Optimizing the Compactness of Political Districts in Oklahoma

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Introduction

Political districting is the process of dividing an area (such as a state) into a certain number of districts, which are used for election purposes. There are several requirements for these districts, including population equality, contiguity, and compactness. There are also other regulations on districts such as the Voting Rights Act, but these are beyond the scope of this work.

My project focuses on a method to mathematically determine an optimally compact district map for Oklahoma. Compactness in this case is measured using the "moment of inertia" (Hess et al., 1965) of district population about the district center. This function is called the objective function, and minimizing it is the goal of this model.

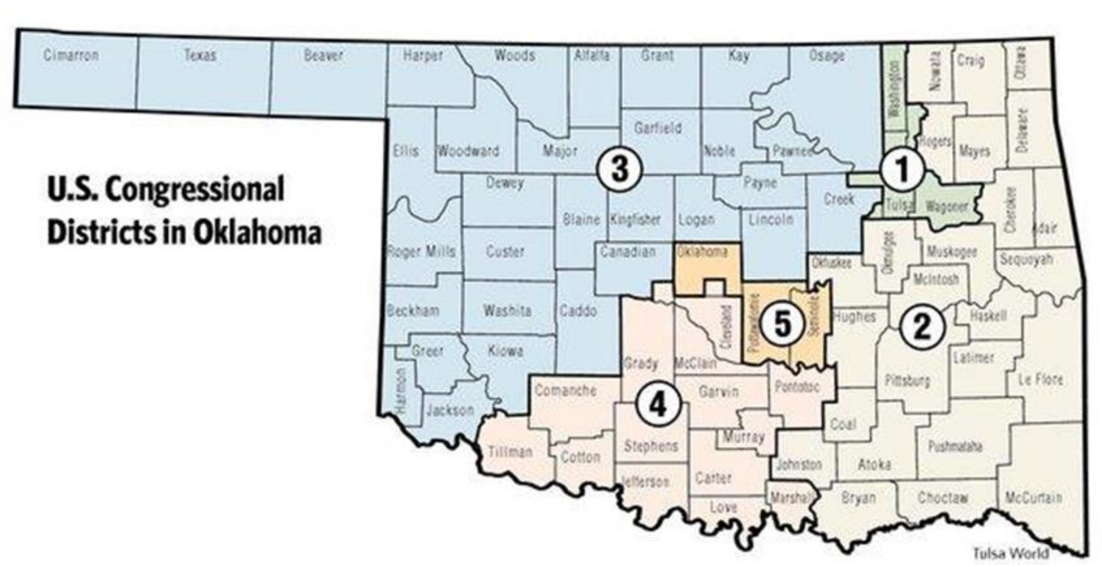

Figure 1: Oklahoma's Current Congressional Districts (tulsaworld.com)

Objectives

- Create a Python and Gurobi program that applies the Hess districting model to a state, creating optimally compact political districts using counties as indivisible territorial units
- Apply this model to the state of Oklahoma, and create a map of the resulting districts to visualize the results; The tools needed for this are ArcGIS and a map-creating code created for use in this project by Jessey Yeager
- Experiment with the model: observe effect on compactness created by varying population bounds and objective function

Experimentation

Optimization Model

- The objective function contains the moment of inertia, the product of population of counties and the squared distance between the county centers

Constraints impose the following conditions:

- Each county must be assigned to one district
- There is a fixed number of districts (five in the case of Oklahoma)
- The population of each district must be between the set upper and lower population bounds

Variables

- x_{ij} is a decision variable; $x_{ij}=1$ if a county i is assigned to a district center j; 0 otherwise
- x_{jj} is a decision variable; $x_{jj}=1$ if a county j is selected as a district center; 0 otherwise

Parameters

- d_{ij} is the distance between the centers of two counties i and j, $(i, j = 1, 2, \ldots, n)$
- p_i is the population of a county i, $(i = 1, 2, \ldots, n)$
- k is the set number of districts
- n is the number of counties (territorial units)
- $\bar{P}$ is the ideal district population, defined as the total state population divided by the number of districts. (Hess et al, 1965)
- a and b are percentages of population difference

$$min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}^2 \, p_i x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{jj} = k$$

$$a\bar{P}x_{jj} \le \sum_{i=1}^{n} p_i x_{ij} \le b\bar{P}x_{jj}, \qquad j = 1, \ldots, n$$

$$x_{ij} \in \{0, 1\} \qquad i, j = 1, \ldots, n$$

Population Bound Experimentation

- I started my experimentation by running experiments in which I incrementally changed the upper and lower population bounds for a district
- Population equality between districts is an important factor in their creation, in order to ensure the one-person, one-vote rule is applied. In practice, population difference is usually within 1%
- I changed the a and b values first within an interval that would result in population differences between 1% and 10% (the larger difference was due to the limitations imposed by the assumption of indivisible counties)

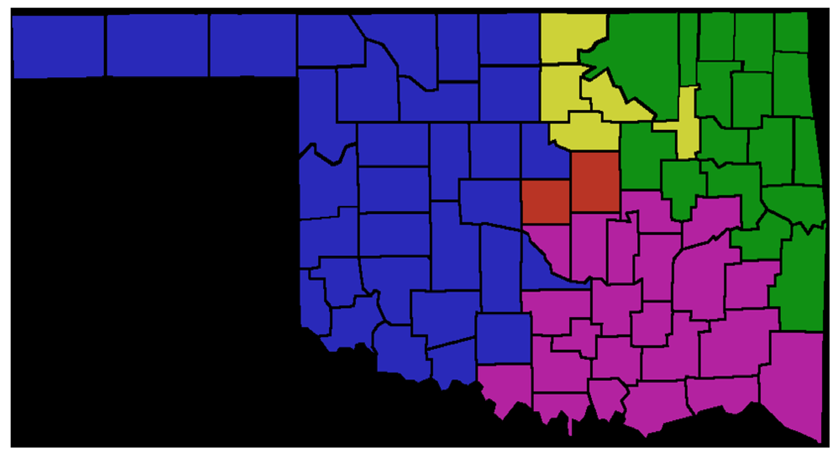

Figure 2: District assignments when the difference between the upper and lower population bounds is 2%

- I also ran tests for which the population bounds were within 1%
- However, none of the maps created by these tests contained connected districts
- This is because the Hess model does not impose contiguity

Population-Weighted vs. Visual Compactness

The Hess districting model results in district maps that are compact in respect to the people in the district.

In order for the maps to appear geographically compact, compactness in respect to the counties themselves should be considered. I ran the same tests as before but removed population from the objective function in order to observe this effect.

- The resulting maps had disconnected districts as a result of the need for population balance, since the counties are indivisible

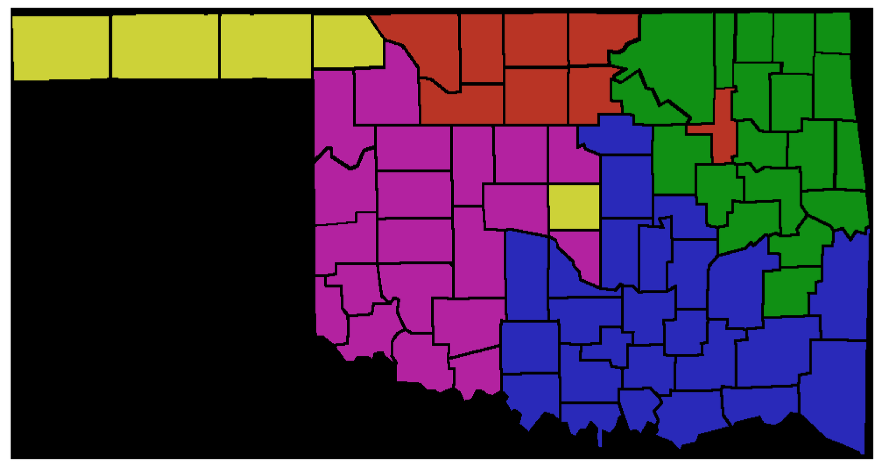

Figure 3: District assignments when population is not included in the objective function.

Conclusions

- The Hess Districting Model can be used to find optimally compact congressional districts for Oklahoma
- In order to satisfy connectivity constraints for realistic districts, a population unit smaller than a county might be beneficial
- If the counties could be split between districts instead of the entire county included in one district, it might be possible to create connected district maps with tighter upper and lower population bounds
- When population is included in the objective function, the resulting map is compact in respect to the people in the district, but was not connected with realistic population bounds
- When population is excluded from the objective function, districts appear geographically compact, but disconnected due to population bound constraints. This might be aided with a smaller population unit, as well as the implementation of contiguity constraints

References

Hess, S. W., Weaver, J.B., H.J., Whelan, J.N., & Zitlau, P.A. (1965). Nonpartisan political redistricting by computer. *Operations Research, 13*, 998-1006.

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