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AN APPLICATION OF STOCHASTIC FORECASTING TO MONTHLY AVERAGED 700 MB HEIGHTS

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THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

AN APPLICATION OF STOCHASTIC FORECASTING TO MONTHLY AVERAGED 700 MB HEIGHTS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

By

ALBERT JOSEPH KOSCIELNY

Norman, Oklahoma

1981

AN APPLICATION OF STOCHASTIC FORECASTING

TO MONTHLY AVERAGED 700 MB HEIGHTS

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DEDICATION

To Karen

ACKNOWLEDGMENTS

I thank my committee, Drs. Brock, Duchon, Kimpel, LeDuc, and Sasaki, for their review of my work, suggestions for improvements, reading of this dissertation, and their patience. Dr. Duchon initiated my interest in time series analysis and statistical methods. Dr. LeDuc offered some very timely words of encouragement.

This research has not received any direct financial support. However, without the cooperation of many people it could not have been done. First, Robert Born of the Climate Research Group at Scripps Institute of Oceanography promptly provided (at no cost) the data from Jerome Namias' data base. The facilities of the University of Oklahoma Computing Services were used for the first half of this research. Extensive computer and graphics support was provided by the National Climatic Center during the final half of the research. My thanks to Dr. Ward Seguin for his support. In addition, the National Climatic Center facilities were used in producing this dissertation.

I have received much technical support in this research. Drs. Duchon and Goerss shared my interest in time series analysis and modeling and provided some of the computer programs; Danny Fulbright was particularly helpful with the Univac computer system and the details of principal component analysis; Tom Reek and Mel Craddock assisted in producing the graphics in this dissertation.

I thank Roz Ledford, who typed this dissertation for her excellent work and patience.

I extend my gratitude to the people who have offered encouragement. In particular, I thank Marcia Richard, Larry Hennington, Dr. Richard Doviak, and Karen Walker for their friendship and interest in my success. To the many people whom I have not mentioned here, my sincere thanks.

ABSTRACT

A complementary study to the numerical modeling of climate is the statistical modeling of existing climatic records. Such statistical models can be used to forecast climate variations without the physical understanding needed in the dynamical approach. Since most climatic records cover only several decades, these statistical methods are feasible for forecast periods of weeks or months.

The statistical forecast method of multivariate autoregression is applied to a 27 year record of monthly averaged 700 mb heights. According to the theorem of predictive decomposition, a stationary series can be represented as the sum of deterministic and stochastic parts that are uncorrelated. Hence the deterministic annual cycle is removed and the statistical model applied to standardized anomalies of the 700 mb heights.

The field of height anomalies is represented by 504 gridded values, which is far too large a dimension for multivariate autoregressive modeling. Therefore, 95 of these values representing equal areas are selected for modeling. Principal component analysis is then used to further reduce the 95 values to twenty. These twenty principal components are modeled by an objectively selected, third order multivariate autoregressive model.

One month dependent and independent forecasts of the standardized height anomalies are made and compared to similar forecasts made climatologically. The multivariate autoregressive model is found to perform better on the average than climatology. Several month forecasts are also made and examined for retrogression of anomalies. For the single case studied, the model is found to move features both eastward and westward.

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CHAPTER I

INTRODUCTION

Interest in climate and its variation has been renewed in recent years. This renewal of interest is due largely to an increasing awareness of the economic and social influences of climate coupled with the realization that the climate is not static but has changed many times in the past. It is, therefore, likely that future climate vagaries will greatly affect the well-being of the human race. Given that the climate is likely to change, accurate forecasting is needed to provide sufficient lead time for adaptation. At the present time, however, the state of knowledge of the physical processes controlling climate is too meager to allow accurate or, at times, even useful forecasts.

The growing awareness of the uncertainty of our climatic future has encouraged a substantial research effort by the scientific community. The current wisdom is that the climate forecasting problem should be approached through building a hierarchy of numerical models. To successfully model climate, a very large range of both time and space scale phenomena must be included. A particular member of this hierarchy would model an appropriate scale while parameterizing subgrid scale effects. Processes active on a larger scale than can be included in a member are represented by constants. The research effort to produce this hierarchy of models will be enormous and will likely take many years.

A complementary study to the numerical modeling of climate is the application of objective statistical prediction methods to existing climatic series. A climatic series is here loosely defined as a collection over some time interval of meteorological observations. Statistical methods are appropriate here because "a substantial portion of climatic variation derives from stochastic mechanisms internal to the climate system" (Mitchell, 1976, p. 492). The application of statistical prediction methods to climate forecasting will serve several purposes.

First, statistical forecasting is an important interim method until the numerical modeling approach provides a less tenuous physical basis. It is noted in GARP Publications Series No. 16 (1975, p. 28) that

> the unlikelihood that predictability studies based upon numerical integrations will be completed in the immediate future indicates that we should not foresake other methods of investigating predictability. Among these are statistical procedures based on the observed behavior of the climate system. Such procedures appear most feasible at ranges of weeks or months, for which observational records of sufficient length are available....predictions of likely climate change are possible using the classical objective methods of Kolmogorov and Weiner.

Second, the application of statistical forecasting will aid the numerical modeling approach. Again in GARP (op. cit.), it is emphasized that

An important complement to the development of numerical models of climate is the study of the observed statistical properties of climate. From such statistical analysis will come the data against which climate models must be tested.

Last, in applying statistical prediction methods, models are developed for the climatic series. The models describe the response of the climatic system to the fluctuations in the external influences on the climate that have occurred during the time period being analyzed. The models could then be used to predict the response of the climatic system to hypothetical changes in external influences (Bell, 1980).

A limitation to the use of statistical methods is that a climatic record which is long compared to the forecast period is needed. Most climatic series are only several decades in length, so, as noted earlier, these methods are feasible only for forecast periods of weeks or months.

The subject of this dissertation is the application and interpretation of a multivariate autoregressive statistical prediction method, based on the Weiner-Kolmogorov theory to a 27 year climatic series of Northern Hemispheric monthly averaged 700 mb heights. Monthly averaged data are being used because, as noted by Namias (1953), the "natural period" of 5-10 days is removed to reveal the more persistent features of the circulation. Since all phenomena with time scales less than a month have been largely suppressed, the basic time scale for this analysis is one month. The data cover the Northern Hemisphere from 15°N to 80°N, with a grid spacing of 10°, and the basic spatial resolution is about 1100 km.

In the climatic series being considered, 504 observations are available for each month. This is too large a number to practically deal with. Hence, the statistical technique of principal component analysis will be used to reduce the dimensionality of the 700 mb heights. The multivariate autoregression will be done then on a small (compared to 504) number of the principal components of the 700 mb heights.

The theoretical considerations of multivariate autoregressive modeling and of principal component analysis will be outlined in Chapter II. The climatic series of monthly averaged 700 mb heights to be analyzed will be described in Chapter III. Since this series will be dominated by the deterministic annual cycle, a procedure for removing it will be presented in Chapter IV. The deviations from the annual cycle, termed "standardized height anomalies," will be subsequently analyzed. Principal component analysis will be applied in Chapter V and the results interpreted. A multivariate autoregressive model will be fitted in Chapter VI. The predictability of the climatic series will be discussed using this model and forecasts generated. A summary and the conclusions of this study will comprise Chapter VII. Recommendations for further studies of this type of statistical forecasting will be made in Chapter VIII.

CHAPTER II

A STOCHASTIC MODEL FOR CLIMATIC SERIES

A climatic series usually consists of a set of synoptic observations taken at locations irregularly distributed in space. When the time interval between observations is constant, the climatic series forms a multivariate time series. The methods of time series analysis can then be used to describe and predict the evolution of the climate series.

The methods of time series analysis have been applied extensively, of course, to the climate forecasting problem. Many of these previous approaches have generally suffered from one or both of two deficiencies, of presupposing that climate varied in cycles (NAS, 1975, p.41) and of applying time series analysis to a single location. First, it has been recognized within recent decades that there are no cycles on the order of months that explain an appreciable amount of the climate's variation. Second, the larger spatial scale climatic phenomena that are more predictable in time are masked by observational errors and small scale phenomena if only a single location is analyzed. A method is needed, therefore, to apply time series analysis to many synoptic observations distributed in space.

One such method, described by Jones (1964b), uses empirical orthogonal functions to efficiently represent the spatial correlation in a multivariate time series. The methods of empirical orthogonal function (EOF) representation and those of principal component analysis (PCA) are equivalent. Since the more commonly used terminology is PCA, it will be used in the remainder of this dissertation.

PCA serves two important functions. First, it permits a reduction in the dimensionality of the series being modeled, making multivariate autoregressive modeling feasible. Second, some of the observation errors and sub-grid scale phenomena can be removed (Davis, 1976). This chapter will outline principal component analysis (PCA) and multivariate autoregressive (MVAR) modeling and will point out those aspects of both that will be important when they are applied to the 700 mb data series.

Principal Component Analysis

Principal component analysis, as noted by Afifi and Azen (1972), was originally developed by Pearson (1901) as a method for finding a line such that sum of squares of the perpendicular distances from the line to a set of vector observations was minimized. The first published application of PCA of a meteorological nature was that of Lorenz (1956), who used the principal components of sea level pressures for 64 stations covering the continental United States. The principal components were used as predictors of the sea level pressure field 24 hours later. Many subsequent meteorological applications were made, mainly using PCA as a diagnostic tool to analyze data fields, such as daily 500 mb height (Craddock and Flood, 1969), surface pressure, temperature, and precipitation (Kutzbach, 1967), and 800, 500, and 200 mb temperatures to be used as predictors of surface rainfall (Kidson, 1975).

For the application herein, the purpose of PCA is data reduction, since it is impractical to attempt MVAR modeling with more than about twenty random variables. An added benefit is the suppression of the noise and small scale effects obscuring the large scale climate variations (Davis, op. cit). The set of synoptic observations are considered to be a collection of random variables, easily represented as a matrix

$$Y_{NM} = \begin{array}{c} \\ \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1M} \\ y_{21} & y_{22} & \cdots & y_{2M} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ y_{N1} & y_{N2} & \cdots & y_{NM} \end{bmatrix} \\ \text{space} \longrightarrow \end{array} \begin{array}{c} \text{time} \\ \downarrow \end{array}$$

where N is the number of observations and M is the number of random variables. For simplicity, the random variables, Y_j, j=1, 2, ···, M are assumed to have zero means. The covariance matrix is estimated by

$$C_{MM} = N^{-1} Y^T_{NM} Y_{NM} \tag{1}$$

where ^{T} indicates the matrix transpose.

The principal components (PC) of the data matrix are formed by computing weighted sums of the original random variables. This is written in matrix form as

$$P_{NM} = Y_{NM} E_{MM} \tag{2}$$

where P_{NM} is a matrix of principal components and E_{MM} is a matrix of coefficients. By specifying that the PC's have the following properties:

1. they are linear combinations of the original random variables,

2. the first PC has the largest variance of any possible linear combination of the original variables, the second PC the largest amount of the remaining variance, and so on to the M-th PC,

3. that the PC's are mutually orthogonal,

4. that the sum of the variances of PC's equal the sum of the variances of the original random variables,

it can be shown (Afifi and Azen, 1972) that the coefficients E_{MM} are given by the eigenvectors of C_{MM} and that the eigenvalues of C_{MM} are the variances of the corresponding PC's.

As is evident from (2) there are M PC's, the same as the number of original random variables. However, because of the construction of the PC's most of the variance will be contained in the first L PC's where L can be much smaller than M. These L PC's can be used to reconstruct the Y_j by

$$Y_{NM} = P_{NL} E^T_{LM}. \tag{3}$$

There is some error made in this reconstruction unless L=M. However, it can be shown (Davis, op. cit.) that PC's provide the most efficient representation of the data in the sense that no other L orthogonal functions have a lower mean square error. In addition, it can be shown (Davis, op. cit.) that if the random variables are the sum of a signal plus spatially uncorrelated noise, then the low number PC's will represent most of the signal but the noise will spread equally over all PC's. Thus, by performing the reconstruction (3) some noise suppression is achieved.

A troublesome point, however, for this type of data representation is the objective selection of L. Various approaches with varying degrees of complexity have been suggested (Farmer, 1971; Preisendorfer and Barnett, 1977). The simplest is to choose L so that some given percentage of the variance (say, 95 percent) is explained. However, this method lacks intuitive appeal.

The method of Farmer (1971) appears to offer objectivity with minimal complication. In this method, it is noted that for random data, the eigenvalues decrease linearly with number when plotted on a logarithmic scale. For the case of climatic data, the logs of the low numbered eigenvalues do not fall linearly if the corresponding PC's represent some signal. Thus, a plot of log eigenvalue (LEV) versus number diagram can be examined. L is chosen as the largest number before the curve becomes linear.

In summary, working with M=504 is not practical when applying MVAR techniques. PCA will reduce the number of variables to the order of ten and suppress some of the noise in the data field. Hereafter, the data to be analyzed are the L observed principal components P_{NL}.

Multivariate Autoregressive Modeling

As noted in Chapter I, the classical methods of Weiner and Kolmogorov can be used for the prediction of a climatic series. These methods are based on the theorem of predictive decomposition (Wold, 1965, p.61) which states that any real-valued stationary series can be represented as the sum of two uncorrelated parts, one part being deterministic, the other being stochastic. The stochastic part of the series can be represented as a moving average process (a weighted running sum of a completely random sequence) of infinite order. The theorem further specifies the form of the best linear prediction of future values of the series. However, since this form is a weighted sum of the entire past history of the series, the linear prediction could not be applied in practice.

Recent developments described by Jones (1964b) have made the application of the Weiner-Kolmogorov theory to observed series practical. The theory is implemented by modeling the series with a multivariate autoregressive (MVAR) model. Forecasts are made by a weighted sum of the current value of the series and a finite number of past values. This MVAR model can be written in matrix form as

$$P_L(t) = \sum_{j=1}^{K} A_j \, P_L(t-j) + Z_L(t) \tag{4}$$

where $P_L(t)$ is a vector of the L principal components at time t, the A_j are LxL matrices of autoregression coefficients, K is the order of model, and $Z_L(t)$ is a vector of length L of completely random innovations with covariance matrix V_{LL}. Thus, in (4) the current value of P_L is expressed as a weighted sum of K previous values of P_L plus a completely random innovation. In other words, the current value of P_L series is regressed on K previous values, giving the name "autoregressive".

The techniques for estimating the order K and the K matrices A are presented by Jones (1964b, 1974, 1976) and Akaike et al. (1979). An introductory treatment of univariate autoregressive techniques is presented by Box and Jenkins (1970). Using the estimated model order $\hat{K}$ and the estimated autoregression coefficients $\hat{A}$, estimates of $Z_L(t)$, termed residuals, can be made by

$$\hat{Z}_L(t) = P_L(t) - \sum_{j=1}^{\hat{K}} \hat{A}_j P_L(t-j) \tag{5}$$

and the residual covariance matrix V_{LL} estimated by

$$\hat{V}_{LL} = (N-\hat{K})^{-1} \sum_{t=1}^{N} \hat{Z}_L(t)\hat{Z}_L^T(t) \tag{6}$$

Forecasts can be made using (4), with the assumption of a zero innovation vector (i.e., $Z_j(t+k) = 0, j=1,2,\ldots,L$),

$$\hat{P}_L(t+k) = \sum_{j=1}^{\hat{K}} \hat{A}_j P_L(t-j+k) \tag{7}$$

where k=1,2,... is called the forecast step. This forecast can be shown to have minimum error variance (Box and Jenkins, 1970, Ch. 5) over all linear combinations of the current and past values. From (7) it can be shown that MVAR forecasts damp exponentially as the forecast step increases. For a first order (K=1) model this is a simple exponential decay of the current values. For second and higher order models this could be a mixture of exponentially damped sine waves.

Thus, the Weiner-Kolmogorov theory can be used to forecast a climatic series by using (7). The important points are to use a field of data to capture the large space scale predictable features and then to reduce the dimensionality of the data by PCA so that MVAR modeling is practical.

Comparison of MVAR, Climatological, and Persistence Forecasts

It is enlightening to compare one step MVAR, climatological, and persistence forecasts for a simple first order univariate model with unit process variance. Assuming the climatology (i.e., the annual cycle) has been removed, then the climatological forecast for this model is zero. The persistence forecast is simply the current value P(t). The MVAR forecast is given by aP(t) where a is the autoregression coefficient. The error variance of the climatological forecast is the process variance 1, regardless of a. For persistence the variance of the forecast is given by $Var[P(t+1) - P(t)] = 2(1-a)Var[P(t)] = 2(1-a)$. The MVAR forecast error variance is $Var[P(t+1) - aP(t)] = Var[P(t)](1-a^2)$. These results are plotted in Figure 1 for $0 \leq a \leq 1$.

For a=0 (no correlation), climatology and MVAR are equivalent. For a=1 (perfect correlation) persistence and MVAR are equivalent. However, for $0<a<1$, the MVAR model provides better forecasts in the expected sense than either climatology or persistence.

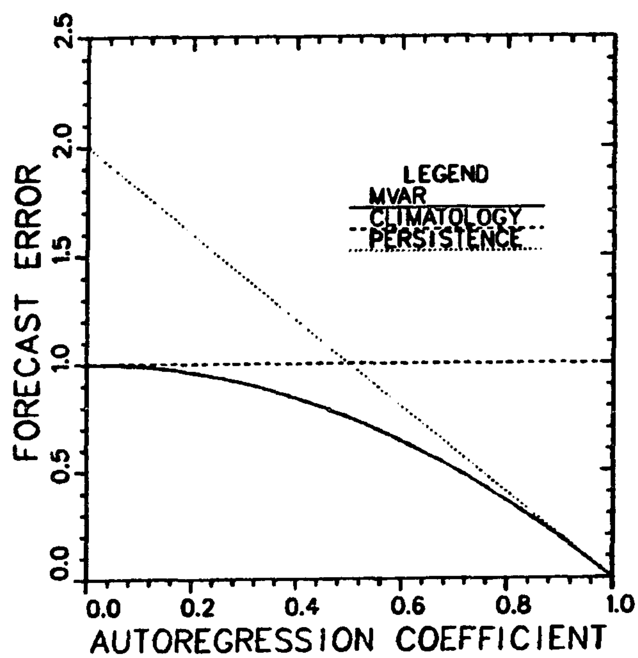


Figure 1. A comparison of forecast error variances for persistence, climatology, and MVAR for a univariate, first order model.

CHAPTER III

THE DATA

The climatic series selected for analysis here is a 29 year record of monthly averaged 700 mb heights. This series is a subset of a record that is part of the historical data base used by Jerome Namias of Scripps Institute, La Jolla, California. These data, originally obtained from the Long Range Prediction Group at the National Meteorological Center (NMC), are provided on a 5° diamond latitude-longitude grid (i.e., 20°N - 140°W, 25°N - 145°W, etc.) The time period covered is from December 1946 to December 1977. The data were prepared at NMC by averaging daily values of gridded 700 mb height from the usual 700 mb upper air analysis prepared by NMC. The techniques for analyzing the gridded daily heights have changed several times over the time period of the record, as described in internal technical notes at NMC (Gilman, 1981). However, one noteworthy change is that of using objective analysis methods, which began in the early 1960's.

Examination of this entire record showed that data were available for the latitudes 15° N to 80°N, inclusive, as indicated in Figure 2 so that there are a total of 504 observations for each month.

Preliminary analysis of the data indicated that many values were missing at low latitudes prior to January 1951. In addition, the quality of the earlier 700 mb heights is much poorer than those after 1951. This is corroborated by Rinne and Frisk (1979) who found a rapid

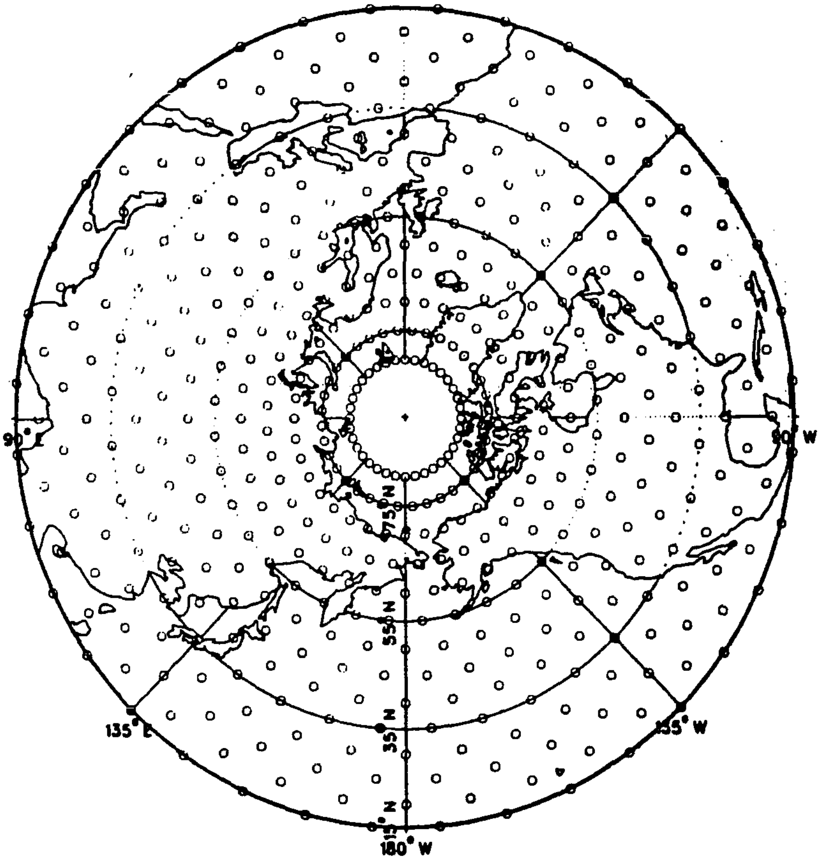


Figure 2. 5° diamond grid for the monthly averaged 700 mb heights.

decrease in observation errors for 500 mb heights from 1946 to 1952. For these reasons, no attempt was made to overcome the difficulties of analyzing the data prior to January 1951, although the data for 1949-50 will be used in Chapter IV.

The 700 mb heights were selected for analysis for several reasons. First, the observations were available in a form convenient for analysis on a digital computer. Second, much research has been done already relating 700 mb patterns to surface variables such as precipitation (Klein, 1965) and crop yields (Steyaert *et al.* 1977). Finally, Namias (1953) showed evidence of month to month correlation of the patterns so forecasts better than climatology could be made with MVAR.

The data set to be analyzed consists of 504 700 mb height values for each of the 324 months from January 1951 to December 1977.

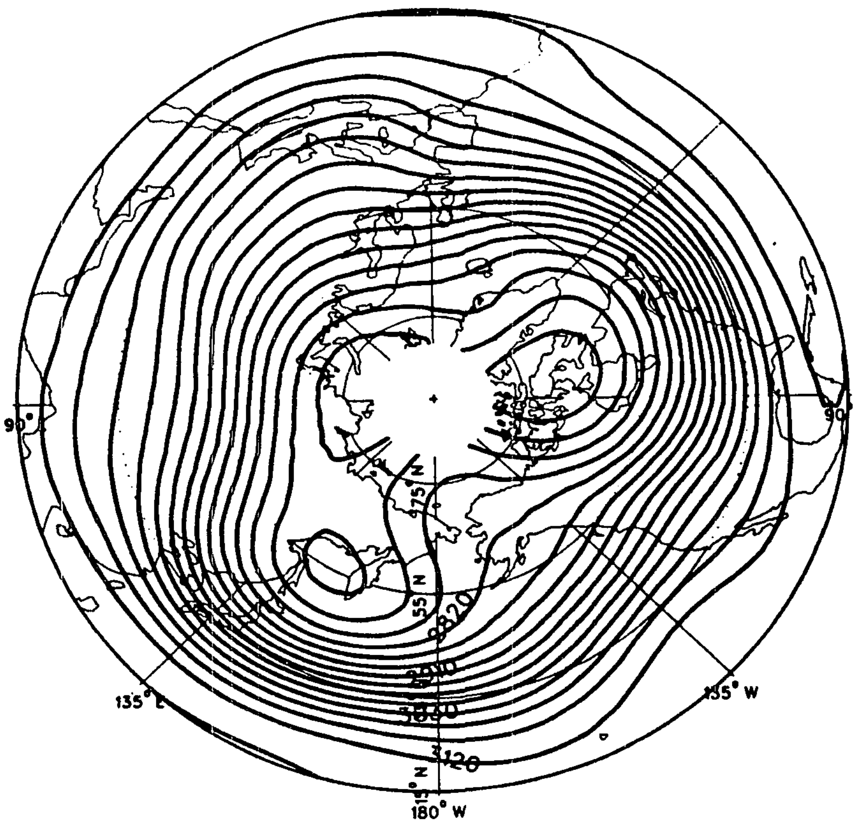
CHAPTER IV

REMOVAL OF THE ANNUAL CYCLE

Atmospheric circulation patterns are dominated, of course, by the annual cycle. This can be expected to be reflected in changes of the interannual means and variances for each of twelve months of the year. Since this variation is deterministic, it will be removed prior to any MVAR modeling. Mean 700 mb values over the 29 years (1949-77) were computed for each of the 504 grid points for each of the twelve months of the year. Some data for November 1959 at 15°N were in error (Born, 1979) and were not used. The mean monthly patterns are presented in Figure 3. Examination of these patterns show the dramatic change in the atmospheric circulation at 700 mb over the course of a year. The transition from the winter pattern to the summer one occurs during March through May. The reverse transition back to winter occurs more sharply in September and October.

In January, the prominent features of the mean 700 mb pattern are troughs at 80°W, 150°E, and 30°E. These trough positions, identical to those by Namias (1978) for winter 700 mb heights, agree with those by Palmen and Newton (1969, Ch. 3) for mean 500 mb heights. In July, the

AVERAGE 700 MB HEIGHT (M) FOR JANUARY



AVERAGE 700 MB HEIGHT (M) FOR FEBRUARY

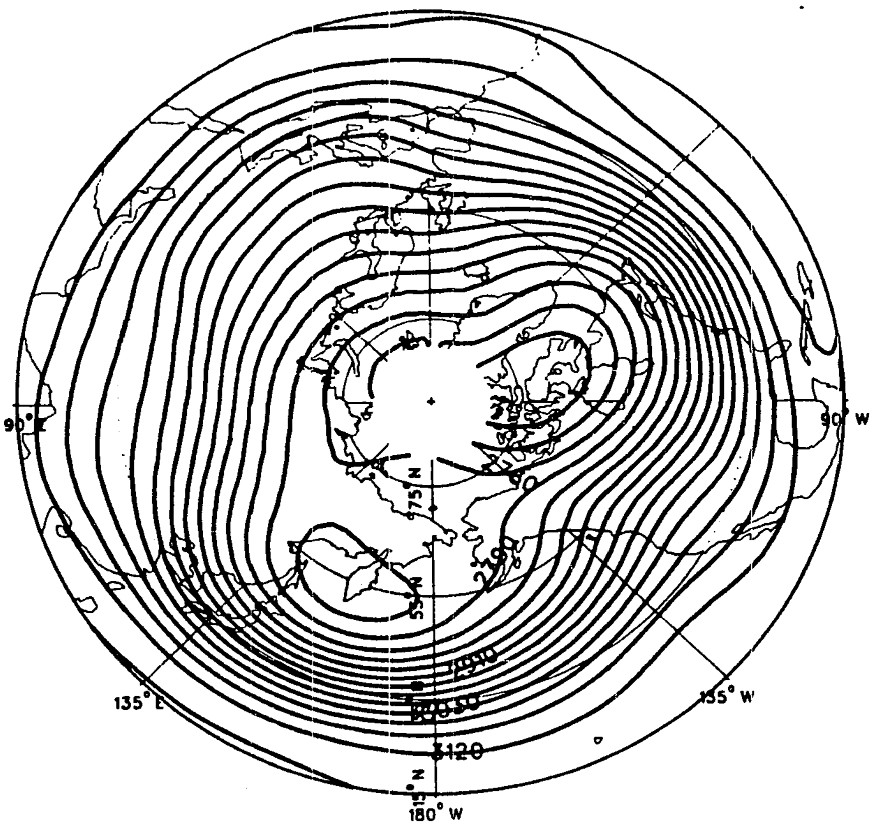
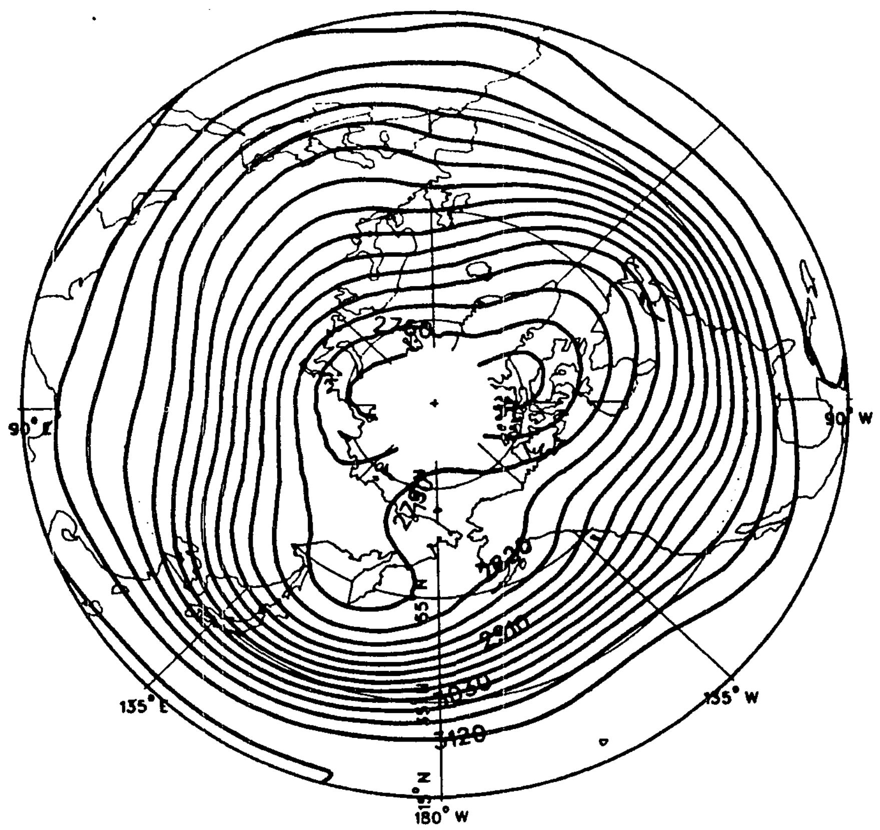
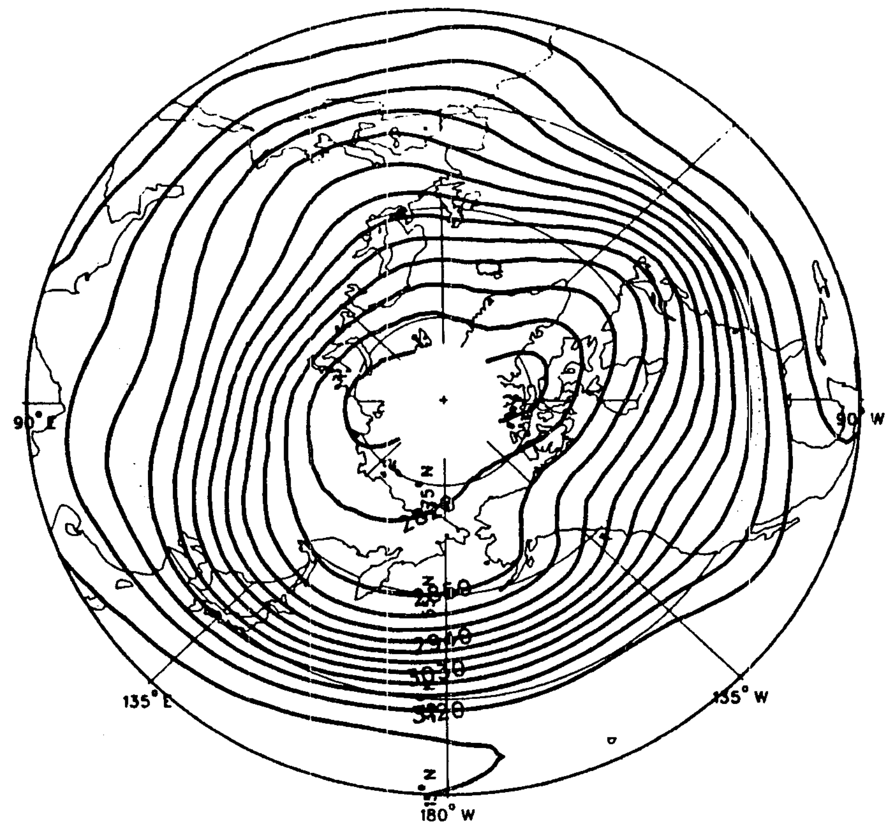


Figure 3. 29 year averages of the monthly averaged 700 mb heights for each of the twelve months. The contouring interval is 30 m.

AVERAGE 700 MB HEIGHT (M) FOR MARCH



AVERAGE 700 MB HEIGHT (M) FOR APRIL



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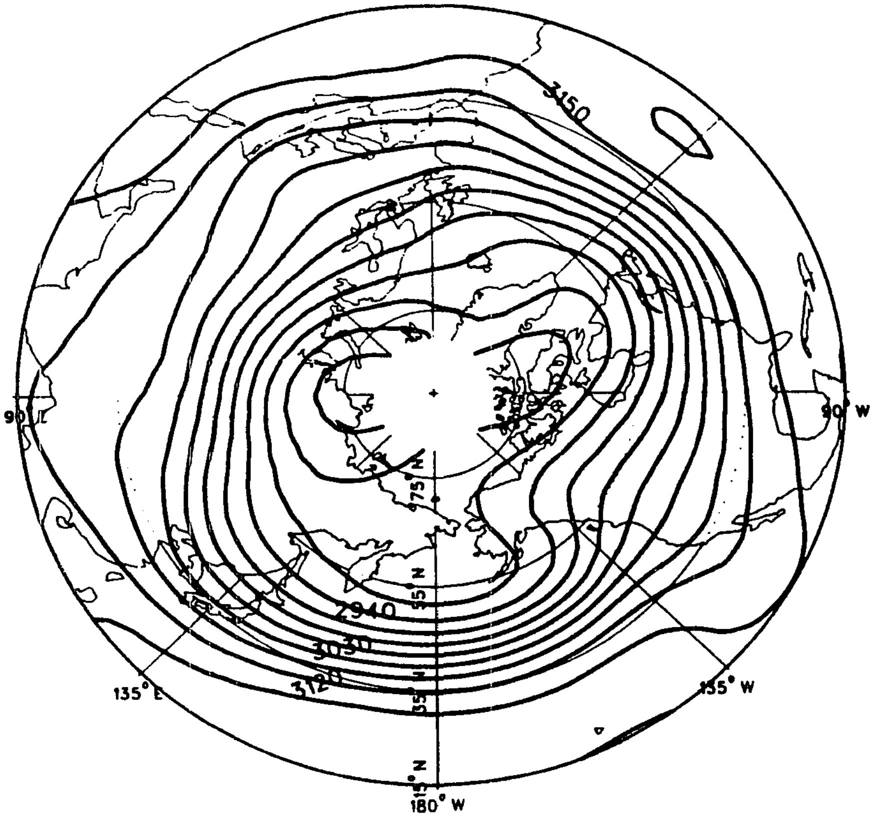

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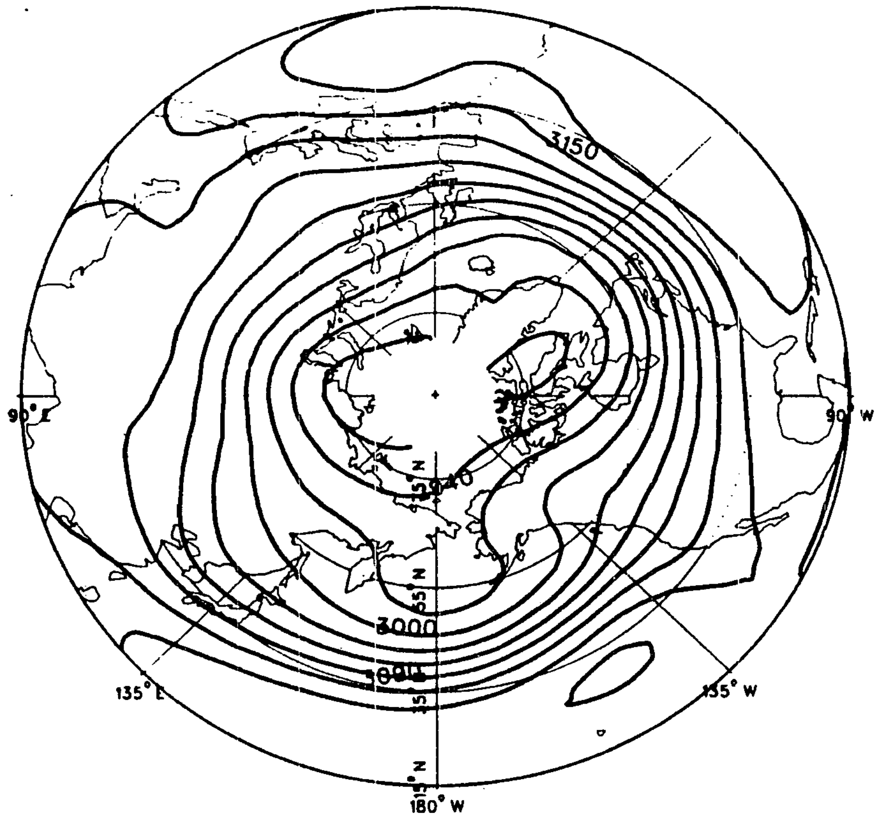
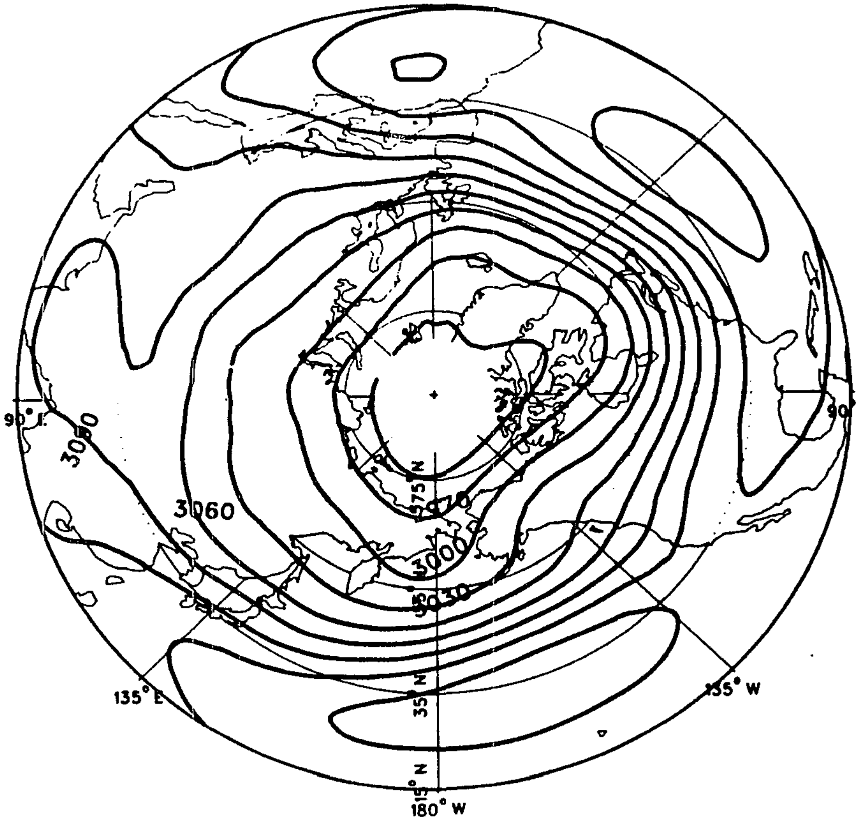
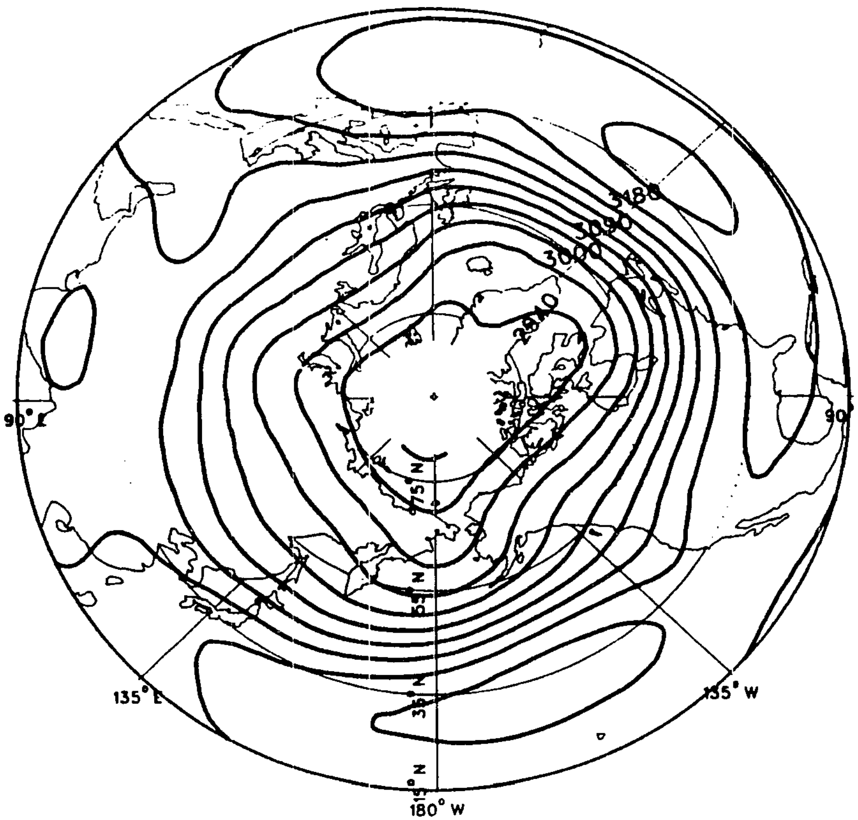


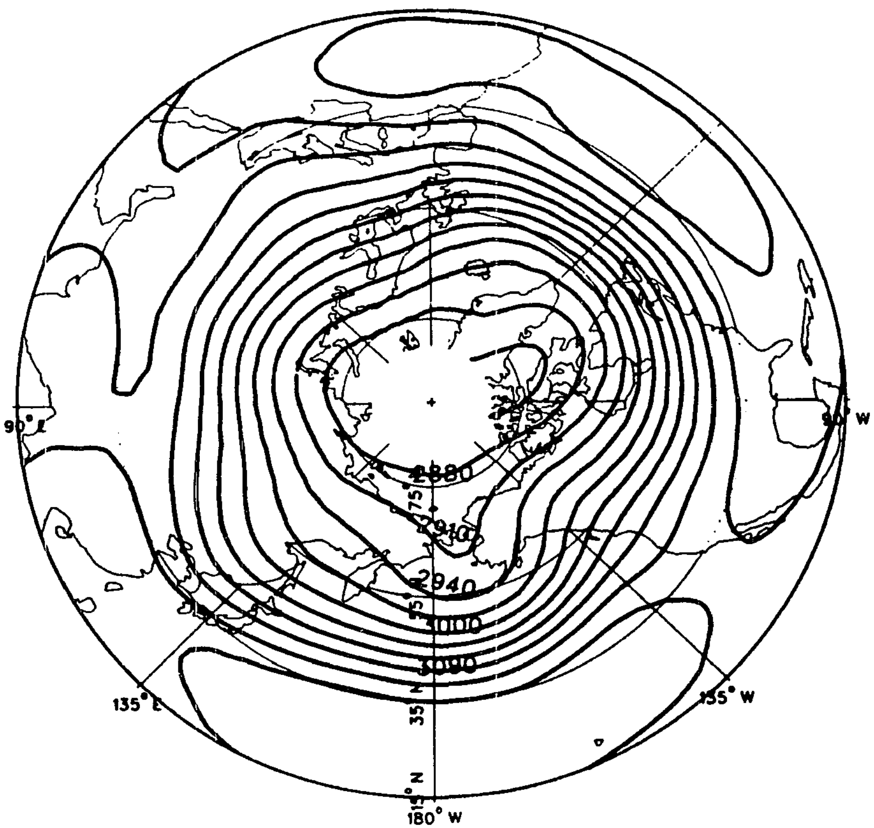
AVERAGE 700 MB HEIGHT (M) FOR JULY



AVERAGE 700 MB HEIGHT (M) FOR AUGUST



AVERAGE 700 MB HEIGHT (M) FOR SEPTEMBER



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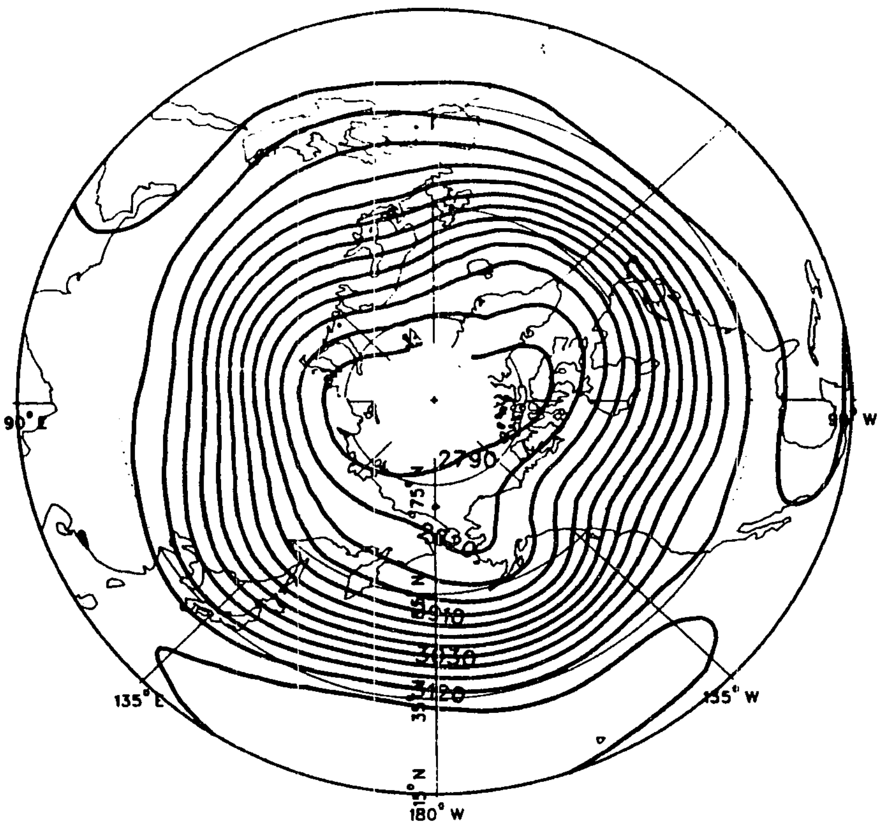
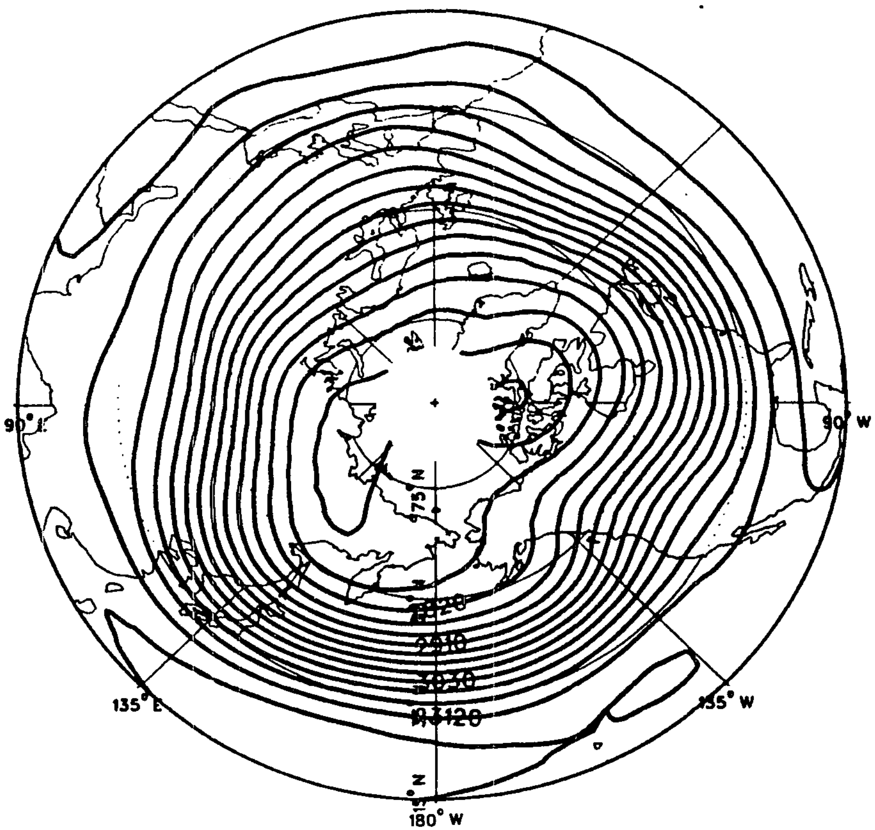
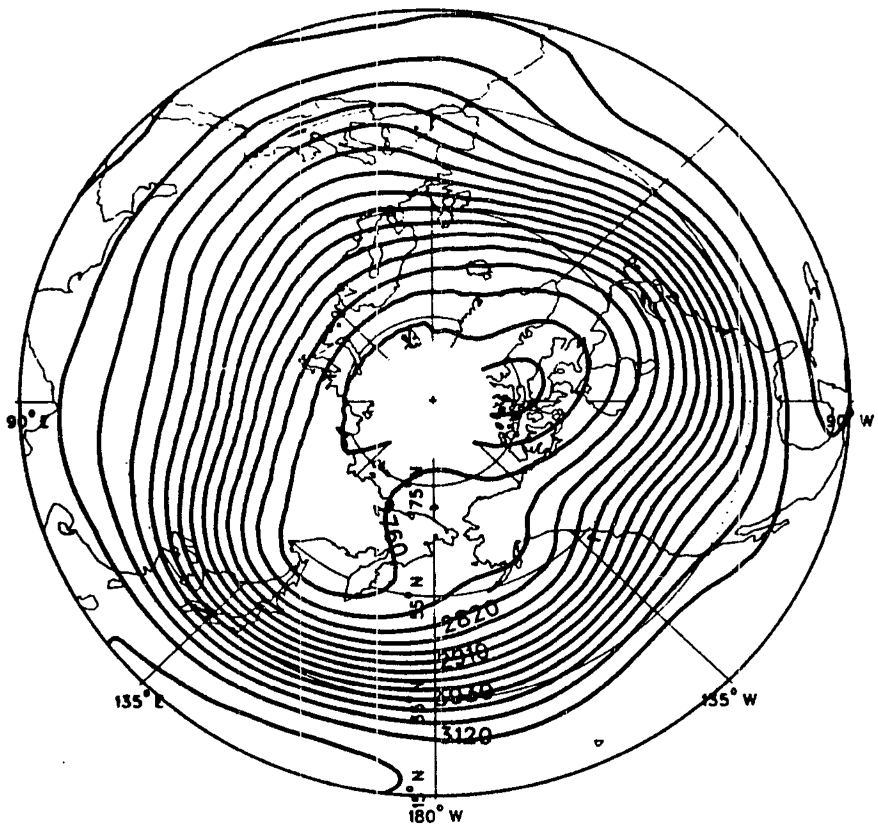


AVERAGE 700 MB HEIGHT (M) FOR NOVEMBER



AVERAGE 700 MB HEIGHT (M) FOR DECEMBER



troughs appear at 170°E and 80°W, also in agreement with the mean 500 mb heights by Palmen and Newton (op. cit.). The locations of the mean troughs positions are thought to be the result of orographic influences (mainly the Rockies and the Himalayas) and of the thermal effects of land and sea (Palmen and Newton, op. cit.)

The patterns in Figure 3 were drawn on a Lambert equal area projection. This projection is not conformal (the scales are not the same in all directions from any point) but it is equal area (DISSPLA, 1978). This projection was selected over conformal projections such as polar stereographic so area relationships would be preserved for subsequent principal component analyses. The patterns in Figure 3 were computer contoured. The data from the diamond grid were interpolated to a regular 27x73 latitude-longitude grid using a weighted interpolation method (DISSPLA, op. cit.). The regular grid was smoothed three times using a five point two dimensional smoothing algorithm from Haltiner (1971) with a weight of 0.5. Since the first and 73rd values for a given row both correspond to 0°E, these values were replaced with their average so the contours are forced to meet at 0°E.

In addition to the annual cycle of mean heights, the annual variation of the atmospheric circulation patterns produces an annual cycle in the interannual variance of the 700 mb heights. The patterns of 700 mb interannual standard deviations are presented in Figure 4. These patterns were computer produced in the same manner as Figure 3. As is expected, the standard deviation is largest in the winter (a maximum of 90 m) and lowest in the summer (a maximum of about 50 m). In addition, the variance increases with latitude, but shows local maxima at 50°N-60°N during the winter months. For the winter months, these

700 MB MONTHLY STD. DEV. (M) FOR JANUARY

700 MB MONTHLY STD. DEV. (M) FOR FEBRUARY

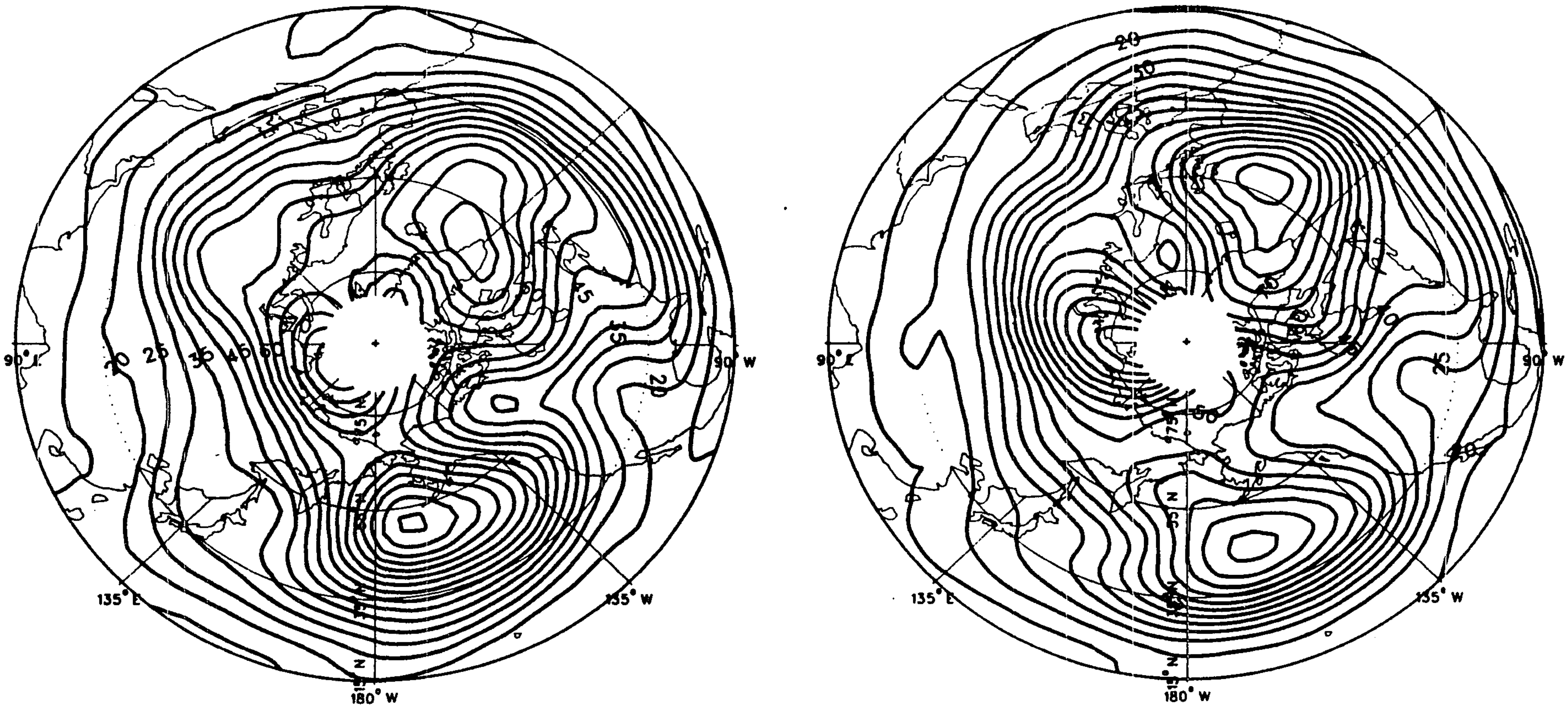
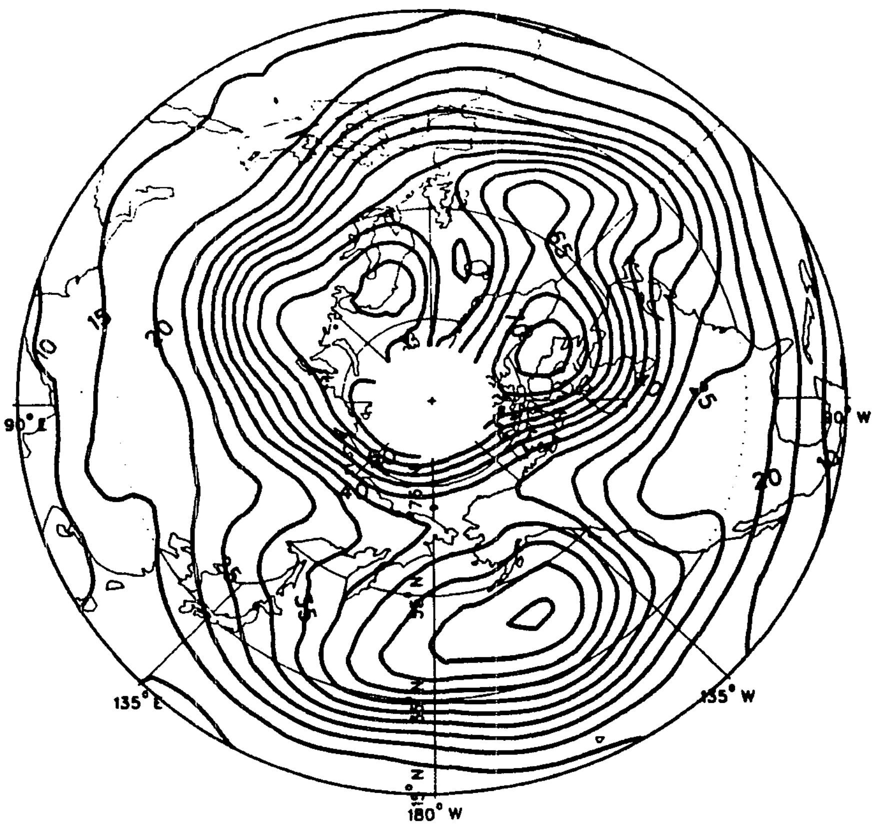
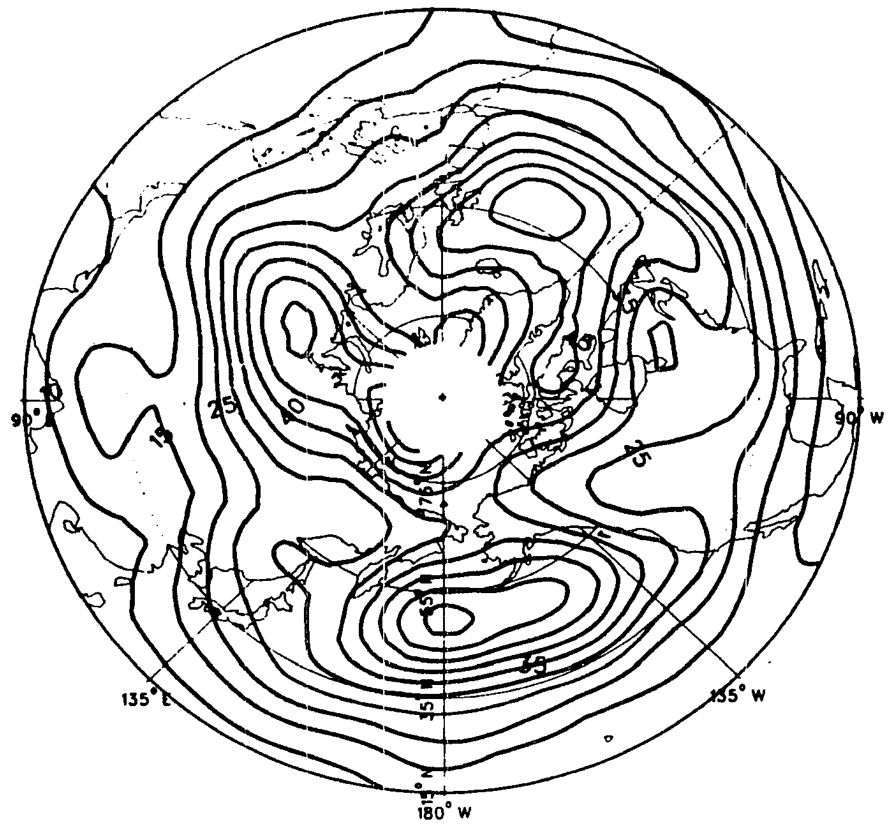


Figure 4. 29 year interannual standard deviations of the monthly averaged 700 mb heights. The contouring interval is 5 m.

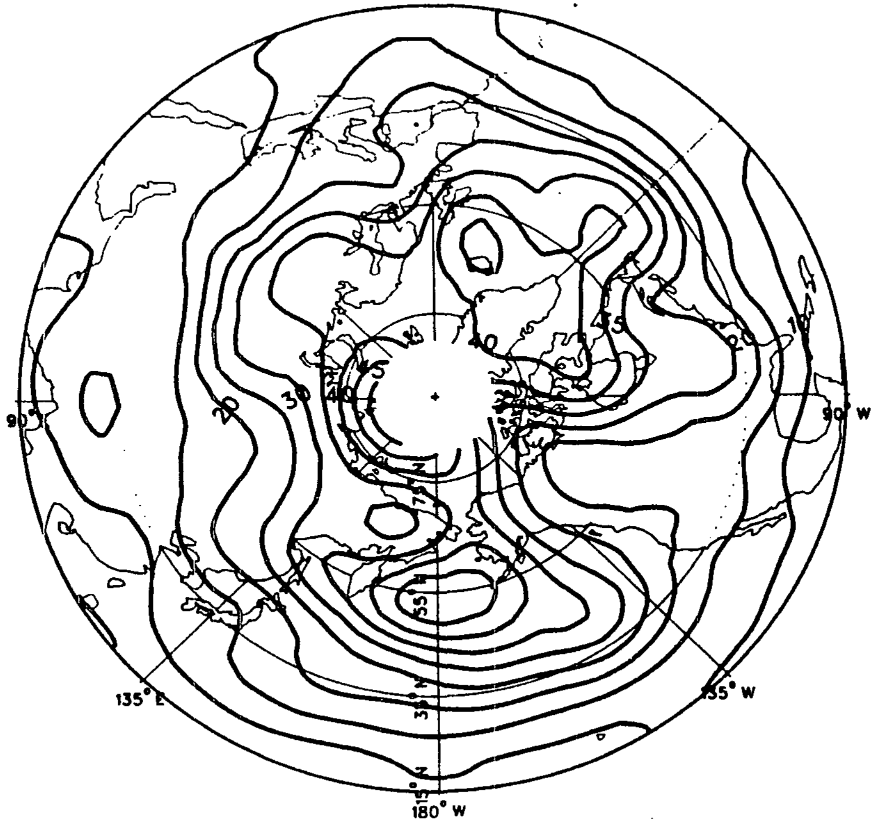
700 MB MONTHLY STD. DEV. (M) FOR MARCH



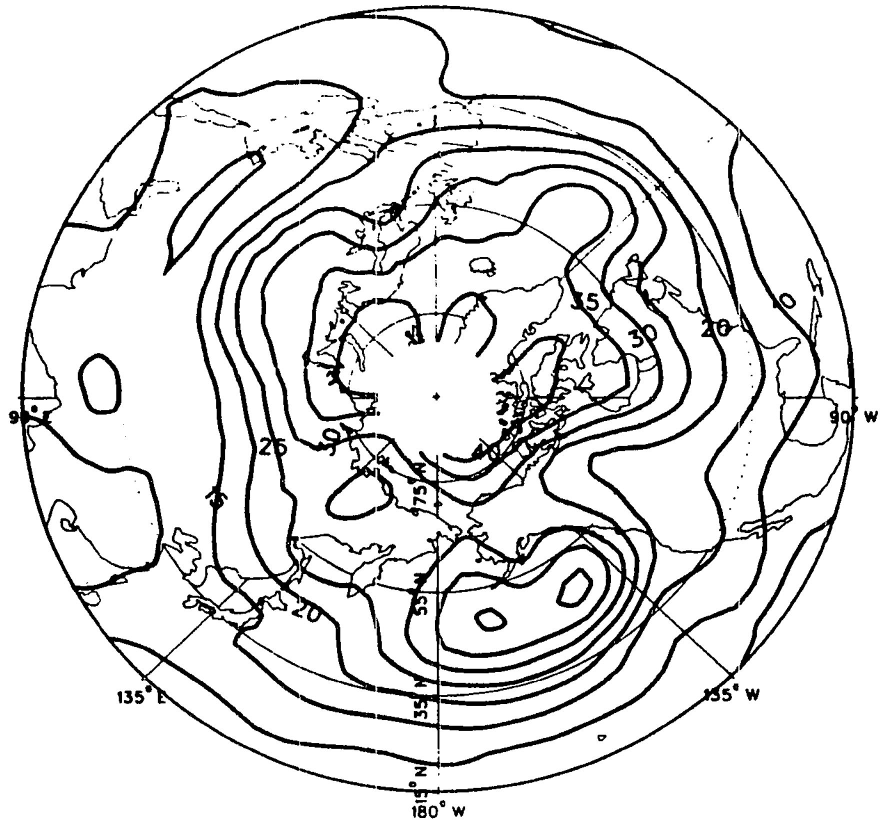
700 MB MONTHLY STD. DEV. (M) FOR APRIL



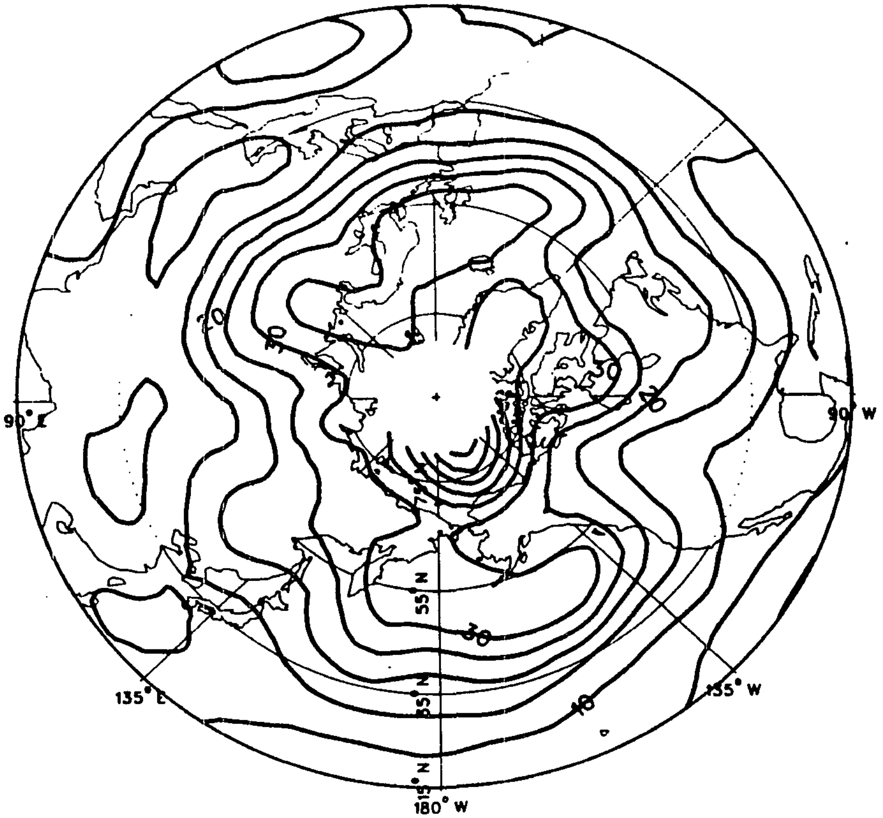
700 MB MONTHLY STD. DEV. (M) FOR MAY



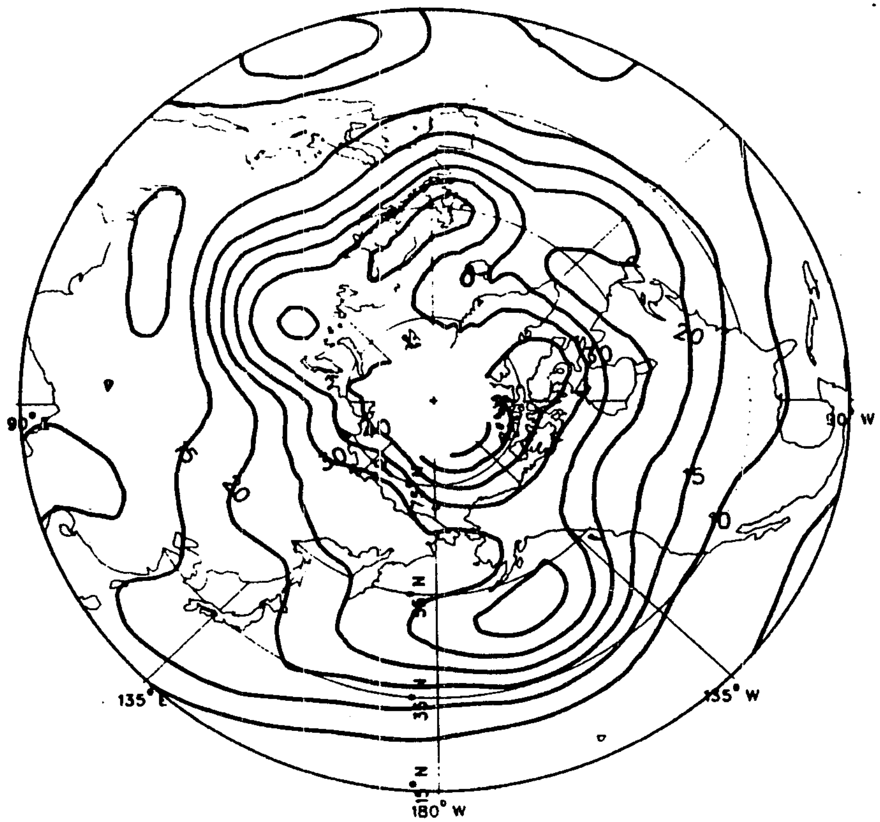
700 MB MONTHLY STD. DEV. (M) FOR JUNE



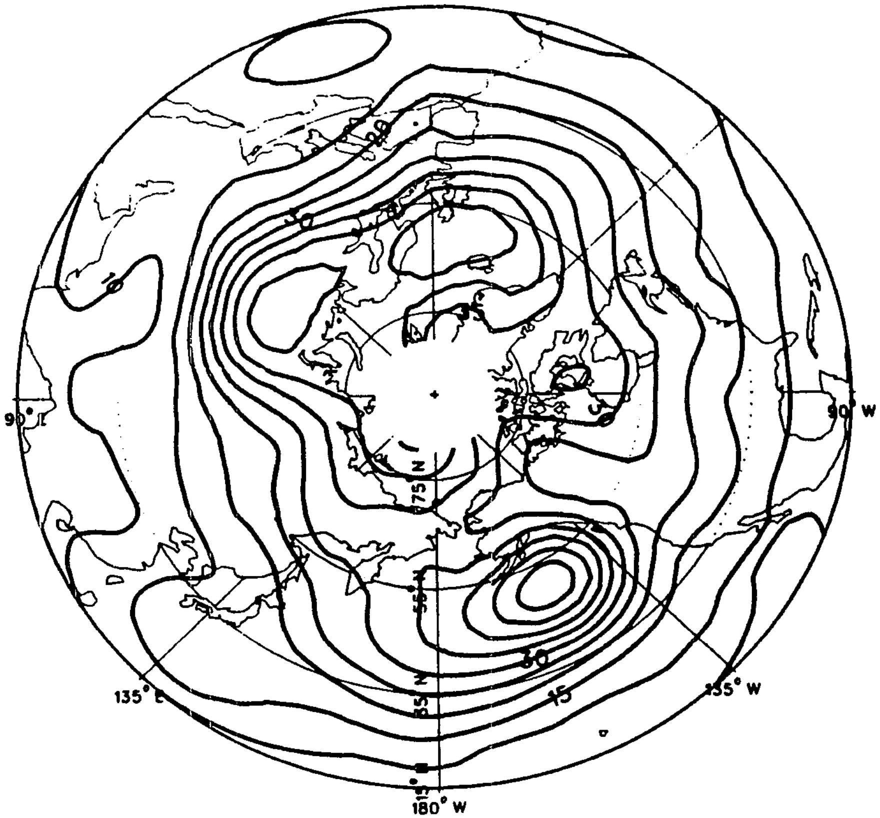
700 MB MONTHLY STD. DEV. (M) FOR JULY



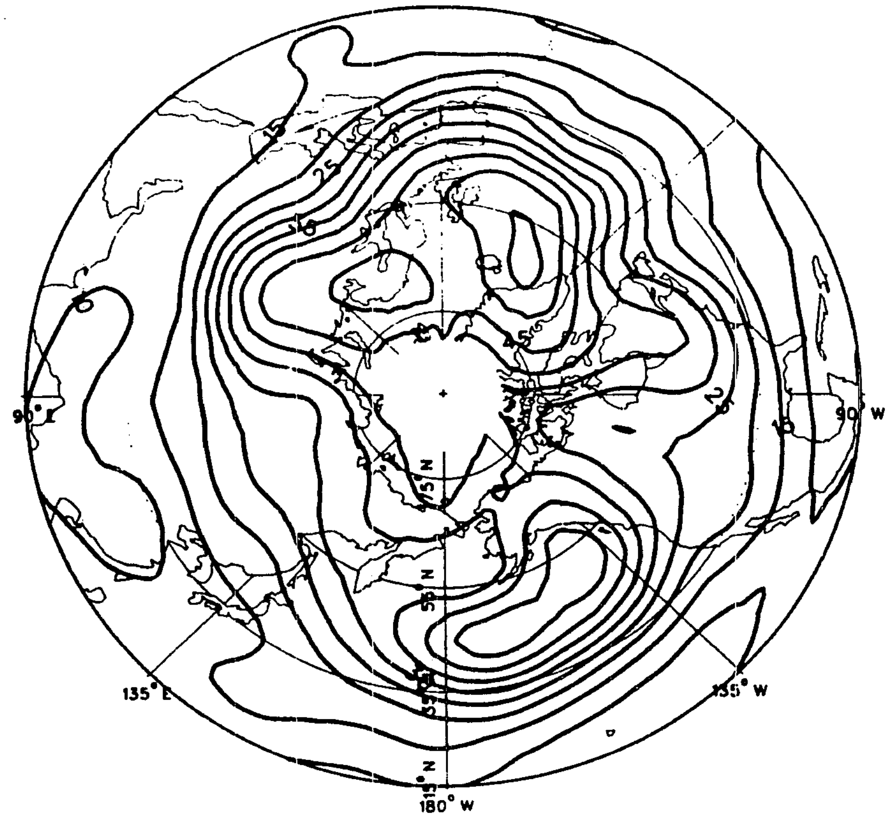
700 MB MONTHLY STD. DEV. (M) FOR AUGUST



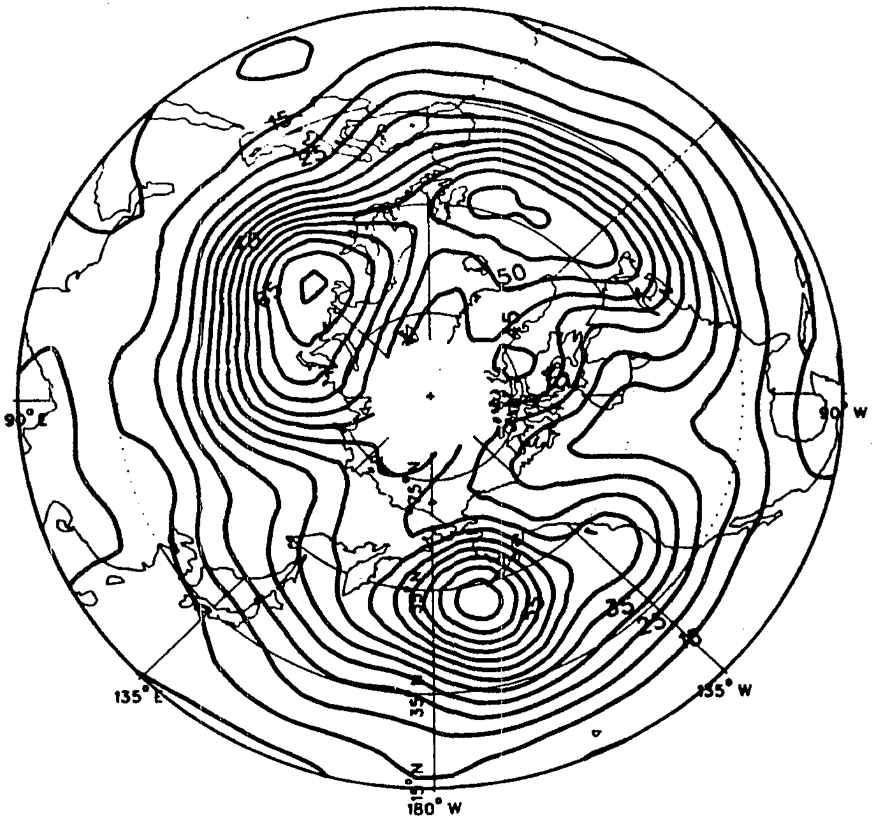
700 MB MONTHLY STD. DEV. (M) FOR SEPTEMBER



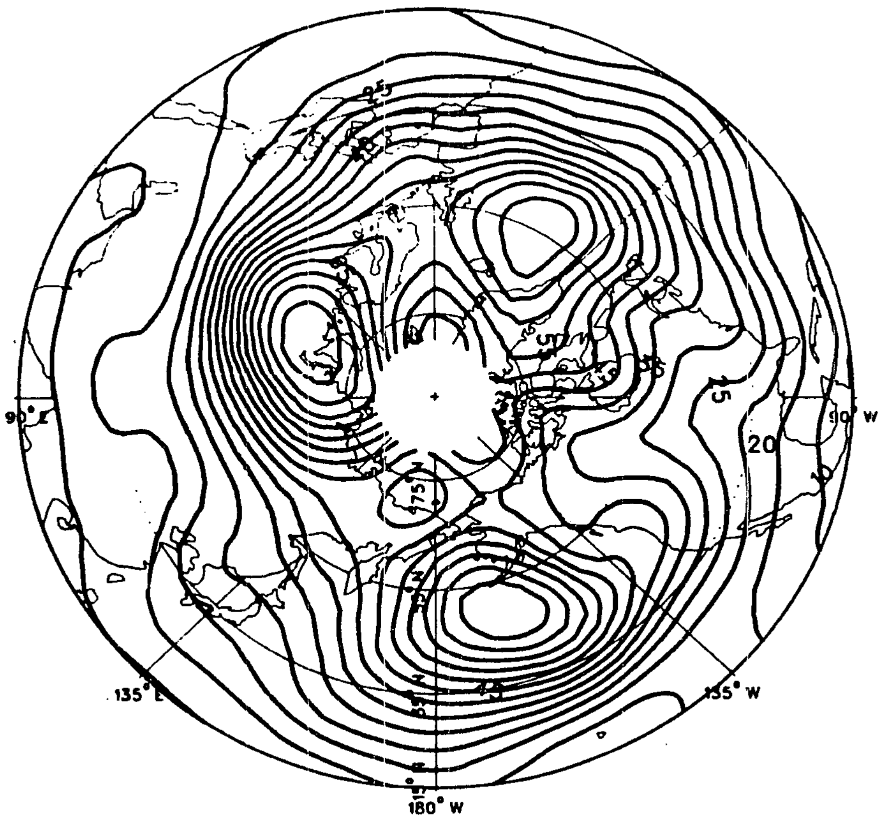
700 MB MONTHLY STD. DEV. (M) FOR OCTOBER



700 MB MONTHLY STD. DEV. (M) FOR NOVEMBER



700 MB MONTHLY STD. DEV. (M) FOR DECEMBER



local maxima occur at 45°W, 170°W, and 60°E. These maxima are ahead (eastward) of the mean trough positions depicted in Figure 3. These high values of interannual variance are partially attributed to yearly variations in the tracks and intensity of mid-latitude cyclones.

These manifestations of the annual cycle were removed from the series by subtracting the monthly pattern for the appropriate month and then dividing by the monthly standard deviation pattern. The effect of subtracting the twelve monthly patterns is to remove the variance of the annual cycle (Jones, 1964a). However, to apply time series methods the annual variation of variance must be removed so that the assumption of stationarity is valid. The procedure of subtracting the monthly average and dividing by the monthly standard deviation is described by Namias (1978), who termed the resultant values "standardized height anamolies". Maps of these height anomalies were verified against those by Namias (1978, 1979) to ensure that the computer programs were working properly. Typical maps of these anomalies are given in Figure 19, in Chapter VI.

In addition, the annual cycle of the atmosphere produces changes in the circulation statistics that would remain even when the mean has been removed and the data standardized. Such variations have been reported by Haney (1979), who considered the correlations between sea surface temperatures and sea level pressures. For these variables the correlations were found to reverse sign over the course of a year. In the present application only 700 mb height anomalies are being considered. Since the eigenvectors are derived from the correlation matrix, such annual variations of correlation will appear in the eigenvectors. For the PCA, it will be assumed that the correlation structure of the standardized height anomalies is approximately invariant over the course of a year. This assumption will be checked later in Chapter V.

As was recommended by Jones (1964a), the deterministic part of the time variation of the 700 mb heights has been removed and the resultant anomalies standardized. In the subsequent analyses, only these standardized anomalies will be considered. If 700 mb heights are of interest they can be recovered simply by multiplying by the monthly standard deviation pattern and adding the monthly average pattern.

CHAPTER V

APPLICATION OF PRINCIPAL COMPONENT ANALYSIS

The annual cycles in the mean and the variance have been removed from the 700 mb heights to give the standardized height anomalies. In this chapter PCA will be applied to the standardized height anomalies, primarily for the purpose of reducing the number of random variables to be forecast by MVAR in Chapter VI. In addition, the assumption of an invariant correlation structure of the standardized height anomalies needs to be examined. This will be done by a separate PCA for each of the twelve months.

The original number of observations (504) for each month is much too large to handle practically on a digital computer. Furthermore, for the first part of the series, these 504 have been analyzed from a smaller number of irregularly distributed upper air observations so that, in reality, there are a smaller number (compared to 504) of independently measured data. Rinne and Frisk (op. cit.) found the number of upper air stations to be 130, 265, 365, and 560 for the years 1946, 1952, 1957, and 1969, respectively. In addition, since the number of observations varies over the time period 1951-77, use of the entire original data set is too formidable a task. Therefore, the number of observations per month was reduced to the order of 100.

An approximately equal area grid was constructed by first computing the ratio of the area of a square degree (latitude x longitude) at the latitudes 20°N to 80°N to the corresponding area at 15°N. This ratio gives the relative number of grid points at these latitudes needed for equal areas. Using these grid points reduced the total number to about 400. Further reduction was achieved by deleting every other latitude and longitude, beginning at 20°N since many data are missing at 15°N prior to 1960. The resulting 95 point sparse grid is depicted in Figure 5.

A comparison of several anomaly maps as represented by the 95 point grid and by the 504 point grid showed insignificant differences. Thus, no attempt was made to spatially filter the data before decimating to the 95 point grid. Such a step is unnecessary since aliasing is apparently negligible and PCA will suppress noise. The basic resolution of the 95 point sparse grid is 2200 km in longitude and about 1100 km in latitude. Histograms of the data at ten of these grid points scattered over the Northern Hemisphere showed negligible deviations from a normal distribution.

Analysis of the Entire Climatic Series

Principal component analysis was performed on the data at the 95 sparse grid points for the time period January 1951 to July 1977. (Five months of data were reserved for an independent data set to be compared to the forecasts in Chapter VI.) The covariance matrix was estimated by (1).

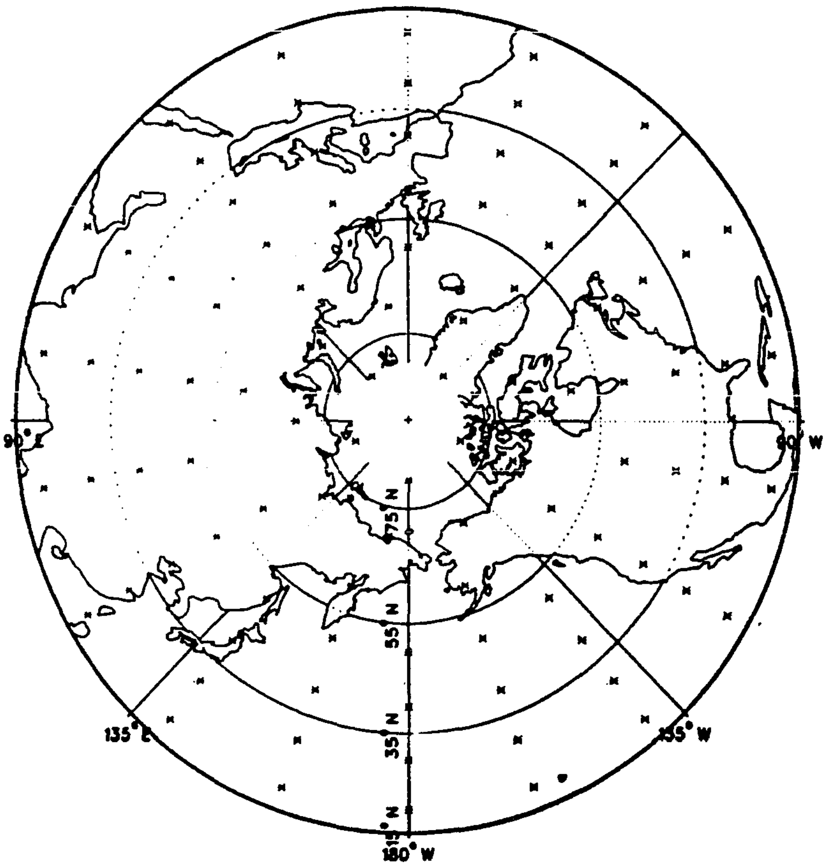


Figure 5. 95 approximately equal area grid points selected for subsequent analysis and modeling.

The eigenvalues and eigenvectors of the covariance matrix were extracted by using the FORTRAN library routine FACTAN (Univac, 1973). Similar extractions of the eigenvectors using the IMSL (1975) routine EIGRS in both single and double precision and by the asymptotic singular decomposition method of Jalickee, et. al. (1975) did not give significantly different results. The log-eigenvalues are plotted in Figure 6. The log-eigenvalues drop off nonlinearly for about the first 30. Thereafter, the slope is constant (except for the last 2 or 3). Clearly, the eigenvectors with numbers above 40 represent noise, as noted in Chapter II. The eigenvectors that are mostly signal are those below 20 to 30. This number of signficant eigenvectors is about the same as that by Craddock and Flood (1969) for a three year period of daily values of Northern Hemisphere 500 mb heights at 130 locations.

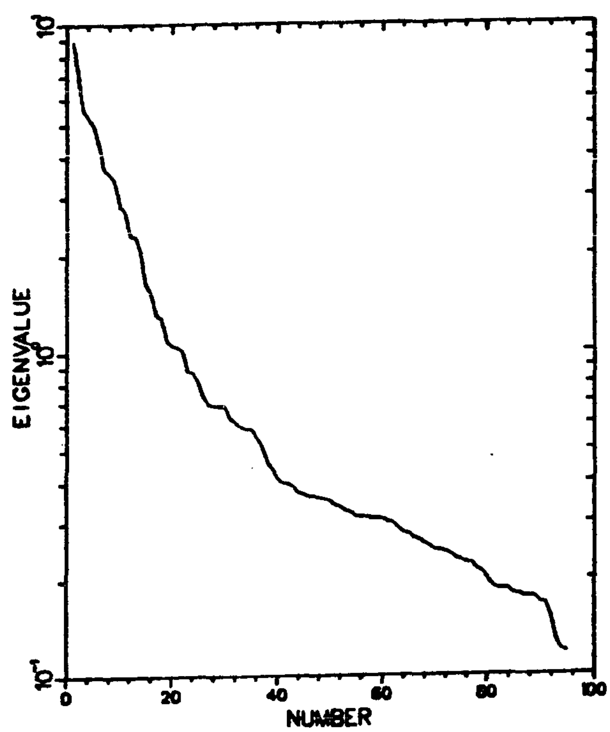


Figure 6. Log eigenvalue versus eigenvector number (LEV) diagram for the 700 mb standardized height anomalies.

The cumulative percentages of variance accounted for by the eigenvectors is presented in Table 1. Ten eigenvectors account for 50% of the total variance, while twenty are needed to account for 70%. Beyond twenty, each eigenvector accounts for no more than 1% of the total variance. Whether these eigenvectors represent observational errors or small scale features can be decided by an independent estimate of the magnitude of the observation errors. The error variance for an observation of 500 mb height was found by Rinne and Frisk (op. cit.) to be about 900 m^2. Assuming this value to be appropriate also for a

700 mb height observation, then the error variance for a 700 mb height standardized by about 30 m would be 1. For a monthly average of 60 values, the error variance would thus be 1/60. Since the eigenvalues of the eigenvectors judged to be noise are much larger than this, the "noise" in the data is primarily spatially uncorrelated small scale effects.

Table 1. Eigenvalues and Cumulative Percentages of Variance for the First Twenty Principal Components

Eigenvector Number	Eigenvalue	Cumulative Percent of Variance
1	8.95	9.4
2	7.27	17.1
3	5.58	27.9
4	5.29	28.5
5	4.98	33.7
6	4.32	38.3
7	3.67	42.2
8	3.56	45.9
9	3.39	49.5
10	2.83	52.4
11	2.74	55.3
13	2.29	60.2
14	2.06	62.3
15	1.64	64.1
16	1.53	65.7
17	1.31	67.1
18	1.28	68.4
19	1.09	69.5
20	1.06	70.7

The number of eigenvectors needed to explain say, 70% of the variance, appears to be large compared to similar statistics found by other studies. This is a result of the removal of the variance of the annual cycle. For example, Heddinghaus and Kung (1980) found that for monthly averaged 700 mb temperatures the first eigenvector accounts for about 80% of the variance. However, this variance also contains the annual variance, and the eigenvectors are used largely to model the deterministic annual cycle.

The first ten eigenvectors are presented in Figure 7. The 95 components of a given eigenvector are associated with the sparse grid points. The bivariate interpolation method by Akima (1978) was used to interpolate the components to a 27x73 regular latitude-longitude grid covering the latitudes 15°N to 80°N. This regular grid was computer contoured on a Lambert equal area projection by the same methods applied to the monthly averages in Chapter IV, except that they were only smoothed once.

The features of the first eigenvector are quite similar to the Namias (1980) winter teleconnection chart (shown later in Figure 13) for standardized height anomalies. (The opposite signs are due to the arbitrariness of the sign of an eigenvector.) Since the PC's are approximately normal with zero means, the eigenvector is equally likely to occur with the opposite sign. To simplify the following discussion, only one sign is considered. The first eigenvector depicts troughs over the east coast of Asia, the mid Pacific, the east coast of North America, and the west coast of Europe; slight ridging over the west coast of North America and central Asia; and a ridge over the pole. Since the eigenvector is constructed to maximize its variance over all possible linear combinations, its components represent the tendency of the height anomalies to covary. For example, if the height anomaly is negative at (45°N, 135°E), then as a first approximation, similar negative anomalies would be expected at (45°N, 170°W), (45°N, 60°W), and (50°N, 0°E) while a positive anomaly would be expected at high latitudes.

The second eigenvector associates high latitude anomalies with those off the east coast of North America. The third associates a ridge

700 MB HEIGHT ANOMALY EIGENVECTOR 1

700 MB HEIGHT ANOMALY EIGENVECTOR 2

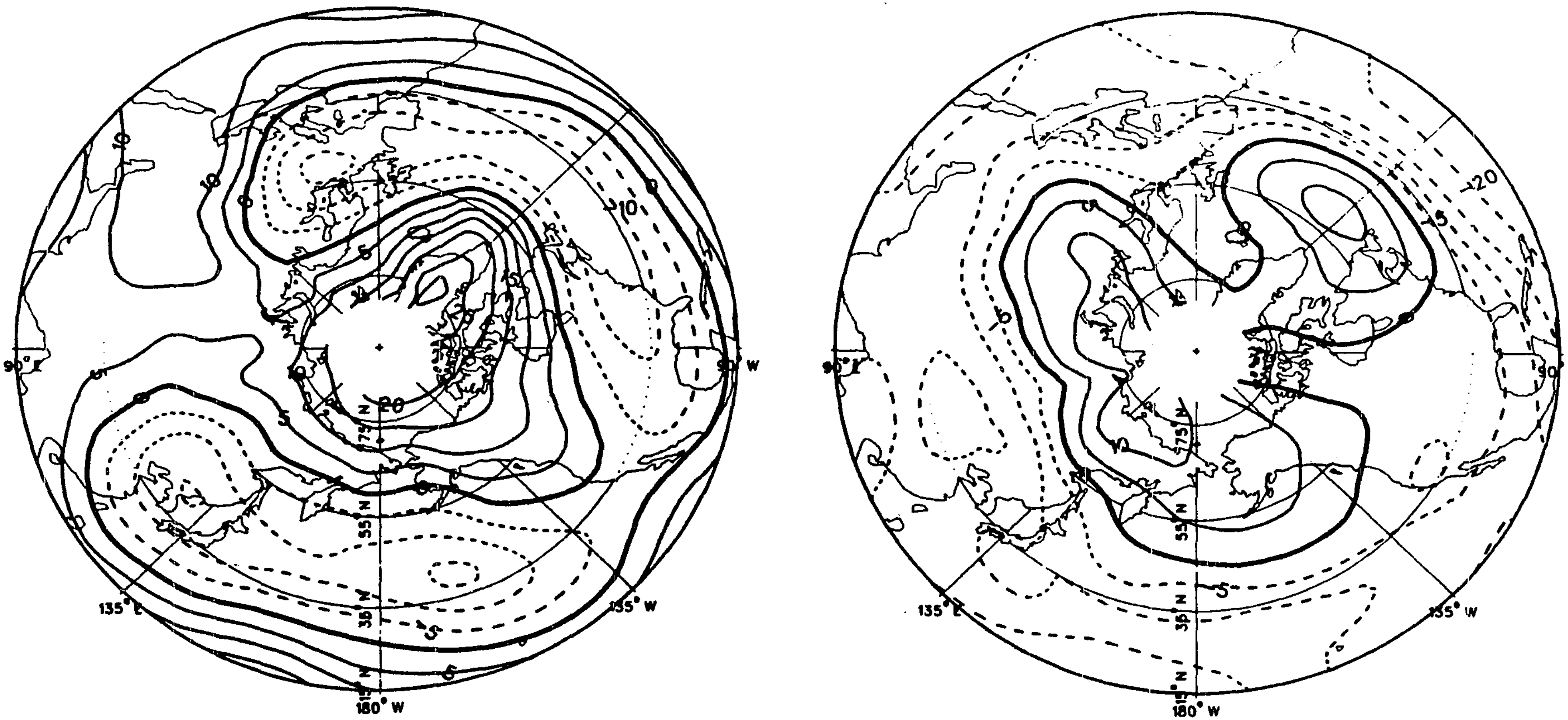
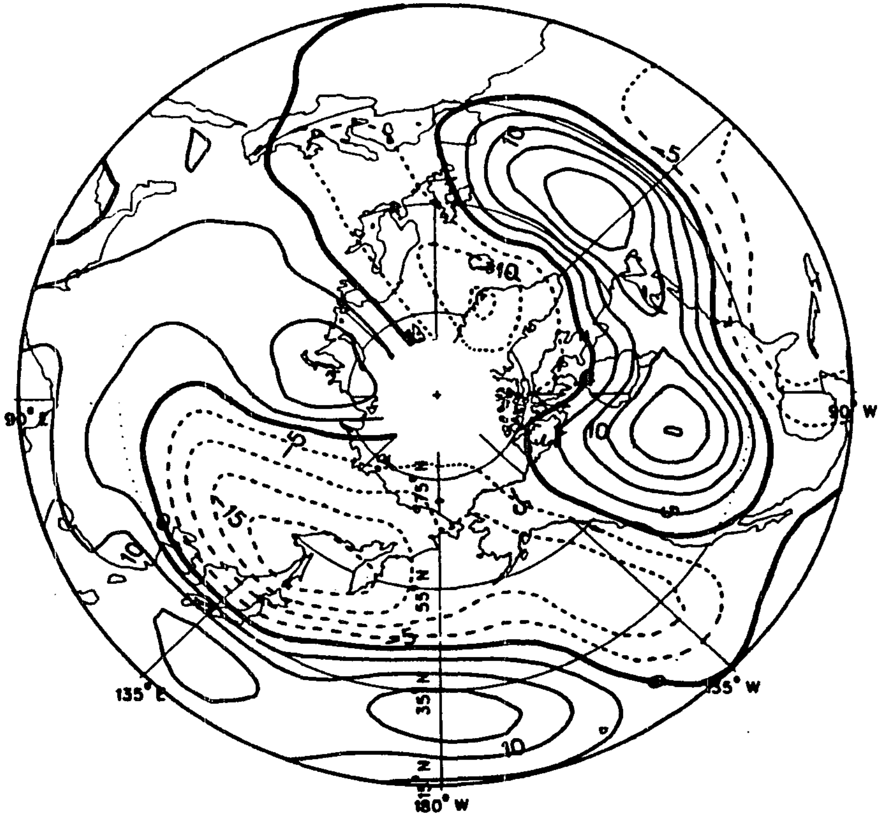
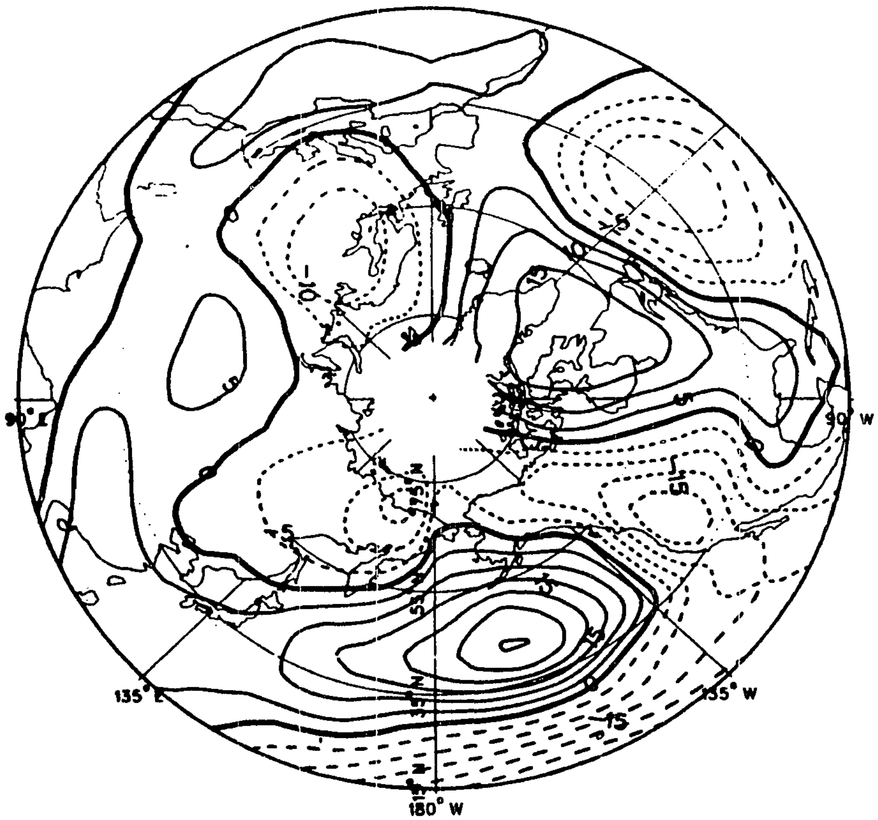


Figure 7. First ten eigenvectors of the 700 mb standardized height anomalies. The contouring interval is 5 m; the eigenvector components have been scaled by 100; contours of negative values are dashed.

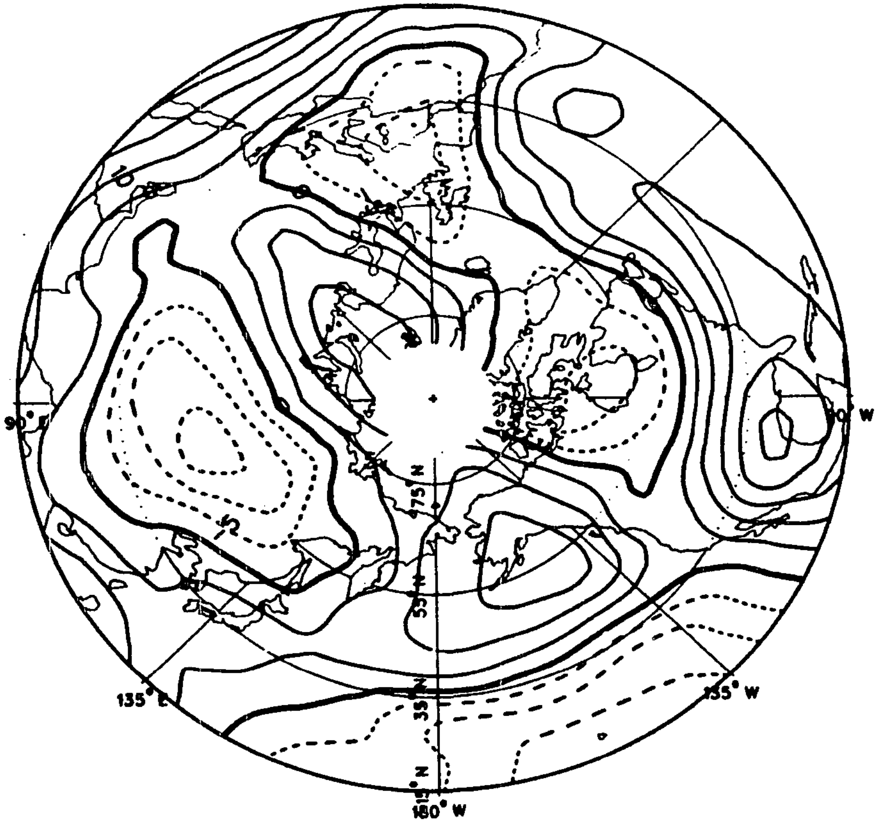
700 MB HEIGHT ANOMALY EIGENVECTOR 3



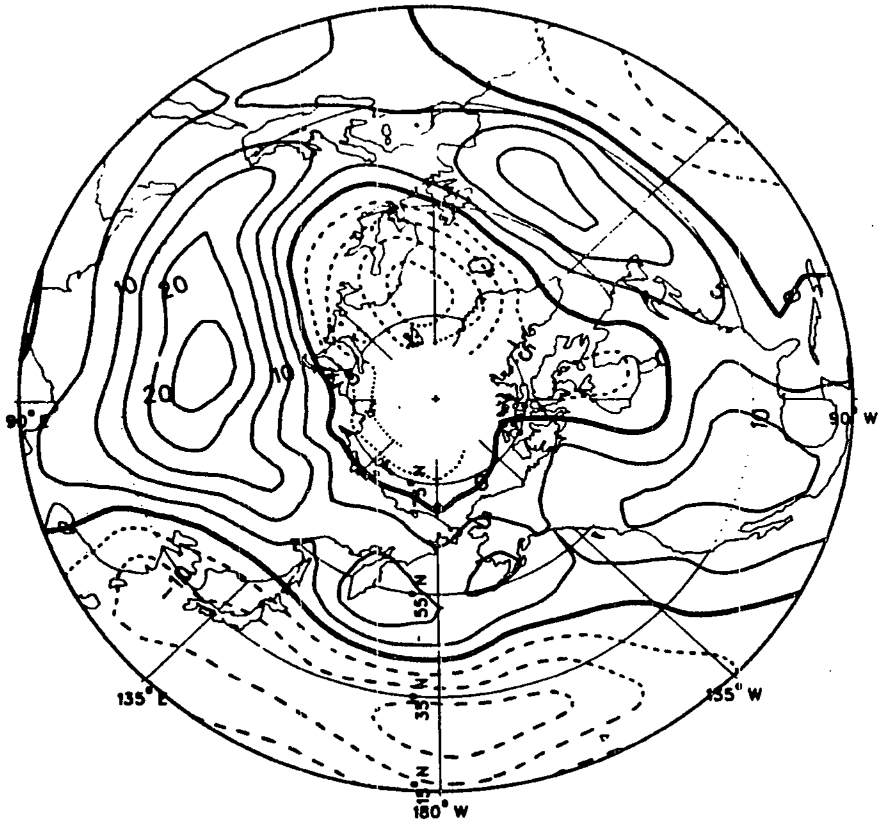
700 MB HEIGHT ANOMALY EIGENVECTOR 4



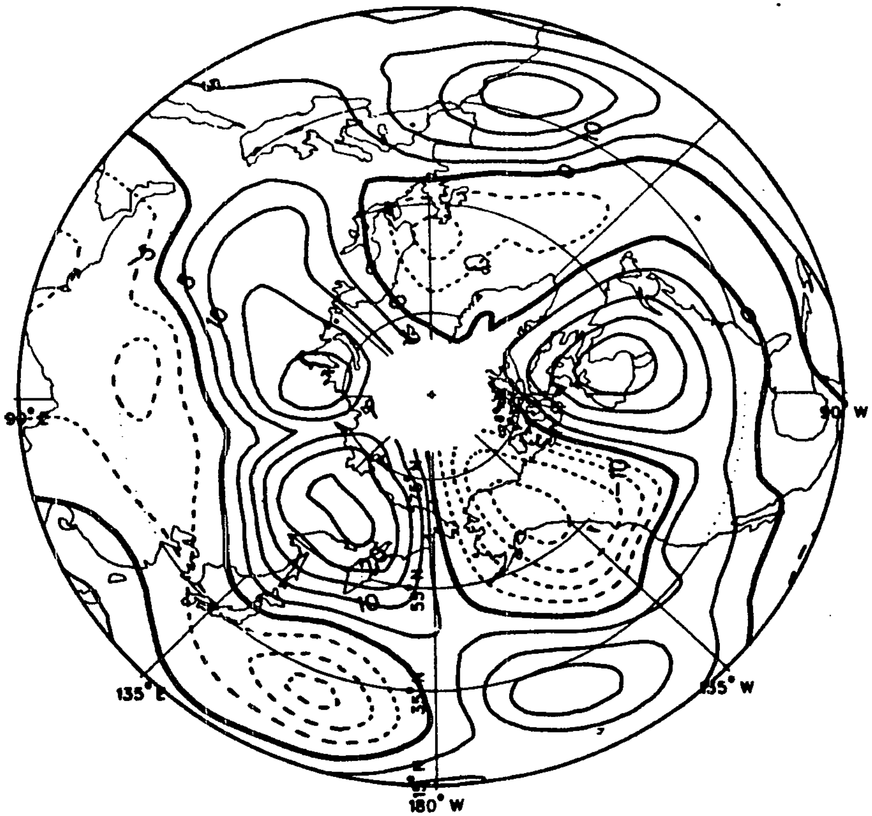
700 MB HEIGHT ANOMALY EIGENVECTOR 5



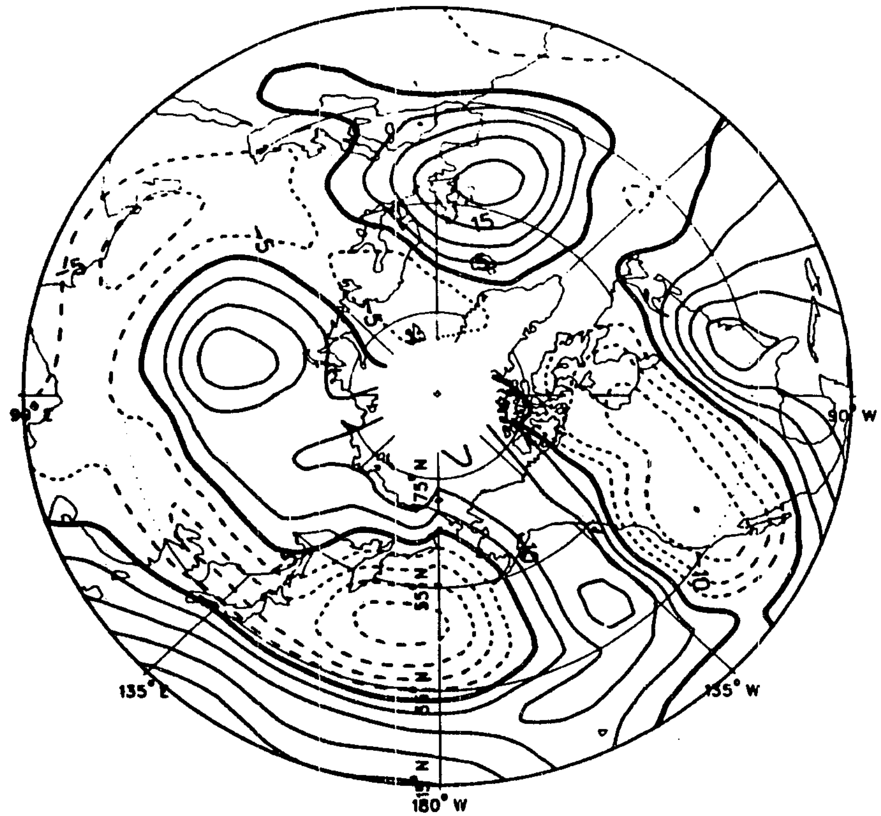
700 MB HEIGHT ANOMALY EIGENVECTOR 6



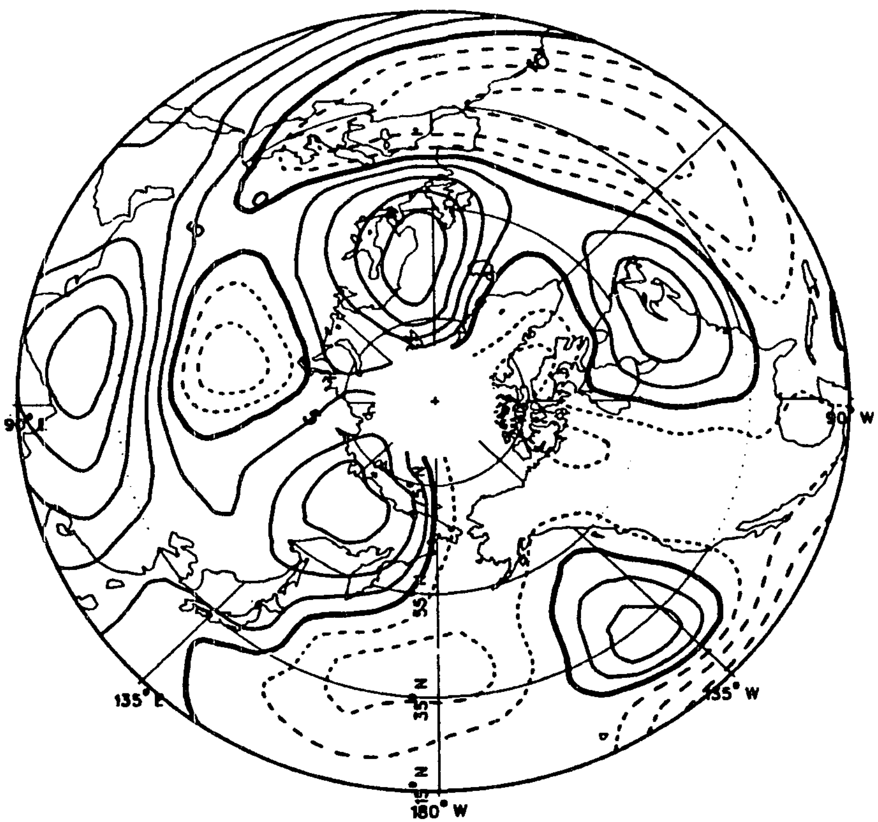
700 MB HEIGHT ANOMALY EIGENVECTOR 7



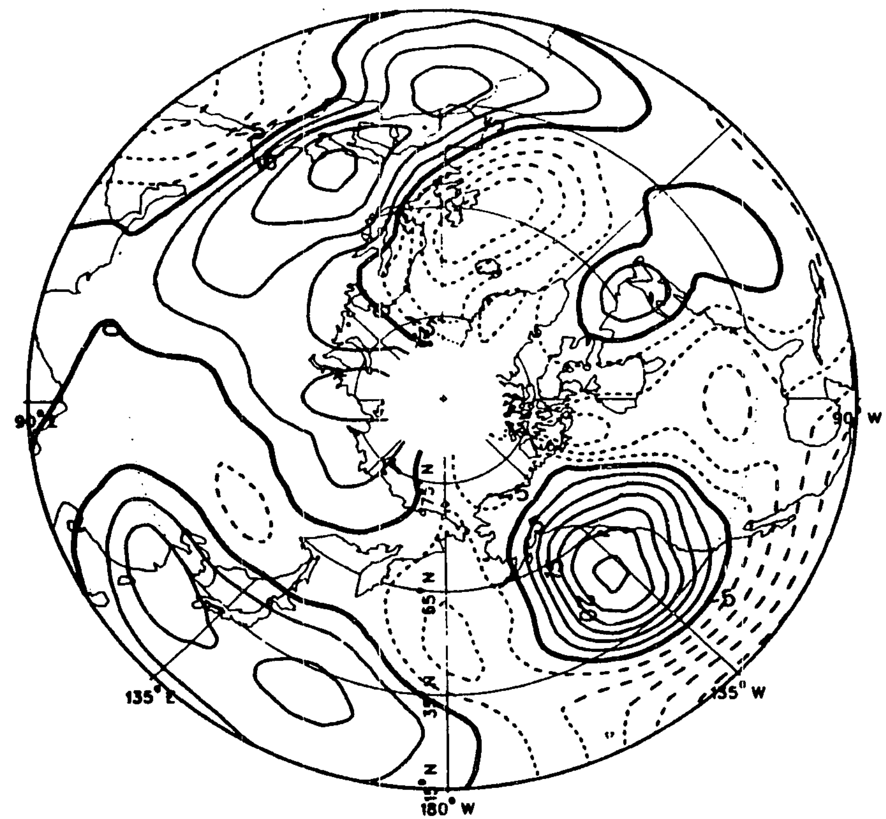
700 MB HEIGHT ANOMALY EIGENVECTOR 8



700 MB HEIGHT ANOMALY EIGENVECTOR 9



700 MB HEIGHT ANOMALY EIGENVECTOR 10



over the western U.S. with a filling of the eastern U.S. coast trough and a deepening of the trough of the east coast of Asia. The fourth eigenvector flattens the U.S. west coast ridge and fills the trough off the U.S. east coast. This particular pattern was found to be particularly active (large negative values of the PC) during the unusual winter of 1976-77. This winter was very cold in the eastern U. S. while drought occurred in the west. When negatively weighted, the fourth eigenvector builds the ridge over the west and deepens the east coast trough. The standardized height anomalies for November 1976 are shown in Figure 8. The similarity (except for sign) to the fourth eigenvector is apparent. The higher numbered eigenvectors contain progressively smaller scale features that are modifications to the basic anomaly patterns. The eigenvectors 11-20 are included in Appendix C.

The observed principal components were computed by (2). Time series plots of the first ten PC's are shown in Figure 9 and the numerical values for the first twenty are given in Appendix A. Also shown in Figure 9 are trends, estimated by second degree least squares polynomials fitted to the PC's (Otnes and Enochson, 1972, p. 61). These trends are, for the most part, small, except for the fifth PC. The trends probably represent changes in the meteorological analysis techniques used to produce the gridded values of 700 mb heights over the 27 years, especially in data sparse areas. The deviations about the trend show no systematic variation, but there is evidence of correlation between adjacent values. The maximum entropy technique (Ulrych and Bishop, 1975) was applied to the time series of the first ten detrended PC's to estimate their spectra. This technique fits an autoregressive (AR) model to a time series and estimates its spectrum by

700 MB HEIGHT ANOMALY FOR NOVEMBER 1976

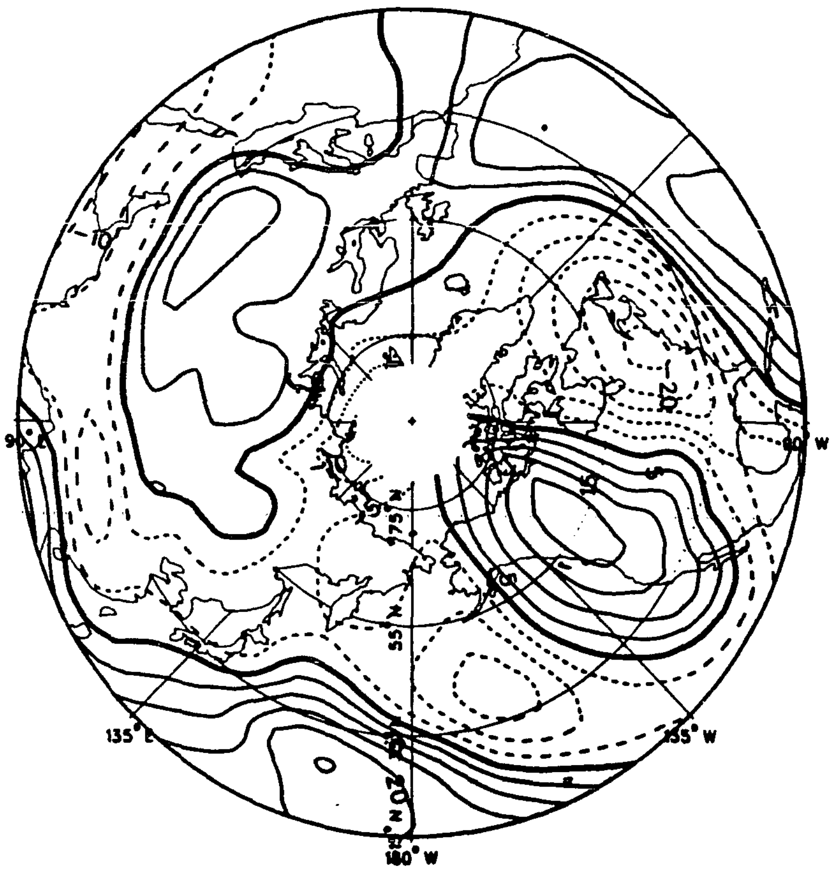


Figure 8. 700 mb standardized height anomalies for November 1976. The contouring interval is 5; values are scaled by 10; negative values are indicated by dashed contours.

the theoretical spectrum of the fitted AR model. For convenience, such spectral estimates will be referred to as AR spectra. The AR spectra of the first ten PC's are presented in Figure 10.

The most important feature of these spectra is that they are not flat. Such spectra would indicate that there was no time correlation in the PC's and that climatology would be the best forecast. Because the PC's are not completely random, an MVAR model can be used to

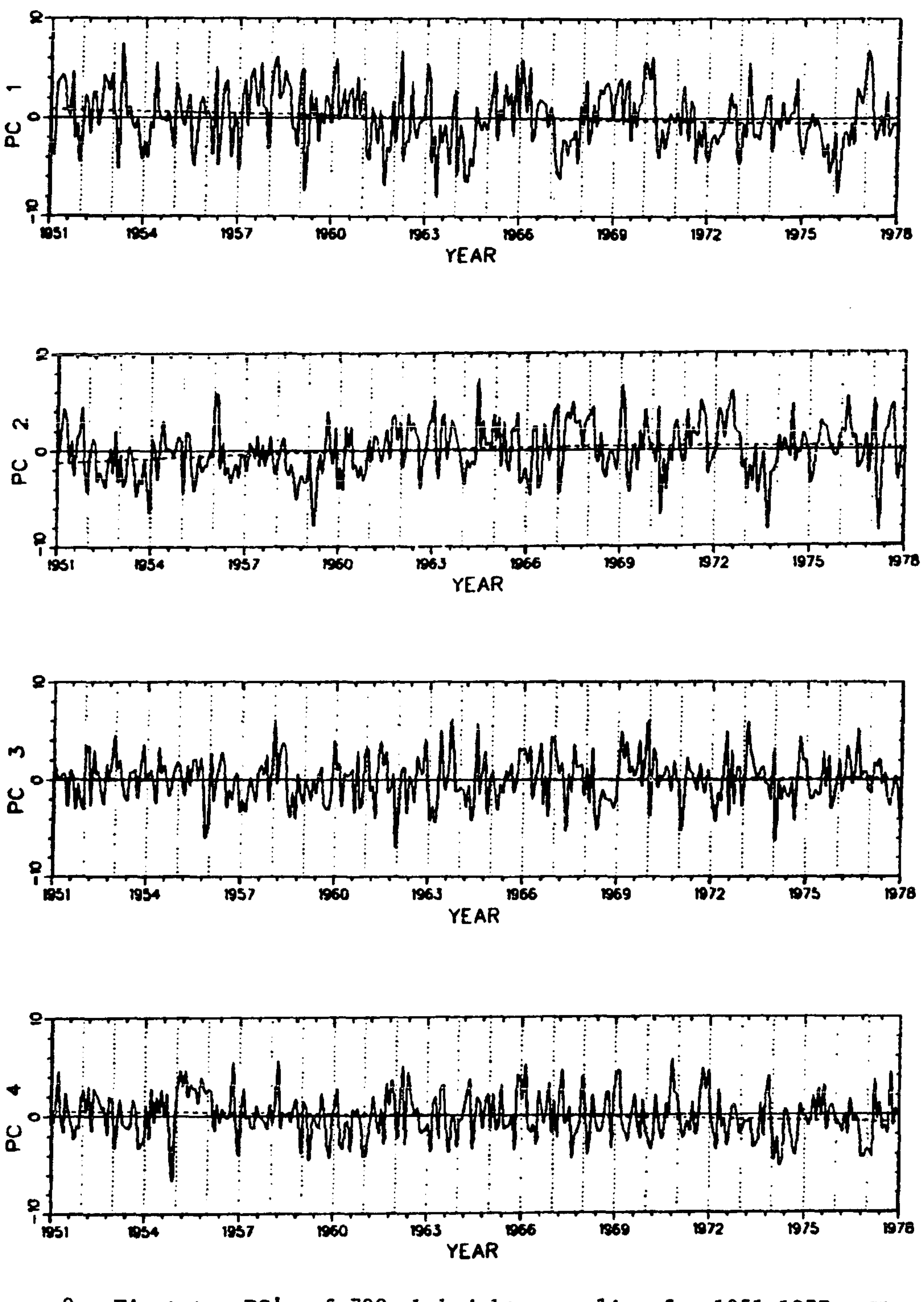
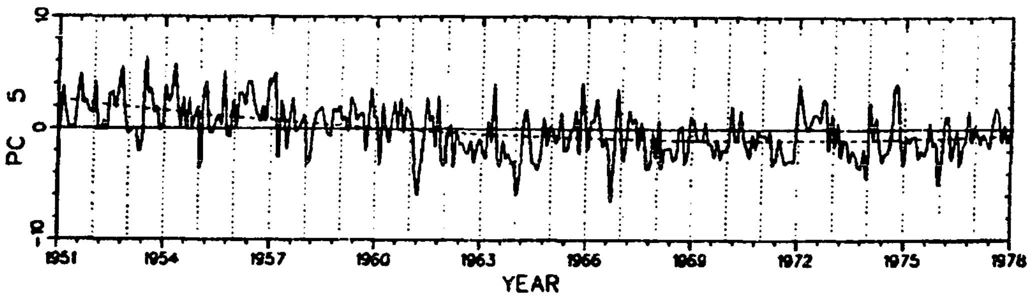
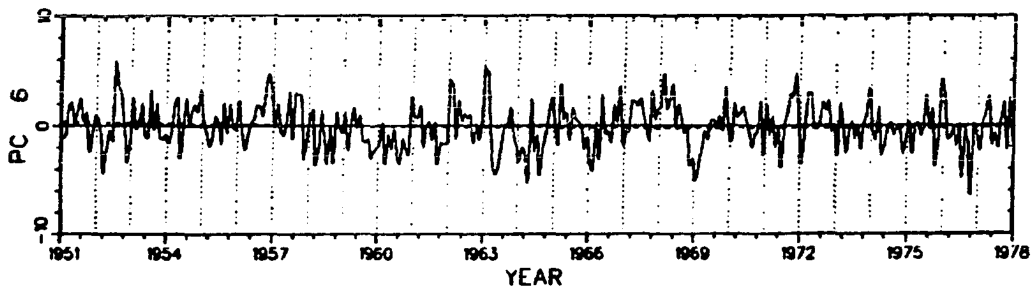


Figure 9. First ten PC's of 700 mb height anomalies for 1951-1977. The dashed lines are second degree polynomials. January of each year is indicated by a vertical dotted line.


PC 5
10
0
-10
1951
1954
1957
1960
1963
1966
1969
1972
1975
1978
YEAR


PC 6
10
0
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1951
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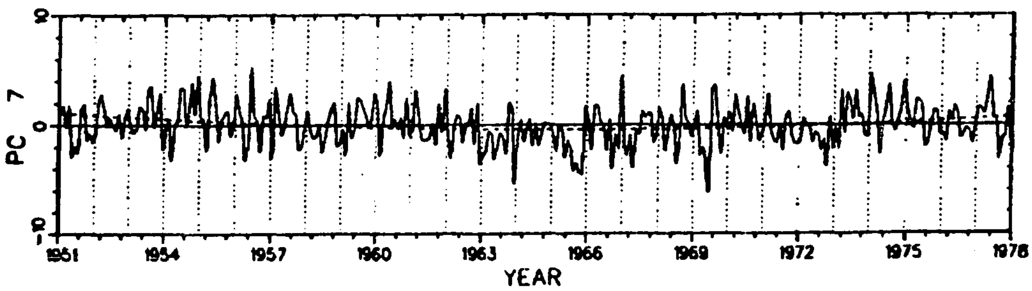
1978
YEAR


PC 7
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0
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1954
1957
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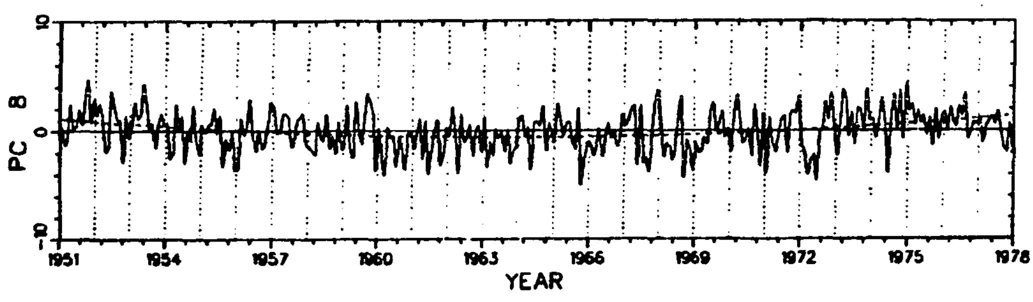
1963
1966
1969
1972
1975
1978
YEAR


PC 8
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1951
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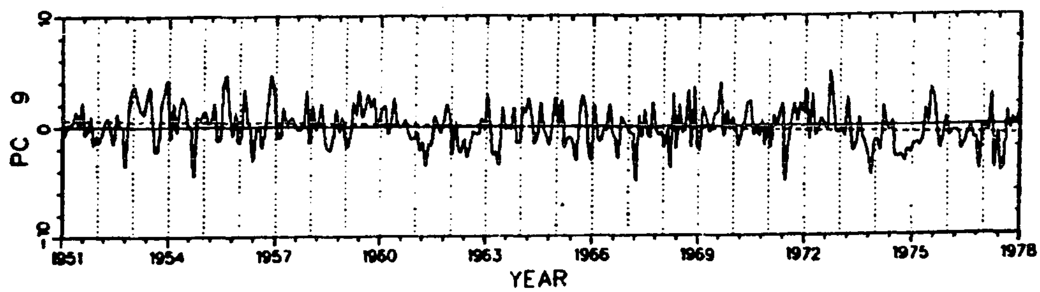
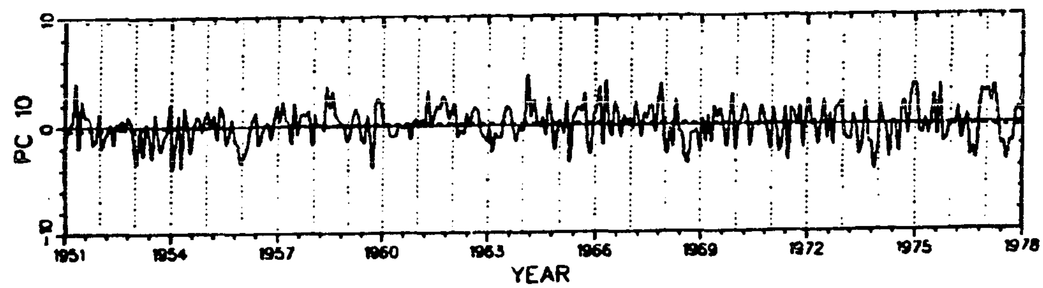
YEAR





make forecasts better (in the expected sense) than climatology. The lag one autocorrelations for the first ten PC's are on the order of 0.20 to 0.34. The PC's are expected to become more contaminated by noise as the number increases. Thus, in Figure 10 the spectra become flatter for the higher numbered PC's.

The AR spectra show the time correlation of the PC's to be much like a simple first order AR. The positive lag one autocorrelation produces a spectrum in which the low frequencies have more of the variance than high frequencies. In addition, these spectra show that the annual cycle has been removed, since the variance density at a frequency of 1/12 cycles/month is not significantly higher than at adjacent frequencies. Thus, when the PC's are modeled with a MVAR, the explained variance will give a measure of the predictability of the standardized height anomalies are from their own past.

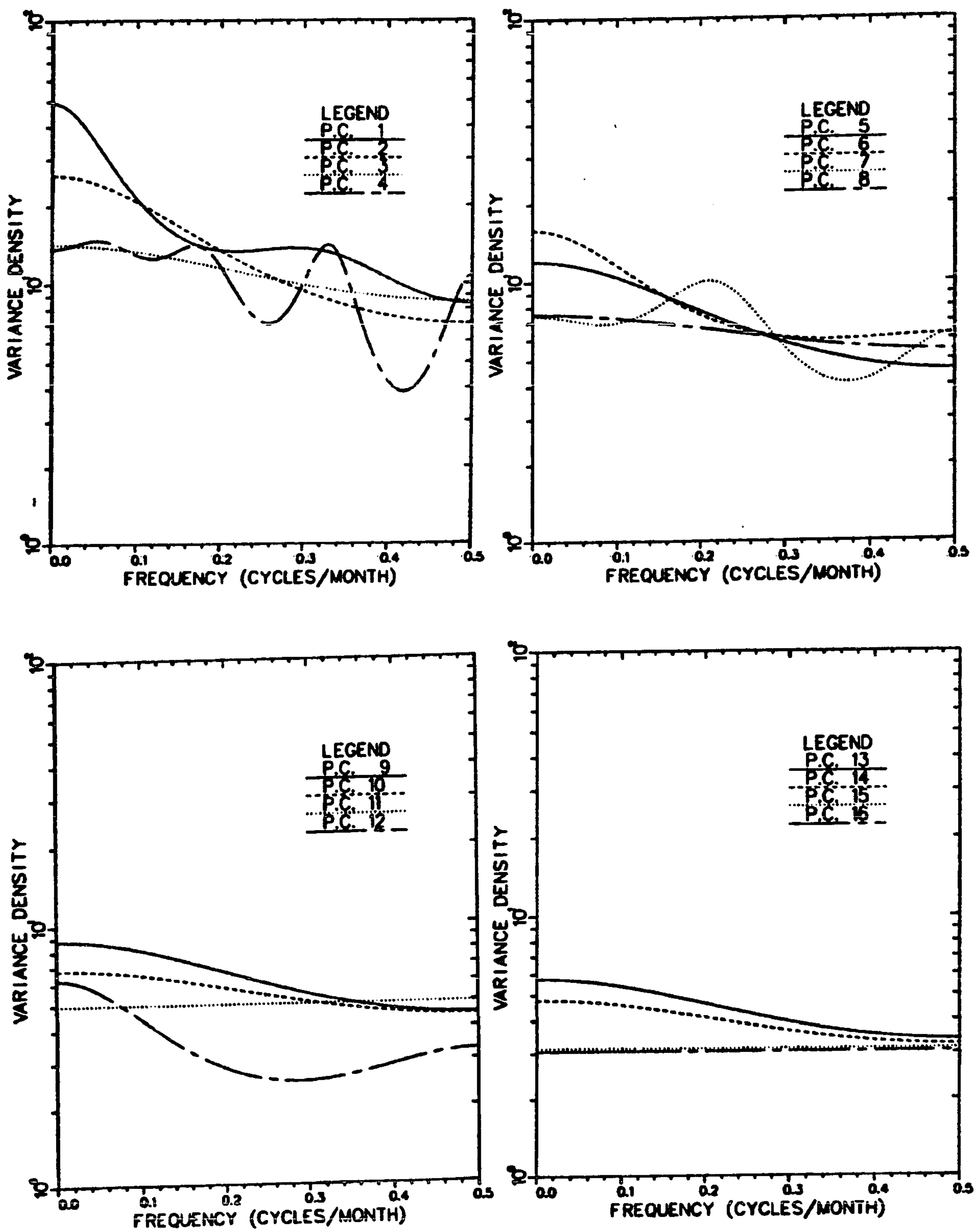
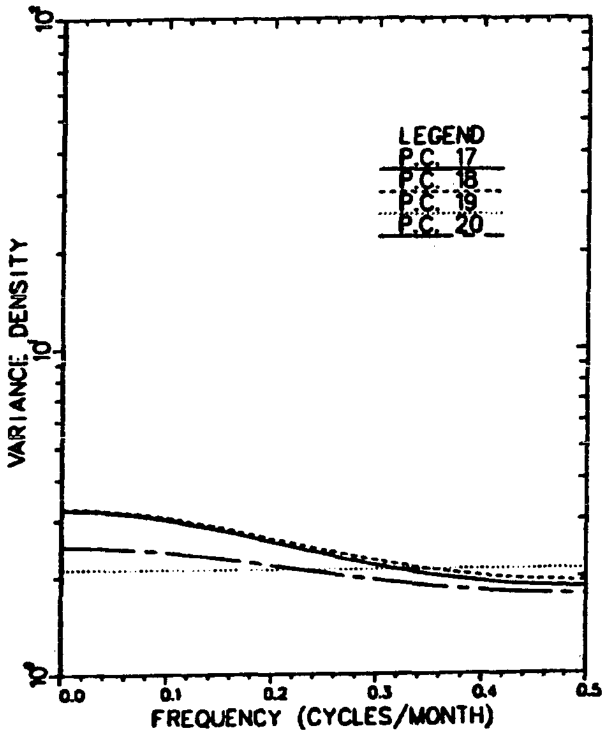


Figure 10. AR spectral estimates for the first twenty detrended PC's.



Analysis of the Climatic Series by Month

The possibility of an annual variation of the correlation structure of the 700 mb height anomalies has been mentioned in the previous chapter. This annual variation can be examined by performing separate analyses for the twelve months. Thus, the data matrix Y_{NM} contains only data for, say, the 27 Januaries. Examination of the first eigenvector for each month will then show how the correlation structure varies over the course of a year.

Twelve separate PCA's were done, one for each month of the year. The log-eigenvalues are presented in Figure 11. For the separate months a smaller number of components account for a given percentage of variance. For January, for example, nine components explain 75% of the variance while twenty were needed when the full data set was analyzed.

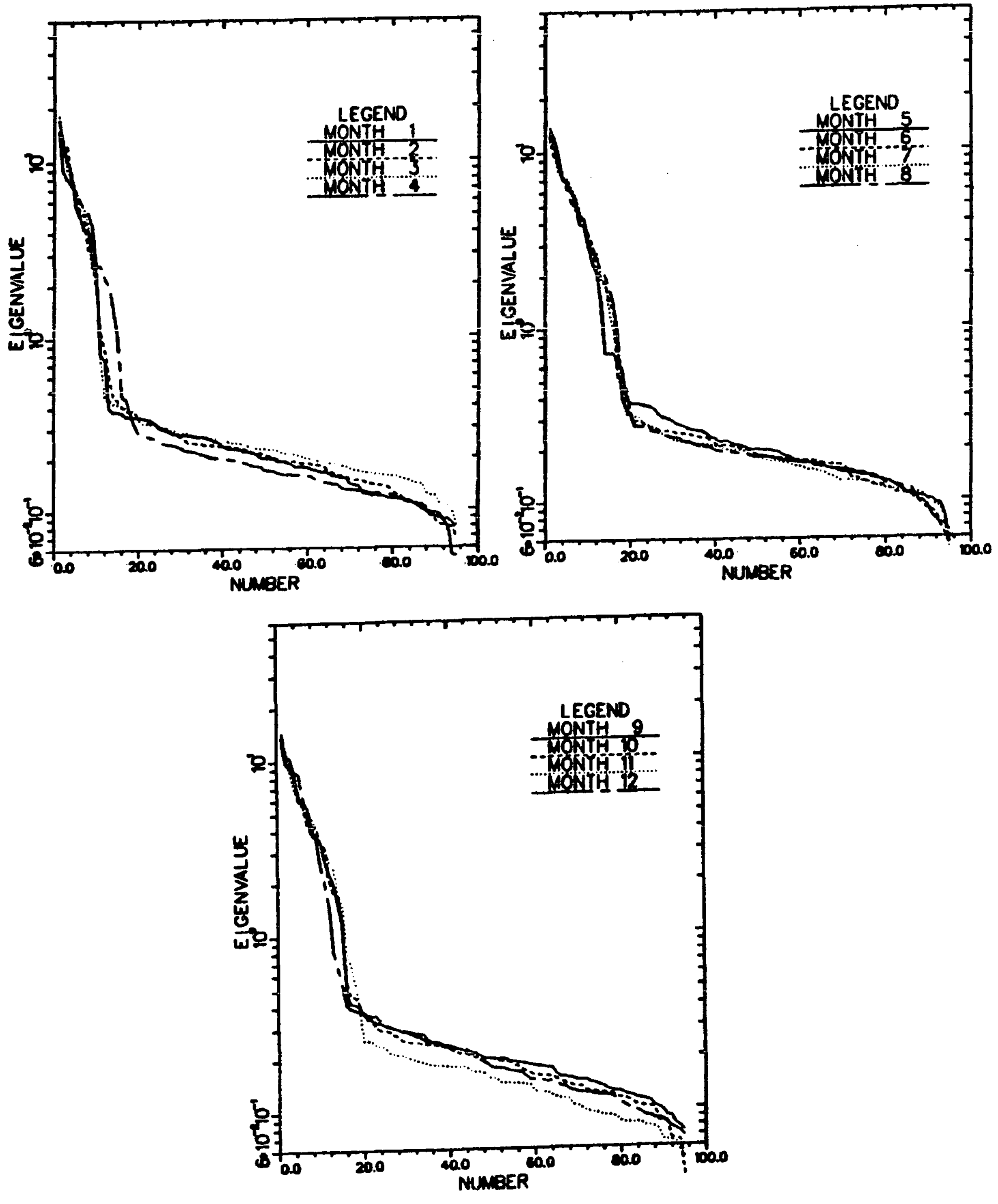


Figure 11. LEV diagrams for the monthly principal components analyses. Eigenvalues above 27 are theoretically zero.

In general, more components are needed to explain a given percentage of the variance for the summer months than for the winter months. For June, twelve components explain 75% of the variance. The first eigenvectors for January, April, July, and October are presented in Figure 12. The changes between these months are fairly systematic, so that these four give a reasonable description of the annual variation of the first eigenvector. For January, the first eigenvector is very similar to the first eigenvector for the full data set. In addition, it is almost identical to the Namias (1980) teleconnection chart for January for (40°N, 140°W). For both January and April the basic pattern has four waves. The July pattern is much less organized than previous months; by October the basic pattern is back to four waves and is more organized than for the summer months.

From this study of the annual variation in the correlation structure of the height anomalies two features of the analysis of the full data set can be explained. First, a relatively large number of eigenvectors is needed to reasonably represent the data. This results because more eigenvectors are required to represent both the winter type pattern and the smaller scale summer type pattern. Second, the winter type pattern is the first eigenvector in the analysis of the full data set because it predominates over a large portion of the year (October through April) and because its features are larger scale than that of the predominant summer pattern.

Relationship of Teleconnections and Eigenvectors

The similarity between teleconnection charts and the eigenvectors has been noted twice previously. A teleconnection chart is made

700 MB ANOMALY EIGENVECTOR 1 FOR JANUARY

700 MB ANOMALY EIGENVECTOR 1 FOR APRIL

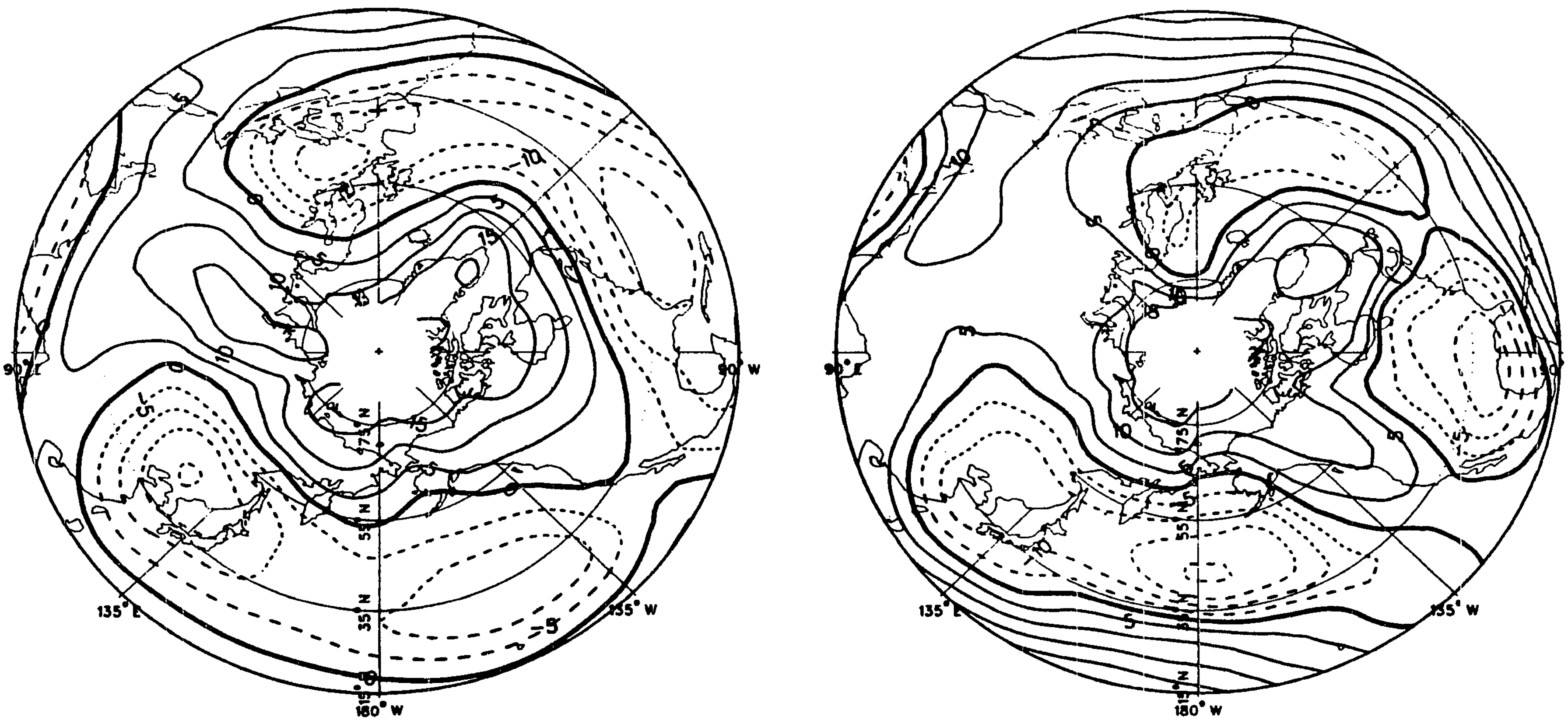
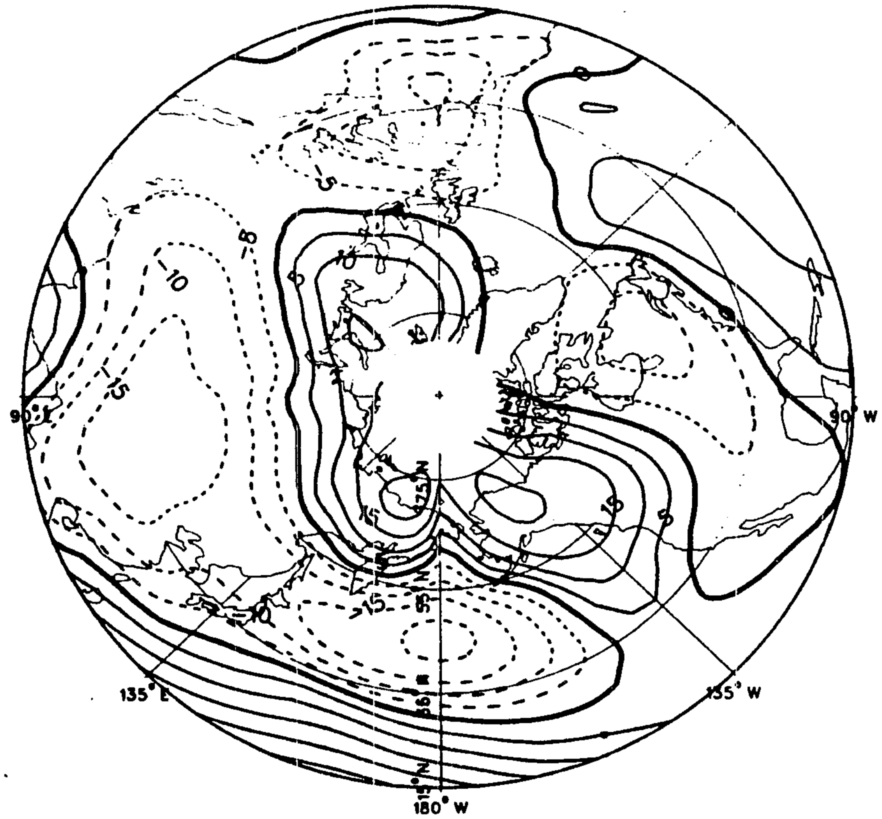


Figure 12. First eigenvectors for January, April, July, and October from the monthly principal component analyses. The contouring interval is 5; values are scaled by 100.

700 MB ANOMALY EIGENVECTOR 1 FOR JULY

700 MB ANOMALY EIGENVECTOR 1 FOR OCTOBER



by computing the correlation between the height anomaly at a given grid location and the anomalies at all the other locations in the grid. Therefore, it represents the tendency of the other grid points to covary with the given grid point. Such charts for January and July are presented in Figure 13, where the correlations are with respect to (40°N, 140°W). This January chart is nearly identical to that by Namias (1980). If the reference grid point for a teleconnection chart is highly weighted in the first eigenvector and the first eigenvector has a substantial fraction of the variance, then the teleconnection chart will resemble the eigenvector as in the charts in Figures 12 and 13 for January. However, if the reference point is not signficantly weighted in the first eigenvector, then the teleconnection chart is not likely to resemble the eigenvector, as in the charts in Figures 12 and 13 for July. Thus, the first eigenvector is easier to apply than a teleconnection chart since it is not referenced to any particular grid point.

In summary, it has been concluded that about twenty eigenvectors are needed to represent the standardized height anomalies. While the correlation structure does change over the course of the year, the change is not great enough to justify the additional complication of monthly eigenvectors. Hereafter, the standardized height anomalies will be represented by twenty PC's, a reduction in the number of random variables of about 80%.

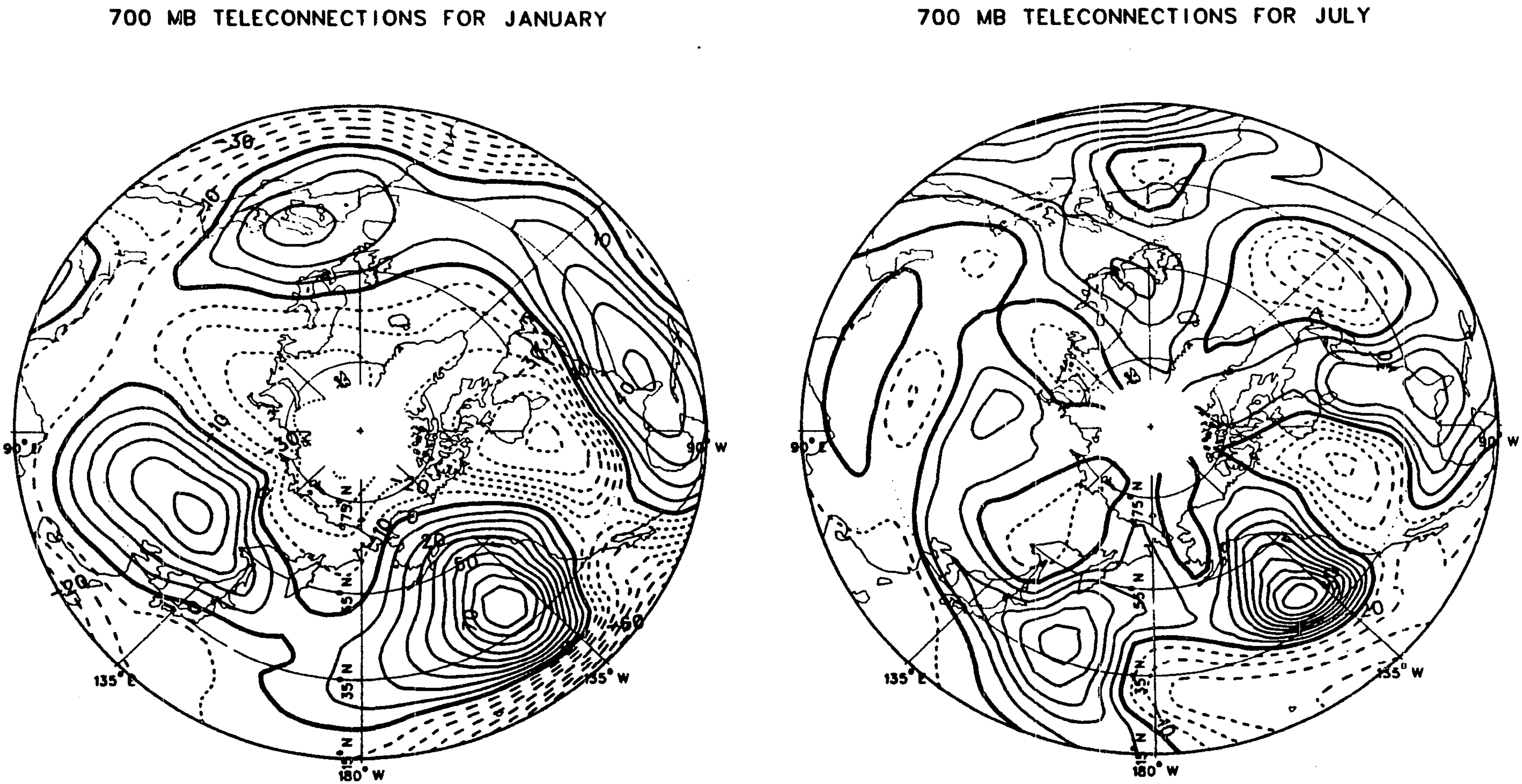


Figure 13. Teleconnection charts for (40°N, 140°W). The contouring interval is 10; values are scaled by 100.

CHAPTER VI

MULTIVARIATE AUTOREGRESSIVE MODELING AND FORECASTING

It was noted by Namias (1953) that the standardized height anomalies possess some month to month continuity, corroborated in the previous chapter by the nonzero autocorrelations of the time series of PC's. Such temporal correlations can be used to improve forecasts of the standardized height anomaly over those made by climatology. In this chapter, an MVAR model will be fitted to the twenty PC's representing the standardized height anomalies. The fitted model parameters (i.e. the order and autoregression coefficients) will be interpreted as to the nature of atmospheric dynamics operating on the monthly time scale, the effects of using more or less than twenty PC's will be discussed, and forecasts will be made using MVAR and its error statistics examined both for single step and multistep forecasts.

MVAR Model Fitting

Prior to modeling the twenty PC's, the time correlations were examined for a systematic annual variation. This was done by computing the correlation of each of the twelve months with the previous month. The hypothesis is that all twelve estimated correlations are actually equal to the same value, estimated by the average $\bar{r}$. The Fisher transformation described by Afifi and Azen (1972, p. 103) was used to transform the estimated correlations to N(0,1) random variables, presented in Table 2. Of the 20x12 = 240 values, 12 are expected to exceed 1.96 in absolute value (5%) and 2 to exceed 3.0 (1%). In Table 2, 13 values exceed 1.96 and 2 exceed 3.0. Thus there is no significant annual variation of the autocorrelations of the twenty principal components.

While the PC's are not cross correlated for zero lag, they may be cross correlated for nonzero lags. Because of this possibility, the multivariate model is used. The superiority of MVAR over univariate AR's will be shown later by a comparison of residual variances.

The computer programs for fitting a MVAR model are available in the TIMSAC (Time Series Analysis and Control) package described by Akaike et. al (1979). The MULMAR program, selected for use here, fits a MVAR model with order selection based on the minimum AIC (Akaike Information Criterion) method. This criterion is a minimization of the one step prediction error variance where the error variance includes that due to the residual variance and to the estimation errors of the autoregression coefficients from a finite sample of data.

The MULMAR program was applied to the 319 values of each of the twenty PC's. A third order model with the coefficients in Appendix B was selected. The residuals from this model, computed by (5), were tested for autocorrelation by using the Q-statistic of Box and Jenkins (op. cit., p. 291). The tests showed no evidence of model inadequacy, that is, the residuals are not significantly different from a completely

Table 2. Values (scaled by 100) of the Fisher transformation of the correlation for each of the twelve months with the previous month.

	Month												$\bar{r}$
P.C.	1	2	3	4	5	6	7	8	9	10	11	12	MEAN
1	25	28	55	-196	-39	187	-67	39	-8	83	-6	-39	.35
2	-271	18	-27	-92	-19	49	26	139	187	1	-12	83	.34
3	-90	31	153	6	33	113	-138	-108	-47	3	-53	120	.11
4	-94	207	-63	-27	-139	-27	114	-58	-63	9	115	73	.17
5	-154	-45	-63	-34	84	-106	230	317	36	34	-104	-9	.41
6	-213	163	-102	153	-86	21	-103	75	120	-10	16	21	.20
7	55	4	-158	-43	175	111	-44	99	51	-11	12	-203	.21
8	-190	-117	250	124	-42	-30	-17	16	-97	34	85	25	.12
9	-94	-48	-92	-123	50	18	26	130	18	-4	12	140	.21
10	-48	84	28	-141	-285	-19	2	189	86	-210	258	97	.10
11	-165	-21	55	91	165	-48	95	103	2	35	-89	-209	.09
12	-188	-78	67	-105	94	136	5	93	40	29	-84	53	.32
13	-107	-35	31	-100	36	-120	-11	160	-63	-52	155	143	.12
14	-46	-364	52	-35	14	27	9	113	86	104	-43	55	.08
15	236	61	-42	77	-46	-178	-25	20	48	-27	68	-172	.06
16	-79	38	39	-85	37	17	-54	41	36	107	38	-136	.03
17	55	88	139	11	43	114	-27	-119	-50	-218	26	-25	.21
18	26	-90	-3	-64	125	36	-133	32	-20	190	-26	-38	.16
19	-43	83	-99	0	194	20	-19	60	-62	-23	-7	-82	.11
20	-169	109	-96	-123	-117	-43	90	116	0	156	23	72	.07

random time series. In addition, since this test is for linear predictability, another test suggested by Granger (1976) for one type of nonlinear predictability was applied. This test of the randomness of the squares of the residuals gave no indication of this type of nonlinear predictability. Hence, the third order MVAR model with the coefficients in Appendix B was accepted as an adequate model of the twenty PC's.

The sum of the variances of the residuals $\hat{Z}_L(t)$ is an estimate of the total variance of the innovations to the model. This sum is given by the trace of the $\hat{V}_{LL}$ matrix and is 56.6. The trace of the modeled output variance matrix is 67.1, so the variance "explained" by the model is [(67.1-56.6)/67.1] = 16%. (Over all 95 components the variance explained is about 9%, if the PC's above twenty are unpredictable.) For a one month forecast the MVAR forecast error variance is then expected to be about 16% smaller than for a climatological forecast, if the PC's above twenty are assumed to represent noise that should not be forecast.

The difference between the original and residual variance, the explained variance, for each of the individual PC's is presented in Table 3. As can be seen, the explained variance is appreciable only for the first ten or fifteen components. As is expected, the high numbered PC's represent smaller scale features that are less predictable in time. Thus, twenty PC's appears to be the point of diminishing returns, in terms of prediction. For this reason, and because of the computational expense (20 min. CPU on a Univac 1100/10), no modeling was done with more than twenty PC's.

Table 3. Explained variances for the twenty individual PC's based on the MVAR model. The percent reduction is the explained variance divided by the original variance.

Principal Component Number	Explained Variance	Percent Reduction
1	1.63	18
2	2.06	28
3	0.22	4
4	1.43	27
5	1.86	38
6	0.54	13
7	0.59	16
8	0.24	7
9	0.58	17
10	0.18	6
11	0.21	8
12	0.71	31
13	0.18	8
14	0.35	17
15	0.13	8
16	0.03	2
17	0.10	8
18	0.07	6
19	0.06	6
20	0.09	9

The residual variance from the MVAR model was compared to the residual variance from univariate AR models. The sum of residual variances for the univariate AR models is 60.4, compared to 56.6 for the MVAR, so the MVAR explains about 50% more variance than univariate AR's.

The MVAR model can be interpreted in terms of the atmospheric dynamics of the monthly time scale. This can be done through the autospectra of the PC's. The autospectra of the PC's from the MVAR, computed by the method by Jones (1974), do not differ greatly from the AR spectra shown in Figure 10. (The MVAR spectrum of the first PC is shown in Figure 14.) Most of these spectra have more variance in the lower frequencies than at higher frequencies, agreeing with the Namias (1953) observation of some month to month continuity of the anomalies.

Recall from Chapter 2 that the simple decay system could be modeled by an AR of order one while higher order AR processes allowed for the possibility of damped oscillation. The autospectra for the first PC from the MVAR and for a simple first order AR are shown in Figure 14. The variance density for the first PC is distributed more towards low frequencies than a simple first order AR. If the assumption is made that the atmosphere itself behaves as a first order system (Madden, 1977), then air-sea influences where the ocean has a much longer time constant, would produce a spectrum like that of the first PC. Obviously, this is not proof of air-sea interaction. The point here is that the MVAR model would not contradict such a hypothesis.

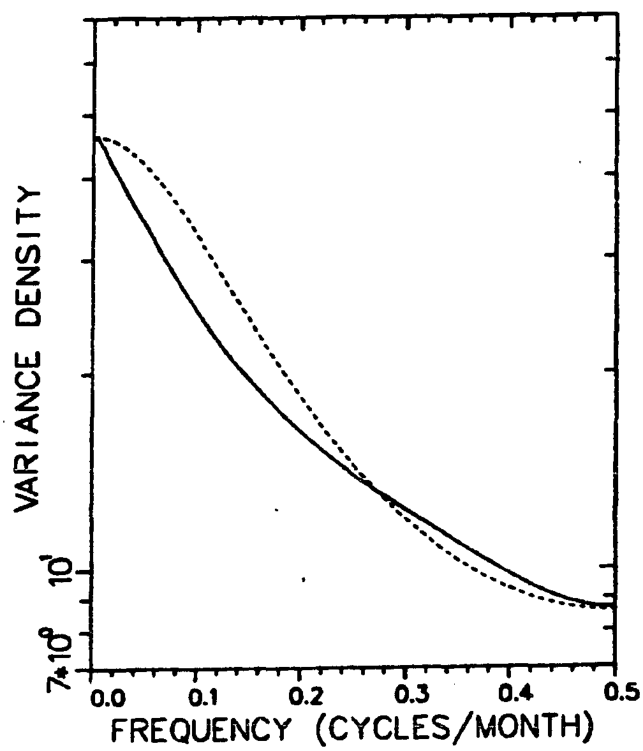


Figure 14. Comparison of the spectrum of the first PC with the spectrum of a first order AR (dashed). The first order AR spectrum coincides at zero and the Nyquist frequencies.

Typical examples of the coherence spectra are presented in Figure 15 for the pairs of PC's (1, 2) and (1, 5). The coherence is a measure of the correlation between two time series as a function of frequency. The existence of coherence between the PC's results in the higher explained variance for the MVAR. However, the physical interpretation of the coherence is not obvious.

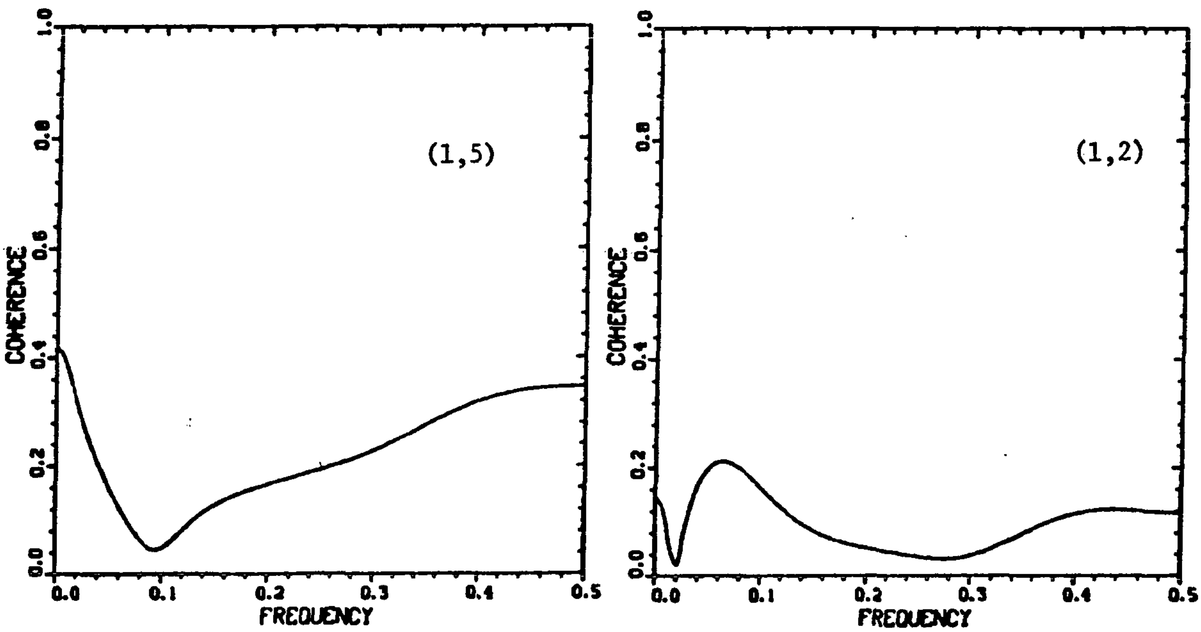


Figure 15. Coherence spectra from the MVAR model for the pair of principal components (1, 5) and (1, 2). Frequency is in cycles/month.

Forecasting With the MVAR Model

Forecasts were generated using (7) for the twenty PC's using the third order MVAR with the coefficients given in Appendix B. The purposes for studying the forecasts were to verify the computer programs, to test the forecasts on the 5 independent samples, and to examine multi-step forecasts for retrogression of features.

One month MVAR forecasts of the twenty PC's were generated for all months having verifications and data for forecasting (some data were missing for August 1954) and were compared to forecasts made from climatology and from persistence. The climatological forecast of a PC is zero, since all the climatology has been removed. The persistence forecast for the PC's was, for convenience, defined as the previous month's values. The appropriate expressions for the squares of the forecast errors are

$$e_C(t) = \sum_{j=1}^{L} P_j^2(t) \tag{8}$$

$$e_P(t) = \sum_{j=1}^{L} [P_j(t) - P_j(t-1)]^2 \tag{9}$$

$$e_M(t) = \sum_{j=1}^{L} [P_j(t) - \hat{P}_j(t-1+1)]^2 \tag{10}$$

for climatology, persistence, and MVAR, respectively, where $\hat{P}_j(t-1+1)$ indicates a one step forecast computed by (7) from time t-1. The average values (over the available data), $\overline{e}_C$, $\overline{e}_P$, and $\overline{e}_M$, are 67.1, 103.1, and 56.9. Thus, the forecast error for the MVAR is about 15% [(67.1 - 56.9)/67.1] lower than climatology, agreeing with the result from the MVAR model fitting. The large persistence forecast error variance, 103.1, is expected from the discussion of Chapter II since the lag one autocorrelations are about 0.2 to 0.3. For this case, $\overline{e}_P = 2\overline{e}_C(1-a) \simeq 2x67.1x(1-0.23) = 103.1$.

Sample values of e_C and e_M are presented in Figure 16 for January 1973 to December 1977. The error for MVAR is generally lower than climatology, but is on occasion higher (for example, May 1975).

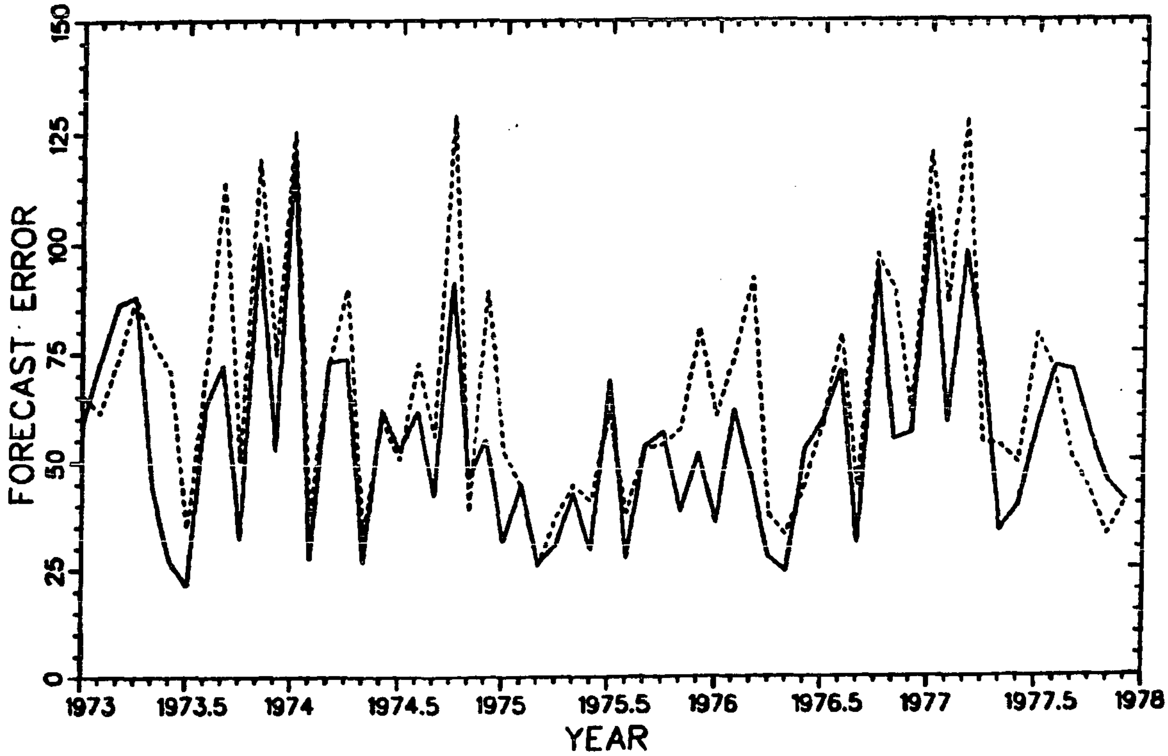


Figure 16. One step forecast errors for the MVAR (solid) and climatology (dashed) for January 1973 to December 1977.

Only the last five values are for the independent samples. Here the MVAR is consistently higher. The entire record (not shown) of e_C and e_M was examined. Similar events (i.e., $e_M > e_C$) were found to occur in the spring of 1965, the fall of 1970, and the winter of 1972-73. Thus, the large MVAR errors are more likely due to chance events rather than nonstationarity of the climatic series. The 700 mb anomalies for the period September to December 1977 were apparently influenced by an unusual North Pacific sea surface temperature (SST) pattern (Namias, 1979). The winter of 1972-73 event was also studied. Namias (1974) presents the 700 mb height and SST anomalies for September 1972 to August 1973. The MVAR fails for December 1972 and January 1973, following the development of a North Pacific SST anomaly in October. Since the MVAR has no direct knowledge of SST anomalies, it must apparently have to wait until the SST anomaly has sufficiently influenced the atmosphere before forecasts again become better than climatology. A one step forecast and verification for September 1977 are included in Figure 17. The forecast height anomalies were reconstructed using (3), giving values at the 95 sparse grid points. These forecast values were interpolated to a regular grid and then computer contoured by the same program used to display eigenvectors in Chapter V. As was indicated by the forecast error statistics, the MVAR forecast is poor.

Namias (1953) observed retrogression of the features of the standardized height anomalies. The MVAR model can be checked for retrogression of features by making multistep forecasts. One month through five month forecasts, shown in Figure 18, were made using (7) for October 1976 through February 1977. This period was studied by Namias (1978) who found retrogression of a trough off the west coast and a Northern Plains ridge from the summer to the fall. Verification anomalies are included in Figure 19.

Comparison of the forecast and verification for October shows that the MVAR performed fairly well, except at 90°E. Comparing the forecasts for October and November, it can be seen that the positive anomaly at (50°N, 90°E) has retrograded to (60°N, 80°E). However, the negative anomaly over the U.S. east coast has moved eastward about 20°. Thus, the MVAR does move features longitudinally. However, examination of other forecasts showed no preference for retrogression. After the second step forecast, the MVAR generally holds features in place and damps them out, as can be seen in Figure 18.

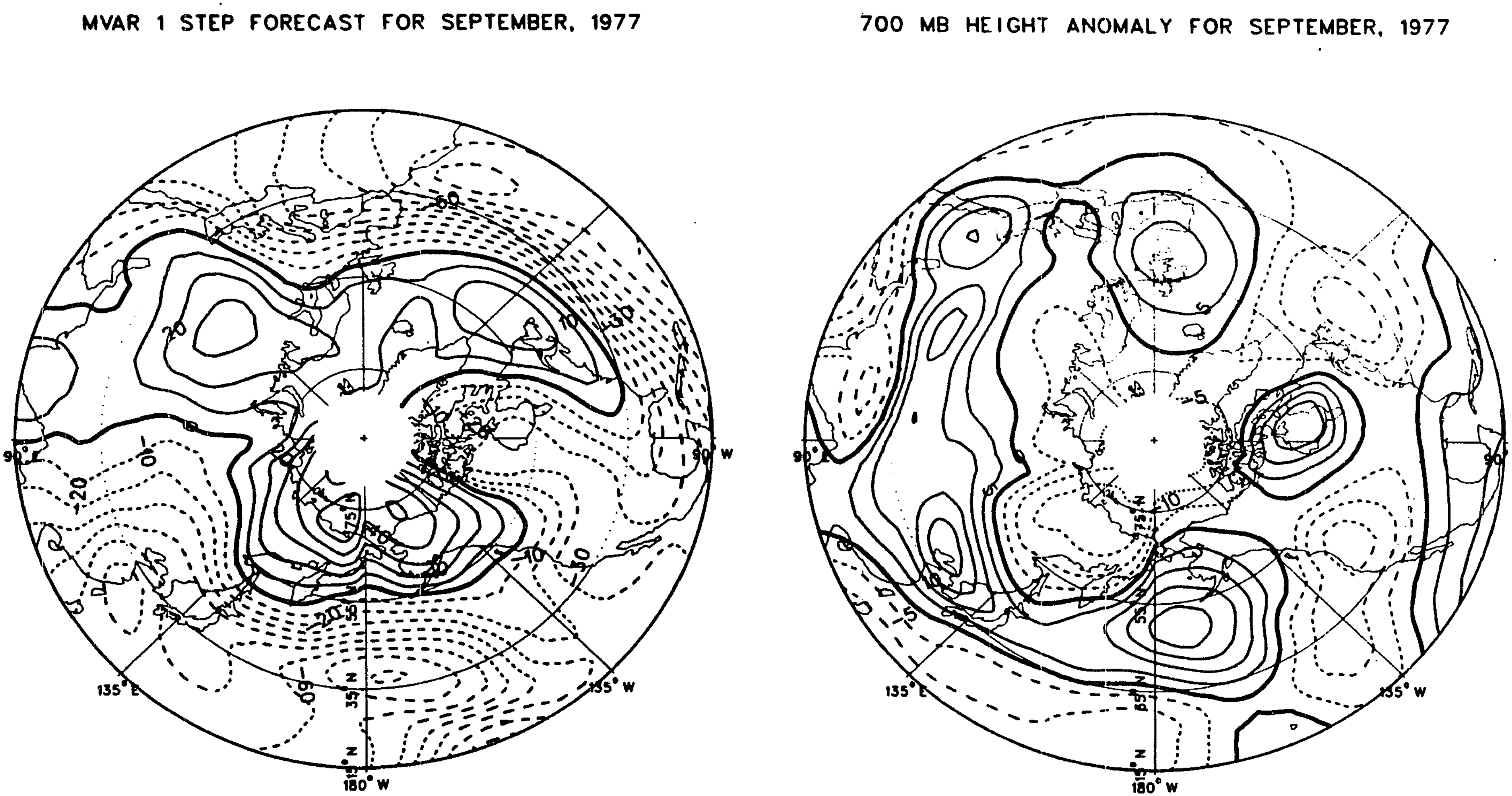


Figure 17. One step MVAR forecast and verification for September, 1977. Forecast values are scaled by 100; anomaly values by 10.

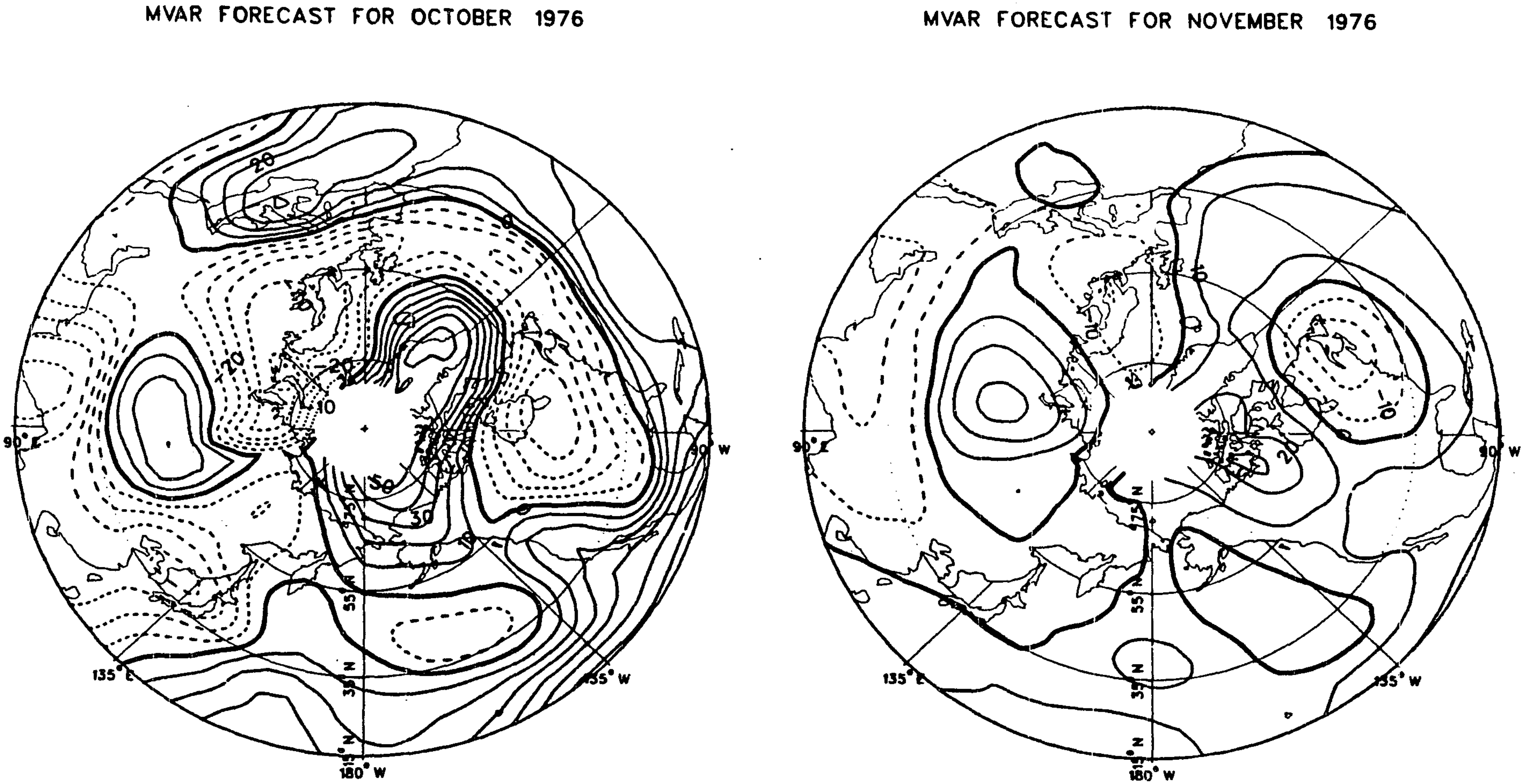
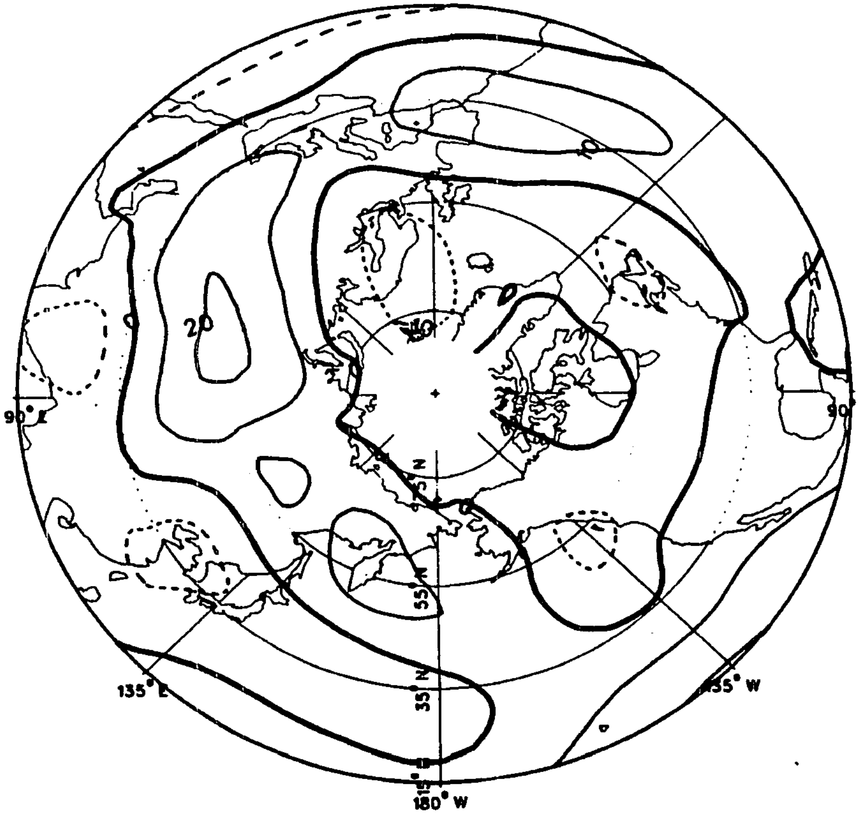
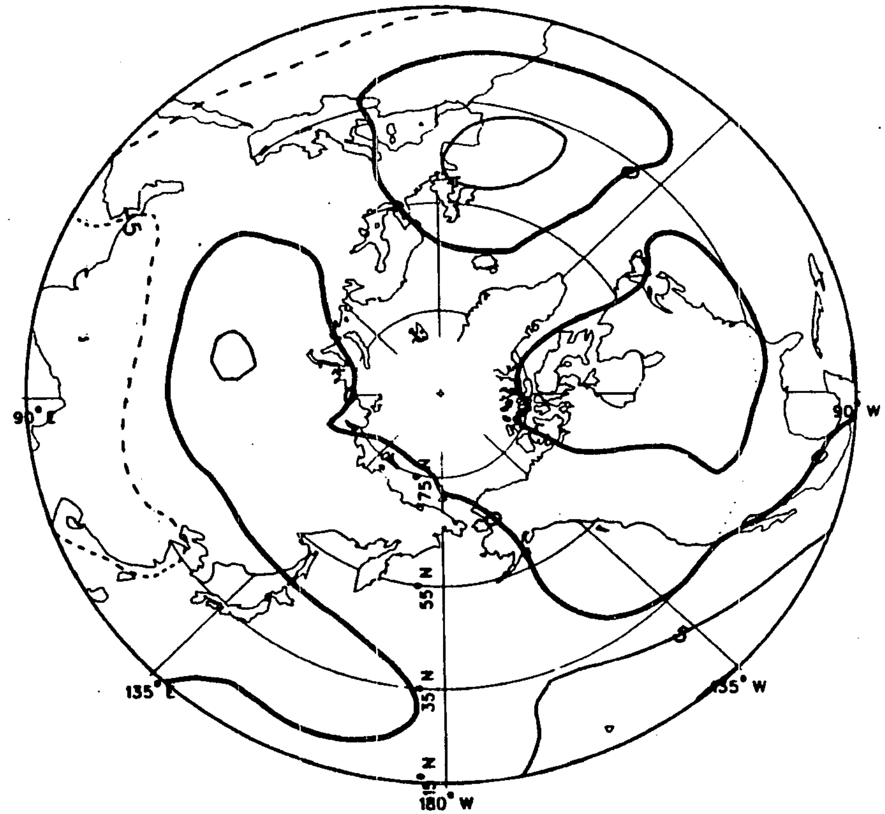


Figure 18. MVAR multi-step forecasts for October 1976 to February 1977 using September as the current month. Values are scaled by 100; the contouring interval is 10 for October to December and 5 for January and February.

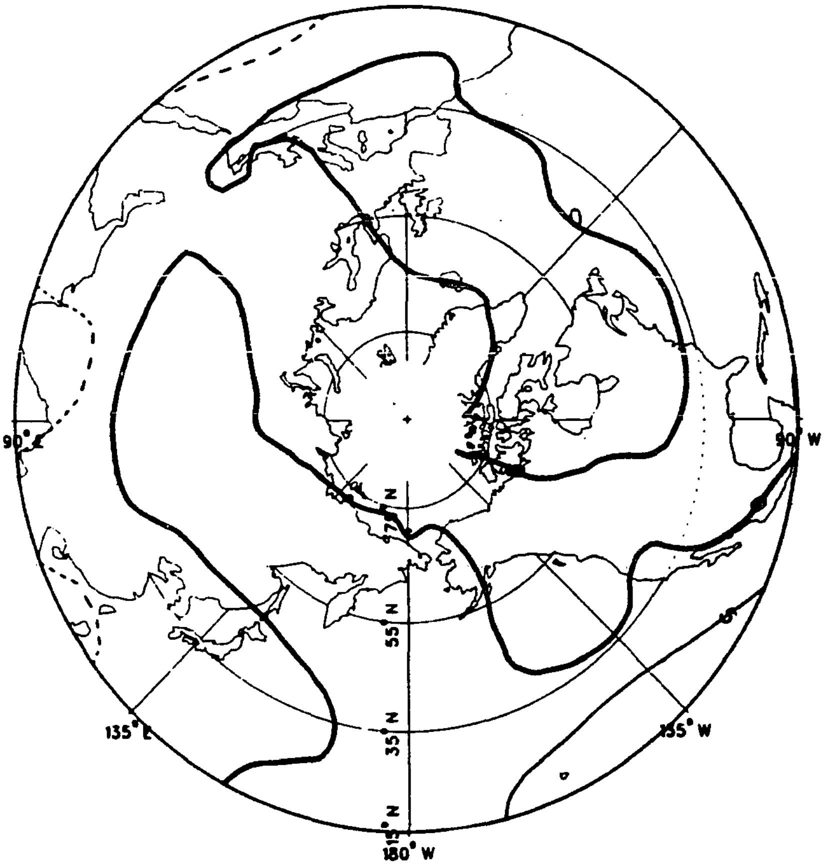
MVAR FORECAST FOR DECEMBER 1976



MVAR FORECAST FOR JANUARY 1977



MVAR FORECAST FOR FEBRUARY 1977



The error variance of a forecast can be estimated using the MVAR model. This has essentially been done for the one step forecast. The theory for the variance computation is presented by Jones (1964b). The forecast error variances so computed are presented in Table 4 for steps 1-5. Since the correlation decreases less rapidly than for a first order process, there is less forecast error at steps 2 and 3 than would be expected from the 16% explained variance for a one step forecast. This is expected from the greater low frequency variance in the PC's than for a simple first order process.

700 MB HEIGHT ANOMALY FOR OCTOBER 1976

700 MB HEIGHT ANOMALY FOR NOVEMBER 1976

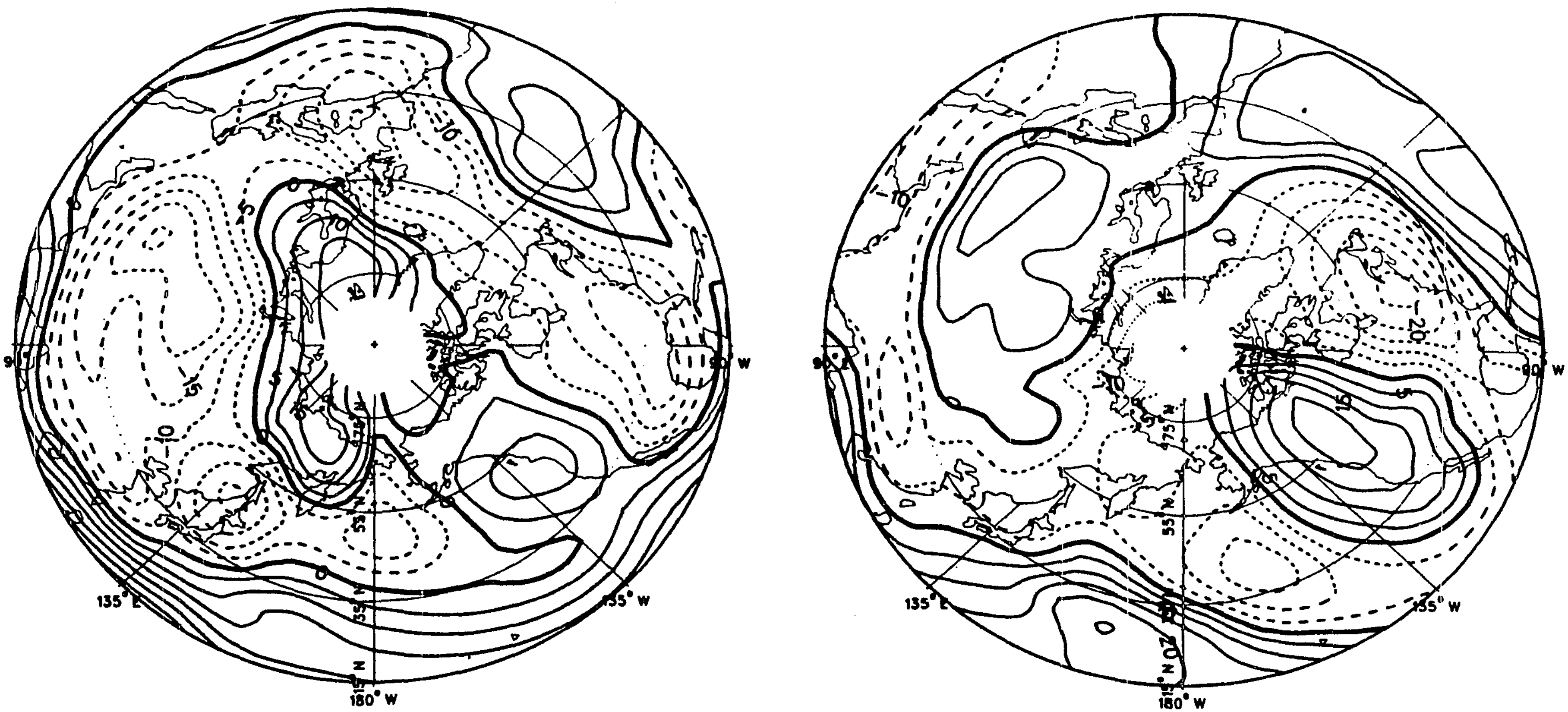
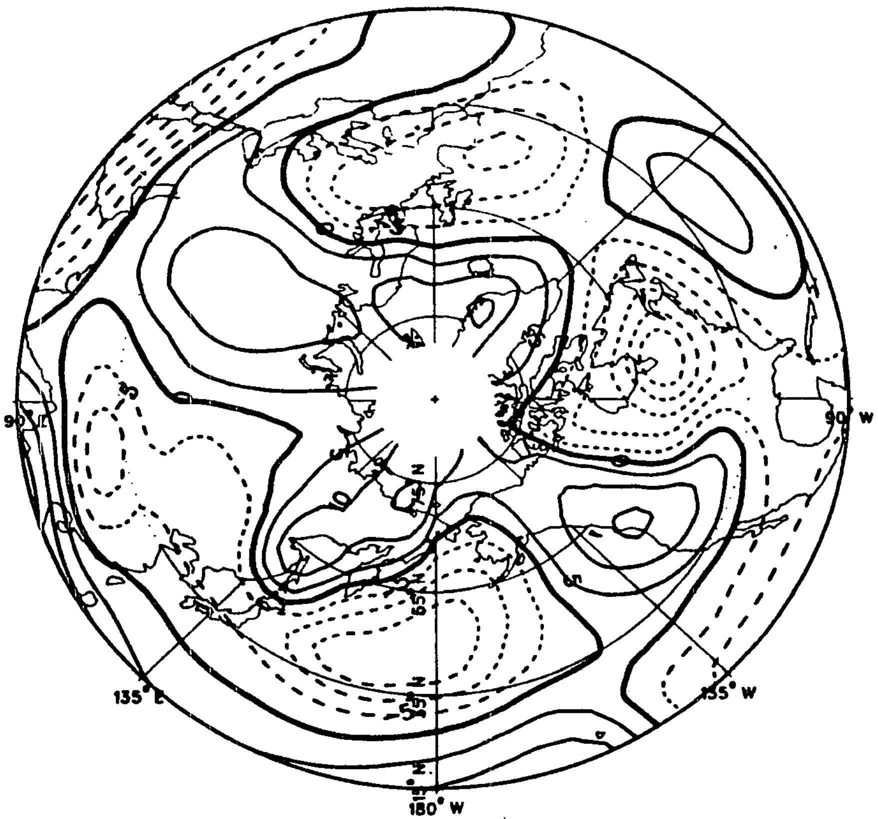
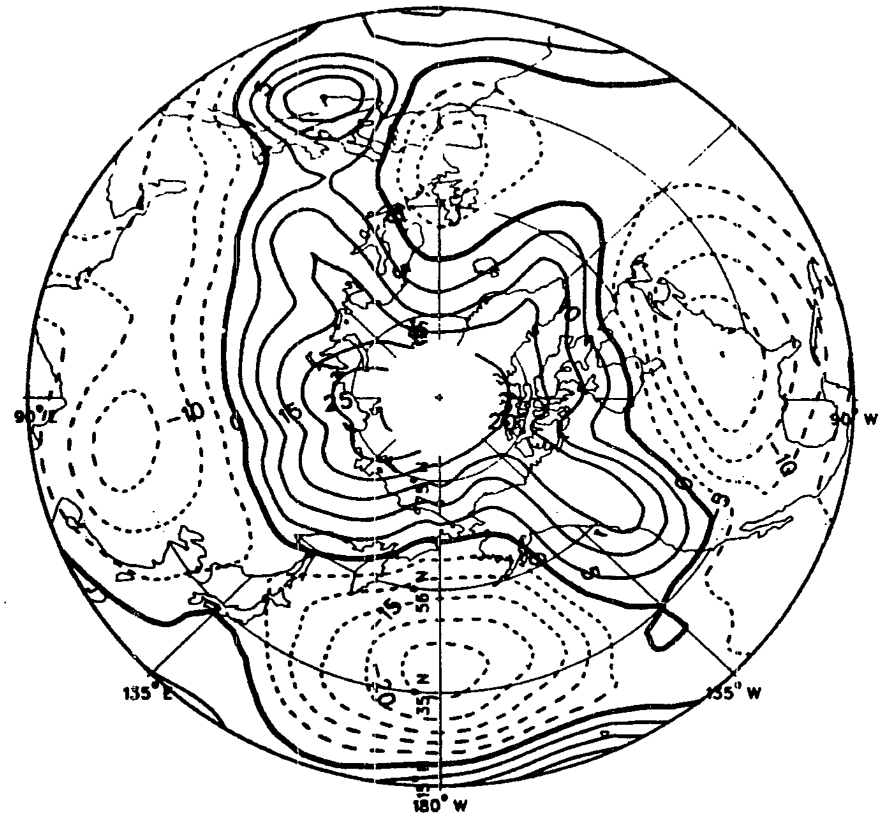


Figure 19. Verifications for the multi-step MVAR forecasts for October 1976 to February 1977. Values are scaled by 10; the contouring interval is 5.

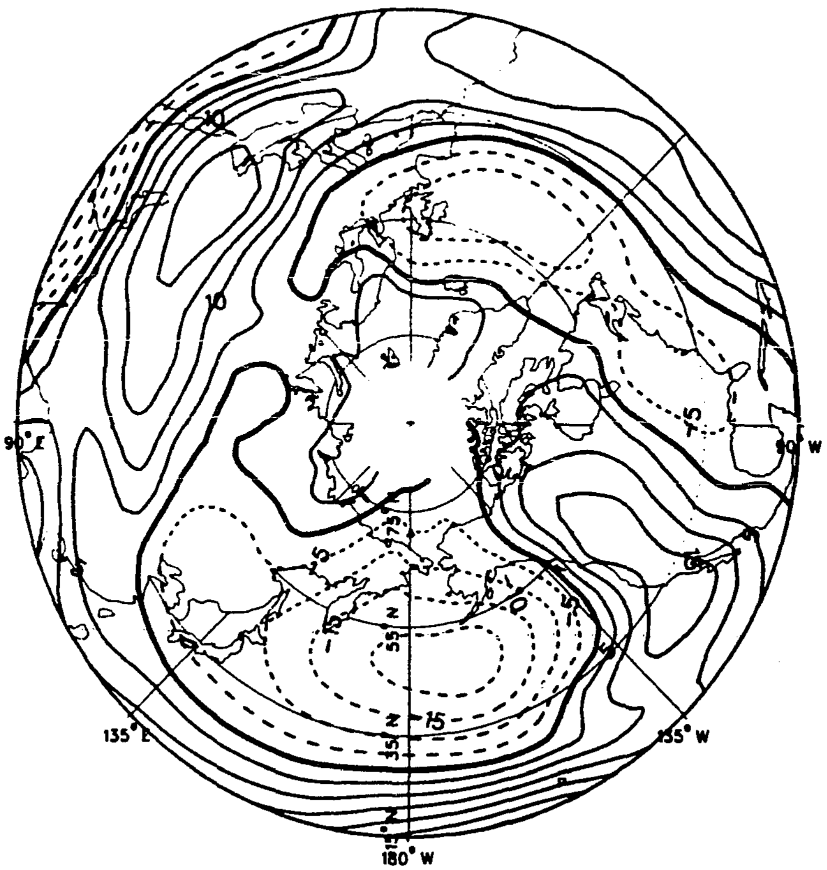
700 MB HEIGHT ANOMALY FOR DECEMBER 1976



700 MB HEIGHT ANOMALY FOR JANUARY 1977



700 MB HEIGHT ANOMALY FOR FEBRUARY 1977



In summary, MVAR modeling has been applied to the twenty PC's representing the larger scale features of the standardized height anomalies. A third order model was objectively selected. Tests applied to the residuals from this model show no indication of predictability. The model explains 16% of the variance and thus can make better forecasts than climatology for 1 to 3 months. Beyond 3 months, the MVAR

Table 4. Forecast error variance and percent explained variance for forecast Step 1-5 for the twenty PC's.

Forecast Step	Forecast Error	Percent Explained
1	56.7	15.6
2	62.3	7.3
3	64.6	3.9
4	65.7	2.2
5	66.7	1.6

forecast is virtually identical to a climatological one. A comparison of multivariate and univariate models showed appreciably more variance explained by the multvariate model. The MVAR model was seen to move features of the anomaly field, but without preference toward retrograding them.

CHAPTER VII

SUMMARY AND CONCLUSIONS

A statistical forecasting method based on the Weiner-Kolmogorov theory has been applied to a climatic series of monthly averaged 700 mb heights. The deterministic portion (i.e., the annual cycle) was removed and the anomalies standardized. The stochastic portion of the series was thus represented by the standardized height anomalies. The statistical forecasting was then performed on the stochastic portion of the climatic series.

Principal component analysis has been used to efficiently represent the height anomalies. Twenty principal components were found to reasonably represent the height anomalies, explaining 70% of the original variance. In addition to analyzing the entire data set, monthly principal component analyses were done, showing an annual variation of the eigenvectors. The first eigenvector for winter was found to have generally larger scale features than that for summer. In addition, a fewer number of principal components is needed to account for a given percentage of the variance for the monthly analyses than for the entire data set.

The relationship of the eigenvectors to teleconnection charts has been discussed. Eigenvectors were found to give a location free description of the intercorrelation structure of the standardized height anomalies and are easier to apply than teleconnections. Multivariate autoregressive modeling was then used to model the time correlations of the twenty principal components representing the entire height anomaly series. A multivariate model was used because the principal components are sufficiently cross correlated at nonzero lags to justify the additional complexity. An objectively selected third order model was found to be an adequate model. That the model is third order indicates that the two previous months provide useful information in making a forecast.

The one step forecast error variance of this model was examined and was found to be about 16% smaller (in the expected sense) than that for a climatological forecast. In addition, MVAR multistep forecast error variances were smaller than those for a first order (simple decay) model. Forecasts of standardized height anomalies showed the ability of the model to move features of the anomaly field, but the model appears to have no distinct preference for retrograding features.

It has thus been found that the large (planetary) scale features of 700 mb height anomalies are somewhat predictable. This is a clear indication of atmospheric predictability on the monthly time scale. Since other variables, such as sea surface temperature, influence 700 mb heights, their incorporation into a statistical model should give added predictability.

Computer programs for principal component analysis and for multivariate autoregressive modeling are readily available for use on most digital computer systems. Thus, multivariate autoregressive forecasting can be implemented with moderate effort for operational forecasting or for research into atmospheric predictability. Because there is an economic need for long range forecasts, statistical methods are a valuable tool for forecasting on the monthly time scale.

The application of statistical forecasting to the 700 mb standardized height anomaly series has been instructive and the statistical techniques have revealed features of the height anomalies that are difficult to observe directly. The encouraging results of this dissertation show that statistical forecasting, properly applied, holds much promise for the climate forecasting problem.

CHAPTER VIII

FURTHER STUDIES

From a practical point of view, the statistical model explains a small portion of the variance of the 700 mb height anomalies. Two cases when the model performs worse than climatological forecasting were studied and attributed to unusual changes in sea surface temperature patterns. Thus, an obvious improvement to the forecasts should be to include sea surface temperatures. This may be a formidable problem since there is evidence of an annual variation in sea-atmosphere correlations (Haney, op.cit.). In addition, studies of cases when the model does not perform well may suggest improvements (i.e., predictors that should be added).

As presented, the MVAR is a statistically adequate model of the variation of the standardized height anomalies. It can therefore be used for studies of atmospheric response as mentioned in Chapter I or for simulating series of 700 mb height anomalies by inputing multivariate random series (Eddy, 1980).

The diagnostic aspect of the stochastic model has been given only a modicum of attention. Much more research can be done on the physical significance, if any, of interactions among the PC's.

Finally, a more efficient representation of the time predictable portion of the height anomalies is possible if the eigenvectors of the lag one covariances are considered. The covariance matrices for lags 1 and -1 could be summed to give a symmetric (though not diagonally dominant matrix) and the eigenvectors of this matrix used. Some preliminary tests with this technique have shown that ten of these "lag one principal components" perform as well for forecasting as the previously described twenty. In addition, their cross correlations at all lags are negligible, so that separate univariate models could be used. This model would be considerably easier to use than the one presented in this dissertation.

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APPENDIX A

The following tables contain the values of the first twenty principal components. The extreme left column gives the month and year and the value in the i-th column is for the i-th principal component, multiplied by 100. Missing values are indicated by ***.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1-51	-23	0	13	-7	3	-11	11	4	-7	-7	16	-3	-1	1	-12	0	3	10	7	1
2-51	-38	26	3	6	38	-9	18	-12	2	2	7	0	-23	24	-5	1	-5	7	16	-17
3-51	36	44	2	46	16	17	-2	-6	2	5	23	6	6	21	-5	-8	-6	-24	5	22
4-51	40	32	6	0	2	22	19	25	5	40	10	33	-23	10	-12	-15	-2	-10	-1	8
5-51	44	-11	6	-16	2	3	-29	2	14	-19	-5	18	0	-10	-4	3	7	-2	4	12
6-51	36	11	-27	24	1	14	-19	4	1	25	9	31	-13	2	4	-2	16	-31	-13	-2
7-51	10	-26	10	6	28	25	-25	20	22	9	12	-13	-19	-5	-13	-10	5	1	5	6
8-51	3	13	5	-7	48	3	14	6	-7	10	26	13	-38	0	4	9	-2	3	-11	6
9-51	46	18	-29	-22	23	13	19	29	-2	6	-6	11	1	-16	-4	-7	2	-18	-21	5
10-51	-20	45	-3	-10	25	-24	-14	47	10	-16	-2	21	-17	3	14	-8	-2	-1	-13	9
11-51	-6	-16	-22	-12	15	-8	-8	8	-16	-13	11	22	-29	4	-27	-14	5	12	-6	-13
12-51	-44	-43	-30	19	15	10	-15	29	-8	12	1	5	-9	0	-19	-13	-14	10	-3	-2
1-52	-11	-5	34	27	42	7	-7	14	-13	-19	13	0	-1	4	-29	15	-3	-9	-5	0
2-52	23	13	34	4	-1	-8	21	26	-5	-9	7	9	10	-6	-2	3	9	-8	-1	4
3-52	17	8	-26	29	-1	-44	28	15	5	-4	-15	14	3	17	-7	-5	5	22	-3	-4
4-52	-15	-32	31	-15	7	-20	15	-18	8	3	-24	16	-9	1	-9	3	-6	-18	-10	-2
5-52	27	-22	6	28	-1	-4	0	-14	-5	-16	1	36	32	-9	14	-38	1	-8	8	-11
6-52	26	-29	8	23	31	-14	7	36	-15	4	-27	4	-5	3	-26	-12	-4	-8	-1	-14
7-52	-8	-38	-13	16	33	59	0	23	13	2	19	20	0	24	19	18	3	-10	-1	-7
8-52	6	-17	-25	7	19	36	-4	10	0	6	19	8	-8	21	-11	-7	5	3	5	12
9-52	43	4	16	10	35	28	11	10	0	-3	16	9	10	18	-16	2	3	-1	-4	10
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11-52	28	21	19	20	16	-34	6	15	14	3	7	31	-13	9	12	-11	18	5	10	-10
12-52	45	-32	45	17	-3	-12	15	-6	33	-19	-1	-1	-13	4	-15	0	11	-18	5	-7
1-53	10	-4	11	-32	-2	26	1	5	36	-33	-1	-1	22	10	-9	-2	3	-6	2	1
2-53	-2	-37	20	-4	0	-3	-7	27	24	1	7	25	31	-19	-3	-7	0	5	0	6
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7-53	12	-47	8	-9	31	31	33	0	1	12	0	-7	-2	6	-8	7	-8	-3	-11	2
8-53	-5	-30	11	17	36	-4	35	7	-22	-11	0	1	-16	-15	30	8	9	-5	-2	-21
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8-54	***	***	***	***	***	***	***	***	***	***	***	***	***	***	***	***	***	***	***	***
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3-58	62	14	29	56	0	-36	-13	-22	5	1	-30	3	-25	5	29	5	-6	16	-3	7
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7-58	36	-28	-38	3	5	-35	-13	15	-22	31	-5	27	-6	14	0	28	1	-3	-3	-9
8-58	30	-51	-13	-6	-4	0	5	-16	-13	10	24	23	4	-8	-11	-4	5	-6	-12	14
9-58	-11	-34	-39	7	-7	-34	14	-5	16	6	-21	5	-13	6	-6	12	7	18	-28	-1
10-58	-11	-25	7	18	17	12	20	-21	-5	2	-8	18	6	-3	13	1	-12	7	-7	0
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2-59	-74	-53	-9	9	11	-23	-27	24	-7	8	-4	21	-15	-8	14	-1	-18	2	-8	21
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4-59	28	-27	-1	-9	28	17	-12	-24	10	10	-14	-22	6	12	-3	28	-4	-3	-5	0
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7-59	-24	4	13	-12	13	-14	22	-13	16	10	2	-6	20	25	8	2	2	-5	-25	4
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1-62	-24	-17	-26	-22	-2	41	-10	-5	-25	20	-7	2	-7	8	8	7	-2	11	-8	-18
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3-62	67	38	12	51	-34	-12	-3	-1	-21	-4	7	-8	6	-1	12	17	11	0	-2	0
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7-74	-3	-13	-2	-17	-21	3	36	10	-29	-26	3	-11	-12	-11	-13	-10	5	9	20	3
8-74	5	-1	-42	-34	-9	-9	2	33	-29	12	-6	-11	-5	28	21	9	-15	-3	14	-11
9-74	6	-4	5	-41	36	-2	-5	0	-27	21	17	-23	3	10	-4	-2	-18	9	0	5
10-74	41	17	44	-18	41	-3	5	37	-33	-10	48	-12	26	6	14	2	27	19	1	0
11-74	-16	11	21	12	-7	-26	15	-7	-21	23	-11	-4	8	-7	13	9	9	-4	10	-14
12-74	-34	-1	21	-1	-32	-13	40	42	-25	37	-2	12	-2	-9	6	-7	7	1	4	20

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1-75	-36	-34	0	-1	-7	3	14	18	-18	35	7	-9	6	6	-9	12	-11	1	-4	-1
2-75	-12	-28	-19	-11	-12	-26	0	25	-16	-8	-16	-4	-3	-15	14	12	-11	-6	-16	-2
3-75	-8	3	-19	-5	-7	0	-3	5	-19	-9	20	1	-20	0	-7	20	-8	12	9	-8
4-75	5	9	-10	19	-6	2	24	18	-11	5	23	-1	-16	-22	6	0	-23	14	7	-5
5-75	-8	30	-15	-4	-26	-10	22	7	15	-10	22	9	-15	5	10	10	-20	-13	0	-5
6-75	-2	24	-14	27	-19	-1	19	14	2	2	29	8	3	0	17	7	-17	-7	9	0
7-75	-11	23	29	-1	-24	27	-20	-3	34	26	0	-20	-7	5	-8	5	19	-10	7	8
8-75	-26	8	-7	30	-17	-2	-16	12	25	-12	6	-5	3	-1	14	7	0	-16	0	8
9-75	-39	2	25	-15	6	16	-1	-13	-7	36	16	8	-2	11	8	-9	-16	-4	3	14
10-75	-26	-6	-30	6	-14	-37	13	21	-21	-17	7	-12	-5	12	4	-16	0	9	-8	5
11-75	-55	-6	-18	11	-11	-9	13	-7	-7	-15	20	3	-5	9	11	17	-3	-12	21	-2
12-75	-43	13	-2	-2	-50	42	-8	13	8	-3	4	20	12	26	-1	-6	-3	14	1	6
1-76	-15	20	20	-10	-26	29	-7	18	-10	1	-14	17	46	1	-10	2	8	3	12	5
2-76	-76	16	-12	-9	11	-13	-15	0	-5	-2	17	-2	3	-5	-9	6	-2	-12	0	4
3-76	-50	55	4	-18	12	-11	13	22	-5	15	17	-8	21	-23	-6	13	12	-20	5	-15
4-76	-15	25	35	1	-27	-3	4	8	-6	-15	2	15	-12	-5	8	8	-2	7	2	4
5-76	-14	11	18	-20	-15	-19	18	3	-23	-3	10	-4	18	7	5	11	-3	1	-4	-14
6-76	-28	14	-5	16	0	1	11	23	-17	9	-9	-19	8	-7	29	19	7	-7	-3	15
7-76	-8	-20	13	-5	-33	-48	-12	13	-6	-31	8	15	-5	-2	-1	9	5	6	1	-1
8-76	-27	-15	53	-1	-16	-2	-8	34	-2	-22	18	-14	-11	8	24	-15	6	12	-21	-4
9-76	30	-6	1	-6	-10	-16	-3	-7	-12	-34	20	32	1	8	-10	2	-1	11	5	-5
10-76	22	15	8	-42	17	-64	-5	0	-12	11	-2	-4	14	0	-1	27	-16	33	-9	10
11-76	15	-25	8	-43	-11	2	-16	8	-41	30	14	20	-2	-26	32	19	-3	8	-17	6
12-76	43	16	-11	-36	3	-8	-1	7	1	28	21	0	2	-16	-13	-13	22	13	-2	-5
1-77	68	52	-13	-35	-4	-23	16	12	-6	30	3	26	-6	-7	-6	-15	-15	5	6	13
2-77	57	-44	18	-42	-13	3	15	-11	-5	19	-3	-12	2	-4	6	-1	8	-3	1	-8
3-77	4	-85	14	37	-4	12	9	4	28	35	20	-29	-6	-4	-10	-9	-4	-16	-7	1
4-77	-21	-7	22	-4	-10	27	24	9	-40	12	5	11	9	10	7	-17	19	12	-10	1
5-77	-12	18	-4	12	-17	-17	44	13	-6	-20	12	-8	-27	-22	5	-3	3	6	4	-9
6-77	3	21	7	-15	2	-7	8	6	-42	-18	29	17	19	13	6	-2	-4	12	-3	9
7-77	-17	41	-17	-11	20	-20	6	19	-39	-35	33	10	5	2	-1	-9	5	-11	14	-1
8-77	27	47	-26	-19	13	5	-31	-1	13	-17	1	-8	-2	1	-8	14	-1	-20	16	-19
9-77	-18	-9	-13	44	-12	22	-12	-16	-4	-17	-2	-15	-11	-19	15	-6	-2	-3	14	-7
10-77	-11	-32	4	-11	3	-22	-10	-19	6	11	-3	9	9	4	-18	-24	-1	-21	20	6
11-77	-5	-2	-9	7	-12	25	16	7	2	14	9	-16	16	19	-8	11	5	-7	12	22
12-77	-8	-1	-38	1	-3	-15	-15	-17	9	10	-16	-2	-12	2	2	-31	-12	1	-5	10

APPENDIX B

The following tables contain the autoregression coefficient matrices for the third order MVAR model for the twenty principal components. The numbers in the extreme left hand column are for the predictand while the numbers in the top row are for the predictor. The values in the table have been multiplied by 100.

AR COEFFICIENTS FOR LAG 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	33	0	12	0	17	0	0	0	0	-14	0	0	0	0	0	0	0	0	0	0
2	-13	33	1	0	2	15	-17	-17	-20	-1	0	-18	0	0	-19	0	0	-24	0	0
3	0	0	13	0	0	-14	0	0	0	0	0	-19	0	0	0	0	0	0	-21	0
4	-7	0	0	22	0	0	0	0	0	0	0	0	0	0	-14	0	17	0	0	0
5	10	-14	-9	0	41	2	-3	6	18	1	0	12	0	0	-3	11	0	11	-16	16
6	0	1	-10	0	-3	23	0	0	8	0	0	-1	0	13	0	0	0	15	1	-1
7	-2	1	0	0	3	1	14	5	-3	0	0	10	0	0	14	0	0	-3	1	-1
8	0	0	0	0	0	13	0	14	0	0	0	8	0	0	0	0	0	0	0	0
9	0	1	0	0	9	12	0	0	18	0	-10	-1	-10	0	0	-1	0	-20	1	-1
10	0	0	0	0	0	-7	-5	0	0	10	-7	0	0	0	0	0	0	0	22	0
11	-5	0	0	0	0	-7	0	11	-5	0	9	0	0	0	0	0	-20	0	0	0
12	-4	3	9	6	9	-3	-1	5	0	0	2	21	1	8	0	-2	-3	-2	-13	12
13	-5	0	0	0	-2	0	0	0	0	-12	0	0	13	0	0	0	0	11	0	0
14	7	-4	0	0	-2	0	0	-1	-2	-1	0	-1	-15	10	8	0	0	-1	0	14
15	0	0	0	0	0	0	12	0	0	0	0	-12	-1	10	0	8	-7	0	0	11
16	4	0	0	0	4	0	0	0	-1	0	0	0	-1	-9	0	0	0	0	0	0
17	0	0	0	0	4	0	-7	-6	-1	0	0	0	0	-8	0	6	15	0	0	0
18	0	0	0	0	-3	0	0	0	0	0	6	-8	0	0	0	0	0	13	0	9
19	-5	3	0	0	0	0	0	0	0	0	0	-5	0	7	0	0	6	0	5	0
20	0	-3	-4	4	0	3	0	0	0	0	0	0	0	1	0	12	-1	0	0	8

AR COEFFICIENTS FOR LAG 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0	0	-23	0	0	24	0	0	0	0
2	-10	0	0	0	0	0	0	0	0	0	14	0	-2	0	0	2	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	-1	0	0	0	0	0	0	0	0	0	-18	15	2	0	0	-2	0	0	0	-20
6	0	0	0	0	0	0	0	0	0	0	1	7	0	0	0	0	0	0	0	1
7	7	-4	0	0	10	0	0	0	0	0	3	-1	16	0	0	0	-15	0	0	1
8	0	0	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	-7	0	0	-7	0	0	7	0	1	-1	0	0	0	0	0	-18	0	22
10	0	0	0	0	0	0	-7	0	0	0	-8	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0
12	-1	0	1	6	4	1	8	0	4	0	5	24	0	0	0	0	1	2	0	0
13	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	9	0	-1
14	-6	0	6	0	0	6	6	0	0	0	1	0	9	0	8	-13	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	1
17	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

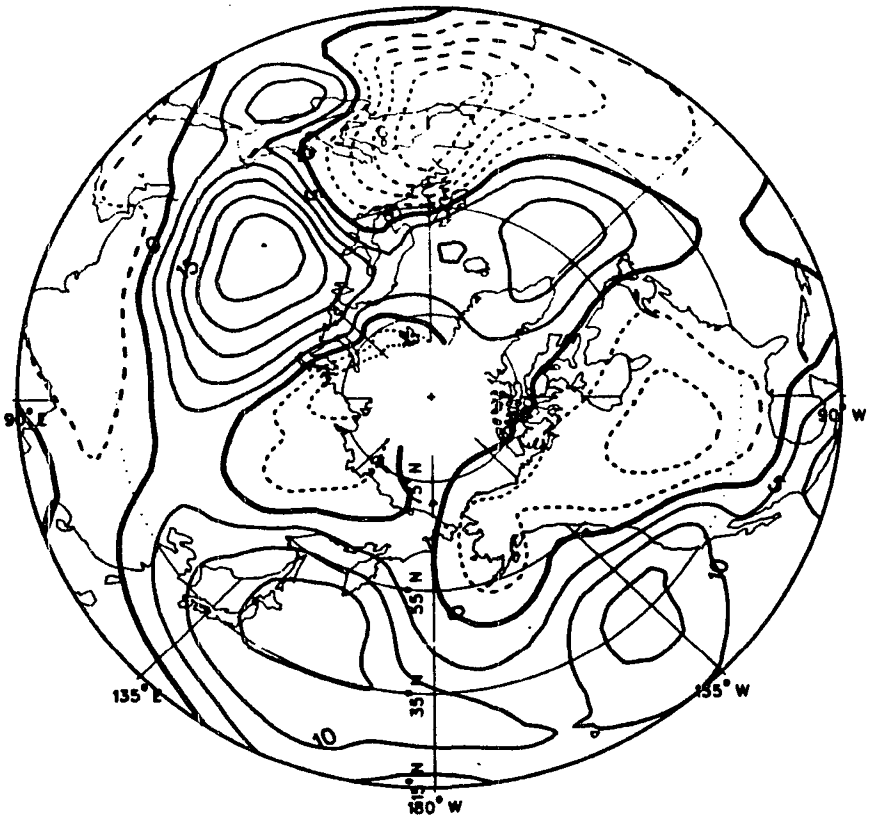
AR COEFFICIENTS FOR LAG 3

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	17	0	0	0	0	0	0	0	0
7	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19
10	0	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	9	0	5	0	0	0	0	-1	0	0	0	0	0	0	0	-2
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

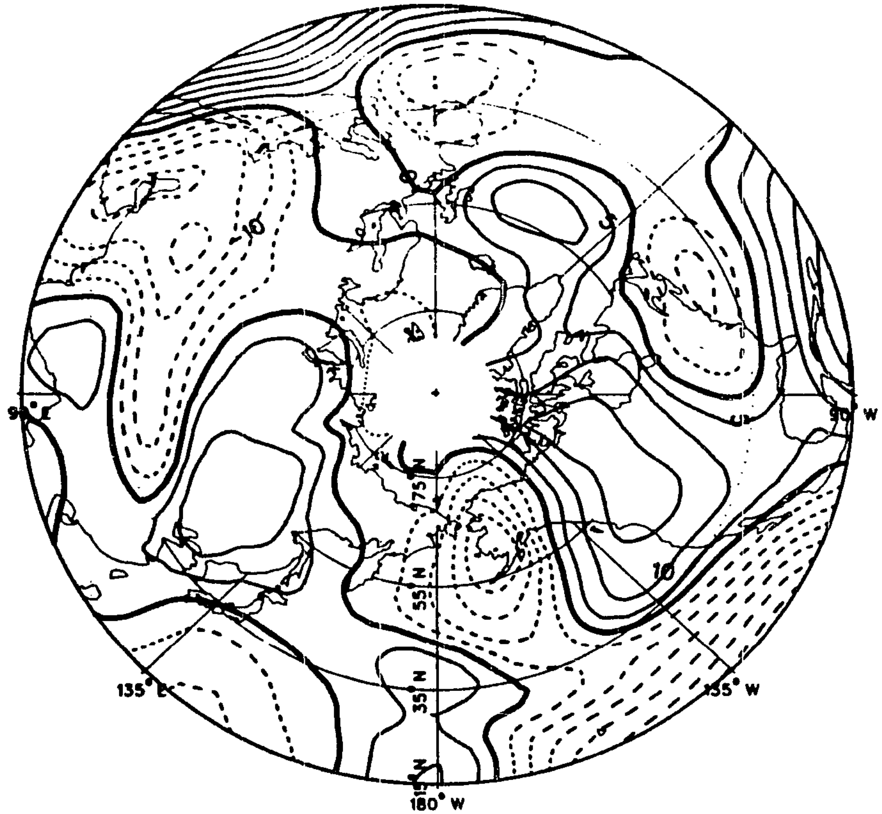
APPENDIX C

The standardized height anomaly eigenvectors 11-20 prepared by the same methods as Figure 7 follow.

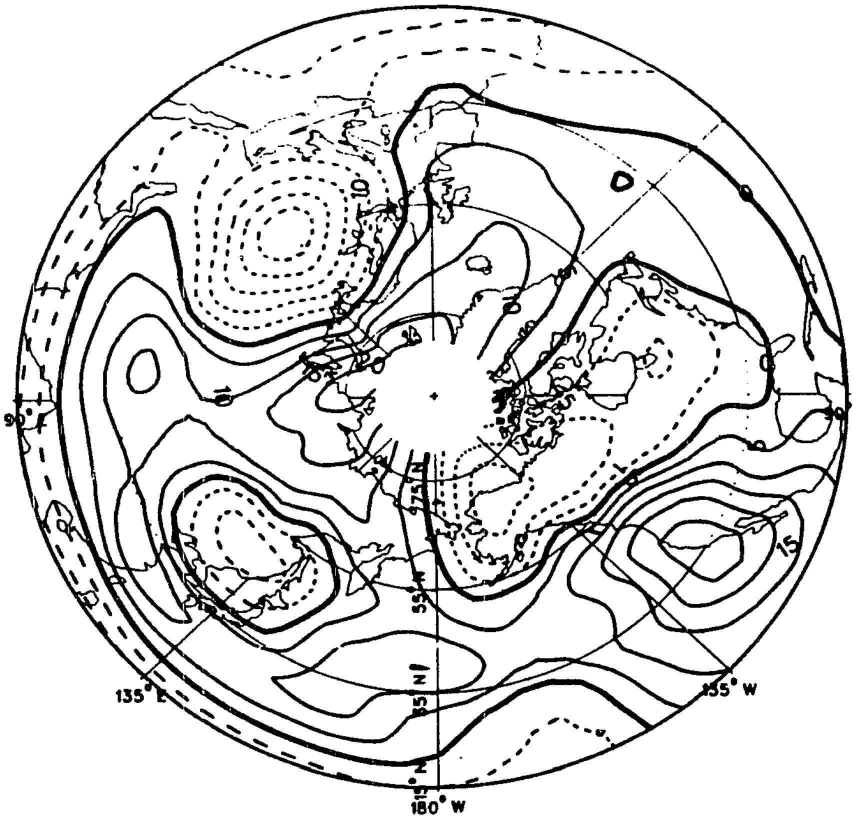
700 MB HEIGHT ANOMALY EIGENVECTOR 11



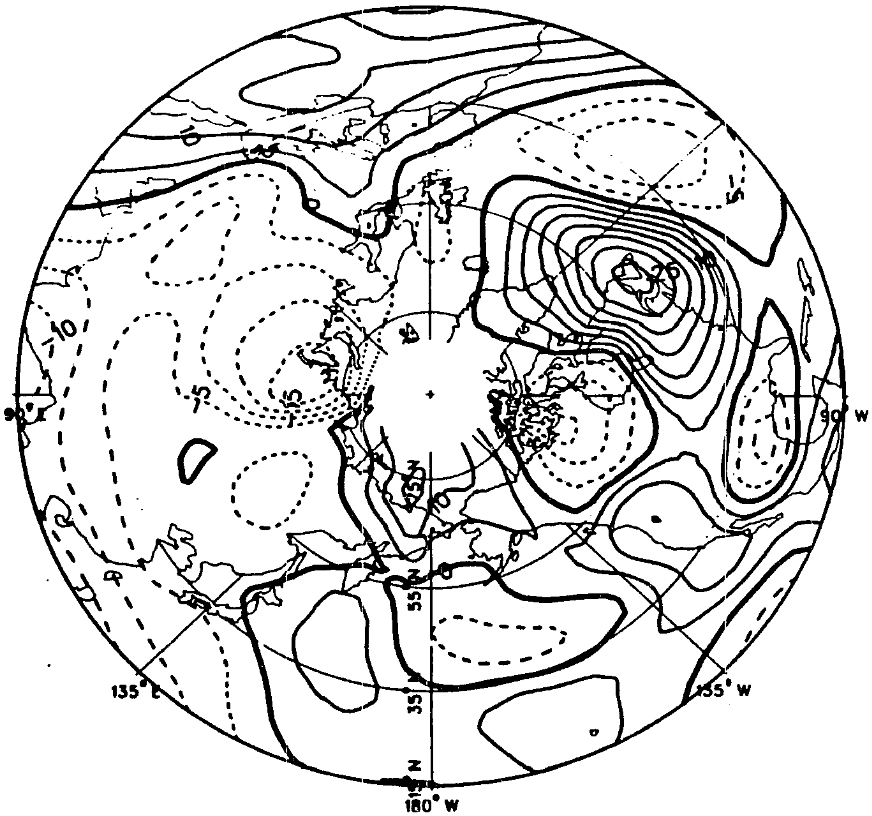
700 MB HEIGHT ANOMALY EIGENVECTOR 12



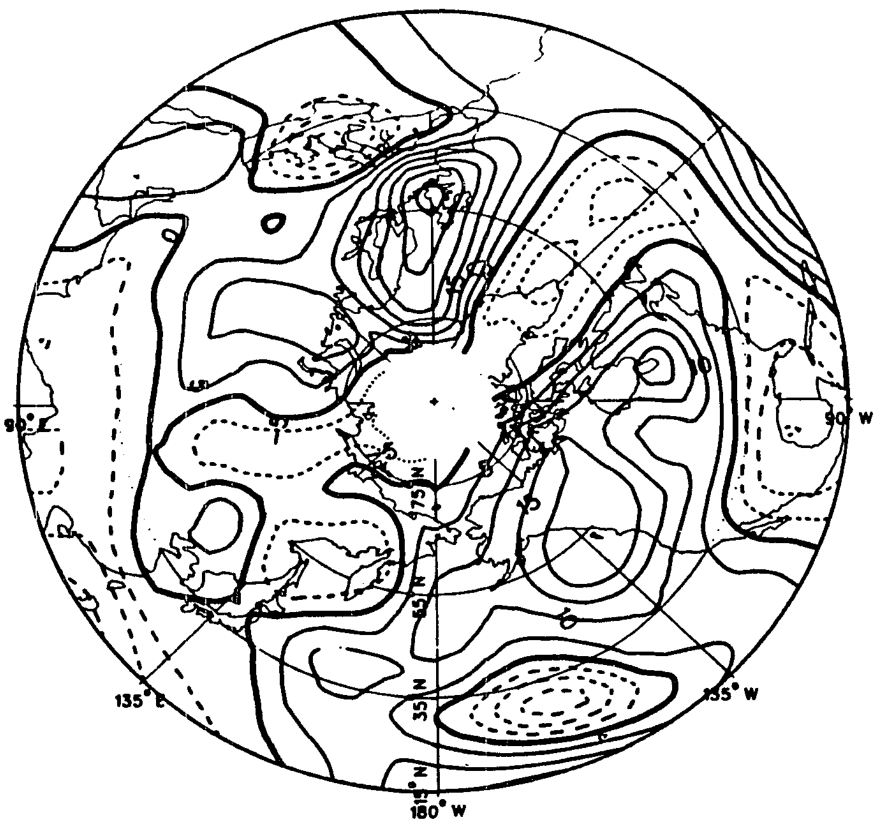
700 MB HEIGHT ANOMALY EIGENVECTOR 13



700 MB HEIGHT ANOMALY EIGENVECTOR 14

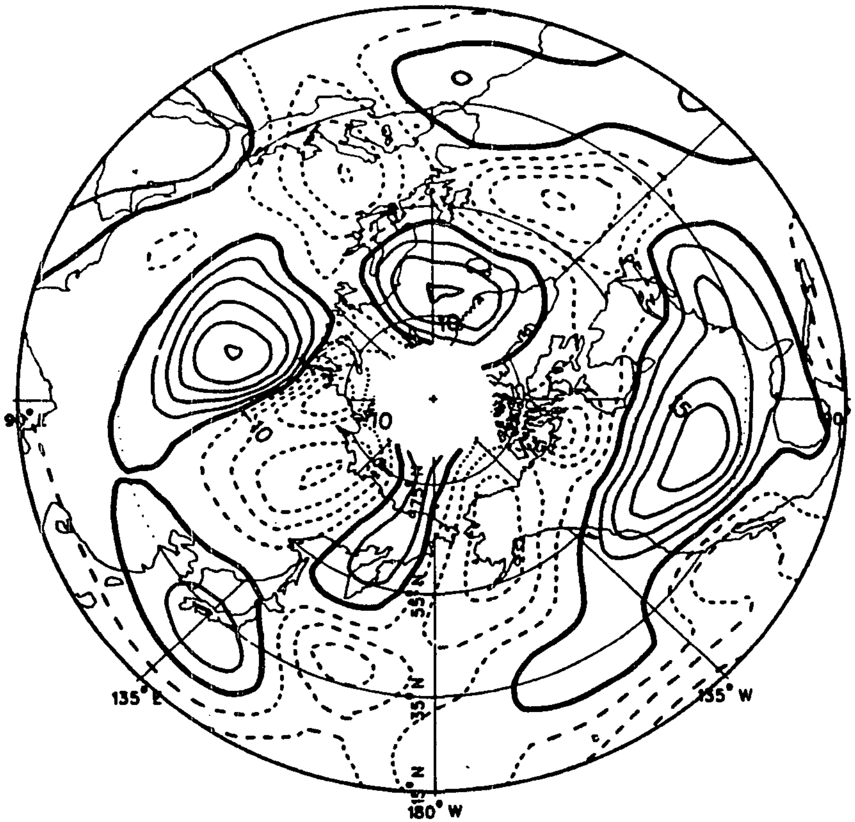


700 MB HEIGHT ANOMALY EIGENVECTOR 15

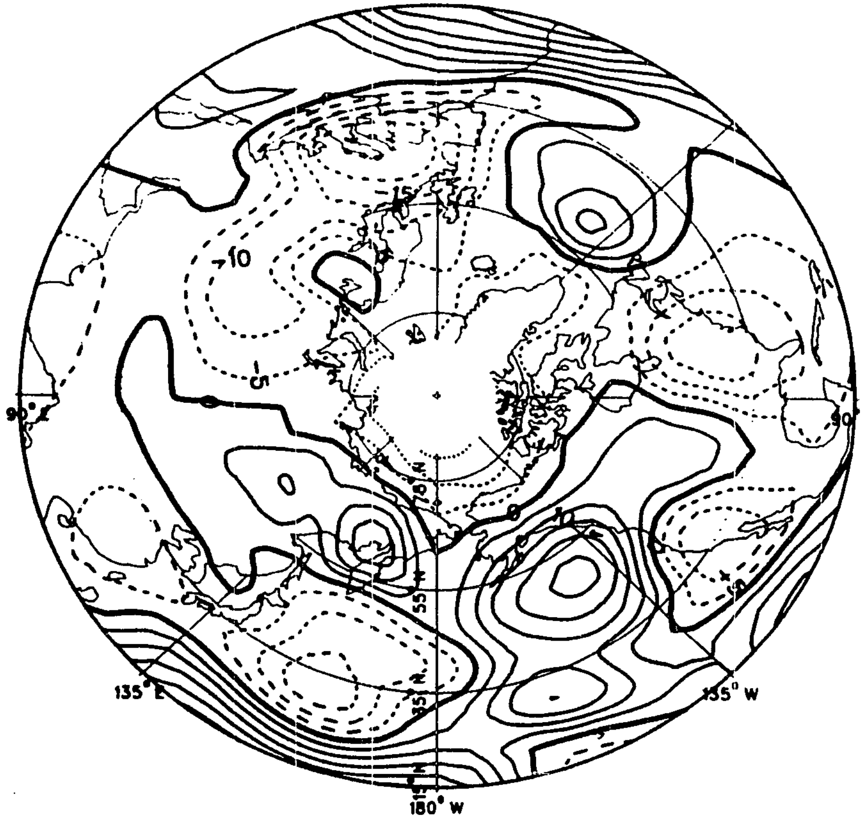


700 MB HEIGHT ANOMALY EIGENVECTOR 16

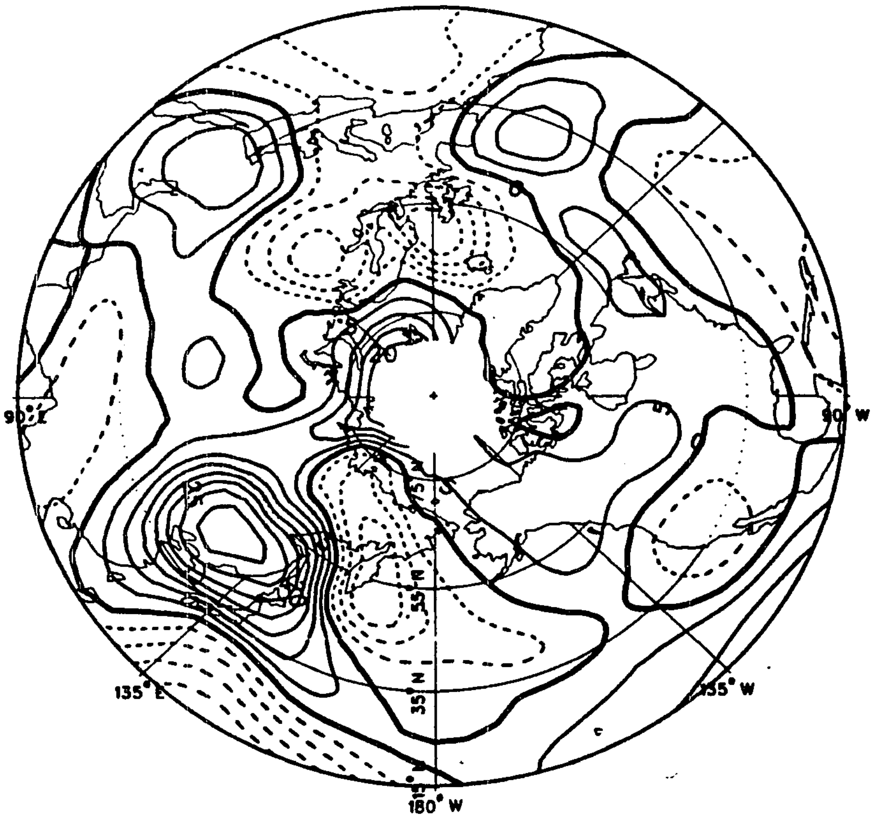
700 MB HEIGHT ANOMALY EIGENVECTOR 17



700 MB HEIGHT ANOMALY EIGENVECTOR 18



700 MB HEIGHT ANOMALY EIGENVECTOR 19



700 MB HEIGHT ANOMALY EIGENVECTOR 20