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SEMI-PARAMETRIC EVALUATION OF RAPID RATE-OF-CHANGE PROPORTIONAL INTENSITY MODELS FOR REPAIRABLE SYSTEMS WITH CENSORING

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SEMI-PARAMETRIC EVALUATION OF RAPID RATE-OF-CHANGE PROPORTIONAL INTENSITY MODELS FOR REPAIRABLE SYSTEMS WITH CENSORING

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Abstract

This research investigates the robustness of four leading proportional intensity (PI) models: PWP-gap time (PWP-GT), PWP-total time (PWP-TT), Andersen-Gill (AG), and Wei-Lin-Weissfeld (WLW), for right-censored recurrent failure event data that follow a Non-homogeneous Poisson Process (NHPP) with log-linear constant or increasing intensity function. The results are beneficial to practitioners in anticipating the more favorable applications domains and selecting appropriate PI models for monitoring failure trends and for decisions in preventive maintenance, service parts inventory, and repair versus replacement. The experimental design has incorporated four levels of censoring severity, three levels of sample size, and seven levels of shape parameter to evaluate these four proposed PI models. The effect of failure event count is also studied. The models of choice are the PWP-GT (for increasing rate of occurrence of failures and low event count) and AG (for constant rate of occurrence of failures), evaluated in terms of three robustness metrics: bias, mean absolute deviation, and mean squared error of covariate regression coefficients. The more favorable engineering application ranges are recommended. Robustness of the PWP-GT for the case of an underlying log-linear increasing intensity function tends to be sensitive to the failure event count. For lower failure counts ($N \leq 4$), the PWP-GT proves to perform well for moderate to severe right-censoring (40% to 80% of units censored), constant and moderately increasing rates of occurrence of failure (log-linear NHPP shape parameter in the range of $0 \leq \theta \leq 0.01$), and small to large sample size ($60 \leq U \leq 180$). The AG model proves to outperform the PWP-TT and WLW for stationary process (HPP) across a wide range of right censorship (0% to 100%) and for sample size of 60 or more. A highly automated SAS macro proved to be a valuable

tool for the research infrastructure in this and future studies.

Keywords: repairable systems reliability, right-censoring, recurrent events, proportional intensity models, log-linear intensity function

Chapter 1

Introduction

A system can be categorized as either repairable or non-repairable. A light bulb is a good example of a non-repairable system; A machine tool, such as a lathe, provides an example of a repairable system. This research addresses statistical modeling of recurrent failure events in repairable systems reliability, by building on previous work of Qureshi (1991), Qureshi et al. (1994), Vithala (1994) and Jiang (2004). They examined the robustness of a semi-parametric Prentice-Williams-Peterson gap-time (PWP-GT) model for estimating the covariate effect where the underlying stochastic process is Non-homogeneous Poisson (NHPP) with power-law or log-linear intensity function, respectively. Both Qureshi and Vithala restricted their studies to the case of uncensored data. Jiang studied the case of censored failure event data drawn from a power-law underlying process.

The structure of this dissertation is organized as follows: Chapter 2 provides a review on the relevant literature with focus on lifetime data with covariate effects, namely the Cox regression model for single failure event (non-repairable) systems, PWP extension to the Cox model, and published engineering applications for multiple failure event (rapairable) systems.

This dissertation addresses the following research question regarding the PI models robustness for the case of an underlying recurrent failure event process that is NHPP with log-linear intensity: How do the PWP-GT, PWP-TT, AG and WLW methods compare in performance under right-censoring?

The research methodology (Chapter 3) is to generate simulated data from a log-linear NHPP process with single two-level covariate and measure the robustness of covariate effect estimates by BIAS, MAD and MSE as a function of censoring severity. The special case of a stationary counting process is examined for the purpose of model validation. For the case of underlying NHPP with log-linear intensity function, the special case of common baseline intensity function (PWP-TT and WLW models) is investigated to compare with the AG model.

Chapter 4 investigates the four semi-parametric PI models under right-censoring for an NHPP and HPP, respectively. The comparison is made between the results from NHPP with power-law and log-linear underlying intensity functions.

The findings in this research provide a handy guide to practitioners in tracking reliability trends for such cases as preventive maintenance schemes, repair versus replacement decisions and service parts inventory management.

Chapter 2

Literature review

Ascher & Feingold (1984) surveyed the theory, methods and engineering applications of reliability methods for repairable systems. They identify the Non-homogeneous Poisson Process (NHPP) as a model for recurrent failure events of a repairable system. They also identify two parametric forms for the NHPP intensity function: power-law and log-linear. Lawless (1987) developed the NHPP proportional intensity models and a Newton-Raphson estimation of the regression coefficients for the covariates. Cox (1972) proposed a semi-parametric proportional hazard (PH) regression model for non-repairable (single failure event) systems. Subsequently, various attempts have been made to adapt the PH model for applications including medical and engineering. The semi-parametric proportional intensity (PI) models relax the assumptions of a single failure event and a parametric NHPP for multiple failure events. The four major PI models are PWP-GT and PWP-TT (Prentice et al., 1981), AG (Andersen & Gill, 1982), and WLW (Wei et al., 1989). Although the PI models were initially proposed for clinical studies in medical applications, they can potentially apply to engineering practice, where the underlying information for a failure process is usually not available. Jiang et al. (2006) provides an exhaustive review on both the available methods for repairable-system (multiple failure events) reliability assessment, and the published engineering application case studies. Jiang's contributions were for the case of underlying NHPP with power-law intensity function. This chapter updates the advancements in this research field.

2.1 Previous robustness studies

Cox (1972) proposed a proportional hazard (PH) model to include explanatory variables (covariates) in life time modeling. Prentice et al. (1981) extended the Cox model to the proportional intensity (PI) of a stochastic processes and applied the approach to model recurrent infections in aplastic anemia and leukemia patients having received bone-marrow transplants. This application involves a small number of events (up to five) for each subject. The paper by PWP did not address the baseline intensity function but rather demonstrated the relative risks for the test and control groups. Researchers in the engineering field have applied the semi-parametric PI/PH models to a variety of industries. Jiang (2004) provided an exhaustive review of the reported engineering applications for Cox-based regression models.

Qureshi (1991) examined the robustness of the PWP-GT model for the case of complete data from a true underlying process that follows the NHPP power-law intensity function. Comparison is made among the PWP-GT estimates, the theoretical parameters, and parametric Lawless (Lawless, 1987) estimates. The study affirms that if there is cause to assume the underlying process is NHPP with power-law baseline intensity function, the Lawless method is preferred over the semi-parametric PWP to model the recurrent failure processes for constant and moderately IROCOF. However, the PWP-GT performs well for an important range of applications, when the underlying process is unknown.

Qureshi et al. (1994) publish the Qureshi (1991) findings for the robustness performance of the PWP-GT method when applied to sample data from a failure process that was actually parametric (specifically the NHPP with power-law intensity function). They conclude that the 2σ bounds of the PWP-GT estimates can cover the true values for a wide range of increasing/decreasing rates of occurrence of failure (ROCOF) with few exceptions. The PWP-GT method performed well, with the exception of small values of the shape parameter ($\delta < 0.6$). The PWP-GT method performs best for larger sample size and for moderately decreasing, constant, and moderately increasing ROCOF. For the PWP-GT estimation of the covariate regression coefficient, the true value of coefficient β lies within the 2σ confidence bounds on the estimate $\hat{\beta}$ for $1.0 \leq \delta \leq 1.4$. The PWP-GT methods tend to underestimate β for a decreasing ROCOF (e.g., BIAS$= -26\%$ at $\delta = 0.5$) and overestimate β for an increasing ROCOF (e.g., BIAS$= 19\%$ at $\delta = 3.0$).

Vithala (1994) studied the PWP-GT method with the NHPP log-linear intensity function for complete data. Vithala conclude that the PWP-GT model performs well in the case of constant and moderately increasing ROCOF, and agrees with Qureshi (1991) that the PWP-GT model is a robust method for many important applications, in which information is not available for the baseline intensity function.

Jiang (2004) assessed the robustness for four Cox-based regression methods (PWP-TT, PWP-GT, AG, and WLW) under right-censoring, when the underlying process follows a NHPP power-law intensity function. Jiang found the PWP-GT and AG methods to be models of choice, evaluated in terms of the bias, mean absolute deviation, and mean squared error of covariate regression coefficients over ranges of sample size, shape parameter, and censoring severity.

2.2 Advancement in methods and applications

There have been few relevant added papers to the literature since Jiang's survey article (Jiang et al., 2006). Huang & Chen (2003) studied the intra-individual correlation that is typically observed in recurrent event data. They suggest and investigate a marginal proportional hazards model for gaps between recurrent events. The inference procedure is based on the establishment of a connection between a subset of the observed gap times and clustered survival data. A novel and general inference procedure is constructed based on a functional formulation of standard Cox regression. Covariates in the model are considered time-independent, although this limitation may be relaxed to some extent, specifically for covariates that depend on time from the earlier episode and have uniform effect across all gaps. Simulation studies suggest that the procedure performs well with practical sample size. Application to the well-known bladder tumor data is given as an illustration, and the results are considered complementary to the existing one.

The connection between the gap times and the clustered survival data is established as follows. Let M denote the number of observed gaps, with the first $M-1$ complete and last one censored at $T^+_{(M)} \equiv C - \sum_{j=1}^{M-1} T_{(j)}$, where $T_{(j)} : j = 1, 2, \cdots$ is the recurrent process. Write $\Delta_i \equiv I(M_i > 1), S_t \equiv \max(M_i - 1, 1)$, and

$$X_{i(j)} \equiv \begin{cases} T_{i(j)} & \text{if } \Delta_i = 1, \\ \\ T^+_{i(j)} & \text{if } \Delta_i = 0 \end{cases}, \quad j = 1, \cdots, S_i$$

the subset consists of $\{X_{i(j)} : j = 1, \cdots, S_i; \Delta_i; \mathbf{Z}_i\}$ which is then passed to the subsequent inference procedure.

Yu et al. (2006) show that the Cox proportional hazards model is applicable in estimating the effects of influential factors on airport runway pavement service life. The research can help in pavement rehabilitation decision-making, overlay design, and budget allocation.

Chapter 8 of Therneau & Grambsch (2000) is devoted to survival analysis to data sets with multiple events per subject, specifically the data formulation and computation algorithm are covered in detail for different Cox-base regression models. Data sets where the multiple events have a distinct ordering and those where they do not are treated separately. An example for the former is multiple sequential infections for a single subject; Paired survival data, such as the subject's two eyes in the diabetic retinopathy data provides an example of unordered outcomes. In terms of the model performance, the variances of covariate effects are assessed.

Chapter 3

Methodology

As an extension of the NHPP power-law model robustness study of Jiang (2004), the four reliability estimates are introduced to handle recurrent event reliability problems with right censoring, where the underlying process is assumed to be NHPP with intensity function of the log-linear form.

Two studies (NHPP and HPP) are conducted for each of the baseline intensity functions (common and event-specific) on PWP-TT and WLW models.

3.1 NHPP

Four Cox-based regression methods (PWP-GT, PWP-TT, AG, and WLW) are used to model recurring failure events from an NHPP with log-linear increasing intensity function, in which right-censoring is explicitly considered. The robustness of each method is evaluated. It is essential to select appropriate baseline hazards and risk interval for adequate modeling. Two classes of baseline intensity functions are considered in this study; namely, the common baseline hazard function and event-specific hazard function. For the risk interval, three types are included: total time model, gap time model, and counting process.

Censorship is a common attribute for most failure event data. In terms of failure history, left-censoring occurs when the early history of failure is not available. Right-censoring arises when the subject or sample is withdrawn from observation. In this research, right-censoring is explicitly modeled. A preset probability controls the proportion of censored sample units in the experiment. The comparison between Cox-based regression methods is made based on the theoretical values of regression coefficients, which measure the covariate effects.

The experiment is designed as follows. Sample units (U) are evenly divided into two groups defined by a single covariate named CLASS. Each sample unit produces up to 10 failure times ($N = 2, \cdots, 10$) generated from an NHPP with a log-linear form, by the Law & Kelton (1991) simulation algorithm as follows:

$$t_n = \frac{1}{\theta}\left[\log(\theta t_i + e^{\mu}) - \mu\right] \tag{3.1}$$

where $t_i = t_{i-1} - \log X_i$ and X_i is a random variate generated from a $(0, 1)$ uniform distribution, n is the failure count, and (θ, μ) are parameters for the log-linear form.

Two covariate levels (CLASS=0 and CLASS=1) are defined by setting the parameter μ to -6.9 and -4.6, thus dividing the observations into two strata.

The data generation algorithm produces complete data; i.e., each sample unit contains an equal number of failures (N). In order to introduce censorship in the model, two groups of sample units were classified, in which one group contains the sample units with complete data and the other group contains the sample units with right-censored data. The portion of the sample units that have right-censored data is defined as censored probability (P_c). It is necessary that the censoring occur randomly across the sample units. A random probability (P_i) is generated to compare with P_c:

the sample unit is specified as a censored unit if $P_i < P_c$; otherwise it is not a censored unit. To generate randomness within the recurring data (failure times in a sequence), another random probability (P_2) is generated. In the censored group, it is a censored time if both of the following conditions are met:

1. the sample unit is a censored unit, and

2. the failure count is greater than F, and $F = \text{floor}(N \times (\text{ranuni(seed)})) + 1$, where *floor* is the function that returns largest integer that is less than or equal to the argument.

The underlying theories for the four Cox-based regression models call for different formulations of the simulated datasets.

For the AG model, the data set is formed from the time interval (T_1, T_2) with respect to the following counting process:

$$\lim_{h \to 0} \frac{1}{h} P\left[N(t+h) - N(t) = 1 \mid T > t\right] = \lambda(t) \tag{3.2}$$

Thus, the logic rule to form the dataset is: $T_2 > T_1$. As a result, all the censored failure times are removed from the dataset since $T_2 = T_1$ is a censored event as stipulated for the AG model.

The theory underlying the PWP method involves conditionality. The later failure times after the n^{th} count cannot be included into the dataset when the intensity function at the n^{th} failure count is estimated. Thus only the censored time for the first censored event count is kept for each censored unit. The record shall be removed if both of the following conditions are met: (1) the current record is marked censored and (2) the previous record is marked censored. The PWP-GT applies to the case of event-specific baseline intensity functions.

The experimental design of this research has three factors: number of the sample units (U), shape parameter (θ), and censoring probability (P_c). I_0 and I_1 represent the number of units in each class. Table 3.1 gives the detailed levels for each of the three factors. The levels are chosen such that they are extensions of the previous relevant works (Qureshi, Vithala, and Jiang) (2) Severe censorship may lead to insufficient data for small sample size (e.g., $U = 20$). P_c, the portion of the censored units, is chosen to reflect three cases: light, moderate, and heavy censoring.

3.2 HPP

The PWP-TT and WLW models apply to the case of a common baseline intensity function. To relax this restriction, four failure events are generated from a stationary, i.e., Homogeneous Poisson Process (HPP) with a right-censoring mechanism and thus the event-specific baseline PWP-TT and WLW models can be studied. As indicated in Jiang (2004), the total time scale (PWP-TT and WLW) has a misspecification problem. For the purpose of capturing the dependence structure that exists among the data, the gap time scale is preferred to the total time scale.

Since stationary data are specified in this study, the simulated data are generated by the same algorithm as in 3.1, except the $\theta = 0$ setting reduces the NHPP to an HPP. The experiment is designed on two factors: sample units (U) and censoring probability (P_c). The levels were selected based on the following consideration: (1) the parameters setting from the previous comparable works (Jiang (2004)) (2) Severe right-censoring may cause the small sample size (e.g., $U = 20$) to have insufficient data to perform model analysis. The P_c levels are chosen to represent light, moderate, and heavy censoring, while the sample units U levels represent the small, median, and large sample sizes.

Table 3.1: Three factor experimental design: (U, θ, P_c)

U	θ	P_c	I_0	I_1	U	θ	P_c	I_0	I_1	U	θ	P_c	I_0	I_1
60	0	0.4	30	30	120	0	0.4	60	60	180	0	0.4	90	90
60	0	0.6	30	30	120	0	0.6	60	60	180	0	0.6	90	90
60	0	0.8	30	30	120	0	0.8	60	60	180	0	0.8	90	90
60	0	1.0	30	30	120	0	1.0	60	60	180	0	1.0	90	90
60	0.001	0.4	30	30	120	0.001	0.4	60	60	180	0.001	0.4	90	90
60	0.001	0.6	30	30	120	0.001	0.6	60	60	180	0.001	0.6	90	90
60	0.001	0.8	30	30	120	0.001	0.8	60	60	180	0.001	0.8	90	90
60	0.001	1.0	30	30	120	0.001	1.0	60	60	180	0.001	1.0	90	90
60	0.002	0.4	30	30	120	0.002	0.4	60	60	180	0.002	0.4	90	90
60	0.002	0.6	30	30	120	0.002	0.6	60	60	180	0.002	0.6	90	90
60	0.002	0.8	30	30	120	0.002	0.8	60	60	180	0.002	0.8	90	90
60	0.002	1.0	30	30	120	0.002	1.0	60	60	180	0.002	1.0	90	90
60	0.004	0.4	30	30	120	0.004	0.4	60	60	180	0.004	0.4	90	90
60	0.004	0.6	30	30	120	0.004	0.6	60	60	180	0.004	0.6	90	90
60	0.004	0.8	30	30	120	0.004	0.8	60	60	180	0.004	0.8	90	90
60	0.004	1.0	30	30	120	0.004	1.0	60	60	180	0.004	1.0	90	90
60	0.008	0.4	30	30	120	0.008	0.4	60	60	180	0.008	0.4	90	90
60	0.008	0.6	30	30	120	0.008	0.6	60	60	180	0.008	0.6	90	90
60	0.008	0.8	30	30	120	0.008	0.8	60	60	180	0.008	0.8	90	90
60	0.008	1.0	30	30	120	0.008	1.0	60	60	180	0.008	1.0	90	90
60	0.01	0.4	30	30	120	0.01	0.4	60	60	180	0.01	0.4	90	90
60	0.01	0.6	30	30	120	0.01	0.6	60	60	180	0.01	0.6	90	90
60	0.01	0.8	30	30	120	0.01	0.8	60	60	180	0.01	0.8	90	90
60	0.01	1.0	30	30	120	0.01	1.0	60	60	180	0.01	1.0	90	90
60	0.02	0.4	30	30	120	0.02	0.4	60	60	180	0.02	0.4	90	90
60	0.02	0.6	30	30	120	0.02	0.6	60	60	180	0.02	0.6	90	90
60	0.02	0.8	30	30	120	0.02	0.8	60	60	180	0.02	0.8	90	90
60	0.02	1.0	30	30	120	0.02	1.0	60	60	180	0.02	1.0	90	90

Table 3.2: Two-factor experimental design: (U, P_c)

$N = 4$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$			
Number of Units U	Censoring Probability P_c	Units per class(I)	
		I_0	I_1
60	0.0	30	30
60	0.4	30	30
60	0.8	30	30
60	1.0	30	30
120	0.0	60	60
120	0.4	60	60
120	0.8	60	60
120	1.0	60	60
180	0.0	90	90
180	0.4	90	90
180	0.8	90	90
180	1.0	90	90

Chapter 4

Robustness study results

4.1 Introduction

A repairable system can fail multiple times in its life cycle. Failure time data on such systems can be viewed as realizations of a stochastic point process, in which the instantaneous rate of occurrence of failures is $\lambda(t)$. Prentice et al. (1981) proposed a semi-parametric (PWP) approach to model recurrent failure event data from a repairable system using two methods: PWP-GT (gap time) and PWP-TT (total time). Alternative modeling methods, among them are AG (Andersen & Gill, 1982) and WLW (Wei et al., 1989) models, were proposed by modifying the risk set (common or event-specific baseline intensity function) and the risk interval (gap time, total time, or counting process).

PWP-GT, PWP-TT, AG, and WLW models are distribution-free (semi-parametric) proportional intensity models based on the proportional hazards (PH) model proposed by Cox (1972). These Cox-based regression models have been applied to recurring events in medical studies (biostatistics field), in which the classical application is the recurrent infection of a patient and to engineering studies (reliability field) for recurrent failure events of repairable systems.

4.2 Models and methods

4.2.1 Cox regression model

Let T denote the random variable representing the time to failure of a system. The Cox PH model takes the following form:

$$h(t; \mathbf{z}) = h_0(t) \exp(\boldsymbol{\beta}'\mathbf{z}), \tag{4.1}$$

where $\boldsymbol{\beta}$ is the vector of regression coefficients and $\mathbf{z}$ is the covariate vector. The PH model can be viewed as a product of a baseline hazard function $h_0(t)$ and an exponential link function $\exp(\boldsymbol{\beta}'\mathbf{z})$, where $\boldsymbol{\beta}$ measures the covariate effect. Thus, the Cox model describes the semi-parametric distribution of time-to-failure for single event systems with covariates. Under proportional hazards, the ratio of the hazard functions of two units (A and B) with covariate vectors $\mathbf{z}_A$ and $\mathbf{z}_B$ is constant over time. The covariates have a multiplicative effect on the baseline hazard function. When the baseline hazard function is fully specified (e.g., Weibull) the analytical procedure is termed a parametric method. Alternatively, the $h_0(t)$ can be left arbitrary, in which case the procedure is termed semi-parametric.

4.2.2 Semi-parametric PWP model

The PWP model generalizes the semi-parametric Cox proportional hazard function to a proportional intensity function $\lambda(t; \mathbf{z})$ for the case of repeated failure events. Under proportional intensities, the ratio of the intensity functions of two units (A and B) with covariate vectors $\mathbf{z}_A$ and $\mathbf{z}_B$ is constant over time. When the baseline intensity function is fully specified (e.g., NHPP with power-law or log-linear) the analytical procedure is termed a parametric method. Alternatively, $\lambda_0(t)$ can be left arbitrary, in which case the procedure is then termed semi-parametric.

Given the counting and covariate processes at time t, the general semi-parametric intensity function takes the following form:

$$\lambda\{t \mid N(t), Z(t)\} = \lim Pr\{t \leq T_{n(t)+1} < t + \Delta \mid N(t), Z(t)\}/\Delta, \tag{4.2}$$

where $N(t)$ denotes the count of failures in $(0, t]$, $Z(t)$ denotes the covariate process up to time t, and Δ limits the time span to zero.

PWP specified two classes of models of the following form:

$$\text{PWP-GT: } \lambda\{t \mid N(t), Z(t)\} = \lambda_{0n}(t - t_{n-1}) \exp[\boldsymbol{\beta}_n' \mathbf{z}(t)] \tag{4.3}$$

$$\text{PWP-TT: } \lambda\{t \mid N(t), Z(t)\} = \lambda_{0n}(t) \exp[\boldsymbol{\beta}_n' \mathbf{z}(t)] \tag{4.4}$$

In the PWP-GT (gap-time) model of (4.3), the time metric is the interval between times of successive failures t_{n-1} and t_n, defined as gap time. The PWP model stratifies a failure data set based on the failure event count. When a unit is placed into operation it has experienced no failures and so resides in stratum 1 ($n = 1$), and when the first failure occurs the unit moves to the second stratum ($n = 2$). In general, the unit moves to stratum n immediately following the $(n-1)^{th}$ failure and remains there until the n^{th} failure.

4.2.3 Semi-parametric AG model

Andersen & Gill (1982) extended the Cox PH model to accommodate recurring events in a counting process. The AG method explains general covariate effects (common baseline function in the concept of risk set), no event-stratifying effects exist since each event count re-starts the failure process. The risk interval of an AG model follows a counting process associated with recurring events, where recurrences $(N_i^{(n)}, Y_i^{(n)}, Z_i^{(n)})$ are independent and identically distributed $(i.i.d)$ replicates of (N, Y, Z), and the probability of the occurrence of two events at a given time is zero, where (N, Y, Z) represents the successive failure count, an at-risk indicator, and covariates. Thus, the risk set of the $(n-1)^{th}$ event is identical to the risk set of the n^{th} event. The AG model is express in the following form:

$$\lambda_i^{(n)}(t) = Y_i^{(n)}(t)\lambda_0(t)\{\boldsymbol{\beta} \times \mathbf{z}_i^{(n)}(t)\}, \tag{4.5}$$

4.2.4 Semi-parametric WLW model

The WLW method takes a marginal approach, expanded from the conditional PWP method, to deal with the recurrent failure data. Depending on the sample size associated with the failure count, the WLW method has a greater or equal risk set compared to the PWP method. The PWP method assumes that the complete history of the subjects is available for estimating the intensity function, while the WLW method additionally considers the subjects that have been withdrawn from observation. The subjects that have been censored remain in the risk set, hence contributing influence on events that are followed after the censoring time. The risk set of each subject is the same regardless of complete data or censoring events since a subject is still at risk when the subject has been withdrawn from the observation.

With the WLW method, for the k^{th} failure type and the i^{th} failure count, the hazards function $\lambda_{ki}(t)$ is assumed to be:

$$\lambda_{ki}(t) = \lambda_{k0}(t) \exp\{\boldsymbol{\beta}' \times \mathbf{z}_{ki}(t)\}, \quad t \geq 0, \tag{4.6}$$

where $\lambda_{k0}(t)$ is an unspecified baseline hazard function and $\boldsymbol{\beta}'_k$ is a vector of failure-specific regression parameters. $\mathbf{z}_{ki}(t)$ denotes a $p \times 1$ vector of covariates for the i^{th} subject at time i with respect to the k^{th} failure type.

4.2.5 Log-linear intensity function

Cox & Lewis (1966) proposed and applied the NHPP log-linear intensity function to aircraft air conditioner failures. Lawless (2003) has used residual analysis to test the adequacy of representation of this model, his results indicate that NHPP with log-linear intensity function provides a good fit to the same data. This model has also been applied by Ascher & Feingold (1969) to the analysis of submarine main propulsion diesel engines.

The log-linear intensity function has the form

$$\lambda(t) = e^{\mu + \theta t} \tag{4.7}$$

where μ and θ are location and shape parameters, respectively. A Cox proportional intensity (PI) model is expressed as the product of a baseline intensity function and a link function, the latter usually takes the exponential form. Thus, the Cox PI model with the log-linear baseline intensity function can be written

$$\lambda(t;\mathbf{z}) = e^{\mu+\theta t} e^{\boldsymbol{\beta}'\mathbf{z}} \tag{4.8}$$

where $\mathbf{z}$ is a vector of covarites and $\boldsymbol{\beta}$ is a vector of corresponding coefficients that measure the covariate effects.

We consider a special case where the covariate vector is constant over the study period. We also define $\mu_0 = \exp(\beta_0 z_0)$, let $z_0 \equiv 1$, and the log-linear intensity function becomes

$$\lambda(t;\mathbf{z}) = e^{\theta t} e^{\boldsymbol{\beta}'\mathbf{z}} \tag{4.9}$$

Note that the case $\mathbf{z} = (1,0)'$ represents no covariate effect, in which case Eq. (4.9) becomes the baseline intensity function:

$$\lambda(t;\mathbf{z}) = e^{\theta t} e^{\beta_0 \cdot 1 + \beta_1 \cdot 0} = e^{\mu_0 + \theta t}$$

Consider the single covariate case. To derive an expression for the covariate coefficient β in the PWP-GT model, we observe that the PWP-GT model has the form

$$\lambda(t;z) = \lambda_0(t - t_{n-1}) \exp(\beta' z) \tag{4.10}$$

where $t - t_{n-1}$ signifies that (4.10) is a function of time to failure n, measured from the immediate preceding event time of failure $n-1$. When $z = 0$, or there is no covariate effect, $\lambda(t,0) = \lambda_0(t - t_{n-1})$; if we let $\beta = 1$, then $\lambda(t;1) = \lambda_1 = \lambda_0(t - t_{n-1}) \exp(\beta)$, resulting in

$$\beta_{PWP} = \ln \frac{\lambda_1(t,1)}{\lambda_0(t,0)} \tag{4.11}$$

For a log-linear intensity function, we have

$$t_n = \frac{1}{\theta} \ln(\frac{\theta n}{e^{\mu}} + 1)$$

where t_n denotes the time of the n-th occurrence of failure. Consequently,

$$t_{n,0} = \frac{1}{\theta} \ln(\frac{\theta n}{e^{\mu_0}} + 1), \qquad t_{n,1} = \frac{1}{\theta} \ln(\frac{\theta n}{e^{\mu_1}} + 1) \tag{4.12}$$

Substituting Eqs. (4.7) and (4.12) into Eq. (4.11), we have

$$\beta_{PWP} = \ln \frac{\theta n + e^{\mu_1}}{\theta n + e^{\mu_0}} \tag{4.13}$$

We note that the covariate coefficient is theoretically failure-count specific for the PWP-GT model. We also observe that for the case of HPP ($\theta = 0$), (4.13) reduces to

$$\beta_{PWP} = \mu_1 - \mu_0,$$

the value we used to generate our simulation data set. We denote this expression as β_{NHPP}, and rearrange (4.13) algebraically as

$$\beta_{PWP} = \ln \left\{ e^{\frac{\theta n}{\mu_0} + e^{\beta_{NHPP}}} \Big/ e^{\frac{\theta n}{\mu_0} + 1} \right\} \tag{4.14}$$

Caution is raised here for the usage of Eqs. (4.14) and (4.13). The PWP model stratifies a failure data set based on the failure event count. At the onset of the experiment, a unit has experienced no failure and so resides in stratum 1, and moves to the second stratum once it has experienced the first failure. Thus, when $n = 0$

(no failures yet occurred), the equations give the corresponding theoretical value for the estimate for stratum 1. In general, the equations give the theoretical value of the estimate for stratum i when $n = i - 1$.

Random variates are generated from the log-linear NHPP, by the algorithm of (Law & Kelton, 1991, pp. 507–510). For nonstationary Poisson process, a recursive approach is to:

1. Generate a random variate from a continuous uniform distribution, i.e., $U \sim U(0,1)$.
2. Set $t'_i = t'_{i-1} - \ln U$.
3. Return $t_i = \Lambda^{-1}(t'_i)$.

where $\Lambda(\cdot)$ is the *expectation function* of the NHPP process defined as

$$\Lambda(t) = \int_0^t \lambda(y)dy$$

For the log-linear intensity function, we replace (4.7) into the above and obtain

$$\begin{aligned}\Lambda(t) &= \int_0^t e^{\mu+\theta y}dy = e^{\mu}\int_0^t e^{\theta y}dy \\ &= e^{\mu}\left[\frac{1}{\theta}e^{\theta y}\right]_{y=0}^{y=t} = e^{\mu}\left[\frac{1}{\theta}e^{\theta t} - \frac{1}{\theta}\right]\end{aligned}$$

The inverse function of $\Lambda(t_i)$ is thus

$$t_n = \frac{1}{\theta}\left[\ln(\theta t'_i + e^{\mu}) - \mu\right],$$

In SAS, the above algorithm is realized with the following code:

```
DATA LOGLIN;
    ** We use three seed numbers 539, 255, and 59 to
       simulate duplications **;
    RETAIN SEED 539;

    FORMAT R Y 16.8;

    ** THETA is the shape parameter of the log-linear
       intensity function **;
    THETA = 1.2;

    ** ITEM is the experiment unit **;
    DO ITEM = 1 TO 60;
        T = 0;
        M = 0;
        R = 0;

        ** For each experiment unit, we generate 10
           recurrent failure times **;
        DO FAILURE = 1 TO 10;

            ** Generate random variates from a continuous
               uniform distribution **;
            X = RANUNI(SEED);

            ** And generate the expectation function **;
            T = T - LOG(X);

            ** Divide the experiment units into two classes,
simulating proportionality **;
            IF ITEM <= 30 THEN MU = -6.9;
            ELSE MU = -4.6;
            IF MU = -6.9 THEN CLASS = 0;
            ELSE CLASS = 1;

            ** Avoid division by zero error in SAS **;
            IF THETA = 0 THEN R = T / EXP(MU);

            ** Solve for the inverse of the expectation
               function **;
            ELSE R = (LOG(THETA*T + EXP(MU)) - MU) / THETA;

            ** Finally, Y becomes the failure time measured
```

```
                  from
the immediate preceding failure **;
             Y = R - M;
             M = R;
             OUTPUT;
             END;
         END;
```

SAS PHREG procedure is applied to the generated dataset to verify the methodology and coding. SAS output of the three seed numbers (539, 255, and 59) is summarized in Table 4.1. Note that the estimates produced from seed number 539 are numerically consistent with those of (Vithala, 1994, pp. 173–182), the discrepancy in the fifth and sixth decimal place likely attributable to the SAS software version change. The average over three seeds does match the graphical presentation that appears on p. 128 of Vithala.

Table 4.1: PWP-GT Estimate of Regression Coefficient

10 failures/unit, 30 units/class, 2 classes $\mu_0 = -6.9, \mu_1 = -4.6$					
Failure	Vithala	Seed 539	Seed 255	Seed 59	Average
1	2.467460	2.46744	2.10776	2.22278	2.26599
2	0.068847	0.06885	0.60108	0.28794	0.31929
3	-0.102998	-0.10300	0.05269	-0.38473	-0.14501
4	-0.050596	-0.05060	-0.06329	0.50464	0.13025
5	-0.662466	-0.66245	0.38282	0.18169	-0.03265
6	0.087593	0.08759	0.03235	-0.13544	-0.00517
7	0.279324	0.27932	0.32019	0.61221	0.40391
8	-0.236752	-0.23675	-0.05163	-0.09089	-0.12642
9	-0.199585	-0.19958	0.03002	0.54392	0.12479
10	-0.039569	-0.03957	0.09075	-0.11363	-0.02082

Effort is also made to duplicate Table 3 of Landers et al. (2001). The results for the calculation of the theoretical value of β is consistent, although a different formula is used. Vithala (refer to Landers et al. (2001)) used Eq. (4.14). Although mathematically equivalent to Eq. (4.13), Eq. (4.14) is easier to code in SAS. Table 4.2 summaries the calculation, with Vithala values (a) in column (6) and Zhou values (b) in column (7) for theoretical coefficient β. Note that Vithala did not include the β estimate for stratum 1 (time to first failure).

Since the intensity function is exponential with respect to time t, the log-linear NHPP can model a repairable system with rapid deterioration in the wear-out phase of the life cycle. The important advantage in survival time modeling is that for any values of μ and θ, the resulting intensity function is always positive. $\lambda(t)$ is strictly decreasing for $\theta < 0$, constant for $\theta = 0$ and strictly increasing for $\theta > 0$. Thus, we have a decreasing rate of occurrence of failure (DROCOF) for $\theta < 0$, a homogeneous Poisson process (HPP) for $\theta = 0$, and an increasing rate of occurrence of failure (IROCOF) for $\theta > 0$ (see Vithala, 1994, p.50)

4.2.6 The range of parameters

When the baseline intensity function is specified as power-law, the parametric proportional intensity (PI) function can be expressed as

$$\lambda(t) = \delta \times t^{\delta-1} \exp(\mathbf{z}\boldsymbol{\beta})$$

where δ is the shape parameter, $z_0 \equiv 1$ and $\nu_0 \equiv \exp(\beta_0)$. Thus when there are no covariate effects, we obtain the baseline intensity function

$$\lambda_0(t) = \nu_0\delta \times t^{\delta-1} \tag{4.15}$$

Table 4.2: PWP estimate of regression coefficient $\boldsymbol{\beta}$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
θ	Failure	$\hat{\boldsymbol{\beta}}(se)$	UB($\hat{\boldsymbol{\beta}}$)	LB($\hat{\boldsymbol{\beta}}$)	$\boldsymbol{\beta}^a$	$\boldsymbol{\beta}^b$
0.03	1	2.02964(0.15073)	2.325070	1.734220	0.255940	2.300000
	2	0.72570(0.11361)	0.948370	0.503020	0.138230	0.255940
	3	0.18971(0.10914)	0.403630	-0.024210	0.094740	0.138230
	4	0.04637(0.10768)	0.257430	-0.164680	0.072080	0.094740
	5	0.16331(0.10876)	0.376480	-0.049850	0.058170	0.072080
	6	0.16838(0.10713)	0.378360	-0.041610	0.048760	0.058170
	7	0.05304(0.10643)	0.261640	-0.155560	0.041970	0.048760
	8	0.20096(0.10725)	0.411170	-0.009250	0.036840	0.041970
	9	0.07409(0.10660)	0.283020	-0.134840	0.032830	0.036840
	10	0.10446(0.10678)	0.313740	-0.104830	0.029600	0.032830
0.2	1	2.02964(0.15073)	2.325070	1.734220	0.044011	2.300000
	2	0.31687(0.10752)	0.527610	0.106120	0.022303	0.044010
	3	0.00744(0.10806)	0.219230	-0.204350	0.014936	0.022300
	4	-0.04742(0.10820)	0.164640	-0.259490	0.011228	0.014940
	5	0.08386(0.10833)	0.296190	-0.128470	0.008994	0.011230
	6	0.11698(0.10710)	0.326890	-0.092940	0.007502	0.008990
	7	-0.00802(0.10625)	0.200230	-0.216270	0.006435	0.007500
	8	0.16234(0.10697)	0.372010	-0.047320	0.005633	0.006430
	9	0.03536(0.10646)	0.244030	-0.173310	0.005009	0.005630
	10	0.07588(0.10684)	0.285280	-0.133520	0.004510	0.005010
1.2	1	2.02964(0.15073)	2.325070	1.734220	0.007502	2.300000
	2	0.19510(0.10735)	0.405510	-0.015320	0.003760	0.007500
	3	-0.04245(0.10803)	0.169300	-0.254190	0.002508	0.003760
	4	-0.06388(0.10836)	0.148500	-0.276260	0.001882	0.002510
	5	0.07423(0.10828)	0.286470	-0.138000	0.001506	0.001880
	6	0.10879(0.10712)	0.318750	-0.101160	0.001255	0.001510
	7	-0.02870(0.10625)	0.179540	-0.236940	0.001076	0.001260
	8	0.15706(0.10695)	0.366690	-0.052560	0.000942	0.001080
	9	0.03062(0.10647)	0.239290	-0.178060	0.000837	0.000940
	10	0.07137(0.10686)	0.280830	-0.138080	0.000753	0.000840

Notes:
a. Vithala
b. Zhou

Likewise, given the baseline intensity function specified as log-linear, the parametric PI function can be expressed as

$$\lambda(t) = \exp(\mu + \theta t)\exp(\mathbf{z}\boldsymbol{\beta})$$

where θ is the shape parameter, let $z_0 \equiv 1$ and define $\exp(\mu_0) = \exp(\beta_0 z_0)$. Thus when there are no covariate, we obtain the baseline intensity function

$$\lambda_0(t) = \exp(\mu_0 + \theta t) \tag{4.16}$$

Figure 4.1 provides intuitive insight on the range of intensity functions for both

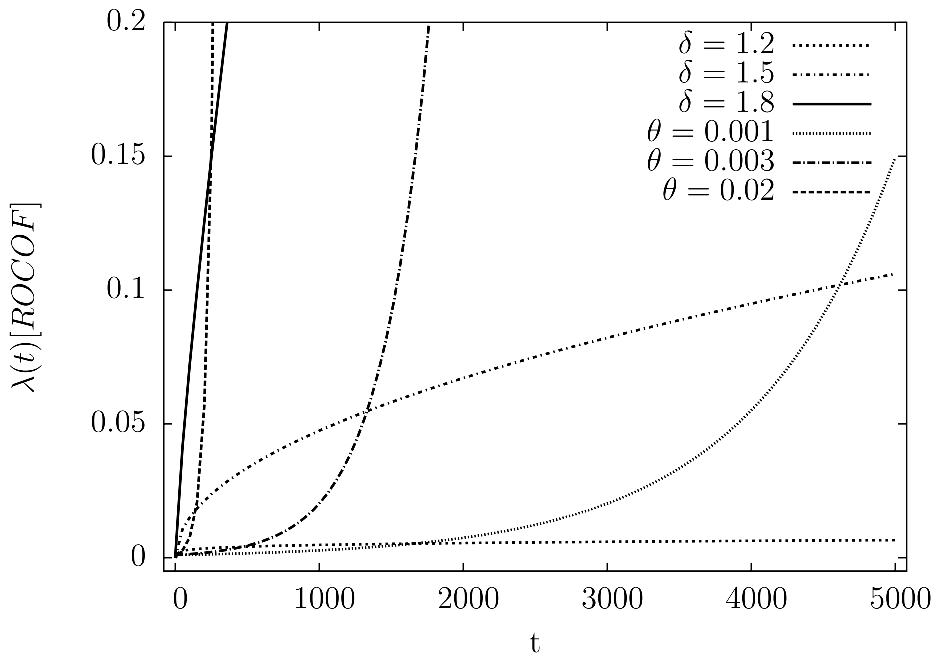


Figure 4.1: Comparison between Power-law and Log-linear intensity functions

power-law and log-linear NHPP processes consistent with the Proschan data set for aircraft air conditioners (Cox & Lewis, 1966, p.6). Figure 4.2 depicts more details over the $(0, 500)$ time interval which approximately covers the Proschan data. Both baseline intensity functions would produce the HPP case (i.e., when $\delta = 1$ in the power-law case and $\theta = 0$ in the log-linear case). The two intensity functions are

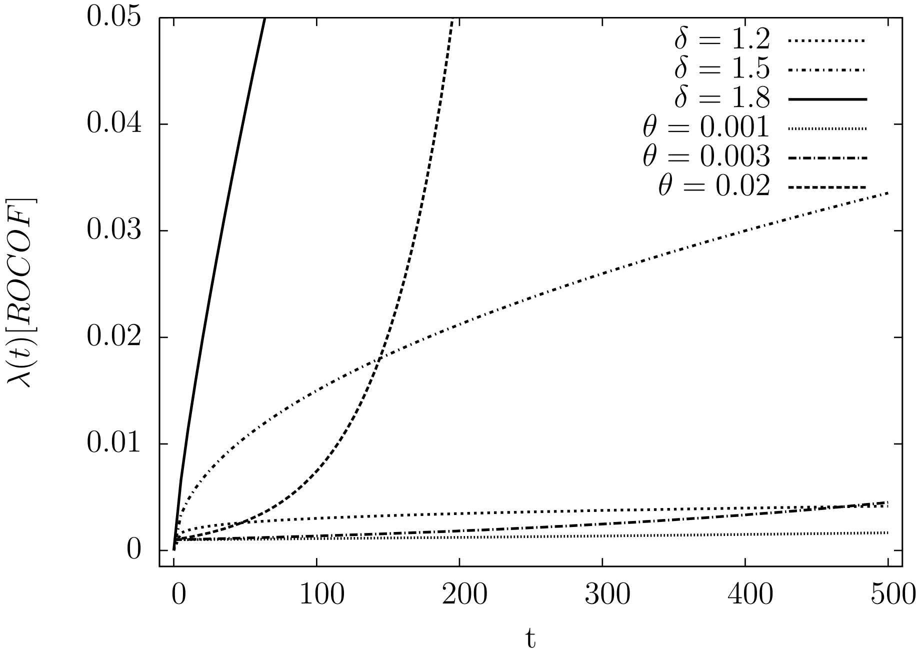


Figure 4.2: Comparison between Power-law and Log-linear intensity functions

seen in Figure 4.1 and 4.2 to exist over a consistent time-intensity regime, but with the power-law $\lambda(t)$ IROCOF convex and the log-linear IROCOF concave. Note the similar ranges for $\delta = 1.2$ (power-law) and $\theta = 0.001$ (log-linear). The ranges are also comparable for power-law with $\delta = 1.5$ and log-linear with $\theta = 0.003$; similarly for power-law with $\delta = 1.8$ and log-linear with $\theta = 0.02$. Overall, the smaller and larger shape parameters δ of the power-law model define bounds, such that with carefully chosen shape parameters θ of the log-linear model, we could obtain intensity functions that are similar in the time-intensity regime, hence the comparison of the performance between the two models becomes feasible. We observe that in order to obtain a comparable range of intensity functions, the range of θ for log-linear is much smaller than that of δ for the power-law. These observations provide guidance for the range selection between different baseline models.

4.2.7 Method

Simulated date with right-censored patterns, where the underlying distribution follows a log-linear NHPP, is generated by the algorithm proposed by Law & Kelton (1991). Two groups of sample units are generated, in which one group contains the sample units with complete data and the other group contains the sample units with right-censored data. In the latter group, the right-censoring pattern is set randomly. The proportion of the sample units that have censored times to total sample units is defined as censored probability (P_c).

A discrete indicator covariate $\mathbf{z}_i$ was used to divide the data into two strata for an arbitrary treatment effect. For consistency with the relevant previous research (Qureshi, Vithala, Jiang), simulated data was generated such that the values of the intensity function are within the same general time-intensity regime. A proportional intensity function dataset is created using two different values for the parameter ($\mu_0 = -6.9, \mu_1 = -4.6$) corresponding to the two values of indicator covariates $\mathbf{z}_1$ ($\mathbf{z}_1 = 0, \mathbf{z}_1 = 1$).

The experiment is designed with three factors: experimental units (U), shape parameter (θ), and censoring probability (P_c). The parameter level selections are: (1) $U = 60, 120$ and 180; (2) $\theta = 0.0, 0.001, 0.002, 0.004, 0.008, 0.01, 0.02$; and (3) $P_c = 0.0, 0.4, 0.6, 0.8,$ and 1.0. The selection of these levels is derived from the following considerations: (1) the parameter settings in the previous relevant research and (2) accounting for the prospect that severe right-censorship may cause the small sample size (e.g., $U = 20$) to have insufficient data. The levels of P_c are representative of light, moderate, and severe censoring. The selection of U levels is taken from the parameter settings in the previous research work, and consideration for the small, medium, and large sample sizes of U. The levels of θ are chosen such that the values of intensity function are comparable with the previous research, also that the resulting estimates are in the comparable time-intensity regime.

The underlying theories of the four Cox-based regression methods call for different formulation of the datasets. For the AG model, the data set is formed from the time interval (T_1, T_2) defined as starting and ending times of an event with respect to the following counting process formulation:

$$\lim_{\Delta \to 0} \frac{1}{\Delta} p\left[N(t+\Delta) - N(t) = 1 \mid T > t\right] = \lambda(t), \tag{4.17}$$

where $\lambda(t)$ is the proportional intensity function of a failure process and $N(t)$ is a random variable denoting the number of failures in $(0, t]$.

Eq. (4.17) defines the instantaneous failure rate between t and $t + \Delta$ under the condition that this individual has survived to time t. Thus, to form the dataset, we must have $T_2 > T_1$. Consequently, all the censored failure times are removed from the dataset since $T_2 = T_1$ for a censored event as stipulated for the AG model. The dataset formed for the PWP method is derived from conditionality theory of probability. The later failure times after the n^{th} failure count cannot be included into the dataset when the intensity function at the n^{th} failure count is estimated. For each censored unit, the censored times are removed from the dataset except for the earliest censored event time. On the other hand, the WLW method is based on the marginality theory of probability, thus the dataset contains full records including all censored events, such that censored units remain in the risk set.

The four semi-parametric methods are implemented using the SAS™ Users Group (SUGI) software code PHREG (Appendix A), which performs the semi-parametric Cox regression method with a blocking option to stratify for a covariate which does not satisfy the proportionality assumption. In this research, the failure event count is stratified (event-specific intensity functions). PHREG uses the product-limit method to estimate the reliability function within all strata and for all values of the covariate. Also PHREG computes the regression coefficients $\boldsymbol{\beta}$ and the covariance matrix by the Newton-Raphson method.

The performance comparison of the four models are based on the three metrics listed:

- relative signed error (BIAS)
- relative mean absolute deviation (MAD), and
- relative mean squared error (MSE).

Comparison is also made between the estimates (PWP-GT, AG and WLW) of regression coefficients $\hat{\beta}$ and the theoretical value β based on failures per unit. Additionally, 95% confidence intervals were constructed on the estimates of $\boldsymbol{\beta}_i$. In the special case of HPP, the other three models having common baseline intensity function (PWP-TT, AG, and WLW), 95% confidence intervals were constructed and compared.

4.3 SAS macro design

The SAS analysis requires lengthy repetitive steps, wherein the same code is executed on numerous groups of experimental parameters and replicates. Additionally, since the SAS procedures often provide more statistical output than is needed for a given analysis, the number of output pages to be reviewed is quite massive and the potential for transcription errors substantial. This section delineates a SAS macro that provides: (1) Automatic iterating SAS code on experimental parameters using *Macro Arrays*, (2) ODS output datasets from multiple procedures, and (3) selected statistics retained from these datasets and merged together to produce one concise summary report. Appendix B contains the SAS macro developed in this research, based on methods proposed by Long & Heaton (2007).

4.3.1 Iteration of SAS code on experimental factors

The experiment has three factors: sample size (U), censoring severity (P_c), and shape parameter (θ), and three seeds are used on each combination of these factors to produce triple replicates. SAS PHREG procedure is then applied to the generated datasets. It is convenient to write a SAS macro and define these factors as *Macro Arrays*, thus when the SAS macro is called, it will iterate PHREG codes on all generated datasets. In the following code listing, sample size (U) is used for illustration of the macro procedures.

```
%MACRO myphregmacro(u     =,
                    p     =,
                    theta =,
                    seed  = );
  data _null_;
    array mysamplesize(3) (&u);
```

```
    ......
    CALL SYMPUTx("dim_u",dim(mysamplesize));
    ......
    do u=1 to dim(mysamplesize);
       CALL SYMPUTx(CATS("u_",u),mysamplesize(u));
    END;
  run;
  ....
```

The syntax `%MACRO myphregmacro` initializes the macro definition and indicates it accepts 4 explicit (named) arguments. A macro array of 3 elements is created with the code `array mysamplesize(3) (&u);`. The code `CALL SYMPUTx("dim_u",dim(mysamplesize));` then writes the value of the dimension (in our case, 3) to a variable called `dim_u`. Finally the elements of the macro array are assigned values through the code `CALL SYMPUTx(CATS("u_",u),mysamplesize(u));`, when the macro is called with argument in the following manner:

```
%myphregmacro(u     = 60 120 180 ,
p     = 0.4 0.6 0.8 1.0 ,
theta = 0.0 0.001 0.002 0.004 0.008 0.01 0.2,
seed  = 539 255 59)
```

The sample size values of 60, 120, and 180 are assigned to the variable `u_1, u_2` and `u_3`, respectively. Similar logic applies to the creation of macro arrays for other experimental factors.

To iterate the SAS PHREG procedure on the macro arrays of variables, we simply create a `DO LOOP` in SAS, and replace all instances of involved SAS variables with macro variables, by prefixing the variable name with an ampersand sign (&):

```
%DO i = 1 %TO &dim_u ;
......
    DO ITEM = 1 TO &&u_&i;
        ......
        END
```

```
%END
```

The first macro %DO LOOP would cycle through three sample sizes, while in the second DO LOOP, the macro variable &&u_&i assumes one of the sample size values (60, 120, and 180) within each iteration.

4.3.2 Trimming SAS output

The output of SAS PHREG procedure contains 7 tables (see Appendix C), including Model Information, Number of Observations, Censored Summary, Convergence Status, Model Fit Statistics, Global Tests, and Parameter Estimates. In computing our desired performance metrics, only the Parameter Estimates are used; in particular, only two variables from that table are needed (Parameter Estimate and Standard Error for each stratum). SAS Output Delivery System (ODS) can be used to select or exclude individual output objects, thus making it possible to generate the desired format report.

The following SAS listing demonstrates the usage of ODS:

```
ODS listing close;
ODS output ParameterEstimates=myPara
 (KEEP=Variable Estimate RENAME=(Estimate=Est_%SCAN(&seed, &
    l)));
```

The code ODS listing close; would suppress all SAS output at first, the next line of code would select the table of Parameter Estimates, and redirect it to a dataset called myPara; only the variable Parameter Estimate is retained in this sample. The output from each iteration is appended to myPara with

```
PROC APPEND base=all_data&&s_&l DATA=myPara FORCE;
```

4.3.3 Report generation

The SAS PHREG does not give the performance metrics directly; instead, only the estimate of β for each stratum is given. Rather than using a spreadsheet software to summarize and post-process the SAS output, this research used an SQL procedure to compute all final statistics, thus eliminating the potential risk of error during transcription.

```
data metrics (DROP = Variable);
    set all_data;
    AVG=mean (of Est_539 Est_255 Est_59);
    EST_TRUE = log( (THETA*(N-1) + exp(-4.6)) / (THETA*(N-1)
        + exp(-6.9)) );
    EN = (AVG - EST_TRUE) / EST_TRUE;

Proc sql;
    Create table abc as
    Select samplesize, probability, theta,
    Avg(EN) as BIAS,
    Avg(abs(EN)) as MAD,
    sum(EN*EN) / (count(*) - 1) as MSE
    From metrics
    Group by samplesize,probability, theta ;
    Quit;
PROC PRINT NOOBS;
    format BIAS MAD MSE  10.5 ;
```

The data set `metrics` contains the mean value over three replicates (represented by seed numbers), as well as the theoretical value of β for each stratum. The relative error is given by `EN = (AVG - EST_TRUE) / EST_TRUE`. A table is created from this dataset, the performance metrics are computed by simple syntax(e.g., `Avg(EN) as BIAS` gives the BIAS). The final output neatly contains only the statistics that we desired.

4.3.4 Research Infrastructure

The parameter analysis in Sections 4.2.6 and 4.4.1 required the capability to perform many SAS runs. Additionally, the analysis in Tables 4.3 - 4.11 required numerous SAS runs. The macro code permits this analysis rapidly and error-free. This research tool will greatly benefit future analysis by this and other investigators.

4.4 Results

4.4.1 PWP-GT model results

This section summarizes the PWP-GT model robustness in estimating the covariate effect $\hat{\beta}$ for failure count $N = 2, \cdots, 10$. Tables 4.3 to 4.11 summarize the robustness across strata defined by ordered failures. Factors involved in the experiment are the sample units (U), shape parameter (θ), and censoring probability (P_c). Refer to Table 4.4 for $n = 3$. In the case of $U = 60$, results for censoring probability P_c from 0.4 to 1.0 are as follows. For the range of the shape parameter, $0.0 \leq \theta \leq 0.02$, with censoring probability $P_c = 0.4$, the PWP-GT estimates have relative MSE in the range of $(1.4\%, 103.8\%)$, relative BIAS in the range of $(-8.4\%, 44.5\%)$, and relative MAD in the range of $(8.4\%, 54.3\%)$. As the value of P_c is increased to 0.6, the PWP-GT estimate have relative MSE in the range of $(1.5\%, 98.7\%)$, relative BIAS in the range of $(-8.9\%, 46.3\%)$, and relative MAD in the range of $(8.9\%, 56.1\%)$. Likewise, when P_c is increased to 0.8, the PWP-GT estimates have relative MSE in the range of $(1.8\%, 100.7\%)$, relative BIAS in the range of $(-10.2\%, 33.3\%)$, and relative MAD in the range of $(10.2\%, 59.4\%)$. When P_c is increased to 1.0, the PWP-GT estimates do not display deterioration, with relative MSE in the range of $(1.8\%, 99.8\%)$, relative BIAS in the range of $(-10.4\%, 37.5\%)$, and relative MAD in the range of $(10.4\%, 55.8\%)$.

As for the case of $U = 120$, for the range of shape parameters $0.0 \leq \theta \leq 0.02$ with $P_c = 0.4$, the PWP-GT estimates have relative MSE in the range of $(0.1\%, 72.4\%)$, relative BIAS in the range of $(-1.7\%, 50\%)$, and relative MAD in the range of $(2.4\%, 52.6\%)$. As the value of P_c is increased to 0.6, the PWP-GT estimates have relative MSE in the range of $(0.13\%, 70.1\%)$, relative BIAS in the range of $(-2.2\%, 48.2\%)$, and relative MAD in the range of $(2.5\%, 50.7\%)$. Likewise, when P_c is increased to 0.8, the PWP-GT estimates have relative MSE in the range of $(0.8\%, 76\%)$, relative BIAS in the range of $(-1.4\%, 53.2\%)$, and relative MAD in the range of $(2.2\%, 55.6\%)$. When P_c is increased to 1.0, the PWP-GT estimates do not display deterioration, with relative MSE in the range of $(0.08\%, 77.8\%)$, relative BIAS in the range of $(-1.1\%, 53.3\%)$, and relative MAD in the range of $(2.1\%, 55.7\%)$.

Increase in sample size significantly improves the performance of the PWP-GT model in terms of the metrics BIAS, MAD, and MSE. As for the case of $U = 180$, for the range of shape parameters $0.0 \leq \theta \leq 0.02$ with $P_c = 0.4$, the PWP-GT estimates have relative MSE in the range of $(0.14\%, 14\%)$, relative BIAS in the range of $(-2.4\%, 17.9\%)$, and relative MAD in the range of $(2.4\%, 18.2\%)$. As the value of P_c is increased to 0.6, the PWP-GT estimates have relative MSE in the range of $(0.23\%, 15.2\%)$, relative BIAS in the range of $(-2.8\%, 12.9\%)$, and relative MAD in the range of $(2.8\%, 22.6\%)$. Likewise, when P_c is increased to 0.8, the PWP-GT estimates have relative MSE in the range of $(0.17\%, 14.5\%)$, relative BIAS in the range of $(-2.3\%, 13.5\%)$, and relative MAD in the range of $(2.3\%, 21.6\%)$. When P_c is increased to 1.0, the PWP-GT estimates do not display deterioration, with relative

MSE in the range of $(0.03\%, 14.9\%)$, relative BIAS in the range of $(-1.1\%, 21.7\%)$, and relative MAD in the range of $(1.2\%, 22.1\%)$.

Table 4.3: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (2 failures/unit)

$N = 2$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	-0.09447	0.10624	0.04042	120	-0.03442	0.03442	0.00427	180	-0.02640	0.02640	0.00207
60	0.4	0.001	-0.10400	0.11577	0.04844	120	0.00645	0.07171	0.01037	180	-0.00116	0.04358	0.00380
60	0.4	0.002	-0.07987	0.09163	0.02955	120	0.05184	0.11709	0.03279	180	0.02688	0.07163	0.01171
60	0.4	0.004	-0.04875	0.06049	0.01207	120	0.14607	0.21133	0.13199	180	0.09624	0.14098	0.05827
60	0.4	0.008	-0.00090	0.01264	0.00032	120	0.25634	0.32159	0.33826	180	0.16475	0.20950	0.14207
60	0.4	0.010	0.02778	0.02778	0.00206	120	0.29488	0.36014	0.43331	180	0.19911	0.24386	0.19823
60	0.4	0.020	0.05529	0.05529	0.00991	120	0.48872	0.55398	1.09148	180	0.34165	0.38640	0.53206
60	0.6	0.000	-0.09623	0.10800	0.04185	120	-0.03823	0.03823	0.00438	180	-0.02817	0.02817	0.00214
60	0.6	0.001	-0.10255	0.11432	0.04717	120	0.00259	0.06785	0.00922	180	-0.00377	0.04098	0.00339
60	0.6	0.002	-0.07743	0.08919	0.02790	120	0.04851	0.11376	0.03059	180	0.02423	0.06898	0.01069
60	0.6	0.004	-0.04203	0.05376	0.00931	120	0.14389	0.20915	0.12889	180	0.09332	0.13806	0.05554
60	0.6	0.008	0.01078	0.01078	0.00023	120	0.25769	0.32295	0.34140	180	0.16381	0.20855	0.14066
60	0.6	0.010	0.04242	0.04242	0.00548	120	0.29594	0.36120	0.43609	180	0.19778	0.24252	0.19587
60	0.6	0.020	0.07764	0.07764	0.02074	120	0.49566	0.56091	1.12060	180	0.34220	0.38695	0.53366
60	0.8	0.000	-0.09937	0.11115	0.04446	120	-0.04222	0.04222	0.00463	180	-0.02570	0.02570	0.00205
60	0.8	0.001	-0.10498	0.11675	0.04930	120	-0.00320	0.06205	0.00772	180	-0.00006	0.04468	0.00399
60	0.8	0.002	-0.07943	0.09119	0.02925	120	0.04391	0.10916	0.02769	180	0.02911	0.07386	0.01261
60	0.8	0.004	-0.04303	0.05477	0.00970	120	0.13714	0.20239	0.11954	180	0.09877	0.14351	0.06070
60	0.8	0.008	0.01226	0.01226	0.00030	120	0.24929	0.31455	0.32218	180	0.17144	0.21618	0.15225
60	0.8	0.010	0.04344	0.04344	0.00579	120	0.28566	0.35091	0.40948	180	0.20708	0.25182	0.21260
60	0.8	0.020	0.08145	0.08145	0.02299	120	0.47697	0.54222	1.04300	180	0.35448	0.39922	0.57006
60	1	0.000	-0.09544	0.10721	0.04120	120	-0.04026	0.04026	0.00449	180	-0.01892	0.02583	0.00205
60	1	0.001	-0.09111	0.10288	0.03777	120	0.00779	0.07305	0.01079	180	0.01363	0.05837	0.00719
60	1	0.002	-0.06016	0.07192	0.01758	120	0.05631	0.12156	0.03590	180	0.04195	0.08669	0.01855
60	1	0.004	-0.01840	0.03014	0.00249	120	0.15074	0.21599	0.13875	180	0.11458	0.15932	0.07702
60	1	0.008	0.04318	0.04318	0.00571	120	0.26781	0.33306	0.36530	180	0.19392	0.23867	0.18913
60	1	0.010	0.07691	0.07691	0.02033	120	0.30526	0.37051	0.46092	180	0.23323	0.27798	0.26334
60	1	0.020	0.11555	0.11555	0.04826	120	0.52075	0.58601	1.22918	180	0.39495	0.43970	0.69864

Table 4.4: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (3 failures/unit)

$N = 3$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	-0.08359	0.08359	0.01410	120	-0.01799	0.02357	0.00104	180	-0.02399	0.02399	0.00144
60	0.4	0.001	-0.02896	0.10562	0.01893	120	0.03410	0.05874	0.00922	180	0.00405	0.03347	0.00236
60	0.4	0.002	0.03303	0.14047	0.04479	120	0.07496	0.09959	0.02450	180	0.02522	0.03378	0.00396
60	0.4	0.004	0.11651	0.21459	0.12008	120	0.14512	0.16975	0.06524	180	0.06643	0.06985	0.01438
60	0.4	0.008	0.18170	0.29499	0.26673	120	0.24970	0.27441	0.19170	180	0.10710	0.11067	0.04393
60	0.4	0.010	0.26023	0.35831	0.41463	120	0.30084	0.32554	0.26526	180	0.12778	0.13135	0.06304
60	0.4	0.020	0.44508	0.54316	1.03864	120	0.50089	0.52560	0.72408	180	0.17863	0.18220	0.13966
60	0.6	0.000	-0.08946	0.08946	0.01477	120	-0.02199	0.02502	0.00126	180	-0.02785	0.02785	0.00230
60	0.6	0.001	-0.04232	0.10561	0.01812	120	0.02467	0.05397	0.00766	180	-0.00495	0.04113	0.00353
60	0.6	0.002	0.02583	0.13664	0.04070	120	0.06247	0.08710	0.02112	180	0.01431	0.04301	0.00443
60	0.6	0.004	0.10795	0.20603	0.11171	120	0.13307	0.15771	0.05748	180	0.05339	0.05681	0.01287
60	0.6	0.008	0.18603	0.28411	0.24995	120	0.23652	0.26123	0.18057	180	0.08988	0.10578	0.04325
60	0.6	0.010	0.26787	0.36595	0.39720	120	0.27936	0.30407	0.24201	180	0.10548	0.12997	0.06287
60	0.6	0.020	0.46273	0.56081	0.98657	120	0.48247	0.50718	0.70101	180	0.12932	0.22575	0.15155
60	0.8	0.000	-0.10219	0.10219	0.01799	120	-0.01362	0.02153	0.00088	180	-0.02342	0.02342	0.00174
60	0.8	0.001	-0.06513	0.12268	0.02361	120	0.03469	0.05933	0.00825	180	-0.00185	0.03942	0.00321
60	0.8	0.002	-0.00957	0.16187	0.04297	120	0.07468	0.09931	0.02306	180	0.01642	0.04151	0.00432
60	0.8	0.004	0.06701	0.23426	0.11832	120	0.14937	0.17401	0.06490	180	0.05349	0.05690	0.01260
60	0.8	0.008	0.12163	0.33744	0.26351	120	0.26224	0.28694	0.19665	180	0.09221	0.09849	0.04093
60	0.8	0.010	0.18670	0.39099	0.39895	120	0.31223	0.33694	0.26865	180	0.10863	0.12199	0.05996
60	0.8	0.020	0.33339	0.59368	1.00732	120	0.53174	0.55645	0.76033	180	0.13483	0.21603	0.14528
60	1	0.000	-0.10409	0.10409	0.01809	120	-0.01136	0.02070	0.00085	180	-0.01180	0.01180	0.00033
60	1	0.001	-0.06159	0.11470	0.02088	120	0.03654	0.06118	0.00870	180	0.01211	0.02651	0.00183
60	1	0.002	-0.00117	0.15208	0.03987	120	0.07681	0.10144	0.02395	180	0.03384	0.03725	0.00401
60	1	0.004	0.07819	0.22407	0.11617	120	0.15107	0.17571	0.06630	180	0.07795	0.08136	0.01572
60	1	0.008	0.13832	0.32488	0.26100	120	0.26368	0.28839	0.19856	180	0.12934	0.13291	0.04698
60	1	0.010	0.21373	0.36835	0.39560	120	0.31310	0.33780	0.27109	180	0.15609	0.15966	0.06833
60	1	0.020	0.37544	0.55755	0.99804	120	0.53263	0.55733	0.77829	180	0.21694	0.22052	0.14859

Table 4.5: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (4 failures/unit)

$N = 4$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	-0.08583	0.08583	0.01248	120	-0.00848	0.01529	0.00046	180	-0.03663	0.04329	0.00347
60	0.4	0.001	-0.04779	0.05183	0.00529	120	0.01473	0.01774	0.00085	180	0.02161	0.04333	0.00589
60	0.4	0.002	-0.00567	0.07247	0.00956	120	0.03553	0.03553	0.00280	180	0.06593	0.07808	0.01472
60	0.4	0.004	0.04589	0.09442	0.01759	120	0.04790	0.07372	0.01369	180	0.12782	0.13998	0.04097
60	0.4	0.008	0.08777	0.11652	0.02778	120	0.02651	0.14981	0.05632	180	0.25835	0.27051	0.14375
60	0.4	0.010	0.14778	0.17652	0.06680	120	0.03608	0.18404	0.08918	180	0.31111	0.32326	0.19769
60	0.4	0.020	0.23331	0.26205	0.15156	120	0.00115	0.41789	0.36746	180	0.49637	0.50853	0.50689
60	0.6	0.000	-0.09410	0.09410	0.01513	120	-0.01265	0.01857	0.00064	180	-0.03943	0.04509	0.00390
60	0.6	0.001	-0.05042	0.07042	0.00796	120	0.01948	0.01964	0.00095	180	0.02516	0.04922	0.00767
60	0.6	0.002	-0.01292	0.08828	0.01350	120	0.04282	0.04282	0.00381	180	0.07065	0.08281	0.01747
60	0.6	0.004	0.03670	0.09996	0.02740	120	0.05666	0.06589	0.01358	180	0.13440	0.14656	0.04523
60	0.6	0.008	0.05234	0.09789	0.01983	120	0.02747	0.15393	0.05752	180	0.26086	0.27302	0.14557
60	0.6	0.010	0.10012	0.14095	0.04607	120	0.02675	0.19982	0.09950	180	0.31028	0.32244	0.19443
60	0.6	0.020	0.13338	0.16218	0.10286	120	0.00648	0.41961	0.37113	180	0.47841	0.49057	0.46175
60	0.8	0.000	-0.08239	0.10125	0.01768	120	-0.00401	0.01591	0.00039	180	-0.03517	0.05687	0.00523
60	0.8	0.001	-0.04385	0.09913	0.01421	120	0.04159	0.04159	0.00362	180	0.03548	0.05994	0.01246
60	0.8	0.002	-0.01324	0.13008	0.02878	120	0.06875	0.06875	0.00977	180	0.08038	0.09254	0.02327
60	0.8	0.004	0.03908	0.16726	0.06449	120	0.09705	0.09705	0.01829	180	0.15193	0.16409	0.06001
60	0.8	0.008	0.05842	0.14935	0.06174	120	0.09832	0.10390	0.04015	180	0.29460	0.30676	0.18962
60	0.8	0.010	0.10802	0.18007	0.11351	120	0.11357	0.14871	0.06628	180	0.34904	0.36120	0.25021
60	0.8	0.020	0.13642	0.32508	0.30288	120	0.17518	0.24809	0.23853	180	0.54099	0.55315	0.57323
60	1	0.000	-0.08043	0.09097	0.01483	120	0.00048	0.00692	0.00008	180	-0.02202	0.04547	0.00298
60	1	0.001	-0.02731	0.09517	0.01440	120	0.05082	0.05082	0.00491	180	0.05336	0.06984	0.01563
60	1	0.002	0.01105	0.12596	0.03267	120	0.07715	0.07715	0.01151	180	0.10086	0.11302	0.03069
60	1	0.004	0.07639	0.17043	0.08557	120	0.10405	0.10405	0.02092	180	0.17722	0.18938	0.07539
60	1	0.008	0.11702	0.18261	0.10504	120	0.10633	0.10963	0.04441	180	0.33790	0.35006	0.23853
60	1	0.010	0.17148	0.23482	0.17559	120	0.11784	0.15707	0.07214	180	0.39546	0.40761	0.31114
60	1	0.020	0.24659	0.35441	0.48724	120	0.17966	0.25608	0.25306	180	0.62086	0.63302	0.72663

Table 4.6: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (5 failures/unit)

$N = 5$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	-0.02040	0.12348	0.01956	120	-0.02579	0.05313	0.00503	180	-0.01978	0.03540	0.00224
60	0.4	0.001	0.05504	0.17500	0.05114	120	0.02376	0.06204	0.00781	180	0.02721	0.05628	0.00958
60	0.4	0.002	0.14148	0.22910	0.10707	120	0.07585	0.09297	0.02334	180	0.06846	0.09912	0.02745
60	0.4	0.004	0.29399	0.36571	0.27494	120	0.16670	0.18289	0.08272	180	0.13742	0.18381	0.08349
60	0.4	0.008	0.58026	0.63337	0.86179	120	0.26164	0.27784	0.20644	180	0.22168	0.32368	0.24586
60	0.4	0.010	0.72255	0.77567	1.26119	120	0.29871	0.32311	0.29612	180	0.25030	0.38001	0.34065
60	0.4	0.020	1.34704	1.40016	4.38553	120	0.42481	0.54434	0.80527	180	0.37575	0.70967	1.18712
60	0.6	0.000	-0.03787	0.12566	0.02074	120	-0.02445	0.05010	0.00523	180	-0.02003	0.03399	0.00260
60	0.6	0.001	0.04244	0.18932	0.05473	120	0.02388	0.06875	0.00748	180	0.02591	0.04312	0.00662
60	0.6	0.002	0.13334	0.23688	0.11510	120	0.07851	0.10888	0.02518	180	0.06505	0.07229	0.01996
60	0.6	0.004	0.28779	0.37930	0.29069	120	0.16738	0.18358	0.08857	180	0.13278	0.15240	0.06708
60	0.6	0.008	0.59848	0.65159	0.96994	120	0.25610	0.27230	0.21559	180	0.21250	0.28027	0.20568
60	0.6	0.010	0.75024	0.80336	1.42774	120	0.29179	0.31641	0.31966	180	0.23940	0.32975	0.28474
60	0.6	0.020	1.39228	1.44540	4.84675	120	0.37816	0.57181	0.80510	180	0.36324	0.61754	1.00073
60	0.8	0.000	-0.00569	0.15444	0.03158	120	-0.01066	0.06120	0.00639	180	-0.01754	0.03773	0.00279
60	0.8	0.001	0.08200	0.22166	0.08351	120	0.03436	0.07574	0.01108	180	0.02009	0.04226	0.00603
60	0.8	0.002	0.17562	0.26847	0.16127	120	0.09241	0.11763	0.03224	180	0.06039	0.07221	0.01963
60	0.8	0.004	0.32705	0.40480	0.35630	120	0.20000	0.21619	0.12554	180	0.13006	0.14719	0.06785
60	0.8	0.008	0.65296	0.70608	1.13161	120	0.31552	0.33172	0.31343	180	0.21757	0.27264	0.20363
60	0.8	0.010	0.82938	0.88250	1.72174	120	0.36269	0.37888	0.45803	180	0.24580	0.31551	0.27255
60	0.8	0.020	1.52701	1.58013	5.72833	120	0.57296	0.64890	1.43847	180	0.40766	0.57511	0.94459
60	1	0.000	0.03841	0.18462	0.05217	120	0.00431	0.06855	0.00913	180	-0.00151	0.04974	0.00441
60	1	0.001	0.06985	0.19961	0.07193	120	0.02405	0.06354	0.00621	180	0.03635	0.05963	0.01469
60	1	0.002	0.13799	0.22056	0.11944	120	0.07148	0.09550	0.01752	180	0.07983	0.10054	0.03743
60	1	0.004	0.23617	0.31563	0.22867	120	0.16684	0.18304	0.07595	180	0.15189	0.17949	0.10199
60	1	0.008	0.50178	0.55490	0.70355	120	0.25547	0.27351	0.19387	180	0.24298	0.32506	0.29138
60	1	0.010	0.63326	0.68638	1.03754	120	0.30300	0.31919	0.28655	180	0.27353	0.37287	0.38253
60	1	0.020	1.14347	1.19659	3.06872	120	0.40489	0.56665	0.79354	180	0.41642	0.65782	1.16306

Table 4.7: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (6 failures/unit)

$N = 6$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	0.04178	0.12712	0.02180	120	-0.01673	0.03783	0.00256	180	0.01495	0.07344	0.00743
60	0.4	0.001	0.06384	0.20449	0.06653	120	0.02043	0.09805	0.01764	180	0.05714	0.11798	0.01826
60	0.4	0.002	0.15868	0.32146	0.15586	120	0.05927	0.15598	0.03816	180	0.11733	0.16021	0.03895
60	0.4	0.004	0.26495	0.50251	0.37921	120	0.10605	0.23734	0.09177	180	0.21018	0.24373	0.09775
60	0.4	0.008	0.35975	0.76406	0.89687	120	0.12723	0.40735	0.28022	180	0.33732	0.37087	0.22921
60	0.4	0.010	0.41583	0.89653	1.24670	120	0.16043	0.50148	0.42848	180	0.38353	0.41708	0.30308
60	0.4	0.020	0.46819	1.34536	2.97305	120	0.18237	0.88319	1.51316	180	0.62454	0.67041	0.84944
60	0.6	0.000	0.07413	0.15690	0.03436	120	0.00220	0.03441	0.00170	180	0.02435	0.07855	0.00821
60	0.6	0.001	0.10948	0.25028	0.09461	120	0.06653	0.11413	0.02226	180	0.08039	0.13021	0.02371
60	0.6	0.002	0.21133	0.38291	0.21925	120	0.12049	0.18054	0.05389	180	0.14656	0.18011	0.05068
60	0.6	0.004	0.34609	0.60396	0.53001	120	0.17670	0.27007	0.12098	180	0.24810	0.28166	0.12094
60	0.6	0.008	0.45752	0.91715	1.23937	120	0.26028	0.46574	0.35145	180	0.39734	0.43089	0.27415
60	0.6	0.010	0.52221	1.07786	1.73403	120	0.31423	0.55879	0.50952	180	0.45368	0.48723	0.35445
60	0.6	0.020	0.60300	1.62276	4.12267	120	0.44895	1.01672	1.76771	180	0.74136	0.77492	0.95686
60	0.8	0.000	0.06828	0.15370	0.03423	120	-0.00059	0.04321	0.00284	180	0.01898	0.08245	0.00881
60	0.8	0.001	0.09608	0.23162	0.08761	120	0.05412	0.12139	0.02385	180	0.06106	0.13387	0.02232
60	0.8	0.002	0.19829	0.36503	0.21277	120	0.10865	0.19743	0.05996	180	0.12380	0.17998	0.04547
60	0.8	0.004	0.34399	0.56749	0.48421	120	0.16538	0.30057	0.14524	180	0.22129	0.26899	0.11333
60	0.8	0.008	0.43160	0.84294	1.07240	120	0.23203	0.51937	0.42984	180	0.38057	0.41412	0.27898
60	0.8	0.010	0.49336	0.99641	1.51195	120	0.29060	0.63106	0.64079	180	0.43542	0.46897	0.36218
60	0.8	0.020	0.54890	1.51974	3.69453	120	0.41892	1.12750	2.20096	180	0.74041	0.77396	1.05880
60	1	0.000	0.08584	0.17316	0.04443	120	0.02452	0.07325	0.00785	180	0.02820	0.08967	0.01170
60	1	0.001	0.12937	0.26746	0.12503	120	0.08732	0.16307	0.03596	180	0.05824	0.13787	0.02418
60	1	0.002	0.23263	0.42286	0.29453	120	0.13867	0.24228	0.08272	180	0.11867	0.17694	0.04398
60	1	0.004	0.36116	0.62090	0.59805	120	0.19509	0.35111	0.18106	180	0.20746	0.26406	0.10450
60	1	0.008	0.46270	0.92757	1.32071	120	0.27057	0.58574	0.52160	180	0.36253	0.39608	0.26021
60	1	0.010	0.51661	1.07997	1.80585	120	0.33838	0.70912	0.77425	180	0.40961	0.44317	0.34097
60	1	0.020	0.48965	1.57191	4.41071	120	0.46054	1.20036	2.34634	180	0.68348	0.71835	0.96412

Table 4.8: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (7 failures/unit)

$N = 7$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$														
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE	
60	0.4	0.000	0.05717	0.09420	0.01179	120	-0.03159	0.04244	0.00284	180	-0.03065	0.05508	0.00514	
60	0.4	0.001	0.10756	0.11588	0.02936	120	0.00373	0.05369	0.00446	180	0.02912	0.06876	0.00851	
60	0.4	0.002	0.12654	0.19252	0.06119	120	0.04993	0.07308	0.01089	180	0.07630	0.10242	0.01690	
60	0.4	0.004	0.18947	0.28190	0.14927	120	0.09585	0.11111	0.02418	180	0.15290	0.16322	0.04554	
60	0.4	0.008	0.24233	0.45714	0.42423	120	0.16585	0.18796	0.06583	180	0.27425	0.28458	0.13228	
60	0.4	0.010	0.25119	0.53952	0.59726	120	0.18501	0.22287	0.09590	180	0.31605	0.32637	0.17379	
60	0.4	0.020	0.30108	0.82809	1.30210	120	0.19525	0.38472	0.24556	180	0.52874	0.53906	0.44910	
60	0.6	0.000	0.04781	0.08548	0.01005	120	-0.03806	0.05095	0.00381	180	-0.03733	0.05830	0.00600	
60	0.6	0.001	0.10853	0.12141	0.02423	120	0.01795	0.07142	0.00718	180	0.02680	0.07843	0.00963	
60	0.6	0.002	0.12333	0.16386	0.03987	120	0.07538	0.11273	0.02824	180	0.07052	0.09700	0.01700	
60	0.6	0.004	0.17756	0.25600	0.11309	120	0.13225	0.15417	0.05213	180	0.12628	0.13661	0.03809	
60	0.6	0.008	0.17339	0.32622	0.18715	120	0.23247	0.25145	0.13147	180	0.21515	0.23691	0.10877	
60	0.6	0.010	0.13758	0.40532	0.27188	120	0.24005	0.31607	0.18451	180	0.24050	0.27469	0.14244	
60	0.6	0.020	-0.00776	0.63187	0.58956	120	0.29639	0.52863	0.47068	180	0.41782	0.42856	0.35610	
60	0.8	0.000	0.05685	0.12415	0.01942	120	-0.05126	0.05710	0.00500	180	-0.03657	0.05156	0.00496	
60	0.8	0.001	0.15091	0.16436	0.06549	120	0.00343	0.07301	0.00781	180	0.02683	0.07921	0.01122	
60	0.8	0.002	0.16490	0.23629	0.10380	120	0.05990	0.11751	0.02811	180	0.07319	0.09989	0.01777	
60	0.8	0.004	0.18724	0.34025	0.22674	120	0.11042	0.16491	0.05119	180	0.12395	0.15650	0.04522	
60	0.8	0.008	0.19381	0.46291	0.43146	120	0.20654	0.26941	0.13390	180	0.19412	0.30777	0.14988	
60	0.8	0.010	0.13458	0.51749	0.55624	120	0.21957	0.33878	0.19560	180	0.20154	0.34604	0.18944	
60	0.8	0.020	-0.06097	0.81949	1.16785	120	0.22237	0.57452	0.53754	180	0.33855	0.55529	0.47168	
60	1	0.000	0.04503	0.11788	0.01932	120	-0.04100	0.05315	0.00400	180	-0.02846	0.04178	0.00327	
60	1	0.001	0.22501	0.23881	0.12748	120	-0.00544	0.06840	0.00651	180	0.02091	0.07205	0.00852	
60	1	0.002	0.27491	0.27491	0.14229	120	0.04190	0.12766	0.02837	180	0.06347	0.08975	0.01358	
60	1	0.004	0.23127	0.24124	0.11765	120	0.10337	0.19843	0.07717	180	0.11566	0.13246	0.03580	
60	1	0.008	0.29551	0.34553	0.23032	120	0.19843	0.35943	0.25118	180	0.18091	0.28526	0.13011	
60	1	0.010	0.22835	0.34105	0.26084	120	0.20485	0.44264	0.35123	180	0.18253	0.32415	0.17604	
60	1	0.020	0.27669	0.59768	0.64608	120	0.29422	0.87772	1.46728	180	0.30705	0.52756	0.46446	

Table 4.9: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (8 failures/unit)

$N = 8$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	0.01707	0.08171	0.00979	120	-0.00887	0.06413	0.00557	180	-0.01674	0.05539	0.00497
60	0.4	0.001	0.14800	0.18253	0.08232	120	0.06946	0.10102	0.01818	180	0.04605	0.07426	0.01476
60	0.4	0.002	0.21304	0.22219	0.13439	120	0.13693	0.16002	0.04506	180	0.10368	0.12675	0.03737
60	0.4	0.004	0.37713	0.38605	0.39003	120	0.25882	0.28190	0.14477	180	0.20486	0.23905	0.11963
60	0.4	0.008	0.64385	0.70161	1.30855	120	0.46759	0.49067	0.41780	180	0.37514	0.42483	0.34401
60	0.4	0.010	0.73978	0.80139	1.75051	120	0.57945	0.60253	0.61101	180	0.47983	0.53446	0.56389
60	0.4	0.020	1.14631	1.38551	5.24338	120	1.02499	1.04807	2.06969	180	0.83185	0.99084	2.04291
60	0.6	0.000	0.03046	0.11469	0.02489	120	-0.00617	0.07777	0.00965	180	-0.01657	0.05126	0.00374
60	0.6	0.001	0.16114	0.21800	0.12573	120	0.06022	0.11483	0.02383	180	0.04437	0.06998	0.01017
60	0.6	0.002	0.25979	0.30060	0.26524	120	0.12559	0.16749	0.05785	180	0.10012	0.11907	0.02446
60	0.6	0.004	0.45947	0.49035	0.74992	120	0.23648	0.28945	0.18075	180	0.19283	0.21179	0.07960
60	0.6	0.008	0.76106	0.84037	2.33484	120	0.39316	0.47876	0.45218	180	0.34084	0.37320	0.22121
60	0.6	0.010	0.82652	0.98024	3.10694	120	0.46263	0.55982	0.58937	180	0.43881	0.47154	0.36674
60	0.6	0.020	1.39418	1.79760	9.80361	120	0.77672	0.93626	1.73348	180	0.74626	0.87438	1.27291
60	0.8	0.000	0.25086	0.34416	0.72703	120	-0.00281	0.08279	0.01043	180	-0.01849	0.05626	0.00444
60	0.8	0.001	0.10677	0.23278	0.13093	120	0.05719	0.12587	0.02591	180	0.02181	0.04882	0.00409
60	0.8	0.002	0.17564	0.33502	0.29159	120	0.11783	0.16499	0.05786	180	0.07462	0.09364	0.01534
60	0.8	0.004	0.33514	0.53240	0.78903	120	0.24506	0.28647	0.19934	180	0.15433	0.17393	0.05581
60	0.8	0.008	0.54578	0.99686	2.51192	120	0.41310	0.51747	0.57525	180	0.31468	0.35042	0.20289
60	0.8	0.010	0.58492	1.21746	3.65931	120	0.50958	0.62893	0.81980	180	0.40313	0.44035	0.33637
60	0.8	0.020	0.81910	2.35594	12.70302	120	0.88394	1.10910	2.60963	180	0.67657	0.84560	1.28916
60	1	0.000	0.25937	0.39233	0.80590	120	0.02891	0.08963	0.01331	180	0.00666	0.06641	0.00764
60	1	0.001	0.09454	0.28137	0.17513	120	0.09537	0.13131	0.03586	180	0.03989	0.06832	0.00870
60	1	0.002	0.13161	0.36347	0.29032	120	0.16246	0.18846	0.07377	180	0.09409	0.11709	0.03409
60	1	0.004	0.24021	0.60195	0.84193	120	0.30292	0.32600	0.22372	180	0.17316	0.20018	0.10586
60	1	0.008	0.36897	1.07902	2.51095	120	0.51061	0.54416	0.57514	180	0.37193	0.39089	0.45971
60	1	0.010	0.33829	1.26566	3.41626	120	0.65813	0.69012	0.92563	180	0.47032	0.48927	0.72628
60	1	0.020	0.53711	2.34572	11.77125	120	1.19027	1.29418	3.20583	180	0.80720	0.91908	3.11933

Table 4.10: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (9 failures/unit)

$N = 9$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	0.02255	0.07744	0.00903	120	-0.00885	0.08273	0.01089	180	-0.01811	0.06248	0.00902
60	0.4	0.001	0.04799	0.07562	0.00853	120	0.01655	0.13861	0.03447	180	0.02277	0.12915	0.02600
60	0.4	0.002	0.06106	0.12206	0.02475	120	0.00386	0.19225	0.08088	180	0.05526	0.19932	0.06193
60	0.4	0.004	0.09820	0.25710	0.11158	120	-0.01387	0.30730	0.25303	180	0.11644	0.31603	0.17071
60	0.4	0.008	0.02403	0.45567	0.56149	120	-0.10778	0.47245	0.77610	180	0.17668	0.54716	0.61195
60	0.4	0.010	0.01806	0.53207	0.86007	120	-0.13878	0.55890	1.11514	180	0.20230	0.65565	0.90200
60	0.4	0.020	-0.23776	1.02842	4.11965	120	-0.39259	1.11746	4.26386	180	0.31620	1.14148	3.00884
60	0.6	0.000	-0.00079	0.08524	0.01233	120	-0.00714	0.08919	0.01302	180	-0.01749	0.07022	0.01063
60	0.6	0.001	0.03320	0.08945	0.01302	120	0.03057	0.15811	0.04250	180	0.03196	0.13852	0.03052
60	0.6	0.002	0.04168	0.15017	0.03366	120	0.01884	0.22007	0.09250	180	0.06381	0.20843	0.07021
60	0.6	0.004	0.01491	0.25502	0.09480	120	0.00421	0.35594	0.28617	180	0.10811	0.31700	0.18403
60	0.6	0.008	-0.13847	0.50602	0.58767	120	-0.08849	0.55979	0.87853	180	0.17207	0.54901	0.65927
60	0.6	0.010	-0.17259	0.56442	0.81839	120	-0.12411	0.66411	1.24647	180	0.20707	0.66881	0.98038
60	0.6	0.020	-0.60824	1.12372	3.65551	120	-0.35623	1.27142	4.76343	180	0.32797	1.15263	3.28369
60	0.8	0.000	-0.00575	0.08009	0.01443	120	-0.00531	0.07896	0.01095	180	-0.01049	0.05840	0.00674
60	0.8	0.001	0.00638	0.10637	0.01688	120	0.05224	0.17153	0.04914	180	0.06846	0.11717	0.02505
60	0.8	0.002	0.00433	0.15492	0.03459	120	0.05784	0.24635	0.10887	180	0.10156	0.16892	0.05305
60	0.8	0.004	-0.13451	0.31852	0.20144	120	0.04314	0.37214	0.30083	180	0.16185	0.25943	0.17121
60	0.8	0.008	-0.46681	0.75220	1.64240	120	-0.06398	0.59312	0.94073	180	0.25923	0.50653	0.61702
60	0.8	0.010	-0.59774	0.90344	2.57632	120	-0.08293	0.71156	1.35680	180	0.30671	0.61321	0.91412
60	0.8	0.020	-1.62728	2.07635	12.92940	120	-0.26755	1.40061	4.98517	180	0.51130	1.14401	3.30716
60	1	0.000	-0.01130	0.10661	0.01963	120	0.01839	0.07180	0.00736	180	0.00680	0.06001	0.00685
60	1	0.001	-0.06938	0.14504	0.03665	120	0.12100	0.17188	0.07448	180	0.07777	0.11739	0.03028
60	1	0.002	-0.15106	0.25605	0.13389	120	0.15808	0.21630	0.11317	180	0.11037	0.17991	0.06198
60	1	0.004	-0.41752	0.62069	1.23870	120	0.21743	0.34542	0.27481	180	0.15238	0.28731	0.18304
60	1	0.008	-0.86606	1.22241	5.24369	120	0.20139	0.53063	0.58982	180	0.21880	0.55992	0.63620
60	1	0.010	-1.04979	1.46325	7.40819	120	0.26483	0.64759	0.93750	180	0.24411	0.68466	0.92272
60	1	0.020	-2.19157	2.80610	25.67192	120	0.29864	1.11029	2.61349	180	0.42265	1.19648	2.95979

Table 4.11: Summary of PWP-GT model result for estimating $\hat{\beta}_i$ (10 failures/unit)

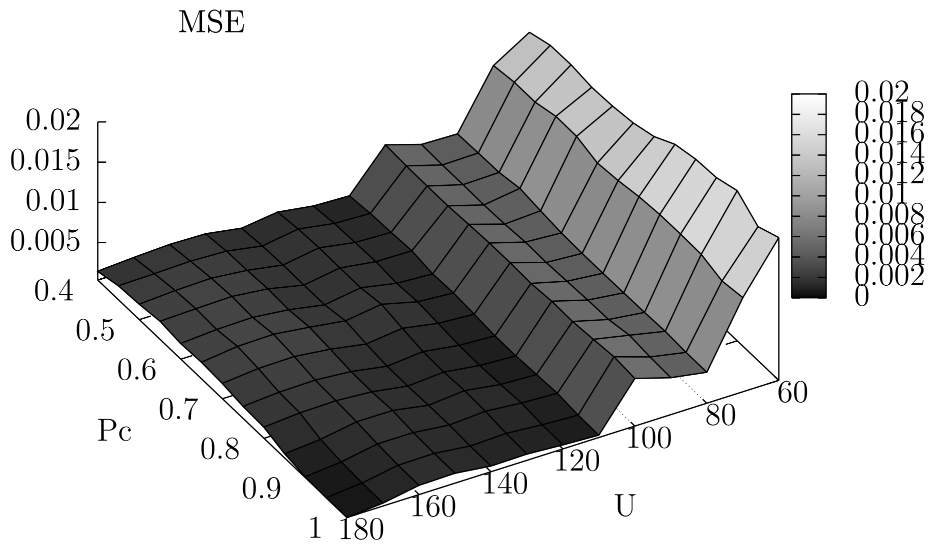
$N = 10$ failure events/unit, $\mu_0 = -6.9, \mu_1 = -4.6$													
U	P_c	θ	BIAS	MAD	MSE	U	BIAS	MAD	MSE	U	BIAS	MAD	MSE
60	0.4	0.000	0.04095	0.08597	0.01117	120	0.00144	0.05907	0.00597	180	-0.00503	0.03337	0.00195
60	0.4	0.001	0.06440	0.17971	0.04966	120	0.03264	0.08274	0.01277	180	0.03965	0.07340	0.00980
60	0.4	0.002	0.13209	0.29479	0.13150	120	0.09165	0.13848	0.03078	180	0.09065	0.12663	0.03316
60	0.4	0.004	0.17537	0.48672	0.37408	120	0.18682	0.25096	0.09680	180	0.18743	0.26367	0.12880
60	0.4	0.008	0.27549	0.82192	1.14755	120	0.29703	0.45979	0.31622	180	0.32097	0.47517	0.42642
60	0.4	0.010	0.35602	1.02450	1.78231	120	0.33229	0.56383	0.48876	180	0.35774	0.56280	0.63713
60	0.4	0.020	0.74930	1.92479	6.54469	120	0.53439	0.95364	1.50239	180	0.60332	1.01840	2.20875
60	0.6	0.000	0.04068	0.11744	0.02024	120	-0.00621	0.05118	0.00470	180	-0.01775	0.04329	0.00363
60	0.6	0.001	0.05337	0.21556	0.07791	120	0.02090	0.07646	0.01178	180	0.01771	0.07953	0.01049
60	0.6	0.002	0.12231	0.32159	0.16130	120	0.07918	0.12513	0.02384	180	0.06001	0.12051	0.02818
60	0.6	0.004	0.11832	0.52309	0.46142	120	0.16454	0.20816	0.06416	180	0.13991	0.20131	0.08852
60	0.6	0.008	0.14528	0.89582	1.46547	120	0.26097	0.37735	0.19455	180	0.23986	0.34614	0.27580
60	0.6	0.010	0.19409	1.10843	2.23882	120	0.28507	0.44627	0.27711	180	0.25794	0.42759	0.42874
60	0.6	0.020	0.41493	2.03250	7.85001	120	0.37976	0.69291	0.70650	180	0.35805	0.85941	1.56322
60	0.8	0.000	-0.00830	0.08909	0.01594	120	0.01200	0.06075	0.00690	180	-0.01011	0.06153	0.00721
60	0.8	0.001	0.02349	0.21117	0.08940	120	-0.00072	0.10152	0.02088	180	-0.01958	0.09618	0.03330
60	0.8	0.002	0.09483	0.33082	0.18257	120	0.07471	0.14354	0.04041	180	-0.02402	0.16117	0.08333
60	0.8	0.004	0.13790	0.49335	0.37664	120	0.14547	0.22067	0.07802	180	-0.03040	0.34019	0.29943
60	0.8	0.008	0.16542	0.88719	1.35125	120	0.27182	0.41752	0.27676	180	-0.12009	0.59180	0.98355
60	0.8	0.010	0.24890	1.10764	2.08746	120	0.28856	0.50766	0.38562	180	-0.18718	0.72919	1.49381
60	0.8	0.020	0.46320	2.29441	9.73531	120	0.38407	0.82279	1.00615	180	-0.50616	1.39658	6.08111
60	1	0.000	0.32260	0.41915	0.63855	120	0.34995	0.38645	1.10221	180	0.00741	0.07137	0.00992
60	1	0.001	-0.70475	0.99517	7.03446	120	0.01577	0.12520	0.02871	180	0.00800	0.09491	0.02214
60	1	0.002	-2.13976	2.59919	57.00721	120	0.13813	0.19013	0.05775	180	0.04106	0.13318	0.02871
60	1	0.004	-3.99949	4.51421	187.04188	120	0.28490	0.34777	0.23743	180	-0.06477	0.29251	0.25216
60	1	0.008	-7.55310	8.33711	640.88087	120	0.46739	0.76534	1.30063	180	-0.17074	0.57378	0.98749
60	1	0.010	-9.24578	10.26145	968.89240	120	0.59215	0.92052	2.05811	180	-0.28937	0.74234	1.75925
60	1	0.020	-17.64636	19.80513	3619.99550	120	1.19821	1.80924	9.23283	180	-0.71731	1.58868	8.10894

To demonstrate the effect of failure event counts on the robustness in estimating the covariate effect, the performance measure of MSE and BIAS are plotted in Figures 4.6 - 4.8, for the case $P_c = 0.8$

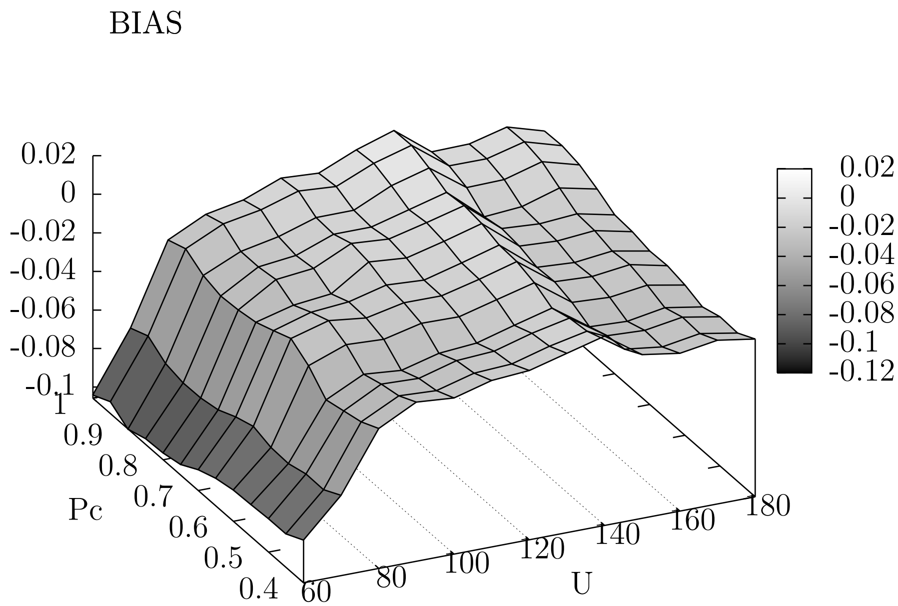
4.4.2 Performance comparison between Power-law and Log-linear

The PWP-GT performance comparison is made on selected parameters for both power-law and log-linear NHPP models. The data for power-law are taken from (Jiang, 2004, p. 124). Severe censoring probabilities are used in this comparison since they tend to produce poorer performance. Refer to Table 4.12, where robustness metrics are compared for a power-law NHPP with $\delta = 1.2/1.8$ and failure event counts $N = 10$, versus a log-linear NHPP with $\theta = 0.001/0.02$ and $N = 3$. Note that the robustness metrics for $\delta = 1.2$ $(N = 10)$ and $\theta = 0.001$ $(N = 3)$ are comparable for larger sample size $(U = 180)$. In Table 4.12, the first four rows are from smaller shape parameters $(\delta = 1.2, \theta = 0.001)$, for both models divided two rows each for $P_c = 0.8$ and $P_c = 1.0$. For this moderately increasing intensity function in the comparable time-intensity regime, observe that the log-linear model is slightly more robust. The bottom four rows are from larger shape parameters $(\delta = 1.8, \theta = -0.02)$ for both models, which produce more rapidly increasing intensity function in the comparable time-intensity regime. Note that the log-linear model is less robust than the power-law model in terms of BIAS, MAD, and MSE. This is attributable to the difference between the two functional forms. The failure intensity of a log-linear model is increasing at an increasing rate with time, while the power-law intensity increases at a decreasing rate. Consequently, the log-linear tends to be robust for small failure event counts but relatively non-robust for larger failure event counts. Figures 4.6 - 4.8 illustrate the effect of failure event count on MAD and MSE for $N = 2, \cdots, 5$ and sample sizes $U = 60$ (Figure 4.6), $U = 120$ (Figure 4.7), and $U = 180$ (Figure 4.8).

Qureshi et al. (1994), Vithala (1994), and Jiang (2004) all found that there is a tendency of the PWP-GT model to over-estimate the regression coefficient β for an increasing intensity (IROCOF). This positive bias increases with IROCOF. Tables 4.3 - 4.11 and Figures 4.3 - 4.5 indicate the same bias pattern for the log-linear with IROCOF. It is noteworthy that the PWP method was developed and first applied for a case of few ($N \leq 5$) recurrent events (see Prentice et al., 1981).

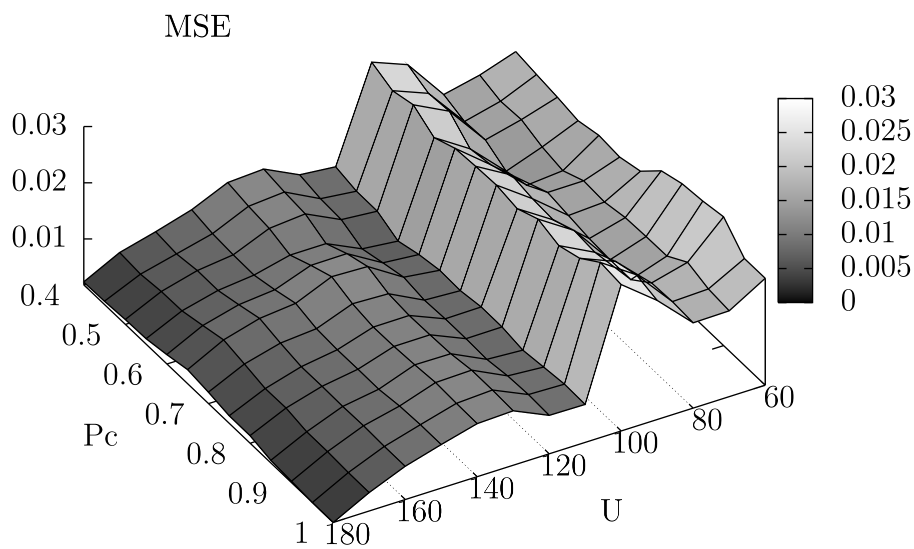


(a)

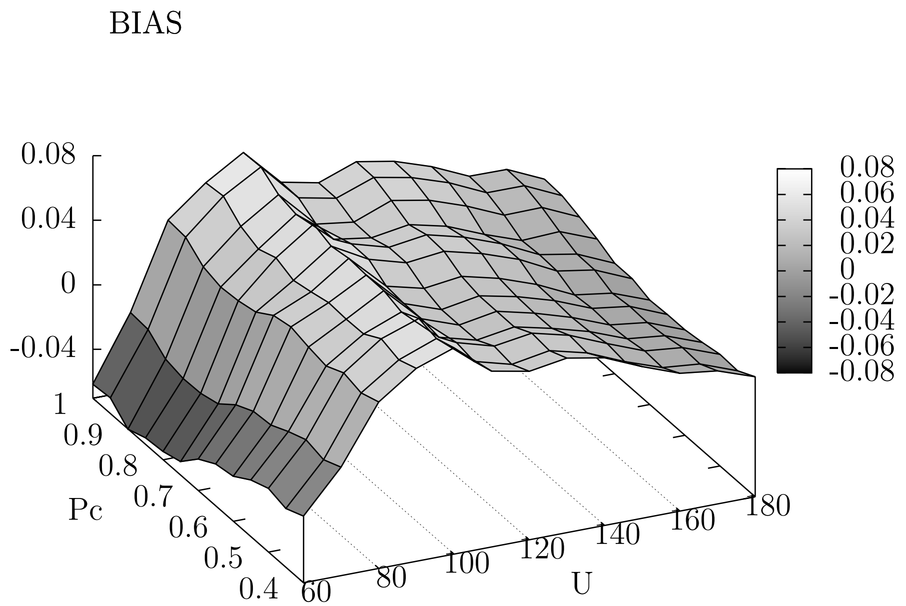


(b)

Figure 4.3: PWP-GT model results for estimating $\hat{\beta}_i$ (3 failures/unit), $\theta = 0$

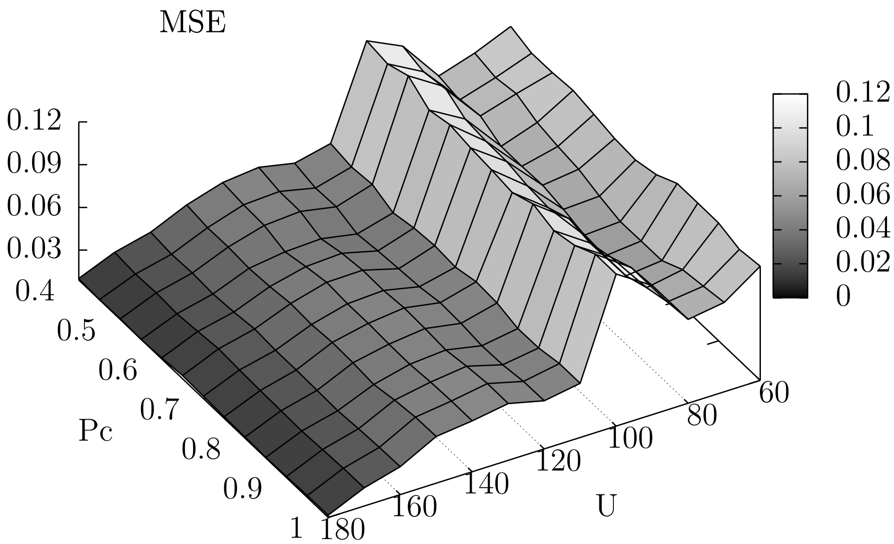


(a)

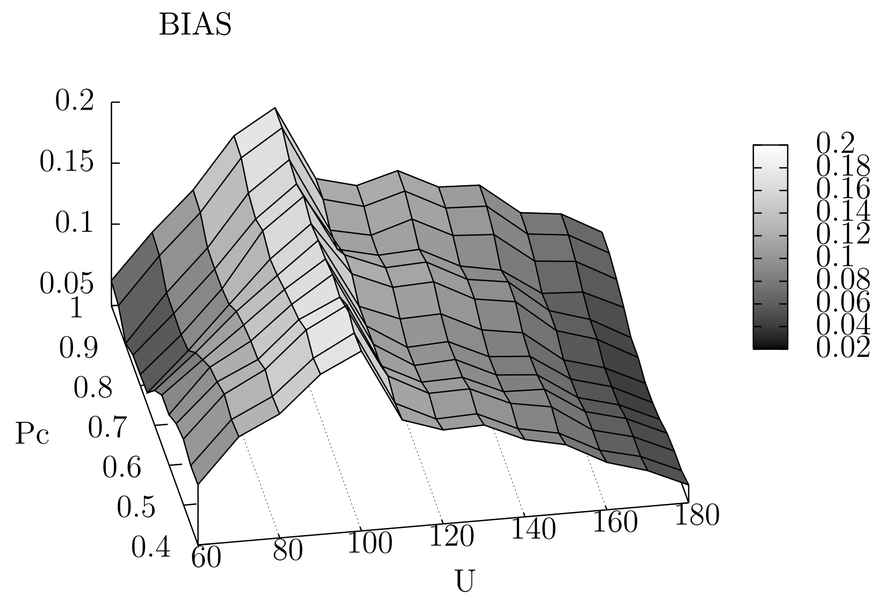


(b)

Figure 4.4: PWP-GT model results for estimating $\hat{\beta}_i$ (3 failures/unit), $\theta = 0.001$



(a)



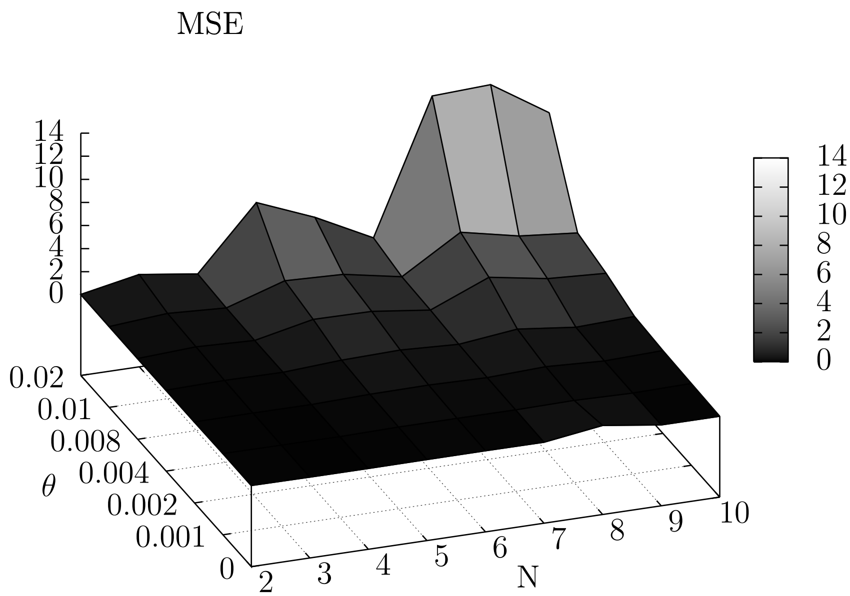
(b)

Figure 4.5: PWP-GT model results for estimating $\hat{\beta}_i$ (3 failures/unit), $\theta = 0.003$

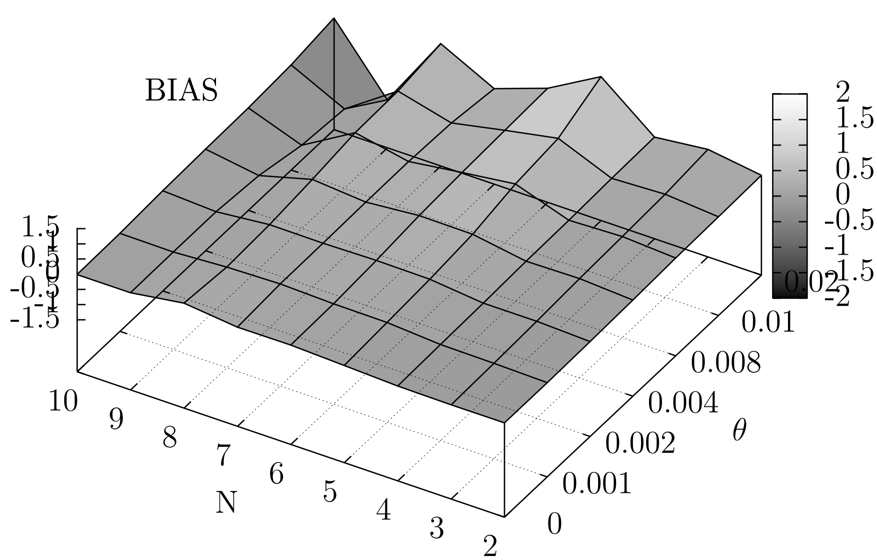
Table 4.12: Performance comparison: Power-law $N = 10$ vs. Log-linear $N = 3$

		U=60			U=120			U=180		
	Pc	BIAS	MAD	MSE	BIAS	MAD	MSE	BIAS	MAD	MSE
PL($\delta = 1.2$)	0.8	0.03604	0.13645	0.03159	0.04122	0.07453	0.00894	0.01806	0.07786	0.11760
LL($\theta = 0.001$)	0.8	-0.06513	0.12268	0.02361	0.03469	0.05933	0.00825	-0.00185	0.03942	0.00321
PL($\delta = 1.2$)	1	0.21679	0.29794	0.32410	0.45838	0.48502	1.68430	0.02747	0.10220	0.01642
LL($\theta = 0.001$)	1	-0.06159	0.11470	0.02088	0.03654	0.06118	0.00870	0.01211	0.02651	0.00183
PL($\delta = 1.8$)	0.8	0.12972	0.27344	0.14486	0.12368	0.15971	0.07195	0.10765	0.16802	0.08241
LL($\theta = 0.02$)	0.8	**0.33390**	**0.59386**	**1.00732**	**0.53174**	**0.55645**	**0.76033**	**0.13483**	**0.21603**	**0.14528**
PL($\delta = 1.8$)	1	-0.22053	0.63335	1.79361	0.16373	0.18841	0.08326	0.12651	0.19201	0.08858
LL($\theta = 0.02$)	1	**0.37544**	0.55755	0.99804	**0.53263**	**0.55733**	**0.77829**	**0.21694**	**0.22052**	**0.14859**

Key: bold face denotes poorer performance domain.

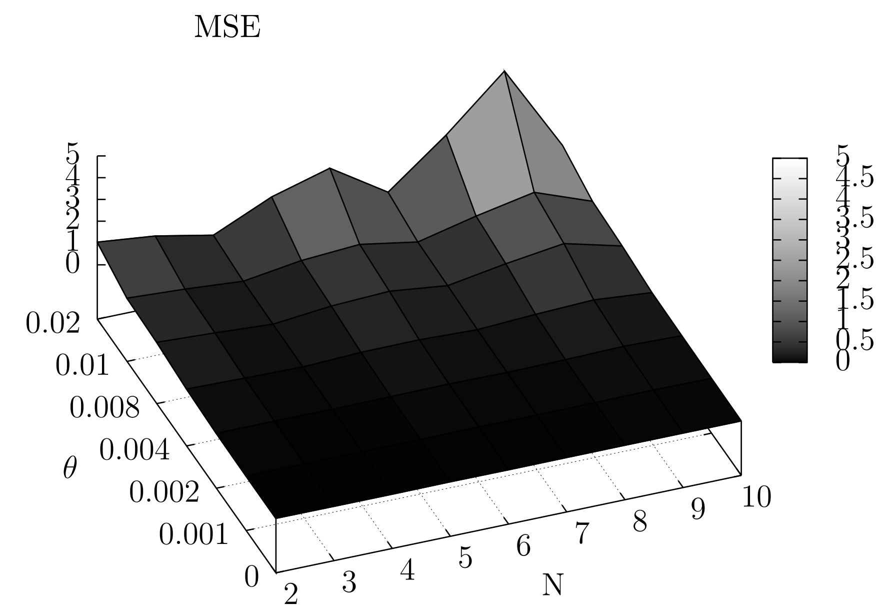


(a)

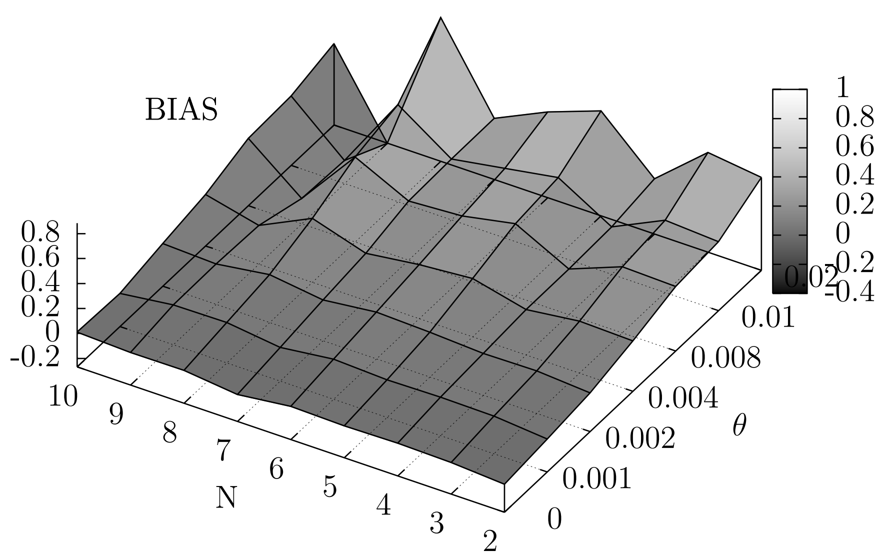


(b)

Figure 4.6: Robustness of PWP-GT estimates as functions of the number of failures: $U = 60$

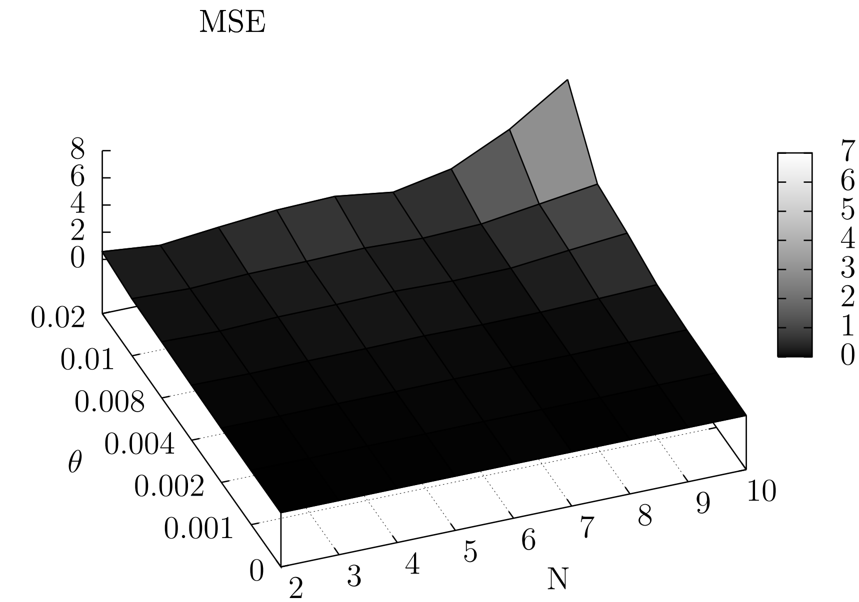


(a)

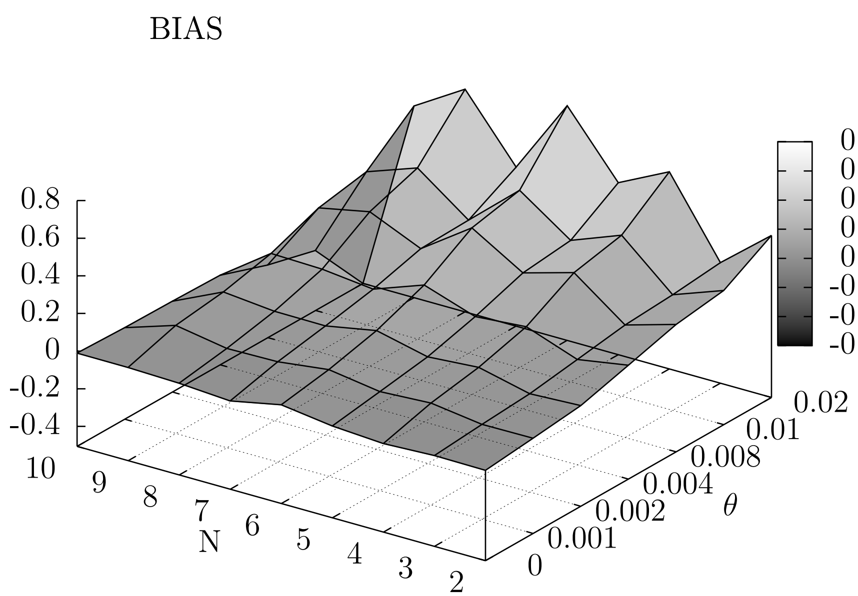


(b)

Figure 4.7: Robustness of PWP-GT estimates as functions of the number of failures: $U = 120$



(a)



(b)

Figure 4.8: Robustness of PWP-GT estimates as functions of the number of failures: $U = 180$

4.4.3 Complete data

This section compares the effects of right-censoring versus the base case of complete data ($P_c = 0$) for three values of shape parameter and three sample sizes. Tables 4.13 and 4.14 summarize the performance metric. Table 4.13 gives the three log-linear intensity functions at sample size $U = 120$, while Table 4.14 examines two other sample size for $\theta = 0.001$.

$N = 10$ failures/unit, $\mu_0 = -6.9, \mu_1 = -4.6$					
U	P_c	θ	BIAS	MAD	MSE
120	0	0	-0.02395	0.05174	0.00388
		0.001	0.00761	0.07212	0.00807
		0.004	0.10338	0.19578	0.06318
120	0.4	0	0.00144	0.05907	0.00597
		0.001	0.03264	0.08274	0.01277
		0.004	0.18682	0.25096	0.09680
120	0.6	0	-0.00621	0.05118	0.00470
		0.001	0.02090	0.07646	0.01178
		0.004	0.16454	0.20816	0.06416
120	0.8	0	0.01200	0.06075	0.00690
		0.001	-0.00072	0.10152	0.02088
		0.004	0.14547	0.22067	0.07802
120	1	0	0.34995	0.38646	1.10221
		0.001	0.01577	0.12520	0.02871
		0.004	0.28490	0.34777	0.23743

Table 4.13: Performance metrics (PWP-GT) in three log-linear intensity functions

4.4.4 95% confidence interval on $\hat{\beta}_i$

To examine the right-censoring effects upon the PWP-GT model, 95% confidence interval were constructed on the covariate estimate $\hat{\boldsymbol{\beta}}_i$ for the HPP, where P_c is set to 1.0 (heavily censored). Three sample sizes are charted in Table 4.15 for $U = 60$, $U = 120$, and $U = 180$.

$N = 10$ failures/unit, $\mu_0 = -6.9, \mu_1 = -4.6$					
U	θ	P_c	BIAS	MAD	MSE
60	0.001	0	0.02657	0.15769	0.04304
		0.4	0.06440	0.17971	0.04966
		0.6	0.05337	0.21556	0.07791
		0.8	0.02349	0.21117	0.08940
		1	-0.70475	0.99517	7.03446
120	0.001	0	0.00761	0.07212	0.00807
		0.4	0.03264	0.08274	0.01277
		0.6	0.02090	0.07646	0.01178
		0.8	-0.00072	0.10152	0.02088
		1	0.01577	0.12520	0.02871
180	0.001	0	0.02470	0.05165	0.00478
		0.4	0.03965	0.07340	0.00980
		0.6	0.01771	0.07953	0.01049
		0.8	-0.01958	0.09618	0.03330
		1	0.00800	0.09491	0.02214

Table 4.14: Performance metrics (PWP-GT) in three sample sizes, $\theta = 0.001$

Table 4.15: 95% C.I on $\hat{\boldsymbol{\beta}}_i, (\theta, P_c) = (0, 1.0)$, for three sample size, 60, 120, 180

n	U	Average	95%LB	95%UB	U	Average	95%LB	95%UB	U	Average	95%LB	95%UB
1	60	2.439(0.260)	1.93	2.95	120	2.199(0.156)	1.89	2.5	180	2.247(0.130)	1.99	2.5
2	60	2.254(0.236)	1.79	2.72	120	2.217(0.163)	1.9	2.54	180	2.261(0.139)	1.99	2.53
3	60	2.422(0.276)	1.88	2.96	120	2.413(0.181)	2.06	2.77	180	2.255(0.145)	1.97	2.54
4	60	2.567(0.309)	1.96	3.17	120	2.490(0.202)	2.09	2.89	180	2.488(0.172)	2.15	2.83
5	60	2.031(0.270)	1.5	2.56	120	2.414(0.208)	2.01	2.82	180	2.484(0.177)	2.14	2.83
6	60	2.603(0.414)	1.79	3.42	120	2.611(0.264)	2.09	3.13	180	2.499(0.200)	2.11	2.89
7	60	6.462(126.222)	-240.93	253.86	120	2.831(0.402)	2.04	3.62	180	2.629(0.264)	2.11	3.15
8	60	2.348(0.509)	1.35	3.35	120	2.101(0.308)	1.5	2.7	180	2.188(0.263)	1.67	2.7
9	60	1.505(0.515)	0.5	2.51	120	2.264(0.451)	1.38	3.15	180	2.306(0.349)	1.62	2.99
10	60	5.789(459.652)	-895.13	906.71	120	9.510(154.255)	-292.83	311.85	180	1.814(0.468)	0.9	2.73

4.4.5 PWP-TT for Log-linear NHPP, with IROCOF

This section reports a pilot study to extend Vithala's work to PWP-TT method while the underlying baseline intensity function remains of NHPP log-linear form. Since Jiang (2004) uses a more recent version of SAS (8.0), the program is conveniently accommodated to perform the robustness study for the PWP-TT method with an underlying intensity function of NHPP log-linear form.

To simulate an NHPP log-linear process, the data generating algorithm requires modification in accordance with Law & Kelton (1991); hence the focal block of code becomes:

```
DATA LOGLIN;
    RETAIN SEED 539;
    FORMAT R Y 16.8;
    THETA = 1.2;
    DO ITEM = 1 TO 60;
        T = 0;
        M = 0;
        R = 0;
        DO FAILURE = 1 TO 10;
            X = RANUNI(SEED);
            T = T - LOG(X);
            IF ITEM <= 30 THEN MU = -6.9;
            ELSE MU = -4.6;
            IF MU = -6.9 THEN CLASS = 0;
            ELSE CLASS = 1;
            R = ((LOG(THETA*T) + EXP(MU)) - MU)/THETA;
            Y = R - M;
            M = R;
            OUTPUT;
            END;
        END;
```

The following tables summarize the pilot study comparisons of robustness using the PWP-GT (Table 4.16) and PWP-TT (Table 4.17) method. Note the values are the averages of three replicates generated by seed numbers 539, 255, and 59. Since the main purpose of this experiment is to validate the program adaptation, only a small range of levels are used in computation. Two sample sizes (60 and 120) and three θ values (0.01, 1, and 2) are chosen. Two values of censoring severity ($P_c = 0$ and $P_c = 0.3$) are selected to represent the presence and absence of censoring.

Table 4.16: The summary of the robustness test using PWP-GT

10 failures/unit, 10 units/class, 2 classes, $\mu_0 = -6.9, \mu_1 = -4.6$							
			Class=0			Class=1	
U	θ	BIAS	MAD	MSE	BIAS	MAD	MSE
60	0.0	0.080086	0.099880	0.014292	-0.010190	0.109233	0.018814
60	1.0	0.108546	0.120144	0.023100	0.056940	0.106137	0.017886
60	2.0	0.104911	0.122469	0.021196	0.060492	0.108437	0.016935
120	0.0	0.034334	0.046789	0.004284	-0.038050	0.062044	0.010009
120	1.0	0.061007	0.061192	0.007688	0.017472	0.041101	0.002549
120	2.0	0.062393	0.062633	0.007916	0.019233	0.038650	0.002428

Table 4.17: The summary of the robustness test using PWP-TT

10 failures/unit, 10 units/class, 2 classes, $\mu_0 = -6.9, \mu_1 = -4.6$							
			Class=0			Class=1	
U	θ	BIAS	MAD	MSE	BIAS	MAD	MSE
60	0.0	0.090011	0.101140	0.014000	0.010010	0.114210	0.017277
60	1.0	0.113721	0.132341	0.027900	0.062410	0.115513	0.017001
60	2.0	0.110230	0.132110	0.020044	0.060997	0.112763	0.018921
120	0.0	0.044321	0.047812	0.003009	-0.039890	0.063977	0.011476
120	1.0	0.072453	0.062276	0.008688	0.019311	0.044067	0.025990
120	2.0	0.110230	0.132110	0.020044	0.021330	0.039350	0.002880

4.4.6 PWP-TT, AG, and WLW models for HPP

This section investigates the estimating performance of the PWP-TT, AG, and WLW models for three sample sizes ($U = 60, 120$ and 180) in an HPP case (i.e., $\theta = 0$). As the censoring severity increases, the PWP-TT (Table 4.19) and WLW (Table 4.20) estimates slightly decrease, while the AG (Table 4.18) estimates remain unchanged. Larger sample size results in narrowed 95% confidence intervals. The total-time mod-

U	P_c	AG estimates	LB	UB
60	0.4	2.29948(0.09691)	2.10952	2.48944
60	0.6	2.27169(0.09983)	2.07601	2.46737
60	0.8	2.28183(0.10185)	2.08220	2.48146
60	1	2.27431(0.10851)	2.06164	2.48699
120	0.4	2.31773(0.06919)	2.18210	2.45335
120	0.6	2.30162(0.07068)	2.16307	2.44017
120	0.8	2.29100(0.07285)	2.14820	2.43380
120	1	2.30808(0.07647)	2.15819	2.45796
180	0.4	2.27175(0.05534)	2.16328	2.38022
180	0.6	2.26054(0.05721)	2.14839	2.37268
180	0.8	2.25604(0.05931)	2.13977	2.37230
180	1	2.26269(0.06255)	2.14010	2.38529

Table 4.18: Summary of semi-parametric AG model results for $\hat{\beta}(\theta = 0, HPP)$

els (PWP-TT, AG, and WLW) are not affected by the shape parameter θ as contrasted to the gap-time model (PWP-GT); that is, when the shape parameter varies, the PWP-TT, AG, and WLW estimate and their variability remain the same. This is because the shape parameter does not influence the likelihood function in the total-time model. Thus, the HPP case is chosen to illustrate the total-time models. Among the three models illustrated, the AG model results in the most robust estimate for right-censoring. The theoretical value of β is 2.3, and Table 4.18 show that this value is covered within the 95% C.I in all combination of experimental units and censoring severity.

U	P_c	PWP-TT Estimates	95%LB	95%UB
60	0.4	2.99786(0.10407)	2.79388	3.20183
60	0.6	2.92181(0.10641)	2.71324	3.13038
60	0.8	2.80021(0.10829)	2.58797	3.01246
60	1	2.62295(0.11336)	2.40076	2.84515
120	0.4	3.04627(0.07435)	2.90054	3.19200
120	0.6	2.93902(0.07549)	2.79105	3.08699
120	0.8	2.82342(0.07775)	2.67102	2.97583
120	1	2.67331(0.08041)	2.51569	2.83093
180	0.4	2.97684(0.05904)	2.86111	3.09257
180	0.6	2.88957(0.06082)	2.77035	3.00879
180	0.8	2.81449(0.06309)	2.69083	2.93816
180	1	2.66378(0.06535)	2.53568	2.79187

Table 4.19: Summary of semi-parametric PWP-TT model results for $\hat{\beta}(\theta = 0, HPP)$

U	P_c	WLW estimates	95%LB	95%UB
60	0.4	3.12759(0.10383)	2.92409	3.33110
60	0.6	3.03081(0.10699)	2.82110	3.24051
60	0.8	2.96469(0.10951)	2.75005	3.17932
60	1	2.80002(0.11477)	2.57506	3.02497
120	0.4	3.11746(0.07465)	2.97114	3.26379
120	0.6	3.04922(0.07610)	2.90004	3.19839
120	0.8	2.95554(0.07886)	2.80096	3.11012
120	1	2.85796(0.08162)	2.69797	3.01795
180	0.4	3.05111(0.05940)	2.93468	3.16755
180	0.6	3.00295(0.06145)	2.88249	3.12341
180	0.8	2.94036(0.06403)	2.81485	3.06587
180	1	2.79545(0.06673)	2.66465	2.92624

Table 4.20: Summary of semi-parametric WLW model results for $\hat{\beta}(\theta = 0, HPP)$

Chapter 5

Conclusions and Future Researches

5.1 Conclusions

Previous studies (Landers & Soroudi (1991), Qureshi et al. (1994)) evaluated the PWP-GT model for the case of an underlying NHPP with power-law intensity function. Jiang (2004) extended the research to model the case of right-censoring and examined other semi-parametric PI models with covariates. Qureshi et al. (1994) examined the PWP-GT models applied to recurrent data without censoring and concluded that the PWP-GT model underestimates the covariate effect in a DROCOF case and overestimates the covariate effect in an IROCOF case. Qureshi et al. (1994) identified the more favorable application range of PWP-GT for the case of complete data from an NHPP power-law intensity function. Jiang (2004) identified the more favorable engineering applications ranges for censored data over the factors of sample size, censoring severity, and power-law shape parameters.

This study investigated the robustness performance of the four proportional intensity models (PWP-GT, PWP-TT, AG, and WLW), when the underlying process has a log-linear increasing intensity function. The PWP-GT is a robust estimator of the covariate regression coefficient β for the case of an underlying NHPP with log-linear increasing intensity function for failure event count $N \leq 4$, shape parameter $\theta \leq 0.01$, censoring probability $P_c \leq 0.8$ and sample size $U \geq 60$.

In the case of HPP, the PWP-GT and AG prove to be models of choice, evaluated in terms of the BIAS, MAD, and MSE of covariate regression coefficients over ranges of sample sizes, shape parameters, and censoring severity as found in engineering applications. The research is conducted over the domain of the three factors of interest: (1) $60 \leq U \leq 180$, (2) $0.0 \leq \theta \leq 0.02$, and (3) $0.0 \leq P_c \leq 1.0$. An interesting result is that small changes in shape parameter from the HPP (i.e., $\theta \geq 0.008$) and failure counts of $N > 4$ result in the performance of PWP-GT deteriorating substantially.

The AG model proves to outperform the WLW for a stationary process (HPP) across a wide range of right-censorship ($0.0 \leq P_c \leq 1.0$) and for sample size of 60 or more.

This research also made an important contribution to the research infrastructure for this and future studies, through an open-source and highly automated system of simulation, data collection and output summarization.

5.2 Future Research

This research has addressed only the NHPP with log-linear underlying process, increasing ROCOF, and single two-level covariates. Future research topics could include applications to: (1) decreasing ROCOF, (2) multi-dimensional covariate, and (3) risk-free intervals. Further studies of the failure event-count effect, for the case of an underlying NHPP with power-law increasing (decreasing) intensity function would also be of interest.

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Appendix A

SAS Program for semi-parametric proportional intensity models

```
/*-----------------------------------------------------------------------*
 |                                                                       |
 |This is the working code without macro automation. Since the           |
 |parameters are hard-coded, each run will generate one dataset for      |
 |recurrent failure events that simulate NHPP with log-linear intensity |
 |function, for given parameters (i.e., seed number, samples size,      |
 |censoring probability, and shape parameters).  Four Cox based          |
 |regression models are used to analyze the generated datasets.         |
 |                                                                       |
*-----------------------------------------------------------------------*/

OPTIONS LS = 75;

DATA LOGLIN;
    RETAIN SEED 539;
    FORMAT T Y 16.8;

    DO ITEM = 1 TO 60;

        P = RANUNI(SEED);
        IF P < 0.4 THEN CENSOR = 0;
        ELSE CENSOR = 1;

        F = FLOOR(10*RANUNI(SEED)) + 1;
        T = 0;
        M = 0;
```

```
        R = 0;

        TSTART = 0;

        DO FAILURE = 1 TO 10;

            RETAIN M 0;
            X = RANUNI(SEED);
            R = R - LOG(X);
            THETA = 2;
            IF ITEM <= 30 THEN MU = -6.9;
            ELSE MU = -4.6;

            IF MU = -6.9 THEN CLASS = 0;
            ELSE CLASS = 1;
            IF(FAILURE > F & CENSOR = 0) THEN STATUS = 0;
            ELSE STATUS = 1;

            IF STATUS = 1 THEN DO;

                IF THETA = 0 THEN T = R / EXP(MU);
                ELSE T = (LOG(THETA*R + EXP(MU)) - MU) / THETA;
                Y = T - M;
                M = T;
                TSTOP = T;
                OUTPUT;
                TSTART = TSTOP;

                END;

            IF STATUS = 0 THEN DO;
                T = 0;
                Y = 0;
                TSTOP = TSTART;
                OUTPUT;
                END;
            END;
        END;


DATA TBFB;
SET LOGLIN;
DROP  M NU THETA;
```

```
PROC PRINT DATA=TBFB;
TITLE1 'RIGHT CENSORING DATA OF TIME BETWEEN FAILURES';

DATA CENSOR;
SET TBFB;
DROP X P F;
IF STATUS=1 THEN DELETE;
PROC PRINT DATA=CENSOR;
TITLE'CENSOR';

DATA UNCENSOR;
SET TBFB;
DROP X P F;
IF STATUS=0 THEN DELETE;
PROC PRINT DATA=UNCENSOR;
TITLE 'UNCENSOR';

DATA CENSOR_AG;
SET TBFB;
IF TSTART=TSTOP THEN DELETE;
PROC PRINT DATA=CENSOR_AG;
TITLE'CENSOR_AG';

DATA CENSOR_PWP(DROP=LSTATUS);
RETAIN LSTATUS;
SET TBFB;
BY ITEM;
IF FIRST.ID THEN LSTATUS=1;
    IF (STATUS=0 AND LSTATUS=0) THEN DELETE;
    LSTATUS=STATUS;
PROC PRINT DATA=CENSOR_PWP;
TITLE'CENSOR_PWP';

DATA CENSOR_WLW;
SET TBFB;
PROC PRINT DATA=CENSOR_WLW;
TITLE'CENSOR_WLW';

PROC PHREG DATA=CENSOR_AG;
MODEL (TSTART,TSTOP)* STATUS(0)= CLASS;
TITLE1' ANDERSEN-GILL SUMMARY';
```

```
DATA CENSOR_PWP1;
SET CENSOR_PWP;
IF FAILURE<11;
CLASS1=CLASS*(FAILURE=1);
CLASS2=CLASS*(FAILURE=2);
CLASS3=CLASS*(FAILURE=3);
CLASS4=CLASS*(FAILURE=4);
CLASS5=CLASS*(FAILURE=5);
CLASS6=CLASS*(FAILURE=6);
CLASS7=CLASS*(FAILURE=7);
CLASS8=CLASS*(FAILURE=8);
CLASS9=CLASS*(FAILURE=9);
CLASS10=CLASS*(FAILURE=10);

PROC PHREG DATA=CENSOR_PWP1;
MODEL Y * STATUS(0)= CLASS1-CLASS10;
STRATA FAILURE;
TITLE1' PWP-GAP TIME SUMMARY';
OUTPUT OUT=SURL_EST_PWP_GAP SURVIVAL=SURL_EST_PWP_GAP;
PROC SORT;
BY FAILURE CLASS1-CLASS10 Y;

/*
PROC PRINT DATA = SURL_EST_PWP_GAP;
TITLE1 'ESTIMATES OF THE SURVIVAL FUNCTION BASED ON THE PWP-GAP TIME METHOD';
*/
PROC PHREG DATA=CENSOR_PWP1;
MODEL TSTOP * STATUS(0)= CLASS;
TITLE1' PWP-TOTAL TIME SUMMARY';
OUTPUT OUT=SURL_EST_PWP_TOTAL SURVIVAL=SURL_EST_PWP_TOTAL;
PROC SORT;
BY CLASS TSTOP;

/*
PROC PRINT DATA = SURL_EST_PWP_TOTAL;
TITLE1 'ESTIMATES OF THE SURVIVAL FUNCTION BASED ON THE PWP-TOTAL TIME METHOD';
*/

DATA CENSOR_WLW1;
SET CENSOR_WLW;
IF FAILURE<11;
CLASS1=CLASS*(FAILURE=1);
CLASS2=CLASS*(FAILURE=2);
```

```
CLASS3=CLASS*(FAILURE=3);
CLASS4=CLASS*(FAILURE=4);
CLASS5=CLASS*(FAILURE=5);
CLASS6=CLASS*(FAILURE=6);
CLASS7=CLASS*(FAILURE=7);
CLASS8=CLASS*(FAILURE=8);
CLASS9=CLASS*(FAILURE=9);
CLASS10=CLASS*(FAILURE=10);

PROC PHREG DATA=CENSOR_WLW1;
MODEL TSTOP * STATUS(0)=CLASS;
TITLE1' WEI-LIN-WEISSFELD SUMMARY';
OUTPUT OUT=SURL_EST_WLW SURVIVAL=SURL_EST_WLW;
PROC SORT;
BY CLASS TSTOP;

/*
PROC PRINT DATA = SURL_EST_WLW;
TITLE1 'ESTIMATES OF THE SURVIVAL FUNCTION BASED ON THE WLW METHOD';
*/
RUN;
```

Appendix B

Sample SAS macro for batch processing and generate single summary

```
/*-----------------------------------------------------------------------*
| This is the macro automated version of SAS code listed in Appendix     |
| A. Since the parameters are replaced by macro variables, each run will |
| iterate on all combinations of parameters. In addition, SAS Output     |
| Delivery System (ODS) is utilized to trim the output, combined with    |
| PROC SQL, the final output neatly contains only the statistics that    |
| are desired.                                                           |
*-----------------------------------------------------------------------*/

OPTIONS LS=75;

** Initialize SAS macro myphregmacros, four arguments are accepted. **;

%MACRO MYPHREGMACRO(U     =,
                    P     =,
                    THETA =,
                    SEED  = );
  DATA _NULL_;

** Create macro arrays, the levels of the experimental design factors **;
** (i.e., sample size, censoring probability, shape parameters) will be **;
** assigned to the elements of corresponding arrays. **;
```

```
    ARRAY MYSAMPLESIZE(3) (&U);
    ARRAY MYP_C(4) (&P);
    ARRAY MYTHETA(7) (&THETA);
    ARRAY MYSEED(3) (&SEED);
    CALL SYMPUTx("DIM_U",DIM(MYSAMPLESIZE));
    CALL SYMPUTx("DIM_P",DIM(MYP_C));
    CALL SYMPUTx("DIM_D",DIM(MYTHETA));
    CALL SYMPUTx("DIM_S",DIM(MYSEED));
    DO U=1 TO DIM(MYSAMPLESIZE);
       CALL SYMPUTx(CATS("U_",U),MYSAMPLESIZE(U));
    END;
    DO P=1 TO DIM(MYP_C);
       CALL SYMPUTx(CATS("P_",P),MYP_C(P));
    END;
    DO D=1 TO DIM(MYTHETA);
       CALL SYMPUTx(CATS("D_",D),MYTHETA(D));
    END;
    DO S=1 TO DIM(MYSEED);
       CALL SYMPUTx(CATS("S_",S),MYSEED(S));
    END;
  RUN;
  %LOCAL I J K L;

** Create temporary dataset to contain intermediate results. **;

  PROC DATASETS;
    DELETE ALL_DATA
       %DO L = 1 %TO &DIM_S ;
               ALL_DATA&&S_&L
           %END; ;
    RUN;
    QUIT;

** Start iteration of data step. The parameters are replaced by  **;
** macro variables. **;

  %DO I = 1 %TO &DIM_U ;
      %DO J = 1 %TO &DIM_P ;
          %DO K = 1 %TO &DIM_D ;
      %DO L = 1 %TO &DIM_S ;

DATA LOGLIN;
RETAIN SEED &&S_&L;
```

```
FORMAT T Y 16.5;

DO ITEM = 1 TO &&U_&I;

P=RANUNI(SEED);
IF P < &&P_&J THEN CENSOR=0;
ELSE CENSOR=1;

F=FLOOR(10*RANUNI(SEED))+1;
T = 0;
M = 0;
R = 0;

TSTART=0;

DO FAILURE = 1 TO 10;

RETAIN M 0;
X = RANUNI(SEED);
R = R - LOG(X);
THETA = &&D_&K;
IF ITEM <= &&U_&I/2 THEN MU = -6.9;
ELSE MU = -4.6;
IF MU = -6.9 THEN CLASS = 0;
ELSE CLASS = 1;

IF(FAILURE>F & CENSOR=0)THEN STATUS=0;
ELSE STATUS=1;

IF STATUS=1 THEN DO;
IF THETA = 0 THEN T = R / EXP(MU);
ELSE T = (LOG(THETA*R + EXP(MU)) - MU) / THETA;
Y = T -M;
M = T;

TSTOP=T;
OUTPUT;
TSTART=TSTOP;

END;

IF STATUS=0 THEN DO;
T=0;
```

```
Y=0;
TSTOP=TSTART;
OUTPUT;
END;

END;

END;

DATA TBFB;
SET LOGLIN;
DROP  M MU THETA X R;
** PROC PRINT DATA=TBFB;  **;
**  TITLE1 'RIGHT CENSORING DATA OF TIME BETWEEN FAILURES'; **;

DATA CENSOR;
SET TBFB;
DROP X P F;
IF STATUS=1 THEN DELETE;
** PROC PRINT DATA=CENSOR; **;
**  TITLE'CENSOR'; **;

DATA UNCENSOR;
SET TBFB;
DROP X P F;
IF STATUS=0 THEN DELETE;
** PROC PRINT DATA=UNCENSOR; **;
** TITLE 'UNCENSOR'; **;

DATA CENSOR_AG;
SET TBFB;
IF TSTART=TSTOP THEN DELETE;
** PROC PRINT DATA=CENSOR_AG; **;
**  TITLE'CENSOR_AG'; **;

DATA CENSOR_PWP(DROP=LSTATUS);
RETAIN LSTATUS;
SET TBFB;
BY ITEM;
IF FIRST.ID THEN LSTATUS=1;
    IF (STATUS=0 AND LSTATUS=0) THEN DELETE;
    LSTATUS=STATUS;
** PROC PRINT DATA=CENSOR_PWP; **;
```

```
**  TITLE'CENSOR_PWP'; **;

DATA CENSOR_WLW;
SET TBFB;
** PROC PRINT DATA=CENSOR_WLW; **;
**  TITLE'CENSOR_WLW'; **;

** PROC PHREG DATA=CENSOR_AG; **;
**  MODEL (TSTART,TSTOP)* STATUS(0)= CLASS; **;
**  TITLE1' ANDERSEN-GILL SUMMARY'; **;


DATA CENSOR_PWP1;
SET CENSOR_PWP;
IF FAILURE<11;
CLASS1=CLASS*(FAILURE=1);
CLASS2=CLASS*(FAILURE=2);
CLASS3=CLASS*(FAILURE=3);
CLASS4=CLASS*(FAILURE=4);
CLASS5=CLASS*(FAILURE=5);
CLASS6=CLASS*(FAILURE=6);
CLASS7=CLASS*(FAILURE=7);
CLASS8=CLASS*(FAILURE=8);
CLASS9=CLASS*(FAILURE=9);
CLASS10=CLASS*(FAILURE=10);

** Suppress SAS output, and redirect the SAS output to a dataset. Keep **;
** only tables and variable that will be used in later computing. **;

ODS LISTING CLOSE;
ODS OUTPUT PARAMETERESTIMATES=MYPARA (KEEP=VARIABLE ESTIMATE
    RENAME=(ESTIMATE=EST_%SCAN(&SEED, &L)));

PROC PHREG DATA=CENSOR_PWP1;
MODEL Y * STATUS(0)= CLASS1-CLASS10;
STRATA FAILURE;
TITLE1' PWP-GAP TIME SUMMARY';
OUTPUT OUT=SURL_EST_PWP_GAP SURVIVAL=SURL_EST_PWP_GAP;
PROC SORT;
BY FAILURE CLASS1-CLASS10 Y;

/*
PROC PRINT DATA = SURL_EST_PWP_GAP;
```

```
TITLE1 'ESTIMATES OF THE SURVIVAL FUNCTION BASED ON THE PWP-GAP TIME METHOD';
*/
** PROC PHREG DATA=CENSOR_PWP1; **;
**  MODEL TSTOP * STATUS(0)= CLASS; **;
**  TITLE1' PWP-TOTAL TIME SUMMARY'; **;
**  OUTPUT OUT=SURL_EST_PWP_TOTAL SURVIVAL=SURL_EST_PWP_TOTAL; **;
** PROC SORT; **;
**  BY CLASS TSTOP; **;

/*
PROC PRINT DATA = SURL_EST_PWP_TOTAL;
TITLE1 'ESTIMATES OF THE SURVIVAL FUNCTION BASED ON THE PWP-TOTAL TIME METHOD';
*/

** DATA CENSOR_WLW1; **;
**  SET CENSOR_WLW; **;
**  IF FAILURE<11; **;
**  CLASS1=CLASS*(FAILURE=1); **;
**  CLASS2=CLASS*(FAILURE=2); **;
**  CLASS3=CLASS*(FAILURE=3); **;
**  CLASS4=CLASS*(FAILURE=4); **;
**  CLASS5=CLASS*(FAILURE=5); **;
**  CLASS6=CLASS*(FAILURE=6); **;
**  CLASS7=CLASS*(FAILURE=7); **;
**  CLASS8=CLASS*(FAILURE=8); **;
**  CLASS9=CLASS*(FAILURE=9); **;
**  CLASS10=CLASS*(FAILURE=10); **;

** PROC PHREG DATA=CENSOR_WLW1; **;
**  MODEL TSTOP * STATUS(0)=CLASS; **;
**  TITLE1' WEI-LIN-WEISSFELD SUMMARY'; **;
**  OUTPUT OUT=SURL_EST_WLW SURVIVAL=SURL_EST_WLW;  **;
** PROC SORT; **;
**  BY CLASS TSTOP; **;

/*
PROC PRINT DATA = SURL_EST_WLW;
TITLE1 'ESTIMATES OF THE SURVIVAL FUNCTION BASED ON THE WLW METHOD';
*/
RUN;

** Manipulate the dataset so that sample size, censoring probability, and **;
** shape parameters are created. Also variable N will be used later to **;
```

```
** compute the theoretical value of beta. **;

PROC PRINT DATA=MYPARA;
DATA MYPARA;
SET MYPARA;
SAMPLESIZE=&&U_&I;
PROBABILITY=&&P_&J;
THETA=&&D_&K;
N=INPUT(SUBSTR(VARIABLE,6),8.0);
RUN;

** The output from each iteration is appended to a single dataset. **;

PROC APPEND BASE=ALL_DATA&&S_&L DATA=MYPARA FORCE;
RUN;

ODS LISTING;
  %END;
  %END;
  %END;
  %END;
  %DO L = 1 %TO &DIM_S ;
PROC SORT DATA=ALL_DATA&&S_&L;
BY SAMPLESIZE PROBABILITY THETA;
RUN;
  %END;
DATA ALL_DATA;
MERGE  %DO L = 1 %TO &DIM_S ;
ALL_DATA&&S_&L
%END; ;
BY SAMPLESIZE PROBABILITY THETA;

RUN;

** Compute intermediate variables. **;

DATA METRICS (DROP = VARIABLE);
SET ALL_DATA;
AVG=MEAN (OF EST_539 EST_255 EST_59);
EST_TRUE = LOG( (THETA*(N-1) + EXP(-4.6)) / (THETA*(N-1)
            + EXP(-6.9)) );
EN = (AVG - EST_TRUE) / EST_TRUE;
```

```
** Compute final statistics. **;

PROC SQL;
CREATE TABLE ABC AS
SELECT SAMPLESIZE, PROBABILITY, THETA,
AVG(EN) AS BIAS,
AVG(ABS(EN)) AS MAD,
SUM(EN*EN) / (COUNT(*) - 1) AS MSE
FROM METRICS
GROUP BY SAMPLESIZE,PROBABILITY, THETA ;
QUIT;
PROC PRINT NOOBS;
FORMAT BIAS MAD MSE  10.5 ;

**   PROC PRINT NOOBS DATA=METRICS; **;
**     BY SAMPLESIZE PROBABILITY THETA; **;
RUN;

** Finish macro definition. **;

%MEND MYPHREGMACRO;

** Run the macro, adjust the parameters as needed. **;

%MYPHREGMACRO(U     = 60 120 180 ,
P     = 0.4 0.6 0.8 1.0 ,
THETA = 0.0 0.001 0.002 0.004 0.008 0.01 0.02,
SEED  = 539 255 59)
```

Appendix C

Sample SAS output with table information

```
                         PWP-GAP TIME SUMMARY                       1

                         The PHREG Procedure

Output Added:
-------------
Name:       ModelInfo
Label:      Model Information
Template:   Stat.Phreg.ModelInfo
Path:       Phreg.ModelInfo
-------------
                          Model Information

              Data Set                   WORK.CENSOR_PWP1
              Dependent Variable         Y
              Censoring Variable         STATUS
              Censoring Value(s)         0
              Ties Handling              BRESLOW

Output Added:
-------------
Name:       NObs
Label:      Observations Summary
Template:   Stat.Phreg.NObs
Path:       Phreg.NObs
-------------
```

```
               Number of Observations Read          368
               Number of Observations Used          368

Output Added:
-------------
Name:       CensoredSummary
Label:      Censored Summary
Template:   Stat.Phreg.CensoredSummary
Path:       Phreg.CensoredSummary
-------------
```

Summary of the Number of Event and Censored Values

Stratum	FAILURE	Total	Event	Censored	Percent Censored
1	1	60	60	0	0.00
2	2	60	54	6	10.00
3	3	54	49	5	9.26
4	4	49	39	10	20.41
5	5	39	35	4	10.26
6	6	35	24	11	31.43
7	7	24	20	4	16.67
8	8	20	15	5	25.00
9	9	15	12	3	20.00
10	10	12	5	7	58.33
Total		368	313	55	14.95

```
Output Added:
```



```
-------------
Name:       ConvergenceStatus
Label:      Convergence Status
Template:   Stat.Phreg.ConvergenceStatus
Path:       Phreg.ConvergenceStatus
-------------
```

```
                      Convergence Status

          Convergence criterion (GCONV=1E-8) satisfied.

Output Added:
-------------
Name:       FitStatistics
Label:      Fit Statistics
Template:   Stat.Phreg.FitStatistics
Path:       Phreg.FitStatistics
-------------


                    Model Fit Statistics

                          Without           With
               Criterion     Covariates     Covariates

               -2 LOG L        1692.153       1482.302
               AIC             1692.153       1502.302
               SBC             1692.153       1539.764

Output Added:
-------------
Name:       GlobalTests
Label:      Global Tests
Template:   Stat.Phreg.GlobalTests
Path:       Phreg.GlobalTests
-------------


               Testing Global Null Hypothesis: BETA=0

       Test                 Chi-Square       DF     Pr > ChiSq

       Likelihood Ratio       209.8508       10         <.0001
       Score                  218.0342       10         <.0001
       Wald                   158.3008       10         <.0001

Output Added:
-------------
```

```
Name:       ParameterEstimates
Label:      Parameter Estimates
Template:   Stat.Phreg.ParameterEstimates
Path:       Phreg.ParameterEstimates
-------------
```

PWP-GAP TIME SUMMARY 3

The PHREG Procedure

Analysis of Maximum Likelihood Estimates

Variable	DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio
CLASS1	1	2.29375	0.40505	32.0685	<.0001	9.912
CLASS2	1	2.22387	0.42252	27.7035	<.0001	9.243
CLASS3	1	1.95692	0.43109	20.6070	<.0001	7.078
CLASS4	1	3.20469	0.65672	23.8125	<.0001	24.648
CLASS5	1	2.23503	0.50084	19.9146	<.0001	9.347
CLASS6	1	2.80052	0.79680	12.3531	0.0004	16.453
CLASS7	1	2.47812	0.79632	9.6844	0.0019	11.919
CLASS8	1	1.88125	0.70294	7.1624	0.0074	6.562
CLASS9	1	1.74099	0.82379	4.4664	0.0346	5.703
CLASS10	1	0.84553	1.16342	0.5282	0.4674	2.329

Appendix D

List of Symbols

$\boldsymbol{\beta}_n$	$(k \times 1)$ vector of stratum-specific regression coefficients $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_k)$
Δ	Indicator of a failure or censored time; limit to time zero
δ	Shape parameter of a power-law NHPP
$\lambda(t; \mathbf{z})$	Proportional intensity function
λ_0	Baseline value of λ for power-law NHPP
$\lambda_0(t)$	baseline intensity function
$\lambda_{0n}(t)$	Stratum-specific baseline function
$\mathbf{z}$	$(k \times 1)$ vector of covariates, $\mathbf{z} = (z_1, z_2, \cdots, z_k)'$
$\mathbf{Z}(t)$	Covariate process up to time t
μ	Scale parameter of a log-linear NHPP
ν	Scale parameter of a power-law NHPP
ν_0	Baseline value of ν, the scale parameter of a power-law NHPP
ν_1	Alternative value of ν, the scale parameter of a power-law NHPP
σ	Standard deviation
θ	Shape parameter of a log-linear NHPP
$\tilde{X}$	Observation time

C_{ki} Censoring time for the i^{th} subject of the k^{th} type of failures

$h(t; z)$ Proportional hazard function

$h_0(t)$ Baseline hazard function

I_0 Number of sample units in class ϕ

I_1 Number of sample units in class 1

$N(t)$ Random variable for the number of failures in $(0, t]$

P_c Censoring probability

T_1, T_2 The beginning and end of an event,; bivariate exponential variables

T_n Random variable for the cumulative time of occurrence of the n^{th} failure

t_n Cumulative time of occurrence of the n^{th} failure; a realization of T_n

U Sample size (number of units)

$Y_i^{(n)}$ An at-risk indicator in the AG model

AG Andersen and Gill model

C.I Confidence interval

DROCOF Decreasing rate of occurrence of failure

HPP Homogeneous Poisson Process

i.i.d Independent and identically distributed

IROCOF Increasing rate of occurrence of failure

MAD Mean absolute deviation

MSE Mean squared error

N Successive failure count

n An integer counting successive failure times; a stratification

NHPP Non-homogeneous Poisson Process

PH Proportional hazards

PI Proportional intensity

PWP Prentice, Williams, and Peterson model

PWP-GT Prentice, Williams, and Peterson-gap time model

PWP-TT Prentice, Williams, and Peterson -total time model

ROCOF Rate of Occurrence of Failures

s.d. Standard deviation

WLW Wei, Lin, and Weissfeld model