Using Extensions of the Residue Theorem for Improper Integrals Computations With CAS

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Abstract

The computation of improper integrals of the first kind (integrals on unbounded domain) are used in different applications in Engineering (for example in Kynetic Energy, electric potential, probability density functions, Gamma (Γ) and Beta (β) functions, Laplace and Fourier Transforms, Differential Equations, ...). Nowadays, Computer Algebra Systems (CAS) are being used for developing such computations. But in many cases, some CAS lack of the appropriate rules for computing some of these improper integrals.

In a previous talk in ESCO 2016 and a later extension in [1], we introduced new rules for computing improper integrals of the first kind using some results from Advanced Calculus Theories (Residue Theorem, Laplace and Fourier Transforms) aimed to improve CAS capabilities on this topic. In this talk, we develop new rules for computing other types of improper integrals using different applications from extended versions of the Residue Theorem.

The type of improper integrals we will compute are:

1. $\displaystyle\int_0^\infty f(x)\,g(x)\,\mathrm{d}x$; $\displaystyle\int_{-\infty}^0 f(x)\,g(x)\,\mathrm{d}x$ and $\displaystyle\int_{-\infty}^\infty f(x)\,g(x)\,\mathrm{d}x$
   where $g(x)=1$ or $g(x)=\sin(ax)$ or $g(x)=\cos(ax)$ and $f(x)=\dfrac{p(x)}{q(x)}$ with grade of $p(x)$ smaller than grade of $q(x)$ and $q(x)$ with no real roots of order greater than 1.
2. $\displaystyle\int_0^\infty x^\alpha f(x)\,\mathrm{d}x$ where $\alpha\in\mathbb{R}\setminus\mathbb{Z}$ or $-1<\alpha<0$

We will show some examples of such improper integrals that current Cas can not compute. Using extensions of the Residue Theorem in Complex Analysis, we will be able to develop new rules schemes for these improper integrals. These new rules will improve the capabilities of Cas, making them able to compute more improper integrals.

References

1. José L.Galán-García and Gabriel Aguilera-Venegas and María Á. Galán-García and Pedro Rodríguez-Cielos and Iván Atencia-Mc.Killop. Improving CAS capabilities: New rules for computing improper integrals. Applied Mathematics and Computation 316 (2018) 525-540.