Sejong Park (University of Southampton)

Title: Butterfly, Möbius, and Double Burnside algebras of noncyclic finite groups

Abstract: The double Burnside ring B(G,G) of a finite group G is the Grothendieck ring of finite (G,G)-bisets with respect to the tensor product of bisets over G. Many invariants of the group G, such as the (single) Burnside ring B(G) and the character ring R_C(G), are modules over B(G,G). The double Burnside ring B(G,G) and its various subrings appear as crucial ingredients in functorial representation theory, homotopy theory, and the theory of fusion systems. It is known to be semisimple over rationals if and only if G is cyclic, and in this case an explicit isomorphism onto a direct product of full matrix algebras is given by Boltje and Danz (2013). But not much is know beyond that on the explicit algebra structure.

We generalize some techniques of Boltje and Danz for cyclic groups to arbitrary finite groups and as a result obtain an explicit isomorphism of the rational double Burnside algebra of a finite group G into a block triangular matrix algebra when all Sylow subgroups (for all primes) of G are cyclic. Such groups can be characterized as groups G where the Zassenhaus Butterfly lemma gives the meet of two sections (H, K) of G with respect to the subsection relation. Key ingredients are a refinement of the inclusion relation among subgroups of G x G and Möbius inversion over various posets of subgroups.

This is a joint work with Goetz Pfeiffer (Galway).