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## 3-D Reconstructions and Numerical Simulations of Precarious Rocks in Southern California

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**Eleventh U.S. National Conference on Earthquake Engineering**  
*Integrating Science, Engineering & Policy*  
June 25-29, 2018  
Los Angeles, California

# 3-D RECONSTRUCTIONS AND NUMERICAL SIMULATIONS OF PRECARIOUS ROCKS IN SOUTHERN CALIFORNIA

C.E. Wittich<sup>1</sup>, T.C. Hutchinson<sup>2</sup>, J. DeSanto<sup>3</sup>, and D. Sandwell<sup>4</sup>

## ABSTRACT

Reliable estimates of seismic hazard are essential for the development of resilient communities; however, estimates of rare, yet high intensity earthquakes are highly uncertain due to a lack of observations and recordings. Lacking this data, seismic hazard analyses may be based on extrapolations from earthquakes with more moderate return periods, which can lead to physically unrealistic earthquake scenarios. However, the existence of certain precariously balanced rocks (PBRs) has been identified as an indicator of an upper bound ground motion, which precludes toppling of the balanced rock, over its lifetime. To this end, a survey of PBRs was conducted in proximity to the Elsinore fault east of San Diego, CA. Each identified PBR is modeled using point clouds derived from ground-based laser scanning and images from an unmanned aerial vehicle. The resultant geometric reconstructions are then used in a probabilistic overturning analysis and compared to the anticipated seismic hazard at the site. Accounting for an estimated age range and 50% probability of overturning for the PBRs, approximately half of the surveyed PBRs indicate a potential overestimation of seismic hazard at the site.

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### ABSTRACT

Reliable estimates of seismic hazard are essential for the development of resilient communities; however, estimates of rare, yet high intensity earthquakes are highly uncertain due to a lack of observations and recordings. Lacking this data, seismic hazard analyses may be based on extrapolations from earthquakes with more moderate return periods, which can lead to physically unrealistic earthquake scenarios. However, the existence of certain precariously balanced rocks (PBRs) has been identified as an indicator of an upper bound ground motion, which precludes toppling of the balanced rock, over its lifetime. To this end, a survey of PBRs was conducted in proximity to the Elsinore fault east of San Diego, CA. Each identified PBR is modeled using point clouds derived from ground-based laser scanning and images from an unmanned aerial vehicle. The resultant geometric reconstructions are then used in a probabilistic overturning analysis and compared to the anticipated seismic hazard at the site. Accounting for an estimated age range and 50% probability of overturning for the PBRs, approximately half of the surveyed PBRs indicate a potential overestimation of seismic hazard at the site.

### Introduction

Reliable estimates of seismic hazard are essential for sound decision-making and the development of resilient communities; however, estimates of infrequent earthquakes become exceedingly uncertain due to a lack of observation going back more than a few hundred years [e.g. 1]. For example, nuclear power plants and nuclear waste repositories must be designed to withstand extremely rare seismic events (return periods in excess of 10,000 – 100,000 years), however there is limited knowledge of the amplitudes of the ground motions resulting from such an infrequent earthquake. Ground motion predictions are generally developed through statistical regression on a large number of earthquake observations as a function of the earthquake magnitude, distance, and

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other site-specific predictors [e.g. 2]. As such, ground motion predictions are most representative of relatively moderate earthquakes with return periods less than 2500 years. Subsequent extrapolations for rare earthquakes are unbounded and can lead to physically unrealizable scenarios [3]. In the absence of significant earthquake observations, the existence of certain ancient, precariously balanced or fragile rock formations provide a way to deduce a maximum possible ground motion amplitude over the lifetime of the formation – that which precludes overturning or toppling [4].

Precariously balanced rocks (PBRs), an example of which is seen in Figure 1a, are individual or groups of rocks which have eroded over time into highly unstable configurations. Tall, slender structures, such as PBRs, behave as systems of rigid bodies during earthquakes; and, as such, they tend to rock, slide, and overturn (or topple) during a sufficiently intense earthquake. Therefore, the existence of a given PBR indicates that an earthquake large enough to overturn it has not occurred over the course of its lifetime. Given that the ages of these formations are estimated in excess of 10,000 years [5], a physically-meaningful upper bound to the seismic hazard can be inferred from a toppling analysis of a particular PBR. To this end, recent studies have presented hundreds of PBRs throughout the seismic southern California region [6]. Studies of these PBRs have been used to test seismic hazard models [7], fault activity rates [8], and ground motion prediction equations [9].

Analyses of the overturning potential of individual or groups of PBRs tend to focus on rigid body dynamics [e.g. 10, and references therein], or more specifically the rigid body rocking problem. Rigid body rocking, as first presented in the context of earthquake engineering by Housner [11], is a theoretical representation for the two-dimensional motion of a rigid block uplifting about one edge and impacting a rigid base. A schematic of this block and relevant geometric parameters is included in Figure 1b. The arbitrarily shaped block is characterized by mass,  $m$ ; mass moment of inertia about its center of mass,  $I_m$ ; total width,  $B$ , which is measured between extreme rocking points; and, total height,  $H$ . Each rocking point is described by a rocking radius,  $R$ ; width,  $b$ ; height to the center of mass,  $h$ ; and, a slenderness, or critical angle,  $\alpha$ . The equation of motion for uplift,  $\theta$ , about a rocking point is provided in Eq. 1 and the reduction in velocity at an impact is provided in Eq. 2.

$$(I_m + mR_i^2)\ddot{\theta} = -mgR_i \sin(\alpha_i - \theta) - m\ddot{x}_g R_i \cos(\alpha_i - \theta) \quad (1)$$

$$r_{i/j} = \frac{\dot{\theta}^+}{\dot{\theta}^-} = \frac{I_m + mR_i R_j \cos(\alpha_j - \alpha_i)}{I_m + mR_i^2} \quad (2)$$

Eq. 1 and Eq. 2 emphasize that rigid body rocking is highly nonlinear with respect to the geometry of the block, namely with respect to the rocking radius and the slenderness angle.

While previous studies incorporate the equations of rigid body rocking [e.g. 12] which carry significant geometric nonlinearities, few have studied the impact of geometric data acquisition of individual PBRs. For example, many studies have relied upon the use of orthophotos for the determination of the PBR's rocking radius and slenderness angle. Given the known geometric nonlinearity and recent technological advances, it is the objective of this paper to compare the efficacy of multiple non-contact geometric data acquisition systems within the context of PBRs. First, a survey of PBRs near Jacumba, CA, is presented, in which both lidar scanning and computer vision approaches are used to document the geometry of the rocks. Then, the resulting point clouds are processed to extract the interface and calculate the necessary geometric

parameters. The geometry is then used in a preliminary probabilistic toppling analysis and compared to the seismic hazard for the site. The final section of this paper presents an analysis of interface variations and the potential impact on the probabilistic toppling analysis and resulting conclusions regarding seismic hazard.

### Survey of Precariously Balanced Rocks

Precariously balanced rocks (PBRs) have been documented throughout the southern California region since the early 1990s [e.g. 4]. A map of known PBRs is presented in Figure 2, as compiled through the SCEC Precariously Balanced Rocks Database [6]. As shown in this map, there is a small subset of PBRs clustered near the Elsinore and Laguna Salada faults near Jacumba, CA (approximately 70 miles east of San Diego, CA). While PBRs in this region were previously identified and included in the database, detailed analyses of this PBR cluster had not previously been presented and forms the basis and motivation of this work.

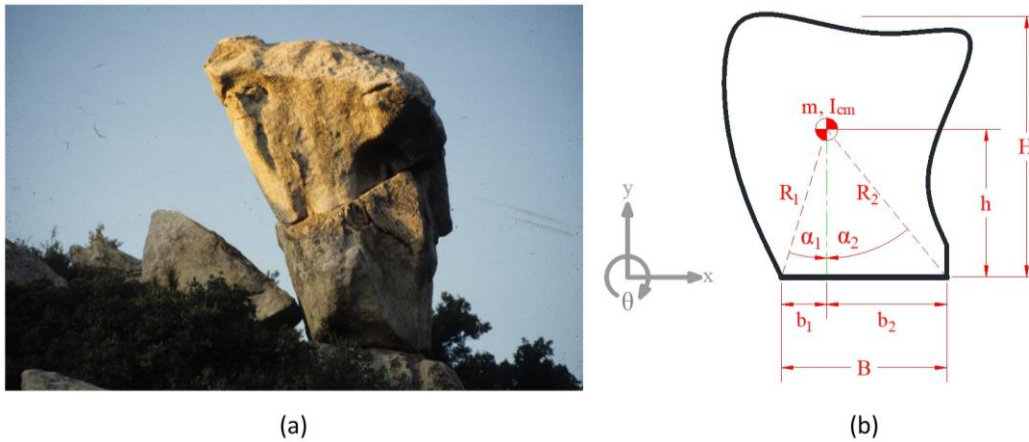


Figure 1. (a) Sample precariously balanced rock (PBR) from the SCEC PBR Archive [6]. (b) Schematic of a two-dimensional rocking block and relevant geometric properties.

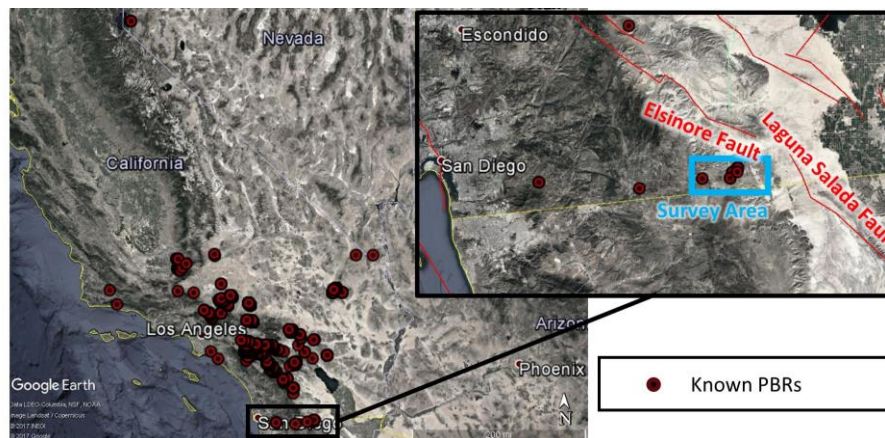


Figure 2. Map of known precariously balanced rocks, as obtained from the SCEC database [6], with a detailed view of the southern California region surveyed in this study including the Elsinore and Laguna Salada fault lines.



## Site Overview

The PBR cluster of interest is located near Jacumba, CA ( $32.654^{\circ}$  N,  $116.106^{\circ}$  W), which is characterized by significant granitic rocky outcrops. Two PBRs were previously identified in this area, specifically R4\_00260 and R4\_00262 from the existing database [6]. Walking surveys of this region in Fall 2016 identified another four precariously balanced rocks, as shown in Figure 3, yielding a total of six PBRs over an approximately  $4000\text{ m}^2$  area. Detailed geometric data was sought for each PBR in the cluster, in an effort to study the overturning probabilities, which are known to be nonlinear with respect to geometry. However, detailed geometric surveys were only conducted for four of the six rocks (a-d in Figure 2) as detailed in the next section, while two rocks were approximately surveyed using photographic documentation. It is noted that while the PBRs are presented with respect to geometry and general location, precise GPS coordinates for the individual rocks are not provided herein, as agreed upon and recommended by the PBR community [13].

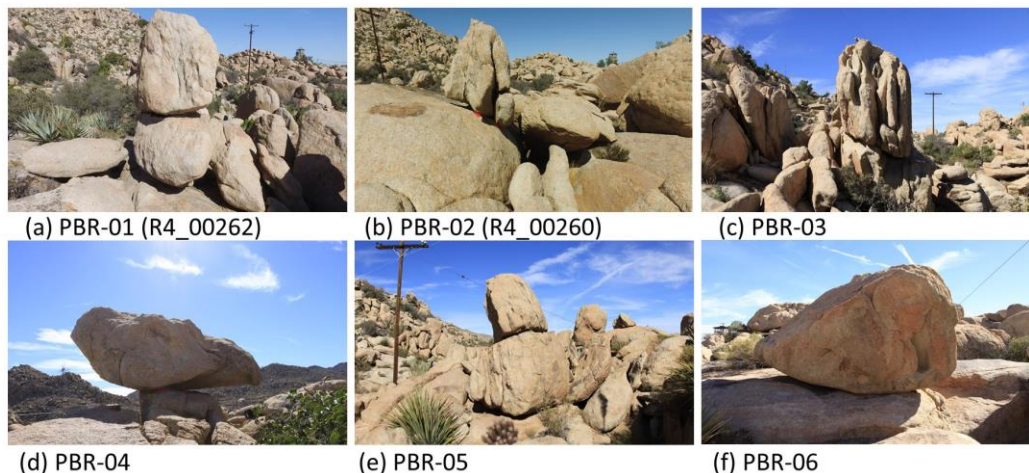


Figure 3. Surveyed PBRs in the Jacumba, CA, vicinity including (a-b) two previously known rocks with PBR record number from the SCEC database in the parenthetical, and (c-f) four newly identified rocks.

## Data Acquisition

### *Lidar Scanning*

Light detection and ranging (lidar) or laser scanning is a technique that provides dense depth maps of objects within the line of sight. The scanners, which are commercially available, emit continuous waves of laser light which are reflected by objects in its path, and upon return yield the distance to the object. As this is a line-of-sight technology, multiple scans of a given PBR are required to minimize occlusion. These scans are then aligned to form a single unified point cloud using iterative cloud-to-cloud comparisons. The result is a 3-D point cloud consisting of a set of points describing the surface of the scene in terms of  $xyz$  coordinates and  $rgb$  color. The resultant cloud is detailed and geometrically accurate with point-to-point distances of 1-mm to 1-cm.

For this field survey, a Faro Focus x120 scanner was utilized and approximately six to seven scans of 15-20 minutes were conducted for each of the PBRs. A sample setup of the scanner in the field is provided in Figure 4a. It is noted that while scan locations were selected to minimize

occlusion, the uneven and non-uniform environment precluded certain viewpoints. The resultant point cloud for PBR-01 is shown in Figure 5a-c. Given the practical limitations of the scanning setup, there is noticeable occlusion in the cloud, particularly along the top of the rock as well as along the interface of the rock and pedestal (or base). Despite the gaps in the data, the detailed point cloud provides significant information with respect to the geometry of the rock compared to manual measurements and photographic documentation.

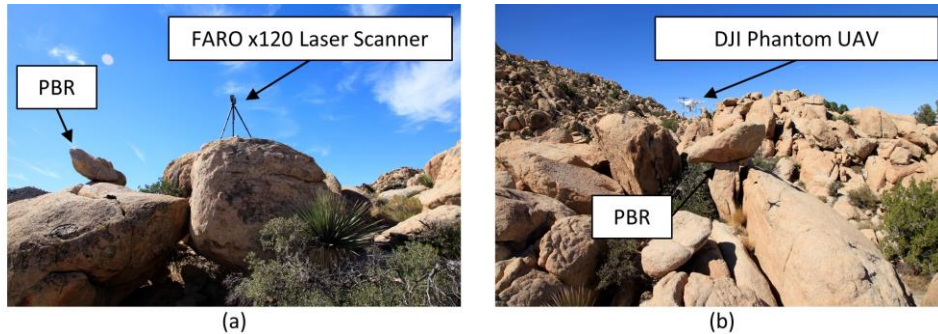


Figure 4. (a) Point cloud acquisition of PBR-04 by laser scanner; and (b) Image acquisition of PBR-04 by unmanned aerial vehicle.

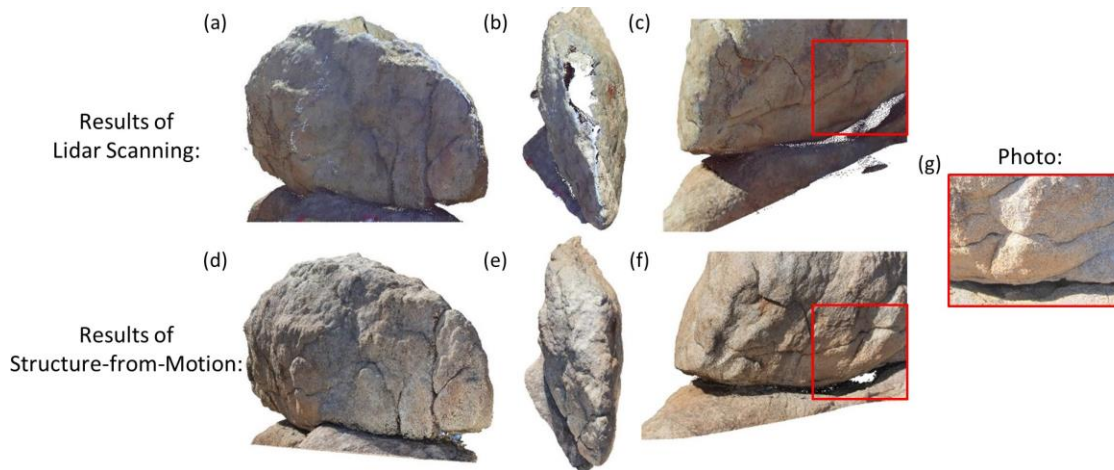


Figure 5. Final point clouds of PBR-01 as generated by (a-c) lidar scanning and (d-f) structure-from-motion, and (g) sample photograph near interface. Note: (b) the missing data near the top of the rock and (c,f) the missing data near the interface.

### ***Structure-from-Motion***

An alternative method for generating 3-D point clouds is a computer vision technique known as structure-from-motion (SfM). This technique relies upon overlapping photographs of the target specimen, which in this case is the PBR. Corresponding features, or clusters of pixels, are identified in pairs of the images and used to estimate the camera's position and orientation. While SfM results in a visually similar point cloud to that of lidar, the resultant point cloud is unscaled and does not correspond to any real world units. For purposes of this survey, images were collected both manually (e.g. for detailed view of the interface) as well as via unmanned aerial vehicles (UAV; e.g. for detailed view of the top of the rock) as shown in Figure 4b. In order to provide scale to the point cloud, reference markers were included for the manual photographs as well as

three global positioning system (GPS) stations for the aerial photographs. For this project, the SfM was constructed using the commercially available Agisoft Photoscan software. The results are compared to that of lidar in Figure 5, which shows enhanced coverage of the top of the rock as well as along the interface. This is likely due to the ability of the photographers and UAV to assume many more viewpoints than the scanner, which was limited to six or seven discrete locations. However, it is noted that coverage along the interface is incomplete due to occlusion, inadequate lighting, and sharp shadows, which hinder the detection and tracking of features during the SfM process. Methods to overcome this incomplete coverage are described in the following section.

The final clouds of lidar and the scaled SfM were compared in terms of point coverage and geometry to gauge the accuracy and efficiency of both methods. For the example PBR of Figure 5, both methods yielded similar point density with a total of 2.4 million points for the lidar cloud compared to 2.7 million points for the SfM cloud. A cloud-to-cloud comparison of both clouds yielded a difference in scale of less than 1% or less than 1 mm in length. It is noted that this geometric and mass difference between the clouds is negligible, given that the point-to-point distances within the lidar cloud are on the millimeter to centimeter scale. Therefore, point clouds generated by SfM are recommended for PBRs given the limited possible viewpoints of the scanner in the complex and non-uniform field conditions. However, it is clarified that lidar may yield superior results in other situations, particularly if ideal data acquisition conditions are present and if the target specimen is less textured than the PBRs of this study [14].

### **3-D Reconstructions**

The goal of the field survey was to obtain highly detailed geometric reconstructions of the PBRs such that accurate geometric and mass properties could be calculated and dynamic analyses conducted. This requires an enclosed triangulated mesh of the PBR, with detailed interface geometry, separate from that of the base or pedestal. However, there is no point data along the interface or bottom of the PBR as both lidar and SfM are line-of-sight techniques. Therefore, a point generation scheme was devised to interpolate data points in this region based upon the interface perimeter of the point clouds. In this scheme, the interface perimeter is first extracted from the point cloud, which consists of a set of 3-D points or a polyline that encloses the unknown interface. At this stage of the research, this perimeter is extracted manually; however, it is envisioned that this will eventually become automated by an analysis of the vertex normals in this region. The manual extraction consists of visually identifying the points that lie at the intersection of the PBR and the pedestal. Photographs of the interface can be used to provide guidance on point selection and shape in areas of low point cloud coverage (e.g. see point cloud in Figure 5f and corresponding photograph in Figure 5g). Once the perimeter is extracted, data points are generated along fiber sections of the interface in a linear fashion using mean points on either end of the fiber. The discretization of the fibers is variable, but was chosen to match the approximate point density of the PBR point cloud. The resultant point cloud with interpolated interface points for PBR-01 is shown in Figure 6a-b, with the original points in color and the generated points in white for both the PBR and the base.

Given a point cloud with sufficient point coverage along the interface, a watertight or fully enclosed triangulated surface mesh of the PBR can be generated. The optimal meshing algorithm in this regard is Poisson Surface Reconstruction [15]. This algorithm considers all points at once rather than marching along the surface, which allows it to produce a single cohesive surface in the presence of significant noise. Triangulated surface meshes can then be generated for both the PBR



and the corresponding pedestal, as seen in Figure 6c-d. At this point in the data processing, the geometric and mass properties can be determined. First, the centroid of the PBR is found by assuming a constant density for the watertight surface mesh. Then, the rocking radii and slenderness are calculated using the interface perimeter, as previously extracted.

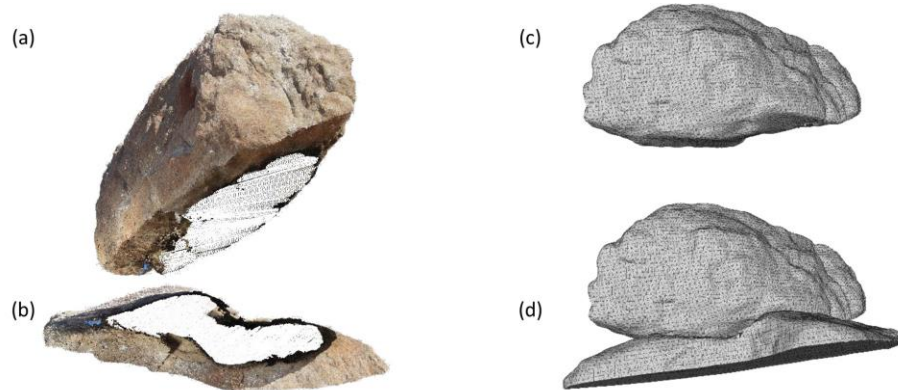


Figure 6. Interpolated points along the interface of (a) PBR-01 and (b) pedestal; and, final meshes using interpolated points for (a) PBR-01 and (b) the PBR-pedestal system.

### Fragility Analysis

While the primary objective of this study was the geometric data acquisition and processing of the complex PBRs, a preliminary study of the probability of overturning is presented herein for the PBR cluster in Jacumba, CA. For this preliminary study, the overturning fragility curves of Dimitrakopoulos and Paraskeva are utilized [16]. These fragility curves were developed using the numerical response of the two-dimensional rocking block (Figure 1b; Eq. 1 – 2) subjected to combined synthetic ground motions for both near and far fault components of an earthquake. The final fragilities incorporate dimensionless-orientationless intensity measures and are representative of the overturning potential of rocking blocks in general, regardless of the size or slenderness. This fragility is presented in Figure 7a, which plots the probability of overturning as a function of the intensity measure,  $IM$ :  $IM = PGA / (g \tan(\alpha))$ , where  $PGA$  is the peak ground acceleration,  $g$  is the acceleration due to gravity, and  $\alpha$  is the slenderness or critical angle of the rocking block (see Figure 1b). Two values for this  $IM$  have been identified from the fragility curve corresponding to 50% and 99% probability of overturning for use in a comparison with the seismic hazard.

Provided the minimum slenderness value for each PBR in the Jacumba cluster from the 3-D reconstructions, a range of  $PGA$  can be readily determined that corresponds to the 50%-99% probability of overturning. This range of  $PGA$  is overlaid on the site-specific seismic hazard curve in Figure 7b, where the frequency of exceedance was determined in accordance with the findings of Bell et al. that the majority of PBRs in the region are between 10,000-30,000 years old [5]. The seismic hazard for the site of the PBR cluster was determined using the 2014 USGS Unified Hazard Tool [17]. Accounting for at least 75% probability of overturning (red circles in the plot) and a range of likely ages for the PBR cluster, it can be seen that the majority of the PBRs are consistent with the seismic hazard with the exception of PBR-02. However, three PBRs show inconsistency when accounting for 50% probability of overturning, namely PBR-01, PBR-02, and PBR-04. Inconsistencies such as these typically indicate that the seismic hazard is overestimated; however, the analyses presented in this section are simplified and do not account for the uneven interface of the PBR, the potential for multiple modes of failure including sliding, and orientation of the PBR

with respect to the ground motions. The impact of this first limitation, regarding the interfaces, is explored in the next section; however, further study of this PBR cluster is warranted given the observed potential for hazard inconsistencies.

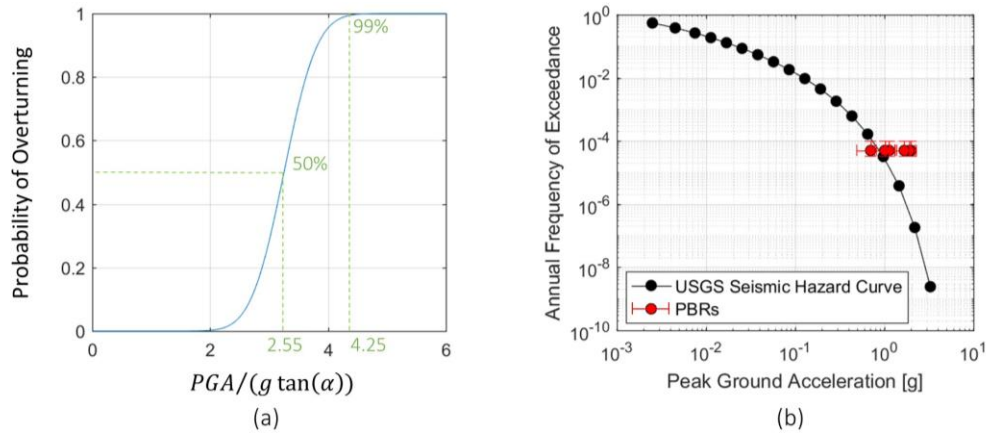


Figure 7. (a) Dimensionless fragility curve [16], and (b) seismic hazard curve for Jacumba, CA, region [17] overlaid with the range of peak ground acceleration that corresponds to 50% - 99% probability of overturning for each PBR.

### Effect of Interface Inconsistencies

The probabilistic overturning analyses of the previous section incorporated a simplified model of the PBR as a two-dimensional rocking block (see Figure 1b). However, the PBRs are characterized by rather arbitrary interfaces including multiple potential rocking points at the edges. This section presents a brief overview of the effect of accounting for additional rocking points at the interface of a rocking block, where this is shown schematically in Figure 8a. The equation of motion for this system is the same as that for the two-point block in Eq. 1, where the rocking radius and slenderness update for the current rocking point throughout the time history. The derivation for this multi-point system and a detailed treatment of its characteristics are included in [18].

For purposes of this study, a simple comparison of overturning spectra for a two-point and three-point block subjected to pulse motions is included in Figure 8b-c. For the average aspect ratio and size of a PBR, the third point of the three-point block corresponds to a height of less than 2 mm (i.e.  $h-h_n < 2$  mm in Figure 8a). Given the PBRs of this study and the resolution of the point cloud data, this height differential corresponds to an approximate minimum value. More significant height differentials as well as multiple rocking point approximations and various aspect ratio blocks, as likely with PBRs, have shown similar trends to that presented herein [18]. Specifically, in the spectra of Figure 8, the rigid block is subjected to a pulse motion of amplitude  $A_p$  and frequency  $\omega_p$ . The amplitude and frequency are presented as normalized quantities with respect to the slenderness,  $\alpha$ , and the frequency parameter,  $p = \sqrt{3g/4R}$ , of the rigid block. The spectra illustrate overturning as either dark red or dark blue depending upon the direction of overturning, as shown in the colorbar on the right. A stark increase in the regions of overturning can be seen in the spectra for the multi-point block with overturning observed across a much broader range of frequencies and amplitudes. This brief comparison emphasizes that the overturning probabilities of the previous section may be much higher under realistic conditions. Therefore, further study is warranted for this PBR cluster, and a more detailed numerical model or fragility relationship is recommended that accounts for the arbitrary interface conditions.

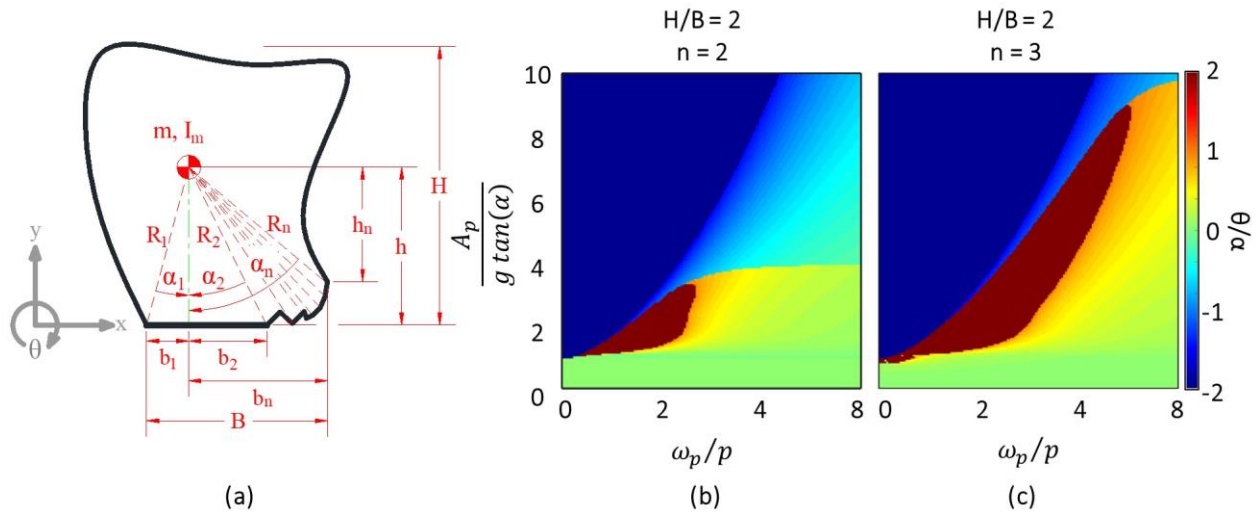


Figure 8. (a) Schematic of a two-dimensional rocking block with an arbitrary number of rocking points; and, overturning spectra for a block of (b) two rocking points and (c) three rocking points, where the third point is characterized by  $0.25B$  and  $0.05\alpha$ .

### Conclusions and Future Work

Precariously balanced rocks (PBRs) are individual or groups of rocks that have eroded over time into rather slender and fragile configurations. PBRs have grown in significance within the seismological and earthquake engineering communities because their existence provides a way to deduce a maximum possible ground motion amplitude over the lifetime of the formation – that which precludes overturning or toppling. The dynamic response of freestanding structures, such as PBRs, is known to be highly nonlinear with respect to the structure’s geometry. Therefore, the primary objective of this study was to compare the efficacy and accuracy of geometric data acquisition for PBRs, and to place the study within the context of southern California seismic hazard. To this end, a survey of PBRs was conducted in the vicinity of Jacumba, CA, within close proximity to the Elsinore and Laguna Salada faults. Four PBRs were documented using lidar scanning as well as structure-from-motion to produce 3-D point clouds. Given the practical limitations of a tripod-based setup in the rocky outcrops, the point clouds derived from lidar contained significant areas of occlusion; and, therefore, the point clouds derived from structure-from-motion were recommended for PBRs. Given accurate point cloud representation of the rocks, an interface interpolation scheme was devised and implemented such that fully enclosed, watertight surface meshes could be generated for the rock and the pedestal separately.

Provided the detailed geometries for the cluster of PBRs in Jacumba, CA, a probabilistic overturning analysis and comparison to seismic hazard was presented. Accounting for 50% probability of overturning, half of the PBRs in the cluster indicated a potential inconsistency with the seismic hazard. However, the preliminary analysis incorporated a two-dimensional simplified rigid block approach; and, as such, a brief presentation on the effect of interface variations was presented. The possibility for multiple points along the interface was shown to significantly increase the likelihood of overturning, which may exacerbate the observed hazard inconsistencies. Therefore, further study is warranted for this PBR cluster using a more detailed numerical model and fragility relationship to examine the potential inconsistencies.

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