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EEG STUDY OF SIMPLE PROBLEM SOLVING

by

MATTHEW COPELLO

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

Bachelors of Arts

in the College of Arts and Sciences

Georgia State University

2017

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Date: _____

EEG STUDY OF SIMPLE PROBLEM SOLVING

by

MATTHEW COPELLO

Under the Direction of Jessica Turner, Ph.D

ABSTRACT

This study was conducted in order to replicate the findings of Earle (1985) in a modern-day setting. Earle (1985) supported that different hemispheres aid in problem solving depending on problem difficulty. These findings were evident by a change in lateral hemispheric inhibition while participants solved "medium" difficulty math problems. Participants were asked to solve multiplication problems in their heads, without the help of a pen, paper, or a calculator. Electroencephalogram (EEG) data was recorded over the Parietal and Temporal lobe during a resting state and while participants solved math problems of "easy", "intermediate", and "hard" conditions. Data was recorded from two matching base pairs across the cortex in order to measure changes in the alpha frequency across the two hemispheres. This study was unable to replicate the findings from Earle (1985), but provides information regarding factors to consider when measuring the alpha band with an EEG.

INDEX WORDS: Alpha Band, Alpha Asymmetry, EEG

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LIST OF ABBREVIATIONS

Analysis of Variance …………………………………………………….. ANOVA

Electroencephalogram…………………………………………………….. EEG

Georgia State University……….……………………………………… GSU

International Review Board…………………………………………….. IRB

1 INTRODUCTION

The alpha wave was the first and most distinguishable wave to be discovered by Hans Berger, the inventor of Electroencephalography (EEG). The alpha wave is a frequency that oscillates around 8 – 13 Hz, and is ubiquitous across the cortex of the brain (Berger, 1929). Very broadly, the alpha wave is reliably observable during periods of relaxation and absence of cognitive activity (Lagopoulos et al., 2009; Teplan, 2002). Due to the accessibility of observable alpha, much research has gone into investigating this inverse relationship of alpha activity and cortical activity. Some recent research into the alpha wave has focused on alpha laterality across the brain hemispheres by measuring differences in power from two matching left and right electrode sites, a measurement known as Alpha Asymmetry.

It was demonstrated by Ahern & Schwartz (1985) that alpha asymmetry is a mediator of emotions, apparent by a decrease in alpha power in the left hemisphere during experiences of negative emotions, and in the right hemisphere during experiences of positive emotions. Moreover, baseline measures of alpha asymmetry have been correlated with the likelihood of an individual's motivation and tendency to withdraw or respond to an emotionally negative stimulus (Harmon-Jones, 2006; Coan & Allen, 2003). More interactions of alpha asymmetry and cognitive functioning include the effects of spatial processing during times of enhanced attention. Lateralized alpha has been shown to aid in either local processing (decreased left-hemispheric alpha) or global processing (decreased right-hemispheric alpha) (Gable, et al., 2013).

Fernández, et al. (1995) argued that when calculating arithmetic problems, one must perceive, comprehend and produce numbers, process the rules of the equation (e.g., multiplication, division, and/or addition), mentally access arithmetical facts, and execute the

retrieved calculation procedure. Based on this overt description of the arithmetic solving process, arithmetic clearly requires the use of working memory. In a similar vein, changes in alpha power has been found to correlate with high-work load working memory tasks (Jensen et al. 2002; Mathewson et al., 2011). Moreover, research supports the laterality of alpha to be elicited during arithmetic problem-solving tasks (Doyle, et al., 1974; Earle, et al., 1985). Based on the literature framework, alpha band lateralization will be measured during problem-solving tasks when the tasks require an adequate amount of working memory.

1.1 The Replicated Paper

By following a related protocol, I seek to obtain results similar to Earle (1985), who used EEG during problem-solving tasks and found more left-hemispheric alpha activity when the participants solved multiplication problems of moderate difficulty level compared to when they solved easy problems, and when they were at rest. In addition, Earle (1985) found evidence of a decrease in alpha asymmetry when participants solved multiplication problems of hard difficulty. Earle (1985) obtained results by using the homologous base pair from 2 electrode sites over the parietal lobe: P3 and P4.

An important aspect regarding the Earle paper pertained to the division of solution-based performance. Earle (1985) imposed a median split of average solution latency in order to divide participants falling above or below the median into two groups of slow and fast latency. The findings from Earle (1985) were dependent on analyses of the participants' data after being split into slow and fast solution latency. This was done in response to research supporting that a fast reaction time is indicative of better arithmetic abilities and faster information processing. Additionally, research supports that faster information processing is positively correlated with exhibited alpha power (Glass & Butler, 1977; Klimesch, 1999).

1.2 The Purpose of this Study

In the present study, an effort is made to contribute evidence for elicited alpha asymmetry during an arithmetic task. In this case, the task will be involving the need to solve arithmetic using only mental calculations. In order to devise a task that will measure the challenges of the participants' working memory, the task will require solving multiplication problems of varying difficulties. By replicating the study conducted by Earle (1985), alpha laterality will be tracked while participants solve multiplication problems in their heads. The Earle paper presents a task that *a priori* meets the criteria for challenging the working memory of participants. This is the case due to imposing the need to retrieve arithmetical facts while mentally carrying and manipulating numbers during the problem-solving procedure.

2 EXPERIMENT

2.1 Participants

This study recruited participants from GSU-SONA and was approved by the GSU IRB. Data from 20 (14 female and 6 male) undergraduate participants with a mean age of 19 was analyzed for this EEG study. Participants were excluded from the study when they were under 18, left hand dominant, taking psychotropic medicines or had neuropsychiatric disorders.

2.2 EEG recording

This study used EMOTIV's EPOC 14-electrode wireless EEG system with a reference on the left mastoid (Emotiv-Epoc® BCI headset). Electrode locations agreed with the standard 10/20 EEG (Badcock, et al., 2013). Both pre and post-task resting state recordings consisted of 8-minute counterbalanced sequences of alternating 2-minute eyes-open and eyes-closed epochs. Following Coan & Allen (2003), the process was followed identically when post-test resting state data was collected immediately following the problem-solving task.

After baseline resting state had been collected, EEG recordings were performed while participants solved three 2-minute long trials of multiplication problems presented in a random order. The trial sequence was documented by appropriately placing labeled markers at the beginning and end of the 2 minutes. Participant data was not analyzed when gross muscle or mechanical artifacts were present.

Data was collected and analyzed from 2 homologous pairs of electrodes located at P7 (left), P8 (right), and T7 (left), T8 (right). P7 and P8 were needed in order to operationalize Earle's (1985) findings from the parietal lobe (P3 and P4). Additionally, the parietal lobe is regarded as a major driving source of alpha power (Haegens, 2014; Klimesch et al., 1993). The T7 and T8 pair was added in order to have a second homologous pair to compare alpha amplitudes. Data from these pairs were analyzed using Fast Fourier Transformations with EEGLAB on Matlab software. These 4 channels were down sampled to 128 points per second. Additionally, in order to minimize higher frequency noise, a bandpass filter was added with a lower bound of 1 Hz and an upper bound of 41 Hz. The alpha band was analyzed by isolating oscillations between 8 and 13 Hz (Berger, 1929; Harmon-Jones, 2006; Gable, et al., 2013; Ehrlichman & Wiener, 1979; Díaz, 2015; Teplan, 2002).

2.3 Measuring Alpha Asymmetry

Obtaining alpha asymmetry was done in two ways. Earle (1985) obtained alpha asymmetry from homologous electrode sites by taking the proportion of difference ratios: (RH+LH)/(RH-LH) x 100. This equation for obtaining cortical asymmetry is somewhat supported by findings from Ahern & Schwartz (1985), differing only by the order of the hemispheres in the equation (i.e., LH+RH/LH-RH x 100). The second equation for obtaining alpha asymmetry was by using the natural log of the alpha power in the right hemisphere and

subtracting it against the natural log of alpha power in the left hemisphere (i.e., ln(r) – ln(l)), an equation with more literature support (Coan & Allen, 2003; Ehrlichman & Wiener, 1979; Harmon-Jones, 2006; Gable et al., 2013).

2.4 Q-Values

Earle (1985) quantified the discrepancy between problem difficulties by following protocol of Thomas' (1963) Constellation Hypothesis of Calculation to generate Q values. In this study, Q values for each multiplication problem ranged from .9– 6.3, and fell within an absolute spectrum of .6 – 7.2 (easiest to hardest, respectively). Q values were generated by measurement of the information content within the arithmetic task. This strategy was used for the current study and is regarded as valid as it takes into account both the size of the problem and the need for carrying digits when calculating problems (Walter, 2014; Spüler, 2016).

2.5 Procedure

Participants were invited into a well-lit, carpeted, and quite room. The participants were asked to sit in a stationary, padded chair with a single desktop computer, a laptop and a full keyboard placed in front of them. Data was collected in a room with a non-laboratory feeling so as to minimize anxiety and discomfort. Following informed consent, participants were given thorough instructions about the task. In order to provide a warm-up, participants solved a set of practice problems during which they could use a pen or pencil. After participants had finished the practice set, the EEG was placed on the participant. Resting state data was then recorded while participants either had their eyes open or eyes closed. Differing from the protocol of Earle (1985), participants were instructed to relax before the resting state condition and to focus on their breathing during each of the 2-minute resting state sequences. This was done in order to keep the mind clear of wondering thought (Doyle et al., 1974). Furthermore, during the eyes

open condition, participants were asked to focus on a fixation point directly in front of them. In order to habituate them to the EEG, and to familiarize them with the program, participants solved a set of practice multiplication problems on their computer following resting state. The practice problems were similarly administrated and were similar in difficulty to the problems they solved for the actual task. During the practice trial, 3 problems from each difficulty were given, with no time limit. Analogous to Earle (1985), participants were instructed to solve the problems as quickly and as accurately as possible, without the help of pen and paper or a calculator during both the practice and real task. Promptly after the practice task, the real task was given with blocks of "easy", "intermediate", and "hard" that were administered in a random order.

The problem-solving task differed from Earle (1985). Rather than using a total of 17 multiplication problems, a pool of 74 problems was used: 45 easy, 20 intermediate, and 10 hard in order to have a 2-minute block for each condition. Arithmetic problems and trial sequences were assigned in a random order for every participant. Participants were allowed to correct their answer if they made a mistake by pressing the "backspace" key. Solution latency was recorded automatically after the participants confirmed their answer by pressing the "enter" key. Pressing the "enter" key would display a new problem for the participants to solve. By allowing participants to input answers into a computer, the current study accommodated to a visual-visual strategy. Much differently, Earle (1985) used a visual-verbal strategy by the presentation of arithmetic problems on paper and submission of oral answers.

Immediately following the end of the task, post-test resting state data was collected following the established resting state protocol. See figure 1 for a visual demonstration for the sequence of the experimental conditions.

Figure 1. *A visual demonstration for the sequence of activates done by the participants during the study.*

Baseline Resting State	→	Practice Sequence	→	Randomly Assigned Block	→	Randomly Assigned Block	→	Randomly Assigned Block	→	Baseline Resting State
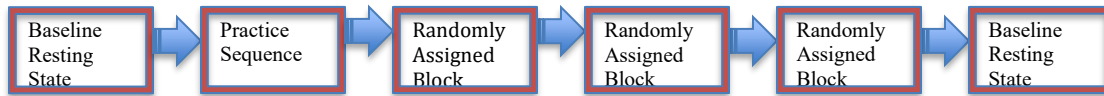
2.7 Behavioral Data Analysis

In order to validate appropriate difficulty level for each condition, performance proportions (accuracy), measured by the proportions of correct vs. completed problems, were entered into a one-way repeated measures ANOVA with a threshold of $p < .05$. Post hoc analyses was done using Tukey's HSD. This method differs from Earle's (1985) validation of difficulty, where solution latencies were used to assess difficulty validity. It is argued, however, that assessment of accuracy eliminates the chance that a trial is statistically seen as easy or hard due only to participants quickly entering answers. Additionally, correlational coefficients were calculated to examine the relationship between the number of completed problems and correct problems for all participants within each condition.

Following criteria from Earle (1985), correlation coefficients were computed for solution latencies and baseline resting state asymmetry scores, task asymmetry scores, and mean difficulty Q-value. Additionally, the same correlation coefficients were computed using accuracy rather than solution latencies.

2.6 Alpha Asymmetry Data Analysis

As done by Earle (1985), participants were split into fast and slow solution latency groups (referred to here as participant groups) by using the median value of average solution latencies. This median split placed participants into fast and slow groups based on their average solution latencies falling above or below the median

Alpha asymmetry scores were calculated ln(R)-ln(L) and (RH+LH)/(RH-LH) x 100 and analyzed separately. Scores were averaged in order to provide one score per participant in each difficulty level. In order to investigate the effects that each difficulty level had on alpha asymmetry, individual scores were entered into a 2 x 4 (participant group x problem difficulty) repeated measures Analysis of Variance (ANOVA). This analysis included resting state as a "zero difficulty" condition.

3   RESULTS

3.1 Validation of Difficulty

An extremely important aspect of this study is ensuring that the difficulty of questions was valid across all trials. Although Thomas' Q value (1963) is cited frequently, the *a priori* notion of difficulty based on information content should certainly be affirmed. Both solution latencies and accuracy were taken into account for each participant in each condition. Detailed in table 2, accuracy of problems was analyzed with a one-way repeated measures ANOVA, a significant main effect was found [$F_{(2,19)}$ = 15.644, $p <$. 0001]. A post hoc Tukey test showed that the groups did not differ significantly at $p < .05$. Figure 2 and table 1 demonstrates this significant decrease in accuracy as the difficulty in each trial increased.

Figure 2 . *This graph shows the decrease in accuracy of participants as the difficulty blocks increased.*
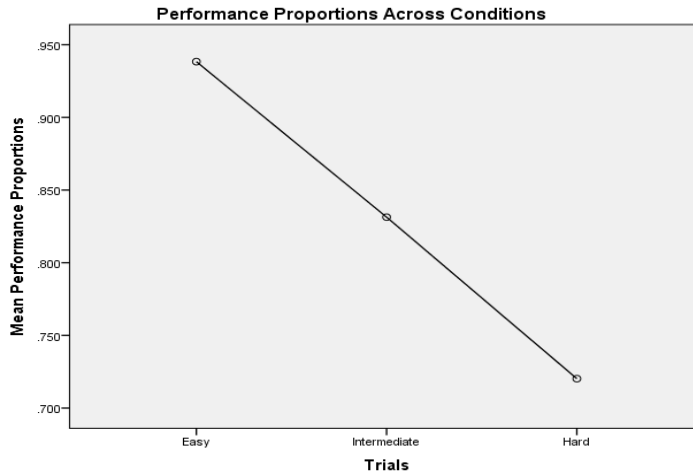

Table 1. *This table adds detail to Figure 2, by providing the means, standard error, and 95% confidence intervals for each problem-solving block. The greatest difference between conditions can be seen between Easy and Hard.*

Blocks	Mean	Std. Error	Lower Bound (95% Confidence Interval)	Upper Bound (95% Confidence Interval)
Easy	.938	.016	.904	.973
Intermediate	.831	.021	.787	.876
Hard	.720	.046	.623	.817

Table 2. *This table details the statistical analysis of difficulty level and accuracy of participants. Accuracy was determined by performance proportions (completed problems / correct problems).*

	Sum of Squares	Df	Mean Square	F	Significance	Eta Squared
Accuracy	.476	2	.238	15.644	.0001	.452

3.2 Participant Groups

When average accuracy means across all conditions within each participant group was

passed through an independent measure T-Test, there was no significant difference in the

accuracy of participant groups [t (18) = 1.079, $p > .05$]. As seen in figure 3, participants in both groups were, on average, 80-85% correct across all conditions. When comparing the accuracy means for each participant group between all problem-solving conditions, no significance was found: easy [t (18) = -.424, $p > .05$], intermediate [t (18) = 1.288, $p > .05$], and hard [t (18) = 1.102, $p > .05$].

Figure 3. *In this graph, the similarities in participant (performance) groups are illustrated in terms of accuracy averaged within each group.*
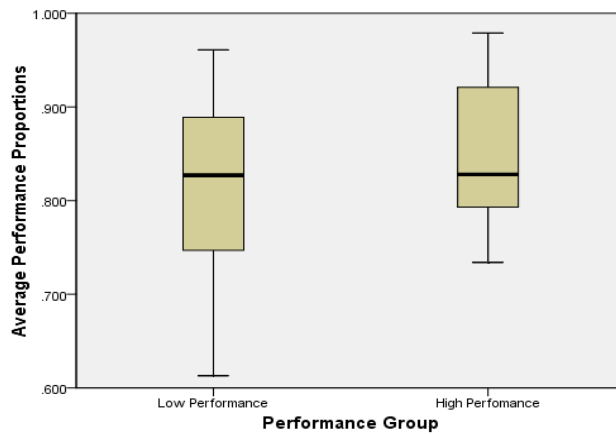
3.3 Alpha Values

The F-test of asymmetry scores for participants in all 4 difficulty conditions did not yield significant effects of either lobe. Mauchly's test indicated that the assumption of sphericity had been violated, [$\chi2$ (5) = 24.40, P < .05]. In response, data was analyzed using the Greenhouse – Geisser estimates of sphericity [F (2.11, 41.11) = .495, $p > .05$]. The F-test showed no significant main effect or interaction, refer to table 3 for more details.

Table 3.*This F-table provides more information regarding the statistically insignificant 2 x 4 (participant group x difficulty condition) repeated measures ANOVA.*

		Sum of Squares	Df	Mean Square	F	Significance	Eta Squared
Difficulty Condition x Participant Group	Greenhouse-Geisser	.077	2.11	.036	.495	.624	.027

The alpha asymmetry scores of each participant group is demonstrated in figures 4 and 5, and tables 4 and 5. Based on the figures, there appears to be an interaction between the two groups. When analyzing the confidence intervals at 95%, however, the overlap indicates that the conditions were possibly not manipulated enough.

Figure 4. *The graph here shows interactions between participant groups in terms of averaged alpha asymmetry scores across all difficulty conditions Parietal Lobe.*

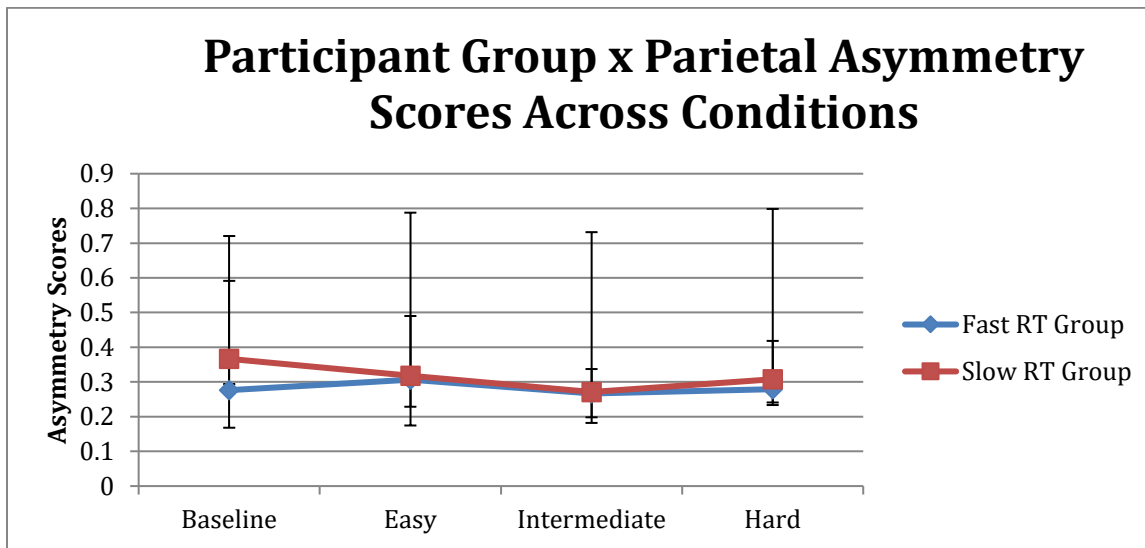**Table 4.** *This table provides information for mean alpha asymmetry scores in the Parietal lobe. Values in the parenthesis indicate the range of Q values within each difficulty condition.*

Reaction Time (Parietal)	Baseline (n/a)	Easy (1.42 – 1.56)	Intermediate (2.84 - 3.43)	Hard (4.70 – 5.57)
Fast	.276	.307	.267	.279
Slow	.366	.318	.271	.308

Figure 5. *The graph here shows interactions between participant groups in terms of averaged alpha asymmetry scores across all difficulty conditions within the Temporal lobe.*
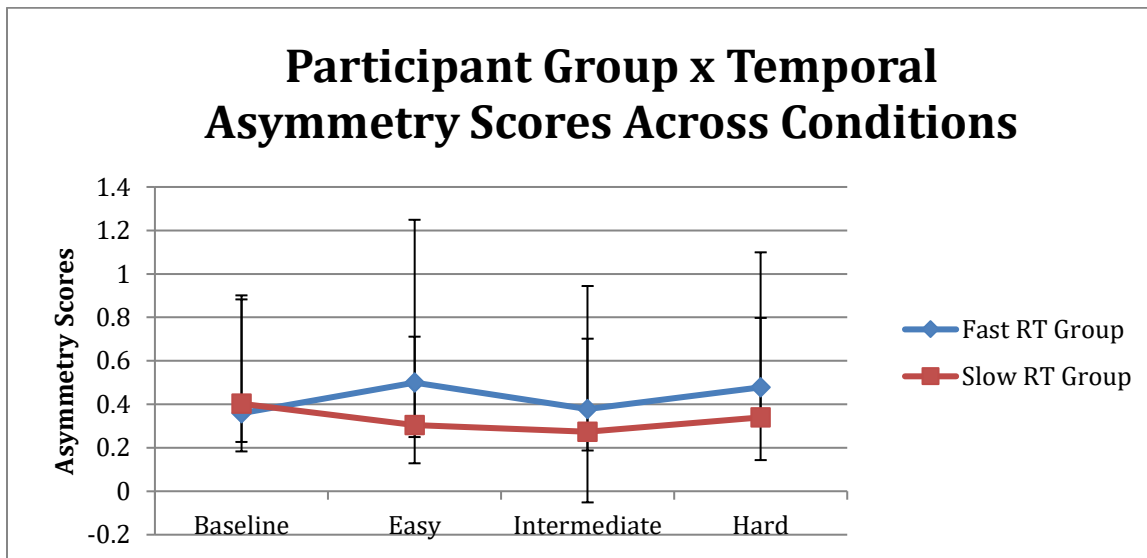
Table 5. *The graph here shows interactions between participant groups in terms of averaged alpha asymmetry scores across all difficulty conditions within the Temporal lobe.*

Reaction Time (temporal)	Baseline (n/a)	Easy (1.42 – 1.56)	Intermediate (2.84 – 3.43)	Hard (4.70 – 5.57)
Fast	.359	.500	.378	.478
Slow	.403	.304	.274	.340

3.4 Asymmetry and Performance Correlations

Correlation coefficients calculated to analyze solution latency means and Q values did not show a significant relationship. Correlation coefficients calculated using accuracy rather than solution latency produced similar null results. Notably, as seen in figure 4, a non-significant

negative trend was seen between accuracy and temporal lobe asymmetry scores within the easy

condition [r (20) = -.407, p = .07].

Figure 6. *The graph here demonstrates the non-significant negative trend between asymmetry scores and accuracy (performance proportions) of each participant in the temporal lobe.*
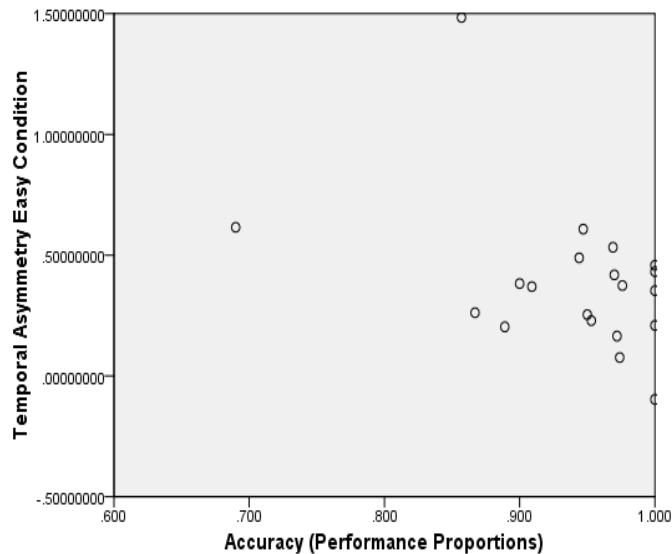
4   CONCLUSION

The present EEG study analyzed recordings of lateralized alpha frequencies during

problem-solving tasks. Problems varied in difficulty for each trial, and difficulty was quantified

based on previous literature. A one-way ANOVA found that accuracy across conditions

decreased with increasing difficulty, and proved that the conditions did in fact increase in

difficulty. The current study however, was unable to replicate findings supported by Earle

(1985). In response, there are multiple facets to examine when studying problem-solving and

signal processing that must be considered in order to explore this question as in depth as

intended.

Earle (1985) findings were contingent upon the imposed solution latency-based median split. This strategy of splitting participants is supported by literature that reported a positive relationship between information processing and alpha asymmetry (Glass & Butler, 1977). The null findings of this study, however, indicated no significant difference in asymmetric scores between the two groups. Moreover, there was no difference in the accuracy of completed problems between the two groups, even when the calculated accuracy was seen to appropriately reflect the difficulty of arithmetic problems. The present analysis, therefore, effectively rendered the judgment of groups based on high and low *performance* meaningless.

This is not to say that the speed of processing information has no effect on the alpha band. Indeed, the literature suggests that people who process information faster have an alpha frequency that is 1 Hz higher than those who process information slower within their same age group (Klimesch, 2013; 1997; 1993). Rather, the participants' solution accuracy must be taken into account in tandem with their solution latency. Otherwise, participants' answers could have been mindlessly provided, causing an idled mind that could have driven the variability in alpha power. An effect that is evident by the well-studied negative correlation between alpha power and cortical activity (Bazanova & Vernon, 2013, Niedermeyer & Lopes da Silva, 2004; Lagopoulos et al., 2009; Teplan, 2002).

An important difference with the current study and the Earle (1985) study is the method of answer submission. Although Earle (1985) recorded answers provided by the participants orally, this study allowed for answers to be submitted on a keyboard. The oral method of answer submission was seen as flawed due to the potential dangers of head or facial artifacts in the EEG data. Indeed, the current study took measures against potential artifacts by providing an external keyboard with a number pad. It was thought that with an external keyboard, participants would

feel comfortable and make fewer head movements when locating keys. Interestingly, Fernandez, et al. (1995) found alpha power from the right posterior areas of the brain to be significantly different during the recognition of arithmetic symbols compared to the actual calculation of the arithmetic task. It can therefore be argued that participants who stared at the screen to examine the arithmetic symbols while solving the problems could have had a different alpha pattern compared to those whom may have looked elsewhere after receiving the problem. Investigations into problem-solving's effect on alpha power should consider removing the stimuli after it has been administered.

A problem with the current study was noted when talking to participants after the conclusion of the study. During this time, participants reported the use of various strategies to solve the arithmetic problems. Two strategies were reported most often: raw procedural calculations, and induction of easier to retrieve values, followed by subtraction or addition to the appropriate answer. For example, instead of procedurally solving (225 x 4 = 900), participants who retrieved values and adjusted their answer would have solved the problem by easily retrieving (225 x 4 = 1,000), then subtracted the answer to the easily retrieved (25 x 4 = 100) in order to get the correct answer. This was made apparent when many of the participants whom performed well on the "hard" conditions reported struggling more so on some of the problems in the "easy" condition. This is understandable for problems such as (7 x 8 = 56) where, unless the participant had memorized the solution, there is no answer that is quickly accessible. Evident by Campbell & Xue (2001), procedural vs. retrieval strategies when solving mental arithmetic are prevalent depending on culture, and age group. This effect very well could have been a major downfall of the study. Indeed, these strategies have an effect on alpha, evident by a decreases in

alpha power across the cortex having a correlation with procedural strategies of problem-solving (Smedt et al., 2009). It is unknown whether this effect could account for the null results.

Finally, research surrounding the alpha band has provided evidence for its peak malleability due to individual human differences (Haegens, 2014; Klimesch, 1999). Alpha peak is typically understood to be observable at 10 Hz on the 8 – 13 Hz band (Berger, 1922). Presumably, 10 Hz is the location where alpha will have the most power on a spectra graph. This may be incorrect, however, during cognitively demanding tasks. As such was the case with Haegens (2014), who found that alpha variability was evident by individual frequency shifts during such tasks. These frequency shifts are partly driven by age, where alpha peak increases until adulthood, then reliably begins to decrease (Aurlien et al., 2004; Klimesch, 1999; Köpruner et al., 1984). Due to the effect that age can have on individual alpha peak, reporting of an age range is important for studies regarding analyses of the alpha band. While this study had an age range of 18-21, the age range of the Earle (1985) paper is unknown.

Furthermore, recent research measuring individual alpha frequency has brought with it new ways of studying signal processing. Rather than setting a meta-standard of 8 – 13 Hz with a 10 Hz peak for every participant, recognition should be given to measuring alpha power after participants have been categorized into groups of high alpha peak (greater than 10 Hz) or a low alpha peak (less than 10 Hz) (Bazanova & Vernon, 2012). Literature examining alpha in terms of high and low peaks have suggested variability to be driven by genetics (Lopes da Silva, 1991), the activity of calcium T-channels (Destexhe & Sejnowski, 2003), and IQ (Jaušovec & Jaušovec, 2000).

The current study focused on engaging participants in an arithmetic task in order to measure hemispheric alpha changes. In order to do this, a study by Earle (1985) was replicated and modified in a modern-day setting. This study was unable to replicate findings proposed by Earle (1985) suggesting that left hemispheric alpha increased during medium difficulty level conditions. In agreement with prior research, findings were contingent upon participants split into fast and slow reaction time groups in order to operationalize information processing speed and arithmetic abilities. Interestingly, when participants were split into groups in the same way for this study, legitimate differences in task-performance was not statistically proven.

There are many differences in the Earle (1985) paper and the current study. By allowing for participants to correct a mistake before moving onto the next trial, the way in which participants were thinking about each problem was unquestionably different from Earle (1985). Moreover, the results in this study could have been different base on how participants were provided as many trials as they could solve within the 2-minutes block. This lassies-faire technique for administering problems allowed for significantly more trials to be solved. The use of a 2-minute block provided more data was per participant compared to the 14-question standard administered by Earle (1985).

Future research on problem solving should allow for participants to solve problems in blocks with a time limit. Imposing a time limit, rather than a set quantity of problems, allows for more problems to be encountered based on performance. This technique decreases the chances of spurious results by increasing the amount of data collected. Additionally, by evidence of modern-day research, the methods for signal processing are quickly innovating. When investigating problem solving and lateral alpha, one should take precocious steps into isolating the individual alpha peak for each participant.

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