

# Optimal Delegation, Unawareness, and Financial Intermediation

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## Abstract

We study the delegation problem between an investor and a financial intermediary. The intermediary has private information about the state of the world that determines the return of the investment. Moreover, he has superior awareness of the available investment opportunities and decides whether to reveal some of them to the investor. We show that the intermediary generally has incentives to make the investor aware of investment opportunities at the extremes, e.g. very risky and very safe projects, while leaving the investor unaware of intermediate investment options. We study how the extent to which the intermediary reveals available investment opportunities to the investor depends on the investor's initial awareness and the degree of competition between intermediaries in the market.

*JEL Codes:* D82, D83, G24.

## 1 Introduction

One of the many striking features of the recent financial crisis was the extreme exposure of investors to risk. Both investment and commercial banks had been selling excessively risky assets to investors, sometimes hiding some of the asset characteristics (e.g., Gerardi et al., 2008). At the same time, despite the impressive amount of new financial instruments and

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the rapidly changing financial world, since the 1950s a large fraction (of approx. 33% in the US) of investment demand has remained on 'safe' assets (e.g., Gordon et al. (2008) and Garcia (2012)).

Most of financial investments are intermediated by professionals. Financial intermediaries are non-neutral brokers and, as such, direct, influence, and distort the demand of assets in the economy. For instance, investment bankers commonly underwrite transactions of newly issued securities, whereby they raise investment capital from investors on behalf of corporations and governments both for equity and debt capital. Such practices give rise to conflicts of interest, sometimes leading to investments that are not necessarily in the best interest of the client. At the same time, investors differ widely in their financial literacy. They not only face limits in their ability to assess the profitability of particular investments, but often also have limited awareness of the available investment opportunities and must therefore rely on professional advice.

This paper studies the implications of such limitations by incorporating unawareness into the canonical delegation problem. Specifically, we consider the problem of an investor (the principal, she) who needs to choose how to invest her savings and delegates the task of picking the right project to a financial intermediary (the agent, he). The intermediary has private information about the payoffs of each investment opportunity and the investor's problem is to determine a set of projects from which the intermediary chooses upon observing the state of the world (see for example Alonso and Matouschek, 2008). We then depart from the traditional framework by considering a situation where the intermediary not only has better information on what the best investment choice is, but also on the actual set of available investment opportunities. This second dimension of asymmetry is captured by the assumption that the investor is only *partially aware of the feasible investment projects*. Before the delegation stage the intermediary has then the possibility to expand the investor's awareness by revealing additional investment opportunities. We are interested in the im-

plications of the investor's unawareness on the interaction with the intermediary. Will the intermediary expand the investor's awareness? If so, which projects will the intermediary disclose/not disclose? What are the properties of the investment projects eventually realized?

We address these questions in an environment with a continuum of states and a continuum of investment projects, some of which the investor is aware of. The intermediary's and investor's preferences are represented by quadratic loss functions, with differing bliss points for the two agents. We view the bliss point to be the investment opportunity which generates the best combination between risk, illiquidity, and return as a function of the state. The divergence between the investor's and intermediary's bliss point can be interpreted as banks being less risk averse, having limited liability, having different liquidity needs, etc. When deciding on the set of projects from which the intermediary can select, the investor then faces the usual tradeoff between granting flexibility to the intermediary so he can react on his private information and precluding the intermediary from choosing projects he is biased towards too often.

We show that the consideration of unawareness has important implications for delegation and investment. In the benchmark case of full awareness, under certain regularity conditions on the distribution of states, the optimal delegation set for the investor is an interval. Effectively, the investor imposes a cap: if, for instance, the intermediary is less risk averse than the investor, the investor places an upper bound on the riskiness of projects the intermediary can select. In our environment, where projects are ordered along the real line, this translates into a threshold below which the intermediary is free to choose and above which he is not allowed to take an action. Imposing the same regularity condition on the distribution of states, we characterize the equilibrium delegation set in an environment where the investor is partially unaware. Our main result shows that the intermediary makes the investor fully aware if and only if the investor is initially aware of the project at the optimal threshold under full awareness. If this is not the case, the intermediary leaves the investor unaware

of an interval of projects around the threshold. The intermediary thus makes the investor *aware of investment opportunities at the extremes (e.g. very safe and very risky projects)*, while he does not disclose intermediate investment opportunities. This makes it optimal for the investor to permit projects at both extremes and thus gives the intermediary the possibility to invest in them.

Next, we introduce competition among intermediaries by incorporating our baseline model into a search environment with imperfect competition. In this framework multiple intermediaries compete for investors over the set of investment opportunities they disclose: the larger this set is, the larger are the chances of attracting an investor. We show that, for intermediate degrees of competition, in equilibrium an intermediary either reveals everything or leaves investors unaware of a significant set of investment options. We thus obtain polarization in the market: some banks fully disclose all investment opportunities, others try to obtain a higher profit by concealing some of them.

Finally, we discuss the policy implications of our findings. Clearly, promoting financial literacy among investors improves their welfare in our model. Interestingly though, our results show that it is not necessary to educate investors about *all possible investment projects*. Instead, we demonstrate that making investors aware of *only one intermediate project* - at the optimal threshold in the benchmark case of full awareness - is enough. Making investors aware of this project gives incentives to intermediaries to reveal all remaining projects as well. We therefore have an interesting complementarity between the regulator and the market, suggesting a surprisingly simple, yet powerful, policy intervention.

The paper makes two main contributions. First, it makes a methodological contribution in that we introduce limited awareness into the model of delegation. The potential applications are much broader than the financial market. Second, the stated framework is able to generate predictions on the equilibrium portfolio differentiation, in particular on the demand

of safe and risky assets, as a function of the nature of misalignment between the investor and the financial intermediary.

**Related Literature:** This paper is first of all related to the literature on optimal delegation. Starting with Holmstrom (1984), who first defines the delegation problem and provides conditions for the existence of its solution, this literature, which includes Melamed and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), Armstrong and Vickers (2010) and Amador and Bagwell (2013), studies optimal delegation problems in environments of increasing generality. None of them consider limited awareness in this framework. Furthermore, this paper is related to a small literature on contract theory and unawareness. The application of the concept of unawareness to contracting problems is still at its beginnings. In contrast to our setting, existing work considers contracting problems where contingent transfers are feasible and where the agent is unaware - either of possible actions (Von Thadden and Zhao, 2012 and 2014) or of possible states (Zhao, 2011; Filiz-Ozbay, 2012; Auster, 2013), while the principal is fully aware.

This paper is also related to the literature on financial intermediation which sees banks as 'efficient' brokers who reduce transaction and information costs. The issue of this role of financial intermediation has been studied by many authors starting from Diamond (1984). A summary of the literature can be found in Bhattacharya and Thakor (1993) and Allen and Santomero (1998). These works focus on the possibility of partially monitoring the financial intermediaries ex-post. Most of the work is on the loan provision side of the banks and on the design of the optimal loan contract, whereas we concentrate on the brokerage role of banks.

## 2 Environment

There is an investor (she) who acts as the principal and a financial intermediary (he) who acts as the agent. The intermediary has access to a set of investment projects  $Y = [y_{min}, y_{max}]$ ,

the return to which depends on the state of the world. Let  $\Theta = [0, 1]$  be the set of states and let  $F(\theta)$  denote the cumulative distribution function on  $\Theta$ , assumed to be twice differentiable in  $[0, 1]$ .<sup>1</sup> Both the investor and the intermediary have a von-Neumann-Morgenstern utility function that takes the quadratic form

$$u(y, \theta) = -(y - \theta)^2 \quad \text{and} \quad v(y, \theta) = -(y - (\theta - \beta))^2.$$

The intermediary's preferred policy is  $y = \theta$ , while the investor's preferred policy is  $y = \theta - \beta$ . We assume  $\beta > 0$ , hence the intermediary has an upward bias of size  $\beta$ .<sup>2</sup> In Appendix A we provide a micro foundation for the assumed utility functions by starting with preferences defined over the mean and variance of the investment. Assuming that the intermediary is less risk averse than the investor, the bias  $b$  then captures the difference between the investor's and the intermediary's preferred investment on the mean-variance frontier. Accordingly, we can interpret low values of  $y$  in  $[y_{min}, y_{max}]$  as relatively safe investments and high values of  $y$  as relatively risky ones.

As in the canonical delegation problem, we assume that the intermediary is informed about the state of the world  $\theta$ , while the investor is not. We rule out monetary transfers and assume that the intermediary's participation constraint is always satisfied. The contracting problem of the investor then reduces to the decision of which projects to let the intermediary choose from.<sup>3</sup>

In contrast to the canonical setting, we assume that the investor is not aware of the whole set  $Y$  but only of a closed subset  $Y_P \subseteq Y$ . The intermediary, on the other hand, is

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<sup>1</sup>For  $\theta = 0$  and  $\theta = 1$ , this condition holds for, respectively, the right and left derivative.

<sup>2</sup>We can interpret  $\beta$  itself as the result of a contracting problem that generates  $\beta$  as the minimal level of conflict of interest between the principal and the agent. An alternative reading is that the intermediary and the investor have portfolios with different correlations across assets.

<sup>3</sup>Formally, the investor commits to a mechanism that specifies the project which will be implemented as a function of the intermediary's message. Alonso and Matouschek (2008) show that this contracting problem is equivalent to delegating a set of projects  $D \subseteq Y$  from which the investor can choose freely after observing the state of the world.

fully aware and has the possibility to make the investor aware of additional projects. After updating her awareness, the investor then makes her delegation choice. The precise timing is as follows:

1. The intermediary reveals a set of projects  $X \subseteq Y$  and the investor updates her awareness to  $\hat{Y} \equiv Y_P \cup X$ .
2. Given  $\hat{Y}$ , the investor chooses a delegation set  $D \in \mathcal{D}(\hat{Y})$ , where  $\mathcal{D}(\hat{Y})$  is the collection of closed subsets of  $\hat{Y}$ .<sup>4</sup>
3. The intermediary observes the state of world  $\theta$  and chooses an action from set  $D$ .
4. Payoffs are realized.

In general, optimal awareness sets  $\hat{Y}$  and optimal delegation sets  $D$  will not be unique since different awareness sets may induce the same delegation set and different delegation sets may induce the same implemented actions for each state of the world. We will assume that if the investor is indifferent between two delegation sets  $D$  and  $D'$  such that  $D' \subset D$ , she chooses the larger set  $D$ . Similarly, we assume that if the intermediary is indifferent between two revelation strategies that yield awareness sets  $\hat{Y}$  and  $\hat{Y}'$  such that  $\hat{Y}' \subset \hat{Y}$ , he expands the investor's awareness to  $\hat{Y}$ . That is, we will consider the sets that yield maximal awareness and maximal discretion.

**Remark:** *It is important to point out that our model is agnostic to the question of whether or not the investor is aware of her unawareness. What matters for the investor's expected payoff is the set of projects she permits the intermediary to implement. The investor may be well aware of the fact that there exist other projects outside her awareness but since she cannot include such projects in the delegation set, their existence does not affect her expected payoff or optimization problem.*

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<sup>4</sup>As discussed in Alonso and Matouscheck, the restriction to closed sets is without loss of generality.

## 3 Equilibrium Analysis

### 3.1 Full Awareness

We will start our analysis by describing optimal delegation under full awareness:  $Y_P = Y$ . For this specification, the existing literature shows that if the density function  $f(\theta) \equiv F'(\theta)$  satisfies the regularity condition  $f'(\theta)\beta + f(\theta) > 0$  for all  $\theta \in [0, 1]$ , the optimal delegation set is an interval (Martimort and Semenov, 2006, and Alonso and Matouschek, 2008). In particular, if the bias is sufficiently small so that the expected best project for the investor  $E[\theta - \beta]^5$  is strictly greater than  $y_{min}$ , the optimal delegation set is given by  $[y_{min}, \hat{y}]$ , where  $\hat{y}$  is such that

$$\hat{y} = E[\theta - \beta | \theta \geq \hat{y}]. \quad (1)$$

Otherwise the optimal delegation set is the singleton  $\{y_{min}\}$ . Hence, only if  $E[\theta - \beta] > y_{min}$ , the implemented project varies with the state of the world and delegation is valuable. In that case, the intermediary chooses his preferred project  $y = \theta$  for all  $\theta < \hat{y}$  and the project  $\hat{y}$  in all remaining states.

To understand why the optimal delegation set takes this form, it is useful to describe the investor's tradeoff in more detail. We can first explain why the optimal delegation set does not have "holes". By adding the missing projects to the delegation set, the intermediary's choice of projects as a function of the state becomes smoother: instead of switching from a low to a high project at some threshold state  $\theta'$ , the implemented action increase continuously with the realized state. Around  $\theta'$  the intermediary's preferred project lies in between the low and the high project, which implies that he switches to higher actions in states below  $\theta'$  and to lower actions in states above  $\theta'$ . Since the agent is upward bias and since the cost of moving away from the bliss point is convexly increasing for the investor, the gain of moving closer to the bliss point in the states above  $\theta'$  outweighs the cost of moving

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<sup>5</sup>All expectations are taken with respect to  $F$ .



away from the bliss point in the states below  $\theta'$ , as long as the probability weight attached to the latter states is not too large. The regularity condition on the state distribution assures that this is indeed the case.

The investor's problem then reduces to finding the optimal upper and lower bound of the delegation interval. Since the intermediary is upward biased it is never optimal to reduce the intermediary's flexibility from below, so the optimal lower bound is  $y_{min}$ . The optimal upper bound is determined by condition (1). Notice that, conditional on the state being greater than  $y$ , the investor's expected preferred project is  $E[\theta - \beta | \theta > y]$ . Condition (1) says that the optimal threshold  $\hat{y}$  is such that the project the intermediary implements in all states above the threshold is exactly the investor's expected preferred project in those states. The assumption  $f'(\theta)\beta + f(\theta) > 0$  assures that there is only one such project.

We will adopt the condition  $f'(\theta)\beta + f(\theta) > 0$  throughout the analysis. Furthermore, we will assume that in each state of the world both the investor's and the intermediary's preferred project is available.<sup>6</sup>

**Assumption 1.**  $f'(\theta)\beta + f(\theta) > 0$  for all  $\theta \in (0, 1)$ .

**Assumption 2.**  $y_{min} < -\beta$  and  $y_{max} > 1$ .

### 3.2 Partial Awareness: Main Result

Our main result shows that, maintaining the regularity condition on the state distribution, it is strictly optimal for the intermediary to leave the investor partially unaware if and only if the investor is initially unaware of the project at the optimal threshold under full awareness,  $\hat{y}$ . In that case, the intermediary optimally reveals projects at the extremes but leaves the investor unaware of intermediate projects.

**Theorem 3.1.** *Let Assumptions 1 and 2 be satisfied.*

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<sup>6</sup>The purpose of the latter assumption is to reduce the number of cases we need to distinguish.

- If  $y \in Y^P$ , the investor becomes fully aware and the optimal delegation set is  $[y_{\min}, \hat{y}]$ .
- If  $y \notin Y^P$ , the investor remains unaware of projects in  $(\hat{y} - \Delta, \hat{y} + \Delta)$  for some  $\Delta > 0$  and the optimal delegation set is  $[y_{\min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}$ .

Theorem 3.1 shows that whether the investor is made aware of all projects by the intermediary is determined only by her awareness of  $\hat{y}$ , the optimal cap under full awareness. If she is unaware of  $\hat{y}$ , the intermediary optimally leaves the investor unaware of an interval of projects around  $\hat{y}$ . As we will show, this makes it optimal for the investor to choose a delegation set that includes a project to the right of  $\hat{y}$ . By leaving the investor unaware of intermediate projects, the intermediary thus incentivizes the investor to permit investment projects that the intermediary is biased towards and that would be precluded under full awareness. As a result, the equilibrium delegation set is no longer an interval, illustrated in Figure 1.

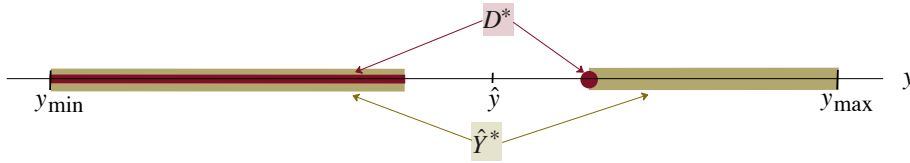


Figure 1: Equilibrium awareness and delegation

The statement of the proposition will be proven in a series of lemmas in the remainder of this section. We proceed recursively by first considering the investor's delegation choice for a given awareness set  $\hat{Y}$ . With the solution to this problem, we then turn to the intermediary's problem of choosing the optimal awareness set.

### 3.3 Delegation Choice

Let  $D^*(\hat{Y})$  denote the optimal delegation set when the awareness set is  $\hat{Y}$ . Alonso and Matouschek (2008) derive conditions under which, in the benchmark case of full awareness,

the optimal delegation set is an interval and therefore has no gaps. They show that, provided these conditions are satisfied, whenever the investor includes two distinct projects in the delegation set, it is strictly optimal to include all projects that lie in between. As we show in the Appendix, their argument perfectly generalizes to generic sets  $\widehat{Y}$  that may be non-connected. In our setting Alonso and Matouschek's (2008) conditions correspond to Assumption 1. We thus obtain the following result.

**Lemma 3.2** (Alonso and Matouschek, 2008). *Let Assumption 1 be satisfied and consider  $y_1, y_2 \in \widehat{Y}$  with  $y_1 < y_2$ . If  $y_1, y_2 \in D^*(\widehat{Y})$ , then all  $y \in \{\widehat{Y} \cap (y_1, y_2)\}$  belong to  $D^*(\widehat{Y})$ .*

*Proof.* See Appendix B.1. □

Lemma 3.2 implies that the optimal delegation set  $\widehat{Y}$  has no "holes" with respect to  $\widehat{Y}$ . With this, we again only need to find the optimal lower and the upper bound of the delegation set. The following lemma shows that, as in the case of full awareness, the investor never finds it optimal to restrict the agent's choice from below.

**Lemma 3.3.** *The optimal delegation set satisfies  $\min D^*(\widehat{Y}) = \min \widehat{Y}$ .*

*Proof.* See Appendix B.1. □

The intuition for Lemma 3.3 is simple. Since the intermediary is biased upwards, whenever he prefers  $\min \widehat{Y}$  over some other project in the delegation set, so does the investor. Thus, in those states where the intermediary chooses  $\min \widehat{Y}$ , the investor strictly prefers  $\min \widehat{Y}$  over any other project the intermediary can select. As a result, the investor optimally includes  $\min \widehat{Y}$  into the delegation set.

We turn next to the optimal upper bound of the delegation set. Here we can show that the rightmost project the investor permits is the project in the investor's awareness closest to  $\hat{y}$ . Importantly, this project may be smaller or greater than  $\hat{y}$  and hence smaller or greater than the optimal upper threshold in the benchmark case of full awareness.

**Lemma 3.4.** *Let Assumption 1 be satisfied. The optimal delegation set is such that*

$$\max D^*(\hat{Y}) = \arg \min_{y \in \hat{Y}} |y - \hat{y}|.$$

*Proof.* See Appendix B.1. □

To gain some intuition, consider the investor's benefit of increasing the upper bound by including an additional project  $y$  into delegation set  $D$  such that  $y > \max D$ . Adding  $y$  to the delegation set changes the investor's payoff in states  $\theta \geq (\max D + y)/2$ , since  $(\max D + y)/2$  is the state at which the intermediary switches from project  $\max D$  to project  $y$ . The expected value of the investor's preferred project in those states is  $E[\theta - \beta | \theta \geq (\max D + y)/2]$ . By definition of  $\hat{y}$  and Assumption 1 this value is greater than  $(\max D + y)/2$  if and only if  $(\max D + y)/2$  is smaller than  $\hat{y}$ . In this case, since  $\max D$  and  $y$  have the same distance to  $(\max D + y)/2$ , the project  $y$  is then in expectation closer to the investor's preferred project than  $\max D$ . Therefore, if  $(\max D + y)/2 < \hat{y}$ , the investor prefers to add  $y$  to the delegation set  $D$ . Clearly, the condition  $(\max D + y)/2 < \hat{y}$  is equivalent to the requirement that the distance between  $\hat{y}$  and  $y$  is smaller than that between  $\hat{y}$  and  $\max D$ . A perfectly symmetric argument implies that if  $(\max D + y)/2 > \hat{y}$ , that is if  $\max D$  is closer to  $\hat{y}$  than  $y$ , the investor strictly prefers not to include project  $y$ .

The fact that the optimal upper bound is the project in  $\hat{Y}$  that is closest to  $\hat{y}$  has two implications: first, the optimal delegation set includes all projects belonging to  $\hat{Y}$  that are weakly smaller than  $\hat{y}$ ; second, it includes at most one project strictly greater than  $\hat{y}$ . In particular, the investor permits the agent to invest in a project  $y > \hat{y}$  if and only if the largest element of  $\hat{Y}$  that is smaller than  $\hat{y}$  has at least the same distance to  $\hat{y}$ .

Taken together, the optimal delegation under partial awareness can be seen as the closest approximation of the optimal delegation set under full awareness,  $[y_{min}, \hat{y}]$ , available to the investor given her restricted awareness. This is illustrated in Figure 2.

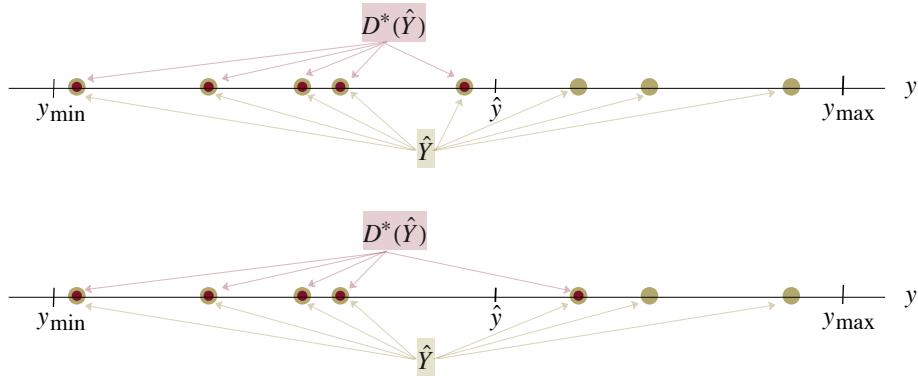


Figure 2: Optimal delegation set  $D^*(\hat{Y})$

### 3.4 Awareness Choice

We can now turn to the intermediary's optimal strategy of expanding the investor's awareness. As a first observation, note that if the investor is aware of the threshold project  $\hat{y}$ , the intermediary optimally reveals all other projects. Since there is no threshold closer to  $\hat{y}$  than  $\hat{y}$  itself, the upper bound of the optimal delegation set will always be  $\hat{y}$ . Revealing projects above  $\hat{y}$  is thereby irrelevant; the investor will never allow the intermediary to implement any of them. On the other hand, revealing projects below the threshold  $\hat{y}$  is strictly optimal since they are always included into the optimal delegation set. The intermediary thus maximizes his choice by revealing all projects to the investor, who then optimally chooses delegation set  $[0, \hat{y}]$ .

The above argument implies that, starting with an arbitrary set  $Y^P$ , the optimal awareness set  $\hat{Y}^*$  has to be such that the upper bound of the corresponding delegation set is at least  $\hat{y}$ . Moreover, provided that the investor is unaware of the project  $\hat{y}$ , the only reason for the intermediary to leave the investor unaware of a set of projects is to incentivize the investor to permit some project strictly to the right of the threshold  $\hat{y}$ . By Lemma 3.4 this is optimal for the investor if and only if the investor is unaware of all projects that are closer

to  $\hat{y}$ . It follows then that the optimal awareness set is of the form

$$\hat{Y}^* = [y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$$

for some  $\Delta \geq 0$ . Excluding projects in  $[y_{min}, \hat{y} - \Delta]$  or the project  $\{\hat{y} + \Delta\}$  strictly reduces the intermediaries choice set, while excluding any projects strictly greater than  $\hat{y} + \Delta$  is irrelevant. Given  $\hat{Y}^*$ , the investor's optimal delegation set is

$$D^*(\hat{Y}^*) = [y_{min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}$$

The intermediary is thus permitted to choose from an interval of projects strictly to the left of the full awareness threshold  $\hat{y}$  and one project to the right. In response to this delegation set, the intermediary's optimal policy is as follows. In states below  $\hat{y} - \Delta$  the intermediary uses his flexibility and implements his preferred project  $y = \theta$ . In states above  $\hat{y} - \Delta$  the preferred project is not available, so the intermediary chooses the one closest to his bliss point. For states in the interval  $(\hat{y} - \Delta, \hat{y})$  this is the project  $\{\hat{y} - \Delta\}$ , for the remaining states it is  $\{\hat{y} + \Delta\}$ . The intermediary's optimal policy can thus be summarized by

$$y^*(\theta; \Delta) = \begin{cases} \theta & \text{if } \theta \leq \hat{y} - \Delta \\ \hat{y} - \Delta & \text{if } \hat{y} - \Delta < \theta < \hat{y} \\ \hat{y} + \Delta & \text{if } \hat{y} \leq \theta \end{cases}$$

We can observe that even though the investor only permits one project to the right of the full awareness threshold  $\hat{y}$ , this project is implemented in all states above  $\hat{y}$ .

Taken together, the previous analysis provides us with a very simple description of the class of delegation and awareness sets that are candidates for an equilibrium in our environment: when deciding which projects to reveal to the investor, the intermediary implicitly chooses an awareness gap, parametrized by  $\Delta$ . To complete the proof of Theorem 3.1 it then remains to show that whenever a gap is feasible, it is also optimal.

To find the optimal value of  $\Delta$ , we can consider the reduced form problem of the intermediary. The feasible values of  $\Delta$  are determined by the initial level of awareness of the investor  $Y^P$ . In particular, the implementable values of  $\Delta$  are weakly smaller than  $\bar{\Delta}(Y_P) := \min_{y \in Y_P} |y - \hat{y}|$ , the minimal distance between a project in the investor's awareness and the threshold  $\hat{y}$ . For each  $\Delta$  in that set, the intermediary then anticipates the investor's optimal delegation set  $D^*$  and his own choice of projects  $y^*(\theta; \Delta)$ . Substituting  $y^*(\theta; \Delta)$  into the intermediary's expected payoff, his optimization problem amounts to

$$\max_{\Delta \in [0, \bar{\Delta}(Y_P)]} - \int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 f(\theta) d\theta - \int_{\hat{y}}^1 (\hat{y} + \Delta - \theta)^2 f(\theta) d\theta \quad (P)$$

**Proposition 3.5.** *Let Assumptions 1 and 2 be satisfied. The solution of problem P is given by  $\text{Min}[\bar{\Delta}(\hat{Y}), \Delta^*]$ , where  $\Delta^*$  solves*

$$(1 - F(\hat{y} - \Delta^*)) [E[\theta | \theta \geq \hat{y} - \Delta^*] - (\hat{y} - \Delta^*)] = 2(1 - F(\hat{y}))\beta. \quad (2)$$

*Proof.* See Appendix B.4. □

Proposition 3.5 characterizes the optimal awareness gap. In the proof we show that the intermediary's payoff as a function of  $\Delta$  is strictly concave and attains its maximum at  $\Delta^*$ , as determined by (2). Crucially, the unconstrained solution  $\Delta^*$  is strictly positive. Formally, this can be seen by noting that at  $\Delta = 0$  the second factor on the left hand side of (2) is equal to  $\beta$  (by definition of  $\hat{y}$ ). At  $\delta = 0$  the left-hand side is thus strictly smaller than the right-hand side. To gain some intuition, consider the net effect on the intermediary's expected payoff when increasing the gap at  $\Delta = 0$ . The marginal benefit of increasing  $\Delta$  is the increase in the intermediary's utility in states  $\theta > \hat{y}$ . Here the cap  $\hat{y}$  forces the intermediary to take an action that is too low from his point of view. By introducing a gap, he can increase the implemented action in these states, thereby getting closer to his bliss point. The marginal cost of increasing  $\Delta$  is the utility loss in the states close to  $\hat{y}$ , where the intermediary moves away from his bliss point. Since, however, the marginal effect on utility is zero at the bliss point, so is the marginal cost of increasing the gap at  $\Delta = 0$ . It follows

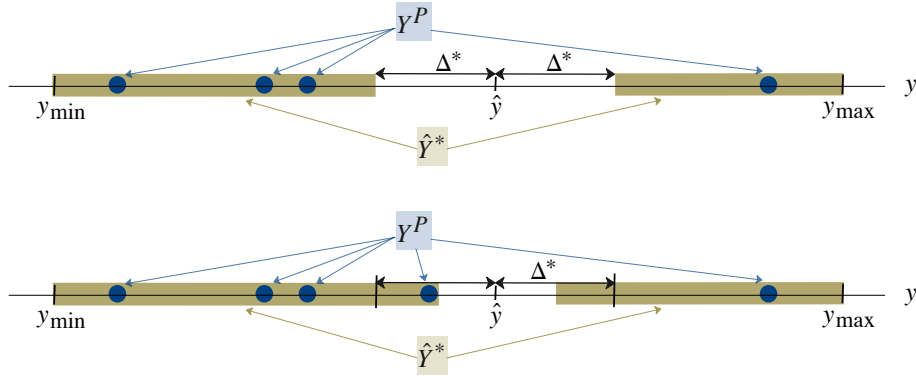


Figure 3: Optimal delegation set  $D^*(\hat{Y})$

that the optimal value of  $\Delta$  is strictly positive.

As can be verified, the unconstrained solution  $\Delta^*$  is increasing in the size of the bias  $\beta$ . That is, the larger the divergence between the investor's and the intermediary's preferred investment is, the more investment projects the intermediary wants to hide from the investor. The solution  $\Delta^*$  is implemented whenever the investor's initial awareness does not constrain the intermediary in his choice of the gap. If, however, the investor is aware of some project in the interval  $(\hat{y} - \Delta^*, \hat{y} + \Delta^*)$ , the intermediary's optimal strategy is to simply choose the largest feasible gap, as shown in Figure 3. The following example illustrates the findings for the uniform distribution.

**Example:** Suppose  $f(\theta) = 1$ . The optimal threshold in the benchmark case of full awareness is then given by  $\hat{y} = 1 - 2\beta$  and the interior solution for the optimal awareness set, characterized by condition (2), is  $\Delta^* = 2\beta(\sqrt{2} - 1)$ . If  $2\beta(\sqrt{2} - 1) \leq \bar{\Delta}(Y^P)$ , the intermediary leaves the investor unaware of all projects in the interval  $(1 - 2\beta\sqrt{2}, 1 + 2\beta(\sqrt{2} - 2))$ . The resulting equilibrium awareness and delegation sets are then given by

$$\begin{aligned}\hat{Y}^* &= [y_{\min}, 1 - 2\beta\sqrt{2}] \cup [1 + 2\beta(\sqrt{2} - 2), y_{\max}], \\ D^*(\hat{Y}^*) &= [y_{\min}, 1 - 2\beta\sqrt{2}] \cup \{1 + 2\beta(\sqrt{2} - 2)\}.\end{aligned}$$



If  $2\beta(\sqrt{2}-1) \leq \bar{\Delta}(Y^P)$  is not satisfied, the intermediary is restricted by the investor's initial awareness and therefore leaves her unaware of all projects in the smaller interval  $(1 - 2\beta - \bar{\Delta}(\hat{Y}^P), 1 - 2\beta + \bar{\Delta}(\hat{Y}^P))$ .

## 4 Competition

The previous section characterized the equilibrium in an environment where the intermediary is a monopolist and, by implication, fully determines the investor's awareness. In reality, investors can seek consult from multiple financial professionals, possibly to expand their choice between different investment options. To capture the interaction between multiple intermediaries, we adopt a simple model of imperfect competition that has been recently proposed by Lester et al. (2017) and is based on the work of Burdett and Judd (1983). Considering a model of imperfect competition accounts for some important features of the financial market, especially over-the-counter trading. It further allows us to consider the effect of the degree of competition on the awareness of investors and the composition of financial products traded in the market.

**Environment:** Following the approach of Lester et al. (2017), we assume that there are two intermediaries and a unit measure of investors.<sup>7</sup> Intermediaries have no capacity constraints and can therefore contract with many investors. There is a friction in that investors do not necessarily have access to both intermediaries. In particular, a fraction of investors is matched with one intermediaries, while the remaining investors are matched with both. As in Lester et al. (2017), we refer to investors that have access to only one intermediary as captive. Whether an investor is captive or non-captive is not observable to the intermediaries. Instead, from the viewpoint of an intermediary, conditional on meeting a particular investor, the investor is non-captive with some probability, denoted by  $\pi$ . The parameter  $\pi$  can then be viewed as a measure of competitiveness in the market: if  $\pi = 0$  we are back in

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<sup>7</sup>As Lester et al. (2017) show, the restriction to two intermediaries can be easily relaxed.

the monopoly case; if  $\pi = 1$  intermediaries engage in Bertrand competition.

To simplify matters we will first assume that investors are initially unaware of all projects. Upon meeting an investor, intermediaries disclose a set of investment projects, as before. If an investor meets with two intermediaries, her updated awareness set is the union of the projects that are revealed by either of the two intermediaries. The investor then needs to decide to which of the intermediaries to delegate her investment. In principle, after updating her awareness, the investor is indifferent between both intermediaries. To make competition matter, we assume here that the investor chooses the intermediary that reveals more investment opportunities. More specifically, if the set revealed by the first intermediary is a strict subset of the set revealed by the second, the investor chooses the second, and viceversa. If both intermediaries choose the same awareness set or if the awareness sets cannot be ordered, the investor chooses either intermediary with equal probability. Once an investor selects an intermediary, the interactions unfolds exactly as in the monopoly case. For a given investor, the timing can be summarized as follows:

1. The investor privately observes whether she meets one or two intermediaries.
2. Each intermediary reveals a set of projects and the investor updates her awareness to the union of both sets.
3. The investor chooses an intermediary and a delegation set.
4. The selected intermediary observes the state of world and chooses an action from the delegation set.
5. Payoffs are realized.

The assumption that an investor delegates to the intermediary that reveals more investment opportunities directly implies that intermediaries optimally choose awareness sets of the form  $[y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta, y_{max}]$ ,  $\Delta \geq 0$ . Intermediaries, therefore, compete over awareness gaps, parameterized by  $\Delta$ : a smaller value of  $\Delta$  increases an intermediaries chances of

being chosen by the investor he meets.

**Remark:** *An alternative assumption that gives rise to the same property is the assumption that the set of projects delegated to a particular intermediary has to be a subset of the set of projects the intermediary reveals. This can be interpreted as the intermediary offering a set of projects to which he has access and the investor restricting the intermediary's choice to a subset of those.*

**Payoffs:** An intermediary's expected payoff is the product of the probability of being chosen by the investor and his conditional expected payoff. Using the intermediary's optimal policy  $y^*(\theta; \Delta)$  as defined in the previous section, his conditional payoff as a function of  $\Delta$  is defined by:

$$U(\Delta) \equiv -\frac{1}{2} \int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 f(\theta) d\theta - \frac{1}{2} \int_{\hat{y}}^1 (\hat{y} + \Delta - \theta)^2 f(\theta) d\theta.$$

From the analysis in Section 3 we know that  $U(\Delta)$  is strictly concave and attains its maximum at  $\Delta^*$ , as determined by (2). The probability that an intermediary is selected by the investor depends on the strategy of the other intermediary. Letting  $H(\Delta)$  denote the probability that the other intermediary chooses an awareness gap smaller than  $\Delta$ , the probability that the investor delegates to the competitor is given by the product of the conditional probability that the investor meets the other intermediary and  $H(\Delta)$ . Letting  $\bar{U}$  denote the outside option of both intermediaries, an intermediary's expected payoff is then given by

$$(1 - \pi H(\Delta))U(\Delta) + \pi H(\Delta)\bar{U}. \quad (3)$$

We assume that intermediation can be profitable, that is:  $\bar{U} < U(\Delta^*)$ .

**Equilibrium:** A symmetric equilibrium in this environment is a (cumulative) distribution function  $H^*(\Delta)$  such that intermediaries are indifferent between all elements in the

support of  $H^*$  and weakly prefer those over any other values of  $\Delta$ . Since  $U(\Delta)$  is strictly decreasing for  $\Delta > \Delta^*$ , the support of  $H^*$  has to be a subset of  $[0, \Delta^*]$ . Standard arguments show that  $H^*$  is continuous. The intermediaries' strategy thus has no mass point, except possibly at  $\Delta = 0$ .

We are now ready to describe the equilibrium distribution  $H^*$  in this environment. While the appendix provides a complete characterization of  $H^*$ , we focus here on some key features of the equilibrium.

**Proposition 4.1.** *Let Assumptions 1 and 2 be satisfied. In the environment with imperfect competition, there exists an equilibrium, characterized by  $H^*$ , with the following properties:*

- if  $\pi \leq \underline{\pi}$ , the support of  $H^*$  is  $[\Delta', \Delta^*]$  for some  $\Delta' \geq 0$ ;
- if  $\underline{\pi} < \pi < \bar{\pi}$ , the support of  $H^*$  is  $\{0\} \cup [\Delta', \Delta^*]$  for some  $\Delta' > 0$ ;
- if  $\bar{\pi} \leq \pi$ , the support of  $H^*$  is  $\{0\}$ ;

where  $0 < \underline{\pi} \leq \bar{\pi} \leq 1$ .

*Proof.* See Appendix B.5. □

Proposition 4.1 shows that there are three parameter regions to be distinguished. If the degree of competition is sufficiently small, then intermediaries choose awareness gaps parameterized by values of  $\Delta$  in the interval  $[\Delta', \Delta^*]$ . As we show in the proof of Proposition 4.1, the size of this interval is strictly increasing in the competition parameter  $\pi$ . Consequently, the more competition there is, the smaller is the minimal awareness gap. At the threshold  $\underline{\pi}$ , the lower bound of the interval,  $\Delta'$ , is zero. When  $\pi$  increases further we observe polarization: there is a strictly positive probability that intermediaries disclose everything ( $\Delta = 0$ ), while otherwise they leave investors unaware of a significant part of the available investment opportunities. As  $\pi$  increases further, the probability that intermediaries leave investors unaware of certain projects becomes smaller, up to the point where this probability is zero, which happens at the second threshold  $\bar{\pi}$ . For all values of  $\pi$  greater than this

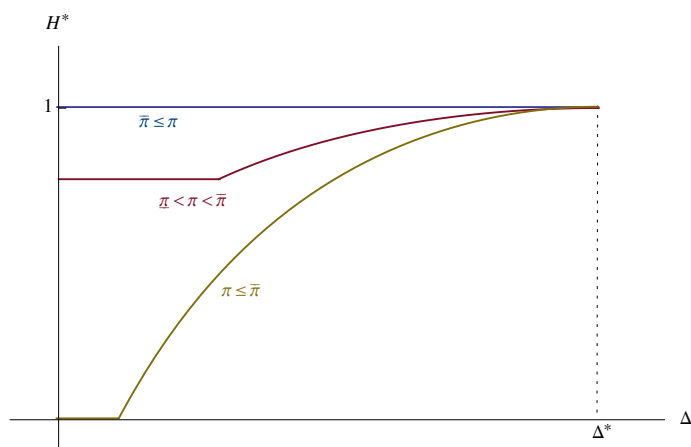


Figure 4: Equilibrium distribution  $H^*$  under imperfect competition

threshold, intermediaries then fully reveal. Figure 4 depicts the equilibrium distribution for the different regimes of  $\pi$ .

We thus find that competition promotes unawareness. In fact we can show that as  $\pi$  increases the equilibrium distribution is shifted towards smaller values of  $\Delta$ .

**Corollary 4.2.** *Let Assumption 1 and 2 be satisfied and let  $0 \leq \pi < \pi' \leq 1$ . In the environment with imperfect competition, the equilibrium distribution  $H^*$  under  $\pi$  first-order stochastically dominates the one under  $\pi'$ .*

*Proof.* See Appendix B.6. □

Summing up, in the proposed environment the disclosure of available investment opportunities is an instrument to compete for customers. An interesting question is how the extent to which investors are left unaware of certain investment opportunities varies over the business cycle. In our framework the state of the economy might be captured by the profitability of investments: when the economy is doing well, financial market investments yield particularly high returns, some of which are appropriated by the financial intermediaries. In our model we thus interpret good times as an upward shift of  $U(\Delta)$  relative to  $\bar{U}$ . We find that

as the gap between  $U(\Delta)$  and  $\bar{U}$  increases, the equilibrium distribution  $H^*$  shifts towards smaller values of  $\Delta$ , illustrated in Figure 5. That is, when the gains from intermediation increase, investors become aware of more and more investment opportunities. Intuitively, when the value of attracting an investor becomes larger, competition for investors increases and this results in smaller awareness gaps. Viceversa, when times are bad and gains from intermediation are small, intermediaries worry less about losing investors to competitors and hence behave more predatory. Our model therefore predicts that in bad times we will observe more banks taking advantage of costumers by hiding certain investment opportunities than in good times.

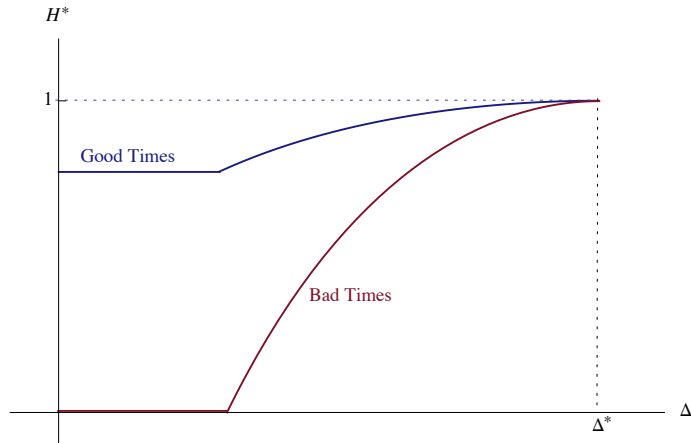


Figure 5: Equilibrium awareness in good and bad times

## 4.1 Heterogenous Investors

Thus far we have assumed that investors are equally sophisticated, in particular that they are all completely unaware.<sup>8</sup> In reality, investors vary widely in their financial literacy, which gives rise to the question of how intermediaries optimally act when investors differ in their

<sup>8</sup>The extension to the case where investors are homogenous but aware of some projects is straight forward. In this case we find the largest feasible value of  $\Delta$  and, if it turns out to be smaller than  $\Delta^*$ , repeat the derivation of the equilibrium above, replacing  $\Delta^*$  with that value.

awareness. Will the presence of more sophisticated investors be beneficial or detrimental to the welfare of the less sophisticated ones?

To address this question in the simplest fashion, we assume that there are two types of investors, sophisticated ones that are aware of all investment opportunities and naive ones that are aware of none. We denote the fraction of naive investors by  $\mu$  and assume that each investor is privately informed about her type. Upon meeting an investor, an intermediary is then confronted with two unknowns. He does not know whether the investor has access to the second intermediary and he does not know whether the investor is sophisticated or naive. If the investor is sophisticated, she delegates the optimal interval under full awareness,  $[0, \hat{y}]$ , no matter how much the intermediary reveals. Nevertheless, she still rewards an intermediary for disclosure by choosing the one that reveals more. If the investor is naive, everything remains as above.

Intermediaries then compete over awareness gaps, parametrized by  $\Delta$ . An intermediary's expected payoff as a function of  $\Delta$  is now given by

$$(1 - \pi H(\Delta))[\mu U(\Delta) + (1 - \mu)U(0)] + \pi H(\Delta)\bar{U}. \quad (4)$$

Following the steps of the equilibrium construction above, we can characterize the equilibrium for this environment and obtain the same properties as described in Proposition 4.1.<sup>9</sup> Of course, the equilibrium distribution  $H^*$  will now depend on  $\mu$ . The following proposition shows that, provided intermediaries can make positive profits with sophisticated investors, an increase in the fraction of sophisticates leads intermediaries to disclose more investment opportunities in equilibrium.

**Proposition 4.3.** *Let Assumption 1 and 2 be satisfied. Assume  $\bar{U} < U(0)$  and let  $0 \leq \mu < \mu' \leq 1$ . In the environment with heterogeneous investors, the equilibrium distribution  $H^*$  under  $\mu$  is first-order stochastically dominated by the distribution under  $\mu'$ .*

<sup>9</sup>For details see the proof of Proposition 4.3.

*Proof.* See Appendix B.7. □

The result in Proposition 4.3 is intuitive. Whenever an intermediary meets a sophisticated investor, revealing additional investment opportunities does not affect the delegation set the investor determines but increases the probability with which the investor delegates to him. The assumption  $U(0) > \bar{U}$  implies that intermediaries make strictly positive profits with sophisticated investors, thus, conditional on meeting a sophisticated investor, it is optimal to fully reveal. By implication, the larger the probability an intermediary attaches to the event of meeting a sophisticated investor is, the more attractive revealing additional projects becomes. The presence of sophisticated investors in the market consequently leads to more disclosure and thereby benefits the naive types.

Suppose now that intermediaries can make positive profits with naive investors but not with sophisticated ones. If intermediaries can reject sophisticated investors, the equilibrium is as if they did not exist and so the previous analysis applies. There are, however, situations where it is reasonable to assume that intermediaries cannot avoid negative profits with certain type of investors. For example, advising and setting up a contract may imply certain opportunity costs. If the investor's type is initially unknown and if the expected profits with sophisticated investors do not compensate the opportunity costs, intermediaries make losses with such investors. As long as these losses are compensated by the profits with other investors, intermediaries may still find it worthwhile to enter the market.

In our framework this situation is captured by the specification  $U(0) < \bar{U} < \mu U(\Delta^*) + (1 - \mu)U(0)$  and the assumption that intermediaries cannot reject any delegation sets. In contrast to the previous case, intermediaries are then no longer interested in attracting sophisticated investors but would rather have them go to competitors. This brings about a new interesting equilibrium property.

**Proposition 4.4.** *Let Assumption 1 and 2 be satisfied. Assume  $U(\Delta) < \bar{U} < \mu U(\Delta^*) + (1 - \mu)U(0)$  and let  $0 \leq \mu < \mu' \leq 1$ . In the environment with heterogeneous investors,*



*the equilibrium distribution  $H^*$  under  $\mu$  first-order stochastically dominates the distribution under  $\mu'$ .*

*Proof.* See Appendix B.7. □

Proposition 4.4 shows that a larger share of sophisticated investors leads to more unawareness among naive investors. Given  $U(0) < \underline{U}$ , revealing all investment opportunities cannot be optimal for intermediaries. Indeed, regardless of how large  $\pi$  is, there is no 'full awareness' equilibrium. Instead, intermediaries randomize across an interval of values of  $\Delta$  bounded away from zero. The losses they make with sophisticates can thereby be compensated with the profits they make with naive investors. The larger the share of sophisticated investors is, the larger this compensation has to be. Hence, as  $\mu$  decreases, the equilibrium distribution shifts towards higher values of  $\Delta$ . The presence of sophisticated investors in the market thus reduces the awareness and, by implication, the welfare of naive investors.

The feature that there is unawareness in equilibrium - no matter how intense competition is - with cross-subsidization across naive and sophisticated investors is reminiscent of the shrouding equilibrium in Gabaix and Laibson (2006). In their work, firms hide costly add-ons, which in equilibrium will be purchased by naive customers only.

## 5 Policy Implications and Conclusion

The paper has implications for bank regulation and brokerage practices. Given that small investors are those more likely to have limited awareness, our results show that banks may have incentives to eliminate investment opportunities with intermediate levels of risk so as to induce investments in risky assets. Of course educating investors about available investment options, thereby expanding  $Y^P$ , benefits investors in our environment. In reality, however, promoting full awareness in that way might not always be feasible or might be very expensive.

The results of our paper suggest that there exists a substantively more effective intervention. We have seen that what determines the final awareness of an investor is not the number of investment products of which she is initially aware but rather how close to the optimal cap under full awareness these products are. In our stylized model, it is sufficient that the investor is aware of  $\hat{y}$  and the intermediary will make him fully aware. This in turn implies that all a regulator must do is promoting awareness of exactly that product, e.g. by issuing and publicly propagating a financial product with the characteristics of  $\hat{y}$ . An intermediary will then find it in his best interest to educate the investor about the remaining investment opportunities. Notably, it is not even crucial that the regulator issues exactly  $\hat{y}$ . As long as the issued product is relatively close, the set of investment opportunities of which the investor remains unaware is very small, as we illustrate in Figure 6. Our findings thus point to an important complementarity between the regulator and private actors, suggesting that a relatively simple policy intervention can lead banks to reveal investment opportunities more suitable to the needs of investors.

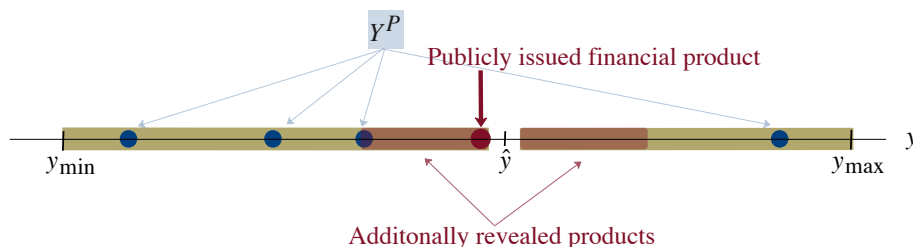


Figure 6: Policy intervention and equilibrium awareness

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## A Microfoundation of Quadratic Loss Preferences

Suppose the investment choice is captured by the classical trade-off between the mean  $\mu$  and the standard deviation  $\sigma$  of the investment and let the mean-standard deviation frontier be determined by:<sup>10</sup>

$$\mu = \gamma(\theta) + \theta\sigma, \tag{5}$$

where  $\theta \in [0, 1]$  is the state of the world. The tradeoff between mean and standard deviation changes stochastically with the state of the world. Investor and intermediary display different aversions to asset volatility. The intermediary's preferences are represented by

$$u(\mu, \sigma) = \mu - \frac{1}{2}\sigma^2,$$

while the investor's preferences are assumed to be:

$$v(\mu, \sigma) = \mu - \beta\sigma - \frac{1}{2}\sigma^2.$$

It is easy to verify that by substituting for  $\mu$  and maximizing the agents' utility functions over the standard variation, we find that the intermediaries preferred policy is  $\sigma = \theta$ , while the investor's is  $\sigma = \theta - \beta$ . The model described here is indeed equivalent to the one we considered in the main part of the paper. We can therefore interpret the choice of  $y$  in the main model as a choice over the riskiness of the investment, where, conditional on the state, the intermediary always prefers a riskier investment product than the agent.

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<sup>10</sup>A simple reference for such restriction is the Capital Allocation Line in the basic model of portfolio selection (Markowitz (1952, 1959), Tobin (1958)). In the general equilibrium version of this model,  $\theta = \frac{s(\theta)}{m(\theta)}$ , where  $s(\theta)$  and  $m(\theta)$  are, respectively, the standard deviation and the mean of the stochastic discount factor. The relevant restriction (the efficient frontier) is linear whenever there is a risk free asset, otherwise the true frontier is concave.

## B Proofs

For the proofs of the following results it is useful to introduce the terms

$$T(y) := F(y) (y - E[\theta - \beta | \theta \leq y]),$$

and

$$S(y) := (1 - F(y)) (y - E[\theta - \beta | \theta \geq y]),$$

in the literature referred to as, respectively, backward bias and forward bias (see Alonso and Matouschek, 2008). By Assumption 1 we have

$$T''(y) = \beta f'(y) + f(y) > 0 \quad \text{and} \quad S''(y) = -(\beta f'(y) + f(y)) < 0 \quad \text{for all } y \in [0, 1]$$

Noticing that  $S(\hat{y}) = S(1) = 0$ , strict concavity of  $S$  implies that  $S(y) > 0$  for all  $y \in (\hat{y}, 1)$ .

### B.1 Proof of Lemma 3.2

Suppose not and let  $y \in \hat{Y}$  be such that  $y \notin D^*(\hat{Y})$  and  $D^*(\hat{Y}) \cap [y_{min}, y] \neq \emptyset$ ,  $D^*(\hat{Y}) \cap [y, y_{max}] \neq \emptyset$ . Further, let  $y^-$  be the largest element of  $D^*(\hat{Y})$  strictly smaller than  $y$  and let  $y^+$  be the smallest element of  $D^*(\hat{Y})$  strictly greater than  $y$ , that is  $y^- = \max\{y' \in D^*(\hat{Y}) : y' < y\}$  and  $y^+ = \min\{y' \in D^*(\hat{Y}) : y' > y\}$ . Define  $s := \frac{y^- + y^+}{2}$  to be the state at which the intermediary is indifferent between choosing project  $y^-$  and project  $y^+$ , and similarly define  $r := \frac{y + y^-}{2}$  and  $t := \frac{y^+ + y}{2}$  be the states in which the intermediary is indifferent, respectively, between choosing  $y^-$  and  $y$  and between  $y^+$  and  $y$ .

Following Alonso and Matouschek (2008), we can write the change in the investor's payoff

when including project  $y$  into the delegation set as follows

$$\begin{aligned}
& \mathbb{E}[v(D^*(\widehat{Y}), \theta)] - \mathbb{E}[v(D^*(\widehat{Y}) \setminus \{y\}, \theta)] \\
&= - \int_r^t (y - \theta + \beta)^2 f(\theta) d\theta + \int_r^s (y^- - \theta + \beta)^2 f(\theta) d\theta + \int_s^t (y^+ - \theta + \beta)^2 f(\theta) d\theta \\
&= (y - y^-) \underbrace{F(r) [r - \mathbb{E}[\theta - \beta | \theta \leq r]]}_{=T(r)} (y^+ - y) \underbrace{F(t) [t - \mathbb{E}[\theta - \beta | \theta \leq t]]}_{=T(t)} \\
&\quad - 2(y^+ - y^-) \underbrace{F(s) [s - \mathbb{E}[\theta - \beta | \theta \leq s]]}_{=T(s)}
\end{aligned}$$

Letting  $y = \lambda y^+ + (1 - \lambda)y^-$  for some  $\lambda \in (0, 1)$  so that  $y - y^- = \lambda(y^+ - y^-)$ ,  $y^+ - y = (1 - \lambda)(y^+ - y^-)$  and  $s = \lambda r + (1 - \lambda)t$ , the payoff difference can be written as

$$\mathbb{E}[v(D^*(\widehat{Y}), \theta)] - \mathbb{E}[v(D^*(\widehat{Y}) \setminus \{y\}, \theta)] = (y^+ - y^-) [\lambda T(r) + (1 - \lambda)T(t) - T(\lambda r + (1 - \lambda)t)],$$

From the strict convexity of  $T$ , it then follows that the payoff difference is strictly positive.

A contradiction.  $\square$

## B.2 Proof of Lemma 3.3

Consider delegation set  $D$  with  $\min D(\widehat{Y}) > \min \widehat{Y}$ . Letting  $y = \min \widehat{Y}$  and  $\underline{y} = \min D(\widehat{y})$ , the state at which the intermediary is indifferent between the two projects is given by  $s := (y + \underline{y})/2$ . If the investor includes  $y$  in the delegation set, the intermediary switches from  $\underline{y}$  to  $y$  in all states  $\theta \leq s$ . The investor's change in expected payoff when including  $y$  is hence given by

$$\begin{aligned}
& - \int_0^s (y - \theta + \beta)^2 f(\theta) d\theta + \int_0^s (\underline{y} - \theta + \beta)^2 f(\theta) d\theta, \\
&= \int_0^s [(\underline{y} - y)(\underline{y} + y) - 2(\underline{y} - y)(\theta - \beta)] f(\theta) d\theta, \\
&= 2(\underline{y} - y)T(s),
\end{aligned}$$

which is strictly positive. Including  $y$  in the delegation set therefore strictly increases the investor's payoff, which implies  $\min D^*(\widehat{Y}) = \min \widehat{Y}$ .  $\square$

### B.3 Proof of Lemma 3.4

Consider delegation set  $D$  and suppose  $\max D < \max \hat{Y}$ . Let  $\bar{y} = \max D$  and consider project  $y > \max D$  such that  $y \in \hat{Y}$ . Let  $t = \frac{y+\bar{y}}{2}$  denote the state at which the intermediary is indifferent between the two projects. When  $t < 1$ , the change in the investor's payoff when including project  $y$  is then given by<sup>11</sup>

$$\begin{aligned} & - \int_t^1 (y - \theta + \beta)^2 f(\theta) d\theta + \int_t^1 (\bar{y} - \theta + \beta)^2 f(\theta) d\theta, \\ = & - \int_t^1 [(y - \bar{y})(y + \bar{y}) - 2(y - \bar{y})(\theta - \beta)] f(\theta) d\theta, \\ = & -2(y - \bar{y})S(t). \end{aligned}$$

This change is weakly positive if  $S(t) \leq 0$ , i.e. if  $t \leq \hat{y}$ . The condition  $t \leq \hat{y}$  is equivalent to

$$\frac{y + \bar{y}}{2} \leq \hat{y} \quad \Leftrightarrow \quad y - \hat{y} \leq \hat{y} - \bar{y}.$$

Since  $y > \hat{y}$ , this condition can only be satisfied if  $\bar{y} < \hat{y}$ . Given this, the inequality is always satisfied when  $y \leq \hat{y}$ . It is also satisfied when  $y > \hat{y}$ , provided that  $d(y, \hat{y}) \leq d(\bar{y}, y)$ . By a perfectly symmetric argument  $-2(y - \bar{y})S(t)$  is strictly negative if  $y - \hat{y} > \hat{y} - \bar{y}$ , which is satisfied if and only if  $d(y, \hat{y}) > d(\bar{y}, \hat{y})$ .  $\square$

### B.4 Proof of Proposition 3.5

Let the intermediary's payoff as a function of  $\Delta$  be defined by

$$U(\Delta) := - \int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 f(\theta) d\theta - \int_{\hat{y}}^1 (\hat{y} + \Delta - \theta)^2 f(\theta) d\theta$$

The first and second derivative of  $U(\Delta)$  are

$$\frac{\partial U(\Delta)}{\partial \Delta} = 2 \int_{\hat{y}-\Delta}^{\hat{y}} [\hat{y} - \Delta - \theta] f(\theta) d\theta - 2 \int_{\hat{y}}^1 [\hat{y} + \Delta - \theta] f(\theta) d\theta, \quad (6)$$

$$\frac{\partial^2 U(\Delta)}{\partial \Delta^2} = -2[1 - F(\hat{y} - \Delta)] < 0 \quad (7)$$

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<sup>11</sup>When  $t \geq 1$  including  $y$  has no effect on the investor's payoff since the intermediary never chooses  $y$ .



$U(\Delta)$  is strictly concave in  $\Delta$  and hence has a unique solution on  $[0, \bar{\Delta}(\hat{Y})]$ . Recalling that  $\hat{y} = E[\theta - \beta | \theta \geq \hat{y}]$ , the first derivative is strictly positive when evaluated at  $\Delta = 0$ :

$$\left. \frac{\partial U(\Delta)}{\partial \Delta} \right|_{\Delta=0} = 2[1 - F(\hat{y})]\beta > 0.$$

The derivative is strictly positive, which implies that whenever  $\bar{\Delta}(Y_P) > 0$ , the optimal value of  $\Delta$  is strictly positive. The interior solution of the intermediary's optimization problem,  $\Delta^*$ , is characterized by the first-order condition that equalizes the expression (6) to zero. After rearranging terms  $\Delta^*$  solves:

$$(1 - F(\hat{y} - \Delta^*)) (E[\theta | \theta \geq \hat{y} - \Delta^*] - (\hat{y} - \Delta^*)) = 2(1 - F(\hat{y}))\beta.$$

## B.5 Proof of Proposition 4.1

Suppose first that both intermediaries choose  $\Delta = 0$  in equilibrium and consider the deviation of one intermediary. Since  $\Delta = 0$  maximizes an investor's payoff, any deviating offer will only be accepted if the investor is captive. The best deviating offer is thus characterized by  $\Delta^*$ . This deviation is not profitable if

$$\pi \bar{U} + (1 - \pi)U(\Delta^*) \leq \left(1 - \frac{1}{2}\pi\right)U(0) + \frac{1}{2}\pi \bar{U}. \quad (8)$$

Letting  $\bar{\pi}$  be the value of  $\pi$  at which (8) holds as equality, it is easy to verify that (8) holds for all  $\pi \geq \bar{\pi}$ .

Suppose now there is a positive probability with which intermediaries choose a non-zero gap. Since  $H$  cannot have a mass point at  $\Delta > 0$ , it follows that there is an interval of values of  $\Delta$  across which intermediaries are indifferent. Differentiation of (3) yields the following first order conditions:

$$\begin{aligned} (1 - \pi H(\Delta))U'(\Delta) &= \pi H'(\Delta)(U(\Delta) - \bar{U}) \quad \text{for } \Delta > 0; \\ (1 - \pi H(\Delta))U'(\Delta) &\geq \pi H'(\Delta)(U(\Delta) - \bar{U}) \quad \text{for } \Delta = 0; \end{aligned}$$

Let  $\hat{H}(\cdot)$  be the solution of the differential equation defined by the first order condition with border condition  $\hat{H}(\Delta^*) = 1$ . We have

$$\hat{H}(\Delta) = \frac{U(\Delta) - [\pi\bar{U} + (1 - \pi)U(\Delta^*)]}{\pi[U(\Delta) - \bar{U}]}.$$

Suppose  $\hat{H}(0) \leq 0$  and consider the following candidate equilibrium distribution:

$$H^*(\Delta) = \begin{cases} 0 & \text{if } \Delta \leq \Delta' \\ \frac{U(\Delta) - [\pi\bar{U} + (1 - \pi)U(\Delta^*)]}{\pi[U(\Delta) - \bar{U}]} & \text{if } \Delta \in (\Delta^{min}, \Delta^*) \\ 1 & \text{if } \Delta \geq \Delta^* \end{cases}$$

with  $\Delta'$  such that  $U(\Delta') = \pi\bar{U} + (1 - \pi)U(\Delta^*)$ . Any value of  $\Delta$  strictly smaller than  $\Delta'$  cannot be optimal as it yields the same trading probability but a lower conditional payoff than  $\Delta'$ . The equilibrium exists if indeed  $\hat{H}(0) \leq 0$ , or equivalently

$$U(0) \leq \pi\bar{U} + (1 - \pi)U(\Delta^*) \quad (9)$$

Letting  $\underline{\pi}$  denote the value of  $\pi$  at which (9) is satisfied with equality, the characterized equilibrium exists if  $\pi \leq \underline{\pi}$ .

Finally, consider an equilibrium where  $H^*$  has a mass point at  $\Delta = 0$ . Notice first that the intermediaries' strategy cannot have a mass point at zero and at the same time positive density arbitrarily close to zero, since  $\Delta = 0$  yields a strictly higher payoff than any  $\Delta$  close to zero. The support of  $H^*(\Delta)$  thus has to have a gap. More precisely, suppose the support of  $H^*(\Delta)$  is given by  $\{0\} \cup [\Delta', \Delta^*]$ , where  $\Delta'$  is such that, given  $H^*(\Delta) = \hat{H}(\Delta)$ ,  $\forall \Delta \geq \Delta'$ , the intermediary is indifferent between offering  $\Delta'$  and  $\Delta = 0$ . That is,

$$\left(1 - \pi\hat{H}(\Delta')\right)U(\Delta') + \pi\hat{H}(\Delta')\bar{U} = \left(1 - \pi\frac{1}{2}\hat{H}(\Delta')\right)U(0) + \pi\frac{1}{2}\hat{H}(\Delta')\bar{U}. \quad (10)$$

The left hand side is equal to  $\pi\bar{U} + (1 - \pi)U(\Delta^*)$  and thus constant in  $\Delta'$ , while the right hand side is strictly decreasing in  $\Delta'$ . At  $\Delta' = 0$  the left hand side is strictly larger than the

right hand side when condition (9) is violated and at  $\Delta' = \Delta^*$  the left hand side is strictly smaller than the right hand side when condition (12) is violated. Hence, whenever neither of the above equilibria exists, condition (10) has a unique solution in  $(0, \Delta^*)$ . We then have

$$H^*(\Delta) = \begin{cases} 0 & \text{if } \Delta < 0 \\ \hat{H}(\Delta') & \text{if } \Delta \in [0, \Delta'] \\ \frac{U(\Delta) - [\pi\bar{U} + (1-\pi)U(\Delta^*)]}{\pi[U(\Delta) - \bar{U}]} & \text{if } \Delta \in (\Delta', \Delta^*) \\ 1 & \text{if } \Delta \geq \Delta^* \end{cases}$$

where  $\Delta'$  is defined by (10).

## B.6 Proof of Proposition 4.2

We can first show that  $\hat{H}(\Delta)$  shifts upwards as  $\pi$  increases. We have indeed:

$$\frac{\partial \hat{H}(\Delta)}{\partial \pi} = \frac{U(\Delta^*) - U(\Delta)}{\pi^2[U(\Delta) - \bar{U}]} > 0 \quad (11)$$

For the case  $\pi \leq \underline{\pi}$ , this immediately yields the stated property. Consider then the case  $\underline{\pi} < \pi < \bar{\pi}$ . Given the above property, it remains to show that the mass point at  $\Delta = 0$  is greater when  $\pi$  is greater. A sufficient condition is that the lower bound of the randomization interval,  $\Delta'$  as determined by (10) is increasing in  $\pi$ . Solving (10) for  $U(\Delta')$  we obtain

$$U(\Delta') = \frac{\frac{1}{2}(\pi\bar{U} + (1-\pi)U(\Delta^*))(U(0) + \bar{U}) - U(0)\bar{U}}{\pi\bar{U} + (1-\pi)U(\Delta^*) - \frac{1}{2}(\bar{U} + U(0))}$$

The first derivative with respect to  $\pi$  is given by

$$\frac{\partial U(\Delta')}{\partial \pi} = \frac{\frac{1}{4}(U(\Delta^*) - \bar{U})(U(0) - \bar{U})^2}{(\pi\bar{U} + (1-\pi)U(\Delta^*) - \frac{1}{2}(\bar{U} + U(0)))^2} > 0$$

Since  $U$  strictly increases in  $\Delta$  on  $[0, \Delta^*]$ , it follows that  $\Delta'$  strictly increases in  $\pi$ . Together with (11) this implies that the statement of Proposition 4.2 is satisfied. Finally, for  $\bar{\pi} \leq \pi$ , a marginal change in  $\pi$  has no effect.

## B.7 Proof of Proposition 4.3

The construction of the equilibrium is analogous to the one in Section B.5. The equilibrium in which both intermediaries choose  $\Delta = 0$  exists if a deviation to  $\Delta = \Delta^*$  is not profitable. This requires

$$\pi\bar{U} + (1 - \pi)[\mu U(\Delta^*) + (1 - \mu)U(0)] \leq \left(1 - \frac{1}{2}\pi\right)U(0) + \frac{1}{2}\pi\bar{U}. \quad (12)$$

When the above condition is not satisfied intermediaries randomize across different values of  $\Delta$ . We can first derive the function  $\hat{H}(\Delta)$ . Differentiating the intermediaries' expected payoff in (4) yields

$$\begin{aligned} (1 - \pi H(\Delta))\mu U'(\Delta) &= \pi H'(\Delta)(\mu U(\Delta) + (1 - \mu)U(0) - \bar{U}) \quad \text{for } \Delta > 0; \\ (1 - \pi H(\Delta))\mu U'(\Delta) &\geq \pi H'(\Delta)(\mu U(\Delta) + (1 - \mu)U(0) - \bar{U}) \quad \text{for } \Delta = 0. \end{aligned}$$

With the condition  $\hat{H}(\Delta^*) = 1$  we obtain

$$\hat{H}(\Delta) = \frac{\mu U(\Delta) - [\pi\bar{U} + (1 - \pi)\mu U(\Delta^*) - \pi(1 - \mu)U(0)]}{\pi[\mu U(\Delta) + (1 - \mu)U(0) - \bar{U}]}.$$

If  $\hat{H}(0) \leq 0$ , or equivalently

$$(1 - \pi)\mu(U(\Delta^*) - U(0)) \geq \pi(U(0) - \bar{U}), \quad (13)$$

there exists an equilibrium with

$$H^*(\Delta) = \begin{cases} 0 & \text{if } \Delta \leq \Delta^{min} \\ \hat{H}(\Delta) & \text{if } \Delta \in (\Delta^{min}, \Delta^*) \\ 1 & \text{if } \Delta \geq \Delta^* \end{cases} \quad (14)$$

where  $\Delta^{min}$  is such that  $\mu U(\Delta^{min}) = [\pi\bar{U} + (1 - \pi)\mu U(\Delta^*) - \pi(1 - \mu)U(0)]$ .

Finally, we consider the case where the equilibrium distribution  $H^*$  with a mass point at  $\Delta = 0$ . As before, let  $\Delta'$  be defined as the value of  $\Delta$  such that, given  $H^*(\Delta) = \hat{H}(\Delta), \forall \Delta \geq$

$\Delta'$ , an intermediary is indifferent between  $\Delta'$  and  $\Delta = 0$ . That is,

$$\left(1 - \pi \hat{H}(\Delta')\right) [\mu U(\Delta') + (1 - \mu)U(0)] + \pi \hat{H}(\Delta') \bar{U} = \left(1 - \pi \frac{1}{2} \hat{H}(\Delta')\right) U(0) + \pi \frac{1}{2} \hat{H}(\Delta') \bar{U}. \quad (15)$$

By the same argument as in Section B.5, condition (15) has a unique solution in  $(0, \Delta^*)$  if and only if neither of the above equilibria exists. Just notice that the left hand side of (15) is equal to  $\pi \bar{U} + (1 - \pi)[\mu U(\Delta^*) + (1 - \mu)U(0)]$ . We then have

$$H^*(\Delta) = \begin{cases} 0 & \text{if } \Delta < 0 \\ \hat{H}(\Delta') & \text{if } \Delta \in [0, \Delta'] \\ \hat{H}(\Delta) & \text{if } \Delta \in (\Delta', \Delta^*) \\ 1 & \text{if } \Delta \geq \Delta^* \end{cases} \quad (16)$$

with  $\Delta'$  defined by (15).

Consider now an increase in  $\mu$ . For values of  $\mu$  sufficiently close to zero condition (12) is always satisfied, so a marginal change of  $\mu$  has no effect on the equilibrium distribution. As  $\mu$  increases condition (12) may be violated and we enter the region where the equilibrium is described by (16). Notice first that  $\Delta'$ , as determined by (15) is decreasing in  $\mu$ . This follows from the fact that the left hand side is shifted upwards as  $\mu$  is increases, whereas the right hand side does not depend on  $\mu$ . The probability that an intermediary chooses  $\Delta = 0$  in equilibrium is thus decreasing in  $\mu$ . Notice further that differentiating  $\hat{H}(\Delta)$  with respect to  $\mu$  we obtain:

$$\frac{\partial \hat{H}(\Delta)}{\partial \mu} = - \frac{(1 - \pi)(U(\Delta^*) - U(\Delta))(U(0) - \bar{U})}{\pi[\mu U(\Delta) + (1 - \mu)U(0) - \bar{U}]^2} \quad (17)$$

The above term is strictly negative for all  $\Delta < \Delta^*$ , which implies that the function  $\hat{H}(\Delta)$  shifts downwards as  $\mu$  increases. Together with the property that the probability with which intermediaries choose  $\Delta = 0$  decreases as  $\mu$  increases this implies that, within the considered parameter region, a higher value of  $\mu$  results in an equilibrium distribution function that first order stochastically dominates the distribution associated to a lower value of  $\mu$ .

Finally, when  $\mu$  increases further, we may enter the parameter region where the equilibrium is characterized by (14). The fact that  $\hat{H}(\Delta)$  shifts downwards when  $\mu$  increases here directly validates the statement of Proposition 4.3.

□

## **B.8 Proof of Proposition 4.4**

Under the assumption  $U(0) < \bar{U}$ , condition (13) is always satisfied, so the equilibrium distribution  $H^*$  is given by (14). We then just need to show that  $\hat{H}$  shifts upwards as  $\mu$  increases. This follows directly from (17).

□