SYSTEMS MODELING AND OPTIMIZATION OF INVESTMENT AND CAPACITY EXPANSION OF AN ELECTRIC UTILITY

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CHAPTER I

INTRODUCTION

The need for electrical energy is growing at an unprecedented rate. The Federal Power Commission has predicted a quadrupling of electric demand and energy consumption between 1970 and 1990 [9]. The investor-owned electric utility, unlike most corporations, operates as a regulated monopoly and bears the responsibility for satisfying this growing demand for its product. The franchised utility does not have the freedom to choose its own level of production output and subsequent growth since this is dictated by demand. This means that expansion of production capacity is fundamentally dependent on the growth of demand. Another important aspect in satisfying electrical energy demand is the non-storable characteristic of electrical energy which eliminates the possibility of meeting demand through production inventory. The electric utility generally has only three alternatives. The utility must continuously meet demand by producing the needed energy or purchasing it from outside sources or both.

Fundamental to the question of purchasing energy or increasing generation capacity is the problem of accumulating large amounts of capital for the expansion of capacity. Raising capital while simultaneously maintaining a desirable market valuation of the company by investors is the task of the financial planner. Therefore, it is apparent that the capacity expansion policy is closely related to long range capital budgeting, and that a financial planner must have a clear understanding of how increased demand for electrical energy propagates through the company's operations and results in capital budgeting requirements. Furthermore, long time delays in construction of new generating plants and distribution equipment, coupled with the frequent difficulty in raising large amounts of capital, require that the utility make a careful study of long-range plans for expansion.

The problem of capacity expansion and the corresponding capital budgeting strategy has historically been divided into two problems. Engineers and planners have addressed themselves to finding the optimal time phasing and expansion of the utility's capacity to meet a growing demand. The financial planner has taken inputs from these expansion plans and converted this to capital requirements. The financial planner then determines the proper mix of sources for raising this capital. Examples of both parts of the problem where each assumes away consideration of the other are presented in the literature review. This research integrates the two problems and seeks a total approach for their solution.

Strategic planning of expansion of the investor-owned electric utility requires that the utility be viewed as a complex economic system in which the various functions of the firm are interconnected through information-feedback relations. Systems analysis provides a formal framework for understanding and quantifying the interactions between the components of a complex system whether it be an electrical network or a socio-economic system. Systems methodology provides the needed tools with which utility management can deal with the diverse nature of the expansion policy and its impact on the other aspects of the company.

The systems approach is based upon the development of a mathematical model which describes the important properties of the system. Therefore, the first step leading to the proper financial and capacity expansion policies is the development of a mathematical model of the relevant processes of the company. Model building abstracts the salient features of the system into mathematical terms.

In the utility case, a model is required which describes and interrelates the capacity expansion process and the financial process. This model should be analytic in nature so that maximum utilization can be made of system analysis and modern optimal control theory. It additionally should contain enough detail to be useful in computer simulation of alternate strategies. Time delays in construction and the intrinsic time dependence of financial variables dictates that the model be dynamic in nature. Once such a model is developed, it then provides a laboratory environment for the analytic development of optimal strategies of expansion, capital budgeting, and purchasing of energy outside the system.

The first objective of this research is to formulate a continuous dynamic model of an investor-owned electric utility which includes capacity expansion, the management of purchased power, and the capital budgeting-market valuation nexus. The second objective is to demonstrate the application of the model to the problem of optimizing, over time, the three management control variables. Two fundamentally different approaches will be taken in the demonstration phase. First, the model will be employed in an analytical study of maximizing the market valuation of the utility by investors. A general solution to the problem is obtained and a geometric presentation of the control space is given. In the second demonstration, an expanded performance index is treated via computer simulation. An important advantage of the model illustrated by both of these problems is the easy formulation of the necessary conditions for the optimal controls.

Chapter II is a review of the relevant literature on the use of mathematical models to describe the electric utility and it points out the unique features of this research compared to past efforts. Chapter III contains the description of the model. This description explicitly states the assumptions underlying the model development and their justification. In Chapter IV, a typical optimal control problem is posed and necessary conditions for the optimal decision variables are developed. These necessary conditions are applied through an analytical approach and the optimal strategies are found and given economic interpretation.

In Chapter V, a modified performance index is considered and the necessary conditions for its optimal trajectories are defined. A computer solution of the resulting two-point boundary value problem is included along with an economic interpretation of the optimal controls. A comparison of this optimal strategy with a typical utility strategy is presented. Chapter VI contains the summary and conclusions of the research along with recommendations for further investigations. An Appendix is included which contains the details of the computer technique utilized to solve the two-point boundary value problem and the listing of the program. Also contained in the Appendix is a glossary of symbols and their corresponding definitions for reference.

CHAPTER II

REVIEW OF RELATED LITERATURE

Introduction

This chapter describes and discusses previous research in the mathematical modeling of the firm and, in particular, the electric utility. Since the modeling of the firm crosses several disciplinary lines, the approaches to the problem are diverse depending upon the ultimate objective of the model's use. Mathematical models of the firm, for the most part, can be divided into three distinct groups. These are microeconomic models, computer corporate models, and financial capital budgeting models. Examples of each type are presented, including their objectives and assumptions. Additionally, there has been research done on related topics to the utility modeling which involve operations research models and the methodology of modeling business management systems. In conclusion, a brief summary of the various modeling approaches is given along with an explanation of how the current research differs from previous work.

Microeconomic Models

In microeconomic terms, the electric utility is defined as a franchised monopoly under regulation. Abstraction of the electric utility into mathematical terms by economists has been motivated by their need to determine the differences in behavior between the regulated and unregulated firm. The objective of this type of economic research is to reveal the proper production output and allocation of input resources to maximize profit. Regulation of the utility takes the form of a maximum limit on the rate-of-return on invested capital. In all other respects, management is permitted to pursue its objective of maximization of profit. These models, like most microeconomic models, are static in nature and are assumed to be in equilibrium; i.e., changes in variables with respect to time are neglected. These models provide economic understanding of the resource allocation behavior of the monopolistic firm under rate-of-return regulation.

Averch [2] was one of the first to model the monopolistic firm under regulation. A similar model was constructed almost simultaneously for the natural gas utility by Wellisz [40]. Baumol [3] provides a clear overview of their research and results. Since the Averch and Johnson model is typical of this type modeling, a brief description is provided.

The A-J model assumes a firm that produces one output product and has two input resources, labor and capital, each of which is available to the firm in unlimited quantities at a fixed price per unit. The production function for the firm's output product and demand function for this product are assumed in general terms and are functions of the input resources and price per unit of the product respectively. The profit equation is defined as total revenue (product of price of output product and quantity of output product) minus the products of input resource prices and their respective quantities used in the production. The rate-of-return constraint takes the form of an inequality and along with the demand function and production function provides the constraints for an algebraic optimization problem. The Lagrange multiplier technique is applied to obtain the optimum quantities of output and input resources of capital and labor. This model, though limited in scope, has yielded one significant hypothesis concerning the allocation of capital resources. This result is that a firm as described will be more capital intensive (large optimum capital-labor quantity ratio) than the unregulated profit maximizing firm.

The A-J model has several obvious limitations in so far as providing any understanding of the proper capacity expansion and capital budgeting strategy. Its most apparent limitation is that the model is concerned only with resource allocation, and it assumes that it can choose any output it desires, while in reality the franchised electric utility is required to meet the demand for its product. The second major limitation is that none of the variables or relations are allowed to vary through time. Another restriction is the assumed objective function. Maximization of profit is the usual objective in microeconomic theory. However, financial management concentrates on the maximization of the firm's value to its stockholders and future investors. The microeconomic model also neglects the financial process of the firm. Though the quantity of capital is determined, no consideration is given as to how this capital is to be obtained. A final restriction of the A-J model is that the production and demand functions are in general terms, and therefore are useless in any actual application of the model to a utility unless a realistic production function and demand function can be hypothesized.

Computer Corporate Models

Another approach to the modeling of the firm which has become popular recently is referred to as computer corporate modeling. Computer corporate models are digital computer programs that simulate the operations of a firm and translate the results of these operations into economic and financial forecasts [43]. The corporate model is distinguished from other management science models in that it attempts to embrace all facets of the operation of the firm. The corporate model ties the production and financial sectors of a company together with the objective of providing the planner and manager an information tool plus a simulation program with which alternate strategies can be evaluated.

Gershefski [17] provides information on the varied development and application of this type modeling to a wide spectrum of different corporations, including the investor-owned electric utility. A survey conducted by the Planning Executives Institute of Oxford Ohio of 323 companies in 1969 as to their activities in corporate modeling yields some interesting observations [17]. The survey uncovered these facts about corporate modeling by the industrial community:

1. Corporate modeling was not initiated to any extent until 1966.
2. One hundred companies out of the reporting 323 will have corporate models in development or operation by the end of 1969.
3. Electric utilities, banks, and the petroleum industry have used corporate modeling most extensively.
4. On the average it takes 3.5 man-years to develop a working version of a corporate model.

5. Ninety-five percent of the models were strictly simulation type while the other five percent were mathematical programming or optimization models.

6. Ninety percent of the models were deterministic.

The proprietary nature of corporate modeling has impeded its disclosure by the developing corporation. For this reason, there are very few detailed examples of corporate models in the literature. However, because of a unique joint effort by two corporations, one example of corporate modeling of an investor-owned electric utility is available for review. This is the corporate model developed jointly for a hypothetical electric utility by General Electric Company and Boston Edison Company, an investor-owned electric utility [8]. Their objective was to develop a complete computer program of the utility so that alternative strategies of operation and financial planning could be evaluated.

This model development was achieved by combining the efforts of accountants and engineers from both companies. The model combines engineering and financial-accounting considerations in a single unified package. The model consists of two major computer programs and an auxiliary program. These are the production simulation program, the economic simulation program, and an auxiliary nuclear fuel management program.

Given an input demand for electrical energy, the production simulation section determines the short-term optimum operating or dispatching policy via a production cost program. This policy reflects the operation characteristics of the various modes of generating electricity; i.e., how much power will be generated from thermal units, pumped-hydro units and nuclear units? This program uses computer optimization techniques and incremental cost criteria for its generator dispatching policy. Additional inputs of contractual buying and selling of electricity can be included. Regression analysis of historical data is used to project monthly maintenance and material costs.

The economic simulation program determines revenue, given demand and several rate classes. Expenses are determined from outputs of the production program. Taxes and depreciation are determined using normal accounting procedures and cash flows are monitored to obtain a realistic picture of a firm's financial operation. Construction of new generating and transmission equipment is initiated in two ways. One way of starting new construction is through input data which is useful for testing a construction strategy. Another way of initiating new construction is through internal plant investment ratios—dollars per kilowatt of peak load for each plant class. As the system load grows, the program automatically constructs new plants to maintain the specified investment ratios. The construction process of projects inputed are simulated and monthly expenditures are charged against the project in a separate work-in-progress account. When the construction phase is completed, the project is transferred to an in-service plant. As in the construction process, financing can be automatic or preplanned or both methods can be used sequentially.

The auxiliary nuclear fuel program provides a means of studying alternative methods of nuclear plant operation. This includes the accounting of nuclear fuel inventories and periodic fuel expenses and the financing of nuclear fuel requirements. With minimum cost as a goal, the program determines the reloading schedule for the nuclear plants.

The output of this model is divided into two parts. One part describes the system characteristics: each unit's maintenance outage schedule, number of start-ups, capacity factor, energy produced, and operating costs. Summations of these quantities across all generating units provides total utility operating characteristics. The second part of the output deals with the financial operation. Four financial reports are produced for each year simulated. These include the income statement, a cash flow report, and a balance sheet of year end assets, capitalization, and liability accounts. These four statements are also available on a monthly interval.

Although a computer corporate model is a beneficial simulation tool because of its completeness, it does not possess a useful mathematical structure for the application of systems analysis or optimal control. This lack of mathematical tractability is a consequence of the use of internal optimization of dispatching generators, the use of regression analysis, and the general heuristic philosophy of modeling utilized. The usefulness of the computer corporate modeling technique in defining the proper capacity expansion, purchasing energy and capital budgeting strategies is at best sub-optimal, since the corporate model can only evaluate strategies but not generate them. It additionally becomes costly to simulate the computer model because of its detail.

Capital Budgeting Models

Capital budgeting models have been developed by financial theorists in an attempt to determine the proper financial decision process. Van Horne [38] describes this decision process in three parts. The first decision is "should we invest?" In the electric utility case, this question can be stated specifically as "should we invest in more capacity or purchase additional energy as needed to meet demand?" The second step in the decision process is determining the mix or structure of desired capital for investment. There are many ways to raise capital, such as, issuance of common stock retain earnings, or incurring long term debt through various debt instruments. The third step in the decision process is the dividend decision. It is difficult to separate the dividend decision from the second part of the financial process since retained earnings and dividends are complimentary. The optimum decisions of step two and three are those that maximize the present market value of the firm. In general terms, this objective function represents the firm's ability to attract the investment of capital.

Davis [10] presents a unique capital budgeting model for a regulated utility in which he formulates a dynamic mathematical model of a utility and determines the optimum capital budgeting strategy for maximization of "capital attraction capability" using modern optimal control theory. A brief description of the model, including its assumptions and objectives follows, since Davis' work represents the only dynamic financial analysis of the firm found in the literature which spans both market valuation and capital expansion considerations.

The Davis model is a second order nonlinear continuous state model of a utility, encompassing operations and investment in an analysis of financial activity. The objective of the model is to determine the optimum amounts of retained earnings and new equity capital allocated to investment and simultaneously determine the impact of rate-of-return regulation on these decision processes.

The formulation is entirely financial in nature, which distinguishes it from economic models of the firm in that it subsumes the production function of the utility and assumes that the utility operates along its optimal expansion path in regard to resource allocation. The model consists of two nonlinear first order differential equations describing the change with respect to time of the stock price and equity per share. These equations include behavioral assumptions pertaining to market valuation and the utility's operation. In the model development, Davis assumes the following:

1. Investors are indifferent between capital gains and dividends.
2. Constant debt-equity ratio.
3. The firm's income is always the maximum rate-of-return on equity times total equity.
4. The rate of growth of assets is constant.
5. The dollar value of new equity subscribed is determined as a proportion of current earnings.

The objective function reflects mathematically the present value of the equity owner's holdings. The control problem therefore is to determine the optimum new equity issue and retained earnings policy to maximize this objective function subject to the differential constraints of the model plus additional inequality constraints of upper bounds on investment growth and rate-of-return on equity.

The solution of the optimal control problem reveals the impact of rate-of-return on the capital budgeting strategy. As Davis [10] points out, "The allowed rate not only affects the values of the strategies, it also affects the very structure of the solution in that totally different solutions result from different ranges in the rate-of-return."

There are several characteristics of the Davis formulation that restricts its general usefulness to the objective of this research and therefore distinguishes the two different research efforts. For example, the exclusion of the production process and factors of production and the assumption that the firm will always earn its maximum rate-of-return precludes the consideration of purchasing or producing energy by the utility to meet demand. Secondly, the assumption that the firm's growth will be constant uncouples the time varying demand for energy from the financial process. This means that Davis has neglected the possibility of an optimum capacity expansion policy. The last restrictive assumption is that new equity issuance is based on current earnings. This makes little sense financially or economically, but does simplify the control problem. These assumptions are relaxed or removed in this research.

Other Models

In addition to the three general types of mathematical modeling already presented, there have been a number of other attempts to model specific aspects of the firm which are related closely to the objectives of this research. They deal with the optimal capacity expansion policy, optimum investment for a monopoly, and methodology of modeling management systems.

Arrow [1] was the first to deal with the general problem of determining the optimum expansion of capacity of a firm's production given a demand for the firm's product as a known function of time for some time interval. Arrow assumed that capacity can never decrease over time and that the rate of increase of capacity is limited by an upper bound. It is further assumed that capital equipment suffers no depreciation, that all maintenance costs are proportional to output production, and that the product is nonstorable. Price of the output product, interest rate, cost per unit of increase in capacity, and production costs are constant over the relevant time interval.

The profit per unit of output is assumed to be unity and therefore the instantaneous total profit is the minimum of demand or capacity. This means that the firm is free to satisfy all or any part of the product demand. The objective of the expansion policy is to maximize the accumulation of the stream of profit through time minus the costs of adding capacity discounted to the beginning of the time interval. The problem is stated in such a way as to make use of the minimax theorem of game theory. The optimal expansion policy then takes the form of the level of capacity for each instant of time in the given time interval. It is shown that the optimal expansion policy involves decomposition of the time interval into smaller intervals in each of which one of the following three policies obtains: no expansion, expansion with capacity equal to demand, or expansion at the maximum permissable rate. Examples of the algorithm are given for various forms of demand function including a sinusoidal form.

Though the objectives of Arrow's efforts and the author's are similar, the assumptions are quite different. Arrow's model allows for providing any part of demand, but does not require that all of the demand be met. It also does not allow for obtaining production capacity from external sources. The assumption that maintenance is a function of output is another restrictive assumption which distinguishes the two formulations.

Berretta [6] provides another example of optimal capacity expansion in which demand is assumed to increase at a geometric rate and the objective is the minimization of the present worth of the total system cost to meet this growing demand. Other research with the same basic assumptions can be found in [14] and [25]. There is the additional possibility of importing products when the internal supply is not adequate to meet demand. Dynamic programming is used to obtain the optimal expansion policy. An illustration of the model is presented for the aluminum industry in Argentina. Berretta shows that a constant period between expansions is not the optimal, but that the time intervals between successive expansions decreases over the planning horizon. The major limitation of Berretta's work to the author's research objectives is the assumption that demand is always increasing at a geometric rate which neglects the possibility of seasonal variations which are a part of the electric utility demand.

Thompson [34] formulates a dynamic continuous time model of a firm encompassing operations and investments which forms an optimal control problem. A somewhat different capacity expansion objective is defined in this case since no assumptions are made concerning the demand for the firm's product. The objective of the firm is to maximize, subject to various constraints, the discounted value of operating profits less the costs of new capacity and the interest on borrowed funds over a fixed decision-making interval plus the discounted value of capacity at the end of the period. The model takes the form of a second order state model in which capacity and long term debt are the state variables. Scale of production and rate of purchase of new capacity are the control variables. Necessary conditions for the optimal solution are determined and the solution is characterized as a conventional bang-bang control.

The objectives of the Thompson model are again similar to the objectives of this research. However, the financial aspects are restricted to only obtaining capital through debt and there are no requirements to meet a demand for the product. The profit per unit of output, output from a unit of capacity per unit of time and price of a unit of capacity are variables which must be hypothezized as functions of time externally.

Thompson [35], using the same general structure as developed in [34], expands the model to include the characterization of a dynamic demand law. This linear demand law for a monopoly is determined by the intercept and slope of the function relating the output level of production to the price per unit of output. The demand law is affected by three state variables (called "stock variables" by Thompson) which are defined as the informative advertising stock variable, brand advertising stock variable and the price per unit of output. In addition to these state variables, capacity of production and net debt are also state variables.

A similar objective to the goal of [34] is defined which is to maximize the discounted value of savings over a finite time interval plus the discounted value of the firm's assets at the end of the interval. These assets include both the actual capacity of production and the stock variables of advertising. The management control variables and the change in price. The firm is viewed as having one financial account for borrowing or saving. The model addresses itself to the general problem of specifying the proper dynamic pricing and output of the product with the additional freedom of changing the demand law for its product through advertising. Optimal control theory and the calculus of variations are employed to obtain the necessary conditions of the optimal trajectories and these conditions are interpreted in economic terms through the observation that the corresponding adjoint variables can be viewed as marginal products of the state variables to the objective function.

The appropriateness of this model to the present research objectives is limited since as in [34], the financial aspects of the model are too restrictive. The advertising considerations, though interesting, are not relevant if one has assumed the future demand for electrical energy. Furthermore the electric utility does not enjoy the freedom of setting its price since this is the function of a regulatory commission.

Krouse [23] presents a methodology for dealing with multi-stage decision processes of financial planning. This modeling technique consists of abstracting and aggregating the important characteristics of the corporate financial process into a linear discrete set of state equations. Linearity is easily maintained in accounting relations and regression analysis is applied to behavioral relations. Also provision is made for random disturbances in the accuracy of describing these dynamic relations and thus noise sources are included in the formulation.

The objective function allows for the more realistic situation of a corporation in that it reflects a multi-objective criterion. This is accomplished through the use of a quadratic performance index which has two different types of terms. These terms represent "target" objectives, such as a debt equity ratio of 1/2, and other terms which are to be absolutely extremized, such as, profitability or sales. This multiple-objective criterion make it possible to consider both short-term and long-term financial goals. Mathematically, the model then takes the form of a linear discrete stochastic state system with a quadratic performance criterion. The maximum principle and dynamic programming both can be applied to models of this classic format to yield optimal financial strategies.

The primary difference in the methodology suggested by Krouse and the methods used in this research are the modeling approximation of linear models to describe non-linear behavior, and secondly the difference in formulation of an objective function. The quadratic criterion requires that you know what you want the target variables to attain. The linearization makes the model only valid for small perturbations around its nominal state values and thus gives very little global description of behavior. Here again, Krouse's work, like the models previously presented, considers the financial process, but not the capacity expansion policies.

Summary

Various modeling efforts of the firm and, in some cases, electric utilities have been reviewed. Their assumptions and objectives have been described along with the distinguishing differences between these efforts and the author's research. These objectives will be briefly summarized in order to provide a clear understanding of how this research fits into the overall picture of mathematical modeling of the investor-owned electric utility.

Microeconomic models were introduced first and it was shown that they are concerned primarily with static resource allocation and the output level for maximization of profit. Computer corporate models are digital computer programs that simulate the operations of a firm. The objective of a corporate model development is to provide a detailed simulation tool which can be used to test alternative strategies of operating the company. Capital budgeting models attempt to determine the proper financial decision process of raising capital. These financial decisions include the optimum dividend decision, issuance of equity decision and incurrence of debt decision which maximizes the market valuation of the firm's equity.

The other models described represent a collection of specialized operations research and management science research developments. These include models for determining the optimum expansion of capacity for maximum profit or minimum cost. Thompson [35] also includes the consideration of advertising and pricing in finding the optimum capacity expansion policy. Krouse [23] presents a methodology for handling the financial process with linear systems and a multi-dimensional objective. None of the models presented offers a total framework for the general problem of determining the optimum expansion of capacity, the corresponding capital budgeting strategy, and purchasing of energy from outside sources to maximize the capital attraction capability of the utility.

CHAPTER III

MODEL DESCRIPTION

Introduction

One of the objectives of this research is to determine the interrelationships between financial decisions and capacity expansion policies for the investor-owned electric utility. To accomplish this task, a mathematical model of the financial and capacity expansion processes has been developed. This model possesses mathematically tractable characteristics for the implementation of optimal control techniques and simultaneously contains adequate detail for use as a simulation tool.

To abstract a large complex economic system into a meaningful mathematical model, one must make a number of simplifying assumptions. These assumptions are included in the model description as needed to further explain the model. The validity of the assumptions cannot be judged entirely on their agreement with the real world, but must be weighed according to their relative importance to the objectives for which the model was formulated. The essence of mathematical modeling lies in the ability to make assumptions which exclude the irrelevant and focus on the relevant.

The utility model established by this research contains three subsections. These are the capacity process, financial process, and internal and external constraints. Figure 1 presents the general structure of the

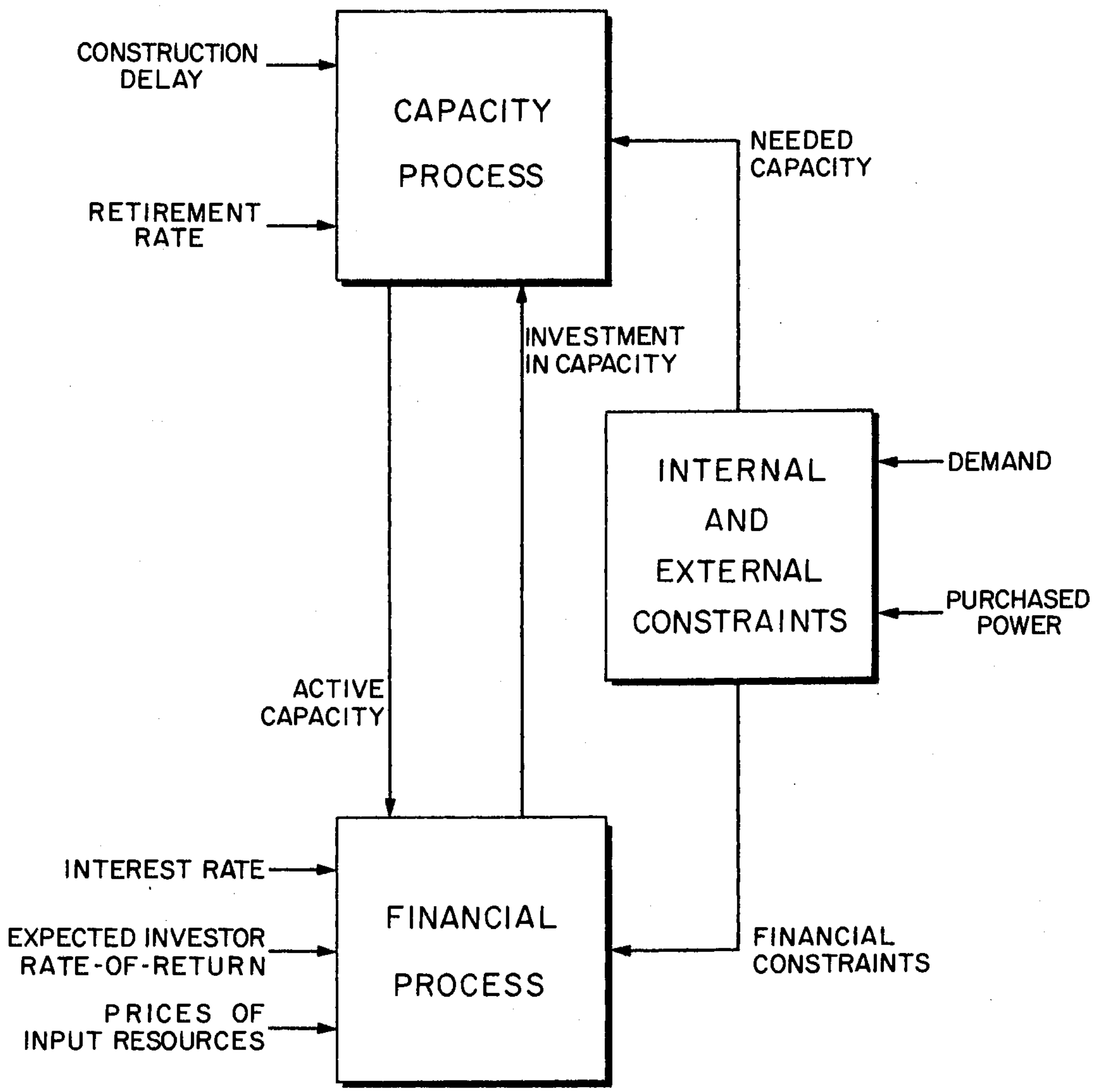

Figure 1. General Structure of the Utility Model

model showing the three subsections, their variables of interconnection and the exogeneous system input variables. The following chapter text is functionally organized, each section describing the capacity process, financial process and internal and external constraints in order.

Capacity Process

This subsection of the model contains the equations which describe the growth or retirement of the generation capacity of the electric utility. Capacity is defined as the maximum rate at which electrical energy can be generated, transmitted, and distributed to the consumer. It has the dimension of power, normally in megawatts. The two most important factors affecting the dynamics of capacity are (1) the construction of new capacity and subsequent time delays in this construction and (2) the retirement of capacity. The term retirement refers to discontinuance of the use of generators when they become physically inoperable or otherwise too costly to operate.

The capacity process is modeled with a set of three continuous linear state equations. The three state variables are capital invested in capacity, total capacity installed, and the total of all capacity retired. There is an additional dependent variable defined as the actual capacity available for use. The model has the form:

$$\dot{A} = Ku \tag{3.1}$$

$$\dot{TC} = c[(1/K)\ A - TC] \tag{3.2}$$

$$\dot{RC} = \beta(TC - RC) \tag{3.3}$$

$$AC = TC - RC \tag{3.4}$$

where

$\dot{A}$ = dA/dt (convention will be used throughout thesis)

A - Dollars invested in capacity

TC - Total capacity (historical sum of all capacity installed including capacity already retired)

RC - Retired capacity

AC - Actual usable capacity

K - Dollars/unit of capacity (construction cost)

u - New capacity/unit of time

c - Represents time delay in construction (1/c is time constant of construction)

β - Represents the rate of retirement (fraction of capacity retired/unit of time).

Equation (3.1) describes the rate of change through time of the assets invested in capacity and it is shown to be solely dependent on the capacity expansion variable u and the construction cost parameter, K. The parameter K is based on the assumption that for any planning period, one mode of generating electrical energy would be utilized. This could be either nuclear, coal or some other method and therefore a single cost of construction characterizes that mode. Since long range financial and capacity expansion planning is the objective, this assumption eliminates the question of what mode of generation should be constructed. Additionally, the question of economies of size as to cost of constructing large plants or smaller plants is handled through the assignment of one parameter for the construction cost.

The second equation relates the change in total historical capacity installed to the dollars invested in capacity, the total capacity installed, and parameters K and c (time constant of construction). The parameter c like K represents the construction delay for the given mode of generation under consideration and is justified for the same reason as the cost of construction parameter. This equation has the characteristic of a first order time delay since total capacity continues to change as long as dollars invested in capacity is not equal to total capacity, but will cease to change when they are equal. The speed of adjustment of the capacity installed to the dollars allocated to expanding capacity is completely dependent on the time constant of construction, c.

The retired capacity Equation (3.3) is based on the assumption that a certain percentage of active capacity is rendered unusable per unit of time. There is no attempt to determine vintage years or life expectancy of equipment. This percentage of retirement per unit of time, β, is generally a very small number.

The final Equation (3.4) is not a state equation, but an auxiliary equation defining a useful dependent variable which is a function of total capacity, TC, and retired capacity, RC. The difference in these two capacity variables yields the instantaneous level of actual usuable capacity on line available to satisfy demand.

To provide a better understanding of the behavior of these equations, the response of the third order system will be given for two different inputs of the management control variable, u. The system can be rewritten in matrix form as:

$$\begin{bmatrix} \dot{A} \\ \dot{TC} \\ \dot{RC} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ c/K & -c & 0 \\ 0 & \beta & -\beta \end{bmatrix} \begin{bmatrix} A \\ TC \\ RC \end{bmatrix} + \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix} u. \qquad (3.5)$$

The unforced solution $(u=0)$ of this system yields

$$A(t) = A(0) \tag{3.6}$$

$$TC(t) = TC(0) \tag{3.7}$$

$$RC(t) = TC(0)[1 - e^{-\beta t}] \tag{3.8}$$

$$AC(t) = TC(0)e^{-\beta t} \tag{3.9}$$

where it is assumed that the initial conditions on A and RC are A(0), and zero, respectively, and the initial condition, TC(0), is equal to $\frac{A(0)}{K}$. This response is to be expected, since if there is no increase in capacity, the actual capacity will decay at the exponential rate of β (retirement rate).

The capacity expansion variable, u, is a decision variable. After careful consideration and planning, the committment for expansion is made over a near zero time interval which mathematically can be characterized as a delta function. If $u(t) = \delta(t)$, a delta function, then the unit impulse response of the system is easily determined. The eigenvalues are 0, $-c$, and $-\beta$ and the response is:

$$A(t) = A(0) + K \tag{3.10}$$

$$TC(t) = TC(0) + (1 - e^{-ct}) \tag{3.11}$$

$$TC(t) = TC(0)[(1 - e^{-\beta t})] + 1\frac{-\beta}{\beta-c} e^{-ct} + \frac{c}{\beta-c} e^{-\beta t} \tag{3.12}$$

$$AC(t) = TC(0)e^{-\beta t} + \frac{c}{\beta-c}[e^{-ct} - e^{-\beta t}] \ . \tag{3.13}$$

It is assumed that capacity is constructed much quicker than it degenerates into retired capacity which means that $c \gg \beta$. Equation (3.10) indicates that the assets invested in capacity will be characterized by a step increase of K from its initial condition, and where Equation (3.11) shows the TC increases one unit in capacity through the first order delay with time constant $1/c$. The equation for RC satisfies the zero initial condition and approaches TC(0) + 1 as time approaches infinity. Equation (3.13) analytically describes the response of actual capacity, AC, and demonstrates a concave behavior with it increasing to a value approaching TC(0) + 1 at a rate similar to the time constant of construction. AC then decreases monotonically from TC(0) + 1 to approach zero as time becomes large at a rate near the retirement rate, β. This behavior is observed because of the assumption that $c \gg \beta$. Figure 2 illustrates the response of actual capacity to a unit impulse in the control variable u.

This model therefore captures the two most important features of the dynamics of generation capacity: (1) the construction delay between committing capital to new capacity and the realization of that capacity, and (2) the inevitable retirement of capacity.

Financial Process

This segment of the model describes the behavior of the capital structure of the investor-owned electric utility. The internal financial variables relevant to the long-range capital budgeting for construction of new capacity are the total equity, the long-term debt, price of a share of common stock, number of shares of stock, and the net income flow. Exogeneous factors affecting these internal variables are the interest rate, investors' expected rate of return, cost of marketing common stock, the price of electrical energy, and the costs of input resources to the production of energy. The following description focuses on the relations governing the behavior of these internal

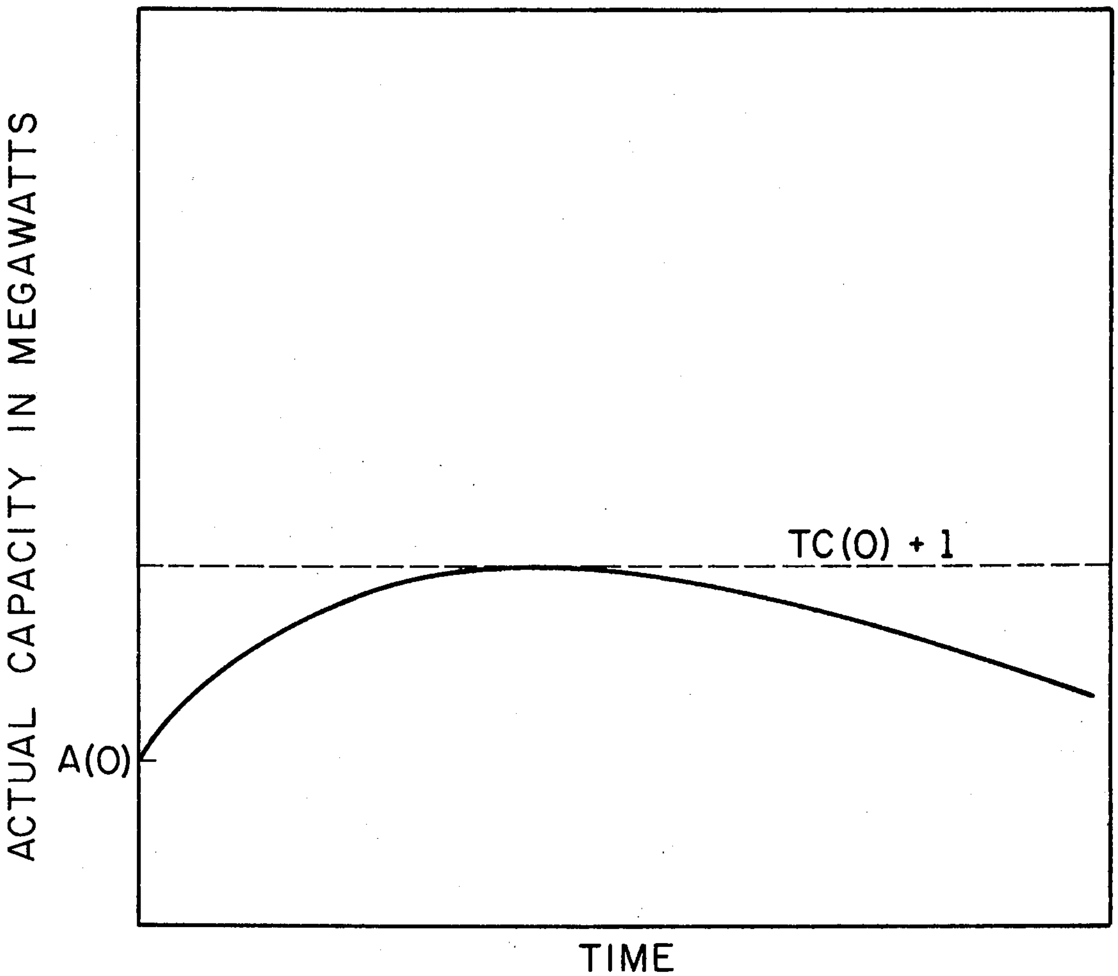

Figure 2. Actual Capacity Response to Unit Impulse of Capacity Expansion

variables and specifies their relationship to the external economic factors.

The equity of the utility is defined as the net worth of the company's capital stocks, capital surplus, and earned surplus (or retained earnings). This definition is taken from Weston [41] and excludes preferred stockholder holdings, since it is assumed that common stock is the only instrument of equity. Equity therefore has two components: retained earnings and revenue from the sale of common stock. Retained earnings represents that portion of the net income that is not paid in dividends to the stockholders. From these assumptions, the level of equity can be changed in only two ways, and the rate of equity change can be represented as

$$\dot{E} = [I - d \cdot N] + [(1-\delta)\, P \cdot \dot{N}] \tag{3.14}$$

where

E - Total equity of the company

I - Net income per unit of time

d - Dividend per share per unit of time

N - Number of outstanding shares of common stock

P - Market price of one share of common stock

δ - the percentage cost of marketing a share of common stock.

The first bracketed term represents the change in equity due to retention of earnings while the second bracketed term represents the change in equity due to the issuance of new stock. It will be noted that the cost of marketing new stock has been included and that $\dot{N}$ designates the time rate of change in the number of shares of common stock.

Long-term debt is assumed to be non-maturing and is implemented in the model as a dependent variable through a debt-equity ratio parameter. The debt-equity ratio, as its name denotes, is the ratio of long-term debt to the total equity of the company. Davis [10] points out that in the current literature there is no conclusive agreement for describing the mechanism underlying changes in debt-equity ratio in a firm to be found. However, once a debt-equity ratio is determined, there is a tendency to maintain this ratio in future expansion decisions. This has been verified through conversations with financial planners of Oklahoma Gas and Electric Company and Public Service Company of Oklahoma. Historically, their ratios have not varied more than a few percent. Therefore, it will be assumed that this ratio is constant and not dependent on any other financial variables. This assumption motivates the following expression for long-term debt:

$$Q(t) = h \cdot E(t) \tag{3.15}$$

where

$Q(t)$ - long-term debt

h - debt equity ratio.

Unlike the equity and long-term debt equation, the equation describing the price of a share of common stock is exclusively behavioral. The necessary basic assumptions required to express the price of a share of common stock are "perfect markets," and "rational behavior," as defined by Miller [27].

In "perfect markets," no single buyer or seller of stock is large enough for his transactions to have an appreciable impact on the current trading price and all traders have equal access to relevant information concerning the characteristics of the shares. Also, there are assumed to be no tax differential and/or preferences between dividends and capital gains in the market.

"Rational behavior" means that investors always prefer more wealth to less and are indifferent as to whether a given increment to their wealth takes the form of dividends or an increase in the market value of their holding of shares.

Under these assumptions Miller states that the value of shares is governed by the following fundamental principle.

$$P(t+1) - P(t) + d(t) = \rho P(t) \tag{3.16}$$

where

$P(t+1)$ - The expected price of a share of common stock at the end of a unit trading period

$P(t)$ - The present price of a share of common stock

$d(t)$ - The present dividend per share

ρ - Expected rate-of-return by investor $(0 \leq \rho \leq 1)$.

Equation (3.16) states that in any trading period the market will adjust the price so that dividends plus expected capital gains equal the rate-of-return the investor requires on an investment $P(t)$. Following the same reasoning as Davis [10], a dynamic relation for the stock price can be developed. If

$$P(t+1) - P(t) + d(t) - \rho P(t) > 0 \tag{3.17}$$

then the market will react to increase the price of the stock. On the other hand, if

$$P(t+1) - P(t) + d(t) - \rho P(t) < 0 \tag{3.18}$$

then the market will respond to decrease the current price of a share of stock. Modeling this concept with continuous variables yields

$$\dot{P} = c_0[d(t) - \rho P], \; c_0 > 0 \tag{3.19}$$

where C_0 represents the trading activity factor and denotes how quickly the market responds to changes in dividend rates. As defined, ρ represents the rate-of-return expected by prospective investors. McDonald [24] has developed an econometric model to estimate this rate-of-return for the equities of electric and gas utilities and estimates are given for these returns for 1969 in four regions of the United States.

The number of shares of common stock is strictly dependent on the management's decision on new stock issuance, $u_s(t)$, as follows:

$$\dot{N} = u_s(t) \tag{3.20}$$

where

$u_s(t)$ - Management control variable (number of shares issued/unit of time).

$u_s(t)$ is always greater than zero, and since the number of shares of stock has a positive initial condition, N is a nondecreasing function of time. It is noted that $u_s(t)$ can be substituted for $\dot{N}$ in the equity Equation (3.14).

The final variable needed to complete the financial process is the net income flow. This is the accounting segment which determines the net income per unit of time and involves the calculation of the total revenue per unit of time and subsequent subtraction of fixed and variable costs of production.

The algebraic equation specifying the net income is

$$I = [R \cdot D(t) \cdot LF] - F[(D(t) - u_p)] - G(AC) - D(AC) - [P_p \cdot u_p] - [I_n \cdot h \cdot E] \tag{3.21}$$

where

I - Net income per unit of time

R - Average price per unit of energy

$D(t)$ - Specified peak power demand (energy per unit of time)

LF - Load Factor (energy produced/peak demand X time)

$F[D(t)-u_p]$ - Fuel cost per unit of time as function of energy produced internally

$G(AC)$ - Maintenance cost per unit of time as function of active capacity available

$D(AC)$ - Depreciation cost per unit of time as function of active capacity available

P_p = Price per unit of purchased energy

u_p - Power purchased from outside sources

I_n - Interest rate on long-term debt.

The other variables have been previously defined.

The first bracketed term on the right of Equation (3.21) is the total revenue per unit of time calculated as the product of the average price per unit of energy, peak power demand, and load factor. It is assumed that peak demand for electrical energy is specified and is completely deterministic and expressed as a continuous function of time. This demand includes residential, commercial, and industrial demands. It also includes the demand for company energy contracted by other utilities through purchase agreements. It is further assumed that the load factor (energy produced/peak demand•time) is specified.

These assumptions are justified since a typical utility supports a group of analysts to estimate the future growth of demand using trend analysis and other economic forecasting techniques. They consider industrial growth within the utility's service area and technological changes which will affect the demand for electrical energy. The load factor of the utility's system is a measure of the utilization of the generation capacity and is estimated through similar analysis of historical data and technological forecasts. Since demand is an aggregation of residential, commercial and industrial consumers, a single price for a unit of energy is assumed. Since the load factor is dimensionless, the dimensions of the bracketed term $[R \cdot D(t) \cdot LF]$ are \$/time.

The last five terms on the right side of Equation (3.21) represent the most significant variable and fixed costs of production. The first of these terms is the fuel cost per unit of time and is a variable cost since it varies with the generated power which is the difference between the peak demand and purchased power. De Salvia [12] contends that the variable cost per unit of output is almost constant. This hypothesis is based on the fact that fuel cost is the most significant component of variable cost. De Salvia presents data on the variation of the incremental fuel cost (mills/kwhr) for fossil fuel steam generators of various sizes. This data indicates a maximum variation of the incremental fuel cost of only 15% on a 100 megawatt generator operated over its minimum to maximum output range. For this reason, it is assumed that the fuel cost is a linear function of energy output. It is noted that this is a fuel cost per unit of time since energy per unit of time, power, is the argument of the fuel function.

The third term in Equation (3.21) is the maintenance cost, denoted G(AC). This is a fixed cost, not dependent on generated output. Maintenance cost is assumed independent of output based on De Salvia's findings. He states that most maintenance activity is programmed and not directly dependent on the level of output. The maintenance cost is therefore only a function of the actual operating capacity.

In Chapter IV it is postulated that the maintenance cost per unit of time is a linear function of actual capacity.

The depreciation cost denoted D(AC) is the cost covering the dissipation of the durable goods of plant and equipment defined as actual capacity of production. Mihalasky [26] defines straight-line depreciation as:

> Method of depreciation whereby the amount to be recovered (written off) is spread uniformly over the estimated life of the asset in terms of time periods or units of output. May be designated "percent of initial value."

Straight-line depreciation is assumed for D(AC) with no salvage value.

The fourth term on the right side of Equation (3.21) represents the variable cost of purchasing power from external sources. It is the product of the price of purchased energy, P_p, and purchased power, u_p. This product gives the cost of purchased energy per unit of time in \$/unit of time.

The last cost term, $[I_n \cdot h \cdot E]$, is the cost of long-term debt. Long-term debt is expressed as the product of the debt-equity ratio and total equity through Equation (3.15). It is assumed that the interest rate is an exogeneous parameter and is not dependent on any of the internal variables of the model. Since interest rate is a ratio of percentage of principal per unit of time and $h \cdot E$ is the principal in dollars, the product of the two is \$/unit of time, a cost flow. This completes the discussion of the separate terms of the income flow equation and indicates the detail of the model in specifying the costs of production of electrical energy.

Internal and External Constraints

Additional constraints are needed to complete the description of the utility model. These constraints are imposed from both within and outside the utility. Mathematically, they take the form of equality and inequality relations. These constraints insure that the company always meets its demand for energy and the total amount paid in dividends is less than the current net income. Also, these relations tie the capacity process and financial process together by insuring that the total equity plus long-term debt equals the total assets invested in capacity. Finally, constraints are used to fix maximum limits on the level of purchased power and issuance of new stock.

The fundamental responsibility of the franchised electric utility is to satisfy the demand for electrical energy. However, the peak demand D(t) is a function of time, displaying a seasonal variation. This is easily explained since in the southwestern United States, greater electrical energy consumption takes place in the summer than in the winter due to air conditioning. The opposite situation occurs in the south central part of the United States where there is extensive use of electrical energy for heating. This seasonal variation plus the overall growth of energy demand characterizes the demand mathematically as a sinusoidal function superimposed on a ramp function. In order to meet the time-varying peak demand, the active capacity on line, AC, plus the power purchased u_p must always be greater than or equal to the peak demand. Thus,

$$AC(t) + u_p(t) - D(t) \geq 0, \quad t \in [0,T] \tag{3.22}$$

where

T - Planning horizon (time interval of interest).

This constraint is illustrated in Figure 3 and it is noted that $AC(t)$ is greater than $D(t)$ at 0, but is less than $D(t)$ at $t = .5$ which requires the purchase of power, $u_p(.5)$, to satisfy the equality of the inequality condition. This figure demonstrates why the constraint cannot be an equality relation.

Basic to the next two constraints is the assumption that the assets defined as total equity and long-term debt does not include working capital. This assumption subsumes the cash flow of the utility and concentrates on the capital needed for expansion. Elimination of the possibility of reserve funds being used for operating costs or dividends dictates that the dividends be paid from the current net income flow. The total dividend flow, therefore, cannot be greater than the income flow and the following constraint is imposed on the model:

$$I - d \cdot N \geq 0, \quad t \in [0,T] . \tag{3.23}$$

Again, it must be remembered that net income flow is rate of dollars per unit of time, and d is the dividend per unit of time.

The capacity and financial processes are coupled together by invoking the condition that the capital needed for capacity expansion must come from the two modes of raising capital; equity and long term debt. Thus,

$$A(t) = E(t) + Q(t), \quad t \in [0,T] . \tag{3.24}$$

Making use of the debt-equity ratio Equation (3.15), (3.24) can be written as

$$A(t) = E(t)[1 + h] . \tag{3.25}$$

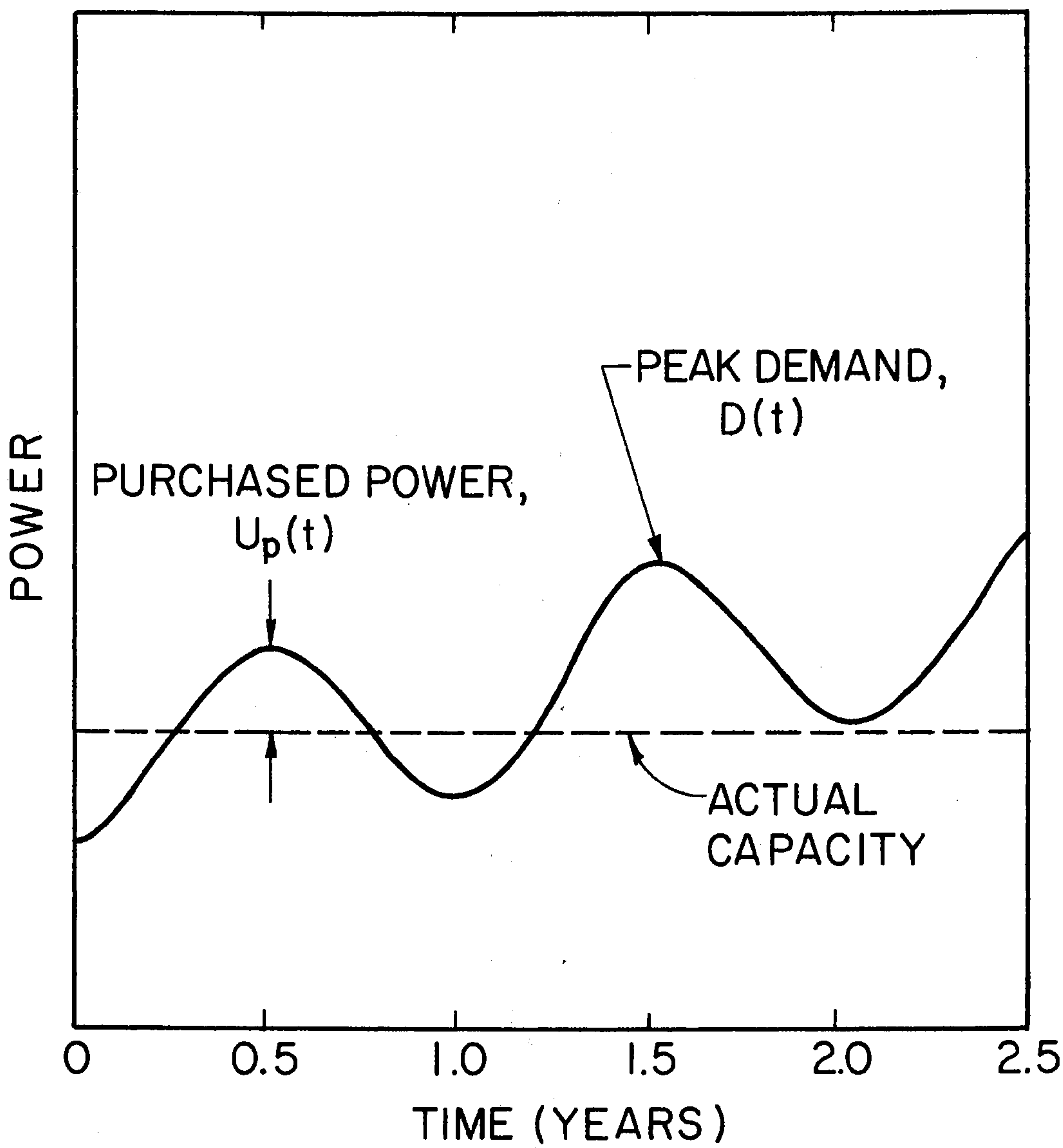

Figure 3. Peak Demand - Actual Capacity - Purchased Power Plot

This equality insures that capital assets will track the needed assets for expansion of capacity of production. This allows the elimination of the state variable A(t) in favor of the term E(t)[1 + h] in the differential Equation (3.2), reducing the total utility model to a fifth order system. The expansion of capacity is then initiated through the dividend management control variable d(t) and/or issuance of new equity, $u_s(t)$.

A limit is placed on the amount of capacity that can be obtained from outside the utility and this limitation takes the form of an upper bound on $u_p(t)$.

$$u_p(t) \leq u_{pMAX} \,. \tag{3.26}$$

Also, a limitation is placed on the maximum amount of new stock which can be issued per unit of time

$$u_s(t) \leq u_{sMAX} \,. \tag{3.27}$$

Finally, all variables are constrained to be non-negative.

The total mathematical model of the utility which describes the interconnected dynamics of the capacity and financial processes can now be presented in its entirety.

$$\dot{TC} = c\left[\left(\frac{1+h}{K}\right)E - TC\right] \tag{3.28}$$

$$RC = \beta[TC - RC] \tag{3.29}$$

$$\dot{E} = [I - d \cdot N] + [1 - \delta]P.u_s \tag{3.30}$$

$$\dot{P} = C_0[d - \rho P] \tag{3.31}$$

$$\dot{N} = u_s \tag{3.32}$$

$$I = [R \cdot LF \cdot D(t)] - F[D(t) - u_p)] - G(AC)$$

$$-D(AC) - [P_p \cdot U_p] - [I_n \cdot h.E] \tag{3.33}$$

$$AC = TC - RC \tag{3.34}$$

$$AC + u_p - D(t) \geq 0 \tag{3.35}$$

$$I - d \cdot N \geq 0 \tag{3.36}$$

$$u_p \leq u_{pMAX} \tag{3.37}$$

$$u_s \leq u_{sMAX} \tag{3.38}$$

with initial conditions on the state variables

$$\begin{bmatrix} TC(0) \\ RC(0) \\ E(0) \\ P(0) \\ N(0) \end{bmatrix} = \begin{bmatrix} TC_0 \\ 0 \\ E_0 \\ P_0 \\ N_0 \end{bmatrix} . \tag{3.39}$$

Figure 4 illustrates the model and its structure with a Forrester diagram. Forrester [15] suggests a methodology of describing industrial systems. The rectangular figures represent "levels" or states of the system. The "valve" shaped figures control the flow into and out of the states of the systems. The solid arrowed lines represent the flow of a physical quantity while the unlabeled irregular forms are external "sources" or "sinks" depending on the direction of the flow emanating from these reservoirs. The dotted lines are information channels. For example, the flow into the total capacity state is dependent on equity through an information channel. Dotted circles are dependent variables which can be calculated from the state variables and other internal and external parameters. Exogeneous inputs and parameters are denoted by their name under horizontal lines connected to information channels. Forrester's diagram of the utility model shows the information-feedback structure of the model and the coupling of various variables to each other.

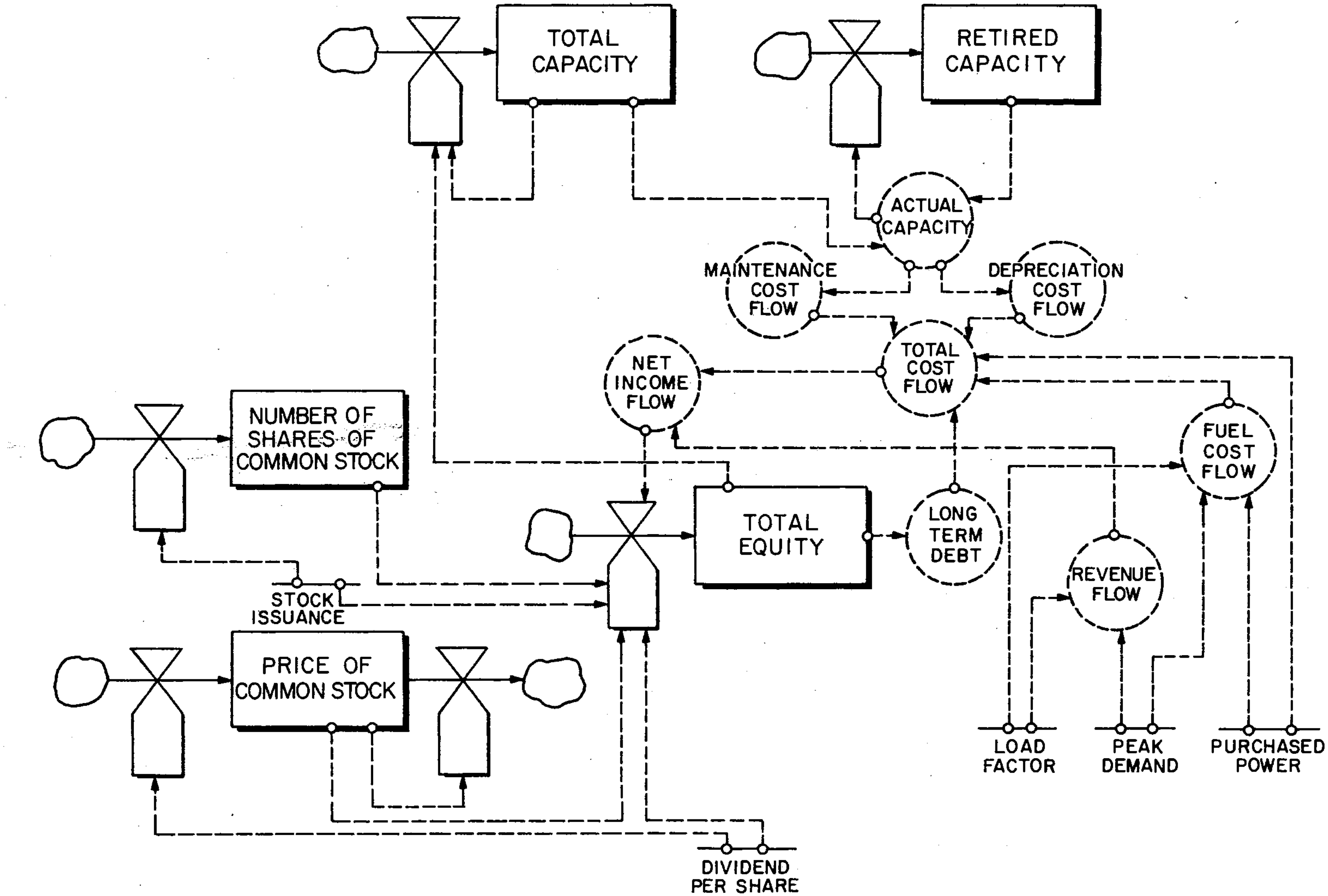

Figure 4. Forrester Diagram of Model

Summary

The mathematical model of the electric utility has been presented. It has been shown that the model includes three sections. These include the capacity process, financial process, and internal and external constraints.

The capacity process describes the expansion and retirement of capacity including the time delay in construction. The financial process contains the behavior of the total equity, long term debt, price of a share of common stock, number of shares of outstanding stock, and the net income flow of the utility. Internal and external constraints mathematically describe the restrictions on the states of the system and management control variables imposed from internal and external conditions. A Forrester diagram is shown and interpreted to illustrate the general structure of the model and the couplings between subsections.

The model offers a mathematical representation useful for both analysis and simulation. It has a mathematical form amenable to the application of modern control theory. It also has enough detail to be useful as a simulation tool. It should be noted that the parameters defined in the model can either be constants or functions of time or of other variables when the model is used in simulation. This provides the necessary flexibility for useful computer simulation.

In order to provide a clear understanding of the model and the assumptions underlying its development, all basic assumptions are listed below. For easy reference, they are classified by the model section to which they apply.

Capacity Process

1. One mode of generation capacity is assumed for the planning period. The mode might be nuclear, coal, natural gas, or any other method of generating electrical energy. This single mode can be characterized by a parameter K, cost of construction of new capacity per unit of capacity, and a time constant of construction, C.

Financial Process

2. It is assumed that there are only two components of equity; retained earnings and common stock. Retained earnings represent that amount of net income not paid in dividends to stockholders.

3. A debt-equity ratio is specified which eliminates the debt decision from the financial decision process.

4. It is assumed that all debt is non-maturing.

5. Common stock is the only instrument of equity ownership. This excludes preferred stock, convertible instruments, etc.

6. It is assumed that investors are indifferent between dividends and capital gains, and there are no tax differentials that could influence preference between dividends and capital gains. No single investor's transactions affect the price of a share of stock.

7. Expected rate-of-return by investors in common stock is assumed to be constant with time.

8. The costs to the utility include fuel costs, maintenance costs, financial depreciation of plant and equipment, purchased energy costs, and interest on long-term debt.

   a. Fuel cost is assumed to be a linear function of energy produced.

   b. Maintenance costs include both capital and labor of maintaining the capacity and are assumed to be a function of actual capacity.

   c. Straight line depreciation is assumed on actual capacity with no salvage value.

9. It is assumed that peak demand for electric energy is specified and is completely deterministic and in function form.

   a. It includes residential, commercial, and industrial demand.

b. It includes energy promised to other electric utilities through purchase agreements.

10. It is assumed that the load factor (energy produced/peak demand time) is specified and in functional form.

11. The price of electric energy will be represented by one value which is an aggregation of all the rate classes.

12. The interest rate is assumed to be specified and not dependent on any other financial variables.

Internal and External Constraints

13. The franchised firm is required to meet peak demand for energy by producing it or purchasing it from outside the utility.

14. It is assumed that the utility is in a growth period, i.e., net income is always greater than zero.

15. The total assets of the utility do not include working capital but only capital invested in plant and equipment.

16. The purchase agreement for energy is constrained only by an upper limit on the maximum power than can be demanded.

17. The issuance of new stock per unit of time is bounded above by a maximum limit, u_{sMAX}.

CHAPTER IV

OPTIMIZATION OF FINANCIAL OBJECTIVE FUNCTION

Introduction

The model developed in the previous chapter will now be used to analytically determine optimum planning strategies for the electric utility. These strategies include decisions on dividend levels, issuance of new stock and purchases of power from external sources subject to the constraints of the mathematical model. Fundamental to the optimization process is a carefully defined objective function or performance criterion which is to be extremized. This chapter defines a suitable objective function and poses the related optimal control problem. The analytic solution of the control problem is obtained and interpreted. The analytic approach provides insight into the sensitivity of the optimum strategies to the variables and parameters of the utility model. This is in contrast to a computer optimization solution which yields only the strategies, but gives no understanding of what affected the decision process.

Financial Objective Function

As stated in the introductory chapter, the objective of this research is to develop optimal planning strategies for the maximization of a financial planner's goal. This goal has been described as the market valuation of the electric utility. Market valuation is a measure of the company's ability to attract investment by prospective investors via common stock sales. This financial goal is suggested by several writers on financial theory. The objective is based on the idea that the optimal investment program is the one that is most beneficial to the suppliers of investment funds. Williams [42] first suggested a mathematical expression for the market valuation of common equity. This expression has been used in later research efforts and financial texts such as [10], [18], and [38]. The expression takes the mathematical form:

$$PI = P(T)e^{-\rho T} + \int_{t_0}^{T} d(t)e^{-\rho t}\, dt \tag{4.1}$$

where

PI - Performance Index

d(t) - Dividend per share per unit of time.

As $T \rightarrow \infty$, Equation (4.1) becomes the classical definition of the present price of a share of common stock.

$$P_0 = \int_{t_0}^{\infty} d(t)e^{-\rho t}\, dt \tag{4.2}$$

where

P_0 - The present value of a share of common stock.

Equation (4.1) reflects the present worth of the discounted stream of dividends from t_0 to T plus the discounted final value of the price of a share of common stock. This mathematical form assumes that the investor is indifferent between dividends and capital gains; an assumption already made in the development of the dynamic model for the share price given by Equation (3.19). It further assumes that ρ, the rate-of-return expected by an investor, is constant through time. This assumption is the result of the general financial stability of a regulated utility and a relatively fixed debt-equity ratio.

Optimal Control Problem

With the statement of the performance index complete, the optimal control problem can now be formally posed.

Maximize: u_p, u_s, d
$$PI = P(T)e^{-\rho T} + \int_0^T d(t)e^{-\rho t}\,dt \tag{4.3}$$

Subject to:

$$\dot{TC} = C\left[\frac{1+h}{K}E - TC\right] \tag{4.4}$$

$$\dot{RC} = \beta[TC - RC] \tag{4.5}$$

$$\dot{E} = [I - d \cdot N] + [(1-\delta)P\,u_s] \tag{4.6}$$

$$\dot{P} = C_0[d(t) - \rho P] \tag{4.7}$$

$$\dot{N} = u_s$$

$$AC = TC - RC \tag{4.9}$$

$$\begin{aligned} I = {} & [R \cdot LF \cdot D(t)] - F[D(t) - u_p] - G(AC) \\ & - D(AC) - P_p\,u_p - [I_n \cdot h \cdot E] \end{aligned} \tag{4.10}$$

$$AC(t) + u_p(t) - D(t) \geq 0 \tag{4.11}$$

$$I - d \cdot N \geq 0 \tag{4.12}$$

$$0 \leq u_p \leq u_{pMAX} \tag{4.13}$$

$$0 \leq u_s \leq u_{sMAX} \tag{4.14}$$

$$d \geq 0 \; . \tag{4.15}$$

Relations (4.4)-(4.15) mathematically asks the question - What are the time functions of purchased power u_p, issuance of new equity u_s, and the dividend per share d, which maximizes the market valuation of the utility stock subject to the constraints?

To expedite the mathematical development of this control problem, the variables are renamed per the following relations. It should be noted that the Appendix has a glossary of variables for reference.

$$TC \longrightarrow x_1$$

$$RC \longrightarrow x_2$$

$$E \longrightarrow x_3$$

$$P \longrightarrow x_4$$

$$N \longrightarrow x_5$$

$$d \longrightarrow u_d$$

It has also been assumed that the functions for fuel cost, maintenance cost and depreciation cost are linear with respect to their corresponding arguments. These functions are therefore represented by the constant coefficients C_f, C_m, and C_D, respectively. The control problem will be restated in these new terms, noting that the actual capacity AC is expressed as $x_1 - x_2$.

$$\underset{u_p,\, u_s,\, u_d}{\text{Maximize:}} \quad PI = x_4(T)e^{-\rho T} + \int_0^T u_d \; e^{-\rho t} \; dt \tag{4.16}$$

Subject to:

$$\dot{x}_1 = C\left[\frac{1+h}{K} x_3 - x_1\right] \tag{4.17}$$

$$\dot{x}_2 = \beta[x_1 - x_2] \tag{4.18}$$

$$\dot{x}_3 = [I - u_d x_5] + [(1-\delta)x_4 u_s] \tag{4.19}$$

$$\dot{x}_4 = c_0[u_d - \rho x_4] \tag{4.20}$$

$$\dot{x}_5 = u_s \tag{4.21}$$

$$\begin{aligned} I = {} & R \cdot LF \cdot D(t) - C_f D(t) + C_f u_p - C_m x_1 \\ & + C_m x_2 - C_D x_1 + C_D x_2 - P_p u_p - I_n h\, x_3 \end{aligned} \tag{4.22}$$

$$x_1 - x_2 + u_p - D(t) \geq 0 \tag{4.23}$$

$$I - u_d x_5 \geq 0 \tag{4.24}$$

$$0 \leq u_p \leq u_{pMAX} \tag{4.25}$$

$$0 \leq u_s \leq u_{sMAX} \tag{4.26}$$

$$u_d \geq 0 \, . \tag{4.27}$$

It should be noted that since all the control variables are non-negative and the states have non-negative initial conditions then $x_i > 0$ for $i = 1, 2, 3, 4, 5$.

Consider a system whose state at time t is represented by a real n-dimensional vector $\underline{x}(t) = [x_1(t), \ldots, x_n(t)]$. This state vector is determined by a system of differential equations with initial conditions.

$$\frac{dx_i}{dt} = f_i(t, \underline{x}, \underline{u}), \quad x_i(t_0) = x_{i0} \qquad \text{for} \quad i = 1, \ldots, n \tag{4.28}$$

where

$$\underline{u}(t) = [u_1(t), \ldots, u_m(t)] \, .$$

The m-dimensional vector $\underline{u}(t)$ is called the control function vector. Additional constraints are imposed and take the form

$$g_j(t,\underline{x},\underline{u}) \geq 0 \qquad j = 1, \ldots, r \,. \tag{4.29}$$

The problem of optimal control is to choose the control $\underline{u}(t)$ so as to bring the system from the given initial state to some terminal state satisfying all constraints in such a way as to extremize a functional

$$J(\underline{u}) = g[t_1,\underline{x}(t_1)] + \int_{t_0}^{t_1} \Phi(t,\underline{x},\underline{u})dt \,. \tag{4.30}$$

The integral in Equation (4.30) can be evaluated for a given control vector function $\underline{u}(t)$. It is further assumed that $\underline{u}(t)$ is piecewise continuous on the time interval $[t_0,t_1]$.

The problem presented, excluding the constraints of Equation (4.29) is a special form of the Bolza problem of the calculus of variations. In solving problems with constraints, several avenues have been explored. Sage [32] considers this general problem and transforms the constraints of Equation (4.29) into r additional real state variables, $\underline{y}(t)$ where

$$\frac{dy_j^2}{dt} = g_j(t,\underline{x},\underline{u}), \quad y_j(0) = 0 \qquad \text{for} \quad j = 1, \ldots, r \,. \tag{4.31}$$

This was first suggested by Valentine [37]. This insures that the constraints are satisfied since $\underline{y}(t)$ is squared. This approach leads to a set of differential equations of $\underline{x}$ and $\underline{y}$ and corresponding adjoint variables which increases the order of the system. Hestenes [19] and Berkovitz [5] developed similar necessary conditions for this problem by transforming the problem into a problem of calculus of variations, and appending the inequality constraints to the Hamiltonian with additional

Labrange parameters for the constraints. This method, like Valentine's, increases the complexity and number of necessary conditions to be satisfied.

Another approach is given by the "Maximum Principle" developed by Pontryagin [29]. The "Maximum Principle" provides a solution to the optimal control problem when the control functions are restricted to a given set of admissible controls. The assumptions and the "Maximum Principle" are presented without proof for the fixed final time problem of Equations (4.28) through (4.30).

It is assumed that the m-dimensional vector function $\underline{u}(t)$ is piecewise continuous and its range is in a closed convex m-dimensional subspace, U. A scalar function defined as the Hamiltonian is

$$H(\underline{x},\underline{u},\underline{\lambda},t) = \Phi(t,\underline{x},\underline{u}) + \sum_{i=1}^{n} \lambda_i f_i(t,\underline{x},\underline{u}) \qquad (4.32)$$

where λ_i is a continuous function called the adjoint variable of state x_i. With these definitions, the necessary conditions for maximizing the performance index are:

$$\dot{\underline{\lambda}} = -H_x \qquad (4.33)$$

$$\dot{\underline{x}} = H_\lambda \qquad (4.34)$$

with boundary conditions

$$\underline{x}(t_0) = \underline{x}_0 \qquad (4.35)$$

and

$$\underline{\lambda}(t_1) = g_x[t_1,\underline{x}(t_1)] \qquad (4.36)$$

$\underline{x}(t_1)$ and $\underline{\lambda}(t_0)$ are free and

$$H(\underline{x}^*,\underline{u}^*,\underline{\lambda}^*,t) \geq \underset{u \in U}{H}(\underline{x},\underline{u},\underline{\lambda},t) \qquad (4.37)$$

where subscripts on H and g indicate partial derivatives with respect to the subscript variable. Further conditions are implied by Equation (4.37), since the Hamiltonian must be maximized for the optimum control function $\underline{u}^*(t)$, the following conditions hold

$$H_{u_i} > 0 \qquad \text{implies} \qquad u_i^* = u_{iMAX} \tag{4.38}$$

$$H_{u_i} = 0 \qquad \text{implies} \qquad u_{iMIN} \leq u_i^* \leq u_{iMAX} \tag{4.39}$$

$$H_{u_i} < 0 \qquad \text{implies} \qquad u_i^* = u_{iMIN} \tag{4.40}$$

where H_{u_i} is the partial derivative of the Hamiltonian with respect to u_i and is called the "Switching function." These conditions insure that the Hamiltonian is maximized with respect to $\underline{u}$. It also shows that the control solution lies on the surface of the admissible control space U.

The maximum principle can be applied to the electric utility control problem given in Equations (4.16) through (4.27). The first consideration is the admissible control subspace, U. Relations (4.23) through (4.27) define the admissible control region. The control region must be shown to be closed and convex. The region is closed since the boundaries are included through the inclusion of the equality sign in all the inequalities defining the region. The control space in this problem is three dimensional with u_s, u_p, and u_d the control variables.

The boundaries in the u_s coordinate are clearly determined with an upper and lower bound.

$$0 \leq u_s \leq u_{sMAX} \; . \tag{4.41}$$

The lower limits on u_p are more complex than would appear from (4.25), since u_p is also bounded by (4.23). Considering both relations, if $D(t) - (x_1-x_2) < 0$ then the lower bound of (4.25) is invoked and $u_p \geq 0$. If $D(t) - (x_1-x_2) > 0$ then $u_p \geq D(t) - (x_1-x_2)$, which implies that

$u_p \geq MAX[D(t) - (x_1 - x_2), 0]$. This insures that peak demand is met, but prevents u_p from becoming negative. The upper limit on u_p is dependent on both (4.24) and (4.25). Since u_p is contained in I, the income flow equation, u_p and u_d are related through (4.24). Expanding (4.24) yields

$$x_s u_d + (P_p - C_f) u_p \leq f(D(t), x_1, x_2, x_3) \tag{4.42}$$

where

$$f(D(t), x_1, x_2, x_3) = I + (P_p - C_f) u_p \ .$$

It is further assumed that other parameters and coefficients in the income equation are constant and therefore the function, f, is only a function of D(t), x_1, x_2, and x_3. It has been assumed that I is always positive, and if $P_p > C_f$, then $f[D(t), x_1, x_2, x_3] > 0$.

In general terms (4.42) can be written

$$a_1 u_d + a_2 u_p \leq b \tag{4.43}$$

where

$$a_1, a_2, b > 0$$

so that u_d and u_p are related through a linear equation with the u_d and u_p intercepts of the plane a function of D(t), x_1, x_2, x_3, and x_5. The upper limit on u_p from (4.25) is a u_{pMAX} so that $u_p \leq MIN\left[u_{pMAX} \ \frac{b - a_1 u_d}{a_2}\right]$. u_d has a lower bound of zero. This completes the description of the boundaries of the control subspace. Figure 5 shows the admissible control region in the control space. The "pie" shaped figure represents the control subspace and is a function of the states of the system. The dotted boundaries are invoked when through time the values of D(t) and the states of system require it. The admissible control space is the intersection of convex sets and therefore is convex. The subspace is described analytically with the following relations:

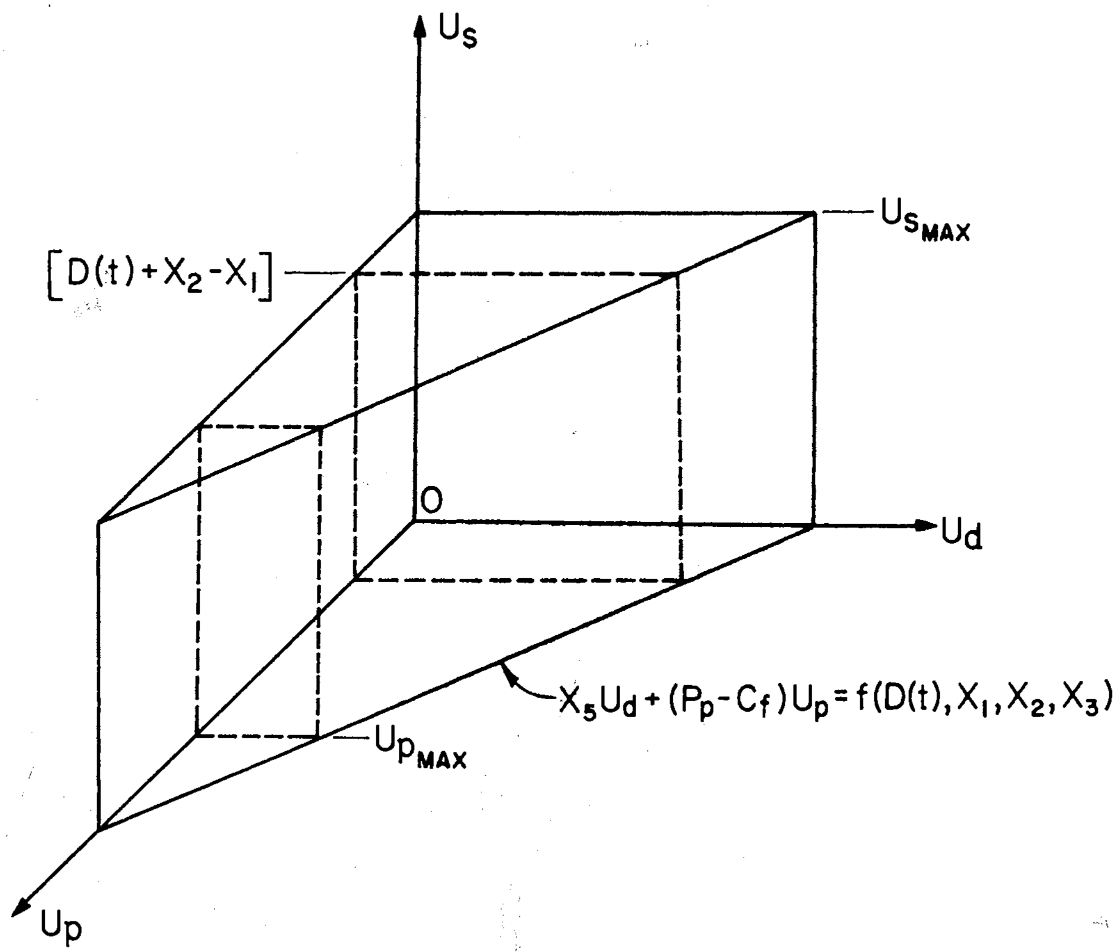

Figure 5. The Admissible Control Subspace

$$0 \le u_s \le u_{sMAX} \tag{4.44}$$

$$\mathrm{MAX}[D(t)+x_2-x_1,0] \le u_p \le \mathrm{MIN}\left[u_{pMAX}, \frac{b-a_1u_d}{a_2}\right] \tag{4.45}$$

$$0 \le u_d \le \frac{b-a_2u_p}{a_1} . \tag{4.46}$$

The Hamiltonian for the problem is

$$\begin{aligned} H(x,u,\lambda,t) = & u_d e^{-\rho t} + \lambda_1[C_1x_3-Cx_1] + \lambda_2[\beta x_1-\beta x_2] \\ & + \lambda_3[R\cdot LF\cdot D(t)-C_fD(t)-(P_p-C_f)u_p+(C_m+C_D)x_2 \\ & -(C_m+C_X)x_1-I_nhx_3-u_dx_5+(1-\delta)x_4u_s] \\ & + \lambda_4[C_0u_d-C_0\rho x_4] + \lambda_5u_s \end{aligned} \tag{4.47}$$

where

$$C_1 = \frac{(1+h)C}{K} . \tag{4.48}$$

The necessary conditions of the optimal trajectory are:

$$\dot{\lambda}_1 = C\lambda_1 - \beta\lambda_2 + \lambda_3(C_m+C_D) \tag{4.49}$$

$$\dot{\lambda}_2 = \beta\lambda_2 - \lambda_3(C_m+C_D) \tag{4.50}$$

$$\dot{\lambda}_3 = -C_1\lambda_1 + I_n h \lambda_3 \tag{4.51}$$

$$\dot{\lambda}_4 = -(1-\delta)u_s\lambda_3 + C_0 \rho \lambda_4 \tag{4.52}$$

$$\dot{\lambda}_5 = u_d\lambda_3 \tag{4.53}$$

and

$$\dot{x}_1 = C_1x_3 - Cx_1 \tag{4.54}$$

$$\dot{x}_2 = \beta(x_1-x_2) \tag{4.55}$$

$$\begin{aligned} \dot{x}_3 = & R\cdot LF\cdot D(t) - C_fD(t) - (P_p-C_f)u_p + (C_m+C_D)x_2 \\ & - (C_m+C_D)x_1 - I_n h x_3 - u_dx_5 + (1-\delta)x_4u_s \end{aligned} \tag{4.56}$$

$$\dot{x}_4 = C_0[u_d - \rho x_4] \tag{4.57}$$

$$\dot{x}_5 = u_s \, . \tag{4.58}$$

The initial condition for $\underline{x}$ is given as $\underline{x}_0$. The transversality condition provides the final values of the adjoint vector $\underline{\lambda}$.

$$\underline{\lambda}(T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-\rho T} \\ 0 \end{bmatrix} \, . \tag{4.59}$$

The partial derivatives of the Hamiltonian with respect to the three control variables are:

$$H_{u_d} = e^{-\rho t} - \lambda_3 x_5 + C_0 \lambda_4 \tag{4.60}$$

$$H_{u_p} = \lambda_3 [C_f - P_p] \tag{4.61}$$

$$H_{u_s} = (1-\delta) x_4 \lambda_3 + \lambda_5 \tag{4.62}$$

and are the "switching" functions of the Hamiltonian, since the sign of these functions determine the corresponding control function as described in Equations (4.38), (4.39), and (4.40).

In order to determine the switching functions, one must first solve the ten differential Equations (4.49) through (4.58). This is generally a difficult task when the equations are nonlinear and the boundary conditions are split. However, in this case, it is noted that the adjoint equations and state equations are uncoupled. It should be further noted that the equations for $\dot{\lambda}_1$, $\dot{\lambda}_2$, and $\dot{\lambda}_3$ do not contain λ_4

and λ_5, or the control functions. It is, therefore, possible to solve these three homogeneous linear equations with the variation of constants formula. If

$$\dot{W} = \underline{A}\, W \tag{4.63}$$

$$W(0) = W_0 \tag{4.64}$$

where $\underline{W}$ is an n-dimensional state vector of a linear system and $\underline{A}$ is a constant $n \times n$ matrix, then

$$W(t) = e^{\underline{A}t}\, W_0 . \tag{4.65}$$

The $\dot{\lambda}_1$, $\dot{\lambda}_2$, and $\dot{\lambda}_3$ equations can be solved backward in time by changing the sign on their state functions and since

$$\begin{bmatrix} \lambda_1(T) \\ \lambda_2(T) \\ \lambda_3(T) \end{bmatrix} = \underline{0} \quad . \tag{4.66}$$

Equation (4.65) implies that $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$ are uniformly zero.

$$\begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \lambda_3(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} . \tag{4.67}$$

This result reduces Equations (4.52) and (4.53) to

$$\begin{bmatrix} \dot{\lambda}_4 \\ \dot{\lambda}_5 \end{bmatrix} = \begin{bmatrix} C_0\rho & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_4 \\ \lambda_5 \end{bmatrix} . \tag{4.68}$$

Applying the proper boundary conditions from (4.59) gives

$$\begin{bmatrix} \lambda_4(t) \\ \lambda_5(t) \end{bmatrix} = \begin{bmatrix} e^{[-(1+C_0)\rho T + C_0\rho t]} \\ 0 \end{bmatrix} . \tag{4.69}$$

Thus, all of the adjoint variables are uniformly zero over the interval [O, T] with the exception of $\lambda_4(t)$. Substitution of (4.67) and (4.69) into the switching functions (4.60) through (4.62) gives:

$$H_{u_d} = C_O\left[e^{\left[-(1+C_O)\rho T+\rho C_O t\right]}\right] + e^{-\rho t} \tag{4.70}$$

$$H_{u_p} = 0 \tag{4.71}$$

$$H_{u_s} = 0 \; . \tag{4.72}$$

From (4.38) through (4.40), it follows that only u_d is determined from (4.70) through (4.72). Since $H_{u_d}(t) > 0$ for $t \in [O, T]$, u_d will equal its maximum value over the time interval by Equation (4.38). A feedback control law for u_d is the result of Equation (4.38) since the upper bound of u_d given by (4.43) and (4.46) is a function of certain states of the system and D(t). Now,

$$u_d = \frac{I(D(t), x_1, x_2, x_3)}{x_5} \quad . \tag{4.73}$$

The remaining control variables u_p and u_s become independent of the Hamiltonian due to the zero values of H_{u_s} and H_{u_p}. This condition is defined as a "singular" condition [7]. A singular condition arises in optimal control problems when the control variable enters the Hamiltonian in a linear fashion and the corresponding control switching function becomes zero for a non-zero time interval. Bryson [7] presents a chapter on the solution of singular control problems where all the techniques considered assume that at least one state variable enters the Hamiltonian in a nonlinear manner. This nonlinearity does not include bilinear terms such as the products of control and state variables found in (4.47). This essential assumption cannot be made in this control problem, and for this reason, traditional methods of handling singular conditions are of no help.

To resolve the singular condition problem on H_{u_s} and H_{u_p}, the original problem can be restated after eliminating u_d from the control vector by using (4.73). This is justified since it has been determined that u_d will maintain its upper bound independent of the other controls. The modified control problem is as follows:

$$\underset{u_p,\, u_s}{\text{Maximize:}} \quad PI = x_4(T)e^{-\rho T} + \int_0^T \Big[(R\cdot LF - C_f)D(t) - (P_p - C_f)u_p - (C_D + C_m)x_1 + (C_D + C_m)x_2 - I_n h\, x_3\Big] \frac{e^{-\rho t}}{x_5}\, dt \tag{4.74}$$

Subject to:

$$\dot{x}_1 = C_1 x_3 - C x_1 \tag{4.75}$$

$$\dot{x}_2 = \beta(x_1 - x_2) \tag{4.76}$$

$$\dot{x}_3 = (1-\delta)u_s x_4 \tag{4.77}$$

$$\dot{x}_4 = C_0\Big[(R\cdot LF - C_f)D(t) - (P_p - C_f)u_p - (C_D + C_m)x_1 + (C_D + C_m)x_2 - I_n h\, x_3\Big]/x_5 - C_0\, \rho\, x_4 \tag{4.78}$$

$$\dot{x}_5 = u_s \tag{4.79}$$

$$x_1 - x_2 + u_p - D(t) \geq 0 \tag{4.80}$$

$$0 \leq u_p \leq u_{pMAX} \tag{4.81}$$

$$0 \leq u_s \leq u_{sMAX} \tag{4.82}$$

$$x_i > 0 \qquad i = 1, 2, 3, 4, 5 \tag{4.83}$$

The Hamiltonian for this new problem is

$$H(x,\lambda,u,t) = [(R\cdot LF-C_f)D(t)-(P_p-C_f)u_p-(C_m+C)x_1+(C_m+C_D)x_2$$
$$-I_n h x_3] \frac{e^{-\rho t}}{x_5}+\lambda_1[C_1x_3-C x_1]+\lambda_2[\beta x_1-\beta x_2]$$
$$+\lambda_3[(1-\delta)u_s x_4]+\lambda_4 C_0[[(R\cdot LF-C_f)D(t)-(P_p-C_f)u_p$$
$$-(C_m+C_D)x_1+(C_m+C_D)x_2-I_n h x_3]/x_5-\rho x_4]+\lambda_5 u_s \,. \tag{4.84}$$

The "Maximum Principle" can be used again to develop the necessary conditions of the extremal trajectory. The adjoint differential equations are:

$$\dot{\lambda}_1 = \frac{(C_m+C_D)e^{-\rho t}}{x_5} + C\lambda_1 - \beta\lambda_2 + \lambda_4 C_0 \frac{(C_m+C_D)}{x_5} \tag{4.85}$$

$$\dot{\lambda}_2 = \frac{-(C_m+C_D)e^{-\rho t}}{x_5} + \beta\lambda_2 - \lambda_4 C_0 \frac{(C_m+C_D)}{x_5} \tag{4.86}$$

$$\dot{\lambda}_3 = \frac{I_n h e^{-\rho t}}{x_5} - C_1\lambda_1 + \lambda_4 \frac{C_0 I_n h}{x_5} \tag{4.87}$$

$$\dot{\lambda}_4 = -(1-\delta)u_s\lambda_3 + C_0 \rho \lambda_4 \tag{4.88}$$

$$\dot{\lambda}_5 = [(R\cdot LF-C_f)D(t)-(P_p-C_f)u_p-(C_m+C_D)x_1$$
$$+(C_D+C_m)x_2-I_n h x_3] \frac{e^{-\rho t}}{x_5^2} + \frac{C_0\lambda_4}{x_5^2} [(R\cdot LF-C_f)D(t)$$
$$-(P_p-C_f)u_p-(C_m+C_D)x_1+(C_m+C_D)x_2-I_n h x_3] \tag{4.89}$$

and the state equations are:

$$\dot{x}_1 = C_1x_3-C x_1 \tag{4.90}$$

$$\dot{x}_2 = \beta(x_1-x_2) \tag{4.91}$$

$$\dot{x}_3 = (1-\delta)u_s x_4 \tag{4.92}$$

$$\dot{x}_4 = \frac{C_0}{x_5}[(R\cdot LF-C_f)D(t)-(P_p-C_f)u_p-(C_D+C_m)x_1 +(C_D+C_m)x_2-I_n h\, x_3]-C_0\,\rho\, x_4 \tag{4.93}$$

$$\dot{x}_5 = u_s \; . \tag{4.94}$$

The initial condition on $\underline{x}$ is $\underline{x}_0$ and from the transversality condition, the final condition of the adjoint vector $\underline{\lambda}$ is the same as in Equation (4.59) and is

$$\underline{\lambda}(T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-\rho T} \\ 0 \end{bmatrix} . \tag{4.95}$$

The switching functions of this new Hamiltonian given by Equation (4.84) are

$$H_{u_p} = \frac{-(P_p-C_f)}{x_5}[e^{-\rho t}+\lambda_4 C_0] \tag{4.96}$$

$$H_{u_s} = \lambda_3(1-\delta)x_4+\lambda_5 \quad . \tag{4.97}$$

It is now necessary to determine the signs of H_{u_p} and H_{u_s} over the interval [0, T] since they determine the control variables.

To accomplish this, the solution trajectory will be synthesized by the reverse time construction. This method is similar to that found in [11] and [13]. The synthesis solution consists of starting at $t = T$ where the adjoint variables are known from the transversality condition. The state variables are known to be positive by Equation (4.83) at the terminal time. The switching function can be evaluated at $t = T$.

If $H_{u_p}(T)$ or $H_{u_s}(T)$ is zero, then they must be tested to see if they are zero for a non-zero time interval. The test is made by time differentiation of the switching function. If the switching function is zero for a non-zero time interval, then all derivatives of that switching function with respect to time must be zero. If the singular condition is not sustained, then the switching function at $t = T^-$ must be either positive or negative where $T^- = T - \Delta t$ as $\Delta t \to 0$. The corresponding control function is then known by Equations (4.38), (4.39), and (4.40). This control value is maintained until its switching function becomes zero at which time, another test for a singular condition will be required. The procedure is repeated until the solution is carried back to $(t = 0)$.

Evaluating H_{u_p} and H_{u_s} at $t = T$ with the values of $\lambda(T)$ and noting that $x_i > 0$ for $i = 1,2,3,4,5$ gives

$$H_{u_p}\Big|_{t=T} = \frac{-(P_p - C_f)}{x_5(T)} [e^{-\rho T} + C_0 e^{-\rho T}] \qquad (4.98)$$

$$H_{u_s}\Big|_{t=T} = 0 \qquad (4.99)$$

Since $P_p > C_f$ and $C_0 > 0$, the control variable $u_p(T)$ must be equal to its lower bound $MAX[D(t) - x_1 + x_2, 0]$ by Equation (4.40). H_{u_s} must be tested for a non-zero time interval singular condition. Taking the time derivative of H_{u_s} yields

$$\dot{H}_{u_s} = \dot{\lambda}_3(1-\delta)x_4 + \lambda_3(1-\delta)\dot{x}_4 + \dot{\lambda}_5 \qquad (4.100)$$

substituting Equations (4.87), (4.89), and (4.93) in for $\dot{\lambda}_3$, $\dot{\lambda}_5$, and $\dot{x}_4$, respevtively, and remembering that $\lambda_1(T)$, $\lambda_3(T) = 0$ while $\lambda_4(T) = e^{-\rho T}$

$$H_{u_s}\Big|_{t=T} = \frac{I_n h(1-\delta)x_4}{x_5}\left[e^{-\rho T} + e^{-\rho T} C_0\right] + \frac{Ie^{-\rho T}}{x_5^2}\left[1+C_0\right] \qquad (4.101)$$

where I is the Net Income flow, and I, I_n, h, $(1-\delta)$, x_4, x_5, and C_0 are all positive and, therefore,

$$\dot{H}_{u_s}\Big|_{t=T} \neq 0 \quad . \qquad (4.102)$$

Equation (4.102) shows that the singular condition on H_{u_s} is not sustained, and since $\dot{H}_{u_s}\Big|_{t=T} > 0$ and $H_{u_s}\Big|_{t=T} = 0$, then $H_{u_s}\Big|_{t=T^-} < 0$. Maximization of the Hamiltonian requires that for $H_{u_s} < 0$ then $u_s = 0$.

The next step in the control synthesis process is finding the next switching time (value of time between 0 and T where $H_{u_s} = 0$ or $H_{u_p} = 0$), if it exists. To find the time response of H_{u_p}, requires the time response of λ_4 and x_5. x_5 is a positive constant since $\dot{x}_5 = u_s$ and $u_s = 0$. The backward time response of

$$\dot{\lambda}_4 = C_0 \rho \lambda_4 \qquad (4.103)$$

necessitates a change of variable. Let $\tau = T-t$ and then for some state variable y(t)

$$\frac{dy}{dt} = \frac{dy}{d\tau} \cdot \frac{d(T-t)}{dt} = -\frac{dy}{d\tau} \quad . \qquad (4.104)$$

This change of variable allows the integration of a state variable equation backwards in terms of τ instead of t, and it only requires that the state function be multiplied by -1. Equation (4.103) in terms of τ becomes

$$\dot{\lambda}_4 = -C_0 \rho \lambda_4 \qquad (4.105)$$

with initial condition of $\lambda_4(\tau=0)=e^{-\rho T}$. The solution is

$$\lambda_4(\tau) = e^{-\rho(C_0\tau+T)} \quad . \tag{4.106}$$

Note that $\lambda_4(\tau)$ monotonically decreases from $e^{-\rho T}$ as $\tau \to T$, and is always positive for $0 \leq \tau \leq T$. Therefore, H_{u_p} is negative for the complete time interval and there are no switching times for H_{u_p} in $[0, T]$.

The backward time response of H_{u_s} is dependent on the backward time response of $\lambda_3(\tau)$ and $\lambda_5(\tau)$. The adjoint differential equations can be solved in the following order: $\lambda_4 \to \lambda_5$ and $\lambda_4 \to \lambda_2 \to \lambda_1 \to \lambda_3$. The solution of λ_4 has been found to be positive so that

$$\dot{\lambda}_5(\tau) = \frac{-I}{x_5^2} e^{-\rho(T-\tau)} - \frac{I}{x_5^2} [C_0 e^{-\rho(C_0\tau+T)}] \tag{4.107}$$

with $\lambda_5(\tau=0)=0$ and $\dot{x}_5 = 0$ and, therefore, x_5 is constant. The net income flow, I, is always positive so that $\dot{\lambda}_5(\tau)$ is negative according to (4.107). Since $\lambda_5=0$ at $\tau=0$ and $\dot{\lambda}_5(\tau) < 0$, the solution must remain negative on the interval $0 < \tau \leq T$.

To obtain the solution of $\lambda_3(\tau)$, requires that $\lambda_2(\tau)$ and $\lambda_1(\tau)$ must first be solved. The solution for $\lambda_2(\tau)$ can be stubstituted into the $\dot{\lambda}_1(\tau)$ equation. $\dot{\lambda}_2(\tau)$ is obtained by changing the sign on the state function of Equation (4.86) and replacing t with $T-\tau$. The solution of this resulting equation is

$$\lambda_2(\tau) = \frac{a}{\rho+\beta} e^{\rho\tau} - \frac{aC_0}{\rho C_0-\beta} e^{-\rho C_0\tau} - \left[\frac{a}{\rho+\beta} - \frac{aC_0}{\rho C_0-\beta}\right] e^{-\beta\tau} \tag{4.108}$$

where

$$a = \frac{(C_m+C_D)}{x_5} e^{-\rho T}$$

substituting this equation into $\dot{\lambda}_1(\tau)$ yields

$$\dot{\lambda}_1(\tau) = a\left[\frac{\beta}{\beta+\rho} - 1\right]e^{\rho\tau} - \left[\frac{\rho C_0}{\rho C_0 - \beta}\right] a C_0 e^{-\rho C_0 \tau}$$

$$- \beta\left[\frac{a}{\rho+\beta} - \frac{aC_0}{\rho C_0 - \beta}\right]e^{-\beta\tau} - C\lambda_1 \qquad (4.109)$$

with

$$\lambda_1(\tau=0) = 0 \ .$$

The first three terms on the right hand side of Equation (4.109) are forcing functions. It has been assumed in Chapter III that $\beta \ll 1$, therefore the sum of these three terms is negative in the interval from $[0, T]$. A first order differential equation with zero initial condition and a negative forcing function has a non-positive solution, and therefore $\lambda_1(\tau) < 0$ for $0 < \tau \leq T$.

Considering now the backward differential equation

$$\dot{\lambda}_3(\tau) = -\frac{I_n h}{x_5} e^{-\rho(T+\tau)} - \frac{C_0 I_n h}{x_5} e^{-\rho(C_0\tau+T)} + C_1\lambda_1 \qquad (4.110)$$

where

$$\lambda_3(\tau=0) = 0 \ .$$

The $\dot{\lambda}_3(\tau)$ is driven by negative forcing functions since $\lambda_1 < 0$, and has a zero initial condition. Therefore, $\lambda_3(\tau) < 0$ for $0 < \tau \leq T$. It has been shown that λ_3 and λ_5 are both negative on the half closed interval $[0, T)$ of t, and so there does not exist a switching time for H_{u_p} in that interval.

This completes the solution of the control vector function $\underline{u} = [u_d, u_p, u_s]$. These optimum control functions are:

$$u_d(t) = \frac{I(D(t), x_1, x_2, x_3)}{x_5} \qquad (4.111)$$

$$u_p(t) = \mathrm{MAX}[D(t) + x_2 - x_1, 0] \qquad (4.112)$$

$$u_s(t) = 0 \; . \qquad (4.113)$$

Interpretation of the Control Functions

The interpretation of this control stategy is as follows: For the maximization of the market valuation performance criterion, the utility should; (1) retain no income, but pay the maximum amount of dividends available, (2) purchase energy only when the demand exceeds the available capacity of the utility, and (3) issue no new shares of common stock. Each of these management decisions will be discussed below.

Heuristically, it might be argued that the utility could benefit; i.e., maximize the market valuation objective function by giving up dividends to retain earnings. These retained earnings would be used to expand capacity. Since it is generally assumed that the utility could produce the energy at a cheaper rate than it could purchase energy, it seems reasonable that expansion of capacity would increase the income flow and subsequently higher future dividends could be paid from this increased income. However, the optimal control solution shows such a strategy to be non-optimal. The fact that the dividend is the most observable measure of the utility's performance to the investor dominates the performance criterion, and it makes any other possible dividend decisions sub-optimal.

If the price of purchased energy is greater than the variable fuel cost, then energy should only be purchased for "peaking" demands (D(t) exceeds the actual capacity of the utility). This has been the traditional policy of the electric utility industry. It is supported by the idea that as long as the utility has fixed costs in its plant and equipment, this capacity should be utilized to meet demand. Another result of the analytic solution of the purchased power decision is the reverse decision to purchase the maximum available amount of power to meet demand if $P_p < C_f$. This is observed in Equation (4.98) for H_{u_p} where $H_{u_p} > 0$ if $P_p < C_f$. This means that the generate or buy decision is not a comparison of P_p against all the variable and fixed costs of producing energy, but it is based on a comparison of P_p and the variable cost of production. This phenomena has been observed in the electric utility industry. For example, hydroelectric facilities operated by the federal government has sold energy under flood conditions at prices below private utility fuel costs. Electric utilities have responded to this condition by purchasing all of this available "dump energy."

The stock issuance decision has been found to be uniformly zero over the planning period [0, T]. This supports the intuitive concept that any issuance of stock dilutes the ownership of equity and directly reduces the market value per share. This is seen from the integrand of the modified performance criterion of Equation (4.74) which is the income per share. Since this integrand is inversely proportional to the number of shares of outstanding stock, it becomes apparent that the optimum stock issuance strategy is to not issue new stock.

There are other interesting results of the analytic solution to be observed in the form of the adjoint variables' response. The adjoint variable can be considered the marginal change in the objective criterion to a unit change in its corresponding state variable. There are two sets of adjoint variables - they are the original set described in

Equations (4.49) through (4.53) and the adjoint variables of the modified performance index, Equations (4.85) through (4.89).

The original $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, and $\lambda_5(t)$ were found to be uniformly zero across the planning period. This means that changes in the total capacity, retired capacity, equity, and number of shares of stock had no affect on the market valuation. The marginal product of the price of a share of stock, $\lambda_4(t)$, was always positive and thus explains the maximum dividend decision.

The second set of adjoint variables corresponding to the modified index represent the marginal products of the accumulation of the discounted stream of income per share over the planning period plus the final value of the price of a share discounted. The marginal product, $\lambda_4(t)$, was again found to be positive and therefore positive changes in the price of a share increased the performance criterion. The marginal product of retired capacity, $\lambda_2(t)$, was also positive. As retired capacity grows, fixed costs of maintenance and depreciation decreased and income increases. The marginal products, $\lambda_1(t)$ and $\lambda_3(t)$, are always negative which means that total capacity and equity increases diminish the income flow through higher maintenance, depreciation, and long term debt costs. The number of shares marginal product, $\lambda_5(t)$, is negative which follows from the discussion of the stock issuance decision. Increases in the number of shares of stock inversely decreases the income per share.

As it was stated in the beginning of this chapter, the importance of these control strategies depends on the importance of electric utility places on market valuation. It is obvious that a utility would not strictly adopt these strategies as a philosophy of capacity expansion and investment, since the utility has other objectives equally as important as market valuation. However, the results of this optimal control problem give the planner a greater insight into what does and does not increase or decrease the market valuation of the utility's common stock. The next chapter presents a multiple objective criterion which more closely reflects the goals of the utility.

CHAPTER V

COMPUTER OPTIMIZATION OF AN EXPANDED FINANCIAL PERFORMANCE CRITERION

Introduction

The optimal control strategies for a selected market valuation criterion were developed in Chapter IV. It was shown that, in order to maximize the market value of the common stock equity, the utility should not invest in new capacity (retained earnings and new equity issuances were zero over the planning horizon). Since capacity is continuously being retired, the net effect of this strategy is a decreasing level of actual capacity available for generation. The increasing demand for energy is met through increasing levels of purchased power.

In order to further demonstrate applications of the model in expansion planning, a more general performance measure of utility expansion is considered in this chapter. The financial performance criterion posed in Chapter IV is modified to also reflect the goal of the utility to expand its own capacity to meet the demand for electrical energy. The necessary conditions for the maximization of this modified performance index are easily developed using the maximum principle and a two-point boundary value problem is obtained. However, instead of pursuing an analytical solution as in the previous chapter, numerical values for a typical utility are assigned to the parameters of the model and a computer technique is used to solve the split boundary value problem. The technique used to obtain the solution is described in the Appendix.

The computer solution provides full time trajectories of the three optimal control variables namely power purchase, stock issuance and dividend payment, along with the five state variables. For comparison purposes, the utility model is simulated assuming a typical utility strategy and the time responses of the utility stock price and capacity expansion variables are presented and discussed.

As mentioned earlier, one purpose of the model is to allow a rapid analysis, synthesis and optimization of long range expansion and financial planning strategies by utility management. The analytical model developed in this research has a form which will facilitate this use. This chapter illustrates the ease with which the model can be used to develop necessary conditions for optimizing a given performance criterion.

Modified Performance Criterion

The performance index described in Chapter IV represents one goal of the financial planner. In order to include the additional goal of constructing generating capacity to meet a growing energy demand, the original performance index must be modified. The modification is the addition of the discounted level of actual capacity at the final time, T, thus producing the new index:

$$PI^1 = P(T)e^{-\rho T} + AC(T)e^{-\rho T} + \int_0^T d(t)e^{-\rho t}\,dt \; . \tag{5.1}$$

Thompson [34, 35], in defining the optimal investments and operations of a firm, includes the discounted final level of production capacity. Since the optimal solution is found for a fixed planning horizon will begin with an adequate production capacity to meet future demands. Since AC is measured in kilowatts and the original performance index is dimensioned in dollars, weighting constants are added to yield the following form:

$$PI^1 = w_1[P(T)e^{-\rho T} + \int_0^T d(t)e^{-\rho t}\, dt] + w_2[AC(T)e^{-\rho T}] \tag{5.2}$$

where

$$w_1 = \frac{AC(0)}{AC(0)+P(0)} \quad \text{and} \quad w_2 = \frac{P(0)}{AC(0)+P(0)} .$$

The constants w_1 and w_2 will equally weight the importance of the level of capacity and the market valuation of the utility's common equity based on initial values of these variables. The new control problem can now be stated using the notation of Equations (4.15) through (4.26).

$$\underset{u_p, u_s, u_d}{\text{Maximize:}} \quad PI^1 = w_1[x_4(T)e^{-\rho T} + \int_0^T u_d e^{-\rho t}\, dt] + w_2[x_1(T) - x_2(T)]e^{-\rho T} \tag{5.3}$$

Subject to:

$$\dot{x}_1 = C_1 x_3 - C x_1 \tag{5.4}$$

$$\dot{x}_2 = \beta[x_1 - x_2] \tag{5.5}$$

$$\dot{x}_3 = [I - u_d x_5] + [(1-\delta)x_4 u_s] \tag{5.6}$$

$$\dot{x}_4 = C_0[u_d - \rho x_4] \tag{5.7}$$

$$\dot{x}_5 = u_s \tag{5.8}$$

$$I = [R \cdot LF - C_f]D(t) - (P_p - C_f)u_p - (C_m + C_D)x_1$$

$$+ (C_m + C_D)x_2 - I_n h\, x_3 \tag{5.9}$$

$$x_1 - x_2 + u_p - D(t) \geq 0 \tag{5.10}$$

$$I - u_d x_5 \geq 0 \tag{5.11}$$

$$0 \leq u_p \leq u_{pMAX} \tag{5.12}$$

$$0 \leq u_s \leq u_{sMAX} \tag{5.13}$$

$$u_d \geq u_{dMIN} \; . \tag{5.14}$$

Development of Necessary Conditions and Solution

The maximum principle presented in Chapter IV can be applied in a direct fashion to establish the necessary conditions for this new control problem. Since the admissible control subspace is similar to the one in the previous problem, the admissible control space is closed and convex.

The Hamiltonian is very nearly the same as in the previous control problem, and therefore the necessary conditions for the maximization of the modified performance criterion are similar with the exception of the boundary conditions on the adjoint variables at the final time are different. The Hamiltonian is:

$$\begin{aligned} H(\underline{x},\underline{\lambda},\underline{u},t) = {} & w_1 u_d e^{-\rho t} + \lambda_1[C_1 x_3 - C x_1] + \lambda_2[\beta x_1 - \beta x_2] \\ & + \lambda_3[R \cdot LF \cdot D(t) - C_f D(t) - (P_p - C_f)u_p + (C_m + C_D)x_2 - (C_m + C_D)x_1 \\ & - I_n h\, x_3 - u_d x_5 + (1-\delta)x_4 u_s] + \lambda_4[C_0 u_d - C_0 \rho\, x_4] + \lambda_5 u_s \end{aligned} \tag{5.15}$$

and the adjoint differential equations are:

$$\dot{\lambda}_1 = C\lambda_1 - \beta\lambda_2 + \lambda_3(C_m+C_D) \tag{5.16}$$

$$\dot{\lambda}_2 = \beta\lambda_2 - \lambda_3(C_m+C_D) \tag{5.17}$$

$$\dot{\lambda}_3 = -C_1\lambda_1 + I_n h \lambda_3 \tag{5.18}$$

$$\dot{\lambda}_4 = -(1-\delta)u_s\lambda_3 + C_0\rho\lambda_4 \tag{5.19}$$

$$\dot{\lambda}_5 = u_d\lambda_3 \quad . \tag{5.20}$$

The state equations are:

$$\dot{x}_1 = C_1x_3 - Cx_1 \tag{5.21}$$

$$\dot{x}_2 = \beta(x_1-x_2) \tag{5.22}$$

$$\begin{aligned}\dot{x}_3 = {} & (R\cdot LF-C_f)D(t) - (P_p-C_f)u_p - (C_m+C_D)x_1 \\ & + (C_m+C_D)x_2 - u_dx_5 + (1-\delta)x_4u_s\end{aligned} \tag{5.23}$$

$$\dot{x}_4 = C_0(u_d-\rho x_4) \tag{5.24}$$

$$\dot{x}_5 = u_s \quad . \tag{5.25}$$

Boundary conditions on the adjoint variables are given by Equation (4.36) as

$$\underline{\lambda}(T) = g_x[T,\underline{x}(T)] = \begin{bmatrix} w_2e^{-\rho T} \\ -w_2e^{-\rho T} \\ 0 \\ w_1e^{-\rho T} \\ 0 \end{bmatrix} \tag{5.26}$$

and the initial conditions given on the state variables are

$$\underline{X}(0) = \underline{x}_0 \; . \tag{5.27}$$

The switching functions of the new control problem are as follows:

$$H_{u_d} = w_1 e^{-\rho t} - \lambda_3 x_5 + C_0 \lambda_4 \tag{5.28}$$

$$H_{u_p} = \lambda_3 [C_f - P_p] \tag{5.29}$$

$$H_{u_s} = (1-\delta) x_4 \lambda_3 + \lambda_5 \; . \tag{5.30}$$

By examination of Equations (5.16) through (5.20), the adjoint variables λ_1, λ_2, and λ_3 are uncoupled from the other adjoint variables and independent of the state and control variables. Therefore, their solution is simply a linear homogeneous solution. However, unlike the solution presented in Chapter IV, the boundary conditions on λ_1, λ_2, and λ_3 are not all zero. This means that λ_1, λ_2, and λ_3 are not uniformly zero on the interval $[0, T]$. Furthermore, the adjoint equations for λ_4 and λ_5 contain the control functions which are in turn dependent on the switching functions. Thus, the solution to the adjoint and state equations must be found simultaneously and satisfy the boundary conditions before the optimal controls are obtained. The solution is direct via a computer simulation - optimization technique.

Before the computer solution can proceed, numerical values must be assigned to the parameters of the model and the demand function specified. Through conversations with local investor-owned electric utility executives, approximate values for the parameters were determined for a typical utility. Also, typical peak demand data for a utility was obtained and approximated by a time function. The solution to the optimal control problem was obtained for this typical utility for a planning horizon of ten years. All parameters are assumed to be constant for the planning period, although the computer technique is not limited to this assumption and time-varying parameters could be implemented. Table I lists these parameters along with their corresponding values used in the simulation. Table II contains the initial values of the state variables for the utility. w_1 and w_2 are evaluated from their definition given in Equation (5.2) and the values of Table II with the exception that w_2 is one-half the value found from this formula. D(t) is given the functional form $16.6t - (4.17t + 500)\cos(\pi t/6) + 1500$.

A description of the computer technique used to solve the two-point boundary value problem is presented in the Appendix including a listing of the computer program.

Comparison of Optimal and Typical Strategies

The optimum strategy variables of purchasing power, issuing new equity, and dispensing dividends were found for the modified performance index. This is compared with a typical strategy expected to be followed by the utility.

The usual dividend decision for an electric utility is that two-thirds of the income will be paid in dividends while a third is retained for reinvestment. The typical utility will purchase power to satisfy demand only when necessary. The issuance of stock by a utility generally takes place once a year and it has been characterized in this comparison to be an issue of 400,000 shares in June of every year for the ten year planning period. These typical strategy variables are based on conversations with utility executives.

TABLE I

MODEL PARAMETER VALUES FOR TYPICAL UTILITY

Parameter	Value for Typical Utility
β - retirement rate	3%/year
C - construction rate	1/c = time constant = 3 years
h - debt-equity ratio	50%
K - construction cost	$100/KW
C_O - market activity factor	8.35%/year
ρ - expected rate-of-return by investor	9%/year
δ - cost of marketing stock	10% of market price
R - price of energy sold	1.2¢/KW•Hr
LF - load factor	50%
C_f - cost of fuel	2.6 mills/KW•Hr
C_D - cost of depreciation	3%/year
C_m - cost of maintenance	2% of initial investment/year
P_p - price of energy purchased	3 mills/KW•Hr
I_n - interest rate	5.5%/year

TABLE II

INITIAL VALUES OF STATE VARIABLES
FOR TYPICAL UTILITY

State Variable	Initial Value
TC(0) - Installed Capacity	1,800 megawatts
RC(0) - Retired Capacity	0
E(0) - Total Equity	$120 million
P(0) - Price of a share	$12/share
N(0) - Number of shares	10 million shares

Figure 6 shows the optimum dividend decision denoted u_d^* and the typical dividend decision variable, u_d plotted against time for the ten year period. It should be noted that constraints on dividend payment have been applied to the optimum case; namely a minimum of 10¢ a share per month and a maximum of 80% of the income per share per month. The optimum follows the typical curve until t = 2.75 where it switches to its minimum until approximately t = 8.75 years where it resumes its upper bound.

Figure 7 shows the optimum power purchases, denoted by u_p^* and the typical purchase, u_p plotted against time for the ten year period. Since both u_p^* and u_p are used only for peaking, the comparison shows that the optimum actual capacity exceeds the future demand at all times after the sixth year. The resulting actual capacity from the typical strategy is never greater than the peak demand in the summer months for the planning period, since u_p is non-zero for every summer.

Figure 8 illustrates the optimum and typical control variables of the issuance of common stock, u_s. The typical issuance, u_s, is characterized as a sequence of pulses with amplitude 400,000 shares. The optimum issuance, u_s^*, is a constant 3460 shares per month.

The response of the stock price per share for the two cases is shown in Figure 9. The optimum price trajectory, P*, is greater than the price for the typical strategy for the first three years. However, after three years, the optimum price falls below the typical price for the rest of the planning period. This is due to the fact that the optimum dividend variable u_d^* goes to 10¢/month near the end of the second year and remains there until the beginning of the ninth year. The P* response begins to rise in the ninth year. It is also noted that

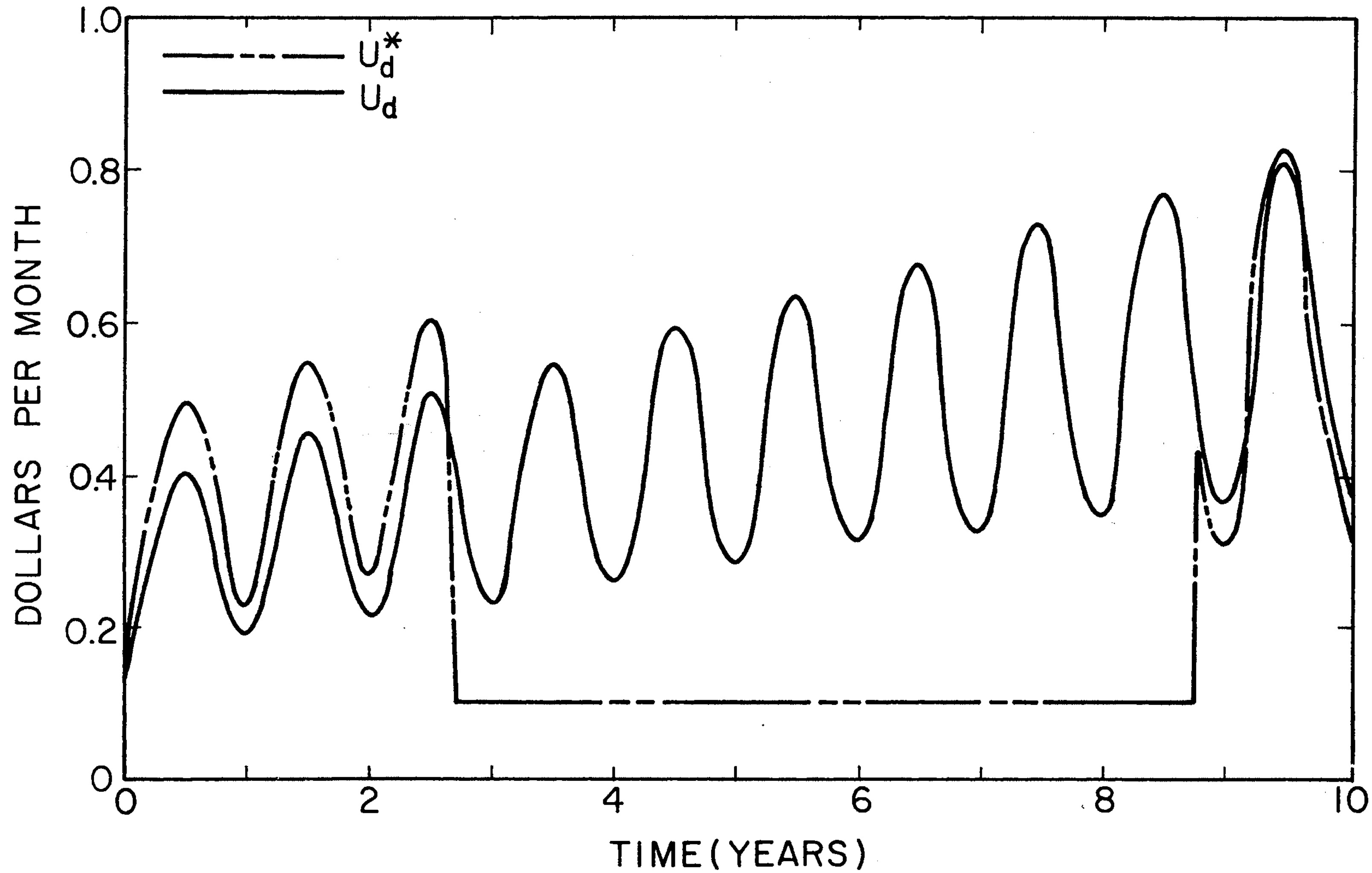

Figure 6. Comparison of Optimum to Typical Dividend Decision

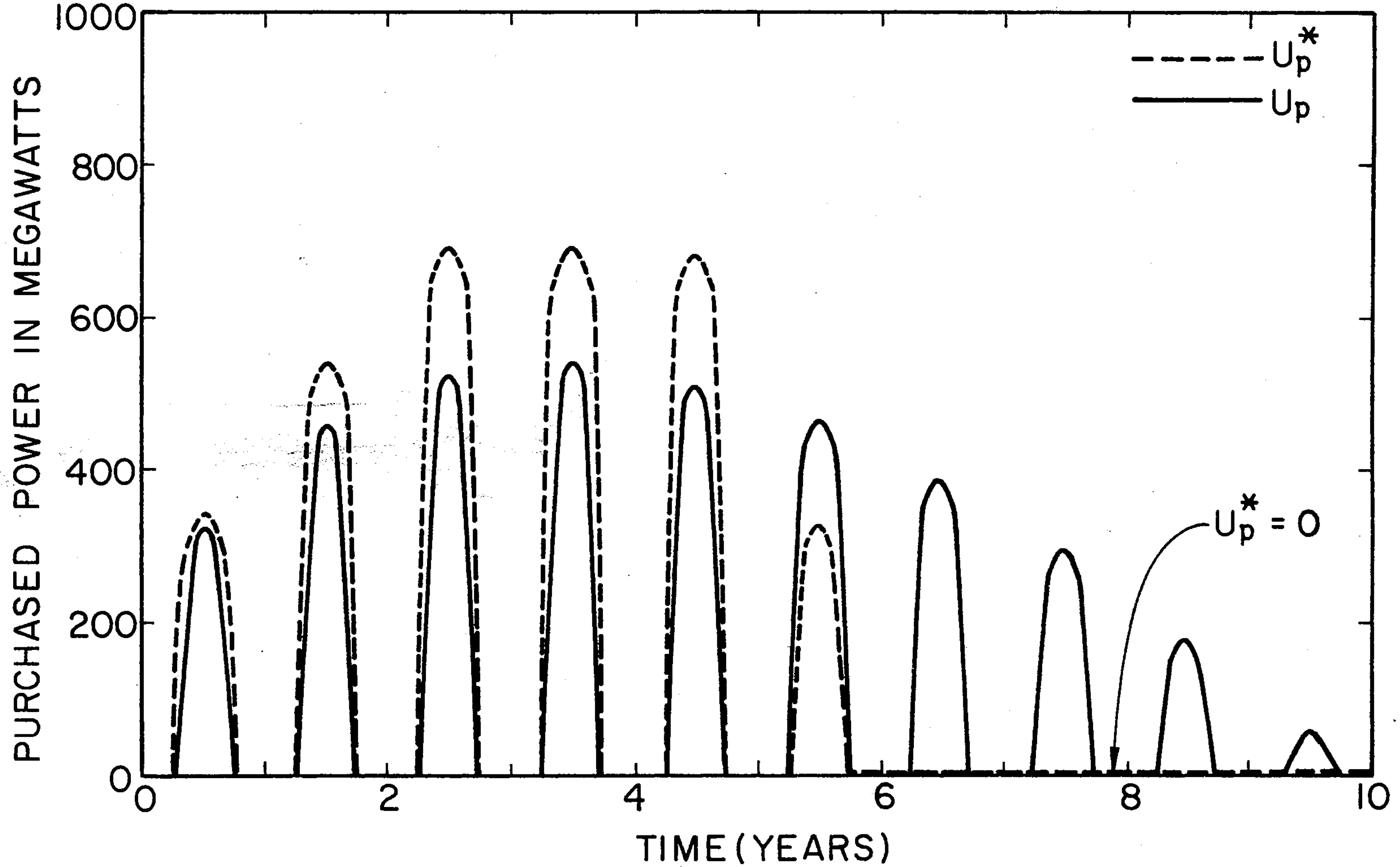

Figure 7. Comparison of Optimum to Typical Purchased Power Decision

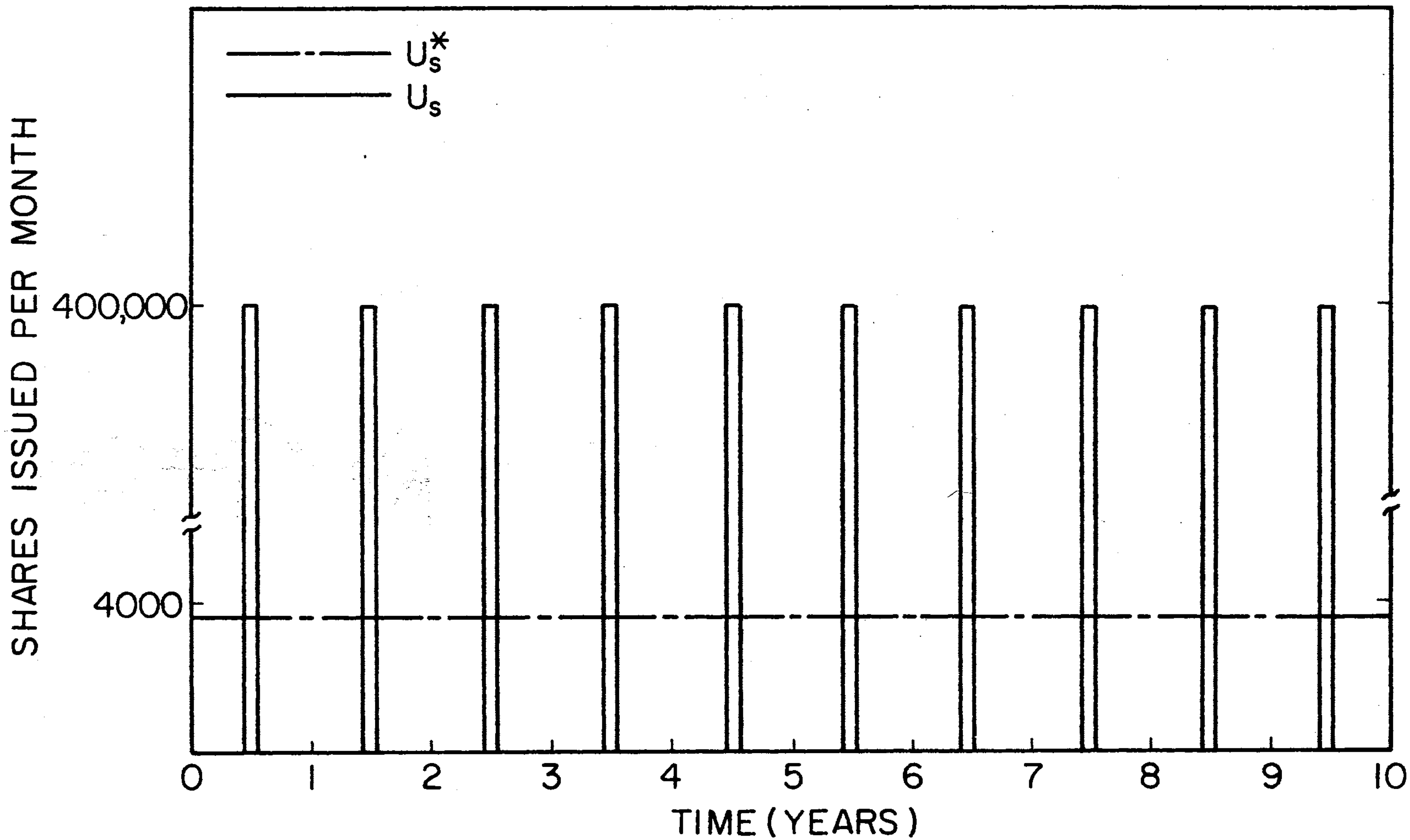

Figure 8. Comparison of Optimal to Typical New Stock Issuance Decision

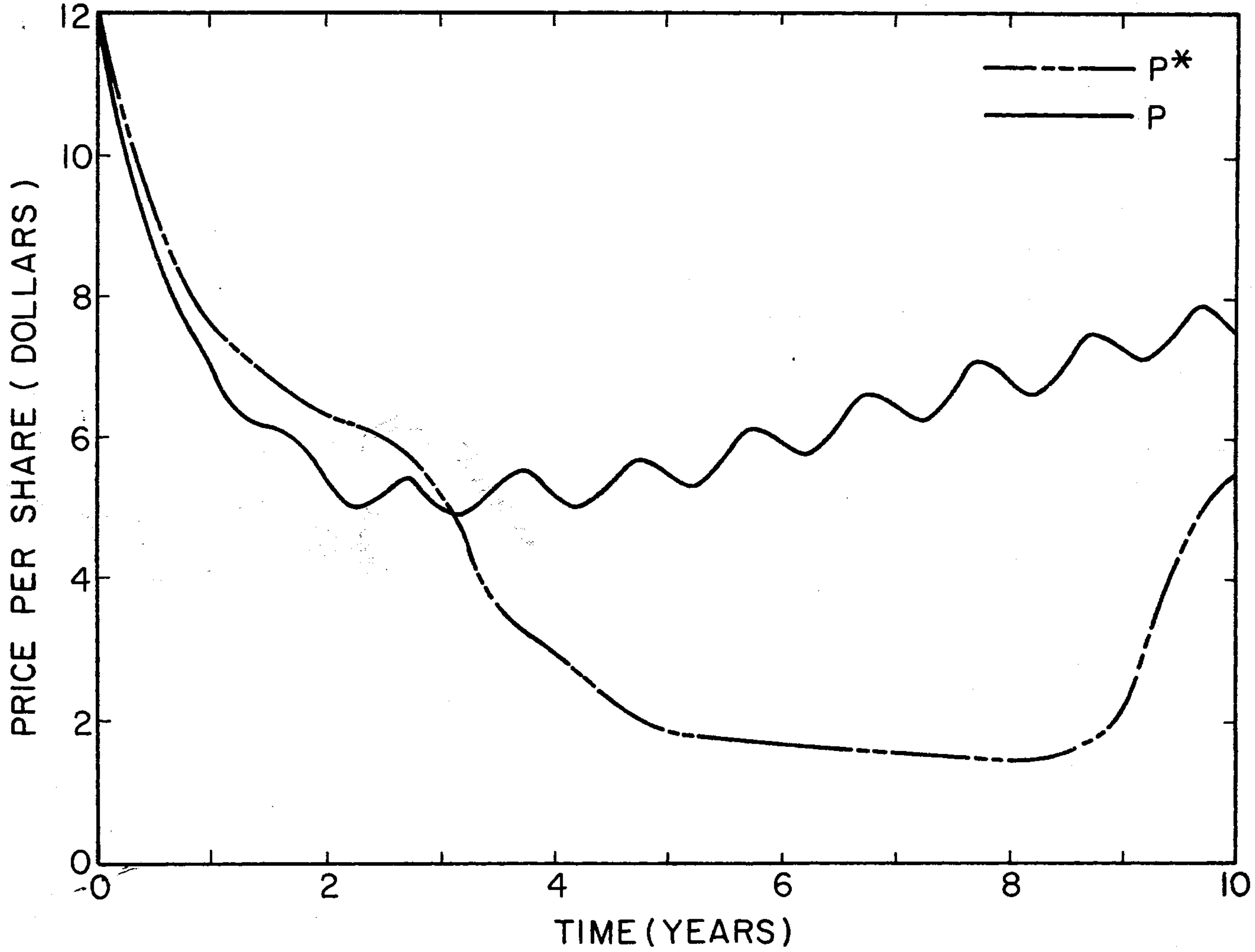

Figure 9. Typical and Optimum Price Responses

P displays a seasonal variation which is delayed in phase from u_d. This variation is due to the seasonal variation in company income I and the delay is dependent on the trading activity factor C_0'.

The response of the actual capacity variable for the optimum and typical controls is shown in Figure 10. Additionally, the peak demand function D(t) is also plotted to illustrate the relation between the two expansion programs and the peak demand. The optimum expansion policy AC* is below the typical capacity trajectory for the first several years. AC* begins to increase when the retained earnings are increased (u_d^* = minimum value). At the end of the planning period, AC* is 2500 megawatts greater than AC. It should be noted that when D(t) is greater than AC or AC*, then u_p and u_p^* are non-zero. This plot explains the fact that u_p^* is zero after the sixth year.

The last consideration is the value of the performance index for the two cases. The optimum case yields a value of 4.58 for the performance index, while the typical case gives a value of 4.13. This indicates that for the modified performance index the optimum strategy obtained is superior to the typical strategy. It becomes obvious by looking at Figures 9 and 10, that there is a trade-off between maximization of capacity and maximization of market value. By maintaining dividends, the typical strategy keeps the price of stock and thus the market value above the optimum price response. However, at the cost of lower market value, the optimum strategy diverts dividends back into investment in capacity. This results in a greater level of capacity at the final time for the optimum strategy as compared to the typical case.

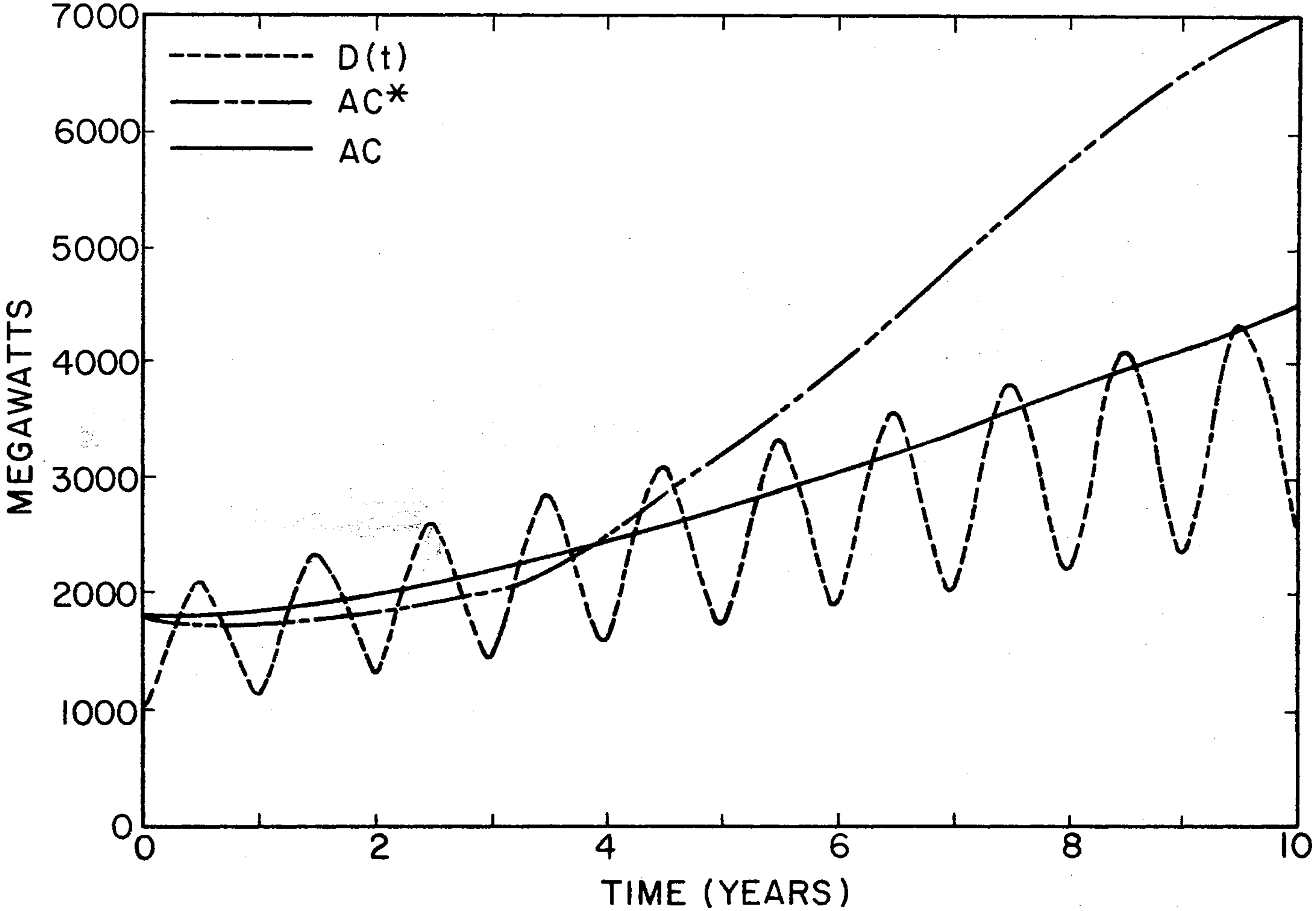

Figure 10. Typical and Optimum Actual Capacity Responses and D(T)

Summary

To illustrate the ease with which the utility model can be used to develop optimal strategies via computer simulation, an expanded performance criterion has been considered in this chapter. The necessary conditions for optimality are seen to be easily written by direct application of the maximum principle and subsequently implemented on the computer.

The particular utility problem solved utilized typical parameters for an investor-owned company and the results of the optimization were compared with an assumed typical strategy for the company. The optimum dividend decision was shown to switch to a low level during the planning interval to provide revenue for capacity construction. This construction ultimately led to a much higher capacity available at the end of the planning period than was produced by the non-optimal typical utility strategy of continuously paying dividends proportional to income. Although only one optimization case was considered the value of the model in the investigation of many different planning situations has been demonstrated.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

This thesis has been devoted to the mathematical modeling and optimization of an investor-owned electric utility. The objective was to develop from basic economic and accounting principles, a tractable and efficient dynamic model which could be used for the analysis and optimization of long-range management decisions. Specific decisions considered are: (1) when should energy be purchased from outside sources as opposed to investing in new generation capacity, (2) when should capacity expansion be initiated, if desirable, and (3) how should capital be raised for such expansion.

This research was motivated by the fact that the electric utility industry faces a growing demand for electrical energy. Each utility company has three options in fulfilling this demand. It may either increase its capacity of production or purchase energy from external sources or both. Fundamental to the question of expanding capacity is problem of accumulating large amounts of capital to support this expansion. Therefore, the proper expansion policy and capital budgeting strategy are interrelated. Historically, the problem of expanding capacity and determining the corresponding capital budgeting strategy has been divided into two separate problems. This research combines the two into a unified planning problem.

In reviewing the literature on modeling of the electric utility, it was found that diverse approaches had been taken, depending upon the ultimate objective of the model's use. These included resource allocation, computer simulation, and capital budgeting models, as well as other models satisfying specific objectives. In each case, these models failed to meet the objectives of this research in one or more of the following ways: (1) The resource allocation models were static and exclusively designed to determine the optimum production input mix of capital and labor for maximization of profit. (2) The computer corporate models encompass the utility's operations in detail, but lack the analytical framework for the application of optimal control theory. (3) The capital budgeting models give the structure for determining the optimum financial decisions of dispensing dividends and issuing stock, but excludes the production process. (4) The other models reviewed which include optimal capacity expansion models and investment models partially meet the objectives of the research, but none of these models combine the capacity expansion with the investment structure in one model.

The model developed in this research includes both the capacity expansion and financial processes. The mathematical form of the model is a nonlinear fifth order system of differential equations with additional inequality constraints on the state and control variables. The model consists of three sections; the capacity process, financial process, and internal and external constraints. The capacity process section models the flow of capital for new capacity with corresponding construction time delay and the gradual retirement of capacity. The financial process section models the behavior of the company's total equity, long term debt, price of a share of common stock, number of outstanding shares of stock, and the net income flow of the utility. The internal and external constraints section mathematically describes the physical and financial restrictions on the states and management control variables imposed by desired operational conditions.

The value of the model to a utility financial planner was demonstrated in two distinct ways. First, the model was employed in an analytical study of the optimum management strategy to maximize the market valuation of the company's common stock. The general solution of the optimal control problem was obtained and the fundamental mathematical relations between the management control variables and the system state variables were developed. A geometric presentation of the control space was obtained and interpreted. The following strategy was determined: (1) convert all net income into dividends, (2) purchase energy only when the demand exceeds the available capacity of the utility, and (3) issue no new shares of common stock.

The interpretation of this strategy is that the dividend decision is the dominant factor in the market valuation criteria and should, therefore, be at its upper bound. Furthermore, power should be purchased only to handle peak loads. This decision was found to be dependent upon the relative size of the variable cost of fuel and the price of purchased energy, and not on the combined fixed and variable costs of producing the energy and the purchase price. If the price of purchased energy falls below the fuel cost then the optimum strategy switches to purchase the maximum amount of energy available from external sources. The non-issuance of stock is supported by the fact that any issuance reduces the market valuation since dividends are reduced per share.

In the second demonstration of the model, an expanded performance index was proposed which balanced the desired use of revenue between dividend payment and capacity expansion. Typical model parameter values and a time function for the peak power demand were postulated. A computer optimization of the performance index was obtained and a comparison was made between the optimum management strategy found and a typical strategy.

Conclusions

The following specific contributions of this research can be cited:

1. A new mathematical model of the investor-owned electric utility has been developed and demonstrated. The unique features of the model are:
   a. The model brings together the financial and capacity expansion processes for the utility and interrelates their individual dynamic characteristics.
   b. The model provides for three distinct management control variables for long-range planning; external power purchase, common stock dividend payment, and new equity issuance.
   c. The analytical form of the model allows the application of modern control theory and standard computer simulation methods to rapidly obtain optimal management strategies for a wide range of postulated future conditions and operational constraints during a given planning interval.
2. Two optimization problems have been solved using the model. One problem demonstrates the analytical properties of the

model and corresponding optimal controls and the second problem demonstrates the flexibility and ease with which the model can be simulated in a computer-aided optimization.

In summary, the modeling and optimization studies carried out during the research have yielded a new type of approach to long-range utility planning. The approach is at a conceptual and mathematical level where the planner can rapidly find and/or evaluate rather general policies of financial management. The model lacks the detail of large-scale accounting-type models but trades this detail for the very desirable feature of rapid optimization. Furthermore, the model allows the planner to examine general concepts of planning through analytical studies or obtain specific results through computer optimization.

Recommendations for Further Research

There are a number of desirable investigations and extensions related to this research that might be considered. These include refinements in the model as follow:

1. Provide for the economies of size in the construction and maintenance of capacity.
2. Include other instruments of equity such as preferred stock and other convertible instruments.
3. Provide for accelerated depreciation.
4. Include rate-of-return regulation on income.
5. Consider making the interest rate a function of other financial variables to reflect the bond rating mechanism.

Other extensions to this research certainly should include the investigation of other performance criterions. These might include the multiple objective of maximizing the market valuation while minimizing the variation between the demand function and the available capacity. Another possible criterion is the maximization of profit or minimization of costs while expanding capacity according to a given demand function.

Since the present model considers only one mode of electrical generation, a more general model could be developed by viewing the present model as a submodel connected to similar submodels for each mode of generation. Also, a separate submodel describing transmission and distribution facilities could be defined thus yeilding a totally integrated systems model. This would require a separate capacity process section for each generation mode and one central financial process section. The model could also be modified to include a variation in demand for energy as a function of the rate structure. This modification would make it possible to investigate various pricing strategies.

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APPENDIX

TWO-POINT BOUNDARY VALUE PROBLEM SOLUTION

Recirculation Algorithm

The two-point boundary value problem can be characterized in the following manner. Given the equations

$$\dot{\underline{x}} = f(\underline{x},\underline{\lambda}) \tag{A.1}$$

$$\dot{\underline{\lambda}} = g(\underline{x},\underline{\lambda}) \tag{A.2}$$

with boundary conditions

$$\underline{x}(t_0) = \underline{x}_{t_0} \tag{A.3}$$

$$\underline{\lambda}(t_f) = \underline{\lambda}_{t_f} \tag{A.4}$$

find the solution of (A.1) and (A.2) which satisfies (A.3) and (A.4). The recirculation algorithm is an iterative technique for the solution of this problem [43]. The algorithm is as follows:

1. Guess a value for $\underline{\lambda}(t_0)$ and integrate the system of Equations (A.1) and (A.2) forward to the terminal time, t_f, with $\underline{x}(t_0)$ given by Equation (A.3).
2. Substitute the correct value for $\underline{\lambda}(t_f)$ and let $\underline{x}(t_f)$ be the value found from step 1., then integrate (A.1) and (A.2) backward in time from t_f to t_0.

3. Replace $\underline{x}(t_0)$ with correct value at the initial time and let $\underline{\lambda}(t_0)$ be the value found from step 2.

The process is a repetition of these three steps until (A.3) and (A.4) are satisfied. It should be noted that the algorithm could have been started at the final time with guesses for $\underline{x}(t_f)$ with the same results. This algorithm does not converge in all cases, depending upon the nonlinear functions, f and g, the initial guess for $\underline{\lambda}(t_0)$ and the total time interval $t_f - t_0$. However, in many cases the algorithm converges rapidly through only a few iterations.

Solution of Necessary Conditions

The Solution to the necessary conditions of the optimal control problem of Chapter V is found using the recirculation algorithm with a slight modification. It was shown that λ_1, λ_2, λ_3 are uncoupled from the other adjoint variables, the state variables, and the control variables. These equations can be integrated from T backward to $t = 0$ to obtain their initial conditions exactly. This leaves only λ_4 and λ_5 with missing initial conditions. Guesses are made for these two adjoint initial conditions and the total system of ten state and adjoint equations are integrated forward to T. The values of the control variables are produced by their respective switching functions and constraints. Thus, step 1 of the algorithm is now complete.

The second step proceeds after adjusting $\lambda_4(T)$ and $\lambda_5(T)$ to their correct values. The state variables and λ_1, λ_2 and λ_3 start step 2 with the final values obtained from step 1. After the completion of step 2, the state variables are set equal to their given initial vector, $\underline{x}_{t_0}$ and step 3 proceeds. After the forward integration in step 3, $\lambda_4(T)$

and $\lambda_5(T)$ were found to be in close agreement with the boundary values of the necessary conditions and the solution process was stopped.

The Computer Program

The Continuous System Modeling Program (CSMP), developed by IBM [20] is utilized for the numerical integration of the optimal control necessary conditions. The model variables of the original problem presented in Chapter V have been maintained where possible. However, some of the parameters have been combined to form new parameters. Table III lists these new parameters used in the program and their definitions in terms of the original parameters.

The solution is found for a ten year planning period with the basic unit of time being one month. The values given in Table I in Chapter V which are expressed as yearly rates have been adjusted to monthly values. These can be located in the "Initial" section of the program listing. A sample output of the actual capacity, AC, is given in Figure 11 for reference. It is noted that AC has the variable name X6.

TABLE III

COMPUTER VARIABLE NAMES

Computer Variable	Model Variables
Ai	λi
Rho	$\rho/12$
CO	$C_O/12$
C	$C/12$
C1	$C_O \frac{(1+h)}{k \cdot 12}$
C2	$I_n \cdot h/12$
C3	$(1-\delta)$
RLF	$R \cdot LF \cdot 724$
CD	$K \cdot C_D/12$
CM	$C_m \cdot K/12$
CF	$C_f \cdot LF \cdot 724$
PP	$P_p \cdot LF \cdot 724$
B	$\beta/12$
TMAX	T
X6	AC
PR	I

```
****CONTINUOUS SYSTEM MODELING PROGRAM****

*** VERSION 1.3 ***

INITIAL
CONSTANT N=0
CONSTANT  D1=0.0, D2=1.0, D4=-1.0
INCON  X10=1.80E+06, X20=0.0, X30=120.E+06, X40=12., X50=10.E+06
INCON A10=1.315E-06,A20=-1.315E-06,A30=0.0, A40=.406, A50=0.0
CONSTANT  W1=1.0,  W2=3.23E-06
CONSTANT D1=1.0, D2=1.0, D4=0.0
CONSTANT C0=1.0, C=.0835, C1=.00125
CONSTANT C2=.0022, C3=.9, RHO=.075
CONSTANT RLF=4.33, PP=1.08, B=2.5E-03
CONSTANT CF=.936, CM=.167, CD=.25
CONSTANT USMAX=3460. , UPMAX=.90E+06, TMAX=120.
DYNAMIC
       X6=X1-X2
       D3=1.0+D4
       TAU=TMAX-TIME
     DT1=D3*(16.67*TIME-(4.17*TIME+500.)*COS(3.1416*TIME/6.)+1500.)*1000.
     DT2=D4*(16.67*TAU-(4.17*TAU+500.)*COS(3.1416*TAU/6.)+1500.)*1000.
       DT=DT1+DT2
       PR=(RLF-CF)*DT-(PP-CF)*UP-(CD+CM)*(X1-X2)-C2*X3
       X1D=D1*(C1*X3-C*X1)
       X2D=D1*(B*(X1-X2))
       X3D=D1*(PR-X5*UD+C3*US*X4)
       X4D=D1*(C0*UD-RHO*C0*X4)
       X5D=D1*US
       A1D=D2*(C*A1-B*A2+A3*(CD+CM))
       A2D=D2*(B*A2-A3*(CD+CM))
       A3D=D2*(-C1*A1+C2*A3)
       A4D=D1*(-C3*A3*US+RHO*C0*A4)
       A5D=D1*UD*A3
        X1=INTGRL(X10,X1D)
        X2=INTGRL(X20,X2D)
        X3=INTGRL(X30,X3D)
        X4=INTGRL(X40,X4D)
        X5=INTGRL(X50,X5D)
       A1=INTGRL(A10,A1D)
       A2=INTGRL(A20,A2D)
       A3=INTGRL(A30,A3D)
       A4=INTGRL(A40,A4D)
       A5=INTGRL(A50,A5D)
       PID=UD*EXP(-RHO*TIME)
        P=INTGRL(0.0,PID)
       PI=(X4*W1+W2*(X1-X2))*EXP(-RHO*120.)+P
PROCED   DEM,UP=DEMAND(DT,X1,X2,A3)
       IF(A3*(CF-PP))10,10,5
10          DEM=DT+X2-X1
            IF(DEM)100,100,200
100         UP=0.0
            GO TO 300
200         UP=DT+X2-X1
       GO TO 300
5      UP=UPMAX
300         CONTINUE
ENDPRO
PROCED    UD=DIVI(A3,X5,A4,PR)
       IF(EXP(-RHO*TIME)-A3*X5+A4*C0)15,15,25
15     UD=.1
       GO TO 35
25     UD=.8*PR/X5
35     CONTINUE
```

```
ENDPRO
PROCED    US=STOCK(A3,X4,A5)
       IF(A3*C3*X4+A5)1,1,2
1      US=0.0
       GO TO 3
2      US=USMAX
3      CONTINUE
ENDPRO
TERMINAL
       N=N+1
       IF(N.GE.2)GO TO 30
       A10=A1
       A20=A2
       A30=A3
       A40=5.06
       A50=0.0
       D1=1.0
       D2=1.0
       D4=0.0
       CALL RERUN
       GO TO 70
30     IF(N.GE.3)GO TO 40
       A10=+1.315E-06
       A20=-1.315E-06
       A30=0.0
       A40=.406
       A50=0.0
       X10=X1
       X20=X2
       X30=X3
       X40=X4
       X50=X5
       D1=-1.0
       D2=-1.0
       D4=-1.0
       CALL RERUN
       GO TO 70
40     IF(N.GE.4)GO TO 70
       A10=A1
       A20=A2
       A30=A3
       A40=A4
       A50=A5
       X10=1.8E+06
       X20=0.0
       X30=120.E+06
       X40=12.
       X50=10.F+06
       D1=1.0
       D2=1.0
       D4=0.0
       CALL RERUN
70      CONTINUE
METHOD RKSFX
TIMER  FINTIM=120., DELT=.0333,PRDEL=1.0, OUTDEL=3.0
END
PRINT X6,X4,UD,UP,US,PI
PRTPLT X6,X4,UD,UP,US,A1,A2,A3,A4,A5
       END
       STOP
```

```
                                                                                PAGE    1

                                   MINIMUM                X6      VERSUS TIME              MAXIMUM
                                  1.7857E 06                                              7.0945E 06
 TIME              X6              I                                                         I
0.0             1.8000E 06         +
3.0000E 00      1.7888E 06         +
6.0000E 00      1.7860E 06         +
9.0000E 00      1.7927E 06         +
1.2000E 01      1.8041E 06         +
1.5000E 01      1.8165E 06         +
1.8000E 01      1.8331E 06         +
2.1000E 01      1.8562E 06         +
2.4000E 01      1.8812E 06         +
2.7000E 01      1.9050E 06         -+
3.0000E 01      1.9314E 06         -+
3.3000E 01      1.9639E 06         -+
3.6000E 01      2.0175E 06         --+
3.9000E 01      2.0867E 06         --+
4.2000E 01      2.1871E 06         ---+
4.5000E 01      2.3294E 06         -----+
4.8000E 01      2.4860E 06         ------+
5.1000E 01      2.6359E 06         --------+
5.4000E 01      2.8011E 06         ---------+
5.7000E 01      2.9971E 06         -----------+
6.0000E 01      3.1965E 06         -------------+
6.3000E 01      3.3794E 06         ---------------+
6.6000E 01      3.5710E 06         ----------------+
6.9000E 01      3.7897E 06         ------------------+
7.2000E 01      4.0074E 06         --------------------+
7.5000E 01      4.2035E 06         ----------------------+
7.8000E 01      4.4058E 06         ------------------------+
8.1000E 01      4.6346E 06         --------------------------+
8.4000E 01      4.8603E 06         ----------------------------+
8.7000E 01      5.0614E 06         ------------------------------+
9.0000E 01      5.2679E 06         --------------------------------+
9.3000E 01      5.5013E 06         ----------------------------------+
9.6000E 01      5.7308E 06         -------------------------------------+
9.9000E 01      5.9335E 06         ---------------------------------------+
1.0200E 02      6.1413E 06         -----------------------------------------+
1.0500E 02      6.3773E 06         -------------------------------------------+
1.0800E 02      6.5952E 06         ---------------------------------------------+
1.1100E 02      6.7621E 06         ----------------------------------------------+
1.1400E 02      6.8945E 06         ------------------------------------------------+
1.1700E 02      7.0055E 06         -------------------------------------------------+
1.2000E 02      7.0945E 06         -------------------------------------------------+
```

Figure 11. CSMP Output for Actual Capacity

Glossary of Symbols

A	- Dollars invested in capacity
AC, x_6	- Actual usable capacity
β	- Rate of retirement
C	- $1/C$ represents time constant of construction
C_D	- Coefficient of depreciation cost
C_f	- Coefficient of fuel cost
C_m	- Coefficient of maintenance cost
C_O	- Represents the trading activity factor of common stock
δ	- The percentage cost of marketing a share of common stock
d, u_d	- Dividend per share per unit of time
$D(t)$	- Specified power demand
$D(AC)$	- Depreciation cost per unit of time as function of actual capacity
E, x_3	- Total equity of company
$F[D(t)-u_p]$	- Fuel cost per unit of time as function of energy produced internally
$G(AC)$	- Maintenance cost per unit of time as function of actual capacity
h	- Debt-equity ratio
I	- Net income per unit of time
I_n	- Interest rate on long-term debt
K	- Construction cost per unit of capacity
λ_i	- Adjoint variable for the x_i state variable
LF	- Load factor
N, x_5	- Number of outstanding shares of common stock
P, x_4	- Market price of one share of common stock
PI	- Performance index of market valuation

PI^1	- Modified performance index of market valuation and terminal actual capacity
P_p	- Price per unit of purchased energy
Q	- Long-term debt
ρ	- Expected rate-of-return by investor
R	- Average price per unit of energy
RC, x_2	- Retired capacity
T	- Planning horizon
TC, x_1	- Historical sum of all capacity installed
u	- New capacity per unit of time
u_p	- Purchased power
u_s	- Number of shares issued per unit of time
w_1	- Weighting constant on market valuation in modified performance index
w_2	- Weighting constant on terminal actual capacity in modified performance index

2

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