# AN INVESTIGATION OF COST RATIOS FOR USE WITH A MODIFIED GUTHRIE-JOHNS MODEL FOR ACCEPTANCE SAMPLING

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## PREFACE

The research described in this paper is aimed at making an economically-based acceptance sampling model easy to use. Toward this end, cost ratios have been introduced to replace actual dollar value costs in the modified Guthrie-Johns model.

It is hoped that the introduction of these cost ratios will ignite a spark of interest among government and industry practitioners so that there will be at least one plausible alternative to the risk-based acceptance sampling plans which have dominated the field for over 60 years.

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# CHAPTER I

# INTRODUCTION AND RESEARCH OBJECTIVES

## Introduction

### The Nature of Acceptance Sampling

Attributes acceptance sampling is the most universally used quantitative tool in quality control. Largely unchanged since its origination in the 1920's, acceptance sampling is used extensively by small and large companies for checking incoming material, in-process items, and finished goods. It is also used in life and reliability testing. In single acceptance sampling for a single attribute, a sample of size n is taken from a lot of size N, and each item in the sample is inspected and classified as either non-defective or defective based upon conformance or non-conformance to some specified quality characteristic or attribute. A count, x, of the number of defectives is maintained. If the count does not exceed a value c, called the acceptance number, the entire lot is accepted; otherwise, the lot is rejected. Acceptance sampling for attributes may also involve two or more samples taken from the lot (double and multiple acceptance sampling) or in the case of sequential sampling, the sample size is not specified and items are examined sequentially until certain conditions are met. In some cases of single, double, multiple and sequential acceptance sampling, two or more quality characteristics are measured in the samples. This procedure is called multi-attribute acceptance sampling. Single acceptance sampling for a single attribute is prevalent in government and industry. In all sections of this paper, acceptance sampling will be understood to mean single acceptance sampling for a single attribute.

An acceptance sampling plan can be identified by the lot size, sample size, and acceptance number. (N,n,c) is often used to denote a single acceptance sampling plan. N is usually treated as a fixed constant and in such cases it suffices to identify sampling plans by the pair, (n,c). Selection of an acceptance sampling plan, i.e., specifying a (n,c) pair, results from either considering the probabilities of rejecting lots of good quality and accepting lots of bad quality (risk-based sampling) or considering the costs associated with sampling, inspecting, testing, accepting and rejecting manufactured lots (economically based sampling).

Risk-Based Acceptance Sampling Plans

Risk-based sampling plans can be developed from several viewpoints. Common among such plans, however, are criteria such as achieving an acceptable quality level (AQL), not accepting lots whose quality level is beyond a certain value, such as the lot tolerance percent defective (LTPD), maintaining a desired average outgoing quality level (AOQL), achieving a limiting quality protection (LQP), etc. The probabilities of failing to meet these criteria are also specified. Hence, the term "risk-based". Foremost among the risk-based plans is MIL-STD-105D whose international designation is ABC-STD-105D. Excellent explanations of this plan are given by Duncan [14] and by Grant and Leavenworth [15]. The Dodge-Romig sampling tables also enjoy widespread use and are discussed in [14] and [15] as well. There are many other risk-based sampling plans prevalent in the literature. Examples are found in [11] and [12]. Certain plans also make use of the process distribution and/or the distribution of defectives in the lots formed from the process (the prior distributions) and hence are Bayesian in nature. Examples of such plans are found in [18], [28], [32], and [36]. The choice of a prior distribution is an important issue. In Bayesian treatments of acceptance sampling, there are two priors of interest. The first is the prior (to forming the lot) distribution of the process fraction defection, p. We denote this distribution by f(p). Popular choices for f(p) include the constant (f(p)=w), the k-point ($f(p)=w_1$, $p=p_1$; $f(p)=w_2$, $p=p_2$, ..., $f(p)=w_k$, $p=p_k$), and the beta. The second prior is the prior (to taking a sample) distribution of defectives in the lot, and it is denoted by $f_N(X)$. The form of $f_N(X)$ depends of f(p) since

$$f_N(X) = \begin{cases} \sum\limits_{\text{all } p} y(X|p)f(p) & \text{or} \\ \int\limits_{\text{all } p} y(X|p)f(p)\,dp \end{cases} \tag{1.1}$$

where $y(X|p)$ is the distribution of defectives in the lot for a given value of p. The mass function $y(X|p)$ is usually taken to be binomial, although the Poisson is sometimes used. For the three f(p) choices mentioned above and a binomial $y(X|p)$, the $f_N(X)$ priors are binomial, mixed binomial, and Polya, respectively. The mixed binomial is a realistic prior. It is applicable, for example, when two or more different machine/material/operator sources supply parts. Practitioners are beginning to report evidence of mixtures in their analysis of quality data (see e.g., [15]). The mixed binomial is a special case of the Polya. Each will be discussed in more detail in Chapter IV.

Mood [31] developed a theorem which implies, among other things, that for the binomial form of $f_N(X)$, sampling is of no value whatsoever. Unfortunately, the work of Mood has escaped a few researchers and they continue to use a binomial $f_N(X)$ in their modeling efforts. Case and Keats [8] have recently illustrated the implications of Mood's theorem both analytically and graphically for five forms of $f_N(X)$, including the binomial.

Throughout this paper, f(p) will be called the process distribution and $f_N(X)$ will be called the prior distribution.

## Economically-Based Sampling Plans

Economically-based acceptance sampling plans select the (n,c) pair which minimizes a cost function. The form of the cost function reflects the economic modeler's beliefs concerning which costs are critical as well as assumptions about matters such as the production process itself, the policies for disposition of rejected lots, handling of returned items under warranty, etc. As in the case of risk-based sampling, economically-based sampling plans are either Bayesian or non-Bayesian. Papers by Breakwell [2], Caplen [4], and Martin [30] are representative of the literature of the non-Bayesian economically-based acceptance sampling plans. Examples of Bayesian economically-based plans are given in [1], [21], [24], [27], and [35]. A review of economically-based plans of both types is presented by Wetherill and Chiu [41]. Some economically-based plans are controlled by risk factors. For example, the (n,c) pair resulting from an economically-based plan will be used only if it affords the protection guaranteed under a risk-based plan subject to one or more statistical constraints. Such plans have been labeled "semi-economic" or "restricted Bayesian". Studies by Hornsell [25] and Hald [22] are illustrative of this concept.

A Brief History and Perspective

Formalized treatments of risk-based acceptance sampling are generally achknowledged to begin with the work of Dodge and Romig in 1920. The first widely published account of their work occurred at the end of that decade [13]. MIL-STD 105D evolved from sampling tables developed for the Navy by the Statistical Research Group at Columbia University in 1945. The Air Force had been using similar tables and after the unification of the armed forces, the Navy tables were adopted by the Department of Defense in 1949 as Joint Army-Navy (JAN) Standard 105. MIL-STD 105A superceded JAN-STD 105 in 1950. Subsequent changes resulted in 105B (1958), 105C (1961), and finally 105D (1963). The working group responsible for 105D consisted of scientists from America, Britain, and Canada and hence the international designation, ABC-STD 105D.

The first papers involving economically-based sampling were published in 1951 [2], [37]. Bayesian applications in acceptance sampling also began to appear in 1951 [40]. Thus, risk-based acceptance sampling preceded its economically-based counterpart by three decades and industrial use of risk-based plans was quite prevalent before economically-based plans were ever introduced. Many of the economically-based plans also offered the use of published tables which were convenient to industry users. The number of published papers in economically-based sampling has increased dramatically over the last ten years. The majority of these papers use the

Bayesian approach. Zonnenshain and Dietrich [42] present a scheme for analyzing acceptance sampling plans from a consumer's as well as a producer's viewpoint. Given a plan and a prior distribution, both consumer and producer costs and risks are used in the analysis.

Users of risk-based plans must decide upon probabilities or assumed risks of accepting bad lots or having good lots rejected. The choice of these probabilities or risks is often the result of a mental assessment of the economic consequences of the undesirable results associated with accepting bad lots or having good lots rejected. Typically, this "mental assessment" is not sophisticated and the measures of good and bad lot quality are standard values which are established sometime in the past. Proponents of Bayesian economically-based plans have argued that since the risks are implicitly determined by sampling costs and the "downstream" costs, these costs should be identified and used in acceptance sampling plans. They further state that the quality level resulting from the production process is a random variable whose distribution should be incorporated in the acceptance sampling model. These arguments seem plausible to large segments of the academic and industrial communities. Furthermore, the use of high-speed electronic digital computers makes the economically-based plans readily available to nearly every potential user. Yet, in government and industry, the use of risk-based plans continues to predominate. Today, one finds little more than token use of economically-based plans in the governmental or industrial setting.

## The Problem

There are good reasons for the reluctance of the government and industry users to adopt economically-based plans. There does not exist a single comprehensive cost model which will accommodate virtually any real-world sampling scenario. Such a model must provide precise definitions of cost parameters. It must cover, for example, situations such as the return of lots to the vendor without screening, identification of the nature of scrap losses, and the assignment of fixed costs associated with sampling, rejection, and inspection. Nearly 25 years have elapsed since Guthrie and Johns [19] developed the generic model for economically-based acceptance sampling. The model used variable costs only and an asymtotic approach to optimization. Only a few refinements to the Guthrie-Jones models have been attempted. These will be reviewed in Chapter II. Many other developments and improvements must be made if the Guthrie-Johns model is to enjoy widespread use. It is extremely difficult for practitioners to obtain reliable values for costs in an economically-based model. In fact, the better models require as many as nine different cost parameters. In practice, only a few costs can be measured with sufficient accuracy to be useful in any model.

If progress is to be made during the 1980's, there must be fundamental changes by government and industry in both the philosophy and actual conduct of attributes acceptance sampling. MIL-STD 105D, the Dodge-Romig tables, and other statistically-based sampling schemes, built upon techniques held sacred for 50 years, must either be replaced or supplemented by economically-based sampling. Practitioners are eager to implement a procedure for providing the right sampling risks which minimize the total costs of inspecting, rejecting good lots, and passing poor lots. The effect of proper sampling efforts upon productivity alone can amount to tens of millions of dollars per year.

## Research Objectives

This paper provides much of the necessary research to close the gap between theory and practice and will aid in establishing economically-based acceptance sampling as the new quality assurance tool for the 1980's. As such, the principal objective of this research is to remove many of the barriers which limit widespread use of one of the best tools available to those engaged in acceptance sampling, the Guthrie-Johns model. This implies the construction of definitions which can be clearly interpreted, the identification of critical cost ratios, and the development of a user-oriented computer program which identifies the optimal plan. In order to accomplish this objective, the following subobjectives have been realized:

1. The establishment of clear definitions and elaborations of each of the cost factors in the modified model to cover virtually any acceptance sampling situation encountered in government or industry.
2. An exact, iterative search for the optimal (n,c) pair using a mixed-Polya prior and all cost factors of the modified model.
3. A thorough sensitivity analysis of the modified model to each of the cost parameters, singly as well as in logical combinations.
4. The development of critical ratios between cost parameters of the modified model. This is an important step as these

ratios will replace cost estimates which are often difficult or impossible to obtain.

5. A validation of the critical ratios by examining the effectiveness of the proposed sets of cost ratios.

6. The development of a flexible, well-documented, interactive computer program suited for use in a wide range of acceptance sampling situations.

The results of this study should make economically-based acceptance sampling the innovative quality assurance technique of this decade. It is anticipated that many users of risk-based acceptance sampling plans will convert to economically-based plans as such plans directly involve the most relevant entities associated with acceptance sampling--the costs of inspection, testing, sampling, and accepting or rejecting a lot. The proposed study has been accomplished in three phases--generic model development, optimization and modeling with cost ratios, and sensitivity with cost ratios. These topics will be treated in Chapters III, IV, and V, respectively.

Without research of this kind, implementation of any form of the Guthrie-Johns model would be extremely difficult. Economically-based acceptance sampling in government and industry can become a reality with the results that this paper is expected to provide. All of the questions cannot be answered--e.g., sensitivity of (n,c) values to the form and parameter values of the prior are not investigated in sufficient detail. However, using the results of this study, practitioners will be able to involve costs directly in the decision-making process. At last there will be a viable alternative to risk-based acceptance sampling.

## Summary

Risk-based acceptance sampling procedures continue to predominate in industrial applications in spite of the intuitive appeal of plans which consider the economic consequences of the decision to accept or reject the lot. The principal reason why economically-based plans do not enjoy widespread use is the difficulty in obtaining estimates for the costs associated with sampling and then accepting or rejecting the lot. The research described in this paper is directed toward overcoming this difficulty by proposing the use of a few easy to obtain cost ratios in lieu of actual dollar costs.

In the process of developing these ratios, clear definitions of all cost factors will be made, a set of candidate ratios will be tested, and sensitivity analyses will be performed using a versatile interactive computer program.

# CHAPTER II

# REVIEW OF RELEVANT LITERATURE

## The Modified Guthrie-Johns Model

### The Guthrie-Johns Model

The basic model from which the model of the present study is developed is due to Guthrie and Johns. The model is given by

$$\begin{aligned} TC(N,n,X,x,c) &= S_0 + nS_1 + xS_2 + A_0 + (N-n)A_1 + (X-x)A_2, \quad x \le c \\ &= S_0 + nS_1 + xS_2 + R_0 + (N-n)R_1 + (X-x)R_2, \quad x > c \end{aligned} \tag{2.1}$$

where $S_0$ = fixed cost of sampling, inspecting, and testing per lot,

$S_1$ = cost per item of sampling, inspection, and testing,

$S_2$ = additional cost per item found defective during sampling, inspection, and testing,

$A_0$ = fixed cost of accepting a lot containing defective items to be found downstream,

$A_1$ = cost per item associated with the N-n items not inspected in an accepted lot (frequently zero),

$A_2$ = cost associated with a defective item found downstream after having been in an accepted lot (may be quite large),

$R_0$ = fixed cost of rejection per lot,

$R_1$ = cost per item associated with the N-n items remaining in a rejected lot, and

$R_2$ = cost associated with a defective item in a rejected lot.

The fixed costs, $S_0$, $A_0$, and $R_0$ have been added by Case [5]. Hence, equation (2.1) henceforth will be referred to as the Modified Guthrie-Johns (MGJ) model. The above definitions are weak and must be elaborated upon and many examples must be provided before practitioners can make effective use of the model. Hence, one of the objectives of this study involves redefining and elaborating the definitions as well as illustrating by example the kinds of costs associated with each cost parameter. The above formulations assume that sampling is performed. Special cases involving no sampling and 100 percent inspection will be treated in Chapter IV.

Guthrie and Johns developed asymtotic solutions for large N which were optimal in the Bayes sense. This means that the Bayes risk--the expected value of (2.1) using the distribution of a random variable providing some measure of lot quality is minimized by selecting a particular sample size and decision procedure. The process distributions specified were members of the exponential family. No examples were provided.

Smith [38] explained the Guthrie-Johns model in readable terms and suggested the beta distribution as the density for process fraction defective. He also used a property developed by Hald [20]. In what must be regarded as a classic paper, Hald showed that certain distributions are reproducible to hypergeometric sampling. This means that with hypergeometric sampling, the form of the posterior is the same as the form of the prior. In other words, the number of defectives in a sample of size n drawn from a lot of size N will be distributed as if the sample were drawn directly from the process. Hald's paper also presented asymtotic solutions using an economic model with only two cost parameters. Smith used Hald's expression for the optimal acceptance number, c*, (which was developed using the reproducibility concept) and the Guthrie-Johns characterization of the optimal sample size, n*, with some realistic numerical examples. The parameters of the beta process distribution were estimated using the method of moments.

Guenther [17] used the Guthrie-Johns model (identifying it as Hald's model) with a constant, a beta, and a two-point process distribution. He obtained solutions to several variations of a sample problem using only standard tables and a desk calculator. The use of a pattern search routine in the (n,c) plane was illustrated by Moskowicz [33] using the Guthrie-Johns model. Examples of the pattern search procedure were applied with normal, skewed, and bimodal process distributions. The method was not efficient for use in single applications as it failed to converge on the optimal sampling plan.

Chen [10] investigated double sampling plans using Case's revision of the Guthrie-Johns model and a three-point process distribution. Results indicated that the optimal double sampling plans were only one to two percent more efficient than their single sampling counterparts.

## A Solution Procedure

Case and Jones [6] described an interactive computer program which allows the user to choose the number of parameters and values of a mixed binomial prior. The user may also elect to include or exclude the two common types of inspection error--Type 1, classifying a good item as bad, and Type 2, classifying a bad item as good. Case and Keats [7] have illustrated the solution procedure for the MGJ model. The procedure is repeated here. The MGJ model may be thought of as a function of lot size, sample size, defectives in the lot, defectives in the sample, and the acceptance number, i.e., TC(N,n,X,x,c). The posterior expected cost may be obtained by rewriting the second and fourth terms of equation (2.1), multiplying by the appropriate conditional probability, and summing over X:

$$TC(N,n,X,x,c) = \sum_{X=x}^{N-n+x} TC(N,n,X,x,c)h_N(X|x)$$

$$= \sum_{X=x}^{N-n+x} [S_0 + nS_1 + x(S_2-A_2) + A_0 + (N-n)A_1 + XA_2]h_N(X|x), \quad x \leq c$$

$$= \sum_{X=x}^{N-n+x} [S_0 + nS_1 + x(S_2-R_2) + R_0 + (N-n)R_1 + XR_2]h_N(X|x), \quad x > c \qquad (2.2)$$

where $h_N(X|x)$ is the probability of X defectives in the lot given x defectives in the sample. (2.2) may be written as

$$TC(N,n,x,c) = S_0 + nS_1 + x(S_2-A_2) + A_0(1-h_N(X=x|x)) + (N-n)A_1 + A_2E(X|x), \quad x < c$$

$$= S_0 + nS_1 + x(S_2-R_2) + R_0 + (N-n)R_1 + R_2E(X|x), \quad x > c \qquad (2.3)$$

where $E(X|x) = \sum_{X=x}^{N-n+x} X \cdot h_N(X|x)$.

Note $\sum_{X=x}^{N-n+x} h_N(X=x|x) = 1$. If X-x = 0 (no defectives downstream) then fixed cost $A_0$ and variable cost $A_2$ are not incurred. Hence, the factor $(1-h_N(X=x|x))$ for $A_0$ in the first portion of the equation (2.3).

It is reasonable to assume that if the number of defectives observed in the sample, x, causes the expected acceptance cost term to be less than the expected rejection cost term, then the logical decision is to accept the lot. Conversely, the lot should be rejected for any value of x causing the expected rejection cost term to be less than the expected acceptance cost term. However, acceptance is of primary interest in the present study. Denoting the acceptance form $(x \leq c)$ of equation (2.3) by $TC_A$ and the rejection form $(x > c)$ by $TC_R$, we require that $TC_A \leq TC_R$. Using this inequality and moving all terms to the left, we have

$$x(R_2-A_2) + (N-n)(A_1-R_1) + (A_2-R_2)E(X|x) - R_0 + A_0(1-h_N(X=x|x)) \leq 0. \tag{2.4}$$

The acceptance number, c, is the largest value of x satisfying the inequality (2.4). Expressions for $E(X|x)$ and $h_N(X=x|x)$ must be developed. Hald has shown that

$$E(X|x) = \frac{(N-n)(x+1)}{(n+1)} \frac{g_{n+1}(x+1)}{g_n(x)} + x \tag{2.5}$$

where $g_n(x)$ is the marginal or unconditional distribution of defectives in the sample. The form of $g_n(x)$ depends upon $f_N(X)$. As mentioned earlier, $g_n(x)$ will be of the same form as $f_N(X)$ for certain cases as shown by Hald. $h_N(X=x|x)$ also depends on $f_N(X)$. The mixed Polya, and a special case of the mixed Polya, the mixed binomial distribution, are of special interest in this study. Expressions for $f_N(X)$, $g_n(x)$, and $h_N(X=x|x)$ based on the mixed Polya distribution will be developed in Chapter IV.

The inequality (2.4) is used to find the "break points" of the solution space. A break point is a value of n which for a fixed value $x = c = 0, 1, 2, \ldots, n$ causes the total cost associated with the plan (n,c) to be approximately equal to the total cost associated with the plan (n,c+1). The total cost is obtained by summing equation (2.3) over x. Thus,

$$TC(N,n,c) = \sum_{x=0}^{n} TC(N,n,x,c)g_n(x)$$

$$= \sum_{x=0}^{c} [S_0 + nS_1 + x(S_2-A_2) + A_0(1-h_N(X=x|x)) + (N-n)A_1 + A_2E(X|x)]g_n(x) + \sum_{x=c+1}^{n} [S_0 + nS_1 + x(S_2-R_2) + R_0 + (N-n)R_1 + R_2E(X|x)]g_n(x) \qquad (2.6)$$

n is varied in increments of one and at each step (2.4) and (2.6) are evaluated. It is known from personal observations and from published results [33] that the surface of (2.6) is not convex. Yet it is reasonably well behaved as shown in Figure 1. As Figure 1 indicates, the value of TC(N,c) makes successive dips, each dip associated with a particular acceptance number. Also, the minimum TC point of each dip gets lower and lower, up to a certain point, at which time it begins to increase. It has been observed that the locus of TC values associated with a given acceptance number, c, is (nearly) convex, having but one local minimum. Case claimed that the locus of each local minimum is itself convex, having but one global optimum in the range of n=1 to n=N. This did not appear to be the case in the study by Chen which involved double sampling. Another observed property is that the sample size, n, at the global minimum TC occurs approximately midway between the sample sizes at which the next lower or higher acceptance numbers become optimum. With these properties, a heuristic search procedure has been devised to find the optimum n and corresponding c.

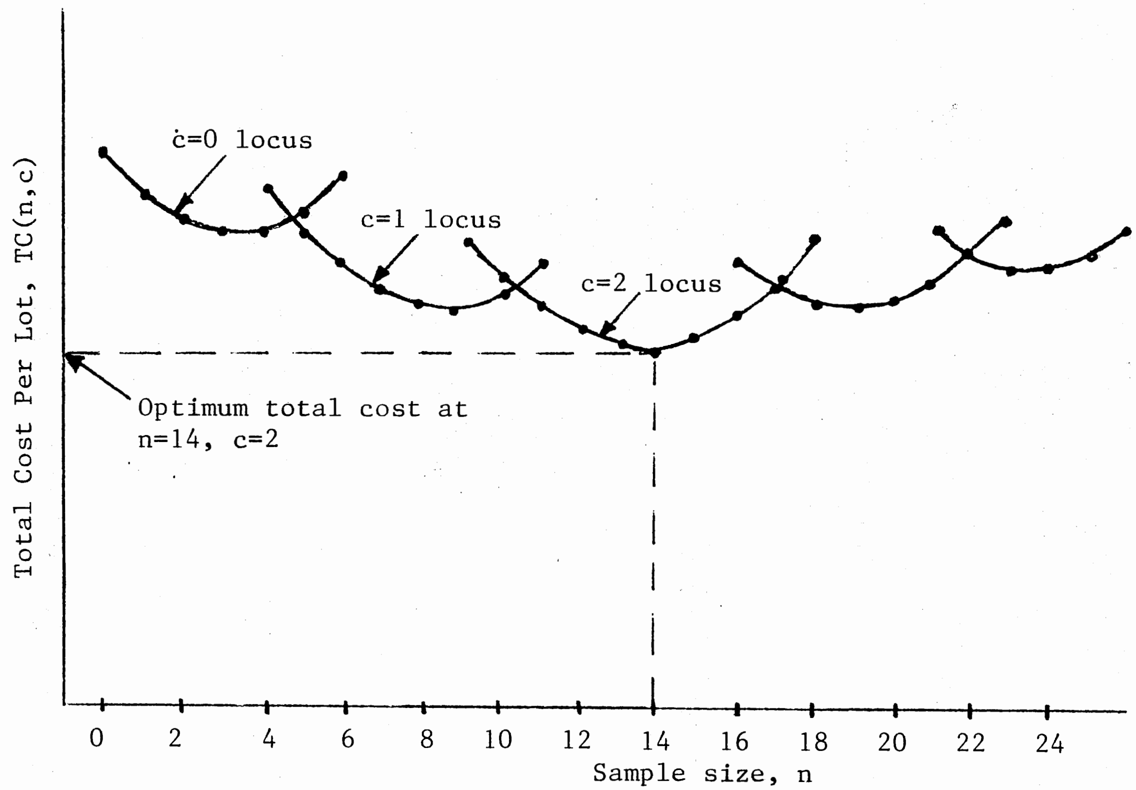


Figure 1. Example Curve of Total Expected Cost Per Lot Versus Sample Size (n)

The basic procedure developed is to find the midpoint of the range of sample sizes for which c=0 is optimum. This range usually occurs between n=0 and the first break point. For this mid-point sample size, the total cost is determined. For the same acceptance number, one by one, the lower sample sizes are searched and evaluated in (2.6) until costs begin to rise. Then the search proceeds to the higher sample sizes for c=0. The minimum cost value of n for c=0 and its corresponding TC are then remembered. The same procedure next takes place for c=1 by examining costs associated with the values of n between break points. As long as lower local minima continue to be found, the search continues. When a higher local minimum is encountered that the previous lowest TC, the search is halted. The minimum cost (n,c) pair is then specified as optimum. As mentioned earlier, this solution procedure is illustrated for a beta process distribution ($f_N(X)$ is a Polya) by Case and Keats [7].

## Cost Ratios

The present study uses the nine cost parameter Guthrie-Johns model as mentioned above. Experiments with the Guthrie-Johns model in industrial settings have indicated that obtaining reasonable estimates for only six cost parameters is unrealistic. The present study identifies the sensitivity of the revised Guthrie-Johns model to ranges of hypothetical values of the nine cost parameters and then investigates the use of cost ratios which will require only a few estimated cost values.

The advantage of ratios in lieu of actual costs is that while the user may be unable to estimate actual dollar values, the user can often obtain a good approximation of the ratio of two costs. Furthermore, it is more likely that a group of quality practitioners would agree on a particular ratio moreso than on the actual dollar values of the costs involved.

The literature on the use of cost ratios in quality cost modeling is rather sparse. A few recent studies will be cited. Stewart, Montgomery, and Heikes [39] developed an economic model for use with double sampling plans. The cost parameters included a fixed ($k_I$) and a unit ($k_i$) sampling cost, the unit costs associated with rejected items ($k_s$) and the unit cost of accepting a defective item ($k_a$). Using a beta process distribution, the optimal plan, ($n_1^*, c_1^*, n_2^*, c_2^*$), was determined to be a function only of the ratios $k_i/k_a$ and $k_s/k_a$. Varying one of these costs and holding the others fixed, it was discovered that increasing $k_a$ makes the plans more discriminating, i.e., it is more difficult to accept lots of equal quality as $k_a$ increases. In general, increasing $k_i$ results in a reduction in the optimal values of $n_1$, $c_1$, $n_2$, and $c_2$. Increasing $k_s$ causes $n_1^*$ and $c_1^*$ to decrease and $n_2^*$ and $c_2^*$ to increase. Hoadley [26] used a ratio of incremental audit costs to incremental field costs in a model for use in a specific company's quality assurance audit. The procedure was non-Bayesian and sensitivity of the optimal plan to the cost parameter values was not investigated. In a follow-up study, Buswell and Hoadley [3] compared this quality audit procedure with MIL-STD 105D. Lee [29] used a failure cost to unit inspection ratio in a simple model to develop sampling plans with a zero acceptance number. No cost sensitivity analysis was performed in this non-Bayesian approach.

## Summary

The MGJ model uses meaningful costs and lot history to specify sampling schemes. Recently added fixed costs provide a more realistic approach. The solution procedure is well-established and requires the use of a computer. No published accounts involving the use of cost ratios in economically-based Bayesian acceptance sampling have been discovered.

# CHAPTER III

# GENERIC MODEL DEVELOPMENT

## Introduction

The key elements of the MGJ model are the cost values. Although the main thrust of this paper is the development of cost ratios for use in the model, it is of paramount importance to present clear and concise explanations of the components of each cost value so that there are no ambiguities present at the time that ratios are to be selected. Given a set of cost component explanations and illustrations of how each is associated with one or more of the nine cost values, the user will be in a position to select appropriate ratios without doubt as to which cost elements belong in the ratio formation. Common conceptions about sampling costs will enhance communications among users and will speed the adoption of economically based acceptance sampling.

## Lot Disposition Policies

During and following the sampling process, there are a number of decisions that must be made concerning the disposition of defective and non-defective items in the sample and the rest of the lot for instances where the lot is either accepted or rejected. Table I on the next page presents the matrix of decision possibilities. The matrix is intended to delineate the alternatives that are available when deciding what to do with defectives found during sampling or found

TABLE I

ITEM DISPOSITION DECISION MATRIX

| | | | Non-Destructive Test | | | | | | Destructive Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Replenish | | | No Replenish | | | Replenish | | | No Replenish | | |
| | | | Scrap/ Sell | Rework/ Repair | Return | Scrap/ Sell | Rework/ Repair | Return | Scrap/ Sell | Rework/ Repair | Return | Scrap/ Sell | Rework/ Repair | Return |
| ACC. LOTS | Sample | Good | X | X | X | | X | | | X | | | X | |
| | | Bad | | | | | | | | X | | | X | |
| | Rest of Lot | Good | X | X | X | | X | | | X | | | X | |
| | | Bad | | | | | | | | X | | | X | |
| REJ. LOTS | Sample | Good | | X | | | X | | | X | | | X | |
| | | Bad | | | | | | | | X | | | X | |
| | Rest of Lot | Good | | X | | | X | | | X | | | X | |
| | | Bad | | | | | | | | X | | | X | |

later during screening of rejected lots or after a lot has been accepted. Likewise, there are alternatives regarding disposition of non-defectives found in the sample and in the rest of the lot. Non-feasible alternatives are marked with an "X". The decision to screen or not to screen the rest of a rejected lot is another matter which must be resolved. It is not a part of Table I. The decision matrix is appropriate at each of the three possible stages where acceptance sampling is used--incoming, in-process, and final inspection. Decisions made concerning disposition of defective and non-defective items from the lot affect several of the components which will now be discussed.

## Cost Components

Each of the cost components introduced in this section will be a part of one or more of the nine cost values. The cost components may be thought of as the contribution to the total lot or individual unit costs due to the use of labor, materials, energy, or the expenditure of capital. The cost is incurred during sampling, or immediately after a lot is rejected or in some stage subsequent to the acceptance of a lot. The cost components will be identified according to whether they are associated with the lot itself (fixed costs) or individual units within the lot (unit costs).

### Fixed Cost Components

The following components involve costs associated with the lot as a whole or costs which cannot be directly identified with individual units. They are used exclusively in forming the costs $S_0$, $A_0$, and $R_0$.

<u>SET-UP(F)</u> - includes the cost of all activities required to prepare a lot for sampling or additional costs to prepare a lot for screening after rejection. The cost of moving inspection equipment to the sampling or screening site should be included here. Includes time required to review drawings and specifications prior to sampling or screening.

<u>HANDLING(H)</u> - involves the cost of moving lots to the inspection or screening site, transporting accepted lots which have been judged defective downstream, and moving rejected lots to a screening area or some other area of the plant to await disposition. Include storage costs whenever applicable.

<u>PAPERWORK(P)</u> - associated with routine, non-administrative tasks involving written reports or the completion of forms. Examples include recording results of inspection and testing, writing rejection tags or special treatment tages for non-conforming lots, and time to enter information at a computer terminal as part of a data base.

<u>ADMINISTRATIVE(A)</u> - involves activities performed by managerial and supervisory personnel such as the cost of the time required to decide on the disposition of a non-conforming lot which has been detected downstream, or the cost of the quality control supervisor's validation, or the cost of time spent to appease or negotiate with buyers because of downstream defective lots. Other examples include the cost of planning programs to update lot history information, the cost of time required to complete corrective action write-ups, and the cost of dealing with vendors concerning quality problems in rejected lots.

LIABILITY(L) - exclusively used with $A_0$, this cost includes monetary concessions made to buyers, legal fees, court awards, liability insurance premiums and the loss of existing and potential customers due to downstream quality problems.

RECALL/RE-INSPECTION(M) - exclusively used with $A_0$, this is the whole lot cost of recall or re-inspection of downstream lots.

## Unit Cost Components

Attention here is directed to costs associated with individual items in the sample and in the unsampled portion of the lot for both accepted and rejected lots. Each of the components described below will be used as part of one or more of the MGJ unit costs--$S_1$, $S_2$, $A_1$, $A_2$, $R_1$, and $R_2$.

VALUE ADDED(V) - the purchase price of an item from a vendor and/or the cost of prior inspections, raw materials, subassemblies, direct labor, direct materials, and overhead (on a unit basis) which have been added to each unit until it reaches this sampling stage. At inspection stations within the plant beyond incoming inspection it can be measured as the charging rate used by the previous cost center.

INSPECTION/TEST(I) - labor, consumable testing materials, energy and capital expended during original, in-process, or final inspection are included here. Likewise, these costs when applied to re-inspection or screening are appropriate.

PAPERWORK(P) - associated with the preparation of individual reports concerning defectives.

HANDLING(H) - the handling, packaging, and/or shipping charges per unit when prepared for sale or for subsequent operations.

SALES(S) - the sale or discounted price of an item or the value of an item prior to the next manufacturing operation. Sale is a negative cost.

CREDIT(C) - involves the return credit paid by vendor or other cost center to the company for defective, questionable, or good items. Includes credit awarded from another source for doing own repair. Credit is a negative cost.

AWARD(A) - return credit paid by the company for defective, questionable, or good items as a result of one or more defectives in accepted lots.

REPAIR/REWORK(R) - labor, material, energy, and capital expended on a non-conforming item to restore it to acceptable status or to prepare it for disposition as a discounted item.

REPLACEMENT(N) - the additional cost of replacing defective items with items known to be conforming ($N \geq V+I$).

RETURN(T) - cost incurred when provisions call for an unsatisfactory item to be returned to the vendor. Includes handling, packaging, storage, and shipping costs whenever applicable.

REMOVAL(O) - a scrap cost. The cost of handling items which cannot be sold, other than as scrap. This cost could be negative when money paid to the company for a scrapped unit exceeds the cost of preparing it for disposal.

DAMAGE(D) - weighted average of potential damage to equipment and/or personal injury as a result of a defective unit downstream.

## Cost Diagrams

Before presenting scenarios which illustrate the use of cost components in forming fixed and unit costs in the MGJ model, cash flow diagrams will be introduced. They are helpful in converting cost components to the total dollar value costs required in the MGJ model.

Each of the nine costs is depicted as an entity from which costs flow. The number of units affected is also included. Sale and Credit values (and sometimes Removal values) are negative costs and flow inward. A representative diagram is shown in Figure 2 which is shown on the next page. The use of a prime indicates a different value for a cost component of the same type. Note that the components V, H, and S are common to $S_1$, $A_1$, and $R_1$. Cost components common to any of the unit costs with the same subscript may be removed without changing the optimal (n,c) value. It is not unusual for the components of A to be present in both S and R. Hence it is often convenient to treat A as zero (after adjusting $S_1$ and $R_1$). More will be mentioned about this situation in Chapter V. In the scenarios which follow, $A_1$ will be adjusted to zero.

## Illustrative Scenarios

The following four scenarios are developed to illustrate the use of cost components to obtain dollar values for each of the nine costs in the MGJ model. They are intended to be representative of actual situations encountered in industrial sampling applications.

### Scenario 1--Incoming Inspection, Purchased Parts

Part A is purchased in lots of 200 at a price of $85 each, F.O.B. vendor. Shipping charges are $600 per lot. The cost of moving a lot

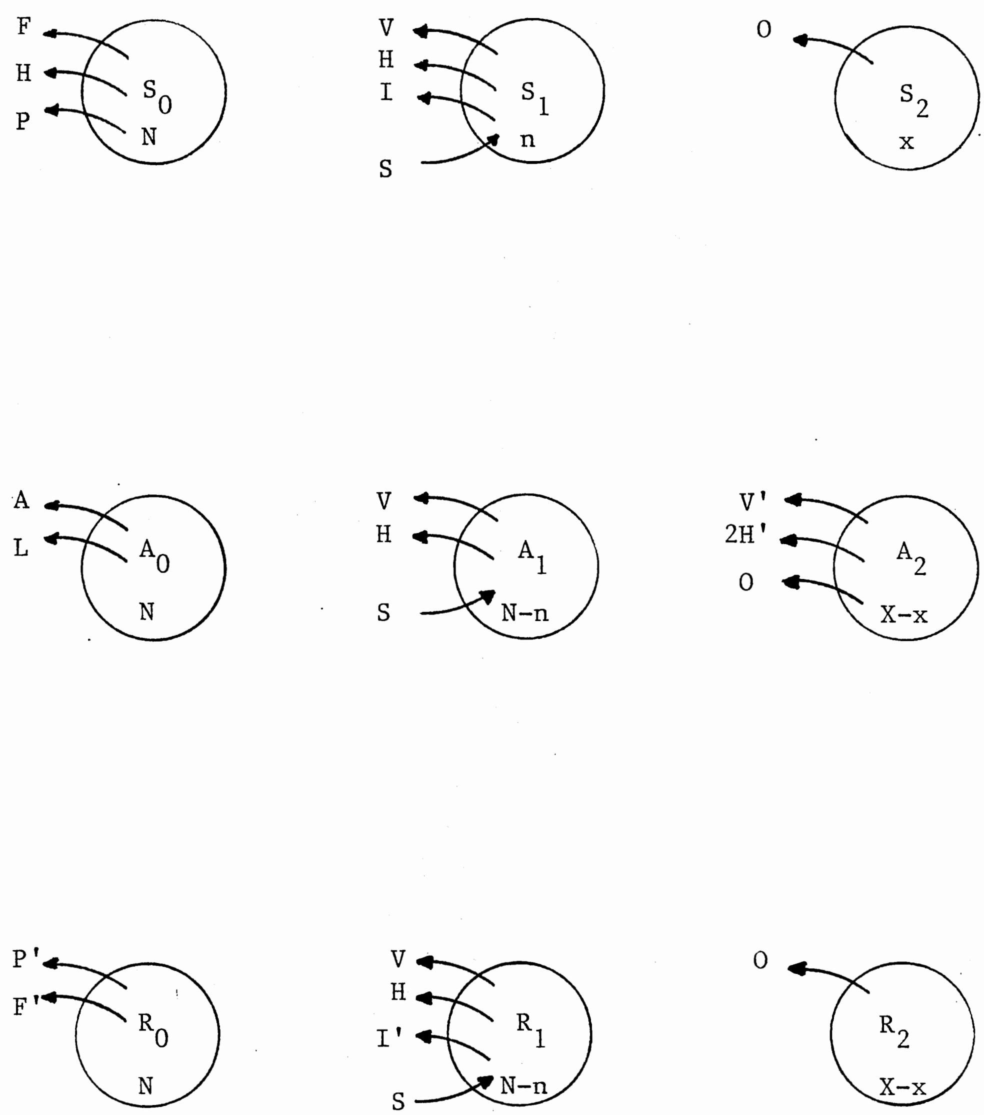


Figure 2. Representative Cost Diagram

from the receiving area to the inspection site is $16. The expense associated with preparing a lot for inspection is $75. The labor and energy cost associated with inspection of each item is $6. Following inspection, the defective items found in the sample are returned to the vendor for a credit of $85. However, the company must pay the handling, packaging, and shipping cost which average $4 for each defective item. The paperwork associated with the sampling process is approximately $30 per lot. Whenever a lot is rejected, it is screened at a cost of $5 per unit. After screening, the results are discussed among a Discrepant Material Committee. The associated cost is $500. Additional preparation charges for screening are negligible as screening is done at the inspection site. The incoming inspection cost center charges the first manufacturing cost center $120 for each Part A. This includes the purchase price, unit shipping and/or handling charges, pro-rated inspection costs and overhead incurred at incoming inspection. Defectives found during screening are returned to the vendor for credit, but the non-defective items are kept. Defectives found during inspection and screening are replenished from a stock of items earlier inspected and kept for relenishment purposes. The replenished items are valued at $105.

Part A items from accepted lots are then subjected to a series of manufacturing operations and subassembly with other parts. There are inspections after each subassembly and sampling is done before the final product is shipped to customers. As the inspections are quite rigorous and the sampling before shipment is generally effective, it is unlikely that a non-conforming product in the hands of a customer will be due to a defective Part A. Thus, nearly all of the defective

Part A's are detected before the final product is shipped. Studies have indicated that subassemblies containing defective Part A's have, on the average, an additional $110 of value added. This is value above and beyond the value of Part A. It includes all value added to the other parts used in subassemblies with Part A. When a subassembly with a defective Part A is discovered, no repairs can be made and the part cannot be returned to the vendor for credit as it has been altered by the manufacturing and assembly operations. Thus, the subassembly is scrapped and must be replaced. Company policy dictates that whenever defective Part A's are found in any subassembly, all subassemblies containing Part A's with the same lot number are segregated and screened at a cost of $50 each. Administrative and paperwork costs associated with this activity are $200 and $100, respectively.

Figure 3 on the next page presents the cost diagrams. Note that the $600 lot shipping charge has been converted to a unit cost ($3) and combined with value added ($85). $S_1$, $A_1$, and $R_1$ have common V and S values and they may be removed so that $A_1$ is set to zero. The following costs would be used as inputs to the MGJ model:

$$S_0 = 121$$
$$A_0 = 10{,}300$$
$$R_0 = 500$$
$$S_1 = 6$$
$$A_1 = 0$$
$$R_1 = 5$$
$$S_2 = 24$$
$$A_2 = 215$$
$$R_2 = 24$$

## Scenario 2--In-Process Inspection

After a welding operation, castings in lots of 50 are sampled and the sample items are subjected to a destructive test to determine the

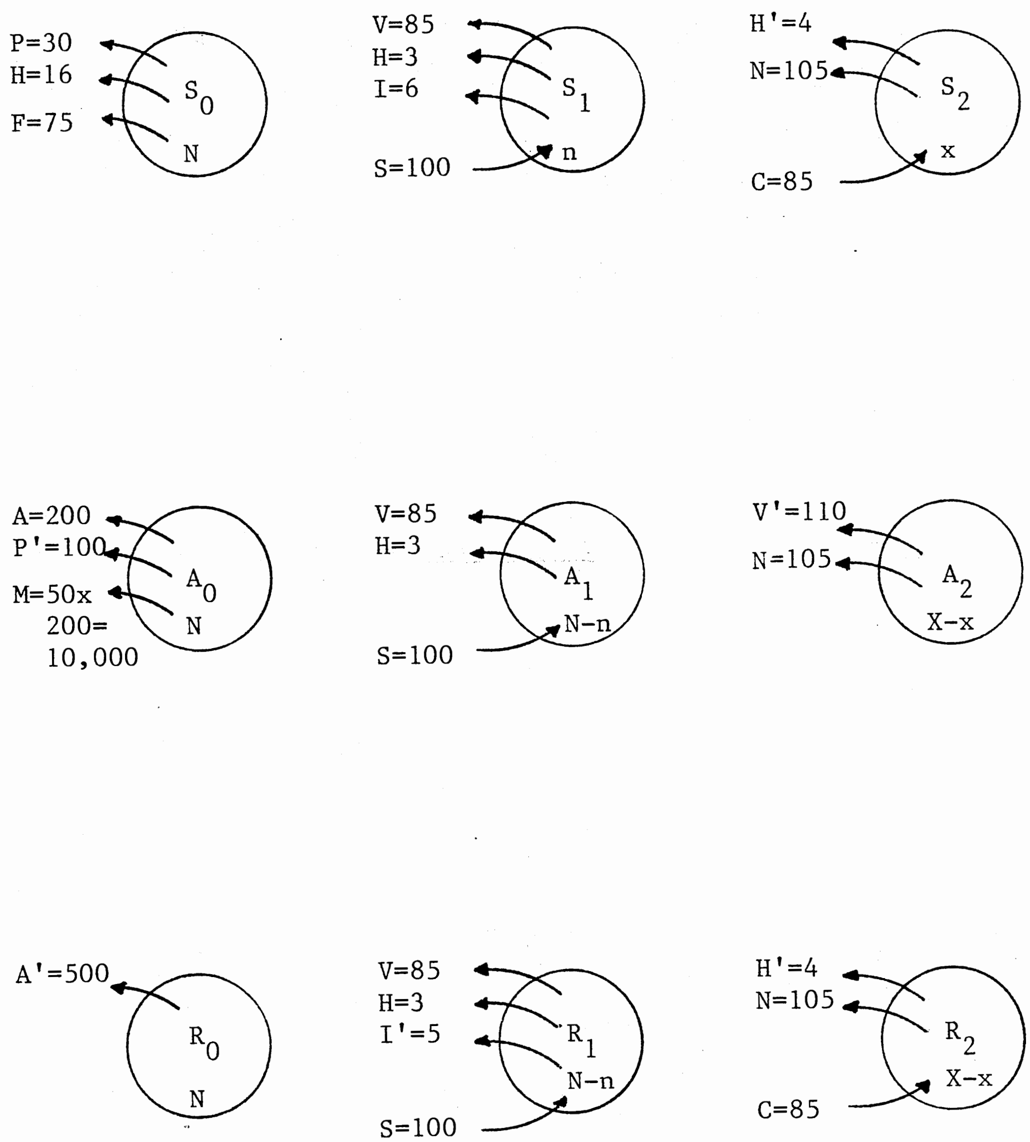


Figure 3. Cost Diagram--Scenario 1

strength of the weld. Handling of the lot prior to testing is estimated to cost \$300. Paperwork associated with sampling is \$25 per lot. The sample items are not replaced, and if a lot is rejected, all of the remaining lot items are tested to failure; i.e., if the sample is bad, the whole lot is destroyed to provide more information. Paperwork associated with rejected lots is an additional \$200. Whenever a lot is rejected, a troubleshooting team is formed and their expenses average \$5000 per rejected lot. Junk dealers purchase the destroyed castings at \$15 each. The value of each part prior to sampling is \$75. After sampling, parts in accepted lots are sold to the next cost center for \$95 each. Set-up for the destructive test is \$100. Labor to perform the test is \$2 per part. There are two manufacturing operations following welding, but it is unlikely that any defective welds will be identified until after sale to customers.

Whenever a weld fails in use, serious damage could result. The manufacturer could be held liable for personal injury and damage to equipment. Although difficult to estimate, a weighted average of \$10,000 per failure will be used. Administrative and liability costs associated with one or more failure in use from the same lot are \$5,000 and \$20,000, respectively.

The cost diagram is presented in Figure 4 on the next page. There are a few interesting anomalies associated with this particular destructive testing situation. Since all items in the sample and all items in the rest of the sample are destroyed, there are no additional costs or revenues associated with defectives in the sample or defectives in the rest of the lot ($S_2 = R_2 = 0$). In most scenarios, revenue (S) produced by sale of the item to the next cost center or to

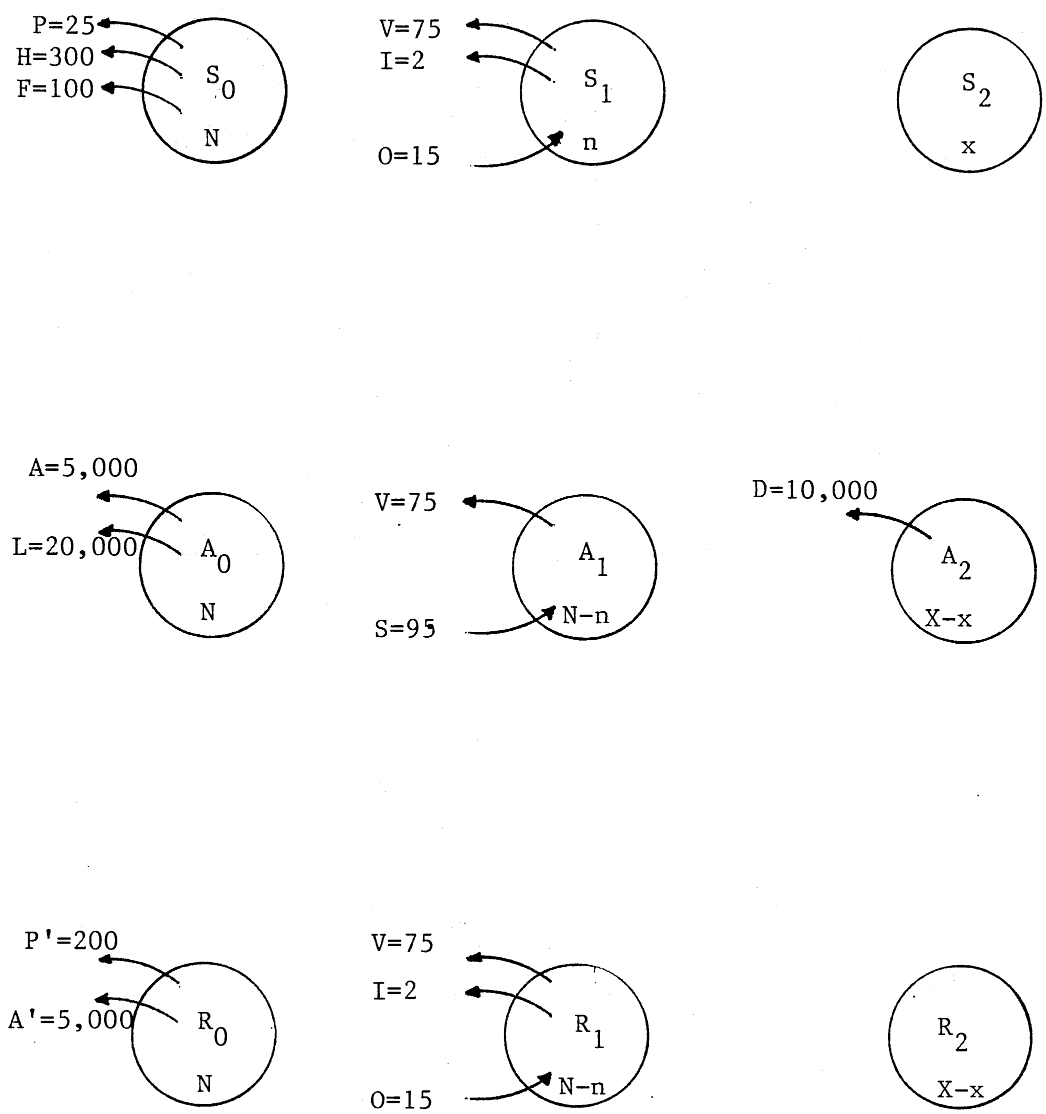


Figure 4. Cost Diagram--Scenario 2

the customer is present in $S_1$, $A_1$, and $R_1$ and hence can be removed. However, in this case, only items in accepted lots are available for sale and S is a part of $A_1$ only. After removing V from $S_1$, $A_1$, and $R_1$, \$95 is added to each of these costs so that $A_1 = 0$. A zero value for $R_2$ will require special treatment with the ratio models discussed in Chapter V, since $R_2$ appears as a denominator in several terms. The following costs would be used as inputs to the MGJ model:

$$
\begin{aligned}
S_0 &= 425 \\
A_0 &= 25{,}000 \\
R_0 &= 5{,}200 \\
S_1 &= 82 \\
A_1 &= 0 \\
R_1 &= 82 \\
S_2 &= 0 \\
A_2 &= 10{,}000 \\
R_2 &= 0
\end{aligned}
$$

<u>Scenarios 3--Final Inspection I</u>

Prior to sale to customers, a manufacturing organization uses sampling to discriminate between good and bad lots. The product is worth \$68 to the company at this point and will be sold for \$99. Handling and set-up prior to sampling are \$120 and \$300, respectively. The labor and energy charges per unit inspected is \$4. Paperwork associated with sampling is \$15. Defectives found during sampling are repaired on site. The cost of repair averages \$15 per unit. Handling and storage of defectives prior to repair is \$3 per unit. Rejected lots are screened for defectives, which are repaired. The screening cost is \$5 per unit. Items found defective in the hands of customers are returned for repair. The company assumes a charge of \$12 per returned unit. There are no fixed administrative, paperwork, or liability costs associated with items returned, but there are paperwork costs of \$6 for each returned item and the company assumes a \$5 "damage to reputation" cost on each returned item.

The cost diagram for this scenario is shown in Figure 5 on the next page. The analysis is rather straightforward. The \$12 return cost includes, in addition to shipping, the \$3 handling and storage cost incurred prior to repair. Dollar value inputs (after $A_1$ if converted to 0) are as follows:

$$
\begin{aligned}
S_0 &= 435 \\
A_0 &= 0 \\
R_0 &= 0 \\
S_1 &= 4 \\
A_1 &= 0 \\
R_1 &= 5 \\
S_2 &= 18 \\
A_2 &= 38 \\
R_2 &= 18
\end{aligned}
$$

Scenario 4--Final Inspection II

A few changes will be made to Scenario 3 which will result in a decidedly different cash flow pattern. Assume now that the remaining items in rejected lots are not screened, but are sold at a discount price of \$70. The company spends \$200 per rejected lot promoting the sale of discounted items. All other parts of Scenario 3 remain unchanged. Figure 6 on the following page presents the cost diagram. Inputs to the MGJ model for this scenario (once again, $A_1$ is converted to 0) would be:

$$
\begin{aligned}
S_0 &= 435 \\
A_0 &= 0 \\
R_0 &= 200 \\
S_1 &= 4 \\
A_1 &= 0 \\
R_1 &= 29 \\
S_2 &= 18 \\
A_2 &= 38 \\
R_2 &= 0
\end{aligned}
$$

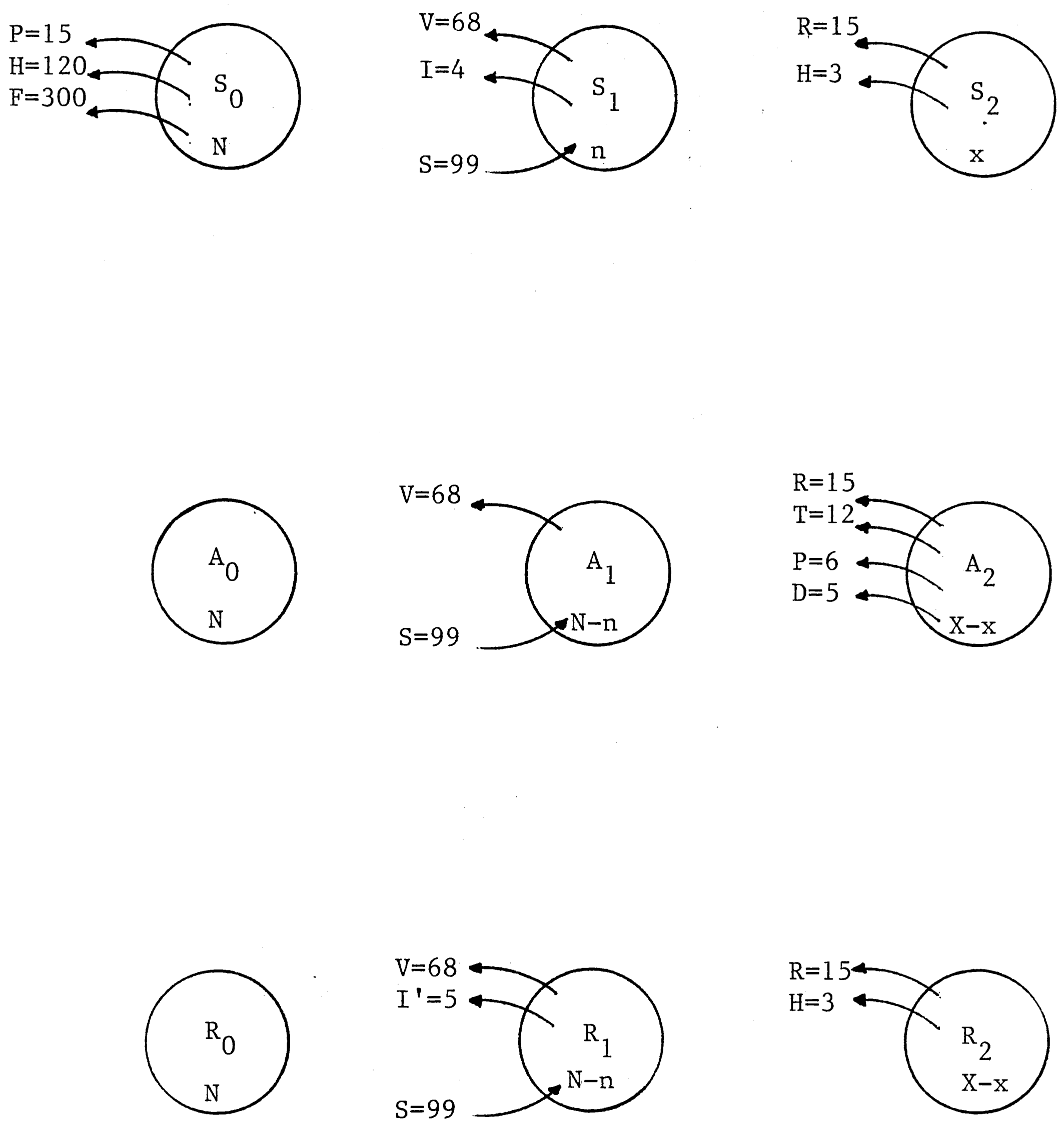


Figure 5. Cost Diagram--Scenario 3

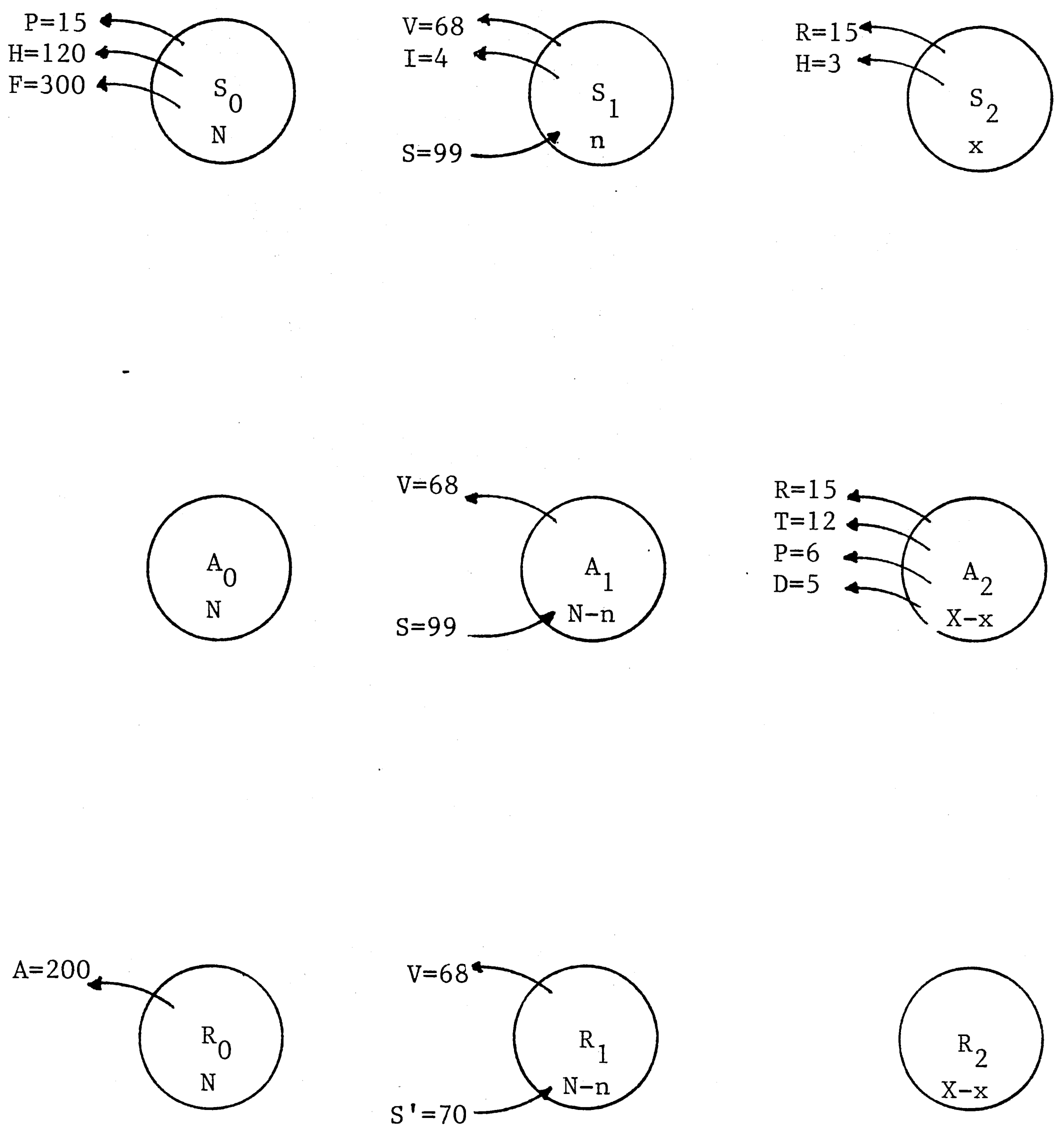


Figure 6. Cost Diagram--Scenario 4

The primary difference between the inputs of Scenarios 3 and 4 is that in Scenario 3, $S_1 \approx R_1$ and $S_2 = R_2$, and this is not the case for Scenario 4. Two fundamental assumptions used in the Two, Three, and Four-Ratio Schemes developed in Chapter V is that $S_1 \approx R_1$ and $S_2 \approx R_2$. Since this is not true for Scenario 4, these ratio approaches would not be valid. However, it can be assumed (with justification) that a vast majority of sampling situations will meet these assumptions and thus can be modeled under the Two, Three, and Four-Ratio Schemes.

## Summary

Fixed and unit cost components have been introduced. Clear explanations and examples of applications of each cost component have been provided. The cost components are the building blocks for the nine cost parameters used as inputs to the MGJ model and may be applied to virtually any sampling situation. Representative scenarios for incoming, in-process, and final inspection were developed to illustrate use of the cost components in forming cost parameters.

Communications among users of economically-based acceptance sampling plans should be improved as a result of agreements concerning the constituency of each cost component and knowledge of how particular cost components are used to build the cost parameters of the model.

Cost ratios, not dollar values of cost, are the focal point of this study. Nevertheless, knowledge of the make-up of each cost parameter will aid in the formulation of realistic ratios and realistic ratios will generate sampling plans in close agreement with those which would result if all cost parameters were known.

CHAPTER IV

# MATHEMATICAL DEVELOPMENTS

## Prior Distributions

### The Mixed Polya Distribution

The prior distribution chosen for all modeling in the present study is the mixed Polya. The Polya family of prior distributions has been used to describe lot quality in numerous situations of theoretical and practical interest. The Polya mass function may take on a wide variety of shapes to describe past data. The mixed Polya allows for distinctly different lots resulting from the use of different machines, operators, vendors, etc. The mixed Polya prior distribution of defectives in the lot is given by

$$f_N(X) = \sum_{i=1}^{k} w_i \binom{N}{X} \frac{\Gamma(s_i+t_i)}{\Gamma(s_i)\Gamma(t_i)} \frac{\Gamma(X+s_i)\Gamma(N-X+t_i)}{\Gamma(N+s_i+t_i)} , \quad X = 0, 1, \ldots, N \tag{4.1}$$

where $w_i$ is the weighting factor for the ith source, $\sum_{i=1}^{k} w_i = 1$ and $s_i$ and $t_i$ are the shape parameters for the Polya distribution associated with the ith source. Owing to reproducibility under hypergeometric sampling, the marginal distribution of defectives in the sample is

$$g_n(x) = \sum_{i=1}^{k} w_i \binom{n}{x} \frac{\Gamma(s_i+t_i)}{\Gamma(s_i)\Gamma(t_i)} \frac{\Gamma(x+s_i)\Gamma(n-x+t_i)}{\Gamma(n+s_i+t_i)} , \quad x = 0, 1, \ldots, n \tag{4.2}$$

The following relationship may be used to obtain the expression for $h_N(X=x|x)$, the posterior distribution describing the probability of having X defectives in the lot (of size N) given that x defectives were observed in the sample (of size n):

$$J(X=x,x) = f_N(X=x)\,l_n(x|X=x) = g_N(x)\,h_N(X=x|x) \tag{4.3}$$

where $J(X=x,x)$ is the joint probability that the number of defectives in the lot and the sample are equal, and $l_n(x|X=x)$ is the hypergeometric probability that all X of the lot defectives appear in the sample. Solving (4.3) for $h_N(X=x|x)$ using equations (4.1) and (4.2) and letting X=x yields

$$h_N(X=x|x) = \frac{\sum_{i=1}^{k} w_i \frac{\Gamma(s_i+t_i)}{\Gamma(s_i)\Gamma(t_i)} \frac{\Gamma(x+s_i)\Gamma(N-x+t_i)}{\Gamma(N+s_i+t_i)}}{\sum_{i=1}^{k} w_i \frac{\Gamma(s_i+t_i)}{\Gamma(s_i)\Gamma(t_i)} \frac{\Gamma(x+s_i)\Gamma(n-x+t_i)}{\Gamma(n+s_i+t_i)}} \tag{4.4}$$

The conditional expectation of defectives in the lot is found by substituting equation (4.2) in equation (2.5). Thus,

$$E(X|x) = \frac{(N-n)\sum_{i=1}^{k} w_i \frac{\Gamma(s_i+t_i)\Gamma(x+1+s_i)\Gamma(n-x+t_i)}{\Gamma(s_i)\Gamma(t_i)\Gamma(n+1+s_i+t_i)}}{\sum_{i=1}^{k} w_i \frac{\Gamma(s_i+t_i)\Gamma(x+s_i)\Gamma(n-x+t_i)}{\Gamma(s_i)\Gamma(t_i)\Gamma(n+s_i+t_i)}} + x \tag{4.5}$$

Equations (4.2), (4.4), and (4.5) are used with the total cost expression, (2.6), and (4.4) and (4.5) are also used in the break point inequality, (2.4).

## The Mixed Binomial Distribution

The mixed binomial distribution has been a frequently chosen prior in earlier modeling efforts of the Guthrie-Johns type. Reasons for choosing this distribution include its mathematical tractability and appropriateness for use with industrial data. Since all modeling efforts were to be performed with the mixed Polya, the mixed binomial parameters were converted to mixed Polya parameters. The mixed binomial prior can be written as

$$f_N(X) = \sum_{i=1}^{k} w_i \binom{N}{X} p_i^{X} (1-p_i)^{N-X}, \quad X = 0, 1, \ldots, N \tag{4.6}$$

where $w_i$ is the weighting for the ith source, $\sum_{i=1}^{K} w_i = 1$, and $p_i$, $0 \leq p_i \leq 1$, is the fraction defective from the ith source. Note that $\bar{p} = \sum_{i=1}^{K} w_i p_i$.

Hald [20] and others have shown that the limiting form of the Polya distribution as s and t approach infinity is the binomial distribution. It remained to discover just how large the s and t values should be for practical use in a computer program which will accept the p and w values as inputs and convert each p value to corresponding Polya s and t parameters. A program, listed in Appendix A, received as inputs, N, n, X, x, and p and then computed $f_N(X)$ and $h_N(X|x)$ for the binomial. These results were compared with the Polya $f_N(X)$ and $h_N(X|x)$ values for a set of s+t values. For a chosen s+t value (large), s and t were computed using $s = p(s+t)$ and $t = (s+t) - s$ since $\hat{p} = s/(s+t)$. After testing numerous and varied N, n, X, x, and p combinations, the best s+t value appeared to be approximately $6 \times 10^8$. This resulted in differences between binomial and Polya values of $f_N(X)$ and $h_N(X|x)$ which were smaller than $1 \times 10^{-6}$. It is of interest to note that s+t values larger than $6 \times 10^8$ resulted in divergence of the $f_N(X)$ and $h_N(X|x)$ values from their binomial counterparts. It would seem that the best s+t value would be the largest value which could be stored in the computer ($\approx 1 \times 10^{75}$ for the IBM 3081D). Numerical methods used in computing log factorials are apparently responsible for the divergence. Whenever mixed binomial inputs ($w_i$ and $p_i$) are supplied to the computer optimization programs $s_i+t_i$ is always $6\times10^8$ and $s_i = p_i(s_i+t_i)$ and $t_i = (s_i+t_i) - s_i$.

## The Modified Guthrie-Johns Model

### A New Expression for Total Cost

Examining equation (2.6), one can see from the second term that a summation over x from c+1 to n will involve a large number of calculations for small values of c. These calculations have been the greatest obstacle in the development of a rapid computer solution. One remedy for this problem is to terminate the summation when the contribution to the partial sum becomes negligible. This occurs with small values of $g_n(x)$. $g_n(x)$ will become small when x is large. An arbitrary stopping rule which has been applied in the past is to terminate the summation when $g_n(x)$ becomes smaller than 0.001. However, this time-saving approach nevertheless resulted in Central Processing Unit (CPU) times of 20-25 seconds on the IBM 3081D.

A 75-80 percent reduction in CPU time has been effected by re-writing the total cost expression (2.6) so that a maximum of c+1 additions are involved in any term which contains additions. The developments are detailed below. Equation (2.6) is written in a different form:

$$TC(N,n,c) = S_0 + nS_1 + \sum_{x=0}^{c} (N-n)A_1 g_n(x) + \sum_{x=c+1}^{n} (N-n)R_1 g_n(x) \qquad \text{(part 1)}$$

$$+ \sum_{x=0}^{c} A_2 E(X|x) g_n(x) + \sum_{x=c+1}^{n} R_2 E(X|x) g_n(x) \qquad \text{(part 2)}$$

$$+ \sum_{x=0}^{c} x(S_2-A_2)g_n(x) + \sum_{x=c+1}^{n} x(S_2-R_2)g_n(x) \qquad \text{(part 3)}$$

$$+ \sum_{x=0}^{c} A_0(1-h_N(X=x|x))g_n(x) + \sum_{x=c+1}^{n} R_0 h_n(x) \qquad \text{(part 4)}$$

(4.7)

In part 1, $\sum_{x=0}^{c} g_n(x)$ is defined as $G_n(c)$. Thus $\sum_{x=c+1}^{n} g_n(x) = 1-G_n(c)$. Making these substitutions and combining terms results in

$$S_0 + nS_1 + (N-n)[R_1+G_n(c)(A_1-R_1)] \qquad (4.7.1)$$

In simplifying part 2, a partial expected value is introduced. $E_{p,f}(v)$ is the sum of the first t terms in the expression for the expected value of random variable v. The "p" denotes a partial expected value. For example, $E_{p,c}(x) = \sum_{x=0}^{c} x \cdot g_n(x)$ and $E_{p,c+1}(x+1) = \sum_{x+1=1}^{c+1} (x+1)g_{n+1}(x+1)$. Making these substitutions and using equation (2.5) for $E(X|x)$, and after some lengthy algebraic manipulations, we have

$$R_2 \left[\frac{N-n}{n+1} E(x+1) + E(x)\right] + (A_2-R_2) \left[\frac{N-n}{n+1} E_{p,c+1}(x+1) + E_{p,c}(x)\right] \qquad (4.7.2)$$

Part 3 is easily simplified using the partial expected value notation. The result is

$$E(x)(S_2-R_2) + E_{p,c}(x)(R_2-A_2) \qquad (4.7.3)$$

In part 4, the $G_n(c)$ substitution plays a major role in the simplification resulting in

$$R_0 + (A_0-R_0)G_n(c) - A_0H_NG_n(c) \qquad (4.7.4)$$

where $H_NG_n(c) = \sum_{x=0}^{c} h_N(X=x|x) \cdot g_n(x)$.

Combining (4.7.1) through (4.7.4) and simplifying results in the new formulation of equation (2.6). It is given by

$$TC(N,n,c) = S_0 + R_0(1-G_n(c)) + A_0(G_n(c) - H_N G_n(c)) + n(S_1+\bar{p}S_2) + (N-n)[R_1+\bar{p}R_2+E_{p,c+1}(x+1)(A_2-R_2)/(n+1) + G_n(c)(A_1-R_1)] \tag{4.8}$$

<u>No Sampling and 100 Percent Inspection</u>

Viable alternatives to taking random samples and inspecting each item in the sample are: (1) avoid sampling and (2) inspect every item in the lot. These alternatives which are called no sampling and 100 percent inspection here must be considered in every economically-based sampling scheme. Total cost expressions for each are now developed. For the no sampling case, consider equation (4.8) with n=0 and c=0:

$$TC(N,0,0) = S_0 + R_0(1-G_0(0)) + A_0(G_0(0) - H_N G_0(0)) + N[R_1+\bar{p}R_2+E_{p,c+1}(x+1)(A_2-R_2) + G_0(0)(A_1-R_1)] \tag{4.9}$$

It is easily seen that $G_0(0) = g_0(0) = 1$, $E_{p,c+1}(x+1) = g_1(1) = \bar{p}$. If no sampling takes place, $S_0 = 0$ and all lots are accepted, i.e., none are rejected. Hence $R_0 = R_1 = R_2 = 0$. Making these substitutions in (4.9) results in

$$TC(N,0,0) = A_0(1-H_N G_0(0)) + NA_1 + N\bar{p}A_2. \tag{4.10}$$

To develop an expression for 100 percent inspection, we begin again with equation (4.8) using n=N and c=0.

$$TC(N,N,0) = S_0 + R_0(1-G_N(0)) + A_0(G_N(0) - H_N G_0(0)) + N(S_1+\bar{p}S_2) \tag{4.11}$$

Since $A_0 = 0$ for 100 percent inspection, (4.11) becomes

$$TC(N,N,0) = S_0 + R_0(1-G_N(0)) + N(S_1+\bar{p}S_2) \tag{4.12}$$

## Summary

The mixed Polya and binomial priors have been introduced. A method for allowing the mixed Polya to approximate a mixed binomial has been developed. This paper introduces a new expression for total cost in the MGJ model which will drastically reduce the computer-based computations and hence reduce the run time to obtain optimal sampling plans. Expressions for no sampling and for 100 percent inspection are given. These alternatives must be considered in every economically-based sampling scheme.

## CHAPTER V

## MODELING AND OPTIMIZATION WITH RATIOS

### A Six-Ratio Scheme

In the process of experimenting with the total cost equation, it was discovered that dividing both sides of equation (4.8) by a non-negative constant did not affect the optimal (n,c) pair. This property may be verified by dividing equation (2.3) by k and noting the $x \leq c$ and $x > c$ portions are changed by the same amount. The cost associated with each defective in a rejected lot, $R_2$, was chosen as the divisor used to form cost ratios as it was thought that expressing other costs as multiples of R would not be extremely difficult. Thus,

$$\frac{TC(N,n,c)}{R_2} = \frac{S_0}{R_2} + \frac{R_0}{R_2}(1-G_n(c)) + \frac{A_0}{R_2}(G_n(c)-H_N G_n(c)) + n\left(\frac{S_1}{R_2} + \bar{p}\,\frac{S_2}{R_2}\right) + (N-n)\left[\frac{R_1}{R_2} + \bar{p} + \frac{E_{p,c+1}(x+1)}{n+1}\left(\frac{A_2}{R_2} - 1\right) + G_n(c)\left(\frac{A_1}{R_2} - \frac{R_1}{R_2}\right)\right] \tag{5.1}$$

There are eight ratios in equation (5.1). However, it will suffice to use six. $S_0/R_2$ is a constant term and unless its value is extremely large, it will not affect the optimal (n,c) pair. Thus, it may be removed. $A_1$ may be removed by combining its additive inverse with $S_1$, $A_1$, and $R_1$. It has been shown that the optimization process is not affected by the addition of a constant to $S_1$, $A_1$, and $R_1$ or the addition of a constant to $S_2$, $A_2$, and $R_2$, or the simultaneous addition of constants to each set of three unit costs. For example, if a constant, k, is added to each of the costs of equation (2.3) having a "1" subscript, the same quantity, Nk, is added to both the $x \leq c$ and $x > c$ portions. Treating $A_1$ as zero, $A_1/R_2$ may be removed. Only one $(A_2/R_2)$ of the six remaining ratios was used as input to the ratio model. The other inputs, chosen on the basis of necessity and practicality were: $A_0/R_0$, $R_0/R_1$, $R_2/R_1$, $R_1/S_1$, and $R_2/S_2$. The following relationships were used to obtain the ratios needed in equation (5.1):

$$S_1/R_1 = (R_1/S_1)^{-1}$$
$$R_1/R_2 = (R_2/R_1)^{-1}$$
$$S_2/R_2 = (R_2/S_2)^{-1}$$
$$R_0/R_2 = R_0/R_1 \cdot R_1/R_2$$
$$A_0/R_2 = A_0/R_0 \cdot R_0/R_1 \cdot R_1/R_2$$
$$S_1/R_2 = S_1/R_1 \cdot R_1/R_2$$

The no sampling and 100 percent inspection costs used in ratio modeling were developed from equations (4.10) and (4.12) they are:

$$TC(N,0,0) = A_0/R_2(1-H_N G_0(0)) + N\bar{p}(A_2/R_2) \tag{5.2}$$

$$TC(N,N,0) = R_0/R_2(1-G_N(0)) + N(S_1/R_2) + N\bar{p}(S_2/R_2) \tag{5.3}$$

Note that the constant, $S_0$, has been removed from (4.12). The break point inequality (equation (2.4)) was changed to

$$x(1-A_2/R_2) - (N-n)(R_1/R_2) + (A_2/R_2-1)E(X|x) - R_0/R_2 + A_0/R_2(1-h_N(X=x|x)) \leq 0 \tag{5.4}$$

When the nine cost values used in the MGJ model are converted to the six ratios and used in the ratio model, the optimal (n,c) pair is identical to that of the MGJ. Without knowledge of dollar values of the nine costs, the user must be provided with a range of values for each ratio. It was decided to use geometric progressions above zero with a multiplier of two. Zero ratios were added to values of $A_0/R_0$ and $R_0/R_1$. Table II presents these ranges.

TABLE II

VALUES USED IN THE SIX-RATIO SCHEME

| Ratio | Values | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $A_0/R_0$ | 0 | - | - | - | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| $R_0/R_1$ | 0 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| $R_1/S_1$ | - | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| $R_2/S_2$ | - | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| $R_2/R_1$ | - | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| $A_2/R_2$ | - | - | - | - | 1 | 2 | 4 | 8 | 16 | 32 | 64 |

The ratio computer program accepted as inputs one value from each row of Table II. Tests of the efficacy of the ratios were performed as follows: Scenarios depicting in-process, incoming, and final inspection were used to develop dollar values for inputs to the MGJ model. An optimal plan $(n_t^*, c_t^*)$ was determined for each scenario. Likewise, an optimal ratio plan $(n_r^*, c_r^*)$ was found using the ratio program to approximate the corresponding complete dollar value scenario. $TC(n_t^*, c_t^*)$ represented the total cost when the $(n_t^*, c_t^*)$ pair was substituted in equation (4.8) and $TC(n_r^*, c_r^*)$ represented the total cost when the $(n_r^*, c_r^*)$ pair was substituted in the same equation. The performance measure employed is the fractional increase in cost incurred by the use of ratios in lieu of actual dollar values. It is given by

$$\delta = \frac{TC(n_r^*, c_r^*) - TC(n_t^*, c_t^*)}{TC(n_t^*, c_t^*)} \tag{5.5}$$

The measure $\delta$ reflects the ratio model's ability to design good sampling plans, even when the cost parameters have been varied. From this measure, it was possible to determine which cost ratios are critical in the sense that $\delta$ may be increased drastically by minor shifts in the selection of a ratio.

Experimentation using the ratio model program revealed that whether ratios were chosen to be as close as possible to the "true" ratios used in the exact cost model, i.e., the proper ratios were chosen, a small value of $\delta$ resulted. The $A_0/R_0$ and $R_0/R_1$ ratios, unless extremely large, could be changed dramatically (holding other ratios constant) without more than a minimal change in $\delta$. When these ratios were removed (treated as zero in the ratio model), the optimal ratio plan either changed very little or did not change at all. When attention was directed to the other ratios it was discovered that, for many scenarios, a ratio could be varied as many as three or four positions in one direction and one or two positions in the other direction (from the proper position) without a large change in $\delta$ (the other three ratios were held constant). For certain scenarios, with three ratios held constant, a movement one ratio value away in the wrong direction from the proper position would result in a very high $\delta$ value. Typically, high $\delta$ values are a result of the ratio model specifying zero or 100 percent inspection when, in fact, a sampling plan (n unequal to 0 or N) is indicated by the MGJ optimization.

It is not unrealistic to expect that from time to time two or more incorrect choices would be made in selecting values for each of the four ratios. An attempt to investigate this situation was made by allowing two or more ratios to vary simultaneously away from the proper position. No generalizations could be made as a result of these efforts. With four ratios changing at the same time, there are too many possible interactions among costs to make predictions concerning the outcome resulting from a particular combination of choices. For this reason and the reason that six ratios are too many to realistically employ, it was decided to abandon the use of a six-ratio (or four, if fixed costs can be treated as zero) model and direct attention to the use of two, three, and four-ratio models.

## Two, Three, and Four-Ratio Schemes

### Variable Cost Assumptions

The variable cost assumptions are based upon what is believed to be prevalent in actual use and upon practical modeling considerations. Each of the three assumptions which follow will hold throughout all subsequent developments in this chapter. (1) $S_1 \approx R_1$; this assumption is realistic as one often finds the cost of sampling, inspecting/testing at about the same level as screening or making some decision about unsampled items in rejected lots, (2) $S_2 \approx R_2$; these costs are expected to be quite similar in that they both involve unit costs associated with defective items, and (3) $A_1 = 0$; if $A_1 \neq 0$, it may be adjusted to zero by adding a constant $(-A_1)$ to $S_1$ and $R_1$.

Fixed Cost Assumptions

Unlike the variable cost assumptions, which hold simultaneously, and are in effect for all cases, the fixed cost assumptions are mutually exclusive and each will hold only for a specific case. These assumptions are the result of experimentation with a large number of cost schedules. This experimentation is discussed later in the chapter. (1) The base case assumes that $S_0 = A_0 = R_0 = 0$. In practice, each is usually non-zero. However, experimentation has shown that whenever $S_0/S_1$, $A_0/S_1$, and $R_0/S_1$ are less than 500, they may be treated as zero for modeling purposes. (2) $S_0/S_1 = 1{,}000$ and other two fixed costs are zero. (3) $S_0/S_1 = 10{,}000$ and the other two fixed costs are zero. (4) $A_0/S_1 = 1{,}000$ and the other two fixed costs are zero. (5) $A_0/S_1 = 10{,}000$ and the other two fixed costs are zero. (6) $A_0/S_1 = 1{,}000$, $R_0/S_1 = 100$, and $S_0 = 0$. In practice, a user would select a fixed cost ratio of 1,000 if the ratio is believed to exceed 500 but not exceed 5,000. If the ratio is greater than 5,000 then 10,000 would be used. For case (6), $R_0/S_1$ should be between 50 and 500.

These six assumptions, along with the variable cost assumptions, which always hold, determine six conditions available for user selection.

Cost Equations

Using the variable cost assumptions and dividing both sides of equation (4.8) by $S_1 = R_1$, a new total cost-ratio equation is obtained.

$$TC(N,n,c)/S_1 = S_0/S_1 + R_0/S_1(1-G_n(c)) + A_0/S_1(G_n(c) - H_N G_n(c)) + n(1+\bar{p}R_2/R_1) + (N-n)[1+\bar{p}R_2/R_1+E_{p,c+1}(x+1)/(n+1) \cdot (A_2/R_1-R_2/R_1) - G_n(c)] \quad (5.6)$$

Using the same assumptions and dividing (4.10) and (4.12) by $S_1$, the no sampling and 100 percent inspection total cost-ratio equations are given by

$$TC(N,0,0)/S_1 = A_0/S_1(1-H_0(0)) + N\bar{p}A_2/R_1 \quad (5.7)$$

$$TC(N,N,0)/S_1 = S_0/S_1 + R_0/S_1(1-G_N(0)) + N(1+\bar{p}R_2/R_1) \quad (5.8)$$

In the same manner, the break-point inequality (equation 2.4) becomes

$$A_0/A_1(1-h_N(X=x|x)) - R_0/S_1 + (E(X|x)-x)(A_2/R_1 - R_2/R_1) + n-N \leq 0 \quad (5.9)$$

The optimization process now involves five ratios--$S_0/S_1$, $A_0/S_1$, $R_0/S_1$, $A_2/R_2$, and $R_2/R_1$. However, it is seen that under fixed cost assumption (1) only two ratios are needed and under (2), (3), (4), and (5), three ratios are needed. Fixed cost assumption (6) required four ratios. Note that $A_2/R_2$ can be obtained from the product of $A_2/R_2$ and $R_2/R_1$. As it is much more convenient for users to supply $A_2/R_2$, it will be used as input in place of $A_2/R_1$.

Experimentation

The experimentation which led to the development of six conditions (corresponding to the six fixed cost assumptions) from which the user can select the one appropriate to any particular sampling scenario is now outlined.

Three prior distributions were used in the analysis. Each is a mixed binomial. Prior 1 used $p_1$ = .02, $p_2$ = .10, and $p_3$ = .30 with $w_1$ = .60, $w_2$ = .25, and $w_3$ = .15. Prior 2 used $p_1$ = .01 and $p_2$ = .30 with $w_1$ = .70 and $w_2$ = .30. Prior 3 used $p_1$ = .07 and $p_2$ = .13 with $w_1$ = .60 and $w_2$ = .40. These priors were combined with 28 cost schedules. For most schedules, only one or two priors were applied. The lot size was 1,000 for all cases. The cost schedules are given in Table III on the next pages. The approach in identifying meaningful cost ratios is based on the development of several 7x10 matrices for patterns of $A_2/R_2$ and $R_2/R_1$. Only one of the matrices is appropriate for a partciular cost scenario. The $A_2/R_2$ and $R_2/R_1$ values used were the same as those of the six-ratio scheme (Table II). The pairing of a prior and a cost schedule yielded an optimal sampling plan when applied to the computer program, OPTI.FORT, given in Appendix B. The n*, c*, and total cost values for OPTI.FORT were used as inputs to the computer program, LANIF.FORT (listed in Appendix C) which generated the matrices. The base case assumption for fixed cost (assumption (1)) was used first with each of the 28 cost schedules. OPTI.FORT then performed 70 Optimizations. LANIF.FORT did the same, yielding a plan and an associated ratio-based total cost for each of 70 $A_2/R_2$ and $R_2/R_1$ conbinations. These total costs were compared with corresponding dollar-value total costs (from OPTI.FORT) using the measure $\delta$ of equation (5.5). Table IV on the following page presents the matrix developed for cost schedule L of Table III using Prior 2. Table IV reveals that a user whose costs are those of Schedule L, who is unaware of the dollar values, but correctly estimates the $A_2/R_2$ and $R_2/R_1$ values to be 4, will use the plan n=28 and c=2 and will be extremely close to the

TABLE III

COST SCHEDULES USED IN THE EXPERIMENTATION

| | Schedule | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
| $S_0$ | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 220 |
| $A_0$ | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 |
| $R_0$ | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 160 | 5000 | 10000 |
| $S_1$ | 1 | 30 | 2 | 35 | 35 | 60 | 100 | 16 | 32 | 32 | 10 | 6 | 6 | 6 |
| $S_2$ | 32 | 45 | 45 | 45 | 45 | 60 | 120 | 32 | 32 | 64 | 55 | 36 | 36 | 36 |
| $A_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $A_2$ | 1024 | 40 | 1200 | 50 | 1200 | 450 | 1024 | 1024 | 1024 | 1024 | 400 | 128 | 128 | 128 |
| $R_1$ | 1 | 30 | 2 | 35 | 35 | 20 | 1 | 16 | 32 | 32 | 10 | 8 | 8 | 8 |
| $R_2$ | 32 | 50 | 60 | 40 | 40 | 60 | 32 | 32 | 32 | 64 | 55 | 32 | 32 | 32 |

TABLE III (Continued)

| | Schedule | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | O | P | Q | R | S | T | U | V | W | X | Y | Z | AA | BB |
| $S_0$ | 220 | 220 | 5000 | 220 | 220 | 2000 | 2000 | 2000 | 7000 | 10000 | 45 | 310 | 45 | 45 |
| $A_0$ | 470 | 5000 | 470 | 10000 | 20000 | 10000 | 10000 | 10000 | 8000 | 3000 | 470 | 2600 | 470 | 470 |
| $R_0$ | 19000 | 160 | 160 | 10000 | 20000 | 3000 | 5000 | 3000 | 6000 | 2000 | 70 | 300 | 70 | 70 |
| $S_1$ | 6 | 6 | 6 | 6 | 6 | 30 | 6 | 3 | 30 | 30 | 3 | 50 | 3 | 3 |
| $S_2$ | 55 | 36 | 36 | 36 | 36 | 60 | 36 | 60 | 60 | 60 | 60 | 100 | 60 | 60 |
| $A_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $A_2$ | 400 | 128 | 128 | 128 | 128 | 450 | 128 | 100 | 450 | 450 | 96 | 420 | 100 | 6 |
| $R_1$ | 10 | 8 | 8 | 8 | 8 | 20 | 8 | 1.5 | 20 | 20 | 1.5 | 20 | 1.5 | 1.5 |
| $R_2$ | 55 | 32 | 32 | 32 | 32 | 60 | 32 | 60 | 60 | 60 | 12 | 100 | 60 | .75 |

# TABLE IV

## COST RATIO DECISION MATRIX--SCHEDULE L AND PRIOR 2

| | | | 0.0 | .18 | .35 | .71 | 1.41 | 2.83 | 5.66 | 11.31 | 22.63 | 45.25 | 90.51 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
| | | | | | | | | | | | | | 0.00 |
| | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 1 | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | |
| | | | | | | | | | | | | | 1.41 |
| | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 19 | 26 | 30 | 33 | 37 | |
| A | 2 | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | |
| | | DELTA | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.0329 | 0.0015 | 0.0000 | 0.0011 | 0.0035 | |
| | | | | | | | | | | | | | 2.83 |
| | | SAMP.SIZE | 0 | 0 | 0 | 0 | 24 | 28 | 32 | 35 | 53 | 1000 | |
| 2 | 4 | ACC. NBR. | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 3 | 0 | |
| | | DELTA | 0.8674 | 0.8674 | 0.8674 | 0.8674 | 0.0049 | 0.0002 | 0.0006 | 0.0022 | 0.0083 | 0.4307 | |
| | | | | | | | | | | | | | 5.66 |
| | | SAMP.SIZE | 0 | 0 | 7 | 25 | 29 | 32 | 36 | 1000 | 1000 | 1000 | |
| R | 8 | ACC. NBR. | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | |
| | | DELTA | 0.8674 | 0.8674 | 0.2865 | 0.0029 | 0.0000 | 0.0006 | 0.0028 | 0.4307 | 0.4307 | 0.4307 | |
| | | | | | | | | | | | | | 11.31 |
| | | SAMP.SIZE | 0 | 17 | 26 | 29 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | |
| 2 | 16 | ACC. NBR. | 0 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.8674 | 0.0600 | 0.0015 | 0.0000 | 0.0011 | 0.0035 | 0.4307 | 0.4307 | 0.4307 | 0.4307 | |
| | | | | | | | | | | | | | 22.63 |
| | | SAMP.SIZE | 18 | 26 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | 32 | ACC. NBR. | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.0447 | 0.0015 | 0.0000 | 0.0011 | 0.0035 | 0.4307 | 0.4307 | 0.4307 | 0.4307 | 0.4307 | |
| | | | | | | | | | | | | | 45.25 |
| | | SAMP.SIZE | 26 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | 64 | ACC. NBR. | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.0015 | 0.0000 | 0.0011 | 0.0035 | 0.4307 | 0.4307 | 0.4307 | 0.4307 | 0.4307 | 0.4307 | |
| | | | | | | | | | | | | | 90.51 |

optimal plan. It is seen from Table IV that minor incorrect estimates of each ratio in the either direction are not critical. Most critical would be underestimating each ratio by one ratio value (i.e., $A_2/R_2 = 2$ and $R_2/R_1 = 2$) which would indicate no sampling is recommended. This would result in an 87 percent over-expenditure. Examination of Schedule V of Table III and Table V appearing on the next page, which is based on Schedule V, reveals that a decision matrix which assumes that each ratio of fixed costs to some variable is small (less than 1,000) is not appropriate for this schedule. It was through examples such as this that it became apparent that additional matrices were necessary to handle situations similar to Schedule V where one or more of the fixed costs is extremely high.

$S_1$ was chosen as a convenient denominator for the three fixed cost ratios. It was felt that users would be able to relate each fixed cost to $S_1$ with little difficulty. Trial runs were made with (1) $S_1/S_1$ assuming values of 10, 100, 1,000, and 10,000 while $A_0/S_1$ and $R_0/S_1$ were held at zero, (2) $A_0/S_1$ having values of 10, 100, 1,000, and 10,000 while $S_0/S_1$ and $R_0/S_1$ were kept at zero, and (3) $A_0/S_1$ assuming values of 10, 100, 1,000, and 10,000 while $R_0/S_1$ assumed values $\leq A_0/S_1$ with $S_0/S_1$ held at zero.

$R_0/S_1$ ratios were not tested alone (with $S_0/S_1$ and $A_0/S_1 = 0$) nor was $S_0/S_1$ tested in combination with $S_0/S_1$, nor were all three fixed cost ratios tested in combinations. These conditions were considered to be impractical.

Situations (1), (2), and (3) above define 18 matrices. Each situation was tested under Prior 1 and under Prior 2 using, at one time

# TABLE V

## COST RATIO DECISION MATRIX--SCHEDULE V AND PRIOR 2

0.0 .18 .35 .71 1.41 2.83 5.66 11.31 22.63 45.25 90.51

| | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 |
| | 1 | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | |
| | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 19 | 26 | 30 | 33 | 37 | 1.41 |
| A | 2 | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | |
| | | DELTA | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.6864 | 0.6668 | 0.6648 | 0.6644 | 0.6643 | |
| | | SAMP.SIZE | 0 | 0 | 0 | 0 | 24 | 28 | 32 | 35 | 53 | 1000 | 2.83 |
| 2 | 4 | ACC. NBR. | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 3 | 0 | |
| | | DELTA | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 0.6692 | 0.6655 | 0.6645 | 0.6643 | 0.6695 | 0.3931 | |
| | | SAMP.SIZE | 0 | 0 | 7 | 25 | 29 | 32 | 36 | 1000 | 1000 | 1000 | 5.66 |
| R | 8 | ACC. NBR. | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | |
| | | DELTA | 0.9859 | 0.9859 | 0.8326 | 0.6678 | 0.6651 | 0.6645 | 0.6643 | 0.3931 | 0.3931 | 0.3931 | |
| | | SAMP.SIZE | 0 | 17 | 26 | 29 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 11.31 |
| 2 | 16 | ACC. NBR. | 0 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.9859 | 0.7024 | 0.6668 | 0.6651 | 0.6644 | 0.6643 | 0.3931 | 0.3931 | 0.3931 | 0.3931 | |
| | | SAMP.SIZE | 18 | 26 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 | 22.63 |
| | 32 | ACC. NBR. | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.6934 | 0.6668 | 0.6648 | 0.6644 | 0.6643 | 0.3931 | 0.3931 | 0.3931 | 0.3931 | 0.3931 | |
| | | SAMP.SIZE | 26 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 45.25 |
| | 64 | ACC. NBR. | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | DELTA | 0.6668 | 0.6648 | 0.6644 | 0.6643 | 0.3931 | 0.3931 | 0.3931 | 0.3931 | 0.3931 | 0.3931 | |
| | | | | | | | | | | | | | 90.51 |

or another Schedules P, Q, R, S, T, U, V, and W. As a result of these experiments, the following generalizations were made:

(a) Matrices for $S_0/S_1 = 10$ and $S_0/S_1 = 100$ were only slightly different from the matrix where $S_0/S_1 = 0$.

(b) Matrices for $A_0/S_1 = 10$ and $A_0/S_1 = 100$ were only slightly different from the matrix where $A_0/S_1 = 0$.

(c) Whenever $A_0/S_1 = R_0/S_1$, the corresponding matrix is identical to the base case ($S_0/S_1 = A_0/S_1 = R_0/S_1 = 0$). It is rather simple to show mathematically that when $A_0$ and $R_0$ start at zero and increase by the same amount with all other costs held constant, the total costs associated with no sampling and with 100 percent inspection increase by that amount ($A_0$ or $R_0$) and the total cost associated with the optimal sampling plan increases by approximately that amount.

(d) With the exception of $A_0/S_1 = 1{,}000$ and simultaneously $R_0/S_1 = 100$, each of the other $A_0/S_1$ and $R_0/S_1$ matrix combinations tested was identical to the $A_0/S_1$ alone matrix (i.e., $R_0/S_1 = 0$).

Conclusions

The generalizations above indicated that an appropriate set of decision matrices would include (1) the zero fixed cost case (for the convenience of the user, it is titled "$S_0/S_1$, $A_0/S_1$, and $R_0/S_1 < 1{,}000$"), (2) $S_0/S_1 = 1{,}000$, (3) $S_0/S_1 = 10{,}000$, (4) $A_0/S_1 = 1{,}000$, (5) $A_0/S_1 = 10{,}000$, and (6) $A_0/S_1 = 1{,}000$ and $R_0/S_1 = 100$. It should now become obvious that for the zero fixed cost case, the user need only estimate $A_2/R_2$ and $R_2/R_1$ (two ratios). The next four sets of decision matrices require estimates of three ratios and the final matrix is associated with four ratios. The user supplies only the parameter estimates of the prior distribution. The program LANIF.FORT generates six decision matrices based upon that prior. The user identifies the one matrix appropriate for his cost situation and then selects the cell associated with the estimated $A_2/R_2$ and $R_2/R_1$ values. Table VI on the next three pages presents the decision matrices associated with Prior 1. Decision matrices associated with Prior 2 and 3 are found in Appendices D and E, respectively. Examining these tables, it becomes clear that the decision processes specified in this paper outline many conditions where either no sampling or 100 percent inspection is recommended. Very few risk-based plans consider these alternatives.

The experimentation with various cost schedules provided some insight as to the extent that the variable cost assumptions may be violated without a large resulting value of $\delta$. Most of the schedules of Table III satisfy $S_1 \approx R_1$ and $S_2 \approx R_2$. Notable exceptions are Schedules F, G, Y, Z, and BB. Table VII shows the $\delta$ values for each schedule and associated prior. For Schedule F, $S_1$ is three times $R_1$ and the penalty is a 13 percent extra cost, whereas for Schedule Z and either prior with $S_1$ two and one half times $R_1$, the additional cost is one percent or less. For Schedule BB, $S_2$ is 80 times $R_2$ and yet the ratio plan cost almost matches the dollar value plan. With Schedule G, neither pair of costs has similar dollar values and the result of using a ratio plan is catastrophic.

A valid conclusion for this topic is that it is difficult to predict the effects of severe violations of the variable cost assumptions. However, whenever the violations were small, the plan selected

# TABLE VI

## DECISION MATRICES--PRIOR 1

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1, AO/S1, AND RO/S1 < 1000
COST RATIO DECISION MATRIX

| A2R2 | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| 1 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| 2 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 18 | 35 | 79 | 139 | 1000 | |
| | A | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 4 | 5 | 4 | 6 | 0 | 2.83 |
| 4 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 35 | 45 | 112 | 193 | 1000 | 1000 | |
| | 2 | ACC. NBR. | 0 | 0 | 0 | 0 | 6 | 3 | 5 | 7 | 0 | 0 | 5.66 |
| 8 | | SAMP.SIZE | 0 | 0 | 0 | 33 | 72 | 117 | 1000 | 1000 | 1000 | 1000 | |
| | R | ACC. NBR. | 0 | 0 | 0 | 5 | 4 | 5 | 0 | 0 | 0 | 0 | 11.31 |
| 16 | | SAMP.SIZE | 0 | 0 | 34 | 76 | 137 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | 2 | ACC. NBR. | 0 | 0 | 5 | 4 | 6 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| 32 | | SAMP.SIZE | 0 | 34 | 78 | 138 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 5 | 4 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| 64 | | SAMP.SIZE | 35 | 78 | 139 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 5 | 4 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1 = 1000
COST RATIO DECISION MATRIX

| A2R2 | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| 1 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| 2 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 139 | 1000 | |
| | A | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 2.83 |
| 4 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 112 | 193 | 1000 | 1000 | |
| | 2 | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 7 | 0 | 0 | 5.66 |
| 8 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 117 | 1000 | 1000 | 1000 | 1000 | |
| | R | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 11.31 |
| 16 | | SAMP.SIZE | 0 | 0 | 0 | 0 | 137 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | 2 | ACC. NBR. | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| 32 | | SAMP.SIZE | 0 | 0 | 0 | 138 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| 64 | | SAMP.SIZE | 0 | 0 | 139 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

TABLE VI (Continued)

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1 = 10000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | | 0.0 – .18<br>1/8 | .18 – .35<br>1/4 | .35 – .71<br>1/2 | .71 – 1.41<br>1 | 1.41 – 2.83<br>2 | 2.83 – 5.66<br>4 | 5.66 – 11.31<br>8 | 11.31 – 22.63<br>16 | 22.63 – 45.25<br>32 | 45.25 – 90.51<br>64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | 0.00 |
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| A | 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 2 | 4 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| R | 8 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 2 | 16 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 1000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | | 0.0 – .18<br>1/8 | .18 – .35<br>1/4 | .35 – .71<br>1/2 | .71 – 1.41<br>1 | 1.41 – 2.83<br>2 | 2.83 – 5.66<br>4 | 5.66 – 11.31<br>8 | 11.31 – 22.63<br>16 | 22.63 – 45.25<br>32 | 45.25 – 90.51<br>64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | 0.00 |
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| A | 2 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 2 | 4 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| R | 8 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 2 | 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

# TABLE VI (Continued)

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 10000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| | 1 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| A | 2 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 2 | 4 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| R | 8 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 2 | 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 1000; RO/S1 = 100
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| A | 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 2 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 2 | 4 | SAMP.SIZE | 0 | 9 | 10 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| R | 8 | SAMP.SIZE | 0 | 1 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 2 | 16 | SAMP.SIZE | 1 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

was a good one. The program which generates the decision matrices, in its present form, should not be used when it is known that severe violations of the variable cost assumptions are present. However, it would be an easy task to develop a new program where $S_1 = m \cdot R_1$ and/or $S_2 = n \cdot R_2$ for any values m and n. The resulting decision matrices could be used with confidence for those particular situations.

TABLE VII

SENSITIVITY TO VIOLATIONS OF THE VARIABLE COST ASSUMPTIONS SELECTED CASES

| Schedule | $S_1$ | $R_1$ | $S_2$ | $R_2$ | Prior | $\delta$ |
|---|---|---|---|---|---|---|
| F | 60 | 20 | 60 | 60 | 1 | 0.128 |
| G | 100 | 1 | 120 | 32 | 1 | 5.679 |
| G | 100 | 1 | 120 | 32 | 2 | 7.415 |
| Y | 3 | 1.5 | 60 | 12 | 2 | 0.031 |
| Z | 50 | 20 | 100 | 100 | 1 | 0.017 |
| Z | 50 | 20 | 100 | 100 | 2 | 0.005 |
| BB | 3 | 1.5 | 60 | 0.75 | 1 | 0.002 |

## The $R_2 = 0$ Case

Scenarios 2 and 4 of Chapter III illustrate the possibility of a zero value for $R_2$. The theoretically appropriate decision matrix ratios when this is the case are $A_2/R_2 = \infty$ and $R_2/R_1 = 0$. A procedure has been developed so that this situation may be handled without altering the decision matrix. The user must first estimate $A_2/R_1$. Then the largest value of $A_2/R_2$ and the smallest value of $R_2/R_1$ are chosen such that $A_2/R_2 \cdot R_2/R_1 \approx A_2/R_1$. The rationale for this approach is associated with the fact that a constant, $\Delta$, may be added to the costs $S_2$, $A_2$, and $R_2$ without changing n* and c*. Say, for example, $A_2$ = 10,000, $R_1$ = 80, and $R_2$ = 0. Then

$$\frac{A_2 + \Delta}{R_2 + \Delta} \cdot \frac{R_2 + \Delta}{R_1} = \frac{10{,}000 + \Delta}{0 + \Delta} \cdot \frac{0 + \Delta}{80} = \frac{10{,}000 + \Delta}{80}$$

Note that with the addition of $\Delta$, division by zero is avoided and if $\Delta$ is chosen to be small then $(A_2 + \Delta)/R_1 \approx A_2/R_1$. For this example, since 10,000/80 = 125, $A_2/R_2$ is selected to be 64 and $R_2/R_1$ will be 2. This procedure will be illustrated for use with Scenarios 2 and 4 in the "Examples" section of this chapter.

Computer Programs

OPTI.FORT and LANIF.FORT, listed in Appendices B and C, respectively, were used extensively in the experimentation with cost ratios. Each has been coded so that the program may be run interactively or in the batch mode. Descriptions of the principal variables are included with the listings. Instructions for use are also included. LANIF.FORT generates six decision matrices. Each of the 70 cells of a matrix is the result of an optimization process. Thus, 420 optimizations are performed. Approximately 13 minutes of CPU time on the IBM 3081D are required to generate the six decision matrices in the batch mode. More time is required in the interactive mode. This waiting period would be extremely inconvenient for an interactive user and 112 columns of output are used in the matrix, which is many more columns than are provided at most video display units. For these reasons, the interactive user will not receive matrix outputs. In the interactive mode, the user is required to input estimated $A_2/R_2$, $R_2/R_1$, and appropriate fixed cost ratio(s) (if any). Output consists of a single recommended plan.

Examples

Scenarios 1, 2, 3, and 4 of Chapter III were applied to OPTI.FORT and LANIF.FORT for illustrative purposes. It should be mentioned that, in practice, dollar values of the costs are unknown and thus only LANIF.FORT would be used. By using the dollar values with OPTI.FORT the "best" plan and associated cost is obtained so that $\delta$ may be calculated. Table VIII on the next page presents the costs associated with each scenario and compares the best dollar value plan with the best ratio plan using the measure $\delta$. Prior 1 was used in each case, so that the matrices of Table VI are appropriate.

In obtaining ratios to use with the matrices of Table VI, perfect knowledge of the costs was assumed. For Scenario 1, $A_0/S_1 = 1{,}716$, $A_2/R_2 = 8.96$, and $R_2/R_1 = 4.80$. Thus, the $A_0/S_1 = 1{,}000$ matrix of Table VI was selected and the $A_2/R_2 = 8$ and $R_2/R_1 = 4$ entries were used to obtain the plan $n_r^* = 1{,}000$, $c_r^* = 0$. Scenario 2 has an $R_2$ value of zero. Follwoing the procedure developed earlier, $A_2/R_1 = 122$. Thus, $A_2/R_2 = 64$ and $R_2/R_1 = 2$. None of the fixed cost ratios was near 1,000, so the base case is again appropriate and ratios $A_2/R_2 = 2$ and $R_2/R_1 = 4$ were used. Scenario 4 has a zero $R_2$ value. $A_2/R_1$ is 1.31. The largest and smallest, respectively, values of $A_2/R_2$ and $R_2/R_1$ are (1,1). Fixed costs are not high and the base case matrix indicated the correct choice of "no sampling". In fact, two of the four cases resulted in a choice of the perfect ($\delta = 0$) plan. The other two $\delta$ values are rather high. In over 100 runs during the experimentation phase, none of the $\delta$ values was above 10 percent. Many were zero or near zero. The 13 percent value for Scenario 3 may be regarded as an outlier.

TABLE VIII

COMPARISON OF COST AND RATIO PLANS USING SCENARIOS OF CHAPTER III

| | Scenario | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Cost | | | | |
| $S_0$ | 121 | 425 | 435 | 435 |
| $A_0$ | 10300 | 25000 | 0 | 0 |
| $R_0$ | 500 | 5200 | 0 | 200 |
| $S_1$ | 6 | 82 | 4 | 4 |
| $S_2$ | 24 | 0 | 18 | 18 |
| $A_1$ | 0 | 0 | 0 | 0 |
| $A_2$ | 215 | 10000 | 38 | 38 |
| $R_1$ | 5 | 82 | 5 | 29 |
| $R_2$ | 24 | 0 | 18 | 0 |
| Plan | | | | |
| $n_t$ | 237 | 1000 | 0 | 0 |
| $c_t$ | 0 | 0 | 0 | 0 |
| $n_r$ | 1000 | 1000 | 18 | 0 |
| $c_r$ | 0 | 0 | 4 | 0 |
| $\delta$ | .091 | .000 | .128 | .000 |

## Summary

The use of ratio-based decision matrices for economically-based acceptance sampling is recommended. Ratios can often be estimated when actual costs cannot. A group of quality experts are more likely to agree about a cost ratio than about the costs which form the ratio. In most practical applications, only two or three ratios are involved. The four-ratio case involves the joint selection of two fixed cost ratios to accompany the two variable cost ratios. All assumptions used in the development of the decision matrices are quite realistic.

The plans selected by the cost-ratio decision matrices compared most favorably with those which used nine dollar value costs. In over 100 applications, the error in selecting a ratio-based plan was almost always less than 10 percent (i.e., an over-expenditure of less than 10 percent). In many cases, the error was zero or near zero.

An important feature of the ratio-based decision matrix approach is that "no sampling" and "100 percent inspection" are included as viable alternatives. Conversely, many risk-based plans blindly lead the user into a random sampling situation which can result in unnecessary expenditures.

As a result of the developments detailed in this chapter, there is now an easy to use alternative to risk-based acceptance sampling which is based upon readily obtainable cost-ratios.

## CHAPTER VI

## SUMMARY AND CONCLUSIONS

The principal objective of this research was to remove many of the barriers which have been limiting widespread use of the Guthrie-Johns model. In order to accomplish this objective, the following subobjectives have been achieved:

1. The establishment of clear definitions and elaborations of each of the cost factors in the MGJ model.
2. An exact, iterative search for the optimal (n,c) pair using a mixed-Polya prior and all cost factors of the MGJ model.
3. A thorough sensitivity analysis of the MGJ model to each of the cost parameters, alone and in logical combinations.
4. The development of critical ratios between cost parameters of the MGJ model.
5. A validation of the critical ratios.
6. The development of a flexible, well-documented computer program suited for use in a wide range of acceptance sampling situations.

Based on the results obtained through this research, the following statements may be made:

a. Near-optimal sampling plans may be obtained using easily estimated cost ratios, provided that a few realistic assumptions are met.

b. Using the cost components developed in this paper, the ratio model will accommodate virtually any acceptance sampling scenario.

c. Ease of use has been facilitated with the introduction of decision matrices.

d. "No sampling" and "100 percent inspection" are offered for consideration in the decision matrices as well as the random sampling plans.

e. The computer program allows a choice between interactive and batch modes.

f. Modeling has been achieved through the use of a single prior distribution--the versatile mixed-Polya.

The following suggestions are offered as either topics for future research or as conditions which will encourage government and industry adaptation of this ratio-based economic sampling model:

1. It appears that the MGJ model in its present form cannot handle situations such as the return of good items taken from the sample as a rejected lot. To accommodate this and other similar situations, it may be necessary to differentially treat good items in the sample according to whether or not the lot is accepted or rejected.

2. During the process of searching for local minima between break points, prior research has started the search at a point midway between break points, proceeding left and right until the total cost increased. Recognizing that the locus of points between break points is in asymmetric loop, this research has introduced a quadratic fit to the points in the loop and then

found each "minimum" using the first derivative of the fitted curve and then searched left and right from this "minimum". This procedure has been observed to be slower than the mid-point approach in several applications. However, many of the computer runs using the quadratic fit approach were extremely fast. It would be a simple matter to compare the two procedures over a range of cost conditions and priors.

3. The most difficult task facing the practitioner will involve the selection of a prior distribution of lot defectives. Recent communications with practitioners indicate that many are gathering and using lot history data. Computer programs for estimating the form of the prior and estimating its parameters are available in the public domain. A logical development following the research of this paper would be to incorporate a program for obtaining mixed Polya priors (such as that of Parkhideh [34]) into the ratio-based program, LANIF.FORT, so that the user can proceed from lot defective data to sampling plan in one step.

4. The age of microprocessors is upon us, yet the programs associated with this research now require large-scale computer systems. Two major obstacles toward the objective of converting these programs for microprocessor use are the time required to obtain optimizations and the lack of a log gamma function in most microprocessor software. Nevertheless, the possible conversion should be investigated.

5. An alternative to practitioners running their own ratio-based computer programs involves the development of sets of

decision matrices based upon a wide range of mixed-Polya priors. The complete set would be offered to prospective users. A histogram for each prior in the set would be included in the package. The user would then select the set whose prior histogram most closely matches his own histogram of lot fraction defectives. Instructions for developing this histogram would be included in the package.

As experimentation and implementation of the ratio-based decision matrices for the MGJ model continues, more questions will be asked and more suggestions will be proposed. It is hoped that the research described in this paper will serve as a starting block for additional developments in economically-based acceptance sampling.

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APPENDICES

# APPENDIX A

## LISTING OF POLMIX.FORT

```
$JOB          ,TIME=5
C**** BINOMIAL-POLYA COMPARISON PROGRAM.  PROGRAM ACCEPTS AS INPUTS
C**** LOT SIZE, SAMPLE SIZE, DEFECTIVES IN THE LOT, DEFECTIVES IN
C**** THE SAMPLE, FRACTION DEFECTIVE AND A TRIAL VALUE OF (S+T)
C**** WHICH REPRESENTS THE SUM OF THE TWO POLYA PARAMETERS.  IT
C**** THEN COMPUTES AND COMPARES MASS FUNCTIONS FOR THE BINOMIAL
C**** AND POLYA (FNXB AND FNXP, RESPECTIVELY) AND THE CONDITIONAL
C**** PROBABILITY OF HAVING BIGX DEFECTIVES IN THE LOT GIVEN
C**** SMALL X DEFECTIVES IN THE SAMPLE FOR THE BINOMIAL AND THE
C**** POLYA (HNXB AND HNXP, RESPECTIVELY).
C****
      IMPLICIT REAL*8(A-H,O-Z)
  100 WRITE(6,1)
    1 FORMAT(' INPUT SL,SS,BIGX,SMALX,P')
      READ(5,*)SL,SS,BIGX,SMALX,P
      WRITE(6,2)
    2 FORMAT(' INPUT S+T VALUE')
      READ(5,*) SPT
      S=P*SPT
      T=SPT-S
      C=COMBO(SL,BIGX)+BIGX*DLOG(P)+(SL-BIGX)*DLOG(1.DO-P)
      IF(C.LT.-90.DO)C=-90.DO
      FNXB=DEXP(C)
      Y=SL-SS
      Z=BIGX-SMALX
      D=COMBO(Y,Z)+Z*DLOG(P)+(Y-Z)*DLOG(1.DO-P)
      IF(D.LT.-90.DO)D=-90.DO
      HNXB=DEXP(D)
      FNXP=POLYA(S,T,SL,BIGX)
      HNXP=POLYA(S+SMALX,T+SS-SMALX,SL-SS,BIGX-SMALX)
      WRITE(6,3) SL,SS,BIGX,SMALX,P,SPT,S,T
    3 FORMAT(' LOT SIZE = ',F9.0/' SAMPLE SIZE = ',F6.0/' BIGX = ',
     AF13.0/' SMAL X = ',F12.0/' P = ',F18.10/' S+T = ',F18.2/' S = ',
     BF18.2/' T = ',F18.2//)
      WRITE(6,4)FNXB,FNXP,HNXB,HNXP
    4 FORMAT(' FNXB = ',F16.14,4X,'FNXP = ',F16.14/' HNXB = ',F16.
     C14,4X,'HNXP = ',F16.14)
      WRITE(6,5)
    5 FORMAT(' DO YOU WISH TO CONTINUE ?; 1=YES     0=NO')
      READ(5,*) MORE
      IF(MORE.EQ.1) GO TO 100
      STOP
      END
C****
C**** THE FUNCTION COMBO COMPUTES THE DOUBLE-PRECISION LOG OF
C**** A COMBINATION OF Y THINGS TAKEN R AT A TIME.
C****

      FUNCTION COMBO(Y,R)
      IMPLICIT REAL*8(A-H,O-Z)
      COMBO=DLGAMA(Y+1.DO)-DLGAMA(R+1.DO)-DLGAMA(Y-R+1.DO)
      RETURN
      END
C****
C**** THE FUNCTION POLYA COMPUTES THE POLYA MASS FUNCTION
C**** FOR PARAMETERS S AND T WITH COMBO VALUES A AND B.
C****

      FUNCTION POLYA(S,T,A,B)
      IMPLICIT REAL*8(A-H,O-Z)
      TEMP=COMBO(A,B)+DLGAMA(S+B)+DLGAMA(T+A-B)+DLGAMA(S+T)-
     ADLGAMA(S)-DLGAMA(T)-DLGAMA(S+T+A)
      IF(TEMP.LT.-90.DO)TEMP=-90.DO
      POLYA=DEXP(TEMP)
      RETURN
      END
```

APPENDIX B

LISTING OF OPTI.FORT

```
$JOB            ,TIME=5
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION B(1500),HB(1500),TOT(1500),JAC(1500),NC(3),NX(3),P(5),
     RC(1005)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      COMMON /BLK3/ CSO,CRO,CAO,CS1S2,CR1R2,CA2R2,CA1R1
      COMMON /BLK4/ GNCO,SMHGO,PEXPTO
      KIP=0
      KNT=0
      KLT=0
      KNEG=0
      READ(5,*) NTYPE
      READ(5,*) SO,AO,RO
      READ(5,*) S1,S2,A1,A2,R1,R2
      READ(5,*) NP
      READ(5,*) (W(I),I=1,NP)
      IF(NTYPE.EQ.1) GO TO 10
      READ(5,*) (P(I),I=1,NP)
      SPT=0.6D09
      PBAR=0.D0
      DO 12 I=1,NP
      IF(P(I).LE..1D-03.OR.P(I).GE..9999D0) SPT=0.1D13
      S(I)=P(I)*SPT
      T(I)=SPT-S(I)
      PBAR=PBAR+W(I)*P(I)
   12 CONTINUE
      GO TO 11
   10 READ(5,*) (S(I),T(I),I=1,NP)
      PBAR=0.D0
      DO 639 I=1,NP
      P(I)=S(I)/(S(I)+T(I))
  639 PBAR=PBAR+W(I)*P(I)
   11 READ(5,*) XLS
C*****ITERATIVE PROCEDURE FOR DETERMINING OPTIMUM SAMPLING PLAN
C*****DETERMINE BREAK POINTS; A BREAK POINT IS A SAMPLE SIZE VALUE
C*****FOR WHICH THE OPTIMAL ACCEPTANCE NUMBER, C*, INCREASES BY ONE
      LFLAG=0
      Y1=AO*(1.D0-HNXEX(XLS,0.D0,0.D0))+A1*XLS+A2*XLS*PBAR
      Y2=SO+RO*(1.D0-GNC(XLS,0))+XLS*S1+XLS*PBAR*S2
      XY=GNC(XLS,0)
      XX=1.D0-(1.D0-PBAR)**XLS
      XZ=HNXEX(XLS,0.D0,0.D0)
      XW=GNC(0.D0,0)
      XT=HNXEX(XLS,XLS,0.D0)
      PRINT,'Y1 = ',Y1,'Y2 = ',Y2
      Q5=GNC(XLS,0)
      PRINT,' GNC(XLS,0) = ',Q5
      CST1=.999D20
      CST2=.999D20
      SS=0.D0
      X=0.D0
   80 SS=SS+1.D0
      EXGX=(XLS-SS)*(X+1.D0)/(SS+1.D0)*POLMIX(SS+1.D0,X+1.D0)/
     3POLMIX(SS,X)+X
      Y=(X-EXGX)*(R2-A2)+(XLS-SS)*(A1-R1)+AO*(1.D0-HNXEX(XLS,SS,X))-RO
      IF(Y.GT.0.D0) GO TO 80
      B(1)=SS-1.D0
      PRINT,' B(1) = ',B(1)
      IF(B(1).GT.0.D0)KIP=1
      IF(B(1).GE.500.D0)KLT=1
      IF(B(1).GE.500.D0)GO TO 796
      JAC(1)=X
      IF(X.GT.SS-1.D0) GO TO 30
   20 X=X+1.D0
      IF(SS.LE.X) SS=X
      IF(SS.GT.XLS) GO TO 624
      EXGX=(XLS-SS)*(X+1.D0)/(SS+1.D0)*POLMIX(SS+1.D0,X+1.D0)/
     APOLMIX(SS,X)+X
      PRINT,X,SS
      IF(DABS(SS-X).LE..01D0)KNT=KNT+1
      IF(KNT.EQ.10)GO TO 624
      YY=(X-EXGX)*(R2-A2)+(XLS-SS)*(A1-R1)+AO*(1.D0-HNXEX(XLS,SS,X))-RO
      IF(YY.GT.0.D0) GO TO 30
      LFLAG=1
      GO TO 20
   30 IF(LFLAG.EQ.0) GO TO 33
      B(1)=SS-1.D0
      JAC(1)=X-1.D0
   33 I=1
  322 SS=B(I)
   35 X=JAC(I)+1.D0
      I=I+1
   40 SS=SS+1.D0
      IF(SS.GT.XLS) GO TO 34
      EXGX=(XLS-SS)*(X+1.D0)/(SS+1.D0)*POLMIX(SS+1.D0,X+1.D0)/
     APOLMIX(SS,X)+X
      Y=(X-EXGX)*(R2-A2)+(XLS-SS)*(A1-R1)+AO*(1.D0-HNXEX(XLS,SS,X))-RO
      IF(Y.GT.0.D0) GO TO 40
   34 B(I)=SS-1
      JAC(I)=X
      HB(I-1)=IDINT((B(I)+B(I-1))/2.D0)
      IF(HB(I-1).LE.0.D0) HB(I-1)=1.D0
      C(I-1)=COST(XLS,HB(I-1),JAC(I-1),S1,S2,A1,A2,R1,R2,SO,AO,RO)
      IF(C(I-1).LT.0.D0)KNEG=1
      IF(KNEG.EQ.1) GO TO 624
      JJ=I-1
      PRINT,' I-1 AND C(I-1) AT THIS POINT ARE ',JJ,C(I-1)
      IF(B(I).GE.500.D0) GO TO 624
      PRINT,' B(I) = ',B(I)
      IF(C(I-1).GT.CST2) GO TO 995
```

```
      CST1=CST2
      CST2=C(I-1)
      GO TO 322
  995 WRITE(6,936)
  936 FORMAT(' BREAK POINTS - UNTIL PROCEDURE STOPPED'/)
      PRINT,'B(1) = ',B(1)
      NBK=I
      DO 18 I=1,NBK
      WRITE(6,*) B(I)
   18 CONTINUE
      N=NBK-1
      DO 374 I=1,N
      TOT(I)=COST(XLS,HB(I),JAC(I),S1,S2,A1,A2,R1,R2,SO,AO,RO)
  374 CONTINUE
      BEST=TOT(1)
      L=1
      IF (N.EQ.1) GO TO 502
      DO 19 I=2,N
      IF(TOT(I).GE.BEST) GO TO 19
      BEST=TOT(I)
      L=I
   19 CONTINUE
      WRITE(6,241)
  241 FORMAT('1')
      WRITE(6,500) BEST,L
  500 FORMAT(//2X,'LOWEST TOTAL COST OF ALL MID-LOOP SAMPLE SIZES = ',F1
     10.2//2X,'THIS OCCURS IN THE ',I6,' TH LOOP')
      IF(DABS(B(L)-XLS).LE..001DO) GO TO 726
  502 IF(L.NE.1) GO TO 727
      IF(N.EQ.1) GO TO 508
      LS=2
      LF=3
      IF(KIP.EQ.1) GO TO 233
  796 IF(KLT.EQ.O)GO TO 797
      NBK=1
      HB(1)=B(1)/2.DO
      L=1
      JAC(1)=O
      B(2)=B(1)
      B(1)=O.DO
      N=1
      GO TO 508
  797 IF(NBK.EQ.1) GO TO 233
      IF(KIP.EQ.O) GO TO 233
      IF(B(1).LE.1.01DO) GO TO 233
      HB(NBK)=HB(NBK-1)
      DO 667 I=1,NBK
      J=NBK+1-I
      B(J+1)=B(J)
  667 HB(J+1)=HB(J)
      B(1)=O.DO
      HB(1)=IDINT(B(2)+B(1))/2.DO
      NBK=NBK+1
      GO TO 233
  726 LS=1
      LF=2
      GO TO 233
  727 LS=1
      LF=3
      GO TO 233
  508 LS=2
      LF=2
  233 DO 99 I=LS,LF
      NX(I)=HB(L+I-2)
      NC(I)=JAC(L+I-2)
      IF(B(L+I-1)-HB(L+I-2).LE.1.DO) GO TO 728
      J=B(L+I-1)-HB(L+I-2)
      IF(J.LT.O) GO TO 99
      IF(J.LE.10) GO TO 858
      IF(B(L+I-2).EQ.O.DO) XN1=1.DO
      XN1=B(L+I-2)
      XN2=HB(L+I-2)
      XN3=B(L+I-1)
      TC1=COST(XLS,XN1,NC(I),S1,S2,A1,A2,R1,R2,SO,AO,RO)
      TC2=COST(XLS,XN2,NC(I),S1,S2,A1,A2,R1,R2,SO,AO,RO)
      TC3=COST(XLS,XN3,NC(I),S1,S2,A1,A2,R1,R2,SO,AO,RO)
      D=(XN1-XN2)*(XN1-XN3)*(XN2-XN3)
      AA=(TC1*(XN2-XN3)+TC2*(XN3-XN1)+TC3*(XN1-XN2))/D
      BB=(TC1*(XN3-XN2)*(XN3+XN2)+TC2*(XN1-XN3)*(XN1+XN3)+
     3TC3*(XN2-XN1)*(XN2+XN1))/D
      HB(L+I-2)=IDINT(-1.DO*BB/(2.DO*AA))
      TOT(L+I-2)=COST(XLS,HB(L+I-2),JAC(L+I-2),S1,S2,A1,A2,R1,R2,SO,AO,R
     $O)
C*****LEFT SIDE OF THE LOOP
  858 KFLAG=O
      DO 66 K=1,J
      V=HB(L+I-2)-K
      M=JAC(L+I-2)
      IF(V.LT.DFLOAT(M)) PRINT,' V = ',V,'  M = ',M
      IF(V.LT.DFLOAT(M)) GO TO 624
      Y=COST(XLS,V,M,S1,S2,A1,A2,R1,R2,SO,AO,RO)
      WRITE(6,*) V,Y,M
      IF(Y.GE.TOT(L+I-2)) GO TO 77
      IF(V.GE.XLS) GO TO 624
      KFLAG=1
      TOT(L+I-2)=Y
      NC(I)=M
      NX(I)=V
   66 CONTINUE
C*****RIGHT SIDE OF LOOP
      GO TO 99
   77 IF(KFLAG.EQ.1) GO TO 99
```

```
      DO 88 II=1,J                                                      00002010
      D=HB(L+I-2)+II                                                    00002020
      Z=COST(XLS,D,M,S1,S2,A1,A2,R1,R2,SO,AO,RO)                        00002030
      WRITE(6,*) D,Z                                                    00002040
      IF(D.GE.XLS)GO TO 624                                             00002050
      IF(Z.GE.TOT(L+I-2)) GO TO 99                                      00002060
      TOT(L+I-2)=Z                                                      00002070
      NX(I)=D                                                           00002080
      NC(I)=M                                                           00002090
   88 CONTINUE                                                          00002100
   99 CONTINUE                                                          00002110
      IF(N.EQ.1) GO TO 729                                              00002120
      IF(L.EQ.1) GO TO 728                                              00002130
      WRITE(6,100)                                                      00002140
  100 FORMAT('1'///20X,'THREE BEST LOOPS'///15X,'LEFT',7X,'MIDDLE',7X,'R00002150
     1IGHT'//)                                                          00002160
      WRITE(6,101)(TOT(L+I-2),I=1,3),(NX(I),I=1,3),(NC(I),I=1,3)        00002170
  101 FORMAT(' TOTAL COST',F9.2,F13.2,F12.2//' SAMPLE SIZE',I8,I13,I12//00002180
     1' ACCEPT. NO.',I8,I13,I12///)                                     00002190
      BEST=DMIN1(TOT(L-1),TOT(L),TOT(L+1))                              00002200
      DO 102 I=1,3                                                      00002210
      IF(BEST.NE.TOT(L+I-2)) GO TO 102                                  00002220
      TC=TOT(L+I-2)                                                     00002230
      XSS=NX(I)                                                         00002240
      NAC=NC(I)                                                         00002250
  102 CONTINUE                                                          00002260
      GO TO 666                                                         00002270
  728 BEST=DMIN1(TOT(L),TOT(L+1))                                       00002280
      DO 701 I=2,3                                                      00002290
      IF(BEST.NE.TOT(L+I-2)) GO TO 701                                  00002300
      TC=TOT(L+I-2)                                                     00002310
      XSS=NX(I)                                                         00002320
      NAC=NC(I)                                                         00002330
  701 CONTINUE                                                          00002340
      GO TO 666                                                         00002350
  729 TC=TOT(1)                                                         00002360
      XSS=NX(2)                                                         00002370
      NAC=NC(2)                                                         00002380
  666 WRITE(6,22)                                                       00002390
   22 FORMAT('1'////20X,'GUTHRIE-JOHNS COST MODEL'////)                 00002400
      WRITE (6,9) XLS,XSS,NAC                                           00002410
    9 FORMAT(20X,'LOT SIZE = ',F13.0/20X,'SAMPLE SIZE = ',F9.0/20X,'ACCE00002420
     1PTANCE NUMBER =',I4//)                                            00002430
      TT=COST(XLS,XSS,NAC,S1,S2,A1,A2,R1,R2,SO,AO,RO)                   00002440
      KBAR=1                                                            00002450
  623 WRITE(6,758) (W(I),I=1,NP)                                        00002460
  758 FORMAT(20X,'WEIGHT(S) ',4F25.10)                                  00002470
      IF(NTYPE.EQ.1) GO TO 771                                          00002480
      WRITE(6,751) (P(I),I=1,NP)                                        00002490
  751 FORMAT(/20X,'P VALUE(S)',4F25.10)                                 00002500
  771 WRITE(6,752) (S(I),I=1,NP)                                        00002510
  752 FORMAT(/20X,'S VALUE(S)',4F25.2)                                  00002520
      WRITE(6,753) (T(I),I=1,NP)                                        00002530
  753 FORMAT(/20X,'T VALUE(S)',4F25.2)                                  00002540
      WRITE (6,32) SO,AO,RO,S1,S2,A1,A2,R1,R2                           00002550
   32 FORMAT(/20X,'SO = ',F25.2/20X,'AO = ',F25.2/20X,'RO = ',F25.2     00002560
     1/20X,'S1 = ',F25.2/20X,'S2 = ',F25.2/20X,'A1 = ',F25.2/20X,       00002570
     1'A2 = ',F25.2/20X,'R1 = ',F25.2/20X,'R2 = ',F25.2)                00002580
      WRITE(6,277)CSO,CRO,CAO,CS1S2,CR1R2,CA2R2,CA1R1                   00002590
  277 FORMAT(//20X,'CSO = ',F25.2/20X,'CRO = ',F25.2/20X,'CAO = ',F25.2/00002600
     220X,'CS1S2 = ',F23.2/20X,'CR1R2 = ',F23.2/20X,'CA2R2 = ',F23.2/   00002610
     320X,'CA1R1 = ',F23.2/)                                            00002620
      WRITE(6,278)GNCO,SMHGO,PEXPTO                                     00002630
  278 FORMAT(//20X,'GNCO = ',F24.20/20X,'SMHGO = ',F23.20/20X,'PEXPTO = 00002640
     2',F22.20/)                                                        00002650
      IF(KBAR.EQ.0) GO TO 624                                           00002660
      WRITE(6,44) TT                                                    00002670
   44 FORMAT(//20X,'TOTAL COST = ',F12.2,'  PER LOT')                   00002680
  624 WRITE(6,211) Y1                                                   00002690
  211 FORMAT(/20X,'TOTAL COST - NO SAMPLING = ',F25.2)                  00002700
      WRITE(6,212) Y2                                                   00002710
  212 FORMAT(/20X,'TOTAL COST - 100 % SAMPLING = ',F25.2)               00002720
      IF(KNEG.EQ.1)PRINT,HB(I-1),JAC(I-1)                               00002730
  441 WRITE(6,242)                                                      00002740
  242 FORMAT('1')                                                       00002750
      STOP                                                              00002760
      END                                                               00002770

      FUNCTION POLMIX(A,B)                                              00002780
      IMPLICIT REAL*8(A-H,O-Z)                                          00002790
      COMMON /BLK1/ W(5),S(5),T(5)                                      00002800
      COMMON /BLK2/ PBAR,NP                                             00002810
      POLMIX=0.D0                                                       00002820
      DO 7 I=1,NP                                                       00002830
      TEMP=COMBO(A,B)+DLGAMA(S(I)+B)+DLGAMA(T(I)+A-B)+DLGAMA(S(I)+T(I   00002840
     1))-DLGAMA(S(I))-DLGAMA(T(I))-DLGAMA(S(I)+T(I)+A)                  00002850
      IF(TEMP.LT.-90.D0) TEMP=-90.D0                                    00002860
    7 POLMIX=POLMIX+W(I)*DEXP(TEMP)                                     00002870
      RETURN                                                            00002880
      END                                                               00002890

      FUNCTION COMBO(Y,R)                                               00002900
      IMPLICIT REAL*8(A-H,O-Z)                                          00002910
      COMBO=DLGAMA(Y+1.D0)-DLGAMA(R+1.D0)-DLGAMA(Y-R+1.D0)              00002920
      RETURN                                                            00002930
      END                                                               00002940
```

```
      FUNCTION COST(XLS,XSS,NAC,S1,S2,A1,A2,R1,R2,SO,AO,RO)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK2/ PBAR,NP
      COMMON /BLK3/ CSO,CRO,CAO,CS1S2,CR1R2,CA2R2,CA1R1
      COMMON /BLK4/ GNCO,SMHGO,PEXPTO
      GNCO=GNC(XSS,NAC)
      SMHGO=SMHG(XLS,XSS,NAC)
      PEXPTO=PEXPT(XSS,NAC)/(XSS+1.DO)
      CSO=SO
      CRO=RO*(1.DO-GNCO)
      CAO=AO*(GNCO-SMHGO)
      CS1S2=XSS*(S1+PBAR*S2)
      CR1R2=(XLS-XSS)*(R1+PBAR*R2)
      CA2R2=(XLS-XSS)*PEXPTO*(A2-R2)
      CA1R1=(XLS-XSS)*GNCO*(A1-R1)
      COST=CSO+CRO+CAO+CS1S2+CR1R2+CA2R2+CA1R1
      RETURN
      END

      FUNCTION PEXPT(XSS,NAC)
      IMPLICIT REAL*8(A-H,O-Z)
      K=NAC+1
      PEXPT=O.DO
      DO 7 I=1,K
      X=DFLOAT(I)-1.DO
      PEXPT=PEXPT+(X+1.DO)*POLMIX(XSS+1.DO,X+1.DO)
    7 CONTINUE
      RETURN
      END

      FUNCTION SMHG(XLS,XSS,NAC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      SMHG=O.DO
      K=NAC+1
      DO 7 I=1,K
      X=DFLOAT(I-1)
      DO 7 J=1,NP
      A=DLGAMA(S(J)+T(J))+DLGAMA(X+S(J))-DLGAMA(S(J))-DLGAMA
     1(T(J))+DLGAMA(XLS-X+T(J))-DLGAMA(XLS+S(J)+T(J))+DLOG(W(J))+
     2COMBO(XSS,X)
      IF(A.LT.-150.DO)A=-150.DO
      A=DEXP(A)
      SMHG=SMHG+A
    7 CONTINUE
      RETURN
      END

      FUNCTION GNC(XSS,NAC)
      IMPLICIT REAL*8(A-H,O-Z)
      K=NAC+1
      GNC=O.DO
      DO 7 I=1,K
      X=DFLOAT(I-1)
    7 GNC=GNC+POLMIX(XSS,X)
      RETURN
      END

      FUNCTION HNXEX(XLS,SS,X)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      SUM=O.DO
      TOT=O.DO
      DO 7 I=1,NP
      A=DLGAMA(S(I)+T(I))+DLGAMA(X+S(I))-DLGAMA(S(I))-DLGAMA
     4(T(I))+DLGAMA(XLS-X+T(I))-DLGAMA(XLS+S(I)+T(I))
      IF(A.LT.-90.DO)A=-90.DO
      A=DEXP(A)
      SUM=SUM+W(I)*A
      B=DLGAMA(S(I)+T(I))+DLGAMA(X+S(I))-DLGAMA(S(I))-DLGAMA
     3(T(I))+DLGAMA(SS-X+T(I))-DLGAMA(SS+S(I)+T(I))
      IF(B.LT.-90.DO)B=-90.DO
      B=DEXP(B)
    7 TOT=TOT+W(I)*B
      HNXEX=SUM/TOT
      RETURN
      END
```

# APPENDIX C

## LISTING AND INSTRUCTIONS FOR LANIF.FORT

```
$JOB            ,TIME=31
C**** THIS PORGRAM COMPUTES A MATRIX OF SAMPLING PLANS INCLUDING NO
C**** SAMPLING AND 100 PERCENT INSPECTION FOR A RANGE OF A2/R2 AND
C**** R2/R1 VALUES.  INPUTS TO THIS PROGRAM ARE NUMBER AND VALUE OF
C**** POLYA PRIOR S AND T PARAMETERS AND WEIGHTS OR NUMBER AND VALUE
C**** OF BINOMIAL FRACTION DEFECTIVES AND WEIGHTS.  IF ALL NINE COSTS
C**** OF THE MGJ MODEL ARE ASSUMED TO BE KNOWN, AN OPTION EXISTS TO
C**** INPUT THESE COSTS,  THE OPTIMAL (N,C) PAIR, AND THE ASSOCIATED
C**** TOTAL COSTS SO THAT THE PERCENT ERROR IN TOTAL COST INCURRED
C**** THROUGH THE USE OF RATIOS IN LIEU OF COST VALUES MAY BE
C**** DETERMINED.
C****
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*1 Q
      CHARACTER*30 TITLE
      DIMENSION B(1500),HB(1500),TOT(1500),JAC(1500),NC(3),NX(3),P(5),
     RC(1005),NSAMP(10),NACC(10),DELT(10)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      COMMON /BLK4/ GNCO,SMHGO,PEXPTO
      TITLE ='SO/S1, AO/S1, AND RO/S1 < 1000'
 9133 NRTMS=6
      N1=7
      N2=10
      TOT(1)=1.OD70
      WRITE(6,50)
   50 FORMAT(' INPUT A "1" IF OUTPUT IS TO BE AT A CRT'/' INPUT A "O"
     2 IF OUTPUT IS TO BE PRINTED ON PAPER')
      READ(5,*) NOUT
      IF(NOUT.EQ.1)NRTMS=1
      IF(NOUT.EQ.1)N1=1
      IF(NOUT.EQ.1)N2=1
      WRITE(6,51)
   51 FORMAT(' INPUT A "O" IF ONLY THE RATIO PLANS ARE TO BE GENERATED'
     2/' INPUT A "1" IF DELTA IS TO BE CALCULATED')
      READ(5,*) NOPT
      WRITE(6,52)
   52 FORMAT(' INPUT A "O" IF THE PRIOR IS IN MIXED BINOMIAL FORM'
     3/' INPUT A "1" IF PRIOR IS A MIXED POLYA')
      READ(5,*) NTYPE
      IF(NOPT.EQ.O) GO TO 1
      WRITE(6,53)
   53 FORMAT(' INPUT THE FIXED COSTS - SO, AO, AND RO')
      READ(5,*) SO,AO,RO
      WRITE(6,54)
   54 FORMAT(' INPUT THE UNIT COSTS - S1,S2,A1,A2,R1,AND R2')
      READ(5,*) S1,S2,A1,A2,R1,R2
    1 WRITE(6,55)
   55 FORMAT(' INPUT THE NUMBER OF POINTS IN THE PRIOR')
      READ(5,*) NP
      WRITE(6,56)
   56 FORMAT(' INPUT THE WEIGHTS ASSIGNED TO EACH POINT IN THE PRIOR')
      READ(5,*) (W(I),I=1,NP)
      IF(NTYPE.EQ.1) GO TO 10
      WRITE(6,57)
   57 FORMAT(' INPUT EACH OF THE MIXED BINOMIAL P VALUES')
      READ(5,*) (P(I),I=1,NP)
C**** CONVERT MIXED BINOMIAL TO MIXED POLYA
      SPT=O.6D09
      PBAR=O.DO
      DO 12 I=1,NP
      IF(P(I).LE..1D-O3.OR.P(I).GE..9999DO) SPT=O.1D13
      S(I)=P(I)*SPT
      T(I)=SPT-S(I)
      PBAR=PBAR+W(I)*P(I)
   12 CONTINUE
      GO TO 2851
   10 WRITE(6,58)
   58 FORMAT(' INPUT EACH OF THE MIXED POLYA S AND T PAIRS')
      READ(5,*) (S(I),T(I),I=1,NP)
      PBAR=O.DO
```

```
      DO 639 I=1,NP
      P(I)=S(I)/(S(I)+T(I))
  639 PBAR=PBAR+W(I)*P(I)
 2851 WRITE(6,59)
   59 FORMAT(' INPUT THE LOT SIZE')
   11 READ(5,*) XLS
      IF(NOUT.EQ.0)GO TO 1663
      SOS1=0.D0
      AOS1=0.D0
      ROS1=0.D0
      WRITE(6,1598)
 1598 FORMAT(' INPUT THE RATIO A2/R2; INCLUDE DECIMAL')
      WRITE(6,1599)
 1599 FORMAT(' SELECT FROM 1,2,4,8,16,32, OR 64')
      READ(5,*) A2R2
      WRITE(6,1600)
 1600 FORMAT(' INPUT THE RATIO R2/R1; INCLUDE DECIMAL')
      WRITE(6,1601)
 1601 FORMAT('SELECT FROM .125, .250, .50, 1, 2, 4, 8, 16, 32, OR 64')
      READ(5,*) R2R1
      WRITE(6,1602)
 1602 FORMAT(' DO YOU WISH TO INCLUDE ANY FIXED COST RATIOS ?; 1=YES
     20=NO')
      READ(5,*) LFIX
 1603 IF(LFIX.EQ.0) GO TO 1663
      WRITE(6,1604)
 1604 FORMAT(' SELECT ONE OF THE FOLLOWING: ')
      WRITE(6,1605)
 1605 FORMAT(' 1   SO/S1 = 1000')
      WRITE(6,1606)
 1606 FORMAT(' 2   SO/S1 = 10000')
      WRITE(6,1607)
 1607 FORMAT(' 3   AO/S1 = 1000')
      WRITE(6,1608)
 1608 FORMAT(' 4   AO/S1 = 10000')
      WRITE(6,1609)
 1609 FORMAT(' 5   AO/S1 = 1000 AND RO/S1=100')
      READ(5,*) LTFIX
      GO TO (1610,1611,1612,1613,1614),LTFIX
 1610 SOS1=1000.D0
      GO TO 1663
 1611 SOS1=10000.D0
      GO TO 1663
 1612 AOS1=1000.D0
      GO TO 1663
 1613 AOS1=10000.D0
      GO TO 1663
 1614 AOS1=1000.D0
      ROS1=100.D0
 1663 IF(NOPT.EQ.0) GO TO 1239
      WRITE(6,60)
   60 FORMAT(' INPUT OPTIMAL SAMPLE SIZE (REAL), ACC. NO. (INTEGER), AND
     4 THE TOTAL COST (REAL)')
      READ(5,*) SSO,IAN,TCO
 1239 DO 1240 II=1,NRTMS
C**** SOS1=SO/S1, AOS1=AO/S1 AND ROS1=RO/S1
C**** IF INTERACTIVE AT CRT, SKIP DEVELOPMENT OF DECISION MATRICES
      IF(NOUT.EQ.1) GO TO 1664
      SOS1=0.D0
      AOS1=0.D0
      ROS1=0.D0
C**** PRINTOUT COST PAGE ONLY IF NOPT=1
 1664 IF(NOPT.EQ.0) GO TO 2
      WRITE(6,22)
   22 FORMAT('1'////20X,'GUTHRIE-JOHNS COST MODEL'////)
      WRITE (6,9) XLS,SSO,IAN
    9 FORMAT(20X,'LOT SIZE = ',F13.0/20X,'SAMPLE SIZE = ',F9.0/20X,'ACCE
     1PTANCE NUMBER =',I4//)
      KBAR=1
      WRITE(6,758) (W(I),I=1,NP)
  758 FORMAT(20X,'WEIGHT(S) ',4F15.5)
      IF(NTYPE.EQ.1) GO TO 771
```

```
      WRITE(6,751) (P(I),I=1,NP)
  751 FORMAT(/2OX,'P VALUE(S)',4F15.5)
  771 WRITE(6,752) (S(I),I=1,NP)
  752 FORMAT(/2OX,'S VALUE(S)',4F15.O)
      WRITE(6,753) (T(I),I=1,NP)
  753 FORMAT(/2OX,'T VALUE(S)',4F15.O)
      WRITE (6,32) SO,AO,RO,S1,S2,A1,A2,R1,R2
   32 FORMAT(/2OX,'SO = ',F25.2/2OX,'AO = ',F25.2/2OX,'RO = ',F25.2
     1/2OX,'S1 = ',F25.2/2OX,'S2 = ',F25.2/2OX,'A1 = ',F25.2/2OX,
     1'A2 = ',F25.2/2OX,'R1 = ',F25.2/2OX,'R2 = ',F25.2)
C**** CALCULATE NO SAMPLING AND 100 PERCENT INSPECTION COSTS')
      Z1=AO*(1.DO-HNXEX(XLS,O.DO,O.DO))+A1*XLS+A2*XLS*PBAR
      Z2=SO+RO*(1.DO-GNC(XLS,O))+XLS*S1+XLS*PBAR*S2
      WRITE(6,44) TCO
   44 FORMAT(//2OX,'TOTAL COST = ',F12.2,'  PER LOT')
  624 WRITE(6,211) Z1
  211 FORMAT(/2OX,'TOTAL COST - NO SAMPLING = ',F25.2)
      WRITE(6,212) Z2
  212 FORMAT(/2OX,'TOTAL COST - 100 % SAMPLING = ',F25.2)
    2 IF(NOUT.EQ.1) GO TO 1665
      IF(II.EQ.1)GO TO 111
      IF(II.EQ.2)GO TO 2222
      IF(II.EQ.3)GO TO 3333
      IF(II.EQ.4)GO TO 4444
      IF(II.EQ.5)GO TO 5555
      ROS1=100.DO
      AOS1=1000.DO
      TITLE='AO/S1 = 1000; RO/S1 = 100'
      GO TO 111
 2222 SOS1=1000.DO
      TITLE='  SO/S1 = 1000  '
      GO TO 111
 3333 SOS1=10000.DO
      TITLE='  SO/S1 = 10000'
      GO TO 111
 4444 AOS1=1000.DO
      TITLE='  AO/S1 = 1000'
      GO TO 111
 5555 AOS1=10000.DO
      TITLE = '  AO/S1 = 10000 '
  111 A2R2=O.5ODO.
C**** TD AND TV ARE USED TO CALCULATE BOUNDARY VALUES FOR RATIO ENTRIES
      TD=-O.5DO
      TV=O.DO
      WRITE(6,645)
  645 FORMAT('1',5OX,'GUTHRIE-JOHNS MODEL')
      WRITE(6,646)
  646 FORMAT(42X,'SINGLE ATTRIBUTE ACCEPTANCE SAMPLING')
      WRITE(6,612) TITLE
  612 FORMAT(48X,A3O)
      WRITE(6,647)
  647 FORMAT(48X,'COST RATIO DECISION MATRIX'/)
      WRITE(6,648)
  648 FORMAT(19X,'O.O',6X,'.18',6X,'.35',6X,'.71',5X,'1.41',5X,'2.83',
     35X,'5.66',5X,'11.31',4X,'22.63',4X,'45.25',4X,'9O.51')
      WRITE(6,649)
  649 FORMAT(2OX,'!',1O(8X,'!'))
      WRITE(6,65O)
  650 FORMAT(13X,'R2R1',3X,'!',3X,'1/8',2X,'!',3X,'1/4',2X,'!',3X,'1/2'
     3,2X,'!',3X,' 1 ',2X,'!',3X,' 2 ',2X,'!',3X,' 4 ',2X,'!',3X,' 8 ',
     42X,'!',3X,'16 ',2X,'!',3X,'32 ',2X,'!',3X,'64 ',2X,'!')
C**** A2R2=A2/R2 AND R2R1=R2/R1
C**** GENERATE THE ROWS OF THE DECISION MATRIX
 1665 DO 1257 IB=1,N1
      IF(N1.EQ.1) GO TO 1666
      A2R2=A2R2*2.DO
      LL=A2R2
      R2R1=O.O625DO
      WRITE(6,385)TV
  385 FORMAT(3X,'-----------------------------------------------------
     3----------------------------------------------------',F5.2)
      TD=TD+1.ODO
      TV=2.DO**TD
C**** GENERATE N2 COLUMNS FOR EACH ROW IN THE DECISION MATRIX
```

```
 1666 DO 1254 IE=1,N2
      IF(N2.EQ.1) GO TO 1256
      R2R1=R2R1*2.DO
C**** INITIALIZE VARIABLES
C**** TC = TOTAL COST ASSOCIATED WITH (N,C) PAIR - RATIO PLAN
C**** KIP=1 MEANS FIRST BREAK POINT BEYOND O;
C**** KNT = COUNT OF NUMBER OF TIMES SAMPLE SIZE AND MAXIMUM
C****       DEFECTIVES ARE EQUAL
C**** KLT=1 MEANS FIRST BREAK POINT IS BEYOND N=499
C**** KST=1 MEANS SAMPLE SIZE IS LESS THAN ACCEPTANCE NUMBER
C**** KFORK=1 MEANS FIRST BREAK POINT IS BEYOND N=499
C**** KBAR=1 MEANS (N,C) PAIR OTHER THAN (O,O) OR (N,O)
C****         HAS BEEN FOUND
C**** LFLAG=1 MEANS BREAK POINT INEQUALITY HAS BEEN SATISFIED
C**** KNEG=1 MEANS TOTAL COSTS IS FOR EITHER A (O,O) OR (N,O) PLAN
C**** KBIG=1 MEANS MID-LOOP N VALUE IS BEYOND 600
C****         TOTAL COST IS VERY LARGE
C**** VD = TOTAL COST FOR THE 9 COST (DOLLAR VALUE) PLAN
C**** VC = TOTAL COST FOR THE RATIO PLAN
 1256 TC=1.OD70
      KIP=O
      KNT=O
      KLT=O
      KST=O
      SAVE=1.OD70
      STOR=1.OD70
      KBAR=O
      VD=1.OD70
      KNEG=O
      KBIG=O
      VC=1.OD70
      KFORK=O
      LFLAG=O
C**** CALCULATE NO SAMPLE (Y1) AND 100 PERCENT (Y2) TOTAL COSTS
      A2R1=A2R2*R2R1
      Y1=A2R1*XLS*PBAR+AOS1*(1.DO-GNC(XLS,O))
      Y2=XLS+R2R1*XLS*PBAR+SOS1+ROS1*(1.DO-GNC(XLS,O))
C**** CST1 AND CST2 ARE TEMPORARY VALUES FOR TOTAL COST - RATIO PLAN
      CST1=.999D20
      CST2=.999D20
C**** DETERMINE FIRST BREAK POINT (B(1))
      SS=O.DO
      X=O.DO
   80 SS=SS+1.DO
C**** EXGX = THE EXPECTED VALUE OF BIG X GIVEN SMALL X
      EXGX=(XLS-SS)*(X+1.DO)/(SS+1.DO)*POLMIX(SS+1.DO,X+1.DO)/
     3POLMIX(SS,X)+X
      Y=(EXGX-X)*(A2R1-R2R1)+SS-XLS+AOS1*(1.DO-HNXEX(XLS,SS,X))-ROS1
      IF(Y.GT.O.DO) GO TO 80
      B(1)=SS-1.DO
      IF(B(1).GT.O.DO)KIP=1
      IF(B(1).GE.500.DO) KLT=1
      IF(B(1).GE.500.DO) GO TO 796
      JAC(1)=X
      IF(X.GT.SS-1.DO) GO TO 30
C**** FIRST BREAK POINT IS LEFT OF ORIGIN (<O)
   20 X=X+1.DO
      IF(SS.LE.X) SS=X
      IF(SS.GT.XLS) GO TO 623
       EXGX=(XLS-SS)*(X+1.DO)/(SS+1.DO)*POLMIX(SS+1.DO,X+1.DO)/
     APOLMIX(SS,X)+X
      IF(DABS(SS-X).LE..O1DO)KNT=KNT+1
      IF(KNT.EQ.10)GO TO 623
      YY=(EXGX-X)*(A2R1-R2R1)+SS-XLS+AOS1*(1.DO-HNXEX(XLS,SS,X))-ROS1
      IF(YY.GT.O.DO) GO TO 30
      LFLAG=1
      GO TO 20
   30 IF(LFLAG.EQ.O) GO TO 33
      B(1)=SS-1.DO
      JAC(1)=X-1.DO
C**** DETERMINE OTHER BREAK POINTS
```

```
        33 I=1
       322 SS=B(I)
        35 X=JAC(I)+1.DO
           I=I+1
        40 SS=SS+1.DO
           IF(SS.GT.XLS) GO TO 34
           EXGX=(XLS-SS)*(X+1.DO)/(SS+1.DO)*POLMIX(SS+1.DO,X+1.DO)/
          APOLMIX(SS,X)+X
           Y=(EXGX-X)*(A2R1-R2R1)+SS-XLS+AOS1*(1.DO-HNXEX(XLS,SS,X))-ROS1
           IF(Y.GT.O.DO) GO TO 40
     C**** B(I) = I TH BREAK POINT; JAC(I) = LOOP I-1
        34 B(I)=SS-1
           JAC(I)=X
           HB(I-1)=IDINT((B(I)+B(I-1))/2.DO)
           IF(HB(I-1).LE.O.DO) HB(I-1)=1.DO
     C**** DETERMINE MID-LOOP TOTAL COST; ACT ONLY IF COST LESS THAN O
           C(I-1)=COST(XLS,HB(I-1),JAC(I-1),A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
           IF(C(I-1).LT.O.DO)KNEG=1
           IF(KNEG.EQ.1) GO TO 623
           JJ=I-1
           IF(B(I).GE.500.DO) KFORK=1
           IF(C(I-1).GT.CST2) GO TO 995
           CST1=CST2
           CST2=C(I-1)
           GO TO 322
     C**** NBK = NUMBER OF BREAK POINTS
       995 NBK=I
           N=NBK-1
     C**** GET ALL MID-LOOP TOTAL COSTS
           DO 374 I=1,N
           TOT(I)=COST(XLS,HB(I),JAC(I),A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
       374 CONTINUE
           BEST=TOT(1)
           L=1
           IF (N.EQ.1) GO TO 502
     C**** FIND MINIMUM MID-LOOP TOTAL COST
           DO 19 I=2,N
           IF(TOT(I).GE.BEST) GO TO 19
           BEST=TOT(I)
     C**** L = NUMBER OF LOOPS
           L=I
        19 CONTINUE
     C**** BEGIN SEARCH FOR OPTIMUM COST AND ASSOCIATED N AND C VALUES
           IF(DABS(B(L)-XLS).LE..001DO) GO TO 726
           IF(BEST.GT.Y1.OR.BEST.GT.Y2) GO TO 623
       502 IF(L.NE.1) GO TO 727
           IF(N.EQ.1) GO TO 508
     C**** LS AND LF ARE THE STARTING AND FINISHING LOOPS TO BE USED IN THE
     C**** SEARCH PROCESS.   IF LS=1 AND LF=3, THEN SEARCH ONE LOOP LEFT
     C**** AND ONE LOOP RIGHT OF THE "BEST" LOOP AFTER SEARCHING "BEST"
     C**** LOOP.  IF LS=2 AND LF=3, THEN SEARCH 1 LOOP RIGHT ONLY AFTER
     C**** SEARCHING "BEST" LOOP. IF LS=1 AND LF=2, THEN SEARCH 1 LOOP LEFT
     C**** ONLY, AFTER SEARCHING "BEST" LOOP.  IF LS=2 AND LF=2, THEN
     C**** SEARCH "BEST" LOOP ONLY.
           LS=2
           LF=3
           IF(KIP.EQ.O) GO TO 233
       796 IF(KLT.EQ.O)GO TO 797
           NBK=1
           N=1
           HB(1)=B(1)/2.DO
           L=1
           JAC(1)=O
           B(2)=B(1)
           B(1)=O.DO
           GO TO 508
       797 IF(NBK.EQ.1) GO TO 233
           IF(KIP.EQ.O) GO TO 233
           IF(B(1).LE.1.01DO) GO TO 233
           HB(NBK)=HB(NBK-1)
           DO 667 I=1,NBK
           J=NBK+1-I
```

```
      B(J+1)=B(J)
  667 HB(J+1)=HB(J)
      B(1)=O.DO
      HB(1)=IDINT(B(2)+B(1))/2.DO
      NBK=NBK+1
      GO TO 233
  726 LS=1
      LF=2
      GO TO 233
  727 LS=1
      LF=3
      GO TO 233
  508 LS=2
      LF=2
  233 DO 99 I=LS,LF
      NX(I)=HB(L+I-2)
      NC(I)=JAC(L+I-2)
      IF(B(L+I-1)-HB(L+I-2).LE.1.DO) GO TO 728
      J=B(L+I-1)-HB(L+I-2)
      IF(J.LT.O) GO TO 99
C**** USE QUADRATIC FIT MINIMUM RATHER THAN MID-LOOP COST VALUE
C**** AS STARTING SEARCH POINT ONLY IF THERE ARE MORE THAN
C**** TEN POINTS BETWEEN A BREAK POINT AND THE MID-LOOP SAMPLE
C**** SIZE VALUE.
      IF(J.LE.10) GO TO 858
      IF(B(L+I-2).EQ.O.DO) XN1=1.DO
      XN1=B(L+I-2)
      XN2=HB(L+I-2)
      XN3=B(L+I-1)
      TC1=COST(XLS,XN1,NC(I),A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
      TC2=COST(XLS,XN2,NC(I),A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
      TC3=COST(XLS,XN3,NC(I),A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
      D=(XN1-XN2)*(XN1-XN3)*(XN2-XN3)
      AA=(TC1*(XN2-XN3)+TC2*(XN3-XN1)+TC3*(XN1-XN2))/D
      BB=(TC1*(XN3-XN2)*(XN3+XN2)+TC2*(XN1-XN3)*(XN1+XN3)+
     3TC3*(XN2-XN1)*(XN2+XN1))/D
      IF(DABS(AA).LT..10D-10)GO TO 858
      HB(L+I-2)=IDINT(-1.DO*BB/(2.DO*AA))
      IF(HB(L+I-2).LE.O.DO) HB(L+I-2)=1.DO
      IF(HB(L+I-2).LT.JAC(L+I-2)) HB(L+I-2)=JAC(L+I-2)
      TOT(L+I-2)=COST(XLS,HB(L+I-2),JAC(L+I-2),A2R1,R2R1,A2R2,SOS1,AOS1,
     3ROS1)
C*****LEFT SIDE OF THE LOOP
  858 KFLAG=O
      DO 66 K=1,J
      V=HB(L+I-2)-K
      M=JAC(L+I-2)
      IF(V.LT.DFLOAT(M)) KST=1
      IF(V.LT.DFLOAT(M)) GO TO 676
      Y=COST(XLS,V,M,A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
      IF(DABS(SAVE-Y).LE..1D-4.AND.V.GT.600.DO)KBIG=1
      IF(KBIG.EQ.1)GO TO 623
      IF(Y.GE.TOT(L+I-2)) GO TO 77
      SAVE=Y
      KFLAG=1
      TOT(L+I-2)=Y
      NC(I)=M
      NX(I)=V
   66 CONTINUE
C*****RIGHT SIDE OF LOOP
      GO TO 99
   77 IF(KFLAG.EQ.1) GO TO 99
      DO 88 IR=1,J
      D=HB(L+I-2)+IR
      Z=COST(XLS,D,M,A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
      IF(DABS(STOR-Z).LE..1D-4.AND.D.GT.600.DO)KBIG=1
      IF(KBIG.EQ.1)GO TO 623
      IF(Z.GE.TOT(L+I-2)) GO TO 99
      STOR=Z
      TOT(L+I-2)=Z
      NX(I)=D
      NC(I)=M
   88 CONTINUE
   99 CONTINUE
```

```
      IF(N.EQ.1) GO TO 729
      IF(L.EQ.1) GO TO 728
C**** BEST = COST ASSOCIATED WITH THE OPTIMAL PLAN
      BEST=DMIN1(TOT(L-1),TOT(L),TOT(L+1))
      DO 102 I=1,3
      IF(BEST.NE.TOT(L+I-2)) GO TO 102
C**** TC = OPTIMAL COST; XSS = OPTIMAL SAMPLE SIZE
C**** NAC = OPTIMAL ACCEPTANCE NUMBER
      TC=TOT(L+I-2)
      XSS=NX(I)
      NAC=NC(I)
  102 CONTINUE
      GO TO 666
  728 BEST=DMIN1(TOT(L),TOT(L+1))
      DO 701 I=2,3
      IF(BEST.NE.TOT(L+I-2)) GO TO 701
      TC=TOT(L+I-2)
      XSS=NX(I)
      NAC=NC(I)
  701 CONTINUE
      GO TO 666
  729 TC=TOT(1)
      XSS=NX(2) .
      NAC=NC(2)
  666 KBAR=1
  623 IF(KNEG.EQ.1.OR.KBIG.EQ.1.OR.SS.GT.XLS) GO TO 676
C**** VC = MIM(TC,NO SAMPLING COST, 100 PERCENT INSPECTION COST)
      VC=TC
  676 IF(Y1.LT.VC)VC=Y1
      IF(Y2.LT.VC)VC=Y2
      IF(NOPT.EQ.O) GO TO 4
  714 IF(KBAR.EQ.O)GO TO 448
C**** VD = DOLLAR VALUE TOTAL COST USING RATIO PLAN
C**** V1 = DOLLAR VALUE ASSOCIATED WITH NO SAMPLING
C**** V2 = DOLLAR VALUE ASSOCIATED WITH 100 PERCENT INSPECTION
C**** NSAMP(IE) = OPTIMAL SAMPLE SIZE THIS RUN (IE TH)
C**** NACC(IE)  = OPTIMAL ACCEPTANCE NUMBER THIS RUN (IE TH)
      VD=EVAL(XLS,XSS,NAC,S1,S2,A1,A2,R1,R2,SO,AO,RO)
  448 V1=AO*(1.DO-HNXEX(XLS,O.DO,O.DO))+A1*XLS+A2*XLS*PBAR
      V2=SO+RO*(1.DO-GNC(XLS,O))+XLS*S1+XLS*PBAR*S2
    4 IF(KNEG.EQ.1)GO TO 972
      IF(VC.NE.TC)GO TO 712
      IF(KST.EQ.1)GO TO 712
      NSAMP(IE)=XSS
      NACC(IE)=NAC
      IF(NOPT.EQ.O)GO TO 1255
      GO TO 713
  712 IF(VC.EQ.Y1.OR.KST.EQ.1)NSAMP(IE)=O
      IF(NOPT.EQ.O) GO TO 5
      IF(VC.EQ.Y1)VD=V1
    5 IF(VC.EQ.Y2)NSAMP(IE)=XLS
      NACC(IE)=O
      IF(NOPT.EQ.O) GO TO 1255
      IF(VC.EQ.Y2)VD=V2
  713 DELTA=(VD-TCO)/TCO
      DELT(IE)=DELTA
 1255 IF(NSAMP(IE).GT.IDINT(XLS)) NSAMP(IE)=XLS
      IF(DABS(Y2-VC).LT..1ODO)NSAMP(IE)=XLS
      IF(DABS(Y1-VC).LT..1ODO)NSAMP(IE)=O
 1254 CONTINUE
      IF(NOUT.EQ.1) GO TO 972
      Q=' '
      IF(IB.EQ.2) Q='A'
      IF(IB.EQ.3) Q='2'
      IF(IB.EQ.4) Q='R'
      IF(IB.EQ.5) Q='2'
      WRITE(6,421)(NSAMP(JJ),JJ=1,10)
  421 FORMAT(3X,'!',4X,'! SAMP.SIZE !',10(I7,1X,'!'))
      IF(NOPT.EQ.O) GO TO 3
      WRITE(6,124)
  124 FORMAT(3X,'!',4X,'!',11X,'!',10(8X,'!'))
      WRITE(6,422)Q,LL,(NACC(JJ),JJ=1,10)
  422 FORMAT(1X,A1,1X,'!',1X,I2,1X,'! ACC. NBR. !',10(I7,1X,'!'))
      WRITE(6,124)
      WRITE(6,423)(DELT(JJ),JJ=1,10)
```

```
  423 FORMAT(3X,'!',4X,'! DELTA      !',10(F7.4,1X,'!'))
      WRITE(6,124)
      GO TO 1257
    3 WRITE(6,126) LL
  126 FORMAT(3X,'!',1X,I2,1X,'!',11X,'!',10(8X,'!'))
      WRITE(6,127)Q,(NACC(JJ),JJ=1,10)
  127 FORMAT(1X,A1,1X,'!',4X,'! ACC. NBR. !',10(I7,1X,'!'))
      WRITE(6,124)
 1257 CONTINUE
      WRITE(6,385)TV
      WRITE(6,8)
    8 FORMAT('1')
 1240 CONTINUE
  972 PRINT,'KNEG = ',KNEG,'  KFORK = ',KFORK
      IF(NOUT.EQ.0) GO TO 9994
      WRITE(6,1667)
 1667 FORMAT('1'//20X,'MODIFIED GUTHRIE-JOHNS MODEL; INTERACTIVE RATIO
     2VERSION')
      WRITE(6,1668) SOS1,AOS1,ROS1
 1668 FORMAT(/40X,'SO/S1 = ',F7.0/40X,'AO/S1 = ',F7.0/40X,'RO/S1 = ',
     2F7.0)
      WRITE(6,1669) A2R2,R2R1
 1669 FORMAT(/40X,'A2/R2 = ',F7.0/40X,'R2/R1 = ',F7.3)
      WRITE(6,1670) NSAMP(1),NACC(1)
 1670 FORMAT(//35X,'SAMPLE SIZE = ',I7/30X,'ACCEPTANCE NUMBER = ',I7)
      IF(NOPT.EQ.0) GO TO 1690
      WRITE(6,1671) DELT(1)
 1671 FORMAT(/41X,'DELTA = ',F7.4)
 1690 WRITE(6,1691)
 1691 FORMAT(' DO YOU WISH TO TRY ANOTHER PLAN?; 1=YES 0=NO')
      READ(5,*) MORE
      IF(MORE.EQ.1) GO TO 9133
 9994 STOP
      END

      FUNCTION POLMIX(A,B)
C**** THIS FUNCTION EVALUATES THE MIXED POLYA DISTRIBUTION
C**** A = LOT SIZE OR SAMPLE SIZE
C**** B = DEFECTIVES IN THE LOT OR DEFECTIVES IN THE SAMPLE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      POLMIX=0.D0
      DO 7 I=1,NP
      TEMP=COMBO(A,B)+DLGAMA(S(I)+B)+DLGAMA(T(I)+A-B)+DLGAMA(S(I)+T(I
     1))-DLGAMA(S(I))-DLGAMA(T(I))-DLGAMA(S(I)+T(I)+A)
      IF(TEMP.LT.-90.D0) TEMP=-90.D0
    7 POLMIX=POLMIX+W(I)*DEXP(TEMP)
      RETURN
      END

      FUNCTION EVAL(XLS,XSS,NAC,S1,S2,A1,A2,R1,R2,SO,AO,RO)
C**** THIS FUNCTION EVALUATES THE DOLLAR VALUE COST ASSOCIATED
C**** WITH THE BEST RATIO PLAN
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK2/ PBAR,NP
      COMMON /BLK4/ GNCO,SMHGO,PEXPTO
      GNCO=GNC(XSS,NAC)
      SMHGO=SMHG(XLS,XSS,NAC)
      PEXPTO=PEXPT(XSS,NAC)/(XSS+1.D0)
      CSO=SO
      CRO=RO*(1.D0-GNCO)
      CAO=AO*(GNCO-SMHGO)
      CS1S2=XSS*(S1+PBAR*S2)
      CR1R2=(XLS-XSS)*(R1+PBAR*R2)
      CA2R2=(XLS-XSS)*PEXPTO*(A2-R2)
      CA1R1=(XLS-XSS)*GNCO*(A1-R1)
      EVAL=CSO+CRO+CAO+CS1S2+CR1R2+CA2R2+CA1R1
      RETURN
      END
```

```
      FUNCTION COMBO(Y,R)
C**** THIS FUNCTION COMPUTES THE DOUBLE-PRECISION LOG OF
C**** A COMBINATION OF Y THINGS TAKEN R AT A TIME
      IMPLICIT REAL*8(A-H,O-Z)
      COMBO=DLGAMA(Y+1.D0)-DLGAMA(R+1.D0)-DLGAMA(Y-R+1.D0)
      RETURN
      END

      FUNCTION COST(XLS,XSS,NAC,A2R1,R2R1,A2R2,SOS1,AOS1,ROS1)
C**** THIS FUNCTION COMPUTES THE RATIO-UNITS COST ASSOCIATED
C**** WITH THE PLAN (XLS,XSS,NAC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK2/ PBAR,NP
      Y=GNC(XSS,NAC)
      SMHGO=SMHG(XLS,XSS,NAC)
      COST=XSS*(1.D0+PBAR*R2R1)+(XLS-XSS)*(1.D0+PBAR*R2R1+PEXPT(XSS,NAC)
     1/(XSS+1.D0)*(A2R1-R2R1)-Y)+SOS1+AOS1*(Y-SMHGO)+ROS1*(1.D0-Y)
      RETURN
      END

      FUNCTION PEXPT(XSS,NAC)
C**** THIS FUNCTION COMPUTES THE "PARTIAL EXPECTED VALUE"
C**** USING XSS AND NAC
      IMPLICIT REAL*8(A-H,O-Z)
      K=NAC+1
      PEXPT=0.D0
      DO 7 I=1,K
      X=DFLOAT(I)-1.D0
      PEXPT=PEXPT+(X+1.D0)*POLMIX(XSS+1.D0,X+1.D0)
    7 CONTINUE
      RETURN
      END

      FUNCTION SMHG(XLS,XSS,NAC)
C**** THIS FUNCTION OBTAINS THE SUM AS SMALL X RANGES FROM ZERO
C**** TO NAC OF THE PRODUCT OF H(SUBN) OF BIG X = SMALL X GIVEN
C**** SMALL X AND G(SUB SMALL N) OF SMALL X.
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      SMHG=0.D0
      K=NAC+1
      DO 7 I=1,K
      X=DFLOAT(I-1)
      DO 7 J=1,NP
      A=DLGAMA(S(J)+T(J))+DLGAMA(X+S(J))-DLGAMA(S(J))-DLGAMA
     1(T(J))+DLGAMA(XLS-X+T(J))-DLGAMA(XLS+S(J)+T(J))+DLOG(W(J))+
     2COMBO(XSS,X)
      IF(A.LT.-150.D0)A=-150.D0
      A=DEXP(A)
      SMHG=SMHG+A
    7 CONTINUE
      RETURN
      END

      FUNCTION GNC(XSS,NAC)
C**** THIS FUNCTION COMPUTES THE PARTIAL SUM (ZERO TO NAC) OF
C**** G(SUB SMALL N) OF SMALL X
      IMPLICIT REAL*8(A-H,O-Z)
      K=NAC+1
      GNC=0.D0
      DO 7 I=1,K
      X=DFLOAT(I-1)
    7 GNC=GNC+POLMIX(XSS,X)
      RETURN
      END

      FUNCTION HNXEX(XLS,SS,X)
C**** THIS FUNCTION COMPUTES H(SUBN) OF BIG X = SMALL X GIVEN SMALL X
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK1/ W(5),S(5),T(5)
      COMMON /BLK2/ PBAR,NP
      SUM=0.D0
      TOT=0.D0
      DO 7 I=1,NP
```

```
      A=DLGAMA(S(I)+T(I))+DLGAMA(X+S(I))-DLGAMA(S(I))-DLGAMA
     4(T(I))+DLGAMA(XLS-X+T(I))-DLGAMA(XLS+S(I)+T(I))
      IF(A.LT.-90.DO)A=-90.DO
      A=DEXP(A)
      SUM=SUM+W(I)*A
      B=DLGAMA(S(I)+T(I))+DLGAMA(X+S(I))-DLGAMA(S(I))-DLGAMA
     3(T(I))+DLGAMA(SS-X+T(I))-DLGAMA(SS+S(I)+T(I))
      IF(B.LT.-90.DO)B=-90.DO
      B=DEXP(B)
    7 TOT=TOT+W(I)*B
      HNXEX=SUM/TOT
      RETURN
      END
```

# INSTRUCTIONS

RUNNING LANIF.FORT IN INTERACTIVE MODE:

1. REMOVE "CHARACTER*1 Q" AND "CHARACTER*30 TITLE" STATEMENTS

2. REMOVE ALL "TITLE = " AND " Q = " STATEMENTS

3. REMOVE ALL "PRINT," STATEMENTS

NOTE: 1., 2., and 3 are easily accomplished by placing a "C" in column 1 of each statement to be "removed".

4. WHEN LOGGING ON, INCLUDE "SIZE(1200)

   e.g., LOGON U12345A/PSWD SIZE (1200)

5. USE THE FOLLOWING STATEMENT IN READY MODE:

   %RUNVFORT LANIF.FORT OPTIONS('LANGLVL(66) NOSOURCE NOSRCFLG')

THE FOLLOWING PROMPTS WILL APPEAR IN INTERACTIVE MODE WHEN DOLLAR COSTS AND THE OPTIMAL PLAN ARE KNOWN (NOPT=1)

```
INPUT A "1" IF OUTPUT IS TO BE AT A CRT
INPUT A "O"    IF OUTPUT IS TO BE PRINTED ON PAPER
INPUT A "O" IF ONLY THE RATIO PLANS ARE TO BE GENERATED
INPUT A "1" IF DELTA IS TO BE CALCULATED
INPUT A "O" IF THE PRIOR IS IN MIXED BINOMIAL FORM
INPUT A "1" IF PRIOR IS A MIXED POLYA
INPUT THE FIXED COSTS - SO, AO, AND RO
INPUT THE UNIT COSTS - S1,S2,A1,A2,R1,AND R2
INPUT THE NUMBER OF POINTS IN THE PRIOR
INPUT THE WEIGHTS ASSIGNED TO EACH POINT IN THE PRIOR
INPUT EACH OF THE MIXED BINOMIAL P VALUES
INPUT THE LOT SIZE
INPUT THE RATIO A2/R2; INCLUDE DECIMAL
SELECT FROM 1,2,4,8,16,32, OR 64
INPUT THE RATIO R2/R1; INCLUDE DECIMAL
ELECT FROM .125, .250, .50, 1, 2, 4, 8, 16, 32, OR 64
DO YOU WISH TO INCLUDE ANY FIXED COST RATIOS ?; 1=YES    O=NO
SELECT ONE OF THE FOLLOWING:
1    SO/S1 = 1000
2    SO/S1 = 10000
3    AO/S1 = 1000
4    AO/S1 = 10000
5    AO/S1 = 1000 AND RO/S1=100
INPUT OPTIMAL SAMPLE SIZE (REAL), ACC. NO. (INTEGER), AND THE TOTAL COST (REAL)
```

IF THE DOLLAR COSTS AND OPTIMAL PLAN ARE NOT KNOWN, THEN THE PROMPTS FOR ENTERING THE FIXED COSTS, UNIT COSTS, AND OPTIMAL SAMPLE SIZE, ACCEPTANCE NUMBER, AND TOTAL COSTS WILL NOT APPEAR.

WHEN RUNNING IN BATCH MODE, (UNDER WATFIV), EACH PROMPT MUST BE ANSWERED SEQUENTIALLY IN ADVANCE JUST AFTER THE $ENTRY LINE. FOR EXAMPLE, SUPPOSE THAT THE FIXED COSTS, UNIT COSTS AND OPTIMAL PLAN ARE KNOWN. THE SEQUENCE WOULD BE AS FOLLOWS:

```
$ENTRY
0
1
0
220. 470. 160.
6. 36. 0. 128. 8. 32.
3
.60 .25 .15
.02 .10 .30
1000.
85. 5 7793.26
//
```

IF COSTS AND THE OPTIMAL PLAN ARE NOT KNOWN, THE SEQUENCE WOULD BE:

```
$ENTRY
0
0
0
3
.60 .25 .15
.02 .10 .30
1000.
//
```

IN BATCH MODE, ALLOW 30 MINUTES OF CPU TIME.

IT IS SUGGESTED THAT ALL RUNS BE MADE IN CLASS=4.

APPENDIX D

DECISION MATRICES--PRIOR 2

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING

COST RATIO DECISION MATRIX

| A2R2 | R2R1 | 1/8 (0.0–.18) | 1/4 (.18–.35) | 1/2 (.35–.71) | 1 (.71–1.41) | 2 (1.41–2.83) | 4 (2.83–5.66) | 8 (5.66–11.31) | 16 (11.31–22.63) | 32 (22.63–45.25) | 64 (45.25–90.51) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 (0.00–1.41) | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 (1.41–2.83) | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 19 | 26 | 30 | 33 | 37 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 |
| 4 (2.83–5.66) | SAMP.SIZE | 0 | 0 | 0 | 0 | 24 | 28 | 32 | 35 | 53 | 1000 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 3 | 0 |
| 8 (5.66–11.31) | SAMP.SIZE | 0 | 0 | 7 | 25 | 29 | 32 | 36 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 0 | 0 | 0 |
| 16 (11.31–22.63) | SAMP.SIZE | 0 | 17 | 26 | 29 | 33 | 37 | 1000 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 0 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 |
| 32 (22.63–45.25) | SAMP.SIZE | 18 | 26 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 |
| 64 (45.25–90.51) | SAMP.SIZE | 26 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1 = 1000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | 1/8 (0.0–.18) | 1/4 (.18–.35) | 1/2 (.35–.71) | 1 (.71–1.41) | 2 (1.41–2.83) | 4 (2.83–5.66) | 8 (5.66–11.31) | 16 (11.31–22.63) | 32 (22.63–45.25) | 64 (45.25–90.51) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 (0.00–1.41) | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 (1.41–2.83) | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 30 | 33 | 37 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 |
| 4 (2.83–5.66) | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 32 | 35 | 53 | 1000 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 3 | 0 |
| 8 (5.66–11.31) | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 32 | 36 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 |
| 16 (11.31–22.63) | SAMP.SIZE | 0 | 0 | 0 | 29 | 33 | 37 | 1000 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 |
| 32 (22.63–45.25) | SAMP.SIZE | 0 | 0 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 |
| 64 (45.25–90.51) | SAMP.SIZE | 0 | 30 | 33 | 37 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | ACC. NBR. | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1 = 10000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | |
| 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 – 1.41 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 – 2.83 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 4 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 2.83 – 5.66 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 8 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 5.66 – 11.31 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 16 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 11.31 – 22.63 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 32 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 22.63 – 45.25 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 64 | SAMP.SIZE | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 45.25 – 90.51 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 1000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | |
| 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 – 1.41 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 2 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1.41 – 2.83 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 4 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 2.83 – 5.66 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 8 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 5.66 – 11.31 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 11.31 – 22.63 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 22.63 – 45.25 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 45.25 – 90.51 |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 10000
COST RATIO DECISION MATRIX

0.0 .18 .35 .71 1.41 2.83 5.66 11.31 22.63 45.25 90.51

| | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | 0.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| | 2 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| A | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| | 4 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| 2 | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| | 8 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| R | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| | 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| 2 | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 1000; RO/S1 = 100
COST RATIO DECISION MATRIX

0.0 .18 .35 .71 1.41 2.83 5.66 11.31 22.63 45.25 90.51

| | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | 0.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| | 2 | SAMP.SIZE | 0 | 0 | 4 | 15 | 18 | 14 | 1000 | 1000 | 1000 | 1000 | |
| A | | ACC. NBR. | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| | 4 | SAMP.SIZE | 0 | 13 | 16 | 19 | 4 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| 2 | | ACC. NBR. | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| | 8 | SAMP.SIZE | 14 | 17 | 13 | 5 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| R | | ACC. NBR. | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| | 16 | SAMP.SIZE | 17 | 14 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| 2 | | ACC. NBR. | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 14 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

APPENDIX E

DECISION MATRICES--PRIOR 3

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1, AO/S1, AND RO/S1 < 1000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| A | 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 2 | 4 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 244 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 22 | 0 | 0 | 0 | 0 | 5.66 |
| R | 8 | SAMP.SIZE | 0 | 0 | 0 | 0 | 373 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 31 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 2 | 16 | SAMP.SIZE | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1 = 1000
COST RATIO DECISION MATRIX

| A2R2 | R2R1 | | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| A | 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 2 | 4 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| R | 8 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 2 | 16 | SAMP.SIZE | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| | 32 | SAMP.SIZE | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| | 64 | SAMP.SIZE | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
SO/S1 = 10000
COST RATIO DECISION MATRIX

A2R2

| R2R1 | | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 4 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| 8 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 16 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| 32 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| 64 | SAMP.SIZE | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 1000
COST RATIO DECISION MATRIX

A2R2

| R2R1 | | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 – .18 | .18 – .35 | .35 – .71 | .71 – 1.41 | 1.41 – 2.83 | 2.83 – 5.66 | 5.66 – 11.31 | 11.31 – 22.63 | 22.63 – 45.25 | 45.25 – 90.51 | 0.00 |
| 1 | SAMP.SIZE | 0 | 499 | 499 | 499 | 499 | 499 | 499 | 499 | 499 | 499 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.41 |
| 2 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.83 |
| 4 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.66 |
| 8 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.31 |
| 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22.63 |
| 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 45.25 |
| 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | |
| | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90.51 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 10000
COST RATIO DECISION MATRIX

R2R1 column boundaries: 0.0, .18, .35, .71, 1.41, 2.83, 5.66, 11.31, 22.63, 45.25, 90.51

A2R2 row boundaries: 0.00, 1.41, 2.83, 5.66, 11.31, 22.63, 45.25, 90.51

| A2R2 | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | 2 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 4 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| R | 8 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

GUTHRIE-JOHNS MODEL
SINGLE ATTRIBUTE ACCEPTANCE SAMPLING
AO/S1 = 1000; RO/S1 = 100
COST RATIO DECISION MATRIX

R2R1 column boundaries: 0.0, .18, .35, .71, 1.41, 2.83, 5.66, 11.31, 22.63, 45.25, 90.51

A2R2 row boundaries: 0.00, 1.41, 2.83, 5.66, 11.31, 22.63, 45.25, 90.51

| A2R2 | | R2R1 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | SAMP.SIZE | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | 2 | SAMP.SIZE | 0 | 0 | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 4 | SAMP.SIZE | 0 | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| R | 8 | SAMP.SIZE | 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 16 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 32 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 64 | SAMP.SIZE | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| | | ACC. NBR. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

VITA

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