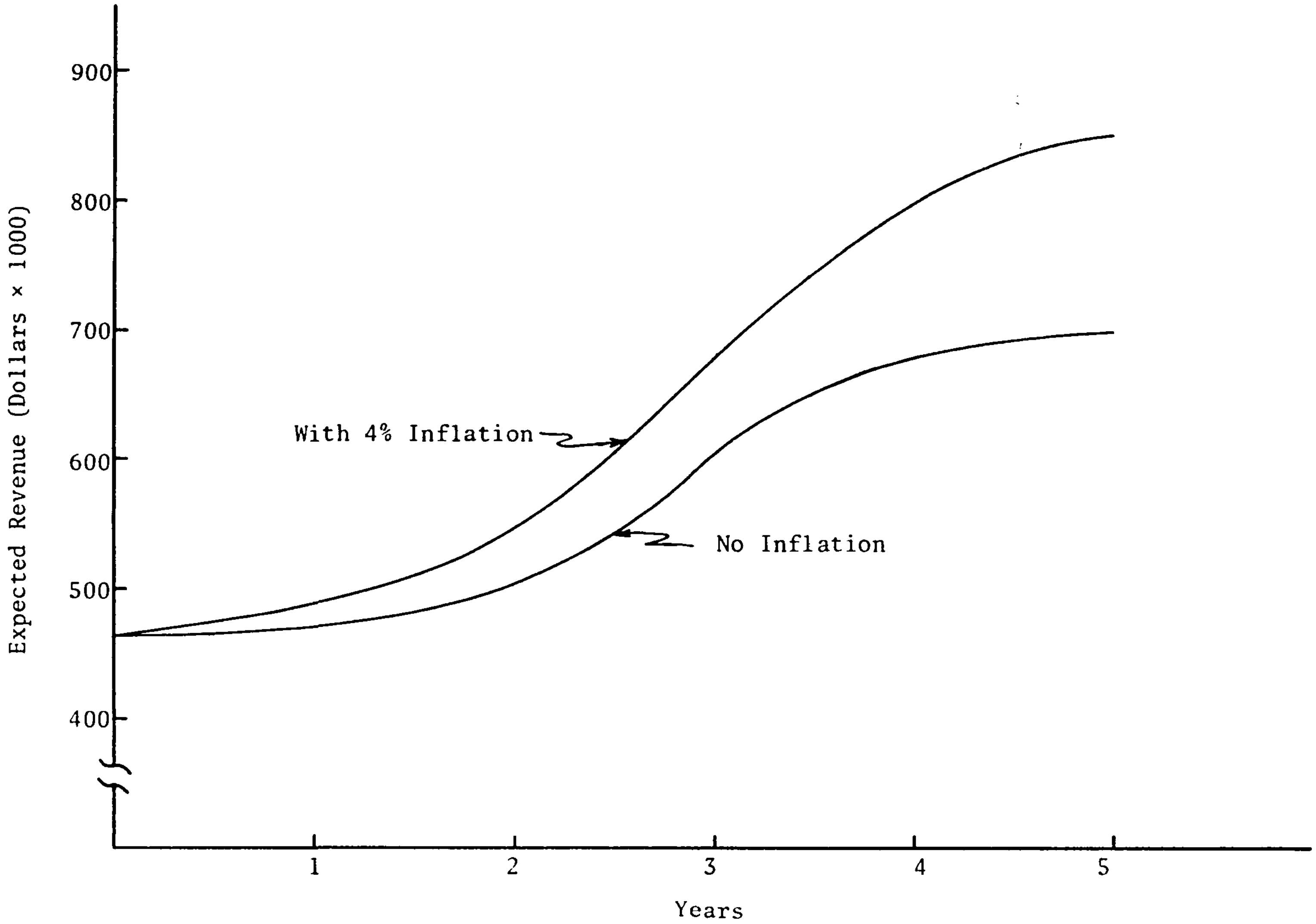

Figure 4-2. Expected Revenue Curves.

Figure 4-3 indicates the number of courses that would be needed to maintain approximately the same class size for the planning period. This was computed by requiring the enrollments per class (x_4/x_1) to remain a constant. Small class size is often projected as politically desirable, yet is said to increase system cost. The effect of changing class size was tested later in the chapter.

Figure 4-4 indicated the number of teachers needed to maintain the classes per teacher x_1/x_2 at the same constant ratio as at the beginning of the planning horizon.

After developing the preceeding relationships from the typical school data of Appendix A, it was necessary to determine the value of the time trended variable x_3 (subjects). In the previous chapter the identity $C_1x_1 = C_3x_3$ was found. The costs C_1 and C_3 were known at the beginning of the planning horizon; however, inflation had to be considered. If it were assumed that the constant rate of inflation were θ, then the amount P at the start of a period would be increased by an amount θP due to the effects of inflation during the period. Hence, the amount P at the beginning of the period to be equivalent to the amount needed at the end of a period was $P+\theta\cdot P$ or $P(1+\theta)$. Substituting in the inflation terms to the above identity yielded $(C_1+\theta C_1)\cdot x_1 = (C_3+\theta C_3)\cdot X_3$.

Simplifying this equality yielded

$$C_1\cdot x_1\cdot(1+\theta) = C_3\cdot x_3\cdot(1+\theta)$$

$$C_1x_1 = \frac{(C_3x_3)(1+\theta)}{(1+\theta)}$$

$$C_1x_1 = C_3x_3 .$$

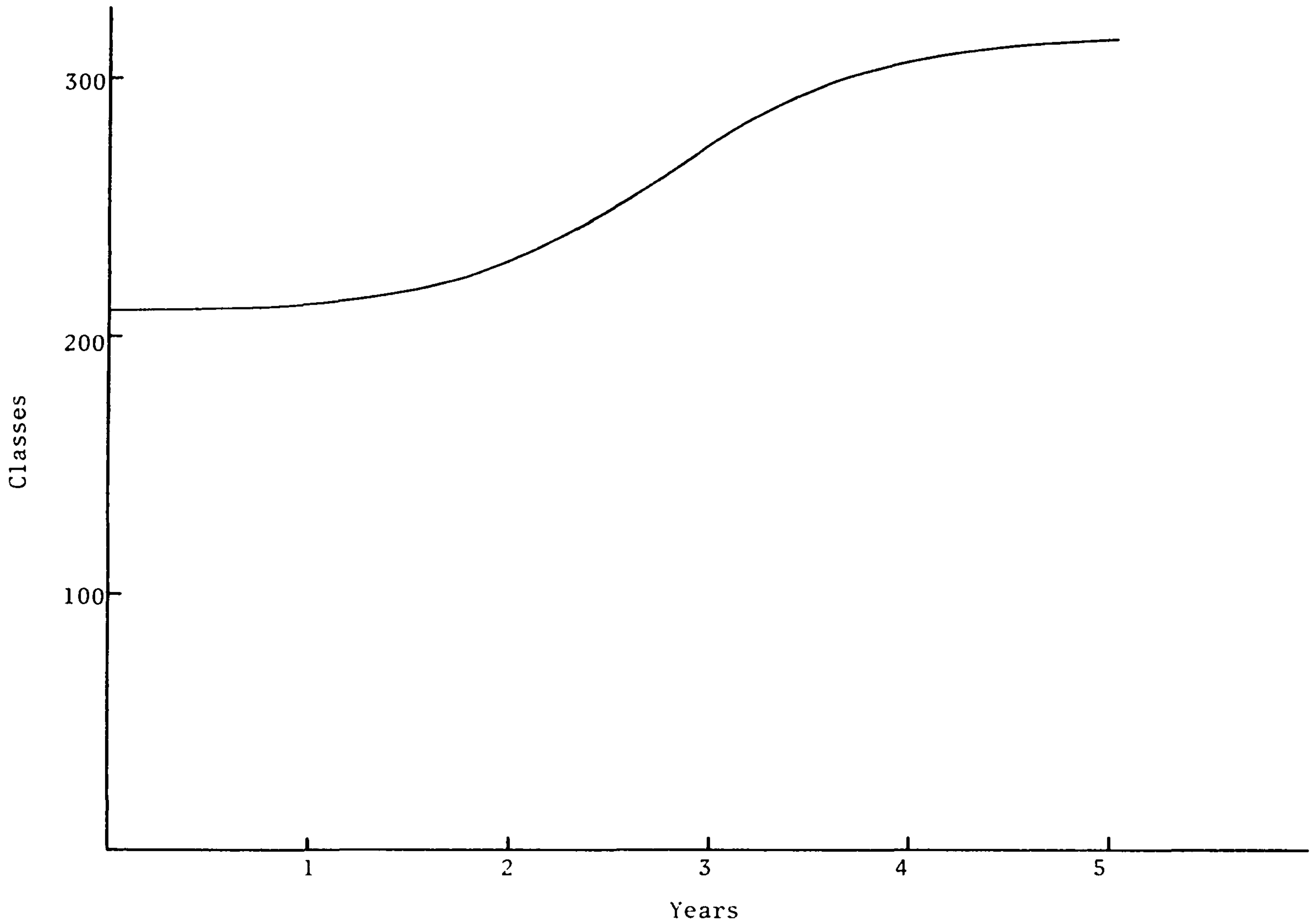

Figure 4-3. Projection of Needed Classes.

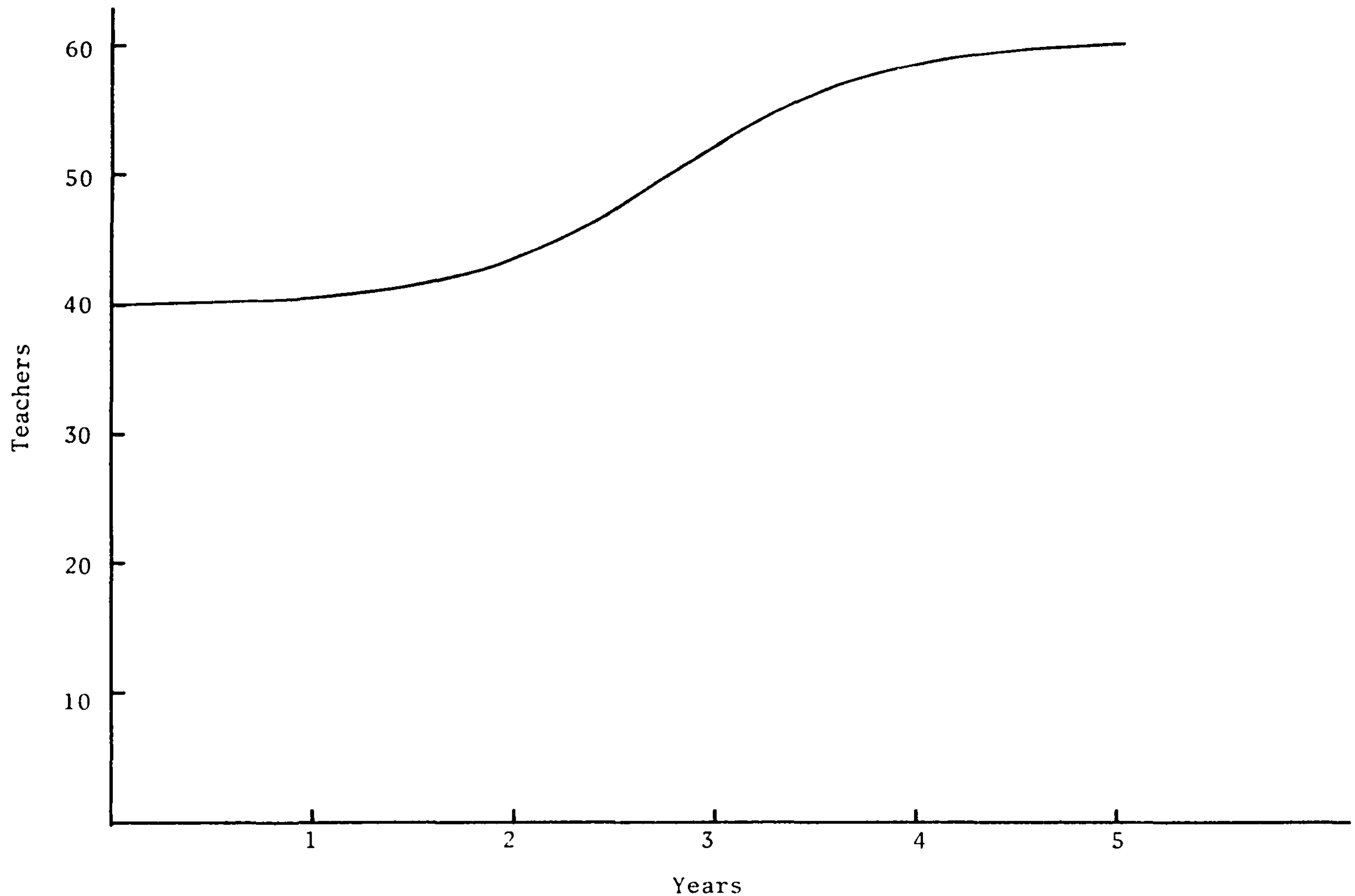

Figure 4-4. **Projection of Needed Teachers.**

Hence, as long as the stated inflation rate applied equally to all sources of prospective revenues and expenses, namely class and subject costs, the original identity held. Thus the projection of x_3 could be determined and is given in Fig. 4-5.

Next the costs for each of the variables $(x_1,...,x_4)$ was projected using a constant inflation rate of 4% per year.

Figure 4-6 and Fig. 4-7 give the projected inflationary increases in the class cost and the teacher cost respectively. Figure 4-8 and Fig. 4-9 give the projected inflationary increases in the subject cost and the enrollment cost respectively.

Hence, the projections of the expected revenues and the changes in the variables x_1 (classes), x_2 (teachers), x_3 (subjects), and x_4 (enrollment) had been determined. In addition, the cost associated with each of the variables C_1, C_2, C_3, and C_4 had been projected. Thus, the next step was to check the index model and see if its requirements were still satisfied.

Of principal interest, at this point, was to verify that the expected revenues were sufficient to finance the projected education plan. This was verified as shown in Table 4-1.

Table 4-1. Comparison of Expected Revenue and Needed Revenue.

Year	Expected Revenue	Needed Revenue
0	$ 465,285.00	$ 465,284.40
1	489,944.90	489,944.60
2	547,286.40	547,286.30
3	680,396.30	680,396.20
4	796,063.50	796,063.40
5	849,134.40	849,133.60

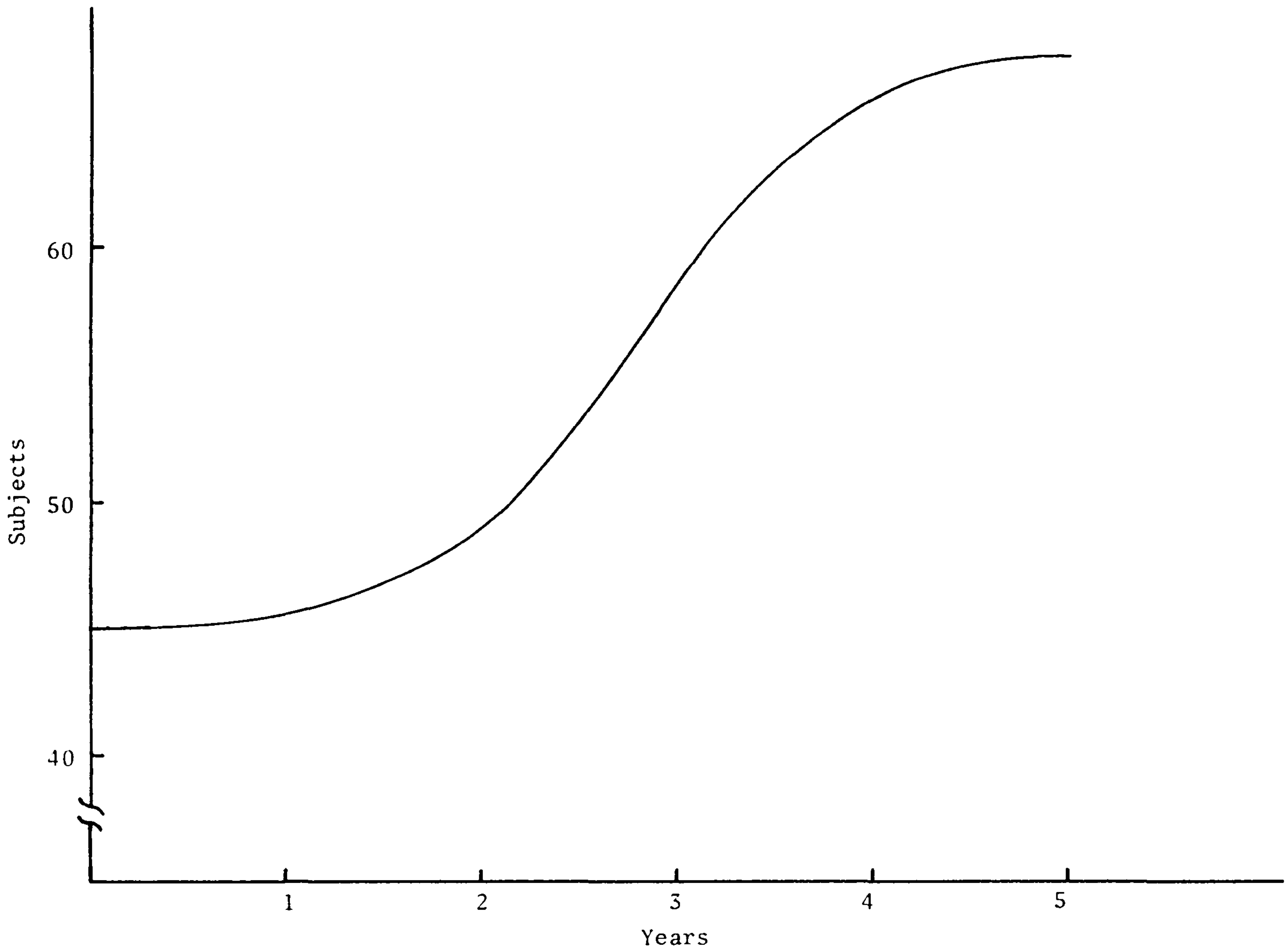

Figure 4-5. Projection of Needed Subjects

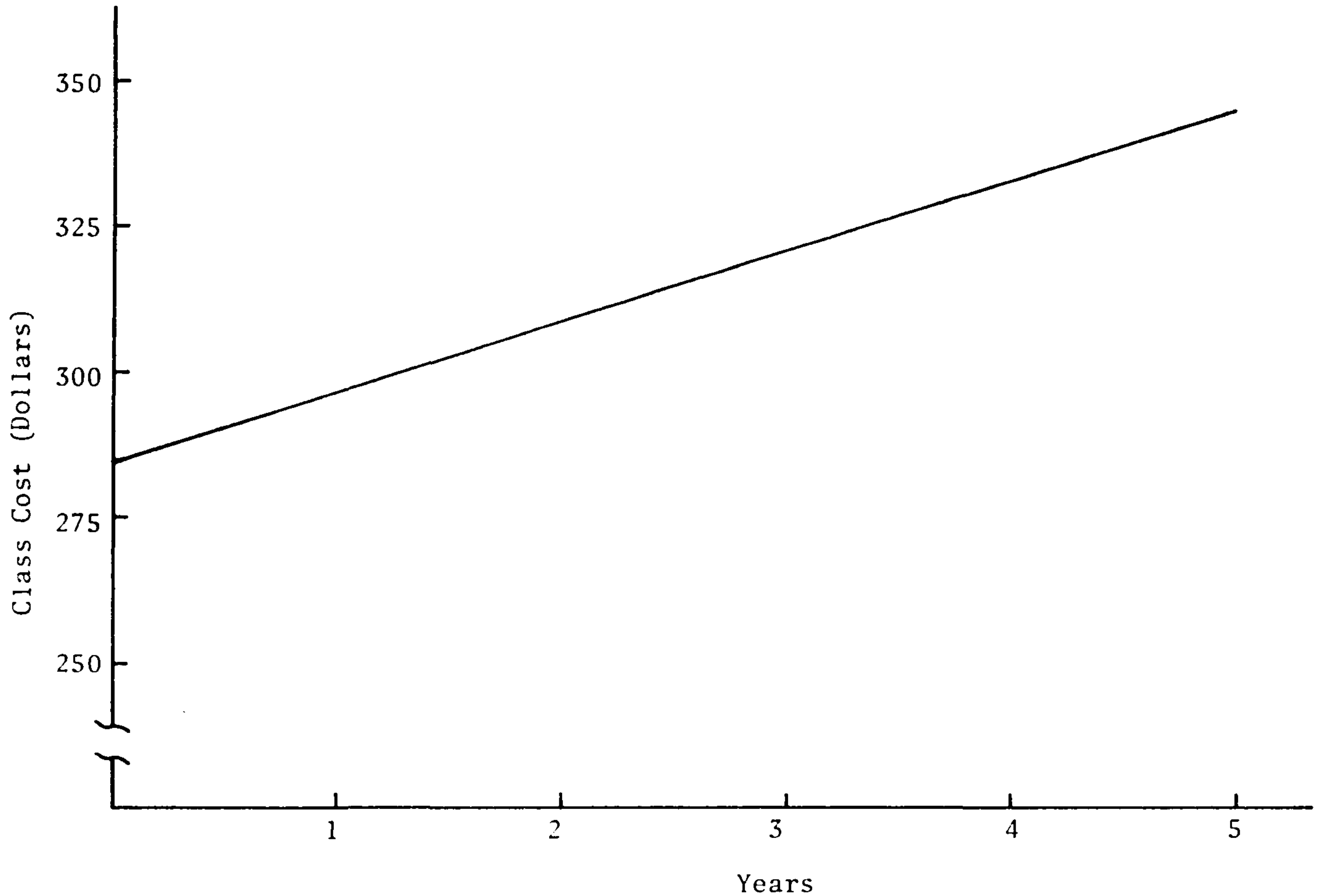

Figure 4-6. Projection of Expected Class Costs.

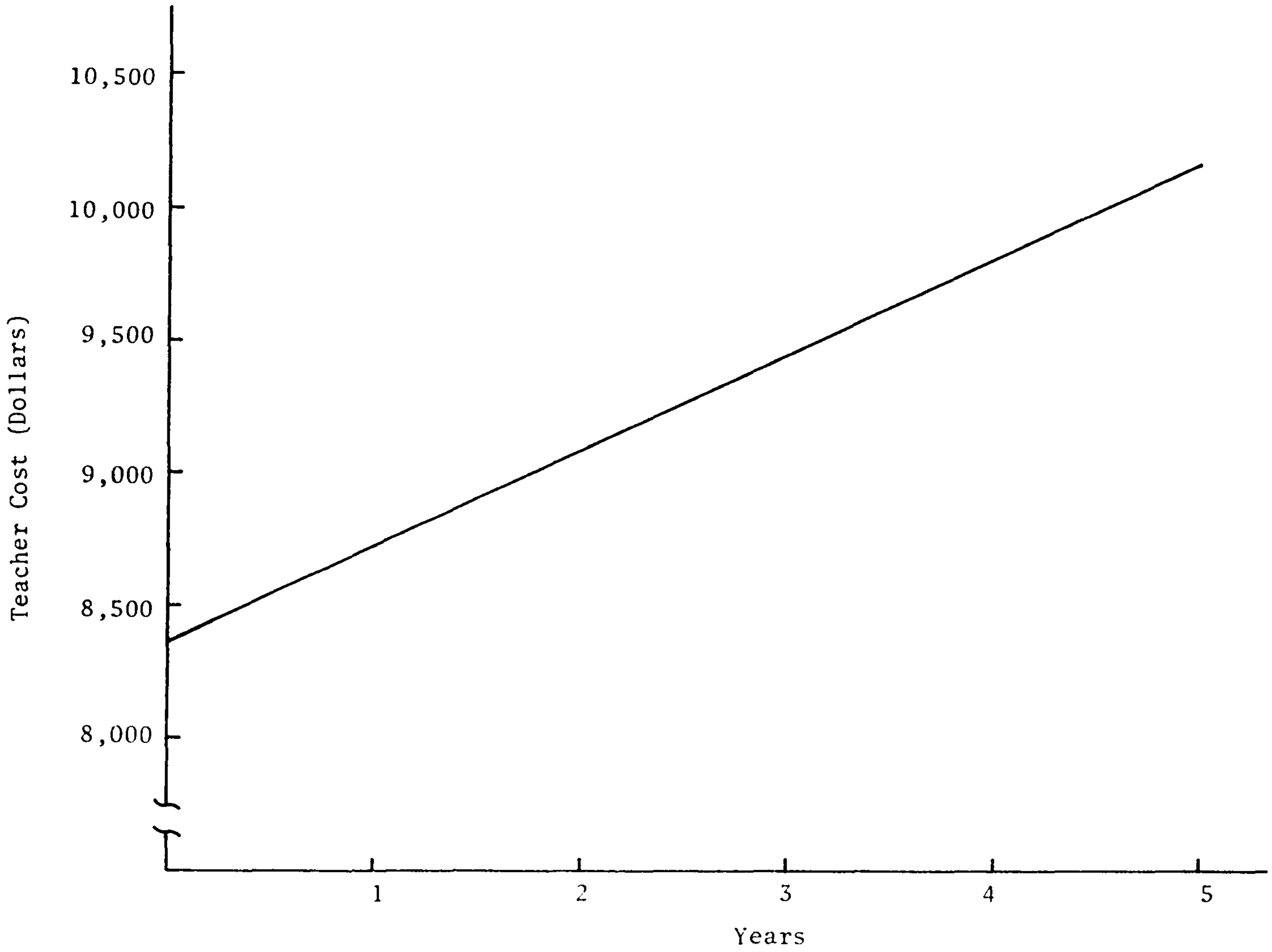

Figure 4-7. Projection of Expected Teacher Cost.

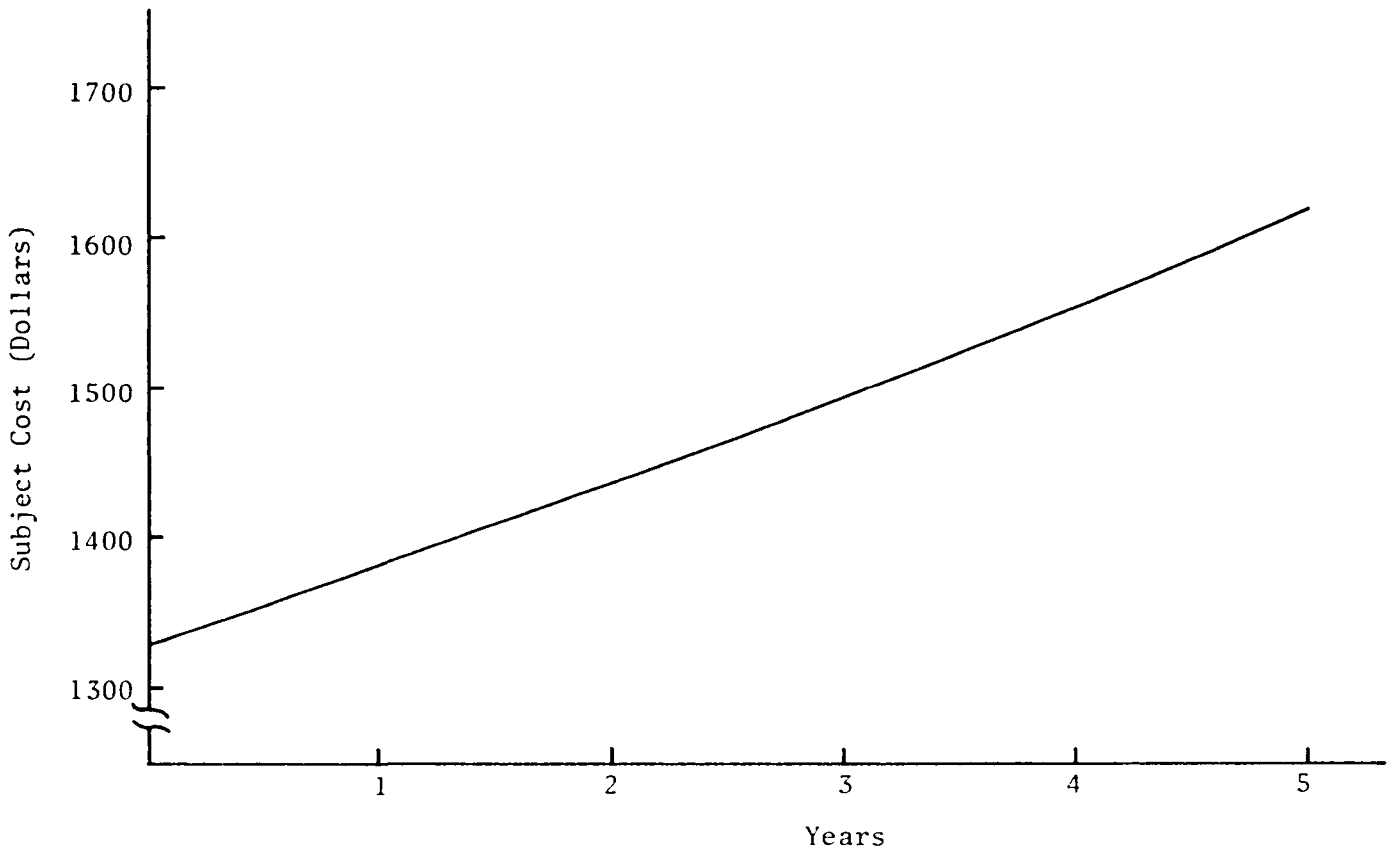

Figure 4-8. Projection of Expected Subject Cost.

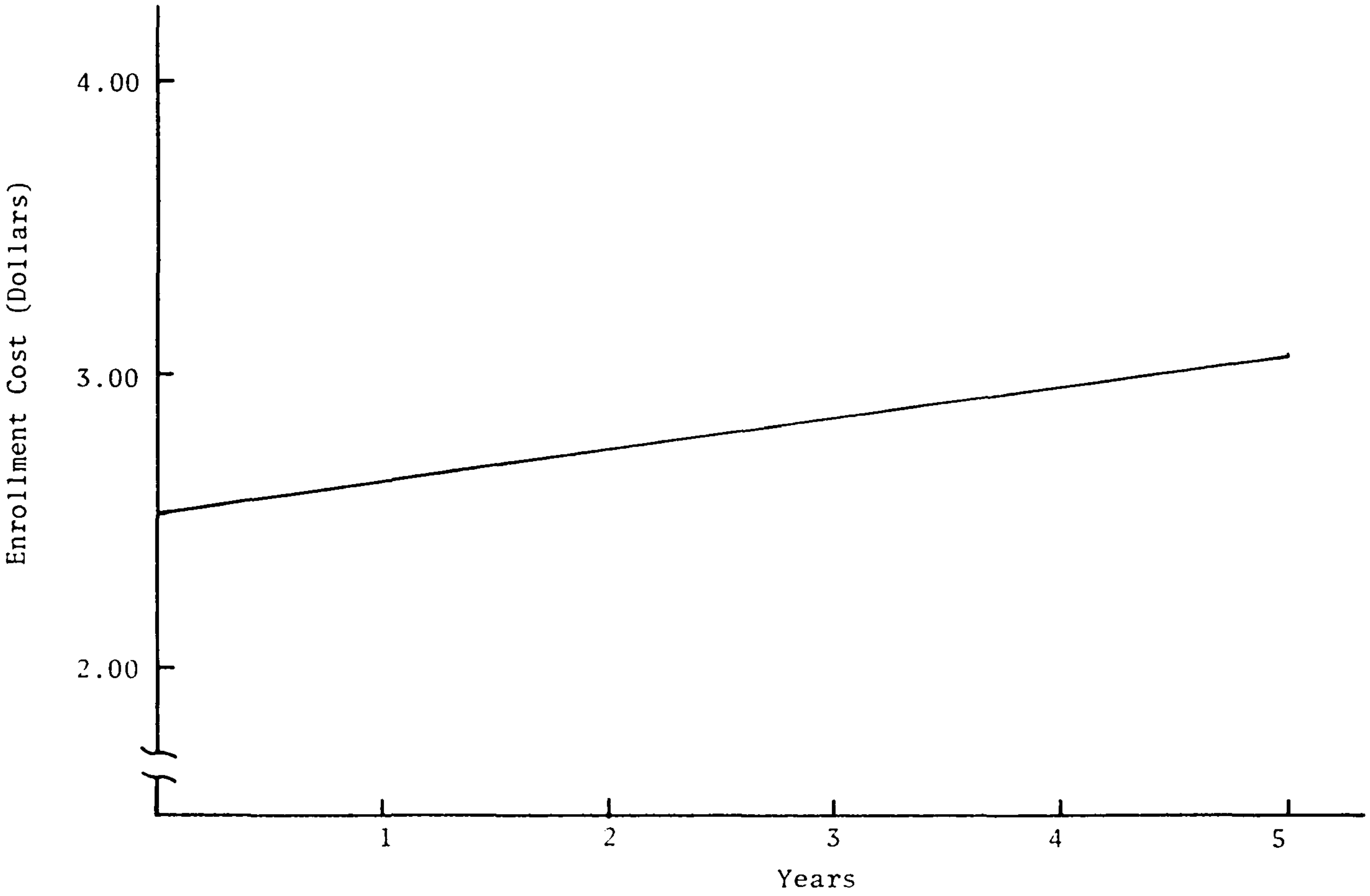

Figure 4-9. Projection of Expected Enrollment Cost.

The equality of expected revenue and needed revenue was anticipated because the only change to the system was due to inflation and enrollment increase. However, the enrollment increase was compensated for by increasing the number of classes, teachers and subjects. The effects of inflation were compensated for by corresponding increases in the total revenue made available.

The ideal situation as described in Case 1 is seldom encountered in the typical application. In fact, a large percentage of states experience deficiencies in needed revenue. In 1966 twenty states suffered a total expenditure gap of over $657 million. (3, p. 195) This figure included operating expenditures only and did not consider capital construction costs.*

III. Case 2

Case 2 considered a "state of nature" where there was a financial deficit. In other words, the projected needed revenue exceeded the expected revenue. One way that this could have occurred was for the teachers to demand salary increases which exceeded the increases given to compensate for an inflationary economy.

Another possibility for the higher rate of increase could have been the desire to increase the overall quality of the teachers. In

*This is, in part, the basis for the arguments in favor of expanded use of federal funds for support of public education. The 20 states needing equalization aid had only 28.5 per cent of the national average daily attendance in 1966. Thus the poor states are the less populated states (3, p. 196).

Case 2 an annual increase of 2% per year over the entire planning horizon of the five years was considered. This was in addition to fixed inflation rate 4% per year.

Case 2 did not assume that a compensating increase was made in the expected revenue; thus C_t was expected to be greater than the expected revenue. Hence, in Case 2, C_t became again the projected needed revenue required to finance the proposed educational plan.

Figure 4-10 summarizes the data for the new teacher's pay schedule and compares it with the old salary schedule. This shows the additional increase over the originally projected teacher cost.

The expected revenue was then compared to the needed revenue in Fig. 4-11. The expected financial deficit was evident. Obviously, there were two pure alternatives available to accommodate the difference between the expected and needed funds. Either revise the educational plan in a manner that will increase the revenues or decrease the expenditures. One could also face a combination of the two extreme conditions.

When they were examined in detail, the data indicated that one way to decrease expenditures was to decrease classes which might be followed by a decrease in the subject expenditure because of the identity $C_1x_1 = C_3x_3$. Several possibilities existed. One could have decreased X_1, thereby increasing the class size and decreasing the class load for the teacher (x_1/x_2). This would have required a decrease in either the subject cost (C_3) or the number of subjects (x_3). Another possibility might have been to decrease the class cost (C_1) and the number of subjects x_3, and so on. The point is that this one simple

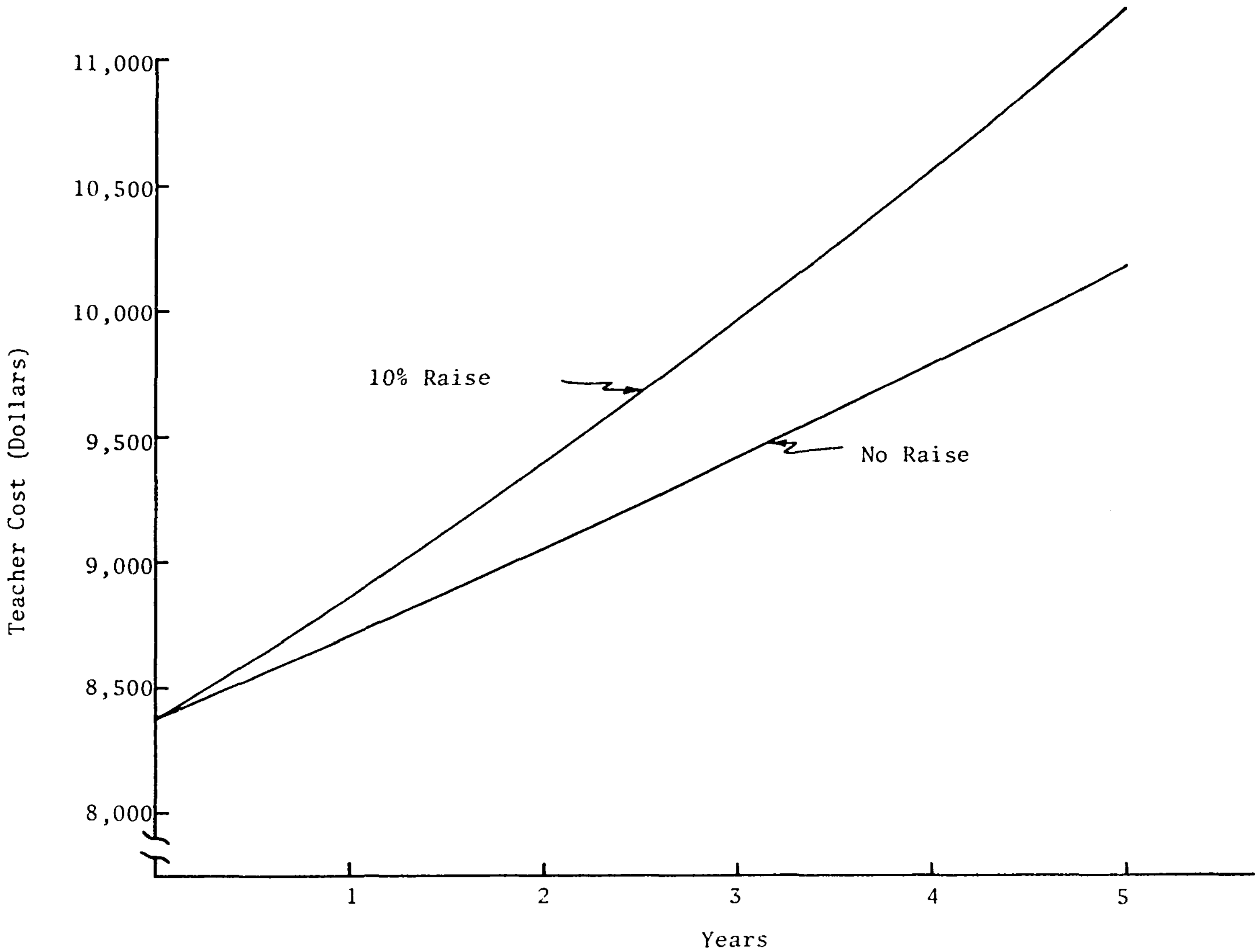

Figure 4-10. Comparison of Teacher Cost Before and After Raise.

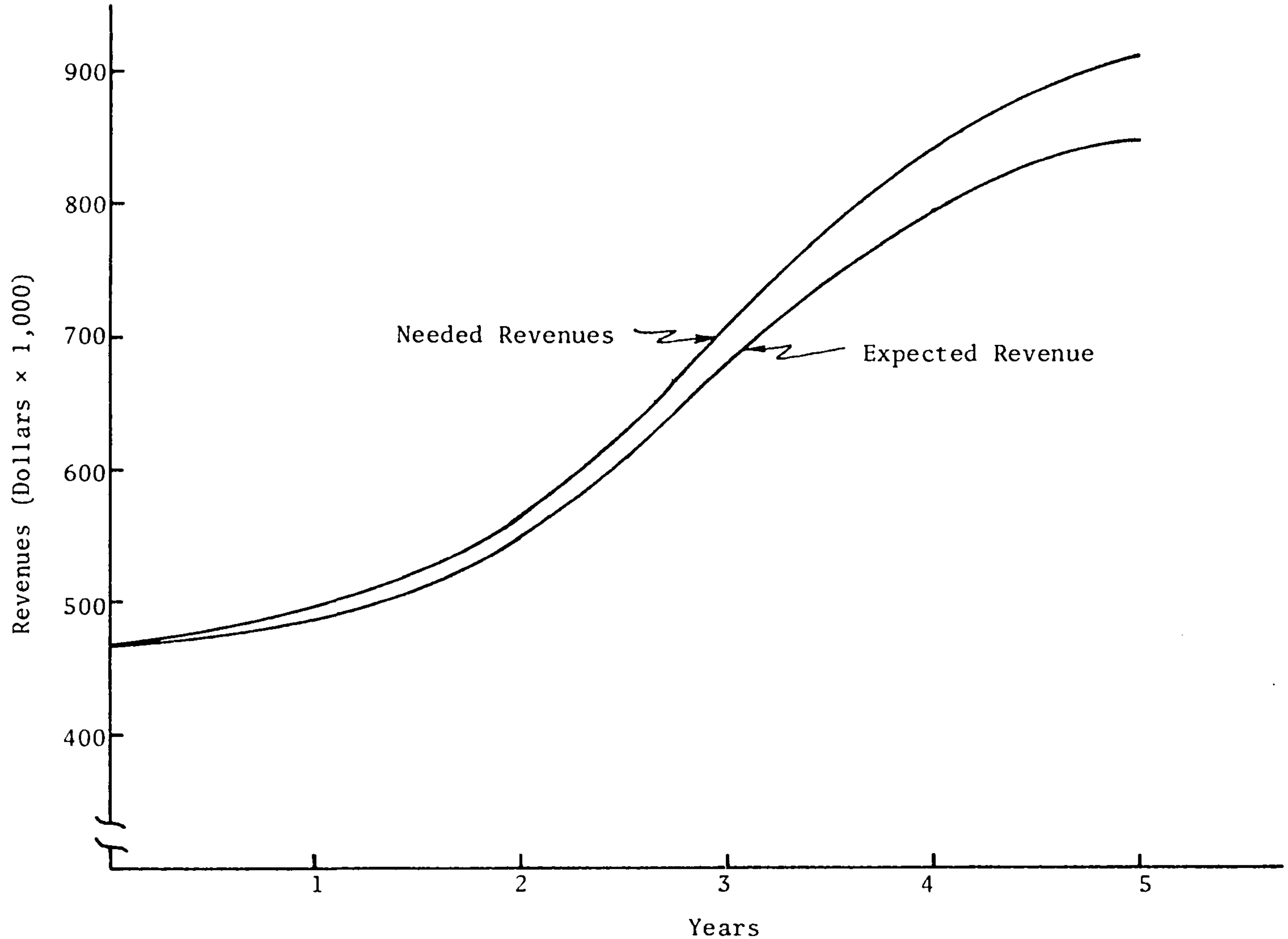

Figure 4-11. Comparison Between Expected and Needed Revenues.

identity could have been used to develop a number of different alternatives.

For Case 2 two different alternatives were tested. One of the alternatives tested was decreasing the number of classes (x_1) and making the necessary revisions in the number of subjects (x_3), while all the C_i values remained fixed. Figure 4-12 shows the variation in the total needed expenditure as the number of classes was decreased for the fifth year. This, of course, tended to increase the class size. Decreasing the number of subjects, of course, decreases the curriculum breadth. It was noted, however, that the class size increased from 20.11 enrollments per class to 21.12 as the needed expenditures decreased from $861,363.00 to $850,973.00 which was within range of the expected revenue of $849,133.60. The number of subjects (x_3) decreased from 67.5 subjects to a little less than 50 subjects. The ratio of classes to teachers (x_1/x_2) decreased from 5.25 classes per teacher to a little less than 4 classes per teacher. Thus this alternative decreased the needed expenditures to the level of the expected revenues, while teacher salaries were raised by decreasing the number of classes from 315 classes to a little less than 230 classes and decreasing the number of subjects from 67.5 subjects to a little less than 50 subjects.

The next alternative that was tested using the model was that of decreasing the number of classes and decreasing the subject cost. Again the fifth year data were used from the index check as C_2 was increased (See Appendix D). Figure 4-13 shows the variation in the needed expenditure as the subject cost was varied. The number of

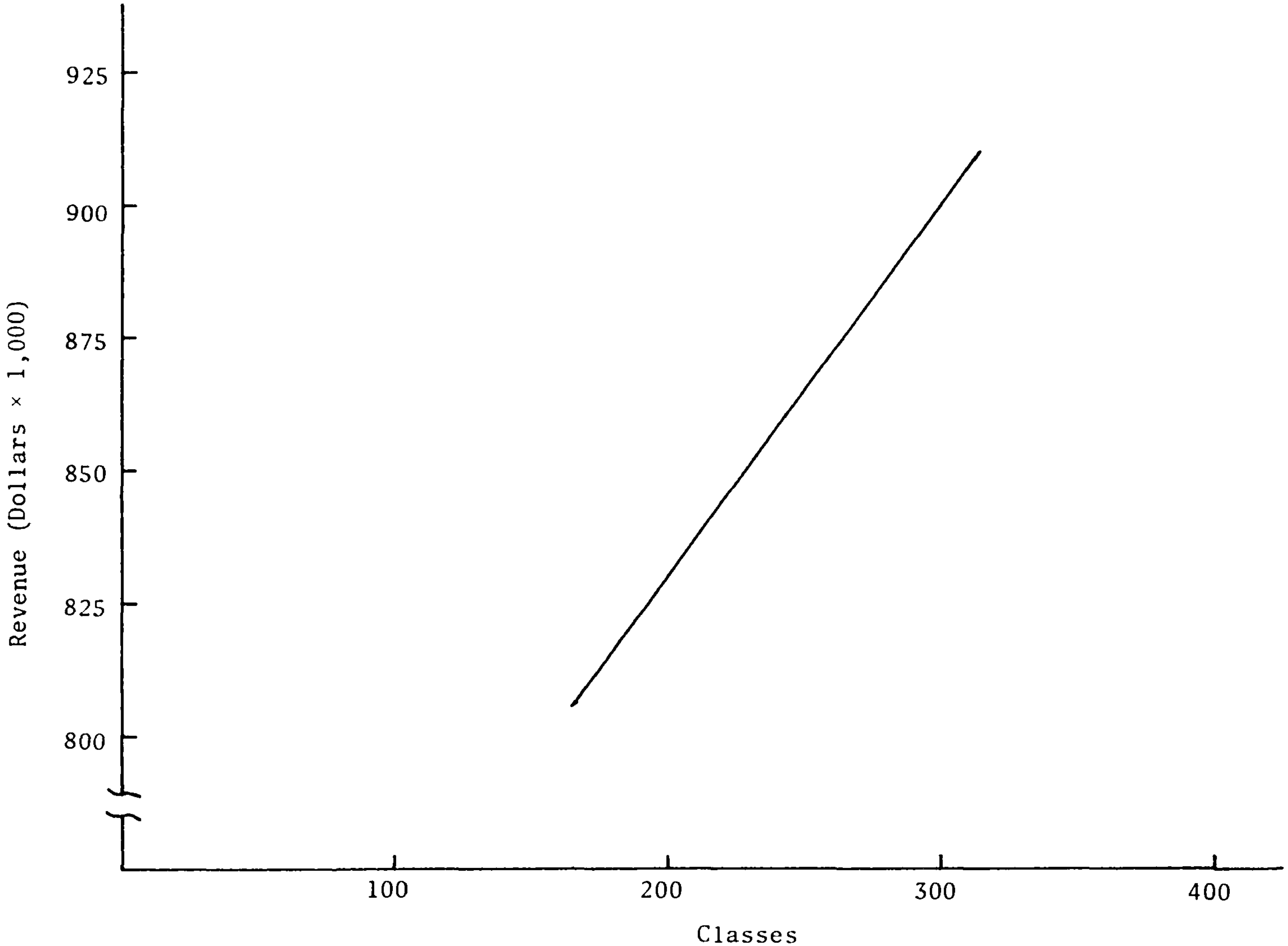

Figure 4-12. Variations in Needed Revenue As The Number of Classes Vary.

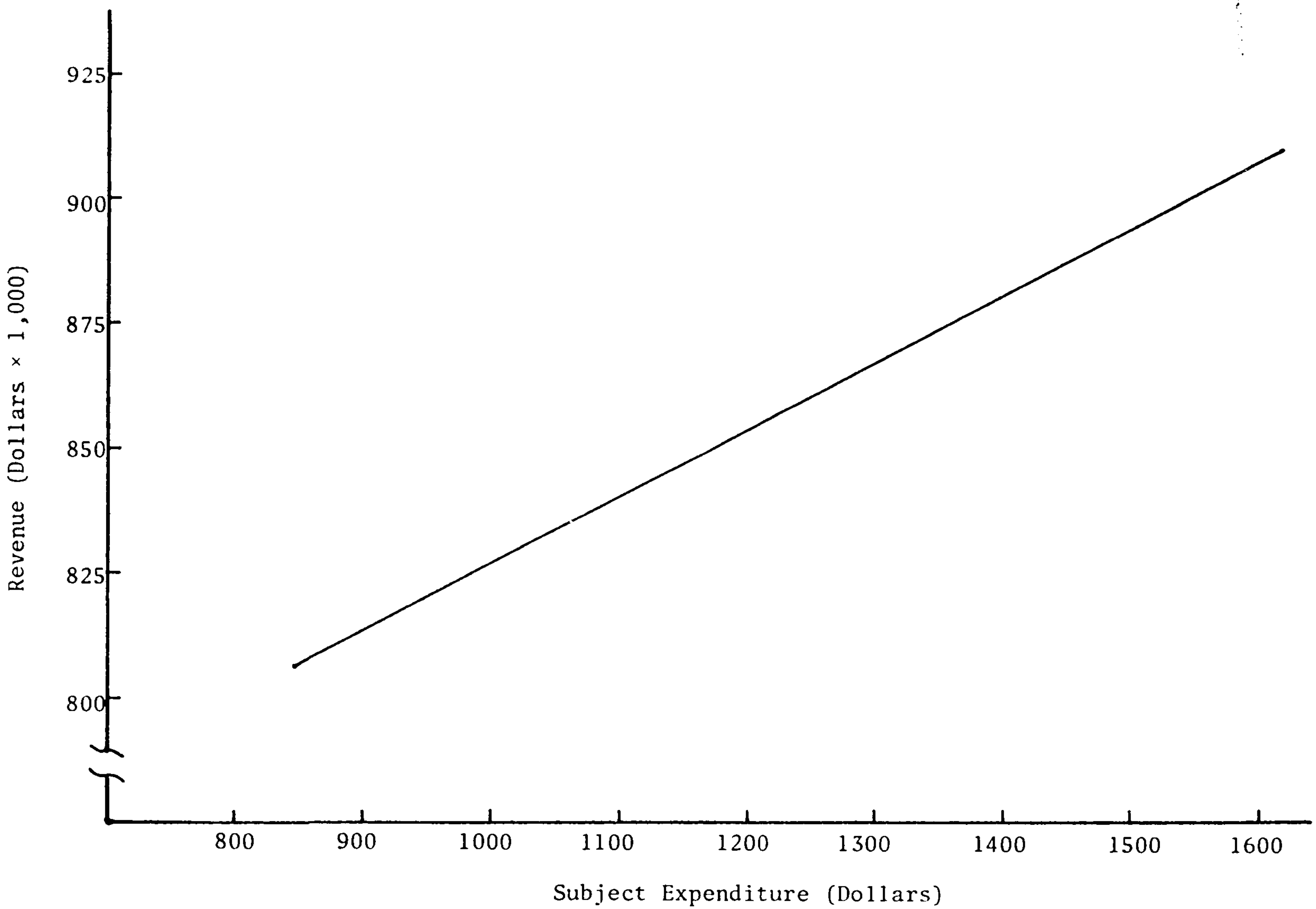

Figure 4-13. Variations in Needed Revenues As the Subject Expenditure Varied.

classes also varied; however, the number of subjects remained fixed at 67.50. Again the needed expenditure decreases to within the range of the expected revenue as the number of classes decreased from 315 classes to a little less than 230 classes. This resulted in a decrease in the subject expenditure from $1,616.20 to a little less than $1,200.00. The class size and classes per teacher were the same as those found in the first alternative tested. Thus, the needed revenue was decreased to the range of the expected revenue by decreasing the number of classes. Instead of decreasing the number of subjects the expenditures per subject was decreased. This would probably mean less equipment and class room aids for the teacher.

The above examples were for the purpose of demonstrating the flexibility of the planning index model. Its use in planning activities were then investigated.

First, with a knowledge of the C_i's, a secondary school can quantitatively appraise its present program and determine the priorities that it has directly or indirectly assigned to the variable x_1, x_2, x_3, or x_4. Thomas (46), as well as others, has demonstrated that the manner in which money has been allocated has been more important than the level of expenditures. This model then yields the information to aid in determining the manner in which resources could be allocated by the administrator.

Burkhead stated that,

> Given the strong tradition in most school systems of central authority for budget preparation, an authority typically lodged in the hands of the superintendent and his budget officers, it would appear that any major budgetary innovation must

> serve the superintendent's needs if it is to be viable. (6, p. 98)

Once the C_i's were determined, information concerning the requirements to realize future goals or expected change could be generated. Such items as the amount of revenue needed, the number of classes, and so on, could be readily obtained.

In addition to quantifying the present program and furnishing the necessary information to implement future plans the model would provide the methodology to test various programs that would strengthen the present program.

> One possibility to improve the conditions of teaching in low-income schools would be greatly reducing class size . . . and pay teachers a bonus of $1,000 - $2,000 annually for their willingness to accept assignments in difficult schools. (6, p. 93)

Because teacher cost along with enrollment and class costs could be determined by the model, sufficient information would be available to determine the feasibility of such change and the model would indicate alternatives that might be implemented so that certain educational targets could be realized.

Finally, the model could provide the state education departments with additional information to aid them in resource allocation decisions. The traditional educational financial standard has been the measure of expenditure per pupil or average daily attendance. This measure, in reality, only indicates the level of economic support and not the manner in which it is used in the system. Greater amounts of detail information could be generated using the model developed in this paper. Not only would total expenditures be indicated by C_t but they would be distributed in the manner indicated by the individual values of the C_i's.

Unfortunately the above mentioned allocation information is often not utilized in the most effective manner even when it is available. For example, program budgeting has tended to turn budget-making into a routine computational exercise that supports prior determination of programs. In other cases performance budgets are used to help "sell" a program in particular circumstances, and this is not unimportant. However, attractive brochures might be more effective in these cases. In most cases program and performance structures have been ignored by legislatures, as is the case with the United States Congress. (6, p.96-97)

In most cases, especially when the decision maker is not familiar with or does not possess detail knowledge about certain programs, there is a strong tendency to select a convenient criterion, such as a single number, upon which to base their decisions.* There would probably be a strong tendency on the part of unitiated managers to misuse the value of the objective function f(x) for the model developed in this paper.

The planning index model is of interest to the educational manager for several reasons. It provides the with a landmark to identify where his system is, has been, or is going with regards to the system's resources. The model also provides the manager with individual indicators of the manner in which the resources are distributed. However, doubling

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*A very common example of this type of phenomenon can be found in most institutions of higher education. Grade point averages are normally used, not only to rank students scholastically, but also are used to judge the total person. This is especially true when other details concerning the individual are not available.

the value of f(x) would not necessarily indicate that the system's "goodness" has doubled. Increasing the value of f(x) would indicate that the manner in which resources were allocated had improved. But, intra-school comparisons, based solely on values of their f(x)'s, might be misleading. The reason for this is that the optimality of the problem was accepted earlier because of the individuality of the school and its unique environment, which included the nonquantitative variables as well as the quantified variables. In other words, the model is dealing with a particular school, a particular policy maker, and the forces acting on both.

The C_i's, however, might well lend themselves for comparison on a limited basis, as was discussed earlier in the chapter, where they were indicators of the manner in which the resources are distributed. The actual values of the individual C_i's may not be as significant as the comparisons of the values between the C_i's for the school or perhaps intra-school comparisons.*

There are many examples similar to the above example that could be examined. The ones that were discussed were chosen to demonstrate a comprehensive and yet transparent system that would be symbolic of the models flexibility. This next section will discuss an indispensable part of any application endeavor. The following deals with data types

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*It is interesting to compare the values that were obtained for the C_i's in Appendix B that were subjectively made after consulting the sources in Appendix A and the values of the C_i's obtained from the model which are in Appendix D. Appendix B's values were made completely independently of any knowledge of the values found in Appendix D, and yet there is a surprising similarity between them.

and sources needed in an actual application of the model.

IV. Data Collection and Sources

In every organization there are people who are responsible for, and have at their fingertips, a great deal of present operations data as well as historical data. Quite often, the information is available and potentially very useful. However, decision makers usually ignore these sources because the data are difficult to "dig out" and even if it were readily available, most managerial personnel are not in a position to analyze the data properly. (35, p. 190)

In order to locate appropriate starting data for a school study, the first step might be to visit the state department of education. The reason for this is that every school district must submit a standard budget in order to receive financial support and these budgets are kept on file for a number of years. In addition each school must submit an application for accreditation which contains the full schedule of classes, details on the teaching staff, particulars on the supporting personnel, and enrollments in each class. Hence, this is a very good starting place to get an overview for any school. Not only is all the information in one report but it is also tallied so that it is fast to retrieve.

The budget, in turn, is usually not adequate for a reliable data source, other than some gross estimate for the following reasons. First most school districts do not budget by schools. If they do construct budgets for the individual schools, they are consolidated into one report for the school district and then sent to the state department of education. Secondly, the information contained in the budget may or may not represent actual expenditures. Individual entries are merely guideposts and do not indicate the expenditures from the various accounts. Finally, the budgets submitted to the state department of education are projected expenditure needs for the forthcoming year and hence are subject to modification and revision.

As a second step, it is important to plan the data gathering carefully so that efforts are not expended on data that is of little value while other items of prime importance are neglected. It is typically necessary to limit the scope of the data gathering process because of economic tradeoffs. The amount of resources one is willing to expend on an item of information must be weighed against the economic benefits to be realized from such an effort.

The next step is to organize a conference of "in-house" specialists for the school, such as operations and maintenance personnel, teaching staff representatives, perhaps a school board member, and individuals of the administration. It might be advisable to include representatives of the student body.* The number of participants should be kept small, somewhere between five to ten people, for this minimizes the problem of managing such a conference and of analyzing the data.

The high cost of utilizing the time of such specialists represents another practical reason for keeping the group small (student time exempted). It would be advisable to communicate to each of the participants before the conference so that each can prepare for the conference

*It has been all too common a practice to completely disregard any consideration of including secondary student comments and ideas. In a typically list of priorities, students have been traditionally placed close to the bottom.

by doing some homework. This is where careful planning can be utilized by letting each individual know what is expected from him. After a well planned conference, the "state of nature" for the particular school should be predictable or at least limited to just a few of all the possible states which could occur during the period of time under preview.

At this point, one should be ready to test the mathematical model to establish its applicability and its underlying assumptions. If positive results are indicated, one can proceed with the full planning study for the system.

Finally, the results are communicated and explained sufficiently so that the users of the information will feel comfortable applying the results and yet understand the model's limitations.

It is clear that this type of planning study can be very costly. However, if the expenditures are high, the resulting program that will be the result of a good model application will more than compensate the expenses incurred.*

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*The above discussion was adapted to educational systems section concerning information gathering and organization for rational decision-making industry of Reisman's Book (36, section 8.2-1). Correa (13, Chapter 4) presents a detailed description of the main elements in the analysis of an educational system. Although Correa is discussing micro models, his comments could be easily applied to the individual school.

CHAPTER V

SUMMARY AND CONCLUSIONS

I. Summary

This research dealt with the allocation of scarce economic resources in a secondary educational system. It also was concerned with a methodology for the analysis of the effects that resource allocation had on the variables used to measure the operation of a secondary school.

The first portion of the research dealt with the similarities and differences of resource allocation in the educational environment and the traditional mercantile environment. This portion of the investigation identified three basic improvements needed by educational resource management. They were, 1) a quantitative process formula relating inputs to outputs, 2) better organization and analysis of existing data, and 3) a resource planning model for the local school.

The review of past research in the area of educational resources indicated that a planning model with these improvements was needed and that the development of such a model would be feasible. While it seemed feasible to develop an overall model for resource allocation, it became evident that a critical factor in the model development would be the formulation of the weighting coefficients used in analyzing individual factors measuring educational operations. The formulation of the weighting coefficients in previous research did not appear to be satisfactory because the techniques involved making subjective judgments by the principal.

A model was then developed that eliminated the arbitrary determination of the weighting coefficients. This was done by assuming that the present operating policy was optimal. In addition, the weighting coefficients were assumed to be given by ratios of the various costs associated with the problem's variables. This allowed the application of certain mathematical techniques to the problem, such that, the weighting coefficients were found analytically without involving any subjective judgment. Essentially, this procedure could be thought of as a "reverse optimization."

> Most often, profit [or cost] improvements stem from executives possessing a deeper understanding of the problem area, and hence developing a keener sense for taking correct actions and maintaining control in an uncertain and competitive environment. . . . In a p eponderence of successful applications, the applications, the beneficial effects are truly manifest in the altered decision behavior of executives and managers . . .
>
> Second, although an operations research model often uses the mathematics of optimization, the resultant solution should not be viewed a *necessarily* yielding an optimal answer to the real problem. After all, as the text has stressed throughout, a model is *inherently* an approximation to reality, and therefore an optimal solution to this approximation need not be the "final" answer to the actual decision problem. The important issue, however, is not whether a proposed solution is *optimal*, but whether the solution yields a significant enough *improvement* over the alternatives to make it worthy of acceptance. (49, p. 928)

The model development in this research would provide the following results when applied to a secondary school.

1. It would enable the secondary school to quantitatively identify program costs for its present resource allocation.

2. It would furnish the administrator with important information concerning the economic requirements for implementation of future change or to realize future goals.

3. It would indicate what areas might be strengthened in the present system by identifying the manner in which present resources were allocated.

4. It would provide the state educational department with additional information to aid them in their allocative decisions.

The feasibility of the model was demonstrated by applying it to hypothetical data for a typical secondary school in a typical urban area. The methods to plan quantitatively for future targets and/or changes were also described for a planning horizon of five years.

Finally the thesis discussed procedures for data gathering and the various types of sources that could be utilized to obtain these data. This data collection phase would be critical for the actual application of the planning model.

II. Recommended Future Research

Continued efforts must be made in program budgeting in education. This model should be helpful in estimating the various costs, especially if the variables were broken up into the general areas of the curriculum, such as, language arts, science, and so on. This would decrease the amount of gross averaging of the costs of teaching in radically different deciplines. Program budgeting, however, would require that the building cost and space consideration by incorporated into the model. This would involve adding a minimum of one variable, perhaps area per pupil. Adding this variable and its associated cost would require another balancing ratio in order to establish an equilibrium as was shown for the present model in Fig. 2-2.

Another area of potential research that is related to the above mentioned area would be the testing of the validity of the values for the costs found by the reverse optimization (RO) model. The values found for the C_i's should reflect tangible expenditures that could be categorized into a system similar to the proposed organization of the budget given in Appendix B. If the C_i's could be determined by another method, then they could be used in the RO model, where the RO model would become a regular nonlinear programming problem (NLP) and could be solved by applying one of the standard NLP algorithms.

The investigation of the costs might also verify whether or not the costs (C_i's) are linear. The costs were assumed linear in this study for all ranges of x_i for simplicity. However, studies concerning the economies of scale indicate that the costs might be nonlinear and that there exists an optimal size that would be the most efficient operating level.*

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*See Nels W. Hanson "Economy of Scale as a Cost Factor in Financing Public Schools," *National Tax Journal*, Vol. 17, No. 1 (March 1964), p. 92-95, for an interesting study of economies of scale at the district level. For an exploratory study at the high school level see Gerald T. Kowitz and William C. Sayres, *Size, Cost and Educational Opportunity in Secondary Schools* (Albany: New York State Education Department, 1959).

Finally continued research is needed to relate economic inputs to quantifiable educational outputs. This will first require development activities such as:

1. A clear and precise statement of educational objectives,

2. Techniques for recognizing and measuring the degree of attainment of the objectives, and

3. Techniques to perform discriminative analysis to see what efforts are good and effective, and what are bad and inefficient.

III. Conclusions

Countless small communities across the U.S. are experiencing wanted tax increases while school administrators are considering dropping courses and putting the schools on double sessions to economize and to cope with defeated bonds or tax increases. It is important then, that the money spent for education be spent wisely.

Resource allocation studies in education are presently needed and that need will grow as inflation raises the cost of education each year. The RO model developed in this study offers not only the methodology for determining the level of program expenditures that can be expected for a given level of operations, but also indicates the manner in which the money will be spent. With additional development, the model could be used to report the effects of different combinations of goods and services upon the school system.

One of the largest obstacles that schools must overcome is their past and present operational mode. Educators must contend for a place in the hierachies of American power and influence. They have become so embeded in an economic and political second-class citizenship that most educators can, at best, exercise indirect influence in educational policy-making. Garvue termed this the "Greyhound bus theory." "You educators do the teaching and leave the decisions to us." "Us" being the rest of society. Only if they can emerge as a powerful profession will educators be able to make their political and economic interest understood. This emergence would certainly sharpen up political debate, and would heat up the processes of allocating resources in educational budget-making sessions. However, if educational needs continue to go unmet, the world's greatest social innovation may be destroyed bit by bit. (20, p. 321)

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APPENDIX A

DATA FOR PLANSVILLE HIGH SCHOOL

Appendix A

The following constructed data was obtained by scaling the basic data for an average daily attendance of 800. The basic data was obtained from the following sources:

"Accreditation Report of Clinton High School, 1971-72," Instruction Division, Oklahoma State Department of Education, State Capitol, Oklahoma City, Oklahoma.

Mr. Bumgarner, Superintendent of Buildings and Grounds, private interview held at the offices of the Norman School District, Norman, Oklahoma, December 1, 1971.

Mr. Cecil Folks, Assistant Director, private interview held at the offices of the Finance Division, Oklahoma State Department of Education, State Capitol, Oklahoma City, Oklahoma, November 30, 1971.

Mr. Bill Harris, Instructional Program Coordinator, private interview held at the offices of the Instructional Division, Oklahoma State Department of Education, State Capitol, Oklahoma City, Oklahoma, November 30, 1971.

Orlando F. Furno and Paul K. Cureo, "Cost of Education Index, 1970-71," *School Management*, Vol. 15, No. 1, January 1971, pp. 10-63.

Orlando F. Furno and James E. Doherty, "Eleventh Annual Cost of Education Index, 1969-70," *School Management*, Vol. 14, No. 1, January 1970, pp. 35-43.

"Master Schedule of Norman High School, 1971-72," Norman High School, Norman, Oklahoma.

Oklahoma State Department of Education Annual Report, 1969-70, Oklahoma State Department of Education, State Capitol, Oklahoma City, Oklahoma.

Dr. Wallace R. Smith, Superintendent of Buildings and Grounds, private interview held at the offices of the Oklahoma City School District, Oklahoma City, Oklahoma, November 30, 1971.

"Summary Report of Subjects Offered in Oklahoma Junior and Senior High Schools, 1970-71," Instructional Division, Oklahoma State Department of Education, State Capitol, Oklahoma City, Oklahoma.

"Tenth Annual Cost of Building Index," *School Management*, Vol. 15, No. 6, June, 1971, pp. 12-16.

Mr. Young, Assistant Principal, private interview held at the offices of the Norman High School, Norman, Oklahoma, December 1, 1971.

Plansville Senior High School

Plansville is an imaginary mid-western town with a population of 16,000. Most of the population is employed at a nearby metropolitan area. Plansville Senior High School is the only high school in the school district and has had a good relationship with the community; however, its achievements in sports has been the concern of some of the community's fathers. Next year has been promised to be better, especially for the girls' basketball team.

During an interview with Mr. Hope, principal of Plansville Senior High School, the following information was obtained:*

Average daily attendance = 800

Number of equivalent full time teachers = 40

Number of different subjects = 45

Total number of classes = 210

Total number of courses = 80

Average number of classes per pupil = 5.28

Average number of sections per course = 2.625

Average teacher salary = $7,284.00.

---

*Mr. Hope had to consult his records and assistant for some of the information.

Therefore,

$$x_1 = 210$$

$$x_2 = 40$$

$$x_3 = 45$$

$$x_4 = 4224 \; .$$

Plansville school district had budgets for each individual school. The following is the budget for Plansville Senior High School:*

Administration--		$ 18,088.00
Professional Salaries	$8,320	
Clerks and Secretaries	5,232	
Other Expenditures	4,536	
Instruction---		$ 359,384.00
Classroom Teachers	$291,360	
Other Professionals	36,832	
Clerks and Secretaries	9,480	
Textbooks	4,600	
Other Teaching Material	12,528	
Other Expenditures	4,584	
Health--		$ 2,928.00
Professional Salaries	$2,568	
Other Expenditures	360	
Operation---		$ 40,728.00
Custodial Salaries	$22,856	
Heat	5,744	
Utilities Other Than Heat	8,400	
Other Expenditures	3,728	
Maintenance---		$ 13,752.00
Maintenance Salaries	$5,160	
Other Expenditures	8,592	
Fixed Charges---		$ 29,133.00
Retirement Fund	$20,245	
Other Expenditures	8,888	
Other Services--		$ 1,272.00
TOTAL CURRENT EXPENDITURES		$ 465,285.00
Current Expenditures----------------------------------		$ 465,285.00
Capital Outlay--		$ 10,360.00
Debt Service--		$ 43,336.00
TOTAL EXPENDITURE**		$ 518,981.00

*The budget is based on a total expenditure of $648.73 per average daily attendant (excluding transportation cost).

**Excludes transportation.

APPENDIX B

REORGANIZED BUDGET OF PLANSVILLE HIGH SCHOOL

Appendix B

This is to demonstrate how the budget given in Appendix A might be organized into different categories. The various divisions of expenditures were subjectively made after consulting the sources in Appendix A.*

Total Teacher Cost----------------------------------		$ 337,172.30
Salaries	$291,360.00	
Fixed Charges**	24,765.00	
10% Administration	1,924.00	
35% Operation	14,866.60	
30% Maintenance	4,256.70	
Total Cost of Enrollment---------------------------		$ 41,738.50
Health	$ 3,219.00	
Other Services	1,272.00	
80% Administration	15,286.40	
35% Operation	14,866.60	
50% Maintenance	7,094.50	
Total Building Cost---------------------------------		$ 58,916.00
Debt Service	$ 43,336.00	
30% Operation	12,742.80	
20% Maintenance	2,837.80	
Total Curriculum Cost------------------------------		$ 81,167.80
Other Professionals	$ 36,832.00	
Clerks and Secretaries	10,353.00	
Textbooks	4,600.00	
Other Expenditures	4,584.00	
Capital Outlay	10,360.00	
10% Administration	1,910.80	

---

*These divisions of expenditures are for demonstration purposes only.

**The fixed charges were distributed by percentages of salaries in each of the budget categories.

If the above were the actual case, then

$$C_2 = 337{,}172.30/x_2 = \$8{,}429.31 \text{ per teacher}$$

$$C_4 = 41{,}738.50/x_4 = \$9.88 \text{ per enrollment}$$

$$C_1x_1 + C_3x_3 = \$81{,}167.80 \text{ per school}$$

and,

$$C_* = \$58{,}916.60 \text{ (Excluding transportation)}$$

APPENDIX C

COMPUTER PROGRAM LISTING OF THE MODEL

```
// JOB    0C01

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0C01        0C01        0000

V2 M09   ACTUAL  8K  CONFIG  8K
```

```
// JOB T                                                ER360302   88668      WALTERS

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0C01        0C01        0000

V2 M09   ACTUAL  8K  CONFIG  8K

// FOR
**  ER360302,DOUGLAS H. WALTERS,RESEARCH,PROGRAM 03572
*EXTENDED PRECISION
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
```

```
      SUBROUTINE FIGUR
      DIMENSION H(4,4)
      COMMON CT,C1X1,C2X2,C3X3,C4X4,X1,X2,X3,X4,IW,PC2X2
      COMMON C1,C2,C3,C4
C*****************************************************************************
C
C     SUBROUTINE FIGUR CALCULATES THE VALUES OF THE LAMBDAS AND OTHER
C     INDICATORS OF THE RESULTS OF CHANGES
C     FUNX IS THE VARIABLE NAME FOR THE OBJECTIVE FUNCTION OR FUNCTION OF X
C     XL1 DENOTES LAMBDA 1, XL2 DENOTES LAMBDA 2, XL3 DENOTES LAMBDA 3
C     AND SO ON. XL1X1 DENOTES THE PRODUCT OF LAMBDA 1 AND X1 AND SO ON
C     WITH THE OTHER VARIABLES AND LAMBDAS
C
C*****************************************************************************
      FUNX = (C4X4 / C3X3) +C3X3 / C2X2 - C1X1/C2X2 - C4X4/ C1X1
      XL1X1 = - C1X1 / C2X2 + C4X4 / C1X1
      XL1 = XL1X1 / X1
      XL2X2 = - C3X3 / C2X2 + C1X1 / C2X2
      XL2 = XL2X2 / X2
      XL3X3 = -C4X4 / C3X3 + C3X3 / C2X2
      XL3 = XL3X3 / X3
      XL4X4 = C4X4 / C3X3 - C4X4 / C1X1
      XL4 = XL4X4 / X4
      XL5 = (C1X1 - C3X3)/(C2X2)**2 + C4X4/(C1X1)**2 - C4X4/(C3X3)**2
     1 + 1./C3X3 - 1./C1X1
C*****************************************************************************
C
C     THE HESSIAN MATRIX IS COMPUTED. IT IS NOTED THAT THE HESSIAN MATRIX
C     IS SYMETRICAL.
C
C*****************************************************************************
      H(1,1) = -(2.*C4X4)/( C1X1*X1**2)
      H(1,2) = C1 / (C2X2*X2)
      H(1,3) = 0.0
      H(1,4) = C4 / (C1X1*X1)
      H(2,2)= (-2./(C2X2*X2**2)) *(-C3X3 + C1X1)
      H(2,3) = - C3 / (C2X2*X2)
      H(2,4) = 0.0
      H(3,3) = (2. * C4X4) / (C3X3 * X3**2)
      H(3,4) = - C4 / (C3X3*X3)
      H(4,4) = 0.0
      DO 10 I=1,3
      K = I + 1
      DO 10 J=K,4
   10 H(J,I) = H(I,J)
C*****************************************************************************
C
C     THE FRACTION OF EXPENDITURE FOR EACH AREA IS NOW COMPUTED
C
C*****************************************************************************
      P1 = C1X1 / CT
      P2 = C2X2 / CT
      P3 = C3X3 / CT
      P4 = C4X4 / CT
      TP = P1 + P2 + P3 + P4
C*****************************************************************************
C
C     THE PRINCIPLE DETERMINANTS OF THE HESSIAN MATRIX IS NOW COMPUTED
C
C*****************************************************************************
      A1 =H(1,1)
      A2=H(1,2)
      A3=H(1,3)
      A4=H(1,4)
      A5= H(2,2)
      A6 = H(2,3)
      A7 = H(2,4)
      A8 = H(3,3)
      A9 = H(3,4)
      A10 = H(4,4)
      AA1 = A1
      AA2 = A1*A5 - A2**2
      AA3 = A1*(A5*A8-A6**2)-A2*( A2*A8 - A6*A3) +A3 * (A2*A6-A3*A5)
      AA4 = A1 * (A5*(A8*A10 - A9**2) - A6*(A6*A10-A9*A7) + A7*(A6*A9-A8
     1*A7))-A2*(A2*(A8*A10-A9**2)-A6*(A3*A10-A9*A4)+A7*(A3*A9-A8*A4))
     2+ A3*(A2*(A6*A10-A9*A7) - A5*(A3*A10-A9*A4) + A7*(A3*A7-A6*A4))
     3- A4*(A2*(A6*A9-A8*A7)-A5*(A3*A9-A8*A4)+A6*(A3*A7-A6*A4))
```

```
C*****************************************************************************
C
C     CS DENOTES CLASS SIZE AND TL DENOTES TEACHER LOAD
C
C*****************************************************************************
      CS = X4 / X1
      TL = X1 / X2
C*****************************************************************************
C
C     THE ABOVE RESULTS ARE OUTPUTED USING THE FOLLOWING VARIABLE NAMES
C     FUNX = THE PLANNING INDEX FUNCTION
C     C1X1 = TOTAL COST OF CLASSES
C     C2X2 = TOTAL COST OF TEACHERS
C     C3X3 = TOTAL COST OF SUBJECTS
C     C4X4 = TOTAL COST OF THE ENROLLMENT
C     X1 = NUMBER OF CLASSES
C     X2 = NUMBER OF TEACHERS
C     X3 = NUMBER OF DIFFERENT SUBJECTS
C     X4 = NUMBER OF ENROLLMENTS
C     CT = TOTAL EXPENDITURE LESS BUILDING AND TRANSPORATION COSTS
C     C1 = COST PER CLASS
C     C2 = AVERAGE COSTS PER TEACHER
C     C3 = COST PER SUBJECT
C     C4 = COST PER ENROLLMENT
C     XL1 = LAMBDA 1
C     XL2 = LAMBDA 2
C     XL3 = LAMBDA 3
C     XL4 = LAMBDA 4
C     XL5 = LAMBDA 5
C     XL1X1 = TOTAL EFFECT OF X1
C     XL2X2 = TOTAL EFFECT OF X2
C     XL3X3 = TOTAL EFFECT OF X3
C     XL4X4 = TOTAL EFFECT OF X4
C     THE MATRIX IS THE HESSIAN MATRIX OF SECOND PARTIAL DERIVATIVES
C     AA1 = THE FIRST PRINCIPLE DETERMINANT
C     AA2 = THE SECOND PRINCIPLE DETERMINANT
C     AA3 = THE THIRD PRINCIPLE DETERMINANT
C     AA4 = THE FOURTH PRINCIPLE DETERMINANT
C     P1 = FRACTION OF CT FOR C1X1
C     P2 = FRACTION OF CT FOR C2X2
C     P3 = FRACTION OF CT FOR C3X3
C     P4 = FRACTION OF CT FOR C4X4
C     TP = SUM OF THE FRACTIONS
C
C*****************************************************************************
      WRITE(IW,20)FUNX,C1X1,C2X2,C3X3,C4X4,C1,C2,C3,C4,CT
   20 FORMAT(1H ,'FUNX =',F15.7///' C1X1 =',F10.2,5X,'C2X2 = ',F10.2,//
     1' C3X3 = ',F10.2,5X,'C4X4 = ',F10.2// ' C1 = ',F9.2,5X,'C2 = ',F9.
     22//' C3 = ',F9.2,5X,'C4 = ',F9.2//' CT = ',F10.2//)
      WRITE(IW,21)X1,X2,X3,X4
   21 FORMAT(1X, 'X1 = ',F10.2,5X,'X2 = ',F10.2,5X,'X3 = ',F10.2,5X,'X4
     1= ',F10.2//)
      WRITE(IW,25)CS,TL
   25 FORMAT(1X,'CLASS SIZE = ',F10.2,5X,'TEACHER LOAD = ',F10.2//)
      WRITE(IW,22)XL1,XL2,XL3,XL4,XL5,XL1X1,XL2X2,XL3X3,XL4X4
   22 FORMAT(1X,'XL1 = ',F12.7,5X,'XL2 = ',F12.7//' XL3 = ',F12.7,5X,'XL
     14 = ',F12.7 //' XL5 = ',F12.7//' XL1X1 = ',F12.7,5X,'XL2X2 = ',F12
     2.7//' XL3X3 = ',F12.7,5X,'XL4X4 = ',F12.7//)
      WRITE(IW,23)((H(I,J),J=1,4),I=1,4)
   23 FORMAT(1H ,4F15.7)
      WRITE(IW,24)AA1,AA2,AA3,AA4,P1,P2,P3,P4,TP
   24 FORMAT(1X/' AA1 = ',F12.7,5X,'AA2 = ',F12.7,5X,'AA3 = ',F12.7,5X,
     1'AA4 = ',F12.7//' P1 = ',F9.5,5X,'P2 = ',F9.5,5X,'P3 = ',F9.5,5X,'
     2P4 = ',F9.5,5X,'TP = ',F9.5/)
      RETURN
      END

FEATURES SUPPORTED
 ONE WORD INTEGERS
 EXTENDED PRECISION

CORE REQUIREMENTS FOR FIGUR
 COMMON      44  VARIABLES     224  PROGRAM    1144

RELATIVE ENTRY POINT ADDRESS IS 01EF (HEX)

END OF COMPILATION
```

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// DUP

*STORE      WS  UA  FIGUR
CART ID 0C01   DB ADDR  439C   DB CNT   0058

// FOR
** ER360302,DOUGLAS H. WALTERS,RESEARCH,PROGRAM 03572
*IOCS(CARD,1403 PRINTER)
*EXTENDED PRECISION
*ONE WORD INTEGERS
*LIST SOURCE PROGRAM

      DIMENSION PADA(6),XX1(6),XX2(6),XX3(6),XX4(6),YEPP(6),XTCE(6)
      DIMENSION YTCE(6),YC1(6),YC2(6),YC3(6),YC4(6)
      DIMENSION NA(60)
      COMMON CT,C1X1,C2X2,C3X3,C4X4,X1,X2,X3,X4,IW,PC2X2
      COMMON C1,C2,C3,C4
      IR = 2
      IW = 5
C*****************************************************************************
C
C     THE NAME OF THE SCHOOL IS READ IN ALONG WITH THE DATA FOR THE
C     PARTICULAR SCHOOL , WHERE THE VARIABLES NAMES ARE DENOTED
C     BY THE FOLLOWING
C     ADA = AVERAGE DAILY ATTENDANCE
C     X2 = NUMBER OF EQUIVALENT FULL TIME TEACHERS
C     X3 = NUMBER OF DIFFERENT SUBJECTS
C     X1 = TOTAL NUMBER OF CLASSES
C     X5 = TOTAL NUMBER OF COURSES
C     X1ADA = AVERAGE NUMBER OF ENROLLMENTS PER PUPIL
C     SEC = AVERAGE NUMBER OF SECTION PER COURSE
C     AC2 = AVERAGE TEACHER'S SALARY
C     FC2 = ADDITIONAL EXPENDITURE PER TEACHER EXPRESSED
C     AS A PERCENTAGE OF SALARY FOR OVERHEAD COSTS
C     TCE = TOTAL CURRENT EXPENDITURE
C     EPP = EXPENDITURE PER PUPIL
C     AEPC= AVERAGE ENROLLMENT PER CLASS = X4/X1
C     ACPT = AVERAGE NUMBER OF CLASSES PER TEACHER
C
C*****************************************************************************
      READ(IR,100)(NA(I),I=1,60)
  100 FORMAT(60A1)
      READ(IR,101)ADA,X2,X3,X1,X5,X1ADA,SEC,AC2,FC2,TCE
  101 FORMAT(7F10.4/2F10.4,F15.2    )
C*****************************************************************************
C
C     THE INITIAL DATA AND INFORMATION IS OUTPUTTED TO VARIFY THE INITIAL
C     INFORMATION AND STARTING POINT FOR THE PLANNING HORIZON
C
C*****************************************************************************
      WRITE(IW,104)
  104 FORMAT(1H1,'THIS IS THE INITIAL INFORMATION OBTAINED FOR'//)
      WRITE(IW,105)(NA(I),I=1,60)
  105 FORMAT(1X,60A1)
      WRITE(IW,106)ADA,X2,X3,X1,X5,X1ADA,SEC,AC2,FC2,TCE
  106 FORMAT(1X//' AVERAGE DAILY ATTENDANCE =',F10.2//' NUMBER OF EQUIVA
     1LENT FULL TIME TEACHERS = ',F10.2//' NUMBER OF DIFFERENT SUBJECTS
     2= ',F10.2//' TOTAL NUMBER OF CLASSES = ',F10.2//' TOTAL NUMBER OF
     3COURSES = ',F10.2//' AVERAGE NUMBER OF ENROLLMENTS PER PUPIL = ',
     4F10.2//' AVERAGE NUMBER OF SECTIONS PER COURSE = ',F10.2//' AVERAG
     5E TEACHERS SALARY = ',F10.2/)
      WRITE(IW,107)FC2,TCE
  107 FORMAT(1X//' PERCENT ADDED TO TEACHERS SALARIES FOR OVERHEAD = ',F
     110.2//' TOTAL CURRENT EXPENDITURE = ',F10.2//)
      EPP = TCE / ADA
      X4 = X1ADA * ADA
      ACPT = X1 / X2
      AEPC = X4 / X1
      WRITE(IW,111)AEPC,ACPT
  111 FORMAT(1X,'CLASS SIZE = ',F10.2//' TEACHER LOAD = ',F10.2//)
      CT = TCE
      PC2X2 = (X2 *( AC2 + AC2 * (FC2 / 100. ))) / TCE
```

```
C*****************************************************************************
C
C      THE PLANNING INDEX MODEL IS APPLIED ASSUMING THE FOLLOWING
C      THE PRESENT OPERATION IS OPTIMAL
C      C1X1 = C3X3
C      AND THE BUDGET CONSTRAINT IS ACTIVE
C
C*****************************************************************************
      WRITE(IW,13)PC2X2
   13 FORMAT(1H1,'***********'/' ***********'//' THIS IS THE INITIAL BASE
     1LINE DATA'/' PC2X2 = ',F10.4//' ***********'/' ***********'///)
      C2X2 = PC2X2 * CT
      C1X1 = (-PC2X2 + SQRT(PC2X2))* CT
      C3X3 = C1X1
      C4XA = CT * (1.0 + PC2X2 -2. * SQRT(PC2X2))
      C1 = C1X1 / X1
      C2 = C2X2/X2
      C3 = C3X3 /X3
      C4 = C4X4 / X4
      CALL FIGUR
      CONTINUE
      WRITE(IW,80)
   80 FORMAT(1H1,' *****'//'  THE FOLLOWING DESCRIBES THE TERMS USED IN
     1THIS PROGRAM'//' *****'//' FUNX = THE PLANNING INDEX FUNCTION'/
     2              ' C1X1 = TOTAL COST OF CLASSES'/' C2X2 = TOTAL COST O
     3F TEACHERS'/' C3X3 = TOTAL COST OF SUBJECTS'/' C4X4 = TOTAL COST O
     4F THE ENROLLMENT'/' X1 = NUMBER OF CLASSES'/' X2 = NUMBER OF TEACH
     5ERS'/' X3 = NUMBER OF SUBJECTS'/' X4 = NUMBER OF ENROLLMENTS')
      WRITE(IW,81)
   81 FORMAT(1X,'CT = TOTAL EXPENDITURE LESS BUILDING AND TRANSPORATION
     1COSTS'/' C1 = COST PER CLASS'/' C2 = COST PER TEACHER'/' C3 = COST
     2 PER SUBJECT'/' C4 = COST PER ENROLLMENT'/' XL1 = LAMBDA 1'/' XL2
     3= LAMBDA 2'/' XL3 = LAMBDA 3'/' XL4 = LAMBDA 4 '/' XL5 = LAMBDA 5'
     4/' PC2X2 = FRACTION OF CT USED FOR C2X2'/' XL1X1 =TOTAL EFFECT FRO
     5M X1')
      WRITE(IW,82)
   82 FORMAT(1X,'XL2X2 = TOTAL EFFECT FROM X2'/' XL3X3 = TOTAL EFFECT FR
     1OM X3'/' XL4X4 = TOTAL EFFECT FROM X4'/' THE MATRIX IS THE HESSIAN
     2 MATRIX'/' AA1 = THE FIRST PRINCIPLE DETERMINANT'/' AA2 = THE SECO
     3ND PRINCIPLE DETERMINANT'/' AA3 = THE THIRD PRINCIPLE DETERMINANT'
     4/' AA4 = THE FOURTH PRINCIPLE DETERMINANT'/' P1 = FRICTION OF CT F
     5OR C1X1'/' P2 = FRACTION OF CT FOR C2X2')
      WRITE(IW,83)
   83 FORMAT(1X,' P3 = FRACTION OF CT FOR C3X3'/' P4 = FRACTION OF CT FO
     1R C4X4'/' TP = SUM OF THE FRACTIONS')
C*****************************************************************************
C
C      THE EXPECTED CHANGES IN THE ENROLLMENT IS NOW READ IN USING PADA AS
C      THE VARIABLE NAME
C
C*****************************************************************************
      WRITE(IW,108)
  108 FORMAT(1H1,'THE FOLLOWING PLANNING MODEL IS BASED ON INPUTED CHANG
     1ES IN THE AVERAGE DAILY ATTENDANCE'///)
      READ(IR,109)(PADA(I),I=1,6)
  109 FORMAT(6F10.2)
C*****************************************************************************
C
C      THE EXPECTED RATE OF INFLATION IS READ IN USING THE VARIABLE NAME INFLA
C
C*****************************************************************************
      READ(IR,110)INFLA
  110 FORMAT(I5)
C*****************************************************************************
C
C      THE EXPECTED VALUES OF THE REST OF THE VARIABLE IS THEN COMPUTED AND
C      THE RESULTS ARE THEN PRINTED OUT
C
C*****************************************************************************
      YTCE(1) = TCE
      XX1(1) = X1
      XX2(1) = X2
      XX3(1) = X3
      XX4(1) = X4
      YC1(1) = C1
      YC2(1) = C2
      YC3(1) = C3
      YC4(1) = C4
```

```
      XTCF(1) = TCE
      YEPF(1) = EPP
      XIN = INFLA
      XINF = XIN / 100.
      DO 200 NN=1,5
      N = NN + 1
      XX4(N) = PADA(N) * XIADA
      XX1(N) = XX4(N) / AEPC
      XX2(N) = XX1(N) / ACPT
      YEPP(N) = YEPP(NN) + YEPP(NN) * XINF
      XTCE(N) = EPP * PADA(N)
      YTCE(N) = YEPP(N) * PADA(N)
      YC1(N) = YC1(NN) + YC1(NN) * XINF
      YC2(N) = YC2(NN) + YC2(NN) * XINF
      YC3(N) = YC3(NN) + YC3(NN) * XINF
      YC4(N) = YC4(NN) + YC4(NN) * XINF
      XX3(N) = ( C1 * XX1(N)) / C3
  200 CONTINUE
C**************************************************************************
C
C      THE FOLLOWING PLANNING MODEL IS BASED ON INPUTED CHANGES IN THE AVERAGE
C      DAILY ATTENDANCE
C      THE FOLLOWING IS USED TO OBTAIN AND OUTPUT A LISTING OF THE PROJECTED
C      ENROLLMENT AND THE NEEDED REVENUE IF CT/ADA WOULD REMAIN THE SAME
C
C**************************************************************************
      WRITE(IW,115)
  115 FORMAT(1H ,'THE FOLLOWING IS A LISTING OF THE PROJECTED ENROLLMENT
     1'//' AND THE NEEDED REVENUE IF CT/ADA WOULD REMAIN THE SAME'//' YE
     2AR',5X,'AVERAGE DAILY ATTENDANCE',5X,'NEEDED REVENUE'/)
      DO 140 N=1,6
      NN = N - 1
  140 WRITE(IW,116)NN,PADA(N),XTCE(N)
  116 FORMAT(1H0,I3,13X,F8.2,13X,F10.2)
C**************************************************************************
C
C      THE FOLLOWING IS USED TO OBTAIN AND OUTPUT A LISTING OF THE
C      PROJECTED ENROLLMENT AND NEEDED REVENUE IF CT/ADA REMAINS THE SAME
C      AND AN INFLATION RATE IS INCLUDED IN THE PROJECTION
C
C**************************************************************************
      WRITE(IW,120)INFLA
  120 FORMAT(1H0,'THE FOLLOWING IS A LISTING OF THE PROJECTED ENROLLMENT
     1'//' AND NEEDED REVENUE IF CT/ADA WOULD REMAIN THE SAME'//' WITH A
     2N ANNUAL INFLATION RATE OF ',I5,'  PERCENT'//' YEAR',5X,'AVERAGE D
     3AILY ATTENDANCE',5X,'NEEDED REVENUE'/)
      DO 141 N=1,6
      NN = N - 1
  141 WRITE(IW,116)NN,PADA(N),YTCE(N)
C**************************************************************************
C
C      THE FOLLOWING IS USED TO OBTAIN AND OUTPUT THE PROJECTED VALUES OF
C      X1, X2, X3, AND X4
C
C**************************************************************************
      WRITE(IW,125)
  125 FORMAT(1H1,'THE FOLLOWING IS A LISTING OF THE PROJECTED VALUES OF
     1X1 X2 X3 AND X4 '                    //' YEAR',10X,'X1',10X,'X2',
     210X,'X3',10X,'X4'/)
      DO 142 N=1,6
      NN = N - 1
  142 WRITE(IW,126)NN,XX1(N),XX2(N),XX3(N),XX4(N)
  126 FORMAT(1H0,I3,8X,F5.1,6X,F7.1,5X,F7.1,6X,F7.1)
C**************************************************************************
C
C      THE FOLLOWING IS USED TO OBTAIN AND WRITE OUT THE PROJECTED COSTS
C      C1, C2, C3, AND C4 WITH AN ANNUAL INFLATION RATE OF INFLA
C
C**************************************************************************
      WRITE(IW,127)INFLA
  127 FORMAT(1H1,'THE FOLLOWING IS A LISTING OF THE PROJECTED COST VALUE
     1S'//' BASED ON AN ANNUAL INFLATION RATE OF ',I5,'  PERCENT'//' YEA
     2R',10X,'C1',11X,'C2',10X,'C3'  ,12X,'C4'/)
      DO 143 N=1,6
      NN = N - 1
  143 WRITE(IW,126)NN,YC1(N),YC2(N),YC3(N),YC4(N)
```

```
C****************************************************************************
C
C      THE VARIOUS EFFECTS OF THE EXPECTED CHANGES ARE CHECKED BY CALLING FIGUR
C      AND CHECKING THE VARIOUS INDICATORS
C
C****************************************************************************
       WRITE(IW,131)
  131 FORMAT(1H1,'THE FOLLOWING IS A CHECK ON THE PLANNING INDEX'//' AND
     1 THE PROJECTED INCOMES AND EXPENDITURES'/)
       JJ = 1
C****************************************************************************
C
C      THE FIRST TIME THROUGH THIS SERIES THE PROGRAM IS USING ONLY THE INITIAL
C      PROJECTED VALUES OF THE VARIABLES FOUND IN THE ABOVE.
C      THE SECOND TIME THE PROGRAM GOES THROUGH THIS SERIES IT IS INCREASING
C      THE TEACHER SALARIES AT A RATE OF TWO PERCENT PER YEAR IN TERMS
C      OF BASE YEAR DOLLARS SO THAT AT THE END OF THE PLANNING PERIOD THE TEACHER
C      SALARY WILL HAVE INCREASED TEN PERCENT IN TERMS OF BASE YEAR DOLLARS
C
C****************************************************************************
  132 CONTINUE
      DO 150 I=1,6
      II = I-1
      GO TO(144,145),JJ
  144 WRITE(IW,134)II
  134 FORMAT(1H1,'THIS IS THE INITIAL INDEX CHECK FOR YEAR ',I3//)
      GO TO 146
  145 WRITE(IW,128)II
  128 FORMAT(1H1,'THIS IS THE INDEX CHECK FOR INCREASE IN C2 '//' FOR YE
     1AR',I3//)
  146 CONTINUE
      X1 = XX1(I)
      C1 = YC1(I)
      C1X1 = C1 * X1
      X2 = XX2(I)
      SC2 = C2
      GO TO(136,135),JJ
  135 IF(I-1)139,139,138
  139 C2 = YC2(I)
      GO TO 137
  138 C2 = SC2 + SC2 *    ( 0.02  + XINF   )
      GO TO 137
  136 C2 = YC2(I)
  137 CONTINUE
      C2X2 = C2 * X2
      X3 = XX3(I)
      C3 = YC3(I)
      C3X3 = C3 * X3
      X4 = XX4(I)
      C4 = YC4(I)
      C4X4 = C4 * X4
      CT = C1X1 + C2X2 + C3X3 + C4X4
      CALL FIGUR
  150 CONTINUE
      GO TO (155,160),JJ
  155 WRITE(IW,156)JJ
  156 FORMAT(1H1,'JJ = ',I3//' THE FOLLOWING IS A PROJECTION AND CHECK O
     1N THE PLANNING MODEL WHEN TEACHER SALARIES ARE RAISED '//' TEN PER
     2CENT IN TERMS OF BASE YEAR DOLLARS'//' AT A RATE OF TWO PERCNET PE
     3R YEAR'//)
      JJ = JJ + 1
      GO TO 132
  160 CONTINUE
C****************************************************************************
C
C      THIS NEXT SECTION WILL TRY TO REDUCE THE NEEDED REVENUE BY INCREASING
C      THE CLASS SIZE AND DECREASING THE NUMBER OF SUBJECTS.
C
C****************************************************************************
       WRITE(IW,170)
  170 FORMAT(1H1,'THE FOLLOWING IS TO TEST THE EFFECT OF REDUCING '//' T
     1HE EXPENDITURE BY INCREASING THE CLASS SIZE WHICH MUST EITHER '//'
     2 DECREASE THE NUMBER OF SUBJECTS, SUBJECT COST, OR INCREASE CLASS
     3COST'//' WHICH WOULD NOT REDUCE THE BUDGET BECAUSE C1X1 = C3X3'//)
       SX3 = X3
       SC3 =C3
```

```
      DO 180 N=1,15
      X1 = X1 - 10.
      C3 = SC3
      X3 = (C1 * X1) /  C3
      CT = C1  * X1 + C2 * X2 + C3 * X3 + C4 * X4
      WRITE(IW,175)
  175 FORMAT(1H1,' THE FOLLOWING IS AN ATTEMPT TO BALANCE '//' THE BUDGE
     1T BY INCREASING CLASS SIZE AND'//' DECREASING NUMBER OF SUBJECTS'/
     2/)
      C1X1 = C1 * X1
      C2X2 = C2 * X2
      C3X3 = C3 * X3
      C4X4 = C4 * X4
      CALL FIGUR
      CONTINUE
C***************************************************************************
C
C     THIS PORTION OF THE PROGRAM TRIES TO REDUCE THE NEEDED REVENUE BY
C     REDUCING THE SUBJECT EXPENDITURE AND INCREASING THE CLASS SIZE
C
C***************************************************************************
      WRITE(IW,176)
  176 FORMAT(1H1,' THE FOLLOWING IS AN ATTEMPT TO BALANCE'//' THE BUDGET
     1 BY REDUCING SUBJECT EXPENDITURE AND'//' INCREASING CLASS SIZE'//)
      X3 = SX3
      C3 = C1X1 / X3
      CT = C1  * X1 + C2 * X2 + C3 * X3 + C4 * X4
      C1X1 = C1 * X1
      C2X2 = C2 * X2
      C3X3 = C3 * X3
      C4X4 = C4 * X4
      CALL FIGUR
      CONTINUE
  180 CONTINUE
      CALL EXIT
      END

FEATURES SUPPORTED
 ONE WORD INTEGERS
 EXTENDED PRECISION
 IOCS

CORE REQUIREMENTS FOR
 COMMON       44  VARIABLES      340  PROGRAM    2954

END OF COMPILATION

// XEQ
```

APPENDIX D

COMPUTER OUTPUT USED TO CONSTRUCT FIGURES

IN CHAPTER III AND IV

SECTION 1

COMPUTER OUTPUT USED TO CONSTRUCT FIGURES 3-1 THROUGH 3-4

THE FOLLOWING GENERATED DATA WAS USED FOR THE
CONSTRUCTION OF FIGURES 3.1 THROUGH 3.4 IN CHAPTER THREE

WHERE

C1X1/CT = FRACTION OF THE TOTAL BUDGET SPENT FOR CLASSES
C2X2/CT = FRACTION OF THE TOTAL BUDGET SPENT FOR TEACHERS
C3X3/CT = FRACTION OF THE TOTAL BUDGET SPENT FOR SUBJECTS
C4X4/CT = FRACTION OF THE BUDGET SPENT FOR THE ENROLLMENT
TOTAL = C1X1/CT + C2X2/CT + C3X3/CT + C4X4/CT

C2X2/CT = 0.0000	C1X1/CT = 0.0000	C3X3/CT = 0.0000	C4X4/CT 1.0000	TOTAL = 1.0000
C2X2/CT = 0.0100	C1X1/CT = 0.0900	C3X3/CT = 0.0900	C4X4/CT 0.8100	TOTAL = 1.0000
C2X2/CT = 0.0200	C1X1/CT = 0.1214	C3X3/CT = 0.1214	C4X4/CT 0.7371	TOTAL = 0.9999
C2X2/CT = 0.0300	C1X1/CT = 0.1432	C3X3/CT = 0.1432	C4X4/CT 0.6835	TOTAL = 1.0000
C2X2/CT = 0.0400	C1X1/CT = 0.1600	C3X3/CT = 0.1600	C4X4/CT 0.6399	TOTAL = 0.9999
C2X2/CT = 0.0500	C1X1/CT = 0.1736	C3X3/CT = 0.1736	C4X4/CT 0.6027	TOTAL = 1.0000
C2X2/CT = 0.0600	C1X1/CT = 0.1849	C3X3/CT = 0.1849	C4X4/CT 0.5701	TOTAL = 1.0000
C2X2/CT = 0.0700	C1X1/CT = 0.1945	C3X3/CT = 0.1945	C4X4/CT 0.5408	TOTAL = 1.0000
C2X2/CT = 0.0800	C1X1/CT = 0.2028	C3X3/CT = 0.2028	C4X4/CT 0.5143	TOTAL = 0.9999
C2X2/CT = 0.0900	C1X1/CT = 0.2100	C3X3/CT = 0.2100	C4X4/CT 0.4899	TOTAL = 1.0000
C2X2/CT = 0.1000	C1X1/CT = 0.2162	C3X3/CT = 0.2162	C4X4/CT 0.4675	TOTAL = 0.9999
C2X2/CT = 0.1100	C1X1/CT = 0.2216	C3X3/CT = 0.2216	C4X4/CT 0.4466	TOTAL = 1.0000
C2X2/CT = 0.1200	C1X1/CT = 0.2264	C3X3/CT = 0.2264	C4X4/CT 0.4271	TOTAL = 0.9999
C2X2/CT = 0.1300	C1X1/CT = 0.2305	C3X3/CT = 0.2305	C4X4/CT 0.4088	TOTAL = 1.0000
C2X2/CT = 0.1400	C1X1/CT = 0.2341	C3X3/CT = 0.2341	C4X4/CT 0.3916	TOTAL = 1.0000
C2X2/CT = 0.1500	C1X1/CT = 0.2372	C3X3/CT = 0.2372	C4X4/CT 0.3754	TOTAL = 0.9999
C2X2/CT = 0.1600	C1X1/CT = 0.2400	C3X3/CT = 0.2400	C4X4/CT 0.3599	TOTAL = 0.9999
C2X2/CT = 0.1700	C1X1/CT = 0.2423	C3X3/CT = 0.2423	C4X4/CT 0.3453	TOTAL = 1.0000
C2X2/CT = 0.1800	C1X1/CT = 0.2442	C3X3/CT = 0.2442	C4X4/CT 0.3314	TOTAL = 1.0000
C2X2/CT = 0.1900	C1X1/CT = 0.2458	C3X3/CT = 0.2458	C4X4/CT 0.3182	TOTAL = 0.9999

C2X2/CT = 0.2000	C1X1/CT = 0.2472	C3X3/CT = 0.2472	C4X4/CT 0.3055	TOTAL = 0.9999
C2X2/CT = 0.2100	C1X1/CT = 0.2482	C3X3/CT = 0.2482	C4X4/CT 0.2934	TOTAL = 1.0000
C2X2/CT = 0.2200	C1X1/CT = 0.2490	C3X3/CT = 0.2490	C4X4/CT 0.2819	TOTAL = 1.0000
C2X2/CT = 0.2300	C1X1/CT = 0.2495	C3X3/CT = 0.2495	C4X4/CT 0.2708	TOTAL = 0.9999
C2X2/CT = 0.2400	C1X1/CT = 0.2498	C3X3/CT = 0.2498	C4X4/CT 0.2602	TOTAL = 0.9999
C2X2/CT = 0.2500	C1X1/CT = 0.2500	C3X3/CT = 0.2500	C4X4/CT 0.2500	TOTAL = 1.0000
C2X2/CT = 0.2600	C1X1/CT = 0.2499	C3X3/CT = 0.2499	C4X4/CT 0.2401	TOTAL = 0.9999
C2X2/CT = 0.2700	C1X1/CT = 0.2496	C3X3/CT = 0.2496	C4X4/CT 0.2307	TOTAL = 0.9999
C2X2/CT = 0.2800	C1X1/CT = 0.2491	C3X3/CT = 0.2491	C4X4/CT 0.2216	TOTAL = 1.0000
C2X2/CT = 0.2900	C1X1/CT = 0.2485	C3X3/CT = 0.2485	C4X4/CT 0.2129	TOTAL = 0.9999
C2X2/CT = 0.3000	C1X1/CT = 0.2477	C3X3/CT = 0.2477	C4X4/CT 0.2045	TOTAL = 0.9999
C2X2/CT = 0.3100	C1X1/CT = 0.2467	C3X3/CT = 0.2467	C4X4/CT 0.1964	TOTAL = 0.9999
C2X2/CT = 0.3200	C1X1/CT = 0.2456	C3X3/CT = 0.2456	C4X4/CT 0.1886	TOTAL = 0.9999
C2X2/CT = 0.3300	C1X1/CT = 0.2444	C3X3/CT = 0.2444	C4X4/CT 0.1810	TOTAL = 0.9999
C2X2/CT = 0.3400	C1X1/CT = 0.2430	C3X3/CT = 0.2430	C4X4/CT 0.1738	TOTAL = 1.0000
C2X2/CT = 0.3500	C1X1/CT = 0.2416	C3X3/CT = 0.2416	C4X4/CT 0.1667	TOTAL = 0.9999
C2X2/CT = 0.3600	C1X1/CT = 0.2400	C3X3/CT = 0.2400	C4X4/CT 0.1599	TOTAL = 0.9999
C2X2/CT = 0.3700	C1X1/CT = 0.2382	C3X3/CT = 0.2382	C4X4/CT 0.1534	TOTAL = 0.9999
C2X2/CT = 0.3800	C1X1/CT = 0.2364	C3X3/CT = 0.2364	C4X4/CT 0.1471	TOTAL = 0.9999
C2X2/CT = 0.3900	C1X1/CT = 0.2344	C3X3/CT = 0.2344	C4X4/CT 0.1410	TOTAL = 1.0000
C2X2/CT = 0.4000	C1X1/CT = 0.2324	C3X3/CT = 0.2324	C4X4/CT 0.1350	TOTAL = 0.9999
C2X2/CT = 0.4100	C1X1/CT = 0.2303	C3X3/CT = 0.2303	C4X4/CT 0.1293	TOTAL = 0.9999
C2X2/CT = 0.4200	C1X1/CT = 0.2280	C3X3/CT = 0.2280	C4X4/CT 0.1238	TOTAL = 0.9999
C2X2/CT = 0.4300	C1X1/CT = 0.2257	C3X3/CT = 0.2257	C4X4/CT 0.1185	TOTAL = 1.0000
C2X2/CT = 0.4400	C1X1/CT = 0.2233	C3X3/CT = 0.2233	C4X4/CT 0.1133	TOTAL = 1.0000
C2X2/CT = 0.4500	C1X1/CT = 0.2208	C3X3/CT = 0.2208	C4X4/CT 0.1083	TOTAL = 0.9999
C2X2/CT = 0.4600	C1X1/CT = 0.2182	C3X3/CT = 0.2182	C4X4/CT 0.1035	TOTAL = 0.9999

C2X2/CT = 0.4700	C1X1/CT = 0.2155	C3X3/CT = 0.2155	C4X4/CT 0.0988	TOTAL = 0.9999
C2X2/CT = 0.4800	C1X1/CT = 0.2128	C3X3/CT = 0.2128	C4X4/CT 0.0943	TOTAL = 1.0000
C2X2/CT = 0.4900	C1X1/CT = 0.2100	C3X3/CT = 0.2100	C4X4/CT 0.0899	TOTAL = 0.9999
C2X2/CT = 0.5000	C1X1/CT = 0.2071	C3X3/CT = 0.2071	C4X4/CT 0.0857	TOTAL = 1.0000
C2X2/CT = 0.5100	C1X1/CT = 0.2041	C3X3/CT = 0.2041	C4X4/CT 0.0817	TOTAL = 0.9999
C2X2/CT = 0.5200	C1X1/CT = 0.2011	C3X3/CT = 0.2011	C4X4/CT 0.0777	TOTAL = 1.0000
C2X2/CT = 0.5300	C1X1/CT = 0.1980	C3X3/CT = 0.1980	C4X4/CT 0.0739	TOTAL = 1.0000
C2X2/CT = 0.5400	C1X1/CT = 0.1948	C3X3/CT = 0.1948	C4X4/CT 0.0703	TOTAL = 1.0000
C2X2/CT = 0.5500	C1X1/CT = 0.1916	C3X3/CT = 0.1916	C4X4/CT 0.0667	TOTAL = 1.0000
C2X2/CT = 0.5600	C1X1/CT = 0.1883	C3X3/CT = 0.1883	C4X4/CT 0.0633	TOTAL = 1.0000
C2X2/CT = 0.5700	C1X1/CT = 0.1849	C3X3/CT = 0.1849	C4X4/CT 0.0600	TOTAL = 1.0000
C2X2/CT = 0.5800	C1X1/CT = 0.1815	C3X3/CT = 0.1815	C4X4/CT 0.0568	TOTAL = 0.9999
C2X2/CT = 0.5900	C1X1/CT = 0.1781	C3X3/CT = 0.1781	C4X4/CT 0.0537	TOTAL = 0.9999
C2X2/CT = 0.6000	C1X1/CT = 0.1745	C3X3/CT = 0.1745	C4X4/CT 0.0508	TOTAL = 0.9999
C2X2/CT = 0.6100	C1X1/CT = 0.1710	C3X3/CT = 0.1710	C4X4/CT 0.0479	TOTAL = 1.0000
C2X2/CT = 0.6200	C1X1/CT = 0.1674	C3X3/CT = 0.1674	C4X4/CT 0.0451	TOTAL = 0.9999
C2X2/CT = 0.6300	C1X1/CT = 0.1637	C3X3/CT = 0.1637	C4X4/CT 0.0425	TOTAL = 1.0000
C2X2/CT = 0.6400	C1X1/CT = 0.1600	C3X3/CT = 0.1600	C4X4/CT 0.0399	TOTAL = 0.9999
C2X2/CT = 0.6500	C1X1/CT = 0.1562	C3X3/CT = 0.1562	C4X4/CT 0.0375	TOTAL = 1.0000
C2X2/CT = 0.6600	C1X1/CT = 0.1524	C3X3/CT = 0.1524	C4X4/CT 0.0351	TOTAL = 0.9999
C2X2/CT = 0.6700	C1X1/CT = 0.1485	C3X3/CT = 0.1485	C4X4/CT 0.0329	TOTAL = 1.0000
C2X2/CT = 0.6800	C1X1/CT = 0.1446	C3X3/CT = 0.1446	C4X4/CT 0.0307	TOTAL = 1.0000
C2X2/CT = 0.6900	C1X1/CT = 0.1406	C3X3/CT = 0.1406	C4X4/CT 0.0286	TOTAL = 1.0000
C2X2/CT = 0.7000	C1X1/CT = 0.1366	C3X3/CT = 0.1366	C4X4/CT 0.0266	TOTAL = 1.0000
C2X2/CT = 0.7100	C1X1/CT = 0.1326	C3X3/CT = 0.1326	C4X4/CT 0.0247	TOTAL = 1.0000
C2X2/CT = 0.7200	C1X1/CT = 0.1285	C3X3/CT = 0.1285	C4X4/CT 0.0229	TOTAL = 1.0000
C2X2/CT = 0.7300	C1X1/CT = 0.1244	C3X3/CT = 0.1244	C4X4/CT 0.0211	TOTAL = 0.9999

C2X2/CT =	0.7400	C1X1/CT =	0.1202	C3X3/CT =	0.1202	C4X4/CT	0.0195	TOTAL =	0.9999
C2X2/CT =	0.7500	C1X1/CT =	0.1160	C3X3/CT =	0.1160	C4X4/CT	0.0179	TOTAL =	0.9999
C2X2/CT =	0.7600	C1X1/CT =	0.1117	C3X3/CT =	0.1117	C4X4/CT	0.0164	TOTAL =	0.9999
C2X2/CT =	0.7700	C1X1/CT =	0.1074	C3X3/CT =	0.1074	C4X4/CT	0.0150	TOTAL =	1.0000
C2X2/CT =	0.7800	C1X1/CT =	0.1031	C3X3/CT =	0.1031	C4X4/CT	0.0136	TOTAL =	0.9999
C2X2/CT =	0.7900	C1X1/CT =	0.0988	C3X3/CT =	0.0988	C4X4/CT	0.0123	TOTAL =	0.9999
C2X2/CT =	0.8000	C1X1/CT =	0.0944	C3X3/CT =	0.0944	C4X4/CT	0.0111	TOTAL =	0.9999
C2X2/CT =	0.8100	C1X1/CT =	0.0899	C3X3/CT =	0.0899	C4X4/CT	0.0099	TOTAL =	0.9999
C2X2/CT =	0.8200	C1X1/CT =	0.0855	C3X3/CT =	0.0855	C4X4/CT	0.0089	TOTAL =	0.9999
C2X2/CT =	0.8300	C1X1/CT =	0.0810	C3X3/CT =	0.0810	C4X4/CT	0.0079	TOTAL =	1.0000
C2X2/CT =	0.8400	C1X1/CT =	0.0765	C3X3/CT =	0.0765	C4X4/CT	0.0069	TOTAL =	0.9999
C2X2/CT =	0.8500	C1X1/CT =	0.0719	C3X3/CT =	0.0719	C4X4/CT	0.0060	TOTAL =	0.9999
C2X2/CT =	0.8600	C1X1/CT =	0.0673	C3X3/CT =	0.0673	C4X4/CT	0.0052	TOTAL =	1.0000
C2X2/CT =	0.8700	C1X1/CT =	0.0627	C3X3/CT =	0.0627	C4X4/CT	0.0045	TOTAL =	0.9999
C2X2/CT =	0.8800	C1X1/CT =	0.0580	C3X3/CT =	0.0580	C4X4/CT	0.0038	TOTAL =	1.0000
C2X2/CT =	0.8900	C1X1/CT =	0.0533	C3X3/CT =	0.0533	C4X4/CT	0.0032	TOTAL =	1.0000
C2X2/CT =	0.9000	C1X1/CT =	0.0486	C3X3/CT =	0.0486	C4X4/CT	0.0026	TOTAL =	0.9999
C2X2/CT =	0.9100	C1X1/CT =	0.0439	C3X3/CT =	0.0439	C4X4/CT	0.0021	TOTAL =	1.0000
C2X2/CT =	0.9200	C1X1/CT =	0.0391	C3X3/CT =	0.0391	C4X4/CT	0.0016	TOTAL =	0.9999
C2X2/CT =	0.9300	C1X1/CT =	0.0343	C3X3/CT =	0.0343	C4X4/CT	0.0012	TOTAL =	1.0000
C2X2/CT =	0.9400	C1X1/CT =	0.0295	C3X3/CT =	0.0295	C4X4/CT	0.0009	TOTAL =	0.9999
C2X2/CT =	0.9500	C1X1/CT =	0.0246	C3X3/CT =	0.0246	C4X4/CT	0.0006	TOTAL =	1.0000
C2X2/CT =	0.9600	C1X1/CT =	0.0197	C3X3/CT =	0.0197	C4X4/CT	0.0004	TOTAL =	1.0000
C2X2/CT =	0.9700	C1X1/CT =	0.0148	C3X3/CT =	0.0148	C4X4/CT	0.0002	TOTAL =	1.0000
C2X2/CT =	0.9800	C1X1/CT =	0.0099	C3X3/CT =	0.0099	C4X4/CT	0.0001	TOTAL =	1.0000
C2X2/CT =	0.9900	C1X1/CT =	0.0049	C3X3/CT =	0.0049	C4X4/CT	0.0000	TOTAL =	1.0000
C2X2/CT =	1.0000	C1X1/CT =	0.0000	C3X3/CT =	0.0000	C4X4/CT	0.0000	TOTAL =	1.0000

SECTION 2

COMPUTER OUTPUT USED TO CONSTRUCT FIGURES 4-1 through 4-13

THE FOLLOWING IS A CHECK ON THE PLANNING INDEX
AND THE PROJECTED INCOMES AND EXPENDITURES

THIS IS THE INITIAL INFORMATION OBTAINED FOR

PLANSVILLE SENIOR HIGH SCHOOL MR HOPE, PRINCIPAL

AVERAGE DAILY ATTENDANCE = 800.00

NUMBER OF EQUIVALENT FULL TIME TEACHERS = 40.00

NUMBER OF DIFFERENT SUBJECTS = 45.00

TOTAL NUMBER OF CLASSES = 210.00

TOTAL NUMBER OF COURSES = 80.00

AVERAGE NUMBER OF ENROLLMENTS PER PUPIL = 5.28

AVERAGE NUMBER OF SECTIONS PER COURSE = 2.62

AVERAGE TEACHERS SALARY = 7264.00

AVERAGE DAILY ATTENDANCE = 15.00

NUMBER OF EQUIVALENT FULL TIME TEACHERS = 465285.00

NUMBER OF DIFFERENT SUBJECTS =

PERCENT ADDED TO TEACHERS SALARIES FOR OVERHEAD = 15.00

TOTAL CURRENT EXPENDITURE = 465285.00

CLASS SIZE = 20.11

TEACHER LOAD = 5.25

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*****

THE FOLLOWING DESCRIBES THE TERMS USED IN THIS PROGRAM

*****

FUNX = THE PLANNING INDEX FUNCTION
C1X1 = TOTAL COST OF CLASSES
C2X2 = TOTAL COST OF TEACHERS
C3X3 = TOTAL COST OF SUBJECTS
C4X4 = TOTAL COST OF THE ENROLLMENT
X1 = NUMBER OF CLASSES
X2 = NUMBER OF TEACHERS
X3 = NUMBER OF SUBJECTS
X4 = NUMBER OF ENROLLMENTS
CT = TOTAL EXPENDITURE LESS BUILDING AND TRANSPORATION COSTS
C1 = COST PER CLASS
C2 = COST PER TEACHER
C3 = COST PER SUBJECT
C4 = COST PER ENROLLMENT
XL1 = LAMBDA 1
XL2 = LAMBDA 2
XL3 = LAMBDA 3
XL4 = LAMBDA 4
XL5 = LAMBDA 5
PC2X2 = FRACTION OF CT USED FOR C2X2
XL1X1 =TOTAL EFFECT FROM X1
XL2X2 = TOTAL EFFECT FROM X2
XL3X3 = TOTAL EFFECT FROM X3
XL4X4 = TOTAL EFFECT FROM X4
THE MATRIX IS THE HESSIAN MATRIX
AA1 = THE FIRST PRINCIPLE DETERMINANT
AA2 = THE SECOND PRINCIPLE DETERMINANT
AA3 = THE THIRD PRINCIPLE DETERMINANT
AA4 = THE FOURTH PRINCIPLE DETERMINANT
P1 = FRICTION OF CT FOR C1X1
P2 = FRACTION OF CT FOR C2X2
 P3 = FRACTION OF CT FOR C3X3
P4 = FRACTION OF CT FOR C4X4
TP = SUM OF THE FRACTIONS
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**********
**********

THIS IS THE INITIAL BASE LINE DATA
PC2X2 =      0.7201

**********
**********


FUNX =       0.0000000

C1X1 =  59778.06      C2X2 =  335064.00

C3X3 =   59778.06      C4X4 =   10664.87

C1 =    284.65      C2 =   8376.60

C3 =   1328.40      C4 =      2.52

CT =  465285.00

X1 =      210.00      X2 =       40.00      X3 =       45.00      X4 =    4224.00

CLASS SIZE =       20.11      TEACHER LOAD =        5.25

XL1 =    -0.0000000      XL2 =     0.0000000

XL3 =     0.0000000      XL4 =     0.0000000

XL5 =     0.0000000

XL1X1 =    -0.0000000      XL2X2 =     0.0000000

XL3X3 =     0.0000000      XL4X4 =     0.0000000

     -0.0000080       0.0000212       0.0000000       0.0000002
      0.0000212       0.0000000      -0.0000991       0.0000000
      0.0000000      -0.0000991       0.0001762      -0.0000009
      0.0000002       0.0000000      -0.0000009       0.0000000

AA1 =    -0.0000080      AA2 =    -0.0000000      AA3 =    -0.0000000      AA4 =     0.0000000

P1 =    0.12847      P2 =    0.72012      P3 =    0.12847      P4 =    0.02292      TP =    0.99999
```

THE FOLLOWING PLANNING MODEL IS BASED ON INPUTED CHANGES IN THE AVERAGE DAILY ATTENDANCE

THE FOLLOWING IS A LISTING OF THE PROJECTED ENROLLMENT

AND THE NEEDED REVENUE IF CT/ADA WOULD REMAIN THE SAME

YEAR	AVERAGE DAILY ATTENDANCE	NEEDED REVENUE
0	800.00	465285.00
1	810.00	471101.06
2	870.00	505997.43
3	1040.00	604870.50
4	1170.00	680479.31
5	1200.00	697927.50

THE FOLLOWING IS A LISTING OF THE PROJECTED ENROLLMENT

AND NEEDED REVENUE IF CT/ADA WOULD REMAIN THE SAME

WITH AN ANNUAL INFLATION RATE OF 4 PERCENT

YEAR	AVERAGE DAILY ATTENDANCE	NEEDED REVENUE
0	800.00	465285.00
1	810.00	489945.10
2	870.00	547286.82
3	1040.00	680397.04
4	1170.00	796064.54
5	1200.00	849135.51

THE FOLLOWING IS A LISTING OF THE PROJECTED VALUES OF X1 X2 X3 AND X4

YEAR	X1	X2	X3	X4
0	210.0	40.0	45.0	4224.0
1	212.6	40.5	45.5	4276.8
2	228.3	43.5	48.9	4593.6
3	273.0	52.0	58.5	5491.2
4	307.1	58.5	65.8	6177.6
5	315.0	60.0	67.5	6336.0

THE FOLLOWING IS A LISTING OF THE PROJECTED COST VALUES

BASED ON AN ANNUAL INFLATION RATE OF 4 PERCENT

YEAR	C1	C2	C3	C4
0	284.6	8376.6	1328.4	2.5
1	296.0	8711.6	1381.5	2.6
2	307.8	9060.1	1436.7	2.7
3	320.2	9422.5	1494.2	2.8
4	333.0	9799.4	1554.0	2.9
5	346.3	10191.4	1616.2	3.0

THIS IS THE INITIAL INDEX CHECK FOR YEAR 0

FUNX = 0.0000000

C1X1 = 59778.06 C2X2 = 335064.00

C3X3 = 59778.06 C4X4 = 10664.87

C1 = 284.65 C2 = 8376.60

C3 = 1328.40 C4 = 2.52

CT = 465284.99

X1 = 210.00 X2 = 40.00 X3 = 45.00 X4 = 4224.00

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = -0.0000000 XL2 = 0.0000000

XL3 = 0.0000000 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = -0.0000000 XL2X2 = 0.0000000

XL3X3 = 0.0000000 XL4X4 = 0.0000000

-0.0000080	0.0000212	0.0000000	0.0000002
0.0000212	0.0000000	-0.0000991	0.0000000
0.0000000	-0.0000991	0.0001762	-0.0000009
0.0000002	0.0000000	-0.0000009	0.0000000

AA1 = -0.0000080 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.12847 P2 = 0.72012 P3 = 0.12847 P4 = 0.02292 TP = 1.00000

THIS IS THE INITIAL INDEX CHECK FOR YEAR 1

FUNX = -0.0000000

C1X1 = 62946.29 C2X2 = 352822.39

C3X3 = 62946.29 C4X4 = 11230.11

C1 = 296.04 C2 = 8711.66

C3 = 1381.53 C4 = 2.62

CT = 489945.10

X1 = 212.62 X2 = 40.50 X3 = 45.56 X4 = 4276.80

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = -0.0000000 XL2 = -0.0000000

XL3 = 0.0000000 XL4 = -0.0000000

XL5 = -0.0000000

XL1X1 = -0.0000000 XL2X2 = -0.0000000

XL3X3 = 0.0000000 XL4X4 = -0.0000000

-0.0000078	0.0000207	0.0000000	0.0000001
0.0000207	0.0000000	-0.0000966	0.0000000
0.0000000	-0.0000966	0.0001718	-0.0000009
0.0000001	0.0000000	-0.0000009	0.0000000

AA1 = -0.0000078 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.12847 P2 = 0.72012 P3 = 0.12847 P4 = 0.02292 TP = 1.00000

THIS IS THE INITIAL INDEX CHECK FOR YEAR 2

FUNX = 0.0000000

C1X1 = 70313.34 C2X2 = 394115.67

C3X3 = 70313.34 C4X4 = 12544.45

C1 = 307.88 C2 = 9060.13

C3 = 1436.79 C4 = 2.73

CT = 547286.82

X1 = 228.37 X2 = 43.50 X3 = 48.93 X4 = 4593.60

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = -0.0000000 XL2 = 0.0000000

XL3 = 0.0000000 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = -0.0000000 XL2X2 = 0.0000000

XL3X3 = 0.0000000 XL4X4 = 0.0000000

-0.0000068	0.0000179	0.0000000	0.0000001
0.0000179	0.0000000	-0.0000838	0.0000000
0.0000000	-0.0000838	0.0001489	-0.0000007
0.0000001	0.0000000	-0.0000007	0.0000000

AA1 = -0.0000068 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.12847 P2 = 0.72012 P3 = 0.12847 P4 = 0.02292 TP = 1.00000

THIS IS THE INITIAL INDEX CHECK FOR YEAR 3

FUNX = 0.0000000

C1X1 = 87414.84 C2X2 = 489971.86

C3X3 = 87414.84 C4X4 = 15595.49

C1 = 320.20 C2 = 9422.53

C3 = 1494.27 C4 = 2.84

CT = 680397.04

X1 = 273.00 X2 = 52.00 X3 = 58.50 X4 = 5491.20

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = -0.0000000 XL2 = -0.0000000

XL3 = 0.0000000 XL4 = -0.0000000

XL5 = -0.0000000

XL1X1 = -0.0000000 XL2X2 = -0.0000000

XL3X3 = 0.0000000 XL4X4 = -0.0000000

-0.0000047	0.0000125	0.0000000	0.0000001
0.0000125	0.0000000	-0.0000586	0.0000000
0.0000000	-0.0000586	0.0001042	-0.0000005
0.0000001	0.0000000	-0.0000005	0.0000000

AA1 = -0.0000047 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = -0.0000000

P1 = 0.12847 P2 = 0.72012 P3 = 0.12847 P4 = 0.02292 TP = 1.00000

THIS IS THE INITIAL INDEX CHECK FOR YEAR 4

FUNX = 0.0000000

C1X1 = 102275.36 C2X2 = 573267.07

C3X3 = 102275.36 C4X4 = 18246.73

C1 = 333.00 C2 = 9799.43

C3 = 1554.04 C4 = 2.95

CT = 796064.54

X1 = 307.12 X2 = 58.50 X3 = 65.81 X4 = 6177.60

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = -0.0000000 XL2 = 0.0000000

XL3 = 0.0000000 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = -0.0000000 XL2X2 = 0.0000000

XL3X3 = 0.0000000 XL4X4 = 0.0000000

-0.0000037	0.0000099	0.0000000	0.0000000
0.0000099	0.0000000	-0.0000463	0.0000000
0.0000000	-0.0000463	0.0000823	-0.0000004
0.0000000	0.0000000	-0.0000004	0.0000000

AA1 = -0.0000037 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.12847 P2 = 0.72012 P3 = 0.12847 P4 = 0.02292 TP = 1.00000

THIS IS THE INITIAL INDEX CHECK FOR YEAR 5

FUNX = 0.0000000

C1X1 = 109093.72   C2X2 = 611484.88

C3X3 = 109093.72   C4X4 = 19463.18

C1 = 346.32   C2 = 10191.41

C3 = 1616.20   C4 = 3.07

CT = 849135.51

X1 = 315.00   X2 = 60.00   X3 = 67.50   X4 = 6336.00

CLASS SIZE = 20.11   TEACHER LOAD = 5.25

XL1 = -0.0000000   XL2 = 0.0000000

XL3 = 0.0000000   XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = -0.0000000   XL2X2 = 0.0000000

XL3X3 = 0.0000000   XL4X4 = 0.0000000

-0.0000035	0.0000094	0.0000000	0.0000000
0.0000094	0.0000000	-0.0000440	0.0000000
0.0000000	-0.0000440	0.0000783	-0.0000004
0.0000000	0.0000000	-0.0000004	0.0000000

AA1 = -0.0000035   AA2 = -0.0000000   AA3 = 0.0000000   AA4 = 0.0000000

P1 = 0.12847   P2 = 0.72012   P3 = 0.12847   P4 = 0.02292   TP = 1.00000

JJ =   1

THE FOLLOWING IS A PROJECTION AND CHECK ON THE PLANNING MODEL WHEN TEACHER SALARIES ARE RAISED

TEN PERCENT IN TERMS OF BASE YEAR DOLLARS

AT A RATE OF TWO PERCNET PER YEAR

THIS IS THE INDEX CHECK FOR INCREASE IN C2

FOR YEAR 0

FUNX = 0.0000000

C1X1 = 59778.06 C2X2 = 335064.00

C3X3 = 59778.06 C4X4 = 10664.87

C1 = 284.65 C2 = 8376.60

C3 = 1328.40 C4 = 2.52

CT = 465284.93

X1 = 210.00 X2 = 40.00 X3 = 45.00 X4 = 4224.00

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = -0.0000000 XL2 = 0.0000000

XL3 = 0.0000000 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = -0.0000000 XL2X2 = 0.0000000

XL3X3 = 0.0000000 XL4X4 = 0.0000000

-0.0000080	0.0000212	0.0000000	0.0000002
0.0000212	0.0000000	-0.0000991	0.0000000
0.0000000	-0.0000991	0.0001762	-0.0000009
0.0000002	0.0000000	-0.0000009	0.0000000

AA1 = -0.0000080 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.12847 P2 = 0.72012 P3 = 0.12847 P4 = 0.02292 TP = 1.00000

THIS IS THE INDEX CHECK FOR INCREASE IN C2

FOR YEAR 1

FUNX = -0.0000000

C1X1 = 62946.29   C2X2 = 359607.43

C3X3 = 62946.29   C4X4 = 11230.11

C1 = 236.04   C2 = 8879.19

C3 = 1381.53   C4 = 2.62

CT = 496730.15

X1 = 212.62   X2 = 40.50   X3 = 45.56   X4 = 4276.80

CLASS SIZE = 20.11   TEACHER LOAD = 5.25

XL1 = 0.0000158   XL2 = -0.0000000

XL3 = -0.0000736   XL4 = -0.0000000

XL5 = -0.0000000

XL1X1 = 0.0033661   XL2X2 = -0.0000000

XL3X3 = -0.0033661   XL4X4 = -0.0000000

-0.0000078	0.0000203	0.0000000	0.0000001
0.0000203	0.0000000	-0.0000948	0.0000000
0.0000000	-0.0000948	0.0001718	-0.0000009
0.0000001	0.0000000	-0.0000009	0.0000000

AA1 = -0.0000078   AA2 = -0.0000000   AA3 = 0.0000000   AA4 = 0.0000000

P1 = 0.12672   P2 = 0.72394   P3 = 0.12672   P4 = 0.02260   TP = 1.00000

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THIS IS THE INDEX CHECK FOR INCREASE IN C2

FOR YEAR 2

FUNX = 0.0000000

C1X1 = 70313.34 C2X2 = 409419.72

C3X3 = 70313.34 C4X4 = 12544.45

C1 = 307.88 C2 = 9411.94

C3 = 1436.79 C4 = 2.73

CT = 562590.87

X1 = 228.37 X2 = 43.50 X3 = 48.93 X4 = 4593.60

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = 0.0000292 XL2 = 0.0000000

XL3 = -0.0001362 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0066688 XL2X2 = 0.0000000

XL3X3 = -0.0066688 XL4X4 = 0.0000000

-0.0000068	0.0000172	0.0000000	0.0000001
0.0000172	0.0000000	-0.0000806	0.0000000
0.0000000	-0.0000806	0.0001489	-0.0000007
0.0000001	0.0000000	-0.0000007	0.0000000

AA1 = -0.0000066 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.12498 P2 = 0.72773 P3 = 0.12498 P4 = 0.02229 TP = 0.99999

```
THIS IS THE INDEX CHECK FOR INCREASE IN C2

FOR YEAR  3

FUNX =       0.0000000

C1X1 =  87414.84      C2X2 =  518786.55

C3X3 =  87414.84      C4X4 =  15595.49

C1 =    320.20      C2 =    9976.66

C3 =   1494.27      C4 =       2.84

CT =  709211.74

X1 =      273.00     X2 =      52.00     X3 =      58.50     X4 =    5491.20

CLASS SIZE =      20.11      TEACHER LOAD =       5.25

XL1 =     0.0000362      XL2 =    -0.0000000

XL3 =   -0.0001693      XL4 =    -0.0000000

XL5 =   -0.0000000

XL1X1 =    0.0099092      XL2X2 =    -0.0000000

XL3X3 =   -0.0099092      XL4X4 =    -0.0000000

     -0.0000047      0.0000118      0.0000000      0.0000001
      0.0000118      0.0000000     -0.0000553      0.0000000
      0.0000000     -0.0000553      0.0001042     -0.0000005
      0.0000001      0.0000000     -0.0000005      0.0000000

AA1 =   -0.0000047      AA2 =   -0.0000000      AA3 =   -0.000000C      AA4 =    0.0000000

P1 =   0.12325      P2 =   0.73149      P3 =   0.12325      P4 =   0.02198      TP =   1.00000
```

THIS IS THE INDEX CHECK FOR INCREASE IN C2

FOR YEAR 4

FUNX = 0.0000000

C1X1 = 102275.36 C2X2 = 618652.97

C3X3 = 102275.36 C4X4 = 18246.73

C1 = 333.00 C2 = 10575.26

C3 = 1554.04 C4 = 2.95

CT = 841450.44

X1 = 307.12 X2 = 58.50 X3 = 65.81 X4 = 6177.60

CLASS SIZE = 20.11 TEACHER LOAD = 5.25

XL1 = 0.0000426 XL2 = 0.0000000

XL3 = -0.0001988 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0130884 XL2X2 = 0.0000000

XL3X3 = -0.0130884 XL4X4 = 0.0000000

-0.0000037	0.0000092	0.0000000	0.0000000
0.0000092	0.0000000	-0.0000429	0.0000000
0.0000000	-0.0000429	0.0000823	-0.0000004
0.0000000	0.0000000	-0.0000004	0.0000000

AA1 = -0.0000037 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.12154 P2 = 0.73522 P3 = 0.12154 P4 = 0.02168 TP = 1.00000

```
THIS IS THE INDEX CHECK FOR INCREASE IN C2
FOR YEAR  5

FUNX =      0.0000000

C1X1 = 109093.72     C2X2 =  672586.82
C3X3 =  109093.72     C4X4 =   19463.18
C1 =    346.32     C2 =  11209.78
C3 =   1616.20     C4 =      3.07
CT =  910237.45

X1 =     315.00     X2 =      60.00     X3 =      67.50     X4 =    6336.00

CLASS SIZE =      20.11     TEACHER LOAD =       5.25

XL1 =    0.0000514     XL2 =    0.0000000
XL3 =   -0.0002401     XL4 =    0.0000000
XL5 =    0.0000000
XL1X1 =    0.0162076     XL2X2 =    0.0000000
XL3X3 =   -0.0162076     XL4X4 =    0.0000000

     -0.0000035      0.0000085      0.0000000      0.0000000
      0.0000085      0.0000000     -0.0000400      0.0000000
      0.0000000     -0.0000400      0.0000783     -0.0000004
      0.0000000      0.0000000     -0.0000004      0.0000000
AA1 =   -0.0000035     AA2 =   -0.0000000     AA3 =    0.0000000     AA4 =    0.0000000
P1 =   0.11985     P2 =   0.73891     P3 =   0.11905     P4 =   0.02138     TP =   1.00000
```

THE FOLLOWING IS TO TEST THE EFFECT OF REDUCING

THE EXPENDITURE BY INCREASING THE CLASS SIZE WHICH MUST EITHER

DECREASE THE NUMBER OF SUBJECTS, SUBJECT COST, OR INCREASE CLASS COST

WHICH WOULD NOT REDUCE THE BUDGET BECAUSE C1X1 = C3X3

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

```
FUNX =      0.0000000

C1X1 = 105630.43      C2X2 =  672586.82
C3X3 =  105630.43      C4X4 =    19463.18
C1 =     346.32      C2 =  11209.78
C3 =    1616.20      C4 =       3.07
CT =  903310.87

X1 =     305.00     X2 = ,     60.00     X3 =       65.35     X4 =    6336.00

CLASS SIZE =       20.77     TEACHER LOAD =        5.08

XL1 =     0.0000892      XL2 =     0.0000000
XL3 =    -0.0004162      XL4 =     0.0000000
XL5 =     0.0000000
XL1X1 =     0.0272063      XL2X2 =     0.0000000
XL3X3 =    -0.0272063      XL4X4 =     0.0000000

     -0.0000012      0.0000085      0.0000000      0.0000000
      0.0000085      0.0000000     -0.0000400      0.0000000
      0.0000000     -0.0000400      0.0000862     -0.0000004
      0.0000000      0.0000000     -0.0000004      0.0000000

AA1 =    -0.0000039      AA2 =    -0.0000000      AA3 =    -0.0000000      AA4 =    -0.0000000

P1 =    0.11693      P2 =    0.74457      P3 =    0.11693      P4 =    0.02154      TP =    1.00000
```

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 98703.84 C2X2 = 672586.82

C3X3 = 98703.84 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 889457.69

X1 = 285.00 X2 = 60.00 X3 = 61.07 X4 = 6336.00

CLASS SIZE = 22.23 TEACHER LOAD = 4.75

XL1 = 0.0001769 XL2 = 0.0000000

XL3 = -0.0008258 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0504350 XL2X2 = 0.0000000

XL3X3 = -0.0504350 XL4X4 = 0.0000000

-0.0000048	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0001057	-0.0000005
0.0000001	0.0000000	-0.0000005	0.0000000

AA1 = -0.0000048 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.11097 P2 = 0.75617 P3 = 0.11097 P4 = 0.02188 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 91777.26 C2X2 = 672586.82

C3X3 = 91777.26 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 875604.52

X1 = 265.00 X2 = 60.00 X3 = 56.78 X4 = 6336.00

CLASS SIZE = 23.90 TEACHER LOAD = 4.41

XL1 = 0.0002853 XL2 = 0.0000000

XL3 = -0.0013315 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0756155 XL2X2 = 0.0000000

XL3X3 = -0.0756155 XL4X4 = 0.0000000

-0.0000060	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0001315	-0.0000005
0.0000001	0.0000000	-0.0000005	0.0000000

AA1 = -0.0000060 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.10481 P2 = 0.76813 P3 = 0.10481 P4 = 0.02222 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE

THE BUDGET BY INCREASING CLASS SIZE AND

DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 84850.67 C2X2 = 672586.82

C3X3 = 84850.67 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 861751.35

X1 = 245.00 X2 = 60.00 X3 = 52.50 X4 = 6336.00

CLASS SIZE = 25.06 TEACHER LOAD = 4.08

XL1 = 0.0004213 XL2 = 0.0000000

XL3 = -0.0019662 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.1032258 XL2X2 = 0.0000000

XL3X3 = -0.1032258 XL4X4 = 0.0000000

-0.0000076	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0001664	-0.0000006
0.0000001	0.0000000	-0.0000006	0.0000000

AA1 = -0.0000076 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.09846 P2 = 0.78048 P3 = 0.09846 P4 = 0.02258 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = -0.0000000

C1X1 = 81387.38 C2X2 = 672586.82

C3X3 = 81387.38 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 854824.76

X1 = 235.00 X2 = 60.00 X3 = 50.35 X4 = 6336.00

CLASS SIZE = 26.96 TEACHER LOAD = 3.91

XL1 = 0.0005027 XL2 = -0.0000000

XL3 = -0.0023459 XL4 = -0.0000000

XL5 = 0.0000000

XL1X1 = 0.1181359 XL2X2 = -0.0000000

XL3X3 = -0.1181359 XL4X4 = -0.0000000

-0.0000086	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0001886	-0.0000007
0.0000001	0.0000000	-0.0000007	0.0000000

AA1 = -0.0000086 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.09520 P2 = 0.78681 P3 = 0.09520 P4 = 0.02276 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 77924.09 C2X2 = 672586.82

C3X3 = 77924.09 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 847898.18

X1 = 225.00 X2 = 60.00 X3 = 48.21 X4 = 6336.00

CLASS SIZE = 28.16 TEACHER LOAD = 3.75

XL1 = 0.0005951 XL2 = -0.0000000

XL3 = -0.0027774 XL4 = -0.0000000

XL5 = -0.0000000

XL1X1 = 0.1339137 XL2X2 = -0.0000000

XL3X3 = -0.1339137 XL4X4 = -0.0000000

-0.0000098	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0002148	-0.0000008
0.0000001	0.0000000	-0.0000008	0.0000000

AA1 = -0.0000098 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = -0.0000000

P1 = 0.09190 P2 = 0.79324 P3 = 0.09190 P4 = 0.02295 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 70997.50 C2X2 = 672586.82

C3X3 = 70997.50 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 834045.01

X1 = 205.00 X2 = 60.00 X3 = 43.92 X4 = 6336.00

CLASS SIZE = 30.90 TEACHER LOAD = 3.41

XL1 = 0.0008223 XL2 = -0.0000000

XL3 = -0.0038375 XL4 = -0.0000000

XL5 = -0.0000000

XL1X1 = 0.1685800 XL2X2 = -0.0000000

XL3X3 = -0.1685800 XL4X4 = -0.0000000

-0.0000130	0.0000085	0.0000000	0.0000002
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0002841	-0.0000009
0.0000002	0.0000000	-0.0000009	0.0000000

AA1 = -0.0000130 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.08512 P2 = 0.80641 P3 = 0.08512 P4 = 0.02333 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 64070.91 C2X2 = 672586.82

C3X3 = 64070.91 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 820191.84

X1 = 185.00 X2 = 60.00 X3 = 39.64 X4 = 6336.00

CLASS SIZE = 34.24 TEACHER LOAD = 3.08

XL1 = 0.0011271 XL2 = 0.0000000

XL3 = -0.0052598 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.2085151 XL2X2 = 0.0000000

XL3X3 = -0.2085151 XL4X4 = 0.0000000

-0.0000177	0.0000085	0.0000000	0.0000002
0.0000085	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0003865	-0.0000012
0.0000002	0.0000000	-0.0000012	0.0000000

AA1 = -0.0000177 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.07811 P2 = 0.82003 P3 = 0.07811 P4 = 0.02373 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY INCREASING CLASS SIZE AND
DECREASING NUMBER OF SUBJECTS

FUNX = 0.0000000

C1X1 = 57144.33 C2X2 = 672586.82

C3X3 = 57144.33 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1616.20 C4 = 3.07

CT = 806338.66

X1 = 165.00 X2 = 60.00 X3 = 35.35 X4 = 6336.00

CLASS SIZE = 38.39 TEACHER LOAD = 2.75

XL1 = 0.0015493 XL2 = 0.0000000

XL3 = -0.0072300 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.2556348 XL2X2 = 0.0000000

XL3X3 = -0.2556348 XL4X4 = 0.0000000

-0.0000250	0.0000085	0.0000000	0.0000003
0.0000035	0.0000000	-0.0000400	0.0000000
0.0000000	-0.0000400	0.0005448	-0.0000015
0.0000003	0.0000000	-0.0000015	0.0000000

AA1 = -0.0000250 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = -0.0000000

P1 = 0.07086 P2 = 0.83412 P3 = 0.07086 P4 = 0.02413 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE

THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND

INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 105630.43 C2X2 = 672586.82

C3X3 = 105630.43 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1564.89 C4 = 3.07

CT = 903310.87

X1 = 305.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 20.77 TEACHER LOAD = 5.08

XL1 = 0.0000892 XL2 = 0.0000000

XL3 = -0.0004030 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0272063 XL2X2 = 0.0000000

XL3X3 = -0.0272063 XL4X4 = 0.0000000

-0.0000039	0.0000085	0.0000000	0.0000000
0.0000085	0.0000000	-0.0000387	0.0000000
0.0000000	-0.0000387	0.0000808	-0.0000004
0.0000000	0.0000000	-0.0000004	0.0000000

AA1 = -0.0000039 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = -0.0000000

P1 = 0.11693 P2 = 0.74457 P3 = 0.11693 P4 = 0.02154 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND
INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 98703.84 C2X2 = 672586.82

C3X3 = 98703.84 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1462.27 C4 = 3.07

CT = 889457.69

X1 = 285.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 22.23 TEACHER LOAD = 4.75

XL1 = 0.0001769 XL2 = 0.0000000

XL3 = -0.0007471 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0504350 XL2X2 = 0.0000000

XL3X3 = -0.0504350 XL4X4 = 0.0000000

-0.0000046	0.0000085	0.0000000	0.0000001
0.0000085	-0.0000000	-0.0000362	0.0000000
0.0000000	-0.0000362	0.0000865	-0.0000004
0.0000001	0.0000000	-0.0000004	0.0000000

AA1 = -0.0000048 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.11097 P2 = 0.75617 P3 = 0.11097 P4 = 0.02188 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND
INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 91777.26 C2X2 = 672586.82

C3X3 = 91777.26 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1359.66 C4 = 3.07

CT = 875604.52

X1 = 265.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 23.90 TEACHER LOAD = 4.41

XL1 = 0.0002853 XL2 = 0.0000000

XL3 = -0.0011202 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.0756155 XL2X2 = 0.0000000

XL3X3 = -0.0756155 XL4X4 = 0.0000000

-0.0000060	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000336	0.0000000
0.0000000	-0.0000336	0.0000930	-0.0000004
0.0000001	0.0000000	-0.0000004	0.0000000

AA1 = -0.0000060 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.10481 P2 = 0.76813 P3 = 0.10481 P4 = 0.02222 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND
INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 84850.67 C2X2 = 672586.82

C3X3 = 84850.67 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1257.04 C4 = 3.07

CT = 861751.35

X1 = 245.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 25.86 TEACHER LOAD = 4.08

XL1 = 0.0004213 XL2 = 0.0000000

XL3 = -0.0015292 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.1032258 XL2X2 = 0.0000000

XL3X3 = -0.1032258 XL4X4 = 0.0000000

-0.0000076	0.0000085	0.0000000	0.0000001
0.0000085	-0.0000000	-0.0000311	0.0000000
0.0000000	-0.0000311	0.0001006	-0.0000005
0.0000001	0.0000000	-0.0000005	0.0000000

AA1 = -0.0000076 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = -0.0000000

P1 = 0.09846 P2 = 0.78048 P3 = 0.09846 P4 = 0.02258 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE

THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND

INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 81387.38 C2X2 = 672586.82

C3X3 = 81387.38 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1205.73 C4 = 3.07

CT = 854824.76

X1 = 235.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 26.96 TEACHER LOAD = 3.91

XL1 = 0.0005027 XL2 = 0.0000000

XL3 = -0.0017501 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.1181359 XL2X2 = 0.0000000

XL3X3 = -0.1181359 XL4X4 = 0.0000000

-0.0000086	0.0000085	0.0000000	0.0000001
0.0000095	0.0000000	-0.0000298	0.0000000
0.0000000	-0.0000298	0.0001049	-0.0000005
0.0000001	0.0000000	-0.0000005	0.0000000

AA1 = -0.0000086 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = -0.0000000

P1 = 0.09520 P2 = 0.78681 P3 = 0.09520 P4 = 0.02276 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND
INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 77924.09 C2X2 = 672586.82

C3X3 = 77924.09 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1154.43 C4 = 3.07

CT = 847898.18

X1 = 225.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 28.16 TEACHER LOAD = 3.75

XL1 = 0.0005951 XL2 = 0.0000000

XL3 = -0.0019839 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.1339137 XL2X2 = 0.0000000

XL3X3 = -0.1339137 XL4X4 = 0.0000000

-0.0000098	0.0000085	0.0000000	0.0000001
0.0000085	0.0000000	-0.0000286	0.0000000
0.0000000	-0.0000286	0.0001096	-0.0000005
0.0000001	0.0000000	-0.0000005	0.0000000

AA1 = -0.0000098 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.09190 P2 = 0.79324 P3 = 0.09190 P4 = 0.02295 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE

THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND

INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 70997.50 C2X2 = 672586.82

C3X3 = 70997.50 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 1051.81 C4 = 3.07

CT = 834045.01

X1 = 205.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 30.90 TEACHER LOAD = 3.41

XL1 = 0.0008223 XL2 = 0.0000000

XL3 = -0.0024974 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.1685800 XL2X2 = 0.0000000

XL3X3 = -0.1685800 XL4X4 = 0.0000000

-0.0000130	0.0000085	0.0000000	0.0000002
0.0000085	0.0000000	-0.0000260	0.0000000
0.0000000	-0.0000260	0.0001203	-0.0000006
0.0000002	0.0000000	-0.0000006	0.0000000

AA1 = -0.0000130 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = 0.0000000

P1 = 0.08512 P2 = 0.80641 P3 = 0.08512 P4 = 0.02333 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND
INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 64070.91 C2X2 = 672586.82

C3X3 = 64070.91 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 949.19 C4 = 3.07

CT = 820191.84

X1 = 185.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 34.24 TEACHER LOAD = 3.08

XL1 = 0.0011271 XL2 = 0.0000000

XL3 = -0.0030891 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.2085151 XL2X2 = 0.0000000

XL3X3 = -0.2085151 XL4X4 = 0.0000000

-0.0000177	0.0000085	0.0000000	0.0000002
0.0000085	0.0000000	-0.0000235	0.0000000
0.0000000	-0.0000235	0.0001333	-0.0000007
0.0000002	0.0000000	-0.0000007	0.0000000

AA1 = -0.0000177 AA2 = -0.0000000 AA3 = -0.0000000 AA4 = 0.0000000

P1 = 0.07811 P2 = 0.82003 P3 = 0.07811 P4 = 0.02373 TP = 1.00000

THE FOLLOWING IS AN ATTEMPT TO BALANCE
THE BUDGET BY REDUCING SUBJECT EXPENDITURE AND
INCREASING CLASS SIZE

FUNX = 0.0000000

C1X1 = 57144.33 C2X2 = 672586.82

C3X3 = 57144.33 C4X4 = 19463.18

C1 = 346.32 C2 = 11209.78

C3 = 846.58 C4 = 3.07

CT = 806338.66

X1 = 165.00 X2 = 60.00 X3 = 67.50 X4 = 6336.00

CLASS SIZE = 38.39 TEACHER LOAD = 2.75

XL1 = 0.0015493 XL2 = 0.0000000

XL3 = -0.0017871 XL4 = 0.0000000

XL5 = 0.0000000

XL1X1 = 0.2556348 XL2X2 = 0.0000000

XL3X3 = -0.2556348 XL4X4 = 0.0000000

-0.0000250	0.0000085	0.0000000	0.0000003
0.0000085	-0.0000000	-0.0000209	0.0000000
0.0000000	-0.0000209	0.0001495	-0.0000007
0.0000003	0.0000000	-0.0000007	0.0000000

AA1 = -0.0000250 AA2 = -0.0000000 AA3 = 0.0000000 AA4 = -0.0000000

P1 = 0.07086 P2 = 0.83412 P3 = 0.07086 P4 = 0.02413 TP = 1.00000

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