Predicting the Electricity Demand Response via Data-driven Inverse Optimization

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Javier Saez-Gallego [1] and **Juan M. Morales** [2]

[1]Siemens Gamesa, Copenhagen, Denmark
[2]Department of Applied Mathematics, University of Málaga, Spain

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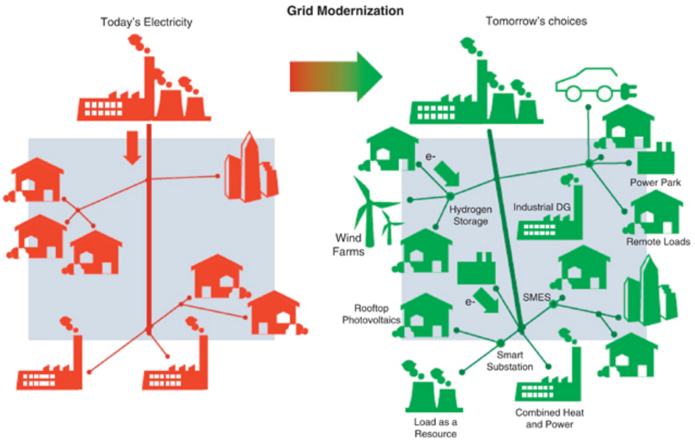
Towards the decentralization of the electricity grid

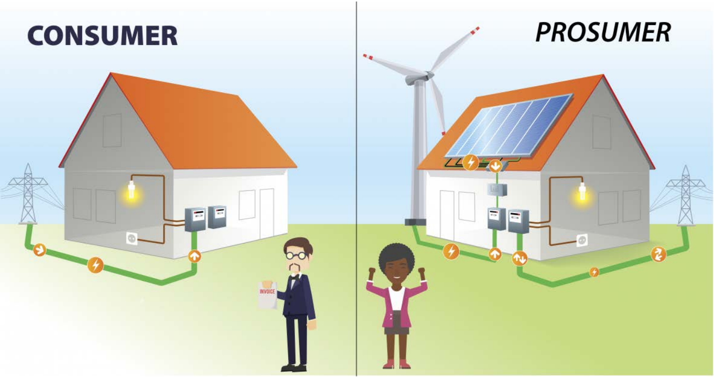


↑Complexity

↑Information

↑ Robustness, resilience, reliability, security

Involving the small consumer

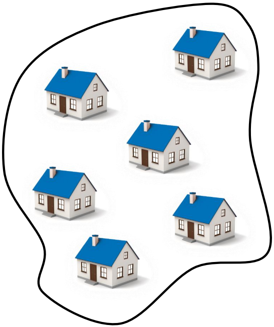


Outline

- Motivation
- Forecasting the price-responsive costumers' demand
- Defining the estimation problem
- Solving the estimation problem
- Case study: HVAC system of a pool of buildings
- Conclusions

Assumptions

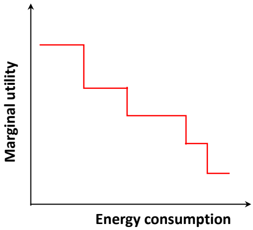
A cluster of **price-responsive** consumers is considered



This cluster is expected to **consume** more **at a favorable price**

We describe the pool of price-responsive consumers as a **utility maximizer agent**

Step-wise marginal utility function



Consumers' price-response model

$$\begin{aligned} \underset{x_{b,t},\forall b}{\text{maximize}} \quad & \sum_{b=1}^{B} x_{b,t}(u_b - p_t) \\ \text{subject to} \quad & \underline{P} \leq \sum_{b=1}^{B} x_{b,t} \leq \overline{P} \qquad (\underline{\lambda}_t, \overline{\lambda}_t) \\ & 0 \leq x_{b,t} \leq E_b \qquad (\underline{\phi}_{b,t}, \overline{\phi}_{b,t}) \end{aligned}$$

It is a **linear optimization problem (LOP)**.

Unknown variables:

- **Marginal utilities** u_b
- **Power bounds** $\overline{P}$, $\underline{P}$

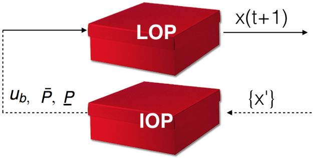
We seek values of u_b, $\overline{P}$, and $\underline{P}$ based on observations of $x'_{b,t}$ and p_t, given E_b. We use the estimated utility maximizer problem to predict x_{t+1}.

The estimation problem: Optimality condition

$$
\begin{aligned}
\underset{\Omega}{\text{minimize}} \quad & \sum_{t=1}^{T} \epsilon_t \\
\text{subject to} \quad & \overline{P}\,\overline{\lambda}_t - \underline{P}\underline{\lambda}_t + \sum_{b=1}^{B} E_b \overline{\phi}_{b,t} - \epsilon_t = \sum_{b=1}^{B} x_{b,t}(u_b - p_t), \quad \forall t \\
& \overline{\phi}_{b,t} - \underline{\phi}_{b,t} + \overline{\lambda}_t - \underline{\lambda}_t = u_b - p_t, \quad \forall t \\
& \overline{\phi}_{b,t}, \underline{\phi}_{b,t}, \overline{\lambda}_t, \underline{\lambda}_t, \epsilon_t \geq 0, \quad \forall t
\end{aligned}
$$

$$\Omega = \left\{ \epsilon_t, \overline{P}, \underline{P}, u_b, \overline{\lambda}_t, \underline{\lambda}_t, \overline{\phi}_{b,t}, \underline{\phi}_{b,t} \right\}$$

Inverse optimization (IOP) is used to determine the **parameters** of the model to make **predictions** of the load.



Leveraging auxiliary information

Model parameters $\overline{P}_t$, $\underline{P}_t$ and $u_{b,t}$, might vary over time. We assume a number of time varying regressors Z such that

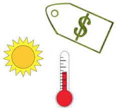
$$\underline{P}_t = \underline{\mu} + \sum_{r=1}^{R} \underline{\alpha}_r Z_{r,t} \tag{1}$$

$$\overline{P}_t = \overline{\mu} + \sum_{r=1}^{R} \overline{\alpha}_r Z_{r,t} \tag{2}$$

$$u_{b,t} = \mu_b^u + \sum_{r=1}^{R} \alpha_r^u Z_{r,t} \tag{3}$$

Regressors relate to **time** and **weather**:

- Temperature of the air outside
- Solar irradiance
- Hour indicator
- Past price and load



Leveraging auxiliary information

The price-response model must make sense for any plausible value of the features, in particular,

- The minimum consumption limit must be lower than or equal to the maximum consumption limit
- The minimum consumption limit must be non-negative

Leveraging auxiliary information

For example,

$$\underline{P}_t = \underline{P} + \sum_{r \in R} \underline{\alpha}_r Z_{r,t} \leq \overline{P} + \sum_{r \in R} \overline{\alpha}_r Z_{r,t} = \overline{P}_t, \quad t \in \mathcal{T}, \text{for all } Z_{r,t}$$

Assume that $Z_{r,t} \in [\overline{Z}_r, \underline{Z}_r]$, then

$$\underline{P} - \overline{P} + \underset{\substack{Z'_{r,t} \\ \text{s.t. } \underline{Z}_r \leq Z'_{r,t} \leq \overline{Z}_r, \quad r \in R}}{\text{Maximize}} \left\{ \sum_{r \in R} (\underline{\alpha}_r - \overline{\alpha}_r) Z'_{r,t} \right\} \leq 0, \quad t \in \mathcal{T}.$$

which is equivalent to

$$\begin{aligned} &\overline{P} - \underline{P} + \sum_{r \in R} (\overline{\phi}_{r,t} \overline{Z}_r - \underline{\phi}_{r,t} \underline{Z}_r) \leq 0 && t \in \mathcal{T} \\ &\overline{\phi}_{r,t} - \underline{\phi}_{r,t} = \overline{\alpha}_r - \underline{\alpha}_r && r \in R, t \in \mathcal{T} \\ &\overline{\phi}_{r,t}, \underline{\phi}_{r,t} \geq 0 && r \in R, t \in \mathcal{T}. \end{aligned}$$

Solving the estimation problem

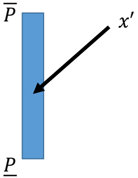
- The estimation problem is **non-linear and non-convex**.
- We statistically **approximate its solution** by solving two **linear programming problems** instead.
  1. A **feasibility** problem (estimation of power bounds).
  2. An **optimality** problem (estimation of marginal utilities).
- A **two-step data-driven estimation procedure** to achieve **optimality** and **feasibility** of x' in a statistical sense.

Feasibility problem: Estimation of power bounds

$$\underset{\underline{P},\overline{P},\boldsymbol{\xi},\boldsymbol{\mu},\boldsymbol{\alpha}}{\text{Minimize}} \sum_{t=1}^{T} \left((1-\mathrm{K}) \left(\overline{\xi}_t^{+} + \underline{\xi}_t^{+} \right) + \mathrm{K} \left(\overline{\xi}_t^{-} + \underline{\xi}_t^{-} \right) \right)$$

subject to

$$\begin{aligned}
&\overline{P}_t - x'_t = \overline{\xi}_t^{+} - \overline{\xi}_t^{-} && \forall t \\
&x'_t - \underline{P}_t = \underline{\xi}_t^{+} - \underline{\xi}_t^{-} && \forall t \\
&\underline{P}_t \leq \overline{P}_t && \forall t \\
&\underline{P}_t = \underline{\mu} + \sum_{r=1}^{R} \underline{\alpha}_r Z_{r,t} && \forall t \\
&\overline{P}_t = \overline{\mu} + \sum_{r=1}^{R} \overline{\alpha}_r Z_{r,t} && \forall t \\
&0 \leq \overline{\xi}_t^{+}, \overline{\xi}_t^{-}, \underline{\xi}_t^{+}, \underline{\xi}_t^{-} && \forall t
\end{aligned}$$

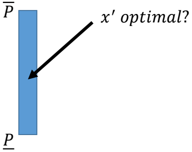


$\widehat{\overline{P}}_t, \widehat{\underline{P}}_t, \widehat{\underline{\mu}}, \widehat{\overline{\mu}}, \widehat{\underline{\alpha}}_r, \widehat{\overline{\alpha}}_r$

Optimality problem: Estimating marginal utilities

$$
\begin{aligned}
\underset{\boldsymbol{\Omega}}{\text{Minimize}} \ & \sum_{t=1}^{T} \epsilon_t \\
\text{subject to } & \widehat{\overline{P}}_t \overline{\lambda}_t - \widehat{\underline{P}}_t \underline{\lambda}_t + \sum_{b=1}^{B} E_b \overline{\phi}_{b,t} - \epsilon_t = \\
& \qquad \sum_{b=1}^{B} \tilde{x}'_{b,t} \left(u_{b,t} - p_t \right) && \forall t \\
& -\underline{\phi}_{b,t} + \overline{\phi}_{b,t} - \underline{\lambda}_t + \overline{\lambda}_t = u_{b,t} - p_t && \forall b, t \\
& u_{b,t} = \mu_b^u + \sum_r \alpha_r^u Z_{r,t} && \forall b, t \\
& \mu_b^u \geq \mu_{b+1}^u && \forall b < B \\
& \mu_1^u \geq 200 + \mu_2^u \\
& 0 \leq \overline{\lambda}_t, \underline{\lambda}_t, \underline{\phi}_{b,t}, \overline{\phi}_{b,t} && \forall b, t.
\end{aligned}
$$

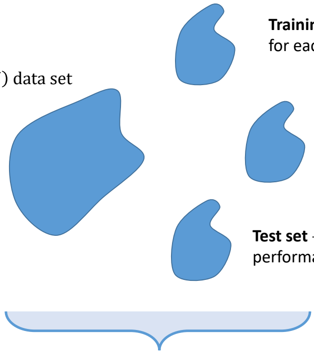
$\hat{u}_{b,t}, \hat{\mu}_b^u, \hat{\alpha}_r^u$



Solving the estimation problem

In the **bound estimation problem**, the **penalty parameter K** is statistically tuned through **validation**:

(x, p, Z) data set



Training set → Used for parametter fitting for each posible value of K

Validation set → Used to tune parameter K

Test set → Used to assess forecasting performance

We choose K so that the **out-of-simple prediction error** is minimized

K as indicator of the **price-responsiveness of the load**:

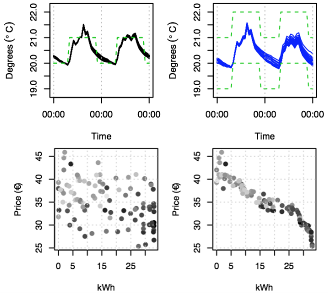
K

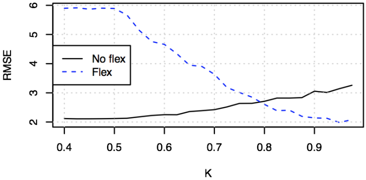
- 0 **Narrow** interval → **Small variability** of the load explained by the price.
- 1 **Wide** interval → **High variability** of the load explained by the price.

Case study (one-hour ahead prediction)

We simulate the price-response behavior of a pool of **100 buildings** equipped with **heat pumps** (assuming economic MPC is in place).

Two classes of buildings are considered, depending on the **comfort bands of the indoor temperature**.





Case study

We conduct a benchmark of the methodology against **simple persistence** forecasting and **autoregressive moving average with exogenous inputs**.

- **Simple persistence model**: The forecast load at time t is set to be equal to the observed load at $t-1$.
- **ARMAX**: The aggregate load x is a linear combination of the past values of the load, past errors and regressors.

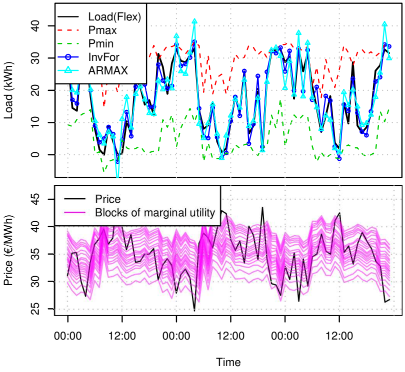
$$x_t = \mu + \epsilon_t + \sum_{p=1}^{P} \varphi_p x_{t-p} + \sum_{r=1}^{R} \gamma_r Z_{t-r} + \sum_{q=1}^{Q} \theta_q \epsilon_{t-q}$$

Forecasting performance is evaluated according to MAE and

$$NRMSE = \frac{1}{x^{max} - x^{min}} \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left(\sum_{b=1}^{B} \widehat{x}_{b,t} - x'_t \right)^2}$$

$$\mathrm{MASE} = \frac{\sum_{t=1}^{T} \left| \sum_{b=1}^{B} \widehat{x}_{b,t} - x'_t \right|}{\frac{T}{T-1} \sum_{t=2}^{T} \left| x'_t - x'_{t-1} \right|}$$

Case study



		MAE	NRMSE	MASE
No Flex	*Persistence*	4.092	0.155	-
	ARMAX	2.366	0.097	0.578
	InvFor	2.275	0.096	0.556
Flex	*Persistence*	8.366	0.326	-
	ARMAX	2.948	0.112	0.352
	InvFor	2.369	0.097	0.283

For an inflexible pool of loads, *InvFor* $\approx$ *ARMAX*. When the load aggregation is sensitive to the price, however, *InvFor* substantially outperforms *ARMAX*.

Conclusions

What we have done:

- A new method to **forecast price-responsive electricity consumption one step ahead.**
- A **two-step algorithm** to statistically approximate the exact inverse-optimization solution.
- A **validation scheme** to minimize the out-of-sample prediction error.
- A methodology evaluation on a data set corresponding to a cluster of price-responsive buildings equipped with a heat pump.

The **non-linearity between price and aggregate load** is well described by our methodology.

Future Work

- Dealing with corrupted measurements.
- Examining more flexible functional forms between model parameters and regressors.
- Investigating statistically consistent set-valued functions (feasibility set as a function of regressors)
- Testing the methodology on other data sets.

Contacts

Any questions?

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Juan Miguel Morales
juan.morales@uma.es
OASYS Webpage: oasys.uma.es

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Full paper
Short-term forecasting of price-responsive loads using inverse optimization
is available online at IEEExplore
http://ieeexplore.ieee.org/document/7859377/