Spatial Price Equilibrium Analyses of the Livestock Economy

3. Spatial Price Equilibrium Models of the Pork Marketing System

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Preface

This is the third in a series of bulletins reporting the results of applying spatial price equilibrium analysis to the livestock sector of the economy. The first report dealt with the methodology of spatial analysis and application was made to several facets of the beef sector. The second study extended this research to include quarterly models for beef and to assess the impact of a wider range of disturbances on the beef sector.

In this study, spatial models were established annually for the years 1954 and 1955 for slaughter pork and for the year 1955 for live hogs for slaughter. Models were also derived for slaughter pork for each of the four quarters of 1955. Subsequent to the models depicting actual conditions, the spatial model was employed as an answer mechanism to assess the effects of changes in the basic data on the spatial and price equilibrium market structure for pork.

A complete listing of the studies reported in this series is given below:

Spatial Price Equilibrium Analyses of the Livestock Economy

1. Methodological Development and Annual Spatial Analyses of the Beef Marketing Economy
2. Application of Spatial Analysis to Quarterly Models and Particular Problems Within the Beef Marketing System
3. Spatial Price Equilibrium Models of the Pork Marketing Sector

Spatial Price Equilibrium Analyses of the Livestock Economy

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I. General Questions

The pork sector of the economy is a complex of production, farm marketing, slaughter, distribution and consumption. The levels of the activities in the producing-consuming chain vary spatially and are conditioned by many factors such as population, income, transport costs and factor availability or costs. Analyses resulting in answers to most questions relevant to the spatial aspects of the pork sector requires that the complex be simplified. In this task, the question or set of questions for which answers are sought conditions the simplification process. One set of questions might lead to construction of a particular model, while another question-set might require that a different answer mechanism be established. This is not to imply that the relationship of question-sets to answer mechanisms is single valued. But a particular answer mechanism obviously cannot be used to solve all problems for a particular segment of the economy. Therefore, any research effort should logically begin with a statement of the problems for which answers are sought. However, a statement of objectives seldom exhausts the set of questions that can be answered once a model is established.

The general questions that guided this research are: How do regional levels of income, population, and regional prices of beef affect regional consumption levels for pork? Given regional slaughter and regional consumption, what are the optimum (minimum cost) flows of pork and what are the equilibrium regional prices of pork that are consistent with the optimum flows and existing transport costs among regions? To handle these questions, spatial and price equilibrium models for pork were established for 1955 on both annual and quarterly bases, and annually for 1954.[1]

[1] For the assumptions and methodology that apply to spatial price equilibrium models, see Bulletin T-78 of this series (5).

A corollary set of particular questions included:

(1) What is the efficiency of the existing locational array of slaughtering plants with respect to hog production? Toward this end, a spatial model for optimum flows of live hogs to slaughter was established, as well as a model depicting optimum flows for pork, assuming that slaughter was production oriented. Comparing the production-oriented model with the live hog and carcass pork models provided a means of realizing this objective.

(2) What is the impact of particular disturbances on the equilibrium solution? To obtain information relevant to questions of this type, three if-then type of analyses were considered. These included assessing the effects on the basic regional factors of (1) a 20 percent increase in all interregional costs of shipping pork, (2) a 90 percent of parity price support program for pork, and (3) an extrapolation of regional hog slaughter, population and incomes to reflect 1963 conditions provided that these factors change at the same rates as they did from 1947 to 1955.

(3) Does the specification of regions affect the equilibrium solution? Realizing that any regional demarcation is largely subjective, a model was established for 1955 that involved an alternative set of regions. By then comparing the two analyses for 1955, some inferences are drawn concerning the importance of sub-division of regions in spatial analyses.

It is hoped that analyses of these questions will generate information which is basic to choice at the various decision-making levels of the pork sector and suggest the wide possibilities of spatial analysis as a mechanism for other research.

II. Regional Demarcation and Data

Criteria for the breakdown of a geographic area into regions are not presently available and, thus the procedure is largely subjective. However, the specification that results is conditioned by the problem, availability of data, and restricted by the high rate of increase in computational burden with respect to increases in model size.[2] Isard (4) points out that the ultimate spatial model of the Beckman-Samuelson

[2] For example, all possible transport rates needed for an N region model are N(N-1)/2.

variety would consider infinitesimally small producing and consuming regions. However, without some aggregation, there would be no abstraction in the spatial sense; therefore, an infinitesimal model would offer no locational information not directly observable.

For the majority of the research reported in this bulletin, the same 21-region model was used that was given in the prior reports of this series. (See Table 2.1.) However, a 29-region model was also postulated for pork in 1955 to provide an example of the consequences of alternative demarcations on the results. The regional breakdown for the larger model is presented in the results chapter.

Table 2.1—The Regional Demarcation and Values of the Predetermined Variables for 1955

Region	States Included	Basing Cities	Total Pork Supply[1]	Per Capita Disposable Income ($)[2]	Population[3] (Thousands)
			(1,000 lbs.)		
1	Vt., N.H., Maine, Mass., Conn., R.I.	Boston	130,908	1,817	9,619
2	N. Y.	N. Y.	219,117	1,970	16,021
3	Md., Del., Wash., D.C., Pa., N.J.	Phil.	709,710	1,785	20,213
4	W. Va., Va., N.C.	Roanoke	423,691	1,180	9,907
5	Ky., Tenn.	Bowling Green	426,140	1,087	6,425
6	Mich., Ohio	Toledo	800,893	1,823	16,271
7	Ill., Ind.	Chicago	1,503,949	1,865	13,630
8	Minn., Wisc.	St. Paul	1,324,867	1,512	6,892
9	Nebr., Iowa	Omaha	2,356,886	1,362	4,065
10	Ka., Mo.	Kansas City	937,673	1,523	6,261
11	Ala., Ga., S.C.	Atlanta	432,747	1,066	9,080
12	Fla.	Tampa	76,271	1,441	3,580
13	Ark., Miss., La.	Vicksburg	157,765	994	6,869
14	Okla., Tex.	Ft. Worth	411,657	1,386	10,958
15	N. Dak., S. Dak.	Bismarck	390,642	1,137	1,326
16	Wash., Ore.	Portland	157,246	1,677	4,292
17	Mont., Idaho	Butte	56,918	1,442	1,241
18	Wyo., Colo.	Denver	100,255	1,534	1,859
19	Utah, Nev.	Ely	36,875	1,528	1,032
20	Ariz., N. M.	Gallup	28,514	1,317	1,800
21	Calif.	Fresno	299,837	1,978	12,961
	United States		10,982,560	1,608	164,302

[1] Agricultural Marketing Service, "The Livestock and Meat Situation," U. S. Department of Agriculture, August 1948 and March 1956; Agricultural Marketing Service, "Livestock Slaughter," U. S. Department of Agriculture, May 1956, p. 4; and Agricultural Marketing Service, "Meat Animal, Farm Production, Disposition and Income, by States," U. S. Department of Agriculture, Bul. 184, 1956, pp. 6-10. For 1955, commercial slaughter in liveweight was added to farm slaughter and the total divided by the appropriate ratio to obtain carcass weight production by states.

[2] U. S. Bureau of Foreign and Domestic Commerces, "Supplement to Survey of Current Business," U. S. Department of Commerce, 1956, p. 141. Per capita disposable income is not available for 1955 on a state basis, so it was necessary to adjust these data on the basis of state personal income payment to obtain estimates of this series.

[3] U. S. Bureau of Foreign and Domestic Commerces, "Supplement to Survey of Current Business," U. S. Department of Commerce, 1956, p. 145. These data apply to population estimates as of July 1.

Basic data to represent the regional variables, population, income, and pork slaughter, were obtained from publications of the U. S. Departments of Agriculture and Commerce. The data are presented for 1955 in Table 2.1, along with sources and a description of the adjustments made by the authors to more closely reflect the variables that the data represent. Data for other time periods were obtained from the same sources and subjected to similar adjustments.

Transport costs for shipping fresh pork and live hogs are essentially the same as for fresh beef and live cattle, respectively, according to information proferred by authorities in the Transportation and Storage Services Division, Commodity Stabilization Service, U.S.D.A. Therefore, the same least-squares relationships were used to generate transport cost estimates that were used in the prior reports of this series.[3]

The following equation (2.1) was used to generate transport cost data among regions for live hogs.

$$T_{ij} = .0005M_{ij} + .0280\sqrt{M_{ij}} \qquad R^2 = .987 \qquad (2.1)$$

The symbol T_{ij} is the estimated rate in cents per pound for shipping live hogs between regions i and j, and M_{ij} represents either rail or highway mileage between i and j depending upon which mode of shipping is cheaper.

The following equation (2.2) was used to estimate rail rates for fresh pork.

$$C^*_{ij} = .0008M^*_{ij} + .0464\sqrt{M^*_{ij}} \qquad R^2 = .970 \qquad (2.2)$$

The symbol C^*_{ij} is used to represent the rail rate in cents per pound for shipping fresh pork between regions i and j, and M^*_{ij} represents the corresponding rail mileage involved.

Equation (2.3) was employed to reflect truck rates for fresh pork.

$$C^{**}_{ij} = .0015\,M^{**}_{ij} + .0226\sqrt{M^{**}_{ij}} \qquad R^2 = .969 \qquad (2.3)$$

For this case, C^{**}_{ij} is the estimated cost in cents per pound of shipping fresh pork by truck and M^{**}_{ij} is highway mileage.

The alternative rates, C^*_{ij} and C^{**}_{ij}, were generated for all possible

[3] For a description of the manner in which these relationships were established and reasons for the decision to reflect rates from smooth functions, rather than using actual rates, see Bulletin T-78 of this series (5). Pp 21-22.

shipments among regions, and the minimum was chosen as relevant in each case.

Since no comprehensive secondary data are available to reflect the regional consumption of pork, a structural demand equation for the U. S. was used to generate this variable. The original equation was of a linear-logarithmic form and contained the following parameter estimates:

$$Y_1 = .3513Y_2 - .9770Y_3 + .8628Z_1 + K, \qquad (2.4)\ ^4$$

where Y_1 represents per capita consumption of pork in the United States; Y_2 is the real, retail price of beef; Y_3 is the real, retail price of pork; and Z_1 is real per capita disposable income. K represents the equation constant. Since the equation is linear in logs, each coefficient can be interpreted as an estimate of elasticity.

Equation (2.4) was transformed to the following linear equation for 1955, using the definition of elasticity at a point and the 1955 U.S. levels of the variables involved.

$$Y_{1i} = .3468Y_{2i} - 1.1917Y_{3i} + .0359Z_{1i} + 50.9829 \qquad (2.5)$$

The above function (2.5) was assumed valid for each region in the spatial model, thus the subscript i refers to the i^{th} region. Other linear relationships were similarly derived from equation (2.4) for other analyses undertaken in this research.

Choice of the particular structural equation to reflect demand for pork from the many available (8, Appendix D) was due to several considerations. Most other sector analyses for this commodity occurred prior to the availability of postwar data. Also, the estimated elasticities of the equation used here were statistically significant and agreed with theoretical preconception. The consequences of using an alternative demand relationship would, of course, be quite broad. The inference is that future improvements in spatial models of the type used in this report will depend a good deal on improvements in sector analyses, as well as upon availability of regional data.

III. Results of the Study

For all analyses reported in this section, the methodology outlined in the first report of this series was used (5). Briefly, this process included approximating a set of price differentials for pork among regions rela-

[4] See Wallace and Judge (8).

tive to a base; determining price in the base region using a demand equation for pork (2.5), levels of income, population, etc. for the period under consideration and the approximate set of differentials; establishing regional prices from price in the base region and the approximate differentials; again using the demand equation (2.5) to estimate regional consumption; differencing regional consumption and production to obtain estimates of excess supplies and demands; using the simplex method to find an optimum flow solution among regions; and, finally, repeating the process, using new price differentials for pork consistent with the optimum flow solution.

Although repetitious, it is necessary to again remind the reader that the degree to which the results that follow describe reality is dependent upon the trichotomy of assumptions, data and method.

A. Spatial Model for Slaughter Pork, 1955

For 1955, the United States average retail price of pork (excluding lard) was 54.8 cents per pound and average per capita pork consumption (production) was about 66.8 pounds. For the 10-year post-war period, 1946-55, per capita pork consumption averaged 67.5 pounds, therefore 1955 represented a typical post-war year for pork. Conversely, beef production was at an all-time high (82 pounds per capita) in 1955.

Regional incomes in 1955 ranged from $994 per person in Region 13 (Arkansas, Mississippi and Louisiana) to $1,978 for Region 21 (California). Regional beef prices ranged from 64.5 cents in Region 16 (Washington and Oregon) to 69.6 cents in Region 1 (New England).[5] Using income and beef prices along with equation (2.5), regional pork prices and consumption were estimated (Table 3.1) by employing the techniques described earlier. Regional surpluses and deficits and minimum cost flows for pork are also given in Table 3.1, as well as the U_i (price differentials for the surplus regions) and V_j (price differentials for the deficit regions) and the indirect costs.

The so-called "indirect costs" appear in light type in the body of Table 3.1. They are computed by:

$$C'_{ij} = C_{ij} - (V_j - U_i) \qquad (3.1)$$

where C_{ij} is the unit transport cost for shipping pork between surplus

[5] These regional beef prices were estimated by spatial analysis in a prior bulletin of this series (5), Pp. 25-34.

Table 3.1—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork (1955)

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.)						
				5	7	8	9	10	15	V_j
1	55.84	709,437	— 578,529	.57	.14	**578,529**	.05	.16	.47	2.93
2	55.67	1,271,668	—1,052,551	.33	**29,168**	**296,714**	**726,669**	.03	.49	2.76
3	55.53	1,472,946	— 763,236	.32	0	0	**763,236**	.03	.51	2.62
4	55.21	510,031	— 86,340	**86,340**	.14	.16	.09	.06	.80	2.30
5	54.00	316,597	109,543							
6	54.47	1,222,776	— 421,883	.66	**421,883**	.08	.11	.28	.75	1.56
7	53.77	1,052,898	451,051							
8	52.99	449,624	875,243							
9	52.91	243,570	2,113,316							
10	53.08	410,475	527,198							
11	54.89	432,844	— 97	**97**	.56	.52	.31	.11	1.04	1.98
12	55.83	215,673	— 139,402	**23,106**	.51	.37	.09	**116,296**	.77	2.92
13	54.61	310,568	— 152,803	.62	.95	.72	.22	**152,803**	1.27	1.70
14	54.42	647,824	— 236,168	1.39	**1.57**	.79	.14	**236,168**	1.01	1.51
15	52.87	68,584	322,058							
16	55.71	288,071	— 130,825	2.65	1.95	.66	.53	.96	**130,825**	2.80
17	54.48	74,996	— 18,078	3.01	2.28	.90	.87	1.27	**18,078**	1.57
18	54.26	119,352	— 19,097	2.26	**1.79**	.70	**19,097**	.32	.31	1.35
19	55.50	64,223	— 27,348	2.18	**1.67**	.65	.03	.48	**27,348**	2.59
20	55.21	100,015	— 71,501	1.83	1.52	.66	**71,501**	.01	.32	2.30
21	56.41	1,000,391	— 700,554	2.01	**1.48**	.53	**532,816**	**21,931**	**145,807**	3.50
			U_i	1.07	.86	.08	0	.17	—.04	

Total shipments = 4,398,412.
Total costs = $109,337,827.

region i and deficit region j, V_j and U_i are as defined previously and C'_{ij} is the indirect cost.

Formerly, it was stated that the price of pork in Region i is determined by the price of pork in the base region plus the price differential between the base and i^{th} region. Since for a given set of price differentials, Y_{30} (price in the base region) is constant, equation (3.1) can be rewritten as:

$$C'_{ij} = C_{ij} - [\ (Y_{30}+V_j)-(Y_{30}+U_i)\ ] \qquad (3.2)$$

and since $(Y_{30}+V_j)$ is the price of pork in the j^{th} deficit region and $(Y_{30}+U_i)$ is the price of pork in the i^{th} surplus region, equation (3.2) reduces to:

$$C'_{ij} = C_{ij} - (Y_{3j} - Y_{3i}) \qquad (3.3)$$

Therefore, the so-called "indirect costs" are nothing more than the cost of shipping a pound of pork between two regions less the difference in prices for the two regions that are consistent with a particular set of shipments. Obviously, if C'_{ij} was negative for any i and j, the associated shipment pattern would not be optimum.

Since all the C'_{ij} appearing in Table 3.1 are positive, the shipment program is optimum. The positive C'_{ij} values can be interpreted as the amount that transport costs would have to be reduced between any two regions before a shipment would be favorable. Alternatively, these values represent the distortion from equilibrium price that would have to exist in order to induce shipment. Where C'_{ij} is zero, an alternative optimum shipment exists.

In regard to surplus and deficit regions, note that the majority of surpluses occur in the mid-west where large supplies of corn are available. The high degree of locational concentration of pork slaughter contrasts with beef slaughter[6] which takes place over a wide geographical area. One of the most striking differences in the locational intensities of beef and pork slaughter occurs in Region 21 (California). According to these estimates, California imported about 70 percent of the total pork consumed there in the form of slaughtered pork. Hassler (3) estimated the same figure at about 61 percent for 1954 by using United States per capita consumption data for pork applied to existing popula-

[6] See a prior report of this series (7) for the locational nature of beef production and slaughter.

tion estimates in California[7]. However, in an earlier study of this series (7), p. 32, it was estimated that in 1955, California slaughtered about 66 million pounds more beef than she consumed. For some of the other regions, the classification of surplus and deficit is not so clear cut. For example, Region 11 (Alabama, Georgia and South Carolina) was estimated to be only 97 thousands pounds deficit in pork slaughter. In such instances, regions are difficult to classify without reservation due to inaccuracies in the data used, etc. Region 11 was divided into individual states in the 29 region model that is presented later in this chapter.

The concentration of surplus slaughter in the mid-west is reflected in the regional price differences. Retail pork price ranged from 52.9 cents per pound in Region 9 (Iowa and Nebraska) and 15 (North and South Dakota) to 56.4 cents in California. East Coast prices were also higher relative to the mid-west. For example, the retail price of pork in Region 1 (New England) was estimated at 55.9 cents and in Region 2 (New York) at 55.7 cents.

Because of the regional differences in pork and beef slaughter, the differences in retail prices for these commodities varied from region to region (Table 3.2).

The differences in regional beef and pork prices are significantly less in the western areas than in other sections of the United States since the West was generally surplus in beef slaughter and deficit in pork slaughter. Oklahoma and Texas were also surplus in beef slaughter and deficit in pork slaughter, but the price difference is less marked for this region (14) due to its being centrally located.

Total transport costs for the optimum pork shipment program were estimated at about $109 millions. Total shipments were approximately 4,398 million pounds. Therefore, the average cost of shipping was about $2.48 per hundred pounds for the model considered.

Figure 1 illustrates the position of surplus and deficit regions in pork slaughter and the optimum shipment pattern for 1955.

The figures in the breaks in the lines of Figure 1 denote the amounts of pork in million pounds shipped among regions in an optimum shipment program. For example, Kansas and Missouri would have shipped 236 million pounds of slaughtered pork to Oklahoma and Texas as part of an optimum shipment program in 1955.

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[7] In the study cited, regional effects of the price of beef and income on consumption were ignored for the regional breakdown considered in this study, California had the largest per capita income. Thus the discrepancy is probably due to the income factor as well as the different time periods considered.

Table 3.2—Regional Retail Price Estimates of Beef and Pork, 1955

Region	Re'ail Price of Beefa (cents/lb.)	Retail Price of Pork (cents/lb.)	Retail Price Difference (cents/lb.)
1	69.6	55.9	13.7
2	69.4	55.7	13.7
3	69.3	55.5	13.8
4	69.1	55.2	13.9
5	68.2	54.0	14.2
6	68.3	54.5	13.8
7	67.6	53.8	13.8
8	66.8	53.0	13.8
9	66.7	52.9	13.8
10	66.9	53.1	13.8
11	68.8	54.6	14.2
12	69.5	55.8	13.7
13	68.2	54.6	13.6
14	67.1	54.4	12.7
15	66.2	52.9	13.3
16	64.5	55.7	8.8
17	65.3	54.5	10.8
18	65.9	54.3	11.6
19	65.1	55.1	10.6
20	66.7	55.2	11.5
21	64.8	56.4	8.4
U. S.	67.7	54.8	12.9

a These estimates obtained from Bulletin T-78 (5) of this series.

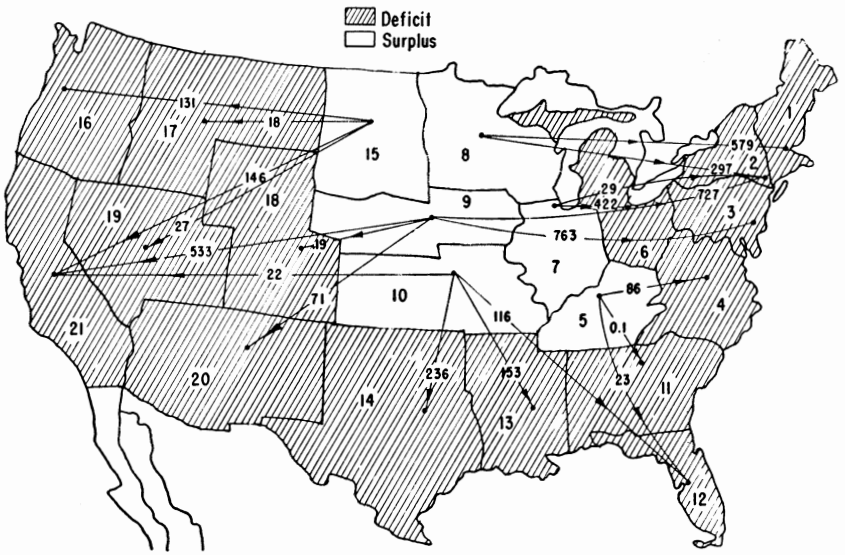

Fig 1 Optimum Shipment Program for Slaughter Pork (Million Pounds), 1955

B. Optimum Flows of Live Hogs for Slaughter, 1955

A considerable number of live hogs are shipped interregionally prior to slaughter. By differencing regional estimates of farm production of hogs for slaughter[8] and regional slaughter, each region was designated as being either surplus or deficit in live hogs for slaughter. An optimum live hog shipment program was then derived for 1955, using live hog shipment costs among regions (Figure 2).

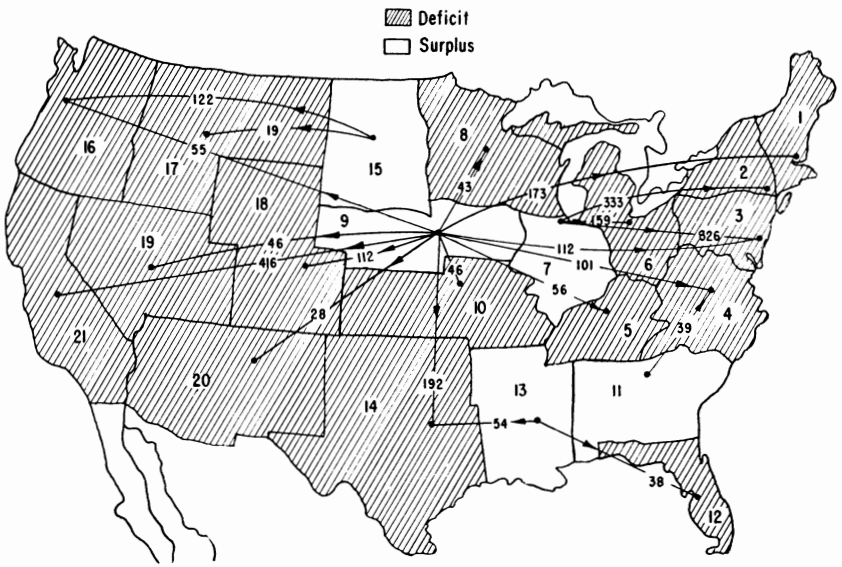

Fig. 2 Optimum Shipment Program of Hogs for Slaughter (Million Pounds), 1955.

Only 5 regions were estimated as being surplus in live hogs for slaughter in 1955. Comparing Figures 1 and 2, live hogs are exported from Regions 11 and 13 and both regions import slaughtered pork. Conversely, Regions 5, 8 and 10 exported pork but imported live hogs for slaughter. The remaining regions either exported or imported consistently. Region 9 (Iowa and Nebraska) provided the source for the bulk of the live hogs for slaughter. This probably indicates that local slaughtering facilities were unable to handle the large amount of hogs produced in this region in 1955.

Regions 11 (Alabama, Georgia and South Carolina) and 13 (Arkansas,

[8] This series was estimated by multiplying estimates of regional farm production of hogs times the ratio of total U. S. slaughter to total farm producton for 1955. To convert from live weight to carcass weight estimates, the dressing ratio of 1:1.75 pounds was used.

Mississippi and Louisiana) were also estimated as being surplus in live cattle and deficit in slaughter beef.[9] This, coupled with the results for pork, provides some evidence that slaughtering facilities have not kept pace with increased production of livestock in the Southeast, at least up to 1955.

For the optimum live hog program, a total of about 2,970 million pounds (live weight) was shipped interregionally at a total transport cost of approximately $41 millions. Thus, the average live weight cost of shipment was $1.38 per hundred pounds of live hogs. However, on a carcass weight base, the average cost was $2.42 per hundred pounds.

C. Spatial Equilibrium Models for Pork, Assuming Slaughter Was Production Oriented in 1955

From the first two analyses presented, it was estimated that about 28 percent of all interregional shipments of pork were in the form of live hogs. To evaluate the indirect movements inherent in the two-stage system of shipping, an analysis was carried out for 1955, assuming that all slaughter of hogs occurred in the producing regions. The results of this analysis are presented in Table 3.3.

The only change in the classification of regions from the initial 1955 analysis was that Region 11 (Alabama, Georgia, South Carolina) became surplus when it was assumed that slaughter was production oriented. Region 11 was a borderline case in both analyses, however. Although surplus and deficit classifications remained virtually unchanged between the initial 1955 analyses and this model, one cannot infer that pork slaughter was production oriented in 1955, since the level of slaughter in some of the surplus regions was not as large as needed to take care of all local hog production.

Although the classification of regions for this analysis is much the same as for the initial 1955 analysis, the levels of surpluses and deficits were such that the optimum shipment programs differed slightly. This resulted in some differences in regional prices between the two analyses. These differences are minute, however.

Total shipments for the production-oriented slaughter program were 5,958 million pounds and total transport costs were about $144 millions. Average shipment cost for this program was, then, $2.41 per hundred pounds.

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[9] See Bulletin T-79 of this series (7).

Table 3.3—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, Assuming Hogs were Slaughtered where Produced, 1955

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)							
	cents/ pound	1,000 pounds	1,000 pounds	5	7	8	9	10	11	15	V_j
1	55.83	709,606	— 677,800	.66	.14	**677,800**	.05	.25	1.26	.47	2.93
2	55.66	1,271,949	—1,242,850	.42	**690,483**	**172,496**	**379,871**	.12	1.05	.49	2.76
3	55.52	1,473,301	—1,299,215	.41	0	0	**1,299,215**	.12	1.02	.51	2.62
4	55.29	509,142	— 165,308	**77,878**	.05	.07	**87,430**	.06	.53	.71	2.39
5	54.08	316,020	77,878								
6	54.46	1,223,061	— 512,780	.75	**512,780**	.08	.11	.37	1.86	.75	1.56
7	53.76	1,053,137	1,203,263								
8	52.98	449,744	805,296								
9	52.90	243,641	2,902,180								
10	53.16	409,913	501,693								
11	54.71	434,842	20,201								
12	55.91	215,351	— 160,634	0	.42	.28	**114,035**	**26,398**	**20,201**	.68	3.01
13	54.69	309,951	— 99,781	.62	.86	.63	.13	**99,781**	1.21	1.18	1.79
14	54.50	646,841	— 375,514	1.39	1.48	.70	.05	**375,514**	2.19	.92	1.60
15	52.86	68,607	402,647								
16	55.70	288,146	— 232,107	2.74	1.95	.66	.53	1.05	3.65	**232,107**	2.80
17	54.47	75,018	— 29,114	3.10	2.28	.91	.87	1.36	4.08	**29,114**	1.57
18	54.25	119,384	— 83,261	2.35	**1.79**	.70	**83,261**	.41	3.46	.31	1.35
19	55.49	64,241	— 53,672	**2.27**	**1.67**	.65	.03	.57	3.36	**53,672**	2.59
20	55.20	100.046	— 87,700	1.92	1.52	.66	**87,700**	.10	2.77	.32	2.30
21	56.40	1,000,618	— 938,425	**2.10**	**1.48**	.53	**850,671**	.09	2.60	**87,754**	3.50
			U'_i	1.18	.86	.08	0	.26	1.81	—.04	

Total shipments (1,000 lbs.) = 5,958,161.
Total cos.s = $143,657,198.

To illustrate the optimum flow solution, assuming that hogs were slaughtered where produced, the following figure (Figure 3) is presented.

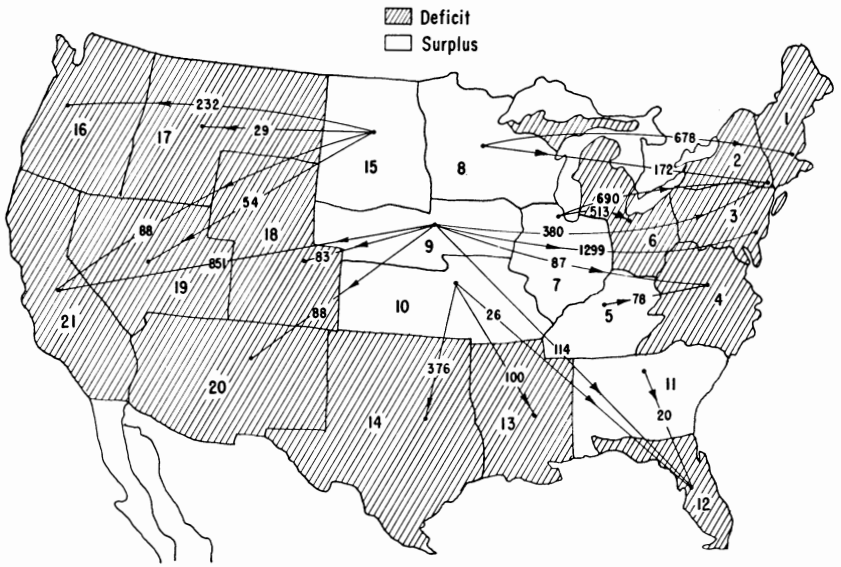

Fig. 3 Optimum Shipment Program for Pork (Million Pounds), 1955, Assuming Hogs were Slaughtered where Produced.

Optimum shipments are indicated by the lines emanating from the unshaded regions. The numbers appearing in the breaks of the lines are the amounts of pork involved in the optimum shipment program.

D. Evaluation of the Locational Efficiency of Pork Slaughter, 1955

By comparing the costs of the three programs that have been presented, some insight may be obtained into the degree of inefficiency of pork slaughter location in 1955. The relevant costs and associated shipments are presented in Table 3.4.

As indicated in the table, a production-oriented slaughtering program would have eliminated about 137 million pounds of excess shipments of pork in 1955, subject to the regional breakdown considered. This would have reduced total transport costs by about $6.7 millions. For example, the optimum shipment program for live hogs (Figure 1) indicated that Region 9 (Iowa and Nebraska) shipped 43 million pounds of live hogs to Region 8 (Wisconsin and Minnesota) and the optimum

Table 3.4—Total and Average Costs and Total Shipments of Pork, Actual and Assumed Conditions, 1955

Optimum Program	Total Shipments (1,000 lbs.)	Total Costs ($)	Average Cost ($/100 lbs.)
Pork Shipments	4,393,412	109,337,827	2.49
Live Hog Shipments for Slaughter	1,697,189[1]	41,055,799	2.42
Total Pork and Live Hog Shipments	6.095,601	150,393,626	2.43
Pork Shipments, Assuming Slaughter was Production Oriented	5,958,161	143,657,193	2.41

[1] Converted to carcass weight equivalent.

shipment program for pork (Figure 1) indicated that Region 8 shipped 678 million pounds of pork to Region 1 (New England). This two-stage shipment involved a transport cost of 6.03 cents per pound. If the pork involved in the indirect movement had been slaughtered in Region 9 and shipped directly to Region 1, a cost of only 2.98 cents per pound would have been incurred.

Unlike beef, no direct crosshauls were evident from the analyses for pork. In general, pork slaughter was production oriented to a larger degree than beef slaughter in 1955. These same comparisons for beef indicated that for the same year and the same regional breakdown, total transport costs for beef could have been reduced by about $30 millions by a production-oriented slaughter program.[10] Since total shipments for both beef and pork are about the same, this represents a substantially larger cost difference than the same figure ($6.7 millions) for pork.

E. Less Aggregative Spatial Models for Pork, 1955

For a particular spatial problem there are many alternatives relative to level of aggregation. The admissible alternatives depend to a large extent upon the interpretation of the basic assumptions that underlie the spatial model. For example, the assumption of homogeneity of product would be quite restrictive under a literal interpretation and perhaps would require that only a particular grade and cut of pork be considered. Also, time is excluded as a variable in the Samuelson-

[10] See Bulletin T-79 (7) of this series for a comparable breakdown of costs for beef shipment.

Beckman formulation; therefore, shorter than annual time periods would more closely reflect this assumption. The assumption that regional demands and supplies are concentrated at a point, if strictly interpreted, would require that an infinite number of regions be included Obviously, the assumptions cannot be strictly met. But the number of feasible alternatives is large and the importance of choice among these alternatives cannot be minimized.

As a step toward assessing the importance of regional aggregation in spatial models, an additional analysis of the pork sector was made for 1955 that included a larger number of regions. Also, to reflect disaggregation in the time dimension, separate spatial analysis was accomplished for each quarter of 1955. Estimation of these models should yield empirical evidence of the distortion that may be present in the more aggregative pork analyses.

1. **A 29-Region Model for Pork, 1955:** The initial 21-region model for pork was expanded to 29 regions by breaking up the originally surplus regions and Region 11 (Alabama, Georgia and South Carolina) into single states. Basing points were chosen for the new system of regions and transport costs for pork were established among the new set of points by using equations 2.2 and 2.3 in the manner described in Chapter 2.

The regional structure, basing points and results of the analysis are presented in Table 3.5.

All of the states that were included among the surplus regions for the 21-region model remained surplus for the new classification. However, by breaking up Region 11 (Alabama, Georgia and South Carolina) into states, Alabama and South Carolina were classified as deficit and Georgia was surplus.

Pork price estimates for the 29-region models ranged from 52.9 cents per pound in Region 10 (Minnesota) to 56.7 cents per pound in Region 29 (California). This represents a spread of 4 cents whereas for the 21-region model the price range was estimated to be 3.5 cents. For both models, price in the base region was estimated to be 52.9 cents per pound.

Total shipments for the 29-region models were about 4,453 million pounds and total costs were estimated to be $103 millions. Therefore, average costs were $2.32 per hundred pounds shipped. For the 21-region model, total shipments (4,398 million pounds) were slightly lower

Table 3.5—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, 29-Region Model, 1955

Region	States Included	Basing Points	Equil. Price	Equil. Cons.	Surplus and Deficit		
			cents/ pound	1,000 pounds	1,000 pounds	5	
1	New England	Boston	55.60	705,863	— 574,955	.42	.82
2	New York	New York	55.43	1,266,827	—1,047,710	.17	.57
3	Md., Del., Pa., N. J.	Philadelphia	55.29	1,467,540	— 757,830	.16	.55
4	W. Va., Va., N. C.	Roanoke	55.00	507,714	— 84,023	27,802	.12
5	Kentucky	Louisville	53.82	147,525	27,802		
6	Tennessee	Nashville	53.92	168,700	82,114		
7	Mich. and Ohio	Detroit	54.35	1,221,740	— 420,847	.46	.94
8	Illinois	Chicago	53.57	754,404	219,769		
9	Indiana	Indianapolis	53.74	301,179	228,597		
10	Minnesota	St. Paul	52.75	205,446	665,840		
11	Wisconsin	Oshkosh	53.21	245,953	207,628		
12	Nebraska	Grand Island	53.32	82,281	496,920		
13	Iowa	Des Moines	52.92	161,995	1,615,690		
14	Kansas	Hutchinson	53.45	128,144	254,134		
15	Missouri	Jefferson City	53.12	283,021	272,374		
16	Alabama	Birmingham	54.57	144,018	— 28,563	.32	.01
17	Georgia	Atlanta	54.43	187,662	59,837		
18	South Carolina	Columbia	55.11	100,178	— 30,385	.01	30,385
19	Florida	Tampa	55.63	214,296	— 138,025	.25	51,729
20	Ark., Miss., La.	Vicksburg	54.52	310,123	— 152,358	.78	.55
21	Okla. and Tex.	Ft. Worth	54.59	647,901	— 236,245	1.28	1.06
22	North Dakota	Bismarck	53.44	34,471	15,165		
23	South Dakota	Pierre	53.18	34,132	306,874		
24	Wash., and Ore.	Portland	56.07	290,999	— 133,753	2.02	
25	Mont., and Idaho	Butte	55.05	75,188	— 18,270	2.19	2
26	Wyo., Colo.	Denver	54.39	120,227	— 19,972	1.91	1
27	Utah, Nev.	Ely	55.62	65,008	— 28,133	1.95	2.24
28	Ariz., N.M.	Gallup	55.16	100,749	— 72,235	1.70	1.67
29	California	Fresno	56.68	1,009,277	— 709,440	1.35	1.29
					U_i	.90	1.00

Total costs = $103,137,300.
Total shipments (1,000 lbs) = 4,452,744.

and total costs ($109 millions) were higher. However, the consistency of the alternative estimates is more worth noting than the divergences.

In order to facilitate comparisons of flows between the 21- and 29-region models, Figure 4 is presented.

As a result of the increase in number of regions, the total numbers of shipments are greater for this analysis. One of the requirements necessary to the optimum flow solution is that n + m — 1 activities occur. Thus, for the 29-region model there are a total of 28 shipments involved.

For the 21-region model, Iowa and Nebraska taken as a single region, shipped west to California and to Region 18 (Colorado and

Table 3.5—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, 29-Region Model, 1955

					Origins and Quantities of Shipments (1,000 lbs.)						
	9	10	11	12	13	14	15	17	22	23	V_j
.18	.26	**574,955**	.14	.93	.14	1.10	.27	1.21	1.28	.98	2.68
.04	.01	**90,885**	.01	.90	**956,825**	1.07	.09	1.00	1.30	1.00	2.51
.04	**172,376**	0	**6,550**	.90	**578,904**	1.07	.09	.97	1.32	1.01	2.37
.15	**56,221**	.13	.12	.96	.08	1.07	.07	.59	1.58	1.14	2.08
219,769	.18	.03	**201,078**	.97	.04	1.27	.30	1.79	1.52	1.15	1.43
.61	.47	.52	.58	1.05	.28	.88	**28,563**	.40	1.84	1.14	1.65
.32	.13	.28	.28	1.06	.20	1.04	.03	.01	1.61	1.11	2.19
.51	.37	.33	.46	.95	.24	.85	**26,459**	**59,837**	1.54	.99	2.71
.84	.83	.57	.79	.92	.26	.70	**152,358**	1.10	1.93	1.24	1.60
1.20	1.27	.38	1.05	.32	.07	**171,251**	**64,994**	1.82	1.41	.74	1.67
?	1.90	.06	.92	.42	.42	.81	.89	3.00	.21	**133,753**	3.15
1	1.95	.09	1.00	.51	.58	.87	.99	3.22	**15,165**	**3,105**	2.13
.6	1.70	.33	1.16	**19,972**	.16	.24	.52	3.04	.75	.19	1.47
1.35	1.80	.29	1.15	.07	.18	.40	.67	2.95	.45	**28,133**	2.70
1.37	1.62	.47	1.28	.19	.29	**72,235**	.33	2.53	.94	.40	2.24
1.01	1.23	.02	.85	**476,948**	**79,961**	**10,648**	.17	2.04	.30	**141,883**	3.76
.65	.82	—.17	.29	.40	0	.53	.20	1.51	.52	.26	

Wyoming) and shipped east to Region 2 (New York) and Region 3 (Pennsylvania, Maryland, Delaware and New Jersey). For the 29-region model, Nebraska and Iowa entered as separate regions and as a result, Nebraska shipped to the west and Iowa shipped mostly to the east. Similarly, Kentucky and Tennessee as a single region shipped to the southeast and to North Carolina, Virginia and West Virginia (Figure 1) and as separate regions, Kentucky shipped to the Virginias and North Carolina while Tennessee shipped southeast (Figure 4). The examples mentioned are what one would expect, but some differences between the two solutions are not so obvious. For example, Wisconsin and Minnesota taken as a single region, shipped only to New York and New England (Figure 1) but taken separately, Wisconsin shipped to the

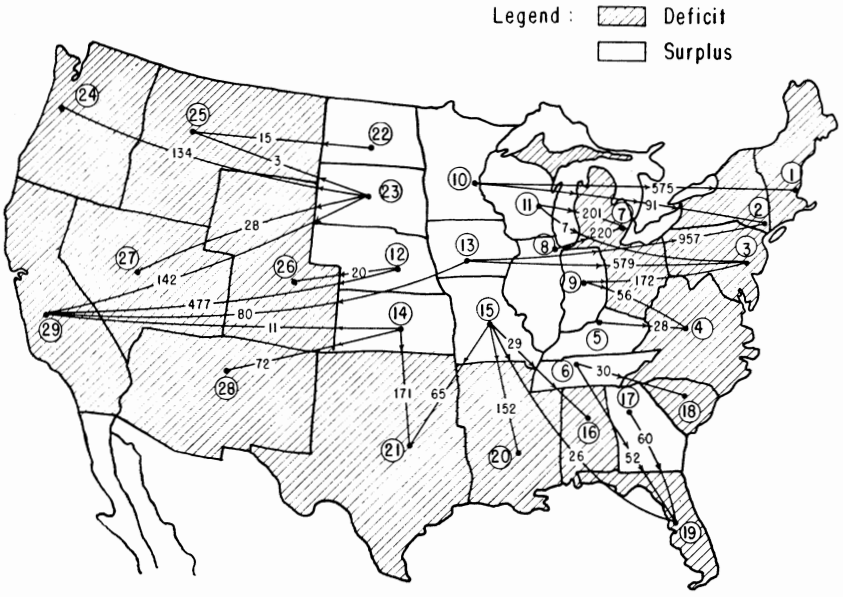

Fig 4 Optimum Flows of Pork, (Million Pounds), 1955, 29 Region Model.

region consisting of Maryland, Delaware, Pennsylvania and New Jersey and to the region that includes Michigan and Ohio (Figure 4).

One major difference between the 21- and 29-region models was due to the separation of Alabama, Georgia and South Carolina into individual regions for the larger model. Alabama and South Carolina remained deficit but Georgia was estimated to have slaughtered about 20 percent more pork than she consumed. Thus, information is provided that was obscured by the more aggregative analysis. This indicates that one perhaps should group regions according to their surplus or deficit classification to prevent this type of obscurity.

The computational burden for the 29-region model was about double that of the 21-region analysis. There were 406 possible transport costs for the 29-region case, compared with 210 for 21 regions. Using the Northwest Corner Rule and the Simplex Technique, 26 iterations were required to establish the minimum cost solution, whereas only 11 to 14 iterations were required for all 21-region problems of this series.[11]

[11] The Simplex procedure has been programmed for the IBM 650 by S. Poley (6).

2. **Quarterly Analyses, 1955:** A spatial and price equilibrium analysis was carried out for pork for each of the four quarters of 1955 in an effort to assess the degree of distortion present in the annual models due to aggregation over time. Tables containing the complete results of the quarterly models are given in Appendix A.

Hog slaughter varied considerably from quarter to quarter in 1955. Estimates of hog slaughter and other predetermined variables used in the quarterly models are presented in Table 3.6.

Table 3.6—United States Estimates of the Predetermined Variables Used in the Quarterly Models for Pork[12]

Quarter	Per Capita Hog* Slaughter	Per Capita Disposable* Income	Population	Retail Price of Beef
	(lbs.)	($)	(thousands)	(cents/lb.)
1. (Jan.-Mar.)	70.1	1568	163,218	69.6
2. (April-June)	57.0	1598	163,858	67.7
3. (July-Sept.)	56.4	1621	164,548	67.0
4. (Oct.-Dec.)	83.9	1645	165,321	65.8
ANNUAL	66.8	1608	164,302	67.7

* Annual Equivalents.

Using the estimates presented in Table 3.6, the logarithmic form of the demand relationship (2.1) was transformed to linear functions for each quarter. The method outlined previously was then used to establish regional equilibrium prices, consumption, surpluses and deficits and optimum flows of pork for each quarter.

(a) *First Quarter Model (Jan.-March), 1955*[13]: For the first quarter, Region 11 (Alabama, Georgia and South Carolina) was classified as surplus, rather than deficit as in the annual analysis. All other regions retained the same classification as for the annual model. Except for the changes induced by the change in classification of Region 11, the directions of flows were the same as for the annual model.

Despite hog slaughter (adjusted to annual) for the first quarter, being slightly higher than for the year and incomes being lower, regional retail pork prices were somewhat higher than for the annual analysis.

[12] See Table 2.1 for sources.

[13] See Table A-1, Appendix A for a presentation of the results.

This is due to beef prices being higher for the first quarter than for any other quarter. For example, price in the base region was estimated to be 52.9 cents per pound for the annual analysis and 53.4 cents per pound for the first quarter, 1955.

Total shipments were estimated to be 1,124 million pounds for the first quarter and total transport costs were $27.9 millions. Therefore, the average transport cost was $2.48 per hundred pounds of pork shipped. Both total shipments and total costs were slightly greater than 1/4 of the annual estimates, due to the relatively larger slaughter for the first quarter.

(b) *Second Quarter Model (April-June), 1955*[14]: Classification of regions for the second quarter was the same as for the annual analysis. However, the magnitudes of surpluses and deficits were such that the shipment program differed slightly from the annual model. Region 9 (Nebraska and Iowa) shipped to Region 12 (Florida) in the model for the second quarter and Region 10 (Kansas and Missouri) shipped to Region 21 (California) in the annual model. This latter shipment did not occur in the model for the second quarter.

In general, regional prices were higher for the second quarter than for annual. For example, Region 14 (Oklahoma and Texas) was estimated to have a retail price of 55.1 cents per pound for the second quarter compared to the annual estimate of 54.4 cents per pound. The higher quarterly prices are due to the low hog slaughter for the second quarter (Table 3.6).

Total shipments for the second quarter were estimated at about 920 million pounds and total shipment costs were $23 millions. Average shipment costs were $2.46 per hundred pounds. Because of the relatively low slaughter, total shipments and costs for the second quarter were only 21 percent of annual.

(c) *Third Quarter Model (July-Sept.), 1955*[15]: Classification of regions remained the same for the third quarter as for the annual and second quarter models. The optimum shipment program for pork for the third quarter was the same as for the second quarter in terms of direction of shipments. Thus, the regional price differentials were also the same as for the second quarter. However, regional prices were higher for the third quarter due to lower slaughter and higher incomes

[14] See Table A-2, Appendix A for the complete results of this analysis.

[15] See Table A-3, Appendix A for the complete results of this analysis.

(Table 3.6). For example, the price in Region 14 (Oklahoma and Texas) was estimated as 55.1 cents per pound for the second quarter and 56.8 cents per pound for the third quarter.

Total shipments for the third quarter were 919 million pounds and total costs of shipping were $22.8 millions. This represents an average cost of shipping of $2.48 per hundred pounds for the third quarter which is approximately the same as the annual average shipment cost ($2.49 per hundred pounds). Again, due to the low production, total shipments and costs for the third quarter were only 21 percent of the same estimates obtained from the annual model.

(*d*) *Fourth Quarter Model (Oct.-Dec.), 1955*[16]: For the fourth quarter, Region 11 (Alabama, Georgia and South Carolina) was dropped from the spatial model as a self-sufficient region.[17] Otherwise, the classification of regions remained the same as for the annual and second and third quarter models. However, because of the magnitudes involved, differences in the shipping patterns existed between the annual and fourth quarter models. For the fourth quarter, Region 8 (Minnesota and Wisconsin) shipped to Region 6 (Michigan and Ohio) and Region 10 (Kansas and Missouri) shipped to Region 4 (Virginia, West Virginia and North Carolina). Neither of these shipments occurred in the annual program. Also, the annual program shipments of Region 5 (Kentucky and Tennessee) to Region 12 (Florida) and Region 7 (Illinois and Indiana) to Region 2 (New York) did not occur in the program for the fourth quarter.

Regional equilibrium prices for pork were lower in the fourth quarter than for the annual or any other quarterly analysis. This is due to the high seasonal slaughter of pork for the fourth quarter as well as the lower beef prices prevailing during this time period (Table 3.6). For example, the price of pork in the base region (9) for the fourth quarter was 49.3 cents per pound compared with 52.9 cents in this region in the annual analysis.

Total shipments for the fourth quarter were 1,440 million pounds and total transport costs were $36.1 millions. Because of the high seasonal slaughter of pork in the fourth quarter, these estimates are about 33 percent of the same annual figures. Average shipment costs for the fourth quarter were $2.51 per hundred pounds.

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[16] See Table A-4, Appendix A for the results of this analysis.

[17] See the first bulletin of this series, T-78, (5), p. 38, for a discussion of this.

(e) *Summary of quarterly shipments*: The following table (3.7) is a presentation of the quarterly shipment programs, and is given to facilitate comparisons with the annual shipment program for pork (Table 3.1).

Table 3.7—Summation of Quarterly Shipments Programs for Pork, 1955

Receiving Regions	Shipping Regions						
	5	7	8	9	10	11	15
1			578,605				
2		79,986	246,189	726,641			
3				753,330			
4	73,045				(12,595)		
6		370,793	(50,696)				
11	4,733						
12	32,013			(30,561)	72,472	(4,260)	
13					152,674		
14					235,945		
16							131,469
17							18,109
18				19,163			
19							27,360
20				71,542			
21				502,011	53,443		145,124

Region 11 (Alabama, Georgia and South Carolina) appears as both surplus and deficit in the summation of quarterly shipments. This represents an obscurity in the annual model that might have been avoided by a proper grouping of regions. All the shipment activities that are in parentheses did not occur in the more aggregative, annual analysis for 1955. Since one of the requisites for a minimum solution for a given problem is that only $n + m - 1$ shipments occur, no annual analysis could have yielded the set of flows that appear in Table 3.7. Thus, there may be some obscurity in annual models that in many cases could be impossible to detect.

Total costs for all four quarterly programs were $109.4 millions and summing the total shipments for the four quarters yielded an estimate of 4,403 million pounds. Both these estimates are slightly higher than the total costs ($109.3 millions) and total shipments (4,398 million

pounds) for the annual analysis (Table 3.1). In general, there is a consistency between the quarterly and annual estimates in regard to the magnitudes of total shipments and costs. This suggests that for the pork sector the aggregative annual model for 1955 affords a good approximation relative to the time assumption.

F. Spatial and Price Equilibrium Analysis for Pork, 1954

A spatial model was derived for slaughter pork for 1954 in order to assess the effect of a low production year on the regional factors. The per capita production of pork estimate for 1954 (60.0 pounds) represents the lowest per capita production level since 1938. Average per capita disposable income was $1,567 for the United States for 1954, which is slightly less than for 1955 ($1,608).

Using the aggregate estimates mentioned above, a linear demand function for pork was derived and a spatial analysis was accomplished in the manner described earlier.[18] Table 3.8 contains the results of the analysis.

Because of the low production, regional equilibrium prices were generally about 10 cents per pound higher in 1954 than for the 1955 analysis. For example, the equilibrium price of pork was estimated to be 64.4 cents per pound in Region 14 (Oklahoma and Texas) for 1954 compared to 54.4 cents for 1955.

Region 11 (Alabama, Georgia and South Carolina) was classified as surplus for 1954. It was estimated that this region (11) had available for export about 8 percent of its regional slaughter in 1954. Classification of all other regions was the same as for 1955. In addition to differences in the shipment programs for 1954 and 1955 caused directly by the reclassification of Region 11, Region 9 (Iowa and Nebraska) shipped to Region 12 (Florida) and Region 10 (Kansas and Missouri) did not ship to Region 21 (California) in the 1954 analysis.

Total shipments for 1954 were estimated at 3,934 million pounds and total shipment costs were $97.2 millions. The average cost was $2.47 per hundred pounds. Thus, because of low production, total shipments of pork were approximately 11 percent less for 1954 than for 1955.

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[18] For 1954, estimates of regional beef prices were not available. However, the average effect of this variable was taken into account in the constant term of the demand relationship.

Table 3.8—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, 1954

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)							
	cents/ pound	1,000 pounds	1,000 pounds	5	7	8	9	10	11	15	V_j
1	65.75	636,636	— 515,233	.66	.14	**515,233**	.05	.25	1.26	.47	2.93
2	65.58	1,143,399	— 950,107	.42	**103,803**	**284,998**	**561,306**	.12	1.05	.49	2.76
3	65.44	1,323,941	— 656,104	.41	0	0	**656,104**	.12	1.02	.51	2.62
4	65.21	453,746	— 64,567	**64,567**	.05	.07	0	.06	.58	.71	2.39
5	64.00	286,176	76,667								
6	64.38	1,078,595	— 364,281	.75	**364,281**	.03	.11	.37	1.86	.75	1.56
7	63.68	945,770	468,084								
8	62.90	408,728	800,231								
9	62.82	241,046	1,834,086								
10	63.08	374,525	420,419								
11	64.63	377,015	30,752								
12	65.83	184,972	— 109,842	**12,100**	.42	.28	**6,010**	**60,980**	**30,752**	.68	3.01
13	64.61	277,855	— 130,143	.62	.86	.63	.13	**130,143**	1.21	1.18	1.79
14	64.42	585,338	— 229,296	1.39	1.48	.70	.05	**229,296**	2.19	.92	1.60
15	62.78	63,085	303,596								
16	65.62	266,962	— 144,503	2.74	1.95	.65	.53	1.05	3.65	**144,503**	2.80
17	64.39	68,534	— 19,344	3.10	2.28	.90	.87	1.36	4.08	**19,344**	1.57
18	64.17	107,441	— 15,907	2.35	1.79	.70	**15,907**	.41	3.46	.31	1.35
19	65.41	57,019	— 23,905	2.27	1.67	.65	.03	.57	3.36	**23,905**	2.59
20	65.12	90,409	— 66,965	1.92	1.52	.66	**66,965**	.10	2.27	.32	2.30
21	66.32	898,815	— 643,637	2.10	1.48	.53	**527,793**	.09	2.60	**115,844**	3.50
			U_i'	1.18	.86	.03	0	.26	1.81	—.04	

Total shipments = 3,933,834.
Total costs = $97,192,198.

G. Conditional Models for Pork

The spatial model is a useful mechanism for answering questions of the if—then variety. Three such analyses are presented in this section. Included are models for 1955 assuming that: (1) transport costs for shipping pork among regions increased by 20 percent and (2) a 90 percent of parity price support for pork existed. Also included is an analysis for 1963 assuming that the basic factors change from 1955 to 1964 similar to factor changes from 1947 to 1955.

1. **Effects of a 20 Percent Increase in Transport Costs:** To obtain a rough estimate of the elasticity of demand for transportation services for pork, as well as the effects of a rate increase on regional prices, consumption, surpluses and deficits and optimum flows for pork, an analysis was made assuming a 20 percent increase in transport costs. The complete results of the analyses appear in Table B-1, Appendix B.

Comparing the model resulting from increasing transport costs by 20 percent and the initial 1955 analysis (Table 3.1), the only difference in either the classification of regions or optimum flow pattern was that Region 11 became self-sufficient. However, all regional prices were higher due to the increased transport costs. For example, price in the base region (9) was 54.9 cents under the original 1955 conditions. All regional price differentials were increased by 20 percent. For example, the price differential for pork between Region 9 (Iowa and Nebraska) and Region 21 (California) became 4.2 cents per pound, assuming a 20 percent increase in shipping costs. The differential between these regions was estimated at 3.5 cents per pound for the actual 1955 conditions.

Total transport costs for this model were $130.7 millions and total shipments were 4,385 million pounds. Thus, a 20 percent increase in transport costs decreased total shipments by only three tenths of one percent and total payments for transport services for pork increased by 19.5 percent. Therefore, for the range considered, the demand for transport services for pork is highly inelastic. The elasticity for the range considered was estimated to be —.015. The inelastic demand for the range considered is due to the insignificance of transport costs relative to the retail price of pork.

2. **Effects of a 90% of Parity Support Price for Pork, 1955:** In 1955 there were mandatory price supports for six basic agricultural products.[19] At one time or another, virtually all major farm products have been supported either directly or indirectly by government programs. Thus

[19]These include corn, rice, wheat, peanuts, cotton and tobacco.

the importance of assessing the effects of support programs on regional consumption, prices and interregional flows and of estimating the optimum location of government storage facilities cannot be minimized.

The average price received by farmers for hogs in 1955 was $15.40 per hundred pounds. The average price that would have been received under a 90 percent of parity support was estimated at $19.08 per hundred pounds. Multiplying the difference in farm prices of $3.68 per hundred pounds times the dressing ratio of 1:2.13, it was then estimated that retail prices for pork would have been increased by $7.84, assuming no changes in marketing margins. Therefore, the retail price of pork in 1955, assuming a 90 percent of parity support price, would have been $62.64 per hundred pounds. Using the estimates of elasticity given in equation (2.1), it was then estimated that consumption of pork would have decreased from 67 pounds per capita to 57 pounds and the retail price of beef would have been increased from 68 to 71 cents per pound. Using these estimates, a linear demand equation for pork was established, consistent with the assumption of a 90 percent of parity support price for pork.

Table 3.9 contains the results of the analysis, assuming a 90 percent of parity price support for hogs, 1955.

Since government purchases of the surplus pork would probably take place prior to slaughter, the analysis was carried out using estimates of farm production for slaughter rather than regional slaughter estimates. Thus, all interregional shipments indicated by the analysis are also conditioned by the assumption that slaughter was production oriented for 1955.

The optimum flow pattern indicates that Region 9 (Iowa and Nebraska) and Region 15 (North and South Dakota) "shipped" to the surplus vector. Thus government purchases of the surplus pork would have taken place at the levels indicated in these two regions in order to have kept transport costs to a minimum.

Because the surplus pork accumulated in Regions 9 and 15, price was the same for the two regions (62.0 cents per pound). Price differences for the other regions were approximately the same as for the analysis for 1955 assuming that slaughter was production oriented (Table 3.3).

Total interregional shipments for the program were 4,792 million pounds at a total shipment cost of $111.7 millions. Thus, average shipment costs were $2.33 per hundred pounds.

Total receipts to farmers for slaughter hogs in 1955 were approxi-

Table 3.9—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, 1955, Assuming a 90 Percent of Parity Price Support Program with Production-Oriented Slaughter

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)							
	cents/ pound	1,000 pounds	1,000 pounds	5	7	8	9	10	11	15	V_j
1	64.90	606,588	— 574,782	.57	.14	**574,782**	.05	.16	1.17	.51	2.93
2	64.73	1,086,811	—1,057,712	.33	**714,493**	**343,219**	0	.03	.96	.53	2.76
3	64.59	1,258,638	—1,084,552	.32	**312,971**	0	**771,581**	.03	.93	.55	2.62
4	64.27	435,663	— 91,829	**91,829**	.14	.16	.09	.06	.58	.84	2.30
5	63.06	269,456	124,442								
6	63.53	1,042,631	— 332,350	.66	**332,350**	.08	.11	.28	1.77	.79	1.56
7	62.83	896,586	1,359,814								
8	62.05	382,039	918,001								
9	61.97	206,836	2,938,985								
10	62.14	348,853	562,753								
11	63.69	371,489	83,554								
12	64.89	184,481	— 129,764	**32,613**	.51	.37	.09	**13,597**	**83,554**	.81	2.92
13	63.67	264,795	— 54,625	.62	.95	.72	.22	**54,625**	1.21	1.31	1.70
14	63.48	552,193	— 280,866	1.39	1.57	.79	.14	**280,866**	2.19	1.05	1.51
15	61.97	58,141	413,113								
16	64.81	246,130	— 190,091	2.61	1.91	.62	.54	.92	3.52	**190,091**	2.84
17	63.58	63,891	— 17,987	2.97	2.24	.86	.83	1.23	3.95	**17,987**	1.61
18	63.32	101,705	— 65,582	2.26	1.79	.70	**65,582**	.32	3.37	.35	1.35
19	64.59	54,859	— 44,290	2.15	1.64	.62	**44,290**	.45	3.24	.01	2.62
20	64.27	85,425	— 73,079	1.83	1.52	.66	**73,079**	.01	2.68	.36	2.30
21	65.47	856,220	— 794,027	2.01	1.48	.53	**580,362**	**213,665**	2.51	.04	3.50
	Surplus Vector			0	0	0	**1,404,091**	0	0	**205,035**	0
			U_i	1.09	.86	.08	0	.17	1.72	0	

Total shipments (1,000 lbs.) = 4,791,536.
Total costs = $111,731,428.

mately $2,960 millions. Under a 90 percent of parity price support program, farm receipts would have been $3,670 millions. The government would have had to purchase a surplus of 1,609 million pounds of pork (carcass weight), or about 2,816 million pounds live weight at a cost of about $537 millions. Additionally, there would have been processing, storage and distributing costs for the surplus pork.

3. **A Model for 1963:** By estimating the future values of the predetermined variables, income, slaughter and population, an analysis was made for pork for 1963.

Regional pork slaughter for 1963 was projected by assuming that the percentage change in slaughter from 1955 to 1963 would be the same as the percentage change from 1947 to 1955. Regional population data was extrapolated in the same manner and regional incomes were estimated by using a linear projection based on 1947 to 1955 changes in regional per capita incomes. The estimates for the total economy obtained in this manner for 1963 were 62.1 pounds of slaughtered pork per capita; average per capita income equal to $2,057; and a population of 189 million. Admittedly, the methods of projection were naive, but the resulting analysis should provide some insight into future conditions and serves as an example of how the spatial model may be used in conjunction with forecasting.

The results of the analysis for 1963 are given in Table 3.10.

Because per capita slaughter was estimated to be about 5 pounds less for 1963 than for 1955 and per capita incomes increased by approximately $450, the price (regional retail) estimates for 1963 are on the average about 17 cents per pound higher than for 1955. For example, price in the base region (9) was estimated at 70.5 cents per pound for 1963 compared to 52.9 cents in 1955.

The classification of regions was the same for 1963 as for the 1955 analysis, but due to shifts in magnitudes of surpluses and deficits, the shipment program was altered (compare Tables 3.1 and 3.10).

Total shipments for the 1963 program were estimated to be 5,043 million pounds at a total cost of $125 millions. Since interregional shipment costs are assumed the same as for 1955, the average shipment cost for 1963 was estimated as $2.48, the same as for 1955. Total shipments were larger than for 1955 because of the larger estimated total production of pork for 1963. Although per capita slaughter was less for 1963 than for 1955, total slaughter was estimated to be larger by about 800 million pounds.

Table 3.10—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, 1963

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 5	7	8	9	10	15	V_j
1	73.41	715,210	— 605,803	.66	.22	**605,803**	.05	.30	.47	2.93
2	73.24	1,315,395	—1,067,982	.42	.08	**188,271**	**879,711**	.17	.49	2.76
3	73.10	1,536,981	— 800,056	.41	.08	0	**800,056**	.17	.51	2.62
4	72.87	528,295	— 47,677	0	.13	.07	**47,677**	.11	.71	2.39
5	71.66	318,495	217,961							
6	72.12	1,368,010	— 505,464	.67	**330,424**	**175,040**	.03	.34	.67	1.64
7	71.42	1,093,786	330,424							
8	70.56	465,725	969,114							
9	70.48	232,391	2,721,969							
10	70.79	416,012	442,486							
11	72.55	455,749	— 88,479	**88,479**	.55	.43	.22	.16	.95	2.07
12	73.49	287,840	— 213,773	**129,482**	.50	.28	**84,291**	.05	.68	3.01
13	72.32	310,662	— 220,751	.57	.89	.58	.08	**220,751**	1.13	1.84
14	72.13	699,389	— 315,330	1.34	1.51	.65	**93,595**	**221,735**	.87	1.65
15	70.44	57,113	361,307							
16	73.28	306,052	— 145,945	2.74	2.03	.66	.53	1.10	**145,945**	2.80
17	72.05	76,334	— 10,593	3.10	2.36	.90	.87	1.41	**10,593**	1.57
18	71.83	132,261	— 45,127	2.35	1.87	.70	**45,127**	.46	.31	1.35
19	73.07	77,705	— 36,569	2.27	1.75	.65	.03	.62	**36,569**	2.59
20	72.78	136,185	— 94,821	1.92	1.60	.66	**94,821**	.15	.32	2.30
21	73.98	1,199,839	— 844,893	2.10	1.56	.53	**676,693**	.14	**168,200**	3.50
			U_i	1.18	.94	.08	0	.31	—.04	

Total shipments (1,000 lbs.) = 5,043,263.
Total costs = $125,028,594.

IV. Summary and Implications

Specific implications accompany the presentation of each analysis in the preceding section. Also, the two initial reports of this series contain comments regarding the general implications of spatial analysis. Therefore, this section will be devoted to reiterating some of the more significant results of this study and to pointing up some of the broader implications and limitations of spatial analysis not mentioned in the initial reports.

In this study, spatial and price equilibrium models were established for pork annually for 1954 and 1955. A model also was established for live hogs for slaughter for 1955. By combining the two analyses of pork and live hog shipments and contrasting the results with a model wherein pork slaughter was assumed to be market oriented, it was estimated that the locational levels of slaughter for 1955 were inefficient. The savings in transport costs that would have been realized in 1955 in the pork industry if slaughter had been production oriented were estimated as $6.7 millions.

Quarterly models for pork also were established for 1955. Although some seasonal information was gained over and above the annual analysis, the results of the quarterly analyses suggested that the distortion induced by aggregation over time was not too significant. Likewise, a more detailed specification of regions, resulting in a 29-region model for pork for 1955, failed to turn up any gross errors due to aggregation over space.

As an example of the use of spatial analysis in policy planning, a model for pork was established assuming a 90 percent of parity support price for pork in 1955. The results of this analysis indicated that a surplus of 2.8 billion pounds of pork would have resulted and government purchases of the surplus would have cost about $537 millions. It was estimated that the optimum location of government purchases and storage facilities would have been in Iowa, Nebraska and the Dakotas. It was further estimated that farm receipts for hogs would have been larger by about $700 millions in 1955 if a support price at 90 percent of parity had been in effect.

In general, the results from this and other analyses serve to point up two major uses of spatial models as tools for economic research. First, spatial models can be used to simplify and to describe certain sectors of the economy, and secondly, they provide a means for analysis of actual or conceptual situations. Analytically, the spatial model can be used

to evaluate the effects of changes in either one or many factors at a time. Thus, institutional peculiarities or rigidities can be examined and contrasted with the normative framework.

As better methods of aggregative forecasting become available, spatial analysis should prove invaluable as an aid to budgeting and planning. For example, the slaughtering industry could better plan their expansion or contraction in the various geographical areas by using the spatial model along with forecasted levels of regional supplies of livestock and demands for livestock products.

The type of model used in this research supplies only point estimates of the various factors involved. Reliability is left unmeasured. As a step in this direction, Dantzig (1, 2) has developed an uncertain demand model of transportation in which regional demands can be represented by discrete probability functions. In classifying and grouping surplus and deficit regions it may prove useful to devise a test that will measure the reliability of such classification.

The operationality of the spatial model can be preserved throughout a wide range of specifications. This flexibility permits application to numerous problems and allows considerable freedom for choosing the "proper" levels of abstraction within a particular problem.

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*For a more complete listing of literature relevant to spatial and price equilibrium analysis see reference (5).

APPENDIX A: Tables of Results of Quarterly Analyses

The four tables presented in this Appendix contain the results of the spatial analyses made for each of the four quarters of 1955. A general discussion of each of the analysis may be found in Chapter III of the text.

Table A-1—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, First Quarter (January-March), 1955

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 5	7	8	9	10	11	15	V_j
1	56.37	184,618	—148,749	.57	.14	**148,749**	.05	.16	1.17	.47	2.93
2	56.20	331,082	—276,467	.33	**19,922**	**74,098**	**182,447**	.03	.96	.49	2.76
3	56.06	383,310	—195,060	.32	.86	.08	**195,060**	.03	.93	.51	2.62
4	55.74	132,670	— 21,769	**21,769**	.14	.16	.09	.06	.58	.80	2.30
5	54.53	82,412	27,532								
6	55.00	318,215	—105,378	.66	**105,378**	.08	.11	.28	1.77	.75	1.56
7	54.30	273,998	125,300								
8	53.52	116,981	222,847								
9	53.44	63,377	543,521								
10	53.61	106,821	119,158								
11	55.16	113,339	4,260								
12	56.36	56,137	— 32,607	**5,763**	.51	.37	.09	**22,582**	**4,260**	.77	2.92
13	55.14	80,833	— 35,949	.62	.95	.72	.22	**35,949**	1.21	1.27	1.70
14	54.95	168,646	— 58,425	1.39	1.57	.79	.14	**58,425**	2.19	1.01	1.51
15	53.40	17,834	81,487								
16	56.24	75,022	— 35,514	2.65	1.95	.66	.53	.96	3.56	**35,514**	2.80
17	55.01	19,532	— 5,815	3.01	2.28	.90	.87	1.27	3.99	**5,815**	1.57
18	54.79	31,068	— 3,874	2.26	1.79	.70	**3,874**	.32	3.37	.31	1.35
19	56.03	16,727	— 6,826	2.18	1.67	.65	.03	.48	3.27	**6,826**	2.59
20	55.74	26,040	— 18,862	1.83	1.52	.66	**18,862**	.01	2.68	.32	2.30
21	56.94	260,518	—178,812	2.01	1.48	.53	**143,278**	**2,202**	2.51	**33,332**	3.50
			U_i	1.09	.86	.08	0	.17	1.72	—.04	

Total shipments (1,000 lbs.) = 1,124,107.
Total costs = $27,897,231.

Table A-2—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, Second Quarter (April-June), 1955

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 5	7	8	9	10	15	V_j
1	56.46	151,043	—123,847	.66	.14	123,847	.05	.25	.47	2.93
2	56.29	270,751	—220,957	.42	19,252	56,734	144,971	.12	.49	2.76
3	56.15	313,531	—154,201	.41	0	0	154,201	.12	.51	2.62
4	55.92	108,366	— 10,599	10,599	.05	.07	0	.06	.71	2.39
5	54.71	67,240	30,985							
6	55.09	260,277	— 87,885	.75	87,885	.08	.11	.37	.75	1.56
7	54.39	224,108	107,137							
8	53.61	95,654	180,581							
9	53.53	51,828	434,869							
10	53.79	87,202	95,176							
11	55.60	91,987	— 4,680	4,680	.47	.43	.22	.11	.95	2.07
12	56.54	45,837	— 33,634	15,706	.42	.28	10,718	7,210	.68	3.01
13	55.32	65,913	— 31,757	.62	.86	.63	.13	31,757	1.18	1.79
14	55.13	137,614	— 56,209	1.39	1.48	.70	.05	56,209	.92	1.60
15	53.49	14,585	70,868							
16	56.33	61,322	— 26,374	2.74	1.95	.66	.53	1.05	26,374	2.80
17	55.10	15,967	— 2,221	3.10	2.28	.90	.87	1.36	2,221	1.57
18	54.88	25,410	— 2,852	2.35	1.79	.70	2,852	.41	.31	1.35
19	56.12	13,674	— 5,239	2.27	1.67	.65	.03	.57	5,239	2.59
20	55.83	21,296	— 14,652	1.92	1.52	.66	14,652	.10	.32	2.30
21	57.03	213,020	—144,507	2.10	1.48	.53	107,473	.09	37,034	3.50
			U_i	1.18	.86	.08	0	.26	—.04	

Total shipments (1,000 lbs.) = 919,616.
Total costs = $22,660,271.

Table A-3—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, Third Quarter (July-September), 1955

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 5	7	8	9	10	15	V_j
1	58.16	150,105	—125,298	.66	.14	**125,298**	.05	.25	.47	2.93
2	57.99	269,013	—216,535	.42	**40,812**	**49,438**	**126,285**	.12	.49	2.76
3	57.85	311,619	—160,628	.41	0	0	**160,628**	.12	.51	2.62
4	57.62	107,674	— 13,297	**13,297**	.05	.07	0	.06	.71	2.39
5	56.41	66,767	23,894							
6	56.79	258,472	— 73,666	.75	**73,666**	.08	.11	.37	.75	1.56
7	56.09	222,406	114,478							
8	55.31	94,874	174,736							
9	55.23	51,396	441,782							
10	55.49	86,488	96,593							
11	57.30	91,363	— 53	**53**	.47	.43	.22	.11	.95	2.07
12	58.24	45,567	— 31,438	**10,544**	.42	.28	**19,843**	**1,051**	.68	3.01
13	57.02	65,509	— 34,193	.62	.86	.63	.13	**34,193**	1.18	1.79
14	56.83	136,682	— 61,349	1.39	1.48	.70	.05	**61,349**	.92	1.60
15	55.19	14,466	67,960							
16	58.03	60,924	— 27,116	2.74	1.95	.66	.53	1.05	**27,116**	2.80
17	56.80	15,852	— 1,879	3.10	2.28	.90	.87	1.36	**1,879**	1.57
18	56.58	25,231	— 4,879	2.35	1.79	.70	**4,879**	.41	.31	1.35
19	57.82	13,594	— 5,684	2.27	1.67	.65	.03	.57	**5,684**	2.59
20	57.53	21,161	— 15,095	1.92	1.52	.66	**15,095**	.10	.32	2.30
21	58.73	211,674	—148,334	2.10	1.48	.53	**115,053**	.09	**33,281**	3.50
			U_i	1.18	.86	.08	0	.26	—.04	

Total shipments (1,000 lbs.) = 919,444.
Total costs = $22,774,557.

Table A-4—Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows for Pork, Fourth Quarter, (October-December), 1955

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 5	7	8	9	10	15	V_j
1	52.24	223,758	—180,711	.63	.22	**130,711**	.05	.16	.47	2.93
2	52.07	401,104	—338,857	.39	.08	**65,919**	**272,938**	.03	.49	2.76
3	51.93	464,603	—253,441	.38	.08	0	**253,441**	.03	.51	2.62
4	51.67	160,632	— 39,975	**27,380**	.16	.10	.03	**12,595**	.74	2.36
5	50.46	99,936	27,380							
6	50.95	385,636	—154,560	.64	**103,864**	**50,696**	.03	.20	.67	1.64
7	50.25	332,296	103,864							
8	49.39	142,207	297,326							
9	49.31	77,035	693,082							
10	49.48	129,806	216,202							
11	51.35	136,360	—							
12	52.23	68,042	— 41,629	.06	.59	.37	.09	**41,629**	.77	2.92
13	51.01	98,191	— 50,775	.68	1.03	.72	.22	**50,775**	1.27	1.70
14	50.82	204,774	— 59,962	1.45	1.65	.79	.14	**59,962**	1.01	1.51
15	49.27	21,697	101,747							
16	52.11	91,452	— 42,465	2.71	2.03	66	.53	.96	**42,465**	2.80
17	50.88	23,677	— 8,194	3.07	2.36	.90	.87	1.27	**8,194**	1.57
18	50.66	37,717	— 7,563	2.32	1.87	.70	**7,563**	.32	.31	1.35
19	51.90	20,250	— 9,611	2.24	1.75	.65	.03	.48	**9,611**	2.59
20	51.61	31,561	— 22,933	1.89	1.60	.66	**22,933**	.01	.32	2.30
21	52.81	315,217	—228,925	2.07	1.56	.53	**136,207**	**51,241**	**41,477**	3.50
			U_i	1.15	.94	.08	0	.17	—.04	

Total shipments (1,000 lbs.) = 1,439,601.
Total costs = $36,102,430.