Application of Huygens' Metasurfaces to the Arbitrary Design of a Leaky-Wave Antenna

Elena Abdo-Sánchez[(1)], Ariel Epstein[(2)], George V. Eleftheriades[(3)]

elenaabdo@ic.uma.es, epsteina@ee.technion.ac.il, gelefth@ece.utoronto.ca

[(1)]Dpto. Ing. Comunicaciones, E.T.S.I. Telecomunicación, Universidad de Málaga, Andalucía Tech, E-29071 Málaga, Spain.

[(2)]Andrew and Erna Viterbi Faculty of Electrical Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel.

[(3)]The Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Toronto, Canada.

Abstract—**Leaky-wave antennas are guiding structures that leak power along their length. Their radiation is mainly characterized by the propagation constant (leakage factor and phase constant) of the traveling wave. In this contribution, a leaky-wave antenna based on parallel-plate waveguide is proposed. Arbitrary control of the leakage factor and the phase constant is achieved by replacing the top plate by an omega-type bianisotropic Huygens' metasurface, which implements the desired field transformation. The theoretical derivation and design methodology are briefly described. Several designs with different pointing angles (phase constants) and leakage rates have been carried out. Electromagnetic simulation results validate the theoretical derivation, highlight the capabilities of the structure and confirm the flexibility in the design parameters.**

I. Introduction

In the recent years, there is an increasing demand on directive antennas with low profile and cost, especially for applications such as automotive radars or satellite communications. Good performance is obtained with phased arrays; however, their feeding networks are complicated, leading to high costs. Leaky-wave antennas (LWAs), on the contrary, have very simple feeding, since they consist on a guiding structure that leaks power while the wave is being propagated along it [1]. Planar LWAs have received an increasing attention lately, especially after the introduction of metamaterials and metasurfaces, which allowed the enhancement of their characteristics (such as the mitigation of the broadside effect) [2]–[5]. In order to obtain a certain radiation pattern, it is necessary to be able to design the propagation constant of the leaky mode, $\gamma = \beta - j\alpha$. The phase constant, β, determines the pointing angle, whereas the leakage factor, α, controls the rate of the power leakage, which sets the amplitude distribution. Independent control of these two parameters is a challenge in the design of LWAs.

Control on the radiation pattern could be achieved if we are able to arbitrarily transform the field inside the guiding structure into the desired radiated field. In this regard, Huygens' metasurfaces have been recently proposed as a powerful tool for arbitrary field manipulation [6]–[8]. They consist of subwavelength electrically- and magnetically-polarizable particles and allow the fulfillment of the required boundary conditions, so that the desired field transformation is achieved. Therefore, by placing a Huygens' metasurface on the top of the guiding structure, the required boundary conditions to transform the guiding mode into the desired leaky mode can be implemented. It has been recently discovered that by employing omega-type bianisotropic metasurfaces (O-BMSs)
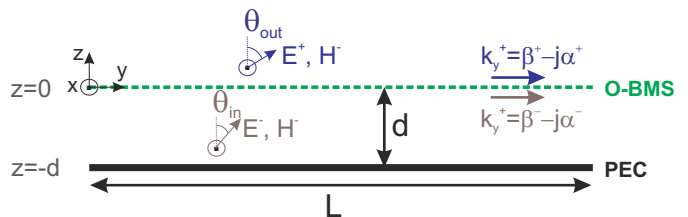

Fig. 1. Transversal section of the proposed LWA configuration, which consists of a parallel-plate waveguide with the top plate being a bianisotropic metasurface (O-BMS).

just one condition in the stipulation of the fields must be met to achieve arbitrary field transformation using passive and lossless particles: local power conservation along the metasurface [9]. This is possible due to the additional (magnetoelectric) degree of freedom provided by the O-BMSs. The fact that only one condition for arbitrary field transformation is required allows the control of the reflection and transmission coefficients from the guiding structure to air through the metasurface. In this contribution, a parallel-plate waveguide which incorporates an O-BMS as the top plate is proposed as a novel LWA with outstanding flexibility of the design parameters and systematic design, as highlighted in [10].

II. Theory

The explored structure (Fig. 1) is a parallel-plate waveguide in which the top plate is replaced by an O-BMS (at $z = 0$). For simplicity, the problem considered here is 2D ($\partial/\partial x = 0$). The O-BMS has a length in the y-coordinate of L and the excitation of the resulting parallel-plate waveguide is located at $y = 0$. d stands for the waveguide height. A transverse electric (TE) polarized field is used as field excitation ($E_y = E_z = H_x = 0$). Then, the transverse field components above (E_x^+ and H_y^+) and below (E_x^- and H_y^-) the O-BMS are related through the bianisotropic sheet transition conditions [11]:

$$\frac{1}{2}(E_x^+ + E_x^-) = -Z_{se}(H_y^+ - H_y^-) - K_{em}(E_x^+ - E_x^-)$$
$$\frac{1}{2}(H_y^+ + H_y^-) = -Y_{sm}(E_x^+ - E_x^-) + K_{em}(H_y^+ - H_y^-)$$

$$(1)$$

where Z_{se} stands for the electric surface impedance, Y_{sm} for the magnetic surface admittance and K_{em} for the magnetoelectric coupling coefficient.

The first step of the theoretical derivation of the problem is to find an expression of the electromagnetic field inside

the waveguide that fulfills Maxwell's equations. Since the metasurface design will force the boundary conditions at $z = 0$ to be met, the only restriction for the field below the O-BMS is to vanish at the PEC ($z = -d$). Therefore, the following electric field has been stipulated:

$$E_x^- = |E_{in}|(e^{jk_z^-(z+d)} - e^{-jk_z^-(z+d)})e^{-jk_y^- y} \qquad (2)$$

where the propagation constants are complex, in order to let the structure radiate:

$$k_y^- = \beta^- - j\alpha^-; \; k_z^- = \beta_z^- - j\alpha_z^-; \; k^{-2} = k_y^{-2} + k_z^{-2}. \qquad (3)$$

The desired field for the region above the metasurface is stipulated as a leaky mode, simply as

$$E_x^+ = |E_{out}|e^{-jk_z^+ z}e^{-jk_y^+ y}, \qquad (4)$$

where

$$k_y^+ = \beta^+ - j\alpha^+; \; k_z^+ = \beta_z^+ - j\alpha_z^+; \; k^{+2} = k_y^{+2} + k_z^{+2}. \qquad (5)$$

We assume constant β and α along y.

It can be demonstrated that for the power conservation condition to be met ($P_z^-(y) = P_z^+(y)$ [9]), the field must have the same decay along y above and below the O-BMS, i.e. $\text{Re}[k_y^+] = \text{Re}[k_y^-] = \alpha$, where α is, indeed, the leakage factor. Moreover, the constant $|E_{out}|$ is given by the rest of the parameters [10].

Once we have expressions to stipulate the fields given desired α and β, we can calculate the metasurface parameters $\{K_{em}, Y_{sm}, Z_{se}\}$ to achieve the required field transformation. The only condition is that α must be the same below and above the metasurface, but the rest of the parameters (β^+, which determines θ_{out}; β^-, which determines θ_{in}; and the waveguide height d) are completely free to be set.

It can be shown that, for constant α, the metasurface parameters $\{K_{em}, Y_{sm}, Z_{se}\}$ result periodic, with a period given by

$$p = \frac{2\pi}{|\beta^+ - \beta^-|}. \qquad (6)$$

III. REALIZATION AND SIMULATION RESULTS

To be able to prove the concept through electromagnetic simulation, the metasurface parameters must be discretized along y (we have used a length of $\lambda_0/6$). To realize the O-BMS being compatible with standard fabrication techniques, we use asymmetric three-layer stack of impedance sheets [9], [12]. In this way, we transform the local $\{K_{em}, Y_{sm}, Z_{se}\}$ into the required Z-matrix for each three-layer unit-cell using (1) and the relations between the tangential fields below and above the metasurface [13]:

$$\begin{pmatrix} E_x^- \\ E_x^+ \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} H_y^- \\ -H_y^+ \end{pmatrix}. \qquad (7)$$

Then, by applying the transmission line model of the three-layer stack of impedance sheets, we transform the matrix $[\mathbf{Z}]$ into the required values of the impedance sheets (which result to be lossless) $\{X_{bot}, X_{mid}, X_{top}\}$ [9].

The resulting structure to simulate using the electromagnetic software $HFSS$ is shown in Fig. 2. The O-BMS consists, as can be observed, of three layers of reactance sheets (which are simulated using impedance boundary conditions)
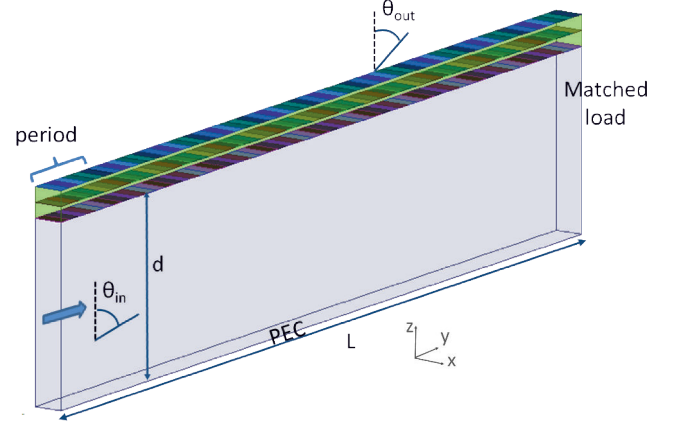

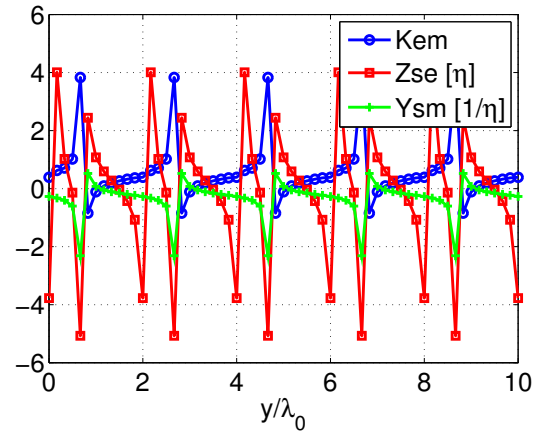Fig. 2. Schematic of the structure to simulate.

Fig. 3. Metasurface parameters for the *Design 1* (with a unit-cell length of $\lambda_0/6$ and a period of $2\lambda_0$).

with certain periodicity. To emulate infinite plates along the x-axis, PEC boundary conditions are used on the faces at the zy-planes. The structure is excited and terminated by waveports.

To illustrate the design procedure and the capabilities of the proposed methodology, a first design ($Design 1$) has been carried out, in which the waveguide height d has been set to $0.6\lambda_0$, the pointing angle θ_{out} has been arbitrarily chosen to $20°$, the period to $2\lambda_0$ (which determines the phase constant inside the waveguide through (6) resulting in $\theta_{in} = 57°$) and α to $0.02k_0$. The length of the metasurface has been set to $10\lambda_0$ in order to radiate 90% of the power with the chosen α. For these parameters, the resulting metasurface parameters to implement are shown in Fig. 3.

The 2D directivity comparison between theory and simulation is plotted in Fig. 4. It can be observed that the structure radiates at the aimed direction ($20°$) and excellent agreement is found between the analytical prediction and the HFSS result.

Fig. 5 shows a comparison of the magnitude of the electric field. Excellent agreement is observed not only in the propagating mode inside the waveguide but also in the radiating field. The exponential power decay along the $y-axis$, which corresponds to a constant α, can be noticed. As can be observed, the propagating mode is gradually leaking power
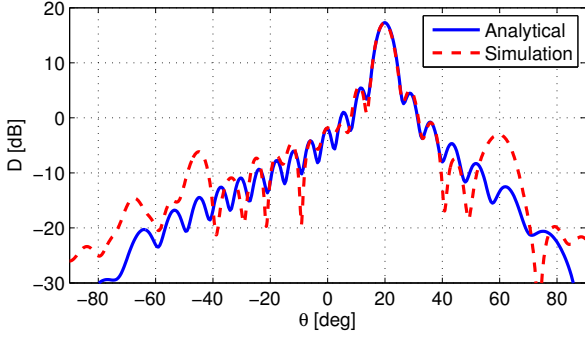
Fig. 4. 2D directivity for the *Design 1* (with $\alpha = 0.02k_0$ and $\theta_{out} = 20°$).

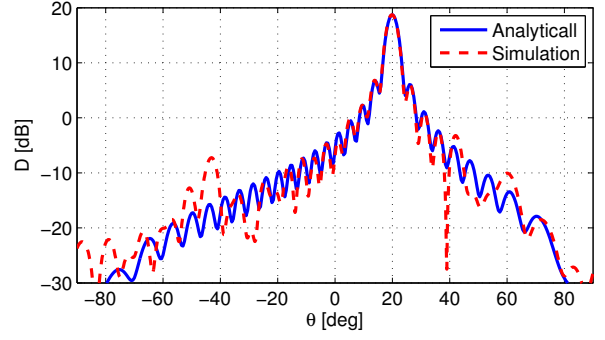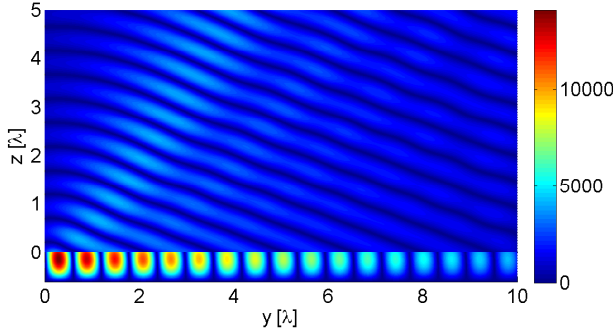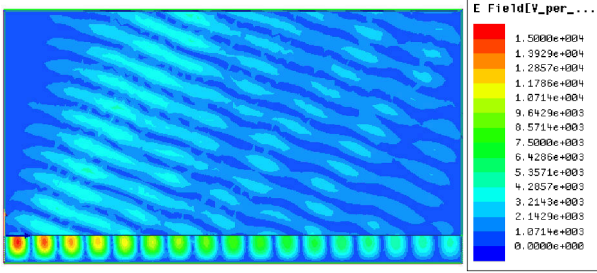Fig. 6. 2D directivity for the *Design 2* (with $\alpha = 0.014k_0$ and $\theta_{out} = 20°$).

(a) Analytical

(a) Analytical

(b) Simulation

(b) Simulation

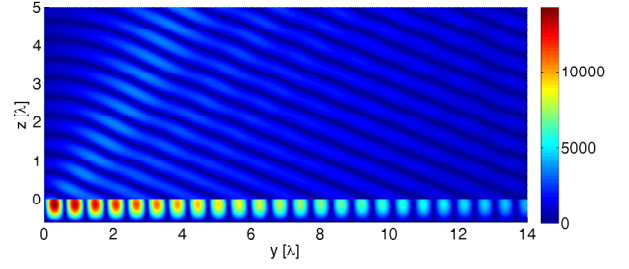Fig. 7. Electric field pattern for the *Design 2* (with $\alpha = 0.014k_0$ and $\theta_{out} = 20°$).

Fig. 5. Electric field pattern for the *Design 1* (with $\alpha = 0.02k_0$ and $\theta_{out} = 20°$).
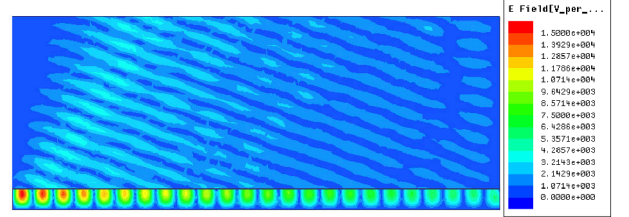
through the metasurface.

In order to illustrate that we can set a desired α, another design (*Design 2*) has been carried out by keeping all the parameters the same as in *Design 1* but reducing α to $0.014k_0$ and increasing the length $L = 14\lambda_0$ to radiate the same amount of power as in *Design 1* but more gradually. Fig. 6 shows the 2D directivity comparison for this case, in which the predicted increase in the directivity and decrease in the beamwidth is observed with respect to *Design 1*, with excellent agreement between theory and simulation. Fig. 7 shows the comparison between theory and simulation of the magnitude of the electric field for the *Design 2*. The field has practically the same pattern as for *Design 1*, since the parameters have been kept the same. The only difference is noticed in the more gradual power leakage, as expected.

The LWA can be also designed to radiate into an arbitrary angle, by choosing the required β^+ ($\beta^+ \approx k_0 \sin(\theta_{out})$). Then, either β^- (θ_{in}) or the period p can be arbitrarily set as well. To illustrate this flexibility, two additional designs with

different θ_{out} have been carried out. In both cases, θ_{in} has been set to $30°$, so p is different in the two cases, according to (6). A more extreme angle has been chosen for *Design 3*, $\theta_{out} = -50°$, whereas broadside radiation will be illustrated with *Design 4*, with resulting periods of $0.8\lambda_0$ and $2\lambda_0$, respectively. Due to the low period for *Design 3* with respect to the unit-cell length of $\lambda_0/6$, some degradation with respect to the analytical results for this case is expected, due to the sampling of the metasurface parameters. In both designs, the length L, waveguide height d, and leakage factor α have been kept to $10\lambda_0$, $0.6\lambda_0$ and $0.02k_0$, respectively.

Fig. 8 shows the 2D directivity comparison between theory and simulation for the *Design 3* and *Design 4*. Some discrepancies in the directivity levels between the analytical and simulation results are found for *Design 3* ($\theta_{out} = -50°$), which are attributed to the low period ($p = 0.8\lambda_0$), as previously mentioned. In fact, this period leads to even less than five different unit-cells per period, which might not be enough to capture the behavior of the continuous metasurface. However, this example was chosen to highlight that the field pattern inside the metasurface can be maintained in designs with different pointing angles by using a different period (see Fig. 9). The pointing angles from the simulations agree with
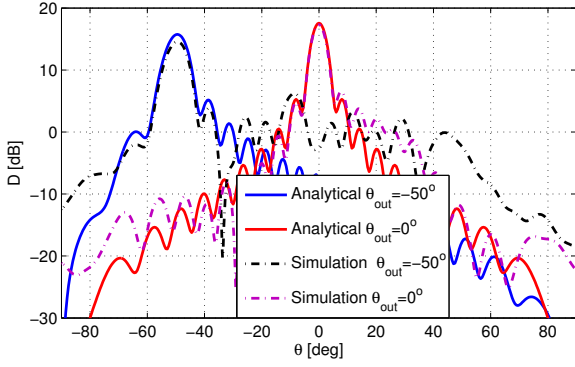
Fig. 8. 2D directivity for *Design 3* and *Design 4* (with $\theta_{out} = -50°$ and $\theta_{out} = 0°$, respectively).

(a) $\theta_{out} = -50°$
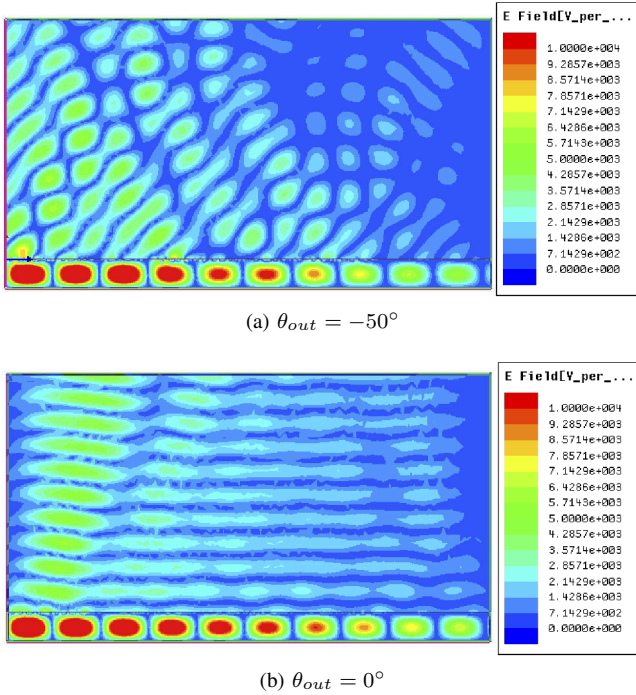

(b) $\theta_{out} = 0°$

Fig. 9. Simulated electric field pattern for the *Design 3* and *Design 4*.

the theoretical prediction. Moreover, it is highlighted that no issue with broadside radiation is found in this structure, unlike traditional LWAs.

IV. CONCLUSIONS

Application of Huygens' metasurface to the design of a LWA has been explored. In order to have enough degrees of freedom to control the leaky mode parameters, omega-type bianisotropy must be introduced into the metasurface. The theoretical derivation to calculate the metasurface parameters for arbitrary leaky mode with constant leakage factor has been shown. Simulation results have been obtained by implementing the O-BMS using three-layer stack of reactance sheets.

The flexibility in the design has been highlighted through four designs. It has been shown that different leakage factors can be set while keeping an arbitrary pointing angle. Moreover, designs with more extreme pointing direction and even broadside radiation have been illustrated. Therefore, independent control of the leakage factor and the pointing direction

has been successfully achieved. The design methodology is very powerful since it allows a systematic design with almost all possible degrees of freedom (constant leakage factor, θ_{in}, θ_{out} and even the waveguide height d).

Further research will focus on extending the derivation and design procedure for a modulated leakage factor to have a better control on the radiation pattern and on getting a physical realization of the metasurface with experimental verification. Moreover, the scanning capabilities of the proposed LWA will be explored.

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