The Fluctuating Two-Ray Fading Model: Exact and Approximate Statistical Characterization

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Abstract— We introduce the Fluctuating Two-Ray (FTR) fading model, a new statistical channel model that consists of two fluctuating specular components with random phases plus a diffuse component. The PDF and MGF are expressed in closed-form, having a functional form similar to other state-of-the-art fading models. We also provide an approximate closed-form expressions for the PDF, which allow for a simple evaluation of these statistics to an arbitrary level of precision. We show that the FTR fading model provides a much better fit than Rician fading for recent small-scale fading measurements in 28 GHz outdoor millimeter-wave channels.

I. INTRODUCTION

Very recently [1], the small-scale fading statistics obtained from a 28 GHz outdoor measurement campaign showed that Rician fading was more suited than Rayleigh even in NLOS environments. However, a deeper look into the results of [1] indicates that conventional fading models in the literature fall short in accurately modeling the random fluctuations suffered by the received signal. We here propose a new model to capture this behavior: the Fluctuating Two-Ray (FTR) fading model, as a natural generalization of the TWDP fading model by allowing the constant-amplitude specular waves associated to LOS propagation to randomly fluctuate. Remarkably, this larger flexibility does not come at the price of an increased mathematical complexity, but instead facilitates a simpler statistical characterization than the TWDP model.

II. STATISTICAL CHARACTERIZATION OF THE FTR MODEL

Let us consider that the complex baseband received signal can be written as

$$V_r = \sqrt{\zeta}V_1 \exp\left(j\phi_1\right) + \sqrt{\zeta}V_2 \exp\left(j\phi_2\right) + X + jY. \quad (1)$$

where $\phi_1, \phi_2 \sim \mathcal{U}[0, 2\pi]$, $X, Y \sim \mathcal{N}(0, \sigma^2)$ and ζ is a unit-mean Gamma distributed random variable with PDF

$$f_\zeta(u) = \frac{m^m u^{m-1}}{\Gamma(m)} e^{-mu}. \quad (2)$$

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This model will be subsequently denoted as the Fluctuating Two-Ray (FTR) and is conveniently expressed in terms of the parameters K and Δ, defined as

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \qquad \Delta = \frac{2V_1 V_2}{V_1^2 + V_2^2}. \quad (3)$$

Lemma 1: Let us consider the FTR fading model as described in (1). Then, the MGF of the received SNR γ (or, equivalently, the power envelope) will be given by

$$M_\gamma(s) = \frac{m^m(1+K)(1+K-\bar{\gamma}s)^{m-1}}{\left(\sqrt{\mathcal{R}(m,k,\Delta;s)}\right)^m}$$
$$\times P_{m-1}\left(\frac{m(1+K)-(m+K)\bar{\gamma}s}{\sqrt{\mathcal{R}(m,k,\Delta;s)}}\right), \quad (4)$$

where $\mathcal{R}(m,k,\Delta;s)$ is a polynomial in s defined as

$$R(m,k,\Delta;s) = \left[(m+K)^2 - \Delta^2 K^2\right]\bar{\gamma}^2 s^2$$
$$- 2m(1+K)(m+K)\bar{\gamma}s + m^2(1+K)^2, \quad (5)$$

$\bar{\gamma}$ is the average SNR and $P_\mu(\cdot)$ is the Legendre function of the first kind of degree μ.

Proof: See [2] ∎

The FTR fading model introduced here is well-suited to recreate the propagation conditions in a wide variety of wireless scenarios, ranging from very favorable ones to worse-than Rayleigh fading. It also includes many important well-known statistical fading models as particular cases, i.e., TWDP, Rician shadowed, Rician, Rayleigh. one-sided Gaussian, Nakagami-*m* and Nakagami-*q* (Hoyt).

Lemma 2: When $m \in \mathbb{Z}^+$, the PDF of the SNR γ in a FTR fading channel can be expressed in terms of the confluent hypergeometric function $\Phi_2(\cdot)$ defined in [3, p. 34, (8)], as given in (7).

Proof: See [2] ∎

Lemma 3: When $m \in \mathbb{Z}^+$, the PDF of the SNR γ in a FTR fading channel can be approximated by a finite sum of elementary functions, as given, in (8), where $M > \lceil K\Delta \rceil$, $\beta = \frac{K+1}{\bar{\gamma}}$ and the coefficients α_i and δ_i are defined as

$$\alpha_i = \frac{2(-1)^i}{(2M-1)(2M-i)!(i-1)!} \int_0^{2M-1} \prod_{\substack{k=1 \\ k=i}}^{2M}(u-k+i)\,du,$$

$$\delta_i = \Delta \cos\left(\frac{(i-i)\pi}{2M-1}\right). \quad (6)$$

Proof: See [2] ∎

The PDF of the received signal envelope r can be easily derived by a simple change of variables. Specifically, $f_r(r) =$

$$f_\gamma(x) = \frac{1}{2^{m-1}} \frac{1+K}{\bar\gamma} \left(\frac{m}{\sqrt{(m+K)^2 - K^2\Delta^2}} \right)^m \sum_{q=0}^{\lfloor(m-1)/2\rfloor} (-1)^q C_q^{m-1} \left(\frac{m+K}{\sqrt{(m+K)^2 - K^2\Delta^2}} \right)^{m-1-2q}$$

$$\times \ \Phi_2^{(4)} \left(1+2q-m, m-q-\frac{1}{2}, m-q-\frac{1}{2}, 1-m; 1; \right.$$

$$\left. -\frac{m(1+K)}{(m+K)\bar\gamma}x, -\frac{m(1+K)}{(m+K(1+\Delta))\bar\gamma}x, -\frac{m(1+K)}{(m+K(1-\Delta))\bar\gamma}x, -\frac{1+K}{\bar\gamma}x \right). \tag{7}$$

$$\widehat{f}_\gamma(x) \approx \sum_{i=1}^{M} \frac{\alpha_i}{2} \left\{ \mathcal{G}_m\left(x; \beta, K(1-\delta_i)\right) + \mathcal{G}_m\left(x; \beta, K(1+\delta_i)\right) \right\}, \tag{8}$$

$$\mathcal{G}_m(x; \beta, K) = \left(\frac{m}{K+m} \right)^m \beta e^{-\beta x} \sum_{n=0}^{m-1} \binom{m-1}{n} \left(\frac{K\beta x}{K+m} \right)^n \frac{1}{n!}; \tag{9}$$

$2r f_\gamma(r^2)$ and replacing $\bar\gamma$ by Ω, where $\Omega = E\{r^2\}$. In Fig. 1 we show the effect of the FTR fading model parameters K, Δ and m on the shape of the signal envelope PDF.
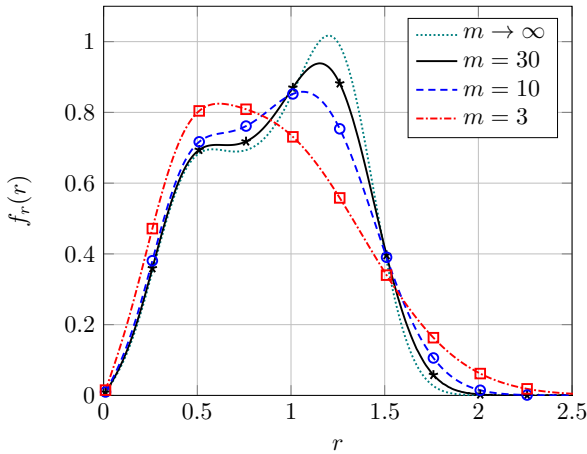

Fig. 1. FTR signal envelope for different values of m, with $K = 15$, $\Delta = 0.9$ and $\Omega = 1$. Solid lines: exact PDF. Markers: approximate PDF.

III. EMPIRICAL VALIDATION

We use the empirical results presented in [1] to validate the FTR fading model in the context of small-scale fading modeling of mmWave outdoor communications in the 28 GHz band. A modified version of the Kolmogorov-Smirnov (KS) statistic has been used to define the error factor ϵ used to quantify the goodness of fit between the empirical and theoretical CDFs, denoted by $\hat{F}_r(\cdot)$ and $F_r(\cdot)$ respectively, i.e,

$$\epsilon \triangleq \max_x \left| \log_{10} \hat{F}_r(x) - \log_{10} F_r(x) \right|. \tag{10}$$

In Fig. 2 we compare the set of measurements corresponding to the NLOS cross-polarized scenarios described in [1, Fig. 6]. For this set of measurements, the empirical CDFs lie within the theoretical CDFs corresponding to a Rician distribution with values of K ranging from 2 to 7 (i.e. 3 to 8 dB).

According to the KS statistic, the values of K that provide the
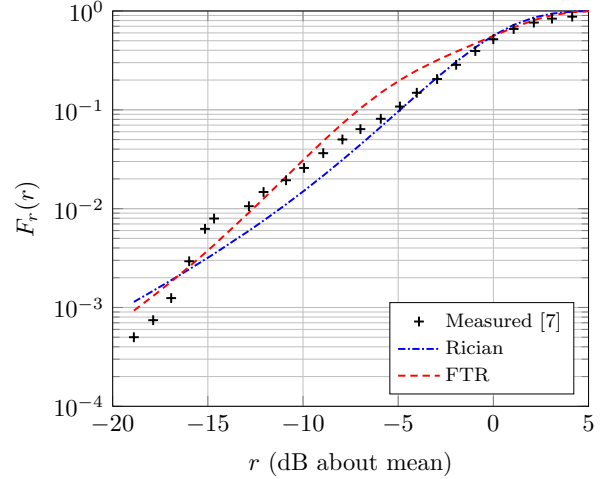

Fig. 2. Empirical vs theoretical CDFs of the received signal amplitude for NLOS scenario. Parameter values are $K_{\text{Rice}} = 4.78$ and $K_{\text{FTR}} = 32.7$, $\Delta = 0.8331$, $m = 10$. Measured data obtained from [1, Fig. 6, NLOS].

best fit to the Rician distribution is $K_{\text{NLOS}}^{\text{Rice}} = 4.78$. Such value of K yield an error factor value of $\epsilon_{\text{NLOS}}^{\text{Rice}} = 0.3571$. Now, using the proposed FTR fading model, we obtain the following set of parameters: $\text{FTR}_{\text{NLOS}} = (K = 32.7, \Delta = 0.8331, m = 10)$. The error factor value obtained by the FTR fit is $\epsilon_{\text{NLOS}}^{\text{FTR}} = 0.2681$. Thus, a remarkable improvement is attained when using the FTR fading model with respect to the Rician model.

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