STRUCTURAL ANALYSIS BY DIRECT AND INDIRECT MODEL METHODS

By

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CHAPTER I

INTRODUCTION

Models have been used for structural analysis for many decades. It is now more than forty years since useful accounts of model analysis of structures were first published. There were some isolated examples of the use of models even before that time, such as the investigation of stresses in dams by rubber models as long ago as 1908.

Although model analysis has many advantages, it has never been granted the importance it deserves. Certainly there are some elements that hamper this method, and keep it from being popular in both practical calculation and classroom teaching. Possibly the main reason for the neglect of model methods is the fear of expense involved in obtaining the proper tools and apparatus for making and testing models. Also, the amount of time involved in achieving an acceptable technique is sometimes considered excessive.

Most of the work done recently in indirect model testing has had satisfactory results. Still, this has not encouraged popular use of the model method, since it emphasizes accurate construction, mounting, and testing of the models with expensive and delicate tools and apparatus. As for direct model testing, most of the models have been built from the designs before the structures were erected to determine any unexpected destructive factors. There is no formulation derived for structural analysis by direct model method.

Great accuracy, which was always emphasized, was not necessary, especially when the method was used for checking of the mathematical results, or for classroom teaching where the purpose of the model was to link the student's mathematical concept of the structure with the visible behavior of the model.

It was this idea that led the writer to try to find some type of model that can be built without special tools, is easy for inexperienced persons to construct, and is simple to operate and measure without expensive apparatus, yet which can obtain results close enough for practical use or for checking purposes, and to try to find some general simple formulas that would connect the relation of behavior of the model and its prototype for structural analysis. Also, the writer hopes that this kind of model would give observers a clear impression of the behavior of the structure.

CHAPTER II

THEORY AND METHOD

In dealing with model structural analysis the first thing involved is model theory, or theory of similarity. Model theory is a theory dealing with the relationship of two objects for which, because of having some particular relations between their dimensions and material properties, determining the behavior of either one under some condition, will predict the behavior of the other under the same condition. A thorough understanding of the theory must begin from the study of dimensional analysis. By this analysis, the complicated relationships among material properties and structural shape and behavior, like those of an airplane or a ship, can be found. As for an architectural structure, these relationships are so simple that the application of this analysis is not necessary. In this chapter only a simple model theory, understandable with common structural knowledge, will be shown as an example.

Model structural analysis generally is classified in two branches: (1) indirect model method, (2) direct model method. In the direct model method, the model is loaded by true weights, so that its deformations modified by the scale factors will become the deformations of its prototype. In the indirect method there is no loading similar to that of the prototype, and the deformations of the model modified by the scale factors will become the influence lines for certain stresses of the prototype.

Indirect Model Method

There are many ways to obtain the influence lines of a structure by analyses of its scale model which are merely different experimental techniques of utilizing an idea based on the Muller-Breslau principle. The principle may be stated as follows: "The ordinates of the influence line for any stress elements such as moment, shear, or axial force of any structure are proportional to those of the deflection curve which is obtained by removing the restraint corresponding to that element from the structure and introducing in its place a displacement in the primary structure". This principle is limited to the structure constructed with

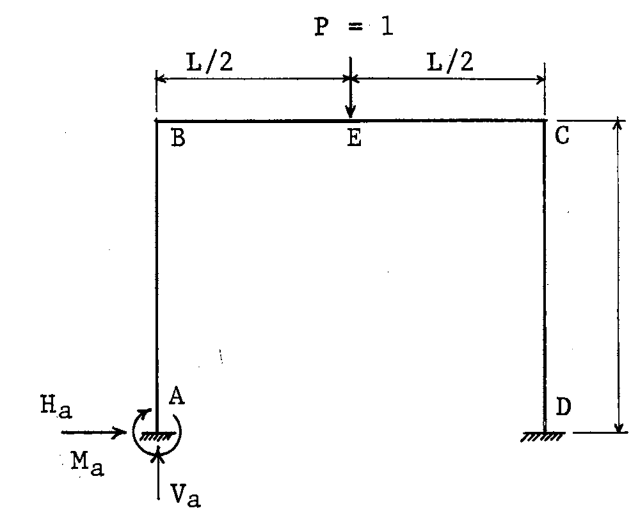


Figure 1. Portal Frame Fixed at A and D

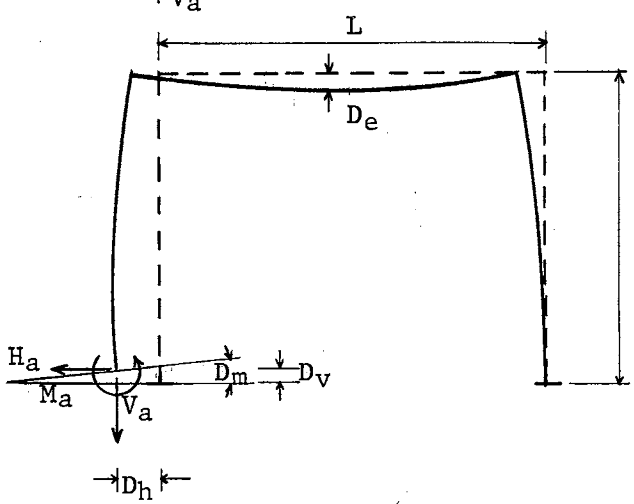


Figure 2. Apply Forces and Moment at A After Restraint at A and Load at E Being Removed

materials that follow Hooke's Law. This principle can be explained by Maxwell's reciprocal theorem.

For instance, in the prototype structure as shown in Fig. I, in order to obtain the influence lines of the stresses at A, the load P and the restraint at A should be removed. A system of forces at A as shown in Fig. 2 is applied, which causes the displacements at A. The displacements correspond to forces H_a', V_a', and M_a' are D_h, D_v, and D_m, and the deflection at the middle point in P direction is D_e. According to Maxwell's reciprocal theorem:

$$P \times D_e + H_a \times D_h + V_a \times D_v + M_a \times D_m$$
$$= H_a' \times 0 + V_a' \times 0 + M_a' \times 0$$

where H_a, V_a, and M_a are the shear, axial force, and moment of the structure at A under the load P, and zeroes in the equation mean A is fixed.

The equation becomes:

$$P \times D_e = -(H_a \times D_h + V_a \times D_v + M_a \times D_m) \quad \ldots\ldots\ldots (1)$$

If H_a', V_a', and M_a' are so arranged that the deformations D_v and D_m equal to 0, D_h not equal to zero, and P is a unit, then:

$$P \times D_e = -H_a \times D_h$$
$$H_a = -D_e/D_h \quad \ldots\ldots\ldots (2)$$

Similarly if D_h and D_v are equal to zero and D_m is not equal to zero, then:

$$M_a = -D_e/D_m \quad \ldots\ldots\ldots (3)$$

If D_h and D_m are equal to zero and D_v is not equal to zero, then:

$$V_a = -D_e/D_v \quad \ldots\ldots (4)$$

If there has been more than one force acting on the structure, there would be as many D_e's representing the deformations in the direction of each force involved in the equation (2), (3), and (4). It becomes clear that the deformations of a restraint-removed structure divided by D_h, D_v, and D_m would be the influence lines of H_a, V_a, and M_a.

It is impractical and uneconomical to try to analyze a structure by applying deformations to the prototype. Therefore, if the structure is to be analyzed by such a method, then a model must be used.

If Fig. 1 is a prototype, and subscript p is used for clearness, then:

$$D_{hp} = (L^3{}_p/E_pI_p) \times (H_a{}'_pK' + V_a{}'_pK'' + M_a{}'_pK'''/L_p) \quad \ldots\ldots (A)$$

$$D_{vp} = (L^3{}_p/E_pI_p) \times (H_a{}'_p\alpha' + V_a{}'_p\alpha'' + M_a{}'_p\alpha'''/L_p) = 0 \quad \ldots\ldots (B)$$

$$D_{mp} = (L^2{}_p/E_pI_p) \times (H_a{}'_p\beta' + V_a{}'_p\beta'' + M_a{}'_p\beta'''/L_p) = 0 \quad \ldots\ldots (C)$$

$$D_{ep} = (L^3{}_p/E_pI_p) \times (H_a{}'_pR' + V_a{}'_pR'' + M_a{}'_pR'''/L_p) \quad \ldots\ldots (D)$$

If Fig. 1 is a model, and subscript m is used, then:

$$D_{hm} = (L^3{}_m/E_mI_m) \times (H_a{}'_mK' + V_a{}'_mK'' + M_a{}'_mK'''/L_m) \quad \ldots\ldots (E)$$

$$D_{vm} = (L^3{}_m/E_mI_m) \times (H_a{}'_m\alpha' + V_a{}'_m\alpha'' + M_a{}'_m\alpha'''/L_m) = 0 \quad \ldots\ldots (F)$$

$$D_{mm} = (L^2{}_m/E_mI_m) \times (H_a{}'_m\beta' + V_a{}'_m\beta'' + M_a{}'_m\beta'''/L_m) = 0 \quad \ldots\ldots (G)$$

$$D_{em} = (L^3{}_m/E_mI_m) \times (H_a{}'_mR' + V_a{}'_mR'' + M_a{}'_mR'''/L_m) \quad \ldots\ldots (H)$$

From Equations A, B, C, D, E, F, G and H:

$$D_{ep}/D_{hp} = D_{em}/D_{hm}$$

therefore, from equation (2), (3), and (4):

$$H_{ap} = -D_{em}/D_{hm} \quad \ldots\ldots (5)$$

Similarly:

$$M_{ap} = (-D_{e'm}/D_{mm}) \times (L_p/L_m) \quad \ldots\ldots (6)$$

$$V_{ap} = -D_{e''m}/D_{vm} \quad \ldots\ldots (7)$$

K and R are constants; their values depend on the shape of the structure. L, E, and I are length, elastic modulus, and moment of inertia of the cross-section of the prototype or its model. The length proportion and moment of inertia proportion of each part of the model must be the same as that of the prototype. From equation (5), (6), and (7), the scale factors of E and I will not be involved in the calculation; the ordinate of the influence line for shear and axial forces of the model are also ordinates of the influence lines of the corresponding forces of the prototype. The position of the forces in the model times the scale factor $\alpha (\alpha = L_p/L_m)$ will be the position of corresponding forces in the prototype. The ordinate of the influence line of moment in the model times the scale factor α will be the ordinate of the influence line of moment in the prototype; loading position in the model times the scale factor α will be the corresponding position in the prototype.

Equations (5), (6), and (7) show how to obtain influence lines of a structure by analyzing its model.

Direct Model Method

Instead of solving simultaneous equations, model analysis can determine the unknowns of the equations by measurement, and easier calculation. This can be demonstrated with a steel cantilever, dimensions as shown in Fig. 3, loaded by a concentrated load at its free end.

By structural theory, the deflection at B will be

$$\delta = PL^3/3EI$$

Suppose that this δ is to be found from the model; plexiglas model is built as shown in Fig. 4:

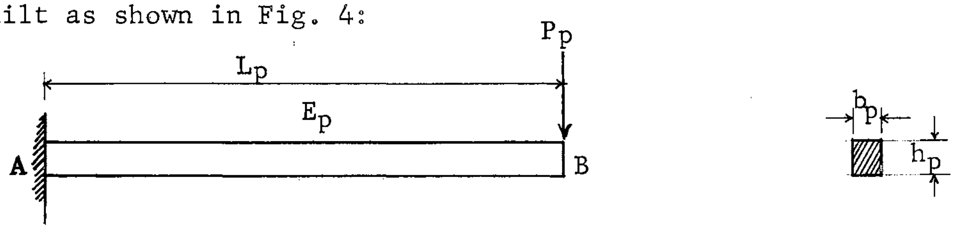


Figure 3. Steel Cantilever

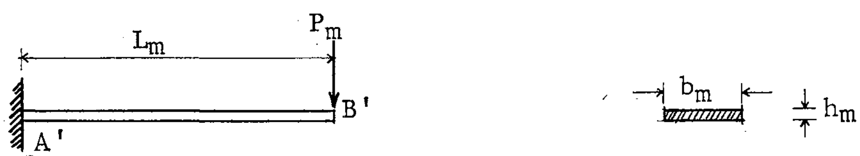


Figure 4. Model

$$\delta_B = P_pL_p^3/3E_pI_p = 4\ P_pL_p^3/E_pb_ph_p^3$$

$$\delta_{B'} = P_mL_m^3/3E_mI_m = 4\ P_mL_m^3/E_mb_mh_m^3$$

$$\delta_B/\ _{B'} = P_p/P_m \cdot L_p^3/L_m^3 \cdot E_m/E_p \cdot b_m/b_p \cdot h_m^3/h_p^3 \quad . \ . \ (8)$$

P_p = 10kip, P_m = 0.001kip, L_p = 100", L_m = 10", $E_p = 3 \times 10^6$, $E_m = 0.3 \times 10^6$, b_p = 10", b_m = 1", h_p = 15", h_m = 0.3"

Therefore, if the measured deflection δ_B' of the model were 1" then by Eq. (8) the deflection of the prototype would be 0.8". It is impractical to use a model to find out the deflection of a cantilever, as it can be calculated easily by a known formula. This example is simply meant to illustrate how the dimensional relation of two structures would predict the relation of their behavior. If a portal frame is subjected to a concentrated load acting at F as shown in Fig. 5, then from slope-deflection equations:

$$M_{AB} = \frac{2EI}{L}(\theta_B - 3\rho)$$

$$M_{BA} = \frac{2EI}{L}(2\theta_B - 3\rho)$$

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C) - \frac{P(aL)(bL)^2}{L^2}$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_b) + \frac{P(aL)^2(bL)}{L^2}$$

$$M_{CD} = \frac{2EI}{L}(2\theta_C - 3\rho)$$

$$M_{DC} = \frac{2EI}{L}(\theta_C - 3\rho)$$

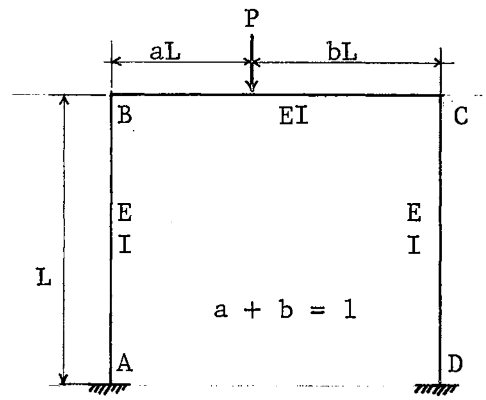


Figure 5. A Simple Portal Frame

$EM_B = 0$

$$4\theta B + \theta_C - 3\rho = ab\frac{2PL^2}{2EI} \quad \ldots\ldots (1)$$

$EM_C = 0$

$$\theta_B + 4\theta_C - 3\rho = -a^2b\frac{PL^2}{2EI} \quad \ldots\ldots (2)$$

$EH = 0$

$$\theta_B + \theta_C - 4\rho = 0 \quad \ldots\ldots (3)$$

Solve (1), (2) and (3) simultaneously: $(\rho = \frac{3\delta}{L})$

$$\theta_B = F' \frac{PL^2}{EI} \quad \ldots \quad (4)$$

$$\theta_C = F'' \frac{PL^2}{EI} \quad \ldots \quad (5)$$

$$\delta = F''' \frac{PL^3}{EI} \quad \ldots \quad (6)$$

F', F'' and F''' are constants, dependents on the shape and loading condition of the structure.

If the structure shown in Fig. 5 were a model, a subscript m is used:

$$\theta_{bm} = F' \frac{P_m L^2_m}{E_m I_m} \quad \ldots \quad (7)$$

$$\theta_{cm} = F'' \frac{P_m L^2_m}{E_m I_m} \quad \ldots \quad (8)$$

$$\delta_m = F''' \frac{P_m L^3_m}{E_m I_m} \quad \ldots \quad (9)$$

If the structure is a prototype, subscript p is used:

$$\theta_{bp} = F' \frac{P_p L^2_p}{E_p I_p} \quad \ldots \quad (10)$$

$$\theta_{cp} = F'' \frac{P_p L^2_p}{E_p I_p} \quad \ldots \quad (11)$$

$$\delta_p = F''' \frac{P_p L^3_p}{E_p I_p} \quad \ldots \quad (12)$$

If the dimensional relation between the model and its prototype are as follows,

$$\frac{\delta p}{\delta m} = \alpha \quad , \quad \frac{\theta p}{\theta m} = \sigma \quad , \quad \frac{P_p}{P_m} = N \quad ,$$

$$\frac{E_p}{E_m} = K \quad , \quad \frac{I_p}{I_m} = \beta \quad , \quad \frac{L_p}{L_m} = \lambda \quad ,$$

then from (7), (8), (9), (10), (11), and (12):

$$\frac{\theta_{bp}}{\theta_{bm}} = \frac{\theta_{cp}}{\theta_{cm}} = \frac{F'P_pL^2_p}{E_pI_p} \cdot \frac{E_mI_m}{F'P_mL^2_m}$$

$$\frac{\theta_p}{\theta_m} = \frac{E_m}{I_p} \cdot \frac{I_m}{I_p} \cdot \frac{P_p}{P_m} \cdot \frac{L^2_p}{L^2_m}$$

$$\sigma = \frac{I}{K} \cdot \frac{I}{\beta} \cdot N \cdot \lambda^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\frac{\delta_p}{\delta_m} = \frac{F'''P_pL^3_p}{E_pI_p} \cdot \frac{E_pI_p}{F'''P_mL^3_m}$$

$$\frac{\delta_p}{\delta_m} = \frac{E_m}{E_p} \cdot \frac{I_m}{I_p} \cdot \frac{P_p}{P_m} \cdot \frac{L^3_p}{L^3_m}$$

$$\alpha = \frac{I}{K} \cdot \frac{I}{\beta} \cdot N \cdot \lambda^3 . \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Equations (13) and (14) were derived by the writer. These two equations can be applied to any complicated portal frame structure when the effect of bending moment only is considered.

In Equations (13) and (14) it is clear that any of four factors could be chosen arbitrarily, and the fifth factor would be expressed in terms of the other four factors.

In most cases, K would be known when the model material is decided, λ would be known when the length scale of the structural member is decided, β would be known when cross-sectional dimension of the model is chosen, and the angular rotational factor σ would be picked according to experience.

After the four factors K, λ, β, and σ, are fixed one by one as described above, the load factor N would be fixed in terms of K, λ, β, and σ, and then the linear deformation factor α would be fixed in terms of K, λ, β, and N.

The load on the prototype divided by N would be the weight to be applied to the model. After loading of the model, the rotation and deflection of the model at any point can be measured. These values multiplied by σ and α would become the rotation and deflection of the prototype at corresponding points. With these deformation values the stresses of the structure at any point can be calculated easily by slope-deflection equations.

In case the factor N is such as to make the calculated loads on the model become too large or too small, the model would be stressed so much as to have large configuration or to run out of its elastic range and make the measurements far from accurate, or the deformation of the model would be so small that measurement with a commercial scale would be impossible, then at least two of the scale factors must be revised; in such occasion, usually σ would be revised and accordingly revise the factors N and α. In this way a proper loading weight of the model could be found by trial and error.

CHAPTER III

DESIGN AND MAKING OF MODELS AND MOUNTING FRAME

Many kinds of materials have been used in the making of models. Among these are concrete, plaster, metal wire, plexiglas, balsa wood, and even paper board. Almost any kind of material that is homogeneously composed and obeys Hooke's Law could be used as model material.

In selecting the model material in this project, some special factors had to be taken into consideration: in this project it was necessary for (1) the model to be cut, trimmed, and connected by common school workshop tools, (2) model deformations to be measurable by commercial scale, (3) the model to be tested both by direct and indirect methods. These requirements implied that the model material must be easy to cut, trim, and connect. It must be flexible enough to make the application of large scale deformations possible. In addition, it must be easy to procure and not expensive.

First paper board was chosen and a portal frame was made. However, several indirect tests proved that paper board was not a very homogenous material and, therefore, it was not considered an ideal one.

After this, the writer found one kind of rather flexible plexiglas in the school workshop. After some simple testings of its properties, the material proved desirable. Although it creeps under load, this drawback can be overcome if the character of creep is known.

Most plastic models made before for structural analysis were cut out of a single sheet. Therefore, the thickness of the sheet became the width of the structural members, and because they were cut from solid plate as shown in Fig. 6, no artificial connection was needed.

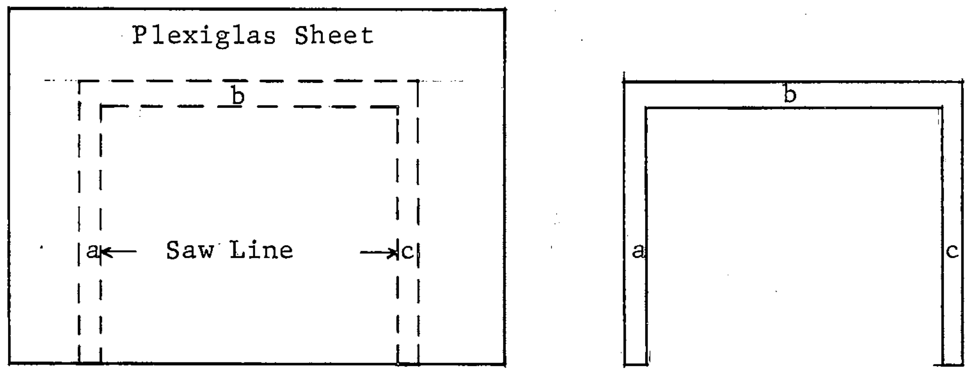


Figure 6. Solid Cut of Plexiglas Model

This kind of cut, though assuring an ideal connection, requires much time and skill. The cut requires great accuracy, because the width of the strip, which is decided by the cut, will become the depth of the model structural member and affect moment of inertia of the cross section of the member very much. This type of model, when tested, tends to buckle, because the lateral dimension of its member is small in comparison with the transverse dimension. The deformations of the model under test always will be very small. Therefore, expensive precision apparatus must be used for the measurement.

Considering all these facts, the type of model described above was not used in this test. Instead the type of model which needs some artificial connections was used. In constructing this kind of model, model members were cut strip by strip from the sheet and connected by plastic cement to form the structure. The model is so constructed that the thickness of the plastic plate becomes the depth of the model member, and the width of the strip which is determined by the cut becomes the width of the model member, as shown in Fig. 7.

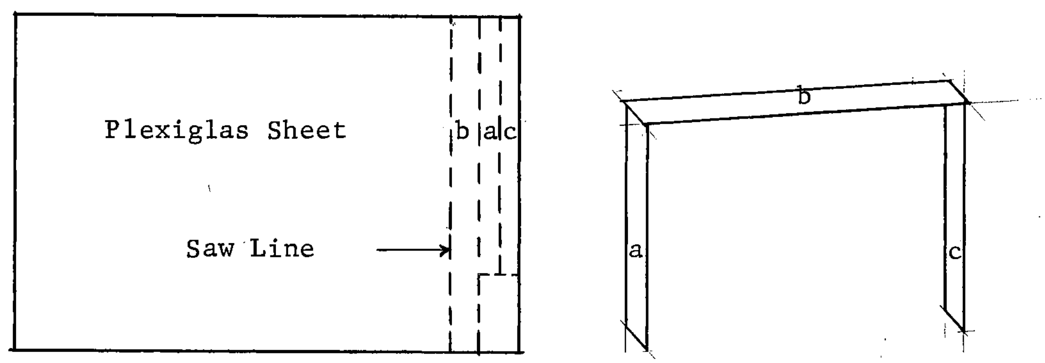


Figure 7. Method of Cut of Plexiglas Model in this Test

In this type of model, an inaccurate cut will not seriously affect the moment of inertia of member cross section, which plays an important role in structural analysis. Besides, this type of model will not easily buckle under the transverse load, and the application of large scale deformations. Therefore measurements with a commercial scale are possible.

The 1/16" thick plexiglas plate supplied by Rohm and Haas Company was used in this test. The cut of the material was first done by a band saw. If great accuracy is needed, the cut edge may be machined, but hand sanding proves accurate enough for this use. An accuracy within 1/32" could be achieved by inexperienced persons by careful trimming of the strip on a sanding block. For connections, Pleximent cement by Cope Plastics, Inc. was used. This is a straight solvent type cement for plexiglas, which requires two days to set to assure a firm connection.

It was later discovered that the thickness of this kind of plexiglas is not exactly 1/16" as termed, and even within the same sheet the thickness varies from one point to another. Although this variation is small, it is significant since sheet thickness represents member depth. It is necessary to modify values for moments of inertia to take these variations into account.

To avoid the friction that a horizontally mounted model always has, and to make the loading of a true weight possible, the model must be mounted vertically, although the weight of the model itself may distort the structure a little when it is vertically mounted. Since the weight of the plexiglas is small, the distortion caused by its own weight is not great.

It was expected that during the measurement of the deformations, the model should not be loaded by hands or the scale; also it was expected that the mounting frame should be stable enough to hold the model in an immovable condition during the test. Concerning all these factors a mounting frame consisting of a wood frame and a calibrated plastic plate was constructed, as shown in Fig. 8. The calibrated transparent plastic plate made the measurement from behind the model possible. The holes on the wooden board were for the holding of the aluminium plates which in turn held the models, and the calibration on the same board was for the applications of deformations to the models in the indirect model method. In the direct model method, models would be held by the same aluminium plates in the desired positions. The type of anchorage was an imitation of the fixed base of prototype structure. Hinges or roller bases can not be reproduced in this mounting frame.

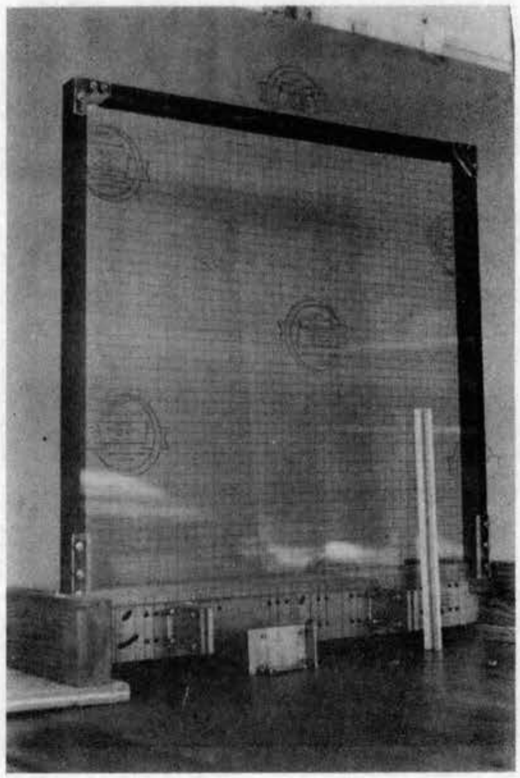


Figure 8. Mounting Frame

CHAPTER IV

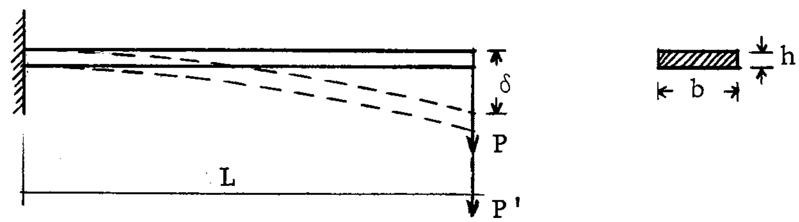
TEST OF THE PROPERTIES OF PLEXIGLAS

In the indirect model method, the use of Muller-Breslau's principle is restricted to materials that obey Hooke's Law. In the direct method, in addition to the material obeying Hooke's Law, its elastic modulus must also be known. The material should be submitted to tests to determine its elastic modulus and type of creep.

For this purpose, three strips of plexiglas were sawed and trimmed. Their measurements are shown in Table 1. Each strip was mounted as a cantilever and loaded at its end by many different weights, as shown in Fig. 9, and its end deflections were measured several times at intervals of 18 minutes.

This test showed clearly (Table 2 and Fig. 10) that the material obeys Hooke's Law. Table 3 and Fig. 11 show the characteristic of the creep of the material, and the various elastic modulii within 18 minutes after loading. The average elastic modulus of three strips at 18 minutes after loading would be used throughout the whole test. Fig. 11 shows that the elastic modulus became nearly constant after 18 minutes.

Cantilever test of properties of plexiglas



$\delta_T = PL^3/3E_TI$

δ_T is end deflection at time T.

E_T is elastic modulus at time T.

$E_T = PL^3/3 \ _TI$

Beam size is known. P is known. By measurement of δ_T, E_T can be calculated.

TABLE 1

DATAS OF THREE CANTILEVERS

Cantilever	Height of Section h in	Width of Section b in	Length L in	Mom. of inertia of Section 1 in^4
A	0.0632	9.6/32	10	6.22×10^{-6}
B	0.0632	17.8/32	10	11.54×10^{-6}
C	0.0632	31.5/32	10	20.43×10^{-6}

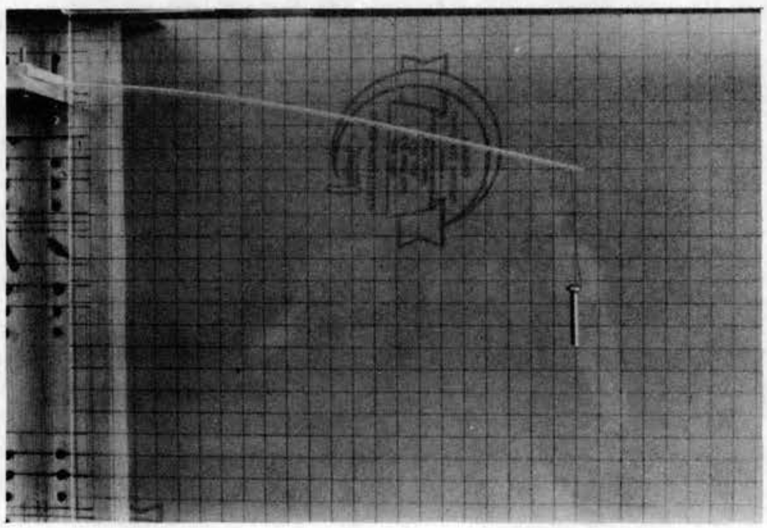


Figure 9. Test of Cantilever C

TABLE 2

LOADS AND DEFLECTIONS OF CANTILEVER C

Weight lb	Deflection at 18 Minutes After Loading in
9.69×10^{-3}	18.5/50
11.00×10^{-3}	21.0/50
19.30×10^{-3}	36.5/50
28.10×10^{-3}	54.0/50
43.80×10^{-3}	81.0/50

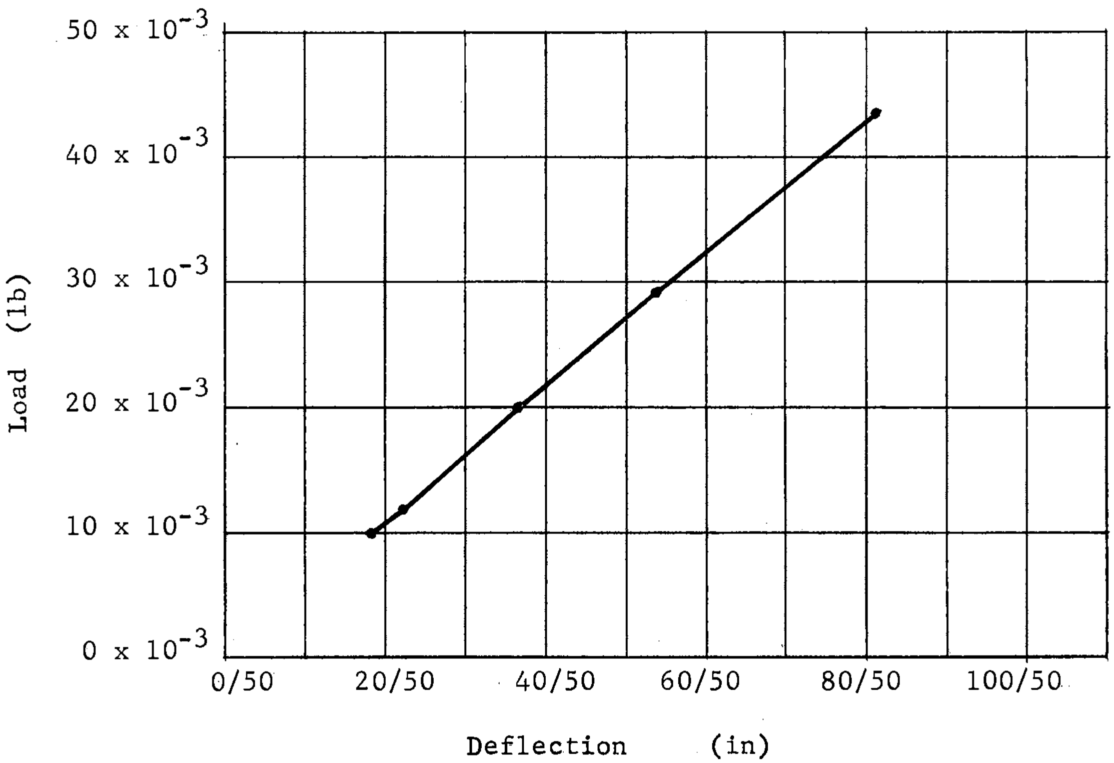


Figure 10. Deflection of Cantilever C by Load

TABLE 3

VARIOUS E OF THREE CANTILEVERS

Cantilever	Weight P lb	Time T Minute	Deflection in	Elastic Modulus E lb/in^2
A	9.69×10^{-3}	0.25 3 9 18	57.0/50 60.5/50 62.5/50 64.0/50	0.451×10^6 0.429×10^6 0.415×10^6 0.406×10^6
	11.00×10^{-3}	0.25 3 9 18	66.0/50 68.0/50 70.0/50 70.5/50	0.446×10^6 0.433×10^6 0.421×10^6 0.417×10^6
B	19.38×10^{-3}	0.25 3 9 18	60.0/50 64.0/50 66.0/50 66.5/50	0.465×10^6 0.437×10^6 0.424×10^6 0.421×10^6
	28.10×10^{-3}	0.25 3 9 18	89.0/50 92.5/50 95.5/50 97.0/50	0.455×10^6 0.438×10^6 0.426×10^6 0.419×10^6
C	28.10×10^{-3}	0.25 3 9 18	50.5/50 52.0/50 53.0/50 54.0/50	0.453×10^6 0.440×10^6 0.431×10^6 0.424×10^6
	43.80×10^{-3}	0.25 3 9 18	73.0/50 78.0/50 79.5/50 81.0/50	0.488×10^6 0.452×10^6 0.449×10^6 0.440×10^6

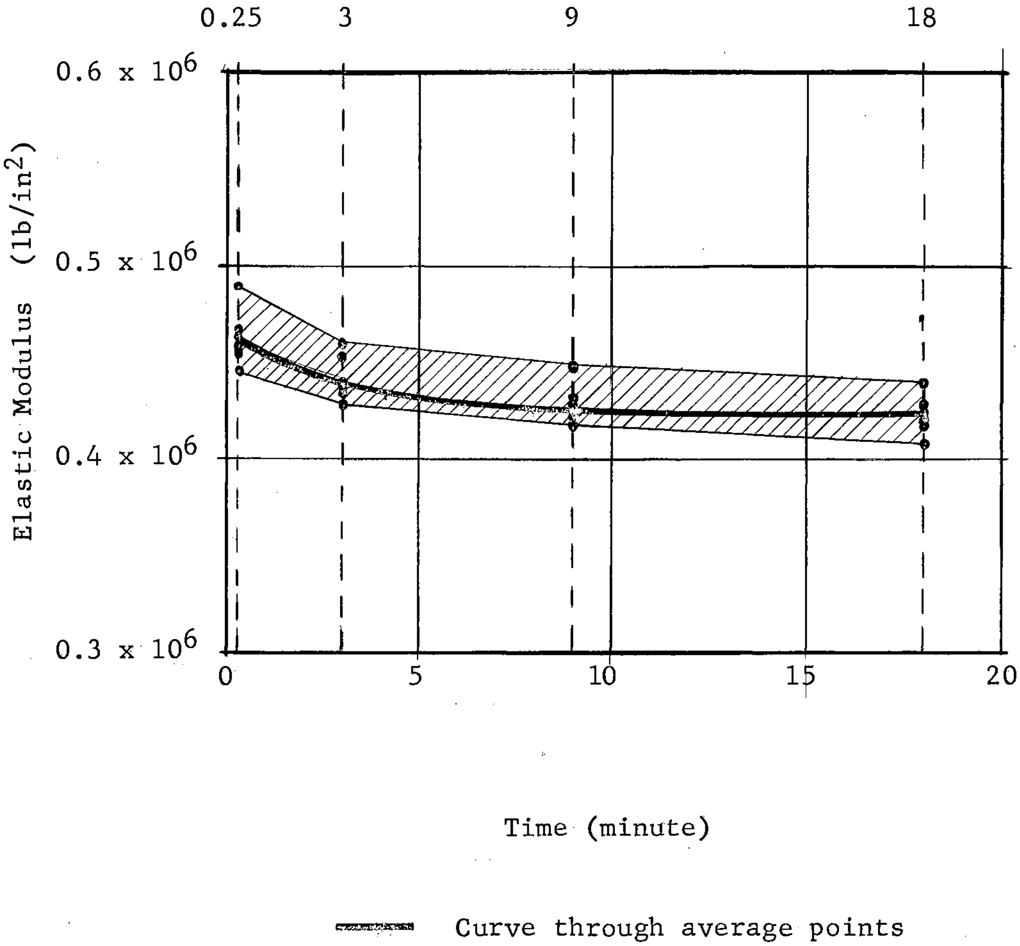


Figure 11. Elastic Modulus of Plexiglas by Time

Time (minute)	0.25	3	9	18
E (lb/in^2)	0.461×10^6	0.440×10^6	0.428×10^6	0.422×10^6

CHAPTER V

ANALYSIS OF STRUCTURES BY DIRECT AND INDIRECT MODEL METHODS

5-1 An analysis of a simple portal frame, with uniform cross section, by indirect model method was performed in order to determine if the various techniques employed in this test would work.

The prototype is shown in Fig. 12 and its model (No. 1) is shown in Fig. 13 and Fig. 14.

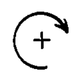
The model was mounted and deformation applied at D, as shown in Fig. 14, in order to find M_d, H_d, and V_d of the prototype under load. 2.5-radians clock-wise angular rotation, 1" to right horizontal displacement, and 1" up vertical displacement were applied one by one at D by adjusting the aluminium plate, which held the model at D, along the calibrated lines. At each adjustment the vertical deflection at E was measured. The calculated stresses and their comparison with those from the moment distribution method was shown in Table 4.

The percentage of difference shown in Table 4 is quite large. Apparently, the application of large scale deformations had distorted the model so much that the results were also distorted. Some former tests suggested that on such occasions application of a pair of equal and opposite deformations should be made in each test; then the influence value would be the average of those measured from the two opposite deformations.

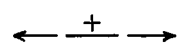
The 2.5 radian counter-clockwise angular rotation, 1" to the left horizontal, and 1" down vertical displacements which were in opposite directions to the three deformations applied before were applied one by one. The stresses calculated by taking the average of the two influence values are shown in Table 5. These results came out surprisingly close to those values from the moment distribution method. From these results, it is realized that the application of two opposite equal deformations will cancel most of the effect of distortion. In the indirect model method, if the large-scale deformation technique is used, two opposite and equal deformations must be applied in each operation in order to assure dependable results.

Sign convention follows:

I. Stress

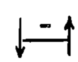
moment — clockwise, plus 

counter-clockwise, minus 

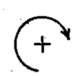
axial force — tension, plus 

compression, minus 

shear — clockwise, plus 

counter-clockwise, minus 

2. Applied deformation

angular rotation — clockwise, plus 

counter-clockwise, minus 

horizontal dis-		
placement	to the left, plus	← +
	to the right, minus	− →
vertical displace-		
ment	down, plus	+ ↓
	up, minus	− ↑

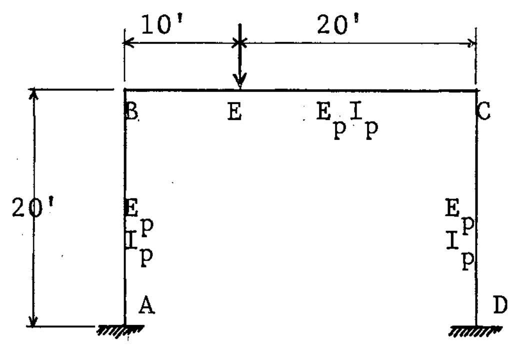


Figure 12. Prototype No. 1.

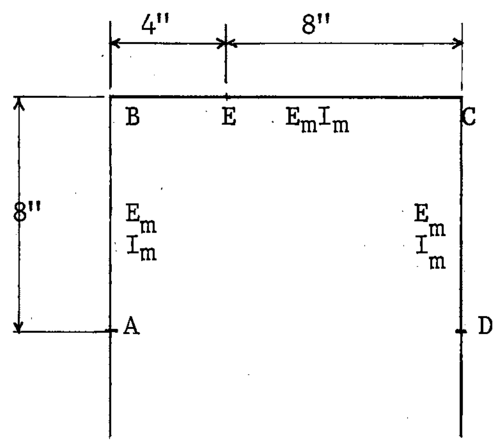


Figure 13. Model No. 1.

TABLE 4

RESULTS FROM TEST OF MODEL NO. 1

(Deformations, Each Applied in One Direction Only)

Applied Deformation at D	Vertical Deflection at E in	Influence Value	Load kip	Stress	Stress from Moment Distribution Method	Percentage of Difference
↻ 0.25 Radians	-0.181	-21.73"	18	M_D -32.6 K - Ft	M_D -26.54 K - Ft	22.8%
→ - 1"	-0.196	0.196	18	H_D 3.53 K	H_D 3.38 K	5.07%
↑ - 1"	+0.332	-0.332	18	V_D -5.98 K	V_D -5.75 K	4.0%

TABLE 5

RESULTS FROM INDIRECT TEST OF MODEL NO. 1

(Deformations Applied in Equal and Opposite Directions)

Applied Deformation at D	Vertical Deflection at E in	Influence Value	Average Influence Value	Load kip	Stress	Stress from Moment Distribution Method	Percentage of Difference
↻ + 0.25	-0.181	-21.73"	-18.09"	18	M_D -27.10 K-Ft	M_D -26.54 K-Ft	2.07%
↺ - 0.25	+0.121	-14.45"					
→ - 1"	-0.196	+ 0.196	+0.189	18	H_D +3.4 K	H_D +3.38 K	0.59%
← + 1"	+0.181	+ 0.181					
↑ - 1"	+0.332	- 0.332	-0.326	18	V_D -5.87 K	V_D -5.75 K	2.09%
↓ + 1"	-0.320	- 0.320					

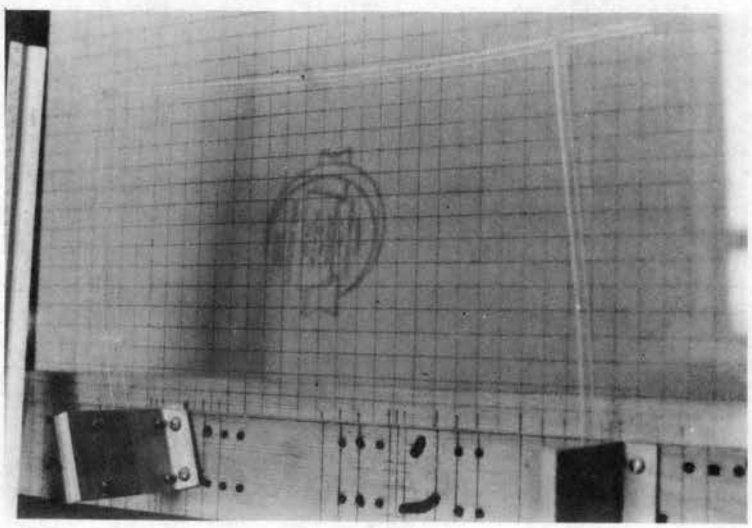


Figure 14. Model No. 1 Under Test
(-0.25 Radians Rotation at D)

5-2 An analysis of a simple portal frame with members of different cross sections was made by indirect model method.

Connections of members of different sizes probably would not function as well as the ideal rigid connection which is usually assumed in mathematical analysis. As shown in Fig. 15, in a connection with two different size members, at points A and B there would probably be some stress concentrations and in the shaded area, as shown, there would probably be reduced stress. These would make the connection different from the structural assumption.

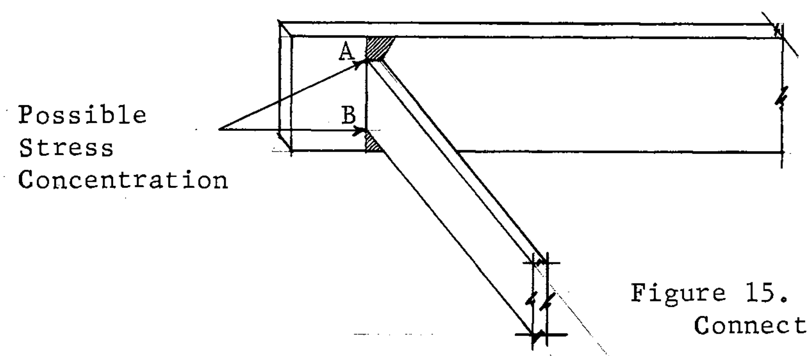


Figure 15. Improper Connection

The main purpose of this test was to determine to what extent this type of connection would affect the results. Model (No. 2) was constructed with the three strips which previously had been used in the properties test. The model and its prototype are shown in Fig. 17 and 18. The prototype shown is imaginary.

The model was mounted and submitted to test. The results are shown in Table 6. Though the results were close to those obtained from the moment distribution, they were not as good as in the first test.

As the same test was applied again and again, the percentage of error became larger and larger. After a careful inspection, it became clear that the connection was weakening. This model was tested one day after construction, while the model in the first test had more than two days to set before being tested. It was then the writer found that if Pleximent cement was used, at least two days must be given for the connection to set to assure a firm connection.

The connections of the model were reconstructed with two pieces of plexiglas reinforcement on both sides of the smaller member, as shown in Fig. 16, and the model was given two days to set.

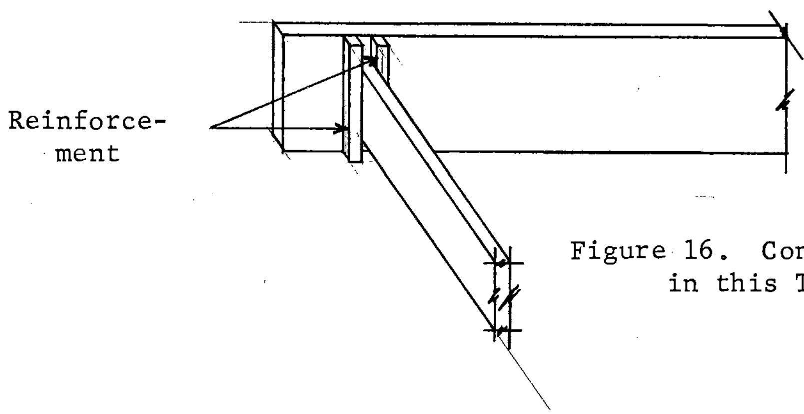


Figure 16. Connection used in this Test

By doing this the writer hoped that two days would be enough to assure a firm connection, and two pieces of reinforcement would transmit the stresses properly between two members to avoid any stress concentration and therefore make the connections closer to the assumption.

This kind of connection might make the portions near the connection stronger than assumed but since the reinforcements were so small the writer hoped that this would not affect the results much.

Two days after construction the reinforced model (No. 2) was mounted and tested, as shown in Fig. 19. The measured and calculated results are shown in Table 7. The results could be termed as satisfactory as to the performance of this type of connection.

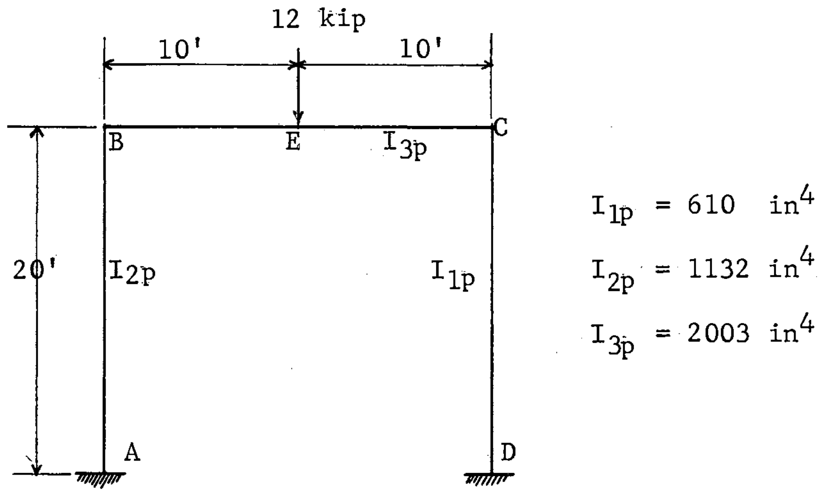


Figure 17. Prototype No. 2

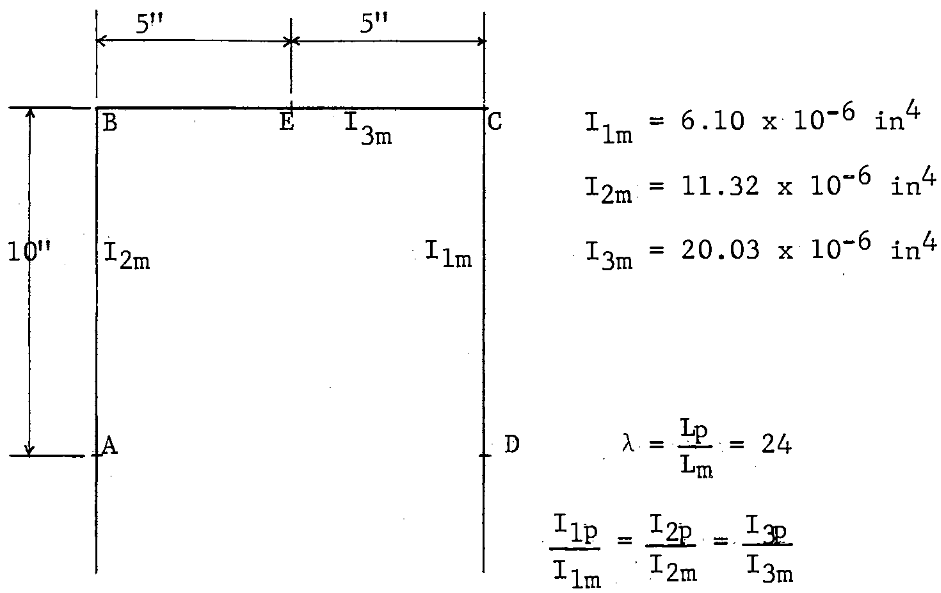


Figure 18. Model No. 2

TABLE 6

RESULTS FROM ANALYSIS OF MODEL NO. 2,

BEFORE PROPER CONNECTIONS WERE MADE

Applied Defor-mation at A	Vertical Deforma-tion at E in	Influence Value	Average Influ-ence Value	Load at E kip	Stress	Stress from Moment Distribution Method	Percentage of Difference
↺ + 0.25	+0.016	+1.536"	5.958"	12	M_A 5.958 K-Ft	M_A 5.23 K-Ft	14.5%
↻ - 0.25	-0.108	+10.380"					
→ - 1"	+0.065	-0.065	-0.076	12	H_A -0.912 K	H_A -0.982 K	7.12%
← + 1"	-0.087	-0.087					
↑ - 1"	+0.486	-0.486	-0.509	12	V_A -6.11 K	V_A -6.101 K	0.15%
↓ + 1"	-0.531	-0.531					

TABLE 7

RESULTS FROM ANALYSIS OF MODEL NO. 2,

AFTER PROPER CONNECTIONS WERE MADE

Applied Defor-mation at A	Vertical Deforma-tion at E in	Influence Value	Average Influ-ence Value	Load at E kip	Stress	Stress from Moment Distribution Method	Percentage of Difference
↺ + 0.25	+0.0125	+1.2	4.88"	12	M_A 4.88 K-Ft	M_A 5.23 K-Ft	6.7%
↻ - 0.25	-0.0891	+8.55					
→ - 1"	+0.0594	-0.0594	-0.0765	12	H_A -0.918 K	H_A -0.982 K	6.9%
← + 1"	-0.0937	-0.0937					
↑ - 1"	+0.484	-0.484	-0.521	12	V_A -6.25 K	V_A -6.101 K	2.4%
↓ + 1"	-0.558	-0.558					

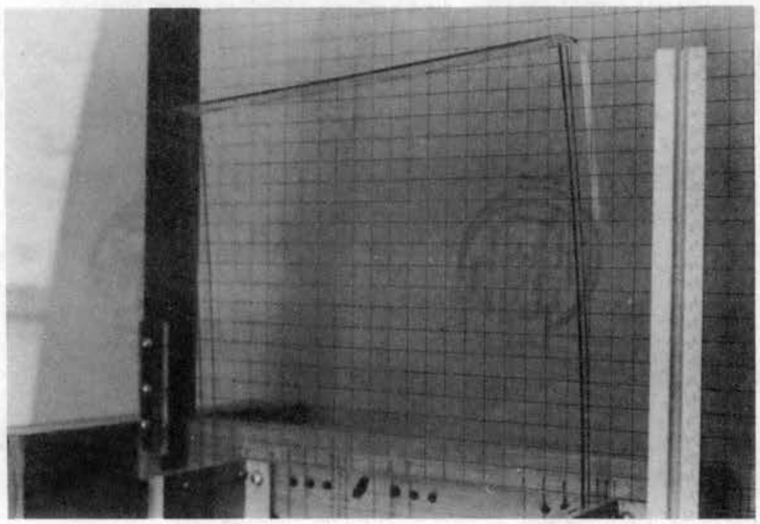


Figure 10. Model No. 2 Under Test
(1" Down Displacement at A)

5-3 An analysis of a simple portal frame with uniform cross section was made by direct model method.

The prototpye and its model (No. 1) used in this test were the same ones that had been used in test 5-1, shown in Figs. 20 and 21.

If K is the elastic modulus scale factor

β is the moment of inertia scale factor

N is the load scale factor

λ is the length scale factor of the member

σ is the angular rotation scale factor

α is the linear deformation scale factor

From data in Figs. 20 and 21:

$$K = E_p/E_m = 2.9 \times 10^6/0.422 \times 10^6 = 6.90$$

$$\beta = I_p/I_m = 911/7.60 \times 10^{-6} = 120 \times 10^6$$

$$\lambda = L_p/L_m = 30 \times 12/12 = 30$$

$$N = P_p/P_m \,,\ P_p = 18 \times 10^3 \text{ lb}$$

From equations (13) and (14):

$$\sigma = N\,{}^2/K$$

$$\alpha = N\,{}^3/K$$

Then if the weight load on the model is 97 gm (0.214 lb)

$$N = 18 \times 10^3/0.214 = 8.4 \times 10^4$$

$$\sigma = 8.4 \times 10^4 \times 30^2/6.9 \times 120 \times 10^6 = 1/10.9$$

$$\alpha = 8.4 \times 10^4 \times 30^3/6.9 \times 120 \times 10^6 = 2.75$$

This means that the angular deformation at any point of the prototype will be 1/10.9 times the angular deformation at the corresponding point of the model, and the linear deformation of the prototype at any point will be 2.75 times the linear deformation of the model at the corresponding point.

If the weight load on the model is 27.21 gm:

$N = 30 \times 10^4$

$\sigma = 1/3.06$

$\alpha = 9.78$

The two different weights were composed of screws, nuts, washers, and wire, all of which had been carefully weighed on a beam balance. These various items were bound together by wire to become hang-weights. At the loading point on the model a hole was drilled through the center of the member to provide a hang point for the weight. The drill hole was small; its effect on moment of inertia could be neglected.

First the 97 gm weight was loaded, as shown in Fig. 22 and the angular rotation at points B and C were measured directly from the rotations of the extensions of both ends of the beam, as shown in Fig. 22. The sidesways were measured from the horizontal displacements at B and C. The value used in the calculation was the average of the two. The measured and calculated results are shown in Table 8.

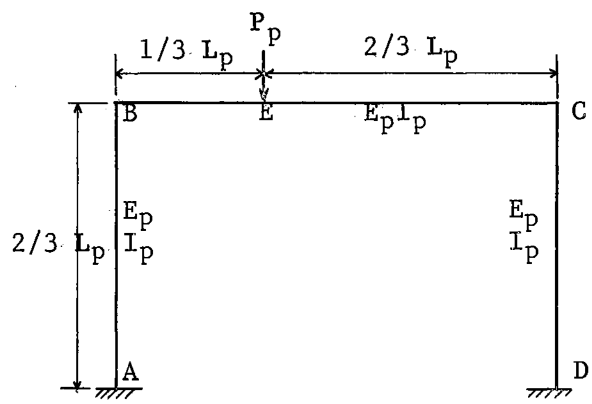
The percentage of difference in this test was rather large. It was understandable since, because the weight was too large for the model, during the test the model was being distorted very much, as shown in Fig. 22.

From previous tests it was observed that measurement of angular rotation by measuring of the rotation of beam extensions not only gives trouble but also is not accurate enough. The use of a moment indicator in this test would be necessary. This is described in the following paragraph.

As shown in Fig. 24, B and C are the points on a member of a structure where one wants to know the rotations when the structure is loaded.

One bar is attached to each point, as shown, so that when the points B and C rotate, the distances between two attached bars at certain points will change. By measurement of the change of distances, the angular rotations at B and C can be found by certain calculation. This is applicable only when rotations are small.

In this test balsa wood was used to construct the moment indicator because of its lightness and the ease of construction. Thin balsa wood strips were attached to the connections B and C of the model, and the 27.21 gm weight was loaded. This loaded model with moment indicator is shown in Fig. 23. The measured and final calculated results are shown in Table 9. The results are sufficiently accurate for this study.



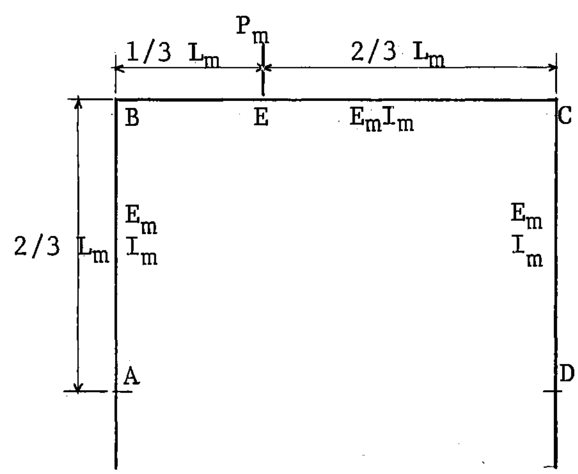
$E_p = 2.9 \times 10^6$ lb/in^2

$I_p = 911$ in^4

$L_p = 30 \times 12$ in

$P_p = 18 \times 10^3$ lb

Figure 20. Prototype No. 1



$E_m = 0.422 \times 10^6$ lb/in^2

$I_m = 7.60 \times 10^{-6}$ in^4

$L_m = 12$ in

$P_m = 97,\ 27.21$ gm

Figure 21. Model No. 1

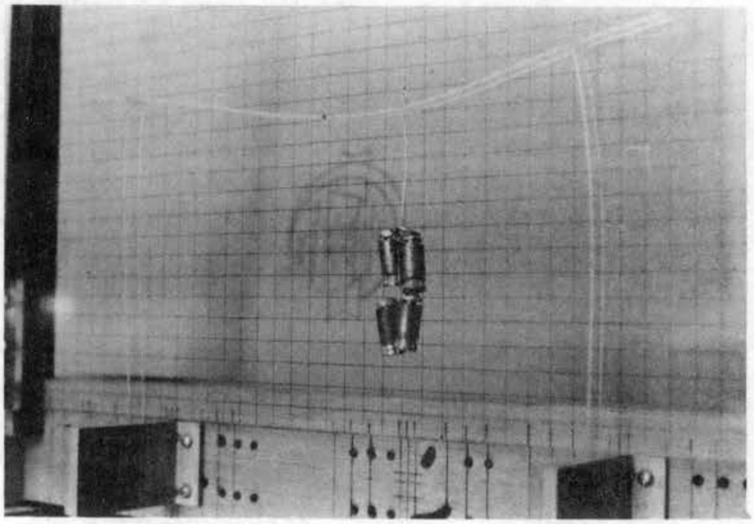


Figure 22. Model No. 1 Under 97 gm Weight

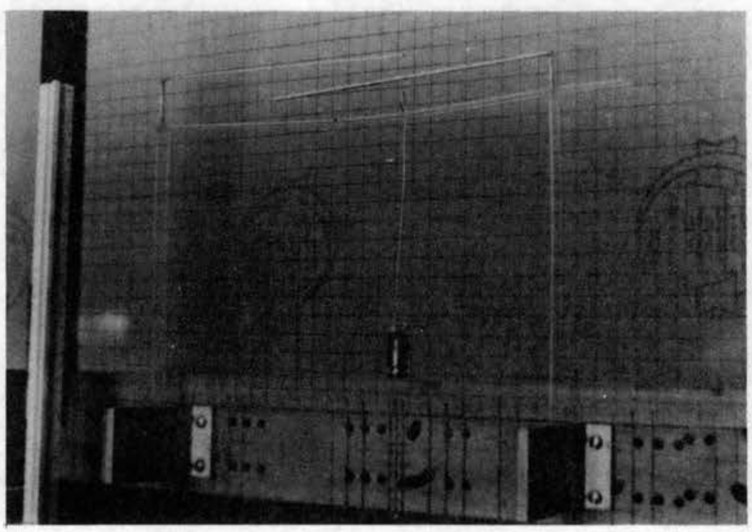


Figure 23. Model No. 1 Under 27.2. gm Weight

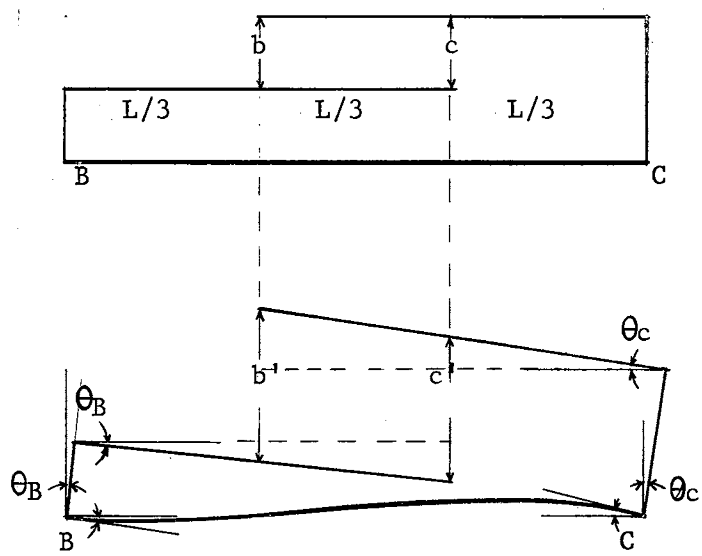


Figure 24. Moment Indicator

Let b" and c" be changes of the distance of the bars

$$b'' = b' - b = (2L/3) \times \theta_C + \theta_B L/3 \quad \text{. . . . . (1)}$$

$$c'' = c' - c + (2L/3) \times \theta_B + \theta_C L/3 \quad \text{. . . . . (2)}$$

From (1), (2)

$$2\theta_B + \theta_C = 3c''/L \quad \text{. . . . (3),} \qquad 2\theta_C + \theta_B = 3b''/L \quad \text{. . . . . . . (4)}$$

Solve (3), (4)

$$\theta_B = \frac{1}{L}(2c'' - b'') \quad \text{. . . . . (5)}$$

$$\theta_C = \frac{1}{L}(2b'' - c'') \quad \text{. . . . . (6)}$$

When the left hand side bar is above the right hand side bar then:

$$\theta_B = -\frac{1}{L}(2c'' - b'') \quad \text{. . . . . (7)}$$

$$\theta_C = -\frac{1}{L}(2b'' - c'') \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (8)$$

These equations are applicable only when θ_B and θ_C are very small.

$P_m = 97$ gm , $\sigma = 1/10.9$, $\alpha = 2.75$

$\theta_B = 0.02$ Radian, $\theta_C = -0.01$ Radian, $\delta = 0.591''$

TABLE 8

RESULTS FROM ANALYSIS OF MODEL NO. 1

BY LOADING 97 gm WEIGHT

Mem.	E	I	L	Mom.	Moment from Model Analysis k-ft	Moment From Slope Deflection Method	Percentage of Difference
AB	2.9 x 10^6 lb/in^2	911 in^4	20'	M_{AB}	$\frac{2EI}{L}(\theta_B - \frac{3}{L}) = 20.55$	18.5	11.1%
				M_{BA}	$\frac{2EI}{L}(2\theta_B - \frac{3\delta}{L}) = 57.1$	49.1	16.3%
BC	2.9 x 10^6 lb/in^2	911 in^4	30'	M_{BC}	$= -M_{BA} = -57.1$	-49.1	16.3%
				M_{CB}	$= -M_{CD} = 52.8$	41.0	28.8%
CD	2.9 x 10^6 lb/in^2	911 in^4	20'	M_{CD}	$\frac{2EI}{L}(2\theta_C - \frac{3\delta}{L}) = -52.8$	-41.0	28.8%
				M_{DC}	$\frac{2EI}{L}(\theta_C - \frac{3\delta}{L}) = -34.4$	-26.54	29.6%

$P_m = 27.21$ gm, $\sigma = 1/3.06$, $\alpha = 9.78$.

$\theta_{Bm} = 0.0515$ Radian, $\theta_{Cm} = -0.0227$ Radian, $\delta_m = 0.0485''$

$\theta_B = \theta_{Bm} \times \sigma = 0.01685$ Radian.

$\theta_C = \theta_{Cm} \times \sigma = -0.00742$ Radian.

$\delta = \delta_m \times \alpha = 0.475''$

Calculated stresses and their comparison with those from slope-deflection method are shown on next page.

TABLE 9

STRESSES FROM ANALYSIS OF MODEL NO. 1 BY LOADING OF 27.21 gm WEIGHT

MEM.	E	I	L	Stress	Stress From Model	Stress From Slope Deflection Method	Percentage of Difference
AB	2.9 x 10^6 lb/in^2	911 in^4	20'	M_{AB}	$\frac{2EI}{L}(\theta_B - \frac{3\delta}{L}) = 19.9$ K-Ft	18.50 K-Ft	7.57%
				M_{BA}	$\frac{2EI}{L}(2\theta_B - \frac{3\delta}{L}) = 50.8$ K-Ft	49.10 K-Ft	3.46%
				V_{AB}	$V_{AB} = V_{BA} = -12.23$ K	-12.25 K	0.16%
				V_{BA}	$V_{BA} = -V_{BC} = -12.23$ K	-12.25 K	0.16%
				H_{AB}	$-\frac{6EI}{L^2}(\theta_B - \frac{2\delta}{L}) = -3.54$ K	-3.38 K	4.73%
				H_{BA}	$-\frac{6EI}{L^2}(\theta_B - \frac{2\delta}{L}) = -3.54$ K	-3.38 K	4.73%
BC	2.9 x 10^6 lb/in^2	911 in^4	30'	M_{BC}	$\frac{2EI}{L}(2\theta_B + \theta_C) - \frac{Pab^2}{L^2} = -47.9$ K-Ft	49.1 K-Ft	2.44%
				M_{CB}	$\frac{2EI}{L}(2\theta_C + \theta_B) + \frac{Pa^2b}{L^2} = 42.46$ K-Ft	41.06 K-Ft	3.56%
				V_{BC}	$-[\frac{6EI}{L^2}(\theta_B + \theta_C) - \frac{Pb}{L} - \frac{Pab}{L^3}(b - a)] = 12.23$ K	12.25 K	0.16%
				H_{BC}	$H_{BC} = H_{BA} = -3.54$ K	-3.38 K	4.73%
				H_{CB}	$H_{CB} = -H_{CD} = -3.13$ K	-3.38 K	7.40%
				V_{CB}	$-[\frac{6EI}{L^2}(\theta_B + \theta_C) + \frac{Pa}{L} - \frac{Pab}{L^3}(b - a)] = 5.77$ K	5.75 K	0.35%
CD	2.9 x 10^6 lb/in^2	911 in^4	20'	M_{CD}	$\frac{2EI}{L}(2\theta_C - \frac{3\delta}{L}) = -38.1$ K-Ft	-41.06 K-Ft	7.22%
				M_{DC}	$\frac{2EI}{L}(\theta_C - \frac{3\delta}{L}) = -23.81$ K-Ft	-26.54 K-Ft	10.30%
				V_{CD}	$V_{CD} = -V_{CB} = -5.77$ K	-5.75 K	0.35%
				V_{DC}	$V_{DC} = V_{CD} = -5.77$ K	-5.75 K	0.35%
				H_{CD}	$\frac{6EI}{L^2}(\theta_C - \frac{2\delta}{L}) = 3.13$ K	3.38 K	7.40%
				H_{DC}	$\frac{6EI}{L^2}(\theta_C - \frac{2\delta}{L}) = 3.13$ K	3.38 K	7.40%

5-4 An analysis of a two-bay one-story frame was made by indirect and direct model methods.

The tests of simple portal frames all had desirable results. The following test was made to see if a more complicated structure would do as well. A two-bay one-story portal frame was chosen for analysis. The prototype and the model are shown in Figs. 25 and 26 and Fig. 27.

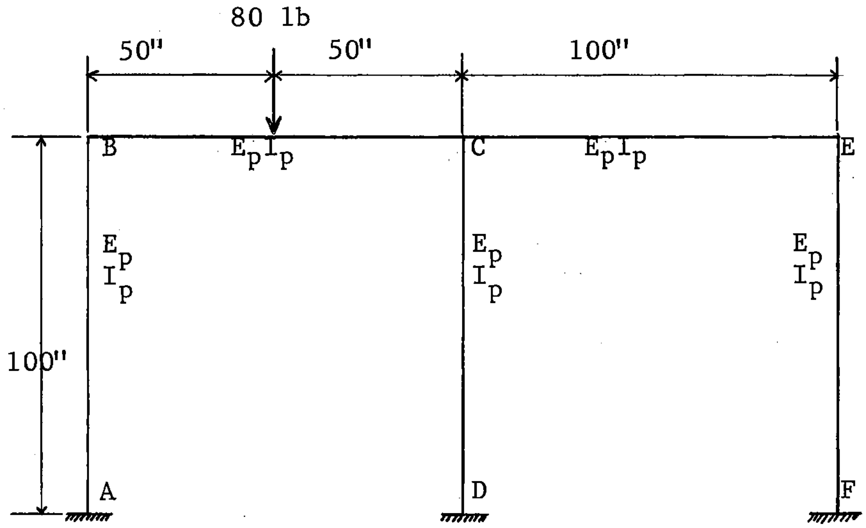
Indirect method:

The model was mounted and deformations were applied at A and D. The measured values and final calculated results are shown in Table 10. The results were close to those from the slope-deflection method.

Direct method:

The balsa wood moment indicators were attached to the model at the three points B, C, and E. Then the model was mounted and submitted to the loading of a 26.25 gm weight at 0 as shown in Fig. 28. The measured values and calculated final results are shown in Table II. The results generally were close, but for some, especially M_E, the difference ran as high as 47.9%. After a careful study of the measured angular rotations, sidesway, and the same deformations from the slope-deflection method with Table 11, it seems that in the condition of loading that made large and comparatively small stresses exist in a structure, the model (direct) method could obtain rather close values for those large stresses and probably rather distorted values for small stresses. This is explained by the fact that a slight difference in angular rotation and sidesway by model method would not greatly affect the calculation of large stresses, but would affect the calculation of small stresses very much. To show this effect the model was loaded again with two 26.25 gm weights at 0 and 0', as shown in Fig. 29 and Fig. 30. The measured value and results are shown in Table 12. Results generally came out fairly close but in the two extremely small moments, M_{BC} and M_{CD}, each equal to zero, the percentage of difference became infinite.

This effect can only be eliminated when the measured rotations and sidesways are almost exactly the same values as those obtained from mathematical methods; this is impossible to achieve since no precision apparatus is used in this method. In design, however, these extremely small stresses generally do not control the determination of member sizes. For instance, for the member CD of the structure, from Table 12, the calculated values are M_{CD} = -19.75 in-lb, M_{DC} = 9.87 in-lb, V_{CD} = V_{DC} = 95.27 lb, and H_{CD} = H_{DC} = 0.297 lb. It is clear that V_{CD} and V_{DC} govern the design of this member. The values of V_{CD} and V_{DC} differ only slightly from the mathematical results.



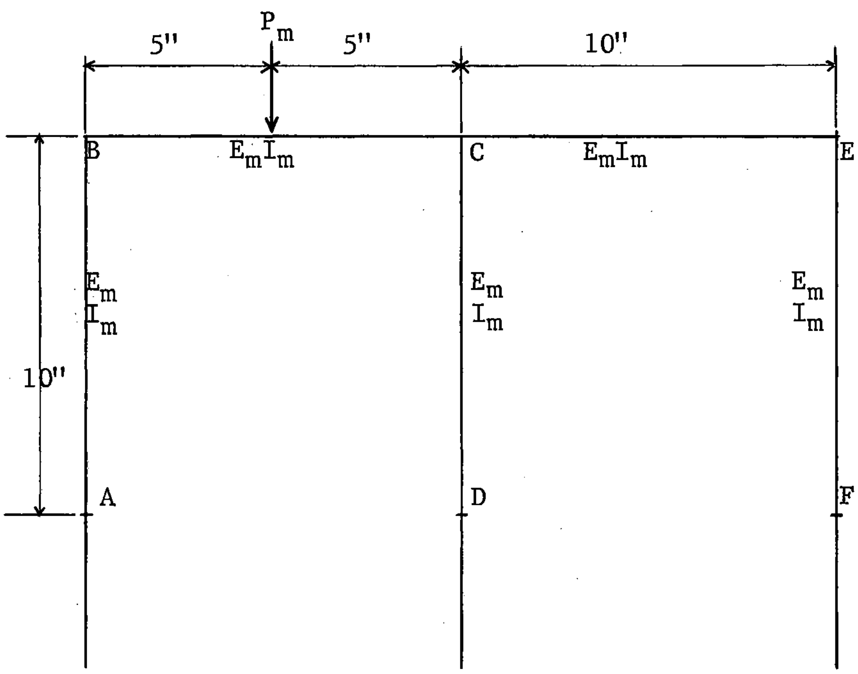
$E_p = 2.9 \times 10^6$ lb/in

$I_p = 8.51$ in^4

$P_p = 80$ lb

$L_p = 100$ in

Figure 25. Prototype No. 3



$E_m = 0.422 \times 10^6$ lb/in

$I_m = 8.51 \times 10^{-6}$ in^4

$P_m = 26.25$ gm

$L_m = 10$ in

Figure 26. Model No. 3

TABLE 10

RESULTS OF PROTOTYPE NO. 3 BY INDIRECT ANALYSIS OF ITS MODEL

Applied Deformation at A and D		Vertical Deflection at 0 in	Influence Value	Average Influence Value	Load lb	Stress	Stress from Slope Deflection Method	Percentage of Difference
At A	↻ + 0.25	+0.0485	+1.941"	2.9705 in	80	M_A 236 lb-ft	M_A 234.4 lb-ft	0.68%
	↺ - 0.25	-0.1000	+4.000"					
	→ - 1"	+0.0844	-0.0844	-0.09695	80	H_A -7.76 lb	H_A -7.85 lb	1.15%
	← + 1"	-0.1095	-0.1095					
	↑ - 1"	_0.445	-0.445	-0.4560	80	V_A -36.5 lb	V_A -36.72 lb	0.61%
	↓ + 1"	-0.467	-0.467					
At D	↻ + 0.25	-0.1188	-4.75"	-3.7200 in	80	M_D -297.5 lb-ft	M_D -312.5 lb-ft	4.80%
	↺ - 0.25	+0.0672	-2.69"					
	→ - 1"	-0.1172	+0.1172	+0.10315	80	H_D +8.25 lb	H_D 8.44 lb	2.25%
	← + 1"	+0.0891	-0.0891					
	↑ - 1"	+0.567	-0.567	-0.582	80	V_D -46.6 lb	V_D 47.49 lb	1.8%
	↓ + 1"	-0.597	-0.597					

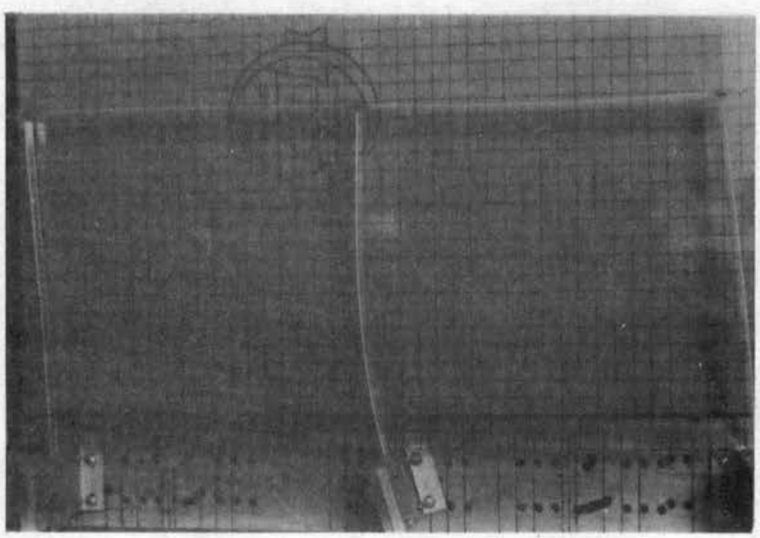


Figure 27. Indirect Test of Model No. 3

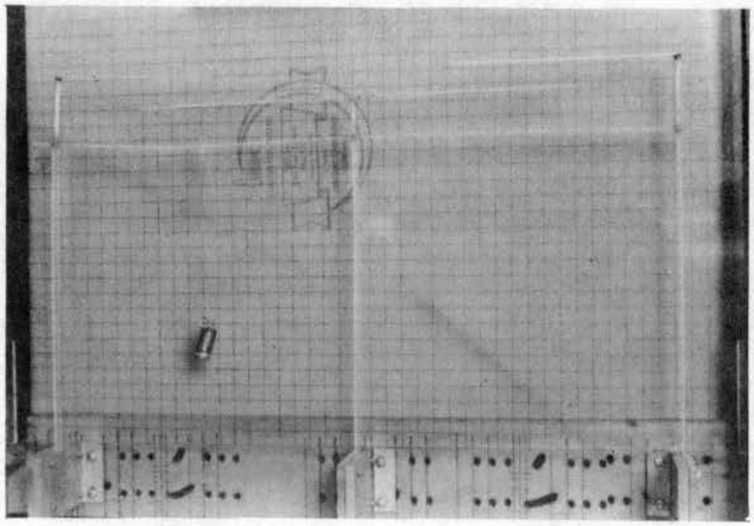


Figure 28. Model No. 3 Under Direct Test

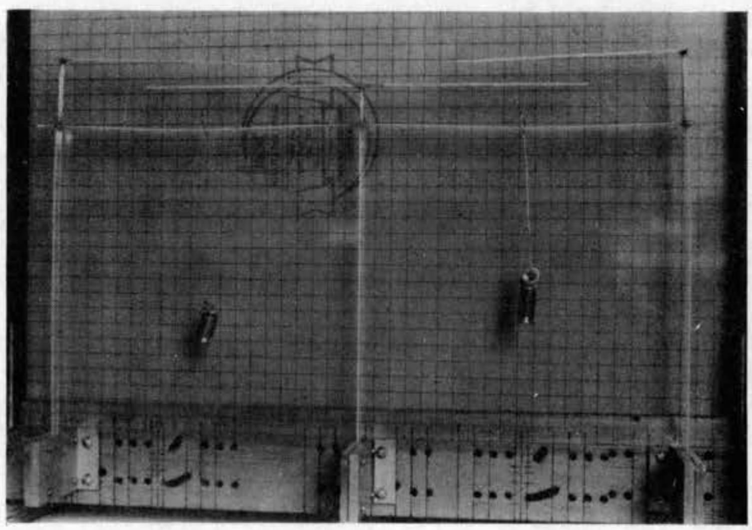


Figure 29. Model No. 3 Under Symmetrical Loading

From Fig. 25, 26:

$E_p = 2.9 \times 10^6$ lb/in^2, $I_p = 8.51$ in^4, $P_p = 80$ lb, $L_p = 100''$.

$E_m = 0.422 \times 10^6$, $I_m - 8.51 \times 10^{-6}$ in^4, $L_m - 10''$.

If $P_m = 26.25$ gm

$$\alpha = \frac{1}{5}, \qquad \sigma = \frac{1}{100}.$$

By model analysis the rotations and sidesway of prototype are as following:

$$\theta_B = 0.00066, \ \theta_C = 0.000466, \ \theta_E - 0.0002,$$

$$\delta = 0.0078''$$

TABLE 11

RESULTS OF PROTOTYPE NO. 3 BY ANALYZING ITS MODEL WITH 26.25 gm WEIGHT

Moment	Moment from Model Analysis (lb-ft)	Moment from Slope-Deflection Method (lb-ft)	Percentage of Difference
M_A	$\frac{2EI}{L}(\theta_B + 0 - \frac{3\delta}{L}) = 210$	234.4	10.4%
M_B	$\frac{2EI}{L}(2\theta_B + \theta_C) - 1000 = -579$	562.5	2.9%
M_{CB}	$\frac{2EI}{L}(\theta_B + 2\theta_C) + 1000 = 866$	890	2.7%
M_{CD}	$\frac{2EI}{L}(2\theta_C + 0 - \frac{3\delta}{L}) = -576$	531.2	8.4%
M_{CE}	$\frac{2EI}{L}(2\theta_C + \theta_E) = 352$	359.3	2.0%
M_E	$\frac{2EI}{L}(2\theta_E + \theta_C) = 32.6$	62.5	47.9%
M_D	$\frac{2EI}{L}(\theta_C - \frac{3\delta}{L}) = -345$	312.5	10.4%
M_F	$\frac{2EI}{L}(\theta_E - \frac{3\delta}{L}) = 16.8$	15.6	7.7%

With an additional 80 lb load on point ' in Fig. 30, the structure is in symmetrical loading condition. For analysis, the model is also loaded at corresponding points 0 and 0'.

The scale factors of previous test are used. The obtained rotations and sidesway are shown below.

$\theta_B = 0.000522$

$\theta_C = -0.000020$

$\theta_E = -0.000506$

$\delta = 0$

The calculated stresses and their comparisons with those from slope-deflection method are shown on next page.

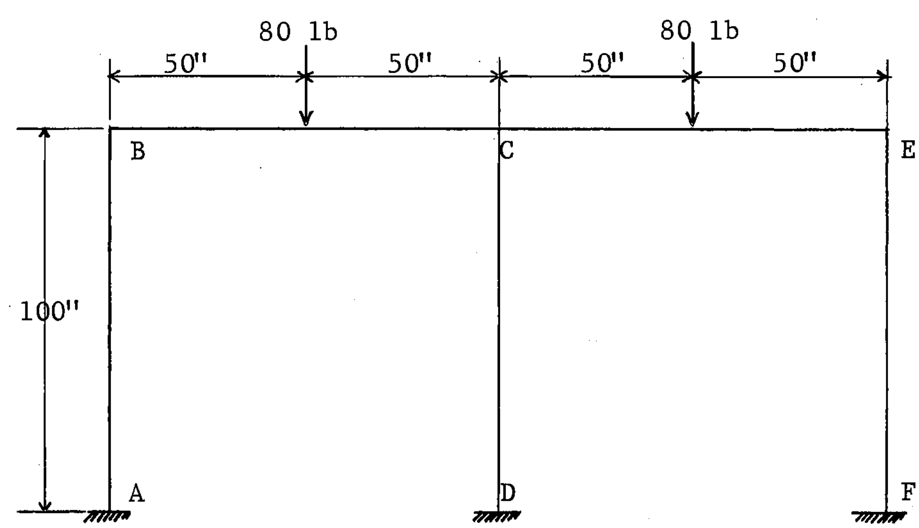


Figure 30. Symmetrical Loading

TABLE 12

RESULTS OF SYMMETRICAL LOADING OF MODEL NO. 3

BY 26.25 gm WEIGHT

Stress	Value of Stress From Model Analysis	Value of Stress From Slope Deflection Method	Percentage of Difference
M_{AB}	$\frac{2EI}{L}(\theta_B + 0) = 257.5$	250.00	3.00%
V_{AB}	$V_A = V_B = -32.54$	-32.507	0.12%
H_{AB}	$-\frac{6EI}{L^2}(\theta_B + 0) = -7.75$	-7.97	2.76%
M_B	$\pm\frac{2EI}{L}(2\theta_B + 0) = \pm 32.54$	±500.00	3.00%
V_B	$\pm[\frac{P}{2} - \frac{6EI}{L^2}(\theta_B + \theta_C)] = \pm 32.54$	±32.507	0.12%
H_B	$H_B = \pm H_A = \pm 7.75$	± 7.97	2.76%
M_{CB}	$\frac{2EI}{L}(2\theta_C + \theta_B) + \frac{PL}{B} = 1237$	1249.30	1.00%
V_{CB}	$\frac{P}{2} + \frac{6EI}{L^2}(\theta_B + \theta_C) = 47.46$	47.47	0.06%
H_{CB}	$H_{CB} = H_{BC} = -7.75$	-7.97	2.76%
M_{CE}	$\frac{2EI}{L}(2\theta_C + \theta_E) - 1000 + -1230$	-1249.30	1.53%
V_{CE}	$\frac{P}{2} - \frac{6EI}{C^2}(\theta_C + \theta_E) = 47.81$	47.49	0.70%
H_{CE}	$H_{CE} = H_{EC} = -7.51$	-7.97	5.80%
M_{CD}	$\frac{2EI}{L}(2\theta_C + 0) = -19.75$	0	
V_{CD}	$-(V_{CE} + V_{CB}) = -(47.81 + 47.46) = 95.27$	94.98	0.34%
H_{CD}	$\frac{6EI}{L^2}(\theta_C + 0) = 0.297$	0	
M_E	$\frac{2EI}{L}(2\theta_F + 0) = \pm 499$	±500.00	0.20%
V_E	$\pm[\frac{P}{2} + \frac{6EI}{L^2}(\theta_E + \theta_C)] = \pm 32.19$	±32.50	0.98%
H_E	$H_E = \pm H_{CE} = \pm 7.51$	±7.97	5.80%
M_D	$\frac{2EI}{L}(\theta_C + 0) = 9.87$	0	
V_D	$V_D = V_{CD} = -95.27$	-94.98	0.34%
H_D	$\frac{6EI}{L^2}(\theta_C + 0) = 0.297$	0	
M_F	$\frac{2EI}{L}(\theta_E + 0) = -250$	250.00	0
V_F	$V_F = V_{EF} = -32.19$	32.507	0.98%
H_F	$H_F = H_{EF} = 7.51$	7.97	5.80%

M - kip-ft; V - kip; H - kip.

5-5 Intermediate deformations were applied using the indirect model method.

All the deformations of the previous tests were applied to fixed ends of the structures. This is satisfactory for analyzing one story frames only. To find stresses in a particular part of a frame which rises more than two stories, a cut-point within the frame must be made for the application of various deformations.

A one-bay two-story portal frame with uniform load as shown in Fig. 32 was chosen for the test. A model of one 12th scale (length ratio) was constructed, as shown in Fig. 33 and Fig. 34.

Point G is the cut-point for applying the deformations. The methods of applying deformations at this point are as shown in Fig. 31.

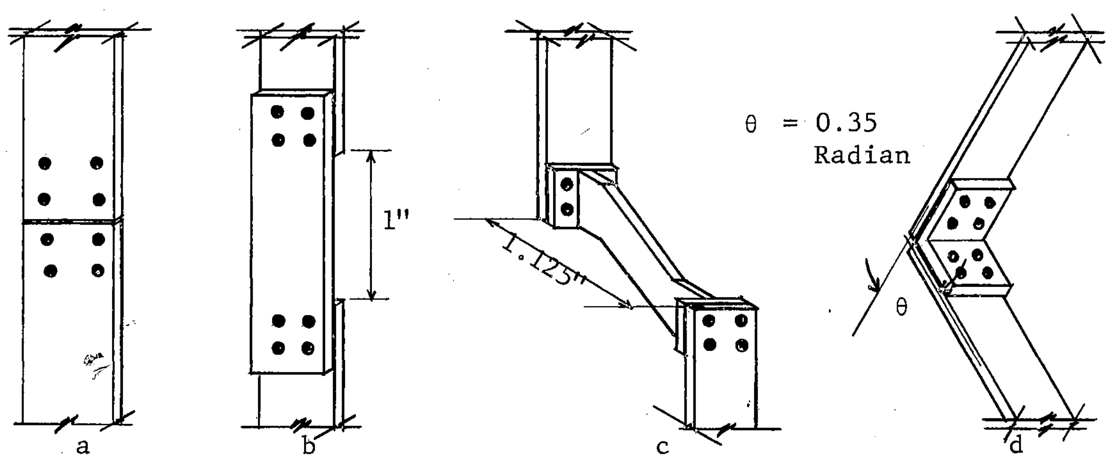
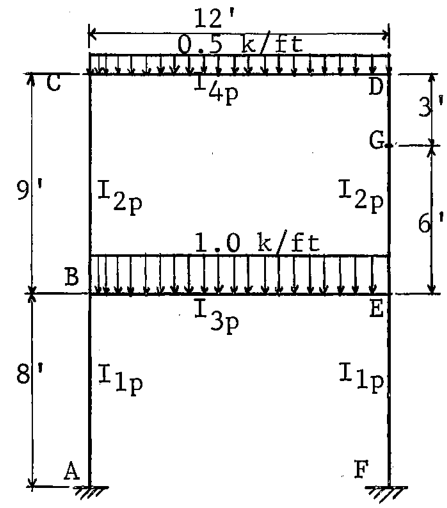


Figure 31. Methods for Applying Intermediate Deformations

On both sides of the cut, four small holes were drilled as shown in Fig. 31a. For application of axial deformation a straight piece of plexiglas with drill holes matching those on both sides of the cut was attached to the model at point G, as in Fig. 31b, to make a 1" axial deformation (other deformations equal zero). The deformation would be the influence line of the axial force at G. Similarly, relative horizontal displacement and relative angular rotation were applied by attaching small pieces of plexiglas to point G, as shown in Fig. 31c and 31d, in order to find out the influence lines of the shearing force and moment at G. Fig. 34 shows the model during application of horizontal displacement.

In this test, the horizontal and angular deformations were applied in two opposite directions to cancel the error caused by the configurations of the structure. The uniform loads were divided into several concentrated loads to make the calculation easier. The results of the test and its comparison with the results from mathematical calculation are shown in Table 13.



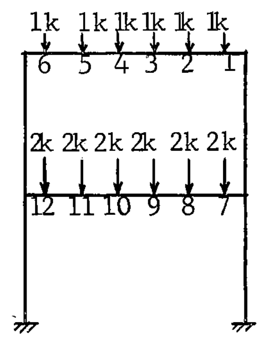
$$I_{1p} = 8.08 \times 10^2 \text{ in}^3$$

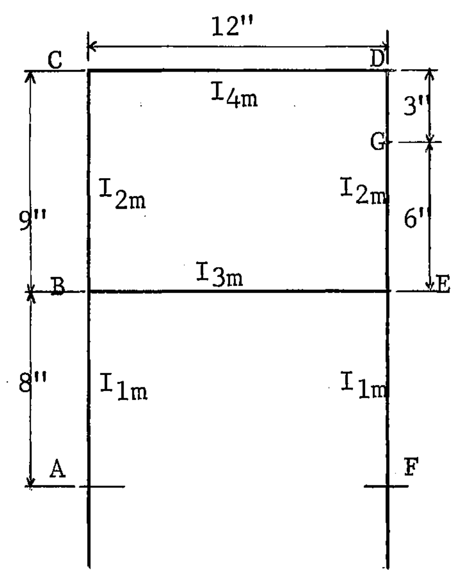
$$I_{2p} = 6.84 \times 10^2 \text{ in}^3$$

$$I_{3p} = 6.95 \times 10^2 \text{ in}^3$$

$$I_{4p} = 3.96 \times 10^2 \text{ in}^3$$

Figure 32. Prototype No. 4





$$I_{1m} = 8.08 \times 10^{-6} \text{ in}^3$$

$$I_{2m} = 6.84 \times 10^{-6} \text{ in}^3$$

$$I_{3m} = 6.95 \times 10^{-6} \text{ in}^3$$

$$I_{4m} = 3.96 \times 10^{-6} \text{ in}^3$$

$$\lambda = 12$$

$$\frac{I_{1m}}{I_{1p}} = \frac{I_{2m}}{I_{2p}} = \frac{I_{3m}}{I_{3p}} = \frac{I_{4m}}{I_{4p}}$$

Figure 33. Model No. 4

TABLE 13

RESULTS OF INDIRECT TEST OF MODEL NO. 4

Points	Apply Relative Vertical Displacement at G (-1")		Apply Relative Horizontal Displacements at G			Apply Relative Angular Rotations at G			Load (kip)
	Deflections (in)	Influence Value δ_{vi}	Deflection Due to (+1.125") at G (in)	Deflection Due to (-1.125") at G (in)	Average Influence Value δ_{hi}	Deflection Due to (-0.35 RAD.) at G (in)	Deflection Due to (+0.35 RAD.) at G (in)	Average Influence Value δ_{mi} (in)	
1	59.5/64	-59.5/64	6.1/64	-1.4/64	3.36/64	-8.8/64	0.4/64	$4.6 \times \frac{12}{64 \times 0.35}$	1
2	48.5/64	-48.5/64	10.3/64	-8.0/64	8.14/64	-15.8/64	6.2/64	$11.0 \times \frac{12}{64 \times 0.35}$	1
3	37.1/64	-37.1/64	12.6/64	-11.9/64	10.8/64	-16.9/64	10.6/64	$13.75 \times \frac{12}{64 \times 0.35}$	1
4	26.1/64	-26.1/64	13.2/64	-12.6/64	11.47/64	-13.8/64	13.3/64	$13.55 \times \frac{12}{64 \times 0.35}$	1
5	13.8/64	-13.8/64	9.6/64	-10.8/64	9.06/64	-8.8/64	11.9/64	$10.35 \times \frac{12}{64 \times 0.35}$	1
6	4.7/64	-4.7/64	2.6/64	-5.4/64	3.56/64	-3.5/64	4.2/64	$3.85 \times \frac{12}{64 \times 0.35}$	1
7	0	0	1.7/64	-1.1/64	1.24/64	2.0/64	-1.3/64	$-1.65 \times \frac{12}{64 \times 0.35}$	2
8	0	0	3.3/64	-3.2/64	2.89/64	3.3/64	-3.5/64	$-3.4 \times \frac{12}{64 \times 0.35}$	2
9	0	0	4.4/64	-4.5/64	3.96/64	2.9/64	-3.5/64	$-3.2 \times \frac{12}{64 \times 0.35}$	2
10	0	0	4.1/64	-4.5/64	3.82/64	1.2/64	-2.2/64	$-1.7 \times \frac{12}{64 \times 0.35}$	2
11	0	0	3.1/64	-3.9/64	3.11/64	0.2/64	-1.0/64	$-0.6 \times \frac{12}{64 \times 0.35}$	2
12	0	0	1.6/64	-1.3/64	1.29/64	0.2/64	-0.8/64	$-0.5 \times \frac{12}{64 \times 0.35}$	2

	V_g	H_g	M_g
By Model Analysis	$\sum_{i=1}^{6} \delta_{vi} \times 1.000$ = 2.960 kip	$\sum_{i=1}^{6} \delta_{hi} \times 1.0 + \sum_{i=7}^{12} \delta_{hi} \times 2.0$ = 1.234 kip	$\sum_{i=1}^{6} \delta_{mi} \times 1.0 + \sum_{i=7}^{12} \delta_{mi} \times 2.0$ = 1.875 k'
By Mathematical Calculation	3.000 kip	1.1933 kip	1.680 k'
Percentage of Difference	1.33%	3.41%	11.6%

CHAPTER VI

SUMMARY AND CONCLUSIONS

The results are better from the indirect method than from the direct method. However, the indirect method is more laborous, especially when the frame is more than one-story. In addition, if accuracy is desired, deformations must be applied to every point where stresses are to be known. This means that solving the redundants and reducing the frame to a statically determinate structure by the model method, then determining the forces in other parts of the structure by statical methods would sometimes yield erroneous results, because a slight error in force by the model method would cause unpredictable errors in forces calculated by the statical method with the value of the former. The comparison shown makes this clear (from Table 5).

Stress by model method	M_D	H_D	V_D
	27.1	3.4	-5.87
Percentage of difference	2.07%	0.59%	2.09%
Stresses calculated by static with values of M_D, H_D, and V_D	M_A	H_A	V_A
	22.64	3.4	12.15
Percentage of difference	23.3%	0.59%	0.82%

Therefore, in order to find reliable forces at A, deformations must be applied at A.

However, in case of complicated loading, the indirect method could be of great advantage, because in this method all the displacements in the direction of loads can be found in one operation.

Percentage of difference in stress of some structures by the direct model method is somewhat larger than that from other methods. Yet, except in complicated loading, the direct method generally has advantages of simplicity. In the direct method, once deflections and rotations of joints are found, all the stresses at connections or fixed ends can be easily calculated by slope deflection equations and the deflection of any part of the structure can be measured at the same time. Also, in this kind of test the model shows clearly how the structure behaves under loads.

The results obtained from this testing are satisfactory considering the simplicity of procedure and equipment used.

If plexiglas strips of various sizes could be accurately produced in the factories and a larger, proper mounting frame could be designed, the model method could become very advantageous as a classroom supplement. Provided reasonable specifications are framed, application of this method to practical use would be possible. Not only as a time-saving method but also as an economical one, this method has definite possibilities.

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