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QUASILINEAR CONTROL OF SYSTEMS WITH TIME-DELAYS AND NONLINEAR ACTUATORS AND SENSORS

A Thesis Presented

by

Wei-Ping Huang

 to

The Faculty of the Graduate College

of

The University of Vermont

In Partial Fulfillment of the Requirements for the Degree of Master of Science Specializing in Electrical Engineering

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Defense Date: July 25th, 2018 Thesis Examination Committee:

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Abstract

This thesis investigates Quasilinear Control (QLC) of time-delay systems with nonlinear actuators and sensors and analyzes the accuracy of stochastic linearization for these systems. QLC leverages the method of stochastic linearization to replace each nonlinearity with an equivalent gain, which is obtained by solving a transcendental equation. The idea of QLC is to stochastically linearize the system in order to analyze and design controllers using classical linear control theory. In this thesis, the existence of the equivalent gain for a closed-loop time-delay system is discussed. To compute the equivalent gain, two methods are explored. The first method uses an explicit but complex algorithm based on delay Lyapunov equation to study the timedelay, while the second method uses Padé approximant. It is shown that, under a suitable criterion, Padé approximant can be effectively applied for QLC of time-delay systems. Furthermore, the method of Saturated-Root Locus (S-RL) is extended to nonlinear time-delay systems. It turns out that, in a time-delay system, S-RL always terminates prematurely as opposed to a delay-free system, which may or may not terminate prematurely. Statistical experiments are performed to investigate the accuracy of stochastic linearization compared to a system without time-delay. The impact of increasing the time-delay in the approach of stochastic linearization is also investigated. Results show that stochastic linearization effectively linearizes a nonlinear time-delay system, even though delays generally degrade accuracy. Overall, the accuracy remains relatively high over the selected parameters. Finally, this approach is applied to pitch control in a wind turbine system as a practical example of a nonlinear time-delay system, and its performance is analyzed to demonstrate the efficacy of the approach.

I dedicate this thesis to my family, who fully support me with their selflessness and love.

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TABLE OF CONTENTS

	Ded	ication	ii
	Acki	nowledgements	.11
	List	of Figures	
	List	of Tables	.11
1	Intr	oduction	1
	1.1	Motivation	1
	1.2	Problem Statement	5
	1.3	Literature review	6
		1.3.1 Literature review of QLC	6
		1.3.2 Literature review of Time-delay systems	7
	1.4	Overview of Research Contribution	8
	1.5	Thesis Outline	9
2	Rev	riew of Conventional QLC 1	.0
	2.1	Open-loop System	0
	2.2	Examples of Typical Nonlinear Instrumentations	1
		2.2.1 Saturation Nonlinearity	2
		2.2.2 Quantization Nonlinearity	3
	2.3	Closed-loop system	4
		2.3.1 Reference tracking with nonlinear actuator	6
		2.3.2 Reference tracking with nonlinear sensor	20
	2.4	Controller Design for Reference	
		Tracking	22
3	QL	C with Time-delays 2	:6
-	3.1	Closed-loop system	26
	3.2	Computation issue - Time-delay	29
	3.3	Numerical Example of Time-Delay OLC System	34
	3.4	Controller design	37
	0.1	3 4 1 Saturated-Boot Locus (S-BL)	37
		3.4.2 QLC-Based Design of Optimal Controllers	39
4	Acc	uracy of QLC with Time-delay 4	2
	4.1	Overview - Monte Carlo Method	12
	4.2	First experiment - Accuracy of SL	13
	4.3	Second experiment - Effect of the time-delay	46
		1	

5	Prac	ctical Example - Wind turbine	48			
	5.1	Overview	48			
	5.2	Model construction	49			
	5.3	Simulation	52			
6	Con	clusion	54			
	6.1	Contributions	54			
	6.2	Future Work	55			
Bi	Bibliography					

LIST OF FIGURES

1.1	Linear system and LPNI system	2
1.2	Stochastically linearized system. Note that the reference and distur-	
	bance are Gaussian random processes.	3
1.3	LPNI time-delay system and its stochastic linearization, where e^{-sT} is	
	a time-delay of length T seconds.	4
2.1	Stochastic linearization of an isolated nonlinearity	10
2.2	Saturation nonlinearity	12
2.3	Quasilinear Gain vs σ_u	13
2.4	Saturation nonlinearity	14
2.5	Closed-loop LPNI system and its stochastic linearization	15
2.6	Illustration of reference tracking in time domain	17
2.7	Quasilinear gain and tracking performance	19
2.8	Illustration of reference tracking	20
2.9	Illustration of disturbance rejection in time domain	21
2.10	K_e vs K of the system without time-delay $\ldots \ldots \ldots \ldots \ldots \ldots$	24
2.11	Unsaturated and saturated root locus	25
3.1	Reference tracking with closed-loop time-delay system having nonlin-	
	ear actuator and sensor and its stochastic linearization	27
3.2	MSE of v and y vs N_a	28
3.3	Bode plot of different orders of Padé approximant	32
3.4	Comparison of the real delay and its Padé approximant	33
3.5	Quasilinear gain and tracking performance with different delays \ldots	34
3.6	Illustration of reference tracking in time domain	35
3.7	K_e vs K of the time-delay system $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	39
3.8	$K_e \text{ vs } T$	39
3.9	Saturated-Root Locus with decreasing K_e	40
4.1	Convergence of the RMS error and the error in σ_y	44
4.2	Histogram of statistic results	45
4.3	Simulation result	47
5.1	Power Curve of a Wind Turbine System, figure is taken from [1]	49
5.2	Wind Turbine System Feedback Control System Model, figure is taken	
	from [2]	49
5.3	Simulation of Pitch Control of Wind Turbine System	52

5.4 Output of the pitch control time-delay system with nonlinear actuator and the corresponding stochastically linearized time-delay system . . 53

LIST OF TABLES

5.1 The parameters of the actuator model in the wind turbine [2] 50

CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

For the past century, classical control theory and methods have focused on the closedloop control system described by a linear controller and a linear plant shown in Figure 1.1a. However, every control system contains nonlinear instrumentation, e.g., actuator saturation due to physical limitation and sensor quantization, which usually are ignored in the linear controller design. In Figure 1.1b, the blocks $f(\cdot)$ and $g(\cdot)$ are nonlinear mathematical functions, which represent the actuator and sensor, respectively. The signals r, d, e, u, v, y, and y_m are the reference, disturbance, error, controller output, actuator output, plant output, and measured output, respectively. Unlike nonlinear plants, which can often be linearized to operate at a desired operating point in a well-designed control system, the actuators and sensors cannot, especially when required to operate far from their initial conditions due to large inputs to the system. We refer to this class of systems as Linear Plant/Nonlinear Instrumentation (LPNI) systems.



Figure 1.1: Linear system and LPNI system

Recently, [3] developed the Quasilinear Control theory (QLC), which extended the classical linear control theory to LPNI systems. The analysis and synthesis equations in QLC remain essentially the same as in the linear control except for additional transcendental equations, which are used for computing an equivalent gain in place of the nonlinear instrumentation. This approach for computing the equivalent gain is based on "stochastic linearization". The stochastically linearized system is shown in Figure 1.2, where all signals are denoted by the same symbols as in Figure 1.1 but with a " ^ ". Note that these notations are used throughout this thesis.

In Figure 1.2, compared to the standard LPNI system shown in Figure 1.1b, each static nonlinearity is replaced with an equivalent gain, i.e., N_a and N_s , where the constant gains are obtained from approximating f(u(t)) by $N_a\hat{u}(t)$ and g(y(t)) by $N_s\hat{y}(t)$, so that the linearized system is "close" to the nonlinear system in a stochastic sense (more details are presented in chapter 2). These equivalent gains are referred

Figure 1.2: Stochastically linearized system. Note that the reference and disturbance are Gaussian random processes.

to as the quasilinear gains of f(u) and g(y). The essential concept of QLC is to use stochastic linearization as opposed to the traditional Jacobian linearization to design the controller using linear control theory for the LPNI system.

Stochastic linearization was developed over 50 years ago and since then applied in numerous engineering fields, including feedback control. This method requires external signals, such as reference and disturbance shown in Figure 1.2, to be random. We can easily find a random disturbance signal, but there are also many reference signals that are random in different applications, for instance, aircraft landing gear control [4], pitch control in a wind turbine [5], and wind farm power systems [6].

In addition to instrumentation nonlinearity, there is another important effect that is often neglected: Time-delay. Figure 1.3 represents a closed-loop nonlinear system with a time-delay. In various situations, such as electronics [7], pneumatic and hydraulic networks [8], chemical processes [9], long transmission lines [10], robotics [11], etc., time-delays usually exist due to transmission delay, material transport, propagation delay, or computation delay. The time-delay may cause unexpected system response or even instability [12]. In less severe cases, time-delays tend to degrade system performance, e.g., reference tracking and disturbance rejection. Therefore, it



Figure 1.3: LPNI time-delay system and its stochastic linearization, where e^{-sT} is a timedelay of length T seconds.

is important that the control system engineer is equipped with the necessary tools to analyze and design nonlinear control systems with time-delays. Standard QLC theory does not take into account that most practical systems have time-delays, which can affect performance in terms of reference tracking and disturbance rejection. Thus, in this thesis, we extend the standard QLC theory to LPNI systems with time-delays. We develop a stochastic linearization theory for systems with time-delays, analyze the accuracy of their stochastic linearization, and study the root locus-based and optimal control-based design of time-delay LPNI systems.

1.2 PROBLEM STATEMENT

Consider an LPNI system with a time-delay, as shown in Figure 1.3a. The stochastic linearization of this system is shown in Figure 1.3b. In this thesis, we address the following four problems:

- 1. Stochastic linearization requires computing the \mathcal{H}_2 norm of transfer functions. The first problem is investigating methods for computing \mathcal{H}_2 norm of transfer functions with time-delays.
- 2. Stochastic linearization provides approximation of the statistical properties (i.e., standard deviation) of the signals in the original LPNI system. Thus, the second problem addressed in this thesis is to determine the accuracy of stochastic linearization for systems with time-delays. Since an analytical investigation of the accuracy of this class of system is impossible to carry out, we numerically perform this study using Monte Carlo experiments.
- 3. In the delay-free case, the standard QLC theory extended the root locus technique to LPNI systems with saturation. The resulting root locus is referred to as the Saturated-Root Locus (S-RL). The third problem addressed in this thesis is to determine the effects of time-delays on the S-RL.
- Finally, in order to show the capacity of the developed QLC theory in practice, a practical example of a pitch control in a wind turbine is presented in Chapter 5.

1.3 LITERATURE REVIEW

In this section, first, we present the literature review on QLC and the method of stochastic linearization. Then, we introduce the literature on time-delay systems with stochastic inputs.

1.3.1 LITERATURE REVIEW OF QLC

The stability of control systems with nonlinear actuators and sensors has been studied in control theory for over 70 years. Although the theory of absolute stability [13–20] and numerous subsequent developments [21, 22] have given rise to effective methods to analyze the stability and domains of attraction for such systems, fewer references have concentrated on performance analysis (i.e., with respect to reference tracking and disturbance rejection) of these systems. The remaining publications generally consider specific nonlinearities for instrumentations, i.e., actuators and sensors. A system with saturating actuator in the framework of absolute stability is considered in [23]. Semi-global stability of LPNI systems with saturating actuators and linear feedback had been studied in [24–26]. The papers [27, 28] consider the problems of stability of systems with sensor nonlinearities. The survey paper [29] presents a thorough review of LPNI systems with saturating actuators.

Recently, the theory of Quasilinear Control was developed using stochastic linearization [3] to address the issues of performance analysis and design [30–38] of controllers for systems with static nonlinearities in actuators and sensors driven by stochastic signals. QLC leverages the method of stochastic linearization [39–42], which uses statistical measures of the stochastic inputs to linearize the system. This approach considers every component in the system so that it provides a more faithful picture of the entire system. Moreover, QLC was extended to systems with *asymmetric* nonlinearities [30, 31, 36–38] and applied to two applications, i.e., wind farm power control and semi-active suspension control [32, 34, 35].

Stochastic linearization, which is the main mathematical tool in this research, was developed in 1954 [39,42]. Afterward, many researchers inaugurated using stochastic linearization to study the behavior of nonlinear systems with stochastic inputs. Some of the earlier applications of stochastic linearization to feedback systems was presented by [40,43]. In [41], a complete description and detailed interpretation of stochastic linearization are presented, where stochastic linearization has been referred to as statistical linearization.

1.3.2 LITERATURE REVIEW OF TIME-DELAY SYSTEMS

Often, time-delays affect practical systems, such as electric power systems, pneumatic systems, and hydraulic systems [44–46]. The transmission or so-called communication delay in an electric power grid causes power losses and poor performance in regulating the power demand and supply [47], especially in a renewable energy system. In other cases, voluntary introduction of delays can aid the control by damping and stabilization [48]. In general, delays have complex effects on stability [49, 50].

Time-delays introduce new characteristics in the mathematical description of systems, and have been modeled in various ways in the literature. In general, delays require functional differential equations (FDEs) and, more specifically, delay differential equations (DDEs) [51], instead of ordinary differential equations (ODEs). Other models involve the behavioral equations [52], the Lambert W function [53, 54], or rational approximations like the Padé approximant [55].

Several techniques exist for analysis and control of nonlinear time-delay systems with deterministic inputs [56]. For example, Smith predictor based-control methods eliminate time-delay from the characteristic equation of the closed-loop system [57]. The problem of local stabilization of nonlinear discrete-time systems with time-varying delay and saturating actuator is studied in [58]. In [59], the theory of non-commutative rings is proposed for the analysis of time-delay systems. Fuzzy control-based approaches are discussed in [60, 61]. However, there is less literature available on nonlinear time-delay systems with stochastic inputs. In [62], the stability and robustness of deterministic and stochastic linear time-delay systems have been discussed. There are few analytical methods based on solving stochastic DDEs [63] and the Fokker-Planck approach [64], but they are not amenable to control system design.

1.4 Overview of Research Contribution

This thesis provides QLC as an additional toolbox for analysis and design of nonlinear time-delay systems with stochastic inputs. Conventional QLC does not take timedelays into account. In order to consider time-delays, QLC requires computation of the \mathcal{H}_2 norm of transfer functions. An explicit approach, based on the delay Lyapunov equation, is not computationally attractive. Alternatively, the time-delay can be approximated by a Padé approximant, which is computationally advantageous. We present a criterion for a selection of the order of the Padé approximant based on the system bandwidth. We investigate the accuracy of stochastic linearization and the effect of a time-delay through Monte Carlo simulations (there is no analytical method to quantify the accuracy for this class of systems). Statistical results indicate that stochastic linearization with Padé approximant leads to an accurate characterization of the performance of the nonlinear system, even though adding a time-delay generally degrades accuracy. This thesis also investigates the design of controllers for nonlinear time-delay systems. The root locus method is extended for time-delay systems with saturating actuators. It turns out that for such systems the saturated-root locus (or S-RL) always terminates prematurely. In addition, a QLC-based design of optimal controllers is presented, and applied to a practical example of pitch control of wind turbines. The simulation of the pitch control illustrates that stochastic linearization is capable of handling time-delay system with 30% increase of tracking performance compared to the literature.

1.5 THESIS OUTLINE

The outline of this thesis is as follows. Chapter 2 presents a review of conventional QLC of systems without time-delay, and two typical nonlinearities in actuators and sensors are introduced. In Chapter 3, the QLC theory for time-delay systems is developed, and an analysis of the Padé approximant given. Chapter 4 presents two Monte Carlo experiments to examine the accuracy of stochastic linearization. Chapter 5 applies the above ideas to a practical example of pitch control in a wind turbine system. The conclusions and future work are outlined in Chapter 6.

CHAPTER 2

REVIEW OF CONVENTIONAL QLC

This chapter presents a brief review of QLC for systems with nonlinear actuators and sensors, without time-delay. The reader is referred to the book [3] for details.

2.1 Open-loop System



Figure 2.1: Stochastic linearization of an isolated nonlinearity

Following the standard stochastic linearization approach [41], consider Figure 2.1, where u(t) is a zero-mean wide-sense stationary (WSS) Gaussian process, f(u) is an odd piece-wise differentiable function, and N is a constant such that: $\hat{v}(t) = Nu(t)$. The problem of stochastic linearization is to approximate f(u) by Nu(t), so that the functional:

$$\varepsilon(N) = E[(v(t) - \hat{v}(t))^2]$$
(2.1)

is minimized, where $E\left[\cdot\right]$ denotes expectation.

The solution to this problem is given by:

$$N = E[f'(u)] = \mathcal{F}(\sigma_u) \tag{2.2}$$

where

$$\mathcal{F}(\sigma_u) = \int_{-\infty}^{\infty} \left[\frac{d}{dx} f(x) \right] \frac{1}{\sqrt{2\pi\sigma_u}} \exp\left(-\frac{x^2}{2\sigma_u^2}\right) \,\mathrm{d}x \tag{2.3}$$

The gain N is referred to as the quasilinear gain of f(u). Since u(t) is a WSS Gaussian process, N is only a function of the standard deviation, σ_u , of u(t). Note that $\mathcal{F}(\sigma_u)$ can be evaluated explicitly as a function of σ_u for a given nonlinearity. Clearly, the technique of stochastic linearization depends on the statistical properties of the input u(t), unlike Jacobian linearization, wherein gains are evaluated as derivatives of f(u), i.e., f'(u), at the operating point [65].

2.2 Examples of Typical Nonlinear In-Strumentations

While the results in this thesis are applicable to all static nonlinearities, below we focus on saturating actuators and sensor quantization, which are the typical nonlinear instrumentations in a control system.



Figure 2.2: Saturation nonlinearity

2.2.1 SATURATION NONLINEARITY

Consider Figure 2.2, which shows the saturation nonlinearity defined by the following function:

$$f(u) = \operatorname{sat}_{\alpha}(u) := \begin{cases} +\alpha, & u > +\alpha \\ u, & -\alpha \le u \le +\alpha \\ -\alpha, & u < -\alpha \end{cases}$$
(2.4)

where $\alpha > 0$ is the saturation boundary level of the actuator. It can be shown [3] that for this nonlinearity, the quasilinear gain is given by:

$$N = E\left[\left.\frac{df(u)}{du}\right|_{u=u(t)}\right] = \mathcal{F}\left(\sigma_u\right) = \operatorname{erf}\left(\frac{\alpha}{\sqrt{2}\sigma_u}\right)$$
(2.5)

where $\operatorname{erf}(\cdot)$ is the error function defined by:

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} dt$$
 (2.6)

As shown in Figure 2.3, $\mathcal{F}(\sigma_u)$ is a decreasing function of σ_u . When σ_u is small, $N \approx 1$, and when σ_u is large, $N \approx \sqrt{\frac{2}{\pi}} \left(\frac{\alpha}{\sigma_u}\right)$. When N = 1, the system is almost linear, which implies that the value of N represents the degree of linearity in the system.



Figure 2.3: Quasilinear Gain vs σ_u

2.2.2 QUANTIZATION NONLINEARITY

The typical nonlinearity in the sensor is quantization, which is usually used in sampling and mapping a analog signal. Consider Figure 2.4, which shows the quantization nonlinearity defined as

$$f(u) = qn_{\Delta}(u) := \begin{cases} +\Delta \lfloor +u/\Delta \rfloor, & u \ge 0\\ -\Delta \lfloor -u/\Delta \rfloor, & u < 0 \end{cases}$$
(2.7)



Figure 2.4: Saturation nonlinearity

where Δ is the quantization step size and $\lfloor u/\Delta \rfloor$ denotes the floor function, which is the function that takes as input a real number u and gives as output the greatest integer less than or equal to u.

The quasilinear gain is given by:

$$N = \mathcal{F}(\sigma_u) = \frac{2\Delta}{\sqrt{2\pi\sigma_u^2}} \exp\sum_{k=1}^{\infty} \left(-\frac{\Delta^2}{2\sigma_u^2}k^2\right)$$
(2.8)

For a small $\sigma_u < \Delta$, N is similar to dead zone, and N approaches 1 as $\sigma_u \to \infty$.

2.3 CLOSED-LOOP SYSTEM

Consider the closed-loop system of Figure 2.5a, where P(s) and C(s) are the plant and the controller respectively, and $f(\cdot)$ and $g(\cdot)$ are odd, piece-wise differentiable functions representing the actuator and sensor. $F_{\Omega_r}(s)$ and $F_{\Omega_d}(s)$ are coloring filters with 3dB bandwidths Ω_r and Ω_d , with DC gains selected so that σ_r is as desired. ω_r and ω_d are standard Gaussian white noise processes, and the scalars r(t), e(t), u(t),



(a) Closed-loop system with nonlinear actuator and sensor



(b) Closed-loop stochastically linearized system

Figure 2.5: Closed-loop LPNI system and its stochastic linearization

v(t), and y(t) represent the reference, error signal, control signal, actuator output, and the plant output respectively. The goal is to obtain the stochastic linearization approximation by (2.2) and replace the nonlinear actuator and sensor with quasilinear gains to obtain the system of Figure 2.5b, where

$$N_a = E \left[\left. \frac{df(\hat{u})}{d\hat{u}} \right|_{\hat{u}=\hat{u}(t)} \right]$$
(2.9)

and

$$N_s = E\left[\left.\frac{df(\hat{y})}{d\hat{y}}\right|_{\hat{y}=\hat{y}(t)}\right].$$
(2.10)

Since the control action, u(t), depends on the output of the nonlinearities (because of feedback), u(t) is not Gaussian, unlike the open-loop case. Furthermore, the signals u(t) and $\hat{u}(t)$ are not the same, unlike the open-loop case. Because of these two obstacles, the quasilinear gain formula in (2.2) is no longer an optimal gain for replacing the nonlinearity. However, if the plant is low-pass filtering, the signal u(t) is close to Gaussian, which addresses the first obstacle. Furthermore, quasilinear control theory assumes that u(t) and $\hat{u}(t)$ are the same, which addresses the second obstacle. It has been shown in previous studies and in Chapter 4 that the accuracy of stochastic linearization in closed-loop systems with or without time-delays is generally less then 10%.

Below, we will introduce the reference tracking problem with nonlinear actuator and sensor. The development for disturbance rejection is similar and is hence omitted.

2.3.1 Reference tracking with nonlinear actuator

Consider a reference tracking system with an actuator nonlinearity shown in Figure 2.6a. Assuming that the system is operating in the stationary regime and $\sigma_u = \sigma_{\hat{u}}$ (the validity of this assumption is addressed in Chapter 4), the standard deviation $\sigma_{\hat{u}}$ can be computed by using the \mathcal{H}_2 norm of the transfer function from ω to \hat{u} :

$$\sigma_{\hat{u}} = \left\| \frac{F_{\Omega}(s)C(s)}{1 + P(s)N_aC(s)} \right\|_2 \tag{2.11}$$

Note that, for an LTI system driven by Gaussian white noise, the \mathcal{H}_2 norm function provides the steady-state variance of the output The \mathcal{H}_2 norm of a continuous system

$$\omega \longrightarrow F_{\Omega}(s) \xrightarrow{r} \underbrace{e}_{-} C(s) \xrightarrow{u}_{-} f(\cdot) \xrightarrow{v}_{-} P(s) \xrightarrow{y}_{-}$$

(a) Closed-loop system with nonlinear actuator

$$\omega \longrightarrow F_{\Omega}(s) \xrightarrow{r} \overset{\hat{e}}{\longrightarrow} C(s) \xrightarrow{\hat{u}} N_a = E\left[\frac{d}{d\hat{u}}f(\hat{u})\right] \xrightarrow{\hat{v}} P(s) \xrightarrow{\hat{y}}$$

(b) Closed-loop stochastically linearized system

Figure 2.6: Illustration of reference tracking in time domain

with transfer function H(s) is defined by:

$$\|H\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\omega)|^2 \, \mathrm{d}\omega}$$
(2.12)

Hence, from (2.2), N is a root of the following transcendental equation:

$$N_a - \mathcal{F}\left(\left\|\frac{F_{\Omega}(s)C(s)}{1 + P(s)N_aC(s)}\right\|_2\right) = 0, \qquad (2.13)$$

where

$$\mathcal{F}(\sigma_{\hat{u}}) = \int_{-\infty}^{\infty} \left[\frac{d}{dx} f(x) \right] \frac{1}{\sqrt{2\pi\sigma_{\hat{u}}}} \exp\left(-\frac{x^2}{2\sigma_{\hat{u}}^2}\right) \,\mathrm{d}x.$$
(2.14)

Numerical Example

Consider the nonlinear system of Figure 2.6a with:

$$P(s) = \frac{10}{s(s+10)}, \ C(s) = 5,$$

$$F_{\Omega}(s) = \frac{\sqrt{3}}{s^3 + 2s^2 + 2s + 1}, \ \text{and} \ f(u) = \operatorname{sat}_{\alpha}(u)$$
(2.15)

Note that F_{Ω} is a 3rd order Butterworth filter with bandwidth $\Omega = 1$ and DC gain selected so that $\sigma_r = 1$. The actuator nonlinearity is selected as standard saturation. Using (2.5), (2.13) becomes:

$$N_a - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2} \left\|\frac{5\sqrt{3}s(s+10)}{(s^3+2s^2+2s+1)(s^2+10s+50N_a)}\right\|_2}\right) = 0.$$
(2.16)

We solve (2.16) with the saturation boundary level $\alpha \in [0,3]$. Figure 2.7a shows that for $\alpha \in (0,0.5)$, N is nearly linear with slope 0.2 and Figure 2.7b shows that the standard deviation of tracking error $\sigma_{\hat{e}}$ decreases with slope -0.7. For $\alpha > 2$, N can be considered to be 1, which means that saturation is ignored and the system is almost linear.

To illustrate the tracking performance of the stochastically linearized system of Figure 2.6b, traces of r(t), $\hat{y}(t)$ and y(t), which are the reference, output of stochastically linearized system, and output of LPNI system, are obtained from the simulation of both the nonlinear system and the stochastically linearized system. As shown in Figure 2.8a, for the saturation boundary level $\alpha = 1$, the tracking behavior of the stochastically linearized system is good and similar to the nonlinear system. For $\alpha = 0.5$, as shown in Figure 2.8b, the system becomes more nonlinear, compared to



Figure 2.7: Quasilinear gain and tracking performance

 $\alpha = 1$, due to the saturation, so that the tracking performance is poor: the output of the stochastically linearized system $\hat{y}(t)$ approximates the output of the nonlinear system y(t) with a lag.



Figure 2.8: Illustration of reference tracking

2.3.2 Reference tracking with nonlinear sensor

Consider the reference tracking system with a sensor nonlinearity shown in Figure 2.9a. The quasilinear gain, which replaces the nonlinear sensor, is obtained from the same procedure as the nonlinear actuator defined by (2.10). The standard deviation



(a) Closed-loop system with nonlinear actuator



(b) Closed-loop stochastically linearized system

Figure 2.9: Illustration of disturbance rejection in time domain

 $\sigma_{\hat{y}}$ can be computed using \mathcal{H}_2 norm of the transfer function from ω to \hat{y} :

$$\sigma_{\hat{y}} = \left\| \frac{F_{\Omega}(s)P(s)C(s)}{1 + P(s)N_sC(s)} \right\|_2 \tag{2.17}$$

Similar to (2.13), N_s is the root of the transcendental equation:

$$N_s - \mathcal{G}\left(\left\| \frac{F_{\Omega}(s)C(s)P(s)}{1 + P(s)N_sC(s)} \right\|_2 \right) = 0,$$
(2.18)

where

$$\mathcal{G}(\sigma_{\hat{y}}) = \int_{-\infty}^{\infty} \left[\frac{d}{dx} g(x) \right] \frac{1}{\sqrt{2\pi} \sigma_{\hat{y}}} \exp\left(-\frac{x^2}{2\sigma_{\hat{y}}^2} \right) \, \mathrm{d}x. \tag{2.19}$$

Numerical Example

Consider the nonlinear system of Figure 2.9a with the same parameters as in (2.15):

$$P(s) = \frac{10}{s(s+10)}, \ C(s) = 5, \ F_{\Omega}(s) = \frac{\sqrt{3}}{s^3 + 2s^2 + 2s + 1}$$
(2.20)

with linear actuator f(u) and quantized sensor g(y):

$$f(u) = u \text{ and } g(y) = qn_{\Delta}(y). \tag{2.21}$$

Thus, for this system, using (2.19) and (2.8), the equation of the quasilinear gain becomes

$$N_s - Q\left(\frac{\Delta}{\left\|\frac{F_{\Omega}(s)C(s)P(s)}{1+P(s)N_sC(s)}\right\|_2}\right) = 0,$$
(2.22)

where

$$Q(z) = \frac{\sqrt{2}z}{\sqrt{\pi}} \sum_{k=1}^{\infty} \exp\left(-\frac{z^2}{2}k^2\right).$$
 (2.23)

2.4 Controller Design for Reference Tracking

The root locus is a useful tool in control theory for designing controllers. A similar root locus technique, referred to as the Saturated-Root Locus (S-RL), is developed in [3] for analyzing systems with saturating actuators. It is shown in [3] that S-RL is a subset of the standard root locus, but may terminate prior to the open loop zeros.

Consider the nonlinear system of Figure 2.6a, where the controller is now KC(s)instead of C(s) and $K \in (0, \infty)$ is a parameter. The quasilinear gain is now a function of K, defined by

$$N_a(K) = \operatorname{erf}\left(\frac{\alpha}{\sqrt{2} \left\|\frac{F_{\Omega}(s)KC(s)}{1+P(s)N_a(K)KC(s)}\right\|_2}\right).$$
(2.24)

In the above equation, the dependence of N_a on K is explicitly shown by $N_a(K)$. Note that, since $N_a(K)$ appears on both sides of the equation, (2.24) is a transcendental equation that must be solved numerically.

Using (2.13), quasilinear gain $N_a(K)$ is a solution of the following equation:

$$N_a(K) - \mathcal{F}\left(\left\|\frac{KF_{\Omega}(s)C(s)}{1 + N_a(K)KC(s)P(s)}\right\|_2\right) = 0$$
(2.25)

The effective gain, K_e , of the stochastically linearized system is defined as $K_e(K) := KN_a(K)$. From (2.5), $K_e(K)$ can be obtained from the equation:

$$K_e(K) = K \operatorname{erf}\left(\frac{\alpha}{\sqrt{2}K \left\|\frac{F_{\Omega}(s)C(s)}{1+K_e(K)C(s)P(s)}\right\|_2}\right)$$
(2.26)

If $K_e(\infty) = \infty$, the S-RL behaves the same as an unsaturated system. If $K_e(\infty) < \infty$, the S-RL terminates at points prior to the open loop zeros. Hence, $K_e(\infty)$ is the saturated-termination gain. Methods for computing $K_e(\infty)$ are provided in [3]. In short, the methods start from the following equation

$$\beta - \left\| \frac{F_{\Omega}(s)C(s)}{1 + \left(\frac{\alpha\sqrt{2/\pi}}{\beta}\right)P(s)C(s)} \right\|_{2} = 0.$$
(2.27)

If a unique solution $\beta^* > 0$ satisfies the above equation, then the saturatedtermination gain is defined by

$$\lim_{K \to \infty} K_e(K) = \frac{\alpha \sqrt{2/\pi}}{\beta^*}$$
(2.28)

If $\beta^* = 0$ is the only solution, then $K_e(\infty)$ is infinite.

Numerical Example

Consider a delay-free closed-loop system as shown in Figure 2.6a with

$$C(s) = 1, \ P(s) = \frac{s+15}{s(s+2.5)}, \ \text{and} \ F_{\Omega}(s) = \frac{\sqrt{3}}{s^3 + 2s^2 + 2s + 1}$$
 (2.29)

and the actuator saturation defined by

$$f(u) = sat_{\alpha}(u) \text{ and } \alpha = 0.16.$$
(2.30)



Figure 2.10: K_e vs K of the system without time-delay

In this case, (2.27) admits a unique solution $\beta^* = 2.58$ for K > 0 and the equivalent gain $K_e(\infty) = 0.3389$. Figure 2.10 shows K_e as a function of K. When K is small, we see that $K_e(K)$ is close to K, because the system does not saturate. As Kincreases, $K_e(K)$ terminates to 0.3389.



Figure 2.11: Unsaturated and saturated root locus

Figure 2.11 shows the results of both the unsaturated and saturated root locus. The blue and green lines are the original root locus. The saturated root locus, shown by the black line, starts from the open-loop poles, shown by red dots, and ends at the termination points, which are at $s = -1.4150 \pm 1.7169i$ shown by green dots. For this example, S-RL may not enter a pre-specified admissible domain and, hence, the tracking performance may be limited.

In this chapter, we reviewed QLC of systems with nonlinear actuators and sensors with two numerical examples and the approach of stochastic linearization. In the review of controller design, S-RL was presented, where it was shown that S-RL may or may not terminate prematurely.

CHAPTER 3

QLC WITH TIME-DELAYS

This chapter, first, introduces QLC with time-delays and stochastic linearization of time-delay systems. Second, the issue of computing \mathcal{H}_2 norm is addressed. Finally, the method of root locus in nonlinear time-delay systems and a QLC-based optimal controller are investigated.

3.1 Closed-loop system

Consider a time-delay system, shown in Figure 3.1a, and its stochastic linearization, shown in Figure 3.1b. Because time-delays in LPNI systems have not been addressed in the literature, the effects of time-delays on stochastic linearization are unknown. Time-delays may cause instability and significantly degrade performance, and moreover, the nonlinearity may increase the negative effects of the time-delay. This section focuses on the analysis of stochastic linearization of an LPNI system with time-delays.

Note that the time-delay does not change the properties of linearity or timeinvariance of the system, so the stochastically linearized system remains LTI. Hence,

$$\omega \longrightarrow F_{\Omega}(s) \xrightarrow{r} \underbrace{e}_{-} C(s) \xrightarrow{e}_{-} f(\cdot) \xrightarrow{v}_{-} P(s) \xrightarrow{e^{-st}} \underbrace{y}_{-}$$

(a) Closed-loop time-delay system with nonlinear actuator and sensor

$$\omega \longrightarrow F_{\Omega}(s) \xrightarrow{r} \overset{\hat{e}}{\longrightarrow} C(s) \xrightarrow{\hat{u}} N_a \xrightarrow{\hat{v}} P(s) \xrightarrow{e^{-st}} \overset{\hat{y}}{\longrightarrow}$$

(b) Stochastically linearization of the above system

Figure 3.1: Reference tracking with closed-loop time-delay system having nonlinear actuator and sensor and its stochastic linearization

the quasilinear gain of the actuator can still be computed using the standard deviation, $\sigma_{\hat{u}}$, which can be obtained from the \mathcal{H}_2 norm of the transfer function from ω to \hat{u} (as shown in (2.11)):

$$\sigma_{\hat{u}} = \left\| \frac{F_{\Omega}(s)C(s)}{1 + P(s)N_aC(s)e^{-sT}} \right\|_2$$
(3.1)

Similar to (2.13), N_a is a root of the following equation:

$$N_a - \mathcal{F}\left(\left\|\frac{F_{\Omega}(s)C(s)}{1 + P(s)N_aC(s)e^{-sT}}\right\|_2\right) = 0$$
(3.2)

A sufficient condition for the existence of N_a is mentioned in the following theorem:

Theorem 1. Assume that the system of Figure 3.1b is asymptotically stable for $N_a \in \mathcal{N}$, where $\mathcal{N} \subset \mathbb{R}$ is the range of function \mathcal{F} in (2.2), and that \mathcal{N} is a closed interval. Then, equation (3.2) has a solution.



Figure 3.2: MSE of v and y vs N_a

Proof. According to the first assumption, the second term of (3.2) is a continuous function of N_a . Moreover, its range covers the range of N_a . Because of this and the second assumption, the existence of a solution is guaranteed by the Brouwer fixed-point theorem [66].

As mentioned in Section 2.3, despite the two assumptions made for the closed loop environment (i.e., the Gaussianity of u and the fact that $u = \hat{u}$), N_a obtained from (3.2) still provides a good approximation of the minimization of (2.1). To illustrate this, we simulate the system of Figure 3.1a and Figure 3.1b with $N_a \in [0, 0.8]$ and all other parameters defined as in the numerical example of Section 2.3.2, with $\alpha = 1$ and a time-delay T = 0.3. We plot the mean squared error (MSE) of v(t) and \hat{v} , i.e., Equation (2.1), and MSE of y(t) and \hat{y} as a function of N, as shown in Figure 3.2. The quasilinear gain N_a^* computed from (3.2) is 0.36 and is plotted in the same figure. As can be seen, N_a^* is located very close to the lowest value in both the MSE plots in Figure 3.2. Hence, the method of stochastic linearization achieves our objective of minimizing (2.1) successfully and is indeed a good approximation of the true minimum. Note that, while QLC finds the approximate location for the minimum of MSEs, the minimum values are rather high in this system (about 0.3), implying that the accuracy of stochastic linearization in predicting the variance of the signals in the nonlinear system may be low. The reason for this low accuracy is not due to the two assumptions mentioned in Section 2.3; rather, it is because there exists no linear approximation that would yield a high accuracy. Accuracy is thoroughly investigated in Chapter 4.

3.2 Computation issue - Time-Delay

In this thesis, we use \mathcal{H}_2 norm to evaluate the standard deviation of the actuator input, σ_u . A well known method to compute the \mathcal{H}_2 norm of linear time-invariant delay-free systems is using the solution of the Lyapunov equation. Let $\{A, B, C\}$ be a minimal realization of an asymptotically stable single-input single-output (SISO) system without time-delays. Consider the Lyapunov equation

$$UA + A'U + C'C = 0 (3.3)$$

where \prime denotes the transpose of a matrix. Then, \mathcal{H}_2 norm can be explicitly computed by the solution of equation (3.3) as follows:

$$\|H\|_{2}^{2} = Tr(BUB') \tag{3.4}$$

However, in time-delay systems, this approach requires delay Lyapunov equations [67]. Consider an asymptotically stable SISO system with a single time-delay described by,

$$\dot{x} = A_0 x + A_1 x (t - T) + B u$$

$$y = C x,$$
(3.5)

where, $A_0, A_1 \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{1 \times n}$, $C \in \mathbb{R}^{n \times 1}$, and T is delay time.

The \mathcal{H}_2 norm of this time-delay system is defined as:

$$||H||_2^2 = Tr\left(BU(0)B'\right) \tag{3.6}$$

where U(0) is the unique solution of the delay Lyapunov equations:

$$U'(t)B = U(t)A_0 + U(t - T)A_1$$
$$U(-t) = U^T(t)$$
(3.7)
$$-C'C = B'U(-T)A_0 + A'_0U'(-TB) + B'U(-T)A_1 + A'_1U'(-TB).$$

A simplified formulation, provided in [67], is as follows:

$$\|H\|_{2}^{2} = -\operatorname{vec}(BB')' [A'_{0} \otimes I + I \otimes A'_{0} + (A'_{1} \otimes I + (I \otimes A'_{1})B_{12})B_{22}^{-1}(I - B_{21}) + (I \otimes A'_{1})B_{11}]^{-1}\operatorname{vec}(C'C), \qquad (3.8)$$

where

$$\begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} = \exp\left(T\begin{pmatrix} A'_0 \otimes I & A'_1 \otimes I \\ -I \otimes A'_1 & -I \otimes A'_0 \end{pmatrix}\right)$$
(3.9)

Note that vec is the vectorization operation that is defined as the stacking of the columns into a vector and \otimes is Kronecker product.

However, this explicit approach requires computations that involve large matrices depending on the system dimension n, with a complexity of order $\mathcal{O}(n^6)$. An alternative but simple way to compute \mathcal{H}_2 norm of time-delay systems is using Padé approximant, which is a rational model of a pure delay e^{-sT} . The accuracy of the Padé approximant in replacing a time-delay is commonly known in the field of control theory [55]. As a review, Figure 3.3 shows the Bode plot of different orders of the Padé approximant of a delay of one second. From the Bode plot, it can be seen that the Padé approximant has a high accuracy at low frequencies, but at high frequencies, the phase of the Padé approximant does not roll-off as a real delay. Furthermore, highorder Padé approximants contains clustered poles in the transfer function. Because this clustered poles tend to be very sensitive to perturbations, Padé approximants with order n > 10 should be avoided.

In order to determine a proper order of the Padé approximant for our purposes, we take into account the differences in system bandwidth and the \mathcal{H}_2 norm between the



Figure 3.3: Bode plot of different orders of Padé approximant

approximation and the real time-delay. We consider the n^{th} order Padé approximant D(s,T,n) to be a good approximation of the real time-delay, e^{-sT} , if the following criterion holds:

$$\angle e^{-j\omega_{\rm BW}T} - \angle D(j\omega_{\rm BW}, T, n) < \delta^{\circ}, \tag{3.10}$$

where

$$\angle e^{-j\omega_{\rm BW}T} = -\omega_{\rm BW} \cdot T \tag{3.11}$$

and the general formula for Padé approximant is

$$e^{-sT} \approx D(s, T, n) = \frac{\sum_{i=0}^{n} {n \choose i} \frac{(2n-1)!}{(2n)!} (-Ts)^{i}}{\sum_{i=0}^{n} {n \choose i} \frac{(2n-1)!}{(2n)!} (Ts)^{i}}.$$
(3.12)

In Equation (3.10), $\delta > 0$ is a desired accuracy. ω_{BW} is the largest 3dB bandwidth of the system of Figure 3.1b $\forall N_a \in \mathcal{N}$, and \angle denotes the phase. Note that this condition takes into account the phase lag of the time-delay, as shown in Figure 3.3.

To compare the explicit method based on the delay Lyapunov equations and the



Figure 3.4: Comparison of the real delay and its Padé approximant

Padé approximant, we consider the example in Section 2.3.1 with a time-delay T, which is varied from 0 to 0.12. Figure 3.4 shows the relative difference between $\sigma_{\hat{u}}$ and $\tilde{\sigma}_{\hat{u}}$ in percentage, where $\sigma_{\hat{u}}$ is computed by the delay Lyapunov method and $\tilde{\sigma}_{\hat{u}}$ is computed by the Padé approximant. As can be seen, for this example, if T < 0.01, even the 1st order Padé approximant is a good approximation for computing \mathcal{H}_2 norm. For larger delays, the 3rd order Padé approximant generally performs well. Note that for delays larger than 0.115, the quasilinear system is unstable and, hence, the difference between $\sigma_{\hat{u}}$ and $\tilde{\sigma}_{\hat{u}}$ tends to infinity.

In this work, we choose the order of all Padé approximant n = 6 under the criterion with $\delta = 5^{\circ}$.



Figure 3.5: Quasilinear gain and tracking performance with different delays

3.3 Numerical Example of Time-Delay QLC System

Consider the system of Figure 3.1a with all elements defined as in Section 2.3.2 and the time-delay given by T, which we vary below. The quasilinear gain is the root of the equation below:

$$N_a - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2} \left\|\frac{F_{\Omega}(s)C(s)}{1 + P(s)N_aC(s)e^{-sT}}\right\|_2}\right) = 0.$$
(3.13)



Figure 3.6: Illustration of reference tracking in time domain

Thus,

$$N_a - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2} \left\|\frac{5\sqrt{3}s(s+10)}{(s^3+2s^2+2s+1)(s^2+10s+50N_ae^{-sT})}\right\|_2}\right) = 0.$$
(3.14)

To illustrate, we solve (3.14) with the actuator boundary level $\alpha \in [0,3]$ and a time-delay $T \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$. As shown in Figure 3.5, for each α , the value of N_a degrades as a function of delay and $\sigma_{\hat{e}}$ increases as a function of delay. This phenomenon is formalized by Theorem 2 below for the general case.

Both the nonlinear and the stochastically linearized systems are simulated with a time-delay of 0.5 s. The traces of r(t), $\hat{y}(t)$ and y(t) from the simulation results are shown in Figure 3.6. From the figure, it can be seen that the output of the nonlinear time-delay system, y(t), lags the reference r(t) by the specified time-delay, as expected. For $\alpha = 1$, the error between the standard deviation of y(t) and that of $\hat{y}(t)$ is 0.0475, and for $\alpha = 0.5$, the error is 0.1993, which is significantly higher. This is because of an increase in the nonlinearity of the system due to saturation. The accuracy of stochastic linearization is studied further in Chapter 4.

Theorem 2. Assume that, for the system shown in Figure 3.1b, the controller is asymptomatically stable, the limit of $N_a(T)$ as $T \to \infty$ in the Equation (3.13) exists, and the nonlinearity is the saturation with fixed α . Then, the solution of Equation (3.13) satisfies $N_a \to 0$ as $T \to \infty$. Moreover, the stochastically linearized system is always stable for $T \in [0, \infty)$.

Proof. The proof is by contradiction. Assume $N_a(T) \neq 0$ as $T \to \infty$. Then \mathcal{H}_2 norm function in Equation (3.13) tends to infinity because of the instability caused by the time-delay. Therefore, the second term of (3.13) tends to 0, which contradicts the assumption that $N_a(T) \neq 0$. This proves that $N_a(T) \to 0$. Next, we prove by contradiction that the quasilinear system is asymptomatically stable $\forall T$. Suppose there exists a T such that the system is unstable. This implies that the \mathcal{H}_2 norm in (3.13) is infinite. Thus, $N_a = 0$. However, this leads a contradiction because with $N_a = 0$, the \mathcal{H}_2 norm in (3.13) satisfies:

$$\left\|\frac{F_{\Omega}(s)C(s)}{1+P(s)N_{a}C(s)e^{-sT}}\right\| = \left\|F_{\Omega}(s)C(s)\right\|,$$

which is bounded because C(s) is assumed to be stable. This contradicts the fact that the \mathcal{H}_2 norm is unbounded.

3.4 Controller design

3.4.1 SATURATED-ROOT LOCUS (S-RL)

To find the Saturated-termination points in a time-delay system, (2.26) is modified to:

$$K_e(K,T) = K \operatorname{erf}\left(\frac{\alpha}{\sqrt{2}K \left\|\frac{F_{\Omega}(s)C(s)}{1 + K_e(K,T)C(s)P(s)e^{-sT}}\right\|_2}\right)$$
(3.15)

where $K_e(K,T)$ is now a function of the parameter K and time-delay T.

Unlike the S-RL without time-delay, which may or may not terminate prematurely as shown in Section 2.4, when we apply the time-delay in (2.26) and evaluate $\lim_{K\to\infty} K_e(K,T)$ for T > 0, the S-RL always terminates prematurely. This is explained in the following Theorem.

Theorem 3. In equation (3.15), there exists $0 < K_e^* < \infty$ such that $K_e(K,T) \leq K_e^*$ $\forall T > 0$ and, hence, S-RL with time-delay terminates prematurely. Furthermore, the S-RL with time-delay always belongs to the left half plane, implying that the quasilinear system of Figure 3.1b is always stable.

Proof. Similar to Theorem 5.2 in [3], an auxiliary transfer function is defined as:

$$T_{\gamma}(s) := \frac{F(s)C(s)}{1 + \gamma P(s)C(s)e^{-sT}}, \ \gamma \in \mathbb{R}^+, \ T > 0$$

Because of the time-delay, large K destabilizes the system, which implies that $T_{\gamma}(s)$ is asymptotically stable only for $\gamma \in [0, \Gamma)$, for some $\Gamma < \infty$. The following approach is by contradiction. Assume there exists $K^* > 0$ such that $K_e(K^*) \geq \Gamma$. Then, it follows from (3.15) that:

$$K_e(K^*) = K \operatorname{erf}\left(\frac{\alpha}{\sqrt{2}K^* \left\|\frac{F_{\Omega}(s)C(s)}{1+K_e(K^*)C(s)P(s)e^{-sT}}\right\|_2}\right)$$
(3.16)

This leads to a contradiction because the LHS of (3.16) is positive, while the RHS is 0. Hence, our assumption is incorrect, and $K_e(K,T) \leq \Gamma$. The K_e^* in the theorem statement is exactly Γ . Because this auxiliary transfer function is stable $\forall T$, the \mathcal{H}_2 norm in (3.16) is finite, which implies that the closed-loop quasilinear system is asymptotically stable.

Theorem 3 states that the quasilinear system of Figure 3.1b always remains stable, even in the presence of time-delays. This is in contrast with linear systems with delays, which become unstable for large K. This result makes sense, as the original nonlinear system contains saturation, which ensures bounded-input bounded-output stability of the closed-loop system.

Note that Theorem 3 holds for both a real delay and its Padé approximant, because, similar to a real delay, the non-minimum zeros of the Padé approximant distabilize the system for sufficient large K. In this thesis, we apply the Padé approximant with sufficient orders for solving (3.15).

Numerical Example of S-RL with time-delays

To illustrate the difference with the case without a time-delay discussed in Section 2.4, consider the system:

$$P(s) = \frac{s+15}{s(s+2.5)}, \quad \alpha = 0.16$$
$$C(s) = 1, \ F_{\Omega}(s) = \frac{\sqrt{3}}{s^3 + 2s^2 + 2s^2 + 1}$$



Figure 3.7: K_e vs K of the time-delay system



Figure 3.7 shows $K_e(K)$ as a function of K for different T. Overall, regardless of the time-delay, when K is small, $K_e(K)$ is practically linear, because the actuator is not saturated. As K increases, $K_e(K)$ terminates at the termination points. Furthermore, as shown in Figure 3.8, as T increases, the termination points decrease bec ause N decreases according to (3.2). This effect turns out to be a problem for controller design. To illustrate, suppose we would like to design a controller such that the closed-loop poles are within an admissible domain denoted by the yellow region in Figure 3.9. When the time-delay increases, the poles may be out of the admissible domain due to decreasing K_e , which degrades the tracking performance and may cause difficulties for controller design.

3.4.2 QLC-BASED DESIGN OF OPTIMAL CONTROLLERS

QLC also can be used to design optimal controllers. Unlike in conventional controller design, where the performance of, for example, the step response is considered, QLC takes into account the statistical properties of stochastic inputs.



Figure 3.9: Saturated-Root Locus with decreasing K_e .

Consider the nonlinear time-delay system of Figure 3.1a. The problem is to design an optimal controller C(s) to ensure effective reference tracking performance. Since the analysis of this nonlinear system is difficult, we stochastically linearize it to obtain the system of Figure 3.1b. To ensure good tracking performance with less control effort, the combined standard deviation of the error signal, i.e., $\sigma_{\hat{e}}$, and a scaled version of that of the input to the actuator, i.e., $\sigma_{\hat{u}}$, is minimized. Clearly, $\sigma_{\hat{e}}$ and $\sigma_{\hat{u}}$ depend on C(s). This is in accordance with standard practice in optimal control, for example in designing an LQR controller, where the combined state and control costs are minimized.

The optimization problem may be formulated as:

$$\min \qquad \rho \sigma_{\hat{u}}^2 + \sigma_{\hat{e}}^2, \tag{3.17}$$

subject to (3.2)

where ρ is a control penalty, $\sigma_{\hat{u}}$ calculated using (3.1), and $\sigma_{\hat{e}}$ using:

$$\sigma_{\hat{e}} = \left\| \frac{F_{\Omega}(s)}{1 + P(s)NC(s)e^{-sT}} \right\|_2.$$
(3.18)

For the sake of numerical optimization, we replace the delay e^{-sT} by an n^{th} order Padé approximant, based on the criterion in Section 3.2. This optimization problem is not convex and its solution may not be unique, because $\sigma_{\hat{e}}$ and $\sigma_{\hat{u}}$ are coupled with N_a through transcendental equations. Therefore, the optimization algorithm must be started from multiple initial conditions in order to find the best solution.

Furthermore, a practical application of this method is given in Chapter 5 for the design of a PID controller for the pitch control of wind turbines.

CHAPTER 4

Accuracy of QLC with Time-delay

In this chapter, we introduced two Monte Carlo experiments. The first experiment is to examine the accuracy of stochastic linearization with time-delays. The second experiment shows the effect of time-delays in the accuracy of stochastic linearization.

4.1 Overview - Monte Carlo Method

Since it is not possible to analytically determine the statistical measures of a signal in a general stochastic nonlinear system, there is no analytical technique to characterize the accuracy of stochastic linearization for such dynamic systems. However, accuracy can be determined numerically [3,30]. In this section, we introduce two statistical experiments based on the Monte Carlo method. Monte Carlo method, in general, is the a statistic algorithm that use repeatedly and randomly generated calculation to obtain numerical results. The essential idea is using randomness to solve problems that might not have other approaches. The first experiment demonstrates the accuracy of stochastic linearization and second one studies the effect of the time-delay.

4.2 FIRST EXPERIMENT - ACCURACY OF SL

In order to statistically examine the accuracy, the following Monte Carlo experiment is performed. We simulated 5000 time-delay systems with the block diagram of Figure 3.1. In 2500 of these systems, P(s) is assumed to be a first order system defined by $P(s) = \frac{K_p}{Ts+1}$, and in the remaining 2500, P(s) is assumed to be a second order system defined by $P(s) = \frac{K_p \omega_n^2}{s^2+2\zeta \omega_n s+\omega_n^2}$. The controller and the nonlinear actuator are assumed to be

$$C(s) = K, \quad f(\cdot) = \operatorname{sat}_{\alpha}(\cdot)$$

where K is the controller gain, and $f(\cdot)$ is a symmetric saturation with actuator authority α . The parameters T_d , ζ , α , K, and K_p are randomly and uniformly selected from the following intervals: $\zeta \in [0, 2]$, $T_d \in [0, 1]$, $\alpha \in [0.01, 5]$, $K_p \in [0.01, 10]$, $K \in$ [0.01, 50]. The parameters T and ω_n are selected randomly and logarithmically from $T, \omega_n \in [0.01, 10]$. Also, we assume that F(s) is the third-order Butterworth filter:

$$F(s) = \sqrt{3} \left(\frac{1}{s^3 + 2s^2 + 2s + 1} \right)$$

From the resulting simulations, all unstable systems and systems in which the phase margin for the stochastically linearized systems is lower than 10 degrees are discarded, because they are not practical. For each of the rest of the systems, σ_y and $\sigma_{\hat{y}}$ and RMSE of y and \hat{y} are evaluated by simulations.

Accuracy is defined by the RMS error and the error in σ_y as follows:

RMSE =
$$\sqrt{\frac{\sum_{t=t_0}^{n} (\hat{y}_t - y_t)^2}{n}}, \quad e_y = \frac{|\sigma_y - \sigma_{\hat{y}}|}{\sigma_y}$$
 (4.1)



Figure 4.1: Convergence of the RMS error and the error in σ_y

where y_t and \hat{y}_t refer to the plant output at time t for the nonlinear system and the corresponding stochastically linearized system, respectively, and t_0 is a time after which the system is in the stationary regime.

To show that 2500 systems are indeed sufficient for this experiment, we compute RMSE and e_y in (4.1) for every randomly generated 1st and 2nd order systems. Then, we find the running average of the computed RMSE and e_y values as a function of the number of systems simulated. The results are shown in Figure 4.1. From these figures, we determine that after 1000 simulations, the error metrics have sufficiently converged. For this reason, we choose 1000 systems for the second Monte Carlo experiment.

Results

The results are shown in Figure 4.2. Figure 4.2a shows that the quasilinear gain, which represents the degree of linearity in the systems, is mostly close to 0 and 1. Since nearly half of the systems are highly nonlinear and the rest linear, this is a valid experiment for analyzing the relevance of the nonlinearity from low to high.



Figure 4.2: Histogram of statistic results

In Figure 4.2b, it can be seen that in all cases the RMS error is lower than 0.6, with mean 0.0901. Hence, we can conclude that RMS error remains low in all cases. In Figure 4.2c, the error in σ_y , i.e., e_y , has mean 0.0501, i.e., 5.01% average error. Overall, the data clearly show that accuracy in all cases remains relatively high, even in the presence of time-delays.

4.3 Second experiment - Effect of the Time-delay

Another Monte Carlo experiment is conducted to find out the effect of the time-delay on the accuracy of stochastic linearization. A total of 1000 first order systems and 1000 second order systems are randomly generated with parameters selected as in Section 4.2. For each random system, the time-delay parameter T_d is varied from 0 to 0.2 seconds in increments of 0.05 and, for each delay, the response of the system is simulated.

Results

The results are shown in Figure 4.3a and Figure 4.3b. Here,

$$\Delta \text{RMSE}(\%) = (\text{RMSE} - \text{RMSE}_0) / \text{RMSE}_0) \times 100\%,$$

$$\Delta e_y(\%) = (e_y - e_{y_0}) / e_{y_0} \times 100\%,$$
(4.2)

where RMSE_0 and e_{y_0} are the RMS error and the error in σ_y , e_y , respectively, for the system without time-delay, i.e., $T_d = 0$. In these figures, each line corresponds to



Figure 4.3: Simulation result

a fixed system with varying levels of time-delay. The increasing $\Delta \text{RMSE}(\%)$ describes the RMS error increases as gradually increasing delay time T. Similarly, the increasing $\Delta e_y(\%)$ indicates the decrease of the tracking performance. It can be seen that overall, accuracy worsens as time-delay increases, although in few situations it can be seen to improve as well.

In this chapter, two Monte Carlo experiments are introduced. Overall, the accuracy of stochastic linearization with time-delays remains relatively high, even though increasing time-delays degrades the accuracy.

CHAPTER 5

PRACTICAL EXAMPLE - WIND TURBINE

5.1 Overview

Pitch control is a method commonly used in modern wind turbine systems to control the pitch angle of the turbine blades in order to keep the electrical power output at the rated power and prevent damage to the turbine due to varying wind speed [68]. Compared to a fixed-pitch system, a pitch control system produces a stable power output to keep the generator of the wind turbine operating at the rated power.

The power curve of a wind turbine system is shown in Figure 5.1 [1]. It shows the relationship between electrical power output of a wind turbine and the wind speed. As wind speed exceeds the cut-in speed, torque control is activated to generate maximum power. When output power achieves its rated value, pitch control is activated instead of torque control, until the wind speed is greater than the cut-out speed.



Figure 5.1: Power Curve of a Wind Turbine System, figure is taken from [1]

5.2 MODEL CONSTRUCTION



Figure 5.2: Wind Turbine System Feedback Control System Model, figure is taken from [2]

Figure 5.2 shows the block diagram of typical wind turbine system [2]. Below, the model of pitch actuator and drive-train model will be introduced.

Pitch actuator model

The pitch actuator is used to change the blade angle. The actuator model describes the dynamic between the pitch demand β_d and the pitch angle measures from the blade. The change of pitch angle is as follows:

$$\frac{d\beta}{dt} = \left(\beta_d - \beta\right) / T_\beta,\tag{5.1}$$

where $T_{\beta} = 0.5$ is the time constant of pitch actuator, which can be calculated by Newton's second law of motion with the parameters of the wind turbine shown in the table below. Construction detail is provided in [2].

Time constant of pitch actuator, T_{β}	0.5
Reference pith angle, β_d	0 to 90 deg
Rate of change of pitch angle	3 deg/sec
Damping coefficient, B	2 N.m/rad/sec
Driven-train inertia, J_T	$0.3 \deg$

Table 5.1: The parameters of the actuator model in the wind turbine [2]

Thus, the Laplace transform of (5.1) is

$$\frac{\beta(s)}{\beta_d(s)} = \frac{1}{sT_d + 1} = \frac{1}{0.5s + 1} \tag{5.2}$$

Drive train model

Similar to the actuator model, the dynamic of drivetrain can be described as the transfer function from the wind turbine torque T_w to turning speed of the motor W_T

$$\frac{W_T}{T_w} = \frac{(1/B)}{(J_T/B)s + 1} = \frac{0.5}{0.375s + 1.5}$$
(5.3)

In a pitch control system, the turbine blade turning rate is saturated and the hydraulic system has a propagation delay. Hence, the pitch control system is a nonlinear time-delay system. Specifically, the pitch control system can be modeled as a time-delay in the electric drive along with a first order inertia. The transfer function for our example can be expressed as:

$$\frac{\beta(s)}{\beta_r(s)} = \frac{0.5}{0.37s + 1.5} \cdot \frac{1}{5s + 1} e^{-2s}$$
(5.4)

where $\beta(s)$ is the pitch angle, and $\beta_r(s)$ is the angle of the pitch demand. In order to control the input to the saturating actuator, a PID controller is used, as shown in the overall block diagram of the system along with its stochastically linearized version in Figure 5.3.

The parameters used for simulation are taken from the literature [68]. The actuator is saturated by the rate of change of pitch angle in the hydraulic system, which allows a range of $-3^{\circ}/s \sim +3^{\circ}/s$, and the time-delay in the hydraulic system is T = 2s.

5.3 SIMULATION



Figure 5.3: Simulation of Pitch Control of Wind Turbine System

The PID controller gains are selected using the QLC-based optimization described in Section 3.4.2, with $\rho = 0.01$. The optimization is performed using several random initial sets of PID parameters. Since the optimization problem is non-convex, different solutions are obtained for different initial values. The best out of them is selected for the optimal PID gains, which are listed below:

 $K_p = 2.4458, \qquad K_i = 0.1923, \qquad K_d = 1.4559$

Note that F(s) is defined by

$$\frac{0.3062}{s^3 + s^2 + 0.5s + 0.125}\tag{5.5}$$

so that the input bandwidth is 0.5 rad/s and $\sigma_r = 1$.

Compared to a baseline set of PID parameters from the literature [68]: $K_p = K_i = 0.5$ and $K_d = 1$, which result in $\sigma_{\hat{e}} = 1.0935$ and the value of the cost function equal to 1.0558. The optimized values of K_p , K_i and K_d mentioned above result in $\sigma_{\hat{e}} = 0.6882$ and the value of the cost function equal to 0.5141. With our method, the standard deviation of tracking error, $\sigma_{\hat{e}}$, is decreased by 28.65%.



Figure 5.4: Output of the pitch control time-delay system with nonlinear actuator and the corresponding stochastically linearized time-delay system

The quasilinear gain obtained from (3.2) is 0.8792, which shows that the actuator is moderately saturated because of the variation in wind speed. To determine the accuracy of stochastic linearization for this example, the system is simulated in Simulink[®] using these parameters. The result of the simulation is shown in Figure 5.4. The RMS error of the output is found to be 0.1311 and the accuracy e_y to be 0.0019.

From Figure 5.4, it can be seen that the tracking performance of the nonlinear time-delay system and that of the corresponding stochastically linearized system, represented in the output signals y(t) and $\hat{y}(t)$ respectively, are similar, implying a good statistical accuracy. Thus, the method of stochastic linearization provides a fairly accurate approximation of the nonlinear system.

CHAPTER 6

CONCLUSION

6.1 Contributions

In this research, quasilinear control of time-delay systems has been investigated. A sufficient condition for the existence of a quasilinear gain has been included, along with the criterion for a Padé approximant of appropriate order.

In the controller design problem, the method of root locus was extended to the LPNI time-delay system and the result shows that the root locus terminates prematurely because of the actuator saturation and time-delay. A QLC-based optimal control design has been also investigated.

Statistical results show that even by taking the time-delay property into account, stochastic linearization produces a fairly accurate representation of the nonlinear system.

Finally, QLC was applied to a pitch control system for regulating and maintaining the electric power output of a wind turbine under varying wind speed.

6.2 FUTURE WORK

Future work includes combining this approach with the disturbance rejection, numerical stability and robustness of the QLC design, and considering different nonlinearities in actuators and sensors for more analysis.

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