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Effect of Recent Curriculum Studies on the Content of Ninth Grade Algebra Textbooks

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EFFECT OF RECENT CURRICULUM STUDIES

ON THE CONTENT OF NINTH GRADE

ALGEBRA TEXTBOOKS

being

A Thesis Presented to the Graduate Faculty

of the Fort Hays Kansas State College in

Partial Fulfillment of the Requirements for

the Degree of Master of Arts

by

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Fort Hays Kansas State College

Date July 22, 1960 Approved E. O. Laalson

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THESIS ABSTRACT

Mildrexler, D. J. Effect of Recent Curriculum Studies on the Content of Ninth Grade Algebra Textbooks

The specific problem of this thesis was to discover how much new material has been included in certain ninth grade algebra textbooks. The author hoped to find some of this material in every textbook and to show a relationship between the amount of new material in a textbook and its date of publication.

The recommendations of two study groups, The Commission of Mathematics and The School Mathematics Study Group, was the basis for this survey. The topics selected for this survey were chosen because they were left out in older textbooks and, in the opinion of the writer as a mathematics teacher and student, are topics essential to algebra textbook content. They are, in most cases, a combination of the individual topics listed by the two study groups.

The following topics were chosen for this study: (a) teaching students to understand principles, (b) presentation of equations and inequalities, (c) treatment of the nature of number systems, (d) functional relationship, and (e) statistical measures. Several laws, rules, and concepts were investigated to determine their inclusion in textbooks.

There seemed to be very little correlation between the method of presenting material and the date of publication. The recommendations of the study groups as to content seem to have been followed more closely

than the suggestions for method of presentation. This conclusion was drawn because of an apparent positive correlation between inclusion of certain materials and date of publication of the textbooks.

The results of this survey were surprising to the author since he expected to find much more of this material included in the books. There was some relationship found between the publication date and the amount of new material included in the textbooks. It seems that most of the books published after 1957 included much new material and in particular, those published after 1958 included many of the recommendations of the study groups.

In order to facilitate the changes needed to meet the needs of all secondary school students, the content of textbooks must be amplified and reorganized. It is the conclusion of the author that such changes are slow in appearing.

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CHAPTER I

INTRODUCTION

Mathematics is a living, growing subject. The vitality and vigor of present-day mathematical research quickly dispels any notion that mathematics is a subject long since embalmed in textbooks. Mathematics today is in many respects entirely different from what it was at the turn of the century. A large number of concepts have been introduced in recent years.

Prominent mathematicans and leaders in the field of education believe that these new developments should be included in our school programs as soon as possible. Several study groups have been created to report on ways to promote this end. The Commission on Mathematics of the College Entrance Examination Board, The School Mathematics Study Group, and the participants in The University of Maryland Mathematics Project are some groups established for this purpose.

In view of the fact that the textbook is the usual tool for teaching in our schools, and that the pupil's knowledge is often limited by what the textbook contains, it is essential that any new developments and concepts be included in as many textbooks as possible, and as soon as possible.

The study groups have strongly recommended that certain topics be included in the mathematics courses offered at the ninth grade level. In order to include this material in a ninth grade course it is necessary to include some of it in the textbooks.

The specific problem of this thesis was to discover how much new material has been included in certain ninth grade algebra textbooks. The author hoped to find some of this material in every textbook and to show a relation of the amount of new material in a textbook and its date of publication. The textbooks used for this study were thirteen in number and at the present time they are on the approved list of algebra textbooks in Kansas. A list of these textbooks follows:

1. Howard Fehr, Walter Carnahan, and Max Beberman, Algebra Course One, Boston, 1955.
2. Rolland Smith and Francis Lankford, Jr., Algebra One, New York, 1955.
3. Virgil Mallory, First Algebra, Chicago, 1956.
4. A. M. Welchons, W. R. Krickenberger, and Helen Pearson, Algebra Book One, Boston, 1956.
5. Julius Freilich, Simon Berman, and Elsie Johnson, Algebra for Problem Solving, Boston, 1957.
6. Walter Hart, Veryl Schult, and Henry Swain, First Year Algebra, Boston, 1957.
7. John Mayor and Marie Wilcox, Algebra First Course, New York, 1957.
8. Myron White, Elementary Algebra, Boston, 1957.
9. William Gager, Mildred Mahood, Carl Shuster, and Franklin Kokomoor, Functional Mathematics Book One, New York, 1958.
10. E. Justin Hills and Estelle Mazziotta, Algebra Accelerated Book One, Peoria, 1959.
11. N. J. Lennes, J. W. Maucker, and John Kinsella, A First Course in Algebra, New York, 1959.
12. Daymond Aiken, Kenneth Henderson, and Robert Pingry, Algebra: Its Big Ideas and Basic Skills, New York, 1960.
13. William Shute, William Kline, William Shirk, and Leroy Willson, Elementary Algebra, New York, 1960.

These textbooks were listed according to their date of publication. In all tables which follow, reference to these books will be made by the number as indicated above.

The purpose of this study was not to determine which is the best textbook of the thirteen, because there is no one best textbook; but rather to determine the amount of "new mathematics" contained in each text. It was the intention of this writer that the study will be so constructed that it will be of value to teachers, particularly those teaching high school algebra and using any of the thirteen books used in this summary.

The recommendations of two study groups, The Commission on Mathematics and The School Mathematics Study Group, was the basis for this survey.

The Commission on Mathematics suggests that the following material be included in Ninth Grade Algebra:

Notion of a set
Use of symbols
Description and evaluation of expressions
Operational laws
Number scale
Inverse operations
Integers and rational numbers
Variables as place holders
Informal solution of linear equations
Direct and inverse variation
Systems of linear equations and inequalities
Notion of a polynomial
Factoring based on distributive law
Rational expressions and solutions
Informal deduction in algebra
Simple theorems on odd and even integers
Solution of quadratic equations
Informal discussion of rational numbers
Statistical data

Collection and organization of data
Averages
Dispersion
Numerical trigonometry of the right triangle
Ratio and proportion ([14], p. 36)

The following is a list of the material recommended by the School Mathematics Study Group:

Truth sets of open sentences
Graphs of open sentences in two variables
System of equations and inequalities
Quadratic polynomials
Functions
Operational inverses
Statistics
Ratios
Solution of rational equations
Operational laws
Sets
Variation
Trigonometry ([15], pp. vii-xi)

Many of the topics listed by these groups were included in all thirteen textbooks. In order to conform with the purpose of this paper it is necessary to reorganize these materials. The topics selected for this survey were chosen because they were left out in older textbooks and, in the opinion of this writer as a mathematics teacher and student, are topics essential to algebra textbook content. They are, in most cases, a combination of the individual topics listed by the Commission on Mathematics and the School Mathematics Study Group.

The following topics were chosen for this study:

Teaching students to understand principles
Presentation of equations and inequalities
Treatment of the nature of number systems
Functional relationship
Statistical measures

No attempt was made to list these in rank or order of importance, but rather in their relationships with one another.

CHAPTER II

DEVELOPMENT OF FUNDAMENTAL PRINCIPLES

In the author's opinion, the goal of instruction in algebra should not be the development of manipulative skills. Instruction should be oriented toward the development of the properties of a number field. However, the author does not advocate the outright presentation of elementary algebra from an abstract point of view.

It is not, however, an alternative of either skill or understanding, since both should be included in every algebra course. The Commission on Mathematics expressed a need for both skills and concepts when it stated:

> The Commission recommends increased attention to algebra as a part of the secondary school curriculum, but couples it with an equally earnest recommendation that the point of view from which the material is presented be that of contemporary mathematics . . .
>
> The Commission fully realizes the necessity of teaching appropriate manipulative skills. ([14], p. 20)

Manipulative skills are needed, but they must be based on understanding and not merely on rote memorization. Once meaning has been achieved, then drill should be provided to establish and improve these skills.

A student who understands the subject is more likely to solve problems that present an element of novelty than one who lacks this understanding. The ability to solve such problems involves more than the application of rules or techniques to typical problems pre-classified as to form.

Probably the best method of teaching mathematics for understanding is through the use of deductive reasoning. Often, teaching by deductive reasoning is confined to geometry alone. The School Mathematics Study Group suggested that this method be used throughout ninth grade mathematics:

> Important mathematical skills and facts are stressed, but equal attention is paid to the basic concepts and mathematical structures which give meaning to these skills and provide a logical framework for these facts. ([15], p. 6)

A deductive approach, in many situations, can best illustrate the basic mathematical structures.

The author has attempted to rate the textbooks according to how well the methods of deductive reasoning was used in each text. In order to have some basis for this rating an investigation was made into eight different mathematical concepts.

It is the suggestion of the Illinois Committee on School Mathematics that each new principle in mathematics be presented in such a way that the students discover the fundamental law, rule, or concept before actually naming it.

It was with this idea in mind that the eight concepts were studied. The question was asked, "Which textbooks present in some manner, a discovery approach for the students, before specifying the law, rule, or concept?"

The eight topics investigated were: (a) law for multiplication of numbers with exponents, (b) division of radicals, (c) subtraction of polynomials, (d) addition of natural numbers, (e) multiplication of negative numbers, (f) trigonometric functions, (g) properties of proportions, and (h) addition-subtraction method of solving simultaneous equations.

These eight principles were chosen arbitrarily. It is believed that accurate results have been obtained, because they were chosen at random and make good examples. Care was taken that the principles chosen were included in all books in order to have a comparison.

Tables I and II contain the results of this survey. If a particular textbook contained some method of discovering the principle, the tables contain "yes" in the appropriate place. If the principle is merely stated, the tables contain a "no."

TABLE I

TEXTBOOKS WHICH USE INDUCTIVE APPROACH (Part a)

Textbook	Law of Multiplication of Numbers with Exponents	Division of Radicals	Subtraction of Polynomials	Addition of Natural Numbers
1.	yes	yes	no	yes
2.	no	no	no	yes
3.	no	no	no	yes
4.	no	no	no	yes
5.	no	no	no	yes
6.	yes	no	no	no
7.	yes	yes	no	yes
8.	yes	no	no	no

TABLE I (continued)

Textbook	Law of Multiplication of Numbers with Exponents	Division of Radicals	Subtraction of Polynomials	Addition of Natural Numbers
9.	yes	no	no	yes
10.	yes	no	no	no
11.	no	no	yes	yes
12.	no	no	no	yes
13.	no	no	no	yes

TABLE II

TEXTBOOKS WHICH USE INDUCTIVE APPROACH (Part b)

Textbook	Multiplication of Signed Numbers	Trigonometric Functions	Properties of Proportions	Solving Simultaneous Equations
1.	yes	yes	no	yes
2.	yes	yes	no	no
3.	no	no	no	no
4.	yes	no	no	no
5.	no	no	no	yes
6.	yes	no	no	no
7.	yes	no	no	no
8.	yes	no	no	yes
9.	yes	yes	yes	yes
10.	yes	no	no	no

TABLE II (continued)

Textbook	Multiplication of Signed Numbers	Trigonometric Functions	Properties of Proportions	Solving Simultaneous Equations
11.	yes	yes	no	yes
12.	yes	no	yes	no
13.	yes	no	no	no

Table III contains the results of Tables I and II compiled according to the number of textbooks which first present a discovery approach to each principle studied.

TABLE III

NUMBER OF TEXTBOOKS WHICH PRESENT A DISCOVERY APPROACH

Principles Studied	Number of Textbooks
Law for Multiplication of Numbers with Exponents	6
Division of Radicals	2
Subtraction of Polynomials	1
Addition of Natural Numbers	10
Multiplication of Signed Numbers	11
Trigonometric Functions	4
Properties of Proportions	2
Solving Simultaneous Equations	5

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Many good methods of using the discovery approach are incorporated by the authors. The "Do You See," "Discovery Exercises," and "Preparation" features are a few examples:

> The "Do You See" feature applies the discovery method of teaching. Instead of being taught a principle directly, the pupil is led through a series of graded exercises to discover certain generalizations for himself. ([12], p. vi)

> It has been the intention of the authors to make possible for pupils and teacher the discovery approach to the learning of algebra. Some teachers may miss the generalizations and rules, so set out in type in many texts that the pupil can scarcely make an observation of his own. ([7], p. v)

> In this edition, greater use is made of arithmetic to introduce and develop algebraic concepts and operations. Discovery of relationships is sought; sheer "telling" is less frequent. ([11], p. xi)

> The basic ideas and operations of algebra have been thoroughly developed. Each new topic is approached through a series of carefully arranged steps or questions. Following the inductive development comes a rule in boldface type or a definition in italics. ([2], p. iii)

The principles previously studied may seem more inductive than deductive. To examine the textbooks from the standpoint of how much deduction is used another survey was made. The amount of derivation by reasoning incorporated in the development of the number system was the basis for this study.

A rating scale was devised to grade the amount of logical reasoning that was used in: (a) developing the negative integers from the counting numbers (positive integers), (b) going from negative integers to rationals, and (c) from rationals to irrationals. (The complex numbers were not studied in the preparation of this paper.) The rating scale used was: 4--a direct line of informal reasoning, 3--an explanation tying the sets of numbers together, 2--some correlation shown between the numbers, and 1--little or no connection shown. Table IV contains the results of this investigation.

TABLE IV

LOGICAL REASONING USED IN DEVELOPING NUMBER SYSTEMS

Textbook	Counting Numbers to Negative Integers	Negative Integers to Rationals	Rationals to Irrationals
1.	3	2	2
2.	1	2	2
3.	2	1	1
4.	3	1	2
5.	2	1	2
6.	2	1	2
7.	2	1	2
8.	3	3	2
9.	3	2	2
10.	4	3	2
11.	1	1	1
12.	3	1	3
13.	2	1	1

It may be seen from this table that the majority of the authors attempted to use deduction in developing new number systems from known systems.

In reading the textbooks, it was also found that a deductive approach was avoided by some authors, since the material took no logical form. For example, many authors insert two or three chapters on formulas and ratios between the positive integers and the negative integers. This is not necessarily a poor characteristic of the textbooks since in some cases, it promotes easier learning.

It must be remembered that this is not an evaluation of the textbooks, but rather a summary of content. Admittedly, the best manner in which to present material in textbooks is subject to debate.

There is very little connection between the date of publication of the textbooks and the method of presenting the eight topics studied in this chapter. It is felt that authors who have presented an inductive approach in their textbooks were not following any particular suggestions, since many textbooks which were printed before the recommendations of the study groups contained this approach.

CHAPTER III

PRESENTATION OF EQUALITIES AND INEQUALITIES

For many, the ability to solve word problems is the most important immediate outcome of the study of algebra. The student must be able to supply the techniques of algebra to real practical problems to make him functionally competent in his use of algebra.

Since the solution of equations is basic to problem solving in algebra, it is important that this topic be presented in a manner that is precise and clearly understood. The Commission on Mathematics and the School Mathematics Study Group have suggested certain topics which should be covered for this purpose. The understanding of some mathematical terms is essential to mastery of equation solving; but probably most important is the method of presenting the topic.

An idea of the School Mathematics Study Group was investigated to determine the extent of using new methods of presentation. This idea was to present an equation as a mathematical sentence. Although many textbooks make use of this method without actually naming it, it is of great benefit to the student to recognize an equation as a sentence.

Therefore, the number of times the term "sentence" was used in connection with solutions of equations in a textbook was investigated. Table V shows the number of times "sentence" was used in connection with equations.

Those textbooks marked "x" presented equations in terms of mathematical sentences, but did not actually name them sentences. Some books used the term "statement:" "An Equation is a statement that two number expressions are equal." ([1], p. 52) A "statement" was not considered a sentence in Table V.

TABLE V

NUMBER OF TIMES SENTENCE WAS USED
IN CONNECTION WITH EQUATION

Textbook	Number of Appearances
1.	0
2.	0
3.	x
4.	x
5.	1
6.	2
7.	x
8.	0
9.	1
10.	3
11.	0
12.	x
13.	2

Some textbooks incorporated the use of the word "sentence" in other ways. As an example, one author distinguished an equation from a formula by classifying two different "sentences:"

> A formula such as $C = 2\pi r$ is a declarative sentence since C does equal $2\pi r$ no matter what value r may have . . . . Many equations are like interrogative sentences. Thus: $4x + 3 = 11$ is true only for the right value of x . . . . ([6], p. 41)

In another example an author explained the difference between algebraic expressions and equations by use of "sentences."

> The algebraic expressions such as $3x - 2y + z$ are much like headlines in a newspaper. They are descriptive but they do not tell a full story or make a complete statement. In algebra, "sentences" are always statements that two quantities are equal. ([10], p. 21)

Systems of equations which are sometimes troublesome to students are dependent and inconsistent systems of linear equations. Many times the definitions of such systems are not clear, causing the two types to be confused.

One possible method to avoid this confusion is to define both consistent and inconsistent systems together and to define dependent and independent systems together. One might then ask the question: "Are the terms consistent, inconsistent, dependent, and independent defined in pairs in each textbook?" The answer to this question is given in Table VI.

TABLE VI

TEXTBOOKS WHICH INCLUDE DEFINITION OF SYSTEMS OF EQUATIONS

Textbook	Consistent	Inconsistent	Dependent	Independent
1.	yes	yes	no	no
2.	no	yes	yes	no
3.	no	no	no	no

TABLE VI (continued)

Textbook	Consistent	Inconsistent	Dependent	Independent
4.	yes	yes	yes	yes
5.	yes	yes	yes	yes
6.	no	yes	yes	no
7.	yes	yes	yes	no
8.	no	no	no	no
9.	no	no	yes	yes
10.	yes	yes	yes	no
11.	no	yes	no	no
12.	no	yes	no	no
13.	no	yes	yes	no

It is evident from Table VI that the conventional method of defining only inconsistent systems or of defining inconsistent and dependent systems is still being used.

The authors of books (4) and (5) give a definition of each system and these are the only authors who define consistent and inconsistent systems as opposites and also dependent and independent systems as opposites.

It is possible to confuse the definition of indeterminate system of equations and that of an indeterminate equation. Examples of this are found in the following:

> A system of equations whose graphs coincide is indeterminate. Any pair or values of x and y that satisfies one equation satisfies the other also. ([1], p. 208)

A definition of indeterminate equations was given in the following examples:

> It is seen that an equation having two unknowns has a great number of solutions, just as formulas do; hence such equations are called indeterminate equations. ([3], p. 136)

> Indeterminate equation--as equation for which there are an indefinite number of solutions, i.e., no unique solution is determined. ([10], p. 302)

Indeterminate systems of equations are essential to those students in ninth grade mathematics who are planning to take advanced courses in high school and college. Again some authors mention the term and others do not. Table VII shows which textbooks contain a definition of an indeterminate system of equations and of an indeterminate equation.

TABLE VII

TREATMENT OF DEFINITION OF INDETERMINATE EQUATIONS

Textbook	Indeterminate System of Equations	Indeterminate Equation
1.	yes	yes
2.	no	no
3.	yes	no
4.	no	no
5.	no	no
6.	no	no
7.	no	no

TABLE VII (continued)

Textbook	Indeterminate System of Equations	Indeterminate Equation
8.	no	no
9.	yes	no
10.	yes	yes
11.	no	yes
12.	no	no
13.	no	no

It is evident, then, that indeterminate is a word seldom used by authors of high school algebra textbooks. Only the more recently published textbooks contain this term and only two authors actually distinguish between indeterminate systems of equations and indeterminate equations.

Because of the growing need for solution of systems of equations in working with modern computers and in many new fields of applied mathematics the study groups have recommended that more time be devoted to the solution of simultaneous equations. The amount of time a teacher will devote to a subject can be measured by the amount of printed matter which is devoted to a subject. Table VIII contains the number of pages devoted to solution of simultaneous equations by each textbook.

The pages contained in Table VIII are only those pages needed to introduce and explain each method, and do not include pages of exercises or supplementary material.

TABLE VIII

NUMBER OF PAGES DEVOTED TO SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

Textbook	Solving by Substitution	Solving by Addition	Solving by Graphs	Total
1.	2	2	1	5
2.	1	1	2	4
3.	2	1	1	4
4.	1	2	3	6
5.	1	2	2	5
6.	2	2	3	7
7.	3	3	2	8
8.	3	2	2	7
9.	3	2	4	8
10.	2	2	3	7
11.	1	2	4	7
12.	2	3	4	9
13.	6	4	3	13

By comparing the bibliography with Table VIII, it was seen that there is a definite relationship between the number of pages an author devoted to solution of simultaneous equations and the date of publication. Evidently authors have only recently recognized the need for increased explanation of this topic.

The Commission on School Mathematics strongly suggested that a good vocabulary be developed in connection with the solution of equations. The two main ideas to be introduced into the algebra textbooks in connection with this topic were variable as a place holder and concept of identity.

Only one author defined variable as a placeholder. He did so in a supplementary chapter:

> From our work thus far in this section, we can now give a more precise definition for the word, variable, than that on pg. 281. A variable is a placeholder for an element of a set. In your work in algebra, the set usually considered is all the real numbers, and the variable holds a place for an unspecified element of the set. ([13], p. 49)

It is not surprising to find variable defined as a placeholder in only one textbook since only two other textbooks contain any material on sets and it is necessary to use the concept of set to define a variable in this manner.

Identity is usually defined in high school textbooks in this way:

> An identity is an equation which is satisfied by all the values of the literal numbers in it. ([6], p. 144)

But, from the notion of a set a new definition is given:

> When an equation selects all of the numbers of the universal set, we say that it is an equation of identity or, simply, identity. ([13], p. 493)

The idea of identity is a very helpful concept for students to use in solving equations. With this tool students have less trouble in solving equations containing one variable. It was felt, therefore, that authors should define the term identity.

Each textbook was studied to see if it contained a definition for identity. Because of its relationship to identity, the use of the term "conditional equation" was also investigated. Table IX contains the result of this study. In the table, if a textbook contains the definition in question it is marked "yes."

TABLE IX

TEXTBOOKS WHICH CONTAIN DEFINITION OF IDENTICAL AND CONDITIONAL EQUATIONS

Textbook	Definition of Identity	Definition of Conditional Equation
1.	no	no
2.	yes	no
3.	no	no
4.	yes	yes
5.	no	no
6.	yes	no
7.	no	no
8.	yes	yes
9.	no	no
10.	no	yes
11.	yes	no
12.	yes	no
13.	yes	no

The author of textbook (13) was the only one who defined identity in terms of sets. Textbooks (4) and (8) were the ones which contained a definition for both conditional equations and identity. These definitions are related:

> The equation $3(x + 2) = 3x + 6$ is true for all values of x since $3(x + 2)$ and $3x + 6$ are always equal. An equation like this is called an identical equation or an identity . . . .
>
> The condition for $3x + 2$ to equal 14 is that x shall equal 4. An equation such as $3x + 2 = 14$ is called a conditional equation. ([4], p. 186)

Again, a positive correlation between date of publication and inclusion of a definition of conditional equation or identity was noted. With the exception of textbooks (4) and (8), conditional equation was not defined in any textbooks published before 1958; and a definition for identity was not found in any textbook published before 1957 except textbooks (2) and (4).

It is suggested by both the Commission on Mathematics and the School Mathematics Study Group that an introduction to inequalities be included in mathematics on the ninth grade level. A study of inequalities was made and included here.

Two of the thirteen textbooks, (10) and (12), contained material on inequalities and presented graphical examples. Textbook (10) not only gave an explanation and a graphical interpretation, but also included operations on inequalities and solution of "inequations." A clear definition of inequations was given by textbook (12): "A sentence which contains one or more variables and an inequality symbol is called an inequation." ([12], p. 188)

Authors of both textbooks evidently felt that inequalities were only for the gifted student since the presentation of this topic was included in supplementary sections rather than in the regular sections.

It is evident, then, that inequalities have been treated very little by any authors and only the very latest textbooks contain any treatment of them. It was felt that this recent inclusion of inequalities is a direct result of the suggestions of the study groups.

CHAPTER IV

THE FUNDAMENTAL LAWS AND THE NUMBER SYSTEM

A study of the nature of the number systems is part of the new material recommended by the Commission of Mathematics and the School Mathematics Study Group. The Commission particularly emphasized this:

> The new emphasis in the study of algebra is upon the understanding of the fundamental ideas and concepts of the subject, such as the nature of number systems and the basic laws for addition and multiplication. ([14], p. 21)

Topics such as group, ring, integral domain, and field were not included in the textbooks studied. Therefore, this chapter will be devoted to the operations of the real number system and the basic laws for performing these operations.

In order to develop the properties and to gain an understanding of any number system, it is necessary for students to know and apply the three basic laws: commutative, associative, and distributive. These laws should not only be defined, but should be applied to different examples within the system, such as integers, fractions, negative numbers, and irrational numbers.

In surveying the textbooks for any effect of the curriculum studies, it was important to keep the application of these laws in mind. The question was asked: "Do the authors present any information about the commutative law for addition, commutative law for multiplication, the associative law for addition, the associative law for multiplication, and the distributive law of multiplication with respect to addition?" The answer to this question is found in Table X. If a textbook contained information about the law a "yes" was placed in the space.

TABLE X

TEXTBOOKS WHICH CONTAIN A DISCUSSION OF OPERATIONAL LAWS (PART a)

Textbook	Associative Law of Addition	Associative Law of Multiplication	Commutative Law of Addition
1.	no	no	no
2.	yes	yes	yes
3.	no	no	yes*
4.	no	no	no
5.	no	no	no
6.	yes	yes	yes
7.	no	no	yes
8.	no	no	no
9.	yes	yes	yes
10.	no	no	no
11.	no	no	no
12.	yes	yes	yes
13.	no	no	no

TABLE XI

TEXTBOOKS WHICH CONTAIN A DISCUSSION OF OPERATIONAL LAWS (PART b)

Textbook	Commutative Law of Multiplication	Distributive Law
1.	no	yes*
2.	yes	yes
3.	yes*	no
4.	no	no
5.	no	yes
6.	yes	yes
7.	no	no
8.	no	no
9.	yes	yes
10.	no	no
11.	no	no
12.	yes	yes*
13.	no	no

Those answers marked "*" did contain laws similar to the ones in question, but were given a different name. Some textbooks contained an explanation of the use of the laws, but in no way were the laws defined or named. As an example: "Multiply 7 a^2b by 5 ab^2. Here we have 7 $\cdot$ $a^2 \cdot b \cdot 5 \cdot a \cdot b^2$. There are six numbers to be multiplied. We rearrange them thus: $7 \cdot 5 \cdot a^2 \cdot a \cdot b \cdot b^2 = 35\ a^3b^3$." ([1], p. 133) Examples of this kind were given a "no" rating.

As indicated in Table X and XI, four authors included the fundamental laws. These four textbooks were investigated further to determine if the laws were used in the development of fraction, signed numbers, and irrational numbers. Table XII contains the results of this study.

TABLE XII

FUNDAMENTAL LAWS EXTENDED TO OTHER NUMBERS

Textbook	Fractions	Signed Numbers	Irrational Numbers
2.	no	no	yes
6.	yes	yes	yes
9.	yes	yes	no
12.	no	yes	no

It seems that only one of these textbooks included the laws of operation for all numbers and the author of this text cleverly organized these laws for the students to better understand them through association:

> The real numbers of mathematics include zero, the positive and negative integers and fractions, and other numbers called irrational numbers. The laws for operating with them start with basic laws that are assumed to be true. These basic laws are:
>
> 1. The commutative law for addition.
> $a + b = b + a$
> 2. The associative law for addition.
> $(a + b) + c = a + (b + c)$
> 3. The commutative law for multiplication.
> $ab = ba$
> 4. The associative law for multiplication.
> $(ab)c = a(bc)$

5. The distributive law for multiplication.
$$a(b + c) = ab + ac$$
6. The law of symmetry.
If $a = b$, then $b = a$
7. The law of transitivity.
If $a = b$ and $b = c$, then $a = c$ ([6], p. 358)

Only textbook (6) contained the law of symmetry and the law of transitivity.

Evidently, the laws of operation on numbers have been included in some textbooks. There seemed to be no relationship between the inclusion of these laws and the date of publication. It is believed that this is due to the fact that the importance of these laws has been recognized by most authors for some time.

Another important topic in the development of the number system, suggested by the study groups, was the inverse operation. These groups felt that subtraction and division must be defined as inverse operations of addition and multiplication, respectively, to develop a clear understanding of the number system. Many authors do not subscribe to this idea as is shown in Table XIII. Those textbooks whose authors presented inverse operations are marked "yes" and those who did not are marked "no".

Only a few textbooks did not contain any discussion of the concept of operational inverses. It is felt that most authors did include this idea, not because of the suggestions of the mathematical study groups, but merely because it is an effective aid to teaching.

The additive inverses should be included in order for students to clearly understand subtraction and later to give more meaning to negative

TABLE XIII

TEXTBOOKS WHICH INCLUDE OPERATIONAL INVERSES

Textbook	Subtraction	Division
1.	yes	yes
2.	yes	yes
3.	no	no
4.	no	no
5.	yes	yes
6.	yes	yes
7.	yes	yes
8.	no	no
9.	yes	yes
10.	no	no
11.	no	no
12.	yes	yes
13.	no	no

numbers. Some authors use other methods of trying to obtain this end: "In algebra each of the signs + and - serve a double purpose and may be used either as a sign of operation or as a sign of quality." ([13], p. 58)

Other authors clearly identify the additive inverse: "In addition, when operations such as $4 + (-4) = 0$ or $-5 + 5 = 0$ are performed, then -4 is said to be the additive inverse or 4 and 5 the additive inverse of -5 and so on. ([12], p. 77)

Little connection between the inclusion of inverses and date of publication was found. It seems that the suggestions of the study groups had little bearing on the presentation of inverses by authors of the thirteen books.

From the result of this study of fundamental laws and the real number system it is clear that in the minds of some authors, it is questionable whether or not to include inverse and a study of the real number system. It is necessary, therefore, that authors understand the reasons for wanting to include this material before it will be included in textbooks.

CHAPTER V

FUNCTIONAL RELATIONSHIPS

The core of the proposed work in advanced mathematics, as given by the Commission on Mathematics, consists of a modification of the traditional advanced algebra course with stress placed upon some of the more advanced functions. Therefore, it is felt that the concept of function should be included in a ninth grade course.

The two study groups recommended that "function" be defined first, then that the concept be extended to include linear, quadratic, exponential, logarithmic, and trigonometric functions.

Many definitions of a function have been found, including the first modern definition of Dirichlet (1805-1859): "f(x) is a real function of a real variable x if, to every real number x, there corresponds a real number f(x)." ([16], p. 22) A more recent definition was given by the Commission on Mathematics: "It is desirable to define a function as a set or ordered pairs." ([16], p. 23)

In reading the textbooks used in this survey, it was found that only one author examined a definition involving the concept of set when the author stated:

> Any given set of ordered pairs of numbers such that for every first number in a pair there is one and only one second number called a function. A function, thus, is a set of ordered pairs of numbers. ([12], p. 151)

The first part of this study was to determine which textbooks contained any kind of definition of function. Table XIV contains the

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results. If a textbook contains a definition of function it is marked with a "yes". If it does not it is marked with a "no".

TABLE XIV

TEXTBOOKS WHICH CONTAIN A DEFINITION OF FUNCTION

Textbook	Contain Definition of Function
1.	no
2.	no
3.	yes
4.	yes
5.	yes
6.	yes
7.	yes
8.	yes
9.	yes
10.	no
11.	yes
12.	yes
13.	yes

Three of the textbooks listed in Table XIV, (1), (2), and (10), contained no mention of function what-so-ever. Five textbooks, (3), (5), (6), (7), and (13), merely gave a definition, but made no further comment. One of these fine descriptions is very concise: "In the formula d = rt, instead of saying d depends on t we may say d is a function of t." ([3], p. 323) Another example is almost as concise: "In c = 80 n, for each value of n, there is a definite value of c, determined by c = 80 n. We say: 'c is a function of n'." ([6], p. 322) The other three of these five books contained similar examples.

An investigation was then made of the extent to which the concept of function was developed in the remaining five textbooks (4), (8), (9), (11), and (12). To adhere to the purpose of this paper, the use of the suggestions of the Committee on Mathematics were investigated. The committee suggested that the following topics be presented from the standpoint of functional relationship: (a) linear equations, (b) quadratic equations, (c) exponents, (d) logarithms, and (e) trigonometry. Table XV contains the results of this study. If an author of a textbook used functional relationship to present the topics, it was marked "yes". If not, it was marked "no".

TABLE XV

TOPICS PRESENTED BY FUNCTIONAL RELATIONSHIP

Textbook	Linear	Quadratic	Exponential	Logarithmic	Trigonometry
4.	yes	yes	no	no	yes
8.	yes	yes	no	no	yes
9.	yes	yes	no	no	yes
11.	yes	yes	no	no	yes
12.	yes	yes	yes	no	yes

It was found that no textbook contained logarithms of any kind and most authors seem to agree that logarithms should be presented in a higher course in mathematics.

All five textbooks used in Table XV defined trigonometry in terms of a function. One case is cited as an example: "Since the values of the trigonometric ratios--sine, cosine, and tangent--depend on the size of an angle, they are called trigonometric functions." ([12], p. 411) A clear definition of a quadritic function was found:

> If the equation consists of three terms, one containing the square of the unknown, one containing the first power of the unknown, and the third not containing the unknown at all, the equation is said to be a complete quadratic equation and the algebraic sum of the three terms is said to be a function of the unknown. ([8], p. 274)

The Commission also recommends that the concept of a function be presented in four ways: verbal statements, equations, tables, and graphs.

Two textbooks carried out this recommendation. Textbook (12) presented them in the ways suggested by the Commission, while textbook (8) reversed the order of using a table and a graph. The Commission, also, suggested that these four ways of expressing functions be pointed out to the students. Only textbook (12) followed this suggestion as indicated by the following quotation: "You have now studied four ways of stating the relationship between two variables: (1) a verbal statement, (2) a formula or equation, (3) a table of values, (4) a graph. ([12], p. 202)

It was evident that five of the authors intended to carry out the suggestions of the study groups in presenting the concept of function.

Two of the authors carried the suggestions more closely than the rest. One author points out explicitly that the recommendations of the Commission were followed in his presentation:

> Algebra: Its Big Ideas and Basic Skills, Book I, Modern Mathematics Edition, is modern in content and modern in teaching method. We have been guided by the publications of the Commission on Mathematics of the College Entrance Examination Board. ([12], p. v)

Again, it was found that the materials recommended by the study groups were included in the more recent textbooks indicating that authors are being influenced by the work of these groups.

CHAPTER VI

STATISTICS

Just as mathematics deals with situations in which a fact can be determined, it should also provide ways to study and control uncertainty. Many of the newer applications of mathematics use the theories of probability and statistical reasoning. It is essential, therefore, that some form of statistics be presented at the ninth grade level. The Commission on Mathematics agrees with this philosophy:

> The Commission believes that it is desirable that material in these areas be introduced into the high school curriculum. Statistical thinking is playing more and more a part in the daily lives of educated men and women. An introduction to statistical thinking is an important supplement to an introduction to deductive thinking. This introduction may well begin in the ninth grade or earlier with a unit on descriptive statistics. ([14], p. 20)

It was first necessary to discover which textbooks contained any statistics whatever. It was found that only four textbooks, (5), (9), (10), and (11) contained any statistics and only two of these, (5) and (11), merely mentioned the subject.

In accordance with the suggestions of the study groups, the use of these terms were investigaged: mean, median, mode, and range. Out of the thirteen textbooks studied, two of them, (9) and (10), defined these four basic concepts of statistics. These textbooks contained complete chapters on the subject of statistics. Textbook (10) contains a discussion of measures of dispersion:

> The average deviation of the mean--The difference between each item and the mean of the group of items is called its deviation.

> The standard deviation--When you wish a more reliable measure of dispersion, square the deviations from the mean, giving more weight to extreme deviations and the statistic obtained is called the standard deviation. ([10], p. 269)

Although textbook (9) did not define deviation; it did give a clear definition of statistics which is worth repeating: "You might say that statistics are classified numerical data." ([9], p. 34)

The Commission on Mathematics, also, suggests that some presentation of frequency tables or graphs be presented on the ninth grade level. Only textbook (9) and (10) contained such material in their chapters on statistics.

Textbook (10), in its twenty-nine pages devoted to statistical measures contained twenty-three frequency tables, while textbook (9) contained twenty-four frequency tables in its thirty-four pages of material.

A discussion of statistics was not included in all thirteen textbooks. Those which do contain some statistics do not contain as much as is recommended by the study groups. The author believes that only recently have authors understood the need for a study of statistics in the ninth grade. The influence of the study groups is shown by the fact that the two textbooks which do contain a large amount of statistics were published in 1958 and 1959.

CHAPTER VII

SUMMARY

This survey was a summary of content and not an evaluation. While no conclusion was drawn as to which textbook contained the greatest total amount of material suggested by the study groups, the appearance of individual topics was checked thoroughly.

In many cases, it seems that authors have avoided presenting the material of the study groups because of the problem of relating the new material to the older material. It was expected that textbooks which were published before 1958 would not contain many of the suggestions of the study groups, since many study groups were not organized until after 1957.

There seemed to be very little correlation between the <u>method</u> of presenting material and the date of publication. The study shows that in general the authors of the thirteen textbooks do not incorporate the suggestions of the study groups in the method of presentation. For example, the method of defining systems of equations has not changed in the thirteen books during the past five years and although the laws of operating with numbers were included in only a few textbooks, they are included in as many older textbooks as new ones. Other examples of low correlation were also found in the presentation of operational inverses and number systems.

Only the more recently published textbooks contained a reasonable amount of new material. The recommendations of the study groups as to content seem to have been followed more closely than the suggestions for method of presentation. This conclusion was drawn because of an apparent positive correlation between inclusion of certain materials and date of publication of the textbooks. This was exemplified by the inclusion of such concepts as sets, place holders, sentences, identities, statistics, function, and simultaneous equations.

The results of this survey were surprising to the author since he expected to find much more of this material included in the books. There was some relationship found between the publication date and the amount of new material included in the textbooks. It seems that most of the books published after 1957 included much new material and in particular, those published after 1958 included many of the recommendations of the study groups.

There is a definite concern by the public about the program of secondary school mathematical instruction. It is important to recognize that such a general concern exists and this concern supports the argument that the time is ripe for the improvement of the high school curriculum in mathematics.

In order that the school curriculum meet the needs of all secondary school students, and in particular the needs of those who are interested in pure mathematics or the applications of mathematics, there must be a change in course content and presentation. In order to facilitate these changes, the content of textbooks must be amplified and reorganized. It is the conclusion of the author that such changes are slow in appearing.

1. [illegible]

2. [illegible]

3. [illegible]

4. [illegible]

5. [illegible]

6. [illegible]

7. [illegible]

8. [illegible]

9. [illegible]

10. [illegible]

11. [illegible]

12. [illegible]

13. [illegible]

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