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PROPER P-COMPATIBLE HYPERSUBSTITUTIONS

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Abstract

An identity $s \approx t$ is called a hyperidentity in a variety V if by substituting terms of appropriate arity for the operation symbols in $s \approx t$, one obtains an identity satisfied in V. If every identity in V is a hyperidentity, the variety V is called solid. All solid varieties of a given type τ form a complete sublattice $\mathcal{S}(\tau)$ of the lattice $\mathcal{L}(\tau)$ of all varieties of type τ . The concept of an M-solid variety generalizes that of a solid variety. An equation $s \approx t$ of terms of type τ is called P-compatible where P is a partition of the set $F = \{f_i | i \in I\}$ of operation symbols of type τ if it has the form $x_i \approx x_i$ or $f_i(t_1, \ldots, t_{n_i}) \approx f_j(t_1', \ldots, t_{n_j}')$ with $f_j \in [f_i]_P$, where $[f_j]_P$ is the block of P containing f_j. A variety is called P-compatible if it contains only P-compatible identities. All P-compatible varieties of type τ form also a sublattice of the lattice of all varieties of type τ. We ask for the intersection of both lattices, i.e. we want to characterize solid varieties which are P-compatible or M-solid varieties which are P-compatible.

1 Preliminaries

Our informal definition of a hyperidentity shows that we are interested in a map which associates to each n_i-ary operation symbol f_i an n_i-ary term $\sigma(f_i)$.

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Any such map is called a hypersubstitution. Let $W_\tau(X)$ be the set of all terms of type τ on an alphabet $X = \{x_1, x_2, \ldots, x_n, \ldots\}$. Using a hypersubstitution σ we can define a uniquely determined mapping $\hat{\sigma}$ defined on terms by
(i) $\hat{\sigma}[x] := x$,
(ii) $\hat{\sigma}[f_i(t_1, \ldots, t_{n_i})] := \sigma(f_i)(\hat{\sigma}[t_1], \ldots, \hat{\sigma}[t_{n_i}])$.
By $Hyp(\tau)$ we denote the set of all these hypersubstitutions. If we define a multiplication $\circ_h$ on the set $Hyp(\tau)$ by $\sigma_1 \circ_h \sigma_2 := \hat{\sigma_1} \circ \sigma_2$ where $\circ$ is the usual composition of functions, together with $\sigma_{id}(f_i) := f_i(x_1, \ldots, x_{n_i})$ we obtain a monoid $\underline{Hyp(\tau)} = (Hyp(\tau); \circ_h; \sigma_{id})$. If $\underline{M}$ is a submonoid of the monoid of all hypersubstitutions of type τ then an equation $s \approx t$ of terms of type τ is called M-hyperidentity in the variety V of groupoids if for all $\sigma \in M$ the equations $\hat{\sigma}[s] \approx \hat{\sigma}[t]$ are satisfied as identities of V. Hyperidentities are M-hyperidentities for $M = Hyp(\tau)$. A variety V of type τ is called *M-solid* if each of its identities is an M-hyperidentity for $M = Hyp(\tau)$. All M-solid varieties of type τ form a complete sublattice $S_M(\tau)$ of the lattice $\mathcal{L}(\tau)$ of all varieties of type τ with

$$M_1 \subseteq M_2 \Rightarrow S_{M_1}(\tau) \supseteq S_{M_2}(\tau).$$

To test whether an identity $s \approx t$ of a variety V is an M-hyperidentity of V our definition requires that we check, for each hypersubstitution in M, that $\hat{\sigma}[s] \approx \hat{\sigma}[t]$ is an identity of V. Indeed, we can restrict our testing to certain "special hypersubstitutions". We recall of two concepts, both introduced by J. Płonka ([4]).

Definition 1.1 *Let V be a variety of type τ. A hypersubstitution σ is called V-proper if for every identity $s \approx t$ in V, the identity $\hat{\sigma}[s] \approx \hat{\sigma}[t]$ also holds in V. We use $P(V)$ for the set of all V-proper hypersubstitutions.*

It is clear that $(P(V); \circ_h; \sigma_{id})$ is a submonoid of $Hyp(\tau) = (Hyp(\tau); \circ_h; \sigma_{id})$ and that a variety V is M-solid for $M = P(V)$ and $P(V)$ is the largest M for which V is M-solid.

Definition 1.2 *Let V be a variety of type τ. Two hypersubstitutions σ_1, σ_2 are called V-equivalent ($\sigma_1 \sim_V \sigma_2$) if $\sigma_1(f_i) \approx \sigma_2(f_i)$ are identities in V for all $i \in I$.*

This relation can be extended to arbitrary terms t, i.e.

$$\sigma_1 \sim_V \sigma_2 \Leftrightarrow \hat{\sigma}_1[t] \approx \hat{\sigma}_2[t]$$

is an identity in V. Then one can prove: If $\sigma_1 \sim_V \sigma_2$ and $\hat{\sigma}_1[s] \approx \hat{\sigma}_1[t]$ then $\hat{\sigma}_2[s] \approx \hat{\sigma}_2[t]$ is an identity in V.

We will use the following denotations:

Id V - the set of all identities satisfied in the variety V,
$C_P(\tau)$ - the set of all P-compatible equations of type τ,
$C_P(V) = C_P(\tau) \cap \mathrm{Id}\ V$ - the set of all P-compatible identities of V,
$Ex(\tau)$ - the set of all externally compatible equations of type τ, i.e. P-compatible for $P = \{\{f_i\} | i \in I\}$,
$Ex(V) = Ex(\tau) \cap \mathrm{Id}\ V$,
$N(\tau)$ - the set of all normal identities of type τ, i.e. P-compatible for $P = \{\{f_i\} | i \in I\}$,
$N(V) = N(\tau) \cap \mathrm{Id}\ V$.

It is easy to see that $C_P(\tau)$ and $C_P(V)$ are equational theories, i.e. closed under the rules of consequences for identities.

2 $C_P(V)$-proper hypersubstitutions

Definition 2.1 *A hypersubstitution $\sigma \in Hyp(\tau)$ is called $C_P(V)$-proper if for all $s \approx t \in C_P(V)$ we have $\hat{\sigma}[s] \approx \hat{\sigma}[t] \in C_P(V)$ (i.e. $\hat{\sigma}[s] \approx \hat{\sigma}[t] \in Id\ V$ and $\hat{\sigma}[s] \approx \hat{\sigma}[t] = x_i \in X$ or $ex(\hat{\sigma}[s]) \in [ex(\hat{\sigma}[t])]_P$ where $ex(\hat{\sigma}[t])$ denotes the first operation symbol occurring in the term $\hat{\sigma}[t]$).*

Let $M_{C_P}(V)$ be the set of all $C_P(V)$- proper hypersubstitutions of type τ. Then we have

Lemma 2.2 *$M_{C_P}(V)$ forms a submonoid of $Hyp(\tau)$.*

Proof. If $s \approx t \in C_P(V)$ then $\hat{\sigma}_{id}[s] = s \approx t = \hat{\sigma}_{id}[t] \in C_P(V)$, thus $\sigma_{id} \in M_{C_P}(V)$. If $\sigma_1, \sigma_2 \in M_{C_P}(V)$ then for all $s \approx t \in C_P(V)$ we have $\hat{\sigma}_2[s] \approx \hat{\sigma}_2[t] \in C_P(V)$ and then $\hat{\sigma}_1[\hat{\sigma}_2[s]] \approx \hat{\sigma}_1[\hat{\sigma}_2[t]] \in C_P(V)$, i.e. $(\sigma_1 \circ_h \sigma_2)\hat{}\,[s] \approx (\sigma_1 \circ_h \sigma_2)\hat{}\,[t] \in C_P(V)$. Therefore $\sigma_1 \circ_h \sigma_2 \in M_{C_P}(V)$. □

Remark that there are different possibilities to define sets of hypersubstitutions which are connected with P-compatible identities of the variety V. For instance we could also define a hypersubstitution to be $C_P(V)$*-generating* if for all $s \approx t \in \mathrm{Id}\ V$ it follows that $\hat{\sigma}[s] \approx \hat{\sigma}[t] \in C_P(V)$. If we denote by $G_{C_P}(V)$ the set of all $C_P(V)$-generating hypersubstitutions we have

Lemma 2.3 *$G_{C_P}(V)$ is a semigroup of hypersubstitutions which in general is not a monoid.*

Proof. If $\sigma_1, \sigma_2 \in G_{C_P}(V)$ then for all $s \approx t \in \text{Id } V$ we get $\hat{\sigma}_2[s] \approx \hat{\sigma}_2[t] \in C_P(V)$ and thus $\hat{\sigma}_1[\hat{\sigma}_2[s]] \approx \hat{\sigma}_1[\hat{\sigma}_2[t]] \in C_P(V)$. This means $G_{C_P}(V)$is closed under the product $\circ_h$. But in general $G_{C_P}(V)$ is not a monoid since $s \approx t \in \text{Id } V$, but $s \approx t \notin C_P(V)$ and then $\hat{\sigma}_{id}[s] \approx \hat{\sigma}_{id}[t] \notin C_P(V)$. □

Remarks:

1. If V is an idempotent variety, (i.e. $(f_i(x, \dots, x) = x) \in \text{Id } V$) then a hypersubstitution belonging to $G_{C_P}(V)$ has to map each f_i to one of the variables $x_1, \dots, x_{n_i}$.

2. Clearly, $G_{C_P}(V)$ is a subsemigroup of the monoid $P(V)$ of all proper hypersubstitutions of type τ and $M_{C_P}(V)$ is the monoid of all proper hypersubstitutions of the variety $V_{C_P} := Mod(C_P(V))$ which is defined by all P-compatible identities of the variety V.
   That means, the variety V_{C_P} is M-solid for the monoid $M_{C_P}(V)$ and $M_{C_P}(V)$ is the greatest monoid of hypersubstitutions such that V_{C_P} is M-solid .

Theorem 2.4 *Let V be a variety of type τ and let P be a partition of the set $\{f_i | i \in I\}$ of operation symbols. Let $M_{C_P}(V)$ be the monoid of all $C_P(V)$-proper hypersubstitutions. If $M_{C_P}(V) = Hyp(\tau)$ then $P = \{\{f_i\} | i \in I\}$ or $P = \{f_i | i \in I\}$.*

Proof. Let $s \approx t$ be an arbitrary identity of $C_P(V)$ and assume that $P \neq \{\{f_i\} | i \in I\}$. Then we can assume that $s = f_i(s_1, \dots, s_{n_i})$ and $t = f_j(t_1, \dots, t_{n_j})$ with $f_j \in [f_i]_P, f_i \neq f_j$. (Such an identity exists since $P \neq \{\{f_i\} | i \in I\}$). Consider now a hypersubstitution which maps f_i to $f_i(x_1, \dots, x_{n_i})$ and f_j to $h(f_j(x_1, \dots, x_{n_j}), \dots, f_j(x_1, \dots, x_{n_j}))$, where h is an arbitrary operation symbol of $P = \{f_i | i \in I\}$. We may assume that h is not nullary, otherwise we change the role of f and g. Since $Hyp(\tau) = C_P(V)$ we obtain $\hat{\sigma}[s] \approx \hat{\sigma}[t] \in C_P(V)$ and $h \in [f_i]_P$ and $P = \{f_i | i \in I\}$. □

Note that Theorem 2.4 is a reformulation of [2,Theorem 8] which says that if $Mod(C_P(V))$ is solid and $Mod(C_P(V)) \neq Mod(Ex(V))$ then $Mod(C_P(V))$ is normal. The proof is also only a reformulation of the proof of [2,Theorem 8].

We consider some more examples. We will call a hypersubstitution σ of type τ a pre-hypersubstitution if for every $i \in I$ the term $\sigma(f_i)$ is not a variable ([3]). Let $Pre(\tau)$ be the set of all pre-hypersubstitutions of type τ. Let T be the trivial variety of type τ, i.e. $T = Mod\{x \approx y\}$ and let Id T be the set of all identities of type τ. Therefore $C_P(T) = C_P(\tau)$ and $M_{C_P}(T) = Pre(\tau) \cap \{\sigma | f_i \in [f_j]_P \Rightarrow ex(\sigma(f_i)) \in [ex(\sigma(f_j))]_P\}$. If $V = Alg(\tau)$ is the class of all algebras of type τ, i.e. $V = Mod\{x \approx x\}$ then V is solid. The set Id V consists of all equations where the terms on the left and on the right hand side are the same. Clearly, $C_P(V) = Hyp(\tau)$ for any partition P of $\{f_i | i \in I\}$.

Since $M_{C_P}(V)$ is a monoid we can apply the theory of M-hypersubstitutions and M-solid varieties developed in [1]. We can apply hypersubstitutions $\sigma \in M_{C_P}(V)$ to both, to equations and to algebras. If $s \approx t$ is an equation of terms of type τ then we can form $\hat{\sigma}[s] \approx \hat{\sigma}[t]$. and define an operator $\mathcal{X}^E_{C_P}$ by
$\mathcal{X}^E_{C_P}[\sum] := \{\hat{\sigma}[s] \approx \hat{\sigma}[t] | \sigma \in M_{C_P}(V), s \approx t \in \sum, \sum \subseteq W_\tau(X)^2\}$.
The application of hypersubstitutions to an algebra $\underline{A} = (A; (f_i{}^A)_{i \in I})$ of type τ is defined by
$\mathcal{X}^A_{C_P}[K] := \{\sigma[\underline{A}] | \sigma \in M_{C_P}(V), \underline{A} \in K, K \subseteq Alg(\tau)\}$, where $\sigma[\underline{A}] := (A; (\sigma(f_i)^{\underline{A}})_{i \in I})$.
It is easy to see that both operators have the properties of closure operators which are defined for arbitrary non-empty sets as union of the results which we obtain if we apply them to one-element sets, i.e.

$$\mathcal{X}^E_{C_P}[\sum] = \bigcup_{s \approx t \in \sum} \mathcal{X}^E_{C_P}(\{ s \approx t\})$$

$$\mathcal{X}^A_{C_P}[K] = \bigcup_{\underline{A} \in K} \mathcal{X}^A_{C_P}(\{\underline{A}\})$$

Such operators are called *additive*. Further they are connected by the property

$$s \approx t \in \text{Id } \mathcal{X}^A_{C_P}[K] \Leftrightarrow \mathcal{X}^E_{C_P}(\{ s \approx t\}) \in \text{Id } K.$$

Because of this property we speak of a *conjugate pair* of additive closure operators.
Further we use the following denotations:
$H_{M_{C_P}(V)}\text{Id } (V)$ - the set of all $M_{C_P}(V)$-hyperidentities satisfied in the variety V and
$H_{M_{C_P}(V)}Mod(\sum)$ - the class of all algebras of type τ, such that every equation of $\sum$ is an $M_{C_P}(V)$-hyperidentity of this algebra.

Further, we say that a variety K is $M_{C_P}(V)$ -solid if $\mathcal{X}^A_{M_{C_P}(V)}[K] = K$. From the properties of the pair $(\mathcal{X}^A_{M_{C_P}(V)}, \mathcal{X}^E_{M_{C_P}(V)})$ as a conjugate pair of additive closure operators we obtain the following characterization of $M_{C_P}(V)$-solid varieties ([1]).

Theorem 2.5 *For all varieties K of type τ and for all equational theories Σ of type τ the following conditions (i)-(iv) and the conditions (i')-(iv') are equivalent:*

(i) $K = H_{M_{C_P}(V)} Mod H_{M_{C_P}(V)} Id\,(K)$ *(K is an $M_{C_P}(V)$-hyperequational class),*

(ii) $\chi^A_{M_{C_P}(V)}[K] = K$, *i.e. K is $M_{C_P}(V)$-solid,*

(iii) $\chi^E_{M_{C_P}(V)}[Id\,(K)] = Id\,(K)$,

(iv) $Id\,(K) = H_{M_{C_P}(V)} Id\,(K)$, *and*

(i') $\sum = H_{M_{C_P}(V)} Id\; H_{M_{C_P}(V)} Mod(\sum)$,

(ii') $\chi^E_{M_{C_P}(V)}[\sum] = \sum$,

(iii') $\chi^A_{M_{C_P}(V)}[Mod(\sum)] = Mod(\sum)$,

(iv') $Mod(\sum) = H_{M_{C_P}(V)} Mod(\sum)$.

We have already mentioned that the variety $V_{C_P} = Mod(C_P(V))$ is $M_{C_P}(V)$-solid. Therefore V_{C_P} satisfies the equivalent conditions (i),(ii),(iii),(iv).

From the general theory (see [1]) it follows also that the class of all $M_{C_P(V)}$-solid varieties forms a complete lattice which is a complete sublattice of the lattice of all varieties of type τ.

3 P-compatible relations on hypersubstitutions

In analogy to the relation $\sim_V$ we define the following binary relation on the set $Hyp(\tau)$ and on submonoids of $Hyp(\tau)$.

Definition 3.1 *Let V be a variety of type τ and let P be a partition of the set of operation symbols $\{f_i | f \in I\}$ of V. Let $C_P(V)$ be the set of all P-compatible identities satisfied in V. Then for any two hypersubstitutions $\sigma_1, \sigma_2 \in Hyp(\tau)$ we define*

$$\sigma_1 \sim_{C_P(V)} \sigma_2 :\Leftrightarrow \forall i \in I(\sigma_1(f_i) \approx \sigma_2(f_i) \in C_P(V)).$$

We notice that $\sim_{C_P(V)}$ is an equivalence relation on $Hyp(\tau)$. It can be easily shown that for all terms $t \in W_\tau(X)$, if $\sigma_1 \sim_{C_P(V)} \sigma_2$ then $\hat{\sigma}_1[t] \approx \hat{\sigma}_2[t] \in C_P(V)$.

The relation $\sim_{C_P(V)}$ can also be restricted to the monoid $M_{C_P(V)}$.

Theorem 3.2 *The monoid $M_{C_P(V)}$ is a union of full equivalence classes with respect to the relation $\sim_{C_P(V)}$.*

Proof. We have to show that if $\sigma_1 \in M_{C_P(V)}$ and if $\sigma_1 \sim_{C_P(V)} \sigma_2$ then $\sigma_2 \in M_{C_P(V)}$. Indeed, $\sigma_1 \in M_{C_P(V)}$ means that for each $s \approx t \in C_P(V)$ the identity $\hat{\sigma}_1[s] \approx \hat{\sigma}_1[t]$ belongs also to $C_P(V)$. The relation $\sigma_2 \sim_{C_P(V)} \sigma_1$ implies $\hat{\sigma}_2[t] \approx \hat{\sigma}_1[t] \in C_P(V)$ for all $t \in W_\tau(X)$. But then, by transitivity we get $\hat{\sigma}_2[s] \approx \hat{\sigma}_2[t] \in C_P(V)$, and this means $\sigma_2 \in M_{C_P(V)}$. □

Theorem 3.2 shows also that, if we want to check whether an identity is an $M_{C_P(V)}$-hyperidentity, we can restrict our checking to one representative from each equivalence class with respect to $\sim_{C_P(V)}$. We can also show

Corollary 3.3 *The restriction of the relation $\sim_{C_P(V)}$ to the submonoid $M_{C_P(V)}$ is a congruence relation on the monoid $M_{C_P(V)}$.*

Proof. We show that the restricted relation $\sim_{C_P(V)} |M_{C_P(V)}$ is a right and a left congruence on $M_{C_P(V)}$. Assume that $\sigma_1 \sim_{C_P(V)|_{M_{C_P(V)}}} \sigma_2$ and that $\sigma \in M_{C_P(V)}$. Then for the term $\sigma(f_i)$ we have $\hat{\sigma}_1[\hat{\sigma}(f_i)] \approx \hat{\sigma}_2[\hat{\sigma}(f_i)] \in C_P(V)$ and therefore $\sigma_1 \circ_h \sigma \sim_{C_P(V)|_{M_{C_P(V)}}} \sigma_2 \circ_h \sigma$. From $\sigma_1 \sim_{C_P(V)|_{M_{C_P(V)}}} \sigma_2$ it follows $\sigma_1(f_i) \approx \sigma_2(f_i) \in C_P(V)$ and for every $\sigma \in M_{C_P(V)}$ also $\hat{\sigma}[\sigma_1(f_i)] \approx \hat{\sigma}[\sigma_2(f_i)] \in C_P(V)$, i.e. $\sigma \circ_h \sigma_1 \sim_{C_P(V)} |_{M_{C_P(V)}} \sigma \circ_h \sigma_1 \in C_P(V)$. □

If we consider the class of the identity hypersubstitution, we notice that it forms a submonoid of $M_{C_P(V)}$ since if $\sigma_1 \sim_{C_P(V)} \sigma_{id}$ and $\sigma_2 \sim_{C_P(V)} \sigma_{id}$ then we have $\sigma_1(f_i) \approx \sigma_{id}(f_i) = f_i(x_1, \ldots, x_{n_i}) \in C_P(V)$ and $\sigma_2(f_i) \approx \sigma_{id}(f_i) = f_i(x_1, \ldots, x_{n_i}) \in C_P(V)$ and then also $(\sigma_1 \circ \sigma_2)(f_i) = \hat{\sigma}_1[\sigma_2(f_i)] \approx \hat{\sigma}_1[\sigma_{id}(f_i)] \in C_P(V)$, i.e. $(\sigma_1 \circ_h \sigma_2)(f_i) = \hat{\sigma}_1[\sigma_2(f_i)] \approx \sigma_1(f_i) = \sigma_{id} \in C_P(V)$, that means $\sigma_1 \circ_h \sigma_2 \sim_{C_P(V)} \sigma_{id}$.

Finally we want to remark that the previous definitions and theorems can be generalized in the following way:
Let V be a variety of type τ and let $T(\tau)$ be the set of all identities of type τ which fulfils a given property. For example the property could also be that

the set of variables occurring on both sides of the identity agree, so that $T(\tau)$ is the set of all regular identities of type τ. We will also assume that $T(\tau)$ is an equational theory. We set $T(V) := T(\tau) \cap \mathrm{Id}\, V$ and if Σ is an equational theory we set $T(\Sigma) := T(\tau) \cap \Sigma$.

Then we define a hypersubstitution to be $T(V)$-proper if for all $s \approx t \in T(V)$ we obtain $\sigma[s] \approx \sigma[t] \in T(V)$.

Let $M_{T(V)}$ be the set of all $T(V)$-proper hypersubstitutions of type τ. Clearly $M_{T(V)}$ is a submonoid of $Hyp(\tau)$ and we get a theorem similar to Theorem 2.6. The relation $\sim_{C_P(V)}$ can be also generalized and we define

$$\sigma_1 \sim_{T(V)} \sigma_2 :\Leftrightarrow \forall i \in I(\sigma_1(f_i) \approx \sigma_2(f_i) \in T(V)).$$

Using this definition we obtain theorems similar to Theorem 3.2 and to Corollary 3.3.

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[2] B. L. Osofsky, *A generalization of quasi-Frobenius rings,* J. Algebra, 4 (1966), 373-389.

For a contribution to a contributed volume:

[3] G. Azumaya *Locally split submodules and modules with perfect endomorphism rings,* in S.K. Jain and S. R. Lopez-Permouth, eds., Non-Commutative Ring Theory, Lecture Notes in Mathematics, Vol. 1448, Springer-Verlag, Berlin - New York (1990), 1-6.

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