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Elements of Difficulty in Arithmetic Experienced By High School Mathematics Students

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ELEMENTS OF DIFFICULTY IN ARITHMETIC EXPERIENCED BY HIGH SCHOOL MATHEMATICS STUDENTS

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INTRODUCTION

Society has become exceedingly complex and technical with the passing of time. Daily problems bring arithmetic into the lives of each individual. Buying, selling, and trading involve the distribution of money and articles; thereby bringing into use the basic functions of computation. Consequently, each man, woman, and child should have a well-grounded knowledge of the basic skills and concepts of arithmetic.

REVIEW OF THE PROBLEM

Many pupils experience a more-than-ordinary difficulty with the fundamental processes of arithmetic. Not only is the slow student confused by the subject material at hand, but many times the average and above-average students are "lost" when certain problems are presented for discussion. Since it is imparative than an understanding of the basic skills of arithmetic be imparted to all pupils, the difficulties must be discovered and conclusions drawn to aid the teacher in the process of instruction.

These statements lead directly to the problem of this thesis, which is entitled:

<u>Elements Of Difficulty In Arithmetic Experienced By High School Mathematics Students</u>

The survey method of research was used to secure data for the investigation of the problem. The "Hundred-Problem Arithmetic Test", a standardized test developed by Raleigh Schorling, John R. Clark, and Mary A. Porter, was administered to the students concerned. This test contains one hundred items arranged in five sections--Addition (10 items), Subtraction (10 items), Multiplication (15 items), Division (15 items), and Fractions, Decimals, and Per Cents (50 items).

The test was administered to approximately four hundred high school mathematics students in eight different high schools located in the southwestern section of Kansas. The tests were graded and the data for the study was taken from the test results thus obtained.

REVIEW OF RESULTS

The response to each test item was analyzed by the writer to determine the errors evident in computation. This analysis discovered the following errors commonly made by the students: (1) failure to carry the proper number to the preceding column in the addition process; (2) borrowing incorrectly in the subtraction process; (3) difficulty multiplying by zero when the zero digit is placed within a three or more digit number; (4) failure to include the zero digit in the difision problem answer; (5) failure to invert when necessary to solve a fractional division problem; (6) inverting the wrong member of a fractional division problem; (7) omit placing a decimal in the problem answer; (8) failure to place the decimal in its correct place in the answer; (9) failure to add the zero digit to the answer after moving the decimal point to the right; (10) improper movement of the decimal when changing a decimal value to a per cent value; (11) poor conception of the relative magnitude of decimal values; (12) difficulty obtaining a common denominator for fractional values; (13) failure to change the numerator when changing fractions to a common denominator; (14) difficulty changing a mixed number to an improper fraction; (15) difficulty changing fractions to a decimal value especially when the denominator does not divide into one hundred evenly; (16) incorrect borrowing from a whole number and adding it to the fractional value of a mixed number in the subtraction process; (17) faulty or lack of understanding of the procedure of moving the decimal point when changing a decimal value to per cent; (18) students are unsure of the procedure of computation necessary for finding a fractional part of one per cent of a quanity; (19) students are unsure of the procedure for finding what per cent of one number is equal to another specified number; (20) students are unsure of the procedure for solving problems which state a given number is equal to what per cent of another specific value; and (21) many students do not know that per cent and hundredths are analogous.

CONCLUSIONS

The foregoing errors are numerous, but an analysis of the errors leads the writer to the following general conclusions; (1) the average student has little difficulty with the computation of whole numbers involved in the various arithmetic processes; (2) a functional knowledge of the decimal system is lacking in the skills attained by high school mathematics students; (3) students have difficulty using fractions in the various processes of arithmetic; (4) many students do not understand the function of per cent and are unable to make use of the concept of per cent; and (5) a certain amount of carelessness is apparent in computation by high school students.

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CHAPTER I

INTRODUCTION

Generally speaking there is a fundamental belief that arithmetic is a necessity for everybody, as society has become exceedingly complex. Consequently, a basic knowledge of arithmetic is essential for every person in the pursuit of happiness. Thus the need for computational skills is of major importance.

Daily problems bring arithmetic into the lives of practically all individuals. Buying, selling, and trading involve the distribution of money and commodities thus bringing into use the basic functions of computation. Consequently, all should have a well-grounded knowledge of the basic skills and concepts of arithmetic.

Civilization has become complex with many changes in our social institutions. Thus, we have come to consider the mathematical ability of our youth as never before. The years covering World War II made increasing demands upon science and mathematics. During this period accuracy carried great significance. Methods of teaching have also changed. Thorndyke pointed out that, "The newer methods emphasize the processes which life will require and the problems which life will offer."[1]

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[1] Edward L. Thorndyke, Thorndyke, The New Methods In Arithmetic (New York: Rand Mc Nally and Co., 1911), p. 1.

The Problem and Its Importance

It has been the writer's experience as a teacher in the secondary schools of Kansas, that many pupils experience a more-than-ordinary difficulty with the fundamental processes of arithmetic. Not only is the slow student confused by the subject material at hand but many times the average and above-average students are "lost" when certain problems are presented to them for discussion and solution.

As further evidence of difficulties encountered by pupils in solving the various problems of arithmetic, teachers continuously remind people of pupil weakness in solving arithmetic problems. Teaching high school mathematics leads to reflection and the belief that such pupil weakness is more evident than is generally realized. Out of experience and reflection the writer determined to explore the extent of this alledged weakness on the part of high school pupils in arithmetic. Since we have a complex society, it is imparative that an understanding of the basic skills of arithmetic be well taught to all pupils. The difficulties need to be isolated and valid assumptions made to be of help to the teacher in the process of arithmetic instruction.

The preceding statements lead directly to the problem of this thesis, which is to find the computational errors in arithmetic as experienced by high school mathematics students.

Limitations on the Problem

Research problems may be projected in several different directions. However, most of them need to be limited in scope otherwise there results an ever broadening area of investigation. This thesis is no exception in its coverage of certain aspects of arithmetic difficulties.

The problem is confined to difficulties of arithmetic as found in the computational process by using a test. All test items (with the exception of the last eight examples of the test) are of a nature that eliminates the verbal factor and concentrates strictly on the difficulties resulting in computation. The study is further limited in that only pupils in the ninth, tenth, eleventh, and twelfth years of the public schools and currently enrolled in a mathematics course offered by their respective high school were tested for data pertinent to the study. A mathematics course is defined as general mathematics, algebra, geometry and any other course which is classified as being under the direction of the mathematics department and for which credit in the field of mathematics is given.

A sampling of the student population of the participating schools was used. Since the total participating school population was approximately one thousand one hundred students, only students enrolled in a mathematics course (approximately four hundred students) were tested.

The Method

The survey method of research was used to secure data for the investigation. The "Hundred-Problem Arithmetic Test", a standardized test developed by Raleigh Schorling, University of Michigan, John R. Clark, Columbia University, and Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin was administered to the students concerned.

This test contains one hundred items arranged in five sections--Addition (10 items), Subtraction (10 items), Multiplication (15 items), Division (15 items), and Fractions, Decimals, and Per Cents (50 items). The test items are grouped by process (i.e. addition only in one section, subtraction in another and etc.) to remove the necessity for pupils having to shift back and forth from one mental process to another.

The test is designed primarily for Grades 7 to 12 inclusive. While the test is not a pure measure of numerical facility, it is so constructed that it minimizes other factors such as verbal facility, and consequently does give a practical measure of the computational skills which experts agree are basic.

The test was administered to approximately four hundred high school mathematics students in eight different high schools located in the southwestern section of Kansas. The tests were scored, and the data for the study was taken from the test results thus obtained.

Definition of Terms

Frequently a thesis problem carries certain terms and limitations. These need to be pointed out and defined to promote understanding of the nature of the problem and eliminate possible misunderstanding on the part of those who may read the report of the investigation. The following are terms and limitations that need to be defined: (1) the term arithmetic refers to the art of computation by the use of real numbers; (2) mathematics is a general term used when referring to the broad field of computation with special emphasis being placed upon those subjects, in the high school curriculum, (i.e. general mathematics, algebra, geometry, and trigonometry); (3) mean is that score obtained by adding all pertinent values and dividing the total by the number of values added. It is the same as average; (4) median designates a middle point in a series which allows half of the scores on one side and half on the other side of the midpoint; (5) mode is the term used to designate the item in a series of statistical data which occurs most often.

Related Research

Much research has been done pretaining to the fundamental operations of arithmetic. Arithmetic not only makes up a large part of the elementary school arithmetic curricula, especially its basic functions, but it is likewise the foundation for all other development in the field of mathematics.

A very large amount of research in arithmetic has been on the elementary school level. Many of these studies have been directed toward methods, vocabulary, and special phases of arithmetic, but a scarcity exists in actual computational ability. At this point attention is directed to certain research in arithmetic which has relationship or bearing on the problem with which this thesis is concerned.

(1) Van Engen of Iowa State Teachers College, in 1948, made a study of research on the learning of arithmetic. Evidence that teaching is gradually changing its conception of what constitutes problem solving in arithmetic was some of the interesting evidence that he found. The so-called problems found in the textbook are now, at times, being called examples; and other names for problems, such as items, are also being used somewhat infrequently by some authors. There is a growing realization that at best the book problems are exercises in using the language of arithmetic

of arithmetic and that very little problem solving activity may accompany the procurement of answers to a page of textbook problems.[2]

(2) "How Can We Improve Instruction And Achievement In Arithmetic," was a study conducted by Dolphus Williams in 1948, in the high school at Songer, California, in which it was found that various ideas were presented and tested in the completing of the investigation. Williams concluded that grammar was the best means of improving the situation. If, frequently, in all the grammar schools throughout the country, on a Friday afternoon for an hour and a half, time would be given to spelling and ciphering matches, we should see improvement in handling the fundamentals of arithmetic.[3]

(3) A study carried out by Esther Swenson at Ball State Teachers College, Muncie, Indiana, in which 332 second grade children were involved, showed difficulty in addition was related to the learning method.

Three types of teaching methods were used in this study. The first method was based, presumably, upon the drill learning theory, with stress upon repetition. The

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[2] H. Van Engen, "A Summary Of Research And Investigations And Their Implications For The Organization And Learning Of Arithmetic," Mathematics Teacher, 41:262, October, 1948.

[3] Dalphus Williams, "How Can We Improve Instruction and Achievement In Arithmetic?" Mathematics Teacher, 42:167, October, 1947.

second method was based upon the generalization or meaning theory of arithmetic, which stresses discovery of relationships among facts and organization of those facts around number relationships.[4] A third method was an intermediate one corresponding as nearly as possible to "common practice" in the teaching of arithmetic. There was much emphasis upon drill, but it was accompanied by concrete presentation of each additional fact and the facts were presented in an organized pattern, that of "size of sum".[5]

Examination of the data from this experiment points to the conclusion that instruction in the one hundred addition facts by methods varying chiefly in their degree of emphasis upon organization and generalization seemed to result in significantly lower correlations of inter-method difficulty ratings for the addition facts. A logical implication is that research workers in the field of arithmetic should plan their research and interpret their results in terms of the type of learning situation in which children are doing their learning.

Research which aims toward establishing the difficulty of arithmetic skills and processes should probably do so in terms of a clearly defined teaching and learning method. It

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[4] Esther J. Swenson, "Difficulty Ratings Of Addition Facts As Related To Learning Method," Journal Of Educational Research, 38:82, October, 1944.

[5] Ibid., p. 83.

may well be that many of the laboratory difficulty studies are not so widely applicable under varied classroom situations as has been supposed.[6]

(4) Ernest Hoopes, in a master's thesis in 1947 entitled "Arithmetical Vocabulary A Factor In Verbal Problem Solving In Sixth Grade Arithmetic" by using a vocabulary test and a problem test, derived certain conclusions to his problem.

From findings in the study, it is concluded there is a close positive relationship between a pupils knowledge of the technical vocabulary of arithmetic and his ability to solve verbal problems in the subject.[7]

(5) A study of retention of percentage was made by Fred Dellett in 1946 at Fort Hays Kansas State College. Dellett used a test containing problems in percentage. The test was given to a sample of seventh grade students as the means of obtaining his data. He concluded: many errors in solving the problems were due to carelessness on the part of the pupils in number rotation;[8] an occassional student paid no attention to the fundamental process used with concrete

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[6] Loc. Cit., p. 83.

[7] Ernest A. Hoopes, "Arithmetical Vocabulary A Factor In Verbal Problem Solving In Sixth Grade Arithmetic," (unpublished Master's Thesis, Fort Hays Kansas State College, Hays, Kansas, 1947), p. 48.

[8] Fred Dellett, "A Study Of Retention Of Percentage," (unpublished Master's thesis, Fort Hays Kansas State College, Hays, Kansas, 1946), p. 44.

numbers and a sense of relationship of numbers need be established, a great number of errors could have been eliminated had the students made an estimate of the number, pupils are aware of the "two-place" change of the decimal point but are lost as to knowing whether it should be moved to the right or the left.[9]

(6) Merle Ohlsen at the State College of Washington conducted the problem, a study of which was as follows, "Control Of Fundamenta Mathematical Skills And Concepts By High School Students". The purpose of this study was to determine reasonable answer to certain questions in mathematical achievements of the students in grades ten, eleven, and twelve in forty three selected Iowa high schools.[10]

In this test the performance on the problems which involved percentage was considerably lower than on those dealing with any other arithmetic concepts. These high school students exhibited an average proficiency of 57.8% on all the arithmetic items in the tests.[11]

The common errors which were made in solving the arithmetic problems fell into six major categories. These high school students confused related mathematical terms and

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[9] Ibid., p. 45.

[10] Merle M. Ohlsen, "Control Of Fundamental Mathematical Skills And Concepts By High School Students," Mathematics Teacher, 38:365, December, 1946.

[11] Ibid., p. 368.

in particular they confused square with square root, sphere with circle, and compute with estimate. They were unable to distinguish between terms in the metric system, and also between various business terms. Either inefficiency or carelessness in reading the problem frequently resulted in failure to select the correct data or in missing the objective of the problem. Lack of understanding of the significance of place value in numbers, errors in placing the decimal point, and incorrect use of the metric system all suggest inadequate knowledge of the decimal system. These students were deficient in the skill of rounding numbers off to the defined degree of accuracy. Miscellaneous mistakes in computation, such as errors in number combinations (particularly those involving zero), carrying, and borrowing, and reducing improper fractions to mixed numbers were made. Three important factors were largely responsible for the low performance on the percentage problems: inadequate understanding of the meaning of per cent, selecting the wrong number as the base, and incorrect interpretation of the time factor in interest problems. Grade differences were not significant for those individuals who had approximately the same mathematical background in high school."[12]

(7) A study at Miami University entitled, "Difficulties Encountered By College Freshmen In Decimals," was made by

[12] Ibid., p. 369.

G. T. Guiler. It was important in that it tested students from a great variety of places and situations. Thus it gives a broad view of the problem.

From this study, it was found that a large proportion of the college freshmen manifested weakness in certain phases of work with decimals. Approximately one fifth of the students encountered difficulty in subtracting decimal numbers, more than one fourth in adding decimal numbers; and more than two fifths in changing mixed numbers to decimals.[13]

Placement of the decimal point in the division of decimal numbers constituted an outstanding source of difficulty. Nearly two fifths of the college freshmen were deficient in this particular ability. A lack of understanding of the procedures involved in changing fractions to decimals and in changing mixed numbers to decimals was a source of difficulty for many of the students. More than one eighth did not know how to change fractions to decimals and approximately three tenths did not know how to change mixed numbers to decimals.

Many of the freshmen did faulty work in computation. Approximately one tenth experienced computational difficulties in changing fractions to decimals, one seventh

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[13] Walter Scribner Guiler, "Difficulties Encountered By College Freshmen In Decimals," Journal Of Educational Research, 40:12, September, 1946.

in changing mixed numbers to decimals; and one fifth in adding decimal numbers.

The students who encountered difficulty in placing the decimal point in the multiplication of decimal numbers displayed a marked tendency to place the point too far to the right and those who encountered difficulty in placing the decimal point in the division of decimal numbers tended strongly to place the point too far to the left.[14]

(8) Guiler made another study of the difficulties encountered in percentage by college freshmen. In this study he found that a very large proportion of the college freshmen exhibited weakness in the various abilities measured.

Most of the difficulties encountered were those peculiar to percentage situations. A large group of errors was due to faulty computation. The largest number of computational faults occured in performing the operation with a lack of understanding of the procedure involved. Dealing with the decimal aspects of the work in percentage constituted another potent source of difficulty. Changing fractional and decimal quotients to per cent equivalents was a source of difficulty for a small proportion of the students.[15]

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[14] Ibid., p. 13.

[15] Walter Scribner Guiler, "Difficulties Encountered In Percentage By College Freshmen, "Journal Of Educational Research, 40:94, October 1946.

CHAPTER II

DIFFICULTIES IN ADDITION

In an analysis of the data provided by the Hundred-Problem Arithmetic Test, it is necessary that certain explanations be made in handling the results. Each problem in addition is studied closely because any one may contain a characteristic different from any other problem in the process of addition. For example: problem number one contains only whole numbers arranged in one column form while problem two is made up of whole numbers having more than one digit. Both problems deal with whole numbers, but the second problem is more complicated in its solution than problem number one.

The problems in the test are arranged in order of increasing difficulty. This gives opportunity for each pupil to do his best. Then, too, each is a separate unit. The problems in the complete test are analyzed and set up in tabulated form indicating the number of students that failed to obtain the correct answer and probably use incorrect processes as well (i.e. subtracted when division should have been used). This means that all incorrect answers will be closely examined to determine if possible the difficulty in not getting the correct answer.

The value of the problems from the standpoint of diagnosis lies in the fact that a comparison of the number of pupils "missing" each test item can be made to obtain information to discover which problems cause the most difficulty in getting the correct response.

An overall view of pupil performance on a single item in a given problem in addition is presented in Table I.

TABLE I

RESPONSE TO ADDITION TEST ITEMS

I Problem Number	II Correct Response	III Incorrect Response
1	342	47
2	328	61
3	259	130
4	323	66
5	327	62
6	318	71
7	311	78
8	213	176
9	276	113
10	296	93
Total	2993	897

In Table I, column I, the problems are listed in 1, 2, 3 order as they appear in the standard test in the section on addition. The second column above reveals the number of correct responses for each example in addition in the third column.

In the case of problem number 1, reading from the upper left across the table, there were 342 correct responses and 47 incorrect ones. Problem one is composed of seven one digit numbers written in column form one above the other in proper order. However the simplicity of the problem left no means by which one might discover reasons for failure on the part of 47 pupils to obtain the correct answer.

Problem 2 containing four 3-digit numbers was incorrectly answered by 61 of the 389 individual pupils taking the test. Thirteen incorrect responses were due to errors resulting from not moving the number to be carried from the preceding column to the next column at the left. The remaining errors were so scattered that it was impossible to determine just what the pupils did. At any rate the answers were incorrect.

The third example was solved incorrectly 130 times by approximately one third of the students. The problem contained seven values expressed as dollars and cents having each value placed directly below the preceding one with the decimal points placed one above the other in proper order. No need was present for moving the decimal point out of regular order. Twenty-one students failed to obtain the correct answer because they did not carry the correct addend to the preceding column. Nine pupils failed to place a decimal point in their otherwise correct answer.

In problem 4 the sum of two simple fractions is involved. Out of 389 answers 66 pupils gave incorrect answers of which 14 appeared to be due to not using the smallest common denominator, and 12 failed to change the numerator in proportion to the change in the denominator.

The fifth problem is similar to item four but different in that the two fractions are arranged in horizontal order. Again 11 pupils failed because they did not use the smallest common denominator, and 12 did not have the numerator changed correctly. Sixty two pupils answered this problem incorrectly.

Problem 6 contains two mixed numbers arranged in common vertical order. Seventy one students made errors in computing the answer to this problem. Six pupils did not have the correct numerator for the fractional portion of the problem and five failed to carry the correct number to the preceding column.

The seventh problem in addition contains three simple fractions arranged in horizontal order that are to be added. Incorrect responses were made by 78 pupils. It is apparent nine pupils did not use the simplest common denominator and twelve used an incorrect numerator.

Problem number 8 in addition contains three mixed numbers (numbers containing whole and fractions; i.e. $21\frac{1}{2}$) arranged in vertical or column order. To correctly solve this problem, each fraction must be changed to a common denominator and the results of the addition properly placed at the point where the answer should be. The fraction need not be reduced; therefore practically no error for reducing fractions should be considered. Eight students failed to carry the correct number to the preceding column and eleven did not find the correct numerator when changin to a common denominator. There were 176 students who failed to compute the problem correctly.

One hundred thirteen pupils had incorrect responses to problem number 7 which in form is four decimal numbers arranged in a horizontal manner. Each number in the problem contains a decimal point of two places which eliminates a knowledge of placing the decimal points directly under each other because these fall in this position naturally when arranged in a vertical column. Sixteen students apparently did not carry the correct value to the preceding column and five failed to place a decimal point in the answer.

Problem 10 was missed by 93 students of which 8 did not carry the correct value to the preceding column. This example contained two values in terms of dollars and cents arranged in horizontal order.

In this study pupils were not required to show their computation of the problem; therefore many of the incorrect answers cannot be determined as to the type of error made in causing incorrect solutions. Other problems counted incorrect were omitted by the student; consequently no conclusion can be reached as to why the item was missed. Since the thesis is concerned with only the computational errors, the reasons for pupils omitting problems are not considered in this piece of research.

A grouping of the errors of computation present in each of the test items of addition shows weaknesses as follows: (1) pupils fail to carry the proper number to the preceding column; (2) the decimal is omitted in the answer by a number of students; (3) pupils have difficulty obtaining the common denominator when adding fractions; (4) a few students do not use a simple common denominator which tends to make a simple problem difficult; (5) in a number of cases the pupil fails to change the numerator correctly in proportion to the change in the denominator.

The problem of difficulty in addition on the part of pupils requires complete study of each pupil's performance as a whole. Consequently the number of correct answers obtained by each student is considered in this observation as well as his errors. Table II provides an overall view of performance.

TABLE II

PUPIL PERFORMANCE IN ADDITION

Number of Students	2	2	8	5	7	30	42	53	77	91	72
Correct Items	0	1	2	3	4	5	6	7	8	9	10

Computations of the data in Table II yields the following results:

Mean 7.69
Median 8
Mode 9

These statistical measures provide an insight into the extend of difficulty in addition. Although the median is 8 and the mode is 9, the mean is lower or 7.69 which indicates that the major portion of the pupils are doing a commendable amount of work in addition, but some students are falling far below the standards which one could consider as a minimum.

CHAPTER III

DIFFICULTIES IN SUBTRACTION

Examples covering subtraction are found in Section II of the Hundred-Problem Arithmetic Test. The section contains ten problems. They are similar to the addition problems in Section I of the test. Each subtraction problem contains the knowledge and use of some particular situation found in principles of subtraction. For example: a pupil solving problem number 14 needs to recognize and understand that the two fractional elements are to be converted into fractions having the same denominator before the subtraction takes place with the integers. These items will be analyzed as to the difficulties they present to high school mathematics students. Since subtraction is the usual basic process of arithmetic introduced to pupils after the fundamentals of addition, at this point, data concerning subtraction will be set forth and reviewed. The results of problems dealing with subtraction will be handled in the same manner as the problems in addition were presented and interpreted in the body of Chapter II, each problem being analyzed as a unit separate and apart from every other problem.

Ten problems are in the unit of the test concerned with subtraction. In the solving of these problems are found the elements of difficulty encountered by high school pupils. Examples 11 to 20 inclusive are accepted as the basis for the following findings.

The results of performance in subtraction are tabulated and presented in Table III thus enableing the reader to readily discern the correct and incorrect responses made by the high school pupils in subtraction. The table is of simple construction and in the left hand column under the word "Test Item" is recorded the problems by number from 11 to 20 inclusive in the "Hundred-Problem Arithmetic Test". The central column in the table labeled "Correct Response" yields the number of correct responses for each problem. The third column to the right records the number of incorrect responses for each of the ten problems in subtraction. To read the table, therefore, problem number 11 was solved correctly by 367 pupils and incorrectly by 22 pupils.

A table is constructed to give the reader an opportunity to find data easily and with little effort. It facilitates the study of certain values which may be of interest. Each table in this thesis is constructed with that view in mind and is not given as a "picture" of the entire section. The explanation with each table is necessary for full interpretation of the data.

TABLE III

RESPONSE TO SUBTRACTION TEST ITEMS

I Problem Number	II Correct Response	III Incorrect Response
11	367	22
12	258	131
13	338	51
14	297	92
15	309	80
16	355	34
17	348	41
18	334	55
19	352	37
20	306	83
Total	3264	626

Item 11 was incorrectly answered by 22 of the 389 pupils tested. The problem consists of two whole numbers to be subtracted. Very few pupils missed this example and no definite patterns of difficulty were discernible as to why the problem was incorrectly solved.

The twelfth problem was a definite obstacle to the pupils tested. One hundred thirty one students had incorrect answers. The example consisted of two decimal values, the smaller to be subtracted from the larger. The main feature of this problem was the necessity of a continuous process of borrowing from each preceding digit. Eighty four pupils errored in the borrowing process and sixty six of the borrowing errors were due to borrowing when the zero was used as one of the digits.

Example 12 consists of two simple fractions to be subtracted, and it was incorrectly answered by 51 pupils. No work was shown; therefore it was impossible to determine the difficulty.

Problem 14 is composed of two mixed numbers and incorrect responses were given by 92 students. The process of borrowing from a whole number and adding it to a fraction accounted for 23 of the mistakes.

Problem 15 was incorrectly answered by 80 pupils. The problem involves a fraction subtracted from a whole number. Due to no work being shown by the pupils, the difficulty cannot be determined.

Two values in terms of dollars and cents, one using the decimal and the other value given in terms of cents comprise the sixteenth item. Thirty four pupils made errors on this problem. Three students showing computation did not place the decimal in its correct position when changing cents to a decimal part of a dollar. Other errors could not be defined.

Decimal values are subtracted in example 17. Forty one errored in regard to this item. No work was shown by the pupils but an examination of the responses revealed faulty use of the process of borrowing.

Problem 18 consists of values in terms of dollars and cents. It was a source of error to 55 pupils, but no definite point of difficulty could be discerned.

The nineteenth example composed of two decimal values was answered incorrectly by 37 students. The errors revealed only faulty computation and no definite item or error.

Example 20 presented two whole numbers to be subtracted. The item was incorrectly answered by 83 students. Upon examination of each response, it is logical to believe that most of the errors were due to the use of the zero as a digit in each value as in the number 9,006.

The incorrect responses in subtraction, as revealed by the various test items, were caused by the following difficulties: (1) a lack of understanding of the borrowing process; (2) incorrect borrowing from the zero digit; (3) incorrect borrowing from a whole number and adding it to a fraction; and (4) inability to place the decimal point in its correct place.

The distribution of students in regard to the number of errors committed is shown in Table IV. This table will enable the reader to obtain an overall view of pupil performance in relation to computation of subtraction problems. The first row of Table IV presents the number of students having a certain number of correct items while directly below, the second row lists the number of correct subtraction items. The corresponding values are given one above the other for quick reference.

TABLE IV

PUPIL PERFORMANCE IN SUBTRACTION

Number of Students	0	1	0	3	6	15	22	37	85	116	104
Correct Items	0	1	2	3	4	5	6	7	8	9	10

Brief statistical treatment of Table IV yields the following data:

Mean 8.37
Median 9
Mode 9

These measures present gratifying results. The subtraction process as a whole, is giving high school mathematics students very little difficulty. Although the average 8.37 is somewhat lower than the median (9) and mode (9), it is sufficiently high to provide reason for believing that a good understanding of the fundamentals of subtraction has been taught in the elementary schools. Very few students are having extreme difficulty in this phase of arithmetic.

It is concluded that subtraction troubles are few, but the process of borrowing and the use of decimals are factors of concern to some pupils in the computation of subtraction problems.

CHAPTER IV

DIFFICULTIES IN MULTIPLICATION

The third area of study in this piece of research is the process of multiplication. A means of grouping items rapidly is becoming more important each day, and an understanding of multiplication may be of value to each person in his daily work.

Section III of the "Hundred-Problem Arithmetic Test" consists of fifteen problems involving the various procedures of multiplication. Each of the fifteen examples will be examined to find the reasons for pupils failing to answer them correctly. Problems 21 to 35 of the test are the problems used for the findings of this topic.

It is to be understood that Table V is similar in construction to each of the preceding ones and is read in the same general manner. The correct and incorrect responses for each multiplication problem are shown in the table. To be clearly understood the table gives the reader a brief review of the correct and incorrect responses. To facilitate understanding this example is cited--problem number 21 involving the multiplication process was answered correctly by 317 of the participating 389 students, and 72 students failed to find the correct solution and gave a response that was not correct. The total correct and incorrect responses are given in the table for future reference of the reader.

TABLE V

RESPONSE TO MULTIPLICATION TEST ITEMS

I Problem Number	II Correct Responses	III Incorrect Responses
21	317	72
22	344	45
23	251	138
24	335	54
25	329	60
26	320	69
27	353	36
28	229	160
29	213	176
30	201	188
31	328	61
32	315	74
33	319	70
34	321	68
35	278	111
Total	4453	1382

Each column of Table V serves a definite purpose. Column one indicates the number of each problem in consecutive order; the middle column indicates correct pupil response and column three the number of incorrect responses to each example.

Problem 21 consisting of two two-digit numbers to be multiplied was not answered correctly by 72 of the pupils tested. Many varied errors accounted for the mistakes, but no definite type of error could be determined; although a number of students were careless in adding the two values when completing the multiplication problem.

Example 22 which is concerned with the multiplication of whole numbers and was incorrectly answered by 45 pupils was given to varied mistakes from which no conclusion could be drawn.

A great amount of difficulty was presented by example 23. One hundred thirty eight pupils made mistakes in computing the answer. Although many of the errors were due to varied reasons, 26 students had a mistake related to multiplying by zero.

Fifty four students gave an incorrect response to item 24 which has a whole number multiplied by a simple fraction; example 25 consisting of two improper fractions to be multiplied was missed by 60 students; an improper fraction to be multiplied by a simple fraction produced mistakes by 69 students on problem 26; and item 27 missed by 36 students consisted of a simple fraction multiplied by a whole number. Each of these examples had incorrect answers due to many varied reasons, but no one factor of difficulty emerged as a real element of trouble. Carelessness appeared to be the only reason for many of the mistakes.

Problem 28 was difficult for many students. One hundred sixty pupils had incorrect answers for this item which is composed of a whole number multiplied by a mixed number. A review of the answers given disclosed that at least fifty three students were unable to change the mixed number to an improper fraction before multiplying. Other mistakes were of a variable and careless nature.

The inability to change a mixed number to an improper fraction was evidenced by 61 pupils in the computation of example 29. This item consists of two mixed numbers and a fraction to be multiplied, and it was answered incorrectly by 176 students.

Item number 30 was the most often missed multiplication problem used in Section III of the test. One hundred eighty eight pupils did not arrive at the correct solution. The example to be multiplied consisted of two decimal values. The points of error were: (1) the pupil failed to place a decimal in his answer, (3) the pupil failed to place the decimal in its correct position, and (3) the pupil was careless in the multiplication and addition processes necessary to arrive at the correct solution.

Problems 31, 32, 33, and 34 were answered incorrectly respectively by 61, 74, 70, and 68 pupils. Each item had been multiplied previously and the answer given without the decimal point being placed in the answer, and the pupil being tested was required to put the decimal in the correct position. A review of the responses indicates a lack of understanding on the part of the pupil in regard to the use of decimal values.

The last multiplication test item (35) required the student to pick the correct answer for the problem from a group of four possibilities which were alike in digits but had the decimal point in a different location. One hundred eleven students failed to select the correct solution. This indicates a lack of understanding of decimal functions.

Difficulties in multiplication were for the most part found in pupil performance and are pointed out as follows: (1) students have trouble multiplying by zero when it is placed within a three or more digit number; (2) carelessness in computation; (3) inability to change a mixed number to an improper fraction; (4) failure to place decimal in the answer; (5) failure to place decimal in its correct position; and (6) an overall lack of understanding of the decimal concept in relation to multiplication.

The number of correct responses given in relation to the number of pupils is shown by Table VI which is similar to the preceding tables.

TABLE VI

PUPIL PERFORMANCE IN MULTIPLICATION

Students	1	2	2	6	6	9	15	16	15	13	31	26	55	70	72	50
Correct Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

By use of the foregoing table, the following statistical measures are revealed.

Mean 11.6
Median 12
Mode 14

These statistical measures lend an overall view to the multiplication process in respect to the ability of high school mathematics students. It is worthy of note that the most often recurring score or mode was 14 correct answers from a possible 15 correct responses. The median is 12 which is an indication of average work, but it is desirable to have a higher median. The mean of 11.6 is equal to approximately 75 per cent of the test items and is too low. This whould be considered as a minimum instead of an average score. Improvement in the ability of handling multiplication processes is needed at the secondary school level.

CHAPTER V

DIFFICULTIES IN DIVISION

Division is the fourth basic process of arithmetic with which this thesis is concerned. The fundamentals of division and their basic understanding is a very essential need on the part of every pupil and adult as well. For an example a given problem or situation may involve an equal distribution of money among a group or any number of similar needs desiring an equal amount of materials to be given to a number of people.

The "Hundred-Problem Arithmetic Test" includes fifteen problems in division to be used to test the proficiency of the student in this process of arithmetic. Examples 36 to 50, grouped in Section IV of the test, are used to provide information that will reveal to the writer difficulties encountered in division by high school mathematics students.

By using Table VII, the reader is able to find the number of correct and incorrect answers given for each test item in division. The table provides an overall view of pupil response to each example. As an aid to the reader, pupil performance on a single item in division is presented by Table VII. The number of correct and incorrect responses for each problem concerning division are listed for reference.

TABLE VII

RESPONSE TO DIVISION TEST ITEMS

I Problem Number	II Correct Responses	III Incorrect Responses
36	348	41
37	290	99
38	263	126
39	304	85
40	282	107
41	242	147
42	307	82
43	308	81
44	261	128
45	273	116
46	284	105
47	246	143
48	250	139
49	294	95
50	212	177
Total	4164	1671

Table VII is composed of three columns which have a definite relation to each other. Column one contains the number of the problem corresponding to the number on the test, column two relates the number of correct answers given for a particular item, and column three lists the number of incorrect responses recorded for each example.

Item 36 was answered incorrectly by 41 pupils. The problem involved the division of one whole number by another whole number, but the errors provided no information relative to the difficulties that may have been involved.

Ninety nine students answered example 37 incorrectly.

A review of the responses given by those tested revealed one outstanding error. Forty six pupils failed to place the zero in their answer which should have been 106.

In example 38, as in example 37, students failed to include the zero in their answer which should have been one hundred five and one third. Forty eight pupils made errors in this respect; although 126 students had an incorrect response to this problem. Other answers were of a varied and non-informative nature.

Two decimal values constituted example 39. This example was missed by 85 students. Examination of the various responses revealed that most errors were due to the student not placing the decimal in its correct position in the answer, or the student did not put a decimal in the answer.

Problem number 40 contains two decimal values and incorrect answers were advanced by 107 pupils. The solution required the student to add a zero to the dividend when moving the decimals in preparation for solving, but 35 students failed to add the zero; consequently they had as an answer thirty one instead of the correct answer three hundred ten. Four students moved the decimal correctly, but they failed to place the zero in the answer giving them the incorrect response of thirty one.

Problems 41, 42, and 43 consisted of two decimal values each. The examples were solved, but the student had to place the decimal point in its correct position in the answer. Item 41 required that the decimal be moved three places to the right and a zero added to the dividend. One hundred forty seven incorrect solutions were given to this problem. Items 42 and 43 required the decimal to be moved two places to the right, and they were missed by 82 and 81 pupils, respectively. A review of the incorrect answers revealed the following difficulties---the pupils had difficulty placing the decimal in the correct position in the answer; students failed to add a zero to the dividend when a decimal movement to the right warrented such an addition; and a number of pupils failed to place a zero as the last digit in the answer thus having too small of an answer.

Item 44 was composed of a whole number to be divided by ten and had four possibilities given for answers. The pupil was to pick the correct solution. One hundred twenty eight students gave an incorrect response to this example. Since no work had to show, it must be assumed that students do not have a good knowledge of decimal functions; or they would have been able to arrive at the correct solution by approximation.

One hundred sixteen pupils had incorrect solutions for example 45 which consisted of two mixed numbers to be divided. The answers provided information showing that some pupils were unable to change mixed numbers to improper fractions, and 39 students inverted the left hand member of the problem instead of the right hand member which is proper.

Item 46 which is composed of a mixed number divided by a fraction was answered incorrectly by 105 students. As in example 45, difficulties in changing a mixed number to an improper fraction and inverting the wrong member of the problem were the prevailing errors.

Problem 47 consisting of a common fraction divided by a whole number was given an incorrect response by 143 pupils. Two outstanding errors were observed in the answers to this example. The students did not invert the correct member of the problem and others did not know how to invert the whole number.

Division of a mixed number by a whole number was the body of example 48. The difficulties observed in the incorrect answers to this problem were---failure to invert the correct member of the example; failure to invert the whole number; and failure to change the mixed number to the correct improper fraction. One hundred thirty nine pupils gave an incorrect response to this item.

The difficulties resulting in errors in answering item 49 were of a varied nature and no definite trouble could be discovered. Ninety five of the pupils tested failed to give the correct response.

Item 50 was incorrectly answered by 177 students. As in example 49, it consisted of two whole numbers to be divided. The errors were of a miscellaneous nature, but 89 students failed to give a response to the problem. It is assumed that those students failing to answer the item did not have enough time to do so.

A review of the errors evident in the various examples dealing with division problems presents a concise list of these difficulties as follows---failure to include the zero digit in the problem answer; failure to place a decimal in the answer when necessary; failure to place the decimal in the correct place in the answer; failure to add the zero digit when necessary in moving the decimal point; failure to change a mixed number to an improper fraction correctly; failure to invert when necessary to solve the problem; and failure to invert the correct member of the example.

To aid the reader in observing the overall performance of the pupils in regard to division, Table VIII is presented giving the number of students in relation to the number of correct responses.

Table VIII is read in the following manner---the number of students having a definite number of correct answers is listed in the first row and directly below, in the second row, the exact number of correct answers is given.

TABLE VIII

PUPIL PERFORMANCE IN DIVISION

Students	0	3	2	8	11	12	14	17	24	25	34	49	37	58	62	34
Correct Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

As an aid to the reader, the statistical measures previously employed in appraising the division process of arithmetic are used. The mean, median, and mode computed from values in Table VIII are these:

Mean 10.82
Median 11
Mode 14

These statistical measures provide the information necessary to show that the mean and median are very near alike which is as it should be; although better pupil performance would be desirable. The mode which is 14 is good, and it is to be noted that many students are doing good work in handling the various procedures in the process of division. An observation of Table VIII will show that many students are doing poor work in the computation of division problems and much work must be done to help them become more proficient in the uses of the division process.

CHAPTER VI

DIFFICULTIES IN DECIMALS, FRACTIONS, AND PER CENTS

The preceding chapters have observed difficulties in the use of decimals and fractions. This topic will endeavor to find the limitations of those difficulties and other errors that prevail in the computational work involving decimals, fractions, and per cents.

Using fractional parts of a value has become of increasing importance with the development of technical aspects of living. It will continue to be important as long as mankind wants to better his standard of living.

Examples 51 to 100 of the "Hundred-Problem Arithmetic Test" are used as the basis for the findings concerning decimals, fractions, and per cents. These problems are grouped under various headings in Section IV of the standardized test.

Various tables are used to clarify the extent of the difficulties observed by the writer. The reader will find these tables located in a position relative to the findings of the particular areas discussed.

The first area to be reviewed in this chapter is that of changing fractions and decimal values to per cents. Ten items constitute the basis for this area of study. They are examples 51 to 60, and Table IX is used to produce the number of correct and incorrect responses given for each problem. The table is read in the same manner as similar tables in the previous chapters.

TABLE IX

RESPONSE TO ITEMS INVOLVING THE CHANGING OF FRACTIONS AND DECIMAL VALUES TO PER CENTS

I Problem Number	II Correct Responses	III Incorrect Responses
51	229	160
52	300	89
53	233	156
54	310	79
55	267	122
56	290	99
57	333	56
58	241	148
59	263	126
60	276	113
Total	2742	1148

Problem 51 in the test is a fraction with one hundred for a denominator and was answered incorrectly by 160 students. Eighty nine of the pupils tested answered example 52 incorrectly. This problem was a fraction with five for the denominator. Problem 53 was a fraction having eight for the denominator and was given an incorrect response by 156 of the pupils.

Seventy nine students gave incorrect responses to example 54 which was a decimal value in hundreths. Item 55 was another decimal value but in terms of thousandths and was missed by 122 pupils.

Students gave 99 incorrect responses to problem 56 which was a fraction having five for a denominator. Example 57, which was the fraction one third to be changed to a per cent, was missed by fifty six of the tested pupils. One hundred forty eight students gave an incorrect answer to item 58, a fraction having a denominator of eight.

Two tenths was the decimal value to be changed to a per cent in item 59, and it was given an incorrect response by 126 students. Example 60 was a decimal value in terms of thousandths, and it was a point of error to 112 of the pupils tested.

Difficulty in changing fractions and decimal values to per cent is an obstacle to many students. Problem 51 shows that many pupils do not know that per cent and hundredths are analogous. Items 52 and 56 use five as a denominator which divides into one hundred evenly. They were not missed quite as often as problems 53 and 58 which use eight as the denominator and eight does not divide into one hundred evenly but must have a zero digit added to the dividend to complete the process, and the introduction of a decimal will cause trouble in the computation process. Example 57 caused few students difficulty; although the denominator three does not divide into one hundred evenly. This is explained by the fact that one third is a very common fraction and its per cent equivalent evidently has been memorized by the students.

Item 54 was not too difficult for most students because seventy five hundredths is a very common decimal value to be changed to per cent, but problems 55, 59, and 60 are not decimal values in terms of hundredths and are not understood as well by the pupils; therefore they responded incorrectly more often to these items. This is explained by the fact that students have a poor knowledge of the function of decimals.

By review, the writer may say that difficulty is apparent in changing fractions and decimal values to per cent. The evident difficulties are---students error in changing fractions to decimals, especially if the denominator does not divide into one hundred evenly; students do not understand how to move the decimal correctly when changing a decimal value to a per cent value; and students do not know that per cent and hundredths are analogous.

Examples 61 to 70 constitute the basis for findings in the second area of this chapter to be studied. This area of study is concerned with the processes involved in changing common fractions and per cents to an equivalent decimal value.

Table X gives the pupil performance in respect to each item of the test concerned with this area of study. The table is to be read in the same manner as exercised in using Table IX.

TABLE X

RESPONSE TO ITEMS INVOLVING THE CHANGING OF FRACTIONS AND PER CENTS TO A DECIMAL VALUE

I Problem Number	II Correct Responses	III Incorrect Responses
61	303	86
62	313	76
63	276	113
64	330	59
65	194	195
66	255	134
67	285	104
68	207	182
69	220	169
70	212	177
Total	2595	1295

Eighty six students had incorrect answers to item 61; seventy six students had an error in problem 62; one hundred thirteen students gave incorrect answers to item 63. The three examples required the pupil to change a fraction to its equivalent decimal value.

Problems 64 and 65 were values in per cent to be changed to a decimal value and were given incorrect responses by 59 and 195 students respectively.

One hundred thirty four pupils answered example 66 incorrectly. It was a common fraction with one hundred for a denominator, and it should have been changed to a decimal fraction. A fraction with a denominator of five composed problem 67, and it was given an incorrect response by 104 pupils. Item 68 was a fraction with eight for a denominator, and it was missed by 182 of the students tested. Examples 69 and 70 were per cent values to be changed to a decimal value. Each per cent was a mixed number value and was incorrectly answered by 169 and 177 students respectively.

Problems 61 and 62 were incorrectly answered less than were examples 63, 66, 67, and 68. This is due to the first two items being fractions of common usage and students have the answers in hand, but the other four items require a working knowledge of the process of changing fractions to a decimal and the students appear to lack this knowledge.

Example 64 is a whole number in terms of per cent and was incorrectly responded to by comparitively few of the pupils, but problems 65, 69, and 70 are mixed numbers in terms of per cent and caused many errors on the part of the students. These errors were due to the student apparently not understanding the process of changing the fractional portion of the mixed number to a decimal and then changing the new term to a decimal value.

Observation of the various errors provided the writer with the information that high school mathematics students have difficulty in changing fractions and per cents to decimal values because---many students do not have a working knowledge of the process of changing fractions to decimal values; many students do not know how to move the decimal point when changing a decimal value to an equivalent value in terms of per cent; and the lack of a functional knowledge of decimals is apparent.

The third area for observation is that of writing values of per cent in the form of a common fraction. Examples 71 to 75 are used as the basis for findings on this topic. Table XI provides an overall view of pupil performance on each item used in this area.

TABLE XI

RESPONSE TO ITEMS INVOLVING THE WRITING OF PER CENT VALUES AS A COMMON FRACTION

I Problem Number	II Correct Responses	III Incorrect Responses
71	316	73
72	188	201
73	329	60
74	204	185
75	290	99
Total	1327	618

Seventy three pupils gave incorrect responses to item 71. Two hundred one errors were recorded for example 72. Problem 73 was missed by 60 pupils, and one hundred eighty five students answered example 74 incorrectly. Item 75 was incorrectly answered by 99 of the students tested.

Each of the problems required that a per cent value be changed to an equivalent common fraction. Examples 71, 73, and 75 were common values of everyday usage; consequently students had less difficulty in giving the correct response because they evidently knew the answer from memory. Problem 72 was nine per cent to be changed to a common fraction, but many students made errors in giving an answer as they do not understand how to arrive at the correct denominator of one hundred which is analogous to per cent. The seventy fourth example was a mixed number given as a per cent and again the pupil did not know how to determine a denominator for the mixed number and thus change the value to a common fraction. Many students did not realize that the fractional portion could be written as a decimal and then the entire amount changed to a common fraction.

The foregoing information points out the difficulty in changing a per cent to a common fraction as being the inability to determine what to use as the value of the denominator for the common fraction. Again it must be stated that students do not realize per cent and hundredths are analogous.

Problems 76 to 85 involve three variations of problems in which the student is to make use of per cent values. The pupil performance relative to each item of concern to this area of review is given in Table XII. The table is to be read in the same manner as previous tables which contain similar types of information.

TABLE XII

RESPONSE TO ITEMS IN WHICH THE STUDENT IS
TO MAKE USE OF PER CENT VALUES

I Problem Number	II Correct Responses	III Incorrect Responses
76	269	120
77	157	232
78	228	161
79	81	308
80	131	258
81	267	122
82	97	292
83	136	253
84	166	223
85	254	135
Total	1786	2104

Item 76 consists of finding 25 per cent of 120 and was incorrectly answered by 120 of the pupils tested. The difficulty involved could not be determined because the actual work was not shown.

Two hundred thirty two students failed to arrive at the correct solution to problem 77 which consisted of finding 2 3/10 per cent of 40. The major error occured in the pupil's inability to place the decimal point in the correct place in the answer.

One hundred twenty per cent of twenty composed item 78, and it was answered incorrectly by 161 of the pupils tested. Many students had difficulty in proper placement of the decimal in the answer.

Example 79 required the student to find 2/3 of 1 per cent of 3,000. This item was missed by 308 of the 389 students tested; indicating students do not understand the procedure of computation necessary for finding a fractional part of one per cent of a quanity.

Two hundred fifty eight students gave an incorrect answer for example 80 which asked the pupil to find what per cent of 24 is equal to 8. Problem 81 wanted to know what per cent of 60 was equal to 6 and was given an incorrect response by 122 pupils. Two hundred ninety two students were unable to arrive at a correct solution for item 82 which required the pupils to find what per cent of 20 would be equal to 25.

These results reveal that students do not understand the procedure for finding a correct solution to problems requiring students to find what per cent of one number will give another specified number.

Four is equal to what per cent of twenty formed the body of example 83. This item was a source of error to 253 students. Problem 84, missed by 223 of the pupils tested, asked the student to solve the problem--"nine is equal to what per cent of eighteen". Item 85 which stated the problem, "eight is what per cent of eighty", was answered incorrectly by 135 students.

These items clearly reveal that students do not have a good basic knowledge of the procedure for solving problems which state a given number is equal to what per cent of another specific value.

Review of items 76 to 85 reveals the following difficulties---pupils are uncertain when attempting to place a decimal point in its correct position in the answer to a problem; students are unsure of the procedure of computation necessary for finding a fractional part of one per cent of a quanity; pupils are unsure of the procedure for finding what per cent of one number is equal to another specified member; students are unsure of the procedure for solving problems which state a given number is equal to what per cent of another specified value.

Pupil performance on items 86 to 90 is given in Table XIII which is read in the same manner as preceding tables.

TABLE XIII

RESPONSE TO ITEMS REQUIRING THE STUDENTS TO WRITE DECIMALS AS A PER CENT AND ARRANGING DECIMAL VALUES IN ORDER OF SIZE

I Problem Number	II Correct Responses	III Incorrect Responses
86	273	116
87	269	120
88	275	114
89	196	193
90	122	267
Total	1135	810

Problems 86, 87, and 88 required the student to write a given decimal value as a per cent. Observation of the number of incorrect answers given by the pupils tested leads to the assumption that high school students do not understand the procedure of moving the decimal point when changing a decimal value to per cent. A functional knowledge of the decimal system is lacking in the skills attained by high school mathematics students.

Problems 89 and 90 consisted of three decimal values each. The values were to be arranged in order of their size with the largest value first and the smallest last. The number of incorrect responses indicates that students have no conception of the magnitude of decimal values.

Table XIV is presented to show pupil performance on problems 91 to 100. The table is read in the same manner as Table XIII.

Items 91 and 92 consists of common fractions which are to be changed to a decimal fraction with the answer carried to three places and rounded off to two places. Examples 93 to 100 are word problems involving the use of per cent.

Approximately ninety per cent of the errors concerning these items were due to the student failing to give an answer; therefore the writer is unable to determine the reasons for difficulties in this area.

TABLE XIV

RESPONSE TO ITEMS INVOLVING THE CHANGE OF COMMON FRACTIONS TO A DECIMAL VALUE AND RESPONSE TO WORD PROBLEMS

I Problem Number	II Correct Responses	III Incorrect Responses
91	87	302
92	80	309
93	114	275
94	132	257
95	72	317
96	81	308
97	72	317
98	54	335
99	78	311
100	73	316
Total	843	3047

An overall view of the number of correct responses given to problems concerning decimals, fractions, and per cents in relation to the number of pupils is shown in Table XV. The table gives the reader an overall view of the number of students having any specific number of problems from Section V of the standardized test correct.

This table is designed to give the reader a comprehensive view of pupil response in relation to the problem and the number of correct answers given for each problem. It is intended to save time and effort involved in reviewing the preceding tables in this chapter except for those who need a detailed explanation of each problem or group of problems.

TABLE XV

PUPIL PERFORMANCE IN DECIMALS, FRACTIONS, AND PER CENTS

I Number of Students	II Number of Correct Items	III Number of Students	IV Number of Correct Items
1	0	14	26
0	1	11	27
1	2	9	28
4	3	10	29
2	4	14	30
7	5	13	31
7	6	9	32
5	7	13	33
1	8	14	34
3	9	9	35
4	10	16	36
9	11	10	37
7	12	8	38
6	13	7	39
9	14	7	40
8	15	9	41
10	16	4	42
9	17	3	43
4	18	6	44
7	19	6	45
7	20	8	46
10	21	8	47
10	22	5	48
12	23	2	49
10	24	2	50
19	25		

Use of the foregoing table enabled the author to obtain various statistical measures. The following statistical measures will aid in the analysis of the proficiency of students working with problems concerning decimals, fractions, and per cents.

Mean 27.03
Median 27
Mode 36

These statistical measures reveal that the tested high school mathematics students are doing very poor in the computation of problems involving decimals, fractions, and per cents. The average pupil is able to work only approximately one half of the problems that are concerned with fractional parts of numbers. A better understanding of procedures needed to solve examples of this type is a must for most of the secondary school mathematics students tested.

CHAPTER VII

VALIDATION APPLIED

"Data collected from tests and experiments are often a series of numbers with little meaning or significance until they have been rearranged or classified in a systematic way."[16] This statement by Garrett applies no doubt to this study; although a review of each problem has been made. The major elements of difficulty in the four fundamental processes of arithmetic made by high school mathematics students have been discovered and assembled in a concise manner, but the findings have little meaning unless the data is of a sound and reliable nature. This chapter procedes to evaluate the responses of the pupils tested and determines the soundness of the study as a whole. The process of measuring the soundness of the data will be carried out by accepted standard statistical formulas which have been derived for use in education and psychology.

The formulas used are found in, Statistics In Psychology and Education, by Henry E. Garrett, Associate Professor of Psychology, Columbia Univerisity.

The first statistic to be employed is that of central tendency. Measures of central tendency used are the mean

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[16] Henry E. Garrett, Statistics In Psychology and Education. (New York: Longmans, Green and Co., 1939), p. 4.

and median. By use of a frequency table the writer will compute the numerical values of the mean and median, which applies to the test scores as a group.

TABLE XVI

FREQUENCY DISTRIBUTION OF STUDENTS

Step-Interval Scores	Midpoint X	III f	IV fX
95--99	97	12	1164
90--94	92	23	2116
85--89	87	20	1740
80--84	82	40	3280
75--79	77	43	3311
70--74	72	39	2808
65--69	67	36	2412
60--64	62	31	1922
55--59	57	38	2166
50--54	52	33	1716
45--49	47	18	846
40--44	42	24	1008
35--39	37	7	259
30--34	32	12	672
25--29	27	7	189
20--24	22	2	44
15--19	17	2	34
10--14	12	2	24
		N = 389	ΣfX = 25711

$$\text{Mean} = \frac{\Sigma fX}{N} = \frac{25711}{389} = 66.1$$

$$\text{Median} = L - \left(\frac{N/2 - F}{f}\right) i = 64.5 - \left(\frac{389/2 - 176}{36}\right) 5 = 67.07$$

In Table XVI the step interval of five is used, and the step intervals are listed in column one. Column two points out the midpoint of the step interval. The number of scores falling in each step interval is recorded in column three, and the product of the frequency of scores, and the midpoint is stated in column four.

As a rule, the mean is regarded as the best average. The mean, however, is greatly influenced by extreme scores. The median is in such situations the best average to use. The writer will compute both averages since it is difficult to determine which value is most important to this study. The computation following Table XVI gives the arithmetic mean as being 66.1 and the median or middle score is 67.07.

These measures of central tendency indicate that the average high school mathematics student has a score of approximately sixty six per cent in regard to the computation of the fundamental problems of arithmetic. These measures are typical or representative of the set of scores as a whole.

The next step is to find some measure of the variability of the scores or the "spread" of the separate scores around the central tendency. The measure of variability that avoids being unduly influenced by a few extreme scores is the quartile deviation, or Q. This is one half the distance between the first and third quartiles. Twenty five per cent of the scores fall below the first quartile, or Q_1, and twenty five per cent of the scores exceed the third quartile or Q_3; the interquartile range is the range of the middle fifty per cent of the scores. The variability of the scores, Q, are computed with the aid of Table XVI, page 56.

$$\frac{N}{4} = 97.25, \text{ hence,} \qquad \frac{3N}{4} = 291.75, \text{ hence,}$$

$$Q_1 = 49.5 + \frac{23}{33} \times 5 = 52.98 \qquad Q_3 = 74.5 + \frac{41}{43} \times 5 = 79.27$$

$$Q = \frac{Q_3 - Q_1}{2} = \frac{79.27 - 52.98}{2} = 13.15$$

The quartile deviation measures the average distance of the quartile points from the median, and it is a measure of the density with which the scores are clustered around the midpoint of the distribution. The computed quartile deviation, Q, is 13.15 which indicates that the scores are widely distributed.

The last thing to find is a measure of the reliability of the test measures. By a true measure of an individual trait we mean the average of a very large number of measurements of the given trait made under precisely the same conditions. In actual practice, one never deals with a true measure as thus defined, but it is possible to estimate the probable amount by which an individual's score varies from its corresponding true score.

The following statistical measures will show the probable error of the median and of the quartile deviation.

The standard error of the median:

$$\sigma_M = \frac{i\sqrt{N}}{2F} = \frac{5\sqrt{389}}{2(36)} = \frac{5(19.73)}{2(36)} = \frac{98.65}{72} = \pm 1.37$$

The standard error of Q in terms of Q of the distribution:

$$\sigma_Q = \frac{1.65Q}{\sqrt{2N}} = \frac{1.65(13.15)}{27.89} = \frac{21.70}{27.89} = \pm .78$$

The various measures point out that all measures are comparitively close to the true score which could be found by using an extremely large set of values. This is an indication that this study has a sufficient number of values and is of a reliable nature.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Society has become exceedingly complex and technical in the passing of time. Daily problems bring arithmetic into the life of most every person. Buying, selling, and trading involve the distribution of money and articles; thereby bringing into use the basic functions of computation. Consequently, each man, woman, and child must have a well-grounded knowledge of the basic skills and concepts of arithmetic.

Review of the Problem

It has been the writer's experience as a teacher in the secondary schools of Kansas, that many pupils experience a more-than-ordinary difficulty with the fundamental processes of arithmetic. Not only is the slow student confused by the subject material at hand, but many times the average and above-average students are "lost" when certain problems are presented for discussion. Since it is imparative that an understanding of the basic skills of arithmetic be imparted to all pupils, the difficulties must be discovered and conclusions drawn to aid the teacher in the process of instruction.

These statements lead directly to the problem of this thesis, which is entitled:

Elements Of Difficulty In Arithmetic Experienced By High School Mathematics Students

The survey method of research was used to secure data for the investigation of the problem. The "Hundred-Problem Arithmetic Test", a standardized test developed by Raleigh Schorling, John R. Clark, and Mary A. Potter, was administered to the students concerned. This test contains one hundred items arranged in five sections--Addition (10 items), Subtraction (10 items), Multiplication (15 items), Division (15 items), and Fractions, Decimals, and Per Cents (50 items).

The test was administered to approximately four hundred high school mathematics students in eight different high schools located in the southwestern section of Kansas. The tests were graded and the data for the study was taken. from the test results thus obtained.

Review of Results

The response to each test item was analyzed by the author to determine the errors evident in computation of the various problems. This analysis discovered the following errors commonly made by the students: (1) failure to carry the proper number to the preceding column in the addition process; (2) borrowing incorrectly in the subtraction process;

(3) difficulty multiplying by zero when the zero digit is placed within a three or more digit number; (4) failure to include the zero digit in the division problem answer; (5) failure to invert when necessary to solve a fractional division problem; (6) inverting the wrong member of a fractional division problem; (7) omit placing a decimal in the problem answer; (8) failure to place the decimal in its correct place in the answer; (9) failure to add the zero digit to the answer after moving the decimal point to the right; (10) improper movement of the decimal when changing a decimal value to a per cent value; (11) poor conception of the relative magnitude of decimal values; (12) difficulty obtaining a common denominator for fractional values; (13) failure to change the numerator when changing fractions to a common denominator; (14) difficulty changing a mixed number to an improper fraction; (15) difficulty changing fractions to a decimal value, especially when the denominator does not divide into one hundred evenly; (16) incorrect borrowing from a whole number and adding it to the fractional value of a mixed number in the subtraction process; (17) the faulty or lack of understanding of the procedure of moving the decimal point when changing a decimal value to per cent; (18) students are unsure of the procedure of computation necessary for finding a fractional part of one per cent of a quanity; (19) students are unsure of the procedure for

finding what per cent of one number is equal to another specified number; (20) students are unsure of the procedure for solving problems which state a given number is equal to what per cent of another specific value; and (21) many students do not know that per cent and hundredths are analogous.

Conclusions

The errors made are many, but the analysis of these errors leads to the following general conclusions: (1) the average student has little difficulty with the computation of whole numbers involved in the various arithmetic processes; (2) a functional knowledge of the decimal system is lacking in the skills attained by high school mathematics students; (3) students have difficulty using fractions in the various processes of arithmetic; (4) many students do not understand the function of per cent and are unable to make use of the concept of per cent; and (5) a certain amount of carelessness is apparent in computation by high school students.

Suggestions

Although the writer is unable to propose solutions that might eliminate the difficulties in arithmetic that are experienced by high school students, certain suggestions are made that might aid in reducing the difficulties to some extent. These are: (1) the high school mathematics instructor needs to spend very little class time concentrating upon the use of whole numbers, but more time should be used in working with fractional numbers; (2) concentrated effort upon the meaning and use of decimals is recommended for most students: (3) the use and development of a more comprehensive knowledge of the concept of per cent no doubt would benefit the majority of high school mathematics students, and common fractions should be closely studied to reduce the misconceptions apparent in the computation involving fractions.

The majority of the advancement to be made in reducing difficulties encountered in the computation of arithmetic problems will depend upon the mathematics instructor. Teachers need to be farsignted and patient in the teaching of mathematical subjects and, above all have the interest of the student in mind at all time.

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