RESPONSE OF A CLOSED-LOOP PIPING SYSTEM
HAVING MULTIPLE INLETS
AND OUTLETS

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INTRODUCTION

The topic of this thesis is an outgrowth of research currently being carried out at the School of Civil Engineering, Oklahoma State University, on liquidborne noise in hydraulic piping systems. One of the goals of this research project is to be able to determine a frequency spectrum for the gain in amplitude of small deviations from mean flow conditions between two points in a complex piping system.

Thus far, most of the work has been restricted to single pipes or to series of pipes connected end to end. Some work has been done with parallel piping systems, and the effect of small "stub" lines (small closed-end pipe segments connected to a series system) has been investigated.

The purpose of this thesis is to develop a procedure for determining similar relationships for a closed-loop piping system with many inlets and outlets. As in the previous cases, an attempt is made to study such a system for both deterministic and stochastic inputs. An example problem is presented to illustrate the actual procedure involved.

CHAPTER I

BACKGROUND

1.1. Literature.

An account of the bulk of the work that has been done at Oklahoma State University in regard to liquidborne noise may be found in a publication by Waller and Hove [4]*. This includes a compilation of the pertinent material of previous publications by Waller [5, 6, 7, 8, 9], Hove [2], and Childs [1]. Surveys of other literature associated with the general problem is presented in these works.

1.2. Hydraulic Transients [4].

The system-describing equations for the wave propagation of slight variations of pressure and flow rate, p and q, from the mean pressure and flow rate, $\overline{p}$ and $\overline{q}$, using Fig. 1.1 as a defining sketch, are:

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} + L\frac{\partial q}{\partial t} + Rq &= 0 \quad , \\ -\frac{\partial q}{\partial x} + C\frac{\partial p}{\partial t} &= 0 \quad , \end{aligned}\right\} \qquad (1)$$

where

$$R = \frac{n\,\overline{p}_f}{\ell\,\overline{q}}$$

---

* Numbers in brackets refer to listing in the Bibliography.

$L = \frac{\rho}{A}$

$C = \frac{A}{K'}$

n = an exponent for mean flow

$\bar{P}_f$ = pressure required to overcome pipe friction

ρ = density of fluid medium

A = cross-sectional area of pipe

K' = bulk modulus of fluid-elastic pipe system.

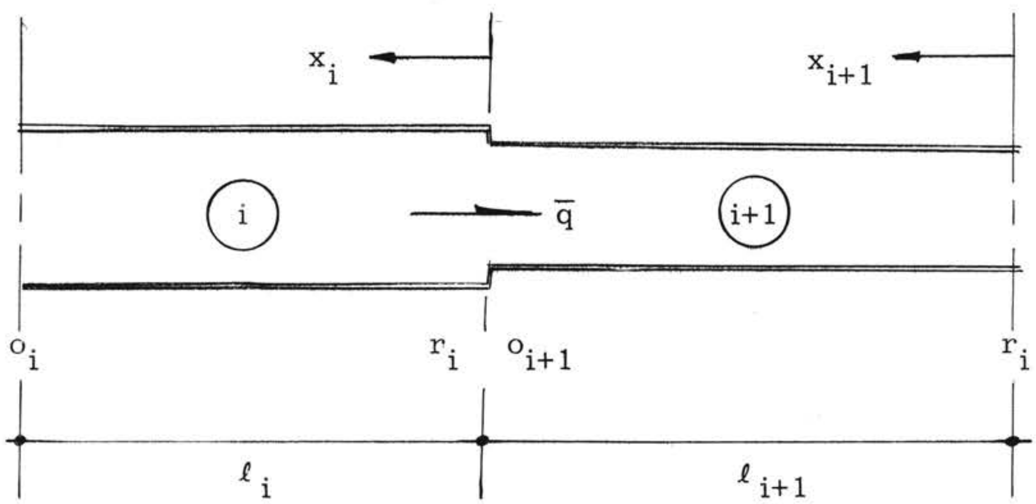

Figure 1. 1. Defining sketch for differential equations.

Using Laplace transform methods, the solution of (1) is:

$$\left.\begin{aligned} P(x,\ s) &= P(o,\ s)\cosh\gamma x - Z_c\, Q(o,\ s)\sinh\gamma x \quad , \\ Q(x,\ s) &= Q(o,\ s)\cosh\gamma x - \frac{1}{Z_c} P(o,\ s)\sinh\gamma x \quad , \end{aligned}\right\} \qquad (2)$$

or

$$\left.\begin{aligned} P(x, s) &= P(r, s)\cosh\gamma x + Z_c\, Q(r, s)\sinh\gamma x \quad , \\ Q(x, s) &= Q(r, s)\cosh\gamma x + \frac{1}{Z_c} P(r, s)\sinh\gamma x \quad , \end{aligned}\right\} \qquad (3)$$

where G(x, s) is the Laplace transform (with respect to time) of g(x, t). That is,

$$G(x, s) = \int_0^{\infty} g(x, t)\, e^{-st}\, dt. \qquad (4)$$

Also,

$$\gamma^2 = (\alpha + j\beta)^2 = s\, C(R + s\, L) \quad , \qquad (5)$$

$$Z_c^2 = \frac{R + sL}{sC} \quad . \qquad (6)$$

If only the frequency response is desired, which is indeed the case here, $j\omega$ may be substituted for the Laplace variable $s (j = \sqrt{-1})$. Considering this,

$$\left.\begin{aligned} \alpha &= \left[\frac{\omega C}{2}\left(\sqrt{R^2 + \omega^2 L^2} - \omega L\right)\right]^{\frac{1}{2}} \quad , \\ \beta &= \left[\frac{\omega C}{2}\left(\sqrt{R^2 + \omega^2 L^2} + \omega L\right)\right]^{\frac{1}{2}} \quad , \\ Z_c &= \frac{1}{\omega C}(\beta - j\alpha) \quad . \end{aligned}\right\} \qquad (7)$$

It has been shown that for most values of ω, the above expressions are sufficiently approximated by:

$$\alpha = \frac{R}{2La} ,$$

$$\beta = \frac{\omega}{a} ,$$

$$Z_c = La ,$$

where

$$a = \sqrt{K'/\rho} = \text{velocity of wave propagation.}$$

Letting the (') indicate the opposite end of the pipe from the reference, the pressure-flow relationships across the joint of pipe, i, can be written:

$$\left.\begin{aligned} P_i' &= B_i P_i + T_i Q_i , \\ Q_i' &= B_i Q_i + M_i P_i , \end{aligned}\right\} \quad (8)$$

where

$$\left.\begin{aligned} B_i &= \cosh \gamma_i \ell_i \\ T_i &= \pm Z_{ci} \sinh \gamma_i \ell_i \\ M_i &= \pm \frac{1}{Z_{ci}} \sinh \gamma_i \ell_i \end{aligned}\right\} \quad \begin{aligned} &\text{+ or - depending upon} \\ &\text{direction of mean flow.} \end{aligned} \quad (9)$$

It should be evident that one need only know the transformed pressure and flow at one end of a series of pipes to use the above formulae to predict the corresponding values at the other end.

CHAPTER II

ANALYSIS OF THE CLOSED-LOOP SYSTEM

2.1. Physical Description.

A schematic sketch of a typical closed-loop piping system with multiple inlets and outlets is shown below in Fig. 2.1. For the purpose of the discussion presented here, the known conditions of pressure and flow variation appear at the outlets; i.e., the outlet pressure (or flow rate) becomes the "input" to the system, with the inlet pressure (or flow rate) becoming the "output."

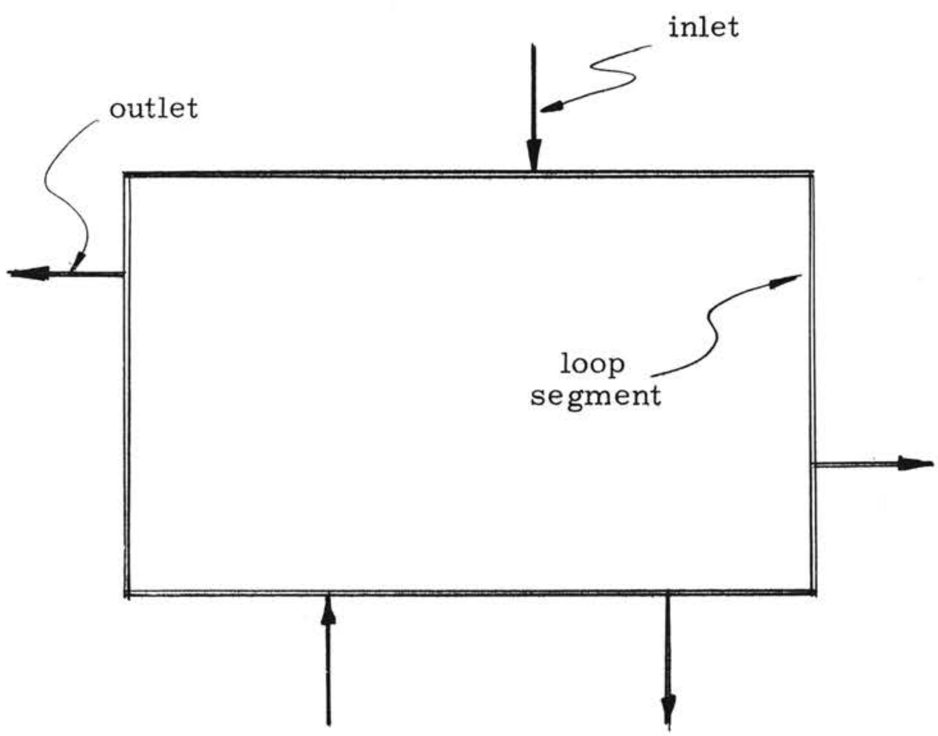

Figure 2.1. Typical closed-loop system.

The directions and rates of mean flow must be known before the transient analysis can proceed, as these quantities figure in the noise propagation properties.

2.2. Analysis of the System.

To begin, in order to establish positive directions so that a set of equations may be built up systematically, the mean flow is assumed to be in a clockwise direction throughout the loop, and to be flowing in from all projecting lines, as indicated in Fig. 2.2. Adjustments must be made before the solution is completed, as explained later in this thesis. The pipe sections (between junctions) in the loop will be designated merely by an Arabic numeral, and the projecting lines by I_i $(i = 1, 2, 3, \cdots, n)$.

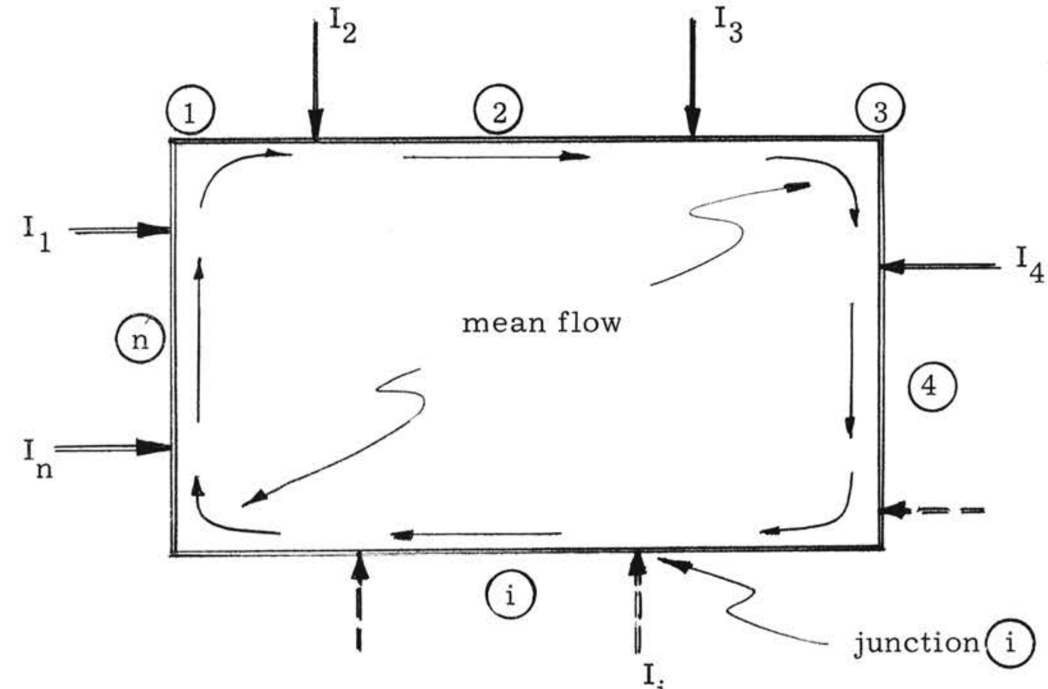

Figure 2.2. System configuration, indicating positive directions.

The quantities P_i and Q_i refer to Laplace-transformed (with respect to time) pressure and flow rate variations at the far end of a loop pipe section (in a clockwise sense), while P_i' and Q_i' refer to similar quantities at the opposite end of the same pipe section. Thus, coming into junction (i) are Q_{i-1} and QI_i, and flowing out is Q_i. Evidently, then, the following equations may be written:

$$\left.\begin{aligned} P_i &= P'_{i+1} = PI_{i+1} \quad , \\ &\text{and} \\ Q_i + QI_{i+1} &= Q'_{i+1} \quad . \end{aligned}\right\} \tag{9}$$

Using equations (7), these become:

$$\left.\begin{aligned} P_i &= PI_{i+1} = B_{i+1}\, P_{i+1} + T_{i+1}\, Q_{i+1} \quad , \\ &\text{and} \qquad\qquad (i = 1,\ 2,\ \cdots,\ n) \\ Q_i + QI_{i+1} &= B_{i+1}\, Q_{i+1} + M_{i+1}\, P_{i+1} \quad . \end{aligned}\right\} \tag{10}$$

Consider the partial loop in Fig. 2.3. Knowing nothing about the system, the P's and Q's are written at their corresponding stations as "potential" unknowns. Those quantities located at the near end (in a clockwise sense) of each loop pipe may be eliminated by equations (8); thus, the (/) mark. Now, the P's and Q's are assumed known at the outlets; this, along with the first of equations (9), justifies the (\) mark. If m equals the number of actual inlets, this leaves m unknown P's and 2m unknown Q's at the inlet junctions, and (n-m) unknown Q's at the outlet junctions - all told, there are (2m + n) unknowns.

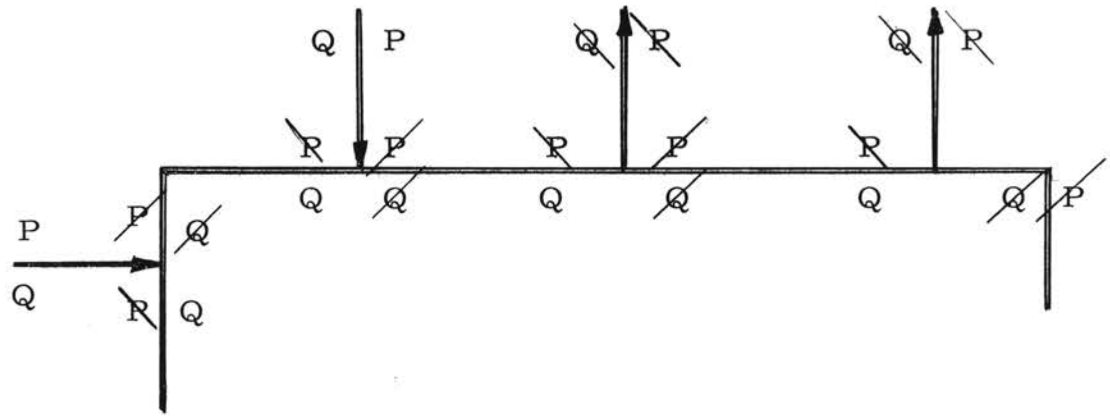

Figure 2. 3. Part of a loop system.

Thus far, only 2n equations have been developed, with (n-m) of these no longer independent due to known conditions at the outlets. This leaves (n+m) equations.

Some way, then, must be found to write m more independent equations to completely describe the transformed pressure and flow variations at the junctions of the system. If the impedances (the ratios Z = P/Q) are known at the inlets, then the required m relationships are immediately at hand.

If, however, nothing is known about the inlet impedances, or if they cannot be obtained by any reasonable calculations, the problem becomes considerably more complicated. One possible solution is to arrive at these values by an iteration process, as set out in Section 2. 5 of this thesis.

2. 3. Solution with Known Inlet Impedances.

It has been shown that there are (2m + n) unknowns to solve for, where n = the number of lines projecting from the loop, and m = the number of these lines which happen to be inlets.

From the equations of flow and pressure continuity (10), 2n expressions may be written; in matrix form, these are (Eq. 11, Fig. 2.4):

$$
\left[\begin{array}{cccccccccc|cccccccccc|cccccccccc}
1 & -B_2 & & \cdot & & & & \cdot & & & & -T_2 & & \cdot & & & & \cdot & & & & & \cdot & & & & \cdot & & & \\
& 1 & -B_3 & \cdot & & & & \cdot & & & & & -T_3 & \cdot & & & & \cdot & & & & & \cdot & & & & \cdot & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
& & & \cdot & 1 & -B_{i+1} & & \cdot & & & & & & \cdot & & -T_{i+1} & & \cdot & & & & & \cdot & & & & \cdot & & & \\
& & & \cdot & & 1 & -B_{i+2} & \cdot & & & & & & \cdot & & & -T_{i+2} & \cdot & & & & & \cdot & & & & \cdot & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
& & & \cdot & & & & \cdot & 1 & -B_n & & & & \cdot & & & & \cdot & & -T_n & & & \cdot & & & & \cdot & & & \\
& & & \cdot & & & & \cdot & & 1 & -T_1 & & & \cdot & & & & \cdot & & & & & \cdot & & & & \cdot & & & \\
\hline
& -M_2 & & \cdot & & & & \cdot & & & 1 & -B_2 & & \cdot & & & & \cdot & & & 1 & & \cdot & & & & \cdot & & & \\
& & -M_3 & \cdot & & & & \cdot & & & & 1 & -B_3 & \cdot & & & & \cdot & & & & 1 & \cdot & & & & \cdot & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
& & & \cdot & & -M_{i+1} & & \cdot & & & & & & \cdot & 1 & -B_{i+1} & & \cdot & & & & & \cdot & & 1 & & \cdot & & & \\
& & & \cdot & & & -M_{i+2} & \cdot & & & & & & \cdot & & 1 & -B_{i+2} & \cdot & & & & & \cdot & & & 1 & \cdot & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
& & & \cdot & & & & \cdot & & -M_n & & & & \cdot & & & & \cdot & 1 & -B_n & & & \cdot & & & & \cdot & & & 1 \\
-M_1 & & & \cdot & & & & \cdot & & & & & & \cdot & & & & \cdot & & 1 & 1 & & \cdot & & & & \cdot & & &
\end{array}\right]
\left\{\begin{array}{c}
P_1 \\ P_2 \\ P_3 \\ \cdot \\ P_i \\ P_{i+1} \\ P_{i+2} \\ \cdot \\ P_{n-1} \\ P_n \\ \hline
Q_1 \\ Q_2 \\ Q_3 \\ \cdot \\ Q_i \\ Q_{i+1} \\ Q_{i+2} \\ \cdot \\ Q_{n-1} \\ Q_n \\ \hline
QI_1 \\ QI_2 \\ QI_3 \\ \cdot \\ QI_i \\ QI_{i+1} \\ QI_{i+2} \\ \cdot \\ QI_{n-1} \\ QI_n
\end{array}\right\} = 0 .
$$

Figure 2.4. Equation (11)

or

$$[H^*] \left\{z^*\right\} = \left\{0\right\} \tag{11}$$

To these must be added m equations of the form:

$$PI_{i+1} = P_i = ZI_{i+1} \, QI_{i+1} \quad , \tag{12}$$

where ZI_i = impedance at inlet ⓘ.

Then, those equations like

$$P_j = B_{j+1} P_{j+1} + T_{j+1} Q_{j+1} \quad ,$$

where j refers to an outlet junction, are deleted. In the other equations, the multipliers of known quantities (outlet P's and Q's) are transfered to the right side of equations (11). Also, care must be taken to assign a negative sign to all outlet QI's, and the proper sign to the M's and T's, according to the actual direction of mean flow in the loop (as indicated in Section 1.2, the sign is changed if the mean flow is actually counterclock wise).

After these operations are performed, the right hand vector (column matrix on right side of equation) contains the outlet P's and Q's multiplied by certain B's and M's. If both P and Q are known at an outlet, then Z is easily found. Thus, the right hand vector can then be written in terms of various B's, M's, and ($\frac{1}{Z}$)'s multiplied by the outlet P's. If these P's are separated, the general matrix equation may be written:

$$[H] \left\{z\right\} = \sum_k \left\{R_k\right\} P_k \quad , \tag{13}$$

where k refers to outlet numbers. (It is to be noted that, in the symbols employed, $P_i = PI_{i+1}$, so that those k values are one less than those values of ⓘ at the outlet junctions. See Fig. 2.2.)

2.4. Digital Computer Application.

The numerical operations involved in the foregoing procedure are quite extensive. And the whole thing must be gone through for each value of frequency. However, it would not be difficult, if large enough computer facilities were available, to set up a program to handle everything with a minimum of input data consisting of two stages: matrix assembly and matrix inversion.

The first step would be to set up the H* matrix, as in equation (11), assigning the correct signs to the M's and T's. This would take up a $2n \times 3n$ space; adding n rows, and expanding for the complex inversion by the first method in Appendix A, the array would appear thus:

$$\overbrace{}^{3n}\ \overbrace{}^{3n}\ \ 1$$

$$\begin{array}{c} n \\ n \\ n \\ n \\ n \\ n \end{array} \left[\begin{array}{c|c} [\ A\ ] & [\ -a\ ] \\ [\ B\ ] & [\ -b\ ] \\ [\ C\ ] & [\ -c\ ] \\ \hline [\ a\ ] & [\ A\ ] \\ [\ b\ ] & [\ B\ ] \\ [\ c\ ] & [\ C\ ] \end{array}\right] \left\{\begin{array}{c} x \\ \hline y \end{array}\right\} = \left\{ 0 \right\} , \qquad (14)$$

where

$$\begin{bmatrix} A + ja \\ B + jb \end{bmatrix} = \begin{bmatrix} H^* \end{bmatrix},$$

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} z \end{Bmatrix} .$$

The next step might be to place equations (12) into the first m rows of the [C + jc] spaces. Then, the (n-m) equations describing the pressure variation at the outlets could be taken out of the [A + ja] spaces, and the remaining rows "squeezed" upward so that there are no gaps. This would leave a $2(2m + n) \times 6n$ array. Now those columns associated with the known (n-m) values of P_i and QI_{i+1} could be taken out and added to the right side of the equation, along with their respective multipliers, with a change in sign. Then, when the array is "squeezed" to the left so that there are no gaps, there is a $2(2m + n) \times 2(2m + n)$ array on the left, and a series of vectors on the right associated with the various P_i's. This arrangement would represent equation (13).

The foregoing procedure is very straight forward, and could easily be programmed if a large enough computer is available; this program would be able to handle any single-loop system, the limit of projecting lines being set by the computer storage capacity. All that would be required for read-in data would be enough information to compute the attenuation coefficient and B's, M's, and T's for each line.

All of the preceding discussion applies to one value of frequency. On a modern-day large, fast computer, for a not-too-large system (say, a dozen or so inlets and outlets), it would be a matter of seconds, after the program is read in, to feed in a value of frequency, assemble the matrix, invert it, and multiply it through the right-hand vector(s). Even yet, this author envisions hours of computer time to run out a fairly complete frequency spectrum analysis.

According to the findings of the group here at Oklahoma State

University working on the liquidborne noise reduction project, the original equations are, in general, good for frequencies up to 10,000 cps. It is this writer's estimation that, to make a realistic analysis over any extended frequency range, it would be necessary to examine the whole range at increments of 3 to 5 cps, and then to use the spectrum plot obtained as a guide for closer examination of significant frequency bands.

2.5. Solution with Unknown Inlet Impedances.

The situation here is quite unlike that of the previous sections; the system itself is now in complete control of the inlet impedances. But now there are only (m+n) equations to handle (2m+n) unknowns. A way out of this dilemma, as mentioned earlier, would be to use an iterative procedure to arrive at these impedances. This should yield quite good results, the only draw back being the quite extensive amount of time needed for these calculations, even on the fastest digital computers available at the present time.

In such an analysis, a procedure similar to that outlined in the previous section must be gone through several times for each value of frequency. One can see that the time involved would quickly amount to ridiculous proportions if there were many inlets. The method will be included here, however, for completeness.

The way to proceed is to assume some sort of impedance, based on an educated guess, for all the inlets but one, then to investigate the resulting effect on the system, considering the source of the fluid disturbance to be at that one inlet (call it I_1). A ratio of P:Q could then be found at this point. Next, proceed to another inlet (I_2) and repeat, using the previously determined value of impedance at I_1 - then on to I_3, and so on around the loop, all the while substituting values of impedance "determined" as the procedure continues. This to keep going until the impedance values converge within preset limits.

One can visualize that, after a few initial values are arrived at rather blindly and are plotted for the computer operator to observe, subsequent guesses for starting values could be made fairly close to the actual value, speeding up the convergence.

It is to be noted that the relative values of the input (outlet) values figure in the inlet impedance values.

2.6. Stochastic Inputs [2, Appendix B].

Suppose a system with multiple inputs, X_i $(i = 1, n)$, and multiple outputs, Y_k $(k = 1, m)$, can be described in the frequency domain by the following relationship:

$$Y_{Tk}(j\omega) = \sum_{i=1}^{n} H_{ik}(j\omega)\, X_{Ti}(j\omega) \quad . \tag{15}$$

then,

$$Y_{Tk}(-j\omega) = \sum_{i=1}^{n} H_{ik}(-j\omega)\, X_{Ti}(-j\omega) \quad . \tag{16}$$

Multiplying the above two together, dividing by 2T, and taking the ensemble average,

$$\phi_{Y_k Y_\ell} = \sum_{i=1}^{n} \sum_{j=1}^{n} H_{ik}(j\omega)\, H_{j\ell}(-j\omega)\, \phi_{X_i X_j} \quad , \tag{17}$$

where ϕ is the spectral density of a random function.

For this problem, the H terms are the elements of the inverted square matrix of the left side of equation (13). Obviously, to obtain $H(j\omega)$ elements and $H(-j\omega)$ elements, two inversions are necessary. Once these terms are found, it is an easy task to compute the output spectral densities by means of equation (17). The procedure, of course, is considerably longer than that for deterministic inputs; but this should be no terrific obstacle for a large, fast modern computer.

This does, however, increase the work involved for the case of the unknown inlet impedances quite radically.

CHAPTER III

ILLUSTRATIVE EXAMPLE

3. 1. The System.

To illustrate the technique involved in the analysis of a loop system, a system with three inlets and four outlets was chosen, as depicted in Fig. 3. 1. The mean flow pattern indicated was worked out for water at 60° F. flowing through smooth pipe, considering friction losses only. The pipe is six inches in diameter throughout, with the lengths indicated on the sketch.

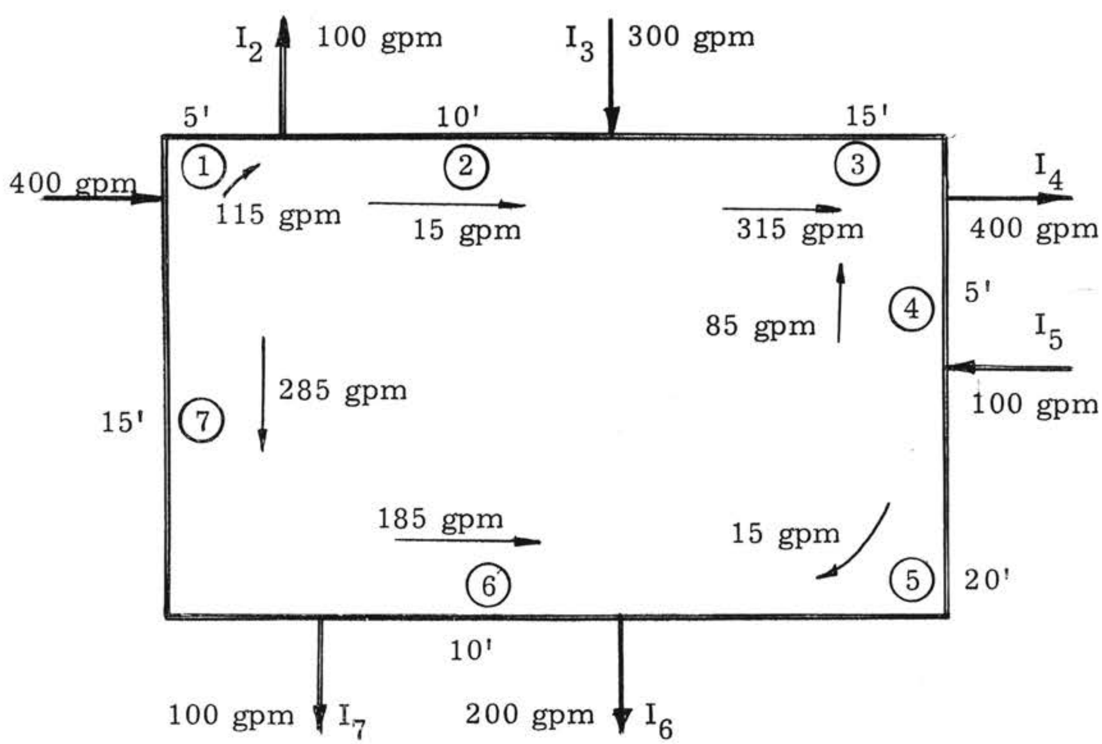

Figure 3. 1. The system.

With the sizes of the lines and the quantities of flow, some attempt has been made to set up a system which might remotely resemble a collection system for sea-water for use in heat exchangers aboard a nuclear submarine of the SS(N) 600 series. (This system is the goal of the research project mentioned before.)

3.2. System Parameters.

K = bulk modulus for water @60° F. = 4.48×10^7 lb/ft^2

E = elastic modulus of cooper-nickel = 2×10^9 lb/ft^2

b = wall thickness of pipe = .200 in.

D = I. D. of pipe = 6.0 in.

$K' = \frac{KbE}{KD + bE}$ = modulus of pipe-water system [5]

$= 2.68 \times 10^7$ lb/ft^2

ρ = density of water at 60° F. = 1.94 slugs/ft^3

a = acoustic velocity of water in pipe = 3720 ft/sec

α = attenuation coefficient = $\frac{nf\bar{v}}{4aD}$ [4]

where f = Darcy - Weisbach friction factor

$\bar{v}$ = mean velocity of water in pipe

β = phase constant

$= \frac{2\pi c}{a} = \frac{c}{592}$, c = frequency in cycles per second.

$$\cosh \gamma \ell = \cosh (\alpha + j\beta)\ell = \cosh \alpha\ell \cos \beta\ell + j \sinh \alpha\ell \sin \beta\ell$$

$$\sinh \gamma \ell = \sinh \alpha\ell \cos \beta\ell + j \cosh \alpha\ell \sin \beta\ell$$

$$\left.\begin{aligned} B &= \cosh \gamma \ell \\ M &= Z_c \sinh \gamma \ell \\ T &= \frac{1}{Z_c} \sinh \gamma \ell \end{aligned}\right\} \quad \text{Transfer functions}$$

$Z_c = \frac{\rho a}{A}$ = system characteristic impedance

$= 36,600 \text{ lb-sec/ft}^5$

Values of α -

Pipe no.	$\alpha \times 10^{-6}$ (ft^{-1})
1	6.41
2	1.38
3	14.30
4	5.09
5	1.38
6	9.40
7	13.18

The next step is to set up 2n equations in matrix form, as in Eq. (11), then to make the necessary corrections, additions, and deletions to achieve the form of Eq. (13).

All the impedances, for both inlets and outlets, are assumed to be the same, due to a discharge into (or flow from) a semi-infinite body of water. The expression for these impedances is assumed to be the same as for a rigid piston radiating into a semi-infinite medium, as set forth in Kinsler and Frey [3]. Contained in this book are expressions for $Z = R + jX$ as infinite series of the form:

$$R(x) = \rho a A \left\{ \frac{x^2}{2 \cdot 4} - \frac{x^4}{2 \cdot 4^2 \cdot 6} + \frac{x^6}{2 \cdot 4^2 \cdot 6^2 \cdot 8} - \cdots \right\}$$

$$X(x) = \rho a A \frac{4}{\pi} \left\{ \frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + \frac{x^5}{3^2 \cdot 5^2 \cdot 7} - \cdots \right\} , \qquad (18)$$

where a = acoustic velocity of semi-infinite medium

= 4800 for water @ 60 F.,

A = cross-sectional area of pipe

$x = \frac{\omega D}{a}$, ω = frequency in cps,

D = diameter of pipe

a = acoustic velocity of water in semi-infinite medium.

For convenience, the two foregoing expressions have been expressed as shorter polynomials, with a fair degree of accuracy [4]:

$$R(x) = \rho a A \left\{ 0.121324\, x^2 - 5.051 \times 10^{-3}\, x^4 + 9.618 \times 10^{-5}\, x^6 - 9.199 \times 10^{-7}\, x^8 + 3.621 \times 10^{-9}\, x^{10} \right\} ,$$

$$X(x) = \rho a A \left\{ 0.4220302\, x^2 - 2.73694 \times 10^{-2}\, x^3 + 7.08794 \times 10^{-4}\, x^5 - 8.4511 \times 10^{-6}\, x^7 + 3.9175 \times 10^{-8}\, x^9 \right\} . \qquad (19)$$

For x between 0 and 8.0, the parts of R(x) and X(x) in the braces lies between 0 and 2.0 - thus it might be convenient to define

$$Z^* = \frac{P}{Q^*} ,$$

where

$$Z^* = Z/\rho aA \quad ,$$

so that the matrix will be better suited for inversion. Here,

$$\rho aA = 49,870 \text{ lb-sec/ft}^5.$$

3.3. Matrix Assembly.

Due to the rather limited computer facilities available (IBM 1620 with 20K storage capacity) this problem was broken down into two parts for solution: a matrix assembly phase and a matrix inversion phase. Furthermore, a little preliminary work had to be done on the matrix assembly before putting it on the computer, as follows.

Taking Equations (11) for this particular example, the results are Equations (20), Fig. 3.2.

Since P_1, P_3, P_5, and P_6 are known, the first, third, fifth, and sixth equations may be thrown out. Then what remains of the first, third, fifth, and sixth columns may be shifted over to the right hand side of the equations, along with their multipliers; the corresponding QI columns (2, 4, 6, and 7) are shifted also, and divided by the impedance so that only P's appear on the right hand side. Next, the three inlet impedance equations are added. Finally, the signs are changed for M and T of lines 4, 6, and 7, whose mean flow is counterclockwise. The results are shown in Equations (21), Fig. 3.3.

The absolute value of P at the outlets is generally specified, so that is the input used in this example. The problem immediately arises as to what to use for the phase angle of the outlet P's, since this transformed quantity is surely complex. For the purposes of this problem

$$
\left[\begin{array}{ccccccc|ccccccc|ccccccc}
1 & -B_2 & & & & & & & -T_2 & & & & & & & & & & & & \\
& 1 & -B_3 & & & & & & & -T_3 & & & & & & & & & & & \\
& & 1 & -B_4 & & & & & & & -T_4 & & & & & & & & & & \\
& & & 1 & -B_5 & & & & & & & -T_5 & & & & & & & & & \\
& & & & 1 & -B_6 & & & & & & & -T_6 & & & & & & & & \\
& & & & & 1 & -B_7 & & & & & & & -T_7 & & & & & & & \\
-B_1 & & & & & & 1 & -T_1 & & & & & & & & & & & & & \\
\hline
& -M_2 & & & & & & 1 & -B_2 & & & & & & 1 & & & & & & \\
& & -M_3 & & & & & & 1 & -B_3 & & & & & & 1 & & & & & \\
& & & -M_4 & & & & & & 1 & -B_4 & & & & & & 1 & & & & \\
& & & & -M_5 & & & & & & 1 & -B_5 & & & & & & 1 & & & \\
& & & & & -M_6 & & & & & & 1 & -B_6 & & & & & & 1 & & \\
& & & & & & -M_7 & & & & & & 1 & -B_7 & & & & & & 1 & \\
-M_1 & & & & & & & -B_1 & & & & & & 1 & 1 & & & & & &
\end{array}\right]
\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ QI_1 \\ QI_2 \\ QI_3 \\ QI_4 \\ QI_5 \\ QI_6 \\ QI_7 \end{Bmatrix} = 0.
$$

Figure 3.2. Equation (20)

$$
\begin{bmatrix}
1 & & & & & -T_3 & & & & & & & \\
& 1 & & & & & & -T_5 & & & & & \\
& & 1 & -T_1 & & & & & & & & & \\
-M_2 & & & 1 & -B_2 & & & & & & & & \\
& & & & 1 & -B_3 & & & & & & 1 & \\
& +M_4 & & & & 1 & -B_4 & & & & & & \\
& & & & & & 1 & -B_5 & & & & & 1 \\
& & & & & & & 1 & -B_6 & & & & \\
& & +M_7 & & & & & & 1 & -B_7 & & & \\
& & & -B_1 & & & & & & 1 & 1 & & \\
1 & & & & & & & & & & -ZI_1 & & \\
& 1 & & & & & & & & & & -ZI_3 & \\
& & 1 & & & & & & & & & & -ZI_5
\end{bmatrix}
\begin{Bmatrix} P_2 \\ P_4 \\ P_7 \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ QI_1 \\ QI_3 \\ QI_5 \end{Bmatrix}
= P_1 \begin{Bmatrix} 0 \\ 0 \\ B_1 \\ -1/ZI_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
+ P_3 \begin{Bmatrix} B_3 \\ 0 \\ 0 \\ 0 \\ M_3 \\ -1/ZI_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
+ P_5 \begin{Bmatrix} 0 \\ B_5 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_5 \\ -1/ZI_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
+ P_6 \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -M_6 \\ -1/ZI_7 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} .
$$

Figure 3.3. Equation (21)

it is assumed that the flow variation is largely resistive (making Q largely real), so that P would very nearly take on the phase angle of the impedance - here it is given the same value.

Letting ϕ equal the value of this phase angle for any given frequency, the right side of Equations (21) becomes:

$$\begin{Bmatrix} 0 \\ 0 \\ (B_1 \cos\phi - BJ_1 \sin\phi) + j(B_1 \sin\phi + BJ_1 \cos\phi) \\ -1/\sqrt{R^2 + X^2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ (M_1 \cos\phi - MJ_1 \sin\phi) + j(M_1 \sin\phi + MJ_1 \cos\phi) \\ 0 \\ 0 \\ 0 \end{Bmatrix} |P_1|, \text{ etc.},$$

where

$$B_i = (B_i + jBJ_i) \quad ,$$

$$M_i = (M_i + jMJ_i) \quad ,$$

$$Z = (R + jX) \quad ,$$

$$\phi = \tan^{-1}(X/R) \quad .$$

The solutions appear in the form:

$$|P_i| = \sum_k G_{ik} \, |P_k| \tag{22}$$

where P_i refers to an inlet pressure, and P_k refers to an outlet pressure.

3.4. Results.

After several matrices were assembled for different frequencies, solutions were obtained on the computer by the inversion process listed in Section A. 3. of the Appendix, which utilizes an ordinary real inversion procedure, modified with complex number operations. Values of frequency examined were 5000, 5005, 5010, 5015, 5020, 5025 cps. It took over one hour to perform these calculations on the 1620 computer; thus, for a complete analysis over a frequency range of several thousand cycles per second, the time would have run up into days. Since the calculation of the gains at the frequencies chosen accomplish the purpose of the example problem (namely, to illustrate the method) those few points were deemed satisfactory.

The results are presented in Table 3. 1. Also, these values are plotted in Figure 3. 4.

One hesitates to draw too many conclusions about the behavior of the system from so little data, but there seems to be no relation between the proximity of an outlet to an inlet and the effect of outlet pressure on inlet pressure. As indicated previously, one would generally need to find the response at considerably more values of frequency.

TABLE 3.1.

GAIN vs. FREQUENCY

f (cps)	G_{21}	G_{23}	G_{25}	G_{26}	G_{41}	G_{43}	G_{45}	G_{46}	G_{71}	G_{73}	G_{75}	G_{76}
5000	.278	.144	.332	.234	.332	.561	.305	.280	.207	.712	.472	.334
5005	.274	.310	.485	.219	.326	.543	.490	.261	.294	.808	.635	.288
5010	.234	.526	.620	.183	.287	.470	.691	.218	.420	.915	.752	.223
5015	.153	.750	.715	.111	.201	.315	.882	.143	.555	1.002	.801	.124
5020	.039	.955	.722	.026	.054	.070	.996	.036	.676	1.020	.740	.027
5025	.084	1.060	.610	.052	.129	.171	.943	.081	.725	.912	.573	.049

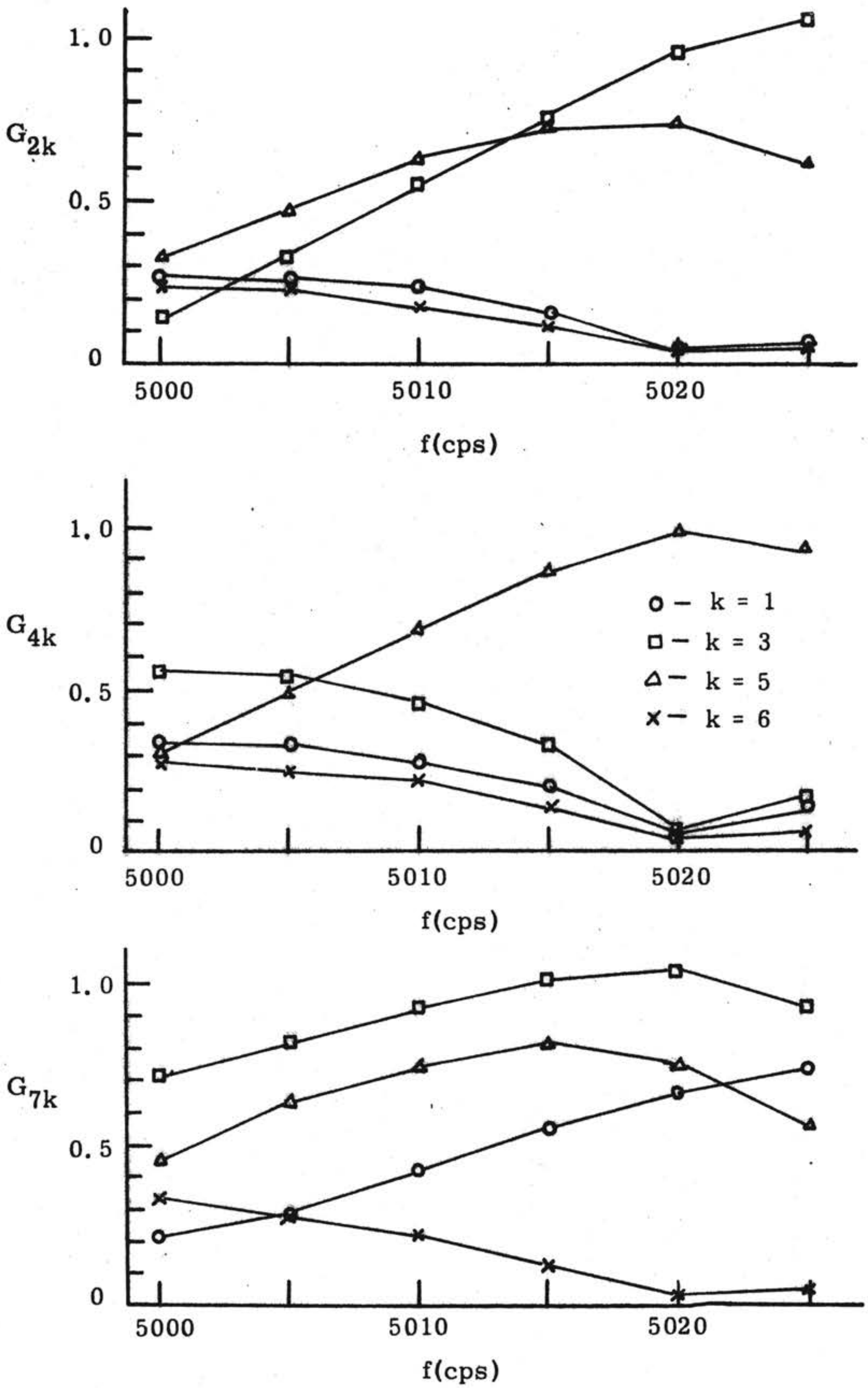

Figure 3.4. Gain vs. Frequency.
(k refers to an outlet pressure)

CHAPTER IV

SUMMARY AND CONCLUSION

In this thesis an attempt has been made to present a workable method for the prediction, at a discrete frequency, of the amplitude of small pressure variations at the inlet lines of a loop piping system, given an allowable or known noise (pressure) level at the outlets. As outlined in Chapter II, a quite systematic procedure was developed for doing just that, provided the inlet impedances are known.

Based on theory developed at Oklahoma State University regarding the relationships between Laplace-transformed fluid pressure and flow fluctuations at opposite ends of an elastic cylinder, a set of equations (13) was drawn up, the solution of which, via matrix operation, presents no problem. Due to the nature of those equations, it was found to be impractical to write algebraic transfer functions relating to inlet and outlet pressures (that is, to solve the system of equations algebraically in order to obtain direct relationships in terms of system parameters and the transform frequency variable. Rather, numbers must be introduced, and the problem must be solved on a digital computer. The calculations involved are quite extensive, as a matrix (containing complex coefficients) inversion must be performed for each value of frequency. Also, for a complete analysis, a large number of frequencies would generally need to be examined. However, this should prove no great hurdle for a large, fast modern-day computer.

For cases where the inlet impedances cannot be determined from external conditions (i.e., when the system itself has complete control over these values), the problem is complicated (rather, compounded) considerably. An iterative procedure for determining these impedances was presented. It was shown, though, that the amount of computation required mounts rapidly in this instance.

The theory involved for the handling of non-deterministic inputs was discussed. The only contribution of these inputs was to increase the length of calculation, especially so in the case of unknown inlet impedances. The sheer weight of the number of operations which must be carried out might even thwart the mightiest computers at hand today as the system increases in size and complexity, particularly in the number of inlets.

In conclusion, it is the author's contention that the method presented is quite practical and practicable, and may be used with good results, the available computational capacity being the only factor which might discourage a comprehensive analysis of the system.

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APPENDIX A

INVERSION OF A COMPLEX MATRIX

A.1. Method I [3].

Suppose n linear simultaneous equations in n unknowns have been written, containing complex terms. In matrix form:

$$C\,z = c \quad , \tag{A-1}$$

where C is the coefficient matrix, z and c are vectors, z containing the unknown quantities.

Denoting:

$$C = A + jB \quad ,$$

$$z = x + jy \quad ,$$

$$c = a + jb \quad ;$$

$$[A + jB]\ \{x + jy\} = \{a + jb\} \quad ,$$

or

$$Ax - By = a$$

and (A-2)

$$Bx + Ay = b \quad .$$

Let

$$\overline{z} = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_n) \quad ,$$

$$\overline{c} = (a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n) \quad ,$$

$$C = \begin{bmatrix} A & -B \\ B & A \end{bmatrix} .$$

Then equations (A-2) can be rewritten:

$$C \bar{z} = \bar{c} . \qquad \text{(A-3)}$$

Obviously, then

$$\bar{z} = C^{-1} \bar{c} , \qquad \text{(A-4)}$$

where

$$C^{-1} C = I .$$

A. 2. Method II [3].

Premultiplying the first of equations (A-2) by A^{-1},

$$A^{-1} Ax - A^{-1} By = A^{-1} a ,$$

or

$$x = A^{-1} By + A^{-1} a . \qquad \text{(A-5)}$$

Then, from the second of equations (A-2),

$$B(A^{-1}By + A^{-1}a) + Ay = b ,$$

or

$$(BA^{-1}B + A)y = b - BA^{-1}a . \qquad \text{(A-6)}$$

Let

$$A_1 = (A + BA^{-1}B)^{-1} .$$

Then

$$y = A_1 b - A_1 BA^{-1} a . \qquad \text{(A-7)}$$

Letting:

$$C^{-1} = \overline{A} + j\overline{B} \quad ;$$

$$(x + jy) = (\overline{A} + j\overline{B})(a + jb) \quad ,$$

or

$$\begin{aligned} x &= \overline{A}a - \overline{B}b \quad , \\ y &= \overline{A}b + \overline{B}a \quad . \end{aligned} \qquad \text{(A-8)}$$

From equation (A-7),

$$\overline{A} = A_1 \quad , \qquad \overline{B} = -A_1 B A^{-1} \quad .$$

Denote

$$A_1 BA^{-1} = B_1 \quad .$$

Then

$$C^{-1} = A_1 - jB_1 \quad ,$$

or

$$\begin{aligned} x &= A_1 a + B_1 b \quad , \\ y &= A_1 b - B_1 a \quad , \end{aligned} \qquad \text{(A-9)}$$

where

$$A_1 = (A + BA^{-1}B)^{-1} \quad ,$$

$$B_1 = A_1 BA^{-1} \quad .$$

A. 3. IBM FORTRAN Program Used to Invert Complex Matrix of Equation (21) .

```
      DIMENSION D(14,14),E(14,14),G(13),H(13),X(13),Y(13)
6     READ 101,N,NZ
      DO 5 I=1,N
      DO 5 J=1,N
      D(I,J)=0.0
5     E(I,J)=0.0
      DO 4 K=1,NZ
4     READ 101,I,J,D(I,J),E(I,J)
      DET=1.0
      DETJ=0.0
      D(1,N+1)=1.0
      E(1,N+1)=0.0
      DO 7 I=1,N
      D(I+1,N+1)=0.0
7     E(I+1,N+1)=0.0
      DO 10 K=1,N
      DET=DET*D(1,1)-DETJ*E(1,1)
      DETJ=DET*E(1,1)+DETJ*D(1,1)
      Q=D(1,1)**2+E(1,1)**2
      PRINT 102,DET,DETJ,Q
      DO 8 J=1,N
      D(N+1,J)=(D(1,J+1)*D(1,1)+E(1,J+1)*E(1,1))/Q
8     E(N+1,J)=(D(1,1)*E(1,J+1)-D(1,J+1)*E(1,1))/Q
      DO 9 I=2,N
      DO 9 J=1,N
      D(I-1,J)=D(I,J+1)-D(I,1)*D(N+1,J)+E(I,1)*E(N+1,J)
9     E(I-1,J)=E(I,J+1)-D(I,1)*E(N+1,J)-E(I,1)*D(N+1,J)
      DO 10 J=1,N
      D(N,J)=D(N+1,J)
10    E(N,J)=E(N+1,J)
      IF (SENSE SWITCH 1) 85,80
80    DO 11 I=1,N
      DO 11 J=1,N
11    PUNCH 101,I,J,D(I,J),E(I,J)
      IF (SENSE SWITCH 2) 85,6
```

```
85      READ 101,NZ,L
        DO 86 I=1,N
        G(I)=0.0
86      H(I)=0.0

        DO 87 K=1,NZ
87      READ 103,I,G(I),H(I)

        DO 89 I=1,N
        X(I)=0.0
        Y(I)=0.0
        DO 88 K=1,N
        X(I)=X(I)+D(I,K)*G(K)-E(I,K)*H(K)
88      Y(I)=Y(I)+D(I,K)*H(K)+E(I,K)*G(K)
89      PUNCH 103,L,X(I),Y(I)

        IF (SENSE SWITCH 2) 85,6

101     FORMAT (I4,I4,E15.8,E15.8)
102     FORMAT (E9.3,E10.3,E10.3)
103     FORMAT (I4,E15.8,E15.8)

        END
```

VITA

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