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REVEALING ADDITIONAL INFORMATION ABOUT ELECTRICITY MARKET UNDERLYING POWER SYSTEM USING POWER SYSTEM PRINCIPLES AND PUBLISHED MARKET RESULTS

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REVEALING ADDITIONAL INFORMATION ABOUT ELECTRICITY MARKET UNDERLYING POWER SYSTEM USING POWER SYSTEM PRINCIPLES AND PUBLISHED MARKET RESULTS

A DISSERTATION APPROVED FOR THE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

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Dedicated to my immediate family

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Abstract

In this dissertation we provide a novel estimation framework designed and structured on duality theory for estimation of some key parameters in electricity markets. Specifically, the framework uses financial transmission right (FTR) auction outcomes such as clearing prices and quantities to estimate implicit parameters of the underlying optimization problem such as power transfer distribution factors (PTDF) without the need to know the auction bid/offers. The application of the proposed framework is not limited to electricity markets and can be used in any optimization problem with similar characteristics. We show successful estimation of parameters by simulating the proposed method on small systems and on a large power system extracted from actual US eastern interconnection network model. We also provide discussion on special estimation cases and proposed methods to address them, and future works to improve the algorithm and applicability of the framework.

Chapter 1: Introduction

Transmission congestion is a recurring event that impacts power system and market operation. Congestion prevents satisfying electrical energy demand in some areas of the system from cheap generation units thus leading to high power prices (locational marginal prices, abbreviated to LMP) at the affected areas.

To hedge against the congestion price risk, financial transmission rights (FTR) are used that entitle the holder to a payoff which is based on the congestion component difference of LMPs between an injection and a withdrawal location. One method to acquire such contracts is to submit bids to buy them in FTR auctions hold by independent system operators (ISO). ISO which has responsibility of reliable and economical operation of power system uses an optimization framework to clear the bids and offers submitted by auction participants. The auction optimization used by ISO tries to ensure obtaining maximum value from submitted bids and offers while satisfying system flow capacity requirements. To make the auction fair to the participants, ISO publishes the physical network model (that is the base model to be modified and used in auction optimization) before holding the auction which can be used by participants to analyze and adjust their to-be-submitted bids/offers. Although ISO uses an updated network model which is set up based on the published base model, the network structural and operational characteristics may be different with the base model such as inclusion of additional planned/forced line outages and line reactance changes. This impacts the values of some implicit structural-related variables such as power transfer distribution factors (PTDF) which have major impact on auction clearance. A PTDF is the sensitivity of active power flow in a reference direction on a line with respect to a power change in injection and withdrawal locations. PTDF changes impact different aspects of FTR applications such as use of FGRs (contracts similar to FTR) for hedging and the FTR auction clearance. Therefore, having a method to track the PTDF variations based on publicly available auction outcome is worthwhile.

A brute-force method to estimate PTDFs is based on extensive stochastic simulations of the FTR auction. In this case, the uncertain inputs such as bids/offer and uncertain system parameters such as line flow limits and reactances are assumed to have distributions and using stochastic draws of the unknown variables the FTR auction optimization is solved to replicate the actual auction result. Extensive and computationally heavy simulation and the need for some priori information are some drawbacks of the brute-force simulation method.

In this dissertation we propose a different estimation method, designed based on the duality theory, for estimation of the underlying parameters in FTR auction and electricity market optimization problems. The objective of the method is to use actual system output to infer the values of the parameters that characterizes the system. The duality in basic definition describes the relationship between primal and dual problems optimal objective values. The application of the proposed estimation method will be shown in PTDF estimation in FTR auctions. This will be of interest to regulators and auditors, market participants, investors, and retail customers as it helps obtain more insights about the market which would lead to a more transparent and efficient market. We would like to mention that the application of our proposed method is not limited to FTR auctions and electricity markets and can be modified to be suitable for any optimization problem with similar characteristics.

The dissertation is organized as follows: Chapter 2 provides a brief overview of electricity markets and the optimization problem solved for clearing bids/offers in the market. We also discuss the locational marginal pricing which has the goal of managing the congestion using market based prices. In Chapter 3, basic attributes and characteristics of financial transmission rights are discussed followed by a formulation of FTR auction optimization problem. Chapter 4 discusses some reasons for estimating PTDFs and the challenges that one may face in doing so. Chapter 5 introduces our proposed estimation method and shows its application in estimating PTDFs in FTR auctions and electricity markets. Simulation results for two small systems and a large power system extracted from actual US eastern interconnection grid are provided in Chapter 6 and Appendices. Chapter 7 provides discussions on some of the special estimation cases and methods used to address them. Simultaneous estimation of multiple parameters is also discussed in this chapter. Finally, Chapter 8 lists some concluding remarks.

Chapter 2: Electricity Market Overview

In this chapter, we provide a brief overview of electricity markets, locational marginal pricing, and a basic cooptimization model which is used in the market for clearing the energy and ancillary services together.

2.1 A brief overview of electricity market

Electricity markets provide a framework for trade of several electricity related commodities. The main product traded in the market is the electrical energy, that is, delivery of a specified amount of MWh energy over a specified time period. Other commodities traded include ancillary services such as Regulation, Spin, Non-Spin and financial contracts such as transmission rights which are means to hedge locational price risk due to congestion in transmission system.

Major market participants include generating companies who use their portfolio of generation units to sell energy such as merchant generators or utilities, transmission companies who own transmission network components, retailers who provide electricity to consumers but do not own generation, and large consumers.

Day ahead (DA) and Real time (RT) are the two general decision time frames for electricity markets. DA market is a forward market typically held the day before RT operating day. These markets are interrelated since DA decisions can impact RT decisions and financial outcomes. For example if a retailer purchased 5 MWh energy in DA for 30$/MWh but used 7 MWh in RT at RT price of 40$/MWh, then will be charged for 80 $ for 2 MWh that used in excess of its DA purchase. Finally, transmission right auction provides another decision time frame interrelated with DA market which is typically held a month or more in advance of the operating DA market.

The DA market is cleared hourly using a process that co-optimizes energy and ancillary services. The process is a multi-hour mixed integer programming algorithm that tries to maximize bid-based revenues minus offer-based costs (social welfare) over the operating day, subject to security, ancillary services requirements, and other constraints [1]. The RT market is typically cleared every 5 minutes using a security constrained economic dispatch process (SCED) which tries to produce a least cost dispatch of online resources while respecting transmission and generation constraints.

ISO is the independent system operator that coordinates transmission right auction, DA, and RT markets together for the sake of reliable and economical operation of power system and market. An ISO in general must ensure the balance between supply and demand for every operating hour, must respond quickly to major disruptions such as line or generation outages, and must ensure economical and efficient operation of the market. There are several ISOs throughout the US as shown in Figure 2-1. Each ISO holds its own markets based on its regulations and typically market implementations are different for each ISO, however the general market concepts are similar.

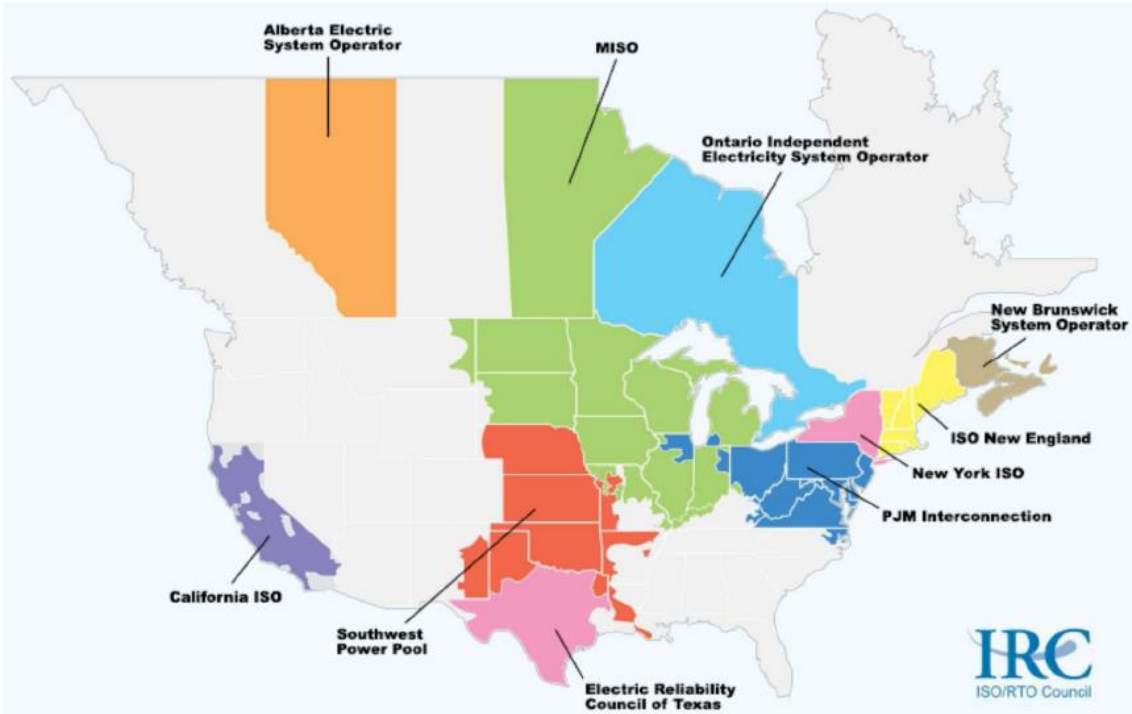

Figure 2-1: North America ISO/RTO map [2]

2.2 Energy Locational Marginal Price (LMP)

Locational marginal pricing is a mechanism for using market based prices for managing transmission congestion [3]. The LMP is the marginal cost of serving the next increment of demand (1 unit) at a specific location taking into account the balance of supply with demand and respecting physical limits of the transmission system. When transmission congestion does not exist, the LMPs are the same throughout the system since an energy demand in any location can be satisfied by increasing the output of cheapest generation units. In this case, the offer price of the unit providing the next increment of demand would be reported as the LMP throughout the system. When congestion exists, it may not be possible to meet the demand in some locations by generation from cheap units and thus the cost of serving the next increment of demand depends on the location of the demand. In this case, LMPs can be different throughout the system although buses with close LMPs can still be found.

Each LMP can be decomposed to three components of energy, congestion, and loss. The energy component is considered as the marginal cost of producing energy at slack bus which is the same for all system buses and changes with different choices of slack bus. The congestion component of LMP is considered as the marginal cost of transmission congestion when transferring energy from slack bus to the desired bus. The loss component is considered as the marginal cost of losses when transferring energy from slack bus to desired bus. Intuitively, LMP at a bus can be considered as the cost of producing energy at slack bus and withdrawing it from the desired bus, while this production of energy changes the system cost by the congestion component due to its effects on transmission congestion, and by the loss component due to its effect on system loss.

2.3 Basic market cooptimization problem

In a very basic form, the RT cooptimization of energy and ancillary services under a DC OPF framework can be summarized as follows:

$$\min_{E_i,R_i} \sum_{i=1}^{K} a_i E_i + b_i R_i \tag{2.1}$$

Subject to $$\sum_{i=1}^{K} E_i - \sum_{i=1}^{N} D_i - loss = 0 \quad (\lambda) \tag{2.2}$$

$$P_{i,min} \le E_i + R_i \le P_{i,max} \quad (\gamma_i, \mu_i) \quad \forall\, i \in \{1,..,K\} \tag{2.3}$$

$$R_{i,min} \le R_i \le R_{i,max} \quad ({\vartheta_1}_i^1, {\vartheta_1}_i^2) \quad \forall\, i \in \{1,..,K\} \tag{2.4}$$

$$\sum_{i=1}^{K} R_i \ge OR \quad (\vartheta_2) \tag{2.5}$$

$$-RR_{down,i} \le E_i - E_{i,t-5} \le RR_{up,i} \quad ({\vartheta_3}_i^1, {\vartheta_3}_i^2) \quad \forall\, i \in \{1,..,K\} \tag{2.6}$$

$$-f_{j,max} \leq f_j \leq f_{j,max} \quad (\theta_j, \beta_j) \quad \forall\, j \in \{1, .., L\} \tag{2.7}$$

where

$$f_j = \sum_{i=1}^{N} GSF_{j,i}(p_i - D_i). \tag{2.8}$$

Note that in this optimization problem:

- The system has N buses, K generators, and L transmission lines.
- In (2.1), a_i and E_i denote marginal offer of generator i and its energy dispatch at time t, and b_i and R_i denote marginal offer for reserve of generator i and its allocated reserve at time t respectively.
- In (2.2), D_i denotes the demand at bus i at time t.
- Equation (2.4) represents the unit reserve capability.
- Equation (2.5) represents the system-wide reserve constraint where OR denotes the system-wide reserve requirement.
- In (2.6), $E_{i,t-5}$ denotes the energy dispatch of unit i at the previous solution of the optimization problem (which here is assumed to be 5 minutes ago). $RR_{down,i}$, and $RR_{up,i}$ are the unit ramping limits.
- In (2.7), f_j represents flow of line j for a fixed direction
- In (2.8), $GSF_{j,i}$ denotes the generation shift factor of bus i on line j.
- Lagrange multipliers associated to every constraint is shown in parenthesis following the constraint.

The Lagrangian function $\mathcal{L}$ can be written as follows:

$$\mathcal{L} = \sum_{i=1}^{K} a_i E_i + b_i R_i + \lambda(\sum_{i=1}^{K} E_i - \sum_{i=1}^{N} D_i - loss)$$

$$+\sum_{j=1}^{L}\beta_j(\sum_{i=1}^{N}GSF_{j,i}(E_i - D_i) - f_{j,max})$$

$$+\sum_{j=1}^{L}\theta_j(-f_{j,max} - \sum_{i=1}^{N}GSF_{j,i}(E_i - D_i)) + Other\ terms\ not\ having\ D_i$$

Finally, the LMP of bus i can be calculated as follows,

$$LMP_i = \frac{\partial\mathcal{L}}{\partial D_i} = -\lambda\left(1 + \frac{\partial loss}{\partial D_i}\right) - \sum_{j=1}^{L}(\beta_j - \theta_j)GSF_{j,i}$$

Chapter 3: Financial Transmission Rights Auction

This chapter is dedicated to introduction of financial transmission rights (FTR) and the auction held by ISO and used by participants to acquire these rights.

3.1 FTR basics

A FTR is a financial contract that entitles the holder to a payoff (positive or negative) which is based on the congestion component difference of LMPs between an injection and a withdrawal location [4]. The FTRs defined between an injection and a withdrawal location are typically called Point-to-Point (PTP) FTRs. Transmission rights can also be found in the form of flowgate rights (FGR) with positive revenue based on the shadow price of the underlying flowgate in market optimization problem. A flowgate is a key transmission network element or bundle of several elements (such as several lines) which is monitored for congestion. From now on until Section 3.4 we would use FTRs to denote both FTRs and FGRs to explain general properties of these transmission rights.

A FTR is specified using several attributes such as

- Account holder/Owner
- Category: PTP or FGR
- Type: Obligation or Option
- Injection and Withdrawal location (or Flowgate name for FGRs)
- FTR term and start/end dates: Monthly, Quarterly, etc.
- Time of Use (TOU): On Peak, Off Peak, 24 Hr, On Peak Weekend, On Peak Weekday
- Bid/Offer price and quantity

FTRs can be used for hedging or investment. When LMPs of two locations are different due to congestion, a market participant holding a transaction between these two locations that adds to the congestion severity has to pay a congestion cost which is the difference between congestion components of LMPs at location of withdrawal and at location of injection. In this case, a PTP FTR can be used to hedge this congestion cost since its payoff is the difference between congestion component of LMPs at withdrawal and injection locations. The FTRs can also be used for investment if the participant trades the FTR for the sake of its positive expected payoff and not necessarily for hedging a transaction risk.

FTRs can be acquired by allocating where the ISO in special cases allocates the FTR to some market entities or transmission operators; auctioning where ISO holds several auctions to let market participants submit bids/offers for FTRs; and trading where participants trade their FTRs bilaterally.

In FTR auctions, participants submit their bids and offers and ISO solves an optimization problem to clear the competitive submissions and to calculate FTR market clearing price (MCP). Note that FTR prices cannot be calculated until the auction optimization is solved. The optimization objective is to award FTRs to participants that value them the most. The optimization constraints are in general FTR quantity requirements and simultaneous feasibility test (SFT) requirements which are used to ensure transmission capacity is not oversold in auction. SFT is used to ensure that the network flows due to introduction of FTRs does not violate operating constraints such as line flow limits. It confirms that the transmission system can support an awarded set of FTRs during normal system conditions [4].

ISO collects congestion charges in the day-ahead market and adds it to a FTR balancing account. This account which is also used to collect some other charges will be used to pay FTR settlement revenues (or collect FTR settlement charges). When the ISO collects equal or more than what it pays as FTR revenue, then it is revenue adequate. The auction optimization problem is designed to ensure revenue adequacy although in practice there are chances of revenue inadequacy. Revenue adequacy is ensured (sufficient condition) if the network model and the constraints that are used in the FTR auction are the same as those of DA market [4]. However, this is not the case in practice as the network model and constraints change as time passes. We will discuss later that power transfer distribution factors which are major components of SFT change between FTR auction and DA market and thus would lead to different set of feasibility constraints.

To further ensure revenue adequacy, the auction optimization is designed so that FTRs that are of option type does not provide counterflow. A counterflow is a flow in the opposite direction of main flow which would reduce the congestion severity and thus could lead to more FTRs being cleared in the main direction of flow. We do not delve into details of this design and refer the reader to [4] for further illustrations. However, we will consider this design element in our models.

3.2 Network model and PTDFs

The network model is a key component of FTR auction and electricity market operations which is prepared in AC form and is converted to a DC model for FTR auction. The DA market network model choice varies from AC to DC-with-losses for various ISOs. However, most of ISOs use a DC network model for their FTR auction.

The AC network model is published to market participants to help them decide their FTR bid/offer submissions however the DC model used in FTR auction is not available and thus is uncertain to participants. The AC network model is generally published for different seasons and loading conditions which are different in active and reactive load pattern, line ratings, and line and generation planned outages. The AC network model includes loads, generators, full transmission line model, voltage transformers and phase shifters, shunt devices, HVDC lines and FACTs and etc. This AC model is converted to a DC network model to be used in FTR auction. To do so, the transmission line resistances are set to zero and all voltage magnitudes are set at 1 p.u. Moreover, all the generation, loads, and shunt and reactive devices are eliminated. Then each FTR will be modeled such as a generator at its injection location and a load at its withdrawal location. This ensures that the sum of FTR injections and withdrawals are balanced in the network.

The converted DC model will be used along with SFT conditions and other constraints to clear the feasible and competitive FTRs. Some of major components of SFT constraints are power transfer distribution factors (PTDFs) which specifies the flow on transmission lines due to FTR injection and withdrawals. A PTDF is specified by a line with reference direction, an injection and withdrawal bus, and a value. The value of PTDF is the sensitivity of active power flow in a reference direction on a line with respect to a power change in injection and withdrawal locations [4]. The PTDFs are calculated from network model and are used in the SFT conditions. For an AC network model, PTDFs change when system operating points are changed such as losses and reactive power flow; system structure is changed such as outages in some lines; line characteristics are changed such as line reactance. For a DC network model, PTDFs only change with changes in system structure and line reactances and they are independent of injections to and withdrawals from network.

3.3 Market Participants interested in FTR for hedging

In this section, using some examples we show how some of the electricity market participants may use FTRs to hedge against locational price risk. Specifically we consider an independent retailer, a merchant generator, and a utility company [5].

An independent retailer does not own generation and thus buys energy at electricity spot price to deliver it to consumers. Suppose the retailer has to supply fixed load of L at location B at a fixed rate of R. The retailer has to buy the required energy from market at location B at spot price of S_B. Thus, its profit is given by

$$P = LR - LS_B. \tag{3.1}$$

The retailer is exposed to spot price risk and its profit will reduce if S_B increases. Suppose the retailer decides to enters an obligation contract to hedge the price risk however the contract is available for spot price at a different location S_A. The profit function considering the obligation contract is given by

$$P = LR - LS_B + Q(\,S_A - K) \tag{3.2}$$

where Q is the hedge quantity and K is the strike price of the contract. In this case if the spot price at location A increases above the K then the hedge contract payoff is positive. Finally, the retailer may enter into a FTR contract with the same hedge quantity between locations A and B to hedge against the spot and locational price risk. In this case the profit is (ignoring cost of acquiring FTR)

$$P = LR - LS_B + Q(\, S_A - K) + Q(S_B - S_A). \tag{3.3}$$

To hedge against the spot and locational price risk, the hedge quantity should be chosen so that

$$-LS_B + QS_A + Q(S_B - S_A) = 0$$

or

$$Q = L.$$

Then, the profit would be given by (3.4) which has no uncertainty assuming L is known and fixed. Note that the example introduced here is only for illustration and is very simplified compared to practical cases.

$$P = LR - LK \tag{3.4}$$

A merchant generator owns generation and sells the electricity at spot price. Consider a merchant generator that decides to sell L amount of energy at the spot price S_B however enters into an obligation contract at a different location A to hedge its price risk. Also suppose the merchant generator enters into a FTR to hedge against locational price risk. Then the profit (ignoring generation cost) is given by

$$P = LS_B - Q(\, S_A - K) + Q(S_A - S_B)$$

To hedge against the spot and locational price risk, the hedge quantity should be chosen so that

$$LS_B - Q\, S_A + Q(S_A - S_B) = 0$$

or

$$Q = L.$$

Then, the profit would be given by

$P = LK.$

Consider a utility company (UC) that has its generation units at location A and its consumers at location B. Suppose it decides to sell L MWh at location A and purchase L MWh at B to deliver to consumers. Then UC is exposed to spot and locational price risk of A and B. To hedge the risks, the UC enters into a FTR with quantity $Q = L$. Then the profit is given by

$$P = \underbrace{L(S_A - S_B)}_{Spot\ sale\ and\ purchase} + \underbrace{L(S_B - S_A)}_{FTR\ payoff} = 0$$

which is certain. Note that the UC collects revenue from consumers by selling them electricity at a fixed rate.

3.4 Auction formulation

In this section we provide a general formulation of FTR auction optimization problem. This formulation is meant to cover major characteristics of the FTR auction and is deduced and modified based on references [6]-[21].

3.4.1 Basic Nomenclature

b/s	Participant => b for buyer and s for seller
t	Time of use (TOU) => Peak or Off-peak or 24hr
w	FTR type => Obligation (Obl), Option (Opt), and FGR
km	Injection bus k and withdrawal bus m
ij	Index of line $i - j$
rs	Index of contingency line $r - s$
r	Auction round
m_r	Fraction of capacity auctioned in round r

β	Bid/Offer price
BL	Bid/Offer Block
F_{ij}^{max}	Flow limit of transmission line between bus i and j
$PTDF_{ij,km}$	PTDF of line $i-j$ w.r.t. injection at k and withdrawal from m
$LODF_{ij,rs}$	LODF of line $i-j$ w.r.t. outage of line $r-s$
$Q_{w,t,km,BL}^{b/s,max}$	Maximum quantity of offer/bid for right w, TOU t, buses km, and block BL
$Q_{w,t,km,BL}^{b/s,min}$	Minimum quantity of offer/bid for right w, TOU t, buses km, and block BL
$MCP_{km,t-(24hr),w}^{r}$	Market clearing price of the transmission right for round r, right w, buses km, and Peak or Off-peak TOU
$MCP_{km,t=24hr,w}^{r}$	Market clearing price of the transmission right for round r, right w, buses km, and 24hr TOU

3.4.2 Auction formulation

The auction optimization problem for round r can be summarized as follows:

$$\max_{Q_{w,t,km,BL}^{b/s}} \left\{ \sum_{b/s,w,t,km,BL} \left[\beta_{w,t,km,BL}^{b} \cdot Q_{w,t,km,BL}^{b} - \beta_{w,t,km,BL}^{s} \cdot Q_{w,t,km,BL}^{s} \right] \right\} \quad (3.5)$$

subject to

$$\sum_{b/s,t,km,BL} \{ PTDF_{ij,km} \cdot [(Q_{t,km,BL,w=Obl}^{b} - Q_{t,km,BL,w=Obl}^{s})$$

$$+\sum_{\rho=1}^{r-1}\left(Q_{t,km,BL,w=Obl}^{b,\rho}-Q_{t,km,BL,w=Obl}^{s,\rho}\right)]$$

$$+max\left(0,PTDF_{ij,km}\right)\cdot[\left(Q_{t,km,BL,w=Opt}^{b}-Q_{t,km,BL,w=Opt}^{s}\right)$$

$$+\sum_{\rho=1}^{r-1}\left(Q_{t,km,BL,w=Opt}^{b,\rho}-Q_{t,km,BL,w=Opt}^{s,\rho}\right)]$$

$$+[\left(Q_{t,ij,BL,w=FGR}^{b}-Q_{t,ij,BL,w=FGR}^{s}\right)$$

$$+\sum_{\rho=1}^{r-1}\left(Q_{t,ij,BL,w=FGR}^{b,\rho}-Q_{t,ij,BL,w=FGR}^{s,\rho}\right)]\}$$

$$\leq m_r F_{ij}^{max} \quad \forall\, ij \qquad \left(\mu_{ij}^{+}\right) \tag{3.6}$$

where $\Sigma_{\rho=1}^{r-1}(\,)$ denotes the net of FTRs cleared in the previous rounds and $Q_{t,ij,BL,w=FGR}$ denotes the bid/offer quantity for flow on flowgate from $i-j$.

$$\sum_{b/s,t,km,BL}\{-PTDF_{ij,km}\cdot[\left(Q_{t,km,BL,w=Obl}^{b}-Q_{t,km,BL,w=Obl}^{s}\right)$$

$$+\sum_{\rho=1}^{r-1}\left(Q_{t,km,BL,w=Obl}^{b,\rho}-Q_{t,km,BL,w=Obl}^{s,\rho}\right)]$$

$$+max\left(0,-PTDF_{ij,km}\right)\cdot[\left(Q_{t,km,BL,w=Opt}^{b}-Q_{t,km,BL,w=Opt}^{s}\right)$$

$$+\sum_{\rho=1}^{r-1}\left(Q_{t,km,BL,w=Opt}^{b,\rho}-Q_{t,km,BL,w=Opt}^{s,\rho}\right)]$$

$$+[\left(Q_{t,ji,BL,w=FGR}^{b}-Q_{t,ji,BL,w=FGR}^{s}\right)$$

$$+\sum_{\rho=1}^{r-1}\left(Q_{t,ji,BL,w=FGR}^{b,\rho}-Q_{t,ji,BL,w=FGR}^{s,\rho}\right)]\}$$

$$\leq m_r F_{ji}^{max} \quad \forall\, ij \qquad \left(\mu_{ij}^{-}\right) \tag{3.7}$$

The $n-1$ contingency constraints are as follows:

$$\sum_{b/s,t,km,BL} \{[PTDF_{ij,km} + LODF_{ij,rs}PTDF_{rs,km}]$$

$$\cdot [\left(Q^{b}_{t,km,BL,w=Obl} - Q^{s}_{t,km,BL,w=Obl}\right)$$

$$+ \sum\nolimits_{\rho=1}^{r-1} \left(Q^{b,\rho}_{t,km,BL,w=Obl} - Q^{s,\rho}_{t,km,BL,w=Obl}\right)]$$

$$+ max\left(0, PTDF_{ij,km} + LODF_{ij,rs}PTDF_{rs,km}\right)$$

$$\cdot [\left(Q^{b}_{t,km,BL,w=Opt} - Q^{s}_{t,km,BL,w=Opt}\right)$$

$$+ \sum\nolimits_{\rho=1}^{r-1} \left(Q^{b,\rho}_{t,km,BL,w=Opt} - Q^{s,\rho}_{t,km,BL,w=Opt}\right)]$$

$$+ [\left(Q^{b}_{t,ij,BL,w=FGR} - Q^{s}_{t,ij,BL,w=FGR}\right)$$

$$+ \sum\nolimits_{\rho=1}^{r-1} \left(Q^{b,\rho}_{t,ij,BL,w=FGR} - Q^{s,\rho}_{t,ij,BL,w=FGR}\right)]\}$$

$$\leq m_r F^{max}_{ij} \ \ \forall\ ij \ and \ rs \qquad \left(\nu^{+}_{ij,rs}\right) \tag{3.8}$$

$$\sum_{b/s,t,km,BL} \{-[PTDF_{ij,km} + LODF_{ij,rs}PTDF_{rs,km}]$$

$$\cdot [\left(Q^{b}_{t,km,BL,w=Obl} - Q^{s}_{t,km,BL,w=Obl}\right)$$

$$+ \sum\nolimits_{\rho=1}^{r-1} \left(Q^{b,\rho}_{t,km,BL,w=Obl} - Q^{s,\rho}_{t,km,BL,w=Obl}\right)]$$

$$+ max\left(0, -[PTDF_{ij,km} + LODF_{ij,rs}PTDF_{rs,km}]\right)$$

$$\cdot [\left(Q^{b}_{t,km,BL,w=Opt} - Q^{s}_{t,km,BL,w=Opt}\right)$$

$$+ \sum\nolimits_{\rho=1}^{r-1} \left(Q^{b,\rho}_{t,km,BL,w=Opt} - Q^{s,\rho}_{t,km,BL,w=Opt}\right)]$$

$$+[(Q^{b}_{t,ji,BL,w=FGR} - Q^{s}_{t,ji,BL,w=FGR})$$

$$+\sum_{\rho=1}^{r-1}(Q^{b,\rho}_{t,ji,BL,w=FGR} - Q^{s,\rho}_{t,ji,BL,w=FGR})]\}$$

$$\leq m_r F_{ji}^{max} \;\; \forall\, ij \text{ and } rs \qquad (\nu^{-}_{ij,rs}) \tag{3.9}$$

Moreover, the bid/offer quantities should be within appropriate or requested limits.

$$Q^{b/s}_{w,t,km,BL} \leq Q^{b/s,max}_{w,t,km,BL} \;\; \forall\, km \text{ and } BL \qquad (\pi^{+}_{w,t,km,BL}) \tag{3.10}$$

$$-Q^{b/s}_{w,t,km,BL} \leq -Q^{b/s,min}_{w,t,km,BL} \; \forall\, km \text{ and } BL \;\; (\pi^{-}_{w,t,km,BL}) \tag{3.11}$$

3.4.3 Notes

1. Each auction round r is solved separately.

2. The bid prices for obligation FTRs can be negative.

3. Transmission rights awarded in previous rounds $\rho = 1, \dots, r-1$ are already known and thus are modeled as fixed injections or flows. This has been addressed in the above formulation by terms in $\Sigma_{\rho=1}^{r-1}$ argument.

4. The optimization problem is solved separately for Peak and Off-Peak periods.

5. A 24hr bid/offer is modeled as a combination of a Peak and an Off-Peak bid/offer.

6. Because option holders do not have the obligation to relieve the congestion if the flow direction is not in their benefit, options are modeled such that they do not provide counterflows.

7. Note that the FGRs modeled here are naturally an option since they have a positive payment if the shadow price is positive in their direction and zero payment if shadow price is positive in negative direction.

8. The optimization is first solved without contingency constraints (3.8) and (3.9). Then a contingency analysis is performed to determine the important set of contingent elements and lines or flowgates. Finally, the optimization is resolved considering the additional contingency constraints (3.8) and (3.9) for identified contingencies.

3.5 MCP

The market clearing price (MCP) of an obligation right for 1 MW injection at k and withdrawal from m is equal to the congestion cost of a transaction between these locations and can be calculated as [6], [8], [9]:

$$MCP_{km} = \sum_{ij} [(\mu_{ij}^{+} - \mu_{ij}^{-}) \cdot PTDF_{ij,km}].$$

For the auction in (3.5)-(3.11), the MCP of FTRs can be summarized as follows:

$$MCP_{km,t-(24hr),w=Obl} = \sum_{ij} [(\mu_{ij}^{+} - \mu_{ij}^{-}) \cdot PTDF_{ij,km}]$$

$$+ \sum_{ij,rs} [(\nu_{ij,rs}^{+} - \nu_{ij,rs}^{-}) \cdot (PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km})] \quad (3.12)$$

$$MCP_{km,t-(24hr),w=Opt} = \sum_{ij} [\mu_{ij}^{+}\, max(0, PTDF_{ij,km})]$$

$$+ \sum_{ij} [\mu_{ij}^{-}\, max(0, -PTDF_{ij,km})]$$

$$+ \sum_{ij,rs} [\nu_{ij,rs}^{+}\, max(0, PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km})]$$

$$+ \sum_{ij,rs} [\nu_{ij,rs}^{-}\, max(0, -[PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}])] \quad (3.13)$$

$$MCP_{km,t-(24hr),w=FGR} = \begin{cases} \mu_{km}^{+} + \sum_{rs} \nu_{km,rs}^{+} & if \quad km = ij \\ \mu_{km}^{-} + \sum_{rs} \nu_{km,rs}^{-} & if \quad km = ji \end{cases} \tag{3.14}$$

The MCP for 24hr transmission right is weighted average of the Peak and Off-peak rights:

$$MCP_{km,t=24hr,w} = \frac{\left(MCP_{km,t=Peak,w} \cdot T_{Peak} + MCP_{km,t=Off-Peak,w} \cdot T_{Off-Peak}\right)}{T_{Peak} + T_{Off-Peak}} \tag{3.15}$$

where T_{Peak} and $T_{Off-Peak}$ denote the number of Peak and Off-Peak hours in FTR term, respectively.

3.6 Example 3.1

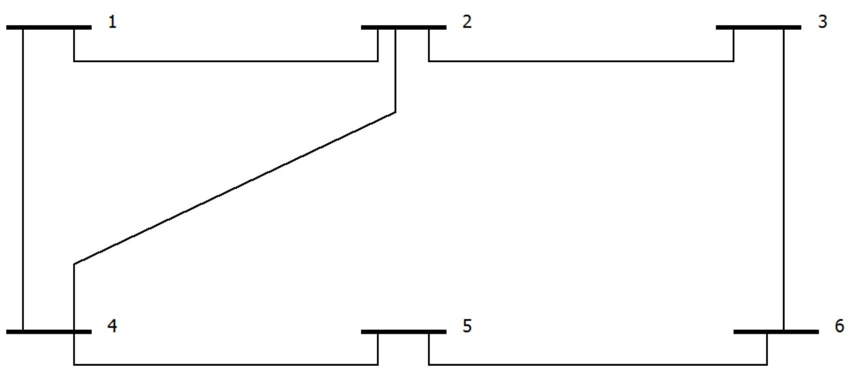

Figure 3-1: 6-bus system

Consider the network model shown in figure 3-1. The network data, FTR bids/offers, and PTDF data are given in the following tables.

Table 3.1: Transmission Line Data				
#	From bus Number	To bus Number	X (PU)	Limit(MW)
1	1	2	0.02	$F_{12}^{max} = F_{21}^{max} = 100$
2	1	4	0.02	$F_{14}^{max} = F_{41}^{max} = 100$

3	2	4	0.02	$F_{24}^{max} = F_{42}^{max} = 200$
4	3	2	0.02	$F_{32}^{max} = F_{23}^{max} = 100$
5	4	5	0.02	$F_{45}^{max} = F_{54}^{max} = 200$
6	6	3	0.02	$F_{63}^{max} = F_{36}^{max} = 100$
7	6	5	0.02	$F_{65}^{max} = F_{56}^{max} = 100$

Table 3.2: FTR Data						
#	Type	Bid/Offer	Injection bus	Withdrawal bus	Max Quantity (MW)	Price ($/MW)
1	Obl	Bid	1	5	$Q_{15}^{b,max} = 150$	$\beta_{15}^{b} = 15$
2	Obl	Bid	6	2	$Q_{62}^{b,max} = 150$	$\beta_{62}^{b} = 12$
3	Opt	Bid	4	5	$Q_{45,Opt}^{b,max} = 100$	$\beta_{45,Opt}^{b} = 6$
4	Opt	Bid	2	4	$Q_{24,Opt}^{b,max} = 200$	$\beta_{24,Opt}^{b} = 16$
5	FGR	Bid	2	4	$Q_{24,FGR}^{b,max} = 200$	$\beta_{24,FGR}^{b} = 20$
6	Opt	Offer	6	5	$Q_{65,Opt}^{s,max} = 50$	$\beta_{65,Opt}^{s} = 8$

Table 3.3: PTDF Data					
$PTDF_{12,15}$	0.4286	$PTDF_{14,15}$	0.5714	$PTDF_{24,15}$	0.1429
$PTDF_{32,15}$	-0.2857	$PTDF_{45,15}$	0.7143	$PTDF_{63,15}$	-0.2857
$PTDF_{65,15}$	0.2857	$PTDF_{12,62}$	0.1429	$PTDF_{14,62}$	-0.1429
$PTDF_{24,62}$	-0.2857	$PTDF_{32,62}$	0.5714	$PTDF_{45,62}$	-0.4286
$PTDF_{63,62}$	0.5714	$PTDF_{65,62}$	0.4286	$PTDF_{12,45}$	0.0714
$PTDF_{14,45}$	-0.0714	$PTDF_{24,45}$	-0.1429	$PTDF_{32,45}$	-0.2143
$PTDF_{45,45}$	0.7857	$PTDF_{63,45}$	-0.2143	$PTDF_{65,45}$	0.2143
$PTDF_{12,24}$	-0.2857	$PTDF_{14,24}$	0.2857	$PTDF_{24,24}$	0.5714
$PTDF_{32,24}$	-0.1429	$PTDF_{45,24}$	-0.1429	$PTDF_{63,24}$	-0.1429
$PTDF_{65,24}$	0.1429	$PTDF_{12,65}$	-0.0714	$PTDF_{14,65}$	0.0714
$PTDF_{24,65}$	0.1429	$PTDF_{32,65}$	0.2143	$PTDF_{45,65}$	0.2143
$PTDF_{63,65}$	0.2143	$PTDF_{65,65}$	0.7857		

The auction optimization, considering a single round, single TOU, single bid/offer blocks, and no contingency constraints, can be formulated as follows:

$$max\,\{\beta_{15}^{b}Q_{15}^{b}+\beta_{62}^{b}Q_{62}^{b}+\beta_{45,Opt}^{b}Q_{45,Opt}^{b}+\beta_{24,Opt}^{b}Q_{24,Opt}^{b}+\beta_{24,FGR}^{b}Q_{24,FGR}^{b}-\beta_{65,Opt}^{s}Q_{65,Opt}^{s}\} \quad (3.16)$$

subject to

$$PTDF_{12,15}Q_{15}^{b}+PTDF_{12,62}Q_{62}^{b}+max(0,PTDF_{12,45})Q_{45,Opt}^{b}+$$

$$max(0,PTDF_{12,24})Q_{24,Opt}^{b}-max(0,PTDF_{12,65})Q_{65,Opt}^{s}\le F_{12}^{max} \quad (\mu_{12}^{+})$$

$$PTDF_{14,15}Q_{15}^{b}+PTDF_{14,62}Q_{62}^{b}+max(0,PTDF_{14,45})Q_{45,Opt}^{b}+$$

$$max(0,PTDF_{14,24})Q_{24,Opt}^{b}-max(0,PTDF_{14,65})Q_{65,Opt}^{s}\le F_{14}^{max} \quad (\mu_{14}^{+})$$

$$PTDF_{24,15}Q_{15}^{b}+PTDF_{24,62}Q_{62}^{b}+max\big(0,PTDF_{24,45}\big)\,Q_{45,Opt}^{b}+$$

$$max\big(0,PTDF_{24,24}\big)\,Q_{24,Opt}^{b}+Q_{24,FGR}^{b}-max\big(0,PTDF_{24,65}\big)\,Q_{65,Opt}^{s}\le F_{24}^{max}$$

$$(\mu_{24}^{+})$$

$$PTDF_{32,15}Q_{15}^{b}+PTDF_{32,62}Q_{62}^{b}+max(0,PTDF_{32,45})Q_{45,Opt}^{b}+$$

$$max(0,PTDF_{32,24})Q_{24,Opt}^{b}-max(0,PTDF_{32,65})Q_{65,Opt}^{s}\le F_{32}^{max} \quad (\mu_{32}^{+})$$

$$PTDF_{45,15}Q_{15}^{b}+PTDF_{45,62}Q_{62}^{b}+max(0,PTDF_{45,45})Q_{45,Opt}^{b}+$$

$$max(0,PTDF_{45,24})Q_{24,Opt}^{b}-max(0,PTDF_{45,65})Q_{65,Opt}^{s}\le F_{45}^{max} \quad (\mu_{45}^{+})$$

$$PTDF_{63,15}Q_{15}^{b}+PTDF_{63,62}Q_{62}^{b}+max(0,PTDF_{63,45})Q_{45,Opt}^{b}+$$

$$max(0,PTDF_{63,24})Q_{24,Opt}^{b}-max(0,PTDF_{63,65})Q_{65,Opt}^{s}\le F_{63}^{max} \quad (\mu_{63}^{+})$$

$$PTDF_{65,15}Q_{15}^{b}+PTDF_{65,62}Q_{62}^{b}+max(0,PTDF_{65,45})Q_{45,Opt}^{b}+$$

$$max(0,PTDF_{65,24})Q_{24,Opt}^{b}-max(0,PTDF_{65,65})Q_{65,Opt}^{s}\le F_{65}^{max} \quad (\mu_{65}^{+})$$

Line flow constraints on opposite direction:

$$-PTDF_{12,15}Q_{15}^{b}-PTDF_{12,62}Q_{62}^{b}+max(0,-PTDF_{12,45})Q_{45,Opt}^{b}+$$

$$max(0,-PTDF_{12,24})Q_{24,Opt}^{b}-max(0,-PTDF_{12,65})Q_{65,Opt}^{s}\le F_{21}^{max} \quad (\mu_{12}^{-})$$

$$-PTDF_{14,15}Q_{15}^{b} - PTDF_{14,62}Q_{62}^{b} + max(0, -PTDF_{14,45})Q_{45,Opt}^{b} +$$

$$max(0, -PTDF_{14,24})Q_{24,Opt}^{b} - max(0, -PTDF_{14,65})Q_{65,Opt}^{s} \leq F_{41}^{max} \quad (\mu_{14}^{-})$$

$$-PTDF_{24,15}Q_{15}^{b} - PTDF_{24,62}Q_{62}^{b} + max\left(0, -PTDF_{24,45}\right) Q_{45,Opt}^{b} +$$

$$max\left(0, -PTDF_{24,24}\right) Q_{24,Opt}^{b} - max\left(0, -PTDF_{24,65}\right) Q_{65,Opt}^{s} \leq F_{42}^{max} \quad (\mu_{24}^{-})$$

$$-PTDF_{32,15}Q_{15}^{b} - PTDF_{32,62}Q_{62}^{b} + max(0, -PTDF_{32,45})Q_{45,Opt}^{b} +$$

$$max(0, -PTDF_{32,24})Q_{24,Opt}^{b} - max(0, -PTDF_{32,65})Q_{65,Opt}^{s} \leq F_{23}^{max} \quad (\mu_{32}^{-})$$

$$-PTDF_{45,15}Q_{15}^{b} - PTDF_{45,62}Q_{62}^{b} + max(0, -PTDF_{45,45})Q_{45,Opt}^{b} +$$

$$max(0, -PTDF_{45,24})Q_{24,Opt}^{b} - max(0, -PTDF_{45,65})Q_{65,Opt}^{s} \leq F_{54}^{max} \quad (\mu_{45}^{-})$$

$$-PTDF_{63,15}Q_{15}^{b} - PTDF_{63,62}Q_{62}^{b} + max(0, -PTDF_{63,45})Q_{45,Opt}^{b} +$$

$$max(0, -PTDF_{63,24})Q_{24,Opt}^{b} - max(0, -PTDF_{63,65})Q_{65,Opt}^{s} \leq F_{36}^{max} \quad (\mu_{63}^{-})$$

$$-PTDF_{65,15}Q_{15}^{b} - PTDF_{65,62}Q_{62}^{b} + max(0, -PTDF_{65,45})Q_{45,Opt}^{b} +$$

$$max(0, -PTDF_{65,24})Q_{24,Opt}^{b} - max(0, -PTDF_{65,65})Q_{65,Opt}^{s} \leq F_{56}^{max} \quad (\mu_{65}^{-})$$

The bid/offer quantity constraints:

$$Q_{15}^{b} \leq Q_{15}^{b,max} \quad (\pi_{15}^{+})$$

$$Q_{62}^{b} \leq Q_{62}^{b,max} \quad (\pi_{62}^{+})$$

$$Q_{45,Opt}^{b} \leq Q_{45,Opt}^{b,max} \quad \left(\pi_{45,Opt}^{+}\right)$$

$$Q_{24,Opt}^{b} \leq Q_{24,Opt}^{b,max} \quad \left(\pi_{24,Opt}^{+}\right)$$

$$Q_{24,FGR}^{b} \leq Q_{24,FGR}^{b,max} \quad \left(\pi_{24,FGR}^{+}\right)$$

$$Q_{65,Opt}^{s} \leq Q_{65,Opt}^{s,max} \quad \left(\pi_{65,Opt}^{+}\right)$$

$$-Q_{15}^{b} \leq 0 \quad (\pi_{15}^{-})$$

$$-Q_{62}^{b} \leq 0 \quad (\pi_{62}^{-})$$

$-Q^b_{45,Opt} \leq 0 \quad (\pi^-_{45,Opt})$

$-Q^b_{24,Opt} \leq 0 \quad (\pi^-_{24,Opt})$

$-Q^b_{24,FGR} \leq 0 \quad (\pi^-_{24,FGR})$

$-Q^s_{65,Opt} \leq 0 \quad (\pi^-_{65,Opt})$

The solution of auction optimization problem is as follows:

Table 3.4: Auction optimization solution – Part 1

#	FTR cleared quantity	Shadow prices	
1	$Q^b_{15} = 150$	$\pi^+_{15} = 3$	$\pi^-_{15} = 0$
2	$Q^b_{62} = 150$	$\pi^+_{62} = 6$	$\pi^-_{62} = 0$
3	$Q^b_{45,Opt} = 58.3$	$\pi^+_{45,Opt} = 0$	$\pi^-_{45,Opt} = 0$
4	$Q^b_{24,Opt} = 137.5$	$\pi^+_{24,Opt} = 0$	$\pi^-_{24,Opt} = 0$
5	$Q^b_{24,FGR} = 150$	$\pi^+_{24,FGR} = 0$	$\pi^-_{24,FGR} = 0$
6	$Q^s_{65,Opt} = 50$	$\pi^+_{65,Opt} = 17$	$\pi^-_{65,Opt} = 0$

Table 3.5: Auction optimization solution – Part 2

#	Line flow constraint shadow prices	
1	$\mu^+_{12} = 0$	$\mu^-_{12} = 0$
2	$\mu^+_{14} = 2$	$\mu^-_{14} = 0$
3	$\mu^+_{24} = 20$	$\mu^-_{24} = 0$
4	$\mu^+_{32} = 0$	$\mu^-_{32} = 0$
5	$\mu^+_{45} = 0$	$\mu^-_{45} = 0$
6	$\mu^+_{63} = 0$	$\mu^-_{63} = 0$
7	$\mu^+_{65} = 28$	$\mu^-_{65} = 0$

Finally, the MCPs can be calculated using the following formulas and auction results:

$$MCP_{15} = (\mu^+_{12} - \mu^-_{12})PTDF_{12,15} + (\mu^+_{14} - \mu^-_{14})PTDF_{14,15}$$

$$+ (\mu^+_{24} - \mu^-_{24})PTDF_{24,15} + (\mu^+_{32} - \mu^-_{32})PTDF_{32,15} + (\mu^+_{45} - \mu^-_{45})PTDF_{45,15}$$

$$+ (\mu^+_{63} - \mu^-_{63})PTDF_{63,15} + (\mu^+_{65} - \mu^-_{65})PTDF_{65,15}$$

$MCP_{62} = (\mu_{12}^{+} - \mu_{12}^{-})PTDF_{12,62} + (\mu_{14}^{+} - \mu_{14}^{-})PTDF_{14,62}$

$+ (\mu_{24}^{+} - \mu_{24}^{-})PTDF_{24,62} + (\mu_{32}^{+} - \mu_{32}^{-})PTDF_{32,62} + (\mu_{45}^{+} - \mu_{45}^{-})PTDF_{45,62}$

$+ (\mu_{63}^{+} - \mu_{63}^{-})PTDF_{63,62} + (\mu_{65}^{+} - \mu_{65}^{-})PTDF_{65,62}$

$MCP_{45,Opt} = \mu_{12}^{+} max(0, PTDF_{12,45}) + \mu_{12}^{-} max(0, -PTDF_{12,45})$

$+\mu_{14}^{+} max(0, PTDF_{14,45}) + \mu_{14}^{-} max(0, -PTDF_{14,45})$

$+\mu_{24}^{+} max(0, PTDF_{24,45}) + \mu_{24}^{-} max(0, -PTDF_{24,45})$

$+\mu_{32}^{+} max(0, PTDF_{32,45}) + \mu_{32}^{-} max(0, -PTDF_{32,45})$

$+\mu_{45}^{+} max(0, PTDF_{45,45}) + \mu_{45}^{-} max(0, -PTDF_{45,45})$

$+\mu_{63}^{+} max(0, PTDF_{63,45}) + \mu_{63}^{-} max(0, -PTDF_{63,45})$

$+\mu_{65}^{+} max(0, PTDF_{65,45}) + \mu_{65}^{-} max(0, -PTDF_{65,45})$

$MCP_{24,Opt} = \mu_{12}^{+} max(0, PTDF_{12,24}) + \mu_{12}^{-} max(0, -PTDF_{12,24})$

$+\mu_{14}^{+} max(0, PTDF_{14,24}) + \mu_{14}^{-} max(0, -PTDF_{14,24})$

$+\mu_{24}^{+} max(0, PTDF_{24,24}) + \mu_{24}^{-} max(0, -PTDF_{24,24})$

$+\mu_{32}^{+} max(0, PTDF_{32,24}) + \mu_{32}^{-} max(0, -PTDF_{32,24})$

$+\mu_{45}^{+} max(0, PTDF_{45,24}) + \mu_{45}^{-} max(0, -PTDF_{45,24})$

$+\mu_{63}^{+} max(0, PTDF_{63,24}) + \mu_{63}^{-} max(0, -PTDF_{63,24})$

$+\mu_{65}^{+} max(0, PTDF_{65,24}) + \mu_{65}^{-} max(0, -PTDF_{65,24})$

$MCP_{24,FGR} = \mu_{24}^{+}$

$MCP_{65,Opt} = \mu_{12}^{+} max(0, PTDF_{12,65}) + \mu_{12}^{-} max(0, -PTDF_{12,65})$

$+\mu_{14}^{+} max(0, PTDF_{14,65}) + \mu_{14}^{-} max(0, -PTDF_{14,65})$

$+\mu_{24}^{+} max(0, PTDF_{24,65}) + \mu_{24}^{-} max(0, -PTDF_{24,65})$

$$+\mu_{32}^{+} max(0, PTDF_{32,65}) + \mu_{32}^{-} max(0, -PTDF_{32,65})$$

$$+\mu_{45}^{+} max(0, PTDF_{45,65}) + \mu_{45}^{-} max(0, -PTDF_{45,65})$$

$$+\mu_{63}^{+} max(0, PTDF_{63,65}) + \mu_{63}^{-} max(0, -PTDF_{63,65})$$

$$+\mu_{65}^{+} max(0, PTDF_{65,65}) + \mu_{65}^{-} max(0, -PTDF_{65,65})$$

Using the given values for the variables, the MCPs are calculated and the results are shown in Table 3.6.

Table 3.6: FTR MCP

#	MCPs
1	12
2	6
3	6
4	16
5	20
6	25

Comparing the calculated MCP values in Table 3.6 with bid/offer prices in Table 3.2, we see that:

- A bid to buy a FTR is cleared if the bid price is larger than or equal to the MCP
- An offer to sell a FTR is cleared if the offer price is less than or equal to the MCP

The MCPs in Table 3.6 will be used to calculate the auction settlements. For example if FTR #1 was requested for 300 hours in TOU, then the bidder must pay a $Cost = Q_{15}^{b} \cdot MCP_1 \cdot 300 = 150 \cdot 12 \cdot 300 = \$540000$ to hold the right.

3.7 Settlements based on LMP and Shadow prices

FTRs cleared in the monthly auctions will generally be settled hourly using DA market congestion component of LMP. FGRs are settled based on the DA market shadow prices of the underlying flowgate. For obligation FTRs, the payment/charge (P/C) is

$$P/C_{Obl} = Quantity(MW) \cdot (LMP_{Cong/with} - LMP_{Cong/inj})$$

where $LMP_{Cong/inj}$ and $LMP_{Cong/with}$ denote congestion components of hourly DA LMP at injection and withdrawal locations. A positive value for P/C would be a payment to FTR holder and a negative value would denote a charge. For option FTRs there is no charge and the payment is

$$P_{Opt} = Quantity(MW) \cdot max(0, LMP_{Cong/with} - LMP_{Cong/inj}).$$

We do not provide settlement formulas for FTR sellers as we assume a participant can only sell FTRs that are already purchased in previous auctions and thus no sell FTR will exist for DA settlement.

3.8 Sample Monthly auction results

Figure 3.2 shows part of the PJM monthly FTR auction results for the auction held for July 2014. We observe for example that the auction is cleared for three TOUs (OnPeak, OffPeak, and 24 HR), different monthly or quarterly time periods (Jul, Aug, Sep, Q2, Q3, and Q4), and for Option and Obligation FTRs. FTRs in PJM monthly auction can have a term of one month for any of the next three individual months in the planning period or one quarter for any full remaining quarters where Q1 covers June, July, August; Q2 covers September, October, November; Q3 covers December, January, February; and Q4 covers March, April, and May [20]. Thus, the FTR terms for July auction can be July, August, September, Q2, Q3, and Q4.

A	B	C	D	E	F	G	H	I	J	K	L	M
FTRID	Class Type	Period Type	Participant	Source Node	Source PNODEID	Sink Node	Sink PNODEID	Trade Type	Hedge Type	Cleared MW	Obligation MCP	Option MCP
102565590	OnPeak	JUL	MCPI	09CARGIL138 KV BK-1	48934021	YELLOWSP69 KV BK-1	40243733	Buy	Obligation	0.5	-3.9	
102188376	OffPeak	JUL	VSNPWR	09CLDWTR69 KV BK-1	40243277	INDUSTRI34 KV BASS	32411655	Buy	Obligation	0.6	112.49	
102188377	OffPeak	JUL	VSNPWR	09CLDWTR69 KV BK-1	40243277	IUPURDUE34 KV T1	32411687	Buy	Obligation	0.6	135.56	
102188378	OffPeak	JUL	VSNPWR	09CLDWTR69 KV BK-1	40243277	SPYRUN 138 KV T1	32412675	Buy	Obligation	0.6	118.48	
102189830	OnPeak	JUL	VSNPWR	09CVNGTN69 KV BK-2	40243287	COVNTRY 138 KV T1	32410997	Buy	Obligation	0.3	276.21	
102187367	24H	JUL	VSNPWR	09CVNGTN69 KV BK-2	40243287	SATURN 138 KV INDIANCR	93353447	Buy	Obligation	0.3	459.57	
102188381	OffPeak	JUL	VSNPWR	09DIXIET69 KV BK-1	40243289	COLUMBIA138 KV T1	32410947	Buy	Obligation	0.3	183.96	
101919553	OnPeak	JUL	CLRPWR	09DIXIET69 KV BK-1	40243289	SABINA 69 KV BK-1	40243663	Buy	Obligation	1	-10.58	
102075548	OnPeak	AUG	DCEMA	09GRNVIL13.8 KV GT1	40243881	09SIDNEY69 KV DIESEL	32418669	Sell	Obligation	8	45.54	
101880440	OnPeak	Q2	SOLPMA	09GRNVIL13.8 KV GT1	40243881	09SIDNEY69 KV DIESEL	32418669	Sell	Obligation	1.1	33.5	
102596674	OffPeak	AUG	EDFFT2	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-30.14	17.11
102219918	OffPeak	AUG	EDFFTR	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Option	3	-30.14	17.11
102199739	OffPeak	AUG	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.3	-30.14	17.11
102199254	OnPeak	AUG	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	27.94	103.76
102200405	OnPeak	AUG	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.1	27.94	103.76
102219919	OffPeak	JUL	EDFFTR	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Option	14.9	-16.16	25.57
102196520	OffPeak	JUL	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-16.16	25.57
102611772	OffPeak	JUL	EDFFT2	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.9	-16.16	25.57
102195487	OffPeak	JUL	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.3	-16.16	25.57
102596835	OffPeak	Q2	EDFFT2	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-26.57	46.27
102219920	OffPeak	Q2	EDFFTR	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Option	1.6	-26.57	46.27
102211667	OffPeak	Q2	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.3	-26.57	46.27
102211127	OnPeak	Q2	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-22.1	106.83
102219921	OffPeak	Q3	EDFFTR	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Option	1.6	-24.98	35.06
102216100	OffPeak	Q3	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.3	-24.98	35.06
102217179	OffPeak	Q3	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-24.98	35.06
102596994	OffPeak	Q3	EDFFT2	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-24.98	35.06
102259258	OffPeak	Q4	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.1	-44.44	52.28
102254997	OffPeak	Q4	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-44.44	52.28
102219922	OffPeak	Q4	EDFFTR	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Option	1.6	-44.44	52.28
102597375	OffPeak	Q4	EDFFT2	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-44.44	52.28
102257752	OnPeak	Q4	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.1	-103.82	95.38
102252287	OnPeak	Q4	RAICOM	09GRNVIL13.8 KV GT1	40243881	DAY	34508503	Sell	Obligation	0.2	-103.82	95.38

Figure 3-2: PJM monthly FTR auction results [18]

3.9 Some notes on ISOs transmission right auctions

In this section, we summarize some additional information about variations in FTR auctions for different ISOs [19]-[22].

3.9.1 General notes

- FTRs are financial positions.
    - There are annual, seasonal, and monthly auctions.
- A base FTR network model is released to participants typically a few weeks before auction.
- FTR MW amount and MCP is the same for all hours within the TOU block and for the whole FTR term.
- Bid prices can be negative for FTR obligations.
- FTRs settled with congestion components of DA LMP.

3.9.2 CAISO auction

- CAISO auctiones congestion revenue rights (CRRs) for hedging congestion risks.
- CRR options are dedicated only to transmission operators.
- Bid/Offers can be piecewise linear.
- Annual auction:
    - Yearly
    - 75% of network capacity
    - Four seasons, two TOU (Peak and Off-Peak): As an example can be awarded a CRR for the 1st season Peak hours. The CRR is going to be applied to all Peak hours in season 1 with fixed price.

    - Eight independent optimizations for two TOU and four seasons.
- Monthly auction: 12 time a year in advance of each month
- DC network model is used for auction.

3.9.3 ERCOT auction

- CRRs auctioned in 1-month strips and three TOU (Off-Peak, Peak weekday (PeakWD), and Peak weekend (PeakWE))
- 24hr bids treated as single bid across all TOUs. The MCP of a 24hr CRR is weighted average of different TOU MCPs.
- Bids in form of not-to-exceed price (single segment).
- Annual auction:
    - Two-years
    - 55% of network capacity for the first year
    - 15% of network capacity for the second year
    - Each year optimization executed separately.
    - Only obligations and options.
    - CRRs could be for more than 1-month strips.
- Monthly auction:
    - 1-month strip
    - 90% of network capacity
    - Obligations, options, and FGRs for designated directional network elements
- DC network model is used for auction.

3.9.4 MISO auction

- Annual auction consists of several rounds.
- Monthly auction:
    - Two separate optimization for Peak and Off-Peak

3.9.5 PJM auction

- Annual auction consists of several rounds.
- Monthly auction:
    - FTR terms of 1 month or 1 quarter
    - Obligations and Options

Chapter 4: Estimation Challenges, Applications, and Contribution

As illustrated in previous chapters, PTDFs play an important role in FTR auction clearance, and FTR hedge. In practice PTDFs change under different network topology, loading pattern, and controllable transmission system elements limits [23]-[25]. For example, line outage, line addition, and phase shifters reaching their limits are among system events impacting PTDF values. PTDF variations impact different aspects of FTR applications such as use of FGRs for hedging and the FTR auction clearance. In the following we provide several general application categories for which it would be beneficial to know the PTDF variations and thus an estimation framework such as ours can be of significant application. However, the use of our proposed estimation framework is not limited to these applications. Finally we would summarize contribution of our proposed framework to the state of art and its benefits to the electricity market and power system industry.

4.1 Estimation of actual PTDFs from auction/market outcome for additional insights and for regulatory auditing

In practice, the FTR auction or DA market network model is published in advance of the auction/market operating day, and it is based on the forecasted network operation. Thus, the PTDFs that are used in the DA market may be different than those published earlier. These PTDFs are not typically publicly available. Thus, an estimation framework which can help estimate PTDF values from publicly available auction/market outcome would have great applications for regulatory audition of market operation such as for market abuse as well as for identifying additional auction/market insights. For example, a generation unit could withhold capacity to cause congestion in several transmission lines. Then, estimation of line flow sensitivities or PTDFs could help auditors to identify the units that have large impact on transmission flows and investigate them carefully for market power abuse.

4.2 The need for reliable PTDFs to construct FGR portfolios

As discussed in earlier chapters, the congestion cost between injection location k and withdrawal location m for a transaction between these locations can be calculated as

$$\text{Congestion Cost} = \sum_{ij}[(\mu_{ij}^{+} - \mu_{ij}^{-}) \cdot PTDF_{ij,km}] \tag{4.1}$$

where $\mu_{ij}^{+}, \mu_{ij}^{-}$ denote shadow prices of line $i-j$ flow limit constraints. Since the settlement of a FGR is equal to the shadow price $(\mu_{ij}^{+}, \mu_{ij}^{-})$ of the underlying flowgate, it is possible to construct a FGR portfolio with $PTDF_{ij,km}$ as portfolio weights to hedge the congestion risk for a 1 MW transaction between the two locations. However, due to changes in PTDFs, the FGR portfolios that are set up based on an initial set of PTDFs cannot fully hedge the underlying position if the PTDFs at the time of settlement are different than the initial set [23]. Thus an estimation framework which can help update the trader's knowledge of PTDF values from publicly available data would be valuable. Unlike point-to-point FTRs which can perfectly hedge the LMP differences due to congestion, when FGR portfolios are used to hedge congestion risk of a position, the FGR holder should construct a portfolio that replicates the congestion LMP differences between two desired location of injection and withdrawal. Such a portfolio insures the holder against risks in flowgate shadow prices and flowgate capacity however not against variations in PTDFs. So it is the responsibility of the FGR holder to track the changes in PTDFs to make sure the portfolio revenue would be enough to cover the congestion charges due to LMP differences [25], [26]. Moreover, [26] argues that a portfolio of FGRs can be constructed for any point to point transaction using the maximal PTDFs under any $n-1$ contingencies. Such a portfolio for sure pays at least the congestion charge between two locations LMPs under $n-1$ contingencies. Thus our estimation framework would be very valuable in estimating the PTDFs and keep track of their changes.

Note that our estimation framework in its current form can be used for estimating PTDFs in FTR auctions from auction outcome. This is also true for estimating DA market PTDFs from DA market outcome if a DA market network is published in advance. However, typically the DA network model is not publicly available and thus cannot be used by a market outsider to estimate PTDFs. In this case, since the DA network model is set up based on the FTR auction network model as the base model, our proposal is to use the DA market results and FTR auction network model to estimate the DA market PTDFs. To do so, we would need to do some modifications to our estimation framework such as inclusion of network losses. We leave this as a topic for our future research.

4.3 PTDF estimation for generation investment

Identifying the appropriate location is one of the earliest steps in investing for a new generation unit. It is crucial to choose a location that does not add to congestion severity of the network otherwise it would impose additional congestion costs to the transactions involving the new unit. In this case, our proposed framework will be very useful as it could give a track of PTDF variations for different injection and withdrawal locations and thus help the investor choose the locations that have small sensitivities on typically congested transmission lines for further investigation for final investment.

4.4 Other applications and contribution

Other than the applications discussed above, there are other uses to our proposed framework for estimation of unknown parameters summarized as follows: the proposed framework can also be beneficial to market participants and retail customers since it would improve market transparency due to making the value of underlying system parameters observable to all market participants and thus will lead to more efficient pricing with the hope of reducing the final rate for retail consumers. Moreover, it can help participants to verify their transaction cleared quantities, prices, and settlement charges in FTR auctions and electricity markets by providing the information for underlying parameters that were unknown to them. Specifically, having a track of PTDF changes would also help them to predict and identify the transactions that are subject to curtailment in the market because of violating the flow limit constraints and thus help them to take appropriate preventive actions.

Finally, the contribution of our research work to the state of art can be seen from two perspectives: from the industrial perspective, as explained using several examples introduced above, the proposed method can be useful for regulators and auditors, market participants, investors, and retail customers. From the academic perspective, it provides a novel method for estimating part of the unknown system parameters in an optimization problem without the need to know the uncertain system inputs that are modeled as part of the optimization parameters. This is accomplished by using the duality theory and by introducing additional optimization constraints from system output data which altogether help shrink the optimization feasible region and thus reveal the true value of the unknown parameters. Specifically for the field of electricity markets and power systems, the application of this framework would be extensive for gaining additional knowledge and conducting further research on FTR auction and electricity market optimization problems since the duality theory holds for these optimizations. Finally, it is worth mentioning that the method can also be used for other optimization problems with similar characteristics.

Chapter 5: Proposed Estimation Method for FTR auctions and electricity market

In this chapter, we propose an estimation method that is designed and structured based on duality theory in optimization [27]-[28]. The duality in basic definition describes the relationship between primal and dual problems optimal objective values. The proposed estimation method, intuitively, tries to estimate the true value of parameters by shrinking the underlying optimization problem feasible region. We perform this by introducing additional constraints from readily available information. The following sections are used to illustrate the steps in the estimation method.

5.1 FTR Auction dual problem

One of the equality constraints used in the proposed estimation method is the strong duality of primal and dual. We derive the dual of the FTR auction optimization introduced in equations (3.5)-(3.11) as follows:

$$\min_{\mu_{ij}^{+},\mu_{ij}^{-},\nu_{ij,rs}^{+},\nu_{ij,rs}^{-},\pi_{w,t,km,BL}^{+},\pi_{w,t,km,BL}^{-}} \{\sum_{ij}\left[(m_r F_{ij}^{max} - \Omega^{+})\mu_{ij}^{+}\right.$$
$$\left. + (m_r F_{ji}^{max} - \Omega^{-})\mu_{ij}^{-}\right]$$
$$+ \sum_{ij,rs}\left[(m_r F_{ij}^{max} - \Theta^{+})\nu_{ij,rs}^{+} + (m_r F_{ji}^{max} - \Theta^{-})\nu_{ij,rs}^{-}\right]$$
$$+ \sum_{km,BL}\left[Q_{w,t,km,BL}^{b/s,max}\pi_{w,t,km,BL}^{+} - Q_{w,t,km,BL}^{b/s,min}\pi_{w,t,km,BL}^{-}\right]\} \quad (5.1)$$

where

$$\Omega^{+} = \sum_{b/s,t,km,BL}\{PTDF_{ij,km} \cdot \sum_{\rho=1}^{r-1}\left(Q_{t,km,BL,w=Obl}^{b,\rho} - Q_{t,km,BL,w=Obl}^{s,\rho}\right)$$

$$+ max\left(0, PTDF_{ij,km}\right) \cdot \sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Opt}^{b,\rho} - Q_{t,km,BL,w=Opt}^{s,\rho}\right)$$

$$+ \sum_{\rho=1}^{r-1} \left(Q_{t,ij,BL,w=FGR}^{b,\rho} - Q_{t,ij,BL,w=FGR}^{s,\rho}\right) \}$$

$$\Omega^{-} = \sum_{b/s,t,km,BL} \{ -PTDF_{ij,km} \cdot \sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Obl}^{b,\rho} - Q_{t,km,BL,w=Obl}^{s,\rho}\right)$$

$$+ max\left(0, -PTDF_{ij,km}\right) \cdot \sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Opt}^{b,\rho} - Q_{t,km,BL,w=Opt}^{s,\rho}\right)$$

$$+ \sum_{\rho=1}^{r-1} \left(Q_{t,ji,BL,w=FGR}^{b,\rho} - Q_{t,ji,BL,w=FGR}^{s,\rho}\right) \}$$

$$\Theta^{+} = \sum_{b/s,t,km,BL} \{ [PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}] \cdot$$

$$[\sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Obl}^{b,\rho} - Q_{t,km,BL,w=Obl}^{s,\rho}\right)]$$

$$+ max\left(0, PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}\right)$$

$$\cdot [\sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Opt}^{b,\rho} - Q_{t,km,BL,w=Opt}^{s,\rho}\right)]$$

$$+ \sum_{\rho=1}^{r-1} \left(Q_{t,ij,BL,w=FGR}^{b,\rho} - Q_{t,ij,BL,w=FGR}^{s,\rho}\right) \}$$

$$\Theta^{-} = \sum_{b/s,t,km,BL} \{ -[PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}] \cdot$$

$$[\sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Obl}^{b,\rho} - Q_{t,km,BL,w=Obl}^{s,\rho}\right)]$$

$$+ max\left(0, -\left[PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}\right]\right)$$

$$\cdot \left[\sum_{\rho=1}^{r-1} \left(Q_{t,km,BL,w=Opt}^{b,\rho} - Q_{t,km,BL,w=Opt}^{s,\rho}\right)\right]$$

$$+ \sum_{\rho=1}^{r-1} \left(Q_{t,ji,BL,w=FGR}^{b,\rho} - Q_{t,ji,BL,w=FGR}^{s,\rho}\right)\}$$

The dual optimization (5.1) is subject to the following constraints:

$$\mu_{ij}^{+}, \mu_{ij}^{-}, \nu_{ij,rs}^{+}, \nu_{ij,rs}^{-}, \pi_{w,t,km,BL}^{+}, \pi_{w,t,km,BL}^{-} \geq 0 \tag{5.2}$$

and

- If the FTR type is a buy and Obligation ($b\ and\ Obl$),

$$\sum_{ij} PTDF_{ij,km}\left(\mu_{ij}^{+} - \mu_{ij}^{-}\right)$$

$$+ \sum_{ij,rs} [PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}]\left(\nu_{ij,rs}^{+} - \nu_{ij,rs}^{-}\right)$$

$$+\pi_{w,t,km,BL}^{+} - \pi_{w,t,km,BL}^{-} \geq \beta_{w,t,km,BL}^{b}$$

for all FTRs of type ($b\ and\ Obl$) and all BL (5.3)

- If FTR is ($s\ and\ Obl$),

$$\sum_{ij} PTDF_{ij,km}\left(\mu_{ij}^{-} - \mu_{ij}^{+}\right)$$

$$+ \sum_{ij,rs} [PTDF_{ij,km} + LODF_{ij,rs} PTDF_{rs,km}]\left(\nu_{ij,rs}^{-} - \nu_{ij,rs}^{+}\right)$$

$$+\pi_{w,t,km,BL}^{+} - \pi_{w,t,km,BL}^{-} \geq -\beta_{w,t,km,BL}^{s}$$

for all FTRs of type ($s\ and\ Obl$) and all BL (5.4)

- If FTR is ($b\ and\ Opt$),

$$\sum_{ij}\left[\mu_{ij}^{+}\,max\left(0,PTDF_{ij,km}\right)+\mu_{ij}^{-}\,max\left(0,-PTDF_{ij,km}\right)\right]$$

$$+\sum_{ij,rs}[\nu_{ij,rs}^{+}\,max\left(0,PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km}\right)]$$

$$+\sum_{ij,rs}[\nu_{ij,rs}^{-}\,max\left(0,-\left[PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km}\right]\right)]$$

$$+\pi_{w,t,km,BL}^{+}-\pi_{w,t,km,BL}^{-}\geq\beta_{w,t,km,BL}^{b}$$

for all FTRs of type ($b\ and\ Opt$) and all BL (5.5)

- If FTR is ($s\ and\ Opt$),

$$-\sum_{ij}\left[\mu_{ij}^{+}\,max\left(0,PTDF_{ij,km}\right)+\mu_{ij}^{-}\,max\left(0,-PTDF_{ij,km}\right)\right]$$

$$-\sum_{ij,rs}[\nu_{ij,rs}^{+}\,max\left(0,PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km}\right)]$$

$$-\sum_{ij,rs}[\nu_{ij,rs}^{-}\,max\left(0,-\left[PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km}\right]\right)]$$

$$+\pi_{w,t,km,BL}^{+}-\pi_{w,t,km,BL}^{-}\geq-\beta_{w,t,km,BL}^{S}$$

for all FTRs of type ($s\ and\ Opt$) and all BL (5.6)

- If FTR is ($b\ and\ FGR$),

$$\sum_{ij}\left(\mu_{ij}^{+}-\mu_{ij}^{-}\right)+\sum_{ij,rs}\left(\nu_{ij,rs}^{+}-\nu_{ij,rs}^{-}\right)+\pi_{w,t,km,BL}^{+}-\pi_{w,t,km,BL}^{-}\geq\beta_{w,t,km,BL}^{b}$$

for all FTRs of type ($b\ and\ FGR$) and all BL (5.7)

- If FTR is ($s \; and \; FGR$),

$$\sum_{ij}(\mu_{ij}^{-} - \mu_{ij}^{+}) + \sum_{ij,rs}(\nu_{ij,rs}^{-} - \nu_{ij,rs}^{+}) + \pi_{w,t,km,BL}^{+} - \pi_{w,t,km,BL}^{-} \geq -\beta_{w,t,km,BL}^{s}$$

for all FTRs of type ($s \; and \; FGR$) and all BL (5.8)

Note that in the discussed primal or dual optimization problems, one of each constraint and its negative duplicate can be binding simultaneously meaning that the following dual variables cannot be non-zero simultaneously:

One of μ_{ij}^{+} or μ_{ij}^{-} $\forall \; ij$

One of $\nu_{ij,rs}^{+}$ or $\nu_{ij,rs}^{-}$ $\forall \; ij \; and \; rs$

One of $\pi_{w,t,km,BL}^{+}$ or $\pi_{w,t,km,BL}^{-}$ $\forall \; km \; and \; BL$

5.2 Involving auction outcome and grid structure in estimation

To estimate the parameters we would need to include additional information. One set of information that is publicly available in all ISOs is the MCPs which are calculated from auction results. We would use MCPs to construct additional set of constraints in estimation. These constraints are constructed by setting equations (3.12)-(3.14) equal to their auction outcomes. Thus, considering $MCP_{km,t-(24hr),w=Obl}$, $MCP_{km,t-(24hr),w=Opt}$, and $MCP_{km,t-(24hr),w=FGR}$ as known auction results the following will be additional equality constraints used in estimation.

$$MCP_{km,t-(24hr),w=Obl} = \sum_{ij}[(\mu_{ij}^{+} - \mu_{ij}^{-}) \cdot PTDF_{ij,km}]$$

$$+\sum_{ij,rs}[(\nu^{+}_{ij,rs}-\nu^{-}_{ij,rs})\cdot(PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km})] \tag{5.9}$$

$$MCP_{km,t-(24hr),w=Opt}=\sum_{ij}[\mu^{+}_{ij}\,max(0,PTDF_{ij,km})]$$

$$+\sum_{ij}[\mu^{-}_{ij}\,max(0,-PTDF_{ij,km})]$$

$$+\sum_{ij,rs}[\nu^{+}_{ij,rs}\,max(0,PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km})]$$

$$+\sum_{ij,rs}[\nu^{-}_{ij,rs}\,max(0,-[PTDF_{ij,km}+LODF_{ij,rs}PTDF_{rs,km}])] \tag{5.10}$$

$$MCP_{km,t-(24hr),w=FGR}=\begin{cases}\mu^{+}_{km}+\sum_{rs}\nu^{+}_{km,rs} & if\quad km=ij\\ \mu^{-}_{km}+\sum_{rs}\nu^{-}_{km,rs} & if\quad km=ji\end{cases} \tag{5.11}$$

In addition to using market results in estimation, power grid structure will also be used in the optimization. For example in applying the estimation framework to PTDF estimation, the grid structure is implicitly considered since PTDFs are present in simultaneous feasibility conditions in the auction formulation and its dual.

5.3 Proposed estimation method

We propose a core optimization framework to be used in the estimation procedure. Note that the proposed estimation method can be applied to estimate various parameters in the auction and market optimizations. For example in addition to PTDF estimation, it can also be used to estimate line flow limits one at a time (That means only one line limit is uncertain). We do not elaborate on this and only provide the simulation results for line flow estimation whenever necessary. The objective function of the core optimization used in the estimation is minimization of a L_1 norm of estimated auction bid/offer prices and their initial guesses. The L_1 norm is chosen since in many estimation situations it is less sensitive to data outliers and has been used widely as a robust norm. The equality constraints are strong duality of primal-dual problems and the MCP constraints introduced in (5.9)-(5.11). Finally, the inequality constraints are the dual inequalities in (5.2)-(5.8). The proposed core optimization framework can be summarized as follows:

$$\min_{\beta_{w,t,km,BL}^{b/s},\mu_{ij}^{+},\mu_{ij}^{-},\nu_{ij,rs}^{+},\nu_{ij,rs}^{-},\pi_{w,t,km,BL}^{+},\pi_{w,t,km,BL}^{-}} \left\|\beta_{w,t,km,BL}^{b/s}-\beta_{w,t,km,BL}^{b/s,0}\right\|_1 \qquad (5.12)$$

subject to

Strong Duality:	$arg\{(3.5)\} - arg\{(5.1)\} = 0$	(5.13)
Dual inequalities:	(5.2) to (5.8)	(5.14)
MCP equalities:	(5.9) to (5.11)	(5.15)
Sign requirement	$\beta_{w,t,km,BL}^{b/s} \geq 0$ or $\beta_{w,t,km,BL}^{b/s} \geq \tau$ for obligation bids where τ denotes a negative bound.	(5.16)

where $\beta_{w,t,km,BL}^{b/s}$ is the vector of unknown bid/offer prices to be estimated and $\beta_{w,t,km,BL}^{b/s,0}$ is the vector of initial guesses.

The optimization framework in (5.12)-(5.16) will be the core of the estimation method. The proposed estimation method for estimation of a single parameter starts by sweeping through an appropriate range of the parameter values while solving the core optimization in (5.12)-(5.16) and focusing on the feasibility/infeasibility flag of the optimization. The true value of the parameter will be identified as the values for which the optimization in (5.12)-(5.16) is feasible and optimum. Later we will apply the proposed estimation framework to estimate PTDFs and line flow limits in FTR auction. We will also show in simulation results that for some PTDFs, the optimization is only feasible at their true value, for a second group of PTDFs the true value will be the border of feasibility-infeasibility region, and for a third group of PTDFs a range can be found which includes the true value while the optimization is feasible for that range.

5.3.1 Example 5.1

In this section we provide the optimization framework for the FTR auction introduced in example 3.1.

The auction optimization dual is as follows:

$$\begin{aligned}\min_{\Delta_1}\{&F_{12}^{max}\mu_{12}^{+} + F_{14}^{max}\mu_{14}^{+} + F_{24}^{max}\mu_{24}^{+} + F_{32}^{max}\mu_{32}^{+} + F_{45}^{max}\mu_{45}^{+} \\ &+ F_{63}^{max}\mu_{63}^{+} + F_{65}^{max}\mu_{65}^{+} + F_{21}^{max}\mu_{12}^{-} + F_{41}^{max}\mu_{14}^{-} \\ &+ F_{42}^{max}\mu_{24}^{-} + F_{23}^{max}\mu_{32}^{-} + F_{54}^{max}\mu_{45}^{-} + F_{36}^{max}\mu_{63}^{-} \\ &+ F_{56}^{max}\mu_{65}^{-} + Q_{15}^{b,max}\pi_{15}^{+} + Q_{62}^{b,max}\pi_{62}^{+} + Q_{45,Opt}^{b,max}\pi_{45,Opt}^{+} \\ &+ Q_{24,Opt}^{b,max}\pi_{24,Opt}^{+} + Q_{24,FGR}^{b,max}\pi_{24,FGR}^{+} + Q_{65,Opt}^{s,max}\pi_{65,Opt}^{+}\}\end{aligned} \tag{5.17}$$

where

$\Delta_1 = \{\mu_{12}^{+}, \mu_{14}^{+}, \mu_{24}^{+}, \mu_{32}^{+}, \mu_{45}^{+}, \mu_{63}^{+}, \mu_{65}^{+}, \mu_{12}^{-}, \mu_{14}^{-}, \mu_{24}^{-}, \mu_{32}^{-}, \mu_{45}^{-}, \mu_{63}^{-}, \mu_{65}^{-}, \pi_{15}^{+}, \pi_{62}^{+},$

$\pi_{45,Opt}^{+}, \pi_{24,Opt}^{+}, \pi_{24,FGR}^{+}, \pi_{65,Opt}^{+}\}$

The dual optimization is subject to the following constraints:

$$(\mu_{12}^{+} - \mu_{12}^{-})PTDF_{12,15} + (\mu_{14}^{+} - \mu_{14}^{-})PTDF_{14,15}$$
$$+ (\mu_{24}^{+} - \mu_{24}^{-})PTDF_{24,15} + (\mu_{32}^{+} - \mu_{32}^{-})PTDF_{32,15}$$
$$+ (\mu_{45}^{+} - \mu_{45}^{-})PTDF_{45,15} + (\mu_{63}^{+} - \mu_{63}^{-})PTDF_{63,15} \quad (5.18)$$
$$+(\mu_{65}^{+} - \mu_{65}^{-})PTDF_{65,15} + \pi_{15}^{+} - \pi_{15}^{-} \geq \beta_{15}^{b}$$

$$(\mu_{12}^{+} - \mu_{12}^{-})PTDF_{12,62} + (\mu_{14}^{+} - \mu_{14}^{-})PTDF_{14,62}$$
$$+ (\mu_{24}^{+} - \mu_{24}^{-})PTDF_{24,62} + (\mu_{32}^{+} - \mu_{32}^{-})PTDF_{32,62}$$
$$+(\mu_{45}^{+} - \mu_{45}^{-})PTDF_{45,62} + (\mu_{63}^{+} - \mu_{63}^{-})PTDF_{63,62} \quad (5.19)$$
$$+(\mu_{65}^{+} - \mu_{65}^{-})PTDF_{65,62} + \pi_{62}^{+} - \pi_{62}^{-} \geq \beta_{62}^{b}$$

$$\mu_{12}^{+}max(0, PTDF_{12,45}) + \mu_{12}^{-}max(0, -PTDF_{12,45})$$
$$+\mu_{14}^{+}max(0, PTDF_{14,45}) + \mu_{14}^{-}max(0, -PTDF_{14,45})$$
$$+\mu_{24}^{+}max(0, PTDF_{24,45}) + \mu_{24}^{-}max(0, -PTDF_{24,45})$$
$$+\mu_{32}^{+}max(0, PTDF_{32,45}) + \mu_{32}^{-}max(0, -PTDF_{32,45})$$
$$+\mu_{45}^{+}max(0, PTDF_{45,45}) + \mu_{45}^{-}max(0, -PTDF_{45,45})$$
$$+\mu_{63}^{+}max(0, PTDF_{63,45}) + \mu_{63}^{-}max(0, -PTDF_{63,45})$$
$$+\mu_{65}^{+}max(0, PTDF_{65,45}) + \mu_{65}^{-}max(0, -PTDF_{65,45})$$
$$+\pi_{45,Opt}^{+} - \pi_{45,Opt}^{-} \geq \beta_{45}^{b} \quad (5.20)$$

$$\mu_{12}^{+}max(0, PTDF_{12,24}) + \mu_{12}^{-}max(0, -PTDF_{12,24})$$
$$+\mu_{14}^{+}max(0, PTDF_{14,24}) + \mu_{14}^{-}max(0, -PTDF_{14,24})$$
$$+\mu_{24}^{+}max(0, PTDF_{24,24}) + \mu_{24}^{-}max(0, -PTDF_{24,24})$$

$$+\mu_{32}^{+}max(0, PTDF_{32,24}) + \mu_{32}^{-}max(0, -PTDF_{32,24})$$

$$+\mu_{45}^{+}max(0, PTDF_{45,24}) + \mu_{45}^{-}max(0, -PTDF_{45,24})$$

$$+\mu_{63}^{+}max(0, PTDF_{63,24}) + \mu_{63}^{-}max(0, -PTDF_{63,24})$$

$$+\mu_{65}^{+}max(0, PTDF_{65,24}) + \mu_{65}^{-}max(0, -PTDF_{65,24})$$

$$+\pi_{24,Opt}^{+} - \pi_{24,Opt}^{-} \geq \beta_{24,Opt}^{b} \qquad (5.21)$$

$$\mu_{24}^{+} + \pi_{24,FGR}^{+} - \pi_{24,FGR}^{-} \geq \beta_{24,FGR}^{b} \qquad (5.22)$$

$$-\mu_{12}^{+}\, max\left(0, PTDF_{12,65}\right) - \mu_{12}^{-}max(0, -PTDF_{12,65})$$

$$-\mu_{14}^{+}\, max\left(0, PTDF_{14,65}\right) - \mu_{14}^{-}max(0, -PTDF_{14,65})$$

$$-\mu_{24}^{+}\, max\left(0, PTDF_{24,65}\right) - \mu_{24}^{-}max(0, -PTDF_{24,65})$$

$$-\mu_{32}^{+}\, max\left(0, PTDF_{32,65}\right) - \mu_{32}^{-}max(0, -PTDF_{32,65})$$

$$-\mu_{45}^{+}\, max\left(0, PTDF_{45,65}\right) - \mu_{45}^{-}max(0, -PTDF_{45,65})$$

$$-\mu_{63}^{+}\, max\left(0, PTDF_{63,65}\right) - \mu_{63}^{-}max(0, -PTDF_{63,65})$$

$$-\mu_{65}^{+}\, max\left(0, PTDF_{65,65}\right) - \mu_{65}^{-}max(0, -PTDF_{65,65})$$

$$+\pi_{65,Opt}^{+} - \pi_{65,Opt}^{-} \geq -\beta_{65}^{s} \qquad (5.23)$$

$$\Delta_1 \geq 0 \qquad (5.24)$$

The core optimization can be summarized as follows:

$$\min_{\Delta_1,\Delta_2}\left\{\left|\beta_{15}^{b}\right| + \left|\beta_{62}^{b}\right| + \left|\beta_{45,Opt}^{b}\right| + \left|\beta_{24,Opt}^{b}\right| + \left|\beta_{24,FGR}^{b}\right| + \left|\beta_{65,Opt}^{s}\right|\right\} \qquad (5.25)$$

where $\Delta_2 = \{\beta_{15}^{b}, \beta_{62}^{b}, \beta_{45,Opt}^{b}, \beta_{24,Opt}^{b}, \beta_{24,FGR}^{b}, \beta_{65,Opt}^{s}\}$. The optimization is subject to

Strong Duality: $arg\{(3.16)\} - arg\{(5.17)\} = 0$ (5.26)

Dual inequalities: (5.18) to (5.24) (5.27)

and

$$MCP_{15} = (\mu_{12}^{+} - \mu_{12}^{-})PTDF_{12,15} + (\mu_{14}^{+} - \mu_{14}^{-})PTDF_{14,15}$$
$$+ (\mu_{24}^{+} - \mu_{24}^{-})PTDF_{24,15} + (\mu_{32}^{+} - \mu_{32}^{-})PTDF_{32,15}$$
$$+(\mu_{45}^{+} - \mu_{45}^{-})PTDF_{45,15} + (\mu_{63}^{+} - \mu_{63}^{-})PTDF_{63,15}$$
$$+(\mu_{65}^{+} - \mu_{65}^{-})PTDF_{65,15} \quad (5.28)$$

$$MCP_{62} = (\mu_{12}^{+} - \mu_{12}^{-})PTDF_{12,62} + (\mu_{14}^{+} - \mu_{14}^{-})PTDF_{14,62}$$
$$+ (\mu_{24}^{+} - \mu_{24}^{-})PTDF_{24,62} + (\mu_{32}^{+} - \mu_{32}^{-})PTDF_{32,62}$$
$$+(\mu_{45}^{+} - \mu_{45}^{-})PTDF_{45,62} + (\mu_{63}^{+} - \mu_{63}^{-})PTDF_{63,62}$$
$$+(\mu_{65}^{+} - \mu_{65}^{-})PTDF_{65,62} \quad (5.29)$$

$$MCP_{45} = \mu_{12}^{+}max(0, PTDF_{12,45}) + \mu_{12}^{-}max(0, -PTDF_{12,45})$$
$$+\mu_{14}^{+}max(0, PTDF_{14,45}) + \mu_{14}^{-}max(0, -PTDF_{14,45})$$
$$+\mu_{24}^{+}max(0, PTDF_{24,45}) + \mu_{24}^{-}max(0, -PTDF_{24,45})$$
$$+\mu_{32}^{+}max(0, PTDF_{32,45}) + \mu_{32}^{-}max(0, -PTDF_{32,45})$$
$$+\mu_{45}^{+}max(0, PTDF_{45,45}) + \mu_{45}^{-}max(0, -PTDF_{45,45})$$
$$+\mu_{63}^{+}max(0, PTDF_{63,45}) + \mu_{63}^{-}max(0, -PTDF_{63,45})$$
$$+\mu_{65}^{+}max(0, PTDF_{65,45}) + \mu_{65}^{-}max(0, -PTDF_{65,45}) \quad (5.30)$$

$$MCP_{24,Opt} = \mu_{12}^{+}max(0, PTDF_{12,24}) + \mu_{12}^{-}max(0, -PTDF_{12,24})$$

$$+\mu_{14}^{+}max(0, PTDF_{14,24}) + \mu_{14}^{-}max(0, -PTDF_{14,24})$$

$$+\mu_{24}^{+}max(0, PTDF_{24,24}) + \mu_{24}^{-}max(0, -PTDF_{24,24})$$

$$+\mu_{32}^{+}max(0, PTDF_{32,24}) + \mu_{32}^{-}max(0, -PTDF_{32,24})$$

$$+\mu_{45}^{+}max(0, PTDF_{45,24}) + \mu_{45}^{-}max(0, -PTDF_{45,24})$$

$$+\mu_{63}^{+}max(0, PTDF_{63,24}) + \mu_{63}^{-}max(0, -PTDF_{63,24})$$

$$+\mu_{65}^{+}max(0, PTDF_{65,24}) + \mu_{65}^{-}max(0, -PTDF_{65,24}) \quad (5.31)$$

$$MCP_{24,FGR} = \mu_{24}^{+} \quad (5.32)$$

$$MCP_{65} = \mu_{12}^{+}\, max\left(0, PTDF_{12,65}\right) + \mu_{12}^{-}max(0, -PTDF_{12,65})$$

$$+\mu_{14}^{+}\, max\left(0, PTDF_{14,65}\right) + \mu_{14}^{-}max(0, -PTDF_{14,65})$$

$$+\mu_{24}^{+}\, max\left(0, PTDF_{24,65}\right) + \mu_{24}^{-}max(0, -PTDF_{24,65})$$

$$+\mu_{32}^{+}\, max\left(0, PTDF_{32,65}\right) + \mu_{32}^{-}max(0, -PTDF_{32,65})$$

$$+\mu_{45}^{+}\, max\left(0, PTDF_{45,65}\right) + \mu_{45}^{-}max(0, -PTDF_{45,65})$$

$$+\mu_{63}^{+}\, max\left(0, PTDF_{63,65}\right) + \mu_{63}^{-}max(0, -PTDF_{63,65})$$

$$+\mu_{65}^{+}\, max\left(0, PTDF_{65,65}\right) + \mu_{65}^{-}max(0, -PTDF_{65,65}) \quad (5.33)$$

$\Delta_2 \geq 0$ and $\beta_{w,t,km,BL}^{b/s} \geq \tau$ for obligation bids and possibly a negative bound τ (5.34)

Chapter 6: Simulation Results

In this chapter we apply the proposed estimation framework to PTDF and line limit estimation for three cases. For each case, we consider one unknown variable at one time and solve the core optimization for a range around the true value of the variable and provide the feasibility behavior of the optimization problem.

6.1 Some special cases

There are a few special cases in estimation of PTDFs that are listed as follows:

First: The method cannot be used to estimate parameters (PTDF or line flow limits) associated with transmission lines that are not binding in the auction optimization. This is because the shadow prices of these transmission lines are zero in equations (5.14) and (5.15) which makes the PTDFs and flow limits of these lines unobservable through output.

Second: For a similar reason, the PTDF for some of Opt FTRs and FGRs cannot be estimated. For Opt FTRs, the terms with maximum function in (5.14) and (5.15) such as $\mu_{ij}^{+}\, max\left(0, PTDF_{ij,km}\right)$ and $\mu_{ij}^{-}\, max\left(0, -PTDF_{ij,km}\right)$ can be zero depending on the sign of PTDF and the line shadow price. This behavior in some cases makes the Opt FTRs unobservable through output which will lead to not-so-accurate estimation results. For FGR type FTRs, the PTDFs are not observable in (5.14) and (5.15) since the assumption in these equations is that the FGR would imply a full flow on the associated line. We would propose a method in Section 7.1.1 to address FGR and Opt FTR estimation special cases.

Due to these special cases and for brevity, we only provide the estimation results associated with Obl FTRs and associated with binding transmission lines. Moreover, we would not consider contingency constraints in our example and would leave it for our future work.

6.2 Estimation of PTDFs and transmission line limits in different scenarios

6.2.1 Case 1-Example 3.1 and 5.1

Tables 6.1 and 6.2 provide the estimation results for the system introduced in examples 3.1 and 5.1. As discussed earlier, we only provide the estimation results of PTDFs that are associated to obligation (Obl) FTRs and binding transmission lines. Moreover, for brevity the results of PTDFs with absolute values of less than 0.1 are omitted. Note that the estimation results for each PTDF or line limit are separated with blank rows in both tables. Moreover, we are interested in the feasibility flag column to identify the true value of PTDF or line limit.

Table 6.1: PTDF estimation results

Line From Number	Line To Number	From Inj. Bus	To Inj. Bus	Estimated value	True value	Feasibility flag (feasible=1)
1	4	1	5	0.45712	0.5714	-2
1	4	1	5	0.468548	0.5714	-2
1	4	1	5	0.479976	0.5714	-2
1	4	1	5	0.491404	0.5714	-2
1	4	1	5	0.502832	0.5714	-2
1	4	1	5	0.51426	0.5714	-2
1	4	1	5	0.525688	0.5714	-2
1	4	1	5	0.537116	0.5714	-2
1	4	1	5	0.548544	0.5714	-2
1	4	1	5	0.559972	0.5714	-2
1	4	1	5	0.5714	0.5714	1
1	4	1	5	0.582828	0.5714	1
1	4	1	5	0.594256	0.5714	1
1	4	1	5	0.605684	0.5714	1
1	4	1	5	0.617112	0.5714	1
1	4	1	5	0.62854	0.5714	1
1	4	1	5	0.639968	0.5714	1
1	4	1	5	0.651396	0.5714	1

1	4	1	5	0.662824	0.5714	1
1	4	1	5	0.674252	0.5714	1
1	4	1	5	0.68568	0.5714	1
2	4	1	5	0.11432	0.1429	-2
2	4	1	5	0.117178	0.1429	-2
2	4	1	5	0.120036	0.1429	-2
2	4	1	5	0.122894	0.1429	-2
2	4	1	5	0.125752	0.1429	-2
2	4	1	5	0.12861	0.1429	-2
2	4	1	5	0.131468	0.1429	-2
2	4	1	5	0.134326	0.1429	-2
2	4	1	5	0.137184	0.1429	-2
2	4	1	5	0.140042	0.1429	-2
2	4	1	5	0.1429	0.1429	1
2	4	1	5	0.145758	0.1429	1
2	4	1	5	0.148616	0.1429	1
2	4	1	5	0.151474	0.1429	1
2	4	1	5	0.154332	0.1429	1
2	4	1	5	0.15719	0.1429	1
2	4	1	5	0.160048	0.1429	1
2	4	1	5	0.162906	0.1429	1
2	4	1	5	0.165764	0.1429	1
2	4	1	5	0.168622	0.1429	1
2	4	1	5	0.17148	0.1429	1
6	5	1	5	0.22856	0.2857	-2
6	5	1	5	0.234274	0.2857	-2
6	5	1	5	0.239988	0.2857	-2
6	5	1	5	0.245702	0.2857	-2
6	5	1	5	0.251416	0.2857	-2
6	5	1	5	0.25713	0.2857	-2
6	5	1	5	0.262844	0.2857	-2
6	5	1	5	0.268558	0.2857	-2
6	5	1	5	0.274272	0.2857	-2
6	5	1	5	0.279986	0.2857	-2
6	5	1	5	0.2857	0.2857	1
6	5	1	5	0.291414	0.2857	1
6	5	1	5	0.297128	0.2857	1
6	5	1	5	0.302842	0.2857	1
6	5	1	5	0.308556	0.2857	1

6	5	1	5	0.31427	0.2857	1
6	5	1	5	0.319984	0.2857	1
6	5	1	5	0.325698	0.2857	1
6	5	1	5	0.331412	0.2857	1
6	5	1	5	0.337126	0.2857	1
6	5	1	5	0.34284	0.2857	1
1	4	6	2	-0.11432	-0.1429	-2
1	4	6	2	-0.11718	-0.1429	-2
1	4	6	2	-0.12004	-0.1429	-2
1	4	6	2	-0.12289	-0.1429	-2
1	4	6	2	-0.12575	-0.1429	-2
1	4	6	2	-0.12861	-0.1429	-2
1	4	6	2	-0.13147	-0.1429	-2
1	4	6	2	-0.13433	-0.1429	-2
1	4	6	2	-0.13718	-0.1429	-2
1	4	6	2	-0.14004	-0.1429	-2
1	4	6	2	-0.1429	-0.1429	1
1	4	6	2	-0.14576	-0.1429	-2
1	4	6	2	-0.14862	-0.1429	-2
1	4	6	2	-0.15147	-0.1429	-2
1	4	6	2	-0.15433	-0.1429	-2
1	4	6	2	-0.15719	-0.1429	-2
1	4	6	2	-0.16005	-0.1429	-2
1	4	6	2	-0.16291	-0.1429	-2
1	4	6	2	-0.16576	-0.1429	-2
1	4	6	2	-0.16862	-0.1429	-2
1	4	6	2	-0.17148	-0.1429	-2
2	4	6	2	-0.22856	-0.2857	-2
2	4	6	2	-0.23427	-0.2857	-2
2	4	6	2	-0.23999	-0.2857	-2
2	4	6	2	-0.2457	-0.2857	-2
2	4	6	2	-0.25142	-0.2857	-2
2	4	6	2	-0.25713	-0.2857	-2
2	4	6	2	-0.26284	-0.2857	-2
2	4	6	2	-0.26856	-0.2857	-2
2	4	6	2	-0.27427	-0.2857	-2
2	4	6	2	-0.27999	-0.2857	-2
2	4	6	2	-0.2857	-0.2857	1
2	4	6	2	-0.29141	-0.2857	-2

2	4	6	2	-0.29713	-0.2857	-2
2	4	6	2	-0.30284	-0.2857	-2
2	4	6	2	-0.30856	-0.2857	-2
2	4	6	2	-0.31427	-0.2857	-2
2	4	6	2	-0.31998	-0.2857	-2
2	4	6	2	-0.3257	-0.2857	-2
2	4	6	2	-0.33141	-0.2857	-2
2	4	6	2	-0.33713	-0.2857	-2
2	4	6	2	-0.34284	-0.2857	-2
6	5	6	2	0.34288	0.4286	-2
6	5	6	2	0.351452	0.4286	-2
6	5	6	2	0.360024	0.4286	-2
6	5	6	2	0.368596	0.4286	-2
6	5	6	2	0.377168	0.4286	-2
6	5	6	2	0.38574	0.4286	-2
6	5	6	2	0.394312	0.4286	-2
6	5	6	2	0.402884	0.4286	-2
6	5	6	2	0.411456	0.4286	-2
6	5	6	2	0.420028	0.4286	-2
6	5	6	2	0.4286	0.4286	1
6	5	6	2	0.437172	0.4286	-2
6	5	6	2	0.445744	0.4286	-2
6	5	6	2	0.454316	0.4286	-2
6	5	6	2	0.462888	0.4286	-2
6	5	6	2	0.47146	0.4286	-2
6	5	6	2	0.480032	0.4286	-2
6	5	6	2	0.488604	0.4286	-2
6	5	6	2	0.497176	0.4286	-2
6	5	6	2	0.505748	0.4286	-2
6	5	6	2	0.51432	0.4286	-2

Table 6.2: Line limit estimation results

Line From Number	Line To Number	Estimated value (MW)	True value (MW)	Feasibility flag (feasible=1)
1	4	80	100	1
1	4	82	100	1
1	4	84	100	1
1	4	86	100	1
1	4	88	100	1
1	4	90	100	1

1	4	92	100	1
1	4	94	100	1
1	4	96	100	1
1	4	98	100	1
1	4	100	100	1
1	4	102	100	-2
1	4	104	100	-2
1	4	106	100	-2
1	4	108	100	-2
1	4	110	100	-2
1	4	112	100	-2
1	4	114	100	-2
1	4	116	100	-2
1	4	118	100	-2
1	4	120	100	-2
2	4	160	200	1
2	4	164	200	1
2	4	168	200	1
2	4	172	200	1
2	4	176	200	1
2	4	180	200	1
2	4	184	200	1
2	4	188	200	1
2	4	192	200	1
2	4	196	200	1
2	4	200	200	1
2	4	204	200	-2
2	4	208	200	-2
2	4	212	200	-2
2	4	216	200	-2
2	4	220	200	-2
2	4	224	200	-2
2	4	228	200	-2
2	4	232	200	-2
2	4	236	200	-2
2	4	240	200	-2
6	5	80	100	1
6	5	82	100	1
6	5	84	100	1

6	5	86	100	1
6	5	88	100	1
6	5	90	100	1
6	5	92	100	1
6	5	94	100	1
6	5	96	100	1
6	5	98	100	1
6	5	100	100	1
6	5	102	100	-2
6	5	104	100	-2
6	5	106	100	-2
6	5	108	100	-2
6	5	110	100	-2
6	5	112	100	-2
6	5	114	100	-2
6	5	116	100	-2
6	5	118	100	-2
6	5	120	100	-2

We see from Tables 6.1 and 6.2 that the true values of all parameters are either identified on the border of feasibility-infeasibility region (namely “Border” behavior) or the optimization is feasible only for the true values of the parameters (namely “Exact” behavior).

6.2.2 Case 2 (18 buses, 23 transmission lines, 8 FTRs) with a negative bid

The system in Case 2 is a 18 bus system connected with 23 transmission lines. There are 8 FTRs defined for the auction. The case data are as follows:

Table 6.3: Transmission Line Data

#	From bus Number	To bus Number	X (PU)	Limit(MW)
1	1	18	0.025	1000
2	2	1	0.0411	600
3	2	3	0.0151	505
4	3	4	0.0213	500
5	4	5	0.0128	600
6	4	10	0.0129	500
7	5	8	0.0112	303.6

8	5	6	0.0026	1200
9	6	9	0.025	900
10	6	7	0.0092	900
11	7	8	0.0046	900
12	10	9	0.0101	371.2
13	11	10	0.0217	600
14	12	14	0.0195	400
15	12	11	0.0094	600
16	13	3	0.013	500
17	13	12	0.0089	600
18	14	9	0.025	900
19	15	12	0.0059	600
20	16	17	0.0323	600
21	16	2	0.0086	500
22	17	13	0.0147	500
23	18	8	0.025	900

Table 6.4: FTR input data and auction results

#	Type	Bid Offer	Inj. bus	With. bus	Max Quantity (MW)	Price ($/MW)	Auction MCP ($/MW)	Auction Quantity cleared (MW)
1	Obl	bid	2	8	1000	25	25	519.0366
2	Obl	bid	3	4	400	-2	-2	235.1622
3	Obl	offer	9	12	50	3	0.499799	0
4	Opt	bid	9	5	120	6	0.208813	120
5	Opt	bid	2	18	400	30	18.1402	400
6	FGR	bid	5	8	310	20	56.18553	0
7	FGR	bid	2	3	500	3	3	104.9485
8	FGR	bid	14	12	400	5	0	400
Optimum Objective function value = 27540.43								

Out of 184 PTDFs that can be estimated, we only provide the estimation results of PTDFs that their absolute value is larger than 0.1 and they are associated to obligation FTRs and binding transmission lines. The estimation results are provided in Tables 6.5 and 6.6.

Table 6.5: PTDF estimation results						
Line From Number	Line To Number	From Inj. Bus	To Inj. Bus	Estimated value	True value	Feasibility flag (feasible=1)
2	3	2	8	0.424	0.53	-2
2	3	2	8	0.4346	0.53	-2
2	3	2	8	0.4452	0.53	-2
2	3	2	8	0.4558	0.53	-2
2	3	2	8	0.4664	0.53	-2
2	3	2	8	0.477	0.53	-2
2	3	2	8	0.4876	0.53	-2
2	3	2	8	0.4982	0.53	-2
2	3	2	8	0.5088	0.53	-2
2	3	2	8	0.5194	0.53	-2
2	3	2	8	0.53	0.53	1
2	3	2	8	0.5406	0.53	-2
2	3	2	8	0.5512	0.53	-2
2	3	2	8	0.5618	0.53	-2
2	3	2	8	0.5724	0.53	-2
2	3	2	8	0.583	0.53	-2
2	3	2	8	0.5936	0.53	-2
2	3	2	8	0.6042	0.53	-2
2	3	2	8	0.6148	0.53	-2
2	3	2	8	0.6254	0.53	-2
2	3	2	8	0.636	0.53	-2
3	4	2	8	0.33896	0.4237	-2
3	4	2	8	0.347434	0.4237	-2
3	4	2	8	0.355908	0.4237	-2
3	4	2	8	0.364382	0.4237	-2
3	4	2	8	0.372856	0.4237	-2
3	4	2	8	0.38133	0.4237	-2
3	4	2	8	0.389804	0.4237	-2
3	4	2	8	0.398278	0.4237	-2
3	4	2	8	0.406752	0.4237	-2
3	4	2	8	0.415226	0.4237	-2
3	4	2	8	0.4237	0.4237	1
3	4	2	8	0.432174	0.4237	-2
3	4	2	8	0.440648	0.4237	-2
3	4	2	8	0.449122	0.4237	-2
3	4	2	8	0.457596	0.4237	-2

3	4	2	8	0.46607	0.4237	-2
3	4	2	8	0.474544	0.4237	-2
3	4	2	8	0.483018	0.4237	-2
3	4	2	8	0.491492	0.4237	-2
3	4	2	8	0.499966	0.4237	-2
3	4	2	8	0.50844	0.4237	-2
5	8	2	8	0.31488	0.3936	-2
5	8	2	8	0.322752	0.3936	-2
5	8	2	8	0.330624	0.3936	-2
5	8	2	8	0.338496	0.3936	-2
5	8	2	8	0.346368	0.3936	-2
5	8	2	8	0.35424	0.3936	-2
5	8	2	8	0.362112	0.3936	-2
5	8	2	8	0.369984	0.3936	-2
5	8	2	8	0.377856	0.3936	-2
5	8	2	8	0.385728	0.3936	-2
5	8	2	8	0.3936	0.3936	1
5	8	2	8	0.401472	0.3936	-2
5	8	2	8	0.409344	0.3936	-2
5	8	2	8	0.417216	0.3936	-2
5	8	2	8	0.425088	0.3936	-2
5	8	2	8	0.43296	0.3936	-2
5	8	2	8	0.440832	0.3936	-2
5	8	2	8	0.448704	0.3936	-2
5	8	2	8	0.456576	0.3936	-2
5	8	2	8	0.464448	0.3936	-2
5	8	2	8	0.47232	0.3936	-2
2	3	3	4	-0.09824	-0.1228	-2
2	3	3	4	-0.1007	-0.1228	-2
2	3	3	4	-0.10315	-0.1228	-2
2	3	3	4	-0.10561	-0.1228	-2
2	3	3	4	-0.10806	-0.1228	-2
2	3	3	4	-0.11052	-0.1228	-2
2	3	3	4	-0.11298	-0.1228	-2
2	3	3	4	-0.11543	-0.1228	-2
2	3	3	4	-0.11789	-0.1228	-2
2	3	3	4	-0.12034	-0.1228	-2
2	3	3	4	-0.1228	-0.1228	1
2	3	3	4	-0.12526	-0.1228	-2

2	3	3	4	-0.12771	-0.1228	-2
2	3	3	4	-0.13017	-0.1228	-2
2	3	3	4	-0.13262	-0.1228	-2
2	3	3	4	-0.13508	-0.1228	-2
2	3	3	4	-0.13754	-0.1228	-2
2	3	3	4	-0.13999	-0.1228	-2
2	3	3	4	-0.14245	-0.1228	-2
2	3	3	4	-0.1449	-0.1228	-2
2	3	3	4	-0.14736	-0.1228	-2
3	4	3	4	0.50664	0.6333	-2
3	4	3	4	0.519306	0.6333	-2
3	4	3	4	0.531972	0.6333	-2
3	4	3	4	0.544638	0.6333	-2
3	4	3	4	0.557304	0.6333	-2
3	4	3	4	0.56997	0.6333	-2
3	4	3	4	0.582636	0.6333	-2
3	4	3	4	0.595302	0.6333	-2
3	4	3	4	0.607968	0.6333	-2
3	4	3	4	0.620634	0.6333	-2
3	4	3	4	0.6333	0.6333	1
3	4	3	4	0.645966	0.6333	-2
3	4	3	4	0.658632	0.6333	-2
3	4	3	4	0.671298	0.6333	-2
3	4	3	4	0.683964	0.6333	-2
3	4	3	4	0.69663	0.6333	-2
3	4	3	4	0.709296	0.6333	-2
3	4	3	4	0.721962	0.6333	-2
3	4	3	4	0.734628	0.6333	-2
3	4	3	4	0.747294	0.6333	-2
3	4	3	4	0.75996	0.6333	-2
3	4	9	12	-0.17208	-0.2151	-2
3	4	9	12	-0.17638	-0.2151	-2
3	4	9	12	-0.18068	-0.2151	-2
3	4	9	12	-0.18499	-0.2151	-2
3	4	9	12	-0.18929	-0.2151	-2
3	4	9	12	-0.19359	-0.2151	-2
3	4	9	12	-0.19789	-0.2151	-2
3	4	9	12	-0.20219	-0.2151	-2
3	4	9	12	-0.2065	-0.2151	-2

3	4	9	12	-0.2108	-0.2151	-2
3	4	9	12	-0.2151	-0.2151	1
3	4	9	12	-0.2194	-0.2151	-2
3	4	9	12	-0.2237	-0.2151	-2
3	4	9	12	-0.22801	-0.2151	-2
3	4	9	12	-0.23231	-0.2151	-2
3	4	9	12	-0.23661	-0.2151	-2
3	4	9	12	-0.24091	-0.2151	-2
3	4	9	12	-0.24521	-0.2151	-2
3	4	9	12	-0.24952	-0.2151	-2
3	4	9	12	-0.25382	-0.2151	-2
3	4	9	12	-0.25812	-0.2151	-2

Table 6.6: Line limit estimation results

Line From Number	Line To Number	Estimated value (MW)	True value (MW)	Feasibility flag (feasible=1)
2	3	404	505	1
2	3	414.1	505	1
2	3	424.2	505	1
2	3	434.3	505	1
2	3	444.4	505	1
2	3	454.5	505	1
2	3	464.6	505	1
2	3	474.7	505	1
2	3	484.8	505	1
2	3	494.9	505	1
2	3	505	505	1
2	3	515.1	505	-2
2	3	525.2	505	-2
2	3	535.3	505	-2
2	3	545.4	505	-2
2	3	555.5	505	-2
2	3	565.6	505	-2
2	3	575.7	505	-2
2	3	585.8	505	-2
2	3	595.9	505	-2
2	3	606	505	-2
3	4	400	500	1
3	4	410	500	1
3	4	420	500	1

3	4	430	500	1
3	4	440	500	1
3	4	450	500	1
3	4	460	500	1
3	4	470	500	1
3	4	480	500	1
3	4	490	500	1
3	4	500	500	1
3	4	510	500	-2
3	4	520	500	-2
3	4	530	500	-2
3	4	540	500	-2
3	4	550	500	-2
3	4	560	500	-2
3	4	570	500	-2
3	4	580	500	-2
3	4	590	500	-2
3	4	600	500	-2
5	8	242.88	303.6	1
5	8	248.952	303.6	1
5	8	255.024	303.6	1
5	8	261.096	303.6	1
5	8	267.168	303.6	1
5	8	273.24	303.6	1
5	8	279.312	303.6	1
5	8	285.384	303.6	1
5	8	291.456	303.6	1
5	8	297.528	303.6	1
5	8	303.6	303.6	1
5	8	309.672	303.6	-2
5	8	315.744	303.6	-2
5	8	321.816	303.6	-2
5	8	327.888	303.6	-2
5	8	333.96	303.6	-2
5	8	340.032	303.6	-2
5	8	346.104	303.6	-2
5	8	352.176	303.6	-2
5	8	358.248	303.6	-2
5	8	364.32	303.6	-2

A similar estimation behavior is seen as that of Case 1 in which all the true values are successfully estimated and parameters show either a Border or an Exact estimation behavior.

6.2.3 Case 3 (618 buses, 500 transmission lines, 100 FTRs)

Finally we test the proposed estimation method in a large actual system extracted from US Eastern Interconnection Grid. The Case input data and estimation results are provided in appendix A-D. As discussed earlier, we only provide the estimation results of PTDFs that their absolute value is larger than 0.1 and they are associated to obligation FTRs and binding transmission lines. The results show the successful estimation of all the true values for PTDF and line limits. We also see either the Border or Exact estimation behaviors.

Chapter 7: Discussions

In this chapter first we will discuss the introduction of Pseudo obligations to address the special cases in estimation of PTDFs for Opt and FGR type FTRs. Then a Brute-Force method for simultaneous estimation of multiple PTDFs will be introduced.

7.1 Pseudo obligations

7.1.1 Addressing the special cases of some Options and FGRs

As discussed in Chapter 6, there are some special cases in estimating the PTDFs associated to some FTRs of type Opt and FGR. However, the method can successfully estimate the PTDFs associated with obligation FTRs. This encourages us to introduce additional obligation FTRs in the auction optimization problem for every FTR of type Opt and FGR that is a special case. Therefore, for every special Opt or FGR type FTR, a pseudo obligation FTR is designed for the same injection and withdrawal locations with maximum bid quantity of 0.05 MW and bid price of 0.05 $/MW. Then the PTDF that could not be estimated using Opt or FGR type FTRs would be estimated using the Pseudo obligation FTR. We would name these FTRs as pseudo obligations since they are not actual FTRs offered/bid by market participants and their impact on the solution is small. Note that we would need to provide a MCP for Pseudo obligations in the proposed estimation method. One way to obtain these MCPs, is replicating the auction optimization using the published auction MCPs and quantities. To do so, for each actual FTR we would use the cleared quantity as the max quantity and would change MCP by a small number and use it as the offer/bid prices for replicating the auction. Then we would use the new auction solution in estimating PTDFs. These additional FTRs should be designed such that the auction optimization solution after introduction of these FTRs remains close to the original auction solution.

7.1.2 Revisiting Case 2

In Chapter 6, we used the proposed estimation method for estimating PTDFs of Case 2 and provided the estimation results for Obl type FTRs. In this section, we introduce pseudo obligations for PTDFs associated with Opt and FGR type FTRs. We chose Case 2 rather than 1 and 3 for better illustration. Table 7.1 shows the original FTRs, the pseudo FTRs denoted by # b, and the auction optimization solution considering pseudo FTRs. Note that the offer/bids quantities are the cleared quantities from Table 6.4 with zeros replaced by ones and the prices are MCPs of Table 6.4 disturbed by about 0.05.

Table 7.1: FTR input data and auction results

#	Type	Bid Offer	Inj. bus	With. bus	Max Quantity (MW)	Price ($/MW)	Auction MCP ($/MW)	Auction Quantity cleared (MW)
1	Obl	bid	2	8	519.0366	25.05	24.9079	519.0366
2	Obl	bid	3	4	235.1622	-1.95	-2.1509	235.1622
3	Obl	offer	9	12	1	0.549	0.549	0.02
4	Opt	bid	9	5	120	0.259	0.1929	120
4-b	Obl	bid	9	5	0.05	0.05	-4.6303	0.05
5	Opt	bid	2	18	400	18.19	18.0734	400
5-b	Obl	bid	2	18	0.05	0.05	18.0734	0
6	FGR	bid	5	8	1	56.136	56.1356	0
6-b	Obl	bid	5	8	0.05	0.05	31.5198	0
7	FGR	bid	2	3	104.9485	3.05	3.05	104.9126
7-b	Obl	bid	2	3	0.05	0.05	-0.7888	0.05
8	FGR	bid	14	12	400	0.05	0	400
8-b	Obl	bid	14	12	0.05	0.05	-0.266	0.05

Finally, in Appendix E, we provide the estimation results of PTDFs that their absolute value is larger than 0.05 and they are associated to obligation FTRs and binding transmission lines.

7.2 Simultaneous estimation of multiple PTDFs

7.2.1 Brute-Force Method

So far we have used the proposed estimation method for estimation of one PTDF at a time. This implies that the value of all other PTDFs are known but the value of the single estimated PTDF. However, there might be cases that values of several PTDFs are unknown or are disturbed from the original value. We can use the proposed estimation method for simultaneous estimation of multiple uncertain PTDFs **if all of them have Exact estimation behavior** (Refer to Section 6.2.1 for definition of Exact behavior). To do so, we use a brute-force estimation approach. This method is the same as the estimation method introduced in Section 5.3 for estimation of a single PTDF with the difference that we would consider all possible combinations of values for multiple PTDFs by sweeping through appropriate ranges for each PTDF and solve the core optimization. The true value of parameters would be the set for which the optimization is feasible. Finally, note that if the PTDF group that we desire to estimate includes both Exact and Border PTDF types, then

- If the PTDF is of type Exact, the optimization will be feasible only at the true value of PTDF.
- If the PTDF is of type Border, the optimization will be feasible for more than one value for the PTDF including its true value.

7.2.2 Revisiting Case 1

We provide the simultaneous estimation of a few sets of PTDFs estimated in Case 1 and reported in Table 6.1. Note that we only provide simultaneous estimation results of sets of two and three PTDFs as it is adequate to draw our conclusion. Table 7.2 summarizes the PTDFs estimated in Table 6.1 and their estimation behavior.

Table 7.2: Estimated PTDFs and their behavior

#	Line From Number	Line To Number	From Inj. Bus	To Inj. Bus	True value	Estimation Behavior
1	1	4	1	5	0.5714	Border
2	2	4	1	5	0.1429	Border
3	6	5	1	5	0.2857	Border
4	1	4	6	2	-0.1429	Exact
5	2	4	6	2	-0.2857	Exact
6	6	5	6	2	0.4286	Exact

The brute-force method is used for simultaneous estimation of PTDF sets introduced in Table 7.3. The PTDF numbers in Table 7.3 are the numbers assigned in Table 7.2.

Table 7.3: PTDF sets

Set #	PTDF numbers included	Estimation Behaviors (B = Border, E = Exact)
1	(1,2)	(B,B)
2	(2,4)	(B,E)
3	(2,6)	(B,E)
4	(1,2,3)	(B,B,B)
5	(1,2,4)	(B,B,E)
6	(1,5,6)	(B,E,E)
7	(4,5,6)	(E,E,E)

The simulation results of simultaneous estimation of above PTDF sets are provided in Appendix F.

The results show the following patterns:

- In simultaneous estimation of PTDFs, all PTDFs in a set can be estimated exactly if all have Exact estimation behaviors.
- If there is at least one PTDF in the set with Border estimation behavior, then estimation would results in some feasibility ranges which include the true PTDF values. However still the true value of Exact PTDF types can be uniquely determined. Identifying the true values of Border PTDFs in a unique manner is not possible at this stage.

We revisit the Case 2 in the following to further examine the above patterns.

7.2.3 Revisiting Case 2

The PTDFs estimated in Case 2 and reported in Table 6.5 are summarized in Table 7.4.

Table 7.4: Estimated PTDFs and their behavior

#	Line From Number	Line To Number	From Inj. Bus	To Inj. Bus	True value	Estimation Behavior
1	2	3	2	8	0.53	Exact
2	3	4	2	8	0.4237	Exact
3	5	8	2	8	0.3936	Exact
4	2	3	3	4	-0.1228	Exact
5	3	4	3	4	0.6333	Exact
6	3	4	9	12	-0.2151	Exact

The brute-force method is used for simultaneous estimation of PTDF sets introduced in Table 7.5.

Table 7.5: PTDF sets

Set #	PTDF numbers included	Estimation Behaviors (B = Border, E = Exact)
1	(1,2)	(E,E)
2	(1,4)	(E,E)
3	(4,5)	(E,E)
4	(1,2,3)	(E,E,E)
5	(1,4,6)	(E,E,E)

6	(4,5,6)	(E,E,E)

The simulation results of simultaneous estimation of above PTDF sets are provided in Appendix G. Since all the PTDFs have Exact estimation behaviors, their true values can be uniquely estimated simultaneously.

Chapter 8: Concluding Remarks

In this dissertation, a mathematical framework is provided for estimation of unknown system parameters in an optimization problem without the need to know the uncertain system inputs that are modeled as part of the optimization parameters. This is accomplished by using the duality theory and by introducing additional optimization constraints from system output data which altogether help shrink the optimization feasible region and thus reveal the true value of the unknown parameters. Specifically for the field of electricity markets and power systems, the application of this framework would be extensive for gaining additional knowledge and conducting further research on FTR auction and electricity market optimization problems. From the industrial perspective, the contribution of our research work is to help regulators and auditors, market participants, investors, and retail customers obtain more insights about the market which would lead to a more transparent and efficient market. As elaborated in Chapter 4, the framework is beneficial to regulators and auditors as it can be used by them to identify market abuse due to generations withholding capacity. It is beneficial to market participants and retail consumers as it can improve market transparency due to making underlying system parameters known to all participants and thus lead to more efficient pricing with the hope of reducing the rates for retail consumers. It is also beneficial since it can be used by participants to construct FGR/FTR portfolios for hedging congestion risk and update the portfolio weights more frequently. Moreover, it can be used for post analysis of settlement charges to assure correct settlement calculations. It is beneficial to investors as it can help recognize the competitive injection locations in power grid for transmission congestion opportunities as well as for reducing the risk of congestion due to transactions involving a new unit by choosing an appropriate unit location. Finally, the proposed method can be modified to be suitable for any optimization problem with similar characteristics to FTR auction and DA market.

There are still some questions that need to be answered in our future research. It would be valuable to understand the impact of $n - 1$ contingency constraints on the estimation framework. These constraints are used to model the impact of transmission line outages on the flow of other transmission lines. According to (3.8) and (3.9), when a contingency constraint is binding in auction solution, the estimated PTDF would include the LODF and PTDF factors associated to the contingent transmission line. Further analysis and modification to the framework is required to eliminate the impact of contingent line LODF and PTDF factors from the target PTDF of non-contingent transmission line.

More research steps are needed to fully tailor the framework for identifying Day-ahead market power system parameters using market published results. This is because, unlike FTR auction, the day-ahead market is an optimization problem that co-optimizes energy and ancillary services simultaneously and the market outcomes are cleared quantities and prices for the two interrelated commodities. Moreover, the transmission system losses have to be considered in the day-ahead market estimation problem which adds to the complexity of the problem due to dependency of loss allocation factors and PTDFs on the choice of slack bus or distributed slack bus weights.

Although the general concepts of FTR auction and electricity markets are the same for all US ISOs, the implementation and design details are rather different. This requires us to modify the proposed framework for each US ISO implementation. For example, CAISO uses multi-point CRRs which are transmission rights defined between multiple injection and withdrawal points. The injection points are prioritized by the participant based on their estimated cost benefits to serve the withdrawal MW. The allocation processes starts from the first injection point unless there is a constraint violation. In this case, the injection point with the next priority will be used for allocation. Further investigation is needed to quantify the impact of this process on our framework. Another example is the differences in published auction results for various ISOs. For example ERCOT publishes the binding flowgate limits whereas PJM only publishes the shadow prices. Thus, for PJM implementation, we would need to consider the transmission line limits as part of the unknown variables in the core optimization problem introduced in Section 5.3.

Further research is needed to understand the impact of special estimation cases on system congestion and auction results. In our initial investigation, we found out that in none of the special cases observed in option contract PTDF estimation did the FTR contract have any business value or importance as it was used for an opposite direction in regard to the congestion direction. However, we still need to carefully investigate this on several test cases for various ISO implementations. To address these special cases, we suggested the use of pseudo obligations which are additional obligation contracts introduced to replace the special option contracts in PTDF estimation. One drawback of this idea is the need to calculate the MCPs for pseudo obligations which requires replication of FTR auction from published outputs. Thus, further research is needed to measure the reliability of replicating the auction for pseudo obligation MCPs and to quantify the sensitivity of auction solution to these obligations. Finally, it is important to characterize the impact of Exact and Border type PTDFs on congestion patterns and their sensitivity to injection/withdrawal locations.

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Appendix A: Case 3-Transmission line input

Table A.1: Transmission Line Data

#	From bus Number	To bus Number	X (PU)	Limit (MW)
1	335348	503304	0	20
2	336154	336153	0.01	600
3	336037	336080	0.08	115
4	337138	337139	0.02	136
5	337909	337910	0.02	945
6	337125	338400	0.07	195
7	338514	338502	0.36	36
8	338514	338501	0.36	36
9	338514	338503	0.36	36
10	335830	335831	0.01	1151
11	336820	336821	0.01	1650
12	336154	336153	0.01	600
13	337651	337653	0.01	910
14	336220	336221	0.03	270
15	338145	338146	0.01	910
16	337665	337666	0.1	50
17	337414	337419	0.14	110
18	337693	337694	0.65	12
19	337005	337007	0.09	193
20	337005	337006	0.09	193
21	337686	337690	0	598
22	337909	337911	0.01	1130
23	337692	337690	0.01	600
24	335825	335830	0	1194
25	337651	337652	0.01	910
26	337602	338514	0.07	114
27	336220	336222	0.02	465
28	337005	337008	0.07	223
29	337415	337449	0.13	84
30	337718	337720	0.23	34
31	335569	335571	0.08	215
32	335569	335570	0.08	215
33	337124	338400	0.12	115
34	335094	335102	0.11	38
35	337430	337427	0.08	215

36	337429	337427	0.08	215
37	337428	337427	0.08	215
38	337710	337712	0.1	50
39	337710	337711	0.1	50
40	335610	335612	0.04	256
41	334458	334455	0.06	217
42	337718	337719	0.25	34
43	337414	337418	0.24	60
44	337414	337417	0.24	60
45	334433	334430	0.01	495
46	336462	336464	0.01	640
47	335569	335572	0.08	215
48	335576	335577	0.33	53.5
49	335576	335578	0.33	53.5
50	337765	337763	0.06	312
51	337764	337763	0.06	312
52	336175	336177	0.11	150
53	336175	336176	0.11	150
54	336190	336191	0.02	650
55	338515	338506	0.96	12
56	338515	338504	0.96	12
57	338515	338505	0.96	12
58	334070	334072	0.03	312
59	335781	335782	0.09	93
60	337646	337648	0.02	253
61	334431	334430	0.05	256
62	334432	334430	0.05	256
63	337581	337582	0.01	112
64	334029	334030	0.04	200
65	334029	334031	0.04	200
66	337921	337927	0.04	223
67	338170	505420	0.04	148
68	334441	334434	0.02	583
69	337647	337648	0.04	255
70	337935	338482	0.43	50
71	336880	336882	0.04	143
72	336444	336446	0.03	300
73	335610	335611	0.05	200
74	338152	338143	0.01	530
75	338151	338734	0.04	148
76	335200	335203	0.06	192

77	335787	335788	0.09	20
78	337619	337647	0.05	239
79	334071	334072	0.03	330
80	334391	334392	0.08	210
81	334440	334434	0.02	680
82	335190	335206	0.01	706
83	338142	338143	0.01	530
84	335030	335031	0.01	39
85	335074	335075	0.07	225
86	335074	335076	0.07	225
87	337927	338424	0	223
88	337016	337019	0.06	120
89	337322	337326	0.24	52
90	337322	337323	0.24	52
91	337322	337325	0.24	52
92	337322	337324	0.24	52
93	337930	337931	0.03	283
94	338734	338879	0.06	148
95	334298	334326	0.15	112.5
96	334299	334326	0.15	112.5
97	337645	337646	0.04	266
98	337718	337746	0.08	111
99	336918	336919	0.02	161
100	336080	336081	0.04	115
101	336945	336919	0.03	600
102	335135	335190	0	532
103	338201	345534	0.74	22
104	337425	337420	0.09	220
105	337423	337420	0.09	220
106	337421	337420	0.09	220
107	336220	336230	0.02	320
108	337426	337420	0.14	140
109	337424	337420	0.14	140
110	337422	337420	0.14	140
111	335190	335204	0.02	680
112	335644	335643	0.08	205
113	337320	337322	0.22	219
114	336871	336880	0.03	231
115	337432	337427	0.07	250
116	335136	335137	0.1	200
117	335136	335137	0.1	200

118	337950	338016	0.03	239
119	334231	334233	0.1	120
120	334231	334232	0.1	120
121	337766	337763	0.06	390
122	334463	334467	0.1	107
123	337644	337645	0.02	266
124	336035	503305	0	120
125	335815	335782	0.06	200
126	334001	334002	0.09	33
127	334737	334741	0.01	72
128	335640	335639	0.13	112
129	335837	335771	0.01	1200
130	337005	337009	0	806
131	338200	344722	0.43	35
132	338813	505460	0.03	162
133	334412	334413	0.01	282
134	335569	335573	0	685
135	336220	336223	0.04	290
136	337740	337742	0.04	201
137	337949	338755	0.07	106
138	334000	334001	0.1	33
139	335573	335574	0	685
140	337331	337390	0.1	115
141	337137	337138	0.05	108
142	335135	335136	0	470
143	337162	360594	0	1172
144	336170	336166	0.04	490
145	336839	336880	0.03	560
146	337620	337644	0.01	239
147	303131	335336	0.03	19
148	338188	338388	0.02	1025
149	338229	338230	0.06	70
150	336280	336283	0.06	175
151	335574	335575	0	685
152	337815	337816	0.01	298
153	337449	337450	0.1	84
154	334413	334430	0.03	287
155	336938	336939	0	239
156	337420	337415	0.02	598
157	337665	337667	0.08	50
158	337686	337740	0	266

159	336555	336559	0.07	231
160	337561	337539	0.04	448
161	337620	337626	0.01	194
162	337625	337626	0.01	194
163	335576	335579	0	685
164	337600	337602	0.1	106
165	336553	336554	0.04	199
166	336554	336555	0.02	199
167	335521	335522	0.06	50
168	336167	336166	0.07	272
169	336168	336166	0.07	272
170	336169	336166	0.07	272
171	336069	336154	0.02	641
172	337931	337932	0	283
173	303000	335827	0	478
174	335022	335023	0	42
175	337804	337815	0.01	319
176	335217	335250	0.1	39
177	338151	338173	0.01	335
178	337180	337150	0.02	392
179	335642	335643	0	243
180	336839	336840	0.03	560
181	336839	336840	0.03	560
182	335647	335646	0.12	143
183	337826	337840	0.03	159
184	337808	337799	0.04	448
185	336778	336870	0.02	159
186	337808	337804	0.04	448
187	337827	337930	0.05	239
188	337382	337414	0.09	228
189	337023	337054	0.07	70
190	336870	336871	0.02	231
191	337583	338642	0.01	176
192	303000	335536	0.03	524
193	334414	334430	0.02	287
194	335513	335514	0	72
195	335513	335508	0.1	72
196	336907	336938	0.01	320
197	337620	337962	0.08	106
198	335336	335337	0.13	56
199	338006	338016	0.01	266

200	338151	338154	0.06	223
201	334026	334039	0.11	206
202	334026	334060	0.08	206
203	337392	337415	0	262
204	336882	336890	0.04	143
205	337162	337163	0.01	462
206	336917	336918	0.02	161
207	334084	334208	0.01	243
208	338044	338879	0.03	148
209	336033	336034	0.03	159
210	335610	335628	0.03	216
211	338172	338173	0	335
212	335713	335771	0.05	200
213	338004	338755	0.06	106
214	337818	337821	0.01	298
215	337929	337930	0.02	239
216	337009	337000	0.02	560
217	337650	337648	0.04	420
218	337650	337648	0.04	420
219	337650	337651	0	840
220	337020	337021	0.07	53
221	335642	335664	0	339
222	337414	337443	0.08	239
223	303325	337331	0.06	115
224	337150	337151	0.02	120
225	337962	338738	0.01	106
226	336230	336231	0.01	360
227	337420	337415	0.02	598
228	336800	336960	0.04	161
229	337742	337744	0.02	201
230	337162	337180	0	797
231	300115	338202	0.03	240
232	334228	334412	0.01	151
233	337922	337923	0.02	600
234	336558	336559	0.04	239
235	337918	337932	0.01	239
236	335713	335771	0.05	200
237	334396	334398	0.02	270
238	336906	336907	0.01	320
239	336939	336941	0.47	20
240	335750	335782	0.07	93

241	334397	334430	0.01	282
242	334560	334561	0.04	22.7
243	334500	334501	0.01	121
244	336925	336926	0.01	231
245	336925	336940	0.01	231
246	336919	336939	0.01	260
247	337826	337827	0.02	255
248	336043	336037	0.03	336
249	336280	336284	0.03	229
250	336919	336940	0.01	260
251	338022	338044	0.09	148
252	335785	335786	0.06	37.6
253	338140	338016	0.03	600
254	338228	338229	0.14	70
255	337028	337032	0.05	108
256	335575	335576	0	685
257	335346	335347	0.12	51
258	303021	334325	0	1215
259	335825	335827	0.01	673
260	336939	336941	0.48	20
261	335258	335259	0.14	33
262	335259	335266	0.1	33
263	335217	335253	0.2	33
264	336842	336890	0.02	392
265	336223	336224	0.03	174
266	336961	336976	0.02	111
267	334396	334397	0.04	282
268	336777	336778	0.02	161
269	336212	336213	0	572
270	334398	334430	0.03	287
271	336830	336800	0.03	560
272	336130	336131	0.01	1200
273	338154	338155	0.04	223
274	335699	335724	0.05	72
275	337800	337801	0.02	159
276	337340	337343	0.15	111
277	335815	335825	0.01	593
278	337304	500170	0.02	382
279	336552	336553	0.04	199
280	337816	337817	0	298
281	335786	335787	0.04	37.6

282	337909	337922	0.01	1732
283	335525	335527	0.06	72
284	335527	335508	0.1	72
285	337918	338484	0.04	255
286	335455	500720	0.04	287
287	336804	336962	0.01	231
288	337747	338659	0.14	106
289	335599	335600	0	287
290	337937	337930	0.03	420
291	337964	338738	0.01	106
292	336967	336968	0.08	87
293	337923	337925	0.05	303
294	338033	338883	0.03	148
295	336557	336558	0.03	239
296	335211	335217	0.05	72
297	336526	336559	0.04	161
298	337343	337341	0.03	300
299	337161	337163	0.01	462
300	335815	335825	0.01	593
301	335713	335771	0.06	200
302	335677	335678	0.03	121
303	336840	336841	0.01	433
304	337665	338852	0.06	159
305	335663	335664	0	361
306	335720	335721	0.05	93
307	336154	336250	0.03	1038
308	335366	335375	0.03	225
309	334058	334060	0.02	206
310	334226	334228	0.03	151
311	335749	335750	0.03	93
312	336084	336280	0.08	174
313	336042	336154	0.03	535
314	337382	337383	0.02	228
315	334193	334319	0.04	239
316	335411	335412	0.08	39
317	337857	337858	0.04	266
318	334641	334642	0.01	69
319	336840	336856	0.01	519
320	338188	360021	0	2546
321	334334	334335	0.01	145
322	335771	336140	0.04	703

323	336130	336562	0.02	1270
324	337685	337734	0.02	176
325	337928	338422	0.02	224
326	334333	334334	0.1	141
327	337674	337675	0.03	68
328	337804	337820	0	319
329	338200	344720	0.44	36
330	337733	337734	0.01	176
331	337858	337859	0.02	283
332	337516	337519	0.02	177
333	334204	334434	0.04	685
334	335829	503303	0	20
335	334588	334600	0.13	72
336	337581	337583	0.02	176
337	336800	336804	0.03	239
338	335836	388700	0.03	1800
339	335347	335348	0.06	51
340	334008	334020	0.2	50
341	338033	338155	0.04	223
342	338682	506932	0.02	210
343	334413	334414	0.01	287
344	337746	337747	0.01	118
345	335805	335815	0.02	300
346	337304	337341	0.03	319
347	338147	338148	0.01	246
348	334226	334227	0.06	151
349	335524	335525	0.02	72
350	335805	335815	0.02	300
351	334411	334412	0	241
352	337040	337042	0.03	392
353	338515	338744	0.01	223
354	338171	338172	0.06	363
355	336190	336210	0.01	1038
356	334039	334040	0.01	206
357	337304	500200	0.02	764
358	337116	337126	0.13	108
359	335742	335743	0.01	93
360	337032	337033	0.06	231
361	336556	336557	0.01	239
362	336007	336154	0.02	459
363	337064	337065	0.08	159

364	336442	336444	0.02	320
365	338215	338216	0.25	50
366	336525	336526	0.03	161
367	335627	335628	0.01	265
368	335796	335805	0.1	100
369	335782	335805	0.1	100
370	336019	336042	0.03	300
371	338142	338151	0.03	416
372	336213	336214	0.01	572
373	334628	334640	0.01	121
374	337009	337000	0.02	615
375	335375	335376	0.03	225
376	337136	337137	0.03	108
377	338170	338171	0.03	335
378	336154	336155	0	797
379	337144	337150	0.02	217
380	336418	336420	0.01	217
381	337744	337745	0.04	201
382	337967	337972	0.06	106
383	335618	335568	0.02	672
384	338162	338161	0.03	450
385	337928	337929	0.04	239
386	338138	338142	0.04	310
387	337624	337625	0.01	194
388	338006	338756	0.02	266
389	337860	337859	0.04	447
390	338123	505460	0	223
391	335428	500490	0.02	39
392	335409	335428	0.1	39
393	337937	337930	0.03	420
394	337937	337930	0.03	420
395	337740	337741	0.02	239
396	337950	337951	0.02	194
397	337799	337825	0.05	266
398	337686	337718	0.05	239
399	338148	338149	0.03	223
400	335023	335024	0.01	42
401	337391	337415	0	320
402	334680	334396	0.12	75
403	336960	336961	0.04	176
404	337390	337391	0.06	298

405	337050	337051	0.02	392
406	335125	335200	0.01	260
407	334316	334317	0.08	134
408	334072	334090	0.05	411
409	337821	337822	0.01	298
410	338202	338230	0.05	112
411	337662	338852	0.01	159
412	337964	338741	0.04	106
413	337664	337685	0.08	159
414	303321	337442	0	198
415	334152	334193	0.03	233
416	334501	334502	0.01	121
417	336081	336082	0.09	115
418	337051	337052	0.05	161
419	337817	337818	0.01	255
420	303321	337444	0.03	176
421	336554	336770	0.05	161
422	334093	334100	0.03	384
423	335600	335601	0.01	339
424	335376	335377	0	225
425	337824	337825	0.02	255
426	336015	336016	0.09	115
427	337415	338642	0.14	239
428	337685	337686	0.01	239
429	303005	335835	0	2048
430	335775	335776	0.03	203
431	335035	335045	0.01	121
432	337717	337718	0.02	159
433	336158	336166	0	917
434	336157	336166	0	917
435	334025	334026	0.06	206
436	334024	334025	0.03	206
437	337686	337695	0.01	201
438	336233	336234	0.03	239
439	334151	334152	0.04	233
440	336194	336196	0.01	524
441	335535	336015	0.05	121
442	334225	334227	0.06	151
443	334413	334628	0.1	100
444	334413	334627	0.1	100
445	337516	337518	0.06	159

446	334413	334626	0.1	100
447	338104	338121	0.07	156
448	334072	334086	0.01	382
449	337697	337705	0.03	201
450	337695	337697	0.04	201
451	336855	336856	0	519
452	336084	336085	0.08	176
453	334639	334640	0.01	121
454	334330	334331	0.01	72
455	335776	335805	0.03	241
456	335699	335710	0.04	121
457	337705	338739	0.02	159
458	336010	336011	0.02	800
459	337361	507788	0.01	80
460	303325	337330	0.04	113
461	338151	338152	0.03	416
462	338151	338152	0.03	416
463	334752	334753	0.02	121
464	336269	336462	0.01	640
465	337731	337733	0.01	176
466	335094	335095	0.03	65
467	337909	337914	0.02	672
468	337912	337914	0	672
469	337741	337800	0.06	239
470	337930	338760	0.07	255
471	337011	337032	0.02	392
472	335030	335035	0.01	121
473	336034	336035	0.02	286
474	337000	337011	0	393
475	335836	335837	0	2598
476	336236	336406	0	221
477	334250	334251	0.05	29.3
478	337371	337372	0	120
479	337663	337664	0.06	159
480	335752	335796	0.09	93
481	337310	500070	0.05	93
482	337392	337393	0.05	320
483	337530	338823	0.02	146
484	336236	336237	0.01	286
485	338875	503912	0.08	157
486	337957	338162	0.02	1732

487	336154	336190	0.03	580
488	338204	338706	0.02	247
489	337921	337923	0.01	558
490	337140	337139	0.02	392
491	334202	334203	0	749
492	337642	337745	0.08	201
493	334100	334234	0	750
494	334200	334234	0.02	750
495	336154	336190	0.03	580
496	336963	336965	0.02	151
497	337028	337031	0.12	108
498	337619	337621	0.01	219
499	338216	338217	0.02	42
500	335598	335599	0	287

Appendix B: Case 3 – FTR input data and auction optimization results

Table B.1: FTR input data and auction results

#	Type	Bid/Offer	Injection bus	Withdrawal bus	Max Quantity (MW)	Price ($/MW)	Auction MCP ($/MW)	Auction Quantity cleared (MW)
1	obl	bid	337692	336154	600	39.8098	39.8098	598.0
2	obl	bid	334433	336154	600	44.8186	44.8186	495.0
3	obl	bid	337765	336220	270	1.9843	33.4428	0.0
4	obl	bid	337764	337686	598	45.2357	45.2357	312.0
5	obl	offer	334070	336220	465	29.7798	29.7798	312.0
6	obl	bid	334431	335610	256	0.3647	30.7148	0.0
7	obl	bid	334432	336462	640	10.3174	29.1727	0.0
8	obl	bid	334441	336190	650	25.0785	29.3367	0.0
9	obl	offer	334071	337646	253	47.6629	25.9525	0.0
10	obl	bid	334440	337647	255	48.0689	25.9315	255.0
11	obl	bid	337930	336444	300	3.6687	33.6030	0.0
12	obl	bid	336945	338152	530	48.3826	27.5739	530.0
13	obl	bid	336220	335190	706	47.6442	28.7405	706.0
14	obl	bid	337766	338142	530	21.6957	30.2145	0.0
15	obl	offer	335569	337645	266	39.0154	24.8351	0.0
16	obl	bid	336220	335135	532	2.8037	28.7354	0.0
17	obl	bid	335573	335190	680	18.1969	27.5705	0.0
18	obl	bid	335135	337644	266	45.3655	26.7281	266.0
19	obl	bid	336170	334412	282	38.5714	38.3822	282.0
20	obl	offer	336839	334413	287	47.7721	29.5164	0.0
21	obl	bid	335574	336069	641	31.0657	28.3613	641.0
22	obl	bid	337815	303000	478	-3.0359	35.2912	0.0
23	opt	bid	337420	334414	287	41.7021	31.0578	287.0
24	opt	bid	337686	336907	320	46.3696	41.5657	320.0
25	opt	offer	337561	338006	266	32.3304	29.0782	0.0
26	opt	bid	335576	337392	262	36.6757	29.0964	262.0
27	opt	bid	336167	338172	335	35.8723	40.2490	0.0
28	opt	bid	336168	334396	270	16.5725	39.4594	0.0
29	opt	bid	336169	336906	320	31.0513	45.6228	0.0
30	obl	bid	337931	334397	282	4.4153	33.2231	0.0
31	obl	bid	337804	337826	255	33.8325	29.0959	255.0
32	obl	offer	338151	334396	282	-3.2492	31.4734	282.0
33	obl	bid	337180	334398	287	10.2308	11.4196	0.0

34	obl	bid	336839	335815	593	-2.4606	31.5680	0.0
35	obl	bid	336839	335455	287	0.3422	30.5362	0.0
36	obl	bid	337808	335599	287	40.2902	36.3462	287.0
37	obl	bid	337808	337343	300	33.2156	33.2390	0.0
38	obl	bid	303000	337161	462	12.4405	41.1815	0.0
39	opt	bid	337162	335815	593	47.2622	33.8218	593.0
40	opt	bid	337818	335663	361	-3.1055	35.1110	0.0
41	opt	bid	337009	336042	535	19.1309	32.5449	0.0
42	opt	offer	337650	337857	266	15.9857	29.6694	266.0
43	obl	bid	337650	337858	283	37.1034	29.3628	283.0
44	obl	bid	335642	334204	685	38.736	27.0203	685.0
45	obl	bid	336230	334413	287	5.278	28.4867	0.0
46	obl	bid	337420	335805	300	21.937	32.9747	0.0
47	obl	offer	337162	335805	300	19.5072	12.7209	0.0
48	obl	bid	337922	338171	363	30.5472	31.0458	0.0
49	obl	bid	336919	337304	764	34.0151	34.0151	35.3
50	obl	bid	336043	336007	459	36.5078	29.1737	459.0
51	obl	bid	336919	336442	320	10.1814	34.6190	0.0
52	obl	bid	338140	335627	265	32.3836	35.1515	0.0
53	obl	bid	335575	336019	300	31.0304	28.4847	300.0
54	obl	offer	335825	338170	335	3.9436	24.7715	335.0
55	obl	bid	336842	336154	797	1.5449	29.6560	0.0
56	obl	bid	336212	338138	310	22.41	22.4100	99.7
57	obl	bid	336830	337391	320	47.7859	47.7859	314.9
58	obl	bid	337304	337390	298	13.7212	40.9664	0.0
59	obl	bid	337816	335125	260	27.1897	33.4593	0.0
60	obl	bid	337918	334093	384	7.3097	32.9118	0.0
61	obl	bid	337937	335600	339	36.3197	36.3197	188.3
62	opt	bid	337923	337824	255	9.0302	29.1834	0.0
63	opt	bid	336840	336855	519	22.8276	29.0729	0.0
64	opt	bid	336840	338151	416	33.4492	29.4269	416.0
65	opt	bid	335771	338151	416	43.9997	29.7415	416.0
66	obl	bid	337804	336269	640	47.761	33.5767	640.0
67	obl	bid	337304	337912	672	25.0969	25.0969	164.7
68	obl	offer	337040	336236	286	2.6243	29.4295	286.0
69	obl	bid	338142	337921	558	3.2112	27.6509	0.0
70	obl	bid	336213	334202	749	9.163	25.1024	0.0
71	obl	bid	337009	334100	750	41.2394	30.2814	750.0
72	obl	bid	335618	335598	287	8.9855	29.8754	0.0
73	obl	bid	336462	336464	640	39.7857	34.3583	640.0
74	obl	bid	335825	335830	1194	8.3939	24.1947	0.0

75	FGR	offer	337910	337909	945	46.1095	0.0000	0.0
76	FGR	bid	335831	335830	1151	14.2491	7.1683	1151.0
77	FGR	bid	336821	336820	1650	5.8127	2.8882	1650.0
78	FGR	bid	337653	337651	910	8.8096	4.4139	910.0
79	FGR	bid	338146	338145	910	28.8825	14.4572	910.0
80	FGR	bid	337911	337909	1130	21.0309	10.5967	1130.0
81	FGR	bid	335830	335825	1194	14.3413	4.8317	1194.0
82	FGR	bid	337652	337651	910	40.6956	20.2751	910.0
83	FGR	bid	335837	335771	1200	27.1895	0.0000	1200.0
84	FGR	bid	337005	337009	806	25.2348	25.2348	706.0
85	FGR	bid	360594	337162	1172	45.4457	45.4457	1155.6
86	FGR	bid	338388	338188	1025	10.7211	5.3833	1025.0
87	FGR	offer	337651	337650	840	36.646	0.0000	0.0
88	FGR	offer	303021	334325	1215	36.4551	0.0000	0.0
89	FGR	offer	336130	336131	1200	15.9245	0.0000	0.0
90	FGR	bid	337909	337922	1732	26.2302	0.0000	1732.0
91	FGR	bid	336154	336250	1038	-0.828	0.0000	0.0
92	FGR	bid	338188	360021	2546	-2.0327	0.0000	0.0
93	FGR	bid	336562	336130	1270	24.1939	0.0000	1270.0
94	FGR	bid	388700	335836	1800	37.8542	37.8542	1663.7
95	FGR	bid	336190	336210	1038	46.3706	4.1415	1038.0
96	FGR	bid	303005	335835	2048	2.1448	0.0000	2048.0
97	FGR	offer	336166	336158	917	26.2853	0.0000	0.0
98	FGR	bid	336166	336157	917	20.8165	20.8165	776.0
99	FGR	bid	335836	335837	2598	-4.3454	0.0000	0.0
100	FGR	bid	337957	338162	1732	13.5417	13.5417	1585.8

Appendix C: Case 3 – PTDF estimation results

Table C.1: PTDF estimation results						
Line From Number	Line To Number	From Inj. Bus	To Inj. Bus	Estimated value	True value	Feasibility flag (feasible=1)
337686	337690	337692	336154	-0.8	-1	-2
337686	337690	337692	336154	-0.82	-1	-2
337686	337690	337692	336154	-0.84	-1	-2
337686	337690	337692	336154	-0.86	-1	-2
337686	337690	337692	336154	-0.88	-1	-2
337686	337690	337692	336154	-0.9	-1	-2
337686	337690	337692	336154	-0.92	-1	-2
337686	337690	337692	336154	-0.94	-1	-2
337686	337690	337692	336154	-0.96	-1	-2
337686	337690	337692	336154	-0.98	-1	-2
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337686	337690	337692	336154	-1	-1	1
337957	338162	337692	336154	0.098416	0.12302	-2
337957	338162	337692	336154	0.100876	0.12302	-2
337957	338162	337692	336154	0.103337	0.12302	-2
337957	338162	337692	336154	0.105797	0.12302	-2
337957	338162	337692	336154	0.108257	0.12302	-2
337957	338162	337692	336154	0.110718	0.12302	-2
337957	338162	337692	336154	0.113178	0.12302	-2
337957	338162	337692	336154	0.115639	0.12302	-2
337957	338162	337692	336154	0.118099	0.12302	-2
337957	338162	337692	336154	0.120559	0.12302	-2
337957	338162	337692	336154	0.12302	0.12302	1

337957	338162	337692	336154	0.12548	0.12302	1
337957	338162	337692	336154	0.127941	0.12302	1
337957	338162	337692	336154	0.130401	0.12302	1
337957	338162	337692	336154	0.132861	0.12302	1
337957	338162	337692	336154	0.135322	0.12302	1
337957	338162	337692	336154	0.137782	0.12302	1
337957	338162	337692	336154	0.140243	0.12302	1
337957	338162	337692	336154	0.142703	0.12302	1
337957	338162	337692	336154	0.145163	0.12302	1
337957	338162	337692	336154	0.147624	0.12302	1
334433	334430	334433	336154	0.8	1	-2
334433	334430	334433	336154	0.82	1	-2
334433	334430	334433	336154	0.84	1	-2
334433	334430	334433	336154	0.86	1	-2
334433	334430	334433	336154	0.88	1	-2
334433	334430	334433	336154	0.9	1	-2
334433	334430	334433	336154	0.92	1	-2
334433	334430	334433	336154	0.94	1	-2
334433	334430	334433	336154	0.96	1	-2
334433	334430	334433	336154	0.98	1	-2
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
334433	334430	334433	336154	1	1	1
337957	338162	337765	336220	0.105076	0.131345	-2
337957	338162	337765	336220	0.107703	0.131345	-2
337957	338162	337765	336220	0.11033	0.131345	-2
337957	338162	337765	336220	0.112957	0.131345	-2
337957	338162	337765	336220	0.115584	0.131345	-2
337957	338162	337765	336220	0.118211	0.131345	-2
337957	338162	337765	336220	0.120838	0.131345	-2
337957	338162	337765	336220	0.123465	0.131345	-2

337957	338162	337765	336220	0.126092	0.131345	-2
337957	338162	337765	336220	0.128718	0.131345	-2
337957	338162	337765	336220	0.131345	0.131345	1
337957	338162	337765	336220	0.133972	0.131345	-2
337957	338162	337765	336220	0.136599	0.131345	-2
337957	338162	337765	336220	0.139226	0.131345	-2
337957	338162	337765	336220	0.141853	0.131345	-2
337957	338162	337765	336220	0.14448	0.131345	-2
337957	338162	337765	336220	0.147107	0.131345	-2
337957	338162	337765	336220	0.149734	0.131345	-2
337957	338162	337765	336220	0.152361	0.131345	-2
337957	338162	337765	336220	0.154987	0.131345	-2
337957	338162	337765	336220	0.157614	0.131345	-2
337764	337763	337764	337686	0.8	1	-2
337764	337763	337764	337686	0.82	1	-2
337764	337763	337764	337686	0.84	1	-2
337764	337763	337764	337686	0.86	1	-2
337764	337763	337764	337686	0.88	1	-2
337764	337763	337764	337686	0.9	1	-2
337764	337763	337764	337686	0.92	1	-2
337764	337763	337764	337686	0.94	1	-2
337764	337763	337764	337686	0.96	1	-2
337764	337763	337764	337686	0.98	1	-2
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
337764	337763	337764	337686	1	1	1
336190	336210	334432	336462	0.114817	0.143521	-2
336190	336210	334432	336462	0.117687	0.143521	-2
336190	336210	334432	336462	0.120558	0.143521	-2
336190	336210	334432	336462	0.123428	0.143521	-2
336190	336210	334432	336462	0.126299	0.143521	-2

336190	336210	334432	336462	0.129169	0.143521	-2
336190	336210	334432	336462	0.132039	0.143521	-2
336190	336210	334432	336462	0.13491	0.143521	-2
336190	336210	334432	336462	0.13778	0.143521	-2
336190	336210	334432	336462	0.140651	0.143521	-2
336190	336210	334432	336462	0.143521	0.143521	1
336190	336210	334432	336462	0.146391	0.143521	-2
336190	336210	334432	336462	0.149262	0.143521	-2
336190	336210	334432	336462	0.152132	0.143521	-2
336190	336210	334432	336462	0.155003	0.143521	-2
336190	336210	334432	336462	0.157873	0.143521	-2
336190	336210	334432	336462	0.160744	0.143521	-2
336190	336210	334432	336462	0.163614	0.143521	-2
336190	336210	334432	336462	0.166484	0.143521	-2
336190	336210	334432	336462	0.169355	0.143521	-2
336190	336210	334432	336462	0.172225	0.143521	-2
336190	336210	337930	336444	0.112491	0.140614	-2
336190	336210	337930	336444	0.115304	0.140614	-2
336190	336210	337930	336444	0.118116	0.140614	-2
336190	336210	337930	336444	0.120928	0.140614	-2
336190	336210	337930	336444	0.12374	0.140614	-2
336190	336210	337930	336444	0.126553	0.140614	-2
336190	336210	337930	336444	0.129365	0.140614	-2
336190	336210	337930	336444	0.132177	0.140614	-2
336190	336210	337930	336444	0.13499	0.140614	-2
336190	336210	337930	336444	0.137802	0.140614	-2
336190	336210	337930	336444	0.140614	0.140614	1
336190	336210	337930	336444	0.143426	0.140614	-2
336190	336210	337930	336444	0.146239	0.140614	-2
336190	336210	337930	336444	0.149051	0.140614	-2
336190	336210	337930	336444	0.151863	0.140614	-2
336190	336210	337930	336444	0.154676	0.140614	-2
336190	336210	337930	336444	0.157488	0.140614	-2
336190	336210	337930	336444	0.1603	0.140614	-2
336190	336210	337930	336444	0.163112	0.140614	-2
336190	336210	337930	336444	0.165925	0.140614	-2
336190	336210	337930	336444	0.168737	0.140614	-2
337957	338162	337930	336444	0.108578	0.135723	-2
337957	338162	337930	336444	0.111293	0.135723	-2

337957	338162	337930	336444	0.114007	0.135723	-2
337957	338162	337930	336444	0.116722	0.135723	-2
337957	338162	337930	336444	0.119436	0.135723	-2
337957	338162	337930	336444	0.122151	0.135723	-2
337957	338162	337930	336444	0.124865	0.135723	-2
337957	338162	337930	336444	0.12758	0.135723	-2
337957	338162	337930	336444	0.130294	0.135723	-2
337957	338162	337930	336444	0.133008	0.135723	-2
337957	338162	337930	336444	0.135723	0.135723	1
337957	338162	337930	336444	0.138437	0.135723	-2
337957	338162	337930	336444	0.141152	0.135723	-2
337957	338162	337930	336444	0.143866	0.135723	-2
337957	338162	337930	336444	0.146581	0.135723	-2
337957	338162	337930	336444	0.149295	0.135723	-2
337957	338162	337930	336444	0.15201	0.135723	-2
337957	338162	337930	336444	0.154724	0.135723	-2
337957	338162	337930	336444	0.157439	0.135723	-2
337957	338162	337930	336444	0.160153	0.135723	-2
337957	338162	337930	336444	0.162867	0.135723	-2
337957	338162	337766	338142	0.127558	0.159447	-2
337957	338162	337766	338142	0.130747	0.159447	-2
337957	338162	337766	338142	0.133936	0.159447	-2
337957	338162	337766	338142	0.137125	0.159447	-2
337957	338162	337766	338142	0.140314	0.159447	-2
337957	338162	337766	338142	0.143503	0.159447	-2
337957	338162	337766	338142	0.146692	0.159447	-2
337957	338162	337766	338142	0.149881	0.159447	-2
337957	338162	337766	338142	0.15307	0.159447	-2
337957	338162	337766	338142	0.156258	0.159447	-2
337957	338162	337766	338142	0.159447	0.159447	1
337957	338162	337766	338142	0.162636	0.159447	-2
337957	338162	337766	338142	0.165825	0.159447	-2
337957	338162	337766	338142	0.169014	0.159447	-2
337957	338162	337766	338142	0.172203	0.159447	-2
337957	338162	337766	338142	0.175392	0.159447	-2
337957	338162	337766	338142	0.178581	0.159447	-2
337957	338162	337766	338142	0.18177	0.159447	-2
337957	338162	337766	338142	0.184959	0.159447	-2
337957	338162	337766	338142	0.188148	0.159447	-2
337957	338162	337766	338142	0.191337	0.159447	-2

336157	336166	336170	334412	-0.4	-0.5	-2
336157	336166	336170	334412	-0.41	-0.5	-2
336157	336166	336170	334412	-0.42	-0.5	-2
336157	336166	336170	334412	-0.43	-0.5	-2
336157	336166	336170	334412	-0.44	-0.5	-2
336157	336166	336170	334412	-0.45	-0.5	-2
336157	336166	336170	334412	-0.46	-0.5	-2
336157	336166	336170	334412	-0.47	-0.5	-2
336157	336166	336170	334412	-0.48	-0.5	-2
336157	336166	336170	334412	-0.49	-0.5	-2
336157	336166	336170	334412	-0.5	-0.5	1
336157	336166	336170	334412	-0.51	-0.5	-2
336157	336166	336170	334412	-0.52	-0.5	-2
336157	336166	336170	334412	-0.53	-0.5	-2
336157	336166	336170	334412	-0.54	-0.5	-2
336157	336166	336170	334412	-0.55	-0.5	-2
336157	336166	336170	334412	-0.56	-0.5	-2
336157	336166	336170	334412	-0.57	-0.5	-2
336157	336166	336170	334412	-0.58	-0.5	-2
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336190	336210	337040	336236	0.124824	0.117759	-2
336190	336210	337040	336236	0.12718	0.117759	-2
336190	336210	337040	336236	0.129535	0.117759	-2
336190	336210	337040	336236	0.13189	0.117759	-2
336190	336210	337040	336236	0.134245	0.117759	-2
336190	336210	337040	336236	0.1366	0.117759	-2
336190	336210	337040	336236	0.138955	0.117759	-2

336190	336210	337040	336236	0.141311	0.117759	-2
337957	338162	338142	337921	-0.11542	-0.14428	-2
337957	338162	338142	337921	-0.11831	-0.14428	-2
337957	338162	338142	337921	-0.12119	-0.14428	-2
337957	338162	338142	337921	-0.12408	-0.14428	-2
337957	338162	338142	337921	-0.12696	-0.14428	-2
337957	338162	338142	337921	-0.12985	-0.14428	-2
337957	338162	338142	337921	-0.13274	-0.14428	-2
337957	338162	338142	337921	-0.13562	-0.14428	-2
337957	338162	338142	337921	-0.13851	-0.14428	-2
337957	338162	338142	337921	-0.14139	-0.14428	-2
337957	338162	338142	337921	-0.14428	-0.14428	1
337957	338162	338142	337921	-0.14716	-0.14428	-2
337957	338162	338142	337921	-0.15005	-0.14428	-2
337957	338162	338142	337921	-0.15293	-0.14428	-2
337957	338162	338142	337921	-0.15582	-0.14428	-2
337957	338162	338142	337921	-0.15871	-0.14428	-2
337957	338162	338142	337921	-0.16159	-0.14428	-2
337957	338162	338142	337921	-0.16448	-0.14428	-2
337957	338162	338142	337921	-0.16736	-0.14428	-2
337957	338162	338142	337921	-0.17025	-0.14428	-2
337957	338162	338142	337921	-0.17313	-0.14428	-2
336190	336210	336213	334202	-0.6287	-0.78588	-2
336190	336210	336213	334202	-0.64442	-0.78588	-2
336190	336210	336213	334202	-0.66014	-0.78588	-2
336190	336210	336213	334202	-0.67586	-0.78588	-2
336190	336210	336213	334202	-0.69157	-0.78588	-2
336190	336210	336213	334202	-0.70729	-0.78588	-2
336190	336210	336213	334202	-0.72301	-0.78588	-2
336190	336210	336213	334202	-0.73873	-0.78588	-2
336190	336210	336213	334202	-0.75444	-0.78588	-2
336190	336210	336213	334202	-0.77016	-0.78588	-2
336190	336210	336213	334202	-0.78588	-0.78588	1
336190	336210	336213	334202	-0.8016	-0.78588	-2
336190	336210	336213	334202	-0.81731	-0.78588	-2
336190	336210	336213	334202	-0.83303	-0.78588	-2
336190	336210	336213	334202	-0.84875	-0.78588	-2
336190	336210	336213	334202	-0.86447	-0.78588	-2
336190	336210	336213	334202	-0.88018	-0.78588	-2

336190	336210	336213	334202	-0.8959	-0.78588	-2
336190	336210	336213	334202	-0.91162	-0.78588	-2
336190	336210	336213	334202	-0.92734	-0.78588	-2
336190	336210	336213	334202	-0.94305	-0.78588	-2
335600	335601	335618	335598	-0.53495	-0.66868	-2
335600	335601	335618	335598	-0.54832	-0.66868	-2
335600	335601	335618	335598	-0.56169	-0.66868	-2
335600	335601	335618	335598	-0.57507	-0.66868	-2
335600	335601	335618	335598	-0.58844	-0.66868	-2
335600	335601	335618	335598	-0.60182	-0.66868	-2
335600	335601	335618	335598	-0.61519	-0.66868	-2
335600	335601	335618	335598	-0.62856	-0.66868	-2
335600	335601	335618	335598	-0.64194	-0.66868	-2
335600	335601	335618	335598	-0.65531	-0.66868	-2
335600	335601	335618	335598	-0.66868	-0.66868	1
335600	335601	335618	335598	-0.68206	-0.66868	-2
335600	335601	335618	335598	-0.69543	-0.66868	-2
335600	335601	335618	335598	-0.7088	-0.66868	-2
335600	335601	335618	335598	-0.72218	-0.66868	-2
335600	335601	335618	335598	-0.73555	-0.66868	-2
335600	335601	335618	335598	-0.74893	-0.66868	-2
335600	335601	335618	335598	-0.7623	-0.66868	-2
335600	335601	335618	335598	-0.77567	-0.66868	-2
335600	335601	335618	335598	-0.78905	-0.66868	-2
335600	335601	335618	335598	-0.80242	-0.66868	-2
336462	336464	336462	336464	0.8	1	-2
336462	336464	336462	336464	0.82	1	-2
336462	336464	336462	336464	0.84	1	-2
336462	336464	336462	336464	0.86	1	-2
336462	336464	336462	336464	0.88	1	-2
336462	336464	336462	336464	0.9	1	-2
336462	336464	336462	336464	0.92	1	-2
336462	336464	336462	336464	0.94	1	-2
336462	336464	336462	336464	0.96	1	-2
336462	336464	336462	336464	0.98	1	-2
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1

336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
336462	336464	336462	336464	1	1	1
335825	335830	335825	335830	0.8	1	-2
335825	335830	335825	335830	0.82	1	-2
335825	335830	335825	335830	0.84	1	-2
335825	335830	335825	335830	0.86	1	-2
335825	335830	335825	335830	0.88	1	-2
335825	335830	335825	335830	0.9	1	-2
335825	335830	335825	335830	0.92	1	-2
335825	335830	335825	335830	0.94	1	-2
335825	335830	335825	335830	0.96	1	-2
335825	335830	335825	335830	0.98	1	-2
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1
335825	335830	335825	335830	1	1	1

Appendix D: Case 3 – Line limit estimation results

Table D.1: Line limit estimation results				
Line From Number	Line To Number	Estimated value (MW)	True value (MW)	Feasibility flag (feasible=1)
335830	335831	920.8	1151	1
335830	335831	943.82	1151	1
335830	335831	966.84	1151	1
335830	335831	989.86	1151	1
335830	335831	1012.88	1151	1
335830	335831	1035.9	1151	1
335830	335831	1058.92	1151	1
335830	335831	1081.94	1151	1
335830	335831	1104.96	1151	1
335830	335831	1127.98	1151	1
335830	335831	1151	1151	1
335830	335831	1174.02	1151	-2
335830	335831	1197.04	1151	-2
335830	335831	1220.06	1151	-2
335830	335831	1243.08	1151	-2
335830	335831	1266.1	1151	-2
335830	335831	1289.12	1151	-2
335830	335831	1312.14	1151	-2
335830	335831	1335.16	1151	-2
335830	335831	1358.18	1151	-2
335830	335831	1381.2	1151	-2
336820	336821	1320	1650	1
336820	336821	1353	1650	1
336820	336821	1386	1650	1
336820	336821	1419	1650	1
336820	336821	1452	1650	1
336820	336821	1485	1650	1
336820	336821	1518	1650	1
336820	336821	1551	1650	1
336820	336821	1584	1650	1
336820	336821	1617	1650	1
336820	336821	1650	1650	1
336820	336821	1683	1650	-2
336820	336821	1716	1650	-2

336820	336821	1749	1650	-2
336820	336821	1782	1650	-2
336820	336821	1815	1650	-2
336820	336821	1848	1650	-2
336820	336821	1881	1650	-2
336820	336821	1914	1650	-2
336820	336821	1947	1650	-2
336820	336821	1980	1650	-2
337651	337653	728	910	1
337651	337653	746.2	910	1
337651	337653	764.4	910	1
337651	337653	782.6	910	1
337651	337653	800.8	910	1
337651	337653	819	910	1
337651	337653	837.2	910	1
337651	337653	855.4	910	1
337651	337653	873.6	910	1
337651	337653	891.8	910	1
337651	337653	910	910	1
337651	337653	928.2	910	-2
337651	337653	946.4	910	-2
337651	337653	964.6	910	-2
337651	337653	982.8	910	-2
337651	337653	1001	910	-2
337651	337653	1019.2	910	-2
337651	337653	1037.4	910	-2
337651	337653	1055.6	910	-2
337651	337653	1073.8	910	-2
337651	337653	1092	910	-2
338145	338146	728	910	1
338145	338146	746.2	910	1
338145	338146	764.4	910	1
338145	338146	782.6	910	1
338145	338146	800.8	910	1
338145	338146	819	910	1
338145	338146	837.2	910	1
338145	338146	855.4	910	1
338145	338146	873.6	910	1
338145	338146	891.8	910	1

338145	338146	910	910	1
338145	338146	928.2	910	-2
338145	338146	946.4	910	-2
338145	338146	964.6	910	-2
338145	338146	982.8	910	-2
338145	338146	1001	910	-2
338145	338146	1019.2	910	-2
338145	338146	1037.4	910	-2
338145	338146	1055.6	910	-2
338145	338146	1073.8	910	-2
338145	338146	1092	910	-2
337686	337690	478.4	598	1
337686	337690	490.36	598	1
337686	337690	502.32	598	1
337686	337690	514.28	598	1
337686	337690	526.24	598	1
337686	337690	538.2	598	1
337686	337690	550.16	598	1
337686	337690	562.12	598	1
337686	337690	574.08	598	1
337686	337690	586.04	598	1
337686	337690	598	598	1
337686	337690	609.96	598	-2
337686	337690	621.92	598	-2
337686	337690	633.88	598	-2
337686	337690	645.84	598	-2
337686	337690	657.8	598	-2
337686	337690	669.76	598	-2
337686	337690	681.72	598	-2
337686	337690	693.68	598	-2
337686	337690	705.64	598	-2
337686	337690	717.6	598	-2
337909	337911	904	1130	1
337909	337911	926.6	1130	1
337909	337911	949.2	1130	1
337909	337911	971.8	1130	1
337909	337911	994.4	1130	1
337909	337911	1017	1130	1
337909	337911	1039.6	1130	1

337909	337911	1062.2	1130	1
337909	337911	1084.8	1130	1
337909	337911	1107.4	1130	1
337909	337911	1130	1130	1
337909	337911	1152.6	1130	-2
337909	337911	1175.2	1130	-2
337909	337911	1197.8	1130	-2
337909	337911	1220.4	1130	-2
337909	337911	1243	1130	-2
337909	337911	1265.6	1130	-2
337909	337911	1288.2	1130	-2
337909	337911	1310.8	1130	-2
337909	337911	1333.4	1130	-2
337909	337911	1356	1130	-2
335825	335830	955.2	1194	1
335825	335830	979.08	1194	1
335825	335830	1002.96	1194	1
335825	335830	1026.84	1194	1
335825	335830	1050.72	1194	1
335825	335830	1074.6	1194	1
335825	335830	1098.48	1194	1
335825	335830	1122.36	1194	1
335825	335830	1146.24	1194	1
335825	335830	1170.12	1194	1
335825	335830	1194	1194	1
335825	335830	1217.88	1194	-2
335825	335830	1241.76	1194	-2
335825	335830	1265.64	1194	-2
335825	335830	1289.52	1194	-2
335825	335830	1313.4	1194	-2
335825	335830	1337.28	1194	-2
335825	335830	1361.16	1194	-2
335825	335830	1385.04	1194	-2
335825	335830	1408.92	1194	-2
335825	335830	1432.8	1194	-2
337651	337652	728	910	1
337651	337652	746.2	910	1
337651	337652	764.4	910	1
337651	337652	782.6	910	1

337651	337652	800.8	910	1
337651	337652	819	910	1
337651	337652	837.2	910	1
337651	337652	855.4	910	1
337651	337652	873.6	910	1
337651	337652	891.8	910	1
337651	337652	910	910	1
337651	337652	928.2	910	-2
337651	337652	946.4	910	-2
337651	337652	964.6	910	-2
337651	337652	982.8	910	-2
337651	337652	1001	910	-2
337651	337652	1019.2	910	-2
337651	337652	1037.4	910	-2
337651	337652	1055.6	910	-2
337651	337652	1073.8	910	-2
337651	337652	1092	910	-2
334433	334430	396	495	1
334433	334430	405.9	495	1
334433	334430	415.8	495	1
334433	334430	425.7	495	1
334433	334430	435.6	495	1
334433	334430	445.5	495	1
334433	334430	455.4	495	1
334433	334430	465.3	495	1
334433	334430	475.2	495	1
334433	334430	485.1	495	1
334433	334430	495	495	1
334433	334430	504.9	495	-2
334433	334430	514.8	495	-2
334433	334430	524.7	495	-2
334433	334430	534.6	495	-2
334433	334430	544.5	495	-2
334433	334430	554.4	495	-2
334433	334430	564.3	495	-2
334433	334430	574.2	495	-2
334433	334430	584.1	495	-2
334433	334430	594	495	-2
336462	336464	512	640	1

336462	336464	524.8	640	1
336462	336464	537.6	640	1
336462	336464	550.4	640	1
336462	336464	563.2	640	1
336462	336464	576	640	1
336462	336464	588.8	640	1
336462	336464	601.6	640	1
336462	336464	614.4	640	1
336462	336464	627.2	640	1
336462	336464	640	640	1
336462	336464	652.8	640	-2
336462	336464	665.6	640	-2
336462	336464	678.4	640	-2
336462	336464	691.2	640	-2
336462	336464	704	640	-2
336462	336464	716.8	640	-2
336462	336464	729.6	640	-2
336462	336464	742.4	640	-2
336462	336464	755.2	640	-2
336462	336464	768	640	-2
337764	337763	249.6	312	1
337764	337763	255.84	312	1
337764	337763	262.08	312	1
337764	337763	268.32	312	1
337764	337763	274.56	312	1
337764	337763	280.8	312	1
337764	337763	287.04	312	1
337764	337763	293.28	312	1
337764	337763	299.52	312	1
337764	337763	305.76	312	1
337764	337763	312	312	1
337764	337763	318.24	312	-2
337764	337763	324.48	312	-2
337764	337763	330.72	312	-2
337764	337763	336.96	312	-2
337764	337763	343.2	312	-2
337764	337763	349.44	312	-2
337764	337763	355.68	312	-2
337764	337763	361.92	312	-2
337764	337763	368.16	312	-2

337764	337763	374.4	312	-2
337005	337009	644.8	806	1
337005	337009	660.92	806	1
337005	337009	677.04	806	1
337005	337009	693.16	806	1
337005	337009	709.28	806	1
337005	337009	725.4	806	1
337005	337009	741.52	806	1
337005	337009	757.64	806	1
337005	337009	773.76	806	1
337005	337009	789.88	806	1
337005	337009	806	806	1
337005	337009	822.12	806	-2
337005	337009	838.24	806	-2
337005	337009	854.36	806	-2
337005	337009	870.48	806	-2
337005	337009	886.6	806	-2
337005	337009	902.72	806	-2
337005	337009	918.84	806	-2
337005	337009	934.96	806	-2
337005	337009	951.08	806	-2
337005	337009	967.2	806	-2
337162	360594	937.6	1172	1
337162	360594	961.04	1172	1
337162	360594	984.48	1172	1
337162	360594	1007.92	1172	1
337162	360594	1031.36	1172	1
337162	360594	1054.8	1172	1
337162	360594	1078.24	1172	1
337162	360594	1101.68	1172	1
337162	360594	1125.12	1172	1
337162	360594	1148.56	1172	1
337162	360594	1172	1172	1
337162	360594	1195.44	1172	-2
337162	360594	1218.88	1172	-2
337162	360594	1242.32	1172	-2
337162	360594	1265.76	1172	-2
337162	360594	1289.2	1172	-2
337162	360594	1312.64	1172	-2

337162	360594	1336.08	1172	-2
337162	360594	1359.52	1172	-2
337162	360594	1382.96	1172	-2
337162	360594	1406.4	1172	-2
338188	338388	820	1025	1
338188	338388	840.5	1025	1
338188	338388	861	1025	1
338188	338388	881.5	1025	1
338188	338388	902	1025	1
338188	338388	922.5	1025	1
338188	338388	943	1025	1
338188	338388	963.5	1025	1
338188	338388	984	1025	1
338188	338388	1004.5	1025	1
338188	338388	1025	1025	1
338188	338388	1045.5	1025	-2
338188	338388	1066	1025	-2
338188	338388	1086.5	1025	-2
338188	338388	1107	1025	-2
338188	338388	1127.5	1025	-2
338188	338388	1148	1025	-2
338188	338388	1168.5	1025	-2
338188	338388	1189	1025	-2
338188	338388	1209.5	1025	-2
338188	338388	1230	1025	-2
336938	336939	191.2	239	1
336938	336939	195.98	239	1
336938	336939	200.76	239	1
336938	336939	205.54	239	1
336938	336939	210.32	239	1
336938	336939	215.1	239	1
336938	336939	219.88	239	1
336938	336939	224.66	239	1
336938	336939	229.44	239	1
336938	336939	234.22	239	1
336938	336939	239	239	1
336938	336939	243.78	239	-2
336938	336939	248.56	239	-2
336938	336939	253.34	239	-2

336938	336939	258.12	239	-2
336938	336939	262.9	239	-2
336938	336939	267.68	239	-2
336938	336939	272.46	239	-2
336938	336939	277.24	239	-2
336938	336939	282.02	239	-2
336938	336939	286.8	239	-2
335836	388700	1440	1800	1
335836	388700	1476	1800	1
335836	388700	1512	1800	1
335836	388700	1548	1800	1
335836	388700	1584	1800	1
335836	388700	1620	1800	1
335836	388700	1656	1800	1
335836	388700	1692	1800	1
335836	388700	1728	1800	1
335836	388700	1764	1800	1
335836	388700	1800	1800	1
335836	388700	1836	1800	-2
335836	388700	1872	1800	-2
335836	388700	1908	1800	-2
335836	388700	1944	1800	-2
335836	388700	1980	1800	-2
335836	388700	2016	1800	-2
335836	388700	2052	1800	-2
335836	388700	2088	1800	-2
335836	388700	2124	1800	-2
335836	388700	2160	1800	-2
336190	336210	830.4	1038	1
336190	336210	851.16	1038	1
336190	336210	871.92	1038	1
336190	336210	892.68	1038	1
336190	336210	913.44	1038	1
336190	336210	934.2	1038	1
336190	336210	954.96	1038	1
336190	336210	975.72	1038	1
336190	336210	996.48	1038	1
336190	336210	1017.24	1038	1
336190	336210	1038	1038	1

336190	336210	1058.76	1038	-2
336190	336210	1079.52	1038	-2
336190	336210	1100.28	1038	-2
336190	336210	1121.04	1038	-2
336190	336210	1141.8	1038	-2
336190	336210	1162.56	1038	-2
336190	336210	1183.32	1038	-2
336190	336210	1204.08	1038	-2
336190	336210	1224.84	1038	-2
336190	336210	1245.6	1038	-2
335782	335805	80	100	1
335782	335805	82	100	1
335782	335805	84	100	1
335782	335805	86	100	1
335782	335805	88	100	1
335782	335805	90	100	1
335782	335805	92	100	1
335782	335805	94	100	1
335782	335805	96	100	1
335782	335805	98	100	1
335782	335805	100	100	1
335782	335805	102	100	-2
335782	335805	104	100	-2
335782	335805	106	100	-2
335782	335805	108	100	-2
335782	335805	110	100	-2
335782	335805	112	100	-2
335782	335805	114	100	-2
335782	335805	116	100	-2
335782	335805	118	100	-2
335782	335805	120	100	-2
337391	337415	256	320	1
337391	337415	262.4	320	1
337391	337415	268.8	320	1
337391	337415	275.2	320	1
337391	337415	281.6	320	1
337391	337415	288	320	1
337391	337415	294.4	320	1
337391	337415	300.8	320	1

337391	337415	307.2	320	1
337391	337415	313.6	320	1
337391	337415	320	320	1
337391	337415	326.4	320	-2
337391	337415	332.8	320	-2
337391	337415	339.2	320	-2
337391	337415	345.6	320	-2
337391	337415	352	320	-2
337391	337415	358.4	320	-2
337391	337415	364.8	320	-2
337391	337415	371.2	320	-2
337391	337415	377.6	320	-2
337391	337415	384	320	-2
335600	335601	271.2	339	1
335600	335601	277.98	339	1
335600	335601	284.76	339	1
335600	335601	291.54	339	1
335600	335601	298.32	339	1
335600	335601	305.1	339	1
335600	335601	311.88	339	1
335600	335601	318.66	339	1
335600	335601	325.44	339	1
335600	335601	332.22	339	1
335600	335601	339	339	1
335600	335601	345.78	339	-2
335600	335601	352.56	339	-2
335600	335601	359.34	339	-2
335600	335601	366.12	339	-2
335600	335601	372.9	339	-2
335600	335601	379.68	339	-2
335600	335601	386.46	339	-2
335600	335601	393.24	339	-2
335600	335601	400.02	339	-2
335600	335601	406.8	339	-2
336157	336166	733.6	917	1
336157	336166	751.94	917	1
336157	336166	770.28	917	1
336157	336166	788.62	917	1
336157	336166	806.96	917	1

336157	336166	825.3	917	1
336157	336166	843.64	917	1
336157	336166	861.98	917	1
336157	336166	880.32	917	1
336157	336166	898.66	917	1
336157	336166	917	917	1
336157	336166	935.34	917	-2
336157	336166	953.68	917	-2
336157	336166	972.02	917	-2
336157	336166	990.36	917	-2
336157	336166	1008.7	917	-2
336157	336166	1027.04	917	-2
336157	336166	1045.38	917	-2
336157	336166	1063.72	917	-2
336157	336166	1082.06	917	-2
336157	336166	1100.4	917	-2
337957	338162	1385.6	1732	1
337957	338162	1420.24	1732	1
337957	338162	1454.88	1732	1
337957	338162	1489.52	1732	1
337957	338162	1524.16	1732	1
337957	338162	1558.8	1732	1
337957	338162	1593.44	1732	1
337957	338162	1628.08	1732	1
337957	338162	1662.72	1732	1
337957	338162	1697.36	1732	1
337957	338162	1732	1732	1
337957	338162	1766.64	1732	-2
337957	338162	1801.28	1732	-2
337957	338162	1835.92	1732	-2
337957	338162	1870.56	1732	-2
337957	338162	1905.2	1732	-2
337957	338162	1939.84	1732	-2
337957	338162	1974.48	1732	-2
337957	338162	2009.12	1732	-2
337957	338162	2043.76	1732	-2
337957	338162	2078.4	1732	-2

Appendix E: Modified Case 2 with Pseudo obligations

Table E.1: PTDF estimation results						
Line From Number	Line To Number	From Inj. Bus	To Inj. Bus	Estimated value	True value	Feasibility flag (feasible=1)
2	3	2	8	0.424	0.53	-2
2	3	2	8	0.4346	0.53	-2
2	3	2	8	0.4452	0.53	-2
2	3	2	8	0.4558	0.53	-2
2	3	2	8	0.4664	0.53	-2
2	3	2	8	0.477	0.53	-2
2	3	2	8	0.4876	0.53	-2
2	3	2	8	0.4982	0.53	-2
2	3	2	8	0.5088	0.53	-2
2	3	2	8	0.5194	0.53	-2
2	3	2	8	0.53	0.53	1
2	3	2	8	0.5406	0.53	-2
2	3	2	8	0.5512	0.53	-2
2	3	2	8	0.5618	0.53	-2
2	3	2	8	0.5724	0.53	-2
2	3	2	8	0.583	0.53	-2
2	3	2	8	0.5936	0.53	-2
2	3	2	8	0.6042	0.53	-2
2	3	2	8	0.6148	0.53	-2
2	3	2	8	0.6254	0.53	-2
2	3	2	8	0.636	0.53	-2
3	4	2	8	0.33896	0.4237	-2
3	4	2	8	0.347434	0.4237	-2
3	4	2	8	0.355908	0.4237	-2
3	4	2	8	0.364382	0.4237	-2
3	4	2	8	0.372856	0.4237	-2
3	4	2	8	0.38133	0.4237	-2
3	4	2	8	0.389804	0.4237	-2
3	4	2	8	0.398278	0.4237	-2
3	4	2	8	0.406752	0.4237	-2
3	4	2	8	0.415226	0.4237	-2
3	4	2	8	0.4237	0.4237	1
3	4	2	8	0.432174	0.4237	-2

3	4	2	8	0.440648	0.4237	-2
3	4	2	8	0.449122	0.4237	-2
3	4	2	8	0.457596	0.4237	-2
3	4	2	8	0.46607	0.4237	-2
3	4	2	8	0.474544	0.4237	-2
3	4	2	8	0.483018	0.4237	-2
3	4	2	8	0.491492	0.4237	-2
3	4	2	8	0.499966	0.4237	-2
3	4	2	8	0.50844	0.4237	-2
5	8	2	8	0.31488	0.3936	-2
5	8	2	8	0.322752	0.3936	-2
5	8	2	8	0.330624	0.3936	-2
5	8	2	8	0.338496	0.3936	-2
5	8	2	8	0.346368	0.3936	-2
5	8	2	8	0.35424	0.3936	-2
5	8	2	8	0.362112	0.3936	-2
5	8	2	8	0.369984	0.3936	-2
5	8	2	8	0.377856	0.3936	-2
5	8	2	8	0.385728	0.3936	-2
5	8	2	8	0.3936	0.3936	1
5	8	2	8	0.401472	0.3936	-2
5	8	2	8	0.409344	0.3936	-2
5	8	2	8	0.417216	0.3936	-2
5	8	2	8	0.425088	0.3936	-2
5	8	2	8	0.43296	0.3936	-2
5	8	2	8	0.440832	0.3936	-2
5	8	2	8	0.448704	0.3936	-2
5	8	2	8	0.456576	0.3936	-2
5	8	2	8	0.464448	0.3936	-2
5	8	2	8	0.47232	0.3936	-2
2	3	3	4	-0.09824	-0.1228	-2
2	3	3	4	-0.1007	-0.1228	-2
2	3	3	4	-0.10315	-0.1228	-2
2	3	3	4	-0.10561	-0.1228	-2
2	3	3	4	-0.10806	-0.1228	-2
2	3	3	4	-0.11052	-0.1228	-2
2	3	3	4	-0.11298	-0.1228	-2
2	3	3	4	-0.11543	-0.1228	-2
2	3	3	4	-0.11789	-0.1228	-2

2	3	3	4	-0.12034	-0.1228	-2
2	3	3	4	-0.1228	-0.1228	1
2	3	3	4	-0.12526	-0.1228	-2
2	3	3	4	-0.12771	-0.1228	-2
2	3	3	4	-0.13017	-0.1228	-2
2	3	3	4	-0.13262	-0.1228	-2
2	3	3	4	-0.13508	-0.1228	-2
2	3	3	4	-0.13754	-0.1228	-2
2	3	3	4	-0.13999	-0.1228	-2
2	3	3	4	-0.14245	-0.1228	-2
2	3	3	4	-0.1449	-0.1228	-2
2	3	3	4	-0.14736	-0.1228	-2
3	4	3	4	0.50664	0.6333	-2
3	4	3	4	0.519306	0.6333	-2
3	4	3	4	0.531972	0.6333	-2
3	4	3	4	0.544638	0.6333	-2
3	4	3	4	0.557304	0.6333	-2
3	4	3	4	0.56997	0.6333	-2
3	4	3	4	0.582636	0.6333	-2
3	4	3	4	0.595302	0.6333	-2
3	4	3	4	0.607968	0.6333	-2
3	4	3	4	0.620634	0.6333	-2
3	4	3	4	0.6333	0.6333	1
3	4	3	4	0.645966	0.6333	-2
3	4	3	4	0.658632	0.6333	-2
3	4	3	4	0.671298	0.6333	-2
3	4	3	4	0.683964	0.6333	-2
3	4	3	4	0.69663	0.6333	-2
3	4	3	4	0.709296	0.6333	-2
3	4	3	4	0.721962	0.6333	-2
3	4	3	4	0.734628	0.6333	-2
3	4	3	4	0.747294	0.6333	-2
3	4	3	4	0.75996	0.6333	-2
5	8	3	4	-0.0508	-0.0635	-2
5	8	3	4	-0.05207	-0.0635	-2
5	8	3	4	-0.05334	-0.0635	-2
5	8	3	4	-0.05461	-0.0635	-2
5	8	3	4	-0.05588	-0.0635	-2
5	8	3	4	-0.05715	-0.0635	-2

5	8	3	4	-0.05842	-0.0635	-2
5	8	3	4	-0.05969	-0.0635	-2
5	8	3	4	-0.06096	-0.0635	-2
5	8	3	4	-0.06223	-0.0635	-2
5	8	3	4	-0.0635	-0.0635	1
5	8	3	4	-0.06477	-0.0635	-2
5	8	3	4	-0.06604	-0.0635	-2
5	8	3	4	-0.06731	-0.0635	-2
5	8	3	4	-0.06858	-0.0635	-2
5	8	3	4	-0.06985	-0.0635	-2
5	8	3	4	-0.07112	-0.0635	-2
5	8	3	4	-0.07239	-0.0635	-2
5	8	3	4	-0.07366	-0.0635	-2
5	8	3	4	-0.07493	-0.0635	-2
5	8	3	4	-0.0762	-0.0635	-2
3	4	9	12	-0.17208	-0.2151	-2
3	4	9	12	-0.17638	-0.2151	-2
3	4	9	12	-0.18068	-0.2151	-2
3	4	9	12	-0.18499	-0.2151	-2
3	4	9	12	-0.18929	-0.2151	-2
3	4	9	12	-0.19359	-0.2151	-2
3	4	9	12	-0.19789	-0.2151	-2
3	4	9	12	-0.20219	-0.2151	-2
3	4	9	12	-0.2065	-0.2151	-2
3	4	9	12	-0.2108	-0.2151	-2
3	4	9	12	-0.2151	-0.2151	1
3	4	9	12	-0.2194	-0.2151	-2
3	4	9	12	-0.2237	-0.2151	-2
3	4	9	12	-0.22801	-0.2151	-2
3	4	9	12	-0.23231	-0.2151	-2
3	4	9	12	-0.23661	-0.2151	-2
3	4	9	12	-0.24091	-0.2151	-2
3	4	9	12	-0.24521	-0.2151	-2
3	4	9	12	-0.24952	-0.2151	-2
3	4	9	12	-0.25382	-0.2151	-2
3	4	9	12	-0.25812	-0.2151	-2
3	4	9	5	0.05464	0.0683	-2
3	4	9	5	0.056006	0.0683	-2
3	4	9	5	0.057372	0.0683	-2

3	4	9	5	0.058738	0.0683	-2
3	4	9	5	0.060104	0.0683	-2
3	4	9	5	0.06147	0.0683	-2
3	4	9	5	0.062836	0.0683	-2
3	4	9	5	0.064202	0.0683	-2
3	4	9	5	0.065568	0.0683	-2
3	4	9	5	0.066934	0.0683	-2
3	4	9	5	0.0683	0.0683	1
3	4	9	5	0.069666	0.0683	-2
3	4	9	5	0.071032	0.0683	-2
3	4	9	5	0.072398	0.0683	-2
3	4	9	5	0.073764	0.0683	-2
3	4	9	5	0.07513	0.0683	-2
3	4	9	5	0.076496	0.0683	-2
3	4	9	5	0.077862	0.0683	-2
3	4	9	5	0.079228	0.0683	-2
3	4	9	5	0.080594	0.0683	-2
3	4	9	5	0.08196	0.0683	-2
5	8	9	5	-0.06744	-0.0843	-2
5	8	9	5	-0.06913	-0.0843	-2
5	8	9	5	-0.07081	-0.0843	-2
5	8	9	5	-0.0725	-0.0843	-2
5	8	9	5	-0.07418	-0.0843	-2
5	8	9	5	-0.07587	-0.0843	-2
5	8	9	5	-0.07756	-0.0843	-2
5	8	9	5	-0.07924	-0.0843	-2
5	8	9	5	-0.08093	-0.0843	-2
5	8	9	5	-0.08261	-0.0843	-2
5	8	9	5	-0.0843	-0.0843	1
5	8	9	5	-0.08599	-0.0843	-2
5	8	9	5	-0.08767	-0.0843	-2
5	8	9	5	-0.08936	-0.0843	-2
5	8	9	5	-0.09104	-0.0843	-2
5	8	9	5	-0.09273	-0.0843	-2
5	8	9	5	-0.09442	-0.0843	-2
5	8	9	5	-0.0961	-0.0843	-2
5	8	9	5	-0.09779	-0.0843	-2
5	8	9	5	-0.09947	-0.0843	-2
5	8	9	5	-0.10116	-0.0843	-2

2	3	2	18	0.30768	0.3846	-2
2	3	2	18	0.315372	0.3846	-2
2	3	2	18	0.323064	0.3846	-2
2	3	2	18	0.330756	0.3846	-2
2	3	2	18	0.338448	0.3846	-2
2	3	2	18	0.34614	0.3846	-2
2	3	2	18	0.353832	0.3846	-2
2	3	2	18	0.361524	0.3846	-2
2	3	2	18	0.369216	0.3846	-2
2	3	2	18	0.376908	0.3846	-2
2	3	2	18	0.3846	0.3846	1
2	3	2	18	0.392292	0.3846	-2
2	3	2	18	0.399984	0.3846	-2
2	3	2	18	0.407676	0.3846	-2
2	3	2	18	0.415368	0.3846	-2
2	3	2	18	0.42306	0.3846	-2
2	3	2	18	0.430752	0.3846	-2
2	3	2	18	0.438444	0.3846	-2
2	3	2	18	0.446136	0.3846	-2
2	3	2	18	0.453828	0.3846	-2
2	3	2	18	0.46152	0.3846	-2
3	4	2	18	0.24592	0.3074	-2
3	4	2	18	0.252068	0.3074	-2
3	4	2	18	0.258216	0.3074	-2
3	4	2	18	0.264364	0.3074	-2
3	4	2	18	0.270512	0.3074	-2
3	4	2	18	0.27666	0.3074	-2
3	4	2	18	0.282808	0.3074	-2
3	4	2	18	0.288956	0.3074	-2
3	4	2	18	0.295104	0.3074	-2
3	4	2	18	0.301252	0.3074	-2
3	4	2	18	0.3074	0.3074	1
3	4	2	18	0.313548	0.3074	-2
3	4	2	18	0.319696	0.3074	-2
3	4	2	18	0.325844	0.3074	-2
3	4	2	18	0.331992	0.3074	-2
3	4	2	18	0.33814	0.3074	-2
3	4	2	18	0.344288	0.3074	-2
3	4	2	18	0.350436	0.3074	-2
3	4	2	18	0.356584	0.3074	-2

3	4	2	18	0.362732	0.3074	-2
3	4	2	18	0.36888	0.3074	-2
5	8	2	18	0.22848	0.2856	-2
5	8	2	18	0.234192	0.2856	-2
5	8	2	18	0.239904	0.2856	-2
5	8	2	18	0.245616	0.2856	-2
5	8	2	18	0.251328	0.2856	-2
5	8	2	18	0.25704	0.2856	-2
5	8	2	18	0.262752	0.2856	-2
5	8	2	18	0.268464	0.2856	-2
5	8	2	18	0.274176	0.2856	-2
5	8	2	18	0.279888	0.2856	-2
5	8	2	18	0.2856	0.2856	1
5	8	2	18	0.291312	0.2856	-2
5	8	2	18	0.297024	0.2856	-2
5	8	2	18	0.302736	0.2856	-2
5	8	2	18	0.308448	0.2856	-2
5	8	2	18	0.31416	0.2856	-2
5	8	2	18	0.319872	0.2856	-2
5	8	2	18	0.325584	0.2856	-2
5	8	2	18	0.331296	0.2856	-2
5	8	2	18	0.337008	0.2856	-2
5	8	2	18	0.34272	0.2856	-2
5	8	5	8	0.45216	0.5652	-2
5	8	5	8	0.463464	0.5652	-2
5	8	5	8	0.474768	0.5652	-2
5	8	5	8	0.486072	0.5652	-2
5	8	5	8	0.497376	0.5652	-2
5	8	5	8	0.50868	0.5652	-2
5	8	5	8	0.519984	0.5652	-2
5	8	5	8	0.531288	0.5652	-2
5	8	5	8	0.542592	0.5652	-2
5	8	5	8	0.553896	0.5652	-2
5	8	5	8	0.5652	0.5652	1
5	8	5	8	0.576504	0.5652	-2
5	8	5	8	0.587808	0.5652	-2
5	8	5	8	0.599112	0.5652	-2
5	8	5	8	0.610416	0.5652	-2
5	8	5	8	0.62172	0.5652	-2

5	8	5	8	0.633024	0.5652	-2
5	8	5	8	0.644328	0.5652	-2
5	8	5	8	0.655632	0.5652	-2
5	8	5	8	0.666936	0.5652	-2
5	8	5	8	0.67824	0.5652	-2
2	3	2	3	0.59816	0.7477	-2
2	3	2	3	0.613114	0.7477	-2
2	3	2	3	0.628068	0.7477	-2
2	3	2	3	0.643022	0.7477	-2
2	3	2	3	0.657976	0.7477	-2
2	3	2	3	0.67293	0.7477	-2
2	3	2	3	0.687884	0.7477	-2
2	3	2	3	0.702838	0.7477	-2
2	3	2	3	0.717792	0.7477	-2
2	3	2	3	0.732746	0.7477	-2
2	3	2	3	0.7477	0.7477	1
2	3	2	3	0.762654	0.7477	-2
2	3	2	3	0.777608	0.7477	-2
2	3	2	3	0.792562	0.7477	-2
2	3	2	3	0.807516	0.7477	-2
2	3	2	3	0.82247	0.7477	-2
2	3	2	3	0.837424	0.7477	-2
2	3	2	3	0.852378	0.7477	-2
2	3	2	3	0.867332	0.7477	-2
2	3	2	3	0.882286	0.7477	-2
2	3	2	3	0.89724	0.7477	-2
3	4	2	3	-0.0696	-0.087	-2
3	4	2	3	-0.07134	-0.087	-2
3	4	2	3	-0.07308	-0.087	-2
3	4	2	3	-0.07482	-0.087	-2
3	4	2	3	-0.07656	-0.087	-2
3	4	2	3	-0.0783	-0.087	-2
3	4	2	3	-0.08004	-0.087	-2
3	4	2	3	-0.08178	-0.087	-2
3	4	2	3	-0.08352	-0.087	-2
3	4	2	3	-0.08526	-0.087	-2
3	4	2	3	-0.087	-0.087	1
3	4	2	3	-0.08874	-0.087	-2
3	4	2	3	-0.09048	-0.087	-2

3	4	2	3	-0.09222	-0.087	-2
3	4	2	3	-0.09396	-0.087	-2
3	4	2	3	-0.0957	-0.087	-2
3	4	2	3	-0.09744	-0.087	-2
3	4	2	3	-0.09918	-0.087	-2
3	4	2	3	-0.10092	-0.087	-2
3	4	2	3	-0.10266	-0.087	-2
3	4	2	3	-0.1044	-0.087	-2
5	8	2	3	-0.04024	-0.0503	-2
5	8	2	3	-0.04125	-0.0503	-2
5	8	2	3	-0.04225	-0.0503	-2
5	8	2	3	-0.04326	-0.0503	-2
5	8	2	3	-0.04426	-0.0503	-2
5	8	2	3	-0.04527	-0.0503	-2
5	8	2	3	-0.04628	-0.0503	-2
5	8	2	3	-0.04728	-0.0503	-2
5	8	2	3	-0.04829	-0.0503	-2
5	8	2	3	-0.04929	-0.0503	-2
5	8	2	3	-0.0503	-0.0503	1
5	8	2	3	-0.05131	-0.0503	-2
5	8	2	3	-0.05231	-0.0503	-2
5	8	2	3	-0.05332	-0.0503	-2
5	8	2	3	-0.05432	-0.0503	-2
5	8	2	3	-0.05533	-0.0503	-2
5	8	2	3	-0.05634	-0.0503	-2
5	8	2	3	-0.05734	-0.0503	-2
5	8	2	3	-0.05835	-0.0503	-2
5	8	2	3	-0.05935	-0.0503	-2
5	8	2	3	-0.06036	-0.0503	-2
3	4	14	12	-0.07536	-0.0942	-2
3	4	14	12	-0.07724	-0.0942	-2
3	4	14	12	-0.07913	-0.0942	-2
3	4	14	12	-0.08101	-0.0942	-2
3	4	14	12	-0.0829	-0.0942	-2
3	4	14	12	-0.08478	-0.0942	-2
3	4	14	12	-0.08666	-0.0942	-2
3	4	14	12	-0.08855	-0.0942	-2
3	4	14	12	-0.09043	-0.0942	-2
3	4	14	12	-0.09232	-0.0942	-2

3	4	14	12	-0.0942	-0.0942	1
3	4	14	12	-0.09608	-0.0942	-2
3	4	14	12	-0.09797	-0.0942	-2
3	4	14	12	-0.09985	-0.0942	-2
3	4	14	12	-0.10174	-0.0942	-2
3	4	14	12	-0.10362	-0.0942	-2
3	4	14	12	-0.1055	-0.0942	-2
3	4	14	12	-0.10739	-0.0942	-2
3	4	14	12	-0.10927	-0.0942	-2
3	4	14	12	-0.11116	-0.0942	-2
3	4	14	12	-0.11304	-0.0942	-2

Appendix F: Simultaneous PTDF estimation for Case 1

Table F.1: Simultaneous estimation of PTDF set # 1				
1st PTDF estimated value	2nd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value
0.45712	0.11432	-2	0.5714	0.1429
0.51426	0.11432	-2	0.5714	0.1429
0.5714	0.11432	-2	0.5714	0.1429
0.62854	0.11432	-2	0.5714	0.1429
0.68568	0.11432	-2	0.5714	0.1429
0.45712	0.12861	-2	0.5714	0.1429
0.51426	0.12861	-2	0.5714	0.1429
0.5714	0.12861	-2	0.5714	0.1429
0.62854	0.12861	-2	0.5714	0.1429
0.68568	0.12861	-2	0.5714	0.1429
0.45712	0.1429	-2	0.5714	0.1429
0.51426	0.1429	-2	0.5714	0.1429
0.5714	0.1429	1	0.5714	0.1429
0.62854	0.1429	1	0.5714	0.1429
0.68568	0.1429	1	0.5714	0.1429
0.45712	0.15719	1	0.5714	0.1429
0.51426	0.15719	1	0.5714	0.1429
0.5714	0.15719	1	0.5714	0.1429
0.62854	0.15719	1	0.5714	0.1429
0.68568	0.15719	1	0.5714	0.1429
0.45712	0.17148	1	0.5714	0.1429
0.51426	0.17148	1	0.5714	0.1429
0.5714	0.17148	1	0.5714	0.1429
0.62854	0.17148	1	0.5714	0.1429
0.68568	0.17148	1	0.5714	0.1429

Note that Table F.1 cannot be used to single out the true values however it provides a range of values including the true values of PTDFs.

Table F.2: Simultaneous estimation of PTDF set # 2				
1st PTDF estimated value	2nd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value
0.11432	-0.11432	-2	0.1429	-0.1429
0.12861	-0.11432	-2	0.1429	-0.1429
0.1429	-0.11432	-2	0.1429	-0.1429

0.15719	-0.11432	-2	0.1429	-0.1429
0.17148	-0.11432	-2	0.1429	-0.1429
0.11432	-0.12861	-2	0.1429	-0.1429
0.12861	-0.12861	-2	0.1429	-0.1429
0.1429	-0.12861	-2	0.1429	-0.1429
0.15719	-0.12861	-2	0.1429	-0.1429
0.17148	-0.12861	-2	0.1429	-0.1429
0.11432	-0.1429	-2	0.1429	-0.1429
0.12861	-0.1429	-2	0.1429	-0.1429
0.1429	-0.1429	1	0.1429	-0.1429
0.15719	-0.1429	1	0.1429	-0.1429
0.17148	-0.1429	1	0.1429	-0.1429
0.11432	-0.15719	-2	0.1429	-0.1429
0.12861	-0.15719	-2	0.1429	-0.1429
0.1429	-0.15719	-2	0.1429	-0.1429
0.15719	-0.15719	-2	0.1429	-0.1429
0.17148	-0.15719	-2	0.1429	-0.1429
0.11432	-0.17148	-2	0.1429	-0.1429
0.12861	-0.17148	-2	0.1429	-0.1429
0.1429	-0.17148	-2	0.1429	-0.1429
0.15719	-0.17148	-2	0.1429	-0.1429
0.17148	-0.17148	-2	0.1429	-0.1429

Table F.3: Simultaneous estimation of PTDF set # 3

1st PTDF estimated value	2nd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value
-0.11432	-0.22856	-2	-0.1429	-0.2857
-0.12861	-0.22856	-2	-0.1429	-0.2857
-0.1429	-0.22856	-2	-0.1429	-0.2857
-0.15719	-0.22856	-2	-0.1429	-0.2857
-0.17148	-0.22856	-2	-0.1429	-0.2857
-0.11432	-0.25713	-2	-0.1429	-0.2857
-0.12861	-0.25713	-2	-0.1429	-0.2857
-0.1429	-0.25713	-2	-0.1429	-0.2857
-0.15719	-0.25713	-2	-0.1429	-0.2857
-0.17148	-0.25713	-2	-0.1429	-0.2857
-0.11432	-0.2857	-2	-0.1429	-0.2857
-0.12861	-0.2857	-2	-0.1429	-0.2857
-0.1429	-0.2857	1	-0.1429	-0.2857
-0.15719	-0.2857	-2	-0.1429	-0.2857

-0.17148	-0.2857	-2	-0.1429	-0.2857
-0.11432	-0.31427	-2	-0.1429	-0.2857
-0.12861	-0.31427	-2	-0.1429	-0.2857
-0.1429	-0.31427	-2	-0.1429	-0.2857
-0.15719	-0.31427	-2	-0.1429	-0.2857
-0.17148	-0.31427	-2	-0.1429	-0.2857
-0.11432	-0.34284	-2	-0.1429	-0.2857
-0.12861	-0.34284	-2	-0.1429	-0.2857
-0.1429	-0.34284	-2	-0.1429	-0.2857
-0.15719	-0.34284	-2	-0.1429	-0.2857
-0.17148	-0.34284	-2	-0.1429	-0.2857

Table F.4: Simultaneous estimation of PTDF set # 4

1st PTDF estimated value	2nd PTDF estimated value	3rd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value	3rd PTDF true value
0.45712	0.11432	0.22856	-2	0.5714	0.1429	0.2857
0.51426	0.11432	0.22856	-2	0.5714	0.1429	0.2857
0.5714	0.11432	0.22856	-2	0.5714	0.1429	0.2857
0.62854	0.11432	0.22856	-2	0.5714	0.1429	0.2857
0.68568	0.11432	0.22856	-2	0.5714	0.1429	0.2857
0.45712	0.11432	0.25713	-2	0.5714	0.1429	0.2857
0.51426	0.11432	0.25713	-2	0.5714	0.1429	0.2857
0.5714	0.11432	0.25713	-2	0.5714	0.1429	0.2857
0.62854	0.11432	0.25713	-2	0.5714	0.1429	0.2857
0.68568	0.11432	0.25713	-2	0.5714	0.1429	0.2857
0.45712	0.11432	0.2857	-2	0.5714	0.1429	0.2857
0.51426	0.11432	0.2857	-2	0.5714	0.1429	0.2857
0.5714	0.11432	0.2857	-2	0.5714	0.1429	0.2857
0.62854	0.11432	0.2857	-2	0.5714	0.1429	0.2857
0.68568	0.11432	0.2857	-2	0.5714	0.1429	0.2857
0.45712	0.11432	0.31427	-2	0.5714	0.1429	0.2857
0.51426	0.11432	0.31427	1	0.5714	0.1429	0.2857
0.5714	0.11432	0.31427	1	0.5714	0.1429	0.2857
0.62854	0.11432	0.31427	1	0.5714	0.1429	0.2857
0.68568	0.11432	0.31427	1	0.5714	0.1429	0.2857
0.45712	0.11432	0.34284	1	0.5714	0.1429	0.2857
0.51426	0.11432	0.34284	1	0.5714	0.1429	0.2857
0.5714	0.11432	0.34284	1	0.5714	0.1429	0.2857
0.62854	0.11432	0.34284	1	0.5714	0.1429	0.2857

0.68568	0.11432	0.34284	1	0.5714	0.1429	0.2857
0.45712	0.12861	0.22856	-2	0.5714	0.1429	0.2857
0.51426	0.12861	0.22856	-2	0.5714	0.1429	0.2857
0.5714	0.12861	0.22856	-2	0.5714	0.1429	0.2857
0.62854	0.12861	0.22856	-2	0.5714	0.1429	0.2857
0.68568	0.12861	0.22856	-2	0.5714	0.1429	0.2857
0.45712	0.12861	0.25713	-2	0.5714	0.1429	0.2857
0.51426	0.12861	0.25713	-2	0.5714	0.1429	0.2857
0.5714	0.12861	0.25713	-2	0.5714	0.1429	0.2857
0.62854	0.12861	0.25713	-2	0.5714	0.1429	0.2857
0.68568	0.12861	0.25713	-2	0.5714	0.1429	0.2857
0.45712	0.12861	0.2857	-2	0.5714	0.1429	0.2857
0.51426	0.12861	0.2857	-2	0.5714	0.1429	0.2857
0.5714	0.12861	0.2857	-2	0.5714	0.1429	0.2857
0.62854	0.12861	0.2857	-2	0.5714	0.1429	0.2857
0.68568	0.12861	0.2857	-2	0.5714	0.1429	0.2857
0.45712	0.12861	0.31427	1	0.5714	0.1429	0.2857
0.51426	0.12861	0.31427	1	0.5714	0.1429	0.2857
0.5714	0.12861	0.31427	1	0.5714	0.1429	0.2857
0.62854	0.12861	0.31427	1	0.5714	0.1429	0.2857
0.68568	0.12861	0.31427	-3	0.5714	0.1429	0.2857
0.45712	0.12861	0.34284	1	0.5714	0.1429	0.2857
0.51426	0.12861	0.34284	1	0.5714	0.1429	0.2857
0.5714	0.12861	0.34284	1	0.5714	0.1429	0.2857
0.62854	0.12861	0.34284	1	0.5714	0.1429	0.2857
0.68568	0.12861	0.34284	1	0.5714	0.1429	0.2857
0.45712	0.1429	0.22856	-2	0.5714	0.1429	0.2857
0.51426	0.1429	0.22856	-2	0.5714	0.1429	0.2857
0.5714	0.1429	0.22856	-2	0.5714	0.1429	0.2857
0.62854	0.1429	0.22856	-2	0.5714	0.1429	0.2857
0.68568	0.1429	0.22856	-2	0.5714	0.1429	0.2857
0.45712	0.1429	0.25713	-2	0.5714	0.1429	0.2857
0.51426	0.1429	0.25713	-2	0.5714	0.1429	0.2857
0.5714	0.1429	0.25713	-2	0.5714	0.1429	0.2857
0.62854	0.1429	0.25713	-2	0.5714	0.1429	0.2857
0.68568	0.1429	0.25713	-2	0.5714	0.1429	0.2857
0.45712	0.1429	0.2857	-2	0.5714	0.1429	0.2857
0.51426	0.1429	0.2857	-2	0.5714	0.1429	0.2857
0.5714	0.1429	0.2857	1	0.5714	0.1429	0.2857
0.62854	0.1429	0.2857	1	0.5714	0.1429	0.2857
0.68568	0.1429	0.2857	1	0.5714	0.1429	0.2857

0.45712	0.1429	0.31427	1	0.5714	0.1429	0.2857
0.51426	0.1429	0.31427	1	0.5714	0.1429	0.2857
0.5714	0.1429	0.31427	1	0.5714	0.1429	0.2857
0.62854	0.1429	0.31427	1	0.5714	0.1429	0.2857
0.68568	0.1429	0.31427	1	0.5714	0.1429	0.2857
0.45712	0.1429	0.34284	1	0.5714	0.1429	0.2857
0.51426	0.1429	0.34284	1	0.5714	0.1429	0.2857
0.5714	0.1429	0.34284	1	0.5714	0.1429	0.2857
0.62854	0.1429	0.34284	1	0.5714	0.1429	0.2857
0.68568	0.1429	0.34284	1	0.5714	0.1429	0.2857
0.45712	0.15719	0.22856	-2	0.5714	0.1429	0.2857
0.51426	0.15719	0.22856	-2	0.5714	0.1429	0.2857
0.5714	0.15719	0.22856	-2	0.5714	0.1429	0.2857
0.62854	0.15719	0.22856	-2	0.5714	0.1429	0.2857
0.68568	0.15719	0.22856	-2	0.5714	0.1429	0.2857
0.45712	0.15719	0.25713	-2	0.5714	0.1429	0.2857
0.51426	0.15719	0.25713	-2	0.5714	0.1429	0.2857
0.5714	0.15719	0.25713	-2	0.5714	0.1429	0.2857
0.62854	0.15719	0.25713	-2	0.5714	0.1429	0.2857
0.68568	0.15719	0.25713	-2	0.5714	0.1429	0.2857
0.45712	0.15719	0.2857	1	0.5714	0.1429	0.2857
0.51426	0.15719	0.2857	1	0.5714	0.1429	0.2857
0.5714	0.15719	0.2857	1	0.5714	0.1429	0.2857
0.62854	0.15719	0.2857	1	0.5714	0.1429	0.2857
0.68568	0.15719	0.2857	1	0.5714	0.1429	0.2857
0.45712	0.15719	0.31427	1	0.5714	0.1429	0.2857
0.51426	0.15719	0.31427	1	0.5714	0.1429	0.2857
0.5714	0.15719	0.31427	1	0.5714	0.1429	0.2857
0.62854	0.15719	0.31427	1	0.5714	0.1429	0.2857
0.68568	0.15719	0.31427	1	0.5714	0.1429	0.2857
0.45712	0.15719	0.34284	1	0.5714	0.1429	0.2857
0.51426	0.15719	0.34284	1	0.5714	0.1429	0.2857
0.5714	0.15719	0.34284	1	0.5714	0.1429	0.2857
0.62854	0.15719	0.34284	1	0.5714	0.1429	0.2857
0.68568	0.15719	0.34284	1	0.5714	0.1429	0.2857
0.45712	0.17148	0.22856	-2	0.5714	0.1429	0.2857
0.51426	0.17148	0.22856	-2	0.5714	0.1429	0.2857
0.5714	0.17148	0.22856	-2	0.5714	0.1429	0.2857
0.62854	0.17148	0.22856	-2	0.5714	0.1429	0.2857
0.68568	0.17148	0.22856	-2	0.5714	0.1429	0.2857
0.45712	0.17148	0.25713	-2	0.5714	0.1429	0.2857

0.51426	0.17148	0.25713	-2	0.5714	0.1429	0.2857
0.5714	0.17148	0.25713	-2	0.5714	0.1429	0.2857
0.62854	0.17148	0.25713	-2	0.5714	0.1429	0.2857
0.68568	0.17148	0.25713	1	0.5714	0.1429	0.2857
0.45712	0.17148	0.2857	1	0.5714	0.1429	0.2857
0.51426	0.17148	0.2857	1	0.5714	0.1429	0.2857
0.5714	0.17148	0.2857	1	0.5714	0.1429	0.2857
0.62854	0.17148	0.2857	1	0.5714	0.1429	0.2857
0.68568	0.17148	0.2857	1	0.5714	0.1429	0.2857
0.45712	0.17148	0.31427	1	0.5714	0.1429	0.2857
0.51426	0.17148	0.31427	1	0.5714	0.1429	0.2857
0.5714	0.17148	0.31427	1	0.5714	0.1429	0.2857
0.62854	0.17148	0.31427	1	0.5714	0.1429	0.2857
0.68568	0.17148	0.31427	1	0.5714	0.1429	0.2857
0.45712	0.17148	0.34284	-2	0.5714	0.1429	0.2857
0.51426	0.17148	0.34284	-2	0.5714	0.1429	0.2857
0.5714	0.17148	0.34284	-2	0.5714	0.1429	0.2857
0.62854	0.17148	0.34284	-2	0.5714	0.1429	0.2857
0.68568	0.17148	0.34284	-2	0.5714	0.1429	0.2857

Table F.5: Simultaneous estimation of PTDF set # 5

1st PTDF estimated value	2nd PTDF estimated value	3rd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value	3rd PTDF true value
0.45712	0.11432	-0.11432	-2	0.5714	0.1429	-0.1429
0.51426	0.11432	-0.11432	-2	0.5714	0.1429	-0.1429
0.5714	0.11432	-0.11432	-2	0.5714	0.1429	-0.1429
0.62854	0.11432	-0.11432	-2	0.5714	0.1429	-0.1429
0.68568	0.11432	-0.11432	-2	0.5714	0.1429	-0.1429
0.45712	0.11432	-0.12861	-2	0.5714	0.1429	-0.1429
0.51426	0.11432	-0.12861	-2	0.5714	0.1429	-0.1429
0.5714	0.11432	-0.12861	-2	0.5714	0.1429	-0.1429
0.62854	0.11432	-0.12861	-2	0.5714	0.1429	-0.1429
0.68568	0.11432	-0.12861	-2	0.5714	0.1429	-0.1429
0.45712	0.11432	-0.1429	-2	0.5714	0.1429	-0.1429
0.51426	0.11432	-0.1429	-2	0.5714	0.1429	-0.1429
0.5714	0.11432	-0.1429	-2	0.5714	0.1429	-0.1429
0.62854	0.11432	-0.1429	-2	0.5714	0.1429	-0.1429
0.68568	0.11432	-0.1429	-2	0.5714	0.1429	-0.1429
0.45712	0.11432	-0.15719	-2	0.5714	0.1429	-0.1429

0.51426	0.11432	-0.15719	-2	0.5714	0.1429	-0.1429
0.5714	0.11432	-0.15719	-2	0.5714	0.1429	-0.1429
0.62854	0.11432	-0.15719	-2	0.5714	0.1429	-0.1429
0.68568	0.11432	-0.15719	-2	0.5714	0.1429	-0.1429
0.45712	0.11432	-0.17148	-2	0.5714	0.1429	-0.1429
0.51426	0.11432	-0.17148	-2	0.5714	0.1429	-0.1429
0.5714	0.11432	-0.17148	-2	0.5714	0.1429	-0.1429
0.62854	0.11432	-0.17148	-2	0.5714	0.1429	-0.1429
0.68568	0.11432	-0.17148	-2	0.5714	0.1429	-0.1429
0.45712	0.12861	-0.11432	-2	0.5714	0.1429	-0.1429
0.51426	0.12861	-0.11432	-2	0.5714	0.1429	-0.1429
0.5714	0.12861	-0.11432	-2	0.5714	0.1429	-0.1429
0.62854	0.12861	-0.11432	-2	0.5714	0.1429	-0.1429
0.68568	0.12861	-0.11432	-2	0.5714	0.1429	-0.1429
0.45712	0.12861	-0.12861	-2	0.5714	0.1429	-0.1429
0.51426	0.12861	-0.12861	-2	0.5714	0.1429	-0.1429
0.5714	0.12861	-0.12861	-2	0.5714	0.1429	-0.1429
0.62854	0.12861	-0.12861	-2	0.5714	0.1429	-0.1429
0.68568	0.12861	-0.12861	-2	0.5714	0.1429	-0.1429
0.45712	0.12861	-0.1429	-2	0.5714	0.1429	-0.1429
0.51426	0.12861	-0.1429	-2	0.5714	0.1429	-0.1429
0.5714	0.12861	-0.1429	-2	0.5714	0.1429	-0.1429
0.62854	0.12861	-0.1429	-2	0.5714	0.1429	-0.1429
0.68568	0.12861	-0.1429	-2	0.5714	0.1429	-0.1429
0.45712	0.12861	-0.15719	-2	0.5714	0.1429	-0.1429
0.51426	0.12861	-0.15719	-2	0.5714	0.1429	-0.1429
0.5714	0.12861	-0.15719	-2	0.5714	0.1429	-0.1429
0.62854	0.12861	-0.15719	-2	0.5714	0.1429	-0.1429
0.68568	0.12861	-0.15719	-2	0.5714	0.1429	-0.1429
0.45712	0.12861	-0.17148	-2	0.5714	0.1429	-0.1429
0.51426	0.12861	-0.17148	-2	0.5714	0.1429	-0.1429
0.5714	0.12861	-0.17148	-2	0.5714	0.1429	-0.1429
0.62854	0.12861	-0.17148	-2	0.5714	0.1429	-0.1429
0.68568	0.12861	-0.17148	-2	0.5714	0.1429	-0.1429
0.45712	0.1429	-0.11432	-2	0.5714	0.1429	-0.1429
0.51426	0.1429	-0.11432	-2	0.5714	0.1429	-0.1429
0.5714	0.1429	-0.11432	-2	0.5714	0.1429	-0.1429
0.62854	0.1429	-0.11432	-2	0.5714	0.1429	-0.1429
0.68568	0.1429	-0.11432	-2	0.5714	0.1429	-0.1429
0.45712	0.1429	-0.12861	-2	0.5714	0.1429	-0.1429
0.51426	0.1429	-0.12861	-2	0.5714	0.1429	-0.1429

0.5714	0.1429	-0.12861	-2	0.5714	0.1429	-0.1429
0.62854	0.1429	-0.12861	-2	0.5714	0.1429	-0.1429
0.68568	0.1429	-0.12861	-2	0.5714	0.1429	-0.1429
0.45712	0.1429	-0.1429	-2	0.5714	0.1429	-0.1429
0.51426	0.1429	-0.1429	-2	0.5714	0.1429	-0.1429
0.5714	0.1429	-0.1429	1	0.5714	0.1429	-0.1429
0.62854	0.1429	-0.1429	1	0.5714	0.1429	-0.1429
0.68568	0.1429	-0.1429	1	0.5714	0.1429	-0.1429
0.45712	0.1429	-0.15719	-2	0.5714	0.1429	-0.1429
0.51426	0.1429	-0.15719	-2	0.5714	0.1429	-0.1429
0.5714	0.1429	-0.15719	-2	0.5714	0.1429	-0.1429
0.62854	0.1429	-0.15719	-2	0.5714	0.1429	-0.1429
0.68568	0.1429	-0.15719	-2	0.5714	0.1429	-0.1429
0.45712	0.1429	-0.17148	-2	0.5714	0.1429	-0.1429
0.51426	0.1429	-0.17148	-2	0.5714	0.1429	-0.1429
0.5714	0.1429	-0.17148	-2	0.5714	0.1429	-0.1429
0.62854	0.1429	-0.17148	-2	0.5714	0.1429	-0.1429
0.68568	0.1429	-0.17148	-2	0.5714	0.1429	-0.1429
0.45712	0.15719	-0.11432	-2	0.5714	0.1429	-0.1429
0.51426	0.15719	-0.11432	-2	0.5714	0.1429	-0.1429
0.5714	0.15719	-0.11432	-2	0.5714	0.1429	-0.1429
0.62854	0.15719	-0.11432	-2	0.5714	0.1429	-0.1429
0.68568	0.15719	-0.11432	-2	0.5714	0.1429	-0.1429
0.45712	0.15719	-0.12861	-2	0.5714	0.1429	-0.1429
0.51426	0.15719	-0.12861	-2	0.5714	0.1429	-0.1429
0.5714	0.15719	-0.12861	-2	0.5714	0.1429	-0.1429
0.62854	0.15719	-0.12861	-2	0.5714	0.1429	-0.1429
0.68568	0.15719	-0.12861	-2	0.5714	0.1429	-0.1429
0.45712	0.15719	-0.1429	1	0.5714	0.1429	-0.1429
0.51426	0.15719	-0.1429	1	0.5714	0.1429	-0.1429
0.5714	0.15719	-0.1429	1	0.5714	0.1429	-0.1429
0.62854	0.15719	-0.1429	1	0.5714	0.1429	-0.1429
0.68568	0.15719	-0.1429	1	0.5714	0.1429	-0.1429
0.45712	0.15719	-0.15719	-2	0.5714	0.1429	-0.1429
0.51426	0.15719	-0.15719	-2	0.5714	0.1429	-0.1429
0.5714	0.15719	-0.15719	-2	0.5714	0.1429	-0.1429
0.62854	0.15719	-0.15719	-2	0.5714	0.1429	-0.1429
0.68568	0.15719	-0.15719	-2	0.5714	0.1429	-0.1429
0.45712	0.15719	-0.17148	-2	0.5714	0.1429	-0.1429
0.51426	0.15719	-0.17148	-2	0.5714	0.1429	-0.1429
0.5714	0.15719	-0.17148	-2	0.5714	0.1429	-0.1429

0.62854	0.15719	-0.17148	-2	0.5714	0.1429	-0.1429
0.68568	0.15719	-0.17148	-2	0.5714	0.1429	-0.1429
0.45712	0.17148	-0.11432	-2	0.5714	0.1429	-0.1429
0.51426	0.17148	-0.11432	-2	0.5714	0.1429	-0.1429
0.5714	0.17148	-0.11432	-2	0.5714	0.1429	-0.1429
0.62854	0.17148	-0.11432	-2	0.5714	0.1429	-0.1429
0.68568	0.17148	-0.11432	-2	0.5714	0.1429	-0.1429
0.45712	0.17148	-0.12861	-2	0.5714	0.1429	-0.1429
0.51426	0.17148	-0.12861	-2	0.5714	0.1429	-0.1429
0.5714	0.17148	-0.12861	-2	0.5714	0.1429	-0.1429
0.62854	0.17148	-0.12861	-2	0.5714	0.1429	-0.1429
0.68568	0.17148	-0.12861	-2	0.5714	0.1429	-0.1429
0.45712	0.17148	-0.1429	1	0.5714	0.1429	-0.1429
0.51426	0.17148	-0.1429	1	0.5714	0.1429	-0.1429
0.5714	0.17148	-0.1429	1	0.5714	0.1429	-0.1429
0.62854	0.17148	-0.1429	1	0.5714	0.1429	-0.1429
0.68568	0.17148	-0.1429	1	0.5714	0.1429	-0.1429
0.45712	0.17148	-0.15719	-2	0.5714	0.1429	-0.1429
0.51426	0.17148	-0.15719	-2	0.5714	0.1429	-0.1429
0.5714	0.17148	-0.15719	-2	0.5714	0.1429	-0.1429
0.62854	0.17148	-0.15719	-2	0.5714	0.1429	-0.1429
0.68568	0.17148	-0.15719	-2	0.5714	0.1429	-0.1429
0.45712	0.17148	-0.17148	-2	0.5714	0.1429	-0.1429
0.51426	0.17148	-0.17148	-2	0.5714	0.1429	-0.1429
0.5714	0.17148	-0.17148	-2	0.5714	0.1429	-0.1429
0.62854	0.17148	-0.17148	-2	0.5714	0.1429	-0.1429
0.68568	0.17148	-0.17148	-2	0.5714	0.1429	-0.1429

Table F.6: Simultaneous estimation of PTDF set # 6

1st PTDF estimated value	2nd PTDF estimated value	3rd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value	3rd PTDF true value
0.45712	-0.22856	0.34288	-2	0.5714	-0.2857	0.4286
0.51426	-0.22856	0.34288	-2	0.5714	-0.2857	0.4286
0.5714	-0.22856	0.34288	-2	0.5714	-0.2857	0.4286
0.62854	-0.22856	0.34288	-2	0.5714	-0.2857	0.4286
0.68568	-0.22856	0.34288	-2	0.5714	-0.2857	0.4286
0.45712	-0.22856	0.38574	-2	0.5714	-0.2857	0.4286
0.51426	-0.22856	0.38574	-2	0.5714	-0.2857	0.4286
0.5714	-0.22856	0.38574	-2	0.5714	-0.2857	0.4286

0.62854	-0.22856	0.38574	-2	0.5714	-0.2857	0.4286
0.68568	-0.22856	0.38574	-2	0.5714	-0.2857	0.4286
0.45712	-0.22856	0.4286	-2	0.5714	-0.2857	0.4286
0.51426	-0.22856	0.4286	-2	0.5714	-0.2857	0.4286
0.5714	-0.22856	0.4286	-2	0.5714	-0.2857	0.4286
0.62854	-0.22856	0.4286	-2	0.5714	-0.2857	0.4286
0.68568	-0.22856	0.4286	-2	0.5714	-0.2857	0.4286
0.45712	-0.22856	0.47146	-2	0.5714	-0.2857	0.4286
0.51426	-0.22856	0.47146	-2	0.5714	-0.2857	0.4286
0.5714	-0.22856	0.47146	-2	0.5714	-0.2857	0.4286
0.62854	-0.22856	0.47146	-2	0.5714	-0.2857	0.4286
0.68568	-0.22856	0.47146	-2	0.5714	-0.2857	0.4286
0.45712	-0.22856	0.51432	-2	0.5714	-0.2857	0.4286
0.51426	-0.22856	0.51432	-2	0.5714	-0.2857	0.4286
0.5714	-0.22856	0.51432	-2	0.5714	-0.2857	0.4286
0.62854	-0.22856	0.51432	-2	0.5714	-0.2857	0.4286
0.68568	-0.22856	0.51432	-2	0.5714	-0.2857	0.4286
0.45712	-0.25713	0.34288	-2	0.5714	-0.2857	0.4286
0.51426	-0.25713	0.34288	-2	0.5714	-0.2857	0.4286
0.5714	-0.25713	0.34288	-2	0.5714	-0.2857	0.4286
0.62854	-0.25713	0.34288	-2	0.5714	-0.2857	0.4286
0.68568	-0.25713	0.34288	-2	0.5714	-0.2857	0.4286
0.45712	-0.25713	0.38574	-2	0.5714	-0.2857	0.4286
0.51426	-0.25713	0.38574	-2	0.5714	-0.2857	0.4286
0.5714	-0.25713	0.38574	-2	0.5714	-0.2857	0.4286
0.62854	-0.25713	0.38574	-2	0.5714	-0.2857	0.4286
0.68568	-0.25713	0.38574	-2	0.5714	-0.2857	0.4286
0.45712	-0.25713	0.4286	-2	0.5714	-0.2857	0.4286
0.51426	-0.25713	0.4286	-2	0.5714	-0.2857	0.4286
0.5714	-0.25713	0.4286	-2	0.5714	-0.2857	0.4286
0.62854	-0.25713	0.4286	-2	0.5714	-0.2857	0.4286
0.68568	-0.25713	0.4286	-2	0.5714	-0.2857	0.4286
0.45712	-0.25713	0.47146	-2	0.5714	-0.2857	0.4286
0.51426	-0.25713	0.47146	-2	0.5714	-0.2857	0.4286
0.5714	-0.25713	0.47146	-2	0.5714	-0.2857	0.4286
0.62854	-0.25713	0.47146	-2	0.5714	-0.2857	0.4286
0.68568	-0.25713	0.47146	-2	0.5714	-0.2857	0.4286
0.45712	-0.25713	0.51432	-2	0.5714	-0.2857	0.4286
0.51426	-0.25713	0.51432	-2	0.5714	-0.2857	0.4286
0.5714	-0.25713	0.51432	-2	0.5714	-0.2857	0.4286
0.62854	-0.25713	0.51432	-2	0.5714	-0.2857	0.4286

0.68568	-0.25713	0.51432	-2	0.5714	-0.2857	0.4286
0.45712	-0.2857	0.34288	-2	0.5714	-0.2857	0.4286
0.51426	-0.2857	0.34288	-2	0.5714	-0.2857	0.4286
0.5714	-0.2857	0.34288	-2	0.5714	-0.2857	0.4286
0.62854	-0.2857	0.34288	-2	0.5714	-0.2857	0.4286
0.68568	-0.2857	0.34288	-2	0.5714	-0.2857	0.4286
0.45712	-0.2857	0.38574	-2	0.5714	-0.2857	0.4286
0.51426	-0.2857	0.38574	-2	0.5714	-0.2857	0.4286
0.5714	-0.2857	0.38574	-2	0.5714	-0.2857	0.4286
0.62854	-0.2857	0.38574	-2	0.5714	-0.2857	0.4286
0.68568	-0.2857	0.38574	-2	0.5714	-0.2857	0.4286
0.45712	-0.2857	0.4286	-2	0.5714	-0.2857	0.4286
0.51426	-0.2857	0.4286	-2	0.5714	-0.2857	0.4286
0.5714	-0.2857	0.4286	1	0.5714	-0.2857	0.4286
0.62854	-0.2857	0.4286	1	0.5714	-0.2857	0.4286
0.68568	-0.2857	0.4286	1	0.5714	-0.2857	0.4286
0.45712	-0.2857	0.47146	-2	0.5714	-0.2857	0.4286
0.51426	-0.2857	0.47146	-2	0.5714	-0.2857	0.4286
0.5714	-0.2857	0.47146	-2	0.5714	-0.2857	0.4286
0.62854	-0.2857	0.47146	-2	0.5714	-0.2857	0.4286
0.68568	-0.2857	0.47146	-2	0.5714	-0.2857	0.4286
0.45712	-0.2857	0.51432	-2	0.5714	-0.2857	0.4286
0.51426	-0.2857	0.51432	-2	0.5714	-0.2857	0.4286
0.5714	-0.2857	0.51432	-2	0.5714	-0.2857	0.4286
0.62854	-0.2857	0.51432	-2	0.5714	-0.2857	0.4286
0.68568	-0.2857	0.51432	-2	0.5714	-0.2857	0.4286
0.45712	-0.31427	0.34288	-2	0.5714	-0.2857	0.4286
0.51426	-0.31427	0.34288	-2	0.5714	-0.2857	0.4286
0.5714	-0.31427	0.34288	-2	0.5714	-0.2857	0.4286
0.62854	-0.31427	0.34288	-2	0.5714	-0.2857	0.4286
0.68568	-0.31427	0.34288	-2	0.5714	-0.2857	0.4286
0.45712	-0.31427	0.38574	-2	0.5714	-0.2857	0.4286
0.51426	-0.31427	0.38574	-2	0.5714	-0.2857	0.4286
0.5714	-0.31427	0.38574	-2	0.5714	-0.2857	0.4286
0.62854	-0.31427	0.38574	-2	0.5714	-0.2857	0.4286
0.68568	-0.31427	0.38574	-2	0.5714	-0.2857	0.4286
0.45712	-0.31427	0.4286	-2	0.5714	-0.2857	0.4286
0.51426	-0.31427	0.4286	-2	0.5714	-0.2857	0.4286
0.5714	-0.31427	0.4286	-2	0.5714	-0.2857	0.4286
0.62854	-0.31427	0.4286	-2	0.5714	-0.2857	0.4286
0.68568	-0.31427	0.4286	-2	0.5714	-0.2857	0.4286

0.45712	-0.31427	0.47146	-2	0.5714	-0.2857	0.4286
0.51426	-0.31427	0.47146	-2	0.5714	-0.2857	0.4286
0.5714	-0.31427	0.47146	-2	0.5714	-0.2857	0.4286
0.62854	-0.31427	0.47146	-2	0.5714	-0.2857	0.4286
0.68568	-0.31427	0.47146	-2	0.5714	-0.2857	0.4286
0.45712	-0.31427	0.51432	-2	0.5714	-0.2857	0.4286
0.51426	-0.31427	0.51432	-2	0.5714	-0.2857	0.4286
0.5714	-0.31427	0.51432	-2	0.5714	-0.2857	0.4286
0.62854	-0.31427	0.51432	-2	0.5714	-0.2857	0.4286
0.68568	-0.31427	0.51432	-2	0.5714	-0.2857	0.4286
0.45712	-0.34284	0.34288	-2	0.5714	-0.2857	0.4286
0.51426	-0.34284	0.34288	-2	0.5714	-0.2857	0.4286
0.5714	-0.34284	0.34288	-2	0.5714	-0.2857	0.4286
0.62854	-0.34284	0.34288	-2	0.5714	-0.2857	0.4286
0.68568	-0.34284	0.34288	-2	0.5714	-0.2857	0.4286
0.45712	-0.34284	0.38574	-2	0.5714	-0.2857	0.4286
0.51426	-0.34284	0.38574	-2	0.5714	-0.2857	0.4286
0.5714	-0.34284	0.38574	-2	0.5714	-0.2857	0.4286
0.62854	-0.34284	0.38574	-2	0.5714	-0.2857	0.4286
0.68568	-0.34284	0.38574	-2	0.5714	-0.2857	0.4286
0.45712	-0.34284	0.4286	-2	0.5714	-0.2857	0.4286
0.51426	-0.34284	0.4286	-2	0.5714	-0.2857	0.4286
0.5714	-0.34284	0.4286	-2	0.5714	-0.2857	0.4286
0.62854	-0.34284	0.4286	-2	0.5714	-0.2857	0.4286
0.68568	-0.34284	0.4286	-2	0.5714	-0.2857	0.4286
0.45712	-0.34284	0.47146	-2	0.5714	-0.2857	0.4286
0.51426	-0.34284	0.47146	-2	0.5714	-0.2857	0.4286
0.5714	-0.34284	0.47146	-2	0.5714	-0.2857	0.4286
0.62854	-0.34284	0.47146	-2	0.5714	-0.2857	0.4286
0.68568	-0.34284	0.47146	-2	0.5714	-0.2857	0.4286
0.45712	-0.34284	0.51432	-2	0.5714	-0.2857	0.4286
0.51426	-0.34284	0.51432	-2	0.5714	-0.2857	0.4286
0.5714	-0.34284	0.51432	-2	0.5714	-0.2857	0.4286
0.62854	-0.34284	0.51432	-2	0.5714	-0.2857	0.4286
0.68568	-0.34284	0.51432	-2	0.5714	-0.2857	0.4286

Table F.7: Simultaneous estimation of PTDF set # 7

1st PTDF estimated	2nd PTDF estimated	3rd PTDF estimated	Feasibility flag	1st PTDF true value	2nd PTDF true value	3rd PTDF true value

value	value	value	(feasible=1)			
-0.11432	-0.22856	0.34288	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.22856	0.34288	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.22856	0.34288	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.22856	0.34288	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.22856	0.34288	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.22856	0.38574	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.22856	0.38574	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.22856	0.38574	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.22856	0.38574	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.22856	0.38574	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.22856	0.4286	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.22856	0.4286	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.22856	0.4286	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.22856	0.4286	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.22856	0.4286	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.22856	0.47146	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.22856	0.47146	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.22856	0.47146	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.22856	0.47146	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.22856	0.47146	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.22856	0.51432	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.22856	0.51432	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.22856	0.51432	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.22856	0.51432	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.22856	0.51432	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.25713	0.34288	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.25713	0.34288	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.25713	0.34288	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.25713	0.34288	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.25713	0.34288	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.25713	0.38574	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.25713	0.38574	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.25713	0.38574	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.25713	0.38574	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.25713	0.38574	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.25713	0.4286	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.25713	0.4286	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.25713	0.4286	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.25713	0.4286	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.25713	0.4286	-2	-0.1429	-0.2857	0.4286

-0.11432	-0.25713	0.47146	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.25713	0.47146	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.25713	0.47146	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.25713	0.47146	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.25713	0.47146	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.25713	0.51432	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.25713	0.51432	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.25713	0.51432	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.25713	0.51432	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.25713	0.51432	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.2857	0.34288	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.2857	0.34288	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.2857	0.34288	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.2857	0.34288	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.2857	0.34288	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.2857	0.38574	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.2857	0.38574	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.2857	0.38574	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.2857	0.38574	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.2857	0.38574	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.2857	0.4286	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.2857	0.4286	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.2857	0.4286	1	-0.1429	-0.2857	0.4286
-0.15719	-0.2857	0.4286	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.2857	0.4286	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.2857	0.47146	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.2857	0.47146	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.2857	0.47146	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.2857	0.47146	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.2857	0.47146	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.2857	0.51432	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.2857	0.51432	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.2857	0.51432	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.2857	0.51432	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.2857	0.51432	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.31427	0.34288	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.31427	0.34288	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.31427	0.34288	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.31427	0.34288	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.31427	0.34288	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.31427	0.38574	-2	-0.1429	-0.2857	0.4286

-0.12861	-0.31427	0.38574	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.31427	0.38574	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.31427	0.38574	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.31427	0.38574	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.31427	0.4286	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.31427	0.4286	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.31427	0.4286	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.31427	0.4286	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.31427	0.4286	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.31427	0.47146	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.31427	0.47146	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.31427	0.47146	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.31427	0.47146	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.31427	0.47146	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.31427	0.51432	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.31427	0.51432	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.31427	0.51432	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.31427	0.51432	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.31427	0.51432	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.34284	0.34288	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.34284	0.34288	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.34284	0.34288	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.34284	0.34288	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.34284	0.34288	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.34284	0.38574	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.34284	0.38574	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.34284	0.38574	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.34284	0.38574	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.34284	0.38574	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.34284	0.4286	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.34284	0.4286	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.34284	0.4286	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.34284	0.4286	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.34284	0.4286	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.34284	0.47146	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.34284	0.47146	-2	-0.1429	-0.2857	0.4286
-0.1429	-0.34284	0.47146	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.34284	0.47146	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.34284	0.47146	-2	-0.1429	-0.2857	0.4286
-0.11432	-0.34284	0.51432	-2	-0.1429	-0.2857	0.4286
-0.12861	-0.34284	0.51432	-2	-0.1429	-0.2857	0.4286

-0.1429	-0.34284	0.51432	-2	-0.1429	-0.2857	0.4286
-0.15719	-0.34284	0.51432	-2	-0.1429	-0.2857	0.4286
-0.17148	-0.34284	0.51432	-2	-0.1429	-0.2857	0.4286

Appendix G: Simultaneous PTDF estimation for Case 2

Table G.1: Simultaneous estimation of PTDF set # 1				
1st PTDF estimated value	2nd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value
0.424	0.33896	-2	0.53	0.4237
0.477	0.33896	-2	0.53	0.4237
0.53	0.33896	-2	0.53	0.4237
0.583	0.33896	-2	0.53	0.4237
0.636	0.33896	-2	0.53	0.4237
0.424	0.38133	-2	0.53	0.4237
0.477	0.38133	-2	0.53	0.4237
0.53	0.38133	-2	0.53	0.4237
0.583	0.38133	-2	0.53	0.4237
0.636	0.38133	-2	0.53	0.4237
0.424	0.4237	-2	0.53	0.4237
0.477	0.4237	-2	0.53	0.4237
0.53	0.4237	1	0.53	0.4237
0.583	0.4237	-2	0.53	0.4237
0.636	0.4237	-2	0.53	0.4237
0.424	0.46607	-2	0.53	0.4237
0.477	0.46607	-2	0.53	0.4237
0.53	0.46607	-2	0.53	0.4237
0.583	0.46607	-2	0.53	0.4237
0.636	0.46607	-2	0.53	0.4237
0.424	0.50844	-2	0.53	0.4237
0.477	0.50844	-2	0.53	0.4237
0.53	0.50844	-2	0.53	0.4237
0.583	0.50844	-2	0.53	0.4237
0.636	0.50844	-2	0.53	0.4237

Table G.2: Simultaneous estimation of PTDF set # 2				
1st PTDF estimated value	2nd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value
0.424	-0.09824	-2	0.53	-0.1228
0.477	-0.09824	-2	0.53	-0.1228
0.53	-0.09824	-2	0.53	-0.1228
0.583	-0.09824	-2	0.53	-0.1228

0.636	-0.09824	-2	0.53	-0.1228
0.424	-0.11052	-2	0.53	-0.1228
0.477	-0.11052	-2	0.53	-0.1228
0.53	-0.11052	-2	0.53	-0.1228
0.583	-0.11052	-2	0.53	-0.1228
0.636	-0.11052	-2	0.53	-0.1228
0.424	-0.1228	-2	0.53	-0.1228
0.477	-0.1228	-2	0.53	-0.1228
0.53	-0.1228	1	0.53	-0.1228
0.583	-0.1228	-2	0.53	-0.1228
0.636	-0.1228	-2	0.53	-0.1228
0.424	-0.13508	-2	0.53	-0.1228
0.477	-0.13508	-2	0.53	-0.1228
0.53	-0.13508	-2	0.53	-0.1228
0.583	-0.13508	-2	0.53	-0.1228
0.636	-0.13508	-2	0.53	-0.1228
0.424	-0.14736	-2	0.53	-0.1228
0.477	-0.14736	-2	0.53	-0.1228
0.53	-0.14736	-2	0.53	-0.1228
0.583	-0.14736	-2	0.53	-0.1228
0.636	-0.14736	-2	0.53	-0.1228

Table G.3: Simultaneous estimation of PTDF set # 3

1st PTDF estimated value	2nd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value
-0.09824	0.50664	-2	-0.1228	0.6333
-0.11052	0.50664	-2	-0.1228	0.6333
-0.1228	0.50664	-2	-0.1228	0.6333
-0.13508	0.50664	-2	-0.1228	0.6333
-0.14736	0.50664	-2	-0.1228	0.6333
-0.09824	0.56997	-2	-0.1228	0.6333
-0.11052	0.56997	-2	-0.1228	0.6333
-0.1228	0.56997	-2	-0.1228	0.6333
-0.13508	0.56997	-2	-0.1228	0.6333
-0.14736	0.56997	-2	-0.1228	0.6333
-0.09824	0.6333	-2	-0.1228	0.6333
-0.11052	0.6333	-2	-0.1228	0.6333
-0.1228	0.6333	1	-0.1228	0.6333
-0.13508	0.6333	-2	-0.1228	0.6333
-0.14736	0.6333	-2	-0.1228	0.6333

-0.09824	0.69663	-2	-0.1228	0.6333
-0.11052	0.69663	-2	-0.1228	0.6333
-0.1228	0.69663	-2	-0.1228	0.6333
-0.13508	0.69663	-2	-0.1228	0.6333
-0.14736	0.69663	-2	-0.1228	0.6333
-0.09824	0.75996	-2	-0.1228	0.6333
-0.11052	0.75996	-2	-0.1228	0.6333
-0.1228	0.75996	-2	-0.1228	0.6333
-0.13508	0.75996	-2	-0.1228	0.6333
-0.14736	0.75996	-2	-0.1228	0.6333

Table G.4: Simultaneous estimation of PTDF set # 4

1st PTDF estimated value	2nd PTDF estimated value	3rd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value	3rd PTDF true value
0.424	0.33896	0.31488	-2	0.53	0.4237	0.3936
0.477	0.33896	0.31488	-2	0.53	0.4237	0.3936
0.53	0.33896	0.31488	-2	0.53	0.4237	0.3936
0.583	0.33896	0.31488	-2	0.53	0.4237	0.3936
0.636	0.33896	0.31488	-2	0.53	0.4237	0.3936
0.424	0.33896	0.35424	-2	0.53	0.4237	0.3936
0.477	0.33896	0.35424	-2	0.53	0.4237	0.3936
0.53	0.33896	0.35424	-2	0.53	0.4237	0.3936
0.583	0.33896	0.35424	-2	0.53	0.4237	0.3936
0.636	0.33896	0.35424	-2	0.53	0.4237	0.3936
0.424	0.33896	0.3936	-2	0.53	0.4237	0.3936
0.477	0.33896	0.3936	-2	0.53	0.4237	0.3936
0.53	0.33896	0.3936	-2	0.53	0.4237	0.3936
0.583	0.33896	0.3936	-2	0.53	0.4237	0.3936
0.636	0.33896	0.3936	-2	0.53	0.4237	0.3936
0.424	0.33896	0.43296	-2	0.53	0.4237	0.3936
0.477	0.33896	0.43296	-2	0.53	0.4237	0.3936
0.53	0.33896	0.43296	-2	0.53	0.4237	0.3936
0.583	0.33896	0.43296	-2	0.53	0.4237	0.3936
0.636	0.33896	0.43296	-2	0.53	0.4237	0.3936
0.424	0.33896	0.47232	-2	0.53	0.4237	0.3936
0.477	0.33896	0.47232	-2	0.53	0.4237	0.3936
0.53	0.33896	0.47232	-2	0.53	0.4237	0.3936
0.583	0.33896	0.47232	-2	0.53	0.4237	0.3936

0.636	0.33896	0.47232	-2	0.53	0.4237	0.3936
0.424	0.38133	0.31488	-2	0.53	0.4237	0.3936
0.477	0.38133	0.31488	-2	0.53	0.4237	0.3936
0.53	0.38133	0.31488	-2	0.53	0.4237	0.3936
0.583	0.38133	0.31488	-2	0.53	0.4237	0.3936
0.636	0.38133	0.31488	-2	0.53	0.4237	0.3936
0.424	0.38133	0.35424	-2	0.53	0.4237	0.3936
0.477	0.38133	0.35424	-2	0.53	0.4237	0.3936
0.53	0.38133	0.35424	-2	0.53	0.4237	0.3936
0.583	0.38133	0.35424	-2	0.53	0.4237	0.3936
0.636	0.38133	0.35424	-2	0.53	0.4237	0.3936
0.424	0.38133	0.3936	-2	0.53	0.4237	0.3936
0.477	0.38133	0.3936	-2	0.53	0.4237	0.3936
0.53	0.38133	0.3936	-2	0.53	0.4237	0.3936
0.583	0.38133	0.3936	-2	0.53	0.4237	0.3936
0.636	0.38133	0.3936	-2	0.53	0.4237	0.3936
0.424	0.38133	0.43296	-2	0.53	0.4237	0.3936
0.477	0.38133	0.43296	-2	0.53	0.4237	0.3936
0.53	0.38133	0.43296	-2	0.53	0.4237	0.3936
0.583	0.38133	0.43296	-2	0.53	0.4237	0.3936
0.636	0.38133	0.43296	-2	0.53	0.4237	0.3936
0.424	0.38133	0.47232	-2	0.53	0.4237	0.3936
0.477	0.38133	0.47232	-2	0.53	0.4237	0.3936
0.53	0.38133	0.47232	-2	0.53	0.4237	0.3936
0.583	0.38133	0.47232	-2	0.53	0.4237	0.3936
0.636	0.38133	0.47232	-2	0.53	0.4237	0.3936
0.424	0.4237	0.31488	-2	0.53	0.4237	0.3936
0.477	0.4237	0.31488	-2	0.53	0.4237	0.3936
0.53	0.4237	0.31488	-2	0.53	0.4237	0.3936
0.583	0.4237	0.31488	-2	0.53	0.4237	0.3936
0.636	0.4237	0.31488	-2	0.53	0.4237	0.3936
0.424	0.4237	0.35424	-2	0.53	0.4237	0.3936
0.477	0.4237	0.35424	-2	0.53	0.4237	0.3936
0.53	0.4237	0.35424	-2	0.53	0.4237	0.3936
0.583	0.4237	0.35424	-2	0.53	0.4237	0.3936
0.636	0.4237	0.35424	-2	0.53	0.4237	0.3936
0.424	0.4237	0.3936	-2	0.53	0.4237	0.3936
0.477	0.4237	0.3936	-2	0.53	0.4237	0.3936
0.53	0.4237	0.3936	1	0.53	0.4237	0.3936
0.583	0.4237	0.3936	-2	0.53	0.4237	0.3936
0.636	0.4237	0.3936	-2	0.53	0.4237	0.3936

0.424	0.4237	0.43296	-2	0.53	0.4237	0.3936
0.477	0.4237	0.43296	-2	0.53	0.4237	0.3936
0.53	0.4237	0.43296	-2	0.53	0.4237	0.3936
0.583	0.4237	0.43296	-2	0.53	0.4237	0.3936
0.636	0.4237	0.43296	-2	0.53	0.4237	0.3936
0.424	0.4237	0.47232	-2	0.53	0.4237	0.3936
0.477	0.4237	0.47232	-2	0.53	0.4237	0.3936
0.53	0.4237	0.47232	-2	0.53	0.4237	0.3936
0.583	0.4237	0.47232	-2	0.53	0.4237	0.3936
0.636	0.4237	0.47232	-2	0.53	0.4237	0.3936
0.424	0.46607	0.31488	-2	0.53	0.4237	0.3936
0.477	0.46607	0.31488	-2	0.53	0.4237	0.3936
0.53	0.46607	0.31488	-2	0.53	0.4237	0.3936
0.583	0.46607	0.31488	-2	0.53	0.4237	0.3936
0.636	0.46607	0.31488	-2	0.53	0.4237	0.3936
0.424	0.46607	0.35424	-2	0.53	0.4237	0.3936
0.477	0.46607	0.35424	-2	0.53	0.4237	0.3936
0.53	0.46607	0.35424	-2	0.53	0.4237	0.3936
0.583	0.46607	0.35424	-2	0.53	0.4237	0.3936
0.636	0.46607	0.35424	-2	0.53	0.4237	0.3936
0.424	0.46607	0.3936	-2	0.53	0.4237	0.3936
0.477	0.46607	0.3936	-2	0.53	0.4237	0.3936
0.53	0.46607	0.3936	-2	0.53	0.4237	0.3936
0.583	0.46607	0.3936	-2	0.53	0.4237	0.3936
0.636	0.46607	0.3936	-2	0.53	0.4237	0.3936
0.424	0.46607	0.43296	-2	0.53	0.4237	0.3936
0.477	0.46607	0.43296	-2	0.53	0.4237	0.3936
0.53	0.46607	0.43296	-2	0.53	0.4237	0.3936
0.583	0.46607	0.43296	-2	0.53	0.4237	0.3936
0.636	0.46607	0.43296	-2	0.53	0.4237	0.3936
0.424	0.46607	0.47232	-2	0.53	0.4237	0.3936
0.477	0.46607	0.47232	-2	0.53	0.4237	0.3936
0.53	0.46607	0.47232	-2	0.53	0.4237	0.3936
0.583	0.46607	0.47232	-2	0.53	0.4237	0.3936
0.636	0.46607	0.47232	-2	0.53	0.4237	0.3936
0.424	0.50844	0.31488	-2	0.53	0.4237	0.3936
0.477	0.50844	0.31488	-2	0.53	0.4237	0.3936
0.53	0.50844	0.31488	-2	0.53	0.4237	0.3936
0.583	0.50844	0.31488	-2	0.53	0.4237	0.3936
0.636	0.50844	0.31488	-2	0.53	0.4237	0.3936
0.424	0.50844	0.35424	-2	0.53	0.4237	0.3936

0.477	0.50844	0.35424	-2	0.53	0.4237	0.3936
0.53	0.50844	0.35424	-2	0.53	0.4237	0.3936
0.583	0.50844	0.35424	-2	0.53	0.4237	0.3936
0.636	0.50844	0.35424	-2	0.53	0.4237	0.3936
0.424	0.50844	0.3936	-2	0.53	0.4237	0.3936
0.477	0.50844	0.3936	-2	0.53	0.4237	0.3936
0.53	0.50844	0.3936	-2	0.53	0.4237	0.3936
0.583	0.50844	0.3936	-2	0.53	0.4237	0.3936
0.636	0.50844	0.3936	-2	0.53	0.4237	0.3936
0.424	0.50844	0.43296	-2	0.53	0.4237	0.3936
0.477	0.50844	0.43296	-2	0.53	0.4237	0.3936
0.53	0.50844	0.43296	-2	0.53	0.4237	0.3936
0.583	0.50844	0.43296	-2	0.53	0.4237	0.3936
0.636	0.50844	0.43296	-2	0.53	0.4237	0.3936
0.424	0.50844	0.47232	-2	0.53	0.4237	0.3936
0.477	0.50844	0.47232	-2	0.53	0.4237	0.3936
0.53	0.50844	0.47232	-2	0.53	0.4237	0.3936
0.583	0.50844	0.47232	-2	0.53	0.4237	0.3936
0.636	0.50844	0.47232	-2	0.53	0.4237	0.3936

Table G.5: Simultaneous estimation of PTDF set # 5

1st PTDF estimated value	2nd PTDF estimated value	3rd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value	3rd PTDF true value
0.424	-0.09824	-0.17208	-2	0.53	-0.1228	-0.2151
0.477	-0.09824	-0.17208	-2	0.53	-0.1228	-0.2151
0.53	-0.09824	-0.17208	-2	0.53	-0.1228	-0.2151
0.583	-0.09824	-0.17208	-2	0.53	-0.1228	-0.2151
0.636	-0.09824	-0.17208	-2	0.53	-0.1228	-0.2151
0.424	-0.09824	-0.19359	-2	0.53	-0.1228	-0.2151
0.477	-0.09824	-0.19359	-2	0.53	-0.1228	-0.2151
0.53	-0.09824	-0.19359	-2	0.53	-0.1228	-0.2151
0.583	-0.09824	-0.19359	-2	0.53	-0.1228	-0.2151
0.636	-0.09824	-0.19359	-2	0.53	-0.1228	-0.2151
0.424	-0.09824	-0.2151	-2	0.53	-0.1228	-0.2151
0.477	-0.09824	-0.2151	-2	0.53	-0.1228	-0.2151
0.53	-0.09824	-0.2151	-2	0.53	-0.1228	-0.2151
0.583	-0.09824	-0.2151	-2	0.53	-0.1228	-0.2151
0.636	-0.09824	-0.2151	-2	0.53	-0.1228	-0.2151

0.424	-0.09824	-0.23661	-2	0.53	-0.1228	-0.2151
0.477	-0.09824	-0.23661	-2	0.53	-0.1228	-0.2151
0.53	-0.09824	-0.23661	-2	0.53	-0.1228	-0.2151
0.583	-0.09824	-0.23661	-2	0.53	-0.1228	-0.2151
0.636	-0.09824	-0.23661	-2	0.53	-0.1228	-0.2151
0.424	-0.09824	-0.25812	-2	0.53	-0.1228	-0.2151
0.477	-0.09824	-0.25812	-2	0.53	-0.1228	-0.2151
0.53	-0.09824	-0.25812	-2	0.53	-0.1228	-0.2151
0.583	-0.09824	-0.25812	-2	0.53	-0.1228	-0.2151
0.636	-0.09824	-0.25812	-2	0.53	-0.1228	-0.2151
0.424	-0.11052	-0.17208	-2	0.53	-0.1228	-0.2151
0.477	-0.11052	-0.17208	-2	0.53	-0.1228	-0.2151
0.53	-0.11052	-0.17208	-2	0.53	-0.1228	-0.2151
0.583	-0.11052	-0.17208	-2	0.53	-0.1228	-0.2151
0.636	-0.11052	-0.17208	-2	0.53	-0.1228	-0.2151
0.424	-0.11052	-0.19359	-2	0.53	-0.1228	-0.2151
0.477	-0.11052	-0.19359	-2	0.53	-0.1228	-0.2151
0.53	-0.11052	-0.19359	-2	0.53	-0.1228	-0.2151
0.583	-0.11052	-0.19359	-2	0.53	-0.1228	-0.2151
0.636	-0.11052	-0.19359	-2	0.53	-0.1228	-0.2151
0.424	-0.11052	-0.2151	-2	0.53	-0.1228	-0.2151
0.477	-0.11052	-0.2151	-2	0.53	-0.1228	-0.2151
0.53	-0.11052	-0.2151	-2	0.53	-0.1228	-0.2151
0.583	-0.11052	-0.2151	-2	0.53	-0.1228	-0.2151
0.636	-0.11052	-0.2151	-2	0.53	-0.1228	-0.2151
0.424	-0.11052	-0.23661	-2	0.53	-0.1228	-0.2151
0.477	-0.11052	-0.23661	-2	0.53	-0.1228	-0.2151
0.53	-0.11052	-0.23661	-2	0.53	-0.1228	-0.2151
0.583	-0.11052	-0.23661	-2	0.53	-0.1228	-0.2151
0.636	-0.11052	-0.23661	-2	0.53	-0.1228	-0.2151
0.424	-0.11052	-0.25812	-2	0.53	-0.1228	-0.2151
0.477	-0.11052	-0.25812	-2	0.53	-0.1228	-0.2151
0.53	-0.11052	-0.25812	-2	0.53	-0.1228	-0.2151
0.583	-0.11052	-0.25812	-2	0.53	-0.1228	-0.2151
0.636	-0.11052	-0.25812	-2	0.53	-0.1228	-0.2151
0.424	-0.1228	-0.17208	-2	0.53	-0.1228	-0.2151
0.477	-0.1228	-0.17208	-2	0.53	-0.1228	-0.2151
0.53	-0.1228	-0.17208	-2	0.53	-0.1228	-0.2151
0.583	-0.1228	-0.17208	-2	0.53	-0.1228	-0.2151
0.636	-0.1228	-0.17208	-2	0.53	-0.1228	-0.2151
0.424	-0.1228	-0.19359	-2	0.53	-0.1228	-0.2151

0.477	-0.1228	-0.19359	-2	0.53	-0.1228	-0.2151
0.53	-0.1228	-0.19359	-2	0.53	-0.1228	-0.2151
0.583	-0.1228	-0.19359	-2	0.53	-0.1228	-0.2151
0.636	-0.1228	-0.19359	-2	0.53	-0.1228	-0.2151
0.424	-0.1228	-0.2151	-2	0.53	-0.1228	-0.2151
0.477	-0.1228	-0.2151	-2	0.53	-0.1228	-0.2151
0.53	-0.1228	-0.2151	1	0.53	-0.1228	-0.2151
0.583	-0.1228	-0.2151	-2	0.53	-0.1228	-0.2151
0.636	-0.1228	-0.2151	-2	0.53	-0.1228	-0.2151
0.424	-0.1228	-0.23661	-2	0.53	-0.1228	-0.2151
0.477	-0.1228	-0.23661	-2	0.53	-0.1228	-0.2151
0.53	-0.1228	-0.23661	-2	0.53	-0.1228	-0.2151
0.583	-0.1228	-0.23661	-2	0.53	-0.1228	-0.2151
0.636	-0.1228	-0.23661	-2	0.53	-0.1228	-0.2151
0.424	-0.1228	-0.25812	-2	0.53	-0.1228	-0.2151
0.477	-0.1228	-0.25812	-2	0.53	-0.1228	-0.2151
0.53	-0.1228	-0.25812	-2	0.53	-0.1228	-0.2151
0.583	-0.1228	-0.25812	-2	0.53	-0.1228	-0.2151
0.636	-0.1228	-0.25812	-2	0.53	-0.1228	-0.2151
0.424	-0.13508	-0.17208	-2	0.53	-0.1228	-0.2151
0.477	-0.13508	-0.17208	-2	0.53	-0.1228	-0.2151
0.53	-0.13508	-0.17208	-2	0.53	-0.1228	-0.2151
0.583	-0.13508	-0.17208	-2	0.53	-0.1228	-0.2151
0.636	-0.13508	-0.17208	-2	0.53	-0.1228	-0.2151
0.424	-0.13508	-0.19359	-2	0.53	-0.1228	-0.2151
0.477	-0.13508	-0.19359	-2	0.53	-0.1228	-0.2151
0.53	-0.13508	-0.19359	-2	0.53	-0.1228	-0.2151
0.583	-0.13508	-0.19359	-2	0.53	-0.1228	-0.2151
0.636	-0.13508	-0.19359	-2	0.53	-0.1228	-0.2151
0.424	-0.13508	-0.2151	-2	0.53	-0.1228	-0.2151
0.477	-0.13508	-0.2151	-2	0.53	-0.1228	-0.2151
0.53	-0.13508	-0.2151	-2	0.53	-0.1228	-0.2151
0.583	-0.13508	-0.2151	-2	0.53	-0.1228	-0.2151
0.636	-0.13508	-0.2151	-2	0.53	-0.1228	-0.2151
0.424	-0.13508	-0.23661	-2	0.53	-0.1228	-0.2151
0.477	-0.13508	-0.23661	-2	0.53	-0.1228	-0.2151
0.53	-0.13508	-0.23661	-2	0.53	-0.1228	-0.2151
0.583	-0.13508	-0.23661	-2	0.53	-0.1228	-0.2151
0.636	-0.13508	-0.23661	-2	0.53	-0.1228	-0.2151
0.424	-0.13508	-0.25812	-2	0.53	-0.1228	-0.2151
0.477	-0.13508	-0.25812	-2	0.53	-0.1228	-0.2151

0.53	-0.13508	-0.25812	-2	0.53	-0.1228	-0.2151
0.583	-0.13508	-0.25812	-2	0.53	-0.1228	-0.2151
0.636	-0.13508	-0.25812	-2	0.53	-0.1228	-0.2151
0.424	-0.14736	-0.17208	-2	0.53	-0.1228	-0.2151
0.477	-0.14736	-0.17208	-2	0.53	-0.1228	-0.2151
0.53	-0.14736	-0.17208	-2	0.53	-0.1228	-0.2151
0.583	-0.14736	-0.17208	-2	0.53	-0.1228	-0.2151
0.636	-0.14736	-0.17208	-2	0.53	-0.1228	-0.2151
0.424	-0.14736	-0.19359	-2	0.53	-0.1228	-0.2151
0.477	-0.14736	-0.19359	-2	0.53	-0.1228	-0.2151
0.53	-0.14736	-0.19359	-2	0.53	-0.1228	-0.2151
0.583	-0.14736	-0.19359	-2	0.53	-0.1228	-0.2151
0.636	-0.14736	-0.19359	-2	0.53	-0.1228	-0.2151
0.424	-0.14736	-0.2151	-2	0.53	-0.1228	-0.2151
0.477	-0.14736	-0.2151	-2	0.53	-0.1228	-0.2151
0.53	-0.14736	-0.2151	-2	0.53	-0.1228	-0.2151
0.583	-0.14736	-0.2151	-2	0.53	-0.1228	-0.2151
0.636	-0.14736	-0.2151	-2	0.53	-0.1228	-0.2151
0.424	-0.14736	-0.23661	-2	0.53	-0.1228	-0.2151
0.477	-0.14736	-0.23661	-2	0.53	-0.1228	-0.2151
0.53	-0.14736	-0.23661	-2	0.53	-0.1228	-0.2151
0.583	-0.14736	-0.23661	-2	0.53	-0.1228	-0.2151
0.636	-0.14736	-0.23661	-2	0.53	-0.1228	-0.2151
0.424	-0.14736	-0.25812	-2	0.53	-0.1228	-0.2151
0.477	-0.14736	-0.25812	-2	0.53	-0.1228	-0.2151
0.53	-0.14736	-0.25812	-2	0.53	-0.1228	-0.2151
0.583	-0.14736	-0.25812	-2	0.53	-0.1228	-0.2151
0.636	-0.14736	-0.25812	-2	0.53	-0.1228	-0.2151

Table G.6: Simultaneous estimation of PTDF set # 6

1st PTDF estimated value	2nd PTDF estimated value	3rd PTDF estimated value	Feasibility flag (feasible=1)	1st PTDF true value	2nd PTDF true value	3rd PTDF true value
-0.09824	0.50664	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.11052	0.50664	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.1228	0.50664	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.13508	0.50664	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.14736	0.50664	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.09824	0.50664	-0.19359	-2	-0.1228	0.6333	-0.2151

-0.11052	0.50664	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.1228	0.50664	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.13508	0.50664	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.14736	0.50664	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.09824	0.50664	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.11052	0.50664	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.1228	0.50664	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.13508	0.50664	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.14736	0.50664	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.09824	0.50664	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.11052	0.50664	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.1228	0.50664	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.13508	0.50664	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.14736	0.50664	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.09824	0.50664	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.11052	0.50664	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.1228	0.50664	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.13508	0.50664	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.14736	0.50664	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.09824	0.56997	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.11052	0.56997	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.1228	0.56997	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.13508	0.56997	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.14736	0.56997	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.09824	0.56997	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.11052	0.56997	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.1228	0.56997	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.13508	0.56997	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.14736	0.56997	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.09824	0.56997	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.11052	0.56997	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.1228	0.56997	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.13508	0.56997	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.14736	0.56997	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.09824	0.56997	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.11052	0.56997	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.1228	0.56997	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.13508	0.56997	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.14736	0.56997	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.09824	0.56997	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.11052	0.56997	-0.25812	-2	-0.1228	0.6333	-0.2151

-0.1228	0.56997	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.13508	0.56997	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.14736	0.56997	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.09824	0.6333	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.11052	0.6333	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.1228	0.6333	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.13508	0.6333	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.14736	0.6333	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.09824	0.6333	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.11052	0.6333	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.1228	0.6333	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.13508	0.6333	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.14736	0.6333	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.09824	0.6333	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.11052	0.6333	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.1228	0.6333	-0.2151	1	-0.1228	0.6333	-0.2151
-0.13508	0.6333	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.14736	0.6333	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.09824	0.6333	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.11052	0.6333	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.1228	0.6333	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.13508	0.6333	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.14736	0.6333	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.09824	0.6333	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.11052	0.6333	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.1228	0.6333	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.13508	0.6333	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.14736	0.6333	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.09824	0.69663	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.11052	0.69663	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.1228	0.69663	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.13508	0.69663	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.14736	0.69663	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.09824	0.69663	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.11052	0.69663	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.1228	0.69663	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.13508	0.69663	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.14736	0.69663	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.09824	0.69663	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.11052	0.69663	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.1228	0.69663	-0.2151	-2	-0.1228	0.6333	-0.2151

-0.13508	0.69663	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.14736	0.69663	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.09824	0.69663	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.11052	0.69663	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.1228	0.69663	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.13508	0.69663	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.14736	0.69663	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.09824	0.69663	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.11052	0.69663	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.1228	0.69663	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.13508	0.69663	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.14736	0.69663	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.09824	0.75996	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.11052	0.75996	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.1228	0.75996	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.13508	0.75996	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.14736	0.75996	-0.17208	-2	-0.1228	0.6333	-0.2151
-0.09824	0.75996	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.11052	0.75996	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.1228	0.75996	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.13508	0.75996	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.14736	0.75996	-0.19359	-2	-0.1228	0.6333	-0.2151
-0.09824	0.75996	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.11052	0.75996	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.1228	0.75996	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.13508	0.75996	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.14736	0.75996	-0.2151	-2	-0.1228	0.6333	-0.2151
-0.09824	0.75996	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.11052	0.75996	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.1228	0.75996	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.13508	0.75996	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.14736	0.75996	-0.23661	-2	-0.1228	0.6333	-0.2151
-0.09824	0.75996	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.11052	0.75996	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.1228	0.75996	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.13508	0.75996	-0.25812	-2	-0.1228	0.6333	-0.2151
-0.14736	0.75996	-0.25812	-2	-0.1228	0.6333	-0.2151