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THE CHILD'S UNDERSTANDING OF THREE INTERPRETATIONS

OF CERTAIN UNIT FRACTIONS PRIOR TO

FORMAL INSTRUCTION

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CHAPTER I

INTRODUCTION

Background of the Problem

Teachers at grades five or six many times complain about students' lack of ability to perform basic operations with fractions. This deficiency could be caused by an incorrect concept of fraction or by no concept at all. Davis says that a child cannot learn things that are so remote from his understanding that they are meaningless to him.[1] Teaching concepts incorrectly is pedagogically unsound, and the teacher needs to make an effort to see that his students understand a concept correctly when it is first encountered.[2]

During the past ten years there have been numerous articles written in The Arithmetic Teacher and other professional literature suggesting concepts to be taught in grades K-2. Early experiences with the concept of fraction have been suggested for each level K-3. The Cambridge Conference Report of 1963 recommended that "use of fractions with small denominators to make additional points on the number line"

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[1] Robert B. Davis, "The Next Few Years," The Arithmetic Teacher, XIII (May, 1966), 358.

[2] Bill Bompart, "Teaching Concepts Incorrectly," The Arithmetic Teacher, IX (February, 1972), 138.

be included in the mathematics curriculum from K-3.[3] After surveying the achievement of pupils entering the first grade for the first time in upstate New York in 1953, Priori made the following comment with regard to the fractions "one-half," "one-fourth," and "one-third":

> This concept of fractions is considered to be difficult and is not introduced into the grades for a while. However, if the children come to school with the ability to identify these fractions, they may be ready to learn a little more about fractions.[4]

One conclusion reached in a study by Gunderson[5] substantiates that early acquaintance with fractions is needed. This study also suggests that a long acquaintance period is needed between the child's first introduction to fractions and the time he is expected to work fractions by use of algorithms.

Early introduction of basic concepts, as pointed out by Davis, provides a framework of basic structural ideas that make future learnings possible.[6] He further concludes:

> In the case of elementary school mathematics, if the child has a good collection of basic mathematical ideas readily available in his intellectual kit of tools, then he will relate his new mathematical learning to these ideas.[7]

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[3]Educational Services Incorporated, Goals for School Mathematics: The Report of the Conference on School Mathematics (Boston: Houghton Mifflin Co., 1963), p. 32.

[4]Angela Priori, "Achievement by Pupils Entering the First Grade," The Arithmetic Teacher, IV (March, 1957), 58.

[5]Ethel Gunderson, "Fractions - Seven-Year-Olds Use Them," The Arithmetic Teacher, V (November, 1958), 238.

[6]Davis, op. cit., 359.

[7]Ibid.

A theory of learning mathematics concepts must be one part of a theoretical framework in mathematics education. No well-developed theory of this sort is available.[8] This possibly accounts for the difficulty of determining why, what, how, to whom, and when topics in mathematics are taught. Bruner's famous hypothesis, which suggests a possible theory in the future, supports the idea that the number of concepts that could be taught in the primary grades is unlimited.[9] This hypothesis suggests that there is an honest way to present the concept of fraction so that it can be understood and appreciated by children at kindergarten level and beyond. If this hypothesis is true, in what way would a child in the primary grades understand the concept of fraction?

Researchers are pleading with teachers to use strategies which take into account the knowledge of numbers possessed by the child before he enters school.[10] Deans, as reported by Dutton,[11] points out that kindergarten children need arithmetical concepts "in order to carry on their small affairs."

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[8]E. G. Begle, "Curriculum Research in Mathematics," The Journal of Experimental Education, XXXVII (Fall, 1968), 46.

[9]Jerome S. Bruner, The Process of Education (New York: Vintage Books, 1960), p. 33.

[10]Anita P. Riess, "Pre-First Grade Arithmetic," The Arithmetic Teacher, IV (March, 1957), 50.

[11]Wilbur H. Dutton, "Growth in Number Readiness in Kindergarten Children." The Arithmetic Teacher, X (May, 1963), 251.

A look into the history of the mathematics curriculum suggests changes in attitudes toward teaching fractions in the primary grades. During the latter part of the 1800's arithmetic passed from the secondary school to the elementary school.[12] Colburn's First Lessons in Arithmetic on the Plan of Pestalozzi, which was revised in 1884, suggested such problems as: What is 3/4 of 2/5? Such problems were to be reasoned out.[13] Colburn's "natural aversion to every kind of rule"[14] suggests that the concept of fractions was to be thought out inductively by young children. Colburn advocated the use of concrete and manipulative materials for "object lessons."[15] This technique follows the suggestion of Pestalozzi and allows arithmetic instruction at an early stage.[16]

Arithmetic in the early grades did not maintain its popularity during the period from 1932-52. Wilson's Teaching

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[12]Philip S. Jones and Arthur F. Coxford, Jr. "Mathematics in the Evolving Schools," A History of Mathematics Education in the United States and Canada, Thirty-second Yearbook of The National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1970), p. 25.

[13]Ibid., p. 26.

[14]Ibid., p. 25.

[15]M. Vere De Vault and J. Fred Weaver, "Forces and Issues Related to Curriculum and Instruction, K-6," A History of Mathematics Education in the United States and Canada, Thirty-second Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1970), p. 105.

[16]Ibid.

the New Arithmetic was regarded as a popular methods text of this period.[17] His "grade curriculum" does not include any arithmetic for grades one and two.[18] Deferring work upon various arithmetic processes to the middle and upper grades became widespread after the Committee of Seven under the chairmanship of Carleton W. Washburne made its report in 1930. This committee recommended a minimal mental age of nine years for teaching the meaning of fractions (non grouping) and eleven years seven months for meaning of fractions (grouping) if eighty per cent retention is desired for three-fourths of the students.[19]

In surveying the literature concerning the mathematics curriculum in the early grades, the Cambridge Report appears to be pronounced in its advocation of the study of the real number system in the early grades. Use of fractions to name numbers on the number line, division with fractional answers, and use of the number line to introduce decimals by change of scale are among some of the items suggested for the earlier grades, K through 2.[20] No suggestion

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[17] Ibid., p. 120.

[18] Guy M. Wilson, Mildred B. Stone, and Charles O. Dalrymple, Teaching the New Arithmetic (New York: McGraw-Hill Company, Inc., 1939), p. 36.

[19] Carleton W. Washburne, "The Grade Placement of Arithmetic Topics: A Committee of Seven Investigation," Research in Arithmetic, Twenty-Ninth Yearbook of the National Society for the Study of Education, Part II (Bloomington, Ill., 1930), p. 670.

[20] Educational Services Incorporated, op. cit., p. 32.

is made in this proposal of how these concepts should be presented.

Among the key ideas to be included in the grade one program of a well-known arithmetic series are the following: constructing one-half of an object, one-fourth of an object, one-half of an object.[21] The formal introduction of the symbols "1/2" and "1/4," along with the idea that a number may be named in different ways, is introduced in the second semester of grade two.[22] Upon examination of the geometric representation, it was observed that one-half of a whole was used throughout. Is it possible for a student at this level to understand one-half of a set?

The Modern Mathematics Through Discovery Beginners Book (a well-known book used in kindergarten) suggests that prior to any systematic classroom instruction in mathematics, most students have a variety of informal pre-school experiences with the concept of number that should be taken into account.[23] In the beginner's edition, however, the only material that hints at the concept of fraction is the

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[21]Robert L. Morton, et. al., Modern Mathematics Through Discovery, Book One (Morristown, N.J.: Silver Burdett Company, 1970), pp. 87-202.

[22]Morton, et. al., Modern Mathematics Through Discovery, Book Two (Morristown, N.J.: Silver Burdett Company, 1970), p. 171.

[23]Merle Gray and Antoinette K. Sinard, Modern Mathematics Through Discovery, Beginner's Book Teacher's Edition (Morristown, N.J.: Silver Burdett Company, 1970), p. iii.

comparison of size and shape.[24] Because no work on unit fractions is included, is it to be assumed that the authors feel that children at this age have had no informal experience with them? Perhaps work on the unit fractions was excluded for other reasons. However, if the kindergarten child has any perception of a unit fraction, how does he think about it? The concern of researchers in elementary school focuses upon three major questions: (1) What to teach and to whom? (2) How to teach? (3) When to teach?[25] These questions should be answered with respect to the concept of fraction.

Mathematical Background

The word "concept" seems to have many definitions.[26] One definition is the following:

> A concept is a way of grouping an array of objects or events in terms of those characteristics that distinguish this array from other objects or events in the universe.[27]

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[24] Ibid., p. 52T.

[25] C. Alan Riedesel, "Topics for Research Studies in Elementary School Mathematics," The Arithmetic Teacher, XIV (December, 1967), 679.

[26] Myron F. Rosskopf, "Strategies for Concept Attainment in Mathematics," The Journal of Experimental Education, XXXVII (Fall, 1968), 78.

[27] Jerome S. Bruner, Jacqueline J. Goodnow, and George A. Austin, A Study of Thinking (New York: John Wiley and Sons, Inc., 1957), p. 275.

This definition lends itself well to explanation of mathematical concepts.[28] Concepts serve as mediators in problem solving. A student's success in problem solving is limited until a firm grasp of mediating concepts is attained.[29] Rosskopf seems to be reflecting the importance of concepts in the following:

> Mathematical education speaks in terms of a student mastering a concept so that it is part of himself; a psychologist would speak in terms of a student internalizing a concept. No matter what language one uses, it is certain that some of the failures in mathematics instruction are due to an instructor assuming his students have understood a mathematical concept at a high level of operational thinking when in reality they have a much lower level of mastery.[30]

The concept of fraction must be internalized or become a part of the student from several different aspects. The word "fraction" has taken on a number of meanings throughout history. A fraction was originally thought of as part of a unit and therefore less than one.[31] In a later period of time, "fraction" was used to name any rational number, large or small, whole or part.[32] A fraction which was first regarded as a "broken number" came to be

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[28] Rosskopf, *op. cit.*, p. 79.

[29] *Ibid.*, p. 85.

[30] *Ibid.*

[31] B. R. Buckingham, "The Social Point of View in Arithmetic," *Teaching of Arithmetic*, The Fiftieth Yearbook of the National Society for the Study of Education (Chicago: The University of Chicago Press, 1951), p. 271.

[32] *Ibid.*

understood as a way of dividing one by another.[33] The idea of a fraction played a part in developing such fields as percentage, ratio and proportion, permutation, combinations, and probability.[34]

Botts identified three common uses of the term "fraction" as follows: fractions as numbers, fractions as pairs of numbers, and fractions as symbols. He says that his research points out that recent books are not in agreement on their usage of the term "fraction."[35] These various usages may explain why many students think that fractions are rather deceptive. Botts considers all three of the common usages of the term fraction as "right" and urges teachers to employ all of them without making a production of the matter.[36]

Fractions are closely associated with the rational number system. Botts outlines several interpretations of the rational numbers as follows:

(1) 3/2 is 3 divided by 2:

$3/2 = 3 \div 2$ (arithmetic operation of division, so that $2 \times 3/2 = 3$.)

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[33] Ibid.

[34] Ibid.

[35] Truman Botts, "Fractions in the New Elementary Curriculum," The Arithmetic Teacher, XV (March, 1968), 219.

[36] Ibid.

(2) 3/2 is half of 3:

3/2 = 1/2 x 3 (corresponds on the number line to cutting length 3 in half.)

(3) 3/2 is 3 halves:

3/2 = 3 x 1/2 (corresponds on the number line to laying off length 1/2 three times.)

(4) 3/2 = 6/4 = 9/6 = 12/8 = ...[37]

The last of these interpretations is especially important in the computational process of adding and subtracting rational numbers, where common denominators must be found. Which of the above interpretations should the teacher be focusing upon in the early grades? What does the child know about these interpretations of fractions before instruction is begun?

Sometimes a study of history will help us ferret out the simplest ideas which formulate the beginnings of conceptual thought.[38] The Egyptians[39] invented symbolic representations for the concept of fraction; in so doing they

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[37] Ibid., p. 218.

[38] H. L. Larson, "The Structure of a Fraction," The Arithmetic Teacher, XVI (April, 1966), 206.

[39] Ibid.

endeavored to avoid some of the computational difficulties.[40] Unit fractions were denoted in hieroglyphs by placing an elliptical symbol above the denominator number.[41] All fractions except "two-thirds" were represented as the sum of unit fractions by the Egyptians.[42] The ideas used by the Egyptians tie in very closely with the third interpretation that Botts has explained. Is this the simplest interpretation of fraction?

Just because a concept is fundamental in a mathematical sense does not imply that it develops first in the child's mind. Almy seems to be addressing herself to this question in the following paragraph:

> It is by no means certain that because some relations are fundamental to the logical analysis of mathematical properties, these same relations underlie the psychological evolution of the recognition of these properties.[43]

Need for the Study

Mathematics educators must proceed with caution in recommending topics to be taught in the primary grades. The child should be provided with the experiences that will be

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[40] Howard Eves, An Introduction to the History of Mathematics (New York: Holt, Rinehart and Winston, 1969), p. 38.

[41] Ibid., p. 39.

[42] Ibid., p. 38.

[43] Millie Almy, Young Childrens' Thinking, Studies of Some Aspects of Piaget's Theory (New York: Teachers College Press, 1966), p. 201.

productive.

In 1953 Morton stated:

> . . . the evidence suggests that the earlier curriculums which assigned all the addition facts and related matters to grade one went much too far. Number ideas grow in the minds of first grade children but they grow slowly. These children have need for numbers but their needs are simple. To go beyond those needs and to attempt to proceed at a faster pace than the developmental rates the children permit is to inhibit growth which should be fostered.[44]

The Cambridge Report suggests some concepts to be taught in grades K-3 that may prove inappropriate.

> Some of them can probably be introduced in nursery school and no doubt should be. Others may prove impossible by the second grade, either because of their intrinsic difficulty or because of the large amount of material to be covered.[45]

The mathematics educator knows that the full development of some of the more difficult concepts means that some experiences must be assimilated several years before the concepts are actually formed.[46]

More research is needed to determine children's knowledge of number concepts in the primary grades. Schwartz points out that knowledge about five-year-olds is limited. He states:

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[44]R. L. Morton, Teaching Arithmetic, What Research Says to the Teacher, No. 2 (Washington, D.C.: Department of Classroom Teachers, American Educational Research Association of the National Education Association, 1953), p. 17.

[45]Educational Services Incorporated, op. cit., p. 35.

[46]Z. P. Dienes, "The Growth of Mathematical Concepts in Children Through Experience," Educational Research, II (November, 1959), 9-28.

> Neither child growth and development specialist, mathematician, nor professional educator seems to be in accord as to what mathematical concepts a child has developed at the preschool level or what the curriculum should encompass at this age. Research has been limited in scope and the results do not lead one to make conclusive decisions.[47]

Knowledge of children's number concepts should be explored so that instructional programs can be adapted to their level of understanding.[48]

Researchers in mathematics education need to provide more insight into teaching the concept of fraction in the early grades. They need to draw upon several disciplines in pursuing their studies.[49] They must be concerned with the process of teaching and learning and the mathematics content.[50] This study seeks the answer to four questions. They are as follows:

1. Prior to formal instruction, what understanding does a five-, six-, or seven-year-old child have of the interpretations--part of a whole; part of a set; operation of division such that the whole consists of two one-halves, three one-thirds, and four one-fourths?

2. Will a child's understanding of a fraction of a set be affected by changing the sizes of the objects in the set?

3. Which unit fraction, one-half, one-fourth, or

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[47] Anthony N. Schwartz, "Assessment of Mathematical Concepts of Five-Year-Old Children," The Journal of Experimental Education, XXXVII (Spring, 1969), 67.

[48] Emma E. Holmes, "First Graders' Number Concepts," The Arithmetic Teacher, X (April, 1963), 95-96.

[49] Riedesel, op. cit., p. 679.

[50] Ibid.

one-third, does the young child know the most about before formal instruction?

4. Does the young child understand the interpretation of one-half, one-fourth, and one-third better when physical or pictorial representations are used?

Statement of the Problem

At three age levels and prior to formal instruction, what are the child's intuitive understandings of three interpretations of the fractions one-half, one-fourth, and one-third--as part of a whole, as part of a set, and as the arithmetic operation of division?

Related Research

Research studies substantiate the fact that children come to kindergarten with considerable knowledge on which the school can build an interesting, challenging, and sequential curriculum.[51] Schwartz's study showed that 41 per cent of the kindergarten students in his study could mark a jar one-half full of water, 12 per cent of the students could mark the jar one-fourth full of water, 35 per cent could mark the frame that showed one-fourth or one-quarter of a pie, 24 per cent could mark the frame that showed one-third of a pie.[52]

There have been attempts to find out what concepts

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[51] Schwartz, *op. cit.*, p. 67.

[52] *Ibid.*, p. 74.

and ideas young children have about fractions.[53] In a study with a group of children in Grade Two, Gunderson and Gunderson found these results:

1. Young children are interested in and like work with fractions. They showed no frustration, but were confident in their approach to the problems.

2. These children showed a good understanding of fractions when using manipulative materials. These children can obviously profit from planned systematic instruction in the meaning and use of fractions.

3. This study indicates a need for an arithmetic program which introduces systematic work with fractions as early as Grade Two. At this level the teaching must be oral, with manipulative and semi-concrete materials available for children to use.[54]

Their study indicates that a planned systematic program for developing the meaning of fractions is essential for readiness or preparation for working with fractions and symbols.[55] In the Gunderson and Gunderson study, fractional parts of circles were used because it was easier for young children to recognize fractional parts of circular wholes than fractional parts of other figures.[56] This raises a question as to whether this is the only way that a child at this age can perceive this concept.

Bruner did related research with the concept of

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[53]Agnes E. Gunderson and Ethel Gunderson, "Fraction Concepts Held by Young Children," The Arithmetic Teacher, XI (October, 1957), 168.

[54]Ibid., 173.

[55]Ibid., 177.

[56]Ibid., 178.

proportion. Children of the five-year, six-year, and seven-year age range do have some concept of proportion. Bruner says that by age five, a definite idea of proportion, although an incorrect one, is already present, and that by the age of seven, the children still have not grasped the idea in its proper mathematical sense.[57]

Bruner concluded from his research the following:

> . . . any idea can be represented honestly and usefully in the thought forms of children of school age, and that these first representations can later be made more powerful and precise the more easily by virtue of the early learning.[58]

Bruner is not advocating teaching the structure of mathematics as an assortment of facts. The key concepts are not to be taught directly to the children. These concepts are meant to be formulated in the child's mind through meaningful educational experiences offered throughout the school grades.[59]

Influenced by such thinking as that of Bruner, writers in childhood education have urged mathematics educators to take a fresh look at the possibilities of developing mathematics concepts at the kindergarten

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[57] Jerome S. Bruner and Henry J. Kenney, Studies in Cognitive Growth (New York: John Wiley and Sons, Inc., 1966), p. 168.

[58] Bruner, op. cit., p. 33.

[59] Helen F. Robinson and Bernard Spodek, New Directions in the Kindergarten (New York: Teachers College Press, 1965), pp. 12-13.

level.[60] Concepts and content identified as both feasible and appropriate for the kindergarten children include fractional parts one-half, one-third, and one-fourth.[61]

Kindergarten children do recognize some fractions. Suydam and Riedesel summarize the research, as follows:

> About half of the kindergarten children tested were able to recognize half of an item; 89% thirds; and 66% fourths (Bjonerud, 1960). In Priori's sample (1957) 78% recognized halves; 51% thirds; and 50% fourths. Wittich (1942) reported comparable percentages for halves and fourths, but found few understood thirds. Woody (1931) indicated that about two-thirds of his sample had some knowledge of fractions.[62]

A study to determine what primary children know about fractions from their everyday experience was made by Polkinghorne. She concluded:

> a. The children knew more about a unit fraction as applied to a single object than about any other fractions used in the test: 1/2 of 1, etc.
>
> b. They knew a unit fraction when it was used in the comparison of two objects. "This is 1/2 as big as that," etc.
>
> c. They knew less about the unit fraction as applied to a group of objects than about the unit fraction as applied to a single object. 1/2 of 1 is easier than 1/2 of 4.
>
> d. They could use fractions in comparing two objects better than in comparing two groups of objects.

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[60] George W. Schlinsog, "Mathematics in the Kindergarten," The Arithmetic Teacher, XIV (April, 1967), 292.

[61] Ibid., 294.

[62] Marilyn N. Suydam and C. Alan Riedesel, "An Interpretative Study of Research and Development of Elementary School Mathematics," Childhood Development, XLVII (January, 1971), 226.

"This is 1/2 of that" is easier than "3 is 1/2 of 6."[63]

Reid found that the conversation of first grade children during the first three months of school suggested that they had some knowledge of fractions. Students in the study made remarks as follows:

1. "I've got mine half sewed."
2. "Two short ones would be the same as one long one wouldn't it?"
3. "We've just a half day left to practice."[64]

These remarks suggest that six-year-olds have some understanding of certain fractions.

Related research was done by Gunderson[65] to determine how many concepts of different numbers and fractions that seven-year-olds understood prior to formal instruction. These children showed some understanding of half a loaf of bread, four quarters make a dollar, and one-third of a cup.

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[63]Ada R. Polkinghorne, "Young Children and Fractions," Childhood Education, XI (May, 1935), 357.

[64]Florence E. Reid, "Incidental Number Situations in First Grade," Journal of Educational Research, XXX (September, 1936), 42.

[65]Agnes G. Gunderson, "Number Concepts Held by Seven-Year-Olds," The Arithmetic Teacher, XXXIII (January, 1940), 22.

CHAPTER II

DESIGN AND EXECUTION OF THE STUDY

The Sample

The population to be studied was defined as the students of the Lincoln Parish School System. This system is one of the sixty-six public school districts in the state of Louisiana. The geographic location of Lincoln Parish Public School System is in the North Central part of the state. On September 13, 1972 the school system had an enrollment of 6,172 students in grades K through 12. This number reflects the approximate enrollment at the time the study was conducted.

All six of the schools in the system having kindergarten, first-year and second-year students were used in the study. All the kindergarten, first-year, and second-year classes in each of the six schools were included in the sample. The six schools were Choudrant School, Cypress Springs Elementary, Hico Elementary, Hillcrest Elementary, Ruston Elementary, and Simsboro Elementary.

In general, the schools of this study are typical of those in this area. They are located in small rural areas around one population center. Each of the schools is within commuting distance (twenty miles) of Louisiana Tech University and Grambling College. In the past, their program of studies has been shaped for the college-capable student. Presently, however, the curriculum is being shifted toward career education. (See Table 2-1 for state and parish median education level for 1970.)

TABLE 2-1

STATE AND PARISH EDUCATIONAL LEVEL FOR 1970[1]

Race or Group	Region	Median Education Level in Years
White	State	12.0
White	Parish	13.7
Black	State	7.9
Black	Parish	8.9
Composite	State	10.8
Composite	Parish	12.0

The student bodies of these schools are very similar. They range from those who come from homes where education is respected and books are plentiful to those where education is of no significance. Children of kindergarten age (five-year-olds) are not required by law to attend school. However, a kindergarten is provided in each school. This probably accounts for the difference in enrollment of five-year-olds and six-year-olds. (See Tables 2-2 and 2-3.)

The superintendent of schools, principals, and classroom teachers were enthusiastic about the study and

[1]Simsboro High School Staff and Administration, Simsboro High School (Louisiana) Southern Association Evaluation," Simsboro, 1973, p. 17. (Mimeographed.)

TABLE 2-2

INFORMATION ON THE PARTICIPATING SCHOOLS

School	Division	Enrollment	Accreditation	Size of Faculty
Choudrant School	K-12	474	Louisiana Dept. of Education	27.5
Cypress Springs Elementary	K-5	476	Louisiana Dept. of Education	20
Hico Elementary	K-6	319	Louisiana Dept. of Education	12
Hillcrest Elementary	K-5	534	Louisiana Dept. of Education	18
Ruston Elementary	K-5	437	Louisiana Dept. of Education	15
Simsboro School	K-12	414	Louisiana Dept. of Education and Southern Association of Schools and Colleges	22.5

TABLE 2-3

THE NUMBER OF CHILDREN FROM WHICH THE RANDOM SELECTION WAS MADE, GIVEN BY SCHOOL AND BY AGE LEVEL

School	Five Years	Six Years	Seven Years
Choudrant School	19	34	29
Cypress Springs Elementary	38	91	86
Hico Elementary	20	49	41
Hillcrest Elementary	49	86	87
Ruston Elementary	42	78	76
Simsboro School	16	32	29

cooperated fully. This was evidenced by their questions before and during the study. They provided a quiet space for the interview in each school and information about each child.

Students who were approximately seven-, six-, and five-years of age were determined from each teacher's class roll. More precisely, seven years of age was defined as being between eighty-three and ninety-three months. Six years was defined as being between seventy-three and eighty-two months of age. Five years was defined as being between sixty-one and seventy-one months of age. (See Table 2-4.)

TABLE 2-4

THE AVERAGE AGE IN MONTHS OF THE SAMPLE AT THE TIME OF THE INTERVIEW, GIVEN BY SCHOOL AND BY AGE LEVEL

School	Five Years	Six Years	Seven Years
Choudrant School	68	77	86
Cypress Springs Elementary	67	75	85
Hico Elementary	66	77	86
Hillcrest Elementary	67	77	85
Ruston Elementary	69	75	86
Simsboro School	65	77	88

The selection of the subjects was made from each age level as follows: Four students were randomly selected from eight kindergarten classes. Since one of the classes

(Simsboro School) had only sixteen children, only two students were used in the sample from this class. A total of thirty-four kindergarten children were interviewed. Similarly, two students were randomly selected from fifteen classes of six-year-olds. Three students were selected from the largest class at Cypress Springs. A total of thirty-three six-year-olds were interviewed. Finally, two students were randomly selected from twelve classes of seven-year-olds. Three seven-year-olds were selected from the largest class at Cypress Springs, Hillcrest and Ruston Elementary. A total of thirty-three seven-year-olds were interviewed. One hundred interviews were conducted.

TABLE 2-5

THE NUMBER OF CLASSES FROM WHICH THE RANDOM SELECTION WAS MADE, GIVEN BY SCHOOL AND BY AGE LEVEL

School	Five Years	Six Years	Seven Years
Choudrant School	1	2	1
Cypress Springs Elementary	2	4	4
Hico Elementary	1	2	2
Hillcrest Elementary	2	4	4
Ruston Elementary	2	3	3
Simsboro Elementary	1	1	1
TOTAL	9	16	15

Since formal instruction in fractions for the second-year students (seven-year-olds) in the Lincoln Parish System begins during the latter part of November, these interviews were done first. Interviews for seven-year-olds were done from October 18, 1972, to November 1, 1972. Interviews for the six-year-olds were done from November 6, 1972, to December 15, 1972. Interviews for the kindergarten students were done from January 15, 1973, to February 9, 1973. No formal instruction in the fractions one-half, one-third, and one-fourth had been received by any of the students before they were interviewed.

Description of Participating Schools

All schools that were used in this study had non-graded programs in arithmetic and language arts. Excluding kindergarten, the first seven years of work in arithmetic is broken into fourteen levels. First-year students (the year following kindergarten) ordinarily progress through four levels and second-year students through two levels. Behavioral objectives were used to determine the child's progress through these levels.

Ruston Elementary, Cypress Springs Elementary, and Hillcrest Elementary Schools are located in the city of Ruston, Louisiana. Whereas, Choudrant, Hico and Simsboro Schools are located in small communities surrounding Ruston. A detailed description of the participating schools is included in Appendix E. This information was obtained from handbooks prepared by the individual schools, observations made by the investigator while serving as supervisor of instruction, and from discussions with building principals.

The Tasks

Introduction

In order to determine what insights a child has about three interpretations of a fraction--part of a whole, part of a set, and operation of division--a series of tasks was used. These tasks were designed to allow the child to demonstrate his understanding of each of these interpretations.

Preliminary tasks and six multi-stage tasks were used in the investigation. The preliminary tasks were used to determine if the subject understood the terminology. The six multi-stage tasks were these: Apple Task--Interpretation of One-Half; Apple Task--Interpretation of One-Fourth; Apple Task--Interpretation of One-Third; Glass Task--Interpretation of One-Half; Glass Task--Interpretation of One-Fourth; Glass Task--Interpretation of One-Third. A detailed description of each task is in Appendix A. The Apple Tasks deal with concrete representations of the interpretations of the fractions, while the Glass Tasks deal with pictorial representations.

The Apple Tasks and Glass Tasks underwent a number of revisions. Important in these revisions and final development were discussions with colleagues in mathematics education and with other doctoral students. The tasks were pilot-tested and administered in individual interviews. All interviews and subsequent ratings were done solely by the investigator.

Pilot Study

The pilot study was designed to provide the investigator with experience in the technique of interviewing children, in evaluating the appropriateness of his choice of words in questioning, in appraising the amount of time and space that is needed, and finally in refining the tasks.

The pilot study sample consisted of twelve children from Choudrant School. The teachers selected four of the seven-year-olds, four of the six-year-olds, and four of the five-year-olds. The tasks were presented to children within each age level who had been selected on the basis of varying abilities.

Adjustments in the interview schedule were made when it was detected that certain precautions were not necessary. For example, the initial interview schedule would not allow the investigator to say, "Draw a circle around." Instead, he was to say, "Draw a line around." However, it was detected that all of the younger children knew what was meant by "draw a circle around."

The results of the pilot study indicated several procedural changes, such as:

1. All gestures made by the investigator should be the same in each interview.

2. Interview should be done only in a quiet place.

3. Plenty of time should be allowed for each interview. An acceptable amount of time was twenty-five minutes.

Validity of the Tasks

Content validity was established by a procedure suggested by Fred N. Kerlinger. Kerlinger defines content validity as "the representativeness or sampling adequacy of the content--the substance, the matter, the topics--of a measuring instrument."[2] This is a theoretical ideal, according to Kerlinger, and is impossible to achieve. Instead, the validation of content is essentially that of expert judgment.

> Alone or with others, one judges the representativeness of the item...
> ...each item must be judged for its presumed relevance to the property being measured,...[3]

The investigator sought the judgment of other competent individuals in mathematics education to determine content validity. The investigator's judgment was exercised before and during the development and refinement of the tasks.

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[2]Fred N. Kerlinger, Foundations of Behavioral Research. (New York: Holt, Rhinehart and Winston, Inc., 1964), p. 446.

[3]Ibid.

Description of the Tasks

Appendix A contains a detailed description of the tasks that were used in the study. Included in this description is (1) a brief statement of how the tasks were presented to the subject; (2) the materials, drawings, and concrete objects that were needed in order to administer the tasks; (3) a detailed transcript of questions asked and movements made by the investigator. However, certain features of each task are not evident in this description, and these characteristics will now be explained.

Preliminary Tasks. In order to assure that the child understood the terminology used in the interview session, several tasks were designed. The child's understanding of the expression "draw a circle around one" was determined by use of a pictorial representation of two trees (see Appendix B-1) on a 8 1/2" x 11" sheet of white paper. The child was asked to draw a circle around one of the trees. Similarly, understanding of the expression "draw a circle around two things, three things, four things" was determined by use of a pictorial representation of three rows of stars. (See Appendix B-2.) A pictorial representation of three rows of stars with five stars on each row on a 8 1/2" x 11" sheet of white paper was placed on a table before the child. The child was asked to draw a circle around two of the stars on the first row, three of the stars on the second row, and four of the stars on the third row.

Awareness of the words "full," "empty," "one-half," "one-third," and "one-fourth" were also determined by use of the preliminary tasks. A pictorial representation of five glasses of the same size (see Appendix B-3) was placed before the child. The first glass was full, the second was half full, the third was one-third full, the fourth was one-fourth full, and the fifth glass was empty. The child was asked to draw a circle around the "full" glass, the "empty" glass, the glass that was "one-half" full, the glass that was "one-third" full, the glass that was "one-fourth" full, in that specific order. (See Appendix A for the detailed description of the preliminary tasks.)

Apple Task--Interpretation of One-Half. This multi-stage task sought information about the child's understanding of what is meant by half of a whole, half of a set, and the operation of division such that a whole consists of two "one-halves." Plastic apples and prepared halves of plastic apples were used to explore the child's understanding of these interpretations of one-half with concrete objects.

The child was asked to mark the plastic apple where he would cut it so that the investigator could have half the apple. He was asked to pick up half the apples when various combinations were presented to him. He was asked to pick up enough halves to make a whole when three pieces were placed on the table. (See Appendix A.) In order to have a written record, the investigator marked on a pictorial representation the observed responses. (See Appendix C-1.)

Understanding of what is meant by half of a set was detected by presenting four different combinations of plastic apples--first, two plastic apples the same size, then two of a different size, then four the same size, and finally three of the same size. The investigator wished to determine if varying the size of the objects in the set was a factor relating to understanding. Three plastic apples were used in one of the combinations in order to determine if the child understood what half of a set was if the set consisted of an odd number of objects.

Three halves of plastic apples the same size were prepared to determine if the child understood that a whole consisted of two "one-halves." Caution was taken to make sure that each of the halves appeared the same size and shape. Evidence that the halves were prepared correctly came from the remarks often heard from subjects, such as, "Are these really halves of apples?"

Apple Task--Interpretation of One-Fourth. This multi-stage task was designed to determine understanding of what is meant by one-fourth of a whole, one-fourth of a set, and the operation of division such that the whole consists of four "one-fourths." Plastic apples and fourths of plastic apples were used to determine the child's understanding of these interpretations of one-fourth by use of concrete objects.

A detailed description of the way the task was administered is found in Appendix A; however, some distinguishing features of this task should be pointed out. When the child was asked to mark where he would cut the plastic apple so the investigator could have one-fourth of it, the place where the child marked was noted on a pictorial representation by the investigator. (See Appendix C-2.)

Understanding of one-fourth of a set was examined by three different arrangements of plastic apples. These were the following: four apples the same size, four apples of different sizes, and two apples the same size. The apples of different sizes were always placed on the table in the order of largest to smallest, and at this time the student was informed that some were larger than others.

The investigation of the interpretations of one-fourth was initiated before that of one-third because it appeared in the pilot study to be the order of increasing difficulty. However, this could not be determined conclusively.

Apple Task--Interpretations of One-Third. The Apple Task--Interpretation of One-Third is similar to that for one-half and one-fourth in that it probes the child's understanding of the three interpretations--one-third of a whole, one-third of a set, and operation of division such that the whole consists of three "one-thirds." Also, it is comparable in that this is done with concrete objects.

The investigator's record of the student's responses was recorded on a pictorial representation just as was done for one-half and one-fourth. (See Appendix C-3.) The same precautions that were made for one-half and one-fourth were made for one-third.

Glass Task--Interpretations of One-Half. This task was also multi-staged and designed to allow the child to reveal his understanding of three interpretations of fractions by use of pictorial representations. (See Appendixes A and B.) A task comparable to each of the Apple Tasks was developed in pictorial form. For instance, a picture of two glasses the same size, a drawing of two glasses of different sizes, a drawing of four glasses the same size, and a drawing of three glasses the same size were used to detect the child's understanding of one-half of a set.

Understanding of what is meant by half of a set was explored by presenting pictorial representations of glasses. The child was asked to draw a circle around half of the glasses in each of the pictorial representations.

In determining the understanding of the operation of division such that two "one-halves" make a whole, it was always pointed out that the glasses were the same size. This was pointed out before the child was instructed to draw a circle around the half filled glasses that were needed to fill the empty glass. (See Appendixes A and B.)

Glass Task--Interpretations of One-Fourth. This multi-stage task was presented after the comparable tasks for one-half and before one-third. With the rationale again being to arrange the tasks in the order of probable increasing difficulty.

Again, the Glass Task--Interpretations of One-Fourth had a pictorial representation comparable to each of the stages of the Apple Task which had been used to probe the understanding of one-fourth. (See Appendixes A and B for details and pictorial representations.)

Glass Task--Interpretations of One-Third. This was also a multi-staged task designed to detect the understanding of three interpretations of the fraction one-third. Understanding of one-third of a whole, one-third of a set, and the operation of division such that a whole consists of three "one-thirds" was revealed by asking the child to react to questions about each interpretation. The child was asked to mark his response on pictorial representations of glasses. Here again, tasks comparable to each of the Apple Tasks for one-third were used. (See Appendixes A and B for detailed procedure and pictorial representations used.)

Administration of the Tasks

The investigator was working as a Supervisor of Instruction in the Lincoln Parish School System during the time the interviews were conducted. The children and teachers were accustomed to seeing him on their campus and in their classroom. This arrangement made it easy to schedule interview times that were convenient to the teacher and student. All facilities of each school were available. This, in effect, made it easy to find a quiet place for the interviews.

Each teacher was asked to make her roll available. Random numbers were selected, and the corresponding student on the teacher's roll was selected as a subject. The teacher was asked to introduce the investigator to the student. The student subsequently was asked if he/she would like to play a game.

On one occasion the child showed an unwillingness to cooperate. Another student was selected when this occurred. Usually, every child in class wanted to "play the game."

There was no particular procedure for selecting the order of schools. Work proceeded with the seven-year-olds, six-year-olds, and five-year-olds in that order so that the investigation could be completed before formal instruction on fractions began for the seven-year-olds.

In order that reliable conclusions could be drawn, tasks were presented in the same order, and questioning was essentially the same for all children interviewed. However, repeating questions on occasion and probing a little deeper in some instances appeared necessary to allow the child to reveal his understanding. Some children asked questions that required an answer in order to get a meaningful interview underway.

The child was assured that there were no wrong answers. He was told that what he thought was what was being sought. On occasion, a child being interviewed would ask, "Is that right?" The answer that was given always assured the child that he was doing well.

Teachers were never told the nature of the study other than that it pertained to arithmetic. Since the investigator was serving as a supervisor at the time, the teachers looked upon this as part of his regular work. The common response the teachers received from the children was, "We played a game with some apples and things." Because of these circumstances, it appeared that very good conditions existed throughout the period of interviewing.

Analysis of Interviews

The rating scheme that was used in the study was one that had been used previously by three investigators. (See Rating Instrument in Appendix D.) It was used by Almy[4] in a study of the understanding among children. Taback[5] refined the rating scheme and used it in a study of children's

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[4]Millie Almy, Young Children's Thinking (New York: Teachers College Press, Columbia University, 1966), p. 67.

[5]Stanley Frederick Taback, "The Child's Concept of Limit," (unpublished Ph.D. dissertation, Columbia University, 1969), p. 48.

understanding of the concept of limit. Thiessen[6] used this same scheme in parts of a study of a child's concept of convexity. The child's response on each task was rated in one of five categories:

1. Clear Evidence of understanding
2. Some evidence of understanding
3. Uncertain evidence of understanding
4. Clear evidence of not understanding
5. Evidence lacking

Analysis of tape recordings and markings on the protocols were used to rate the child's level of understanding.

Reliability of the Rating Scheme

The investigator did all the interviewing and rating. In order to determine the reliability of the rating scheme, a fellow mathematics educator independently rated fifteen of the interviews. Five samples were selected randomly from each of the three age levels. Excluding the preliminary task, there were 480 responses that were rated. The two independent ratings were in agreement on 88 per cent of the items. When the responses on the preliminary task were included, the two ratings were in agreement on 80 per cent of all the items.

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[6] Richard Eugene Thiessen, "The Child's Concept of Convexity," (unpublished Ph.D. dissertation, University of Oklahoma, 1971), p. 40.

CHAPTER III

RESULTS OF THE STUDY

The responses of the children used in this study were recorded on paper and/or taped for subsequent analysis and rating. For any task that allowed the subject to make no verbal or written response, the investigator verbalized the response and made a written record.

Major Results

Results for the Preliminary Tasks

The tasks which allowed the child to reveal his understanding of "Draw a circle around one tree, two stars, three stars, and four stars" were not rated. Also, those on which he showed his understanding of "full" and "empty" were not rated. All the children understood these items and were able to perform the tasks correctly.

The child's understanding of the terminology used in the tasks involving one-half, one-fourth, and one-third was explored with three tasks. A pictorial representation of five glasses--one empty, one full, one one-half full, one one-third full, and one one-fourth full--was presented to the child. He was asked, "How much is in this glass? (See Appendix B-3.) The investigator pointed to the glass in question. This procedure was repeated for each glass. The subject was asked to draw a circle around the glass that was one-half full, one-third full, and one-fourth full. If the child used the term "one-half," "one-third," or "one-fourth" correctly and subsequently drew a circle around the proper representation, he was rated as showing "Clear evidence of understanding." If he did not use the terms correctly or at all, but drew a circle around the correct representation, he was rated as showing "Some evidence of understanding." If the child did not use the terms and showed hesitancy in drawing a circle around the correct glass, he was rated as showing "Uncertain evidence of understanding." "Clear evidence of not understanding" was marked if the child was unsuccessful on any part of the task. "Evidence lacking" was marked one time on the preliminary tasks. This child indicated that two of the glasses were one-half full and did not respond to the question that pertained to one-fourth.

These preliminary tasks possibly reveal understanding of one-half an object as well as use of the terminology. This statement also applies to the preliminary tasks for one-fourth and one-third.

Fifty-eight per cent of the sample was rated as having "Clear understanding" of the terminology to be used in the tasks involving one-half. (See Table 3-1.)

Eighty-six per cent of the children interviewed were rated as having "Some evidence" or "Clear evidence of understanding." Eight of the children that were not rated in either of these categories were five-year-olds. As evidenced by the data, the terminology used in the task involving one-half was generally understood.

TABLE 3-1

RESULTS OF THE PRELIMINARY TASK IN WHICH CHILD IS ASKED ABOUT THE TERMINOLOGY TO BE USED IN THE TASKS INVOLVING ONE-HALF

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	13 (38%)	20 (61%)	25 (76%)
Some evidence of understanding	13 (38%)	9 (27%)	6 (18%)
Uncertain evidence of understanding	6 (18%)	4 (12%)	1 (3%)
Clear evidence of not understanding	2 (6%)		1 (3%)
TOTAL	34	33	33

Seventy-two per cent of the sample was rated as having "Some evidence" or "Clear evidence of understanding" of the terminology to be used in the task involving one-fourth. Sixty-eight per cent of this number did not use the term "one-fourth" when the investigator pointed to the glass and asked, "How much is in this glass?" However, these children could draw a circle around the proper glass when they were instructed, "Draw a circle around the glass which looks like it is one-fourth full." Table 3-2 is a summary of the ratings on the preliminary tasks concerning the terminology to be used relative to one-fourth.

TABLE 3-2

RESULTS OF THE PRELIMINARY TASK IN WHICH CHILD IS ASKED ABOUT THE TERMINOLOGY TO BE USED IN THE TASKS INVOLVING ONE-FOURTH

Performance	Age		
	5 Years	6 Years	7 Years
Clear evidence of understanding	1 (3%)		3 (9%)
Some evidence of understanding	23 (67.5%)	25 (76%)	20 (61%)
Uncertain evidence of understanding	3 (9%)	7 (21%)	8 (24%)
Clear evidence of not understanding	7 (20.5%)	1 (3%)	1 (3%)
Evidence lacking			1 (3%)
TOTAL	34	33	33

One response was rated as "Evidence lacking." The child seemed to be confused about the drawing. He remarked that the glass, which represented one that was full, had a top on it. Because of this insistence, no questioning about the fractions one-third and one-fourth was done.

Table 3-3 is a summary of the levels of understanding of the terminology to be used in the tasks involving one-third. Sixty-seven per cent of the sample was rated as having "Some understanding" or "Clear understanding" of the terminology to be used in these tasks. Ten of the seven-year-olds were rated as showing "Uncertain evidence of understanding."

As evidenced by the data presented, the child's familiarity with one-half is noticeably greater than with one-fourth or one-third. According to these data the terminology used regarding each of these fractions was satisfactory. The majority of the children showed some understanding of the terminology.

TABLE 3-3

RESULTS OF THE PRELIMINARY TASK IN WHICH CHILD IS ASKED ABOUT THE TERMINOLOGY TO BE USED IN THE TASKS INVOLVING ONE-THIRD

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	1 (3%)		1 (3%)
Some evidence of understanding	21 (61.7%)	25 (76%)	19 (58%)
Uncertain evidence of understanding	4 (11.7%)	7 (21%)	10 (30%)
Clear evidence of not understanding	7 (20.6%)	1 (3%)	2 (6%)
Evidence lacking	1 (3%)		1 (3%)
TOTAL	34	33	33

Results of the Apple Tasks--Half of a Whole (Concrete Level)

This task sought to determine the child's understanding of the interpretation of one-half as one-half of a whole. The "whole" was a bright red plastic apple. The child was asked to mark on the apple with a felt-tip pen where he would cut the apple so that one could have one-half of it. (See Appendix A for interview schedule and

Appendix C-1 for investigator's response record sheet.)

The results of this task are summarized in Table 3-4.

TABLE 3-4

RESULTS OF THE APPLE TASK--HALF OF A WHOLE (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	33 (97%)	32 (97%)	32 (97%)
Some evidence of understanding	1 (3%)		
Uncertain evidence of understanding		1 (3%)	1 (3%)
TOTAL	34	33	33

Ninety-seven per cent of the children were successful in marking the plastic apple and were rated as having "Clear evidence of understanding." One five-year-old was rated as having "Some understanding" after she made a tiny mark at the stem of the apple. One six-year-old and one seven-year-old made marks in such a position that it was uncertain that they had shown any evidence of understanding.

<u>Results of the Apple Task with Two Apples the Same Size--Half of a Set (Concrete Level)</u>

The Apple Task with two apples the same size sought information about the child's understanding of one-half of a set. This task made use of objects with which the child was familiar, and was the first of a series of four tasks designed to test the child's understanding of one-half of a set. Two apples the same size were used here, whereas subsequent tasks utilized various numbers and sizes of apples.

Table 3-5 summarizes the results of this task.

TABLE 3-5

RESULTS OF THE APPLE TASK WITH TWO APPLES
THE SAME SIZE--HALF OF A SET
(CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	21 (62%)	29 (88%)	28 (85%)
Some evidence of understanding	3		3
Uncertain evidence of understanding	2	2	
Clear evidence of not understanding	8 (24%)	2	2
TOTAL	34	33	33

Seventy-eight per cent of the children were rated as having "Clear evidence of understanding." This implies that these children picked up one apple when two had been placed upon the table before them.

Three five-year-old children were rated as having "Some evidence of understanding." These children indicated that they must cut each of the apples in "half" in order to display one-half of the apples. Two of the children in this group were rated as showing "Uncertain evidence of understanding." These two children indicated that they must cut one of the apples in order to produce one-half the set. Of the eight that were rated as having "Clear evidence of not understanding" in the five-year-old group, seven picked up both the apples and the eighth said, "I don't know how."

"Uncertain evidence of understanding" was the rating received by two of the six-year-olds. One of them picked up both apples and said he needed two more in order to have one-half the apples. The other child indicated that he would cut one of the apples in "half" in order to have one-half the apples. There were two of the six-year-olds that were rated as showing "Clear evidence of not understanding." Both of these children picked up the two apples.

Three of the seven-year-olds were rated as having "Some evidence of understanding" on this task. Two of the children indicated that they would cut the apples and the third picked up one apple but was hesitant in doing so. The two seven-year-olds that were rated as showing "Clear evidence of not understanding" picked up both the apples.

Results of the Apple Task with Two Apples, Different Sizes--Half of a Set (Concrete Level)

Table 3-6 is a summary of results of the Apple Task using two apples of different sizes which were placed on a table before the child, who was then instructed to "Pick up one-half of those apples."

Seventy-eight per cent of the total sample was rated as having "Clear evidence of understanding." An additional three per cent was rated as having "Some evidence of understanding." The three children that were rated in the latter category were all five-year-olds. After some hesitancy, they responded by picking up one apple.

TABLE 3-6

RESULTS OF THE APPLE TASK WITH TWO APPLES, DIFFERENT SIZES--HALF OF A SET (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	21 (62%)	29 (88%)	28 (85%)
Some evidence of understanding	3 (8%)		
Uncertain evidence of understanding	2 (6%)	1 (3%)	
Clear evidence of not understanding	8 (24%)	3 (9%)	5 (15%)
TOTAL	34	33	33

Two children that had been rated as having "Clear understanding" when the same-sized apples were used were rated "Clear evidence of not understanding." Of the three seven-year-olds rated as having "Some evidence of understanding" when same-sized apples are used (see Table 3-5), two were rated as having "Clear evidence of not understanding" in this task.

There was only one six-year-old whose rating on this task was different from that in which the same-sized apples were used. When asked how many apples were needed to have half of them in the first task he picked up both apples and commented, "Two more." He picked up both apples in the second task but would offer no comment.

Thirty-eight per cent of the five-year-old children were rated less than "Clear evidence of understanding." With the exception of two children, each was rated at the same level of understanding on the task using apples the same size and the task using different sizes. One child, T.C., was rated "Clear evidence of not understanding" when two apples the same size were used. He was rated as having "Clear evidence of understanding" when two apples of different sizes were used. The following is the protocol:

> T.C.(age 5). Here is an apple and another apple the same size. How many apples do you see? "Two." Would you pick up half of those apples? "Half?" Yes, get half of them. "How do you do that?" I want you to show me. "I don't know what it means."
> Here is an apple and here is another apple a little smaller. How many apples is that? "Two." Would you pick up half of those apples? You picked up the smallest.

<u>Results of the Apple Task with Four Apples, Same Size--Half of a Set (Concrete Level)</u>

Four plastic apples the same size were placed on a table before the child and he was instructed, "Pick up 'half' of the apples."

Forty-one per cent of the sample was rated as showing "Clear evidence of understanding," (see Table 3-7). An additional six per cent was rated as having "Some understanding."

Thus, 47 per cent was rated as having at least "Some understanding." Fifty-two per cent of the sample was rated "Clear evidence of not understanding." One child was rated as showing "Uncertain evidence of understanding."

TABLE 3-7

RESULTS OF THE APPLE TASK WITH FOUR APPLES, SAME SIZE--HALF OF A SET (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	8 (24%)	15 (45%)	18 (55%)
Some evidence of understanding	4 (11%)	1 (3%)	1 (3%)
Uncertain evidence of understanding	1 (3%)		
Clear evidence of not understanding	21 (62%)	17 (52%)	14 (42%)
TOTAL	34	33	33

Four of the five-year-old children were rated as having "Some understanding." Three of these children indicated that they should cut each apple to give the investigator "one-half" the apples. The fourth child picked up two apples; however, he insisted that there were five apples on the table.

Results of the Apple Task with Three Apples, Same Size--Half of a Set (Concrete Level)

Eighty-two per cent of the sample showed "No evidence of understanding." This group was not successful in any way in performing the task. Fifteen per cent showed at least "Some evidence of understanding," and the remaining three per cent was rated as showing "Uncertain evidence of understanding." Table 3-8 summarizes the ratings on this task.

TABLE 3-8

RESULTS OF THE APPLE TASK WITH THREE APPLES, SAME SIZE--HALF OF A SET (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding			5 (15%)
Some evidence of understanding	3 (9%)	5 (15%)	2 (6%)
Uncertain evidence of understanding		2 (6%)	1 (3%)
Clear evidence of not understanding	31 (91%)	26 (79%)	25 (76%)
TOTAL	34	33	33

In order to give a feeling of what response was rated as "Some evidence of understanding," the following is presented:

> JP(age 6). Here is an apple, here is an apple, and here is an apple. How many apples do you see? "Three." Would you pick up half of those apples? (pause) Is something wrong? "It's not enough." Could you take half of those apples? "No."

Results of the Apple Task--Operation of Division Such That the Whole Consists of Two One-Halves (Concrete Level)

This task was designed to test whether a child understood that two "one-halves" were needed to make a whole. Three halves of plastic apples the same size were placed on a table before the child. He was asked to pick up the halves that were needed to have a whole apple.

The results of this task are reported in Table 3-9.

TABLE 3-9

RESULTS OF THE APPLE TASK--OPERATION OF DIVISION SUCH THAT THE WHOLE CONSISTS OF TWO ONE-HALVES (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	25 (73%)	25 (76%)	30 (91%)
Some evidence of understanding	3 (9%)	2 (6%)	2 (6%)
Uncertain evidence of understanding		3 (9%)	
Clear evidence of not understanding	6 (18%)	3 (9%)	1 (3%)
TOTAL	34	33	33

Eighty-seven per cent of the children showed at least "Some understanding," with 80 per cent being rated as having "Clear understanding." Only ten per cent of the sample was rated as showing "Clear evidence of not understanding."

"Some evidence of understanding" was the rating assigned if there was hesitancy in responding. The common response on this task was that a child picked up two of the halves and fitted them together. This is illustrated in the following:

> PB(age 6). Here is half of an apple, here is half of an apple, and here is half of an apple. How many halves would you pick up to have a whole apple? (Picks up two halves and puts them together.) Why is that a whole apple? "Because you can put them together."

Table 3-9 substantiates that most children in this sample were selecting two halves and confirming that they had a whole. Table 3-4 is evidence that a majority of this sample understood what is meant by dividing an apple in half. In comparing Tables 3-5 and 3-6, it is noticeable that it made little difference whether same or different sized apples were used.

Table 3-8 shows that few children in this sample understood the meaning of half a set of three. Children were somewhat more successful in performing this task when four apples were used (see Table 3-7).

Results of the Apple Task--
One-Fourth of a Whole
(Concrete Level)

Table 3-10 summarizes the ratings of the levels of understanding. Seventy-two per cent of the subjects showed "No evidence of understanding." Only 17 per cent showed that they "Clearly understood." Evidence appears here which differentiates the five-year-olds from the seven-year-olds.

Only 6 per cent of the five-year-olds were rated at the highest level of understanding; whereas, 30 per cent of the seven-year-olds were rated at this level. Also, 85 per cent of the younger children were rated as having "No understanding"; whereas, 61 per cent of the older children were rated in this same category.

TABLE 3-10

RESULTS OF THE APPLE TASK--ONE-FOURTH
OF A WHOLE (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	2 (6%)	5 (15%)	10 (30%)
Some evidence of understanding	1 (3%)	2 (6%)	1 (3%)
Uncertain evidence of understanding	2 (6%)	3 (9%)	2 (6%)
Clear evidence of not understanding	29 (85%)	23 (70%)	20 (61%)
TOTAL	34	33	33

"Uncertain evidence" was the rating assigned when the apple was marked in a place that made it impossible to determine if understanding existed. Some responses were marks around the stem or simply a circle drawn on an apple.

An illustration of "Clear understanding" follows:

> CW(age 5). Here is an apple. Take the pen and mark where you would cut it so that I could have one-fourth of it. (pause) (Divides the apple into four equal parts.) "There." How many pieces would you have? "Four."

Results of the Apple Task with Four Apples, Same Size--One-Fourth of a Set (Concrete Level)

This task sought information relative to the child's understanding of "one-fourth" of a set. A set of four apples the same size was placed on a table before the child, and he was asked to pick up one-fourth of them.

Table 3-11 summarizes the ratings of the responses.

TABLE 3-11

RESULTS OF THE APPLE TASK WITH FOUR APPLES, SAME SIZE--ONE-FOURTH OF A SET (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	12 (35%)	21 (64%)	21 (64%)
Some evidence of understanding	2 (6%)	2 (6%)	
Uncertain evidence of understanding	1 (3%)	2 (6%)	3 (9%)
Clear evidence of not understanding	19 (56%)	8 (24%)	9 (27%)
TOTAL	34	33	33

Fifty-four per cent of all children showed "Clear understanding." Thirty-six per cent were rated as having "No understanding." Ten per cent were rated either as having "Some understanding" or "Uncertain understanding."

A clear break in the level of understanding of the five-year-olds and six-year-olds is observable. Thirty-five per cent of the five-year-olds were rated as having "Clear understanding"; whereas, 64 per cent of the six-year-olds received the same rating. Fifty-six per cent of the younger children were rated as having "No understanding," and 24 per cent of the six-year-olds received this rating.

A common response that was rated "No understanding" was that of immediately picking up two or four of the apples. An illustration of a response that was rated as "Uncertain evidence" follows:

> PC(age 7). Here is an apple, here is an apple, here is an apple, and here is an apple. All the apples are the same size. Would you pick up one-fourth of those apples? (Long pause) Tell me what you think. (Pause) Pick up one-fourth of them. "Cut half of it off." How would you pick up one-fourth of them? (Points to each of parts that were indicated to be cut off.)

<u>Results of the Apple Task with Four Apples, Different Sizes--One-Fourth of a Set (Concrete Level)</u>

This task was designed to test the child's understanding of one-fourth of a set of different sized apples. Table 3-12 summarizes the results of this task.

Analyzing the results of the seven-year-olds, it was found that of the four subjects that were rated as having "Some understanding" when different sized apples were used, three had been rated as having "Clear understanding" when the same sized apples were used. However, three subjects that were rated as having "No understanding" when apples the same size were used were marked "Clear understanding" on this task.

TABLE 3-12

RESULTS OF THE APPLE TASK WITH FOUR APPLES, DIFFERENT SIZE--ONE-FOURTH OF A SET (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	15 (44%)	20 (61%)	19 (58%)
Some evidence of understanding		3 (9%)	4 (12%)
Uncertain evidence of understanding	1 (3%)	2 (6%)	1 (3%)
Clear evidence of not understanding	18 (53%)	8 (24%)	9 (27%)
TOTAL	34	33	33

Results of the Apple Task with Two Apples, Same Size--One-Fourth of a Set (Concrete Level)

Data presented in Table 3-13 substantiates that most five-, six-, or seven-year-olds have no understanding of what is meant by one-fourth of a set of two elements. Ninety-one per cent of the sample was rated as having "No understanding." None of the five-year-olds showed any evidence of understanding when two apples were placed on a table and were asked to pick up "One-fourth" of them. Only two children showed at least "Some evidence of understanding." Both of these children were six-year-olds.

The majority of five-year-olds have no understanding of the meaning of one-fourth of a set; however, the use of four objects of a different size produced a better result.

The child may have been responding to the sizes of the objects rather than to one-fourth of the set. The majority of the six-year-olds and seven-year-olds understood the meaning of one-fourth of a set. It made little difference whether the sizes varied or not, as long as there were four objects being considered.

TABLE 3-13

RESULTS OF THE APPLE TASK WITH TWO APPLES, SAME SIZE--ONE-FOURTH OF A SET (CONCRETE LEVEL)

Performance	Age: 5 Years	6 Years	7 Years
Clear evidence of understanding		1 (3%)	
Some evidence of understanding		1 (3%)	
Uncertain evidence of understanding		3 (9%)	2 (6%)
Clear evidence of not understanding	34 (100%)	28 (85%)	29 (88%)
Evidence lacking			2 (6%)
TOTAL	34	33	33

<u>Results of the Apple Task--Operation of Division Such that the Whole Consists of Four One-Fourths (Concrete Level)</u>

This Apple Task was designed to test the child's understanding that a whole consists of four "one fourths." Five fourths of plastic apples the same size were placed before the child, and he was asked to pick up those needed to have a whole apple.

Most of the subjects began immediately to try to fit the fourths together. Sixty-nine per cent of the children were successful in selecting four of the one-fourths and confirmed that they had picked up a whole apple (Table 3-14). However, 11 per cent did so with some hesitation. Twenty-five per cent of the sample demonstrated "No evidence of understanding."

TABLE 3-14

RESULTS OF THE APPLE TASK--OPERATION OF DIVISION SUCH THAT THE WHOLE CONSISTS OF FOUR ONE-FOURTHS (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	18 (53%)	20 (61%)	20 (61%)
Some evidence of understanding	4 (12%)	3 (9%)	4 (12%)
Uncertain evidence of understanding	1 (3%)	2 (6%)	1 (3%)
Clear evidence of not understanding	9 (26%)	8 (24%)	8 (24%)
Evidence lacking	2 (6%)		
TOTAL	34	33	33

Results of the Apple Task--
One-Third of a Whole
(Concrete Level)

A plastic apple was placed before the subject, and he was asked to mark it where it should be cut in order to produce one-third of the apple. (See Appendix A for interview schedule and Appendix C-3 for investigator's response sheet.)

Table 3-15 is a summary of the responses of the children. As age increases in this sample a corresponding higher level of understanding was recorded. Eighty-five per cent of the five-year-olds showed "Clear evidence of not understanding" and 52 per cent of the seven-year-olds obtained the same rating. Twenty-three per cent of the sample was rated as having at least "Some understanding."

TABLE 3-15

RESULTS OF THE APPLE TASK--ONE-THIRD OF A WHOLE (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding		2 (6%)	5 (15%)
Some evidence of understanding	3 (9%)	6 (18%)	7 (21%)
Uncertain evidence of understanding	2 (6%)	1 (3%)	3 (9%)
Clear evidence of not understanding	29 (85%)	24 (73%)	17 (52%)
Evidence lacking			1 (3%)
TOTAL	34	33	33

Tables 3-4, 3-10, and 3-15 show a decline in the level of understanding as the children were questioned concerning one-half, one-third, and one-fourth of a whole. Ninety-eight per cent of the sample showed at least "Some understanding" of one-half of a whole; 23 per cent showed the same understanding of one-third of a whole; 21 per cent showed evidence of the same understanding of one-fourth of a whole. No child received a rating of "No understanding" for the fraction one-half; 72 per cent received this rating for one-fourth of a whole, and 70 per cent received this rating for one-third of a whole. This is contrary to a statement made earlier with regard to the observation made in the pilot study. It appeared then that the child had slightly more trouble with one-third than with one-fourth.

Results of the Apple Task with Three Apples the Same Size-- One-Third of a Set (Concrete Level)

Table 3-16 summarizes the ratings received by the children on this task.

Sixty per cent of the subjects immediately picked up one plastic apple. The three apples had been placed on a table, and the subjects had been asked to pick up one-third of them. Forty-seven per cent of the five-year-olds, 58 per cent of the six-year-olds, and 76 per cent of the seven-year-olds were rated as having "Clear understanding."

Thirty-four per cent of the children showed "No evidence of understanding." Fifty per cent of the five-year-olds, 30 per cent of the six-year-olds, and 21 per cent of the seven-year-olds were rated in this category.

TABLE 3-16

RESULTS OF THE APPLE TASK WITH THREE APPLES
THE SAME SIZE--ONE-THIRD OF A SET
(CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	16 (47%)	19 (58%)	25 (76%)
Some evidence of understanding		1 (3%)	
Uncertain evidence of understanding	1 (3%)	3 (9%)	1 (3%)
Clear evidence of not understanding	17 (50%)	10 (30%)	7 (21%)
TOTAL	34	33	33

Results of the Apple Task with Three Apples, Different Sizes--One-Third of a Set (Concrete Level)

Three plastic apples of different sizes were placed on a table and the child was asked to pick up "one-third" of them.

Sixty-seven per cent of the subjects were rated as having "Some evidence" or "Clear evidence of understanding." Twenty-eight per cent were rated as showing "Clear evidence of not understanding." Five per cent of the sample could not be rated with certainty. Table 3-17 summarizes the results obtained on this task.

There was a slight increase in success when apples of different sizes were used (see Tables 3-16 and 3-17).

TABLE 3-17

RESULTS OF THE APPLE TASK WITH THREE APPLES, DIFFERENT SIZES--ONE-THIRD OF A SET (CONCRETE LEVEL)

Performance	Age: 5 Years	6 Years	7 Years
Clear evidence of understanding	18 (53%)	20 (61%)	27 (82%)
Some evidence of understanding		1 (3%)	1 (3%)
Uncertain evidence of understanding	1 (3%)	3 (9%)	1 (3%)
Clear evidence of not understanding	15 (44%)	9 (27%)	4 (12%)
TOTAL	34	33	33

Similar results were obtained when exploring the child's understanding of one-fourth of a set (see Tables 3-11 and 3-12).

Considerable difference in levels of success was observed between the five-, six-, and seven-year-olds. "Clear understanding" was the rating received by 53 per cent, 61 per cent, and 81 per cent of these respective age levels. "No understanding" was the rating of 44 per cent, 27 per cent, and 12 per cent of these respective age levels.

Results of the Apple Task with Two Apples the Same Size--One-Third of a Set (Concrete Level)

Two apples were placed on a table before the child and he was asked to pick up "one-third" of them. Table 3-18 summarizes the levels of understanding revealed by this task.

TABLE 3-18

RESULTS OF THE APPLE TASK WITH TWO APPLES
THE SAME SIZE--ONE-THIRD OF A SET
(CONCRETE LEVEL)

	Age		
Peformance	5 Years	6 Years	7 Years
Uncertain evidence of understanding		2 (6%)	2 (6%)
Clear evidence of not understanding	34 (100%)	30 (91%)	31 (94%)
Evidence lacking		1 (3%)	
TOTAL	34	33	33

Most of the subjects exhibited "No understanding." Ninety-five per cent of the sample were rated as showing "Clear evidence of not understanding." All five-year-olds were rated in this category. Due to an interruption, one subject was rated as "Evidence lacking" on his task.

Results of the Apple Task--
Operation of Division Such That
the Whole Consists of Three One-
Thirds (Concrete Level)

This Apple Task was designed to determine the child's understanding of the idea that a whole consists of three one-thirds.

Table 3-19 summarizes the ratings of levels of understanding. Fifty-five per cent of the subjects were rated as having "Clear understanding," and 28 per cent were rated as having "No understanding." As in the comparable task with one-fourth, the subjects immediately began fitting the one-thirds together to make a whole. The subjects rated as having "Some understanding" were hesitant in picking up three of the one-thirds.

An overall 69 per cent of the sample was rated as having "Some understanding" of the interpretation that the whole consists of three one-thirds. The same results were obtained for one-fourth. Seventy-six per cent of the five-year-olds were rated with at least "Some understanding," while 71 per cent of the seven-year-olds received this rating.

TABLE 3-19

RESULTS OF THE APPLE TASK--OPERATION OF DIVISION SUCH THAT THE WHOLE CONSISTS OF THREE ONE-THIRDS (CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	22 (64.7%)	18 (55%)	15 (45.5%)
Some evidence of understanding	4 (11.7%)	5 (15%)	5 (15%)
Uncertain evidence of understanding		1 (3%)	2 (6%)
Clear evidence of not understanding	8 (23.5%)	9 (27%)	11 (33.3%)
TOTAL	34	33	33

Results of the Glass Task with Pictorial Representation of One Glass--Half of a Whole (Semi-Concrete Level)

The Glass Tasks are pictorial representations which were designed to reveal the child's understanding of three interpretations of certain fractions. This particular Glass Task is a pictorial representation of one empty glass. (See Appendix B-4) The child was asked to mark on the pictorial representation the place he would fill the glass so that it would be one-half full.

Table 3-20 summarizes the ratings on this task.

TABLE 3-20

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF ONE GLASS--HALF OF A WHOLE (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	13 (38%)	15 (45.5%)	19 (58%)
Some evidence of understanding	11 (32%)	8 (24%)	3 (9%)
Uncertain evidence of understanding	2 (6%)	8 (24%)	5 (15%)
Clear evidence of not understanding	8 (24%)	2 (6%)	6 (18%)
TOTAL	34	33	33

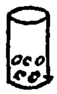
Forty-seven per cent of the sample was rated as having "Clear understanding" of one-half of a whole. An additional 22 per cent exhibited "Some evidence of understanding," and 16 per cent of the sample showed "Clear evidence of not understanding."

Tables 3-4 and 3-20 show a marked difference in the levels of understanding. Ninety-seven per cent of the subjects were rated "Clear understanding" on the Apple Task; only 47 per cent received the same rating on the corresponding Glass Task. The child was more successful when concrete representation was used than when semi-concrete was used.

"Clear evidence of understanding" was assigned when the subject marked on the pictorial representation clearly at the "one-half" point. However, "Some understanding" was assigned to various markings. For clarity, the following are examples of responses that were rated at this level:

DA (Age 5): 

YR (Age 5): 

JT (Age 5): 

CG (Age 6): 

TC (Age 7): 

The child demonstrated some uncertainty in his/her response on this task in each of these interviews.

Results of the Glass Task with Pictorial Representation of Two Glasses the Same Size--Half of a Set (Semi-Concrete Level)

This Glass Task was the first of four which sought to determine the child's understanding of one-half a set.

A pictorial representation of two glasses was presented to the child, (see Appendix B-5), and he was asked to draw a circle around one-half of them.

Table 3-21 is a summary of the ratings on this task.

TABLE 3-21

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF TWO GLASSES THE SAME SIZE--HALF OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	12 (35%)	24 (73%)	24 (73%)
Some evidence of understanding	11 (32.3%)	7 (21%)	8 (24%)
Uncertain evidence of understanding	2 (6%)		
Clear evidence of not understanding	9 (26.4%)	2 (6%)	1 (3%)
TOTAL	34	33	33

Eighty-six per cent of the sample was rated as having at least "Some evidence of understanding." Twelve per cent was rated as showing "Clear evidence of not understanding." Five-year-olds were rated as having "No understanding" roughly 26 per cent of the time. A distinct break in the level of understanding was found between the five-year-olds and the six-year-olds. Only 35 per cent of the five-year-olds received a rating of "Clear understanding"; while this same rating was assigned to 73 per cent of the six-year-olds.

Also, 73 per cent of the seven-year-olds received this rating.

Results of the Glass Task with Pictorial Representation of Two Glasses, Different Sizes--Half of a Set (Semi-Concrete Level)

A pictorial representation of two glasses of different sizes (see Appendix B-6) was placed on a table before the subject, and he was asked to draw a circle around one-half of the glasses. Table 3-22 is a summary of the ratings on this task.

TABLE 3-22

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF TWO GLASSES, DIFFERENT SIZES--HALF OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	14 (41%)	25 (76%)	23 (70%)
Some evidence of understanding	8 (24%)	6 (18%)	7 (21%)
Uncertain evidence of understanding	2 (6%)		
Clear evidence of not understanding	10 (29%)	2 (6%)	2 (6%)
Evidence lacking			1 (3%)
TOTAL	34	33	33

Eighty-three per cent of the sample showed at least "Some understanding," with 62 per cent receiving the highest rating. Only 14 per cent was rated as having "No understanding."

Examining Tables 3-21 and 3-22, two important findings are noted: (1) Six-year-olds are rated markedly higher than five-year-olds in both cases. (2) Slightly higher percentage of the subjects were rated clear understanding when different size glasses were used than when same sized were used.

Tables 3-21 and 3-22, when compared with Tables 3-5 and 3-6 respectively, show that by removing the concrete objects and replacing them with pictorial representations, the level of understanding was reduced considerably. Also, the difference in the level of understanding between the five- and six-year-olds may be observed in Tables 3-5 and 3-6.

Results of the Glass Task with Pictorial Representation of Four Glasses, Same Size--Half of a Set (Semi-Concrete Level)

A pictorial representation of four empty glasses (see Appendix B-7) was placed on a table before the subject, who was asked to draw a circle around one-half of them.

Fifty-seven per cent of the sample showed "Clear evidence of no understanding." Table 3-23 summarizes the ratings. Forty per cent were rated as having at least "Some understanding." The data indicates an increase in the level of understanding with age. The greatest difference in the level of understanding is noted between the five- and six-year-olds.

TABLE 3-23

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF FOUR GLASSES THE SAME SIZE--HALF OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	2 (6%)	8 (24%)	9 (27.3%)
Some evidence of understanding	7 (20.5%)	5 (15%)	9 (27.3%)
Uncertain evidence of understanding	1 (3%)	2 (6%)	
Clear evidence of not understanding	24 (70.5%)	18 (55%)	15 (45.4%)
TOTAL	34	33	33

A common response that was rated as "Some understanding" was one in which the subject drew a circle around one-half of each glass. All of the five-year-olds receiving this rating responded this way. This was not a factor when the concrete objects were used. This accounts for the differences in the level of understanding between the two tasks.

Results of the Glass Task with Pictorial Representation of Three Glasses the Same Size--Half of a Set (Semi-Concrete Level)

A pictorial representation of three glasses was placed on a table before the subject (see Appendix B-8), and he was asked to draw a circle around one-half of the glasses.

Table 3-24, which summarizes the results, shows that the majority of the children have "No understanding."

Seventy-four per cent of the sample was rated as having "No understanding." Twenty-one per cent of the sample was rated as having at least "Some understanding." Only four per cent was rated at the highest level.

TABLE 3-24

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF THREE GLASSES THE SAME SIZE--HALF OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding		1 (3%)	3 (9%)
Some evidence of understanding	5 (15%)	5 (15%)	7 (21%)
Uncertain evidence of understanding	1 (3%)	3 (9%)	1 (3%)
Clear evidence of not understanding	28 (82%)	24 (73%)	22 (67%)
TOTAL	34	33	33

The subjects performed slightly better as age increased. Subjects were rated somewhat higher on this task than on the corresponding task with apples (see Table 3-8). This is because a rating of "Some understanding" was assigned when the subject marked one-half of each glass. Such a response did not occur with the corresponding Apple Task.

Results of the Glass Task with Pictorial Representation--Operation of Division Such That the Whole Consists of Two One-Halves (Semi-Concrete Level)

A pictorial representation of four glasses the same size was placed before the child. Three glasses were half full and one was empty (see Appendix B-9). The child was asked to draw a circle around the glasses that could be used to fill the empty glass. The results of this task are summarized in Table 3-25.

TABLE 3-25

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION--OPERATION OF DIVISION SUCH THAT THE WHOLE CONSISTS OF TWO ONE-HALVES (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	9 (26%)	10 (30%)	21 (64%)
Some evidence of understanding		6 (18%)	1 (3%)
Uncertain evidence of understanding	1 (3%)		1 (3%)
Clear evidence of not understanding	24 (71%)	17 (52%)	10 (30%)
TOTAL	34	33	33

Fifty-one per cent of the sample showed "Clear evidence of no understanding." Forty per cent was rated as having "Clear understanding." Thirty per cent of the six-year-olds received the highest rating, while 64 per cent of the seven-year-olds were given this rating. The greatest difference in level of understanding existed between these two age levels.

Apparently, the use of concrete objects suggested the correct solution. Seventy-three per cent of the five-year-olds received the highest rating on the comparable Apple Task, but, only 26 per cent of this same group received this rating on the Glass Task. There was some difference in the other age levels, but not as dramatic.

Results of the Glass Task with Pictorial Representation--One-Fourth of a Whole (Semi-Concrete Level)

A pictorial representation of one glass was placed on a table before the child (see Appendix B-4). He was asked to show where the glass should be filled so that it would be one-fourth full.

Table 3-26 shows two important findings: (1) only two five-year-olds, no six- and no seven-year-olds were rated as having "Clear understanding," and (2) 32 per cent of the entire sample was rated as showing "Uncertain evidence of understanding." The two five-year-olds clearly marked the glass at one-fourth. Of the 28 per cent of the sample that was rated "Some understanding," the general response was a mark somewhere near "one-fourth." A rating of "Uncertain evidence" was awarded when a mark was placed on the paper in a position that made it impossible to determine if "Some evidence" or "No evidence" was being exhibited.

Table 3-10 shows that 21 per cent of the sample was rated as having at least "Some understanding" when concrete objects were used. In contrast, Table 3-26 discloses that 30 per cent received this rating when semi-concrete representations were used. Fifteen per cent of the six- and seven-year-olds were rated at the highest level when concrete objects were used.

TABLE 3-26

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION--
ONE-FOURTH OF A WHOLE (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	2 (5.8%)		
Some evidence of understanding	6 (17.6%)	7 (21.2%)	15 (45.5%)
Uncertain evidence of understanding	6 (17.6%)	13 (39.4%)	13 (39.4%)
Clear evidence of not understanding	20 (58.8%)	13 (39.4%)	5 (15.1%)
TOTAL	34	33	33

Results of the Glass Task with Pictorial Representation of Four Glasses, Same Size--One-Fourth of a Set (Semi-Concrete Level)

This Glass Task was the first of three designed to reveal the child's understanding of one-fourth of a set. A pictorial representation of four empty glasses was placed on a table before the child (see Appendix B-7). The subject was asked to draw a circle around one-fourth of the glasses.

Table 3-27 summarizes the results of this task.

TABLE 3-27

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF FOUR GLASSES, SAME SIZE--ONE-FOURTH OF A SET (SEMI-CONCRETE LEVEL)

Performance	Age 5 Years	Age 6 Years	Age 7 Years
Clear evidence of understanding	12 (35%)	21 (64%)	19 (58%)
Some evidence of understanding	1 (3%)		1 (3%)
Uncertain evidence of understanding	3 (9%)	2 (6%)	5 (15%)
Clear evidence of not understanding	18 (53%)	10 (30%)	7 (21%)
Evidence lacking			1 (3%)
TOTAL	34	33	33

Fifty-four per cent of the sample was rated as having at least "Some understanding." Thirty-five per cent of the sample had "No understanding." "Clear evidence of understanding" was shown by 35 per cent, 64 per cent, and 58 per cent of the five-, six-, and seven-year-olds, respectively.

Tables 3-11 and 3-27 suggest that it apparently makes little difference whether concrete or semi-concrete materials are used. More than 50 per cent of the sample had a "Clear understanding" in either case.

Thirty-five per cent of the five-year-olds were rated as having "Clear understanding," while 64 per cent of the six-year-olds received this rating. This appears to be a distinct increase in level of understanding.

Results of the Glass Task with Pictorial Representation of Four Glasses, Different Sizes--One-Fourth of a Set (Semi-Concrete Level)

A pictorial representation of four glasses of various sizes (see Appendix B-10) was shown to the child, and he was asked to draw a circle around one-fourth of them.

Table 3-28, as compared to Table 3-27, shows that it makes little difference whether the same or different sized glasses are used. The greatest variation here was with the seven-year-olds, who were rated at a higher level when different sized glasses were used.

TABLE 3-28

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF FOUR GLASSES, DIFFERENT SIZES--ONE-FOURTH OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	13 (38%)	20 (61%)	19 (58%)
Some evidence of understanding	1 (3%)	1 (3%)	3 (9%)
Uncertain evidence of understanding	3 (9%)	1 (3%)	4 (12%)
Clear evidence of not understanding	17 (50%)	11 (33%)	4 (12%)
Evidence lacking			3 (9%)
TOTAL	34	33	33

Fifty-seven per cent of the sample was rated as having at least "Some understanding" when different sized glasses were used, and 54 per cent received this rating when the same sized glasses were used. Fifty-eight per cent received this rating on the comparable Apple Task with same sized apples, and 61 per cent when different sizes were used. Generally, more children understood the interpretation of one-fourth of four objects if they were concrete and varied in size.

Results of the Glass Task with Pictorial Representation of Two Glasses, Same Size--One-Fourth of a Set (Semi-Concrete Level)

A pictorial representation of two glasses (see Appendix B-11) was placed on a table before the child, who was then asked to draw a circle around one-fourth of them.

Most children in this study showed "No understanding" of one-fourth of a set when it contains two elements. This statement is substantiated by Tables 3-29 and 3-13. When a pictorial representation of two glasses was presented, 89 per cent of the sample revealed "No understanding" of the meaning of one-fourth of a set. When two apples were presented, 91 per cent of the sample failed to show any understanding.

One six-year-old and one seven-year-old showed signs of "Some understanding." Because of the vagueness in the response, "Uncertain evidence" was assigned to seven subjects.

JH (age 7) was one subject that was rated as having "Some understanding" in this task. His response was to draw a circle carefully around approximately one-fourth of each glass.

A slight increase in the level of understanding was observed as ages increased. One hundred, 91, and 76 per cent of the five-, six-, and seven-year-olds were respectively rated as having "No understanding." One hundred, 85, and 88 per cent of the respective groups received the same rating on the Apple Task. However, only six per cent of the sample was rated with at least "Some understanding" on both the Apple and Glass Tasks.

TABLE 3-29

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF TWO GLASSES, SAME SIZE--ONE-FOURTH OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Some evidence of understanding		1 (3%)	1 (3%)
Uncertain evidence of understanding		2 (6%)	5 (15%)
Clear evidence of not understanding	34 (100%)	30 (91%)	25 (76%)
Evidence lacking			2 (6%)
TOTAL	34	33	33

Results of the Glass Task with Pictorial Representation--Operation of Division Such That the Whole Consists of Four One-Fourths (Semi-Concrete Level)

A pictorial representation of six glasses the same size was presented to the child (see Appendix B-12). Five of the glasses were one-fourth full. The child was asked to draw a circle around the glasses that could be used to fill the empty glass.

Table 3-30 is a summary of the results. Seventy-eight per cent of the sample demonstrated "Clear evidence of not understanding." Only nine per cent of the subjects were rated as having at least "Some understanding." A moderate increase in the level of understanding was recorded with age increase.

TABLE 3-30

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION--OPERATION OF DIVISION SUCH THAT THE WHOLE CONSISTS OF FOUR ONE-FOURTHS (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	2 (6%)	1 (3%)	1 (3%)
Some evidence of understanding		2 (6%)	3 (9%)
Uncertain evidence of understanding		5 (15%)	8 (24%)
Clear evidence of not understanding	32 (94%)	25 (76%)	21 (64%)
TOTAL	34	33	33

Tables 3-14 and 3-30 show a dramatic difference between the use of concrete and semi-concrete representations of one-fourths. Fifty-eight per cent of the sample was rated "Clear understanding" on the Apple Task, while only four per cent received this rating on the corresponding Glass Task. The child could put the "one-fourths" together and determine when they had a "whole" with the apples, but could not mentally manipulate the glasses to do the same.

Results of the Glass Task with Pictorial Representation of One Glass--One-Third of a Whole (Semi-Concrete Level)

A pictorial representation of one empty glass was placed before the child (see Appendix B-4) and he was asked to mark where he would fill it so that it would be one-third full.

A moderate increase in level of understanding with increase in age was noted (see Table 3-31). Thirty-two per cent of the sample was rated as having at least "Some understanding." Forty-six per cent of the seven-year-olds received this rating. Thirty-eight per cent of the sample was rated as having "No understanding." Roughly sixty-four per cent of the youngest children received this rating. No seven-year-olds were rated at the highest level, whereas one five-year-old and one six-year-old received this rating.

Comparing Tables 3-15 and 3-31, three observations can be made:

(1) The use of concrete objects produced better results with the five-year-olds than with the seven-year-olds.

(2) More of the subjects received a rating of "Some understanding" with the pictorial representation than with the concrete objects.

(3) Roughly 42 per cent of the six-year-olds were rated as showing "Uncertain evidence of understanding" on the Glass Task, and three per cent received this rating on the Apple Task. Most of the subjects receiving this rating on the Glass Task had been rated as having "No understanding" on the former task.

TABLE 3-31

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF ONE GLASS--ONE-THIRD OF A WHOLE (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	1 (3%)	1 (3%)	
Some evidence of understanding	7 (20.6%)	8 (24.2%)	15 (46%)
Uncertain evidence of understanding	4 (11.7%)	14 (42.4%)	12 (36%)
Clear evidence of not understanding	22 (64.7%)	10 (30.3%)	6 (18%)
TOTAL	34	33	33

<u>Results of the Glass Task with Pictorial Representation of Three Glasses, Same Size--One-Third of a Set (Semi-Concrete Level)</u>

Table 3-32 summarizes the results of this task in which the child was asked to draw a circle around one-third of the glasses in a pictorial representation of three glasses.

TABLE 3-32

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF THREE GLASSES, SAME SIZE--ONE-THIRD OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	11 (32%)	18 (55%)	21 (64%)
Some evidence of understanding	1 (3%)		2 (6%)
Uncertain evidence of understanding	3 (9%)	3 (9%)	5 (15%)
Clear evidence of not understanding	19 (56%)	11 (33%)	5 (15%)
Evidence lacking		1 (3%)	
TOTAL	34	33	33

Half of the children were rated as having "Clear understanding" of one-third of a set. Thirty-two per cent of the five-year-olds, 55 per cent of the six-year-olds, and 64 per cent of the seven-year-olds drew a circle around one of the glasses. Thus, quite a difference was recorded between the five- and seven-year-olds.

Thirty-five per cent of the sample was rated as having "No understanding." Fifty-six per cent, 33 per cent, and 15 per cent of the five-, six-, and seven-year-olds received this rating, respectively. Clearly, most of the six- and seven-year-olds were successful at this task.

Results of the Glass Task with Pictorial Representation of Three Glasses, Different Sizes--One-Third of a Set (Semi-Concrete Level)

Table 3-33 summarizes the results of the second task designed to study the child's understanding of one-third of a set. A pictorial representation of three glasses of different sizes (see Appendix B-13) was placed on a table, and the child was asked to draw a circle around one-third of them.

TABLE 3-33

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF THREE GLASSES, DIFFERENT SIZES--ONE-THIRD OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	13 (38.9%)	20 (61%)	23 (70%)
Some evidence of understanding	1 (3%)		3 (9%)
Uncertain evidence of understanding	3 (9%)	2 (6%)	5 (15%)
Clear evidence of not understanding	17 (50%)	11 (33%)	2 (6%)
TOTAL	34	33	33

Sixty per cent of the subjects were rated as having at least "Some understanding." The rating was higher than that on the comparable task with glasses of the same size (see Table 3-32). Fifty-three per cent received this rating when the same sized glasses were used. Fewer subjects were rated with "No understanding" on this task than when same sized glasses were used. Fifty per cent of the five-year-olds received this rating when different sized glasses were used. Generally, the subjects showed slightly more evidence of understanding when different sized glasses were used in pictorial representations.

Comparing Tables 3-16, 3-17, 3-32, and 3-33, four important findings may be noted:

(1) At least one-half of all subjects were rated as having "Clear understanding" on each of these tasks.

(2) Highest levels of understanding on both the concrete and semi-concrete tasks were obtained when different sized objects were used.

(3) More than 50 per cent of the five-year-olds were rated "Clear understanding" when concrete objects of different sizes were used.

(4) Highest levels of understanding for all the subjects were recorded when concrete objects of different sizes were used.

Results of the Glass Task with Pictorial Representation of Two Glasses, Same Size--One-Third of a Set (Semi-Concrete Level)

A pictorial representation of two glasses (see Appendix B-11) was placed before the child and he was asked to draw a circle around one-third of the glasses.

Table 3-34 shows that ninety-two per cent of all the subjects demonstrated "No evidence" of understanding. Only one seven-year-old showed "Some evidence of understanding." This closely correlates with the results obtained from the corresponding Apple Task (see Table 3-18). Seven-year-olds were rated slightly higher on the Glass Task than on the Apple Task; however, the six-year-olds were rated about the same. All five-year-olds showed "No evidence of understanding" on both tasks.

TABLE 3-34

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION OF TWO GLASSES, SAME SIZE--ONE-THIRD OF A SET (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Some evidence of understanding			1 (3%)
Uncertain evidence of understanding		2 (6%)	5 (15%)
Clear evidence of not understanding	34 (100%)	31 (94%)	27 (82%)
TOTAL	34	33	33

Results of the Glass Task with Pictorial Representation-- Operation of Division Such That the Whole Consists of Three One-Thirds (Semi-Concrete Level)

A pictorial representation of five glasses the same size, four of which were one-third filled and the other empty, was placed before the child (see Appendix B-14). He was asked to draw a circle around the glasses that would be used to fill the empty one.

Table 3-35 summarizes the results of this task. Only 18 per cent of the sample was rated with at least "Some understanding." Fourteen per cent received a rating of "Uncertain evidence." Generally, higher levels of understanding were recorded with greater age. Ninety-one per cent of the five-year-olds were rated with "No understanding," while roughly 48 per cent of the seven-year-olds received this rating.

TABLE 3-35

RESULTS OF THE GLASS TASK WITH PICTORIAL REPRESENTATION--OPERATION OF DIVISION SUCH THAT THE WHOLE CONSISTS OF THREE ONE-THIRDS (SEMI-CONCRETE LEVEL)

	Age		
Performance	5 Years	6 Years	7 Years
Clear evidence of understanding	2 (6%)	6 (18%)	5 (15.2%)
Some evidence of understanding	1 (3%)		4 (12.1%)
Uncertain evidence of understanding		6 (18%)	8 (24.2%)
Clear evidence of not understanding	31 (91%)	21 (64%)	16 (48.5%)
TOTAL	34	33	33

Comparing Tables 3-19 and 3-35, a noticeable difference in the level of understanding was recorded on the two tasks. When familiar concrete objects were used (see Table 3-19), children at each age level were rated at a much higher level than when semi-concrete representations were used (see Table 3-35).

Evidence reported in these tables suggests that:

(1) When pictorial representations are used, most five-, and six-year-olds exhibit "No understanding" that a whole consists of three one-thirds.

(2) When concrete objects are used, five-, six-, and seven-year-olds reveal a higher level of understanding that a whole consists of three one-thirds than that a whole consists of four one-fourths.

CHAPTER IV

DISCUSSION AND CONCLUSIONS

The present study sought information about the child's understanding of three interpretations of the unit fractions one-half, one-fourth, and one-third prior to formal instruction. To this end, a series of concrete and semi-concrete tasks involving each of these fractions was devised. Understanding of the interpretations--part of a whole, part of a set, and the operation of division such that the whole consists of two one-halves, three one-thirds, and four one-fourths--was investigated with five-, six-, and seven-year-old children. Answers to three other related questions were sought: (1) Will a child's understanding of a fraction of a set be affected by changing the size of the objects in the set? (2) Which fraction--one-half, one-fourth, or one-third--does the child know the most about before formal instruction?, and (3) Does the child show more understanding when physical objects are used or when pictorial representations are used?

The population was defined as the students of Lincoln Parish School System, Ruston, Louisiana. The six schools in this system housing kindergarten, first and second year students were used. Thirty-four kindergarten, thirty-three first-year, and thirty-three second-year students were randomly selected from these schools.

The investigator designed and pilot-tested the tasks. Each task was presented in individual interviews. Tape recordings of the interviews were made to aid in subsequent evaluations. Levels of understanding were rated as follows: (1) Clear evidence of understanding, (2) Some evidence of understanding, (3) Uncertain evidence of understanding, (4) Clear evidence of not understanding, and (5) Evidence lacking.

Results of each individual task are presented in Chapter III, with results for each level tabulated separately. The following section contains a discussion of the performance of children at each level with regard to: (1) understanding of three interpretations of one-half, one-fourth, and one-third; (2) changing sizes and number of elements in the set; (3) use of concrete and semi-concrete representations; and (4) intuitive understanding of one-half, one-fourth, and one-third prior to formal instruction.

DISCUSSION OF PERFORMANCE

Interpretation of One-Half

Ninety-seven per cent of all five-, six-, and seven-year-olds show "Clear understanding" of what is meant by one-half of a whole when concrete objects are used. Eighty per cent of these children are able to determine correctly that two "one-halves" make a whole when physical objects are used. A majority of the seven-year-olds exhibit a "Clear understanding" of the latter task when pictorial representations are used.

More than 60 per cent of the five-, six-, and seven-year-olds seem to have a "Clear understanding" of the interpretation of one-half of a set when two concrete objects are used. It makes little difference whether the objects are the same size or not. Increased understanding with age is also observable.

A majority of the seven-year-olds understand the interpretation of "one-half" of a set when it consists of four concrete elements. When pictorial representations are used, "Clear understanding" is cut in half. An increase in levels of understanding is observed with increased age.

Seventy per cent or more of the five-, six-, and seven-year-olds exhibit "No understanding" of one-half of a set when it consists of three elements. This is true whether concrete or semi-concrete representations are used.

Five-, six-, and seven-year-old children understand clearly that two one-halves can be put together to make a whole when concrete objects are used. Roughly one-fourth of the five-year-olds, one-third of the six-year-olds, and two-thirds of the seven-year-olds have this level of understanding when pictorial representations are used for this interpretation.

Rated by the percent of the entire sample receiving at least some understanding, the interpretations of one-half are ranked from least to most difficult for the five-, six-, and seven-year-olds as follows:

(1) Half of a whole (Concrete Level)
(2) Whole consists of two halves (Concrete Level)
(3) One-half of a set of two objects, same size (Semi-Concrete Level)
(4) One-half of a set of two objects, same size (Concrete Level)
(5) One-half of a set of two objects, different size (Concrete Level)
(6) One-half of a set of two objects, different size (Semi-Concrete Level)
(7) Whole consists of two halves (Semi-Concrete Level)
(8) One-half of a set of four (Concrete Level)
(9) One-half of a set of four (Semi-Concrete Level)
(10) Whole consists of two halves (Semi-Concrete Level)
(11) Half of a set of three (Semi-Concrete Level)
(12) Half of a set of three (Concrete Level)

Interpretations of One-Fourth

When concrete objects are used, more than half of the five-, six-, and seven-year-olds are successful in determining that four one-fourths make a whole. If pictorial representations are used 78 per cent of these children reveal "No understanding."

One-fourth of a whole is understood very little by any of the age groups represented. Slightly more understanding of this interpretation is exhibited when concrete objects are used. There is an increase in level of understanding with age when concrete objects are used; however, this increase is not present when pictorial representations are used.

Six- and seven-year-olds generally understand the meaning of one-fourth of a set of four elements and little difference is made if same or different sized objects are used. Five-year-olds show little understanding of this interpretation regardless of how the task is designed.

Generally, five-, six-, or seven-year-olds do not understand the interpretation of one-fourth of a set of two elements, and concrete or semi-concrete representations make little difference.

The interpretations of one-fourth are ranked from least to most difficult for this sample as follows:

(1) Four one-fourths make a whole (Concrete Level)
(2) One-fourth of a set of four, different sizes (Concrete Level)
(3) One-fourth of a set, same size (Concrete Level)
(4) One-fourth of a set of four, different sizes (Semi-Concrete Level)
(5) One-fourth of a set of four, same size (Semi-Concrete Level)
(6) One-fourth of a whole (Semi-Concrete Level)
(7) One-fourth of a whole (Concrete Level)
(8) A whole consists of four "one-fourths" (Semi-Concrete Level)
(9) One-fourth of a set of two, same size (Semi-Concrete Level)
(10) One-fourth of a set of two, same size (Concrete Level)

Interpretations of One-Third

Slightly more than half of the five-, six-, and seven-year-olds understand the interpretation that three "one-thirds" make a whole when concrete objects are used. Levels of understanding of this concept as exhibited by each age group are higher than those recorded for the comparable interpretation for one-fourth. Five-year-olds appear more successful in this interpretation with concrete objects than either the six- or seven-year-olds. (This is not true for the corresponding interpretations of one-fourth.) Most five-year-olds immediately pick up three "one-thirds" and put them together and verify they had a whole. When pictorial representations are used to test understanding of this interpretation, most children at all age levels demonstrate that they have "No understanding."

Considerable confusion is exhibited by five-, six-, and seven-year-olds when questioned about one-third of a whole. Very little "Clear understanding" is exhibited by any of the children. However, a few seven-year-olds show "Some understanding." Concrete or semi-concrete representations seem to make little difference, and in general, five-, six-, or seven-year-olds reveal little understanding of one-third of a whole.

More than 54 per cent of the six- and seven-year-olds have a "Clear understanding" of the interpretation one-third of a set of three elements the same size. A slightly higher percentage of each age shows this level of understanding when concrete (as opposed to semi-concrete) representations are used. Approximately half of the five-year-olds show "No understanding" of this interpretation. However, "Clear understanding" is exhibited by more five-year-olds when concrete objects are used than when pictorial representations are used. The highest level of understanding for this interpretation of one-third occurs when objects of different sizes are used. This is true for concrete as well as for semi-concrete representations.

The interpretations of one-third, ranked from least to most difficult for this sample follow:

(1) Three "one-thirds" make a whole (Concrete Level)
(2) One-third of a set of three elements, different sizes (Concrete Level)
(3) One-third of a set of three elements, same size (Concrete Level)
(4) One-third of a set of three elements, different sizes (Semi-Concrete Level)
(5) One-third of a set of three elements, same size (Semi-Concrete Level)
(6) One-third of a whole (Semi-Concrete Level)
(7) One-third of a whole (Concrete Level)
(8) Three "one-thirds" make a whole (Semi-Concrete Level)
(9) One-third of a set of two elements (Concrete Level)
(10) One-third of a set of two elements (Semi-Concrete Level)

Varying Sizes and Numbers in a Set

Five-year-olds understand the meaning of one-half of a set of two (Concrete) elements. No less than 58 per cent of the six- or seven-year-olds understand the meaning of one-third of a set of three or one-fourth of a set of four (Concrete Level). A slightly higher level of understanding for one-third of a set is shown when different sizes of concrete objects are used than when same sizes are used.

Varying the number of elements in the set does affect the level of understanding. Children of all three ages generally do not understand the interpretation one-third, or one-fourth of a set of two elements, or one-half of a set of three elements. Most five-year-olds do not understand what is meant by one-fourth of a set of four elements. About half of the six-year-olds and slightly more than half of the seven-year-olds understand this interpretation.

Concrete and Semi-Concrete Representations

Five-year-olds generally exhibit a higher level of understanding of one-half, one-third, and one-fourth when concrete representations are used. Six- and seven-year-olds are not affected as much by the use of concrete representations as are the five-year-olds. However, in comparing the results of the tasks involving the various interpretations of one-half, one-fourth, and one-third, it was noted that the task designed with concrete objects generally was rated higher than its corresponding task in which semi-concrete representations were used.

Understanding of One-Half, One-Third, and One-Fourth Prior to Formal Instruction

Five-, six-, and seven-year-olds generally exhibit more understanding of one-half than either one-third or one-fourth. Generally, these children have at least some understanding of: (1) half of a whole (concrete level), (2) the whole consists of two halves (concrete level), (3) one-half of a set of two objects, same or different sizes (concrete or semi-concrete level). Generally, there is an increase in level of understanding with age.

Five-, six-, and seven-year-olds show a higher level of understanding of some interpretations of one-third than corresponding interpretations of one-fourth. Generally, these children have at least some understanding of:

(1) three "one-thirds" make a whole (concrete level), (2) one-third of a set of three elements, different and same size (concrete level), (3) one-third of a set of three elements, different sizes (semi-concrete level). Again, there is a general increase in level of understanding with age.

Prior to formal instruction, children show slightly less understanding of the fraction one-fourth than of one-third. Generally, however, they show at least some understanding of: (1) four "one-fourths" make a whole (concrete level), and (2) one-fourth of a set of four, different sizes (concrete level).

CONCLUSIONS

The conclusions reached in the present study may be summarized as follows:

(1) Prior to formal instruction, "half" of a whole (concrete representation) is understood by more five-, six-, and seven-year-olds than either of the other two interpretations.

(2) Prior to formal instruction, a five-, six-, or seven-year-old understands how many fractional parts (concrete) needed to make a whole. That is, he understands how many "one-thirds," "one-fourths," or "one-halves" that it takes to make a whole. The level of understanding of this interpretation is exceeded only by the interpretation of "half" of a whole.

(3) Prior to formal instruction, level of understanding of each interpretation increases with age. More difference in level of understanding seems to exist between five- and six-year-olds than between six- and seven-year-olds.

(4) Prior to formal instruction, children generally show more understanding of the interpretation of fractions when concrete representations are used than when semi-concrete representations are used. Five-year-olds exhibit a greater difference in level of understanding than do six- or seven-year-olds.

(5) Prior to formal instruction, a child's level of understanding of the interpretations of one-half is higher than for those of one-third. Also, his level of understanding of the interpretations of one-third is higher than for those of one-fourth.

(6) Prior to formal instruction, a child's understanding of one-third or one-fourth of a set is greater if the objects in the set are varied in size than when the objects are the same size. The reverse appears to be true for one-half of a set.

IMPLICATIONS FOR EDUCATION

This study provides information concerning the levels of understanding of three different interpretations of the fractions one-half, one-third, and one-fourth. This information should be of interest to mathematics educators concerned with curriculum development for the early years, and in particular in writing instructional materials involving these fractions.

In general, the results appear favorable for presenting some work with fractions with kindergarten children. The results suggest that with concrete objects, the interpretations--two "one-halves," three "one-thirds," and four "one-fourths" each make a whole--can be successfully discussed with kindergarten children. Also, one-half of a whole can be discussed with these children.

Generally, the five-year-olds are more bound to use of concrete objects than are the six- or seven-year-olds. It may be concluded then that beginning entirely with concrete representations and gradually working into the use of semi-concrete with older children is plausible.

Perhaps the most significant result of this study is the fact that most of the children were successful in putting the fractional parts together and making a whole. Also significant was the fact that generally the children had less trouble with the fraction one-third than with one-fourth. These aspects should be of especial interest to mathematics educators.

Finally, these findings contribute to a growing body of knowledge about children's understanding of fractions before formal instruction is begun. Woody[1] and Polkinghorne[2] both acknowledged in the 1930's that children possess much ability in processes of arithmetic before the time of formal instruction. Contributing to the existing body of knowledge is important, as recognized by Lovell. He suggests:

> ...now we know--thanks to the Piaget-type-research--much more about the profound aspects of the deceptively simple material in mathematics that children are called upon to learn. Again, if we take the trouble we can analyze in far greater detail the difficulties that children have in approaching such material. We also know that the development of the general ways of knowing will determine the manner in which the mathematical ideas are assimilated. Of course, we have only just made a beginning in these matters, and far more knowledge is required.[3]

IMPLICATIONS FOR FURTHER RESEARCH

There are a number of areas that need follow-up study as a result of this investigation. These areas are as follows:

1. The children that participated in the present study were from a rural, southern community. Similar studies are needed to provide information about the under-

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[1]Clifford Woody, "The Arithmetic Background of Young Children," Journal of Educational Research, XXIV (October, 1931), 195.

[2]Ada R. Polkinghorne, "Young Children and Fractions," Childhood Education, XI (May, 1935), 354.

[3]Kenneth Lovell, Intellectual Growth and Understanding Mathematics, Science and Math Education Information Report, February, 1971 (Columbus, Ohio: ERIC Information Analysis Center for Science and Mathematics Education, 1971), p. 12.

standing of (1) children from other geographic locations, (2) children according to sex, (3) children according to social and economic background, and (4) children representative of a large population.

2. The present study made use of concrete and semi-concrete representations that were familiar to the child. This study needs to be replicated with tasks designed with less familiar objects to be manipulated. Also, not only should the representations vary in size but they should vary in shape. Varying size, shape, and familiarity would tend to produce stronger results.

3. A child's understanding, prior to formal instruction, that one-half has the same value as two one-fourths needs to be explored. Tasks on the concrete and semi-concrete level should be developed, and understanding studied in the manner in which this investigation was made.

4. The results of this study need to be tested in the classroom. Studies in which experimental groups of children are exposed to unit fractions in the order suggested in this study are needed. These studies should conclude whether following the order of difficulty does improve the child's competency in working with fractions.

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APPENDIXES

APPENDIX A

DESCRIPTION OF TASKS

PRELIMINARY TASK

General Situation:

Various drawings were placed on a table before the child. The subject was asked to perform tasks to check his understanding of the following: draw a circle around one, around two, around three, and around four; and empty. Some of the questioning related to whether the child had heard of one-half, one-fourth, and one-third.

Materials:

1. One drawing of two trees of different sizes on 8 1/2" by 11" white paper.
2. One drawing of three sets of five stars the same size on 8 1/2" x 11" white paper.
3. One drawing of five glasses--one glass full, one glass half full, one glass one-third full, one glass one-fourth full, and one glass empty.
4. One table and two chairs.
5. One pencil.

Procedure and Questioning:

1. <u>Draw a circle around one tree.</u>
   "Here is a picture of two trees. (Place a drawing of two trees and a pencil on the table before the child.) Draw a circle around 'one' of the trees."

2. <u>Draw a circle around two, three, and four stars.</u>
   "Here is a picture of some stars. (Place a drawing of three rows of stars with five stars in each row on 8 1/2" x 11" white paper on the table before the child.) Draw a circle around 'two' of the stars in this row. (Point at the top row.) Draw a circle around 'three' stars on this row. (Point at the second row.) Draw a circle around 'four' stars on this row." (Point at the third row.)

3. <u>Draw a circle around the glass that is full, empty, half full, one-fourth full, and one-third full.</u>
   "Here is a picture of some glasses. (Place a drawing of five glasses the same size on the table before the child. One glass is full, one is half full, one is one-third full, one is one-fourth full and one is empty.) Draw a circle around the glass that is 'full.' Draw a circle around the glass that

is 'empty.' Draw a circle around the glass that is 'one-half' full. Draw a circle around the glass that is 'one-third' full. Draw a circle around the glass that is 'one-fourth' full."

APPLE TASK FOR THE INTERPRETATIONS OF ONE-HALF

General Situation:

Various arrangements of the whole and parts of plastic apples were placed on a table at which the subject was seated. The questioning and tasks relate to three interpretations of the fraction one-half--half of a whole, half of a set, and the operation of division such that the whole consists of two one-halves.

Materials:

1. Four plastic apples the same size.
2. Three halves of plastic apples the same size.
3. One plastic apple of a different size from the others.
4. One table and two chairs.
5. One felt-tip pen.

Procedure and Questioning:

1. <u>Half of a Whole.</u>
   "Here is a whole plastic apple. (Place one plastic apple and a felt-tip pen on the table before the child.) Would you mark the apple where you would cut it so that I could have one-half of it"?

2. <u>Half of a Set.</u>
   "Suppose that I place two whole apples on the table. (Place two plastic apples the same size on the table before the child.) Now, would you pick up half of the apples? Now I will place four apples of the same size on the table. (Place four apples the same size on the table before the child.) Would you pick up half of the apples? Suppose that I place three apples which are all the same size on the table. (Place three apples which are the same size on the table before the child.) Would you pick up half of these apples"?

3. <u>Operation of Division Such That the Whole Consists of Two One-Halves.</u>
   "Now, this is one-half of an apple, this is one-half

of an apple, and this is one-half of an apple the same size. (Place three one-halves of an apple the same size on the table.) Would you pick up enough one-halves so that you would have a whole apple? Why do you think that you have picked up a whole apple"?

APPLE TASK FOR THE INTERPRETATIONS OF ONE-FOURTH

General Situation:

Various arrangements of whole and parts of plastic apples were placed on the table at which the subject was seated. The questioning and tasks relate to three interpretations of the fraction one-fourth--one-fourth of a whole, one-fourth of a set, and the operation of division such that the whole consists of four one-fourths.

Materials:

1. Four plastic apples the same size.
2. Four plastic apples all of a different size.
3. Five one-fourths of plastic apples the same size.
4. One table and two chairs.
5. One felt-tip pen.

Procedure and Questioning:

1. <u>One-Fourth of a Whole.</u>
   "Let us talk about one whole apple again. (Place one plastic apple and a felt-tip pen on the table before the child.) Here we have a whole apple. (Point at apple.) Would you mark where you would cut the apple so that I could have one-fourth of it"?

2. <u>One-Fourth of a Set.</u>
   "Suppose that I place four whole apples the same size on the table. (Place four plastic apples the same size on the table before the child.) These four apples are the same size. (Point at each of the plastic apples.) Now, would you pick up one-fourth of the apples? Suppose that I place four apples of a different size on the table. (Place four plastic apples in order of largest to smallest on the table before the child.) Pick up one-fourth of those apples. Now, here are two apples the same size on the table. (Place two plastic apples the

same size on the table before the child.) Would you pick up one-fourth of the apples"?

3. Operation of Division Such That the Whole Consists of Four One-Fourths.
"This is one-fourth of an apple, this is one-fourth of an apple, this is one-fourth of an apple, this is one-fourth of an apple, and this is one-fourth of an apple. (Place five one-fourths of plastic apples the same size on the table before the child.) Pick up enough one-fourths so that you would have a whole apple. Why do you think that you have a whole apple in your hands"? (Point at the one-fourths that the child has in his hands.)

APPLE TASK FOR THE INTERPRETATIONS OF ONE-THIRD

General Situation:

Various arrangements of whole and parts of plastic apples were placed on a table at which the subject was seated. The questioning and tasks relate to three interpretations of the fraction one-third--one-third of a whole, one-third of a set, and the operation of division such that the whole consists of three one-thirds.

Materials:

1. Three plastic apples the same size.
2. Four one-thirds of plastic apples the same size.
3. Three plastic apples of different sizes.
4. One table and two chairs.
5. One felt-tip pen.

Procedure and Questioning:

1. One-Third of a Whole.
"Let us talk about one whole apple again. (Place one plastic apple and a felt-tip pen on the table before the child.) Here we have a whole apple. Would you mark where you would cut the apple so that I could have one-third of it"?

2. One-Third of a Set.
"Suppose that I place three whole apples the same size on the table. (Place three plastic apples the same size on the table before the child.) Now, would you pick up one-third of the apples? Now,

suppose that I place three apples of a different size on the table. (Place three plastic apples in order of largest to smallest on the table in front of the child.) Would you pick up one-third of these apples? Suppose that I place two plastic apples the same size on the table. (Place two plastic apples the same size on the table before the child.) Would you pick up one-third of these apples"?

3. Operation of Division Such That a Whole Consists of Three One-Thirds.
"This is one-third of an apple, this is one-third of an apple, this is one-third of an apple, and this is one-third of an apple. (Place four one-thirds of plastic apples the same size on the table before the child.) Pick up enough one-thirds so that you would have a whole apple. Why do you think you have a whole apple in your hands"? (Point at the one-thirds that the child has in his hands.)

GLASS TASK FOR INTERPRETATIONS OF ONE-HALF

General Situation:

A series of drawings of different numbers of glasses were presented to the child during an interview. The drawings represent partially filled glasses and empty glasses. The questioning and tasks related to three interpretations of the fraction one-half--half of a whole, half of a set, and the operation of division such that the whole consists of two "one-halves."

Materials:

1. One drawing of an empty glass on 8 1/2" x 11" white paper.
2. One drawing of two empty glasses the same size on 8 1/2" x 11" white paper.
3. One drawing of two empty glasses of different sizes on 8 1/2" x 11" white paper.
4. One drawing of four empty glasses the same size on 8 1/2" x 11" white paper.
5. One drawing of three empty glasses the same size on 8 1/2" x 11" white paper.
6. One drawing of four glasses the same size, three half filled and one empty on 8 1/2" x 11" white paper.
7. One pencil.
8. One table and two chairs.

Procedure and Questioning:

1. Half of a Whole.
"Here is a picture of one empty glass. (Place a drawing of one glass and a pencil on the table before the child.) Put a mark where you would fill the glass so that it would be one-half full." (Point to the glass.)

2. Half of a Set.
"Here is a picture of two glasses the same size. (Place a drawing of two glasses the same size and a pencil on the table before the child.) Would you draw a circle around one-half of the glasses? Now, here is a picture of two glasses but different sizes. (Place a drawing of two glasses of different sizes and a pencil on the table before the child.) Draw a circle around one-half of the glasses. (Point at the picture.) Here is a picture of four glasses the same size. (Place a drawing of four glasses the same size on the table before the child.) Draw a circle around one-half of the glasses in this picture. Now, here is a picture of three glasses the same size. (Place a drawing of three glasses the same size on the table before the child.) Draw a circle around half of the glasses. Why can't you draw a circle around one-half of the glasses"?

3. Operation of Division Such That a Whole Consists of Two One-Halves.
"Here is a picture of four glasses the same size. (Place a drawing of four glasses, three of which are half full and one empty, and a pencil on the table before the child.) This one is half full, this one is half full, this one is half full, and this one is empty. (Point at each glass.) All the glasses are the same size. Draw a circle around the glasses that you would use to fill this glass. (Point at the empty glass.) Why did you draw a circle around these glasses"?

GLASS TASK FOR THE INTERPRETATIONS OF ONE-FOURTH

General Situation:

A series of drawings of different numbers of glasses were presented to the child during an interview. The drawings represent partially filled glasses and empty glasses. The questioning and tasks relate to three interpretations of the fraction one-fourth--one-fourth of a whole, one-fourth of a set, and the operation of division such that the whole consists of four one-fourths.

Materials:

1. One drawing of an empty glass on 8 1/2" x 11" white paper.
2. One drawing of four empty glasses the same size on 8 1/2" x 11" white paper.
3. One drawing of four empty glasses of different sizes on 8 1/2" x 11" white paper.
4. One drawing of two empty glasses the same size on 8 1/2" x 11" white paper.
5. One drawing of six glasses, five filled one-fourth and one empty, the same size on 8 1/2" x 11" white paper.
6. One pencil.
7. One table and two chairs.

Procedure and Questioning:

1. <u>One-Fourth of a Whole.</u>
   "Here is a picture of one glass. (Place a drawing of one glass and a pencil on the table before the child.) Put a mark where you would fill the empty glass so that it would be one-fourth full." (Point at the drawing of the empty glass.)

2. <u>One-Fourth of a Set.</u>
   "Here is a picture of four glasses the same size. (Place a drawing of four glasses and a pencil on the table before the child.) Draw a circle around one-fourth of the glasses. Now, here is a picture of four glasses but different sizes. (Place a drawing of four glasses, different sizes arranged from the largest to the smallest before the child.) Draw a circle around one-fourth of the glasses. Here is a picture of two glasses the same size. (Place a drawing of two glasses the same size on the table before the child.) Would you draw a circle around one-fourth of the glasses? Why can't you draw a circle around one-fourth of the glasses"?

3. <u>Operation of Division Such That the Whole Consists of Four One-Fourhts.</u>
   "Here is a picture of some glasses that are one-fourth full. (Place a drawing of six glasses, five of which are one-fourth full and one empty, and a pencil on the table before the child.) This one

is one-fourth full, this one is one-fourth full, this one is one-fourth full, this one is one-fourth full, this one is one-fourth full, and this one is empty. (Point at each glass.) All the glasses are the same size. Draw a circle around the glasses that you would use to fill this glass, which is the same size, full. (Point to the empty glass.) Why did you draw a circle around these glasses"?

GLASS TASK FOR THE INTERPRETATIONS OF ONE-THIRD

General Situation:

A series of drawings of different numbers of glasses were presented to the child during the interview. The drawings represent partially filled glasses and empty glasses. The questioning and tasks relate to three interpretations of the fraction one-third--one-third of a whole, one-third of a set, and the operation of division such that the whole consists of three one-thirds.

Materials:

1. One drawing of an empty glass on 8 1/2" x 11" white paper.
2. One drawing of three empty glasses the same size on 8 1/2" x 11" white paper.
3. One drawing of three empty glasses of different sizes on 8 1/2" x 11" white paper.
4. One drawing of two empty glasses the same size on 8 1/2" x 11" white paper.
5. One drawing of five glasses, four filled one-third and one empty, the same size on 8 1/2" x 11" white paper.
6. One pencil.
7. One table and two chairs.

Procedure and Questioning:

1. <u>One-Third of a Whole.</u>
   "Here is a picture of one glass. (Place a drawing of one glass and a pencil on the table before the child.) Put a mark where you would fill this glass so that it would be one-third full."

2. <u>One-Third of a Set.</u>
   "Here is a picture of three glasses the same size. (Place a drawing and a pencil on the table before

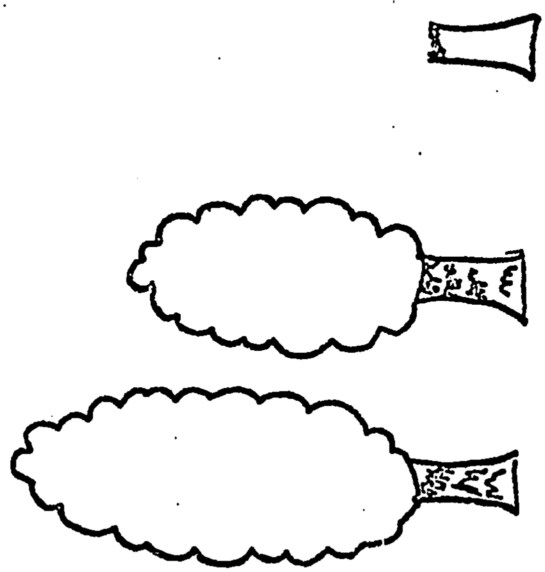
the child.) Would you draw a circle around one-third of the glasses? Now, here is a picture of three glasses. They are all different sizes. (Place a drawing and a pencil on the table before the child.) Draw a circle around one-third of the glasses in this picture. Here is a picture of two glasses. (Place the drawing and a pencil on the table before the child.) Draw a circle around one-third of the glasses."

3. <u>Operation of Division Such That a Whole Consists of Three One-Thirds.</u>
"Here is a picture of five glasses the same size. (Place a drawing of five glasses, four of which are one-third full and one empty, and a pencil on the table before the child.) This one is one-third full, this one is one-third full, this one is one-third full, this one is one-third full, and this one is empty. (Point at each glass.) All the glasses are the same size. Draw a circle around the glasses that you would use to fill this glass. (Point at the empty glass.) Why did you draw a circle around these glasses"?

APPENDIX B

PICTORIAL REPRESENTATIONS PRESENTED TO SUBJECTS

APPENDIX B-1



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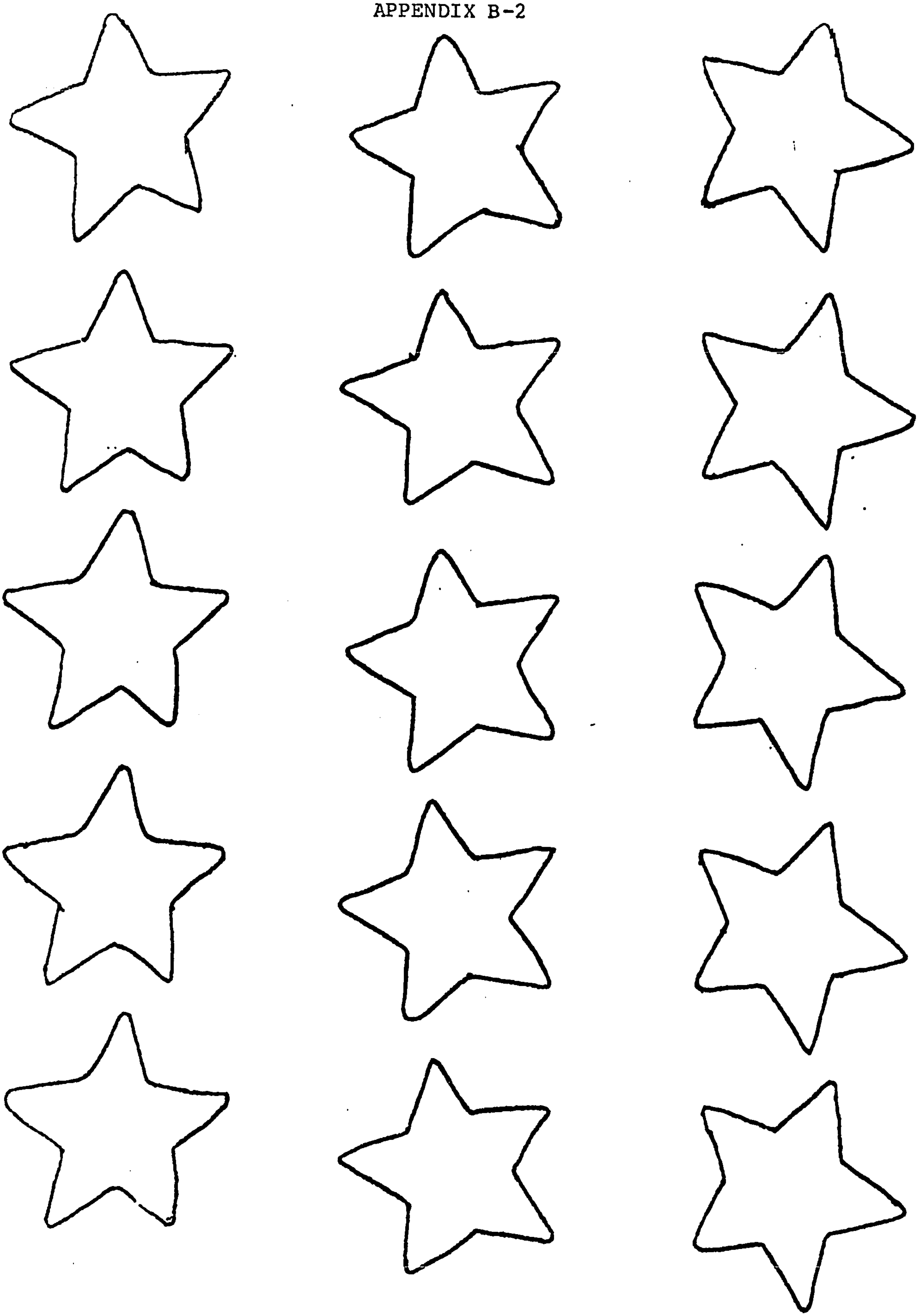

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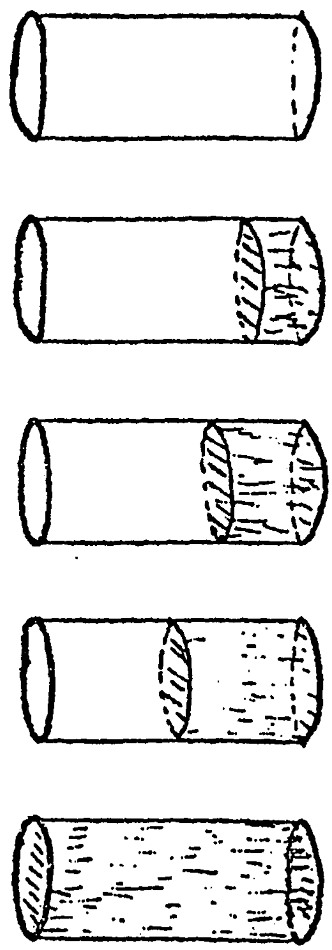

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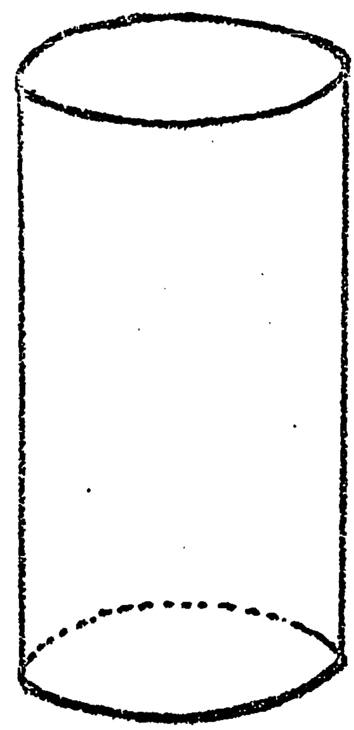

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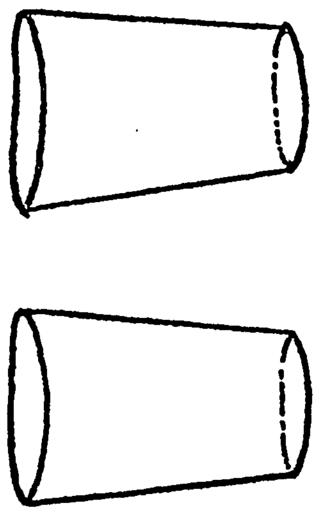

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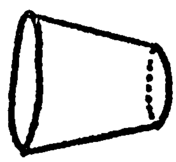

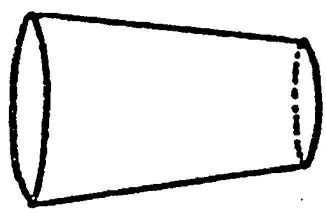

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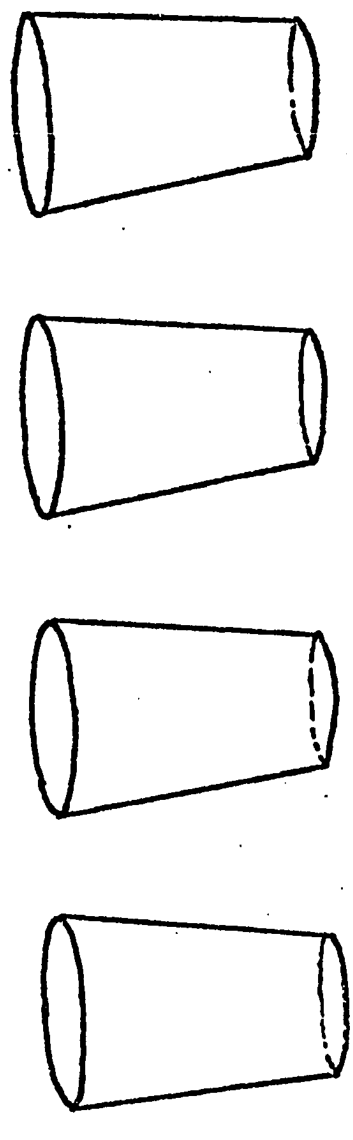

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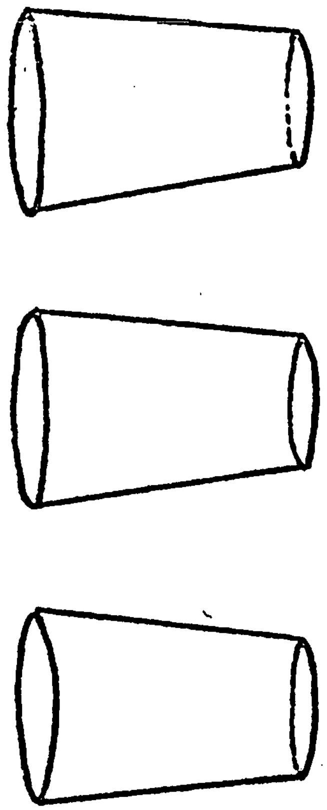

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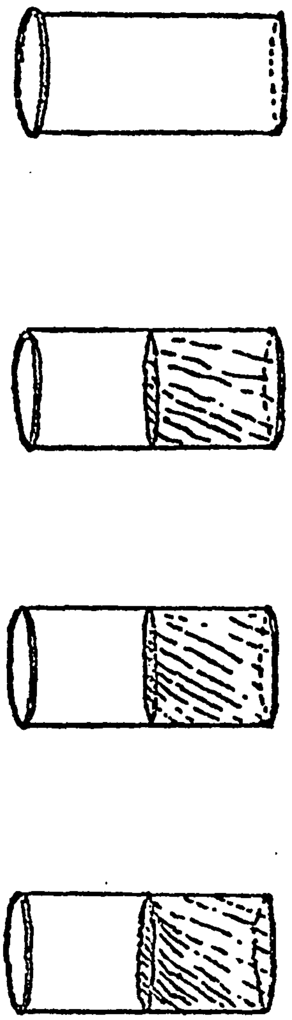

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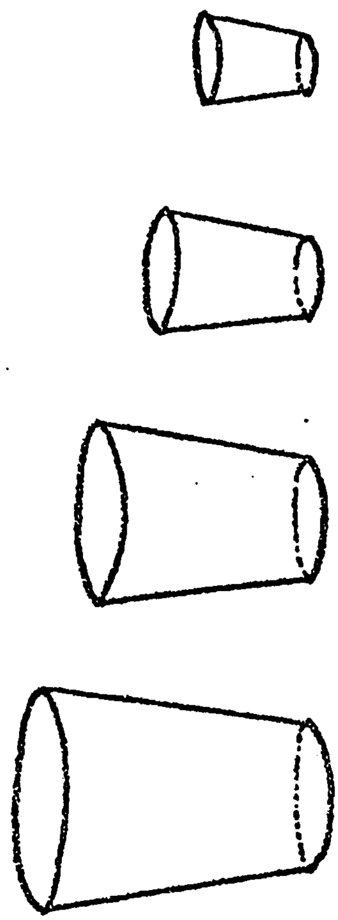

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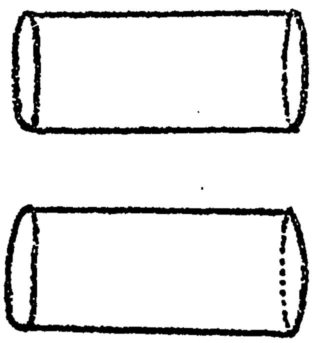

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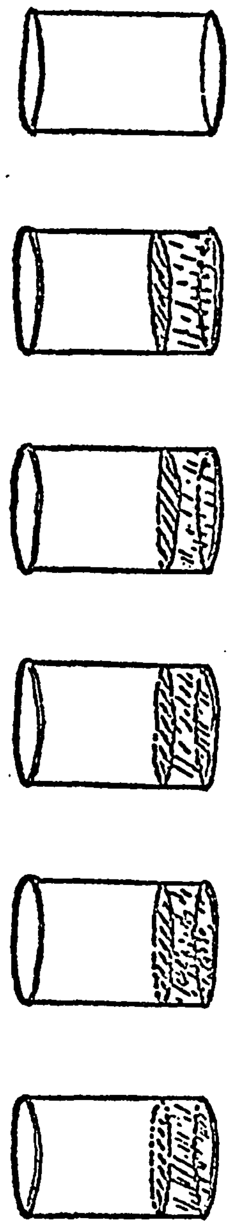

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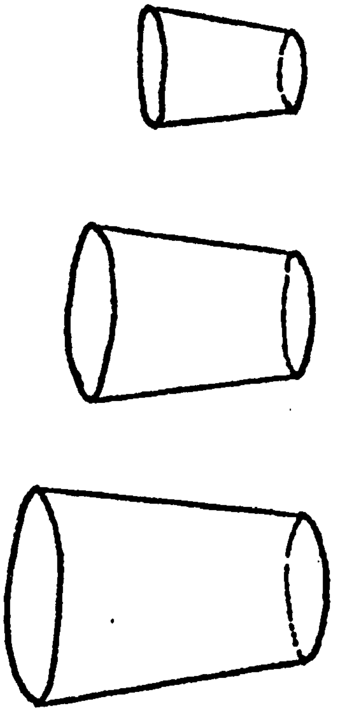

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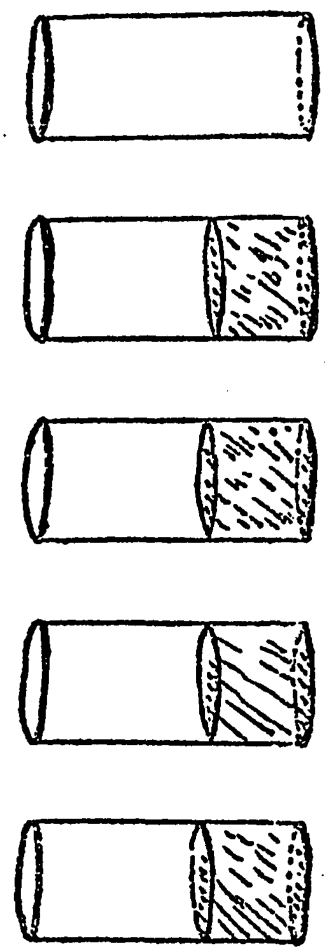


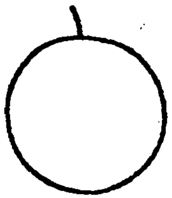
APPENDIX C

RESPONSE RECORD SHEETS

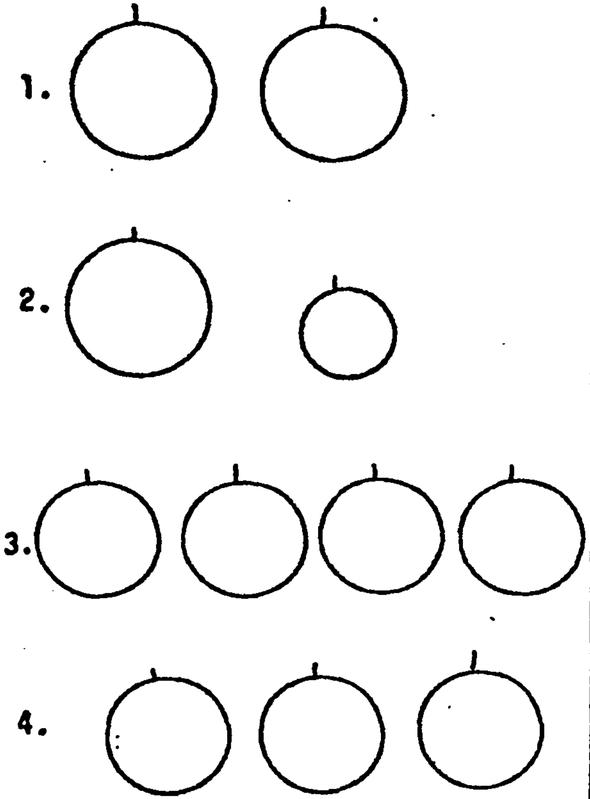
APPLE TASK

INTERPRETATIONS OF ONE-HALF

Half of a Whole



Half of a Set

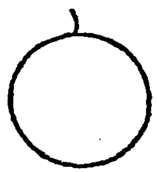


Arithmetic Operation of Division such that Whole Consists of two one-halves

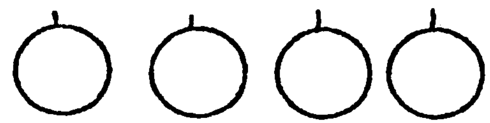
APPLE TASK

INTERPRETATIONS OF ONE-FOURTH

One-fourth of a Whole



One-fourth of a Set

1. 

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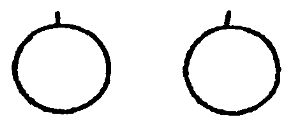
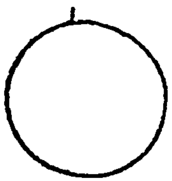
3. 

Arithmetic Operation of Division such that the Whole Consists of Four One-Fourths

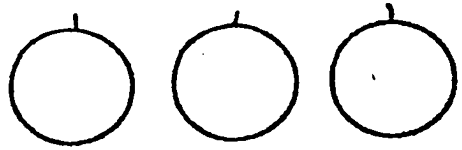
APPLE TASK

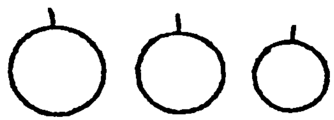
INTERPRETATIONS OF ONE-THIRD

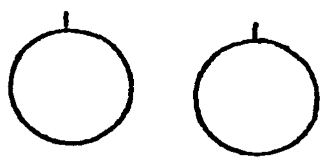
One-third of a Whole



One-third of a set

1. 

2. 

3. 

Arithmetic Operation of Division such that the Whole Consists of Three One-Thirds

APPENDIX D

RATING INSTRUMENT

RATING INSTRUMENT

SUBJECT____________________ SCHOOL____________________

AGE in MONTHS____________________ DATE of INTERVIEW____________________

LENGTH of INTERVIEW____________________ ____________________

SEX____________________ ____________________

DATE of RATING____________________

I. Preliminary task in which child is asked about the terminology to be used in the tasks involving one-half.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

II. Preliminary task in which child is asked about the terminology to be used in the tasks involving one-fourth.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

III. Preliminary task in which child is asked about the terminology to be used in the tasks involving one-third.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

APPLE TASK

INTERPRETATION of ONE-HALF

I. Rating for the interpretation -- half of a whole.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

II. Rating for the interpretation -- half of a set.

	two apples-same size	two apples-different size	four apples-same size	three apples-same size	
1. Clear evidence of understanding					
2. Some evidence of understanding					
3. Uncertain evidence of understanding					
4. Clear evidence of not understanding					
5. Evidence lacking					

III. Rating for the interpretation -- arithmetic operation of division such that the whole consists of two one-halves.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

APPLE TASK

INTERPRETATION of ONE-FOURTH

I. Rating for the interpretation -- one-fourth of a whole.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

II. Rating for the interpretation -- one-fourth of a set.

	four apples-same size	four apples-different size	two apples-same size	
1. Clear evidence of understanding				
2. Some evidence of understanding				
3. Uncertain evidence of understanding				
4. Clear evidence of not understanding				
5. Evidence lacking				

III. Rating for the interpretation -- arithmetic operation of division such that the whole consists of four one-fourths.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

APPLE TASK

INTERPRETATION OF ONE-THIRD

I. Rating for the interpretation -- one-third of a whole.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

II. Rating for the interpretation -- one-third of a set.

	three apples-same size	three apples-different size	two apples-same size	
1. Clear evidence of understanding				
2. Some evidence of understanding				
3. Uncertain evidence of understanding				
4. Clear evidence of not understanding				
5. Evidence lacking				

III. Rating for the interpretation -- arithmetic operation of division such that the whole consists of three one-thirds.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

GLASS TASK

INTERPRETATION of ONE-HALF

I. Rating for the interpretation -- half of a whole.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

II. Rating for the interpretation -- half of a set.

	two glasses-same size	two glasses-different size	four glasses-same size	three glasses-same size	
1. Clear evidence of understanding					
2. Some evidence of understanding					
3. Uncertain evidence of understanding					
4. Clear evidence of not understanding					
5. Evidence lacking					

III. Rating for the interpretation -- arithmetic operation of division such that the whole consists of two one-halves.

______1. Clear evidence of understanding
______2. Some evidence of understanding
______3. Uncertain evidence of understanding
______4. Clear evidence of not understanding
______5. Evidence lacking

GLASS TASK

INTERPRETATION of ONE-FOURTH

I. Rating for the interpretation -- one-fourth of a whole.

_____1. Clear evidence of understanding
_____2. Some evidence of understanding
_____3. Uncertain evidence of understanding
_____4. Clear evidence of not understanding
_____5. Evidence lacking

II. Rating for the interpretation -- one-fourth of a set.

	four glasses-same size	four glasses-different size	two glasses-same size	
1. Clear evidence of understanding				
2. Some evidence of understanding				
3. Uncertain evidence of understanding				
4. Clear evidence of not understanding				
5. Evidence lacking				

III. Rating for the interpretation -- arithmetic operation of division such that the whole consists of four one-fourths.

_____1. Clear evidence of understanding
_____2. Some evidence of understanding
_____3. Uncertain evidence of understanding
_____4. Clear evidence of not understanding
_____5. Evidence lacking

GLASS TASK

INTERPRETATION of ONE-THIRD

I. Rating for the interpretation -- one-third of a whole.

_____1. Clear evidence of understanding
_____2. Some evidence of understanding
_____3. Uncertain evidence of understanding
_____4. Clear evidence of not understanding
_____5. Evidence lacking

II. Rating for the interpretation -- one-third of a set.

	three glasses-same size	three glasses-different size	two glasses--same size	
1. Clear evidence of understanding				
2. Some evidence of understanding				
3. Uncertain evidence of understanding				
4. Clear evidence of not understanding				
5. Evidence lacking				

III. Rating for the interpretation -- arithmetic operation of division such that the whole consists of three one-thirds.

_____1. Clear evidence of understanding
_____2. Some evidence of understanding
_____3. Uncertain evidence of understanding
_____4. Clear evidence of not understanding
_____5. Evidence lacking

APPENDIX E

DESCRIPTION OF PARTICIPATING SCHOOLS

DESCRIPTION OF PARTICIPATING SCHOOLS

Simsboro--"Education has always maintained a high degree of importance in the community of Simsboro. The people have been concerned with providing the best education possible for their children, and have a history of supporting the school and its teachers. Simsboro has the distinction of being the only school district in Lincoln Parish which has never voted against a school tax proposal."[1]

Simsboro, being a combination of elementary and high school, had its first graduating class in 1903. The school district, in 1972, contained 6.7 per cent of the parish population, of which 46.2 per cent was White, 53.7 per cent was Black, and 0.1 per cent was Other.

At this time the description of employment of persons sixteen years of age or older is as follows: 14.4 per cent professionals, 12.8 per cent clerical, 11.1 per cent craftsmen, 21.9 per cent manufacturing and transportation, 7.6 per cent farm and construction, and 32.6 per cent service workers. The representative education level of the school district in 1973 was as follows: no education--4.1 per cent, 1-6 years--20.4 per cent, 9-11 years--22.5 per cent, 12 years--18.4 per cent, 13-15 years--7.1 per cent, 16 years--5.6 per cent, beyond 16 years--2.5 per cent.[2]

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[1]Simsboro High School Staff and Administration, "Southern Association Evaluation," Simsboro, 1973, p. 17. (Mimeographed.)

[2]Ibid., p. 19.

The stated objectives of the Simsboro School are these:

1. To develop the ability to think independently, objectively, creatively, and rationally.

2. To instill effective work habits and self-discipline.

3. To increase proficiency in fundamental skills: reading, writing, speaking, and problem-solving.

4. To instill a pride in our heritage and the American way of life.

5. To promote a sense of personal opportunity and responsibility as a citizen.

6. To emphasize the significance and importance of good family life.

7. To cultivate self-respect and respect for others.

8. To offer opportunities for students to develop leadership, followship, and social competence.

9. To help students explore and test various vocational interests.

10. To develop salable occupational skills.

11. To provide a wide variety of experiences to enable students to find their special interests for the wise use of leisure time.

12. To encourage the acquisition and practice of good health and safety habits.

13. To stimulate an interest in and appreciation of art, music, literature, and nature.

14. To implant a realization of the value of our environment and natural resources and of the need for conservation.

15. To prepare students to adapt to change in social and cultural patterns in our ever-changing world.[3]

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[3] Ibid., pp. 6-7.

Choudrant--Choudrant is also a combination elementary and high school. The stated philosophy of Choudrant School is as follows:

> We, the faculty of Choudrant School, believe that education is for all the children of all the people and that our responsibilities are to guide the pupil in his moral, intellectual, spiritual, social, cultural, and physical growth. It is our belief that all agencies of a community should participate in this educational program; thus the program of the school should be planned to encompass the activities and relationships of the school.
>
> We believe that the school exists for the pupil and that we should provide varied instructional and co-curricular experiences that will help pupils to obtain personal satisfaction, to solve problems, to judge values, to assume responsibilities, and to cooperate with others.
>
> We believe that there are many good methods of education and instruction, that the correct method for any situation will depend upon the school, the community, and the pupils' ability to learn, and that we, as a group, should be alert for new and more effective methods.
>
> We believe that the pupil, through the school program should be guided into a definite realization of his obligation to himself and to others. He should be so trained that when he leaves our school he will be prepared to govern himself intelligently and to assume the responsibilities of life.[4]

In the 1966 "Five-Year Report," the faculty of Choudrant School reported that 50 per cent of the parents of the children in this school graduated from high school and 10 per cent were college graduates. Farming, petroleum industry, trucking, and construction were noted as the major sources of income of the parents. Fifty per cent of the 1965 senior class entered college, 15 per cent entered business

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[4] Choudrant High School Staff and Administration, "Choudrant (Louisiana) High School Five-Year-Report," Choudrant, 1966, p. 4. (Mimeographed.)

schools, and 10 per cent married upon graduation. At this time there was an increase in the number of students entering college.[5]

Hico School--Hico School is the only elementary feeder school of Dubach High School. According to the "Southern Association Report, Dubach High School, 1973,"[6] the educational intentions of the students at the high school were these: 66 per cent planned to attend four-year college, 20 per cent planned to attend other post-secondary school, 7 per cent planned to stop formal education upon graduation, and 7 per cent were undecided. This survey reflects the educational aspirations of the clientel of Hico School. According to this same report, the educational status of the adults in this community was this: 52 per cent were high school drop outs; 22 per cent completed high school; 6 per cent were not accounted for; 6 per cent completed college; and 14 per cent attended college. The occupational status of the parents in 1973 shows that 49 per cent were skilled, 5 per cent were professional, 10 per cent were self-employed, 3 per cent were in military, 4 per cent were engaged in selling, 8 per cent were deceased, and 21 per cent were unemployed.

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[5] Ibid., p. 2.

[6] Dubach High School Staff and Administration, "Southern Association Evaluation," Dubach, 1973, pp. 13-19. (Mimeographed.)

The stated objectives of the Hico Elementary School are as follows:

1. To develop skills, understanding, respect, attitudes, responsibilities, and cooperation that will help one to make a constructive contribution to his community.

2. To develop good health and physical fitness habits that will have carry-over value throughout life.

3. To encourage students to develop ethical and esthetic values.

4. To instill respect for organized authority and to cause the student to understand his role in our system of law and order.

5. To help students learn social responsibilities through participation in work which has social value.

6. To increase an appreciation of our culture and heritage through literature, music, history, and student participation in art works for special projects and bulletin boards.

7. To develop powers of logical thinking through skillfully designed experiences.

8. To work with each child as much as possible in terms of his own abilities and to develop him to the limit of his potentialities.

9. To develop better study habits and work methods that will contribute to self-directed learning.

10. To develop his ability to think creatively and rationally as well as to express his thoughts clearly.

11. To improve the students' communication through reading, writing, listening, and speaking.

12. To provide opportunities for growth through student participation in school activities.[7]

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[7] _Ibid._, p. 17.

Cypress Springs School, Hillcrest School, and Ruston Elementary School all draw their students from the Ruston School District. The clientel of each school is quite similar. In 1972, an Educational Goals Committee prepared a report which effectively describes the aspirations of the parents of the children attending these three schools:

> In order to function as an effective citizen, it is essential that each pupil live through mathematical experiences that will enable him to develop basic skills and concepts which are necessary for adjustment in a changing society. This will encompass the fundamental operations in whole numbers and fractions. Further, emphasis should be placed upon understanding the structure of mathematics, its sequence and order, its laws and principles, and the way in which mathematics as a system expands to meet new needs; and finally, there should be concern for specific skills in computer mathematics.[8]

The specific goals in the area of mathematics suggested by this committee were the following:

A. Basic computational skills which are necessary for functioning as an effective citizen in an American democracy.
B. Adequate mathematical background for those persons entering college to matriculate in desired areas of concentration.
C. Appreciation of contributions of mathematics to our culture.[9]

According to the principal of the Ruston Elementary, 97 per cent of the students attending this school ride buses, 80 per cent live inside the Ruston city limits. This is typical of each of these three schools.

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[8]Education Goals Committee, "Education Goals for Lincoln Parish Schools (Louisiana)," Ruston, 1972, p. 6. (Mimeographed.)

[9]*Ibid*., p. 5.